```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import itertools
```

A

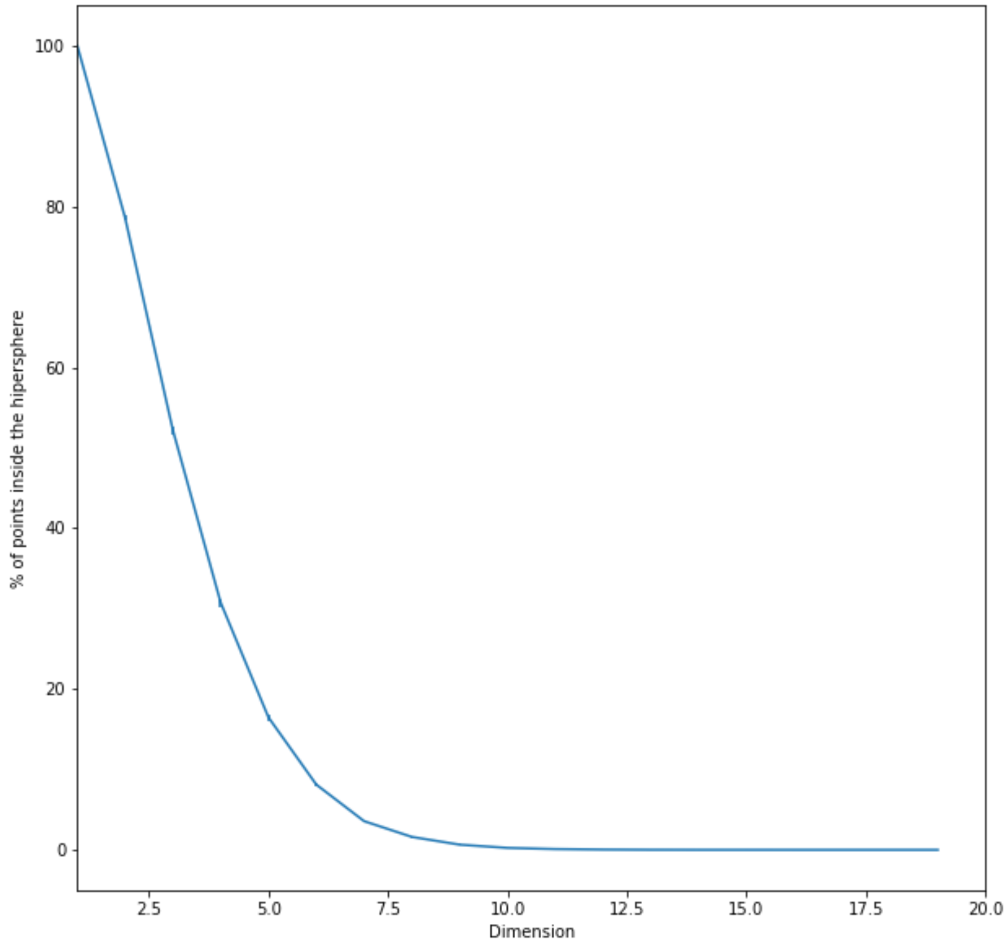
```
In [2]: def is_in_hipersphere(point,radius,center_point):
            return np.square(point - center_point).sum() <= np.square(radius)
```

```
In [3]: def generate_random_point(min_val,max_val,dim):
            np.random.rand(dim)
            return np.random.rand(dim)*(max_val-min_val) - abs(min_val)
```

```
In [4]: X = 1
        def space(dim,density):
            return np.array([generate_random_point(-X,X,dim) for _ in np.arange(dens
        ity)])
```

```
In [5]: density= 10000
        dims = np.arange(1,20)
        aggr_counts=[]
        counts = []

        for _ in range(10):
            for dim in dims:
                spc = space(dim,density)
                mask = np.array([is_in_hipersphere(point,X,np.zeros(dim)) for point
        in spc])
                counts += [len(spc[mask])/density*100]
            aggr_counts += [np.array(counts)]
            counts = []
```

In [6]:
```
mean_counts = np.mean(aggr_counts,axis=0)
deviation= np.std(aggr_counts,axis=0)
fig,ax = plt.subplots(figsize=(10,10))
ax.errorbar(dims,mean_counts,yerr=deviation)
plt.xlim(1,20)
ax.set_xlabel('Dimension')
ax.set_ylabel('% of points inside the hipersphere')
plt.show()
```



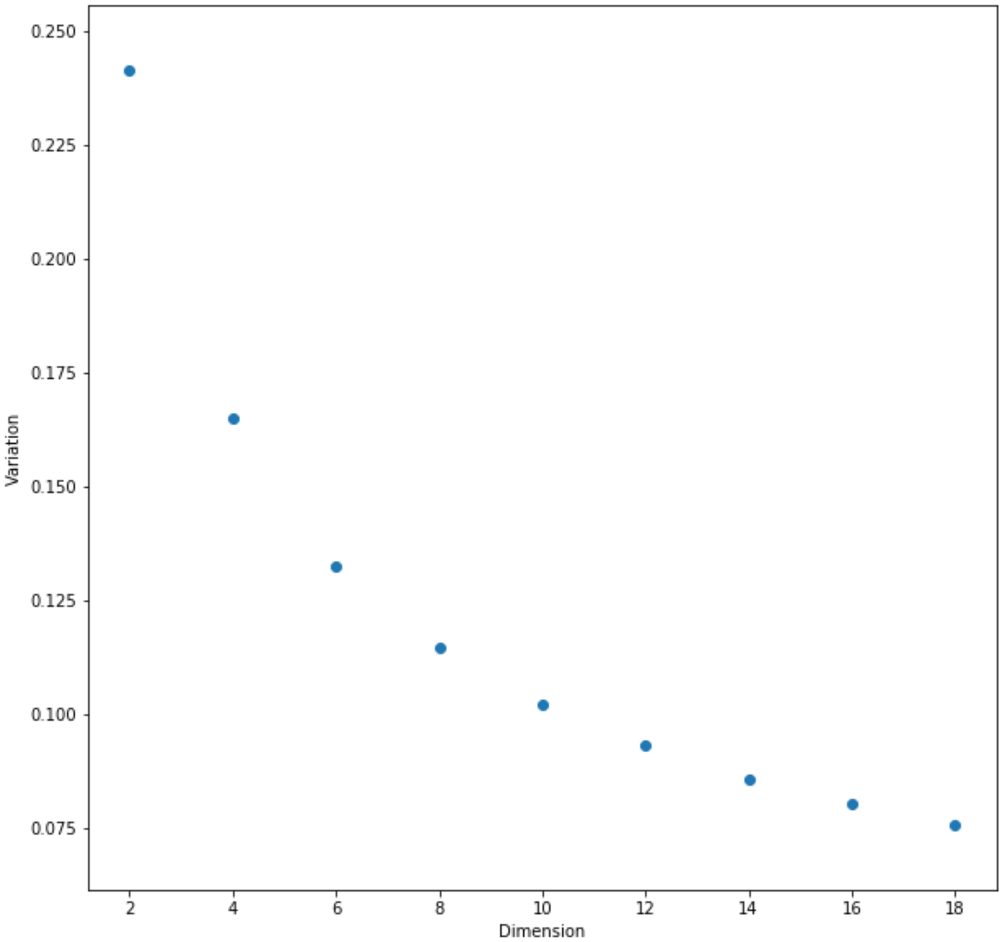
Można zauważyć że wraz ze wzrostem wymiaru przestrzeni spada objętość hiperkuli względem objętości sześcianu, co jest zjawiskiem oczekiwanym

B

In [7]:
```
def space(density,dim):
    return np.array([generate_random_point(0,1,dim) for _ in np.arange(densi
ty)])
```

```
In [8]: dims = np.arange(2,20,2)
        coeff = []
        for dim in dims:
            tmp_coeff = []
            for _ in range(10):
                spc = space(1000,dim)
                combs = np.array(list(itertools.combinations(spc,2)))
                distances = np.sqrt(np.sum(np.sum(np.square(combs),axis=2),axis=1))
                tmp_coeff += [np.std(distances)/np.mean(distances)]
            coeff += [np.mean(tmp_coeff)]
            tmp_coeff = []
```

```
In [9]: fig,ax = plt.subplots(figsize=(10,10))
        ax.scatter(np.arange(2,20,2),coeff)
        ax.set_xlabel('Dimension')
        ax.set_ylabel('Variation')
        plt.show()
```



Widzimy że spada wariancja odległości dwóch losowo wybranych punktów, wynika to z tego że odległość punktów zależy od położenia a to zależy od losowy wybranych punktów. Zatem zwiększając poziom wymiaru, upodabniamy punkty względem odległości => odległość jest słabą metryką różnicującą w przestrzeni hiperwymiarowej.

C

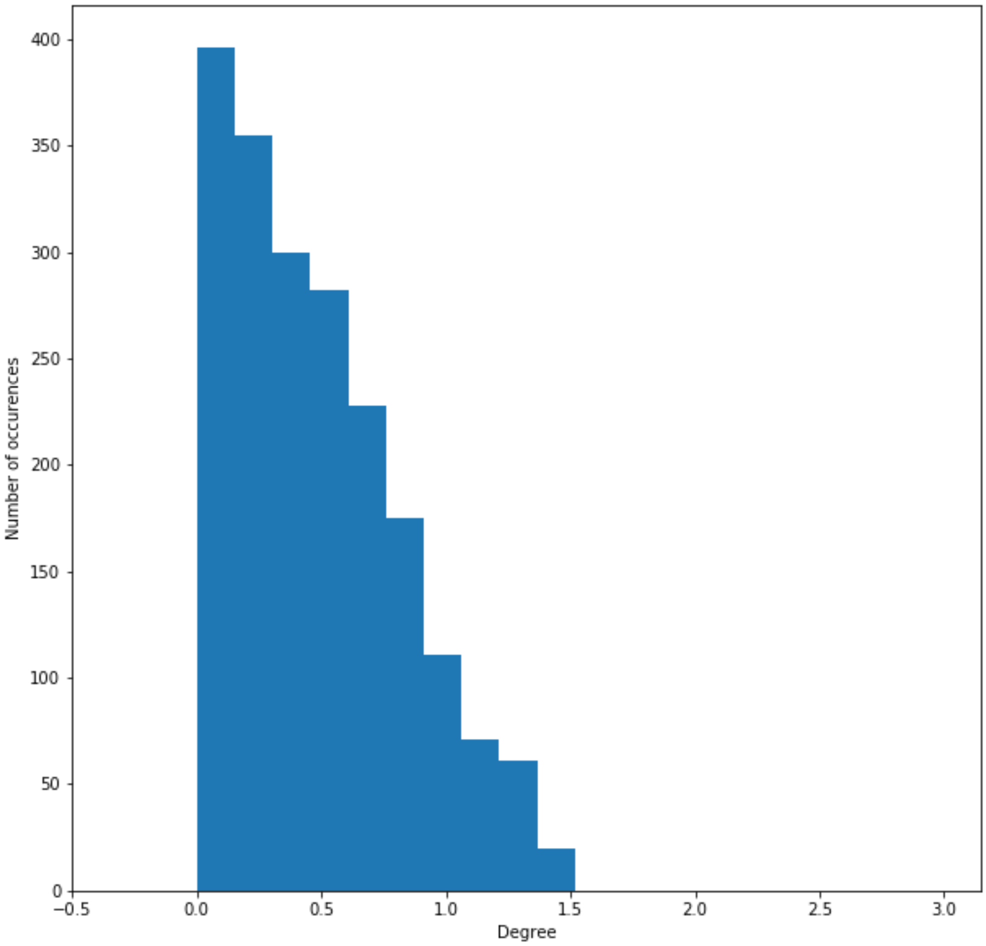
In [10]:
```python
def unit_vector(vector):
    return vector / np.linalg.norm(vector)

def angle_between(v1, v2):
    v1_u = unit_vector(v1)
    v2_u = unit_vector(v2)
    return np.arccos(np.clip(np.dot(v1_u, v2_u), -1.0, 1.0))
```

In [11]:
```python
def show_histogram(spc):
    size = spc.shape[0]
    shuffled = np.random.permutation(spc)
    shuffled_pairs = [angle_between(*i) for i in zip(shuffled[:size-1],shuff
led[1:size])]
    fig,ax = plt.subplots(figsize=(10,10))
    ax.hist(shuffled_pairs)
    ax.set_xlabel('Degree')
    ax.set_ylabel('Number of occurences')
    plt.xlim(-.5, 3.15)
    print(f'Current dim {dim}')
    plt.show()
```

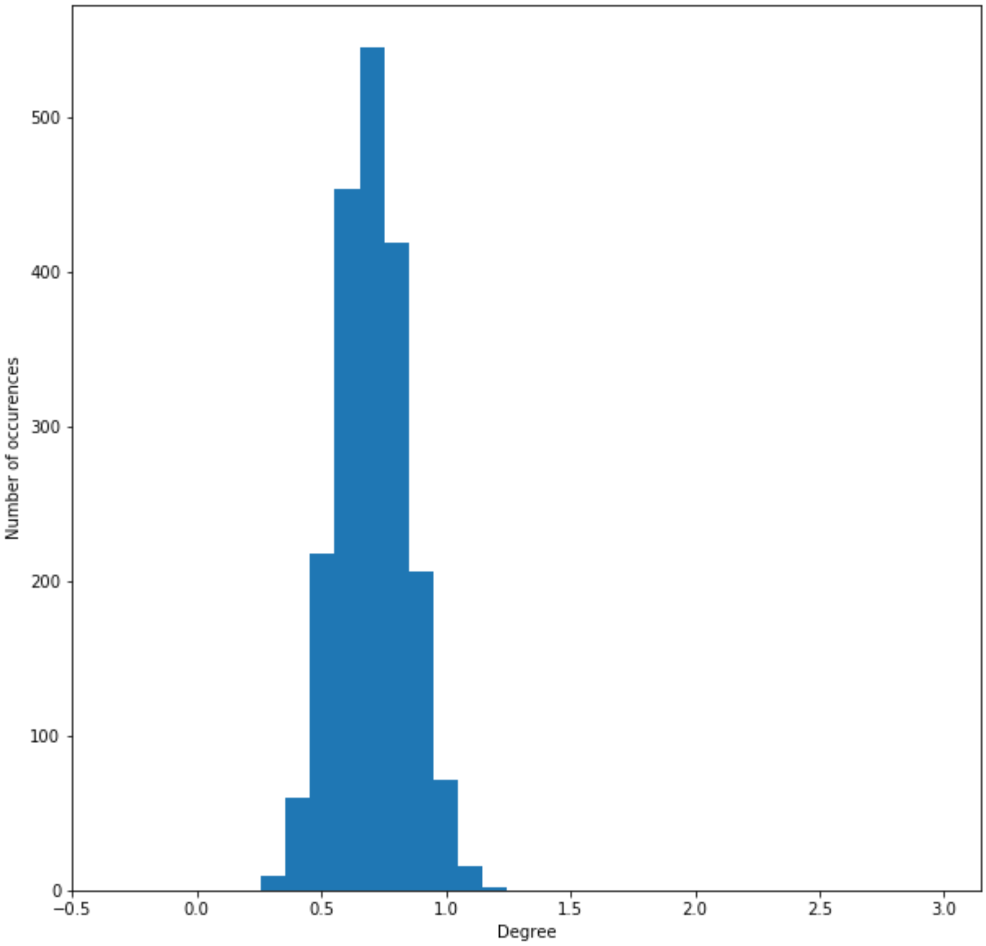
In [12]:

```
for dim in range(2,100,10):
    spc = space(2*1000,dim)
    show_histogram(spc)
```

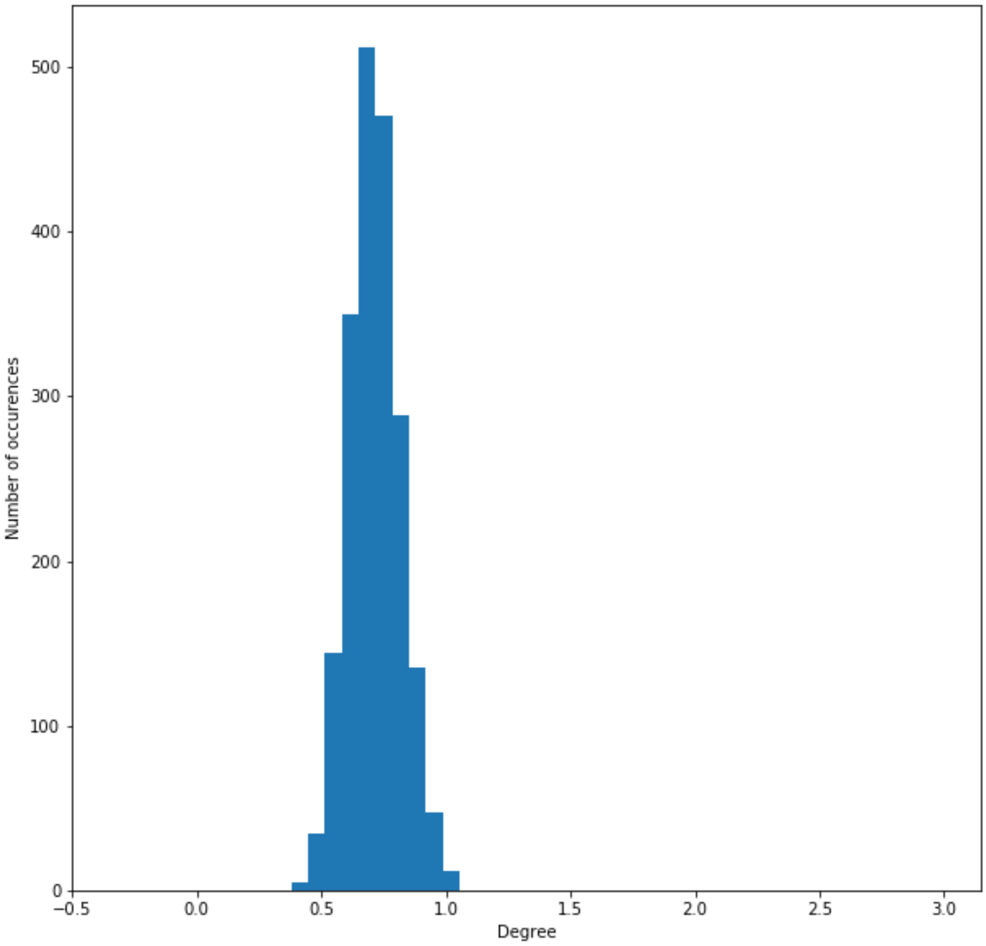
Current dim 2



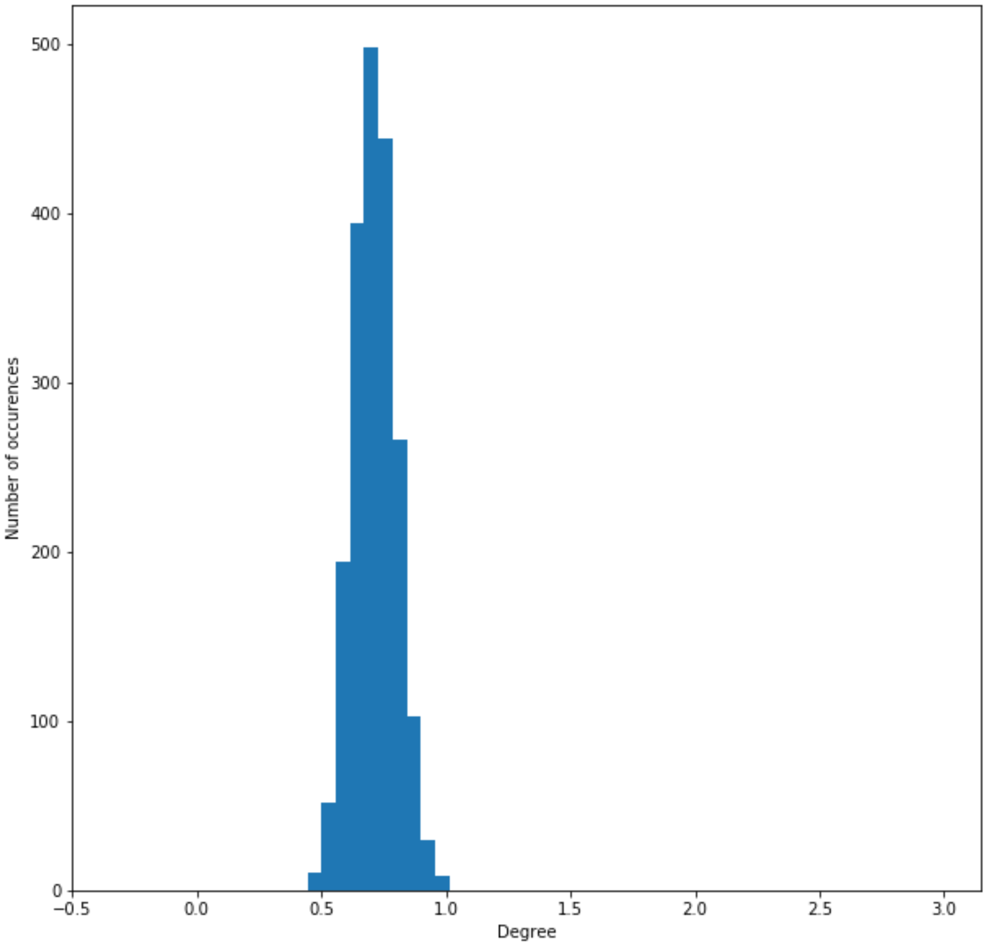
Current dim 12



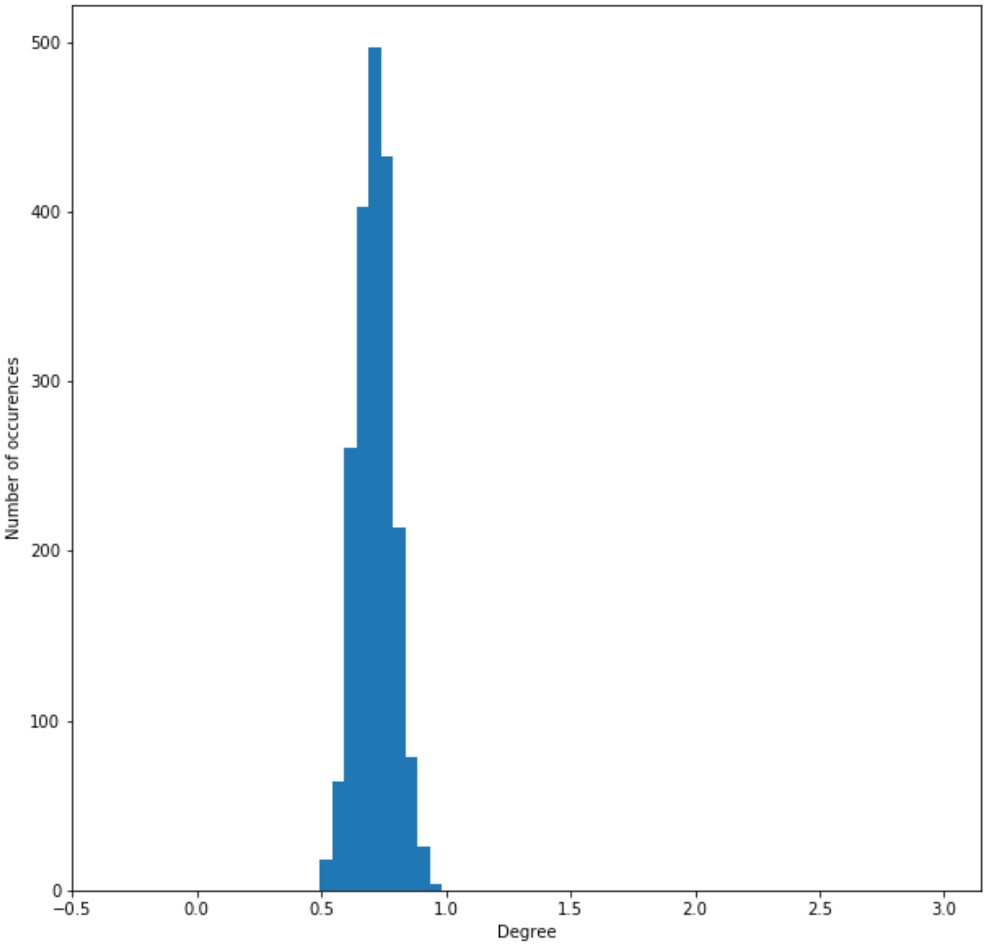

```
Current dim 22
```



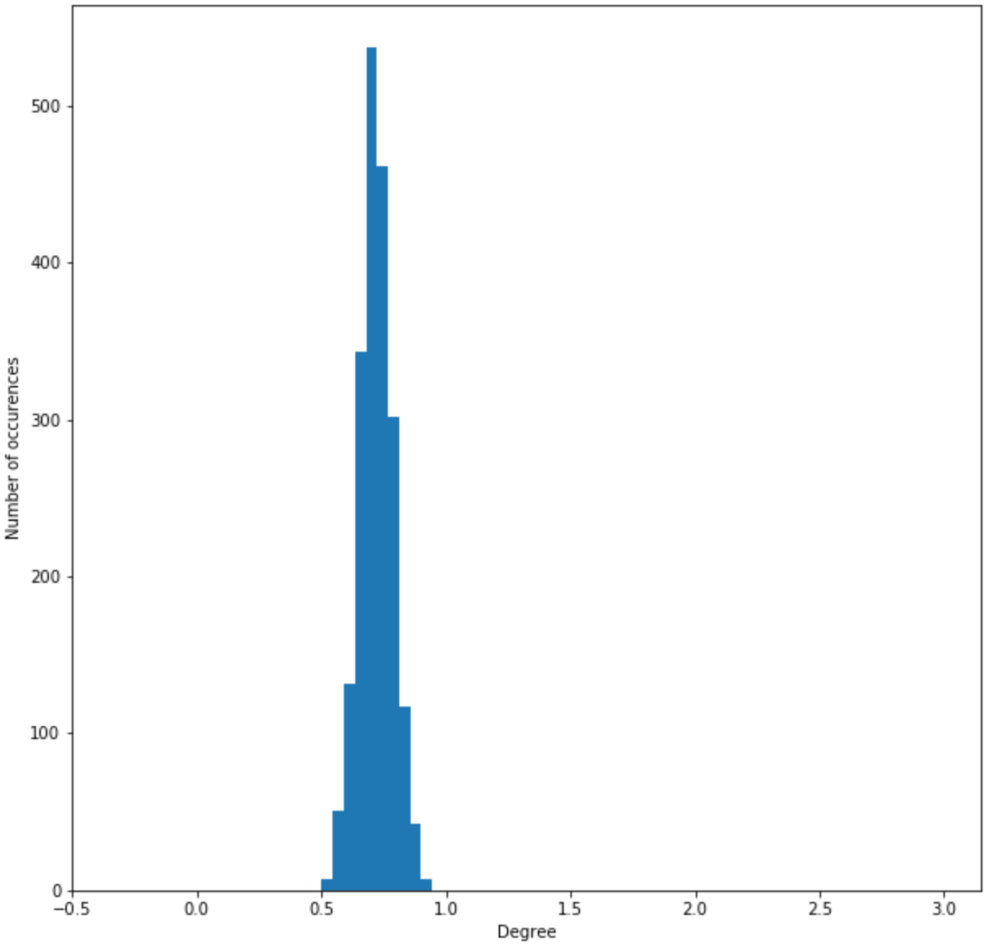
Current dim 32



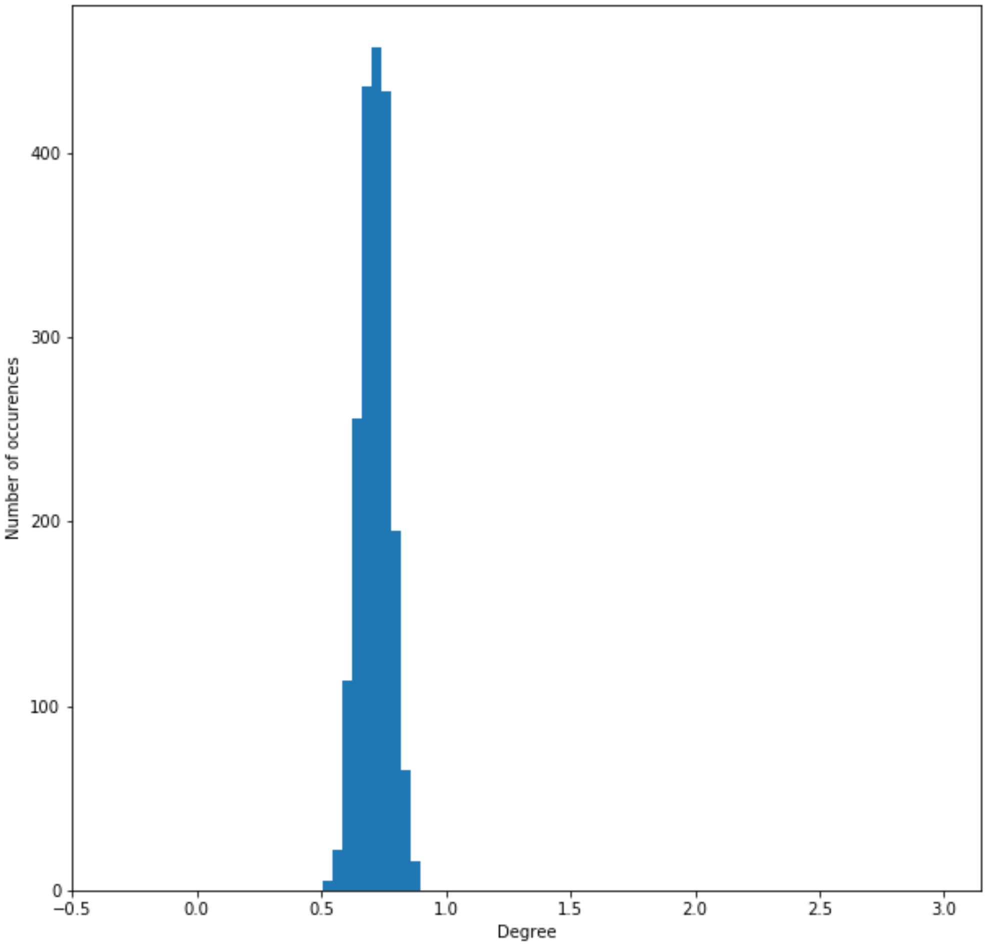
Current dim 42



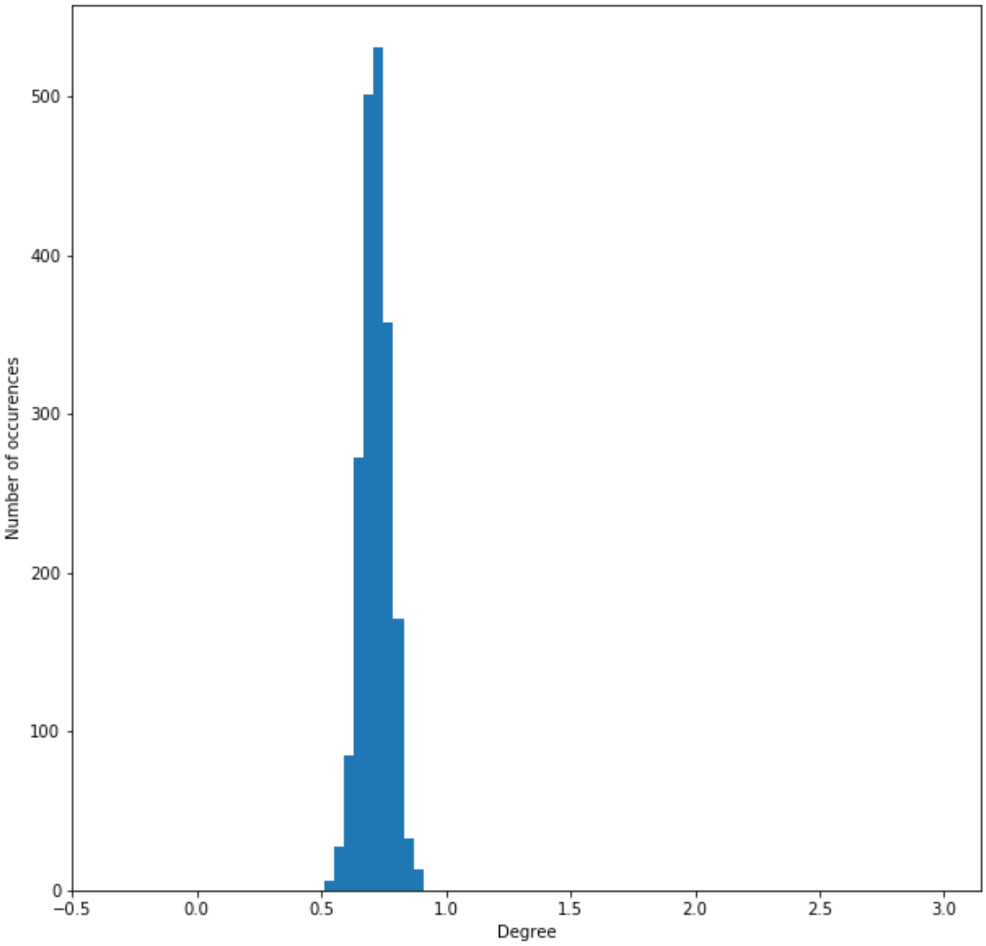
Current dim 52



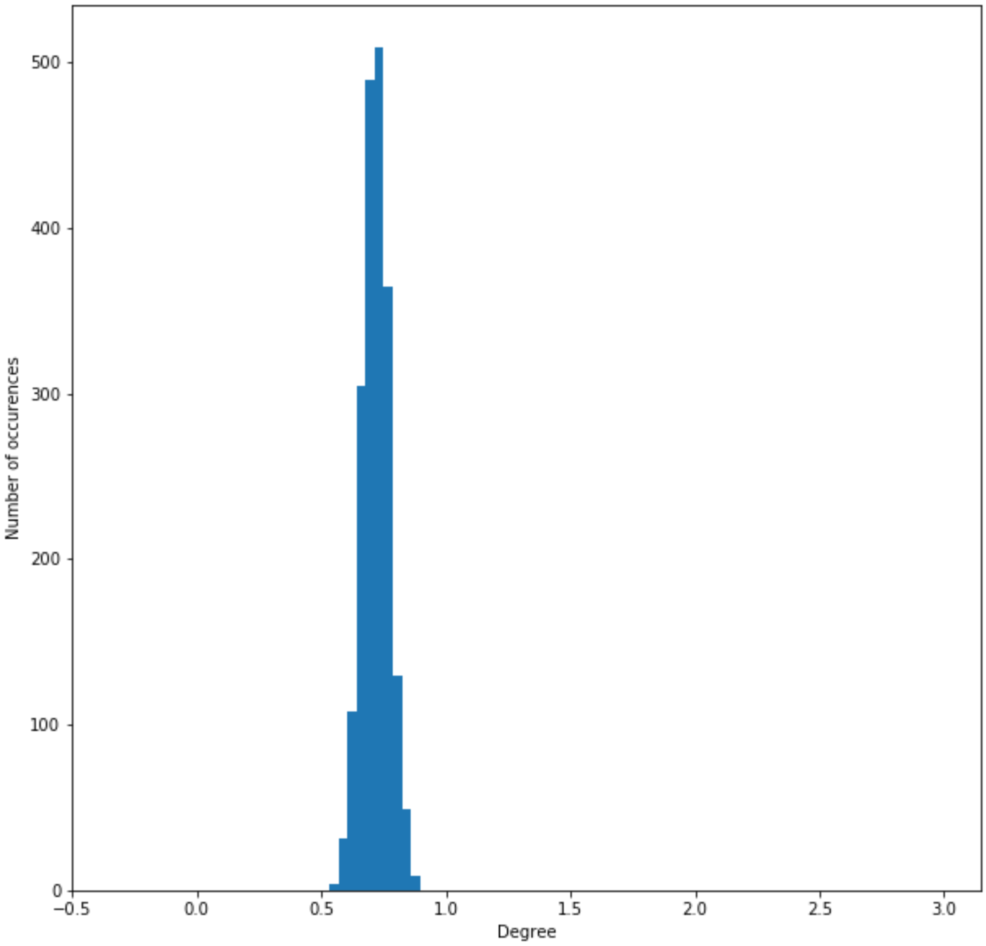

```
Current dim 62
```



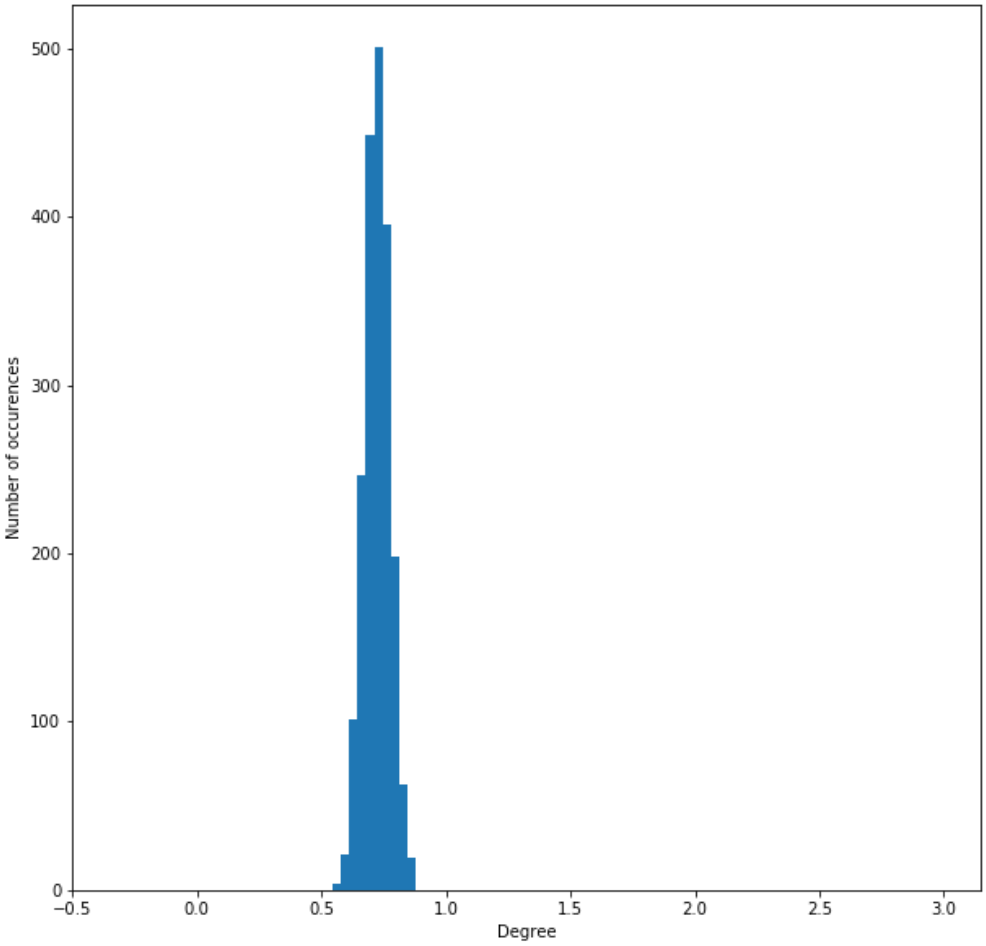
Current dim 72



Current dim 82



Current dim 92



Widzimy że miara kąta losowo stworzonego wektora dąży do ok 40 stopni co pokazuje że miara tego kąta również nie będzie dobrym rozróżnikiem przy identyfikacji wektorów w przestrzeniach wielo wymiarowych.