## **Analysis: Oversized Pancake Flipper**

## **Oversized Pancake Flipper: Analysis**

Let's start with two simple but key observations. First, the flips are commutative. That is, the order in which you make the flips doesn't matter: flipping at the same positions in any order always yields the same result. Second, you never need to flip at the same position more than once: since flips are commutative, you may rearrange the flips so that two flips at the same position happen consecutively, and then it is clear that they cancel each other out and you might as well not do either.

## **Small dataset**

Applying the observations above to the Small dataset leaves only a really small number of combinations to try. Since there are N-K+1 possible flips, the number of flip subsets is  $2^{N-K+1}$ , which is at most  $2^9 = 512$  for this dataset. We can just try all of these combinations, discard the ones that leave at least one pancake blank side up, and get the minimum number of flips or determine that it is impossible.

Even if you don't have the observations above, a <u>breadth-first search</u> of all possible pancake states, using all possible flips as transitions, is also easily fast enough.

## Large dataset

Of course, the approach above will take way too long for the Large dataset, which can have up to 1000 pancakes. There is a simple greedy strategy that is not too hard to find. Notice that the left-most pancake  $p_1$  is only affected by the left-most flip  $f_1$ . That means that the initial status of  $p_1$  completely determines what do we need to do with  $f_1$  if we want to have any chance of leaving all pancakes happy side up: use  $f_1$  if and only if  $p_1$  is initially blank side up. After deciding on  $f_1$ , we can notice that there is only one remaining flip  $f_2$  that affects the second to the left pancake  $p_2$ . So, after we have used  $f_1$  (or not), the current state of  $p_2$  completely determines whether to use  $f_2$ .

Continuing with this reasoning, we can use the leftmost **N-K**+1 pancakes to determine all flips. After doing that, we only need to check whether the last **K**-1 pancakes are happy side up or not. If they all are, then the answer is the number of flips we used. Otherwise, the answer is IMPOSSIBLE. Notice that this reasoning also implies that at most one of the subsets of flips from our Small-only solution above would have worked.

If we carry out the above strategy literally and actually flip all of the pancakes in memory each time, the complexity is  $O(N^2)$ . We can reduce this to O(N) by only flipping one pancake at a time, and keeping a running count of the number of flips that we "owe" the current pancake. For instance, suppose that N = 10 and K = 5, and the first pancake is blank side up. We start with the first pancake, and because we must flip it, we know we must flip the first five pancakes. We increase our flip count to 1, and also make a memo to decrease the flip count by 1 once we reach the sixth pancake. We continue from left to right, and whenever a pancake is blank side up and the flip count is even, or a pancake is happy side up and the flip count is odd, we increase the flip count and make another memo. These memos can be easily stored in another array of length N that is checked before handling each pancake. This optimization is not necessary to solve the Large dataset, but it is a useful trick in programming contest problems

(and at least one of your Code Jam engineers has used it in day-to-day software engineering work!)