Medical Image Processing for Diagnostic Applications

Parallel Beam – Preparation

Online Course – Unit 30

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Topics

Important Methods

Central Slice Theorem

Summary

Take Home Messages

Further Readings

Fourier Transform

- Fourier transform in 1-D:

$$P(\omega) = \int_{-\infty}^{\infty} p(s) e^{-2\pi i s \omega} \mathrm{d}s$$

- Inverse Fourier transform in 1-D:

$$p(s) = \int_{-\infty}^{\infty} P(\omega) e^{2\pi i s \omega} \mathrm{d}\omega$$

$\longrightarrow$ Fourier pairs

Convolution

- Convolution:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,\mathrm{d}\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)\,\mathrm{d}\tau$$

- Convolution theorem:

$$q(s) = f(s) * g(s) \quad \Leftrightarrow \quad Q(\omega) = F(\omega) \cdot G(\omega)$$

Hilbert Transform

- The spatial representation of the Hilbert transform:

$$(f * h)(t) = \text{p.v.} \int_{-\infty}^{\infty} f(t-\tau)h(\tau)\,\mathrm{d}\tau,$$

  where “p.v.” denotes the principal value, has the transformation kernel

$$h(\tau) = \frac{1}{\pi\tau}.$$

convolution with this

- Its Fourier representation is:

$$H(\omega) = -i\,\mathrm{sgn}(\omega).$$

is the same as this in frequency domain

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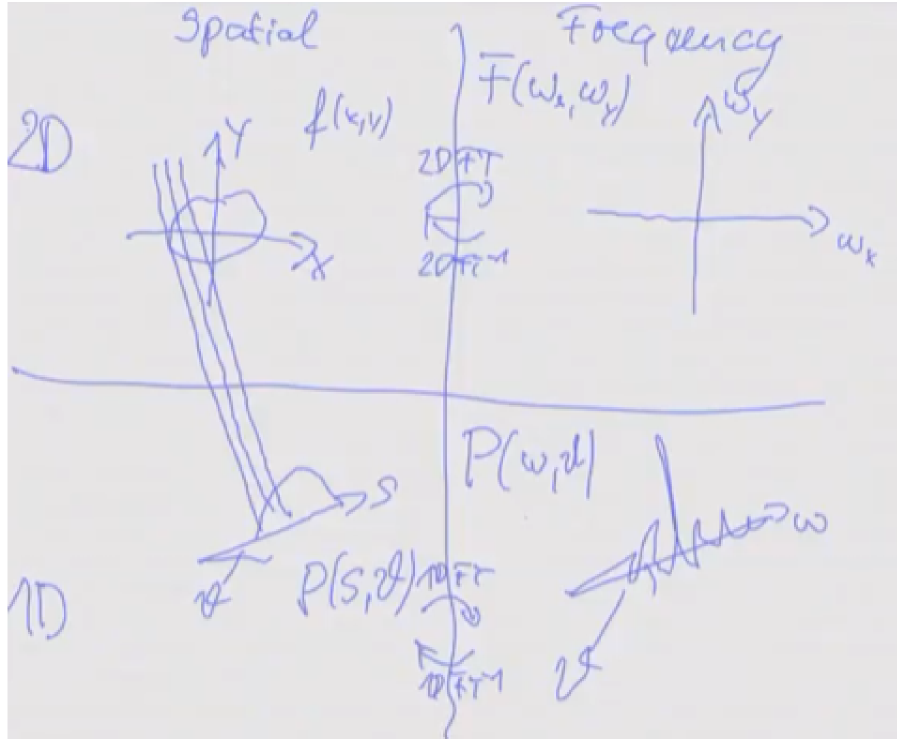
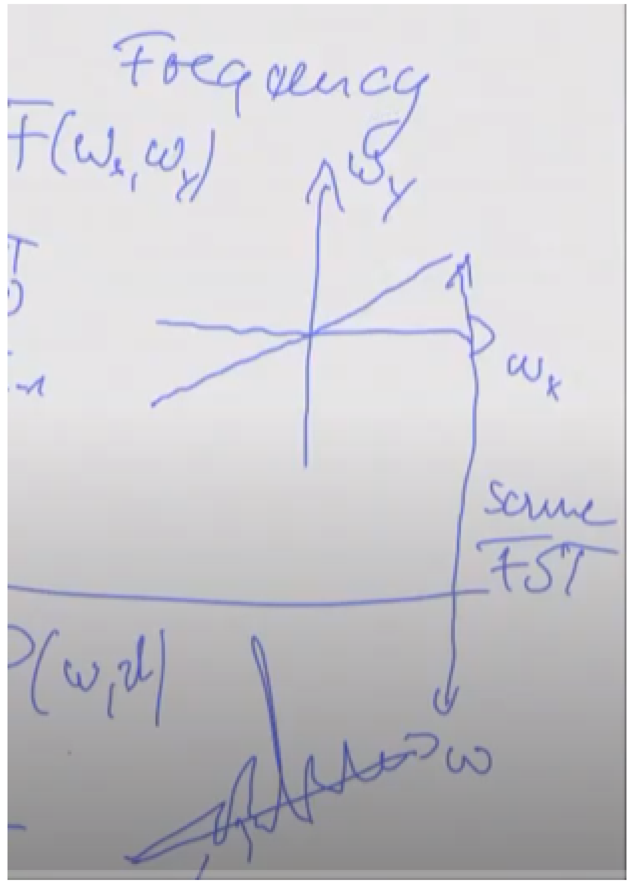




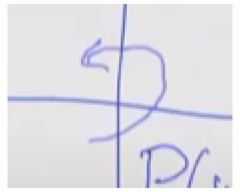
this slice cutting throu the center of k space is the same as the 1D projection

-> Furier slice theorem

cental slice theorem?

the only way of going from 1D to 2D in spacial domain is via inverse and we dont want that.

So we go from 2D to 1D in spacial domain, transform into FD and now the 1D FD projection is just a sample line of the 2D FD image
This way we can sample the entire k-space and then transform back into SD :O :)



we go this way around

Central Slice Theorem

$$P(\omega, \theta) = F(\omega\cos\theta, \omega\sin\theta)$$

1D

2D

all in FD

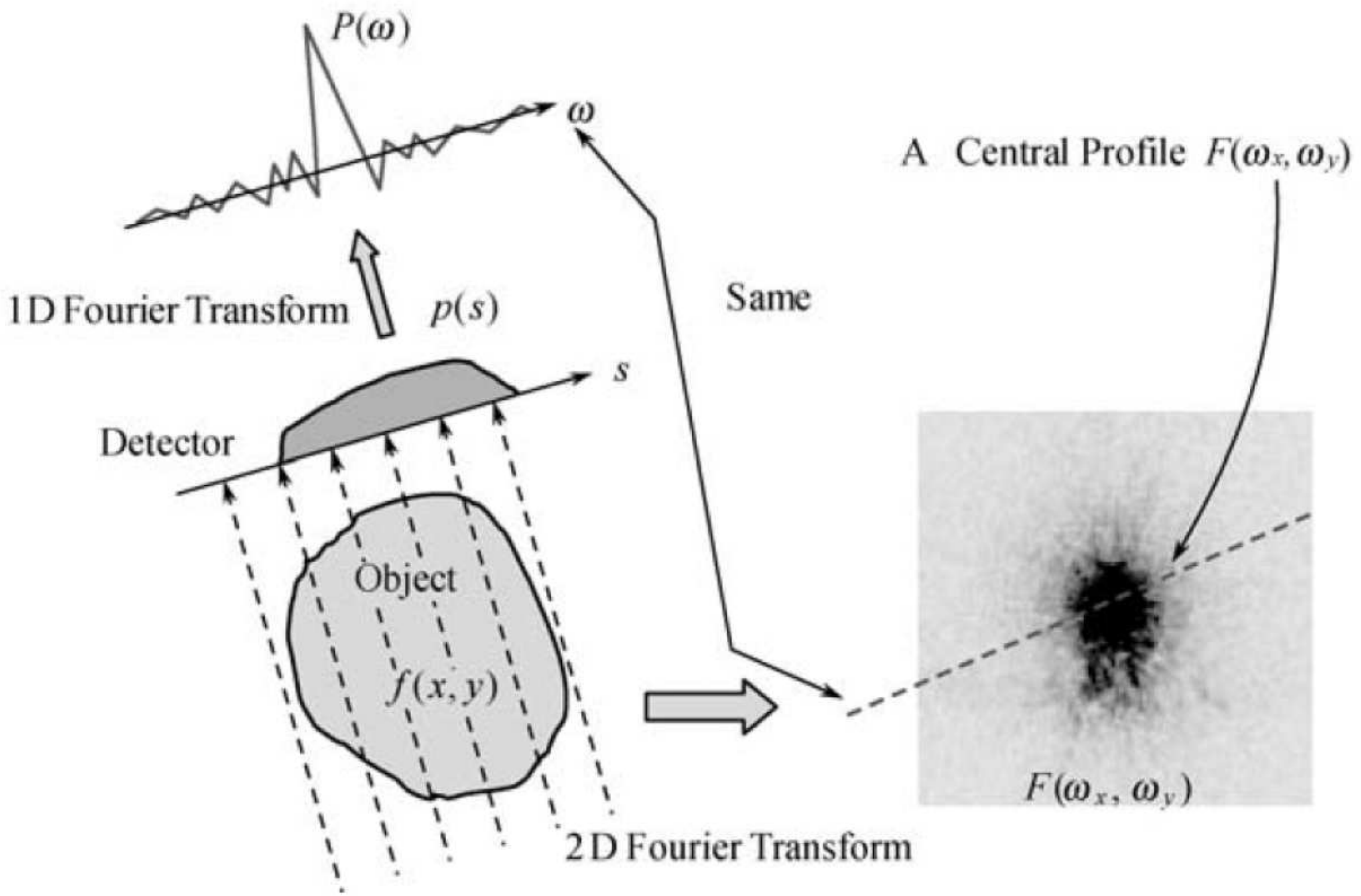


Figure 1: Central slice theorem (Zeng, 2009)

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proof

$$P(\omega,\vartheta) = \int_{-\infty}^{\infty} p(s,\vartheta)\, e^{-2\pi i \omega s}\, ds$$

$$P(\omega,\vartheta) = \int_{-\infty}^{\infty} \iint f(x,y)\, \delta(x\cos\vartheta + y\sin\vartheta - s)\, e^{-2\pi i \omega s}\, dx\, dy\, ds =$$

$$= \iint f(x,y) \int \delta(\underbrace{x\cos\vartheta + y\sin\vartheta - s}_{s = x\cos\vartheta + y\sin\vartheta})\, e^{-2\pi i \omega s}\, ds\, dx\, dy$$

$$= \iint f(x,y)\, e^{-2\pi i (\omega x\cos\vartheta + \omega y\sin\vartheta)}\, dx\, dy$$

$$= \iint f(x,y)\, e^{-2\pi i (x \underbrace{\omega\cos\vartheta}_{u} + y \underbrace{\omega\sin\vartheta}_{v})}\, dx\, dy$$

$$= F(\omega\cos\vartheta, \omega\sin\vartheta)$$

Take Home Messages

- The Fourier transform, the convolution theorem, and the Hilbert transform are important tools for image reconstruction.
- The central slice theorem (also: Fourier slice theorem) is one of the most fundamental concepts in reconstruction theory!

Further Readings

The derivation of the Fourier slice theorem can also be found here (bibsource):

Joachim Hornegger, Andreas Maier, and Markus Kowarschik. “CT Image Reconstruction Basics”. In: *MR and CT Perfusion and Pharmacokinetic Imaging: Clinical Applications and Theoretical Principles*. Ed. by Roland Bammer. 1st ed. Alphen aan den Rijn, Netherlands: Wolters Kluwer, 2016, pp. 01–09

The concise reconstruction book from ‘Larry ’Zeng:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

If you want to learn more about applications of the Fourier transform:

Ronald N. Bracewell. *The Fourier Transform and Its Applications*. 3rd ed. Electrical Engineering Series. Boston: McGraw-Hill, 2000