

Analysis: Gridception

Test set 1

This test set can be solved using complete search. We can try enumerating every possible connected pattern. For each possible connected pattern, we check whether that pattern exists in the grid which has been deepened twice. Among all connected patterns which exist in the grid which has been deepened twice, we take the one with the largest number of cells. The correctness of this algorithm will be proved later in this section.

Note that it is not sufficient to check the pattern in the grid which has been deepened only once. The first sample case in the problem description shows that there is a pattern which does not exist in the grid which has been deepened once, but the pattern exists in the grid which has been deepened more than once.

Why is it sufficient to check the pattern in the grid which has been deepened twice? We can first observe that a grid which has been deepened X times consists of blocks of cells. Each block is a square of cells of the same color with side length 2^X . This does mean any pattern of size at most 3×4 can fit in the same block in the grid which has been deepened twice, whereas this might not be the case in the grid which has been deepened once.

Moreover, any pattern of size at most 3×4 overlaps with at least one and at most four blocks in the grid which has been deepened twice. This is also the case in the grid which has been deepened more than twice. Therefore, the set of patterns of size at most 3×4 which exists in the grid which has been deepened twice is equivalent to the set of patterns of size at most 3×4 which exists in the grid which has been deepened X times for any $X > 2$. Therefore, it is sufficient to check the pattern in the grid which has been deepened twice.

There are at most $O(2^{R \times C})$ patterns. Therefore, there are not more than $O(2^{R \times C})$ possible connected patterns. For each pattern, we can first check whether it is connected, and if it is, we can then check whether that pattern exists in the grid which has been deepened twice in $O(R \times C)$ time. Therefore, the total complexity of this solution is $O(2^{R \times C} \times R \times C)$ which is fast enough to solve test set 1.

Test set 2

Since R and C can be up to 20, the exponential solution will not run in time. Therefore, we must not enumerate every possible connected pattern. To solve this test set, we can first observe that a block of cells in the grid which has been deepened at least a googol times will have a side length larger than the size of any possible pattern.

The observation in the previous paragraph ensures that every possible pattern can overlap with at most four blocks of cells in the deepened grid. This means that Codd's pattern needs to be divisible into four quadrants by a horizontal and a vertical line where each cell of the pattern in the same quadrant has the same color. Moreover, that particular combination of four colors has to exist in the original grid.

Therefore, we can consider every possible quadrant center and combination of colors (there are up to $O(2^4 \times R \times C)$). For each quadrant center and combination of colors, we want to get the largest connected component where each cell in this connected component has the same color as the color assigned to the quadrant it belongs to.

For each possible quadrant center and combination of colors, we need $O(\mathbf{R} \times \mathbf{C})$ time to find the largest connected component. Therefore, this solution will run in $O(2^4 \times \mathbf{R}^2 \times \mathbf{C}^2)$ time.