Vectors

What is a Vector?
A vector is an array of numbers, that have a magnitude and a direction. It is important to not confuse a vector with a position in space, although they can be viewed this way. Think of a position as a point, and a vector as the steps that get you to that point.

Magnitude - The length of a vector.
Direction - The way the vector is pointing in space.

An example of a vector, put into words, would be "Take *three steps forward*", assuming forward is up on the y axis, this could be represented as [0, 3]. *Three Steps* is the magnitude and *forward* is the direction.

What is a Scalar?
A scalar is simply an ordinary number. Examples of this could be displacement or velocity that could be applied to a vector.

"I am moving *northeast* at *50mph*", where northeast could be considered the direction and 50 mph the speed/velocity. Represented as vector * scalar: [1.5, 1.5] * 50, assuming 1.5x, and 1.5y indicate northeast. The numbers are arbitrary.

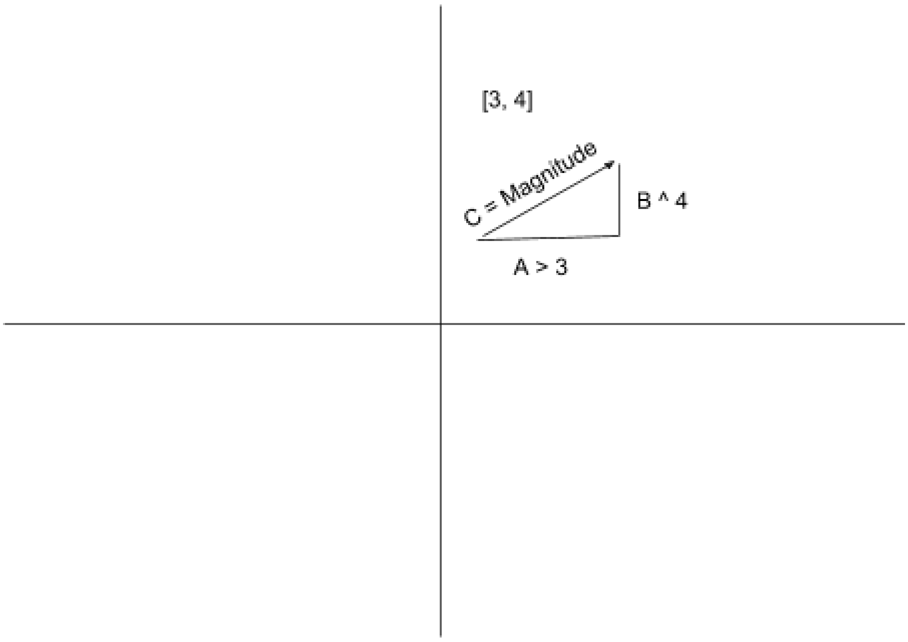
Calculating Magnitude
The magnitude is the length of a vector but it isn't expressed in the vector itself. For example, the magnitude of [3, 4] isn't 3 or 4, it is actually 5. Here's how:

Magnitude is noted as ||V|| where V is [3, 4].

$$||V|| = \sqrt{3^2} + 4^2$$
$$||V|| = \sqrt{25}$$
$$||V|| = 5$$

It is essentially pythagoras. The equation should be filled in with all components of the vector, so if there was a z component, we would just add on the square of the value of z.

To help visualise this, let's use a triangle.



Using pythagoras, $A^2 + B^2 = C^2$
And the magnitude of the vector is C.

So the square root of $A^2 + B^2$ as demonstrated above gets us the magnitude.

Vector Multiplication
Two vectors cannot be multiplied together, so here we are talking about *Scalar * Vector*.

You simply multiply each component of the vector by the scalar, and the result is a new vector.

*2 * [3, 8, 2] = [3 * 2, 8 * 2, 2 * 2] = [6, 16, 4]*

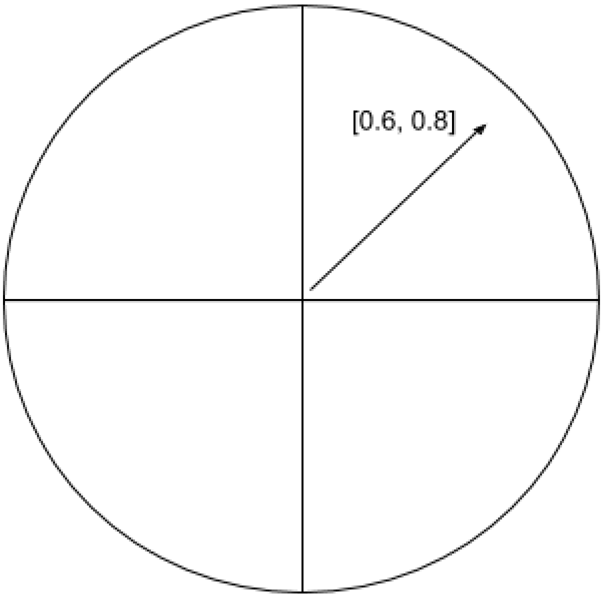
Vector Normalisation
If we wanted to know the direction of a vector, we can give it a magnitude (length) of 1, which will give us just the direction it is facing. This is also known as a unit or normalised vector.

To normalise a vector, we would divide the vector by its magnitude. Division works the same as multiplication, where the magnitude is a scalar.

Let's find the magnitude of [3, 4], we know it is 5 from earlier.

So then: *5 / [3, 4] = [3 / 5, 4 / 5] = [0.6, 0.8]*

[0.6, 0.8] is the normalised vector of [3, 4] and shows the direction the vector is facing. This happens because a unit vector sits within a circle with a radius of 1.



Vector Addition and Subtraction
Addition and subtraction works by performing the calculation on each component, provided they have the same dimension.

A = [6, 4, 8], B = [2, -2, 4]

$$A + B = [6 + 2, 4 + -2, 8 + 4] = [8, 2, 12]$$
$$A - B = [6 - 2, 4 - (-2), 8 - 4] = [4, 6, 4]$$

Addition and subtraction only works between 2 vectors of the same dimension, it cannot occur between a scalar and a vector.

The Distance Formula
To find the distance between two points, we must find the vector from one of the points to the other. We do this by subtracting them from one another.

Point B - Point A gives Vector V.

The distance between the points is the magnitude of the resulting vector, meaning:

$$||V|| = distance\ between\ B\ and\ A.$$

Vector Dot Product
The dot product of two vectors is the the sum of each component multiplied by one another, the dot product will result in a scalar.

The dot product is noted as: $v \bullet v$ where v are vectors

A = [3, 5] and B = [8, 2]

$A \bullet B = (3 * 8) + (5 * 2)$
$= 24 + 10$
$= 34$

The equation is then, as follows:
$A \bullet B = (Ax * Bx) + (Ay * By) + (Az * Bz)$

Removing and/or adding components depending upon the dimension of the vectors.

The dot product can be used for many things, but it is particularly useful when trying to find the angle between two vectors, which will be demonstrated later.

Vector Cross Product
The cross product of a vector can only be used on 3D vectors and it will result in a 3D vector. The cross product is noted as *a x b*.

The cross product is a lengthy equation, but it isn't complicated.

If we have two vectors, V and N.

$V = [Vx, Vy, Vz], N = [Nx, Ny, Nz]$

The equation would be as follows:
$x = (Vy * Nz) - (Vz * Ny)$
$y = (Vz * Nx) - (Vx * Nz)$
$z = (Vx * Ny) - (Vy * Nx)$

Here's an example:
a = [1, 3, 4] and b = [2, -5, 8]

$x = (3 * 8) - (4 * -5)$
$y = (4 * 2) - (1 * 8)$
$z = (1 * -5) - (3 * 2)$

Which becomes,

$$x = 24 - 20$$
$$y = 8 - 8$$
$$z = -5 - 6$$

Resulting in the cross product of a and b:
$[44, 0, -11]$

Finding the angle between two vectors
The equation is as follows, where Θ is the angle and *a* and *b* are the two vectors:
$Cos(\Theta) = a \bullet b \div ||a|| * ||b||$

So we can deconstruct this using an easy example.

a = [2, -3, 1] and b = [1, 0, 0]

First let's work out the dot product of the two vectors,

$$a \bullet b = (2 * 1) + (-3 * 0) + (1 * 0)$$
$$a \bullet b = 2$$

The equation becomes,

$$Cos(\Theta) = 2 \div ||a|| * ||b||$$

Let's work out the magnitude of both of the vectors,

$$||a|| = \sqrt{2^2 + -3^2 + 1^2} = \sqrt{14} = 3.741$$
$$||b|| = \sqrt{1^2 + 0^2 + 0^2} = \sqrt{1} = 1$$

Let's substitute the values into the equation,

$$Cos(\Theta) = 2 \div 3.741 * 1$$
$$Cos(\Theta) = 2 \div 3.741$$
$$Cos(\Theta) = 0.534$$

Now we need to get the inverse cosine of 0.534 to find the angle and convert it to degrees,
$acos(0.534) = 1.007$

$$\Theta = (1.007 * 180) / PI = 57.753°$$