Kick Start 2017 - Round D

Analysis: Trash

Trash Throwing: Analysis

In this problem, we will use binary search multiple times to transform an optimization problem into a decision problem. The problem asks us to find the maximal radius for a thrown circle with a center position satisfying $f(x)=ax(x-\mathbf{P})$, without touching the ceiling or any obstacles.

First of all, if there is an *a* so that a circle with radius **R** could pass, then for all $\mathbf{R'} \le \mathbf{R}$, **R'** is also a valid solution, as we can choose the same *a*. In other words, we can binary search on **R**, and for each **R**, check whether there is a valid *a*. If there is, **R** is an acceptable answer.

How can we determine whether there is a valid parabola for a given **R**?

Let's convert the problem into a simpler form. Bob wants to throw a circle without touching any obstacles. That is, for any point on the parabola (x, f(x)), the distance from that point to the obstacle ($\mathbf{X_i}$, $\mathbf{Y_i}$) must be larger than **R**. We can reframe the problem as follows: given some obstacle circles, with the i-th circle centered at ($\mathbf{X_i}$, $\mathbf{Y_i}$) with radius **R**, Bob will throw a point such that the point does not touch/enter any circles. (We must also lower the ceiling by exactly **R** units.) This conversion does not change the final answer.

In the Small dataset, there is only one obstacle, as shown in example, and there are two ways to throw it into the trash can. One way is to throw the point over the obstacle, and the other one is to throw the point under the obstacle. Let's consider the former situation. If there is an *a* for which the parabola does not touch the obstacle circle, then for all $a' \le a$, the parabola with parameter *a'* is a valid solution. Therefore, we can binary search again (this time on *a*) and get an interval of real numbers that are valid value of *a*. The latter situation is similar. After we have considered both situations, we will have two (or maybe one, in a corner case) intervals of possible values for *a*.

After we binary search to find **R** and *a*, the problem becomes, given the parabola parameter *a* and an obstacle circle with radius **R**, determine whether the parabola will intersect the circle. We can list the equations:

$y = ax(x-\mathbf{P})$

$(x - \mathbf{X_i})^2 + (y - \mathbf{Y_i})^2 = \mathbf{R^2}$

Since *a*, **P**, $\mathbf{X_i}$, $\mathbf{Y_i}$ and **R** are all known variables, we can solve the equations by [Newton's method]() or [Ferrari's algorithm](). Each real root represents an intersection point of the parabola and the circle, so an *a* is acceptable if the equations do not have real roots. As for the ceiling case, since the parabola gets the maximal f(x) when $x=\mathbf{P}/2$, if the maximal f(x) is lower than **H-R**, the trash will not touch the ceiling. This is enough to solve the Small dataset.

Finally, let us handle the case of multiple obstacles. We can use the same binary search mechanism. Note that in the previous solution for one obstacle, after binary search for **R**, we used another binary search to find the valid intervals for *a*. What we need to do is to find a value of *a* that is valid for all the obstacles. We can do this by masking all invalid intervals, and seeing what remains. If the invalid intervals cover all real numbers, then the value of **R** that we are considering is not acceptable.