Analysis: Bribe the Prisoners

This problem is solved by dynamic programming. For each pair of cells $a \le b$, we want to compute dp[a][b], the best answer if we only have prisoners in cells from a to b, inclusive. Once we decide the location of c, the first prisoner between a and b to be released, we face the smaller sub-problems dp[a][c-1] and dp[c+1][b]. The final answer we want is dp[1][P].

It is clear we only need to solve those dp[a][b]'s where both a and b are either 1, P, or adjacent to a prisoner to be released. Thus the number of sub-problems we need to solve is just $O(Q^2)$.

Here is the annotated judge's solution.

```
int p[200]; // prisoners to be released.
map<pair<int, int>, int> dp;
// Finds the minimum amount of gold needed,
// if we only consider the cells from a to b, inclusive.
int Solve(int a, int b) {
  // First, look up the cache to see if the
  // result is computed before.
  pair<int, int> pr(a, b);
  if(mp.find(pr) != mp.end()) return mp[pr];
  // Start the computation.
  int r = 0;
  for(int i=0; i<Q; i++) {
    if(p[i] >= a \&\& p[i] <= b) {
      int tmp = (b-a) + Solve(a, p[i]-1) + Solve(p[i]+1, b);
      if (!r || tmp<r) r=tmp;
  }
  mp[pr]=r;
  return r;
}
```