

Q.1. A _____ in a tree is a vertex of degree one.

Q.2. From the following formulae find the tautology, contingency and contradiction

i) $\neg (A \rightarrow B) \vee (\neg A \vee (A \wedge B))$

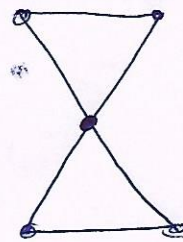
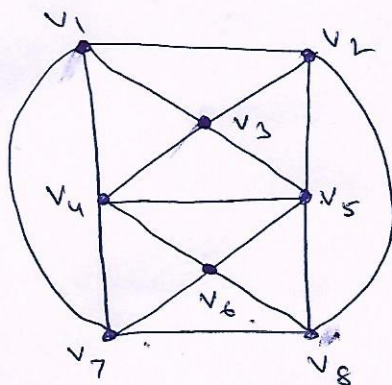
ii) $(H \rightarrow (I \wedge J)) \rightarrow (H \rightarrow I)$

iii) $(P \leftrightarrow Q) \rightarrow (P \wedge Q) \vee (\neg P \wedge Q)$

Q.3. Determine whether the following are equivalent, using biconditional statement.

$(p \rightarrow q) \rightarrow t \Leftrightarrow (p \wedge \neg q) \rightarrow t$.

Q.4. Show that $E \geq 3V - 6$ for the connected planar graph shown below.



Q.5. Consider the binary operation $*$ on \mathbb{Q} , the set of rational numbers, defined by

$$a * b = a^2 + b^2 \quad \forall a, b \in \mathbb{Q}$$

Determine whether $*$ is commutative

Q.6. Consider the binary operation $*$ on \mathbb{Q} , defined by

$$a * b = a + b - ab \quad \forall a, b \in \mathbb{Q}$$

Determine whether $*$ is associative

Q.7. Define ~~Group~~ Groupoid, Semigroup and Monoid with example.

Q.8. When Monoid becomes group. Write the conditions satisfied by a group.

Q.9 Define coset. ~~show that~~.

Let G be an additive group of integers i.e.

$$G = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

and H be the subgroup of G obtained by multiplying each element of G by 3. ~~Then show that the three right coset H , $H+1$ and $H+2$ are all distinct.~~

Show that exists three disjoint right cosets.
(Hint: H , $H+1$, $H+2$).

Q.10. Find the product of permutations and show that it is not commutative.

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \text{ and } g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$$

Q.11. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 4 & 3 & 1 \end{pmatrix}$ as a ~~two~~ product of transpositions

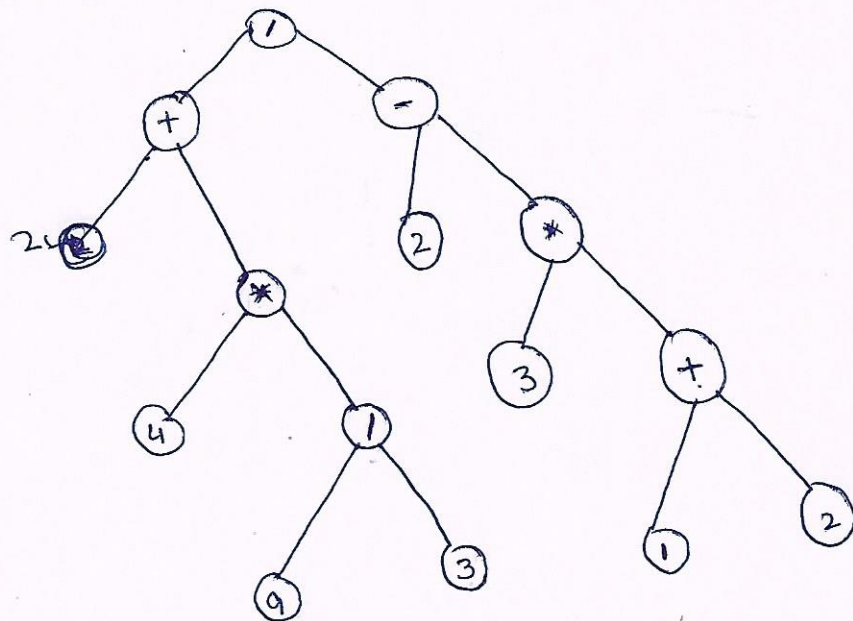
Q.12. How many generators are there of the cyclic group G of order 8?

Q.14. ^{Show that} the multiplicative group $\{1, \omega, \omega^2\}$ is a cyclic group.

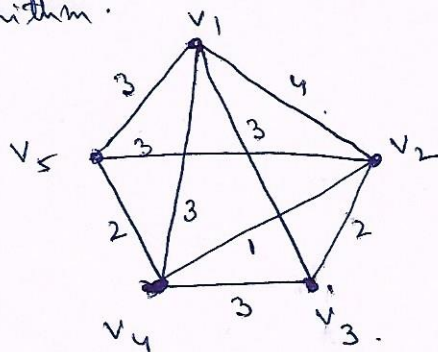
Q.15. Give an example of a ring with zero divisor.

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Q.16. For the set $I_4 = \{0, 1, 2, 3\}$, show that the modulo 4 system is a ring.

Q.17. Determine the value of the expression represented in a binary tree shown in figure.



Q.18 Find the minimal ~~path~~ spanning tree of the weighted graph of the following figure using Prim's algorithm, and also by Kruskal algorithm.



Q.19 Prove that the maximum number of vertices on level n of binary tree is 2^n .

Q. 20 Prove - that - the maximum number of vertices in a binary tree of depth d is $2^d - 1$ where $d \geq 1$

Q. 21 Given an ~~ex~~ example of complete binary tree .

