Medical Image Processing for Diagnostic Applications

Basic Principles of Reconstruction

Online Course – Unit 28

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Topics

Image Reconstruction

Simple Example

Reconstruction Steps

Backprojection

Simple Example

Mathematical Formulation

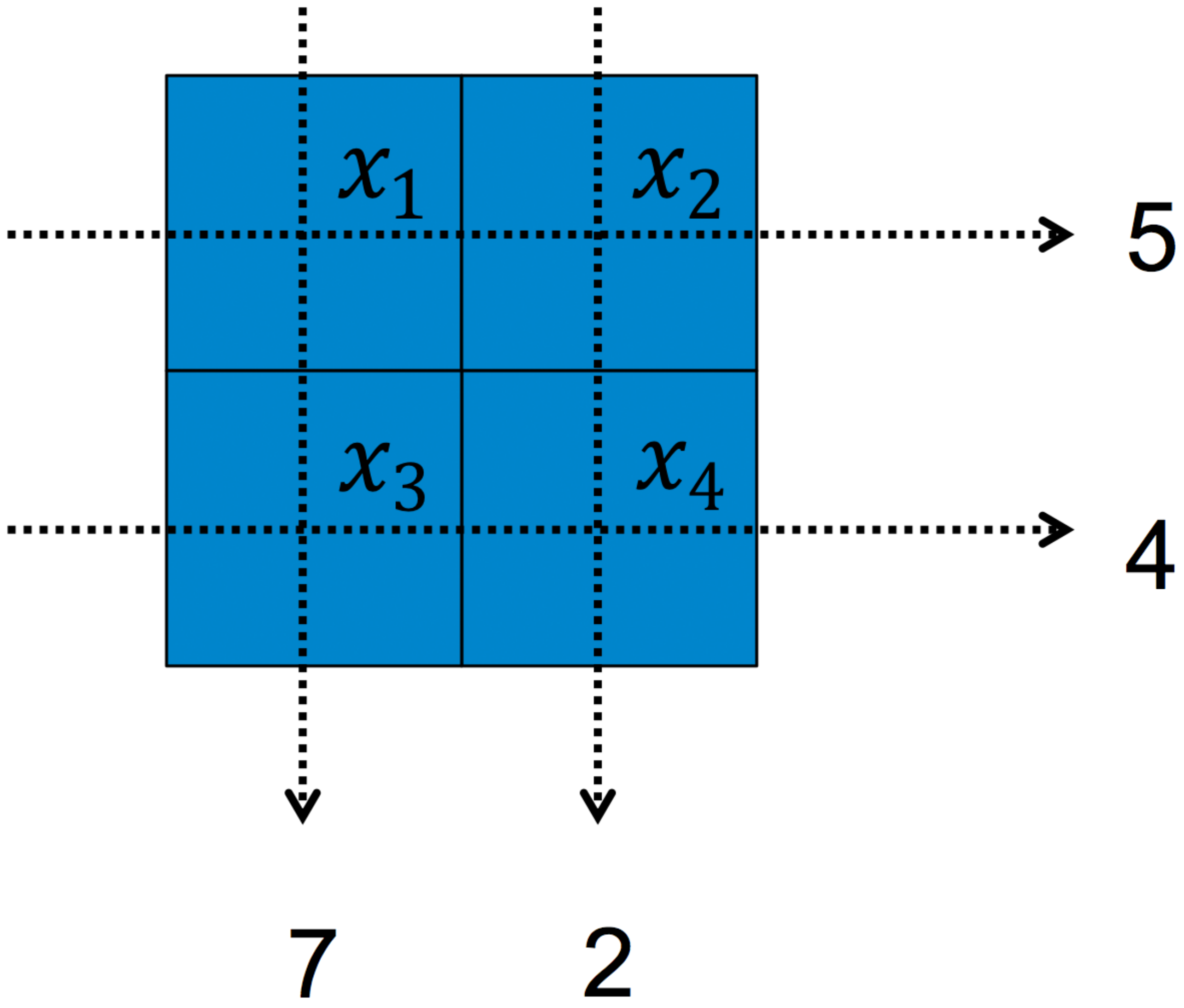
Summary

Take Home Messages

Further Readings

Reconstruction: Simple Example

Solve the puzzle:



$$
\begin{aligned}
x_1 + x_3 &= 7\\
x_2 + x_4 &= 2\\
x_1 + x_2 &= 5\\
x_3 + x_4 &= 4
\end{aligned}
$$

$$
\Rightarrow \quad
\begin{aligned}
x_1 &= 3\\
x_2 &= 2\\
x_3 &= 4\\
x_4 &= 0
\end{aligned}
$$

Reconstruction: Simple Example

- The projection process can be formulated in matrix notation:

$$\boldsymbol{P} = \boldsymbol{A}\boldsymbol{X},$$

where

$$\boldsymbol{P} = \begin{pmatrix} 7 \\ 2 \\ 5 \\ 4 \end{pmatrix}, \quad \boldsymbol{A} = \underbrace{\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}}_{\text{the beams}}, \quad \boldsymbol{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}.$$

- Can this be solved using the matrix inverse?

$$\boldsymbol{A}^{-1}\boldsymbol{P} = \boldsymbol{X}$$

- **Consider:** A common problem size is:

$$\boldsymbol{A} \in \mathbb{R}^{512^3 \times 512^2 \times 512},$$

which implicates

$$512^6 \cdot 4\ \text{Byte} = 2^{9 \cdot 6} \cdot 2^2\ \text{B} = 2^6 \cdot 2^{50}\ \text{B} = 64\ \text{PB} = 65536\ \text{TB}.$$:(too big -> solution via psudoinverse not posible

since A is space we could do this smarter

Reconstruction Steps: Projection

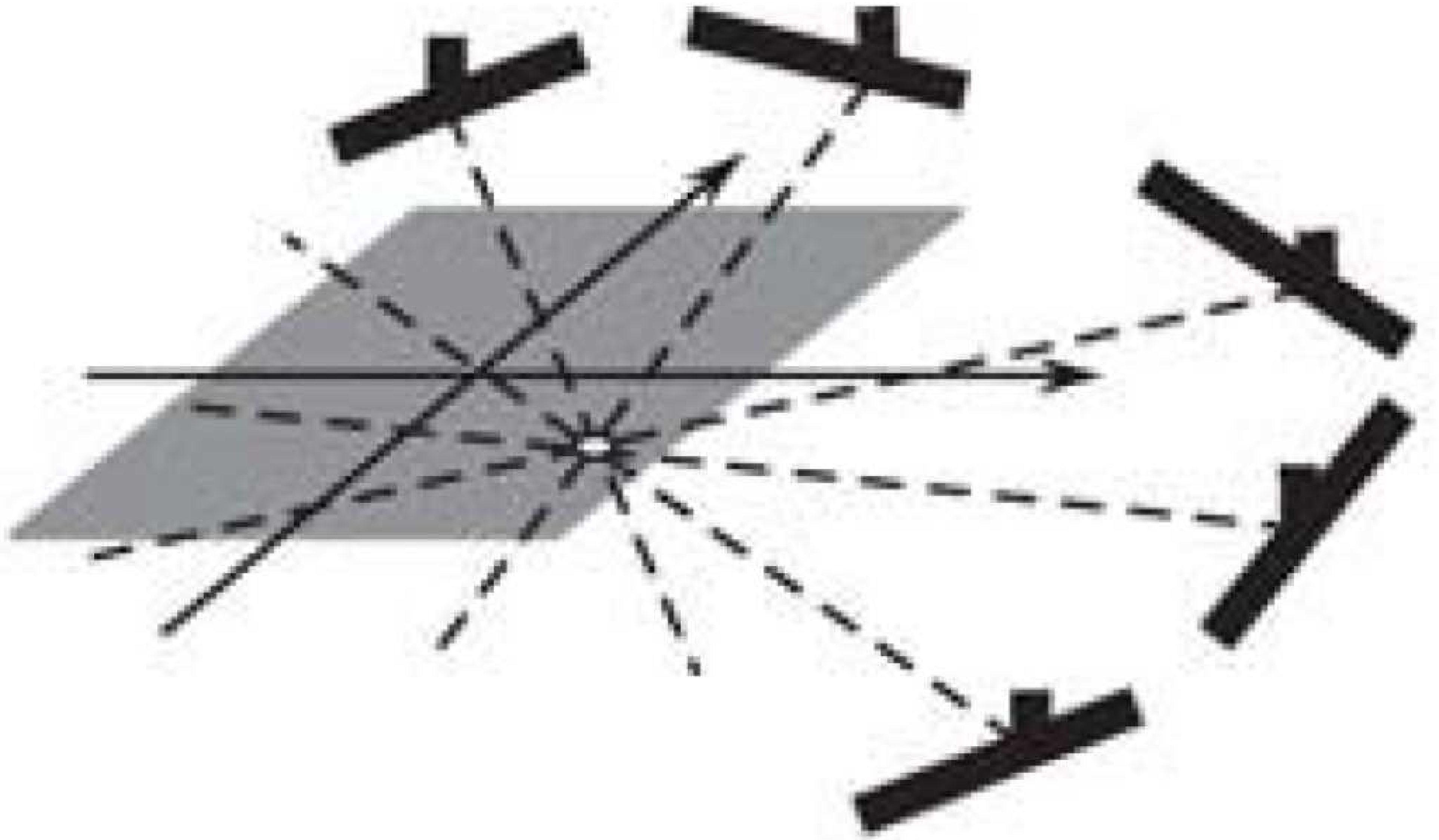


Figure 1: Schematic example for a set of projections (Zeng, 2009)

Reconstruction Steps: Backprojection (1)

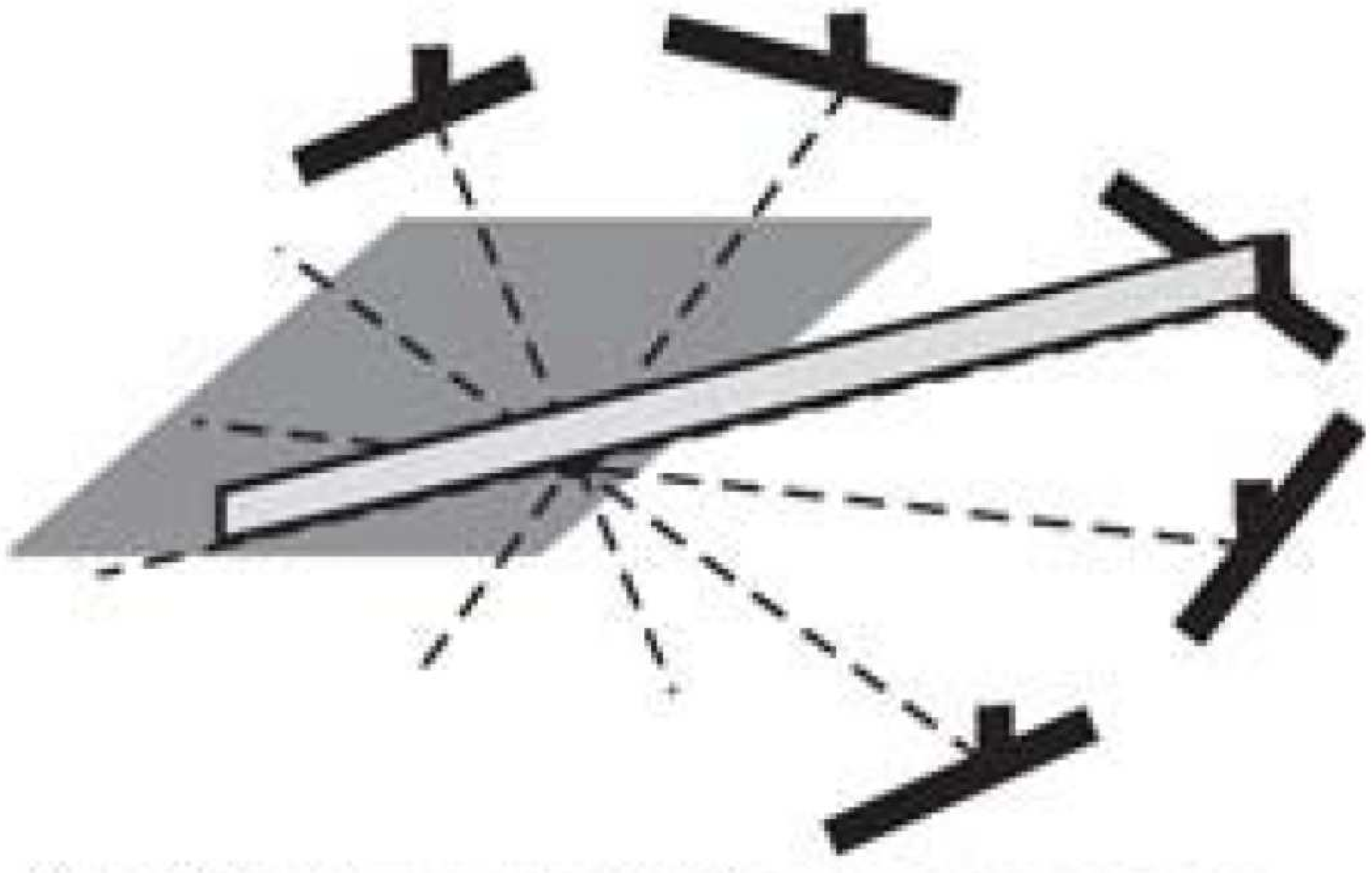


Figure 2: Schematic example for the backprojection process - one projection (Zeng, 2009)

Reconstruction Steps: Backprojection (2)

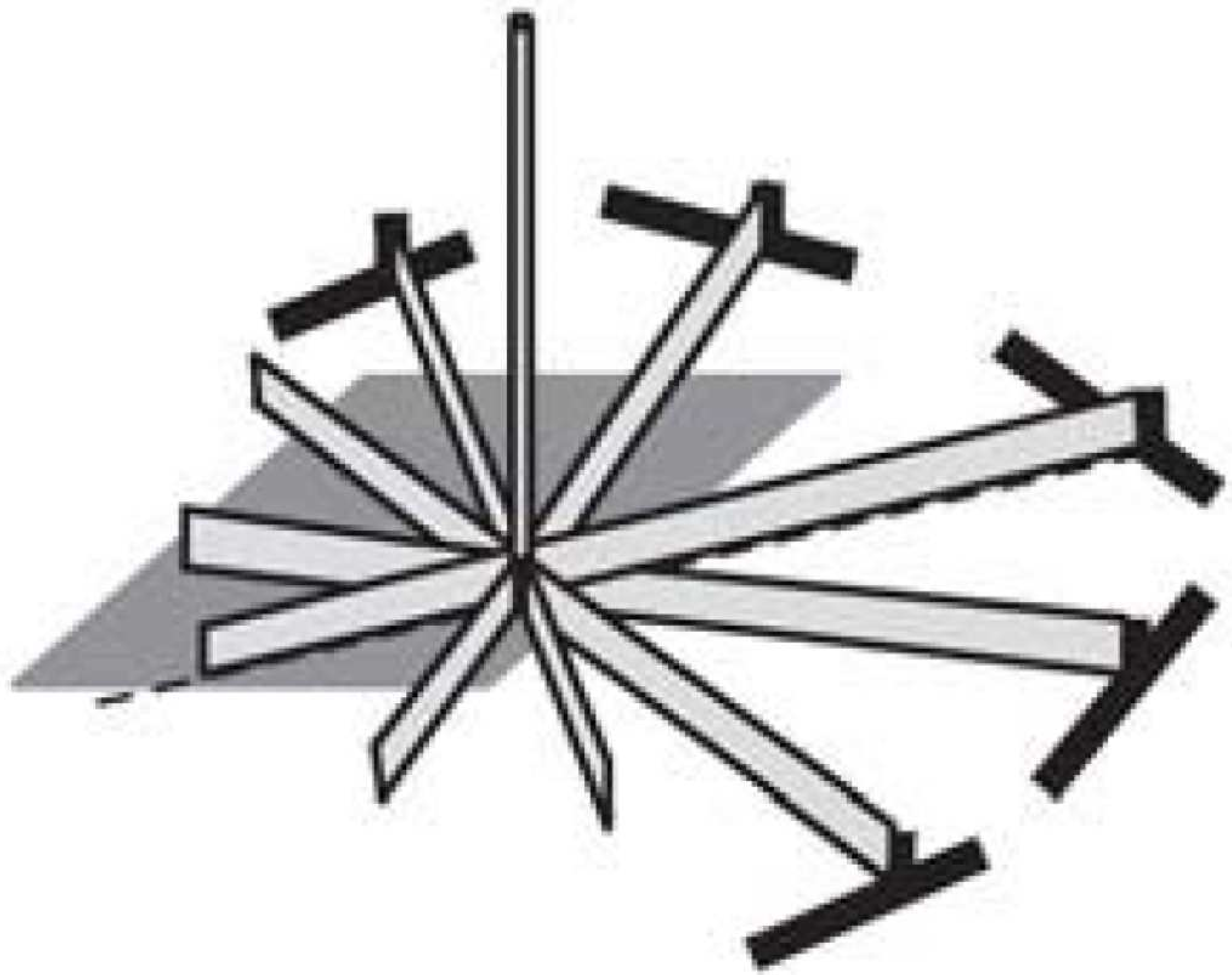
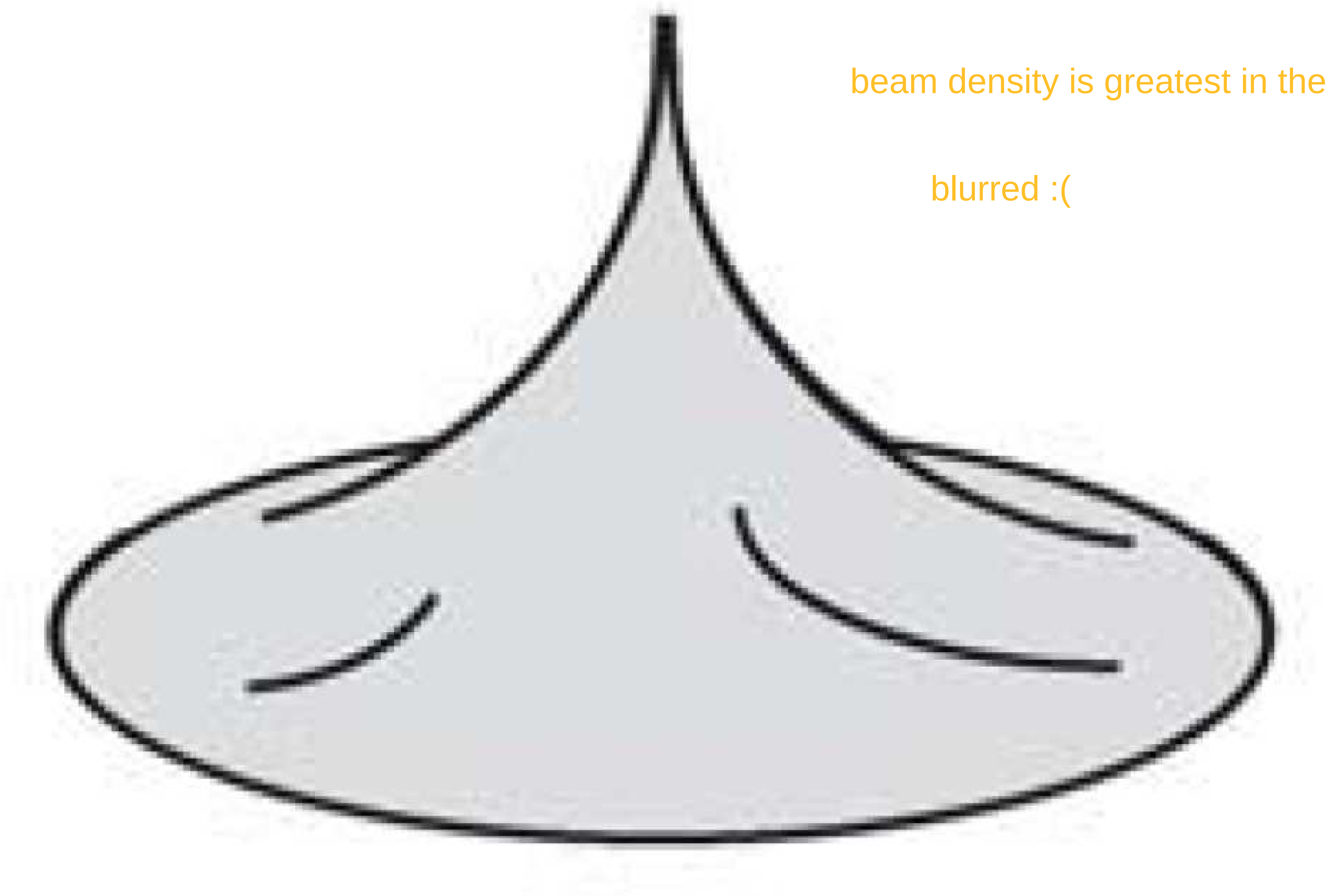


Figure 3: Schematic example for the backprojection process - multiple projections (Zeng, 2009)

Reconstruction Steps: Backprojection (3)



beam density is greatest in the middle

blurred :(

Figure 4: Schematic example for the backprojection process - infinitely many projections (Zeng, 2009)

Reconstruction Steps: “Negative Wings”

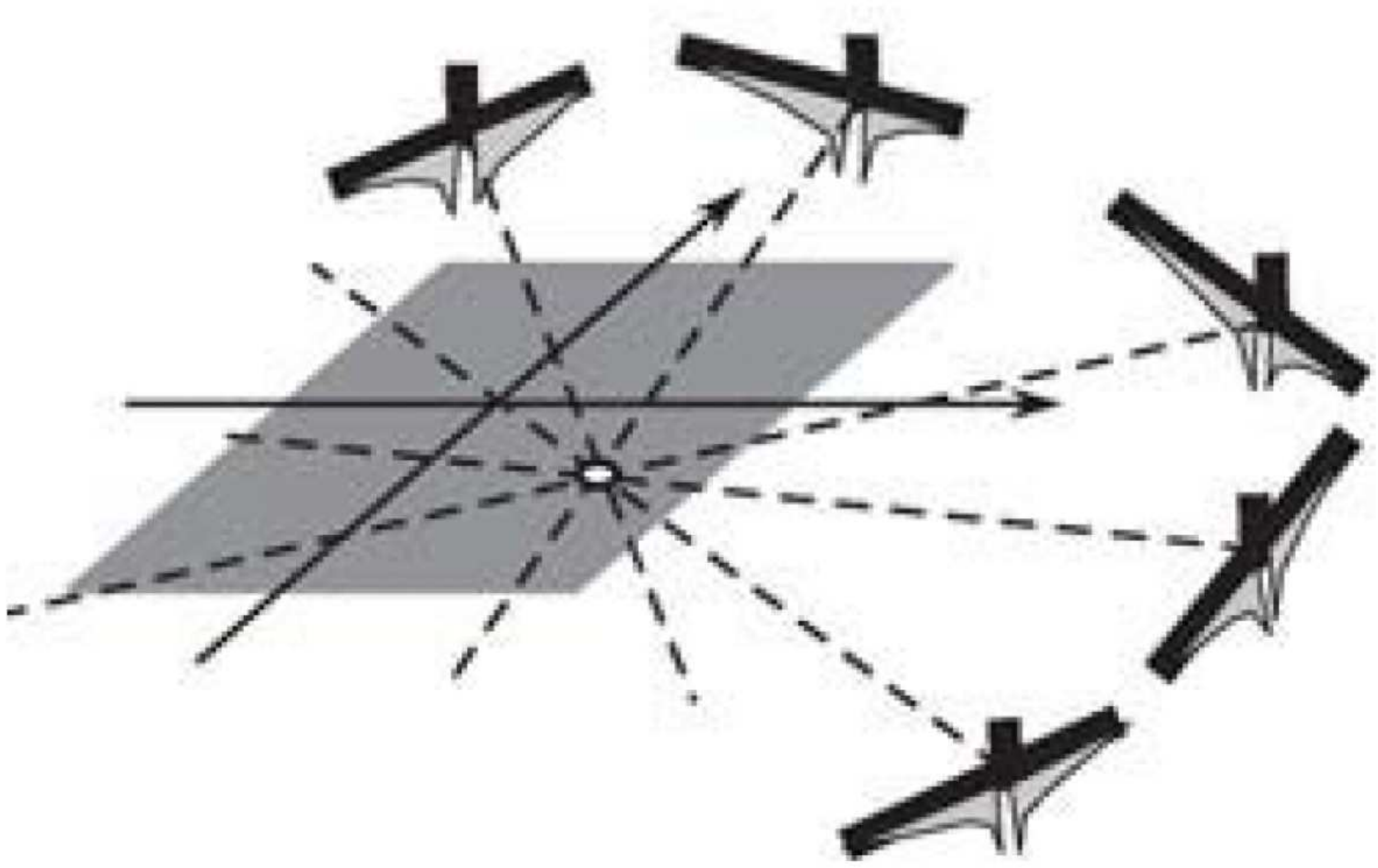
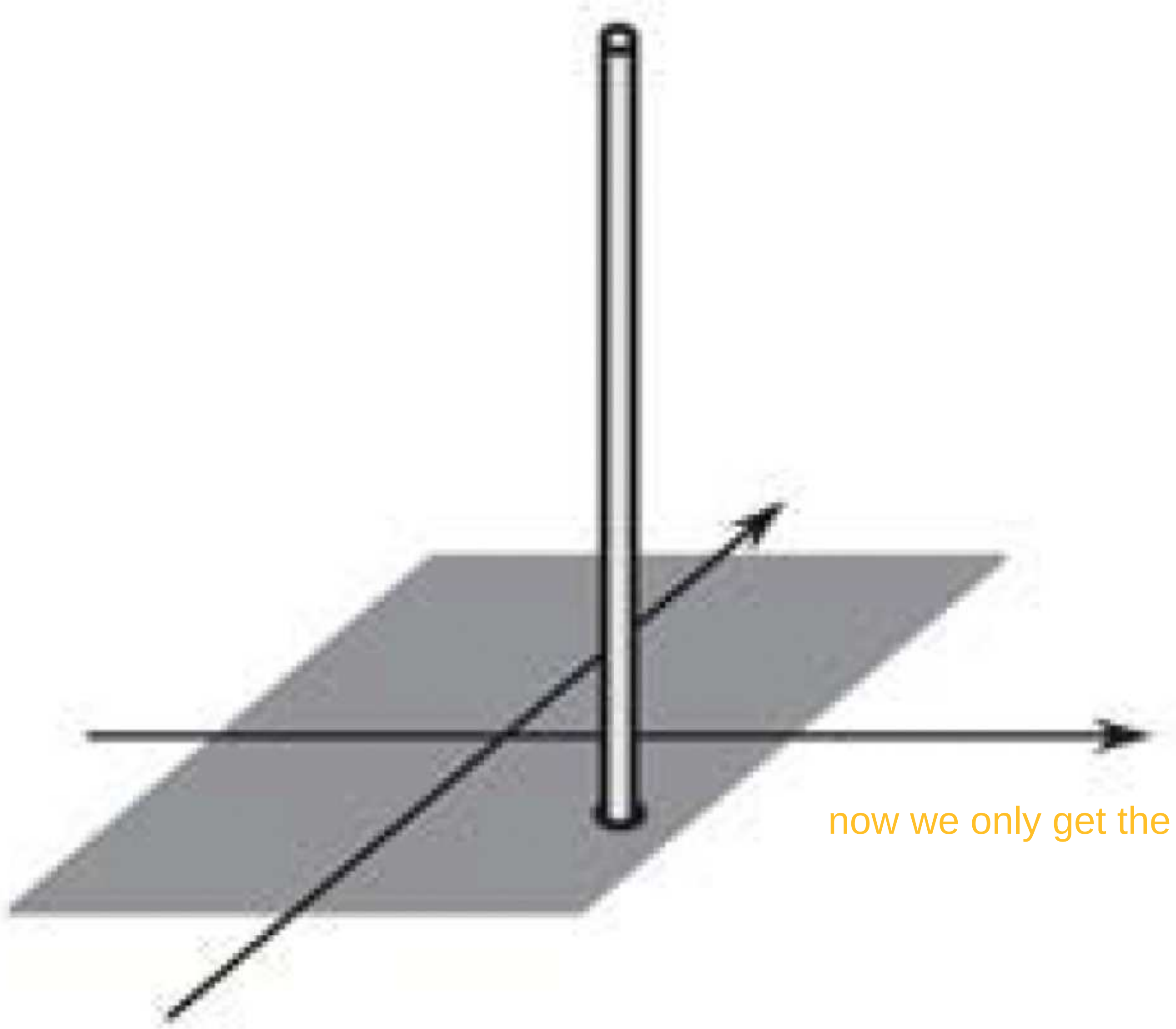


Figure 5: Schematic example for corrective filtering (Zeng, 2009)

Reconstruction Steps: Reconstruction Result



now we only get the point we want

Figure 6: Schematic example for the reconstruction output (Zeng, 2009)

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- Simple Example
- Reconstruction Steps

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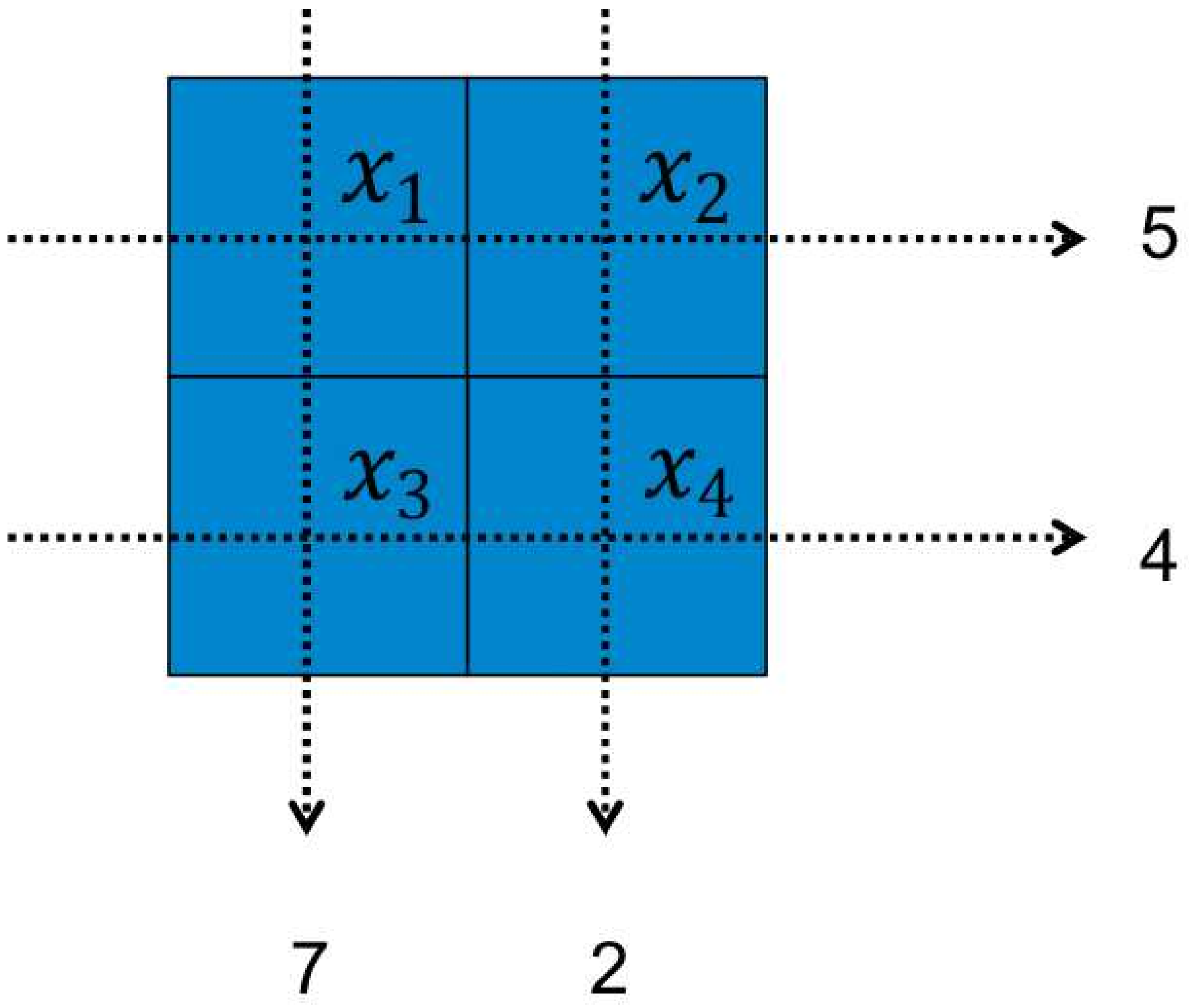
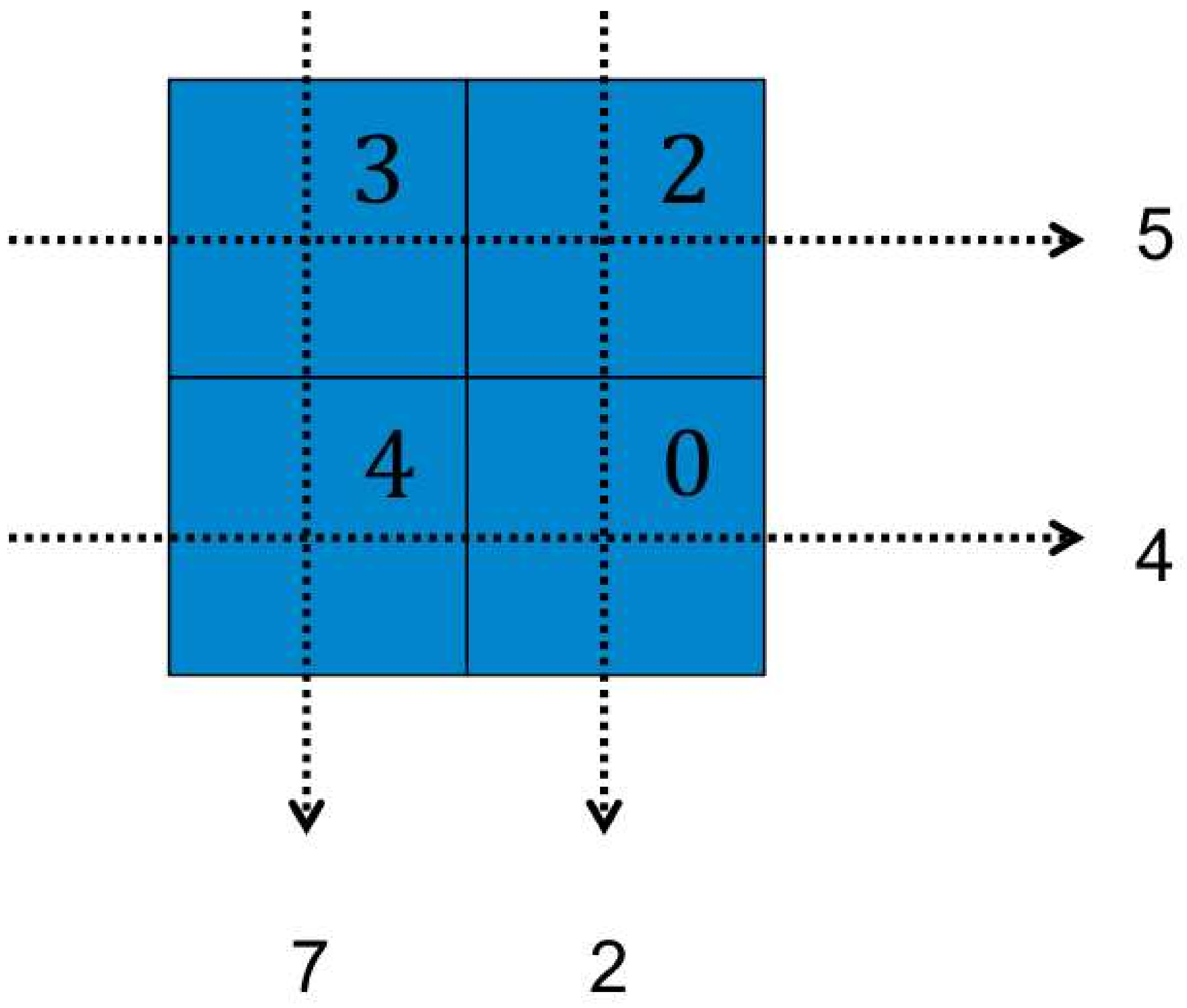
- Simple Example
- Mathematical Formulation

Summary
- Take Home Messages
- Further Readings

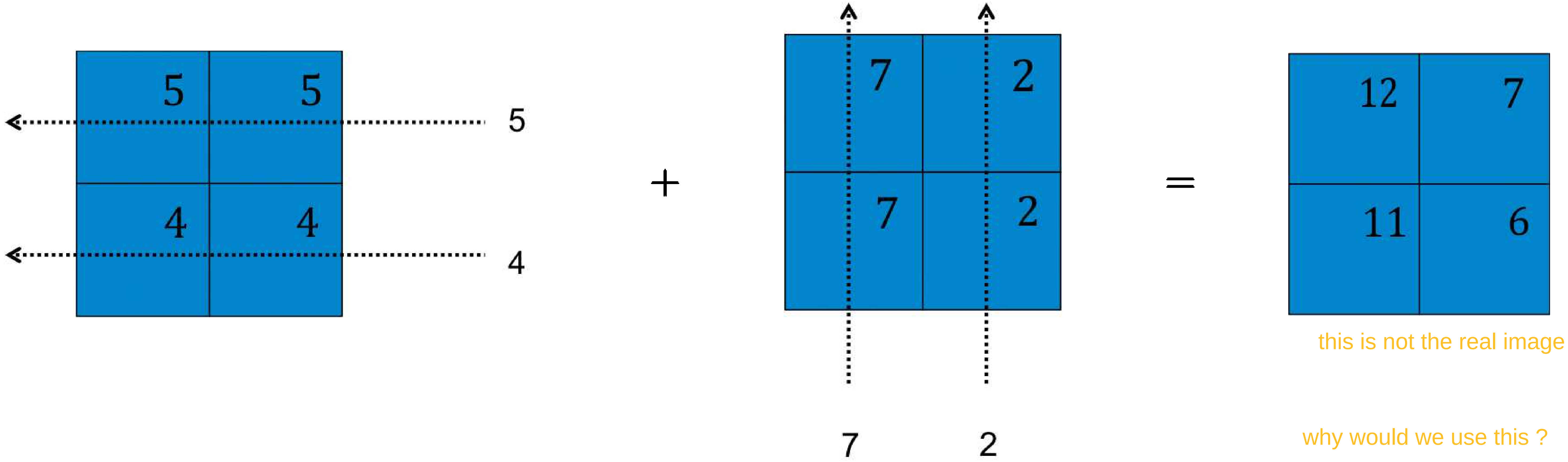
Backprojection: Simple Example



$\rightarrow$



Backprojection: Simple Example



this is not the real image :/

why would we use this ?

Backprojection: Simple Example

- Backprojection is not the inverse of projection!
- In matrix notation, it is simply the matrix transpose:

$$\boldsymbol{B} = \boldsymbol{A}^T \boldsymbol{P},$$

better than nothing but not as good as inverse solution

where

$$\boldsymbol{A}^T = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{P} = \begin{pmatrix} 7 \\ 2 \\ 5 \\ 4 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 12 \\ 7 \\ 11 \\ 6 \end{pmatrix}.$$

Backprojection: Mathematical Formulation

The following equivalent formulations are employed in literature:

$$b(x,y) = \int_0^{\pi} p(s,\theta)\big|_{s = x\cos\theta + y\sin\theta}\,\mathrm{d}\theta,$$

$$b(x,y) = \int_0^{\pi} p(s,\theta)\big|_{s = \boldsymbol{x}\cdot\boldsymbol{\theta}}\,\mathrm{d}\theta,$$

$$b(x,y) = \int_0^{\pi} p(\boldsymbol{x}\cdot\boldsymbol{\theta},\theta)\,\mathrm{d}\theta,$$

$$b(x,y) = \frac{1}{2}\int_0^{2\pi} p(x\cos\theta + y\sin\theta,\theta)\,\mathrm{d}\theta.$$

same as before, but now continous

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Take Home Messages

- Reconstruction involves several steps: projection, backprojection, and filtering.
- Backprojection is not the inverse of projection, but just the transpose.

Further Readings

Students learning about reconstruction should have a look at one of the following books:

- Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9
- Avinash C. Kak and Malcolm Slaney. *Principles of Computerized Tomographic Imaging*. Classics in Applied Mathematics. Accessed: 21. November 2016. Society of Industrial and Applied Mathematics, 2001. DOI: 10.1137/1.9780898719277. URL: http://www.slaney.org/pct/
- Thorsten Buzug. *Computed Tomography: From Photon Statistics to Modern Cone-Beam CT*. Springer Berlin Heidelberg, 2008. DOI: 10.1007/978-3-540-39408-2
- Willi A. Kalender. *Computed Tomography: Fundamentals, System Technology, Image Quality, Applications*. 3rd ed. Publicis Publishing, July 2011