Kick Start 2022 - Round G

Analysis: Curling

Let us denote the score of the red team and the yellow team as s_{red} and s_{yellow}, respectively.

Test Set 1

For this test set, $\mathbf{M}$ = 0, i.e., there are no stones remaining on the curling sheet for the yellow team. In this case:

- $s_{red} =$ number of stones which are in the house.
- $s_{yellow} = 0$.

For a house of radius $\mathbf{R_h}$ centered at $(0, 0)$, a stone centered at (x, y) with radius $\mathbf{R_s}$ is:

1. in the house iff: $\sqrt{x^2 + y^2} \leq \mathbf{R_h} + \mathbf{R_s}$ (Equality is for the case when the stone and the house are tangent to each other. Figure 1 shows such a case).
2. outside the house iff: $\sqrt{x^2 + y^2} > \mathbf{R_h} + \mathbf{R_s}$.

To count the stones in the house, we can iterate over the stones and count those which satisfy condition 1, i.e., $x^2 + y^2 \leq (\mathbf{R_h} + \mathbf{R_s})^2$.

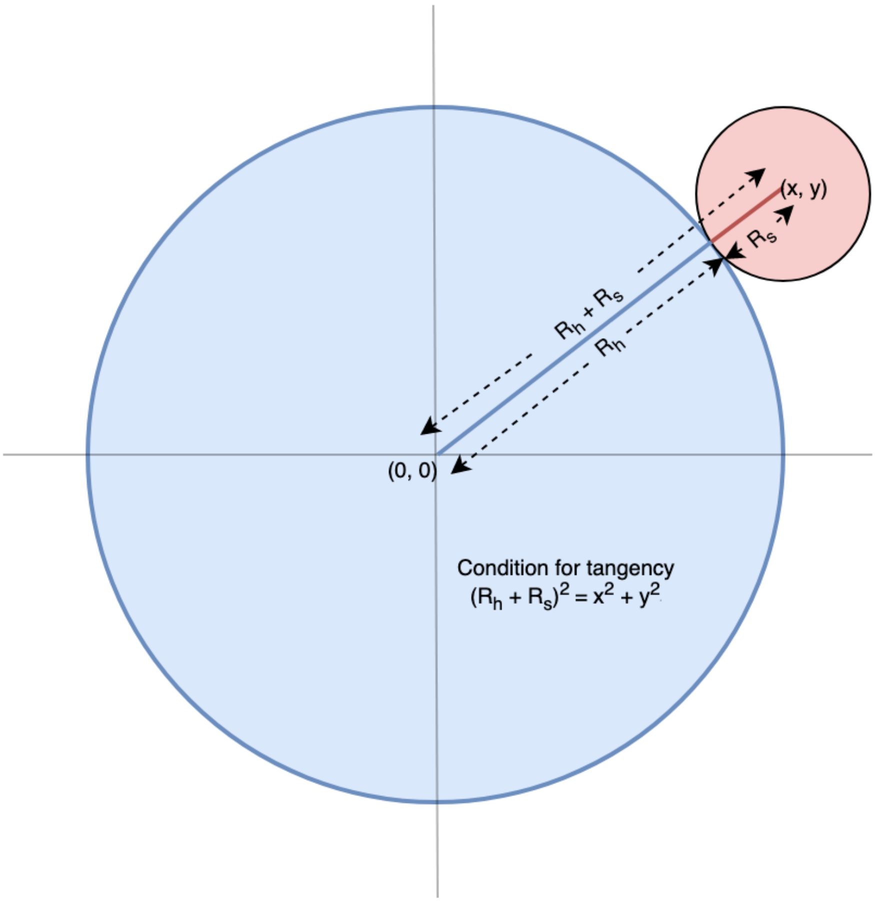

Figure 1: Stone touches the house externally.

Here is a sample code in C++:

```
int s_red = 0, s_yellow = 0;
for(int i = 1; i <= N; i++) {
  s_red += (x[i] * x[i] + y[i] * y[i]) <= (rh + rs) * (rh + rs);
}
```

Test Set 2

In this test set, we can have non-zero number of stones remaining for both teams. Figure 2 shows some examples of scoring in the game. To calculate the score of a team, we can count the number of stones which contribute to the score of a team. A stone contributes to the score of a team iff:

1. It is in the house.
2. It is closer to the center $(0, 0)$ than all of the stones of the opponent team.

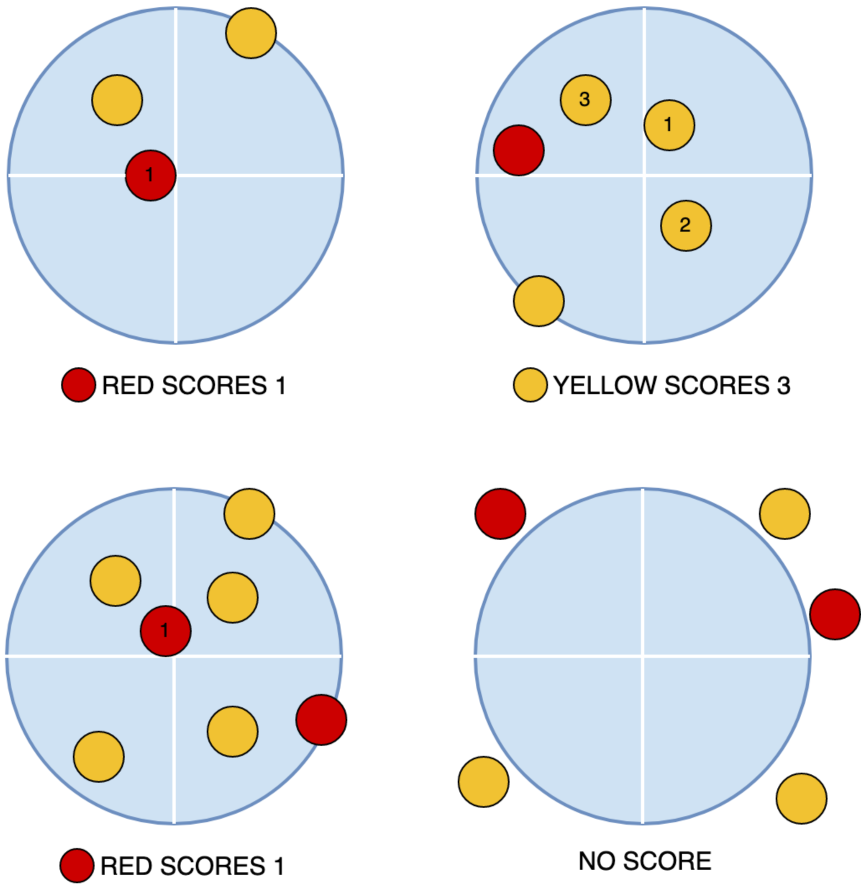

Figure 2: Examples of scoring in curling. Numbered circles are the only scoring ones.

Here is a sample code in C++:

```
int dist(int x, int y) { return x * x + y * y; }

void solve() {
  int s_red = 0;
  for(int i = 1; i <= N; i++) {
   bool is_scoring = dist(x[i], y[i]) <= (rs + rh) * (rs + rh); // Inside house.
   for(int j = 1; j <= M; j++) {
    is_scoring &= dist(x[i], y[i]) < dist(z[j], w[j]);
   }
   s_red += is_scoring;
  }

  int s_yellow = 0;
  for(int i = 1; i <= M; i++) {
   bool is_scoring = dist(z[i], w[i]) <= (rs + rh) * (rs + rh); // Inside house.
   for(int j = 1; j <= N; j++) {
    is_scoring &= dist(z[i], w[i]) < dist(x[j], y[j]);
   }
   s_yellow += is_scoring;
  }
}
```

The overall time complexity of the above solution would be $O(\mathbf{N} \times \mathbf{M})$.

Another solution

Note that the score of at least one team must be 0. If a team does not have any stones on the curling sheet, their score is 0. If both teams have at least one stone still in play, the opponent of the team that has the stone closest to the center will have a 0 score.

- Case 1: $\mathbf{N} = 0$ or $\mathbf{M} = 0$, i.e. there is at least one team that does not have any stones remaining on the curling sheet.
  - In this case, the score of the team which does not have any stones left is 0 and the score of the other team is the number of stones in the house.

- Case 2: $\mathbf{N} > 0$ and $\mathbf{M} > 0$, i.e. each team has at least one stone on the curling sheet.

- Let m_{red} be the least squared distance of a stone of the red team, and m_{yellow} be the least squared distance of a stone of the yellow team.

$$m_{red} = \min_{1 \le i \le \mathbf{N}} \left(\mathbf{X_i^2} + \mathbf{Y_i^2}\right)$$
$$m_{yellow} = \min_{1 \le i \le \mathbf{M}} \left(\mathbf{Z_i^2} + \mathbf{W_i^2}\right)$$

  Note: $m_{red} \neq m_{yellow}$, as no two stones can be equally close to the center $(0,0)$.

  The score of teams:

$$s_{red} = \begin{cases} \text{number of stones such that } \mathbf{X_i}^2 + \mathbf{Y_i}^2 < m_{yellow} \text{ and } \mathbf{X_i}^2 + \mathbf{Y_i}^2 \le (\mathbf{R_h} + \mathbf{R_s})^2, & m_{red} < m_{yellow} \\ 0, & m_{red} > m_{yellow} \end{cases}$$

$$s_{yellow} = \begin{cases} \text{number of stones such that } \mathbf{Z_i}^2 + \mathbf{W_i}^2 < m_{red} \text{ and } \mathbf{Z_i}^2 + \mathbf{W_i}^2 \le (\mathbf{R_h} + \mathbf{R_s})^2, & m_{yellow} < m_{red} \\ 0, & m_{yellow} > m_{red} \end{cases}$$

The overall time complexity of the above solution would be $O(\mathbf{N} + \mathbf{M})$.