Medical Image Processing for Diagnostic Applications

Implementation Issues

Online Course – Unit 12

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Topics

Remarks on Parameterization

- Regress Carefully
- Interpolation, Regression, and Overfitting

Scaling of Input Data

Summary

- Take Home Messages
- Further Readings

Remarks on Parameterization

y = mx+t -> m needs to be infinity :(

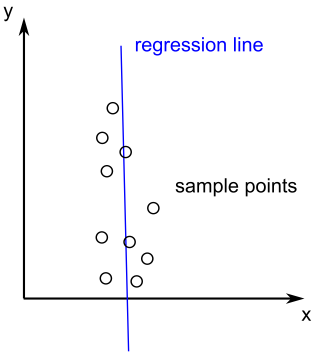

Figure 1: Regression line with infinite slope

Remarks on Parameterization

solution: rotate coordinate system

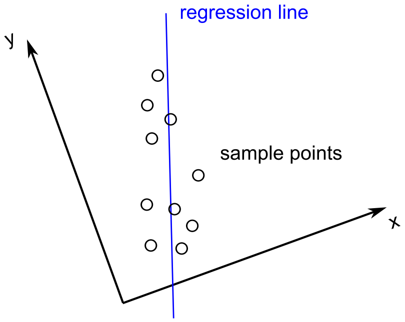

Figure 2: Regression line (rotated reference coordinate system)

Remarks on Parameterization

The parameterization of the straight line decides on the **sensitivity** of estimated parameters **to variations in input data**.

A **well-conditioned problem might appear ill-conditioned** if the parameterization of the problem is not done properly.

Remarks on Parameterization

For straight lines we observe:

impossible m

- The line representation $y = mx + t$ has **singularities**: the more parallel the regression line to the y-axis, the larger m. For lines parallel to the y-axis, we observe the singularity $m = \infty$ (infinite slope).
- A **fair** representation of straight lines is

  here vertical lines are not a problem

  $$x \cos \alpha + y \sin \alpha = d,$$

  where $\alpha \in [0, 2\pi]$, $d \in \mathbb{R}$.

Conclusion: Select a parameterization that is independent from orthogonal transforms of the reference coordinate system.

Interpolation, Regression, and Overfitting

Definition

estimate points in an area where there are no samples

Interpolation defines the estimation of unknown data between observed data. In addition, we require the interpolation curve to fit all the training data.

Definition

Like interpolation, ***regression*** defines a technique to discover a mathematical relationship between multiple variables using a set of data points, i. e., training data. In regression it is not required that the regression curve fits the training data perfectly.

Note: The regression function is usually estimated using a least square approach. The transition from interpolation to regression is smoothly, and some authors do not differentiate between these two techniques explicitly.

Interpolation, Regression, and Overfitting

Definition

Overfitting is defined as training a model, (e. g., a parametric model), so that it well fits the training data, but fails to predict well in between and outside the data.

Overfitting can occur, if a complex model (e. g., a model with many parameters) is trained with a sparse set of data, i. e., too few training examples.

Interpolation, Regression, and Overfitting

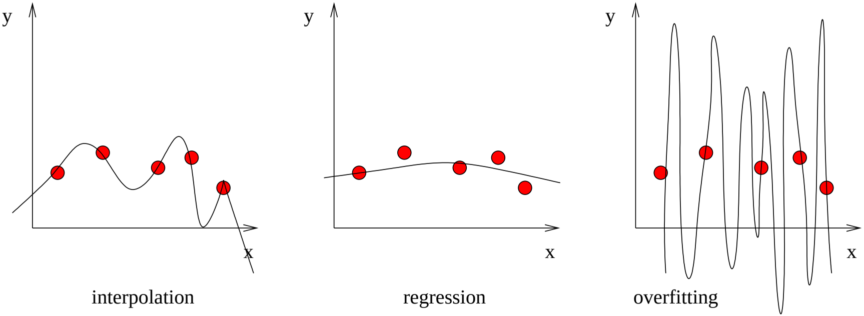

Figure 3: Interpolation vs. regression vs. overfitting

Remarks on Parameter Estimation

Problem: Compute the sensitivity of the estimated parameters from a set of *N* point correspondences.

- The parameters shall fit for all data that is processed by the used algorithm.
- How can we figure if the estimated parameters are sufficient for the observed data in practice? We might get different data as input for the algorithm.
- To compute the sensitivity (robustness) of the estimated parameters, we need many data samples.
- If we do not have many samples, we can try a bootstrapping approach, but we will not go into detail here.

model with all but one point -> set of params for each point
-> hight variance ? -> bad modality

Topics

Scaling of Input Data

- Proper scaling of data is **crucial** for the quality of the output, a fact that is often overseen.
- Limited numerical accuracy requires certain ranges.
- “*Data normalization must not be considered optional!*” (Richard Hartley)
- Select the optimal scaling by minimization of the condition number to **minimize sensibility** and to find a **proper data range**:

$$\kappa(\boldsymbol{A}^\mathsf{T}\boldsymbol{A}) \rightarrow \min\,.$$

Scaling of Input Data

Example

- Use a polynomial of total degree 5 to undistort images.
- The dimensions of the input images are 1024×1024 pixels.
- The $x-$ and $y-$coordinates are represented in pixels, i. e., $x, y \in \{1, 2, \ldots 1024\}$.

we are reaching the limit of floating point operations

- The monomials range from 1 to $1024^5 = 1125899906842624$.
- The result has to be between 0 and 1023!!!

for even higer order polynomials we would have an even bitter problem

$\rightarrow$ Think about it! Do you have a good feeling in doing this?

Minimization of κ how to find good scaling ?

The Gramian matrix $\boldsymbol{A}^\mathsf{T}\boldsymbol{A}$ can be used to test for linear independence of functions. Any decrease of the condition number will be useful, even if it is not a global optimum!

Method to compute a proper scaling:

1. Select two constants k and l.
2. Scale all data points (x_i, y_i) to (kx_i, ly_i).
3. Rewrite the linear system for solving for the calibration coefficients from the last unit.
4. Compute the new measurement matrix $\boldsymbol{A}$.
5. Compute the condition number $\kappa(\boldsymbol{A}^\mathsf{T}\boldsymbol{A})$.
6. Minimize κ with respect to k and l (e. g., by gradient descent).
7. Finally, recover the original coefficients $u_{i,j}, v_{i,j}$ and invert the scaling process.

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Scaling of Input Data

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Take Home Messages

- A parameterization has to be chosen wisely.
- Know the differences between interpolation and regression.
- Also be aware of overfitting, i. e., that a model can adapt too much to its training data.
- Data normalization is mandatory.

Further Readings

A book that covers many image preprocessing methods applied in medical imaging systems is:

Jiří Jan. *Medical Image Processing, Reconstruction, and Restoration: Concepts and Methods*. Signal Processing and Communications. CRC Press, Taylor & Francis Group, Nov. 2005

For the original article about the bootstrapping method see

Bradley Efron. “Bootstrap Methods: Another Look at the Jackknife”. In: *The Annals of Statistics* 7.1 (1979), pp. 1–26. DOI: doi:10.1214/aos/1176344552