CQF Module 1.1 Exercises: Random Behaviour of Assets & Maths Revision

1. A stock price has an expected return of 12% per annum (with continuous compounding) and a volatility of 20% per annum. Changes in the share price satisfy $dS = \mu S dt + \sigma S dX$. What is the distribution of the price increase for the share movement?

2. Using a different set of stock prices, repeat the Excel based computational exercise conducted in class.

3. Differentiate $y = (x^x)^x$.

4. Express the complex number $\dfrac{7-2i}{5+3i}$ in the form $a + ib$. Hence find its modulus.

5. Calculate the following indefinite integrals

$$\int \frac{x+2}{x^2+4x-5}dx$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$$

$$\int xe^{2x}dx$$

6. Use the transformation $x = e^t$ to convert the Cauchy-Euler equation

$$x^2y'' - 4xy' + 6y = 3x^4$$

to a constant coefficient differential equation and then solve this to obtain a solution of the original equation. If a and b are arbitrary constants, show that this solution is

$$y = ax^2 + bx^3 + \tfrac{3}{2}x^4.$$

7. Consider the real vector space $\mathbb{R}^4$ and vectors

$$\mathbf{u}_1 = (-1,1,1,1)\,, \mathbf{u}_2 = (1,-1,1,1)\,, \mathbf{u}_3 = (1,1,-1,1)\,, \mathbf{u}_4 = (1,1,1,-1)\,.$$

Test these for linear independence. Repeat this test for the set

$$\mathbf{v}_1 = (-1,-1,1,1)\,, \mathbf{v}_2 = (1,-1,-1,1)\,, \mathbf{v}_3 = (1,1,-1,-1)\,, \mathbf{v}_4 = (-1,1,1,-1)\,.$$