Code Jam 2015 - Round 3

Analysis: River Flow

Assume the farmers are not cheating. The river flow data is periodic, with period 2D.

Let x_i be d_i - d_{i-1} when i>1, and $x_1 = d_1 - d_{2D}$. x_i represents the difference in flow between day i and the previous day in the 2D-day cycle.

Each farmer's actions change x_i in the following way. Let the first time a farmer changes between diverting water and allowing it to flow, or vice-versa, be T, and let the number of days for which that farmer performs the same action be P. At times T, T+P, T+2P, etc., the farmer will alternate between adding 1 to x_i and subtracting 1 from x_i. (Whether it adds 1 or subtracts 1 first depends on whether the farmer is diverting water or allowing it to flow at the start of the 2D-day cycle). For example, a farmer that starts diverting water at time 3, and changes every 8 days, contributes -1 to x_3, +1 to x_{11}, -1 to x_{19}, +1 to x_{27}, etc.

We can assume that if two farmers share the same T and P, they either both start the cycle diverting water or both start the cycle allowing it to flow. If they were performing different actions, we could remove the two farmers and their tributaries, and get the same data.

Now consider the quantity $F(T,P) = x_T - x_{T+P} + x_{T+2P} - x_{T+3P} + X_{T+4P} - \ldots$ for some T and P. Any farmers with those values of T and P will each contribute 2D/P or -2D/P to F(T,P), depending on which action they perform first. Any farmers with a different value of T or P will contribute zero to F(T,P). So the number of farmers for T and P is |F(T,P)| * P / 2D, and the sign of F(T,P) tells us their initial action.

We can try every valid value of T and P to find out how many farmers of each type there are. Once we know this information, we can check that the original data matches it for some number of tributaries. If it does not, we output `"CHEATERS!"`, otherwise we output the number of farmers.