Code Jam 2014 - Qualification Round

Analysis: Deceitful War

Ken's best possible game of War

First, let's think about the best possible outcome for Ken in a game of War. Let's suppose the maximum number of points Ken can get is **k**; then after a little work convincing yourself, you should see that Ken can achieve that outcome if Ken's **k** heaviest blocks are played in decreasing order against Naomi's **k** lightest blocks, also in decreasing order.

That exact pairing likely won't happen, and without knowing Naomi's blocks' weights, Ken doesn't even know what the pairing would be; but Ken can follow a simple strategy to score **k** points anyway.

Ken's strategy

Ken's strategy is simple: when Naomi plays a block, Ken beats it with the lightest possible block if he can; and if he can't beat it, he plays his lightest block. It is clear that using such strategy will maximize Ken's winning potential for the following rounds because Ken not only wins one point for this round, but also preserves his heavier blocks for the following rounds.

The following block of text proves that this strategy will earn Ken the maximum possible number of points, **k**, no matter what Naomi does. If that's obvious to you, or you aren't into formal proofs, go ahead and skip it.

Proof

In this strategy, to win **k** points Ken wants to maintain an invariant: after Ken has scored **i** points, Ken's **k-i** heaviest blocks will still beat Naomi's **k-i** lightest blocks. When **k-i** is zero, Ken cannot win any more points because his heaviest block is lighter than any of Naomi's blocks. Therefore, if that invariant is maintained Ken must have **k** points when there are no more blocks left.

Proof of invariant:
Suppose Ken has **i** points. We'll refer to "pairs" of blocks later: Ken has **k - i** heaviest blocks remaining and his **j**-th heaviest remaining block is "paired" with Naomi's **(k-i-j)**-th lightest remaining block. There are three types of blocks Naomi might play:

1. If Naomi plays a block that's heavier than all of Ken's, it isn't from Naomi's lightest **k-i** blocks. Ken will play his lightest block which isn't from his heaviest **k-i** blocks, and the invariant is maintained.
2. If Naomi plays a block that's lighter than one of Ken's blocks but isn't from Naomi's lightest **k-i** blocks, Ken will either beat it with his heaviest blockâ€”in which case Naomi's lightest **k-i-1** blocks will lose to Ken's heaviest remaining **k-i-1** blocks, since nothing has changed about how the remaining blocks are pairedâ€”or Ken will beat it with something lighter, in which case his position is obviously no worse. Either way Ken has **i+1** points and the invariant is maintained.
3. If Naomi plays a block from her lightest **k-i** blocks, Ken will either beat it with the block it's paired with, in which case Ken now has **i+1** points and the remaining **k-i-1** pairs are maintained; or Ken will beat it with something lighter, in which case Ken is clearly no worse off. Either way Ken has **i+1** points and the invariant is maintained.

Naomi's best possible game of War

As we saw in the proof above, Ken can force his maximum possible number of points in War no matter what Naomi does. No wonder Naomi is tired of playing it!

Naomi's best possible game of Deceitful War

In Deceitful War, however, it turns out that the situation is exactly reversed: as we'll show in the following paragraphs. In War, Ken got the best possible pairings for himself but Naomi will get the best possible pairings for herself in Deceitful War.

Naomi's strategy for Deceitful War

There are several strategies for Naomi to play Deceitful War optimally. We present one here. Naomi's strategy for that is almost trivial. Assume **k** is the best possible score Naomi could achieve. Naomi can do score **k** points by pairing her **k** heaviest blocks with Ken's **k** lightest blocks.

Naomi will take her **i**-th heaviest block and tell Ken that it is heavier than all of Ken's blocks. Ken will believe her and play his lightest block, which is what Naomi wanted. Now Naomi simply has to repeat the process of playing her remaining heaviest blocks from lightest to heaviest and inciting Ken to play his lightest block from lightest to heaviest. After Naomi scores **k** points, her heaviest block will be lighter than any of Ken's available blocks. Now, since Naomi cannot lie anymore Naomi plays her remaining blocks without lying about their mass.

Conclusion

It's a particularly beautiful piece of symmetry that Ken can achieve his optimal pairing by being reactive, despite being honest and without information; and Naomi can reverse Ken's advantage by beating Ken's reactiveness with dishonesty and perfect information.