# Estimation of Stress-Strength Reliability of Xgamma Exponential Distribution with Generalized Progressive Hybrid Censoring

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**Abstract**

This article focuses on the estimation of the stress-strength reliability parameter $R = P(Y < X)$, where both the stress $Y$ and strength $X$ are assumed to follow a newly proposed extension of the one-parameter Xgamma distribution. The model's structural and distributional properties are studied, and inference is carried out under generalized progressive hybrid censoring, which provides a unified framework encompassing both time and failure-based censoring mechanisms. Parameter estimation is performed under both classical and Bayesian approaches. In the classical setup, only the maximum likelihood estimation (MLE) method is considered. To quantify uncertainty, asymptotic confidence intervals for $R$ are constructed using both the Fisher information and nonparametric bootstrap methods. In the Bayesian framework, the estimation is conducted using a gamma prior under the squared error loss function. The Bayes estimates are computed via Lindley's approximation and Markov Chain Monte Carlo (MCMC) simulation. Credible intervals for the stress-strength reliability parameter are also obtained, and their coverage probabilities are evaluated. A Monte Carlo simulation study is carried out to compare the performance of the MLE and Bayesian estimates under various censoring scenarios in terms of bias, mean squared error, and coverage probability. The effectiveness and flexibility of the proposed model and estimation methods are demonstrated through two real data examples.

**Keywords:** Stress-strength reliability; Xgamma distribution; Generalized progressive hybrid censoring; Bayesian estimation; Markov Chain Monte Carlo; Bootstrap confidence interval.

## 1 Introduction and genesis

The CDF of the Xgamma-E (Xg-E) model is given by

$$F_{\text{Xg-E}}(x;\lambda) = 1 - \frac{1}{2}\mathrm{e}^{-\lambda\,x}\left[2 + \lambda\,x + \frac{1}{2}\left(\lambda\,x\right)^2\right]\ |_{\ (x>0,\lambda>0)}, \tag{1}$$

The PDF corresponding to the above CDF, given by

$$f_{\text{Xg-E}}(x;\lambda) = \frac{1}{2}\lambda\mathrm{e}^{-\lambda\,x}\left[1 + \frac{1}{2}\left(\lambda\,x\right)^2\right]. \tag{2}$$

In recent years, the estimation of stress-strength reliability $R = P(Y < X)$, which quantifies the probability that a system's strength exceeds the applied stress, has garnered significant attention in reliability and life-testing studies. While various classical and Bayesian approaches have been employed for this purpose under different lifetime distributions, most existing works focus on standard censoring schemes such as Type-I, Type-II, or progressive censoring. However, in practical applications, censoring often arises in more complex forms, and generalized progressive hybrid censoring has emerged as a more realistic and flexible alternative by accommodating both time and failure constraints. Additionally, although the Xgamma distribution exhibits flexible hazard shapes and has potential applicability in reliability modeling, its role in stress-strength analysis under complex censoring schemes remains largely unexplored. To address these gaps, this article proposes a new extension of the one-parameter Xgamma distribution and investigates the estimation of the stress-strength reliability parameter under generalized progressive hybrid censored data. The unknown parameters are estimated using the classical maximum likelihood method and Bayesian approaches with a gamma prior under the squared error loss function. The Bayesian estimates are obtained via Lindley's approximation and Markov Chain Monte Carlo (MCMC) simulation. Confidence intervals are constructed using asymptotic theory and nonparametric bootstrap techniques, while Bayesian credible intervals are also developed and evaluated. A comparative Monte Carlo simulation study is conducted to assess the performance of classical and Bayesian estimators in terms of bias, mean squared error, and coverage probability. Furthermore, a modified goodness-of-fit test based on the Nikulin–Rao–Robson (NRR) statistic is introduced to evaluate the fit of the proposed model under both complete and censored data. The proposed methods are illustrated through two real-life data sets involving complete and right-censored observations.

The remainder of this article is organized as follows: Section 2 introduces the proposed distribution and its properties; Section 3 describes the generalized progressive hybrid censoring scheme; Section 4 presents classical estimation and interval procedures; Section 5 discusses Bayesian estimation techniques; Section 6 reports the results of simulation studies; Section 7 presents the goodness-of-fit test; Section 8 applies the methodology to real data; and Section 9 concludes with key findings and suggestions for future work.

## 1.1 Stress-strength reliability

In mechanical engineering, stress-strength reliability is very frequently used to measure the performances of the equipment in use. Let $X$ and $Y$ denote the strength-stress RVs observed from the population Xg-E($\lambda_1$) and Xg-E($\lambda_2$), respectively. Then the probability $P[Y < X]$ is called as stress-strength reliability parameter. It is denoted by $R$. The same is evaluated for the proposed model and is given by;

$$\begin{aligned} R = \Pr[Y < X] &= \int_{x=0}^{\infty} \int_{y=0}^{x} f(x, \lambda_1)\, f(y, \lambda_2) dx\, dy \\ &= \int_{x=0}^{\infty} f(x, \lambda_1)\, F(x, \lambda_2) dx, \end{aligned} \tag{3}$$

using the PDF and CDF of the proposed model, $R$ is calculated as;

$$
\begin{aligned}
R &= \frac{1}{2}\lambda_1 \int_{x=0}^{\infty} \left\{ \, \mathrm{e}^{-\lambda_1 x} \left[1 + \tfrac{1}{2} (\lambda_1 x)^2\right] \times \left[1 - \tfrac{1}{2}\mathrm{e}^{-\lambda_2 x} \left( \begin{array}{c} 2 + \lambda_2 x \\ +\tfrac{1}{2} (\lambda_2 x)^2 \end{array} \right)\right] \, \right\} dx \\
&= 1 - \frac{\lambda_1}{2(\lambda_1 + \lambda_2)} \left[ \, 1 + \tfrac{\lambda_2}{2(\lambda_1+\lambda_2)} + \tfrac{2\lambda_1^2+\lambda_2^2}{2(\lambda_1+\lambda_2)^2} + \tfrac{3\lambda_1^2\lambda_2}{2(\lambda_1+\lambda_2)^3} + \tfrac{3\lambda_1^2\lambda_2^2}{(\lambda_1+\lambda_2)^4} \, \right].
\end{aligned}
\tag{4}
$$

# 2 Method of maximum likelihood estimation

# 3 Bootstrap confidence interval

# 4 Bayesian estimation

# 5 Monte Carlo simulation

# 6 Real data illustration

# 7 Concluding remark

# References


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