Kick Start 2021 - Round D

Analysis: Primes and Queries

Let us define $F(a, s) = V(a^s - (a \ mod \ \mathbf{P})^s)$.
The main idea is to use segment trees as there are range queries and point updates. If we can calculate the $F(\mathbf{A_i}, \mathbf{S})$ efficiently, then we can query the range sum and also update the values using segment tree operations. Calculating the $F(a, s)$ part needs different approaches for different test sets.

Test Set 1

As $\mathbf{S}$ can only be at most *4*, we can maintain individual segment trees for each $\mathbf{S}$. So now $\mathbf{S}$ is fixed for a single segment tree. The values of $\mathbf{A_i}$s are also small, so we can calculate $\mathbf{A_i}^{\mathbf{S}}$ without overflowing. Let us say we have *2* segment tree operations, $sum(start, end)$ which gives us the sum from index *start* to *end* and $update(idx, val)$ which updates the index idx with *val*. We are maintaining *maxS* different segment trees. So initially in the *j'th* tree, we have the values $F(\mathbf{A_i}, \mathbf{j})$s. When there is a query like $\mathbf{2\ S\ L\ R}$, we call $sum(\mathbf{L}, \mathbf{R})$ on the $\mathbf{S}$th tree. And when there is an update like $\mathbf{1\ pos\ val}$, we have to update each tree, so we call $update(\mathbf{pos}, F(\mathbf{val}, j))$ on the *j'th* tree.

Building the trees initially takes $O(maxS \cdot \mathbf{N})$.
Type 1 query (update) takes $O(\mathbf{S} \cdot \log \mathbf{N})$.
Type 2 query takes $O(\log \mathbf{N})$.
The time complexity of this solution is $O(maxS \cdot \mathbf{N} + \mathbf{Q} \cdot \mathbf{S} \cdot \log \mathbf{N})$.

Test Set 2

As $\mathbf{S}$ and $\mathbf{A_i}$s are huge now, we can't maintain a segment tree for each $\mathbf{S}$ and also can't calculate $\mathbf{A_i}^{\mathbf{S}}$ without overflowing. So a different approach is needed.

The idea originates from [lifting-the-exponent-lemma](). The lemma states that, if *P* is a prime, and *P* divides *a-b* but divides neither *a* nor *b*, then $V(a^n - b^n) = V(a - b) + V(n)$. But it has a special case.
When $P = 2$ and *n* is even, then $V(a^n - b^n) = V(a - b) + V(a + b) + V(n) - 1$.
Here $V(x)$ carries the same meaning as defined in the problem statement.
Another observation is $\mathbf{A_i} - (\mathbf{A_i} \ mod \ \mathbf{P})$ is always divisible by $\mathbf{P}$, which makes it possible for us to use the lemma.

We will handle everything in 2 cases.

Case 1 - $\mathbf{A_i}$ or $\mathbf{val}$ is divisible by $\mathbf{P}$:
We will have a segment tree for this. Initially we will update the indices having $\mathbf{A_i}$s divisible by $\mathbf{P}$ with $V(\mathbf{A_i})$s. When there is an update like $\mathbf{1\ pos\ val}$, we will update index $\mathbf{pos}$ with $V(\mathbf{val})$. When we have a query like $\mathbf{2\ S\ L\ R}$, we will call $sum(\mathbf{L}, \mathbf{R})$ and multiply that with $\mathbf{S}$, because $F(\mathbf{A_i}, \mathbf{S}) = \mathbf{S} \cdot V(\mathbf{A_i})$ in this case.

Case 2 - $\mathbf{A_i}$ or $\mathbf{val}$ is not divisible by $\mathbf{P}$:
This has *2* subcases because of the special case, so we will have *2* separate segment trees. Initially we will update the indices having $\mathbf{A_i}$s not divisible by $\mathbf{P}$ with $V(\mathbf{A_i} - (\mathbf{A_i} \ mod \ \mathbf{P}))$ in one tree and with $V(\mathbf{A_i} + (\mathbf{A_i} \ mod \ \mathbf{P})) - 1$ in the other. When there is an update like $\mathbf{1\ pos\ val}$, we will update index $\mathbf{pos}$ with $V(\mathbf{val} - (\mathbf{val} \ mod \ \mathbf{P}))$ in the first tree and with

$V(\mathbf{val} + (\mathbf{val} \mod \mathbf{P})) - 1$ on the other .
We will also maintain another segment tree that will help us query the number of values that are not divisible by $\mathbf{P}$ in a given range.
When we have a query like $\mathbf{2}\ \mathbf{S}\ \mathbf{L}\ \mathbf{R}$, if $\mathbf{P} = 2$ and $\mathbf{S}$ is even, then we call $sum(\mathbf{L}, \mathbf{R})$ on both segment trees and add them. Otherwise we call it only on the first one. Also if there are *X* values not divisible by $\mathbf{P}$ in the range $\mathbf{L}$ to $\mathbf{R}$, we will add $X \cdot V(\mathbf{S})$ to the answer.

The final answer is the summation of the queries from the *2* above cases.

$V(\mathbf{S})$ can be calculated in $O(\log \mathbf{S})$. And when $\mathbf{A_i}$ is divisible by $\mathbf{P}$, the value of $V(\mathbf{A_i}^{\mathbf{S}} - (\mathbf{A_i} \mod \mathbf{P})^{\mathbf{S}})$ is just $V(\mathbf{A_i}^{\mathbf{S}})$, which is $\mathbf{S} \cdot V(\mathbf{A_i})$, that can be calculated with brute force with complexity of $O(\log \mathbf{A_i})$.

The time complexity of this solution is $O(\mathbf{N} \log(\max(\mathbf{A_i})) + \mathbf{Q} \cdot (\log \mathbf{N} + \log(\max(\mathbf{S}, \mathbf{val}))))$.