Introduction to Mathematical Research

Instructor

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Course Overview

In this course, we will discuss topics in differential topology, differential geometry, probabilistic methods, algorithms, and data structures. We will see how they can be applied to machine learning, and we will go on to discuss fundamental concepts of deep learning, different deep neural network models, and mathematical interpretations of why deep neural networks are effective from a calculus viewpoint. We will conclude the course with a gentle introduction to cryptography, introducing some of the iconic topics: Yao's Millionaires' problem, zero-knowledge proof, the multi-party computation algorithm, and its proof.

The program hopes to provide several research mentors from various disciplines who will give some of the course lectures. Students will have the opportunity to work with one of the research mentors offered by the program.

Location: 20 Garden Street Room G10, Cambridge
Zoom ID: 988 9485 0001 (Password: 824783)

Schedule

- Dates: June 16 – August 4
- Schedule: Mondays and Thursdays at 7:00-8:30 pm EST

Textbook

- Guillemin, Victor, and Alan Pollack. Differential topology. Vol. 370. American Mathematical Soc., 2010.
- Taubes, Clifford Henry. Differential geometry: Bundles, connections, metrics and curvature. Vol. 23. OUP Oxford, 2011.
- Do Carmo, Manfredo P. Differential geometry of curves and surfaces: revised and updated second edition. Courier Dover Publications, 2016.
- Cormen, Thomas H., et al. Introduction to algorithms. MIT press, 2022.

Participation

There will be homework assignments given on a weekly basis to help your learning and a final write-up to hand in at the end of the course. You will have to complete the final write-up to receive the full stipend for attending the course.

Course Module

Module I (Week 1-2) Introduction to differential geometry. We will talk about basic concepts of differential topology and differential geometry, and a gentle introduction on Euler Characteristic, Gauss-Bonnet and applications, and uniformization.

Module II (Week 2-4) Introduction to Theoretical Computer Science. This module discusses topics on algorithm, data structure, object-oriented programming, and probabilistic methods in combinatorics.

Module III (Week 4-6) Numerical analysis and machine learning. We will discuss classical geometry, projective geometry, and differential geometry implemented via the finite element method. And we will go over a few examples and use hands-on projects to show how to utilize advanced mathematical concepts in research. Basic concepts of deep learning, different deep learning models, and mathematical interpretations of why deep neural networks are effective.

Module IV (Week 7) Quantum information and quantum simulation. Introduction to cryptography. The goal of this part is an introduction to theoretical computer science, where mathematical proofs play a central role in this field. In this section, I will introduce the concepts of zero-knowledge proof, the multi-party computation algorithm, and its proof.

Course Schedule

Date	Module	Topic
June 16AB	Introduction To Smooth Manifolds I	Manifold, submanifolds, Grassmaniann, parametrization, derivatives, tangents.
June 20A	Introduction To Smooth Manifolds II	Inverse and implicit function theorem, Immersion, embedding, submersion.
June 20B	Geometry of Surfaces I	1st + 2nd fundamental form, connection, curvature
June 23A	Geometry of Surfaces II	Euler Characteristic, Gauss-Bonnet and applications
June 23B	Geometry of Surfaces III	Gauss-Bonnet continued, Uniformization
June 27AB	Differential Topology	Transversality, homotopy, Sard's theorem

Date	Module	Topic
June 30A	Differential Geometry	Bundles
June 30B	Graph Theory I	Lists, stacks, queues, sets, and dictionaries, queues, stacks. Asymptotic Efficiency. Algorithm and Graph Theory. Graph, directed graph, complete graph, graph isomorphism.
July 5A	Graph Theory II	Fleury's algorithm, Hierholzer's algorithm. Spanning tree, MST, and MST property. Prim's algorithm and proof of correctness, Kruskal's algorithm and proof of correctness.
July 5B	Graph Theory III	Network Flow
July 7A	Probabilistic methods in combinatorics I	Random graphs: Basic models and first moment method.
July 7B	Probabilistic methods in combinatorics II	Thresholds in random graphs: Basic phenomena and the second moment method.
July 11A	Probabilistic methods in combinatorics III	Random greedy algorithms.
July 11B	Introduction to Theoretical Computer Science I	Recursion, dynamic programming, and problems: Fibonacci numbers, Catalan number, Bell numbers, Binomial coefficient, permutation coefficient, coin change problem.
July 14A	Graph Theory IV	Dynamic graph data structures.
July 14B	Numerical Analysis and Machine Learning	Forward, backward, central differencing scheme. Runge- Kutta methods. Fourier transform. Machine learning taxonomy, supervised and unsupervised learning, reinforcement learning. Feature space, linear regression and examples.
July 18A	Deep Learning Basics	Basic concepts of deep learning, different deep learning models, forward and backward propagation. Linear regression using deep learning. Fully-connected layers. Convolutional neural networks, reinforcement learning basic concepts.
July 18B	Game Theory	Nash equilibria, approximating Nash
July 21A	Multiagent Learning	Price of Anarchy, learning to share
July 21B	SVD	Singular value decomposition, generalized eigenvalue problem, EigenGame
July 25A	Quantum Simulation	
July 25B	Sensing Using Solid State Defects in Diamond	
July 28A	Graph Theory V	Continuous graph algorithms
July 28B	Introduction to Theoretical Computer Science II	Complexity and Cryptography. Verifiability, definition of NP, how to prove problems in NP, NP-Hardness and NP-Completeness.

Date	Module	Topic
August 1A	Introduction to Theoretical Computer Science III	Security and combinatorics, submodular function and submodular problems. Yao's Millionaires' problem, zero- knowledge proof, the multi-party computation algorithm and its proof.
August 1B	Student Presentation	
August 4AB	Student Presentation	