CQF Module 2.2 Solutions

1. Consider a 12-month period over which the variable continuously compounded interest rate r is given by

$$r(t) = \begin{cases} 5.3\% & 0 \le t < 3 \text{ months} \\ 4\,\% & 3 \le t < 6 \text{ months} \\ 6.1\% & 6 \le t \le 12 \text{ months} \end{cases}$$

Calculate the value of £1000 in 8 months' time.

Future value S at time T is given by

$$S(T) = S(0)\exp\left(\int_0^T r(t)\,dt\right)$$

where $r(t)$ is the interest rate (per annum), and $S(0)$ the money at $t = 0$ (i.e. £1000). Note, we are working in months (as a fraction of a year). So

$$S(2/3) = 1000\exp\left(\int_0^{\frac{2}{3}} r(t)\,dt\right) = 1000\exp\left(\int_0^{0.25} r(t)\,dt + \int_{0.25}^{0.5} r(t)\,dt + \int_{0.5}^{\frac{2}{3}} r(t)\,dt\right)$$

$$= 1000\exp\left(0.25 \times 5.3\% + 0.25 \times 4\% + \frac{1}{6} \times 6.1\%\right) \simeq £1034$$

2. How much money would I invest today to guarantee £5000 in six months' time if the continuously compounded rate of interest is 5%?

The money today is less than its value in 6 months time, so we use the discount factor (with a minus sign) to calculate the present worth of £5k $S(0) = S(0.5)\exp\left(-\int_0^T r(t)\,dt\right) \to S(0.5) = 5000\exp\left(-\int_0^{0.5} 5\%\,dt\right) \to$

$5000\exp(-0.5 \times 5\%) = £4876.54$

3. Find the values of the following portfolios of options at expiry, as a function of the share price:

(a) Long one share, long one put with exercise price E.

$$\Pi(S) = \begin{cases} E & \text{if } 0 \le S \le E \\ S & \text{if } \quad S > E \end{cases}$$

(b) Long one call and one put, with exercise price E.

$$\Pi(S) = \begin{cases} E - S & \text{if } 0 \le S \le E \\ S - E & \text{if } \quad S > E \end{cases}$$

(c) Long one call, exercise price E_1, short one call, exercise price E_2, where $E_1 < E_2$.

$$\Pi(S) = \begin{cases} 0 & \text{if } \ 0 \le S \le E_1 \\ S - E_1 & \text{if } E_1 < S \le E_2 \\ E_2 - E_1 & \text{if } \quad S > E_2 \end{cases}$$

(d) Long one call, exercise price E_1, long one put, exercise price E_2. There are three cases to consider.

When $E_1 > E_2$ $\quad \Pi(S) = \begin{cases} E_2 - S & \text{if } \ 0 \le S \le E_2 \\ 0 & \text{if } E_2 < S \le E_1 \\ S - E_1 & \text{if } \quad S > E_1 \end{cases}$

When $E_1 = E_2$ $\quad \Pi(S) = \begin{cases} E_1 - S & \text{if } 0 \le S \le E_1 \\ S - E_1 & \text{if } \quad S > E_1 \end{cases}$

When $E_1 < E_2$ $\quad \Pi(S) = \begin{cases} E_2 - S & \text{if } \ 0 \le S \le E_1 \\ E_2 - E_1 & \text{if } E_1 < S \le E_2 \\ S - E_1 & \text{if } \quad S > E_2 \end{cases}$

(e) Long two calls, one with exercise price E_1 and one with exercise price E_2, short two calls, both with exercise price E, where $E_1 < E < E_2$.

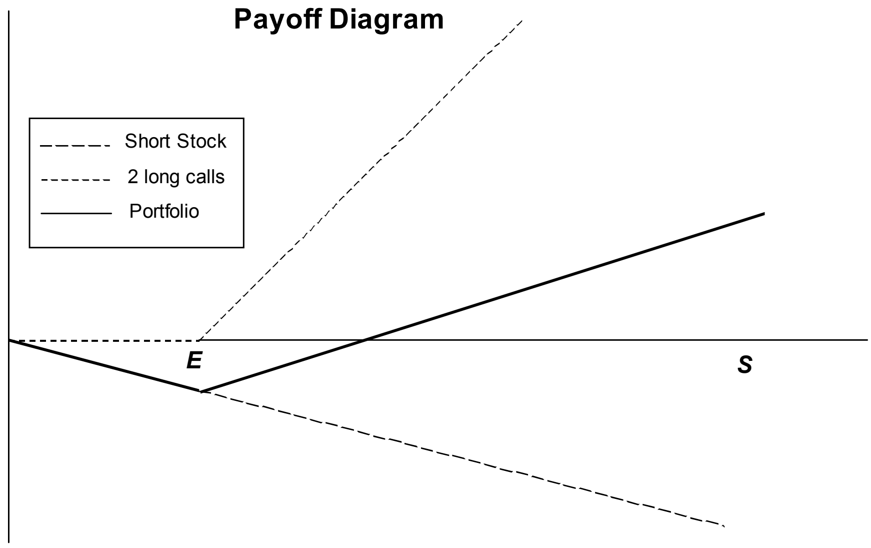
$$\Pi(S) = \begin{cases} 0 & \text{if} \quad 0 \le S \le E_1 \\ S - E_1 & \text{if} \quad E_1 < S \le E \\ 2E - E_1 - S & \text{if} \quad E < S \le E_2 \\ 2E - E_1 - E_2 & \text{if} \quad S > E_2 \end{cases}$$

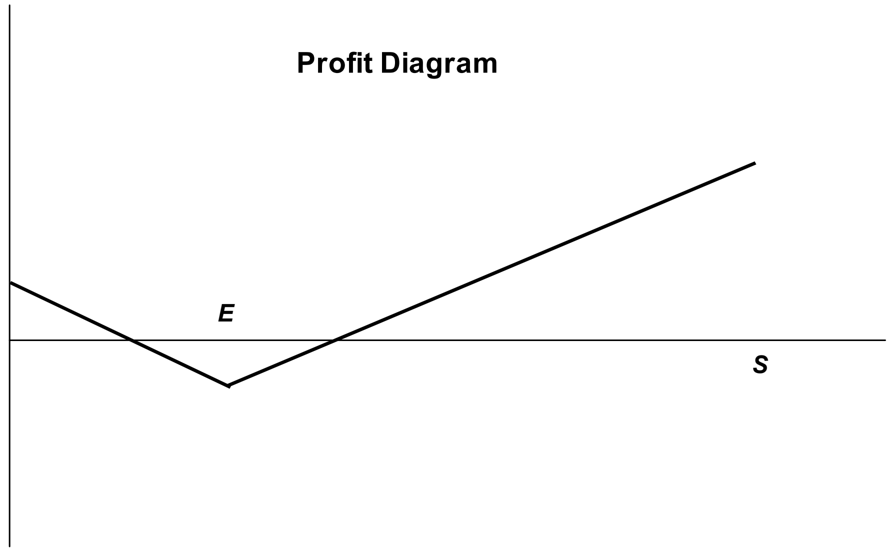
4. What is the difference between a payoff diagram and a profit diagram? Illustrate with a portfolio of short one share, long two calls with exercise price E.

A payoff diagram shows the value of a portfolio as a function of the share price ($\Pi(S)$) at a specified time (usually the expiry date of an options contract in the portfolio).
A profit diagram takes into account the initial cost of setting up the portfolio, by including the present value of this initial cost. To find the value of a portfolio using Black-Scholes, our final condition should be represented by a payoff diagram.
As an example consider the following portfolio which has payoff

$$\Pi(S) = \begin{cases} -S & \text{if} \quad 0 \le S \le E \\ S - 2E & \text{if} \quad S > E \end{cases}$$

In the diagram the payoff & profit diagrams are given for this portfolio





which differ significantly due to the large (negative) cost associated with going short the share.

5. At its simplest: Suppose I (the investor) wish to short sell (1000) shares (from say, BT). I would instruct my broker to sell this number of shares from his/her portfolio - i.e. 1000 BT shares from another client's account. This sale would be done in the usual way. To close out the position the investor must purchase the shares. In the meantime the proceeds of the sale would be paid into my account. I would then be obliged to pay the broker (and hence owner of shares) any cash flows associated with the shares. If BT paid a dividend whilst I was short the BT shares, then I would be obliged to pay the broker the dividend (which would then be paid to the original owner of the sold shares).

At some point, normally agreed at the time of the initial short sale, there would be an obligation for me to buy the shares back (so the original owner of the sold shares owned them again).

6. The contract must be altered so that the fraction of the company that you have the option to sell is unchanged. In a three-for-one stock split, each new share is worth one quarter of the old share. The contract is therefore adjusted to read:

A put option to sell 800 shares of the company for £6.25 per share. The option price is unaffected.

7. A trader is *hedging* when he/she has an exposure to the price of an asset and takes a position in a derivative to offset the exposure. In a *speculation* the trader has no exposure to offset - they are betting on the future movements in the price of the asset. *Arbitrage* involves taking a position in two or more different markets to lock in profit.

8. Writing a call option involves selling an option to someone else. It gives a payoff of

$$\min\left(E - S_T,\ 0\right)$$

where E is the strike and S_T the spot price at expiry.

Buying a put option involves buying an option from someone else. It gives a payoff of

$$\max\left(E - S_T,\ 0\right)$$

In both cases the potential payoff is $E - S_T$. When you write a call option, the payoff is negative or zero. When you buy a put option, the payoff is zero or positive.