

## Kick Start 2020 - Round G

# Merge Cards

### Problem

Panko is playing a game with  $N$  cards laid out in a row. The  $i$ -th card has the integer  $A_i$  written on it.

The game is played in  $N - 1$  rounds. During each round Panko will pick an adjacent pair of cards and *merge* them. Suppose that the cards have the integers  $X$  and  $Y$  written on them. To merge two cards, Panko creates a new card with  $X + Y$  written on it. He then removes the two original cards from the row and places the new card in their old position. Finally Panko receives  $X + Y$  points for the merge. During each round Panko will pick a pair of adjacent cards uniformly at random amongst the set of all available adjacent pairs.

After all  $N - 1$  rounds, Panko's total score is the sum of points he received for each merge. What is the [expected value](#) of Panko's total score at the end of the game?

### Input

The first line of the input gives the number of test cases,  $T$ .  $T$  test cases follow. Each test case begins with a line containing the integer  $N$ . A second line follows containing  $N$  integers, describing the initial row of cards. The  $i$ -th integer is  $A_i$ .

### Output

For each test case, output one line containing `Case #x: y`, where  $x$  is the test case number (starting from 1) and  $y$  is the expected total score at the end of the game.

$y$  will be considered correct if it is within an absolute or relative error of  $10^{-6}$  of the correct answer. See the [FAQ](#) for an explanation of what that means, and what formats of real numbers we accept.

### Limits

Time limit: 40 seconds.

Memory limit: 1 GB.

$1 \leq T \leq 100$ .

$1 \leq A_i \leq 10^9$  for all  $i$ .

#### Test Set 1

$2 \leq N \leq 9$ .

#### Test Set 2

$2 \leq N \leq 100$ .

#### Test Set 3

$2 \leq N \leq 5000$ .

## Sample

### Sample Input

```
2
3
2 1 10
5
19 3 78 2 31
```

### Sample Output

```
Case #1: 20.000000
Case #2: 352.33333333
```

In sample case #1,  $N = 3$ . The initial row of cards is  $[2, 1, 10]$ . In the first round, Panko has two choices, of which he will choose one at random.

- If Panko merges the first pair  $(2, 1)$ , then the row of cards becomes  $[3, 10]$ , adding  $2 + 1 = 3$  points to his total score. In the second round, there is only one pair remaining  $(3, 10)$ . If he merges them, the row of cards becomes  $[13]$ , adding  $3 + 10 = 13$  points to his total score. Panko ends the game with  $3 + 13 = 16$  points.
- If Panko merges the second pair  $(1, 10)$ , then the row of cards becomes  $[2, 11]$ , adding  $1 + 10 = 11$  points to his total score. In the second round, there is only one pair remaining  $(2, 11)$ . If he merges them, the row of cards becomes  $[13]$ , adding  $2 + 11 = 13$  points to his total score. Panko ends the game with  $11 + 13 = 24$  points.

Thus, the expected number of points Panko ends the game with is  $(16 + 24)/2 = 20$ .

In sample case #2,  $N = 5$ . The initial row of cards is  $[19, 3, 78, 2, 31]$ . There are too many possibilities to list here, so we will only go through one possible game:

- In the first round, if Panko merges the pair  $(78, 2)$ , then the row of cards becomes  $[19, 3, 80, 31]$ , adding  $78 + 2 = 80$  to his score.
- In the second round, if Panko merges the pair  $(80, 31)$ , then the row of cards becomes  $[19, 3, 111]$ , adding  $80 + 31 = 111$  to his score.
- In the third round, if Panko merges the pair  $(19, 3)$ , then the row of cards becomes  $[22, 111]$ , adding  $19 + 3 = 22$  to his score.
- In the fourth round, if Panko merges the pair  $(22, 111)$ , then the row of cards becomes  $[133]$ , adding  $22 + 111 = 133$  to his score.

At the end of the game explained above, Panko's total score is  $80 + 111 + 22 + 133 = 346$ .