Searching in Rotated Sorted Array

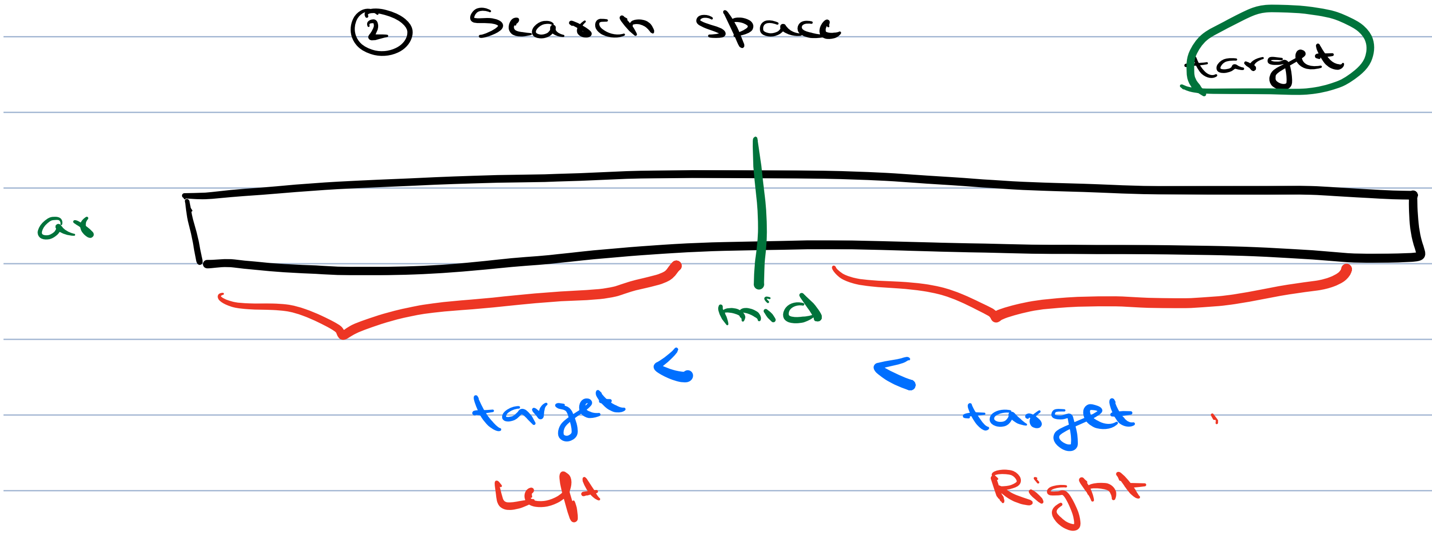
Find square root of a number

A^{th} Magical Number

Median of 2 sorted Arrays

① Target

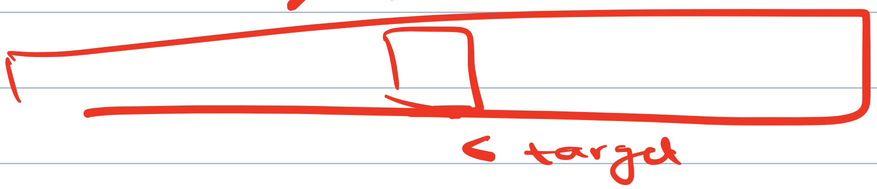
② Search space



1 2 3 4 5 6

6 4 5 3 2 1

L > mid > S



1. Find the target in a rotated sorted array (elements are distinct)

2 4 8 10 15

↓ rotated 2 times

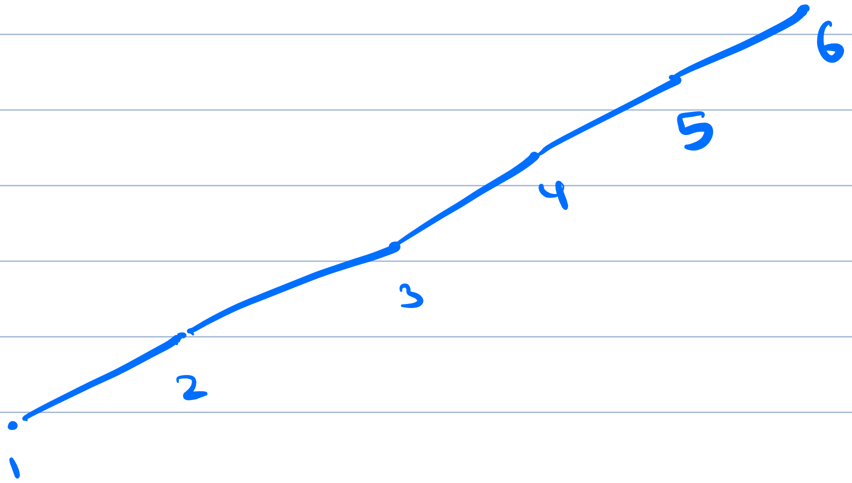
Input

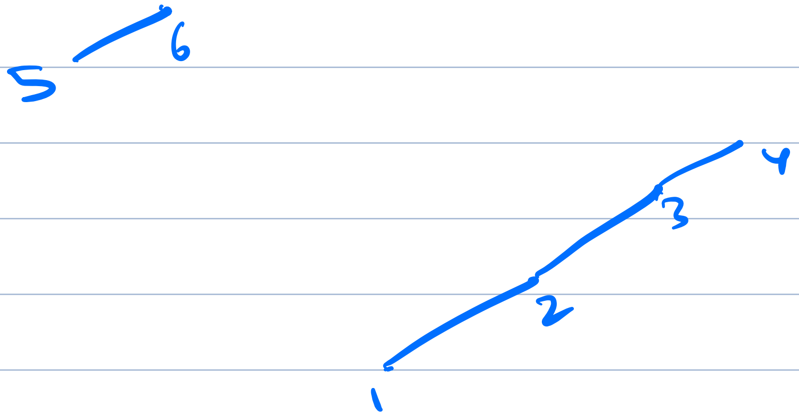
0	1	2	3	4
10	15	2	4	8

Target = 2

Ans 2

Sorted



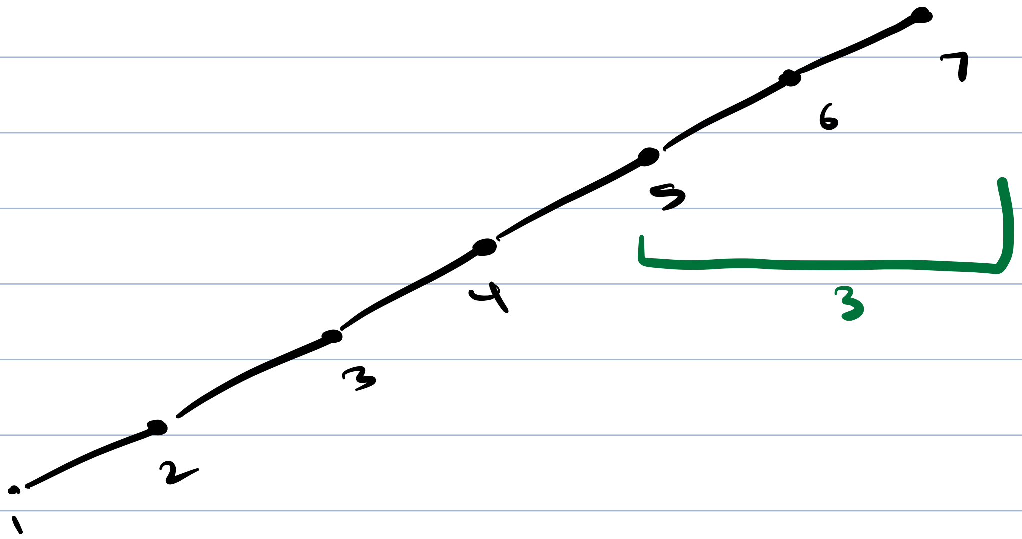


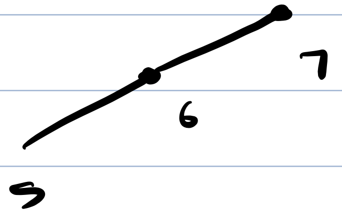
if (A[0] < A[N-1])

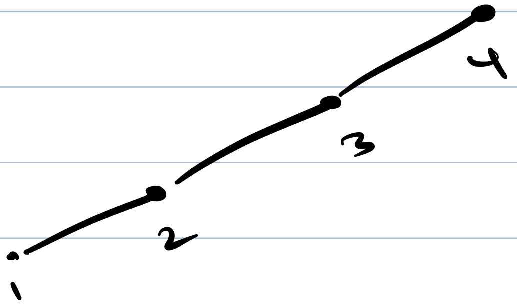
not rotated

else

rotated



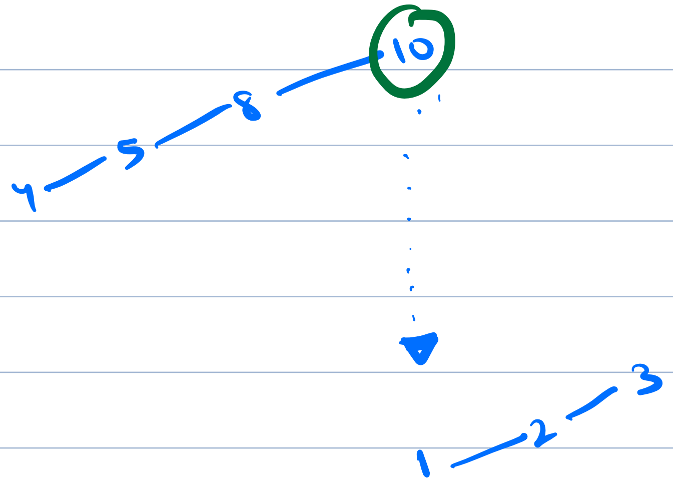




ar → 1 2 3 4 5 8 10

4 5 8 10   1 2 3

Part I   Part 2



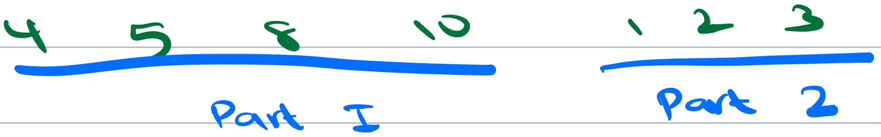
Find local maxima using BS

Then apply BS on part 1 and part 2

↓

TC : O(log n) → 3 log N

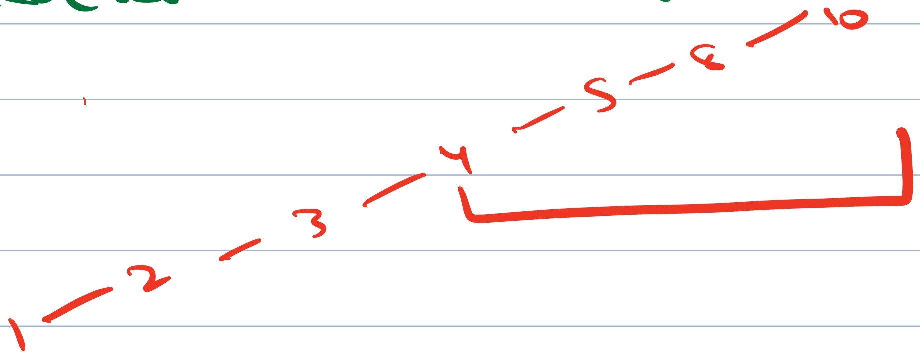
Approach 3 : Use BS only once



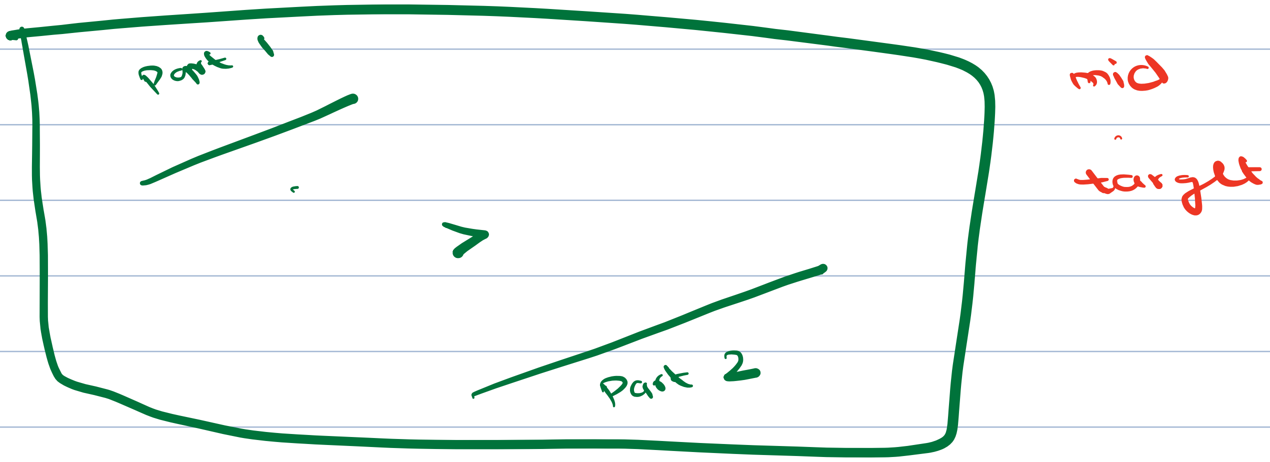
① 4 <= part I   4 > Part 2

② Part 1 > Part 2

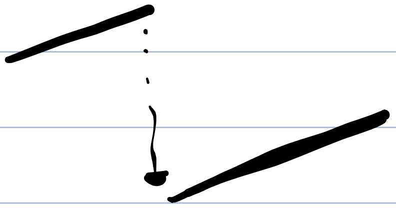
Rotated   Original



```
if ( A[0] <=ele)
        Part 1
else
        part 2
```



Case 1	Mid is in part 1	Target is in part 2	Go right
Case 2	Mid in part 2	Target in part 1	Left
Case 3	Mid part 1	Target part 1	BS in part 1
Case 4	Mid part 2	Target part 2	BS in part 2



0	1	2	3	4	5	6	7	8	9	10	11
10	20	30	1	2	3	4	5	6	7	8	9

Part 1 7 Part 2

Target = 20

s	e	mid	Mid → Part	Target → Part	
0	11	5	Part 2	Part 1	Left
0	4	2	Part 1	Part 1	Left
0	1	0	Part 1	Part 1	Right
1	1	1	—	—	

A[mid] = A[target]
20 == 20

return mid

TC : log (search space)

```
s = 0, e = n-1
while (s <= e) {

    mid = $\frac{(s+e)}{2}$

    if (A[mid] == target) {
            return mid
    }

    if (target >= A[0]) { // part 1
        if (mid >= A[0]) { // part 1
            if (A[mid] < target) {
                    s = mid+1     //right
            }
            else {
                    e = mid-1     // left
            }
        }
        else {    //mid in part 2
                e = mid-1     //left
        }
    }
else {        //target        part 2
    if (mid  <  A[0]) {    // part 2
        if (A[mid] < target) {
                    s = mid+1        //right
        }
        else {
                    e = mid-1        // left
        }
    }
    else {        //mid  in  part 1
                s = mid+1
                                      //right
    }
}
}
```

TC : $O(\log_2 n)$

SC : $O(1)$

2. Given a positive no. N, find square root of N.

↓

floor (square root (N))

N	Ans
25	5
20	4
10	3

$N \rightarrow x$

$x^2 = N$

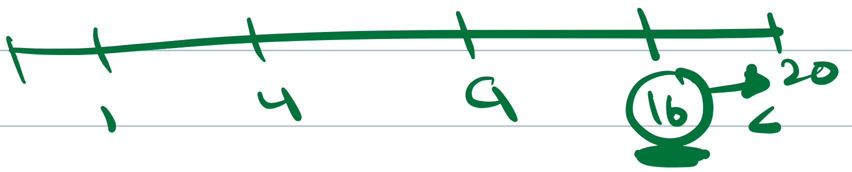
	N	Ans
min →	1	1

i	N
	25
1	
2	
3	
4	
5	

```
i=1
while (i*i <= N) {
    ans = i
    i++
}
```

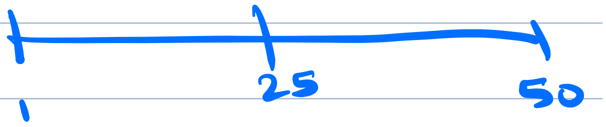
TC : $O(\sqrt{N})$ SC: $O(1)$

i	20
$1^2 < 20$ ✓	
$2^2 < 20$ ✓	
$3^2 < 20$ ✓	
$4^2 < 20$ ✓	
ans = → $5^2 > 20$	



ans
= 47

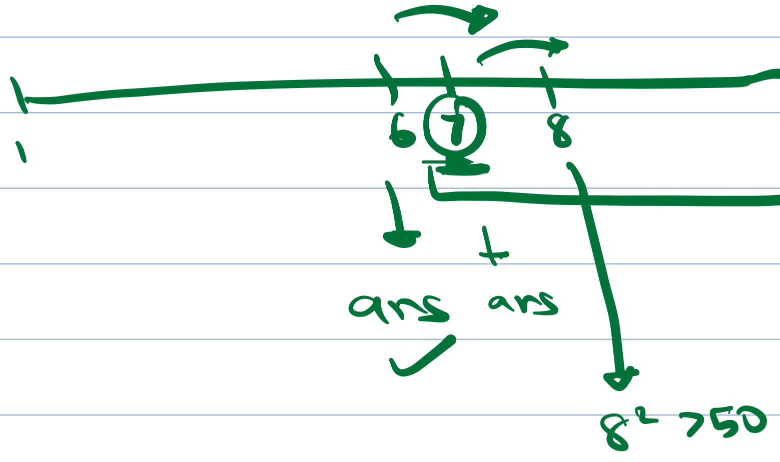
N = 50



s	e	mid		
1	50	25	25 * 25 > 50	left
1	24	12	12 * 12 > 50	left
1	11	6	6 * 6 < 50 ans = 6	right
7	11	9	9 * 9 > 50	left
7	8	7	7 * 7 < 50 ans = 7	right
8	8	8	8 * 8 > 50	left
8	7	break		

TC : O ($\log_2 N$)

SC : O(1)



50

$x^2 < 50$

$6^2 < 50$

$7^2 < 50$

```
if (N == 0  ||  N == 1)
        return  N

s = 1 , e = N , ans = 0
while (s <= e) {

    mid = (s + e)/2

    if (mid * mid == N)
                    return  mid
    else  if  (mid * mid  < N) {
                    ans = mid
                    s = mid + 1           //right
    }

    else  {           // mid * mid  > N
                    e = mid - 1           //left
    }
}
return  ans
```

10:33

Median → Middle element in sorted data

4 5 $\overset{\downarrow}{10}$ 13 17   4 5 $\overset{\downarrow}{10}$ $\overset{\downarrow}{13}$ 17 20

Median = 10   Median $= \frac{10+13}{2} = \frac{23}{2} = 11.5$

3. Median of 2 sorted arrays

A → 1, 4, 5
B → 2, 3   ans = 3

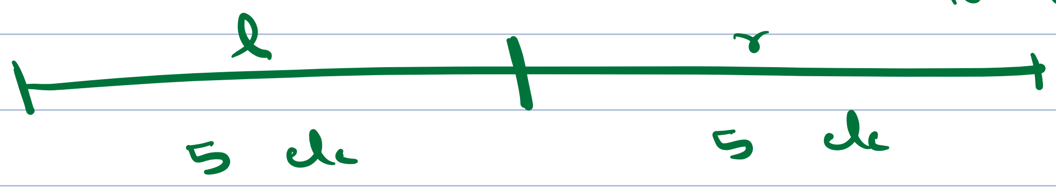
A → 1, 2, 3
B → 4   ans = 2.5

3 1 2
↓
1 2 3

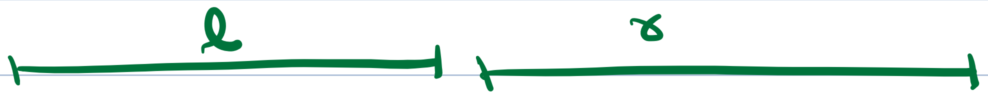
Median → 2

A → 1, 3, 4, 7, 10, 12
B → 2, 3, 6, 15   ] 10 elements

10 elements



l <= r

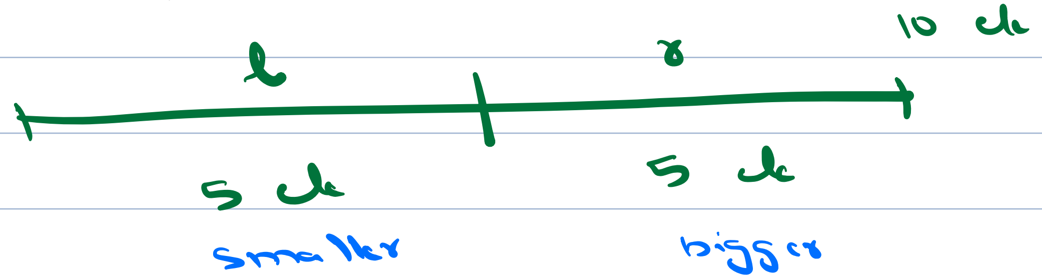


1, 2, 3, 3, 4, 6, 7, 10, 12, 15

1, 2, 3, 3, 4   6, 7, 10, 12, 15

A → 1, 3, 4, 7, 10, 12

B → 2, 3, 6, 15



Case 1 : 4 elements from A

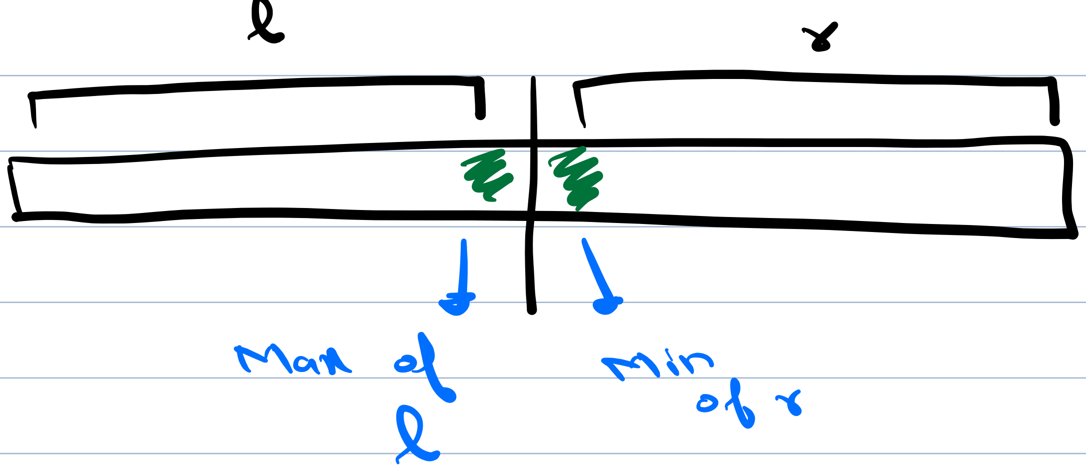
	l	r	
A	1, 3, 4, 7	10, 12	✗
B	2	3, 6, 15	

Case 2 : 2 ele from A

	l	r	
A	1, 3	4, 7, 10, 12	✗
B	2, 3, 6	15	

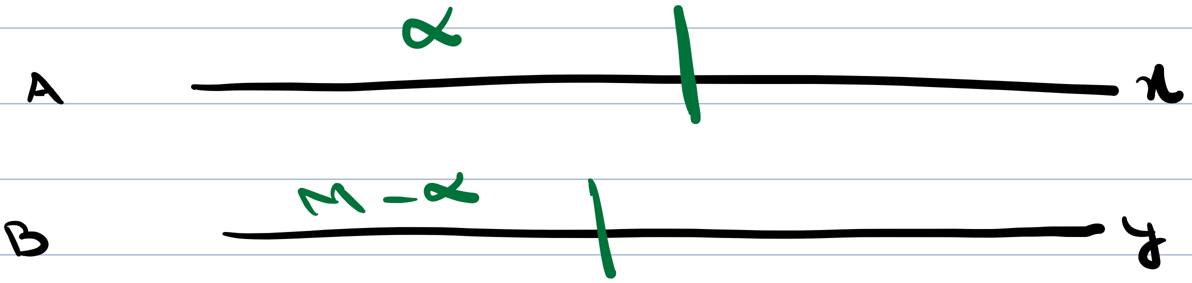
Case 3 : 3 ele from A

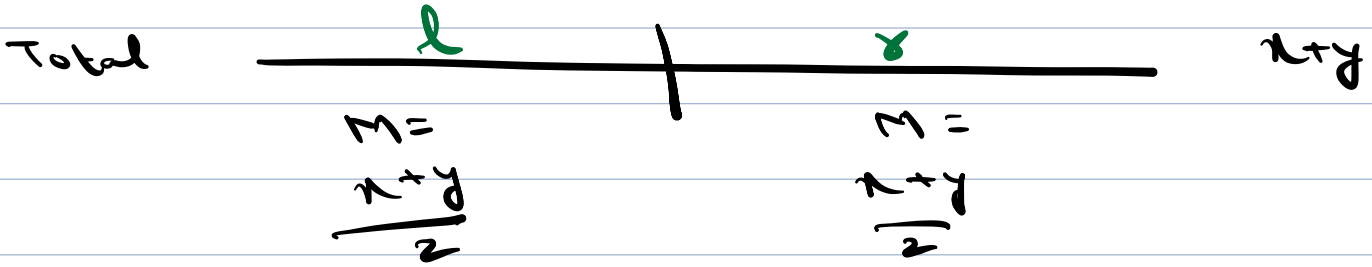
	l	r	
A	1, 3, 4	7, 10, 12	✓
B	2, 3	6, 15	



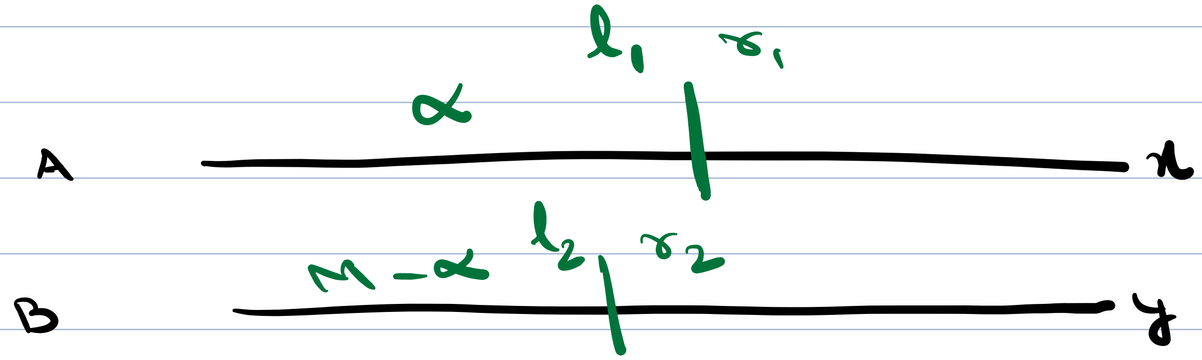
4 , 6

$$\text{Median} = \frac{4+6}{2} = 5$$



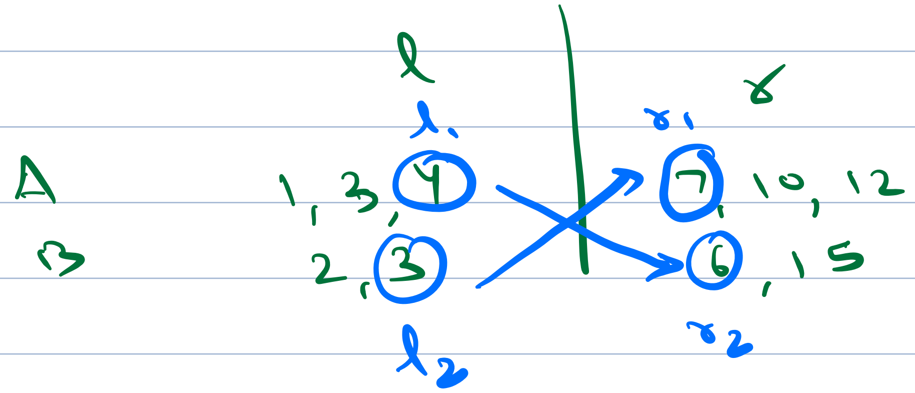


		l	r
A	→	α	$x-\alpha$
B	→	$M-\alpha$	$y-(M-\alpha)$



A → 1, 3, 4, 7, 10, 12

B → 2, 3, 6, 15



$l_1 \to r_2$

$l_2 \to r_1$

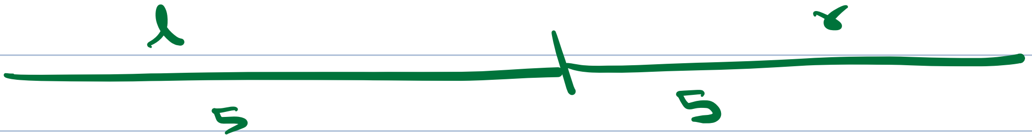
$l <= r$

Check ($l_1 <= r_2$ and $l_2 <= r_1$)

Median (even) $$\frac{\max(l_1, l_2) + \min(r_1, r_2)}{2}$$

A → 7 12 14 15

B → 1 2 3 4 9 11] 10 elements



BS → no. of elements that you should pick from A to put on l half

s e mid

0 4 2

l | r

A 7 l_1 12 | r_1 14 15

B 1 2 l_2 3 | r_2 4 9 11

$l_1 <= r_2$ → Left $l_1 > r_2$

✗ $12 <= 4$

0 1 0

l | r

A 0 | r_1 7 12 14 15

B 1 2 3 4 l_2 9 | r_2 11

$l_1 <= r_2$ ✓

$-\infty$

$l_2 <= r_1$

$9 <= 7$ ✗

right $l_2 > r_1$

	l	r
	l_1	r_1
A →	7	12 14 15
B →	1 2 3 4	9 11
	l_2	r_2

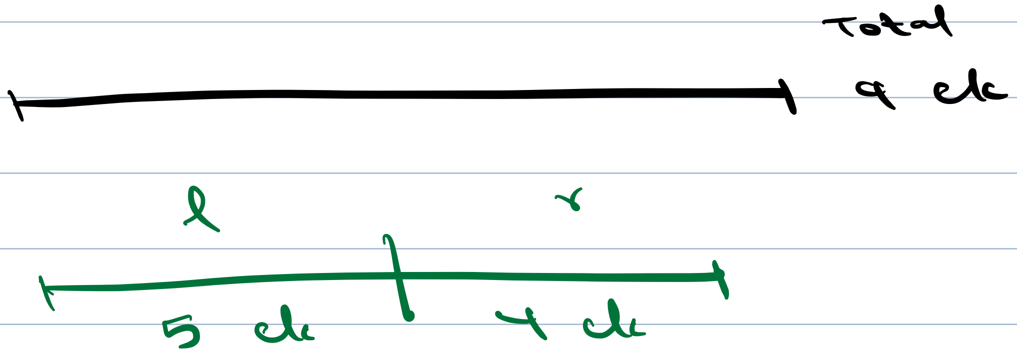
$l_1 <= r_2$ and $l_2 <= r_1$

$7 <= 9$ ✓ $4 <= 12$ ✓

$\max(l_1, l_2)$	$\min(r_1, r_2)$
↓	↓
7	9

$$\text{Median} = \frac{7+9}{2} = 8$$

odd

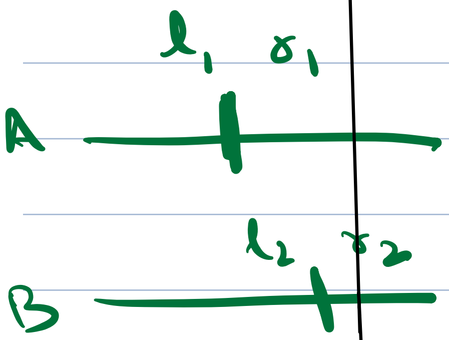


$\text{Median} = \max(l_1, l_2)$

```
                                       smaller arr        larger arr
findMedian ( int [] A , int [] B ) {
    if ( B.size < A.size ) {
        return findMedian ( B, A )
    }

    int  m = A . size
    ina  n = B. size
    s = 0 , e = m
    lhalf cnt = (n + m + 1 )/2

    while ( s <= e ) {
        mid = (s + e)/2
        ↓
        no. of ele picked from A
                for left half

        cnt A = mid
        cnt B = lhaft cnt - cntA
  -∞ ←  l1 = A [cnt A - 1]
  -∞ ←  l2 = B [cnt B - 1]
     ←  r1 = cnt A == m ? ∞ : A [ cnt A]
        r2 = cnt B == n ? ∞ : B [cnt B]
```

no. of ele are even

```
if (l1 <= r2 && l2 <= r1) {
    if (m+n % 2 == 0) {
        return max(l1, l2) +
               min (r1, r2)
               ------------
                    2
    }
    else {
        return max(l1, l2)
    }
}
else if (l1 > r2) {
        e = mid - 1      // left
}
else {      // l2 > r1
        s = mid + 1      // right
}
```



$l_1 = \text{cntA} - 1 < 0 \; ? \; -\infty : A[\text{cntA}-1]$

TC : $O(\log(\min(N, M)))$

SC : $O(1)$