Engineering Mathematics - II
RAS- 203

Lecture NO. - 1 (Introduction)

Differential Equation :- A differential equation is an equation involving differentials or differential coefficients.

eg. - ① $\frac{dy}{dx} = \cot x$, ② $\frac{d^2y}{dx^2} + y = 0$, ③ $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$,

④ $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$, ⑤ $\frac{d^2y}{dx^2} + \sqrt{1+\left(\frac{dy}{dx}\right)^3} = 0$

⑥ $$\rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$$

all are differential equations.

Ordinary differential equation :- A differential equation which involves only one independent variable is called "ordinary differential equation."

Partial differential equation :- A differential equation which involves two or more independent variables and partial derivatives with respect to them is called Partial differential equation.

Order of a differential equations :- The order of a differential equation is the order of the highest ordered derivative occurring in the differential equation.

eg - ① have order - 1.

② have order - 2.

③ have order - 1.

④ have order - 1.

⑤ have order - 2.

⑥ have order - 2.

Degree of a differential equation :- The degree of a differential equation is the degree of the highest ordered derivative present in the differential equation when it is made free from radical signs and fractional powers.

eg - ① have degree - 01.

② have degree - 01.

③ have degree - 03.

④ have degree - 01.

⑤ have degree - 02.

⑥ have degree - 02.

Solution of differential equation :- A Solution (Integral) of a differential equation is a relation free from derivatives, between the variables which satisfy the given equation is Called the solution of differential equation.

eg- $y = C_1 \cos x + C_2 \sin x$ is the solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$.

General Solution :- The general solution of a differential eqn is the solution in which the number of arbitrary constant is equal to the order of the given equation.

Thus $y = C_1 \cos x + C_2 \sin x$ have two arbitrary constant C_1 & C_2 is the general solution of the differential eqn $\frac{d^2y}{dx^2} + y = 0$ of second order.

Particular Solution :- A particular solution of a differential equation is the solution which is obtained from its general solution by giving particular values to the arbitrary constants.

The Operator D :-

$$\frac{d}{dx} \equiv D, \quad \frac{d^2}{dx^2} \equiv D^2, \quad \frac{d^3}{dx^3} \equiv D^3, \quad \ldots\ldots\ldots \frac{d^n}{dx^n} \equiv D^n$$

Thus the symbol D is a differential operator.

eg -
$$\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - y = 0$$

$$(D^4 + 3D^3 + 4D^2 + 2D - 1)\, y = 0$$

Linear differential equation with Constant Coefficient :-

An equation is of the form

$$a_0 \frac{d^ny}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}y}{dx^{n-2}} + \cdots\cdots + a_n y = Q$$

where $a_0, a_1, a_2, \ldots\ldots a_n$ are all constants and Q is a function of x alone is called a linear differential equation of n^{th} order with constant coefficient.

Homogeneous linear differential equation with Constant Coefficient (Euler - Cauchy Equation)

An equation is of the form

$$x^n \frac{d^ny}{dx^n} + a_1 x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2}y}{dx^{n-2}} + \cdots\cdots + a_{n-1} x\frac{dy}{dx} + a_n y = Q$$

where $a_1, a_2, \ldots\ldots a_n$ are all constants and Q is a function of x - alone is called Homogeneous linear differential equation with constant coefficient.

Linear differential equation of Second order :- A differential equation is of the form $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ is known as linear diff^n equation of second order, where P, Q and R are the function of x alone.

Complementary Function (C.F.) :- Consider the differential equation

$$f(D)\,y = Q$$

Complementary function is actually the solution of the given differential eq^n, when its right hand side member ie Q is replaced by zero. To find CF, we first find Auxiliary equation (A.E.).

The Inverse operator $\frac{1}{f(D)}$:- $\frac{1}{f(D)} Q$ is that function of x, free from arbitrary constants, which when operated upon by $f(D)$ gives Q.

NOTE- $\frac{1}{D} Q = \int Q\,dx$.

Particular Integral :- Consider the diff^n eq^n $f(D)\,y = Q$, then.

$$\boxed{PI = \frac{1}{f(D)} Q}$$

Complete Solution :- $\boxed{y = CF + PI}$