

Flip Array

Max sum value

Ways to form Max Heap

- Given an array A of +ve elements, flip sign of some elements such that resultant sum of array should be minimum non negative (0/+ve) (as close to 0 as possible)

Return min. no. of elements whose sign needs to be flipped such that resultant sum is minimum non negative

$$tsum = 20$$

$$A = [6, 2, 3, 7, 2]$$

ans  
2

cnt	-	+	sum
	2	6, 3, 7, 2	16
	2, 6	3, 7, 2	4
3 →	2, 6, 2	3, 7	0
2 →	7, 3	2, 6, 2	0

$$A = [15, 10, 6]$$

ans  
1

cnt	-	+	(0/+ve) sum
	15, 10	6	-19 X
1 →	15	10, 6	1
			0 X

A = [ - - - - ]

tsum = 20

-ve	+ve	sum
sum → 10	sum → 10	0

A = [ - - - - ]

tsum = N

-ve	+ve	sum
N/2	N/2	0

A = [ - - - - ]

tsum = 20

-ve	+ve	sum
sum → 10	sum → 10	0 x
9	11	2
8	12	4
7	13	6

some -ve elements

sum → 10, 9, 8, 7 . . . .

sum of -ve ele as closest to 10 as possible

Pick -ve elements whose sum is as closest to  $\frac{\text{totalsum}}{2}$

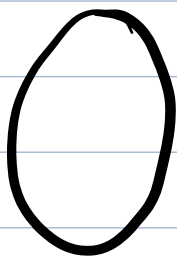
-ve  
↓

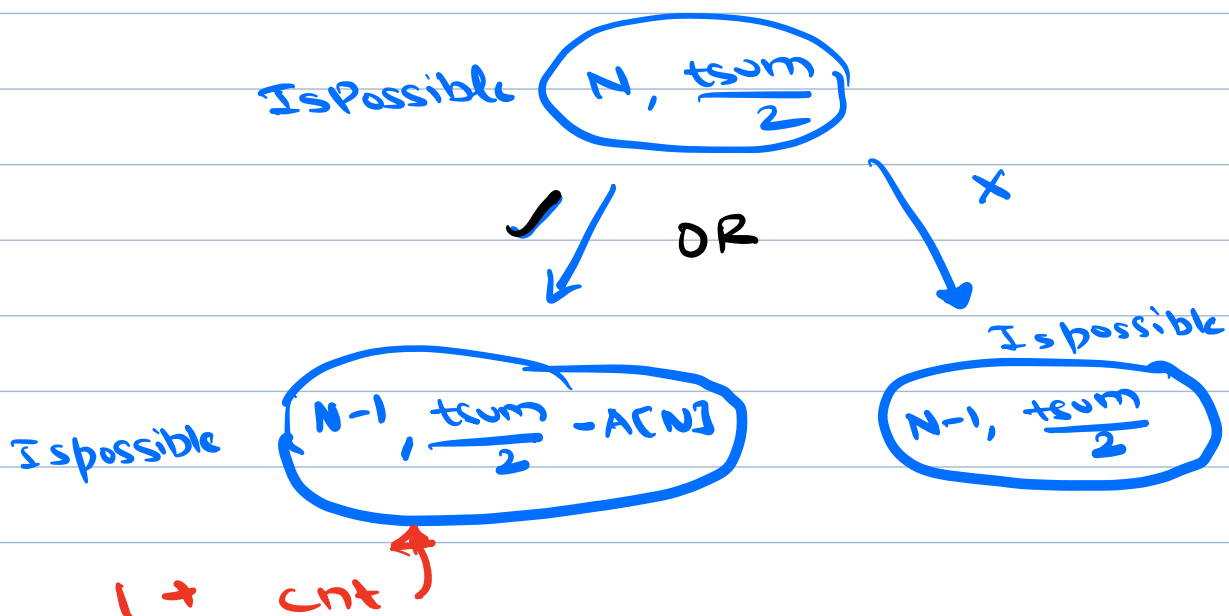
Pick some ele such that  $\text{sum} \leq \frac{\text{totalsum}}{2}$

↓  
min cnt of ele

- ① Constraint  $\rightarrow \text{sum} \leq \frac{\text{ts}}{2}$
- ② Pick a subset
- ③ Min / Max  $\rightarrow \text{cnt of ele}$

$A \rightarrow [1, 2, \dots, N]$  tsum

-ve  




cnt of ele, possible to make sum or not

Is Possible

$(N, \frac{ts}{2})$

ans  
↓  
2

$$A = [4^0 \quad 3^1 \quad 1^2 \quad 4^3]$$

$$N = 4$$

$$\frac{ts}{2} = \frac{12}{2} = 6$$

-ve	+ve	com
6 x	6	0 x
5	7	2
↓		
<4, 17	<3, 4>	

Is Possible

$(4, 6)$

$$dp [N+1] [ts/2 + 1]$$

$$op [4+1] [6 + 1]$$

$$\downarrow$$

$$\downarrow$$

$$A = [4^0 \quad 4^1 \quad 3^2 \quad 1^3]$$

$$\begin{matrix} 3 & 3 \\ \swarrow & \searrow \\ 2 & 0 \end{matrix}$$

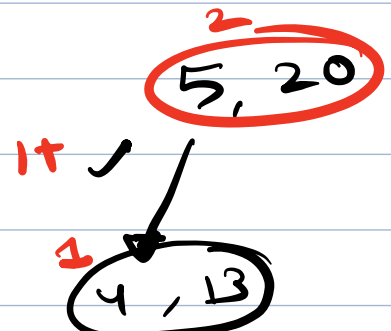
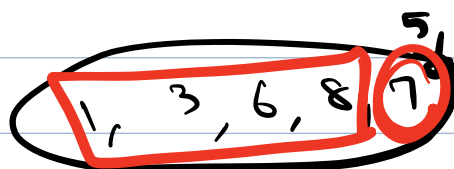
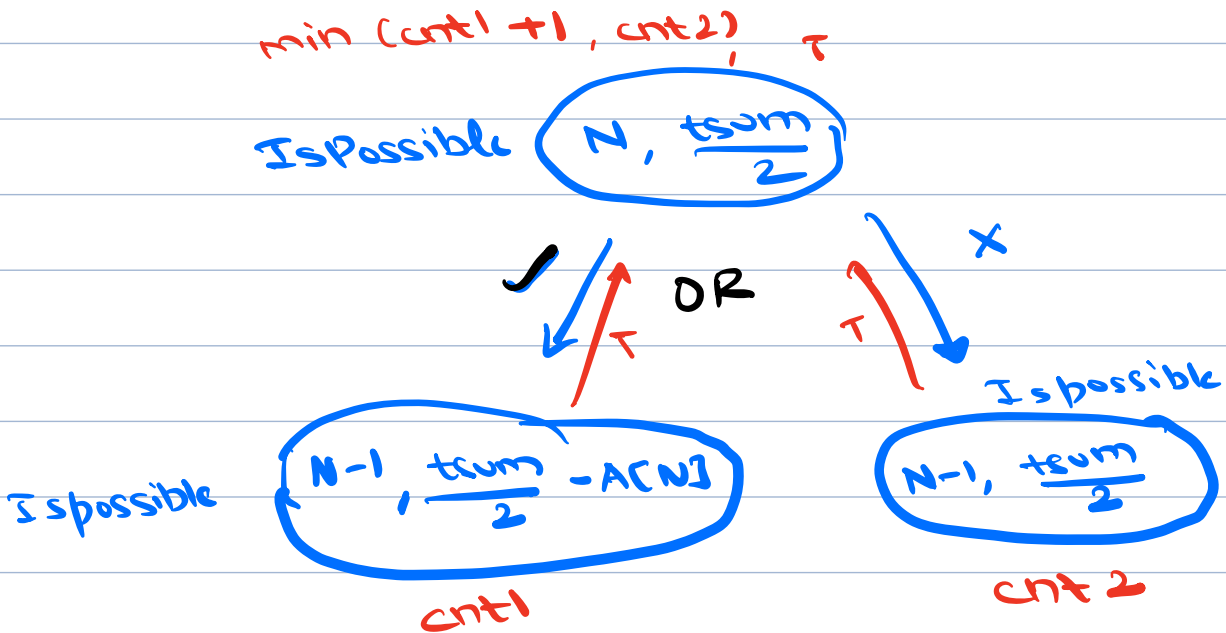
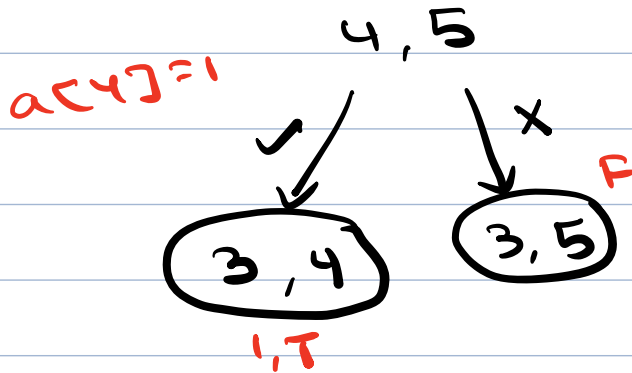
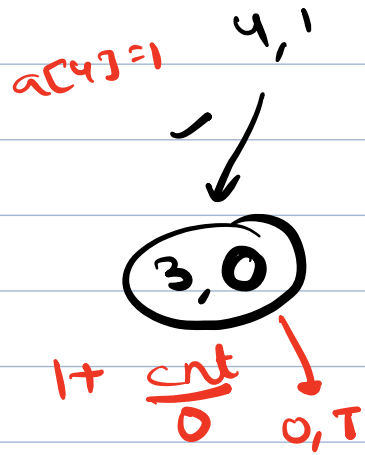
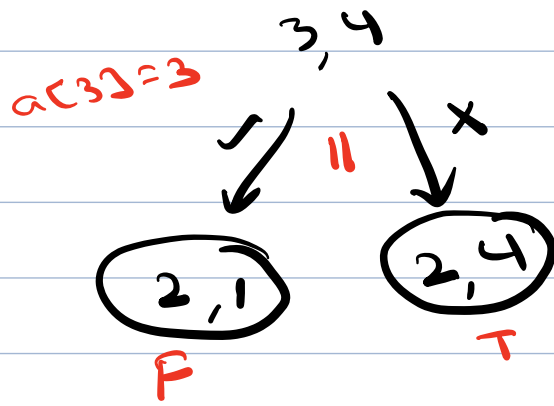
sum →

		0	1	2	3	4	5	6
0		0, T	∞, F	∞, F	∞, F	∞, F	∞, F	∞, F
4	1	0, T	∞, F	∞, F	∞, F	1+0, T	∞, F	∞, F
4	2	0, T	∞, F	∞, F	∞, F	1, T	∞, F	∞, F
3	3	0, T	∞, F	∞, F	1, T	1, T	∞, F	∞, F
1	4	0, T	1, T	∞, F	1, T	1, T	2, T	∞, F

$$\begin{matrix} 1 & 5 \\ \swarrow & \searrow \\ 0 & 1 \end{matrix}$$

$$\begin{matrix} 4 > 2 \\ \swarrow & \searrow \\ 1 & 2 \\ \swarrow & \searrow \\ 0 & 2 \end{matrix}$$

$$\begin{matrix} 1 & 4 \\ \swarrow & \searrow \\ 0 & 0 \\ \swarrow & \searrow \\ 0 & 4 \end{matrix}$$



2. Given an array A of N integers and 3 integers B, C and D.

Maximize value of  $A[i] * B + A[j] * C + A[k] * D$   
where  $1 \leq i \leq j \leq k \leq N$

$A = [1, 5, -3, 4, -2]$        $B = 2$      $C = 1$      $D = -1$

$i = 1$   
 $j = 1$   
 $k = 2$

$$5 \times 2 + 5 \times 1 + (-3) \times (-1) = 18$$

$i, j, k$

$B \rightarrow i$

$C \rightarrow j$

$D \rightarrow k$

for ( $i = 0$  ;  $i < N$  ;  $i++$ ) <

for ( $j = i$  ;  $j < N$  ;  $j++$ ) <

for ( $k = j$  ;  $k < N$  ;  $k++$ ) <

TC:  $O(N^3)$

SC:  $O(1)$

$1 \leq N \leq 10^5$

$N^3$  works?

**NO**

$$C = [-10, 1, 4, -20, 6], B$$

Max

$$A[i] * B$$

$$B = 5$$

$$A[i] = 6$$

$$B = -5$$

$$A[i] = -20$$

$$B > 0$$

$$A[i] \rightarrow \text{max}$$

$$B < 0$$

$$A[i] \rightarrow \text{min}$$

$$C = [-, -, 0, -, -], \text{Max } (A[i] * B + A[j] * C)$$

$\downarrow \quad \quad \downarrow$   
 $A[j] * C \quad A[4] * C$   
 $\downarrow$   
 Max  $A[i] * B$  till ind  $j$

$i \leq j$

$$C = [0, 1, 2, 3, 4]$$

$\downarrow \quad \quad \downarrow$   
 $0-0 \quad \quad 0-1$   
 Max  $A[i] * B \quad \quad \text{Max } A[i] * B$

$$0 - i \rightarrow \text{Max } A[i] * B$$



$$A = \begin{bmatrix} & 0 & 1 & 2 & 3 & 4 \\ 1 & 5 & -3 & 4 & -2 \end{bmatrix} \quad B = 2 \quad C = 1 \quad D = -1$$

1st  $\rightarrow AC[i]$

$$AC[i] * B$$

$$\text{val } [i] = [2, 10, -6, 8, -4]$$

$$\text{premax}[i] = [2, 10, 10, 10, 10]$$

$\downarrow$   
0 - i

2nd  $\rightarrow AC[j]$

$$AC[i] * B + AC[j] * C$$

$$\text{val } [j] = \begin{bmatrix} 3 & 15 & 7 & 14 & 8 \end{bmatrix}$$

$2 + 1 * 1 \quad 10 + 5 * 1 \quad 10 + (-3) \quad 10 + 4 \quad 10 + -2$

$$\text{premax}[j] = [3, 15, 15, 15, 15]$$

$\downarrow$   
0 - i

3rd  $\rightarrow A[k]$

$$AC[i] * B + AC[j] * C + A[k] * D$$

Mark

$$\text{val } [k] \rightarrow \begin{bmatrix} 2 & 10 & 18 & 11 & 17 \end{bmatrix}$$

$3 + -1 \quad 15 + -5 \quad 15 + 3 \quad 15 + -4 \quad 15 + 2$

$$\text{premax}[k] \rightarrow [2, 10, 18, 18, 18]$$

// A[N], B, C, D

val [N] = <0>, premax [N] = <0>

for (cnt = 1 ; cnt ≤ 3 ; cnt++) <

// val

for (idx = 0 ; idx < N ; idx++) <

// contri of <sup>idx</sup>th de

if (cnt == 1)

cnt == 2

cnt == 3

contri = A[idx] \* B

A[idx] \* C

A[idx] \* D

val [idx] = contri + premax [idx]

// premax

return premax [N-1]

TC: O(N)

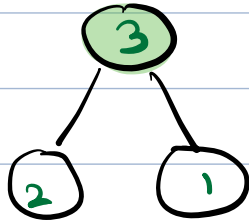
SC: O(N)

10:54

3. Find the no. of distinct Max Heap that can be made from  $A$  distinct integers.

$1 \leq A \leq 100$

$A = 3$

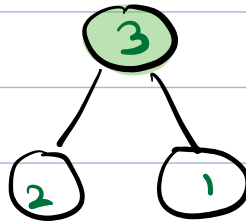
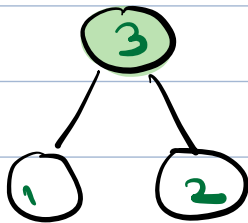


Max Heap

① CBT

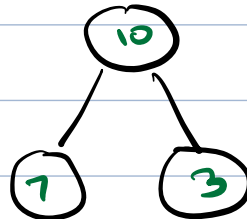
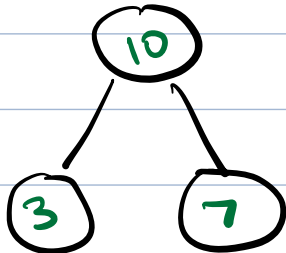
②  $\text{Par} > \text{child}$

1, 2, 3



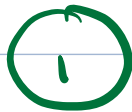
ans  
2

3, 7, 10  
1<sup>st</sup> 2<sup>nd</sup> 3<sup>rd</sup>



ans  
2

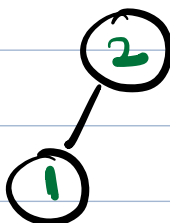
$A = 1$



ans  
1

1

$A = 2$



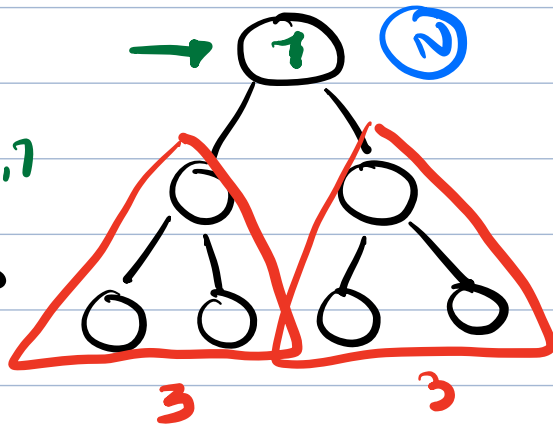
ans  
1

1, 2

A = 7

1, 2, 3, 4, 5, 6, 7

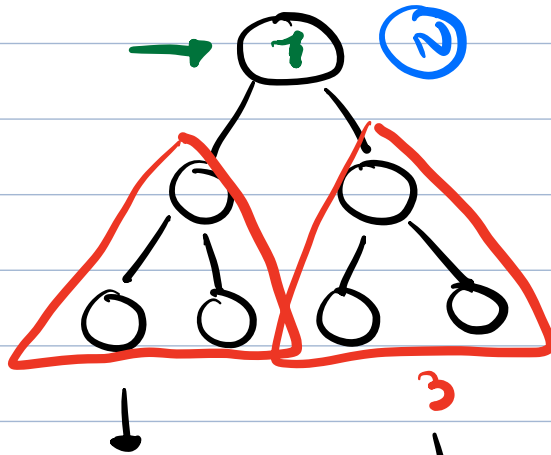
5, 4, 3



6 / N-1  
1, 2, 6

No relationship b/w elements in  
LST and RST

5, 4, 3



1, 2, 6

heaps → LST  
ways (3)

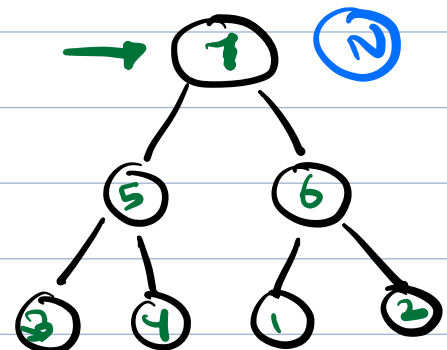
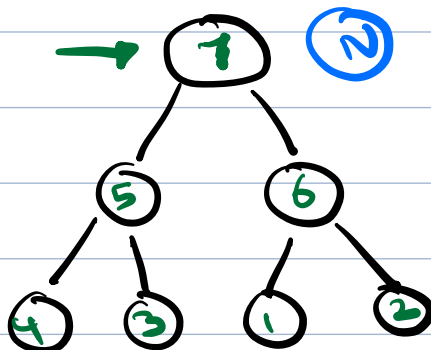
↓  
2

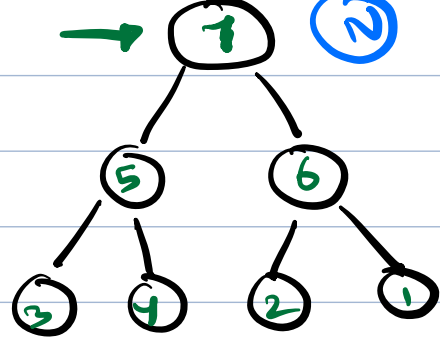
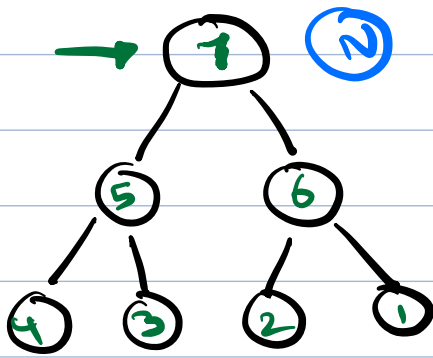
\*

heaps → RS  
ways (3)

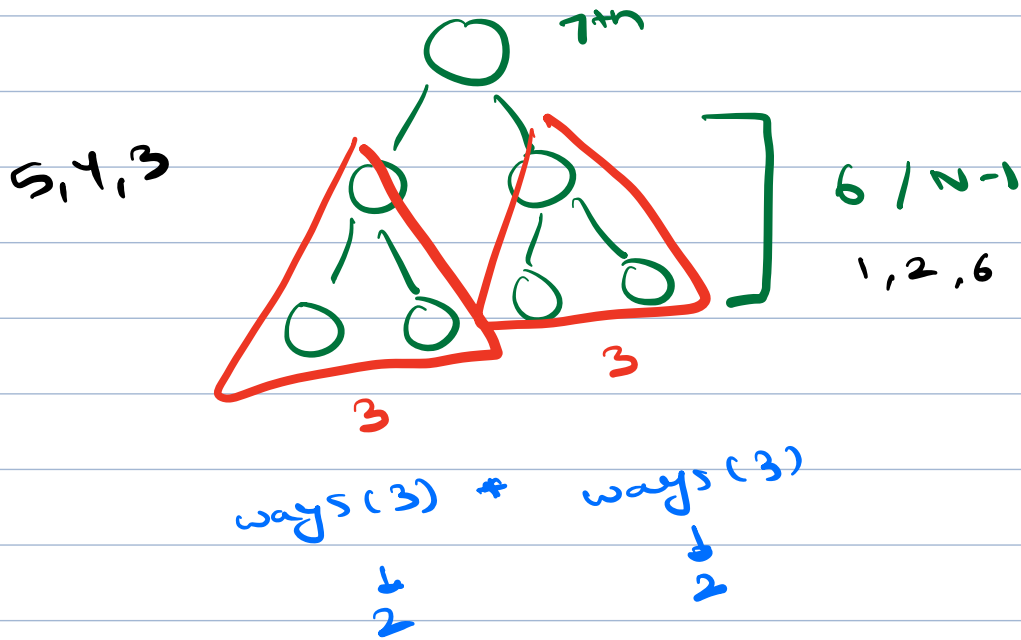
↓  
2

5, 4, 3





$$\text{ways}(A) = {}^{N-1}C_L \times \text{ways}(L) + \text{ways}(R)$$



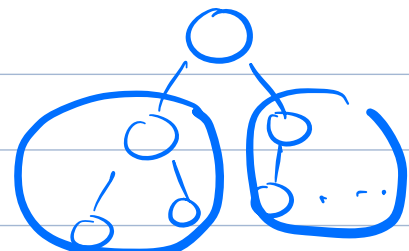
$$A = N$$

$$\text{ways}(A) = {}^{A-1}C_L \times \text{ways}(L) + \text{ways}(R)$$

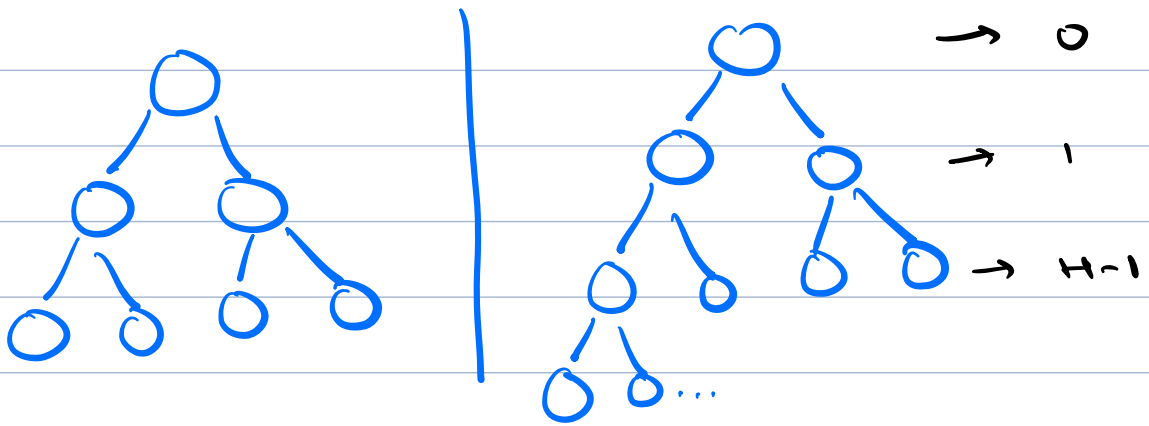
$$L = ? \quad R = ?$$

$$\textcircled{1} \quad A = L + R + 1$$

$$\textcircled{2} \quad L \geq R \quad (\text{bcz of CBT})$$



③  $H = \lfloor \log_2 A \rfloor$  (bcuz of CBT)



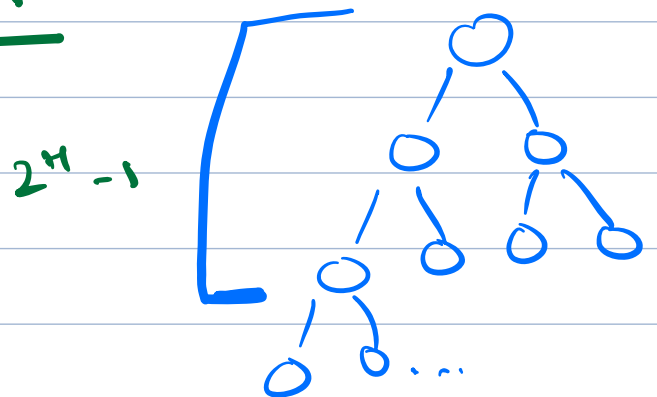
Last level might not be filled

All levels from  $0 \rightarrow n-1$  will be filled.

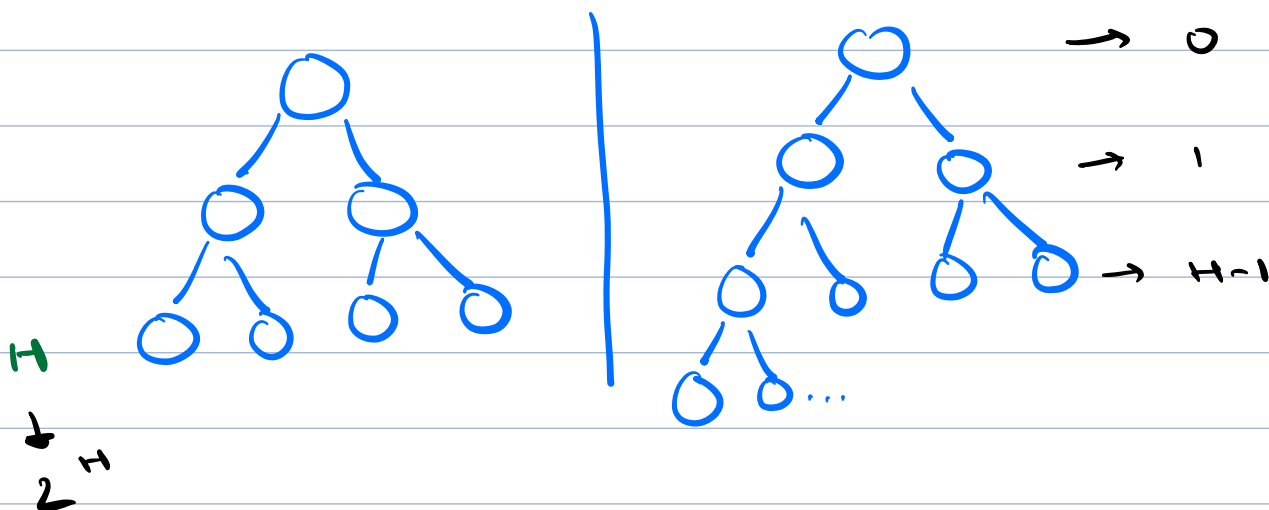
Total nodes from  $0 \rightarrow H-1$  level

$$= 2^0 + 2^1 + 2^2 + \dots + 2^{H-1}$$
$$= 2^H - 1$$

$$\text{LST} \rightarrow 0 \rightarrow \text{H-1} = \frac{2^H - 1 - 1}{2}$$



LST  $\rightarrow$  nodes in last level



$$LST = \frac{2^H}{2} \text{ (Best case)}$$

$$\text{Actual nodes in last level} = A - (2^H - 1)$$

$$LST \rightarrow \text{nodes in last level} = \min\left(\frac{2^H}{2}, A - (2^H - 1)\right)$$

$$\text{Total nodes in LST} = \frac{2^H - 1 - 1}{2} +$$

( $0 \rightarrow H$  level)

$$\min\left(\frac{2^H}{2}, A - (2^H - 1)\right)$$

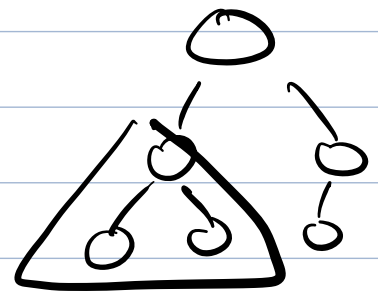
$$A = L + R + 1$$
$$\Rightarrow R = A - L - 1$$

$A \rightarrow$

$H \rightarrow \text{floor}(\log_2 A)$

$L \rightarrow$  using formula

$R \rightarrow A - L - 1$



```
int dp[A+1] = {-1}
```

```
ways(A) <
```

```
if (A == 1 || A == 2)
```

```
    return 1
```

```
if (dp[A] != -1) return dp[A]
```

```
    H = floor(log2 A)
```

$$L = \frac{2^H - 1 - 1}{2} + \min\left(\frac{2^H}{2}, A - (2^H - 1)\right)$$

$$R = A - L - 1$$

$$dp[A] = A-1 \cdot C_L \times \text{ways}(L) \times \text{ways}(R)$$

```
    return dp[A]
```

7

TC :  $O(A)$

SC :  $O(A)$