CQF Exercises 4.3 Calibration

1. Very briefly outline the difference between (one factor) *equilibrium* and *no-arbitrage* models for the spot rate.

2. Substitute the fitted function for $A(t\,;T)$, using the Ho & Lee model, back into the solution of the bond pricing equation for a zero-coupon bond,

$$Z(r,\ t\,;T) = \exp\left(A(t\,;T) - r(T-t)\right).$$

   The form for $A(t\,;T)$ can be found on page 11 of the lecture notes. What do you notice when $t = t^*$?

3. Differentiate Equation (2) on page 16 of the lecture notes, twice to solve for the value of $\eta^*(t)$. What is the value of a zero-coupon bond with a fitted Vasicek model for the interest rate?

4. Use spot rate data to find ν and β if we assume that interest rate movements are of the form

$$dr = u(r)\,dt + \nu r^\beta dX.$$

   Does your estimated value of β lie close to that of any of the standard models? (Use any finance based website to download interest rate data for this question).

5. In problem sheet 4.2 we derived a BPE which gave zero coupon bonds of the form

$$V(r,t) = \exp\left(A(t) + rB(t)\right)$$

   where $A(t)$ was

$$A(t) = -\int_t^T \left[a(s)(s-T)\right] ds - \frac{(t-T)^3}{6}.$$

   Suppose at time t^* bond prices are given for a continuous range of maturities, T, so that

$$V(r^*, t^*; T)$$

   is known as a function of T. r^* is the spot rate at time t^*.
   Hence determine $a(T)$ in terms of

$$\frac{\partial^2}{\partial T^2}\left(\log V(r^*, t^*; T)\right).$$