

CQF 2009
Module 2.6
Live Lecture: February 23
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Fundamentals of Optimization and Application to Portfolio Selection

In this lecture:

- how to formulate an optimization problem
- how to use calculus to solve unconstrained optimization problems
- the method of Lagrange
- the Kuhn-Tucker conditions
- how to select a portfolio in the Markowitz world
- the minimum variance portfolio
- portfolio selection with a risk-free asset
- how to derive the CAPM.

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