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1. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{4x}$

<u>Ans</u> Hence we have

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{4x} \longrightarrow (i)$$

Now, first finding the Solution of the homogeneous System (i)

put $y = e^{mx}$

$$m^2 - 2m - 3 = 0$$

$$m^2 - 3m + m - 3 = 0$$

$$(m-3)(m+1) = 0$$

$$\therefore m_1 = 3, \; m_2 = -1$$

$$y_h = C_1 e^{3x} + C_2 e^{-x}$$

Now, to find particular Solution y_p we see on the right side of the function which is equal to $2e^{4x}$ So we assume the particular Solution of the form

$$y_p = Ae^{4x}$$

$$y'_p = 4Ae^{4x} \quad , \quad y_p'' = 16Ae^{4x}$$

Putting y_p, y_p' and y_p'' in eq.(i) we get

$$16Ae^{4x} - 8Ae^{4x} - 3Ae^{4x} = 2e^{4x}$$

$$5Ae^{4x} = 2e^{4x}$$

$A = \frac{2}{5}$

So $y_p = \frac{2}{5} e^{4x}$

Therefore the general Solution is

$$y = y_h + y_p$$

$$y = C_1 e^{3x} + C_2 e^{-x} + \frac{2}{5} e^{4x}$$

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2. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{x} - 10 \sin x$

Ans Here we have

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{x} - 10 \sin x \longrightarrow (i)$$

$$m^2 - 2m - 3 = 0$$

$$m^2 - 3m + m - 3 = 0$$

$$m = 3, -1$$

$$y_h = C_1 e^{3x} + C_2 e^{-x}$$

The non homogeneous term is the linear combination $2e^{x} - 10 \sin x$

So we form a linear Combination

$$y_p = Ae^{x} + B \sin x + C \cos x$$

$$y_p' = Ae^{x} + B\cos x - C \sin x$$

$$y_p'' = Ae^{x} - B \sin x - C \cos x$$

From eq. (i)

$$(Ae^{x} - B \sin x - C \cos x) - 2(Ae^{x} + B\cos x - C \sin x) - 3(Ae^{x} + B \sin x + C \cos x) = 2e^{x} - 10 \sin x$$

$$-4Ae^{x} + (-4B + 2C) \sin x + (-4C - 2B) \cos x = 2e^{x} - 10 \sin x$$

Equating the coefficients of like terms we obtain

$-4A=2$, $-4B+2C=-10$, $-4C-2B=0$

Now,

$$-4B+2C=-10 \rightarrow \text{(ii)}$$
$$-2B-4C=0 \rightarrow \text{(iii)}$$

solving eq. (ii) and eq. (iii) simultaneously we get

$$-4B+2C=-10 \rightarrow \text{(ii)} \times 1$$
$$-2B-4C=0 \rightarrow \text{(iii)} \times +2$$

$$-4B+2C=-10$$
$$\underset{(+)}{-4B}\ \underset{(+)}{-8C} = \underset{(-)}{0}$$

$$10C=-10$$
$$C=-1$$

putting the value of C in (ii) we get

$$B=2$$

$\therefore A=-\frac{1}{2}$, $B=2, C=-1$

$\therefore y_p = -\frac{1}{2}e^{x}+2\sin x-\cos x$

So, $y = y_h + y_p$

$$y=C_1e^{3x}+C_2e^{-x}-\frac{1}{2}e^{x}+2\sin x-\cos x$$

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problem sum

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1. A 32 - lb weight is attached to the lower end of a coil spring suspended from the Ceiling. The weight comes to rest in its equilibrium position, thereby stretching the String 2 ft. The weight is then pulled down 6 in. below its equilibrium position and released at $t=0$. No external

forces are present; but the resistance of the medium in ~~pounds~~ Peunds is numerically equal to $4\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instantaneous velocity in feet per second. Determine the resulting motion of the weight on the spring.

Ans This is free damped motion

The equation here

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 16x = 0$$

with initial conditions

$$x(0) = \frac{1}{2} \quad , \quad x'(0) = 0$$

$$m\frac{d^2x}{dt^2} + a\frac{dx}{dt} + Kx = 0$$ [Henckes law] then as strain]

$$m = \frac{w}{g} = \frac{32}{32} = 1, \quad a = 4$$

$f = Ks$

$32 = K \times 2$

$K = 16$

$$x = e^{-3t}\left(\frac{\sqrt{3}}{6}\sin(2\sqrt{3}t) + \frac{1}{2}\cos 2\sqrt{3}t\right)$$

A circuit has in series an electromotive force given by $E = 100 \sin 60t$, a resistor of 2Ω, an inductor of 0.1 H and a capacitor of $\frac{1}{260}$ farads. If the initial current and the initial charge on the capacitor are both zero, find the charge on the capacitor at any time $t>0$.

<u>Ans</u> Here $L = \frac{1}{10}$, $C = \frac{1}{260}$, $R = 2$

Applying Kirchoff's law we have

$$\frac{1}{10}\frac{d^2q}{dt^2} + 2\frac{dq}{dt} + 260q = 100 \sin 60t \longrightarrow (i)$$

with initial condition $q(0) = 0$, $q'(0) = 0$

$$\frac{d^2q}{dt^2} + 20\frac{dq}{dt} + 2600q = 1000 \sin 60t \longrightarrow (ii)$$

$$r^2 + 20r + 2600 = 1000 \sin 60t$$

$$r^2 + 20r + 2600 = 0$$

$$r = \frac{-20 \pm \sqrt{400 - 10400}}{2}$$

$$r = \frac{-20 \pm \sqrt{-10000}}{2} = \frac{-20 \pm 100i}{2} = -10 \pm 50i$$

$$q_h = e^{-10t}(c_1 \sin 50t + c_2 \cos 50t)$$

Let $q_p = A \sin 60t + B \cos 60t$

$$q_p' = 60A \cos 60t - 60B \sin 60t$$

$$q_p'' = -3600 A \sin 60t - 3600 B \cos 60t$$

putting q_p, q_p' and q_p'' in (ii) we get

$$-3600A\sin 60t - 3600B\cos 60t + 1200A\cos 60t - 1200B\sin 60t + 2600A\sin 60t + 2600B\cos 60t = 1000\sin 60t$$

$$-1000A\sin 60t + 1200A\cos 60t - 1000B\cos 60t - 1200B\sin 60t = 1000\sin 60t$$

$$(-1000A - 1200B)\sin 60t + (1200A - 1000B)\cos 60t = 1000\sin 60t$$

equating the coefficients of like terms we obtain

$$-1000A - 1200B = 1000$$

$$-100(10A + 12B) = 1000$$

$$10A + 12B = -10 \longrightarrow \text{(iii)}$$

$$1200A - 1000B = 0$$

$$100(12A - 10B) = 0$$

$$12A - 10B = 0 \longrightarrow \text{(iv)}$$

solving eq. (iii) and eq. (iv) simultaneously we get

$$10A + 12B = -10 \longrightarrow \text{(iii)} \times 6$$

$$12A - 10B = 0 \longrightarrow \text{(iv)} \times 5$$

$$60A + 72B = -60$$

$$\underset{(-)}{60A} \underset{(+)}{-} 50B = \underset{(-)}{0}$$

$$122B = -60$$

$$B = -\frac{60}{122} = -\frac{30}{61}$$

putting the value of B in eq. (iii) we get

$$10A - \frac{360}{61} = -10$$

$$10A = -10 + \frac{360}{61}$$

$$10A = \frac{-610 + 360}{61}$$

$$10A = -\frac{250}{61}$$

$$A = -\frac{25}{61}$$

$$\therefore A = -\frac{25}{61}, \quad B = -\frac{30}{61}$$

$$\therefore q = e^{-10t}(c_1 \sin 50t + c_2 \cos 50t) - \frac{25}{61}\sin 60t - \frac{30}{61}\cos 60t \rightarrow (v)$$

Differentiating (v) w.r.t t we get

$$q' = e^{-10t}(50c_1 \cos 50t - 50c_2 \sin 50t) - 10e^{-10t}(c_1 \sin 50t + c_2 \cos 50t) - \frac{1500}{61}\cos 60t + \frac{1800}{61}\sin 60t \rightarrow (vi)$$

Now, $q(0) = 0$ and $q'(0) = 0$

putting $t = 0$, $q = 0$ in (v) we get

$$0 = c_2 - \frac{30}{61}$$

$$c_2 = \frac{30}{61}$$

Again, putting $t = 0$, $q' = 0$ ~~and we get~~ and $c_2 = \frac{30}{61}$ in (vi) we get

$$0 = 50c_1 - \frac{300}{61} - \frac{1500}{61}$$

$$C_1 = \frac{1800}{61 \times 50} = \frac{36}{61}$$

$$\therefore C_1 = \frac{36}{61} \text{ and } C_2 = \frac{30}{61}$$

$$\therefore q = \frac{6e^{-10t}}{61}\left(6 \sin 50t + 5 \cos 50t\right) - \frac{5}{61}\left(5 \sin 60t + 6 \cos 60t\right)$$