**Unit 1 Error Analysis**

**Strucure**

* 1. **Introduction**
  2. **Reason of Numerical Errors**
  3. **Measurement of Errors**
  4. **Summary**
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**1.1 Introduction:**

The process of solving physical problems can be roughly divided into three phases. The first consists of constructing a mathematical model for the corresponding physical problem. This model could be in the form of differential equations or algebraic equations. In most cases, this mathematical model cannot be solved analytically, and hence a numerical solution is required. In which case, the second phase in the solution process usually consists of constructing an appropriate numerical model or approximation to the mathematical model. For example, an integral or a differential equation in the mathematical formulation will have to be approximated appropriately for numerical solution. A numerical model is one where everything in principle can be calculated using a finite number of basic arithmetic operations. The third phase of the solution process is the actual implementation and solution of the numerical model.

**1.2 Reason of numerical Errors**

It can be the combined effect of two kinds of error in a calculation.

* the first is caused by the finite [precision](https://en.wikipedia.org/wiki/Precision_(computer_science)) of computations involving [floating-point](https://en.wikipedia.org/wiki/Floating-point) or integer values
* the second usually called truncation error is the difference between the exact mathematical solution and the approximate solution obtained when simplifications are made to the mathematical equations to make them more amenable to calculation. The term truncation comes from the fact that either these simplifications usually involve the truncation of an [infinite series](https://en.wikipedia.org/wiki/Infinite_series) expansion so as to make the computation possible and practical, or because the least significant bits of an arithmetic operation are thrown away.

**1.3 Measurement of Errors:**

Numerical Errors usually measured in two ways. One is Absolute Error, other is Percentage Error.

**Absolute Error:** Absolute Error is the magnitude of the difference between the true value and the approximate value , Therefore absolute erroris defined as the error between two values is defined as , where denotes the exact value and denotes the approximation.

**Relative Error:** The relative error of  {\displaystyle {\tilde {x}}}is the absolute error relative to the exact value. Look at it this way: if your measurement has an error of ± 1 inch, this seems to be a huge error when you try to measure something which is 3 in. long. However, when measuring distances on the order of miles, this error is mostly negligible. The definition of the relative error is .

Note: Consider you try to measure a rod of length 10 cm, and found lenghth as 9.98 cm from your scale. Here True value or actual value of the rod 10 cm and approximate value of the length of the rod is 9.98 cm. So, the absolute error will be and the relative error will be .

One can express this error in percentage as , which gives the value for the example taken here. This is called percentage error.

**Example :** If is approximated as 3.14, find the absolute error, relative error and relative percentage error.

Solution: Absolute error

= = 0.002857

Relative error

= 0.0009

Relative percentage error =

=

=

**Example :** Compute the percentage error in the time period for if the error in the measurement of .

Solution: Given the

Taking of both sides we have,

Now We will discuss some important types of Numerical Errors

* [**Loss of significance**](https://en.wikipedia.org/wiki/Loss_of_significance)
* **Inherent errors**
* [**Round-off error**](https://en.wikipedia.org/wiki/Round-off_error)
* **Truncation errors:**

**(i) Loss of significance** is an undesirable effect in calculations using finite-precision arithmetic such as floating-point arithmetic. It occurs when an operation on two numbers increases relative error substantially more than it increases absolute error, for example in subtracting two nearly equal numbers (known as **catastrophic cancellation**). The effect is that the number of significant digits in the result is reduced unacceptably. Ways to avoid this effect are studied in numerical analysis.

Example:  As an example, consider the behavior of {\displaystyle f(x)={\sqrt {x^{2}+1}}-1}as approaches . Evaluating this function at using Matlab incorrectly returns the answer 0, which shows that too many significant digits have cancelled.

(ii) **Inherent errors:** This type of errors is present in the statement of the problem itself,

before determining its solution. Inherent errors occur due to the simplified assumptions

made in the process of mathematical modelling of a problem. It can also arise when the

data is obtained from certain physical measurements of the parameters of the proposed

problem.

Inherent errors can be minimized by taking better data on by using high precision computing aids. High precision regers to the number of decimal positions, i.e. the order of magnitude of the last digit in a value. For example the number 46.398 has a precision of 0.001 or

**Example:** Which of the following numbers have greatest precision?

3.1201, 2.42, 5.320205.

Solution : In 3.1202, the precision is

In 2.42, the precision is ,

In 5.320205, the precision is

Hence the 5.320205 has the greatest precision.

(iii) **Round-off errors:** Generally, the numerical methods are carried out using calculator

or computer. In numerical computation, all the numbers are represented by

decimal fraction. Some numbers such as 1/3, 2/3, 1/7 etc. can not be represented by

decimal fraction in finite numbers of digits. Thus, to get the result, the numbers should

be rounded-off into some finite number of digits.

Again, most of the numerical computations are carried out using calculator and computer.

These machines can store the numbers up to some finite number of digits. So in

arithmetic computation, some errors will occur due to the finite representation of the

numbers; these errors are called round-off error. Thus, round-off errors occur due to the

finite representation of numbers during arithmetic computation. These errors depend

on the word length of the computational machine.

**Example**: Round off the following numbers, to four significant digits and then calculate absolute error, relative error and percentage error.

1. 54762 ii) 437.261 iii) 19.36235

**Solution:** i) The given number is 54762 (=N)

After round off to four significant figures,

the given number would be 54760 (=

Absolute error

= 2

Relative error

=

Relative percentage error =

=

=

**Exercise**: Do (ii) and (iii)

(iv) **Truncation errors:** These errors occur due to the finite representation of an

inherently infinite process. For example, the use of a finite number of terms in the

infinite series to compute the value of *, ,,* etc.

The Taylor’s series expansion of is

This is an infinite series expansion. If only first five terms are taken to compute the

value of for a given , then we obtain an approximate result. Here, the error occurs

due to the truncation of the series. Suppose, we retain the first *n* terms, the truncation

error (*E*trunc) is given by

It may be noted that the truncation error is independent of the computational machine.

**Example**: Find the number of terms of the exponential series such that their sum gives the value correct to six decimal places at

**Solution**: We know,

Where

Maximum absolute error (at and maximum relative error is

Hence at

For a six decimal accuracy at we have

or,

which gives

**1.4 Summary**

In this unit, the concept of Numerical errors, measurement of errors like absolute errors, relative errors, percentage error, loss of significats, inherent , roundoff and truncations errors are discussed with different examples.

**1.5 Exercises**

1. If 0.333 is the approximate valure of , find absolute, relative and percentage errors. (Ans: .00033, 0.00099, 0.99)
2. If and error in Compute the relative error in when (Ans: .14)
3. Find the difference of correct to three digits. (Ans: 3.53
4. If find the relative error in the computation of . (Ans: 0.001 (approx.))
5. Use the series of to compute the value of correct to seven deciamal places and find the number of terms retained.

(Ans:

1. What do you understand by Inherent errors occurs in numerical computation?