Analysis: Stable Wall

Test set 1

Each of the polyominos contains only one letter. We can take each of the <u>permutations</u> of the distinct letters, and check if the resulting wall is stable by following the order of the letters in the permutation to place the polyominos, one by one. There are **N** distinct letters in the input, so we will have **N**! permutations.

Let's say W is the given input wall letters. There are various ways to check if a wall is stable for a given permutation. One way is to create a new 2D array of integers, I. I[i][j] will contain the index of the letter W[i][j] in the permutation. Each of the letters at(i, j) in the wall must be placed after the letter below them, i.e. at(i+1, j), if they are not part of the same polyomino. We can check if it's true by checking if I[i][j] \geq I[i+1][j], for all positions (i, j). Since I contains relative indexing of the order of the polyominos that were placed, this check works as intended. We can check this in O($\mathbb{R} \times \mathbb{C}$) time, so the runtime complexity of this approach for each test case is O($\mathbb{N}! \times \mathbb{R} \times \mathbb{C}$). This is sufficient for test set 1.

Test set 2

Given the picture of the wall, if two polyominos are positioned one above another, and they touch each other, then to make a stable wall, the bottom one must be placed before the top one. This gives us a set of orderings of pairs of letters which must be followed while placing the polyominos.

We can make a graph with the set of orderings just mentioned. Each polyomino/letter can be considered as a vertex, and each of the orderings can be considered as a directed edge of the graph. If the resulting graph contains any cycle, it is not possible to have a stable wall. Otherwise, we can use topological sort to find a suitable order that fulfills all the orderings. We can find all the orderings in $O(R \times C)$ time. We can find a topological sorting of the vertices of a directed acyclic graph by using depth first search. Depth first search gives us a runtime complexity of O(number of edges + number of vertices). The number of edges in the graph is at most $(R-1) \times C$, as each cell of the grid adds at most one edge, to the cell right above it. So the total runtime complexity of this approach for each test case is $O(R \times C) + O(N + R \times C) = O(R \times C)$, since N can be at most $R \times C$. This is sufficient for test set 2.