Table of Contents

The Instability Of Liquid Surfaces When Accelerated In A Direction Perpendicular To Their Planes

By Sir Geoffrey Taylor (Royal Society Publishing, 1949)

Introduction

The two superposed fluid with different densities are initially maintained one above the other and they are provided an acceleration perpendicular to their planes. This paper focused on determining the relationship between the rate of development of the instability and the length of disturbances, the acceleration and the densities.

Problem Statement and Assumptions

The aim of the analysis is to produce a relation between acceleration, wave growth rate, densities and wave number. The final expression based on the paper is as follows:

$$n^2 = -K(g+g_1)\frac{(\rho_2-\rho_1)}{\rho_2+\rho_1} \quad (1)$$

It is mentioned clearly in the paper that the equations of motion relative to the accelerating axes are identical to those of a fluid at rest except an additional acceleration term.

In this paper, normal mode analysis is carried out to find the solutions to the perturbations.

Let the upper fluid parameter be denoted by $(.)_1$ and the lower fluid parameter be denoted by $(.)_2$

For the two fluids let the densities are denoted as: ρ_1, ρ_2

For the two fluids let the velocity potentials are denoted as: ϕ_1, ϕ_2

Initial pressure is denoted as: p

For the two fluids let the pressures in each fluid are denoted as: p_1, p_2

Let the external acceleration provided is denoted as: g_1

Let the interfacial deflection from unperturbed location is denoted as: η

Let the wave number is denoted by: K

Let the growth rate is denoted by: n

The assumptions taken in the analysis are:

a) The viscous diffusion length is very small compared to the capillary length that means; we neglect the interfacial surface tension i.e; $\gamma \rightarrow 0$
b) The two fluids are immiscible.
c) Each fluid is isotropic and homogenous.
d) Both the fluids are deep compared with the wavelength of the disturbance of the interface
e) There is no vorticity on the interface of the fluids.

Derivation Using General Physics

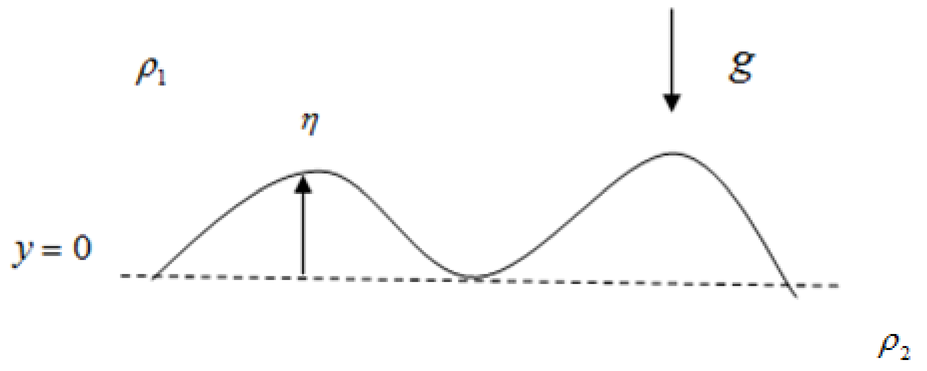

Fig 1: Schematic of Fluids at the interface

We know that,

Volume per unit width of the wave $\propto \lambda^2 \;\; (\lambda = wavelength)$

$\therefore$ mass of the fluid due to perturbed wave $\propto \rho_1\lambda^2 + \rho_2\lambda^2$

$$\Rightarrow (\rho_1 + \rho_2)\lambda^2$$

$$\because k = \frac{2\pi}{\lambda}$$

So, the mass of the fluid can be written as:

Mass of the fluid $\Rightarrow (\rho_1 + \rho_2)\dfrac{4\pi}{K^2}$

Since, we are not looking into the exact solutions, the constant value of 4π can be ignored.

Mass of the fluid $\Rightarrow (\rho_1 + \rho_2)\dfrac{1}{K^2}$

Let the amplitude of the wave be a

Let the vertical upward acceleration be g_1

Net Buoyancy Force per width of the wave is as: $\dfrac{(\rho_1 - \rho_2)(g + g_1)a}{K}$

Looking at our fluid system as a spring mass system, the oscillation can be taken as the following:

$$n^2 \sim \frac{k_{sp}}{mass}$$

Where, (k_{sp}) is the parameter which acts like the spring constant which is given by:

$$(k_{sp}) = \frac{(\rho_1 - \rho_2)(g + g_1)}{K}$$

(n) is the frequency of oscillation of the perturbed wave.

So, the oscillation frequency can be written as:

$$n^2 \sim \frac{\dfrac{(\rho_1 - \rho_2)(g + g_1)}{K}}{(\rho_1 + \rho_2)\dfrac{1}{K^2}}$$

$$n^2 \sim \frac{-(\rho_2 - \rho_1)(g + g_1)K}{(\rho_1 + \rho_2)}$$

$$n = \sqrt{\frac{-K(\rho_2 - \rho_1)(g + g_1)}{(\rho_1 + \rho_2)}} \qquad \text{(A)}$$

The above equation matches the expression in equation (1) derived in the paper.

Normal Mode Analysis For The Interface Of An Interface Between Two Fluids

Case I: Simple Velocity Potential

Assuming that the fluids are accelerated vertically upwards with the acceleration g_1, we expect to see some changes in the equation of motion.

The velocity potentials of the motion are taken as:

For the base state; $(j = 1, 2)$

$\bar{\eta} = 0$

$\bar{U}_j = 0$

Now, for the pressure;

$\bar{P}_j(y) = p - \rho_j g y - \rho_j g_1 y$

$\bar{P}_j(y) = p - \rho_j (g + g_1) y$

So, $\bar{P}_1(y) = p - \rho_1 (g + g_1) y$ (2)

$\bar{P}_2(y) = p - \rho_2 (g + g_1) y$ (3)

For perturbed case:

$\bar{U}_j$	$= \underline{\bar{U}_j}$	$+ \bar{u}_j{}'$	(providing a small corrugation)
(total)	(base)	(perturbation)	

At base state, $\bar{U}_j = 0$

$P_j = \bar{P}_j + p_j{}'$

$\eta = \bar{\eta} + \eta'$

<u>At base state</u>: $\bar{\eta} = 0$

Considering no vorticity,

$\bar{u}_j' = \nabla \phi_j'$

Considering 2-D perturbation i.e; (x,y)

$\nabla.\underline{u} = 0$

$\because \underline{u} = \nabla\phi$

$\because \nabla.\underline{u} = \nabla^2\phi_j^{'} = 0$

$$\Rightarrow \frac{\partial^2\phi_j^{'}}{\partial x^2} + \frac{\partial^2\phi_j^{'}}{\partial y^2} = 0 \quad (j = 1,2) \tag{4}$$

Since the inviscid fluids are taken so, no tangential velocity continuity exists also it is not possible to apply the tangential stress continuity.

Total stress:

$$\underline{\underline{T}} = -pI \qquad \text{(for the inviscid fluid)} \tag{5}$$

Normal stress continuity depicts the pressure equality at the perturbed interface.

To calculate the pressure, we have to use the unsteady Bernoulli equation given as:

$$\underbrace{\rho_j \frac{\partial\phi_j^{'}}{\partial t}}_{(unsteady)} + \underbrace{\frac{1}{2}\rho_j \nabla\phi_j^{'}\nabla\phi_j^{'}}_{(Kinetic\ energy\ term)} + \underbrace{(\bar{p} + p_j^{'})}_{(pressure)} + \underbrace{\rho_j g y}_{(gravity)} = C_j \tag{6}$$

Applying Bernoulli at base state,

$\bar{p}_j + \rho_j g y = C_j$

$\Rightarrow p - \rho_j g y + \rho_j g y = C_j$

$$\Rightarrow p = C_j \tag{7}$$

After substituting the base state quantities in the total Bernoulli equation, we have;

$$\rho_j \frac{\partial\phi_j^{'}}{\partial t} + \frac{1}{2}\rho_j \nabla\phi_j^{'}\nabla\phi_j^{'} + p_j^{'} = 0 \tag{8}$$

Kinematic condition

Considering a functional H

$H = y - \eta(x,t) = 0$ is the equation of the interface

$$\frac{DH}{Dt} = 0$$

$$\frac{\partial H}{\partial t} + \underline{u}.\nabla H = 0 \qquad (9)$$

Where, $\underline{u}$ is the velocity of a point on the interface.

$$-\frac{\partial H}{\partial t} = \underline{u}.\nabla H$$

$$-\frac{1}{|\nabla H|}\frac{\partial H}{\partial t} = \underline{u}.\frac{\nabla H}{|\nabla H|}$$

$$\underline{u}.\underline{n} = \frac{\partial H}{\partial t}.\frac{1}{|\nabla H|} \qquad (10)$$

$$H(x,y,t) = y - \eta(x,t)$$

Differentiating the above wrt time;

$$\frac{\partial H}{\partial t} = -\frac{\partial \eta}{\partial t}$$

So,

$\underline{u}.\underline{n} = \frac{\partial H}{\partial t}.\frac{1}{|\nabla H|}$ can be written as follows:

$$\underline{u}.\underline{n} = \frac{\partial n}{\partial t}.\frac{1}{|\nabla H|}$$

We know;

$\nabla H = \frac{\partial H}{\partial x} e_x + \frac{\partial H}{\partial y} e_y$ (unit vectors along x,y are e_x, e_y)

$$\Rightarrow \nabla H = \frac{\partial H}{\partial x} e_x + (1) e_y$$

$$(\nabla H.\nabla H)^{1/2} = |\nabla H| = \left[1+\left(\frac{\partial \eta^{'}}{\partial x}\right)^2\right]^{1/2}$$

$$\underline{u}.\underline{n} = \frac{\dfrac{\partial \eta^{'}}{\partial t}}{\left[1+\left(\dfrac{\partial \eta^{'}}{\partial x}\right)^2\right]^{1/2}} \tag{11}$$

The normal velocity continuity can be shown as:

$$\underline{U_j}.\underline{n} = \underline{u}.\underline{n} \tag{12}$$

Also,

$$\underline{n} = \frac{\nabla H}{|\nabla H|} = \frac{\dfrac{\partial H}{\partial x} e_x + e_y}{\left(1+\left(\dfrac{\partial \eta'}{\partial x}\right)^2\right)^{1/2}}$$

$$\underline{n} \Rightarrow \frac{-\dfrac{\partial \eta^{'}}{\partial x} e_x + e_y}{\left(1+\left(\dfrac{\partial \eta'}{\partial x}\right)^2\right)^{1/2}}$$

Since, we are doing the linear analysis, $\left(\frac{\partial \eta^{'}}{\partial x}\right)^2$ can be neglected so we are remained with the following:

$$\underline{n} = -\frac{\partial \eta^{'}}{\partial x} e_x + e_y$$

Now,

$$U_j^{'} = \frac{\partial \phi_j^{'}}{\partial x} e_x + \frac{\partial \phi_j^{'}}{\partial y} e_y$$

From <u>normal velocity continuity</u>,

$$U_j^{'}.\underline{n} = \underline{u}.\underline{n}$$

$$\Rightarrow \frac{\partial \phi_j^{'}}{\partial x}\left(\frac{-\partial \eta^{'}}{\partial x}\right) + \frac{\partial \phi_j^{'}}{\partial y} = \frac{-\partial \eta^{'}}{\partial t}$$

For the upper fluid;

$$\Rightarrow \frac{\partial \phi_1'}{\partial x}\left(\frac{-\partial \eta'}{\partial x}\right)+\frac{\partial \phi_1'}{\partial y}=\frac{-\partial \eta'}{\partial t}$$

For the lower fluid;

$$\Rightarrow \frac{\partial \phi_2'}{\partial x}\left(\frac{-\partial \eta'}{\partial x}\right)+\frac{\partial \phi_2'}{\partial y}=\frac{-\partial \eta'}{\partial t}$$

Since linear analysis is considered, we can neglect $\frac{\partial \phi_j'}{\partial x}\left(\frac{-\partial \eta'}{\partial x}\right)$

$$\Rightarrow \frac{\partial \phi_1'}{\partial y}=\frac{-\partial \eta'}{\partial t} \tag{13}$$

$$\Rightarrow \frac{\partial \phi_2'}{\partial y}=\frac{-\partial \eta'}{\partial t} \tag{14}$$

Now, <u>for the Stress Continuity</u>

We have;

$$\left(\underline{n}.\underline{\underline{T}}.\underline{n}\right)_2-\left(\underline{n}.\underline{\underline{T}}.\underline{n}\right)_1=\gamma(\nabla.\underline{n}) \tag{15}$$

We know, from eqn(5)

$$\left(\underline{\underline{T}}\right)_2=-p_2 I$$
$$\left(\underline{\underline{T}}\right)_1=-p_1 I$$

So,

$$\left(\underline{n}.\underline{\underline{T}}.\underline{n}\right)_2-\left(\underline{n}.\underline{\underline{T}}.\underline{n}\right)_1=\gamma(\nabla.\underline{n})$$

$$-p_2 I-(-p_1 I)=\gamma.(\nabla.\underline{n})$$

$$\Rightarrow p_1-p_2=\gamma.(\nabla.\underline{n}) \tag{16}$$

$$\underline{n}=-\frac{\partial \eta'}{\partial x}e_x+e_y$$

And, for linear order

$$\nabla.\underline{n} = \frac{-\partial^2 \eta}{\partial x^2}$$

So,

$$p_1 - p_2 = \gamma.\left(\frac{-\partial^2 \eta}{\partial x^2}\right) \tag{17}$$

For linearity in Bernoulli, $\nabla\phi_j^{'}\nabla\phi_j^{'}$ can be ignored because this expression is non-linear

$$\rho_j \frac{\partial \phi_j^{'}}{\partial t} + p_j^{'} \;=\; 0$$

$$p_j^{'} = -\rho_j \frac{\partial \phi_j^{'}}{\partial t} \tag{18}$$

Based on the velocity potential

$$\phi_j^{'}(x, y, t) = \hat{\phi}_j.e^{nt}$$

$$\frac{\partial \phi_j}{\partial t} = n\hat{\phi}_j.e^{nt} = n\phi_j^{'}$$

So, $p_j^{'} = -\rho_j \dfrac{\partial \phi_j^{'}}{\partial t}$

$$\Rightarrow p_j^{'} = -\rho_j(n\phi_j^{'})$$

$$\Rightarrow p_j^{'} = -n\rho_j\phi_j^{'}$$

So, $P_1(y) = p - \rho_1(g + g_1)y + \rho_1.\phi_1^{'}$ $\quad\quad \because P = \bar{P} + p'$ (19)

Similarly, $P_2(y) = p - \rho_2(g + g_1)y + \rho_2.\phi_2^{'}$ (20)

The velocity potential can be written in the following form:

$$\phi_j = A_j e^{-Ky} + B_j e^{Ky} \tag{21}$$

So, $\phi_1 = A_1 e^{-Ky} + B_1 e^{Ky}$ (22)

$$\phi_2 = A_2 e^{-Ky} + B_2 e^{Ky} \tag{23}$$

Using the boundary conditions,

As $y \to +\infty$, $\hat{\phi}_1 = B_1 e^{-Ky}$ (As the first term decays in ϕ_1)

And, as $y \to -\infty$, $\hat{\phi}_2 = A_2 e^{Ky}$ (As the second term decays in ϕ_2)

If we Taylor expand about $y = 0$ and assume only linear terms, we get for any function $f(y)$

$$f(y = \eta') = f(0) + \left.\frac{df}{dy}\right|_{y=0} \eta' + \text{H.O.T} \tag{24}$$

So,

$$f = \overline{f} + f' \tag{B}$$

$$f = \overline{f}(y=0) + f'(y=0) + \left.\frac{d\overline{f}}{dy}\right|_{y=0} \eta' + \left.\frac{df'}{dy}\right|_{y=0} \eta'$$

Since linearity is considered, $\left.\frac{df'}{dy}\right|_{y=0} \eta'$ can be neglected, then

$$\underbrace{\overline{f}(y=0)}_{\text{(base state)}} + \overbrace{f'(y=0)}^{\text{(perturbation)}}$$

$$f = \overline{f}(y=0) + f'(y=0) + \left.\frac{df'}{dy}\right|_{y=0} \eta' \tag{25}$$

From the above understanding, $\frac{\partial \phi_1}{\partial x}\left(\frac{\partial \eta'}{\partial x}\right)$ can be neglected.

We will be remained with

$$-\left.\frac{\partial \eta'}{\partial t}\right|_{y=0} = \left.\frac{\partial \phi_j'}{\partial y}\right|_{y=0} \tag{26}$$

Using Fourier Mode Analysis, we can write the above as:

$$-\left.\frac{\partial \hat{\eta}}{\partial t}\right|_{y=0} = \left.\frac{\partial \hat{\phi}_j}{\partial y}\right|_{y=0}$$

So, for the functions, the above relation can be represented as:

$$-\left.\frac{\partial\eta}{\partial t}\right|_{y=0} = \left.\frac{\partial\phi_j}{\partial y}\right|_{y=0} \tag{27}$$

Now for the pressure difference at $y=\eta$ which is as follows, we implement the Taylor expansion about $y=0$

$$p_1 - p_2 = \gamma.\left(\frac{-\partial^2\eta'}{\partial x^2}\right) \tag{28}$$

Taylor expanding about $y=0$, we get;

$$\overline{p}_1 + p_1^{'} - (\overline{p}_2 + p_2^{'}) = \gamma.\left(\frac{-\partial^2\eta'}{\partial x^2}\right)$$

$$p - \rho_1(g+g_1)y + p_1^{'} - p - \rho_2(g+g_1)y - p_2^{'} = \gamma.\left(\frac{-\partial^2\eta'}{\partial x^2}\right)$$

$$p_1^{'}(0) + \left.\frac{dp_1^{'}}{dy}\right|_{y=0}\eta' - \frac{d}{dy}\rho_1(g+g_1)y\Big|_{y=0}\eta' + \frac{d}{dy}\rho_2(g+g_1)y\Big|_{y=0}\eta' - p_2^{'}(0) = \gamma.\left(\frac{-\partial^2\eta'}{\partial x^2}\right)$$

$$p_1^{'}(0) - \rho_1(g+g_1)\eta' + \rho_2(g+g_1)\eta' - p_2^{'}(0) = \gamma.\left(\frac{-\partial^2\eta'}{\partial x^2}\right)$$

The above can be written as:

$$(\hat{p}_1 - \rho_1(g+g_1)\hat{\eta}) - (\hat{p}_2 - \rho_2(g+g_1)\hat{\eta}) = \gamma^2\hat{\eta} \tag{29}$$

Let's list down all the parameters that might be needed to get an expression for n

We have,

$$\eta = AKn^{-1}e^{nt}\cos Kx$$

$$\hat{\phi}_1 = B_1 e^{Ky}$$

$$\hat{\phi}_2 = A_2 e^{-Ky}$$

$$\frac{\partial\hat{\phi}_1}{\partial y} = K.B_1 e^{Ky}$$

$$-\frac{\partial \eta}{\partial t}\Big|_{y=0} = \frac{\partial \phi_j}{\partial y}\Big|_{y=0}$$

Here,

$$\frac{\partial \eta}{\partial t} = \frac{\partial}{\partial t}\left(AKn^{-1}e^{nt}\cos Kx\right) = n\eta = \frac{-\partial \phi_j}{\partial y}\ (j = 1,2)$$

From $\frac{\partial \eta}{\partial t}$ and $\frac{\partial \phi_j}{\partial y}$ at $y = 0$

We get,

$$B_1 = -\frac{n\eta}{K} \text{ and } A_2 = \frac{n\eta}{K} \tag{C}$$

So,

$$\hat{\phi}_1 = B_1 e^{Ky}$$

$$\hat{\phi}_2 = A_2 e^{-Ky}$$

For the upper fluid: $\hat{\phi}_1 = B_1 e^{Ky}$ is $\phi_1 = Ae^{-Ky+nt}\cos Kx$ (30)

For the lower fluid: $\hat{\phi}_2 = A_2 e^{-Ky}$ is $\phi_2 = -Ae^{Ky+nt}\cos Kx$ (31)

Substituting the above in the governing equation

$$(\hat{p}_1 - \rho_1(g+g_1)\hat{\eta}) - (\hat{p}_2 - \rho_2(g+g_1)\hat{\eta}) = \gamma^2\hat{\eta}$$

Since, $\gamma \to 0$

$$-n\rho_1\phi_1 - \rho_1(g+g_1)\eta + n\rho_2\phi_2 + \rho_2(g+g_1)\eta = 0$$

$$-n\rho_1\left(B_1e^{-Ky}\right) - \rho_1(g+g_1)\eta + n\rho_2(A_2e^{Ky}) + \rho_2(g+g_1)\eta = 0$$

$$\because y = 0, e^{Ky} = 1, e^{-Ky} = 1$$

$$-n\rho_1\left(-\frac{n\eta}{K}\right) - \rho_1(g+g_1)\eta + n\rho_2\left(\frac{n\eta}{K}\right) + \rho_2(g+g_1)\eta = 0$$

Taking η common;

$$-n\rho_1\left(-\frac{n\eta}{K}\right)-\rho_1(g+g_1)\eta+n\rho_2(\frac{n\eta}{K})+\rho_2(g+g_1)\eta=0$$

$$n\rho_1\left(\frac{n}{K}\right)-\rho_1(g+g_1)+n\rho_2(\frac{n}{K})+\rho_2(g+g_1)=0$$

$$n^2\rho_1-\rho_1(g+g_1)K+n^2\rho_2+\rho_2(g+g_1)K=0$$

$$n^2(\rho_1+\rho_2)-\rho_1(g+g_1)K+\rho_2(g+g_1)K=0$$

$$n^2(\rho_1+\rho_2)+(\rho_2-\rho_1)(g+g_1)K=0$$

$$n^2(\rho_1+\rho_2)=-K(\rho_2-\rho_1)(g+g_1)$$

$$n^2=\frac{-K(\rho_2-\rho_1)(g+g_1)}{(\rho_1+\rho_2)}$$

$$n=\sqrt{\frac{-K(\rho_2-\rho_1)(g+g_1)}{(\rho_1+\rho_2)}} \tag{32}$$

From the above expression we can conclude that, the rate of development of instability is directly proportional to $\sqrt{\frac{(\rho_2-\rho_1)}{(\rho_1+\rho_2)}}$.

If $g+g_1$ is negative, the downward acceleration is greater than g and n^2 becomes positive.

Case II: Varied velocity potential

From the paper, when $(g+g_1)<0$, the velocity potential changes accordingly.

$\eta_o = C\cos Kx$ is the initial displacement from the plane $y=0$

And,

$$\eta = C\cosh(nt)\cos Kx \tag{33}$$

Let $\phi_j(y) = (Ae^{nt-Ky} + Be^{-nt-Ky})\cos Kx$ (34)

$$-\frac{\partial \eta}{\partial t} = -nc\sinh(nt)\cos Kx = -n\eta_o \sinh(nt)$$

$$\frac{\partial \phi_1}{\partial t} = -K(Ae^{nt-Ky} + Be^{-nt-Ky})\cos Kx$$

Since we have established that $-\left.\frac{\partial \eta}{\partial t}\right|_{y=0} = \left.\frac{\partial \phi_j}{\partial y}\right|_{y=0}$

$$-nc\sinh(nt)\cos Kx = -K(Ae^{nt} + Be^{-nt})\cos Kx$$

Multiplying both sides by e^{-ky}

$$-nce^{-Ky}\sinh(nt)\cos Kx = -Ke^{-Ky}(Ae^{nt} + Be^{-nt})\cos(Kx)$$

$$\frac{nc}{K}e^{-Ky}\sinh(nt)\cos Kx = e^{-Ky}(Ae^{nt} + Be^{-nt})\cos(Kx)$$

$$\phi_1 = \frac{nc}{K}e^{-Ky}\sinh(nt)\cos Kx \tag{35}$$

Similarly $\phi_2 = -\frac{nc}{K}e^{Ky}\sinh(nt)\cos Kx$ (36)

Using the equation, the pressure can be defined as;

$$\hat{p}_1 = -\rho_1\frac{\partial \phi_1}{\partial t} \quad \text{and} \quad \hat{p}_2 = -\rho_2\frac{\partial \phi_2}{\partial t} \tag{37}$$

$$\hat{p}_1 = -\rho_1\left(\frac{n^2 c}{K}\right)e^{-Ky}\cosh(nt)\cos Kx \tag{38}$$

$$\hat{p}_2 = -\rho_2\left(\frac{n^2 c}{K}\right)e^{Ky}\cosh(nt)\cos Kx \tag{39}$$

Substituting the values in the equation

$$(\hat{p}_1 - \rho_1(g+g_1)\hat{\eta}) - (\hat{p}_2 - \rho_2(g+g_1)\hat{\eta}) = 0$$

$$-\rho_1\left(\frac{n^2 c}{K}\right)e^{-Ky}\cosh(nt)\cos Kx - \rho_1(g+g_1)\eta - \left(-\rho_2\left(\frac{n^2 c}{K}\right)e^{Ky}\cosh(nt)\cos Kx\right) + \rho_2(g+g_1)\eta = 0$$

$$-(\rho_1+\rho_2)n^2 C\cosh(nt)\cos Kx - (\rho_2-\rho_1)(g+g_1)K\eta = 0$$

$$-(\rho_1+\rho_2)n^2\eta - (\rho_2-\rho_1)(g+g_1)K\eta = 0$$

$$n^2 = \frac{-(\rho_2-\rho_1)(g+g_1)K}{(\rho_2+\rho_1)}$$

$$n = \sqrt{\frac{-K(\rho_2-\rho_1)(g+g_1)}{(\rho_2+\rho_1)}} \tag{40}$$

Growth Rate for Downward Vertical Acceleration

Considering the downwards acceleration, we obtain the same form of the growth rate, the only difference that is seen is in the acceleration part, which is shown by an opposite sign and is given as:

$$n = \sqrt{\frac{-K(\rho_2-\rho_1)(g_1-g)}{(\rho_2+\rho_1)}} \quad \because g_1 > g$$

Amplification Factor

If the amplification factor is defined as the ratio of disturbance at any time to the initial value, then from the above relations, we obtain;

$$\frac{\eta}{\eta_o} = \cosh(nt) \tag{41}$$

The above expression is similar to the expression obtained in eqn (30) of the paper.

Conclusion

The following relations were derived based on the paper.

Parameters	Relations obtained
Velocity potential for fluid 1	$\phi_1 = Ae^{-Ky+nt}\cos Kx$
Velocity potential for fluid 2	$\phi_2 = -Ae^{Ky+nt}\cos Kx$
Pressure in fluid 1	$P_1(y) = p - \rho_1(g+g_1)y + \rho_1.\phi_1^{'}$
Pressure in fluid 2	$P_2(y) = p - \rho_2(g+g_1)y + \rho_2.\phi_2^{'}$
Growth rate	$n = \sqrt{\dfrac{-K(\rho_2-\rho_1)(g+g_1)}{(\rho_1+\rho_2)}}$
Velocity potential for fluid 1 when $(g+g_1)<0$	$\phi_1 = \dfrac{nc}{K}e^{-Ky}\sinh(nt)\cos Kx$
Velocity potential for fluid 2 when $(g+g_1)<0$	$\phi_2 = -\dfrac{nc}{K}e^{Ky}\sinh(nt)\cos Kx$
Amplitude Factor	$\dfrac{\eta}{\eta_o} = \cosh(nt)$