Topics of Computation.

ch-2. Languages (1)

finite set of unit out of which we build structures are called Alphabets.

Set of strings from the alphabets will be called Language.

Those strings that are permissible in the language are called words

eg. A = { a b } → alphabet in the set

L(A) = { a, b, aa, ab, bb, ba, aaa, ... } → lang is collection of words/strings

↓

Language (collection of strings with any combination of given alphabets)

words

empty string / Null string :- A string that have no letters / alphabets

Denoted by the symbol Λ.

Languages:

eg:- $\Sigma = \{ x \}$

then $L_1 = \{ x \quad xx \quad xxx \quad xxxx, \ldots \}$ [all words]

$L_2 = \{ x, \quad xxx \quad xxxxx \ldots \}$ [L_2 contain odd no. of x in length].

$= \{ x^{odd} \}$

functions

①. Length :- Length of a string is the no. of letters in the string

eg:- if a = xxxx. length(a) = 4

c = 428 length(c) = 3

also, we can write length(aaa) = 3.

also length (Λ) = 0

② Reverse

eg:- $a = xxx$

$\Rightarrow$ reverse $(a) = xxx$.

$\Rightarrow$ reverse $(xxxxx) = xxxxx$.

The reversed output are also the words in the language but the ~~sa~~ function does not hold true in situations like :-

$L = \{$ finite string of numbers having length more than one & does not start with a '0' $\}$.

eg:- reverse (145) = 541 (holds true)

reverse (140) = 041 (does not hold true).

eg. Defⁿ a language 'Palindrome' over the alphabet $\Sigma = \{a \ b\}$

Solⁿ. Palindrome $= \{\Lambda$, and all strings x such that reverse $(x) = x\}$.

OR.

Palindrome $= \{\Lambda \quad a \quad b \quad aa \quad bb \quad aaa \quad aba \quad bab \quad bbb \quad aaaa \quad abba \ldots\}$

Kleene Closure :- A language in which any string of letters from Σ is a word & null string is also included. This language is called closure of the alphabet. Denoted by '*' after the name of Alphabet ~~as~~ as a superscript. (Σ^*).

This notation is known as Kleene star.

eg:- $\Sigma = \{x\}$, then

$\Sigma^* = \{\Lambda \quad x \quad xx \quad xxx \quad xxxx \ldots\}$

eg. $\Sigma = \{0 \quad 1\}$

then $\Sigma^* = \{\Lambda \quad 0 \quad 1 \quad 00 \quad 11 \quad 01 \quad 10 \quad 000 \quad 001 \ldots\}$

Lexicographic order :-

means words of shortest length come first and then processed in ascending order.

Examples :-

① $S = \{aa \quad b\}$

then, $S^* = \{\Lambda$ plus all strings of a's & b's in which 'a' occur in even clumps.$\}$

$= \{\Lambda \quad b \quad aa \quad bb \quad aab \quad baa \quad bbb \quad aaaa \quad aabb \ldots$

② $S = \{a \quad ab\}$

then, $S^* = \{\Lambda$ plus strings of a's & b's except those that start with 'b' & those that contain a double 'b'.$\}$

Unique factoring

In above example, show that abaab is in S^*.

factoring the string as (ab)(a)(ab) and this factoring is unique.

eg. $S = \{xx \quad xxx\}$, $S^* = \{\Lambda$ & all strings of more than one $x\}$

factoring the string $(xxxxxxx)$,

we get $(xx)(xx)(xxx)$ or,

$(xx)(xxx)(xx)$ or,

$(xxx)(xx)(xx)$.

which is not unique obviously.

Scanned by CamScanner

Positive closure.

If $\Sigma = \{x\}$, then,

$\Sigma^* = \{\Lambda \quad x \quad xx \quad xxx \quad ----\}$ &

$\Sigma^+ = \{x \quad xx \quad xxx \quad ----\}$

That is,

If Σ is a set of strings not including Λ, then Σ^+ is the language Σ^* without the word Λ.

This 'plus operation' is called <u>positive closure.</u>

Theorem. For any set of strings, we have $S^* = S^{**}$.

Proof:- every word in S^{**} is made up of factors from S^*.

Every word in S^* is made of factors from S.

$\therefore$ every word in S^{**} is also a word in S^*.

$\therefore$ we can write $S^{**} \subset S^*$. — (1)

Now, in general, it is true that for any set A, we know $A \subset A^*$. So, if we consider A to be our set S^*, we have

$S^* \subset S^*$ and then $S^* \subset S^{**}$. — (2)

Together these two inclusions (1 & 2) prove that,

$$\boxed{S^* = S^{**}}$$

Hence Proved.

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Q-11 Prove that for all sets S,

(i) $(S^+)^* = (S^*)^*$

Sol^n:- Every factor from S^+ is made up of factors from S,
LHS. excluding Λ if it is not in the set.
Every factor from $(S^+)^*$ is made up of factors from S^+, including Λ.

R.H.S. Every factor from S^* is made up of factors from S, including Λ.
Every factor from $(S^*)^*$ is made up of factors from S^*, including Λ.

$\therefore$ Every word in $(S^+)^*$ & $(S^*)^*$ is made up of factors ~~of~~ from S.

$\therefore$ Every word in $(S^+)^*$ is also a word in $(S^*)^*$.

(ii) $(S^+)^+ = S^+$

LHS. Every factor from S^+ is made up of factors from S, excluding Λ if it's not in set. Every factor from $(S^+)^+$ is made up of factors from S^+, excluding Λ again.

Therefore, every word in $(S^+)^+$ and S^+ is made up of factors from S excluding Λ. $\therefore$ Every word in $(S^+)^+$ is also a word in S^+.

(iii) Is $(S^*)^+ = (S^+)^*$ for all sets S?

Sol^n. Yes,
Every factor from S^* is made up of factors from S, including Λ.
Every factor from $(S^*)^+$ is made up of factors from S^+, excluding Λ, if it doesn't exist. However, since S^* will always contain Λ, $(S^*)^+$ will also contain Λ.

Every factor from S^+ is made up of factors from S, excluding Λ if it's in the set. Every factor from $(S^+)^*$ is made up of factors from S^+, including Λ even if it doesn't exist in S^+.

$\therefore$ every word in $(S^*)^+$ is made up of factors of S including Λ, & every word in $(S^+)^*$ is made up of factors from S including Λ. $\therefore$ LHS = RHS.