

Analysis: Illumination Optimization

Test Set 1

For Test Set 1, we can use brute force and check all possible combinations of street light locations. For each combination, we need to check if it covers the entire freeway, and if it does, we add it to the list of candidates. We then return the combination which uses the least number of street lights. This will take $O(N \times 2^N)$.

Test Set 2

For the Test Set 2, there are two observations we can make:

- The street light locations are given in sorted order
- The radius of each street light is the same

Since the radius is the same, this ensures that if the left point of a street light starts before the left point of another street light, it must also end before that street light. Since the light locations are given in sorted order, we will be able to iterate through them from left to right without skipping any street lights. Our ultimate goal is to find the minimum number of street lights where the freeway is lit up. Thus we can iterate through the given street light locations from left to right, and when deciding which street light to use, we take the rightmost light that also is able to cover all area (on the left) that has not been covered yet. This greedy strategy is always an optimal one, because it always leaves the rest of the uncovered areas of the street to be as small as possible.

To do this, we can keep a counter of how many street lights we used, as well as a variable to keep track of what area has been covered so far. We can do this by saving a variable *curr_rightmost_covered* for what is covered so far. Specifically, the range $[0, \text{curr_rightmost_covered}]$ denotes what is already covered. Everytime we see a new street light at X_i , we check that it does not leave a gap in the uncovered area by checking if $X_i - R \leq \text{curr_rightmost_covered}$. If it does leave a gap, then it means that no street light can cover that gap, so the answer is impossible. Otherwise, we check the next street light after it, X_{i+1} , to see whether that one also leaves no gap. If so, then we would prefer to use X_{i+1} instead of X_i and so forth. We will need to keep checking the next street light until we see one that leaves a gap X_{k+1} . If the next street light X_{k+1} does leave a gap, that means we must take X_k . In that case, we update *curr_rightmost_covered* to $X_k + R$ where X_k is the rightmost street light that leaves no gap.

When the entire freeway is covered, we can stop and return the count of how many streets lights we used. The time complexity of this solution is $O(N)$ for each test case.