Code Jam 2012 - Round 1A

Analysis: Kingdom Rush

This problem was an interesting one to create. In the actual game Kingdom Rush, there are three stars per level, "challenge" levels, and you can't try level 2 until you've beaten level 1 with at least one star. Coming up with a problem that was solvable, while maintaining the same feeling as the game that inspired it, was a balancing act.

We solved this problem with a **greedy** algorithm. At every step of the algorithm, Ryan will make a decision about which level to play, and his decision will be based simply on the properties of the levels available, and what he's done so far.

First, let's observe that Ryan should only complete a level if he's never completed it before, or if he can go from a one-star rating to a two-star rating. There's simply no point in beating a level otherwise. When we're talking about levels below, we'll ignore levels that he shouldn't complete for this reason.

Second, if Ryan ever reaches a state where he can't complete any of the remaining levels, then he is "TOO BAD" to beat the game. This will happen independent of the order in which he completes the levels.

Third, if Ryan can complete a level with a two-star rating, he should do it immediately. There's no reason for him to wait: he can earn those two stars (or one star) with one level completion either now or later. If there are multiple levels with two-star ratings that Ryan could complete, he should choose one arbitrarily; he can do the other one next.

Now we've covered all situations except for one: when the only levels Ryan can complete are levels that he can complete with a one-star rating. Consider two levels like that, level 0 and level 1:
a_0 b_0
a_1 b_1
The values of a_0 and a_1 don't matter: by assumption, Ryan has at least that many stars already. Let's assume without loss of generality that $b_0 < b_1$. Which level should Ryan complete first?

Let's remember that Ryan's objective is to complete levels the minimum number of times. In the worst case, it will take Ryan 4 completions to finish those two levels: two to get him a one-star rating in both levels, and two more to get him a two-star rating in both levels. But earning stars from these levels (or other levels) might allow him to complete one of them with a two-star rating without having to complete it with a one-star rating first.

Here's a possible series of events. Assume Ryan starts with S stars. We'll decide later whether k is 0 or 1:

1. Ryan completes level k with a 1-star rating and earns 1 star.
2. Ryan completes other levels and earns s stars.
3. Ryan completes level $1-k$ with a 2-star rating.

Which choice of k makes this scenario possible? If $k=0$, then this is possible iff $S + 1 + s \geq b_1$. If $k=1$, then this is possible iff $S + 1 + s \geq b_0$. Since $b_0 < b_1$, then this is possible with $k=0$ only if it's possible with $k=1$. So we might as well simply choose $k=1$, and have Ryan choose the level with the highest value of **b**.

So to summarize, Ryan's strategy should be:

1. While there is any level remaining that Ryan hasn't completed yet, or any level for which he can earn a higher rating than he previously had:
    - If he can earn a two-star rating on any of those levels, he should complete one of those levels (chosen arbitrarily).
    - Otherwise, if there is some set of levels on which he can earn a one-star rating, he should complete the one of those levels with the highest value of **b**.
2. If Ryan has beaten all levels with a two-star rating, he's done. Otherwise he's TOO BAD.

By simulating this strategy, we can see whether Ryan can beat Kingdom Rush, and the smallest number of level completions he can do it in.