

Kick Start 2022 - Round E

Analysis: Coloring Game

Let us use the following notation for the analysis:

- Cells painted by the bot are marked as B .
- Cells painted by John are marked as J .
- Valid cells are marked as V .
- Invalid cells are marked as X .

Let us first look at the case with $N = 5$. All cells are valid in the beginning.

Initial state: $[V, V, V, V, V]$.

The cell numbered with integer i is represented as C_i . As the bot makes the first move, it colors the leftmost valid cell, which is C_1 . This move makes the cells C_1 and C_2 invalid and reduces the number of valid cells by 2.

State after first move: $[B, X, V, V, V]$.

As John plays optimally to minimize the bot's score, he colors a cell which makes as many invalid cells as possible. Coloring the cell C_3 , which is adjacent to an invalid cell, makes the cells C_3 and C_4 invalid, while coloring the cell C_4 makes the cells C_3 , C_4 and C_5 invalid. Thus, John colors the cell C_4 , reducing the number of valid cells by 3 and leaving no more cells for the bot to color in the next turn.

State after John's move: $[B, X, X, J, X]$

Test Set 1

For $N \leq 5$, the bot can color at most 1 cell in its first move and John can make the remaining cells invalid. Thus, maximum achievable score by the bot for $N \leq 5$ is 1.

For $N = 6$, the bot can color another cell in the second move. Thus, maximum achievable score by the bot for $N = 6$ is 2.

Test Set 2

Since the bot always paints the first valid cell from the left, we can show that after each move by the bot, the number of valid cells reduces by at most 2. Also, after each move by John, the number of valid cells reduces by at most 3. We can show that this is achievable if John always paints the second valid cell from the left unless there is only one valid cell remaining. This results in reduction of the number of valid cells by at most 5 between successive turns of the bot.

In other words, the bot paints 1 cell for every 5 cells. Thus, the total number of cells the bot can paint is at least $\lceil N/5 \rceil$.

Time and Space Complexity: The answer for both the test sets can be found in $O(1)$ time and no extra space is required for calculating the answer.