$$\lambda = -\frac{S'(t)}{S(t)} = -\frac{-F(t)}{1-F(t)} = \frac{f(t)}{1-F(t)}$$

$$f(t) = (1-F(t))\lambda$$

$$d\pi = dV - \Delta\, dZ$$

$$dV = \left(\frac{\partial V}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 V}{\partial r^2}\right) dt + \frac{\partial V}{\partial r} dr = \mathcal{L}(V)\,dt + \frac{\partial V}{\partial r} dr$$

$$dZ = \left(\frac{\partial Z}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 Z}{\partial r^2}\right) dt + \frac{\partial Z}{\partial r} dr = \mathcal{L}(Z)\,dt + \frac{\partial V}{\partial r} dr$$

$$d\pi = \left(\mathcal{L}(V) - \Delta \mathcal{L}(Z)\right) dt + \left(\frac{\partial V}{\partial r} - \Delta \frac{\partial Z}{\partial r}\right) dr$$

$$\Delta = \frac{\partial V}{\partial r} \Big/ \frac{\partial Z}{\partial r}$$

$$E(d\pi) = \left(\mathcal{L}(V) - \Delta \mathcal{L}(Z)\right) dt - V p\, dt$$

$$= r\,(V - \Delta Z)\, dt$$

$$\mathcal{L}(V) - (r+p)V = \Delta\left(\mathcal{L}(Z) - rZ\right)$$

$$\mathcal{L}(z) - rz = \left(\frac{\partial z}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 z}{\partial r^2} - rz\right)$$

$$= -(u - \lambda w)\frac{\partial z}{\partial r} \quad \leftarrow$$

$$\frac{\partial z}{\partial t} + \frac{1}{2} w \frac{\partial^2 z}{\partial r^2} + (u - \lambda w)\frac{\partial z}{\partial r} - rz = 0$$

$$\mathcal{L}(v) - (r+p)v = (\mathcal{L}(z) - rz)\,\delta$$

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$$= -(u - \lambda w)\frac{\partial z}{\partial r} \cdot \frac{\partial v}{\partial r} \Big/ \frac{\partial z}{\partial r}$$

$$= -(u - \lambda w)\frac{\partial v}{\partial r}$$

$$\frac{\partial v}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 v}{\partial r^2} + (u - \lambda w)\frac{\partial v}{\partial r} - (r+p)v = 0$$

$$dZ = \mathcal{L}(Z)\,dt + \frac{\partial Z}{\partial r}\,dr$$

$$dV(r,p,t) = \mathcal{L}'(V)\,dt + \frac{\partial V}{\partial r}\,dr + \frac{\partial V}{\partial p}\,dp$$

$$\mathcal{L}'(V) = \frac{\partial V}{\partial t} + \frac{1}{2}w^2\frac{\partial^2 V}{\partial r^2} + \frac{1}{2}\delta^2\frac{\partial^2 V}{\partial r^2} + \rho w \delta \frac{\partial^2 V}{\partial r\,\partial p}$$

$$dV_1 = \mathcal{L}'(V_1)\,dt + \frac{\partial V_1}{\partial r}\,dr + \frac{\partial V_1}{\partial p}\,dp$$

$$d\pi = dV - \Delta\,dZ - \Delta\,dV_1$$

$$\begin{aligned} d\pi = {} & \left(\mathcal{L}'(V) - \Delta\mathcal{L}(Z) - \Delta_1\mathcal{L}'(V_1)\right)dt \\ & + \left(\frac{\partial V}{\partial r} - \Delta\frac{\partial Z}{\partial r} - \Delta_1\frac{\partial V_1}{\partial r}\right)dr \\ & + \left(\frac{\partial V}{\partial p} - \Delta_1\frac{\partial V_1}{\partial p}\right)dp \end{aligned}$$

$$\begin{cases} \dfrac{\partial V}{\partial r} - \Delta\dfrac{\partial Z}{\partial r} - \Delta_1\dfrac{\partial V_1}{\partial r} = 0 \\ \dfrac{\partial V}{\partial p} - \Delta_1\dfrac{\partial V_1}{\partial p} = 0 \end{cases}$$

$$E(d\pi) = \left(\mathcal{L}'(v) - \delta \mathcal{L}(z) - \delta_1 \mathcal{L}'(v_1) \right) dt$$

$$+ \left(-v + \delta_1 v_1 \right) p \, dt$$

$$= r \left(v - \delta z - \delta_1 v_1 \right) dt$$

$$\left(\mathcal{L}'(v) - (r+p) v \right) - \delta \left(\mathcal{L}(z) - r z \right)$$

$$- \delta_1 \left(\mathcal{L}'(v_1) - (r+p) v_1 \right) = 0$$

$$- \delta \left(\mathcal{L}(z) - r z \right) = \frac{\frac{\partial v}{\partial r} - \frac{\partial v_1}{\partial r} \delta_1}{\frac{\partial z}{\partial r}} (u - \lambda w) \frac{\partial z}{\partial r}$$

$$= \left(\frac{\partial v}{\partial r} - \frac{\partial v_1}{\partial r} \delta_1 \right) (u - \lambda w)$$

$$\left(\mathcal{L}'(V) - (r+p)V\right) + \left(\frac{\partial V}{\partial r} - s_1 \frac{\partial V_1}{\partial r}\right)(u - \lambda w)$$

$$- s_1\left(\mathcal{L}'(V_1) - (p+r)V_1\right) = 0$$

$$\mathcal{L}'(V) - (r+p)V + \frac{\partial V}{\partial r}(u-\lambda w) = s_1\Big(\mathcal{L}'(V_1) - (p+r)V_1$$

$$+ (u-\lambda w)\frac{\partial V_1}{\partial r}\Big)$$

$$\frac{\mathcal{L}'(V) + \frac{\partial V}{\partial r}(u-\lambda w) - (r+p)V}{\frac{\partial V}{\partial p}} =$$

$$\frac{\mathcal{L}(V_1) + \frac{\partial V_1}{\partial r}(u-\lambda w) - (r+p)V_1}{\frac{\partial V_1}{\partial p}} = a(r,p,t)$$

$$= -(\gamma - \delta\lambda')$$

$$V(t,T) = E\left[e^{-\int_t^T r_s\,ds} \mathbb{1}_{\{\tau > T\}} \,\middle|\, \mathcal{F}_t \right]$$

$$= E\left[E\left(\underbrace{e^{-\int_t^T r_s\,ds}} \mathbb{1}_{\{\tau > T\}} \,\middle|\, \mathcal{F}_T \right) \,\middle|\, \mathcal{F}_t \right]$$

$$= E\left[e^{-\int_t^T r_s\,ds} \, E\left(\underbrace{\mathbb{1}_{\{\tau > T\}} \,\middle|\, \mathcal{F}_T} \right) \,\middle|\, \mathcal{F}_t \right]$$

$$= E\left[e^{-\int_t^T r_s\,ds} \, e^{-\int_t^T \rho_s\,ds} \,\middle|\, \mathcal{F}_t \right]$$

$$= E\left[e^{-\int_t^T (r_s + \rho_s)\,ds} \,\middle|\, \mathcal{F}_t \right]$$

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial x^2} + \frac{1}{2}\eta^2 \frac{\partial^2 V}{\partial y^2} + \rho\sigma\eta \frac{\partial^2 V}{\partial x \partial y} - ax\frac{\partial V}{\partial x} - by\frac{\partial V}{\partial y} - RV = 0$$

$$V(t,T) = e^{A(t,T) - B(t,T)x - C(t,T)y}$$

$$\frac{\partial V}{\partial t} = (\dot{A} - \dot{B}x - \dot{C}y)V$$

$$\frac{\partial V}{\partial x} = -BV \qquad \frac{\partial V}{\partial y} = -CV$$

$$\frac{\partial^2 V}{\partial x^2} = B^2 V \qquad \frac{\partial^2 V}{\partial y^2} = C^2 V \qquad \frac{\partial^2 V}{\partial x \partial y} = BCV$$

$$(\dot{A} - \dot{B}x - \dot{C}y)V + \frac{1}{2}\sigma^2 B^2 V + \frac{1}{2}\eta^2 C^2 V + \rho\sigma\eta BCV$$
$$+ axBV + byCV - (\phi + x + y)V = 0$$

$\dot{A} + \frac{1}{2}\sigma^2 B^2 \qquad {}^2 \quad + \frac{1}{2}\eta^2 C^2 + \rho\sigma\eta BC - \phi$
$- x(\underline{\dot{B} - aB + 1}) - y(\underline{\dot{C} - bC + 1}) = 0$

$$\begin{cases} \dot{A} + \frac{1}{2}\sigma^2 B^2 + \frac{1}{2}\eta^2 C^2 + \rho\sigma\eta BC - \phi = 0 & (1) \\ \dot{B} - aB + 1 = 0 & (2) \\ \dot{C} - bC + 1 = 0 & (3) \end{cases}$$

(2) $\dfrac{\partial B(t,T)}{\partial t} - aB(t,T) + 1 = 0$

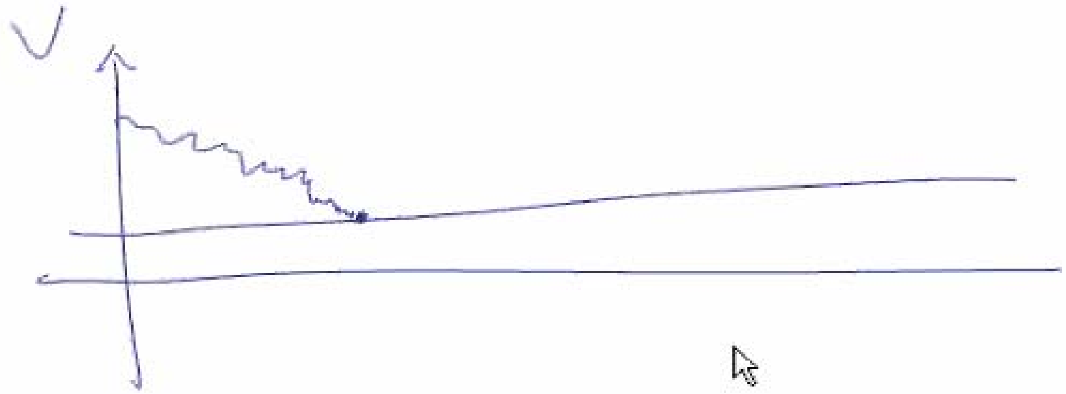
I.F. e^{-at} $\qquad \dfrac{d}{dt}\left(e^{-at} B(t,T)\right) = -e^{-at}$

$e^{-aT} B(T,T) - e^{-at} B(t,T) = -\int_t^T e^{-as}\, ds$

$$e^{-at} B(t,T) = \int_t^T e^{-as}\, ds = -\frac{1}{a}\left(e^{-aT} - e^{-at}\right)$$

$$\begin{cases} B(t,T) = \dfrac{1 - e^{-a(T-t)}}{a} \\[2ex] C(t,T) = \dfrac{1 - e^{-b(T-t)}}{b} \end{cases}$$

$$A(t,T) = \frac{\gamma\sigma}{ab}\left[T - t - B(t,T) - A(t,T) + \frac{1 - e^{-(a+b)(T-t)}}{a+b}\right]$$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