ABESIT Ghaziabad (290)

Course: Engg. Physics (KAS101T)

Unit - VI

wave optics

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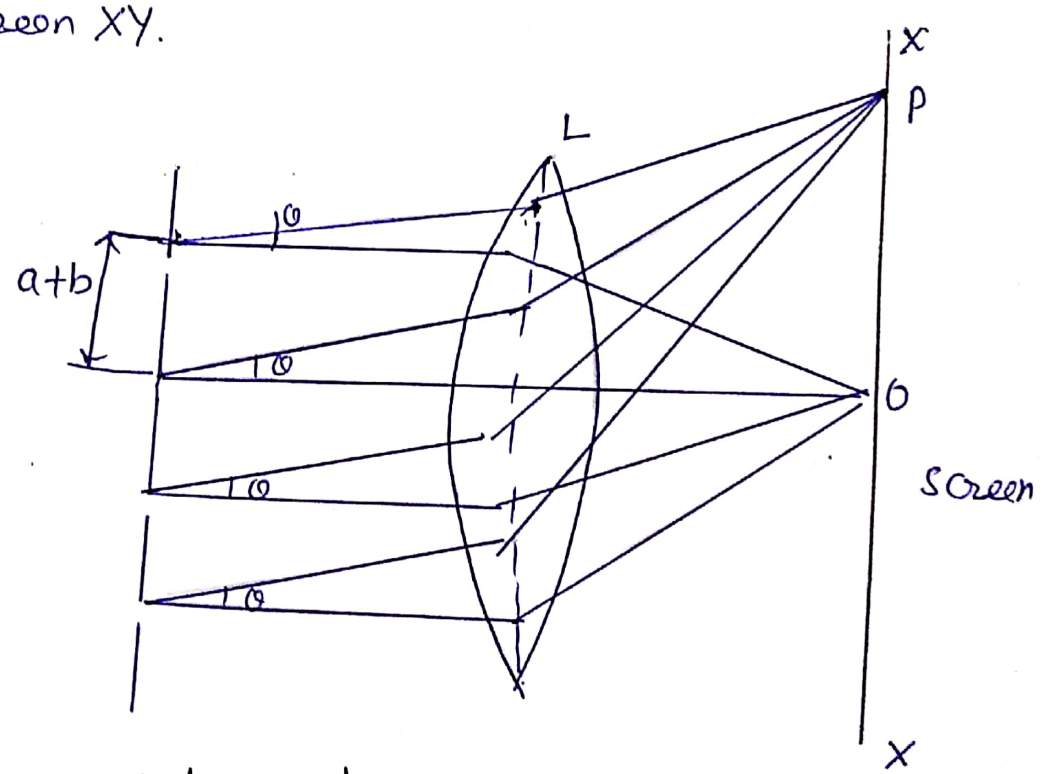
Lecture 43: Spectra with grating, Dispersive Power

Out Come: 1- Explain N slit diffraction, 2- Interpret the Condition of Maxima and Minima

3- Deduce maximum order in diffraction grating

4- Deduce the dispersive power of grating.

Spectra with Grating:- Let a parallel beam of monochromatic light of wavelength λ be incident normally on the grating. The light diffracted through N slits is focussed by a convex lens on the screen XY.



Path difference between two successive wave $= (a+b)\sin\theta$

The corresponding phase difference $= \frac{2\pi}{\lambda}(a+b)\sin\theta$

Theory:- In this Case the resultant amplitude in the direction θ is given by

$$R' = \frac{A \sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}$$

and intensity is $I = R'^2 = \frac{A^2 \sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta}$ ——— (1)

where $\beta = \frac{\pi(a+b)\sin\theta}{\lambda}$ and $\alpha = \frac{\pi a \sin\theta}{\lambda}$

Hence the intensity distribution is the product of two terms. The first term $\frac{A^2\sin^2\alpha}{\alpha^2}$ represents the diffraction pattern due to single slit. Second term $\frac{\sin^2 N\beta}{\sin^2\beta}$ represents the interference pattern due to N-slits.

(i) Position of Principal Maxima

Intensity will be maximum, if $\sin\beta = 0$

$\Rightarrow \beta = \pm n\pi$, when $n = 0, 1, 2, 3 \ldots$

Also, we have

$$\sin N\beta = 0$$

So $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ i.e indeterminate

Applying 'L' Hospital rule i.e

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$\lim_{\beta \to \pm n\pi} \frac{N\cos N\beta}{\cos\beta} = \pm N$$

$\therefore$ ~~Son~~

Therefore, the intensity of principal maxima is proportional to N

Substituting the value of $\frac{\sin N\beta}{\sin \beta} = N$ in equ (1)

$$I = \frac{A^2 \sin^2\alpha}{\alpha^2} N^2 \quad \text{——(2)}$$

These maxima are most intense and are called 'Principal maxima'.

The directions of principal Maxima are

$$\sin\beta = 0, \quad \text{i.e } \beta = \pm n\pi, \text{ where } n = 0, 1, 2 \ldots$$

or $$\frac{\pi(a+b)\sin\theta}{\lambda} = \pm n\pi$$

or $$\boxed{(a+b)\sin\theta = \pm n\lambda} \quad \text{——(3)}$$

This is known as grating equation.

· If we put $n=0$ in equ (3) we get the zero order maximum. for $n = 1, 2, 3 \ldots$ we obtain the first, second, third -- order principal maxima respectively.

Minima :- The intensity is minimum when $\sin N\beta = 0$ but $\sin\beta \neq 0$

Therefore

$$N\beta = \pm m\pi$$

or putting the value of β

$$N \cdot \frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

$$\boxed{N(a+b)\sin\theta = \pm m\lambda} \quad \text{——(4)}$$

where $m \neq 0, N, 2N \ldots nN$ because these values of m make $\sin\beta = 0$ which gives principal maxima.

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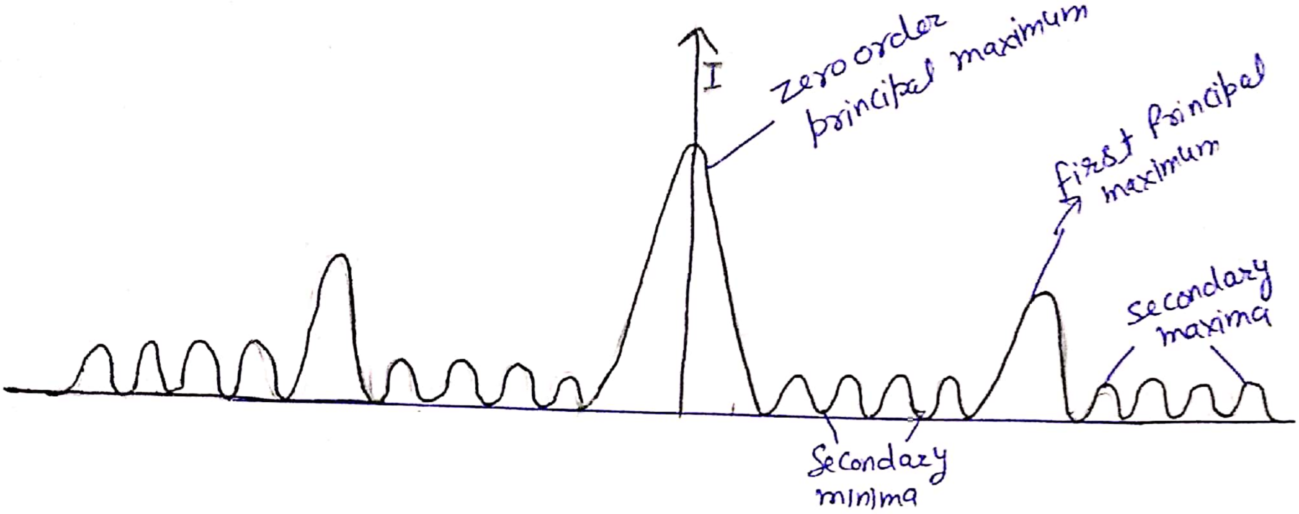
It is Clear $m = 1, 2, 3 \cdots (N-1)$ give minima and then at $m = N$, we get principal maximum of first order. Thus there are $(N-1)$ minima between two successive principal maxima.



Condition for Missing order or Absent Spectra with a Diffraction grating:-

When Condition for principal maximum of n^{th} order is simultaneously satisfied with the condition of m^{th} order minima, then n^{th} order of the principal maximum will be absent from the diffraction pattern. These are called known as absent spectra.

The Condition of principal maximum in the grating is

$$(a+b)\sin\theta = n\lambda \quad \text{——} \quad (1)$$

Condition of minima in single slit

$$a\sin\theta = m\lambda \quad \text{——} \quad (2)$$

Now dividing equ (1) by equ (2)

$$\frac{(a+b)\sin\theta}{a\sin\theta} = \frac{n}{m}$$

$$\frac{(a+b)}{a} = \frac{n}{m} \quad \text{——— (3)}$$

This is the required condition of missing order spectra in the diffraction pattern.

(i) When $b = a$, then from equ. (3)

$$\frac{2a}{a} = \frac{n}{m}$$

$$n = 2m$$

for $m = 1, 2, 3 \ldots$

$n = 2, 4, 6$

Hence 2^{nd}, 4^{th}, 6^{th} ... orders are absent will be absent from diffraction pattern

(ii) If $b = 2a$, then

$$\frac{a+2a}{a} = \frac{n}{m}$$

$$\frac{3a}{a} = \frac{n}{m}, \Rightarrow n = 3m$$

when $m = 1, 2, 3 \ldots$

then $n = 3, 6, 9 \ldots$

Hence, when width of the opacities (b) of the grating is doubled to that of transparencies (a), then 3^{rd}, 6^{th}, 9^{th} order maxima will be absent.

Determination of Grating Element (a+b)

On a grating the number of ruling /inch is given by the manufacture. If N is the number of ruling /inch then

$$N(a+b) = 1'' = 2.54\,cm \Rightarrow (a+b) = 2.54/N\ cm.$$

Maximum numbers of orders with a diffraction Grating :-

The maximum number of spectra available with a diffraction grating in the visible region can be calculated by using the grating equation for normal incidence as

$$(a+b)\sin\theta = n\lambda$$

$$n_{max} = \frac{(a+b)}{\lambda} \quad (\sin\theta = 1)$$

The maximum possible value of angle of diffraction is 90°.
Thus, the grating element determines the maximum possible order

Ex (i) If the grating element $(a+b)$ lies between λ and 2λ i.e $(a+b) < 2\lambda$, then

$$n_{max} < \frac{2\lambda}{\lambda} < 2$$

i.e for normal incidence only first order will be obtained.
Hence, if the width of a grating element is less than twice the wavelength of light, then only first order is possible.

(ii) If $(a+b)$ is in between 2λ and 3λ i.e $(a+b) < 3\lambda$

$$n_{max} < \frac{3\lambda}{\lambda} < 3$$

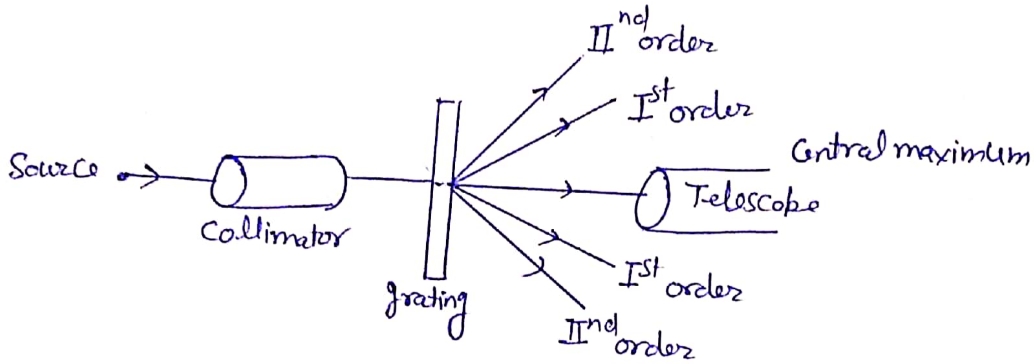
Hence, for normal incidence, when $(a+b) < 3\lambda$ only two orders are available.
Hence if the width of the grating element is less than that thrice the wavelength of light, then only two order are possible.

Ques. Show that only first order spectra is possible if the width of the grating element is less than twice the wavelength of light.

Determination of Wavelength using a plane transmission grating:-

One of main application of diffraction grating is the measurement of wavelength of an unknown source of light. The relation used in a diffraction grating is for the principal maxima obtained in the direction θ given by

$$(a+b)\sin\theta = n\lambda$$



Diffraction grating can be used in the laboratory to determine the wavelength of a monochromatic source.

The experimental procedure is as follows.

(1) The spectrometer (consisting of the collimator and telescope) is adjusted for parallel light which is the condition required for Fraunhofer diffraction

(2) The diffraction grating is mounted on a prism table between the collimator and the telescope and adjusted for normal incidence.

(3) The position of telescope is adjusted so that the cross wire of its eyepiece coincides with the central maximum and the corresponding reading on the vernier scale is noted.

(4) The telescope is moved to one side of the Central maximum and reading are recorded for the first and second order principal maxima.

(5) The telescope is now moved to the other side of the Central maximum and reading are recorded for the first and second order principal maxima.

(6) The angles of the first and second order maxima from the Central maxima are calculated.

(7) The grating element is calculated using the equation

$$(a+b) = \frac{1}{\text{No of lines per unit Length}} = \frac{2.54}{N} \text{ cm}$$

(8) The wavelength is determined from first order using

$$(a+b)\sin\theta = \lambda$$

and from the second order using

$$(a+b)\sin\theta = 2\lambda$$

(9) The mean wavelength is then determined which is the wavelength of the unknown monochromatic sources.

Ques. What do you understand by missing order spectrum?

Ques. Give the theory of plane transmission grating and show how would you use it to determine the wavelength of light.

Ques What is diffraction grating? Derive an expression for dispersive power of grating and explain it.

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Dispersive Power:- The dispersive power of a diffraction grating is defined as the rate of change of the angle of diffraction with the change of the wavelength of light used.

If the wavelength of light is suppose changed from λ to $\lambda+d\lambda$ then if angle of diffraction is changed from θ to $\theta+d\theta$ then $\frac{d\theta}{d\lambda}$ represents the dispersive power.

The direction of principal maxima (grating equation) is given by

$$(a+b)\sin\theta = n\lambda \quad \text{--- (1)}$$

Differentiating equation (1)

$$(a+b)\cos\theta \, d\theta = n \, d\lambda$$

$$\boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}} \Rightarrow \text{dispersive power}$$

Now

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\sqrt{1-\sin^2\theta}}$$

from equation (1) $\sin\theta = \frac{n\lambda}{(a+b)}$

Then

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\sqrt{1-\left(\frac{n\lambda}{a+b}\right)^2}}$$

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{\frac{(a+b)^2}{n^2} - \lambda^2}}$$

Conclusion:-

① The dispersive power is directly proportional to n i.e as the number of order increases, the dispersive power of grating increases.

(2) The dispersive power is inversely proportional to (a+b), i.e grating element, so, smaller is the grating element, higher the dispersive power.

(3) The dispersive power is inversely proportional to cos θ, i.e for large values of θ, the dispersive power increases.

Ques. What do you understand by missing orders spectrum? What particular spectra would be absent if the width of transparencies and opacities of the grating are equal.

Ques. Give the theory of plane transmission grating and show how would you use it to determine the wavelength of light.

Ques. Give the construction and theory of plane transmission grating and explain the formation of spectra by it. Explain what are absent spectra in the grating.