**Unit 5 Numerical differentiation**

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**5.1 Introduction**

Numerical differentiation is connected with the computation of derivatives of a function whose values are known at a tabular points. The fundamental operation of differentiation is applied to the interpolating polynomial to evaluate the derivatives of the given of the given function whose values are known at some tabular points.

**5.2 Newton’s Forward Differentiation Formula**

Let denote a continuously differential function which takes the values for the equidistant values of the independent variables x, then we have from Newton’s Forward Interpolation formula as

Where , ( is the step length, and so that

And so on

In particular for i.e. for

The above formulae are applicable for numerical differentiation at a point near the beginning of the tabulated values.

**5.3 Newton’s Backward Differentiation Formula**

Let denote a continuously differential function which takes the values for the equidistant values of the independent variables x, then we have from Newton’s Forward Interpolation formula as

where , ( is the step length, and so that

and so on

In particular for i.e. for

The above formulae are applicable for numerical differentiation at a point near the end of the tabulated values.

**5.4 Lagrange’s Differentiation Formula**

Let denote a continuously differential function which takes the values corresponding to (n+1) non-equidistant values . Since the (n+1) values of the function are given corresponding to (n+1) values of the independent variable x, we can represent the function to be a polynomial in of degree . Then we have Lagrange’s Interpolation formula as

where

Now

(5.1)

For non tabular points we use the above formula but for the tabular points equation (5.1) is indeterminate. Hence we proceed as

where

**Example:** Compute and for , using following table

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 1 | 8 | 27 | 64 | 125 | 216 |

**Solution:** The difference table is

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 1 | 1 |  |  |  |  |
|  |  | 7 |  |  |  |
| 2 | 8 |  | 12 |  |  |
|  |  | 19 |  | 6 |  |
| 3 | 27 |  | 18 |  | 0 |
|  |  | 37 |  | 6 |  |
| 4 | 64 |  | 24 |  | 0 |
|  |  | 61 |  | 6 |  |
| 5 | 125 |  | 30 |  |  |
|  |  | 91 |  |  |  |
| 6 | 216 |  |  |  |  |

We have

and

and .

**Example :** Find the value of for which is minimum and find the minimum value from the table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 0.60 | 0.65 | 0.70 | 0.75 |
|  | 0.6221 | 0.6155 | 0.6138 | 0.6174 |

Solution: Taking 0.60 as origin, we have

We have the difference table as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 0.60 | 0.6221 |  |  |  |
|  |  | -0.0066 |  |  |
| 0.65 | 0.6155 |  | 0.0049 |  |
|  |  | -0.0017 |  | 0 |
| 0.70 | 0.6138 |  | 0.0049 |  |
|  |  | 0.0032 |  |  |
| 0.75 | 0.6170 |  |  |  |

Putting the values, we have

where

Also

**5.5 Summary**

In this unit numerical differentiation has been done by Using Newton’ Forward, backward, Lagrange’s differentiation formulae. Using this maximum and minimum values are also calculated.

**5.6 Exercises**

1. Find

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 60 | 75 | 90 | 105 | 120 |
| f(x) | 28.2 | 38.2 | 43.2 | 40.9 | 37.7 |

Ans : -0.03627

1. Find the first and second order derivative of at from the following table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 15 | 17 | 19 | 21 | 23 | 25 |
|  | 3.873 | 4.123 | 4.359 | 4.583 | 4.796 | 5.000 |

Ans: 0.1289, -0.004

1. Find the minimum values of from the table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 0 | 2 | 4 | 6 |
|  | 3 | 3 | 11 | 27 |

Ans: 2.25

1. Find the maximum values of from the table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
|  | 0.9320 | 0.9636 | 0.9855 | 0.9975 | 0.9996 |

Ans: 1.58

1. The population of a certain town is given below. Find the rate of growth of the population in 1931, 1971

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Year (x) | 1931 | 1941 | 1951 | 1961 | 1971 |
| Population on thousands(y) | 40.62 | 60.80 | 79.95 | 103.56 | 132.65 |

Ans: 2.36425, 3.10525