

Analysis: Field Trip

Test Set 1 (Visible)

Since the number of kids and the grid dimensions are small in Test set 1, it is sufficient to develop a correct greedy heuristic for where to move the teacher and then run the simulation to completion.

The first observation is that the problem can be solved independently in each dimension (x and y). At each time step, kid i always moves exactly 1 step closer to person $i-1$ in both x and y until both people occupy the same position in that dimension. Diagonal movement happens when the two people have different positions in both dimensions; orthogonal movement happens when the two people have the same position in one dimension but different positions in the other dimension; and no movement happens when the people share the same position in both dimensions.

The second observation is that when we take the one-dimensional perspective, the time is limited by only the most distant kids from the teacher in each dimension and direction. Let K be the number of the most distant kid in a certain dimension and direction; if multiple kids share that same position, pick the one with the lowest number. By assumption, all kids with a number lower than K are either closer to or on the opposite side of the teacher than kid K , including kid $K-1$. Therefore, kid K will move one step in the direction of the teacher. The effect inducts to kids with numbers higher than K ; now that kid K has moved closer to the teacher, the first kid with number greater than K that has a distance to the teacher equal to kid K 's original distance becomes the most distant kid and is guaranteed to step toward the teacher.

Hence, the correct heuristic is to minimize the distance between the teacher and the most distant kid, treating each dimension independently. To do this, calculate the most distant kid in each dimension, and move the teacher toward each of those kids in their respective dimension. If two kids are equally distant but in opposite directions, do not move the teacher.

An equivalent statement of the heuristic is that the teacher always moves toward the average of the minimum and maximum coordinates in each dimension, not moving if the teacher is already at that center point. If the maximum difference in a certain dimension is odd, the teacher can oscillate between the two central spaces in that dimension; this does not affect the correctness of the heuristic.

A few *incorrect* greedy heuristics include:

1. Move the teacher toward the single most distant kid. This is wrong because x and y need to be independent.
2. Move the teacher toward the average or median kid position. This is wrong because it biases the teacher toward a cluster of kids instead of dealing with outliers.
3. Move the teacher toward first kid that is not on their current space. This is wrong because the teacher could waste time making a cluster smaller before moving toward outliers.

Test Set 2 (Hidden)

Running the simulation to completion clearly does not work with the higher bounds.

By the greedy heuristic described above, at each time step, since the most distant student in each direction and dimension moves one step closer to the teacher, in each dimension, the minimum coordinate increases by 1 and the maximum coordinate decreases by 1. The problem

in each dimension is solved once the minimum and maximum coordinates equal each other. Therefore, we can calculate the total time by taking the difference between the minimum and maximum coordinates in each direction and dividing by 2 (rounding up), choosing the larger of the value in the x dimension and the value in the y dimension. This requires looking at each kid once and therefore takes $O(N)$ time.