Code Jam to I/O for Women 2021 - Code Jam to I/O for Women

Analysis: Inconstant Ordering

Test Set 1

It is given that a valid string must start with A.

For $\mathbf{N} = 1$, the only block B_1 should therefore start with a character greater than `A`. To keep building the alphabetically first valid string, we start B_1 with `B` which is just greater than `A`. We keep on incrementing the character as we move right to ensure that B_1 is strictly increasing. Notice that we will eventually end up with a prefix of `BCDE..XYZ` as B_1. As $\mathbf{L_i} \le 25$, we will never run out of characters for this.

For $\mathbf{N} = 2$, we construct the first block B_1 using the process described for $\mathbf{N} = 1$. We now evaluate the possible ways to fill the strictly decreasing second block B_2. We try to start B_2 with the smallest possible character to get the alphabetically first string. This implies that B_2 will end with the smallest character in the alphabet which is `A`. So, we start filling B_2 from the end with `A` and move to the left, incrementing the character.

Let b_1 be the last character in B_1 and b_2 be the first character in B_2. If $b_1 > b_2$, then we will always get a valid string as it ensures that B_2 is strictly decreasing. On the other hand if $b_1 \le b_2$, then the final string is invalid. For example, if $\mathbf{L_1} = 2$ and $\mathbf{L_2} = 4$, the resulting string from the above process will be `ABCDCBA` which is invalid.

To ensure that B_2 is always strictly decreasing, we update b_1 with the character just greater than b_2. This will guarantee that the string is valid and alphabetically the first one. Again as $\mathbf{L_i} \le 25$, we will never run out of characters while filling B_2. In the above example, `ABCDCBA` now transforms to a valid string `ABEDCBA`.

Test Set 2

We generalize the above solution for $\mathbf{N} > 2$.

If $\mathbf{N}$ is even, we can divide the blocks into $\mathbf{N}/2$ pairs as |$\mathbf{L_1}$,$\mathbf{L_2}$|$\mathbf{L_3}$,$\mathbf{L_4}$|...|$\mathbf{L_{N-1}}$,$\mathbf{L_N}$| and solve the $\mathbf{N}/2$ pairs independently using the same approach as for $\mathbf{N} = 2$. We can do this as every such pair of blocks will end with `A`.

If $\mathbf{N}$ is odd, we solve for $\mathbf{N} - 1$ blocks using the above approach and for the last block $\mathbf{L_N}$, we treat it as the case when $\mathbf{N} = 1$.

The time complexity is $O(\mathbf{L_1} + \mathbf{L_2} + \cdots + \mathbf{L_N})$ for each test case.