Project Euler

http://projecteuler.net

Problems/Probleme

English/deutsch

translated by/übersetzt von

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Problem 1

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

Problem 1

Wenn wir alle natürlichen Zahlen auflisten, die kleiner sind als 10 und gleichzeitig Vielfache von 3 oder 5, so erhalten wir 3, 5, 6 und 9. Die Summe dieser Vielfachen ist 23.

Ermittle die Summe aller Vielfachen von 3 oder 5, die kleiner sind als 1000.

Problem 2

Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

Find the sum of all the even-valued terms in the sequence which do not exceed four million.

Problem 2

Jede neue Zahl in der Folge der Fibonacci-Zahlen ergibt sich, wenn man die vorhergehenden zwei Zahlen zusammenzählt. Wenn man mit den beiden Zahlen 1 und 2 beginnt, sind die ersten 10 Fibonacci-Zahlen

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

Ermittle die Summe aller geraden Zahlen in dieser Folge, die vier Millionen nicht übersteigen.

Problem 3

The prime factors of 13195 are 5, 7, 13 and 29.

What is the largest prime factor of the number 600851475143 ?

Problem 3

Die Primfaktoren von 13195 sind 5, 7, 13 und 29.

Welches ist der größte Primfaktor der Zahl 600851475143 ?

Problem 4

A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is

$$9009 = 91 \times 99.$$

Find the largest palindrome made from the product of two 3-digit numbers.

Problem 4

Eine Palindromzahl ist eine Zahl, die von hinten wie von vorn gleich ist. Die größte Palindromzahl, die man als Produkt zweier zweistelliger Zahlen schreiben kann ist

$$9009 = 91 \cdot 99.$$

Finde die größte Palindromzahl, die sich als Produkt zweier dreistelliger Zahlen schreiben lässt.

Problem 5

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

Problem 5

2520 ist die kleinste Zahl, die man durch alle Zahlen von 1 bis 10 ohne Rest teilen kann.

Welches ist die kleinste positive Zahl, die sich ohne Rest durch alle Zahlen von 1 bis 20 teilen lässt?

Problem 6

The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + \dots + 10^2 = 385$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + \dots + 10)^2 = 55^2 = 3025$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is $3025 - 385 = 2640$.

Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

Problem 6

Die Summe der ersten zehn Quadratzahlen ist

$$1^2 + 2^2 + \dots + 10^2 = 385$$

Das Quadrat der Summe der ersten zehn natürlichen Zahlen ist

$$(1 + 2 + \dots + 10)^2 = 55^2 = 3025$$

Die Differenz zwischen der Summe der Quadrate der ersten zehn natürlichen Zahlen und dem Quadrat ihrer Summe ist also $3025 - 385 = 2640$.

Ermittle die Differenz zwischen der Summe der Quadrate der ersten einhundert natürlichen Zahlen und dem Quadrat ihrer Summe.

Problem 7

By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6^{th} prime is 13.

What is the 10001^{st} prime number?

Problem 7

Wir listen die ersten sechs Primzahlen auf: 2, 3, 5, 7, 11 und 13. Wir sehen, dass die sechste Primzahl die 13 ist.

Welches ist die 10001. Primzahl?

Problem 8

Find the greatest product of five consecutive digits in the 1000-digit number.

73167176531330624919225119674426574742355349194934
96983520312774506326239578318016984801869478851843
85861560789112949495459501737958331952853208805511
12540698747158523863050715693290963295227443043557
66896648950445244523161731856403098711121722383113
62229893423380308135336276614282806444486645238749
30358907296290491560440772390713810515859307960866
70172427121883998797908792274921901699720888093776
65727333001053367881220235421809751254540594752243
52584907711670556013604839586446706324415722155397
53697817977846174064955149290862569321978468622482
83972241375657056057490261407972968652414535100474
82166370484403199890008895243450658541227588666881
16427171479924442928230863465674813919123162824586
17866458359124566529476545682848912883142607690042
24219022671055626321111109370544217506941658960408
07198403850962455444362981230987879927244284909188
84580156166097919133875499200524063689912560717606
05886116467109405077541002256983155200055935729725
71636269561882670428252483600823257530420752963450

Problem 8

Finde das größte Produkt von fünf unmittelbar aufeinander folgenden Ziffern der folgenden 1000-stelligen Zahl.

73167176531330624919225119674426574742355349194934
96983520312774506326239578318016984801869478851843
85861560789112949495459501737958331952853208805511
12540698747158523863050715693290963295227443043557
66896648950445244523161731856403098711121722383113
62229893423380308135336276614282806444486645238749
30358907296290491560440772390713810515859307960866
70172427121883998797908792274921901699720888093776
65727333001053367881220235421809751254540594752243
52584907711670556013604839586446706324415722155397
53697817977846174064955149290862569321978468622482
83972241375657056057490261407972968652414535100474
82166370484403199890008895243450658541227588666881
16427171479924442928230863465674813919123162824586
17866458359124566529476545682848912883142607690042
24219022671055626321111109370544217506941658960408
07198403850962455444362981230987879927244284909188
84580156166097919133875499200524063689912560717606
05886116467109405077541002256983155200055935729725
71636269561882670428252483600823257530420752963450

Problem 9

A Pythagorean triplet is a set of three natural numbers, $a < b < c$, for which $a^2 + b^2 = c^2$

For example,

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2.$$

There exists exactly one Pythagorean triplet for which

$$a + b + c = 1000.$$

Find the product *abc*.

Problem 9

Ein pythagoräisches Tripel ist ein Satz von drei natürlichen Zahlen $a < b < c$, für die gilt: $a^2 + b^2 = c^2$

Beispielsweise gilt:

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2.$$

Es gibt ein einziges pythagoräisches Tripel, für das gilt:

$$a + b + c = 1000$$

Ermittle das Produkt dieser Zahlen, *abc*.

Problem 10

The sum of the primes below 10 is 2 + 3 + 5 + 7 = 17.

Find the sum of all the primes below two million.

Problem 10

Die Summe aller Primzahlen, die kleiner sind als 10, ist
2 + 3 + 5 + 7 = 17.

Ermittle die Summe aller Primzahlen, die kleiner sind als zwei Millionen.

Problem 11

In the 20×20 grid below, four numbers along a diagonal line have been marked in red.

08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08
49 49 99 40 17 81 18 57 60 87 17 40 98 43 69 48 04 56 62 00
81 49 31 73 55 79 14 29 93 71 40 67 53 88 30 03 49 13 36 65
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50
32 98 81 28 64 23 67 10 **26** 38 40 67 59 54 70 66 18 38 64 70
67 26 20 68 02 62 12 20 95 **63** 94 39 63 08 40 91 66 49 94 21
24 55 58 05 66 73 99 26 97 17 **78** 78 96 83 14 88 34 89 63 72
21 36 23 09 75 00 76 44 20 45 35 **14** 00 61 33 97 34 31 33 95
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69
04 42 16 73 38 25 39 11 24 94 72 18 08 46 29 32 40 62 76 36
20 69 36 41 72 30 23 88 34 62 99 69 82 67 59 85 74 04 36 16
20 73 35 29 78 31 90 01 74 31 49 71 48 86 81 16 23 57 05 54
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 19 67 48

The product of these numbers is
26 × 63 × 78 × 14 = 1788696.

What is the greatest product of four adjacent numbers in any direction (up, down, left, right, or diagonally) in the 20×20 grid?

Problem 11

In dem untenstehenden 20×20-Gitter wurden vier Zahlen auf einer Diagonalen rot markiert.

08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08
49 49 99 40 17 81 18 57 60 87 17 40 98 43 69 48 04 56 62 00
81 49 31 73 55 79 14 29 93 71 40 67 53 88 30 03 49 13 36 65
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50
32 98 81 28 64 23 67 10 **26** 38 40 67 59 54 70 66 18 38 64 70
67 26 20 68 02 62 12 20 95 **63** 94 39 63 08 40 91 66 49 94 21
24 55 58 05 66 73 99 26 97 17 **78** 78 96 83 14 88 34 89 63 72
21 36 23 09 75 00 76 44 20 45 35 **14** 00 61 33 97 34 31 33 95
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69
04 42 16 73 38 25 39 11 24 94 72 18 08 46 29 32 40 62 76 36
20 69 36 41 72 30 23 88 34 62 99 69 82 67 59 85 74 04 36 16
20 73 35 29 78 31 90 01 74 31 49 71 48 86 81 16 23 57 05 54
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 19 67 48

Das Produkt dieser Zahlen ist 26 • 63 • 78 • 14 = 1788696.

Welches ist das größte Produkt von vier horizontal, vertikal oder diagonal unmittelbar aufeinanderfolgenden Zahlen (hoch, runter, links, rechts oder diagonal) in obigem 20×20-Gitter?

Problem 12

The sequence of *triangle numbers* is generated by adding the natural numbers. So the 7^{th} triangle number would be 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28.

The first ten terms would be:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

Let us list the factors of the first seven triangle numbers:

1: 1
3: 1,3
6: 1,2,3,6
10: 1,2,5,10
15: 1,3,5,15
21: 1,3,7,21
28: 1,2,4,7,14,28

We can see that 28 is the first triangle number to have over five divisors.

What is the value of the first triangle number to have over five hundred divisors?

Problem 12

Die Folge der *Dreieckszahlen* erzeugt man, indem man die natürlichen Zahlen zusammenzählt. So ist die 7. Dreieckszahl 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28.

Die ersten zehn Zahlen in dieser Folge sind also:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

Listen wir für jede dieser Dreieckszahlen einmal ihre Teiler:

1: 1
3: 1,3
6: 1,2,3,6
10: 1,2,5,10
15: 1,3,5,15
21: 1,3,7,21
28: 1,2,4,7,14,28

Wir sehen, dass 28 die erste Dreieckszahl ist, die mehr als fünf Teiler hat.

Welches ist die kleinste Dreieckszahl, die mehr als fünfhundert Teiler hat?

Problem 13

Work out the first ten digits of the sum of the following one hundred 50-digit numbers.

37107287533902102798797998220837590246510135740250
46376937677490009712648124896970078050417018260538
74324986199524741059474233309513058123726617309629
91942213363574161572522430563301811072406154908250
23067588207539346171171980310421047513778063246676
89261670696623633820136378418383684178734361726757
28112879812849979408065481931592621691275889832738
44274228917432520321923589422876796487670272189318
47451445736001306439091167216856844588711603153276
70386486105843025439939619828917593665686757934951
62176457141856560629502157223196586755079324193331
64906352462741904929101432445813822663347944758178
92575867718337217661963751590579239728245598838407
58203565325359399008402633568948830189458628227828
80181199384826282014278194139940567587151170094390
35398664372827112653829987240784473053190104293586
86515506006295864861532075273371959191420517255829
71693888707715466499115593487603532921714970056938
54370070576826684624621495650076471787294438377604
53282654108756828443191190634694037855217779295145
36123272525000296071075082563815656710885258350721
45876576172410976447339110607218265236877223636045
17423706905851860660448207621209813287860733969412
81142660418086830619328460811191061556940512689692
51934325451728388641918047049293215058642563049483
62467221648435076201727918039944693004732956340691
15732444386908125794514089057706229429197107928209
55037687525678773091862540744969844508330393682126
18336384825330154686196124348767681297534375946515
80386287592878490201521685554828717201219257766954
78182833757993103614740356856449095527097864797581
16726320100436897842553539920931837441497806860984
48403098129077791799088218795327364475675590848030
87086987551392711854517078544161852424320693150332
59959406895756536782107074926966537676326235447210
69793950679652694742597709739166693763042633987085
41052684708299085211399427365734116182760315001271
65378607361501080857009149939512557028198746004375
35829035317434717326932123578154982629742552737307
94953759765105305946966067683156574377167401875275
88902802571733229619176668713819931811048770190271
25267680276078003013678680992525463401061632866526
36270218540497705585629946580636237993140746255962
24074486908231174977792365466257246923322810917141
91430288197103288597806669760892938638285025333403
34413065578016127815921815005561868836468420090470
23053081172816430487623791969842487255036638784583
11487696932154902810424020138335124462181441773470
63783299490636259666498587618221225225512486764533
67720186971698544312419572409913959008952310058822
95548255300263520781532296796249481641953868218774
76085327132285723110424803456124867697064507995236
37774242535411291684276865538926205024910326572967
23701913275725675285653248258265463092207058596522
29798860272258331913126375147341994889534765745501
18495701454879288984856827726077713721403798879715
38298203783031473527721580348144513491373226651381
34829543829199918180278916522431027392251122869539
40957953066405232632538044100059654939159879593635
29746152185502371307642255121183693803580388584903
41698116222072977186158236678424689157993532961922
62467957194401269043877107275048102390895523597457
23189706772547915061505504953922979530901129967519
86188088225875314529584099251203829009407770775672
11306739708304724483816533873502340845647058077308
82959174767140363198008187129011875491310547126581
97623331044818386269515456334926366572897563400500
42846280183517070527831839425882145521227251250327
55121603546981200581762165212827652751691296897789
32238195734329339946437501907836945765883352399886
75506164965184775180738168837861091527357929701337
62177842752192623401942399639168044983993173312731
32924185707147349566916674687634660915035914677504
99518671430235219628894890102423325116913619626622
73267460800591547471830798392868535206946944540724
76841822524674417161514036427982273348055556214818
97142617910342598647204516893989422179826088076852
87783646182799346313767754307809363333018982642090
10848802521674670883215120185883543223812876952786
71329612474782464538636993009049310363619763878039
62184073572399794223406235393808339651327408011116
66627891981488087797941876876144230030984490851411
60661826293682836764744779239180335110989069790714
85786944089552990653640447425576083659976645795096
66024396409905389607120198219976047599490197230297
64913982680032973156037120041377903785566085089252
16730939319872750275468906903707539413042652315011
94809377245048795150954100921645863754710598436791
78639167021187492431995700641917969777599028300699
15368713711936614952811305876380278410754449733078
40789923115535562561142322423255033685442488917353
44889911501440648020369068063960672322193204149535
41503128880339536053299340368006977710650566631954
81234880673210146739058568557934581403627822703280
82616570773948327592232845941706525094512325230608
22918802058777319719839450180888072429661980811197
77158542502016545090413245809786882778948721859617
72107838435069186155435662884062257473692284509516
20849603980134001723930671666823555245252804609722
53503534226472524250874054075591789781264330331690

Problem 13

Ermittle die ersten zehn Ziffern der Summe der folgenden einhundert 50-stelligen Zahlen:

37107287533902102798797998220837590246510135740250
46376937677490009712648124896970078050417018260538
74324986199524741059474233309513058123726617309629
91942213363574161572522430563301811072406154908250
23067588207539346171171980310421047513778063246676
89261670696623633820136378418383684178734361726757
28112879812849979408065481931592621691275889832738
44274228917432520321923589422876796487670272189318
47451445736001306439091167216856844588711603153276
70386486105843025439939619828917593665686757934951
62176457141856560629502157223196586755079324193331
64906352462741904929101432445813822663347944758178
92575867718337217661963751590579239728245598838407
58203565325359399008402633568948830189458628227828
80181199384826282014278194139940567587151170094390
35398664372827112653829987240784473053190104293586
86515506006295864861532075273371959191420517255829
71693888707715466499115593487603532921714970056938
54370070576826684624621495650076471787294438377604
53282654108756828443191190634694037855217779295145
36123272525000296071075082563815656710885258350721
45876576172410976447339110607218265236877223636045
17423706905851860660448207621209813287860733969412
81142660418086830619328460811191061556940512689692
51934325451728388641918047049293215058642563049483
62467221648435076201727918039944693004732956340691
15732444386908125794514089057706229429197107928209
55037687525678773091862540744969844508330393682126
18336384825330154686196124348767681297534375946515
80386287592878490201521685554828717201219257766954
78182833757993103614740356856449095527097864797581
16726320100436897842553539920931837441497806860984
48403098129077791799088218795327364475675590848030
87086987551392711854517078544161852424320693150332
59959406895756536782107074926966537676326235447210
69793950679652694742597709739166693763042633987085
41052684708299085211399427365734116182760315001271
65378607361501080857009149939512557028198746004375
35829035317434717326932123578154982629742552737307
94953759765105305946966067683156574377167401875275
88902802571733229619176668713819931811048770190271
25267680276078003013678680992525463401061632866526
36270218540497705585629946580636237993140746255962
24074486908231174977792365466257246923322810917141
91430288197103288597806669760892938638285025333403
34413065578016127815921815005561868836468420090470
23053081172816430487623791969842487255036638784583
11487696932154902810424020138335124462181441773470
63783299490636259666498587618221225225512486764533
67720186971698544312419572409913959008952310058822
95548255300263520781532296796249481641953868218774
76085327132285723110424803456124867697064507995236
37774242535411291684276865538926205024910326572967
23701913275725675285653248258265463092207058596522
29798860272258331913126375147341994889534765745501
18495701454879288984856827726077713721403798879715
38298203783031473527721580348144513491373226651381
34829543829199918180278916522431027392251122869539
40957953066405232632538044100059654939159879593635
29746152185502371307642255121183693803580388584903
41698116222072977186158236678424689157993532961922
62467957194401269043877107275048102390895523597457
23189706772547915061505504953922979530901129967519
86188088225875314529584099251203829009407770775672
11306739708304724483816533873502340845647058077308
82959174767140363198008187129011875491310547126581
97623331044818386269515456334926366572897563400500
42846280183517070527831839425882145521227251250327
55121603546981200581762165212827652751691296897789
32238195734329339946437501907836945765883352399886
75506164965184775180738168837861091527357929701337
62177842752192623401942399639168044983993173312731
32924185707147349566916674687634660915035914677504
99518671430235219628894890102423325116913619626622
73267460800591547471830798392868535206946944540724
76841822524674417161514036427982273348055556214818
97142617910342598647204516893989422179826088076852
87783646182799346313767754307809363333018982642090
10848802521674670883215120185883543223812876952786
71329612474782464538636993009049310363619763878039
62184073572399794223406235393808339651327408011116
66627891981488087797941876876144230030984490851411
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85786944089552990653640447425576083659976645795096
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16730939319872750275468906903707539413042652315011
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78639167021187492431995700641917969777599028300699
15368713711936614952811305876380278410754449733078
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82616570773948327592232845941706525094512325230608
22918802058777319719839450180888072429661980811197
77158542502016545090413245809786882778948721859617
72107838435069186155435662884062257473692284509516
20849603980134001723930671666823555245252804609722
53503534226472524250874054075591789781264330331690

Problem 14

The following iterative sequence is defined for the set of positive integers:

$n \rightarrow n/2$ (n is even)
$n \rightarrow 3n + 1$ (n is odd)

Using the rule above and starting with 13, we generate the following sequence:

$$13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

It can be seen that this sequence (starting at 13 and finishing at 1) contains 10 terms. Although it has not been proved yet (Collatz Problem), it is thought that all starting numbers finish at 1.

Which starting number, under one million, produces the longest chain?

NOTE: Once the chain starts the terms are allowed to go above one million.

Problem 14

Es wird für die Menge der positiven Zahlen die folgende iterative Zahlenfolge definiert:

$n \rightarrow n:2$ (für gerade n)
$n \rightarrow 3n + 1$ (für ungerade n)

Die Startzahl 13 liefert mit der obigen Regel die folgende Zahlenfolge:

$$13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

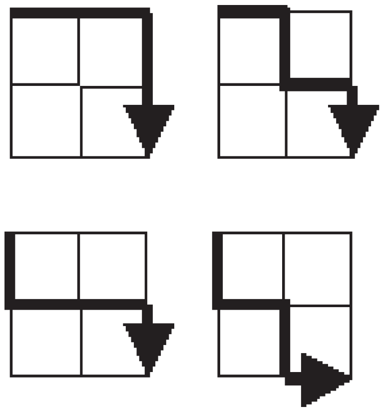
Wie man sieht, enthält diese Folge einschließlich der Startzahl 13 zehn Zahlen. Obwohl es bisher nicht bewiesen werden konnte (Collatz-Vermutung), glaubt man, dass alle Startzahlen irgendwann bei 1 enden.

Welche Startzahl, die kleiner ist als eine Million, liefert die längste Kette?

Anmerkung: Die einzelnen Zahlen in der Kette können sehr wohl größer als eine Million werden.

Problem 15

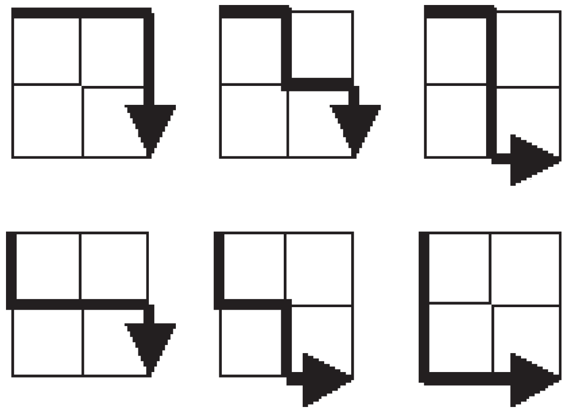
Starting in the top left corner of a 2×2 grid, there are 6 routes (without backtracking) to the bottom right corner.



How many routes are there through a 20×20 grid?

Problem 15

Wenn man in der linken oberen Ecke eines 2×2-Gitters startet, gibt es zur rechten unteren Ecke genau 6 Wege (ohne Umkehren und Zurücklaufen).



Wie viele Wege gibt es durch ein 20×20-Gitter?

Problem 16

$2^{15} = 32768$ and the sum of its digits is

$$3 + 2 + 7 + 6 + 8 = 26.$$

What is the sum of the digits of the number 2^{1000}?

Problem 16

$2^{15} = 32768$, und die Quersumme des Ergebnisses ist

$$3 + 2 + 7 + 6 + 8 = 26.$$

Wie groß ist die Quersumme von 2^{1000}?

Problem 17

If the numbers 1 to 5 are written out in words: one, two, three, four, five, then there are 3 + 3 + 5 + 4 + 4 = 19 letters used in total.

If all the numbers from 1 to 1000 (one thousand) inclusive were written out in words, how many letters would be used?

NOTE: Do not count spaces or hyphens. For example, 342 (three hundred and forty-two) contains 23 letters and 115 (one hundred and fifteen) contains 20 letters. The use of »and« when writing out numbers is in compliance with British usage.

Problem 17

Wenn die Zahlen 1 bis 5 in englischen Worten geschrieben werden (one, two, three, four, five), braucht man dafür insgesamt 3 + 3 + 5 + 4 + 4 = 19 Buchstaben.

Wenn man alle Zahlen von 1 bis einschließlich 1000 englisch schreibt, wie viele Buchstaben braucht man dann dafür?

Anmerkung: Leerzeichen und Bindestriche zählen nicht mit. Die Zahl 342 (three hundred and forty-two) beispielsweise hat 23 Buchstaben, und die Zahl 115 (one hundred and fifteen) hat 20 Buchstaben. Das englische Wort »and«, das man beim Ausschreiben einiger Zahlen benutzt, richtet sich nach der britischen Schreibweise.

Problem 18

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

3
7 4
2 **4** 6
8 5 **9** 3

That is, 3 + 7 + 4 + 9 = 23.

Find the maximum total from top to bottom of the triangle below:

75
95 64
17 47 82
18 35 87 10
20 04 82 47 65
19 01 23 75 03 34
88 02 77 73 07 63 67
99 65 04 28 06 16 70 92
41 41 26 56 83 40 80 70 33
41 48 72 33 47 32 37 16 94 29
53 71 44 65 25 43 91 52 97 51 14
70 11 33 28 77 73 17 78 39 68 17 57
91 71 52 38 17 14 91 43 58 50 27 29 48
63 66 04 68 89 53 67 30 73 16 69 87 40 31
04 62 98 27 23 09 70 98 73 93 38 53 60 04 23

NOTE: As there are only 16384 routes, it is possible to solve this problem by trying every route. However, Problem 67, is the same challenge with a triangle containing *one hundred* rows; it cannot be solved by brute force, and requires a clever method!

Problem 18

Wenn man an der Spitze des folgenden Dreiecks beginnt und zu angrenzenden Zahlen in der nächsttieferen Zeile weitergeht, ergibt sich eine Maximalsumme von 23

3
7 4
2 **4** 6
8 5 **9** 3

Also: 3 + 7 + 4 + 9 = 23.

Finde die Maximalsumme von oben nach unten durch das folgende Dreieck:

75
95 64
17 47 82
18 35 87 10
20 04 82 47 65
19 01 23 75 03 34
88 02 77 73 07 63 67
99 65 04 28 06 16 70 92
41 41 26 56 83 40 80 70 33
41 48 72 33 47 32 37 16 94 29
53 71 44 65 25 43 91 52 97 51 14
70 11 33 28 77 73 17 78 39 68 17 57
91 71 52 38 17 14 91 43 58 50 27 29 48
63 66 04 68 89 53 67 30 73 16 69 87 40 31
04 62 98 27 23 09 70 98 73 93 38 53 60 04 23

Anmerkung: Da es nur 16384 Wege gibt, kann man dieses Problem durch systematisches Probieren lösen. Jedoch stellt sich in Problem Nr. 67 das gleiche Problem, aber dort hat das Dreieck *einhundert* Zeilen. Dann hilft systematisches Probieren nicht mehr weiter, nur eine clevere Methode!

Problem 19

You are given the following information, but you may prefer to do some research for yourself.

- 1 Jan 1900 was a Monday.

- Thirty days has September,
  April, June and November.
  All the rest have thirty-one,
  Saving February alone,
  Which has twenty-eight, rain or shine.
  And on leap years, twenty-nine.

- A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.

How many Sundays fell on the first of the month during the twentieth century (1 Jan 1901 to 31 Dec 2000)?

Problem 19

Du erhältst folgende Information, aber vielleicht willst Du auch selber Nachforschungen anstellen.

- Der 1. Januar 1900 war ein Montag.

- Dreißig Tage hat September,
  April und Juni und November.
  Einunddreißig hat der Rest,
  nur Februar nicht, der Tage lässt;
  er hat nur achtundzwanzig
  und im Schaltjahr neunundzwanzig.

- Ein Schaltjahr ist jedes Jahr, das sich ohne Rest durch 4 teilen lässt, aber nicht Jahrhundertwechsel, es sei denn, das Jahr lässt sich auch ohne Rest durch 400 teilen.

Wie viele Sonntage fielen während des zwanzigsten Jahrhunderts (also vom 1. Januar 1901 bis zum 31. Dezember 2000) auf einen Monatsersten?

Problem 20

n! means $n \times (n - 1) \times \ldots \times 3 \times 2 \times 1$

Find the sum of the digits in the number 100!

Problem 20

n! bedeutet: $n \cdot (n - 1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

Ermittle die Summe der Ziffern (Quersumme) der Zahl 100!

Problem 21

Let *d(n)* be defined as the sum of proper divisors of n (numbers less than n which divide evenly into n).

If *d(a)* = b and *d(b)* = a, where a ≠ b, then a and b are an *amicable pair* and each of a and b are called *amicable numbers.*

For example, the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110; therefore *d(220)* = 284. The proper divisors of 284 are 1, 2, 4, 71 and 142; so *d(284)* = 220.

Evaluate the sum of all the amicable numbers under 10000.

Problem 21

d(n) sei definiert als die Summe der echten Teiler der Zahl n (Zahlen, die kleiner sind als n und glatt in n hineinpassen).

Wenn *d(a)* = b und *d(b)* = a gilt (mit a ≠ b), dann heißen a und b *ein Paar befreundeter Zahlen*, und a und b nennt man dann *befreundete Zahlen*.

Ein Beispiel: die echten Teiler von 220 sind 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110; es gilt also *d(220)* = 284. Die echten Teiler von 284 sind 1, 2, 4, 71 und 142; also gilt: *d(284)* = 220.

Berechne die Summe aller befreundeten Zahlen, die kleiner sind als 10000.

Problem 22

Using names.txt (please extract the file from the pdf file you are reading at the moment or copy and paste from the following three pages), a 46K text file containing over five-thousand first names, begin by sorting it into alphabetical order. Then working out the alphabetical value for each name, multiply this value by its alphabetical position in the list to obtain a name score.

For example, when the list is sorted into alphabetical order, COLIN, which is worth $3 + 15 + 12 + 9 + 14 = 53$, is the 938th name in the list. So, COLIN would obtain a score of $938 \times 53 = 49714$.

What is the total of all the name scores in the file?

Problem 22

Die etwa 46 Kilobyte große Datei names.txt (Datei aus dieser pdf-Datei zu entnehmen oder den drei folgenden Seiten mit Kopieren & Einfügen) enthält mehr als fünftausend Vornamen.

Sortiere diese Namen zunächst alphabetisch. Ermittle dann für jeden Namen seinen Alphawert, nimm diesen Alphawert mit der Position des Namens in der sortierten Liste mal und erhalte so für jeden Namen einen Score.

Ein Beispiel: Sobald die Liste alphabetisch sortiert ist, steht der Name COLIN, der einen Alphawert von $3 + 15 + 12 + 9 + 14 = 53$ hat, an 938. Stelle in der Liste. Damit hätte COLIN einen Score von $938 \cdot 53 = 49714$.

Wie groß ist die Summe aller Namensscores aus der Datei?

Problem 22 Names/Namen, page 1/Seite 1

MARY,PATRICIA,LINDA,BARBARA,ELIZABETH,JE
NNIFER,MARIA,SUSAN,MARGARET,DOROTHY,LI
SA,NANCY,KAREN,BETTY,HELEN,SANDRA,DON
NA,CAROL,RUTH,SHARON,MICHELLE,LAURA,S
ARAH,KIMBERLY,DEBORAH,JESSICA,SHIRLEY,CY
NTHIA,ANGELA,MELISSA,BRENDA,AMY,ANNA,
REBECCA,VIRGINIA,KATHLEEN,PAMELA,MARTH
A,DEBRA,AMANDA,STEPHANIE,CAROLYN,CHRI
STINE,MARIE,JANET,CATHERINE,FRANCES,ANN,
JOYCE,DIANE,ALICE,JULIE,HEATHER,TERESA,DO
RIS,GLORIA,EVELYN,JEAN,CHERYL,MILDRED,KAT
HERINE,JOAN,ASHLEY,JUDITH,ROSE,JANICE,KEL
LY,NICOLE,JUDY,CHRISTINA,KATHY,THERESA,BE
VERLY,DENISE,TAMMY,IRENE,JANE,LORI,RACHEL
,MARILYN,ANDREA,KATHRYN,LOUISE,SARA,AN
NE,JACQUELINE,WANDA,BONNIE,JULIA,RUBY,L
OIS,TINA,PHYLLIS,NORMA,PAULA,DIANA,ANNI
E,LILLIAN,EMILY,ROBIN,PEGGY,CRYSTAL,GLADYS
,RITA,DAWN,CONNIE,FLORENCE,TRACY,EDNA,T
IFFANY,CARMEN,ROSA,CINDY,GRACE,WENDY,V
ICTORIA,EDITH,KIM,SHERRY,SYLVIA,JOSEPHINE,
THELMA,SHANNON,SHEILA,ETHEL,ELLEN,ELAI
NE,MARJORIE,CARRIE,CHARLOTTE,MONICA,ES
THER,PAULINE,EMMA,JUANITA,ANITA,RHONDA
,HAZEL,AMBER,EVA,DEBBIE,APRIL,LESLIE,CLARA
,LUCILLE,JAMIE,JOANNE,ELEANOR,VALERIE,DA
NIELLE,MEGAN,ALICIA,SUZANNE,MICHELE,GAIL
,BERTHA,DARLENE,VERONICA,JILL,ERIN,GERAL
DINE,LAUREN,CATHY,JOANN,LORRAINE,LYNN,S
ALLY,REGINA,ERICA,BEATRICE,DOLORES,BERNI
CE,AUDREY,YVONNE,ANNETTE,JUNE,SAMANTH
A,MARION,DANA,STACY,ANA,RENEE,IDA,VIVIA
N,ROBERTA,HOLLY,BRITTANY,MELANIE,LORETT
A,YOLANDA,JEANETTE,LAURIE,KATIE,KRISTEN,V
ANESSA,ALMA,SUE,ELSIE,BETH,JEANNE,VICKI,C
ARLA,TARA,ROSEMARY,EILEEN,TERRI,GERTRUD
E,LUCY,TONYA,ELLA,STACEY,WILMA,GINA,KRIST
IN,JESSIE,NATALIE,AGNES,VERA,WILLIE,CHARLE
NE,BESSIE,DELORES,MELINDA,PEARL,ARLENE,M
AUREEN,COLLEEN,ALLISON,TAMARA,JOY,GEOR
GIA,CONSTANCE,LILLIE,CLAUDIA,JACKIE,MARCI
A,TANYA,NELLIE,MINNIE,MARLENE,HEIDI,GLEN
DA,LYDIA,VIOLA,COURTNEY,MARIAN,STELLA,C
AROLINE,DORA,JO,VICKIE,MATTIE,TERRY,MAXI
NE,IRMA,MABEL,MARSHA,MYRTLE,LENA,CHRIS
TY,DEANNA,PATSY,HILDA,GWENDOLYN,JENNIE,
NORA,MARGIE,NINA,CASSANDRA,LEAH,PENNY
,KAY,PRISCILLA,NAOMI,CAROLE,BRANDY,OLGA,
BILLIE,DIANNE,TRACEY,LEONA,JENNY,FELICIA,S
ONIA,MIRIAM,VELMA,BECKY,BOBBIE,VIOLET,KR
ISTINA,TONI,MISTY,MAE,SHELLY,DAISY,RAMON
A,SHERRI,ERIKA,KATRINA,CLAIRE,LINDSEY,LIND
SAY,GENEVA,GUADALUPE,BELINDA,MARGARIT
A,SHERYL,CORA,FAYE,ADA,NATASHA,SABRINA,I
SABEL,MARGUERITE,HATTIE,HARRIET,MOLLY,CE
CILIA,KRISTI,BRANDI,BLANCHE,SANDY,ROSIE,J
OANNA,IRIS,EUNICE,ANGIE,INEZ,LYNDA,MADE
LINE,AMELIA,ALBERTA,GENEVIEVE,MONIQUE,J
ODI,JANIE,MAGGIE,KAYLA,SONYA,JAN,LEE,KRIS
TINE,CANDACE,FANNIE,MARYANN,OPAL,ALISO
N,YVETTE,MELODY,LUZ,SUSIE,OLIVIA,FLORA,SH
ELLEY,KRISTY,MAMIE,LULA,LOLA,VERNA,BEULA
H,ANTOINETTE,CANDICE,JUANA,JEANNETTE,PA

M,KELLI,HANNAH,WHITNEY,BRIDGET,KARLA,CE
LIA,LATOYA,PATTY,SHELIA,GAYLE,DELLA,VICKY,L
YNNE,SHERI,MARIANNE,KARA,JACQUELYN,ER
MA,BLANCA,MYRA,LETICIA,PAT,KRISTA,ROXAN
NE,ANGELICA,JOHNNIE,ROBYN,FRANCIS,ADRIE
NNE,ROSALIE,ALEXANDRA,BROOKE,BETHANY,S
ADIE,BERNADETTE,TRACI,JODY,KENDRA,JASMI
NE,NICHOLE,RACHAEL,CHELSEA,MABLE,ERNES
TINE,MURIEL,MARCELLA,ELENA,KRYSTAL,ANGE
LINA,NADINE,KARI,ESTELLE,DIANNA,PAULETTE,
LORA,MONA,DOREEN,ROSEMARIE,ANGEL,DESI
REE,ANTONIA,HOPE,GINGER,JANIS,BETSY,CHRI
STIE,FREDA,MERCEDES,MEREDITH,LYNETTE,TER
I,CRISTINA,EULA,LEIGH,MEGHAN,SOPHIA,ELOI
SE,ROCHELLE,GRETCHEN,CECELIA,RAQUEL,HEN
RIETTA,ALYSSA,JANA,KELLEY,GWEN,KERRY,JEN
NA,TRICIA,LAVERNE,OLIVE,ALEXIS,TASHA,SILVI
A,ELVIRA,CASEY,DELIA,SOPHIE,KATE,PATTI,LORE
NA,KELLIE,SONJA,LILA,LANA,DARLA,MAY,MIND
Y,ESSIE,MANDY,LORENE,ELSA,JOSEFINA,JEANNI
E,MIRANDA,DIXIE,LUCIA,MARTA,FAITH,LELA,JO
HANNA,SHARI,CAMILLE,TAMI,SHAWNA,ELISA,E
BONY,MELBA,ORA,NETTIE,TABITHA,OLLIE,JAIM
E,WINIFRED,KRISTIE,MARINA,ALISHA,AIMEE,RE
NA,MYRNA,MARLA,TAMMIE,LATASHA,BONITA,
PATRICE,RONDA,SHERRIE,ADDIE,FRANCINE,DEL
ORIS,STACIE,ADRIANA,CHERI,SHELBY,ABIGAIL,C
ELESTE,JEWEL,CARA,ADELE,REBEKAH,LUCINDA,
DORTHY,CHRIS,EFFIE,TRINA,REBA,SHAWN,SALL
IE,AURORA,LENORA,ETTA,LOTTIE,KERRI,TRISHA
,NIKKI,ESTELLA,FRANCISCA,JOSIE,TRACIE,MARI
SSA,KARIN,BRITTNEY,JANELLE,LOURDES,LAURE
L,HELENE,FERN,ELVA,CORINNE,KELSEY,INA,BET
TIE,ELISABETH,AIDA,CAITLIN,INGRID,IVA,EUGE
NIA,CHRISTA,GOLDIE,CASSIE,MAUDE,JENIFER,T
HERESE,FRANKIE,DENA,LORNA,JANETTE,LATO
NYA,CANDY,MORGAN,CONSUELO,TAMIKA,ROS
ETTA,DEBORA,CHERIE,POLLY,DINA,JEWELL,FAY,J
ILLIAN,DOROTHEA,NELL,TRUDY,ESPERANZA,PA
TRICA,KIMBERLEY,SHANNA,HELENA,CAROLINA
,CLEO,STEFANIE,ROSARIO,OLA,JANINE,MOLLIE,
LUPE,ALISA,LOU,MARIBEL,SUSANNE,BETTE,SUS
ANA,ELISE,CECILE,ISABELLE,LESLEY,JOCELYN,PAI
GE,JONI,RACHELLE,LEOLA,DAPHNE,ALTA,ESTER,
PETRA,GRACIELA,IMOGENE,JOLENE,KEISHA,LA
CEY,GLENNA,GABRIELA,KERI,URSULA,LIZZIE,KIR
STEN,SHANA,ADELINE,MAYRA,JAYNE,JACLYN,G
RACIE,SONDRA,CARMELA,MARISA,ROSALIND,
CHARITY,TONIA,BEATRIZ,MARISOL,CLARICE,JEA
NINE,SHEENA,ANGELINE,FRIEDA,LILY,ROBBIE,S
HAUNA,MILLIE,CLAUDETTE,CATHLEEN,ANGELI
A,GABRIELLE,AUTUMN,KATHARINE,SUMMER,J
ODIE,STACI,LEA,CHRISTI,JIMMIE,JUSTINE,ELMA,
LUELLA,MARGRET,DOMINIQUE,SOCORRO,REN
E,MARTINA,MARGO,MAVIS,CALLIE,BOBBI,MARI
TZA,LUCILE,LEANNE,JEANNINE,DEANA,AILEEN,
LORIE,LADONNA,WILLA,MANUELA,GALE,SELM
A,DOLLY,SYBIL,ABBY,LARA,DALE,IVY,DEE,WINNI
E,MARCY,LUISA,JERI,MAGDALENA,OFELIA,MEA
GAN,AUDRA,MATILDA,LEILA,CORNELIA,BIANC
A,SIMONE,BETTYE,RANDI,VIRGIE,LATISHA,BARB
RA,GEORGINA,ELIZA,LEANN,BRIDGETTE,RHOD
A,HALEY,ADELA,NOLA,BERNADINE,FLOSSIE,ILA,

GRETA,RUTHIE,NELDA,MINERVA,LILLY,TERRIE,LE
THA,HILARY,ESTELA,VALARIE,BRIANNA,ROSALY
N,EARLINE,CATALINA,AVA,MIA,CLARISSA,LIDIA,
CORRINE,ALEXANDRIA,CONCEPCION,TIA,SHAR
RON,RAE,DONA,ERICKA,JAMI,ELNORA,CHAND
RA,LENORE,NEVA,MARYLOU,MELISA,TABATHA,
SERENA,AVIS,ALLIE,SOFIA,JEANIE,ODESSA,NAN
NIE,HARRIETT,LORAINE,PENELOPE,MILAGROS,E
MILIA,BENITA,ALLYSON,ASHLEE,TANIA,TOMMIE
,ESMERALDA,KARINA,EVE,PEARLIE,ZELMA,MALI
NDA,NOREEN,TAMEKA,SAUNDRA,HILLARY,AMI
E,ALTHEA,ROSALINDA,JORDAN,LILIA,ALANA,G
AY,CLARE,ALEJANDRA,ELINOR,MICHAEL,LORRIE
,JERRI,DARCY,EARNESTINE,CARMELLA,TAYLOR,
NOEMI,MARCIE,LIZA,ANNABELLE,LOUISA,EARL
ENE,MALLORY,CARLENE,NITA,SELENA,TANISHA
,KATY,JULIANNE,JOHN,LAKISHA,EDWINA,MARI
CELA,MARGERY,KENYA,DOLLIE,ROXIE,ROSLYN,K
ATHRINE,NANETTE,CHARMAINE,LAVONNE,ILE
NE,KRIS,TAMMI,SUZETTE,CORINE,KAYE,JERRY,M
ERLE,CHRYSTAL,LINA,DEANNE,LILIAN,JULIANA,
ALINE,LUANN,KASEY,MARYANNE,EVANGELINE,
COLETTE,MELVA,LAWANDA,YESENIA,NADIA,M
ADGE,KATHIE,EDDIE,OPHELIA,VALERIA,NONA,
MITZI,MARI,GEORGETTE,CLAUDINE,FRAN,ALISS
A,ROSEANN,LAKEISHA,SUSANNA,REVA,DEIDRE
,CHASITY,SHEREE,CARLY,JAMES,ELVIA,ALYCE,DEI
RDRE,GENA,BRIANA,ARACELI,KATELYN,ROSAN
NE,WENDI,TESSA,BERTA,MARVA,IMELDA,MARI
ETTA,MARCI,LEONOR,ARLINE,SASHA,MADELYN
,JANNA,JULIETTE,DEENA,AURELIA,JOSEFA,AUG
USTA,LILIANA,YOUNG,CHRISTIAN,LESSIE,AMAL
IA,SAVANNAH,ANASTASIA,VILMA,NATALIA,RO
SELLA,LYNNETTE,CORINA,ALFREDA,LEANNA,C
AREY,AMPARO,COLEEN,TAMRA,AISHA,WILDA,K
ARYN,CHERRY,QUEEN,MAURA,MAI,EVANGELIN
A,ROSANNA,HALLIE,ERNA,ENID,MARIANA,LAC
Y,JULIET,JACKLYN,FREIDA,MADELEINE,MARA,HE
STER,CATHRYN,LELIA,CASANDRA,BRIDGETT,AN
GELITA,JANNIE,DIONNE,ANNMARIE,KATINA,BE
RYL,PHOEBE,MILLICENT,KATHERYN,DIANN,CAR
ISSA,MARYELLEN,LIZ,LAURI,HELGA,GILDA,ADRI
AN,RHEA,MARQUITA,HOLLIE,TISHA,TAMERA,A
NGELIQUE,FRANCESCA,BRITNEY,KAITLIN,LOLIT
A,FLORINE,ROWENA,REYNA,TWILA,FANNY,JAN
ELL,INES,CONCETTA,BERTIE,ALBA,BRIGITTE,ALY
SON,VONDA,PANSY,ELBA,NOELLE,LETITIA,KITT
Y,DEANN,BRANDIE,LOUELLA,LETA,FELECIA,SHA
RLENE,LESA,BEVERLEY,ROBERT,ISABELLA,HERMI
NIA,TERRA,CELINA,TORI,OCTAVIA,JADE,DENICE
,GERMAINE,SIERRA,MICHELL,CORTNEY,NELLY,D
ORETHA,SYDNEY,DEIDRA,MONIKA,LASHONDA
,JUDI,CHELSEY,ANTIONETTE,MARGOT,BOBBY,A
DELAIDE,NAN,LEEANN,ELISHA,DESSIE,LIBBY,KA
THI,GAYLA,LATANYA,MINA,MELLISA,KIMBERLEE
,JASMIN,RENAE,ZELDA,ELDA,MA,JUSTINA,GUS
SIE,EMILIE,CAMILLA,ABBIE,ROCIO,KAITLYN,JESS
E,EDYTHE,ASHLEIGH,SELINA,LAKESHA,GERI,ALL
ENE,PAMALA,MICHAELA,DAYNA,CARYN,ROSAL
IA,SUN,JACQULINE,REBECA,MARYBETH,KRYSTL
E,IOLA,DOTTIE,BENNIE,BELLE,AUBREY,GRISELD
A,ERNESTINA,ELIDA,ADRIANNE,DEMETRIA,DEL
MA,CHONG,JAQUELINE,DESTINY,ARLEEN,VIRGI

NA,RETHA,FATIMA,TILLIE,ELEANORE,CARI,TREV
A,BIRDIE,WILHELMINA,ROSALEE,MAURINE,LAT
RICE,YONG,JENA,TARYN,ELIA,DEBBY,MAUDIE,JE
ANNA,DELILAH,CATRINA,SHONDA,HORTENCIA
,THEODORA,TERESITA,ROBBIN,DANETTE,MARY
JANE,FREDDIE,DELPHINE,BRIANNE,NILDA,DAN
NA,CINDI,BESS,IONA,HANNA,ARIEL,WINONA,V
IDA,ROSITA,MARIANNA,WILLIAM,RACHEAL,GUI
LLERMINA,ELOISA,CELESTINE,CAREN,MALISSA,
LONA,CHANTEL,SHELLIE,MARISELA,LEORA,AGA
THA,SOLEDAD,MIGDALIA,IVETTE,CHRISTEN,AT
HENA,JANEL,CHLOE,VEDA,PATTIE,TESSIE,TERA,
MARILYNN,LUCRETIA,KARRIE,DINAH,DANIELA,
ALECIA,ADELINA,VERNICE,SHIELA,PORTIA,MER
RY,LASHAWN,DEVON,DARA,TAWANA,OMA,VE
RDA,CHRISTIN,ALENE,ZELLA,SANDI,RAFAELA,M
AYA,KIRA,CANDIDA,ALVINA,SUZAN,SHAYLA,LY
N,LETTIE,ALVA,SAMATHA,ORALIA,MATILDE,MA
DONNA,LARISSA,VESTA,RENITA,INDIA,DELOIS,
SHANDA,PHILLIS,LORRI,ERLINDA,CRUZ,CATHRI
NE,BARB,ZOE,ISABELL,IONE,GISELA,CHARLIE,VA
LENCIA,ROXANNA,MAYME,KISHA,ELLIE,MELLIS
SA,DORRIS,DALIA,BELLA,ANNETTA,ZOILA,RETA,
REINA,LAURETTA,KYLIE,CHRISTAL,PILAR,CHARL
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NA,JAYME,CARMEL,VERNELL,TOMASA,MANDI,
DOMINGA,SANTA,MELODIE,LURA,ALEXA,TAME
LA,RYAN,MIRNA,KERRIE,VENUS,NOEL,FELICITA,
CRISTY,CARMELITA,BERNIECE,ANNEMARIE,TIAR
A,ROSEANNE,MISSY,CORI,ROXANA,PRICILLA,K
RISTAL,JUNG,ELYSE,HAYDEE,ALETHA,BETTINA,
MARGE,GILLIAN,FILOMENA,CHARLES,ZENAIDA,
HARRIETTE,CARIDAD,VADA,UNA,ARETHA,PEAR
LINE,MARJORY,MARCELA,FLOR,EVETTE,ELOUIS
E,ALINA,TRINIDAD,DAVID,DAMARIS,CATHARIN
E,CARROLL,BELVA,NAKIA,MARLENA,LUANNE,L
ORINE,KARON,DORENE,DANITA,BRENNA,TATIA
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NA,MIMI,JACQUELIN,GAYE,TONJA,MISTI,JOE,GE
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NTONETTE,SHELLI,MOZELLE,MARIAH,JOELLE,C
ORDELIA,JOSETTE,CHIQUITA,TRISTA,LOUIS,LAQ
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KAROL,GERRY,GABRIELLA,TIANA,ROMA,RICHEL
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HENIE,SABINA,NATHALIE,MARCELLE,GERTIE,DA
RLEEN,THEA,SHARONDA,SHANTEL,BELEN,VEN
ESSA,ROSALINA,ONA,GENOVEVA,COREY,CLEM
ENTINE,ROSALBA,RENATE,RENATA,MI,IVORY,GE
ORGIANNA,FLOY,DORCAS,ARIANA,TYRA,THED
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OSELYN,MELVINA,JANNETTE,GINNY,DEBRAH,C
ORRIE,ASIA,VIOLETA,MYRTIS,LATRICIA,COLLETT
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RISTAL,SUNNY,LEONE,LEILANI,GERRI,DEBI,AND
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OSINA,MARIANN,LEISA,JONNIE,DAWNA,CATHI
E,BILLY,ASTRID,SIDNEY,LAUREEN,JANEEN,HOLLI,
FAWN,VICKEY,TERESSA,SHANTE,RUBYE,MARCE
LINA,CHANDA,CARY,TERESE,SCARLETT,MARTY,
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ALETA,ADRIANNA,ZORAIDA,RONNIE,NICOLA,L
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ANJUANITA,RONA,NANCI,MARILEE,MARANDA,
CORY,BRIGETTE,SANJUANA,MARITA,KASSAND
RA,JOYCELYN,IRA,FELIPA,CHELSIE,BONNY,MIRE
YA,LORENZA,KYONG,ILEANA,CANDELARIA,TO
NY,TOBY,SHERIE,OK,MARK,LUCIE,LEATRICE,LAK
ESHIA,GERDA,EDIE,BAMBI,MARYLIN,LAVON,HO
RTENSE,GARNET,EVIE,TRESSA,SHAYNA,LAVINA,
KYUNG,JEANETTA,SHERRILL,SHARA,PHYLISS,MI
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,JOANIE,TIFFANIE,LASHANDA,KARISSA,ENRIQU
ETA,DARIA,DANIELLA,CORINNA,ALANNA,ABBE
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OELLEN,ERA,CORAL,CARLEEN,TRESA,PEGGIE,N
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BETH,LAURETTE,JERRIE,HERMELINDA,CAROLEE,
TIERRA,MIRIAN,META,MELONY,KORI,JENNETTE,
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ELY,TOMEKA,SHALONDA,MARTI,LACIE,KALA,JA
DA,ILSE,HAILEY,BRITTANI,ZONA,SYBLE,SHERRYL
,RANDY,NIDIA,MARLO,KANDICE,KANDI,DEB,DE
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INGA,HOA,HETTIE,GERALYN,FONDA,ESTRELLA,
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TA,ELNA,DAVINA,CIERRA,ASHLIE,ALBERTHA,TA
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ARUE,FLORENE,DEMETRA,DEDRA,CIARA,CHAN
TELLE,ASHLY,SUZY,ROSALVA,NOELIA,LYDA,LEAT
HA,KRYSTYNA,KRISTAN,KARRI,DARLINE,DARCIE

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,CINDA,CHEYENNE,CHERRIE,AWILDA,ALMEDA,
ROLANDA,LANETTE,JERILYN,GISELE,EVALYN,CY
NDI,CLETA,CARIN,ZINA,ZENA,VELIA,TANIKA,PA
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ONDA,KAYLEE,KATHLENE,JONNA,IRENA,ILONA,
IDALIA,CANDIS,CANDANCE,BRANDEE,ANITRA,
ALIDA,SIGRID,NICOLETTE,MARYJO,LINETTE,HED
WIG,CHRISTIANA,CASSIDY,ALEXIA,TRESSIE,MO
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TANGELA,STEPHINE,NORINE,NELLA,LIANA,LESL
EE,KIMBERELY,ILIANA,GLORY,FELICA,EMOGENE,
ELFRIEDE,EDEN,EARTHA,CARMA,BEA,OCIE,MAR
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AMI,AMADA,ADELIA,VERNITA,TAMAR,SIOBHA
N,RENEA,RASHIDA,OUIDA,ODELL,NILSA,MERYL
,KRISTYN,JULIETA,DANICA,BREANNE,AUREA,A
NGLEA,SHERRON,ODETTE,MALIA,LORELEI,LIN,L
EESA,KENNA,KATHLYN,FIONA,CHARLETTE,SUZI
E,SHANTELL,SABRA,RACQUEL,MYONG,MIRA,M
ARTINE,LUCIENNE,LAVADA,JULIANN,JOHNIE,EL
VERA,DELPHIA,CLAIR,CHRISTIANE,CHAROLETT
E,CARRI,AUGUSTINE,ASHA,ANGELLA,PAOLA,NI
NFA,LEDA,LAI,EDA,SUNSHINE,STEFANI,SHANEL
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RLENE,JULISSA,JETTIE,JENNIFFER,HUI,CORRINA,
CHRISTOPHER,CAROLANN,ALENA,TESS,ROSARI
A,MYRTICE,MARYLEE,LIANE,KENYATTA,JUDIE,JA
NEY,IN,ELMIRA,ELDORA,DENNA,CRISTI,CATHI,Z
AIDA,VONNIE,VIVA,VERNIE,ROSALINE,MARIELA
,LUCIANA,LESLI,KARAN,FELICE,DENEEN,ADINA,
WYNONA,TARSHA,SHERON,SHASTA,SHANITA,
SHANI,SHANDRA,RANDA,PINKIE,PARIS,NELIDA,
MARILOU,LYLA,LAURENE,LACI,JOI,JANENE,DOR
OTHA,DANIELE,DANI,CAROLYNN,CARLYN,BERE
NICE,AYESHA,ANNELIESE,ALETHEA,THERSA,TA
MIKO,RUFINA,OLIVA,MOZELL,MARYLYN,MADIS
ON,KRISTIAN,KATHYRN,KASANDRA,KANDACE,J
ANAE,GABRIEL,DOMENICA,DEBBRA,DANNIELL
E,CHUN,BUFFY,BARBIE,ARCELIA,AJA,ZENOBIA,S
HAREN,SHAREE,PATRICK,PAGE,MY,LAVINIA,KU
M,KACIE,JACKELINE,HUONG,FELISA,EMELIA,ELE
ANORA,CYTHIA,CRISTIN,CLYDE,CLARIBEL,CAR
ON,ANASTACIA,ZULMA,ZANDRA,YOKO,TENISH
A,SUSANN,SHERILYN,SHAY,SHAWANDA,SABIN
E,ROMANA,MATHILDA,LINSEY,KEIKO,JOANA,ISE
LA,GRETTA,GEORGETTA,EUGENIE,DUSTY,DESIR
AE,DELORA,CORAZON,ANTONINA,ANIKA,WILL
ENE,TRACEE,TAMATHA,REGAN,NICHELLE,MICKI
E,MAEGAN,LUANA,LANITA,KELSIE,EDELMIRA,B
REE,AFTON,TEODORA,TAMIE,SHENA,MEG,LINH
,KELI,KACI,DANYELLE,BRITT,ARLETTE,ALBERTINE
,ADELLE,TIFFINY,STORMY,SIMONA,NUMBERS,N
ICOLASA,NICHOL,NIA,NAKISHA,MEE,MAIRA,LO
REEN,KIZZY,JOHNNY,JAY,FALLON,CHRISTENE,B
OBBYE,ANTHONY,YING,VINCENZA,TANJA,RUBI
E,RONI,QUEENIE,MARGARETT,KIMBERLI,IRMGA
RD,IDELL,HILMA,EVELINA,ESTA,EMILEE,DENNIS
E,DANIA,CARL,CARIE,ANTONIO,WAI,SANG,RISA
,RIKKI,PARTICIA,MUI,MASAKO,MARIO,LUVENIA,
LOREE,LONI,LIEN,KEVIN,GIGI,FLORENCIA,DORIA
N,DENITA,DALLAS,CHI,BILLYE,ALEXANDER,TOM
IKA,SHARITA,RANA,NIKOLE,NEOMA,MARGARIT
E,MADALYN,LUCINA,LAILA,KALI,JENETTE,GABRI
ELE,EVELYNE,ELENORA,CLEMENTINA,ALEJAND
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TTA,PIA,PATIENCE,NOELLA,NICKIE,JONELL,DELT
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REW,THANH,SUZANN,SPRING,SHU,MILA,LILLA,
LAVERNA,KEESHA,KATTIE,GIA,GEORGENE,EVELI
NE,ESTELL,ELIZBETH,VIVIENNE,VALLIE,TRUDIE,S
TEPHANE,MICHEL,MAGALY,MADIE,KENYETTA,K
ARREN,JANETTA,HERMINE,HARMONY,DRUCILL
A,DEBBI,CELESTINA,CANDIE,BRITNI,BECKIE,AMI
NA,ZITA,YUN,YOLANDE,VIVIEN,VERNETTA,TRU
DI,SOMMER,PEARLE,PATRINA,OSSIE,NICOLLE,L
OYCE,LETTY,LARISA,KATHARINA,JOSELYN,JONE
LLE,JENELL,IESHA,HEIDE,FLORINDA,FLORENTIN
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,ARDELLA,TWANA,THU,TARAH,SUNG,SHEA,SH
AVON,SHANE,SERINA,RAYNA,RAMONITA,NGA,
MARGURITE,LUCRECIA,KOURTNEY,KATI,JESUS,J
ESENIA,DIAMOND,CRISTA,AYANA,ALICA,ALIA,V
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HANA,HA,ELEASE,CARLETTA,BRITANY,SHONA,S
ALOME,ROSAMOND,REGENA,RAINA,NGOC,NE
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LEY,JOYE,JACQULYN,IGNACIA,HYUN,HIROKO,H
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BABETTE,AYANNA,ANETTE,ALBERTINA,SKYE,SH
AWNEE,SHANEKA,QUIANA,PAMELIA,MIN,MER
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DAISEY,CODY,CASIE,BELIA,BABARA,VERSIE,VAN
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VERLEE,ANJANETTE,VALDA,TRINITY,TAMALA,ST
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,SOO,SIGNE,SALENA,RONNI,REBBECCA,MYRTIE,
MCKENZIE,MALIKA,MAIDA,LOAN,LEONARDA,K
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AKIKO,VENICE,TYESHA,TONIE,TIESHA,TAKISHA,
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INE,TENNIE,SON,SHERLY,SHERLEY,SHARYL,REM
EDIOS,PETRINA,NICKOLE,MYUNG,MYRLE,MOZE
LLA,LOUANNE,LISHA,LATIA,LANE,KRYSTA,JULIE
NNE,JOEL,JEANENE,JACQUALINE,ISAURA,GWE
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HONEY,HEE,GUDRUN,FRANK,DREAMA,CRISSY,
CHANTE,CARMELINA,ARVILLA,ARTHUR,ANNA
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,NANCIE,MONSERRATE,MINH,MELYNDA,MELA
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ANDON,ALLA,VALERY,TONDA,SUEANN,SORAY
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TIN,JULIAN,JIMMY,JANN,ILDA,HILDRED,HILDEG
ARDE,GENIA,FUMIKO,EVELIN,ERMELINDA,ELLY,
DUNG,DOLORIS,DIONNA,DANAE,BERNEICE,AN
NICE,ALIX,VERENA,VERDIE,TRISTAN,SHAWNNA
,SHAWANA,SHAUNNA,ROZELLA,RANDEE,RAN
AE,MILAGRO,LYNELL,LUISE,LOUIE,LOIDA,LISBET
H,KARLEEN,JUNITA,JONA,ISIS,HYACINTH,HEDY,
GWENN,ETHELENE,ERLINE,EDWARD,DONYA,D
OMONIQUE,DELICIA,DANNETTE,CICELY,BRAND
A,BLYTHE,BETHANN,ASHLYN,ANNALEE,ALLINE,
YUKO,VELLA,TRANG,TOWANDA,TESHA,SHERLY
N,NARCISA,MIGUELINA,MERI,MAYBELL,MARLA
NA,MARGUERITA,MADLYN,LUNA,LORY,LORIAN
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HA,LARONDA,KATRICE,KASIE,KARL,KALEY,JAD
WIGA,GLENNIE,GEARLDINE,FRANCINA,EPIFANI
A,DYAN,DORIE,DIEDRE,DENESE,DEMETRICE,DEL
ENA,DARBY,CRISTIE,CLEORA,CATARINA,CARISA
,BERNIE,BARBERA,ALMETA,TRULA,TEREASA,SO
LANGE,SHEILAH,SHAVONNE,SANORA,ROCHEL
L,MATHILDE,MARGARETA,MAIA,LYNSEY,LAWA
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A,GAYLENE,ELVINA,ELANOR,DANUTA,DANIKA,
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RYNN,AZUCENA,AUNDREA,ANGELE,YI,WALTER,
VERLIE,VERLENE,TAMESHA,SILVANA,SEBRINA,S
AMIRA,REDA,RAYLENE,PENNI,PANDORA,NORA
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UMINADA,HILARIA,HANH,GENNIE,FRANCIE,FL
ORETTA,EXIE,EDDA,DREMA,DELPHA,BEV,BARBA
R,ASSUNTA,ARDELL,ANNALISA,ALISIA,YUKIKO,
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ILA,TEMEKA,TAMEIKA,SHIRLEEN,SHENITA,PIED
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ELENA,ALLEN,ALIZA,ADRIENE,ADALINE,XOCHIT
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SY,SIU,RUTHA,ROXY,RHONA,RAYMOND,OTHA,
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,ARIANNE,ALESHIA,YUNG,WILLIEMAE,TROY,TRI
NH,THORA,TAI,SVETLANA,SHERIKA,SHEMEKA,S
HAUNDA,ROSELINE,RICKI,MELDA,MALLIE,LAVO
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SHIRL,SHIRELY,SHERIDAN,SHELLA,SHANTELLE,S
ACHA,ROYCE,REBECKA,REAGAN,PROVIDENCIA,
PAULENE,MISHA,MIKI,MARLINE,MARICA,LORIT
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ATHRIN,JAYMIE,JACK,GRICELDA,GINETTE,ERYN,
ELINA,ELFRIEDA,DANYEL,CHEREE,CHANELLE,BA
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NE,AIDE,YASMINE,VASHTI,VALENTINE,TREASA,T
ORY,TIFFANEY,SHERYLL,SHARIE,SHANAE,SAU,R
AISA,PA,NEDA,MITSUKO,MIRELLA,MILDA,MARY
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UARY,JALISA,JACELYN,IZOLA,IVEY,GREGORY,EU
NA,ETHA,DREW,DOMITILA,DOMINICA,DAINA,
CREOLA,CARLI,CAMIE,BUNNY,BRITTNY,ASHAN
TI,ANISHA,ALEEN,ADAH,YASUKO,WINTER,VIKI,V
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IE,MAN,MAIRE,LYNDIA,LUCI,LORRIANE,LORETA,
LEONIA,LAVONA,LASHAWNDA,LAKIA,KYOKO,K
RYSTINA,KRYSTEN,KENIA,KELSI,JUDE,JEANICE,I
SOBEL,GEORGIANN,GENNY,FELICIDAD,EILENE,
DEON,DELOISE,DEEDEE,DANNIE,CONCEPTION,
CLORA,CHERILYN,CHANG,CALANDRA,BERRY,A
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SSA,STEPHANIA,SIMA,SHYLA,SHONTA,SHERA,S
HAQUITA,SHALA,SAMMY,ROSSANA,NOHEMI,N
ERY,MORIAH,MELITA,MELIDA,MELANI,MARYLY
NN,MARISHA,MARIETTE,MALORIE,MADELENE,L
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RLOS,BULAH,BRITTANIE,BOK,BLONDELL,BIBI,BE
AULAH,BEATA,ANNITA,AGRIPINA,VIRGEN,VALE
NE,UN,TWANDA,TOMMYE,TOI,TARRA,TARI,TAM
MERA,SHAKIA,SADYE,RUTHANNE,ROCHEL,RIV
KA,PURA,NENITA,NATISHA,MING,MERRILEE,ME
LODEE,MARVIS,LUCILLA,LEENA,LAVETA,LARITA,
LANIE,KEREN,ILEEN,GEORGEANN,GENNA,GENE
SIS,FRIDA,EWA,EUFEMIA,EMELY,ELA,EDYTH,DE
ONNA,DEADRA,DARLENA,CHANELL,CHAN,CAT
HERN,CASSONDRA,CASSAUNDRA,BERNARDA,
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,MARTH,MARGENE,MADELAINE,LONDA,LEONT
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A,LAKITA,KIYOKO,KETURAH,KATELIN,KAREEN,J
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NA,ALVERTA,XENIA,WAVA,VANETTA,TORRIE,

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TASHINA,TANDY,TAMBRA,TAMA,STEPANIE,SHILA,SHAUNTA,SHARAN,SHANIQUA,SHAE,SETSUKO,SERAFINA,SANDEE,ROSAMARIA,PRISCILA,OLINDA,NADENE,MUOI,MICHELINA,MERCEDEZ,MARYROSE,MARIN,MARCENE,MAO,MAGALI,MAFALDA,LOGAN,LINN,LANNIE,KAYCE,KAROLINE,KAMILAH,KAMALA,JUSTA,JOLINE,JENNINE,JACQUETTA,IRAIDA,GERALD,GEORGEANNA,FRANCHESCA,FAIRY,EMELINE,ELANE,EHTEL,EARLIE,DULCIE,DALENE,CRIS,CLASSIE,CHERE,CHARIS,CAROYLN,CARMINA,CARITA,BRIAN,BETHANIE,AYAKO,ARICA,AN,ALYSA,ALESSANDRA,AKILAH,ADRIEN,ZETTA,YOULANDA,YELENA,YAHAIRA,XUAN,WENDOLYN,VICTOR,TIJUANA,TERRELL,TERINA,TERESIA,SUZI,SUNDAY,SHERELL,SHAVONDA,SHAUNTE,SHARDA,SHAKITA,SENA,RYANN,RUBI,RIVA,REGINIA,REA,RACHAL,PARTHENIA,PAMULA,MONNIE,MONET,MICHAELE,MELIA,MARINE,MALKA,MAISHA,LISANDRA,LEO,LEKISHA,LEAN,LAURENCE,LAKENDRA,KRYSTIN,KORTNEY,KIZZIE,KITTIE,KERA,KENDAL,KEMBERLY,KANISHA,JULENE,JULE,JOSHUA,JOHANNE,JEFFREY,JAMEE,HAN,HALLEY,GIDGET,GALINA,FREDRICKA,FLETA,FATIMAH,EUSEBIA,ELZA,ELEONORE,DORTHEY,DORIA,DONELLA,DINORAH,DELORSE,CLARETHA,CHRISTINIA,CHARLYN,BONG,BELKIS,AZZIE,ANDERA,AIKO,ADENA,YER,YAJAIRA,WAN,VANIA,ULRIKE,TOSHIA,TIFANY,STEFANY,SHIZUE,SHENIKA,SHAWANNA,SHAROLYN,SHARILYN,SHAQUANA,SHANTAY,SEE,ROZANNE,ROSELEE,RICKIE,REMONA,REANNA,RAELENE,QUINN,PHUNG,PETRONILA,NATACHA,NANCEY,MYRL,MIYOKO,MIESHA,MERIDETH,MARVELLA,MARQUITTA,MARHTA,MARCHELLE,LIZETH,LIBBIE,LAHOMA,LADAWN,KINA,KATHELEEN,KATHARYN,KARISA,KALEIGH,JUNIE,JULIEANN,JOHNSIE,JANEAN,JAIMEE,JACKQUELINE,HISAKO,HERMA,HELAINE,GWYNETH,GLENN,GITA,EUSTOLIA,EMELINA,ELIN,EDRIS,DONNETTE,DONNETTA,DIERDRE,DENAE,DARCEL,CLAUDE,CLARISA,CINDERELLA,CHIA,CHARLESETTA,CHARITA,CELSA,CASSY,CASSI,CARLEE,BRUNA,BRITTANEY,BRANDE,BILLI,BAO,ANTONETTA,ANGLA,ANGELYN,ANALISA,ALANE,WENONA,WENDIE,VERONIQUE,VANNESA,TOBIE,TEMPIE,SUMIKO,SULEMA,SPARKLE,SOMER,SHEBA,SHAYNE,SHARICE,SHANEL,SHALON,SAGE,ROY,ROSIO,ROSELIA,RENAY,REMA,REENA,PORSCHE,PING,PEG,OZIE,ORETHA,ORALEE,ODA,NU,NGAN,NAKESHA,MILLY,MARYBELLE,MARLIN,MARIS,MARGRETT,MARAGARET,MANIE,LURLENE,LILLIA,LIESELOTTE,LAVELLE,LASHAUNDA,LAKEESHA,KEITH,KAYCEE,KALYN,JOYA,JOETTE,JENAE,JANIECE,ILLA,GRISEL,GLAYDS,GENEVIE,GALA,FREDDA,FRED,ELMER,ELEONOR,DEBERA,DEANDREA,DAN,CORRINNE,CORDIA,CONTESSA,COLENE,CLEOTILDE,CHARLOTT,CHANTAY,CECILLE,BEATRIS,AZALEE,ARLEAN,ARDATH,ANJELICA,ANJA,ALFREDIA,ALEISHA,ADAM,ZADA,YUONNE,XIAO,WILLODEAN,WHITLEY,VENNIE,VANNA,TYISHA,TOVA,TORIE,TONISHA,TILDA,TIEN,TEMPLE,SIRENA,SHERRIL,SHANTI,SHAN,SENAIDA,SAMELLA,ROBBYN,RENDA,REITA,PHEBE,PAULITA,NOBUKO,NGUYET,NEOMI,MOON,MIKAELA,MELANIA,MAXIMINA,MARG,MAISIE,LYNNA,LILLI,LAYNE,LASHAUN,LAKENYA,LAEL,KIRSTIE,KATHLINE,KASHA,KARLYN,KARIMA,JOVAN,JOSEFINE,JENNELL,JACQUI,JACKELYN,HYOHIEN,GRAZYNA,FLORRIE,FLORIA,ELEONORA,DWANA,DORLA,DONG,DELMY,DEJA,DEDE,DANN,CRYSTA,CLELIA,CLARIS,CLARENCE,CHIEKO,CHERLYN,CHERELLE,CHARMAIN,CHARA,CAMMY,BEE,ARNETTE,ARDELLE,ANNIKA,AMIEE,AMEE,ALLENA,YVONE,YUKI,YOSHIE,YEVETTE,YAEL,WILLETTA,VONCILE,VENETTA,TULA,TONETTE,TIMIKA,TEMIKA,TELMA,TEISHA,TAREN,TA,STACEE,SHIN,SHAWNTA,SATURNINA,RICARDA,POK,PASTY,ONIE,NUBIA,MORA,MIKE,MARIELLE,MARIELLA,MARIANELA,MARDELL,MANY,LUANNA,LOISE,LISABETH,LINDSY,LILLIANA,LILLIAM,LELAH,LEIGHA,LEANORA,LANG,KRISTEEN,KHALILAH,KEELEY,KANDRA,JUNKO,JOAQUINA,JERLENE,JANI,JAMIKA,JAME,HSIU,HERMILA,GOLDEN,GENEVIVE,EVIA,EUGENA,EMMALINE,ELFREDA,ELENE,DONETTE,DELCIE,DEEANNA,DARCEY,CUC,CLARINDA,CIRA,CHAE,CELINDA,CATHERYN,CATHERIN,CASIMIRA,CARMELIA,CAMELLIA,BREANA,BOBETTE,BERNARDINA,BEBE,BASILIA,ARLYNE,AMAL,ALAYNA,ZONIA,ZENIA,YURIKO,YAEKO,WYNELL,WILLOW,WILLENA,VERNIA,TU,TRAVIS,TORA,TERRILYN,TERICA,TENESHA,TAWNA,TAJUANA,TAINA,STEPHNIE,SONA,SOL,SINA,SHONDRA,SHIZUKO,SHERLENE,SHERICE,SHARIKA,ROSSIE,ROSENA,RORY,RIMA,RIA,RHEBA,RENNA,PETER,NATALYA,NANCEE,MELODI,MEDA,MAXIMA,MATHA,MARKETTA,MARICRUZ,MARCELENE,MALVINA,LUBA,LOUETTA,LEIDA,LECIA,LAURAN,LASHAWNA,LAINE,KHADIJAH,KATERINE,KASI,KALLIE,JULIETTA,JESUSITA,JESTINE,JESSIA,JEREMY,JEFFIE,JANYCE,ISADORA,GEORGIANNE,FIDELIA,EVITA,EURA,EULAH,ESTEFANA,ELSY,ELIZABET,ELADIA,DODIE,DION,DIA,DENISSE,DELORAS,DELILA,DAYSI,DAKOTA,CURTIS,CRYSTLE,CONCHA,COLBY,CLARETTA,CHU,CHRISTIA,CHARLSIE,CHARLENA,CARYLON,BETTYANN,ASLEY,ASHLEA,AMIRA,AI,AGUEDA,AGNUS,YUETTE,VINITA,VICTORINA,TYNISHA,TREENA,TOCCARA,TISH,THOMASENA,TEGAN,SOILA,SHILOH,SHENNA,SHARMAINE,SHANTAE,SHANDI,SEPTEMBER,SARAN,SARAI,SANA,SAMUEL,SALLEY,ROSETTE,ROLANDE,REGINE,OTELIA,OSCAR,OLEVIA,NICHOLLE,NECOLE,NAIDA,MYRTA,MYESHA,MITSUE,MINTA,MERTIE,MARGY,MAHALIA,MADALENE,LOVE,LOURA,LOREAN,LEWIS,LESHA,LEONIDA,LENITA,LAVONE,LASHELL,LASHANDRA,LAMONICA,KIMBRA,KATHERINA,KARRY,KANESHA,JULIO,JONG,JENEVA,JAQUELYN,HWA,GILMA,GHISLAINE,GERTRUDIS,FRANSISCA,FERMINA,ETTIE,ETSUKO,ELLIS,ELLAN,ELIDIA,EDRA,DORETHEA,DOREATHA,DENYSE,DENNY,DEETTA,DAINE,CYRSTAL,CORRIN,CAYLA,CARLITA,CAMILA,BURMA,BULA,BUENA,BLAKE,BARABARA,AVRIL,AUSTIN,ALAINE,ZANA,WILHEMINA,WANETTA,VIRGIL,VI,VERONIKA,VERNON,VERLINE,VASILIKI,TONITA,TISA,TEOFILA,TAYNA,TAUNYA,TANDRA,TAKAKO,SUNNI,SUANNE,SIXTA,SHARELL,SEEMA,RUSSELL,ROSENDA,ROBENA,RAYMONDE,PEI,PAMILA,OZELL,NEIDA,NEELY,MISTIE,MICHA,MERISSA,MAURITA,MARYLN,MARYETTA,MARSHALL,MARCELL,MALENA,MAKEDA,MADDIE,LOVETTA,LOURIE,LORRINE,LORILEE,LESTER,LAURENA,LASHAY,LARRAINE,LAREE,LACRESHA,KRISTLE,KRISHNA,KEVA,KEIRA,KAROLE,JOIE,JINNY,JEANNETTA,JAMA,HEIDY,GILBERTE,GEMA,FAVIOLA,EVELYNN,ENDA,ELLI,ELLENA,DIVINA,DAGNY,COLLENE,CODI,CINDIE,CHASSIDY,CHASIDY,CATRICE,CATHERINA,CASSEY,CAROLL,CARLENA,CANDRA,CALISTA,BRYANNA,BRITTENY,BEULA,BARI,AUDRIE,AUDRIA,ARDELIA,ANNELLE,ANGILA,ALONA,ALLYN,DOUGLAS,ROGER,JONATHAN,RALPH,NICHOLAS,BENJAMIN,BRUCE,HARRY,WAYNE,STEVE,HOWARD,ERNEST,PHILLIP,TODD,CRAIG,ALAN,PHILIP,EARL,DANNY,BRYAN,STANLEY,LEONARD,NATHAN,MANUEL,RODNEY,MARVIN,VINCENT,JEFFERY,JEFF,CHAD,JACOB,ALFRED,BRADLEY,HERBERT,FREDERICK,EDWIN,DON,RICKY,RANDALL,BARRY,BERNARD,LEROY,MARCUS,THEODORE,CLIFFORD,MIGUEL,JIM,TOM,CALVIN,BILL,LLOYD,DEREK,WARREN,DARRELL,JEROME,FLOYD,ALVIN,TIM,GORDON,GREG,JORGE,DUSTIN,PEDRO,DERRICK,ZACHARY,HERMAN,GLEN,HECTOR,RICARDO,RICK,BRENT,RAMON,GILBERT,MARC,REGINALD,RUBEN,NATHANIEL,RAFAEL,EDGAR,MILTON,RAUL,BEN,CHESTER,DUANE,FRANKLIN,BRAD,RON,ROLAND,ARNOLD,HARVEY,JARED,ERIK,DARRYL,NEIL,JAVIER,FERNANDO,CLINTON,TED,MATHEW,TYRONE,DARREN,LANCE,KURT,ALLAN,NELSON,GUY,CLAYTON,HUGH,MAX,DWAYNE,DWIGHT,ARMANDO,FELIX,EVERETT,IAN,WALLACE,KEN,BOB,ALFREDO,ALBERTO,DAVE,IVAN,BYRON,ISAAC,MORRIS,CLIFTON,WILLARD,ROSS,ANDY,SALVADOR,KIRK,SERGIO,SETH,KENT,TERRANCE,EDUARDO,TERRENCE,ENRIQUE,WADE,STUART,FREDRICK,ARTURO,ALEJANDRO,NICK,LUTHER,WENDELL,JEREMIAH,JULIUS,OTIS,TREVOR,OLIVER,LUKE,HOMER,GERARD,DOUG,KENNY,HUBERT,LYLE,MATT,ALFONSO,ORLANDO,REX,CARLTON,ERNESTO,NEAL,PABLO,LORENZO,OMAR,WILBUR,GRANT,HORACE,RODERICK,ABRAHAM,WILLIS,RICKEY,ANDRES,CESAR,JOHNATHAN,MALCOLM,RUDOLPH,DAMON,KELVIN,PRESTON,ALTON,ARCHIE,MARCO,WM,PETE,RANDOLPH,GARRY,GEOFFREY,JONATHON,FELIPE,GERARDO,ED,DOMINIC,DELBERT,COLIN,GUILLERMO,EARNEST,LUCAS,BENNY,SPENCER,RODOLFO,MYRON,EDMUND,GARRETT,SALVATORE,CEDRIC,LOWELL,GREGG,SHERMAN,WILSON,SYLVESTER,ROOSEVELT,ISRAEL,JERMAINE,FORREST,WILBERT,LELAND,SIMON,CLARK,IRVING,BRYANT,OWEN,RUFUS,WOODROW,KRISTOPHER,MACK,LEVI,MARCOS,GUSTAVO,JAKE,LIONEL,GILBERTO,CLINT,NICOLAS,ISMAEL,ORVILLE,ERVIN,DEWEY,AL,WILFRED,JOSH,HUGO,IGNACIO,CALEB,TOMAS,SHELDON,ERICK,STEWART,DOYLE,DARREL,ROGELIO,TERENCE,SANTIAGO,ALONZO,ELIAS,BERT,ELBERT,RAMIRO,CONRAD,NOAH,GRADY,PHIL,CORNELIUS,LAMAR,ROLANDO,CLAY,PERCY,DEXTER,BRADFORD,DARIN,AMOS,MOSES,IRVIN,SAUL,ROMAN,RANDAL,TIMMY,DARRIN,WINSTON,BRENDAN,ABEL,DOMINICK,BOYD,EMILIO,ELIJAH,DOMINGO,EMMETT,MARLON,EMANUEL,JERALD,EDMOND,EMIL,DEWAYNE,WILL,OTTO,TEDDY,REYNALDO,BRET,JESS,TRENT,HUMBERTO,EMMANUEL,STEPHAN,VICENTE,LAMONT,GARLAND,MILES,EFRAIN,HEATH,RODGER,HARLEY,ETHAN,ELDON,ROCKY,PIERRE,JUNIOR,FREDDY,ELI,BRYCE,ANTOINE,STERLING,CHASE,GROVER,ELTON,CLEVELAND,DYLAN,CHUCK,DAMIAN,REUBEN,STAN,AUGUST,LEONARDO,JASPER,RUSSEL,ERWIN,BENITO,HANS,MONTE,BLAINE,ERNIE,CURT,QUENTIN,AGUSTIN,MURRAY,JAMAL,ADOLFO,HARRISON,TYSON,BURTON,BRADY,ELLIOTT,WILFREDO,BART,JARROD,VANCE,DENIS,DAMIEN,JOAQUIN,HARLAN,DESMOND,ELLIOT,DARWIN,GREGORIO,BUDDY,XAVIER,KERMIT,ROSCOE,ESTEBAN,ANTON,SOLOMON,SCOTTY,NORBERT,ELVIN,WILLIAMS,NOLAN,ROD,QUINTON,HAL,BRAIN,ROB,ELWOOD,KENDRICK,DARIUS,MOISES,FIDEL,THADDEUS,CLIFF,MARCEL,JACKSON,RAPHAEL,BRYON,ARMAND,ALVARO,JEFFRY,DANE,JOESPH,THURMAN,NED,RUSTY,MONTY,FABIAN,REGGIE,MASON,GRAHAM,ISAIAH,VAUGHN,GUS,LOYD,DIEGO,ADOLPH,NORRIS,MILLARD,ROCCO,GONZALO,DERICK,RODRIGO,WILEY,RIGOBERTO,ALPHONSO,TY,NOE,VERN,REED,JEFFERSON,ELVIS,BERNARDO,MAURICIO,HIRAM,DONOVAN,BASIL,RILEY,NICKOLAS,MAYNARD,SCOT,VINCE,QUINCY,EDDY,SEBASTIAN,FEDERICO,ULYSSES,HERIBERTO,DONNELL,COLE,DAVIS,GAVIN,EMERY,WARD,ROMEO,JAYSON,DANTE,CLEMENT,COY,MAXWELL,JARVIS,BRUNO,ISSAC,DUDLEY,BROCK,SANFORD,CARMELO,BARNEY,NESTOR,STEFAN,DONNY,ART,LINWOOD,BEAU,WELDON,GALEN,ISIDRO,TRUMAN,DELMAR,JOHNATHON,SILAS,FREDERIC,DICK,IRWIN,MERLIN,CHARLEY,MARCELINO,HARRIS,CARLO,TRENTON,KURTIS,HUNTER,AURELIO,WINFRED,VITO,COLLIN,DENVER,CARTER,LEONEL,EMORY,PASQUALE,MOHAMMAD,MARIANO,DANIAL,LANDON,DIRK,BRANDEN,ADAN,BUFORD,GERMAN,WILMER,EMERSON,ZACHERY,FLETCHER,JACQUES,ERROL,DALTON,MONROE,JOSUE,EDWARDO,BOOKER,WILFORD,SONNY,SHELTON,CARSON,THERON,RAYMUNDO,DAREN,HOUSTON,ROBBY,LINCOLN,GENARO,BENNETT,OCTAVIO,CORNELL,HUNG,ARRON,ANTONY,HERSCHEL,GIOVANNI,GARTH,CYRUS,CYRIL,RONNY,LON,FREEMAN,DUNCAN,KENNITH,CARMINE,ERICH,CHADWICK,WILBURN,RUSS,REID,MYLES,ANDERSON,MORTON,JONAS,FOREST,MITCHEL,MERVIN,ZANE,RICH,JAMEL,LAZARO,ALPHONSE,RANDELL,MAJOR,JARRETT,BROOKS,ABDUL,LUCIANO,SEYMOUR,EUGENIO,MOHAMMED,VALENTIN,CHANCE,ARNULFO,LUCIEN,FERDINAND,THAD,EZRA,ALDO,RUBIN,ROYAL,MITCH,EARLE,ABE,WYATT,MARQUIS,LANNY,KAREEM,JAMAR,BORIS,ISIAH,EMILE,ELMO,ARON,LEOPOLDO,EVERETTE,JOSEF,ELOY,RODRICK,REINALDO,LUCIO,JERROD,WESTON,HERSHEL,BARTON,PARKER,LEMUEL,BURT,JULES,GIL,ELISEO,AHMAD,NIGEL,EFREN,ANTWAN,ALDEN,MARGARITO,COLEMAN,DINO,OSVALDO,LES,DEANDRE,NORMAND,KIETH,TREY,NORBERTO,NAPOLEON,JEROLD,FRITZ,ROSENDO,MILFORD,CHRISTOPER,ALFONZO,LYMAN,JOSIAH,BRANT,WILTON,RICO,JAMAAL,DEWITT,BRENTON,OLIN,FOSTER,FAUSTINO,CLAUDIO,JUDSON,GINO,EDGARDO,ALEC,TANNER,JARRED,DONN,TAD,PRINCE,PORFIRIO,ODIS,LENARD,CHAUNCEY,TOD,MEL,MARCELO,KORY,AUGUSTUS,KEVEN,HILARIO,BUD,SAL,ORVAL,MAURO,ZACHARIAH,OLEN,ANIBAL,MILO,JED,DILLON,AMADO,NEWTON,LENNY,RICHIE,HORACIO,BRICE,MOHAMED,DELMER,DARIO,REYES,MAC,JONAH,JERROLD,ROBT,HANK,RUPERT,ROLLAND,KENTON,DAMION,ANTONE,WALDO,FREDRIC,BRADLY,KIP,BURL,WALKER,TYREE,JEFFEREY,AHMED,WILLY,STANFORD,OREN,NOBLE,MOSHE,MIKEL,ENOCH,BRENDON,QUINTIN,JAMISON,FLORENCIO,DARRICK,TOBIAS,HASSAN,GIUSEPPE,DEMARCUS,CLETUS,TYRELL,LYNDON,KEENAN,WERNER,GERALDO,COLUMBUS,CHET,BERTRAM,MARKUS,HUEY,HILTON,DWAIN,DONTE,TYRON,OMER,ISAIAS,HIPOLITO,FERMIN,ADALBERTO,BO,BARRETT,TEODORO,MCKINLEY,MAXIMO,GARFIELD,RALEIGH,LAWERENCE,ABRAM,RASHAD,KING,EMMITT,DARON,SAMUAL,MIQUEL,EUSEBIO,DOMENIC,DARRON,BUSTER,WILBER,RENATO,JC,HOYT,HAYWOOD,EZEKIEL,CHAS,FLORENTINO,ELROY,CLEMENTE,ARDEN,NEVILLE,EDISON,DESHAWN,NATHANIAL,JORDON,DANILO,CLAUD,SHERWOOD,RAYMON,RAYFORD,CRISTOBAL,AMBROSE,TITUS,HYMAN,FELTON,EZEQUIEL,ERASMO,STANTON,LONNY,LEN,IKE,MILAN,LINO,JAROD,HERB,ANDREAS,WALTON,RHETT,PALMER,DOUGLASS,CORDELL,OSWALDO,ELLSWORTH,VIRGILIO,TONEY,NATHANAEL,DEL,BENEDICT,MOSE,JOHNSON,ISREAL,GARRET,FAUSTO,ASA,ARLEN,ZACK,WARNER,MODESTO,FRANCESCO,MANUAL,GAYLORD,GASTON,FILIBERTO,DEANGELO,MICHALE,GRANVILLE,WES,MALIK,ZACKARY,TUAN,ELDRIDGE,CRISTOPHER,CORTEZ,ANTIONE,MALCOM,LONG,KOREY,JOSPEH,COLTON,WAYLON,VON,HOSEA,SHAD,SANTO,RUDOLF,ROLF,REY,RENALDO,MARCELLUS,LUCIUS,KRISTOFER,BOYCE,BENTON,HAYDEN,HARLAND,ARNOLDO,RUEBEN,LEANDRO,KRAIG,JERRELL,JEROMY,HOBERT,CEDRICK,ARLIE,WINFORD,WALLY,LUIGI,KENETH,JACINTO,GRAIG,FRANKLYN,EDMUNDO,SID,PORTER,LEIF,JERAMY,BUCK,WILLIAN,VINCENZO,SHON,LYNWOOD,JERE,HAI,ELDEN,DORSEY,DARELL,BRODERICK,ALONSO

Problem 23

A *perfect number* is a number for which the sum of its proper divisors is exactly equal to the number. For example, the sum of the proper divisors of 28 would be 1 + 2 + 4 + 7 + 14 = 28, which means that 28 is a perfect number.

A number n is called *deficient* if the sum of its proper divisors is less than *n* and it is called *abundant* if this sum exceeds *n*.

As 12 is the smallest abundant number, 1 + 2 + 3 + 4 + 6 = 16, the smallest number that can be written as the sum of two abundant numbers is 24. By mathematical analysis, it can be shown that all integers greater than 28123 can be written as the sum of two abundant numbers. However, this upper limit cannot be reduced any further by analysis even though it is known that the greatest number that cannot be expressed as the sum of two abundant numbers is less than this limit.

Find the sum of all the positive integers which cannot be written as the sum of two abundant numbers.

Problem 23

Eine *vollkommene Zahl* ist eine Zahl, deren Teilersumme (die Summe aller echten Teiler) wieder die Zahl selbst ergibt.
Ein Beispiel: die Summe aller echten Teile von 28 ist
1 + 2 + 4 + 7 + 14 = 28, also ist 28 eine vollkommene Zahl.

Eine Zahl n nennt man *defizient*, wenn die Summe ihrer echten Teiler kleiner als *n* ist und *abundant*, wenn die Summe *n* übersteigt.

12 ist die kleinste abundante Zahl (1 + 2 + 3 + 4 + 6 = 16), daher ist die kleinste Zahl, die man als Summe zweier abundanter Zahlen schreiben kann, 24. Mathematische Analyse ergibt, dass sich alle natürlichen Zahlen, die größer sind als 28123, als Summe zweier abundanter Zahlen schreiben lassen. Diese obere Grenze lässt sich durch Analyse bisher nicht weiter verringern, obwohl man weiß, dass die größte Zahl, die sich nicht als Summe zweier abundanter Zahlen schreiben lässt, kleiner ist als diese Obergrenze.

Ermittle die Summe aller positiven ganzen Zahlen, die sich nicht als Summe zweier abundanter Zahlen schreiben lassen.

Problem 24

A *permutation* is an ordered arrangement of objects.

For example, 3124 is one possible permutation of the digits 1, 2, 3 and 4. If all of the permutations are listed numerically or alphabetically, we call it *lexicographic order*. The lexicographic permutations of 0, 1 and 2 are:

012 021 102 120 201 210

What is the millionth lexicographic permutation of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9?

Problem 24

Eine *Permutation* ist eine Anordnung von Objekten.

3124 beispielsweise ist eine mögliche Permutation der Ziffern 1, 2, 3 und 4. Wenn man alle Permutationen der Größe nach oder alphabetisch auflistet, nennt man das *lexikographische Ordnung*. Die lexikographischen Permutationen von 0, 1 und 2 sind:

012 021 102 120 201 210

Welches ist die millionste lexikographische Permutation der Ziffern 0, 1, 2, 3, 4, 5, 6, 7, 8 und 9?

Problem 25

The *Fibonacci sequence* is defined by the recurrence relation:

$$\mathbf{F_n = F_{n-1} + F_{n-2}}\quad\text{, where } F_1 = 1 \text{ and } F_2 = 1.$$

Hence the first 12 terms will be:

$F_1 = 1$
$F_2 = 1$
$F_3 = 2$
$F_4 = 3$
$F_5 = 5$
$F_6 = 8$
$F_7 = 13$
$F_8 = 21$
$F_9 = 34$
$F_{10} = 55$
$F_{11} = 89$
$F_{12} = 144$

The 12th term, F_{12}, is the first term to contain three digits.

What is the first term in the Fibonacci sequence to contain 1000 digits?

Problem 25

Die *Fibonacci-Folge* ist durch folgende rekursive Beziehung definiert:

$$\mathbf{F_n = F_{n-1} + F_{n-2}}\quad\text{, mit } F_1 = 1 \text{ und } F_2 = 1.$$

Damit ergeben sich die ersten zwölf Folgeglieder zu:

$F_1 = 1$
$F_2 = 1$
$F_3 = 2$
$F_4 = 3$
$F_5 = 5$
$F_6 = 8$
$F_7 = 13$
$F_8 = 21$
$F_9 = 34$
$F_{10} = 55$
$F_{11} = 89$
$F_{12} = 144$

Das Folgeglied mit der Nummer 12, F_{12}, ist das erste Glied, das dreistellig ist.

Welche Nummer hat das erste Folgeglied, das 1000-stellig ist?

Problem 26

A *unit fraction* contains 1 in the numerator. The decimal representation of the unit fractions with denominators 2 to 10 are given:

$^1/_2$	=	0.5
$^1/_3$	=	0.(3)
$^1/_4$	=	0.25
$^1/_5$	=	0.2
$^1/_6$	=	0.1(6)
$^1/_7$	=	0.(142857)
$^1/_8$	=	0.125
$^1/_9$	=	0.(1)
$^1/_{10}$	=	0.1

Where 0.1(6) means 0.166666..., and has a 1-digit recurring cycle. It can be seen that $^1/_7$ has a 6-digit recurring cycle.

Find the value of $d < 1000$ for which $^1/_d$ contains the longest recurring cycle in its decimal fraction part.

Problem 26

Ein *Stammbruch* ist ein Bruch, der 1 als Zähler hat. Es folgend die Stammbrüche mit Nennern von 1 bis 10 als Kommazahlen:

$^1/_2$	=	$0{,}5$
$^1/_3$	=	$0{,}\overline{3}$
$^1/_4$	=	$0{,}25$
$^1/_5$	=	$0{,}2$
$^1/_6$	=	$0{,}1\overline{6}$
$^1/_7$	=	$0{,}\overline{142857}$
$^1/_8$	=	$0{,}125$
$^1/_9$	=	$0{,}\overline{1}$
$^1/_{10}$	=	0.1

$0{,}1\overline{6}$ bedeutet dabei bedeutet 0,166666..., hat also eine Periodenlänge von 1. Wie man sieht, hat $^1/_7$ eine Periodenlänge von 6.

Ermittle den Wert für d mit $d < 1000$, für den der Bruch $^1/_d$ die längste Periodenlänge in den Nachkommastellen hat.

Problem 27

EULER published the remarkable quadratic formula:

$$n^2 + n + 41$$

It turns out that the formula will produce 40 primes for the consecutive values n = 0 to 39. However, when n = 40,

$$40^2 + 40 + 41 = 40(40 + 1) + 41$$

is divisible by 41, and certainly when n = 41, $41^2 + 41 + 41$ is clearly divisible by 41.

Using computers, the incredible formula $n^2 - 79n + 1601$ was discovered, which produces 80 primes for the consecutive values n = 0 to 79. The product of the coefficients, −79 and 1601, is −126479.

Considering quadratics of the form:

$n^2 + an + b$, where $|a| < 1000$ and $|b| < 1000$

where $|n|$ is the *modulus/absolute value* of n, e.g. $|11| = 11$ and $|-4| = 4$

Find the product of the coefficients, a and b, for the quadratic expression that produces the maximum number of primes for consecutive values of n, starting with n = 0.

Problem 27

EULER veröffentlichte die bemerkenswerte quadratische Formel:

$$n^2 + n + 41$$

Wie sich zeigt, liefert diese Formel 40 Primzahlen für die aufeinanderfolgenden Werte für n = 0 bis 39. Sobald n = 40 wird, ist

$$40^2 + 40 + 41 = 40 \cdot (40 + 1) + 41$$

jedoch ohne Rest durch 41 teilbar, und auch für den Fall, dass n = 41 wird, ist $41^2 + 41 + 41$ klar glatt durch 41 teilbar.

Computer haben die unglaubliche Formel $n^2 - 79n + 1601$ entdeckt, die 80 Primzahlen für aufeinanderfolgende Werte für n = 0 bis 79 liefert. Das Produkt ihrer Koeffizienten −79 und 1601 ist −126479.

Betrachten wir nun quadratische Formeln der Form:

$n^2 + an + b$, wobei $|a| < 1000$ und $|b| < 1000$

wobei $|n|$ is den *Betrag* von n bedeutet, also beispielsweise: $|11| = 11$, und $|-4| = 4$

Ermittle das Produkt der Koeffizienten a und b für den quadratischen Ausdruck, der die größte Anzahl von Primzahlen für aufeinanderfolgende Werte von n liefert, angefangen mit n = 0.

Problem 28

Starting with the number 1 and moving to the right in a clockwise direction a 5 by 5 spiral is formed as follows:

21	22	23	24	**25**
20	**7**	8	**9**	10
19	6	**1**	2	11
18	**5**	4	**3**	12
17	16	15	14	**13**

It can be verified that the sum of the numbers on the diagonals is 101.

What is the sum of the numbers on the diagonals in a 1001 by 1001 spiral formed in the same way?

Problem 28

Beginnt man mit der Zahl 1 und bewegt sich rechts im Uhrzeigersinn, ergibt sich die folgende 5×5-Spirale:

21	22	23	24	**25**
20	**7**	8	**9**	10
19	6	**1**	2	11
18	**5**	4	**3**	12
17	16	15	14	**13**

Wie sich überprüfen lässt, beträgt die Summe aller Zahlen auf den Diagonalen 101.

Welche Summe ergibt sich auf den Diagonalen bei einer ebenso aufgebauten 1001×1001-Spirale?

Problem 29

Consider all integer combinations of a^b for $2 \le a \le 5$ and $2 \le b \le 5$:

$2^2=4$, $2^3=8$, $2^4=16$, $2^5=32$
$3^2=9$, $3^3=27$, $3^4=81$, $3^5=243$
$4^2=16$, $4^3=64$, $4^4=256$, $4^5=1024$
$5^2=25$, $5^3=125$, $5^4=625$, $5^5=3125$

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 15 distinct terms:

4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125

How many distinct terms are in the sequence generated by a^b for $2 \le a \le 100$ and $2 \le b \le 100$?

Problem 29

Wir betrachten alle ganzzahligen Kombinationen von a^b für $2 \le a \le 5$ und $2 \le b \le 5$:

$2^2=4$, $2^3=8$, $2^4=16$, $2^5=32$
$3^2=9$, $3^3=27$, $3^4=81$, $3^5=243$
$4^2=16$, $4^3=64$, $4^4=256$, $4^5=1024$
$5^2=25$, $5^3=125$, $5^4=625$, $5^5=3125$

Wenn man die Ergebnisse nach Größe aufsteigend sortiert und dabei alle doppelten Ergebnisse tilgt, erhält man die folgenden 15 unterschiedlichen Ergebnisse:

4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125

Wie viele unterschiedliche Ergebnisse gibt es in der Folge, die von a^b für $2 \le a \le 100$ und $2 \le b \le 100$ erzeugt wird?

Problem 30

Surprisingly there are only three numbers that can be written as the sum of fourth powers of their digits:

$1634 = 1^4 + 6^4 + 3^4 + 4^4$
$8208 = 8^4 + 2^4 + 0^4 + 8^4$
$9474 = 9^4 + 4^4 + 7^4 + 4^4$

As $1 = 1^4$ is not a sum, it is not included.

The sum of these numbers is 1634 + 8208 + 9474 = 19316.

Find the sum of all the numbers that can be written as the sum of fifth powers of their digits.

Problem 30

Überraschenderweise gibt es nur drei Zahlen, die sich als Summe der vierten Potenzen ihrer Ziffern schreiben lassen:

$1634 = 1^4 + 6^4 + 3^4 + 4^4$
$8208 = 8^4 + 2^4 + 0^4 + 8^4$
$9474 = 9^4 + 4^4 + 7^4 + 4^4$

Da $1 = 1^4$ keine Summe ist, ist 1 nicht in dieser Liste.

Die Summe dieser Zahlen ist 1634 + 8208 + 9474 = 19316.

Ermittle die Summe aller Zahlen, die sich als Summe der fünften Potenzen ihrer Ziffern schreiben lassen.

Problem 31

In England the currency is made up of pound, £, and pence, p, and there are eight coins in general circulation:

1p, 2p, 5p, 10p, 20p, 50p, £1 (100p) and £2 (200p).

It is possible to make £2 in the following way:

1×£1 + 1×50p + 2×20p + 1×5p + 1×2p + 3×1p

How many different ways can £2 be made using any number of coins?

Problem 31

Die englische Währung kennt Pfund, £, und Pence, p.
Im allgemeinen Umlauf sind die folgenden acht Münzen:

1p, 2p, 5p, 10p, 20p, 50p, £1 (100p) und £2 (200p).

£2 lassen sich beispielsweise so zusammensetzen:

1•£1 + 1•50p + 2•20p + 1•5p + 1•2p + 3•1p

Auf wie viele unterschiedliche Weisen lassen sich £2, mit einer beliebigen Anzahl von Münzen, zusammensetzen?

Problem 32

We shall say that an n-digit number is *pandigital* if it makes use of all the digits 1 to n exactly once; for example, the 5-digit number, 15234, is 1 through 5 pandigital.

The product 7254 is unusual, as the identity, 39 × 186 = 7254, containing multiplicand, multiplier, and product is 1 through 9 pandigital.

Find the sum of all products whose multiplicand/multiplier/product identity can be written as a 1 through 9 pandigital.

HINT: Some products can be obtained in more than one way so be sure to only include it once in your sum.

Problem 32

Wir wollen eine n-stellige Zahl *pandigital* nennen, wenn in ihr alle Ziffern von 1 bis n genau ein Mal vorkommen. Die 5-stellige Zahl 15234 beispielsweise ist 1-bis-5-pandigital.

Das Produkt 7254 ist insofern ungewöhnlich, als die Gleichung 39 × 186 = 7254, die aus den beiden Faktoren und dem Produkt besteht, 1-bis-9-pandigital ist.

Ermittle die Summe aller Produkte, deren Gleichung aus den beiden Faktoren und dem Produkt sich 1-bis-9-pandigital schreiben lässt.

Anmerkung: Einige der Produkte kommen auf mehr als eine Weise zustande. Sie sollen aber in die Summe nur ein Mal einfließen.

Problem 33

The fraction $^{49}/_{98}$ is a curious fraction, as an inexperienced mathematician in attempting to simplify it may incorrectly believe that $^{49}/_{98} = {}^{4}/_{8}$, which is correct, is obtained by cancelling the 9s.

We shall consider fractions like, $^{30}/_{50} = {}^{3}/_{5}$, to be trivial examples.

There are exactly four non-trivial examples of this type of fraction, less than one in value, and containing two digits in the numerator and denominator.

If the product of these four fractions is given in its lowest common terms, find the value of the denominator.

Problem 33

Der Bruch $^{49}/_{98}$ ist ein merkwürdiger Bruch: wer wenig von Mathematik versteht, könnte auf die Idee kommen, den Bruch zu $^{49}/_{98} = {}^{4}/_{8}$ zu kürzen, indem er einfach die 9en aus Zähler und Nenner streicht. Und in diesem Fall wäre das Ergebnis sogar richtig.

Es gibt einige solcher Brüche. Beispiele wie $^{30}/_{50} = {}^{3}/_{5}$ sind dabei für uns trivial.

Es gibt exakt vier nicht-triviale Beispiele für diese Art von Bruch. Alle sind dem Wert nach kleiner als eins, und alle haben zweistellige Zähler und Nenner.

Welchen Nenner hat das vollständig gekürzte Produkt dieser vier Brüche?

Problem 34

145 is a curious number, as

$$1! + 4! + 5! = 1 + 24 + 120 = 145.$$

Find the sum of all numbers which are equal to the sum of the factorial of their digits.

Note: as 1! = 1 and 2! = 2 are not sums, they are not included.

Problem 34

145 ist eine merkwürdige Zahl, denn

$$1! + 4! + 5! = 1 + 24 + 120 = 145.$$

Ermittle die Summe aller Zahlen, die gleich der Summe der Fakultäten ihrer Ziffern sind.

Anmerkung: Da 1! = 1 and 2! = 2 keine Summen sind, werden sie nicht mit berücksichtigt.

Problem 35

The number 197 is called a *circular prime* because all rotations of the digits: 197, 971, and 719, are themselves prime.

There are thirteen such primes below 100:

2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79, and 97.

How many circular primes are there below one million?

Problem 35

Die Zahl 197 nennt man zirkukläre Primzahl, weil alle Rotationen ihrer Ziffern, also 197, 971 und 719, auch wieder prim sind.

Es gibt dreizehn solcher Primzahlen, die kleiner sind als 100:

2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79 und 97.

Wie viele zirkuläre Primzahlen gibt es, die kleiner als eine Million sind?

Problem 36

The decimal number 585 = 1001001001_2 (binary), is palindromic in both bases.

Find the sum of all numbers, less than one million, which are palindromic in base 10 and base 2.

Note: Palindromic numbers, in either base, may not include leading zeros.

Problem 36

Die Dezimalzahl 585 = 1001001001_2 (binär) ist palindromisch im Zehner- und im Zweiersytem.

Ermittle die Summe aller Zahlen, die kleiner als eine Million und Palindromzahlen sowohl im Zehner- als auch im Zweiersystem sind.

Anmerkung: Palindromzahlen dürfen in keiner Basis führende Nullen haben.

Problem 37

The number 3797 has an interesting property. Being prime itself, it is possible to continuously remove digits from left to right, and remain prime at each stage: 3797, 797, 97, and 7. Similarly we can work from right to left: 3797, 379, 37, and 3.

Find the sum of the only eleven primes that are both truncatable from left to right and right to left.

NOTE: 2, 3, 5, and 7 are not considered to be truncatable primes.

Problem 37

Die Zahl 3797 hat eine interessante Eigenschaft. Zum einen ist sie prim, aber auch, wenn man von links nach rechts Ziffer um Ziffer streicht, bleiben die Restzahlen immer prim: 3797, 797, 97 und 7. Entsprechend können wir auch von rechts nach links vorgehen: 3797, 379, 37 und 3.

Emittle die Summe der elf einzigen Primzahlen, die sich auf diese Weise, sowohl von links nach rechts als auch von rechts nach links, zusammenstreichen lassen und dabei prim bleiben.

Anmerkung: 2, 3, 5 und 7 sind zwar prim, können aber nicht gestrichen werden. Sie zählen daher nicht mit.

Problem 38

Take the number 192 and multiply it by each of 1, 2, and 3:

$$192 \times 1 = 192$$
$$192 \times 2 = 384$$
$$192 \times 3 = 576$$

By concatenating each product we get the 1 to 9 pandigital, 192384576. We will call 192384576 the *concatenated product* of 192 and (1,2,3)

The same can be achieved by starting with 9 and multiplying by 1, 2, 3, 4, and 5, giving the pandigital, 918273645, which is the concatenated product of 9 and (1,2,3,4,5).

What is the largest 1 to 9 pandigital 9-digit number that can be formed as the concatenated product of an integer with (1,2, ... , n) where n > 1?

Problem 38

Nehmen wir die Zahl 192 und multiplizieren wir sie mit 1, 2 und 3:

$$192 \cdot 1 = 192$$
$$192 \cdot 2 = 384$$
$$192 \cdot 3 = 576$$

Wenn wir alle Produkte nahtlos aneinanderreihen, erhalten wir die 1-bis-9-pandigitale Zahl 192384576. Wir nennen 192384576 das *Reihungsprodukt* von 192 mit (1,2,3).

Das Gleiche erreichen wir, wenn wir mit 9 beginnen und die 9 der Reihe nach mit 1, 2, 3, 4 und 5 malnehmen. Es ergibt sich das pandigitale Produkt 918273645 als Reihungsprodukt von 9 mit (1,2,3,4,5).

Welches ist die höchste 1-bis-9-pandigitale Zahl, die sich als Produkt einer ganzen Zahl mit (1,2, ... , n) bilden lässt, wobei n > 1 gilt?

Problem 39

If p is the perimeter of a right angle triangle with integral length sides, {a,b,c}, there are exactly three solutions for $p = 120$.

{20,48,52}, {24,45,51}, {30,40,50}

For which value of $p \leq 1000$, is the number of solutions maximised?

Problem 39

Wenn p der Umfang eines rechtwinkligen Dreiecks mit ganzzahligen Seitenlängen {a,b,c} ist, dann gibt es genau drei Lösungen für $p = 120$, nämlich

{20,48,52}, {24,45,51}, {30,40,50}

Für welchen Wert von p mit $p \leq 1000$ wird die Anzahl der Lösungen maximal?

Problem 40

An irrational decimal fraction is created by concatenating the positive integers:

$$0.123456789101112131415161718192021...$$

It can be seen that the 12^{th} digit of the fractional part is 1.

If d_n represents the n^{th} digit of the fractional part, find the value of the following expression.

$$d_1 \times d_{10} \times d_{100} \times d_{1000} \times d_{10000} \times d_{100000} \times d_{1000000}$$

Problem 40

Wenn man alle positiven ganzen Zahlen aneinanderreiht, erhält man folgende irrationale Dezimalzahl:

$$0.123456789101112131415161718192021...$$

Wie man sieht, ist die 12. Nachkommastelle 1.

d_n bezeichne die n. Nachkommastelle. Ermittle den Wert des folgenden Ausdrucks:

$$d_1 \times d_{10} \times d_{100} \times d_{1000} \times d_{10000} \times d_{100000} \times d_{1000000}$$

Problem 41

We shall say that an *n*-digit number is *pandigital* if it makes use of all the digits 1 to *n* exactly once. For example, 2143 is a 4-digit pandigital and is also prime.

What is the largest *n*-digit pandigital prime that exists?

Problem 41

Wir nennen eine *n*-stellige Zahl *pandigital*, wenn sie alle Ziffern von 1 bis *n* genau je einmal enthält. 2143 ist zum Beispiel eine 4-stellige Pandigitalzahl und außerdem prim.

Welches ist die höchste *n*-stellige Pandigitalprimzahl?

Problem 42

The n^{th} term of the sequence of triangle numbers is given by, $t_n = \frac{1}{2}n(n+1)$; so the first ten triangle numbers are:

$$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \ldots$$

By converting each letter in a word to a number corresponding to its alphabetical position and adding these values we form a word value. For example, the word value for SKY is $19 + 11 + 25 = 55 = t_{10}$. If the word value is a triangle number then we shall call the word a *triangle word*.

Using words.txt (extract from the following page or as an attachment from this pdf problems file), a 16K text file containing nearly two-thousand common English words, how many are triangle words?

Problem 42

Das n. Glied in der Folge der Dreieckszahlen lässt sich durch $t_n = \frac{1}{2}n(n+1)$ berechnen. Damit ergeben sich die ersten zehn Dreieckszahlen zu:

$$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \ldots$$

Indem wir jedem Buchstaben eines Wortes eine Zahl entsprechend seiner Position im Alphabet zuordnen und diese Alphawerte zusammenzählen, erhalten wir einen Wortwert. Der Wortwert für das englische Wort SKY ist beispielsweise $19 + 11 + 25 = 55 = t_{10}$. Wenn der Wortwert eine Dreieckszahl ist, nennen wir das Wort ein *Dreieckswort*.

Wie viele der knapp 2000 gebräuchlichen englischen Wörter in der etwa 16 Kilobyte großen Datei words.txt (der folgenden Seite oder auch der pdf-Datei als Anhang zu entnehmen) sind Dreieckswörter?

Problem 42 Common English words/gebräuchliche englische Wörter

A, ABILITY, ABLE, ABOUT, ABOVE, ABSENCE, ABSOLUTELY, ACADEMIC, ACCEPT, ACCESS, ACCIDENT, ACCOMPANY, ACCORDING, ACCOUNT, ACHIEVE, ACHIEVEMENT, ACID, ACQUIRE, ACROSS, ACT, ACTION, ACTIVE, ACTIVITY, ACTUAL, ACTUALLY, ADD, ADDITION, ADDITIONAL, ADDRESS, ADMINISTRATION, ADMIT, ADOPT, ADULT, ADVANCE, ADVANTAGE, ADVICE, ADVISE, AFFAIR, AFFECT, AFFORD, AFRAID, AFTER, AFTERNOON, AFTERWARDS, AGAIN, AGAINST, AGE, AGENCY, AGENT, AGO, AGREE, AGREEMENT, AHEAD, AID, AIM, AIR, AIRCRAFT, ALL, ALLOW, ALMOST, ALONE, ALONG, ALREADY, ALRIGHT, ALSO, ALTERNATIVE, ALTHOUGH, ALWAYS, AMONG, AMONGST, AMOUNT, AN, ANALYSIS, ANCIENT, AND, ANIMAL, ANNOUNCE, ANNUAL, ANOTHER, ANSWER, ANY, ANYBODY, ANYONE, ANYTHING, ANYWAY, APART, APPARENT, APPARENTLY, APPEAL, APPEAR, APPEARANCE, APPLICATION, APPLY, APPOINT, APPOINTMENT, APPROACH, APPROPRIATE, APPROVE, AREA, ARGUE, ARGUMENT, ARISE, ARM, ARMY, AROUND, ARRANGE, ARRANGEMENT, ARRIVE, ART, ARTICLE, ARTIST, AS, ASK, ASPECT, ASSEMBLY, ASSESS, ASSESSMENT, ASSET, ASSOCIATE, ASSOCIATION, ASSUME, ASSUMPTION, AT, ATMOSPHERE, ATTACH, ATTACK, ATTEMPT, ATTEND, ATTENTION, ATTITUDE, ATTRACT, ATTRACTIVE, AUDIENCE, AUTHOR, AUTHORITY, AVAILABLE, AVERAGE, AVOID, AWARD, AWARE, AWAY, AYE, BABY, BACK, BACKGROUND, BAD, BAG, BALANCE, BALL, BAND, BANK, BAR, BASE, BASIC, BASIS, BATTLE, BE, BEAR, BEAT, BEAUTIFUL, BECAUSE, BECOME, BED, BEDROOM, BEFORE, BEGIN, BEGINNING, BEHAVIOUR, BEHIND, BELIEF, BELIEVE, BELONG, BELOW, BENEATH, BENEFIT, BESIDE, BEST, BETTER, BETWEEN, BEYOND, BIG, BILL, BIND, BIRD, BIRTH, BIT, BLACK, BLOCK, BLOOD, BLOODY, BLOW, BLUE, BOARD, BOAT, BODY, BONE, BOOK, BORDER, BOTH, BOTTLE, BOTTOM, BOX, BOY, BRAIN, BRANCH, BREAK, BREATH, BRIDGE, BRIEF, BRIGHT, BRING, BROAD, BROTHER, BUDGET, BUILD, BUILDING, BURN, BUS, BUSINESS, BUSY, BUT, BUY, BY, CABINET, CALL, CAMPAIGN, CAN, CANDIDATE, CAPABLE, CAPACITY, CAPITAL, CAR, CARD, CARE, CAREER, CAREFUL, CAREFULLY, CARRY, CASE, CASH, CAT, CATCH, CATEGORY, CAUSE, CELL, CENTRAL, CENTRE, CENTURY, CERTAIN, CERTAINLY, CHAIN, CHAIR, CHAIRMAN, CHALLENGE, CHANCE, CHANGE, CHANNEL, CHAPTER, CHARACTER, CHARACTERISTIC, CHARGE, CHEAP, CHECK, CHEMICAL, CHIEF, CHILD, CHOICE, CHOOSE, CHURCH, CIRCLE, CIRCUMSTANCE, CITIZEN, CITY, CIVIL, CLAIM, CLASS, CLEAN, CLEAR, CLEARLY, CLIENT, CLIMB, CLOSE, CLOSELY, CLOTHES, CLUB, COAL, CODE, COFFEE, COLD, COLLEAGUE, COLLECT, COLLECTION, COLLEGE, COLOUR, COMBINATION, COMBINE, COME, COMMENT, COMMERCIAL, COMMISSION, COMMIT, COMMITMENT, COMMITTEE, COMMON, COMMUNICATION, COMMUNITY, COMPANY, COMPARE, COMPARISON, COMPETITION, COMPLETE, COMPLETELY, COMPLEX, COMPONENT, COMPUTER, CONCENTRATE, CONCENTRATION, CONCEPT, CONCERN, CONCERNED, CONCLUDE, CONCLUSION, CONDITION, CONDUCT, CONFERENCE, CONFIDENCE, CONFIRM, CONFLICT, CONGRESS, CONNECT, CONNECTION, CONSEQUENCE, CONSERVATIVE, CONSIDER, CONSIDERABLE, CONSIDERATION, CONSIST, CONSTANT, CONSTRUCTION, CONSUMER, CONTACT, CONTAIN, CONTENT, CONTEXT, CONTINUE, CONTRACT, CONTRAST, CONTRIBUTE, CONTRIBUTION, CONTROL, CONVENTION, CONVERSATION, COPY, CORNER, CORPORATE, CORRECT, COS, COST, COULD, COUNCIL, COUNT, COUNTRY, COUNTY, COUPLE, COURSE, COURT, COVER, CREATE, CREATION, CREDIT, CRIME, CRIMINAL, CRISIS, CRITERION, CRITICAL, CRITICISM, CROSS, CROWD, CRY, CULTURAL, CULTURE, CUP, CURRENT, CURRENTLY, CURRICULUM, CUSTOMER, CUT, DAMAGE, DANGER, DANGEROUS, DARK, DATA, DATE, DAUGHTER, DAY, DEAD, DEAL, DEATH, DEBATE, DEBT, DECADE, DECIDE, DECISION, DECLARE, DEEP, DEFENCE, DEFENDANT, DEFINE, DEFINITION, DEGREE, DELIVER, DEMAND, DEMOCRATIC, DEMONSTRATE, DENY, DEPARTMENT, DEPEND, DEPUTY, DERIVE, DESCRIBE, DESCRIPTION, DESIGN, DESIRE, DESK, DESPITE, DESTROY, DETAIL, DETAILED, DETERMINE, DEVELOP, DEVELOPMENT, DEVICE, DIE, DIFFERENCE, DIFFERENT, DIFFICULT, DIFFICULTY, DINNER, DIRECT, DIRECTION, DIRECTLY, DIRECTOR, DISAPPEAR, DISCIPLINE, DISCOVER, DISCUSS, DISCUSSION, DISEASE, DISPLAY, DISTANCE, DISTINCTION, DISTRIBUTION, DISTRICT, DIVIDE, DIVISION, DO, DOCTOR, DOCUMENT, DOG, DOMESTIC, DOOR, DOUBLE, DOUBT, DOWN, DRAW, DRAWING, DREAM, DRESS, DRINK, DRIVE, DRIVER, DROP, DRUG, DRY, DUE, DURING, DUTY, EACH, EAR, EARLY, EARN, EARTH, EASILY, EAST, EASY, EAT, ECONOMIC, ECONOMY, EDGE, EDITOR, EDUCATION, EDUCATIONAL, EFFECT, EFFECTIVE, EFFECTIVELY, EFFORT, EGG, EITHER, ELDERLY, ELECTION, ELEMENT, ELSE, ELSEWHERE, EMERGE, EMPHASIS, EMPLOY, EMPLOYEE, EMPLOYER, EMPLOYMENT, EMPTY, ENABLE, ENCOURAGE, END, ENEMY, ENERGY, ENGINE, ENGINEERING, ENJOY, ENOUGH, ENSURE, ENTER, ENTERPRISE, ENTIRE, ENTIRELY, ENTITLE, ENTRY, ENVIRONMENT, ENVIRONMENTAL, EQUAL, EQUALLY, EQUIPMENT, ERROR, ESCAPE, ESPECIALLY, ESSENTIAL, ESTABLISH, ESTABLISHMENT, ESTATE, ESTIMATE, EVEN, EVENING, EVENT, EVENTUALLY, EVER, EVERY, EVERYBODY, EVERYONE, EVERYTHING, EVIDENCE, EXACTLY, EXAMINATION, EXAMINE, EXAMPLE, EXCELLENT, EXCEPT, EXCHANGE, EXECUTIVE, EXERCISE, EXHIBITION, EXIST, EXISTENCE, EXISTING, EXPECT, EXPECTATION, EXPENDITURE, EXPENSE, EXPENSIVE, EXPERIENCE, EXPERIMENT, EXPERT, EXPLAIN, EXPLANATION, EXPLORE, EXPRESS, EXPRESSION, EXTEND, EXTENT, EXTERNAL, EXTRA, EXTREMELY, EYE, FACE, FACILITY, FACT, FACTOR, FACTORY, FAIL, FAILURE, FAIR, FAIRLY, FAITH, FALL, FAMILIAR, FAMILY, FAMOUS, FAR, FARM, FARMER, FASHION, FAST, FATHER, FAVOUR, FEAR, FEATURE, FEE, FEEL, FEELING, FEMALE, FEW, FIELD, FIGHT, FIGURE, FILE, FILL, FILM, FINAL, FINALLY, FINANCE, FINANCIAL, FIND, FINDING, FINE, FINGER, FINISH, FIRE, FIRM, FIRST, FISH, FIT, FIX, FLAT, FLIGHT, FLOOR, FLOW, FLOWER, FLY, FOCUS, FOLLOW, FOLLOWING, FOOD, FOOT, FOOTBALL, FOR, FORCE, FOREIGN, FOREST, FORGET, FORM, FORMAL, FORMER, FORWARD, FOUNDATION, FREE, FREEDOM, FREQUENTLY, FRESH, FRIEND, FROM, FRONT, FRUIT, FUEL, FULL, FULLY, FUNCTION, FUND, FUNNY, FURTHER, FUTURE, GAIN, GAME, GARDEN, GAS, GATE, GATHER, GENERAL, GENERALLY, GENERATE, GENERATION, GENTLEMAN, GET, GIRL, GIVE, GLASS, GO, GOAL, GOD, GOLD, GOOD, GOVERNMENT, GRANT, GREAT, GREEN, GREY, GROUND, GROUP, GROW, GROWING, GROWTH, GUEST, GUIDE, GUN, HAIR, HALF, HALL, HAND, HANDLE, HANG, HAPPEN, HAPPY, HARD, HARDLY, HATE, HAVE, HE, HEAD, HEALTH, HEAR, HEART, HEAT, HEAVY, HELL, HELP, HENCE, HER, HERE, HERSELF, HIDE, HIGH, HIGHLY, HILL, HIM, HIMSELF, HIS, HISTORICAL, HISTORY, HIT, HOLD, HOLE, HOLIDAY, HOME, HOPE, HORSE, HOSPITAL, HOT, HOTEL, HOUR, HOUSE, HOUSEHOLD, HOUSING, HOW, HOWEVER, HUGE, HUMAN, HURT, HUSBAND, I, IDEA, IDENTIFY, IF, IGNORE, ILLUSTRATE, IMAGE, IMAGINE, IMMEDIATE, IMMEDIATELY, IMPACT, IMPLICATION, IMPLY, IMPORTANCE, IMPORTANT, IMPOSE, IMPOSSIBLE, IMPRESSION, IMPROVE, IMPROVEMENT, IN, INCIDENT, INCLUDE, INCLUDING, INCOME, INCREASE, INCREASED, INCREASINGLY, INDEED, INDEPENDENT, INDEX, INDICATE, INDIVIDUAL, INDUSTRIAL, INDUSTRY, INFLUENCE, INFORM, INFORMATION, INITIAL, INITIATIVE, INJURY, INSIDE, INSIST, INSTANCE, INSTEAD, INSTITUTE, INSTITUTION, INSTRUCTION, INSTRUMENT, INSURANCE, INTEND, INTENTION, INTEREST, INTERESTED, INTERESTING, INTERNAL, INTERNATIONAL, INTERPRETATION, INTERVIEW, INTO, INTRODUCE, INTRODUCTION, INVESTIGATE, INVESTIGATION, INVESTMENT, INVITE, INVOLVE, IRON, IS, ISLAND, ISSUE, IT, ITEM, ITS, ITSELF, JOB, JOIN, JOINT, JOURNEY, JUDGE, JUMP, JUST, JUSTICE, KEEP, KEY, KID, KILL, KIND, KING, KITCHEN, KNEE, KNOW, KNOWLEDGE, LABOUR, LACK, LADY, LAND, LANGUAGE, LARGE, LARGELY, LAST, LATE, LATER, LATTER, LAUGH, LAUNCH, LAW, LAWYER, LAY, LEAD, LEADER, LEADERSHIP, LEADING, LEAF, LEAGUE, LEAN, LEARN, LEAST, LEAVE, LEFT, LEG, LEGAL, LEGISLATION, LENGTH, LESS, LET, LETTER, LEVEL, LIABILITY, LIBERAL, LIBRARY, LIE, LIFE, LIFT, LIGHT, LIKE, LIKELY, LIMIT, LIMITED, LINE, LINK, LIP, LIST, LISTEN, LITERATURE, LITTLE, LIVE, LIVING, LOAN, LOCAL, LOCATION, LONG, LOOK, LORD, LOSE, LOSS, LOT, LOVE, LOVELY, LOW, LUNCH, MACHINE, MAGAZINE, MAIN, MAINLY, MAINTAIN, MAJOR, MAJORITY, MAKE, MALE, MAN, MANAGE, MANAGEMENT, MANAGER, MANNER, MANY, MAP, MARK, MARKET, MARRIAGE, MARRIED, MARRY, MASS, MASTER, MATCH, MATERIAL, MATTER, MAY, MAYBE, ME, MEAL, MEAN, MEANING, MEANS, MEANWHILE, MEASURE, MECHANISM, MEDIA, MEDICAL, MEET, MEETING, MEMBER, MEMBERSHIP, MEMORY, MENTAL, MENTION, MERELY, MESSAGE, METAL, METHOD, MIDDLE, MIGHT, MILE, MILITARY, MILK, MIND, MINE, MINISTER, MINISTRY, MINUTE, MISS, MISTAKE, MODEL, MODERN, MODULE, MOMENT, MONEY, MONTH, MORE, MORNING, MOST, MOTHER, MOTION, MOTOR, MOUNTAIN, MOUTH, MOVE, MOVEMENT, MUCH, MURDER, MUSEUM, MUSIC, MUST, MY, MYSELF, NAME, NARROW, NATION, NATIONAL, NATURAL, NATURE, NEAR, NEARLY, NECESSARILY, NECESSARY, NECK, NEED, NEGOTIATION, NEIGHBOUR, NEITHER, NETWORK, NEVER, NEVERTHELESS, NEW, NEWS, NEWSPAPER, NEXT, NICE, NIGHT, NO, NOBODY, NOD, NOISE, NONE, NOR, NORMAL, NORMALLY, NORTH, NORTHERN, NOSE, NOT, NOTE, NOTHING, NOTICE, NOTION, NOW, NUCLEAR, NUMBER, NURSE, OBJECT, OBJECTIVE, OBSERVATION, OBSERVE, OBTAIN, OBVIOUS, OBVIOUSLY, OCCASION, OCCUR, ODD, OF, OFF, OFFENCE, OFFER, OFFICE, OFFICER, OFFICIAL, OFTEN, OIL, OKAY, OLD, ON, ONCE, ONE, ONLY, ONTO, OPEN, OPERATE, OPERATION, OPINION, OPPORTUNITY, OPPOSITION, OPTION, OR, ORDER, ORDINARY, ORGANISATION, ORGANISE, ORGANIZATION, ORIGIN, ORIGINAL, OTHER, OTHERWISE, OUGHT, OUR, OURSELVES, OUT, OUTCOME, OUTPUT, OUTSIDE, OVER, OVERALL, OWN, OWNER, PACKAGE, PAGE, PAIN, PAINT, PAINTING, PAIR, PANEL, PAPER, PARENT, PARK, PARLIAMENT, PART, PARTICULAR, PARTICULARLY, PARTLY, PARTNER, PARTY, PASS, PASSAGE, PAST, PATH, PATIENT, PATTERN, PAY, PAYMENT, PEACE, PENSION, PEOPLE, PER, PERCENT, PERFECT, PERFORM, PERFORMANCE, PERHAPS, PERIOD, PERMANENT, PERSON, PERSONAL, PERSUADE, PHASE, PHONE, PHOTOGRAPH, PHYSICAL, PICK, PICTURE, PIECE, PLACE, PLAN, PLANNING, PLANT, PLASTIC, PLATE, PLAY, PLAYER, PLEASE, PLEASURE, PLENTY, PLUS, POCKET, POINT, POLICE, POLICY, POLITICAL, POLITICS, POOL, POOR, POPULAR, POPULATION, POSITION, POSITIVE, POSSIBILITY, POSSIBLE, POSSIBLY, POST, POTENTIAL, POUND, POWER, POWERFUL, PRACTICAL, PRACTICE, PREFER, PREPARE, PRESENCE, PRESENT, PRESIDENT, PRESS, PRESSURE, PRETTY, PREVENT, PREVIOUS, PREVIOUSLY, PRICE, PRIMARY, PRIME, PRINCIPLE, PRIORITY, PRISON, PRISONER, PRIVATE, PROBABLY, PROBLEM, PROCEDURE, PROCESS, PRODUCE, PRODUCT, PRODUCTION, PROFESSIONAL, PROFIT, PROGRAM, PROGRAMME, PROGRESS, PROJECT, PROMISE, PROMOTE, PROPER, PROPERLY, PROPERTY, PROPORTION, PROPOSE, PROPOSAL, PROSPECT, PROTECT, PROTECTION, PROVE, PROVIDE, PROVIDED, PROVISION, PUB, PUBLIC, PUBLICATION, PUBLISH, PULL, PUPIL, PURPOSE, PUSH, PUT, QUALITY, QUARTER, QUESTION, QUICK, QUICKLY, QUIET, QUITE, RACE, RADIO, RAILWAY, RAIN, RAISE, RANGE, RAPIDLY, RARE, RATE, RATHER, REACH, REACTION, READ, READER, READING, READY, REAL, REALISE, REALITY, REALIZE, REALLY, REASON, REASONABLE, RECALL, RECEIVE, RECENT, RECENTLY, RECOGNISE, RECOGNITION, RECOGNIZE, RECOMMEND, RECORD, RECOVER, RED, REDUCE, REDUCTION, REFER, REFERENCE, REFLECT, REFORM, REFUSE, REGARD, REGION, REGIONAL, REGULAR, REGULATION, REJECT, RELATE, RELATION, RELATIONSHIP, RELATIVE, RELATIVELY, RELEASE, RELEVANT, RELIEF, RELIGION, RELIGIOUS, RELY, REMAIN, REMEMBER, REMIND, REMOVE, REPEAT, REPLACE, REPLY, REPORT, REPRESENT, REPRESENTATION, REPRESENTATIVE, REQUEST, REQUIRE, REQUIREMENT, RESEARCH, RESOURCE, RESPECT, RESPOND, RESPONSE, RESPONSIBILITY, RESPONSIBLE, REST, RESTAURANT, RESULT, RETAIN, RETURN, REVEAL, REVENUE, REVIEW, REVOLUTION, RICH, RIDE, RIGHT, RING, RISE, RISK, RIVER, ROAD, ROCK, ROLE, ROLL, ROOF, ROOM, ROUND, ROUTE, ROW, ROYAL, RULE, RUN, RURAL, SAFE, SAFETY, SALE, SAME, SAMPLE, SATISFY, SAVE, SAY, SCALE, SCENE, SCHEME, SCHOOL, SCIENCE, SCIENTIFIC, SCIENTIST, SCORE, SCREEN, SEA, SEARCH, SEASON, SEAT, SECOND, SECONDARY, SECRETARY, SECTION, SECTOR, SECURE, SECURITY, SEE, SEEK, SEEM, SELECT, SELECTION, SELL, SEND, SENIOR, SENSE, SENTENCE, SEPARATE, SEQUENCE, SERIES, SERIOUS, SERIOUSLY, SERVANT, SERVE, SERVICE, SESSION, SET, SETTLE, SETTLEMENT, SEVERAL, SEVERE, SEX, SEXUAL, SHAKE, SHALL, SHAPE, SHARE, SHE, SHEET, SHIP, SHOE, SHOOT, SHOP, SHORT, SHOT, SHOULD, SHOULDER, SHOUT, SHOW, SHUT, SIDE, SIGHT, SIGN, SIGNAL, SIGNIFICANCE, SIGNIFICANT, SILENCE, SIMILAR, SIMPLE, SIMPLY, SINCE, SING, SINGLE, SIR, SISTER, SIT, SITE, SITUATION, SIZE, SKILL, SKIN, SKY, SLEEP, SLIGHTLY, SLIP, SLOW, SLOWLY, SMALL, SMILE, SO, SOCIAL, SOCIETY, SOFT, SOFTWARE, SOIL, SOLDIER, SOLICITOR, SOLUTION, SOME, SOMEBODY, SOMEONE, SOMETHING, SOMETIMES, SOMEWHAT, SOMEWHERE, SON, SONG, SOON, SORRY, SORT, SOUND, SOURCE, SOUTH, SOUTHERN, SPACE, SPEAK, SPEAKER, SPECIAL, SPECIES, SPECIFIC, SPEECH, SPEED, SPEND, SPIRIT, SPORT, SPOT, SPREAD, SPRING, STAFF, STAGE, STAND, STANDARD, STAR, START, STATE, STATEMENT, STATION, STATUS, STAY, STEAL, STEP, STICK, STILL, STOCK, STONE, STOP, STORE, STORY, STRAIGHT, STRANGE, STRATEGY, STREET, STRENGTH, STRIKE, STRONG, STRONGLY, STRUCTURE, STUDENT, STUDIO, STUDY, STUFF, STYLE, SUBJECT, SUBSTANTIAL, SUCCEED, SUCCESS, SUCCESSFUL, SUCH, SUDDENLY, SUFFER, SUFFICIENT, SUGGEST, SUGGESTION, SUITABLE, SUM, SUMMER, SUN, SUPPLY, SUPPORT, SUPPOSE, SURE, SURELY, SURFACE, SURPRISE, SURROUND, SURVEY, SURVIVE, SWITCH, SYSTEM, TABLE, TAKE, TALK, TALL, TAPE, TARGET, TASK, TAX, TEA, TEACH, TEACHER, TEACHING, TEAM, TEAR, TECHNICAL, TECHNIQUE, TECHNOLOGY, TELEPHONE, TELEVISION, TELL, TEMPERATURE, TEND, TERM, TERMS, TERRIBLE, TEST, TEXT, THAN, THANK, THANKS, THAT, THE, THEATRE, THEIR, THEM, THEME, THEMSELVES, THEN, THEORY, THERE, THEREFORE, THESE, THEY, THIN, THING, THINK, THIS, THOSE, THOUGH, THOUGHT, THREAT, THREATEN, THROUGH, THROUGHOUT, THROW, THUS, TICKET, TIME, TINY, TITLE, TO, TODAY, TOGETHER, TOMORROW, TONE, TONIGHT, TOO, TOOL, TOOTH, TOP, TOTAL, TOTALLY, TOUCH, TOUR, TOWARDS, TOWN, TRACK, TRADE, TRADITION, TRADITIONAL, TRAFFIC, TRAIN, TRAINING, TRANSFER, TRANSPORT, TRAVEL, TREAT, TREATMENT, TREATY, TREE, TREND, TRIAL, TRIP, TROOP, TROUBLE, TRUE, TRUST, TRUTH, TRY, TURN, TWICE, TYPE, TYPICAL, UNABLE, UNDER, UNDERSTAND, UNDERSTANDING, UNDERTAKE, UNEMPLOYMENT, UNFORTUNATELY, UNION, UNIT, UNITED, UNIVERSITY, UNLESS, UNLIKELY, UNTIL, UP, UPON, UPPER, URBAN, US, USE, USED, USEFUL, USER, USUAL, USUALLY, VALUE, VARIATION, VARIETY, VARIOUS, VARY, VAST, VEHICLE, VERSION, VERY, VIA, VICTIM, VICTORY, VIDEO, VIEW, VILLAGE, VIOLENCE, VISION, VISIT, VISITOR, VITAL, VOICE, VOLUME, VOTE, WAGE, WAIT, WALK, WALL, WANT, WAR, WARM, WARN, WASH, WATCH, WATER, WAVE, WAY, WE, WEAK, WEAPON, WEAR, WEATHER, WEEK, WEEKEND, WEIGHT, WELCOME, WELFARE, WELL, WEST, WESTERN, WHAT, WHATEVER, WHEN, WHERE, WHEREAS, WHETHER, WHICH, WHILE, WHILST, WHITE, WHO, WHOLE, WHOM, WHOSE, WHY, WIDE, WIDELY, WIFE, WILD, WILL, WIN, WIND, WINDOW, WINE, WING, WINNER, WINTER, WISH, WITH, WITHDRAW, WITHIN, WITHOUT, WOMAN, WONDER, WONDERFUL, WOOD, WORD, WORK, WORKER, WORKING, WORKS, WORLD, WORRY, WORTH, WOULD, WRITE, WRITER, WRITING, WRONG, YARD, YEAH, YEAR, YES, YESTERDAY, YET, YOU, YOUNG, YOUR, YOURSELF, YOUTH

Problem 43

The number, 1406357289, is a 0 to 9 pandigital number because it is made up of each of the digits 0 to 9 in some order, but it also has a rather interesting sub-string divisibility property.

Let d_1 be the 1st digit, d_2 be the 2nd digit, and so on. In this way, we note the following:

- $d_2d_3d_4$ = 406 is divisible by 2
- $d_3d_4d_5$ = 063 is divisible by 3
- $d_4d_5d_6$ = 635 is divisible by 5
- $d_5d_6d_7$ = 357 is divisible by 7
- $d_6d_7d_8$ = 572 is divisible by 11
- $d_7d_8d_9$ = 728 is divisible by 13
- $d_8d_9d_{10}$ = 289 is divisible by 17

Find the sum of all 0 to 9 pandigital numbers with this property.

Problem 43

Die Zahl 1406357289 ist eine 0-bis-9-Pandigitalzahl, weil sie jede Ziffer von 0 bis 9 an irgendeiner Stelle genau einmal enthält. Außerdem haben ihre Ziffern eine interessante Teilbarkeits-Eigenschaft.

Es bezeichne d_1 die 1. Ziffer, d_2 die 2. Ziffer, und so weiter. Uns fällt folgendes auf:

- $d_2d_3d_4$ = 406 ist ohne Rest teilbar durch 2
- $d_3d_4d_5$ = 063 ist ohne Rest teilbar durch 3
- $d_4d_5d_6$ = 635 ist ohne Rest teilbar durch 5
- $d_5d_6d_7$ = 357 ist ohne Rest teilbar durch 7
- $d_6d_7d_8$ = 572 ist ohne Rest teilbar durch 11
- $d_7d_8d_9$ = 728 ist ohne Rest teilbar durch 13
- $d_8d_9d_{10}$ = 289 ist ohne Rest teilbar durch 17

Ermittle die Summe aller 0-bis-9-Pandigitalzahlen mit dieser Eigenschaft.

Problem 44

Pentagonal numbers are generated by the formula,

$$P_n = n(3n-1) : 2.$$

The first ten pentagonal numbers are:

1, 5, 12, 22, 35, 51, 70, 92, 117, 145, ...

It can be seen that $P_4 + P_7 = 22 + 70 = 92 = P_8$.
However, their difference, $70 - 22 = 48$, is not pentagonal.

Find the pair of pentagonal numbers, P_j and P_k, for which their sum and difference is pentagonal and $D = |P_k - P_j|$ is minimised; what is the value of D?

Problem 44

Pentagonalzahlen berechnet man mit der Formel

$$P_n = n(3n-1) : 2.$$

Die ersten zehn Pentagonalzahlen sind:

1, 5, 12, 22, 35, 51, 70, 92, 117, 145, ...

Wie man sieht, ist $P_4 + P_7 = 22 + 70 = 92 = P_8$.
Ihre Differenz jedoch, $70 - 22 = 48$, ist nicht pentagonal.

Finde das Paar pentagonaler Zahlen, P_j and P_k, für die ihre Summe und ihre Differenz pentagonal sind und für die $D = |P_k - P_j|$ minimal wird. Welches ist der Wert für D?

Problem 45

Triangle, pentagonal, and hexagonal numbers are generated by the following formulae:

Triangle	$T_n = n(n+1) : 2$	1, 3, 6, 10, 15, ...
Pentagonal	$P_n = n(3n-1) : 2$	1, 5, 12, 22, 35, ...
Hexagonal	$H_n = n(2n-1)$	1, 6, 15, 28, 45, ...

It can be verified that $T_{285} = P_{165} = H_{143} = 40755$.

Find the next triangle number that is also pentagonal and hexagonal.

Problem 45

Dreieckszahlen, Pentagonalzahlen und Hexagonalzahlen erzeugt man mit folgenden Formeln:

Dreieckszahlen	$T_n = n(n+1) : 2$	1, 3, 6, 10, 15, ...
Pentagonal	$P_n = n(3n-1) : 2$	1, 5, 12, 22, 35, ...
Hexagonal	$H_n = n(2n-1)$	1, 6, 15, 28, 45, ...

Man kann nachrechnen, dass $T_{285} = P_{165} = H_{143} = 40755$ gilt.

Finde die nächstgrößere Dreieckszahl, die auch pentagonal und hexagonal ist.

Problem 46

It was proposed by Christian Goldbach that every odd composite number can be written as the sum of a prime and twice a square.

$$
\begin{aligned}
9 &= 7 + 2\times1^2\\
15 &= 7 + 2\times2^2\\
21 &= 3 + 2\times3^2\\
25 &= 7 + 2\times3^2\\
27 &= 19 + 2\times2^2\\
33 &= 31 + 2\times1^2
\end{aligned}
$$

It turns out that the conjecture was false.

What is the smallest odd composite that cannot be written as the sum of a prime and twice a square?

Problem 46

Christian Goldbach vermutete, dass sich jede ungerade zusammengesetzte Zahl als die Summe aus einer Primzahl und dem Doppelten einer Quadratzahl schreiben ließe.

$$
\begin{aligned}
9 &= 7 + 2 \bullet 1^2\\
15 &= 7 + 2 \bullet 2^2\\
21 &= 3 + 2 \bullet 3^2\\
25 &= 7 + 2 \bullet 3^2\\
27 &= 19 + 2 \bullet 2^2\\
33 &= 31 + 2 \bullet 1^2
\end{aligned}
$$

Diese Vermutung erwies sich als falsch.

Welches ist die kleinste ungerade zusammengesetzte Zahl, die sich nicht als Summe aus einer Primzahl und dem Doppelten einer Quadratzahl schreiben lässt?

Problem 47

The first two consecutive numbers to have two distinct prime factors are:

$$14 = 2 \times 7$$
$$15 = 3 \times 5$$

The first three consecutive numbers to have three distinct prime factors are:

$$644 = 2^2 \times 7 \times 23$$
$$645 = 3 \times 5 \times 43$$
$$646 = 2 \times 17 \times 19.$$

Find the first four consecutive integers to have four distinct primes factors. What is the first of these numbers?

Problem 47

Die ersten zwei unmittelbar aufeinanderfolgenden Zahlen, die zwei unterschiedliche Primfaktoren haben, sind:

$$14 = 2 \cdot 7$$
$$15 = 3 \cdot 5$$

Die ersten drei unmittelbar aufeinanderfolgenden Zahlen mit je drei unterschiedlichen Primfaktoren sind:

$$644 = 2^2 \cdot 7 \cdot 23$$
$$645 = 3 \cdot 5 \cdot 43$$
$$646 = 2 \cdot 17 \cdot 19.$$

Finde die ersten vier unmittelbar aufeinanderfolgenden ganzen Zahlen, die vier unterschiedliche Primfaktoren haben. Welches ist die erste dieser Zahlen?

Problem 48

The series, $1^1 + 2^2 + 3^3 + ... + 10^{10} = 10405071317$.

Find the last ten digits of the series,

$$1^1 + 2^2 + 3^3 + ... + 1000^{1000}.$$

Problem 48

Die Summe $1^1 + 2^2 + 3^3 + ... + 10^{10} = 10405071317$.

Ermittle die letzten zehn Ziffern der Summe

$$1^1 + 2^2 + 3^3 + ... + 1000^{1000}.$$

Problem 49

The arithmetic sequence, 1487, 4817, 8147, in which each of the terms increases by 3330, is unusual in two ways:

(i) each of the three terms are prime, and,
(ii) each of the 4-digit numbers are permutations of one another.

There are no arithmetic sequences made up of three 1-, 2-, or 3-digit primes, exhibiting this property, but there is one other 4-digit increasing sequence.

What 12-digit number do you form by concatenating the three terms in this sequence?

Problem 49

Die arithmetische Folge 1487, 4817, 8147, in der jede folgende Zahl um 3330 größer ist als die vorhergehende, ist in zweierlei Hinsicht ungewöhnlich:

(i) jede der Zahlen in der Folge ist prim und
(ii) alle vierstelligen Zahlen in der Folge sind Permutationen von einander.

Es gibt keine arithmetischen Folgen aus 1-, 2- oder 3-stelligen Primzahlen mit dieser Eigenschaft, aber es gibt noch eine weitere wachsende Folge aus vierstelligen Zahlen.

Welche 12-stellige Zahl ergibt sich, wenn man die drei Zahlen in der gesuchten Folge nahtlos aneinanderreiht?

Problem 50

The prime 41, can be written as the sum of six consecutive primes:

$$41 = 2 + 3 + 5 + 7 + 11 + 13$$

This is the longest sum of consecutive primes that adds to a prime below one-hundred.

The longest sum of consecutive primes below one-thousand that adds to a prime, contains 21 terms, and is equal to 953.

Which prime, below one-million, can be written as the sum of the most consecutive primes?

Problem 50

Die Primzahl 41 lässt sich als Summe sechs unmittelbar aufeinanderfolgender Primzahlen schreiben:

$$41 = 2 + 3 + 5 + 7 + 11 + 13$$

Dies ist die längste Summe aufeinanderfolgender Primzahlen, die eine Primzahl unter 100 ergibt.

Die längste Summe unmittelbar aufeinanderfolgender Primzahlen unter 1000, die eine Primzahl ergibt, enthält 21 Zahlen und hat einen Wert von 953.

Welche Primzahl unter einer Million lässt sich als Summe der meisten unmittelbar aufeinanderfolgenden Primzahlen schreiben?

Problem 51

By replacing the 1st digit of *3, it turns out that six of the nine possible values: 13, 23, 43, 53, 73, and 83, are all prime.

By replacing the 3rd and 4th digits of 56**3 with the same digit, this 5-digit number is the first example having seven primes among the ten generated numbers, yielding the family: 56003, 56113, 56333, 56443, 56663, 56773, and 56993. Consequently 56003, being the first member of this family, is the smallest prime with this property.

Find the smallest prime which, by replacing part of the number (not necessarily adjacent digits) with the same digit, is part of an eight prime value family.

Problem 51

Wenn man die 1. Ziffer des Musters *3 durch andere Ziffern ersetzt, ergeben sich von den neun Möglichkeiten genau sechs Primzahlen, nämlich 13, 23, 43, 53, 73 und 83.

Wenn man die 3. und die 4. Ziffer des Musters 56**3 durch dieselbe Ziffer ersetzt, ist diese Zahl das erste fünfstellige Beispiel, bei dem unter den zehn erzeugten Zahlen eine Familie aus sieben Primzahlen auftritt: 56003, 56113, 56333, 56443, 56663, 56773 und 56993. Damit ist 56003 als erstes Mitglied dieser Familie die kleinste Primzahl mit dieser Eigenschaft.

Finde die kleinste Primzahl, die Mitglied einer Familie mit acht Primzahlen wird, wenn man einige ihrer Ziffern (die nicht notwendigerweise unmittelbar aneinandergrenzen müssen) durch dieselbe Ziffer ersetzt.

Problem 52

It can be seen that the number, 125874, and its double, 251748, contain exactly the same digits, but in a different order.

Find the smallest positive integer, x, such that 2x, 3x, 4x, 5x, and 6x, contain the same digits.

Problem 52

Betrachtet man die Zahl 125874 und ihr Doppeltes, 251748, so sieht man, dass beide Zahlen exakt die gleichen Ziffern enthalten, nur in unterschiedlicher Reihenfolge.

Finde die kleinste positive ganze Zahl x, so dass 2x, 3x, 4x, 5x und 6x die gleichen Ziffern enthalten.

Problem 53

There are exactly ten ways of selecting three from five, 12345:

123, 124, 125, 134, 135, 145, 234, 235, 245, and 345

In combinatorics, we use the notation, ${}^{5}C_{3} = 10$.

In general,

$$ {}^{n}C_{r} = \frac{n!}{r!(n-r)!}, $$

where $r \le n$, $n! = n \times (n-1) \times \ldots \times 3 \times 2 \times 1$, and $0! = 1$

It is not until $n = 23$, that a value exceeds one-million:

$$ {}^{23}C_{10} = 1144066. $$

How many, not necessarily distinct, values of ${}^{n}C_{r}$, for $1 \le n \le 100$, are greater than one-million?

Problem 53

Es gibt genau zehn Möglichkeiten, drei aus fünf auszuwählen, 12345:

123, 124, 125, 134, 135, 145, 234, 235, 245 und 345

In der Kombinatorik schreiben wir dafür kurz ${}^{5}C_{3} = 10$.

Allgemein gilt:

$$ {}^{n}C_{r} = \frac{n!}{r!(n-r)!}, $$

mit $r \le n$, $n! = n \times (n-1) \times \ldots \times 3 \times 2 \times 1$ und $0! = 1$

Erst ab $n = 23$ überschreitet einer der Werte eine Million:

$$ {}^{23}C_{10} = 1144066. $$

Wie viele, nicht notwendigerweise unterschiedliche, Werte von ${}^{n}C_{r}$, für $1 \le n \le 100$, sind größer als eine Million?

Problem 54

In the card game poker, a hand consists of five cards and are ranked, from lowest to highest, in the following way:

- *High Card:* Highest value card.
- *One Pair:* Two cards of the same value.
- *Two Pairs:* Two different pairs.
- *Three of a Kind:* Three cards of the same value.
- *Straight:* All cards are consecutive values.
- *Flush:* All cards of the same suit.
- *Full House:* Three of a kind and a pair.
- *Four of a Kind:* Four cards of the same value.
- *Straight Flush:* All cards are consecutive values of same suit.
- *Royal Flush:* Ten, Jack, Queen, King, Ace, in same suit.

The cards are valued in the order:
2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace.

If two players have the same ranked hands then the rank made up of the highest value wins; for example, a pair of eights beats a pair of fives (see example 1 below). But if two ranks tie, for example, both players have a pair of queens, then highest cards in each hand are compared (see example 4 below); if the highest cards tie then the next highest cards are compared, and so on.

Consider the following five hands dealt to two players:

Hand	Player 1	Player 2	Winner
1	5H 5C 6S 7S KD Pair of Fives	2C 3S 8S 8D TD Pair of Eights	Player 2
2	5D 8C 9S JS AC Highest card Ace	2C 5C 7D 8S QH Highest card Queen	Player 1
3	2D 9C AS AH AC Three Aces	3D 6D 7D TD QD Flush with Diamonds	Player 2
4	4D 6S 9H QH QC Pair of Queens	3D 6D 7H QD QS Highest card Nine	Player 1
5	2H 2D 4C 4D 4S Full House with 3 Fours	3C 3D 3S 9S 9D Full House with 3 Threes	Player 1

The file, poker.txt (next pages) contains one-thousand random hands dealt to two players. Each line of the file contains ten cards (separated by a single space): the first five are Player 1's cards and the last five are Player 2's cards. You can assume that all hands are valid (no invalid characters or repeated cards), each player's hand is in no specific order, and in each hand there is a clear winner.

How many hands does Player 1 win?

Problem 54

Beim Pokerspiel besteht eine Hand aus fünf Karten. Es gibt unterschiedlich wertvolle Spiele gemäß der folgenden Rangordnung.

- *High Card:* Karte mit dem höchsten Wert.
- *Ein Paar:* Zwei Karten mit demselben Wert.
- *Zwilling:* Zwei unterschiedliche Paare.
- *Drilling:* Drei Karten mit demselben Wert.
- *Straight (Straße):* Alle Karten folgen wertmäßig aufeinander.
- *Flush:* Alle Karten sind von derselben Farbe.
- *Full House:* Drilling und Paar.
- *Vierling:* Vier Karten mit demselben Wert.
- *Straight Flush:* Alle, gleichfarbigen, Karten folgen wertmäßig aufeinander.
- *Royal Flush:* Zehn, Bube, Dame, König, As in derselben Farbe.

Die Karten werden folgendermaßen aufsteigend bewertet:
2, 3, 4, 5, 6, 7, 8, 9, 10 (**T**), Bube (**J**), Dame (**Q**), König (K), As (A).
Es gibt vier Sorten: Pik (**S**), Herz (H), Karo (**D**) und Kreuz (**C**).

Wenn zwei Spieler eine ranggleiche Hand haben, gewinnt der Rang mit dem höchsten Kartenwert: ein Achterpaar schlägt ein Fünferpaar (siehe Beispiel 1 unten). Wenn zwei Ränge den gleichen Wert haben (z. B. haben beide Spieler ein Damenpaar), vergleicht man die höchsten Karten in jeder Hand (siehe Beispiel 4 unten). Sind die höchsten Karten gleich, vergleicht man die zweithöchsten usw.

Betrachten wir die folgenden fünf Hände für Spieler 1 und Spieler 2:

Hand	Spieler 1	Spieler 2	Gewinner
1	5H 5C 6S 7S KD Fünferpaar	2C 3S 8S 8D TD Achterpaar	Spieler 2
2	5D 8C 9S JS AC Höchste Karte As	2C 5C 7D 8S QH Höchste Karte Dame	Spieler 1
3	2D 9C AS AH AC Drilling mit Assen	3D 6D 7D TD QD Karo-Flush	Spieler 2
4	4D 6S 9H QH QC Damen-Paar	3D 6D 7H QD QS Höchste Karte Neun	Spieler 1
5	2H 2D 4C 4D 4S Full House mit drei Vieren	3C 3D 3S 9S 9D Full House mit drei Dreien	Spieler 1

Die Datei poker.txt (folgende Seiten) enthält eintausend zufällige Hände für zwei Spieler. Jede Zeile enthält zehn Karten (durch Leerzeichen getrennt): die ersten fünf gehören Spieler 1, die folgenden fünf Spieler 2. Alle Hände sind gültig (keine ungültigen Zeichen oder Kartendoppelungen). Die Hände sind nicht geordnet, und in jeder Hand gibt es einen eindeutigen Gewinner.

Wie viele Hände gewinnt Spieler 1?

Problem 54 Poker hands/Pokerblätter,

8C TS KC 9H 4S 7D 2S 5D 3S AC
5C AD 5D AC 9C 7C 5H 8D TD KS
3H 7H 6S KC JS QH TD JC 2D 8S
TH 8H 5C QS TC 9H 4D JC KS JS
7C 5H KC QH JD AS KH 4C AD 4S
5H KS 9C 7D 9H 8D 3S 5D 5C AH
6H 4H 5C 3H 2H 3S QH 5S 6S AS
TD 8C 4H 7C TC KC 4C 3H 7S KS
7C 9C 6D KD 3H 4C QS QC AC KH
JC 6S 5H 2H 2D KD 9D 7C AS JS
AD QH TH 9D 8H TS 6D 3S AS AC
2H 4S 5C 5S TC KC JD 6C TS 3C
QD AS 6H JS 2C 3D 9H KC 4H 8S
KD 8S 9S 7C 2S 3S 6D 6S 4H KC
3C 8C 2D 7D 4D 9S 4S QH 4H JD
8C KC 7S TC 2D TS 8H QD AC 5C
3D KH QD 6C 6S AD AS 8H 2H QS
6S 8D 4C 8S 6C QH TC 6D 7D 9D
2S 8D 8C 4C TS 9S 9D 9C AC 3D
3C QS 2S 4H JH 3D 2D TD 8S 9H
5H QS 8S 6D 3C 8C JD AS 7H 7D
6H TD 9D AS JH 6C QC 9S KD JC
AH 8S QS 4D TH AC TS 3C 3D 5C
5S 4D JS 3D 8H 6C TS 3S AD 8C
6D 7C 5D 5H 3S 5C JC 2H 5S 3D
5H 6H 2S KS 3D 5D JD 7H JS 8H
KH 4H AS JS QS QC TC 6D 7C KS
3D QS TS 2H JS 4D AS 9S JC KD
QD 5H 4D 5D KH 7H 3D JS KD 4H
2C 9H 6H 5C 9D 6C JC 2D TH 9S
7D 6D AS QD JH 4D JS 7C QS 5C
3H KH QD AD 8C 8H 3S TH 9D 5S
AH 9S 4D 9D 8S 4H JS 3C TC 8D
2C KS 5H QD 3S TS 9H AH AD 8S
5C 7H 5D KD 9H 4D 3D 2D KS AD
KS KC 9S 6D 2C QH 9D 9H TS TC
9C 6H 5D QH 4D AD 6D QC JS KH
9S 3H 9D JD 5C 4D 9H AS TC QH
2C 6D JC 9C 3C AD 9S KH 9D 7D
KC 9C 7C JC JS KD 3H AS 3C 7D
QD KH QS 2C 3S 8S 8H 9H 9C JC
QH 8D 3C KC 4C 4H 6D AD 9H 9D
3S KS QS 7H KH 7D 5H 5D JD AD
2H 2C 6H TH TC 7D 8D 4H 8C AS
4S 2H AC QC 3S 6D TH 4D 4C KH
4D TC KS AS 7C 3C 6D 2D 9H 6C
8C TD 5D QS 2C 7H 4C 9C 3H 9H
5H JH TS 7S TD 6H AD QD 8H 8S
5S AD 9C 8C 7C 8D 5H 9D 8S 2S
4H KH KS 9S 2S KC 5S AD 4S 7D
QS 9C QD 6H JS 5D AC 8D 2S AS
KH AC JC 3S 9D 9S 3C 9C 5S JS
AD 3C 3D KS 3S 5C 9C 8C TS 4S
JH 8D 5D 6H KD QS QD 3D 6C KC
8S JD 6C 3S 8C TC QC 3C QH JS
KC JC 8H 2S 9H 9C JH 8S 8C 9S
8S 2H QH 4D QC 9D KC AS TH 3C
8S 6H TH 7C 2H 6S 3C 3H AS 7S
QH 5S JS 4H 5H TS 8H AH AC JC
9D 8H 2S 4S TC JC 3C 7H 3H 5C
3D AD 3C 3S 4C QC AS 5D TH 8C
6S 9D 4C JS KH AH TS JD 8H AD
4C 6S 9D 7S AC 4D 3D 3S TC JD
AD 7H 6H 4H JH KC TD TS 7D 6S
8H JH TC 3S 8D 8C 9S 2C 5C 4D
2C 9D KC QH TH QS JC 9C 4H TS
QS 3C QD 8H KH 4H 8D TD 8S AC
7C 3C TH 5S 8H 8C 9C JD TC KD
QC TC JD TS 8C 3H 6H KD 7C TD
JH QS KS 9C 6D 6S AS 9H KH 6H
2H 4D AH 2D JH 6H TD 5D 4H JD
KD 8C 9S JH QD JS 2C QS 5C 7C

4S TC 7H 8D 2S 6H 7S 9C 7C KC
8C 5D 7H 4S TD QC 8S JS 4H KS
AD 8S JH 6D TD KD 7C 6C 2D 7D
JC 6H 6S JS 4H QH 9H AH 4C 3C
6H 5H AS 7C 7S 3D KH KC 5D 5C
JC 3D TD AS 4D 6D 6S QH JD KS
8C 7S 8S QH 2S JD 5C 7H AH QD
8S 3C 6H 6C 2C 8D TD 7D 4C 4D
5D QH KH 7C 2S 7H JS 6D QC QD
AD 6C 6S 7D TH 6H 2H 8H KH 4H
KS JS KD 5D 2D KH 7D 9C 8C 3D
9C 6D QD 3C KS 3S 7S AH JD 2D
AH QH AS JC 8S 8H 4C KC TH 7D
JC 5H TD 7C 5D KD 4C AD 8H JS
KC 2H AC AH 7D JH KH 5D 7S 6D
9S 5S 9C 6H 8S TD JD 9H 6C AC
7D 8S 6D TS KD 7H AC 5S 7C 5D
AH QC JC 4C TC 8C 2H TS 2C 7D
KD KC 6S 3D 7D 2S 8S 3H 5S 5C
8S 5D 8H 4C 6H KC 3H 7C 5S KD
JH 8C 3D 3C 6C KC TD 7H 7C 4C
JC KC 6H TS QS TD KS 8H 8C 9S
6C 5S 9C QH 7D AH KS KC 9S 2C
4D 4S 8H TD 9C 3S 7D 9D AS TH
6S 7D 3C 6H 5D KD 2C 5C 9D 9C
2H KC 3D AD 3H QD QS 8D JC 4S
8C 3H 9C 7C AD 5D JC 9D JS AS
5D 9H 5C 7H 6S 6C QC JC QD 9S
JC QS JH 2C 6S 9C QC 3D 4S TC
4H 5S 8D 3D 4D 2S KC 2H JS 2C
TD 3S TH KD 4D 7H JH JS KS AC
7S 8C 9S 2D 8S 7D 5C AD 9D AS
8C 7H 2S 6C TH 3H 4C 3S 8H AC
KD 5H JC 8H JD 2D 4H TD JH 5C
3D AS QH KS 7H JD 8S 5S 6D 5H
9S 6S TC QS JC 5C 5D 9C TH 8C
5H 3S JH 9H 2S 2C 6S 7S AS KS
8C QD JC QS TC QC 4H AC KH 6C
TC 5H 7D JH 4H 2H 8D JC KS 4D
5S 9C KH 6D 9H 5C TS 3D 7D 2D
5H AS TC 4D 8C 2C TS 9D 3H 8D
6H 8D 2D 9H JD 6C 4S 5H 5S 6D
AD 9C JC 7D 6H 9S 6D JS 9H 3C
AD JH TC QS 4C 5D 9S 7C 9C AH
KD 6H 2H TH 8S QD KS 9D 9H AS
4H 8H 8D 5H 6C AH 5S AS AD 8S
QS 5D 4S 2H TD KS 5H AC 3H JC
9C 7D QD KD AC 6D 5H QH 6H 5S
KC AH QH 2H 7D QS 3H KS 7S JD
6C 8S 3H 6D KS QD 5D 5C 8H TC
9H 4D 4S 6S 9D KH QC 4H 6C JD
TD 2D QH 4S 6H JH KD 3C QD 8C
4S 6H 7C QD 9D AS AH 6S AD 3C
2C KC TH 6H 8D AH 5C 6D 8S 5D
TD TS 7C AD JC QD 9H 3C KC 7H
5D 4D 5S 8H 4H 7D 3H JD KD 2D
JH TD 6H QS 4S KD 5C 8S 7D 8H
AC 3D AS 8D TD 7H KH 5D 6C 8D
9D KS 7C 6D QH TC JD KD AS KC
JH 8S 5S 7S 7D AS 2D 3D AD 2H
2H 5D AS 3C QD KC 6H 9H 9S 2C
9D 5D TH 4C JH 3H 8D TC 8H 9H
6H KD 2C TD 2H 6C 9D 2D JS 8C
KD 7S 3C 7C AS QH TS AD 8C 2S
QS 8H 6C JS 4C 9S QC AD TD TS
2H 7C TS TC 8C 3C 9H 2D 6D JC
TC 2H 8D JH KS 6D 3H TD TH 8H
9D TD 9H QC 5D 6C 8H 8C KC TS
2H 8C 3D AH 4D TH TC 7D 8H KC
TS 5C 2D 8C 6S KH AH 5H 6H KC
5S 5D AH TC 4C JD 8D 6H 8C 6C
KC QD 3D 8H 2D JC 9H 4H AD 2S

TD 6S 7D JS KD 4H QS 2S 3S 8C
4C 9H JH TS 3S 4H QC 5S 9S 9C
2C KD 9H JS 9S 3H JC TS 5D AC
AS 2H 5D AD 5H JC 7S TD JS 4C
2D 4S 8H 3D 7D 2C AD KD 9C TS
7H QD JH 5H JS AC 3D TH 4C 8H
6D KH KC QD 5C AD 7C 2D 4H AC
3D 9D TC 8S QD 2C JC 4H JD AH
6C TD 5S TC 8S AH 2C 5D AS AC
TH 7S 3D AS 6C 4C 7H 7D 4H AH
5C 2H KS 6H 7S 4H 5H 3D 3C 7H
3C 9S AC 7S QH 2H 3D 6S 3S 3H
2D 3H AS 2C 6H TC JS 6S 9C 6C
QH KD QD 6D AC 6H KH 2C TS 8C
8H 7D 3S 9H 5D 3H 4S QC 9S 5H
2D 9D 7H 6H 3C 8S 5H 4D 3S 4S
KD 9S 4S TC 7S QC 3S 8S 2H 7H
TC 3D 8C 3H 6C 2H 6H KS KD 4D
KC 3D 9S 3H JS 4S 8H 2D 6C 8S
6H QS 6C TC QD 9H 7D 7C 5H 4D
TD 9D 8D 6S 6C TC 5D TS JS 8H
4H KC JD 9H TC 2C 6S 5H 8H AS
JS 9C 5C 6S 9D JD 8H KC 4C 6D
4D 8D 8S 6C 7C 6H 7H 8H 5C KC
TC 3D JC 6D KS 9S 6H 7S 9C 2C
6C 3S KD 5H TS 7D 9H 9S 6H KH
3D QD 4C 6H TS AC 3S 5C 2H KD
4C AS JS 9S TC TS 7H 9H JC KS
4H 8C JD 3H 6H AD 9S 4S 5S KS
4C 2C 7D 3D AS 9C 2S QS KC 6C
8S 5H 3D 2S AC 9D 6S 3S 4D TD
QD TH 7S TS 3D AC 7H 6C 5D QC
TC QD AD 9C QS 5C 8D KD 3D 3C
9D 8H AS 3S 7C 8S JD 2D 8D KC
4C TH AC QH JS 8D 7D 7S 9C KH
9D 8D 4C JH 2C 2S QD KD TS 4H
4D 6D 5D 2D JH 3S 8S 3H TC KH
AD 4D 2C QS 8C KD JH JD AH 5C
5C 6C 5H 2H JH 4H KS 7C TC 3H
3C 4C QC 5D JH 9C QD KH 8D TC
3H 9C JS 7H QH AS 7C 9H 5H JC
2D 5S QD 4S 3C KC 6S 6C 5C 4C
5D KH 2D TS 8S 9C AS 9S 7C 4C
7C AH 8C 8D 5S KD QH QS JH 2C
8C 9D AH 2H AC QC 5S 8H 7H 2C
QD 9H 5S QS QC 9C 5H TC 7H 4H
6C 6S 3H 5H 3S 6H KS 8D AC 7S
AC QH 7H 8C 4S KC 6C 3D 3S TC
9D 3D JS TH AC 5H 3H 8S 3S TC
QD KH JS KS 9S QC 8D AH 3C AC
5H 6C KH 3S 9S JH 2D QD AS 8C
6C 4D 7S 7H 5S JC 6S 9H 4H JH
AH 5S 6H 9S AD 3S TH 2H 9D 8C
4C 8D 9H 7C QC AD 4S 9C KC 5S
9D 6H 4D TC 4C JH 2S 5D 3S AS
2H 6C 7C KH 5C AD QS TH JD 8S
3S 4S 7S AH AS KC JS 2S AD TH
JS KC 2S 7D 8C 5C 9C TS 5H 9D
7S 9S 4D TD JH JS KH 6H 5D 2C
JD JS JC TH 2D 3D QD 8C AC 5H
7S KH 5S 9D 5D TD 4S 6H 3C 2D
4S 5D AC 8D 4D 7C AD AS AH 9C
6S TH TS KS 2C QC AH AS 3C 4S
2H 8C 3S JC 5C 7C 3H 3C KH JH
7S 3H JC 5S 6H 4C 2S 4D KC 7H
4D 7C 4H 9S 8S 6S AD TC 6C JC
KH QS 3S TC 4C 8H 8S AC 3C TS
QD QS TH 3C TS 7H 7D AH TD JC
TD JD QC 4D 9S 7S TS AD 7D AC
AH 7H 4S 6D 7C 2H 9D KS JC TD
7C AH JD 4H 6D QS TS 2H 2C 5C
TC KC 8C 9S 4C JS 3C JC 6S AH

AS 7D QC 3D 5S JC JD 9D TD KH
TH 3C 2S 6H AH AC 5H 5C 7S 8H
QC 2D AC QD 2S 3S JD QS 6S 8H
KC 4H 3C 9D JS 6H 3S 8S AS 8C
7H KC 7D JD 2H JC QH 5S 3H QS
9H TD 3S 8H 7S AC 5C 6C AH 7C
8D 9H AH JD TD QS 7D 3S 9C 8S
AH QH 3C JD KC 4S 5S 5D TD KS
9H 7H 6S JH TH 4C 7C AD 5C 2D
7C KD 5S TC 9D 6S 6C 5D 2S TH
KC 9H 8D 5H 7H 4H QC 3D 7C AS
6S 8S QC TD 4S 5C TH QS QD 2S
8S 5H TH QC 9H 6S KC 7D 7C 5C
7H KD AH 4D KH 5C 4S 2D KC QH
6S 2C TD JC AS 4D 6C 8C 4H 5S
JC TC JD 5S 6S 8D AS 9D AD 3S
6D 6H 5D 5S TC 3D 7D QS 9D QD
4S 6C 8S 3S 7S AD KS 2D 7D 7C
KC QH JC AC QD 5D 8D QS 7H 7D
JS AH 8S 5H 3D TD 3H 4S 6C JH
4S QS 7D AS 9H JS KS 6D TC 5C
2D 5C 6H TC 4D QH 3D 9H 8S 6C
6D 7H TC TH 5S JD 5C 9C KS KD
8D TD QH 6S 4S 6C 8S KC 5C TC
5S 3D KS AC 4S 7D QD 4C TH 2S
TS 8H 9S 6S 7S QH 3C AH 7H 8C
4C 8C TS JS QC 3D 7D 5D 7S JH
8S 7S 9D QC AC 7C 6D 2H JH KC
JS KD 3C 6S 4S 7C AH QC KS 5H
KS 6S 4H JD QS TC 8H KC 6H AS
KH 7C TC 6S TD JC 5C 7D AH 3S
3H 4C 4H TC TH 6S 7H 6D 9C QH
7D 5H 4S 8C JS 4D 3D 8S QH KC
3H 6S AD 7H 3S QC 8S 4S 7S JS
3S JD KH TH 6H QS 9C 6C 2D QD
4S QH 4D 5H KC 7D 6D 8D TH 5S
TD AD 6S 7H KD KH 9H 5S KC JC
3H QC AS TS 4S QD KS 9C 7S KC
TS 6S QC 6C TH TC 9D 5C 5D KD
JS 3S 4H KD 4C QD 6D 9S JC 9D
8S JS 6D 4H JH 6H 6S 6C KS KH
AC 7D 5D TC 9S KH 6S QD 6H AS
AS 7H 6D QH 8D TH 2S KH 5C 5H
4C 7C 3D QC TC 4S KH 8C 2D JS
6H 5D 7S 5H 9C 9H JH 8S TH 7H
AS JS 2S QD KH 8H 4S AC 8D 8S
3H 4C TD KD 8C JC 5C QS 2D JD
TS 7D 5D 6C 2C QS 2H 3C AH KS
4S 7C 9C 7D JH 6C 5C 8H 9D QD
2S TD 7S 6D 9C 9S QS KH QH 5C
JC 6S 9C QH JH 8D 7S JS KH 2H
8D 5H TH KC 4D 4S 3S 6S 3D QS
2D JD 4C TD 7C 6D TH 7S JC AH
QS 7S 4C TH 9D TS AD 4D 3H 6H
2D 3H 7D JD 3D AS 2S 9C QC 8S
4H 9H 9C 2C 7S JH KD 5C 5D 6H
TC 9H 8H JC 3C 9S 8D KS AD KC
TS 5H JD QS QH QC 8D 5D KH AH
5D AS 8S 6S 4C AH QC QD TH 7H
3H 4H 7D 6S 4S 9H AS 8H JS 9D
JD 8C 2C 9D 7D 5H 5S 9S JC KD
KD 9C 4S QD AH 7C AD 9D AC TD
6S 4H 4S 9C 8D KS TC 9D JH 7C
5S JC 5H 4S QH AC 2C JS 2S 9S
8C 5H AS QD AD 5C 7D 8S QC TD
JC 4C 8D 5C KH QS 4D 6H 2H 2C
TH 4S 2D KC 3H QD AC 7H AD 9D
KH QD AS 8H TH KC 8D 7S QH 8C
JC 6C 7D 8C KH AD QS 2H 6S 2D
JC KH 2D 7D JS QC 5H 4C 5D AD
TS 3S AD 4S TD 2D TH 6S 9H JH
9H 2D QS 2C 4S 3D KH AS AC 9D

KH 6S 8H 4S KD 7D 9D TS QD QC
JH 5H AH KS AS AD JC QC 5S KH
5D 7D 6D KS KD 3D 7C 4D JD 3S
AC JS 8D 5H 9C 3H 4H 4D TS 2C
6H KS KH 9D 7C 2S 6S 8S 2H 3D
6H AC JS 7S 3S TD 8H 3H 4H TH
9H TC QC KC 5C KS 6H 4H AC 8S
TC 7D QH 4S JC TS 6D 6C AC KH
QH 7D 7C JH QS QD TH 3H 5D KS
3D 5S 8D JS 4C 2C KS 7H 9C 4H
5H 8S 4H TD 2C 3S QD QC 3H KC
QC JS KD 9C AD 5S 9D 7D 7H TS
8C JC KH 7C 7S 6C TS 2C QD TH
5S 9D TH 3C 7S QH 8S 9C 2H 5H
5D 9H 6H 2S JS KH 3H 7C 2H 5S
JD 5D 5S 2C TC 2S 6S 6C 3C 8S
4D KH 8H 4H 2D KS 3H 5C 2S 9H
3S 2D TD 7H 8S 6H JD KC 9C 8D
6S QD JH 7C 9H 5H 8S 8H TH TD
QS 7S TD 7D TS JC KD 7C 3C 2C
3C JD 8S 4H 2D 2S TD AS 4D AC
AH KS 6C 4C 4S 7D 8C KD AH 9S
5S 3C 9S 2C QS KD 4D 4S AC 5D
2D TS 2C JS KH QH 5D 8C AS KC
4D 4H 6C TH 8S 7S KH 6H 9S AC
6H 7S 6C QS AH 2S 2H 4H 5D 5H
5H JC QD 2C 2S JD AS QC 6S 7D
6C TC AS KD 8H 9D 2C 7D JH 9S
2H 4C 6C AH 8S TD 3H TH 7C TS
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6D 7H AH 7S QC 6H 2D TD JD AS
JH 5D 7H TC 9S 7D JC AS 5S KH
2H 8C AD TH 6H QD KD 9H 6S 6C
QH KC 9D 4D 3S JS JH 4H 2C 9H
TC 7H KH 4H JC 7D 9S 3H QS 7S
AD 7D JH 6C 7H 4H 3S 3H 4D QH
JD 2H 5C AS 6C QC 4D 3C TC JH
AC JD 3H 6H 4C JC AD 7D 7H 9H
4H TC TS 2C 8C 6S KS 2H JD 9S
4C 3H QS QC 9S 9H 6D KC 9D 9C
5C AD 8C 2C QH TH QD JC 8D 8H
QC 2C 2S QD 9C 4D 3S 8D JH QS
9D 3S 2C 7S 7C JC TD 3C TC 9H
3C TS 8H 5C 4C 2C 6S 8D 7C 4H
KS 7H 2H TC 4H 2C 3S AS AH QS
8C 2D 2H 2C 4S 4C 6S 7D 5S 3S
TH QC 5D TD 3C QS KD KC KS AS
4D AH KD 9H KS 5C 4C 6H JC 7S
KC 4H 5C QS TC 2H JC 9S AH QH
4S 9H 3H 5H 3C QD 2H QC JH 8H
5D AS 7H 2C 3D JH 6H 4C 6S 7D
9C JD 9H AH JS 8S QH 3H KS 8H
3S AC QC TS 4D AD 3D AH 8S 9H
7H 3H QS 9C 9S 5H JH JS AH AC
8D 3C JD 2H AC 9C 7H 5S 4D 8H
7C JH 9H 6C JS 9S 7H 8C 9D 4H
2D AS 9S 6H 4D JS JH 9H AD QD
6H 7S JH KH AH 7H TD 5S 6S 2C
8H JH 6S 5H 5S 9D TC 4C QC 9S
7D 2C KD 3H 5H AS QD 7H JS 4D
TS QH 6C 8H TH 5H 3C 3H 9C 9D
AD KH JS 5D 3H AS AC 9S 5C KC
2C KH 8C JC QS 6D AH 2D KC TC
9D 3H 2S 7C 4D 6D KH KS 8D 7D
9H 2S TC JH AC QC 3H 5S 3S 8H
3S AS KD 8H 4C 3H 7C JH QH TS
7S 6D 7H 9D JH 4C 3D 3S 6C AS
4S 2H 2C 4C 8S 5H KC 8C QC QD
3H 3S 6C QS QC 2D 6S 5D 2C 9D
2H 8D JH 2S 3H 2D 6C 5C 7S AD
9H JS 5D QH 8S TS 2H 7S 6S AD
6D QC 9S 7H 5H 5C 7D KC JD 4H
QC 5S 9H 9C 4D 6S KS 2S 4C 7C
9H 7C 4H 8D 3S 6H 5C 8H JS 7S
2D 6H JS TD 4H 4D JC TH 5H KC
AC 7C 8D TH 3H 9S 2D 4C KC 4D
KD QS 9C 7S 3D KS AD TS 4C 4H
QH 9C 8H 2S 7D KS 7H 5D KD 4C
9C 2S 2H JC 6S 6C TC QC JH 5C
7S AC 8H KC 8S 6H QS JC 3D 6S
JS 2D JH 8C 4S 6H 8H 6D 5D AD
6H 7D 2S 4H 9H 7C AS AC 8H 5S
3C JS 4S 6D 5H 2S QH 6S 9C 2C
3D 5S 6S 9S 4C QS 8D QD 8S TC
9C 3D AH 9H 5S 2C 7D AD JC 3S
7H TC AS 3C 6S 6D 7S KH KC 9H

3S TC 8H 6S 5H JH 8C 7D AC 2S
QD 9D 9C 3S JC 8C KS 8H 5D 4D
JS AH JD 6D 9D 8C 9H 9S 8H 3H
2D 6S 4C 4D 8S AD 4S TC AH 9H
TS AC QC TH KC 6D 4H 7S 8C 2H
3C QD JS 9D 5S JC AH 2H TS 9H
3H 4D QH 5D 9C 5H 7D 4S JC 3S
8S TH 3H 7C 2H JD JS TS AC 8D
9C 2H TD KC JD 2S 8C 5S AD 2C
3D KD 7C 5H 4D QH QD TC 6H 7D
7H 2C KC 5S KD 6H AH QC 7S QH
6H 5C AC 5H 2C 9C 2D 7C TD 2S
4D 9D AH 3D 7C JD 4H 8C 4C KS
TH 3C JS QH 8H 4C AS 3D QS QC
4D 7S 5H JH 6D 7D 6H JS KH 3C
QD 8S 7D 2H 2C 7C JC 2S 5H 8C
QH 8S 9D TC 2H AD 7C 8D QD 6S
3S 7C AD 9H 2H 9S JD TS 4C 2D
3S AS 4H QC 2C 8H 8S 7S TD TC
JH TH TD 3S 4D 4H 5S 5D QS 2C
8C QD QH TC 6D 4S 9S 9D 4H QC
8C JS 9D 6H JD 3H AD 6S TD QC
KC 8S 3D 7C TD 7D 8D 9H 4S 3S
6C 4S 3D 9D KD TC KC KS AC 5S
7C 6S QH 3D JS KD 6H 6D 2D 8C
JD 2S 5S 4H 8S AC 2D 6S TS 5C
5H 8C 5S 3C 4S 3D 7C 8D AS 3H
AS TS 7C 3H AD 7D JC QS 6C 6H
3S 9S 4C AC QH 5H 5D 9H TS 4H
6C 5C 7H 7S TD AD JD 5S 2H 2S
7D 6C KC 3S JD 8D 8S TS QS KH
8S QS 8D 6C TH AC AH 2C 8H 9S
7H TD KH QH 8S 3D 4D AH JD AS
TS 3D 2H JC 2S JH KH 6C QC JS
KC TH 2D 6H 7S 2S TC 8C 9D QS
3C 9D 6S KH 8H 6D 5D TH 2C 2H
6H TC 7D AD 4D 8S TS 9H TD 7S
JS 6D JD JC 2H AC 6C 3D KH 8D
KH JD 9S 5D 4H 4C 3H 7S QS 5C
4H JD 5D 3S 3C 4D KH QH QS 7S
JD TS 8S QD AH 4C 6H 3S 5S 2C
QS 3D JD AS 8D TH 7C 6S QC KS
7S 2H 8C QC 7H AC 6D 2D TH KH
5S 6C 7H KH 7D AH 8C 5C 7S 3D
3C KD AD 7D 6C 4D KS 2D 8C 4S
7C 8D 5S 2D 2S AH AD 2C 9D TD
3C AD 4S KS JH 7C 5C 8C 9C TH
AS TD 4D 7C JD 8C QH 3C 5H 9S
3H 9C 8S 9S 6S QD KS AH 5H JH
QC 9C 5S 4H 2H TD 7D AS 8C 9D
8C 2C 9D KD TC 7S 3D KH QC 3C
4D AS 4C QS 5S 9D 6S JD QH KS
6D AH 6C 4C 5H TS 9H 7D 3D 5S
QS JD 7C 8D 9C AC 3S 6S 6C KH
8H JH 5D 9S 6D AS 6S 3S QC 7H
QD AD 5C JH 2H AH 4H AS KC 2C
JH 9C 2C 6H 2D JS 5D 9H KC 6D
7D 9D KD TH 3H AS 6S QC 6H AD
JD 4H 7D KC 3H JS 3C TH 3D QS
4C 3H 8C QD 5H 6H AS 8H AD JD
TH 8S KD 5D QC 7D JS 5S 5H TS
7D KC 9D QS 3H 3C 6D TS 7S AH
7C 4H 7H AH QC AC 4D 5D 6D TH
3C 4H 2S KD 8H 5H JH TC 6C JD
4S 8C 3D 4H JS TD 7S JH QS KD
7C QC KD 4D 7H 6S AD TD TC KH
5H 9H KC 3H 4D 3D AD 6S QD 6H
TH 7C 6H TS QH 5S 2C KC TD 6S
7C 4D 5S JD JH 7D AC KD KH 4H
7D 6C 8D 8H 5C JH 8S QD TH JD
8D 7D 6C 7C 9D KD AS 5C QH JH
9S 2C 8C 3C 4C KS JH 2D 8D 4H

7S 6C JH KH 8H 3H 9D 2D AH 6D
4D TC 9C 8D 7H TD KS TH KD 3C
JD 9H 8D QD AS KD 9D 2C 2S 9C
8D 3H 5C 7H KS 5H QH 2D 8C 9H
2D TH 6D QD 6C KC 3H 3S AD 4C
4H 3H JS 9D 3C TC 5H QH QC JC
3D 5C 6H 3S 3C JC 5S 7S 2S QH
AC 5C 8C 4D 5D 4H 2S QD 3C 3H
2C TD AH 9C KD JS 6S QD 4C QC
QS 8C 3S 4H TC JS 3H 7C JC AD
5H 4D 9C KS JC TD 9S TS 8S 9H
QD TS 7D AS AC 2C TD 6H 8H AH
6S AD 8C 4S 9H 8D 9D KH 8S 3C
QS 4D 2D 7S KH JS JC AD 4C 3C
QS 9S 7H KC TD TH 5H JS AC JH
6D AC 2S QS 7C AS KS 6S KH 5S
6D 8H KH 3C QS 2H 5C 9C 9D 6C
JS 2C 4C 6H 7D JC AC QD TD 3H
4H QC 8H JD 4C KD KS 5C KC 7S
6D 2D 3H 2S QD 5S 7H AS TH 6S
AS 6D 8D 2C 8S TD 8H QD JC AH
9C 9H 2D TD QH 2H 5C TC 3D 8H
KC 8S 3D KH 2S TS TC 6S 4D JH
9H 9D QS AC KC 6H 5D 4D 8D AH
9S 5C QS 4H 7C 7D 2H 8S AD JS
3D AC 9S AS 2C 2D 2H 3H JC KH
7H QH KH JD TC KS 5S 8H 4C 8D
2H 7H 3S 2S 5H QS 3C AS 9H KD
AD 3D JD 6H 5S 9C 6D AC 9S 3S
3D 5D 9C 2D AC 4S 2S AD 6C 6S
QC 4C 2D 3H 6S KC QH QD 2H JH
QC 3C 8S 4D 9S 2H 5C 8H QS QD
6D KD 6S 7H 3S KH 2H 5C JC 6C
3S 9S TC 6S 8H 2D AD 7S 8S TS
3C 6H 9C 3H 5C JC 8H QH TD QD
3C JS QD 5D TD 2C KH 9H TH AS
9S TC JD 3D 5C 5H AD QH 9H KC
TC 7H 4H 8H 3H TD 6S AC 7C 2S
QS 9D 5D 3C JC KS 4D 6C JH 2S
9S 6S 3C 7H TS 4C KD 6D 3D 9C
2D 9H AH AC 7H 2S JH 3S 7C QC
QD 9H 3C 2H AC AS 8S KD 8C KH
2D 7S TD TH 6D JD 8D 4D 2H 5S
8S QH KD JD QS JH 4D KC 5H 3S
3C KH QC 6D 8H 3S AH 7D TD 2D
5S 9H QH 4S 6S 6C 6D TS TH 7S
6C 4C 6D QS JS 9C TS 3H 8D 8S
JS 5C 7S AS 2C AH 2H AD 5S TC
KD 6C 9C 9D TS 2S JC 4H 2C QD
QS 9H TC 3H KC KS 4H 3C AD TH
KH 9C 2H KD 9D TC 7S KC JH 2D
7C 3S KC AS 8C 5D 9C 9S QH 3H
2D 8C TD 4C 2H QC 5D TC 2C 7D
KS 4D 6C QH TD KH 5D 7C AD 8D
2S 9S 8S 4C 8C 3D 6H QD 7C 7H
6C 8S QH 5H TS 5C 3C 4S 2S 2H
8S 6S 2H JC 3S 3H 9D 8C 2S 7H
QC 2C 8H 9C AC JD 4C 4H 6S 3S
3H 3S 7D 4C 9S 5H 8H JC 3D TC
QH 2S 2D 9S KD QD 9H AD 6D 9C
8D 2D KS 9S JC 4C JD KC 4S TH
KH TS 6D 4D 5C KD 5H AS 9H AD
QD JS 7C 6D 5D 5C TH 5H QH QS
9D QH KH 5H JH 4C 4D TC TH 6C
KH AS TS 9D KD 9C 7S 4D 8H 5S
KH AS 2S 7D 9D 4C TS TH AH 7C
KS 4D AC 8S 9S 8D TH QH 9D 5C
5D 5C 8C QS TC 4C 3D 3S 2C 8D
9D KS 2D 3C KC 4S 8C KH 6C JC
8H AH 6H 7D 7S QD 3C 4C 6C KC
3H 2C QH 8H AS 7D 4C 8C 4H KC
QD 5S 4H 2C TD AH JH QH 4C 8S

3H QS 5S JS 8H 2S 9H 9C 3S 2C
6H TS 7S JC QD AC TD KC 5S 3H
QH AS QS 7D JC KC 2C 4C 5C 5S
QH 3D AS JS 4H 8D 7H JC 2S 9C
5D 4D 2S 4S 9D 9C 2D QS 8H 7H
6D 7H 3H JS TS AC 2D JH 7C 8S
JH 5H KC 3C TC 5S 9H 4C 8H 9D
8S KC 5H 9H AD KS 9D KH 8D AH
JC 2H 9H KS 6S 3H QC 5H AH 9C
5C KH 5S AD 6C JC 9H QC 9C TD
5S 5D JC QH 2D KS 8H QS 2H TS
JH 5H 5S AH 7H 3C 8S AS TD KH
6H 3D JD 2C 4C KC 7S AH 6C JH
4C KS 9D AD 7S KC 7D 8H 3S 9C
7H 5C 5H 3C 8H QC 3D KH 6D JC
2D 4H 5D 7D QC AD AH 9H QH 8H
KD 8C JS 9D 3S 3C 2H 5D 6D 2S
8S 6S TS 3C 6H 8D 5S 3H TD 6C
KS 3D JH 9C 7C 9S QS 5S 4H 6H
7S 6S TH 4S KC KD 3S JC JH KS
7C 3C 2S 6D QH 2C 7S 5H 8H AH
KC 8D QD 6D KH 5C 7H 9D 3D 9C
6H 2D 8S JS 9S 2S 6D KC 7C TC
KD 9C JH 7H KC 8S 2S 7S 3D 6H
4H 9H 2D 4C 8H 7H 5S 8S 2H 8D
AD 7C 3C 7S 5S 4D 9H 3D JC KH
5D AS 7D 6D 9C JC 4C QH QS KH
KD JD 7D 3D QS QC 8S 6D JS QD
6S 8C 5S QH TH 9H AS AC 2C JD
QC KS QH 7S 3C 4C 5C KC 5D AH
6C 4H 9D AH 2C 3H KD 3D TS 5C
TD 8S QS AS JS 3H KD AC 4H KS
7D 5D TS 9H 4H 4C 9C 2H 8C QC
2C 7D 9H 4D KS 4C QH AD KD JS
QD AD AH KH 9D JS 9H JC KD JD
8S 3C 4S TS 7S 4D 5C 2S 6H 7C
JS 7S 5C KD 6D QH 8S TD 2H 6S
QH 6C TC 6H TD 4C 9D 2H QC 8H
3D TS 4D 2H 6H 6S 2C 7H 8S 6C
9H 9D JD JH 3S AH 2C 6S 3H 8S
2C QS 8C 5S 3H 2S 7D 3C AD 4S
5C QC QH AS TS 4S 6S 4C 5H JS
JH 5C TD 4C 6H JS KD KH QS 4H
TC KH JC 4D 9H 9D 8D KC 3C 8H
2H TC 8S AD 9S 4H TS 7H 2C 5C
4H 2S 6C 5S KS AH 9C 7C 8H KD
TS QH TD QS 3C JH AH 2C 8D 7D
5D KC 3H 5S AC 4S 7H QS 4C 2H
3D 7D QC KH JH 6D 6C TD TH KD
5S 8D TH 6C 9D 7D KH 8C 9S 6D
JD QS 7S QC 2S QH JC 4S KS 8D
7S 5S 9S JD KD 9C JC AD 2D 7C
4S 5H AH JH 9C 5D TD 7C 2D 6S
KC 6C 7H 6S 9C QD 5S 4H KS TD
6S 8D KS 2D TH TD 9H JD TS 3S
KH JS 4H 5D 9D TC TD QC JD TS
QS QD AC AD 4C 6S 2D AS 3H KC
4C 7C 3C TD QS 9C KC AS 8D AD
KC 7H QC 6D 8H 6S 5S AH 7S 8C
3S AD 9H JC 6D JD AS KH 6S JH
AD 3D TS KS 7H JH 2D JS QD AC
9C JD 7C 6D TC 6H 6C JC 3D 3S
QC KC 3S JC KD 2C 8D AH QS TS
AS KD 3D JD 8H 7C 8C 5C QD 6C

Problem 55

If we take 47, reverse and add, 47 + 74 = 121, which is palindromic.

Not all numbers produce palindromes so quickly.
For example,

$$\begin{aligned} 349 + 943 &= 1292, \\ 1292 + 2921 &= 4213 \\ 4213 + 3124 &= 7337 \end{aligned}$$

That is, 349 took three iterations to arrive at a palindrome.

Although no one has proved it yet, it is thought that some numbers, like 196, never produce a palindrome. A number that never forms a palindrome through the reverse and add process is called a *Lychrel number*. Due to the theoretical nature of these numbers, and for the purpose of this problem, we shall assume that a number is Lychrel until proven otherwise. In addition you are given that for every number below ten-thousand, it will either (i) become a palindrome in less than fifty iterations, or, (ii) no one, with all the computing power that exists, has managed so far to map it to a palindrome. In fact, 10677 is the first number to be shown to require over fifty iterations before producing a palindrome: 4668731596684224866951378664 (53 iterations, 28-digits).

Surprisingly, there are palindromic numbers that are themselves Lychrel numbers; the first example is 4994.

How many Lychrel numbers are there below ten-thousand?

NOTE: Wording was modified slightly on 24 April 2007 to emphasise the theoretical nature of Lychrel numbers.

Problem 55

Nehmen wir 47, drehen wir die Zahl um und addieren, dann erhalten wir 47 + 74 = 121, die palindromisch ist.

Nicht alle Zahlen ergeben so schnell Palindrome.
Zum Beispiel:

$$\begin{aligned} 349 + 943 &= 1292, \\ 1292 + 2921 &= 4213 \\ 4213 + 3124 &= 7337 \end{aligned}$$

So benötigt 349 drei Iterationen, um Palindrom zu werden.

Obwohl niemand es bisher beweisen konnte, glaubt man, dass einige Zahlen nie ein Palindrom ergeben. Eine Zahl, die auch nach wiederholtem Umdrehen und Addieren nicht palindromisch wird, nennt man *Lychrel-Zahl*. Da Lychrel-Zahlen bisher nur theoretisch existieren, nehmen wir für dieses Problem an, dass eine Zahl eine Lychrel-Zahl ist, bis das Gegenteil bewiesen ist. Weiterhin steht fest, dass jede Zahl unter 10000 (i) entweder in weniger als 50 Schritten ein Palindrom wird oder (ii) dass keine Rechenpower der Welt die Zahl bisher zu einem Palindrom machen konnte. 10577 ist die erste Zahl, die nachweislich mehr als fünfzig Iterationen benötigt, um ein Palindrom zu werden: 4668731596684224866951378664 (53 Schritte, 28 Ziffern).

Überraschenderweise gibt es Palindromzahlen, die selbst Lychrel-Zahlen sind; das erste Beispiel ist 4994.

Wie viele Lychrel-Zahlen gibt es unter zehntausend?

Anmerkung: Die Formulierung wurde am 24. April 2007 leicht geändert, um die theoretische Natur von Lychrel-Zahlen zu betonen.

Problem 56

A googol (10^{100}) is a massive number: one followed by one-hundred zeros; 100^{100} is almost unimaginably large: one followed by two-hundred zeros. Despite their size, the sum of the digits in each number is only 1.

Considering natural numbers of the form, a^b, where $a, b < 100$, what is the maximum digital sum?

Problem 56

Ein Googol (10^{100}) ist eine massive Zahl: eine Eins gefolgt von einhundert Nullen; 100^{100} ist schier unvorstellbar groß: eine Eins gefolgt von zweihundert Nullen. Trotz ihrer Größe ist ihre *Quersumme* (die Summe ihrer Ziffern) aber nur 1.

Welche natürliche Zahl der Form a^b, mit $a, b < 100$, hat die größte Quersumme?

Problem 57

It is possible to show that the square root of two can be expressed as an infinite continued fraction.

$$\sqrt{2} = 1 + 1/(2 + 1/(2 + 1/(2 + \dots))) = 1.414213\dots$$

By expanding this for the first four iterations, we get:

$$1 + 1/2 = 3/2 = 1.5$$
$$1 + 1/(2 + 1/2) = 7/5 = 1.4$$
$$1 + 1/(2 + 1/(2 + 1/2)) = 17/12 = 1.41666\dots$$
$$1 + 1/(2 + 1/(2 + 1/(2 + 1/2))) = 41/29 = 1.41379\dots$$

The next three expansions are 99/70, 239/169, and 577/408, but the eighth expansion, 1393/985, is the first example where the number of digits in the numerator exceeds the number of digits in the denominator.

In the first one-thousand expansions, how many fractions contain a numerator with more digits than denominator?

Problem 57

Man kann zeigen, dass sich die Quadratwurzel aus zwei als unendlicher Kettenbruch schreiben lässt.

$$\sqrt{2} = 1 + 1/(2 + 1/(2 + 1/(2 + \dots))) = 1.414213\dots$$

Wenn wir vier Iterationen ausmultiplizieren, erhalten wir:

$$1 + 1/2 = 3/2 = 1.5$$
$$1 + 1/(2 + 1/2) = 7/5 = 1.4$$
$$1 + 1/(2 + 1/(2 + 1/2)) = 17/12 = 1.41666\dots$$
$$1 + 1/(2 + 1/(2 + 1/(2 + 1/2))) = 41/29 = 1.41379\dots$$

Die folgenden drei Iterationen ergeben 99/70, 239/169 und 577/408, aber die achte Iteration, 1393/985, ist das erste Beispiel, in dem der Zähler mehr Ziffern hat als der Nenner.

Wie viele der ersten eintausend Iterationen haben mehr Ziffern im Zähler als im Nenner?

Problem 58

Starting with 1 and spiralling anticlockwise in the following way, a square spiral with side length 7 is formed.

37	36	35	34	33	32	**31**
38	**17**	16	15	14	**13**	30
39	18	**5**	4	**3**	12	29
40	19	6	1	2	11	28
41	20	**7**	8	9	10	27
42	21	22	23	24	25	26
43	44	45	46	47	48	49

It is interesting to note that the odd squares lie along the bottom right diagonal, but what is more interesting is that 8 out of the 13 numbers lying along both diagonals are prime; that is, a ratio of 8/13 ≈ 62%.

If one complete new layer is wrapped around the spiral above, a square spiral with side length 9 will be formed. If this process is continued, what is the side length of the square spiral for which the ratio of primes along both diagonals first falls below 10%?

Problem 58

Wenn man mit 1 beginnt und gegen den Uhrzeigersinn spiralförmig nach außen fortschreitet, entsteht eine quadratische Spirale mit Seitenlänge 7.

37	36	35	34	33	32	**31**
38	**17**	16	15	14	**13**	30
39	18	**5**	4	**3**	12	29
40	19	6	1	2	11	28
41	20	**7**	8	9	10	27
42	21	22	23	24	25	26
43	44	45	46	47	48	49

Interessanterweise liegen die ungeraden Quadratzahlen auf der Diagonalen nach rechts unten, aber noch interessanter ist, dass 8 der 13 Zahlen, die auf beiden Diagonalen liegen, prim sind, also ein Anteil von 8:13 ≈ 62%.

Setzt man die obige Spirale mit einer weiteren Wicklung fort, entsteht eine quadratische Spirale mit Seitenlänge 9.
Wenn man diesen Prozess noch weiter fortsetzt, für welche Seitenlänge der quadratischen Spirale sinkt dann der Anteil der Primzahlen, die auf beiden Diagonalen liegen, erstmalig unter 10 %?

Problem 59

Each character on a computer is assigned a unique code and the preferred standard is ASCII (American Standard Code for Information Interchange). For example, uppercase A = 65, asterisk (*) = 42, and lowercase k = 107.

A modern encryption method is to take a text file, convert the bytes to ASCII, then XOR each byte with a given value, taken from a secret key. The advantage with the XOR function is that using the same encryption key on the cipher text, restores the plain text; for example, 65 XOR 42 = 107, then 107 XOR 42 = 65.

For unbreakable encryption, the key is the same length as the plain text message, and the key is made up of random bytes. The user would keep the encrypted message and the encryption key in different locations, and without both "halves", it is impossible to decrypt the message.

Unfortunately, this method is impractical for most users, so the modified method is to use a password as a key. If the password is shorter than the message, which is likely, the key is repeated cyclically throughout the message. The balance for this method is using a sufficiently long password key for security, but short enough to be memorable.

Your task has been made easy, as the encryption key consists of three lower case characters. Using cipher1.txt (extract from this pdf or copy & paste from the follwing page), a file containing the encrypted ASCII codes, and the knowledge that the plain text must contain common English words, decrypt the message and find the sum of the ASCII values in the original text.

Problem 59

Jedem Zeichen auf dem Computer ist ein eindeutiger Code, zugeordnet, der Standard ist der ASCII-Code (American Standard Code for Information Interchange). Darin gilt beispielsweise für das große A = 65, Sternchen (*) = 42 und das kleine k = 107.

Eine moderne Methode zur Verschlüsselung nimmt einen Text, wandelt die Bytes in ASCII-Codes um und verrechnet dann jeden ASCII-Wert durch XOR mit einem gegebenen Wert aus einem geheimen Schlüssel. Die XOR-Funktion hat den Vorteil, dass die Anwendung von XOR mit demselben Schlüsselwert auf den Codetext den Klartext wiederherstellt; beispielsweise gilt: 65 XOR 42 = 107, und dann 107 XOR 42 = 65.

Unknackbar wird die Verschlüsselung, wenn der Schlüssel genau so lang ist wie der Klartext und der Schlüssel dabei aus zufälligen Bytes besteht. Der Benutzer bewahrt dann den Codetext und den Schlüssel an verschiedenen Orten auf, und ohne beide »Hälften« ist die Entschlüsselung der Botschaft unmöglich.

Leider ist diese Methode den meisten Benutzern zu unpraktisch. Eine abgewandelte Methode benutzt ein Passwort als Schlüssel. Wenn das Passwort, was wahrscheinlich ist, kürzer ist als die Botschaft, werden die Schlüsselbytes bei der Verschlüsselung zyklisch wiederverwendet. Dies ist ausreichend sicher, wenn der Schlüssel ausreichend lang ist, aber er muss auch kurz genug sein, damit man ihn sich merken kann.

Deine Aufgabe ist leicht, da der Schlüssel aus nur drei Kleinbuchstaben besteht. Die Datei cipher1.txt (extrahieren aus diesem pdf oder Kopieren & Einfügen von der folgenden Seite) enthält die verschlüsselten ASCII-Codes. Entschlüssele im Wissen, dass der Klartext gewöhnliche englische Wörter enthalten muss, die Botschaft und ermittle die Summe der ASCII-Werte des Klartextes.

79, 59, 12, 2, 79, 35, 8, 28, 20, 2, 3, 68, 8, 9, 68, 45, 0, 12, 9, 67, 68, 4, 7, 5, 23, 27, 1, 21, 79, 85, 78, 79, 85, 71, 38, 10, 71, 27, 12, 2, 79, 6, 2, 8, 13, 9, 1, 13, 9, 8, 68, 19, 7, 1, 71, 56, 11, 21, 11, 68, 6, 3, 22, 2, 14, 0, 30, 79, 1, 31, 6, 23, 19, 10, 0, 73, 79, 44, 2, 79, 19, 6, 28, 68, 16, 6, 16, 15, 79, 35, 8, 11, 72, 71, 14, 10, 3, 79, 12, 2, 79, 19, 6, 28, 68, 32, 0, 0, 73, 79, 86, 71, 39, 1, 71, 24, 5, 20, 79, 13, 9, 79, 16, 15, 10, 68, 5, 10, 3, 14, 1, 10, 14, 1, 3, 71, 24, 13, 19, 7, 68, 32, 0, 0, 73, 79, 87, 71, 39, 1, 71, 12, 22, 2, 14, 16, 2, 11, 68, 2, 25, 1, 21, 22, 16, 15, 6, 10, 0, 79, 16, 15, 10, 22, 2, 79, 13, 20, 65, 68, 41, 0, 16, 15, 6, 10, 0, 79, 1, 31, 6, 23, 19, 28, 68, 19, 7, 5, 19, 79, 12, 2, 79, 0, 14, 11, 10, 64, 27, 68, 10, 14, 15, 2, 65, 68, 83, 79, 40, 14, 9, 1, 71, 6, 16, 20, 10, 8, 1, 79, 19, 6, 28, 68, 14, 1, 68, 15, 6, 9, 75, 79, 5, 9, 11, 68, 19, 7, 13, 20, 79, 8, 14, 9, 1, 71, 8, 13, 17, 10, 23, 71, 3, 13, 0, 7, 16, 71, 27, 11, 71, 10, 18, 2, 29, 29, 8, 1, 1, 73, 79, 81, 71, 59, 12, 2, 79, 8, 14, 8, 12, 19, 79, 23, 15, 6, 10, 2, 28, 68, 19, 7, 22, 8, 26, 3, 15, 79, 16, 15, 10, 68, 3, 14, 22, 12, 1, 1, 20, 28, 72, 71, 14, 10, 3, 79, 16, 15, 10, 68, 3, 14, 22, 12, 1, 1, 20, 28, 68, 4, 14, 10, 71, 1, 1, 17, 10, 22, 71, 10, 28, 19, 6, 10, 0, 26, 13, 20, 7, 68, 14, 27, 74, 71, 89, 68, 32, 0, 0, 71, 28, 1, 9, 27, 68, 45, 0, 12, 9, 79, 16, 15, 10, 68, 37, 14, 20, 19, 6, 23, 19, 79, 83, 71, 27, 11, 71, 27, 1, 11, 3,

68, 2, 25, 1, 21, 22, 11, 9, 10, 68, 6, 13, 11, 18, 27, 68, 19, 7, 1, 71, 3, 13, 0, 7, 16, 71, 28, 11, 71, 27, 12, 6, 27, 68, 2, 25, 1, 21, 22, 11, 9, 10, 68, 10, 6, 3, 15, 27, 68, 5, 10, 8, 14, 10, 18, 2, 79, 6, 2, 12, 5, 18, 28, 1, 71, 0, 2, 71, 7, 13, 20, 79, 16, 2, 28, 16, 14, 2, 11, 9, 22, 74, 71, 87, 68, 45, 0, 12, 9, 79, 12, 14, 2, 23, 2, 3, 2, 71, 24, 5, 20, 79, 10, 8, 27, 68, 19, 7, 1, 71, 3, 13, 0, 7, 16, 92, 79, 12, 2, 79, 19, 6, 28, 68, 8, 1, 8, 30, 79, 5, 71, 24, 13, 19, 1, 1, 20, 28, 68, 19, 0, 68, 19, 7, 1, 71, 3, 13, 0, 7, 16, 73, 79, 93, 71, 59, 12, 2, 79, 11, 9, 10, 68, 16, 7, 11, 71, 6, 23, 71, 27, 12, 2, 79, 16, 21, 26, 1, 71, 3, 13, 0, 7, 16, 75, 79, 19, 15, 0, 68, 0, 6, 18, 2, 28, 68, 11, 6, 3, 15, 27, 68, 19, 0, 68, 2, 25, 1, 21, 22, 11, 9, 10, 72, 71, 24, 5, 20, 79, 3, 8, 6, 10, 0, 79, 16, 8, 79, 7, 8, 2, 1, 71, 6, 10, 19, 0, 68, 19, 7, 1, 71, 24, 11, 21, 3, 0, 73, 79, 85, 87, 79, 38, 18, 27, 68, 6, 3, 16, 15, 0, 17, 0, 7, 68, 19, 7, 1, 71, 24, 11, 21, 3, 0, 71, 24, 5, 20, 79, 9, 6, 11, 1, 71, 27, 12, 21, 0, 17, 0, 7, 68, 15, 6, 9, 75, 79, 16, 15, 10, 68, 16, 0, 22, 11, 11, 68, 3, 6, 0, 9, 72, 16, 71, 29, 1, 4, 0, 3, 9, 6, 30, 2, 79, 12, 14, 2, 68, 16, 7, 1, 9, 79, 12, 2, 79, 7, 6, 2, 1, 73, 79, 85, 86, 79, 33, 17, 10, 10, 71, 6, 10, 71, 7, 13, 20, 79, 11, 16, 1, 68, 11, 14, 10, 3, 79, 5, 9, 11, 68, 6, 2, 11, 9, 8, 68, 15, 6, 23, 71, 0, 19, 9, 79, 20, 2, 0, 20, 11, 10, 72, 71, 7, 1, 71, 24, 5, 20, 79, 10, 8, 27, 68, 6, 12, 7, 2, 31, 16, 2, 11, 74, 71, 94, 86, 71, 45,

17, 19, 79, 16, 8, 79, 5, 11, 3, 68, 16, 7, 11, 71, 13, 1, 11, 6, 1, 17, 10, 0, 71, 7, 13, 10, 79, 5, 9, 11, 68, 6, 12, 7, 2, 31, 16, 2, 11, 68, 15, 6, 9, 75, 79, 12, 2, 79, 3, 6, 25, 1, 71, 27, 12, 2, 79, 22, 14, 8, 12, 19, 79, 16, 8, 79, 6, 2, 12, 11, 10, 10, 68, 4, 7, 13, 11, 11, 22, 2, 1, 68, 8, 9, 68, 32, 0, 0, 73, 79, 85, 84, 79, 48, 15, 10, 29, 71, 14, 22, 2, 79, 22, 2, 13, 11, 21, 1, 69, 71, 59, 12, 14, 28, 68, 14, 28, 68, 9, 0, 16, 71, 14, 68, 23, 7, 29, 20, 6, 7, 6, 3, 68, 5, 6, 22, 19, 7, 68, 21, 10, 23, 18, 3, 16, 14, 1, 3, 71, 9, 22, 8, 2, 68, 15, 26, 9, 6, 1, 68, 23, 14, 23, 20, 6, 11, 9, 79, 11, 21, 79, 20, 11, 14, 10, 75, 79, 16, 15, 6, 23, 71, 29, 1, 5, 6, 22, 19, 7, 68, 4, 0, 9, 2, 28, 68, 1, 29, 11, 10, 79, 35, 8, 11, 74, 86, 91, 68, 52, 0, 68, 19, 7, 1, 71, 56, 11, 21, 11, 68, 5, 10, 7, 6, 2, 1, 71, 7, 17, 10, 14, 10, 71, 14, 10, 3, 79, 8, 14, 25, 1, 3, 79, 12, 2, 29, 1, 71, 0, 10, 71, 10, 5, 21, 27, 12, 71, 14, 9, 8, 1, 3, 71, 26, 23, 73, 79, 44, 2, 79, 19, 6, 28, 68, 1, 26, 8, 11, 79, 11, 1, 79, 17, 9, 9, 5, 14, 3, 13, 9, 8, 68, 11, 0, 18, 2, 79, 5, 9, 11, 68, 1, 14, 13, 19, 7, 2, 18, 3, 10, 2, 28, 23, 73, 79, 37, 9, 11, 68, 16, 10, 68, 15, 14, 18, 2, 79, 23, 2, 10, 10, 71, 7, 13, 20, 79, 3, 11, 0, 22, 30, 67, 68, 19, 7, 1, 71, 8, 8, 8, 29, 29, 71, 0, 2, 71, 27, 12, 2, 79, 11, 9, 3, 29, 71, 60, 11, 9, 79, 11, 1, 79, 16, 15, 10, 68, 33, 14, 16, 15, 10, 22, 73

Problem 60

The primes 3, 7, 109, and 673, are quite remarkable. By taking any two primes and concatenating them in any order the result will always be prime. For example, taking 7 and 109, both 7109 and 1097 are prime. The sum of these four primes, 792, represents the lowest sum for a set of four primes with this property.

Find the lowest sum for a set of five primes for which any two primes concatenate to produce another prime.

Problem 60

Die Primzahlen 3, 7, 109 und 673 sind recht bemerkenswert. Man kann zwei beliebige dieser Primzahlen aneinanderreihen, und das Ergebnis ist ebenfalls stets prim. Nimmt man beispielsweise 7 und 109, sind sowohl 7109 als auch 1097 prim. Die Summe dieser vier Primzahlen, 792, ist die niedrigste Summe für einen Satz von vier Primzahlen mit dieser Eigenschaft.

Ermittle die niedrigste Summe für einen Satz von fünf Primzahlen, von denen je zwei Primzahlen aneinandergereiht eine weitere Primzahl ergeben.

Problem 61

Triangle, square, pentagonal, hexagonal, heptagonal, and octagonal numbers are all figurate (polygonal) numbers and are generated by the following formulae:

Triangle	$P_{3,n}=n(n+1)/2$	1, 3, 6, 10, 15, ...
Square	$P_{4,n}=n^2$	1, 4, 9, 16, 25, ...
Pentagonal	$P_{5,n}=n(3n-1)/2$	1, 5, 12, 22, 35, ...
Hexagonal	$P_{6,n}=n(2n-1)$	1, 6, 15, 28, 45, ...
Heptagonal	$P_{7,n}=n(5n-3)/2$	1, 7, 18, 34, 55, ...
Octagonal	$P_{8,n}=n(3n-2)$	1, 8, 21, 40, 65, ...

The ordered set of three 4-digit numbers: 8128, 2882, 8281, has three interesting properties.

1) The set is cyclic, in that the last two digits of each number is the first two digits of the next number (including the last number with the first).
2) Each polygonal type: triangle (P3,127=8128), square (P4,91=8281), and pentagonal (P5,44=2882), is represented by a different number in the set.
3) This is the only set of 4-digit numbers with this property.

Find the sum of the only ordered set of six cyclic 4-digit numbers for which each polygonal type: triangle, square, pentagonal, hexagonal, heptagonal, and octagonal, is represented by a different number in the set.

Problem 61

Dreiecks-, Quadrat, Pentagonal-, Hexagonal-, Heptagonal- und Oktagonal-Zahlen sind alle figurierte (Polygon-) Zahlen und werden von folgenden Formeln erzeugt:

Dreieck	$P_{3,n}=n(n+1)/2$	1, 3, 6, 10, 15, ...
Quadrat	$P_{4,n}=n^2$	1, 4, 9, 16, 25, ...
Pentagonal	$P_{5,n}=n(3n-1)/2$	1, 5, 12, 22, 35, ...
Hexagonal	$P_{6,n}=n(2n-1)$	1, 6, 15, 28, 45, ...
Heptagonal	$P_{7,n}=n(5n-3)/2$	1, 7, 18, 34, 55, ...
Octagonal	$P_{8,n}=n(3n-2)$	1, 8, 21, 40, 65, ...

Die geordnete Menge der vierstelligen Zahlen 8128, 2882, 8281, hat drei interessante Eigenschaften:

1) Die Menge ist zyklisch, das heißt, die letzten beiden Ziffern jeder Zahl sind zugleich die ersten beiden Ziffern der jeweils folgenden Zahl (einschließlich letzte und erste Zahl).
2) Jeder Polygon-Typ, nämlich Dreieck ($P_{3,127}$=8128), Quadrat ($P_{4,91}$=8281) und Pentagonal ($P_{5,44}$=2882) wird durch eine andere Zahl in der Menge repräsentiert.
3) Diese Menge ist die einzige Menge vierstelliger Zahlen mit dieser Eigenschaft.

Bestimmte die Summe der einzigen geordneten Menge von sechs zyklischen vierstelligen Zahlen, bei denen jeder Polygon-Typ, also Dreieck, Quadrat, Pentagonal, Hexagonal, Heptagonal und Oktagonal, durch eine andere Zahl in der Menge vertreten ist.

Problem 62

The cube, 41063625 (3453), can be permuted to produce two other cubes: 56623104 (3843) and 66430125 (4053). In fact, 41063625 is the smallest cube which has exactly three permutations of its digits which are also cube.

Find the smallest cube for which exactly five permutations of its digits are cube.

Problem 62

Die Kubikzahl 41063625 (345^3) läßt sich permutieren und erzeugt dann zwei weitere Kubikzahlen: 56623104 (384^3) und 66430125 (405^3). Die Zahl 41063625 ist die kleinste Kubikzahl, die genau drei Permutationen hat, die selbst wieder Kubikzahlen sind.

Bestimme die kleinste Kubikzahl, für die genau fünf Permutationen ihrer Ziffern auch wieder Kubikzahlen sind.

Problem 63

The 5-digit number, $16807=7^5$, is also a fifth power. Similarly, the 9-digit number, $134217728=8^9$, is a ninth power.

How many n-digit positive integers exist which are also an n^{th} power?

Problem 63

Die 5-stellige Zahl $16807=7^5$ ist auch eine fünfte Potenz. Entsprechend ist die 9-stellige Zahl $134217728=8^9$ eine neunte Potenz.

Wie viele n-stellige positive ganze Zahlen gibt es, die gleichzeiig eine n-te Potenz sind?

Problem 64

All square roots are periodic when written as continued fractions and can be written in the form:

$$\sqrt{N} = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \dots}}}$$

For example, let us consider √23:

$$\sqrt{23} = 4 + \sqrt{23} - 4 = 4 + \cfrac{1}{\cfrac{1}{\sqrt{23} - 4}} = 4 + \cfrac{1}{1 + \cfrac{\sqrt{23} - 3}{7}}$$

If we continue we would get the following expansion:

$$\sqrt{23} = 4 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{8 + \dots}}}}$$

The process can be summarised as follows:

$$a_0 = 4, \quad \frac{1}{\sqrt{23}-4} = \frac{\sqrt{23}+4}{7} = 1 + \frac{\sqrt{23}-3}{7}$$

$$a_1 = 1, \quad \frac{7}{\sqrt{23}-3} = \frac{7(\sqrt{23}+3)}{14} = 3 + \frac{\sqrt{23}-3}{2}$$

$$a_2 = 3, \quad \frac{2}{\sqrt{23}-3} = \frac{2(\sqrt{23}+3)}{14} = 1 + \frac{\sqrt{23}-4}{7}$$

Problem 64

Alle Quadratwurzeln sind periodisch, wenn man sie als Kettenbruch schreibt, und lassen sich in folgender Form schreiben:

$$\sqrt{N} = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \dots}}}$$

Betrachten wir beispielsweise √23:

$$\sqrt{23} = 4 + \sqrt{23} - 4 = 4 + \cfrac{1}{\cfrac{1}{\sqrt{23} - 4}} = 4 + \cfrac{1}{1 + \cfrac{\sqrt{23} - 3}{7}}$$

Fahren wir damit fort, erhalten wir den folgenden Kettenbruch:

$$\sqrt{23} = 4 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{8 + \dots}}}}$$

Der Prozeß läßt sich folgendermaßen zusammenfassen:

$$a_0 = 4, \quad \frac{1}{\sqrt{23}-4} = \frac{\sqrt{23}+4}{7} = 1 + \frac{\sqrt{23}--3}{7}$$

$$a_1 = 1, \quad \frac{7}{\sqrt{23}-3} = \frac{7(\sqrt{23}+3)}{14} = 3 + \frac{\sqrt{23}-3}{2}$$

$$a_2 = 3, \quad \frac{2}{\sqrt{23}-3} = \frac{2(\sqrt{23}+3)}{14} = 1 + \frac{\sqrt{23}-4}{7}$$

Problem 64, page 2

$$a_3 = 1, \quad \frac{7}{\sqrt{23}-4} = \frac{7(\sqrt{23}+4)}{7} = 8 + \sqrt{23}-4$$

$$a_4 = 8, \quad \frac{1}{\sqrt{23}-4} = \frac{\sqrt{23}+4}{7} = 1 + \frac{\sqrt{23}-3}{7}$$

$$a_5 = 1, \quad \frac{7}{\sqrt{23}-3} = \frac{7(\sqrt{23}+3)}{14} = 3 + \frac{\sqrt{23}-3}{2}$$

$$a_6 = 3, \quad \frac{2}{\sqrt{23}-3} = \frac{2(\sqrt{23}+3)}{14} = 1 + \frac{\sqrt{23}-4}{7}$$

$$a_7 = 1, \quad \frac{7}{\sqrt{23}-4} = \frac{7(\sqrt{23}+4)}{7} = 8 + \sqrt{23}-4$$

It can be seen that the sequence is repeating. For conciseness, we use the notation $\sqrt{23} = [4;(1,3,1,8)]$, to indicate that the block (1,3,1,8) repeats indefinitely.

The first ten continued fraction representations of (irrational) square roots are:

$\sqrt{2}=$ [1;(2)], period=1
$\sqrt{3}=$ [1;(1,2)], period=2
$\sqrt{5}=$ [2;(4)], period=1
$\sqrt{6}=$ [2;(2,4)], period=2
$\sqrt{7}=$ [2;(1,1,1,4)], period=4
$\sqrt{8}=$ [2;(1,4)], period=2
$\sqrt{10}=$[3;(6)], period=1

Problem 64, Seite 2

$$a_3 = 1, \quad \frac{7}{\sqrt{23}-4} = \frac{7(\sqrt{23}+4)}{7} = 8 + \sqrt{23}-4$$

$$a_4 = 8, \quad \frac{1}{\sqrt{23}-4} = \frac{\sqrt{23}+4}{7} = 1 + \frac{\sqrt{23}-3}{7}$$

$$a_5 = 1, \quad \frac{7}{\sqrt{23}-3} = \frac{7(\sqrt{23}+3)}{14} = 3 + \frac{\sqrt{23}-3}{2}$$

$$a_6 = 3, \quad \frac{2}{\sqrt{23}-3} = \frac{2(\sqrt{23}+3)}{14} = 1 + \frac{\sqrt{23}-4}{7}$$

$$a_7 = 1, \quad \frac{7}{\sqrt{23}-4} = \frac{7(\sqrt{23}+4)}{7} = 8 + \sqrt{23}-4$$

Wie man sieht, wiederholt sich diese Folge. Der Kürze halber schreiben wir $\sqrt{23} = [4;(1,3,1,8)]$, um anzuzeigen, daß der Block (1,3,1,8) sich endlos wiederholt.

Die ersten zehn Kettenbruch-Darstellungen (irrationaler) Quadratwurzeln sind:

$\sqrt{2}=$ [1;(2)], Periode=1
$\sqrt{3}=$ [1;(1,2)], Periode=2
$\sqrt{5}=$ [2;(4)], Periode=1
$\sqrt{6}=$ [2;(2,4)], Periode=2
$\sqrt{7}=$ [2;(1,1,1,4)], Periode=4
$\sqrt{8}=$ [2;(1,4)], Periode=2
$\sqrt{10}=$[3;(6)], Periode=1

Problem 64, page 3

√11= [3;(3,6)],	period=2
√12= [3;(2,6)],	period=2
√13= [3;(1,1,1,1,6)],	period=5

Exactly four continued fractions, for N ≤ 13, have an odd period.

How many continued fractions for N ≤ 10000 have an odd period?

Problem 64, Seite 3

√11= [3;(3,6)],	Periode=2
√12= [3;(2,6)],	Periode=2
√13= [3;(1,1,1,1,6)],	Periode=5

Genau vier Kettenbrüche für for N ≤ 13, haben eine ungerade Periodenlänge.

Wie viele Kettenbrüche für N ≤ 10000 haben ungerade Periodenlänge?

Problem 65

The square root of 2 can be written as an infinite continued fraction.

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \dots}}}}$$

The infinite continued fraction can be written, $\sqrt{2} = [1;(2)]$ (2) indicates that 2 repeats ad infinitum. In a similar way, $\sqrt{23} = [4;(1,3,1,8)]$.

It turns out that the sequence of partial values of continued fractions for square roots provide the best rational approximations. Let us consider the convergents for $\sqrt{2}$:

$$1 + \frac{1}{2} = 3/2$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2}} = 7/5$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}} = 17/12$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}} = 41/29$$

Problem 65

Die Quadratwurzel aus 2 läßt sich als Kettenbruch schreiben.

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \dots}}}}$$

Dieser unendliche Kettenbruch läßt sich so schreiben: $\sqrt{2} = [1;(2)]$. Dabei bedeutet (2), daß sich die 2 unendlich oft wiederholt. Ganz ähnlich folgt für $\sqrt{23} = [4;(1,3,1,8)]$.

Wie sich herausstellt, ist die Folge von Teilentwicklungen von Kettenbrüchen für Quadratwurzeln die beste Näherung in Bruchform. Betrachten wir die Näherungen für $\sqrt{2}$:

$$1 + \frac{1}{2} = 3/2$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2}} = 7/5$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}} = 17/12$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}} = 41/29$$

Problem 65, page 2

Hence the sequence of the first ten convergents for √2 are:
1, 3/2, 7/5, 17/12, 41/29, 99/70, 239/169, 577/408, 1393/985, 3363/2378, ...

What is most surprising is that the important mathematical constant,

$$e = [2; \quad 1,2,1, \quad 1,4,1, \quad 1,6,1 , \ldots , \quad 1,2k,1, \quad \ldots].$$

The first ten terms in the sequence of convergents for ***e*** are:
2, 3, 8/3, 11/4, 19/7, 87/32, 106/39, 193/71, 1264/465, 1457/536, ...

The sum of digits in the numerator of the 10th convergent is 1+4+5+7=17.

Find the sum of digits in the numerator of the 100th convergent of the continued fraction for ***e***.

Problem 65, Seite 2

Damit ist die Folge der ersten zehn Näherungen für √2:
1, 3/2, 7/5, 17/12, 41/29, 99/70, 239/169, 577/408, 1393/985, 3363/2378, ...

Dabei überrascht besonders, daß sich die wichtige mathematische Konstante ***e*** folgendermaßen schreiben läßt:

$$e = [2; \quad 1,2,1, \quad 1,4,1, \quad 1,6,1 , \ldots , \quad 1,2k,1, \quad \ldots].$$

Die ersten zehn Terme in der Näherungsfolge für ***e*** sind:
2, 3, 8/3, 11/4, 19/7, 87/32, 106/39, 193/71, 1264/465, 1457/536, ...

Die Summe der Ziffern im Zähler der 10ten Näherung ist 1+4+5+7=17.

Bestimme die Summe der Ziffern im Zähler der 100sten Kettenbruch-Näherung für ***e***.

Problem 66

Consider quadratic Diophantine equations of the form:

$$x^2 - Dy^2 = 1$$

For example, when D=13, the minimal solution in x is

$$649^2 - 13\times180^2 = 1.$$

It can be assumed that there are no solutions in positive integers when D is square.

By finding minimal solutions in x for D = {2, 3, 5, 6, 7}, we obtain the following:

$$3^2 - 2\times2^2 = 1$$
$$2^2 - 3\times1^2 = 1$$
$$9^2 - 5\times4^2 = 1$$
$$5^2 - 6\times2^2 = 1$$
$$8^2 - 7\times3^2 = 1$$

Hence, by considering minimal solutions in x for D ≤ 7, the largest x is obtained when D=5.

Find the value of D ≤ 1000 in minimal solutions of x for which the largest value of x is obtained.

Problem 66

Betrachten wir quadratische diophantische Gleichungen der Form:

$$x^2 - Dy^2 = 1$$

Für D=13 zum Beispiel ist die minimale Lösung in x:

$$649^2 - 13\times180^2 = 1.$$

Wir nehmen an, daß es keine positiven ganzzahligen Lösungen gibt, wenn D eine Quadratzahl ist.

Indem wir minimale Lösungen in x für D = {2, 3, 5, 6, 7} ermitteln, erhalten wir folgendes:

$$3^2 - 2\times2^2 = 1$$
$$2^2 - 3\times1^2 = 1$$
$$9^2 - 5\times4^2 = 1$$
$$5^2 - 6\times2^2 = 1$$
$$8^2 - 7\times3^2 = 1$$

Wenn man also minimale Lösungen in x für D ≤ 7 betrachtet, erhält man das größte x für D=5.

Ermittle den Wert von D ≤ 1000, in deren minimaler Lösung sich der größte Wert für x ergibt.

Problem 67

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

3
7 4
2 **4** 6
8 5 **9** 3

That is, 3 + 7 + 4 + 9 = 23.

Find the maximum total from top to bottom in triangle.txt (right click and 'Save Link/Target As...'), a 15K text file containing a triangle with one-hundred rows.

NOTE: This is a much more difficult version of Problem 18. It is not possible to try every route to solve this problem, as there are 2^{99} altogether! If you could check one trillion (1012) routes every second it would take over twenty billion years to check them all. There is an efficient algorithm to solve it. ;o)

Problem 67

Beginnt man an der Spitze des untenstehenden Dreiecks und geht in den darunterliegenden Zeilen jeweils zu einer benachbarten Zahl weiter, ist die maximale Summe von oben nach unten 23.

3
7 4
2 **4** 6
8 5 **9** 3

Das heißt: 3 + 7 + 4 + 9 = 23.

Ermittle die Maximalsumme von oben nach unten des Dreiecks auf Seite 68 (dort liegt das Dreieck gekippt, so daß die Spitze links ist). Dort steht ein Dreieck mit einhundert Zeilen.

Anmerkung: Dies ist eine viel schwierigere Version von Problem 18. Man kann nicht alle möglichen Wege durch das Dreieck durchprobieren, da es insgesamt 2^{99} davon gibt! Könnte man eine Billion Wege pro Sekunde überprüfen, würde die Rechnung über 20 Milliarden Jahre dauern. Es gibt aber einen effizienten Lösungsalgorithmus. ;o)

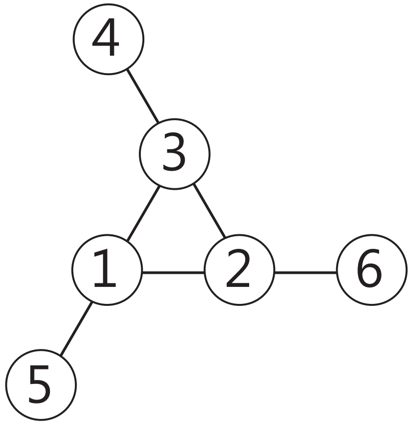
Triangle for Problem 67
Dreieck für Problem 67

```
59
73 41
52 40 09
26 53 06 34
10 51 87 86 81
61 95 66 57 25 68
90 81 80 38 92 67 73
30 28 51 76 81 18 75 44
84 14 95 87 62 81 17 78 58
21 46 71 58 02 79 62 39 31 09
56 34 35 53 78 31 81 18 90 93 15
78 53 04 21 84 93 32 13 97 11 37 51
45 03 81 79 05 18 78 86 13 30 63 99 95
39 87 96 28 03 38 42 17 82 87 58 07 22 57
06 17 51 17 07 93 09 07 75 97 95 78 87 08 53
67 66 59 60 88 99 94 65 55 77 55 34 27 53 78 28
```

[illegible]

Problem 68

Consider the following »magic« 3-gon ring, filled with the numbers 1 to 6, and each line adding to nine:



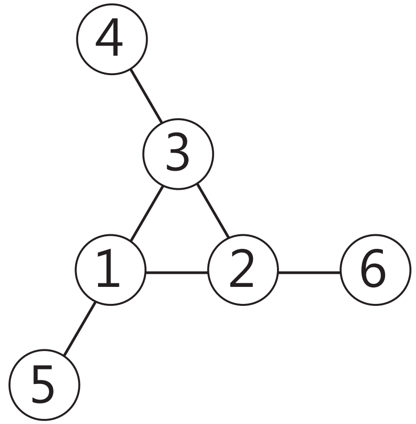
Working **clockwise**, and starting from the group of three with the numerically lowest external node (4,3,2 in this example), each solution can be described uniquely. For example, the above solution can be described by the set: 4,3,2; 6,2,1; 5,1,3.

It is possible to complete the ring with four different totals: 9, 10, 11, and 12. There are eight solutions in total.

Total	**Solution Set**
9	4,2,3; 5,3,1; 6,1,2
9	4,3,2; 6,2,1; 5,1,3
10	2,3,5; 4,5,1; 6,1,3
10	2,5,3; 6,3,1; 4,1,5
11	1,4,6; 3,6,2; 5,2,4
11	1,6,4; 5,4,2; 3,2,6
12	1,5,6; 2,6,4; 3,4,5
12	1,6,5; 3,5,4; 2,4,6

Problem 68

Betrachte den folgenden »magischen« 3-Gon-Ring, mit den Zahlen von 1 bis 6, wobei jede Linie zusammengezählt 9 ergibt:



Wenn man **im Uhrzeigersinn** mit der Dreiergruppe mit dem zahlenmäßig kleinsten Außenknoten beginnt (4,3,2 in diesem Beispiel), kann man jede Lösung eindeutig beschreiben, die Lösung oben etwa ist durch die Menge 4,3,2; 6,2,1; 5,1,3 eindeutig beschrieben.

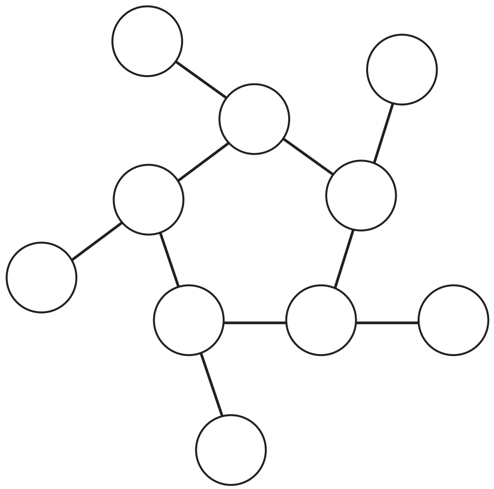
Man kann den Ring nun mit vier verschiedenen Gesamtsummen ausfüllen: 9, 10, 11, and 12. Insgesamt gibt es acht Lösungen.

Summe	**Lösungsmenge**
9	4,2,3; 5,3,1; 6,1,2
9	4,3,2; 6,2,1; 5,1,3
10	2,3,5; 4,5,1; 6,1,3
10	2,5,3; 6,3,1; 4,1,5
11	1,4,6; 3,6,2; 5,2,4
11	1,6,4; 5,4,2; 3,2,6
12	1,5,6; 2,6,4; 3,4,5
12	1,6,5; 3,5,4; 2,4,6

Problem 68, page 2

By concatenating each group it is possible to form 9-digit strings; the maximum string for a 3-gon ring is 432621513.

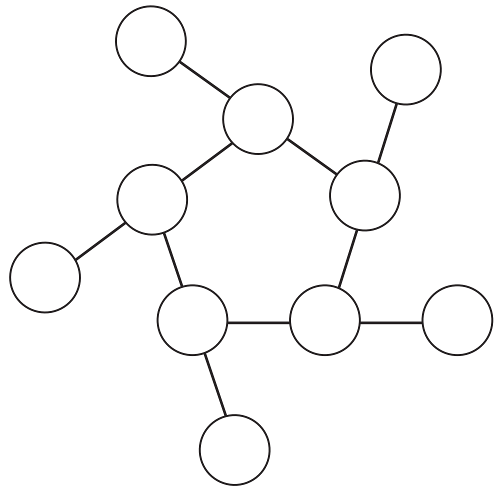
Using the numbers 1 to 10, and depending on arrangements, it is possible to form 16- and 17-digit strings. What is the maximum 16-digit string for a »magic« 5-gon ring?



Problem 68, Seite 2

Wenn man die Gruppen in den Lösungen aneinanderhängt, ergeben sich 9-stellige Ziffernketten. Die höchste Ziffernkette für einen 3-Gon-Ring ist 432621513.

Mit den Zahlen von 1 bis 10, und je nach Anordnung, lassen sich 16- und 17-stellige Ziffernketten erzeugen. Welches ist die maximale 16-stellige Ziffernkette für einen »magischen« 5-Gon-Ring?



Problem 69

Euler's Totient function, φ(n) [sometimes called the phi function], is used to determine the number of numbers less than n which are relatively prime to n. For example, as 1, 2, 4, 5, 7, and 8, are all less than nine and relatively prime to nine, φ(9)=6.

n	Relatively Prime	φ(n)	n/φ(n)
2	1	1	2
3	1,2	2	1.5
4	1,3	2	2
5	1,2,3,4	4	1.25
6	1,5	2	3
7	1,2,3,4,5,6	6	1.1666...
8	1,3,5,7	4	2
9	1,2,4,5,7,8	6	1.5
10	1,3,7,9	4	2.5

It can be seen that n=6 produces a maximum n/φ(n) for n ≤ 10.

Find the value of n ≤ 1,000,000 for which n/φ(n) is a maximum.

Problem 69

Eulers Totient-Funktion, φ(n) [auch Phi-Funktion genannt], benutzt man, um herauszufinden, wie viele Zahlen kleiner als n relativ prim zu n sind. Beispielsweise sind die sechs Zahlen 1, 2, 4, 5, 7 und 8 alle kleiner als neun und relativ prim zu neun; also ist φ(9)=6.

n	Relatively Prime	φ(n)	n/φ(n)
2	1	1	2
3	1,2	2	1.5
4	1,3	2	2
5	1,2,3,4	4	1.25
6	1,5	2	3
7	1,2,3,4,5,6	6	1.1666...
8	1,3,5,7	4	2
9	1,2,4,5,7,8	6	1.5
10	1,3,7,9	4	2.5

Man sieht, daß für für n ≤ 10 genau n=6 den maximalen Quotienten n/φ(n) ergibt.

Bestimme den Wert für n ≤ 1.000.000, für den n/φ(n) ein Maximum wird.

Problem 70

Euler's Totient function, $\varphi(n)$ [sometimes called the phi function], is used to determine the number of positive numbers less than or equal to n which are relatively prime to n. For example, as 1, 2, 4, 5, 7, and 8, are all less than nine and relatively prime to nine, $\varphi(9)=6$.

The number 1 is considered to be relatively prime to every positive number, so $\varphi(1)=1$.

Interestingly, $\varphi(87109)=79180$, and it can be seen that 87109 is a permutation of 79180.

Find the value of n, $1 < n < 107$, for which $\varphi(n)$ is a permutation of n and the ratio $n/\varphi(n)$ produces a minimum.

Problem 70

Eulers Totient-Funktion, $\varphi(n)$ [auch Phi-Funktion genannt], benutzt man, um herauszufinden, wie viele Zahlen kleiner oder gleich n relativ prim zu n sind. Beispielsweise sind die sechs Zahlen 1, 2, 4, 5, 7 und 8 alle kleiner als neun und relativ prim zu neun; also ist $\varphi(9)=6$.

Die Zahl 1 betrachtet man als relatively prim zu jeder positiven Zahl, daher gilt: $\varphi(1)=1$.

Interessanterweise ist $\varphi(87109)=79180$, und man sieht, daß 87109 eine Permutation von 79180 ist.

Bestimme den Wert für n mit $1 < n < 107$, für den $\varphi(n)$ eine Permutation von n ist und das Verhältnis $n/\varphi(n)$ ein Minumum wird.

Problem 71

Consider the fraction, n/d, where n and d are positive integers. If n<d and HCF(n,d)=1, it is called a reduced proper fraction.

If we list the set of reduced proper fractions for d ≤ 8 in ascending order of size, we get:

1/8, 1/7, 1/6, 1/5, 1/4, 2/7, 1/3, 3/8, **2/5**, 3/7, 1/2, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 4/5, 5/6, 6/7, 7/8

It can be seen that 2/5 is the fraction immediately to the left of 3/7.

By listing the set of reduced proper fractions for d ≤ 1,000,000 in ascending order of size, find the numerator of the fraction immediately to the left of 3/7.

Problem 71

Betrachte den Bruch n/d, mit positiven ganzahligen n und d. Wenn n<d und wenn der größte gemeinsame Faktor von n und d, GGF(n,d)=1 ist, nennt man n/d vollständig gekürzt.

Ordnen wir die vollständig gekürzten Brüche für d ≤ 8 wertmäßig aufsteigend an, erhalten wir:

1/8, 1/7, 1/6, 1/5, 1/4, 2/7, 1/3, 3/8, **2/5**, 3/7, 1/2, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 4/5, 5/6, 6/7, 7/8

Man sieht, daß der Bruch 2/5 unmittelbar links von of 3/7 steht.

Ordnen wir nun die vollständig gekürzten Brüche für d ≤ 1.000.000 wertmäßig aufsteigend an. Bestimme den Zähler des Bruches unmittelbar links von 3/7.

Problem 72

Consider the fraction, n/d, where n and d are positive integers. If n<d and HCF(n,d)=1, it is called a reduced proper fraction.

If we list the set of reduced proper fractions for d ≤ 8 in ascending order of size, we get:

1/8, 1/7, 1/6, 1/5, 1/4, 2/7, 1/3, 3/8, 2/5, 3/7, 1/2, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 4/5, 5/6, 6/7, 7/8

It can be seen that there are 21 elements in this set.

How many elements would be contained in the set of reduced proper fractions for d ≤ 1,000,000?

Problem 72

Betrachte den Bruch n/d, mit positiven ganzahligen n und d.
Wenn n<d und wenn der größte gemeinsame Faktor von n und d, GGF(n,d)=1 ist, nennt man n/d vollständig gekürzt.

Ordnen wir die vollständig gekürzten Brüche für d ≤ 8 wertmäßig aufsteigend an, erhalten wir:

1/8, 1/7, 1/6, 1/5, 1/4, 2/7, 1/3, 3/8, 2/5, 3/7, 1/2, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 4/5, 5/6, 6/7, 7/8

Man sieht, daß diese Liste 21 Elemente enthält.

Wie viele Elemente enthielte die Liste vollständig gekürzter Brüche für d ≤ 1.000.000?

Problem 73

Consider the fraction, n/d, where n and d are positive integers. If n<d and HCF(n,d)=1, it is called a reduced proper fraction.

If we list the set of reduced proper fractions for d ≤ 8 in ascending order of size, we get:

1/8, 1/7, 1/6, 1/5, 1/4, 2/7, 1/3, 3/8, 2/5, 3/7, 1/2, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 4/5, 5/6, 6/7, 7/8

It can be seen that there are 3 fractions between 1/3 and 1/2.

How many fractions lie between 1/3 and 1/2 in the sorted set of reduced proper fractions for d ≤ 12,000?

Note: The upper limit has been changed recently.

Problem 73

Betrachte den Bruch n/d, mit positiven ganzahligen n und d. Wenn n<d und wenn der größte gemeinsame Faktor von n und d, GGF(n,d)=1 ist, nennt man n/d vollständig gekürzt.

Ordnen wir die vollständig gekürzten Brüche für d ≤ 8 wertmäßig aufsteigend an, erhalten wir:

1/8, 1/7, 1/6, 1/5, 1/4, 2/7, 1/3, 3/8, 2/5, 3/7, 1/2, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 4/5, 5/6, 6/7, 7/8

Man sieht, daß zwischen 1/3 und 1/2 genau 3 Brüche liegen.

Wie viele Brüche liegen zwischen 1/3 und 1/2 in der wertmäßig aufsteigend sortierten Liste der vollständig gekürzten Brüche für d ≤ 12.000?

Anmerkung: Die Obergrenze wurde kürzlich geändert.

Problem 74

The number 145 is well known for the property that the sum of the factorial of its digits is equal to 145:

$$1! + 4! + 5! = 1 + 24 + 120 = 145$$

Perhaps less well known is 169, in that it produces the longest chain of numbers that link back to 169; it turns out that there are only three such loops that exist:

169 ⇨ 363601 ⇨ 1454 ⇨ 169
871 ⇨ 45361 ⇨ 871
872 ⇨ 45362 ⇨ 872

It is not difficult to prove that EVERY starting number will eventually get stuck in a loop. For example,

69 ⇨ 363600 ⇨ 1454 ⇨ 169 ⇨ 363601 (⇨ 1454)
78 ⇨ 45360 ⇨ 871 ⇨ 45361 (⇨ 871)
540 ⇨ 145 (⇨ 145)

Starting with 69 produces a chain of five non-repeating terms, but the longest non-repeating chain with a starting number below one million is sixty terms.

How many chains, with a starting number below one million, contain exactly sixty non-repeating terms?

Problem 74

Die Zahl 145 ist bekannt für ihre Eigenschaft, daß die Summe der Fakultäten ihrer Ziffern wieder gleich 145 ist:

$$1! + 4! + 5! = 1 + 24 + 120 = 145$$

Vielleicht weniger bekannt ist, daß 169 die längste Kette von Zahlen erzeugt, die wieder zu 169 zurückführt; wie sich herausstellt, gibt es nur drei solche Schleifen:

169 ⇨ 363601 ⇨ 1454 ⇨ 169
871 ⇨ 45361 ⇨ 871
872 ⇨ 45362 ⇨ 872

Es ist nicht schwierig zu zeigen, daß JEDE Startzahl früher oder später ein einer Schleife feststecken wird. Zum Beispiel:

69 ⇨ 363600 ⇨ 1454 ⇨ 169 ⇨ 363601 (⇨ 1454)
78 ⇨ 45360 ⇨ 871 ⇨ 45361 (⇨ 871)
540 ⇨ 145 (⇨ 145)

Wenn man mit 69 anfängt, ergibt sich eine Kette von fünf sich nicht wiederholenden Termen, aber die längste Kette ohne Wiederholungen mit einer Startzahl unter einer Million hat 60 Terme.

Wie viele Ketten für Startzahlen unter einer Million enthalten exakt 60 sich nicht wiederholende Terme?

Problem 75

It turns out that 12 cm is the smallest length of wire that can be bent to form an integer sided right angle triangle in exactly one way, but there are many more examples.

12 cm: (3,4,5)
24 cm: (6,8,10)
30 cm: (5,12,13)
36 cm: (9,12,15)
40 cm: (8,15,17)
48 cm: (12,16,20)

In contrast, some lengths of wire, like 20 cm, cannot be bent to form an integer sided right angle triangle, and other lengths allow more than one solution to be found; for example, using 120 cm it is possible to form exactly three different integer sided right angle triangles.

120 cm: (30,40,50), (20,48,52), (24,45,51)

Given that L is the length of the wire, for how many values of L ≤ 1,500,000 can exactly one integer sided right angle triangle be formed?

Note: This problem has been changed recently, please check that you are using the right parameters.

Problem 75

Es stellt sich heraus, daß 12 cm die kleinste Länge für einen Draht ist, der sich zu einem rechtwinkligen Dreieck mit ganzzahligen Seitenlängen biegen läßt. Es gibt aber noch viel mehr Beispiele:

12 cm: (3,4,5)
24 cm: (6,8,10)
30 cm: (5,12,13)
36 cm: (9,12,15)
40 cm: (8,15,17)
48 cm: (12,16,20)

Im Gegensatz zu diesen Beispielen lassen sich einige Drahtlängen, wie etwa 20 cm, gar nicht zu einem rechtwinkligen Dreieck mit ganzzahligen Seitenlängen biegen, während es für andere Längen mehr als eine Lösung gibt. Mit einem 120-cm-Draht beispielsweise kann man genau drei unterschiedliche rechtwinklige Dreiecke mit ganzzahligen Seitenlängen formen:

120 cm: (30,40,50), (20,48,52), (24,45,51)

Wenn L die Drahtlänge ist, für wie viele Werte für L ≤ 1,500,000 läßt sich dann exakt ein rechtwinkliges Dreieck mit gannzahligen Seitenlängen formen?

Anmerkung: Dieses Problem wurde kürzlich geändert. Bitte darauf achten, daß die richtigen Parameter benutzt werden.

Problem 76

It is possible to write five as a sum in exactly six different ways:

4 + 1
3 + 2
3 + 1 + 1
2 + 2 + 1
2 + 1 + 1 + 1
1 + 1 + 1 + 1 + 1

How many different ways can one hundred be written as a sum of at least two positive integers?

Problem 76

Fünf läßt sich auf genau sechs Weisen als Summe schreiben:

4 + 1
3 + 2
3 + 1 + 1
2 + 2 + 1
2 + 1 + 1 + 1
1 + 1 + 1 + 1 + 1

Auf wie viele Weisen läßt sich einhundert als Summe von mindestens zwei positiven ganzen Zahlen schreiben?

Problem 77

It is possible to write ten as the sum of primes in exactly five different ways:

7 + 3
5 + 5
5 + 3 + 2
3 + 3 + 2 + 2
2 + 2 + 2 + 2 + 2

What is the first value which can be written as the sum of primes in over five thousand different ways?

Problem 77

Zehn läßt sich auf genau fünf Weisen als Summe von Primzahlen schreiben:

7 + 3
5 + 5
5 + 3 + 2
3 + 3 + 2 + 2
2 + 2 + 2 + 2 + 2

Welches ist die kleinste Zahl, die sich auf mehr als fünftausend Weisen als Summe von Primzahlen schreiben läßt?

Problem 78

Let **p(n)** represent the number of different ways in which n coins can be separated into piles. For example, five coins can separated into piles in exactly seven different ways, so p(5)=7:

OOOOO
OOOO O
OOO OO
OOO O O
OO OO O
OO O O O
O O O O O

Find the least value of n for which p(n) is divisible by one million.

Problem 78

p(n) bezeichne die Anzahl von unterschiedlichen Arten, wie man n Münzen auf Haufen verteilen kann. Fünf Münzen lassen sich beispielsweise auf genau sieben Arten auf Haufen verteilen, also gilt: p(5)=7:

OOOOO
OOOO O
OOO OO
OOO O O
OO OO O
OO O O O
O O O O O

Ermittle den kleinsten Wert für n, für den p(n) ohne Rest durch eine Million teilbar ist.

Problem 79

A common security method used for online banking is to ask the user for three random characters from a passcode. For example, if the passcode was 531278, they may ask for the 2nd, 3rd, and 5th characters; the expected reply would be: 317.

The text file, keylog.txt, contains fifty successful login attempts.

Given that the three characters are always asked for in order, analyse the file so as to determine the shortest possible secret passcode of unknown length.

The 50 passwort requests yielded the following user replies:

319, 680, 180, 690, 129, 620, 762, 689, 762, 318, 368, 710, 720, 710, 629, 168, 160, 689, 716, 731, 736, 729, 316, 729, 729, 710, 769, 290, 719, 680, 318, 389, 162, 289, 162, 718, 729, 319, 790, 680, 890, 362, 319, 760, 316, 729, 380, 319, 728, 716

Problem 79

Eine übliche Sicherheitsmaßnahme für das Onlinebanking besteht darin, daß man den Benutzer nach drei zufällig ausgewählten Buchstaben eines Passwortes fragt. Wenn das Passwort beispielsweise 531278 ist, könnte man nach dem zweiten, dritten und fünften Buchstaben fragen. Die erwartete Antwort wäre dann 317.

Unten stehen fünfzig erfolgreiche Login-Versuche.

Wir setzen voraus, daß die drei Buchstaben immer von links nach rechts erfragt werden. Analysiere die Login-Versuche, um das kürzestmögliche Passwort (unbekannter Länge) zu ermitteln.

Die 50 Passwort-Abfragen ergaben folgende Benutzer-Antworten:

319, 680, 180, 690, 129, 620, 762, 689, 762, 318, 368, 710, 720, 710, 629, 168, 160, 689, 716, 731, 736, 729, 316, 729, 729, 710, 769, 290, 719, 680, 318, 389, 162, 289, 162, 718, 729, 319, 790, 680, 890, 362, 319, 760, 316, 729, 380, 319, 728, 716

Problem 80

It is well known that if the square root of a natural number is not an integer, then it is irrational. The decimal expansion of such square roots is infinite without any repeating pattern at all.

The square root of two is 1.41421356237309504880..., and the digital sum of the first one hundred decimal digits is 475.

For the first one hundred natural numbers, find the total of the digital sums of the first one hundred decimal digits for all the irrational square roots.

Problem 80

Man weiß, daß die Quadratwurzel einer natürlichen Zahl irrational ist, wenn sie keine ganze Zahl ist. Als Kommazahl geschrieben sind solche irrationalen Quadratwurzel unendlich ohne irgendwelche sich wiederholenden Muster.

Die Quadratwurzel von zwei ist 1.41421356237309504880 ..., und die Quersumme ihrer ersten einhundert Ziffern ist 475.

Ermittle die Summe aller Quersummen aller irrationalen Quadratwurzeln unter den ersten einhundert natürlichen Zahlen.

Problem 81

In the 5 by 5 matrix below, the minimal path sum from the top left to the bottom right, by only moving to the right and down, is indicated in bold red and is equal to 2427.

131	673	234	103	18
201	**96**	**342**	965	150
630	803	**746**	**422**	111
537	699	497	**121**	956
805	732	524	**37**	**331**

Find the minimal path sum, in the matrix on the next page, 31K of text containing a 80 by 80 matrix (copy & paste from the page), from the top left to the bottom right by only moving right and down.

Problem 81

In der 5x5-Matrix unten ist die minimale Pfadsumme von oben links nach rechts unten, wenn man nur nach rechts oder nach unten gehen darf, 2427 (in rot halbfett markiert).

131	673	234	103	18
201	**96**	**342**	965	150
630	803	**746**	**422**	111
537	699	497	**121**	956
805	732	524	**37**	**331**

Zu ermitteln ist die minimale Pfadsumme der 80x80-Matrix auf der nächsten Seite, 31KB Text (kopieren und einfügen von der Seite), von links oben nach rechts unten. Erlaubt sind nur Bewegungen nach rechts und nach unten.

Matrix for/Matrix für Problem 81

Problem 82

NOTE: This problem is a more challenging version of Problem 81.

The minimal path sum in the 5 by 5 matrix below, by starting in any cell in the left column and finishing in any cell in the right column, and only moving up, down, and right, is indicated in red and bold; the sum is equal to 994.

131	673	**234**	**103**	**18**
201	**96**	**342**	965	150
630	803	746	422	111
537	699	497	121	956
805	732	524	37	331

Find the minimal path sum, in the matrix on the following page (copy & paste from the page), 31K of text containing a 80 by 80 matrix, from the left column to the right column.

Problem 82

Anmerkung: Dieses Problem ist eine schwierigere Fassung von Problem 81.

Die minimale Pfadsumme in der 5x5-Matrix unten, wenn man in irgend einer Zelle der linken Spalte startet und in irgend einer Zelle in der rechten Spalte endet und dabei nur hoch, runter und rechts geht, ist halbfett rot markiert und beträgt 994.

131	673	**234**	**103**	**18**
201	**96**	**342**	965	150
630	803	746	422	111
537	699	497	121	956
805	732	524	37	331

Zu ermitteln ist die minimale Pfadsumme der 80x80-Matrix auf der nächsten Seite (kopieren und einfügen von der Seite), 31K Text, von der linken Spalte zur rechten Spalte.

Matrix for/Matrix für Problem 82

4445,2697,5115,718,2209,2212,654,4348,3079,6821,7668,3276,8874,4190,3785,2752,9473,7817,9137,496,7338,3434,7152,4355,4552,7917,7827,2460,2350,691,3514,5880,3145,7633,7199,3783,5066,7487,3285,1084,8985,760,872,8609,8051,1134,9536,5750,9716,9371,7619,5617,275,9721,2997,2698,1887,8825,6372,3014,2113,7122,7050,6775,5948,2758,1219,3539,348,7989,2735,9862,1263,8089,6401,9462,3168,2758,3748,5870
1096,20,1318,7586,5167,2642,1443,5741,7621,7030,5526,4244,2348,4641,9827,2448,6918,5883,3737,300,7116,6531,567,5997,3971,6623,820,6148,3287,1874,7981,8424,7672,7575,6797,6717,1078,5008,4051,8795,5820,346,1851,6463,2117,6058,3407,8211,17,4822,1317,4377,4434,5925,8341,4800,1175,4173,690,8978,7470,1295,3799,8724,3509,9849,618,3320,7068,9633,2384,7175,544,6583,1908,8983,481,4187,9353,9377
9607,7385,521,6084,1364,8983,7623,1585,6935,8551,2574,8267,4781,3834,2764,2084,2669,4656,9343,7709,2203,9328,8004,6192,5856,3555,2260,5118,6504,1839,9227,1259,9451,1388,7909,5733,6968,8519,9973,1663,5315,7571,3035,4325,4283,2304,6438,8515,9213,9806,9536,196,5542,6907,2475,1159,5820,9075,9470,2179,9248,1828,4592,9167,3713,4640,47,3637,309,7344,6955,346,378,9044,8635,7466,5036,9515,6385,9230
7206,3114,7760,1094,6150,5182,7358,7387,4497,955,101,1478,7777,6966,7010,8417,6453,4955,3496,107,449,8271,131,2948,6185,784,5937,8001,6104,8282,4165,3642,710,2390,575,715,3089,6964,4217,192,5949,7006,715,3328,1152,66,8044,4319,1735,146,4818,5456,6451,4113,1063,4781,6799,602,1504,6245,6550,1417,1343,2363,3785,5448,4545,9371,5420,5068,4613,4882,4241,5043,7873,8042,8434,3939,9256,2187
3620,8024,577,9997,7377,7682,1314,1158,6282,6310,1896,2509,5436,1732,9480,706,496,101,6232,7375,2207,2306,110,6772,3433,2878,8140,5933,8688,1399,2210,7332,6172,6403,7333,4044,2291,1790,2446,7390,8698,5723,3678,7104,1825,2040,140,3982,4905,4160,2200,5041,2512,1488,2268,1175,7588,8321,8078,7312,977,5257,8465,5068,3453,3096,1651,7906,253,9250,6021,8791,8109,6651,3412,345,4778,5152,4883,7505
1074,5438,9008,2679,5397,5429,2652,3403,770,9188,4248,2493,4361,8327,9587,707,9525,5913,93,1899,328,2876,3604,673,8576,6908,7659,2544,3359,3883,5273,6587,3065,1749,3223,604,9925,6941,2823,8767,7039,3290,3214,1787,7904,3421,7137,9560,8451,2669,9219,6332,1576,5477,6755,8348,4164,4307,2984,4012,6629,1044,2874,6541,4942,903,1404,9125,5160,8836,4345,2581,460,8438,1538,5507,668,3352,2678,6942
4295,1176,5596,1521,3061,9868,7037,7129,8933,6659,5947,5063,3653,9447,9245,2679,767,714,116,8558,163,3927,8779,158,5093,2447,5782,3967,1716,931,7772,8164,1117,9244,5783,7776,3846,8862,6014,2330,6947,1777,3112,6008,3491,1906,5952,314,4602,8994,5919,9214,3995,5026,7688,6809,5003,3128,2509,7477,110,8971,3982,8539,2980,4689,6343,5411,2992,5270,5247,9260,2269,7474,1042,7162,5206,1232,4556,4757
510,3556,5377,1406,5721,4946,2635,7847,4251,8293,8281,6351,4912,287,2870,3380,3948,5322,3840,4738,9563,1906,6298,3234,8959,1562,6297,8835,7861,239,6618,1322,2553,2213,5053,5446,4402,6500,5182,8585,6900,5756,9661,903,5186,7687,5998,7997,8081,8955,4835,6069,2621,1581,732,9564,1082,1853,5442,1342,520,1737,3703,5321,4793,2776,1508,1647,9101,2499,6891,4336,7012,3329,3212,1442,9993,3988,4930,7706
9444,3401,5891,9716,1228,7107,109,3563,2700,6161,5039,4992,2242,8541,7372,2067,1294,3058,1306,320,8881,5756,9326,411,8650,8824,5495,8282,8397,2000,1228,7817,2099,6473,3571,5994,4447,1299,5991,543,7874,2297,1651,101,2093,3463,3189,6872,6118,872,1008,1779,2805,9084,4048,2123,5877,55,3075,1737,9459,4535,6453,3644,108,5982,4437,5213,1340,6967,9943,5815,669,8074,1838,6979,9132,9315,715,5048
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6882,1265,1775,3648,4690,959,5837,4520,5394,1378,9485,1360,4018,578,9174,2932,9890,3696,116,1723,1178,9355,7063,1594,1918,8574,7594,7942,1547,6166,7888,354,6932,4651,1010,7759,6905,661,7689,6092,9292,3845,9605,8443,443,8275,5163,7720,7265,6356,7779,1798,1754,5225,6661,1180,8024,5666,88,9153,1840,3508,1193,4445,2648,3538,6243,6375,8107,5902,5423,2520,1122,5015,6113,8859,9370,966,8673,2442
7338,3423,4723,6533,848,8041,7921,8277,4094,5368,7252,8852,9166,2250,2801,6125,8093,5738,4038,9808,7359,9494,601,9116,4946,2702,5573,2921,9862,1462,1269,2410,4171,2709,7508,6241,7522,615,2407,8200,4189,5492,5649,7353,2590,5203,4274,710,7329,9063,956,8371,3722,4253,4785,1194,4828,4717,4548,940,983,2575,4511,2938,1827,2027,2700,1236,841,5760,1680,6260,2373,3851,1841,4968,1172,5179,7175,3509
4420,1327,3560,2376,6260,2988,9537,4064,4829,8872,9598,3228,1792,7118,9962,9336,4368,9189,6857,1829,9863,6287,7303,7769,2707,8257,2391,2009,3975,4993,3068,9835,3427,341,8412,2134,4034,8511,6421,3041,9012,2983,7289,100,1355,7904,9186,6920,5856,2008,6545,8331,3655,5011,839,8041,9255,6524,3862,8788,62,7455,3513,5003,8413,3918,2076,7960,6108,3638,6999,3436,1441,4858,4181,1866,8731,7745,3744,1000
356,8296,8325,1058,1277,4743,3850,2388,6079,6462,2815,5620,8495,5378,75,4324,3441,9870,1113,165,1544,1179,2834,562,6176,2313,6836,8839,2986,9454,5199,6888,1927,5866,8760,320,1792,8296,7898,6121,7241,5886,5814,2815,8336,1576,4314,3109,2572,6011,2086,9061,9403,3947,5487,9731,7281,3159,1819,1334,3181,5844,5114,9898,4634,2531,4412,6430,4262,8482,4546,4555,6804,2607,9421,686,8649,8860,7794,6672
9870,152,1558,4963,8750,4754,6521,6256,8818,5208,5691,9659,8377,9725,5050,5343,2539,6101,1844,9700,7750,8114,5357,3001,8830,4438,199,9545,8496,43,2078,327,9397,106,6090,8181,8646,6414,7499,5450,4850,6273,5014,4131,7639,3913,6571,8534,9703,4391,7618,445,1320,5,1894,6771,7383,9191,4708,9706,6939,7937,8726,9382,5216,3685,2247,9029,8154,1738,9984,2626,9438,4167,6351,5060,29,1218,1239,4785
192,5213,8297,8974,4032,6966,5717,1179,6523,4679,9513,1481,3041,5355,9303,9154,1389,8702,6589,7818,6336,3539,5538,3094,6646,6702,6266,2759,4608,4452,617,9406,8064,6379,444,5602,4950,1810,8391,1536,316,8714,1178,5182,5863,5110,5372,4954,1978,2971,5680,4863,2255,4630,5723,2168,538,1692,1319,7540,440,6430,6266,7712,7385,5702,620,641,3136,7350,1478,3155,2820,9109,6261,1122,4470,14,8493,2095
1046,4301,6082,474,4974,7822,2102,5161,5172,6946,8074,9716,6586,9962,9749,5015,2217,995,5388,4402,7652,6399,6539,1349,8101,3677,1328,9612,7922,2879,231,5887,2655,508,4357,4964,3554,5930,6236,7384,4614,280,3093,9600,2110,7863,2631,6626,6620,68,1311,7198,7561,1768,5139,1431,221,230,2940,968,5283,6517,2146,1646,869,9402,7068,8645,7058,1765,9690,4152,2926,9504,2939,7504,6074,2944,6470,7859
4659,736,4951,8344,1927,6271,8837,8711,3241,6579,7660,5499,5616,3743,5801,4682,9748,8796,779,1833,4549,8138,4026,775,4170,2432,4174,3741,7540,8017,2833,4027,396,811,2871,1150,9809,2719,9199,8504,1224,540,2051,3519,7982,7367,2761,308,3358,6505,2050,4836,5090,7864,805,2566,2409,6876,3361,8622,5572,5895,3280,441,7893,8105,1634,2929,274,3926,7786,6123,8233,9921,2674,5340,1445,203,4585,3837
5759,338,7444,7968,7742,3755,1591,4839,1705,650,7061,2461,9230,9391,9373,2413,1213,431,7801,4994,2380,2703,6161,6878,8331,2538,6093,1275,5065,5062,2839,582,1014,8109,3525,1544,1569,8622,7944,2905,6120,1564,1839,5570,7579,1318,2677,5257,4418,5601,7935,7656,5192,1864,5886,6083,5580,6202,8869,1636,7907,4759,9082,5854,3185,7631,6854,5872,5632,5280,1431,2077,9717,7431,4256,8261,9680,4487,4752,4286
1571,1428,8599,1230,7772,4221,8523,9049,4042,8726,7567,6736,9033,2104,4879,4967,6334,6716,3994,1269,8995,6539,3610,7667,6560,6065,874,848,4597,1711,7161,4811,6734,5723,6356,6026,9183,2586,5636,1092,7779,7923,8747,6887,7505,9909,1792,3233,4526,3176,1508,8043,720,5212,6046,4988,709,5277,8256,3642,1391,5803,1468,2145,370,6301,7767,2359,8487,9771,8785,7520,856,1605,8972,2402,2386,991,1383,5963
1822,4824,5957,6511,9868,4113,301,9353,6228,2881,2966,6956,9124,9574,3233,7641,7740,973,9396,540,4747,8590,9535,3650,7333,7583,4806,3593,2738,8157,5215,8472,2284,9473,3906,6982,5505,6053,7936,6074,7179,6688,1564,1103,6860,5839,2022,8490,910,7551,7805,881,7024,1855,9448,4790,1274,3672,2810,774,7623,4223,4850,6071,9975,4935,1915,9771,6690,3866,517,463,7624,4511,614,6394,3661,7409,1395,8127
8738,3850,9555,3695,4383,2378,87,6256,6740,7682,9546,4255,6105,2000,1851,4073,8957,9022,6547,5189,2487,303,9602,7833,1628,4163,6678,3144,8589,7096,8913,5823,4890,7679,1212,9294,5884,2972,3012,3359,7794,7428,1579,4350,7246,4301,7779,7790,3294,9547,4367,3549,1958,8237,6758,3497,3250,3456,6318,1663,708,7714,6143,6890,3428,6853,9334,7992,591,6449,9786,1412,8500,722,5468,1371,108,3939,4199,2535
7047,4323,1934,5163,4166,461,3544,2767,6554,203,6098,2265,9078,2075,4644,6641,8412,9183,487,101,7566,5622,1975,5726,2920,5374,7779,5631,3753,3725,2672,3621,4280,1162,5812,345,8173,9785,1525,955,5603,2215,2580,5261,2765,2990,5979,389,3907,2484,1232,6359,9053,5871,3304,1138,1616,5114,9199,5072,7442,7245,6472,4760,6359,9053,8776,2564,9404,3043,9206,2261,3374,4460,7306,2326,966,828,3274,1712,3446
3975,4565,8131,5800,4570,2306,8838,4392,9147,11,3911,7118,9645,4994,2028,6062,5431,2279,8752,2658,7836,994,7316,5336,7185,3289,1898,9689,2331,5737,3403,1124,2679,3241,7748,16,2724,5441,6640,9368,9001,5618,858,4969,17,2103,6035,8043,7475,2181,939,415,1617,8500,8253,2155,7843,7974,7859,1746,6336,3193,2617,8736,4079,6324,6645,8891,9396,5522,6103,1857,8979,3835,2475,1310,7422,610,8345,7615
9248,5397,5686,2988,3446,4359,6634,9141,497,9176,6773,7448,1907,8454,916,1596,2241,1626,1384,2741,3649,5362,8791,7170,2903,2475,5325,6451,924,3328,522,90,4813,9737,9557,691,2388,1383,4021,1609,9206,4707,5200,7107,8104,4333,9860,5013,2475,7772,6268,621,4915,9349,5970,706,6539,5801,2208,2107,9114,3836,7503,2383,1977,4870,8035,2379,9704,1037,3992,3642,1016,4303
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6237,4607,7527,9047,522,7371,4883,2540,5867,6366,5301,1570,421,276,3361,527,6637,4861,2401,7522,5808,9371,5298,2045,5096,5447,7755,5115,7060,8529,4078,1943,1697,1764,5453,7085,960,2405,739,2100,5800,728,9737,5704,5693,1431,8979,6428,673,7540,6,7773,5857,6823,150,5869,8486,684,5816,9626,7451,5579,8260,3397,5322,6920,1879,2127,2884,5478,4977,9016,6165,6292,3062,5671,5968,78,4619,4763
9905,7127,9390,5185,6923,3721,9164,9705,4341,1031,1046,5127,7376,6528,3248,4941,1178,7889,3364,4406,5358,9402,9158,8600,1025,874,1839,1783,309,9030,1843,845,8398,1433,7118,70,8071,2977,9904,8866,6722,4299,10,1929,5897,4188,600,1889,3325,2485,6473,4474,7444,6992,4946,6166,4441,2283,2629,4352,7775,1101,2214,9985,2570,9740,2746,8361,7103,5930,8664,9690,4632,344,2077,1372,1880,9550
5825,8517,7769,2405,8204,1060,3603,7025,478,8334,1997,3692,7433,9101,7294,7498,9415,5452,3850,3508,6857,9213,6807,4412,7310,854,5384,686,4978,892,8651,3241,2743,3801,3813,8588,6701,4416,6990,6490,3197,6838,6503,114,8343,5844,4866,8694,65,791,5979,2687,2621,2019,8097,1423,3644,9764,4921,3266,3662,5561,2476,8271,8138,6147,1168,3340,1998,9874,6572,9873,6659,5609,2711,3931,9567,4143,7833,6887
6223,2099,2700,589,4716,8333,1362,5007,2753,2848,4441,8397,7192,8191,4959,9955,6076,3370,6396,6971,3156,249,3911,2488,4930,2458,7183,5455,170,6809,6417,3390,1956,7188,577,7526,2203,968,8164,479,8699,7915,507,6393,4632,1597,7534,3604,618,3280,6061,9793,9238,8347,568,9645,7023,5198,6482,5000,9212,5056,7513,1323,1872,6170,3812,4146,2736,67,3151,5548,2781,9679,7565,5043,8587,1893,4531
5826,3690,6724,2121,9308,6986,8106,6659,2142,1642,1770,2877,5757,6494,8026,6571,8387,9961,6043,9758,9607,6450,8631,8334,7359,5256,8523,2225,7487,1977,9555,8048,5763,2414,4948,4265,2427,8978,8088,8841,9208,9601,5810,9198,8866,9138,4176,5875,7212,3272,6759,5678,7649,4922,5422,1343,8197,3154,3600,687,1028,4579,2084,9467,4492,7262,7296,6538,7657,7134,2077,1505,7332,6890,8964,4879,7603,7400,5973,739
1861,1613,4879,1884,7334,966,2000,7489,2123,4287,1472,3263,4726,9203,1040,4103,6075,6049,330,9253,4062,4268,1635,9960,577,1320,1955,8926,1030,4092,4979,6474,6393,2799,6967,8687,7724,7392,9927,2085,3200,6466,8702,265,7646,8665,7986,7266,4574,6587,612,2724,704,3191,8323,9523,3202,6077,2758,8393,4875,4641,9584,6401,7883,7014,768,443,5490,7506,1852,2005,8850,5776,4487,4269
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2650,3883,4154,5809,315,7756,4430,1788,4451,1631,6461,7230,6017,5751,138,588,5282,2442,9110,9035,6349,2515,1570,6122,4192,4174,3530,1933,4186,4420,4609,5739,4135,2963,6308,1161,8809,8619,2796,3819,6971,8228,4188,1492,909,8048,2328,6772,8467,7671,9068,2226,7579,6422,7056,8042,3296,2272,3006,2196,7320,3238,3490,3102,37,1293,3212,4767,5041,8773,5794,4456,6174,7279,7054,2835,7053,9088,790,6640
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4865,9912,8038,8238,782,3095,8199,1127,4501,7280,2112,2487,3626,2790,9432,1475,6312,8277,4827,2218,5806,7132,8752,1468,7471,6386,739,8762,8323,8120,5169,9078,9058,3370,9560,7987,8585,8531,5347,4032,1058,4271,1159,5286,5404,6925,8606,3741,2415,560,586,4002,2644,1927,2824,768,4901,6267,7979,5140,8643,7553,9438,7320,4938,2666,4609,2778,8158,6730,3748,3867,1866,7181
171,3771,7134,8927,4778,2913,3326,2004,3089,7853,1378,1729,4777,2706,9578,1360,5693,3036,1851,7248,2403,2273,8536,6501,9216,613,9671,7131,7719,6425,73,717,8803,160,1114,7554,7197,753,4513,4322,8499,4533,2609,4226,8710,6627,644,9666,6260,4870,5744,7385,6542,6203,7703,6130,8944,5589,2262,6803,6381,7414,6888,5123,7320,9392,9061,6780,322,8975,7050,5089,1061,2260,3199,1150,1865,5386,9639,6501
3744,8454,6885,8277,919,1923,4001,6864,7854,5519,2491,6057,8794,9645,1776,5714,9786,9281,7538,6916,3215,395,2501,9618,4835,8846,9708,2813,3303,1794,8309,7176,2206,1602,1838,236,4593,2245,8993,4017,10,8215,6921,5206,4023,5932,6997,7801,262,7640,3107,8275,4938,7822,2425,3223,3886,2105,8700,3526,2088,8662,8034,7004,5710,2124,7164,3574,6630,9980,4242,2901,9471,1491,2117,4562,1130,9086,4117,6698
2810,2280,2331,1170,4554,4071,8387,1215,2274,9846,6738,1604,7281,8805,439,1298,8318,7834,9426,8650,8603,6092,7944,1309,8828,303,3157,4638,4439,9175,1921,4695,7716,1494,1015,1772,5913,1127,1952,1950,8905,4064,9890,385,9357,7945,5035,7082,5369,4093,6546,5187,5637,2041,8946,1758,7111,6566,1027,1049,5148,7224,7248,296,6169,375,1656,7993,2816,3717,4279,4675,1609,3317,42,6201,3100,3144,163,9530,4531
7096,6070,1009,4988,3538,5801,7149,3063,2324,2912,7911,7002,4338,7880,2481,7368,3516,2016,7556,2193,1388,3865,8125,4637,4096,8114,750,3144,1938,7002,9343,4095,1392,4220,3455,6969,9647,1321,9048,1996,1640,6626,1788,314,9578,6630,2813,6626,4981,9908,4254,1355,3201,3521,3864,1403,4639,8807,9085,1892,9431,8317,9016,9221,8574,9981,9240,5395,2009,6310,2854,9255
8830,3145,2960,9615,8220,6061,3452,2918,6481,9278,2297,3385,6565,7066,7316,5682,107,7646,4466,68,1952,9603,8615,54,7191,791,6833,2560,693,9733,4168,570,9127,9537,1925,8287,5508,4297,8452,8795,6213,7994,2420,4208,524,5915,8602,8330,2651,8547,6156,1812,6271,7991,9407,9804,1553,6866,1128,2119,4691,9711,8315,5879,9935,6900,482,682,4126,1041,428,6247,3720,5882,7526,2582,4327,7725,3503,2631
2738,9323,721,7434,1453,6294,2957,3786,5722,6019,8685,4386,3066,9057,6860,499,5115,3045,5194,7111,3137,9104,941,586,3066,755,4177,8819,7040,5093,3383,3897,4428,7788,4721,7249,6559,7324,825,7311,3760,6064,6070,9672,4882,584,1365,9739,9331,5783,2624,7889,1604,1303,1555,7125,8312,425,8936,3202,1420,7440,1784,1754,4721,1569,652,3893,4574,9052,4729,1597,6138,9094,8291,9199,7101,3391,8914
6044,2928,9332,3328,8588,447,3830,1176,3523,2705,8365,6136,5442,9049,5526,8575,8869,9031,7280,706,2794,8814,5767,4241,7696,78,6570,556,5083,1426,4502,3336,9518,2292,1885,3740,3153,9348,8331,8051,2759,5407,9028,7840,9255,831,515,2612,9747,7435,8964,4971,2048,4900,5967,8271,1719,9670,2810,6777,1594,6367,6259,8316,3815,1689,6840,9437,4361,822,9619,3065,83,6344,7486,8657,8228,9635,6932,4864
8478,4777,6334,4678,7476,4963,6735,3096,5860,1405,5127,7269,7793,4738,227,9168,2996,8928,765,733,1276,7677,6258,1528,9558,3329,302,8901,1422,8277,6340,645,9125,8869,5952,141,8141,1816,3635,4025,4184,3093,83,2344,2747,9352,7966,1206,1126,1826,218,7939,2957,2729,810,8752,5247,4174,4038,8884,7899,9567,301,5265,5752,7524,4381,1669,3106,8270,6228,6373,754,2547,4240,2313,5514,3022,1040,9738
2265,8192,1763,1369,8469,8789,4836,52,1222,6690,5257,8918,6723,6319,378,4039,2421,8555,8184,9577,1432,7139,8078,5452,9628,7579,4161,7490,5159,8559,1011,61,478,5840,1964,1334,6875,8670,9000,739,1514,8692,522,9316,6995,1345,8132,2277,3193,9773,3923,4171,1283,1236,6747,6575,4874,6003,6409,8187,745,8776,9440,7543,8435,1289,6417,7246,5185,7564,6125,218,7991,6394,3776,7456,5128,5294
2132,8992,8160,5782,4420,3371,3798,5054,552,5631,7546,4716,1332,6486,7892,7441,4370,6231,4579,2121,8615,1145,9391,1524,1185,2400,9437,2454,7896,7467,2928,8400,3299,4025,7458,6033,7206,6358,792,6200,725,4275,4136,7390,5984,4502,7929,5085,8176,4600,119,3568,76,9363,6943,2248,9077,9711,6213,5187,6279,4190,3092,6901,759,2692,8380,1254,9604,3011,9291,5329,9453,9746,2739,6522,3765,5634,1113,5789
5304,5499,564,2801,679,2653,1783,3608,7359,7797,3284,796,3222,437,7185,6135,8571,2778,7488,5746,678,6140,861,7750,803,9859,9918,2425,3734,2698,9005,4864,9818,6743,2475,132,9486,3825,5472,919,292,4411,7213,7699,6435,9019,6769,1388,802,2124,1345,8493,9487,8558,7061,8777,8833,2427,2238,5409,4957,8503,3171,7622,5779,6145,2417,5873,5563,5693,9574,9491,1937,7384,4563,6842,5432,2751,3406,7981

Problem 83

NOTE: This problem is a significantly more challenging version of Problem 81.

In the 5 by 5 matrix below, the minimal path sum from the top left to the bottom right, by moving left, right, up, and down, is indicated in bold red and is equal to 2297.

131	673	**234**	**103**	**18**
201	**96**	**342**	965	**150**
630	803	746	**422**	**111**
537	699	497	**121**	956
805	732	524	**37**	**331**

Find the minimal path sum, in the matrix on the following page (copy & paste from the page), 31K of text containing a 80 by 80 matrix, from the top left to the bottom right by moving left, right, up, and down.

Problem 83

Anmerkung: Dieses Problem ist deutlich schwieriger als Problem 81.

Wenn man in der 5x5-Matrix unten nach links, rechts, oben und unten geht, ist die minimale Pfadsumme von der Zelle oben links zur Zelle rechts unten (hier halbfett in rot markiert) 2297.

131	673	**234**	**103**	**18**
201	**96**	**342**	965	**150**
630	803	746	**422**	**111**
537	699	497	**121**	956
805	732	524	**37**	**331**

Zu ermitteln ist die minimale Pfadsumme der 80x80-Matrix auf der nächsten Seite (kopieren und einfügen von der Seite), 31K Text, von der Zelle links oben zur Zelle links unten. Erlaubt sind Bewegungen nach links, rechts, oben und unten.

Matrix for/Matrix für Problem 83

4445,2697,5115,718,2209,2212,654,4348,3079,6821,7668,3276,8874,4190,3785,2752,9473,7817,9137,496,7338,3434,7152,4355,4552,7917,7827,2460, [illegible]
1096,20,1318,7586,5167,2642,1443,5741,7621,7030,5526,4244,2348,4641,9827,2448,6918,5883,3737,300,7116,6531,567,5997,3971,6623,820,6148, [illegible]
9607,7385,521,6084,1364,8983,7623,1585,6935,8551,2574,8267,4781,3834,2764,2084,2669,4656,9343,7709,2203,9328,8004,6192,5856,3555,2260,5118, [illegible]
7206,3114,7760,1094,6150,5182,7358,7387,4497,955,101,1478,7777,6966,7010,8417,6453,4955,3496,107,449,8271,131,2948,6185,784,5937,8001,6104, [illegible]
3620,8024,577,9997,7377,7682,1314,1158,6282,6310,1896,2509,5436,1732,9480,706,496,101,6232,7375,2207,2306,110,6772,3433,2878,8140,5933, [illegible]
1074,5438,9008,2679,5397,5429,2652,3403,770,9188,4248,2493,4361,8327,9587,707,9525,5913,93,1899,328,2876,3604,673,8576,6908,7659,2544,3359, [illegible]
4295,1176,5596,1521,3061,9868,7037,7129,8933,6659,5947,5063,3653,9447,9245,2679,767,714,116,8558,163,3927,8779,158,5093,2447,5782,3967, [illegible]
510,3556,5377,1406,5721,4946,2635,7847,4251,8293,8281,6351,4912,287,2870,3380,3948,5322,3840,4738,9563,1906,6298,3234,8959,1562,6297,8835, [illegible]

[illegible]

Problem 84

In the game, *Monopoly*, the standard board is set up like this:

GO	A1	CC1	A2	T1	R1	B1	CH1	B2	B3	JAIL
H2										C1
T2										U1
H1										C2
CH3										C3
R4										R2
G3										D1
CC3										CC2
G2										D2
G1										D3
G2J	F3	U2	F2	F1	R3	E3	E2	CH2	E1	FP

A player starts on the GO square and adds the scores on two 6-sided dice to determine the number of squares they advance in a clockwise direction. Without any further rules we would expect to visit each square with equal probability: 2.5%. However, landing on G2J (Go To Jail), CC (community chest), and CH (chance) changes this distribution.

In addition to G2J, and one card from each of CC and CH, that orders the player to go directly to jail, if a player rolls three consecutive doubles, they do not advance the result of their 3rd roll. Instead they proceed directly to jail.

Problem 84

Monopoly wird standardmäßig auf folgendem Brett gespielt:

GO	A1	CC1	A2	T1	R1	B1	CH1	B2	B3	JAIL
H2										C1
T2										U1
H1										C2
CH3										C3
R4										R2
G3										D1
CC3										CC2
G2										D2
G1										D3
G2J	F3	U2	F2	F1	R3	E3	E2	CH2	E1	FP

Ein Spieler startet auf dem GO-Feld und addiert die Augen von zwei 6-seitigen Würfeln, um dann diese Zahl im Uhrzeigersinn vorzurücken. Ohne weitere Regeln würden wir erwarten, daß jedes Feld etwa mit derselben Häufigkeit besucht wird: 2,5%. Wenn man auf den Feldern G2J (geh ins Gefängnis), CC (Gemeinschaftskarte) oder CH (Zufall) landet, ändert sich diese Verteilung.

Neben G2J und je einer Karte von CC- und von CH, die den Spieler direkt ins Gefängnis beordern, darf ein Spieler, der zwei Pasch in Folge würfelt, das Ergebnis seines dritten Wurfes nicht vorrücken. Stattdessen geht man dann direkt ins Gefängnis.

Problem 84, page 2

At the beginning of the game, the CC and CH cards are shuffled. When a player lands on CC or CH they take a card from the top of the respective pile and, after following the instructions, it is returned to the bottom of the pile. There are sixteen cards in each pile, but for the purpose of this problem we are only concerned with cards that order a movement; any instruction not concerned with movement will be ignored and the player will remain on the CC/CH square.

Community Chest (2/16 cards):
  Advance to GO
  Go to JAIL
Chance (10/16 cards):
  Advance to GO
  Go to JAIL
  Go to C1
  Go to E3
  Go to H2
  Go to R1
  Go to next R (railway company)
  Go to next R
  Go to next U (utility company)
  Go back 3 squares.

The heart of this problem concerns the likelihood of visiting a particular square. That is, the probability of finishing at that square after a roll. For this reason it should be clear that, with the exception of G2J for which the probability of finishing on it is zero, the CH squares will have the lowest probabilities, as 5/8 request a movement to another square, and it is the final square that the player finishes at on each roll that we are interested in.

Problem 84, Seite 2

Zu Beginn des Spiels werden die CC- und die CH-Karten gemischt. Wenn ein Spieler auf CC oder CH landet, nehmen sie eine Karte vom jeweiligen Stapel und legen sie, nachdem sie die Anweisungen darauf befolgt haben, unter den Stapel zurück. Jeder Stapel hat sechszehn Karten, aber bei diesem Problem interessieren uns nur Karten, die eine Bewegung anordnen. Alle Nicht-Bewegungskarten werden ignoriert, und der Spieler bleibt auf seinem CC- oder CH-Feld. Für dieses Problem sind nur folgende Karten relevant:

Community Chest (2 von 16 Karten):
  Gehe auf das Feld GO
  Gehe auf das Feld GO
Chance (10 von 16 Karten):
  Gehe auf das Feld GO
  Gehe auf das Feld GO
  Gehe auf das Feld C1
  Gehe auf das Feld E3
  Gehe auf das Feld H2
  Gehe auf das Feld R1
  Gehe zum nächsten R-Feld
  Gehe zum nächsten R-Feld
  Gehe zum nächsten U-Feld
  Gehe 3 Felder zurück.

Im Kern dreht sich dieses Problem um die Wahrscheinlichkeit, mit der ein bestimmtes Feld besucht wird, also die Wahrscheinlichkeit, nach einem Wurf auf einem bestimmten Feld zu landen. Daher sollte klar sein, daß mit Ausnahme des Feldes G2J, auf dem man mit der Wahrscheinlichkeit Null landet, die CH-Felder die geringsten Wahrscheinlichkeiten haben, da 5 von 8 eine Bewegung auf ein anderes Feld anordnen, und es ist eben dies Endfeld, auf dem ein Spieler nach seinem Wurf landet, das uns interessiert.

Problem 84, page 3

We shall make no distinction between »Just Visiting« and being sent to JAIL, and we shall also ignore the rule about requiring a double to “get out of jail”, assuming that they pay to get out on their next turn.

By starting at GO and numbering the squares sequentially from 00 to 39 we can concatenate these two-digit numbers to produce strings that correspond with sets of squares.

Statistically it can be shown that the three most popular squares, in order, are JAIL (6.24%) = Square 10, E3 (3.18%) = Square 24, and GO (3.09%) = Square 00. So these three most popular squares can be listed with the six-digit modal string: 102400.

If, instead of using two 6-sided dice, two 4-sided dice are used, find the six-digit modal string.

Problem 84, Seite 3

Wir machen hier keinen Unterschied zwischen »Nur zu Besuch« und dem direkten Inhaftiertwerden, und wir ignorieren auch die Regel, derzufolge man ein Pasch braucht, um aus dem Gefängnis wieder herauszukommen; wir nehmen an, daß der Spieler für seine Entlassung aus der Haft bei seinem nächsten Wurf bezahlt.

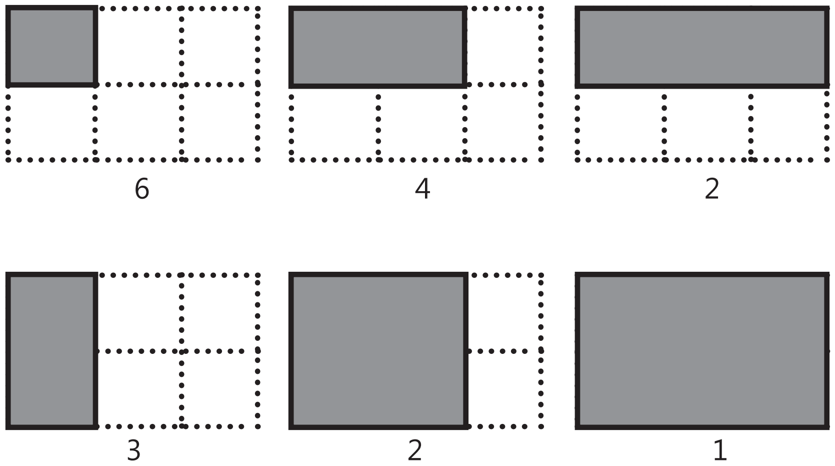
Der Spieler beginnt auf dem GO-Feld. Wir numerieren die Felder der Reihe nach von 00 bis 39 und hängen diese zweistelligen Zahlen aneinander. So erhalten wir Zeichenketten, die Felderfolgen entsprechen.

Statistisch kann man zeigen, daß die drei beliebtesten Felder in der Reihenfolge ihrer Beliebtheit, JAIL (6,24%) = Feldnummer 10, E3 (3,18%) = Feldnummer 24, und GO (3,09%) = Feldnummer 00 sind. Ketten wir diese drei Feldnummern aneinander, so erhalten wir die sechsstellige Modalkette 102400.

Für unsere Spielvarianten benutzen wir statt der beiden sechsseitigen Würfel zwei vierseitige Würfel. Zu bestimmen ist für ein solches Spiel die sechsstellige Modalkette.

Problem 85

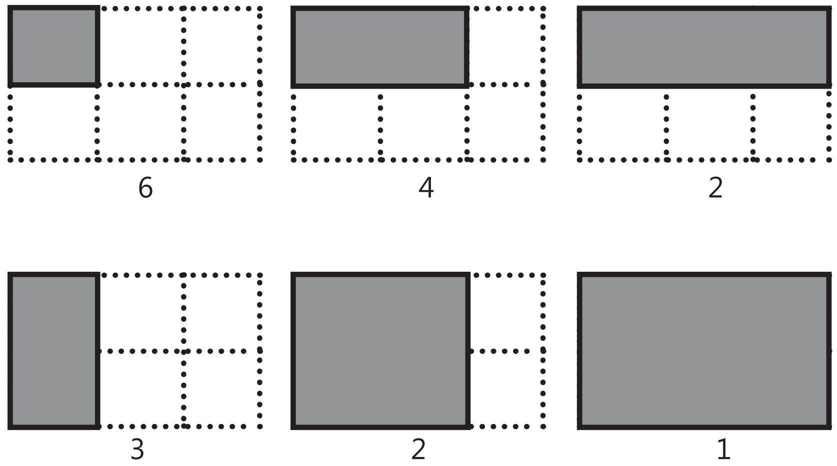
By counting carefully it can be seen that a rectangular grid measuring 3 by 2 contains eighteen rectangles:



Although there exists no rectangular grid that contains exactly two million rectangles, find the area of the grid with the nearest solution.

Problem 85

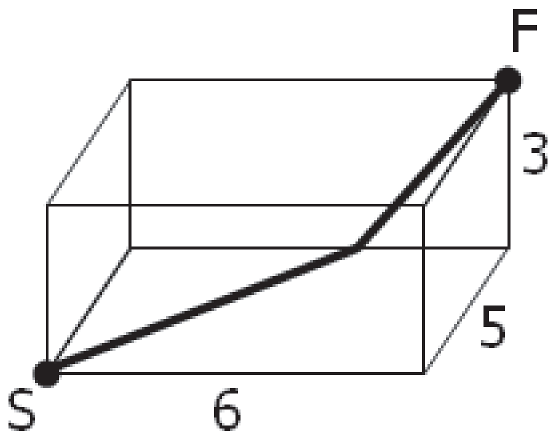
Wenn man aufmerksam zählt, sieht man, daß ein rechteckiges 3x2-Gitter achtzehn Rechtecke enthält:



Es gibt kein rechteckiges Gitter gibt, das genau zwei Millionen Rechtecke enthalten würde. Bestimme den Flächeninhalt des Gitters mit der nächstmöglichen Lösung.

Problem 86

A spider, S, sits in one corner of a cuboid room, measuring 6 by 5 by 3, and a fly, F, sits in the opposite corner. By travelling on the surfaces of the room the shortest “straight line” distance from S to F is 10 and the path is shown on the diagram.



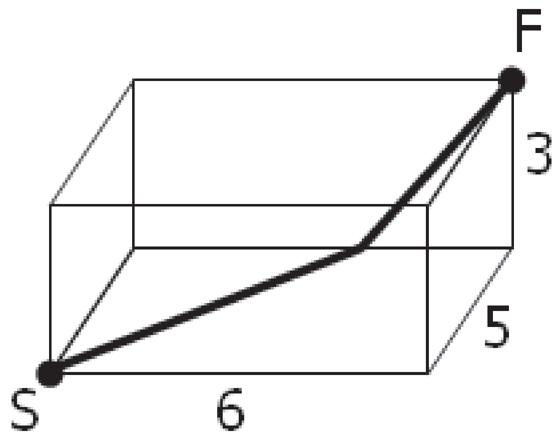
However, there are up to three »shortest« path candidates for any given cuboid and the shortest route is not always integer.

By considering all cuboid rooms with integer dimensions, up to a maximum size of M by M by M, there are exactly 2060 cuboids for which the shortest distance is integer when M=100, and this is the least value of M for which the number of solutions first exceeds two thousand; the number of solutions is 1975 when M=99.

Find the least value of M such that the number of solutions first exceeds one million.

Problem 86

Eine Spinne, S, sitzt in einer Ecke eines quaderförmigen Raumes mit den Maßen 6 x 5 x 3. Eine Fliege, F, sitzt in der gegenüberliegenden Ecke. Wenn man auf den Oberflächen des Raumes wandert, ist die kürzeste »gerade Verbindungslinie« von S zu F 10 Einheiten lang: den Weg zeigt das Diagramm.



Es gibt jedoch für jeden Quader bis zu drei »kürzeste« Weg-Kandidaten, und die kürzeste Route ist nicht immer ganzzahlig.

Wenn wir alle quaderförmigen Räume mit ganzzahligen Kantenlängen betrachten, bis hinauf zu einer Größe von M x M x M, gibt es für M=100 genau 2060 Quader, für welche der kürzeste Abstand eine ganze Zahl ist. Dieser Wert für M ist zugleich der kleinste Wert, für den die Anzahl der Lösungen erstmalig zwei Tausend überschreitet. Für M=99 gibt es 1975 Lösungen.

Zu bestimmen ist der kleinste Wert für M, so daß die Anzahl der Lösungen erstmalig eine Million überschreitet.

Problem 87

The smallest number expressible as the sum of a prime square, prime cube, and prime fourth power is 28. In fact, there are exactly four numbers below fifty that can be expressed in such a way:

$$28 = 2^2 + 2^3 + 2^4$$
$$33 = 3^2 + 2^3 + 2^4$$
$$49 = 5^2 + 2^3 + 2^4$$
$$47 = 2^2 + 3^3 + 2^4$$

How many numbers below fifty million can be expressed as the sum of a prime square, prime cube, and prime fourth power?

Problem 87

Die kleinste Zahl, die sich als Summe eines Primzahlquadrates, eines Primzahlkubus und der vierten Potenz einer Primzahl ausdrücken läßt, ist 28. Es gibt unter fünfzig insgesamt genau vier Zahlen, die sich auf diese Weise ausdrücken lassen:

$$28 = 2^2 + 2^3 + 2^4$$
$$33 = 3^2 + 2^3 + 2^4$$
$$49 = 5^2 + 2^3 + 2^4$$
$$47 = 2^2 + 3^3 + 2^4$$

Wie viele Zahlen unter fünfzig Millionen lassen sich als Summe eines Primzahlquadrates, eines Primzahlkubus und der vierten Potenz einer Primzahl ausdrücken?

Problem 88

A natural number, N, that can be written as the sum and product of a given set of at least two natural numbers, $\{a_1, a_2, \ldots, a_k\}$ is called a *product-sum number*:

$$N = a_1 + a_2 + \ldots + a_k = a_1 \times a_2 \times \ldots \times a_k.$$

For example, $6 = 1 + 2 + 3 = 1 \times 2 \times 3$.

For a given set of size, k, we shall call the smallest N with this property a *minimal* product-sum number. The minimal product-sum numbers for sets of size, k = 2, 3, 4, 5, and 6 are as follows.

k=2: $4 = 2 \times 2 = 2 + 2$
k=3: $6 = 1 \times 2 \times 3 = 1 + 2 + 3$
k=4: $8 = 1 \times 1 \times 2 \times 4 = 1 + 1 + 2 + 4$
k=5: $8 = 1 \times 1 \times 2 \times 2 \times 2 = 1 + 1 + 2 + 2 + 2$
k=6: $12 = 1 \times 1 \times 1 \times 1 \times 2 \times 6 = 1 + 1 + 1 + 1 + 2 + 6$

Hence for $2 \le k \le 6$, the sum of all the minimal product-sum numbers is $4+6+8+12 = 30$; note that 8 is only counted once in the sum.

In fact, as the complete set of minimal product-sum numbers for $2 \le k \le 12$ is {4, 6, 8, 12, 15, 16}, the sum is 61.

What is the sum of all the minimal product-sum numbers for $2 \le k \le 12000$?

Problem 88

Eine natürliche Zahl N, die sich als Summe und auch als Produkt einer gegebenen Menge von mindestens zwei natürlichen Zahlen, $\{a_1, a_2, \ldots, a_k\}$, schreiben läßt, nennt man *Produktsummenzahl*:

$$N = a_1 + a_2 + \ldots + a_k = a_1 \cdot a_2 \cdot \ldots \cdot a_k.$$

Zum Beispiel ist $6 = 1 + 2 + 3 = 1 \cdot 2 \cdot 3$.

Für eine gegebene Menge mit k Elementen nennen wir das kleinste N mit dieser Eigenschaft eine *minimale* Produktsummenzahl. Die minimalen Produktsummenzahlen für Mengen der Größe K = 2, 3, 4, 5 und 6 sind:

k=2: $4 = 2 \times 2 = 2 + 2$
k=3: $6 = 1 \times 2 \times 3 = 1 + 2 + 3$
k=4: $8 = 1 \times 1 \times 2 \times 4 = 1 + 1 + 2 + 4$
k=5: $8 = 1 \times 1 \times 2 \times 2 \times 2 = 1 + 1 + 2 + 2 + 2$
k=6: $12 = 1 \times 1 \times 1 \times 1 \times 2 \times 6 = 1 + 1 + 1 + 1 + 2 + 6$

Für $2 \le k \le 6$, ist die Summe aller minimalen Produktsummenzahlen also $4+6+8+12 = 30$; zu beachten ist, daß die 8 in der Summe nur einmal gezählt wird.

Da die vollständige Menge der Produktsummenzahlen für $2 \le k \le 12$ die Menge {4, 6, 8, 12, 15, 16}, ist, ergibt die Summe 61.

Wie viel ergibt sich, wenn man alle minimalen Produktsummenzahlen für $2 \le k \le 12000$ summiert?

Problem 89

The rules for writing Roman numerals allow for many ways of writing each number. However, there is always a »best« way of writing a particular number.

For example, the following represent all of the legitimate ways of writing the number sixteen:

IIIIIIIIIIIIIIII
VIIIIIIIIIII
VVIIIIII
XIIIIII
VVVI
XVI

The last example being considered the most efficient, as it uses the least number of numerals.

The following page (copy and paste from the page), contains one thousand numbers written in valid, but not necessarily minimal, Roman numerals; that is, they are arranged in descending units and obey the subtractive pair rule.

Find the number of characters saved by writing each of these in their minimal form.

Note: You can assume that all the Roman numerals in the file contain no more than four consecutive identical units.

Problem 89

Die Regeln zur Schreibung römischer Zahlen erlauben eine Reihe von Möglichkeiten Es gibt jedoch immer eine »beste« Möglichkeit, eine gegebene Zahl zu schreiben.

Beispielsweise sind die die folgenden sechs Möglichkeiten, die Zahl sechszehn zu schreiben, allesamt gültig.

IIIIIIIIIIIIIIII
VIIIIIIIIIII
VVIIIIII
XIIIIII
VVVI
XVI

Das letzte Beispiel erachten wir als das effizienteste, weil es die kleinste Zahl von Zahlzeichen benutzt.

Die folgende Seite (kopieren und einfügen) enthält eintausend römische Zahlen in gültiger, wenn auch nicht notwendigerweise minimaler, Schreibweise. In den Zahlen sind die Einheiten ab-steigend sortiert und sie befolgen die Subtraktionsregel für Paare.

Zu ermitteln ist die Anzahl von Zeichen, die man einspart, wenn man alle tausend römischen Zahlen in minimaler Form schreibt.

Anmerkung: Die römischen Zahlen, um die es hier geht, enthalten nicht mehr als vier aufeinanderfolgende identische Einheiten.

Roman numerals for/Römische Zahlen für Problem 89

MMMMDCLXXII
MMDCCCLXXXIII
MMMDLXVIIII
MMMMDXCV
DCCCLXXII
MMCCCVI
MMMCDLXXXVII
MMMMCCXXI
MMMCCXX
MMMMDCCCLXXIII
MMMCCXXXVII
MMCCCLXXXXIX
MDCCCXXIIII
MMCXCVI
CCXCVIII
MMMCCCXXXII
MDCCXXX
MMMDCCCL
MMMMCCLXXXVI
MMDCCXCVI
MMMDCII
MMMCCXII
MMMMDCCCCI
MMDCCCXCII
MDCXX
CMLXXXVII
MMMXXI
MMMMCCCXIV
MLXXII
MCCLXXVIIII
MMMMCCXXXXI
MMDCCCLXXII
MMMMXXXI
MMMDCCLXXX
MMDCCCLXXIX
MMMMLXXXV
MCXXI
MDCCCXXXVII
MMCCCLXVII
MCDXXXV
CCXXXIII
CMXX
MMMCLXIV
MCCCLXXXVI
DCCCXCVIII
MMMDCCCCXXXIV
CDXVIIII
MMCCXXXV
MDCCCXXXII
MMMMD
MMDCCLXIX
MMMMCCCLXXXXVI
MMDCCXLII
MMMDCCCVIIII
DCCLXXXIIII
MDCCCCXXXII
MMCXXVII
DCCCXXX
CCLXIX
MMMXI
MMMMCMLXXXXVIII
MMMMDLXXXVII
MMMMDCCCLX
MMCCLIV
CMIX
MMDCCCLXXXIIII
CLXXXII
MMCCCCXXXXV
MMMMDLXXXVIIII
MMMDCCCXXI
MMDCCCCLXXXVI
MCCCCLXX
MMCDLVIIII
MMMDCCCLIX
MMMMCCCCXIX
MMMDCCCLXXV
XXXI
CDLXXXIII
MMMCXV
MMDCCLXIII
MMDXXX
MMMMCCCLVII
MMMDCI
MMMMCDLXXXIIII

MMMMCCCXVI
CCCLXXXVIII
MMMMCML
MMMMXXIV
MMMCCCCXXX
DCCX
MMMCCLX
MMDXXXIII
CCCLXIII
MMDCCXIII
MMMCCCXLIV
CLXXXXI
CXVI
MMMMCXXXIII
CLXX
DCCCXVIII
MLXVII
DLXXXX
MMDXXI
MMMMDLXXXXVIII
MXXII
LXI
DCCCCXLIII
MMMMDV
MMMMXXXIV
MDCCCLVIII
MMMCCLXXII
MMMMDCCXXXVI
MMMMLXXXIX
MDCCCLXXXI
MMMMDCCCXV
MMMMCCCCXI
MMMMCCCLIII
MDCCCLXXI
MMCCCCXI
MLXV
MMCDLXII
MMMMDXXXXII
MMMMDCCCXL
MMMMCMLVI
CCLXXXIV
MMMDCCLXXXVI
MMCLII
MMMCCCCXV
MMLXXXIII
MMMV
MMMV
DCCLXII
MMDCCCCXVI
MMDCXLVIII
CCLIIII
CCCXXV
MMDCCLXXXVIIII
MMMMDCLXXVIII
MMMMDCCCXCI
MMMMCCCXX
MMCCXLV
MMMDCCCLXIX
MMCCLXIIII
MMMDCCXLIX
MMMMCCCLXIX
CMLXXXXI
MCMLXXXIX
MMCDLXI
MMDCLXXVIII
MMMMDCCLXI
MCDXXV
DL
CCCLXXII
MXVIIII
MCCCCLXVIII
CIII
MMMDCCLXXIIII
MMMDVIII
MMMMCCCLXXXXVII
MMDXXVII
MMDCCLXXXXV
MMMMCXLVI
MMDCCLXXXII
MMMDXXXVI
MCXXII
CLI
DCLXXXIX
MMMCLI

MDCLXIII
MMMMDCCXCVII
MMCCCLXXXV
MMMDCXXVIII
MMMCDLX
MMMCMLII
MMMIV
MMMMDCCCLVIII
MMMDLXXXVIII
MCXXIV
MMMMLXXVI
CLXXIX
MMMMCCCCXXVIIII
DCCLXXXV
MMMDCCCVI
LI
CLXXXVI
MMMMCCCLXXVI
MCCCLXVI
CCXXXIX
MMDXXXXI
MMDCCCXLI
DCCCLXXXVIII
MMMMDCCCIV
MDCCCCXV
MCMVI
MMMMCMLXXXXV
MMDCCLVI
MMMMCCXLVIII
DCCCCIIII
MMCCCCIII
MMMDCCLXXXVIIII
MDCCCLXXXXV
DVII
MMMV
DCXXV
MMDCCCXCV
DCVIII
MMCDLXVI
MCXXVIII
MDCCXCVIII
MMDCLX
MMMDCCLXIV
MMCDLXXVII
MMDLXXXIIII
MMMMCCCXXII
MMMDCCCXLIIII
DCCCCLXVII
MMMCLXXXXIII
MCCXV
MMMMDCXI
MMMMDCLXXXXV
MMMMCCCLII
MMCMIX
MMDCCXXV
MMDLXXXVI
MMMMDCXXVIIII
DCCCCXXVIIII
MMCCXXXIIII
MMDCCLXXVIII
MDCCLXVIIII
MMCCLXXXV
MMMMDCCCLXXX-
VIII
MMCMXCI
MDXLII
MMMMDCCXIV
MMMMLI
DXXXXIII
MMDCCXI
MMMMCCLXXXIII
MMMDCCLXXIII
MDCLVII
MMCD
MCCCXXVII
MMMMDCCIIII
MMMDCCXLVI
MMMCLXXXVII
MMMCCVIIII
MCCCCLXXXIX
DL
DCCCLXXVI
MMDXCI
MMMMDCCCCXXXVI

MMCII
MMMDCCCXXXXV
MMMCDXLV
MMDCXXXXIV
MMD
MDCCCLXXXX
MMDCXLIII
MMCCXXXII
MMDCXXXXVIIII
DCCCLXXI
MDXCVIIII
MMMMCCLXXVIII
MDCLVIIII
MMMCCLXXXIX
MDCLXXXV
MDLVIII
MMMMCCVII
MMMMDCXIV
MMMCCLXIIII
MMIIII
MMMMCCCLXXIII
CCIII
MMMCCLV
MMMDXIII
MMMCCXC
MMMDCCCXXI
MMMMCCCCXXXII
CCCLVI
MMMCCLXXXVI
MXVIIII
MMMCCCCXIIII
CLXVII
MMMCCLXX
CCCCLXIV
MMXXXXII
MMMMCCLXXXX
MXL
CCXVI
CCCCLVIIII
MMCCCII
MCCCLVIII
MMMMCCX
MCDLXXXXIV
MDCCCXIII
MMDCCCXL
MMMMCCCXXIII
DXXXIV
CVI
MMMMDCLXXX
DCCCVII
MMCMLXIIII
MMMDCCCXXXIII
DCCC
MDIII
MMCCCLXVI
MMMCCCLXXI
MMDCCCCXVIII
CCXXXVII
CCCXXV
MDCCCXII
MMMCMV
MMMMCMXV
MMMMDCXCI
DXXI
MMCCXLVIIII
MMMMCMLII
MDLXXX
MMDCLXVI
CXXI
MMMDCCCLIIII
MMMCXXI
MCCIII
MMDCXXXXI
CCXCII
MMMMDXXXV
MMMCCCLXV
MMMMDLXV
MMMCCCCXXXII
MMMCCCVIII
DCCCCLXXXXII
MMCLXIV
MMMMCXI
MLXXXXVII
MMMCDXXXVIII

MDXXII
MLV
MMMMDLXVI
MMMCXII
XXXIII
MMMMDCCCXXVI
MMMLXVIIII
MMMLX
MMMCDLXVII
MDCCCLVII
MMCXXXVII
MDCCCCXXX
MMDCCCLXIII
MMMMDCXLIX
MMMMCMXLVIII
DCCCLXXVIIII
MDCCCLIII
MMMCMLXI
MMMMCCLXI
MMDCCCLIII
MMMDCCCVI
MMDXXXXIX
MMCLXXXXV
MMDXXX
MMMXIII
DCLXXIX
DCCLXII
MMMMDCCLXVIII
MDCXXXXIII
CCXXXI
MMMMDCXXV
MMMMCCXXVIII
MDCVIII
MMMCLXXXXIIII
CLXXXI
MDCCCCXXXIII
MMMMDCXXX
MMMDCXXIV
MMMCCXXXVII
MCCCXXXXIIII
CXVIII
MMDCCCCIV
MMMMDCLXXV
MMMDLXIV
MDXCIII
MCCLXXXI
MMMDCCCXXIV
MCXLIII
MMMDCCCI
MCCLXXX
CCXV
MMDCCLXXI
MMDLXXXIII
MMMMDCXVII
MMMCMLXV
MCLXVIII
MMMMCCLXXVI
MMMDCCLXVIIII
MMMMDCCCIX
DLXXXXIX
DCCCXXII
MMMMIII
MMMMCCCLXXVI
DCCCXCIII
DXXXI
MXXXIIII
CCXII
MMMDCCLXXXIIII
MMMCXX
MMMCMXXVII
DCCCXXXX
MMCDXXXVIIII
MMMMDCCXVIII
LV
MMMDCCCCVI
MCCCII
MMCMLXVIIII
MDCCXI
MMMMDLXVII
MMCCCCLXI
MMDCCV
MMMCCCXXXIIII
MMMMDI
MMMDCCCXCV

MMDCCLXXXXI
MMMDXVI
MMMDCCCLVI
MMDCXXX
MCCCVII
MMMMCCCLXII
MMMMXXV
MMCMXXV
MMLVI
MMDXXX
MMMMCVII
MDC
MCCIII
MMMMDCC
MMCCLXXV
MMDCCCXXXXVI
MMMMCCCLXV
CDXIIII
MLXIIII
CCV
MMMCMXXXI
CCCCLXVI
MDXXXII
MMMMCCCLVIII
MMV
MMMCLII
MCMLI
MMDCCXX
MMMMCCCCXXXVI
MCCLXXXI
MMMCMVI
DCCXXX
MMMMCCCLXV
DCCCXI
MMMMDCCCXIV
CCCXXI
MMDLXXV
CCCCLXXXX
MCCCLXXXXII
MMDCIX
DCCXLIIII
DXIV
MMMMCLII
CDLXI
MMMCXXVII
MMMMDCCCCLXIII
MMMDCLIIII
MCCCCXXXXIII
MMCCCLX
CCCCLIII
MDCCLXXXVI
MCMXXIII
MMMMDLXXVIII
MMDCCCCLX
MMMMCCCLXXXX
MMMCDXXVI
MMMDLVIII
CCCLXI
MMMMDCXXII
MMDCCCXXI
MMDCXIII
MMMMCLXXXVI
MDCCCCXXVI
MDV
MMDCCCCLXXVI
MMMMCCXXXVII
MMMDCCLXXVIIII
MMMMCCCLXVII
DCCXLI
MMCLXXXVIII
MCCXXXVI
MMDCXLVIII
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MMMMDCCLXVI
MMMMCMLI
MMMMCLXV
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MCCLXXVII
LXXVIIII
DCCLII
MMMMCCXCVI
MMMCLV
MMDCCCXXXXVIII
DCCCXV

MXC
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MMDCCCLXXVIII
DXXI
MCCCXLI
DCLXXXXI
MMCCCLXXXXVIII
MDCCCCLXXVIII
MMMMDXXV
MMMDCXXXVI
MMMCMXCVII
MMXVIIII
MMMDCCLXXIV
MMMCXXV
DXXXVIII
MMMMCLXVI
MDXII
MMCCCLXX
CCLXXI
DXIV
MMMCLIII
DLII
MMMCCCXLIX
MMCCCCXXVI
MMDCXLIII
MXXXXII
CCCLXXXV
MDCLXXVI
MDCXII
MMMCCCLXXXIII
MMDCCCCLXXXII
MMMMCCCLXXXV
MMDCXXI
DCCCXXX
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MMMDCCXXII
MMMMCDXCVIII
MMMCCLXVIIII
MMXXV
MMMMCDXIX
MMMMCCCX
MMMCCCCLXVI
MMMMDCLXXVIIII
MMMMDCXXXXIV
MMMCMXII
MMMMXXXIII
MMMMDLXXXII
DCCCLIV
MDXVIIII
MMMCLXXXXV
CCCCXX
MMDIX
MMCMLXXXVIII
DCCXLIII
DCCLX
D
MCCCVII
MMMMCCCLXXXIII
MDCCCLXXIIII
MMMDCCCCLXXXVII
MMMMCCCVII
MMMDCCLXXXXVI
CDXXXIV
MCCLXVIII
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MDCCCXXXXIV
XXXXVII
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MCXXXV
MDCXXVIIII
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DCLXXXXVIIII
LXXXXI
CXXXIII

MCDX
MCCLVII
MDCXXXXII
MMMCXXIV
MMMMLXXXX
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MLXXX
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MCDLIII
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MMMDCVIII
DCCCCXXIII
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MMDCCIV
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CCCLXXI
MCCLXXXII
MCMIII
CCXXXI
DCCXXXVIII
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DCCXCI
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CCCXCV
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MCCCCII
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CCXLIII
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MCXV
MMCCXXV
MLXXIIII
MDCCXXVI
MMMCCCXX
MMDLXX
MMCCCCVI
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XCIV
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MMXVII
MLXXI
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MDCCCCII
MMMCMLVII
MMCLXXXXVIII
MDCCCCLV
MCCCCLXXIIII
MCCCLII
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DCCLXXXIX
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MDCCCCXLIII
CLXXXXV
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DCCCLV
MCDLXX
DCLXXVIII
MMDCXXXIX
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MMDCCLI
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MMMMCMLXII
MMMMCCV
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MMMMCCIII
CLXVII
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MMLXXIX
CMLXIX
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DCLXII
MMMCCCXLVII
MDCCCXXXV
MMMMDCCXCVI
DCXXX
XXVI
MMLXIX
MMCXI
DCXXXVII
MMMMCCCXXXXVIII
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MMMMDCLXXIIII
MMMMVIII
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MDCXCI
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CCCCXXXXV
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MCVI
MMDCCLXVIII
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MLXVIII
CMXXVII
CCCLV
MDCCLXXXIX
MMMCCCCLXV
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MDLXVI
MMMCCCXVIII
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MMMCXCII
MMMMDCLVIII
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MDCCXL
MDCCLVII
MMMMDCCCLXXXVI
DCCXXXIII
MMMMDCCCCLXXXV
MMCCXXXXVIII
MMMCCLXXVIII
MMMDCLXXVIII
DCCCI
MMMMLXXXXVIIII
MMMCCCCLXXII
MMCLXXXVII
CCLXVI
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MMCXXVIII
MDXIV
CCCXCVIII
CLXXVIII
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MMMDCLXXXIV
CMLVIII
MCDLIX
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MMMMDCXXXIIII
MDCXXI
MMMDCXLV
MCLXXVIII
MCDXXII
IV
MCDLXXXIII
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CCLI
MMMMDCCCXXXVIII
DCLXII
MCCCLXVII
MMMMDCCCXXXVI
MMDCCXLI
MLXI
MMMCDLXVIII
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XXXIII
MMMDCLXIII
MMMMDCL

DCCCXXXXIIII
MMDLVII
DXXXVII
MCCCCXXIIII
MCVII
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MCCCCXXIV
MMCLXVIII
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MMMCCIIII
DCCLXXXVIIII
MDCLIV
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CMLXXXI
DCCLXXXVII
XXV
MMMXXXVI
MDVIIII
CLXIII
MMMCDLVIIII
MMCCCCVII
MMMLXX
MXXXXII
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MMDCCCXXVIII
MMMMDCXXXXI
MMMMDCCCXXXXV
MMMXV
MMMMCCXVIIII
MMDCCXIIII
MMMXXVII
MDCCLVIIII
MMCXXIIII
MCCCLXXIV
DCLVIII
MMMLVII
MMMCXLV
MMXCVII
MMMCCCLXXXVII
MMMMCCXXII
DXII
MMMDLV
MCCCLXXVIII
MMMCLIIII
MMMMCLXXXX
MMMCLXXXIIII
MDCXXIII
MMMMCCXVI
MMMMDLXXXIII
MMMDXXXIII
MMMMCCCCLV
MMMDLXXXI
MMMCCLXXVI
MMMMXX
MMMMDLVI
MCCCCLXXX
MMMXXII
MMXXII
MMDCCCCXXXI
MMMDXXV
MMMDCLXXXVIIII
MMMDLXXXXVII
MDLXIIII
CMXC
MMMXXXVIII
MDLXXXVIII
MCCCLXXVI
MMCDLIX
MMDCCCXVIII
MDCCCXXXXVI
MMMMCMIV
MMMMDCIIII
MMCCXXXV
XXXXVI
MMMMCCXVII
MMCCXXIV
MCMLVIIII
MLXXXIX

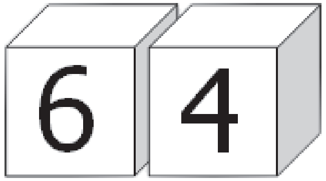
MMMMLXXXIX
CLXXXXIX
MMMDCCCCLVIII
MMMMCCLXXIII
MCCCC
DCCCLIX
MMMCCCLXXXII
MMMCCLXVIIII
MCLXXXV
CDLXXXVII
DCVI
MMX
MMCCXIII
MMMMDCXX
MMMMXXVIII
DCCCLXII
MMMMCCCXLIII
MMMMCLXV
DXCI
MMMMCLXXX
MMMDCCXXXXI
MMMMXXXXVI
DCLX
MMMCCCXI
MCCLXXX
MMCDLXXII
DCCLXXI
MMMCCCXXXVI
MCCCCLXXXVIIII
CDLVIII
DCCLVI
MMMMDCXXXVIII
MMCCCLXXXIII
MMMMDCCLXXV
MMMXXXVI
CCCLXXXXIX
CV
CCCCXIII
CCCCXVI
MDCCCLXXXIIII
MMDCCLXXXII
MMMMCCCCLXXXI
MXXV
MMCCCLXXVIIII
MMMCCXII
MMMMCCXXXIII
MMCCCLXXXVI
MMMDCCCLVIIII
MCCXXXVII
MDCLXXV
XXXV
MMDLI
MMMCCXXX
MMMMCXXXXV
CCCCLIX
MMMMDCCCLXXIII
MMCCCXVII
DCCCXVI
MMMCCCXXXXV
MDCCCCXCV
CLXXXI
MMMMDCCLXX
MMMDCCCIII
MMCLXXVII
MMMDCCXXIX
MMDCCCXCIIII
MMMCDXXIIII
MMMMXXVIII
MMMMDCCCCLXVIII
MDCCCXX
MMMMCDXXI
MMMMDLXXXIX
CCXVI
MDVIII
MMCCLXXI
MMMDCCCLXXI
MMMCCCLXXVI
MMCCLXI
MMMMDCCCXXXIV
DLXXXVI
MMMMDXXXII
MMMXXIIII
MMMMCDIV
MMMMCCCXLVIII

MMMMCXXXVIII
MMMCCCLXVI
MDCCXVIII
MMCXX
CCCLIX
MMMMDCCLXXII
MDCCCLXXV
MMMMDCCCXXIV
DCCCXXXXVIII
MMMDCCCCXXX-
VIIII
MMMMCCXXXV
MDCLXXXIII
MMCCLXXXIV
MCLXXXXIIII
DXXXXIII
MCCCXXXXVIII
MMCLXXIX
MMMMCCLXIV
MXXII
MMMCXIX
MDCXXXVII
MMDCCVI
MCLXXXXVIII
MMMCXVI
MCCCLX
MMMCDX
CCLXVIIII
MMMCCLX
MCXXVIII
LXXXII
MCCCCLXXXI
MMMI
MMMCCCLXIV
MMMCCCXXVIIII
CXXXVIII
MMCCCXX
MMMCCXXVIIII
MCCLXVI
MMMCCCCXXXXVI
MMDCCXCIX
MCMLXXI
MMCCLXVIII
CDLXXXXIII
MMMMDCCXXII
MMMMDCCLXXXVII
MMMDCCLIV
MMCCLXIII
MDXXXVII
DCCXXXIIII
MCII
MMMDCCCLXXI
MMMLXXIII
MDCCCLIII
MMXXXVIII
MDCCXVIIII
MDCCCCXXXVII
MMCCCXVI
MCMXXII
MMMCCCLVIII
MMMMDCCCXX
MCXXIII
MMMDLXI
MMMMDXXII
MDCCCX
MMDXCVIIII
MMMDCCCCVIII
MMMMDCCCCXXXXVI
MMDCCCXXXV
MMCXCIV
MCMLXXXIII
MMMCCCLXXVI
MMMMDCLXXXV
CMLXIX
DCXCII
MMXXVIII
MMMMCCCXXX
XXXXVIIII

Problem 90

Each of the six faces on a cube has a different digit (0 to 9) written on it; the same is done to a second cube. By placing the two cubes side-by-side in different positions we can form a variety of 2-digit numbers.

For example, the square number 64 could be formed:



In fact, by carefully choosing the digits on both cubes it is possible to display all of the square numbers below one-hundred: 01, 04, 09, 16, 25, 36, 49, 64, and 81.

For example, one way this can be achieved is by placing {0, 5, 6, 7, 8, 9} on one cube and {1, 2, 3, 4, 8, 9} on the other cube.

However, for this problem we shall allow the 6 or 9 to be turned upside-down so that an arrangement like {0, 5, 6, 7, 8, 9} and {1, 2, 3, 4, 6, 7} allows for all nine square numbers to be displayed; otherwise it would be impossible to obtain 09.

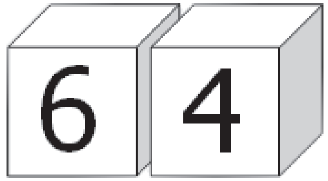
In determining a distinct arrangement we are interested in the digits on each cube, not the order.

{1, 2, 3, 4, 5, 6} is equivalent to {3, 6, 4, 1, 2, 5}
{1, 2, 3, 4, 5, 6} is distinct from {1, 2, 3, 4, 5, 9}

Problem 90

Jede der sechs Flächen eines Würfels trägt eine andere Ziffer (0 bis 9), ebenso ein zweiter Würfel. Stellt man die beiden Würfel nebeneinander in verschiedenen Positionen hin, läßt sich eine Vielzahl zweistelliger Zahlen bilden.

So läßt sich beispielsweise die Quadratzahl 64 bilden:



Wenn man die Ziffern auf beiden Würfel sorgfältig wählt, lassen sich alle Quadratzahlen unter einhundert anzeigen: 01, 04, 09, 16, 25, 36, 49, 64 und 81.

Dies läßt sich beispielsweise dadurch erreichen, daß man {0, 5, 6, 7, 8, 9} auf einen Würfel schreibt und {1, 2, 3, 4, 8, 9} auf den zweiten.

Für dieses Problem erlauben wir zusätzlich, daß die 6 oder 9 auf den Kopf gestellt wird, so daß man auch mit einer Anordnung wie {0, 5, 6, 7, 8, 9} und {1, 2, 3, 4, 6, 7} alle neun Quadratzahlen anzeigen kann, anderenfalls könnte man die 09 nicht erzeugen.

Bei den unterschiedlichen Anordnungen geht es uns um die Ziffern auf den beiden Würfeln, nicht um ihre Reihenfolge.

{1, 2, 3, 4, 5, 6} ist gleichwertig mit {3, 6, 4, 1, 2, 5}
{1, 2, 3, 4, 5, 6} ist gleichwertig mit {1, 2, 3, 4, 5, 9}

Problem 90, page 2

But because we are allowing 6 and 9 to be reversed, the two distinct sets in the last example both represent the extended set {1, 2, 3, 4, 5, 6, 9} for the purpose of forming 2-digit numbers.

How many distinct arrangements of the two cubes allow for all of the square numbers to be displayed?

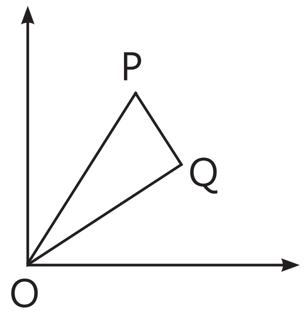
Problem 90, Seite 2

Da wir aber zulassen, daß 6 und 9 umgedreht werden, stellen beide unterschiedliche Mengen aus dem letzten Beispiel zur Bildung zweistelliger Zahlen die erweiterte Menge {1, 2, 3, 4, 5, 6, 9} dar.

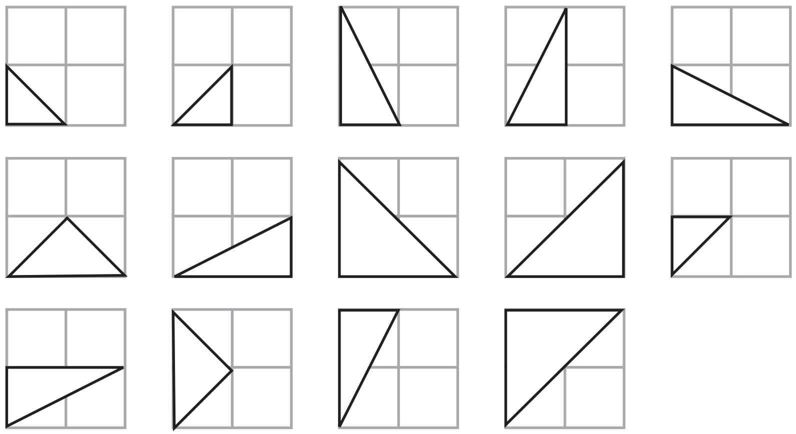
Mit wie vielen unterschiedlichen Anordnungen der beiden Würfel lassen sich alle Quadratzahlen anzeigen?

Problem 91

The points P (x_1, y_1) and Q (x_2, y_2) are plotted at integer co-ordinates and are joined to the origin, O(0,0), to form ΔOPQ.



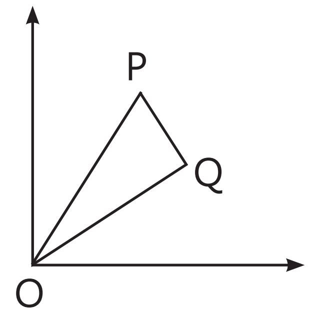
There are exactly fourteen triangles containing a right angle that can be formed when each co-ordinate lies between 0 and 2 inclusive; that is, $0 \le x_1, y_1, x_2, y_2 \le 2$.



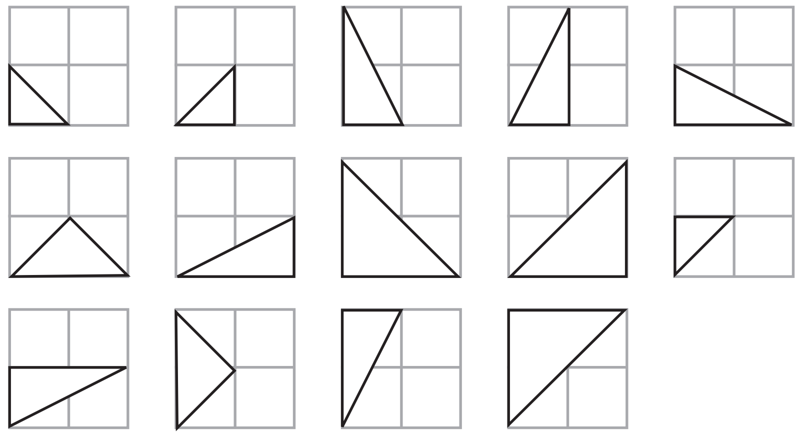
Given that $0 \le x_1, y_1, x_2, y_2 \le 50$, how many right triangles can be formed?

Problem 91

Zeichne die Punkte $P(x_1, y_1)$ und $Q(x_2, y_2)$ auf ganzzahlige Koordinaten, verbinde sie mit dem Ursprung O(0,0), um ΔOPQ zu bilden.



Es entstehen genau vierzehn rechtwinklige Dreiecke, wenn jede Koordinate zwischen (inklusive) 0 und 2 liegt, wenn also gilt: $0 \le x_1, y_1, x_2, y_2 \le 2$.



Wenn gilt, daß $0 \le x_1, y_1, x_2, y_2 \le 50$, wie viele rechtwinklige Dreiecke lassen sich dann bilden?

Problem 92

A number chain is created by continuously adding the square of the digits in a number to form a new number until it has been seen before.

For example,

44 ⇨ 32 ⇨ 13 ⇨ 10 ⇨ **1** ⇨ **1**
85 ⇨ **89** ⇨ 145 ⇨ 42 ⇨ 20 ⇨ 4 ⇨ 16 ⇨ 37 ⇨ 58 ⇨ **89**

Therefore any chain that arrives at 1 or 89 will become stuck in an endless loop. What is most amazing is that EVERY starting number will eventually arrive at 1 or 89.

How many starting numbers below ten million will arrive at 89?

Problem 92

Man bildet eine Zahlenkette, indem man die Quadrate der Ziffern einer Zahl immer wieder addiert und so eine neue Zahl erhält. Damit hört man auf, sobald sich eine Zahl wiederholt.

Zum Beispiel:

44 ⇨ 32 ⇨ 13 ⇨ 10 ⇨ **1** ⇨ **1**
85 ⇨ **89** ⇨ 145 ⇨ 42 ⇨ 20 ⇨ 4 ⇨ 16 ⇨ 37 ⇨ 58 ⇨ **89**

Entsprechend wird jede Kette, die bei 1 oder 89 ankommt, in einer endlosen Schleife verharren. Höchst erstaunlich ist dabei, daß irgendwann JEDE Startzahl einmal bei 1 oder bei 89 ankommt.

Wie viele Startzahlen unter zehn Millionen führen zur 89?

Problem 93

By using each of the digits from the set, {1, 2, 3, 4}, exactly once, and making use of the four arithmetic operations (+, −, *, /) and brackets/parentheses, it is possible to form different positive integer targets.

For example,

$$8 = (4 * (1 + 3)) / 2$$
$$14 = 4 * (3 + 1 / 2)$$
$$19 = 4 * (2 + 3) - 1$$
$$36 = 3 * 4 * (2 + 1)$$

Note that concatenations of the digits, like 12 + 34, are not allowed.

Using the set, {1, 2, 3, 4}, it is possible to obtain thirty-one different target numbers of which 36 is the maximum, and each of the numbers 1 to 28 can be obtained before encountering the first non-expressible number.

Find the set of four distinct digits, $a < b < c < d$, for which the longest set of consecutive positive integers, 1 to n, can be obtained, giving your answer as a string: abcd.

Problem 93

Indem man jede der Ziffern aus der Menge {1, 2, 3, 4} genau ein Mal benutzt und die vier Grundrechenarten (+, −, *, /) sowie Klammern benutzt, kann man unterschiedliche ganze Zielzahlen errechnen.

Zum Beispiel:

$$8 = (4 \cdot (1 + 3)) : 2$$
$$14 = 4 \cdot (3 + 1 : 2)$$
$$19 = 4 \cdot (2 + 3) - 1$$
$$36 = 3 \cdot 4 \cdot (2 + 1)$$

Dabei ist zu beachten, daß man Ziffern nicht aneinanderhängen darf, etwa wie in 12 + 34.

Mit der Menge {1, 2, 3, 4} kann man 31 verschiedenen Zielzahlen bilden, deren höchste 36 ist, und jede der Zahlen von 1 bis 28 ist ausdrückbar, bevor man auf die erste Zahl stößt, die sich mit diesen Ziffern und Grundrechenarten nicht bilden läßt.

Ermittle die Menge mit vier unterschiedlichen Ziffern, $a< b < c < d$, für die man die längste ununterbrochenen Kette positiver ganzer Zahlen von 1 bis n bilden kann. Gib die Antwort als Zeichenkette an: abcd.

Problem 94

It is easily proved that no equilateral triangle exists with integral length sides and integral area. However, the almost equilateral triangle 5-5-6 has an area of 12 square units.

We shall define an *almost equilateral triangle* to be a triangle for which two sides are equal and the third differs by no more than one unit.

Find the sum of the perimeters of all almost equilateral triangles with integral side lengths and area and whose perimeters do not exceed one billion (1,000,000,000).

Problem 94

Es läßt sich leicht zeigen, daß es kein gleichseitiges Dreieck mit ganzzahligen Seitenlängen und ganzzahligem Flächeninhalt gibt. Das fast gleichseitige Dreieck 5-5-6 jedoch hat einen Flächeninhalt von 12 Flächeneinheiten.

Wir definieren ein *fast-gleichseitiges Dreieck*: ein Dreieck, bei dem zwei Seiten gleich lang sind und die dritte höchstens um eine Einheit abweicht.

Emittle die Summe der Umfänge aller fast-gleichseitigen Dreiecke mit ganzzahligen Seitenlängen und ganzzahligem Flächeninhalt, deren Umfänge 1 Milliarde (1.000.000.000) nicht überschreiten.

Problem 95

The proper divisors of a number are all the divisors excluding the number itself. For example, the proper divisors of 28 are 1, 2, 4, 7, and 14. As the sum of these divisors is equal to 28, we call it a *perfect number*.

Interestingly the sum of the proper divisors of 220 is 284 and the sum of the proper divisors of 284 is 220, forming a chain of two numbers. For this reason, 220 and 284 are called an *amicable pair.*

Perhaps less well known are longer chains. For example, starting with 12496, we form a chain of five numbers:

12496 ⇨ 14288 ⇨ 15472 ⇨ 14536 ⇨ 14264 (⇨ 12496 ⇨ ...)

Since this chain returns to its starting point, it is called an *amicable chain.*

Find the smallest member of the longest amicable chain with no element exceeding one million.

Problem 95

Die echten Teiler einer Zahl sind alle ihre Teiler außer der Zahl selbst. Die echten Teiler von 28 sind beispielsweise 1, 2, 4, 7 und 14. Da die Summe dieser Teiler wieder 28 ergibt, nennen wir 28 eine *perfekte Zahl*.

Interessanterweise ist die Summe der echten Teiler von 220 genau 284, und die Summe der echten Teiler von 284 ergibt 220 und damit eine Kette von zwei Zahlen. Deshalb nennen wir 220 und 284 ein *befreundetes Paar*.

Vielleicht weniger bekannt sind längere Ketten. Wenn wir beispielsweise mit 12496 beginnen, bilden wir eine Kette aus fünf Zahlen:

12496 ⇨ 14288 ⇨ 15472 ⇨ 14536 ⇨ 14264 (⇨ 12496 ⇨ ...)

Da diese Kette an ihren Anfangspunkt zurückkehrt, nennen wir sie eine *befreundete Kette.*

Ermittle das zahlenmäßig kleinste Mitglied der längsten befreundeten Kette, in dem kein Element eine Million überschreitet.

Problem 96

Su Doku (Japanese meaning number place) is the name given to a popular puzzle concept. Its origin is unclear, but credit must be attributed to Leonhard Euler who invented a similar, and much more difficult, puzzle idea called Latin Squares. The objective of Su Doku puzzles, however, is to replace the blanks (or zeros) in a 9 by 9 grid in such that each row, column, and 3 by 3 box contains each of the digits 1 to 9. Below is an example of a typical starting puzzle grid and its solution grid.

0	0	3	0	2	0	6	0	0
9	0	0	3	0	5	0	0	1
0	0	1	8	0	6	4	0	0
0	0	8	1	0	2	9	0	0
7	0	0	0	0	0	0	0	8
0	0	6	7	0	8	2	0	0
0	0	2	6	0	9	5	0	0
8	0	0	2	0	3	0	0	9
0	0	5	0	1	0	3	0	0

4	8	3	9	2	1	6	5	7
9	6	7	3	4	5	8	2	1
2	5	1	8	7	6	4	9	3
5	4	8	1	3	2	9	7	6
7	2	9	5	6	4	1	3	8
1	3	6	7	9	8	2	4	5
3	7	2	6	8	9	5	1	4
8	1	4	2	5	3	7	6	9
6	9	5	4	1	7	3	8	2

A well constructed Su Doku puzzle has a unique solution and can be solved by logic, although it may be necessary to employ »guess and test« methods in order to eliminate options. The complexity of the search determines the difficulty of the puzzle; the example above is considered easy because it can be solved by straight forward direct deduction.

The following page (copy & paste from the page), contains fifty different Su Doku puzzles ranging in difficulty, but all with unique solutions (the first puzzle in the file is the example above).

By solving all fifty puzzles find the sum of the 3-digit numbers found in the top left corner of each solution grid; for example, 483 is the 3-digit number found in the top left corner of the solution grid above.

Problem 96

Su Doku (japanisch für Zahlenort) heißt eine populäre Puzzle-Idee. Der Ursprung ist unklar, aber Leonhard Euler gebührt sicher ein Teil des Verdienstes, denn er erfand eine ähnliche, und ungleich schwierigere, Puzzle-Idee namens *Magische Quadrate*. Das Ziel bei einem Su Doku ist es, die Leerstellen (oder Nullen) in einem 9x9-Gitter so auszufüllen, daß in jeder Zeile, Spalte und in jedem 9x9-Kasten jede der Ziffern von 1 bis 9 enthalten ist. Untenstehend ein Beispiel für ein typisches Startgitter und sein Lösungsgitter.

0	0	3	0	2	0	6	0	0
9	0	0	3	0	5	0	0	1
0	0	1	8	0	6	4	0	0
0	0	8	1	0	2	9	0	0
7	0	0	0	0	0	0	0	8
0	0	6	7	0	8	2	0	0
0	0	2	6	0	9	5	0	0
8	0	0	2	0	3	0	0	9
0	0	5	0	1	0	3	0	0

4	8	3	9	2	1	6	5	7
9	6	7	3	4	5	8	2	1
2	5	1	8	7	6	4	9	3
5	4	8	1	3	2	9	7	6
7	2	9	5	6	4	1	3	8
1	3	6	7	9	8	2	4	5
3	7	2	6	8	9	5	1	4
8	1	4	2	5	3	7	6	9
6	9	5	4	1	7	3	8	2

Ein gut konstruiertes Su Doku-Puzzle hat eine eindeutige Lösung und läßt sich durch Logik lösen, wobei es zur Eliminierung von Möglichkeiten nötig sein kann, zu raten und zu testen. Die Komplexität der Suche bestimmt die Schwierigkeit des Puzzles. Das Beispiel oben gilt als leicht, weil es sich durch einfache Deduktion lösen läßt.

Die nächste Seite (kopieren und einsetzen) enthält fünfzig unterschiedlich schwierige verschiedene Su Doku-Puzzle. Alle haben eine eindeutige Lösung (das erste Su Doku ist das oben gezeigte).

Die Lösung dieses Problems ist die Summe aller dreistelligen Zahlen in den oberen linken Ecken der Lösungsgitter; im Lösungsgitter oben beispielsweise ist 483 die dreistellige Zahl.

Su Doku list for/Su Doku-Liste für Problem 98

Grid 01
003020600
900305001
001806400
008102900
700000008
006708200
002609500
800203009
005010300
Grid 02
200080300
060070084
030500209
000105408
000000000
402706000
301007040
720040060
004010003
Grid 03
000000907
000420180
000705026
100904000
050000040
000507009
920108000
034059000
507000000
Grid 04
030050040
008010500
460000012
070502080
000603000
040109030
250000098
001020600
080060020
Grid 05
020810740
700003100
090002805
009040087
400208003
160030200
302700060
005600008
076051090
Grid 06
100920000
524010000
000000070
050008102
000000000
402700090
060000000
000030945
000071006
Grid 07
043080250
600000000
000001094
900004070
000608000
010200003
820500000
000000005
034090710
Grid 08
480006902
002008001
900370060
840010200
003704100
001060049
020085007
700900600
609200018
Grid 09
000900002
050123400
030000160
908000000
070000090
000000205
091000050
007439020
400007000
Grid 10
001900003
900700160
030005007
050000009
004302600
200000070
600100030
042007006
500006800
Grid 11
000125400
008400000
420800000
030000095
060902010
510000060
000003049
000007200
001298000
Grid 12
062340750
100005600
570000040
000094800
400000006
005830000
030000091
006400007
059083260
Grid 13
300000000
005009000
200504000
020000700
160000058
704310600
000890100
000067080
000005437
Grid 14
630000000
000500008
005674000
000020000
003401020
000000345
000007004
080300902
947100080
Grid 15
000020040
008035000
000070602
031046970
200000000
000501203
049000730
000000010
800004000
Grid 16
361025900
080960010
400000057
008000471
000603000
259000800
740000005
020018060
005470329
Grid 17
050807020
600010090
702540006
070020301
504000908
103080070
900076205
060090003
080103040
Grid 18
080005000
000003457
000070809
060400903
007010500
408007020
901020000
842300000
000100080
Grid 19
003502900
000040000
106000305
900251008
070408030
800763001
308000104
000020000
005104800
Grid 20
000000000
009805100
051907420
290401065
000000000
140508093
026709580
005103600
000000000
Grid 21
020030090
000907000
900208005
004806500
607000208
003102900
800605007
000309000
030020050
Grid 22
005000006
070009020
000500107
804150000
000803000
000092805
907006000
030400010
200000600
Grid 23
040000050
001943600
009000300
600050002
103000506
800020007
005000200
002436700
030000040
Grid 24
004000000
000030002
390700080
400009001
209801307
600200008
010008053
900040000
000000800
Grid 25
360020089
000361000
000000000
803000602
400603007
607000108
000000000
000418000
970030014
Grid 26
500400060
009000800
640020000
000001008
208000501
700500000
000090084
003000600
060003002
Grid 27
007256400
400000005
010030060
000508000
008060200
000107000
030070090
200000004
006312700
Grid 28
000000000
079050180
800000007
007306800
450708096
003502700
700000005
016030420
000000000
Grid 29
030000080
009000500
007509200
700105008
020090030
900402001
004207100
002000800
070000090
Grid 30
200170603
050000100
000006079
000040700
000801000
009050000
310400000
005000060
906037002
Grid 31
000000080
800701040
040020030
374000900
000030000
005000321
010060050
050802006
080000000
Grid 32
000000085
000210009
960080100
500800016
000000000
890006007
009070052
300054000
480000000
Grid 33
608070502
050608070
002000300
500090006
040302050
800050003
005000200
010704090
409060701
Grid 34
050010040
107000602
000905000
208030501
040070020
901080406
000401000
304000709
020060010
Grid 35
053000790
009753400
100000002
090080010
000907000
080030070
500000003
007641200
061000940
Grid 36
006080300
049070250
000405000
600317004
007000800
100826009
000702000
075040190
003090600
Grid 37
005080700
700204005
320000084
060105040
008000500
070803010
450000091
600508007
003010600
Grid 38
000900800
128006400
070800060
800430007
500000009
600079008
090004010
003600284
001007000
Grid 39
000080000
270000054
095000810
009806400
020403060
006905100
017000620
460000038
000090000
Grid 40
000602000
400050001
085010620
038206710
000000000
019407350
026040530
900020007
000809000
Grid 41
000900002
050123400
030000160
908000000
070000090
000000205
091000050
007439020
400007000
Grid 42
380000000
000400785
009020300
060090000
800302009
000040070
001070500
495006000
000000092
Grid 43
000158000
002060800
030000040
027030510
000000000
046080790
050000080
004070100
000325000
Grid 44
010500200
900001000
002008030
500030007
008000500
600080004
040100700
000700006
003004050
Grid 45
080000040
000469000
400000007
005904600
070608030
008502100
900000005
000781000
060000010
Grid 46
904200007
010000000
000706500
000800090
020904060
040002000
001607000
000000030
300005702
Grid 47
000700800
006000031
040002000
024070000
010030080
000060290
000800070
860000500
002006000
Grid 48
001007090
590080001
030000080
000005800
050060020
004100000
080000030
100020079
020700400
Grid 49
000003017
015009008
060000000
100007000
009000200
000500004
000000020
500600340
340200000
Grid 50
300200000
000107000
706030500
070009080
900020004
010800050
009040301
000702000
000008006

Problem 97

The first known prime found to exceed one million digits was discovered in 1999, and is a Mersenne prime of the form $2^{6972593}-1$; it contains exactly 2,098,960 digits. Subsequently other Mersenne primes, of the form 2^p-1, have been found which contain more digits.

However, in 2004 there was found a massive non-Mersenne prime which contains 2,357,207 digits: $28433\times2^{7830457}+1$.

Find the last ten digits of this prime number.

Problem 97

Die erste Primzahlen mit mehr als einer Million Ziffern wurde 1999 entdeckt und ist eine Mersenne-Primzahl der Form $2^{6972593}-1$ mit genau 2.098.960 Ziffern. Danach wurden noch weitere Mersenne-Primzahlen der Form 2^p-1 mit noch mehr Ziffern entdeckt.

Im Jahr 2004 hat man jedoch eine gigantische nicht-Mersenne-Primzahl mit 2.357.207 Ziffern entdeckt: $28433\times2^{7830457}+1$.

Zu ermitteln sind die letzten zehn Ziffern dieser Primzahl.

Problem 98

By replacing each of the letters in the word CARE with 1, 2, 9, and 6 respectively, we form a square number: $1296 = 36^2$. What is remarkable is that, by using the same digital substitutions, the anagram, RACE, also forms a square number: $9216 = 96^2$. We shall call CARE (and RACE) a square anagram word pair and specify further that leading zeroes are not permitted, neither may a different letter have the same digital value as another letter.

Using the nearly two-thousand common English words to be found on the following three pagea (copy & paste), find all the square anagram word pairs (a palindromic word is NOT considered to be an anagram of itself).

What is the largest square number formed by any member of such a pair?

NOTE: All anagrams formed must be contained in the given text file.

Problem 98

Wenn man die Buchstaben des Wortes CARE durch 1, 2, 9 und 6 ersetzt, erhalten wir eine Quadratzahl: $1296 = 36^2$. Bemerkenswerterweise ergibt die gleiche Ersetzung durch Ziffern im Anagramm RACE ebenfalls eine Quadratzahl: $9216 = 96^2$. Nennen wir CARE (und RACE) ein Paar von Quadratanagrammen und legen weiterhin fest, daß führende Nullen nicht erlaubt sind und unterschiedliche Buchstaben unterschiedliche Werte haben müssen.

Ermittle in den fast zweitausend gebräuchlichen englischen Wörtern auf den nächsten drei Seiten (kopieren und einfügen) alle Paare von Quadratanagrammen (ein Palindrom-Wort ist hierbei kein Anagramm seiner selbst).

Welches ist die größte Quadratzahl, die von irgendeinem Mitglied eines Quadratanagramm-Paares gebildet wird?

Anmerkung: Alle Anagramme müssen in der Wortliste enthalten sein.

Word list for/Wortliste für Problem 98

”A”,”ABILITY”,”ABLE”,”ABOUT”,”ABOVE”,”ABSENCE”,”ABSOLUTELY”,”ACADEMIC”,”ACCEPT”,”ACCESS”,”ACCIDENT”,”ACCOMPANY”,”ACCORDING”,”ACCOUNT”,”ACHIEVE”,”ACHIEVEMENT”,”ACID”,”ACQUIRE”,”ACROSS”,”ACT”,”ACTION”,”ACTIVE”,”ACTIVITY”,”ACTUAL”,”ACTUALLY”,”ADD”,”ADDITION”,”ADDITIONAL”,”ADDRESS”,”ADMINISTRATION”,”ADMIT”,”ADOPT”,”ADULT”,”ADVANCE”,”ADVANTAGE”,”ADVICE”,”ADVISE”,”AFFAIR”,”AFFECT”,”AFFORD”,”AFRAID”,”AFTER”,”AFTERNOON”,”AFTERWARDS”,”AGAIN”,”AGAINST”,”AGE”,”AGENCY”,”AGENT”,”AGO”,”AGREE”,”AGREEMENT”,”AHEAD”,”AID”,”AIM”,”AIR”,”AIRCRAFT”,”ALL”,”ALLOW”,”ALMOST”,”ALONE”,”ALONG”,”ALREADY”,”ALRIGHT”,”ALSO”,”ALTERNATIVE”,”ALTHOUGH”,”ALWAYS”,”AMONG”,”AMONGST”,”AMOUNT”,”AN”,”ANALYSIS”,”ANCIENT”,”AND”,”ANIMAL”,”ANNOUNCE”,”ANNUAL”,”ANOTHER”,”ANSWER”,”ANY”,”ANYBODY”,”ANYONE”,”ANYTHING”,”ANYWAY”,”APART”,”APPARENT”,”APPARENTLY”,”APPEAL”,”APPEAR”,”APPEARANCE”,”APPLICATION”,”APPLY”,”APPOINT”,”APPOINTMENT”,”APPROACH”,”APPROPRIATE”,”APPROVE”,”AREA”,”ARGUE”,”ARGUMENT”,”ARISE”,”ARM”,”ARMY”,”AROUND”,”ARRANGE”,”ARRANGEMENT”,”ARRIVE”,”ART”,”ARTICLE”,”ARTIST”,”AS”,”ASK”,”ASPECT”,”ASSEMBLY”,”ASSESS”,”ASSESSMENT”,”ASSET”,”ASSOCIATE”,”ASSOCIATION”,”ASSUME”,”ASSUMPTION”,”AT”,”ATMOSPHERE”,”ATTACH”,”ATTACK”,”ATTEMPT”,”ATTEND”,”ATTENTION”,”ATTITUDE”,”ATTRACT”,”ATTRACTIVE”,”AUDIENCE”,”AUTHOR”,”AUTHORITY”,”AVAILABLE”,”AVERAGE”,”AVOID”,”AWARD”,”AWARE”,”AWAY”,”AYE”,”BABY”,”BACK”,”BACKGROUND”,”BAD”,”BAG”,”BALANCE”,”BALL”,”BAND”,”BANK”,”BAR”,”BASE”,”BASIC”,”BASIS”,”BATTLE”,”BE”,”BEAR”,”BEAT”,”BEAUTIFUL”,”BECAUSE”,”BECOME”,”BED”,”BEDROOM”,”BEFORE”,”BEGIN”,”BEGINNING”,”BEHAVIOUR”,”BEHIND”,”BELIEF”,”BELIEVE”,”BELONG”,”BELOW”,”BENEATH”,”BENEFIT”,”BESIDE”,”BEST”,”BETTER”,”BETWEEN”,”BEYOND”,”BIG”,”BILL”,”BIND”,”BIRD”,”BIRTH”,”BIT”,”BLACK”,”BLOCK”,”BLOOD”,”BLOODY”,”BLOW”,”BLUE”,”BOARD”,”BOAT”,”BODY”,”BONE”,”BOOK”,”BORDER”,”BOTH”,”BOTTLE”,”BOTTOM”,”BOX”,”BOY”,”BRAIN”,”BRANCH”,”BREAK”,”BREATH”,”BRIDGE”,”BRIEF”,”BRIGHT”,”BRING”,”BROAD”,”BROTHER”,”BUDGET”,”BUILD”,”BUILDING”,”BURN”,”BUS”,”BUSINESS”,”BUSY”,”BUT”,”BUY”,”BY”,”CABINET”,”CALL”,”CAMPAIGN”,”CAN”,”CANDIDATE”,”CAPABLE”,”CAPACITY”,”CAPITAL”,”CAR”,”CARD”,”CARE”,”CAREER”,”CAREFUL”,”CAREFULLY”,”CARRY”,”CASE”,”CASH”,”CAT”,”CATCH”,”CATEGORY”,”CAUSE”,”CELL”,”CENTRAL”,”CENTRE”,”CENTURY”,”CERTAIN”,”CERTAINLY”,”CHAIN”,”CHAIR”,”CHAIRMAN”,”CHALLENGE”,”CHANCE”,”CHANGE”,”CHANNEL”,”CHAPTER”,”CHARACTER”,”CHARACTERISTIC”,”CHARGE”,”CHEAP”,”CHECK”,”CHEMICAL”,”CHIEF”,”CHILD”,”CHOICE”,”CHOOSE”,”CHURCH”,”CIRCLE”,”CIRCUMSTANCE”,”CITIZEN”,”CITY”,”CIVIL”,”CLAIM”,”CLASS”,”CLEAN”,”CLEAR”,”CLEARLY”,”CLIENT”,”CLIMB”,”CLOSE”,”CLOSELY”,”CLOTHES”,”CLUB”,”COAL”,”CODE”,”COFFEE”,”COLD”,”COLLEAGUE”,”COLLECT”,”COLLECTION”,”COLLEGE”,”COLOUR”,”COMBINATION”,”COMBINE”,”COME”,”COMMENT”,”COMMERCIAL”,”COMMISSION”,”COMMIT”,”COMMITMENT”,”COMMITTEE”,”COMMON”,”COMMUNICATION”,”COMMUNITY”,”COMPANY”,”COMPARE”,”COMPARISON”,”COMPETITION”,”COMPLETE”,”COMPLETELY”,”COMPLEX”,”COMPONENT”,”COMPUTER”,”CONCENTRATE”,”CONCENTRATION”,”CONCEPT”,”CONCERN”,”CONCERNED”,”CONCLUDE”,”CONCLUSION”,”CONDITION”,”CONDUCT”,”CONFERENCE”,”CONFIDENCE”,”CONFIRM”,”CONFLICT”,”CONGRESS”,”CONNECT”,”CONNECTION”,”CONSEQUENCE”,”CONSERVATIVE”,”CONSIDER”,”CONSIDERABLE”,”CONSIDERATION”,”CONSIST”,”CONSTANT”,”CONSTRUCTION”,”CONSUMER”,”CONTACT”,”CONTAIN”,”CONTENT”,”CONTEXT”,”CONTINUE”,”CONTRACT”,”CONTRAST”,”CONTRIBUTE”,”CONTRIBUTION”,”CONTROL”,”CONVENTION”,”CONVERSATION”,”COPY”,”CORNER”,”CORPORATE”,”CORRECT”,”COS”,”COST”,”COULD”,”COUNCIL”,”COUNT”,”COUNTRY”,”COUNTY”,”COUPLE”,”COURSE”,”COURT”,”COVER”,”CREATE”,”CREATION”,”CREDIT”,”CRIME”,”CRIMINAL”,”CRISIS”,”CRITERION”,”CRITICAL”,”CRITICISM”,”CROSS”,”CROWD”,”CRY”,”CULTURAL”,”CULTURE”,”CUP”,”CURRENT”,”CURRENTLY”,”CURRICULUM”,”CUSTOMER”,”CUT”,”DAMAGE”,”DANGER”,”DANGEROUS”,”DARK”,”DATA”,”DATE”,”DAUGHTER”,”DAY”,”DEAD”,”DEAL”,”DEATH”,”DEBATE”,”DEBT”,”DECADE”,”DECIDE”,”DECISION”,”DECLARE”,”DEEP”,”DEFENCE”,”DEFENDANT”,”DEFINE”,”DEFINITION”,”DEGREE”,”DELIVER”,”DEMAND”,”DEMOCRATIC”,”DEMONSTRATE”,”DENY”,”DEPARTMENT”,”DEPEND”,”DEPUTY”,”DERIVE”,”DESCRIBE”,”DESCRIPTION”,”DESIGN”,”DESIRE”,”DESK”,”DESPITE”,”DESTROY”,”DETAIL”,”DETAILED”,”DETERMINE”,”DEVELOP”,”DEVELOPMENT”,”DEVICE”,”DIE”,”DIFFERENCE”,”DIFFERENT”,”DIFFICULT”,”DIFFICULTY”,”DINNER”,”DIRECT”,”DIRECTION”,”DIRECTLY”,”DIRECTOR”,”DISAPPEAR”,”DISCIPLINE”,”DISCOVER”,”DISCUSS”,”DISCUSSION”,”DISEASE”,”DISPLAY”,”DISTANCE”,”DISTINCTION”,”DISTRIBUTION”,”DISTRICT”,”DIVIDE”,”DIVISION”,”DO”,”DOCTOR”,”DOCUMENT”,”DOG”,”DOMESTIC”,”DOOR”,”DOUBLE”,”DOUBT”,”DOWN”,”DRAW”,”DRAWING”,”DREAM”,”DRESS”,”DRINK”,”DRIVE”,”DRIVER”,”DROP”,”DRUG”,”DRY”,”DUE”,”DURING”,”DUTY”,”EACH”,”EAR”,”EARLY”,”EARN”,”EARTH”,”EASILY”,”EAST”,”EASY”,”EAT”,”ECONOMIC”,”ECONOMY”,”EDGE”,”EDITOR”,”EDUCATION”,”EDUCATIONAL”,”EFFECT”,”EFFECTIVE”,”EFFECTIVELY”,”EFFORT”,”EGG”,”EITHER”,”ELDERLY”,”ELECTION”,”ELEMENT”,”ELSE”,”ELSEWHERE”,”EMERGE”,”EMPHASIS”,”EMPLOY”,”EMPLOYEE”,”EMPLOYER”,”EMPLOYMENT”,”EMPTY”,”ENABLE”,”ENCOURAGE”,”END”,”ENEMY”,”ENERGY”,”ENGINE”,”ENGINEERING”,”ENJOY”,”ENOUGH”,”ENSURE”,”ENTER”,”ENTERPRISE”,”ENTIRE”,”ENTIRELY”,”ENTITLE”,”ENTRY”,”ENVIRONMENT”,”ENVIRONMENTAL”,”EQUAL”,”EQUALLY”,”EQUIPMENT”,”ERROR”,”ESCAPE”,”ESPECIALLY”,”ESSENTIAL”,”ESTABLISH”,”ESTABLISHMENT”,”ESTATE”,”ESTIMATE”,”EVEN”,”EVENING”,”EVENT”,”EVENTUALLY”,”EVER”,”EVERY”,”EVERYBODY”,”EVERYONE”,”EVERYTHING”,”EVIDENCE”,”EXACTLY”,”EXAMINATION”,”EXAMINE”,”EXAMPLE”,”EXCELLENT”,”EXCEPT”,”EXCHANGE”,”EXECUTIVE”,”EXERCISE”,”EXHIBITION”,”EXIST”,”EXISTENCE”,”EXISTING”,”EXPECT”,”EXPECTATION”,”EXPENDITURE”,”EXPENSE”,”EXPENSIVE”,”EXPERIENCE”,”EXPERIMENT”,”EXPERT”,”EXPLAIN”,”EXPLANATION”,”EXPLORE”,”EXPRESS”,

Word list for/Wortliste für Problem 98

”EXPRESSION”,”EXTEND”,”EXTENT”,”EXTERNAL”,”EXTRA”,”EXTREMELY”,”EYE”,”FACE”,”FACILITY”,”FACT”,”FACTOR”,”FACTORY”,”FAIL”,”FAILURE”,”FAIR”,”FAIRLY”,”FAITH”,”FAL
L”,”FAMILIAR”,”FAMILY”,”FAMOUS”,”FAR”,”FARM”,”FARMER”,”FASHION”,”FAST”,”FATHER”,”FAVOUR”,”FEAR”,”FEATURE”,”FEE”,”FEEL”,”FEELING”,”FEMALE”,”FEW”,”FIELD”,”FIG
HT”,”FIGURE”,”FILE”,”FILL”,”FILM”,”FINAL”,”FINALLY”,”FINANCE”,”FINANCIAL”,”FIND”,”FINDING”,”FINE”,”FINGER”,”FINISH”,”FIRE”,”FIRM”,”FIRST”,”FISH”,”FIT”,”FIX”,”FLAT”,”FLI
GHT”,”FLOOR”,”FLOW”,”FLOWER”,”FLY”,”FOCUS”,”FOLLOW”,”FOLLOWING”,”FOOD”,”FOOT”,”FOOTBALL”,”FOR”,”FORCE”,”FOREIGN”,”FOREST”,”FORGET”,”FORM”,”FORMAL
”,”FORMER”,”FORWARD”,”FOUNDATION”,”FREE”,”FREEDOM”,”FREQUENTLY”,”FRESH”,”FRIEND”,”FROM”,”FRONT”,”FRUIT”,”FUEL”,”FULL”,”FULLY”,”FUNCTION”,”FUND”,”FU
NNY”,”FURTHER”,”FUTURE”,”GAIN”,”GAME”,”GARDEN”,”GAS”,”GATE”,”GATHER”,”GENERAL”,”GENERALLY”,”GENERATE”,”GENERATION”,”GENTLEMAN”,”GET”,”GIRL”,”GIVE”,
”GLASS”,”GO”,”GOAL”,”GOD”,”GOLD”,”GOOD”,”GOVERNMENT”,”GRANT”,”GREAT”,”GREEN”,”GREY”,”GROUND”,”GROUP”,”GROW”,”GROWING”,”GROWTH”,”GUEST”,”GUI
DE”,”GUN”,”HAIR”,”HALF”,”HALL”,”HAND”,”HANDLE”,”HANG”,”HAPPEN”,”HAPPY”,”HARD”,”HARDLY”,”HATE”,”HAVE”,”HE”,”HEAD”,”HEALTH”,”HEAR”,”HEART”,”HEAT”,”HEAV
Y”,”HELL”,”HELP”,”HENCE”,”HER”,”HERE”,”HERSELF”,”HIDE”,”HIGH”,”HIGHLY”,”HILL”,”HIM”,”HIMSELF”,”HIS”,”HISTORICAL”,”HISTORY”,”HIT”,”HOLD”,”HOLE”,”HOLIDAY”,”HO
ME”,”HOPE”,”HORSE”,”HOSPITAL”,”HOT”,”HOTEL”,”HOUR”,”HOUSE”,”HOUSEHOLD”,”HOUSING”,”HOW”,”HOWEVER”,”HUGE”,”HUMAN”,”HURT”,”HUSBAND”,”I”,”IDEA”,”IDE
NTIFY”,”IF”,”IGNORE”,”ILLUSTRATE”,”IMAGE”,”IMAGINE”,”IMMEDIATE”,”IMMEDIATELY”,”IMPACT”,”IMPLICATION”,”IMPLY”,”IMPORTANCE”,”IMPORTANT”,”IMPOSE”,”IMPO
SSIBLE”,”IMPRESSION”,”IMPROVE”,”IMPROVEMENT”,”IN”,”INCIDENT”,”INCLUDE”,”INCLUDING”,”INCOME”,”INCREASE”,”INCREASED”,”INCREASINGLY”,”INDEED”,”INDEP
ENDENT”,”INDEX”,”INDICATE”,”INDIVIDUAL”,”INDUSTRIAL”,”INDUSTRY”,”INFLUENCE”,”INFORM”,”INFORMATION”,”INITIAL”,”INITIATIVE”,”INJURY”,”INSIDE”,”INSIST”,”INS
TANCE”,”INSTEAD”,”INSTITUTE”,”INSTITUTION”,”INSTRUCTION”,”INSTRUMENT”,”INSURANCE”,”INTEND”,”INTENTION”,”INTEREST”,”INTERESTED”,”INTERESTING”,”INTE
RNAL”,”INTERNATIONAL”,”INTERPRETATION”,”INTERVIEW”,”INTO”,”INTRODUCE”,”INTRODUCTION”,”INVESTIGATE”,”INVESTIGATION”,”INVESTMENT”,”INVITE”,”INVOLV
E”,”IRON”,”IS”,”ISLAND”,”ISSUE”,”IT”,”ITEM”,”ITS”,”ITSELF”,”JOB”,”JOIN”,”JOINT”,”JOURNEY”,”JUDGE”,”JUMP”,”JUST”,”JUSTICE”,”KEEP”,”KEY”,”KID”,”KILL”,”KIND”,”KING”,”KITC
HEN”,”KNEE”,”KNOW”,”KNOWLEDGE”,”LABOUR”,”LACK”,”LADY”,”LAND”,”LANGUAGE”,”LARGE”,”LARGELY”,”LAST”,”LATE”,”LATER”,”LATTER”,”LAUGH”,”LAUNCH”,”LAW”,”L
AWYER”,”LAY”,”LEAD”,”LEADER”,”LEADERSHIP”,”LEADING”,”LEAF”,”LEAGUE”,”LEAN”,”LEARN”,”LEAST”,”LEAVE”,”LEFT”,”LEG”,”LEGAL”,”LEGISLATION”,”LENGTH”,”LESS”,”LET”,
”LETTER”,”LEVEL”,”LIABILITY”,”LIBERAL”,”LIBRARY”,”LIE”,”LIFE”,”LIFT”,”LIGHT”,”LIKE”,”LIKELY”,”LIMIT”,”LIMITED”,”LINE”,”LINK”,”LIP”,”LIST”,”LISTEN”,”LITERATURE”,”LITTLE”,”LI
VE”,”LIVING”,”LOAN”,”LOCAL”,”LOCATION”,”LONG”,”LOOK”,”LORD”,”LOSE”,”LOSS”,”LOT”,”LOVE”,”LOVELY”,”LOW”,”LUNCH”,”MACHINE”,”MAGAZINE”,”MAIN”,”MAINLY”,”M
AINTAIN”,”MAJOR”,”MAJORITY”,”MAKE”,”MALE”,”MAN”,”MANAGE”,”MANAGEMENT”,”MANAGER”,”MANNER”,”MANY”,”MAP”,”MARK”,”MARKET”,”MARRIAGE”,”MARRIE
D”,”MARRY”,”MASS”,”MASTER”,”MATCH”,”MATERIAL”,”MATTER”,”MAY”,”MAYBE”,”ME”,”MEAL”,”MEAN”,”MEANING”,”MEANS”,”MEANWHILE”,”MEASURE”,”MECHANISM”,”
MEDIA”,”MEDICAL”,”MEET”,”MEETING”,”MEMBER”,”MEMBERSHIP”,”MEMORY”,”MENTAL”,”MENTION”,”MERELY”,”MESSAGE”,”METAL”,”METHOD”,”MIDDLE”,”MIGHT”,”MIL
E”,”MILITARY”,”MILK”,”MIND”,”MINE”,”MINISTER”,”MINISTRY”,”MINUTE”,”MISS”,”MISTAKE”,”MODEL”,”MODERN”,”MODULE”,”MOMENT”,”MONEY”,”MONTH”,”MORE”,”M
ORNING”,”MOST”,”MOTHER”,”MOTION”,”MOTOR”,”MOUNTAIN”,”MOUTH”,”MOVE”,”MOVEMENT”,”MUCH”,”MURDER”,”MUSEUM”,”MUSIC”,”MUST”,”MY”,”MYSELF”,”NA
ME”,”NARROW”,”NATION”,”NATIONAL”,”NATURAL”,”NATURE”,”NEAR”,”NEARLY”,”NECESSARILY”,”NECESSARY”,”NECK”,”NEED”,”NEGOTIATION”,”NEIGHBOUR”,”NEITHER”
,”NETWORK”,”NEVER”,”NEVERTHELESS”,”NEW”,”NEWS”,”NEWSPAPER”,”NEXT”,”NICE”,”NIGHT”,”NO”,”NOBODY”,”NOD”,”NOISE”,”NONE”,”NOR”,”NORMAL”,”NORMALLY”,
”NORTH”,”NORTHERN”,”NOSE”,”NOT”,”NOTE”,”NOTHING”,”NOTICE”,”NOTION”,”NOW”,”NUCLEAR”,”NUMBER”,”NURSE”,”OBJECT”,”OBJECTIVE”,”OBSERVATION”,”OBSER
VE”,”OBTAIN”,”OBVIOUS”,”OBVIOUSLY”,”OCCASION”,”OCCUR”,”ODD”,”OF”,”OFF”,”OFFENCE”,”OFFER”,”OFFICE”,”OFFICER”,”OFFICIAL”,”OFTEN”,”OIL”,”OKAY”,”OLD”,”O
N”,”ONCE”,”ONE”,”ONLY”,”ONTO”,”OPEN”,”OPERATE”,”OPERATION”,”OPINION”,”OPPORTUNITY”,”OPPOSITION”,”OPTION”,”OR”,”ORDER”,”ORDINARY”,”ORGANISATION
”,”ORGANISE”,”ORGANIZATION”,”ORIGIN”,”ORIGINAL”,”OTHER”,”OTHERWISE”,”OUGHT”,”OUR”,”OURSELVES”,”OUT”,”OUTCOME”,”OUTPUT”,”OUTSIDE”,”OVER”,”OVERAL
L”,”OWN”,”OWNER”,”PACKAGE”,”PAGE”,”PAIN”,”PAINT”,”PAINTING”,”PAIR”,”PANEL”,”PAPER”,”PARENT”,”PARK”,”PARLIAMENT”,”PART”,”PARTICULAR”,”PARTICULARLY”,”PAR
TLY”,”PARTNER”,”PARTY”,”PASS”,”PASSAGE”,”PAST”,”PATH”,”PATIENT”,”PATTERN”,”PAY”,”PAYMENT”,”PEACE”,”PENSION”,”PEOPLE”,”PER”,”PERCENT”,”PERFECT”,”PERFORM”,
”PERFORMANCE”,”PERHAPS”,”PERIOD”,”PERMANENT”,”PERSON”,”PERSONAL”,”PERSUADE”,”PHASE”,”PHONE”,”PHOTOGRAPH”,”PHYSICAL”,”PICK”,”PICTURE”,”PIECE”,”P
LACE”,”PLAN”,”PLANNING”,”PLANT”,”PLASTIC”,”PLATE”,”PLAY”,”PLAYER”,”PLEASE”,”PLEASURE”,”PLENTY”,”PLUS”,”POCKET”,”POINT”,”POLICE”,”POLICY”,”POLITICAL”,”POLI
TICS”,”POOL”,”POOR”,”POPULAR”,”POPULATION”,”POSITION”,”POSITIVE”,”POSSIBILITY”,”POSSIBLE”,”POSSIBLY”,”POST”,”POTENTIAL”,”POUND”,”POWER”,”POWERFUL”,”P
RACTICAL”,”PRACTICE”,”PREFER”,”PREPARE”,”PRESENCE”,”PRESENT”,”PRESIDENT”,”PRESS”,”PRESSURE”,”PRETTY”,”PREVENT”,”PREVIOUS”,”PREVIOUSLY”,”PRICE”,”PRIMA
RY”,”PRIME”,”PRINCIPLE”,”PRIORITY”,”PRISON”,”PRISONER”,”PRIVATE”,”PROBABLY”,”PROBLEM”,”PROCEDURE”,”PROCESS”,”PRODUCE”,”PRODUCT”,”PRODUCTION”,

Word list for/Wortliste für Problem 98

”PROFESSIONAL”,”PROFIT”,”PROGRAM”,”PROGRAMME”,”PROGRESS”,”PROJECT”,”PROMISE”,”PROMOTE”,”PROPER”,”PROPERLY”,”PROPERTY”,”PROPORTION”,”PROPOS
E”,”PROPOSAL”,”PROSPECT”,”PROTECT”,”PROTECTION”,”PROVE”,”PROVIDE”,”PROVIDED”,”PROVISION”,”PUB”,”PUBLIC”,”PUBLICATION”,”PUBLISH”,”PULL”,”PUPIL”,”PURP
OSE”,”PUSH”,”PUT”,”QUALITY”,”QUARTER”,”QUESTION”,”QUICK”,”QUICKLY”,”QUIET”,”QUITE”,”RACE”,”RADIO”,”RAILWAY”,”RAIN”,”RAISE”,”RANGE”,”RAPIDLY”,”RARE”,”RAT
E”,”RATHER”,”REACH”,”REACTION”,”READ”,”READER”,”READING”,”READY”,”REAL”,”REALISE”,”REALITY”,”REALIZE”,”REALLY”,”REASON”,”REASONABLE”,”RECALL”,”RECEIV
E”,”RECENT”,”RECENTLY”,”RECOGNISE”,”RECOGNITION”,”RECOGNIZE”,”RECOMMEND”,”RECORD”,”RECOVER”,”RED”,”REDUCE”,”REDUCTION”,”REFER”,”REFERENCE”,”REF
LECT”,”REFORM”,”REFUSE”,”REGARD”,”REGION”,”REGIONAL”,”REGULAR”,”REGULATION”,”REJECT”,”RELATE”,”RELATION”,”RELATIONSHIP”,”RELATIVE”,”RELATIVELY”,”REL
EASE”,”RELEVANT”,”RELIEF”,”RELIGION”,”RELIGIOUS”,”RELY”,”REMAIN”,”REMEMBER”,”REMIND”,”REMOVE”,”REPEAT”,”REPLACE”,”REPLY”,”REPORT”,”REPRESENT”,”REPRES
ENTATION”,”REPRESENTATIVE”,”REQUEST”,”REQUIRE”,”REQUIREMENT”,”RESEARCH”,”RESOURCE”,”RESPECT”,”RESPOND”,”RESPONSE”,”RESPONSIBILITY”,”RESPONSIBL
E”,”REST”,”RESTAURANT”,”RESULT”,”RETAIN”,”RETURN”,”REVEAL”,”REVENUE”,”REVIEW”,”REVOLUTION”,”RICH”,”RIDE”,”RIGHT”,”RING”,”RISE”,”RISK”,”RIVER”,”ROAD”,”ROC
K”,”ROLE”,”ROLL”,”ROOF”,”ROOM”,”ROUND”,”ROUTE”,”ROW”,”ROYAL”,”RULE”,”RUN”,”RURAL”,”SAFE”,”SAFETY”,”SALE”,”SAME”,”SAMPLE”,”SATISFY”,”SAVE”,”SAY”,”SCALE”,”S
CENE”,”SCHEME”,”SCHOOL”,”SCIENCE”,”SCIENTIFIC”,”SCIENTIST”,”SCORE”,”SCREEN”,”SEA”,”SEARCH”,”SEASON”,”SEAT”,”SECOND”,”SECONDARY”,”SECRETARY”,”SECTIO
N”,”SECTOR”,”SECURE”,”SECURITY”,”SEE”,”SEEK”,”SEEM”,”SELECT”,”SELECTION”,”SELL”,”SEND”,”SENIOR”,”SENSE”,”SENTENCE”,”SEPARATE”,”SEQUENCE”,”SERIES”,”SERIOU
S”,”SERIOUSLY”,”SERVANT”,”SERVE”,”SERVICE”,”SESSION”,”SET”,”SETTLE”,”SETTLEMENT”,”SEVERAL”,”SEVERE”,”SEX”,”SEXUAL”,”SHAKE”,”SHALL”,”SHAPE”,”SHARE”,”SHE”,”S
HEET”,”SHIP”,”SHOE”,”SHOOT”,”SHOP”,”SHORT”,”SHOT”,”SHOULD”,”SHOULDER”,”SHOUT”,”SHOW”,”SHUT”,”SIDE”,”SIGHT”,”SIGN”,”SIGNAL”,”SIGNIFICANCE”,”SIGNIFICA
NT”,”SILENCE”,”SIMILAR”,”SIMPLE”,”SIMPLY”,”SINCE”,”SING”,”SINGLE”,”SIR”,”SISTER”,”SIT”,”SITE”,”SITUATION”,”SIZE”,”SKILL”,”SKIN”,”SKY”,”SLEEP”,”SLIGHTLY”,”SLIP”,”SLOW
”,”SLOWLY”,”SMALL”,”SMILE”,”SO”,”SOCIAL”,”SOCIETY”,”SOFT”,”SOFTWARE”,”SOIL”,”SOLDIER”,”SOLICITOR”,”SOLUTION”,”SOME”,”SOMEBODY”,”SOMEONE”,”SOMETHIN
G”,”SOMETIMES”,”SOMEWHAT”,”SOMEWHERE”,”SON”,”SONG”,”SOON”,”SORRY”,”SORT”,”SOUND”,”SOURCE”,”SOUTH”,”SOUTHERN”,”SPACE”,”SPEAK”,”SPEAKER”,”SPECI
AL”,”SPECIES”,”SPECIFIC”,”SPEECH”,”SPEED”,”SPEND”,”SPIRIT”,”SPORT”,”SPOT”,”SPREAD”,”SPRING”,”STAFF”,”STAGE”,”STAND”,”STANDARD”,”STAR”,”START”,”STATE”,”STATE
MENT”,”STATION”,”STATUS”,”STAY”,”STEAL”,”STEP”,”STICK”,”STILL”,”STOCK”,”STONE”,”STOP”,”STORE”,”STORY”,”STRAIGHT”,”STRANGE”,”STRATEGY”,”STREET”,”STRENGTH”,
”STRIKE”,”STRONG”,”STRONGLY”,”STRUCTURE”,”STUDENT”,”STUDIO”,”STUDY”,”STUFF”,”STYLE”,”SUBJECT”,”SUBSTANTIAL”,”SUCCEED”,”SUCCESS”,”SUCCESSFUL”,”SUCH”
,”SUDDENLY”,”SUFFER”,”SUFFICIENT”,”SUGGEST”,”SUGGESTION”,”SUITABLE”,”SUM”,”SUMMER”,”SUN”,”SUPPLY”,”SUPPORT”,”SUPPOSE”,”SURE”,”SURELY”,”SURFACE”,”SU
RPRISE”,”SURROUND”,”SURVEY”,”SURVIVE”,”SWITCH”,”SYSTEM”,”TABLE”,”TAKE”,”TALK”,”TALL”,”TAPE”,”TARGET”,”TASK”,”TAX”,”TEA”,”TEACH”,”TEACHER”,”TEACHING”,”TE
AM”,”TEAR”,”TECHNICAL”,”TECHNIQUE”,”TECHNOLOGY”,”TELEPHONE”,”TELEVISION”,”TELL”,”TEMPERATURE”,”TEND”,”TERM”,”TERMS”,”TERRIBLE”,”TEST”,”TEXT”,”THAN
”,”THANK”,”THANKS”,”THAT”,”THE”,”THEATRE”,”THEIR”,”THEM”,”THEME”,”THEMSELVES”,”THEN”,”THEORY”,”THERE”,”THEREFORE”,”THESE”,”THEY”,”THIN”,”THING”,”THIN
K”,”THIS”,”THOSE”,”THOUGH”,”THOUGHT”,”THREAT”,”THREATEN”,”THROUGH”,”THROUGHOUT”,”THROW”,”THUS”,”TICKET”,”TIME”,”TINY”,”TITLE”,”TO”,”TODAY”,”TOGET
HER”,”TOMORROW”,”TONE”,”TONIGHT”,”TOO”,”TOOL”,”TOOTH”,”TOP”,”TOTAL”,”TOTALLY”,”TOUCH”,”TOUR”,”TOWARDS”,”TOWN”,”TRACK”,”TRADE”,”TRADITION”,”TRA
DITIONAL”,”TRAFFIC”,”TRAIN”,”TRAINING”,”TRANSFER”,”TRANSPORT”,”TRAVEL”,”TREAT”,”TREATMENT”,”TREATY”,”TREE”,”TREND”,”TRIAL”,”TRIP”,”TROOP”,”TROUBLE”,”
TRUE”,”TRUST”,”TRUTH”,”TRY”,”TURN”,”TWICE”,”TYPE”,”TYPICAL”,”UNABLE”,”UNDER”,”UNDERSTAND”,”UNDERSTANDING”,”UNDERTAKE”,”UNEMPLOYMENT”,”UNFORT
UNATELY”,”UNION”,”UNIT”,”UNITED”,”UNIVERSITY”,”UNLESS”,”UNLIKELY”,”UNTIL”,”UP”,”UPON”,”UPPER”,”URBAN”,”US”,”USE”,”USED”,”USEFUL”,”USER”,”USUAL”,”USUALL
Y”,”VALUE”,”VARIATION”,”VARIETY”,”VARIOUS”,”VARY”,”VAST”,”VEHICLE”,”VERSION”,”VERY”,”VIA”,”VICTIM”,”VICTORY”,”VIDEO”,”VIEW”,”VILLAGE”,”VIOLENCE”,”VISION”,”V
ISIT”,”VISITOR”,”VITAL”,”VOICE”,”VOLUME”,”VOTE”,”WAGE”,”WAIT”,”WALK”,”WALL”,”WANT”,”WAR”,”WARM”,”WARN”,”WASH”,”WATCH”,”WATER”,”WAVE”,”WAY”,”WE”,”WEA
K”,”WEAPON”,”WEAR”,”WEATHER”,”WEEK”,”WEEKEND”,”WEIGHT”,”WELCOME”,”WELFARE”,”WELL”,”WEST”,”WESTERN”,”WHAT”,”WHATEVER”,”WHEN”,”WHERE”,”WHEREA
S”,”WHETHER”,”WHICH”,”WHILE”,”WHILST”,”WHITE”,”WHO”,”WHOLE”,”WHOM”,”WHOSE”,”WHY”,”WIDE”,”WIDELY”,”WIFE”,”WILD”,”WILL”,”WIN”,”WIND”,”WINDOW”,”WIN
E”,”WING”,”WINNER”,”WINTER”,”WISH”,”WITH”,”WITHDRAW”,”WITHIN”,”WITHOUT”,”WOMAN”,”WONDER”,”WONDERFUL”,”WOOD”,”WORD”,”WORK”,”WORKER”,”WOR
KING”,”WORKS”,”WORLD”,”WORRY”,”WORTH”,”WOULD”,”WRITE”,”WRITER”,”WRITING”,”WRONG”,”YARD”,”YEAH”,”YEAR”,”YES”,”YESTERDAY”,”YET”,”YOU”,”YOUNG”,”YOU
R”,”YOURSELF”,”YOUTH”

Problem 99

Comparing two numbers written in index form like 2^{11} and 3^7 is not difficult, as any calculator would confirm that

$$2^{11} = 2048 < 3^7 = 2187.$$

However, confirming that $632382^{518061} > 519432^{525806}$ would be much more difficult, as both numbers contain over three million digits.

Using the following two pages (copy & paste) containing one thousand lines (in several columns) with a base/exponent pair on each line, determine which line number has the greatest numerical value.

NOTE: The first two lines in the file represent the numbers in the example given above.

Problem 99

Zahlen in Potenzschreibweise wie 2^{11} und 3^7 miteinander zu vergleichen ist nicht schwierig, da jeder Taschenrechner bestätigt, daß

$$2^{11} = 2048 < 3^7 = 2187.$$

Zu bestätigen, daß $632382^{518061} > 519432^{525806}$ ist, wäre jedoch deutlich schwieriger, da beide Zahlen mehr als drei Millionen Ziffern haben.

Zu bestimmen ist auf den folgenden zwei Seiten die Nummer der Zeile (von eintausend Zeilen mit jeweils einem Paar aus Grund- und Hochzahl, in mehreren Spalten) mit dem größten Zahlenwert.

ANMERKUNG: Die ersten beiden Zeilen stellen die im Beispiel oben verwendeten Zahlen dar.

Base & exponent list for/List von Grund- und Hochzahlen für Problem 98

519432,525806	444409,532117	597706,520257	164446,576167	506501,526817	678136,515373	191618,568919	32847,665404	926980,503640	854063,506662	814642,508425
632382,518061	33833,663511	310484,547206	753413,511364	244520,557738	897144,504851	946699,502874	891292,505088	882353,505459	365255,540263	969939,501993
78864,613712	381850,538396	944468,502959	11410,740712	144745,582349	989554,501263	289555,550247	152715,579732	566887,522345	165437,575872	242856,558047
466580,530130	402931,536157	121283,591152	448845,531712	69274,620858	413292,535106	799322,509139	824104,507997	3326,853312	662240,516281	76302,615517
780495,510032	92901,604930	451131,531507	925072,503725	292620,549784	55297,633667	703886,513942	234057,559711	911981,504248	289970,550181	472083,529653
525895,525320	304825,548004	566499,522367	564888,522477	926027,503687	788650,509637	194812,568143	730507,512532	416309,534800	847977,506933	587101,520964
15991,714883	731917,512452	425373,533918	7062,780812	736320,512225	486748,528417	261823,554685	960529,502340	392991,537199	546083,523816	99066,601543
960290,502358	753734,511344	40240,652665	641155,517535	515528,526113	150724,580377	203052,566221	388395,537687	622829,518651	413252,535113	498005,527503
760018,511029	51894,637373	39130,654392	738878,512100	407549,535688	56434,632490	217330,563093	958170,502437	148647,581055	975829,501767	709800,513624
166800,575487	151578,580103	714926,513355	636204,517828	848089,506927	77207,614869	734748,512313	57105,631806	496483,527624	361540,540701	708000,513716
210884,564478	295075,549421	469219,529903	372540,539436	24141,685711	588631,520859	391759,537328	186025,570311	666314,516044	235522,559435	20171,698134
555151,523163	303590,548183	806929,508783	443162,532237	9224,757964	611619,519367	807052,508777	993043,501133	48562,641293	224643,561577	285020,550936
681146,515199	333594,544123	287970,550487	571192,522042	980684,501586	100006,601055	564467,522510	576770,521664	672618,515684	736350,512229	266564,553891
563395,522587	683952,515042	92189,605332	655350,516680	175259,573121	528924,525093	59186,629748	215319,563513	443676,532187	328303,544808	981563,501557
738250,512126	60090,628880	103841,599094	299741,548735	489160,528216	190225,569257	113447,594545	927342,503628	274065,552661	35022,661330	846502,506991
923525,503780	951420,502692	671839,515725	581914,521307	878970,505604	851155,506789	518063,525916	521353,525666	265386,554079	307838,547578	334,1190800
595148,520429	28335,674991	452048,531421	965471,502156	969546,502002	682593,515114	905944,504492	39563,653705	347668,542358	474366,529458	209268,564829
177108,572629	714940,513349	987837,501323	513441,526277	525207,525365	613043,519275	613922,519213	752516,511408	31816,667448	873755,505819	9844,752610
750923,511482	343858,542826	935192,503321	808682,508700	690461,514675	514673,526183	439093,532607	110755,595770	181575,571446	73978,617220	996519,501007
440902,532446	549279,523586	88585,607450	237589,559034	156510,578551	877634,505655	445946,531981	309749,547305	961289,502320	827387,507845	410059,535426
881418,505504	804571,508887	613883,519216	543300,524025	659778,516426	878905,505602	230530,560399	374379,539224	365689,540214	670830,515791	432931,533188
422489,534197	260653,554881	144551,582413	804712,508889	468739,529945	1926,914951	297887,549007	919184,503952	987950,501317	326511,545034	848012,506929
979858,501616	291399,549966	647359,517155	247511,557192	765252,510770	613245,519259	459029,530797	990652,501226	932299,503440	309909,547285	966803,502110
685893,514935	402342,536213	213902,563816	543486,524008	76703,615230	152481,579816	403692,536075	647780,517135	27388,677243	400970,536363	983434,501486
747477,511661	408889,535550	184120,570789	504383,526992	165151,575959	841774,507203	855118,506616	187177,570017	746701,511701	884827,505352	160700,577267
167214,575367	40328,652524	258126,555322	326529,545039	29779,671736	71060,619442	963127,502245	168938,574877	492258,527969	718307,513175	504374,526989
234140,559696	375856,539061	502546,527130	792493,509458	928865,503569	865335,506175	841711,507208	649558,517023	147823,581323	28462,674699	832061,507640
940238,503122	768907,510590	407655,535678	86033,609017	577538,521605	90244,606469	407411,535699	278126,552016	57918,630985	599384,520150	392825,537214
728969,512609	165993,575715	401528,536306	126554,589005	927555,503618	302156,548388	924729,503735	162039,576868	838849,507333	253565,556111	443842,532165
232083,560102	976327,501755	477490,529193	579379,521481	185377,570477	399059,536557	914823,504132	658512,516499	678038,515375	284009,551093	440352,532492
900971,504694	898500,504795	841085,507237	948026,502823	974756,501809	478465,529113	333725,544101	498115,527486	27852,676130	343403,542876	745125,511776
688801,514772	360404,540830	732831,512408	404777,535969	800130,509093	558601,522925	176345,572832	896583,504868	850241,506828	446557,531921	13718,726392
189664,569402	478714,529095	833000,507595	265767,554022	217016,563153	69132,620966	912507,504225	561170,522740	818403,508253	992372,501160	661753,516312
891022,505104	694144,514472	904694,504542	266876,553840	365709,540216	267663,553700	411273,535308	747772,511647	131717,587014	961601,502308	70500,619875
445689,531996	488726,528258	581435,521348	46631,643714	774508,510320	988276,501310	259774,555036	775093,510294	850216,506834	696629,514342	436952,532814
119570,591871	841380,507226	455545,531110	492397,527958	588716,520851	378354,538787	632853,518038	652081,516882	904848,504529	919537,503945	424724,533973
821453,508118	328012,544839	873558,505829	856106,506581	631673,518104	529909,525014	119723,591801	724905,512824	189758,569380	894709,504944	21954,692224
371084,539600	22389,690868	94916,603796	795757,509305	954076,502590	161733,576968	163902,576321	499707,527365	392845,537217	892201,505051	262490,554567
911745,504251	604053,519852	720176,513068	748946,511584	777828,510161	758541,511109	22691,689944	47388,642755	470876,529761	358160,541097	716622,513264
623655,518600	329514,544641	545034,523891	294694,549480	990659,501222	823425,508024	402427,536212	646668,517204	925353,503711	448503,531745	907584,504425
144361,582486	772965,510390	246348,557409	409781,535463	597799,520254	149821,580667	175769,572988	571700,522007	285431,550877	832156,507636	60086,628882
352442,541775	492798,527927	556452,523079	775887,510253	786905,509727	269258,553438	837260,507402	180430,571747	454098,531234	920045,503924	837123,507412
420726,534367	30125,670983	832015,507634	543747,523991	512547,526348	481152,528891	603432,519893	710015,513617	823910,508003	926137,503675	971345,501940
295298,549387	895603,504906	173663,573564	210592,564536	756449,511212	120871,591322	313679,546767	435522,532941	318493,546112	416754,534757	947162,502855
6530,787777	450785,531539	502634,527125	517119,525990	869787,505988	972322,501901	538165,524394	98137,602041	766067,510730	254422,555966	139920,584021
468397,529976	840237,507276	250732,556611	520253,525751	653747,516779	981350,501567	549026,523608	759176,511070	261277,554775	92498,605151	68330,621624
672336,515696	380711,538522	569786,522139	247926,557124	84623,609900	676129,515483	61083,627945	486124,528467	421530,534289	826833,507873	666452,516038
431861,533289	63577,625673	216919,563178	592141,520626	839698,507295	950860,502717	898345,504798	526942,525236	694130,514478	660716,516371	731446,512481
84228,610150	76801,615157	521815,525623	346580,542492	30159,670909	119000,592114	992556,501153	878921,505604	120439,591498	689335,514746	953350,502619
805376,508857	502694,527123	92304,605270	544969,523902	797275,509234	392252,537272	369999,539727	408313,535602	213308,563949	160045,577467	183157,571042

Base & exponent list for/List von Grund- und Hochzahlen für Problem 98

845400,507045	284778,550976	288597,550389	595687,520398	543514,524006	333798,544090	548142,523667	913658,504176	318593,546089
651548,516910	81040,612235	285819,550812	540360,524240	506835,526794	625733,518476	756491,511205	68117,621790	179810,571911
20399,697344	161699,576978	538400,524385	245779,557511	868368,506052	995584,501037	987352,501340	989729,501253	200531,566799
861779,506331	616394,519057	809930,508645	924873,503730	847025,506971	506135,526853	766520,510705	567697,522288	314999,546580
629771,518229	767490,510661	738326,512126	509628,526577	678623,515342	238050,558952	591775,520647	820427,508163	197020,567622
801706,509026	156896,578431	955461,502535	528523,525122	876139,505726	557943,522972	833758,507563	54236,634794	301465,548487
189207,569512	427408,533714	163829,576343	3509,847707	571997,521984	530978,524938	843890,507108	291557,549938	237808,559000
737501,512168	254849,555884	826475,507891	522756,525555	598632,520198	634244,517949	925551,503698	124961,589646	131944,586923
719272,513115	737217,512182	376488,538987	895447,504922	213590,563892	177168,572616	74816,616598	403177,536130	882527,505449
479285,529045	897133,504851	102234,599905	44840,646067	625404,518497	85200,609541	646942,517187	405421,535899	468117,530003
136046,585401	203815,566051	114650,594002	45860,644715	726508,512738	953043,502630	354923,541481	410233,535417	711319,513541
896746,504860	270822,553189	52815,636341	463487,530404	689426,514738	523661,525484	256291,555638	815111,508403	156240,578628
891735,505067	135854,585475	434037,533082	398164,536654	332495,544264	999295,500902	634470,517942	213176,563974	965452,502162
684771,514999	778805,510111	804744,508880	894483,504959	411366,535302	840803,507246	930904,503494	83099,610879	992756,501148
865309,506184	784373,509847	98385,601905	619415,518874	242546,558110	961490,502312	134221,586071	998588,500934	437959,532715
379066,538702	305426,547921	856620,506559	966306,502129	315209,546555	471747,529685	282663,551304	513640,526263	739938,512046
503117,527090	733418,512375	220057,562517	990922,501212	797544,509219	380705,538523	986070,501394	129817,587733	614249,519196
621780,518717	732087,512448	844734,507078	835756,507474	93889,604371	911180,504275	123636,590176	1820,921851	391496,537356
209518,564775	540668,524215	150677,580387	548881,523618	858879,506454	334149,544046	123678,590164	287584,550539	62746,626418
677135,515423	702898,513996	558697,522917	453578,531282	124906,589666	478992,529065	481717,528841	299160,548820	688215,514806
987500,501340	628057,518328	621751,518719	474993,529410	449072,531693	325789,545133	423076,534137	860621,506386	75501,616091
197049,567613	640280,517587	207067,565321	80085,612879	235960,559345	335884,543826	866246,506145	529258,525059	883573,505412
329315,544673	422405,534204	135297,585677	737091,512193	642403,517454	426976,533760	93313,604697	586297,521017	558824,522910
236756,559196	10604,746569	932968,503404	50789,638638	720567,513047	749007,511582	783632,509880	953406,502616	759371,511061
357092,541226	746038,511733	604456,519822	979768,501620	705534,513858	667067,516000	317066,546304	441234,532410	173913,573489
520440,525733	839808,507293	579728,521462	792018,509483	603692,519870	607586,519623	502977,527103	986217,501386	891351,505089
213471,563911	457417,530938	244138,557813	665001,516122	488137,528302	674054,515599	141272,583545	781938,509957	727464,512693
956852,502490	479030,529064	706487,513800	86552,608694	157370,578285	188534,569675	71708,618938	461247,530595	164833,576051
702223,514032	341758,543090	711627,513523	462772,530469	63515,625730	565185,522464	617748,518975	735424,512277	812317,508529
404943,535955	620223,518824	853833,506674	589233,520821	666326,516041	172090,573988	581190,521362	146623,581722	540320,524243
178880,572152	251661,556451	497220,527562	891694,505072	619226,518883	87592,608052	193824,568382	839838,507288	698061,514257
689477,514734	561790,522696	59428,629511	592605,520594	443613,532186	907432,504424	682368,515131	510667,526494	69149,620952
691351,514630	497733,527521	564845,522486	209645,564741	597717,520257	8912,760841	352956,541712	935085,503327	471673,529694
866669,506128	724201,512863	623621,518603	42531,649269	96225,603069	928318,503590	351375,541905	737523,512167	159092,577753
370561,539656	489217,528217	242689,558077	554376,523226	86940,608450	757917,511138	505362,526909	303455,548204	428134,533653
739805,512051	415623,534867	125091,589591	803814,508929	40725,651929	718693,513153	905165,504518	992779,501145	89997,606608
71060,619441	624610,518548	363819,540432	334157,544042	460976,530625	315141,546566	128645,588188	60240,628739	711061,513557
624861,518534	847541,506953	686453,514901	175836,572970	268875,553508	728326,512645	267143,553787	939095,503174	779403,510081
261660,554714	432295,533249	656813,516594	868379,506051	270671,553214	353492,541647	158409,577965	794368,509370	203327,566155
366137,540160	400391,536421	489901,528155	658166,516520	363254,540500	638429,517695	482776,528754	501825,527189	798176,509187
166054,575698	961158,502319	386380,537905	278203,551995	384248,538137	628892,518280	628896,518282	459028,530798	667688,515963
601878,519990	139173,584284	542819,524052	966198,502126	762889,510892	877286,505672	485233,528547	884641,505363	636120,517833
153445,579501	421225,534315	243987,557841	627162,518387	377941,538833	620895,518778	563606,522574	512287,526364	137410,584913
279899,551729	579083,521501	693412,514514	296774,549165	278878,551890	385878,537959	111001,595655	835165,507499	217615,563034
379166,538691	74274,617000	488484,528271	311803,547027	176615,572755	423311,534113	115920,593445	307723,547590	556887,523038
423209,534125	701142,514087	896331,504881	843797,507118	860008,506412	633501,517997	365510,540237	160587,577304	667229,515991
675310,515526	374465,539219	336730,543721	702304,514032	944392,502967	884833,505360	959724,502374	735043,512300	672276,515708
145641,582050	217814,562985	728298,512647	563875,522553	608395,519571	883402,505416	938763,503184	493289,527887	325361,545187
691353,514627	358972,540995	604215,519840	33103,664910	225283,561450	999665,500894	930044,503520	110717,595785	172115,573985
917468,504026	88629,607424	153729,579413	191932,568841	45095,645728	708395,513697	970959,501956	306480,547772	13846,725685

Problem 100

If a box contains twenty-one coloured discs, composed of fifteen blue discs and six red discs, and two discs were taken at random, it can be seen that the probability of taking two blue discs,

$$P(BB) = (15/21)\times(14/20) = 1/2.$$

The next such arrangement, for which there is exactly 50% chance of taking two blue discs at random, is a box containing eighty-five blue discs and thirty-five red discs.

By finding the first arrangement to contain over 10^{12} = 1,000,000,000,000 discs in total, determine the number of blue discs that the box would contain.

Problem 100

Wenn eine Schachtel einundzwanzig farbige Scheiben enthält (davon fünfzehn blaue und sechs rote) und man zwei Scheiben zufällig herausgreift, dann ist die Wahrscheinlichkeit dafür, zwei blaue Scheiben zu greifen:

$$P(BB) = (15/21) \cdot (14/20) = 1/2.$$

Die nächste solche Anordnung, für die es eine genau 50-prozentige Change gibt, zufällig zwei blaue Scheiben herauszugreifen, ist eine Schachtel mit fünfundachtzig blauen und fünfunddreißig roten Scheiben.

Zu ermitteln ist die Anzahl blauer Scheiben in der ersten Anordnung mit mehr als insgesamt 10^{12} = 1,000,000,000,000 Scheiben, bei der die Wahrscheinlichkeit für zwei blaue Scheiben wieder genau 50% beträgt.