Parallel Partition

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Partition

- input: a range and unary predicate
- output: the range with rearranged elements:
  all elements where the predicate is true come first

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2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

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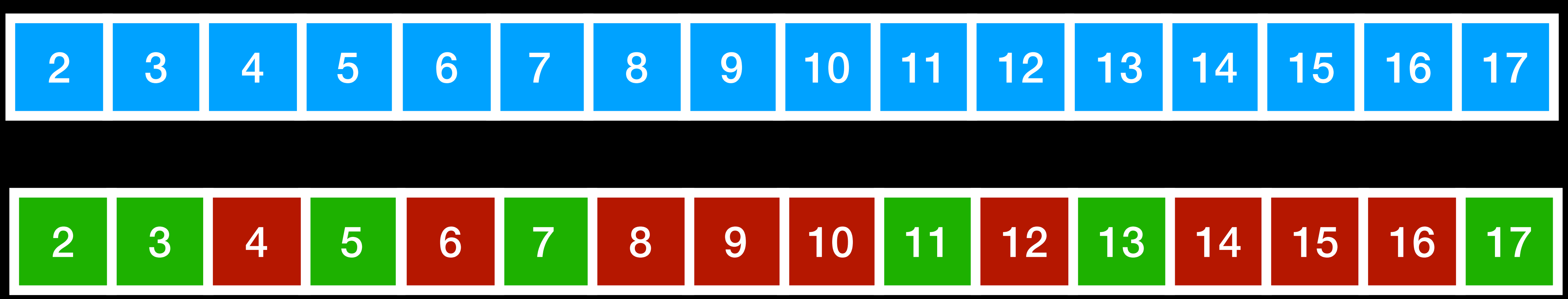

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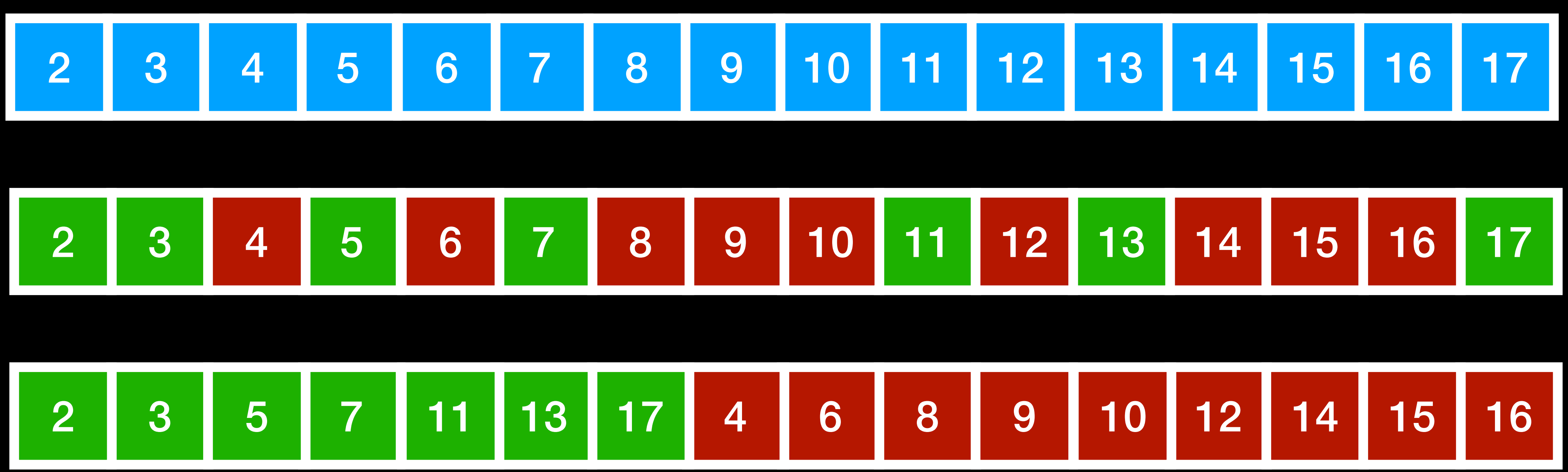

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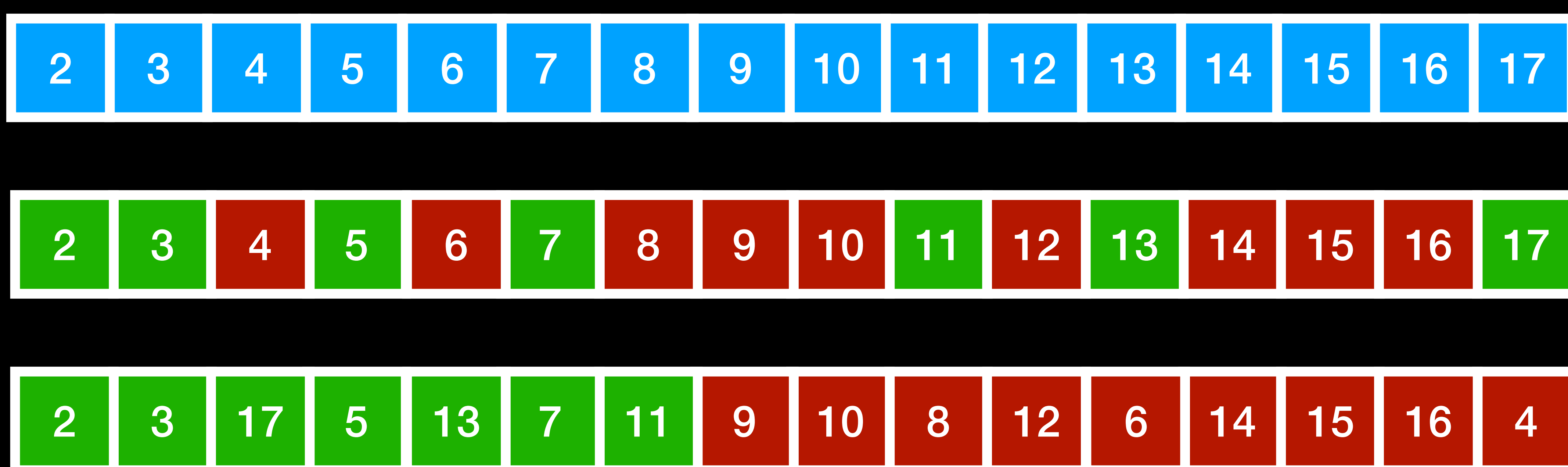
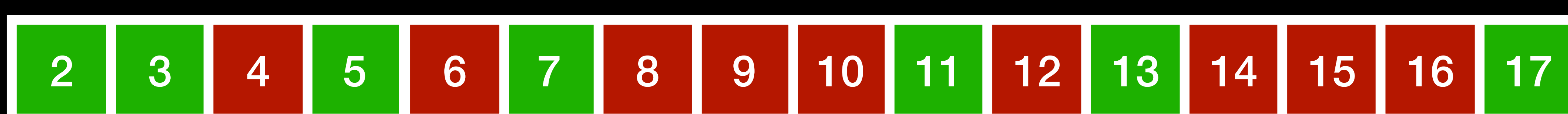


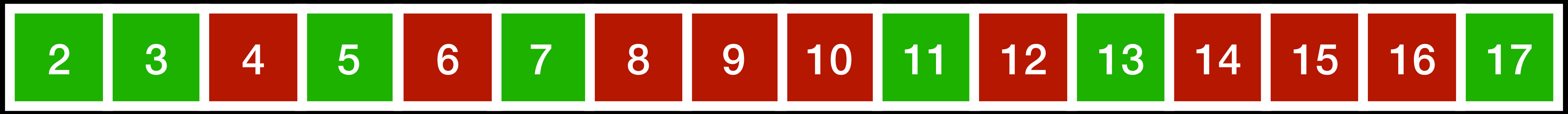
Lomuto’s Partition

```
template <typename FwdIt, typename Predicate>
FwdIt lomuto(FwdIt it, FwdIt end, Predicate predicate) {
   FwdIt to(it);
   for (; it != end; ++it)
       if (predicate(*it))
           std::iter_swap(to++, it);
   return to;
}
```



Hoare's Partition

```
template <typename BiDirIt, typename Predicate>
BiDirIt hoare(BiDirIt it, BiDirIt end, Predicate predicate) {
   while (true) {
       while (it != end && predicate(*it)) { ++it; }
       while (it != end && !predicate(*--end)) {}
       if (it == end) { return it; }
       std::iter_swap(it++, end);
}
```



Sequential

- Lomuto’s and Hoare’s scheme do not parallelise
  - Hoare’s scheme is very effective when sequential
- something operating independently is needed:
  - do most work in parallel on parts of the range
  - clean up where things remain out of order

Blocked Partition

```
template <typename RndIt, typename Predicate>
RndIt blocked(RndIt begin, RndIt end, Predicate pred) {
   BlockQueue<RndIt> q(begin, end); Block<RndIT> f, b;
   while (true) {
      if (f.empty() && (f = q.front()).empty()) { break; }
      if (b.empty() && (b = q.back()).empty()) { break; }
      tie(f, b) = block(f, b, pred);
   }
   return clean_up(f, b, pred);
}
```

Blocked Partition

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template <typename RndIt, typename Predicate>
RndIt blocked(RndIt begin, RndIt end, Predicate pred) {
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      tie(f, b) = block(f, b, pred);
   }
   return clean_up(f, b, pred);
}
```

Block Queue

```
template <typename RndIt> struct BlockQueue {
  RndIt beg, end; int size; constexpr int bs = 123;
  BlockQueue(RndIt b, RndIt e): beg(b), end(e), size(e-b) {}
  Block front() {
    auto s = min(size, bs); size -= s; beg += s;
    return Block<RndIt>(beg - s, beg); }
  Block back() {
    auto s = min(size, bs); size -= s; end -= s;
    return Block<RndIt>(end, end + s); }
};
```

Blocked Partition

```
template <typename RndIt, typename Predicate>
RndIt blocked(RndIt begin, RndIt end, Predicate pred) {
  BlockQueue<RndIt> q(begin, end); Block<RndIT> f, b;
  while (true) {
    if (f.empty() && (f = q.front()).empty()) { break; }
    if (b.empty() && (b = q.back()).empty()) { break; }
    tie(f, b) = block(f, b, pred);
  }
  return clean_up(f, b, pred);
}
```

Block Partition

```
template <typename Blk, typename Predicate>
std::pair<Blk, Blk> block(Blk f, Blk b, Predicate pred) {
   while (true) {
      while (!f.empty() && pred(*f)) { ++f; }
      while (!b.empty() && !pred(*b)) { ++b; }
      if (f.empty() || b.empty()) { return std::make_pair(f, b); }
      using std::swap; swap(*f, *b);
}   }
```

Blocked Partition

```
template <typename RndIt, typename Predicate>
RndIt blocked(RndIt begin, RndIt end, Predicate pred) {
   BlockQueue<RndIt> q(begin, end); Block<RndIt> f, b;
   while (true) {
      if (f.empty() && (f = q.front()).empty()) { break; }
      if (b.empty() && (b = q.back()).empty()) { break; }
      tie(f, b) = block(f, b, pred);
   }
   return clean_up(f, b, pred);
}
```

Clean Up

```
template <typename It, typename Pred>
It clean_up(Block<It> f, Block<It> b, Pred pred) {
   if (b.empty()) {
      return std::partition(f.current(), f.end(), pred);
   }
   auto s = b.current();
   auto p = std::partition(b.current(), b.end(), pred);
   std::swap_ranges(s, p, b.begin());
   return b.begin() + (p - s);
}
```

Parallel Block Partition

- blocks can be processed in parallel
    - make the queue thread-safe to allow concurrent access
- running individual threads is not effective
    - instead schedule jobs with thread pool
- schedule processing blocks
- after partitioning blocks some clean-up is needed

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Block Queue (retake)

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template <typename RndIt> struct BlockQueue {
  RndIt beg, end; static constexpr int bs = 123;
  int size;
  BlockQueue(RndIt b, RndIt e): beg(b), end(e), size(e-b) {}
  Block<RndIt> front() {
     auto s = min(size, bs);
     size -= s;
     beg += s;

     return Block<RndIt>(beg - s, beg); }
};
```

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Block Queue (retake)

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template <typename RndIt> struct BlockQueue {
  RndIt beg, end; static constexpr int bs = 123;
  std::atomic<int> size;
  BlockQueue(RndIt b, RndIt e): beg(b), end(e), size(e-b) {}
  Block<RndIt> front() {
     auto s  = size.fetch_sub(bs);
     s = min(max(0, s), bs);
     beg += s;

     return Block<RndIt>(beg - s, beg); }
};
```

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Block Queue (retake)

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template <typename RndIt> struct BlockQueue {
  std::atomic<RndIt> beg, end; static constexpr int bs=123;
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  BlockQueue(RndIt b, RndIt e): beg(b), end(e), size(e-b) {}
  Block<RndIt> front() {
    auto s  = size.fetch_sub(bs);
    s = min(max(0, s), bs);
    beg += s;

    return Block<RndIt>(beg - s, beg); }
};
```

Block Queue (retake)

```
template <typename RndIt> struct BlockQueue {
  RndIt beg, end; static constexpr int bs = 123;
  std::atomic<int> size, f_off{0}, b_off{0};
  BlockQueue(RndIt b, RndIt e): beg(b), end(e), size(e-b) {}
  Block<RndIt> front() {
     auto s  = size.fetch_sub(bs);
     s = min(max(0, s), bs);
     auto off = f_off.fetch_add(s);

     return Block<RndIt>(beg + off, beg + off + s); }
};
```

Parallel Block Partition

- blocks can be processed in parallel
  - make the queue thread-safe to allow concurrent access
- running individual threads is not effective
  - instead schedule jobs with thread pool
- schedule processing blocks
- after partitioning blocks some clean-up is needed

Thread Pool

```
class thread_pool {




public:
    explicit thread_pool(int count);
    ~thread_pool();
    void stop();
    template <typename Job> void enqueue(Job job);
};
```

Thread Pool

```
class thread_pool {
    std::mutex                                          mutex;




public:
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Thread Pool

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class thread_pool {
    std::mutex                                  mutex;




    void run();
public:
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Thread Pool

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class thread_pool {
    std::mutex                          mutex;
    std::condition_variable             cond;


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Thread Pool

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class thread_pool {
    std::mutex                          mutex;
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    std::deque<std::function<void()>> jobs;

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Thread Pool

```
class thread_pool {
    std::mutex                              mutex;
    std::condition_variable                 cond;
    std::deque<std::function<void()>> jobs;
    std::list<join_thread>                  threads;
    void run();
public:
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public:
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};
```

Joining Threads

- destroying a non-joined, non-detect thread ⇒ terminate

```
struct join_thread
    : std::thread {
    using std::thread::thread;
    ~join_thread() { this->join(); }
};
```

Thread Pool

```
class thread_pool {
    std::mutex                            mutex;
    std::condition_variable               cond;
    std::deque<std::function<void()>> jobs;
    std::list<join_thread>                threads;
    void run();
public:
    explicit thread_pool(int count);
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Thread Pool

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    std::condition_variable                   cond;
    std::deque<std::function<void()>> jobs;
    std::list<join_thread>                    threads;
    void run();
public:
    explicit thread_pool(int count);
    ~thread_pool();
    void stop();
    template <typename Job> void enqueue(Job job);
};
```

Thread Pool

```
void thread_pool::run() {
    for (std::function<void()> fun([]{}); fun; ) {
        fun();
        std::unique_lock<std::mutex> kerberos(this->mutex);
        this->cond.wait(kerberos,
                        [this]{ return !this->jobs.empty(); });
        fun = std::move(this->jobs.front());
        this->jobs.pop_front();
    }
}
```

Thread Pool

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    void run();
public:
    explicit thread_pool(int count);
    ~thread_pool();
    void stop();
    template <typename Job> void enqueue(Job job);
};
```

Thread Pool

```
template <typename Job>
void thread_pool::enqueue(Job job) {
    {
        std::lock_guard<std::mutex> kerberos(this->mutex);
        this->jobs.emplace_back(std::move(job));
    }
    this->cond.notify_one();
}
```

Thread Pool

```
class thread_pool {
    std::mutex                              mutex;
    std::condition_variable                 cond;
    std::deque<std::function<void()>> jobs;
    std::list<join_thread>                  threads;
    void run();
public:
    explicit thread_pool(int count);
    ~thread_pool();
    void stop();
    template <typename Job> void enqueue(Job job);
};
```

Thread Pool

```
thread_pool::thread_pool(int count) {
    while (count—) {
        threads.emplace_back([this]{ this->run(); });
    }
}
thread_pool::~thread_pool(){
    this->stop();
}
```

Thread Pool

```
class thread_pool {
    std::mutex                          mutex;
    std::condition_variable             cond;
    std::deque<std::function<void()>> jobs;
    std::list<join_thread>              threads;
    void run();
public:
    explicit thread_pool(int count);
    ~thread_pool();
    void stop();
    template <typename Job> void enqueue(Job job);
};
```

Thread Pool

```
void thread_pool::stop(){
    {
        std::lock_guard<std::mutex> kerberos(this->mutex);
        for (std::size_t i(0); i != this->threads.size(); ++i) {
            this->jobs.emplace_back();
        }
    }
    this->cond.notify_all();
    this->jobs.clear();
}
```

Thread Pool

```
void thread_pool::run() {
    for (std::function<void()> fun([]{}); fun; ) {
        fun();
        std::unique_lock<std::mutex> kerberos(this->mutex);
        this->cond.wait(kerberos,
                        [this]{ return !this->jobs.empty(); });
        fun = std::move(this->jobs.front());
        this->threads.pop_front();
    }
}
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Partition Job

```
BlockQueue<RndIt>          q(begin, end);
auto job = [&q, pred](auto& remain){
  remain = [&q, pred]()->Block<RndIt>{
    for (Block<RndIt> f, b;;) {
      if (f.empty() && (f = q.front()).empty())
        return  { b.cur(), std::partition(b.cur(), b.end(), pred) };
      if (b.empty() && (b = q.back()).empty())
        return { std::partition(f.cur(), f.end(), pred), f.end() };
      std::tie(f, b) = block(f, b, pred);
    }
  }();
};
```

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        return { std::partition(f.cur(), f.end(), pred), f.end() };
      std::tie(f, b) = block(f, b, pred);
    }
  }();
};
```

Executing the Jobs

```
BlockQueue<RndIt>              q(begin, end);
std::vector<Block<RndIt>> remain(p.size());
latch                          l(remain.size());

auto job = [...](){ ... };

for (auto& block: remain) {
  p.enqueue_job([job, &l, &block](){ job(block); l.arrive(); });
}
l.wait();

// clean-up
```

Executing the Jobs

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BlockQueue<RndIt>             q(begin, end);
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// clean-up
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Latch

```
class latch {
    std::mutex              d_mutex;
    std::condition_variable d_condition; int d_await;
public:
    explicit latch(int await): d_await(await) {}
    void arrive() {
        std::lock_guard<std::mutex> kerberos(this->d_mutex);
        if (!--this->d_await) { this->d_condition.notify_one(); }
    }
    void wait() {
        std::unique_lock<std::mutex> kerberos(this->d_mutex);
        this->d_condition.wait(kerberos, [this]{return !this->d_await; });
    }
};
```

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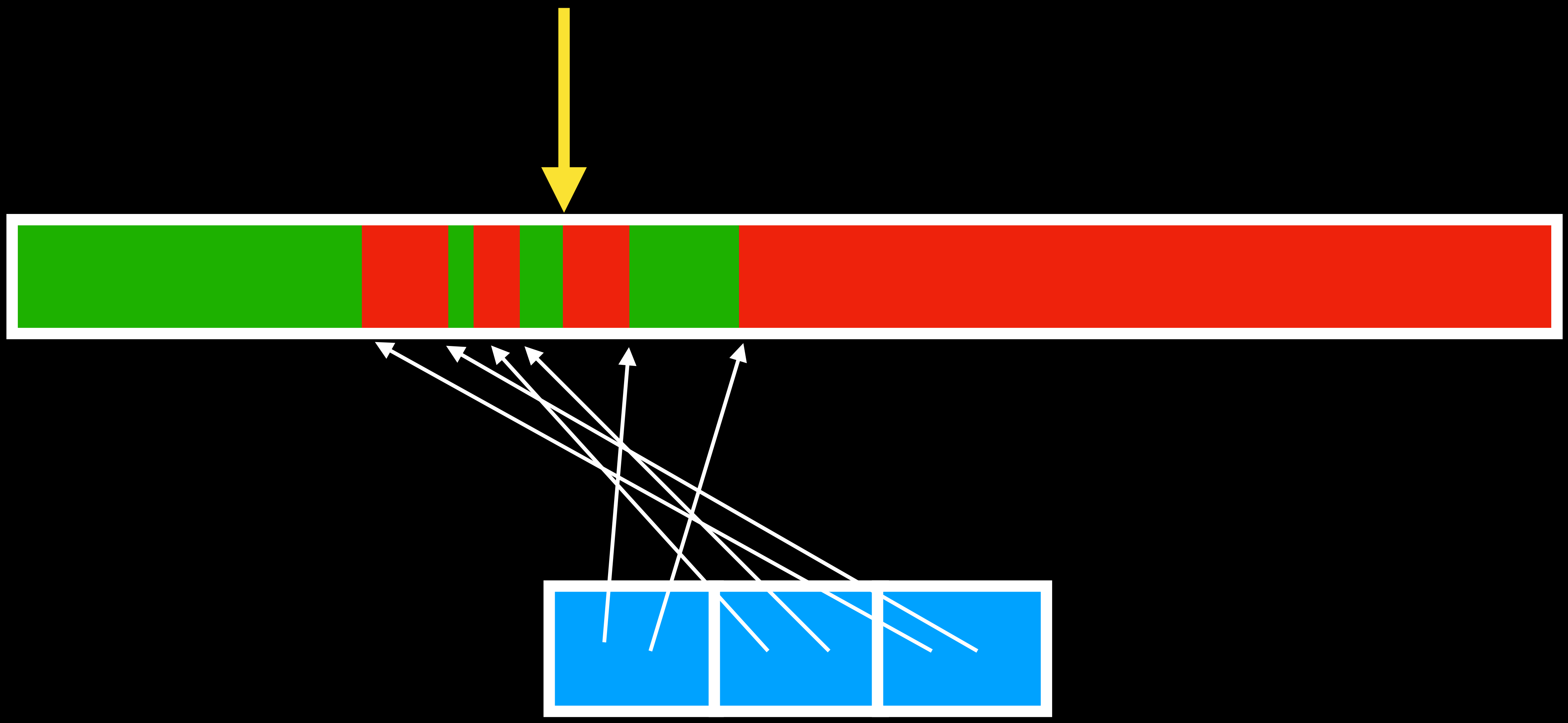
        std::unique_lock<std::mutex> kerberos(this->d_mutex);
        this->d_condition.wait(kerberos, [this]{return !this->d_await; });
    }
};
```

Clean-up

```
RndIt mid = q.midpoint();
auto rp = std::partition(remain.begin(), remain.end(),
                              [mid](auto& b){ return b.begin() < mid; });
std::sort(remain.begin(), rp,
          [](auto& b0, auto& b1){ return b1.begin() < b0.begin(); });
for (auto it(remain.begin()); it != rp; ++it) {
    mid -= it->end() - it->begin();
    swap_ranges_helper(it->begin(), it->end(), mid);
}
std::sort(rp, remain.end(),
          [](auto& b0, auto& b1){ return b0.begin() < b1.begin(); });
for (auto it(rp); it != remain.end(); ++it) {
    swap_ranges_helper(mid, mid + (it->end() - it->begin()), it->begin());
    mid += it->end() - it->begin();
}
return mid;
```

Clean-up



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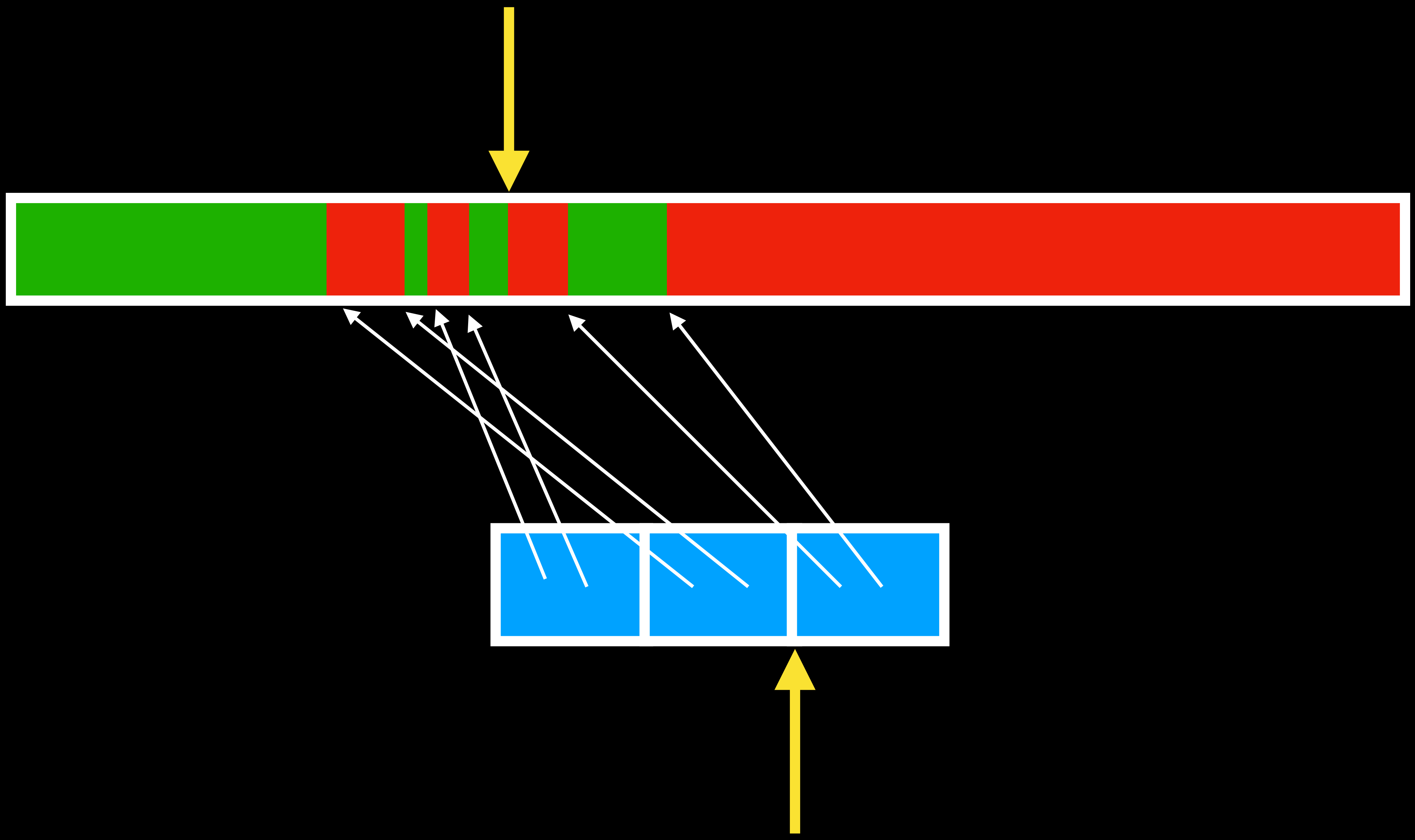

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    mid -= it->end() - it->begin();
    swap_ranges_helper(it->begin(), it->end(), mid);
}
std::sort(rp, remain.end(),
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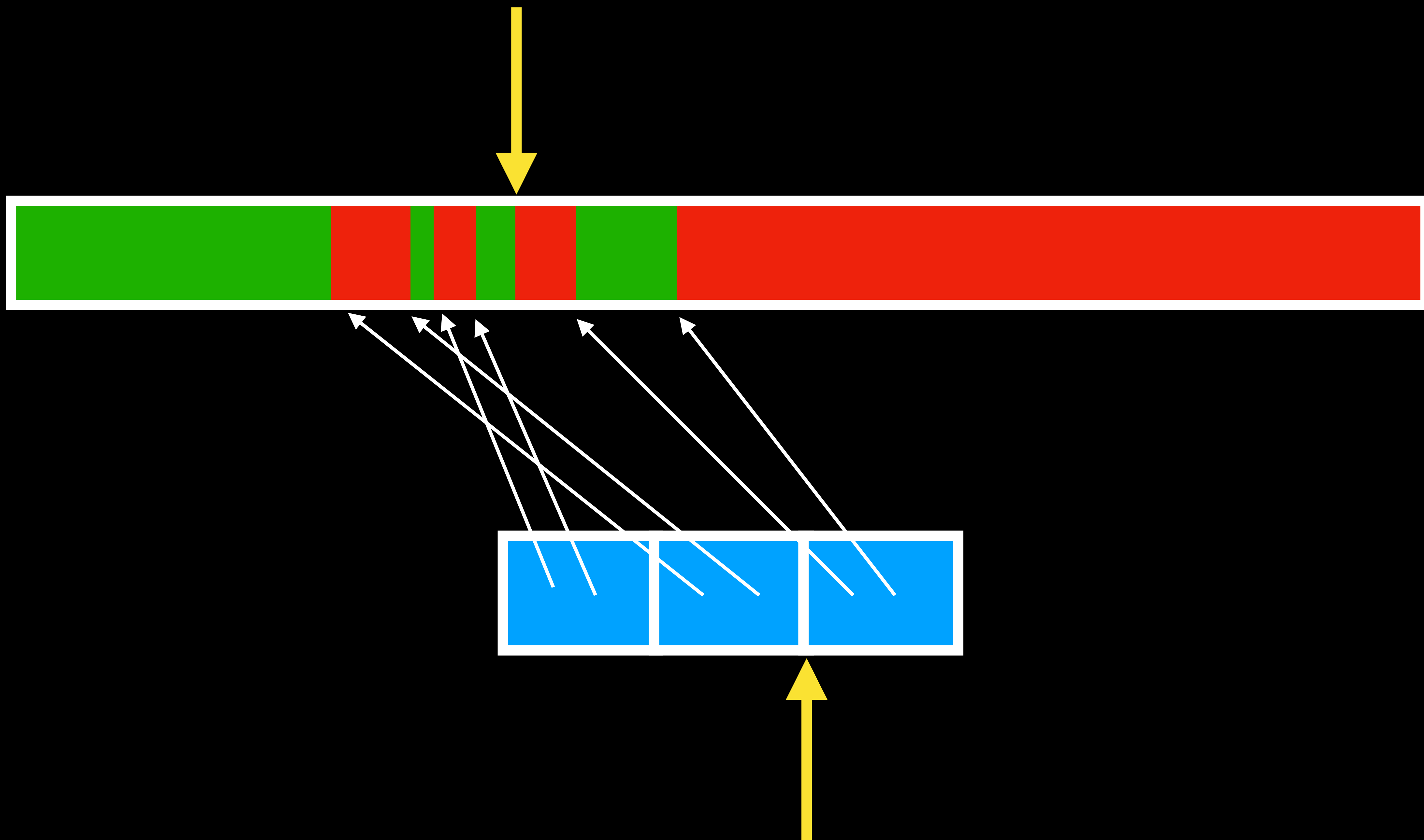
for (auto it(rp); it != remain.end(); ++it) {
     swap_ranges_helper(mid, mid + (it->end() - it->begin()), it->begin());
     mid += it->end() - it->begin();
}
return mid;
```

Clean-up



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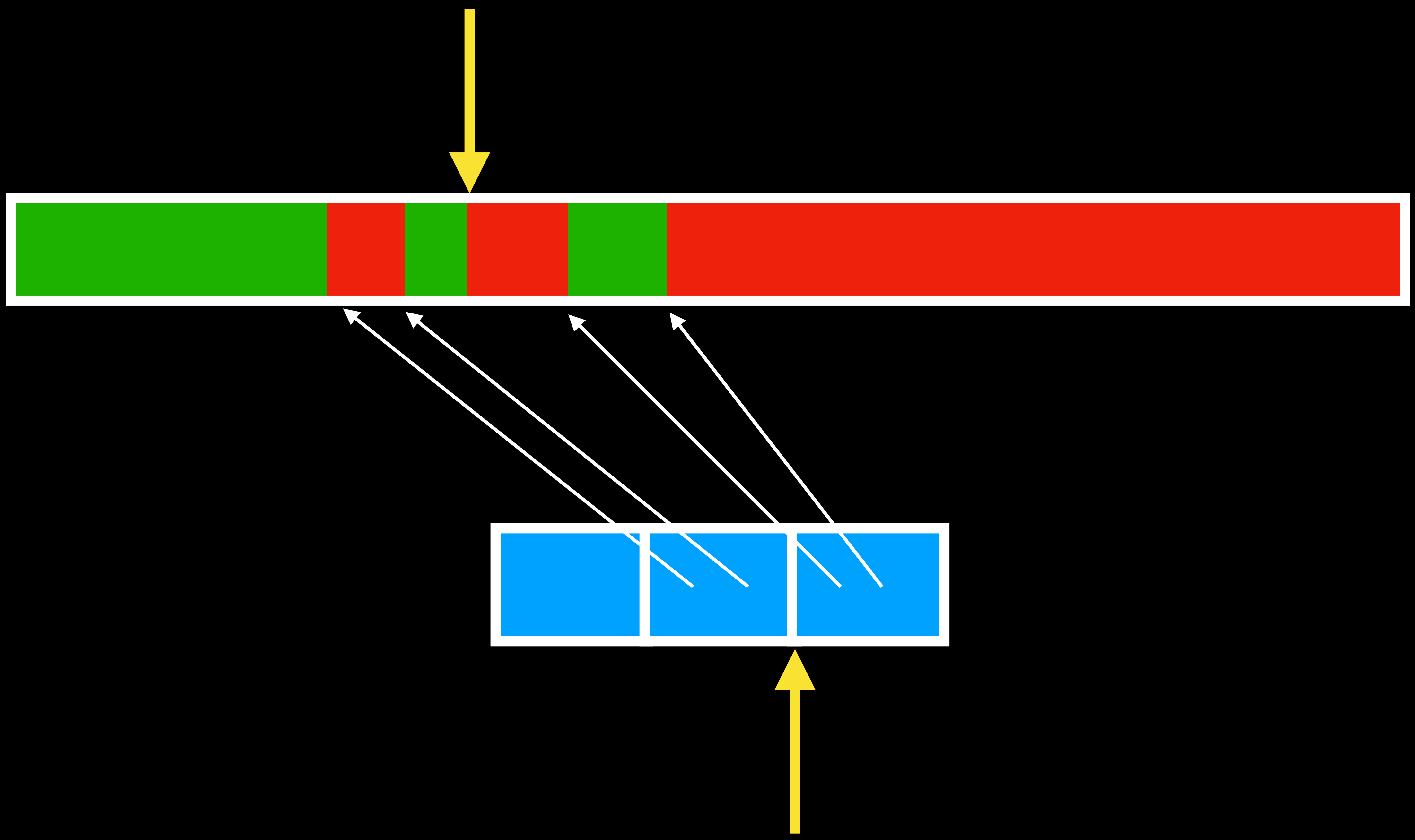

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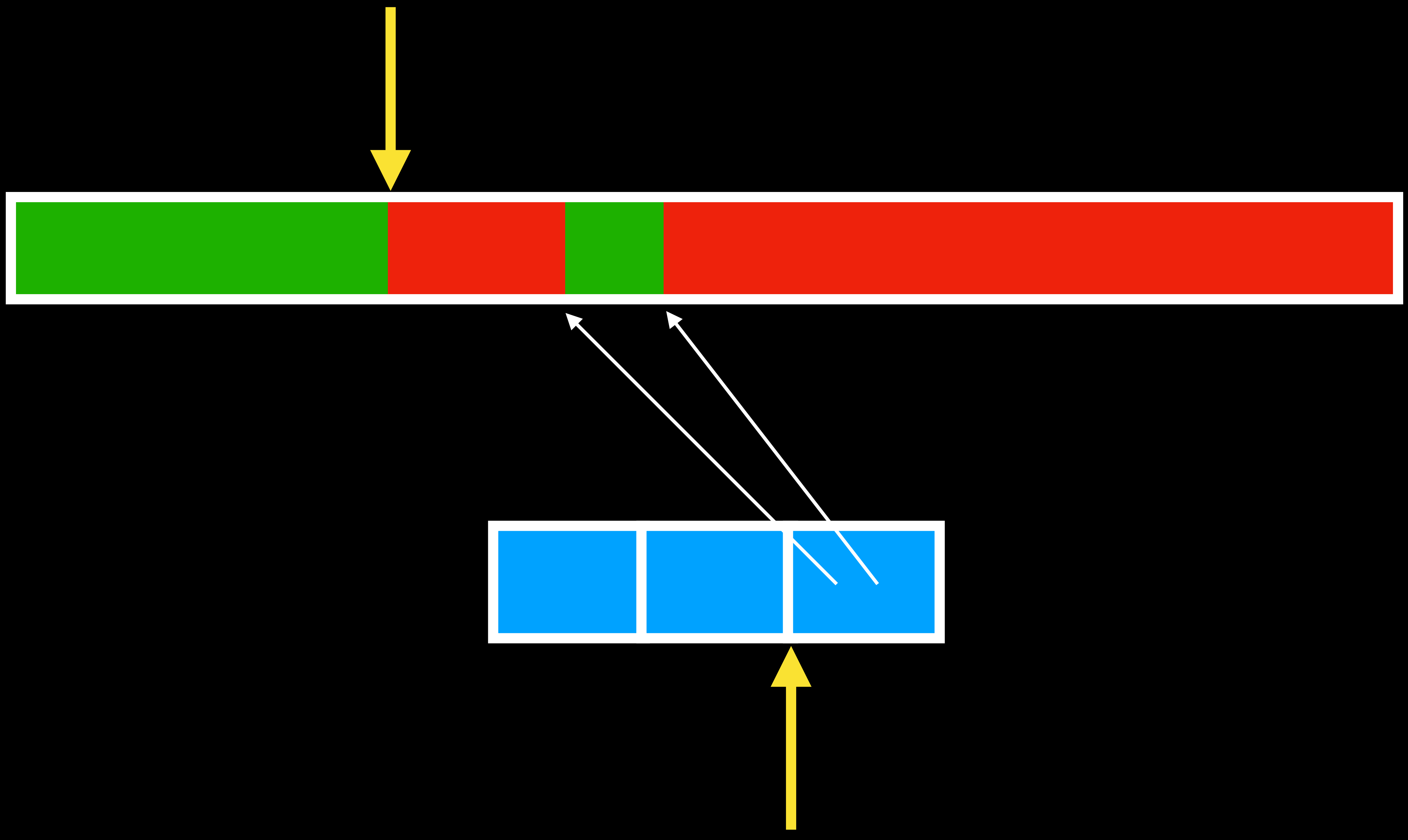

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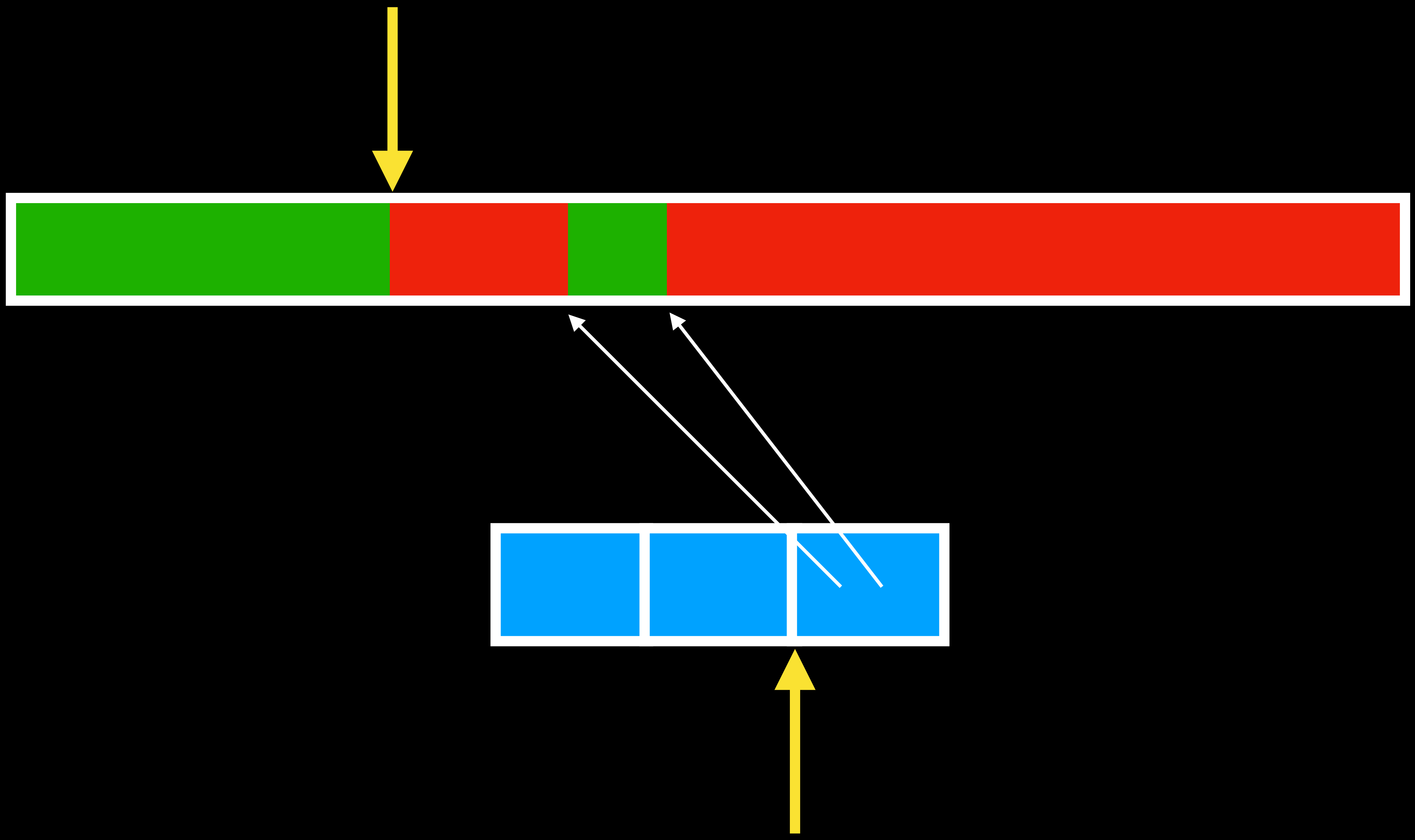
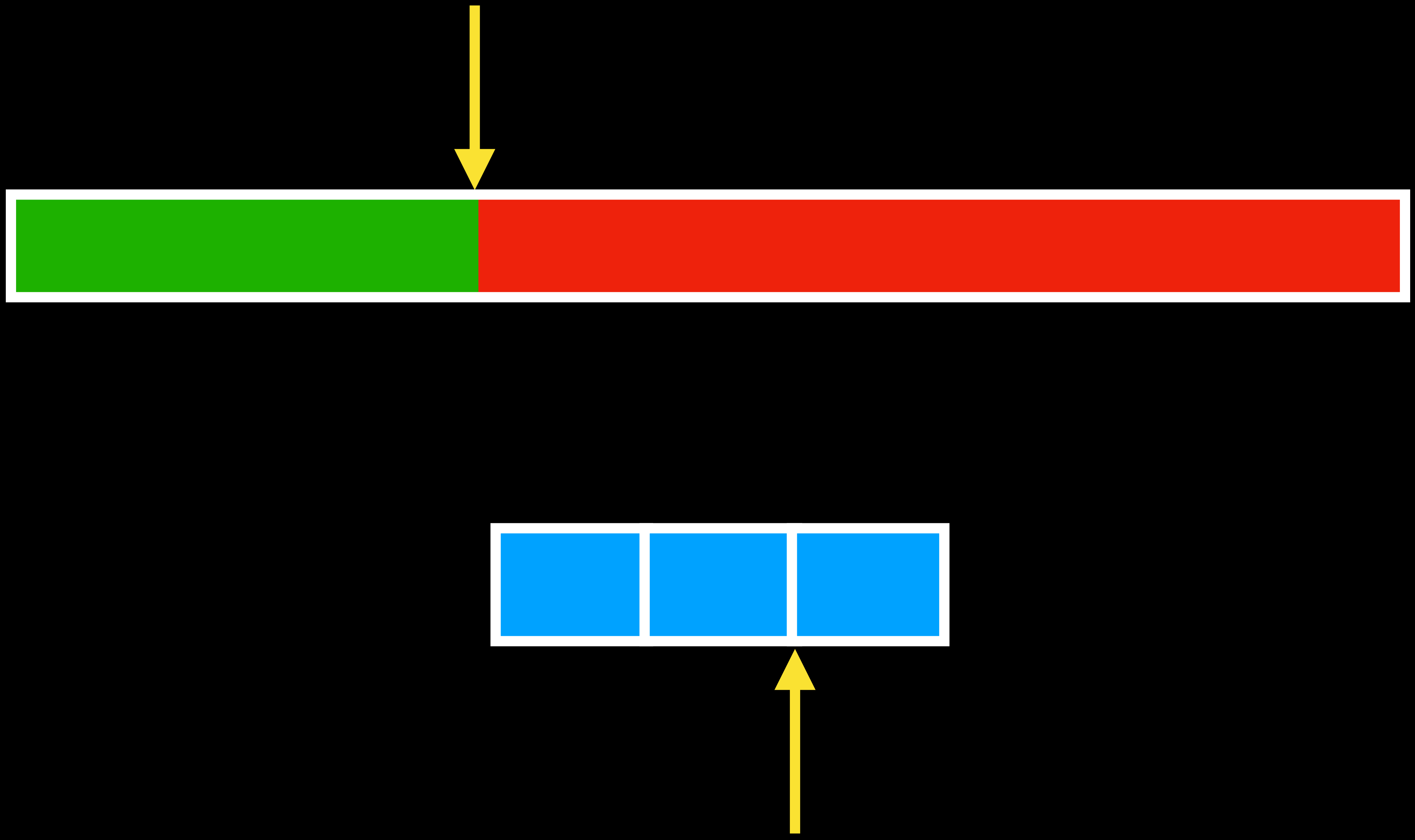
std::sort(remain.begin(), rp,
          [](auto& b0, auto& b1){ return b1.begin() < b0.begin(); });
for (auto it(remain.begin()); it != rp; ++it) {
    mid -= it->end() - it->begin();
    swap_ranges_helper(it->begin(), it->end(), mid);
}
std::sort(rp, remain.end(),
          [](auto& b0, auto& b1){ return b0.begin() < b1.begin(); });
for (auto it(rp); it != remain.end(); ++it) {
    swap_ranges_helper(mid, mid + (it->end() - it->begin()), it->begin());
    mid += it->end() - it->begin();
}
return mid;
```

Clean-up



Clean-up



Clean-up

```
RndIt mid = q.midpoint();
auto rp = std::partition(remain.begin(), remain.end(),
                         [mid](auto& b){ return b.begin() < mid; });
std::sort(remain.begin(), rp,
          [](auto& b0, auto& b1){ return b1.begin() < b0.begin(); });
for (auto it(remain.begin()); it != rp; ++it) {
    mid -= it->end() - it->begin();
    swap_ranges_helper(it->begin(), it->end(), mid);
}
std::sort(rp, remain.end(),
          [](auto& b0, auto& b1){ return b0.begin() < b1.begin(); });
for (auto it(rp); it != remain.end(); ++it) {
    swap_ranges_helper(mid, mid + (it->end() - it->begin()), it->begin());
    mid += it->end() - it->begin();
}
return mid;
```

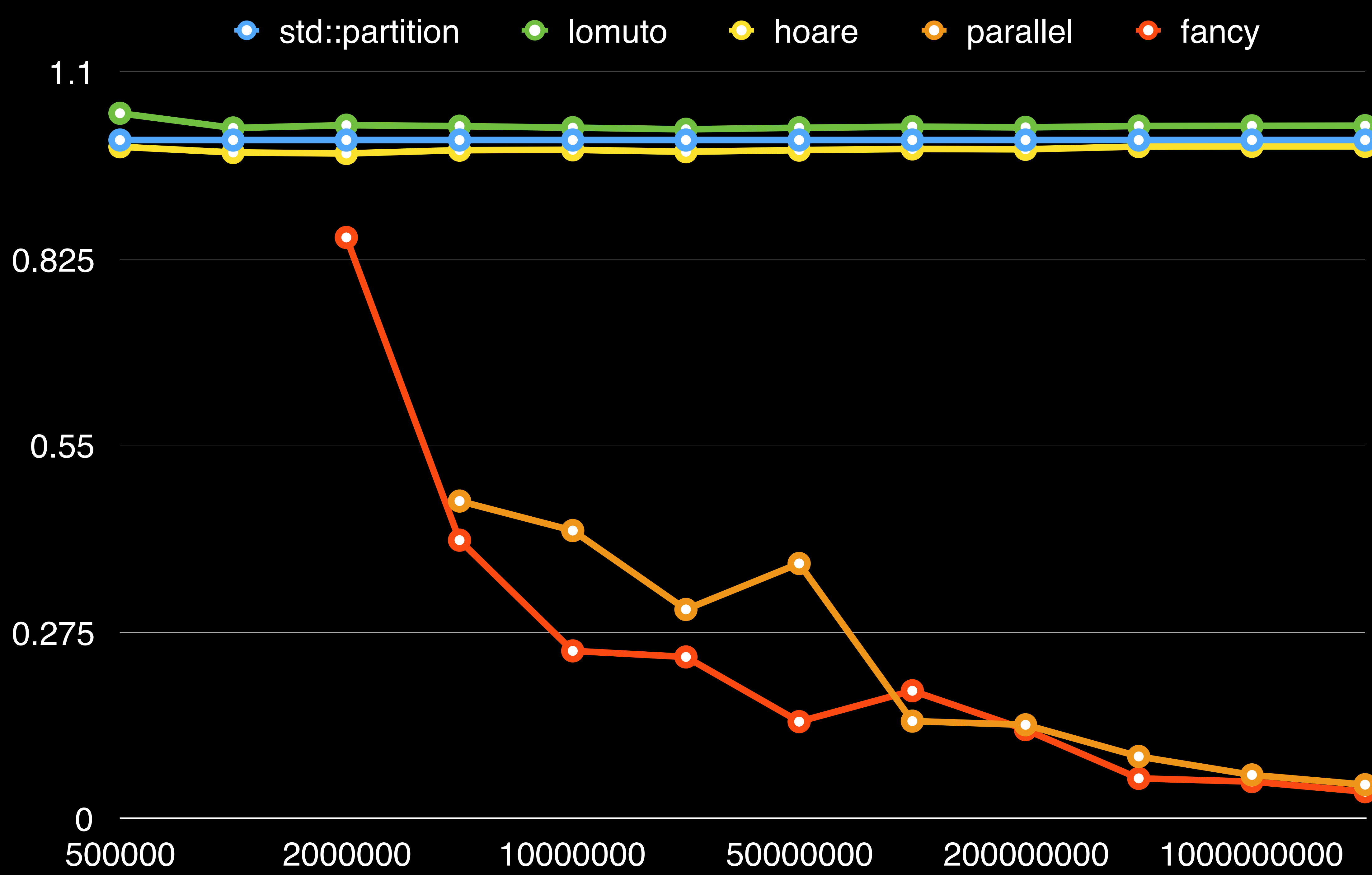
Results

- relative time taken compared to a benchmark
  - benchmark: std::partition - the blue line
- high values ⇒ slow, low values ⇒ fast
- x-axis: roughly log scale
- machine: 64 cores, 96GB memory (+16GB on chip)
  Intel Knights Landing

Results [2]

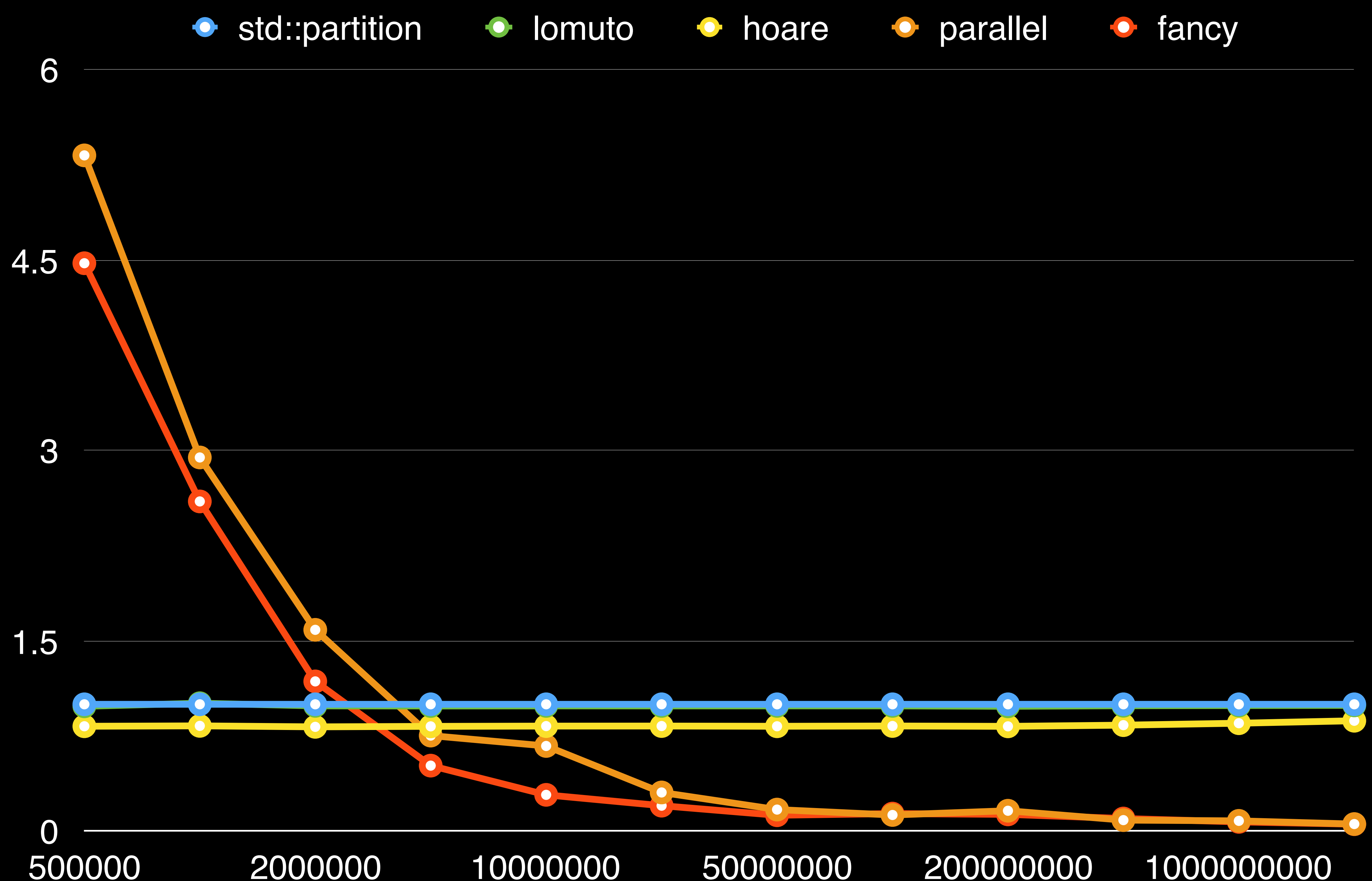
std::partition
lomuto
hoare
parallel
fancy
3
2.25
1.5
0.75
0
500000
2000000
10000000
50000000
200000000
1000000000

Results ₂



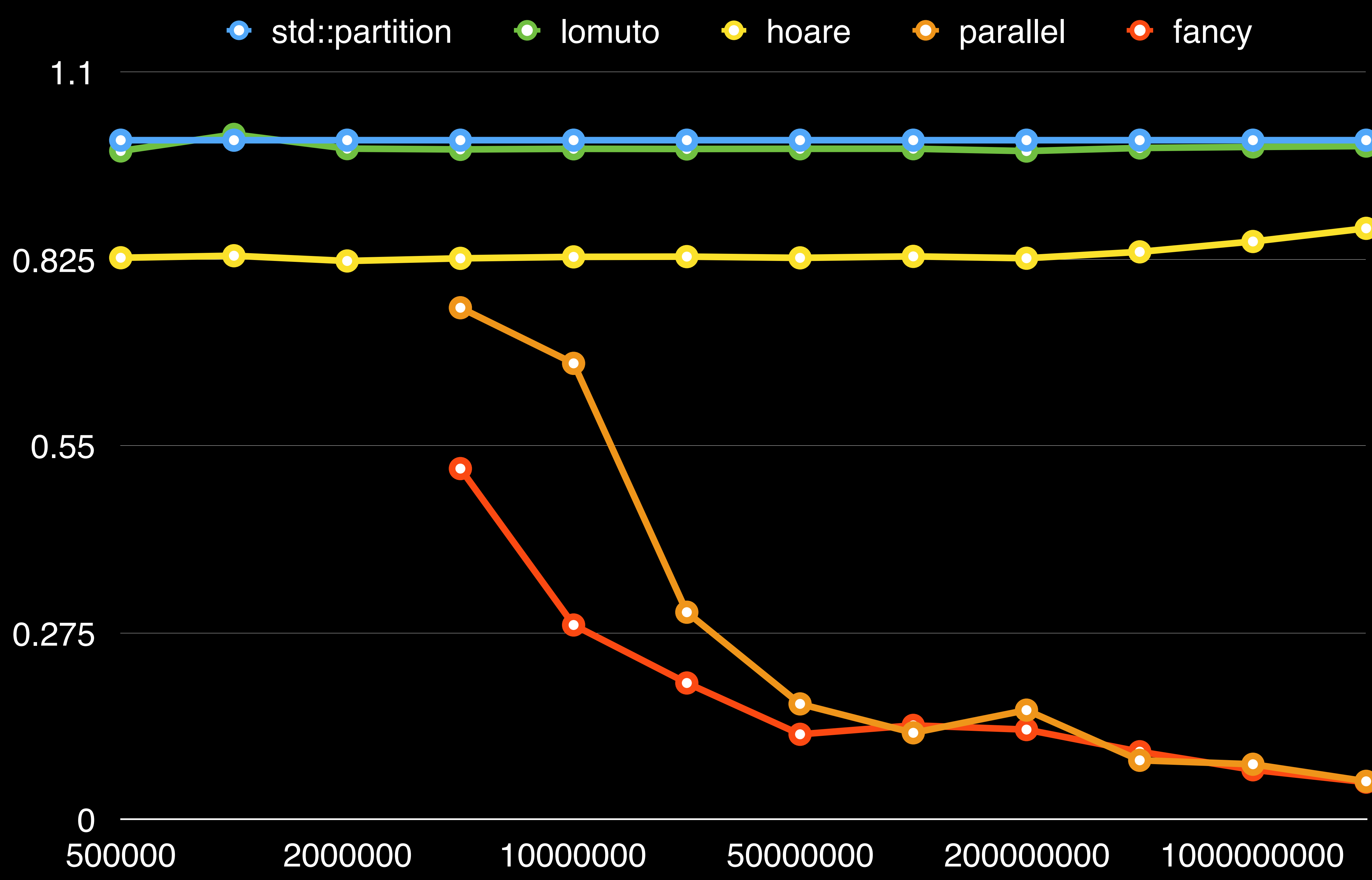
std::partition
lomuto
hoare
parallel
fancy
1.1
0.825
0.55
0.275
0
500000
2000000
10000000
50000000
200000000
1000000000

Results 20



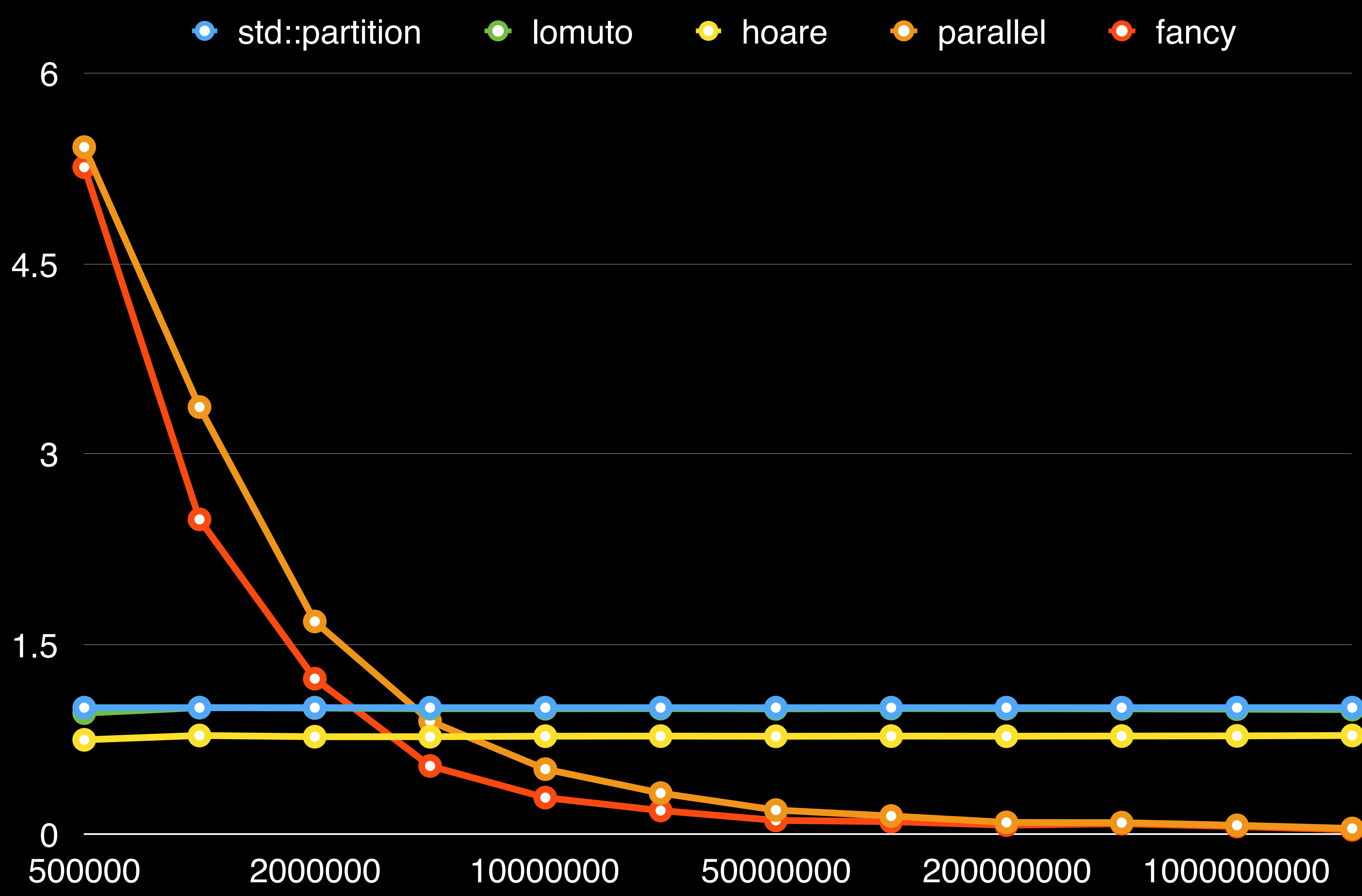
std::partition
lomuto
hoare
parallel
fancy
6
4.5
3
1.5
0
500000
2000000
10000000
50000000
200000000
1000000000

Results [20]

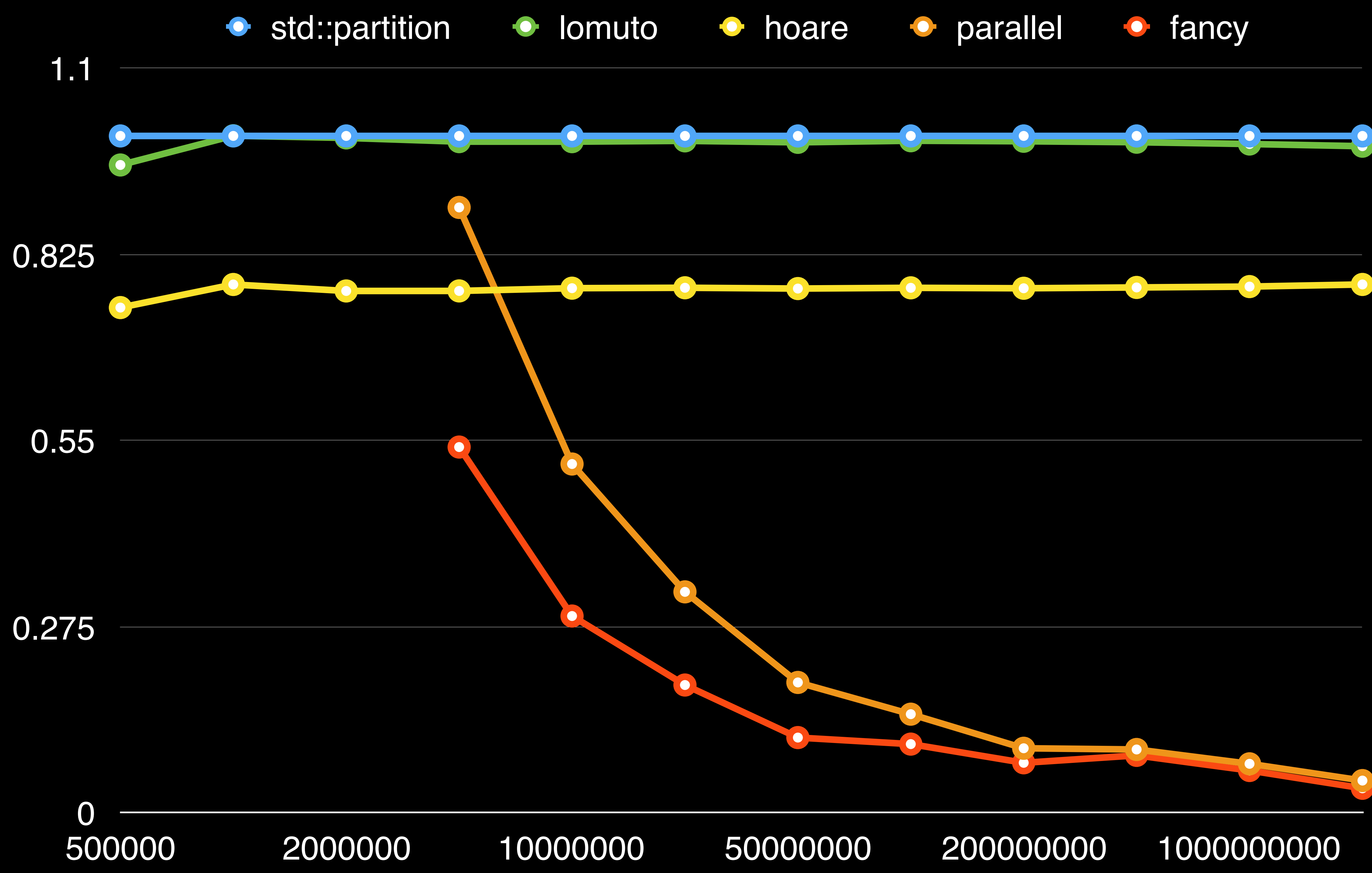


std::partition
lomuto
hoare
parallel
fancy
1.1
0.825
0.55
0.275
0
500000
2000000
10000000
50000000
200000000
1000000000

Results 100


std::partition
lomuto
hoare
parallel
fancy
6
4.5
3
1.5
0
500000
2000000
10000000
50000000
200000000
1000000000

Results 100



std::partition
lomuto
hoare
parallel
fancy
1.1
0.825
0.55
0.275
0
500000
2000000
10000000
50000000
200000000
1000000000

Improvements

- handling of smaller ranges
  - engage fewer threads with sufficiently sized blocks?
  - create tasks more clever?
- do clean-up from multiple threads
- extract a continuation/non-waiting form of the algorithm

Summary

- quite a bit of administrative work around the algorithm
- need an idea how to separate the processing
- overall more total work than sequential algorithm
- on larger ranges faster by using multiple processors