COURSE :- MATHEMATICS-I

COURSE CODE :- KAS-103T

MODULE-1 :- MATRICES

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LECTURE-1 & 2

Course: Mathematics-I | **Course Code: KAS-103T**

Module-1: Matrices | **Teacher: Dr. S. P. Gupta**

Topic: Basics of Matrix Algebra and Types of Matrices.

LO: ***Define*** Matrix, Applications of Matrices and Different types of matrices.

Matrix :- Let us Consider a set of simultaneous eq.

$$x + 2y + 3z + 5t = 0$$
$$4x + 2y + 5z + 7t = 0$$
$$3x + 4y + 2z + 6t = 0$$

Now we write down the Coffn. of x, y, z, t of the above eqn and enclose them with in brackets and then we get

$$A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 4 & 2 & 5 & 7 \\ 3 & 4 & 2 & 6 \end{bmatrix}$$

The above system of number, arranged in a rectangular array in rows and Columns and bounded by the brackets is Called a Matrix.

Various Types of Matrices :-

① Row Matrix :- If a Matrix has only one row and any number of Columns. eg - $\begin{bmatrix} 2 & 7 & 3 & 9 \end{bmatrix}$

② Column Matrix :- A Matrix having one Column and any number of rows. eg - $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

③ Null or Zero Matrix :- Any Matrix in which all the elements are zero. eg - $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

④ Square Matrix :- A matrix in which the number of rows is equal to the number of Column.

eg - $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$

⑤ Diagonal Matrix :- A square matrix is called a diagonal if all its non-diagonal elements are zero.

eg - $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(6) Unit or identity Matrix :- A square Matrix is said to be unit if all the principle diagonal elements are unity and non-daigonal elements are zero.

eg - $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(7) Symmetric Matrix :- A square Matrix is said to be symmetric if

$$\boxed{A' = A}$$

(8) Skew-Symmetric Matrix :- A square Matrix is said to be skew-symmetric if $\boxed{A' = -A}$

(9) Upper-Triangular Matrix :- A square matrix, all of whose element below the leading diagonal are zero is called an upper-triangular matrix.

eg - $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

(10) Lower-Triangular Matrix :- A square Matrix, all of whose element above the leading diagonal are zero is called an lower-triangular Matrix.

eg - $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

(11) Transpose of a Matrix :- If in a given matrix, we interchange the rows and the corresponding columns, the new matrix obtained is called the transpose of the matrix.
It is denoted by A' or A^T.

eg- $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 5 \\ 6 & 7 & 8 \end{bmatrix}$, $A' = \begin{bmatrix} 2 & 1 & 6 \\ 3 & 0 & 7 \\ 4 & 5 & 8 \end{bmatrix}$

(12) Orthogonal Matrix :- A square Matrix A is called an orthogonal, if the product of A and its transpose A' is an identity matrix. $\boxed{A \cdot A' = I}$

(13) Conjugate of a Matrix :- let $A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix}$

Conjugate of Matrix A is $\bar{A}$

$$\therefore \quad \bar{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & i & 3+2i \end{bmatrix}$$

(14) Matrix A^{θ} :- Transpose of the Conjugate of a Matrix A is denoted by A^{θ}.

let $A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix} \Rightarrow \bar{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & i & 3+2i \end{bmatrix}$

$$(\bar{A})' = \begin{bmatrix} 1-i & 7-2i \\ 2+3i & i \\ 4 & 3+2i \end{bmatrix} = A^{\theta}$$

(15) Unitary Matrix :- A square matrix A is said to be unitary if $\boxed{A^{\theta}A = I}$

(16) Hermitian Matrix :- A square matrix A is said to be Hermitian if $\boxed{A = A^{\theta}}$

(17) Skew-Hermitian Matrix :- A square matrix A is said to be skew-Hermitian if $\boxed{A = -A^{\theta}}$

(18) Equal Matrix :- Two Matrices are said to be equal, if

(i) They are of the same order.

(ii) The elements in the Corresponding positions are equal.

eg - $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

Here $A = B$.

Addition of Matrices :- If $A = a_{ij}$ and $B = b_{ij}$ of the same order, then addition is possible

$$A+B = [a_{ij} + b_{ij}]$$

Subtraction of Matrices :- If $A = a_{ij}$ and $B = b_{ij}$ of the same order then subtraction is possible.

$$\therefore \quad A - B = [a_{ij} - b_{ij}]$$

Matrix Multiplication :- The product of two matrices A and B is only possible if the number of Columns in A is equal to the number of Rows in B.

let $A = [a_{ij}]$ be an $m \times n$

$B = [b_{ij}]$ be an $n \times p$.

Then the product AB of these matrices is an $m \times p$ order matrix $C = [c_{ij}]$

$$c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + a_{i3} b_{3j} + \cdots + a_{in} b_{nj}$$

Inverse of the Matrix :- $A^{-1} = \dfrac{\text{adj of } A}{|A|}$