PARUL UNIVERSITYs
FACULTY OF ENGINEERING AND TECHNOLOGY
DEPARTMENT OF APPLIED SCIENCE AND HUMANITIES
4th SEMESTER B. TECH PROGRAMME
PROBABILITY, STATISTICS AND NUMERICAL METHODS (303193252)
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UNIT: 1 CORRELATION, REGRESSION AND CURVE FITTING

CORRELATION

Correlation is the relationship that exists between two or more variables. Two variables are said to be correlated if a change in one variable affects a change in the other variable.

EXAMPLE:

1. Relationship between heights and weights.
2. Relationship between price and demand of commodity.
3. Relationship between rainfall and yield of crops.

Types Of Correlations

1. Positive and Negative correlations.
2. Simple and multiple correlations.
3. Partial and Total correlations.
4. Linear and Non-linear correlations.

POSITIVE AND NEGATIVE CORRELATIONS

POSITIVE CORRELATIONS (Same direction)

If both the variables vary in the same direction. The correlation is said to be positive. In the other words, If the value of one variable increases, the value of another variable also increases. Same decreases.

Height (cm)	120	130	135	140	145
Weight(kg)	50	55	60	65	70

NEGATIVE CORRELATIONS (Opposite direction)

If both the variables vary in the opposite direction. The correlation is said to be negative. In the other words, If the value of one variable increases, the value of another variable is decreases.

Height (cm)	120	130	135	140	145
Weight(kg)	70	65	60	55	50

SIMPLE AND MULTIPLE CORRELATIONS

1. **Simple Correlation**: -

When only two variables are studied, the relationship is described as simple correlation, e.g., the quantity of money and price level, demand and price.

2. **Multiple Correlation**: -

When more than two variables are studied, the relationship is described as multiple correlation, e.g., relationship of price, demand, and supply of a commodity.

PARTIAL AND TOTAL CORRELATIONS

1. **Partial Correlation**

When more than two variables are studied excluding someother variables, the relationship is termed as partial correlation.

2. **Total Correlation**

When more than two variables are studied without excluding any variables, the relationship is termed total correlation.

Linear and Nonlinear Correlations

Linear Correlation: - If the ratio of change between two variables is constant, the correlation is said to be linear. If such variables are plotted on a graph paper, a straightline is obtained.

X	5	10	15	20	25	30
Y	2	4	6	8	10	12

Nonlinear Correlation: - If the ratio of change between two variables is not constant, the correlation is said to nonlinear. The graph of a nonlinear or curvilinear relationship will be a curve.

X	15	22	25	30	35	40
Y	4	5	8	9	10	12

Method of correlation:

There are two different methods.

1. Graphic methods.
2. Mathematical methods.

Graphic methods are

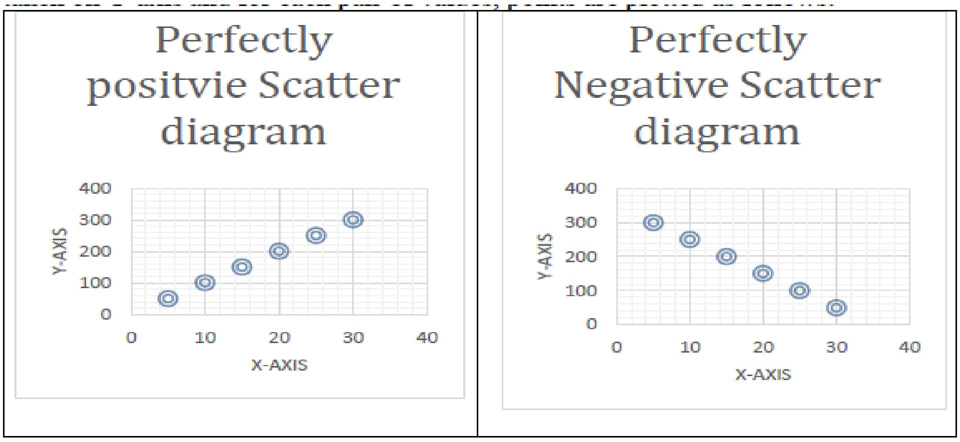
1. Scatter diagram.
2. Simple graph.

Mathematical methods

1. Karl Pearson's coefficient of correlation.
2. Spearman's rank coefficient of correlation.

Scatter diagram:

This is a very simple method studying the relationship between two variables. In this method one variable is taken on X-axis and the other variable is taken on Y-axis and for each pair of values, points are plotted as follows:



Karl Pearson's Coefficient of Correlation

The coefficient of correlation is the measure of correlation between two random variables *X* and *Y*, and is denoted by *r*.

$$r = \frac{\text{cov}(x,y)}{\sigma_x \sigma_y}$$

where $\text{cov}(x, y)$ is covariance of variables *x* and *y*.

σ_x is the standard deviation of variable *x*.

σ_y is the standard deviation of variable *y*.

This expression is known as Karl Pearson's coefficient of correlation or Karl Pearson's product-moment coefficient of correlation.

$$cov(X,Y) = \frac{1}{n}\sum(x-\bar{x})(y-\bar{y})$$

$$\sigma_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}, \qquad \sigma_y = \sqrt{\frac{\sum(y-\bar{y})^2}{n}}$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2}\sqrt{\sum(y-\bar{y})^2}}$$

The above expression can be further modified.

$$r = \frac{\sum xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}\sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}}$$

Properties Of Coefficient of Correlation

1. The Coefficient of Correlation Lies Between -1 And 1.
2. Correlation Coefficient Is Independent of Change of Origin and Change of Scale.
3. Two Independent Variables Are Uncorrelated.

Example: 1

Calculate the correlation coefficient between the following data

X	2	4	5	6	8	11
Y	18	12	10	8	7	5

solution

$n = 6$

x	y	x^2	y^2	xy
2	18	4	324	36
4	12	16	144	48
5	10	25	100	50
6	8	36	64	48
8	7	64	49	56
11	5	121	25	55
$\Sigma x = 36$	$\Sigma y = 60$	$\Sigma x^2 = 266$	Σy^2=706	Σxy=293

$$r = \frac{\sum xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}\sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}}$$

$$r = \frac{293 - \frac{(36)(60)}{6}}{\sqrt{266 - \frac{(36)^2}{6}}\sqrt{706 - \frac{(60)^2}{6}}}$$

$$\boldsymbol{r = -0.9203}$$

Example : 2

Calculate the correlation coefficient between the following data

x	5	9	13	17	21
y	12	20	25	33	35

Solution

$n = 5$

$\bar{x} = \Sigma \frac{x_i}{n} = \frac{65}{5} = 13$

$\bar{y} = \Sigma \frac{y_i}{n} = \frac{125}{5} = 25$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2}\sqrt{\sum(y-\bar{y})^2}}$$

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
5	12	–8	–13	64	169	104
9	20	–4	–5	16	25	20
13	25	0	0	0	0	0
17	33	4	8	16	64	32
21	35	8	10	64	100	80
$\sum x = 65$	$\sum y = 125$	$\sum(x-\bar{x}) = 0$	$\sum(y-\bar{y}) = 0$	$\sum(x-\bar{x})^2 = 160$	$\sum(y-\bar{y})^2 = 358$	$\sum(x-\bar{x})(y-\bar{y}) = 236$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2}\sqrt{\sum(y-\bar{y})^2}}$$

$$r = \frac{236}{\sqrt{160}\sqrt{358}}$$

$$\boldsymbol{r = 0.986}$$

Example 3

Calculate The Correlation Coefficient Between for The Following Values of Demand And The Corresponding Price Of A Commodity:

Demand in quintals	65	66	67	67	68	69	70	72
Price in rupees per kg	67	68	65	68	72	72	69	71

Solution

Let the demand in quintal be denoted by x and the price in rupees per kg be denotedby y.

$n = 8$

$\bar{x} = \Sigma \frac{x_i}{n} = \frac{544}{8} = 68$

$\bar{y} = \Sigma \frac{y_i}{n} = \frac{552}{8} = 69$

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
65	67	–3	–2	9	4	6
66	68	–2	–1	4	1	2
67	65	–1	–4	1	16	4
67	68	–1	–1	1	1	1
68	72	0	3	0	9	0
69	72	1	3	1	9	3
70	69	2	0	4	0	0
72	71	4	2	16	4	8
65	67	–3	–2	9	4	6
$\sum x = 544$	$\sum y = 552$	$\sum(x-\bar{x}) = 0$	$\sum(y-\bar{y})=0$	$\sum(x-\bar{x})^2 = 36$	$\sum(y-\bar{y})^2 = 44$	$\sum(x-\bar{x})(y-\bar{y}) = 24$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2}\sqrt{\sum(y-\bar{y})^2}} = \frac{24}{\sqrt{36}\sqrt{44}} = \mathbf{0.603}$$

Example 4

Given n = 10, $\sigma_x = 5.4$, $\sigma_y = 6.2$, and sum of the product of deviations from the mean of x and y is 66. Find the correlation coefficient.

n = 10, $\sigma_x = 5.4$, $\sigma_y = 6.2$

$$\sigma_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

$$5.4 = \sqrt{\frac{\sum(x-\bar{x})^2}{10}}$$

$$\sum(x-\bar{x})^2 = 291.6$$

$$\sigma_y = \sqrt{\frac{\sum(y-\bar{y})^2}{n}}$$

$$6.2 = \sqrt{\frac{\sum(y-\bar{y})^2}{10}}$$

$$\sum(y-\bar{y})^2 = 384.4$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2}\sqrt{\sum(y-\bar{y})^2}} = \frac{66}{\sqrt{291.6}\sqrt{384.4}} = \mathbf{0.197}$$

Examples

1.Find the Pearson's Correlation Coefficient of the following data:

x	100	101	102	102	100	99	97	98	96	95
y	98	99	99	97	95	92	95	94	90	91

2.Calculate Karl Pearson's coefficient of correlation for the data given below:

x	10	14	18	22	26	30	10
y	18	12	24	6	30	36	18

3.Calculate the correlation coefficient between the following data

x	17	19	21	26	20	28	26	27
y	23	27	25	26	27	25	30	33

4. Calculate Karl Pearson's coefficient of correlation for the data given below:

x	17	19	21	26	20
y	23	27	25	26	27

5. Given n = 10, σ_x = 10.8, σ_y = 12.4, and sum of the product of deviations from the mean of x and y is 132. Find the correlation coefficient.

Spearman's Rank correlation coefficient:

Spearman's rank correlation coefficient, often denoted by the symbol $\boldsymbol{\rho}$ **(rho),** is a non-parametric measure of statistical dependence between two variables.

Here's a brief explanation of how Spearman's rank correlation coefficient is calculated.

Rank the data: For each variable, rank the data from lowest to highest, assigning a rank to each value. If there are ties, assign each tied value the average of the ranks it would have received if there were no tie.

Calculate the differences between ranks: For each pair of data points, find the difference between their ranks.

Spearman's Rank correlation coefficient:

Calculated by following formula: $\boldsymbol{r = 1 - \frac{6\sum d^2}{n(n^2-1)}}$

Where, n = number of pair

In case finding out **rank correlation coefficient** when the observations are paired the above formula can be written as:

$$r = 1 - \frac{6\left\{\sum d^2 + \frac{m}{12}(m^2-1) + \frac{m}{12}(m^2-1) + \dots\dots\dots\dots\dots\dots\right\}}{n(n^2-1)}$$

In $\sum d^2, \frac{m}{12(m^2-1)}$ is added where m is the number of times an item is repeated.

The value of $\boldsymbol{\rho}$ lies between -1 and 1. A positive value indicates a positive monotonic relationship, while a negative value indicates a negative monotonic relationship. A value of 0 indicates no monotonic relationship.

Example:1

Two judges have given ranks to 10 students for their honesty. Find the rank correlation coefficient of the following data:

1^{ST} Judge	3	5	8	4	7	10	2	1	6	9
2^{nd} Judge	6	4	9	8	1	2	3	10	5	7

Solution

$Rank\ given$ $by\ 1st\ judge$	$Rank\ given$ $by\ 2nd\ judge$	$Difference\ in$ $ranks\ \ d$	d^2
3	6	-3	9
5	4	1	1
8	9	-1	1
4	8	-4	16
7	1	6	36
10	2	8	64
2	3	-1	1
1	10	-9	81
6	5	1	1

9	7	2	4
			$\sum d^2$=214

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(214)}{10(100-1)} = 1 - \frac{1284}{990} = 1 - 1.30 = \mathbf{-0.3}$$

Example:2

Ten students got the following percentage of marks in mathematics and physics.

(x)maths	8	36	98	25	75	82	92	62	65	35
(y)physics	84	51	91	60	68	62	86	58	35	49

Find the rank correlation coefficient.

Solution

$$n = 10$$

x	y	Rank in maths(x)	Rank in physics(y)	$d = x - y$	d^2
8	84	10	3	7	49
36	51	7	8	–1	1
98	91	1	1	0	0
25	60	9	6	3	9
75	68	4	4	0	0
82	62	3	5	–2	4
92	86	2	2	0	0
62	58	6	7	–1	1
65	35	5	10	–5	25
35	49	8	9	–1	1
				$\sum d = 0$	$\sum d^2 = 90$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(90)}{10(100-1)} = \mathbf{0.455}$$

Example:3

Find the Coefficient of rank correlation of the following data:

x	35	40	42	43	40	53	54	49	41	55
y	102	101	97	98	38	101	97	92	95	95

Solution:

x	y	Rank in (x)	Rank in (y)	$d = x - y$	d^2
35	102	10	1	9	81
40	101	8.5	2.5	6	36
42	97	6	5.5	0.5	0.25
43	98	5	4	1	1
40	38	8.5	10	-1.5	2.25
53	101	3	2.5	0.5	0.25
54	97	2	5.5	-3.5	10.25
49	92	4	9	-5	25
41	95	7	7.5	-0.5	0.25
55	95	1	7.5	-6.5	42.25
					$= \sum d^2 = 200.25$

$$r = 1 - \frac{6\left\{\sum d^2 + \frac{m}{12}(m^2-1) + \frac{m}{12}(m^2-1) + \frac{m}{12}(m^2-1) + \frac{m}{12}(m^2-1)\right\}}{n(n^2-1)}$$

$$r = 1 - \frac{6\{200.50 + 0.5 + 0.5 + 0.5 + 0.5\}}{990}$$

$$\boldsymbol{r = -0.227}$$

EXAMPLES

1.Compute Spearman's rank correlation coefficient from the following data:

x	18	20	34	52	12
y	39	23	35	52	12

2.The following table gives the scores obtained by 11 students in English and Tamil translation. Find the rank correlation coefficient.

x	40	46	54	60	70	80	82	85	85	90	95
y	45	45	50	43	40	75	55	72	65	42	70

3.Following are the scores of ten students in a class and their IQ:

Score	35	40	25	55	85	90	65	55	45	50
IQ	100	100	110	140	150	130	100	120	140	110

REGRESSION: By studying the correlation, we can know the existence, degree and direction of relationship between two variables but we cannot answer the question of the type if there is a certain amount of change in one variable, what will be the corresponding change in the other variable. The above type of question can be answered if we can establish a quantitative relationship between two related variables. The statistical tool by which it is possible to predict or estimate the unknown values of one variable from known values of another variable is called regression. A line of regression is a straight line.

This equation is called regression line Y on X and b_{yx} is called regression coefficient. The formula can be computed as:

$$(\boldsymbol{y}-\overline{\boldsymbol{y}}) = \boldsymbol{b_{yx}}(\boldsymbol{x}-\overline{\boldsymbol{x}})$$

$$\text{Where } b_{yx} = \frac{n\sum xy-(\Sigma x)(\Sigma y)}{n\Sigma x^2-(\Sigma x)^2}$$

This formula can be used to compute the value of y for given value of x.

Similarly, the regression line Xon Y and b_{xy} is called regression coefficient. The formula can be computed as

$$(\boldsymbol{x}-\overline{\boldsymbol{x}}) = \boldsymbol{b_{xy}}(\boldsymbol{y}-\overline{\boldsymbol{y}})$$

$$\text{Where } b_{xy} = \frac{n\sum xy-(\Sigma x)(\Sigma y)}{n\Sigma y^2-(\Sigma y)^2}$$

This formula can be used to compute the value of x for the given value of y.

NOTE:

(1) b_{xy} and b_{yx} are also computed using the following formula

$b_{xy} = \frac{r\sigma_x}{\sigma_y}andb_{yx} = \frac{r\sigma_y}{\sigma_x}$

Angle between the two regression lines are as follows:

$$\theta = \left|\frac{\left(\frac{r^2-1}{r}\right)\sigma_x\sigma_y}{\sigma^2{}_x+\sigma^2{}_y}\right|$$

When $r=0$ and $\theta=\frac{\pi}{2}$ in this case both the regression lines are perpendicular to each other. If $r=\pm 1$ and $\theta = 0$ in this case both the regression lines are same line because point (x, y) is common point.

When $r = 0$ and $\theta = \frac{\pi}{2}$ in this case both the regression lines are perpendicular to each other. If $r = \pm 1$ and $\theta = 0$ in this case both the regression lines are same line because point (x, y) is common point.

Properties of regression coefficient:

(1) $r = \pm r = \pm\sqrt{b_{xy} * b_{yx}}$, the sign of r should be taken before the square root is that of the regression coefficient.

(2) Since $(b_{xy})(b_{yx}) = r^2 \leq 1$, both the regression coefficient cannot be greater than unity (1).

(3) Arithmetic mean of regression coefficients is greater than or equal to the coefficient of correlation.,$\left(\frac{b_{xy}+b_{yx}}{2}\right) \geq r$

(4) Regression coefficient is independent of origin but not of scale.

Example: The following data regarding the heights (y) and weights (x) of 100 college students are given:$\Sigma x = 15000 \Sigma x^2 = 2272500, \Sigma xy = 1022250 \Sigma y = 6800 \Sigma y^2 = 463025$. Find the coefficient of correlation between height and weight and also the equation of regression of height and weight.

Solution:

$$b_{yx} = \frac{n\sum xy-(\Sigma x)(\Sigma y)}{n\Sigma x^2-(\Sigma x)^2}=0.1$$

$$b_{xy} = \frac{n\sum xy - (\Sigma x)(\Sigma y)}{n\Sigma y^2 - (\Sigma y)^2} = 3.6$$

$$r = \sqrt{b_{xy} * b_{yx}} = 0.6$$

$$\overline{x} = \frac{\sum x}{n} = \frac{15000}{100} = 150$$

$$\overline{y} = \frac{\sum y}{n} = \frac{6800}{100} = 68$$

The equation of the line of regression of y on x is;

$$(y - \overline{y}) = b_{yx}(x - \overline{x})$$

$$(y - 68) = 0.1(x - 150)$$

$$\boldsymbol{y = 0.1x + 53}$$

The equation of the line of regression of x on y is;

$$(x - \overline{x}) = b_{xy}(y - \overline{y})$$

$$x - 150 = 3.6(y - 68)$$

$$\boldsymbol{x = 3.6y - 94.8}$$

Example:1

Find the regression coefficients b_{yx} and b_{xy} and hence, find the correlationcoefficient between x and y for the following data:

X	4	2	3	4	2
Y	2	3	2	4	4

Solution

$n = 5$

x	y	x^2	y^2	xy
4	2	16	4	8
2	3	4	9	6
3	2	9	4	6
4	4	16	16	16
2	4	4	16	8
$\sum x = 15$	$\sum y = 15$	$\sum x^2 = 49$	$\sum y^2 = 49$	$\sum xy = 44$

$$b_{yx} = \frac{n\sum xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2} = -0.25$$

$$b_{xy} = \frac{n\sum xy - (\Sigma x)(\Sigma y)}{n\Sigma y^2 - (\Sigma y)^2} = -0.25$$

$$r = \sqrt{b_{xy} * b_{yx}} = \mathbf{0.25}$$

Examples

1.Find the regression coefficient of y on x for the following data:

$\boldsymbol{x}$	1	2	3	4	5
$\boldsymbol{y}$	160	180	140	180	200

2. Find the equation of regression lines from the following data and also estimate y for x=1 and x for y=4.

x	3	2	-1	6	4	-2	5	7
y	5	13	12	-1	2	20	0	-3

3.Find the equation of regression lines and the correlation coefficient from the following data:

x	28	41	40	38	35	33	46	32	36	33
y	30	34	31	34	30	26	28	31	26	31

4.The following information is obtained for two variables x and y. Find regression equation of y on x. n=10;$\sum x = 130; \sum x^2 = 2288; \sum xy = 3467.$

Curve Fitting

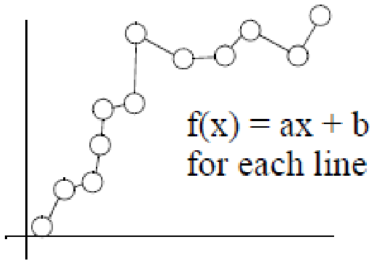
Curve fitting is a statistical technique used to find the best-fitting curve or function that describes a set of data points. The goal is to minimize the difference between the observed data and the values predicted by the model. This process is commonly used in various fields, including physics, engineering, biology, finance, and more. Curve fitting involves the development of a mathematical model that describes the relationship between independent and dependent variables in a dataset. The term "curve" is often used broadly, encompassing various functional forms such as linear, polynomial, exponential, logarithmic, and more complex equations. The "fitting" aspect refers to the optimization of model parameters to minimize the discrepancy between observed data points and the values predicted by the model. In essence, curve fitting aims to uncover the underlying structure of the data and express it in a concise, mathematical form.

METHODS:

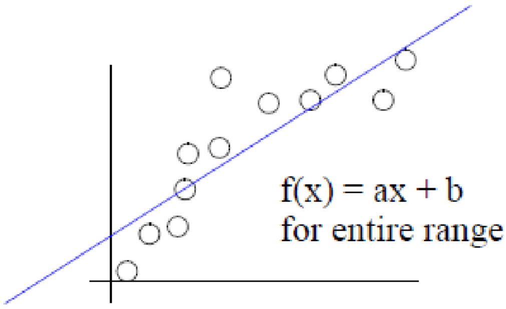
1.Linear Regression: One of the simplest forms of curve fitting, linear regression assumes a linear relationship between the independent and dependent variables. The goal is to find the best-fitting line (or hyperplane in higher dimensions) that minimizes the sum of squared differences between observed and predicted values.$(Y = AX + B \text{ or } X = AY + B)$.

2.Polynomial Regression: Polynomial regression extends linear regression by allowing the model to include higher-degree polynomials. This flexibility enables a better fit for nonlinear relationships in the data.$(Y = AX^2 + BX + C) or (X = AY^2 + BY + C)$.

3.Exponential and Logarithmic: Exponential and logarithmic curve fitting is suitable for datasets exhibiting exponential growth or decay. These models are often used in fields like biology, physics, and finance.$(Y = e^{ax})$



Interpolation

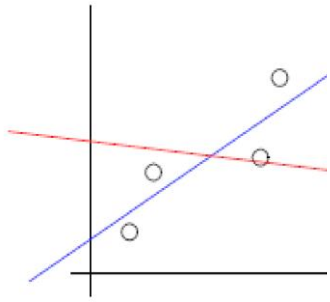


Curve Fitting

Linear Regression:

Given the general form of a straight line

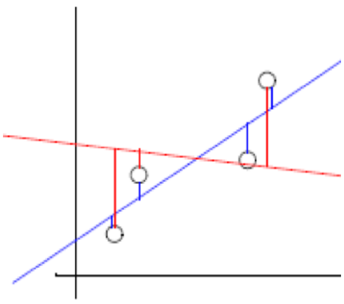
$$\boldsymbol{f(x) = ax + b}$$



How can we pick the coefficients that best fits the line to the data?

First question: What makes a particular straight line a ‘good’ fit?

Why does the blue line appear to us to fit the trend better?



• Consider the distance between the data and points on the line

• Add up the length of all the red and blue verticle lines

• This is an expression of the ‘error’ between data and fitted line

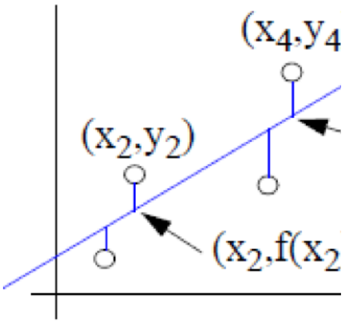
• The one line that provides a minimum error is then the ‘best’ straight line

Quantifying errors in a curve fit

(1) positive or negative error have the same value (data point is above or below the line)

(2) Weight greater errors more heavily

we can do both of these things by squaring the distance denote data values as (x, y) ======>>



denote points on the fitted line as (x, f(x))

sum the error at the four data points

$$err = \sum_{i=1}^{n} d_i^{\,2} = (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 + ...(y_n - f(x_n))^2$$
$$= (y_1 - (ax_1 + b))^2 + (y_2 - (ax_2 + b))^2 + ...+ (y_n - (ax_n + b))^2$$
$$= \sum_{i=1}^{n} (y_i - (ax_i + b))^2$$

Error is minimum if first ordered partial derivatives=0

$$\frac{\partial(err)}{\partial a} = \sum_{i=1}^{n} -2x_i\,(y_i - (ax_i + b)) = 0 \qquad \frac{\partial(err)}{\partial b} = \sum_{i=1}^{n} -2(y_i - (ax_i + b)) = 0$$

$$\therefore \sum_{i=1}^{n} x_i y_i - a\sum_{i=1}^{n} x_i^{\,2} - b\sum_{i=1}^{n} x_i = 0 \qquad \therefore \sum_{i=1}^{n} y_i - a\sum_{i=1}^{n} x_i - b\sum_{i=1}^{n} 1 = 0$$

$$\therefore \sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i^{\,2} + b\sum_{i=1}^{n} x_i \quad \text{and} \quad \therefore \sum_{i=1}^{n} y_i = a\sum_{i=1}^{n} x_i + nb$$

Solve the equations

$$\sum_{i=1}^{n} y_i = a\sum_{i=1}^{n} x_i + nb \qquad (1)$$

$$\sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i \qquad (2)$$

EXAMPLE:1

Fit a straight line to the following data:

x	1	2	3	4	6	8
y	2.4	3	3.6	4	5	6

Solution

Let the straight line to be fitted to the data be

$$y = a + bx$$

$$\sum y = na + b\sum x \qquad (1)$$

$$\sum xy = a\sum x + b\sum x^2 \qquad (2)$$

$$n = 6$$

x	y	x^2	xy
1	2.4	1	2.4
2	3	4	6
3	3.6	9	10.8
4	4	16	16
6	5	36	30
8	6	64	48
$\sum x = 24$	$\sum y = 24$	$\sum x^2 = 130$	$\sum xy = 113.2$

Substituting these values inn Eqs (1) and (2)

$$24 = 6a + 24b \qquad (3)$$

$$113.2 = 24a + 130b \qquad (4)$$

Solving Eqs (3) and (4)

$$a = 1.9764$$

$$b = 0.5059$$

Then the equation of straight line is $\boldsymbol{y = 1.9764 + 0.5059x}$

Example:2

Fit a straight line to the following data. Also, estimate the value of y at $x = 2.5$

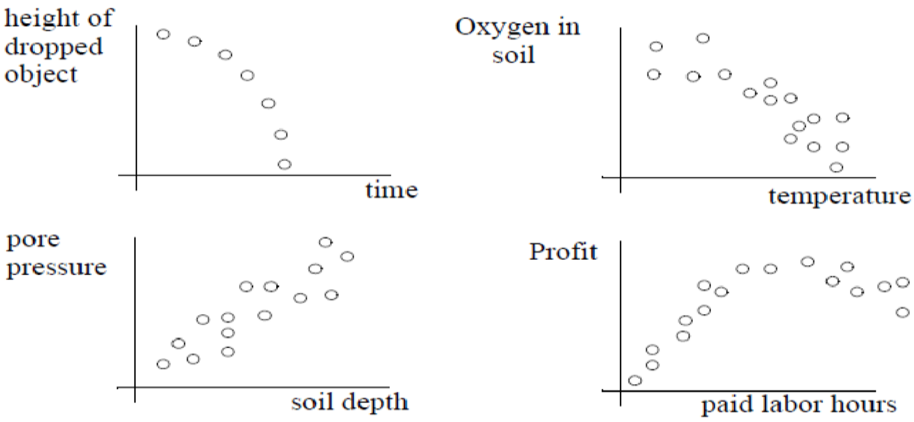
x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

Example:3

Fit a straight line using least square method

x	0	0.5	1	1.5	2	2.5
y	0	1.5	3	4.5	6	7.5

Polynomial Regression: We started the linear curve fit by choosing a generic form of the straight line $f(x) = ax + b$
This is just one kind of function. There are an infinite number of generic forms we could choose from for almost any shape we want. Let's start with a simple extension to the linear regression concept recall the examples of sampled data.



Error - Least squares approach

$$err = \sum_{i=1}^{n} d_i^{\,2} = \left(y_1 - f(x_1)\right)^2 + \left(y_2 - f(x_2)\right)^2 + \ldots\ldots.\left(y_n - f(x_n)\right)^2$$

$$= \left(y_1 - \left(a + bx_1 + cx_1^{\,2}\right)\right)^2 + \left(y_2 - \left(a + bx_2 + +cx_2^{\,2}\right)\right)^2 + \ldots\ldots.. + \left(y_n - \left(a + bx_n + cx_n\right)\right)^2$$

$$= \sum_{i=1}^{n} \left(y_i - \left(a + bx_i + cx_i^{\,2}\right)\right)^2$$

To minimize the error, derivatives with respect to a, b and c equal to 0.

$$\frac{\partial(err)}{\partial a} = \sum_{i=1}^{n} -2\left(y_i - \left(a + bx_i + cx_i^{\,2}\right)\right) = 0$$

$$\frac{\partial(err)}{\partial b} = \sum_{i=1}^{n} -2x_i\left(y_i - \left(a + bx_i + cx_i^{\,2}\right)\right) = 0$$

$$\frac{\partial(err)}{\partial b} = \sum_{i=1}^{n} -2x_i^{\,2}\left(y_i - \left(a + bx_i + cx_i^{\,2}\right)\right) = 0$$

Simplify these equations, we get

$$\sum_{i=1}^{n} y_i = an + b\sum_{i=1}^{n} x_i + c\sum_{i=1}^{n} x_i^2$$

$$\sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 + c\sum_{i=1}^{n} x_i^3$$

$$\sum_{i=1}^{n} x_i^2 y_i = a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i^3 + c\sum_{i=1}^{n} x_i^4$$

Example:1

Fit a second order polynomial equation to following data:

x_i	0	0.5	1.0	1.5	2.0	2.5
y_i	0	0.25	1.0	2.25	4.0	6.25

Solution:

x_i	y_i	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i^2 y_i$
0	0	0	0	0	0	0
0.5	0.25	0.25	0.125	0.0625	0.125	0.0625
1	1	1	1	1	1	1
1.5	2.25	2.25	3.375	5.0625	3.375	5.0625
2	4	4	8	16	8	16
2.5	6.25	6.25	15.625	39.0625	15.625	39.0625
$\sum x_i$ =7.5	$\sum y_i$ =13.75	$\sum x_i^2$ =13.75	$\sum x_i^3$ =28.125	$\sum x_i^4$ =61.1875	$\sum x_i y_i$ =28.125	$\sum x_i y_i$ =61.1875

Substitute these values in equations

$$\sum_{i=1}^{n} y_i = an + b\sum_{i=1}^{n} x_i + c\sum_{i=1}^{n} x_i^2$$

$$\sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 + c\sum_{i=1}^{n} x_i^3$$

$$\sum_{i=1}^{n} x_i^2 y_i = a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i^3 + c\sum_{i=1}^{n} x_i^4$$

Hence, $y = x^2$ is required equation which fits the data.

Curve fitting

Curve fitting - Other nonlinear fits (exponential)

Q: Will a polynomial of any order necessarily fit any set of data?

A: Nope, lots of phenomena don't follow a polynomial form. They may be, for example, exponential

(1) General exponential equation $f(x) = Ce^{Ax}$

Now, take log on both side, we get

$$\ln y = \ln C + Ax$$
$$Y = b + aX; \qquad \text{where } Y = \ln y,\ X = x, \ln C = b \text{ and } a = \ln A$$

Which is equation of line, the original data in xy- plane mapped into XY-plane. This is called *linearization.* The data (x, y) transformed as $(x, \ln y)$.

To find the value of a and b we will use the equations

$$\sum_{i=1}^{n} Y_i = a\sum_{i=1}^{n} X_i + nb \qquad (1)$$

$$\sum_{i=1}^{n} X_i Y_i = a\sum_{i=1}^{n} X_i^2 + b\sum_{i=1}^{n} X_i \qquad (2)$$

After getting values of a and b, $A = \text{antilog } a,\ C = \text{antilog } b$.

Example:1

An experiment gave the following values:

X	1	5	7	9	12
Y	10	15	12	15	21

Fit an exponential curve $y = Ce^{Ax}$

Solution:

$X_I = x_i$	y_i	$Y_i = \ln y_i$	X_i^2	$X_i Y_i$
1	10	2.302585	1	2.302585
5	15	2.70805	25	13.54025
7	12	2.484906	49	17.39435
9	15	2.70805	81	24.37245
12	21	3.044522	144	36.53427
$\sum_{i=1}^{5} X_i$ =34		$\sum_{i=1}^{5} Y_i$ =13.24811	$\sum_{i=1}^{5} X_i^2$ =300	$\sum_{i=1}^{5} X_i Y_I$ =94.1439

$$13.24811 = 34A + 5B$$
$$94.1439 = 300A + 34B$$

A=2.00479, B=2.248664

a=antilog2.00479=7.424536, b=antilog (2.248664) =9.475068

Hence, best fit curve is $y = 9.475068e^{2.248664x}$

(2) $y = bx^a$

Taking $\log_{10}$ on both the side

$$\log_{10} y = \log_{10} b + a\log_{10} x$$
$$Y = B + AX; \qquad \text{where } Y = \log_{10} y,\ X = \log_{10} x \text{ and } a = A, B = \log_{10} b$$

$$\sum_{i=1}^{n} Y_i = nB + A\sum_{i=1}^{n} X_i \qquad (1)$$

$$\sum_{i=1}^{n} X_i Y_i = B\sum_{i=1}^{n} X_i + A\sum_{i=1}^{n} X_i^2 \qquad (2)$$

Example: An experiment gave the following values:

v (ft/min)	350	400	500	600
t (min)	61	26	7	2.6

It is known that v and t are connected by the relation $v = bt^a$, find the best possible values of a and b.

v	t	$y = logv$	$X = logt$	X^2	XY
350	61	2.544068	1.78533	3.18740262	4.542001
400	26	2.60206	1.414973	2.002149575	3.681846
500	7	2.69897	0.845098	0.714190697	2.280894
600	2.6	2.778151	0.414973	0.17220288	1.152859
		$\sum_{i=1}^{4} Y_i$ =10.62325	$\sum_{i=1}^{4} X_i$ =4.460375	$\sum_{i=1}^{4} X_i^2$ =6.075945772	$\sum_{i=1}^{4} X_i^3$ =11.6576

Substitute in given equation,

$$\sum_{i=1}^{n} Y_i = nB + A\sum_{i=1}^{n} X_i \qquad (1)$$

$$\sum_{i=1}^{n} X_i Y_i = B\sum_{i=1}^{n} X_i + A\sum_{i=1}^{n} X_i^2 \qquad (2)$$

$$10.62325 = 4B \qquad + 4.460375A$$

$$11.6575 = 4.460375B + 6.075945772A$$

On solving these equations B=2.845 A=a= - 0.17.

$b = anti\log(2.845) = 699.842$

3)The following values of T and l follow the law $T = al^n$. Test if this is so and find the best values of a and n.

T	1.0	1.5	2.0	2.5
l	25	56.2	100	1.56