

# Functional Dependency

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## Introduction

- **Functional Dependency** (FD) is a *many to one relationship* from one set of attributes to another in a given relation
- Two aspects of dependency
  - ❖ **Case a:** the values of a given relation at a given point in time
  - ❖ **Case b:** the set of all possible values that the given relation may assume at different time

FIRST

s#	status	city	p#	qty
s1	20	London	p1	100
s1	20	London	p2	100
s2	10	Paris	p1	200
s2	10	Paris	p2	200
s3	10	Paris	p2	300
s4	20	London	p2	400
s4	20	London	p4	500
s4	20	London	p5	600
s5	20	Kolkata	p1	200

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## Functional Dependency

- ❖ Let  $r$  be a relation and let  $X$  and  $Y$  be arbitrary subsets of the set of attributes of  $r$ .

- ❖ Then we say  $Y$  is functionally dependent on  $X$ - in symbols

$$X \rightarrow Y$$

- ❖ (or  $X$  functionally determines  $Y$ )- if and only if, each  $X$  value in  $r$  has associated with it precisely one  $Y$  value in  $r$
- ❖ Whenever two tuples of  $r$  agree in their  $X$  value, they also agree on their  $Y$  value

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## Examples of FDs

$$\{s\# \} \rightarrow \{city\}$$

$$\{city\} \rightarrow \{status\}$$

$$\{s\#,p\# \} \rightarrow \{qty\}$$

$$\{s\#,p\# \} \rightarrow \{city\}$$

$$\{s\#,p\# \} \rightarrow \{s\# \}$$

$$\{s\#,p\# \} \rightarrow \{s\#,p\#,city,qty\}$$

FIRST

s#	status	city	p#	qty
s1	20	London	p1	100
s1	20	London	p2	100
s2	10	Paris	p1	200
s2	10	Paris	p2	200
s3	10	Paris	p2	300
s4	20	London	p2	400
s4	20	London	p4	500
s4	20	London	p5	600
s5	20	Kolkata	p1	200

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## Functional Dependency

- ❖ Determinant and Dependent of FD
- ❖ Singleton set
- ❖ FD holds for all time
- ❖ Integrity Constraint

$$X \rightarrow Y$$

$$s\# \rightarrow \text{city}$$

$$\{s\#, p\#\} \rightarrow \text{qty}$$

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## Functional Dependency

- ❖ If X is a candidate key of a relation R  
then all attributes Y of R is functionally  
dependent on X
- ❖  $p\# \twoheadrightarrow \{p\#, \text{pname}, \text{color}, \text{weight}, \text{city}\}$
- ❖ If Relation R satisfies  $A \twoheadrightarrow B$  and  
the  $B$  is not *trivial* and A is not candidate  
key then R will involve some  
**REDUNDANCY**

FIRST

s#	status	city	p#	qty
s1	20	London	p1	100
s1	20	London	p2	100
s2	10	Paris	p1	200
s2	10	Paris	p2	200
s3	10	Paris	p2	300
s4	20	London	p2	400
s4	20	London	p4	500
s4	20	London	p5	600
s5	20	Kolkata	p1	200

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s# → city 11/5/2020

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## Trivial and Nontrivial FDs

**Trivial FD:** R.H.S is the subset of LHS

$$\{s\#,p\#\} \rightarrow \{s\#\}$$

**Nontrivial FD**

$$\{s\#,p\#\} \rightarrow \{city\}$$

$$\{s\#,p\#\} \rightarrow \{s\#,p\#,city,qty\}$$

$$\{s\#\} \rightarrow \{city\}$$

$$\{s\#,p\#\} \rightarrow \{qty\}$$

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## Transitive Dependency

- Let us have R(A,B,C)
- FDs:  $A \rightarrow B$  and  $B \rightarrow C$  holds true for R
- Then we say that  $A \rightarrow C$  also holds for R
- The FD  $A \rightarrow C$  is a **transitive FD**..... **C is transitively dependent on A via B**

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## Transitive Dependency

FIRST

s#	status	city	p#	qty
s1	20	London	p1	100
s1	20	London	p2	100
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s2	10	Paris	p2	200
s3	10	Paris	p2	300
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s4	20	London	p4	500
s4	20	London	p5	600
s5	20	Kolkata	p1	200

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## Closure of a Set of Dependencies

❖ Some FDs might imply other

$$\begin{array}{ccc} \{s\#,p\# \} \rightarrow \{city\} & & \{s\#,p\# \} \rightarrow \{qty\} \\ & \downarrow & \\ \{s\#,p\# \} \rightarrow \{city,qty\} \end{array}$$

❖ The set of all FDs that are implied by a given set F of FDs is called the **closure of F**

Written as  $F^+$

$F^+$  is a superset of F

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## Inference Rules (Armstrong's Axioms)

❖ Let A, B, C, W be arbitrary subsets of the set of attributes of the given relation R

❖ **IR1: REFLEXIVITY:** If B is a subset of A  
Then  $A \rightarrow B$  holds true

FIRST

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s5	20	Kolkata	p1	200

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## Inference Rules (Armstrong's Axioms)

❖ Let A, B, C, W be arbitrary subsets of the set of attributes of the given relation R

❖ **IR1: REFLEXIVITY:** If B is a subset of A Then  $A \rightarrow B$  holds true

❖ **IR2: AUGMENTATION** If  $A \rightarrow B$  then  $AC \rightarrow BC$

(Notation: AC stands for  $A \cup C$ )

❖ **IR3: TRANSITIVE:** If  $A \rightarrow B$  and  $B \rightarrow C$ , then  $A \rightarrow C$

❖ These rules are

➤ **complete** (Given a set S of FDs, all FDs implied by S can be derived from S using the rules)

➤ **sound** (No additional FDs can be derived )

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## Inference Rules (Armstrong's Axioms)

### Some additional inference rules :

- **Self Determination:**  $A \rightarrow A$
- **Decomposition:** If  $A \rightarrow BC$ , then  $A \rightarrow B$  and  $A \rightarrow C$
- **Union:** If  $A \rightarrow B$  and  $A \rightarrow C$ , then  $A \rightarrow BC$
- **Composition:** If  $A \rightarrow B$  and  $C \rightarrow D$ , Then  $AC \rightarrow BD$
- **Pseudotransitivity:** If  $A \rightarrow B$  and  $WB \rightarrow C$ , then  $WA \rightarrow C$

These inference rules, as well as any other inference rules, can be deduced from IR1, IR2, and IR3 (completeness property)

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## Inference Rules (Armstrong's Axioms)

### •Example:

- Consider a relation R with attributes A,B,C,D,E , F and the FDs
  - ❖  $A \rightarrow BC$
  - ❖  $B \rightarrow E$
  - ❖  $CD \rightarrow EF$
- Show that FD  $AD \rightarrow F$  holds for R and is thus a member of the closure of the given set.

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## F Closure ( $F^+$ )

### ❖ Closure of set of Functional Dependencies

- Given a set  $F$  set of functional dependencies, there are certain other functional dependencies that are logically implied by  $F$ 
  - For e.g.: If  $A \rightarrow B$  and  $B \rightarrow C$ , then we can infer that  $A \rightarrow C$
- The set of **all** functional dependencies logically implied by  $F$  is the **closure** of  $F$
- We denote the *closure* of  $F$  by  $F^+$

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## F Closure example

- $R = \{A, B, C, G, H, I\}$
- $F = \{ A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H \}$
- Some members of  $F^+$ 
  - $A \rightarrow H$ 
    - by transitivity from  $A \rightarrow B$  and  $B \rightarrow H$
  - $AG \rightarrow I$ 
    - by augmenting  $A \rightarrow C$  with  $G$ , to get  $AG \rightarrow CG$  and then transitivity with  $CG \rightarrow I$
  - $CG \rightarrow HI$ 
    - by augmenting  $CG \rightarrow I$  to infer  $CG \rightarrow CGI$ , and augmenting of  $CG \rightarrow H$  to infer  $CGI \rightarrow HI$ , and then transitivity

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## F Closure

- To compute the closure of a set of functional dependencies  $F$ :

```

 $F^+ = F$ 
repeat
  for each functional dependency  $f$  in  $F^+$ 
    apply reflexivity and augmentation rules on  $f$ 
    add the resulting functional dependencies to  $F^+$ 
  for each pair of functional dependencies  $f_1$  and  $f_2$  in  $F^+$ 
    if  $f_1$  and  $f_2$  can be combined using transitivity
      then add the resulting functional dependency to  $F^+$ 
until  $F^+$  does not change any further
  
```

**NOTE:** We shall see an alternative procedure for this task later

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## Closure of a set of Attributes

- **Closure of a set of attributes**  $Z^+$  with respect to  $F$  is the set  $Z^+ = \{ \text{all attributes that are functionally determined by } Z \}$
- $Z^+$  can be calculated by repeatedly applying IR1, IR2, IR3 using the FDs in  $F$
- Given a relation  $R$ , a set  $Z$  of attributes  $R$ , and a set  $F$  of FDs that hold for  $R \rightarrow$  can be used to determine the set of all attributes of  $R(Z^+)$  that are functionally dependent on  $Z \rightarrow$  **Closure  $Z^+$  of  $Z$  under  $F$ .**

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## Closure of Attribute Sets( $^+$ )

- Given a relation  $R$ , and set of attributes , **define the closure of** under  $F$  (denoted by  $^+$ ) as the set of attributes that are functionally determined by under  $F$
- Algorithm to compute  $^+$ , the closure of under  $F$ 

```

result := ;
while (changes to result) do
  for each  $X \rightarrow Y$  in  $F$  do
    begin
      if  $X \subseteq \text{result}$  then  $\text{result} := \text{result} \cup Y$ 
    end

```

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## Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H\}$
- Find  $(AG)^+$  of the set of attributes  $\{AG\}$  under the set of FDs
  1.  $\text{result} = AG$
  2.  $\text{result} = ABG$  ( $A \rightarrow B$ )
  3.  $\text{result} = ABCG$  ( $A \rightarrow C$ )
  3.  $\text{result} = ABCGH$  ( $CG \rightarrow H$  and  $CG \subseteq AGBC$ )
  4.  $\text{result} = ABCGHI$  ( $CG \rightarrow I$  and  $CG \subseteq AGBCH$ )

**Is  $AG$  a candidate key?**

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## Example of Attribute Set Closure

### □ Is $AG$ a candidate key?

#### 1. Is $AG$ a super key?

1. Does  $AG \rightarrow R? \iff \text{Is } (AG)^+ \supseteq R$

#### 2. Is any subset of $AG$ a superkey?

1. Does  $A \rightarrow R? \iff \text{Is } (A)^+ \supseteq R$

2. Does  $G \rightarrow R? \iff \text{Is } (G)^+ \supseteq R$

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## Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

### □ Testing for Superkey:

- To test if  $X$  is a superkey, we compute  $X^+$  and check if  $X^+$  contains all attributes of  $R$ .

### □ Testing Functional Dependencies

- To check if a functional dependency  $\alpha \rightarrow \beta$  holds (or, in other words, is in  $F^+$ ), just check if  $\beta \subseteq \alpha^+$ .
- That is, we compute  $\alpha^+$  by using attribute closure, and then check if it contains  $\beta$ .
- Is a simple and cheap test, and very useful

### □ Computing closure of $F$

- For each  $\gamma \subseteq R$ , we find the closure  $\gamma^+$ , and for each  $S \subseteq \gamma^+$ , we output a functional dependency  $\gamma \rightarrow S$ .

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## Example of Attribute Set Closure

- *Example:*
- $R = (A, B, C, D, E, F)$
- $F = \{A \rightarrow BC, E \rightarrow CF, B \rightarrow E, CD \rightarrow EF\}$
- Compute the closure  $(AB)^+$  of the set of attributes  $\{AB\}$  under the set of FDs

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## Cover

- ❖ Let  $S1$  and  $S2$  be two set of FDs.
- ❖ If every FD implied by  $S1$  is implied by  $S2$  – i.e.,  $S1^+$  is a subset of  $S2^+ \rightarrow S2$  is a cover of  $S1$
- ❖ If  $S2$  is a cover of  $S1$  and  $S1$  is a cover of  $S2 \rightarrow$   
 $S1^+ = S2^+ \rightarrow S1$  and  $S2$  are equivalent

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## Canonical Cover

❖ Sets of functional dependencies may have redundant dependencies that can be inferred from the others

✓ For example:  $A \rightarrow C$  is redundant in:  $\{A \rightarrow B, B \rightarrow C\}$

✓ **Parts of a functional dependency may be redundant**

✓ E.g.: on RHS:  $\{A \rightarrow B, B \rightarrow C, A \rightarrow CD\}$  can be simplified to  $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$

✓ E.g.: on LHS:  $\{A \rightarrow B, B \rightarrow C, AC \rightarrow D\}$  can be simplified to  $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$

❖ Intuitively, a canonical cover of F is a “minimal” set of functional dependencies equivalent to F, having no redundant dependencies or redundant parts of dependencies

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## Canonical Cover: RHS

■  $\{A \rightarrow B, B \rightarrow C, A \rightarrow CD\} \rightarrow \{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$

▪ (1)  $A \rightarrow CD \rightarrow A \rightarrow C$  and  $A \rightarrow D$  (2)  $A \rightarrow B, B \rightarrow C \rightarrow A \rightarrow C$

▪  $A^+ = ABCD$

■  $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\} \rightarrow \{A \rightarrow B, B \rightarrow C, A \rightarrow CD\}$

▪  $A \rightarrow B, B \rightarrow C \rightarrow A \rightarrow C$

▪  $A \rightarrow C, A \rightarrow D \rightarrow A \rightarrow CD$

▪  $A^+ = ABCD$

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## Canonical Cover: LHS

- $\{A \rightarrow B, B \rightarrow C, AC \rightarrow D\} \rightarrow \{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$ 
  - $A \rightarrow B, B \rightarrow C \Rightarrow A \rightarrow C \Rightarrow A \rightarrow AC$
  - $A \rightarrow AC, AC \rightarrow D \Rightarrow A \rightarrow D$
  - $A^+ = ABCD$
- $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\} \rightarrow \{A \rightarrow B, B \rightarrow C, AC \rightarrow D\}$ 
  - $A \rightarrow D \Rightarrow AC \rightarrow D$
  - $AC^+ = ABCD$

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## Extraneous Attributes

- Consider a set  $F$  of functional dependencies and the functional dependency  $\alpha \rightarrow \beta$  in  $F$ .
  - Attribute  $A$  is **extraneous** in  $\alpha$  if  $A \in \alpha$  and  $F$  logically implies  $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$ .
  - Attribute  $A$  is **extraneous** in  $\beta$  if  $A \in \beta$  and the set of functional dependencies  $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$  logically implies  $F$ .
- **Note:** implication in the opposite direction is trivial in each of the cases above, since a “stronger” functional dependency always implies a weaker one
  - Example: Given  $F = \{A \rightarrow C, AB \rightarrow C\}$   
 $B$  is **extraneous** in  $AB \rightarrow C$  because  $\{A \rightarrow C, AB \rightarrow C\}$  logically implies  $A \rightarrow C$  (i.e. the result of dropping  $B$  from  $AB \rightarrow C$ ).
  - Example: Given  $F = \{A \rightarrow C, AB \rightarrow CD\}$   
 $C$  is **extraneous** in  $AB \rightarrow CD$  since  $AB \rightarrow C$  can be inferred even after deleting  $C$

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## Testing if an Attribute is Extraneous

- Consider a set  $F$  of functional dependencies and the functional dependency  $\alpha \rightarrow \beta$  in  $F$ .
- To test if attribute  $A \in \alpha$  is extraneous in  $\alpha$ 
  1. compute  $(\{\alpha\} - A)^+$  using the dependencies in  $F$
  2. check that  $(\{\alpha\} - A)^+$  contains  $\beta$ ; if it does,  $A$  is extraneous in  $\alpha$
- To test if attribute  $A \in \beta$  is extraneous in  $\beta$ 
  1. compute  $\alpha^+$  using only the dependencies in  $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ ,
  2. check that  $\alpha^+$  contains  $A$ ; if it does,  $A$  is extraneous in  $\beta$

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## Canonical Cover

- A **canonical cover** for  $F$  is a set of dependencies  $F_c$  such that
  - $F$  logically implies all dependencies in  $F_c$  and
  - $F_c$  logically implies all dependencies in  $F$ , and
  - No functional dependency in  $F_c$  contains an extraneous attribute, and
  - Each left side of functional dependency in  $F_c$  is unique.
- To compute a canonical cover for  $F$ :
  - repeat**
    - Use the union rule to replace any dependencies in  $F$ 
 $\alpha_1 \rightarrow \beta_1$  and  $\alpha_1 \rightarrow \beta_2$  with  $\alpha_1 \rightarrow \beta_1 \beta_2$
    - Find a functional dependency  $\alpha \rightarrow \beta$  with an extraneous attribute either in  $\alpha$  or in  $\beta$
    - If an extraneous attribute is found, delete it from  $\alpha \rightarrow \beta$
  - until**  $F$  does not change
- **Note:** Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied

Minimal Set  
of FDs  
Irreducible  
set of FDs

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## Computing a Canonical Cover

- $R = (A, B, C)$   
 $F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$
- Combine  $A \rightarrow BC$  and  $A \rightarrow B$  into  $A \rightarrow BC$ 
  - Set is now  $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- $A$  is extraneous in  $AB \rightarrow C$ 
  - Check if the result of deleting  $A$  from  $AB \rightarrow C$  is implied by the other dependencies  
 Yes: in fact,  $B \rightarrow C$  is already present!
  - Set is now  $\{A \rightarrow BC, B \rightarrow C\}$
- $C$  is extraneous in  $A \rightarrow BC$ 
  - Check if  $A \rightarrow C$  is logically implied by  $A \rightarrow B$  and the other dependencies  
 Yes: using transitivity on  $A \rightarrow B$  and  $B \rightarrow C$ .  
 Can use attribute closure of  $A$  in more complex cases
- The canonical cover is:  $A \rightarrow B$   
 $B \rightarrow C$

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## Irreducible set of Functional Dependencies

❖  $R(p\#, pname, color, wt, city)$

❖ The set of FDs:

❖  $p\# \rightarrow pname$

❖  $p\# \rightarrow color$

❖  $p\# \rightarrow wt$

❖  $p\# \rightarrow city$

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## Irreducible set of Functional Dependencies

- A set  $F$  of FD is said to be **irreducible** if and only if it satisfies the following
  - The RHS ( the dependent) of every FD in  $F$  involves just one attribute ,i.e., it is a singleton set
  - The LHS (the determinant) of every FD in  $F$  is irreducible in turn- meaning that no attribute can be discarded from the determinant without changing the closure  $F^+ \rightarrow$  Such an FD is called **Left irreducible**
  - No FD in  $F$  can be discarded from  $F$  without changing the closure  $F^+$  ,i.e., without converting  $F$  into some set not equivalent to  $F$

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## Irreducible set of Functional Dependencies

- The RHS (the dependent) of every FD in  $F$  involves just one attribute ,i.e., it is a singleton set

 $p\# \rightarrow \{pname, color\}$ 
 $p\# \rightarrow color$ 
 $p\# \rightarrow wt$ 
 $p\# \rightarrow city$ 
 $p\# \rightarrow pname$ 
 $p\# \rightarrow color$ 
 $p\# \rightarrow wt$ 
 $p\# \rightarrow city$ 

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## Irreducible set of Functional Dependencies

- The LHS (the determinant) of every FD in  $F$  is irreducible in turn—meaning that no attribute can be discarded from the determinant without changing the closure  $F^+$  → Such an FD is called **Left irreducible**

$\{p\#, pname \rightarrow color$

$p\# \rightarrow pname$

$p\# \rightarrow wt$

$p\# \rightarrow city$

**NOT LEFT-IRREDUCIBLE**

$p\# \rightarrow pname$

$p\# \rightarrow color$

$p\# \rightarrow wt$

$p\# \rightarrow city$

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## Irreducible set of Functional Dependencies

- No FD in  $F$  can be discarded from  $F$  without changing the closure  $F^+$ , i.e., without converting  $F$  into some set not equivalent to  $F$

$p\#, \rightarrow p\#$

$p\#, \rightarrow color$

$p\# \rightarrow pname$

$p\# \rightarrow wt$

$p\# \rightarrow city$

$p\# \rightarrow pname$

$p\# \rightarrow color$

$p\# \rightarrow wt$

$p\# \rightarrow city$

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## Irreducible set of Functional Dependencies

- ❖ **Example:** Consider a relation  $R(A, B, C, D)$  and FDs
- ❖  $A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C, AC \rightarrow D$
- ❖ Compute an irreducible set of FDs that is equivalent to this given set

Solution:

$A \rightarrow B$

$B \rightarrow C$

$A \rightarrow D$

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## Equivalence of Sets of Functional Dependencies

- Let  $F$  &  $G$  are two functional dependency sets.
  - These two sets  $F$  &  $G$  are equivalent if  $F^+ = G^+$
  - Equivalence means that every functional dependency in  $F$  can be inferred from  $G$ , and every functional dependency in  $G$  can be inferred from  $F$
- $F$  and  $G$  are equal only if
  - $F$  covers  $G$ : Means that all functional dependency of  $G$  are logically members of functional dependency set  $F \rightarrow F \supseteq G$ .
  - $G$  covers  $F$ : Means that all functional dependency of  $F$  are logically members of functional dependency set  $G \rightarrow G \supseteq F$

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## Practice problem

### Find if a given functional dependency is implied from a set of Functional Dependencies:

1. For:  $A \rightarrow BC, CD \rightarrow E, E \rightarrow C, D \rightarrow AEH, ABH \rightarrow BD, DH \rightarrow BC$ 
  - a. Check:  $BCD \rightarrow H$
  - b. Check:  $AED \rightarrow C$
2. For:  $AB \rightarrow CD, AF \rightarrow D, DE \rightarrow F, C \rightarrow G, F \rightarrow E, G \rightarrow A$ 
  - a. Check:  $CF \rightarrow DF$
  - b. Check:  $BG \rightarrow E$
  - c. Check:  $AF \rightarrow G$
  - d. Check:  $AB \rightarrow EF$
3. For:  $A \rightarrow BC, B \rightarrow E, CD \rightarrow EF$ 
  - a. Check:  $AD \rightarrow F$

### Find Candidate Key using Functional Dependencies:

1. Relational Schema  $R(ABCDE)$ . Functional dependencies:  $AB \rightarrow C, DE \rightarrow B, CD \rightarrow E$
2. Relational Schema  $R(ABCDE)$ . Functional dependencies:  $AB \rightarrow C, C \rightarrow D, B \rightarrow EA$

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## 3NF Decomposition

➤  $R = ABCDEFGH$

➤  $FD = \{ A \rightarrow B, ABCD \rightarrow E, EF \rightarrow GH, ACDF \rightarrow EG \}$

### ➤ Compute Canonical cover:

- Make RHS a single attribute:  $\{ A \rightarrow B, ABCD \rightarrow E, EF \rightarrow G, EF \rightarrow H, ACDF \rightarrow E, ACDF \rightarrow G \}$
- Minimize LHS:  $ACD \rightarrow E$  instead of  $ABCD \rightarrow E$
- Eliminate redundant FDs:
  - ✓ Can  $ACDF \rightarrow E$  be removed?
  - ✓ Can  $ACDF \rightarrow G$  be removed?

➤ Canonical cover =  $\{ A \rightarrow B, ACD \rightarrow E, EF \rightarrow G, EF \rightarrow H \}$

➤ Super key =  $ACDF$

➤ 3NF decomposition:  $\{ AB, ACDE, EFG, EFH, \} \cup \{ ACDF \}$

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## 3NF Decomposition

- $R = CSJDPQV$
- $FD = \{C \rightarrow CSJDPQV, SD \rightarrow P, JP \rightarrow C, J \rightarrow S\}$
- Compute Canonical cover:
  - Make RHS a single attribute:
  - Minimize LHS:
  - Eliminate redundant FDs:
    - ✓ Can be removed?
    - ✓ Can be removed?
- Canonical cover=
- Super key=
- 3NF decomposition:  $\{CJDQV, JPC, JS, SDP\}$

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## BCNF Decomposition

- To check if a non-trivial dependency  $\alpha \rightarrow \beta$  causes a violation of BCNF
  1. compute  $\alpha^+$  (the attribute closure of  $\alpha$ ), and
  2. verify that it includes all attributes of  $R$ , that is, it is a super key of  $R$ .
- **Simplified test:** To check if a relation schema  $R$  is in BCNF, it suffices to check only the dependencies in the given set  $F$  for violation of BCNF, rather than checking all dependencies in  $F^+$ .
  - ❖ If none of the dependencies in  $F$  causes a violation of BCNF, then none of the dependencies in  $F^+$  will cause a violation of BCNF either.
- However, **simplified test using only  $F$  is incorrect when testing a relation in a decomposition of  $R$** 
  - Consider  $R = (A, B, C, D, E)$ , with  $F = \{A \rightarrow B, BC \rightarrow D\}$ 
    - Decompose  $R$  into  $R_1 = (A, B)$  and  $R_2 = (A, C, D, E)$
    - Neither of the dependencies in  $F$  contain only attributes from  $(A, C, D, E)$  so we might be misled into thinking  $R_2$  satisfies BCNF.
    - In fact, dependency  $AC \rightarrow D$  in  $F^+$  shows  $R_2$  is not in BCNF.

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## BCNF Decomposition Algorithm

1. For all dependencies  $A \rightarrow B$  in  $F^+$ , check if  $A$  is a superkey
  - By using attribute closure
2. If not, then
  - Choose a dependency in  $F^+$  that breaks the BCNF rules, say  $A \rightarrow B$
  - Create  $R_1 = A \rightarrow B$
  - Create  $R_2 = A (R - (B - A))$
  - Note that:  $R_1 \cap R_2 = A$  and  $A \rightarrow AB (= R_1)$ , so this is lossless decomposition
3. Repeat for  $R_1$ , and  $R_2$ 
  - By defining  $F_1^+$  to be all dependencies in  $F$  that contain only attributes in  $R_1$
  - Similarly  $F_2^+$

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## BCNF Decomposition Algorithm

```

result := {R};
done := false;
compute  $F^+$ ;

```

```

while (not done) do

```

```

  if (there is a schema  $R_i$  in result that is not in BCNF)

```

```

    then begin

```

```

      let  $\alpha \rightarrow s$  be a nontrivial functional dependency that holds on  $R_i$ 
      such that  $\alpha \rightarrow R_i$  is not in  $F^+$ ,
      and  $\alpha \cap s = \emptyset$ ;

```

```

      result := (result -  $R_i$ )  $\cup$  ( $R_i - s$ )  $\cup$  ( $\alpha, s$ );

```

```

    end

```

```

  else done := true;

```

Note: each  $R_i$  is in BCNF, and decomposition is lossless-join.

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## BCNF Decomposition Example

- $R = (A, B, C)$   
 $F = \{A \rightarrow B, B \rightarrow C\}$
- Key =  $\{A\}$
- $R$  is not in BCNF ( $B \rightarrow C$  but  $B$  is not super key)
- Decomposition  
 $R_1 = (B, C)$   
 $R_2 = (A, B)$

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## BCNF Decomposition Example

- Original relation  $R$  and functional dependency  $F$   
 $R = (\text{branch\_name}, \text{branch\_city}, \text{assets}, \text{customer\_name}, \text{loan\_number}, \text{amount})$   
 $F = \{\text{branch\_name} \rightarrow \text{assets}, \text{branch\_city}, \text{loan\_number} \rightarrow \text{amount}, \text{branch\_name}\}$   
  
 Key =  $\{\text{loan\_number}, \text{customer\_name}\}$
- **Decomposition**
  - $R_1 = (\text{branch\_name}, \text{branch\_city}, \text{assets})$
  - $R_2 = (\text{branch\_name}, \text{customer\_name}, \text{loan\_number}, \text{amount})$
  - $R_3 = (\text{branch\_name}, \text{loan\_number}, \text{amount})$
  - $R_4 = (\text{customer\_name}, \text{loan\_number})$
- Final decomposition  
 $R_1, R_3, R_4$

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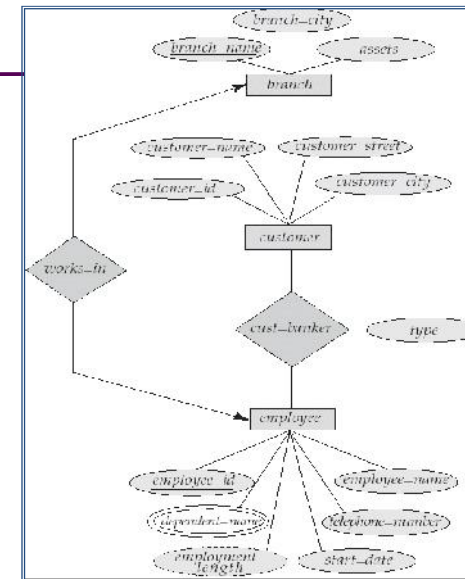
## ERD vs NF

- When an E-R diagram is carefully designed, identifying all entities correctly, the tables generated from the E-R diagram should not need further normalization
- However, in a real (imperfect) design, there can be functional dependencies from non-key attributes of an entity to other attributes of the entity
  - Example: an *employee* entity with attributes *department\_name* and *building*, and a functional dependency *department\_name* → *building*
  - Good design would have made *department* an entity
- Functional dependencies from non-key attributes of a relationship set possible, but rare --- most relationships are binary

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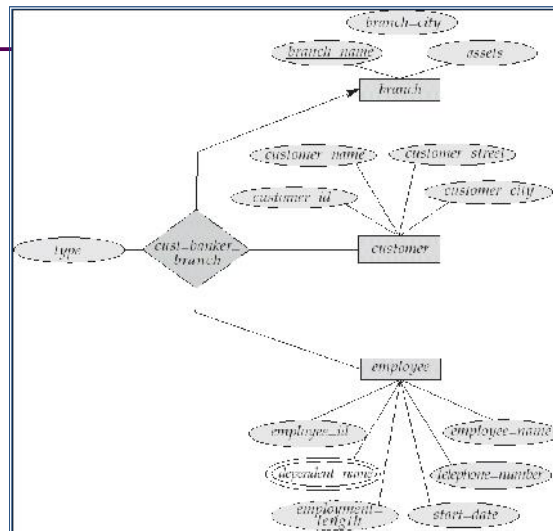
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Thank You

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