Analysis: 3D Printing

The first thing we can notice is that if a printer has u units of ink left of a given color, we cannot use more than u units of that color. Moreover, this is the only restriction imposed by that value. So, we can summarize the input by saying we cannot use more than $C = \min(\mathbf{C_1}, \mathbf{C_2}, \mathbf{C_3})$ units of cyan ink, $M = \min(\mathbf{M_1}, \mathbf{M_2}, \mathbf{M_3})$ units of magenta ink, $Y = \min(\mathbf{Y_1}, \mathbf{Y_2}, \mathbf{Y_3})$ units of yellow ink, or $K = \min(\mathbf{K_1}, \mathbf{K_2}, \mathbf{K_3})$ units of black ink.

If $C + M + Y + K < 10^6$ then the case is impossible and we are done. Otherwise, we may need to use lower amounts of each color. We can simply go one color at a time, lowering the amounts of ink until we make the sum exactly 10^6. Doing it one unit at a time works, but it is very slow. We can do better: in the same way as before, we can consider all the colors one at at a time. Let S be the sum of the current amount of ink for the 3 colors not currently under consideration. If $S \geq 10^6$, we can simply set the amount of the current color to 0 and continue with the next one. If $S < 10^6$ we can lower the current color to $10^6 - S$ and finish immediately. This works because at all times we maintain the invariant that the total amount of ink we are considering is at least 10^6 units.