Lahore University of the Management Sciences
Centre for Advanced Studies in Mathematics

Problem for the Month of February – 2006

Assume that an asset price S evolves according to the stochastic differential equation.

$$\frac{dS}{S} = (\mu - D)dt + \sigma\, dX$$

where μ and σ are constants. In addition *S* pays out a continuous dividend stream equal to *DS dt* during the infinitesimal time interval *dt*, where *D* the dividend yield is constant.

Now suppose a European option is written on this asset with the properties that at expiry the holder receives the asset and prior to expiry the option pays a continuous cash flow *C*(*S*,*t*) *dt* during each time interval of length *dt*. The value *V* of the option satisfies the following Black-Scholes equation.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - D)S\frac{\partial V}{\partial S} - rV = -C(S,t),$$
$$V(S,T) = S$$

Suppose that *C*(*S*,*t*) has the form *C*(*S*,*t*) = *f* (*t*)*S*. By writing $V = \phi(t)S$ find an expression for *V*(*S*,*t*), and hence show that the delta $\Delta(S,t) = \frac{\partial V}{\partial S}$ of the derivative security is

$$\Delta(S,t) = \exp(-D(T-t)) + \int_t^T \exp(-D(\tau - t))\, f(\tau)\, d\tau$$

Please send your solution to: **kashifi@lums.edu.pk** before February 28, 2006 (Tuesday). Best solution and names of all those who submit correct solution will be posted on this website.