Coordinate Systems And Time

Learners' Space Astronomy

Contents

Introduction

In this module we will study astronomical coordinate frames, directions and apparent motions of celestial objects, determination of position from astronomical observations, observational errors, astronomical time systems etc. We shall concentrate mainly on different astronomical coordinate systems, sidereal and synodic time, analemma and different time systems . For simplicity we will assume that the observer is always on the northern hemisphere. Although all definitions and equations are easily generalized for both hemispheres, this might be unnecessarily confusing. In spherical astronomy all angles are usually expressed in degrees; we will also use degrees unless otherwise mentioned.

1.1 Geometry of Sphere

Before we begin with the spherical astronomy, we need to first get started with the trigonometry involved in it. We shall start with a few basic terms which you may be familiar with. If a plane passes through the centre of a sphere, it will split the sphere into two identical hemispheres along a circle called a great circle (Fig.1.1). A line perpendicular to the plane and passing through the centre of the sphere intersects the sphere at the poles P and P'.

If a sphere is intersected by a plane not containing the centre, the intersection curve is a small circle. There is exactly one great circle passing through two given points Q and Q' on a sphere (unless these points are antipodal, in which case all circles passing through both of them are great circles). The arc Q Q' of this great circle is the shortest path on the surface of the sphere between these points. A spherical triangle is not just any three-cornered figure lying on a sphere; its sides must be arcs of great circles.

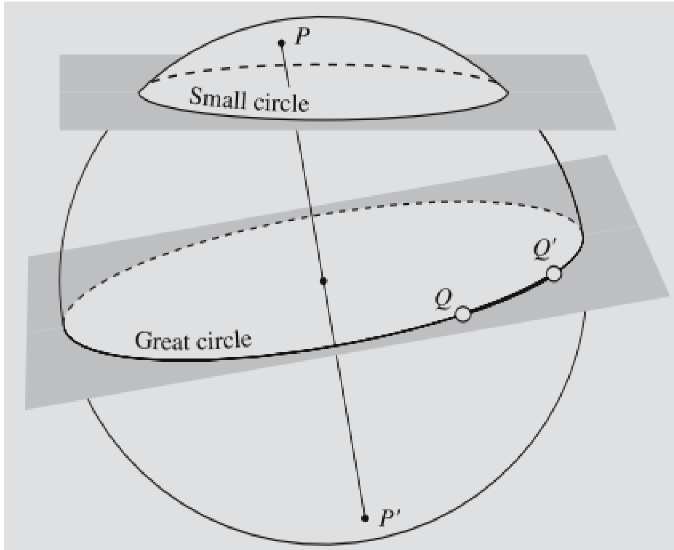

Figure 1.1: The above figure shows difference between small circle and great circle

The spherical triangle ABC has the arcs AB, BC and AC as its sides. If the radius of the sphere is r, the length of the arc AB is $|AB|$ = rc, [c] = rad , where c is the angle subtended by the arc AB as seen from the centre. This angle is called the central angle of the side AB. Because lengths of sides and central angles correspond to each other in a unique way, it is customary to give the central angles instead of the sides. In this way, the radius of the sphere does not enter into the equations of spherical trigonometry.

An angle of a spherical triangle can be defined as the angle between the tangents of the two sides meeting at a vertex, or as the dihedral angle between the planes intersecting the sphere along these two sides.

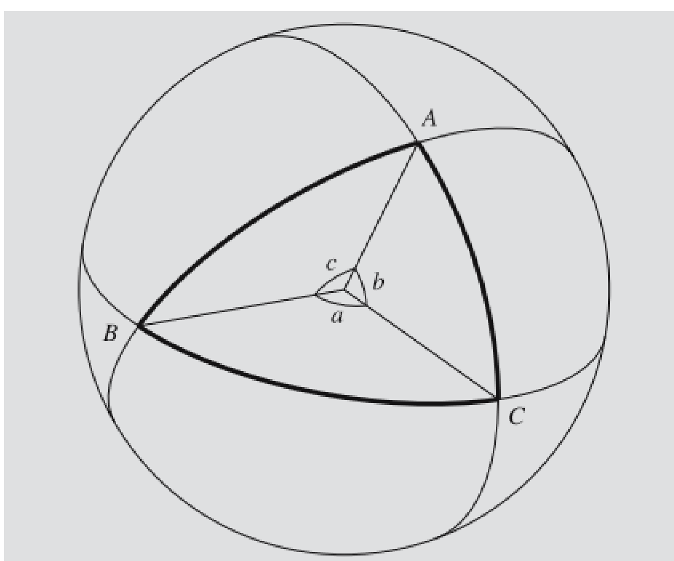

Figure 1.2: The angles A,B,C are spherical angles of the triangle while a,b,c are corresponding central angles

We denote the angles of a spherical triangle by capital letters (A, B, C) and the opposing sides, or, more correctly, the corresponding central angles, by lowercase letters (a, b, c). As in plane trigonometry, we have sin and cosine laws in spherical trigonometry which indeed are much general form of these laws. There are three of these equations which we would require to solve problems at the end of this chapter.

Sine Rule:
$\sin B \sin a = \sin A \sin b$ .....(1)
Cosine Rule:
$\cos a = \cos A \sin b \sin c + \cos b \cos c$ .....(2)
Analogue of Cosine Rule:
$\cos B \sin a = -\cos A \sin b cosc + \cos b \sin c$ .....(3)
Four parts formula:
$\cos a \cos C = \sin a \cot b - \sin C \cot B$ .....(4)
Equations for other sides and angles are obtained by cyclic permutations of the sides a, b, c and the angles A, B, C. All these variations of the sine formula can be written in an easily remembered form:

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \tag{1.1}$$

The equation number (1) and (3) would be most useful to solve the questions.

The Celestial Sphere

The ancient folks had a very simplistic view of the universe. The ancients thought universe to be confined within a finite spherical shell. The stars were fixed to this shell while some objects, called wanderers(planets) moved on this sphere and thus they were all equidistant from the Earth, which was at the centre of the spherical universe. This simple model is still in many ways as useful as it was in antiquity: it helps us to easily understand the diurnal and annual motions of stars, and, more important, to predict these motions in a relatively simple way.
Therefore we will assume for the time being that all the stars are located on the surface of an enormous sphere and that we are at its centre. Because the radius of this celestial sphere is practically infinite, we can neglect the effects due to the changing position of the observer, caused by the rotation and orbital motion of the Earth.

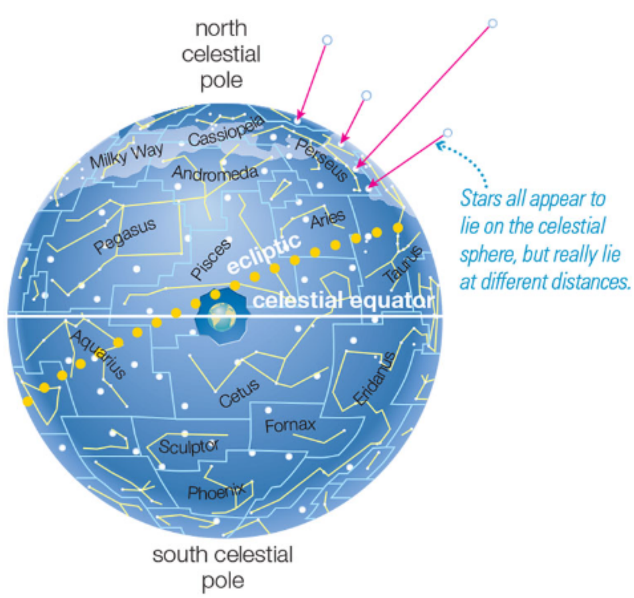

Figure 2.1: The Celestial Sphere, depicting North Celestial Pole, South Celestial Pole, ecliptic and celestial equator(an explanation to all these terms is given under the equatorial system)

Since the distances of the stars are ignored, we need only two coordinates to specify their directions. This is just like the system of latitudes and longitudes on the earth. Each coordinate frame has some fixed reference plane passing through the centre of the celestial sphere and dividing the sphere into two hemispheres. One of the coordinates indicates the angular distance from this reference plane.

$$z = 90^o - a$$

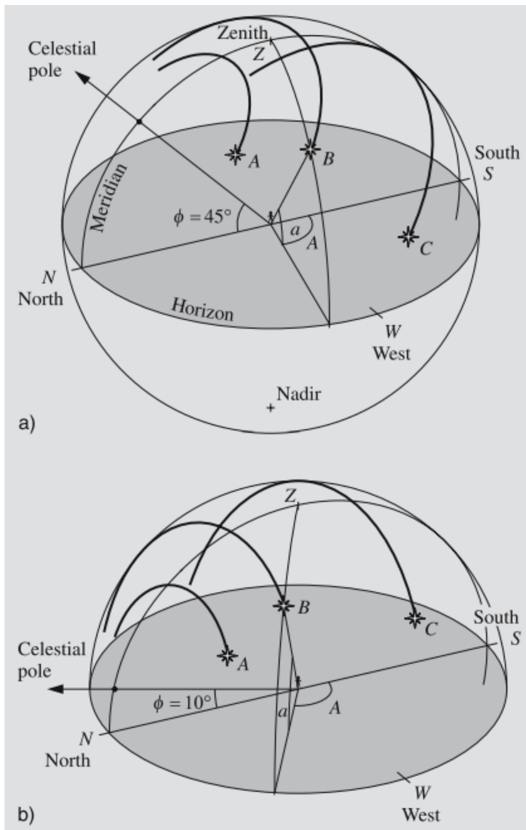

Figure 2.2: (a) The apparent motions of stars during a night as seen from latitude $\phi = 45^o$. (b) The same stars seen from latitude $\phi = 10^o$

There is exactly one great circle going through the object and intersecting this plane perpendicularly; the second coordinate gives the angle between that point of intersection and some fixed direction.
For example if we look at the latitude and longitudes on earth and when we try to define coordinates of a place on it we define latitude as the angular distance between that place and the equator plane(first reference plane) and for defining longitude we take the the longitude $= 0^o$ great circle as second reference plane and similarly measure angular distance from it.

The Horizontal System

The most natural coordinate frame from the observer's point of view is the horizontal frame (Fig. 3.1). Its reference plane is the tangent plane of the Earth passing through the observer; this horizontal plane intersects the celestial sphere along the horizon. The point just above the observer is called the **zenith** and the antipodal point below the observer is the **nadir**. (These two points are the poles corresponding to the horizon.) Great circles through the zenith are called verticals. All verticals intersect the horizon perpendicularly.

By observing the motion of a star over the course of a night, an observer finds out that it follows a track like one of those in Fig. 2.2 . Stars rise in the east, reach their highest point, or **culminate**, on the vertical NZS (North,Zenith,South), and set in the west. The vertical NZS is called the **local meridian**. North and south directions are defined as the intersections of the meridian and the horizon. So we can define **culmination** of star as when it on the local meridian. One of the horizontal coordinates is the **altitude** (or elevation) denoted as 'a', which is measured from the horizon along the vertical passing through the object. The altitude lies in the range [-90^o , $+90^o$]; it is positive for objects above the horizon and negative for the objects below the horizon. The zenith distance, or the angle between the object and the zenith, is obviously

$$z = 90^o - a$$

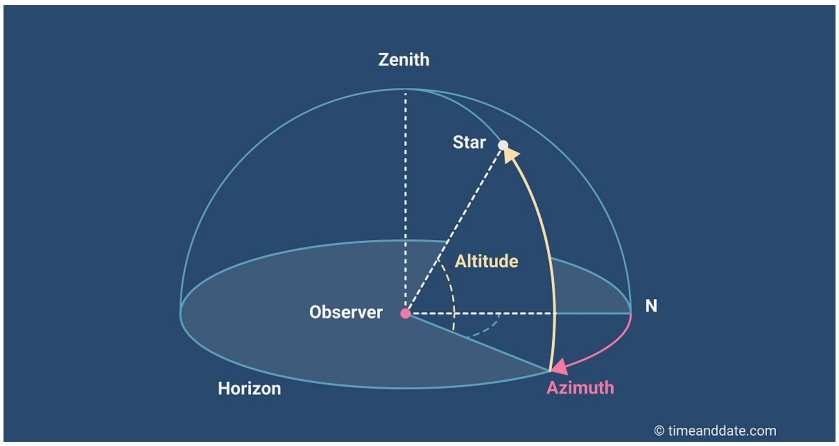

Figure 3.1: The altitude and azimuth of a star in the horizontal system

$$z = 90^o - a$$

The second coordinate is the **azimuth**, A; it is the angular distance of the vertical of the object from some fixed direction. Normally it is measured from the North Pole in the clockwise direction. Unfortunately, in different contexts, different fixed directions are used; thus it is always advisable to check which definition is employed. The azimuth is usually measured from the north or south, and though clockwise is the preferred direction, counterclockwise measurements are also occasionally made.

In this chapter we have adopted a fairly common astronomical convention, measuring the azimuth clockwise from the north. Its values are usually normalized between 0^o and 360^o . Since the horizontal coordinates are time and position dependent, they cannot be used, for instance, in star catalogues. So we need to get a uniform system independent of time and position of the observer on the earth which brings us to the Equatorial System.

The Equatorial System

The equatorial coordinate system is a celestial coordinate system in which the origin is at the centre of the earth, and the coordinates of an object are defined with the help of two coordinates, namely **declination** δ and **right ascension** α. Let's see how these coordinates are defined.

The direction of the rotation axis of the Earth remains almost constant and so does the equatorial plane perpendicular to this axis. Therefore the equatorial plane is a suitable reference plane for a coordinate frame that has to be independent of time and the position of the observer.
The intersection of the celestial sphere and the equatorial plane is a great circle, which is called the **equator** of the celestial sphere. We define first coordinate as the angular separation of a star from the equatorial plane(which is not affected by the rotation of the Earth) as **declination** δ(see fig 4.2).

There is one more significant great circle which is called **ecliptic**. This represents the path of sun on the celestial sphere over the course of the year. The ecliptic passes through twelve constellations on the celestial sphere, and these constellations are collectively called **zodiac constellations**. They are (in order): Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpius, Sagittarius, Capricornus, Aquarius and Pisces. The point of intersection of ecliptic and equator are equinoxes(equal day and night). The one occurring during the movement of sun from southern hemisphere to northern hemisphere is called the **vernal equinox**(usually around 21^{st} March) and **autumnal equinox**(close to 21^{st} September) during its north to south movement . The time of the year when the sun attains its maximum declination, is called the **Summer Solstice**(longest day)(21^{st} June). And **Winter Solstice**(shortest day)(22^{nd} December) occurs at the time of minimum declination of sun.

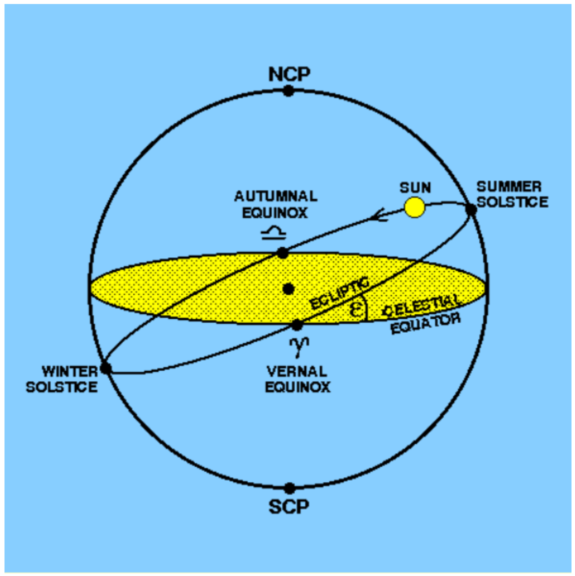

Figure 4.1: Ecliptic on the celestial sphere with vernal and autumnal equinox.

The point where the extension of the Earth's rotational axis meets the celestial sphere is called **North Celestial pole(NCP)** if it lies to the north of equator and similarly **South Celestial pole(SCP)** on its other end. Apart from these two poles we have other two also corresponding to the ecliptic, these are **North Ecliptic pole(NEP)** and **South Ecliptic pole(SEP)**, though these are of less importance . The NCP is at a distance of about one degree (which is equivalent to two full moons) from the moderately bright star *Polaris*. The **local meridian** as discussed earlier is the circle in the sky joining zenith , NCP , nadir and SCP. It is always perpendicular to the horizon. The meridian may be divided into two: north and south meridian depending upon the hemisphere it lies in.

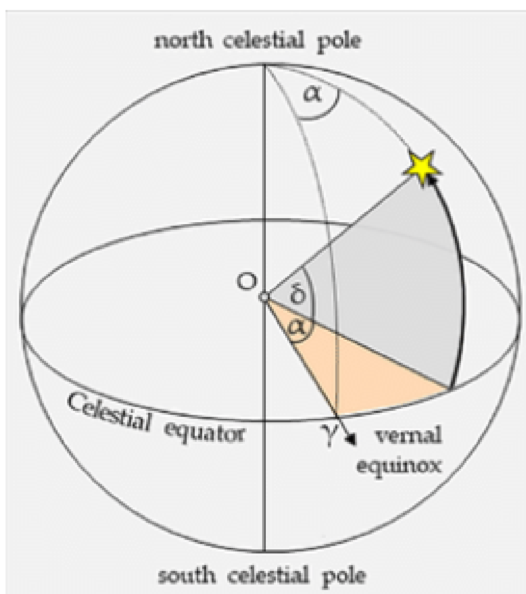

Figure 4.2: The declination and right ascension of a star on the celestial sphere

To define the second coordinate(right ascension), we must again agree on a fixed direction,

unaffected by the Earth's rotation. From a mathematical point of view, it does not matter which point on the equator is selected. However, for later purposes, it is more appropriate to employ a certain point with some valuable properties, like the point of intersection of the celestial equator and ecliptic. This point is called the **vernal equinox**.
Because it used to be in the constellation Aries (the Ram), it is also called the first point of Aries ant denoted by the sign of Aries ♈ . Now we can define the second coordinate as the angle from the vernal equinox measured along the equator. This angle is the **right ascension** α (or R.A.) of the object, measured counterclockwise from ♈.

Even the position of vernal equinox is not fixed!

We know that Earth undergoes rotation about its axis and revolution around the sun. Apart from these two movements, the earth also shows precession. Precession is the change in orientation of earth's rotation axis. This is similar to what we see in a toy top. The top revolves around its axis and its axis in turn revolves around another axis.
Video explanation(animation): https://www.youtube.com/watch?v=qlVgEoZDjok
This is really slow though, but due to this, the vernal equinox, which was earlier defined in Aries has not shifted to Pisces. One cycle of precession take around 26,000 years.
Due to precession, vega(a bright star in Lyra) will be our pole star in next 14,000 years!

Since declination and right ascension are independent of the position of the observer and the motions of the Earth, they can be used in star maps and catalogues. But the zero point of the right ascension seems to move in the sky, due to the diurnal rotation(apparent daily motion of sky w.r.t the observer on earth due to earth's rotation about its axis) of the Earth. So we cannot use the right ascension to find an object unless we know the direction of the vernal equinox.

Since the meridian is a well-defined line in the sky, we use it to establish a local coordinate corresponding to the right ascension. The **hour angle** is defined for the purpose of finding the position of vernal equinox and thus the right ascension. The hour angle is the time elapsed since the last culmination. Which means hour anle of any object say star is zero at culmination and grows with time. Hour angle is measured clockwise from the meridian. The *hour angle* of an object is not a constant, but grows at a steady rate, due to the Earth's rotation. The hour angle of the vernal equinox is called the sidereal time Θ. Figure 3 shows that for any object

$$\Theta = h + \alpha$$

where h is the object's hour angle and α its right ascension.
The link demonstrates the hour angle, sidereal time and its realtion with R.A.
https://www.youtube.com/watch?v=1o7T68LZBJI

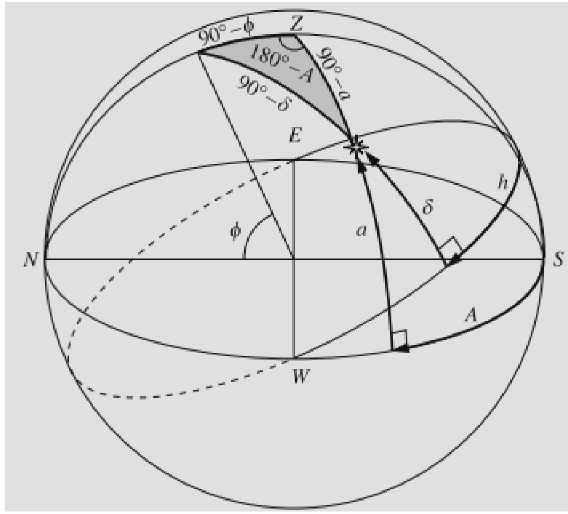

Figure 4.4: The nautical triangle for deriving transformations between the horizontal and equatorial frame

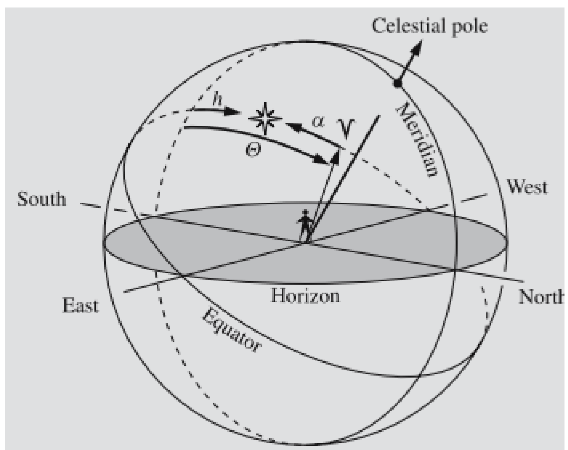

Figure 4.3: The sidereal time Θ (the hour angle of the vernal equinox) equals the hour angle plus right ascension of any object

Since hour angle and sidereal time change with time at a constant rate, it is practical to express them in units of time. Also the closely related right ascension is customarily given in time units. Thus 24 hours equals 360 degrees, 1 hour = 15 degrees, 1 minute of time = 15 minutes of arc, and so on. All these quantities are in the range (0 h, 24 h).
The altitude at upper culmination(the highest point an object attains during the course of a day) is given by

$$a_{max} = 90 - \phi + \delta, \tag{4.1}$$

if the object culminates south of zenith,

$$a_{max} = 90 - \phi - \delta, \tag{4.2}$$

if the object culminates north of zenith.

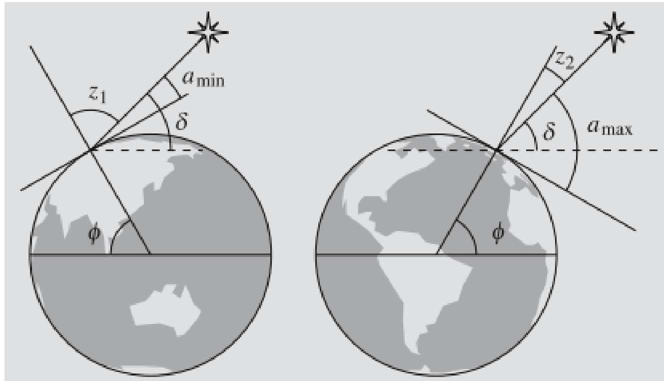

Figure 4.5: . The altitude of a circumpolar star at upper and lower culmination

The altitude is positive for objects with $\delta > \phi - 90^o$. Objects with declinations less than $\phi - 90^o$ can never be seen at the latitude ϕ. Similarly altitude at the lower culmination is

$$a_{min} = \phi + \delta - 90, \tag{4.3}$$

Stars with $\delta > 90^o - \phi$ will never set (just put $a_{min} > 0$). For example, in Helsinki ($\phi = 60^o$), all stars with a declination higher than 30^o are such circumpolar stars. And stars with a declination less than -30^o can never be observed there.
The link illustrates the concept of circumpolar stars:
https://www.youtube.com/watch?v=yMsEjtQ-7n4

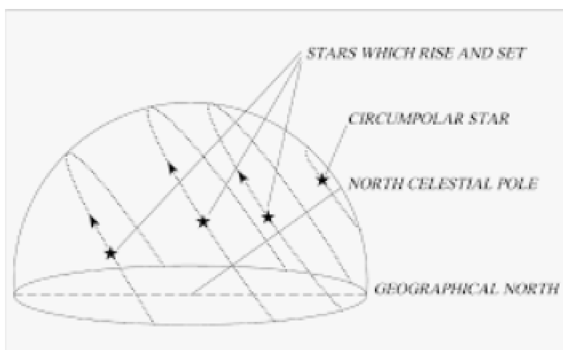

Figure 4.6: The diurnal motion of circumpolar and other stars

We shall now study briefly how the (α, δ) frame can be established by observations. Suppose we observe a circumpolar star at its upper and lower culmination (Fig. 4.6). At the upper transit, its altitude is $a_{max} = 90^o - \phi + \delta$ and at the lower transit, $a_{min} = \delta + \phi - 90^o$. Eliminating the latitude, we get

$$\delta = \frac{1}{2}\left\{a_{min} + a_{max}\right\} \tag{4.4}$$

Thus we get same value of declination independent of the observer's location as discussed earlier.

Other Coordinate Systems

Apart from the Horizontal and the Equatorial Systems we have two other famous coordinate systems, namely the Ecliptic System and the Galactic System. These are useful in certain settings. We shall just go through the basics definition of both the systems not touching the mathematical part of it.

5.1 Ecliptic Coordinate System

The orbital plane of the Earth, the ecliptic, is the reference plane of another important coordinate frame. The ecliptic can also be defined as the great circle on the celestial sphere described by the motion of the Sun in the course of one year. This frame is used mainly for planets and other bodies of the solar system. The orientation of the Earth's equatorial plane remains invariant, unaffected by annual motion.

In spring, the Sun appears to move from the southern hemisphere to the northern one (Fig. 5.1). Almost midway between this transition there is a moment called the vernal equinox. At the vernal equinox, the Sun's right ascension and declination are zero. The equatorial and ecliptic planes intersect along a straight line directed towards the vernal equinox. Thus we can use this direction as the zero point for both the equatorial and ecliptic coordinate frames. The point opposite the vernal equinox is the autumnal equinox, it is the point at which the Sun crosses the equator from north to south.

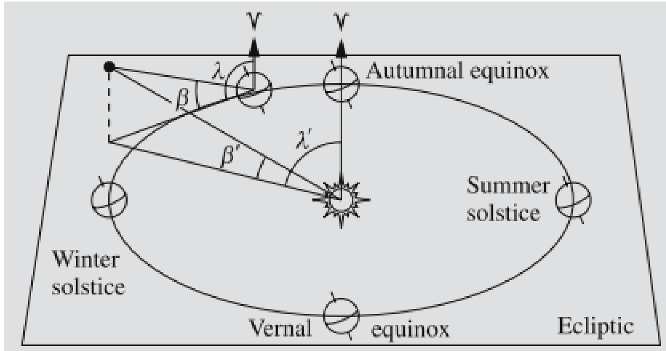

Figure 5.1: . The ecliptic geocentric (λ, β) and heliocentric (λ' , β') coordinates are equal only if the object is very far away. The geocentric coordinates depend also on the Earth's position in its orbit

The ecliptic latitude β is the angular distance from the ecliptic; it is in the range $[-90^o, +90^o]$. The other coordinate is the ecliptic longitude λ, measured counterclockwise from the vernal equinox.
Depending on the problem to be solved, we may encounter heliocentric (origin at the Sun), geocentric (origin at the centre of the Earth) or topocentric (origin at the observer) coordinates. For very distant objects the differences are negligible, but not for bodies of the solar system.

5.2 Galactic Coordinate System

For studies of the Milky Way Galaxy, the most natural reference plane is the plane of the Milky Way (Fig. 5.2). Since the Sun lies very close to that plane, we can put the origin at the Sun. The galactic longitude l is measured counterclockwise (like right ascension) from the direction of the centre of the Milky Way (in Sagittarius, α = 17 h 45.7 min, $\delta = -29^o$). The galactic latitude b is measured from the galactic plane, positive northwards and negative southwards.

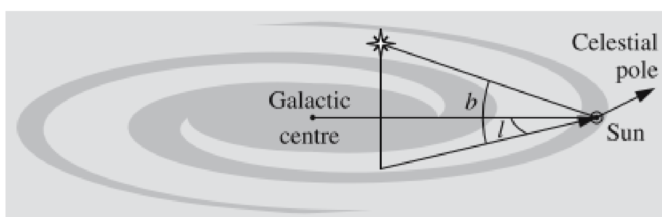

Figure 5.2: The galactic coordinates l and b

Sidereal And Solar Time

Earlier in this chapter we defined the sidereal time as the hour angle of the vernal equinox. A good basic unit is a sidereal day, which is the time between two successive upper culminations of the vernal equinox. After one sidereal day the celestial sphere with all its stars has returned to its original position with respect to the observer. The flow of sidereal time is as constant as the rotation of the Earth. The rotation rate is slowly decreasing, and thus the length of the sidereal day is increasing. In addition to the smooth slowing down irregular variations of the order of one millisecond have been observed.
video explanation : https://www.youtube.com/watch?v=xUQKrUpQeXQ

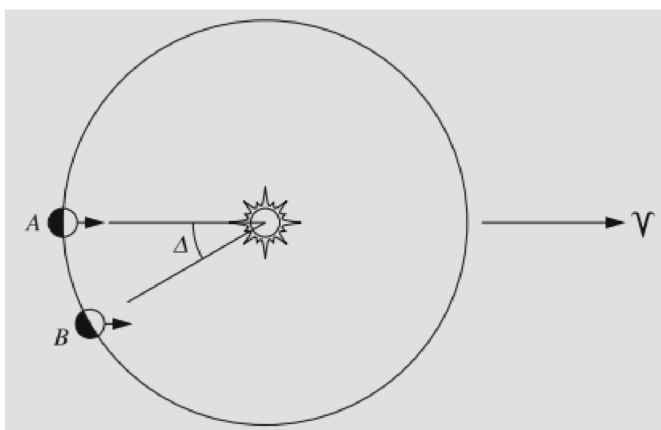

Figure 6.1: . One sidereal day is the time between two successive transits or upper culminations of the vernal equinox. By the time the Earth has moved from A to B, one sidereal day has elapsed. The angle shown is greatly exaggerated; in reality, it is slightly less than one degree

Figure 6.1 shows the Sun and the Earth at vernal equinox. When the Earth is at the point A, the Sun culminates and, at the same time, a new sidereal day begins in the city with the huge black arrow standing in its central square. After one sidereal day, the Earth has moved along its orbit almost one degree of arc to the point B. Therefore the Earth has to turn almost a degree further before the Sun will culminate.
The link for animation:
https://www.youtube.com/watch?v=WWw4JY2dNXM&t=3s
The solar or synodic day is therefore 3 min 56.56 s (sidereal time) longer than the sidereal day.

This means that the beginning of the sidereal day will move around the clock during the course of one year. After one year, sidereal and solar time will again be in phase. The number of sidereal days in one year is one higher than the number of solar days.

When we talk about rotational periods of planets, we usually mean sidereal periods. The length of day, on the other hand, means the rotation period with respect to the Sun. If the orbital period around the Sun is P, sidereal rotation period τ^* and synodic day τ, we now know that the number of sidereal days in time P, P/τ^*, is one higher than the number of synodic days, P/τ:

$$\frac{P}{\tau^*} - \frac{P}{\tau} = 1 \tag{6.1}$$

or

$$\frac{1}{\tau} = \frac{1}{\tau^*} - \frac{1}{P} \tag{6.2}$$

This holds for a planet rotating in the direction of its orbital motion (counterclockwise). If the sense of rotation is opposite, or retrograde, the number of sidereal days in one orbital period is one less than the number of synodic days, and the equation becomes

$$\frac{1}{\tau} = \frac{1}{\tau^*} + \frac{1}{P} \tag{6.3}$$

For the Earth, we have P = 365.2564 d, and τ = 1 d, whence eqn (7) gives τ^* = 0.99727 d = 23 h 56 min 4 s, solar time. Since our everyday life follows the alternation of day and night, it is more convenient to base our timekeeping on the apparent motion of the Sun rather than that of the stars.

Unfortunately, solar time does not flow at a constant rate. There are two reasons for this. First, the orbit of the Earth is not exactly circular, but an ellipse, which means that the velocity of the Earth along its orbit is not constant. Second, the Sun moves along the ecliptic, not the equator. Thus its right ascension does not increase at a constant rate. The change is fastest at the end of December (4 min 27 s per day) and slowest in mid-September (3 min 35 s per day). As a consequence, the hour angle of the Sun (which determines the solar time) also grows at an uneven rate.

To find a solar time flowing at a constant rate, we define a fictitious mean sun, which moves along the celestial equator with constant angular velocity, making a complete revolution in one year. By year we mean here the tropical year, which is the time it takes for the Sun to move from one vernal equinox to the next. That is, in one tropical year the right ascension of the Sun increases exactly 24 hours. The length of the tropical year is 365 d 5 h 48 min 46 s = 365.2422 d.

Since the direction of the vernal equinox moves due to precession, the tropical year differs from the sidereal year, during which the Sun makes one revolution with respect to the background stars. One sidereal year is 365.2564 d. Using our artificial mean sun, we now define an evenly flowing solar time, the mean solar time (or simply mean time) T_M, which is equal to the hour angle h_M of the centre of the mean sun plus 12 hours (so that the date will change at midnight, to annoy astronomers):

$$T_M = h_M + 12h \tag{6.4}$$

The difference between the true solar time T and the mean time T_M is called the equation of time:

$$E.T. = T - T_M \tag{6.5}$$

The greatest positive value of E.T. is about 16 minutes and the greatest negative value about −14 minutes.

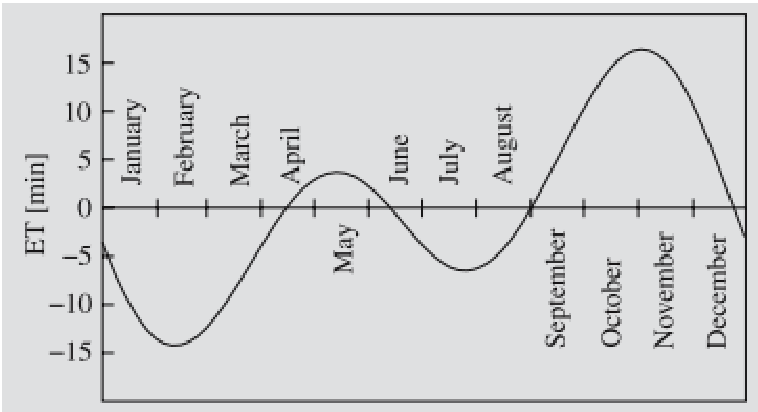

Figure 6.2: Equation of time. A sundial always shows (if correctly installed) true local solar time. To find the local mean time the equation of time must be subtracted from the local solar time

6.1 Analemma

If you looked at the Sun at the same (solar) time each day, from the same place, would it appear at the same location in the sky? If the Earth were not tilted, and if its orbit around the Sun were perfectly circular, then, yes, it would. However, a combination of the Earth's 23.5 degree tilt and its slightly elliptical orbit combine to generate this figure "8" pattern of where the Sun would appear at the same time throughout the year. The pattern is called an analemma.

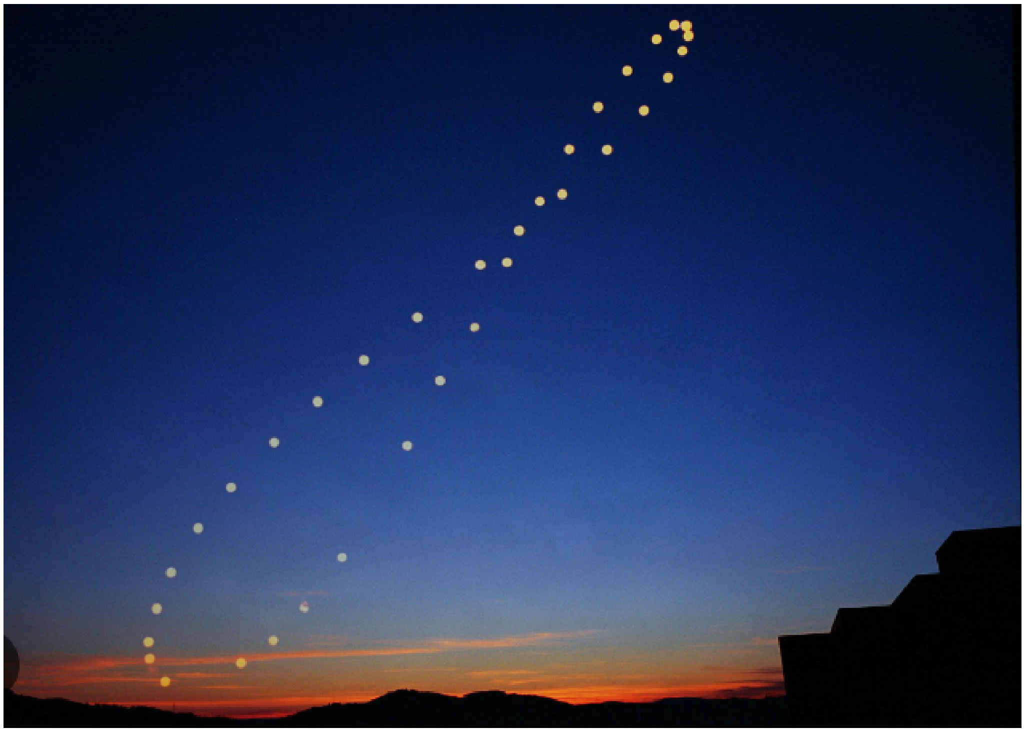

Figure 6.3: An Analemma

The north–south component of the analemma results from the change in the Sun's declination due to the tilt of Earth's axis of rotation. The east–west component results from the non-uniform rate of change of the Sun's right ascension, governed by the combined effects of Earth's axial tilt and its orbital eccentricity.
The Sun will appear at its highest point in the sky, and highest point in the analemma, during summer. In the winter, the Sun is at its lowest point. The in between times generate the rest of the analemma pattern due to above stated reasons. (See Analemma Curve.) Analemmas viewed from different Earth latitudes have slightly different orientation but same shape, as do analemmas created at different times of the day. Analemmas on the other planets have different shapes entirely!

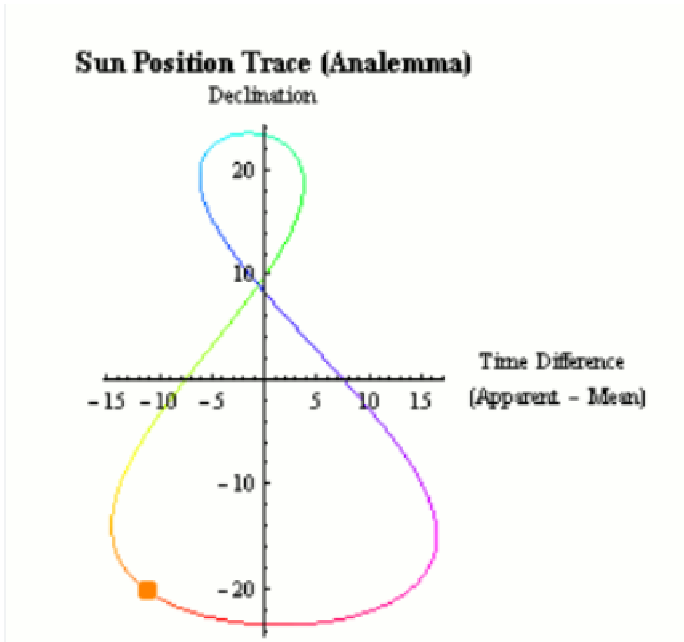

Figure 6.4: The analemma put on Declination vs Equation of Time graph helps us visualize how analemma happens and correlate it with the graph in fig 1.12

It is interesting to note that E.T. $\neq 0$ at equinoxes. This is beacuse the analemma is a result of two factors mentioned earlier, for you to recall they were:
1. 23.5^o tilt of earth's axis
2. Earth's ecliptic orbit

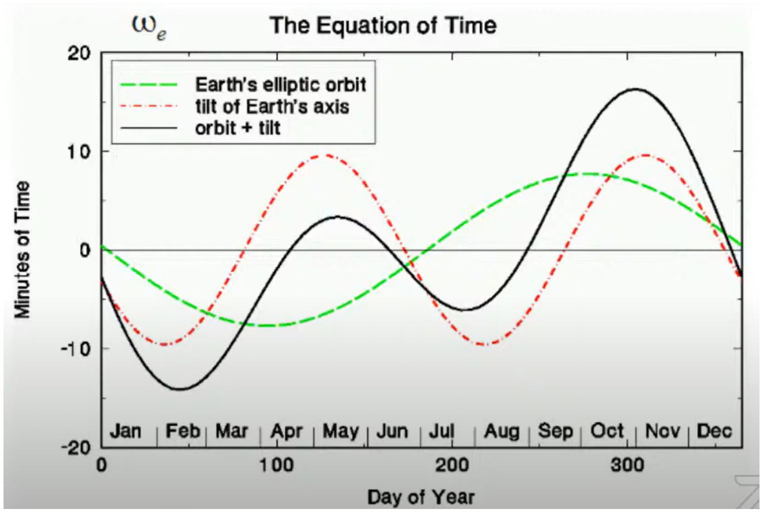

Figure 6.5: The effective graph of E.T. taking into consideration the two factors mentioned above

Taking into account both of these factors results in a formation of "8" which has a smaller upper lobe as E.T. is smaller during May-June and bigger lower lobe is because E.T. has larger value during November-December. Fig 6.5 illustrates it.
This link lets you play around with various parameters for an analemma:
https://www.analemma.com/other-analemmas.html

Astronomical Time Systems

Since the Stone Age, as humanity progressed and became civilized, the need for a dating system (calendar dates ;-)) arose. Historians believe that timekeeping dates back to the Neolithic period, but actual calendars were not developed until the Bronze Age in 3100 BC. Early attempts at creating calendars were made by various civilizations, including the Babylonians, Chinese, Egyptians, and others.
These calendars often consisted of 12 months with alternating 29 and 30-day months to align with the Moon, resulting in a total of 354 days. However, this was approximately 11 days shorter than a solar year. In this module, we will explore different calendars, such as the Indian (Vedic) calendar, Roman calendar, Julian calendar, and Gregorian calendar. But before that, let's understand the basis of these calendars:

- **Lunar calendars** (e.g., Islamic calendar)
  Lunar calendars measure the month based on the lunar cycle, which is approximately 29 and a half days. These calendars start the next year after 12 lunar months, without necessarily aligning with the completion of a full cycle of seasons.

- **Solar calendars** (e.g., Persian/Buddhist/Bangla)
  Solar calendars, on the other hand, aim to have an year consisting of 365 days, with months having 31 or 30 days. They do not focus on starting a month with a fixed lunar phase.

- **Lunisolar calendars** (e.g., Jewish/Vedic)
  Lunisolar calendars are more complex. They determine the month based on lunar phases, ensuring that each month starts on a fixed phase. However, to account for the mismatch between 354 lunar days and 365 solar days in a year, these calendars incorporate an intercalary month every 2-3 years. This extra month helps reconcile the difference between lunar and solar cycles.

7.1 Indian Lunisolar Calendar

In the Indian Lunisolar calendar, months end with the new moon day, and the next month begins on the day following the new moon. The month's name is determined by the lunar mansion (*Nakshatras*) coinciding with the full moon day. Each "tithi" corresponds to a 12-degree interval of lunar elongation. The tithi of any given day is determined by the elongation of moon at sunrise. A tithi can last anywhere between 20 to 27 hours. We know that Sun moves faster(in the sky) in

December as compared to its slow speed in June. Also the orbit of moon is elliptical so its motion is also not constant. This variation in speed of Sun and elliptical orbit of moon leads to some tithis to get repeated and others to get missing from the calendar. Let us take an example to explain it.

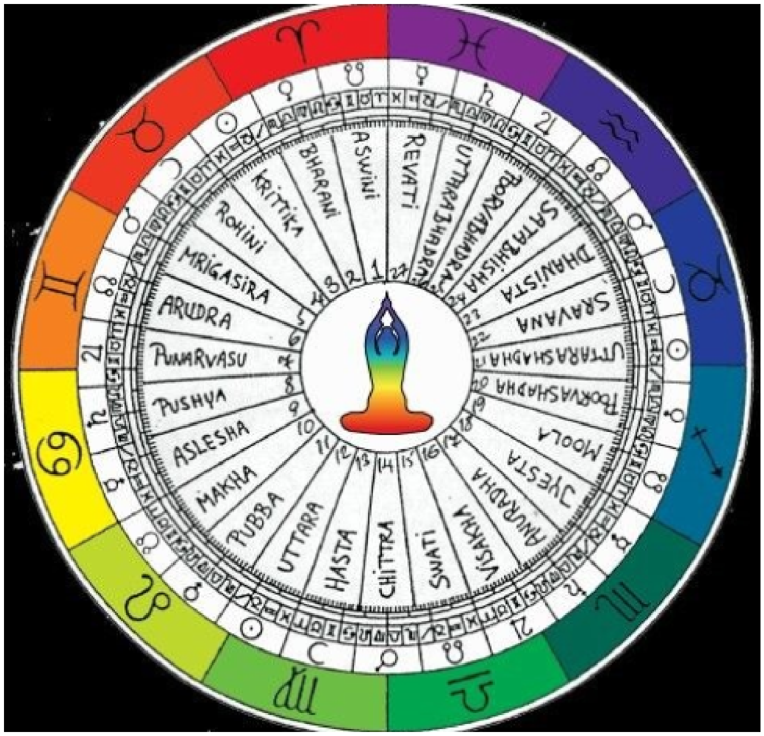

Figure 7.1: In Indian astronomy, the *Nakshatras* are divisions of 13 degrees 20 minutes starting from zero Aries and ending at 30 degrees of Pisces. The *Nakshatras* are referred to as the lunar mansions because the Moon moves approximately $13^o20'$ per day, therefore, resides in one *Nakshatras* per day. A *Nakshatras* is one of 27 (sometimes also 28) sectors along the ecliptic.

Say *dwitiya* starts at 5:25 am and Sun rise at local time is 5:30 am, whole day name is *dwitiya* until next Sun-rise at 5:31 am (say). If *tritiya* starts at 5:29 (of next day), there will be *tritiya* at sun-rise, so whole day named as *tritiya*. If *tritiya* ends at 5:34 am of third day and Sun-rise at 5:32 am. there will be *tritiya* in the next Sun-rise too. In this case, *tritiya* obtains two separate Sun-rises. The *tritiya* has repeated here (double Sun-rise tithi). In the reverse case, sometimes a tithi starts after Sun-rise and ends before next sun-rise. In that case, there will not be any named lunar day (called as kshya).Some tithis in the calendar may be missing or repeated, depending on the sunrise timings. This creates a unique characteristic of the Indian Lunisolar calendar.
The most common surviving Indian Lunisolar eras are:

- Shalivahan Shaka Samvat (starting from 78 CE)
- Vikram Samvat (starting from 56 BCE)

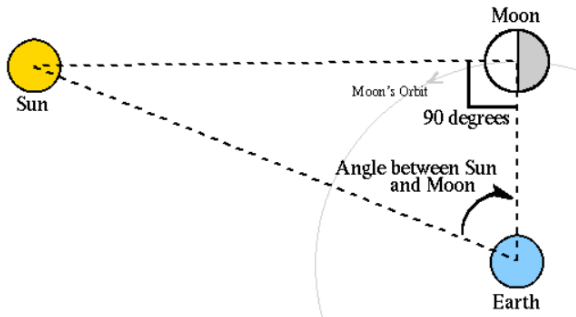

Figure 7.2: The Angle between Sun and Moon shown above is what we refer to as elongation of moon

Meaning to some key words!

- Elongation : The angular distance in celestial longitude separating the Moon or a planet from the Sun
- Dwitiya : The second day of waxing phase or waning phase of moon. There are two of them in a lunar month with a time gap of a fortnight.
- Tritiya : The third day of waxing phase or waning phase of moon. There are two of them in a lunar month with a time gap of a fortnight.

7.2 Roman Calendar

7.2.1 The Old Roman Calendar

The Roman calendar was a lunar calendar with a unique structure. It consisted of 10 months totaling 304 days, titled from March to December. The remaining 50-odd winter days were either added to the last month or considered separately. The decision of when to start the next year was left to the local priest. While this system may seem peculiar, it worked for the people at that time, as they lacked education, and the winter season often confined them to their homes and it didn't matter for them which day of the month it was.

7.2.2 The Roman Republican Calendar

This calendar was slightly improved version of the previous one. It consisted of 12 months totaling 354 days, with two months added at the beginning. This led to September, the 7^th and October, the eighth month to be ninth and tenth respectively, even though their names suggest otherwise. An intercalary month was added every 2-3 years. However, the decision to start the next year still rested with the local priest.

7.3 The Julian Calendar

When Julius Caesar unified the Roman republics and established the Roman Empire, he discovered that different provinces had varying opinions on when to start the new year. This led to chaos

when tax reports from different provinces did not align due to calendar discrepancies.
To resolve this issue, Caesar convened a conference of astronomers and sought their guidance. The astronomers recommended adopting a calendar where an year comprised of 365 days, with an additional leap day every fourth year, similar to what we follow today. This resulted in an effective year length of 365.25 days, while the tropical year length is 365 days 5 hours 48 minutes 46 seconds.

Figure 7.3: Julius Caesar

The difference of 11 minutes 14 seconds per year accumulates to an error of 1 day in every 128 years. However, since life expectancy during that period was around 40 years, this error was considered negligible over three generations. The new calendar started uniformly on January 1, 45 BCE. Then began a chaotic period of about 50 years, due to death of Julius Caesar. During that period different emperors had different views on how to continue the calendar, some asked to take leap year every 3 years, then later realising that they had taken too many leap years then taking a break from leap years for next 10 years to make up for it and so on. Finally this system of one leap year in every 4 years and 365 days a year settled down from 4 CE.

Why do July and August have 31 days while February has 28 days?

Julius Caesar's general and successor, Marcus Antonius, renamed the month of Julius Caesar's birth, "Mensis Quintilis" (fifth month), to "Mensis Julius" (July) in his honor. To make July a full 31-day month instead of 30 days, one day was taken from February. Similarly, Marcus Antonius's successor, Augustus Caesar, renamed "Mensis Sextilis" to "Mensis Augustus" because August was considered auspicious for war, harvest, and other events. To ensure August had 31 days like July, a day was also taken from February and added to August.
You may wonder why February was consistently targeted. Recall that initially, March was the first month, and January and February were added later. Both January and February were considered dreary months with no significant agricultural activities or festivals. February's role was to account for the remaining days after January had been assigned its days, making it less popular among many.

7.4 Gregorian Calendar

Was the calendar issue finally resolved and everything running smoothly? Not quite! The 11-minute error reappeared in 1582 when it accumulated to 10 days, shifting the vernal equinox from March 21st to March 11th. In response, Pope Gregory called a meeting of astronomers to find a solution. The decision was made to skip 10 days and bring the calendar back on track. This was achieved by skipping days between October 4th and October 15th, 1582. To prevent similar anomalies in the future, it was proposed that centuries should only be leap years if divisible by 400. This corrected the error by 3/400 day per year. The accuracy improved from 1 day in 128 years to 1 day in 3300 years. To illustrate this:
The errors written below are for one year
Initial error= $\frac{1}{128}$ day
Final error= $\frac{1}{128} - \frac{3}{400} = \frac{1}{3200}$ day
Since these errors are not exact, therefore we get 3200 instead of 3300, which is indeed the correct answer.

There was also a proposal to further refine the calendar by making years divisible by 4000 non-leap years, achieving an accuracy of 1 day in 20,000 years. However, this proposal is yet to be implemented, and future astronomers may handle it when the time comes.

7.5 Julian Dates

Astronomers devised a simpler way to track dates known as Julian dates, abbreviated as JD. Julian day 0 corresponds to noon on January 1, 4713 BCE. The day changes at noon in UT (Universal Time). Julian dates are calculated using straightforward subtraction to determine the number of days between two events, even if they are months or years apart.
Examples:
January 1, 2000, 00:00 UT: JD 2,451,544.500
January 1, 2020, 12:00 UT: JD 2,458,850.000

Futher Reading

1.Astronomy: Principles and Practice, Fourth Edition (PBK) , Book by Archie Roy and David Clarke (Chapters 7 to 9)
2.Spherical Astronomy in Fundamental Astronomy by "Hannu Karttunen"
3.Textbook on Spherical Astronomy 6^{th} Edition by W.M. Smart