

Analysis: Googlander

Like Power Levels, this problem was inspired by pop culture: the movie Zoolander and its main character, who is unable to turn left. As he puts it: "I'm not an ambi-turner."

One way to approach the problem is to construct all possible paths using, for example, a breadth-first search. Since the answer for a 10x10 grid is only 48620, this works fine for the Small data set, but it's too slow for the Large. Googlander can walk 32247603683100 paths in a 25 x 25 grid, and that's just too many paths to keep track of! So we need a better way.

An important observation is that once Googlander leaves the line of cells he's currently walking along (without loss of generality, we'll say it's a column) by turning right, it's impossible for him to return to it (or to any cell in the grid beyond the row into which he turns). This means that once he has taken his first step out of the column, he is in a new instance of the same problem, but smaller and rotated 90 degrees to the right. This suggests a recursive solution that accounts for the fact that he can make his turn anywhere in the column:

```
answer(R, C) = sum from i = 1 to R of answer(C-1, i)
answer(1, anything) = answer(anything, 1) = 1
```

If you use memoization to avoid solving the same subproblem more than once, this recursive strategy is fast enough for the Large data set. Here's a sample solution:

```
memo = [[-1 for i in range(26)] for j in range(26)]

def solve(r, c):
    if r == 1 or c == 1:
        return 1
    if memo[r][c] != -1:
        return memo[r][c]
    ans = 0
    for i in range(1, r+1):
        ans += solve(c-1, i)
    memo[r][c] = ans
    return ans

t = int(raw_input().strip())
for case in range(1, t+1):
    r, c = [int(a) for a in raw_input().strip().split()]
    print "Case #" + str(case) + ": " + str(solve(r, c))
```

Some of you may have noticed that the solutions to this problem are entries in Pascal's triangle (which are also the binomial coefficients). We leave it as an exercise to the reader to dig into that relationship.