Code Jam 2011 - Round 3

Analysis: Irregular Cakes

Before we can start solving the problem, we first need to know how to calculate the area of the cake, so we know how much cake each party-goer is going to eat. We are guaranteed that this is a non-intersecting polygon in the limits, so we can use the equation:

$2*\text{Area} = \text{sum}(X_i * Y_{i+1} - X_{i+1} * Y_i)$ for i = 0 to N-1.

For this to work the first and last points in the polygon must be the same point. That is, $X_0 = X_N$ and $Y_0 = Y_N$. The list of points must also be in counterclockwise order. This can be achieved by iterating over all points in L, and then iterating over all points in U in reverse order.

Once we know the total area of the cake, we need to determine how much cake to give each guest. Since there are G guests, we can evenly calculate this number as AreaPerGuest = Area / G.

Finally we are ready to determine where to cut the cake. This can be solved a variety of ways, but here we will discuss using a binary search as this is often the simplest algorithm to code. To obtain a piece of cake with an area of AreaPerGuest, we know that the vertical cut will have an X coordinate between 0 and W, so we do a binary search over this range. For each cut point we try during the search, we compute the area of the cake to the left of that cut. If it produces a piece of cake with area greater than AreaPerGuest, we update the upper bound of our search. If we choose a cut that produces a piece of cake with area less than or equal to AreaPerGuest, we update the lower bound of our search. This search will eventually converge to the correct cut point with sufficient accuracy.

If G=2, then we are done. If G > 2, then the binary search can be repeated to find the other cuts. The second cut point should be placed such that the area of cake to the left of that cut is AreaPerGuest * 2, the third cut point should be placed so that the area of cake to the left of that cut is AreaPerGuest * 3, and so on.

At each iteration of the binary search, we need to calculate the area of a polygon using part of L, U, and a line on an arbitrary X coordinate, which requires more work than computing the area of the whole cake. The first thing to note is that we can use all points in L and U that have X coordinates such that $0 <= X_i <= X_{cut}$, where X_i is the X coordinate of point i, and X_{cut} is the X coordinate of the cut. Next we need to determine the Y coordinates of our X_{cut} points on L and U. If we find two consecutive points A and B in L such that $A_x <= X_{cut}$ and $B_x >= X_{cut}$, we can use line intersection to find the intersection of line AB and the vertical line defined by X_{cut}. Once we know these two new points the area of the cut piece of cake can be calculated.

This problem can also be solved with a linear time solution in O(N+G). The basic premise is to split the cake into trapezoids, and iterate from left to right accumulating the total area. Any time a trapezoid needs to be split for a cut, a quadratic equation is used to determine where to cut the cake. This approach only requires the basic formula for the area of a trapezoid, which is (a+b)/2 * h, where a and b are the lengths of the bases, and h is the height of the trapezoid.

More information:
Polygon area
Binary search