Problem Set: Monte Carlo Simulation Applications

Mathematics Initiative in Nepal (MIN)

Note on Using AI for Problem Solving

For these problems, feel free to leverage AI tools like **ChatGPT**, **DeepSheek**, or **Claude** for:

- **Theoretical Understanding:** Ask for explanations on the Monte Carlo method and how it applies to these problems.
- **Code Assistance:** Get help writing code, debugging, or optimizing your simulation.
- **Hints and Optimization:** If you're stuck, AI can offer additional strategies, performance tips, or ways to improve your simulations.

Easy Problem: Estimating the Area of an Irregular Shape

Problem:
You are given a 2D irregular shape defined by a set of boundary points. Use Monte Carlo simulation to estimate its area by randomly generating points within a bounding box that encloses the shape. The area is estimated by the ratio of points that fall inside the shape to the total number of points.

Hint:

- First, define the bounding box for the irregular shape.
- Generate random points within the bounding box.
- For each point, check if it lies inside the shape. If it does, count it as inside.
- The ratio of points inside the shape to the total number of points multiplied by the area of the bounding box gives the estimated area.

Medium Problem: Estimating the Probability of a Traffic Jam

Problem:
Simulate a traffic scenario on a highway with multiple cars, each moving at a random speed. Use Monte Carlo to estimate the probability of a traffic jam occurring based on random simulations of car movement and road conditions.

Hint:

- Define the road length and the number of cars.
- Each car can have a random speed within a predefined range.
- Set a threshold for traffic jam formation (e.g., when cars are less than a certain distance apart).
- Estimate the probability by counting the number of simulations where a traffic jam occurs divided by the total number of simulations.

Hard Problem: Simulating the Spread of an Epidemic

Problem:
Simulate the spread of an epidemic within a population using Monte Carlo methods. Assume that each individual can either be susceptible, infected, or recovered, and that the infection spreads based on probabilities. Estimate the expected number of infected individuals after a certain number of days.

Hint:

- Define a population size and initial conditions (e.g., one infected individual).
- At each step, simulate the interactions between individuals and the probability of transmission from infected to susceptible.
- Track how many individuals remain susceptible, get infected, or recover.
- Repeat the simulation multiple times to estimate the expected outcome.

Very Hard Problem: Predicting the Outcome of a Complex Board Game

Problem:
Consider a complex board game with multiple players. Each player has a set of strategies that can be chosen randomly, and the game progresses with each player making random moves. Use Monte Carlo simulation to predict the probability of winning for each player after a series of random games.

Hint:

- Model the board game as a series of states, where each state represents the position of players and the game board.
- Randomly generate moves for each player based on their available strategies.
- Simulate multiple games and track the winning player in each simulation.
- The probability of each player winning is estimated by the ratio of wins for that player to the total number of simulations.

Good luck, and happy simulating!