

$$* f(x+h) \approx a + bh + ch^2 + dh^3 + e h^4 + \dots$$

1. Put $h=0$.

$$f(x) = a + 0 + 0 + \dots = a$$

$$\boxed{a = f(x)}$$

2. Diff $*$ w.r.t. h :

$$\frac{df(x+h)}{dh} = \frac{df(x+h)}{dh} = 0 + b + 2ch + 3dh^2 + 4eh^3 + \dots$$

$$\text{Set } h=0: \frac{df(x)}{dh} = b + 0 + 0 + \dots$$

$$\boxed{b = \frac{df(x)}{dx}}$$

$$\sum_{i=0}^3 \frac{d^i f(x+h)}{dh^i} = 0 + 2c + \underline{3 \cdot 2 \cdot dh} + \underline{4 \cdot 3 \cdot c h^2} + \dots$$

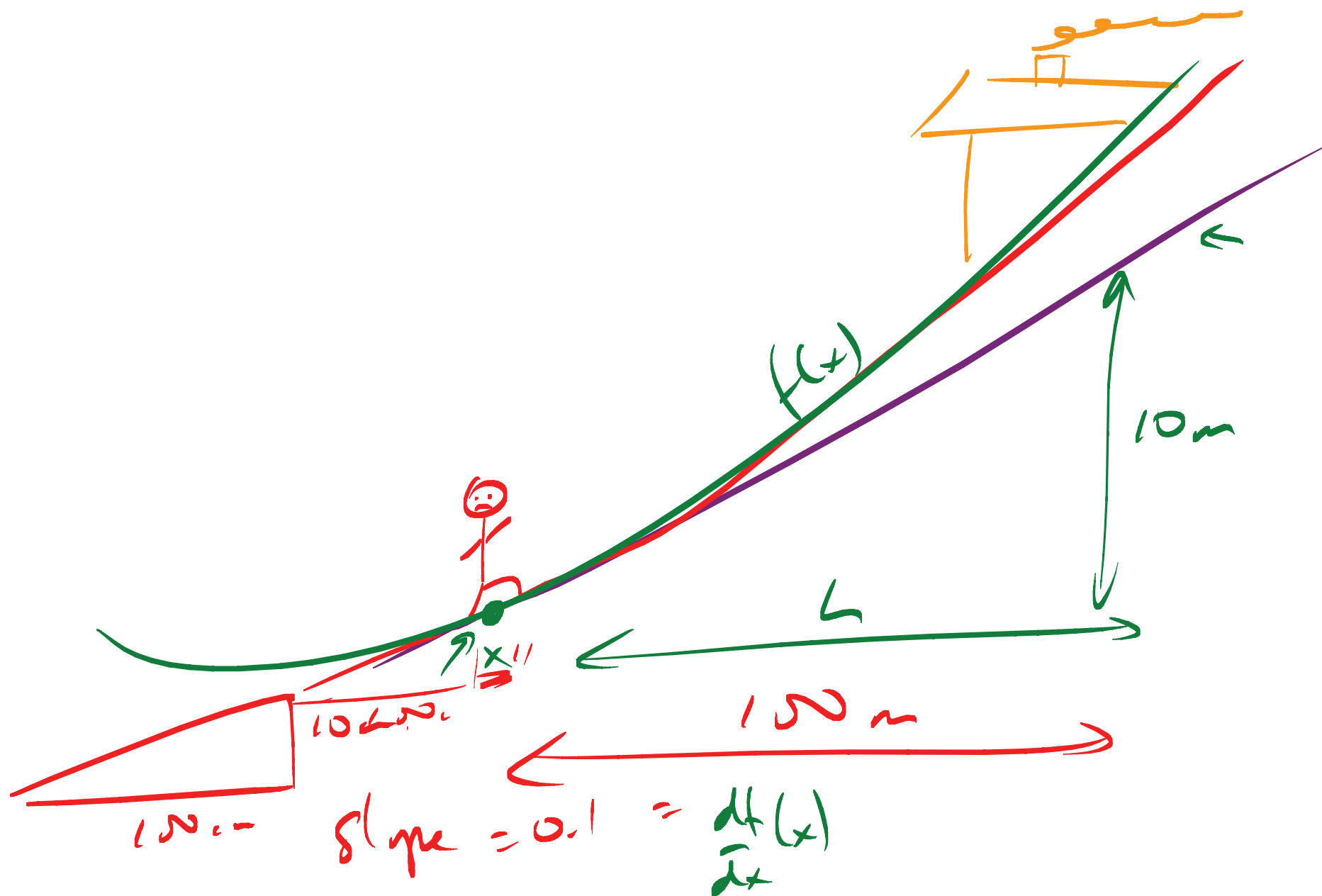
$$\frac{d^2 f(x)}{dx^2} = 2c$$

$$c = \frac{1}{2} \frac{d^2 f(x)}{dx^2}$$

4. D-9 ~~***~~ wrt h , set $L=0 \dots$

$$d = \frac{1}{3!} \frac{d^3 f(x)}{dx^3}$$

$$e = \frac{1}{4!} \frac{d^4 f(x)}{dx^4}$$



$$\delta S = S_{i+1} - S_i = \mu S_i \Delta t + \sigma S_i \phi \sqrt{\Delta t}$$



$$V(\underline{S} + \underline{\delta S}, \underline{t} + \underline{\Delta t}) \approx \underline{V(S, t)}$$

$$+ \frac{\partial V(S, t)}{\partial t} \underline{\Delta t} + \frac{\partial V(S, t)}{\partial S} \underline{\delta S} = o(\sqrt{\Delta t})$$

$$+ \frac{1}{2} \frac{\partial^2 V}{\partial S^2} (\underline{\delta S})^2 + \frac{1}{2} \frac{\partial^2 V}{\partial t^2} (\underline{\Delta t})^2 + \frac{\partial^2 V}{\partial S \partial t} \underline{\delta S} \underline{\Delta t} \propto \Delta t^{3/4}$$

$$+ \frac{1}{3!} \frac{\partial^3 V}{\partial S^3} (\underline{\delta S})^3 + \dots + o(\Delta t)$$

$$\left(w + o(\Delta t^{3/4}) \right)$$

if $\text{RAND}() < \alpha$

$\swarrow + \delta y$

if $\text{RAND}() > 1 - \alpha$

$- \delta y$

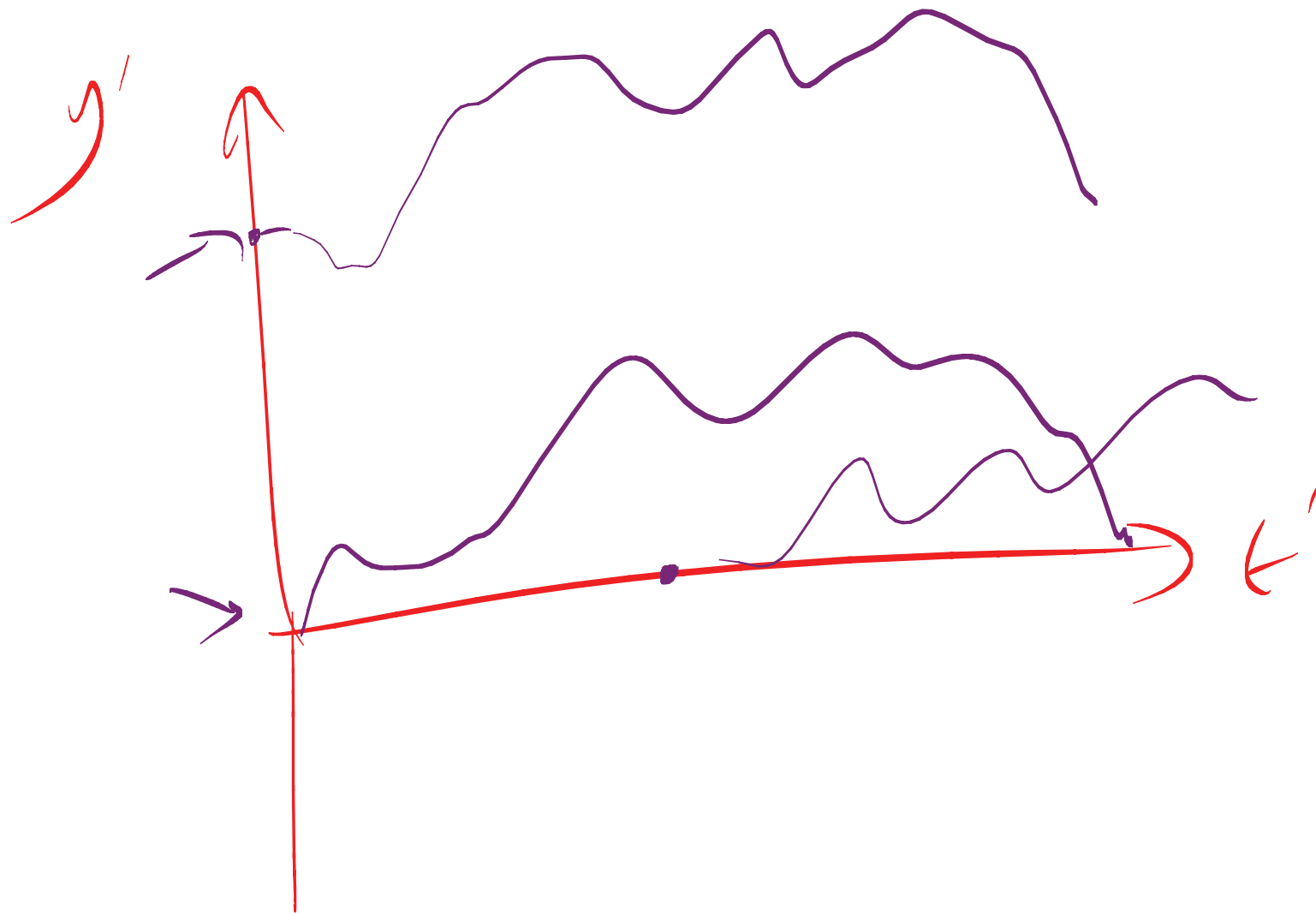
else

0

$$P(\underline{y, t}; \underline{y', t'})$$

backward forward

$$t' > t$$



$$c^2 = \alpha \frac{\delta y^2}{\delta t} \leftarrow \leftarrow$$

$$\frac{\partial p}{\partial t'} = c^2 \frac{\partial^2 p}{\partial y'^2}$$

1. $\frac{\partial p}{\partial t'} = 0$

2. $\frac{\partial^2 p}{\partial y'^2} = 0$

3. $\frac{\partial p}{\partial t'} = c^2 \frac{\partial^2 p}{\partial y'^2}$

1) $\delta y = \Delta t$

$$c^2 = \alpha \frac{\delta y^2}{\delta t} = \alpha \cdot \Delta t$$

$$c \rightarrow 0$$

2)

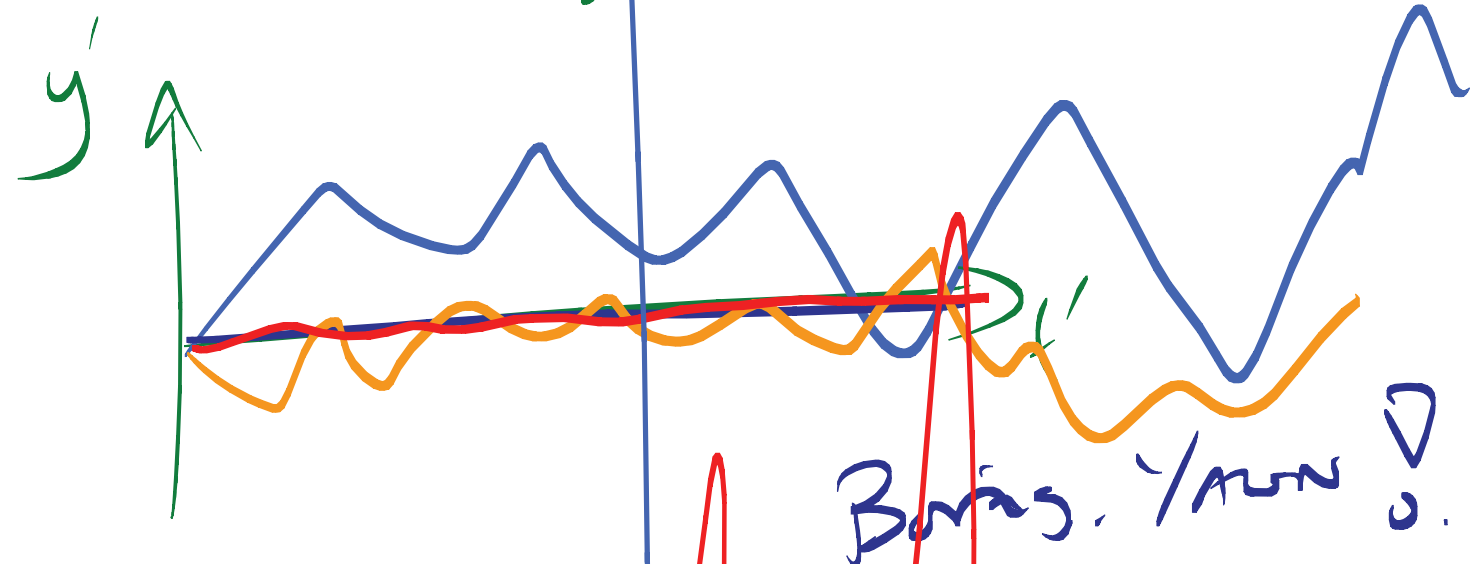
$$c \rightarrow \infty$$

3) $\delta y = O(\sqrt{\Delta t})$

$$c^2 = \text{finite}$$

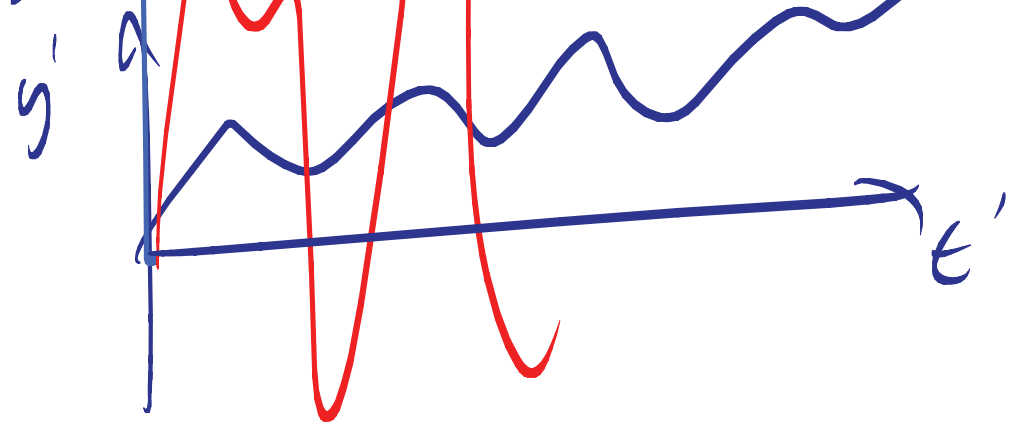
1.

$c \rightarrow 0$. $\delta y \rightarrow 0$ δt , $\delta t \rightarrow 0$.



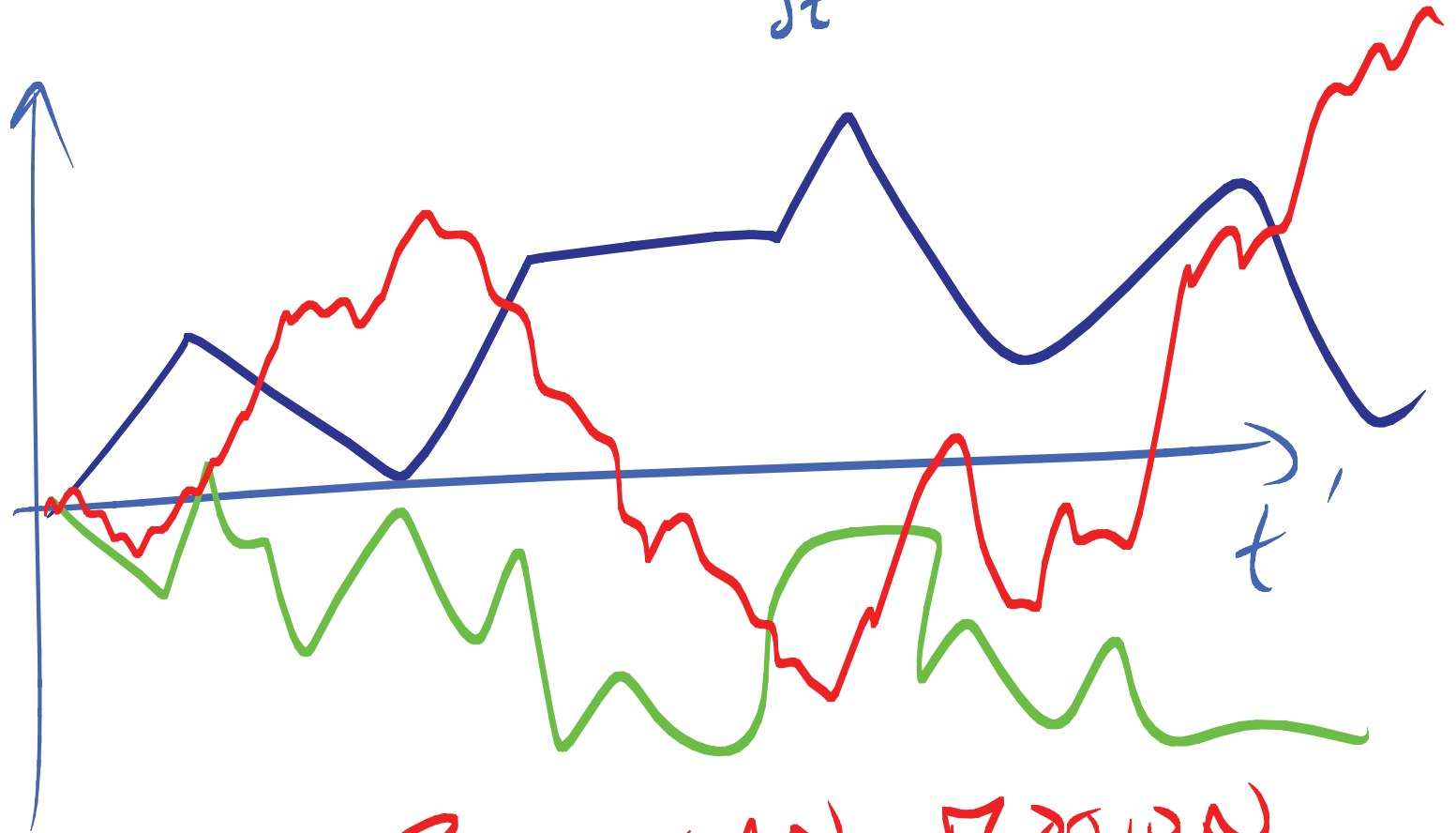
2.

$c \rightarrow \infty$. $\delta t \rightarrow 0$



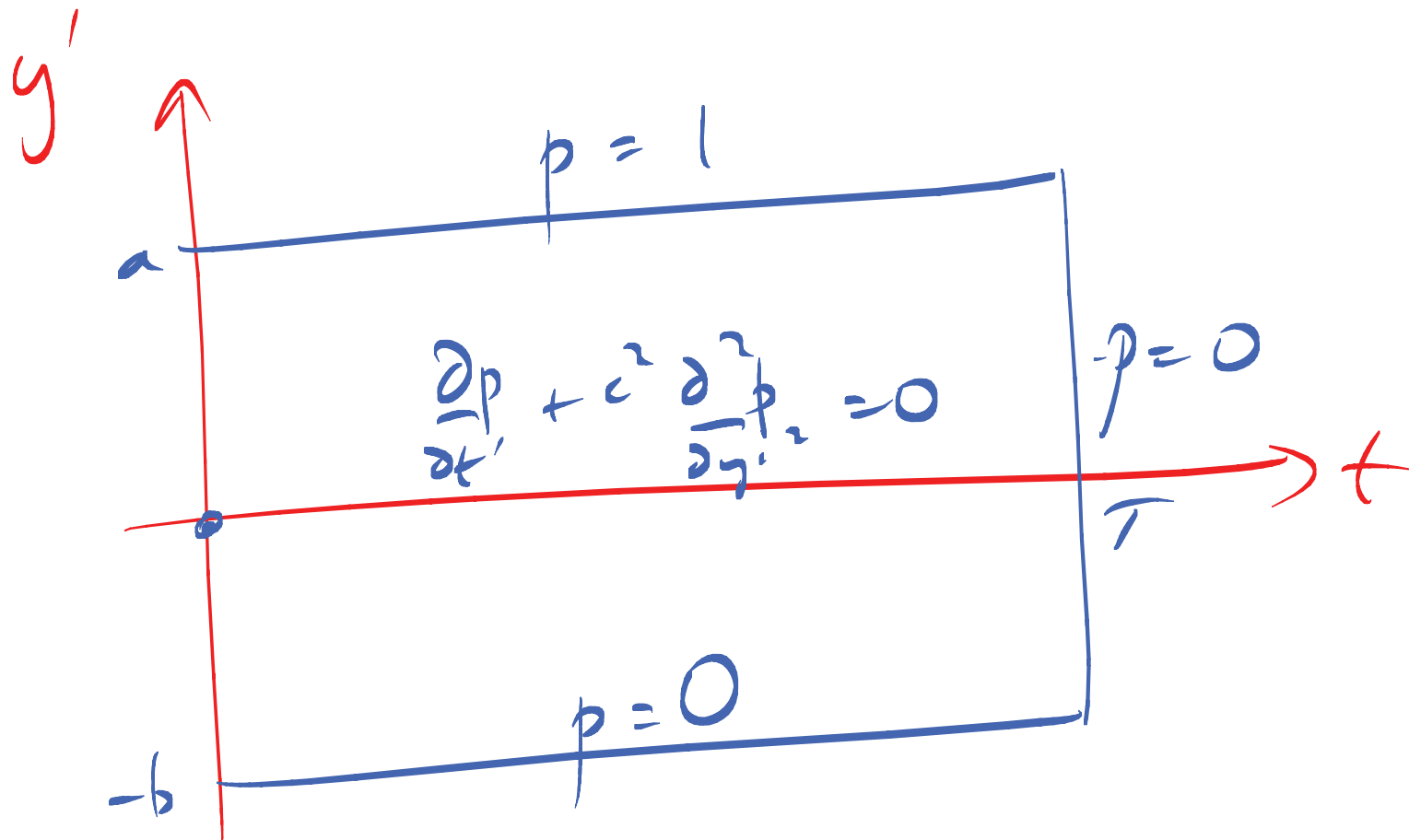
3. Distinguished line

$$\frac{\delta y}{\delta t} = f(x)$$



BROWNIAN MOTION

Sy = $O(\sqrt{st})$



$$p = \alpha + \beta y'$$

$$p = \beta(y' + b)$$

$$1 = \beta(\alpha + b)$$

$$\beta = \frac{1}{\alpha + b}$$

$$p = \frac{y' + b}{\alpha + b}$$

y'
a

$p = 1$

~~$$\frac{\partial p}{\partial \alpha'}$$~~

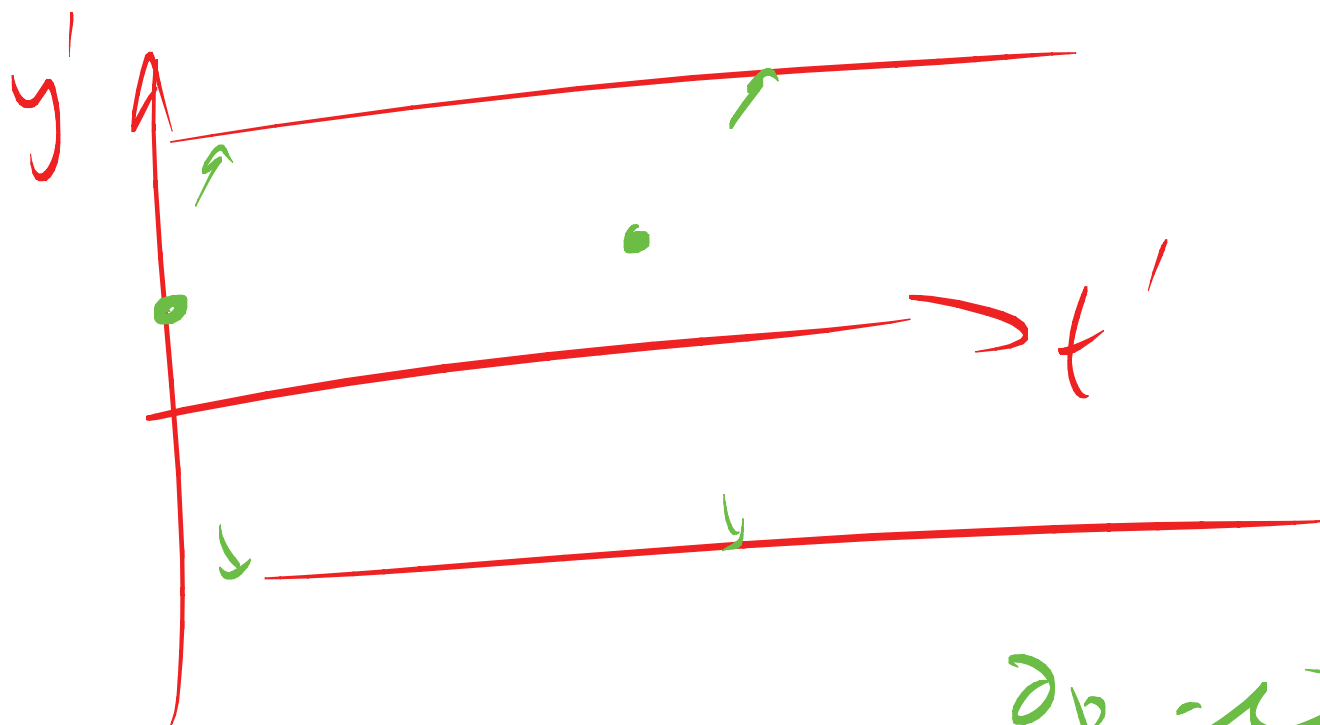
+

$$\frac{\partial^2 p}{\partial y'^2} = 0$$

t'

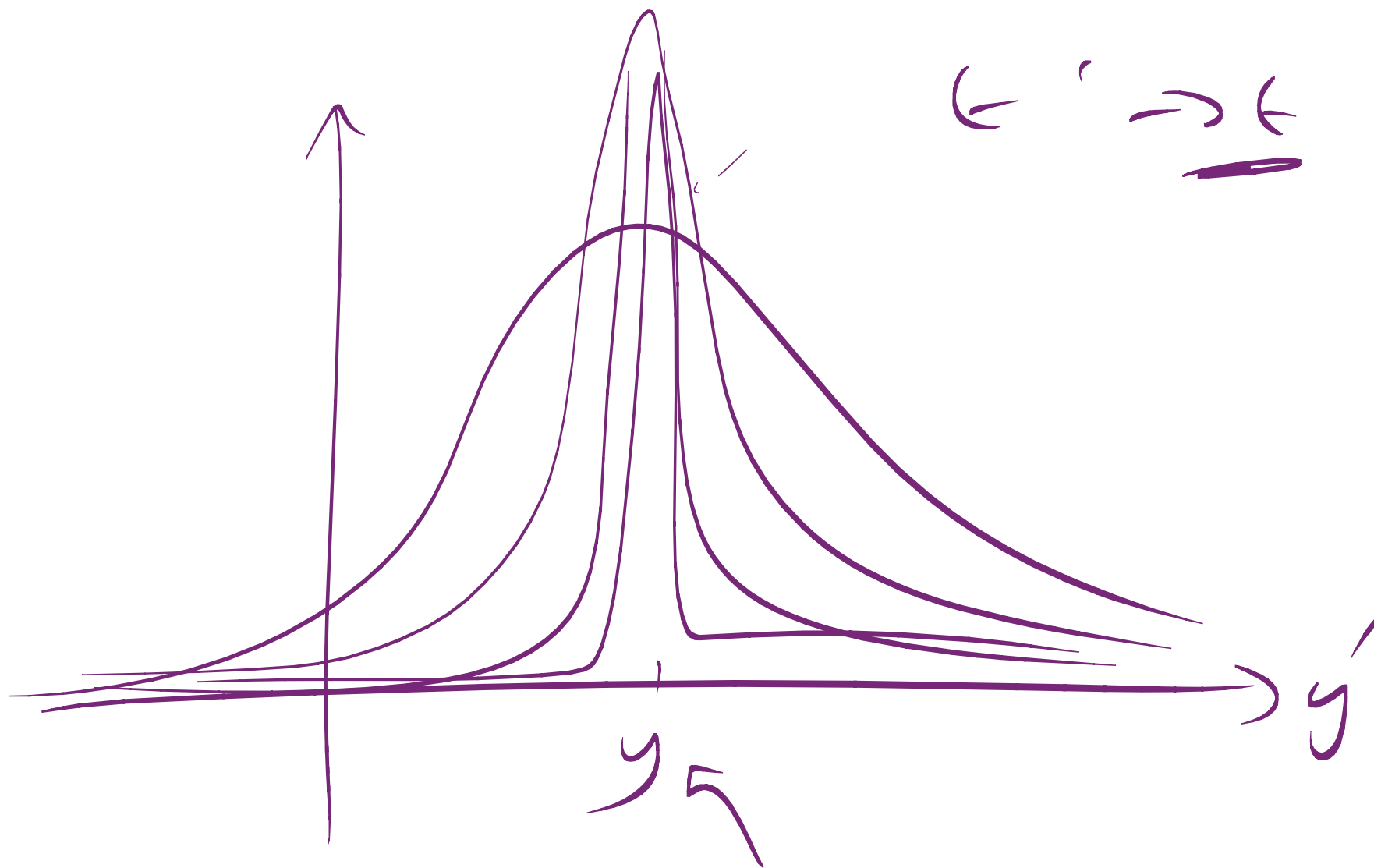
$p = 0$

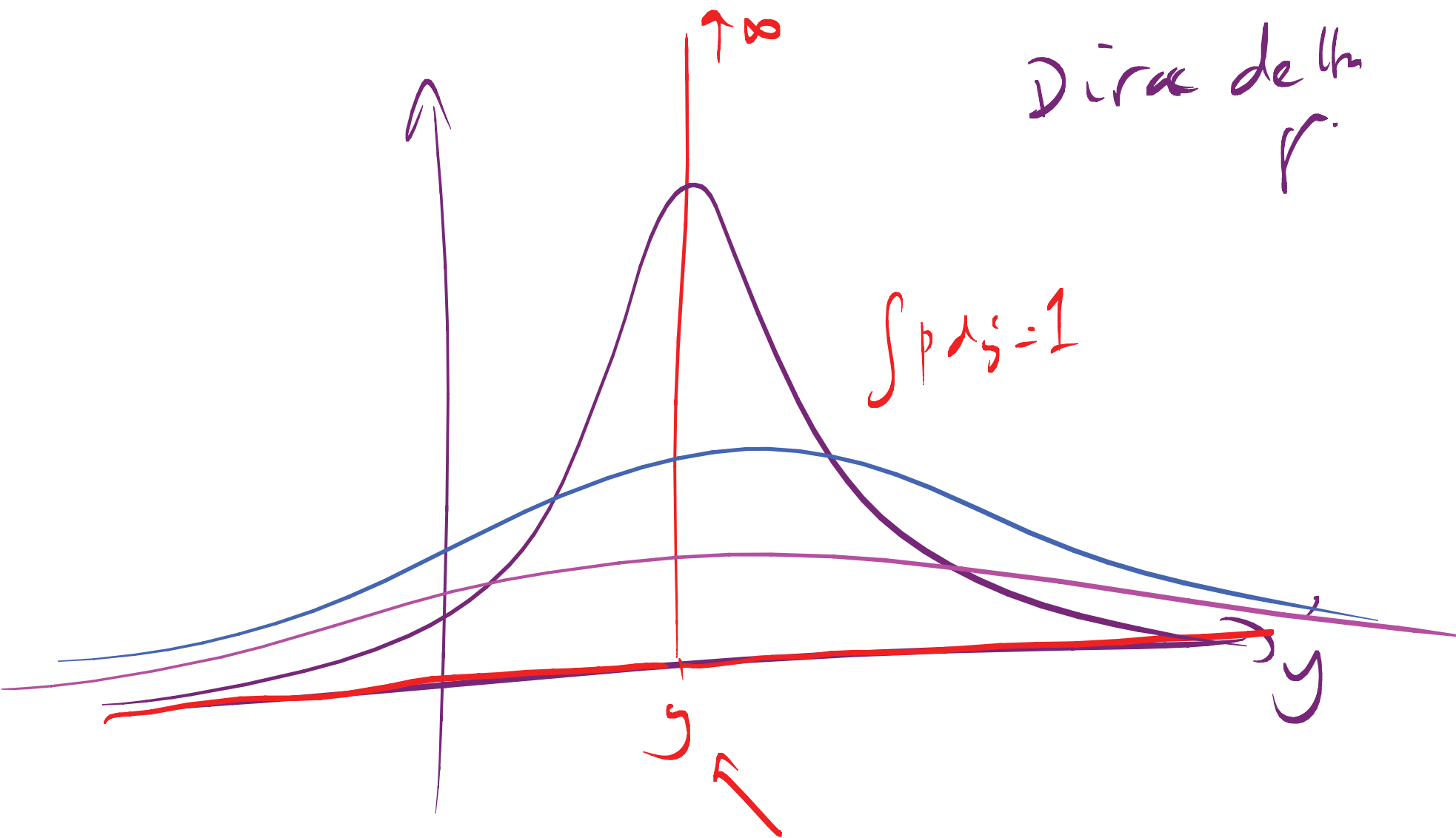
$\frac{b}{\alpha + b}$



$$\frac{\partial p}{\partial t'} = 0$$

"steady-state
soln"





Dirac delta
 δ .

$$\int p \delta y = 1$$



