# LAB4

## 注意事項

1. Lab 的繳交期限為**星期二(3/21)17:00 a.m.**。
2. Lab 的分數分配：Lab 分數 100%，Bonus 20%。
3. 請儘量於 Lab 時段完成練習，完成後請找助教檢查，檢查後即可離開。
4. 檔名規定: 檔名錯誤將記為 0 分
   **i.** Lab: 請用 **學號_LabNumber** 為檔名做一個資料夾 **(e.g.,N96091350_Lab4)**，將 **ipynb 檔**放入資料夾，壓縮後上傳至課程網站**(e.g., N96091350_Lab4.zip)**
   ii. Bonus: 請用 **學號_bonus** 為檔名做一個資料夾**(e.g.,N96091350_bonus)**，將 **ipynb 檔**放入資料夾，壓縮後上傳至課程網站**(e.g., N96091350_bonus.zip)**。
5. **Code 中需有註解。**
6. 未完成者可於**下周一 (3/27) 09:00 a.m.** 前上傳至 Moodle，惟補交的分數將乘以 0.8 計，超過期限後不予補交。
7. Bonus 需於**下周一 (3/27) 09:00 a.m.**前上傳至 Moodle，不予補交。
8. 準時繳交者，請交至「Lab4 準時繳交區」；補交者，請交至「Lab4 補交區」；bonus 請繳交至「bonus 繳交區」
9. 本次 lab 及 bonus 為同一個 template 檔，繳交同一個檔案即可。

**請勿抄襲，抄襲者與被抄襲者本次作業皆 0 分計算**

## Total: 120%

**1.(100%)** Please download the template file `GELU.ipynb`. Please **follow the step1-step3 in the file** `GELU.ipynb` and **meet the requirements** in it. In the file, you need to use numpy array to represent the equations below, and print the coefficients, answers of each equations, and augmented matrix separately. By what you do above, do Gauss-Elimination.

$$5y - 7z = 7$$
$$4x - z = 8$$
$$x - y - z = 9$$

The output is:

```
the augment matrix after gauss eliminatio:
[[ 4.    0.    -1.     8.   ]
 [ 0.    5.    -7.     7.   ]
 [ 0.    0.    -2.15   8.4  ]]
```

**Bonus(20%).** Use the **same file** `GELU.ipynb` as 1., please follow the **step4-step5,** and do LU-decomposition of the equation above and use the module of `scipy.linalg` to check your answer.

The LU decomposition method aims to turn A into the product of two matrices L and U, where L is a lower triangular matrix while U is an upper triangular matrix.

$$Ax = LU_x = y \rightarrow \begin{bmatrix} l_{1.1} & 0 & 0 & 0 \\ l_{2.1} & l_{2.2} & 0 & 0 \\ l_{3.1} & l_{3.2} & l_{3.3} & 0 \\ l_{4.1} & l_{4.2} & l_{4.3} & l_{4.4} \end{bmatrix} \begin{bmatrix} u_{1.1} & u_{1.2} & u_{1.3} & u_{1.4} \\ 0 & u_{2.2} & u_{2.3} & u_{2.4} \\ 0 & 0 & u_{3.3} & u_{3.4} \\ 0 & 0 & 0 & u_{4.4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ m_{2.1} & 1 & 0 & 0 \\ m_{3.1} & m_{3.2} & 1 & 0 \\ m_{4.1} & m_{4.2} & m_{4.3} & 1 \end{bmatrix} \begin{bmatrix} u_{1.1} & u_{1.2} & u_{1.3} & u_{1.4} \\ 0 & u_{2.2} & u_{2.3} & u_{2.4} \\ 0 & 0 & u_{3.3} & u_{3.4} \\ 0 & 0 & 0 & u_{4.4} \end{bmatrix}$$

Where the elements below the diagonal elements ($m_{2,1}, m_{3,1}, m_{4,1}, m_{4,2}, m_{4,3}$) are the multipliers that multiply the pivot equations to eliminate the elements during the calculation. Now, we define $Ux = M$, then the above equations become:

$$LM = y$$

$$\begin{bmatrix} l_{1.1} & 0 & 0 & 0 \\ l_{2.1} & l_{2.2} & 0 & 0 \\ l_{3.1} & l_{3.2} & l_{3.3} & 0 \\ l_{4.1} & l_{4.2} & l_{4.3} & l_{4.4} \end{bmatrix} M = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

After we solve for M, we can use $Ux = M$ to solve for x.

$$U_x = M$$

$$\begin{bmatrix} u_{1.1} & u_{1.2} & u_{1.3} & u_{1.4} \\ 0 & u_{2.2} & u_{2.3} & u_{2.4} \\ 0 & 0 & u_{3.3} & u_{3.4} \\ 0 & 0 & 0 & u_{4.4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix}$$

The output of LU is:

```
L:
[[ 1.    0.    0.  ]
 [ 0.    1.    0.  ]
 [ 0.25 -0.2   1.  ]]
U:
[[ 4.    0.   -1.  ]
 [ 0.    5.   -7.  ]
 [ 0.    0.   -2.15]]
```