Analysis: Inversions Organize

Test Set 1

Since $\mathbf{N} \leq 2$, there will be a maximum of 16 elements in the matrix. We can simply brute force this by trying every possible combination and checking which installation that satisfies the organizational goal (top $\mathbf{N}$ rows has the same number of `I`s as the bottom $\mathbf{N}$ rows, and left $\mathbf{N}$ columns has the same number of `I`s as the right $\mathbf{N}$ columns) involves the minimum number of letter switches. Since there are at most 2^{16} total possible combinations, this is fast enough for Test Set 1.

Test Set 2

To solve Test Set 2, we can first split the matrix into quadrants. Let A, B, C, and D be the number of `I`s in each quadrant, in the following order:

```
A B
C D
```

In Sample Case #1, we would split it as follows:

```
I I   O O
O O   O I


I I   I I
O O   O I
```

Here, $A = 2$, $B = 1$, $C = 2$, and $D = 3$.

Similarly, let A', B', C', and D' be the number of `I`s on each quadrant of the output, in the same order as before. Then, we need A', B', C', and D' such that $A' + B' = C' + D'$ and $A' + C' = B' + D'$.

Adding the two equations, we obtain $A' = D'$, and replacing that in either equation, we obtain $B' = C'$. Notice that having $A' = C'$ and $B' = C'$ are also sufficient conditions for the original equations. Therefore, we can solve the equivalent problem of minimizing the letter touches to get $A' = D'$ and $B' = C'$. We can see now that fulfilling $A' = D'$ and $B' = C'$ are independent problems.

Then, to find the minimum number of letter changes, we need to find the sum of the differences between each pair of sets, A and D, and B and C (to achieve equalization, we can greedily switch that amount of `I`s to `O`s from the side that has more `I`s).

To implement this idea, we iterate through the matrix and keep track of the counts per quadrant, A, B, C, D, and return the summation of absolute differences: $|A - D| + |C - B|$.

This algorithm runs in $O(\mathbf{N}^2)$ time, since we are iterating through the matrix.