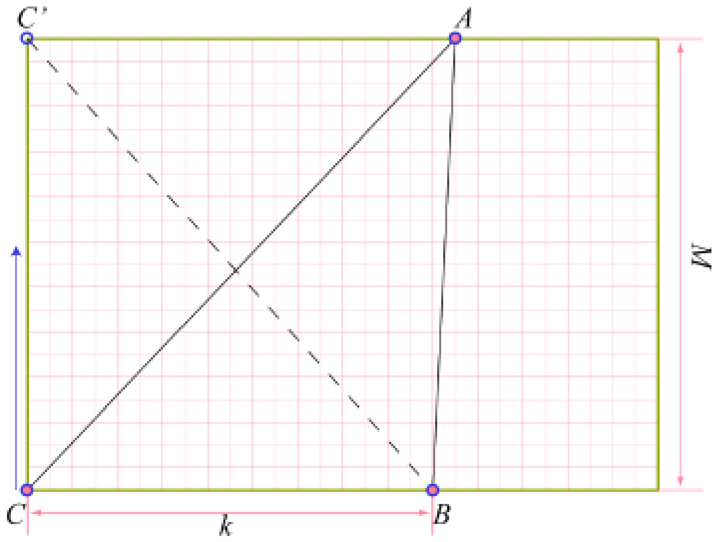
Code Jam 2008 - Round 2

Analysis: Triangle Areas

This problem was originally in the form of a little puzzle: For how many numbers is the area possible? The answer is that there are **MN** such numbers, for any integer 0 < **A** ≤ **MN**, we can find a triangle with area **A**/2 formed by integer points on the graph paper.

The following picture gives the key step in the proof, as well as a solution to our problem.



Assume the integer part of **A**/**M** is k, we have kM ≤ A ≤ (k+1)M. Denote S(XYZ) the area of triangle ΔXYZ. Clearly 2S(ABC) = kM and 2S(ABC') = (k+1)M. Now take a point C* from your pocket, put it on C, and move it up towards C', one unit at a time. What can we say about the quantity 2S(ABC*)?

- It starts with kM and ends with (k+1)M.
- It is monotone increasing because the distance from C* to AB is monotone.
- It is always an integer.
- The journey takes exactly **M** steps.

Putting these together, the simple conclusion is that 2S(ABC*) will hit every integer between kM and (k+1)M, including **A**.

Exercises

(1) For the careful readers, there is one more thing we did not prove yet. There is no way to form a triangle on the graph paper with an area bigger than **MN**/2. Find a simple reason for this.

(2) We argued that 2S(ABC*) must hit **A** because it will hit every integer between kM and (k+1)M. Reason directly that, while C* is moving upward, S(ABC*) will increase by 0.5 every time C* moves one unit higher.