(7.) Maximize, $Z = 5x_1 + 3x_2$

subject to $3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$

$x_1, x_2 \geq 0$

Introducing slack variables, we rewrite the problem in standard format

Max. $Z = 5x_1 + 3x_2 + 0.x_3 + 0.x_4$

S.t $3x_1 + 5x_2 + x_3 + 0.x_4 = 15$

$5x_1 + 2x_2 + 0.x_3 + x_4 = 10$

$x_j \geq 0 \quad j = 1,2,3,4$

Now,
$x_B$ = Basis variable
$B$ = Basis vector
$C_B$ = Coeff of Basis variable in objective fn

Here, $(a_3, a_4)$ form the initial Bases

Table

| | | | $c_j$ | 5 | 3 | 0 | 0 | Ratio | operation | |
|---|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | B | $X_B$ | b | $a_1$ | $a_2$ | $a_3$ | $a_4$ | min | | Remarks |
| 0 | $a_3$ | $x_3$ | 15 | 3 | 5 | 1 | 0 | $15/3$ | | $a_1 \rightarrow$ entering, $a_4 \rightarrow$ departing |
| ← 0 | $a_4$ | $x_4$ | 10 | $\boxed{5}$ ↑ | 2 | 0 | 1 ↓ | $10/5$ min | | $\boxed{5} \rightarrow$ key element |
| $z_j - c_j$ | | | | $-5$ | $-3$ | 0 | 0 | | | |
| ← 0 | $a_3$ | $x_3$ | 9 | 0 | $\boxed{\frac{19}{5}}$ | 1 | $-3/5$ | $\frac{9.5}{19}$ | $R_2' = \frac{1}{5}R_1$ | $a_2 \rightarrow$ entering |
| 5 | $a_1$ | $x_1$ | 2 | 1 | $2/5$ ↑ | 0 ↓ | $1/5$ | $\frac{2.5}{2}$ | $R_1' = R_1 - 3R_2'$ | $a_3 \rightarrow$ departing, $\frac{19}{5} \rightarrow$ key element |
| $z_j - c_j$ | | | | 0 | $-1$ | 0 | 1 | | | |
| 3 | $a_2$ | $x_2$ | $\frac{45}{19}$ | 0 | 1 | $\frac{5}{19}$ | $\frac{-3}{19}$ | | $R_1' = \frac{5}{19}R_1$ | |
| 5 | $a_1$ | $x_1$ | $\frac{20}{19}$ | 1 | 0 | $-\frac{2}{19}$ | $\frac{5}{19}$ | | $R_2' = R_2 - \frac{2}{5}R_1'$ | |
| $z_j - c_j$ | | | | 0 | 0 | $5/19$ | 1 | | | |

Here all $z_j - c_j \geq 0$ and thus optimal condition is achived and optimal soln.s are

$x_1 = \frac{20}{19}, \quad x_2 = \frac{45}{19} \qquad Z_{max} = \frac{100 + 135}{19} = \frac{235}{19}$ Ans.

| | | | $c_j$ | $c_1$ | $c_2$ | 0 | 0 | Ratio | Operation |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | B | $x_B$ | b | $a_1$ | $a_2$ | $a_3$ | $a_4$ | | |
| $c_1$ | $a_1$ | $x_1$ | $b_1'$ | 1 | $a_{12}'$ | $a_{13}'$ | 0 | | |
| 0 | $a_4$ | $x_4$ | $b_2'$ | 0 | $a_{22}'$ | $a_{23}'$ | 1 | | |
| $z_j -$ | $c_j$ | | | | | | | | |

~~Now $a_{11}$~~ Since $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is left ~~b~~, so by raw operation we have to make

$$a_{11} \longrightarrow 1$$
$$a_{21} \longrightarrow 0$$

$$R_1' = \frac{1}{a_{11}} R_1$$

$$R_2' = a_{21} - a_{21} \times R_2$$

Continue the process until all $z_j - c_j \geq 0$

$\downarrow$

Optimality Condition

To calculate departing variable

Consider the minimum ratio from the column of entering variable

ie For $a_1$ if entering

Min Ratio $= b_1/a_{11}$ , $b_2/a_{21}$

Consider the minimum value

If $\frac{b_1}{a_{11}}$ is min.

Then $x_3$ is departing

The Element in the junction of entering and departing variable is called Key element

here $a_{11}$ will be Key element

In next table $x_3$ will be replaced by $a_3$
$C_3$ " " " " $C_1$

Table

| | | | $c_j$ | $c_1$ | $c_2$ | 0 | 0 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | $B$ | $x_B$ | $b$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | Min Ratio | operation | Remark |
| 0 | $a_3$ | $x_3$ | $b_1$ | $\boxed{a_{11}}$ | $a_{12}$ | 1 | 0 | $b_1/a_{11}$ | | |
| 0 | $a_4$ | $x_4$ | $b_2$ | $a_{21}$ | $a_{22}$ | 0 | 1 | $b_2/a_{21}$ | | |
| $Z_j - C_j$ | | | | $-c_1$ | $-c_2$ | | | | | |

Min Ratio is applicable only when the ratio is nonnegative

$$\cancel{\text{Ratio} = \frac{a_{ij}}{b_i} > 0}$$

~~Ratio =~~

First calculate $Z_j - C_j$

$$Z_j - C_j = \left(\sum_i a_{ij} \times c_i\right) - c_j$$

ie, $a_{11} \times 0 + a_{21} \times 0 - c_1 = -c_1$

$a_{12} \times 0 + a_{22} \times 0 - c_2 = -c_2$

Consider the minimum most value

If $-c_1$ is minimum ~~then~~

$a_1$ is the entering vector

ie, $x_1$ " " " variable

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$$\text{Max } z = c_1x_1 + c_2x_2$$

subject to

$$a_{11}x_1 + a_{12}x_2 \le b_1$$

$$a_{21}x_1 + a_{22}x_2 \le b_2$$

$$x_1, x_2 \geq 0$$

$$\text{Max } Z = c_1x_1 + c_2x_2 + 0.x_3 + 0x_4$$

subject to

$$a_{11}x_1 + a_{12}x_2 + x_3 + 0x_4 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + 0x_3 + x_4 = b_2$$

$$x_j \geq 0 \qquad j = 1,2,3,4$$

$X_B \rightarrow$ Basis variables

$B \rightarrow$ Basis vector $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$

or $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$

$C_B$: Coefficient of Basis variables

here Basis variables are $x_3, x_4$

So $C_B = 0, 0$

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# Simplex Table

$$\text{Max } Z = CX$$
$$\text{st } AX \leq (\geq) b$$
$$X \geq 0$$

Note.

For "Max" Prob

Case I

Constraints must be $\leq$ type

For "Min" Problem

Case II

it must be $\geq$ type

Suppose for case I there is a constraint

$$3x_1 + 5x_2 \geq 7$$

Multiply by -ve sign

$$\boxed{-3x_1 - 5x_2 \leq -7}$$

It will be $\leq$ type

Same For Case II also

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Q3/ Three products are processed through three different operations. The times (in minutes) required per unit of each product, the daily capacity of the operations (in minutes per day) and the profit per sold for each product (in Rs) are as follows

| operation | Time / unit Product-I | Product II | Product III | Operation Capacity |
|---|---|---|---|---|
| 1 | 3 | 4 | 3 | 42 |
| 2 | 5 | 0 | 3 | 45 |
| 3 | 3 | 6 | 2 | 41 |
| Profit | 3 | 2 | 1 | |

The zero time indicates that the product doesnot require the given operation. The problem is to determine the optimum daily production for three products that maximize the profit.

Formulate the above problem as a linear programming prob.

Soln/ If $x_1, x_2, x_3$ are the number of units produced daily of products 1, 2, 3 then

$$\text{Max } Z = 3x_1 + 2x_2 + x_3$$

s.t.
$$3x_1 + 4x_2 + 2x_3 \leq 42$$
$$5x_1 \quad + 3x_3 \leq 45$$
$$3x_1 + 6x_2 + 2x_3 \leq 41$$

$$x_1, x_2, x_3 \geq 0$$

Q2 At a cattle breeding firm it is prescribed that food ration for one animal must contain at least 14, 22 and 1 units of nutrients A, B and C respectively. Two different kinds of fodder are available. Each unit weight of these two contains the following amounts of the three nutrients.

| | Fooder 1 | Fooder 2 |
|---|---|---|
| Nutrient A | 2 | 1 |
| Nutrient B | 2 | 3 |
| Nutrient C | 1 | 1 |

It is given that the costs of unit quantity of fooder 1 and 2 are 3 and 2 monetary units respectively. Pose a linear programming prob in terms of minimizing the cost purchasing the fooders for the above cattle breeding firm.

Soln Let $x_1, x_2$ are unit weights of fooder 1,

$\therefore$ Min $Z = 3x_1 + 2x_2$

st $2x_1 + x_2 \geq 14$

$2x_1 + 3x_2 \geq 22$

$x_1 + x_2 \geq 1$ $\quad x_1, x_2 \geq 0$.

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## Formulation of LPP

1) A factory in engaged in manufacturing two products A and B which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1 and 1 hours respectively and for one unit of B are 3, 1 and 3 hours respectively. The profits on each unit of A and B are Rs 2.00 and Rs 3.00 respectively.

Assuming that 300 hours of lathe time, 300 hours of grinding time and 240 hours of assembling time, are avialable, pose a linear Programming Problem in terms of maximizing the profit.

Soln let $x_1$ = no. of units of A

$x_2$ = " " " " B

∴ Maximize the Profit

$$\therefore \text{Max } z = 2x_1 + 3x_2$$

s. t.

$$2x_1 + 3x_2 \le 300$$

$$x_1 + x_2 \le 300$$

$$x_1 + 3x_2 \le 240$$

$$x_1, x_2 \geq 0$$

| | Cutty | grind | Assem |
|---|---|---|---|
| A | 2 | 1 | 1 |
| B | 3 | 1 | 3 |
| hours | 300 | 300 | 240 |

## Formulation of Linear Programming Problem

The structure is

Maximize / minimize → Objective function

subjected to a set of constraints placed either in the form of equality / inequality

The variables are known as decesion variables $x_j \geq 0$.

Example if there are two decesion variables say $x_1, x_2$

Then $\max. \ z = 5x_1 + 3x_2$

subject to
$$5x_1 - x_2 \geq 10$$
$$x_1 + 3x_2 \geq 20$$
$$x_1, x_2 \geq 0.$$

The matrix representation is given by

$$\max z = CX$$
where
$$AX \geq b.$$
$$x_1, x_2 \geq 0$$

where $C = \begin{bmatrix} 5 & 3 \end{bmatrix}$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix} ; \quad b = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

Point A is the intersection of lines $2x+y=2$ and $x+3y=2$

or, $2x+y-2=x+3y-2$

or, $x=2y$

$2(2y)+y=2$

or, $y=0.4$

$\therefore x=2-(3\times0.4)$

$=0.8$

The feasible region is ABCA. The co-ordinates of extreme points of the feasible region are,

$A=(0.8,0.4)$, $B=(1,0)$, $C=(2,0)$

Objective function - max $Z=3x-y$

$Z|_{(0.8,0.4)}=3\times0.8-0.4=2.0$

$Z|_{(1,0)}=3.0$

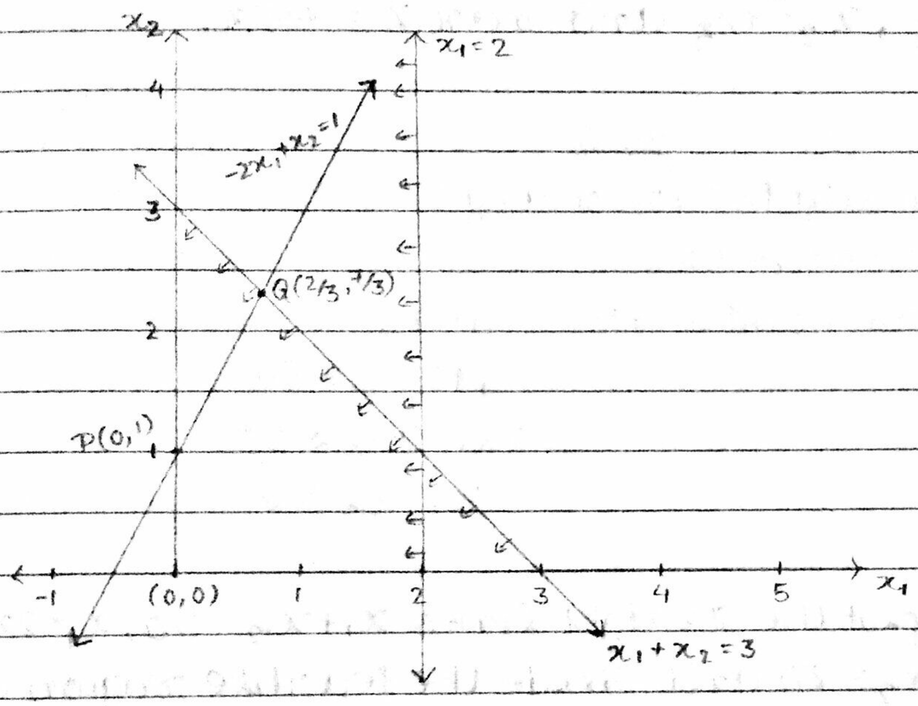
$Z|_{(2,0)}=6.0$

Therefore, the maximum value of $Z=6$ which occurs at point $C(2,0)$, the solution to the given problem is, $x=2$, $y=0$ and max $Z=6$.

2014

(2) Solve graphically the following LPP -
Minimize $Z = 3x_1 + 2x_2$
Subject to Constraints $-2x_1 + x_2 = 1,$
$x_1 + x_2 \leq 3,$
$x_1 \leq 2,$
$x_1, x_2 \geq 0$

Firstly, we plot the straight lines $-2x_1 + x_2 = 1$, $x_1 + x_2 = 3$ and $x_1 = 2$ and shade the feasible region as shown in figure -




Point Q is the intersection of lines $-2x_1 + x_2 = 1$ — (1)
and $x_1 + x_2 = 3$ — (2)
From (2) $x_2 = 3 - x_1$ and putting in equation (1) we get,

$$-2x_1 + 3 - x_1 = 1$$
$$-3x_1 = -2$$
$$x_1 = 2/3$$

$\therefore x_2 = 3 - x_1$
$= 3 - 2/3$
$= 7/3$

So, $Q(2/3, 7/3)$

Point B – Intersection of $3x_1+8x_2=24$
and $x_1+x_2=4$

$\therefore 3x_1+8(4-x_1)=24$
$3x_1+32-8x_1=24$
$-5x_1=-8$
$x_1=8/5=1.6$
Therefore, $x_2=2.4$
$B(2.4, 1.6)$

Point C – Intersection of $10x_1+7x_2=35$
and $x_1+x_2=4$

or, $10x_1+7(4-x_1)=35$
$3x_1=7$
$x_1=7/3$
$\therefore x_2=4-7/3=5/3$
$C(5/3, 7/3)$
The feasible region is OABCDO
The coordinates of extreme points of the feasible region are $O(0,0)$, $A(0,3)$, $B(8/5, 12/5)$, $C(5/3, 7/3)$, $D(7/2, 0)$.
Objective function → $Z=5x_1+7x_2$
$Z|_{0,0}=0$, $Z|_{(0,3)}=21$, $Z|_{(8/5,12/5)}=5.8/5+7.12/5=124/5$,

$Z|_{(5/3,7/3)}=5.5/3+7.7/3=\frac{74}{3}$, $Z|_{(7/2,0)}=35/2$

Since, max value of $z$ occurs at $B(8/5, 12/5)$ which is $Z=124/5$, the solution to the given problem is $x_1=8/5$, $x_2=12/5$ and max $z=124/5$.

Since min values of z occurs at y (15, 1.25) which is 205, the solution to the given problem is $a_1 = 15$, $a_2 = 1.25$ and min $Z = 205$.

2013

(2) Solve the LPP by graphical method -

Maximize $Z = 5x_1 + 7x_2$

Subject to $x_1 + x_2 \leq 4$

$3x_1 + 8x_2 \leq 24$

$10x_1 + 7x_2 \leq 35$

$x_1, x_2 \geq 0$

We plot the straight lines and shade the feasible region as shown in the figure

$x_1 + x_2 = 4 \rightarrow (0,4)$ and $(4,0)$

$3x_1 + 8x_2 = 24 \rightarrow (0,3)$ and $(8,0)$

$10x_1 + 7x_2 = 35 \rightarrow (0,5)$ and $(7/2, 0)$