LECTURE-47

Course: Mathematics-I **Course Code: KAS-103T**

Module-5: Vector Calculus **Teacher: Dr. S. P. Gupta**

Topic: Vector Integration: Volume integral

LO: ***Remember*** the concept of Volume integral and ***apply*** for ***Evaluating*** Volume Integral of vector function.

Any integral which is to be evaluated over a volume is called volume integration.

If V is a volume bounded by surface S, then

$\iiint_V \vec{F}\, dv$ are called or $\iiint_V \phi\, dv$ are also called Volume integral of vector and scalar function.

i.e. $$\iiint_V \phi\, dv = \iiint_V \phi(x,y,z)\, dx\,dy\,dz.$$

and $$\iiint_V \vec{F}\, dv = \hat{i}\iiint_V f_1\, dx\,dy\,dz + \hat{j}\iiint_V f_2\, dx\,dy\,dz + \hat{k}\iiint_V f_3\, dx\,dy\,dz.$$

Example :- ① If $\vec{F} = (2x^2-3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, evaluate $\iiint \nabla\cdot\vec{F}\, dv$, where V is bounded by $x=0, y=0, z=0$ and $2x+2y+z=4$.

Sol^n:- $$\nabla\cdot\vec{F} = \frac{\partial}{\partial x}(2x^2-3z) + \frac{\partial}{\partial y}(-2xy) + \frac{\partial}{\partial z}(-4x)$$

$$= 4x - 2x = \underline{\underline{2x}}$$

The limits are

$z=0$ to $z = 4-2x-2y$.

$y=0$ to $y = 2-x$.

$x=0$ to $x = \underline{2}$

$$\therefore \iiint_V \nabla\cdot\vec{F}\,dv = \int_{x=0}^{2}\int_{y=0}^{2-x}\int_{z=0}^{4-2x-2y} 2x\,dz\,dy\,dx.$$

$$= \int_0^2\int_0^{2-x} 2x\,\Big[z\Big]_0^{4-2x-2y} dy\,dx.$$

$$= \int_0^2\int_0^{2-x} 2x\,(4-2x-2y)\,dy\,dx.$$

$$= \int_0^2\int_0^{2-x} (8x-4x^2-4xy)\,dy\,dx.$$

$$= \int_0^2 \Big[8xy-4x^2y-4x\frac{y^2}{2}\Big]_0^{2-x} dx.$$

$$= \int_0^2 \Big\{8x(2-x)-4x^2(2-x)-2x(2-x)^2\Big\}\,dx.$$

$$= \int_0^2 \Big\{16x-8x^2-8x^2+4x^3-2x(4+x^2-4x)\Big\}\,dx.$$

$$= \int_0^2 (16x-16x^2+4x^3-8x-2x^3+8x^2)\,dx$$

$$= \int_0^2 (8x-8x^2+2x^3)\,dx.$$

$$= \Big[8\frac{x^2}{2}-8\frac{x^3}{3}+2\frac{x^4}{4}\Big]_0^2$$

$$= \frac{\cancel{32}^{16}}{\cancel{2}} - \frac{64}{3} + \frac{\cancel{32}^{8}}{\cancel{4}} = 24-\frac{64}{3} = \frac{72-64}{3}$$

$$= \frac{8}{3} \text{ Ans.}$$

Excercise :- ① $\iiint_V \phi\, dv$; where $\phi = 45x^2y$ and V is the closed region bounded by the planes $4x+2y+z=8$, $x\geq0, y=0, z\geq0$.

② If $\vec{F} = (2x^2-3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, then evaluate $\iiint_V \nabla\times\vec{F}\, dv$; where V is the closed region bounded by the planes $x\geq0$, $y\geq0$, $z\geq0$, and $2x+2y+z=4$.

③ Evaluate $\iiint_V (2x+y)\, dv$, where V is closed region bounded by the cylinder $z=4-x^2$ and the planes $x\geq0, y\geq0, y=2$, and $z\geq0$.