CE 302: Structural Analysis

Project
Report on

Tuned
Mass
Damper

by

Team Name: STrUX

Himanshu Singh 23110134
Karthik Chandra 23110158
Piyush Choudhary 23110247

Indian Institute of Technology Gandhinagar

Declaration

We, **Himanshu Singh**, **Karthik Chandra**, and **Piyush Choudhary**, declare that we have done the following proportion of work towards the project entitled *"Tuned Mass Damper"* of the Structural Analysis (CE 302) course.

Sl. No.	Name	Responsibilities	Percentage of Total Work	Signature
1	Himanshu Singh	3D Printed All Joints, Helped in Creating The Model, Did all the SAP Modelling	33.33 %	Himanshu
2	Karthik Chandra	Major Fabrication work and helped in report(Analytical results, summary and Conclusion)	33.34 %	Karthik
3	Piyush Choudhary	Helped in Fabrication, Wrote the necessary Codes, Compiled the report and wrote the Experimental set-up and Comparison part	33.33 %	Choudhary

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Introduction

1.1 The Problem: Why Do Buildings Sway?

Tall structures such as skyscrapers are inherently flexible, like vertical cantilevers that oscillate when subjected to dynamic loads like wind or seismic activity. These structures possess a **natural frequency** (ω_n), a property determined by their stiffness (k) and mass (m):

$$\omega_n = \sqrt{\frac{k}{m}}$$

When external forces (e.g., wind gusts or earthquakes) apply periodic excitation at a **forcing frequency** (ω_{ext}) matching the structure's natural frequency, **resonance** occurs:

$$\omega_{\text{ext}} \approx \omega_n$$

At resonance, energy transfer to the structure becomes highly efficient, causing displacements to amplify exponentially. For lightly damped systems, this can lead to structural fatigue, occupant discomfort, or even catastrophic failure.

1.1.1 Amplitude of Vibration (Without a TMD)

The displacement response (x) of a building under a harmonic force $F_0 \cos(\omega t)$ is modeled by the damped harmonic oscillator equation:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_0 \cos(\omega t)$$

where c is the damping coefficient. At resonance ($\omega = \omega_n$), the steady-state amplitude A reaches its maximum:

$$A = \frac{F_0}{2m\omega_n\zeta}, \quad \zeta = \frac{c}{2\sqrt{km}}$$

Here, ζ is the damping ratio. For typical civil structures ($\zeta \approx$ 1–5%), the amplitude A can become dangerously large, necessitating mitigation strategies.

1.2 The Solution: Tuned Mass Damper (TMD)

1.2.1 Principle of Operation

A Tuned Mass Damper (TMD) is a passive vibration control device consisting of a secondary mass (m_d), spring (k_d), and damper (c_d) installed atop a structure (see Figure 1.1). The TMD is tuned to the structure's natural frequency ($\omega_d = \omega_n$), enabling it to oscillate out of phase with the building's motion. This generates **destructive interference**, effectively canceling resonant vibrations.

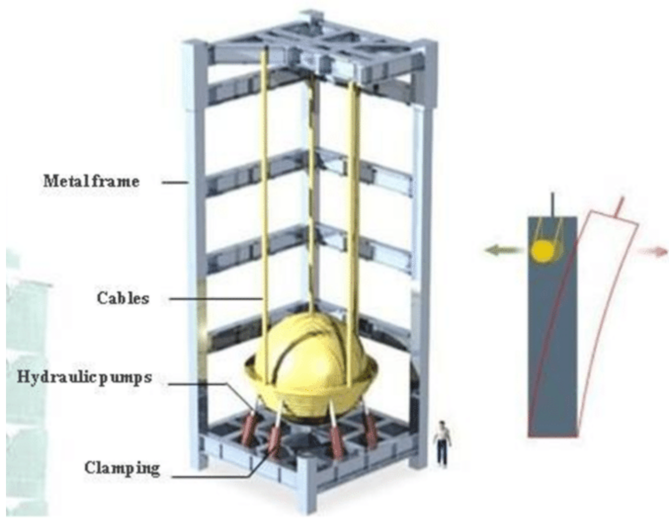

Figure 1.1: TMD in a building

Key Components

Component	Function
Mass	Counterweight (1–5% of structural mass) to absorb kinetic energy
Spring	Tuned to match the building's natural frequency ($\omega_d = \sqrt{k_d/m_d}$)
Damper	Dissipates energy via viscous damping, converting oscillations into heat

Table 1.1: Components of a Tuned Mass Damper

1.2.2 Mathematical Modeling [2]

The coupled building-TMD system is governed by two equations of motion:

Building Dynamics

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx - c_d\left(\frac{dy}{dt} - \frac{dx}{dt}\right) - k_d(y-x) = F_0\cos(\omega t)$$

TMD Dynamics

$$m_d \frac{d^2 y}{dt^2} + c_d \left(\frac{dy}{dt} - \frac{dx}{dt} \right) + k_d (y - x) = 0$$

Here, $x(t)$ and $y(t)$ represent the displacements of the building and TMD, respectively. The TMD exerts opposing forces proportional to the relative displacement $(y - x)$ and velocity $(\dot{y} - \dot{x})$, counteracting the building's motion.

1.2.3 Mechanism of Destructive Interference

When the building sways right $(+x)$, the TMD lags by half a cycle, swaying left $(-y)$. This phase difference of 180° ensures the TMD's inertial force opposes the building's motion:

$$F_{\text{TMD}} = -k_d (y - x) - c_d \frac{d}{dt}(y - x)$$

The net force on the building becomes:

$$F_{\text{net}} = F_{\text{wind}} + F_{\text{TMD}}$$

thereby reducing the effective amplitude x.

1.2.4 Design Considerations

The TMD's effectiveness hinges on three parameters:

1. **Mass Ratio** ($\mu = m_d / m$):
   A larger mass (typically 1–5% of the building's mass) improves energy absorption but increases cost and space requirements.

2. **Tuning Accuracy**:
   Mismatch between ω_d and ω_n drastically reduces performance. Post-construction tuning via adjustable springs/pendulums is critical.

3. **Optimal Damping** (ζ_d):
   Excessive damping restricts TMD motion, while insufficient damping fails to dissipate energy. The optimal damping ratio is derived as:

$$\zeta_d = \sqrt{\frac{3\mu}{8(1 + \mu)}}$$

1.2.5 Practical Applications

The Taipei 101 skyscraper employs a 660-tonne spherical TMD, reducing peak accelerations by 30–40% during typhoons. Similarly, the Citicorp Center in New York uses a 400-ton TMD to mitigate wind-induced oscillations.

2
Analytical results

2.1 Deflection Angle (θ)

The cantilever beam is analyzed under its self-weight (Assuming the roof and the electronic components are mass-less), modeled as a uniformly distributed load (UDL). The following parameters are considered:

- **Length (L)** = 1.2 m
- **Breadth (b)** = 1.5 cm = 0.015 m
- **Depth (d)** = 1 cm = 0.01 m
- **Young's Modulus (E)** = 4 GPa = 4×10^9 Pa
- **Density** (ρ) = 725 kg/m^3
- **Moment of Inertia (I)** = $\dfrac{bd^3}{12} = \dfrac{0.015 \times (0.01)^3}{12} = 1.25 \times 10^{-9}$ m^4
- **Uniformly Distributed Load (w)** = $\frac{\rho \cdot g \cdot V}{L} = 725 \cdot 9.81 \cdot (0.015 \cdot 0.01) = 1.06575$ N/m

The theoretical deflection angle at the free end is given by:

$$\theta = \frac{wL^3}{6EI}$$

Substituting values:

$$\theta = \frac{1.06575 \times (1.2)^3}{6 \times 4 \times 10^9 \times 1.25 \times 10^{-9}} \approx 0.06138 \text{ radians} \approx 3.5168^\circ$$

2.2 Natural Frequency Analysis

To assess the dynamic behavior of the beam, time-domain acceleration data was recorded for two conditions:

- With Tuned Mass Damper (TMD)
- Without Tuned Mass Damper

To evaluate the natural frequency of the beam, we analyzed the time-domain acceleration data and identified the time between the first two peaks of vibration. The time period (T) was calculated as:

$$T = t_2 - t_1$$

where t_1 and t_2 are the time values corresponding to the first and second peaks, respectively.
Using the relationship:

$$f = \frac{1}{T}$$

we estimated the natural frequencies for both cases.
Results:

- **Without TMD:** Natural Frequency $\approx$ 2.88 Hz
- **With TMD:** Natural Frequency $\approx$ 2.50 Hz

3
SAP2000 model

We began by carefully deciding the dimensions for our model to meet the structural and damping requirements. The initial concept was sketched by hand to finalize proportions and design intent.

3.1 Handmade Draft

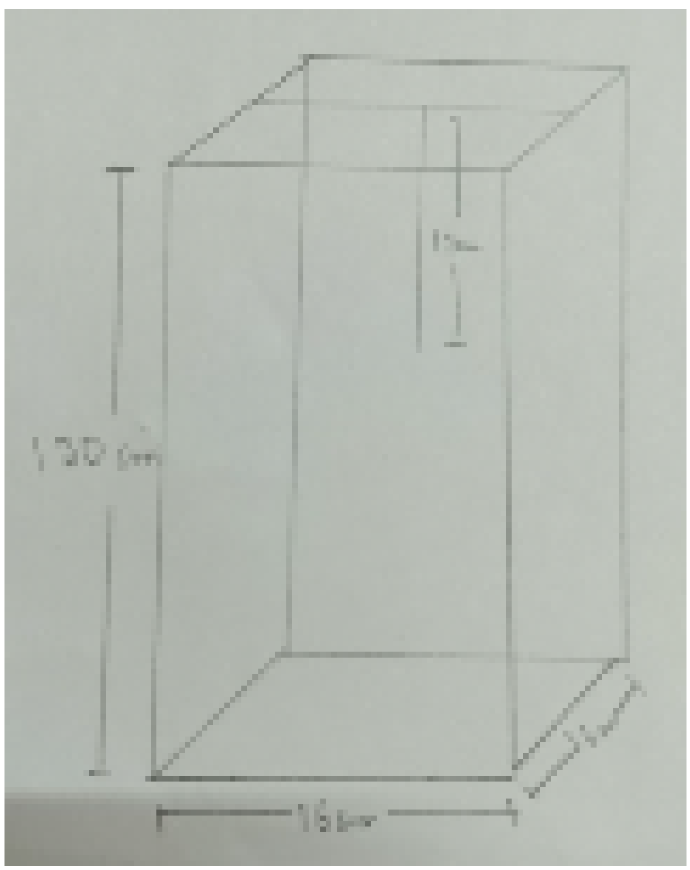

Figure 3.1: Initial handmade draft of the TMD system.

3.2 SAP2000 Modeling

After finalizing the design, we created the structure in SAP2000 using the same dimensions and configuration as the handmade draft.

3.2.1 Top View of the SAP Model

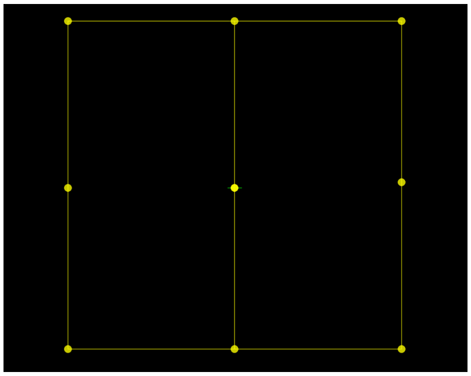

Figure 3.2: Top view of the SAP model.

3.2.2 Side View of the SAP Model

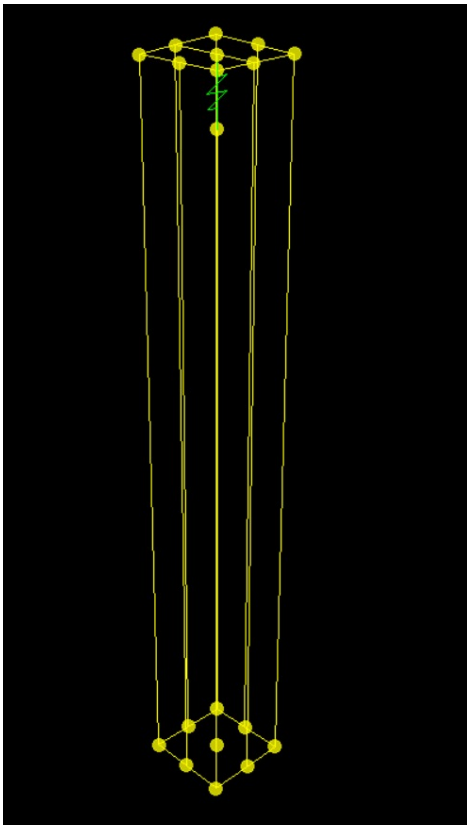

Figure 3.3: Side view of the SAP model.

3.2.3 Full Skyscraper with TMD

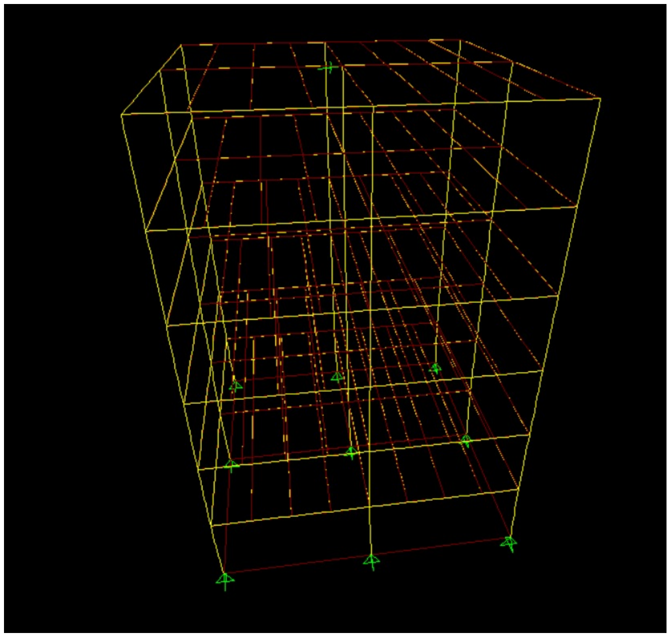

Figure 3.4: 3D model of skyscraper with TMD (green line represents the damper).

3.2.4 Side View of the Skyscraper

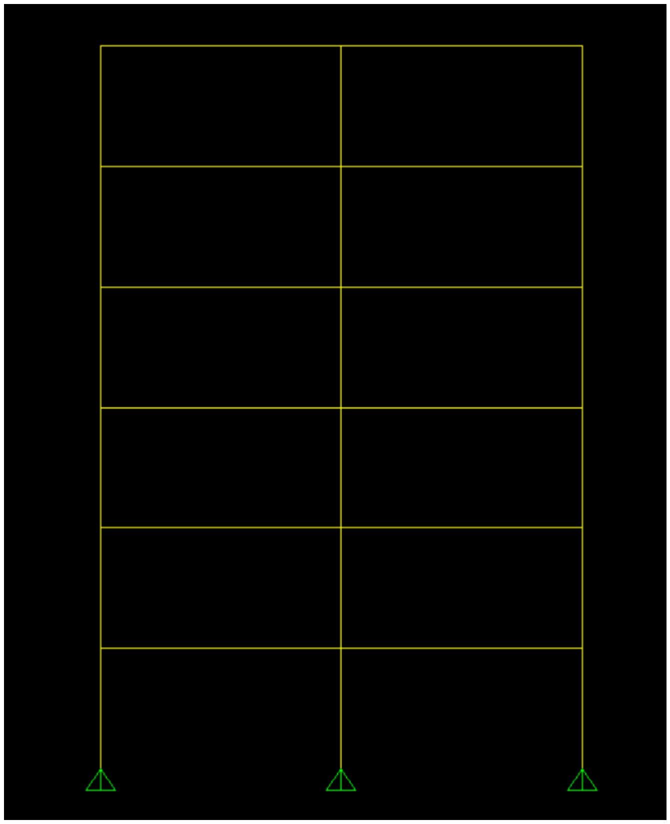

Figure 3.5: Side view of the skyscraper.

3.3 Earthquake Load Simulation

We applied earthquake loads to the structure and visualized the resulting deflections. The model responded realistically, showing clear differences in behavior with and without the TMD.

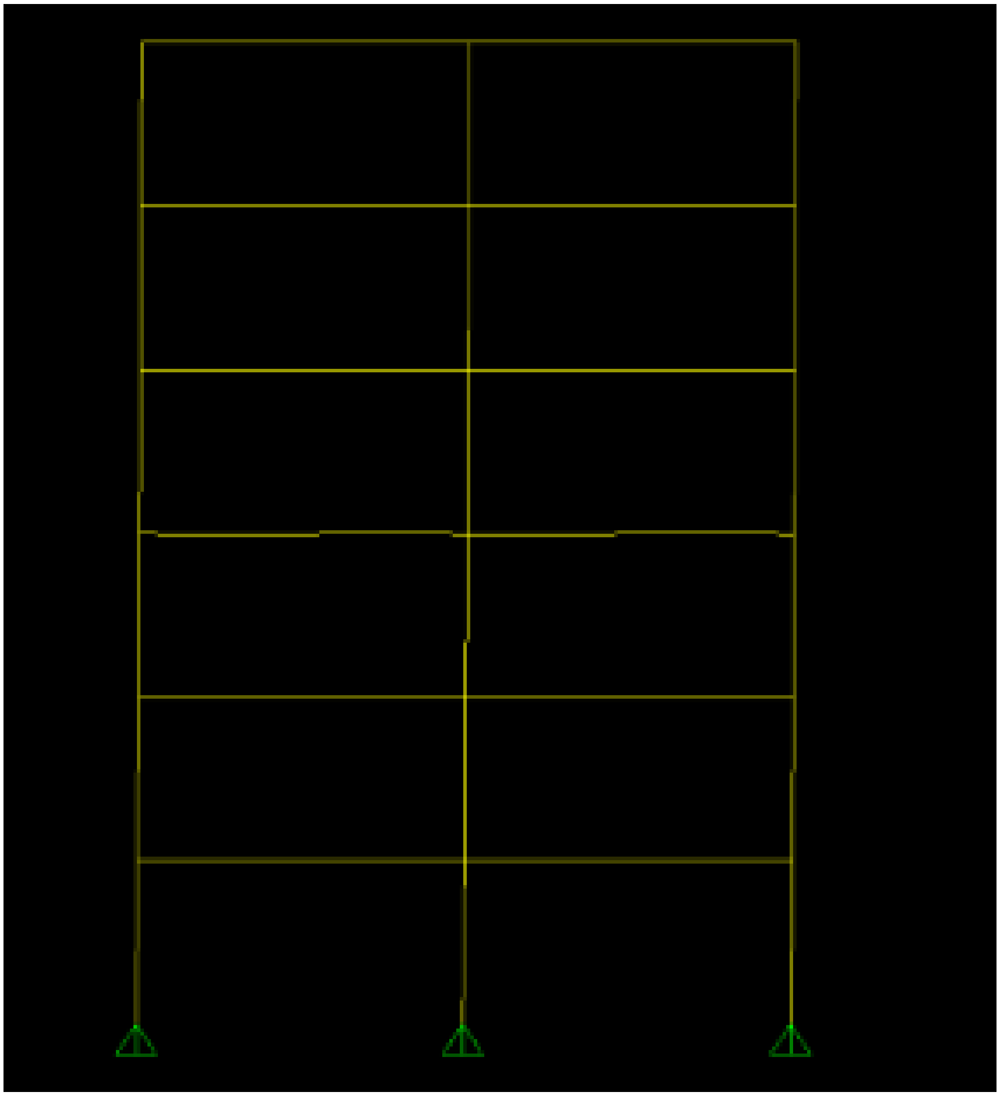

Figure 3.6: Deflection In The Building Due To Applied Earthquake Load.

3.4 Energy Dissipation Comparison

We analyzed the time taken for the building's kinetic energy to reduce to zero, with and without the damper.

3.5 Observation

From the simulation results:

- With TMD: The kinetic energy reduces to zero in approximately **2.6 seconds**.
- Without TMD: The building takes slightly more than **3 seconds** to stabilize.

This confirms that the TMD significantly enhances the damping performance of the structure.

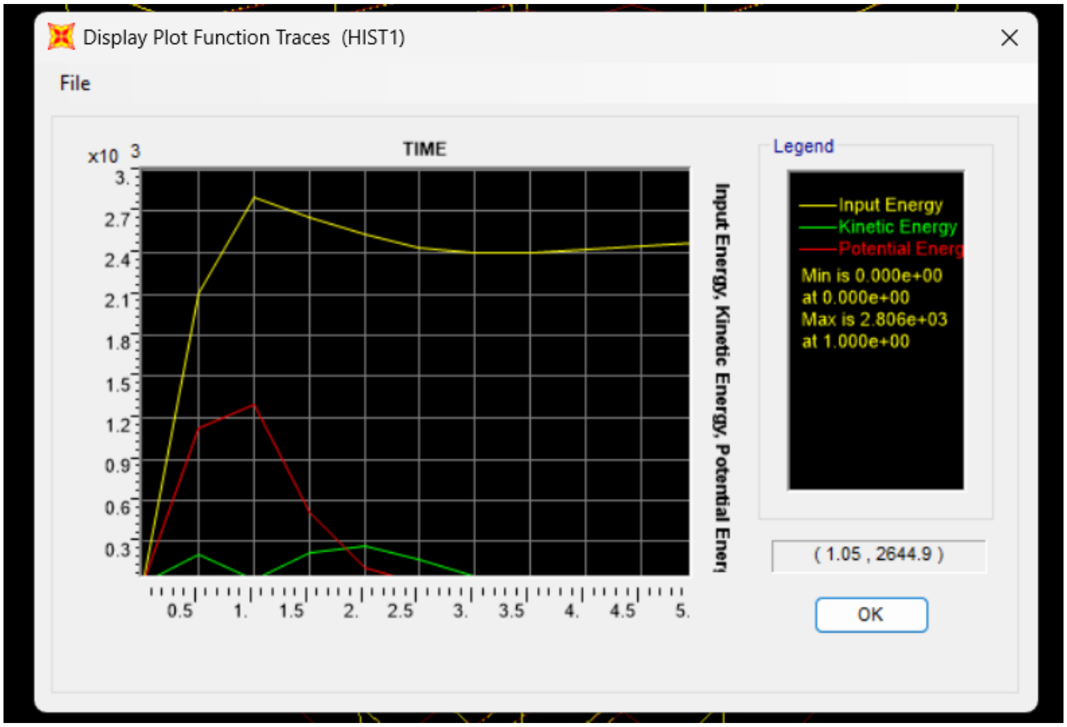

Figure 3.7: Energy dissipation with TMD.

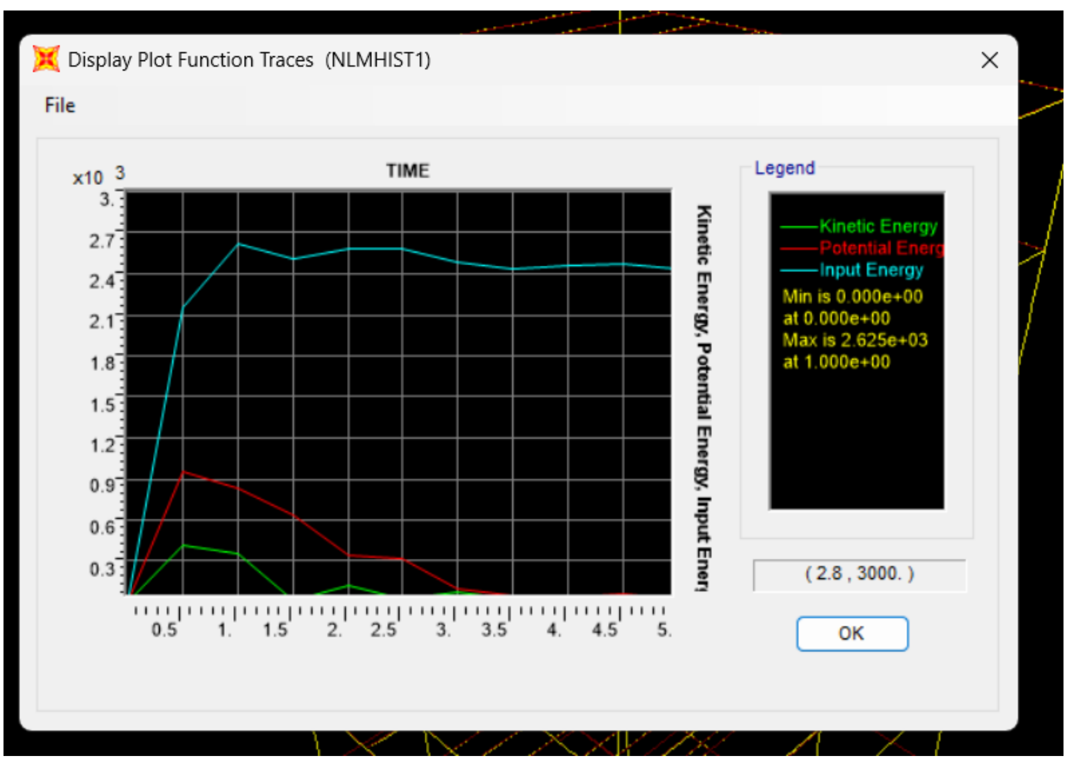

Figure 3.8: Energy dissipation without TMD.

4
Experimental set-up, testing protocol and results

4.1 Experimental Setup

4.1.1 Physical Model

A scaled-down building model was constructed with the following components:

- **Primary structure:** MDF sheet was used and joints were 3D printed.
- **TMD attachment:** Removable mass-damper system (tuned to the model's natural frequency).
- **Sensors:** Accelerometer (MPU6050) mounted at the top floor to measure lateral acceleration.
- **Data acquisition:** Arduino Uno programmed to sample acceleration data.

Figure 4.1: Model without TMD

Figure 4.2: Model with TMD

4.1.2 Software Tools

- **Arduino IDE:** Script was written to print the accelerometer data for all 3 axis. See Section 8.1 for more details.
- **Python:** Then python was used to filter out the data from lateral direction, plot and store the real time data (Section 8.2). Then the final code gives the Logarithmic Decrement constant (δ) and Damping Ratio (ζ) (Section 8.3).

4.1.3 Arduino Connections

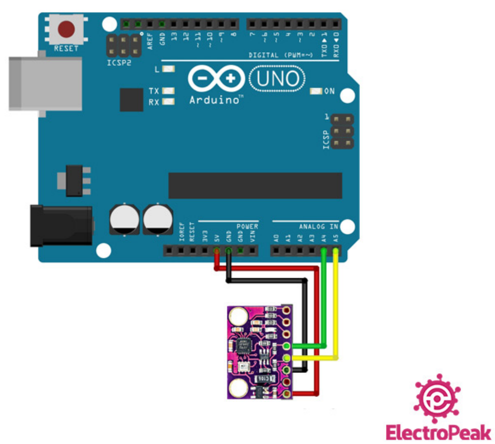

Figure 4.3: Arduino Connections with Accelerometer [1]

4.2 Testing Protocol

4.2.1 Baseline Damping Ratio (Without TMD)

1. **Initial displacement:** Manually displace the model by a fixed x.
2. **Free vibration:** Release the model and record acceleration till the acceleration becomes almost 0.
3. **Peak detection:** Identify consecutive amplitude peaks $x_1, x_2, \ldots, x_n$ from the decaying oscillation (see Figure 4.2.1).
4. **Damping calculation:** Compute damping ratio ζ_{original} using logarithmic decrement:

$$\delta = \frac{\ln\left(\frac{|x_1|}{|x_2|}\right)}{p_2 - p_1}, \quad \zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

where p_1 and p_2 are indices of adjacent peaks.

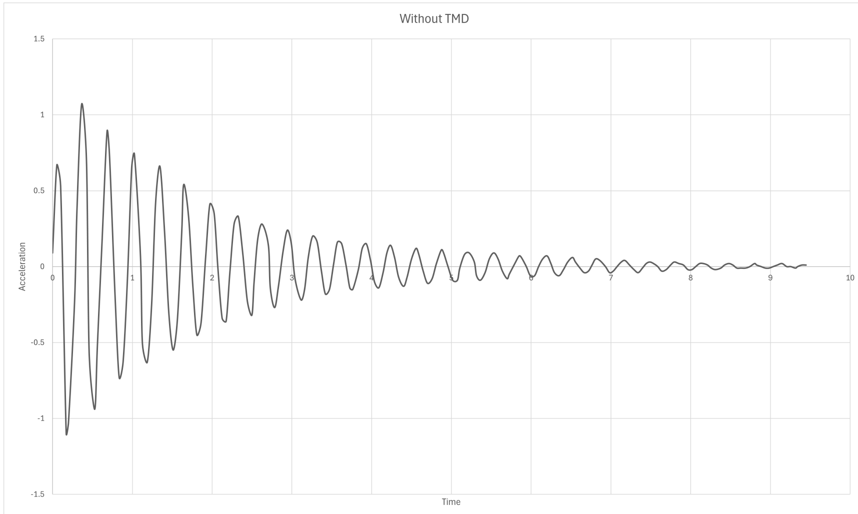

Figure 4.4: Acceleration Decay for Model (Without TMD)

4.2.2 TMD-Enhanced System

1. **TMD installation:** Attach the tuned mass damper to the model.
2. **Repeat test:** Displace the model by the same x, release, and record acceleration data.
3. **Peak analysis:** Extract peaks from the damped oscillation (see Figure 4.2.2).
4. **Damping calculation:** Derive the effective damping ratio ζ_{TMD}.

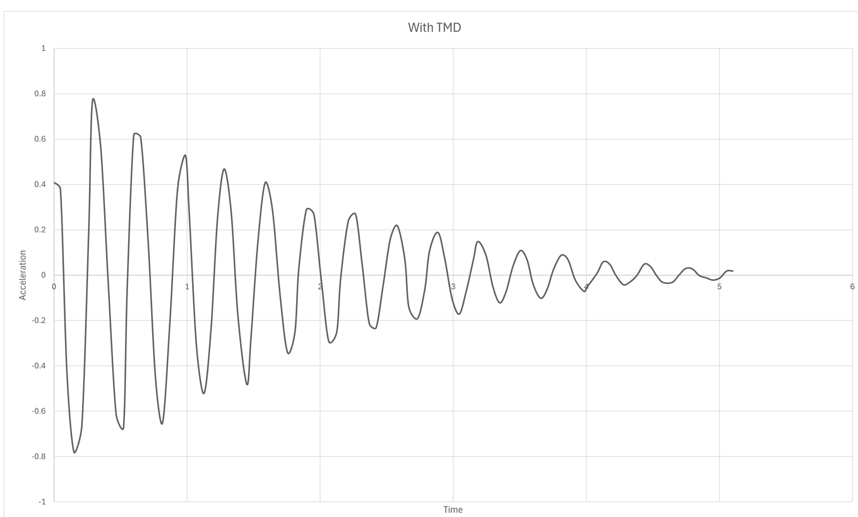

Figure 4.5: Acceleration Decay for Model (With TMD)

4.2.3 Deflection Angle Calculation (θ)

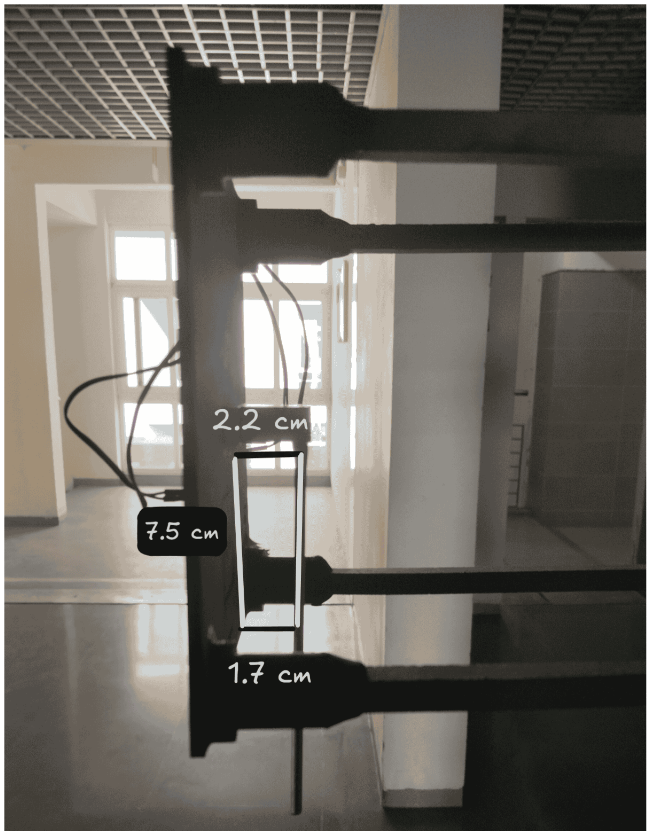

Figure 4.6: Deflection taking the structure as Cantilevered beam

To calculate the deflection angle θ of the TMD model due to gravity, we consider the given measurements: the horizontal displacements at two points along the pendulum, 2.2 cm and 1.7 cm, separated by a vertical distance of 7.5 cm. These measurements form a trapezoid with the parallel sides (horizontal displacements) being 2.2 cm and 1.7 cm, and the height (vertical distance) being 7.5 cm.

1. **Determine the horizontal difference between the two points**:

$$\Delta x = 2.2\text{ cm} - 1.7\text{ cm} = 0.5\text{ cm}$$

2. **Calculate the tangent of the deflection angle**:

$$\tan(\theta) = \frac{\Delta x}{\text{vertical distance}} = \frac{0.5\text{ cm}}{7.5\text{ cm}} = \frac{1}{15} \approx 0.0667$$

3. **Find the** $\tan^{-1}$ **to get the angle**:

$$\theta = \tan^{-1}\left(\frac{1}{15}\right) \approx 3.81^\circ$$

Thus, the deflection angle θ is 3.81°.

4.3 Results

4.3.1 Damping Ratio Comparison

Parameter	Without TMD	With TMD
Peak 1 amplitude (x_1)	0.6703 m/s^2	0.7679 m/s^2
Peak 2 amplitude (x_2)	0.1203 m/s^2	0.0279 m/s^2
Logarithmic decrement (δ)	0.1227	0.2368
Damping ratio (ζ)	1.95%	3.77%

Table 4.1: Experimental damping ratios

4.3.2 Key Observations

- **Amplitude decay:** The TMD reduced the amplitude by **96.4%** between the first and second peaks, compared to 82% without TMD.
- **Damping enhancement:** The effective damping ratio increased from $\zeta_{\text{original}} = 1.95\%$ to $\zeta_{\text{TMD}} = 3.77\%$, demonstrating the TMD's energy dissipation capability.

4.4 Discussion

The TMD introduces two damping mechanisms:

1. **Structural damping:** Intrinsic energy loss in the building materials.
2. **Destructive Interference:** When the building reaches one of its extreme position then TMD is at its other extreme position and vice-versa.

The doubling(almost) of ζ confirms that the TMD actively participates in energy dissipation rather than merely adding mass. This aligns with the theoretical principle of *mass-spring phase opposition*, where the TMD's motion generates destructive interference.

5
Comparison of analytical and experimental results

5.1 Deflection Angle (θ)

Theoretical Value: 3.5168°
Observed Value: 3.8140748°

Percentage Deviation:

$$\frac{3.8140748 - 3.5168}{3.5168} \times 100 \approx 8.45\%$$

The observed and theoretical values are in good agreement, with a small deviation attributed to experimental conditions and assuming roof, arduino and other electronic components to be massless.

5.2 Natural Frequency Analysis

Analytical Results:

- **Without TMD:** Natural Frequency ≈ 2.88 Hz
- **With TMD:** Natural Frequency ≈ 2.50 Hz

Experimental(Manual) Results:

- **Both cases:** Natural Frequency ≈ 3 Hz

Since there is error factor in Manual Calculations of Natural Frequency due to obvious reasons.
In the ideal theoretical model, adding a TMD does not significantly change the primary natural frequency of the structure. Instead, it causes a splitting of modes, resulting in two closely spaced natural frequencies: one slightly below and one slightly above the original frequency. The energy shifts to these new modes, and the response at the original frequency is suppressed.
But, our model shows a noticeable decrease in natural frequency from 2.88 Hz $\rightarrow$ 2.50 Hz, which is a $\sim 13\%$ reduction.

5.3 Damping Ratio

To quantify the damping characteristics of the a sky scraper like system, experiments were conducted in two phases: without and with a Tuned Mass Damper (TMD).

5.3.1 Baseline Damping (Without TMD)

The structure was displaced manually and released to undergo free vibration. Acceleration data was recorded until the system reached equilibrium. From the decaying oscillation curve, the first two consecutive peaks (x_1 and x_2) were extracted.

- Peak 1 Amplitude (x_1): 0.6703 m/s^2
- Peak 2 Amplitude (x_2): 0.1203 m/s^2
- Logarithmic Decrement (δ):

$$\delta = \ln\left(\frac{|x_1|}{|x_2|}\right) = \ln\left(\frac{0.6703}{0.1203}\right) \approx 1.825$$

- Damping Ratio (ζ):

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} = \frac{1.825}{\sqrt{(2\pi)^2 + (1.825)^2}} \approx 0.0195 \text{ or } 1.95\%$$

5.3.2 TMD-Enhanced System

With the TMD installed, the same test procedure was followed. The resulting acceleration data revealed significantly increased damping:

- Peak 1 Amplitude (x_1): 0.7679 m/s^2
- Peak 2 Amplitude (x_2): 0.0279 m/s^2
- Logarithmic Decrement (δ):

$$\delta = \ln\left(\frac{0.7679}{0.0279}\right) \approx 3.278$$

- Damping Ratio (ζ):

$$\zeta = \frac{3.278}{\sqrt{(2\pi)^2 + (3.278)^2}} \approx 0.0377 \text{ or } 3.77\%$$

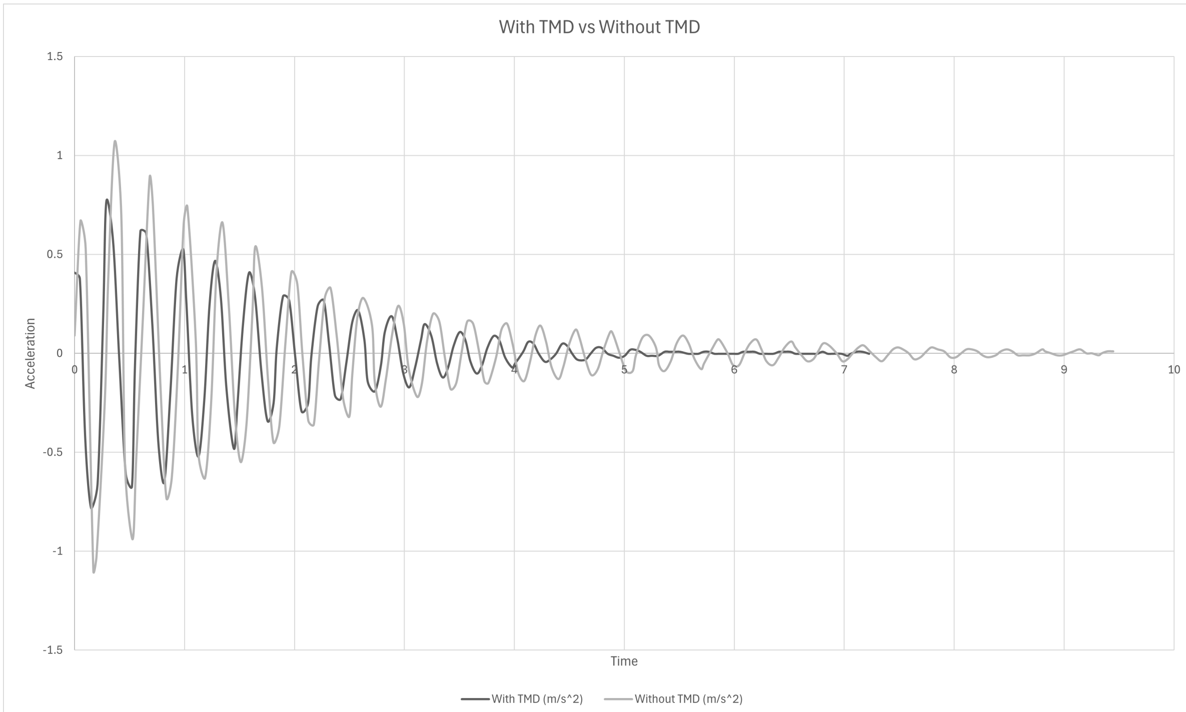

Figure 5.1: Graph with and without TMD (on same scale)

The damping ratio increased from **1.95%** without TMD to **3.77%** with TMD. This confirms the TMD's effectiveness in dissipating vibrational energy, making the system more stable.

6
Summary and conclusions

This project aimed to investigate the effectiveness of a Tuned Mass Damper (TMD) in reducing structural vibrations in a skyscraper like system modeled both physically and analytically.

A physical model was constructed, equipped with accelerometers and a data acquisition system using Arduino. Parallelly, analytical calculations were carried out to determine the deflection angle, natural frequency, and damping ratio of the system with and without a TMD. A SAP2000 model of a simplified skyscraper was created to simulate real-world scenarios, allowing visualization of the structural response under earthquake loads.

The experimental observations were in close agreement with analytical predictions. The theoretical deflection angle was calculated to be approximately 3.5168°, while the observed angle was 3.814°, showing a deviation of about 8.45% from the theoretical value. The natural frequency dropped from 2.88 Hz to 2.50 Hz upon adding the TMD. Most significantly, the damping ratio nearly doubled—from **1.95%** without the TMD to **3.77%** with it—highlighting a clear improvement in energy dissipation capability. Simulations in SAP2000 further reinforced these findings. The skyscraper model with TMD stabilized in 2.6 seconds, while the same structure without TMD took over 3 seconds to dissipate kinetic energy. This validated the damping enhancement and vibration control provided by the TMD in a real-world application.

In conclusion, both experimental and simulation-based results affirmed that the TMD significantly improves the dynamic performance of structures by reducing peak responses and increasing damping. This project not only demonstrated the practicality and efficiency of TMDs in structural systems but also laid a strong foundation for future work involving forced vibration testing, non-linear damping analysis, and optimization of mass-damper configurations for real-world deployment.

7
References

[1] Electropeak. *GY-91 IMU Module Wiring Diagram*. https://electropeak.com/learn/wp-content/uploads/2020/11/GY91-IMU-wire.jpg. Accessed: 2025-04-16. 2020.

[2] Purdue University. *Intro to Structural Motion Control – Chapter 4*. https://engineering.purdue.edu/~ce573/Documents/Intro%20to%20Structural%20Motion%20Control_Chapter4.pdf. Accessed: 2025-04-16. 2020.

8 Appendix

8.1 Arduino Code

```
#include <MPU9250_asukiaaa.h>
#include <Adafruit_BMP280.h>
#ifdef _ESP32_HAL_I2C_H_
#define SDA_PIN 21
#define SCL_PIN 22
#endif

Adafruit_BMP280 bmp;
MPU9250_asukiaaa mySensor;

float aX, aY, aZ, aSqrt;
void setup()
{
  Serial.begin(115200);
  while (!Serial)
    ;

#ifdef _ESP32_HAL_I2C_H_
  Wire.begin(SDA_PIN, SCL_PIN);
  mySensor.setWire(&Wire);
#else
  Wire.begin();
  mySensor.setWire(&Wire);
#endif
  bmp.begin();
  mySensor.beginAccel();
}
```

```
void loop()
{
  if (mySensor.accelUpdate() == 0)
  {aX = mySensor.accelX();
    aY = mySensor.accelY();
    aZ = mySensor.accelZ();
    aSqrt = mySensor.accelSqrt();
    Serial.print(String(aX));
    Serial.print("\t" + String(aY));
    Serial.print("\t" + String(aZ));
    Serial.println();}
  delay(50);}
```

8.2 Plot real time data & storing CSV

```python
import serial
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
import collections
import time
import csv

SERIAL_PORT = "COM5"
BAUD_RATE = 115200
NUM_OFFSET_SAMPLES = 100
Y_AXIS_MIN = -5
Y_AXIS_MAX = 5
DATA_WINDOW_SIZE = 200
CSV_FILE_NAME = "acceleration_data.csv"

def compute_offset(ser, num_samples):
    offset_samples = []
    print(f"Collecting {num_samples} samples  offseted...")
    while len(offset_samples) < num_samples:
        try:
            line = ser.readline().decode('utf-8').strip()
            if line:
                parts = line.split()
                if len(parts) >= 2:
                    y_val = float(parts[1])
                    offset_samples.append(y_val)
        except Exception as e:
            print("Parsing error:", e)

    offset = sum(offset_samples) / len(offset_samples)
    print(f"Computed y-axis offset: {offset:.4f}")
    return offset

def main():
    try:
        ser = serial.Serial(SERIAL_PORT, BAUD_RATE, timeout=1)
        time.sleep(2)
    except Exception as e:
        print("Failed to open serial port:", e)
        return

    y_offset = compute_offset(ser, NUM_OFFSET_SAMPLES)
    y_data = collections.deque([0] * DATA_WINDOW_SIZE, maxlen=DATA_WINDOW_SIZE)
    x_time_data = collections.deque([0] * DATA_WINDOW_SIZE, maxlen=DATA_WINDOW_SIZE)
```

```python
    start_time = time.time()
    csv_file = open(CSV_FILE_NAME, mode='w', newline='')
    csv_writer = csv.writer(csv_file)
    csv_writer.writerow(["Time (ms)", "Acceleration (m/s^2)"])

    fig, ax = plt.subplots()
    line_plot, = ax.plot([], [], lw=1.5)
    ax.set_ylim(Y_AXIS_MIN, Y_AXIS_MAX)
    ax.set_title("Acceleration vs Time")
    ax.set_xlabel("Time (s)")
    ax.set_ylabel("Acceleration (m/s²)")

    def update(frame):
        current_time = time.time() - start_time
        try:
            while ser.in_waiting:
                line_in = ser.readline().decode('utf-8').strip()
                if line_in:
                    parts = line_in.split()
                    if len(parts) >= 2:
                        y_val = float(parts[1])
                        y_val_adj = y_val - y_offset
                        y_data.append(y_val_adj)
                        x_time_data.append(current_time)
                        current_time_ms = int(current_time * 1000)
                        csv_writer.writerow([current_time_ms, y_val_adj])

            line_plot.set_data(x_time_data, y_data)
            ax.set_xlim(max(0, current_time - (DATA_WINDOW_SIZE * 0.05)), current_time)
        except Exception as e:
            print("Error reading/updating data:", e)
        return line_plot,

    ani = FuncAnimation(fig, update, interval=10)
    plt.show()
    ser.close()

if __name__ == "__main__":
    main()
```

8.3 Damping Ratio Calculator

```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import find_peaks
# attach csv file
df = pd.read_csv("test1.csv")
df["Time (s)"] = df["Time"] / 1000.0

time = df["Time"].values
acc = df["Acceleration"].values
peaks, _ = find_peaks(acc, distance=1)

plt.plot(time, acc, label="Acceleration")
plt.plot(time[peaks], acc[peaks], "ro", label="Peaks")
plt.xlabel("Time")
plt.ylabel("Acceleration")
plt.title("Acceleration Peaks")
plt.legend()
plt.grid(True)
plt.show()

if len(peaks) >= 15:
    p1=1
    p2=14
    x1 = acc[peaks[p1]]
    x2 = acc[peaks[p2]]
    delta = np.log(abs(x1 / x2))/(p2-p1)
    zeta = delta / np.sqrt((2 * np.pi)**2 + delta**2)

    print(f"Peak 1 Amplitude: {x1:.4f}")
    print(f"Peak 2 Amplitude: {x2:.4f}")
    print(f"Logarithmic Decrement (): {delta:.4f}")
    print(f"Damping Ratio (): {zeta:.4f}")
else:
    print("Not enough peaks to compute damping.")
```

8.4 Data Tables

8.4.1 Without TMD

Time	Acceleration
0	0.0903
0.054	0.6703
0.101	0.5303
0.171	-1.0997
0.199	-1.0297
0.275	-0.2397
0.307	0.3903
0.365	1.0703
0.425	0.7003
0.46	-0.5797
0.528	-0.9397
0.562	-0.5197
0.62	0.1603
0.683	0.8903
0.713	0.7403
0.784	-0.1997
0.835	-0.7297
0.884	-0.6297
0.938	-0.0897
0.992	0.6503
1.026	0.7403
1.102	0.0703
1.127	-0.5097
1.187	-0.6297
1.242	-0.2497
1.292	0.4203
1.346	0.6603
1.404	0.2303
1.458	-0.3097
1.512	-0.5497
1.565	-0.3497
1.619	0.2203
1.644	0.5403
1.705	0.3303
1.759	-0.1297
1.809	-0.4497
1.862	-0.3697
1.919	0.0603

Time	Acceleration
1.972	0.4103
2.026	0.3503
2.073	-0.0097
2.126	-0.3397
2.176	-0.3597
2.223	-0.0397
2.276	0.2803
2.328	0.3303
2.384	0.0903
2.443	-0.2297
2.499	-0.3197
2.527	-0.0997
2.572	0.1803
2.626	0.2803
2.706	0.1403
2.73	-0.1397
2.784	-0.2697
2.832	-0.1297
2.89	0.1003
2.944	0.2403
2.994	0.1503
3.04	-0.0797
3.116	-0.2197
3.16	-0.1497
3.206	0.0603
3.261	0.2003
3.318	0.1603
3.371	-0.0297
3.42	-0.1797
3.472	-0.1497
3.522	0.0103
3.571	0.1603
3.625	0.1503
3.679	0.0103
3.727	-0.1397
3.764	-0.1497
3.834	-0.0197
3.883	0.1203

Time	Acceleration
3.934	0.1503
3.987	0.0403
4.036	-0.0997
4.09	-0.1397
4.144	-0.0397
4.192	0.0903
4.238	0.1403
4.288	0.0603
4.339	-0.0697
4.401	-0.1297
4.445	-0.0697
4.501	0.0503
4.558	0.1203
4.592	0.0803
4.647	-0.0297
4.701	-0.1097
4.759	-0.0797
4.813	0.0203
4.875	0.1103
4.903	0.0903
4.958	0.0003
5.017	-0.0897
5.076	-0.0897
5.106	-0.0097
5.166	0.0803
5.224	0.0903
5.288	0.0303
5.316	-0.0597
5.364	-0.0897
5.423	-0.0397
5.476	0.0503
5.533	0.0903
5.586	0.0503
5.64	-0.0297
5.7	-0.0797
5.727	-0.0497
5.796	0.0203
5.85	0.0703

Time	Acceleration
5.878	0.0603
5.94	0.0003
5.99	-0.0597
6.044	-0.0597
6.094	0.0003
6.145	0.0503
6.201	0.0703
6.248	0.0203
6.293	-0.0397
6.352	-0.0597
6.406	-0.0197
6.459	0.0303
6.519	0.0603
6.556	0.0303
6.613	-0.0097
6.666	-0.0397
6.719	-0.0297
6.764	0.0103
6.809	0.0503
6.862	0.0403
6.932	0.0003
6.984	-0.0397
7.03	-0.0297
7.078	0.0003
7.131	0.0303
7.177	0.0403
7.234	0.0103
7.29	-0.0197
7.343	-0.0397
7.396	-0.0097
7.444	0.0203
7.489	0.0303
7.535	0.0203
7.588	0.0003
7.638	-0.0297
7.694	-0.0197
7.749	0.0103
7.795	0.0303
7.852	0.0203
7.91	0.0103
7.966	-0.0197

Time	Acceleration
8.016	-0.0197
8.064	0.0003
8.113	0.0203
8.16	0.0203
8.213	0.0103
8.259	-0.0097
8.311	-0.0197
8.379	-0.0097
8.425	0.0103
8.48	0.0203
8.53	0.0103
8.58	-0.0097
8.629	-0.0097
8.689	-0.0097
8.743	0.0003
8.802	0.0203
8.829	0.0103
8.884	0.0003
8.937	-0.0097
8.985	-0.0097
9.036	0.0003
9.087	0.0103
9.146	0.0203
9.205	0.0003
9.255	0.0003
9.314	-0.0097
9.343	0.0003
9.394	0.0103
9.448	0.0103

8.4.2 With TMD

Time	Acceleration
0	0.4079
0.048	0.3779
0.097	-0.4321
0.15	-0.7821
0.209	-0.6621
0.26	0.1779
0.289	0.7679
0.35	0.5679
0.406	-0.0221
0.466	-0.6121
0.522	-0.6721
0.55	-0.0521
0.599	0.6179
0.651	0.6079
0.707	0.1479
0.762	-0.4521
0.815	-0.6521
0.871	-0.2021
0.928	0.3779
0.987	0.5279
1.017	0.2579
1.07	-0.3121
1.126	-0.5221
1.179	-0.2421
1.228	0.2479
1.278	0.4679
1.331	0.2779
1.382	-0.1821
1.449	-0.4821
1.479	-0.2821
1.532	0.1479
1.587	0.4079
1.641	0.2879
1.699	-0.0921
1.755	-0.3421
1.81	-0.2521
1.839	0.0279
1.896	0.2879
1.952	0.2679

Time	Acceleration
2.008	-0.0221
2.064	-0.2921
2.124	-0.2521
2.154	-0.0121
2.211	0.2379
2.266	0.2679
2.315	0.0479
2.368	-0.2121
2.42	-0.2321
2.471	-0.0521
2.526	0.1579
2.579	0.2179
2.636	0.0679
2.666	-0.1421
2.731	-0.1921
2.786	-0.0621
2.822	0.1079
2.884	0.1879
2.934	0.0779
2.99	-0.1021
3.044	-0.1721
3.097	-0.0721
3.151	0.0679
3.184	0.1479
3.245	0.0879
3.298	-0.0521
3.349	-0.1221
3.397	-0.0721
3.448	0.0379
3.504	0.1079
3.554	0.0679
3.601	-0.0421
3.657	-0.1021
3.705	-0.0621
3.755	0.0279
3.812	0.0879
3.862	0.0679
3.917	-0.0221
3.982	-0.0721

Time	Acceleration
4.006	-0.0521
4.08	0.0079
4.126	0.0579
4.175	0.0479
4.222	-0.0021
4.277	-0.0421
4.324	-0.0321
4.379	-0.0021
4.434	0.0479
4.481	0.0379
4.527	-0.0021
4.574	-0.0321
4.645	-0.0321
4.693	-0.0021
4.743	0.0279
4.794	0.0279
4.849	-0.0021
4.9	-0.0121
4.952	-0.0221
5.004	-0.0121
5.054	0.0179
5.1	0.0179

Figure 8.1: All the files, images and pdf version of report can be accessed here