Medical Image Processing for Diagnostic Applications

Efficient Implementation

Online Course – Unit 13
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Topics

Efficient Evaluation of Polynomials

Interpolation of Intensities

Distortion Algorithm: Summary and Further Applications

Summary
    Take Home Messages
    Further Readings

Efficient Evaluation of Polynomials

Problem: Each acquired image requires a distortion correction.

Observations:

- Bivariate polynomials can be understood as univariate polynomials where the coefficients are univariate polynomials (instead of a simple constant value):

$$x = \sum_{i=0}^{d} \left(\sum_{j=0}^{d-i} u_{i,j} y'^j \right) x'^i.$$

- Univariate polynomials are evaluated using Horner's scheme:

$$p(x) = \sum_{i=0}^{d} a_i x^i = (\dots (a_d x + a_{d-1}) x + \dots) x + a_0.$$

sowas hattest du schonmal -> war das in Mathe A3?   -> reduced # of multiplications

Efficient Evaluation of Polynomials

Definition

The **_Horner scheme_**, named after William George Horner, is an algorithm for the efficient evaluation of polynomials in monomial form. Horner's method describes a manual process by which one may approximate the roots of a polynomial equation. The Horner scheme can also be viewed as a fast algorithm for dividing a polynomial by a linear polynomial. (Wikipedia)

- For each line we get a univariate polynomial ($y' = $ const).
- Row and column increments are constant.
- Arithmetic progression $\rightarrow$ reuse of former evaluations

Conclusion: After an initialization, for each pixel only sums have to be computed.

Example: Horner Scheme

Let us assume we have a matrix $\boldsymbol{M}$ and observations $(x', y')^\mathsf{T}$, and we want to find $(x, y)^\mathsf{T}$ such that:

$$\boldsymbol{M} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}.$$

Suppose we implement the necessary operations:

$$\begin{pmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} m_{1,1}x + m_{1,2}y \\ m_{2,1}x + m_{2,2}y \end{pmatrix}$$

for several pairs of x and y in two nested `for`-loops over x and y, we will do some computations more often than necessary.
Using the Horner scheme, we reuse earlier computations constant in the inner loop.

Topics

Interpolation of Intensities

- **Nearest Neighbor Interpolation** is the simplest interpolation method. We assign to an image point in between pixels the intensity value of the closest pixel. Usually the Euclidean distance is applied.

- In **Bilinear Interpolation**, we compute for the new image point a weighted mean of neigboring intensities. Here the intuition is applied that pixels closer to the new image point have an higher impact on the final intensity value.

- Mostly the results of nearest neighbor interpolation are not well appreciated. The images appear crispy and noisy, though the interpolation method is extremely fast.

- In most practical applications where interpolation is required, bilinear interpolation is applied.

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Interpolation of Intensities

Computation of the intensity i

- Nearest neighbor interpolation:

$$i = a$$

- Bilinear interpolation:

$$x = a(1 - d') + dd',$$
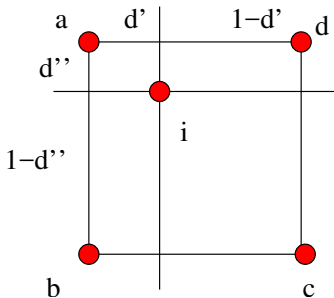$$y = b(1 - d') + cd',$$
$$i = x(1 - d'') + yd''$$

Figure 1: Interpolation: i is interpolated using the known intensities a, b, c, d.

graphics cards automaticly can run interpolations between pixels :)

Topics

Accelerated Distortion Correction Algorithm

FOR $x :=$ first to last line DO

    FOR $y :=$ first to last column DO

        compute coordinate $x = X(x', y')$

        compute coordinate $y = Y(x', y')$

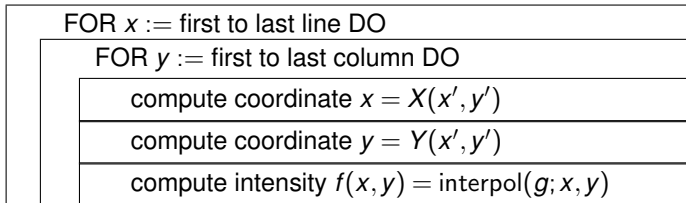   compute intensity $f(x, y) = \text{interpol}(g; x, y)$

Figure 2: Image undistortion routine (*g* distorted image, *f* corrected output image)

Hardware Accelerated Image Warping

summary

- Image is decomposed into squares.
- Map the vertices of the square.
- Undistortion can be implemented using texture mapping unit.
- Bilinear interpolation hardware is supported in GPUs.
- Texture mapping hardware is supported in GPUs.

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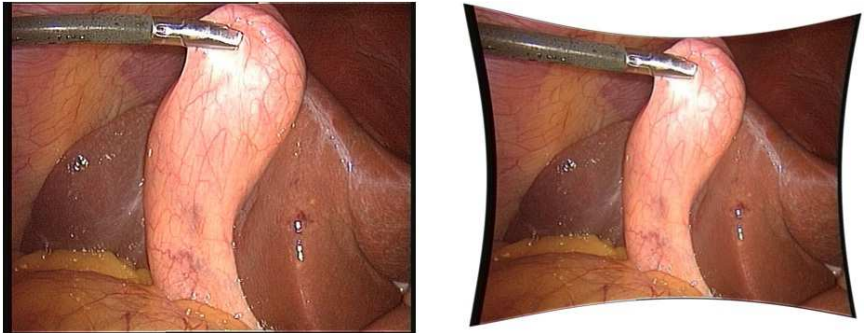ERLANGEN-NÜRNBERG
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Application of Image Undistortion: Endoscopy

Figure 3: Original, distorted endoscope image (left), and the result of distortion correction (right)

Topics

Take Home Messages

- The <mark>Horner scheme</mark> can be used for efficient evaluation of polynomials.
- <mark>Bilinear interpolation</mark> is sufficient for nearly all practical problems.

Further Readings

In case you need to learn more about polynomials and the efficient evaluation of polynomials, you have to read Volume 2 of Prof. Knuth's classic work on The Art of Computer Programming.

A book that covers many image preprocessing methods applied in medical imaging systems is:

Jiří Jan. *Medical Image Processing, Reconstruction, and Restoration: Concepts and Methods*. Signal Processing and Communications. CRC Press, Taylor & Francis Group, Nov. 2005