Family Name/
Surname: ______________________

First Name/
Given Name: ______________________

ID number: |_|_|_|_|_|_|_|_|

Venue: ______________________

Seat Number: __________

No electronic/communication devices are permitted.

Mathematics and Statistics

MOCK

MOCK Test, Semester 1 2024

MATH102-24S1 (C) Mathematics 1A

Time allowed:	**2 hours**
Number of questions:	**5**
Number of pages:	**27**

Instructions to Students:

- You are allowed one double-sided A4 sheet of notes.
- UC-approved calculators are permitted.
- Write your answers in the spaces provided.
- Do not use red ink in answers or diagrams.
- Show full working or give reasons for your answers.
- Marks will be lost for poorly presented or incomplete answers.
- Turn off your mobile phone.
- This test is for practice and revision only.

For Examiner Use Only

Question	Mark
Q1 /16	
Q2 /15	
Q3 /23	
Q4 /19	
Q5 /13	
Total /86	

Blank page for extra working if needed. Please write the question number if used.

The questions start on page 3.

Question 1 (16 marks)

(a) The system of equations

$$\begin{aligned} 2x \quad + 4z &= 6 \\ y \qquad &= 9 \end{aligned}$$

can be row reduced to the following augmented matrix

$$\left[\begin{array}{ccccc} 2 & 0 & 4 & : & 6 \\ 0 & 1 & 0 & : & 9 \end{array}\right]$$

(i) Describe what this system of equations represents geometrically.

(ii) Identify the pivot variable(s) for this system.

(iii) Identify the free variable(s) for this system.

(iv) Find the solution set for this system and give it in the form $(x, y, z) = \ldots$

(b) Consider the following system of equations.

$$\begin{aligned} 2x + 4y + 2z &= 8 \\ 2x + 5y + 3z &= 12 \\ 4x + 10y + 7z &= 25 \end{aligned}$$

(i) Write the system of equations in augmented matrix form.

(ii) This system can be solved to give a single solution. Use Gaussian elimination to solve the system. Show your full working including the row operations you use.

You can use page 5 for your working and answer as well as the space below.

Write your answer in the form $(x, y, z) = \ldots$

This page can be used for answering part (b)(ii).

(c) A system of 3 linear equations in 3 variables has been row reduced to the following augmented matrix

$$\left[\begin{array}{ccc:c} k & k & 0 & 2 \\ 0 & k & 0 & 3 \\ 0 & 0 & k & 2 \end{array}\right]$$

where k is a constant.

Give the value(s) of k that would result in this system having a unique solution.

(d) A system of 3 linear equations in 3 variables has been row reduced to the following augmented matrix

$$\left[\begin{array}{ccc:r} 1 & 0 & 1 & 3p \\ 0 & 1 & 1 & p(p-1) \\ 0 & 0 & 1 & p^2 \end{array}\right]$$

where p is a constant.

Give the value of p that would make this a homogeneous system.

Blank page for extra working if needed. Please write the question number if used.

The questions continue on the following pages.

Question 2 (15 marks)

(a) Consider the matrices A and B below.

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(i) Given that

$$AB = \begin{bmatrix} a & 7 & 2 \\ 1 & b & 1 \\ 2 & 5 & c \end{bmatrix}$$

Find the values of a, b and c in the matrix product AB above.

(ii) Find A^T for the matrix $A = \begin{bmatrix} 9 & 2 \\ 7 & 6 \\ 4 & 0 \end{bmatrix}$.

(b) The inverse, if it exists, of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

(i) Find the value of k for which the matrix $\begin{bmatrix} 2 & 4 \\ k & 8 \end{bmatrix}$ would **not** be invertible. Show your working or reasoning.

(ii) If $A = \begin{bmatrix} 5 & 7 \\ 11 & 13 \end{bmatrix}$ and $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find the matrix B.

(c) Let A, B and C be $n \times n$ matrices.

Show that if $AB = CB$ and B is invertible then $A = C$.

(d) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ by row-reducing $[A \,|\, I]$ to $[I \,|\, C]$.

Show your full working including the row operations you use.

(e) Consider the following linear system.

$$\begin{aligned} x + 4y + 2z &= 1 \\ 2x + 6y + 3z &= 2 \\ x + 3y + z &= 4 \end{aligned}$$

(i) Write the linear system as a matrix equation $A\underline{\mathbf{x}} = \underline{\mathbf{b}}$.

(ii) If $A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 6 & 3 \\ 1 & 3 & 1 \end{bmatrix}$ then its inverse matrix $A^{-1} = \begin{bmatrix} -3 & 2 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -2 \end{bmatrix}$.

Use A^{-1} to solve this linear system and give the solution in the form $(x, y, z) = \ldots$

You must use A^{-1} to solve the system to gain any marks for this question.

Question 3 (23 marks)

(a) Write the following inequalities using interval notation.

(i) $x \leq 9$

(ii) $x \geq -3$

(b) Solve the following inequalities for x. Give your answers using interval notation.

(i) $10 \leq \dfrac{1}{2x} \leq 32$

(ii) $|2 - 3x| \leq 14$

(c) Solve the following equation for x assuming that $b > 1$.

$$b^{12}b^x = \sqrt{b^8}$$

(d) Write the following expression as a single logarithm.

$$\log(9) + \log(3) - \frac{1}{3}\log(8)$$

(e) Consider the function $f(x) = \frac{2x}{x-1}$.

(i) Give the natural domain of this function using interval notation.

(ii) Give the range of this function using interval notation.

(f) Consider the function $g(x) = \ln(3x)$.

(i) Give the natural domain of this function using interval notation.

(ii) Give the range of this function using interval notation.

(iii) Give the inverse $g^{-1}(x)$ of this function $g(x) = \ln(3x)$.

(g) A function $f(x)$ is invertible with domain $[10, \infty)$ and range $(-\infty, 0)$.

Give the domain of its inverse function f^{-1}.

(h) Is the following function odd, even, or neither? Show your working and give the reasons for your conclusion.

$$g(x) = \frac{1}{x(x^2 + x^4)}$$

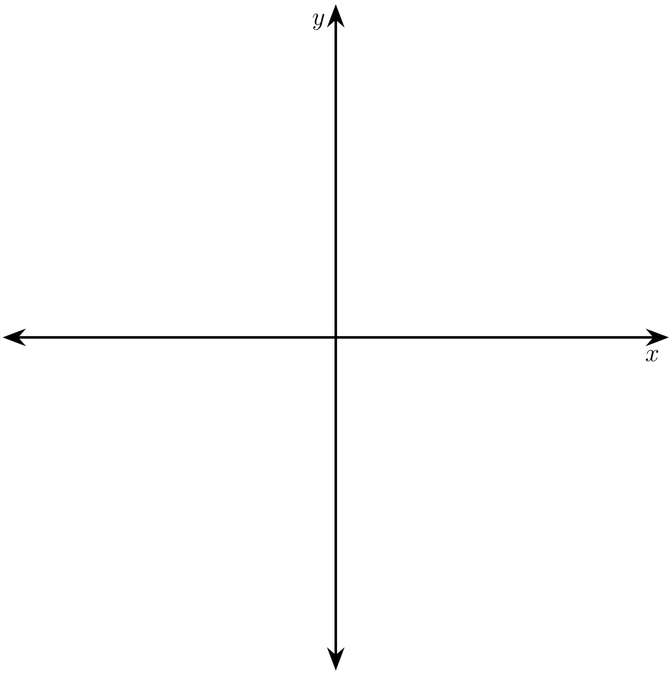
(i) Let $f(x) = x^2 + x$ and $g(x) = x - 1$.

Find and simplify $(f \circ g)(x) = f(g(x))$.

(j) Consider the function $f(x) = b^x$, $b > 1$.

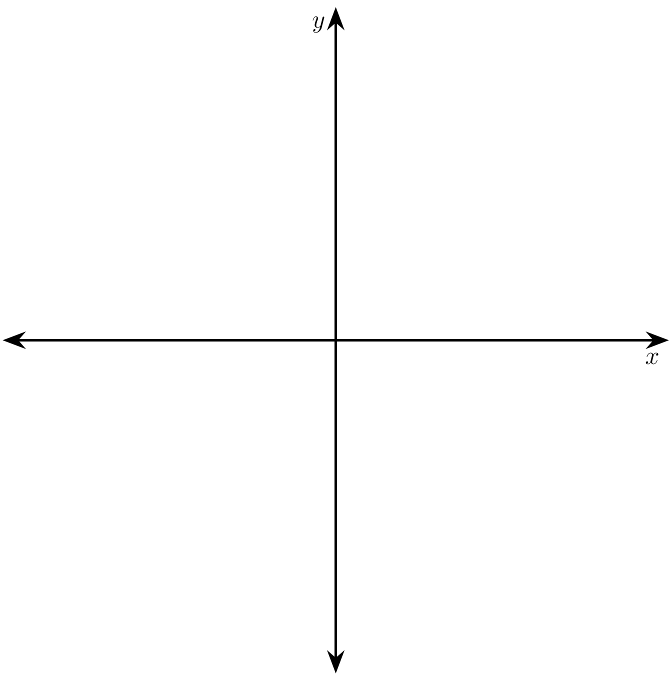
(i) Find the value of the y-intercept of this function.

(ii) Sketch the graph $y = f(x)$ of this function on the axes below.



There are spare axes below if needed. If you use the spare axes please indicate clearly which graph you want us to mark!

Spare axes for part (ii)



(k) Solve the following equation for x.

$$e^x \cosh(x) = 25$$

Give your answer in an exact form.

Question 4 (19 marks)

(a) Consider the following piecewise function.

$$f(x) = \begin{cases} 2x+1 & \text{if } x < 5 \\ kx+k & \text{if } x > 5 \end{cases}$$

Give the value(s) of k for which following limit exists.

$$\lim_{x \to 5} f(x)$$

Remember to show all your working and reasoning clearly.

(b) Find the following limits. If a limit does not exist write "does not exist" or '"DNE".

(i) $\lim\limits_{x \to 2} \dfrac{10x + 1}{(x - 2)(x + 2)}$

(ii) $\lim\limits_{x \to 1} \dfrac{2x - 6}{x^2 + x - 12}$

(iii) $\lim_{x\to\infty} \dfrac{x^3 + 2x + 1}{x^2 + 3}$

(iv) $\lim_{x\to\infty} \dfrac{2x}{x^3 + 9}$

(c) Consider the rational function

$$f(x) = \frac{9x + 3x^2}{(x+1)^2}$$

(i) Find the equation(s) of all the vertical asymptotes of the graph of the function.

(ii) Find the equation(s) of all the horizontal asymptotes of the graph of the function.

(iii) Solve $f(x) = 0$ for x.

(d) (i) Consider the relationship $\cos^{-1}(x) = \theta$ where $0 \le x \le 1$ and $0 \le \theta \le \frac{\pi}{2}$.

Draw a right-angled triangle representing this relationship. Remember to label x and θ on your triangle.

(ii) Use your triangle from part (i) to write $\tan\left(\cos^{-1}(x)\right)$ in terms of x without using trig functions.

(iii) Solve the following equation for x in the interval $[-1, 1]$.

$$\tan\left(\sin^{-1}(x)\right) = \frac{\sqrt{3}}{2}$$

You can use your answer from part (ii) and assume that this is valid for $-1 \le x \le 1$, or you can use any other valid method.

(e) For this question you may use the standard limits given below if needed.

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1 \qquad \text{and} \qquad \lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$$

Find the following limit.

$$\lim_{x\to 0} \left(\frac{\sin(x) - \sin(x)\cos(x)}{x^2} \right)$$

Question 5 (13 marks)

(a) Compute the derrivatives of the following functions.

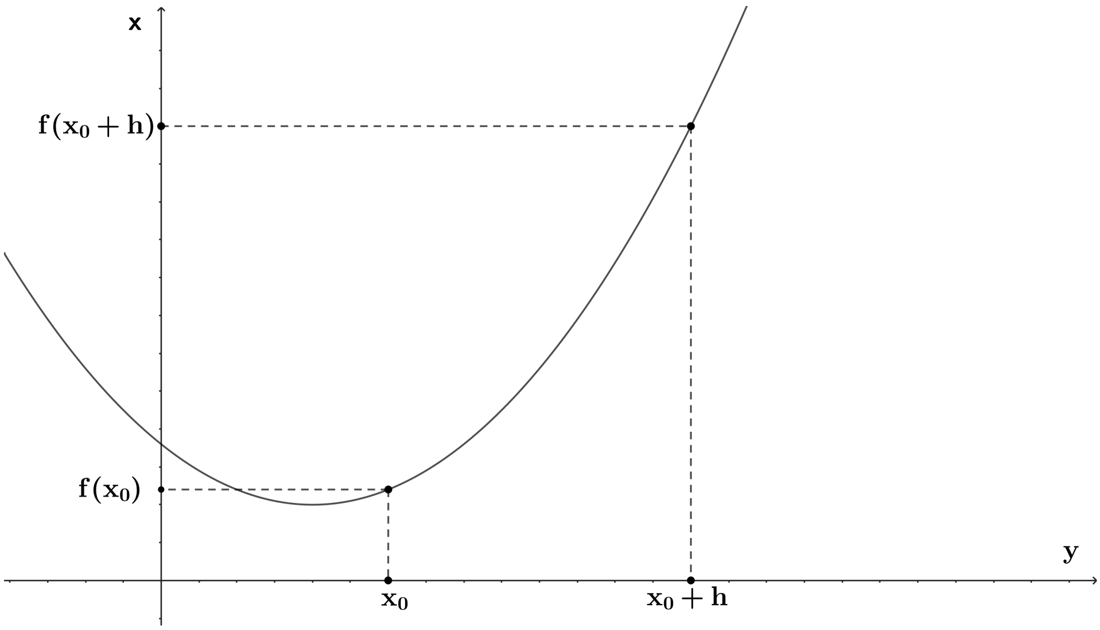
(i) $y = 12x^3 + 2x + 4$

(ii) $y = x\sin(x)$

(iii) $y = \dfrac{\cos x}{x}$

(iv) $y = \tan(x)\ln(x)$

(b) Consider the function $f(x) = x^2 - 6x + 13$, graphed with some slight scaling below.



(i) Sketch the secant line connecting $(x_0, f(x_0))$, and $x_0 + h, f(x_0 + h))$.

(ii) Sketch the tangent line at x_0.

(iii) Consider the case where $x_0 = 3$ and $h = 1$, compute the gradient of the secant line

(iv) Using first principles calculate the derrivative of $f(x) = x^2 - 6x + 13$.

(v) Using your result from (iv), calculate the gradient of the tangent line at $x = 3$.

End of MOCK

Blank page for extra working if needed. Please write the question number if used.