ABESIT Ghaziabad (290)

Course: Engg. Physics (KAS201T)

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Unit - III

Quantum Mechanics

Lecture 34: Compton effect

Outcome: Explain compton effect

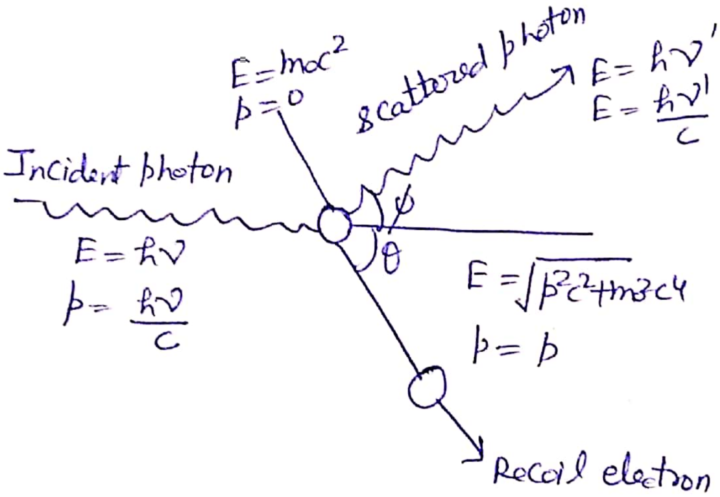
Compton Effect :-

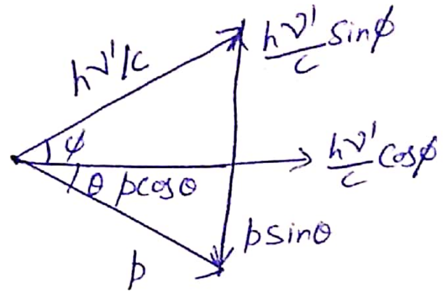
When an X-ray photons strike an electron and is scattered away from its original direction, then the scattered radiation contain photons not only of the same frequency as that of incident photon, but also the photons of lower frequency. The radiation of unchanged frequency is known as un-modified radiation, while the radiation of lower frequency is called as modified radiation. Thus, the phenomenon of scattering with change in frequency is called the Compton effect.

$$\text{Compton shift} = \frac{h}{m_0 c}(1-\cos\phi)$$

$$\boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\phi)}$$

Derivation :- The expression for change in wavelength is derived by applying the law of conservation of energy and momentum between the incident photon and electron.





Consider, a photon of energy $h\nu$ and momentum $\frac{h\nu}{c}$ incident on a target rich electron.

Let the electron be at rest. On collision some of the energy of the photon is transferred to the electron.

Let the scattered photon is emitted at angle ϕ and the electron recoils at angle θ.

Energy of photon before collision $E = h\nu$

Momentum of photon before collision $p = \frac{h\nu}{c}$

Energy of electron before collision $E = m_0c^2$

Momentum of electron before collision $P = 0$

Since photon is scattered in direction ϕ, its component are:

Momentum of photon in X-direction $= \frac{h\nu'}{c}\cos\phi$

Momentum of photon in Y-direction $= \frac{h\nu'}{c}\sin\phi$

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Now, momentum of electron after collision

Along x- direction $= p\cos\theta$

Along y- direction $= p\sin\theta$

According to principle of conservation of momentum

Momentum before collision = Momentum after collision

Along x- axis

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta \quad \text{---} (1)$$

and perpendicular to this direction

$$0 + 0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta \quad \text{---} (2)$$

equ. (1) $\times c$ and Rearranging

$$pc\cos\theta = h\nu - h\nu'\cos\phi \quad \text{---} (3)$$

equ(2) $\times c$ and Rearranging

$$pc\sin\theta = h\nu'\sin\phi \quad \text{---} (4)$$

Squaring equ. (3) and (4) and adding.

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2 \quad \text{---} (5)$$

According to principle of conservation of energy.

Energy before collision = energy after collision

$$h\nu + m_0c^2 = h\nu' + \sqrt{m_0^2c^4 + p^2c^2}$$

$$h\nu - h\nu' + m_0c^2 = \sqrt{m_0^2c^4 + p^2c^2}$$

Squaring both sides

$$\left[h(\nu-\nu')+m_oc^2\right]^2 = m_o^2c^4+p^2c^2$$

$$h^2(\nu-\nu')^2 + m_o^2c^4 + 2h(\nu-\nu')m_oc^2 = m_o^2c^4+p^2c^2$$

$$p^2c^2 = h^2(\nu-\nu')^2 + 2h(\nu-\nu')\,m_oc^2$$

$$p^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2(h\nu)(h\nu') + 2h\nu\, m_oc^2 - 2h\nu'\, m_oc^2 \quad \text{——— (6)}$$

Comparing equ. (5) and (6)

$$h^2\nu^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2 = h^2\nu^2 + h^2\nu'^2 - 2(h\nu)(h\nu') + 2h\nu\, m_oc^2 - 2h\nu'\, m_oc^2$$

$$(h\nu)(h\nu')\cos\phi = (h\nu)(h\nu') - m_oc^2h\nu + m_oc^2h\nu'$$

Dividing both sides by h^2c^2

$$\frac{\nu\nu'}{c^2}\cos\phi = \frac{\nu\nu'}{c^2} - \frac{m_o\nu}{h} + \frac{m_o\nu'}{h}$$

$$\frac{m_o}{h}(\nu-\nu') = \frac{\nu\nu'}{c^2} - \frac{\nu\nu'}{c^2}\cos\phi \quad \text{——— (7)}$$

$$\frac{\nu}{c} = \frac{1}{\lambda} \text{ and } \frac{\nu'}{c} = \frac{1}{\lambda'} \qquad \left(\nu = \frac{c}{\lambda}\right)$$

$$\frac{m_o}{h}\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right) = \frac{1}{\lambda\lambda'} - \frac{1}{\lambda\lambda'}\cos\phi$$

$$\frac{m_oc}{h}\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = \frac{1}{\lambda\lambda'}(1-\cos\phi)$$

$$\lambda'-\lambda = \frac{h}{m_oc}(1-\cos\phi)$$

$$\boxed{\Delta\lambda = \frac{h}{m_oc}(1-\cos\phi)} \quad \text{——— (8)}$$

$\Rightarrow$ This is expression for Compton shift.

It constitutes very strong evidence in support of the quantum theory of radiation. The Compton shift $\Delta\lambda$ depends only on the angle of scattering and is independent of the wavelength of incident photon.

Conclusions :-

(1) The wavelength λ' of scattered photon is greater than the wavelength of λ of the incident photon.

(2) If $\phi = 0$

$\therefore \Delta\lambda = \lambda' - \lambda = 0$ or $\lambda' = \lambda$ i.e. Compton shift is zero it means that no scattering occurs along the direction of radiation.

(3) When $\phi = \frac{\pi}{2}$

$$\Delta\lambda = \frac{h}{m_0 c} = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}$$

$\Delta\lambda = 0.0242\ A^\circ$. This is Compton wavelength.

(4) when $\phi = \pi$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos 180)$$

$$\Delta\lambda = \frac{2h}{m_0 c} = 2\times 0.242 = 0.04852\ \mathring{A} \simeq 0.05 A^\circ$$

Hence, as angle of scattering ϕ varies from 0 to 180°, the wavelength of scattered photon varies from λ to $\lambda + \frac{2h}{m_0 c}$

Thus the maximum change possible is about $0.05\mathring{A}$.

Therefore, from equ. (8) it follows that the Compton effect can most readily detected for radiations of wavelength not greater than a few angstrom unit. for example, for $\lambda = 5\mathring{A}$, the maximum change in wavelength, $(\Delta\lambda)_{max} (0.05\mathring{A})$ is 1% of wavelength λ while for $\lambda = 1 A^\circ$ it is 5%. for visible light. whose wavelength is about (λ_{mean}) 5000 A°, $(\Delta\lambda)_{max}$ is only about .001% of the incident wavelength which is undetectable. [Hence, Compton effect cannot be detected for visible light rays]

Energy of Recoil electron in Compton effect :-

Since the incident photon gives up a part of its kinetic energy to the striking electron and then gets scattered, the kinetic energy of the recoiled electron is the difference between the energies of incident and scattered photon i.e

Kinetic energy of electron $= h\nu - h\nu'$

$$= h\nu\left(1 - \frac{\nu'}{\nu}\right)$$

$$= h\nu\left(1 - \frac{\lambda}{\lambda'}\right) \qquad \left(\because \nu = \frac{c}{\lambda}\right)$$

$$= h\nu\left(\frac{\lambda' - \lambda}{\lambda'}\right) = h\nu\left(\frac{\Delta\lambda}{\lambda'}\right)$$

$$= h\nu\left(\frac{\Delta\lambda}{\lambda + \Delta\lambda}\right) \quad \text{——— (1)}$$

As we know

Compton shift $\Delta\lambda = \lambda' - \lambda$

$$= \frac{h}{m_0 c}(1 - \cos\phi) \quad \text{——— (2)}$$

using equation (2) in equ. (1)

Kinetic energy of electron

$$E = \frac{h\nu \cdot \frac{h}{m_0 c}(1 - \cos\phi)}{\lambda + \frac{h}{m_0 c}(1 - \cos\phi)}$$

Dividing λ on both Numerator and denominator

$$E = \frac{h\nu \cdot \frac{h}{m_0 c\lambda}(1 - \cos\phi)}{1 + \frac{h}{m_0 c\lambda}(1 - \cos\phi)}$$

$$E = \frac{\frac{h\nu^2}{m_0c^2}(1-\cos\phi)}{1+\frac{h\nu}{m_0c^2}(1-\cos\phi)} \qquad (\because c = \nu\lambda)$$

Case – I   if $\phi = 0$, $E = 0$

No recoil electron

(ii) If $\phi = \frac{\pi}{2}$

$$E = \frac{h\nu}{(1+a)}a \qquad \left(\text{where } a = \frac{h\nu}{m_0c^2}\right)$$

(iii) If $\phi = \pi$

$$E = \frac{2h\nu a}{1+2a} \qquad \left(\because a = \frac{h\nu}{m_0c^2}\right)$$

This is the required expression for maximum kinetic energy of recoil electron.

Ques. What is Compton effect? Derive a suitable expression for Compton shift $\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\phi)$

Ques. What is Compton effect? How does it support the photon nature of light.

Ques. Derive an expression for Compton shift. What is Compton wavelength? Explain why Compton shift is not observed with visible light.

Direction of Recoiled Compton Electron :-

To find the direction of recoiled Compton electron, divide equation (4) by equ. (3)

$$\tan\theta = \frac{\nu'\sin\phi}{\nu-\nu'\cos\phi} \quad \text{——— (9)}$$

$$\tan\theta = \frac{\frac{c}{\lambda'}\sin\phi}{\frac{c}{\lambda}-\frac{c}{\lambda'}\cos\phi} \quad \text{or} \quad \tan\theta = \frac{\lambda\sin\phi}{\lambda'-\lambda\cos\phi}$$

for finding the value of ν' rearranging equation (7)

$$\frac{1}{\nu'} = \frac{1}{\nu}+\frac{h}{m_0c^2}(1-\cos\phi)$$

$$\frac{1}{\nu'} = \frac{1}{\nu}+\frac{2h}{m_0c^2}\sin^2\frac{\phi}{2}$$

$$\frac{1}{\nu'} = \frac{1+(2h\nu/m_0c^2)\sin^2\frac{\phi}{2}}{\nu}$$

$$\nu' = \frac{\nu}{1+\left(\frac{h\nu}{m_0c^2}\right)2\sin^2\frac{\phi}{2}} = \frac{\nu}{1+2\alpha\sin^2\frac{\phi}{2}}, \quad \text{where } \alpha=\frac{h\nu}{m_0c^2} \quad \text{——— (10)}$$

Substituting this value of ν' in equ (9)

$$\tan\theta = \frac{\nu\sin\phi/\left[1+2\alpha\sin^2\frac{\phi}{2}\right]}{\nu-\left(\frac{\nu}{1+2\alpha\sin^2\frac{\phi}{2}}\right)\cos\phi}$$

$$\tan\theta = \frac{\nu\sin\phi}{\nu\left(1+2\alpha\sin^2\frac{\phi}{2}\right)-\nu\cos\phi} = \frac{\sin\phi}{(1-\cos\phi)+2\alpha\sin^2\frac{\phi}{2}}$$

$$\tan\theta = \frac{2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{2\sin^2\frac{\phi}{2}+2\alpha\sin^2\frac{\phi}{2}} = \frac{\cos\frac{\phi}{2}}{\sin\left(\frac{\phi}{2}\right)(1+\alpha)} = \frac{\cot\frac{\phi}{2}}{1+\alpha}$$

$$\tan\theta = \frac{\cot\frac{\phi}{2}}{\left[1+\frac{h\nu}{m_0c^2}\right]} \quad \text{——— (11)}$$

Equation(11) reveals that the angle of the recoil electron θ depends on the scattering angle ϕ. If $\phi = 0$, $\theta = 90$ and if $\phi = 180$, $\theta = 0$

It shows that the electron can recoil only in the onward direction at angle less than $90°$, while a photon can scattered in any direction.

<u>Ques</u>. What is the Compton effect? Derive the expression for the direction of recoild Compton electron. Show that the Compton electron can recoil only in onward direction at angles less than $90°$.