CS/COE 1501

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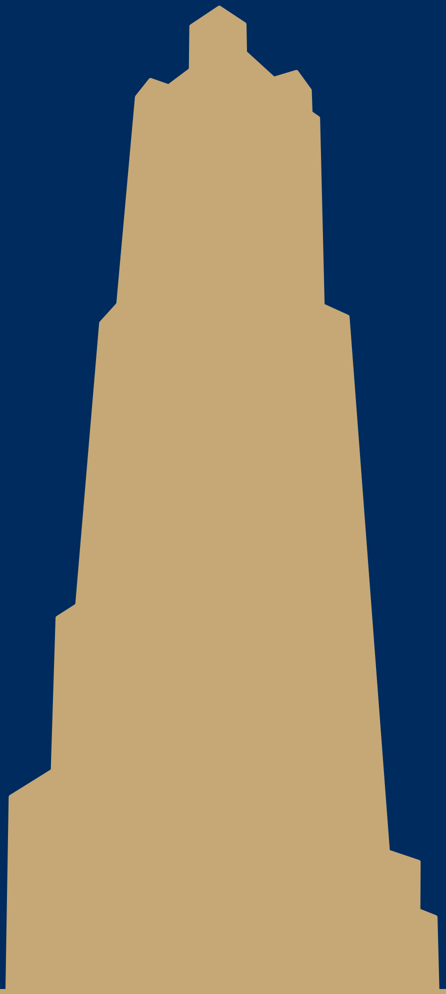
String Pattern Matching



General idea

- Have a pattern string p of length m
- Have a text string t of length n
- Can we find an index i of string t such that each of the m characters in the substring of t starting at i matches each character in p
    - Example: can we find the pattern “fox” in the text “the quick brown fox jumps over the lazy dog”?
        - Yes! At index 16 of the text string!

Simple approach

- BRUTE FORCE
    - start at the beginning of both pattern and text
    - compare characters left to right
    - mismatch?
    - start again at the 2nd character of the text and the beginning of the pattern...

Brute force code

```
public static int bf_search(String pat, String txt) {
    int m = pat.length();
    int n = txt.length();
    for (int i = 0; i <= n - m; i++) {
        int j;
        for (j = 0; j < m; j++) {
            if (txt.charAt(i + j) != pat.charAt(j))
                break;
        }
        if (j == m)
            return i; // found at offset i
    }
    return n; // not found
}
```

Brute force analysis

- Runtime?
  - What does the worst case look like?
    - a = XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXY
    - p = XXXXY
  - m (n - m + 1)
    - Θ(nm) if n >> m
  - Is the average case runtime any better?
    - Assume we mostly miss on the first pattern character
    - Θ(n + m)
      - Θ(n) if n >> m

Where do we improve?

- Improve worst case
    - Theoretically very interesting
    - Practically doesn't come up that often for human language
- Improve average case
    - Much more practically helpful
        - Especially if we anticipate searching through large files

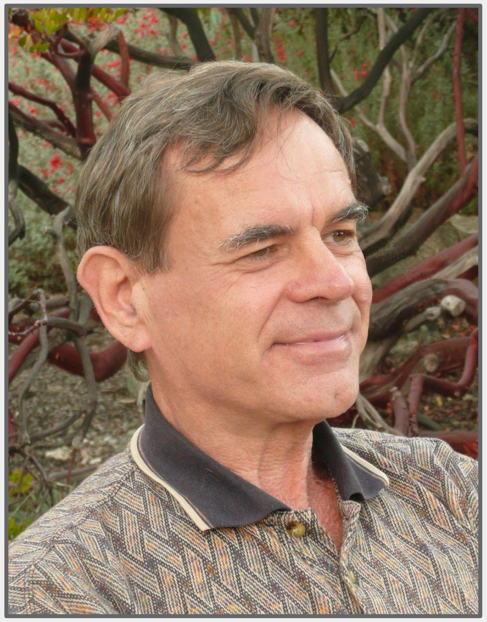
First: improving the worst case

Discovered the same algorithm independently

Knuth

Morris

Pratt

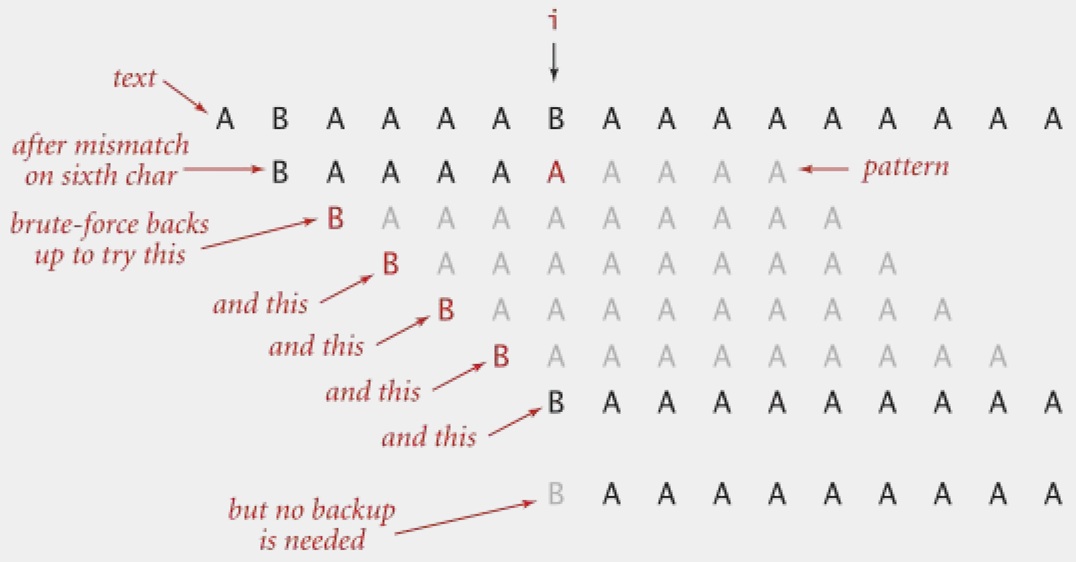


Worked together

Jointly published in 1976

Back to improving the worst case

- Knuth Morris Pratt algorithm (KMP)
- Goal: avoid backing up in the text string on a mismatch
- Main idea: In checking the pattern, we learned something about the characters in the text, take advantage of this knowledge to avoid backing up



i
text
A B A A A A B A A A A A A A A A
after mismatch on sixth char
B A A A A A A A A A
pattern
brute-force backs up to try this
and this
and this
and this
and this
but no backup is needed

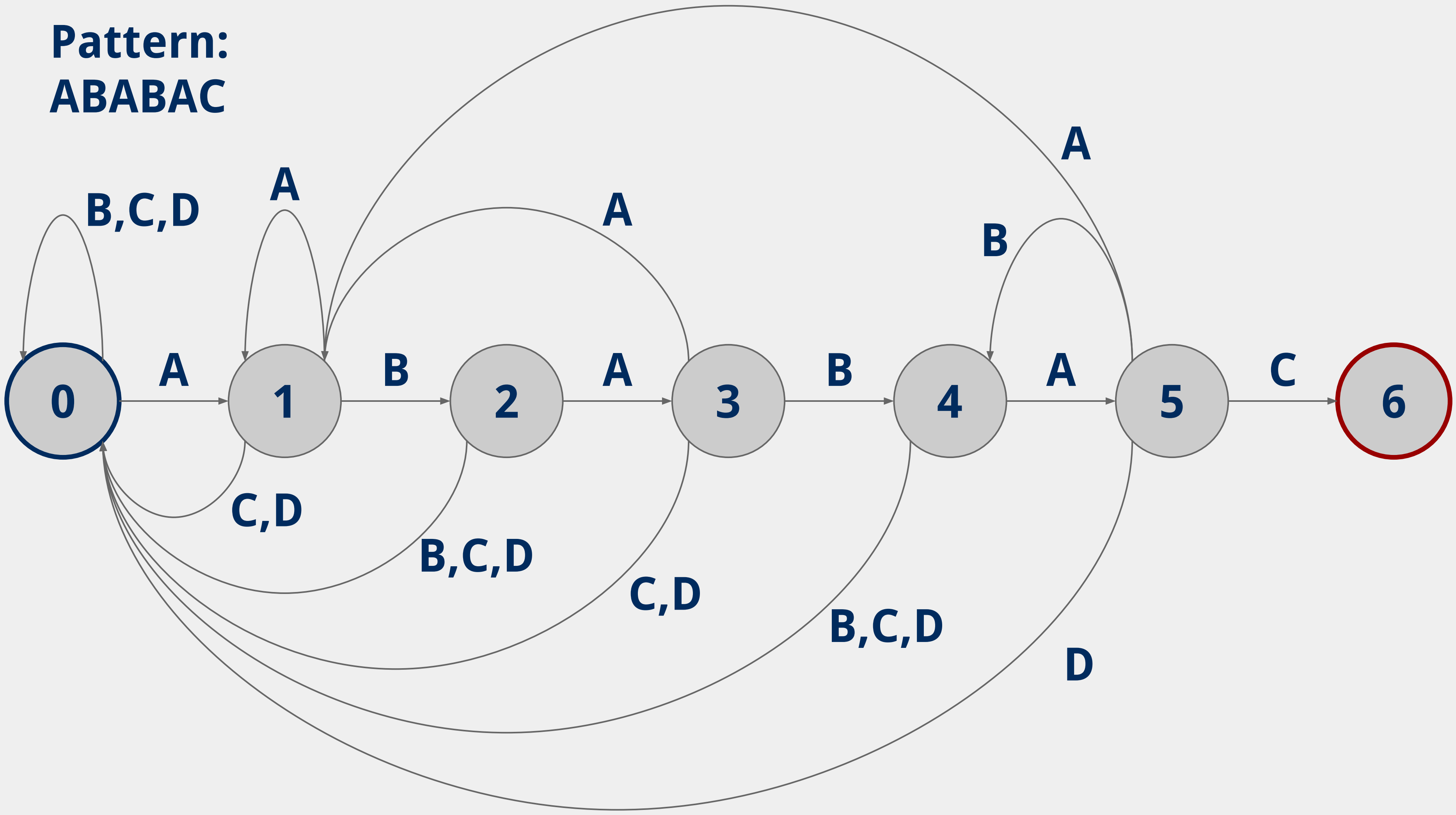
Text pointer backup in substring searching

How do we keep track of text processed?

- Actually, build a deterministic finite-state automata (DFA) storing information about the *pattern*
  - From a given state in searching through the pattern, if you encounter a mismatch, how many characters currently match from the beginning of the pattern

DFA example

Pattern:
ABABAC



Representing the DFA in code

- DFA can be represented as a 2D array:
  - dfa[cur_text_char][pattern_counter] = new_pattern_counter
    - Storage needed?
      - mR

	0	1	2	3	4	5
A						
B						
C						
D						

KMP code

```
public int kmp_search(String pat, String txt) {
    int m = pat.length();
    int n = txt.length();
    int i, j;
    for (i = 0, j = 0; i < n && j < m; i++)
        j = dfa[txt.charAt(i)][j];
    if (j == m) return i - m; // found
    return = n; // not found
}
```

- Runtime?

Another approach: Boyer Moore

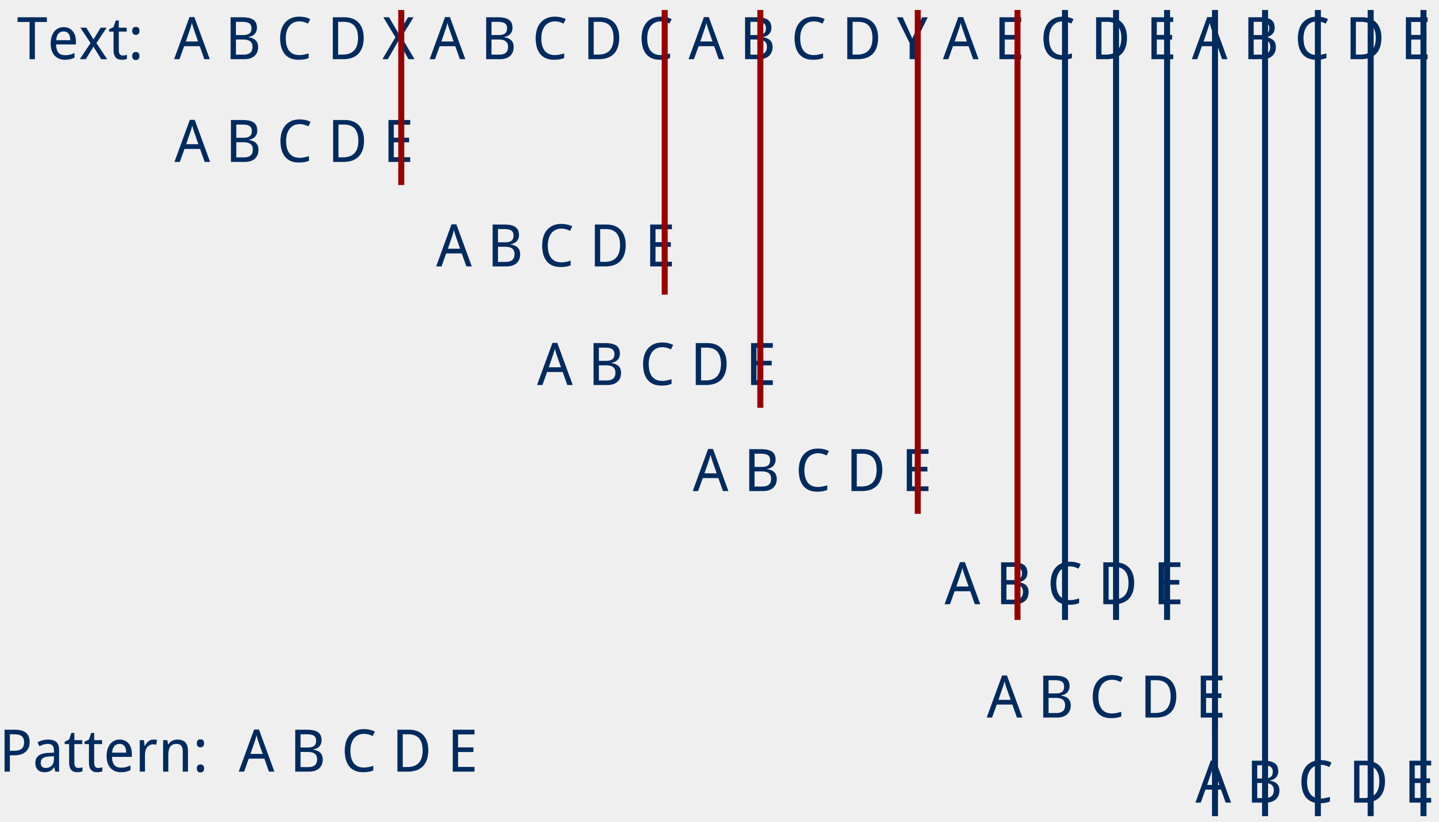
- What if we compare starting at the end of the pattern?
  - a = ABCDVABCDWABCDXABCDYABCDZ
  - p = ABCDE
  - V does not match E
    - Further V is nowhere in the pattern…
    - So skip ahead m positions with 1 comparison!
      - Runtime?
        - In the best case, n/m
- When searching through text with a large alphabet, will often come across characters not in the pattern.
  - One of Boyer Moore's heuristics takes advantage of this fact
    - Mismatched character heuristic

Missed character heuristic

- How well it works depends on the pattern and text at hand
    - What do we do in the general case after a mismatch?
        - Consider:
            - a = XYXYXYZXXXXXXXXXXXXXX
            - p = XYXYZ
        - If mismatched character *does* appear in p, need to “slide” to the right to the next occurrence of that character in p
            - Requires us to pre-process the pattern
                - Create a right array

```
for (int i = 0; i < R; i++)
   right[i] = -1;
for (int j = 0; j < m; j++)
   right[p.charAt(j)] = j;
```

Missed character heuristic example



Pattern: A B C D E

right = [0, 1, 2, 3, 4, -1, -1, …]

Runtime for missed character

- What does the worst case look like?
  - Runtime:
    - Θ(nm)
      - Same as brute force!
- This is why missed character is only one of Boyer Moore's heuristics
  - The works similarly to KMP
- See BoyerMoore.java

Another approach

- Hashing was cool, let's try using that

```
public static int hash_search(String pat, String txt) {
    int m = pat.length();
    int n = txt.length();
    int pat_hash = h(pat);
    for (int i = 0; i <= n - m; i++) {
        if (h(txt.substring(i, i + m)) == pat_hash)
            return i; // found!
    }
    return n; // not found
}
```

Well that was simple

- Is it efficient?
    - Nope! Practically worse than brute force
        - Instead of nm character comparisons, we perform n hashes of m character strings
- Can we make an efficient pattern matching algorithm based on hashing?

Horner’s method

- Brought up during the hashing lecture

```
public long horners_hash(String key, int m) {
    long h = 0;
    for (int j = 0; j < m; j++)
        h = (R * h + key.charAt(j)) % Q;
    return h;
}
```

- horners_hash("abcd", 4) =
  - 'a' * R^3 + 'b' * R^2 + 'c' * R + 'd' mod Q
- horners_hash("bcde", 4) =
  - 'b' * R^3 + 'c' * R^2 + 'd' * R + 'e' mod Q
- horners_hash("cdef", 4) =
  - 'c' * R^3 + 'd' * R^2 + 'e' * R + 'f' mod Q

Efficient hash-based pattern matching

```
text = "abcdefg"
pattern = "defg"
```

- This is Rabin-Karp

What about collisions?

- Note that we're not storing any values in a hash table…
    - So increasing Q doesn't affect memory utilization!
        - Make Q really big and the chance of a collision becomes really small!
            - But not 0…
- OK, so do a character by character comparison on a hash match just to be sure
    - Worst case runtime?
        - Back to brute force esque runtime...

Assorted casinos

- Two options:
  - Do a character by character comparison after hash match
    - Guaranteed correct
    - Probably fast

Las Vegas

  - Assume a hash match means a substring match
    - Guaranteed fast
    - Probably correct

Monte Carlo