Question Bank

Q1.

Question Bank

1) $\left[x^2-4xy-2y^2\right]dx+\left[y^2-4xy-2x^2\right]dy=0$

$$M\,dx+N\,dy=0$$

$M=x^2-4xy-2y^2 \qquad N=y^2-4xy-2x^2$

$\frac{\partial M}{\partial y}=-4x-4y \qquad \frac{\partial N}{\partial x}=-4y-4x$

$$\boxed{\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}}$$

The eq^n is exact

The sol^n is

$$\int M\,dx+\int N\,dy=c$$

$$\cancel{\int\left(x^2-4xy-2y^2\right)dx+\int\left(y^2-4xy-2x^2\right)dy=0}$$

$$\cancel{\frac{x^3}{3}-\frac{4x^2y}{2}-2xy^2+\frac{y^2}{3}-\frac{4xy^2}{2}-2x^2y=0}$$

$$\frac{x^3}{6}-\int\left(x^2-4xy-4y\right)dx+\int y^2\,dy=c$$

$$\frac{x^3}{3}-\frac{4x^2y}{2}-2xy^2+\frac{y^3}{3}=c$$

$$\boxed{\frac{x^3}{3}-2x^2y-2xy^2+\frac{y^3}{3}=c}$$

$$\boxed{x^3-6x^2y-6xy^2+y^3=3c}$$

Q2.

2) $\left[x\sqrt{x^2+y^2}-y\right]dx + \left[y\sqrt{x^2+y^2}-x\right]dy = 0$

$M = x\sqrt{x^2+y^2} - y \qquad N = y\sqrt{x^2+y^2} - x$

$\frac{\partial M}{\partial y} = \frac{x\,2y}{2\sqrt{x^2+y^2}} - 1 \qquad \frac{\partial N}{\partial x} = \frac{2xy}{2\sqrt{x^2+y^2}} - 1$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

This eq^n is exact

The sol^n is

$$\int M\,dx + \int N\,dy = c$$

$$\int \left(x\sqrt{x^2+y^2} - y\right)dx + \int 0\,dy = c$$

Put $x^2+y^2 = t$

$2x = \frac{dt}{dx}$

$x\,dx = \frac{dt}{2}$

$$\int \sqrt{t}\,\frac{dt}{2} - \int y\,dx = c$$

$$\frac{1}{2}\,\frac{t^{3/2}}{3/2} - xy = c$$

$$\boxed{\frac{1}{3}\left(x^2+y^2\right)^{3/2} - xy = c}$$

$$\boxed{\left(x^2+y^2\right)^{3/2} - 3xy = 3c}$$

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9) $[1+\log(xy)]\,dx + [1+\frac{x}{y}]\,dy = 0$

$M = 1+\log(xy)$ $\quad N = 1+\frac{x}{y}$

$\frac{\partial M}{\partial y} = \frac{1}{xy}x$ $\quad \frac{\partial N}{\partial x} = \frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{1}{y}$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

The eq^n exact

The sol^n is

$\int M\,dx + \int N\,dy = c$

$\int [1+\log(xy)]\,dx + \int 1\,dy = c$

$x + \int \log(xy)\,dx + y = c$

$x + \left[\log(xy)\int 1\,dx - \int \frac{d}{dx}\log xy \int 1\,dx\,dx\right] + y = c$

$x + \left[x\log xy - \int \frac{1}{\cancel{xy}}\cancel{y}\cancel{x}\,dx\right] + y = c$

$x + x\log xy - \int 1\,dx + y = c$

$\cancel{x} + x\log xy - \cancel{x} + y = c$

$$\boxed{x\log xy + y = c}$$

Q4.

(9) $(x^3y^4 + x^2y^3 + xy^2 + y)\,dx + (x^4y^3 - x^3y^2 - x^2y + x)\,dy = 0$

$M = x^3y^4 + x^2y^3 + xy^2 + y$

$\frac{\partial M}{\partial y} = 4x^3y^3 + 3x^2y^2 + 2xy + 1$

$N = x^4y^3 - x^3y^2 - x^2y + x$

$\frac{\partial N}{\partial x} = 4x^3y^3 - 3x^2y^2 - 2xy + 1$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

The eq^n is not exact.

$Mx - Ny = x(x^3y^4 + x^2y^3 + xy^2 + y) - y(x^4y^3 - x^3y^2 - x^2y + x)$

$= x^4y^4 + x^3y^3 + x^2y^2 + xy - x^4y^4 + x^3y^3 + x^2y^2 - xy$

$Mx - Ny = 2x^3y^3 + 2x^2y^2$

$I.F = \frac{1}{2x^2y^2(xy+1)}$ $\quad I.F = \frac{1}{Mx - Ny}$

$I.F = \frac{1}{2x^2y^2(xy+1)}$

multiply eq^n ① by I.F

$$\frac{(x^3y^4 + x^2y^3 + xy^2 + y)}{2x^2y^2(xy+1)}\,dx + \frac{(x^4y^3 - x^3y^2 - x^2y + x)}{2x^2y^2(xy+1)}\,dy = 0$$

$$\frac{x^2y^3(xy+1) + y(xy+1)}{2x^2y^2(xy+1)}\,dx + \frac{(x^3y^2(xy-1) - x(xy-1))}{2x^2y^2(xy+1)}\,dy = 0$$

$$\frac{(xy+1)(x^2y^3+y)}{2x^2y^2(xy+1)}\,dx + \frac{(xy-1)(x^3y^2 - x)}{2x^2y^2(xy+1)}\,dy = 0$$

$$\frac{y(x^2y^2+1)}{2x^2y^2}\,dx + \frac{(xy-1)\,x\,(x^2y^2-1)}{2x^2y^2(xy+1)}\,dy = 0$$

$$\frac{y(x^2y^2+1)}{2x^2y^2}\,dx + \frac{x(xy-1)(xy-1)(xy+1)}{2x^2y^2(xy+1)}\,dy = 0$$

$$\frac{(x^2y^2+1)}{2x^2y}\,dx + \frac{(xy-1)^2}{2xy^2}\,dy = 0$$

$$M' = \frac{x^2y^2+1}{2x^2y} \qquad N' = \frac{(xy-1)^2}{2xy^2}$$

$$M' = \frac{x^2y^2}{2x^2y} + \frac{1}{2x^2y} \qquad N' = \frac{x^2y^2}{2xy^2} - \frac{2xy}{2xy^2} + \frac{1}{2xy^2}$$

$$M' = \frac{y}{2} + \frac{1}{2x^2y} \qquad N' = \frac{x}{2} - \frac{1}{y} + \frac{1}{2xy^2}$$

$$\frac{\partial M'}{\partial y} = \frac{1}{2} - \frac{1}{2x^2y^2} \qquad \frac{\partial N'}{\partial x} = \frac{1}{2} - \frac{1}{2x^2y^2}$$

$$\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N}{\partial x}}$$

The eqⁿ is exact

The solⁿ is

$$\int M\,dx + \int N\,dy = C$$

$$\int\left(\frac{y}{2} + \frac{1}{2x^2y}\right)dx + \int -\frac{1}{y}\,dy = C$$

$$\boxed{\frac{xy}{2} - \frac{1}{2xy} - \log y = C}$$

$$\boxed{xy - \frac{1}{xy} - 2\log y = 2C}$$

Q5.

10) $(2xy^4e^y + 2xy^3 + y)\,dx + (x^2y^4e^y - x^2y^2 - 3x)\,dy = 0$ — ①

$M = 2xy^4e^y + 2xy^3 + y \qquad N = x^2y^4e^y - x^2y^2 - 3x$

$\frac{\partial M}{\partial y} = 2x(y^4e^y + e^y 4y^3) + 6xy^2 + 1$

$\frac{\partial N}{\partial x} = 2xy^4e^y - 2xy^2 - 3$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

$\therefore$ the eq^n is not exact

$$Mx - Ny = x(2xy^4e^y + 2xy^3 + y) - y(x^2y^4e^y - x^2y^2 - 3x)$$
$$= 2x^2y^4e^y + 2x^2y^3 + xy - x^2y^5e^y + x^2y^3 + 3xy$$

$$\frac{\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}}{M} = \frac{2xy^4e^y + 8xy^3e^y + 6xy^2 + 1}{-2}$$

$$\frac{\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}}{M} = \frac{2xy^4e^y - 2xy^2 - 3 - 2xy^4e^y - 8xy^3e^y - 6xy^2 - 1}{2xy^4e^y + 2xy^3 + y}$$

$$= \frac{-8xy^2 - 8xy^3e^y - 4}{y(2xy^3e^y + 2xy^2 + 1)}$$

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$$\frac{\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}}{M} = \frac{-4\left(2xy^3e^y + \cancel{2xy^2} + 1\right)}{y\left(2xy^3e^y + \cancel{2xy^2} + 1\right)}$$

$$f(y) = \frac{-4}{y}$$

$$I.F = e^{\int f(y)\,dy}$$

$$= e^{-4\int \frac{1}{y}\,dy}$$

$$= e^{-4\log y}$$

$$= e^{\log \frac{1}{y^4}}$$

$$\boxed{I.F = \frac{1}{y^4}}$$

multiply eqn ① by I.F

$$\left(\frac{2xy^4e^y}{y^4} + \frac{2xy^3}{y^4} + \frac{y}{y^4}\right)dx + \left(\frac{x^2y^4e^y}{y^4} - \frac{x^2y^2}{y^4} - \frac{3x}{y^4}\right)dy$$

$$\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx + \left(x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}\right)dy$$

$$M' = 2xe^y + \frac{2x}{y} + \frac{1}{y^3} \qquad N' = x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}$$

$$\frac{\partial M'}{\partial y} = 2xe^y - \frac{2x}{y^2} - \frac{3}{y^4} \qquad \frac{\partial N'}{\partial x} = 2xe^y - \frac{2x}{y^2} - \frac{3}{y^4}$$

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$$\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}}$$

The eqn is exact.

The soln is

$$\int M\,dx + \int N\,dy = c$$

$$\int\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx + \int 0\,dy = c$$

$$\frac{2x^2e^y}{2} + \frac{2x^2}{2y} + \frac{x}{y^3} = c$$

$$\boxed{x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = c}$$

Q6. A

6

A $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$

$(1+x^2)\,dy = -(2xy - 4x^2)\,dx$

$(2xy - 4x^2)\,dx + (1+x^2)\,dy = 0$

$M = 2xy - 4x^2 \qquad N = 1 + x^2$

$\frac{\partial M}{\partial y} = 2x \qquad \frac{\partial N}{\partial x} = 2x$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

The eq^n is exact

The sol^n is

$\int M\,dx + \int N\,dy = c$

$\int (2xy - 4x^2)\,dx + \int 1\,dy = c$

$\frac{\cancel{2}x^2y}{\cancel{2}} - \frac{4x^3}{3} + y = c$

$$\boxed{x^2y - \frac{4x^3}{3} + y = c}$$

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(B) $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

$(1+y^2)\,dx - (\tan^{-1}y - x)\,dy$

$M = 1+y^2 \qquad N = -\tan^{-1}y + x$

$\frac{\partial M}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = 1$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

This eq^n is not exact

$$\frac{dx}{dy} = \frac{\tan^{-1}y - x}{(1+y^2)}$$

$$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$\frac{dx}{dy} + Px = Q$$

This is linear diff. eq^n.

$$I.F = e^{\int P\,dy}$$

$P = \frac{1}{1+y^2} \qquad Q = \frac{\tan^{-1}y}{1+y^2}$

$$I.F = e^{\int \frac{1}{1+y^2} dy}$$

$$I.F = e^{\tan^{-1} y}$$

The sol^n is

$$x \, I.F = \int g \, I.F \, dy + C$$

$$x \, e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2} \, e^{\tan^{-1} y} \, dy + C$$

put $\tan^{-1} y = t$

$$\frac{1}{1+y^2} dy = dt$$

$$x e^t = \int t e^t \, dt + C$$

$$x e^t = t \int e^t dt - \int \frac{dt}{dt} \int e^t \, dt + c$$

$$x e^t = t e^t - \int e^t \, dt + c$$

$$= t e^t - e^t + C$$

$$x e^t = e^t (t - 1) + C$$

$$\boxed{x \, e^{\tan^{-1} y} = e^{\tan^{-1} y} (\tan^{-1} y - 1) + C}$$

Q7. A

8) (A) $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

$\frac{dy}{dx} = e^{2x-y} - e^{x-y+y}$

$= e^{2x} e^{-y} - e^x$

$\frac{dy}{dx} + e^x = \frac{e^{2x}}{e^y}$

$e^y \frac{dy}{dx} + e^x e^y = e^{2x}$

put, $e^y = v$

diff. w.r.t. x

$e^y \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dv}{dx} + e^x v = e^{2x}$

This is linear diff. eqn

$P = e^x \qquad Q = e^{2x}$

$I.F = e^{\int P dx}$

$= e^{\int e^x dx}$

$\boxed{I.F = e^{e^x}}$

The sol^n is

$$y \; I.F = \int I.F \; Q \; dx + c$$

$$e^y e^{e^x} = \int e^{e^x} e^{2x} \, dx + c$$

$$= \int e^{e^x} e^x e^x \, dx + c$$

put $e^x = t$

diff. w.r.t. x

$$e^x \, dx = dt$$

$$e^y e^t = \int e^t . t \; dt + c$$

$$= t \int e^t - \int \frac{dt}{dt} \int e^t dt + c$$

$$= t e^t - \int e^t \, dt + c$$

$$= t e^t - e^t + c$$

$$e^y e^t = e^t (t - 1) + c$$

$$\boxed{e^y e^{e^x} = e^{e^x} (e^x - 1) + C}$$

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(B) $x \frac{dy}{dx} + y = x^3 y^6$

$\frac{x}{y^6} \frac{dy}{dx} + \frac{y}{y^6} = x^3$

$\frac{1}{y^6} \frac{dy}{dx} + \frac{1}{x y^5} = x^2$

Put $\frac{1}{y^5} = v$

diff. w.r.t x

$-\frac{5}{y^6} \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{1}{y^6} \frac{dy}{dx} = -\frac{1}{5} \frac{dv}{dx}$

$-\frac{1}{5} \frac{dv}{dx} + \frac{1}{x} v = x^2$

$\frac{dv}{dx} - \frac{5}{x} v = -5x^2$

This is linear diff. eq^n

$\frac{dv}{dx} + Pv = Q$

$P = \frac{-5}{x}$ $Q = -5x^2$

$I.F = e^{\int P dx}$

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$$I.F = e^{-5\int \frac{1}{x}}$$

$$= e^{-5\log x}$$

$$= e^{\log \frac{1}{x^5}}$$

$$\boxed{I.F = \frac{1}{x^5}}$$

The sol^n is

$$v . I.F = \int I.F . Q \, dx + c$$

$$\frac{1}{y^5} \frac{1}{x^5} = \int \frac{1}{x^5} \left(-5x^2\right) dx + c$$

$$\frac{1}{y^5 x^5} = -5 \int \frac{1}{x^3} dx + c$$

$$\frac{1}{y^5 x^5} = -5 \left(\frac{-1}{2x^2}\right) + c$$

$$\boxed{\frac{1}{y^5 x^5} = \frac{5}{2x^2} + c}$$

8. A

(12) (A) $(D^2 - 6D + 9)y = 0$

Auxillary Equation is

$D^2 - 6D + 9 = 0$

$D = 3, 3$

$C.F = e^{3x}(C_1 x + C_2)$

$y = C.F$

$\boxed{y = e^{3x}(C_1 x + C_2)}$

B

(13) (A) $(D^4 + k^4)y = 0$

Auxillary eqn is

$D^4 + k^4 = 0$

$D^4 + k^4 + 2D^2k^2 - 2D^2k^2 = 0$

$(D^2 + k^2)^2 - (\sqrt{2}Dk)^2 = 0$

$(D^2 + k^2 - \sqrt{2}Dk)(D^2 + k^2 + \sqrt{2}Dk) = 0$

$D^2 + k^2 - \sqrt{2}Dk = 0$

$D^2 + k^2 + \sqrt{2}Dk = 0$

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$$D^2 - \sqrt{2}Dk + k^2 = 0$$

$$ax^2 + Bx + C = 0$$

$$a = 1, \quad B = -\sqrt{2}k \quad C = k^2$$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{\sqrt{2}k \pm \sqrt{2k^2 - 4k^2}}{2a}$$

$$D = \frac{\sqrt{2}k \pm \sqrt{-2k^2}}{2a}$$

$$D = \frac{\sqrt{2}k \pm i\sqrt{2}k}{2}$$

$$D = \frac{\cancel{\sqrt{2}}(k + ik)}{\cancel{2}}$$

$$\boxed{D = \frac{k + ik}{\sqrt{2}}}$$

$$D = \frac{k}{\sqrt{2}} \pm \frac{k}{\sqrt{2}}$$

$$\alpha = \frac{k}{\sqrt{2}} \qquad \beta = \frac{k}{\sqrt{2}}$$

$$D^2 + \sqrt{2}Dk + k^2 = 0$$

$$\boxed{D = \frac{-k \pm ik}{\sqrt{2}}}$$

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$$D = \frac{-k}{\sqrt{2}} \pm \frac{ik}{\sqrt{2}}$$

$$\alpha = \frac{-k}{\sqrt{2}} \quad \beta = \frac{k}{\sqrt{2}}$$

The solⁿ is

$y = \cancel{e}$

$$\boxed{y = e^{k/2}\left[c_1 \cos\frac{k}{2}x + c_2 \sin\frac{k}{2}x\right] + e^{-k/2}\left[c_3 \cos\frac{k}{\sqrt{2}}x + c_4 \sin\frac{k}{\sqrt{2}}x\right]}$$

C

(B) $(D-1)^2 (D^2+1) y = 0$

Auxillary eqⁿ is

$(D-1)^2 = 0$

$(D^2+1) = 0$

$D^2 - 2D + 1 = 0$

$D = 1, 1$

$D^2 + 1 = 0$

$D^2 = -1$

$D = \pm i$

$y = e^{x}(c_1 x + c_2) + e^{0x}(c_3 \cos x + c_4 \sin x)$

$$\boxed{y = e^{x}(c_1 x + c_2) + c_3 \cos x + c_4 \sin x}$$

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(14) (A) $(D^2+2)y = e^x - \cos 2x$

$AE :- D^2+2=0$

$D^2 = -2$

$D = \pm i\sqrt{2}$

$C.F = \cancel{C_1} e^{0x}(C_1\cos\sqrt{2}x + C_2\sin\sqrt{2}x)$

$$\boxed{C.F = C_1\cos\sqrt{2}x + C_2\sin\sqrt{2}x}$$

$P.I = \frac{1}{f(D)} \times (e^x - \cos 2x)$

$= \frac{1}{D^2+2}(e^x - \cos 2x)$

$= \frac{e^x}{D^2+2} - \frac{\cos 2x}{D^2+2}$

$= \frac{e^x}{1+2} - \frac{\cos 2x}{-(2)^2+2}$

$$\boxed{P.I = \frac{e^x}{3} + \frac{\cos 2x}{\cancel{6}}}$$

The sol^n is

$y = CF + P.I$

$$\boxed{y = C_1\cos\sqrt{2}x + C_2\sin\sqrt{2}x + \frac{e^x}{3} + \frac{\cos 2x}{6}}$$

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(B) $(D^3 - 2D^2 - 5D + 6)y = e^{3x} + 8$

Auxillary eqn is

$D^3 - 2D^2 - 5D + 6 = 0$

$D = -2, 3, 1$

$$\boxed{CF = C_1 e^{-2x} + C_2 e^{3x} + C_3 e^{x}}$$

$$P.I = \frac{1}{D^3 - 2D^2 - 5D + 6} \times (e^{3x} + 8)$$

$$P.I = \frac{e^{3x}}{D^3 - 2D^2 - 5D + 6} + \frac{8e^{0x}}{D^3 - 2D^2 - 5D + 6}$$

$$P.I = \frac{x e^{3x}}{3D^2 - 4D - 5} + \frac{8}{(0)^3 - 2(0)^2 - 5(0) + 6}$$

$$= \frac{x e^{3x}}{3(3)^2 - 4(3) - 5} + \frac{8}{6}$$

$$\boxed{P.I = \frac{x e^{3x}}{10} + \frac{4}{3}}$$

Q10. A

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(15) (A) $\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 4y = e^x + \cos 2x$

$\cancel{D^2y - 4Dy}$ $D^2y - 4Dy + 4y = e^x + \cos 2x$

$(D^2 - 4D + 4)y = e^x + \cos 2x$

Auxillary eqⁿ is

$D^2 - 4D + 4 = 0$

$D = 2, 2$

$$\boxed{C.F = e^{2x}(C_1 x + C_2)}$$

$P.I = \frac{1}{f(D)} e^x + \cos 2x$

$P.I = \frac{e^x}{D^2 - 4D + 4} + \frac{\cos 2x}{D^2 - 4D + 4}$

$P.I = \frac{e^x}{(1)^2 - 4(1) + 4} + \frac{\cos 2x}{-(2)^2 - 4D + 4}$

$P.I = \frac{e^x}{1 - 4 + 4} + \frac{\cos 2x}{-4 - 4D + 4}$

$P.I = \frac{e^x}{1} + \frac{\cos 2x}{-4D}$

$P.I = e^x - \frac{1}{4} \frac{1}{D} \cos 2x$

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$$P.I = e^x - \frac{1}{4}\int \cos 2x \, dx$$

$$P.I = e^x - \frac{1}{4}\left(\frac{\sin 2x}{2}\right)$$

$$\boxed{P.I = e^x - \frac{\sin 2x}{8}}$$

$$y = C.F + P.I$$

$$\boxed{y = e^{2x}(c_1 x + c_2) + e^x - \frac{\sin 2x}{8}}$$

B

(B) $(D^2 + 4D + 4)y = \cosh 2x$

$A.E = D^2 + 4D + 4 = 0$

$D = -2, -2$

$$\boxed{C.F = e^{-2x}(C_1 x + C_2)}$$

$$P.I = \frac{1}{D^2 + 4D + 4} \times \cosh 2x$$

$$= \frac{1}{D^2 + 4D + 4} \times \left(\frac{e^{2x} + e^{-2x}}{2}\right)$$

$$= \frac{1}{2}\left[\frac{e^{2x}}{D^2 + 4D + 4} + \frac{e^{-2x}}{D^2 + 4D + 4}\right]$$

$$= \frac{1}{2}\left[\frac{e^{2x}}{(2)^2 + 4(2) + 4} + \frac{x e^{-2x}}{2D + 4}\right]$$

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$$= \frac{1}{2}\left[\frac{e^{2x}}{16} + \frac{x^2 e^{-2x}}{2}\right]$$

$$\boxed{P.I = \frac{e^{2x}}{32} + \frac{x^2 e^{-2x}}{4}}$$

$y = C.F + P.I$

$$\boxed{y = e^{-2x}(c_1 x + c_2) + \frac{e^{2x}}{32} + \frac{x^2 e^{-2x}}{4}}$$

Q11. i)

(17) Evaluate

(1) $\int_0^\infty e^{-h^2x^2} dx$

Put $h^2x^2 = t$

$x^2 = \frac{t}{h^2}$

$x = \frac{\sqrt{t}}{h}$

$dx = \frac{1}{2h} \frac{1}{\sqrt{t}}$

$= \int_0^\infty e^{-t} \frac{1}{2h} \frac{1}{\sqrt{t}} dt$

$= \frac{1}{2h} \int_0^\infty e^{-t} t^{-\frac{1}{2}} dt$

$= \frac{1}{2h} \int_0^\infty e^{-t} t^{-\frac{1}{2}+1-1} dt$

$\int_0^\infty e^{-x} x^{n-1} dx = \Gamma n$

$= \frac{1}{2h} \Gamma\left(-\frac{1}{2}+1\right)$

$= \frac{1}{2h} \Gamma\frac{1}{2}$

$= \frac{1}{2h} \sqrt{\pi}$

ii)

(3) $\int_0^\infty e^{-x} x^{3/2} \, dx$

$$= \int_0^\infty e^{-x} x^{3/2+1-1} \, dx$$

$$= \int_0^\infty e^{-x} x^{5/2-1} \, dx$$

$$\int_0^\infty e^{-x} x^{n-1} \, dx = \Gamma n$$

$$= \Gamma \frac{5}{2}$$

$$= \left(\frac{5}{2}-1\right) \Gamma \left(\frac{5}{2}-1\right)$$

$$= \frac{3}{2} \Gamma \frac{3}{2}$$

$$= \frac{3}{2} \cdot \frac{1}{2} \Gamma \frac{1}{2}$$

$$\boxed{\int_0^\infty e^{-x} x^{3/2} \, dx = \frac{3}{4} \sqrt{\pi}}$$

iii)

(5) $\int_0^\infty x^2 e^{-x^8} \, dx$

put $x^8 = t$

$x = t^{\frac{1}{8}}$

$dx = \frac{1}{8} t^{-7/8} \, dt$

$= \int_0^\infty t^{\frac{1}{4}} e^{-t} \frac{1}{8} t^{-7/8}$

$= \frac{1}{8} \int_0^\infty e^{-t} \, t^{\frac{1}{4} - \frac{7}{8}} \, dt$

$= \frac{1}{8} \int_0^\infty e^{-t} \, t^{\frac{-5}{8}} \, dt$

$= \frac{1}{8} \int_0^\infty e^{-t} \, t^{-\frac{5}{8}+1-1} \, dt$

$= \frac{1}{8} \int_0^\infty e^{-t} \, t^{3/8 - 1} \, dt$

$\therefore \int_0^\infty e^{-x} x^{n-1} \, dx = \Gamma n$

$$\boxed{\int_0^\infty x^2 e^{-x^8} \, dx = \frac{1}{8} \Gamma \frac{3}{8}}$$

iv)

(4) $\int_0^\infty x^2 e^{-x^4} \, dx$

$x^4 = t$

$x = t^{\frac{1}{4}}$

$dx = \frac{1}{4} t^{-3/4} \, dt$

$= \int_0^\infty t^{\frac{1}{2}} e^{-t} \frac{1}{4} t^{-3/4} \, dt$

$= \frac{1}{4} \int_0^\infty e^{-t} t^{1/2 - 3/4} \, dt$

$= \frac{1}{4} \int_0^\infty e^{-t} t^{-\frac{1}{4}} \, dt$

$= \frac{1}{4} \int_0^\infty e^{-t} t^{-\frac{1}{4}+1-1} \, dt$

$= \frac{1}{4} \int_0^\infty e^{-t} t^{3/4-1} \, dt$

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$\int_0^\infty e^{-x} x^{n-1} \, dx = \Gamma n$

$\boxed{\int_0^\infty x^2 e^{-x^4} \, dx = \frac{1}{4} \Gamma \frac{3}{4}}$

V)

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② Evaluate $\int_0^\infty x^2 \, 7^{-4x^2} \, dx$

$$\int_0^\infty \frac{x^2}{7^{4x^2}} \, dx$$

Put $7^{4x^2} = e^t$

$$\log 7^{4x^2} = t$$

$$4x^2 \log 7 = t$$

$$x^2 = \frac{t}{4\log 7}$$

$$x = \frac{\sqrt{t}}{2\sqrt{\log 7}}$$

$$dx = \frac{1}{2\sqrt{\log 7}} \frac{1}{2\sqrt{t}} \, dt$$

$$dx = \frac{t^{-\frac{1}{2}}}{4\sqrt{\log 7}} \, dt$$

$$I = \int_0^\infty \frac{\frac{t}{(\log 7)\times 4}}{e^{t}} \cdot \frac{t^{-\frac{1}{2}}}{4\sqrt{\log 7}} \, dt$$

$$I = \int_0^\infty \frac{e^{-t} \, t}{4\log 7} \cdot \frac{t^{-\frac{1}{2}}}{4\sqrt{\log 7}} \, dt$$

$$I = \frac{1}{16(\log 7)^{3/2}} \int_0^\infty e^{-t} \, t^{\frac{1}{2}+1-1} \, dt$$

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$$I = \frac{1}{16(\log 7)^{3/2}} \Gamma\frac{3}{2}$$

$$= \frac{1}{16(\log 7)^{3/2}} \cdot \frac{1}{2} \Gamma\frac{1}{2}$$

$$\boxed{I = \frac{\sqrt{\pi}}{32(\log 7)^{3/2}}}$$