(ii) Minimize $z = 4x_1 + 3x_2$

$\therefore \text{Max}(-z) = -4x_1 - 3x_2$

Subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$x_1, x_2 \geq 0$

Introducing slack, surplus, artificial variables we rewrite the problem in standard form as

$\text{Max}(-z) = -4x_1 - 3x_2 - 0.x_3 - 0.x_4 - Mx_5 - Mx_6$ where M is very large +ve number

Subject to $x_1 + 2x_2 - x_3 + 0x_4 + x_5 + 0.x_6 = 8$

$3x_1 + 2x_2 + 0.x_3 - x_4 + 0.x_5 + x_6 = 12$

$x_j \geq 0 \quad j = 1, 2, \ldots, 6.$

$a_5 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, a_6 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Here (a_5, a_6) forms initial basis

x_B = Basis variable, B = Basis vector, C_B = coefficient of basis variable in objective function.

Table:-

			c_j	-4	-3	0	0	$-M$	$-M$	Ratio	Operation	Remark
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	(min)		
$-M$	a_5	x_5	8	1	$\boxed{2}$	-1	0	1	0	4 (min)		$a_2 \to$ entering $a_5 \to$ departing $2 \to$ key element
$-M$	a_6	x_6	12	3	2 ↑	0	-1	0 ↓	1	6		
$z_j - c_j$				$-4M+4$	$-4M+3$	M	M	0	0			
-3	a_2	x_2	4	$1/2$	1	$-1/2$	0	x	0	8	$R_1' = \frac{1}{2}R_1$	$a_1 \to$ entering
$-M$	a_6	x_6	4	$\boxed{2}$ ↑	0	1	-1	x	1 ↓	2 (min)	$R_2' = R_2 - 2R_1'$	$a_6 \to$ departing $2 \to$ key element
$z_j - c_j$				$-2M+5/2$	0	$-M+3/2$	M					
-3	a_2	x_2	3	0	1	$-3/4$	$1/4$	x	x		$R_2' = \frac{1}{2}R_2$	
-4	a_1	x_1	2	1	0	$1/2$	$-1/2$	x	x		$R_1' = R_1 - \frac{1}{2}R_2'$	
$z_j - c_j$				0	0	$1/4$	$5/4$					

Here all $z_j - c_j \geq 0$ and thus the optimal condition is achived

$\therefore$ optimal soln. is $x_1 = 2, x_2 = 3$

$Z_{min} = 4.2 + 3.3 = 17$ Ans.

is achived.

∴ optimal soln is

$x_1 = 5/2$, $x_2 = 3/2$ Max $Z = \frac{7.5}{2} + \frac{3.3}{2}$

$= \frac{35+9}{2} = 22$ Ans.

(16)

ii) Minimize

$Z = 4x_1 + 8x_2 + 3x_3$.

Subject to $x_1 + x_2 \ge 2$

$2x_1 + x_3 \ge 5$

$x_1, x_2, x_3 \ge 0$

Introducing slack and surplus variables, we rewrite the problem in standard form as

Maximize $(-Z) = -4x_1 - 8x_2 - 3x_3 + 0.x_4 + 0.x_5$

Subject to

$x_1 + x_2 + 0.x_3 - x_4 - 0.x_5 = 2$

$2x_1 + 0.x_2 + x_3 + 0x_4 - x_5 = 5$

$x_j \ge 0$ $j = 1,2,3,4,5$

here (a_2, a_3) forms initial basis

x_B = Basis variable, B = Basis vector, C_B = coefficient of basis variable in objective function.

Table:-

		c_j		-4	-8	-3	0	0	Ratio	Operation	Remarks
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	(min)		
-8	a_2	x_2	2	$\boxed{1}$	1	0	-1	-1	2 (min)		$a_1 \to$ entering $a_2 \to$ departing $\boxed{1} \to$ key element
-3	a_3	x_3	5	2	0	1	0	-1	$5/2$		
$z_j - c_j$				-10	0	0	8	11			
-4	a_1	x_1	2	1	1	0	-1	-1	$-$	$R_2' = R_2 - 2R_1'$	$a_4 \to$ entering $a_3 \to$ departing $\boxed{2} \to$ key element
-3	a_3	x_3	1	0	-2	1	$\boxed{2}$	1	$1/2$ (min)		
$z_j - c_j$				0	10	0	-2	1			
-4	a_1	x_1	$5/2$	1	0	$1/2$	0	$-1/2$		$R_2' = \frac{1}{2}R_2$ $R_1' = R_1 + R_2'$	
0	a_4	x_4	$1/2$	0	-1	$1/2$	1	$1/2$			
$z_j - c_j$				0	8	1	0	2			

Here all $z_j - c_j \ge 0$ and thus the optimal condition is achived. ∴ optimal soln is $x_1 = 5/2$, $x_2 = x_3 = 0$. ∴ Min $Z = 10$ Ans

Very large +ve number

s.t

~~$x_1+x_2+x_3$~~

$x_1+2x_2-x_3+0.x_4+0.x_5+0x_6+x_7=3$

$x_1+x_2+0.x_3+x_4+0.x_5+0x_6+0x_7=4$

$x_1+0.x_2+0.x_3+0.x_4+x_5+0x_6+0x_7=5/2$ $\qquad x_j \geq 0$ 16

$0.x_1+x_2+0.x_3+0.x_4+0.x_5+x_6+0.x_7=3/2$ $\qquad j=1,2,\dots,7$ iv)

Here (a_7, a_4, a_5, a_6) forms initial basis

x_B = Basis variable , B = basis vector , C_B = coefficient of basis variables in objective f^n.

Table :-

			c_j	7	3	0	0	0	0	-M	Ratio	Operation	Remarks
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	(min)		
-M	a_7	x_7	3	1	$\boxed{2}$	-1	0	0	0	1	3/2		$a_2 \to$ entering
0	a_4	x_4	4	1	1	0	1	0	0	0	4		$a_7 \to$ departing
0	a_5	x_5	5/2	1	0	0	0	1	0	0	—		$\boxed{2} \to$ key element
0	a_6	x_6	3/2	0	1	0	0	0	1	0	3/2		
z_j-c_j				-M -7	-2M -3	M	0	0	0	0			
3	a_2	x_2	3/2	1/2	1	-1/2	0	0	0	X	3	$R_1'=\frac{1}{2}R_1$	$a_1 \to$ entering
0	a_4	x_4	5/2	1/2	0	1/2	1	0	0	X	5	$R_2'=R_2-R_1'$	$a_5 \to$ departing
0	a_5	x_5	5/2	$\boxed{1}$	0	0	0	1	0	X	5/2 (min)	$R_3'=R_4-R_1'$	$\boxed{1} \to$ key element
0	a_6	x_6	0	-1/2	0	1/2	0	0	1	X	—		
z_j-c_j				-11/2	0	-3/2	0	0	0				
3	a_2	x_2	1/4	0	1	-1/2	0	-1/2	0	X	—	$R_1'=R_1-\frac{1}{2}R_3'$	$a_3 \to$ entering
0	a_4	x_4	7/4	0	0	1/2	1	-1/2	0	X	5/2	$R_2'=R_2-\frac{1}{2}R_3'$	$a_6 \to$ departing
7	a_1	x_1	5/2	1	0	0	0	1	0	X	—		$\boxed{1/2} \to$ key element
0	a_6	x_6	5/4	0	0	$\boxed{1/2}$	0	1/2	1	X	5/2	$R_4'=R_4+\frac{1}{2}R_3'$	
z_j-c_j				0	0	-3/2	0	7	0				
3	a_2	x_2	3/2	0	1	0	0	0	1	X		$R_4'=2R_4$	
0	a_4	x_4	0	0	0	0	1	-1	-1	X		$R_2'=R_2-\frac{1}{2}R_4'$	
7	a_1	x_1	5/2	1	0	0	0	1	0	X			
0	a_3	x_3	5/2	0	0	1	0	1	2	X		$R_1'=R_1+\frac{1}{2}R_4'$	
z_j-c_j				0	0	0	0	7	3				

Here all $z_j-c_j \geq 0$ and thus the optimal condition

Max $z = 2x_1 - 3x_2 + 0\cdot x_3 + 0\cdot x_4 + 0\cdot x_5 - Mx_6$ where M is very large +ve no.

s.t. $x_1 - x_2 + x_3 + 0\cdot x_4 + 0\cdot x_5 + 0\cdot x_6 = 2$

$5x_1 + 4x_2 + 0\cdot x_3 + x_4 + 0\cdot x_5 + 0\cdot x_6 = 46$ $\quad x_j \geq 0 \quad j = 1,\dots,6$

$7x_1 + 2x_2 + 0\cdot x_3 + 0\cdot x_4 - x_5 + x_6 = 32$

here (a_3, a_4, a_6) forms initial basis. x_B = Basis variable

B = basis vector C_B = coefficient of Basis variable in objective f^n

Table :-

			c_j	2	-3	0	0	0	-M	Ratio	Operation	Remark
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	(min)		
0	a_3	x_3	2	$\boxed{1}$	-1	1	0	0	0	2 (min)		$a_1 \to$ entering
0	a_4	x_4	46	5	4	0	1	0	0	46/5		$a_3 \to$ departing
-M	a_6	x_6	32	7	2	0	0	-1	1	32/7		$\boxed{1} \to$ key element
$z_j - c_j$				$-7M-2$	$-2M+3$	0	0	M	0			
2	a_1	x_1	2	1	-1	1	0	0	0	—	$R_2' = R_2 - 5R_1'$	$a_2 \to$ entering
0	a_4	x_4	36	0	9	-5	1	0	0	4	$R_3' = R_3 - 7R_1'$	$a_6 \to$ departing
-M	a_6	x_6	18	0	$\boxed{9}$	-7	0	-1	1	2		$\boxed{9} \to$ key element
$z_j - c_j$				0	$-9M+1$	$7M+2$	0	M	0			
2	a_1	x_1	4	1	0	2/9	0	-1/9	X		$R_3' = \frac{1}{9}R_3$	
0	a_4	x_4	18	0	0	2	1	1	X		$R_2' = R_2 - 9R_3'$	
-3	a_2	x_2	2	0	1	-7/9	0	-1/9	X		$R_1' = R_1 + R_3'$	
$z_j - c_j$				0	0	25/9	0	1/9				

Here all $z_j - c_j \geq 0$ And thus the optimality condition is achived

$\therefore$ optimal sol^n is $x_1 = 4,\ x_2 = 2$

$\therefore$ Max $z = 2.4 - 3.2 = 2$ Ans.

(12) (i) Maximize $z = 7x_1 + 3x_2$

s.t $x_1 + 2x_2 \geq 3$

$x_1 + x_2 \leq 4$

$x_1 \leq 5/2$

$x_2 \leq 3/2$

$x_1, x_2 \geq 0$

Introducing slack surplus and artifitial variables, we rewrite the problem in standard form as

Max $z = 7x_1 + 3x_2 + 0\cdot x_3 + 0\cdot x_4 + 0\cdot x_5 + 0\cdot x_6 - Mx_7$ where M is

Table :-

			c_j	3	-1	0	0	0	-M	Ratio	Operation	Remarks
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	(min)		
← -M	a_6	x_6	2	$\boxed{2}$	1	-1	0	0	1	$\frac{2}{2}=1$		Entering → a_1 departing → a_6 $\boxed{2}$ → key element
0	a_4	x_4	3	1	3	0	1	0	0	3		
0	a_5	x_5	4	0 ↑	0	0	0	1	0 ↓	—		
$z_j - c_j$				-2M -3	-M +1	M	0	0	0			
3	a_1	x_1	1	1	1/2	-1/2	0	0	X	—	$R_1' = \frac{1}{2}R_1$	a_3 → entering a_4 → departing $\boxed{\frac{1}{2}}$ → key element
← 0	a_4	x_4	2	0	5/2	$\boxed{1/2}$	1	0	X	$\frac{2}{1/2}$	$R_2' = R_2 - 0R_1'$	
0	a_5	x_5	4	0	0	0 ↑	0 ↓	1	X	—		
$z_j - c_j$				0	5/2	-3/2	0	0				
3	a_1	x_1	3	1	3	0	1	0	X		$R_2' = 2R_2$	
0	a_3	x_3	4	0	5	1	2	0	X		$R_1' = R_1 + \frac{1}{2}R_2'$	
0	a_5	x_5	4	0	0	0	0	1	X			
$z_j - c_j$				0	1	0	3	0				

Here all $z_j - c_j \geq 0$ and thus the optimal condition is achieved.

$\therefore$ Optimal sol^n is $x_1 = 3, x_2 = 0$

$\therefore Z_{max} = 9$. Ans.

(ii) Maximize $z = 2x_1 - 3x_2$

subject to $-x_1 + x_2 \geq -2$

$5x_1 + 4x_2 \leq 46$

$7x_1 + 2x_2 \geq 32$

$x_1, x_2 \geq 0$

At first we make the rhs of first constraint +ve and then we have

$x_1 - x_2 \leq 2$

$5x_1 + 4x_2 \leq 46$

$7x_1 + 2x_2 \geq 32$ Introducing slack, surplus,

$x_1, x_2 \geq 0$.

artificial variables, we rewrite the problem in standard form

Here (a_4, a_5) forms the initial basis

x_B = Basis variable, B = basis vector, c_B = coefficient of Basis variable in objective f^n

Table :—

			c_j	-1	-1	-1	$-M$	$-M$	Ratio	Operation	Remarks
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	(min)		
$-M$	a_4	x_4	2	1	-1	$\boxed{2}$	1	0	$2/2$		$a_3 \to$ entering, $a_4 \to$ departing, $\boxed{2} \to$ key element
$-M$	a_5	x_5	1	-1	2	-1	0	1	—		
$z_j - c_j$				1	$1-M$	$1-M$	0	0			
-1	a_3	x_3	1	$1/2$	$-1/2$	1	x	0	—	$R_1' = \frac{1}{2}R_1$	$a_2 \to$ entering
$-M$	a_5	x_5	2	$-1/2$	$\boxed{3/2}$	0	x	1	$2/3/2$	$R_2' = R_2 + R_1$	$a_5 \to$ departing, $\boxed{3/2} \to$ key element
$z_j - c_j$				$\frac{M+1}{2}$	$\frac{-3M+3}{2}$	0		0			
-1	a_3	x_3	$5/3$	$1/3$	0	1	x	x		$R_2' = \frac{2}{3}R_2$	
-1	a_2	x_2	$4/3$	$-1/3$	1	0	x	x		$R_1' = R_1 + \frac{1}{2}R_2$	
$z_j - c_j$				1	0	0					

Here all $z_j - c_j \geq 0$, and thus the optimality condition is achieved

$\therefore$ Feasible sol^n is $x_1 = 0, x_2 = \frac{4}{3}, x_3 = \frac{5}{3}$

$Z_{max} = -\frac{4}{3} - \frac{5}{3} = -3$ Ans.

(18.) i) Maximize $\quad z = 3x_1 - x_2$

subject to $\quad 2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

Introducing slack, surplus and artificial variables, we rewrite the problem in standard form as

$\max z = 3x_1 - x_2 + 0.x_3 + 0.x_4 + 0.x_5 - Mx_6$ where M is very large +ve no.

s.t. $2x_1 + x_2 - x_3 + 0.x_4 + 0.x_5 + x_6 = 2$

$x_1 + 3x_2 + 0.x_3 + x_4 + 0.x_5 + 0.x_6 = 3$

$0.x_1 + 0.x_2 + 0.x_3 + 0.x_4 + x_5 + 0.x_6 = 4$

$x_j \geq 0 \quad j = 1,2,3,4,5,6$

(a_6, a_4, a_5) forms initial basis

(15) Maximize $z = 10x_1 + x_2 + 2x_3$

$$x_1 + x_2 - 2x_3 \le 10$$
$$4x_1 + x_2 + x_3 \le 20$$
$$x_1, x_2, x_3 \ge 0$$

Introducing slack variables, we have $\quad$ Max $z = 10x_1 + x_2 + 2x_3 + 0.x_4 + 0.x_5$

$$x_1 + x_2 - 2x_3 + x_4 + 0.x_5 = 10$$
$$4x_1 + x_2 + x_3 + 0x_4 + x_5 = 20$$
$$x_j \ge 0 \quad j = 1, 2, \ldots, 5$$

here (a_4, a_5) forms initial basis

x_B = basis variable, B = Basis vector

C_B = Coefficient of basis variable in objective f^n.

			c_j	10	1	2	0	0	Ratio	Operation	Remarks
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	(min)		
0	a_4	x_4	10	1	1	-2	1	0	$10/1$		$a_1 \to$ entering vector
0	a_5	x_5	20	$\boxed{4}$	1	1	0	1	$\frac{20}{4}$ (min)		$a_5 \to$ departing vector; $4 \to$ key element
$z_j - c_j$				-10	-1	-2	0	0			
0	a_4	x_4	15	0	$3/4$	$-\frac{9}{4}$	1	$-\frac{1}{4}$		$R_2' = \frac{1}{4}R_2$	
10	a_1	x_1	5	1	$1/4$	$1/4$	0	$1/4$		$R_1' = R_1 - R_2'$	
$z_j - c_j$				0	$3/2$	$1/2$	0	0			

Here all $z_j - c_j \ge 0$ and thus optimal condition is achived

$\therefore$ optimal sol^n is $x_1 = 5, x_2 = 0, x_3 = 0$

Max $z = 50$ Ans.

(16) (i) Maximize $z = 7x_1 + 3x_2$

Subject to, $x_1 + 2x_2 \ge 3$
$$x_1 + x_2 \le 4$$
$$x_1 \le 5/2$$
$$x_2 \le 3/2$$
$$x_1, x_2 \ge 0$$

Introducing slack and surplus variables, we rewrite the problem in standard form as.

s.t., $8x_1+3x_2+2x_3+2x_4+x_5+0.x_6+0.x_7=10$

$2x_1+5x_2+x_3+4x_4+0.x_5+x_6+0.x_7=5$

$x_1+2x_2+5x_3+x_4+0.x_5+0.x_6+x_7=6$

$x_j \geq 0 \quad j=1,2,\dots,7$

here (a_5, a_6, a_7) forms initial basis

x_B = Basic variable

B = basis vector

C_B = coeff. basic variable in objective fn.

Table :-

C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	Ratio (min)	Operation	Remarks
			c_j	3	4	1	5	0	0	0			
0	a_5	x_5	10	8	3	2	2	1	0	0	$10/2$		$a_4 \to$ enterin
← 0	a_6	x_6	5	2	5	1	$\boxed{4}$	0	1	0	$5/4$ (min)		$a_6 \to$ departi
0	a_7	x_7	6	1	2	5	1 ↑	0	0 ↓	1	$6/1$		$4 \to$ key element
$z_j - c_j$				-3	-4	-1	-5	0	0	0			
← 0	a_5	x_5	$15/2$	$\boxed{7}$	$1/2$	$3/2$	0	1	$-1/2$	0	$15/14$	$R_2' = \frac{1}{4}R_2$	$a_1 \to$ enteri
5	a_4	x_4	$5/4$	$1/2$	$5/4$	$1/4$	1	0	$1/4$	0	$\frac{5}{4}\cdot\frac{2}{1}=5/2$	$R_1' = R_1 - 2R_2'$	$a_5 \to$ depa
0	a_7	x_7	$19/4$	$1/2$ ↑	$3/4$	$19/4$	0	0 ↓	$-1/4$	1	$\frac{19}{4}\cdot\frac{2}{1}=\frac{19}{2}$	$R_3' = R_3 - R_2'$	$7 \to$ key element
$z_j - c_j$				$-1/2$	$9/4$	$1/4$	0	0	$5/4$	0			
3	a_1	x_1	$\frac{15}{14}$	1	$\frac{1}{14}$	$\frac{3}{14}$	0	$\frac{1}{7}$	$-\frac{1}{14}$	0		$R_1' = \frac{1}{7}R_1$	
5	a_4	x_4	$5/7$	0	$\frac{17}{14}$	$1/4$	1	$\frac{-1}{14}$	$2/7$	0		$R_2' = R_2 - \frac{1}{2}R_1'$	
0	a_7	x_7	$\frac{59}{14}$	0	$5/7$	$\frac{65}{14}$	0	$-\frac{1}{14}$	$\frac{-3}{14}$	1		$R_3' = R_3 - \frac{1}{2}R_1'$	
$z_j - c_j$				0	$\frac{16}{7}$	$\frac{5}{14}$	0	$1/14$	$\frac{17}{14}$	0			

Here all $z_j - c_j \geq 0$ and thus the optimality condition is achieved.

$\therefore$ optimal soln. is

$x_1 = \frac{15}{14}, \; x_2 = 0, \; x_3 = 0, \; x_4 = \frac{5}{7}$

$Z_{max} = \frac{45}{14} + \frac{25}{7} = \frac{45+50}{14} = \frac{95}{14}$ Ans.

(14) Maximize $Z = 4x_1 + 5x_2 + 9x_3 + 11x_4$

Subject to, $x_1 + x_2 + x_3 + x_4 \leq 15$

$7x_1 + 5x_2 + 3x_3 + 2x_4 \leq 120$

$3x_1 + 5x_2 + 10x_3 + 15x_4 \leq 100$

$x_1, x_2, x_3, x_4 \geq 0$

Introducing slack variables we rewrite the problem as is achived.

∴ optimal soln is

$x_1 = 5/2$, $x_2 = 3/2$ Max $Z = \frac{7.5}{2} + \frac{3.3}{2}$

$= \frac{35+9}{2} = 22$ Ans.

16 ii) Minimize

$Z = 4x_1 + 8x_2 + 3x_3.$

Subject to $x_1 + x_2 \geq 2$
$2x_1 + x_3 \geq 5$
$x_1, x_2, x_3 \geq 0$

Introducing slack and surplus variables, we rewrite the problem in standard form as

Maximize $(-Z) = -4x_1 - 8x_2 - 3x_3 + 0.x_4 + 0.x_5$

Subject to

$\cancel{x_1 + x_2 - x_3 = 2}$

$\cancel{2x_1 + 0.x_2 + 0.x_3}$

$x_1 + x_2 + 0.x_3 - x_4 - 0.x_5 = 2$
$2x_1 + 0.x_2 + x_3 + 0.x_4 - x_5 = 5.$
$x_j \geq 0 \quad j = 1,2,3,4,5$

here (a_2, a_3) forms initial basis

~~Introd~~ x_B = Basis variable, B = Basis vector, C_B = coefficient of basis variable in objective function.

Table:-

			c_j	-4	-8	-3	0	0	Ratio	Operation	Remarks
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	(min)		
-8	a_2	x_2	2	$\boxed{1}$	1	0	-1	-1	2 (min)		$a_1 \to$ entering, $a_2 \to$ departing, $\boxed{1} \to$ key element
-3	a_3	x_3	5	2	0	1	0	-1	$5/2$		
$z_j - c_j$				-10	0	0	8	11			
-4	a_1	x_1	2	1	1	0	-1	-1	$-$	$R_2' = R_2 - 2R_1'$	$a_4 \to$ entering, $a_3 \to$ departing, $\boxed{2} \to$ key element
-3	a_3	x_3	1	0	-2	1	$\boxed{2}$	1	$1/2$ (min)		
$z_j - c_j$				0	10	0	-2	1			
-4	a_1	x_1	$5/2$	1	0	$1/2$	0	$-1/2$		$R_2' = \frac{1}{2}R_2$, $R_1' = R_1 + R_2'$	
0	a_4	x_4	$1/2$	0	-1	$1/2$	1	$1/2$			
$z_j - c_j$				0	8	1	0	2			

Here all $z_j - c_j \geq 0$ and thus the optimal condition is achived. ∴ Optimal soln is $x_1 = 5/2$, $x_2 = x_3 = 0$. ∴ Min $Z = 10$

Ans.