Transformation of the factorization problem into a problem of solving a non-linear system

All the numbers can be traced back to this case:

Input N such that N = p * q & (p + q-4) mod 8 = 0 & (q-p + 2) / 4 = y is odd & (p + q-4)/8=x is even

Algorithm A

0 step

$(3 * N-1) / 8 = 3 * x * (x + 1) / 2-3 * y * (y-1) / 2 + (3 * x + 1) * (3 * x + 2) / 2$

,

$p=2*(3*x+1-(x-y+1))+1-(4*y-2)$

,

$q=2*(3*x+1-(x-y+1))+1$

1 step

$(3*N-1)/8 = (3*x*(x + 1))/2 - (3*y*(y - 1))/2 + ((3*x + 1)*(3*x + 2))/2$

,

$M=9*((2*(3*N-1)/8 - 3*y + 1)/24+(y-1)*(y+1)/8)+1$

,

$3*((2*M - 3*z + 1)/24 + (3*x*(x + 1))/2) + 1 = A$

,

$M = (3*x*(x + 1))/2 - (3*z*(z - 1))/2 + ((3*x + 1)*(3*x + 2))/2$

,

$3*((2*A - 3*a + 1)/24 + (3*x*(x + 1))/2) + 1 = B$

,

$a = 1$

,
x>=1
,
y>=1
,
z=x+1
,
A>M
,
B=A
,
p=2*(3*x+1-(x-y+1))+1-(4*y-2)
,
q=2*(3*x+1-(x-y+1))+1

2 step
(3*N-1)/8 = (3*x*(x + 1))/2 - (3*y*(y - 1))/2 + ((3*x + 1)*(3*x + 2))/2
,
M=9*((2*(3*N-1)/8 - 3*y + 1)/24+(y-1)*(y+1)/8)+1
,
3*((2*M - 3*z + 1)/24 + (3*x*(x + 1))/2) + 1 = A
,
M = (3*x*(x + 1))/2 - (3*z*(z - 1))/2 + ((3*x + 1)*(3*x + 2))/2
,
3*((2*A - 3*a + 1)/24 + (3*x*(x + 1))/2) + 1 = B
,
A + (3*a*(a - 1))/2 = (12*x*(x + 1))/2 + 1
,
3*((2*B - 3*b + 1)/24 + (3*x*(x + 1))/2) + 1 = C
,
b = 1
,
x>=1
,
y>=1
,
z=x+1

,
A>M
,
B>A
,
C=B
,
p=2*(3*x+1-(x-y+1))+1-(4*y-2)
,
q=2*(3*x+1-(x-y+1))+1

3 step
(3*N-1)/8 = (3*x*(x + 1))/2 - (3*y*(y - 1))/2 + ((3*x + 1)*(3*x + 2))/2
,
M=9*((2*(3*N-1)/8 - 3*y + 1)/24+(y-1)*(y+1)/8)+1
,
3*((2*M - 3*z + 1)/24 + (3*x*(x + 1))/2) + 1 = A
,
M = (3*x*(x + 1))/2 - (3*z*(z - 1))/2 + ((3*x + 1)*(3*x + 2))/2
,
3*((2*A - 3*a + 1)/24 + (3*x*(x + 1))/2) + 1 = B
,
A + (3*a*(a - 1))/2 = (12*x*(x + 1))/2 + 1
,
3*((2*B - 3*b + 1)/24 + (3*x*(x + 1))/2) + 1 = C
,
B + (3*b*(b - 1))/2 = (12*x*(x + 1))/2 + 1
,
3*((2*C - 3*c + 1)/24 + (3*x*(x + 1))/2) + 1 = D
,
c = 1
,
x>=1
,
y>=1
,
z=x+1

,
A>M
,
B>A
,
C>B
,
D=C
,
p=2*(3*x+1-(x-y+1))+1-(4*y-2)
,
q=2*(3*x+1-(x-y+1))+1

4 step
(3*N-1)/8 = (3*x*(x + 1))/2 - (3*y*(y - 1))/2 + ((3*x + 1)*(3*x + 2))/2
,
M=9*((2*(3*N-1)/8 - 3*y + 1)/24+(y-1)*(y+1)/8)+1
,
3*((2*M - 3*z + 1)/24 + (3*x*(x + 1))/2) + 1 = A
,
M = (3*x*(x + 1))/2 - (3*z*(z - 1))/2 + ((3*x + 1)*(3*x + 2))/2
,
3*((2*A - 3*a + 1)/24 + (3*x*(x + 1))/2) + 1 = B
,
A + (3*a*(a - 1))/2 = (12*x*(x + 1))/2 + 1
,
3*((2*B - 3*b + 1)/24 + (3*x*(x + 1))/2) + 1 = C
,
B + (3*b*(b - 1))/2 = (12*x*(x + 1))/2 + 1
,
3*((2*C - 3*c + 1)/24 + (3*x*(x + 1))/2) + 1 = D
,
C + (3*c*(c - 1))/2 = (12*x*(x + 1))/2 + 1
,
3*((2*D - 3*d + 1)/24 + (3*x*(x + 1))/2) + 1 = E
,
d = 1

,
x>=1
,
y>=1
,
z=x+1
,
A>M
,
B>A
,
C>B
,
D>C
,
E=D
,
p=2*(3*x+1-(x-y+1))+1-(4*y-2)
,
q=2*(3*x+1-(x-y+1))+1

5 step
(3*N-1)/8 = (3*x*(x + 1))/2 - (3*y*(y - 1))/2 + ((3*x + 1)*(3*x + 2))/2
,
M=9*((2*(3*N-1)/8 - 3*y + 1)/24+(y-1)*(y+1)/8)+1
,
3*((2*M - 3*z + 1)/24 + (3*x*(x + 1))/2) + 1 = A
,
M = (3*x*(x + 1))/2 - (3*z*(z - 1))/2 + ((3*x + 1)*(3*x + 2))/2
,
3*((2*A - 3*a + 1)/24 + (3*x*(x + 1))/2) + 1 = B
,
A + (3*a*(a - 1))/2 = (12*x*(x + 1))/2 + 1
,
3*((2*B - 3*b + 1)/24 + (3*x*(x + 1))/2) + 1 = C
,
B + (3*b*(b - 1))/2 = (12*x*(x + 1))/2 + 1

,
3*((2*C - 3*c + 1)/24 + (3*x*(x + 1))/2) + 1 = D
,
C + (3*c*(c - 1))/2 = (12*x*(x + 1))/2 + 1
,
3*((2*D - 3*d + 1)/24 + (3*x*(x + 1))/2) + 1 = E
,
D + (3*d*(d - 1))/2 = (12*x*(x + 1))/2 + 1
,
3*((2*E - 3*e + 1)/24 + (3*x*(x + 1))/2) + 1 = F
,
e = 1
,
x>=1
,
y>=1
,
z=x+1
,
A>M
,
B>A
,
C>B
,
D>C
,
E>D
,
F=E
,
p=2*(3*x+1-(x-y+1))+1-(4*y-2)
,
q=2*(3*x+1-(x-y+1))+1

6 step

.....

.....
.....

Alorithm A work

the logarithm acts on x in this way

If (x + 1 + 1) / 2 is even a = - (x-1) / 2

If (x + 1 + 1) / 2 is odd a = (x + 1) / 2

If (| a | +1) / 2 is even b = - (| a | -1) / 2

If (| a | +1) / 2 is odd b = (| a | +1) / 2

and so on

for | a | I mean unsigned a

Algorithm B

0 step
(3 * N-1) / 8 = 3 * x * (x + 1) / 2-3 * y * (y-1) / 2 + (3 * x + 1) * (3 * x + 2) / 2

1 step
3 * (((2 * (3 * N-1) / 8-3 * y + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = A
,
(3 * N-1) / 8 = 3 * x * (x + 1) / 2-3 * y * (y-1) / 2 + (3 * x + 1) * (3 * x + 2) / 2
,
3 * (((2 * A-3 * a + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = B

2 step

$$3 * (((2 * (3 * N-1) / 8-3 * y + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = A$$

,

$$(3 * N-1) / 8 = 3 * x * (x + 1) / 2-3 * y * (y-1) / 2 + (3 * x + 1) * (3 * x + 2) / 2$$

,

$$3 * (((2 * A-3 * a + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = B$$

,

$$A + 3 * a * (a-1) / 2 = 12 * x * (x + 1) / 2 + 1$$

,

$$3 * (((2 * B-3 * b + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = C$$

3 step

$$3 * (((2 * (3 * N-1) / 8-3 * y + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = A$$

,

$$(3 * N-1) / 8 = 3 * x * (x + 1) / 2-3 * y * (y-1) / 2 + (3 * x + 1) * (3 * x + 2) / 2$$

,

$$3 * (((2 * A-3 * a + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = B$$

,

$$A + 3 * a * (a-1) / 2 = 12 * x * (x + 1) / 2 + 1$$

,

$$3 * (((2 * B-3 * b + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = C$$

,

$$B + 3 * b * (b-1) / 2 = 12 * x * (x + 1) / 2 + 1$$

,

$$3 * (((2 * C-3 * c + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = D$$

,

4 step

.....

.....

and so on

$x>=1$

,

$y>=1$

,

$1<A< B< C< D<E<.....$

Algorithm B

the logarithm acts on y in this way

If (y + 1) / 2 is even a = - (y-1) / 2

If (y + 1) / 2 is odd a = (y + 1) / 2

If (| a | +1) / 2 is even b = - (| a | -1) / 2

If (| a | +1) / 2 is odd b = (| a | +1) / 2

and so on

for | a | I mean unsigned a

A combination of the systems of algorithm A with algorithm B will give us the desired solution

Example

3367 = (3*x*(x + 1))/2 - (3*y*(y - 1))/2 + ((3*x + 1)*(3*x + 2))/2

,

M=9*((2*3367 - 3*y + 1)/24+(y-1)*(y+1)/8)+1

,

3*((2*M - 3*z + 1)/24 + (3*x*(x + 1))/2) + 1 = A

,

M = (3*x*(x + 1))/2 - (3*z*(z - 1))/2 + ((3*x + 1)*(3*x + 2))/2

,

3*((2*A - 3*a + 1)/24 + (3*x*(x + 1))/2) + 1 = B

,

A + (3*a*(a - 1))/2 = (12*x*(x + 1))/2 + 1

,

3*((2*B - 3*b + 1)/24 + (3*x*(x + 1))/2) + 1 = C

,

B + (3*b*(b - 1))/2 = (12*x*(x + 1))/2 + 1

,

3*((2*C - 3*c + 1)/24 + (3*x*(x + 1))/2) + 1 = D

,

C + (3*c*(c - 1))/2 = (12*x*(x + 1))/2 + 1

,

3*((2*D - 3*d + 1)/24 + (3*x*(x + 1))/2) + 1 = E

,

D + (3*d*(d - 1))/2 = (12*x*(x + 1))/2 + 1

,

3*((2*E - 3*e + 1)/24 + (3*x*(x + 1))/2) + 1 = E

,

e = 1

,

x>=1

,

y>=1

,

z=x+1

,

A>M

,

B>A

,

C>B

,

D>C

,

E>D

,

3*(((2*3367-3*y+1)/24)+3*x*(x+1)/2)+1=V

,

3367=3*x*(x+1)/2-3*y*(y-1)/2+(3*x+1)*(3*x+2)/2

,

3*(((2*V-3*v+1)/24)+3*x*(x+1)/2)+1=U

,

V+3*v*(v-1)/2=12*x*(x+1)/2+1

,

3*(((2*U-3*u+1)/24)+3*x*(x+1)/2)+1=T

,

U+3*u*(u-1)/2=12*x*(x+1)/2+1

,

3*(((2*T-3*t+1)/24)+3*x*(x+1)/2)+1=S

,

T+3*t*(t-1)/2=12*x*(x+1)/2+1

,

3*(((2*S-3*s+1)/24)+3*x*(x+1)/2)+1=S

,

s= 1

,

U>V

,

T>U

,

S>T

,

p=2*(3*x+1-(x-y+1))+1-(4*y-2)

,

q=2*(3*x+1-(x-y+1))+1

-> p =73 & q=123

Explanation

we must think of all factorizable numbers such that the sum of their factors is 8 * x + 4

example

for x = 3

they are

195 y odd
187 y even
171 odd y
147 y even
115 y odd
75 y even

of which the relative M = (3 * N-1) / 8 are
73
70
64
55
43
28

let's calculate
Z = (2 * M-3 * y + 1) / 24 if y is odd
and
Z = (2 * M-3 * (1-y) +1) / 24 if y is even

6
6
5
5
3
3

now let's calculate 3 * (Z + 3 * x * (x + 1) / 2) +1
73
73
70
70
64
64

so if we want to factor

115 with y equal to 5

3 * (((2 * 43-3 * y + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = A

A will be 64

3 * (((2 * A-3 * a + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = B

a will be 3 from what has been said about how the logarithm works and B = 70

therefore

A + 3 * a * (a-1) / 2 = 12 * x * (x + 1) / 2 + 1

a = 3 -> b = -1 from what has been said about how the logarithm works

3 * (((2 * B-3 * b + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = C

and C = 73

therefore

B + 3 * b * (b-1) / 2 = 12 * x * (x + 1) / 2 + 1

3 * (((2 * C-3 * c + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = D

c will be 1 from what has been said about how the logarithm works

and D = C = 73

All N can be transformed in our case

an odd number is in the form 4 * k + 1 or 4 * k + 3

our N is in the form 4 * k + 3

now the 4 * k + 3 are divided into (p + q) mod 8 = 0 or (p + q-4) mod 8 = 0

now to go from 4 * k + 1 to 4 * h + 3 multiply by 3

to go from (p + q) mod 8 = 0 to (p + q-4) mod 8 = 0 multiply by 5

so if you have a 4 * k + 1 you have to multiply by 3 and by 15 and you will have one of the two in the form (p + q-4) mod 8

if you have a 4 * k + 3 you have to multiply by 5 and you will have one of the two in the form (p + q-4) mod 8

now if y is odd it is used

3 * (((2 * (3 * N-1) / 8-3 * y + 1) / 24) + 3 * x * (x + 1) / 2) + 1 = A

if y is even it is used

3 * (((2 * (3 * N-1) / 8-3 * (1-y) +1) / 24) + 3 * x * (x + 1) / 2) + 1 = A

I hope I was clear

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