Code Jam 2009 - Round 2

Analysis: Watering Plants

This problem involves finding circles that enclose other circles. The problem of finding a circle surrounding a set of points is the fairly well-known minimal enclosing circle problem, but changing the points to circles makes the problem slightly trickier.

One solution is to do a binary search to find the minimum sprinkler radius. This reduces the problem to the problem of determining whether two sprinklers of a given radius can cover all the plants. To solve this, we can make the assumption that any sprinkler used in the solution either:

- covers exactly one plant, or
- the boundary of the sprinkler touches the boundary of at least two of the plants it covers.

This assumption is safe because if a sprinkler covers more than one plant but does not have two plants on its boundary, the sprinkler can be shifted and rotated, while still covering the same plants, until it does. Given this assumption, we can create a set of candidate sprinkler locations including:

- a sprinkler centered on each plant, and
- for each pair of plants, the set of sprinklers covering those plants and touching their boundary (there are 0, 1, or 2 of these per pair.)

Then we check every pair of candidate sprinklers, and see if any of them together cover every plant.

A second solution is to directly find the minimum sprinkler radius. To do this, we can use a slightly different simplifying assumption -- that every sprinkler either:

- covers exactly one plant (using the same radius as the plant),
- covers two plants which touch the edge of the sprinkler and whose centers are collinear with the center of the sprinkler, or
- covers three plants which touch the edge of the sprinkler.

This assumption is safe because any other sprinkler can be shrunk to a sprinkler of smaller radius that covers the same set of plants. We try each set of plants of size 1,2 or 3, create the corresponding sprinkler from the three cases above, and check its radius and which set of plants it covers. Then we find the pair of sprinklers that covers every plant using the minimum maximum radius.

Finding the circle which touches 3 given circles is harder than the equivalent problem for 3 points. Here are three possible approaches:

1. The set of points where a sprinkler can be centered in order to touch two plants is a hyperbola, so we could algebraically compute the intersection of two of those hyperbolae.
2. We can use a gradient-descent approach to find numerically the point minimizing the function from potential centers of sprinklers to the radius required for a sprinkler centered at that location to cover all three plants.
3. We can subtract from the radius of each of the three plants the radius of the smallest plant, then compute an inversion about that plant's center. Then we find appropriate tangents to the two inverted plants, re-invert to find the corresponding circle, and add back the radius of the smallest plant.