

Nov23_PSP_28Feb

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Agenda

searching in sorted rotated array

floor (\sqrt{a})

Ath magic number

Median of two sorted array

Basics

Binary Search

search space	Target	conditions
Range in which we are performing our search	Key of search	① For answer ② To eliminate left half ③ To eliminate right half
WHERE	WHAT	HOW

Question

Finding an element in a rotated sorted array

Rotated sorted array

Rotated k times

arr =	<table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr></table>	1	2	3	4	5	6	7	8	①	<table><tr><td>8</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr></table>	8	1	2	3	4	5	6	7
1	2	3	4	5	6	7	8												
8	1	2	3	4	5	6	7												
		②	<table><tr><td>7</td><td>8</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr></table>	7	8	1	2	3	4	5	6								
7	8	1	2	3	4	5	6												
		③	<table><tr><td>6</td><td>7</td><td>8</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr></table>	6	7	8	1	2	3	4	5								
6	7	8	1	2	3	4	5												

Brute force

Do a linear Search using single for loop

$$T.C = O(n) \quad S.C = O(1)$$

Optimized Approach

arr =

1	2	3	4	5	6	7	8
---	---	---	---	---	---	---	---

①

8	1	2	3	4	5	6	7
---	---	---	---	---	---	---	---

②

7	8	1	2	3	4	5	6
---	---	---	---	---	---	---	---

③

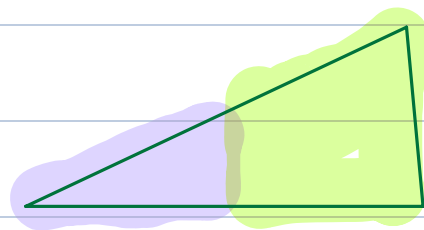
6	7	8	1	2	3	4	5
---	---	---	---	---	---	---	---

Observation 1

In a rotated array we have 2 individual sorted subarray

Observation 2

To identify the section of array we can use $A[0]$ element

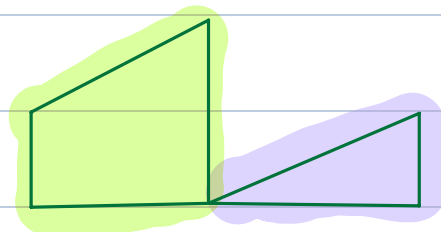


if ($k \geq A[0]$)

// 1st part

else

// 2nd part



Can you find the element if I give the no. of times array is rotated?

6	7	8	1	2	3	4	5
0	1	2	3	4	5	6	7

- ① Identify which part of array the key is located
- ② Apply Binary search on that subarray by modifying start & end index

Give me solution in 1 BS

10	20	30	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9	10	11
part 1			part 2								

Overall Binary search

search space: The entire array

target: Key, value asked by user

conditions:

- ① Identify which part of array is my user input part of

a) condition for ans $a[mid] == k$ return mid

b) eliminate left

Dry Run

0	1	2	3	4	5	6	7	8	9	10	11
10	20	30	1	2	3	4	5	6	7	8	9

Search for 20

part 1

part 2

target in First Half = True

S	E	MID	$a[mid]$	upd on which part	same part	condition
0	11	5	3	2 nd SA	X	$e = m - 1$
0	4	2	30	1 st SA	✓	$e = m - 1$
0	1	0	10	1 st SA	✓	$s = m + 1$
1	1	1	20			

pseudo code

```
int search array (int arr, int t, int n) {
```

```
    int s = 0, e = n - 1;
```

```
    // check if target is in part 1
```

```
    boolean targetInFP = true;
```

```
    if (t < arr[0])
```

```
        targetInFP = false;
```

```
    // perform Binary Search
```

```
    while (s <= e) {
```

```
        mid = s + (e - s) / 2;
```

```
        if (arr[mid] == t) return mid;
```

```
        // check if mid & target are in
```

```
        // same subarray
```

```
        if (targetInFP) {
```

```
            if (arr[mid] >= arr[0]) {
```

```
                // Basic BS
```

```
                if (arr[mid] > t) e = mid - 1;
```

```
                else s = mid + 1;
```

```
            }
```

```
else e = mid - 1;
```

```
}
```

```
else {
```

```
if (arr[mid] >= a[0])
```

```
s = mid + 1;
```

```
else {
```

```
if (arr[mid] > target) {
```

```
e = mid - 1;
```

```
}
```

```
else s = mid + 1;
```

```
}
```

```
}
```

```
}
```

```
}
```

$T.C = O(\log n)$

$S.C = O(1)$

Question

Given an integer n , find $\text{floor}(\text{sqrt}(n))$.

example

Brute force ($n=50$)

n	$\text{floor}(\sqrt{n})$
9	3
12	3
17	4

$$T.C = O(\sqrt{n})$$

i	$i * i$	ans
1	1	1
2	4	2
3	9	3
\vdots	\vdots	\vdots
7	49	7
8	64	return 7

Binary Search

Search space 1 to n

Because my $\text{sqrt}(n)$
value will always be between
1 to n

target : $\text{sqrt}(n)$

condition

for ans :

visualize

if ($\text{mid} * \text{mid} == n$)

only works for perfect square

if ($\text{mid} * \text{mid} \leq N$) \longrightarrow potential ans

$s = \text{mid} + 1$

if ($\text{mid} * \text{mid} > N$) $e = \text{mid} - 1$

Dry Run

$N = 50$

ans = ~~1~~ ~~6~~ 7

s	e	mid	mid * mid	where next?
1	50	25	625	$625 > 50$ L
1	24	12	144	$144 > 50$ L
1	11	6	36	$36 \leq 50$ R
7	11	9	81	$81 > 50$ L
7	8	7	49	$49 \leq 50$ R
8	8	8	64	$64 > 50$ L

8

7

pseudo code

```
int getSqrt (int n) {  
    int s=1; e=n; ans=1;  
    while (s <= e) {  
        mid = s + (e-s) / 2;  
        if (mid * mid <= n)  
            ans = mid; s = mid + 1;  
        else  
            e = mid - 1;  
    }  
    return ans;  
}
```

Break

10:25 pm

$$T.C = O(\log n) \quad S.C = O(1)$$

LCM (Lowest Common Multiple)

$\text{Lcm}(a, b)$ = smallest number divisible by both
a and b

property

$$\text{Lcm}(a, b) * \text{gcd}(a, b) = a * b$$

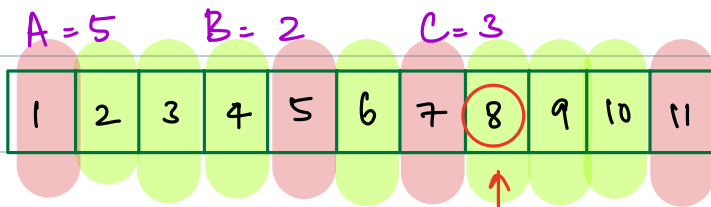
Question

Given A , B and C . Find A th magic number

Magic number is defined as

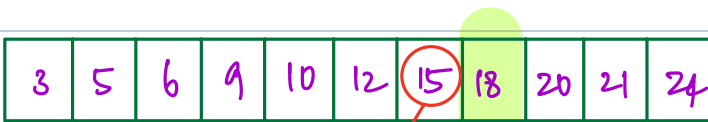
Any number that is divisible by B or C or both

example



5th magic no.

$A=8$, $B=3$, $C=5$, $\text{LCM}(3,5) = 15$



Ans = 18

only counted once

Binary search

Search space :

my search space is between
 $[1, \min(B, C) * A]$

targetting : A^{th} magic number

CALCULATING MAGIC NUMBER

$$A = 8$$

$$B = 2$$

$$C = 5$$

assume

$$\text{mid} = 25$$

multiples of 3 in mid + multiples of 5 in mid - multiples of LCM

$$\text{position} = \frac{25}{3} + \frac{25}{5} - \frac{25}{15}$$

$$= 8 + 5 - 1$$

$$= 12$$

Visualize

$$B = 3 \quad C = 5$$

1	2	3	4	5	6	7	8	9	10	11	12	13
---	---	---	---	---	---	---	---	---	----	----	----	----

$$A = 5$$

$$\text{position} = \frac{10}{3} + \frac{10}{5} - \frac{10}{15}$$

$$= 3 + 2 - 0 = 5$$

$$\text{position} = \frac{11}{3} + \frac{11}{5} - \frac{11}{15}$$

$$= 3 + 2 - 0 = 5$$

many numbers can have the same position

However, we want the smallest among them

Binary search condition

$$s = 1 \quad e = \min(B, C) \neq A$$

$$\text{LCM} = B * C / \text{GCD}(B, C)$$

$$\text{pos} = \frac{\text{mid}}{B} + \frac{\text{mid}}{C} - \frac{\text{mid}}{\text{LCM}}$$

if (pos < A) {

// move right

$$s = \text{mid} + 1$$

}

else {

$$\text{ans} = \text{mid};$$

$$e = \text{mid} - 1;$$

}

$$A = 8, B = 3, C = 5$$

$$\text{LCM} = 15$$

$$\text{ans} = \max(B, C)$$

$$\text{ans} = 18$$

s	e	mid	no. of mul	where to go
1	24	12	$12/3 + 12/5 - 12/15$	6 th move Right
13	24	18	$18/3 + 18/5 - 18/15$	8 th move left
13	17	15	$15/3 + 15/5 - 15/15$	7 th move right
16	17	16	$16/3 + 16/5 - 16/15$	7 th move right
17	17	17	$17/3 + 17/5 - 17/15$	7 th move right
18	17			

$$T.C = O(\lg(\min(B, C) * A))$$

$$S.C = O(1)$$

Question

Given two sorted array, find the median of merged array.

median : Given sorted array, median is middle element

A =

1	3	4	7	10	12
---	---	---	---	----	----

B =

2	3	6	15
---	---	---	----

Merged array

C =

1	2	3	3	4	6	7	10	12	15
---	---	---	---	---	---	---	----	----	----

0 1 2 3 4 5 6 7 8 9

$$\text{median} = \frac{4 + 6}{2} = 5$$

Brute force

Merge two sorted arrays

$O(n+m)$

Find the middle elements

calculate median

$$T.C = O(n+m) \quad S.C = O(n+m)$$

Binary Search

We are calculating median (middle part)

C =

1	2	3	3	4	6	7	10	12	15
0	1	2	3	4	5	6	7	8	9

part 1 part 2

Observations

Both parts have some elements from array A and some elements from Array B

Observation

Elements in part 1 will always be lesser than elements in part 2

TRIAL AND ERROR

A =

1	3	4	7	10	12
---	---	---	---	----	----

B =

2	3	6	15
---	---	---	----

Try 1

let me pick 4 elements from A in part 1

→ 1 element from B in part 1

→ 2 element from A in part 2

→ 3 element from B in part 2

Part 1

1	3	4	7
---	---	---	---

2

Part 2

10	12
----	----

3	6	15
---	---	----

$2 < 10$

However $7 > 3$

↑ Red flag

we should select less elements from
A in part 1

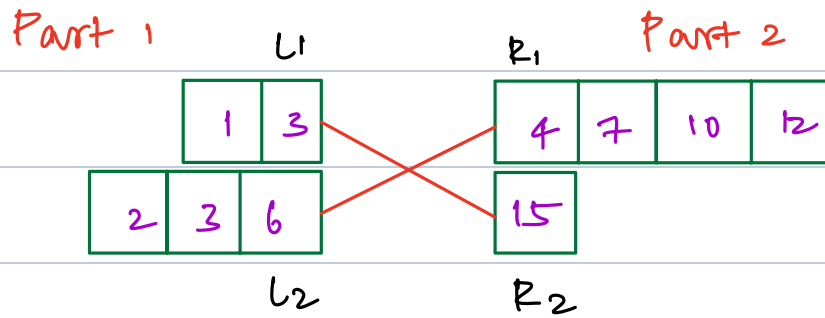
Try 2

let me pick 2 elements from A in part 1

→ 3 element from B in part 1

→ 4 element from A in part 2

→ 1 element from B in part 2



$$L_1 < R_2 (3 < 15) \quad L_2 < R_1 (6 < 4)$$

Select more from Array A

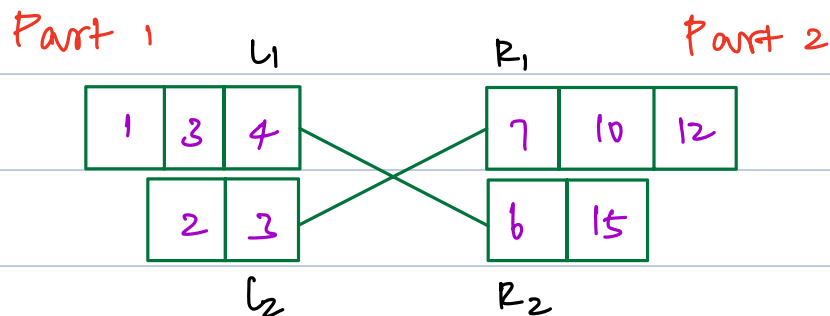
Try 3

let me pick 3 elements from A in part 1

→ 2 element from B in part 1

→ 3 element from A in part 2

→ 2 element from B in part 2



$$4 < 6$$

&&

$$3 < 7$$

$$L_1 < R_2$$

$$L_2 < R_1$$

$$\text{median} = \frac{\max(l_1, l_2) + \min(r_1, r_2)}{2}$$

Binary Search

Search space : Both array A & B

target : median of merged array

conditions :

To be continued in next class