Tutorial - 4

Master Theorem -

The Master Theorem applies to recurrences of the following form : $T(n) = aT(n/b) + f(n)$ where $a \geq 1$ and $b > 1$ are constants and $f(n)$ is an asymptotically positive function.

Q1) $T(n) = 3T(n/2) + n^2$

→ $a = 3, b = 2, f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing : $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (case 3)

∴ according to Master Theorem $T(n) = \Theta(n^2)$

Q2) $T(n) = 4T(n/2) + n^2$

→ $a = 4, \ b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ [case 2]

∴ according to masters theorem $T(n) = \Theta(n^2 \log n)$

Q3) $T(n) = T(n/2) + 2^n$

→ $a = 1, \ b = 2$

$n^{\log_2 1} = n^0 = 1$

$\Rightarrow 1 < 2^n$ (case 3)

∴ According to Masters theorem $T(n) = \Theta(2^n)$

Q4) $T(n) = 2^n T(n/2) + n^n$

→ ∴ Master's theorem is not applicable as a is function of n.

Q5) $T(n) = 16T(n/4) + n$

$\rightarrow$ $a = 16,\ b = 4\ ,\ f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (case 1)

$\therefore T(n) = \Theta(n^2)$

Q6) $T(n) = 2T(n/2) + n\log n$

$\rightarrow$ $a = 2,\ b = 2\ ,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

Now $f(n) > n$

$\therefore$ According to Masters $T(n) = \Theta(n\log n)$

7) $T(n) = 2T(n/2) + \frac{n}{\log n}$

$\rightarrow$ $a = 2,\ b = 2,\ f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

$\therefore$ According to master theorem $T(n) = \Theta(n)$

8) $T(n) = 2T(n/4) + n^{0.51}$

$\rightarrow$ $a = 2,\ b = 4,\ f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

$\therefore$ According master theorem $T(n) = \Theta(n^{0.8})$

9) $T(n) = 0.5T(n/2) + 1/n$

$\rightarrow$ $\therefore$ Masters Not applicable as $a < 1$

10) $T(n) = 16T(n/4) + n!$

$\rightarrow$ $a = 16,\ b = 4,\ f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

$\therefore$ According to masters, $T(n) = \Theta(n!)$

11) [illegible]

→ $a = 4$, $b = 2$, $f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

∴ According to master's, $T(n) = \Theta(n^2)$

12) $T(n) = \text{sqrt}(n) + (n/2) + \log n$

→ ∴ Master's Not applicable as a is not constant.

13) $T(n) = 3T(n/2) + n$

→ $a = 3$, $b = 2$, $f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

∴ According to master's theorem, $T(n) = \Theta(n^{\log_2 3})$

14) $T(n) = 3T(n/3) + \sqrt{n}$

→ $a = 3$, $b = 3$, $f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

∴ According to masters theorem, $T(n) = \Theta(n)$

15) $T(n) = 4T(n/2) + cn$

→ $a = 4$, $b = 2$, $f(n) = c*n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c*n$

∴ According to masters theorem, $T(n) = \Theta(n^2)$

16) $T(n) = 3T(n/4) + n\log n$

→ $a = 3$, $b = 4$, $f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

∴ According to masters theorem, $T(n) = \Theta(n\log n)$