Kick Start 2019 - Round H

Analysis: Elevanagram

Test set 1

We need to divide each digits to two partitions: positive partition and negative partition, where positive partition means the digit is on the odd index (be calculated as *add*), and negative partition means the digit is in the even index (be calculated as *minus*).

We can use dynamic programming to solve test set 1. Let dp[i][j][k] denote if it is possible to achieve the state that when we are considering digits 1, 2, ... *i*, the current number of digits in the positive partition is *j* and the current sum modulo 11 is *k*. Then for each digit i, we can put 0, 1, ..., $\mathbf{A_i}$ digits into the positive partition, and calculate if the current state is possible. We want to calculate dp[9][sum(**A**)/2][0], where sum(**A**) means the total sum of all elements in array **A**.

The time complexity is O(9 * sum(**A**) * 11 * max(**A**)), which fits the time limit for test case 1. Here max(**A**) means the maximum of all elements in array **A**.

Test set 2

Assume the positive number of digits i is P_i, and negative number of digits i is $\mathbf{A_i}$ - P_i. Then, we will have the following three equations:

$$(1)\ \Sigma P_i = \text{ceil}(\text{sum}(\mathbf{A}) / 2)$$
$$(2)\ \Sigma i \times (P_i - (\mathbf{A_i} - P_i)) \% 11 = 0$$
$$(3)\ 0 \le P_i \le \mathbf{A_i}$$

In order to solve this, initially we can put half the number of each digits to be in positive partition (e.g. P_i = $\mathbf{A_i}$ / 2, take care of odd numbers), and then try to adjust each P_i to satisfy equation (2). For each i, we can adjust its P_i from -$\mathbf{A_i}$ / 2 to $\mathbf{A_i}$ / 2.

We can prove the two following conclusions:

1. If there are at least two numbers of $\mathbf{A_i}$ ≥ 10, then the solution must exist.

This is very easy to prove. We can only adjust these two digits from -5 to 5, and each adjustment will result in a different remain value of modulo 11. Thus, we will get 0 finally.

2. If there are at least three numbers of $\mathbf{A_i}$ ≥ 6, then the solution must exist.

To prove this, we can prove that:

For any $1 \le i < j < k \le 9$, $0 \le r \le 10$, the following equations:
$$(1)\ (i * x_1 + j * x_2 + k * x_3) \% 11 = r$$
$$(2)\ x_1 + x_2 + x_3 = 0$$
$$(3)\ -3 \le x_i \le 3$$
will have a valid solution.

This can be proved by iterating all possible situations, where the total number is 9**C**3 * 11 * 7 * 7 = 45276, quite small.

According to these two conclusions, if there are at least two numbers ≥ 10 or at least three numbers ≥ 6, we can return YES immediately. Otherwise, there are at most one value ≥ 10, and at most two values ≥ 6, we can calculate all possible situations, where in the worst case time complexity is $O(6^7 * 10) = O(2799360)$ which fits the time limit for test case 2.