1 Annulus Volume

Given two cylindrical annuli of densities ρ_1 andρ_2 respectively, shared mass m and outer radius R, find inner radii r_1 and r_2 so that each have the same mass. We can calculate the volume as a

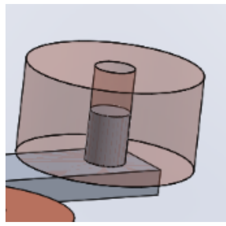

Figure 1: Cylindrical annulus

difference between the volumes of a large cylinder (R) and small cylinder (r).

$$V = \pi R^2 h - \pi r^2 h$$

The mass is then

$$m = \pi h(R^2 - r^2)\rho_{Cu}$$

If we include the aluminium pin as part of the mass,

$$m = \pi(R_o^2 - R_i^2)H\rho_{Cu} + \pi(R_i^2)\frac{2}{3}H\rho_{Al6061}$$

Solving within Sympy for the inner radius,

$$R_i = \sqrt{\frac{3}{\pi}}\sqrt{\frac{m - \pi H\rho_{Cu}R_o^2}{H(2\rho_{Al} - 3\rho_{Cu})}}$$

1.1 Experimental calculations

Determining mass of the copper annulus:

- $H = 0.300$ in $= 0.007620$ m
- $R_{o,Cu} = 0.275$ in $= 0.006985$ m
- $R_{i,Cu} = 0.125$ in $= 0.003175$ m
- $\rho_{Cu} = 8940$ kg/m^3
- $\rho_{Nb} = 8570$ kg/m^3

- $\rho_{Al} = 2700$ kg/m^3

The mass of the test body and the aluminium pin is then

$$m_{Cu} = \pi(0.006985^2 - 0.003175^2) * 0.007620 * 8940 + \pi * 0.003175^2 * \frac{2}{3} * 0.007620 * 2700 = 0.008719 \text{ kg}$$

Testing to confirm our algebra was right for R_i,

$$r = \sqrt{\frac{3m - 3\pi R_o^2 H \rho_{Cu}}{\pi H(2\rho_{Al} - 3\rho_{Cu})}} \tag{1}$$

$$r = \sqrt{\frac{3(0.01039) - 3\pi 0.006985^2(0.007620)(8940)}{\pi 0.006985(2(2700) - 3(8940))}} \tag{2}$$

$$r = 0.003175 \tag{3}$$

Determining the inner radius of the niobium annulus uses this mass as found in (3) above.

$$R_{i,Nb} = 0.002822 \text{ m}$$

If we use a variable outer radius, inner radius held at 0.003175 m,

$$R_{o,Nb} = 0.007104 \text{ m}$$