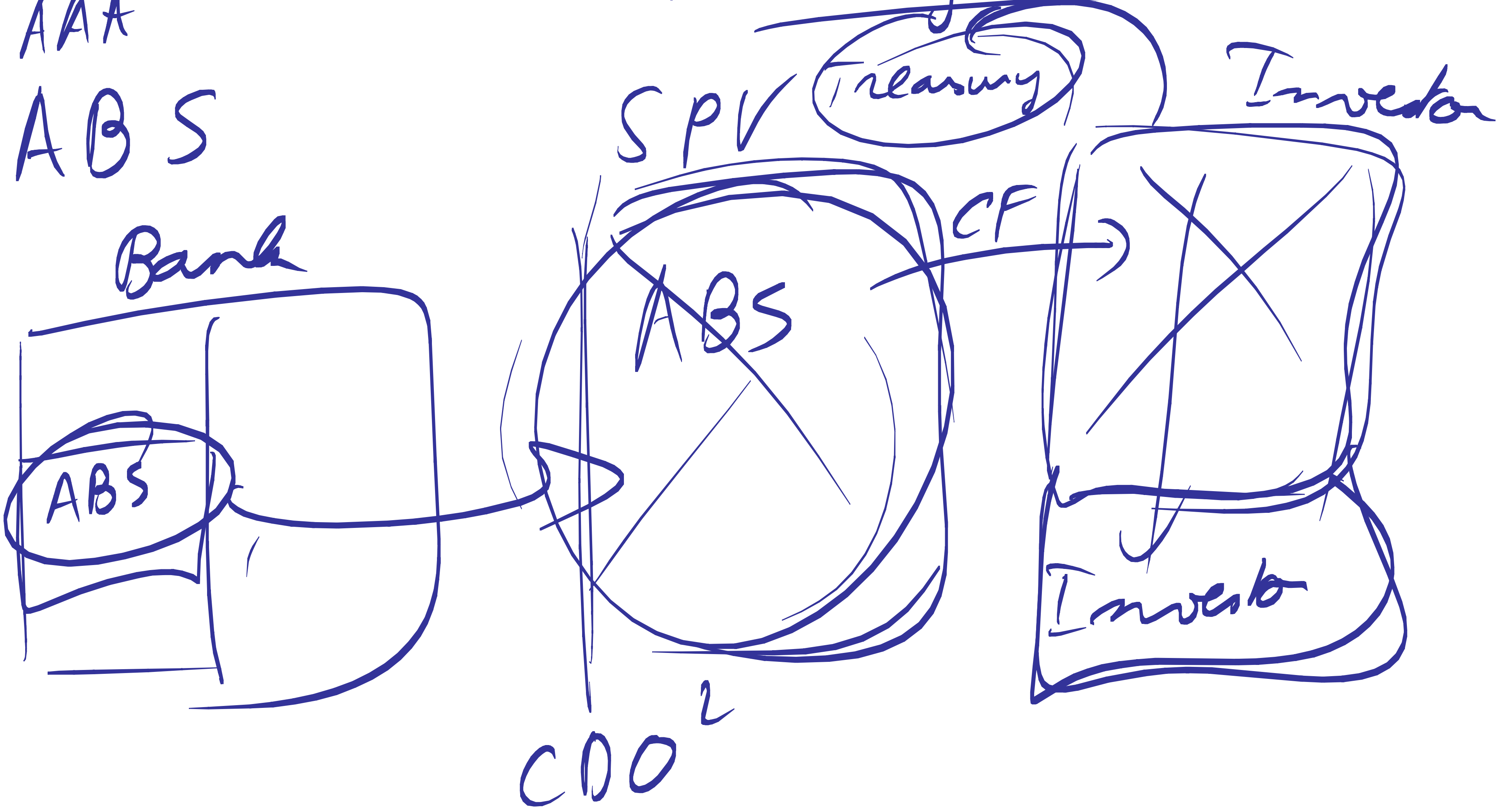
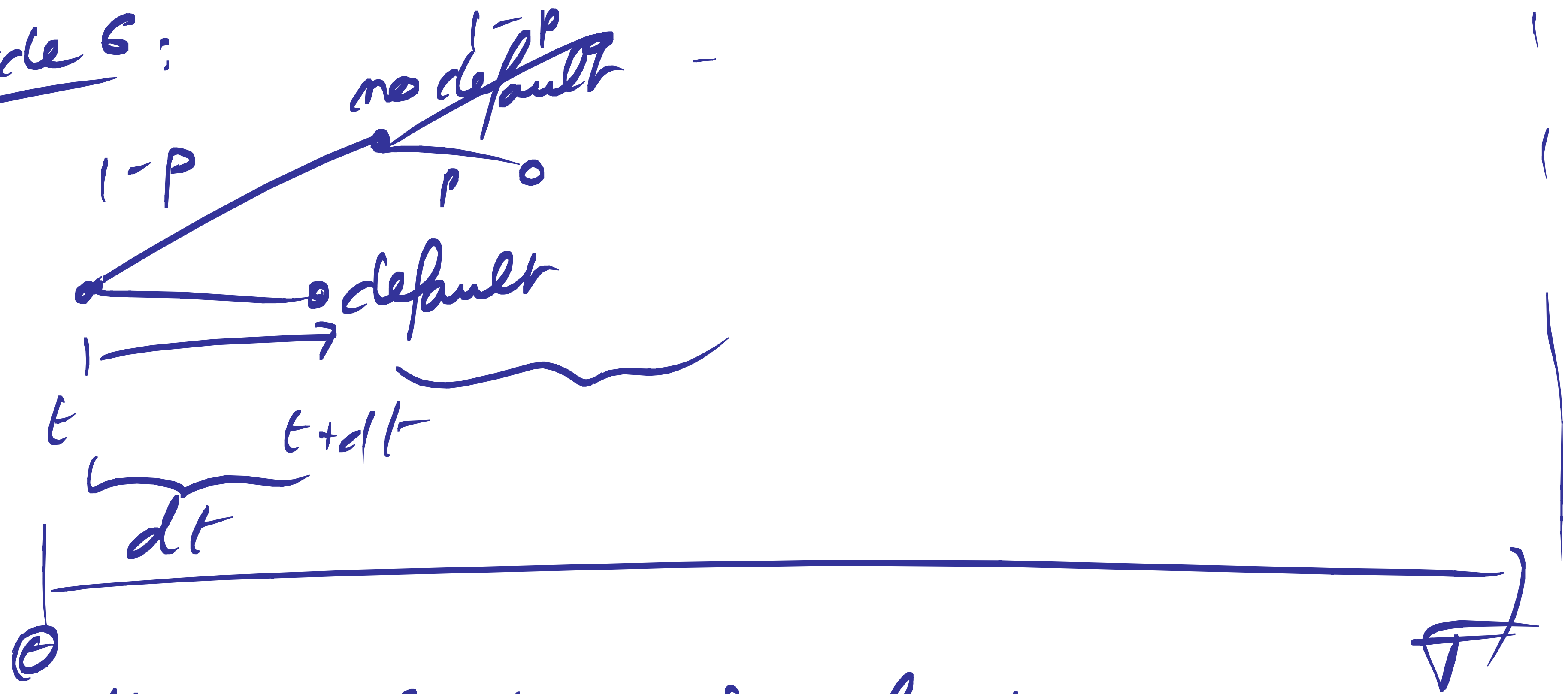
TALF → Term ABS Loan (1 year) Facility

Pay interest.

AAA ABS



Slide 6:

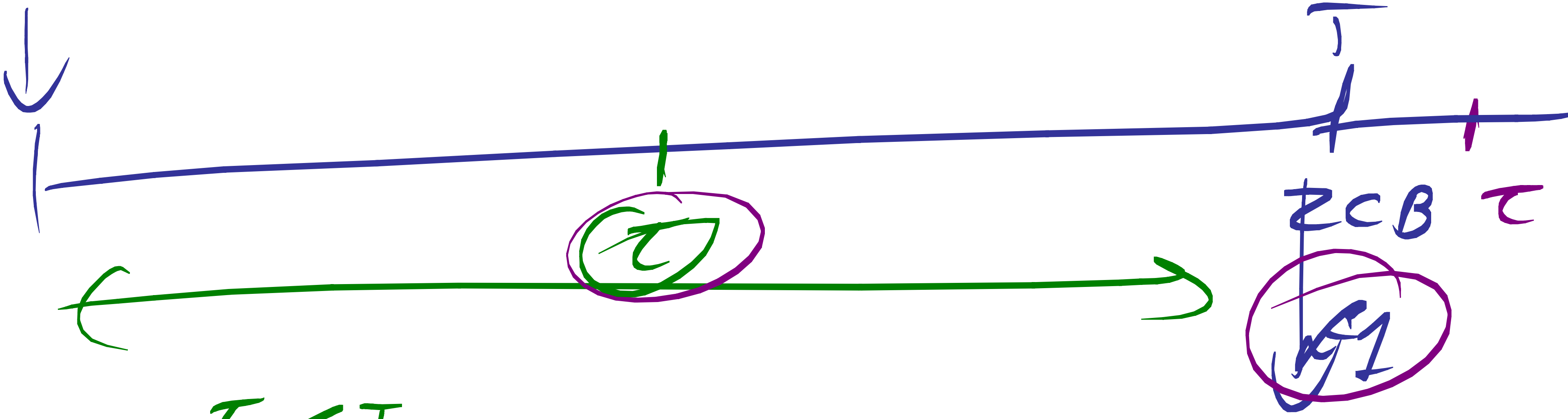


different but equivalent:

default as a time : $\tau \to$ RV

$\tau \in (0, \infty)$

$\tau \to$ default time



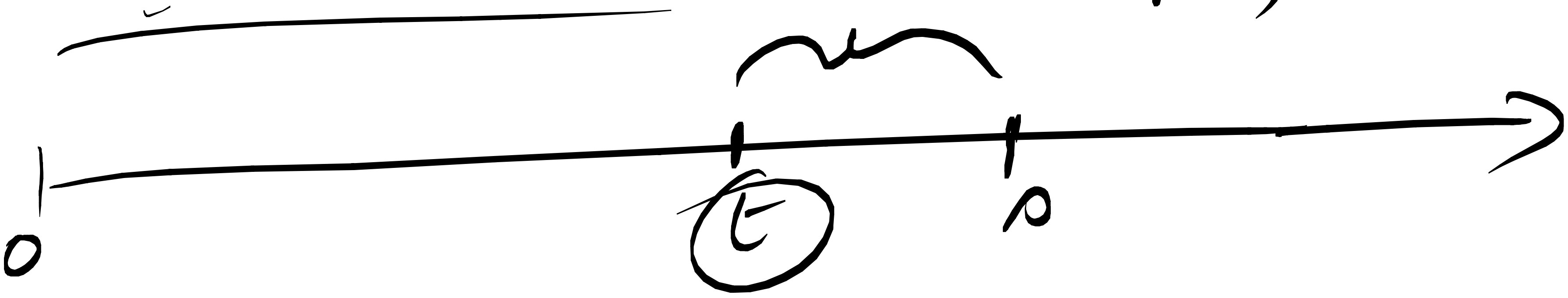
$\tau < T$

issuer default during the life of the Bond

$\tau > T$

issuer defaults but after maturity of the Bond.

$$p(s|t) = \frac{p(s)}{p(t)}$$



<u>Next question</u>: what happens next?

- issuer has survived until time t.
- what is the probability of survival up to time s, $s > t$.

$$p(s \mid t) = \frac{p(\text{"s" and "t"})}{p(t)}$$

$$p(\text{"s" and "t"}) = p(s)$$

$$\boxed{p(s \mid t) = \frac{p(s)}{p(t)}}$$

Slide 8:

$$f(t, s) = 1 - p(s|t) = 1 - \frac{p(s)}{p(t)}$$

Fix t and take the derivative w.r.t s.

$$\frac{df}{ds} = - \frac{\frac{dp(s)}{ds}}{p(t)} \qquad = \lim_{s \to t} \frac{df}{ds} = \frac{-\frac{dp(t)}{dt}}{p(t)}$$

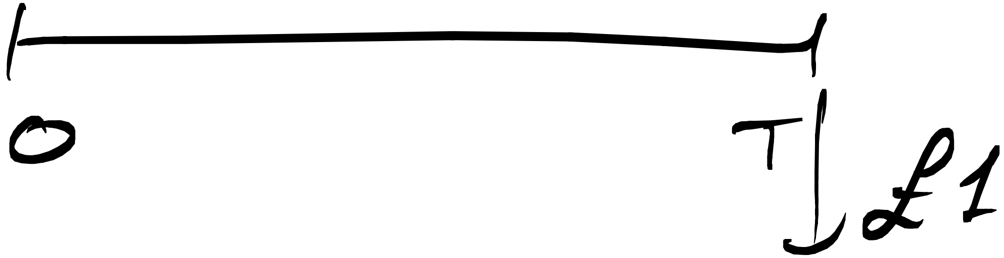
$f(t)$

Take the limit as $s \to t$ and define the instantaneous forward default rate

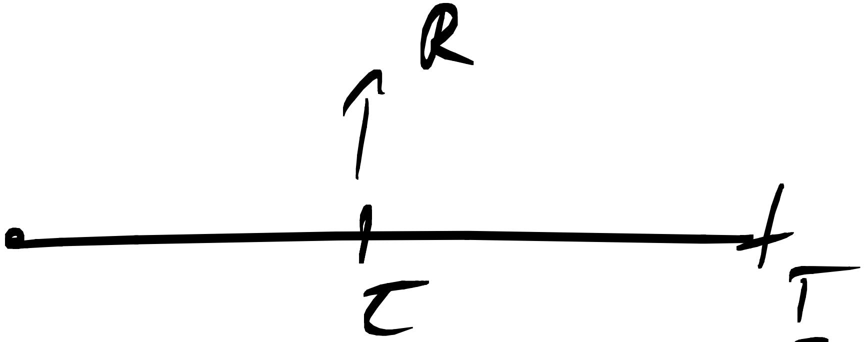
$$f(t) = \lim_{s \to t} \frac{df}{ds} = \frac{-\frac{dp(t)}{dt}}{p(t)}$$

Separable ODE

ZCB



① Case 1 $\tau < T$



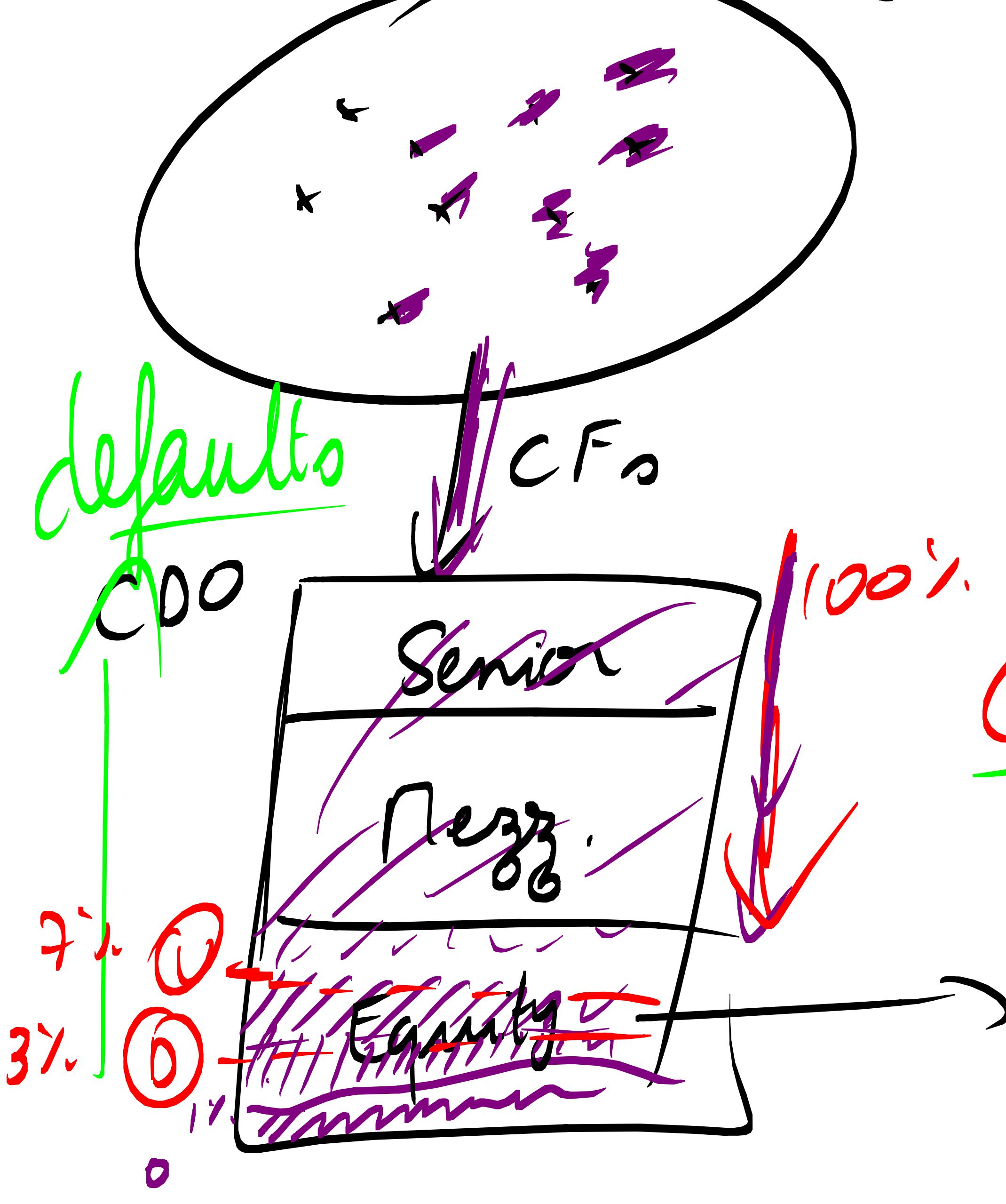
$$V(S_T) = \mathbb{E}^{Q}\left[e^{-\int_0^{\tau} r(s)ds} R \right]$$

that's if $\tau < T$

② $\tau > T$

$$V(S_t, T) = \mathbb{E}^{Q}\left[e^{-\int_0^{T} r(s)ds} 1 \right]$$

Pool of Collateral 80 - 100 securities.



CF

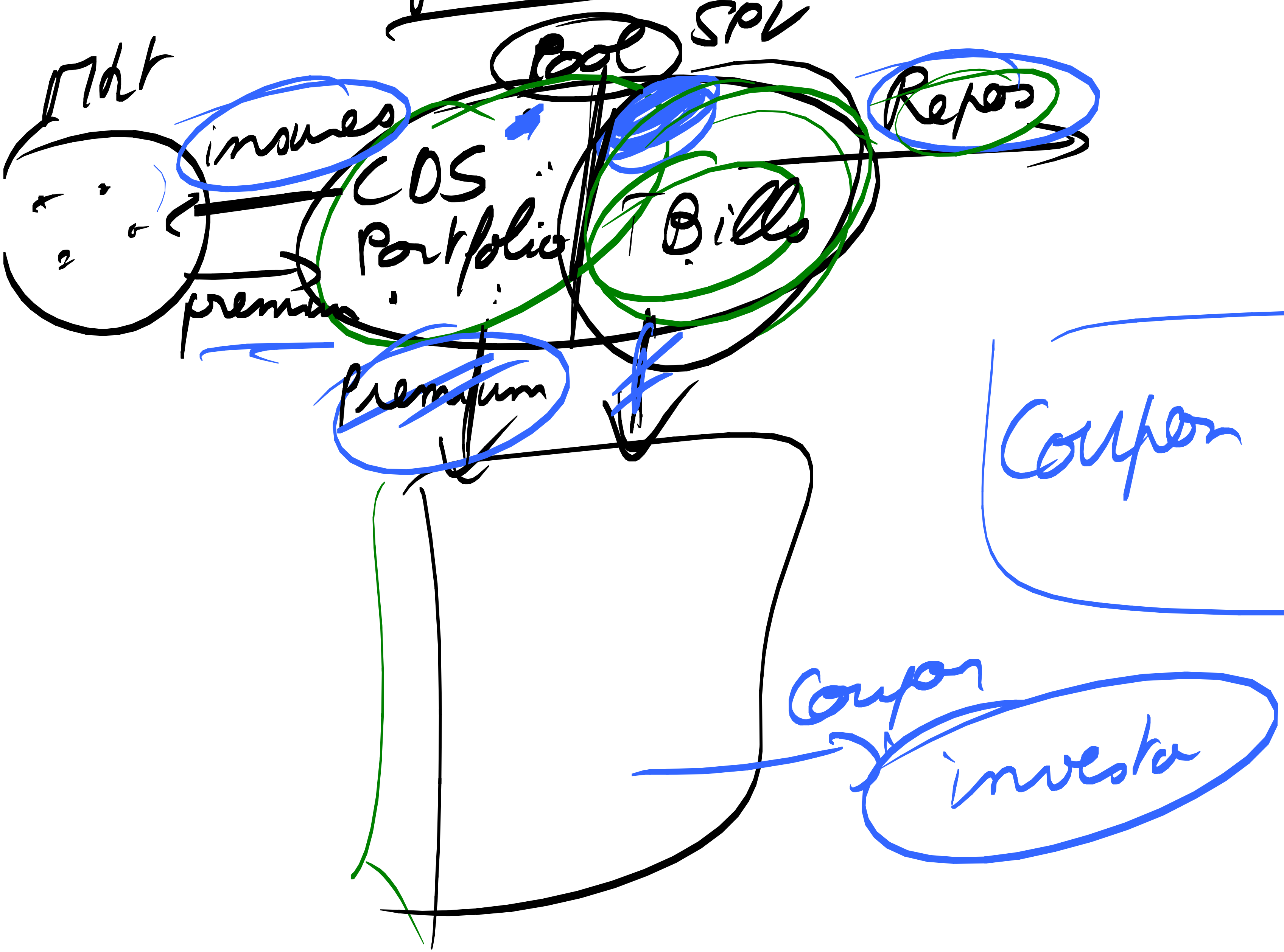
investor

- IR and default rates are independent
- Recoveries
- Each Tranche Pays a coupon:

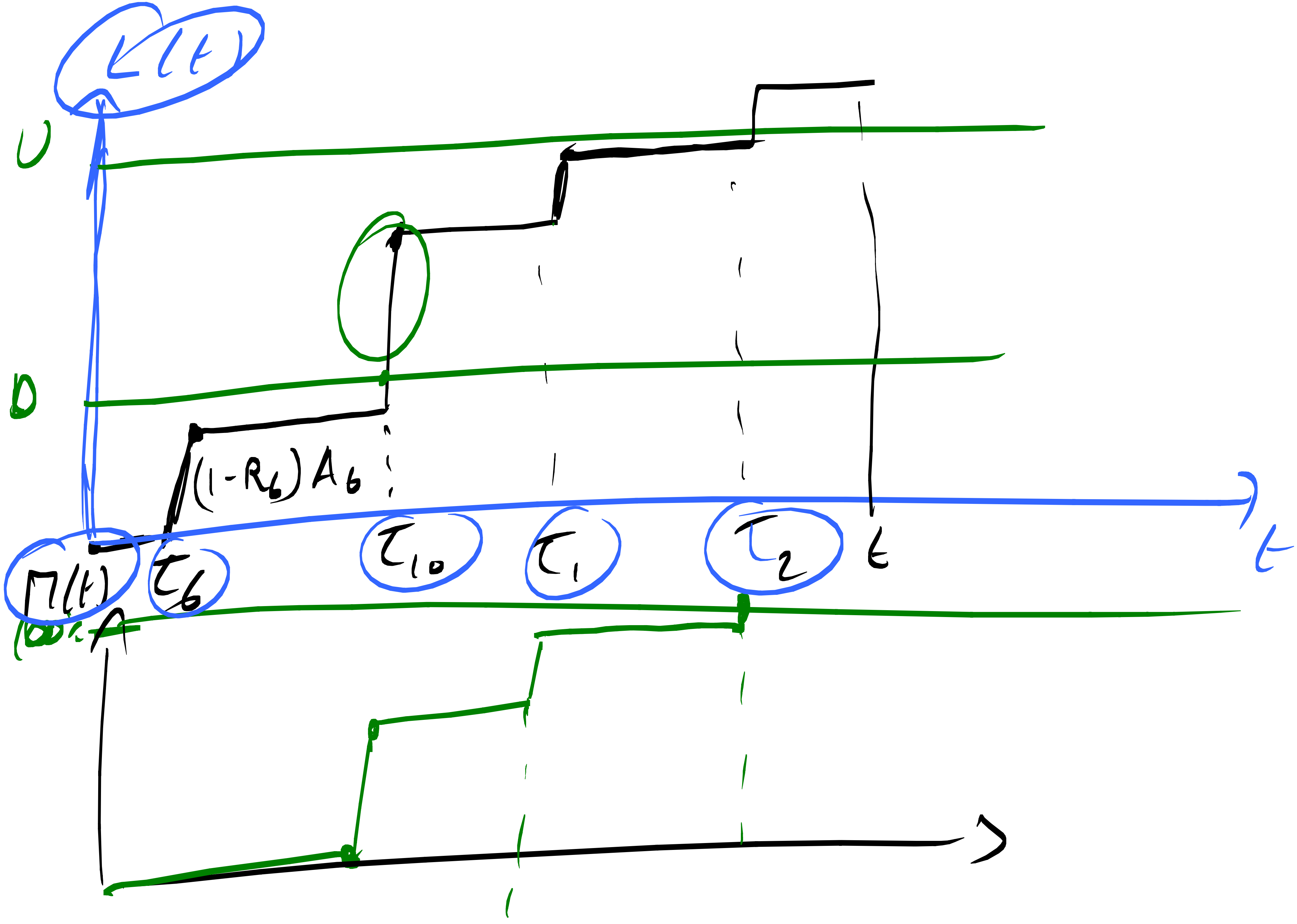
Coupon Rate = LIBOR + Spread

Synthetic CDOs :

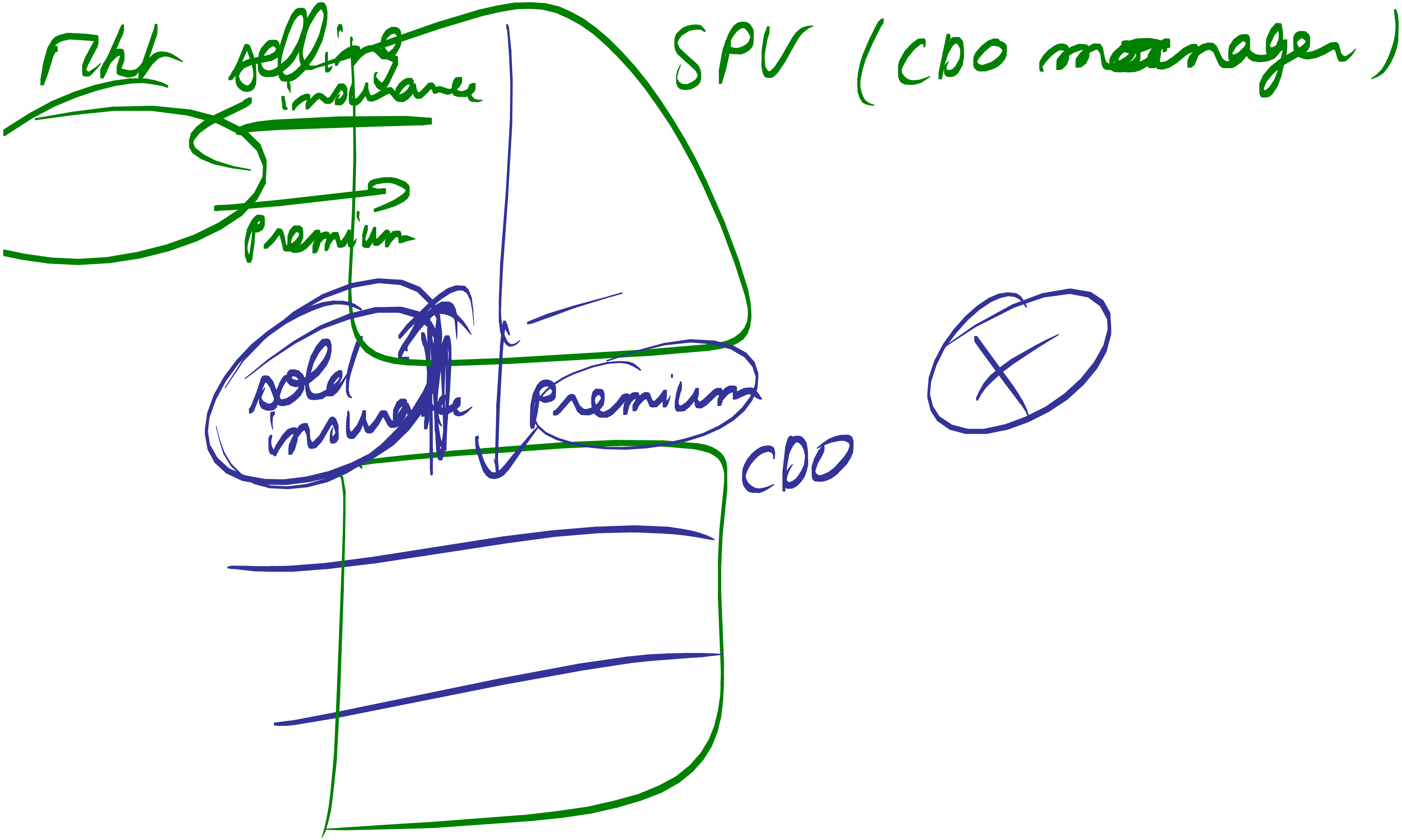
"Risky Bond" = "Treasury note + CDS"



Coupon = LIBOR + Spread



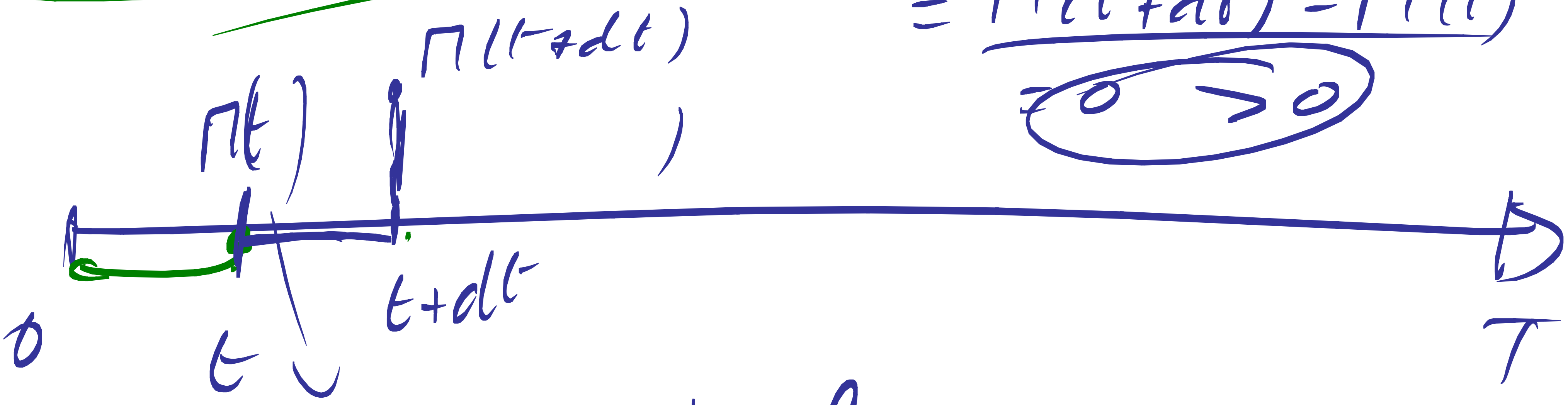
L(t)
U
D
(1-R_6)A_6
t
τ_6
τ_10
τ_1
τ_2
t
t
Π(t)



Fiht
selling insurance
premium
SPV (CDO manager)
sold insurance
premium
CDO

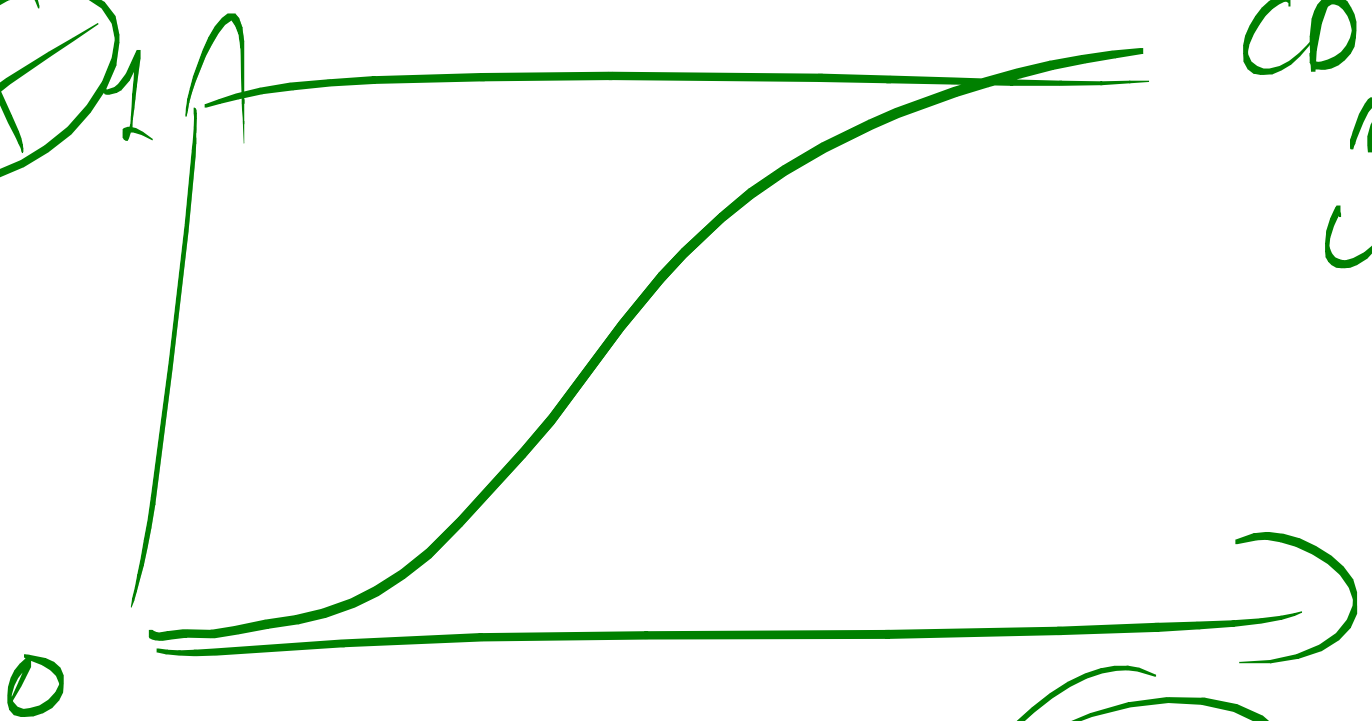
$$\int_0^T B(0,t)\, d\Pi(t) \longleftarrow \sum_0^T B(0,t)\, \Delta\Pi(t, t+dt)$$

$$= \underbrace{\Pi(t+dt) - \Pi(t)}_{\geq 0 \quad > 0}$$



Loss on our tranche

X



CDF

marginal

↳ just 1 R.V.

X, Y

joint → multiple R.V.

$X, Y \qquad X \sim \mathcal{N}(\mu_X, \sigma_X^2) \qquad Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$

CDF for X

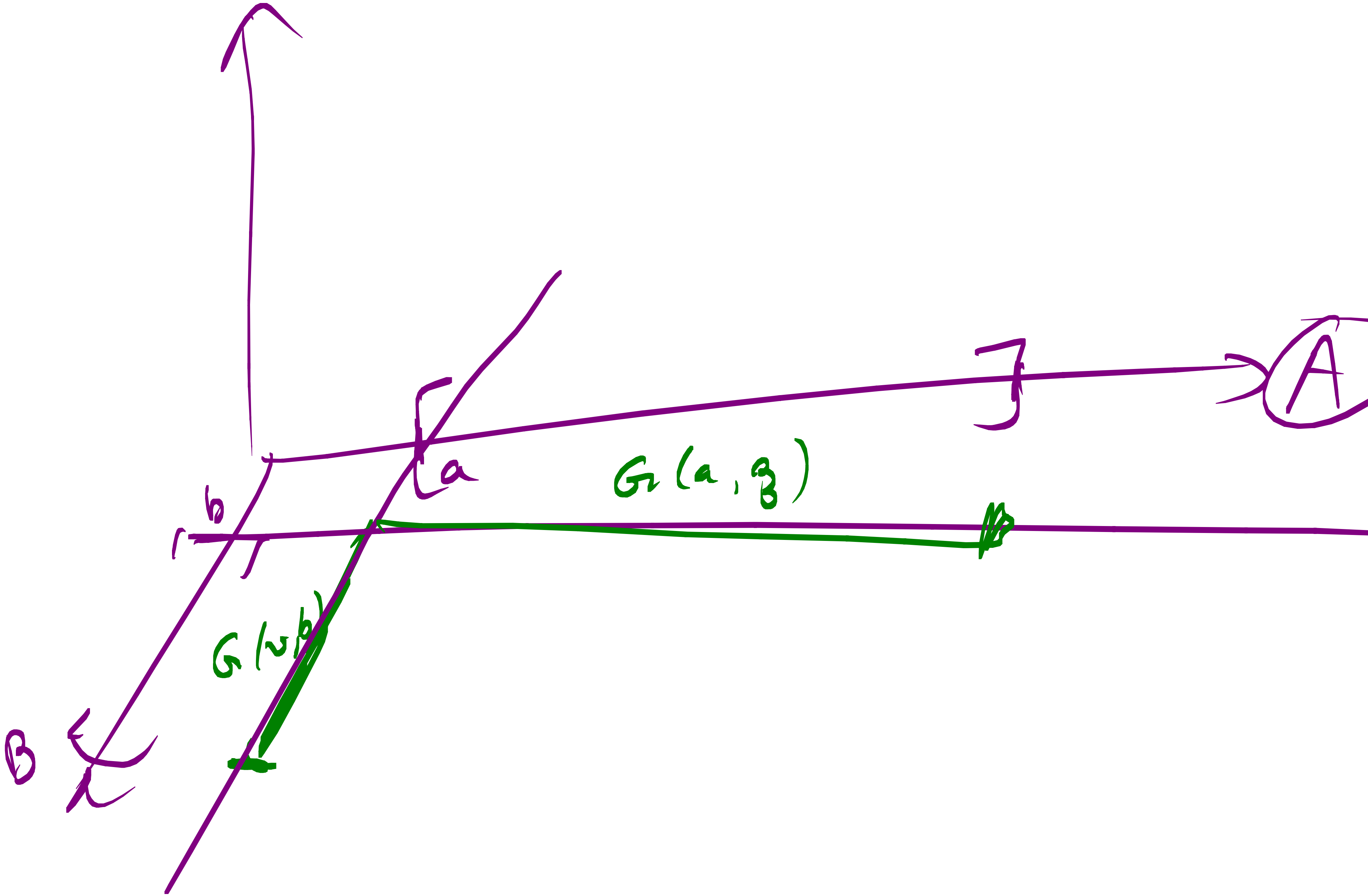
$$P[X \le x] = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\frac{(x-\mu_X)^2}{\sigma_X^2}}\, dx$$

$$P[Y \le y] = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\frac{(y-\mu_Y)^2}{\sigma_Y^2}}\, dy$$

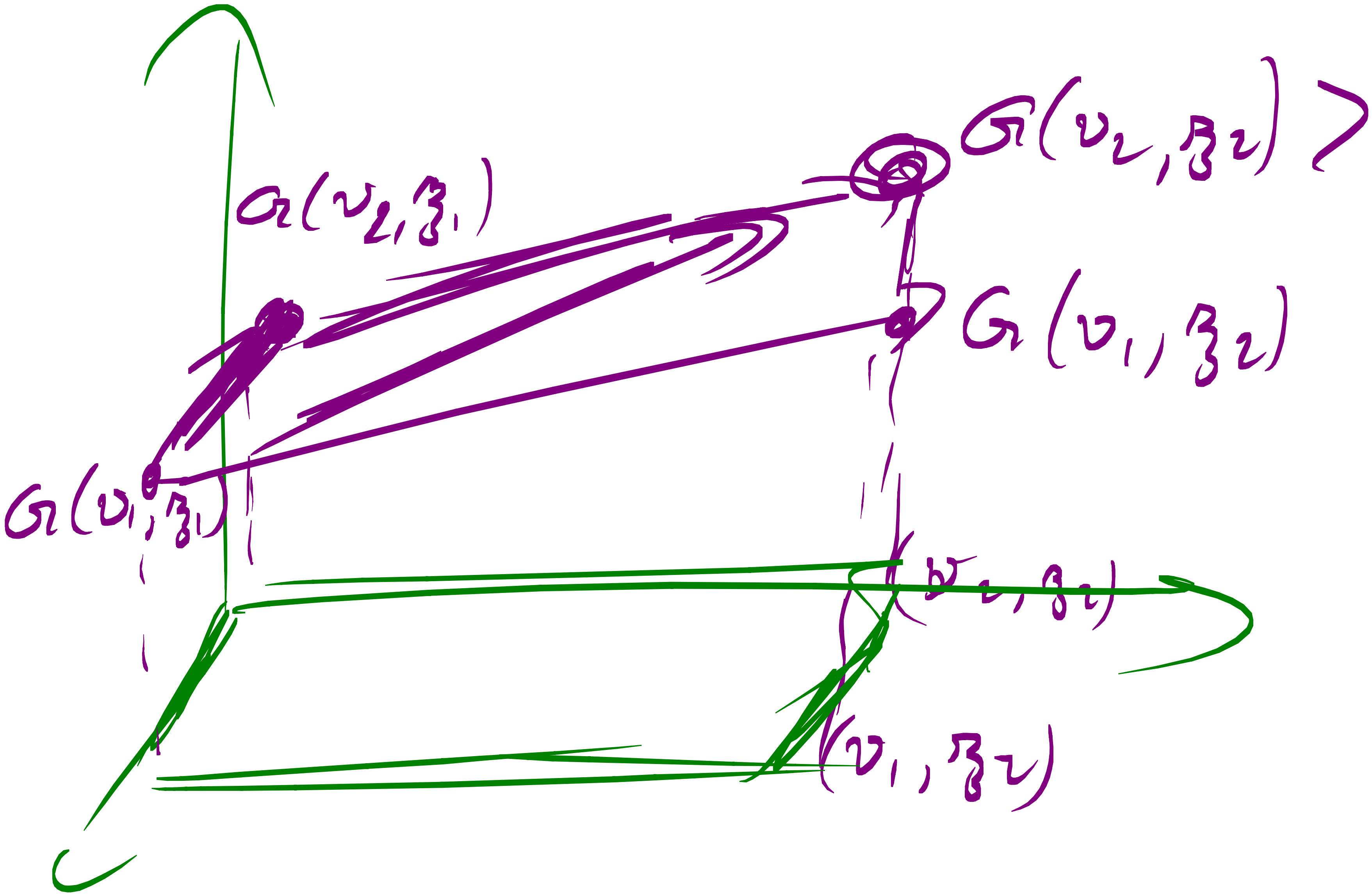
X, Y jointly Normally distributed $\to \rho$

joint CDF

$$P[X \le x, Y \le y] = \int_{-\infty}^{x}\int_{-\infty}^{y} \frac{1}{2\pi\sqrt{1-\rho^2}\,\sigma_X\sigma_Y} \times \exp\left\{-\frac{1}{2}\frac{(x-\mu_X)^2}{\sigma_X^2} - \frac{1}{2}\frac{(y-\mu_Y)^2}{\sigma_Y^2} - \frac{\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y}\right\} dy\,dx$$

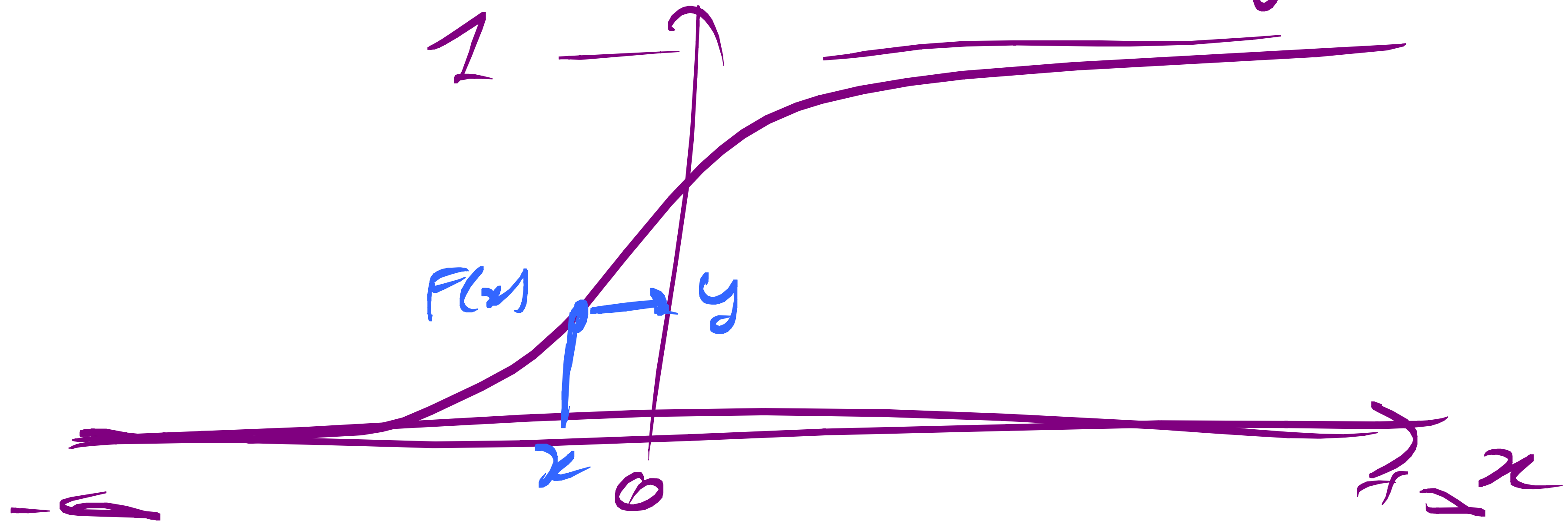


A
B
a
b
G(a,β)
G(α,b)



$G(v_2, \zeta_1)$
$G(v_2, \zeta_2)$
$G(v_1, \zeta_2)$
$G(v_1, \zeta_1)$
(v_2, ζ_2)
(v_1, ζ_2)

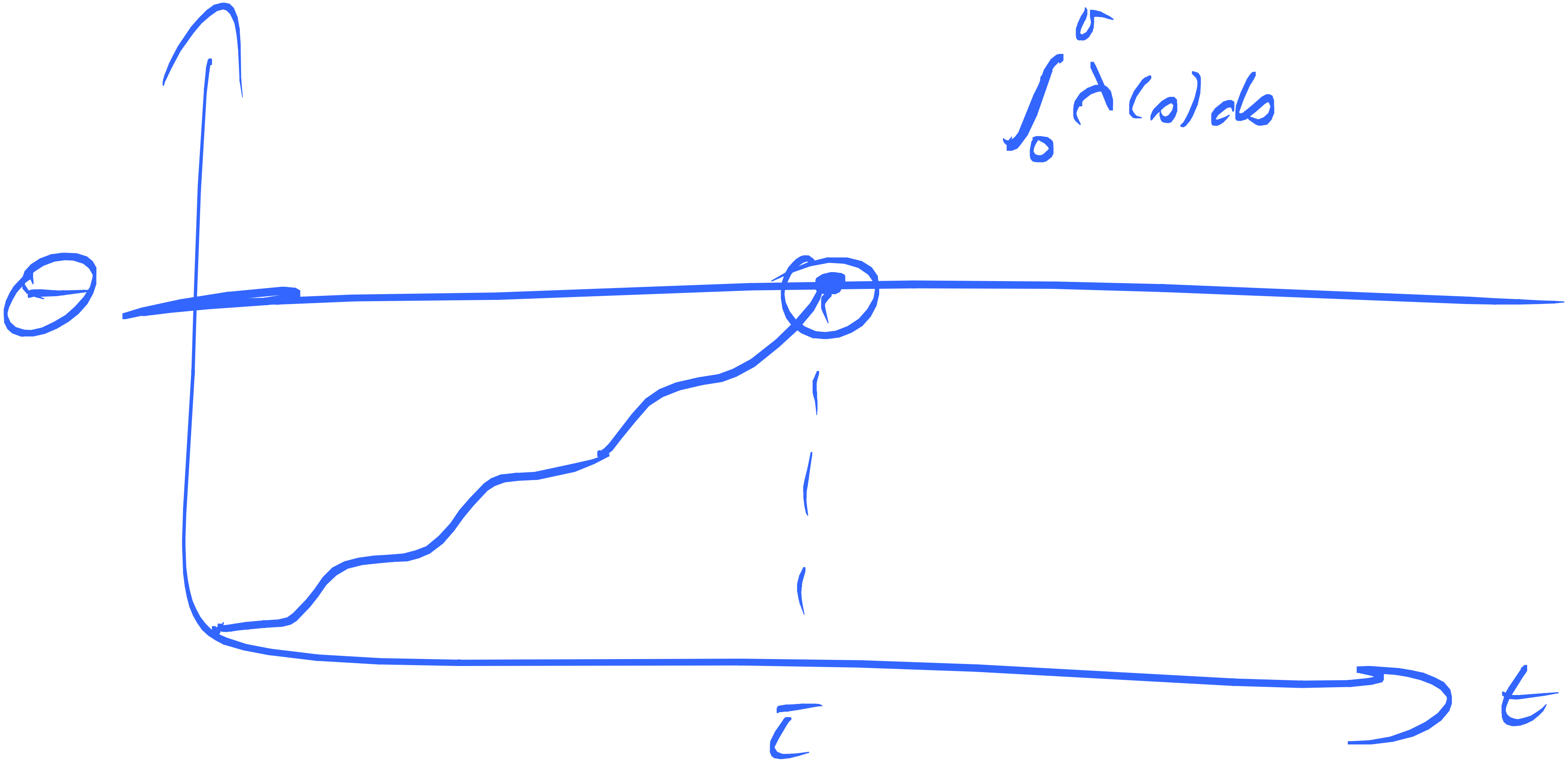
CDF $\quad x \in \mathbb{R} \to y \in [0, 1)$



$X \sim \mathcal{N}(0,1)$ $\qquad P[X \leq x] = F(x)$

$$P(U \leq y) = P\left(F^{-1}(U) \leq F^{-1}(y)\right) = y.$$

$$= P\left(F^{-1}(U) \leq x\right) = P[X \leq x]$$



θ
$\int_0^t \lambda(s)\,ds$
τ
t

$$\tau_1 = 1$$

$$\tau_2 = 0.5$$

$$\tau_3 = 0.75$$