- Decimal Number System
- Binary Number System
- Binary ⇄ Decimal
- Addition of Binary Nos.
- Bitwise Operators
- Binary Representation of -ve nos.
- Range of data types
- Importance of constraints

Contest → Thursday (21 Sept)

Full Intermediate Module

9 - 10:30 PM

1.5 hrs → 3 Ques

1. Revision of Notes
2. Assignment

Passed

Atleast 2 out of 3 ques

Contest Discussion → 10:30 PM

(2) Reattempt contest → Sept 23 and 24
Sat and Sun

1.5 hr → 3 new Qus

Decimal Number System $\rightarrow$ digits 0 – 9
$\rightarrow$ base 10

$342 = 300 + 40 + 2 = 3\times10^2 + 4\times10^1 + 2\times10^0$

3 2 1 0
$2563 = 2000 + 500 + 60 + 3$
$= 2\times10^3 + 5\times10^2 + 6\times10^1 + 3\times10^0$

$\text{digit} \times 10^{\text{pos of digit}}$

3 2 1 0
$4672 = 2\times10^0 + 7\times10^1 + 6\times10^2 + 4\times10^3$
$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$
$2 + 70 + 600 + 4000$

Binary Number System $\rightarrow$ digits/bits 0 – 1
$\rightarrow$ base 2

$\text{bit} \times 2^{\text{pos of bit}}$

2 1 0
$110 = 0\times2^0 + 1\times2^1 + 1\times2^2$
$\downarrow$
$0 + 2 + 4$
$= 6$

$(110)_2 = (6)_{10}$

$$\begin{array}{cccc} 3 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{array} = 1\times2^0 + 1\times2^1 + 0\times2^2 + 1\times2^3$$

$$= 1 + 2 + 0 + 8$$

$$= 11$$

$$(1011)_2 = (11)_{10}$$

$$\begin{array}{ccccc} 4 & 3 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{array} = 1\times2^0 + 0\times2^1 + 1\times2^2 + 0\times2^3 + 1\times2^4$$

$$= 1 + 0 + 4 + 0 + 16$$

$$= 21$$

$$\begin{array}{cccc} 3 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{array} \rightarrow 11$$

$$10101 \rightarrow 21$$

Quiz: $\begin{array}{ccccccc} 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{array} \longrightarrow (90)_{10}$

$$0\times2^0 = 0$$
$$1\times2^1 = 2$$
$$0\times2^2 = 0$$
$$1\times2^3 = 8$$
$$1\times2^4 = 16$$
$$0\times2^5 = 0$$
$$1\times2^6 = 64$$
$$\underline{\underline{90}}$$

Decimal to Binary → Long division

$(20)_{10}$

$$\begin{array}{c|ll} 2 & 20 & - 0 \\ \hline 2 & 10 & - 0 \\ \hline 2 & 5 & - 1 \\ \hline 2 & 2 & - 0 \\ \hline 2 & 1 & - 1 \\ \hline & 0 & \end{array} \uparrow$$

Collect remainders from bottom to top

$$20 \rightarrow \overset{4}{1}\overset{3}{0}\overset{2}{1}\overset{1}{0}\overset{0}{0}$$

0
0
$1 \times 2^2 = 4$
0
$1 \times 2^4 = 16$
$\overline{20}$

① Keep Divide the no. by 2 till the no. becomes 0

② Collect remainders from bottom to top

$$\begin{array}{c|ll} 2 & 45 & - 1 \\ \hline 2 & 22 & - 0 \\ \hline 2 & 11 & - 1 \\ \hline 2 & 5 & - 1 \\ \hline 2 & 2 & - 0 \\ \hline 2 & 1 & - 1 \\ \hline & 0 & \end{array} \uparrow$$

$45 \rightarrow 10\ 1101$

Addition of decimal nos.

$$\begin{array}{rrrr} 0 & 1 & 1 & \\ & 3 & 6 & 8 \\ + & 4 & 5 & 3 \\ \hline 0 & 8 & 2 & 1 \end{array}$$

11 → d, ↓ c

$d = sum \% 10$

$c = sum / 10$

12

$d = 12 \% 10$

$c = 12 / 10$

$$\begin{array}{rr} & 9 \\ 1 & 8 \\ \hline & 17 \end{array}$$

$9 + 8 = 17$ → d, c

8

$d = 8 \% 10 = 8$

$c = 8 / 10 = 0$

$d = 17 \% 10$

$c = 17 / 10$

Addition of binary nos.

$$\begin{array}{rrrrrr} 1 & 1 & 1 & 0 & 1 & \\ & 1 & 0 & 1 & 0 & 1 \\ + & & 1 & 1 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \end{array}$$

sum = 2

$d = sum \% 2$

$c = sum / 2$

sum = 1

$d = 1 \% 2 = 1$

$c = 1/2 = 0$

```
   1 0 0 1 0 0
     1 1 0 1 0 1
+    1 0 0 1 1 0
   1 0 1 1 0 1 1
```

sum

$d = sum \% 2$

$c = sum / 2$

```
   0 1 1 0
   1 0 1 1 0   →  22
+  0 0 1 1 1   →   7
   1 1 1 0 1      29
```

```
4 3 2 1 0
0 0 1 1 1
```

$1 \times 2^0 \quad 1$

$1 \times 2^1 \quad 2$

$1 \times 2^2 \quad 4$

$\quad 7$

```
4 3 2 1 0
1 0 1 1 0
```

$1 \times 2^1 = 2$

$1 \times 2^2 = 4$

$1 \times 2^4 = 16$

$\quad 22$

```
4 3 2 1 0
1 1 1 0 1
```

$1 \times 2^0 = 1$

$1 \times 2^2 = 4$

$1 \times 2^3 = 8$

$1 \times 2^4 = 16$

$\quad 29$

Bitwise Operations (individual bits of binary nos.)

1 → true / set

0 → false / unset

bitwise AND

bitwise OR (+)

bitwise XOR

exclusive OR

A	B	A & B	A | B	A ^ B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

same same puppy shame

NOT ~

~0 → 1

~1 → 0

Bitwise operations on numbers

5 & 6

↓

4

5 → 101

6 → 110 &

100

↓

4

20 & 45 $\rightarrow$ 4

20 $\rightarrow$ 010100
45 $\rightarrow$ 101101 &
000100

20 | 45 $\rightarrow$ 61

20 $\rightarrow$ 010100
45 $\rightarrow$ 101101 |
111101 $\rightarrow$ 61

92 ^ 154 $\rightarrow$ 198

92 $\rightarrow$ 01011100
154 $\rightarrow$ 10011010 ^
11000110

~92 $\rightarrow$ 163

92 $\rightarrow$ 01011100
~92 $\rightarrow$ 10100011 $\rightarrow$ 163

4 $\rightarrow$ 00000100

20 ^ 45 = 57

20 $\rightarrow$ 00010100
45 $\rightarrow$ 00101101 ^
00111001
7 6 5 4 3 2 1 0

$\rightarrow 1\times2^0$
$\rightarrow 1\times2^3$

$2^0 + 2^3 + 2^4 + 2^5$
$1 + 8 + 16 + 32 = 57$

$\rightarrow 1 \times 2^7$

$\rightarrow 1 \times 2^5$

10 : 26

Binary Representation of -ve numbers

Decimal No → 8 bit no.

7	6	5	4	3	2	1	0
1	0	0	0	0	0	0	0

2^7 ↓ 128

7	6	5	4	3	2	1	0
0	1	1	1	1	1	1	1
	2^6	2^5	2^4	2^3	2^2	2^1	2^0

↓ 127

MSB   LSB

7	6	5	4	3	2	1	0

sign bit ↑

	7	6	5	4	3	2	1	0
10 →	0	0	0	0	1	0	1	0
-10 →	1	0	0	0	1	0	1	0

0 → +ve

1 → -ve

$\underbrace{1}_{-}\ \underbrace{0001010}_{10}$

(1)

	7	6	5	4	3	2	1	0
0 →	0	0	0	0	0	0	0	0
−0 = 0 →	1	0	0	0	0	0	0	0

$\underbrace{0\ 0\ 0\ 0\ 0\ 0\ 0}_{0}$

(2)

	7	6	5	4	3	2	1	0
10 →	0	0	0	0	1	0	1	0
+ (−4) →	1	0	0	0	0	1	0	0
	1	0	0	0	1	1	1	0

− 14

10 − 4

↓

10 + (−4)

$10 + (-4) \quad \times \quad -14 \quad ?$

8 bit no.

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
7	6	5	4	3	2	1	0
1	1	1	1	1	1	1	1

$-128 + 127 = -1$

7th bit

↓

1 −ve

0 +ve

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
7	6	5	4	3	2	1	0
0	1	1	1	1	1	1	1

0 127

MSB has -ve base value

Binary of -10

	7	6	5	4	3	2	1	0
10 →	0	0	0	0	1	0	1	0
	1	1	1	1	0	1	0	1

(1) Binary of x

(2) Invert bits

(3) add 1

					0	0	1	
	1	1	1	1	0	1	0	1
+	0	0	0	0	0	0	0	1
	1	1	1	1	0	1	1	0

	7	6	5	4	3	2	1	0
-10 →	1	1	1	1	0	1	1	0

$2^1 = 2$

$2^2 \quad 4$

$2^4 \quad 16$

$2^5 \quad 32$

$2^6 \quad 64$

118

$-2^7 \quad -128$

$-128 + 118$
$= -10$

	-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	7	6	5	4	3	2	1	0
10 →	0	0	0	0	1	0	1	0

2^1

2^3

10

		7	6	5	4	3	2	1	0
1	3 →	0	0	0	0	0	0	1	1
		0	0	0	0	0	0	0	
2		1	1	1	1	1	1	0	0
3	+	0	0	0	0	0	0	0	1
		1	1	1	1	1	1	0	1

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
7	6	5	4	3	2	1	0
1	1	1	1	1	1	0	1

$$\left.\begin{matrix} 2^0 \\ 2^2 \\ 2^3 \\ 2^4 \\ 2^5 \\ 2^6 \end{matrix}\right\} 125$$

$-2^7 \rightarrow -128$

$-128 + 125 = -3$

MSB

4 bits → 3 2 1 0

$-2^3 \; 2^2 \; 2^1 \; 2^0$

Range of data types

2	140	– 0
2	70	– 0
2	35	– 1
2	17	– 1
2	8	– 0
2	4	– 0
2	2	– 0
2	1	– 1
	0	

-2^7

	7	6	5	4	3	2	1	0
140 →	1	0	0	0	1	1	0	0

$-2^7 \quad 2^3 \quad 2^2$

$-128 + 8 + 4$

140 in 8 bit → -116

Range of data types

1 B → 8 bits

7	6	5	4	3	2	1	0
1	0	0	0	0	0	0	0

$-2^7 = -128$

8 bit

↓

[-128 127]

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
7	6	5	4	3	2	1	0
0	1	1	1	1	1	1	1

$2^6 + 2^5 + 2^7 + \ldots 2^0$

$= 127$

-2^{N-1}

N bit $\quad \begin{array}{cccccccc} N-1 & & \cdots & & & 2 & 1 & 0 \\ \underline{1} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} \end{array}$

$\downarrow$

-2^{N-1}

-2^{N-1}

$\begin{array}{cccccccc} N-1 & & \cdots & & & 2 & 1 & 0 \\ \underline{0} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{1} \end{array}$

$\downarrow$

$GP = \dfrac{a(r^N-1)}{r-1}$ $\qquad 2^{N-2} + 2^{N-3} + \cdots\cdots 2^2 + 2^1 + 2^0$

$\text{No.} = \dfrac{1(2^{N-1}-1)}{2-1} = 2^{N-1}-1$

$[0 \to N-2]$

$N-2-0+1$

$= N-1$

N bits $\to [-2^{N-1} \qquad 2^{N-1}-1]$

8 bits $\to [-2^7 \qquad 2^7-1]$

$[-128 \qquad 127]$

Int 4B

$= 32$ bits $\quad [-2^{31} \qquad 2^{31}-1]$

$|$

Int $[-2147483648 - 2147483647]$

INT_MIN INT_MAX

$2^{10} = 1024$ $2^{10} \simeq 10^3$

$10^3 = 1000$

Int 4B

$= 32$ bits $[-2^{31} \quad 2^{31}-1]$

$-2 \times 2^{10} \times 2^{10} \times 2^{10}$

$-2 \times 10^3 \times 10^3 \times 10^3$

$[-2^{31} \rightarrow 2^{31}-1]$

INT

$[-2 \times 10^9 \rightarrow 2 \times 10^9 - 1]$

$-2^{N-1} \quad 2^{N-1}-1$

Long → 8B $[-2^{63} \quad 2^{63}-1]$

64 bits

$-2^3 \times (2^{10})^6$

$2^{10} = 10^3$

$[-8 \times 10^{18} \quad 8 \times 10^{18} - 1]$

Input k

$1 \le k \le 2^{36}$

Importance of Constraints

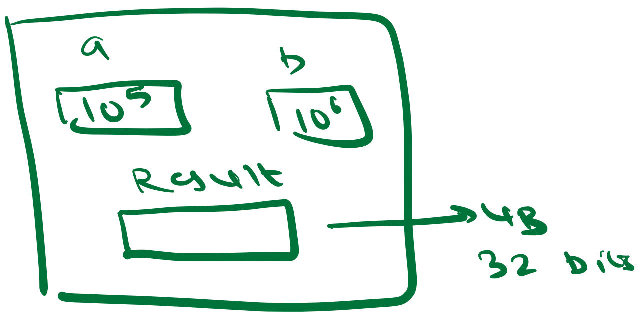
int $a = 10^5$

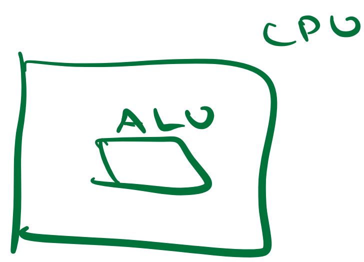
int $b = 10^6$

ALU

Arithmetic Logic Unit

① int $c = a \times b$

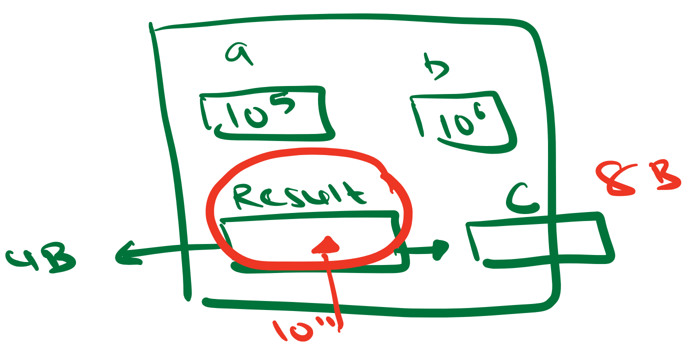




$10^{11} \rightarrow 32$ bits

Overflow

② ~~int~~ long $c = a \times b$



① a and b

② Product → overflow

③ copy Result to c

3 ~~int~~ long $c = \text{long}(\underline{a * b})$ ✗

1 $a * b \rightarrow$ Result 4 B

2 long

4 $\boxed{\text{long } c = (\text{long})\, a * b}$ ✓

	a	b	a	b
1	int	int	8 B long	4 B int
2	long	int		

3 Copy result to c

Result 8 B long $\boxed{10^{11}}$

~~int~~ long sum = 0

for (i=0 ; i < N ; i++)

sum += A[i]

$1 \le N \le 10^5$

$1 \le A[i] \le \underline{10^6}$

[10^6 10^6 10^6 ....]

max sum = $10^6 \times 10^5 = 10^{11}$

int
sum → max 10^9

10^{11}

long a × b × c × d

int b, int c

10^6 10^5

Arrays and string
Hashmaps

1. Base value -ve ?
2. Idea : TC

Prefix → O(N)

subarray | range | sum

Sliding window → size of subarray fixed