

Engineering Management

Transportation Model

AIM OF TRANSPORTATION MODEL

▣ To find out the **optimum transportation schedule** keeping in mind the cost of transportation to be **minimized**.

THE MODEL REQUIRES FEW DATA ELEMENTS

- ☑ **Origin of Supply**
- ☑ **Destination**
- ☑ **Unit Cost of Shipping (Per Unit Cost)**

WHAT IS TRANSPORTATION MODEL?

Transportation Model is a special case of LPP (Linear Programming Problem) in which the main objective is **to transport a product from various sources various destinations at total minimum costs.**

WHAT IS TRANSPORTATION MODEL?

- In transportation models, the **sources** and **destinations** at each source and destination are also known.
- It is desired to find the best arrangement for transportation such that the transportation cost is minimum.

WHAT IS TRANSPORTATION MODEL?

As it is a model, we have to make some assumptions.

These assumptions are:

- Items are homogeneous
- Shipping costs per unit are the same, no matter the quantity delivered
- Only one route is chosen between the origin and destination

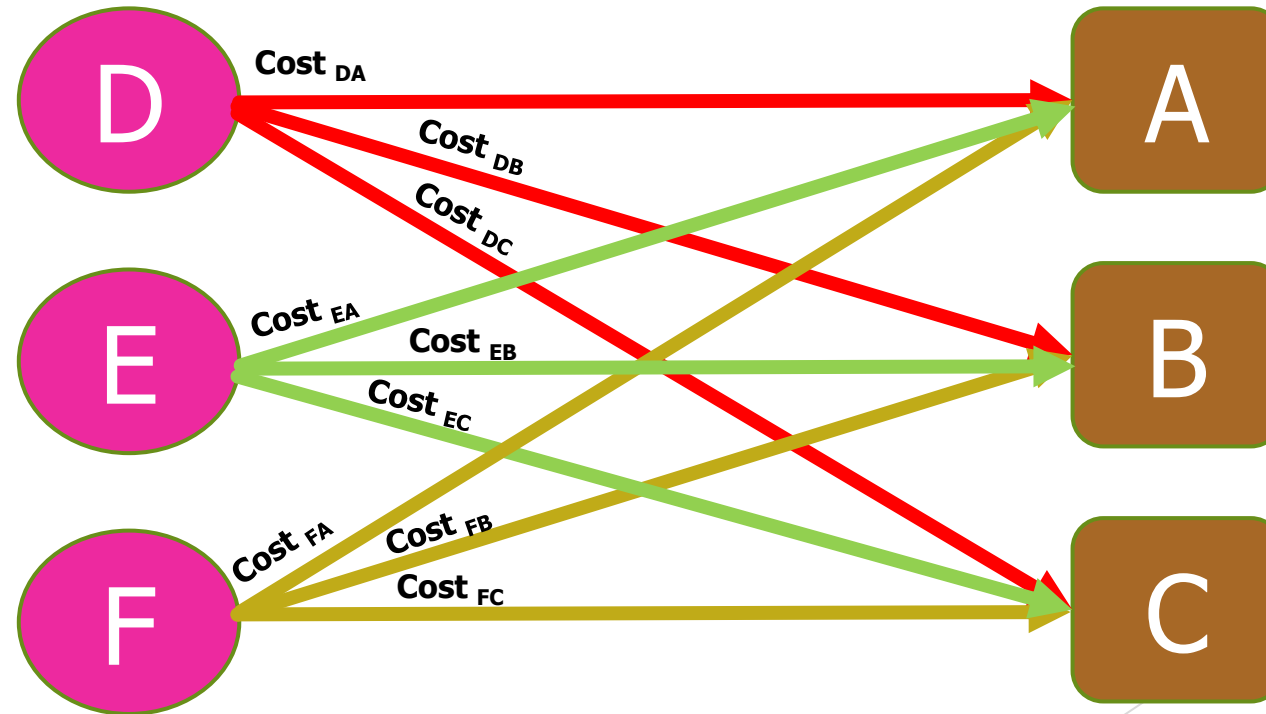
WHAT IS TRANSPORTATION MODEL?

- For example, consider three companies (Company 1, Company 2, and Company 3) which produces mobile phones and are located in different regions.
- Similarly, consider three cities (namely A, B & C) where the mobile phones are transported.
- The companies where mobile phones are available are known as **sources** and the cities where mobile phones are transported are called **destinations**.

WHAT IS TRANSPORTATION MODEL?

Supply Origin

Demand Destination



WHAT IS TRANSPORTATION MODEL?

- Let, Company 1 produces a_1 units,
Company 2 produces a_2 units, and
Company 3 produces a_3 units,
- Let, demand in City A is b_1 units,
demand in City B is b_2 units, and
demand in City C is c_3 units

WHAT IS TRANSPORTATION MODEL?

The cost of transportation from each source to destination is given in table.

DESTINATIONS					
SOURCES		CITY A	CITY B	CITY C	SUPPLY
	COMPANY 1	C_{1A}	C_{1B}	C_{1C}	a1
	COMPANY 2	C_{2A}	C_{2B}	C_{2C}	a2
	COMPANY 3	C_{3A}	C_{3B}	C_{3C}	a3
DEMAND		b1	b2	b3	$\Sigma a = \Sigma b$

The transportation of mobile phones should be done in such a way, that the total transportation cost is **minimum**.

TYPES OF TRANSPORTATION PROBLEMS

There are two types of transportation problems.

i) Balanced Transportation Problems

The sum of supply and sum of demand are same.

$$\Sigma \text{ Supply} = \Sigma \text{ Demand}$$

ii) Unbalanced Transportation Problems

The sum of supply and sum of demand are different.

$$\Sigma \text{ Supply} \neq \Sigma \text{ Demand}$$

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

ii. Least Cost Method



NORTH-WEST CORNER METHOD

METHODS TO SOLVE TRANSPORTATION MODEL

i. **North-West Corner Method**

The North West Corner Method is one of the methods to obtain a basic feasible solution of the transportation problems (special case of LPP).

We will now see how to apply this very simple method to a transportation problem.

We will study steps of this method while applying it in the problem itself.

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

A mobile phone manufacturing company has three branches located in three different regions, say Barishal, Rajshahi, and Rangpur.

The company has to transport mobile phones to three destinations, say Dhaka, Sylhet and Chittagong.

The availability from Barishal, Rajshahi, and Rangpur is 40, 60 and 70 units respectively.

The demand at Dhaka, Sylhet, and Chittagong are 70, 40 and 60 respectively.

The transportation cost is shown in the matrix below (in Tk.).

Use the North-West Corner Method to find a **basic feasible solution (BFS)**.

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Barishal	4	5	1	40
	Rajshahi	3	4	6	60
	Rangpur	6	2	8	70
	DEMAND	70	40	60	170

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 1: Balance the problem

Balance the problem meaning we need to check that if; $\sum \text{Supply} = \sum \text{Demand}$

If this holds true, then we will consider the given problem as a balanced problem.

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 2: Start allocating from North-West corner cell

We will start the allocation from the left hand top most corner (north-west) cell in the matrix and make allocation based on availability and demand.

Now, verify the smallest among the availability (Supply) and requirement (Demand), corresponding to this cell.

The smallest value will be allocated to this cell and check out the difference in supply and demand, representing that supply and demand are fulfilled, as shown below.

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Barishal	4 (40)	5	1	40 (0)
	Rajshahi	3	4	6	60
	Rangpur	6	2	8	70
	DEMAND	70 (30)	40	60	

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Barishal	4 (40)	5	1	40 (0)
	Rajshahi	3	4	6	60
	Rangpur	6	2	8	70
	DEMAND	70 (30)	40	60	

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix

As we have fulfilled the availability or requirement for that row or column respectively, remove that row or column and prepare a new matrix, as shown below.

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Rajshahi	3	4	6	60
	Rangpur	6	2	8	70
	DEMAND	30	40	60	

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 4: Repeat the procedure until all the allocations are over

Repeat the same procedure of allocation of the new North-west corner so generated and check based on the smallest value as shown below, until all allocations are over.

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Rajshahi	3 (30)	4	6	60 (30)
	Rangpur	6	2	8	70
	DEMAND	30 (0)	40	60	

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 4: Repeat the procedure until all the allocations are over

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Rajshahi	3 (30)	4	6	60 (30)
	Rangpur	6	2	8	70
	DEMAND	30 (0)	40	60	

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 4: Repeat the procedure until all the allocations are over

DESTINATIONS				
SOURCES		Sylhet	Chittagong	SUPPLY
	Rajshahi	4 (30)	6	30 (0)
	Rangpur	2	8	70
	DEMAND	40 (10)	60	

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 4: Repeat the procedure until all the allocations are over

DESTINATIONS				
SOURCES		Sylhet	Chittagong	SUPPLY
	Rajshahi	4 (30)	6	30 (0)
	Rangpur	2	8	70
	DEMAND	40 (10)	60	

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 4: Repeat the procedure until all the allocations are over

DESTINATIONS				
SOURCES		Sylhet	Chittagong	SUPPLY
	Rangpur	2 (10)	8 (60)	70 (60) (0)
	DEMAND	10 (0)	60 (0)	

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 4: Repeat the procedure until all the allocations are over

DESTINATIONS				
SOURCES		Sylhet	Chittagong	SUPPLY
	Rangpur	2 (10)	8 (60)	(0)
	DEMAND	(0)	(0)	

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 5: After all the allocations are over, write the allocations and calculate the transportation cost

Once all allocations are over, prepare the table with all allocations marked and calculate the transportation cost as follows.

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

Step 5: After all the allocations are over, write the allocations and calculate the transportation cost

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Barishal	4	5	1	40
	Rajshahi	3	4	6	60
	Rangpur	6	2	8	70
	DEMAND	70	40 30	60	170

METHODS TO SOLVE TRANSPORTATION MODEL

i. North-West Corner Method

Sample Problem# 1

Solution

		DESTINATIONS			
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Barishal	4 (40)	5	1	40
	Rajshahi	3 (30)	4 (30)	6	60
	Rangpur	6	2 (10)	8 (60)	70
	DEMAND	70	40	60	170

$$\begin{aligned}\text{Total Transportation Costs} &= (4 \times 40) + (3 \times 30) + (4 \times 30) + (2 \times 10) + (8 \times 60) \\ &= \text{Tk. 870/-}\end{aligned}$$

LEAST COST METHOD

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

A mobile phone manufacturing company has three branches located in three different regions, say Barishal, Rajshahi, and Rangpur.

The company has to transport mobile phones to three destinations, say Dhaka, Sylhet and Chittagong.

The availability from Barishal, Rajshahi, and Rangpur is 40, 60 and 70 units respectively.

The demand at Dhaka, Sylhet, and Chittagong are 70, 40 and 60 respectively.

The transportation cost is shown in the matrix below (in Tk.).

Use the Least Cost Method to find a **basic feasible solution (BFS)**.

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Barishal	4	5	1	40
	Rajshahi	3	4	6	60
	Rangpur	6	2	8	70
	DEMAND	70	40	60	170

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

Step 1: Balance the problem

The given Transportation problem is balanced as the $\Sigma \text{ Supply} = \Sigma \text{ Demand}$

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Barishal	4	5	1 (40)	40 (0)
	Rajshahi	3	4	6	60
	Rangpur	6	2	8	70
	DEMAND	70	40	60 (20)	

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Barishal	4	5	1 (40)	40 (0)
	Rajshahi	3	4	6	60
	Rangpur	6	2	8	70
	DEMAND	70	40	60 (20)	

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix.

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Rajshahi	3	4	6	60
	Rangpur	6	2	8	70
	DEMAND	70	40	20	38

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

Step 4: Repeat the procedure until all the allocations are over.

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Rajshahi	3	4	6	60
	Rangpur	6	2 (40)	8	70 (30)
	DEMAND	70	40 (0)	20	

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Rajshahi	3	4	6	60
	Rangpur	6	2 (40)	8	70 (30)
	DEMAND	70	40 (0)	20	

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

DESTINATIONS				
SOURCES		Dhaka	Chittagong	SUPPLY
	Rajshahi	3	6	60
	Rangpur	6	8	30
	DEMAND	70	20	

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

DESTINATIONS				
SOURCES		Dhaka	Chittagong	SUPPLY
	Rajshahi	3 (60)	6	60 (0)
	Rangpur	6	8	30
	DEMAND	70 (10)	20	

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

DESTINATIONS				
SOURCES		Dhaka	Chittagong	SUPPLY
	Rajshahi	3 (60)	6	60 (0)
	Rangpur	6	8	30
	DEMAND	70 (10)	20	

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

DESTINATIONS				
SOURCES		Dhaka	Chittagong	SUPPLY
	Rangpur	6	8	30
	DEMAND	10	20	

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

DESTINATIONS				
SOURCES		Dhaka	Chittagong	SUPPLY
	Rangpur	6 (10)	8 (20)	30 (20) (0)
	DEMAND	10 (0)	20 (0)	

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

Step 5: After all allocations are over, write the allocations and calculate the transportation costs.

METHODS TO SOLVE TRANSPORTATION MODEL

ii. Least Cost Method

Sample Problem# 2

Solution

DESTINATIONS					
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY
	Barishal	4	5	1 (40)	40
	Rajshahi	3 (60)	4	6	60
	Rangpur	6 (10)	2 (40)	8 (20)	70
	DEMAND	70	40	60	170

$$\begin{aligned}\text{Total Transportation Costs} &= (1 \times 40) + (3 \times 60) + (6 \times 10) + (2 \times 40) + (8 \times 20) \\ &= \text{Tk. 520/-}\end{aligned}$$

Unbalanced Transportation Problems

METHODS TO SOLVE TRANSPORTATION MODEL

Solve the following problem for BFS using Least Cost Method

DESTINATIONS					
SOURCES		A	B	C	SUPPLY
	P	5	1	7	20
	Q	7	4	6	70
	R	3	2	5	25
	DEMAND	65	30	50	

METHODS TO SOLVE TRANSPORTATION MODEL

Solve the following problem for BFS using Least Cost Method

Sample Problem# 4

Solution

Step 1: Balance the problem

The given Transportation problem is unbalanced as the

$$\Sigma \text{ Supply} \neq \Sigma \text{ Demand}$$

As the $\Sigma \text{ Supply} = 115$ and the $\Sigma \text{ Demand} = 145$

DESTINATIONS					
SOURCES		A	B	C	SUPPLY
	P	5	1	7	20
	Q	7	4	6	70
	R	3	2	5	25
	DEMAND	65	30	50	145//115

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 1: Balance the problem

Therefore, Dummy Row ' D_1 ' is added with the supply of 30 units in order to balance the Transport problem.

DESTINATIONS					
SOURCES		A	B	C	SUPPLY
	P	5	1	7	20
	Q	7	4	6	70
	R	3	2	5	25
	D_1	0	0	0	30
	DEMAND	65	30	50	145

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand

DESTINATIONS					
SOURCES		A	B	C	SUPPLY
	P	5	1 (20)	7	20 (0)
	Q	7	4	6	70
	R	3	2	5	25
	D ₁	0	0	0	30
	DEMAND	65	30 (10)	50	

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand

DESTINATIONS					
SOURCES		A	B	C	SUPPLY
	P	5	1 (20)	7	20 (0)
	Q	7	4	6	70
	R	3	2	5	25
	D ₁	0	0	0	30
	DEMAND	65	30 (10)	50	

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix

DESTINATIONS					
SOURCES		A	B	C	SUPPLY
	Q	7	4	6	70
	R	3	2	5	25
	D ₁	0	0	0	30
	DEMAND	65	10	50	

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix

DESTINATIONS					
SOURCES		A	B	C	SUPPLY
	Q	7	4	6	70
	R	3	2 (10)	5	25 (15)
	D ₁	0	0	0	30
	DEMAND	65	10 (0)	50	

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix

DESTINATIONS					
SOURCES		A	B	C	SUPPLY
	Q	7	4	6	70
	R	3	2 (10)	5	25 (15)
	D ₁	0	0	0	30
	DEMAND	65	10 (0)	50	

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 4: Repeat the procedure until all the allocations are over

DESTINATIONS				
SOURCES		A	C	SUPPLY
	Q	7	6	70
	R	3 (15)	5	15 (0)
	D ₁	0	0	30
	DEMAND	65 (50)	50	

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 4: Repeat the procedure until all the allocations are over

DESTINATIONS				
SOURCES		A	C	SUPPLY
	Q	7	6	70
	R	3 (15)	5	15 (0)
	D ₁	0	0	30
	DEMAND	65 (50)	50	

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 4: Repeat the procedure until all the allocations are over

DESTINATIONS				
SOURCES		A	C	SUPPLY
	Q	7	6 (50)	70 (20)
	D ₁	0	0	30
	DEMAND	50	50 (0)	

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 4: Repeat the procedure until all the allocations are over

DESTINATIONS				
SOURCES		A	€	SUPPLY
	Q	7	6 (-50)	70 (20)
	D ₁	0	0	30
	DEMAND	50	50 (0)	

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 4: Repeat the procedure until all the allocations are over

DESTINATIONS			
SOURCES		A	SUPPLY
	Q	7 (20)	20 (0)
	D ₁	0 (30)	30 (0)
	DEMAND	50 (30) (0)	

METHODS TO SOLVE TRANSPORTATION MODEL

Solution

Step 5: After all allocations are over, write all the allocations and calculate the transportation cost.

		DESTINATIONS			
SOURCES		A	B	C	SUPPLY
	P	5	1 (20)	7	20
	Q	7 (20)	4	6 (50)	70
	R	3 (15)	2 (10)	5	25
	D ₁	0 (30)	0	0	30
	DEMAND	65	30	50	145

$$\begin{aligned}\text{Total Transportation Costs} &= (1 \times 20) + (7 \times 20) + (6 \times 50) + (3 \times 15) + (2 \times 10) + (0 \times 30) \\ &= \text{Tk. 525/-}\end{aligned}$$

METHODS TO SOLVE TRANSPORTATION MODEL

NOTE

If the sum of demand is less than the sum of supply, then we will have to add a **"Dummy Column"**.

DESTINATIONS					
SOURCES		A	B	C	SUPPLY
	P	5	1	7	65
	Q	7	4	6	30
	R	3	2	5	50
	DEMAND	20	70	25	115//145

METHODS TO SOLVE TRANSPORTATION MODEL

NOTE

In order to balance the Transportation problem, we have to add **"0"** in the **"Dummy Column"** having demand of 30

DESTINATIONS						
SOURCES		A	B	C	D ₁	SUPPLY
	P	5	1	7	0	65
	Q	7	4	6	0	30
	R	3	2	5	0	50
	DEMAND	20	70	25	30	145//145

Further, we can solve this by using any of the discussed method (NWCM, LCM or VAM)

PRACTICE PROBLEM...

Problem...

Explanation:

Given three sources O1, O2 and O3 and four destinations D1, D2, D3 and D4.

For the sources O1, O2 and O3, the supply is 300, 400 and 500 respectively.

The destinations D1, D2, D3 and D4 have demands 250, 350, 400 and 200 respectively.

Based on the matrix given in the next slide, calculate the Transportation Costing for **North-West Corner Method** and **Least Cost Method**, and comment on which transportation method is ideal to be chosen?

PRACTICE PROBLEM...

Problem...

DESTINATIONS						
SOURCES		D1	D2	D3	D4	SUPPLY
	Q1	3	1	7	4	300
	Q2	2	6	5	9	400
	Q3	8	3	3	2	500
	DEMAND	250	350	400	200	

END OF THE CHAPTER...