Kick Start 2018 - Round F

Analysis: Specializing Villages

Specializing Villages: Analysis

Small dataset

For the Small dataset, we can enumerate all plans, find the average of the distances for the villages, and count the plans with a minimal average distance.

Large dataset

If each village can get the other food it needs from its nearest neighbor, then the average distance is clearly minimized. We will show that it is possible to construct this situation.

For each village v, let l(v) be the numerical label of v, r(v) be the shortest of the roads that have v as an endpoint, d(v) be the distance of r(v), and f(v) be the other village to which r(v) connects v. We first sort the villages by increasing d(v). Since two villages might share the same d(v), we break ties by increasing l(v).

Then, we handle each village in sorted order. We assign v the opposite food of f(v); if f(v) does not have a food assigned, we can assign either food to v. How do we know this will not cause problems later in the process? Well, this situation can only happen when $d(f(v)) \geq d(v)$. But we know that r(v) connects f(v) and v, so $d(f(v)) \leq d(v)$, and thus $d(f(v)) = d(v)$ in this case, which implies $r(f(v)) = r(v)$ and $l(f(v)) > l(v)$. Since $r(f(v)) = r(v)$, $f(f(v)) = v$, so f(v) will be assigned the opposite food of v when we get to it.

In our approach, some villages may have their food choices forced by the food choices of other villages, whereas some villages may be able to choose either food. So, the count of optimal plans is 2^N, where N is the number of villages that can choose. N is equal to the size of $\{v \mid f(f(v)) = v \text{ and } l(f(v)) > l(v)\}$.

Roads with length 0 pose a special case. If a 0-length road connects villages x and y, any village v other than x and y with f(v) in {x, y} has two choices, since it can get one food through r(v) and the other from a combination of r(v) and the 0-length road. So, with a 0-length road present, the answer will be 2^N, where N is the size of $\{v \mid f(f(v)) = v \text{ and } l(f(v)) > l(v)\} \cup \{v \mid d(v) > 0 \text{ and } d(f(v)) = 0\}$.