SECTION 2: ELECTRICITY & MAGNETISM

'Electricity & Magnetism' contains the following topics:

TOPIC 2.1: Electric Fields

2.1.1 Coulomb's Law

Coulomb's Law

(Required definition)

"*There is a mutual force between any two point charges which is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. This force acts along the line joining them and is repulsive if the charges are alike and attractive if the charges are unlike*".

This law can be represented by the formula:

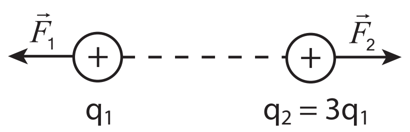
$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 . q_2}{r^2}$$

(The term on the left is called Coulomb's constant and has the value 8.99×10^9 N m^2 C^{-2}).

If the two charges are alike (both positive or both negative), the force will have a positive value and be repulsive, while if the two charges are unlike (one positive and one negative), the force will have a negative value and be attractive.

Newton's Third Law applies to the case of electric forces. This means that each charge will exert an equal and opposite force on the other charge, even if the two charges have different magnitudes. These forces must act along the line connecting the two point charges.

Newton's 3rd Law applies:



$$|\vec{F}_1| = |\vec{F}_2|$$

2.1.2 Changes in the Electric Force

A. Due to a change in charge

The electric force is directly proportional to the product of the two charges. Therefore, if one of two charges that experience an electric force is altered by a certain proportion, the electric force between them will change by the same proportion. For example, if one of the two charges is doubled, the electric force will also be doubled.

i.e. if the two initial charges are Q and q (at a distance r apart), the electric force will be

$$F_1 = \frac{1}{4\pi\varepsilon_0} . \frac{Q.q}{r^2} \quad .$$

If charge Q is replaced by charge 2Q, the new electric force becomes

$$F_2 = \frac{1}{4\pi\varepsilon_0} . \frac{(2Q)q}{r^2} = \frac{1}{4\pi\varepsilon_0} . \frac{2Q.q}{r^2} = 2F_1 .$$

B. Due to a change in distance

The electric force is inversely proportional to the square of the distance between the two point charges. Therefore, if the distance between two point charges that experience an electric force is altered by a certain proportion, the electric force between them will change by the inverse square of that proportion. For example, if the distance between the two charges is doubled, the electric force will be one quarter of the original force.

i.e. if the two charges (Q and q) are initially a distance r apart, the electric force will be

$$F_1 = \frac{1}{4\pi\varepsilon_0} . \frac{Q.q}{r^2} \quad .$$

If the distance is increased to 2r, the new electric force becomes

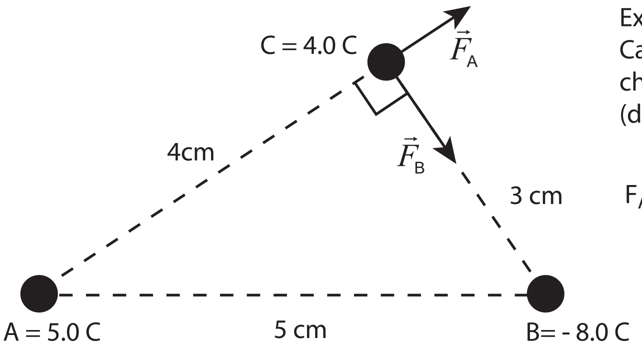
$$F_2 = \frac{1}{4\pi\varepsilon_0} . \frac{Q.q}{(2r)^2} = \frac{1}{4\pi\varepsilon_0} . \frac{Q.q}{4r^2} = \frac{1}{4}F_1 .$$

Student Tip

Coulomb's Law can be applied in the same way as Newton's Law of Universal Gravitation, with charges replacing masses and the constant $\frac{1}{4\pi\varepsilon_0}$ replacing G. Therefore, the proportionality principles apply similarly to both laws. However, remember that the gravitational force is always attractive, while the electric force can be attractive or repulsive.

2.1.3 The Superposition Principle

Coulomb's Law can also be applied to situations where there are more than two point charges. In this case, the overall force acting on any one charge is equal to the vector sum of the individual forces on this charge due to each of the other charges. For example, given three point charges A, B and C, the force on charge A is the vector sum of the force that B exerts on A and the force that C exerts on A.



Example Question:
Calculate the force on charge C due to charges A and B
(don't forget to convert distances to metres)

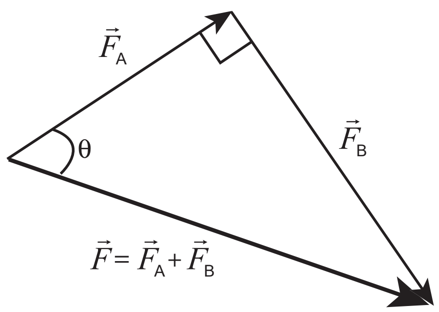
$$F_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A.q_C}{r^2_{AC}}$$

$$= 1.125 \times 10^{14}\ \text{N}$$

$$F_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B.q_C}{r^2_{BC}}$$

$$= 3.2 \times 10^{14}\ \text{N}$$

Answer:



1. $F = \sqrt{(1.125 \times 10^{14})^2 + (3.2 \times 10^{14})^2}$ (Pythagoras)

   $= 3.39 \times 10^{14}$ N

2. $\tan\theta = \dfrac{F_B}{F_A}$

   $\theta = 70.6^{\circ}$

Therefore $\vec{F} = 3.39 \times 10^{14}$ N 70.6° clockwise from $\vec{F}_A$

2.1.4 The Electric Field

Any electric charge will create an electric field in the space around it. Whenever another charge is placed in this space, it will experience an electric force due to the electric field.

(SACE Stage 2 Physics Curriculum Statement, Topic 2.1)
"The electric field at a point is defined as the electric force F per unit charge on a small positive test charge q placed at that point, provided that all charges remain undisturbed".

It is given the symbol E and has the units of newtons per coulomb ($N.C^{-1}$).

$$\vec{E} = \frac{\vec{F}}{q}$$

When a charge is placed in an electric field, it will experience a force in the same direction as the field if the charge is positive, while it will experience a force in the opposite direction to the field if it is negative.

The concept of an electric field can replace the concept of “action at a distance”. Instead of one charge exerting a force on another charge that is some distance away, it is the electric field of the first charge that exerts a force on the second, and correspondingly, the electric field of the second charge exerts a reaction force on the first.

Electric field strength can also be represented by deriving the following formula:

$$E = \frac{F}{q_x} \left(\frac{\text{force on test charge}}{\text{test charge}} \right)$$

$$= \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q.q_x}{r^2}}{q_x} \quad \text{(Coulomb's Law)}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}}$$

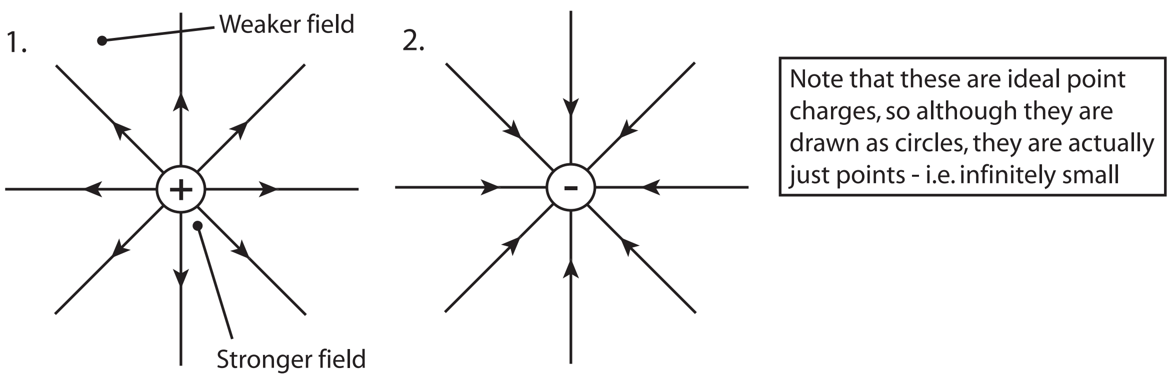
2.1.4 Pictorial Representation of Electric Fields

An electric field can be represented by drawing “electric lines of force”, or “field lines”. These lines indicate the direction of the field at any point (i.e. the direction of the force that a small positive charge would experience if placed at that point).

Electric lines of force are directed away from points of positive charge and towards points of negative charge.

The strength of the electric field is indicated by the number of field lines per unit area. Therefore, an electric field will be strongest in regions where the lines of force are close together and weakest in regions where the lines of force are far apart.

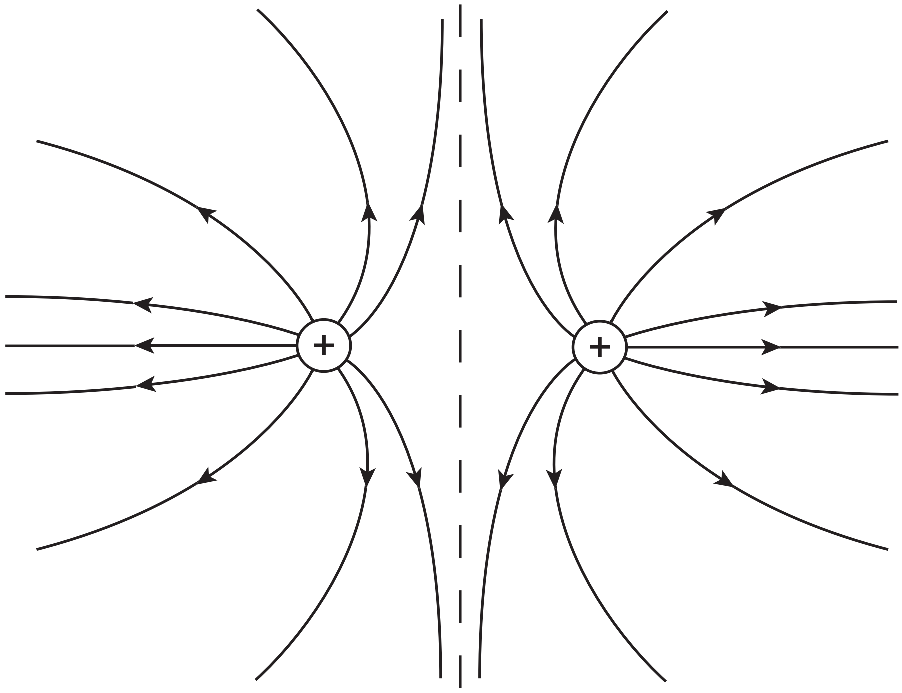
THE ELECTRIC FIELD AROUND A POINT CHARGE



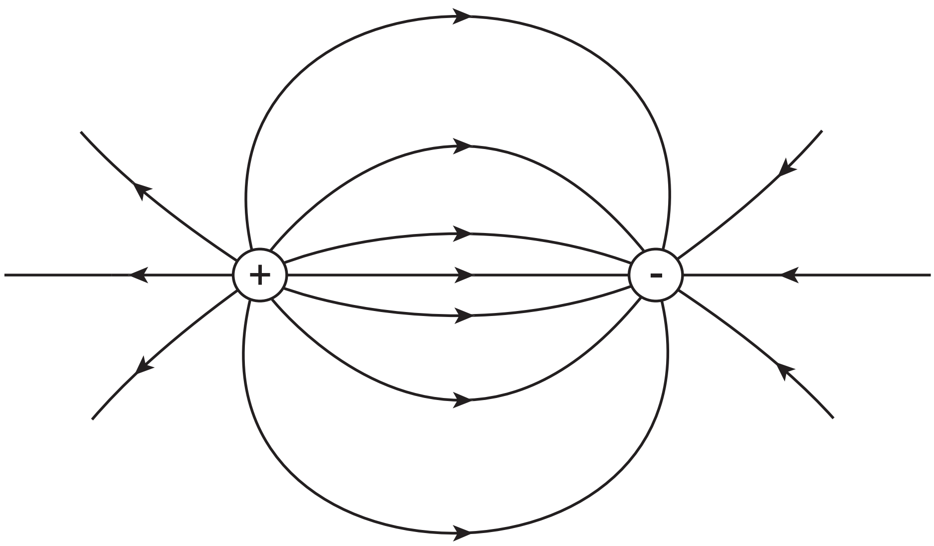
Superposition of Electric Fields

The total electric field strength at a point due to more than one electric field is equal to the vector sum of each electric field at that point.

TWO POINT CHARGES OF THE SAME MAGNITUDE AND THE SAME SIGN



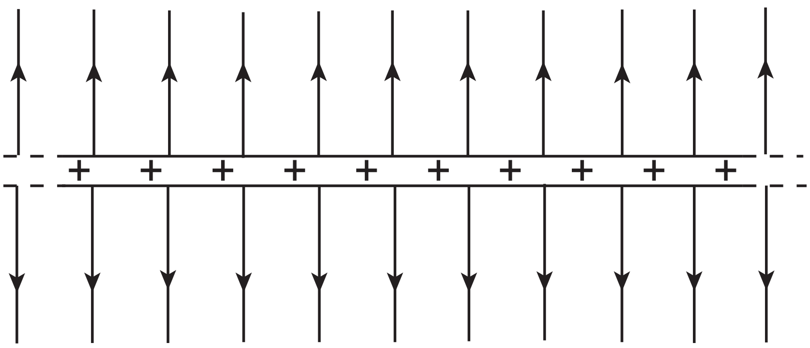
TWO OPPOSITELY CHARGED POINT CHARGES



The Electric Field due to One or Two Charged Plates

If an infinitely large conducting plate is given an overall charge, the charge will be distributed uniformly over the plate. This causes the electric field due to such a plate to be uniform.

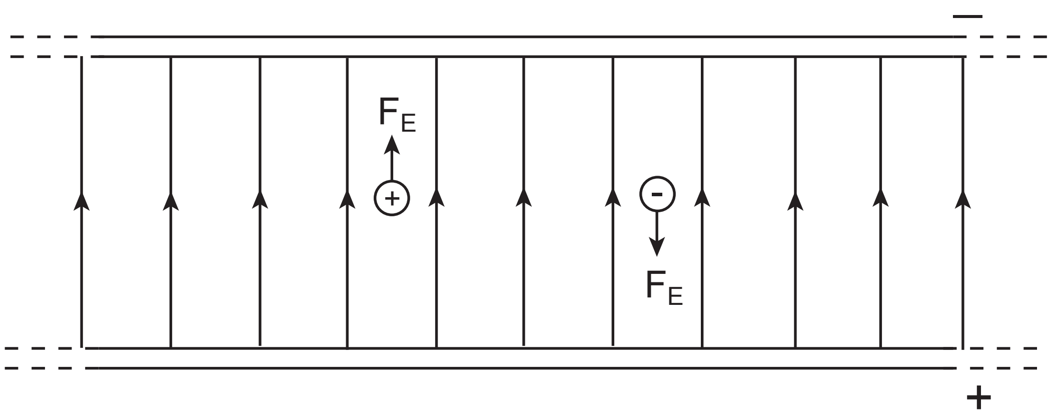
ELECTRIC FIELD PRODUCED BY A SINGLE INFINITE CHARGED PLATE



Uniform Electric Field

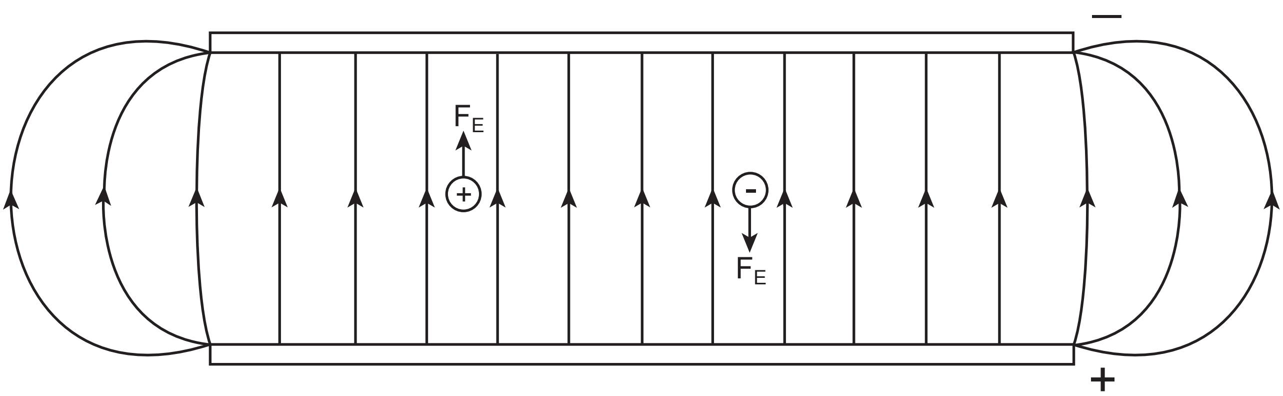
(SACE Stage 2 Physics Curriculum Statement, Topic 2.1)
"The electric field between two infinite parallel conducting plates with equal and opposite charges per unit area in uniform between the plates and zero elsewhere" .

ELECTRIC FIELD PRODUCED BY TWO INFINITE CHARGED PLATES



(SACE Stage 2 Physics Curriculum Statement, Topic 2.1)
"The electric field near and beyond the edges of two finite plates is non-uniform".

ELECTRIC FIELD PRODUCED BY TWO FINITE CHARGED PLATES



2.1.5 Conductors and Electric Fields

“For any conductor, whether charged or uncharged:

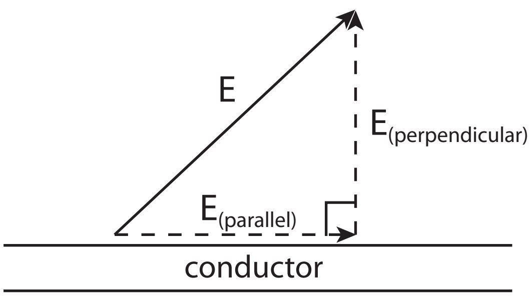
- Electric fields always meet the conducting surface at right angles;
- There is no electric field inside the conducting material

(SACE Stage 2 Physics Curriculum Statement, Topic 2.1)
An uncharged conductor within an external electric field will experience charge polarisation”.

Explanations (Proofs)

Proof 1:

Suppose the electric field line (a vector) left the surface at an angle other than 90°, as shown below:



- This vector can be resolved into horizontal and vertical components, as discussed in *Topic 1.1 Projectile Motion.*
- In this case, there would be a component of the electric field parallel to the conducting surface.
- This would exert a force on electrons on the surface of the conductor and cause them to accelerate.
- However, charged surfaces are in electrostatic equilibrium – that is, there are no charged particles moving on the surface. If the surface was not in equilibrium, the charged particles would move until they reached an equilibrium position.
- Therefore, once the surface is in equilibrium, the component of the electric field parallel to the surface must be zero.
- Hence, the electric field is perpendicular to the surface.

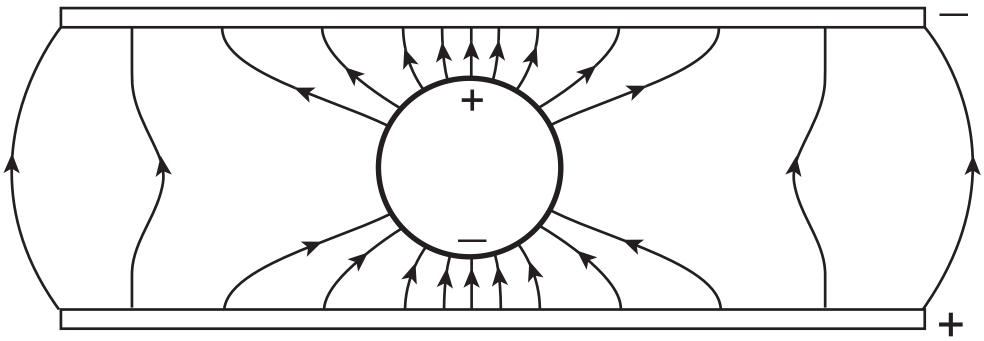
Proof 2:

Suppose there was an electric field inside a conducting material.

- In this case, all free electrons within that material would experience a force ($\vec{F}$ = q.$\vec{E}$)
- This would cause all the free electrons to move
- The electrons would continue to move until they reached positions where the electric force on them became zero.
- When this happens, the overall electric field within the conductor would become zero.
- Hence, there will be no electric field within a conductor.

Hence:

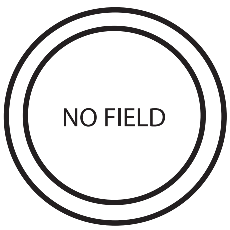
RESULTANT ELECTRIC FIELD WHEN AN UNCHARGED, SOLID CONDUCTING SPHERE IS PLACED IN A UNIFORM ELECTRIC FIELD



Note: The electrons in an uncharged conductor that is placed inside an electric field will be attracted to the end of the conductor that is nearest to the positive plate. That end of the conductor will become slightly negative, while the other end becomes slightly positive – the conductor experiences charge polarisation. This is shown in the diagram above.

The Electric Field Inside a Hollow Conductor

(SACE Stage 2 Physics Curriculum Statement, Topic 2.1)
“There is no electric field inside a hollow conductor of any shape, whether or not the conductor is charged, provided that there is no charge in the cavity”.



There is no electric field inside a hollow conductor (whether charged or uncharged)