

Analysis: Mountain View

This problem gives you a lot of freedom in how you approach it, as it is OK to output any sequence of peak heights matching the input data. What follows is one of the possible solutions, but others are possible.

First notice that if from peak 3 we perceive peak 7 as highest, then from peaks 4, 5 and 6 I can surely see no farther than 7, as the peaks have to lie below the line connecting peaks 3 and 7, and no peak after 7 lies above that line (or it would appear higher from peak 3). If this condition is not satisfied, we can safely return "impossible".

So, begin with peak 1, then go to the one appearing to be highest from it, then the apparent highest from it, and so on. Give all these peaks height of 10^9 . Note this sequence necessarily ends on peak **N** (as it is strictly increasing).

Now sequentially take the first peak **A** that we haven't assigned a height to yet. As it's the first non-assigned, the previous peak (**A**-1) has necessarily been assigned, and we can look at the peak that appears highest from **A**-1. Suppose it's **B**. We know that when we start a sequence of apparently highest peaks from **A**, it has to contain at **B** - it cannot skip **B**, because the whole sequence has to lie below the $[(\mathbf{A}-1), \mathbf{B}]$ line), and it is strictly increasing. If the sequence jumps over **B**, we return "impossible".

Assume the line $[(\mathbf{A}-1), \mathbf{B}]$ has slope **T**; our construction will guarantee that **T** is an integer (notice the slope of the first line is zero). We will want all the peaks on the sequence starting at **A** to lie on a line passing through peak **B** and with slope **T**+1; this determines the line uniquely. If we do it this way, we will not spoil visibilities built before (because the whole line is below the line connecting **A**-1 and **B**), and in the sequence for each peak the apparently highest peak will be the next peak in the sequence (as they are all in one line, no peaks with constructed heights lie between **A** and **B**, and all the peaks after **B** are invisible, because they lie below the line connecting (**A**-1) and **B**, and thus, even more so, below the line connecting **A** and **B** (which has larger slope).

We continue in this fashion. We can check that at any moment of our construction:

- If we assume the peaks that we have not constructed yet do not stand in the way (have, say, a height of zero), then for each peak we already constructed the peak appearing to be the highest is the one we expect to appear highest.
- For any constructed peak **A**, either **A**+1 is constructed as well, or no peak between **A** and the peak visible from **A** has its height constructed.
- Adding a new sequence upholds these invariants, which proves that when we end, the visibilities are all OK.

Thus, the construction works.

Notice that we increase the slope by one for each sequence, so it is at most **N** at the end, and so the lowest we can get is $10^9 - \mathbf{N}^2$; thus the heights are all positive.