Medical Image Processing for Diagnostic Applications

Defect Pixel Interpolation – Utilizing Symmetry

Online Course – Unit 17

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Topics

Defect Pixel Interpolation using Symmetry Properties

Interpolation Algorithm

Summary
- Take Home Messages
- Further Readings

Notation

For simplicity and without loss of generality we limit the following discussion to 1-D signals of length N, and use the following notation:

- discrete ideal signal: $f(n)$,
- binary mask image: $w(n)$,
- observed signal: $g(n)$.

$$g(n) = f(n) \cdot w(n)$$

The respective Fourier transforms are denoted $F(\xi)$, $W(\xi)$, $G(\xi)$.

Frequency Domain Defect Pixel Interpolation

Observations:

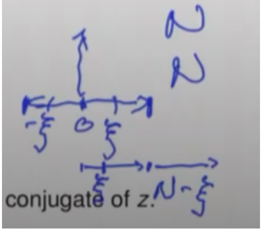
- We consider real valued signals $f(n)$, $g(n)$, $w(n)$. real value -> symmetry holds
- They satisfy the following relationship:

$$g(n) = f(n) \cdot w(n) \quad \Leftrightarrow \quad G(\xi) = F(\xi) * W(\xi).$$

- For the ideal, the mask, and the defect image, the Fourier transform satisfies the symmetry property:

if we map everything to an array and the neg quadrant after the pos quadrant -> conjugated Value at index 0 and N
N is the length of the signal/ furier transformed

$$F(\xi) = \overline{F}(N - \xi),$$
$$G(\xi) = \overline{G}(N - \xi),$$
$$W(\xi) = \overline{W}(N - \xi),$$



where the bar symbol in $\overline{z}$ denotes the complex conjugate of z.

Frequency Domain Defect Pixel Interpolation

Now we make explicit use of the symmetry property of the Fourier transform to derive an interpolation algorithm:

- Select a pair $G(s)$ and $G(N-s)$ of the Fourier transform of the corrupted image showing pixels defects.
- Select a pair $F(s)$ and $F(N-s)$ of the Fourier transform of the ideal image.

Frequency Domain Defect Pixel Interpolation

- Let us assume that the Fourier transform of the ideal image $F(\xi)$ consists only of two lines at s and $N-s$, where $s \neq 0$.
- We can then rewrite the Fourier transform of $f(n)$ using Dirac's δ-function:this is a dirac pulse is 1 if ξ = s or if ξ-N = s -> basicly if, else statement

$$F(\xi) = \widehat{F}(s)\delta(\xi - s) + \widehat{F}(N-s)\delta(\xi - N + s),$$

what we want sifnal only consists of F(s) and F(N-s) -> rest is zero

where $\widehat{F}$ denotes an estimate of F, and the δ-function is defined by

$$\delta(k) = \begin{cases} 1, & \text{if } n = 0, \\ 0, & \text{otherwise.} \end{cases}$$

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Frequency Domain Defect Pixel Interpolation

The convolution of F and the Fourier transform W of the given mask image leads to the Fourier transform of the observed corrupted image:

$$G(s) = \frac{1}{N}\left(\widehat{F}(s)W(0) + \overline{\widehat{F}}(s)W(2s)\right).$$

This can be shown as follows:

$$G(s) = F(s) * W(s) = \frac{1}{N}\sum_{k=0}^{N-1} F(k) * W(s-k).$$

definition of convolution

Due to our assumption, we know that $F \neq 0$ only at $k = s$ or $k = N - s$, hence:

kick out rest of the sum

$$G(s) = \frac{1}{N}\left(\widehat{F}(s)W(0) + \widehat{F}(N-s)W(s-N+s)\right)$$

furier is cyclic -> drop N

$$= \frac{1}{N}\left(\widehat{F}(s)W(0) + \overline{\widehat{F}}(s)W(2s)\right).$$

Frequency Domain Defect Pixel Interpolation

- For the conjugate complex Fourier transform of the observed image we get:

$$\overline{G}(s) = \frac{1}{N}\left(\overline{\widehat{F}}(s)\overline{W}(0) + \widehat{F}(s)\overline{W}(2s)\right).$$

- Since W is known, we get two equations linear in $\widehat{F}(s)$ and $\overline{\widehat{F}}(s)$.
- Hence, the final estimator for the Fourier transform of the ideal image is:

$$\widehat{F}(s) = N\frac{G(s)\overline{W}(0) - \overline{G}(s)W(2s)}{|W(0)|^2 - |W(2s)|^2}, \tag{FT-EST}$$

where $|.|$ denotes the absolute value of the complex number.

Error Spectrum

- An objective function to measure the quality of the interpolated image results from the least square error: in spacial domain

$$\Delta_{\varepsilon} = \frac{1}{N} \sum_{n=0}^{N-1} \left(g(n) - w(n) \widehat{f}(n) \right)^2 .$$

- The spectrum of the error in the i-th iteration step is given by:

error in frequency domain -> now we dont have to transform back :)

$$G^{(i)}(\xi) = G^{(i-1)}(\xi) - \frac{1}{N} \left(\widehat{F}^{(i)}(s) \delta(\xi - s) + \overline{\widehat{F}}^{(i)}(s) \delta(\xi - N + s) \right) * W(\xi).$$

Topics

Interpolation Algorithm

compute FT of input signal $g(n)$

initialize $\widehat{F}^{(0)}(k) = 0$, $G^{(0)}(k) = G(k)$, $i = 1$

- randomly select a line pair $G^{(i-1)}(s)$, $G^{(i-1)}(N-s)$ for $s \neq 0$
- estimate $\widehat{F}^{(i)}(s)$, $\widehat{F}^{(i)}(N-s)$ using (FT-EST)
- update spectrum $\widehat{F}^{(i)}(k)$, compute error Δ_ε
- error Δ_ε above a threshold?
  - `true`: Compute error spectrum $G^{(i)}(k)$, increment i
  - `false`: GOTO END

UNTIL error is below a threshold

END: Compute inverse FT of $\widehat{F}^{(i)}(k)$

Figure 1: Interpolation algorithm according to Aach and Metzler

Topics

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Interpolation Algorithm

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Take Home Messages

- Assuming the spectrum of a signal/function consists of two non-zero lines, then we find an estimate for the Fourier transform.
- The symmetry property of the Fourier transform w. r. t. real valued functions can be used to build a defect interpolation algorithm.

basicly a deconvolution in furier space

iteratively look at all the frequencies (and fix them??)

Further Readings

- The method presented for defect pixel interpolation in the frequency domain was published by Til Aach and Volker Metzler in 2001:

Til Aach and Volker Metzler. "Defect Interpolation in Digital Radiography: How Object-Oriented Transform Coding Helps". In: *Proc. SPIE 4322, Medical Imaging 2001: Image Processing*. Vol. 4322. San Diego, CA: SPIE, Feb. 2001, pp. 824–835. DOI: 10.1117/12.431161

- A recent article about defect pixel interpolation with respect to image quality issues can be found here:

Jan Kuttig et al. "Effects of Defect Pixel Correction Algorithms for X-ray Detectors on Image Quality in Planar Projection and Volumetric CT Data Sets". In: *Measurement Science and Technology* 26.9 (Aug. 2015), 095406 (14pp). DOI: 10.1088/0957-0233/26/9/095406