Code Jam 2009 - Round 1A

Analysis: Crossing the Road

A grid of roads; a person moving from place to place; this problem has all the hallmarks of a graph problem, and a shortest path problem at that. Our pedestrian is trying to get from one corner of the map to the opposite corner. Some steps that she takes can be made at any time, and take 2 minutes; for others, she has to wait until the light turns green, and then take another minute. How can we minimize the time she takes to reach her goal?

To solve this problem, we need to describe the state of the world. The pedestrian can be in any of 2m positions in the x direction, in any of 2n positions in the y direction, and at some time t. Since the pedestrian can always just wait if she needs to, she would always prefer to be at (x, y, t) over (x, y, t+1); so an algorithm for computing the *earliest* time at which the pedestrian can arrive at (x, y) will solve this problem.

This problem differs from a standard graph problem in that, for two neighboring locations across a road from one another, the weight of the edge between them is not fixed. As it turns out this is only a minor complication, and we can use slightly modified versions of some standard shortest-path algorithms.

From each location, the pedestrian can go north, south, east or west, unless the direction in question is off the edge of the map. The amount of time this takes will be 2 minutes if it's along a block, or (the amount of time until the traffic light is green) + 1 minute if it's to cross a road.

Bellman-Ford is probably the easiest algorithm to implement for this. The longest path will take at most O(m+n) steps, so your algorithm will terminate in at most that many stages. At each stage, for each point, you'll try to update its (up to) four neighbors. The running time is O(nm(n+m)).

Dijkstra's algorithm is usually more efficient. With 4nm states and 4 directions in which the pedestrian can move from each state, this algorithm will complete in O(nm log(nm)) time.

The correctness of either algorithm can be proved in almost the same way as its standard counterpart. We leave these proofs as worthy exercises for the reader.

More information:

Dijkstra's algorithm - Bellman-Ford algorithm