Certificate in Quantitative Finance (CQF)
Session 5.4: Credit Default Swaps *
ERRATA

May 18, 2009

1 Session 5.4: Slide 64

The PV of the premium leg is

$$PL_N = S_N \sum_{n=1}^{N} D(0,T_n)P(T_n)\,(\Delta t_n)$$

where Δ_n is the year fraction corresponding to $T_{n-1} - T_n$ and $(P(T_{n-1}) - P(T_n))$ is the probability of the credit default event occuring during period $T_{n-1} - T_n$.

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The PV of the default leg is

$$DL_N = (1-R)\sum_{n=1}^{N} D(0,T_n)\,(P(T_{n-1}) - P(T_n))$$

3 Session 5.4: Slide 67

The spread S_N for an $N-$period credit default swap is given by

$$S_N = \frac{(1-R)\sum_{n=1}^{N} D(0,T_n)\,(P(T_{n-1}) - P(T_n))}{\sum_{n=1}^{N} D(0,T_n)P(T_n)\,(\Delta t_n)}$$

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4 Session 5.4: Slide 71

Step N=1

In the Boostrapping procedure, for T_1 we have

$$P(T_1) = \frac{L}{L + \Delta t_1 S_1}$$

where $L = (1 - R)$.

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Step N=2

For T_2 we have

$$P(T_2) = \frac{D(0,T_1)\left[L(1) - (L + \Delta t_1 S_2)P(T_1)\right]}{D(0,T_2)(L + \Delta t_2 S_2)} + \frac{P(T_1)L}{L + \Delta t_2 S_2}$$

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Step N

$$P(T_N) = \frac{\sum_{n=1}^{N-1} D(0,T_n)\left[LP(T_{n-1}) - (L + \Delta t_n S_N)P(T_n)\right]}{D(0,T_N)(L + \Delta t_n S_N)} + \frac{P(T_{N-1})L}{(L + \Delta t_N S_N)}.$$

7 Session 5.4: Problem Sheet Solutions: Q3

The Credit Triangle

This problem is solved by assuming a continous approximation to the pricing of a CDS.

The premium leg (PL) is

$$PL(0,T) = S\int_0^T Z(0,t)P(0,t)dt$$

where $P(0,t)$ is the survival probability as seen from time zero.

The default leg (DL) is

$$DL(0,T) = (1-R)\int_0^T D(0,t)(-dP(0,t)dt)$$

with $D(0,t)$ the discount factor for time t.

CDS
JP Morgan formulation

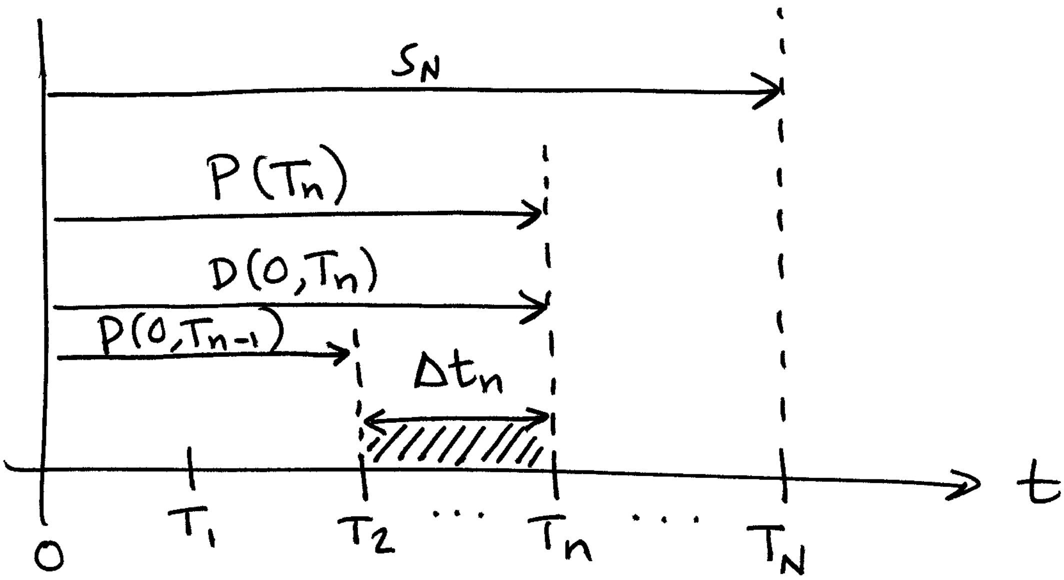
PREMIUM LEG

$$PL_N = S_N \sum_{n=1}^{N} D(0,T_n)\, P(T_n)\, \Delta t_n$$

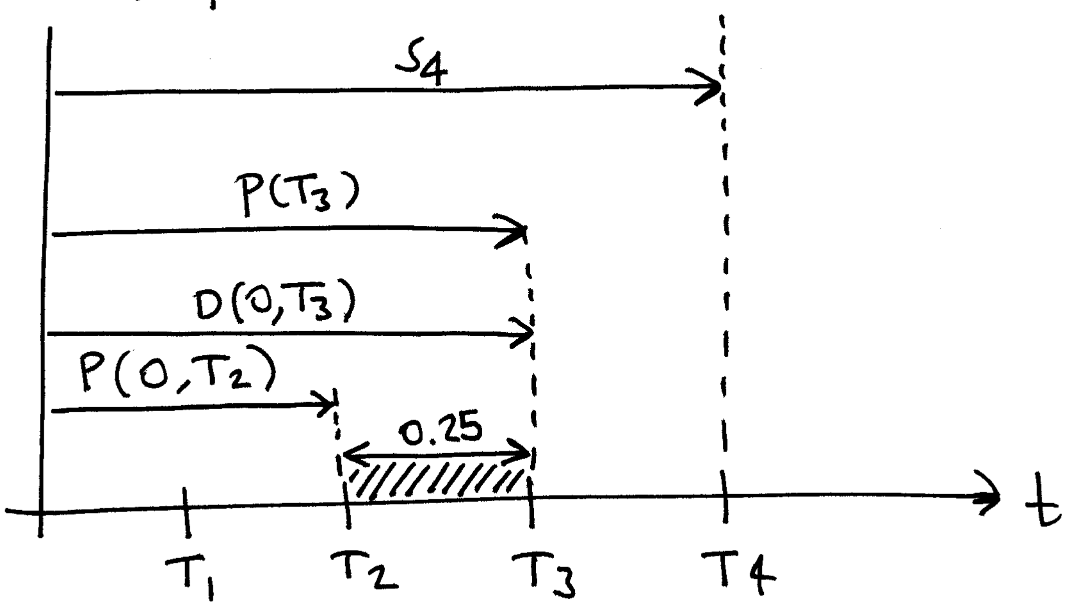
DEFAULT LEG

$$DL_N = (1-R) \sum_{n=1}^{N} D(0,T_n)\Big(P(T_{n-1}) - P(T_n)\Big)$$

based on time-grid:



for a quarterly CDS with maturity 1 year:
→ $N=4$



CDS Bootstrapping

We assume that we have a vector of CDS market spreads for increasing maturities $[S_1, S_2, \dots, S_N]$. We now determine their associated survival probabilities $[P(T_1), P(T_2), \dots, P(T_N)]$.

$\underline{\underline{N=1}}$

$$PL_N = S_N \sum_{n=1}^{N} \left(D(0,T_n)\, P(T_n)\, \Delta t_n \right)$$

$$PL_1 = S_1 \left(D(0,T_1)\, P(T_1)\, \Delta t_1 \right)$$

$$DL_N = (1-R) \sum_{n=1}^{N} \left(D(0,T_n) \left(P(T_{n-1}) - P(T_n) \right) \right)$$

$$DL_1 = (1-R)\, D(0,T_1) \left(P(T_0) - P(T_1) \right)$$

$$PL_1 = DL_1$$

$$S_1\, D(0,T_1)\, P(T_1)\, \Delta t_1 = \underbrace{(1-R)}_{L}\, D(0,T_1) \left[P(T_0) - P(T_1) \right]$$

$$S_1\, D(0,T_1)\, P(T_1)\, \Delta t_1 = L\, D(0,T_1)\, P(T_0) - L\, D(0,T_1)\, P(T_1)$$

$$S_1 D(0,T_1) P(T_1) \Delta t_1 + L D(0,T_1) P(T_1) = L D(0,T_1) P(T_0)$$

$$P(T_1)\left[S_1 D(0,T_1) \Delta t_1 + L D(0,T_1)\right] = L D(0,T_1) P(T_0)$$

$$P(T_1) \cancel{D(0,T_1)} \left[S_1 \Delta t_1 + L\right] = L \cancel{D(0,T_1)} P(T_0)$$

with $P(T_0) = 1$

$$P(T_1) = \frac{L}{S_1 \Delta t_1 + L}$$

$\underline{\underline{N = 2}}$

$$PL_N = S_N \sum_{n=1}^{N} \left(D(0,T_n) P(T_n) \Delta t_n \right)$$

$$PL_2 = S_2 \left[D(0,T_1) P(T_1) \Delta t_1 + D(0,T_2) P(T_2) \Delta t_2 \right]$$

$$DL_N = (1-R) \sum_{n=1}^{N} D(0,T_n) \left(P(T_{n-1}) - P(T_n) \right)$$

$$DL_2 = (1-R) \left[D(0,T_1) \left(P(T_0) - P(T_1) \right) + D(0,T_2) \left(P(T_1) - P(T_2) \right) \right]$$

$$PL_2 = DL_2$$

$$s_2\left[D(0,T_1)P(T_1)\Delta t_1 + D(0,T_2)P(T_2)\Delta t_2\right] = \underbrace{(1-R)}_{L}\left[D(0,T_1)\left(\overset{1}{\cancel{P(T_0)}} - P(T_1)\right) + D(0,T_2)\left(P(T_1)-P(T_2)\right)\right]$$

$$s_2 D(0,T_1)P(T_1)\Delta t_1 + s_2 D(0,T_2)P(T_2)\Delta t_2 = L\,D(0,T_1)\left(1-P(T_1)\right) + L\,D(0,T_2)\times\left(P(T_1)-P(T_2)\right)$$

$$s_2 D(0,T_1)P(T_1)\Delta t_1 + s_2 D(0,T_2)\underline{P(T_2)}\Delta t_2 = L\,D(0,T_1) - L\,D(0,T_1)P(T_1) + L\,D(0,T_2)P(T_1) - L\,D(0,T_2)\underline{P(T_2)}$$

$$s_2 D(0,T_2)P(T_2)\Delta t_2 + L\,D(0,T_2)P(T_2) = L\,D(0,T_1) - L\,D(0,T_1)P(T_1) + L\,D(0,T_2)P(T_1) - s_2 D(0,T_1)P(T_1)\Delta t_1$$

$$P(T_2)\left[s_2 D(0,T_2)\Delta t_2 + L\,D(0,T_2)\right] = \cdots$$

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$$P(T_2)\left[D(0,T_2)\left(S_2\Delta t_2 + L\right)\right] = D(0,T_1)\left(L - L\,P(T_1) - S_2 P(T_1)\Delta t_1\right) + D(0,T_2)\,L\,P(T_1)$$

$$P(T_2)\left[D(0,T_2)\left(S_2\Delta t_2 + L\right)\right] = D(0,T_1)\left[L - P(T_1)\left(L + S_2\Delta t_1\right)\right] + D(0,T_2)\,L\,P(T_1)$$

$$P(T_2) = \frac{D(0,T_1)\left[L - P(T_1)\left(L + S_2\Delta t_1\right)\right]}{D(0,T_2)\left(S_2\Delta t_2 + L\right)} + \frac{\cancel{D(0,T_2)}\,L\,P(T_1)}{\cancel{D(0,T_2)}\left(S_2\Delta t_2 + L\right)}$$

$$P(T_2) = \frac{D(0,T_1)\left[L - P(T_1)\left(L + S_2\Delta t_1\right)\right]}{D(0,T_2)\left(L + S_2\Delta t_2\right)} + \frac{P(T_1)\,L}{L + S_2\Delta t_2}$$

N = 3

$$P(T_N) = \frac{\sum_{n=1}^{N-1} D(0,T_n)\left[L P(T_{n-1}) - (L + \Delta t_n S_N) P(T_n)\right]}{D(0,T_N)(L + \Delta t_n S_N)} + \frac{P(T_{N-1})\, L}{(L + \Delta t_N S_N)}$$

$$P(T_3) = \frac{\sum_{n=1}^{2} D(0,T_n)\left[L P(T_{n-1}) - (L + \Delta t_n S_3) P(T_n)\right]}{D(0,T_3)(L + \Delta t_n S_3)} + \frac{P(T_2)\, L}{(L + \Delta t_3 S_3)}$$

Note: The bootstrapping formulas above are implemented in the XLS file:
ImprovedBootstrappingExample.xls

PROBABILITY OF DEFAULT

Recovery Rate	50%

TIME (Years)	dt	MARKET SPREAD	DF	IMPLIED SURVIVAL PROB	first term	second term	third term	fourth term	sum	quotient	first term	last term
0				100.00%								
1	1	29.00	0.9803	99.42%								
2	1	39.00	0.9514	98.45%	- 0.0010				- 0.0010	0.4794	- 0.0020	0.9865
3	1	46.00	0.9159	97.26%	- 0.0017	0.0003			- 0.0013	0.4622	- 0.0029	0.9755
4	1	52.00	0.8756	95.88%	- 0.0022	- 0.0002	0.0008		- 0.0017	0.4424	- 0.0038	0.9626
5	1	57.00	0.8328	94.37%	- 0.0027	- 0.0007	0.0004	0.0013	- 0.0018	0.4211	- 0.0043	0.9480

$\underline{\underline{N=1}}$

$$P(T_1) = \frac{L}{s_1 \Delta t_1 + L}$$

$\underline{\underline{N=5}}$

$$P(T_5) = \frac{\sum_{n=1}^{4} \left(\blacksquare + \blacksquare + \blacksquare + \blacksquare \right)}{\blacksquare} + \frac{P(T_4)\, L}{L + \Delta t_5 S_5}$$