Search in Sorted Matrix

No. of 1s in a row

Spiral Matrix

sum of all submatrices sum

1. Given a row wise and col wise sorted matrix, check element K is present or not.

bool

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	18

K = 13 True

K = 2 True

K = 15 False

BF Approach : Iterate through all rows and cols

TC : O(N × M)

rows cols

Optimized : Use the property → sorted mat

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	18

TR

BL

3 < 17

U

L $\boxed{3}$ R

D

K = 0

TL $\boxed{-5}$ $\xrightarrow{\text{big}}$ R

big ↓ D

Ambiguous

BR

U ↑ small

L $\xleftarrow{\text{small}}$ $\boxed{18}$

Ambiguous

TR

Smaller

L ← $\boxed{13}$

↓ Bigger

D

BL

U ↑ small

$\boxed{-3}$ → R

Bigger

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	18

k = 0

small

L ← $\boxed{\text{cell}}$

↓ Big

D

	0	1	2	3
0	-5	-2	1	13
1	-4	0	3	14
2	-3	2	6	18

k = 8

N x M

rows cols

row, col

```
int i = 0 , j = M-1

while (i<N && j>=0) {
    if (arr[i][j] == k)
        return true
    else if (k < arr[i][j])
        j--
    else {
        i++
    }
}
return false
```

TC: O(N+M)

SC: O(1)

1 step → 1 row or col

N + M steps → N rows + M cols

2. Given a binary sorted (0,1) matrix A of N×N. Find row with max. number of 1s.

3x3

$$
A: \begin{array}{c|ccc|c}
 & 0 & 1 & 2 & \text{cnt} \\
\hline
\boxed{0} & 0 & 1 & 1 & 2 \\
1 & 0 & 0 & 1 & 1 \\
2 & 0 & 1 & 1 & 2 \\
\end{array}
$$

ans → 0

① Every row is sorted

② If 2 rows have same no. of 1s, pick row with smallest idx

4x4

$$
A: \begin{array}{c|cccc|c}
 & 0 & 1 & 2 & 3 & \text{cnt} \\
\hline
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 \\
2 & 0 & 0 & 1 & 1 & 2 \\
\boxed{3} & 0 & 1 & 1 & 1 & 3 \\
\end{array}
$$

ans → 3

BF : Iterate over each row → cnt no. of 1s

TC : $O(N^2)$ SC : $O(1)$

L ← R

	0	1	2	3	4	5
0	0	0	0	0	1	1
1	0	0	1	1	1	1
2	0	0	0	0	0	1
3	0	0	0	0	1	1
4	0	1	1	1	1	1
5	0	0	0	1	1	1

cnt = ~~2~~ ~~3~~ ~~4~~ 5

ans = ~~0~~ ~~1~~ 4

Left ← 1

0 ↓ Down

0	0	1
0	0	1
0	0	1

```
i=0        j=N-1        cnt=0
ans = -1
while (i<N && j≥0) {
    if (arr[i][j]==1) {
        cnt++        j--
        ans=i
    }
    else {
        i++
    }
}
return ans
```

TC: O(N)

SC: O(1)

1 step → 1 row or col

N+N step   N row + N col

3. Given a mat of N×N, print boundary elements in clockwise direction.

A[5][5]

Ex 1

	0	1	2	3	4
0	1	2	3	4	5
1	6	7	8	9	10
2	11	12	13	14	15
3	16	17	18	19	20
4	21	22	23	24	25

	idx	
1st row	0	L → R
Last col		T → B
Last row		R → L
1st col		B → T

1 2 3 4 5 10 15 20 25 24 23 22
21 16 11 6

Ex 2

	0	1	2
0	1	2	3
1	4	5	6
2	7	8	9

A[3][3]

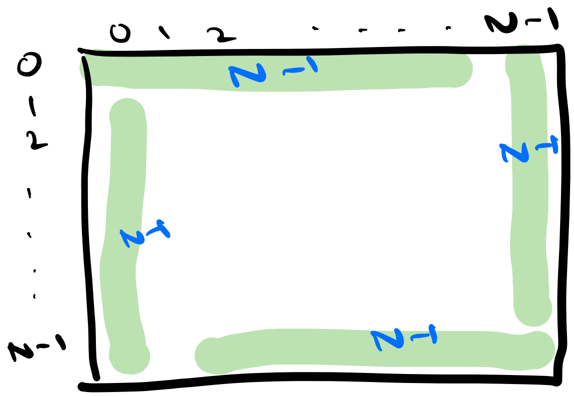
1 2 3 6 9 8 7 4

cnt = 1 ; cnt < 5 <
↳ print —
j++

5×5

cnt	1	2	3	4	5
	0	1	2	3	4
0	1	2	3	4	5
1	6	7	8	9	10
2	11	12	13	14	15
3	16	17	18	19	20
4	21	22	23	24	25

0th row	4 elem
Last col	4
Last row	4
0th col	4



N x N

cnt of ele in each loop = N-1

fn print boundary() {

→ N-1 ele in 0th row

```
i=0  j=0
for (cnt=1 ; cnt < N ; cnt++) {
    print ( arr[i][j])
        j++
}
```

// already i, j → TR cell

N-1 ele in last col

```
for (cnt=1 ; cnt < N ; cnt++) {
    print ( arr[i][j])
        i++
}
```

// print N-1 ele in last row

```
for (cnt=1 ; cnt < N ; cnt++) {
    print ( arr[i][j])
        j--
}
```

```
// print N-1 ele in 0th col
for (cnt =1 ; cnt < N ; cnt ++) {
    print ( arr[i][j])
    i--
}
}
```

TC : O(N)

SC : O(1)

---

10 : 38

4. Given mat[N][N], print elements in spiral order in clockwise direction.

6x6

	0	1	2	3	4	5
0	1	2	3	4	5	6
1	7	8	9	10	11	12
2	13	14	15	16	17	18
3	19	20	21	22	23	24
4	25	26	27	28	29	30
5	31	32	33	34	35	36

	i	j	N
	0	0	6
+1	↓	↓ +1	↓ −2
	1	1	4
+1	↓	↓ +1	↓ −2
	2	2	2

1 2 3 4 5 6 12 18 24 30 36 35
34 33 32 31 25 19 13 7 8 9
10 11 17 23 29 28 27 26 20 14
15 16 22 21

	0	1	2	3	4	5
0	1	2	3	4	5	6
1	7	8	9	10	11	12
2	13	14	15	16	17	18
3	19	20	21	22	23	24
4	25	26	27	28	29	30
5	31	32	33	34	35	36

1, 4 → 24

$$\begin{array}{c|c} & 0 \\ \hline 0 & \boxed{4} \end{array} \rightarrow 4$$

```
fn printboundary() {
    i=0   j=0
    while (N>1 ) {        → N-1 ele in 0th row
        for (cnt=1 ; cnt < N ; cnt++) {
            print ( arr[i][j])
              j++
        }

        // already i, j → TR cell

        for (cnt=1 ; cnt < N ; cnt++) {
            print ( arr[i][j])
              i++
        }

        // print N-1 ele in last row
        for (cnt=1 ; cnt < N ; cnt++) {
            print ( arr[i][j])
              j--
        }

        // print N-1 ele in 0th col
        for (cnt=1 ; cnt < N ; cnt++) {
            print ( arr[i][j])
              i--
        }
        i++   j++   N=N-2
    }
```

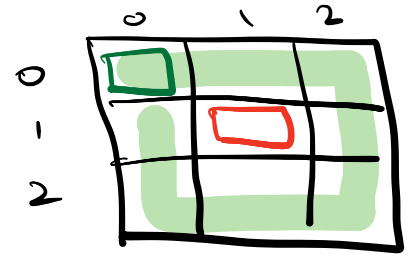
N-1 ele in last col

TC: O(N²)
SC: O(1)

if (N == 1)   print (arr[i][j])

}

i, j	N
0, 0	3
1, 1	1



Submatrix

↓

subarray → continuous part of array

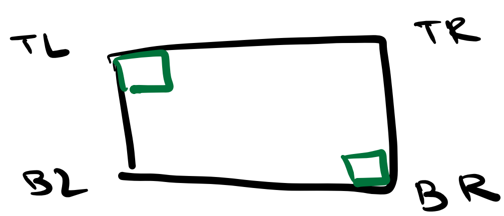
submatrix → continuous part of matrix

	0	1	2	3
0	1	2	3	4
1	5	20	15	17
2	19	18	6	8
3	10	14	21	23

4x4

$$\begin{bmatrix} 15 & 17 \\ 6 & 8 \end{bmatrix}$$

submatrix

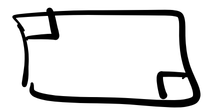


uniquely identify a submatrix → TL and BR

	0	1	2	3
0	1	2	3	4
1	5	20	15	17
2	19	18	6	8
3	10	14	21	23

TL → 1,1

BR → 3,2



5. Given a matrix of N rows, M col.
Determine sum of all possible submatrices.

$$\begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 4 & 9 & 6 \\ 5 & -1 & 2 \end{bmatrix} \end{array}$$

N × M
2 × 3

s e

TL → BR

Fixed TL

BR →

[4] → 4
[9] → 9
[6] → 6
[5] → 5
[-1] → -1
[2] → 2

[4 9] → 13
[9 6] → 15
[5 -1] → 4
[-1 2] → 1

$\begin{bmatrix} 4 \\ 5 \end{bmatrix} \to 9$ $\quad$ $\begin{bmatrix} 9 \\ -1 \end{bmatrix} \to 8$ $\quad$ $\begin{bmatrix} 6 \\ 2 \end{bmatrix} \to 8$

$[4 \quad 9 \quad 6] \rightarrow 19$

$[5 \quad -1 \quad 2] \rightarrow 6$

$\begin{bmatrix} 4 & 9 \\ 5 & -1 \end{bmatrix} \rightarrow 17 \qquad \begin{bmatrix} 9 & 6 \\ -1 & 2 \end{bmatrix} \rightarrow 16$

$\begin{bmatrix} 4 & 9 & 6 \\ 5 & -1 & 2 \end{bmatrix} \rightarrow 15$

Gtotal $= 166$

sum of all subarray sums

$\Downarrow$

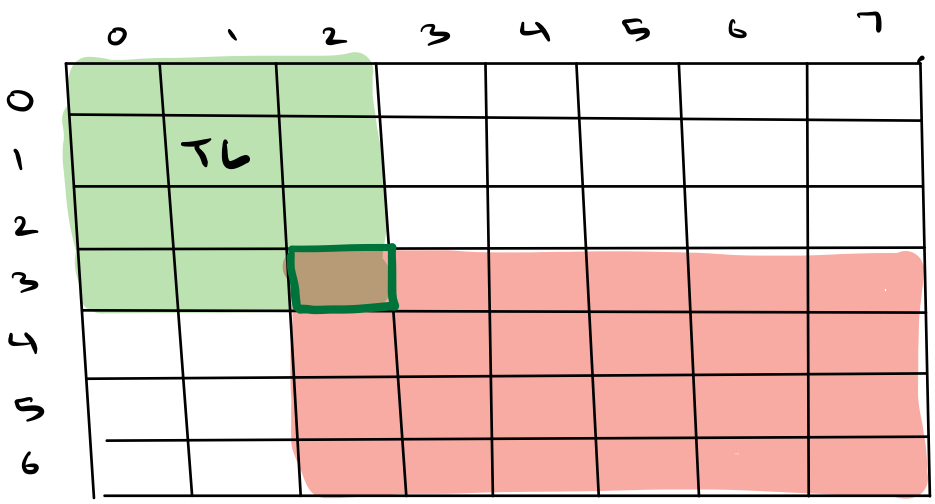
contribution of each Element

mat cell $\rightarrow$ part of how many submatrices

$A[i][j] \rightarrow 20$ submatrices

contribution $= 20 * A[i][j]$

Ele is part of how many submatrices?



TL

BR

Top Left

BR

TL row $0 \to 3$ (4)

col $0 \to 2$ (3)

↓

12 cells

BR row $3 \to 6$ (4)

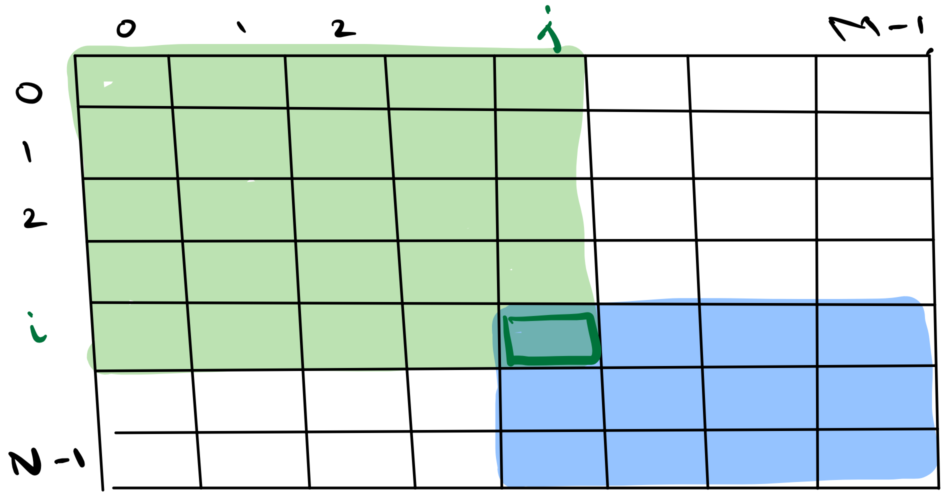
col $2 \to 7$ (6)

↓

24 cells

No. of submatrices in which 3,2 is present $= 12 \times 24$

* Every TL with every BR gives us a unique submatrix



TL

			cnt
TL	row	$0 \to i$	$(i+1)$
	col	$0 \to j$	$(j+1)$

TL cells $\to$ $(i+1)(j+1)$

			cnt
BR	row	$i \to N-1$	$(N-i)$
	col	$j \to M-1$	$(M-j)$

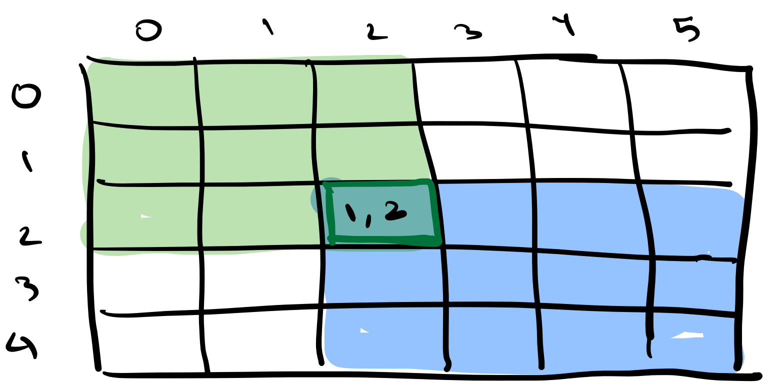
BR cells $\to$ $(N-i)*(M-j)$

count of submatrices in which (i, j) is present = $(i+1)*(j+1)*(N-i)*(M-j)$

contribution of a[i][j] =

a[i][j] × cnt

contri $\boxed{a[i][j] \times (i+1) * (j+1) * (N-i) * (M-j)}$

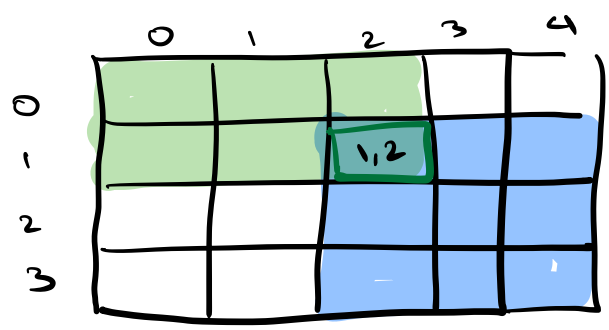


N × M
5 × 6

TL → 3×3
9 cells

BR → (5-2)*
(6-2)
3*4
= 12 cells

cnt = 9×12
= 108 submatrices



N × M
4 × 5

i , j
1 2

TL → (i+1)×(j+1) = 2×3 = 6 cells

BR → (N-i) × (M-j) = 3×3 = 9 cells

(1,2) → 6×9 = 54 submatrices

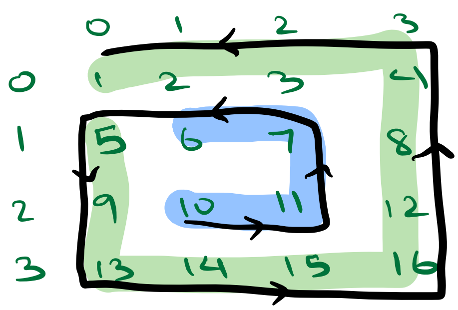
```
total = 0                                         N x M

for (i=0 ; i < N ; i++) {
    for (j= 0 ; j < M ; j++) {
        top left = (i+1) * (j+1)
        bottom right = (N-i) * (M-j)
        contri = A[i][j] * topleft *
                                   bottom right

        total += contri
    }
}
```

TC: O(NM)

SC: O(1)

Doubts

Print in anti clockwise



$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 2 & 3 & 4 \\ 1 & 5 & 6 & 7 & 8 \\ 2 & 9 & 10 & 11 & 12 \\ 3 & 13 & 14 & 15 & 16 \end{array}$$

ele	i	j	N
10	2	~~1~~ 2	2
	-1 ↓	↓ -1	
5	0	1	4

0th col Top → Bottom

Last row L → R

Last col Bottom → Top

0th row R → L

i-- j-- N += 2

ar → [9, 9, 9]

num 999
+1
1000

+ 1
10 00

→ [0, 0, 1, 2, 3]

c=0

sum=4

+ 1
124

123
+1
124

999
+1
1000

_ _ _ _ n digit
+1

1 1
999
+ 1
1000

d
c

9999 4
+ 1
10000

sum=10

d = 10 % 10 = 0

c = 10/10 = 1

789
+ 1
90