Code Jam 2009 - Round 1C

Analysis: Center of Mass

Let P_i and V_i be the initial position and the velocity of the i-th firefly; its position at time t is given by $P_i + tV_i$. Therefore, the center of mass at time t is

$$M(t) = \sum (P_i + t\,V_i) / n = \sum P_i / n + t \sum V_i / n = P_{ave} + t\,V_{ave},$$

where P_{ave} (= M(0)) and V_{ave} are the average initial position and average speed. Thus we show that the center of mass is also moving on a straight ray (a half line) with constant speed. (One special case is that the average speed could be 0, when the center of mass does not move at all.)

So, the problem is actually finding the closest distance from a point (the origin) to a ray, an elementary geometry problem. This is an easy exercise with many possible short solutions. One can use basic manipulations or calculus to derive the exact formula for the answers. One may also notice that the distance function d(t) is convex in t, thus use trinary search to find the best t. As always, we encourage the readers to download correct solutions from the scoreboard.

Note

We did the calculations using vectors. It can also be carried out with 3-dimensional coordinates.

Reference

The exact formula of the distance from a point to a line in 3d space.