Kick Start 2022 - Round C

Palindromic Deletions

Problem

Games with words and strings are very popular lately. Now Edsger tries to create a similar new game of his own. Here is what he came up with so far.

Edsger's new game is called *Palindromic Deletions*. As a player of this game, you are given a string of length $\mathbf{N}$. Then you will perform the following process $\mathbf{N}$ times:

1. Pick an index in the current string uniformly at random.
2. Delete the character at that index. You will then end up with a new string with one fewer character.
3. If the new string is a palindrome, you eat a piece of candy in celebration.

Now Edsger wonders: given a starting string, what is the expected number of candies you will eat during the game?

Input

The first line of the input gives the number of test cases, $\mathbf{T}$. $\mathbf{T}$ test cases follow. Each test case consists of two lines.

The first line of each test case contains an integer $\mathbf{N}$, representing the length of the string.

The second line of each test case contains a string $\mathbf{S}$ of length $\mathbf{N}$, consisting of lowercase English characters.

Output

For each test case, output one line containing `Case #`x`:` E, where x is the test case number (starting from 1) and E is the expected number of candies you will eat during the game.

E should be computed *modulo* the prime 10^9+7 (1000000007) as follows. Represent the answer of a test case as an irreducible fraction $\frac{p}{q}$. The number E then must satisfy the modular equation $E \times q \equiv p \pmod{(10^9+7)}$, and be between 0 and 10^9+6, inclusive. It can be shown that under the constraints of this problem, such a number E always exists and can be uniquely determined.

Limits

Time limit: 30 seconds.
Memory limit: 1 GB.
$1 \leq \mathbf{T} \leq 20$.
String $\mathbf{S}$ consists of only lowercase letters of the English alphabet.

Test Set 1

$2 \leq \mathbf{N} \leq 8$.

Test Set 2

$2 \leq \mathbf{N} \leq 400$.

Sample

Sample Input

```
2
2
ab
3
aba
```

Sample Output

```
Case #1: 2
Case #2: 333333338
```

In the first test case the game can go in one of two ways (character removed at each step is underlined):

1. $\texttt{"a}\underline{\texttt{b}}\texttt{"} \rightarrow \texttt{"}\underline{\texttt{a}}\texttt{"} \rightarrow \texttt{""}$ (where `""` denotes empty string). Both a and `""` are palindromes, so you will eat two candies.
2. $\texttt{"}\underline{\texttt{a}}\texttt{b"} \rightarrow \texttt{"}\underline{\texttt{b}}\texttt{"} \rightarrow \texttt{""}$. Both b and `""` are palindromes, so you will eat two candies.

Overall, the expected number of candies you will eat is $\frac{2+2}{2} = 2$ candies.

In the second test case, the game can go in one of six ways (character removed at each step is underlined):

1. $\texttt{"}\underline{\texttt{a}}\texttt{ba"} \rightarrow \texttt{"}\underline{\texttt{b}}\texttt{a"} \rightarrow \texttt{"}\underline{\texttt{a}}\texttt{"} \rightarrow \texttt{""}$
2. $\texttt{"}\underline{\texttt{a}}\texttt{ba"} \rightarrow \texttt{"b}\underline{\texttt{a}}\texttt{"} \rightarrow \texttt{"}\underline{\texttt{b}}\texttt{"} \rightarrow \texttt{""}$
3. $\texttt{"a}\underline{\texttt{b}}\texttt{a"} \rightarrow \texttt{"}\underline{\texttt{a}}\texttt{a"} \rightarrow \texttt{"}\underline{\texttt{a}}\texttt{"} \rightarrow \texttt{""}$
4. $\texttt{"a}\underline{\texttt{b}}\texttt{a"} \rightarrow \texttt{"a}\underline{\texttt{a}}\texttt{"} \rightarrow \texttt{"}\underline{\texttt{a}}\texttt{"} \rightarrow \texttt{""}$
5. $\texttt{"ab}\underline{\texttt{a}}\texttt{"} \rightarrow \texttt{"}\underline{\texttt{a}}\texttt{b"} \rightarrow \texttt{"}\underline{\texttt{b}}\texttt{"} \rightarrow \texttt{""}$
6. $\texttt{"ab}\underline{\texttt{a}}\texttt{"} \rightarrow \texttt{"a}\underline{\texttt{b}}\texttt{"} \rightarrow \texttt{"}\underline{\texttt{a}}\texttt{"} \rightarrow \texttt{""}$

Overall, the expected number of candies you will eat is $\frac{2+2+3+3+2+2}{6} = \frac{14}{6} = \frac{7}{3}$ candies. 333333338 is a uniquely determined number that satisfies the conditions mentioned in the output section as $333333338 \times 3 \equiv 7 \pmod{10^9+7}$, therefore 333333338 is the answer to this test.