

Today's content

→ Longest prefix

→ Boring substring

→ KMP Algorithm (Knuth-Morris-Pratt)

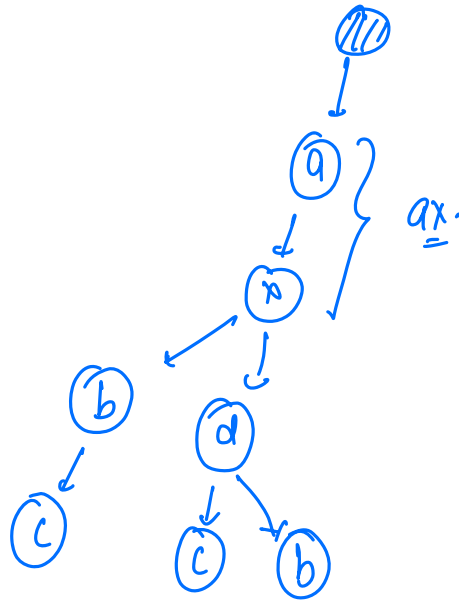
① Given an array of strings. Find the longest string which is prefix of all strings in the array.

["axdc", "axbc", "axdb"]

ans → ax

$N \rightarrow$ no. of words
 $m \rightarrow$ max length of a word.

1 → Trie



$T.C \rightarrow O(N \times m)$
 $S.C \rightarrow O(N \times m)$

2 → Brute force

axdc
axbc
axdb
axab
ax

$T.C \rightarrow O(N \times m)$
 $S.C \rightarrow O(1)$

2. Given a string s , check whether it is possible to re-arrange the characters of the string s such that there is no boring substring in s .

Boring substring \rightarrow length = 2 and consecutive alphabets.

$\rightarrow ab, cd, xy, yz, yx, dc, cb, ba$

$s = "abc"$

abc
acb
bac
bca
cab
cba

false

$s = "abcd"$

abcd X

cadb

bdac

true

$s = "aabccb"$

ans \rightarrow false

baac**cb** X

bb**a**acc X

idea-1 \rightarrow Consider all the permutations of given string & check if they are having boring substring or not.

T.C $\rightarrow O(N! \times N)$

idea-2

s = "aabccdb"

bbdaacc

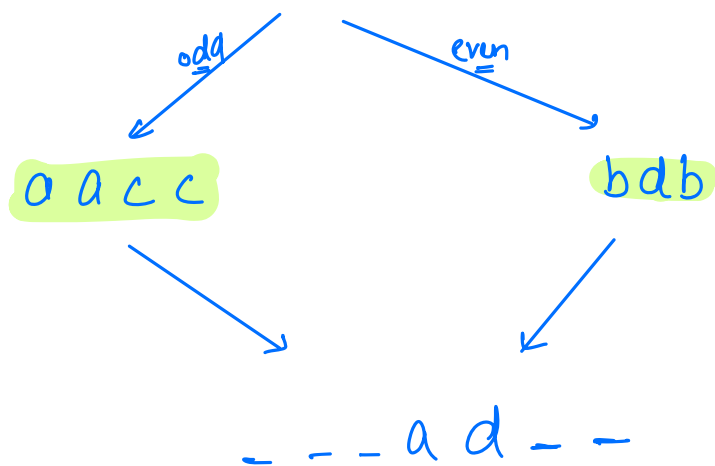
ans → true.

a b c d e f g h i j k _ _ w x y z

a c e g i k _ _ w y
b d f h j l _ _ x z

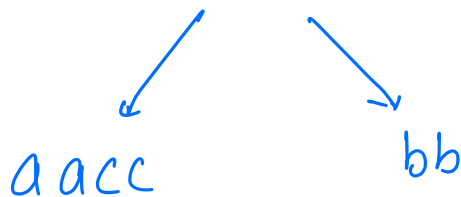
$$\begin{bmatrix} a \rightarrow 97 \\ b \rightarrow 98 \\ c \rightarrow 99 \\ d \rightarrow 100 \\ \vdots \\ z \rightarrow 122 \end{bmatrix}$$

s = "aabccab"



ans = true.

s = "aabccbb"



ans = false.

$$\begin{matrix} & 2 & \\ \swarrow & & \searrow \\ n/2 & & n/2 \end{matrix}$$

T.C $\rightarrow \frac{n}{2} \times \frac{n}{2} \rightarrow \underline{\underline{O(n^2)}}$

idea-2 →

i) only check the unique characters.

a c e _ _ y
13

b d f _ _ x z
13

13 x 13 iterations

T.C → O(1)

ii) HashMap/HashSet to check if odd ASCII characters,
if anything other than (ch-1) or (ch+1) is present.

a
c

b
d

T.C → O(1)

iii) odd set → smallest largest
even set → smallest largest

a a a c c e g g
↑ ↑

b b d f f
↑ ↑

a	b	X
g	f	X
a	f	✓
g	b	✓

true.

$s = " \overset{\checkmark}{c} \overset{\checkmark}{c} \overset{\checkmark}{e} \overset{\checkmark}{b} \overset{\checkmark}{b} \overset{\checkmark}{d} \overset{\checkmark}{f} "$

smallest-odd ASCII char $\rightarrow c$

largest-odd ASCII char $\rightarrow e$

smallest-even ASCII char $\rightarrow b$

largest-even ASCII char $\rightarrow f$

ans \rightarrow true.

$\left[\begin{array}{l} P.C \rightarrow O(N) \\ S.C \rightarrow O(1) \end{array} \right]$

KMP [Knuth-Morris-Pratt Algorithm]

Given a string s of N size.

prefix substrings \rightarrow substrings starting with $idx \rightarrow 0$

suffix substrings \rightarrow substrings ending with $idx \rightarrow N-1$

$\hookrightarrow s = \text{"a b a b"}$
 0 1 2 3

prefix substrings

a
ab
aba
abab

suffix substrings

b
ab
bab
abab

LPS of a string \rightarrow length of longest prefix substring which is also a suffix sub-string.

Note → Except the whole string.

Ex 1: $S = a \ b \ c \ a \ b \rightarrow [2]$

<u>prefix</u>	<u>substrings</u>	<u>suffix</u>	<u>substrings</u>
a			b
ab			ab
abc			cab
abca			bcab

$C_{y2} : s = 'a' \rightarrow \underline{0}$

G_3 : $S = \text{"aaaaa"} \rightarrow \underline{4}$

<u>prefix</u>	<u>suffix</u>
a	a
aa	aa
aaa	aaa
aaaa	aaaa

Ex 4: $S = a a b a a b a \rightarrow \underline{4}$

_{0 1 2 3 4 5 6}

<u>prefix</u>	<u>suffix</u>
a	a
aa	ba
aab	aba
<u>aaba</u>	<u>aaba</u>
aabaa	baaba
aabaaab	abaaaba

Given a string s of length n . Return the $LPS[i]$

$LPS[i] \rightarrow$ LPS value of substring $[0, i]$.

$s \rightarrow a a b a a b a$

_{0 1 2 3 4 5 6}

$LPS[] =$

0	1	0	1	2	3	4
---	---	---	---	---	---	---

$LPS[1] \rightarrow "aa"$

a a

$LPS[2] \rightarrow "aab"$

a b
 aa ab

$LPS[3] \rightarrow "aaba"$

a a
 aa ba
 aab aba

LPS[4] → "aabaa"

a	a
aa	aa
aab	baa
aaba	abaa

LPS[5] → "aabaab"

a	b
aa	ab
aab	aab
aaba	baab
aabaa	abaa

Q →

S = a a b a c a a b a

0 1 2 3 4 5 6 7 8

LPS[] →

0	1	0	1	0	1	2	3	4
---	---	---	---	---	---	---	---	---

Q -

S = a a b a c d

0 1 2 3 4 5

LPS[] →

0	1	0	1	0	0
---	---	---	---	---	---

→ Pattern Matching

Text → a a b a c [n]

pattern → a b a c [m].

B.f. → Go to each idx & compare characters one by one.

T.C → $O(N \times m)$

T → d a b c e a b c

p → a b c

↓
p # T
delimeter

s → a b c # d a b c e a b c

ans →

0	0	0	0	0	1	2	3	0	1	2	3
---	---	---	---	---	---	---	---	---	---	---	---

pattern is matched.

$T \rightarrow a a a a$

p → aa

S: P+T → a a a a a a

LPST \rightarrow

0	1	2	3	4	5
---	---	---	---	---	---

$$S: P + {}^n H^r + T \rightarrow a \ a \ \cancel{H} \ a \ a \ a \ a$$

LPS -

0	1	0	1	2	2	2
---	---	---	---	---	---	---

Ans: 3.

→ Calculate $LPS[N]$

obs → 1 ∴ $\text{LPS}(i) = 5$.

$$S_N = S_0 \quad S_1 \quad S_2 \quad S_3 \quad S_4 \quad - \quad - \quad - \quad S_{i-5} \quad S_{i-4} \quad S_{i-3} \quad S_{i-2} \quad S_{i-1} \quad \underline{S_i} \quad - \quad S_{N-1}$$

5

$$\underline{\text{Obs.}} \rightarrow S_0 \quad S_1 \quad S_2 \quad S_3 \quad S_4 \quad = \quad S_{i-4} \quad S_{i-3} \quad S_{i-2} \quad S_{i-1} \quad S_i$$

x x

$$\begin{array}{ccccccc} & & & \downarrow & & & \\ s_0 & s_1 & s_2 & s_3 & = & s_{i-4} & s_{i-3} & s_{i-2} & s_{i-1} \end{array}$$

LPS [i-1] ≥ 4.

Assuming.

$$LPS(i) = x$$

$$LPS[i-1] \geq x-1$$

$$LPS[i-1] \geq LPS[i] - 1$$

$$LPS[i-1] + 1 \geq LPS[i]$$

$$LPS[i] \leq LPS[i-1] + 1$$

$$LPS[i] = LPS[i-1] + 1 \quad (A1-max)$$

Skip 2.

$s \rightarrow$ a b a y a b a ?

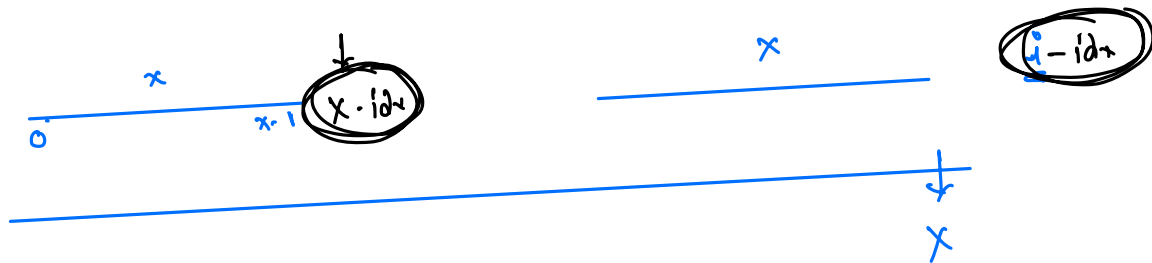
$lps[i] \rightarrow$ 0 0 1 0 1 2 3

if $? == y$.

then $lps[i] \rightarrow 4$.

$s \rightarrow$ b c a d c b c a d ?

$lps[i] \rightarrow$ 0 0 0 0 0 1 2 3 4 $\rightarrow s$.



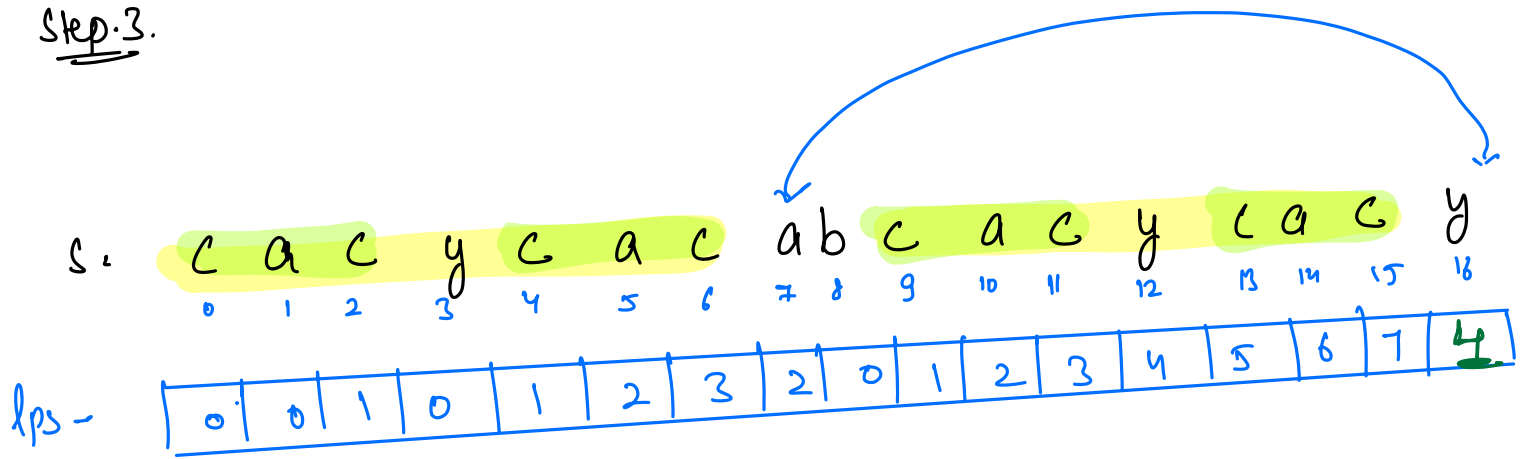
calculating $lps[i]$ \rightarrow

$$x = lps[i-1]$$

if ($s[x] == s[i]$) {

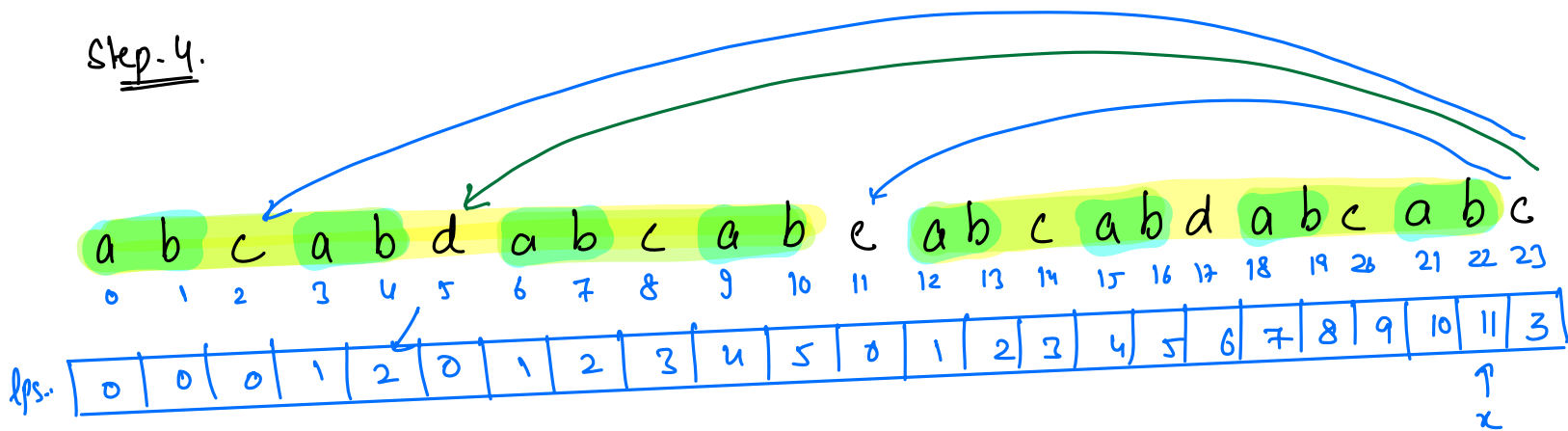
$\left\{ \begin{array}{l} lps[i] = lps[i-1] + 1; \end{array} \right.$

Step-3.



$lps[16]$.

Step-4.



$f = 23$.

for($i = 1$; $i < N$; $i++$) {

$x = lps[i-1];$

 while($s[i] \neq s[x]$) {

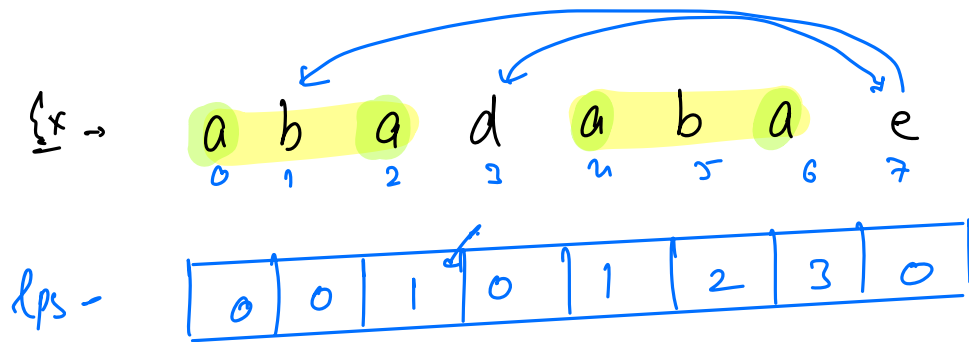
 if($x == 0$) { $x = -1$, break }

$x = lps[x-1];$

 }

$lps[i] = x + 1;$

$x = \cancel{1} \neq 2$



$i = 7$

$n = 8, 10$

$x = \text{lps}[i-1] \quad \times \Rightarrow \quad x = \text{lps}[x-1]$

code \rightarrow

LPS (string s)

```

int n = str.length(), lps[N];

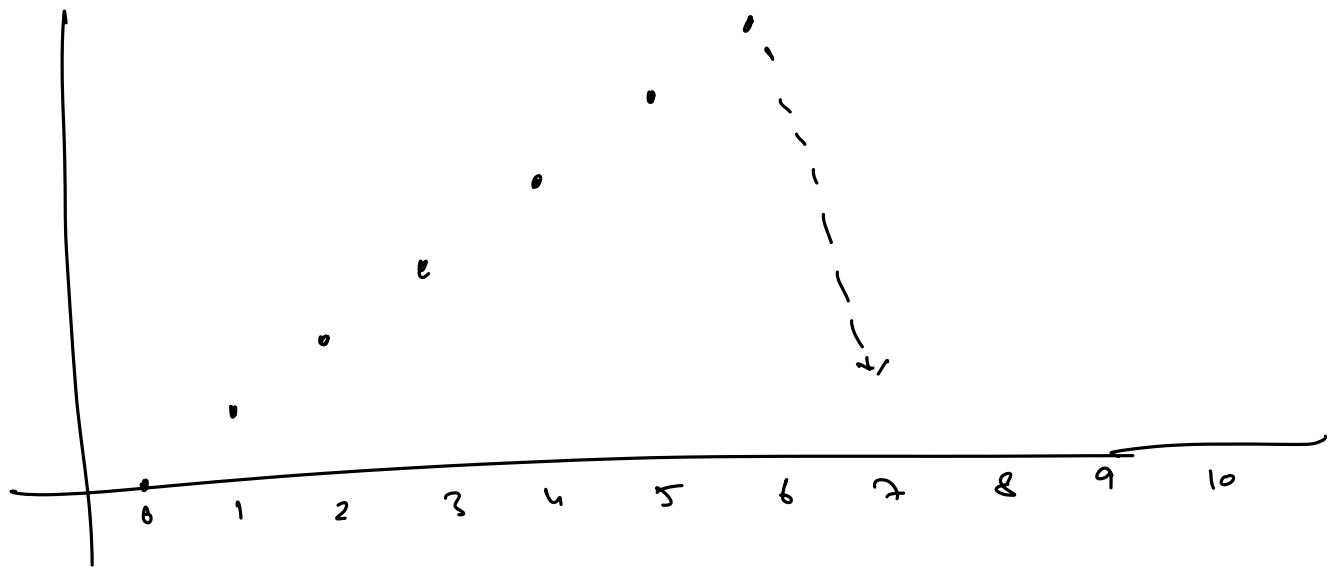
lps[0] = 0;

for( i = 1; i < n; i++) {
    x = lps[i-1];
    while( s[i] != s[x]) {
        if(x == 0) { x = -1; break; }
        x = lps[x-1];
    }
    lps[i] = x + 1;
}

return lps[n];

```

(T.C $\rightarrow O(N)$)



0 1 2 3 4 5 6 -

```

x = lps[i-1];
while( s[i] != s[x] ){
    if(x == 0) { x = -1; break; }
    x = lps[x];
}

```

$lps[i] = x + 1;$

$[T.C \rightarrow O(N)]$

n=6
5
4
3
2
1
0

i=7.

⇒ Online Assessment problem.