Projection Models and Homogeneous Coordinates

Calibration – Part 1

Refresher Course

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Topics

Calibration – Simplified Case

- Definition
- Calibration Patterns
- Calibration of X-ray Systems

Summary

- Take Home Messages
- Further Readings

Calibration of X-ray Systems

Definition

The estimation of the projection parameters is called ***calibration***.

Let us first consider a *simplified case* for calibration, where we consider the following problems in detail:

- Design a calibration pattern whose 3-D geometry is known exactly.
- Capture an X-ray image of the calibration pattern.
- Compute the correspondence of 2-D to 3-D points.
- Compute the focal length f using a least square estimator.

calibration phantom -> known 3D and korrosponding 2D positions -> use for paremeter calibration

Calibration Patterns / phantoms

all beats on one plane :/

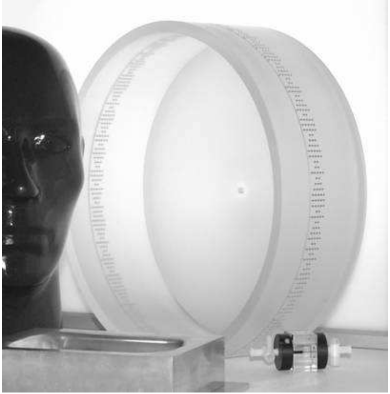


Figure 1: Calibration pattern used for C-arm calibration (image courtesy of Siemens Healthcare)

Figure 2: Observed 2-D point features (image courtesy of Siemens Healthcare)

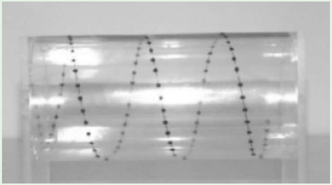
PDS2 Calibration Phantom

beads located in helix -> beats have different size -> make up bitcode for position

Example

Let us assume we have a so-called calibration pattern that is manufactured precisely, and N centroids of the 3-D metal spheres are known in 3-D world coordinates (x_0, x_1, x_2):

$$X = \{\boldsymbol{x}_i = (x_{i,0}, x_{i,1}, x_{i,2})^{\mathrm{T}} \mid i = 1, \ldots, N\}.$$



$$\underline{\vec{p}_{2D}} \cong \underline{P}\, \underline{\vec{p}_{3D}}$$

projection matrix

Figure 3: Calibration pattern used for the computation of the projection geometry of a C-arm

PDS2 Calibration Phantom

Example

Properties of the PDS2 calibration pattern used for today's C-arm systems:

- 108 steel spheres,
- large spheres represent a logical one,
- small spheres represent a logical zero,
- 8 Bit binary encoding,
- position of spheres is known in the world coordinate system,
- the world coordinate system is attached to the phantom.

Calibration of X-ray Systems

Let $(x_{i,0}, x_{i,1}, x_{i,2})^{\mathrm{T}}$, $i = 1, \ldots, N$, be the set of 3-D points of the calibration pattern, and let $(y_{i,0}, y_{i,1})^{\mathrm{T}}$ be the corresponding set of 2-D points.

The projection is defined by:

only equal to scale

$$\begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} x_{i,0} \\ x_{i,1} \\ x_{i,2} \\ 1 \end{pmatrix}}_{\text{3D}} \overset{=\lambda}{\cong} \underbrace{\begin{pmatrix} y_{i,0} \\ y_{i,1} \\ 1 \end{pmatrix}}_{\text{2D}} .$$

Calibration of X-ray Systems

For each pair of points 2-D/3-D points we get the pair of equations:

$$\frac{fx_{i,0}}{x_{i,2}} = y_{i,0}, \tag{1}$$

now they are realy equal, not up to scale :)

$$\frac{fx_{i,1}}{x_{i,2}} = y_{i,1}. \tag{2}$$

Due to image noise and segmentation errors, the equations have to be replaced by the following minimization problem:

$$\hat{f} = \arg\min_{f} \sum_{i=1}^{N} \sum_{k=0}^{1} \left(\frac{fx_{i,k}}{x_{i,2}} - y_{i,k} \right)^2. \tag{OP 1}$$

least square -> find out f (focal lengt?)

Calibration of X-ray Systems

Solution: We compute the zero crossings of the partial derivatives w. r. t. the unknown parameter f and get the following estimator for f:

$$\hat{f} = \frac{\sum_{i=1}^{N} \sum_{k=0}^{1} \frac{y_{i,k} x_{i,k}}{x_{i,2}}}{\sum_{i=1}^{N} \sum_{k=0}^{1} \left(\frac{x_{i,k}}{x_{i,2}} \right)^2}.$$

Alternative Objective Functions

We can multiply (1) and (2) by $x_{i,2}$ and get:

$$fx_{i,0} = x_{i,2}y_{i,0},$$
$$fx_{i,1} = x_{i,2}y_{i,1}.$$

The resulting objective function is:

$$\hat{f} = \arg\min_{f} \sum_{i=1}^{N} \sum_{k=0}^{1} \left(fx_{i,k} - x_{i,2}y_{i,k}\right)^2. \tag{OP 2}$$

Different Objective Functions

There is yet another option to rewrite (1) and (2):

$$f = \frac{x_{i,2} y_{i,0}}{x_{i,0}},$$
$$f = \frac{x_{i,2} y_{i,1}}{x_{i,1}}.$$

Using these equations, the resulting objective function is:

$$\hat{f} = \arg\min_f \sum_{i=1}^{N} \sum_{k=0}^{1} \left(f - x_{i,2} \frac{y_{i,k}}{x_{i,k}} \right)^2 . \qquad \text{(OP 3)}$$

Different Objective Functions

now w have 3 possible objective funktions

Question: Which objective function is the best one? Should we optimize (OP 1), (OP 2) or (OP 3)?

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Exercise: Compute the estimators for (OP 1), (OP 2), and (OP 3) and analyze the sensitivity of estimates by adding noise to the input data.

Topics

Take Home Messages

- In a real imaging system the detector has to be calibrated.
- Calibration can be performed by use of calibration phantoms that contain several markers whose pattern is exactly known.
- We have studied how a projection parameter like the focal length can be estimated by solving an optimization problem with measured 2-D points and known 3-D positions.

the world coordinate system depends on the positioning of the phantom -> if we take it out and put it back differently we will get different results :/

Further Readings

For further details on geometric aspects of imaging see:

1. Richard Hartley and Andrew Zisserman. *Multiple View Geometry in Computer Vision*. 2nd ed. Cambridge: Cambridge University Press, 2004. DOI: 10.1017/CBO9780511811685
2. Olivier Faugeras. *Three-Dimensional Computer Vision: A Geometric Viewpoint*. MIT Press, Nov. 1993