A Conjecture of Mine

Gark Garcia

December 29, 2018

Conjecture:

Let $S : \mathbb{N} \to \mathbb{N}$ be the sum of the digits of a natural number — represented in a *base* δ *posicional system* — and let $\nu \coloneqq \delta - 1$. We have:

$$\forall a, b \in \mathbb{N} : S_{a+b} = S_a + S_b + \nu k, k \in \mathbb{Z}$$

Proof:

For all $a \in \mathbb{N}$:

$$a = a_0 \cdot 10_\delta^0 + a_1 \cdot 10_\delta^1 + \cdots + a_n \cdot 10_\delta^n \Rightarrow S_a = a_0 + a_1 + \cdots + a_n$$

Therefore:

$$\begin{aligned} a = a_0 \cdot (1+\nu)^0 + \cdots + a_n(1+\nu)^n &= a_0 \cdot (1 + \nu k_0) + \cdots + a_n \cdot (1 + \nu k_n) \\ &= a_0 + \cdots + a_n + \nu k = S_a + \nu k, k \in \mathbb{Z} \end{aligned}$$

Thus, for $c \coloneqq a + b$:

$$\begin{cases} c = S_c + \nu k, k \in \mathbb{Z} \\ a + b = (S_a + \nu k) + (S_b + \nu k') = S_a + S_b + \nu k'' \mid k, k', k'' \in \mathbb{Z} \end{cases}$$

Therefore $c = a + b \Rightarrow S_c + \nu k = S_a + S_b + \nu k' \mid k, k' \in \mathbb{Z} \Rightarrow S_c = S_{a+b} = S_a + S_b + \nu k, k \in \mathbb{Z}$
Q.E.D