1. **Use spot rate data to find ϕ and β if we assume that interest rate movements are of the form**

$$dr = \mu(r)dt + \phi r^{\beta} dX.$$

Does your estimated value of β lie close to that of any of the standard models?

Divide the changes in the interest rate, δr, into buckets covering a range of r values. Calculate the average value of $(\delta r)^2$, for each bucket.

We assume

$$E[(\delta r)^2] = \phi^2 r^{2\beta} \delta t$$

to leading order in the timestep δt, and plot $\log(E[(\delta r)^2])$ against $\log r$. This is shown in Figure 38.1.

The slope of this 'line' gives an estimate for 2β. We estimate that

$$\beta = 1.13 \quad \text{and} \quad \phi = 0.126.$$

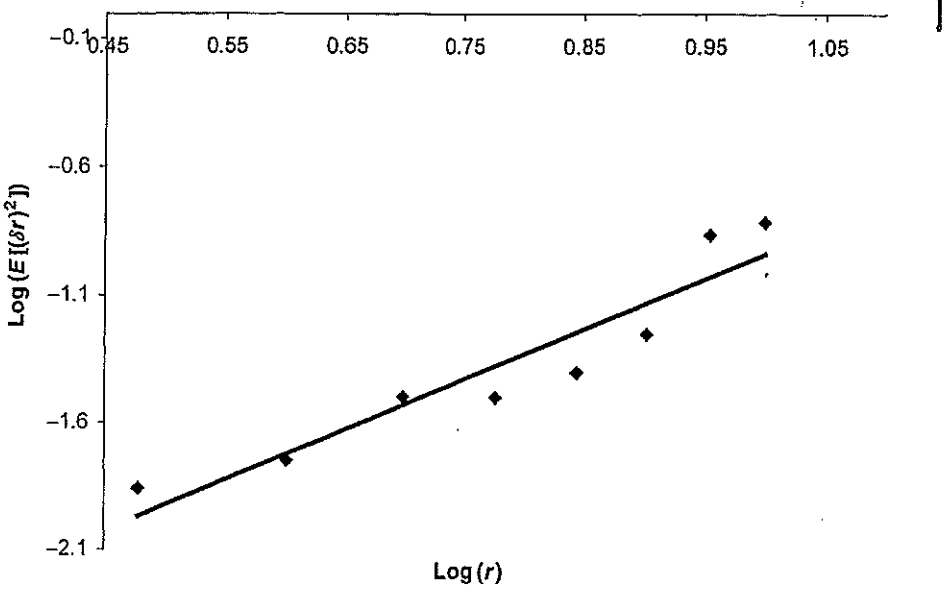

Figure 38.1: Estimation of β.

possible singularities in a phase plane must we try to avoid when we construct our model?

A centre could model an economic cycle since it consists of periodic orbits in the (r, l) plane. We would have to be wary of unstable singularities as they can easily lead to unrealistic results (with l flying off to infinity, while r remains bounded, for instance).