Code Jam 2008 - EMEA Semifinal

Analysis: Rainbow Trees

Pick any vertex **r**. We view the tree as rooted at **r**, and -- as usual -- draw the tree with the root at the top, and draw the nodes of depth **d** at the same level, **d** units below the root.

By a *partial coloring* on a subset of edges we mean the assignment of colors to the edges in the subset so that the "rainbow coloring" constraint is satisfied on that subset. We do not care about the coloring of the edges that are not in the subset.
For each node **x**, we define the value

> f(x) := number of ways to color the subtree rooted at **x**, given any partial coloring for the set of edges that are incident to the parent of **x**.

It is not at all obvious that f(x) is well defined. (Why would the number always be the same for any given partial coloring?). To see that f(x) is indeed well defined, let's look at an algorithm for computing it.

Suppose **z** is the parent of **x**, and the degree of **z** is **D**. Note that the rainbow constraint is a very local condition -- the color of any edge other than those incident to **z** does not affect the coloring for the subtree rooted at **x**.

Assume that **x** has **t** children -- $\mathbf{y}_1$, $\mathbf{y}_2$, ..., $\mathbf{y}_t$. To color all the edges in the subtree rooted at **x**, we do the following.

- Color the edge **x** $\mathbf{y}_1$. There are **k** - **D** choices, since this edge cannot have the same color as any of the **D** edges incident to **z**, and no other edge in the partial coloring puts any constraint on it.
- Color the edge **x** $\mathbf{y}_2$. There are **k** - **D** - 1 choices, since this edge cannot have the same color as any of the **D** edges as above, nor the same color as **x** $\mathbf{y}_1$.
- ...
- Color the edge **x** $\mathbf{y}_t$. There are **k** - **D** - t + 1 choices.
- Now we have a partial coloring where all the edges incident to **x** are colored. We color the subtree rooted at $\mathbf{y}_1$. There are $f(y_1)$ ways.
- ...
- There are $f(y_t)$ ways to color the subtree rooted at $\mathbf{y}_t$.

Indeed, the computation only depends on **D**, the degree of the parent of **x**. f(x) is the product of the numbers above.

There is nothing very special about the root **r**, except that we need to agree **D** = 0 in that case. And the solution to our problem is just f(r).