Code Jam 2019 - Round 1A

Analysis: Alien Rhyme

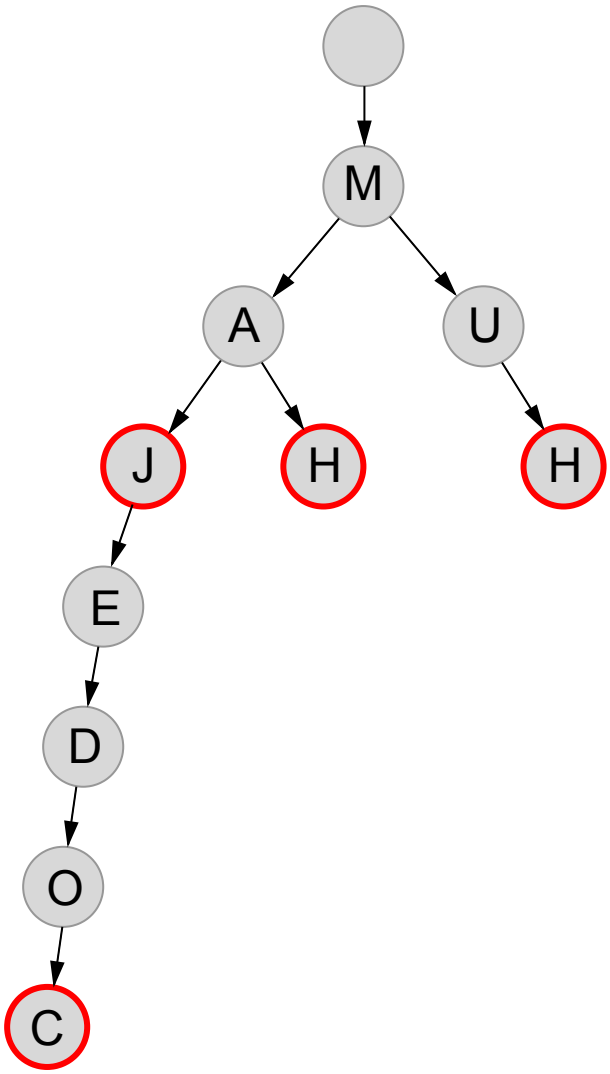
Test set 1

In the first test set there are only up to **N** = 6 words with up to 50 characters in each of them. We can simply use brute force to try all ways of grouping words into pairs (allowing some words not to be in any pair – we will effectively discard these words), try all choices of an accent-suffix for every pair, and check that none of the pairs have the same accent-suffix. Finally, we choose the maximum size across all valid groupings.

Test set 2

Let's notice how the size of an accent-suffix affects the chance of multiple words sharing it. In case we have words `CODEJAM`, `JAM`, `HUM` and `HAM`, accent-suffix `JAM` can be part of only two words, whereas a shorter accent-suffix `M` fits all four words. This leads to the following observation: for two words that we want to pair, it is never suboptimal to choose their longest available common suffix as the accent-suffix – this way we are still making sure that they rhyme, and we are allowing shorter accent-suffixes to be used by other pairs. Notice that any other pair that could use the longer suffix can also use any shorter suffix. For example, if we want words `CODEJAM` and `JAM` rhyme, we should choose `JAM` as their accent-suffix, and allow suffix `M` to be potentially used by `HUM` and `HAM`.

In this problem it is all about common suffixes of the words. In order to better operate with word suffixes, let's actually reverse the words first (so now original word suffixes are prefixes of reversed words), and build a trie (also often called prefix tree) on the reversed words. This is how a trie containing the words `CODEJAM`, `JAM`, `HUM` and `HAM` looks:



Let's also mark the trie nodes where some of the input words end. Since we are guaranteed that all the words are unique, we can use a simple boolean flag. In the picture above, trie nodes where a word ends are marked in red.

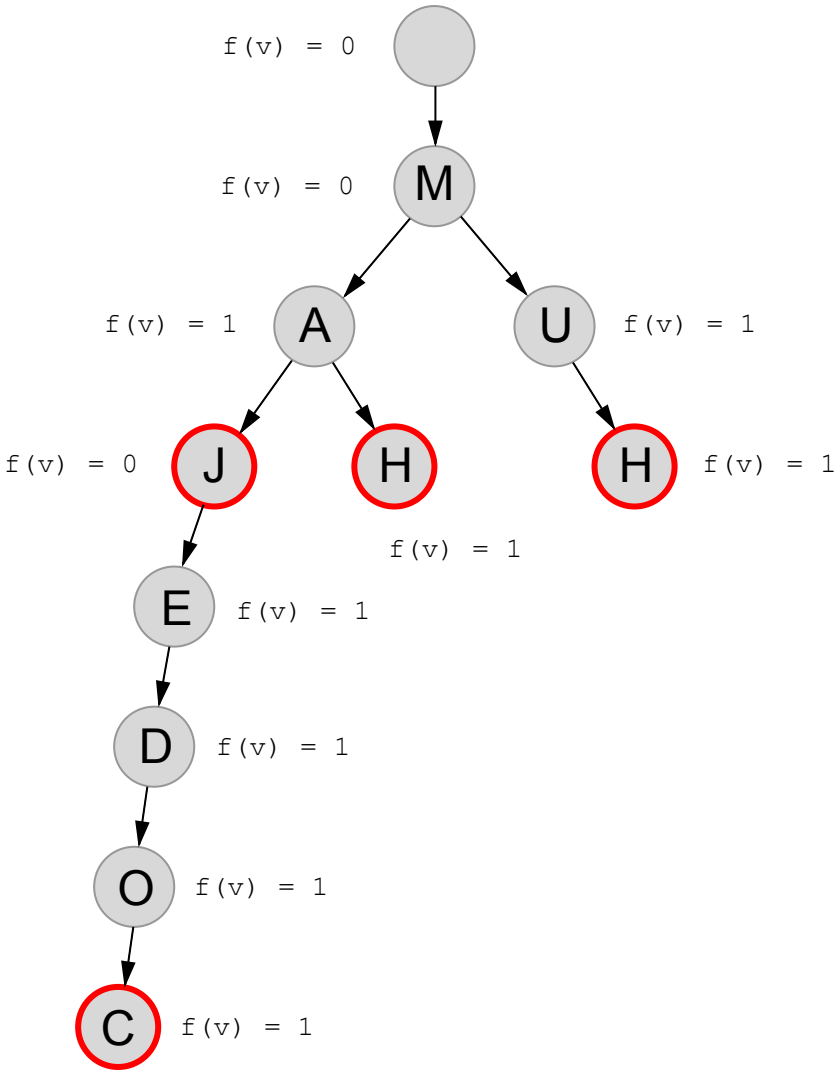
Now we can solve the problem as follows: for a trie node *v*, let `f(v)` be the minimum possible number of unpaired words that use accent-suffixes whose reverses end in the node *v* or the subtree under it. The answer to the problem is then `N - f(root)`, since `f(root)` represents all usable accent-suffixes.

How do we calculate the values of `f(v)`? If *v* does not have any children nodes, we set `f(v)` to be 1, since we know that in our trie all leaf nodes are the end of a word. If node *v* has children, we can calculate `f(v)` with the following algorithm assigning the result to `r`:

```
r = sum(f(c) for all c where c is a child node of v)
if node v is marked (there is a word that ends at v): r = r + 1
if v is not the root and f(v) ≥ 2: r = r - 2
```

First, we simply count the number of unpaired words recursively. Finally, we let two of those words be paired using the prefix that ends in the node *v*, which represents suffixes of original words, as the accent-suffix.

In our example trie we would get the following values of `f(v)`:



Proving the algorithm above correctly calculates f(v) is straightforward. First notice that only words that are represented in the trie at or below v are pairable with the set of accent-suffixes represented by v or its subtree. Then we can proceed by induction: f is pretty clearly correct for a single node tree, as there is no pairing possible. Assume by the inductive hypotheses f works correctly on all proper subtrees under v. The pairing implied by the construction of f(v) — adding any remaining pair of words to the recursive result — is valid: we are only pairing two words with the accent-suffix represented by v, and the rest is valid by the inductive hypotheses. To show that the pairing it is also of maximum size, notice that, by inductive hypothesis, there is no way to pair more than `sum(f(c) for all c where c is a child node of v)` words with accent-suffixes that are represented by the subtree but not by v directly. This is because words in different subtrees of v cannot be matched with a longer accent-suffix than the one represented by v, and the accent-suffix represented by v can add at most a single pair to the total.

Note how we calculate the values of `f(v)` in a recursive manner, and `f(v)` is calculated exactly once for each possible `v`. Since the algorithm itself takes constant time in addition to the time of the recursion, we can calculate all `f(v)` values in O(T) time, where T is a total number of nodes in the trie. We can bound T by the total length of all words, or by **N** × m where m is the maximum word length.

Finally, there are less efficient but simpler implementations that also work. For example, sort the reversed words alphabetically and take any two adjacent words with a longest common prefix, pair them, remove them from the list, and repeat. This simple-to-implement algorithm basically constructs the same pairing our recursive formulation does. This shifts some implementation

complexity onto the correctness proof. If you are faster with proofs than with code, it might be an overall gain in solving speed.