Code Jam 2022 - Round 2

Spiraling Into Control

Problem

As punishment for being naughty, Dante has been trapped in a strange house with many rooms. The house is an $\mathbf{N} \times \mathbf{N}$ grid of rooms, with $\mathbf{N}$ odd and greater than 1. The upper left room is numbered 1, and then the other rooms are numbered 2, 3, ..., $\mathbf{N}^2$, in a clockwise spiral pattern. That is, the numbering proceeds along the top row of the grid and then makes a 90 degree turn to the right whenever a grid boundary or an already numbered room is encountered, and finishes in the central room of the grid. Because $\mathbf{N}$ is odd, there is always a room in the exact center of the house, and it is always numbered $\mathbf{N}^2$.

For example, here are the room numberings for houses with $\mathbf{N} = 3$ and $\mathbf{N} = 5$:

1	2	3
8	9	4
7	6	5

1	2	3	4	5
16	17	18	19	6
15	24	25	20	7
14	23	22	21	8
13	12	11	10	9

Dante starts off in room 1 and is trying to reach the central room (room $\mathbf{N}^2$). Throughout his journey, he can only make moves from his current room to higher-numbered, adjacent rooms. (Two rooms must share an edge — not just a corner — to be adjacent.)

Dante knows that he could walk from room to room in consecutive numerical order — i.e., if he is currently in room x, he would move to room $x + 1$, and so on. This would take him exactly $\mathbf{N}^2 - 1$ moves. But Dante wants to do things his way! Specifically, he wants to reach the central room in exactly $\mathbf{K}$ moves, for some $\mathbf{K}$ strictly less than $\mathbf{N}^2 - 1$.

Dante can accomplish this by taking one or more *shortcuts*. A shortcut is a move between rooms that are not consecutively numbered.

For example, in the 5×5 house above,

- If Dante is at 1, he cannot move to 17, but he can move to 2 or to 16. The move to 2 is not a shortcut, since $1 + 1 = 2$. The move to 16 is a shortcut, since $1 + 1 \neq 16$.
- From 2, it is possible to move to 3 (not a shortcut) or to 17 (a shortcut), but not to 1, 16, or 18.
- From 24, Dante can only move to 25 (not a shortcut).
- It is not possible to move out of room 25.

As a specific example using the 5×5 house above, suppose that $\mathbf{K}$ = 4. One option is for Dante to move from 1 to 2, then move from 2 to 17 (which is a shortcut), then move from 17 to 18, then move from 18 to 25 (which is another shortcut). This is illustrated below (the red arrows represent shortcuts):

Can you help Dante find a sequence of exactly $\mathbf{K}$ moves that gets him to the central room, or tell him that it is impossible?

Input

The first line of the input gives the number of test cases, $\mathbf{T}$. $\mathbf{T}$ test cases follow. Each test case consists of one line with two integers $\mathbf{N}$ and $\mathbf{K}$, where $\mathbf{N}$ is the dimension of the house (i.e. the number of rows of rooms, which is the same as the number of columns of rooms), and $\mathbf{K}$ is the exact number of moves that Dante wants to make while traveling from room 1 to room $\mathbf{N}^2$.

Output

For each test case, output one line containing `Case #`x`:` y, where x is the test case number (starting from 1).

If no valid sequence of exactly $\mathbf{K}$ moves will get Dante to the central room, y must be `IMPOSSIBLE`.

Otherwise, y must be an integer: the number of times that Dante takes a shortcut, as described above. (Notice that because Dante wants to finish in strictly less than $\mathbf{N}^2 - 1$ moves, he must always use at least one shortcut.) Then, output y more lines of two integers each. The i-th of these lines represents the i-th time in Dante's journey that he takes a shortcut, i.e., he moves from some room a_i to another room b_i such that $a_i + 1 < b_i$.

Notice that because these lines follow the order of the journey, $a_i < a_{i+1}$ for all $1 \leq i < y$.

Limits

Memory limit: 1 GB.
$1 \leq \mathbf{T} \leq 100$.
$1 \leq \mathbf{K} < \mathbf{N}^2 - 1$.
$\mathbf{N} \mod 2 \equiv 1$. ($\mathbf{N}$ is odd.)

Test Set 1 (Visible Verdict)

Time limit: 5 seconds.
$3 \leq \mathbf{N} \leq 9$.

Test Set 2 (Visible Verdict)

Time limit: 20 seconds.
$3 \leq \mathbf{N} \leq 39$.

Test Set 3 (Hidden Verdict)

Time limit: 20 seconds.
$3 \le \mathbf{N} \le 9999$.

Sample

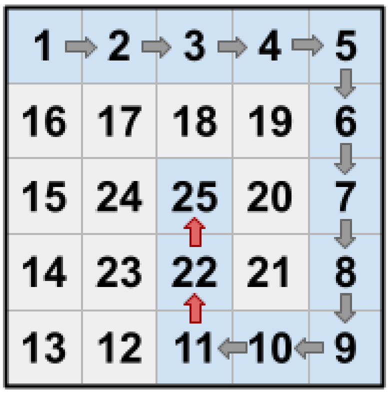
Sample Input	Sample Output

`4`
`5 4`
`5 3`
`5 12`
`3 1`
	`Case #1: 2`
	`2 17`
	`18 25`
	`Case #2: IMPOSSIBLE`
	`Case #3: 2`
	`11 22`
	`22 25`
	`Case #4: IMPOSSIBLE`

Sample Case #1 is described in the problem statement. Dante's route is $1 \to 2 \to 17 \to 18 \to 25$. Because $1 \to 2$ and $17 \to 18$ are moves between consecutively numbered rooms, they are not included in the output. Only the shortcuts ($2 \to 17$ and $18 \to 25$) are included.

In Sample Case #2, there is no solution. (Recall that there is no way for Dante to move diagonally.)

In Sample Case #3, observe that 22 appears both as the end of one shortcut and the start of the next. It would not be valid to include the line `11 22 25` in the output; each line must represent a single shortcut.



There is another solution that uses only one shortcut: Dante can move from $1 \to 2 \to 3 \to 4 \to 5 \to 6$, then move from $6 \to 19$ (a shortcut), then move from $19 \to 20 \to 21 \to 22 \to 23 \to 24 \to 25$. This is also valid; there is no requirement to minimize (or maximize) the number of shortcuts taken.

In Sample Case #4, Dante cannot get to the central room (9, in this case) in just one move.