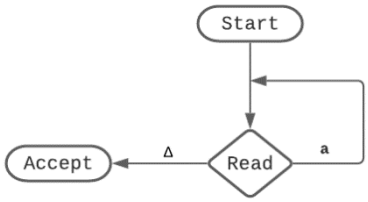
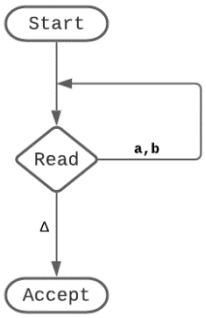
Practice Set 2

Consider alphabet set Σ = {a, b} unless specified. Define Context Free Language (CFL), Context Free Grammar (CFG) and Pushdown Automata (PDA) for the following:

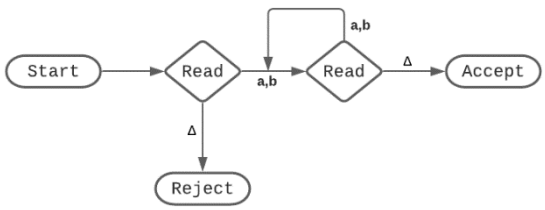
1. L = a*, Σ = {a}
   CFL:
   L = { Λ, a, aa, aaa, aaaa, … }
   CFG:
   S → aS | Λ
   PDA:



2. L = (a + b)*
   CFL:
   L = { Λ, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, … }
   CFG:
   S → aS | bS | Λ
   PDA:



3. L = $(a + b)^+$
   CFL:
   L = { a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb, … }
   CFG:
   S → aS | bS | a | b
   PDA:



4. Language with ‘aa’ in them somewhere.
   CFL:

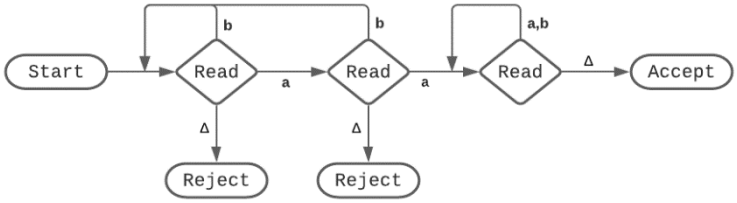
L = { aa, aaa, aab, baa, aaaa, aaab, baaa, baab, … }

CFG:

S → TaaT

T → aT | bT | Λ

PDA:



5. Strings that end with a.

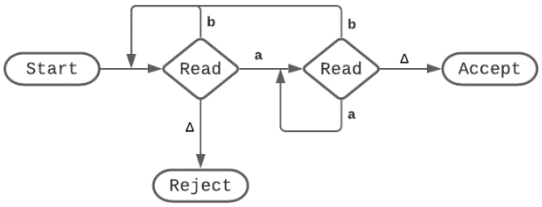
CFL:

L = { a, aa, ba, aaa, aba, baa, bba, … }

CFG:

S → Sa | a | b | Λ

PDA:



6. Strings that start with a.

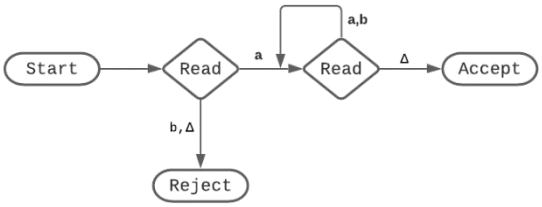
CFL:

L = { a, aa, ab, aaa, aab, aba, abb, … }

CFG:

S → aS | a | b | Λ

PDA:



7. Strings that do not contain the substring ‘aaa’.

CFL:

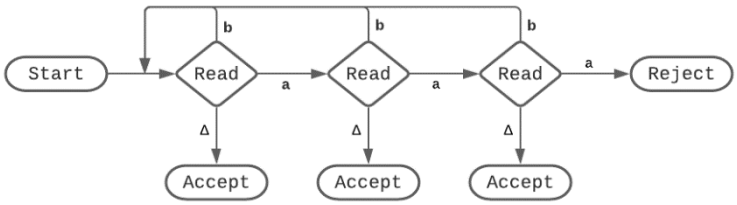
L = { Λ , a, b, aa, ab, ba, bb, aab, aba, abb, baa, bab, bba, bbb, abba, baab … }

CFG:

S → ST | aa | a | Λ

T → bS

PDA: (contd.)



8. $L = a^n b^n, n \geq 0$

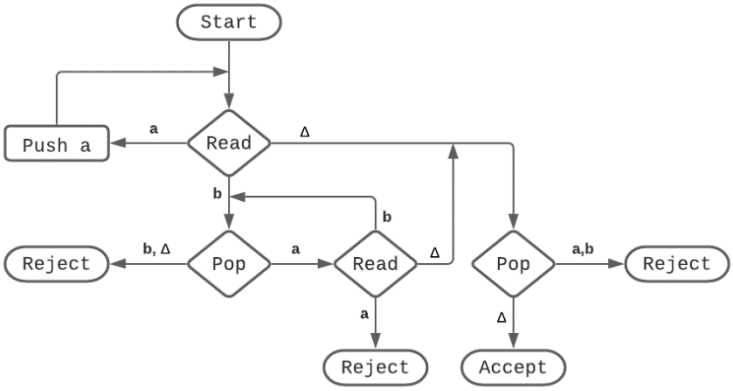
   CFL:

   $L = \{ \Lambda , ab, aabb, aaabbb, \ldots \}$

   CFG:

   $S \rightarrow aSb \mid \Lambda$

   PDA:



9. $L = a^n b^n, n \geq 1$

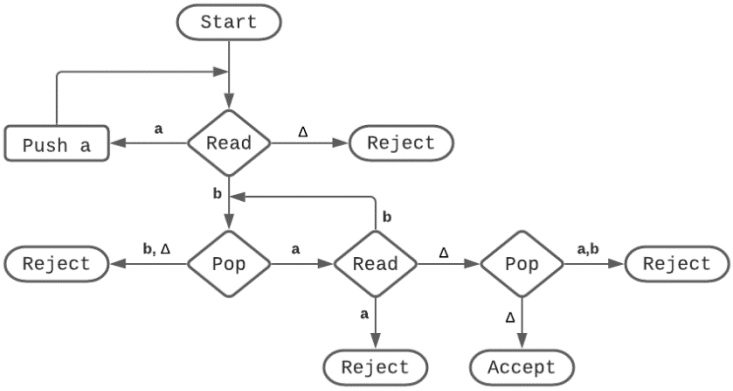
   CFL:

   $L = \{ \Lambda , ab, aabb, aaabbb, \ldots \}$

   CFG:

   $S \rightarrow aSb \mid ab$

   PDA:



10. For the language Even-Palindrome.

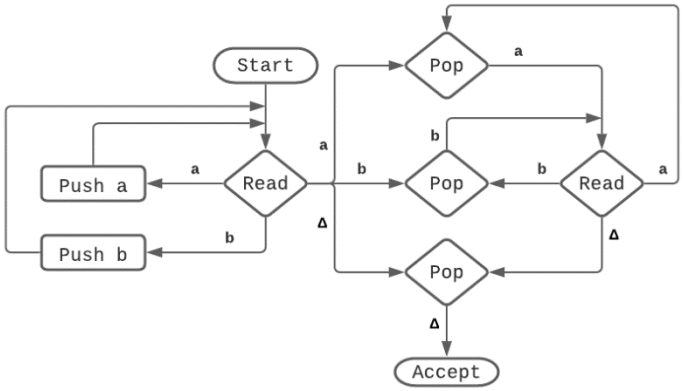
    CFL:

    $L = \{ \Lambda, aa, bb, aaaa, abba, baab, bbbb, \ldots \}$

    CFG:

    $S \rightarrow aSa \mid bSb \mid \Lambda$

    PDA: (contd.)



11. For the language Odd-Palindrome.

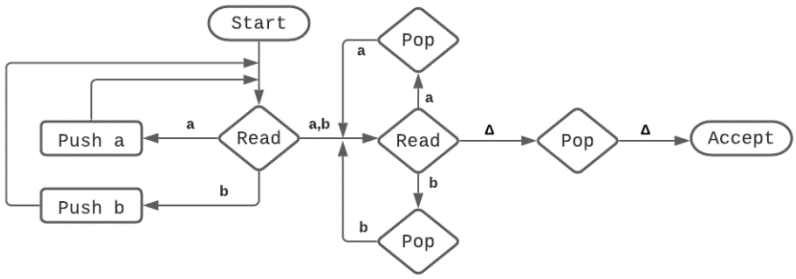
CFL:

L = { a, b, aaa, aba, bab, bbb, … }

CFG:

S → aSa | bSb | a | b

PDA:



12. For the language Palindrome.

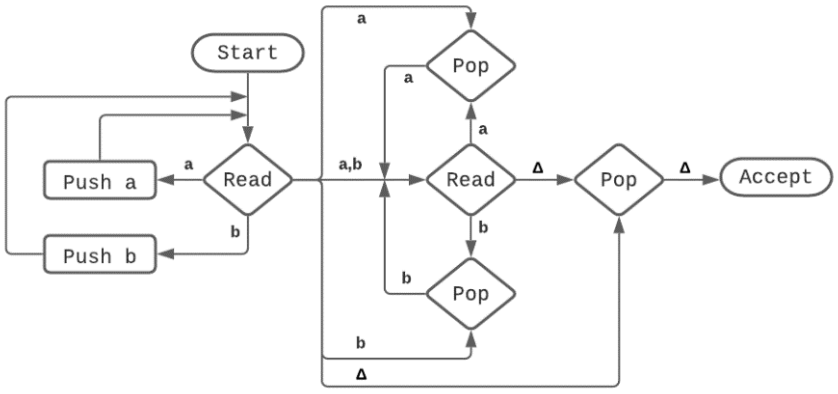
CFL:

L = { Λ, a, b, aa, bb, aaa, aba, bab, bbb, … }

CFG:

S → aSa | bSb | a | b | Λ

PDA:



13. $L = a^n b a^n$, $n \geq 1$

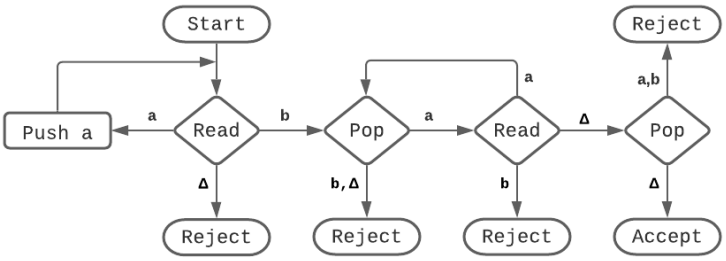
CFL:

L = { aba, aabaa, aaabaaa, … }

CFG:

S → aSa | aba

PDA: (contd.)



14. L = (aa + bb)*

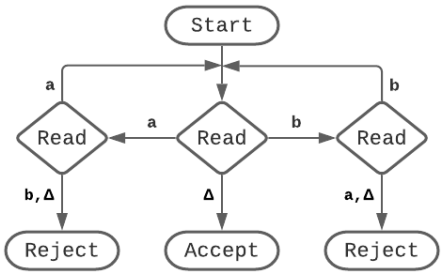
CFL:

L = { Λ, aa, bb, aaaa, aabb, bbaa, bbbb, … }

CFG:

S → aaS | bbS | Λ

PDA:



15. L = $(aa + bb)^+$

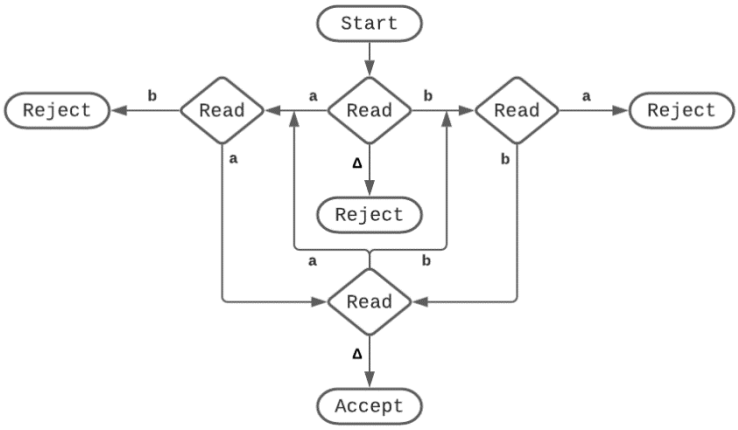
CFL:

L = { aa, bb, aaaa, aabb, bbaa, bbbb, … }

CFG:

S → aaS | bbS | aa | bb

PDA:



16. Even no. of a's. (distributed anywhere)

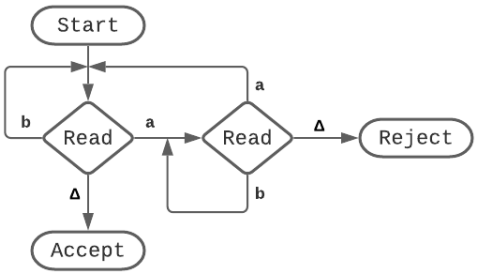
CFL:

L = { Λ, b, aa, bb, aab, baa, aba, aabb, abab, abba, bbaa, bbbb, … }

CFG:

S → aSa | bS | Λ

PDA: (contd.)



17. L= aa*bb*

CFL:

L = { ab, aab, abb, aabb, aaab, abbb, … }

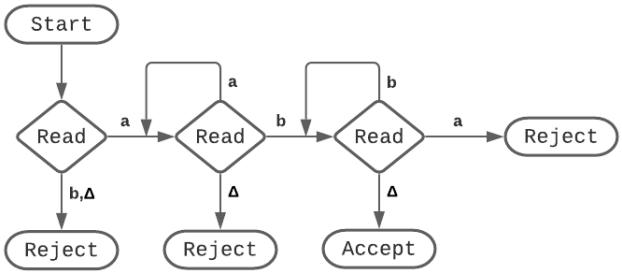
CFG:

S → aTbU

T → aT | Λ

U → bU | Λ

PDA:



18. L= (a + b)* bb (a + b)*

CFL:

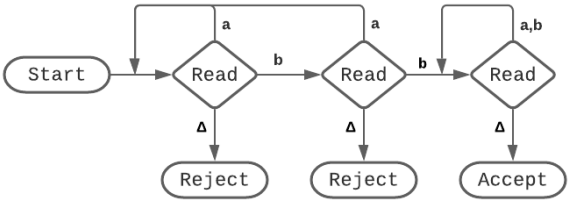
L = { bb, abb, bba, bbb, aabb, abba, abbb, bbaa, bbba, bbbb, … }

CFG:

S → TbbT

T → aT | bT | Λ

PDA:



19. L= a^{4n}, n ≥ 1

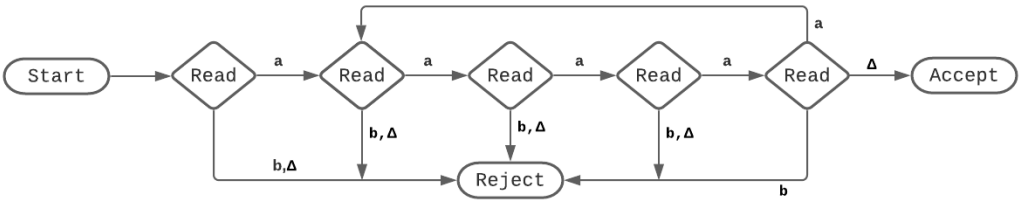
CFL:

L = { aaaa, aaaaaaaa, aaaaaaaaaaaa, … }

CFG:

S → aaaaS | aaaa

PDA: (contd.)



20. $L = a^n b^n a^m$, n, m ≥ 1 (positive independent integers)

CFL:

L = { aba, abaa, aabba, aabbaa, aabbaaa, aaabbba, … }

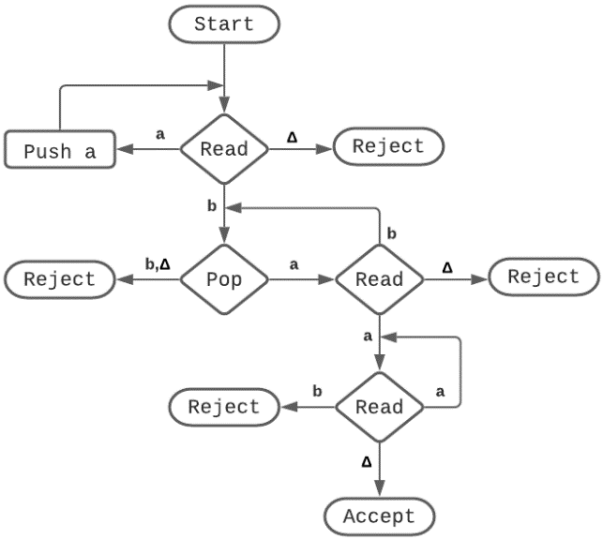
CFG:

S → TU

T → aTb | ab

U → aU | a

PDA:



21. $L = a^n b^m a^m$, n, m ≥ 1 (positive independent integers)

CFL:

L = { aba, aaba, aaaba, abbaa, aabbaa, aaabbaa, abbbaaa, … }

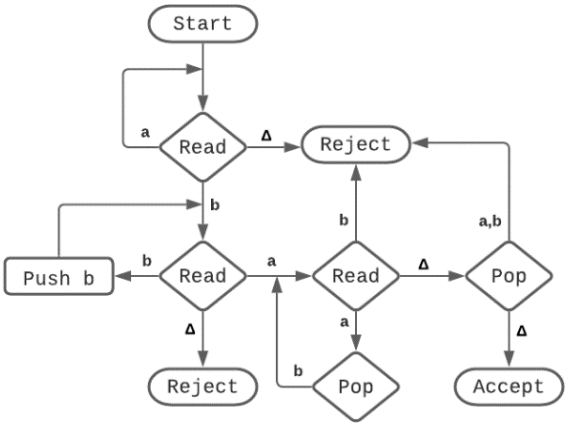
CFG:

S → TU

T → aT | a

U → bUa | ba

PDA: (contd.)



22. L = a^n S, S starts with ‘b’ and length(S) = n

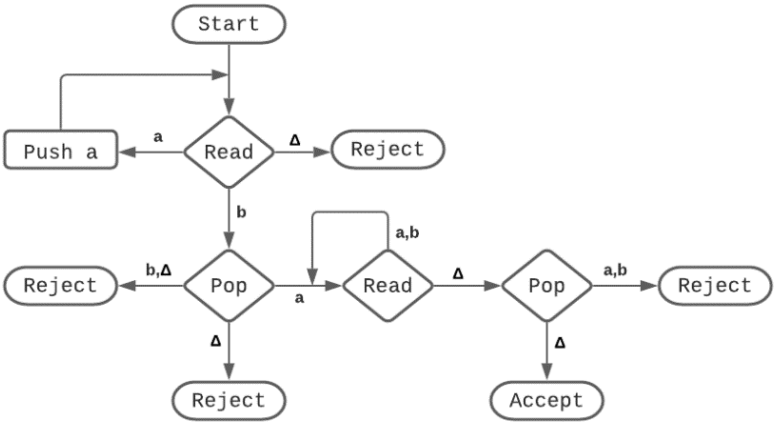
CFL:

L = { ab, aaba, aabb, aaabaa, aaabab, aaabbam aaabbb, … }

CFG:

S → aSa | aSb | ab

PDA:



23. Strings that do not have ‘ab’ as substring.

CFL:

L = { Λ, a, b, aa, ba, bb, aaa, baa, bba, … }

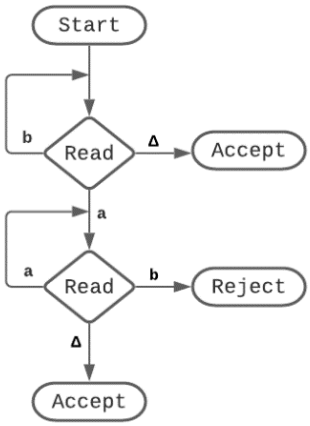
CFG:

S → TU | Λ

T → bT | Λ

U → aU | Λ

PDA: (contd.)



24. Strings that do not contain the substring ‘baa’.

CFL:

L = { Λ, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, bab, bba, bbb, … }

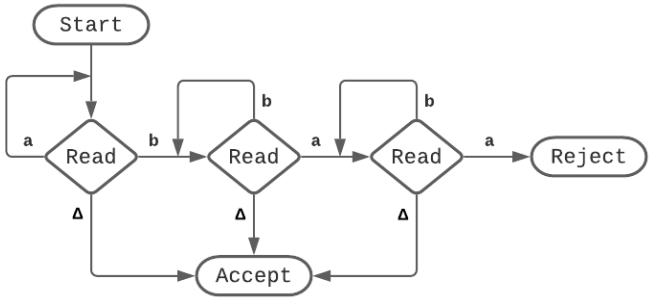
CFG:

S → aS | bT | Λ

T → bT | aU | Λ

U → bU | Λ

PDA:



25. $L = a^i b^j c^k$, i, j, k ≥ 0 and i + j = k, Σ = {a, b, c}

CFL:

L = { Λ, ac, bc, aacc, abcc, bbcc, … }

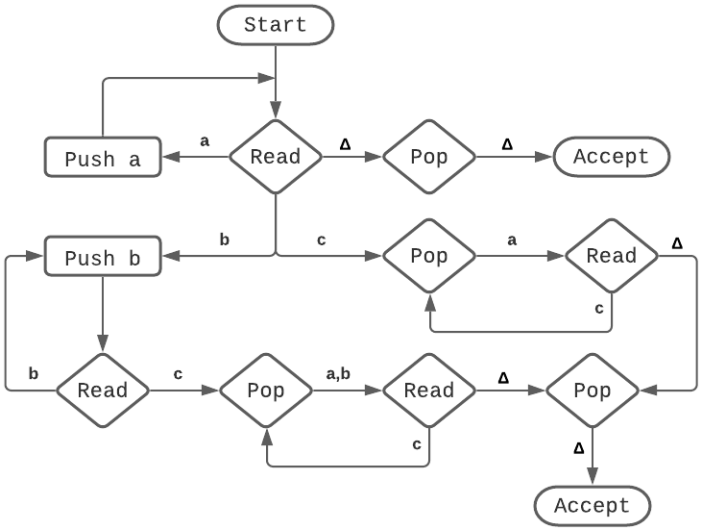
CFG:

S → T | U | Λ

T → aTc | U | Λ

U → bUc | Λ

PDA: (contd.)



26. $L = a^p b^q c^r d^s$, $p, q, r, s \geq 0$ and $p + q = r + s$, $\Sigma = \{a, b, c, d\}$

CFL:

L = { Λ, ac, ad, bc, bd, abcd, aabccd, aabcdd, abbccd, abbcdd, … }

CFG:

S → T | U | Λ

T → aTd | aWc | U | Λ

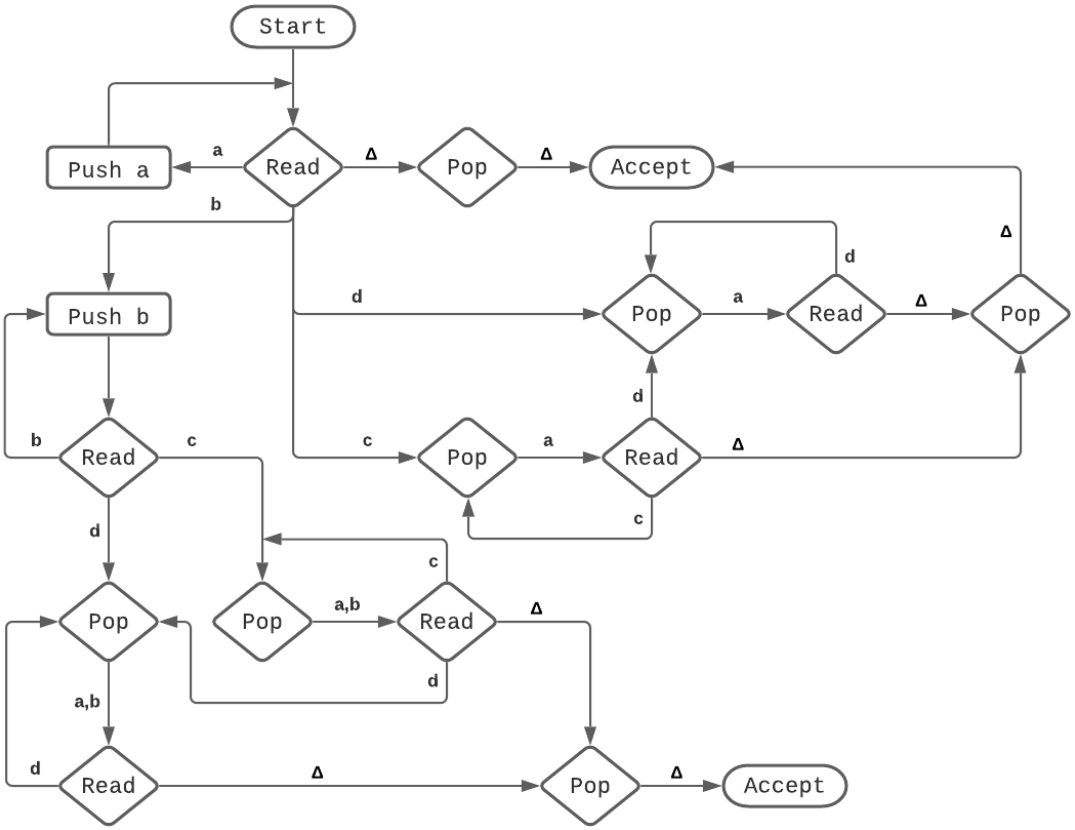
W → aWc | V | Λ

U → bUd | bXc | Λ

X → bXc | Λ

V → bXc

PDA:



27. Language that contains exactly 2 or 3 b's, Σ = {a, b} (distributed anywhere, in clumps)

Distributed Anywhere:

CFL:

L = { bb, abb, bab, bba, abab, abba, baba, … }

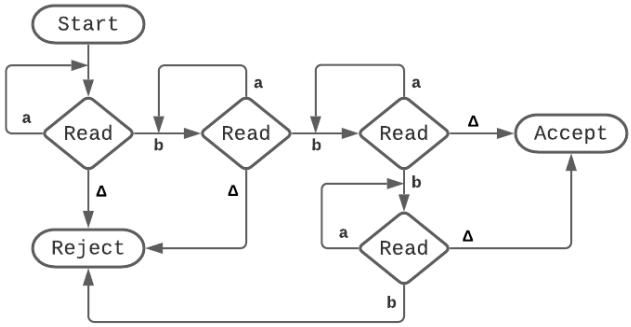
CFG:

S → TTU | TTTU

T → Ub

U → aU | Λ

PDA:



In Clumps:

CFL:

L = { bb, abb abb, bba, bbb, abba, abbb, bbaa, bbba, … }

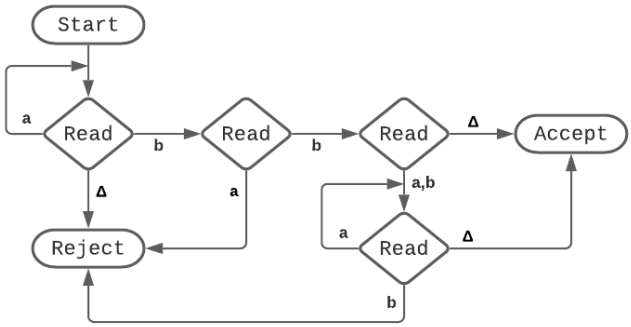
CFG:

S → TUT

T → aT | Λ

U → bb | bbb

PDA:



28. L = (aaa + b)*

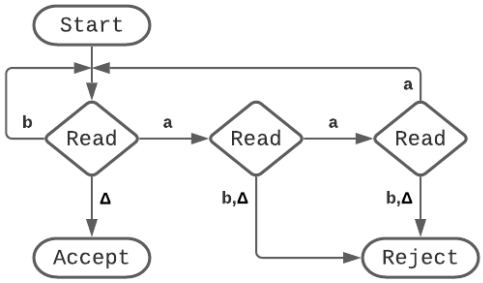
CFL:

L = { Λ, b, bb, aaa, bbb, aaab, baaa, bbbb, … }

CFG:

S → aaaS | bS | Λ

PDA: (contd.)



29. $L = a^m b^n a^{m+n}$, $n, m \geq 1$

CFL:

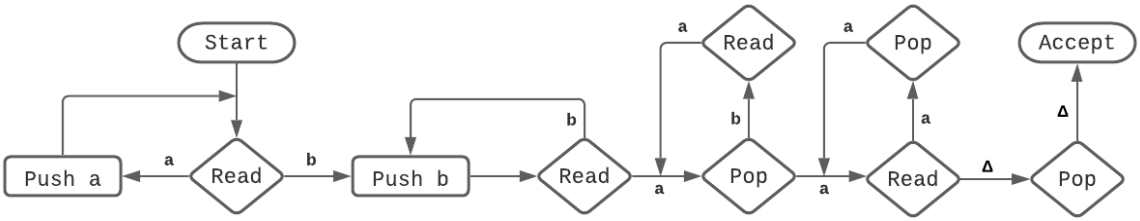
L = { abaa, aabaaa, abbaaa, aaabaaaa, aabbaaaa, abbbaaaa, … }

CFG:

S → aSa | aTa

T → bTa | ba

PDA:



30. $L = a^n b^m$, $n \neq m$

CFL:

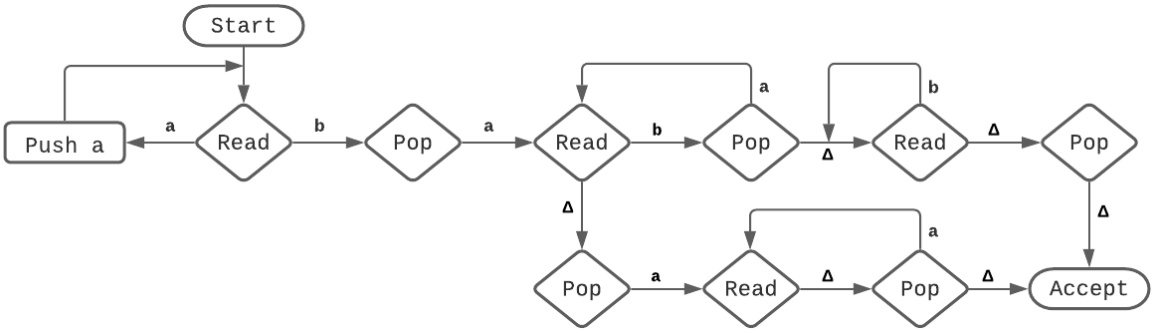
L = { aab, abb, aaab, abbb, aaaab, aabbb, abbbb,… }

CFG:

S → aTb

T → aTb | aT | Tb | a | b

PDA:



31. $L = a^n b^m$, $n > m$

CFL:

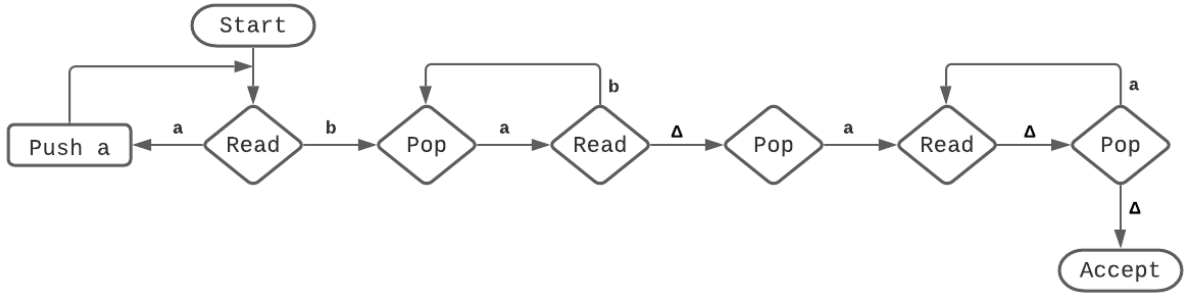
L = { aab, aaab, aaaab, aaabb, aaaaab, aaaabb,… }

CFG:

S → aTb

T → aTb | aT | a

PDA:



32. $L = a^n b^m, n \geq m$

CFL:

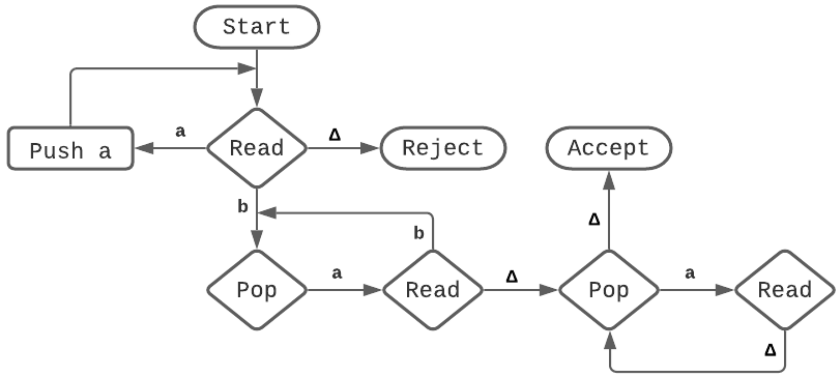
L = { ab, aab, aaab, aabb, aaaab, aaabb, aaaaab, aaaabb, aaabbb, … }

CFG:

S → aTb

T → aTb | aT | Λ

PDA:



33. $L = a^n b^m, n < m$

CFL:

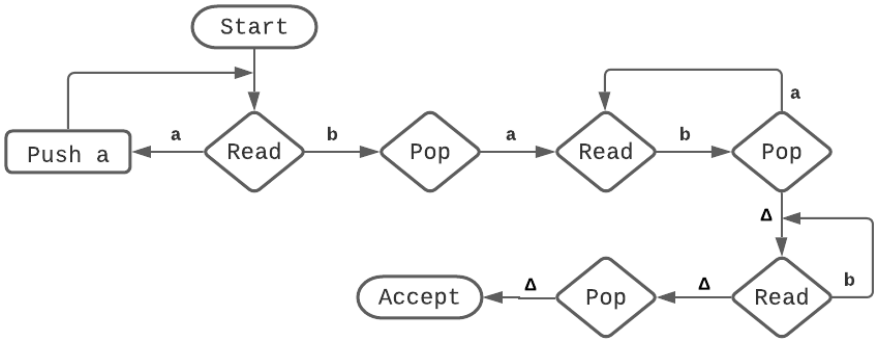
L = { abb, abbb, aabbb, abbbb, aabbbb, abbbbb, … }

CFG:

S → aTb

T → aTb | Tb | b

PDA:



34. $L = a^n b^m, n \leq m$

CFL:

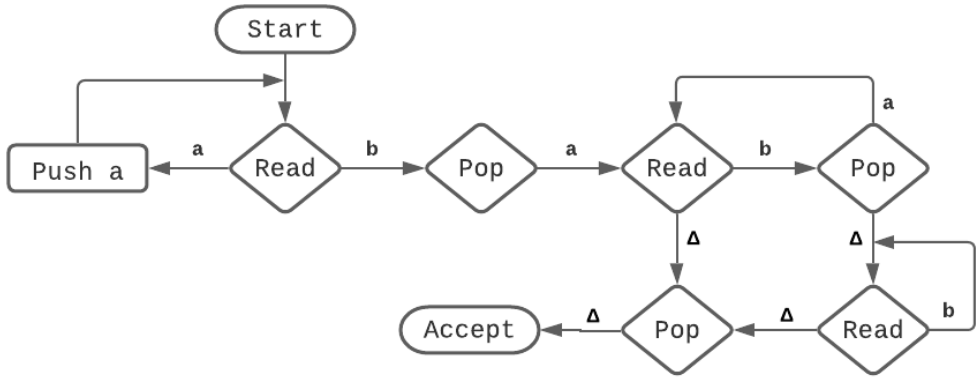
L = { ab, abb, aabb, abbb, abbbb, aabbb, abbbbb, aabbbb, aaabbb, … }

CFG:

S → aTb

T → aTb | Tb | Λ

PDA:



35. L = Non-Palindrome words

CFL:

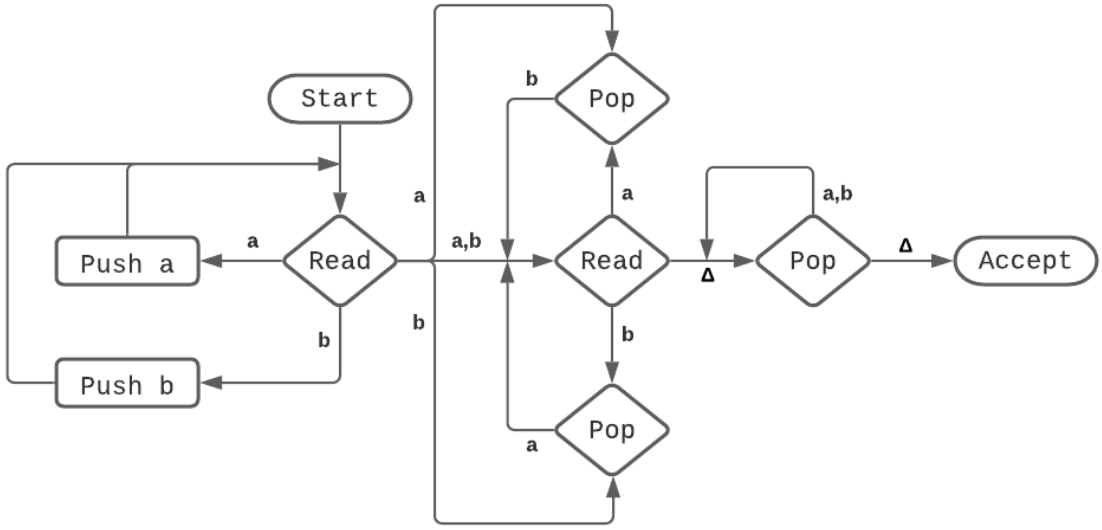
L = { ab, ba, aab, abb, baa, bba, aaab, aaba, abaa, abab, abbb, baba, bbba, … }

CFG:

S → aSa | bSb | aTb | bTa

T → aT | bT | Λ

PDA:



36. L = EQUAL (Equal no. of a's and b's distributed anywhere)

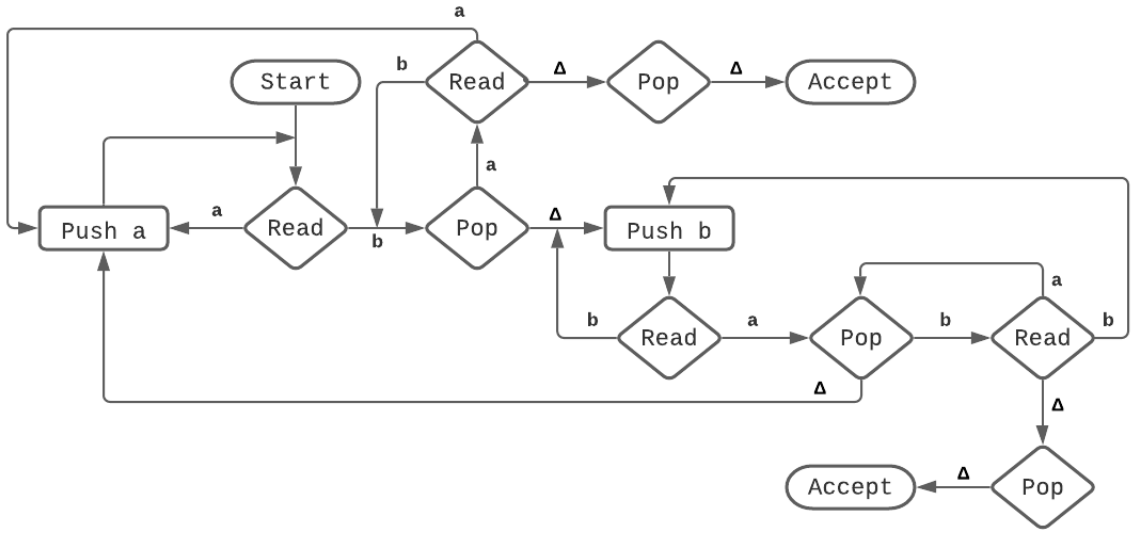
CFL:

L = { ab, ba, aabb, abab, abba, baba, baab, bbaa , … }

CFG:

S → aSb | bSa | SS | Λ

PDA: (contd.)


Start
Read
Pop
Accept
Push a
Read
Pop
Push b
Read
Pop
Read
Accept
Pop
a
b
Δ