

Lecture 01: Absorption of radiation, Spontaneous and Stimulated emission of radiation, Einstein coefficients and Population inversion

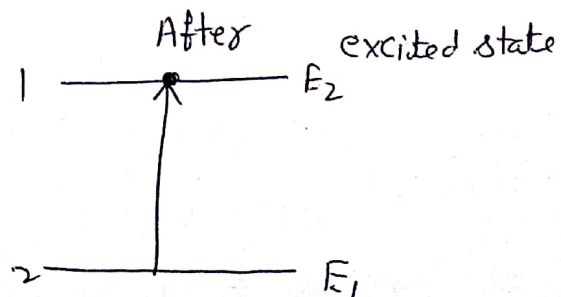
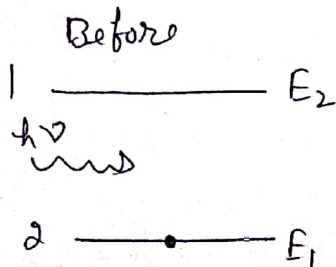
- Outcome: 1. Explain absorption of radiation, spontaneous and stimulated emission of radiation  
2. Compute relation between Einstein's coefficients  
3. Define population inversion

Laser :-

The word laser is an acronym for light amplification by stimulated emission of Radiation.

It is a device to produce a strong, monochromatic, collimated and highly coherent beam of light.

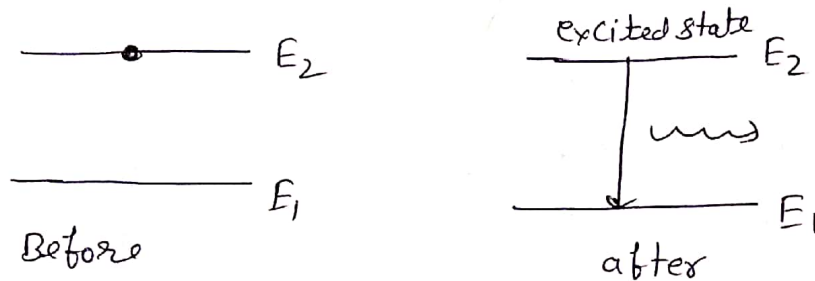
Absorption of light:- Let us consider two energy levels 1 and 2 of an atom with energies  $E_1$  and  $E_2$ .



When the photon of light having energy  $h\nu = (E_2 - E_1)$  is incident on an atom in the lower energy state  $E_1$ . Then the atom

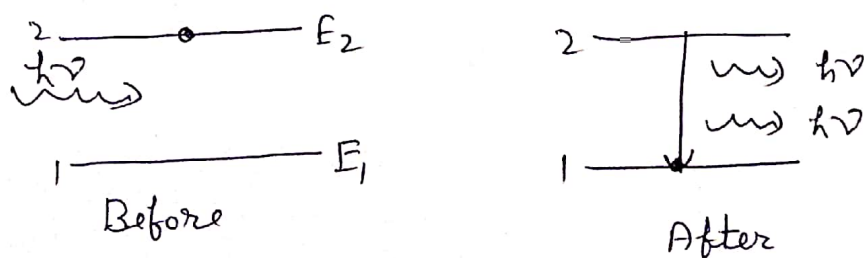
In the lower state  $E_1$  may absorb the photon and jump to higher energy state  $E_2$ . This process is called absorption of radiation.

### Spontaneous emission :



Let us now consider that the atom is in higher (excited) energy state  $E_2$ . The excited state with higher energy is not a stable state. Hence atom in excited state does not stay for longer time and it jumps to the lower energy state by emitting a photon of energy  $h\nu = (E_2 - E_1)$ . This is known as spontaneous emission.

### Stimulated emission of Radiation :



Suppose the atom is in excited energy state  $E_2$  and a photon of energy  $h\nu = E_2 - E_1$  is incident on it.

The incident photon interacts with the atom and then it induces the atom to come down to the ground energy state  $E_1$  by emitting a new photon.

Thus, when an atom ejects a photon due to its interaction with a photon incident on it, the process is called stimulated emission.

The emitted photon has exactly the same phase, direction and energy as the incident photon.

### Characteristics:

- ① For each incident photon, there are two outgoing photons moving in the same direction.
- ② The direction of emitted photon is same as the direction of incident photon, since the emitted photon has exactly the same energy, phase and direction as the incident photon, we can achieve an amplified and coherent beam.

### Einstein's coefficients:

Let us consider a system of atoms having a ground state energy  $E_1$  and excited state energy  $E_2$  with number densities of atoms in these system  $N_1$  and  $N_2$  respectively.

1. The probability of absorption that the number of atoms in state 1 that absorb a photon and rise to state 2 is proportional to Rate

Rate of absorption  $\propto N_1 u(\nu)$

$N_1 \rightarrow$  number of atoms in ground state 1

$u(\nu) \rightarrow$  energy density of incident photon.



Therefore

$$R_{12} = B_{12} N_1 u(\nu)$$

where  $B_{12}$  is a proportionality constant and is known as Einstein's coefficient of absorption.

2. Atoms in excited state  $E_2$  can come down to ground state either through spontaneous emission or through stimulated emission of radiation. In the case of spontaneous emission, the rate of transition of atoms from  $E_2$  to  $E_1$  does not depend on the energy density in the incident radiation and is proportional only to the number density of atoms in the excited state  $E_2$ .

Therefore, Rate of spontaneous emission  $\propto N_2$

$$(R_{21}) \propto N_2$$

$$R_{21} = A_{21} N_2$$

$A_{21} \rightarrow$  Proportionality constant and is known as Einstein's coefficient of spontaneous emission.

- ③ Consider the case of stimulated emission. The rate of stimulated emission  $(R_{21})_{st}$  is proportional to the energy density  $u(\nu)$  and number density  $N_2$ .

$$(R_{21}) \propto N_2 u(\nu)$$

$$R_{21} = B_{21} N_2 u(\nu)$$

where  $B_{21} \rightarrow$  Einstein's coefficient of stimulated radiation of emission

In a state of thermal equilibrium, the rate of transition from  $E_1$  to  $E_2$  must equal the total rate of transition from  $E_2$  to  $E_1$

Rate of absorption = Rate of emission

$$R_{12} = (R_{21})_{sp} + (R_{21})_{st}$$

$$B_{12} N_1 u(\nu) = A_{21} N_2 + B_{21} N_2 u(\nu)$$

$$(N_1 B_{12} - N_2 B_{21}) u(\nu) = N_2 A_{21}$$

dividing by  $N_1$

$$u(\nu) \left[ B_{12} - \frac{N_2}{N_1} B_{21} \right] = A_{21} \frac{N_2}{N_1}$$

$$u(\nu) = \frac{A_{21} \left( \frac{N_2}{N_1} \right)}{B_{12} - B_{21} \left( \frac{N_2}{N_1} \right)}$$

$$u(\nu) = \frac{A_{21}}{B_{12} \left( \frac{N_1}{N_2} \right) - B_{21}}$$

$$\therefore u(\nu) = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\left( \frac{B_{12}}{B_{21}} \right) \left( \frac{N_1}{N_2} \right) - 1} \right]$$

from Boltzmann distribution law

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/KT}$$

$$\frac{N_2}{N_1} = e^{-(h\nu/KT)}$$

then  $\therefore u(\nu) = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\left( \frac{B_{21}}{B_{12}} \right) e^{\frac{h\nu}{KT}} - 1} \right] \quad \text{--- (1)}$

The Planck radiation formula for energy distribution

$$u(\nu) = \frac{8\pi h\nu^3}{c^2} \left[ \frac{1}{e^{\frac{h\nu}{kT}} - 1} \right] \quad \text{--- (2)}$$

Comparing equ. (1) and (2), we get

$$\frac{B_{12}}{B_{21}} = 1$$

$$B_{12} = B_{21} \quad \text{--- (3)}$$

$$\text{and } \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

It is clear that the ratio between spontaneous and stimulated coefficient is proportional to  $\nu^3$ . It means that at thermal equilibrium the probability of spontaneous emission increases with the energy difference between two states.

### Population Inversion :-

In thermal equilibrium condition the many number of atoms are in ground state than in excited state. This is called equilibrium or normal state.

If a photon having energy  $E_2 - E_1$  is incident on such system the probability is more than the probability of stimulated emission.

Let  $N_1$  be the number of atoms in ground state and  $N_2$  to be the number of atoms in excited state. Then if  $N_1 > N_2$ , shows the normal condition

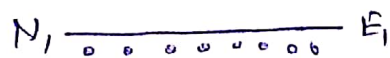
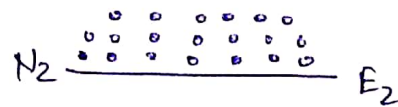
$$N_2 \text{ --- } E_2$$

$$N_1 \text{ --- } E_1$$

Normal state,  $N_1 > N_2$



To increase the probability of stimulated emission, the number of atoms in the higher energy state must be made greater than the number of atoms in the lower energy state. If the situation is such that more atoms are in higher energy state than the lower state, then stimulated emission will take place easily. Such condition is called population inversion.



After population inversion  $N_2 > N_1$

To achieve a state of population inversion, the atoms in the lower state have to be pumped to the higher energy state by providing energy. The process of raising the atoms from a lower energy state to higher, to create population inversion, is called pumping.

Ques:- Differentiate between spontaneous and stimulated emission of radiation. Which one is required for laser?

Ques Why population inversion is necessary for laser action.

Ques. What are Einstein's coefficients? Obtain a relation between them. Also discuss the essential conditions for laser action.

Ques: Define population inversion and pumping.

Ques: What are Einstein's coefficients? Derive Einstein's relation.