Experiment No. 3

Aim:

To Calculate Capacitance of a Two-layered Dielectric Capacitor using EM Simulation Software (COMSOL Multiphysics).

What will you learn by performing this experiment?

This experiment develops an understanding of the potential gradient inside a capacitor. A 3-D model of a two-layered dielectric capacitor is simulated to obtain its capacitance using finite-element based electromagnetic simulation software (COMSOL Multiphysics). The result obtained is validated with theory (numerical calculations).

Software Required:

1. COMSOL Multiphysics (AC/DC Module) or any FEM based EM Simulation Software.

Theory:

Capacitance is the ability of a component or circuit to collect and store energy in the form of an electrical charge. Generally speaking, to have a capacitor we must have two (or more) conductors carrying equal but opposite charges. This implies that all the flux lines leaving one conductor must necessarily terminate at the surface of the other conductor. The conductors are sometimes referred to as the plates of the capacitor. The plates may be separated by free space or a dielectric. Consider the two-conductor capacitor in Figure 6.1.

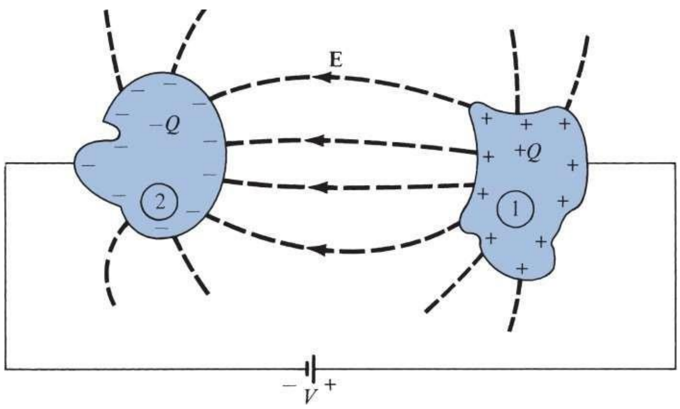


Fig. 6.1 Two conductor capacitor

The conductors are maintained at a potential difference V given by

$$V = V_1 - V_2 = -\int_2^1 \mathrm{E} \cdot d\mathrm{L} \qquad (1)$$

where E is the electric field existing between the conductors and conductor 1 is assumed to

carry a positive charge. (Note that the E field is always normal to the conducting surfaces.)
We define the capacitance C of the capacitor as the ratio of the magnitude of the charge on one of the plates to the potential difference between them; that is,

$$C = \frac{Q}{V} = \frac{\varepsilon \int_S \mathrm{E} \cdot d\mathrm{S}}{\int_L \mathrm{E} \cdot d\mathrm{L}} \tag{2}$$

The negative sign before V has been dropped because we are interested in the absolute value of V. The capacitance C is a physical property of the capacitor and is measured in farads (F). Most capacitances are practically much smaller than a farad and are specified in microfarads (mF) or picofarads (pF). We can use eq. (2) to obtain C for any given two-conductor capacitance.

Parallel-Plate Capacitor: Consider the parallel-plate capacitor of Figure 6.2(a). Suppose that each of the plates has an area S and they are separated by a distance d. We assume that plates 1 and 2, respectively, carry charges +Q and -Q uniformly distributed on them so that

$$\rho_s = \frac{Q}{S} \tag{3}$$

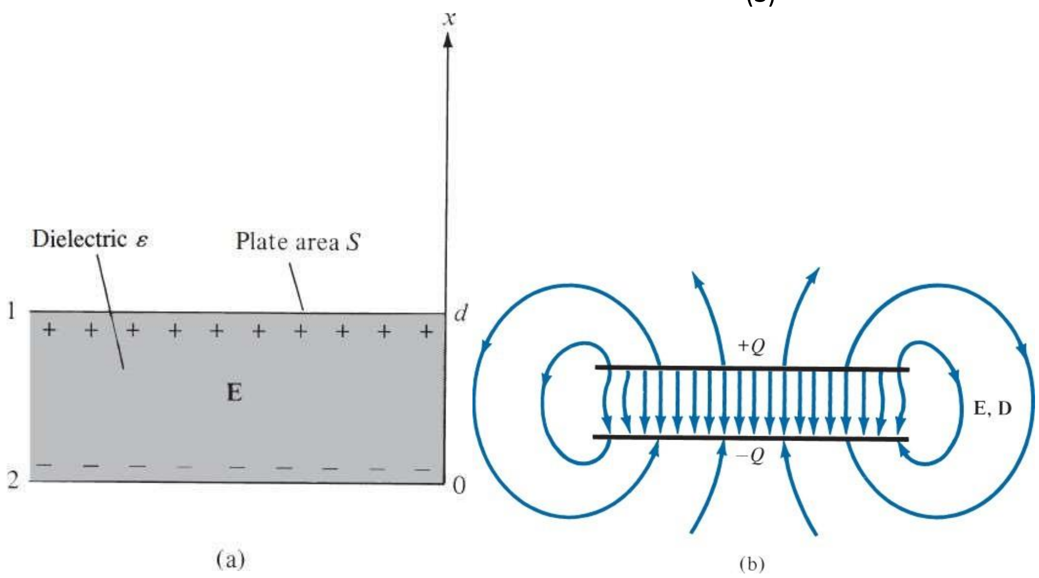


Fig. 6.2: (a) Parallel-plate capacitor. (b) Fringing effect due to a parallel-plate capacitor.

An ideal parallel-plate capacitor is one in which the plate separation d is very small compared with the dimensions of the plate. Assuming such an ideal case, the fringing field at the edge of the plates, as illustrated in Figure 6.2(b), can be ignored so that the field between them is considered uniform. If the space between the plates is filled with a homogeneous dielectric with permittivity ε and we ignore flux fringing at the edges of the plates,

$$\mathrm{E} = \frac{\rho_s}{\varepsilon}(-a_x) = -\frac{Q}{\varepsilon S} a_x \tag{4}$$

Hence,

$$V = -\int_1^2 \mathrm{E} \cdot d\mathrm{L} = \frac{Qd}{\varepsilon S} \tag{5}$$

and thus for a parallel-plate capacitor

$$C = \frac{Q}{V} = \frac{sS}{d} = \frac{s_o s_r S}{d} \qquad (6)$$

the energy stored in a capacitor is given by

$$W_E = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{Q^2}{2C} \qquad (7)$$

<u>Multi-layered Parallel Plate Capacitor:</u> Since D and E are normal to the dielectric interface, the capacitor in Figure 6.3 can be treated as consisting of two capacitors C1 and C2 in series. The total capacitor C is given by

$$C = \frac{C_1 C_2}{C_1 + C_2} \qquad (8)$$

Where

$$C_1 = \frac{s_o s_{r1} S}{d/2} \text{ and } C_2 = \frac{2 s_o s_{r2} S}{d}$$

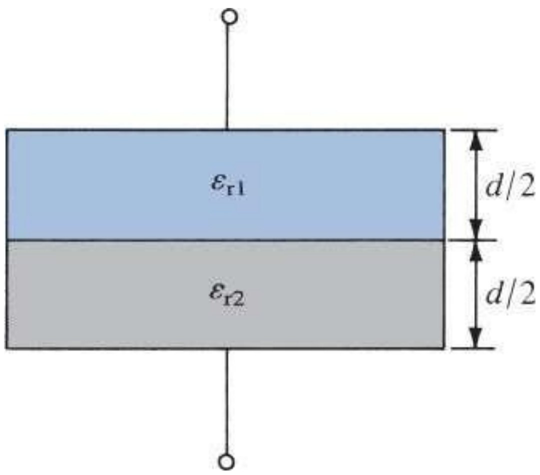


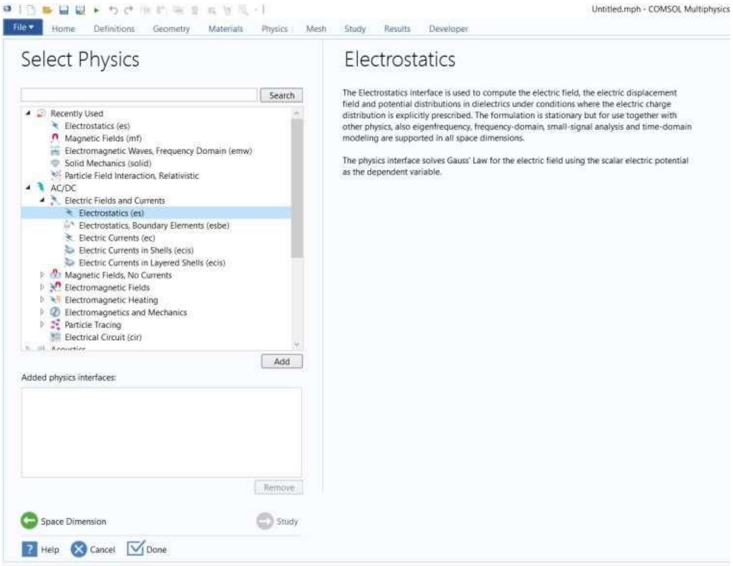
Fig. 6.3: Example Problem

<u>Example:</u> Determine the capacitance of the capacitor in Figure 6.3. Take ε_{r1}=4 and ε_{r2}=6, d = 5mm, S =30 cm^2.

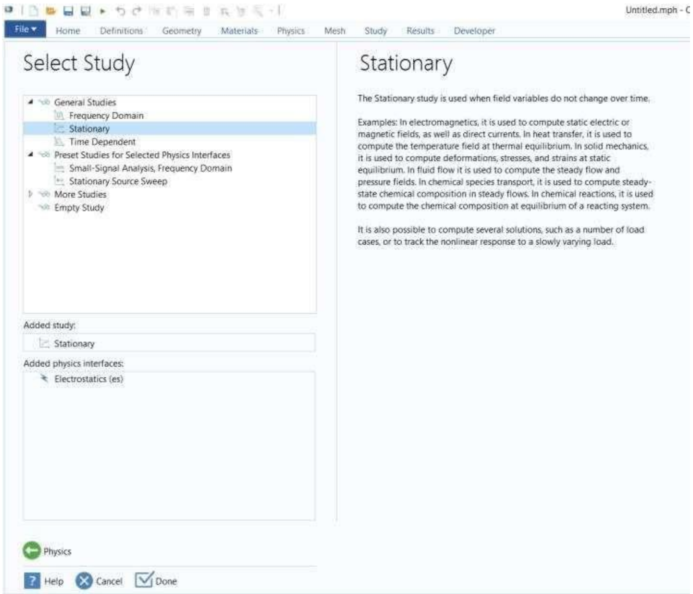
Ans: C = 25.46 pF

Procedure:

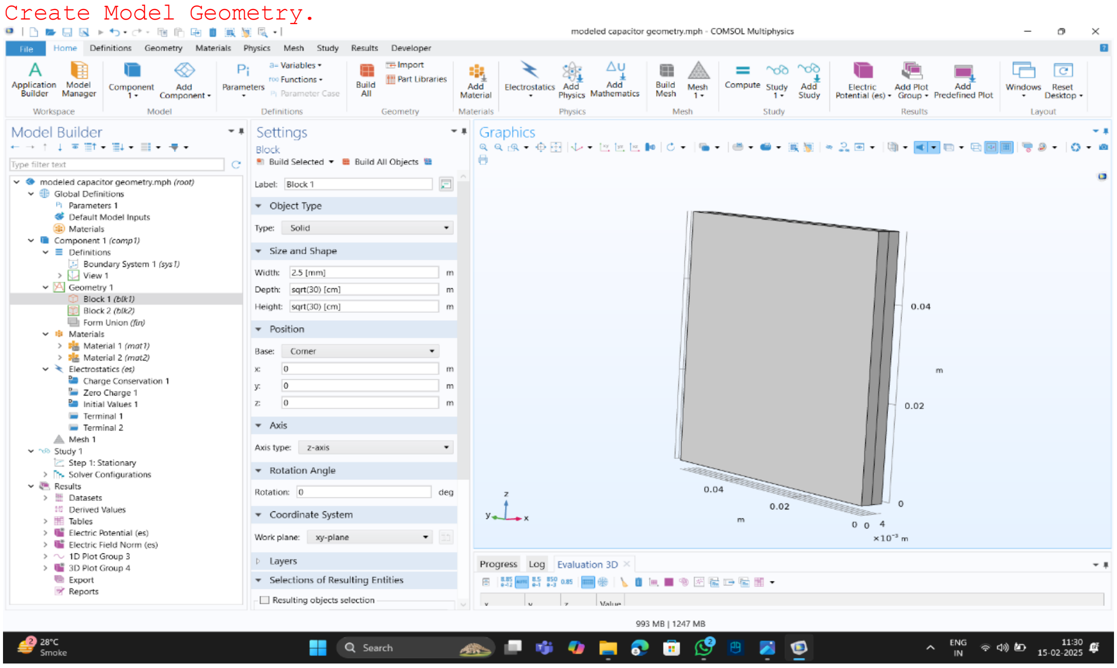
1. Install the EM Simulation Software.
2. Select 3D Modeling->AC/DC Module->Electric Field no Currents->Electrostatics



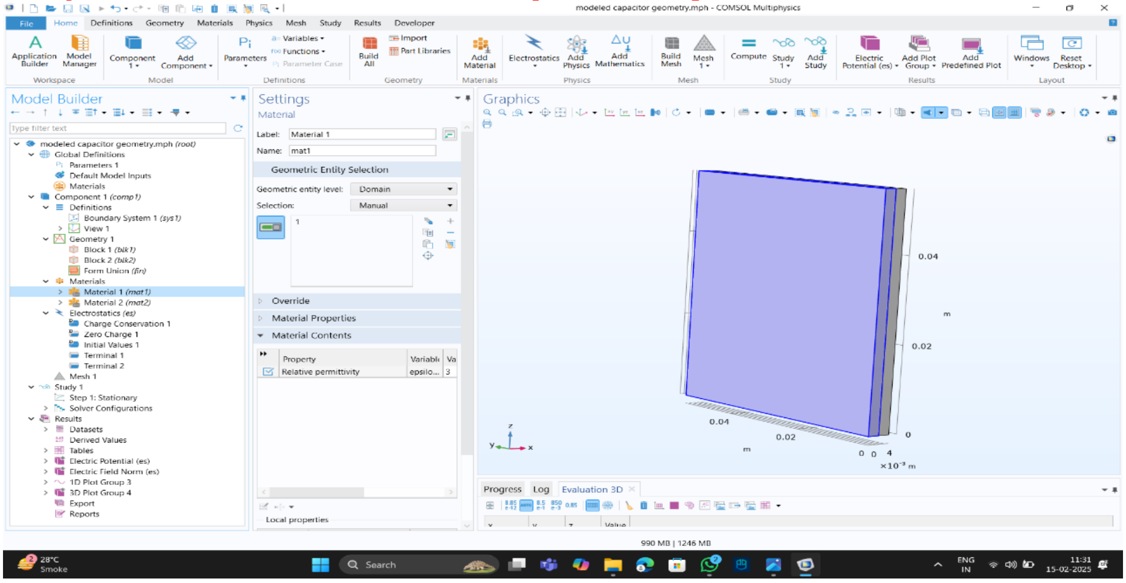
3. Choose Stationary Solver.



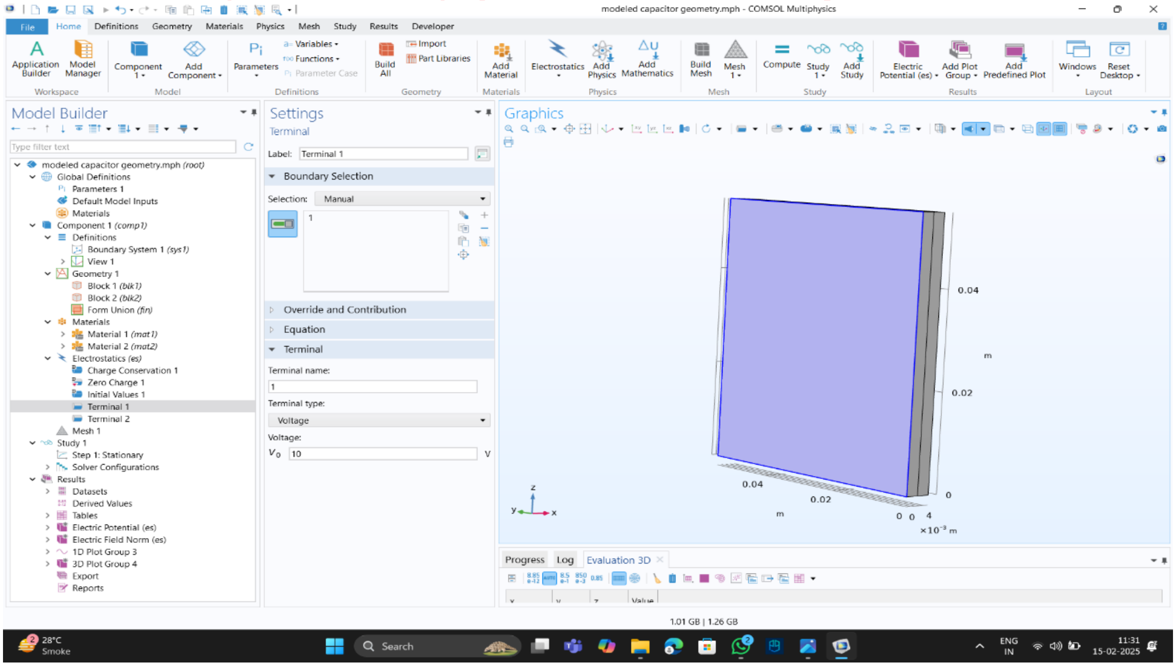
4. Create Model Geometry.



5. Assign blank materials with relative permittivity 3 and 5.



6. Define voltages on the charge plates (terminal boundary conditions).



7. Use Default Mesh, Click on Compute.
8. Observe the Results/Plots.

Results:

Fig. 6.4 shows the modeled geometry for the problem mentioned in Fig. 6.3. Fig. 6.5 shows the voltage distribution between the plates of the capacitor. Fig. 6.6 shows the potential variation observed along a line passing through the middle of the capacitor. It also shows the keyword 'es.C11' which represents the variable used to store the capacitance value. The value comes to be ________________.

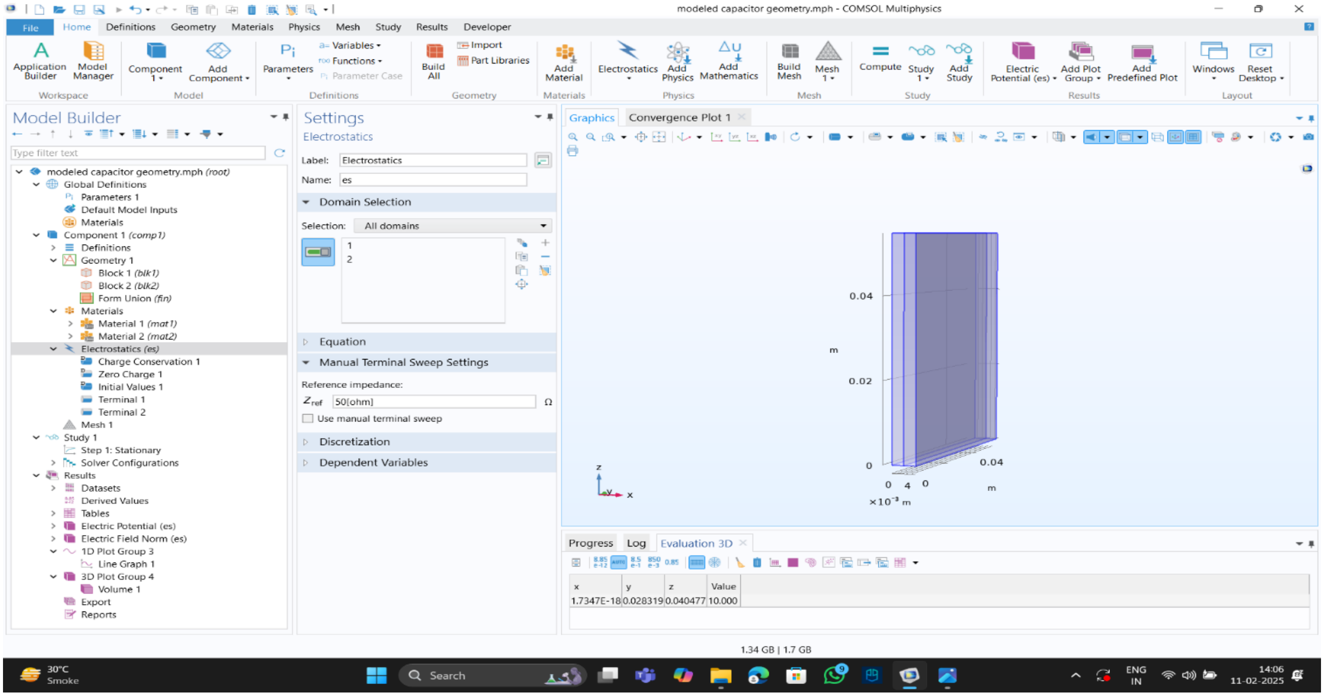


Fig. 6.4: Modeled Capacitor Geometry

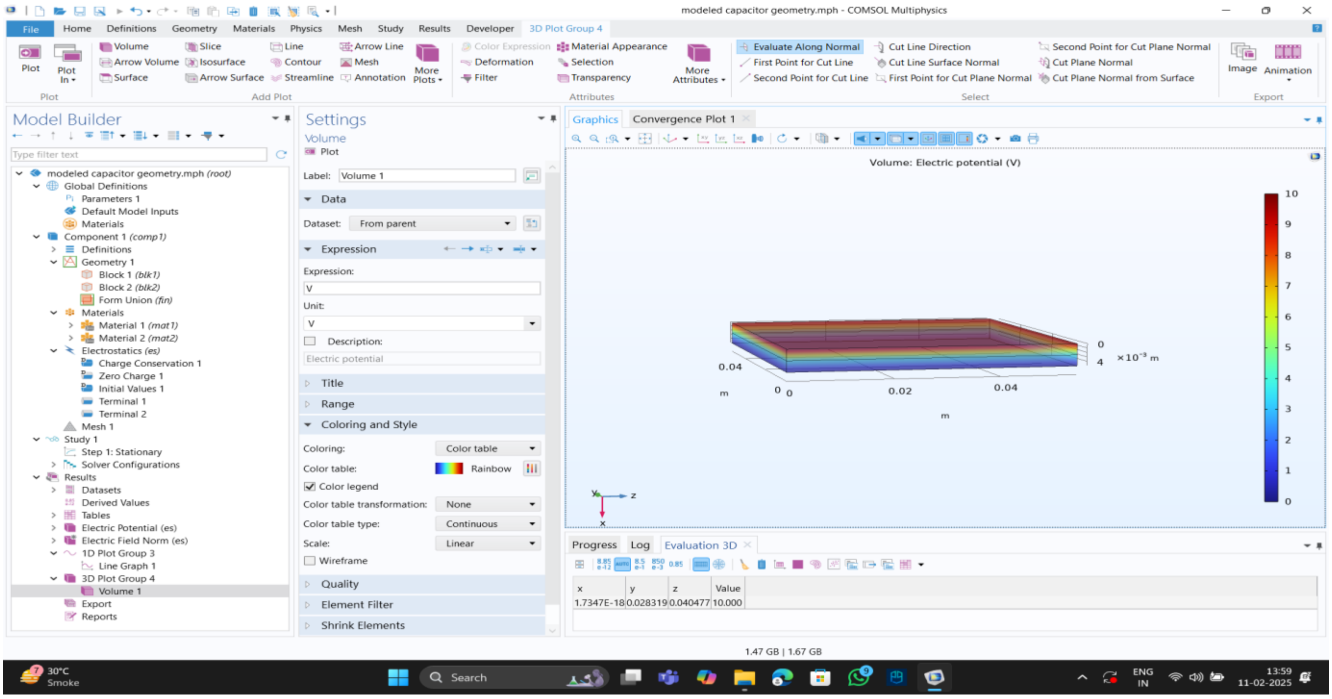


Fig. 6.5: Voltage Distribution

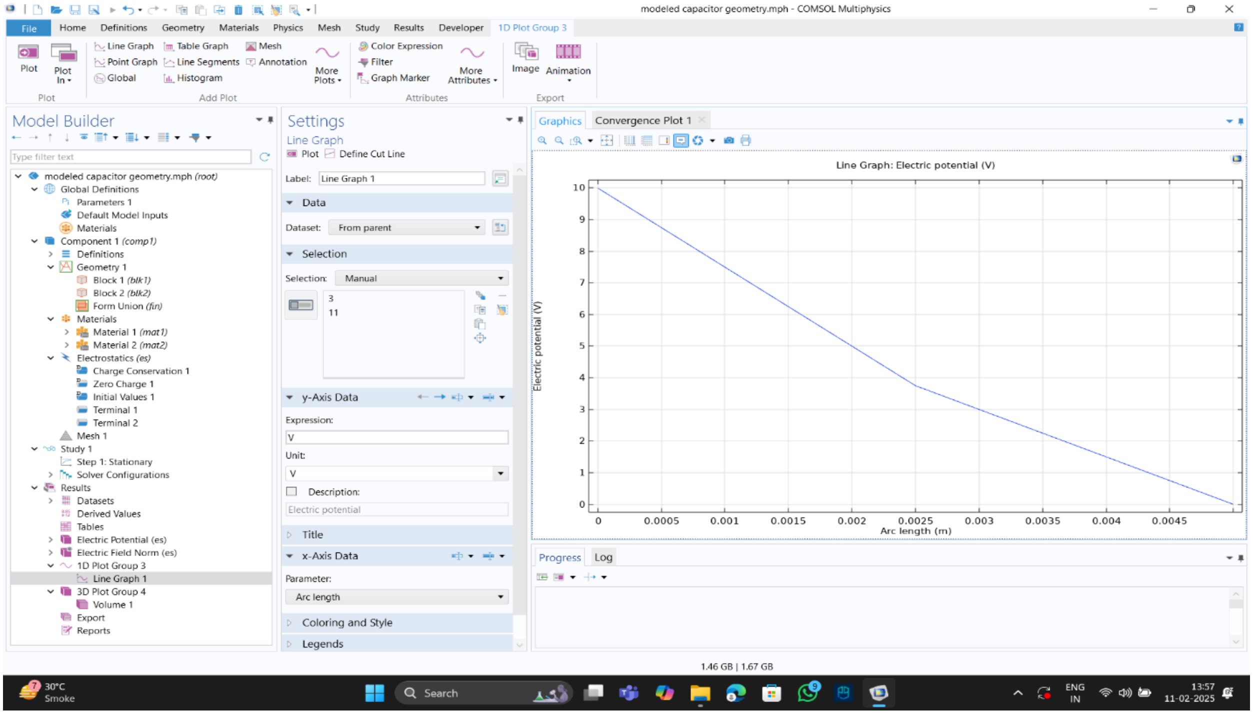


Fig. 6.6: Voltage along a line through middle of capacitor and capacitance calculation.

Conclusion & Discussion:

The simulated capacitance is ______________ which nearly same as the theoretical value of _________________. Voltage variation along the line through the middle of the capacitor represents the change in slope of the voltage curve in materials with different relative permittivity.

Quiz / Viva Questions:

- Calculate the total energy stored in the capacitor.
- Discuss capacitances connected in series and parallel.

References:

1. Mathew N. O. Sadiku, "Principles of Electromagnetics", 4th Edition, Oxford University Press Inc, 2009.
2. https://www.comsol.com/model/download/628691/models.acdc.capacitor_dc.pdf
3. https://www.comsol.com/video/basics-of-capacitor-simulation