Medical Image Processing for Diagnostic Applications

Parallel Beam – Ram-Lak Filter

Online Course – Unit 33

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Topics

How to Implement a Parallel Beam Algorithm - Part 1

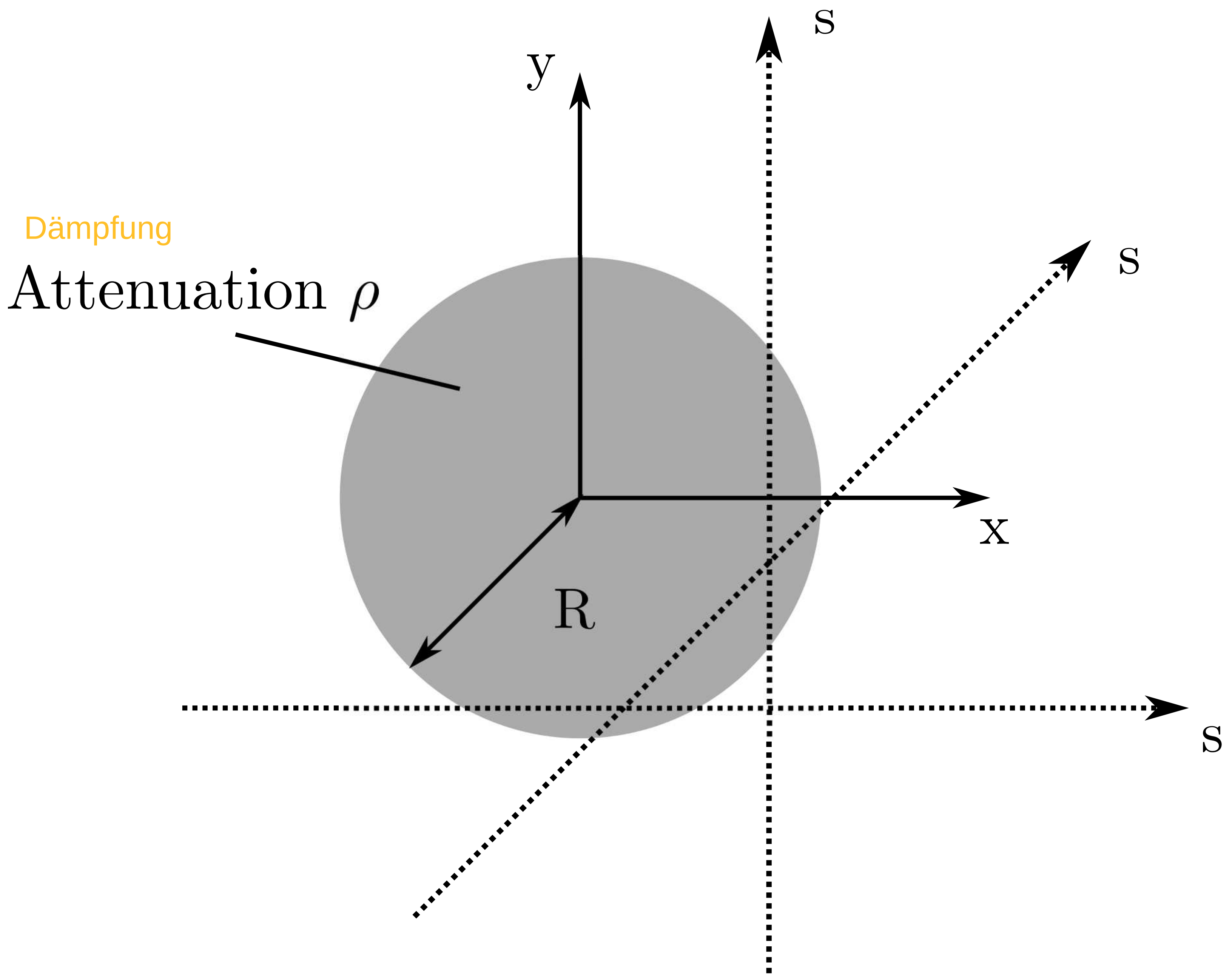
- Example Projection
- Implementation Scheme
- Discrete Spatial Form of the Ramp Filter

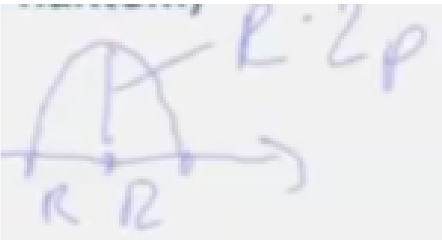
Summary

- Take Home Messages
- Further Readings

staz in a continous setting all the way until the end and the make diskrete. -> the detector only has discrete pixels

Example: Homogeneous Cylinder (My First Phantom)





Disc of radius R is in the coordinate center
$\rightarrow$ projection is the same in all views:

$$p(s) = \begin{cases} 2\rho\sqrt{R^2 - s^2} & s \leq R, \\ 0 & s > R. \end{cases}$$

basicly how long does the ray travel trou the cylinder

(The dotted lines indicate rays from different projection angles.)

Example: Homogeneous Cylinder

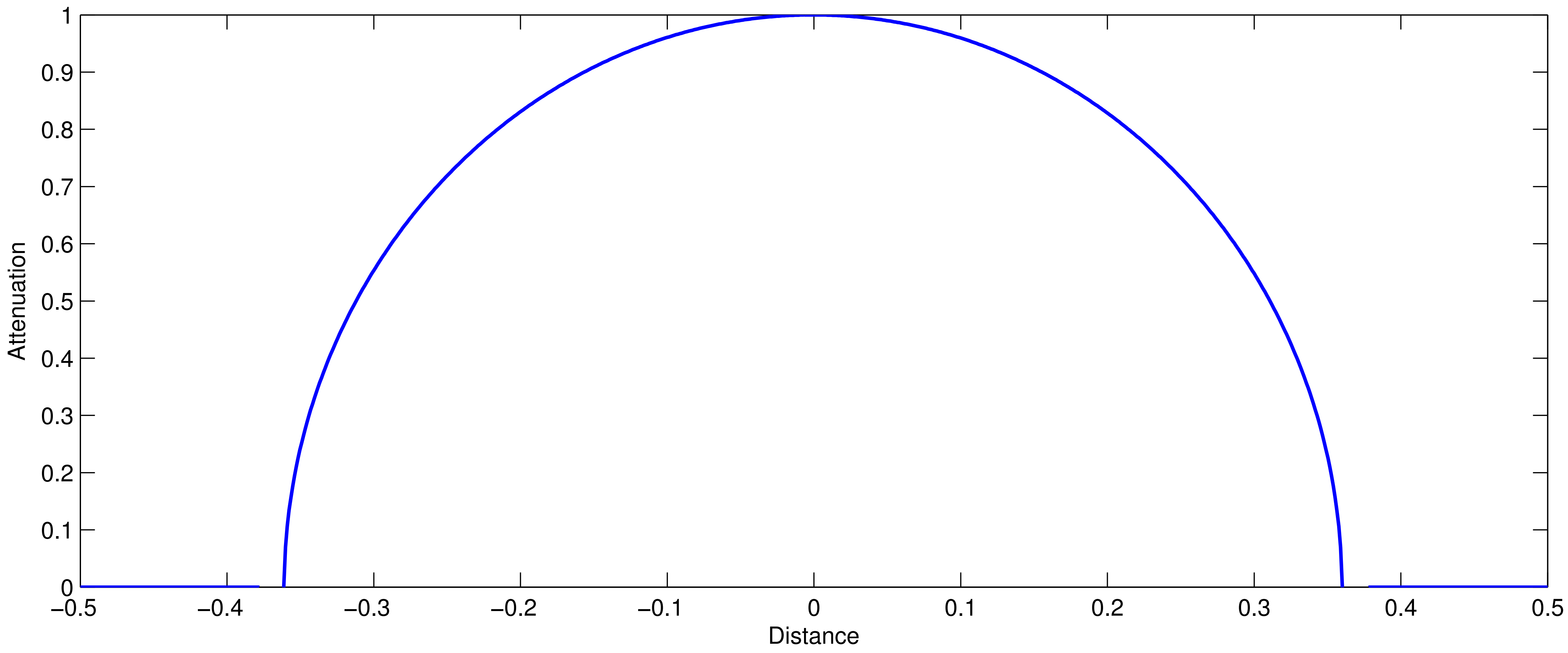


Figure 1: 1-D projection profile of the cylinder object in 2-D

Filtered Backprojection: Implementation Scheme

- Apply filter on the detector row:

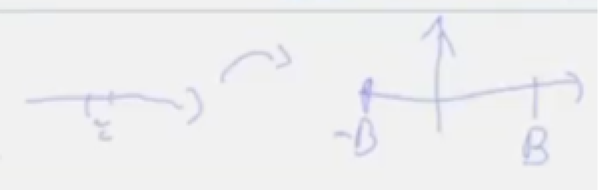
$$q(s,\theta) = h(s) * p(s,\theta),$$

$$h(s) = \int_{-\infty}^{\infty} |\omega| e^{2\pi i \omega s} \mathrm{d}\omega.$$

- Backproject $q(s,\theta)$:

$$f(x,y) = \int_{0}^{\pi} q(s,\theta)|_{s=x\cos\theta+y\sin\theta} \mathrm{d}\theta.$$

Discrete Spatial Form of the Ramp Filter



max and min frequency depends on the length of the signal ??

detector spacing -> distance between detector pixels?

the ram filter cuts off the mean value in the SD (the center value in the FD)

Problem: Find the inverse Fourier transform of $|\omega|$.

Given a detector spacing τ, we know from the Nyquist-Shannon sampling theorem the maximum frequency that can be represented by the DFT:

$$2B = \frac{1}{\tau}.$$

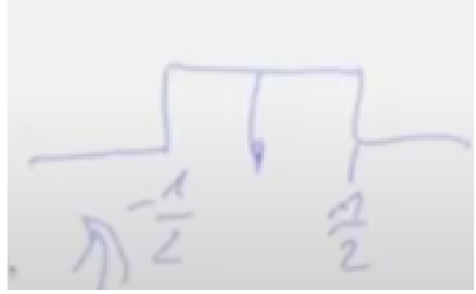
Therefore, we set the cut-off frequency of the ramp filter at $\omega = B$.

So we want to determine

$$h(s) = \int_{-B}^{B} |\omega| e^{2\pi i \omega s} \mathrm{d}\omega = \int_{-\infty}^{\infty} |\omega| \operatorname{rect}\left(\frac{\omega}{2B}\right) e^{2\pi i \omega s} \mathrm{d}\omega,$$

where

$$\operatorname{rect}(t) = \begin{cases} 1, & \text{if } |t| < \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$



now we can go from inf to inf again

Discrete Spatial Form of the Ramp Filter

With the rect-function we can also rewrite $|\omega|$:

$$|\omega| = B - \mathrm{rect}\left(\frac{\omega}{B}\right) * \mathrm{rect}\left(\frac{\omega}{B}\right).$$

The convolution of both rect-functions yields a triangular shaped function with support on $[-B, B]$ and its maximum B at zero.

We now have:

$$\begin{aligned} h(s) &= \mathrm{FT}^{-1}\left(\left(B - \mathrm{rect}\left(\frac{\omega}{B}\right) * \mathrm{rect}\left(\frac{\omega}{B}\right)\right) \mathrm{rect}\left(\frac{\omega}{2B}\right)\right) \\ &= \mathrm{FT}^{-1}\left(B\,\mathrm{rect}\left(\frac{\omega}{2B}\right)\right) - \mathrm{FT}^{-1}\left(\underbrace{\left(\mathrm{rect}\left(\frac{\omega}{B}\right) * \mathrm{rect}\left(\frac{\omega}{B}\right)\right)}_{\text{support on } [-B,B]} \underbrace{\mathrm{rect}\left(\frac{\omega}{2B}\right)}_{=1 \text{ on } [-B,B]}\right) \\ &= \mathrm{FT}^{-1}\left(B\,\mathrm{rect}\left(\frac{\omega}{2B}\right)\right) - \mathrm{FT}^{-1}\left(\mathrm{rect}\left(\frac{\omega}{B}\right)\right) \cdot \mathrm{FT}^{-1}\left(\mathrm{rect}\left(\frac{\omega}{B}\right)\right). \end{aligned}$$

Discrete Spatial Form of the Ramp Filter

The Fourier transform of the rect-function is a sinc-function, and using the appropriate scaling properties of the Fourier transform, we get:

$$\begin{aligned} h(s) &= 2B^2 \operatorname{sinc}(2Bs) - B^2 \operatorname{sinc}^2(Bs) \\ &= \frac{1}{2\tau^2} \frac{\sin\left(\frac{\pi s}{\tau}\right)}{\frac{\pi s}{\tau}} - \frac{1}{4\tau^2} \left(\frac{\sin\left(\frac{\pi s}{2\tau}\right)}{\frac{\pi s}{2\tau}} \right)^2 . \end{aligned}$$

The detector is sampled by $s = n\tau$, $n \in \mathbb{Z}$, hence we find the discrete filter in the spatial domain:

$$h(n\tau) = \begin{cases} \frac{1}{4\tau^2} & n = 0, \\ 0 & n \text{ even}, \\ -\frac{1}{\pi^2 (n\tau)^2} & n \text{ odd}, \end{cases}$$

also known as the **“Ramachandran-Lakshminarayanan” convolver** or shortly the **“Ram-Lak” filter**.

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Take Home Messages

if we ignore discretisation -> artifacts

- In this unit we derived the discrete spatial filter version of the ramp filter. It is also called the Ram-Lak filter.
- By design, the Ram-Lak filter “fits” optimally on the detector grid which enhances the accuracy of the reconstruction algorithm (see next unit).

Further Readings

The original Ram-Lak article is:

G. N. Ramachandran and A. V. Lakshminarayanan. “Three-dimensional Reconstruction from Radiographs and Electron Micrographs: Application of Convolutions instead of Fourier Transforms”. In: *Proceedings of the National Academy of Sciences of the United States of America* 68.9 (Sept. 1971), pp. 2236–2240

The derivation shown in this unit is based on a document by Martin Berger.

The concise reconstruction book from ‘Larry ’Zeng:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Another mathematical examination of filtered backprojection can be found in

Thorsten Buzug. *Computed Tomography: From Photon Statistics to Modern Cone-Beam CT*. Springer Berlin Heidelberg, 2008. DOI: 10.1007/978-3-540-39408-2