Projection Models and Homogeneous Coordinates

Extrinsic and Intrinsic Camera Parameters

Refresher Course
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Topics

Extrinsic Camera Parameters

-> orientation, pos of camera in relation to world coordinate

before we were always projecting along the Z axis ?!?

So far we have described the projection of a 3-D point into the image plane.

We have not considered the motion of the position and orientation of the acquisition device yet:

- an X-ray source can be translated in 3-D,
- an X-ray source can be rotated in 3-D.

Extrinsic Camera Parameters

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Definition

Extrinsic parameters characterize the *pose*, i. e., position and orientation of the camera with respect to a world coordinate system. The position is defined by a 3-D translation vector, the orientation by three rotation angles.

Extrinsic Camera Parameters

Figure 1: C-arm device in different positions and orientations that can be characterized by the extrinsic parameters of the acquisition device (image courtesy of Siemens Healthcare)

Extrinsic Camera Parameters

Mathematical characterization:

Rotation and translation of a 3-D point can be expressed by:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \boldsymbol{R} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \boldsymbol{t},$$

where

- $\boldsymbol{R} \in \mathbb{R}^{3\times 3}$ denotes a rotation matrix (with its known properties), and
- $\boldsymbol{t} \in \mathbb{R}^{3}$ represents a translation in Euclidean space.

This is an affine mapping.

Extrinsic Camera Parameters

Using homogeneous coordinates we can rewrite the affine as a linear mapping:

$$\begin{pmatrix} wx' \\ wy' \\ wz' \\ w \end{pmatrix} = \boldsymbol{D} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \underbrace{\left(\begin{array}{ccc|c} & \boldsymbol{R} & & \boldsymbol{t} \\ \hline 0 & 0 & 0 & 1 \end{array} \right)}_{\text{now linear operation :)}} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} .$$

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Problem: How does the rotation matrix look like?

Extrinsic Camera Parameters

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Problem: How does the rotation matrix look like?

Solution: As we already know, the columns of the linear mapping are the images of the base vectors of the original coordinate system.

Topics

Intrinsic Camera Parameters

Besides the position and orientation of the acquisition device, in a real imaging system we have to take another set of parameters into account. There is a mapping of projected points in the ideal image plane to the used detector.

Intrinsic Camera Parameters

should not change based on position of system, (but with c arm x ray it might change :( )

Besides the position and orientation of the acquisition device, in a real imaging system we have to take another set of parameters into account. There is a mapping of projected points in the ideal image plane to the used detector.

Definition

Intrinsic parameters define the mapping of 2-D coordinates from the ideal image plane to the 2-D detector coordinates.

Intrinsic Camera Parameters

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- The pixels in the detector coordinate system are not necessarily square pixels, but scaled by k_x and k_y.
- There might exist a radial distortion due to the camera optics (not considered here).

Intrinsic Camera Parameters

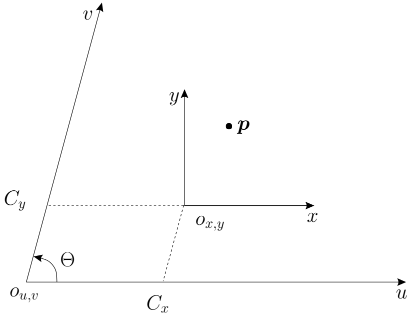

Figure 2: Detector and ideal image coordinate system

(x, y) – ideal image coordinate system:

- used in all formulas so far
- $o_{x,y}$: origin

(u, v) – detector coordinate system:

- real image matrix of measurements
- Θ: skew angle between axes
- k_x, k_y: scaling of u and v axis with respect to units in (x, y)-system
- (C_x, C_y): offset of origins of both coordinate systems

Intrinsic Camera Parameters

Transformation between (u, v)- and (x, y)-coordinate system

At first, we consider the images of base vectors of the detector coordinate system in the image coordinate system:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} \frac{1}{k_x} \\ 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} \frac{1}{k_y}\cos\Theta \\ \frac{1}{k_y}\sin\Theta \end{pmatrix}.$$

The required transform from the (x, y)- to the (u, v)-coordinate system is given by the inverse of the mapping above:

$$\boldsymbol{T} = \begin{pmatrix} \frac{1}{k_x} & \frac{1}{k_y}\cos\Theta \\ 0 & \frac{1}{k_y}\sin\Theta \end{pmatrix}^{-1} = \begin{pmatrix} k_x & -k_x\frac{\cos\Theta}{\sin\Theta} \\ 0 & \frac{k_y}{\sin\Theta} \end{pmatrix}.$$

Intrinsic Camera Parameters

The complete mapping of (x, y)- to (u, v)-coordinates in Euclidean space is thus given by:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} k_x & -k_x \frac{\cos\Theta}{\sin\Theta} \\ 0 & \frac{k_y}{\sin\Theta} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} C_x \\ C_y \end{pmatrix} .$$

Using homogeneous coordinates we get the matrix including the described intrinsic parameters that maps the ideal image coordinates to the detector coordinates:

$$\underbrace{\boldsymbol{K}}_{\text{intrinsic param matrix}} = \left(\begin{array}{cc|c} \multicolumn{2}{c|}{\underbrace{\boldsymbol{T}}_{\text{scew matrix}}} & \begin{matrix} -C_x \\ -C_y \end{matrix} \text{ offset} \\ \hline 0 & 0 & 1 \end{array} \right) \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

3x3 matrix

Topics

Complete Projection

The total perspective transformation is:

Pproj

just

$$\tilde{\boldsymbol{p}}' = \underbrace{\begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\boldsymbol{P}\ ??} \tilde{\boldsymbol{p}}.$$

$$\boldsymbol{P}\tilde{\boldsymbol{p}} = \boldsymbol{K}\boldsymbol{P}_{\text{proj}}\boldsymbol{D}\tilde{\boldsymbol{p}}.$$

f -> focal length

- $\boldsymbol{D}$: extrinsic camera parameters
  - $\rightarrow$ position and orientation of camera w. r. t. the world coordinate system
- $\boldsymbol{P}_{\text{proj}}$: projection model matrix, ideal perspective projection
- $\boldsymbol{K}$: intrinsic camera parameters
  - optical and geometric characteristics of the camera
  - do *not* change with camera movement

Topics

Extrinsic Camera Parameters

Intrinsic Camera Parameters

Complete Projection

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- For projections with a real detector extrinsic and intrinsic camera parameters have to be considered.
- Extrinsic parameters describe the source/camera movement, and can be written as a linear mapping in homogeneous coordinates.
- Intrinsic parameters describe the (usually constant) deviations of the detector from an ideal image plane, and can be written as a linear mapping in homogeneous coordinates as well.

Further Readings

For further details on geometric aspects of imaging see:

1. Richard Hartley and Andrew Zisserman. *Multiple View Geometry in Computer Vision*. 2nd ed. Cambridge: Cambridge University Press, 2004. DOI: 10.1017/CBO9780511811685
2. Olivier Faugeras. *Three-Dimensional Computer Vision: A Geometric Viewpoint*. MIT Press, Nov. 1993