Computer and Robot Vision I

Chapter 3

Binary Machine Vision: Region Analysis

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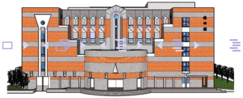

Outline

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- Texture properties (Co-occurrence matrix)
- Spatial gray level moments (Location + Intensity)
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Introduction

Recap on connected components, describing regions

Introduction

- A connected component is a region obtained by a labeling operator. The property defining a connected component is that for every pair of points within the component, there is path whose elements belong to the same component.
- There are two types of connectivity: 4-connectivity, 8-connectivity.
- A region defined by a connected component can be described in terms of its properties (geometry, texture, intensity). And these properties can be used later to describe, classify or compare regions.

Introduction (cont.)

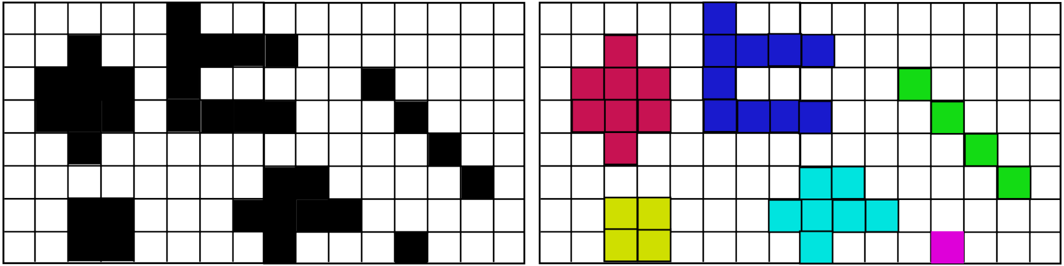


Figure 1: *Example of connected components computation (labeling).*

Introduction (cont.)

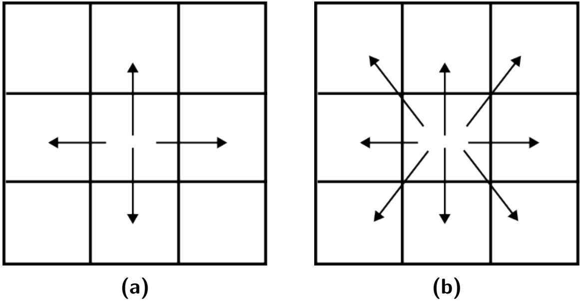
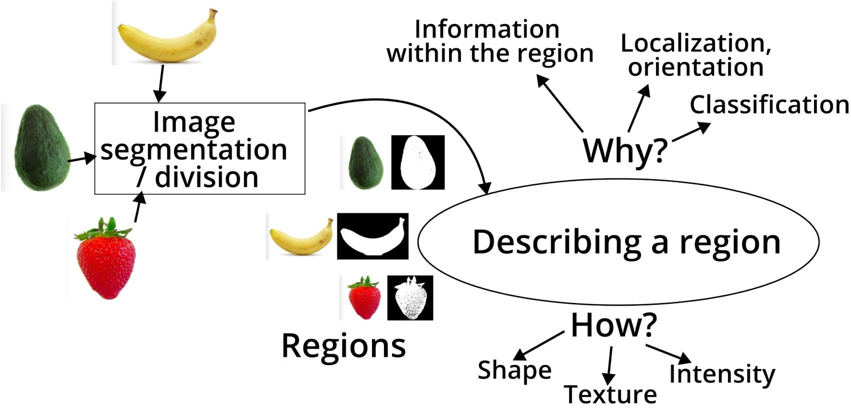


Figure 2: *Types of connectivity or neighborhood employed in region analysis: (a) 4-connectivity; (b) 8-connectivity.*

Introduction (cont.)



Region properties

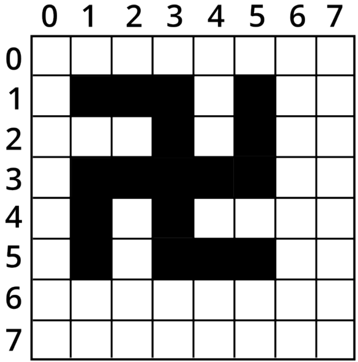
Geometrical properties

Area:

$$A = \sum_{(r,c)\in R} 1 \qquad (1)$$

Centroid:

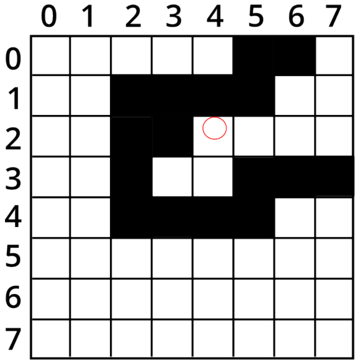
$$\bar{\mathrm{p}} = \begin{bmatrix} \bar{r} \\ \bar{c} \end{bmatrix} = \frac{1}{A} \sum_{(r,c)\in R} \begin{bmatrix} r \\ c \end{bmatrix} \qquad (2)$$



$A = 17 \qquad \bar{p} = [3, 3]^T$

Geometrical properties (cont.)

- The centroid might contain non-integers, and might not belong to the region.
- If the region has no holes and is convex, the centroid is guaranteed to belong to the region.



$A = 16 \qquad \bar{p} = [2.25, 4]^T$

Geometrical properties (cont.)

Boundary of a region

1. Inner pixel criterion: If $p = [r, c]$ s.t. $N_n(p) \subset R$ then p is in the inner part $\mathcal{I}$ of the region R.
2. Border pixel criterion: If $N_n(p) - R \neq \emptyset$, then p is in the border part B of R.
3. When **8-connectivity** is used to determine the border pixel, the resulting set of perimeter pixel is **4-connected**
4. When **4-connectivity** is used to determine the border pixel, the resulting set of perimeter pixel is **8-connected**

$$B_4 = \{(r, c) \in R | N_8(r, c) - R \neq \emptyset\} \tag{3}$$

$$B_8 = \{(r, c) \in R | N_4(r, c) - R \neq \emptyset\} \tag{4}$$

Geometrical properties (cont.)

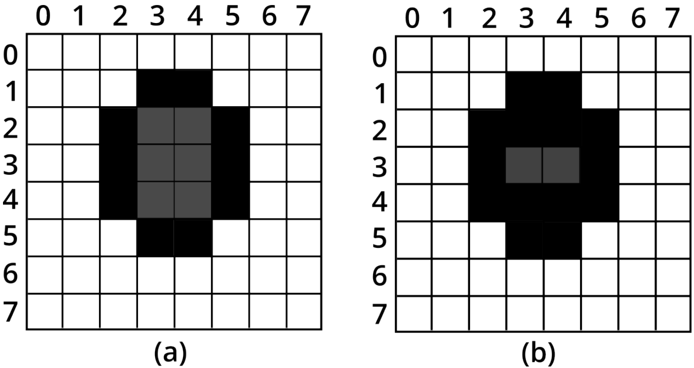


Figure 3: *Inner and border parts* ($\mathcal{I}, B$) *of a region: (a) Regarding 4-connectivity, (b) Regarding 8-connectivity.*

Geometrical properties (cont.)

Algorithm 1: Perimeter (boundary) computation

Input: The set of points contained in the boundary B_n
Result: A vector (sequence) containing the perimeter or boundary P

```
P ← [ ]; K = 0; V ← ∅                          // Initialization
p ← pickAPoint(B_n)
V ← V ∪ {p}
P.append(p)
K = K + 1
𝒯.create();                                  // Neighbor indexing table

while B_n − V ≠ ∅ do
    𝒩 ← N_n(p) ∩ B_n
    if |𝒩| > 1 then
        p ← p_0 ∈ 𝒩 − P_[K−1], min 𝒯.getVal(p, p_0)
    end
    else
        p ← p_0 ∈ 𝒩
    end
    𝒯.updateTable(p, P_[K−1])
    V ← V ∪ {p}
    P.append(p)
    K = K + 1
end
Return P
```

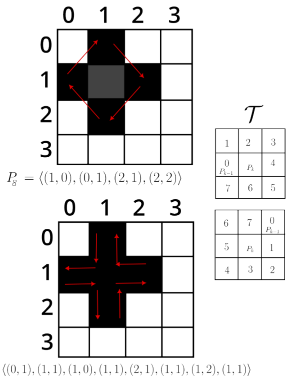


Figure 4: *Example of perimeter (contour) computation.*

Geometrical properties (cont.)

Perimeter's length $|P|$

$$
\begin{aligned}
|P| = &\#\{k | P_k \in N_4(P_{k+1})\} \\
&+ \sqrt{2}\#\{k | P_k \in N_8(P_{k+1}) - N_4(P_{k+1})\}
\end{aligned}
\tag{5}
$$

Compactness

$$
\text{Compactness} = \frac{|P|^2}{A} \tag{6}
$$

Geometrical properties (cont.)

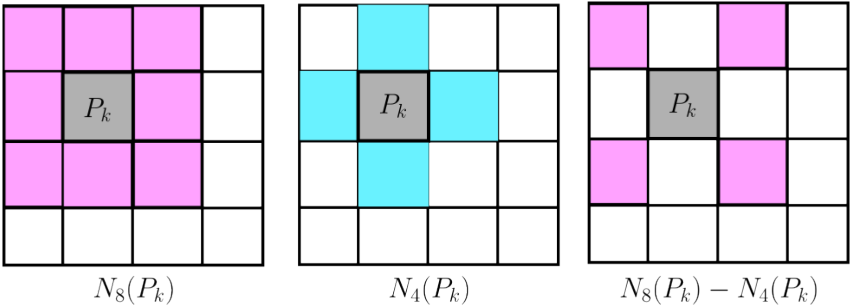


Figure 5: *Neighborhood sets employed to compute the perimeter's length.*

Geometrical properties (cont.)

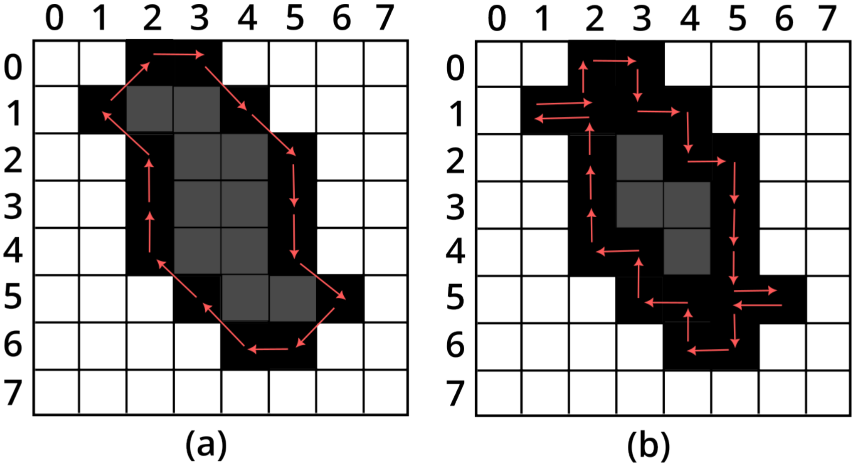


Figure 6: *Perimeter computation example: (a) Using B_8; (b) Using B_4. Note that when using B_4, $N_8(P_k)$-$N_4(P_k) = \emptyset$*

Geometrical properties (cont.)

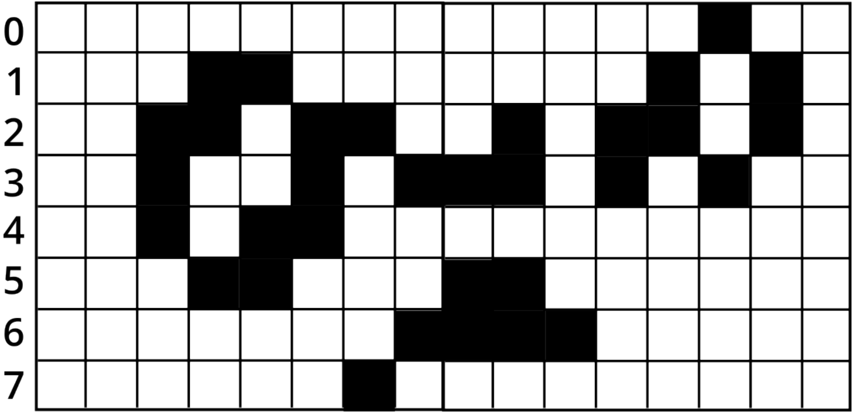


Figure 7: *There are 9 connected components. Which connectivity is used?*

Geometrical properties (cont.)

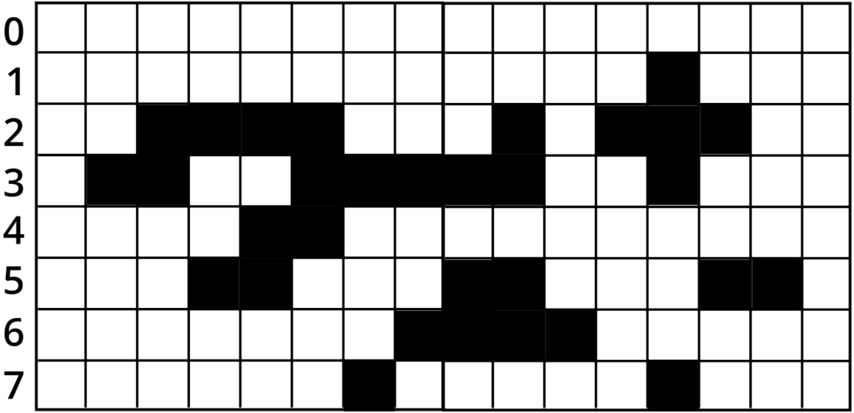


Figure 8: *There are 5 connected components. Which connectivity is used?*

Geometrical properties (cont.)

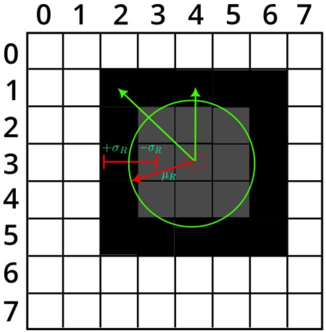
Mean centroid-boundary distance:

$$\mu_R = \frac{1}{K} \sum_{k=0}^{K-1} ||P_k - \bar{p}|| \tag{7}$$

Standard deviation of the centroid-boundary distance:

$$\sigma_R^2 = \frac{1}{K} \sum_{k=0}^{K-1} (||P_k - \bar{p}|| - \mu_R)^2 \tag{8}$$

Geometrical properties (cont.)



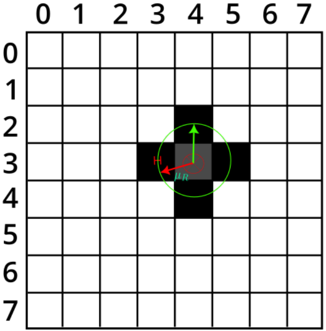
$A = 25$

$\bar{p} = [3, 4]^T$

$\mu_R = 1.87$

$\sigma_R = 0.71$

$\frac{\mu_R}{\sigma_R} = 2.63$



$A = 5$

$\bar{p} = [3, 4]^T$

$\mu_R = 1$

$\sigma_R = 0$

$\frac{\mu_R}{\sigma_R + \epsilon} \to \infty^+$

Geometrical properties (cont.)

Circularity

$$\gamma = \frac{\mu_R}{\sigma_R} \tag{9}$$

The circularity value $\gamma = \frac{\mu_R}{\sigma_R}$ has the following properties:

- Region's shape $\rightarrow$ circular shape. γ increases monotonically to infinity.
- Similar shapes (discrete or continuous) have similar γ values.
- γ is invariant (quasi-invariant) to rotation, translation, and scale.

Geometrical properties (cont.)

The centroid of a region R is $\bar{p} = \begin{bmatrix} \bar{r} \\ \bar{c} \end{bmatrix}$ is regarded as the first-order spatial moment of a region.

The second-order spatial moments around the centroid are:

$$\mu_{rr} = \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})^2 \tag{10}$$

$$\mu_{cc} = \frac{1}{A} \sum_{(r,c)\in R} (c - \bar{c})^2 \tag{11}$$

$$\mu_{rc} = \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})(c - \bar{c}) \tag{12}$$

Gray level properties

Mean gray level value:

$$\mu_I = \frac{1}{A} \sum_{(r,c) \in R} I(r,c) \tag{13}$$

Gray level variance:

$$\sigma_I = \frac{1}{A} \sum_{(r,c) \in R} (I(r,c) - \mu_I)^2 = [\frac{1}{A} \sum_{(r,c) \in R} I(r,c)^2] - \mu_I^2 \tag{14}$$

Normalized Gray level histogram:

$$H_i = \frac{1}{A} \sum_{\{(r,c) \in R | I(r,c) = i\}} 1; \quad s.t.\ i \text{ is a discrete intensity value} \tag{15}$$

Texture properties

- Gray level primitives: Regions with gray level properties (average value, max value, variance)
- Texture: Describes the spatial relationships (distribution) of the gray level primitives within the image.
- If there is a random pattern and the gray level value range is wide (high variance on the gray level primitives), the image evidences a fine-grained texture.
- If there is a well-structured pattern (checkerboard-like, hexagonal patterns) and the variance of the gray level primitives is low, the image depicts a coarse-grained texture.
- Two scales of texture: Macrotexture (gray level primitives have their own shape and organization) , Microtexture (for small gray level primitives).

Texture properties (cont.)

Co-occurrence matrix:

- Let $\mathcal{S}$ be a set of all pairs of pixels (p_a, p_b) in R following a specific spatial relationship (e.g. 4-neighbors, 8-neighbors, within a specific distance).
- The element P$_{ij}$ in ..., defines how "likely" two pixels with values i and j follows the spatial relationship.
- For the mentioned cases, P is symmetric ($P_{ij} = P_{ji}$).

$$P_{ij} = \frac{|\{(p_a, p_b)|I(p_a) = i, I(p_b) = j\}|}{|S|} \tag{16}$$

Texture properties (cont.)

Example of the computation of a GLCM (Gray-level co-occurrence matrix) for a 4×4 image with 4 gray levels.

$$S = \{[(k,l),(m,n)] \in (L \times L) \times (L \times L) \ s.t. \ k = m, |l - n| = 1\} \qquad L = \{0,1,2,3\}$$

$$S = \{[(0,0),(0,1)],[(0,1),(0,0)],[(0,1),(0,2)],[(0,2),(0,1)],[(0,2),(0,3)],[(0,2),(0,3)]\ldots\}$$

$$|S| = 24$$

	0	1	2	3
0	0	0	1	1
1	0	0	1	1
2	2	2	2	2
3	2	2	3	3

$$P_{00} = \frac{|\{[(0,0),(0,1)],[(0,1),(0,0)],[(1,0),(1,1)],[(1,1),(1,0)]\}|}{|S|} = \frac{1}{6}$$

$$P_{01} = \frac{|\{[(0,1),(0,2)],[(1,1),(1,2)]\}|}{|S|} = \frac{1}{12}$$

$$P_{10} = \frac{|\{[(0,2),(0,1)],[(1,2),(1,1)]\}|}{|S|} = \frac{1}{12}$$

$$P = \frac{1}{24}\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 0 \\ 1 & 0 & 6 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Texture properties (cont.)

Let's assume $\mathcal{L} \subset \mathbb{N}$ is the set of available gray levels (e.g. $\{0, 1, 2, ..., 254, 255\}$).

Second moment, Energy uniformity (M):

$$M = \sum_{i,j \in \mathcal{L}} P_{ij}^2 \tag{17}$$

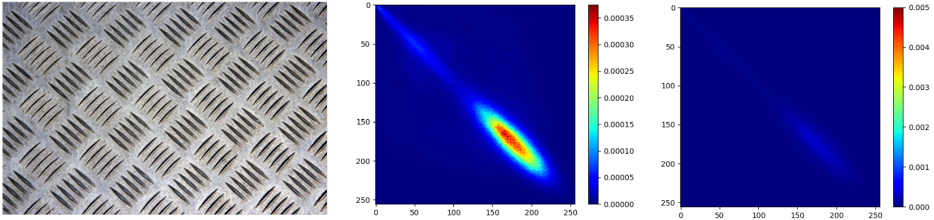


Figure 9: *Image with low energy uniformity.* $M = 12.6 \times 10^{-5}$

Texture properties (cont.)

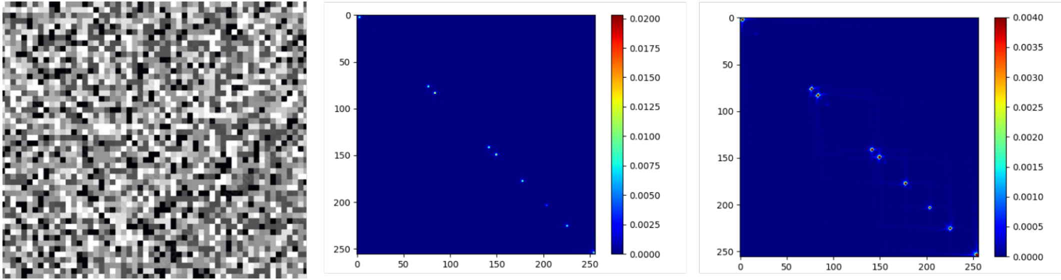


Figure 10: *Image with high energy uniformity.* $M = 373.4 \times 10^{-5}$

Texture properties (cont.)

Entropy (E):

$$E = -\sum_{i,j \in \mathcal{L}} P_{ij} \log(P_{ij}) \quad (18)$$

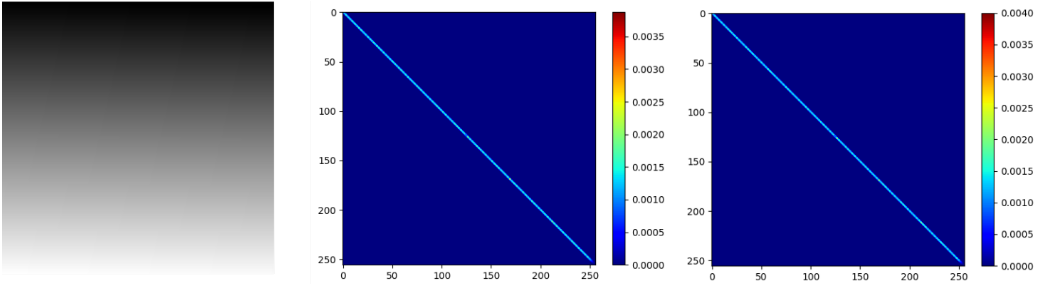


Figure 11: *Image with low entropy.* $E = 6.619$

Texture properties (cont.)

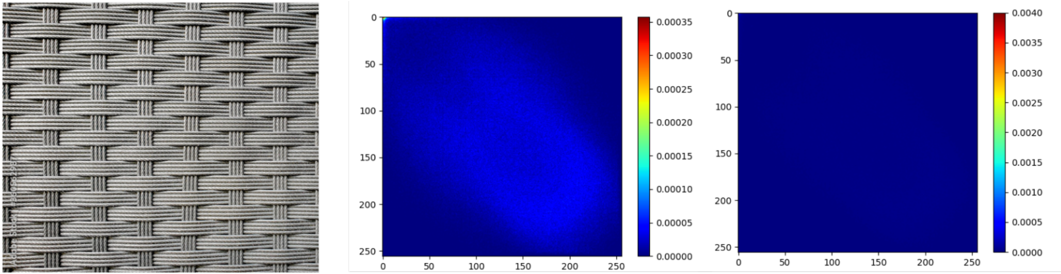


Figure 12: *Image with high entropy.* $E = 10.719$

Texture properties (cont.)

Contrast (C):

$$C = \sum_{i,j \in \mathcal{L}} |i-j| P_{ij} = \sum_{\Delta=0}^{max(\mathcal{L})} \Delta \sum_{|i-j|=\Delta} P_{ij} \tag{19}$$

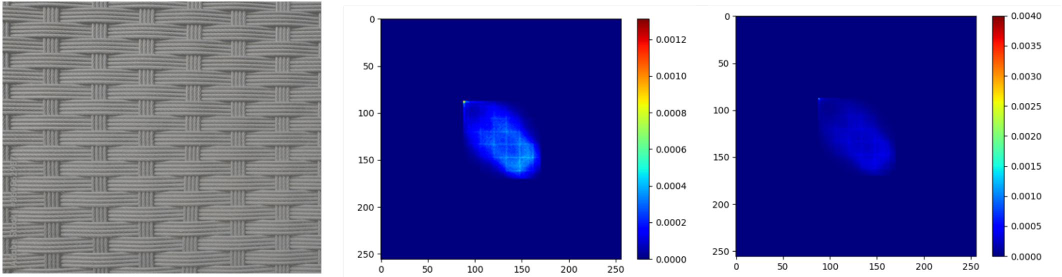


Figure 13: *Image with low contrast, and high homogeneity.* $C = 16.97$, $H = 0.114$

Texture properties (cont.)

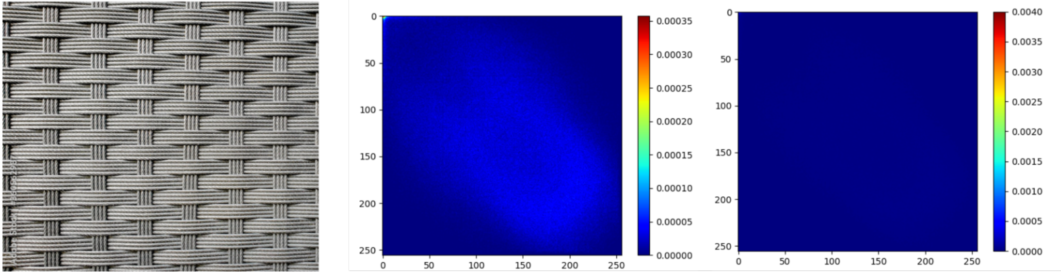
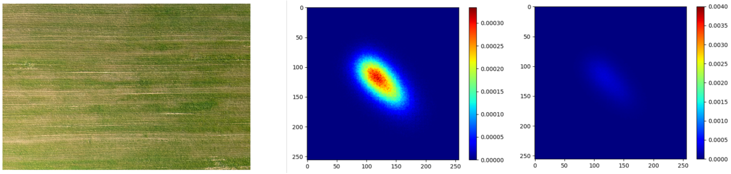
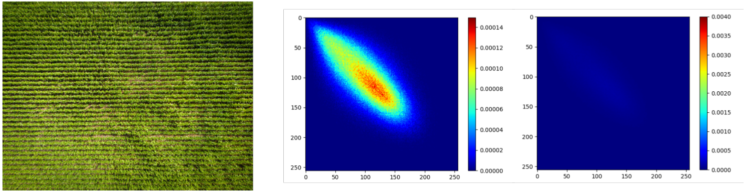


Figure 14: *Image with high contrast, and low homogeneity.* $C = 52.09$, $H = 0.049$

Texture properties (cont.)



(a)



(b)

Figure 15: *Aerial images of crops. (a) Image with* $C = 19.629$, $H = 0.112$, $E = 9.838$*: (b) Image with* $C = 24.123$, $H = 0.098$, $E = 9.838$.

Texture properties (cont.)

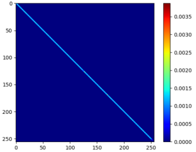
Homogeneity:

$$\sum_{i,j \in \mathcal{L}} \frac{P_{ij}}{1 + |i - j|} \tag{20}$$

Correlation (ρ):
"How linear is the relationship between i and j?"

$$\sum_{i,j \in \mathcal{L}} \frac{(i - \mu_{\mathcal{L}})(j - \mu_{\mathcal{L}}) P_{ij}}{\sigma_{\mathcal{L}}^2}$$
$$\mu_{\mathcal{L}} = \sum_{i,j \in \mathcal{L}} i P_{ij} \quad ; \quad \sigma_{\mathcal{L}}^2 = \sum_{i,j \in \mathcal{L}} (i - \mu_{\mathcal{L}})^2 P_{ij} \tag{21}$$

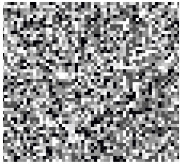
Texture properties (cont.)

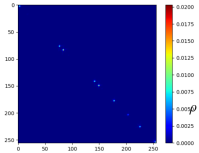


$\mu_{\mathcal{L}} = 125.7$

$\sigma_{\mathcal{L}} = 72.73$

$\rho = 0.999$

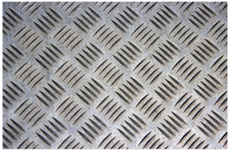


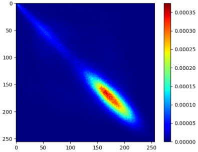


$\mu_{\mathcal{L}} = 140.2$

$\sigma_{\mathcal{L}} = 72.67$

$\rho = 0.9$

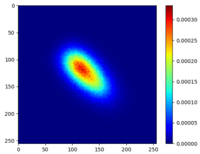




$\mu_{\mathcal{L}} = 150.9$

$\sigma_{\mathcal{L}} = 52.02$

$\rho = 0.79$



$\mu_{\mathcal{L}} = 124.9$

$\sigma_{\mathcal{L}} = 25.89$

$\rho = 0.53$

Texture properties (cont.)

The following matrices are symmetrical and $\sum_{i,j \in \{0,1,2,3,4\}} P_{i,j} = 1.0$.
Which one(s) can be a GLCM (co-occurrence matrix)?.
S represents the set of possible point pairs.

	0	1	2	3	4
0	0.2	0.0	0.0	0.0	0.0
1	0.0	0.2	0.0	0.0	0.0
2	0.0	0.0	0.2	0.0	0.0
3	0.0	0.0	0.0	0.2	0.0
4	0.0	0.0	0.0	0.0	0.2

S represents the set of point pairs having 8-connectivity

(a)

	0	1	2	3	4
0	0.0	0.1	0.05	0.05	0.1
1	0.1	0.0	0.0	0.05	0.0
2	0.05	0.0	0.0	0.05	0.0
3	0.05	0.05	0.05	0.0	0.1
4	0.1	0.0	0.0	0.1	0.0

S represents the set of point pairs having 8-connectivity

(b)

Texture properties (cont.)

The following matrices are symmetrical and $\sum_{i,j \in \{0,1,2,3,4\}} P_{i,j} = 1.0$.
Which one(s) can be a GLCM (co-occurrence matrix)?.
S represents the set of possible point pairs.

	0	1	2	3	4
0	0.0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	1.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0

$|S| = 50 \rightarrow \frac{1}{|S|} = 0.02$

(a)

	0	1	2	3	4
0	0.04	0.0	0.2	0.02	0.0
1	0.0	0.04	0.01	0.15	0.0
2	0.2	0.01	0.04	0.0	0.0
3	0.02	0.15	0.0	0.04	0.02
4	0.0	0.0	0.0	0.02	0.04

$|S| = 50 \rightarrow \frac{1}{|S|} = 0.02$

(b)

Spatial gray level moments

Generic mixed moments of order $(k + l + m)$:

$$M_{klm} = \sum_{(r,c) \in R} r^k c^l I(r,c)^m ; \text{if} m = 0- > \text{Spatial moments} \tag{22}$$

Second-order centered mixed spatial gray level moments:

$$\begin{aligned} \mu_{rg} &= \frac{1}{A} \sum_{(r,c) \in R} (r - \bar{r})[I(r,c) - \mu_I] = \left(\frac{1}{A} \sum_{(r,c) \in R} rI(r,c) \right) - \bar{r}\mu_I \\ \mu_{rg} &= \frac{M_{101}}{M_{000}} - \frac{M_{100} M_{001}}{M_{000}^2} \end{aligned} \tag{23}$$

Spatial gray level moments (cont.)

$$\mu_{cg} = \frac{1}{A} \sum_{(r,c)\in R} (c - \bar{c})[I(r,c) - \mu_I] = \left(\frac{1}{A} \sum_{(r,c)\in R} cI(r,c) \right) - \bar{c}\mu_I$$

$$\mu_{cg} = \frac{M_{011}}{M_{000}} - \frac{M_{010}M_{001}}{M_{000}^2} \tag{24}$$

Signature properties

- Signature extraction can be easily implemented in hardware. Thus its computation is straightforward and fast.
- All the above mentioned spatial properties (area, centroid, second moments, bounding rectangle) can be obtained from the signatures.
- Application: SMD soldering $->$ determining the orientation and center of well-defined shapes (rectangles, circles).

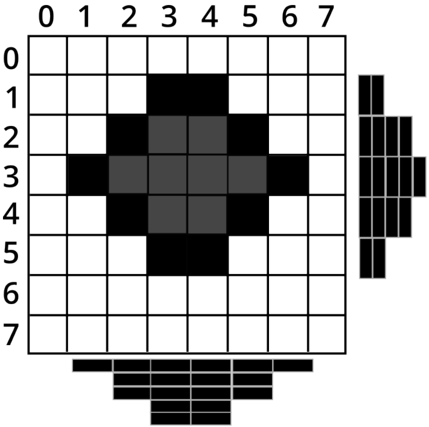
Signature of a region:

$$P_V(c) = \mid \{r \mid (r, c) \in R\} \mid \tag{25}$$

$$P_H(r) = \mid \{c \mid (r, c) \in R\} \mid \tag{26}$$

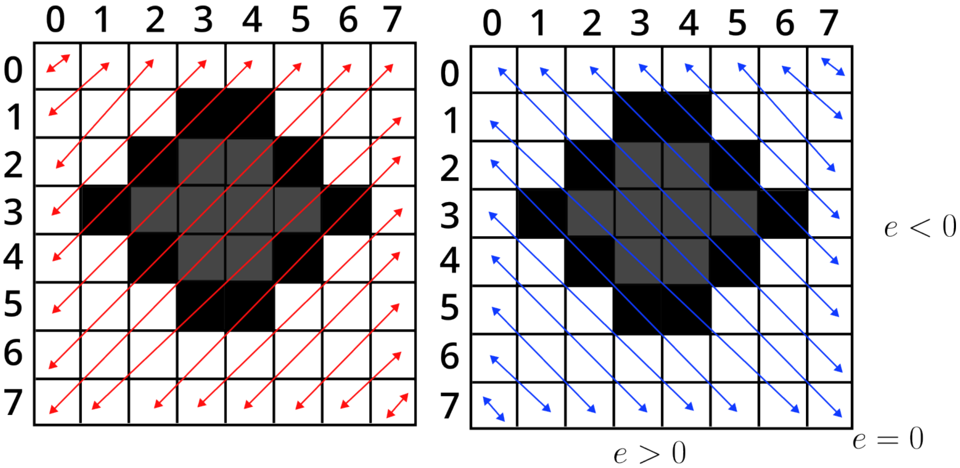
$$P_D(c) = \mid \{(r, c) \in R \mid r + c = d\} \mid \tag{27}$$

$$P_E(e) = \mid \{(r, c) \in R \mid r - c = e\} \mid \tag{28}$$



$$P_V = [0, 1, 3, 5, 5, 3, 1, 0]$$
$$P_H = [0, 2, 4, 5, 4, 2, 0, 0]$$

Figure 16: *Example about the computation of vertical and horizontal signatures.*



$$P_D(d),\ \ d \in [0, 14] \qquad P_E(e),\ \ e \in [-7, +7]$$

$$P_D = [0,0,0,0,3,3,3,3,3,3,0,0,0,0,0] \qquad P_E = [0,0,0,0,3,3,3,3,3,3,0,0,0,0,0]$$

Figure 17: *Example about the computation of diagonal signatures.*

Signature properties (cont.)

Area:

$$A = \sum_{(r,c)\in R} 1 = \sum_r \sum_{\{c|(r,c)\in R\}} 1 = \sum_r P_H(r) \\ = \sum_c \sum_{\{r|(r,c)\in R\}} 1 = \sum_c P_V(c) \tag{29}$$

Centroid:

$$\bar{\mathrm{p}} = \begin{bmatrix} \bar{r} \\ \bar{c} \end{bmatrix} = \frac{1}{A} \sum_{(r,c)\in R} \begin{bmatrix} r \\ c \end{bmatrix} = \frac{1}{A} \begin{bmatrix} \sum_r rP_H(r) \\ \sum_c cP_H(c) \end{bmatrix} \tag{30}$$

Signature properties (cont.)

Diagonal centroid $\bar{e}$:

$$
\begin{aligned}
\bar{e} &= \frac{1}{A}\sum_{e} eP_E(e) = \frac{1}{A}\sum_{e} e \sum_{\{(r,c)\in R | r-c=e\}} 1 = \frac{1}{A}\sum_{e} \sum_{\{(r,c)\in R | r-c=e\}} (r-c) \\
&= \frac{1}{A}\sum_{(r,c)\in R} r-c \\
&= \bar{r} - \bar{c}
\end{aligned} \tag{31}
$$

Signature properties (cont.)

Diagonal centroid $\bar{d}$:

$$
\begin{aligned}
\bar{d} &= \frac{1}{A}\sum_{d} dP_D(d) = \frac{1}{A}\sum_{d} d \sum_{\{(r,c)\in R \mid r+c=d\}} 1 = \sum_{d} \sum_{\{(r,c)\in R \mid r+c=d\}} (r+c) \\
&= \frac{1}{A}\sum_{(r,c)\in R} r+c \\
&= \bar{r}+\bar{c}
\end{aligned}
\tag{32}
$$

Signature properties (cont.)

Second-order spatial moments from signature:

$$
\begin{aligned}
\mu_{rr} &= \frac{1}{A}\sum_{(r,c)\in R}(r-\bar{r})^2 = \frac{1}{A}\sum_{r}\sum_{\{c|(r,c)\in R\}}(r-\bar{r})^2 = \sum_{r}(r-\bar{r})^2\sum_{\{c|(r,c)\in R\}}1 \\
&= \sum_{r}(r-\bar{r})^2 P_H(r)
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
\mu_{cc} &= \frac{1}{A}\sum_{(r,c)\in R}(c-\bar{c})^2 = \frac{1}{A}\sum_{c}\sum_{\{r|(r,c)\in R\}}(c-\bar{c})^2 = \sum_{c}(c-\bar{c})^2\sum_{\{r|(r,c)\in R\}}1 \\
&= \sum_{c}(c-\bar{c})^2 P_V(c)
\end{aligned} \tag{34}
$$

Signature properties (cont.)

Second-order diagonal moment:

$$
\begin{aligned}
\mu_{dd} &= \frac{1}{A}\sum_{d}(d-\bar{d})^2 P_D(d) = \frac{1}{A}\sum_{d}\sum_{\{(r,c)\in R \mid r+c=d\}}(r+c-\bar{r}-\bar{c})^2 \\
&= \frac{1}{A}\sum_{(r,c)\in R}((r-\bar{r})+(c-\bar{c}))^2 \\
&= \frac{1}{A}\sum_{(r,c)\in R}(r-\bar{r})^2 + \frac{2}{A}\sum_{(r,c)\in R}(r-\bar{r})(c-\bar{c}) + \frac{1}{A}\sum_{(r,c)\in R}(c-\bar{c})^2 \\
\mu_{dd} &= \mu_{rr} + 2\mu_{rc} + \mu_{cc}
\end{aligned}
\tag{35}
$$

Signature properties (cont.)

Second-order diagonal moment:

$$\mu_{ee} = \frac{1}{A}\sum_{e}(e-\bar{e})^2 P_D(e) = \frac{1}{A}\sum_{e}\sum_{\{(r,c)\in R|r-c=e\}}(r-c-\bar{r}+\bar{c})^2 \tag{36}$$

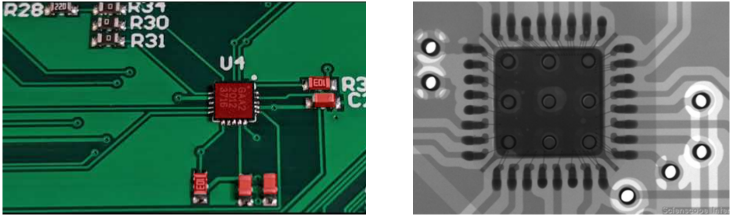
$$\mu_{ee} = \mu_{rr} - 2\mu_{rc} + \mu_{cc}$$

$$\mu_{rc} = \frac{\mu_{dd} - \mu_{ee}}{4} \tag{37}$$

Signature properties (cont.)

Application of signature analysis to SMT (Surface Mount Technology) soldering

- Most of SMT components have a rectangular shape (resistor, capacitor, transistor, diodes, IC, inductances)
- A multilayer SMT PCB presents many holes (screws, inter-layer connections, soldering points) with circular shapes.
- The proper functioning of SMD circuits (motherboards) requires a precise and fast procedure of picking, placing, and soldering of the components within the PCB.
- How can we quickly obtain the location of rectangular and circular regions in an image depicting a SMT circuit.

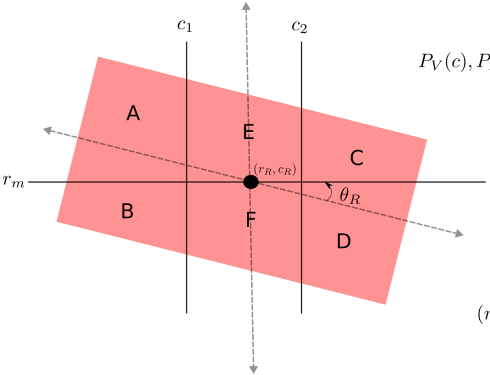


(a) **(b)**

Figure 18: *Examples of images taken from PCBs: (a) Color image; (b) X-ray image.*

Signature Analysis

- Employ region signatures (P_H , P_V).
- Rectangle (position, orientation), Circle (position, area).

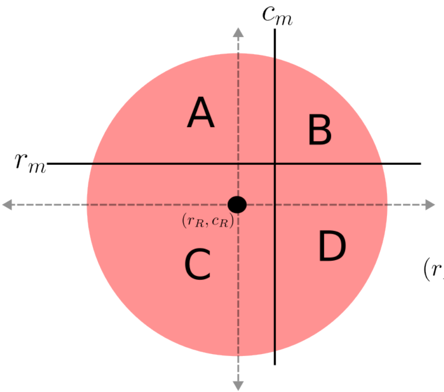


$P_V(c), P_H(r)$ are computed in advance

$$A + B = \sum_{c=0}^{c_1} P_V(c)$$
$$E + F = \sum_{c=c_1}^{c_2} P_V(c)$$
$$C + D = \sum_{c=c}^{W} P_V(c)$$
$$A + E + C = \sum_{r=0}^{r_m} P_H(r)$$
$$B + F + D = \sum_{r=r_m}^{H} P_H(r)$$
$$\downarrow$$
$$(r_R, c_R) = \mathscr{L}(A, B, C, D, E, F)$$
$$\theta_R = \mathscr{O}(A, B, C, D, E, F)$$

Figure 19: *Finding the location and orientation of a rectangle from its signature.*

Signature Analysis



$$A + C = \sum_{c=0}^{c_m} P_V(c)$$

$$A + B = \sum_{r=0}^{r_m} P_H(r)$$

$$C + D = \sum_{r=r_m}^{H} P_H(r)$$

$$B + D = \sum_{c=c_m}^{W} P_V(c)$$

$$\downarrow$$

$$(r_R, c_R) = \mathscr{L}(A, B, C, D, E, F)$$

$$\text{Area} = \sum_c P_V(c) = \sum_r P_H(r)$$

Figure 20: *Finding the location and area of a circle from its signature.*

Extremal points, Line-segment length and orientation

Extremal points, Line segments

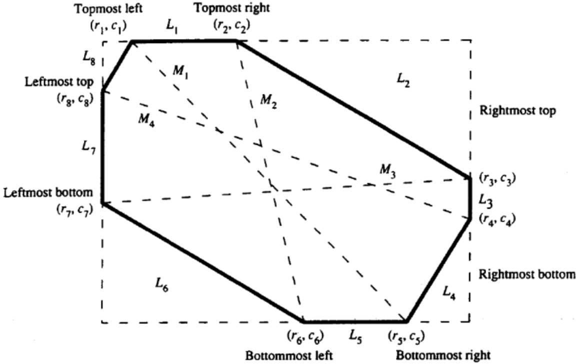


Figure 21: *The 8 extremal points of a region and the axes between them. Note that a extremal point must belong to the region*

Point		Opposite point		Axis
Topmost left	(r_1, c_1)	Bottommost right	(r_5, c_5)	M_1
Topmost right	(r_2, c_2)	Bottommost left	(r_6, c_6)	M_2
Leftmost bottom	(r_7, c_7)	Rightmost top	(r_3, c_3)	M_3
Leftmost top	(r_8, c_8)	Rightmost bottom	(r_4, c_4)	M_4

Extremal points, Line segments (cont.)

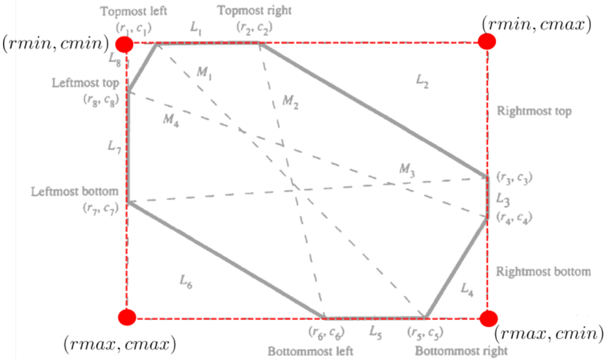


Figure 22: *The points defining the bounding box of a region. Note that the points might not belong to the region*

Extremal edge	Definition
Topmost row ($rmin$)	$\min\{r \mid (r, c) \in R\}$
Bottommost row ($rmax$)	$\max\{r \mid (r, c) \in R\}$
Leftmost column ($cmin$)	$\min\{c \mid (r, c) \in R\}$
Rightmost column ($cmax$)	$\max\{c \mid (r, c) \in R\}$

Extremal points, Line segments (cont.)

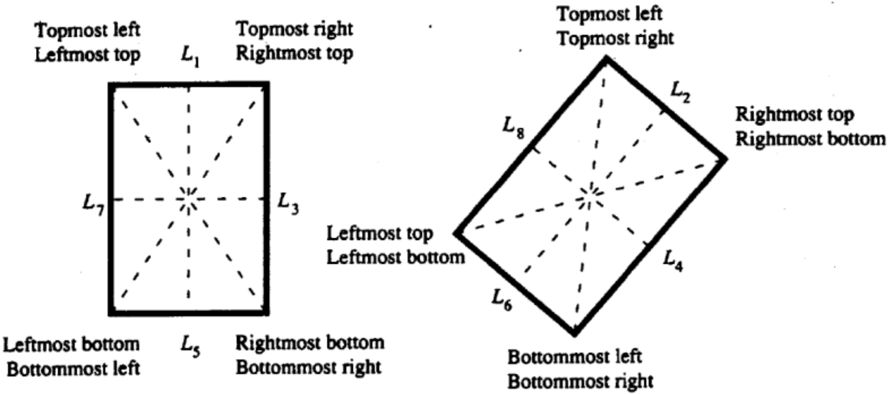


Figure 23: *Special cases when the number of extremal points is reduced to 4.*

Extremal points, Line segments (cont.)

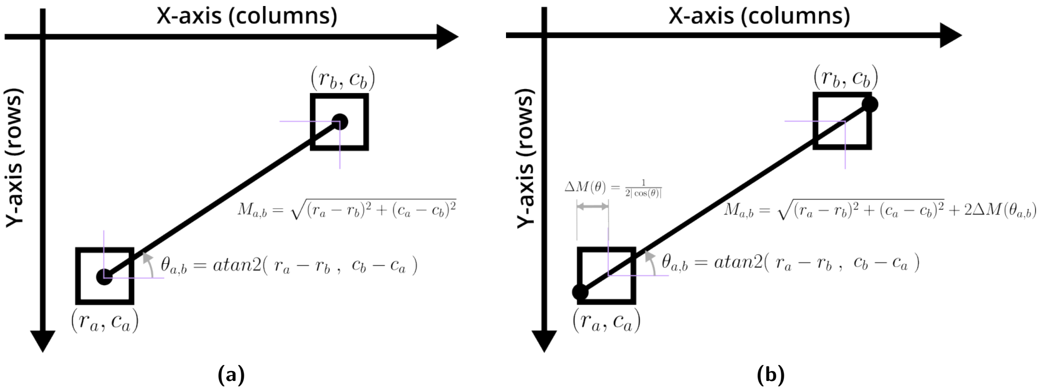


Figure 24: *Length and angle of a segment between two pixels: (a) Regarding the pixels' centers; (b) Regarding the pixels' edges.*

Extremal points, Line segments (cont.)

Length compensation factor:

$$Q(\phi) = \begin{cases} \frac{1}{|\cos(\phi)|} & \bmod (|(\phi)|, 180^o) \leq 45^o \\ \frac{1}{|\sin(\phi)|} & \bmod (|(\phi)|, 180^o) > 45^o \end{cases} \quad (38)$$

Compensation factor's expected value:

$$\mathbb{E}[Q(\phi)] = 1.12 \quad (39)$$

Maximum length error:

$$\begin{gathered} \max(Q(\phi)) = \sqrt{2} \\ \therefore |Q(\phi) - \mathbb{E}[Q(\phi)]| \leq 0.294 \end{gathered} \quad (40)$$

Extremal points, Line segments (cont.)

$$M_1 = \sqrt{(r_1 - r_5)^2 + (c_1 - c_5)^2} + Q(\phi_1)$$

$$M_2 = \sqrt{(r_2 - r_6)^2 + (c_2 - c_6)^2} + Q(\phi_2)$$

$$M_3 = \sqrt{(r_3 - r_7)^2 + (c_3 - c_7)^2} + Q(\phi_3)$$

$$M_4 = \sqrt{(r_4 - r_8)^2 + (c_4 - c_8)^2} + Q(\phi_4)$$

(a)

$$\phi_1 = \tan^{-1} \frac{r_1 - r_5}{-(c_1 - c_5)}$$

$$\phi_2 = \tan^{-1} \frac{r_2 - r_6}{-(c_2 - c_6)}$$

$$\phi_3 = \tan^{-1} \frac{r_3 - r_7}{-(c_3 - c_7)}$$

$$\phi_4 = \tan^{-1} \frac{r_4 - r_8}{-(c_4 - c_8)}$$

(b)

Figure 25: *Properties of the axes formed between mate extremal points: (a) Lengths; (b) Orientation angles (counterclockwise)*

Extremal points, Line segments (cont.)

Examples on regions depicting line, triangle, and rectangle:

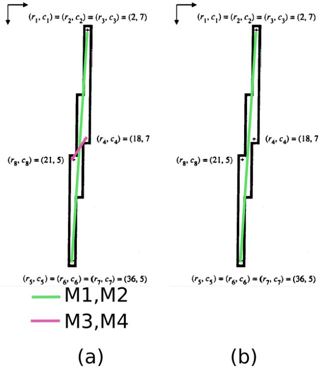


Figure 26: *A line shaped region: (a) Its axes; (b) Relevant segments.*

Extremal points, Line segments (cont.)

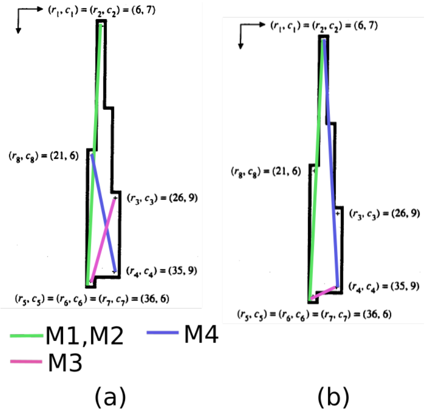


Figure 27: *A triangle shaped region: (a) Its axes; (b) Relevant segments.*

Extremal points, Line segments (cont.)

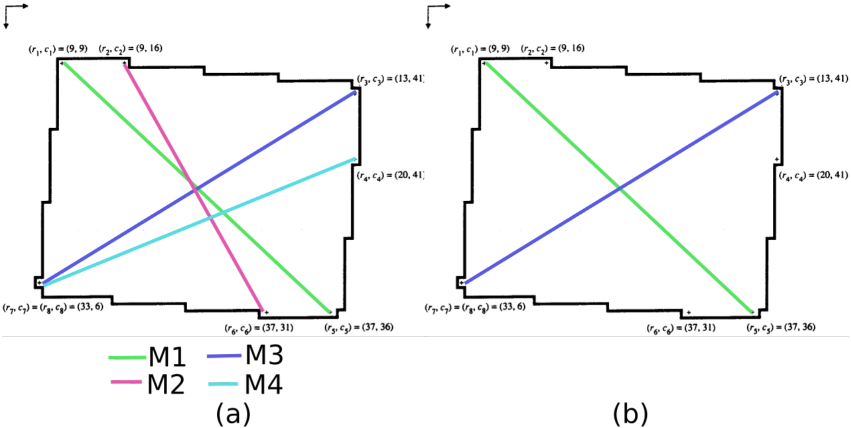


Figure 28: *A rectangle shaped region: (a) Its axes; (b) Relevant segments.*

Additional region properties

Additional region properties

- The previously mentioned region descriptors might not be the best option to know whether two regions are similar.
- The moments M_{klm} of a region capture its shape information to some extent. By increasing the moment order we can obtain more information about the region's shape.
- The region moments are sensitive to affine transformations (scale, rotation, translation).
- Moment invariants: A set of scalars, obtained from the region moments, whose values are invariant to the 3 mentioned affine transformations.

Additional region properties (cont.)

The moments of a region :

$$M_{klm} = \sum_{(r,c)\in R} r^k c^l I(r,c)^m$$
$$M_{kl} = \sum_{(r,c)\in R} r^k c^l I(r,c) \tag{41}$$

$$\bar{p} = \begin{bmatrix} \bar{r} & \bar{c} = \end{bmatrix} \begin{bmatrix} \frac{M_{10}}{M_{00}} & \frac{M_{01}}{M_{00}} \end{bmatrix} \tag{42}$$

Central moments:

$$\mu_{kl} = \sum_{(r,c)\in R} (r-\bar{r})^k (c-\bar{c})^l I(r,c) \tag{43}$$

$$\mu_{00} = M_{00} \qquad \mu_{02} = M_{02} - \bar{c}M_{01}$$
$$\mu_{01} = 0 \qquad \mu_{21} = M_{21} - 2\bar{r}M_{11} - \bar{c}M_{20} + 2\bar{r}^2 M_{01}$$
$$\mu_{10} = 0 \qquad \mu_{12} = M_{12} - 2\bar{c}M_{11} - \bar{r}M_{02} + 2\bar{c}^2 M_{10}$$
$$\mu_{11} = M_{11} - \bar{r}M_{01} \qquad \mu_{30} = M_{30} - 3\bar{r}M_{20} + 2\bar{r}^2 M_{10}$$
$$\mu_{20} = M_{20} - \bar{r}M_{10} \qquad \mu_{03} = M_{03} - 3\bar{c}M_{02} + 2\bar{c}^2 M_{01}$$

Additional region properties (cont.)

- By definition, the central moments are invariant to translation, yet not invariant to scale neither rotation.
- Use the zeroth-order central moment to achieve scale invariance:

$$\eta_{kl} = \frac{\mu_{kl}}{\mu_{00}^{1+\frac{k+l}{2}}}$$

- The Hu's 7 moment invariants I_m:

$$\Theta = \{\eta_{kl} | k, l \in \mathbb{N},\ k + l = 3\}$$

$$I_m = \mathbb{P}_m(\Theta)$$

Additional region properties (cont.)

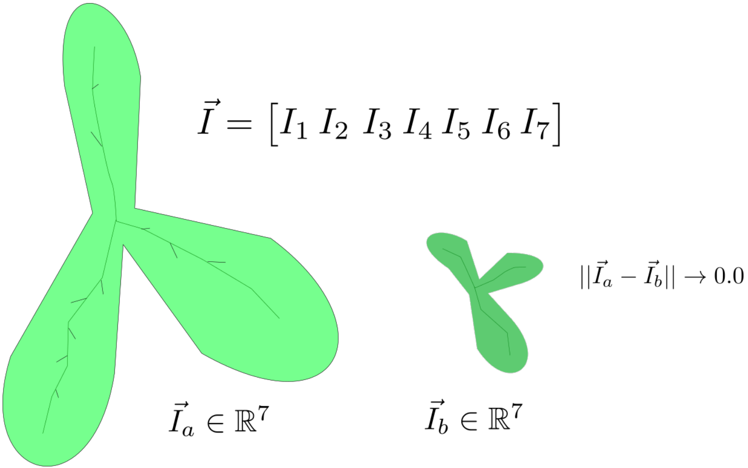


Figure 29: *Comparing two similar regions using Hu moment invariants.*

Additional region properties (cont.)

Fourier descriptors

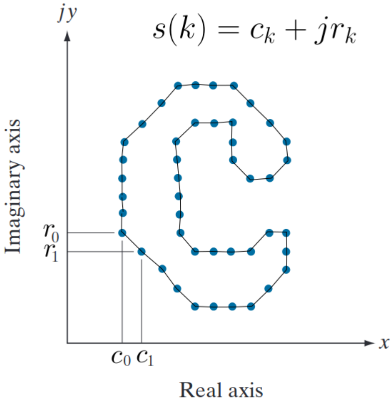
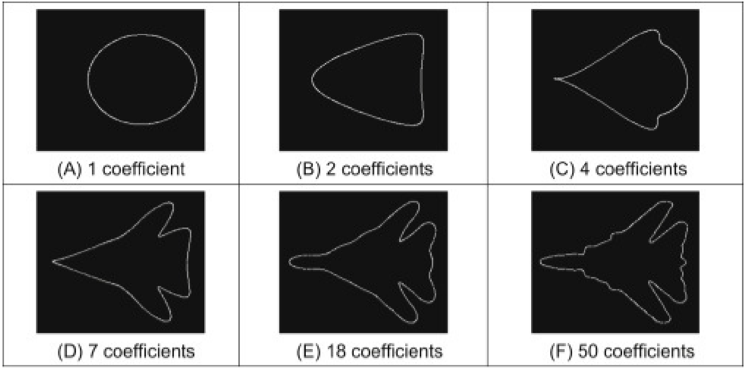


Figure 30: *Arranging the points in the boundary into a sequence of complex numbers.*

$$a(u) = \sum_{k=0}^{K-1} s(k) \exp^{-j2\pi uk/K} \tag{44}$$

$$s(k) = \frac{1}{K} \sum_{u=0}^{K-1} a(u) \exp^{j2\pi uk/K} \tag{45}$$

Additional region properties (cont.)



Source: M. S. Nixon, A. S. Aguado
Feature Extraction and Image Processing for Computer Vision (Fourth Edition)

Figure 31: *Boundary approximation using Fourier descriptors.*

Additional region properties (cont.)

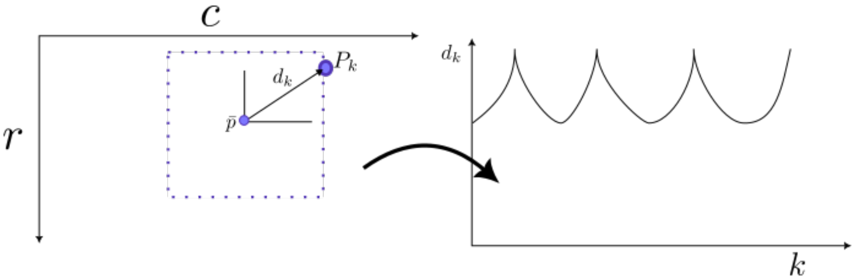


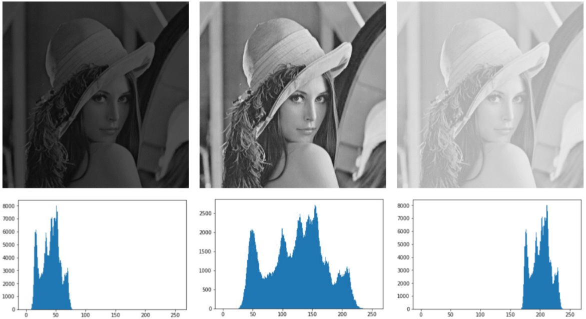
Figure 32: *Another way to describe a shape: Distance boundary-to-centroid.*

$$F = \mathcal{F}(d); d \in \mathbb{R}^K, \; F \in \mathbb{C}^K \tag{46}$$

Homework

Due: October 15, 2024

Histogram Equalization

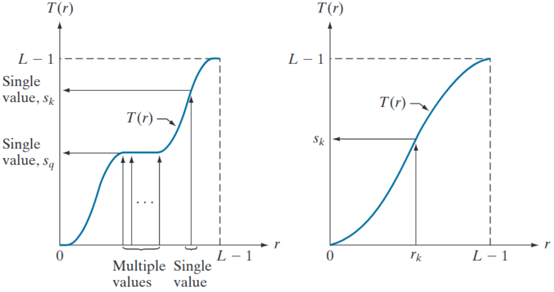


Low brightness, low contrast

Original

High brightness, low contrast

- Histogram equalization comprises a pixel-level transformation, which aims to broaden the width of the histogram thus increasing the contrast.
- A pixel-level transformation entails a function $s = T(r)$, where r is the intensity of a pixel in the original image, and s is the intensity after applying the transformation.
- In order to keep the appearance of objects in the original image, T must be a monotonically increasing single-valued function. In some cases it is better to ensure that $T^{-}1$ is single-valued. Thus $r = T^{-1}(s)$.



- It is proved that if $T(r)$ is a CDF (cumulative distribution function) of any PDF (probability distribution function), the PDF of s will follow a uniform distribution (equalization).
- The equalization works for any continuous PDF. If the PDF is discrete the equalization does not work. However, it has a similar effect.
- The normalized histogram of the original image is

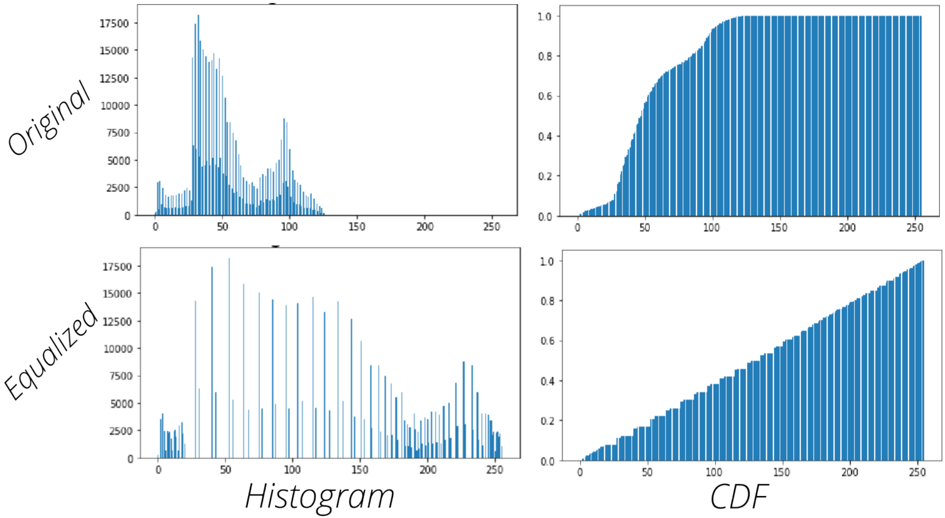
$$p_r(r_k) = \frac{n_k}{MN}$$

.

- The transformation $s = T(r)$ used to perform Histogram Equalization is defined as a discrete CDF:

$$s(r_k) = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j); k = \{0, 1, 2, ..., L-1\}$$

.


Original
Equalized
Histogram
CDF
17500
15000
12500
10000
7500
5000
2500
0
1.0
0.8
0.6
0.4
0.2
0.0
0 50 100 150 200 250

Homework

Histogram Equalization

- Write a program to perform histogram equalization:
    1. Show the original image and its histogram.
    2. Show an image obtained by scaling the intensity of the original image by $1/3$ and plot its histogram.
    3. Perform histogram equalization over the image generated in (2.). Show the equalized image and plot its histogram.
- Due date: **October 08, 2024**