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1. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$

Hence we have

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0 \rightarrow (i)$$

eq. (i) can also be written as

$$(D^2 - 2D - 3)y = 0$$

The auxiliary equation is

$$m^2 - 2m - 3 = 0$$
$$(m+1)(m-3) = 0$$
$$m = -1, 3$$

So the roots are real and different

Complementary function $(C.F) = c_1 e^{-x} + c_2 e^{3x}$

particular Integral $(P.I) = 0$

Hence the complete solution is $y = C.F + P.I$

$$y = c_1 e^{-x} + c_2 e^{3x}$$

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2. $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 0$

Hence we have

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 0 \rightarrow (i)$$

eq. (i) can also be written as

$$(D^2 - 8D + 16)y = 0$$

The auxiliary equation is

$$m^2 - 8m + 16 = 0$$
$$(m-4)(m-4) = 0$$

$m = 4, 4$

So the roots are real and equal

Complementary function $(C.F) = (C_1 + C_2 x) e^{4x}$

particular Integral $(P.I) = 0$

Hence the complete solution is $y = C.F + P.I$

$$y = (C_1 + C_2 x) e^{4x}$$

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3. $\frac{d^2y}{dx^2} + 9y = 0$

Here we have

$$\frac{d^2y}{dx^2} + 9y = 0 \rightarrow (i)$$

eq. (i) can also be written as

$$(D^2 + 9) y = 0$$

The auxiliary equation is

$$m^2 + 9 = 0$$

$$m = \pm 3i$$

So the roots are imaginary

Complementary function $(C.F) = C_1 \cos 3x + C_2 \sin 3x$

particular Integral $(P.I) = 0$

Hence the complete solution is $y = C.F + P.I$

$$y = C_1 \cos 3x + C_2 \sin 3x$$

4. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 25y = 0$

Here we have

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 25y = 0 \rightarrow \text{(i)}$$

eq. (i) can also be written as

$$(D^2 + 6D + 25)y = 0$$

The auxiliary equation is

$$m^2 + 6m + 25 = 0$$

$$m = \frac{-6 \pm \sqrt{36 - 100}}{2}$$

$$= \frac{-6 \pm \sqrt{-64}}{2} = \frac{-6 \pm 8i}{2} = -3 \pm 4i$$

So the roots are imaginary

Complementary function (C.F) $= e^{-3x}(c_1 \cos 4x + c_2 \sin 4x)$

particular Integral (P.I) $= 0$

Hence the complete solution is $y = C.F + P.I$

$$y = e^{-3x}(c_1 \cos 4x + c_2 \sin 4x)$$

Solve the initial value problem

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0$, $y(0) = 3$, $y'(0) = 5$

Here we have

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0 \rightarrow \text{(i)}$$

eq.(i) can also be written as

$$(D^2-D-12)y=0$$

The auxiliary equation is

$$m^2-m-12=0$$

$$(m-4)(m+3)=0$$

$$m=4,-3$$

So the roots are real and different

Complementary function $(C.F) = c_1 e^{4x}+c_2 e^{-3x}$

Particular Integral $(P.I)=0$

Hence the complete solution is $y=C.F+P.I$

$$y=c_1 e^{4x}+c_2 e^{-3x} \rightarrow \text{(ii)}$$

Differentiating eq.(ii) w.r.t x we get

$$y'=4c_1 e^{4x}-3c_2 e^{-3x} \rightarrow \text{(iii)}$$

Given $y(0)=3$ and $y'(0)=5$

Putting $x=0, y=3$ in eq.(ii) we get

$$3=c_1+c_2$$

$$c_1+c_2=3 \rightarrow \text{(iv)}$$

Putting $x=0, y'=5$ in eq.(iii) we get

$$5=4c_1-3c_2$$

$$4c_1-3c_2=5 \rightarrow \text{(v)}$$

Solving eq.(iv) and (v) simultaneously we get

$$c_1+c_2=3 \rightarrow \text{(iv)}\times 4$$

$$4c_1-3c_2=5 \rightarrow \text{(v)}\times 1$$

$$4c_1+4c_2=12$$

$$\underset{(-)}{4c_1}\ \underset{(+)}{-}\ 3c_2 \underset{(-)}{=} 5$$

$$7c_2=7$$

$$c_2=1$$

$\therefore$ putting the value of c_2 in (v) we have

$$c_1=2$$

$\therefore c_1 = 2,\ c_2 = 1$

Hence the complete solution after finding the value of constants is

$$y = 2e^{4x} + e^{-3x}$$

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2. $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 4y = 0,\quad y(0) = 3,\ y'(0) = 7$

Here we have

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0 \rightarrow (i)$$

eq. (i) can also be written as

$$(D^2 + 4D + 4)y = 0$$

The auxiliary equation is

$$m^2 + 4m + 4 = 0$$

$$(m+2)(m+2) = 0$$

$$m = -2, -2$$

So the roots are real and equal

Complementary function $(C.F) = (c_1 + c_2x)e^{-2x}$

particular Integral $(P.I) = 0$

Hence the complete solution is $y = C.F + P.I$

$$y = (c_1 + c_2x)e^{-2x} \longrightarrow (ii)$$

Differentiating (ii) w.r.t x we get

$$y' = -2(c_1 + c_2x)e^{-2x} + c_2e^{-2x} \longrightarrow (iii)$$

Putting $x=0,\ y=3$ in eq.(ii) we get

$$C_1 = 3$$

Now putting the value of C_1, $y'=7$, $x=0$ in eq.(iii) we get

$$7 = -2(3 + C_2 \cdot 0)e^0 + C_2 e^0$$

$$7 = -6 + C_2$$

$$C_2 = 13$$

Hence the complete solution after finding the value of constants is

$$y = (3 + 13x)e^{-2x}$$

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3. $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 29y = 0, \quad y(0) = 0, \quad y'(0) = 5$

Here we have

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 29y = 0 \longrightarrow \text{(i)}$$

eq.(i) can also be written as

$$(D^2 - 4D + 29)y = 0$$

The auxiliary equation is

$$m^2 - 4m + 29 = 0$$

$$m = \frac{4 \pm \sqrt{16 - 116}}{2}$$

$$m = \frac{4 \pm \sqrt{-100}}{2} = 2 \pm \frac{5}{1}i = 2 \pm 5i$$

So the roots are imaginary

Complementary function (c.f) $= e^{2x}(c_1 \cos 5x + c_2 \sin 5x)$

particular Integral (P.I) $= 0$

Hence the complete solution is $y = C.F + P.I$

$$y = e^{2x}(c_1 \cos 5x + c_2 \sin 5x) \longrightarrow \text{(ii)}$$

Differentiating (ii) w.r.t x we get

$$y' = e^{2x}(-5c_1 \sin 5x + 5c_2 \cos 5x) + 2e^{2x}(c_1 \cos 5x + c_2 \sin 5x) \longrightarrow \text{(iii)}$$

putting $y=0$, $x=0$ in (ii) we get

$$0 = c_1$$

$$c_1 = 0$$

putting the value of c_1, $y' = 5$, $x = 0$ in eq. (iii) we get

$$5 = 5c_2 + 2(0)$$

$$c_2 = 1$$

Hence the complete solution after finding the constants is

$$y = e^{2x} \sin 5x$$