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es: $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$$L(S) = \left\{ \lambda_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \right\}$$

Express $(-1,2,4)$ as the linear combination of $\alpha = (-1,2,0)$, $\beta = (0,-1,1)$ and $\gamma = (3,-4,2)$

Let → $(1,2,4) = a(-1,2,0) + b(0,-1,1) + c(3,-4,2)$

$$\Rightarrow \left.\begin{aligned} -a+3c &= 1 \\ 2a-b-4c &= 2 \\ b+2c &= 4 \end{aligned}\right\}$$

solving $a = 4$, $b = 2$, $c = 1$

Show that $(-1,2,1)$, $(3,0,-1)$ and $(-5,4,3)$ are linearly dependent

$a(-1,2,1) + b(3,0,-1) + c(-5,4,3) = (0,0,0)$

$$\left.\begin{aligned} -a+3b-5c &= 0 \\ 2a+0.b+4c &= 0 \\ a-b+3c &= 0 \end{aligned}\right\}$$

Let at least one a_i i.e $a_k \neq 0$

$$\Rightarrow \sum_{\substack{i=1 \\ i\neq k}}^{n} a_i\alpha_i + a_k\alpha_k = 0$$

$$\Rightarrow a_k\alpha_k = -\sum_{\substack{i=1 \\ i\neq k}}^{n} a_i\alpha_i$$

$$\Rightarrow \alpha_k = -\sum_{\substack{i=1 \\ i\neq k}}^{n} \frac{a_i}{a_k}\alpha_i$$

Now $a_k \neq 0 \Rightarrow a_k^{-1} \in F$

so α_k is the linear combination others.

Linear span

Let V be a vector space over the field F and S be a nonempty subset of V. Then the linear span of S is defined as the set of all linear combinations of finite sets of elements of S.

denoted by $L(S)$

If $S = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$.

$a_i \in F$, $\alpha_i \in S$

$$L(S) = \{a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n\}$$