# Introduction to Machine Learning

# Evaluation: Simple Measures for Classification

| | | True Class $y$ | |
|---|---|---|---|
| | | $+$ | $-$ |
| **Pred.** | $+$ | True Positive (TP) | False Positive (FP) |
| $\hat{y}$ | $-$ | False Negative (FN) | True Negative (TN) |

## Learning goals

- Know the definitions of misclassification error rate (MCE) and accuracy (ACC)
- Understand the entries of a confusion matrix
- Understand the idea of costs
- Know defintions of Brier score and log loss

# LABELS VS PROBABILITIES

In classification we predict:

1. Class labels $\rightarrow \hat{h}(\mathbf{x}) = \hat{y}$
2. Class probabilities $\rightarrow \hat{\pi}_k(\mathbf{x})$

$\rightarrow$ We evaluate based on those

# LABELS: MCE

The misclassification error rate (MCE) counts the number of incorrect predictions and presents them as a rate:

$$MCE = \frac{1}{n}\sum_{i=1}^{n}[y^{(i)} \neq \hat{y}^{(i)}] \in [0;1]$$

Accuracy is defined in a similar fashion for correct classifications:

$$ACC = \frac{1}{n}\sum_{i=1}^{n}[y^{(i)} = \hat{y}^{(i)}] \in [0;1]$$

- If the data set is small this can be brittle
- The MCE says nothing about how good/skewed predicted probabilities are
- Errors on all classes are weighed equally (often inappropriate)

# LABELS: CONFUSION MATRIX

True classes in columns.
Predicted classes in rows.

```
           setosa versicolor virginica -err.- -n-
setosa         50          0         0      0  50
versicolor      0         46         4      4  50
virginica       0          4        46      4  50
-err.-          0          4         4      8  NA
-n-            50         50        50     NA 150
```

We can see class sizes (predicted and true) and where errors occur.

# LABELS: CONFUSION MATRIX

## In binary classification

| | | **True Class** $y$ | |
|---|---|---|---|
| | | $+$ | $-$ |
| **Pred.** | $+$ | True Positive (TP) | False Positive (FP) |
| $\hat{y}$ | $-$ | False Negative (FN) | True Negative (TN) |

# LABELS: COSTS

We can also assign different costs to different errors via a cost matrix.

$$Costs = \frac{1}{n}\sum_{i=1}^{n} C[y^{(i)}, \hat{y}^{(i)}]$$

Example:
Predict if person has a ticket (yes / no).
Should train conductor check ticket of a person?

**Costs:**

| | |
|---|---|
| Ticket checking: | 3 EUR |
| Fee for fare-dodging: | 40 EUR |


http://www.oslobilder.no/OMU/OB.%C3%9864/2902

# LABELS: COSTS

Predict if person has a ticket (yes / no).

```
Cost matrix C
     predicted
true   no yes
  no  -37   0
  yes   3   0
```

```
Confusion matrix
     predicted
true  no yes
  no   7   0
  yes 93   0
```

```
Confusion matrix * C
     predicted
true    no yes
  no  -259   0
  yes  279   0
```

**Costs:**

| | |
|---|---|
| Ticket checking: | 3 EUR |
| Fee for fare-dodging: | 40 EUR |

Our model says that we should not trust anyone and check the tickets of all passengers.

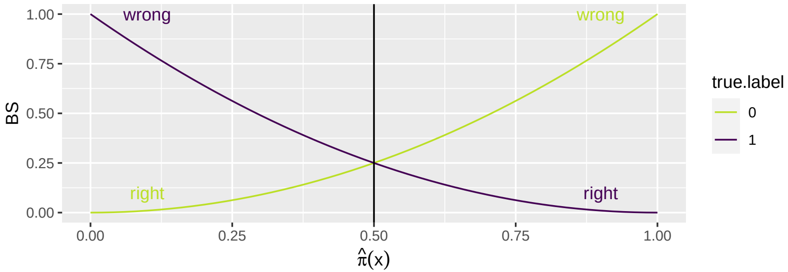
$$
\begin{aligned}
\mathit{Costs} &= \frac{1}{n}\sum_{i=1}^{n} C[y^{(i)}, \hat{y}^{(i)}] \\
&= \frac{1}{100}\left(-37 \cdot 7 + 0 \cdot 0 + 3 \cdot 93 + 0 \cdot 0\right) \\
&= \frac{20}{100} = 0.2
\end{aligned}
$$

# PROBABILITIES: BRIER SCORE

Measures squared distances of probabilities from the true class labels:

$$BS1 = \frac{1}{n} \sum_{i=1}^{n} \left( \hat{\pi}(\mathbf{x}^{(i)}) - y^{(i)} \right)^2$$

- Fancy name for MSE on probabilities
- Usual definition for binary case, $y^{(i)}$ must be coded as 0 and 1.



# PROBABILITIES: BRIER SCORE

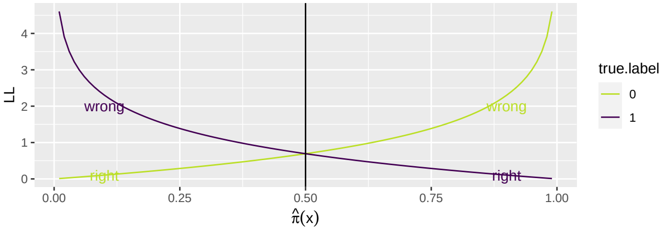
$$BS2 = \frac{1}{n}\sum_{i=1}^{n}\sum_{k=1}^{g}\left(\hat{\pi}_k(\mathbf{x}^{(i)}) - o_k^{(i)}\right)^2$$

- Original by Brier, works also for multiple classes
- $o_k^{(i)} = [y^{(i)} = k]$ is a 0-1-one-hot coding for labels
- For the binary case, BS2 is twice as large as BS1, because in BS2 we sum the squared difference for each observation regarding class 0 **and** class 1, not only the true class.

# PROBABILITIES: LOG-LOSS

Logistic regression loss function, a.k.a. Bernoulli or binomial loss, $y^{(i)}$ coded as 0 and 1.

$$LL = \frac{1}{n} \sum_{i=1}^{n} \left( -y^{(i)} \log(\hat{\pi}(\mathbf{x}^{(i)})) - (1 - y^{(i)}) \log(1 - \hat{\pi}(\mathbf{x}^{(i)})) \right)$$




- Optimal value is 0, “confidently wrong” is penalized heavily
- Multiclass version: $LL = -\frac{1}{n} \sum_{i=1}^{n} \sum_{k=1}^{g} o_k^{(i)} \log(\hat{\pi}_k(\mathbf{x}^{(i)}))$