Acquisition and Analysis of Biosignals DTEK0042

Filtering and Artifact Removal

Iman Azimi , Ph.D. (Tech.)
Email: iman.azimi@utu.fi

Turun yliopisto
University of Turku

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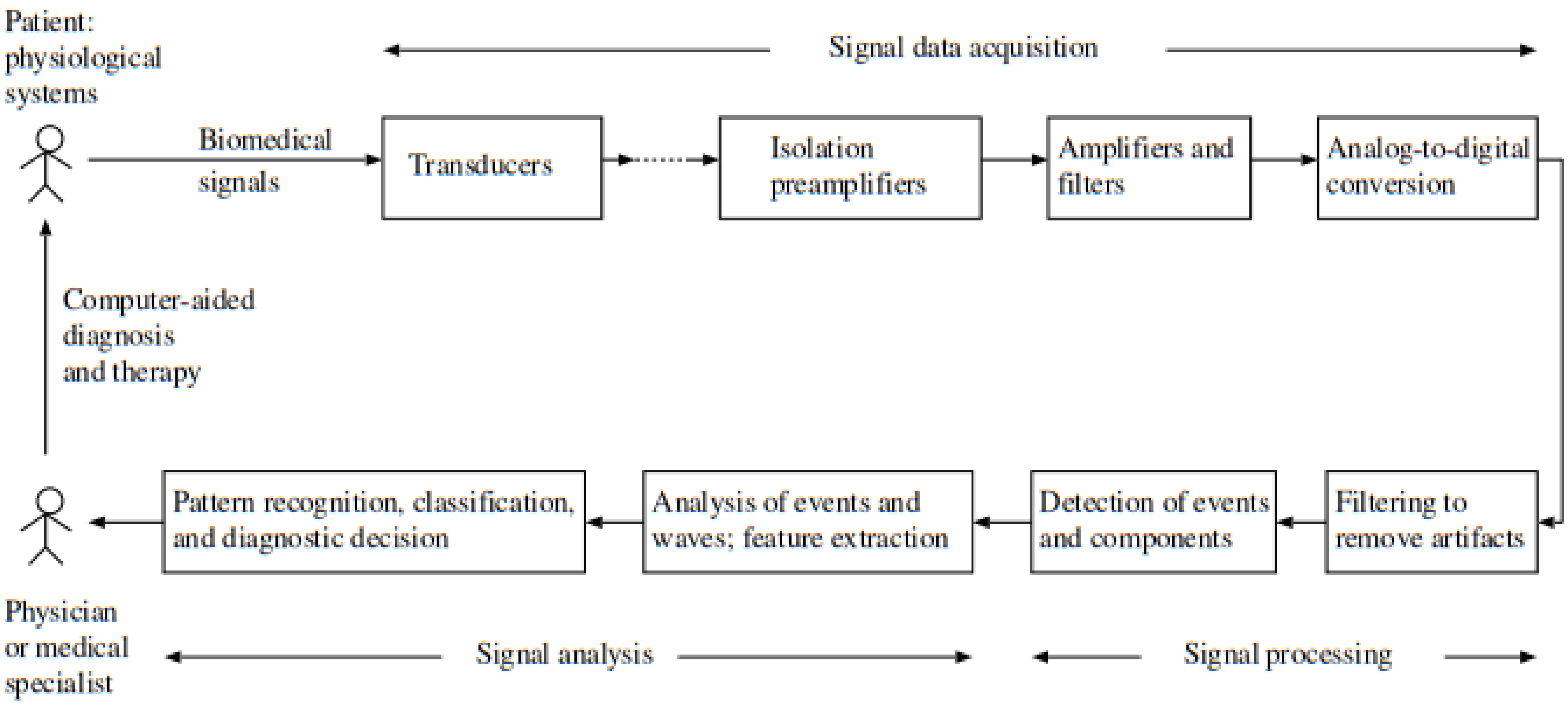
Introduction

So far, we learned:

- ❑ About the origins and acquisitions of biosignals

In this session, we will learn:

- ❑ Noise and artifacts in biosignals
- ❑ Different filtering techniques



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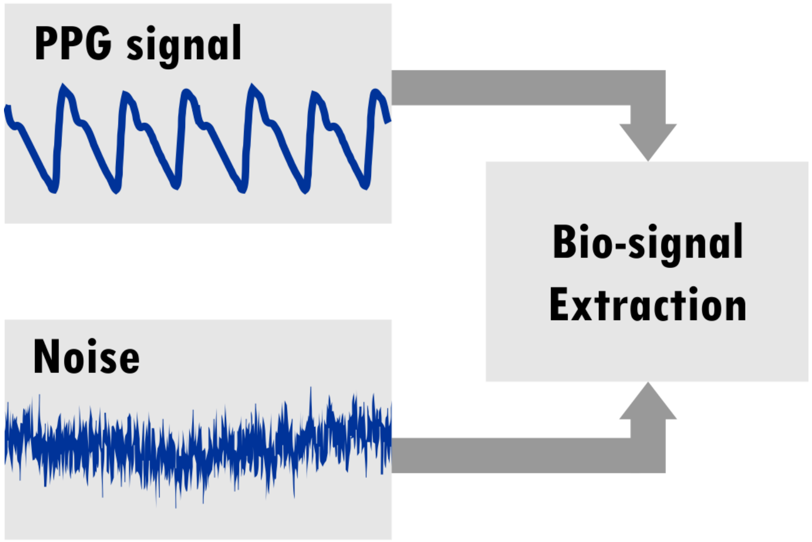
Noise in biosignals

Problem Statement

- Biosignals are weak signals in an environment with many other signals of various origins.
  - E.g., The PPG signal is distorted due to hand movements
- Any signal other than that of interest is **interference**, **artifact**, or ***noise***.



Sources of noise

- **Physiological**:
  - E.g., fetal ECG signal
    - Noise: ECG of mother
- **Instrumentation**:
  - E.g., PPG signal
    - Noise: high frequency noise
- **Environment**:
  - E.g., ECG signal
    - Noise: power line

Types of noise

1. Random noise

2. Structured noise

3. Physiological noise

Random noise (1)

- A signal that does not meet this condition: **nondeterministic** or **random** signal.
- *Deterministic signal:* value at a given instant of time may be computed using a mathematical function of time or predicted from a few past values of the signal.
- Example: Interference that arises from thermal noise in electronic devices
- A test for randomness:
  - Given a signal of *N* samples, the signal may be labeled as being random if

$$\textit{Number of turning points} > \frac{2}{3}(N-2)$$

Random noise (2)

- A random process is characterized by the ***probability density function* (PDF)**, representing the probabilities of occurrence of all possible values => mean of a random noise is zero
- The random noise is stationary
- In most cases, the noise is additive:

$$\boldsymbol{y}(\boldsymbol{n}) = \boldsymbol{x}(\boldsymbol{n}) + \eta(\boldsymbol{n})$$

- In most practical applications, the signal and the noise may be assumed to be *statistically independent processes:*

$$P_{x,\eta}(x,\eta) = P_x(x)P_\eta(\eta)$$

Structured noise

- The typical waveform of the interference is known in advance
- Example: Power line interference at 50Hz and 60Hz
- Phase of the interfering waveform might be not known
- The interfering waveform may not be an exact sinusoid

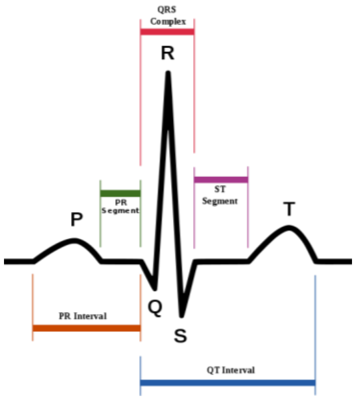
Physiological noise

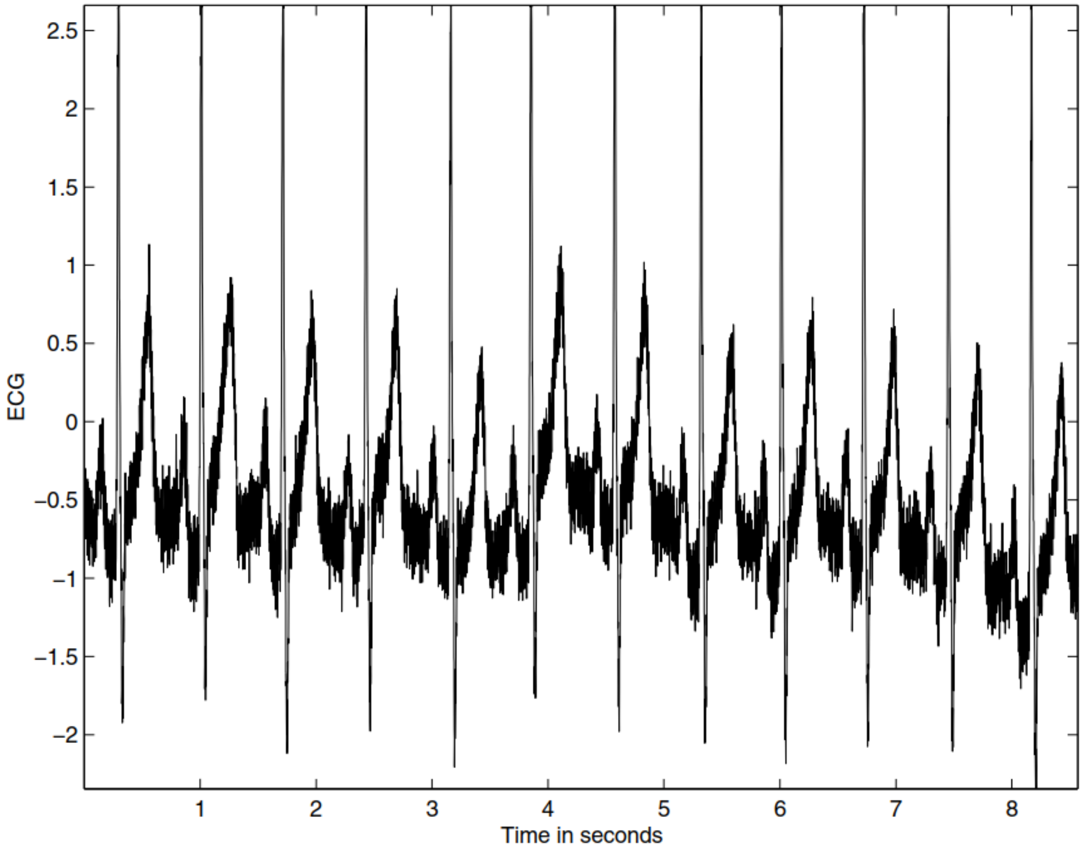
- ❑ Human body: complex of several systems
- ❑ Several physiological processes active at a given time
- ❑ Each one producing many signals of different types
- ❑ Appearance of signals from systems or processes other than those of interest: *physiological interference*
  - EMG related to breathing in ECG
  - Lung and bowel sounds in PCG
- ❑ The waveform is typically dynamic and nonstationary
  - Statistical properties (e.g., mean and variance) change over time

Illustration of the problem

Example 1

- High-frequency noise in the ECG
  - Due to instrumentation amplifiers





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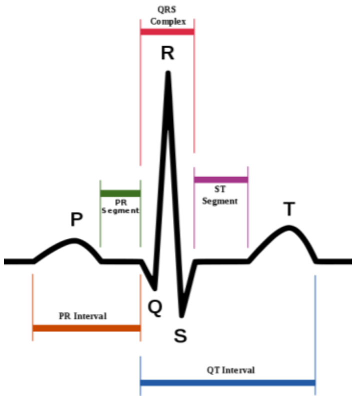
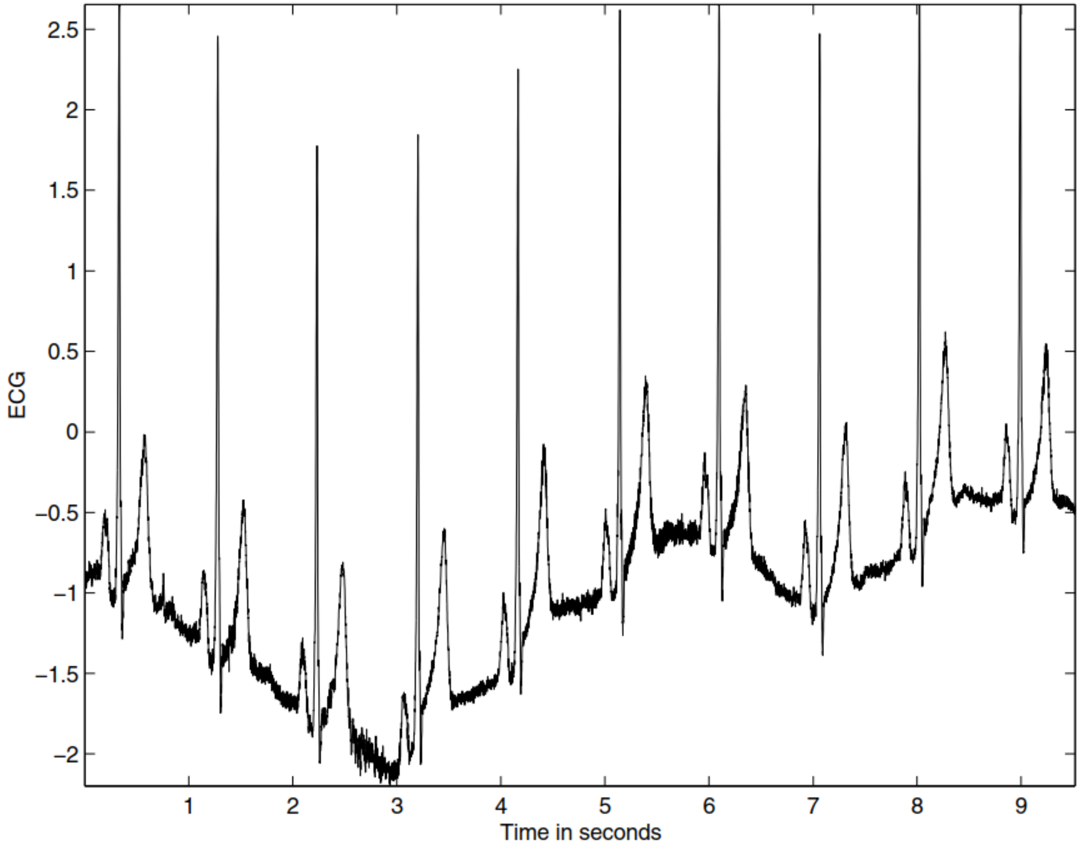
Example 2

- Low-frequency artifacts in the ECG
  - Motion artifact
  - Poor contact





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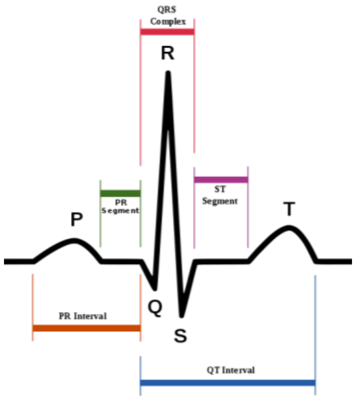
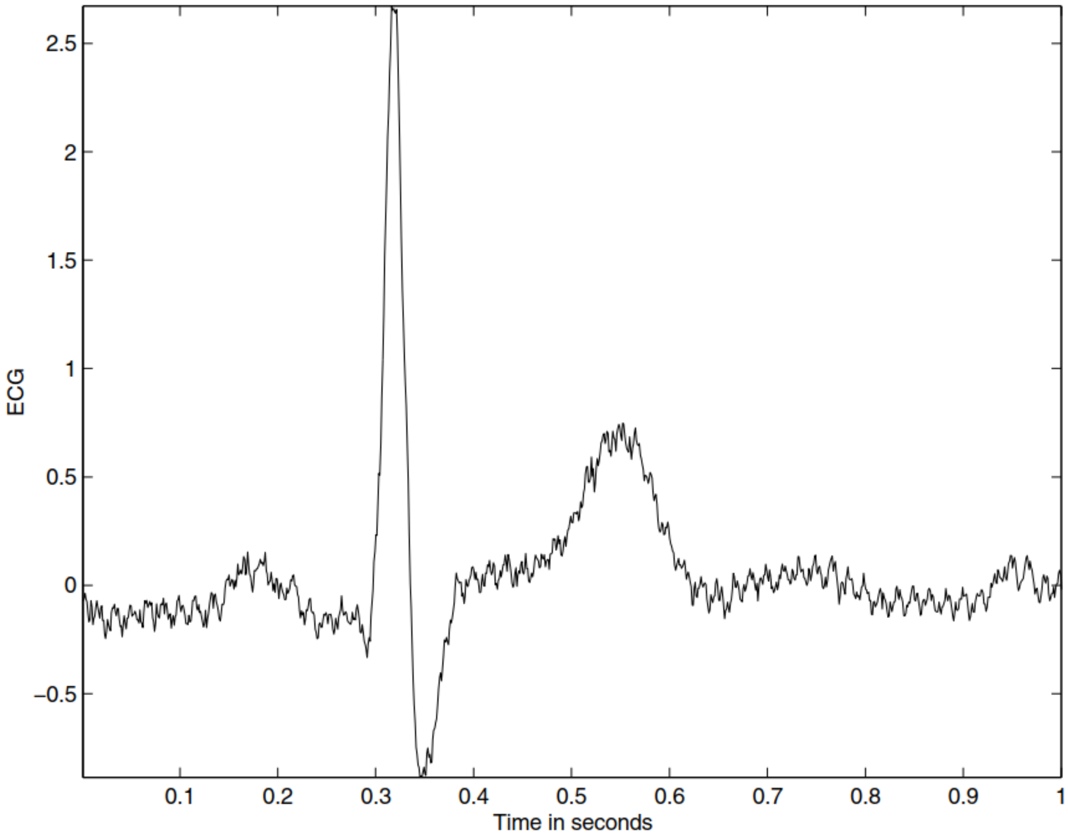
Example 3

- ❑ Power-line interference in the ECG signal
  - 50 Hz or 60 Hz (+ harmonics)





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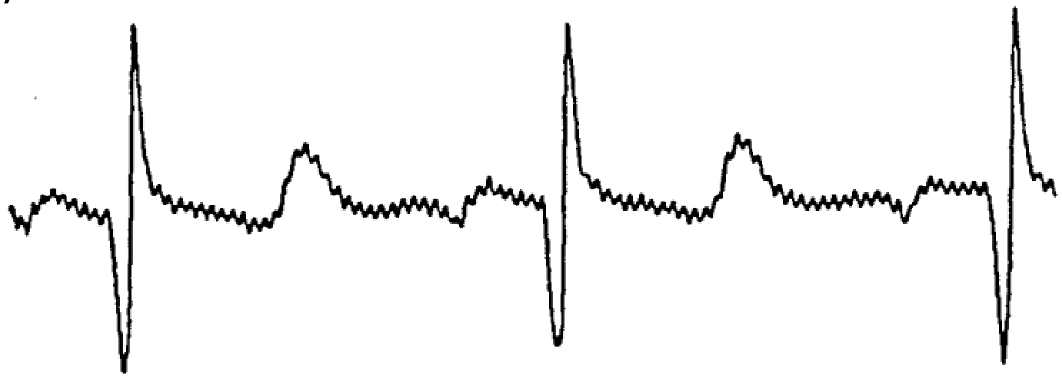
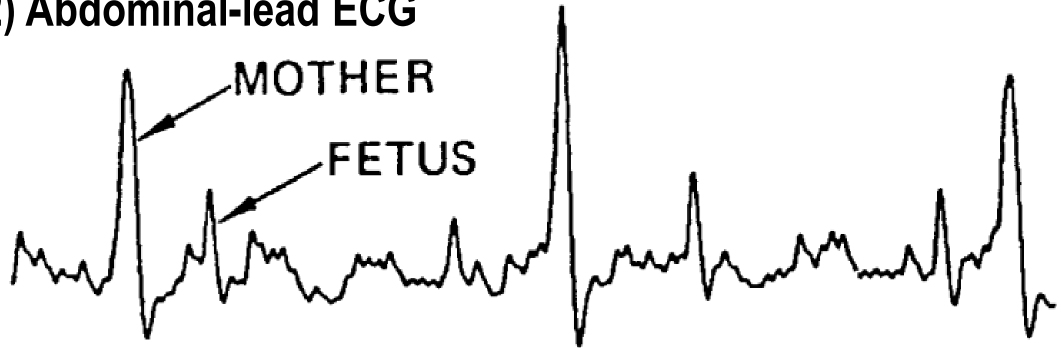
Example 4

- Maternal interference in fetal ECG
  - Abdominal-lead ECG includes the ECG of mother and fetus
  - If the signal of interest is the ECG of fetus, the ECG of the mother is an artifact and should be removed

1) Chest-lead ECG of mother



2) Abdominal-lead ECG



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Fundamental Concepts of Filtering

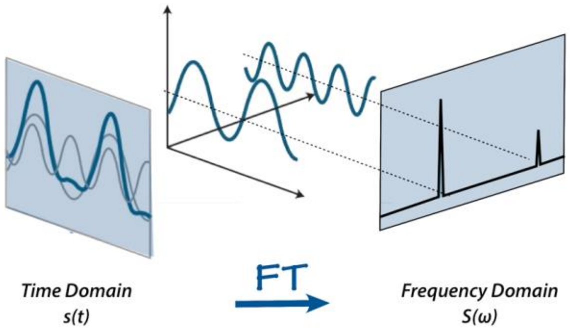
Concepts of filtering

- A filter is a **signal processing system**, **algorithm**, or **method**
  - Can be in hardware or software
    - We focus on software filtering
- Filter is used to modify a given signal in a particular manner
  - To remove undesired components that are referred to as noise or artifacts
- Filters are designed and analyzed by
  - Impulse response
  - Frequency response
  - Zero-pole diagram
  - etc.

Fourier Transform

- **Fourier Transform**: decomposes a function of time (a *signal*) into its frequencies

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t}\, dt$$



http://mriquestions.com/fourier-transform-ft.html

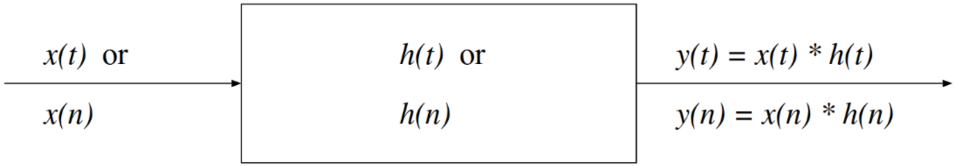
- **Discrete-time Fourier Transform**:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-i\omega n}$$

- **Inverse FT/ Inverse DTFT**: transform a continuous or discrete spectrum into a function for the amplitude with the given spectrum

Linear shift-invariant filters

- Linear time-invariant (LTI) or shift-invariant (LSI) filters are important category of filters.
- In the time domain, a continuous-time or discrete-time LSI filter can be presented as:



- The output of the filter is mathematically expressed as the **convolution** of the input
- Convolution is a mathematical operation: $y(t) = \int_{\tau=0}^{t} x(\tau)h(t-\tau)d\tau$

- **Impulse response:** the output of the filter is mathematically expressed as the convolution of the input with a impulse function (i.e., delta).

Frequency response

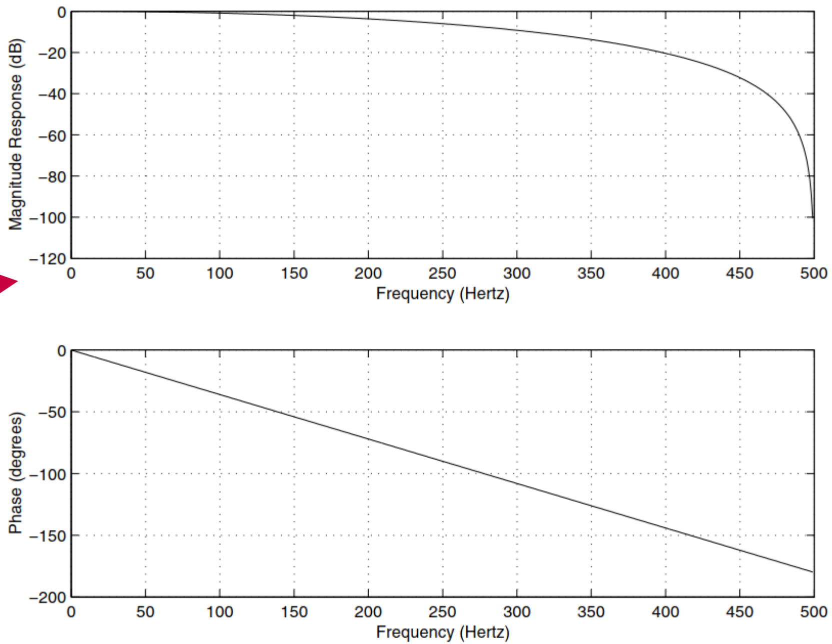
- In the frequency domain, we obtain the Fourier transform of h(t)

$$H(\omega) = \int_0^T h(t)e^{-i\omega t}dt$$

- The output becomes:

$$Y(\omega) = X(\omega)H(\omega)$$

- $H(\omega)$ is a complex quantity:
  - A magnitude $|H(\omega)|$
  - A phase $\angle H(\omega)$



Zero-Pole analysis

- **Z-transform**: converts a discrete-time signal into a complex frequency-domain representation

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} \xrightarrow{\text{Generalization of DTFT}} \boldsymbol{X(z)|_{z\,=\,e^{i\omega}}}$$

- We obtain the z-transform of h(n):

$$H(z) \; = \sum_{n=0}^{N-1} h(n) z^{-n}$$

- The output becomes: $Y(z)$ = $X(z)H(z)$

$$H(z) = \frac{P(z)}{Q(z)}$$

Two polynomials

- Roots of $P(z)$ are **zeros**, and roots of $Q(z)$ are **poles**
- The filter can be designed by defining appropriate **zeros** and **poles** ($P(z)$ and $Q(z)$)

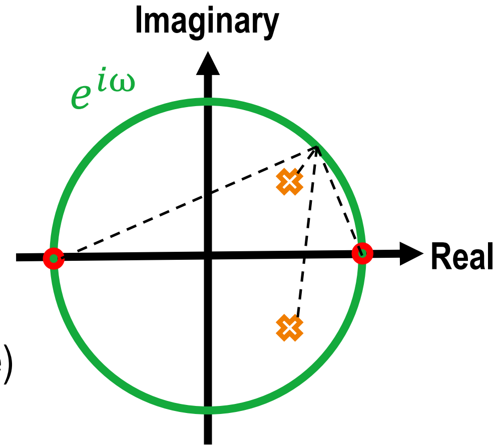
Zero-Pole analysis - Example

$$H(\omega) = H(z)|_{z\ =\ e^{i\omega}} = \frac{P(e^{i\omega})}{Q(e^{i\omega})}$$

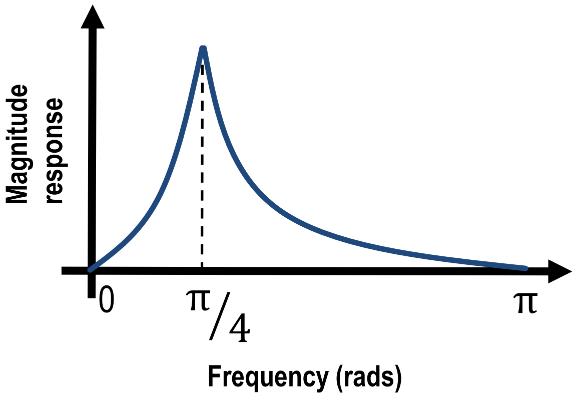
$$|H(\omega)| = \frac{|a_0|\ \prod_{k=1}^{M}|1-a_k e^{-i\omega}|}{|b_0|\ \prod_{k=1}^{N}|1-b_k e^{-i\omega}|}$$

$$= \frac{|a_0|\ \prod_{k=1}^{M}|e^{i\omega}-a_k|}{|b_0|\ \prod_{k=1}^{N}|e^{i\omega}-b_k|}$$

$$= \frac{|a_0|\ \prod_{k=1}^{M} \mathit{dist\ between}\ e^{i\omega} \mathit{and}\ zero_k}{|b_0|\ \prod_{k=1}^{N} \mathit{dist\ between}\ e^{i\omega} \mathit{and}\ pole_k}$$



Complex plane (z-plane)



Time domain Filters

Time domain Filters

- Certain types of noise may be filtered directly in the time domain
- Spectral characterization of the signal and noise is not required
- In the following, we focus on two types of Time-domain filters:
  1. Synchronized averaging
  2. Moving-average filter

Synchronized averaging (1)

- Synchronized averaging can separate a **repetitive** signal (events) from noise:
  - We are interested on patterns that are obtain from different sources
    - E.g., Delta rhythm in EEG
  - We are interested on a part of a signal that is repetitive
    - E.g., heart cycles in ECG
- We first **sync** the events and then **average**.

Synchronized averaging (2)

Uncorrupted signal noise

- Lets assume the observe signal is $$y(n) = \underbrace{x(n)}_{\text{Uncorrupted signal}} + \underbrace{\eta(n)}_{\text{noise}}$$
- We average M (different) copies of the signal:

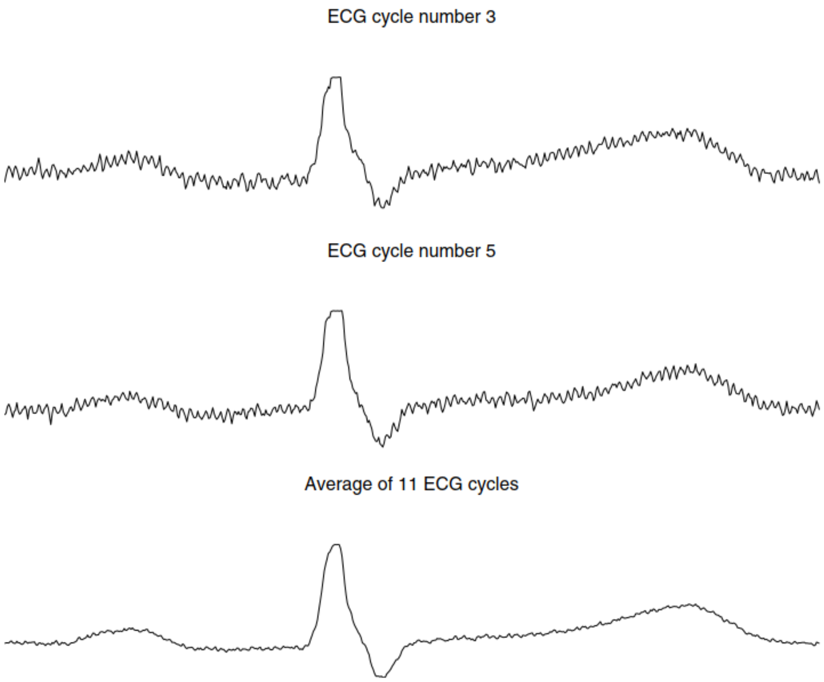
$$\sum_{k=1}^{M} y_k(n) = \sum_{k=1}^{M} x_k(n) + \sum_{k=1}^{M} \eta_k(n)$$

- If noise is random (zero mean) and M increases

$$\sum_{k=1}^{M} y_k(n) \simeq \sum_{k=1}^{M} x_k(n) = Mx(n)$$

Example – Heart cycles in ECG

1. The heart cycles are segmented.
2. The cycles are aligned (by the QRS complex)
3. The average values are calculated



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Moving average filter (MA filter)

- ❑ MA filter removes the noise by a **temporal** averaging
- ❑ A window of preceding points is used
- ❑ Number of preceding points included is the order of the filter

Order of the filter

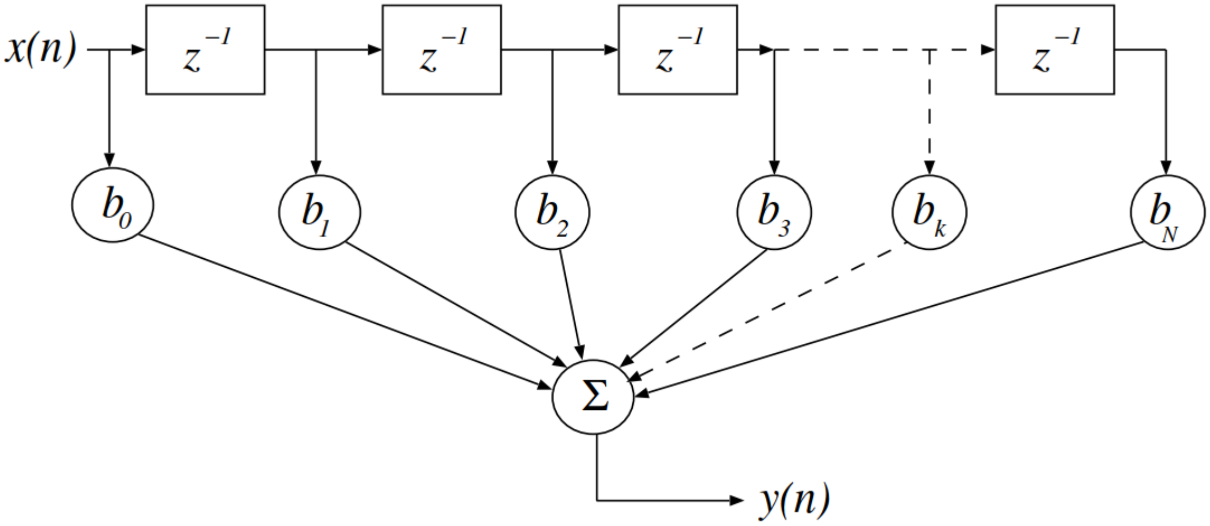
Signal

$$y(n) = \sum_{k=0}^{N} b_k x(n-k)$$

Weights

Signal-flow diagram of an MA filter

Z^{-1} represents a delay of one sample



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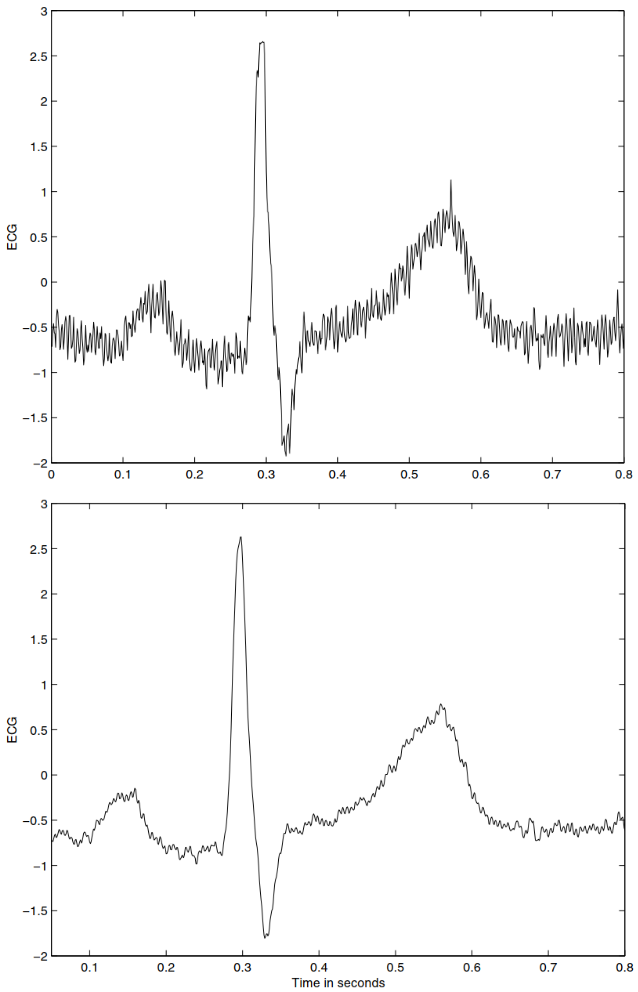
Example

- Sampling frequency = 1000 Hz
- 8-point MA filter
- $y(n) = \frac{1}{8}\sum_{k=0}^{8} x(n-k)$



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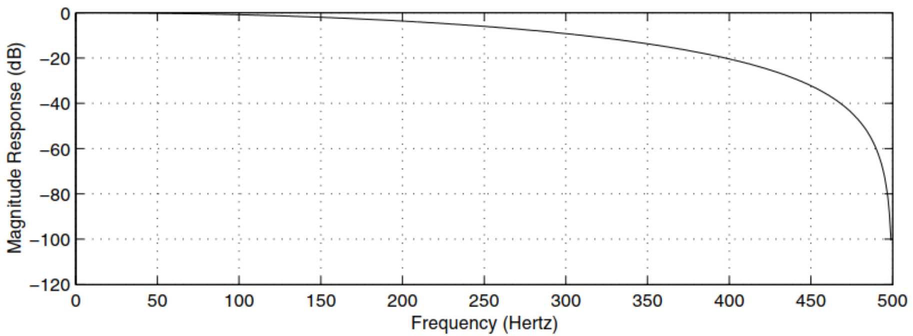
Von Hann filter

- The von Hann (or Hanning) filter is a simple MA filter for filtering noise

$$y(n) = \frac{1}{4}[x(n) + 2x(n-1) + x(n-2)]$$

The frequency response is for a signal with 1000 Hz



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Frequency domain Filters

Frequency domain filters

- Filters are designed to only pass desired frequency of the signal
- Filters in frequency domain are:
  - **Low pass**
  - **High pass**
  - **Band pass**
  - **Band reject (notch)**
- **Butterworth** is a commonly used filter design that have a frequency response as flat as possible in the passband.
- Other filter designs are Chebyshev, elliptic, and Bessel.

Butterworth low pass filter (1)

- **Low pass**: deliver **low** frequencies and eliminate **high** frequencies of a signal
- Frequency response:

$$|H(\omega)|^2 = \frac{1}{1+(\frac{\omega}{\omega_c})^{2n}}$$

Cutoff frequency

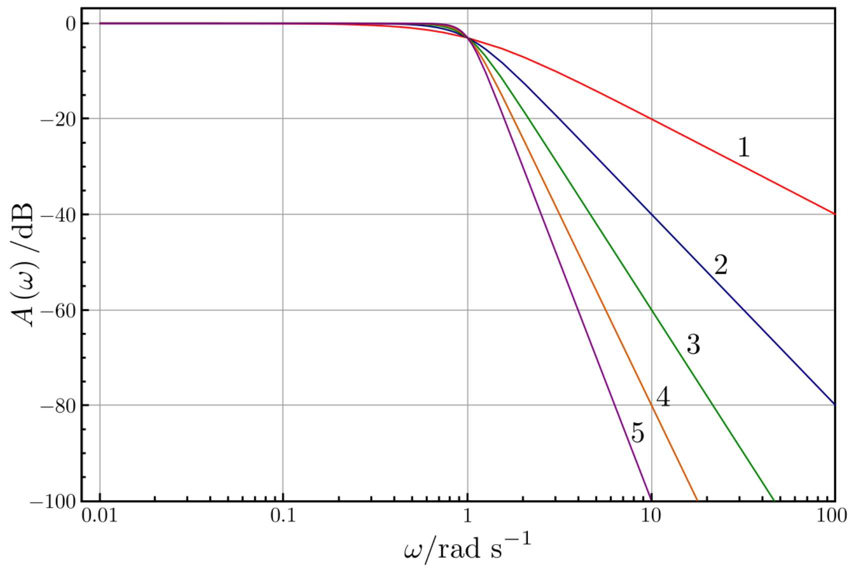
Order of the filter

- ω normalized to $(0, 2\pi)$ for sampled signals
- ω_c should be between $(0, \pi)$ **(Nyquist frequency)**

Butterworth low pass filter (2)

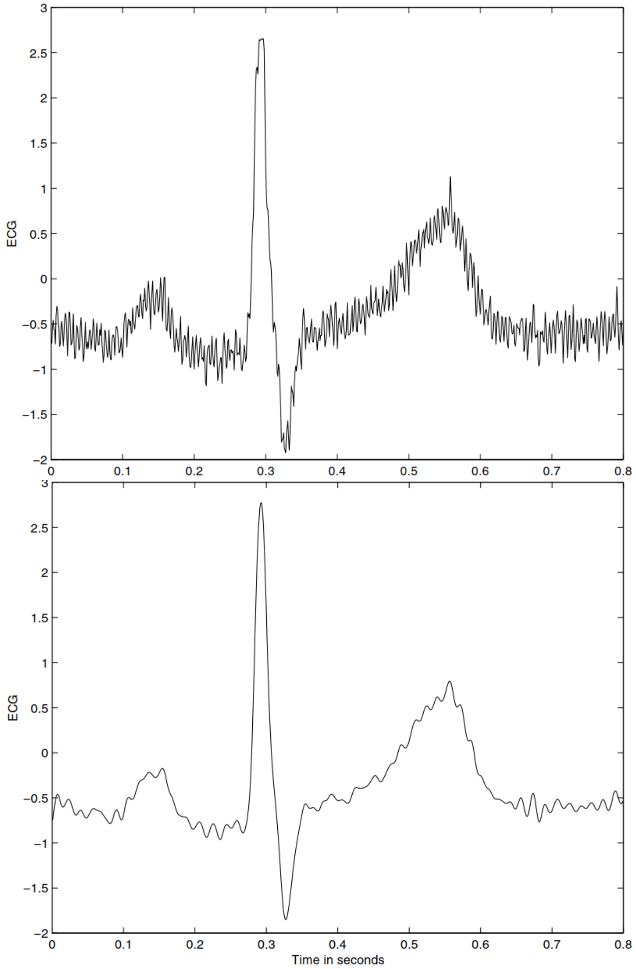
- When increase the order => add more zeros and poles

Gain of the filters with cutoff frequency of 1 and different orders



Example

- ❑ Sampling frequency = 1000 Hz
- ❑ Cutoff frequency = 70 Hz
- ❑ Order of the filter = 8



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Butterworth high pass filter

- **High pass**: deliver **high** frequencies and eliminate **low** frequencies of a signal
- Frequency response:

Cutoff frequency

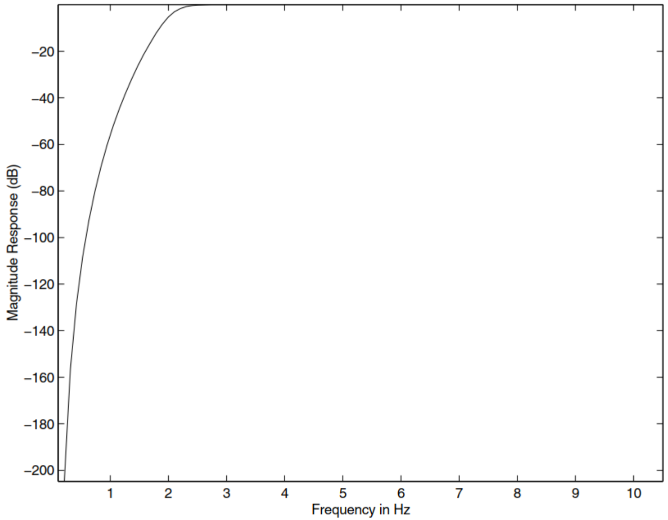
$$|H(\omega)|^2 = \frac{1}{1+(\frac{\omega_c}{\omega})^{2n}}$$

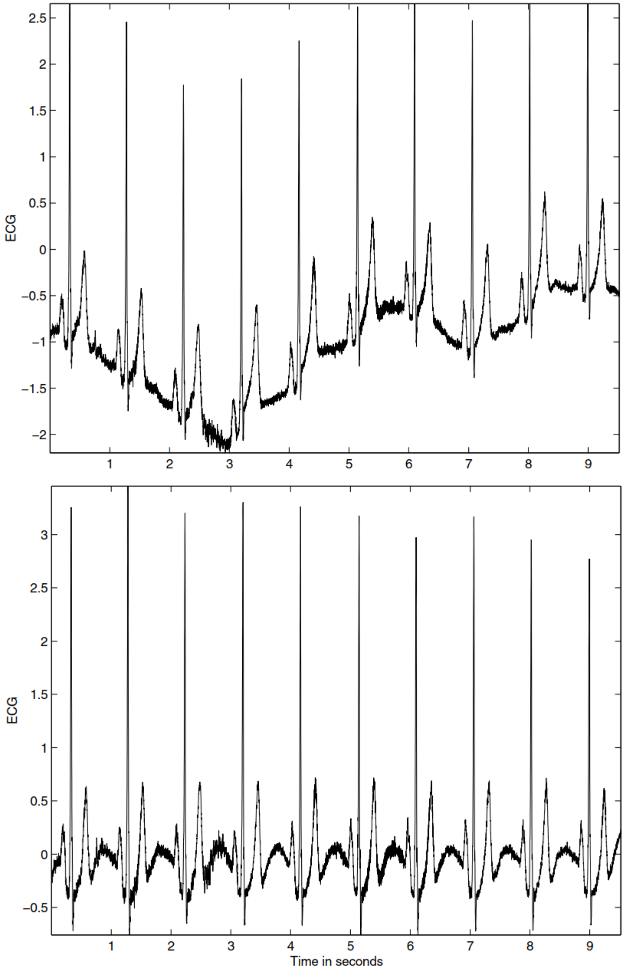
Order of the filter

- ω normalized to (0, 2π) for sampled signals
- ω_c should be between (0, π) **(Nyquist frequency)**

Example

- Cutoff frequency = 2 Hz
- Order of the filter = 8





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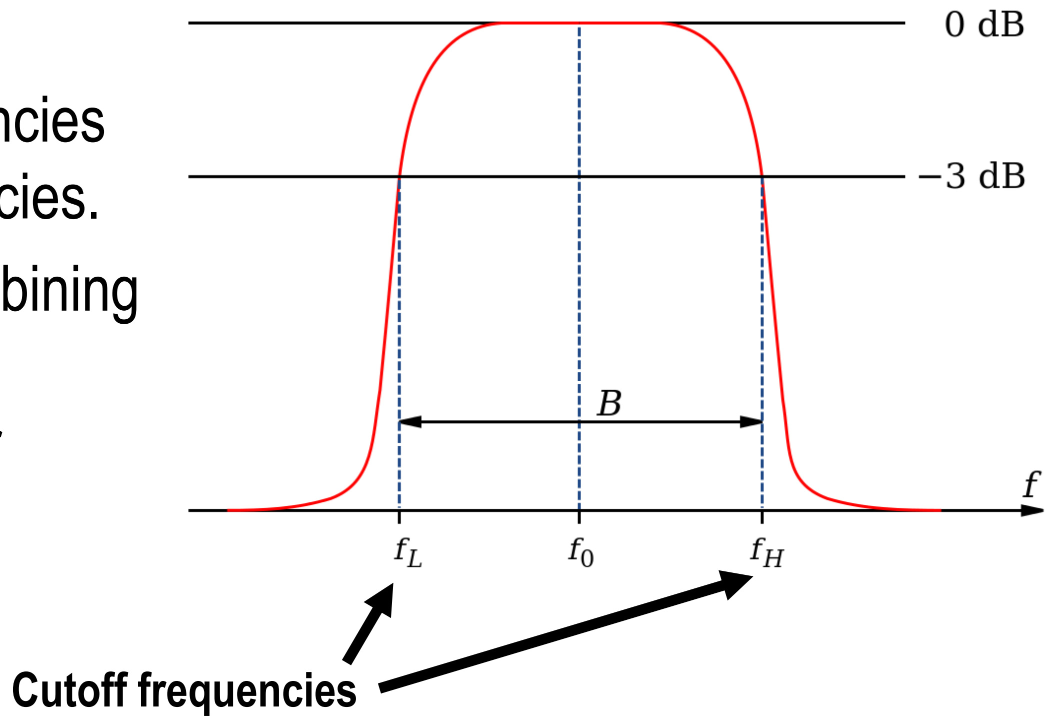
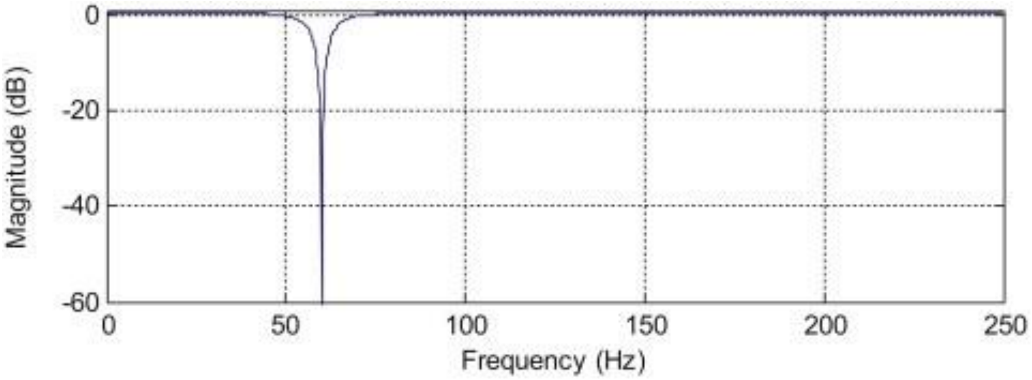
Band pass filter

- **Band pass**: pass some particular range of frequencies and stop all other frequencies.
- It can be obtained by combining or cascading a low pass filter with a high pass filter



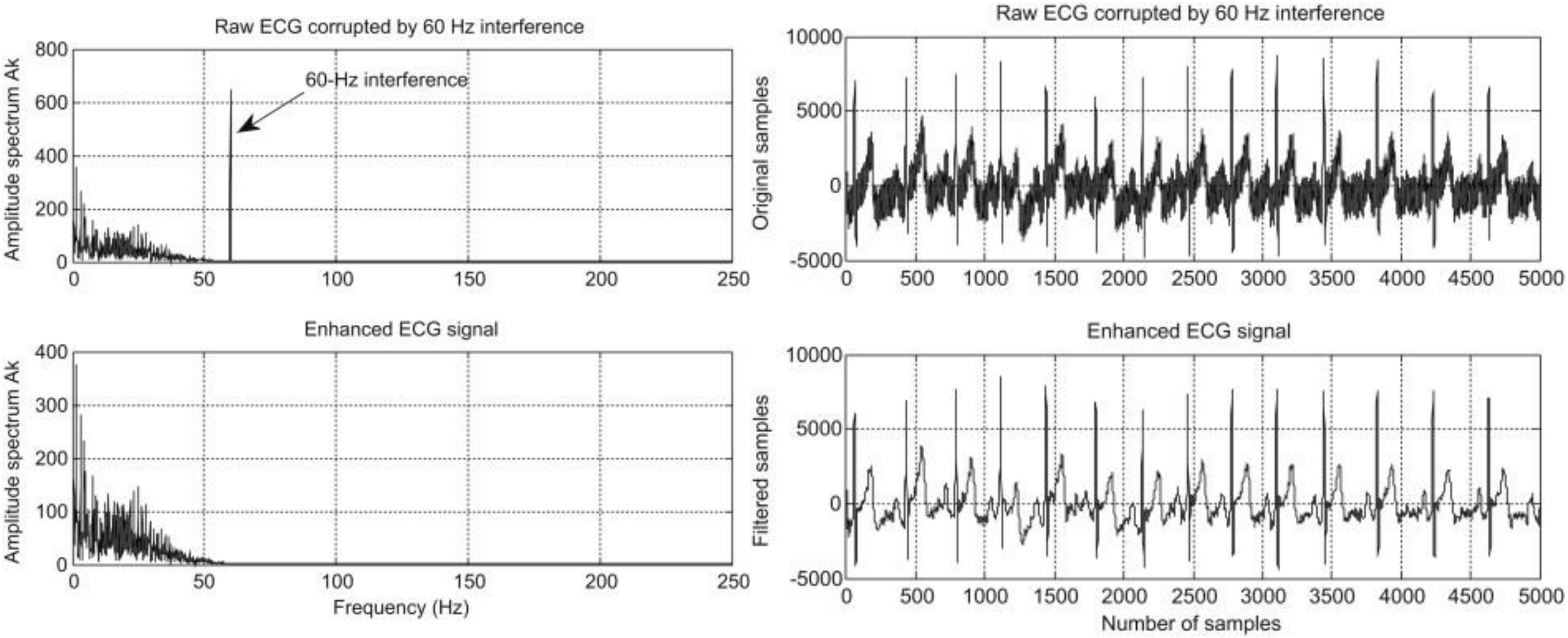
Band reject

- **Band reject**: stop a range of frequencies and pass all other frequencies
- **Notch filter**: stop a very narrow range of frequencies and pass all other frequencies
- Add zeros to the z-plane



Li Tan, Jean Jiang, Digital Signal Processing (Second Edition), Academic Press, 2013,

Example



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Adaptive filter

Problem of fixed filters

- Filters with fixed characteristics (weights) are not applicable when the characteristics of the signal and/or noise vary with time
- They are also not suitable when the spectral contents of the signal and the interference overlap significantly
  - ECG of mother and ECG of fetus
- We assume the recorded signal as:

Signal of interest

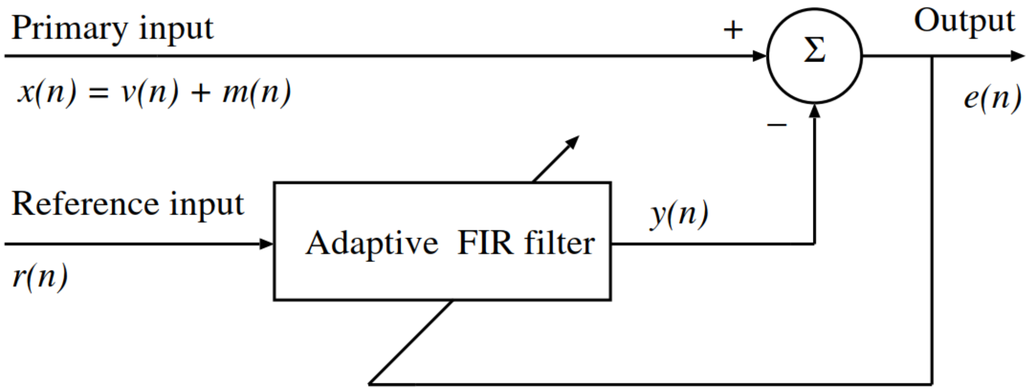
$$x(n) = v(n) + m(n)$$

Noise

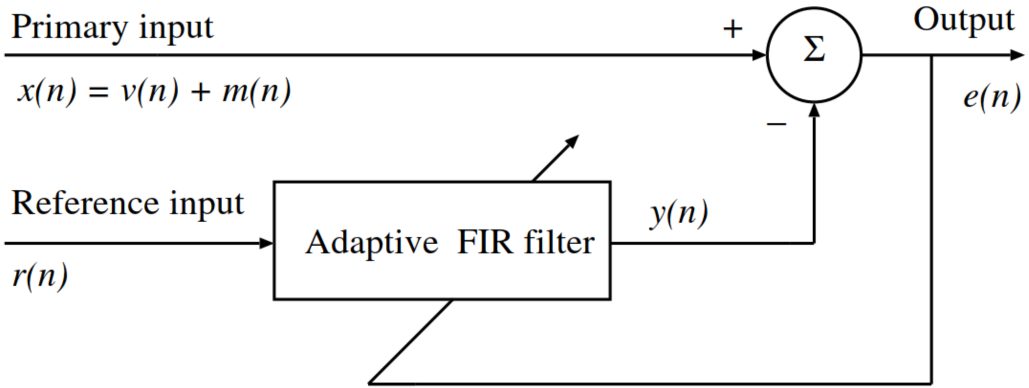
- An adaptive and optimal filter is required
  - To remove $m(n)$

Adaptive filter (1)

- Adaptive filtering requires a second input, known as the "reference input" $r(n)$
  - **Correlated** with the noise and **uncorrelated** with the signal of interest



Adaptive filter (2)



- $\tilde{v}(n) = e(n) = x(n) - y(n)$
- $e(n) = v(n) + \boxed{m(n) - y(n)}$

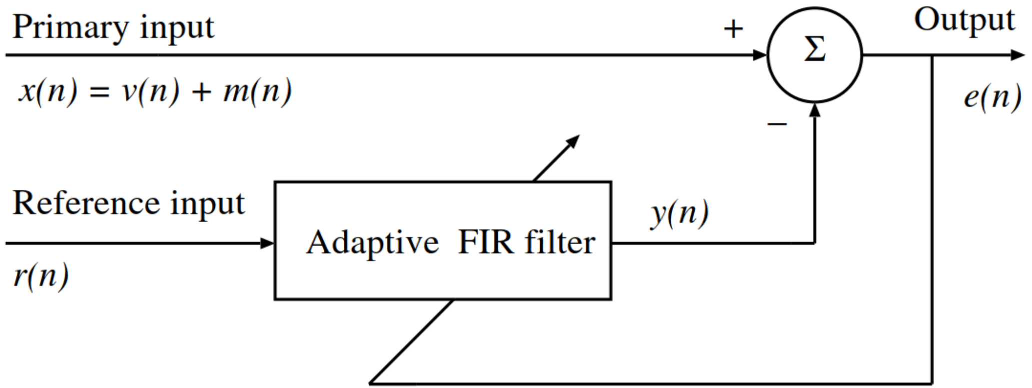
should be minimized

- $E[e^2(n)] = E[v^2(n)] + E[\{m(n) - y(n)\}^2] + \overbrace{2E[v(n)\{m(n) - y(n)\}]}^{0}$

unaffected to the filter

- So, if we minimize $E[e^2(n)]$ we then minimize $E[\{m(n) - y(n)\}^2]$

Adaptive filter (3)



- Adaptive filter should use the $r(n)$ and the feedback from the output to minimize the power of output ($\mathrm{E}[e^2(n)]$)

- $y(n) = \sum_{k=0}^{M-1} w_k r(n-k) = \boldsymbol{w}^T(n)\boldsymbol{r}(n)$
    - w_k are weights and M is the order of the filter

Least-mean-squares adaptive filter

- The goal is to adjust the weight vector to minimize the error

$$e^2(n) = x^2(n) - 2x(n)y(n) + y^2(n)$$

$$e^2(n) = x^2(n) - 2x(n)\boldsymbol{r}^T(n)\boldsymbol{w}(n) + \boldsymbol{w}^T(n)\boldsymbol{r}(n)\boldsymbol{r}^T(n)\boldsymbol{w}(n)$$

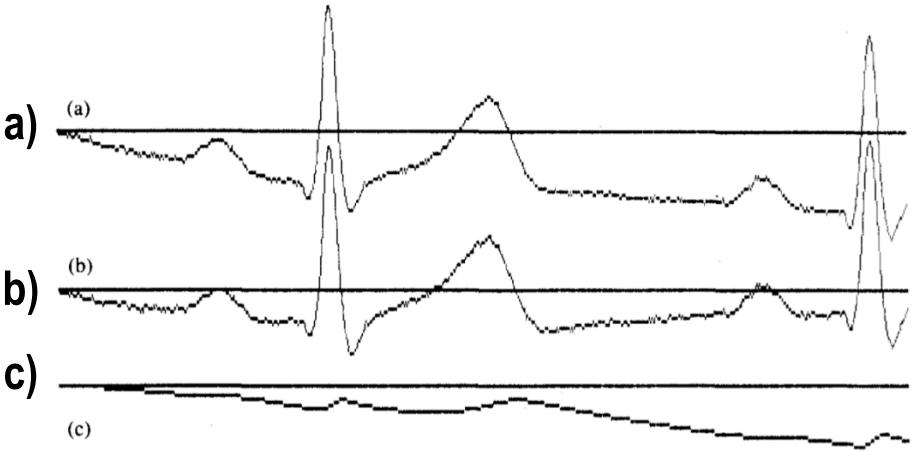
- Gradient-based methods may be used

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) - \nabla\mu e^2(n)$$

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + 2\mu e(n)\boldsymbol{r}(n)$$

Example (1)

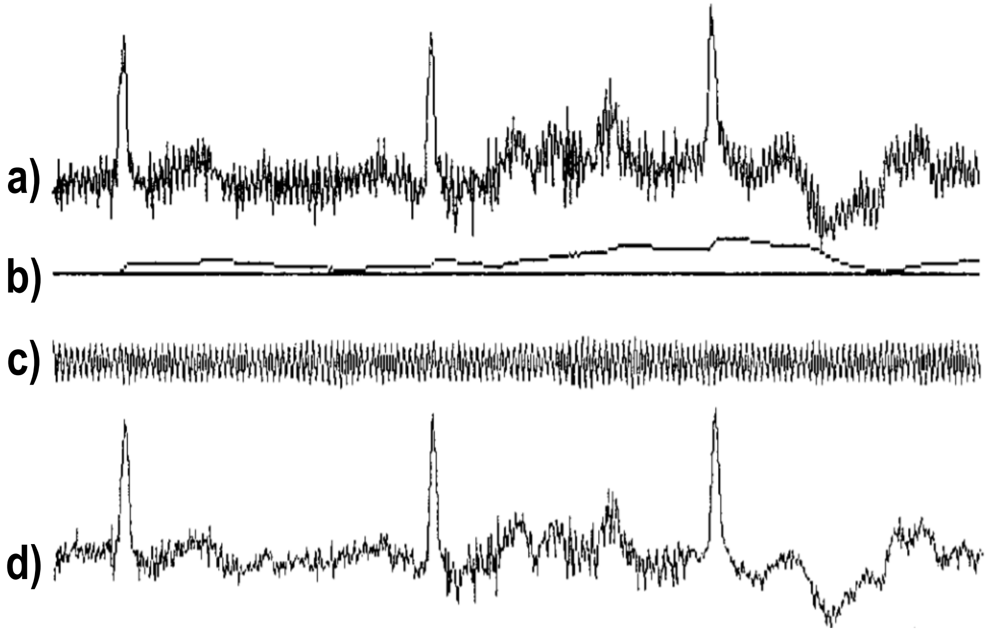
a) Raw ECG
b) Filtered ECG
c) Filter estimate of baseline wander



Thakor, N. V., & Zhu, Y. S. (1991). Applications of adaptive filtering to ECG analysis: noise cancellation and arrhythmia detection. IEEE transactions on biomedical engineering, 38(8), 785-794.

Example (2)

a) Raw ECG
b) Filter estimate of baseline wander
c) Filter estimate of the common mode 60 Hz noise
d) Filtered ECG



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Selection of the filter

Selection of the filter

- Depending on the signal and noise:

  Synchronized averaging: Multiple realizations of signal, and noise has a zero mean.

  MA filtering: The signal is a slow (low-frequency) phenomenon and fast real-time filtering is desired.

  Frequency-domain filtering: Loss of information in the spectral band removed by the filter does not seriously affect the signal.

  Adaptive filtering: The noise is uncorrelated with the signal, and a second source is available to obtain a reference signal strongly correlated with the noise but not with the signal.

Conclusion

In this session, we learned:

- ❑ Noise in biosignals
- ❑ Fundamental Concepts of Filtering
- ❑ Filtering techniques
  - Time domain filters
  - Frequency domain filters
  - Adaptive filter

In the next session, we will learn:

- ❑ Analysis of biosignals
  - E.g., correlation and waveform analysis techniques

Thank You

Questions?

Turun yliopisto
University of Turku