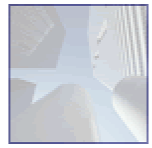



:: The Greek Letters - Delta ::

The Greek Letters or simply the "Greeks" are quantities representing the market sensitivities of the options or other derivatives. Each Greek measures a different aspect of the risk in an option position. Through understanding and managing these Greeks, market makers, traders, financial institutions and portfolio managers can manage their risks appropriately, whether they deal in OTC or exchange-traded options.

Delta

The delta of an option is defined as the rate of change of the option price w.r.t. the price of the underlying asset. The delta of an option dependent on a single asset S is mathematically expressed as:

$$\Delta = \frac{\partial V}{\partial S}$$

For a European call option on a non-dividend-paying underlying, the value of the option is:

$$C(S,t) = S N(d_1) - E e^{-r(T-t)} N(d_2)$$

For a European call option on a non-dividend-paying share,

$$\begin{aligned}\Delta &= \frac{\partial C}{\partial S} \\ &= N(d_1) + S N'(d_1)\frac{\partial d_1}{\partial S} - E e^{-r(T-t)} N'(d_2)\frac{\partial d_2}{\partial S} \\ &= N(d_1) + \left(S N'(d_1) - E e^{-r(T-t)} N'(d_2)\right)\frac{\partial d_1}{\partial S}\end{aligned}$$

Since

$$d_2 = d_1 - \sigma\sqrt{T-t}$$
$$\Rightarrow \frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S}$$

Now

$$N'(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)$$

$$N'(d_1) = \frac{1}{\sqrt{2\pi}} \exp(-d_1^2/2)$$

$$N'(d_2) = \frac{1}{\sqrt{2\pi}} \exp(-d_2^2/2)$$

$$= \frac{1}{\sqrt{2\pi}} \exp(-d_1^2/2) \exp \left(\sigma\sqrt{T-t}\, d_1 - \sigma^2 (T-t)/2\right)$$

$$= N'(d_1) \exp \left(\log(S/E) + \left(r + \frac{1}{2}\sigma^2\right)(T-t) - \sigma^2 (T-t)/2\right)$$

$$= N'(d_1) \frac{S}{E} e^{r(T-t)}$$

i.e.

$$S N'(d_1) = E e^{-r(T-t)} N'(d_2)$$

This gives

$$\Delta = N(d_1)$$

Deltas for call options are always positive, which means that a **long** (buy) **call** should be hedged with a **short** (sell) position in the underlying, and vice versa.

Similarly, for a European put option of the same underlying, delta is given by:

$$\Delta = N(d_1) - 1$$

Deltas for put options are always negative, which means that a **long put** should be hedged with a **long** position in the underlying, and vice versa.

Delta is between 0 and +1 for calls and between 0 and -1 for puts. The delta for the underlying is always 1. A put option with a delta of 0.5 will drop £0.5 in price for each £1 rise in the underlying (i.e. increasingly out-of-the-money), a call option with the same delta will rise £0.5 instead (i.e. increasingly in-the-money).

Delta Hedging

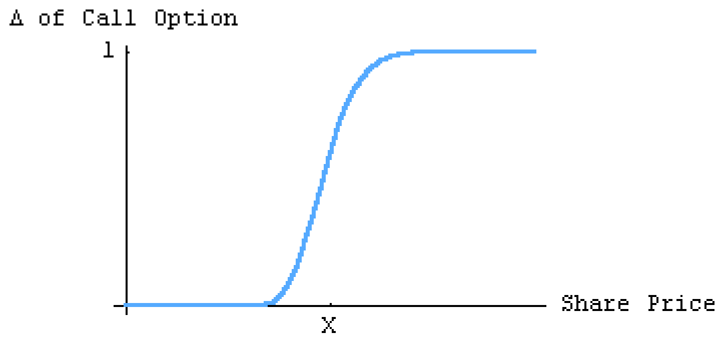
If, for example, the share price is £10 and the call option price is £1 and the delta of the call option is 0.5, an investor who has sold 12 call option contracts (options to buy 1,200 shares) can **delta-hedge** his/her position by buying 0.5 x 1,200 = 600 shares. A rise in share price will produce a loss of 0.5 x 1,200 = £600 on the call options but a gain of £600 on the shares.

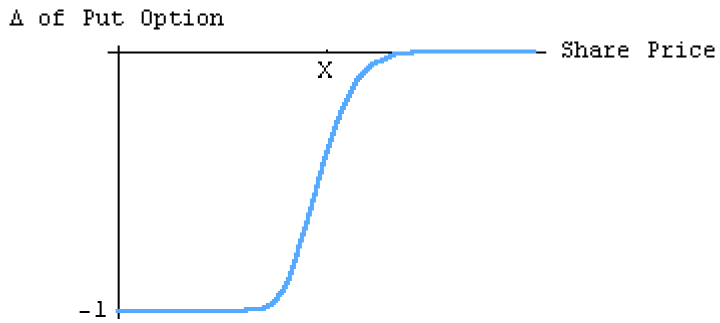
The delta of the portfolio can be determined by adding up all his/her positions.The delta of the short option position is -0.5 x 1,200 = -600 and delta of the long share position is 1 x 600 = 600,thus his/her position has a delta of zero, this is referred as being **delta neutral**.

Unfortunately, delta-hedging only works for a short period of time during when delta of the option is fixed. The hedge will have to be readjusted periodically to reflect changes in delta, which could be affected by the share price, time to expiry, risk-free rate of return and volatility of the underlying. Below we show how delta changes with the underlying share price and time to expiry.

Variation of Delta with Share Price

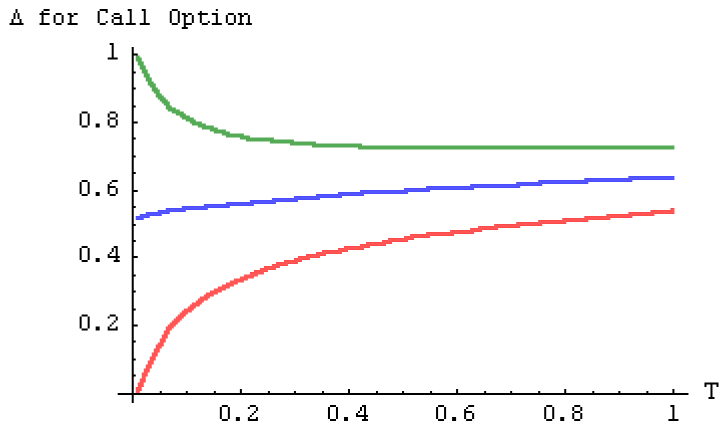
Variation of Delta with share price (S) for European option on a non-dividend-paying share with strike price of X. Here one can see that delta for in-the-money options is very close to one and zero for out-of-the-money options.

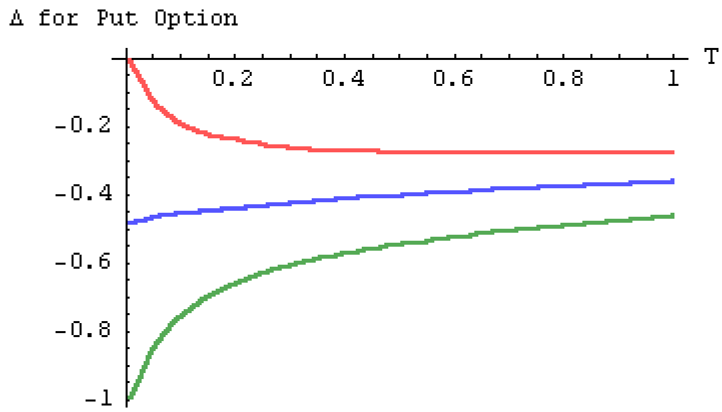




Variation of Delta with Time to Expiry

Variation of Delta with Time to Expiry (T) for European option on a non-dividend-paying share with strike price of X. **Red**, **Blue** and **Green** lines denote out-of-the-money, at-the-money and in-the-money options respectively.





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