Лабораторна робота №2

Півень Денис

Варіант 10

1 Метод Гаусса

1.1 Опис методу

Нехай початкова система виглядає наступним чином:

$$\begin{cases} a_{11}x_1 + ... + a_{1n}x_n = b_1 \\ ... \\ a_{n1}x_1 + ... + a_{mn}x_n = b_n \end{cases}$$

Запишемо її у матричному вигляді:

$$Ax = b$$

де

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

Запишимо доповнену матрицю системи:

$$\left(\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} & b_n \end{array}\right)$$

Взявши a_{11} ведучим елементом, шляхом елементарних перетворень рядків отримаємо нулі в першому стовпчику (зрозуміло за винятком a_{11}). А сам ведучий рядок поділимо на a_{11}.

$$\left(\begin{array}{cccc|c} 1 & \hat{a}_{12} & \dots & \hat{a}_{1n} & \hat{b}_1 \\ 0 & \hat{a}_{22} & \dots & \hat{a}_{2n} & \hat{b}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \hat{a}_{n2} & \dots & \hat{a}_{nn} & \hat{b}_n \end{array}\right)$$

Аналогічним чином робимо з наступним діагональним елементом, тобто $\hat{a}_{22}$.

$$\left(\begin{array}{cccc|c} 1 & 0 & \dots & \breve{a}_{1n} & \breve{b}_1 \\ 0 & 1 & \dots & \breve{a}_{2n} & \breve{b}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \breve{a}_{nn} & \breve{b}_n \end{array}\right)$$

Продовжуємо поки не отримаємо одиничну матрицю.

$$\left(\begin{array}{cccc|c} 1 & 0 & \dots & 0 & \bar{b}_1 \\ 0 & 1 & \dots & 0 & \bar{b}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & \bar{b}_n \end{array}\right)$$

Тоді вектор $\bar{b}$ є розв'язком нашої системи.

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \vdots \\ \bar{b}_n \end{pmatrix}$$

1.2 Розв'язання

Умова задачі:

$$\left(\begin{array}{cccc|c} 4 & 3 & 1 & 0 & 29 \\ -2 & 2 & 6 & 1 & 38 \\ 0 & 5 & 2 & 3 & 48 \\ 0 & 1 & 2 & 7 & 56 \end{array}\right)$$

Розв'яжемо задачу програмно:

Відповідь: $\begin{cases} x_1 = 3 \\ x_2 = 4 \\ x_3 = 5 \\ x_4 = 6 \end{cases}$

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Task: Gauss`s Method
+4.00 +3.00 +1.00 +0.00         +29.00
-2.00 +2.00 +6.00 +1.00         +38.00
+0.00 +5.00 +2.00 +3.00         +48.00
+0.00 +1.00 +2.00 +7.00         +56.00

Step: 1
+1.00 +0.75 +0.25 +0.00         +7.25
+0.00 +3.50 +6.50 +1.00         +52.50
+0.00 +5.00 +2.00 +3.00         +48.00
+0.00 +1.00 +2.00 +7.00         +56.00

Step: 2
+1.00 +0.00 -1.14 -0.21         -4.00
+0.00 +1.00 +1.86 +0.29         +15.00
+0.00 +0.00 -7.29 +1.57         -27.00
+0.00 +0.00 +0.14 +6.71         +41.00

Step: 3
+1.00 +0.00 +0.00 -0.46         +0.24
+0.00 +1.00 +0.00 +0.69         +8.12
+0.00 +0.00 +1.00 -0.22         +3.71
+0.00 +0.00 +0.00 +6.75         +40.47

Step: 4
+1.00 +0.00 +0.00 +0.00         +3.00
+0.00 +1.00 +0.00 +0.00         +4.00
+0.00 +0.00 +1.00 +0.00         +5.00
+0.00 +0.00 +0.00 +1.00         +6.00

Answer:
x1 = 3.00
x2 = 4.00
x3 = 5.00
x4 = 6.00
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2 Метод квадратного кореня

2.1 Опис методу

Якщо в системи лінійних алгебраїчних рівнянь $Ax = b$ матриця A є невиродженою ($detA \neq 0$) та симетричною ($A = A^T$), то розв'язок можна знайти методом квадратного кореня.

Матриця A симетрична, то ми можемо розкласти її на добуток взаэмотранспонованих трикутних матриць $A = T^T T$

$$T = \begin{pmatrix} t_{11} & t_{12} & \dots & t_{1n} \\ 0 & t_{22} & \dots & t_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & t_{nn} \end{pmatrix}, \quad T^T = \begin{pmatrix} t_{11} & 0 & \dots & 0 \\ t_{12} & t_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ t_{1n} & t_{2n} & \dots & t_{nn} \end{pmatrix}$$

Перемножимо матриці T та T^T, отриману матрицю прирівняємо до матриці A. Отримаємо наступні формули для знаходження невідомих t_{ij}:

$$\begin{cases} t_{11} = \sqrt{a_{11}}, \quad t_{1j} = \dfrac{a_{1j}}{t_{11}}, & j > i \\ t_{ii} = \sqrt{a_{ii} - \sum\limits_{k=1}^{i-1} t_{ki}^2}, & 1 < i \leq n \\ t_{ij} = \dfrac{a_{ii} - \sum\limits_{k=1}^{i-1} t_{ki} \cdot t_{kj}}{t_{ii}}, & i < j \\ t_{ij} = 0, & i > j \end{cases}$$

Так як матриця A представлена у вигляді добутку, то систему можна замінити двома системами рівнянь вигляду:

$$T^T \cdot y = b, \quad T \cdot x = y$$

Далі знаходимо розв'язки цих систем, наприклад методом Гаусса.

2.2 Розв'язання

Умова задачі:

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 2 & 2 & 4 & 22 \\ 0 & 4 & 3 & 20 \end{array}\right)$$

Розв'яжемо задачу програмно:

Відповідь: $\begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = 4 \end{cases}$

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Task: Square Root Method
(1.00,0.00) (2.00,0.00) (0.00,0.00)     (5.00,0.00)
(2.00,0.00) (2.00,0.00) (4.00,0.00)     (22.00,0.00)
(0.00,0.00) (4.00,0.00) (3.00,0.00)     (20.00,0.00)

T Matrix:
(1.00,0.00) (2.00,0.00) (0.00,0.00)
(0.00,0.00) (0.00,1.41) (0.00,-2.83)
(0.00,0.00) (0.00,0.00) (3.32,0.00)

T-Transpose Matrix:
(1.00,0.00) (0.00,0.00) (0.00,0.00)
(2.00,0.00) (0.00,1.41) (0.00,0.00)
(0.00,0.00) (0.00,-2.83) (3.32,0.00)

Task: Find y-vector with T-Transpose Matrix
(1.00,0.00) (0.00,0.00) (0.00,0.00)     (5.00,0.00)
(2.00,0.00) (0.00,1.41) (0.00,0.00)     (22.00,0.00)
(0.00,0.00) (0.00,-2.83) (3.32,0.00)    (20.00,0.00)

Step: 1
(1.00,0.00) (0.00,0.00) (0.00,0.00)     (5.00,0.00)
(0.00,0.00) (0.00,1.41) (0.00,0.00)     (12.00,0.00)
(0.00,0.00) (0.00,-2.83) (3.32,0.00)    (20.00,0.00)

Step: 2
(1.00,0.00) (0.00,0.00) (0.00,0.00)     (5.00,0.00)
(0.00,0.00) (1.00,0.00) (0.00,0.00)     (0.00,-8.49)
(0.00,0.00) (0.00,0.00) (3.32,0.00)     (44.00,0.00)

Step: 3
(1.00,0.00) (0.00,0.00) (0.00,0.00)     (5.00,0.00)
(0.00,0.00) (1.00,0.00) (0.00,0.00)     (0.00,-8.49)
(0.00,0.00) (0.00,0.00) (1.00,0.00)     (13.27,0.00)
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Task: Find x-vector with T Matrix
(1.00,0.00) (2.00,0.00) (0.00,0.00)     (5.00,0.00)
(0.00,0.00) (0.00,1.41) (0.00,-2.83)    (0.00,-8.49)
(0.00,0.00) (0.00,0.00) (3.32,0.00)     (13.27,0.00)

Step: 1
(1.00,0.00) (2.00,0.00) (0.00,0.00)     (5.00,0.00)
(0.00,0.00) (0.00,1.41) (0.00,-2.83)    (0.00,-8.49)
(0.00,0.00) (0.00,0.00) (3.32,0.00)     (13.27,0.00)

Step: 2
(1.00,0.00) (0.00,0.00) (4.00,0.00)     (17.00,0.00)
(0.00,0.00) (1.00,0.00) (-2.00,-0.00)   (-6.00,-0.00)
(0.00,0.00) (0.00,0.00) (3.32,0.00)     (13.27,0.00)

Step: 3
(1.00,0.00) (0.00,0.00) (0.00,0.00)     (1.00,-0.00)
(0.00,0.00) (1.00,0.00) (0.00,0.00)     (2.00,0.00)
(-0.00,-0.00) (0.00,0.00) (1.00,0.00)   (4.00,-0.00)

Answer:
x1 = (1.00,-0.00)
x2 = (2.00,0.00)
x3 = (4.00,-0.00)
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