Collin Blinder
cblinder@students.pitzer.edu
Problem 1:

$T(n) = T(n-1)+n+c$
$T(n-1) = T(n-2)+(n-1)+c$
$T(n-2) = T(n-3)+(n-2)+c$

$T(n) = T(n-2)+(n-1)+c+n+c$
$= T(n-3)+(n-2)+c +(n-1)+c+n+c$
$= T(n-3)+(n-2)+(n-1)+n +3c$

$T(n) = T(n-k) + (n-(k+1)) + (n-(k+2)) + .. + n + kc$

$O(n^2)$

Problem 2:

$$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$$
$$0\leq c_1 \log_a(n) \leq f(n)\leq c_2 \log_b(n)$$

$$\log_a(n) = \log_d(n)/\log_d(a)$$
$$\log_b(n) = \log_d(n)/\log_d(b)$$

$$0\leq c_1 (\log_d(n)/\log_d(a)) \leq f(n)\leq c_2 (\log_d(n)/\log_d(b))$$

$$= 0\leq (c_1/\log_d(a)) \times \log_d(n) \leq f(n)\leq (c_2/\log_d(b)) \times \log_d(n)$$

$$= 0\leq C \times \log_d(n) \leq f(n)\leq C \times \log_d(n)$$

$$= 0\leq \log_d(n) \leq f(n)\leq \log_d(n)$$

Problem 4:

Best case

$T(n) = 2T(n/2)+cn$

$T(n/2) = 2T(n/4)+cn/2$

$T(n/3) = 2T(n/6)+cn/3$

$T(n) = 2(2T(n/4)+cn/2)+cn$
$= 4T(n/4)+4cn/2+cn$
$= 4T(n/4)+2cn+cn$

$T(n/k) = 2T(n/2k)+cn/k$
$T(n/n) = 2T(n/2n)+cn/n$
$= 2T(n/2n)+c$
$= cT(n/2n)+c$
$= T(n/2n)$
= Omega(log2(n))

Worst case

$T(n) = T(n-1)+cn$
$T(n-1) = T(n-2)+c(n-1)$
$T(n-2) = T(n-3) + c(n-2)$

$T(n) = T(n-2)+c(n-1)+cn$
$T(n) = T(n-3) + c(n-2) +c(n-1)+cn$
$T(n) = T(n-k)+c(n-k+1) + c(n-k+2) + \ldots + c(n-2) + c(n-1) + cn$