

# **ESTIMATING R/C MODEL AERODYNAMICS AND PERFORMANCE**

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## **I OVERVIEW**

The purpose of this white paper is to enlighten students participating in the SAE Aero Design competition on how to estimate the aerodynamics and performance of their R/C models.

The student needs to understand that the analysis and performance of the R/C model is identical to a full scale airplane such as a Cessna 172. The only differences between the R/C model and the full scale airplane are the wing loading, Reynolds Number and the moments of inertia.

The R/C model wing loading is one to two orders of magnitude less than a full scale airplane (because of the “square-cube law” ... look it up). R/C models typically have wing loadings of 1-3 lb/ft<sup>2</sup> whereas the full scale airplanes are greater than 10 (Cessna 172 is 12.6 lb/ft<sup>2</sup>). The impact is lower stall speeds and lower take-off and landing distances.

The R/C model will typically have Reynolds Numbers less than 500,000 which gives the wing a predominately laminar boundary layer. Full scale airplanes are greater than one million Reynolds Number and have turbulent boundary layer wings. The impact is that the full scale airplanes have higher maximum lift coefficients due to the turbulent boundary layer delaying flow separation over the wing better than the laminar boundary layer. The R/C models and the full scale airplanes are in a Reynolds Number region where the drag coefficients are about the same.

The R/C model will have much smaller moments of inertia than the full scale airplane. The impact is that the time-to-double-amplitude  $t_2$  from a disturbance will be much shorter for the R/C model since  $t_2 = \pi \sqrt{I / (mg \cdot \Delta z)}$ . The R/C pilot will have his hands full with a neutral or unstable model.

## **II DEFINITIONS**

**LIFT:** The aerodynamic force resolved in the direction normal to the free stream due to the integrated effect of the static pressures acting normal to the surfaces.

**DRAG:** The aerodynamic force resolved in the direction parallel to the free stream due to (1) viscous shearing stresses, (2) integrated effect of the static pressures acting normal to the surfaces and (3) the influence of the trailing vortices on the aerodynamic center of the body.

**INVISCID DRAG-DUE-TO-LIFT:** Usually called induced drag. The drag that results from the influence of trailing vortices (shed downstream of a lifting surface of finite aspect ratio) on the wing aerodynamic center. The influence is an impressed downwash at the wing aerodynamic center which induces a downward incline to the local flow. (Note: it is present in the absence of viscosity)

**VISCOUS DRAG-DUE-TO-LIFT:** The drag that results due to the integrated effect of the static pressure acting normal to a surface resolved in the drag direction when an airfoil angle-of-attack is increased to generate lift. (Note: it is present without vortices)

**SKIN FRICTION DRAG:** The drag on a body resulting from viscous shearing stress over its wetted surface.

**PRESSURE DRAG:** Sometimes called form drag. The drag on a body resulting from the integrated effect of the static pressure acting normal to its surface resolved in the drag direction.

**INTERFERENCE DRAG:** The increment in drag from bringing two bodies in proximity to each other. For example, the total drag of a wing-fuselage combination will usually be greater than the sum of the wing drag and fuselage drag independent of one another.

**PROFILE DRAG:** Usually taken to mean the sum of the skin friction drag and the pressure drag for a two-dimensional airfoil.

**TRIM DRAG:** The increment in drag resulting from the aerodynamic forces required to trim the aircraft about its center of gravity. Usually this takes the form of added drag-due-to-lift on the horizontal tail.

**BASE DRAG:** The specific contribution to the pressure drag attributed to a separated boundary layer acting on an aft facing surface.

**WAVE DRAG:** Limited to supersonic flow. This drag is a pressure drag resulting from noncancelling static pressure components on either side of a shock wave acting on the surface of the body from which the wave is emanating.

**COOLING DRAG:** The drag resulting from the momentum lost by the air that passes through the power plant installation (ie; heat exchanger) for purposes of cooling the engine, oil and etc.

**RAM DRAG:** The drag resulting from the momentum lost by the air as it slows down to enter an inlet.

**AIRFOIL:** The two-dimensional wing shape in the X and Z axes. The airfoil gives the wing its basic angle-of-attack at zero lift ( $\alpha_{OL}$ ), maximum lift coefficient ( $C_{lmax}$ ), moment about the aerodynamic center (that point where  $C_{m\alpha} = 0$ ),  $C_l$  for minimum drag and viscous drag-due-to-lift. Two-dimensional airfoil test data is obtained in a wind tunnel by extending the wing span across the tunnel and preventing the formation of trailing vortices at the tip (essentially an infinite aspect ratio wing with zero induced drag). The 2D aerodynamic coefficients of lift, drag and moment are denoted by lower case letters (ie;  $C_l$ ,  $C_d$  and  $C_m$ )

**REYNOLDS NUMBER:** The Reynolds number is a major similarity parameter and is the ratio of the inertia forces to the viscous forces. The equation for Reynolds number is

$$Re = \rho V L / \mu$$

Where  $\rho$  = density (slugs/ ft<sup>3</sup>)

$V$  = flight speed (ft/sec)

$L$  = characteristic length such as wing/tail MAC (mean aerodynamic chord), fuselage length (feet)

$\mu$  = coefficient of viscosity (slugs/ft-sec)

### III APPROACH

We approximate the aircraft drag polar by the expression

$$C_D = C_{Dmin} + K' C_L^2 + K'' (C_L - C_{Lmin})^2 \quad (1)$$

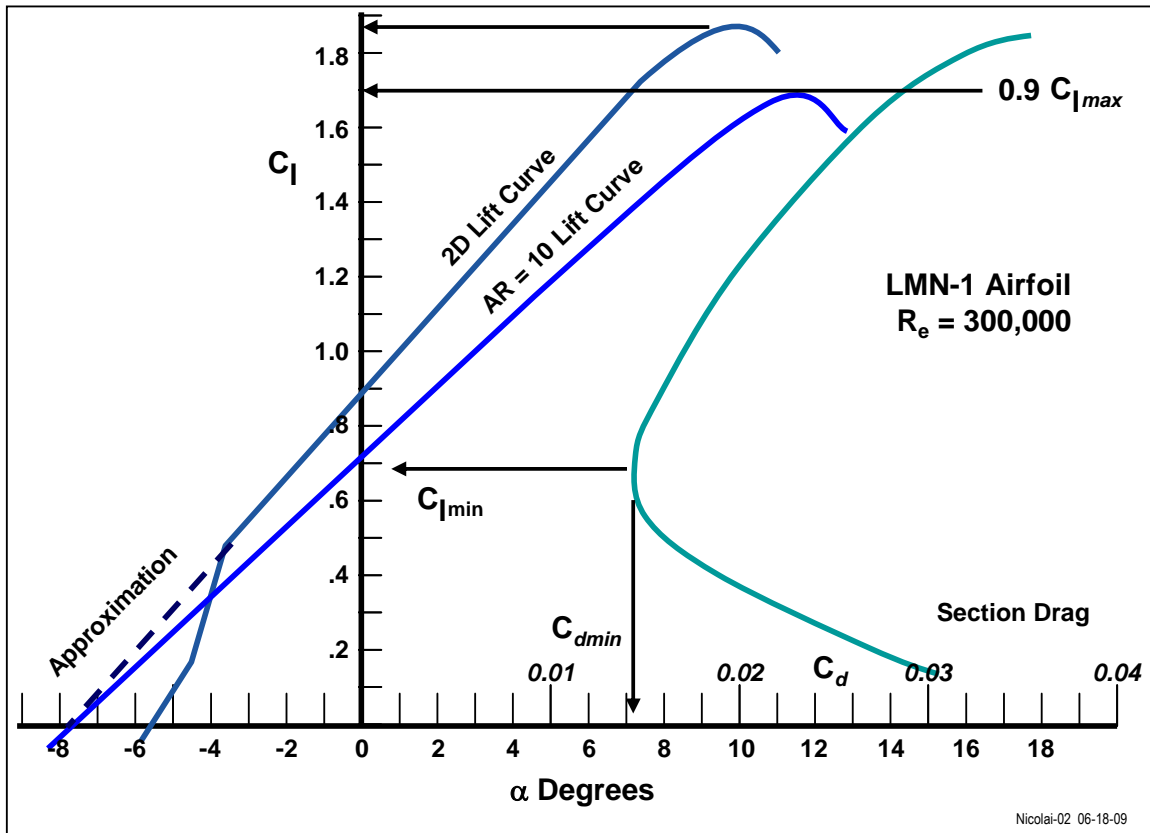
The  $C_{Dmin}$  is made up of the pressure and skin friction drag from the fuselage, wing, tails, landing gear, engine, etc. With the exception of the landing gear and engine, the  $C_{Dmin}$  contributions are primarily skin friction since we take deliberate design actions to minimize separation pressure drag (ie; fairings, tapered aft bodies, high fineness ratio bodies, etc).

The second term in the  $C_D$  equation is the inviscid drag-due-to-lift (or induced drag) and  $K'$  is the inviscid or induced factor =  $1/(\pi AR e)$ . The  $e$  in the  $K'$  factor can be determined using inviscid vortex lattice codes such as AVL. The  $e$  for low speed, low sweep wings is typically 0.9 – 0.95 (a function of the lift distribution).

The third term is the viscous drag-due-to-lift where  $K''$  is the viscous factor =  $fn(LE \text{ radius, } t/c, \text{ camber})$  and  $C_{Lmin}$  is the  $C_L$  for minimum wing drag. Both  $K''$  and  $C_{Lmin}$  are determined from airfoil data. The  $K''$  term is difficult to estimate (see reference 2, page 11-11) and is often omitted. It is usually determined from 2D airfoil test data and will be discussed in Section IV.

#### IV SECTION AND WING DATA

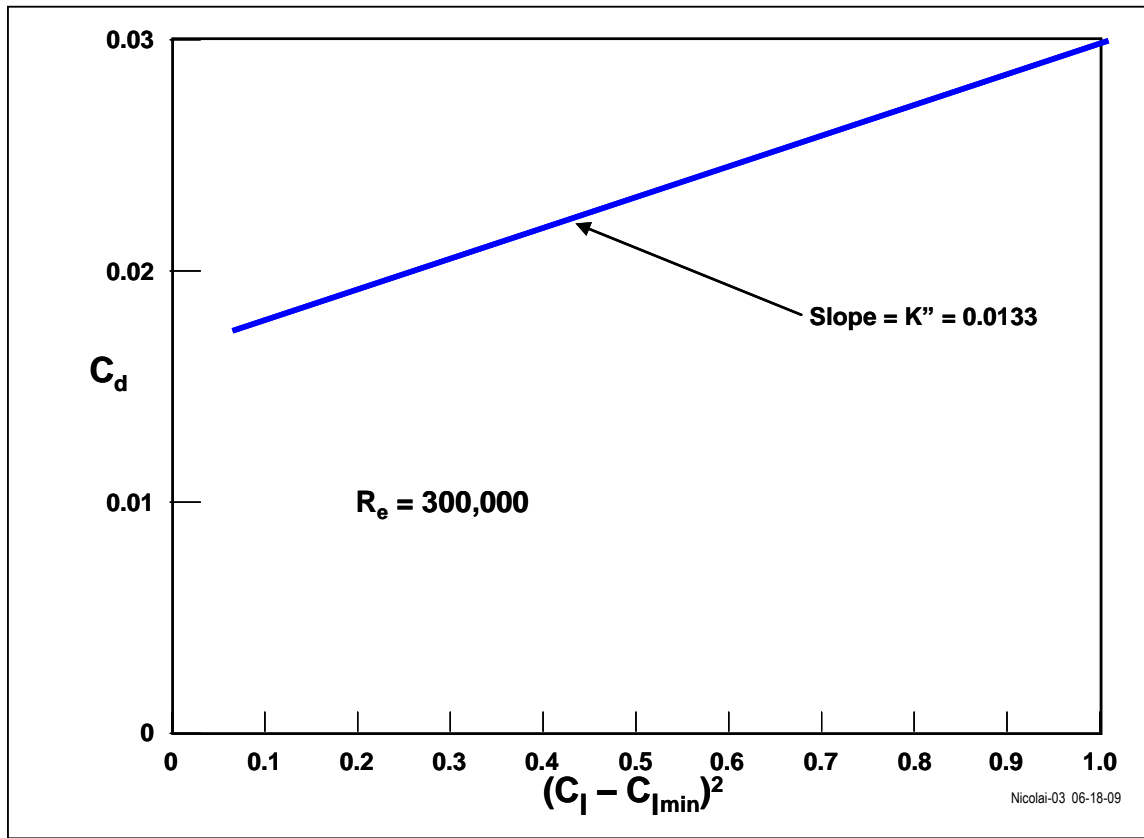
The two dimensional section data needs to be corrected for finite wing effects. These corrections will be discussed below using the notional airfoil (termed the LMN-1) shown in Figure 1. The LMN-1 airfoil is a 17% thick highly cambered shape with its maximum thickness at 35% chord. This airfoil is similar to the shapes used by the SAE Aero Design teams (ie; the Selig 1223, Liebeck LD-X17A, and Wortman FX-74-CL5 1223).



**Figure 1 Notional LMN-1 airfoil data at  $Re = 300,000$**

The first thing the user needs to check is that the data is for the appropriate Reynolds Number. If we assume an altitude of 3000 ft, standard day conditions and a flight speed of 55 ft/sec, the  $\rho = 0.002175$  slugs/ft<sup>3</sup>,  $\mu = 0.3677 \times 10^{-6}$  slugs/ft-sec the Reynolds Number per ft is 325,000. Thus the airfoil data of Figure 1 will be good for a wing having a chord of about one foot.

From the airfoil data in Figure 1 the section  $C_{lmax} = 1.85$  can be determined for a 2D  $\alpha_{stall} = 10^\circ$ . Notice that the airfoil has a nasty inverted stall at  $\alpha \approx -2.5^\circ$  (ie; the lower surface is separated). Since we do not plan on operating at negative  $\alpha$  this is OK. Notice also that the linear lift curve slope has been approximated to an  $\alpha_{OL} = -8^\circ$  on Figure 1.



The section lift data needs to be corrected for 3D, finite wing effects. The low speed unswept finite wing lift curve slope is estimated as follows for  $AR > 3$  (see Reference 1, page 264 or Reference 2, page 8):

$$dC_L/d\alpha = C_{L\alpha} = C_{l\alpha} AR / (2 + (4 + AR^2)^{1/2}) \quad (2)$$

where  $C_{l\alpha}$  = section lift curve slope (typically  $2\pi$  per radian)  
and  $AR$  = wing aspect ratio =  $(\text{span})^2 / \text{wing area}$

Figure 1 shows the construction of a three dimensional  $AR = 10$  wing lift curve using the 2D and the section lift curve slope. The  $\alpha_{OL}$  is an anchor point for constructing the 3D wing lift characteristics since at  $\alpha_{OL}$  the lift is zero and there is no correction to the 3D lift curve for the trailing vortices. Estimate the 3D wing lift curve slope for the model aspect ratio using equation 2 and draw it on the  $C_l$  vs  $\alpha$ . For large  $AR$  (ie;  $AR > 5$ ) low speed, unswept wings, the wing  $C_{Lmax} \approx 0.9 C_{lmax} = 1.67$  (Reference 2, page 9-15). The 3D  $\alpha_{stall}$  is approximated using the 2D stall characteristics and is about  $11.5^\circ$ .

The section drag polar data is used to estimate the following wing data:

$$C_{Lmin} \approx C_{lmin} = 0.7$$

$$C_{Dmin} \approx C_{dmin} = 0.0145$$

and the wing viscous drag-due-to-lift factor  $K'' = 0.0133$  as shown on Figure 2.

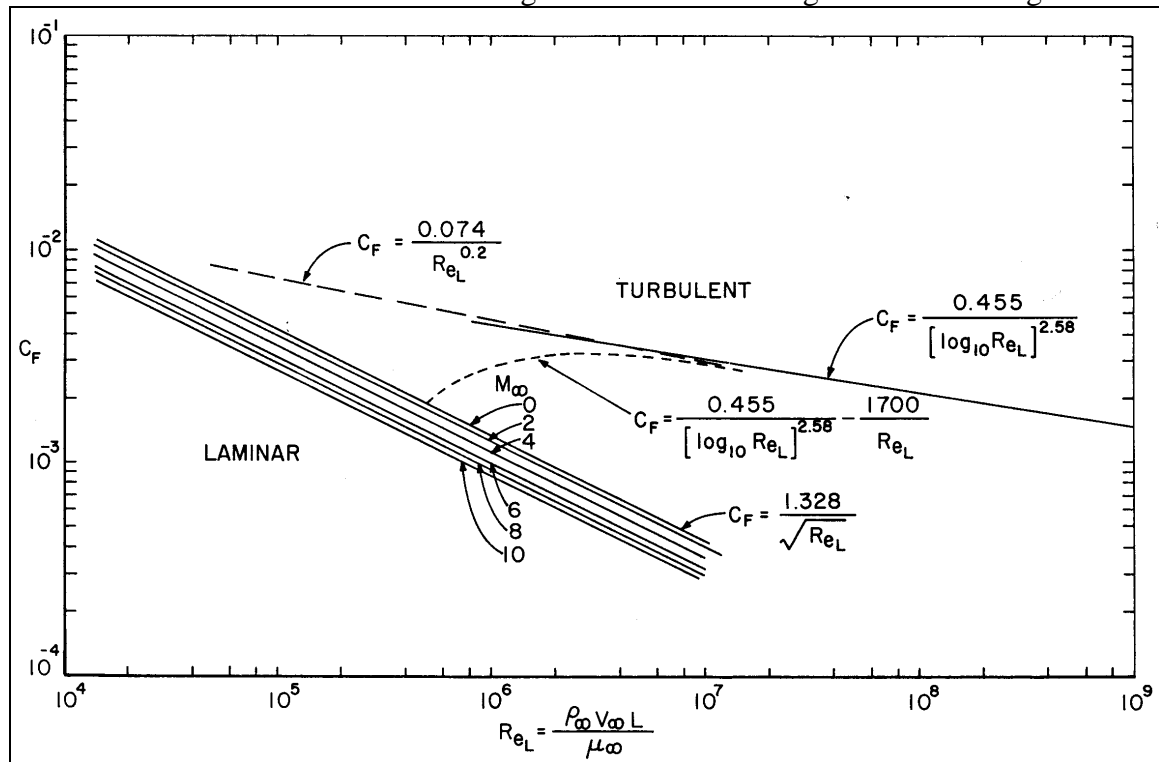
**Figure 2 Viscous drag-due-to-lift factor for the LMN-1 airfoil**

## V ESTIMATING MODEL DRAG

As mentioned earlier we will approximate the aircraft drag polar by the expression

$$C_D = C_{Dmin} + K' C_L^2 + K'' (C_L - C_{Lmin})^2 \quad (1)$$

The  $C_{Dmin}$  term is primarily skin friction and the data on Figure 3 will be used in its estimation. The boundary layer can be one of three types: laminar, turbulent or separated. We eliminate the separated BL (except in the case of stall) by careful design. For  $Re < 10^5$  the BL is most likely laminar. At a  $Re = 5 \times 10^5$  the BL is tending to transition to turbulent with a marked increase in skin friction. By  $Re = 10^6$  the BL is usually fully turbulent. Notice that our model  $Re$  is right in the transition region shown on Figure 3.



**Figure 3 Skin friction coefficient versus Reynolds Number**

We will demonstrate the methodology by estimating the drag of a notional R/C model with the following characteristics:

Configuration: Fuselage/payload pod with a boom holding a horizontal and vertical tail.

Fuselage/boom length = 84 in,  
 Fuselage length = 25 in, Fuselage width = 5 in  
 Wing AR = 10, Wing taper = 0.5  
 Wing area =  $S_{Ref} = 1440 \text{ in}^2 = 10 \text{ ft}^2$  (total planform area)  
 Wing span = 120 in  
 Landing gear: tricycle  
 Take-off weight w/o payload = 12 lb

Item	Planform Area (in <sup>2</sup> )	Wetted Area (in <sup>2</sup> )	Reference Length (in)
Fuselage	151	605	25
Engine /mount	15	100	na
Horiz Tail	126	252	7 (MAC)
Tail Boom	14	28	48 + fuselage
Landing gear	12	24	na
Wing (exposed)	1360	2720	12.4 (MAC)
Vert Tail	0	189	9.8 (MAC)

The drag coefficients for the model components are estimated as follows (all based on  $S_{Ref} = 10 \text{ ft}^2$ ).

#### Fuselage

$Re = 625,000$ , assume BL is turbulent

$$\text{Fuselage } C_{Dmin} = FF_F C_f S_{Wet}/S_{Ref} \quad (3)$$

Where  $FF$  is a form factor (Reference 1, pg 281 or Reference 2, page 11-21) representing a pressure drag contribution. Form factors are empirically based and can be replaced with CFD or wind tunnel data.  $S_{Wet}$  is the wetted area of the component (fuselage) and the  $C_f$  is the skin friction coefficient of the component (fuselage) determined from Figure 3.

$$FF_F = 1 + 60/(FR)^3 + 0.0025 FR \quad (4)$$

For our model the  $FR$  = fuselage fineness ratio = fuselage length/diameter =  $25/5 = 5$  giving a  $FF_F = 1.49$  and a fuselage  $C_{Dmin} = 0.0032$

#### Wing

$Re = 310,000$

$$\text{Wing } C_{Dmin} = FF_W C_f S_{Wet}/S_{Ref} \quad (5)$$

$$\text{Where } FF_W = [1 + L(t/c) + 100(t/c)^4] R \quad (6)$$

and  $L$  is the airfoil thickness location parameter ( $L = 1.2$  for the max  $t/c$  located at  $\geq 0.3c$  and  $L = 2.0$  for the max  $t/c < 0.3c$ ) and  $R$  is the lifting

surface correlation parameter. Thus  $L = 1.2$ .  $R$  is determined from Reference 1, page 281 or Reference 2, page 11-13. For a low speed, unswept wing  $R$  is approximately 1.05.

Since a wing  $Re = 310,000$  could be either laminar or turbulent, we will calculate the minimum drag coefficient both ways and compare with the section  $C_{Dmin} = 0.0145$  (from Figure 1).

If the BL is laminar, the wing  $C_f = 0.00239$  and wing  $C_{Dmin} = 0.0057$ .

If the BL is turbulent, the wing  $C_f = 0.0059$  and wing  $C_{Dmin} = 0.014$ .

Thus the wing boundary layer must be turbulent and we will use wing  $C_{Dmin} = 0.0145$ .

#### Horizontal Tail

The  $Re = 175,000$ , therefore we'll assume the BL is laminar. The tail (both horizontal and vertical)  $C_{Dmin}$  equation is the same as for the wing. For a  $t/c = 0.09$  airfoil with  $L = 1.2$  and  $R = 1.05$ , the horiz tail  $C_{Dmin} = 0.00046$ .

#### Vertical Tail

The  $Re = 245,000$ , therefore assume the BL is laminar. For a  $t/c = 0.09$  airfoil with  $L = 1.2$  and  $R = 1.05$ , the vert tail  $C_{Dmin} = 0.00039$ .

#### Tail Boom

The reference length for the tail boom is the fuselage length plus the boom length since the BL will start on the fuselage and continue onto the boom.

Thus the tail boom  $Re = 1.825 \times 10^6$  and the BL is turbulent. Thus

Tail Boom  $C_{Dmin} = 1.05 C_f S_{Wet}/S_{Ref} = 0.00009$ .

Where the factor 1.05 accounts for tail/boom interference drag.

#### Landing Gear

From Reference 3, page 13.14 a single strut and wheel (4 inch diameter, 0.5 inch wide) has a  $C_{Dmin} = 1.01$  based upon frontal area. Thus the tricycle gear  $C_{Dmin} = (3)(1.01)(2)/1440 = 0.0042$  based upon the wing reference area..

#### Engine

From Reference 3, page 13.4, Figure 13 the engine  $C_{Dmin} = 0.34$  based upon frontal area. For a  $6 \text{ in}^2$  frontal area the engine  $C_{Dmin} = 0.002$  based upon the wing reference area..

#### Total $C_{Dmin}$

The total  $C_{Dmin}$  is the sum of all the components. Thus the total model  $C_{Dmin} = 0.02484$  based upon a wing reference area of  $10 \text{ ft}^2$ .

Note #1: As a sanity check of our  $C_{Dmin}$  we compare it with a Cessna 172 which has a  $C_{Dmin} = 0.026$  based upon a wing reference area of  $175 \text{ ft}^2$ .



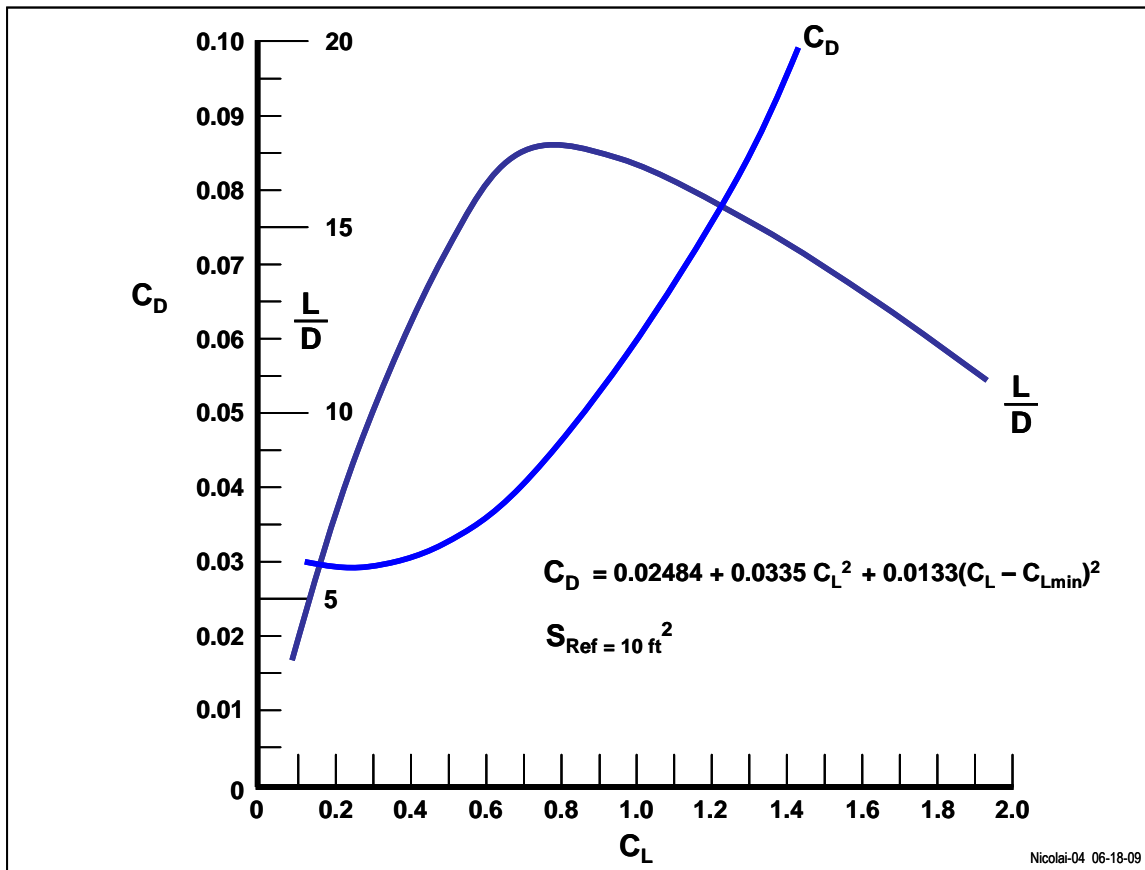
Note #2: The wing is the largest drag item due to its large wetted area. However the second largest drag item is the landing gear which is the case for all full scale airplanes. This drag can be cut in half by putting wheel fairings over the wheels. Is it surprising that airplane designers go to the trouble of designing retractable landing gears?

### Total Drag Expression

Assuming a wing efficiency  $e = 0.95$  gives an induced drag factor  $K' = 1/(\pi AR e) = 0.0335$ . Notice that the often omitted viscous drag factor  $K'' = 0.0133$  is 40% of the induced drag factor. The total drag expression is

$$C_D = 0.02484 + 0.0335C_L^2 + 0.0133(C_L - 0.7)^2$$

The untrimmed (neglecting the horizontal tail drag-due-to lift) model drag polar and L/D are shown on Figure 4.



**Figure 4 Notional model aircraft total drag polar and L/D**

## VI ESTIMATING PERFORMANCE

### Takeoff

The takeoff ground roll distance  $S_G$  is the distance required to accelerate from  $V = 0$  to a speed  $V_{TO}$ , rotate to  $0.8 C_{Lmax}$  and have  $L = W$ . The  $0.8 C_{Lmax}$  is an accepted value to allow some margin for gusts, over rotation, maneuver, etc.

Assuming a  $W = 45$  lb (12 lb model and 33 lb payload), altitude = 3000 feet (standard day) and a  $C_{Lmax} = 1.67$  (from Figure 1) gives the following  $V_{TO}$

$$V_{TO} = [2 W / (S \rho 0.8 C_{Lmax})]^{1/2} = 55.65 \text{ ft/sec} = 38 \text{ mph}$$

The takeoff acceleration will vary during the ground roll and is given by the following expression (see discussion in Reference 2, Chapter 10)

$$a = (g/W)[(T - D) - F_C (W - L)] \quad (7)$$

where  $g$  = gravitational constant =  $32.2 \text{ ft/sec}^2$  and  
 $F_C$  = coefficient of rolling friction =  $0.03$  (typical value for a Cessna 172 on an average runway).

Note: A useful ground test is to measure the coefficient of rolling friction for your airplane. The testing is fairly simple using a fish scale to measure the rolling force for different loading conditions. The value for  $F_C$  can increase dramatically if the landing gear is damaged, the wheels do not track straight or the take-off is in tall grass. The landing gear needs to be sturdy with large diameter wheels.

A useful expression for the ground roll distance  $S_G$  is given by the equation (from reference 2, page 10-7)

$$S_G = V_{TO}^2 / (2 a_{mean}) \quad (8)$$

where  $a_{mean}$  = acceleration at  $0.7 V_{TO}$

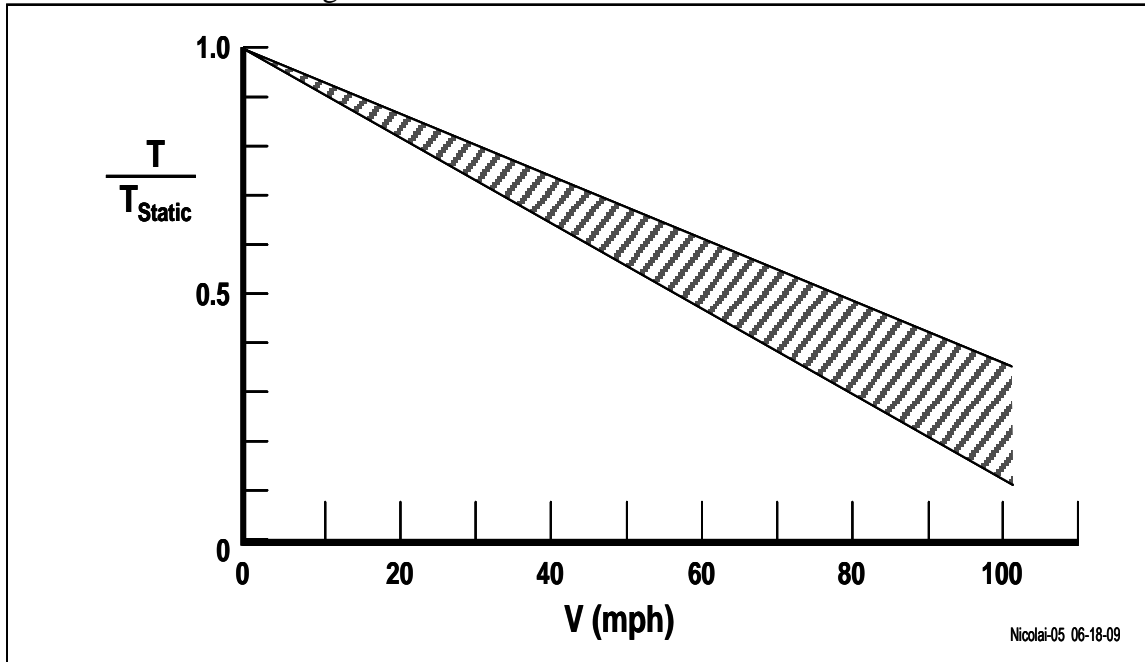
Using the notional model aircraft with the wing at  $0^\circ$  angle of incidence ( $\alpha$  for minimum drag during the ground run) and data from Figures 1 and 4 gives

$$\text{Ground roll } C_L = 0.7 \quad (\text{from Figure 1})$$

$$\text{Ground roll } C_D = 0.0412 \quad (\text{from Figure 4})$$

$$C_{LTO} = 1.34 @ \alpha = 7^\circ \quad (0.8 C_{Lmax})$$

The static thrust available is assumed to be 20 lb. This thrust will degrade with forward speed as shown on Figure 5. The data scatter represents measurements by different SAE Aero Design teams.



**Figure 5 Thrust variation with forward speed for a fixed pitch prop**

A static thrust of 20 lb gives an  $S_G = 151$  feet. It is useful to examine the different pieces of this ground roll distance. The mean acceleration is

$$\text{acceleration @ } 0.7 V_{TO} = (32.2/45)[20(0.8) - 0.68 - 1.02] = 10.25 \text{ ft/sec}^2$$

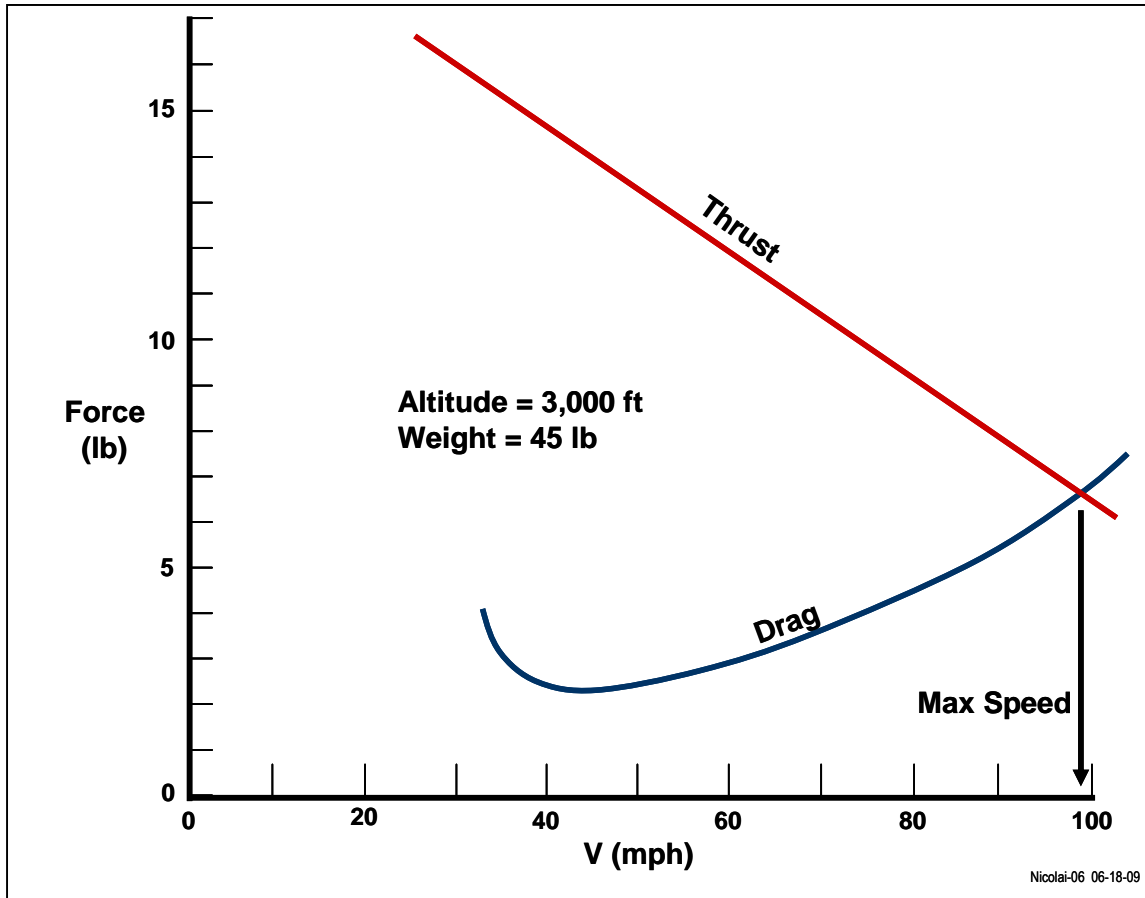
Notice that the ground roll drag (0.68 lb) and the rolling friction force (1.02 lb) are overwhelmed by the available thrust force. If the static thrust was reduced by 20% to 12 lb then  $S_G = 210$  feet. Thus the critical ingredient to lifting a certain payload is having sufficient thrust to accelerate to  $V_{TO}$  in less than 200 feet and having a large useable  $C_{Lmax}$  so that  $V_{TO}$  is small. Having a headwind will reduce  $V_{TO}$  which has a significant effect on  $S_G$  due to the square of the  $V_{TO}$  in the  $S_G$  equation.

After the ground roll, the aircraft rotates to  $0.8 C_{Lmax} = 1.34$  and lifts off. Note that this rotation will take a certain distance (typical rotation time is 1/3 second) and is part of the 200 feet takeoff distance limit. After liftoff the model accelerates and climbs to a safe altitude where the  $\alpha$  is reduced to  $\sim 0^\circ$  ( $C_L = 0.7$ ) and the power reduced for a steady state  $L = W$ ,  $T = D$  cruise.

Bottom line for the notional model in this white paper is a max payload of about 33 lb for a no wind takeoff at 3000 feet standard day.

### Maximum Level Flight Speed

The maximum level flight speed occurs when  $T = D$  at  $L = W$  as shown on Figure 6. For the notional model at  $L = W = 45$  lb and 3,000 ft altitude the maximum speed is 144 ft/sec or 98 mph where  $T = D = 6.6$  lb.



**Figure 6 Thrust available Vs required (drag)**

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