

Matching With a Status Quo: The Agreeable Core

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Objectives

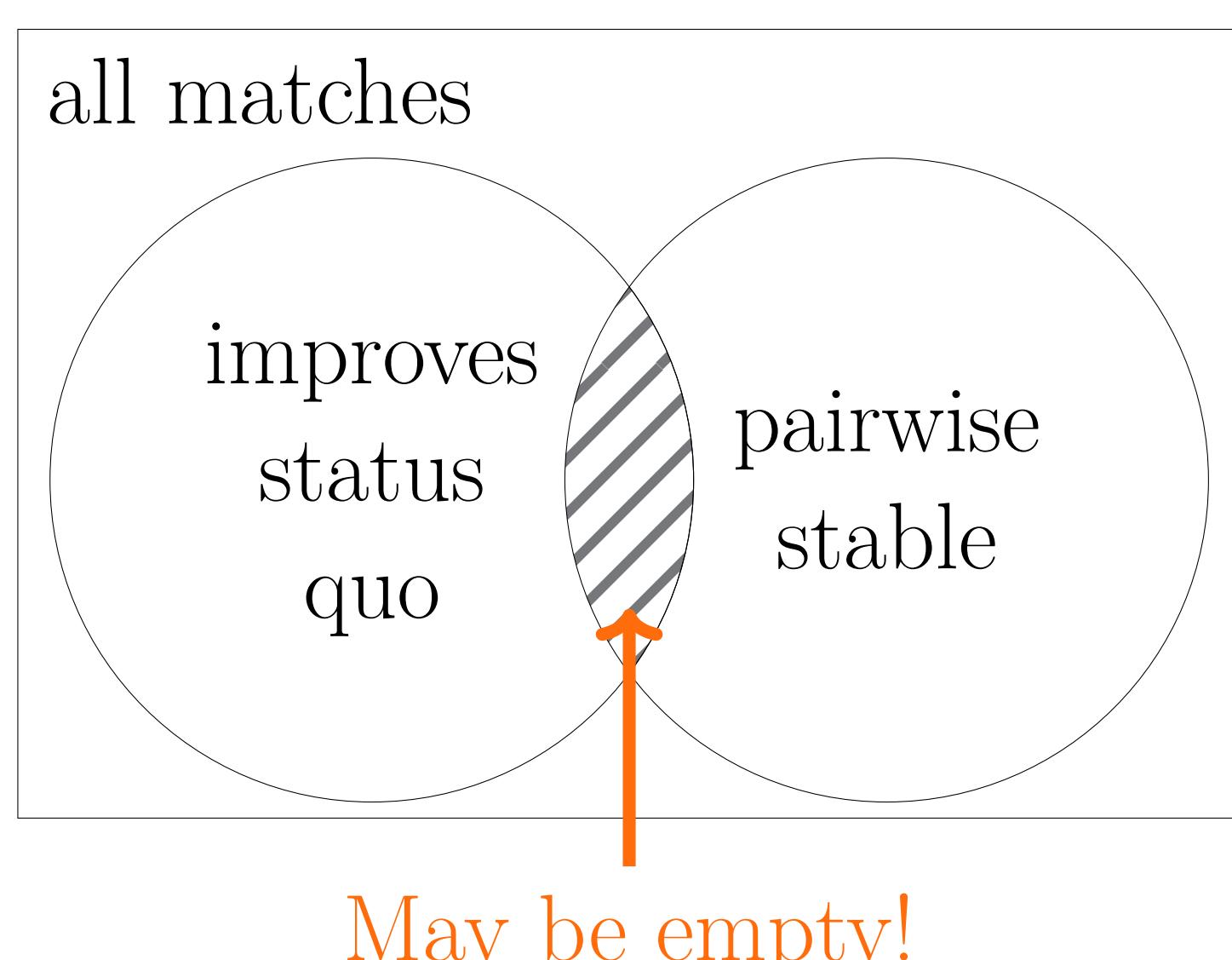
- A theory of **equilibrium** in matching markets with a **status quo**
- A **constructive algorithm** for applications to **market design**

Matching Markets

- Matching markets: students to schools, residents to hospitals, etc.
- **Status quo:** (current contracts) constrains new matches:
 - Ohtani cannot leave the Dodgers and the Dodgers cannot fire Ohtani

No agent can be worse off than his status quo!

- Problem: *pairwise stable* outcomes may harm agents.



- **Insight:** *pairwise stable* = *core* but some blocking coalitions require consent from status quo partners.
- **Solution:** restrict coalitions according to the status quo.

Model Primitives

- Set of agents A partitioned into workers W and firms F
- Strict preference \succ_a for every agent over the other side and himself (being unmatched)
- **Status quo** match μ_0

Definitions

- μ is individually rational if every agent a weakly prefers $\mu(a)$ to $\mu_0(a)$.
- Coalition $C \subseteq A$ blocks μ through ν if
 - $\nu(C) = C$, and
 - $\nu \succsim_C \mu$, and
 - $\nu(a) \succ_a \mu(a)$ for some $a \in C$.
- The **core** is every match *not* blocked by *any* coalition.

Step 2: Exchange Phase

- Finds a *Pareto improvement* that eliminates all *cyclic* blocking coalitions.
- Worker-pointing Top Trading Cycles algorithm except:
 - Only workers and firms *who do not find a better partner in the Propose phase* participate.
 - Worker w can only point to a firm f if f prefers w to her status quo worker $\mu_0(f)$.

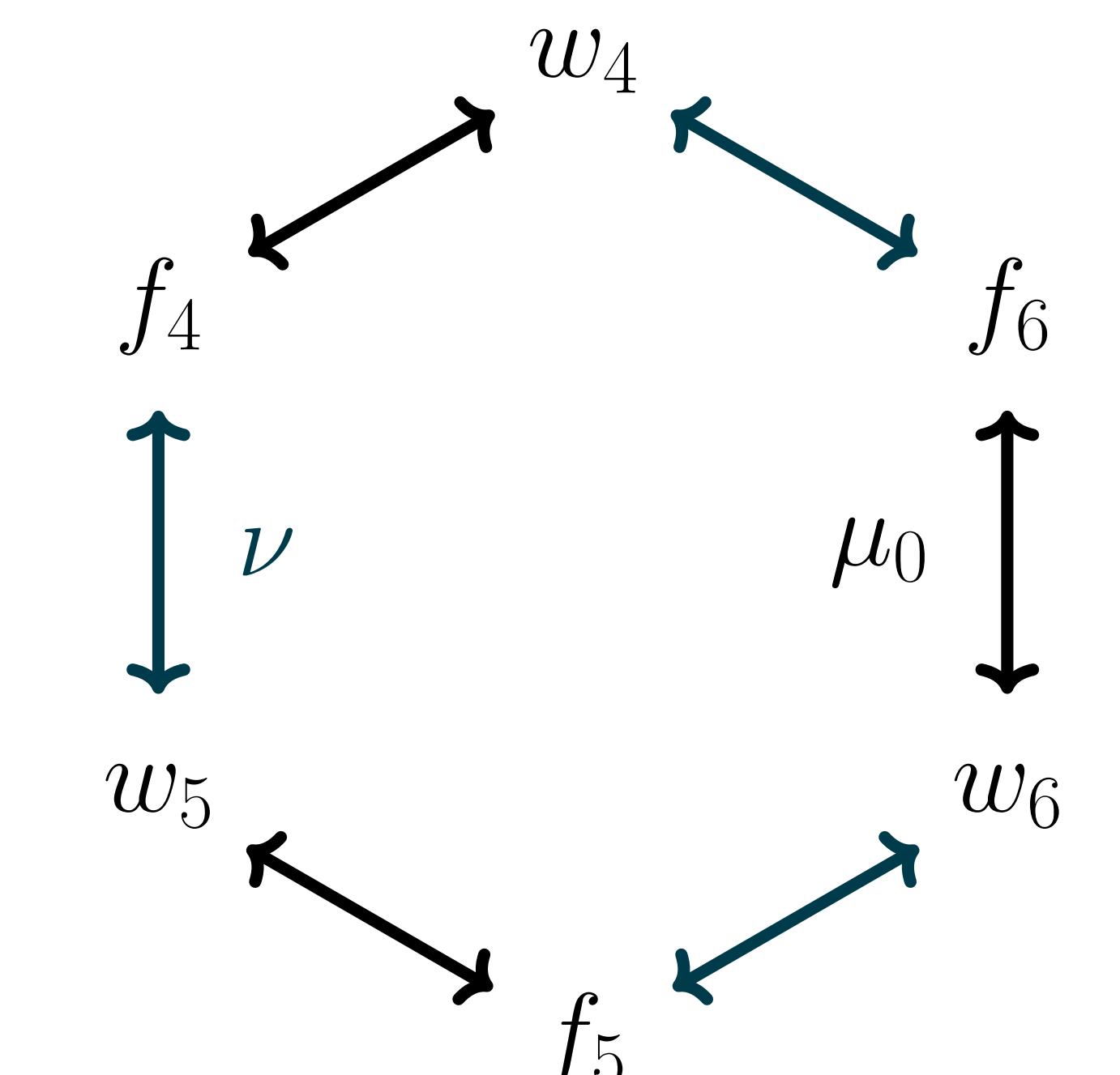


Figure: Cyclic

Key Definitions

- Coalition C is **agreeable** if it does *not* include an agent without including his status quo partner.
- The **agreeable core** is every match not blocked by any *agreeable* coalition.

Interpretation: an agent can veto any coalition that includes his *status quo* partner.

Main Result

I find a match such that no *agreeable* coalition blocks it.

Proof Sketch

- Consider an auxiliary graph with the agents for vertices and an edge between a and b if:
 - ① $\mu_0(a) = b$; or
 - ② $\mu(a) = b$ or a and b are a blocking pair.
- Blocking coalitions of μ correspond to paths that alternate between the two kinds of edges.
- Pareto improvements eliminate paths.

Step 1: Propose Phase

- Eliminates *acyclic* blocking coalitions.
- Worker-proposing Deferred Acceptance algorithm except:
 - Worker w can *only* propose (1) if $\mu_0(w) = w$, or (2) once $\mu_0(w)$ has received a more preferred proposal.
 - Firm f *never* rejects a proposal from $\mu_0(f)$.

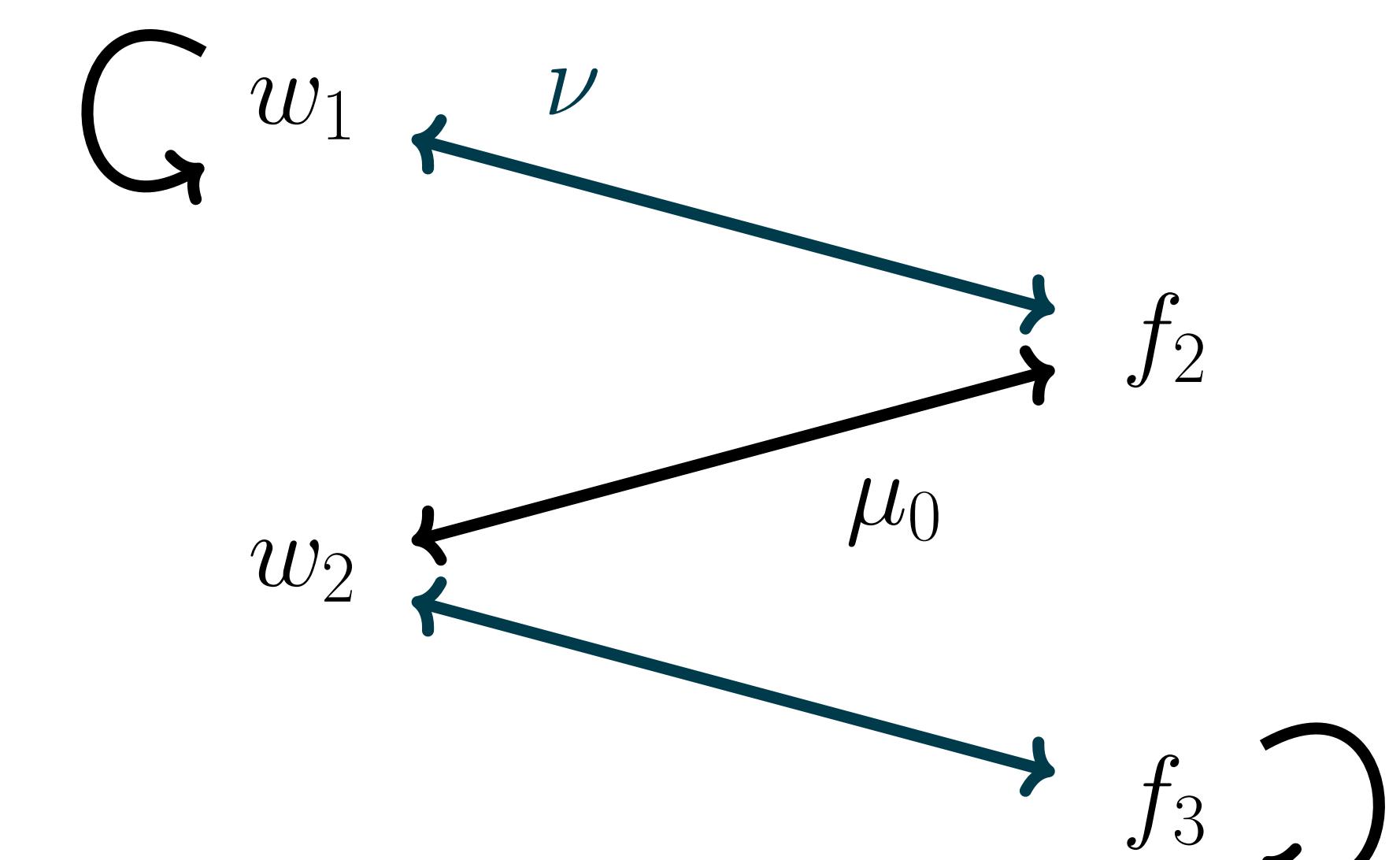


Figure: Acyclic