

## Analysis: The Next Number

Let  $x$  be our input. We want to find the next number  $y$ . We denote  $L(s)$  to be the length of  $s$ , i.e., the number of digits in  $s$ .

*Case 1.* If all the digits in  $x$  are non-increasing, for example  $x = 776432100$ , then  $x$  is already the biggest one among the numbers in the list with  $L(x)$  digits. The next number,  $y$ , must have one more digit, that is, one more 0. Actually  $y$  must be the smallest one with  $L(x)+1$  digits. To get  $y$ , we put the smallest non-zero digit in front, and all the other digits are put in non-decreasing order.

*Case 2.* Otherwise,  $x$  can be written as the concatenation  $x = ab$ , where  $b$  is the longest non-increasing suffix of  $x$ , so  $d$ , the last digit of  $a$  is smaller than the first digit of  $b$ . Let  $d'$  be the smallest among all the digits in  $b$  that are bigger than  $d$ .

Because  $b$  is non-increasing,  $x$  is the biggest number among those who has  $L(x)$  digits and starts with prefix  $a$ . The first  $L(a)$  digits of  $y$  must be bigger than  $a$ . The smallest we can do is to replace  $d$  with  $d'$ , and then for the rest digits, we arrange them in non-decreasing order.

Let us do another example.  $x = 134266530$ . Then  $a = 1342$ ,  $b = 66530$ ,  $d = 2$ , and  $d' = 3$ . The next number is  $y = 134302566$ .

In fact, we can unify the two cases above. Since the number of 0's is not restricted, we can just imagine there is one more 0 in the beginning of  $x$ , thus Case 1 is reduced to Case 2.

The above described is actually exactly the procedure to get the next permutation of a finite sequence in certain languages. Below is a solution that is essentially one line in C++. From the author:

```
deque<char> f;  
...  
f.push_front('0');  
next_permutation(f.begin(), f.end());  
if (f.front() == '0') f.pop_front();
```