Medical Image Processing for Diagnostic Applications

Polynomial Undistortion

Online Course – Unit 11

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Topics

image intensifier has a convex entrance screen -> distortion

Polynomial Undistortion

- Distortion Correction Mapping
- Parameter Estimation/Calibration

Summary

- Take Home Messages
- Further Readings

Distortion Correction Mapping

describe how every pixel moves individually -> lots of parameters

Non-Parametric Mapping: $\rightarrow$ look-up table or displacement vector field

Parametric Mapping: We assume separable base functions.

- $\rightarrow$ univariate base functions $b_k : \mathbb{R} \rightarrow \mathbb{R}$, where $k = 0, \ldots, d \in \mathbb{N}$
- image point of undistorted image: (x', y')
- image point of distorted image: (x, y)
- mapping coefficients $u_{i,j}, v_{i,j} \in \mathbb{R}, i = 0, \ldots, d, j = 0, \ldots, d$

X mapping

$$x = X(x', y') = \sum_{i=0}^{d} \sum_{j=0}^{d-i} u_{i,j} b_j(y') b_i(x')$$

Y mapping

$$y = Y(x', y') = \sum_{i=0}^{d} \sum_{j=0}^{d-i} v_{i,j} b_j(y') b_i(x')$$

v and u are weights

Distortion Correction Mapping

Example

We choose the **standard polynomials (monomials)**:

$$b_i(x) = x^i,$$

but can be something different?

and get the following bi-variate polynomial of degree d for the x-coordinates:

$$\underset{\text{dist.}}{x} = X(x', y') = \sum_{i=0}^{d} \sum_{j=0}^{d-i} u_{i,j} \underset{\text{undist.}}{y'^j x'^i}.$$

i = 0 -> this is the offset

so the result is a summ over different polynomials
weighted by u_i

Us are the unknowns

Example: Solving Steps

same we did in PR when we lifted x into a higher dimension

Let $d = 2$. We can rewrite the mapping in matrix notation for corresponding image points $(x_1, y_1), \dots, (x_n, y_n)$ and $(x'_1, y'_1), \dots, (x'_n, y'_n)$:

$(u_{x,y})$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & y'_1 & (y'_1)^2 & x'_1 & y'_1 x'_1 & (x'_1)^2 \\ \vdots & & & \ddots & & \vdots \\ 1 & y'_n & (y'_n)^2 & x'_n & y'_n x'_n & (x'_n)^2 \end{pmatrix}}_{\text{measurement matrix } \boldsymbol{A}} \begin{pmatrix} u_{0,0} \\ u_{0,1} \\ u_{0,2} \\ u_{1,0} \\ u_{1,1} \\ u_{2,0} \end{pmatrix}.$$

Us are the coefficiants

= $\binom{n+\tilde{d}}{d}$

$$\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!}$$

now its a linear porblem - use SVD to find solution :)

$A^{-1} * x = u$

with pseudoinverse, this is the least square (L2) optimal solution to the problem

in Exam: "construct measurement matrix and write up how to solve this"

A is the measurement matrix

Distortion Correction: Calibration of the Mapping

Definition

We call the estimation process of parameters that define the mapping of real-world objects into the camera image plane ***calibration***.

pattern need to be 10 times more accurate

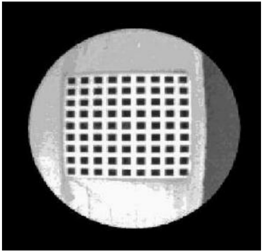
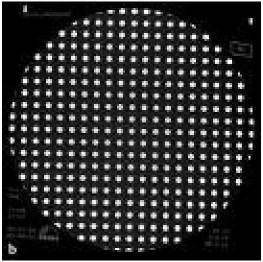
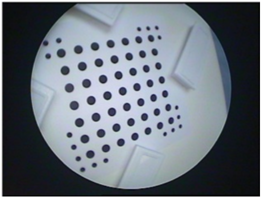

Figure 1: A few examples of calibration patterns with squares, dots, and circles (images courtesy of Siemens Healthcare)

Distortion Correction: Calibration of the Mapping

Calibration problem:

- N 2-D points on the planar calibration pattern are precisely known:

$$(x'_1, y'_1), (x'_2, y'_2), \ldots, (x'_N, y'_N).$$

- N points in the distorted image are observed:

$$(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N).$$

Problem: Estimate the parameters of the distortion function.

Distortion Correction: Calibration of the Mapping

Definition

Least square estimation is a numerical procedure that fits a parametric or non-parametric curve to data points by minimization of the sum of squared distances of data points from the curve.

For the calibration of the mapping, the least square estimation results in the following two optimization problems:

$$\sum_{n=1}^{N} \left(X(x'_n, y'_n) - x_n\right)^2 \rightarrow \min,$$

$$\sum_{n=1}^{N} \left(Y(x'_n, y'_n) - y_n\right)^2 \rightarrow \min.$$

exact solution not possible because of noise

Distortion Correction: Calibration of the Mapping

Using univariate base functions, we have the following $2N$ equations that are linear in $u_{i,j}$ and $v_{i,j}$:

$$x_n = \sum_{i=0}^{d} \sum_{j=0}^{d-i} u_{i,j} b_j(y'_n) b_i(x'_n), \quad n = 1, \ldots, N,$$

$$y_n = \sum_{i=0}^{d} \sum_{j=0}^{d-i} v_{i,j} b_j(y'_n) b_i(x'_n), \quad n = 1, \ldots, N.$$

U and V were bot calculated with the pseudoinverted A - Matrix

Distortion Correction: Calibration of the Mapping

These equations can be rewritten in matrix notation (here shown for the x−correspondences, y−correspondences analogously):

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} u_{0,0} \\ u_{0,1} \\ \vdots \\ u_{d,0} \end{pmatrix}.$$

The matrix $\boldsymbol{A}$ is the measurement matrix for the particular problem of image undistortion.

Distortion Correction: Calibration Parameter Estimation

Estimates of the coefficients are computed by:

$$\begin{pmatrix} u_{0,0} \\ u_{0,1} \\ \vdots \\ u_{d,0} \end{pmatrix} = \boldsymbol{A}^{\dagger} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix},$$

where $\boldsymbol{A}^{\dagger}$ is the pseudoinverse of $\boldsymbol{A}$. As we already know, the pseudoinverse can be computed by using the singular value decomposition.

Topics

Take Home Messages

- A distortion correction method needs a mapping that is estimated from the correspondences of a calibration pattern and the measured data.
- Choosing univariate base functions is a good option. Using monomials, we find the Vandermonde matrix to be the measurement matrix.
  the way we constructed A - matrix
- In the general case, we can use least square estimation to find the mapping parameters.

Further Readings

In case you need to learn more about polynomials and the efficient evaluation of polynomials, you have to read Volume 2 of Prof. Knuth's classic work on The Art of Computer Programming.

A book that covers many image preprocessing methods applied in medical imaging systems is:

Jan2005MIPRaRCaM