Instructions For Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Part A is of 35 marks and all its questions are compulsory. Attempt any four questions from Part B.

3. Assume $\Sigma = \{a, b\}$ as the underlying alphabet set unless mentioned otherwise.

4. Parts of a question must be answered together.

Part A

1. (a) Prove that for all sets S, $(S^+)^* = S^*$ — 2

   (b) Give regular expression for the language of all strings that do not contain 'ab' as substring. — 2

   (c) Does $(a^*b^*)^*$ and $(a + b)^*$ defines the same language. Generate first 6 words of each of the language in the lexicographic order. — 3

   (d) Build deterministic finite automata (DFA) machine that accept all strings that either start with 'ab' or end with 'ba'. — 4

   (e) Build a DFA machine that accepts only those strings that do not end with double letters. — 4

   (f) Find a Context Free Grammar (CFG) for a language of the form $a^x b^y a^z$ where x,y,z >=0 and x+y=z. — 4

   (g) Using pumping lemma for regular languages show that the language $L = \{ a^n b\, a^n \mid n \geq 0 \}$ is not regular. — 4

   (h) Show that if L_1 and L_2 are regular languages then so are $L_1 + L_2$, $L_1 L_2$ and L_1^*. — 4

   (i) Construct a PDA for the language $L = \{a^n b^{2n} \mid n \geq 0\}$. — 4

   (j) Design a right shifting Turing machine. — 4

Part B

2. (a) Define regular expression. — 2

   (b) Build a regular expression for all strings in which b's occur in clumps of an odd number at a time such as ab, ba, abbb, bbba, abaabbb,…….. — 3

   (c) Build an FA that accepts all strings that have an even length that is not divisible by 6. — 5

3. (a) For languages, $L_1 = (a + b)^* a$ and $L_2 = (a + b)^* aa(a + b)^*$, Construct respective DFA's and derive the finite automata that define $L_1 + L_2$. — 6

   (b) Show that the following context free grammar is ambiguous: — 4
$$S \rightarrow aSb \mid Sb \mid Sa \mid a.$$

4. (a) Convert the following NFA to DFA. — 5
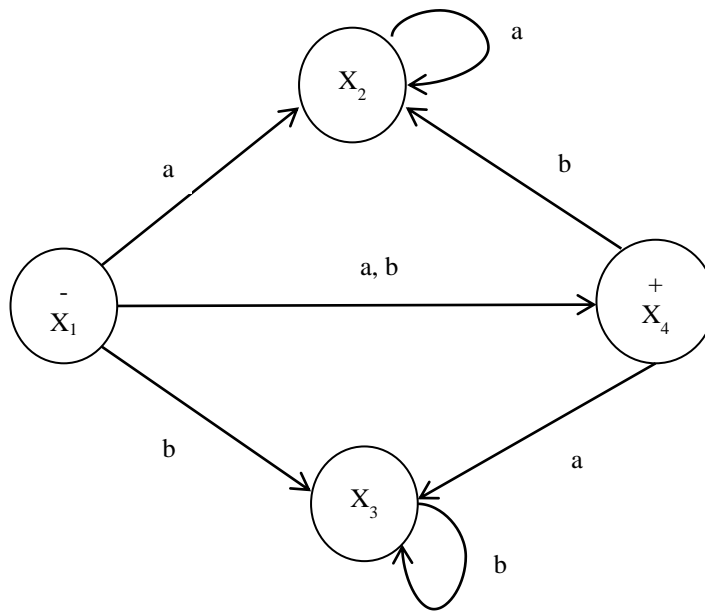
(b) Write a regular expression and construct a DFA for the language of all words that have an even number of substrings 'ab' in them.   5

5 (a) Construct a PDA for the language EQUAL.   6
(Language that contains equal number of a's and b's no matter where they are distributed).

(b) Construct a CFG for the language (ba+ab)*   4

6 (a) Prove that a recursive language is also recursively enumerable.   5

(b) Consider the following CFG in Chomsky Normal Form (CNF)   5

$$S \rightarrow PQ$$
$$Q \rightarrow QS \mid b$$
$$P \rightarrow a$$

Generate the derivation trees for the word
(i) abab
(ii) ababab
State whether the given CFG is ambiguous or not.

7 (a) Design a Turing machine for the language $a^n b^n c^n$ where $n \geq 1$.   6

(b) Describe Universal Turing machine.   4