### Kick Start 2018 - Round B

# Analysis: No Nine

## No Nine: Analysis

### Small dataset

To solve the Small dataset, we can check all numbers in the range [**A**, **B**] and count how many of them are legal.

### Large dataset

Let f(X) be the number of legal numbers in the range [0, X]. Then the answer is f(**B**) - f(**A**) + 1, since **A** and **B** are both legal numbers.

To calculate f(X), let x[0], x[1], ..., x[n-1] be the decimal representation of X, such that X = $\Sigma_{0 \le i < n} x[i] \times 10^i$. For numbers in the range [X - x[0], X], we will check each number individually to see if it is legal.

If we list numbers without the digit 9 in their decimal representations, we can find that their decimal representations are the same as listing numbers in base 9. So for numbers in the range [0, X-x[0]), there are C = $\Sigma_{1 \le i < n} x[i] \times 9^i$ numbers consisting only of digits in the range [0, 8]. According to the formula, C is divisible by 9.

For any integer Y, there is exactly one number divisible by 9 in the set {10Y + 0, 10Y + 1, ..., 10Y + 8}. The C numbers form C/9 such groups, and in each group, there are exactly 8 legal numbers, so there are 8C/9 legal numbers in the range [0, X-x[0]).

**Alternative solution (Dynamic Programming):** To calculate f(X), we can define dp[i][j] to be the number of integers Y such that

- Y < floor($X/10^i$)
- Y does not contain 9 in its decimal representation;
- Y ≡ j (mod 9).

Then f(X) = dp[0][1] + dp[0][2] + ... + dp[0][8] + 1.

The Bellman equation is dp[k-1][j] = sum[dp[k][j'] for 0≤d≤8, 10j'+d ≡ j (mod 9)] + |{g | 10floor($X/10^k$)≤g<floor($X/10^{k-1}$), g ≡ j (mod 9)}|.