

Next Fri
↓
17 Nov

Pairs with given sum - 2 Ass/HW
Pairs with given difference ↓
Subarray with given sum Notes
Container with most water ↓
Recording

Reattempt
date

Contest 3 Nov
Advance DSA : Contest 2

The more you sweat in peace,
the less you bleed in war.

1. Given a sorted integer array A & an integer k, find any pair (i, j) s.t $A[i] + A[j] = k$ and $i \neq j$

$$A = [-5, -2, 1, 8, 10, 12, 15] \quad k = 11$$

$$\begin{array}{ccccccc} i & & j & & k \\ 2 & & 4 & & 11 \\ A[2] + A[4] = 11 \end{array}$$

pair
with
sum = k

BF : Consider all pairs (i, j)
(upper or lower Δ)

```

for (i=0 ; i<n ; i++) {
    for (j=i+1 ; j<n ; j++) {
        if (A[i] + A[j] == k)
            return i, j
    }
}
  
```

$$A = [-5, -2, 1, 8, 10, 12, 15] \quad k = 11$$

$-2 + \underline{\quad} = 11$

$$\begin{array}{cc} i & j \\ 0 & 1 \rightarrow 6 \\ 1 & 2 \rightarrow 6 \end{array}$$

$$\begin{aligned} A[i] + A[j] &= k \\ \downarrow \\ -5 + A[i] &= 11 \end{aligned}$$

$$\begin{aligned} A[j] &= 11 - (-5) \\ &= 16 \end{aligned}$$

$$\begin{array}{ccccc}
 i & j & & & \\
 0 & 1 \rightarrow 6 & \text{search for } 16 [11 - (-5)] \\
 1 & 2 \rightarrow 6 & \text{search for } 13 []
 \end{array}$$

$A[i] + A[j] = k$
 $-2 + A[j] \parallel$
 $A[j] = 11 - (-2)$

$$\begin{array}{ccccc}
 2 & 3 \rightarrow 6 & \text{search for } 10 \\
 & & A[j] = k - A[i]
 \end{array}$$

$A[i] + A[j] = k$

for ($i=0$; $i < n$; $i++$) <
 // search for $k - A[i]$
 in range $[i+1, n-1]$

$T_C: O(n \log n)$
 $S_C: O(1)$

Approach 3: 2 pointers

Directly
eliminate
some pairs

$$A = [-5, -2, 1, 8, 10, 12, 15] \quad k = 11$$

$i \quad j$

$$\begin{array}{ll}
 A[i] + A[j] \\
 A[0] + A[1] \\
 -5 + -2
 \end{array}$$

$$-7 < 11$$

$$x+y < k$$

$$x \boxed{i++ \text{ or } j++}$$

$$A = [-5, -2, 1, 8, 10, 12, 15] \quad k=11$$

$i \downarrow \uparrow j$

$\downarrow A[i]$ or $\downarrow A[j]$

$i--$ or $j--$

\times

$$12 + 15 = 27 > 11$$

$$i+j > k$$

$$\downarrow i \text{ or } \downarrow j$$

$$A = [-5, -2, 1, 8, 10, 12, 15] \quad k=11$$

$i \downarrow \uparrow j$

$$i+j < k$$

$A[i]$	$A[j]$	sum	$i++$	$j--$	$\uparrow i \text{ or } \uparrow j$
-5	15	10 < 11	$i++$		
-2	15	13 > 11		$j--$	
-2	12	10 < 11	$i++$		$-5 + \text{biggest } dc < k$
1	12	13 > 11		$j--$	$-5 + \text{any } dc \quad k$
1	10	11 = k	\checkmark		

$$> i+j > k$$

$$\downarrow i \text{ or } \downarrow j$$

$$i=0, j=N-1$$

while ($i < j$) {

 if ($A[i] + A[j] == k$)
 return (i, j)

 else if ($A[i] + A[j] < k$)
 $i++$

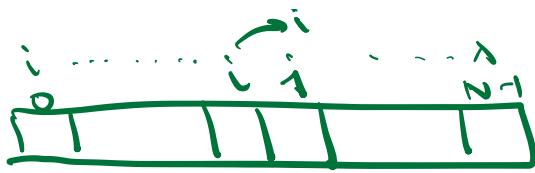
 else
 $j--$

$$\begin{array}{l} \text{cur } \\ \text{smallest } \\ \text{dc} \end{array} + 15 > 11$$

$$+ 15$$

$T_C: O(N)$

$S_C: O(1)$



Find all pairs in distinct array.

1 2 3 4 5 6 8 $k = 10$

i j $4, 6$
 j $2, 8$

$$cnt = \phi \times 2$$

if ($A[i] + A[j] == k$) <
 | $i++$
 | $j--$
 | $cnt++$

$i = 0, j = n - 1$ $cnt = 0$

while ($i < j$) <

| if ($A[i] + A[j] == k$) <
 | | $cnt++$ $i++$ $j--$
 | else if ($A[i] + A[j] < k$)
 | | $i++$
 | else
 | | $j--$

→ SC: O(1)

For duplicates TC: O(N) SC: O(N)

① Freq array

1	3	3	<u>10</u>	10	10	15	K = 13
							$3 + 10 = 13$

\downarrow

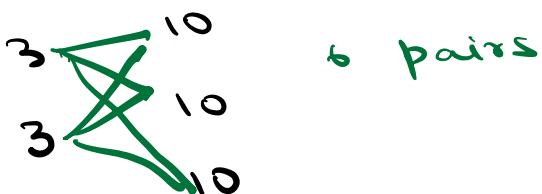
Ax → 1 3 10 15

freq 1 2 3 1

elem: freq 6 pairs

1:1
3:2
10:3
15:1

No. of pairs
 $= \text{freq}[ACi] * \text{freq}[ACj]$



Create a distinct array and store freq of every element. Whenever we find a pair where sum = k , $\text{cnt} += \text{freq}[ACi] * \text{freq}[ACj]$

② without freq array

0	1	2	3	4	5	6	7	8	9	10	10	10	K = 14
2	1	1	4	5	5	7	10	10	10	15			10

i++ j--

copy copy copy

$\text{cnt} = \phi$

count i m de

```

int iCopy = i
while (A[iCopy] == A[i]) <
    | iCopy ++
    |
ele = iCopy - i

```

$\text{cnt} = 0 \quad i = 0 \quad j = n-1$

while ($i < j$) <

```

if (A[i] + A[j] == k) <
    if (A[i] == A[j]) <
        freq = j - i + 1
        cnt += (freq * (freq - 1)) / 2
        break
    |

```

$iCopy = i \quad jCopy = j$

while (A[iCopy] == A[i]) <

| iCopy ++

$iEle = iCopy - i$

while (A[jCopy] == A[j]) <

| jCopy --

$jEle = j - jCopy$

count += iEle * jEle

```
i = icop +  
j = jcop &  
  
else if (AC[i] + AC[j] < k)  
    i++  
  
else  
    j--
```

$T_C = O(N)$ $SC = O(1)$

$$+ C_2$$



$K = 2$

1 1 1 1
↓ ↓ ↓ ↓
3 2 1 0

2. Given a sorted integer array A & an integer k, find any pair (i, j) s.t $A[j] - A[i] = k$ and $i \neq j$

$$k > 0$$

$$A = [-5, -2, 1, 8, 10, 12, 15] \quad k = 11$$

$$\begin{array}{ccccccc} & i & & j & & & \\ & 2 & & 5 & & & \\ \text{idx} & 1 & 12 & 12-1=11 & & & \\ & & & & A[j] - A[i] = k > 0 & & \\ & & & & A[j] - A[i] > 0 & & \end{array}$$

pair with
diff = k

BF : check all idx pairs

$$\begin{array}{lll} i \rightarrow 0 \text{ to } N-1 \\ j \rightarrow i+1 \text{ to } N-1 \end{array}$$

$$A[j] > A[i]$$

j is always
on right of i

$$TC: O(N^2)$$

$$SC: O(1)$$

$$\begin{array}{ccccccc} & i & & & & & \\ & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ A = [-5, -2, 1, 8, 10, 12, 15] & \downarrow & & & & & \\ & & & & & & \\ & i & & j & & & \\ & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{Search for } 6 & & & & & & & \\ & & & & & & & \\ & 1 & 2 & 3 & 4 & 5 & 6 & \\ \text{Search for } 9 & & & & & & & \end{array}$$

$$A[j] - A[i] = k$$

$$\begin{aligned} A[j] - (-5) &= 11 \\ A[j] &= 11 - 5 \\ &= 6 \\ A[j] &= k + A[i] \end{aligned}$$

TC: $O(N \log N)$ SC: $O(1)$

Approach 3: 2 pointers

$$A = [-5, -2, 1, 8, 10, 12, 15] \quad k = 11$$

$i \downarrow \quad j \uparrow$

$$A[j] - A[i]$$
$$15 - (-5) = 20 > 11 \quad j - i > k$$

$\downarrow j \text{ or } \uparrow i$

$\times \quad j-- \text{ or } i++$

$$A = [-5, -2, 1, 8, 10, 12, 15] \quad k = 11$$

$i \downarrow \quad j \uparrow$

$$15 - 12 = 3 < 11 \quad j - i < k$$

largest - $A[i] < k$

etc

any $d - A[i] < k$

$\uparrow j \text{ or } \downarrow i$

$\downarrow i--$

$$A[j] - A[i] = k$$

\downarrow

$$\begin{array}{c} \uparrow \quad \downarrow \\ A[j] - A[i] < k \end{array}$$

$\uparrow \quad \downarrow$

$\uparrow \quad \downarrow$

$$A = [-5, -2, 1, 8, 10, 12, 15] \quad k=11$$

i *j*

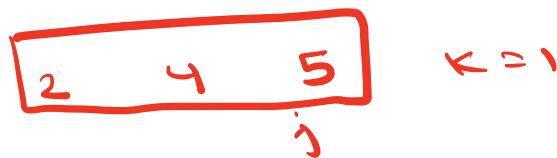
$A[i]$	$A[j]$	Diff	
-2	-5	$-2 - (-5) = 3 < 11$	$j++$
1	-5	$1 - (-5) = 6 < 11$	$j++$
8	-5	$8 - (-5) = 13 > 11$	$j++$
8	-2	$8 - (-2) = 10 < 11$	$j++$
10	-2	$10 - (-2) = 12 > 11$	$j++$
10	1	$10 - 1 = 9 < 11$	$j++$
12	1	$12 - 1 = 11 \checkmark$	

```

i=0  j=1           -2 - smallest < k
while ( j < N ) <         else
    diff = A[j] - A[i]      -2 - 1
    if (diff == k)
        return (i,j)
    else if (diff < k)
        j++
    else
        i++
    }

```

$T C: O(N)$
 $S C: O(1)$



i

10: 45
break

3. Given an integer array of two elements and an integer K . Check if there exists a subarray with sum = K .

$$A = [1, 3, 15, 10, \underbrace{20, 3}_{\text{sum } 23}, 23] \quad K = 33$$

\downarrow
 $N=7 \quad S \quad E$
 $\frac{7+8}{2} = 28$ $3 \quad 5$

$K = 43$
False

BF : Check all subarray sums

$\rightarrow \text{cnt} \rightarrow \frac{N(N+1)}{2}$

using carry forward

$TC: O(N^2) \quad SC: O(1)$

$$A = [1, 3, 15, 10, 20, 3, 23] \quad K = 33$$

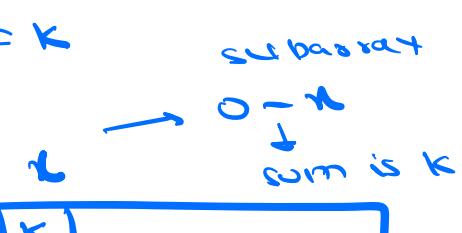
$$pf = []$$

Prefix sum

$$\text{sum } [i \rightarrow j] = pf[j] - pf[i-1] \quad i \neq 0$$

$$\text{sum } [0 \rightarrow j] = pf[j] = K \quad i = 0$$

$$\text{sum of subarray } (i \rightarrow j) = K$$



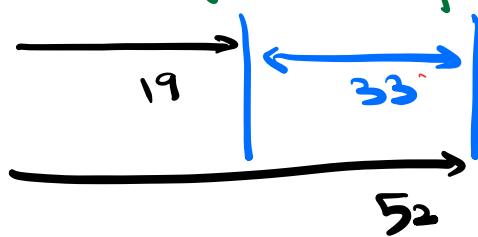
$$\text{sum}[i \rightarrow j] = \text{pf}[j] - \text{pf}[i-1] = k$$

① $\forall i \rightarrow \text{check } \text{pf}[i] = k$ if it is k
 subarray $(0 \rightarrow i) \rightarrow \text{sum} = k$

② $\text{sum}[i \rightarrow j] = \text{pf}[j] - \text{pf}[i-1] = k$
 Find a pair in $\text{pf}[E]$ whose $\text{diff} = k$

$$A = [1, 3, 15, \textcircled{10}, 20, \textcircled{3}, 23] \quad k = 33$$

$$\text{pf} = [1, 4, 19, 29, 49, \underline{52}, 75]$$



Prefix array

TC: $O(N)$

SC: $O(N)$

\downarrow
 $O(1)$
 modify
 same arr
 to store $\text{pf}[C]$

Approach 3 : Dynamic Sliding window Approach

$$A = [1, 3, 15, 10, 20, 3, 23] \quad k = 33$$

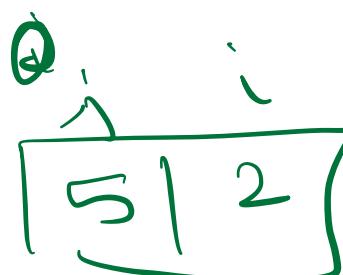
 i j

$i - j$	sum = $\Sigma A[i:j]$
0 - 0	1
0 - 1	4 < 33
0 - 2	19 < 33
0 - 3	29 < 33
0 - 4	49 > 33
1 - 1	j++
1 - 2	j++
1 - 3	j++
i - 4	i++

1 - 4	48 > 33	i++	$i \rightarrow \text{start}$
2 - 4	45 > 33	i++	$j \rightarrow \text{end}$
3 - 4	30 < 33	j++	
3 - 5	33 = 33	j	

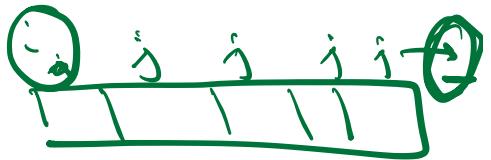
$k = 2$

$$\text{sum} = 5 + 2$$



$i=0 \quad j=0 \quad sum = A[0]$

while ($j < N$) <

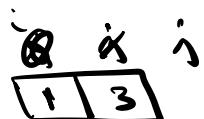


if ($sum == k$)
return true

else if ($sum < k$) <

$j++$
if ($j == n$) break
 $sum += A[i]$

$k=40$

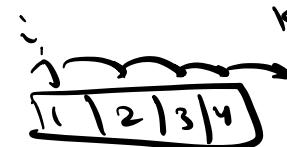


else if ($sum > k$) <

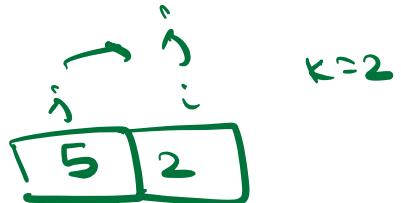
$sum = A[i]$

$i++$
if ($i > j$ or $i \leq n-1$) <
 $j++$
 $sum += A[j]$

$k=40$



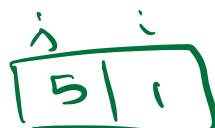
7



$TC: O(N)$

$SC: O(1)$

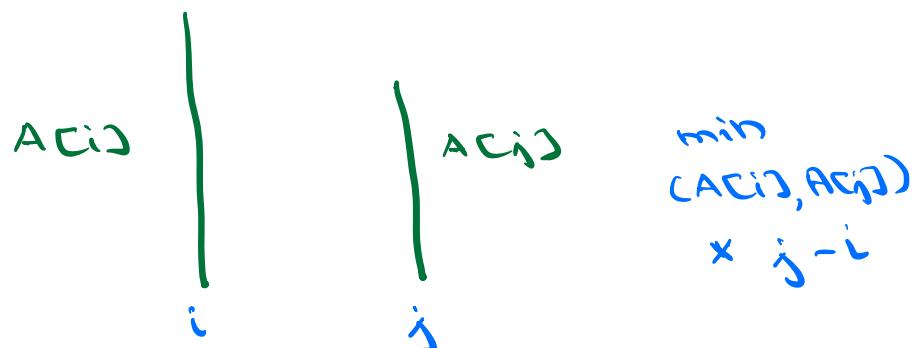
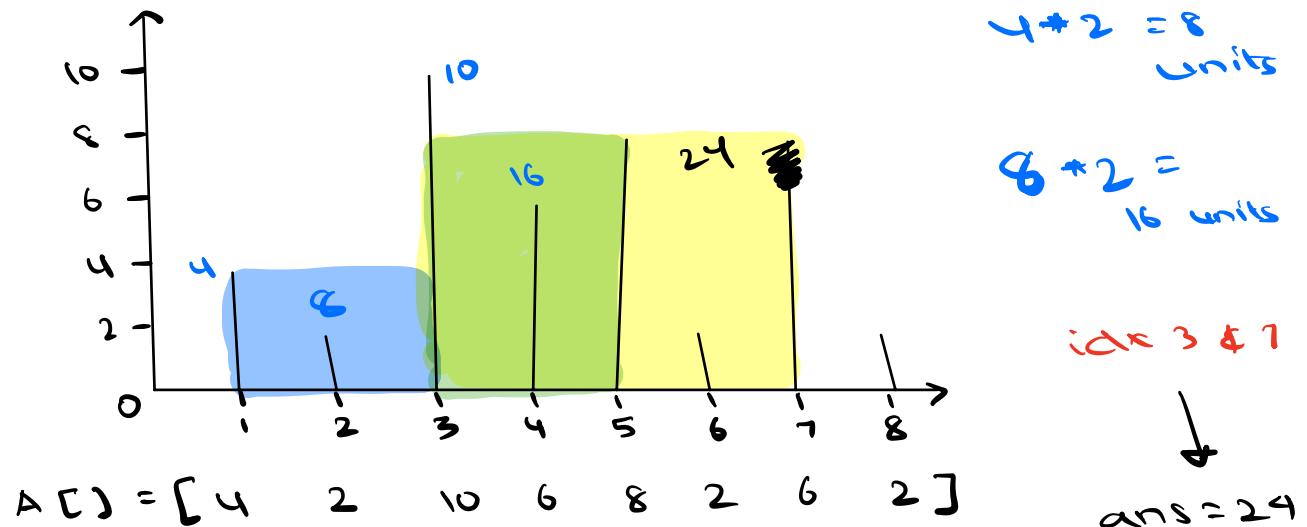
$sum = \cancel{8} \cancel{9} 2$



$k=2$

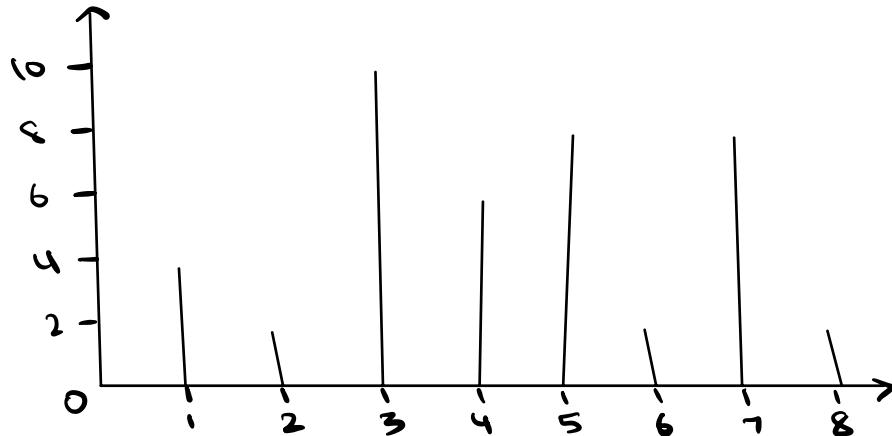
$sum = \cancel{5} 0$

4. Return max water that can be stored b/w any 2 walls



Water stored b/w $AC[i]$ & $AC[j]$
 $= \min(AC[i], AC[j]) * (j - i)$

5. Return max water that can be stored b/w any 2 walls

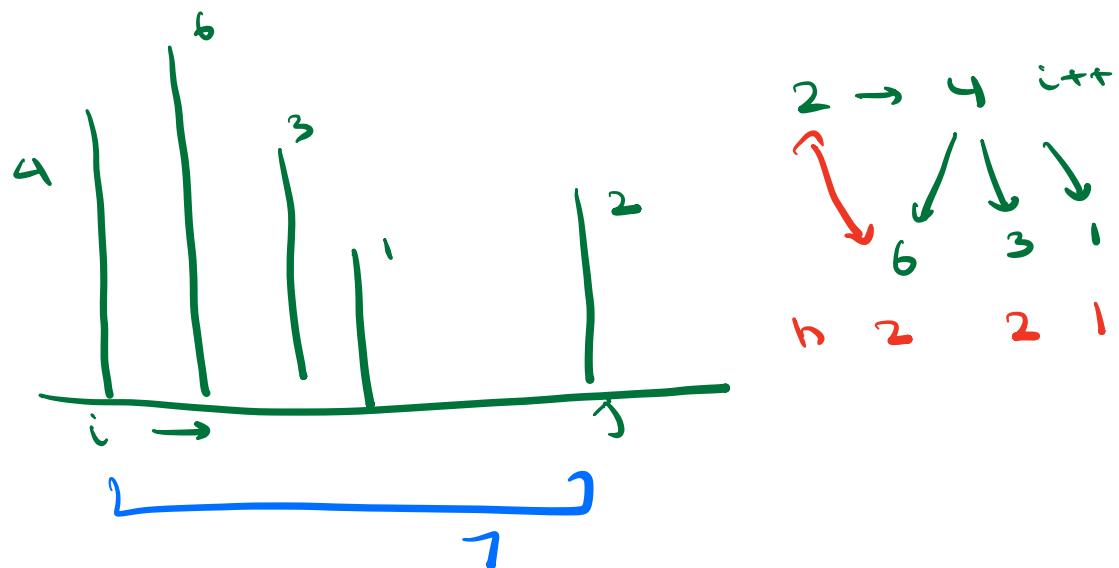


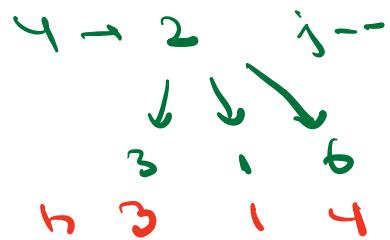
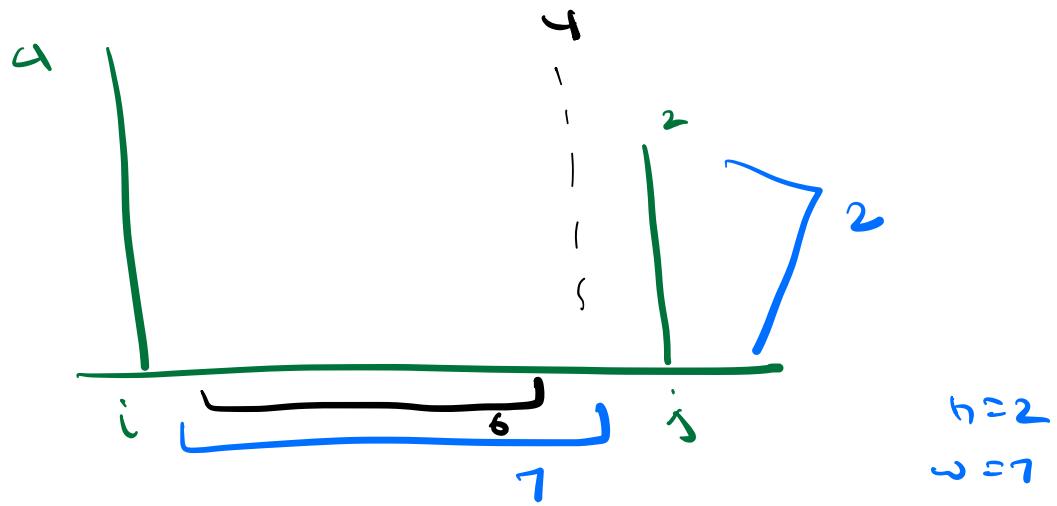
$$\min(4, 2) = 2$$

$A[] = [4 \quad 2 \quad 10 \quad 6 \quad 8 \quad 2 \quad 6 \quad 2]$

$$\text{water} = \\ 2 + 7 \\ = 14 \\ \text{units}$$

$[4 \quad 2 \quad 10 \quad 6 \quad 8 \quad 2 \quad 6 \quad 2]$





Always move smaller wall

$\left[\begin{matrix} 0 & 1 & 2 & 6 & 3 & 4 & 5 & 6 & 7 \\ 5 & 2 & 10 & 6 & 8 & 2 & 1 & 6 & 2 \end{matrix} \right]$

	i	j	water	Max = 0
$j--$	0	7	14	14
$i++$	0	6	24	24
$i++$	1	6	10	24
$j--$	2	6	24	24
$j--$	2	5	6	24
$j--$	2	4	16	24

$j-- \quad 2 \quad 3 \quad 6$

24

$i = 0 \quad j = n-1 \quad ans = 0$
while ($i < j$) <

area = min (A[i], A[j]) * (j - i)

ans = max (ans, area)

if (A[i] < A[j])
 $i++$

else

$j--$

>

TC : O(N)

SC : O(1)

2 ptr \rightarrow pairs

\hookrightarrow starting
pt of
 i and j

ele pair
with
 $sum = k$
 \downarrow
sorted
arr

eliminate pairs and
move in a specific
direction