1. Solve : $(D^2+4D+3)\,y(x) = e^{-2x}$

Piyush Chandra Chandra
ID-1811100001027

Ans Here we have

$$(D^2+4D+3)\,y(x) = e^{-2x}$$

The auxiliary equation is

$$m^2+4m+3=0$$

$$(m+1)(m+3)=0$$

$$m=-1,-3$$

So the roots are real and different

Complementary function $(C.F) = c_1 e^{-x} + c_2 e^{-3x}$

particular Integral (P.I) $= \dfrac{1}{D^2+4D+3} e^{-2x}$

$$= \frac{1}{4-8+3} e^{-2x}$$

$$= -e^{-2x}$$

Hence the Complete Solution is $y = C.F + P.I$

$$y = c_1 e^{-x} + c_2 e^{-3x} - e^{-2x}$$

2. Solve: $(D^2+a^2)y = \sin ax$

<u>Ans</u> Here we have

$$(D^2+a^2)y = \sin ax$$

Here the auxiliary equation is

$$m^2+a^2=0$$

$$m^2=-a^2$$

$$m=\pm ai$$

So the Roots are imaginary

Complementary function $(C.F) = C_1 \cos ax + C_2 \sin ax$

particular Integral $(P.I) = \frac{1}{D^2+a^2} \sin ax$

$$= x\frac{1}{2D} \sin ax$$

$\left[\because f(-a^2)=0\right.$ then $\frac{1}{f(D^2)}\cos ax = x\frac{1}{f'(-a^2)}\cos ax\left.\right]$

$$= \frac{x}{2}\left(\frac{-1}{a}\cos ax\right)$$

$$= -\frac{x}{2a}\cos ax$$

Hence the complete solution is

$$y = C.F + P.I$$

$$y = C_1\cos ax + C_2 \sin ax - \frac{x}{2a}\cos ax$$

3. Solve: $(D^2+2D+1)y=0$

<u>Ans</u> Here we have

$$(D^2+2D+1)y=0$$

Here the auxiliary equation is

$$m^2+2m+1=0$$

$$(m+1)(m+1)=0$$

$$m=-1,-1$$

So the roots are real and equal

Complementary function $(C.F) = (C_1+C_2x)e^{-x}$

particular Integral $(P.I) = 0$

Hence the complete solution is $y = C.F + P.I$

$$y=(C_1+C_2x)e^{-x}$$

---

4. $(D^2+4D-12)y=(x-1)e^{2x}$

<u>Ans</u> Here we have

$$(D^2+4D-12)y=(x-1)e^{-2x}$$

Here the auxiliary equation is

$$m^2+4m-12=0$$

$$(m-2)(m+6)=0$$

$$m=2,-6$$

So the roots are real and different

Complementary function (C.F) $= C_1 e^{2x} + C_2 e^{-6x}$

particular Integral $= \frac{1}{D^2+4D-12} x e^{2x} - \frac{1}{D^2+4D-12} e^{2x}$

where $I_1 = \frac{1}{D^2+4D-12} x e^{2x}$

$$= e^{2x} \frac{1}{(D+2)^2+4(D+2)-12} x$$

$$= e^{2x} \frac{1}{D^2+8D} x$$

$$= e^{2x} \frac{1}{D(D+8)} x$$

$$= e^{2x} \frac{1}{8D\left(1+\frac{D}{8}\right)} x$$

$$= e^{2x} \frac{1}{8D} \left(1+\frac{D}{8}\right)^{-1} x$$

$$= e^{2x} \frac{1}{8D} \left(1 - \frac{D}{8} + \frac{D^2}{64}\right) x$$

$$= \frac{e^{2x}}{8} \left(\frac{1}{D} - \frac{1}{8} + \frac{D}{64}\right) x$$

$$= \frac{e^{2x}}{8} \left(\frac{x^2}{2} - \frac{x}{8} + \frac{1}{64}\right)$$

and $I_2 = -\frac{1}{D^2+4D-12} e^{2x}$

$$= -x \frac{1}{2D+4} e^{2x} \quad \left[\because f(a)=0, \text{ so } \frac{1}{f(D)} e^{ax} = x \frac{1}{f(a)} e^{ax}\right]$$

$$= -x \frac{1}{8} e^{2x}$$

$$= -\frac{x}{8} e^{2x}$$

Hence the complete solution is

$$y = C.F + P.I$$

$$y = C_1 e^{2x} + C_2 e^{-6x} + \frac{x^2 e^{2x}}{16} - \frac{9x e^{2x}}{64}$$

The term $\frac{e^{2x}}{512}$ has been omitted from the P.I

Since $C_1 e^{2x}$ is present in the C.F