Code Jam 2010 - Round 3

Analysis: De-RNG-ed

Finding the next integer in the sequence means figuring out the values of **A**, **B** and **P** that fit the given numbers and using them to generate the next number.

This problem has a few special cases. In all of them, it is important to note that the value of **P** must always be larger than every element of the "randomly" generated sequence. (8 can never be a remainder after dividing by 7.) Also, the problem statement requires that **P** be no larger than $10^{\mathbf{D}}$. Let's call all primes that satisfy both of these bounds "valid primes". Now, let's look at the special cases.

K = 1

First of all, when **K** is 1, the answer is always "I don't know." This is because we can pick any valid prime **P**, set **A** to 0 and **B** to 0 or 1. This will give us two different answers.

K = 2 and the two sequence elements are the same

In this case, the answer is unique because the next element of the sequence depends only on the current element. If two consecutive elements are the same, then the entire sequence consists of a single repeated number.

K > 2 and all the sequence elements are the same

Similarly, the next element must be the same as all other elements.

K = 2 and the two sequence elements are different

Here, the answer is always "I don't know." To see that, pick any valid prime and consider the cases **A**=0 and **A**=1. If we call the first element of the sequence **x** and the second element **y**, we can express **y** as a function of **x**, **A**, **B** and **P**: **y** = (**A*****x** + **B**) % **P**. In both cases, we can solve this equation for **B**. The next element, **z** is then **z** = (**A*****y** + **B**) % **P**, and it must be different in the two cases (**A**=0 and **A**=1) as long as **x** is different from **y**.

K = 3

We are going to brute force all valid primes and solve for **A** and **B**. We will then use these values to generate the next element of the sequence. If all the values we get this way are the same, then the answer is unique. If we get different valid answers, then the answer is "I don't know."

Let's call the 3 elements **x**, **y** and **z**. By writing **y** as a function of **x** and **z** as a function of **y** and subtracting **y** from **z**, we get
z - **y** = (**A*****y** + **B**) - (**A*****x** + **B**) = **A***(**y** - **x**) (mod **P**).

We have already dealt with the case when **x** equals **y**, so we can assume that **x** and **y** are different, so we can divide by their difference. This lets us solve for **A**.
A = (**z** - **y**)*(**y** - **x**)$^{-1}$ (mod **P**).
Computing the inverse of (**y** - **x**) can be done using the Euclidean algorithm, which runs in O(log(**P**)) time.

Once we have **A**, solving for **B** is easy:
B = **y** - **A***x**.

The answer is then **A***z** + **B**.

K > 3

In this case, we brute force **P**, use the first 3 elements of the sequence to solve for **A** and **B**, and check whether the remaining elements fit the sequence generated with these parameters.