NAME - NANDINI SHOREWALA
ID - 1812100001064
DEPARTMENT - B.TECH (C.S.E.)

MATHS ASSIGNMENT.

1.) Round off the following numbers correct to three decimal places.
(a) 0.0035 (b) 2.35007

Ans (a) 0.0035
∴ Round off 0.0035 upto 3 decimal places is 0.004.

(b) 2.35007
∴ Round off 2.35007 upto 3 decimal places is 2.350

2.) Round off the following numbers correct to four significant figures
(a) 5.04594 (b) 0.0076581

Ans a) 5.04594 - It has 6 significant figures
∴ Rounding off 5.04594 upto 4 significant figures is 5.046

b) 0.0076581 - It has 5 significant figures
∴ Round off 0.0076581 upto 4 significant figures is 0.007658

3.) If the value of π is approximated by 3.1412 then calculate the absolute error, relative error and the percentage error.

Ans E_a (absolute error) $= |x - x_a|$

$$= \left|\frac{22}{7} - 3.1412\right|$$

$$= 0.001657$$

Relative error $(E_r) = \frac{E_a}{x} = \frac{0.001657}{22/7} = 0.000527$

Percentage error $= E_r \times 100$

$$= 0.000527 \times 100$$

$$= 0.0527\%$$

4) If $f(x) = 1/x$ then the divided difference $f[a,b,c]$.

Ans $f(a) = \frac{1}{a}$; $f(b) = \frac{1}{b}$; $f(c) = \frac{1}{c}$

$$f[a,b] = \frac{\frac{1}{b} - \frac{1}{a}}{b-a} = -\frac{1}{ab}$$

$$f[b,c] = \frac{\frac{1}{c} - \frac{1}{b}}{c-b} = -\frac{1}{bc}$$

$$f[a,b,c] = \frac{f[a,b] - f[b,c]}{a-c} = \frac{-\frac{1}{ab} - \left(-\frac{1}{bc}\right)}{a-c} = \frac{1}{abc}$$

5) If the absolute error is 0.2×10^{-3} and the relative error is 0.32×10^{-5} then find the true value

Ans $\varepsilon_a = 0.2 \times 10^{-3}$

$\varepsilon_r = 0.32 \times 10^{-5}$

$$\varepsilon_r = \frac{\varepsilon_a}{x}$$

$$x = \frac{\varepsilon_a}{\varepsilon_r} = \frac{0.2 \times 10^{-3}}{0.32 \times 10^{-5}} = \underline{\underline{62.5}}$$

6) From the following table, find the number of students who obtained less than 45 marks.

Marks	30-40	40-50	50-60	60-70	70-80
Number of students	31	42	51	35	31

NAME - NANDINI SHOREWALA

8.

Marks less than (45) x	No. of students	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	c.f
Below 40	31					31
Below 50	42	$\underline{42}$	$\underline{9}$			73
Below 60	51	51	-16	$\underline{-25}$	$\underline{37}$	124
Below 70	35	35	-4	12		159
Below 80	31	31				190

~~$y_{45} = 42 + 0.5\times 9 + \frac{0.5\times -0.5}{2}\times 0$~~

$u = \frac{45-40}{10} = 0.5$

$$y_{45} = 31 + 0.5\times 42 + \frac{0.5\times -0.5}{2}\times 9 + \frac{0.5\times -0.5\times -1.5}{3\times 2}\times -25$$

$$+ \frac{0.5\times -0.5\times -1.5\times -2.5}{4\times 3\times 2}\times 37$$

$$= 31 + 21 - 1.125 - 1.5625 - 1.4453 = \underline{47.86} \simeq \underline{48}$$

7) The value of x and y are given below

x:	5	6	9	11
y:	12	13	14	16

Find the value of y at $x=10$.

Ans $x_0 = 5,\ x_1 = 6,\ x_2 = 9,\ x_3 = 11$

$y_0 = 12,\ y_1 = 13,\ y_2 = 14,\ y_3 = 16$

$$\therefore y_{10} = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}\times 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}\times 13 +$$

$$\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}\times 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}\times 16$$

$$= \frac{4\times 1\times -1}{-1\times -4\times -6}\times 12 + \frac{5\times 1\times -1}{1\times -3\times -5}\times 13 + \frac{5\times 4\times -1}{4\times 3\times -2}\times 14 + \frac{5\times 4\times 1}{6\times 5\times 2}\times 16$$

$$= 2 - 4.33 + 11.67 + 5.33 = \underline{14.67}$$

$$\therefore \Delta_y x^2 = \frac{y^2 - x^2}{y - x} = \frac{(y-x)(y+x)}{(y-x)} = (y+x).$$

8) Using Simpson's 1/3rd rule estimate $\int_1^7 f(t)dt$ from the following

t:	1	2	3	4	5	6	7
f(t):	81	75	80	83	78	70	60
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Ans $h = \frac{b-a}{n} = \frac{7-1}{6} = \frac{6}{6} = 1$

$$\int_1^7 f(t)\,dt = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(81+60) + 4(75+83+70) + 2(80+78)\right]$$

$$= \frac{1}{3}\left[141 + 912 + 316\right]$$

$$= \cancel{351.142}\ \ 456.33$$

9) Derive Newton's backward interpolation formula.

Ans Let $y = f(x)$ and $y_0, y_1, y_2 \ldots\ldots y_n$ are values corresponding to points $x_0, x_0+h, x_0+2h, \ldots\ldots x_0+nh$. Suppose we want to find $f(x) = y$ at point $x = x_n + ph$ (or $p = \frac{x-x_n}{h}$)

we know that by Definition of E

$$E^p f(x) = f(x+ph)$$

$$E^p f(x_n) = f(x_n+ph)$$

$$f(x) = E^p y_n \quad (\because y_n = f(x_n))$$

$$= (E^{-1})^{-p} y_n$$

$$= (1-\nabla)^{-p} y_n$$

$$= \left[1 + p\nabla + \frac{p(p+1)}{\underline{|2}}\nabla^2 + \frac{p(p+1)(p+2)}{\underline{|3}}\nabla^3 + \ldots\right] y_n$$

$$= y_n + p\nabla y_n + \frac{p(p+1)}{\underline{|2}}\nabla^2 y_n + \frac{p(p+1)(p+2)}{\underline{|3}}\nabla^3 y_n + \ldots\ldots$$

$\because E^{-1} = 1-\nabla$

$\because (1+x)^n = 1 + nx + \frac{n(n-1)}{\underline{|2}}x^2 + \ldots\ldots\ldots$

$x = -\nabla$
$n = -p$

2020.02.18 13:04

5) Evaluate $\int_0^1 \frac{x^2 dx}{1+x^3}$ using Trapezoidal rule taking $n=10$.

Ans

$$I = \int_0^1 \frac{x^2\,dx}{1+x^3}$$

$$h = \frac{b-a}{n} = \frac{1-0}{10}$$

Suppose $h = 1/10$

$\therefore n = 10$

$$I = \int_0^1 \frac{x^2}{1+x^3}\,dx$$

$$= \frac{h}{2}\left[y_0 + 2(y_1 + y_2 \dots + y_{n-1}) + y_n\right]$$

x	0	1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	10/10
$y = \frac{x^2}{1+x^3}$	0	0.0099	0.0397	0.0876	0.1504	0.2222	0.2960	0.3648	0.4283	0.4685	0.5

Apply trapezoidal formula

$$\int_0^1 \frac{x^2\,dx}{1+x^3}$$

$$\frac{1}{20}\left[0.5 + 2(0.0099 + 0.0397 + 0.0876 + 0.1504 + 0.2222 + 0.2960 + 0.3648 + 0.4233 + 0.4685)\right]$$

$= \underline{0.23}$ Ans.

11) Evaluate $f(0.5)$ and $f(2.5)$ from the following table:

x	0	1	2	3
$f(x)$	1	2	11	34

Ans

x	$f(x)$	$f(x_0,x_1)$	$f(x_0,x_1,x_2)$	$f(x_0,x_1,x_2,x_3)$
0	1	1		
1	2	9	4	1
2	11	23	7	
3	34			

$f(0.5) = 1 + (0.5-0)\times 1 + (0.5-0)(0.5-1)\times 4 + 0.5\times -0.5\times -1.5\times 1$

$= 1 + 0.5 - 1 + 0.375 = \underline{0.875}$

$f(2.5) = 1 + (2.5-0)\times 1 + 2.5\times(2.5-1)\times 4 + 2.5\times(2.5-1)\times(2.5-2)\times 1$

$= 1 + 2.5 + 15 + 1.875$

$= \underline{20.375}$

12) Using Lagrange's interpolation formula, find the interpolation polynomial from the following table.

x:	0	2	4	8
$f(x)$:	3	8	11	19

Ans. $x_0 = 0$, $x_1 = 2$, $x_2 = 4$, $x_3 = 8$

$\therefore$ By Lagrange's formula,

$$f(x) = \frac{(x-2)(x-4)(x-8)}{(0-2)(0-4)(0-8)}\times 3 + \frac{(x-0)(x-4)(x-8)}{(2-0)(2-4)(2-8)}\times 8 + \frac{(x-0)(x-2)(x-8)}{(4-0)(4-2)(4-8)}\times 11 + \frac{(x-0)(x-2)(x-4)}{(8-0)(8-2)(8-4)}\times 19$$

$$= \frac{3}{-64}(x^3 - 14x^2 + 56x + 64) + \frac{8}{24}(x^3 - 12x^2 + 32x) + \frac{11}{-32}(x^3 - 10x^2 + 16x) + \frac{19}{192}(x^3 - 6x^2 + 8x)$$

$$= 0.041x^3 - 0.492x^2 + 3.312x - 3.008$$

2020.02.18 13:04

The velocity v cm/s of a particle at a time t is given in the table

t:	0	2	4	6	8	10	12
V:	4	6	16	34	60	94	136

Find the distance moved by the particle in 12 second.

Ans

t	v
0	4
2	6
4	16
6	34
8	60
10	94
12	136

$$S = \int_0^{12} v\,dt$$

$$= \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + \cdots + y_{n-1})\right]$$

$$= (4+136) + 2(6+16+34+60+94)$$

$$= \underline{\underline{560 \text{ cm}}}$$

2020.02.18 11:04

Scanned by CamScanner