Module 1.3 Solutions Erratum

3. a) Show that

$$\int_0^t X(\tau)\, dX(\tau) = \frac{1}{2}X^2 - \frac{1}{2}t$$

Solution: We use the stochastic integral formula for $F(X(t))$ given by

$$\int_0^t \frac{dF}{dX} dX(\tau) = F(X(t)) - F(X(0)) - \int_0^t \frac{1}{2}\frac{d^2F}{dX^2} d\tau$$

$$\frac{dF}{dX} = X(t) \Rightarrow F = \frac{1}{2}X^2(t) \;\; \& \;\; F'' = 1$$

substituting in formula gives

$$\int_0^t X(\tau)\, dX = \frac{1}{2}\left[X^2(t) - X^2(0)\right] - \frac{1}{2}\int_0^t 1 d\tau$$

which can simplified because we know $X(0) = 0$ so

$$\int_0^t X(\tau)\, dX = \frac{1}{2}X^2(t) - \frac{t}{2}$$