

- ∵ coordinate of A (1, 0) ; $Z = 3$
 " B (3, 0) ; $Z = 12$
 " C (3, 18) ; $Z = 12 + 36 = 48$
 " D (0, 27) ; $Z = 54$

Since it is a problem of minimization, we move the
 objective line towards origin and lastly it touches
 B(3, 18) in the feasible region

$\therefore \min Z = 48$, optimal soln is $x = 3, y = 18$ Ans.

(7) Minimize $Z = 2x_1 + 3x_2$

s.t $2x_1 + 3x_2 \leq 6$

$x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0$

$$2x_1 + 3x_2 = 6$$

$$\frac{x_1}{3} + \frac{x_2}{2} = 1 \quad \text{--- (1)}$$

$$\frac{x_1}{1} + \frac{x_2}{1} = 1 \quad \text{--- (2)}$$

Considering the constraints as equation, we draw the graph. Now, giving the direction of inequalities and non-negativity restriction we get the common region ABCDA shaded i.e. called feasible region. coordinate of A (1, 0), B(3, 0) C(0, 2), D(0, 1)

Now we draw the objective line by suitable Z over the region

line $Z = 12$

Since the problem is of minimization we move it towards the origin and it touches lastly D(0, 1) in the feasible region

- \therefore At A ; $Z = 3$
 B ; $Z = 9$
 C ; $Z = 12$
 D ; $Z = 1$

$\therefore \min Z = 1$
 Optimal soln is

$$x_1 = 0$$

$$x_2 = 1$$

✓

Coordinate of C is (3, 18)

- ∴ Coordinate of A (12, 0) ; $Z = 48 + 0 = 48$
" " B (30, 0) ; $Z = 120$
" " C (3, 18) ; $Z = 12 + 36 = 48$
" " D (0, 27) ; $Z = 54$

Since it is a problem of minimization, we move the objective line towards origin and easily it touch at point D (0, 27) in the feasible region.

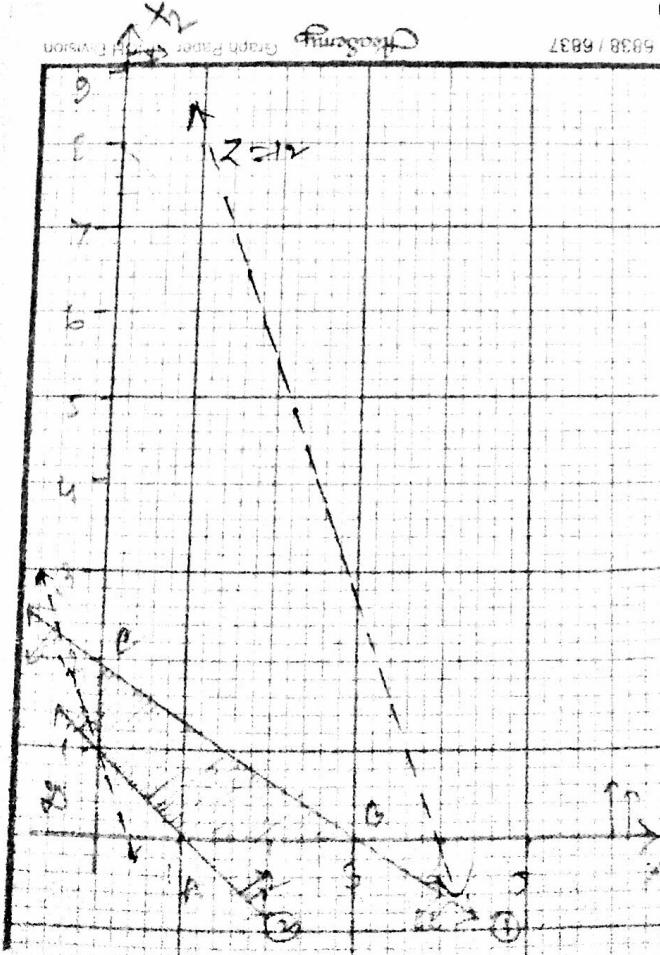
D (0, 27) is the optimal soln in $Z = 3x_1 + 18x_2$ Ans.

$\therefore \min Z = 48$, optimal soln is

(7) Minimize $Z = 3x_1 + 18x_2$
s.t $2x_1 + 3x_2 \leq 6$
 $x_1 + x_2 \geq 1$
 $x_1, x_2 \geq 0$

$$2x_1 + 3x_2 = 6$$
$$\frac{x_1}{3} + \frac{x_2}{2} = 1 \quad \text{--- (1)}$$
$$\frac{x_1}{1} + \frac{x_2}{1} = 1 \quad \text{--- (2)}$$

From the graph, we draw the region of inequalities and the common region ABCDA region. Coordinate of A (1, 0),



Line by suitable Z over the region
of minimization, we move it towards
the origin and easily it touch at
point D (0, 1) in the feasible region.

$\min Z = 1$

Optimal soln is

$$x_1 = 0 \\ x_2 = 1$$

Since, it is a problem of maximization we move the objective line far away from origin and it touches lastly the point C in the feasible region.

$$\therefore \text{At } O, Z = 0$$

$$\text{At } A, Z = 8$$

$$\text{At } B, Z = 18$$

$$\text{At } C, Z = 21$$

$$\text{At } D, Z = 15$$

$$\therefore \text{Max } Z = 21$$

are the feasible solns

$$x=3, y=3$$

⑧

$$\text{Minimize } Z = 4x + 2y$$

$$\text{Subject to } 3x + y \geq 27$$

$$-x - y \leq -21$$

$$x + 2y \geq 30$$

$$x, y \geq 0$$

$$3x + y = 27$$

$$\text{or } \frac{x}{9} + \frac{y}{27} = 1 \dots \textcircled{A}$$

$$x + y = 21$$

$$\text{or } \frac{x}{21} + \frac{y}{21} = 1 \dots \textcircled{B}$$

$$x + 2y = 30$$

$$\text{or } \frac{x}{30} + \frac{y}{15} = 1 \dots \textcircled{C}$$

Considering the constraints as equations we draw the graph. Now giving the direction of inequalities and non-negativity restriction of decision variable. We see that the common region is unbounded and i.e., ABCD

Let, we draw the objective line by suitable Z

$$\text{where } Z = 144$$

$$\therefore 4x + 2y = 144$$

$$\therefore \frac{x}{36} + \frac{y}{72} = 1$$

$$\text{Solving } \textcircled{A}, \textcircled{B} \quad x + y = 21$$

$$x + 2y = 30$$

$$\begin{array}{r} x + y = 21 \\ x + 2y = 30 \\ \hline -y = -9 \end{array} \therefore y = 9 \quad \therefore x = 21 - 9 = 12$$

∴ coordinates of A is (12, 9)

Solving $\textcircled{A}, \textcircled{C}$

$$x + y = 21$$

$$3x + y = 27$$

$$\begin{array}{r} x + y = 21 \\ 3x + y = 27 \\ \hline -2x = -6 \end{array} \therefore x = 3$$

$$\therefore y = 21 - 3$$

$$= 18$$

Since, it is a maximizing problem, we draw the objective line far from origin and lastly it touch point B and pass out from feasible region.

$$\therefore \text{At } O, Z = 0$$

$$\text{At } A, Z = \frac{15}{2} = 7.5$$

$$\text{At } O, Z = \frac{15}{2} + \frac{7}{2} = 11$$

$$\text{At } C, Z = \frac{20}{5} + \frac{42}{5} = \frac{62}{5} = 12.5$$

$$\text{At } D, Z = \frac{21}{2} = 10.5$$

Hence $\max Z = \frac{62}{5}$, and the optimal soln is $x_1 = \frac{4}{5}$
 $x_2 = \frac{6}{5}$

Q) Max $Z = 2x + 5y$

subject to $0 \leq x \leq 4$

$0 \leq y \leq 3$

$x+y \leq 6$.

$x+y = 6$

$x=4, y=0$

(*)

$y=3, x=0$

Considering the constraints as equations, we draw the graphs of the lines. Now giving the direction of inequalities and nonnegativity restriction of decision variables, we have a common region ABCD (shaded) called feasible region with extreme points O, A, B, C, D.

$$x+y=6 \rightarrow ①$$

$$x=4 \rightarrow ②$$

$$y=0$$

$$x+y=6 \rightarrow ①$$

$$y=3$$

$$\therefore x=3$$

\therefore coordinate of O is $(0,0)$

" " A is $(4,0)$

" " B is $(4,2)$

" " C is $(3,3)$

" " D is $(0,3)$

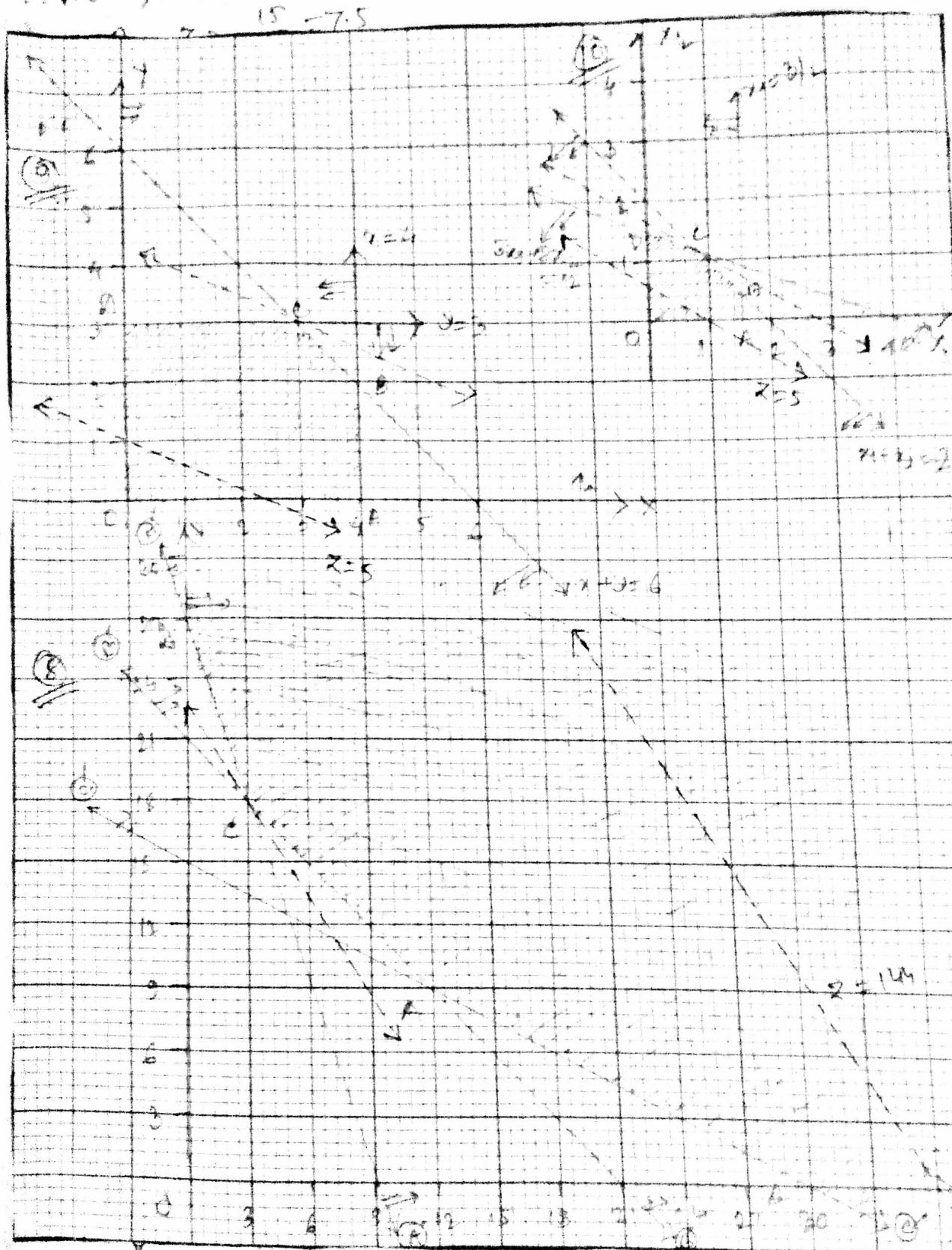
Now we draw the objective line by suitable L, here $Z=5$

$$\therefore 2x+5y=5$$

$$\text{or } \frac{x}{2.5} + \frac{y}{1} = 1$$

Since it is a maximizing problem we draw the objective line far from origin and finally it touch point B and pass out from feasible region.

$$\therefore A \text{ at } 0, Z = 0$$



Z , where $Z = 5$

$$\therefore 2x + 5y = 5$$

$$\text{or } \frac{x}{2.5} + \frac{y}{1} = 1$$

(10)

$$\text{Maximize } Z = 5x_1 + 7x_2$$

$$\text{Subject to } 3x_1 + 8x_2 \leq 12$$

$$x_1 + x_2 \leq 2$$

$$2x_1 \leq 3$$

$$x_1, x_2 \geq 0$$

$$3x_1 + 8x_2 = 12$$

$$\text{or } \frac{x_1}{4} + \frac{x_2}{1.5} = 1$$

$$\frac{x_1}{2} + \frac{x_2}{2} = 1$$

$$x_1 = 3/2$$

Considering the constraints as equations, we draw the graph. Now, giving the directions of inequalities and non-negativity restriction of decision variable, we have a common region OABCDO (shaded) called feasible region with extreme point O(0,0), A(3/2, 0), B(3/2, 1/2), C(4/5, 6/5).

$$x_1 + x_2 = 2 \quad \text{--- (1)}$$

$$2x_1 = 3 \quad \text{--- (2)}$$

$$2x_1 + 2x_2 = 4$$

$$\text{or } 3 + 2x_2 = 4$$

$$\therefore x_2 = \frac{1}{2}$$

\therefore Coordinates of

$$B\left(\frac{3}{2}, \frac{1}{2}\right)$$

$$3x_1 + 3x_2 = 6$$

$$3x_1 + 8x_2 = 12$$

~~$$\begin{array}{rcl} 3x_1 + 3x_2 & = & 6 \\ 3x_1 + 8x_2 & = & 12 \\ \hline -5x_2 & = & -6 \end{array}$$~~

~~$$\therefore x_2 = \frac{6}{5}$$~~

$$\therefore x_1 = 2 - x_2$$

$$= 2 - \frac{6}{5} = \frac{4}{5}$$

$$\text{Coordinate of } C\left(\frac{4}{5}, \frac{6}{5}\right)$$

\therefore Coordinate of O is (0,0)

" " A is $(\frac{3}{2}, 0)$

" " B is $(\frac{3}{2}, \frac{1}{2})$

" " C is $(\frac{4}{5}, \frac{6}{5})$

" " D is $(0, \frac{3}{2})$

Now, we draw the objective line by suitable Z.

$$\text{here } Z = 5 \quad \therefore 5x_1 + 7x_2 = 5$$

$$\frac{x_1}{1} + \frac{x_2}{0.71} = 1$$

$$\text{or } \frac{x_1}{0.71} + \frac{x_2}{1} = 1$$

(1) Maximize

$$Z = 9x + 8y$$

Subject to $4x + 3y \leq 30$

$$2x + 3y \leq 18$$

$$2x + y \leq 10$$

$$x, y \geq 0$$

$$4x + 3y = 30$$

$$\text{or } \frac{x}{7.5} + \frac{y}{10} = 1$$

$$2x + 3y = 18$$

$$\text{or } \frac{x}{9} + \frac{y}{6} = 1$$

$$2x + y = 10$$

$$\text{or } \frac{x}{5} + \frac{y}{10} = 1$$

$$2x + 3y = 18 \quad \text{--- (1)}$$

$$2x + y = 10 \quad \text{--- (2)}$$

Solving (1) and (2), $y = 8$

$$\therefore y = 4 \quad \therefore x = 3$$

$$\therefore (x, y) = (3, 4)$$

Considering the constraints as equations, we draw the graphs of the lines. Now giving the directions of inequalities & on non-negativity restrictions of the decision variable, we see that the feasible region is ABCD (shaded region) with the extreme points $O(0,0); A(5,0); B(3,4); C(0,6)$.

Now, we draw the objective line by suitable Z

here $Z = 36$

$$\therefore 9x + 8y = 36$$

$$\text{or } \frac{x}{4} + \frac{y}{4.5} = 1$$

Since it is a maximising problem, we move the objective line far away from origin and we have

at O, $Z = 0$

At A, $Z = 45$ Hence $\text{Max } Z = 59$

At B, $Z = 39$

and the optimal soln is

At C, $Z = 48$

$x = 3$ Ans.
 $y = 4$

(1) Maximize $Z = 9x + 8y$
 subject to $4x + 3y \leq 30$
 $2x + 6y \leq 18$
 $x, y \geq 0$

$4x + 3y = 30$
 $2x + 6y = 18$
 $\frac{x}{3} + \frac{y}{6} = 1$

Considering the constraints as equations, we draw the graphs of the lines. Now giving the direction of inequalities and non-negativity restriction of the decision variable, we see that feasible soln. is at $OABC$ (shaded region). with extreme points

$$O(0,0), A(7.5,0), B(6,2), C(3,6)$$

$$\begin{aligned} & 4x + 3y = 30 \\ & 2x + 6y = 18 \\ & -3y = -6 \\ & \therefore y = 2 \\ & \therefore 2x = 18 - 6 \\ & = 12 \\ & \therefore x = 6. \end{aligned}$$

$$\begin{aligned} & 4x + 3y = 30 \\ & 2x + 3y = 18 \\ & 2x = 12 \\ & x = 6 \\ & \therefore y = \frac{30 - 24}{3} \end{aligned}$$

$$Z = 9x + 8y$$

Now, we draw the objective line by suitable Z
 where $Z = 9x + 8y$

$$\therefore 9x + 8y = 45$$

$$\text{or } \frac{x}{5} + \frac{y}{8} = 1.$$

$$\text{or } \frac{x}{6} + \frac{y}{6} = 1$$

$$\text{or } \frac{x}{5} + \frac{y}{5.625} = 1$$

$$\text{At } O, Z = 0$$

$$\text{At } A, Z = 7.5$$

Hence $\text{Max } Z = 70$

The optimal soln. is $x = 6, y = 2$

$$A(6,2) \quad Z = 54 + 16$$

$$= 70$$

Since it is a problem of maximization, we move it faraway from origin over the region and it lastly touch point B and rest of the feasible region.

(11) Maximize Z

$$Z = 9x + 8y$$

$$\text{Subject to } 4x + 3y \leq 30$$

$$2x + 3y \leq 18$$

$$2x + y \leq 10$$

$$x, y \geq 0$$

$$4x + 3y = 30$$

$$\text{or } \frac{4x}{7.5} + \frac{y}{10} = 1$$

$$2x + 3y = 18$$

$$\text{or } \frac{x}{9} + \frac{y}{6} = 1$$

