Regular Expressions ch-4 (Cohen)

Def

The symbols that appear in regular expressions are the letters of the alphabet Σ, the symbol for the null string Λ, parentheses, the star operator, and the plus sign.

Set of Regular Expressions is defined by the following rules:

Rule 1 Every letter of Σ can be made into a regular Expression by writing it in boldface; Λ itself is a regular Expression.

Rule 2 If r_1 and r_2 are regular expressions, then so are,

(i) (r_1)

(ii) $r_1 r_2$

(iii) $r_1 + r_2$

(iv) r_1^*

Rule 3. Nothing else is a regular Expression.

egs

1) ab^* $\Rightarrow$ initial 'a' and with some or no b's.

2) x^* $\Rightarrow$ Some or no x's.

3). $(ab)^*$ $\Rightarrow$ Λ or ab or abab or ababab ----.

4). ab^*a $\Rightarrow$ begin & end with 'a' with some or no b's inside.

5) $a^* b^*$ $\Rightarrow$ all the a's (if any) come before all the b's (if any).

6). $x(xx)^*$ $\Rightarrow$ odd no. of x's.

7). $x^* x x^*$ $\Rightarrow$ 6 $\neq$ 7

8) $(a+c)b^*$ $\Rightarrow$ either a or c then some no. of b's.

9) $(a+b)(a+b)(a+b)$ $\Rightarrow$ all strings of a's & b's of length three exactly.

10) $a(a+b)^*$ $\Rightarrow$ start with an 'a' then anything.

11) $a(a+b)^*b$ $\Rightarrow$ a(arbitrary string) b.

12) $(a+b)^* a (a+b)^*$ $\Rightarrow$ words left out are those that have only b's & the word Λ.

13) $(a+b)^* a (a+b)^* a (a+b)^*$ $\Rightarrow$ atleast two a's.

$b^* a b^* a (a+b)^*$ $\Rightarrow$ atleast two a's.

14) $b^* a b^* a b^*$ $\Rightarrow$ exactly two a's.

15) atleast one 'a' & atleast one 'b'.

$$(a+b)^* a (a+b^*) b (a+b)^* \quad + \quad (a+b)^* b (a+b)^* a (a+b)^*.$$

If the language is finite,

$L = \{abba \quad baaa \quad bbbb\}$

then reg. expression is $(\mathbf{abba} + \mathbf{baaa} + \mathbf{bbbb})$ (bold letters)

Product Set.

$S = \{a \quad aa \quad aaa\}$, $T = \{bb \quad bbb\}$

then

$ST = \{abb \quad abbb \quad aabb \quad aabbb \quad aaabb \quad aaabbb\}$.

All finite languages are regular.

\# languages described by regular expressions are regular languages.

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Even-Even Language.

even no. of a's and even no. of b's.

$$E = [\underbrace{aa}_{type 1} + \underbrace{bb}_{type 2} + \underbrace{(ab+ba)(aa+bb)^*(ab+ba)}_{type 3}]^*$$

To check language is Even-Even :-

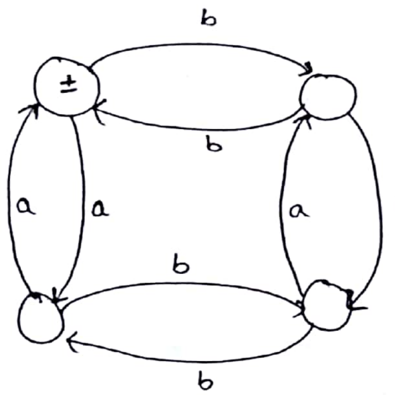
I^{ST} Method :- To check the string is even, we keep two binary flags one for 'a' & one for 'b'. each time an 'a' is read reversed the flag (0 to 1 & 1 to 0). We start both flags at '0' & check to be sure they are both '0' at the end.

II^{nd} Method :- This method uses only one flag. We read two letters at a time. If they are same, discard them. If not, reverse the flag considering them of type 3. So, if we end with a '0', we are ~~in the~~ having EVEN-EVEN languag

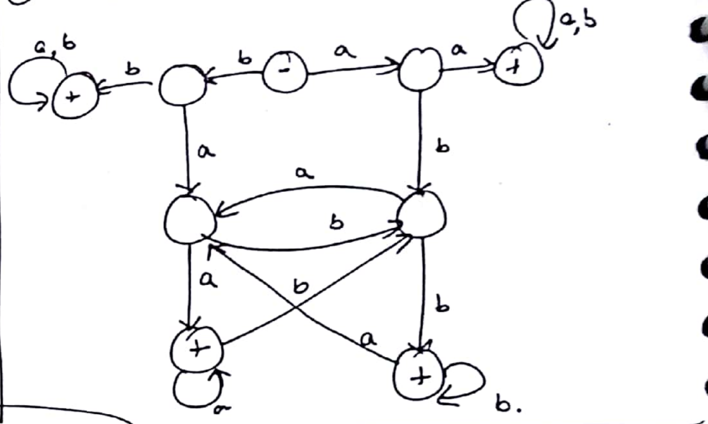
Exercise. [Contd....]. [= → Prb.].

1) (7) strings in which letter 'b' is never tripled. (ie no words contains the substring bbb].
2) (8) words in which 'a' is tripled or 'b' is tripled, but not both.
3) (9) words that do not have substring ab.
3) (10) words that do not have both the substrings bba and abb.
(11) total no. of a's div. by 3, no matter how they are distributed.
2) (12) words in which any b's that occur are found in clumps of an odd no. at a time.
3) (13) strings that have even no. of a's & odd no. of b's.
3) (14) strings that have odd no. of a's & odd no. of b's

eg 15) Finite Automata for EVEN - EVEN.



(7) begin or end with double letter



Exercises (F.A).

(13)

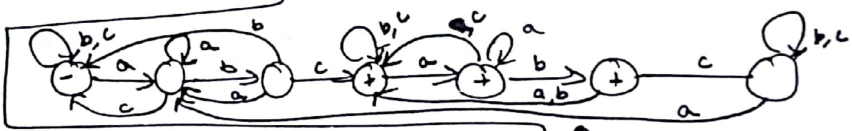
1) words with 'b' as second letter.

2) accepts only baa, ab & abb.

3) words that have more than 4 letters.

4) fewer that four letters.

5) exactly four letters.



6) words do not end with ba { exclude Λ, include Λ)

7) words that begin or end with a double letter

8) words that have both the letters a & b in them in any order.

9) words with only a's or only b's in them.

10) strings of a's & b's such that ~~next~~ Prev. to last letter is an a.

11) lang. of all strings of length 4 or more such that ~~next~~ Prev.-to-last letter is equal to the second letter of the input string.

2) all strings that have an even length that is not divisible by 6.

3) odd no. of occurrences of the substring abc.

Regular Exp)

1) (1) a' appears tripled, if at all. i.e every clumps of a' contain 3, 6, 9, - - - 12 . . a's.

2) (2) words that contain atleast one of the strings S_1, S_2, S_3 or S_4.

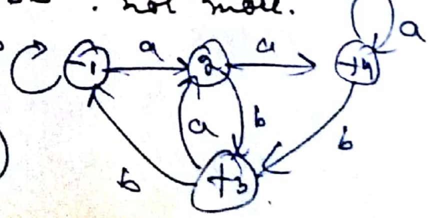
3) (3) that contain exactly two b's or three b's . not more.

) (4) that end in a double letter.

) (5) do not end in a double letter.

) (6) exactly one double letter in them.

(10)



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ch-4. Regular Expressions

$\Sigma = \{a, b\}$

Q.3. All words that contain at least one of the strings S_1, S_2, S_3 or S_4.

$$(a+b)^* (S1 + S2 + S3 + S4)(S1 + S2 + S3 + S4)^* (a+b)^*$$

Q-5. (i) strings that end in double letter

$$(a+b)^* (aa + bb)$$

(ii) strings that do not end in double letter.

$$((a+b)^* (ab+ba)) + a + b + \Lambda$$

Q-7. All strings in which letter 'b' is never tripled. i.e No word contains the substring 'bbb'.

Sol^n

$$(a + ba + bba)^* (b + bb + \Lambda)$$

Q-9. (i) All words that do not have the substring ab.

$$b^* a^*$$

(ii) All words that do not have both the substrings bba & abb.

doubt

?→ $a^* (baa^*)^* b^* + b^* (a^* ab)^* a^*$

?→ $a^* + b^* + (ab)^* + (ba)^* + b(ab)a^* + a^* (ba)^* b$

Q-11. (i) All string in which any b's that occur are found in clumps of an odd no. at a time, such as 'abaabbbab'

$$a^* (b(bb)^* aa^*)^* (\Lambda + b(bb)^*)$$

compulsory a after same no. of odd b's as odd + odd = Even to separate these odd clumps.

(ii) All strings that have even no. of a's & odd no. of b's. (61)

Sol^n: Div. the lang. to two groups

(a) when words that start with b and followed by even no. of a's and even no. of b's. It becomes odd no. of b's and even no. of a's.

(b) when words that start with a and followed by odd no. of a's & odd no. of b's. It also becomes odd no. of b's & even no. of a's.

$$b\left[aa+bb+(ab+ba)(aa+bb)^*(ab+ba)\right]^* +$$

$$a\left[\left[aa+bb+(ab+ba)(aa+bb)^*(ab+ba)\right]^*(ab+ba)\left[aa+bb+(ab+ba)(aa+bb)^*(ab+ba)\right]^*\right]$$

(c) All string that have odd no. of a's & odd no. of b's.

Sol^n. The small string is ab or ba, we can add even letter string left or right or both.

$$\left[aa+bb+(ab+ba)(aa+bb)^*(ab+ba)\right]^*(ab+ba)\left[aa+bb+(ab+ba)(aa+bb)^*(ab+ba)\right]^*$$

R.E. for odd no. of a's. (dist. any where).

$(b^* a b^* a b^*)^* b^* a b^*$ or $b^* a b^* (b^* a b^* a b^*)^*$

Q. R.E. for strings that do not have 'ab' as substring.

$b^* a^*$