Medical Image Processing for Diagnostic Applications

Basic Principles of Tomography

Online Course – Unit 27

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Topics

Tomography

Projection

Summary

- Take Home Messages
- Further Readings

Basic Principles of Tomography

from several projections to a virtual slice

$\tau \acute{o} \mu o \varsigma$ ['tomos] $\rightarrow$ slice

schwarz -> luft im darm ?, weiss -> Knochen

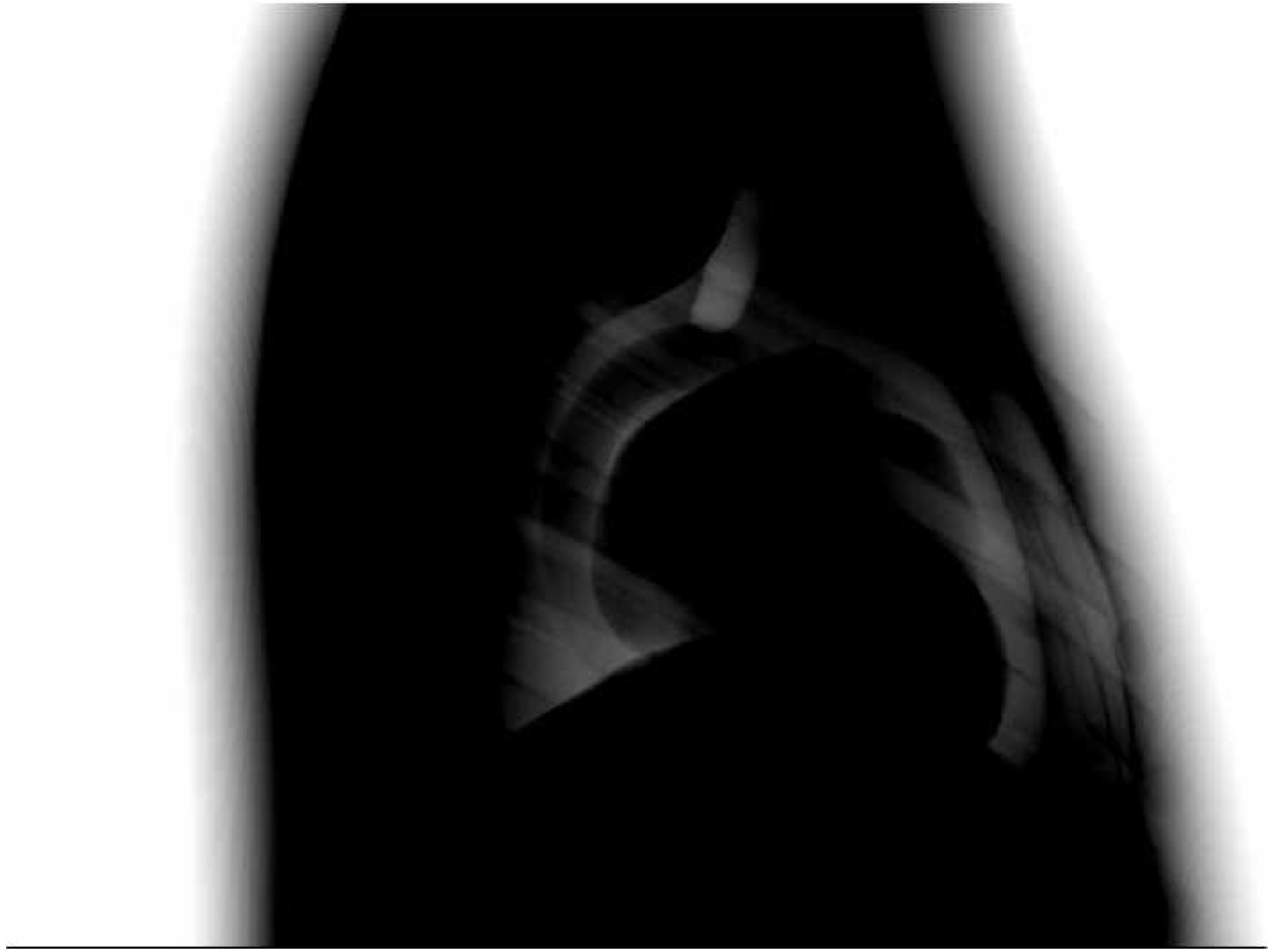


Figure 1: Single chest phantom projection

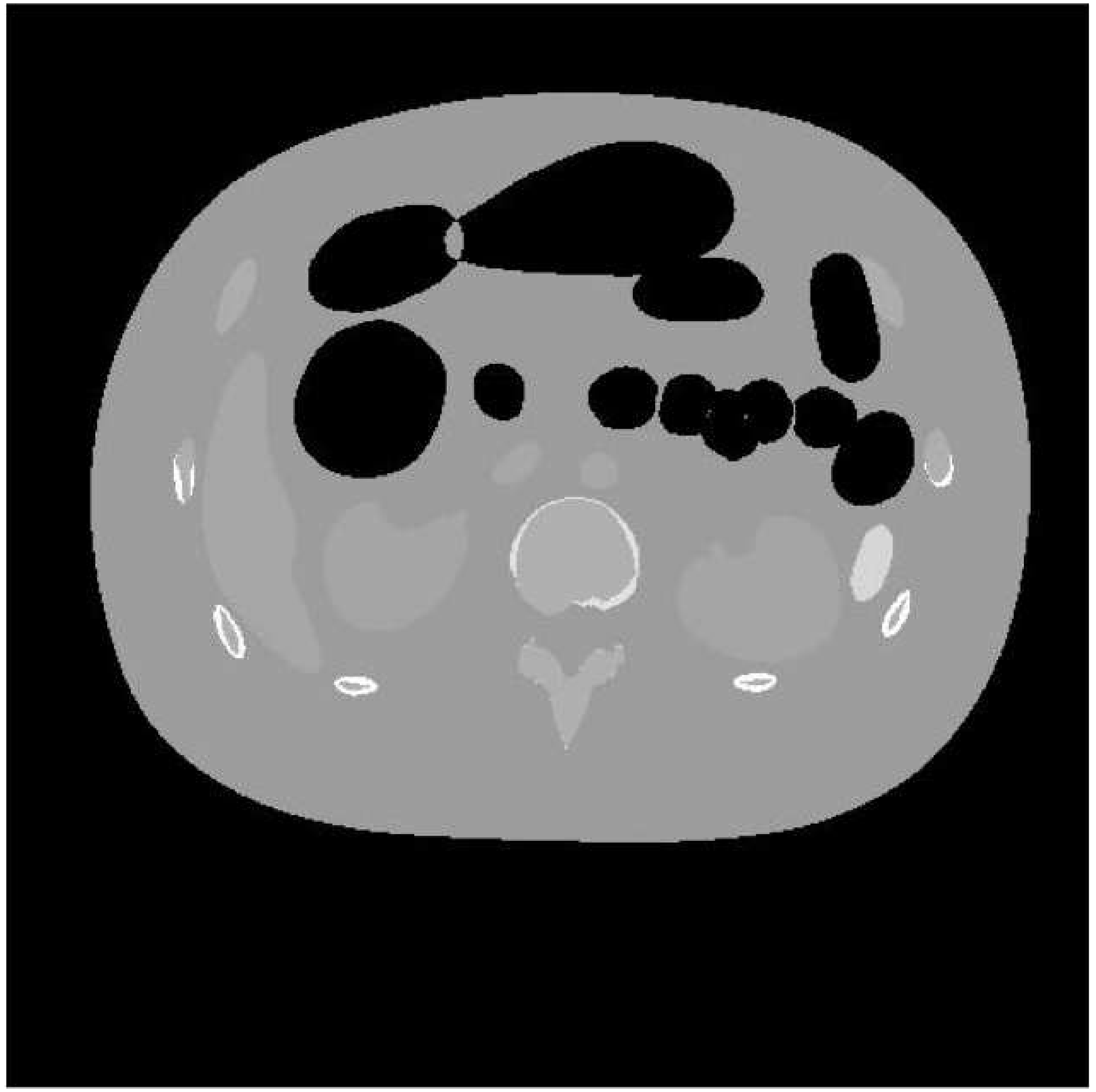


Figure 2: Slice view (click for video)

Basic Principles of Tomography

Idea: Observe the object of interest from multiple sides:

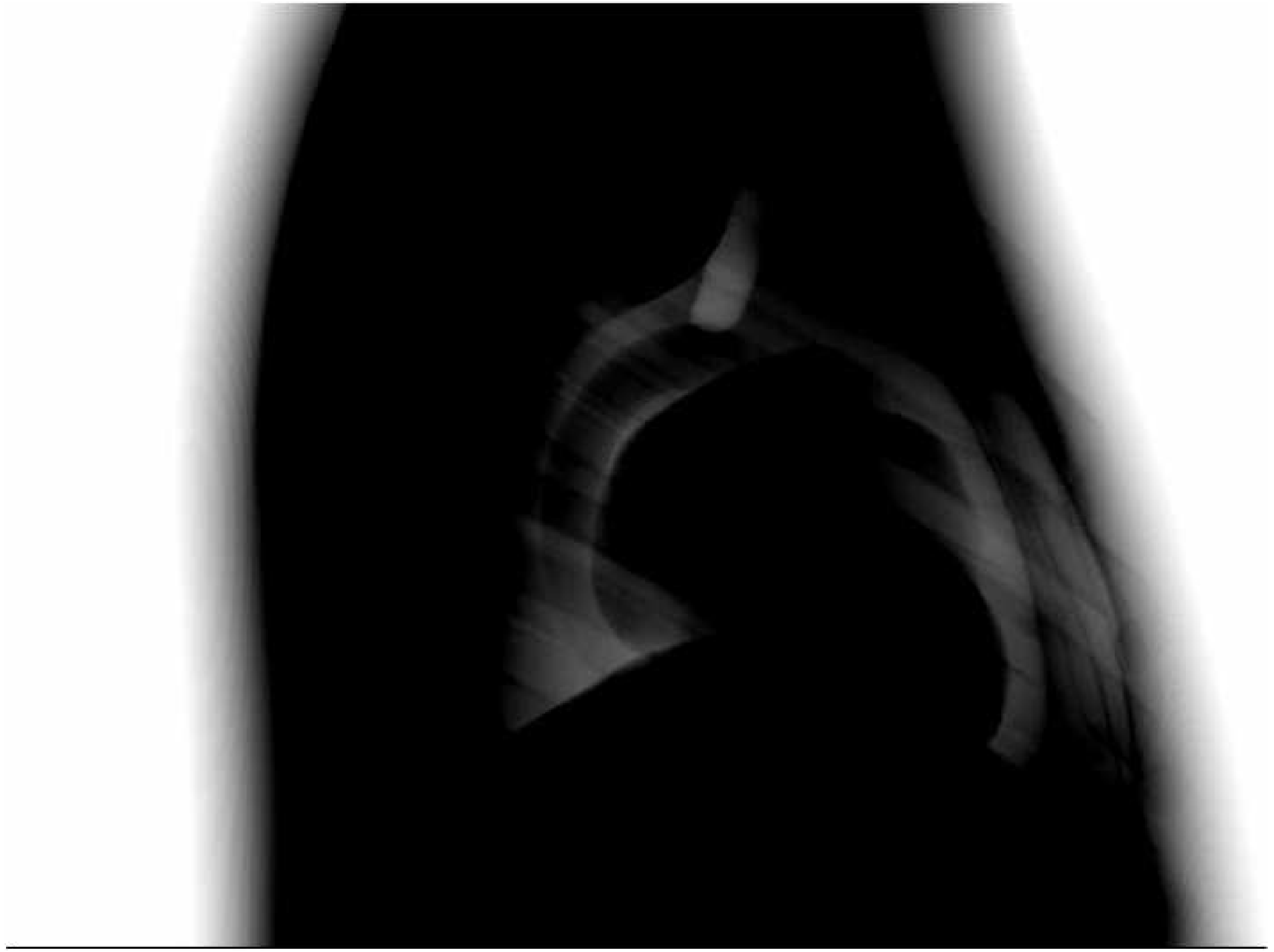
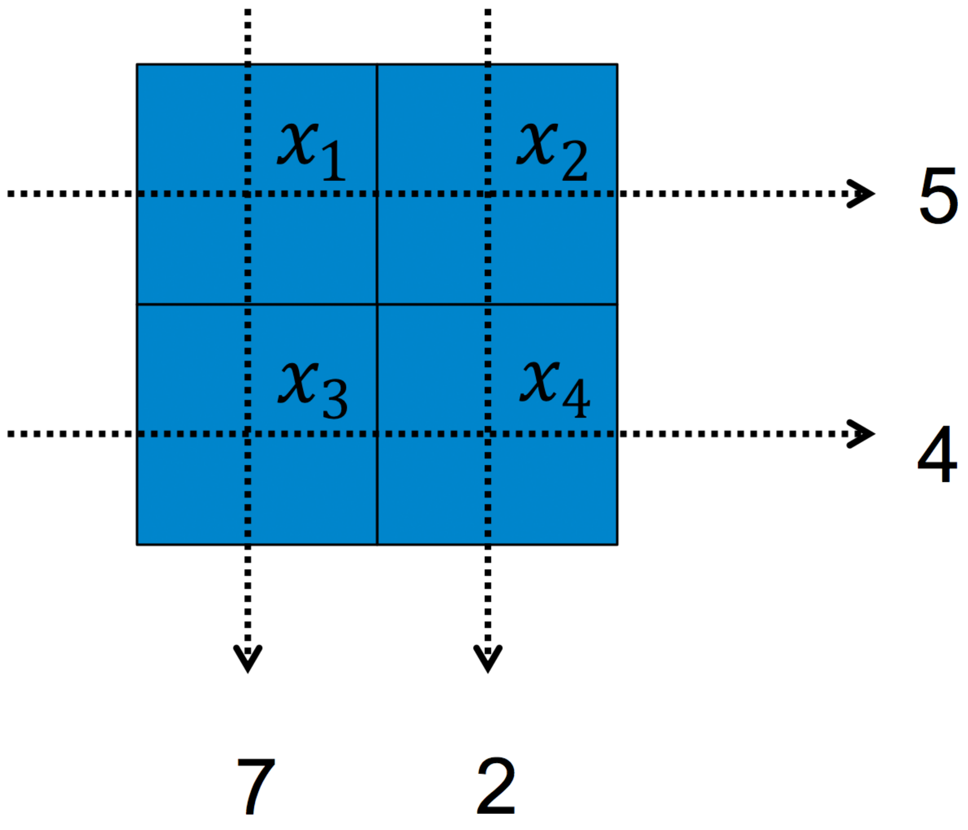


Figure 3: Multiple scan views (click for video)

Basic Principles of Tomography

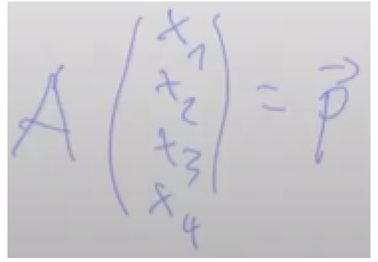
patiennts consisting of 4 pixels

Solve the puzzle:



$$\begin{aligned} x_1 + x_3 &= 7 \\ x_2 + x_4 &= 2 \\ x_1 + x_2 &= 5 \\ x_3 + x_4 &= 4 \end{aligned} \quad \Rightarrow \quad \begin{aligned} x_1 &= 3 \\ x_2 &= 2 \\ x_3 &= 4 \\ x_4 &= 0 \end{aligned}$$

write as matrix
clac pseudo inverse
solution



A describes how the rays pass throu
the patients

- Usually, the problem size in CT is $512 \times 512 \times 512 = 134\,217\,728$. pixel
- **How can this problem be solved?**

this are to many unknowns :(
the matrix would be squared :(

Topics

Projection: X-ray Attenuation

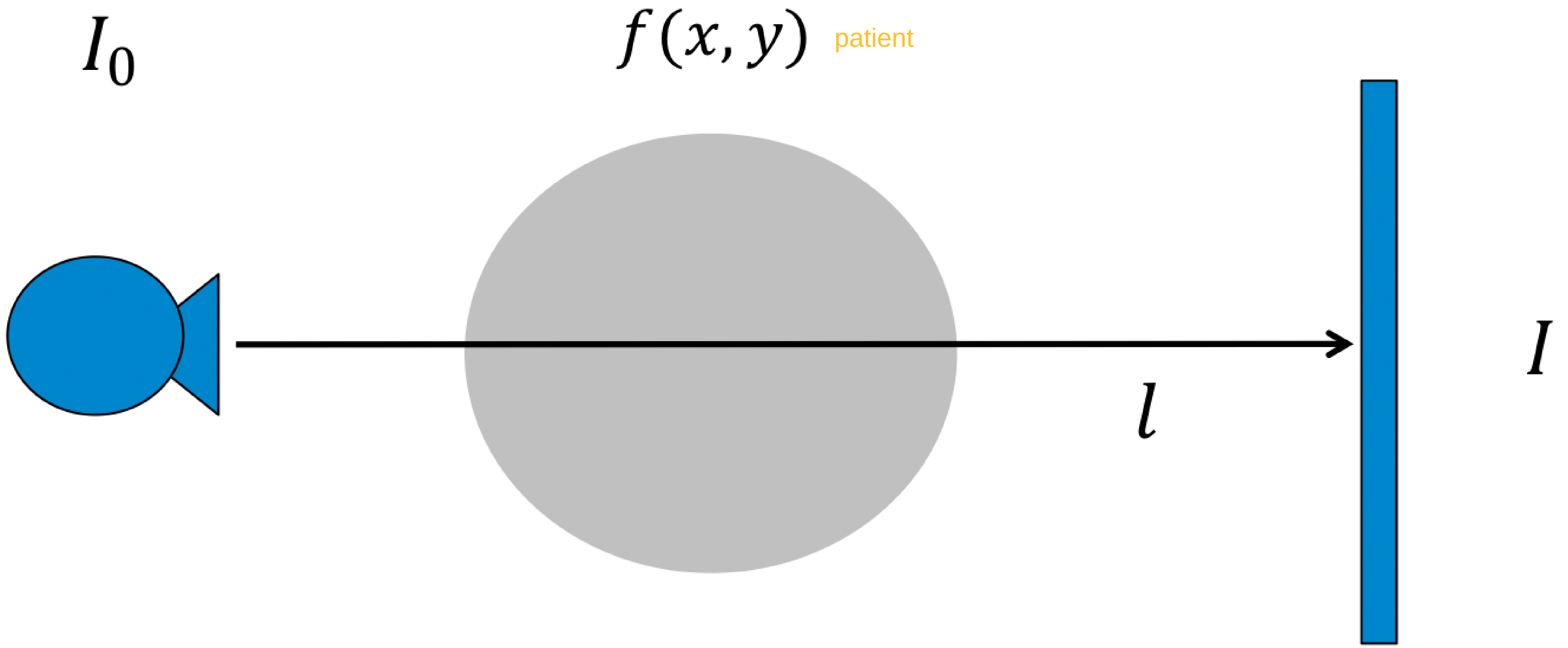
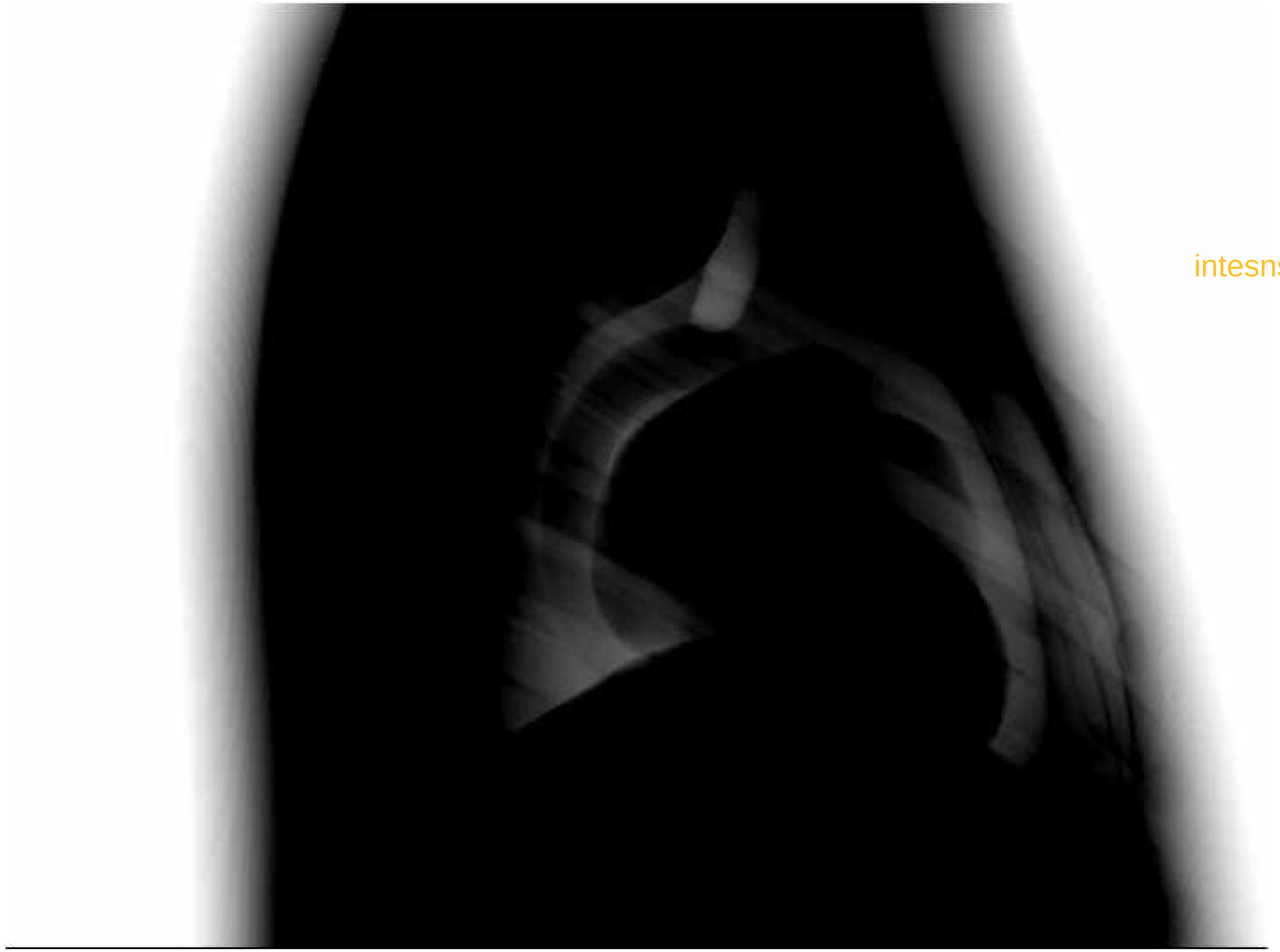


Figure 4: Beer–Lambert law: $I = I_0 e^{-(\int f(x,y)\,\mathrm{d}l)}$

$-\ln \frac{I}{I_0} = \int f(x,y)\,dl$

-> "p"
"line integral image"

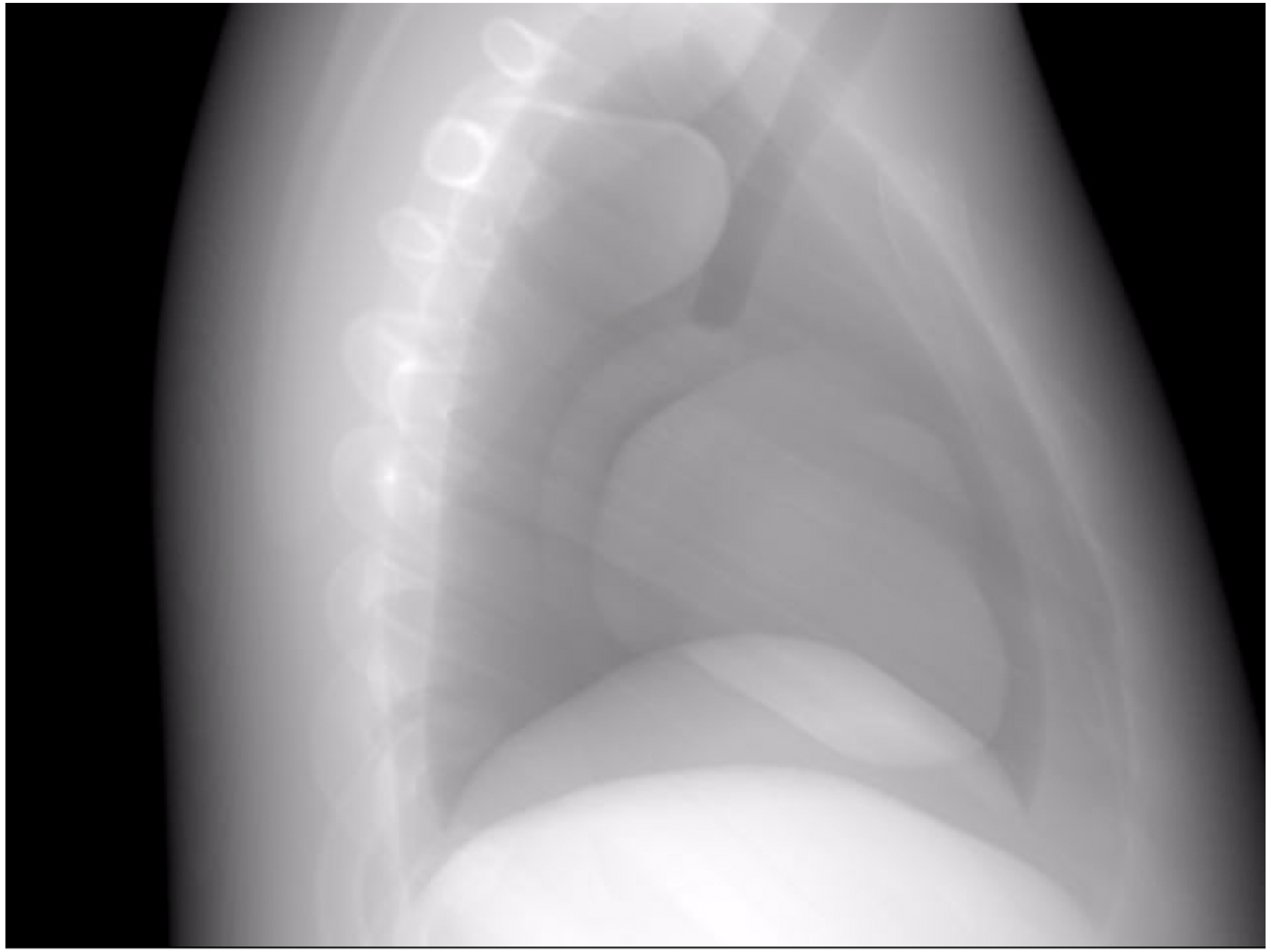
Projection: Physical Observations



intesnsity domain

Figure 5: Observed projection signal (click for video)

Projection: Physical Observations



line integral domain

Figure 6: Line integral data (click for video)

Projection Formation

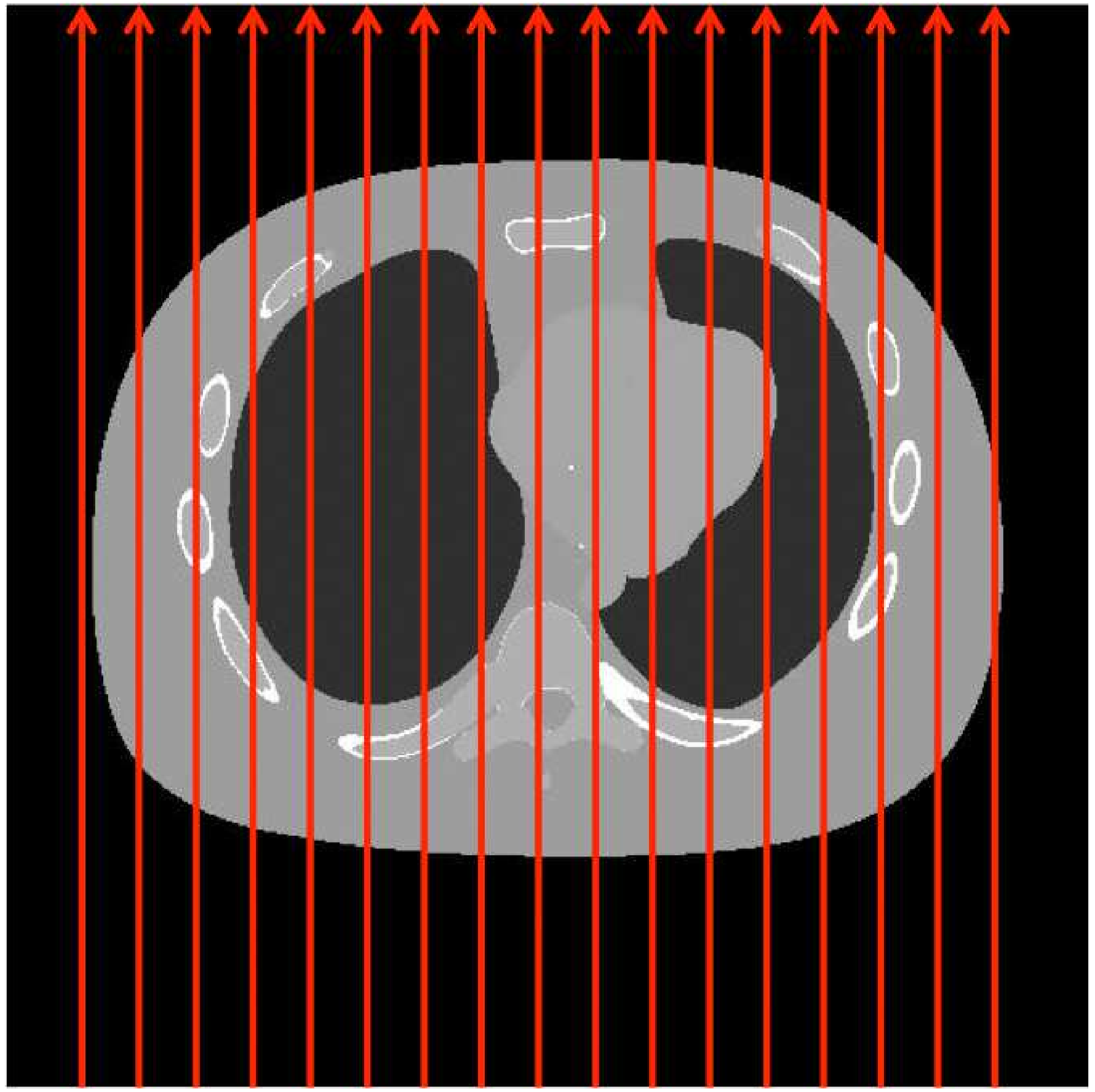


Figure 7: Slice by slice projection (left), projection ray scheme (right)

Projection: Mathematical Formulation

s linear index
theta rotation of system

$$p(s,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - s)\,\mathrm{d}x\,\mathrm{d}y$$

this is a line function

delta function is one where the line touches a pixel
we get the summed intensity over the line

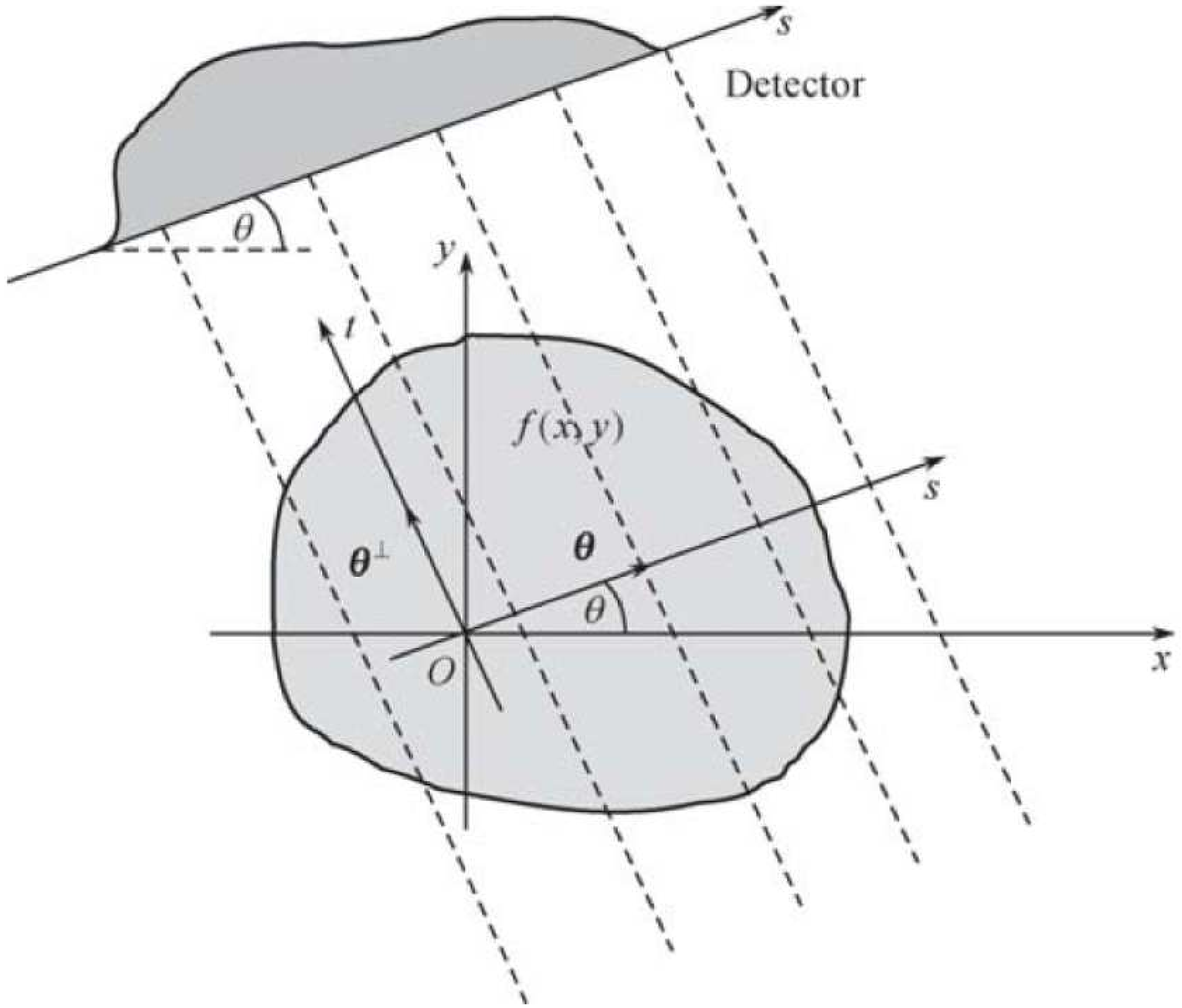


Figure 8: Parallel beam geometry (Zeng, 2009)

Projection: Example Point Object

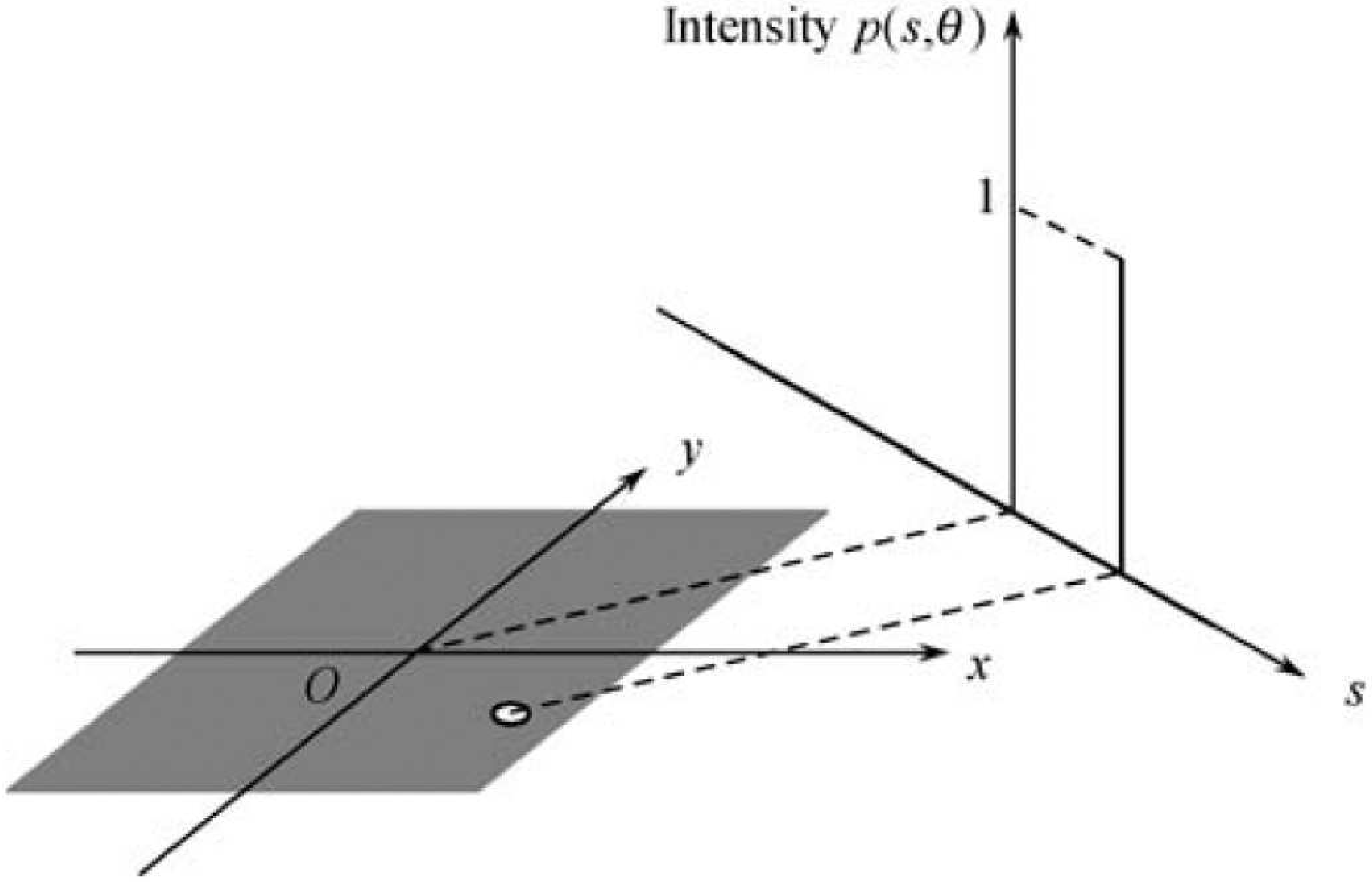


Figure 9: Intensity profile of a point object (Zeng, 2009)

Topics

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Take Home Messages

- Tomography is looking through an object from several angles and recombining the projection views back to a 3-D volume of the object.
- 3-D data is usually represented as a stack of slices.
- The Beer-Lambert law describes the attenuation of X-rays on their path through an object.
- A single projection can be regarded as the integral of projections of every point inside the object.

Further Readings

Students learning about reconstruction should have a look at one of the following books:

- Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9
- Avinash C. Kak and Malcolm Slaney. *Principles of Computerized Tomographic Imaging*. Classics in Applied Mathematics. Accessed: 21. November 2016. Society of Industrial and Applied Mathematics, 2001. DOI: 10.1137/1.9780898719277. URL: http://www.slaney.org/pct/
- Thorsten Buzug. *Computed Tomography: From Photon Statistics to Modern Cone-Beam CT*. Springer Berlin Heidelberg, 2008. DOI: 10.1007/978-3-540-39408-2
- Willi A. Kalender. *Computed Tomography: Fundamentals, System Technology, Image Quality, Applications*. 3rd ed. Publicis Publishing, July 2011