Credit Default Swaps

Abstract

In this lecture we will study the most common type of credit derivative: credit default swaps (CDS). We will first review some instances of CDS in the media. Then, we will study the characteristics of CDS contracts and their pricing in the context of the intensity (reduced form) approach. We will conclude by studying some numerical examples and creating a CDS pricing in an Excel Workshop.

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CQF Module 5: Credit

Session 5.1 Credit Risk: Structural Models (Alonso Peña)

Session 5.2 Credit Risk: Intensity Models (SiYi Zhou)

Session 5.3 Introduction to Credit Derivatives (Moorad Choudhry)

Session 5.4 Credit Default Swaps (Alonso Peña) ✓

Session 5.5 Collateralized Debt Obligations (SiYi Zhou) ✓

Session 5.6 Advanced Credit Derivatives (Seb Lleo)

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Part 1

An Introduction to CDS

CDS in the News 1

Buffett's time bomb goes off on Wall Street

Chicago - On Main Street, insurance protects people from the effects of catastrophes. But on Wall Street, specialized insurance known as a **credit default swaps** are turning a bad situation into a catastrophe.

Recent events suggest Buffett was right. The collapse of Bear Stearns. The fire sale of Merrill Lynch Co Inc. The meltdown at American International Group Inc... In each case, **credit default swaps** played a role in the fall of these financial giants.

Source: Reuters, September 18 2008, *www.reuters.com*

CDS in the News 2

Demonization of CDS mis-states real role in crisis

They've been described as evil, acts of Satan and weapons of mass destruction. The fervor around credit derivatives hasnever been higher. CDSs have been made the fall guy for the collapse of institutions including Lehman Brothers Holdings Inc and bailout of American International Group. They've been called the epicenter of the crisis because they played a role in spreading the risks of bad mortgages and other assets globally, threatening systemic consequences from a large counterparty collapse.

Some people "think of derivatives as being everything toxic about the market they don't like," he said. "That's not true, markets have been toxic for many, many years before derivatives arrived. And we've always thought of having a means to have a hedge as desirable."

Source: Reuters, October 31 2008, *www.reuters.com*

CDS in the News 3

CDS market set to begin switch into clearing format

New York - The complicated and potentially costly process of switching the largest part of the $28,000bn credit derivatives market into a format that can be cleared is expected to start next month.

While the industry has agreed on the new streamlined contract for future trades on individual entities as part of the "Big Bang Protocol", which begins on April 8, the new measures do not apply to outstanding contracts.

Source: Financial Times, March 27 2009, *www.ft.com*

CDS in the News 4

Case Opens New Front on Insider Trading

The Securities and Exchange Commission brought its first-ever case alleging insider trading in credit-default swaps – an opaque derivative investment at the heart of the recent carnage in the financial industry.

"We are looking at a broad array of financial products associated with the financial crisis, including credit-default swaps," Kay Lackey, associate regional director of the SEC's New York office, said in an interview.

Source: Wall Street Journal, May 6, 2009, *www.wsj.com*

CDS in the News 5

Brighter Side Of 'Evil' Swaps

Credit-default swaps have been demonized as having played a role in the struggles of insurer American International Group and in the collapse of Bear Stearns.

But these derivatives can be a force for good. Indeed, demand for credit-default swaps is among the factors spurring the revival in the market for corporate bonds. Large institutional investors, hedge funds in particular, are buying more investment-grade and high-yield corporate bonds of late and are pairing them with credit-default swaps to earn extra return, according to investment bankers.

Source: Wall Street Journal, May 5, 2009, *www.wsj.com*

CDS in the News 6

ISDA: CDS Standard Model

- The International Swaps and Derivatives Association (ISDA) CDS Standard Model is a source code for CDS calculations.
- Can be downloaded freely through ISDA website.
- The source code is copyright of ISDA and available under an Open Source license.

Source: www.isda.org

CDS in the News 6

ISDA: "CDS Standard Model"

As the CDS market evolves to trade single name contracts with a fixed coupon and upfront payment, it is critical for CDS investors to match the upfront payment amounts and to be able to translate upfront quotations to spread quotations and vice versa in a standardized manner.

One of the primary goals in making the code available is to enhance transparency and to optimize use of standard technology for CDS pricing.

Source: www.cdsmodel.org

CDS Definition

CDS = CREDIT + DEFAULT + SWAP

with

CREDIT: debt-linked instrument

DEFAULT: sensitive to default

SWAP: exchange of cashflows

CDS Definition

A credit default swap is a financial derivative which offers protection* against the default of an underlying instrument.

In a CDS the protection* buyer makes a series of payments to the protection* seller and, in exchange, receives a payoff if a credit instrument (e.g. bond, loan) goes into default.

In some contracts, the credit event that triggers the payoff can be a company undergoing restructuring, bankruptcy or even just having its credit rating downgraded.

*Note the word "Protection". Can this be interpreted as a form of insurance?

CDS Are Not Insurance

In a CDS the seller need not be a regulated entity.

In a CDS the seller is not required to maintain any reserves to pay off that particular buyer (apart from bank capital requirements).

Insurers manage risk primarily by setting loss reserves, while dealers in CDS manage risk primarily by means of hedging CDS with other dealers.

The buyer of a CDS does not need to own the underlying security or other form of credit exposure; in fact the buyer does not even have to suffer a loss from the default event. In contrast, to purchase insurance the insured is generally expected to have an insurable interest such as owning a debt.

CDS Example

Consider that an investor buys a CDS from Bank of America, where the reference entity is General Motors.

The investor will make regular payments to Bank of America, and if General Motors defaults on its debt, the investor will receive a one-off payment from Bank of America and the CDS contract is terminated.

If the investor actually owns General Motors debt, the CDS can be thought of as **hedging**. But investors can also buy CDS contracts referencing General Motors debt, without actually owning any General Motors debt. This may be done for **speculative** purposes, to bet against the solvency of General Motors in a gamble to make money if it fails, or to hedge investments in other companies whose fortunes are expected to be similar to those of General Motors.

CDS Example

If the reference entity (General Motors) defaults, one of two things can happen:

Physical settlement: The investor delivers a defaulted asset to Bank of America for a payment of the par value.

Cash settlement: Bank of America pays the investor the difference between the par value and the market price of a specified debt obligation (even if General Motors defaults, there is usually some recovery).

CDS Example

The **spread** of a CDS is the annual amount the protection buyer must pay the protection seller over the length of the contract, expressed as a percentage of the notional amount. For example, if the CDS spread of General Motors is 50 basis points or 0.5%, then an investor buying $10 million worth of protection from Bank of America must pay the bank $50,000 per year. These payments continue until either the CDS contract expires or General Motors defaults.

All things being equal, at any given time, if the **maturity** of two credit default swaps is the same, then the CDS associated with a company with a higher CDS spread is considered more likely to default by the market, since a higher fee is being charged to protect against this happening. However, factors such as liquidity and estimated loss given default can impact the comparison.

CDS Mechanics

As outlined above, in general, we refer to a *credit default swap* (CDS) a credit derivative which enables the investors to isolate the default risk of an obligor. The basic structure is as follows. **B** agrees to pay the default payment to **A** *if a default has happened*. The default payment is structured to replace the loss that a lender would incur upon a credit event of the reference entity **C**. If there is no default of the reference security until the maturity of the default swap, counterparty **B** pays nothing.

CDS Mechanics

Counterparty **A** pays a fee for the default protection. Generally, the fee is a regular fee at intervals until default or maturity. If a default occurs between two fee payment dates, **A** still has to pay the fraction of the next fee payment that has accrued until the time of default.

CDS Mechanics

The credit default swap contains a clean isolation of obligor **C**'s default risk. If the protection buyer (**A**) has an underlying exposure to **C**, he holds the market risk, but he is hedged against the default risk, while the protection seller (**B**) can assume the credit risk alone. By changing the set of reference securities in the CDS, the counterparties can agree to focus more on the default risk of an individual bond issued by **C**, or they can widen the coverage to any of **C**'s obligations, thus covering the obligor's default risk completely.

CDS Mechanics

To specify a credit default swap, then, we need the following information:

[1] The reference obligor and his reference assets;

[2] The definition of the credit event that is to be insured;

[3] The notional of the CDS;

[4] The start of the CDS and the start of protection;

CDS Mechanics

[5] The maturity date;

[6] The CDS spread; or Prob Default

[7] The frequency and day count convention for the spread payments;

[8] The payment at the credit event and its settlement.

CDS Mechanics

The reference obligor in our case is **C**, his default risk is the object of the CDS contract. It is also necessary to specify a set of reference, usually a set of bond of a given seniority class issued by the reference obligor. Reference assets are necessary to:

- Determine some default events (missed payment on the reference assets);
- Specify the set of deliverable assets in default (for physical delivery);
- Determine a basis for the price and recovery determination mechanism in default (for cash settlement).

CDS Mechanics

The event that is to be insured against is a default of the reference obligor, but, because of the large payments involved, the definition of what constitutes a default has to be made more precise, and a mechanism for the determination of the default event must be given. The standard definition of default includes:

bankruptcy, filing for protection, failure to pay, repudiation, moratorium,restructuring.

CDS Mechanics

The **starting date** of most CDSs is three trading days after the trading date. We can specify a later starting date and in this case we speak of *forward credit default swaps*.

Most credit default swaps are quoted for a benchmark **time-to-maturity** of five years, but dealers also quote prices for other times to maturity, ranging from 1 to 10 years. Currently most CDS use rolling dates.

Notional values of CDSs vary from one million USD up to several hundred million, with smaller sizes for lower credit quality.

CDS Mechanics

The credit default swap **spread** is the *price* of the default protection that has to be paid by the protection buyer to the protection seller.

The cash payment amount is the CDS spread multiplied by the notional, adjusted for the day count convention. Typical payment terms are quarterly or semi-annually with an actual/360 day count convention.

The first fee is usually payable at the end of the first period and if a default happens between two fee payment dates, the accrued fee up to the time of default must be paid to the protection seller.

CDS Applications

- Hedging

- Speculation

- Arbitrage

CDS Applications: Hedging

The holder of a corporate bond may hedge their exposure by entering into a CDS contract as the buyer of protection. If the bond goes into default, the proceeds from the CDS contract will cancel out the losses on the underlying bond.

CDS Applications: Speculation

CDS allow investors to speculate on changes in CDS spreads. An investor might believe that an entity's CDS spreads are either too high or too low relative to the entity's bond yields and attempt to profit from that view by entering into a trade, known as a basis trade, that combines a CDS with a cash bond and an interest rate swap.

Finally, an investor might speculate on an entity's credit quality, since generally CDS spreads will increase as credit-worthiness declines, and decline as credit-worthiness increases. The investor might therefore buy CDS protection on a company in order to speculate that the company is about to default.

Alternatively, the investor might sell protection if they think that the company's creditworthiness might improve.

CDS Applications: Arbitrage

Capital Structure Arbitrage (CSA) is an example of an arbitrage strategy which utilises CDS transactions.

This technique relies on the fact that a company's stock price and its CDS spread should exhibit negative correlation; i.e. if the outlook for a company improves then its share price should go up and its CDS spread should tighten, since it is less likely to default on its debt. However if its outlook worsens then its CDS spread should widen and its stock price should fall.

Techniques reliant on this are known as capital structure arbitrage because they exploit market inefficiencies between different parts of the same company's capital structure; i.e. mis-pricings between a company's debt and equity.

Part 2

Default Modelling Toolkit

Setup

All processes and random variables we introduce are defined on a complete filtered probability space $(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$, where Ω is the set of possible states of nature, the filtration $(\mathcal{F}_t)_{t\geq 0}$ represents the information structure of the setup and $\mathbb{P}$ is the probability measure that attaches probabilities to the events in Ω.

Stopping times

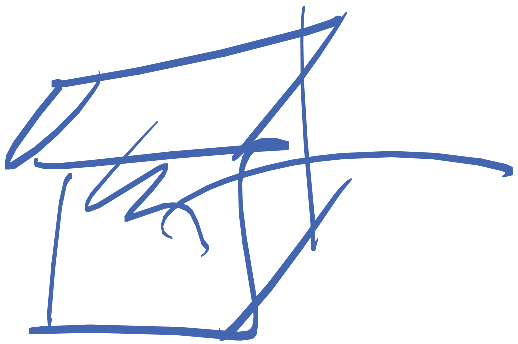
To model the *arrival risk* of a credit event we need to model an unknown random point in time $\tau \in \mathbb{R}_+$. If τ is the time of some event, we want that *at the time of the event it is known that this event has occurred*. This means that at every time t we know if τ has already occurred or not:

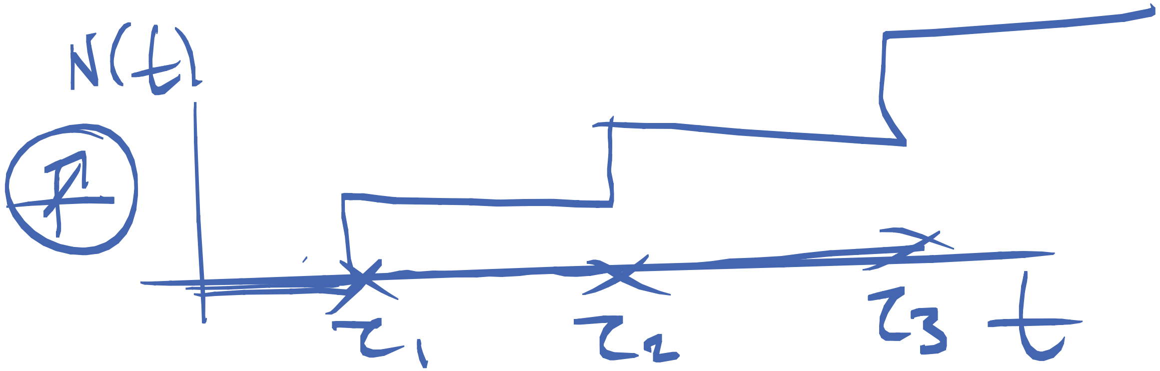
$$\{\tau \leq t\} \in \mathcal{F}_t, \forall t \geq 0.$$

Stopping times

This property defines the random variable τ as a *stopping time*. This equation says that we can observe the event at the time it occurs. In order to represent a stopping time with a stochastic process, we define its *indicator process* that jumps from zero to one at the stopping time:

$$N_\tau(t) := \mathbf{1}_{\tau \leq t}.$$





Stopping times

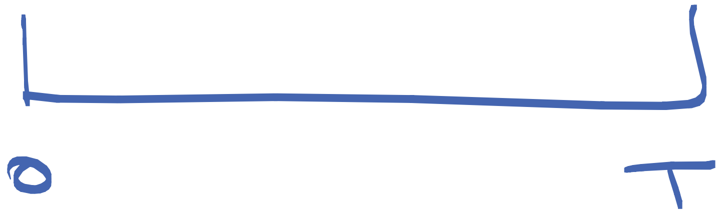
For default risk modeling we use the *default indicator function* (the indicator function of the default event) and *survival indicator function* (one minus the default indicator function).

Survival indicator function

$$I(t) = 1_{\{\tau>T\}} = \begin{cases} 1 \; if \tau > T \\ 0 \; if \tau \leq T \end{cases}$$

no default by T

default by maturity



Hazard Rates

We give now a formal definition of the hazard rate and its connection with the probability of default. Let τ be a stopping time and $F(T) = \mathbb{P}(\tau \leq T)$ its distribution function. Assume that $F(T) \quad 1 \, \forall T$ and that $F(T)$ has a density $f(T)$. The *hazard rate function* h of τ is defined as:

$$h(T) := \frac{f(T)}{1 - F(T)}.$$

Hazard Rates

At later points in time $t > 0$ with $\tau > t$, the *conditional hazard rate* is defined as:

$$h(t,T) := \frac{f(t,T)}{1 - F(t,T)},$$

where $F(t,T) := \mathbb{P}(\tau \leq T | \mathcal{F}_t)$ is the conditional distribution of τ given the information at time t, and $f(t,T)$ is the corresponding density. The hazard rate of default gives the finest possible resolution of the likelihood of default in an infinitesimally small time interval $[t, t + dt]$:

$$h(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \mathbb{P}(\tau \leq t + \Delta t | \tau > t).$$

Hazard Rates

Knowledge of the hazard rate function allows a reconstruction of $F(t)$ and $F(t, T)$:

$$F(t) = 1 - e^{-\int_0^t h(s)ds}$$

and

$$F(t, T) = 1 - e^{-\int_t^T h(t,s)ds}.$$

$\lambda(t)$

probability structure

$p(\lambda)$

Point Processes

A stopping time is the mathematical description of *one* event, a point process is a generalization to multiple events. A *point process* is a collection of points in time:

$$\{\tau_i,\, i \in \mathbb{N}\} = \{\tau_1, \tau_2, \ldots\}.$$

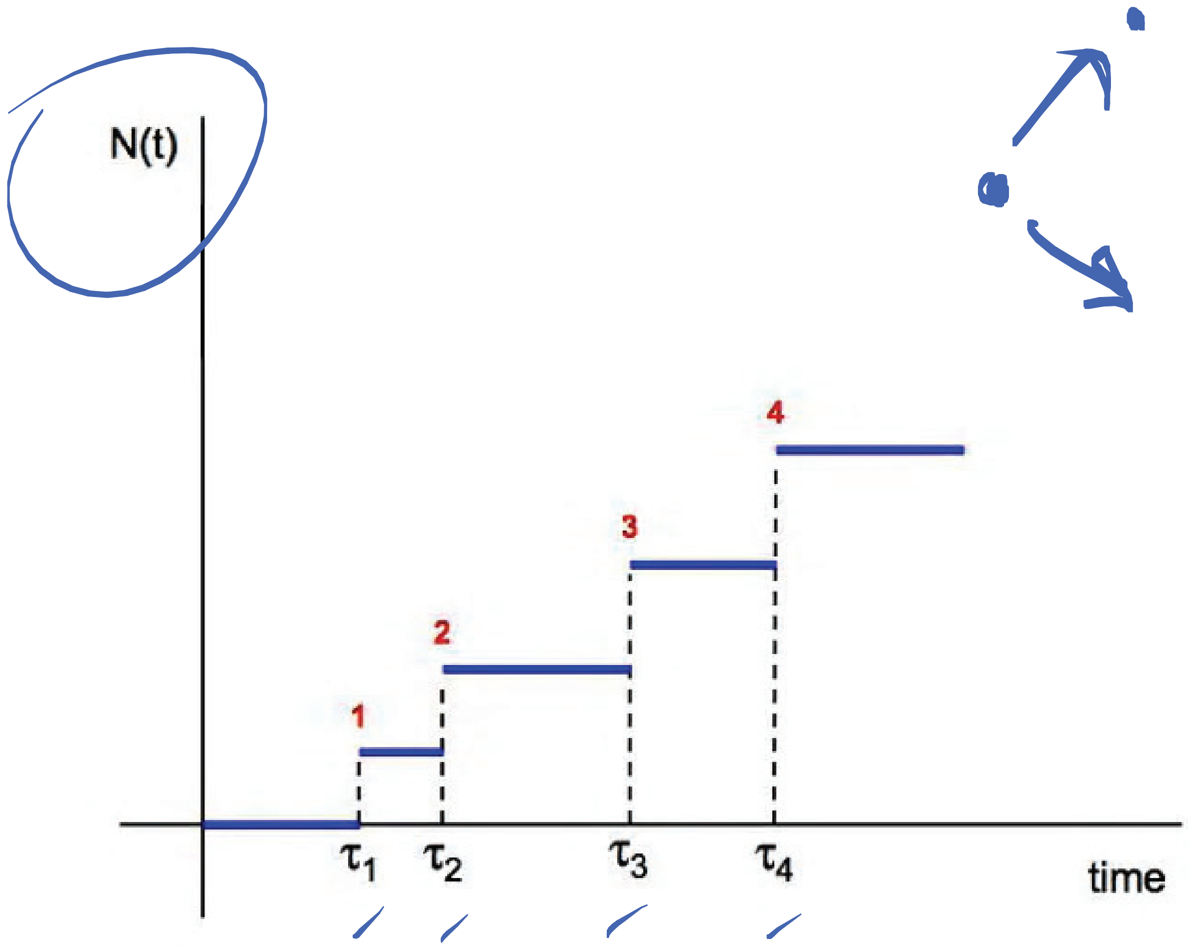
Point Processes

We assume that we have indexed these points in time in ascending order ($\tau_i < \tau_{i+1}$). We further assume that they are all stopping times, that they are all different and that there is only a finite number of such points over any finite time horizon. The point processes provide a good mathematical framework to analyse several events, i.e multiple defaults. We can turn this collection of time points into a stochastic process using the associated *counting process*:

$$N(t) := \sum_i \mathbf{1}_{\tau_i \leq t}.$$

Point Processes

$N(t)$ counts the number of time points of the point process that lie before t. If all τ_i are greater than zero, a sample path of $N(t)$ would be a step function that starts at zero and increases by one at each τ_i. $N(t)$ contains all the information that is contained in the point process $\{\tau_i,\, i \in \mathbb{N}\}$ and viceversa. The advantage of using $N(t)$ is that we now have a *stochastic process*.


N(t)
1
2
3
4
τ1
τ2
τ3
τ4
time

Poisson Processes

In this section we introduce a mathematical framework for modeling default times using a Poisson process. Its main property is that the probability of a jump over a small time step is approximately proportional to the length of this time interval.

Poisson Processes

The following assumption describes the way in which the default arrival risk is modeled in all intensity-based default risk models. Let $N(t)$ be a *counting process** with intensity $\lambda(t)$. The time of default τ is the time of the first jump of N, i.e.

$$\tau = \inf\{t \in \mathbb{R}_+ \,|\, N(t) > 0\}.$$

* A counting process is a non-decreasing, integer-valued process $N(t)$ with $N(0) = 0$.

Poisson Processes

The survival probabilities in this case are given by:

$$P(t,T) = \mathbb{P}(N(T) - N(t) = 0|\mathcal{F}_t).$$

Poisson Processes

A Poisson process $N(t)$ is an increasing process in the integers $0, 1, 2, 3, \ldots$, and to each of them we associate a the following *times of the jumps* $\tau_1, \tau_2, \tau_3, \ldots$ and the probability of a jump in the next instant. Let us assume that the probability of a jump in the next small time interval Δt is proportional to Δt:

$$\mathbb{P}(N(t + \Delta t) - N(t) = 1) = \lambda \Delta t.$$

Poisson Processes

Furthermore we suppose that jumps by more than 1 do not occur and that jumps in disjoint time intervals happen independently of each other. So the probability of the process remaining constant (i.e. not jumping) is $\mathbb{P}(N(t+\Delta t) - N(t) = 0) = 1 - \lambda\Delta t$

and over the interval $[t, 2\Delta t]$ is

$$\mathbb{P}(N(t+2\Delta t) - N(t) = 0) =$$

$$\mathbb{P}(N(t+\Delta t) - N(t) = 0)\cdot$$

$$\mathbb{P}(N(t+2\Delta t) - N(t+\Delta t) = 0) = (1 - \lambda\Delta t)^2.$$

Poisson Processes

Now we subdivide the interval $[t, T]$ into n subintervals of length $\Delta t = (T-t)/n$; in each of these subintervals the process N has a jump with probability $\lambda \Delta t$. If we conduct n independent binomial experiments, the probability of *no* jump at all in $[t, T]$ is given by:

$$\mathbb{P}(N(T) = N(t)) = (1 - \lambda \Delta t)^n = \left(1 - \frac{\lambda(T-t)}{n}\right)^n.$$

Because $(1 + x/n)^n \to e^x$ as $n \to \infty$, this converges to:

$$\mathbb{P}(N(T) = N(t)) \to e^{-\lambda(T-t)}.$$

Poisson Processes

As regards the probability of exactly *one* jump in $[t, T]$, there are n possibilities of having exactly one jump, giving a probability of

$$\begin{aligned}
\mathbb{P}(N(T) - N(t) = 1) &= n \cdot \lambda \Delta t \cdot (1 - \lambda \Delta t)^{n-1} \\
&= n \cdot \lambda \frac{T-t}{n} \cdot \frac{\left(1 - \frac{1}{n}\lambda(T-t)\right)^n}{\left(1 - \frac{1}{n}\lambda(T-t)\right)} \\
&= \frac{\lambda(T-t)}{1 - \frac{1}{n}\lambda(T-t)} \left(1 - \frac{1}{n}\lambda(T-t)\right)^n .
\end{aligned}$$

Again, using the limit result for the exponential function and the fact that the term in the denominator converges to 1 in the limit,

$$\mathbb{P}(N(T) - N(t) = 1) \to \lambda(T - t)e^{-\lambda(T-t)},\ n \to \infty.$$

In the same fashion the limit probabilities of *two* jumps are

$$\mathbb{P}(N(T) - N(t) = 2) = \frac{1}{2}\lambda^2(T - t)^2 e^{-\lambda(T-t)}$$

and for n jumps

$$\mathbb{P}(N(T) - N(t) = n) = \frac{1}{n!}\lambda^n(T - t)^n e^{-\lambda(T-t)}.$$

Poisson Processes

A Poisson process with intensity $\lambda > 0$ is a non-decreasing, integer-valued process with initial value $N(0) = 0$ whose increments are independent and satisfy, for all $0 \leq t < T$,

$$\mathbb{P}(N(T) - N(t) = n) = \frac{1}{n!}\lambda^n (T - t)^n e^{-\lambda(T-t)}.$$

Poisson Processes

Poisson processes are usually used to model rare events and discretely countable events such as defaults. Usually one models the time of default of a firm as *the time of the first jump of a Poisson process*. The parameter λ in the construction of the Poisson process is called the *intensity* of the process.

Poisson Processes

Other properties of the Poisson process are:

- The Poisson process has no memory. The probability of n jumps in $[t, t+s]$ is independent of $N(t)$ and the history of N before t.

- The inter-arrival times of a Poisson process $(\tau_{n+1} - \tau_n)$ are exponentially distributed with density of the time of the next jump of N given by
$$\mathbb{P}((\tau_{n+1} - \tau_n) \in t dt) = \lambda e^{-\lambda t} dt.$$

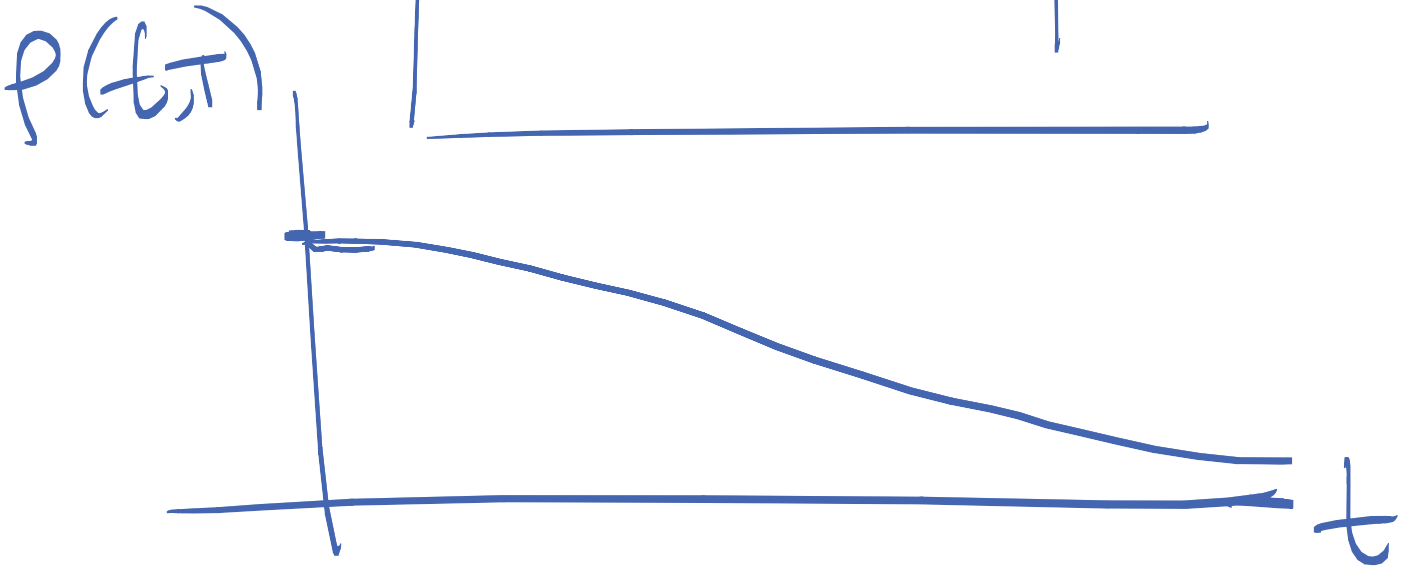
- Two or more jumps at exactly the same time have probability zero.

Poisson Processes

If we use the preceding development we can arrival of default as the first jump of $N(t))$ with a Poisson process with intensity λ, the survival probability can be obtained as:

$1 - P(t,T)$

$$P(t,T) = e^{-\lambda(T-t)}.$$



Inhomogeneous Poisson Processes

If we let the intensity λ of the Poisson process be a function of time $\lambda(t)$, we reach an *inhomogeneous* Poisson process. Starting from the jump probability $\mathbb{P}(N(t+\Delta t) - N(t) = 1) = \lambda(t)\Delta t$ we calculate the probability of *no* jump in $[t, T]$:

$$
\begin{aligned}
\mathbb{P}(N(T) - N(t) = 0) &= \prod_{i=1}^{n} (1 - \lambda(t + i\Delta t)\, \Delta t), \\
\ln \mathbb{P}(N(T) - N(t) = 0) &= \sum_{i=1}^{n} \ln(1 - \lambda(t + i\Delta t)\, \Delta t) \\
&\approx \sum_{i=1}^{n} -\lambda(t + i\Delta t)\, \Delta t \\
&\to -\int_t^T \lambda(s)ds \text{ as } \Delta t \to 0, \\
\mathbb{P}(N(T) - N(t) = 0) &\to \exp\left(-\int_t^T \lambda(s)ds\right) \text{ as } \Delta t \to 0.
\end{aligned}
$$

Inhomogeneous Poisson Processes

Similarly we can derive the general formula for the probability of n jumps, thus obtaining the following:

An inhomogeneous Poisson process with intensity function $\lambda(t) > 0$ is a non-decreasing, integer-valued process with initial value $N(0) = 0$ whose increments are independent and satisfy

$$\mathbb{P}(N(T) - N(t) = n) = \frac{1}{n!}\left(\int_t^T \lambda(s)ds\right)^n exp\left(-\int_t^T \lambda(s)ds\right).$$

$$P(t,T) = \exp\left(-\int_t^T \underline{\lambda(s)}\, ds\right)$$

Inhomogeneous Poisson Processes

The intensity $\lambda(t)$ is a non-negative function of time only. Again, we can derive the survival probability for an inhomogeneous Poisson process using:

$$P(t,T) = e^{-\int_t^T \lambda(s)ds}$$

and the corresponding hazard rate of default:

$$h(t,T) = \lambda(T).$$

Now the default hazard rate depends on the time horizon T: the term structure of hazard rates is not flat, but given by $\lambda(T)$, so we can reach every term structure we desire.

Part 3

CDS Pricing

Mathematical Setup

In this section we provide a simplified pricing model for credit default swaps.

Let us suppose that there are N periods, indexed by $n = 1, \ldots, N$. Without loss of generality, each period is of length Δt, expressed in units of years. Thus, time intervals are:

$$\Theta = \{(0, \Delta t), (\Delta t, 2\Delta t), \ldots, ((N-1)\Delta t, N\Delta t)\}$$

The corresponding end of period maturities are:

$$T_n = n\Delta t$$

.

Mathematical Setup

Risk free forward interest rates are denoted $r((n-1)\Delta t, n\Delta t) \equiv r(T_{n-1}, T_n)$, i.e. the rate over the n^{th} period. We write these one-period forward rates in short form as r_n, as the forward rate applicable to the n^{th} time interval. The discount factors may be written as functions of forward rates, i.e.

$$\boxed{D(0, T_n) = \exp\left(-\sum_{k=1}^{n} r_k \Delta t\right).}$$

$r(T_{n-1}, T_n)$

$n=2$

$$= \exp\left(-\sum_{k=1}^{2} r_k \Delta t\right)$$

$$D(0, T_2) = \exp\left(-r_1\Delta t - r_2\Delta t\right)$$

Mathematical Setup

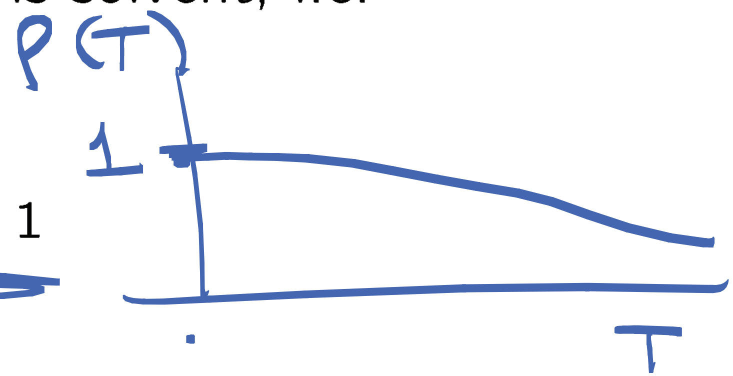
For a given obligor, we suppose that default is likely with an hazard rate $\lambda_n \equiv \lambda(T_{n-1}, T_n)$, constant over forward period n. Given these default intensities, the survival function of the obligor is defined as

$$n=2 \quad P(T_2) = \exp\left(-\lambda_1 \Delta t - \lambda_2 \Delta t\right)$$

$$P(T_n) = \exp\left(-\sum_{k=1}^{n} \lambda_k \Delta t\right),$$

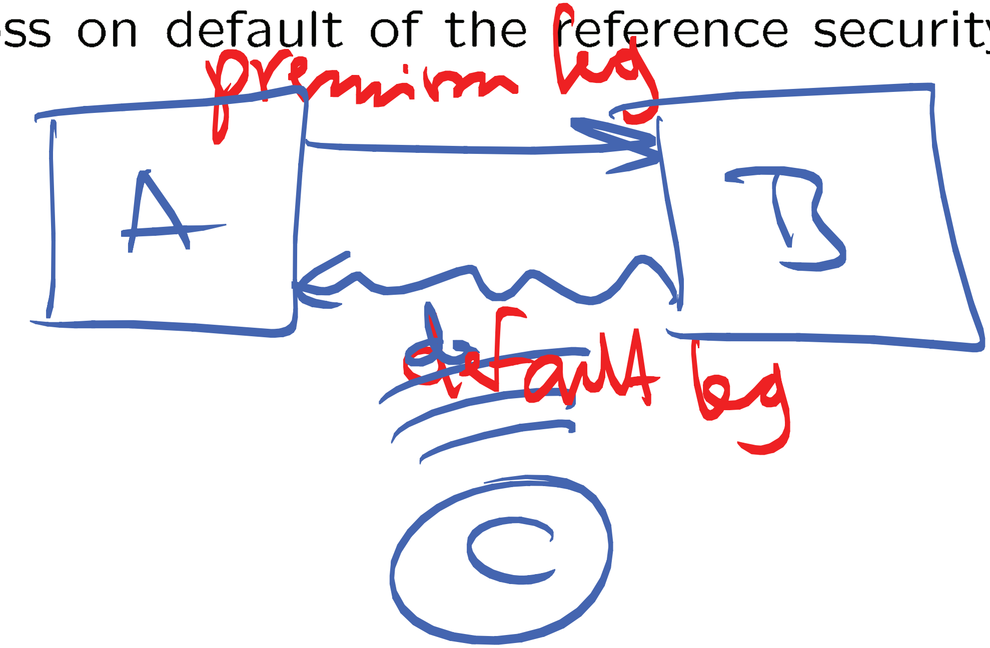
assuming that at time zero the obligor is solvent, i.e.

$$P(T_0) = P(0) = 1$$



Mathematical Setup

In our framework, the buyer of the CDS (**A**) purchases credit protection against the default of the reference security and, in return, pays a periodic payment to the seller (**B**) (*Premium leg*). These periodic payments continue until maturity or until the reference instrument default, in which event the seller pays to the buyer the loss on default of the reference security (*Default leg*).



$(T_0, T_1, T_2, \ldots, T_n)$

$N=5$ $T=5Y$ $\Delta t = 1Y$

CDS Pricing: Premium Leg

We denote the $N-$period CDS spread as S_N, stated as an annualized percentage of the nominal value of the contract. Without loss of generality, we set the nominal value to 1 €. We assume that defaults occur only at the end of the period, so the premiums will be paid until the end of the period. Since the premium payments are made as long as the reference security survives, the expected present value of the premiums paid (PL_N) is as follows:

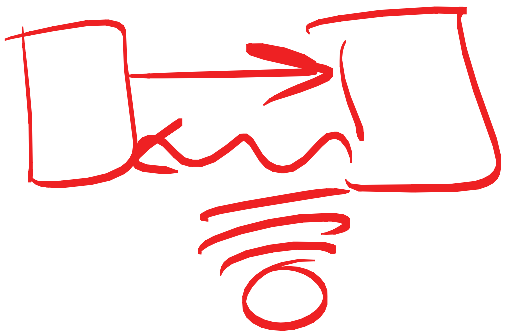
$$PL_N = S_N \Delta t \sum_{n=1}^{N} P(T_{n-1}) D(0, T_n). \qquad (1)$$

This accounts for the expected present value of payments made from the buyer **A** to the seller **B**.

T_1 T_2 T_3 0 S_N

CDS Pricing: Default Leg

The other possible payment of the CDS arises in the event of default, and goes from the seller to the buyer. The expected present value of this payment depends on the recovery rate in the event of default, which we denote as R. The loss payment on default is then equal to $(1 - R)$ for every 1 € of notional principal. This implicitly assumes that the recovery of par convention is used.



$R = 30\%$

$R = 50\%$

CDS Pricing: Default Leg

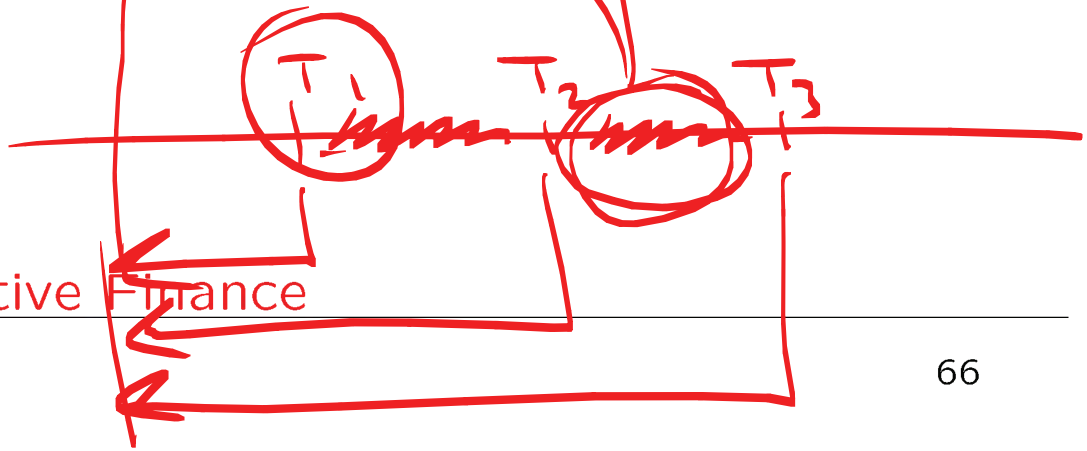
(Contingent)

The expected loss payment in period n is based on the probability of default in period n, conditional on no default in a prior period. This probability is given by the probability of surviving until period $n-1$ and then defaulting in period n:

$$P(T_{n-1})(1 - e^{-\lambda_n \Delta t}).$$

Therefore, the expected present value of loss payments (DL_N) equals the following:

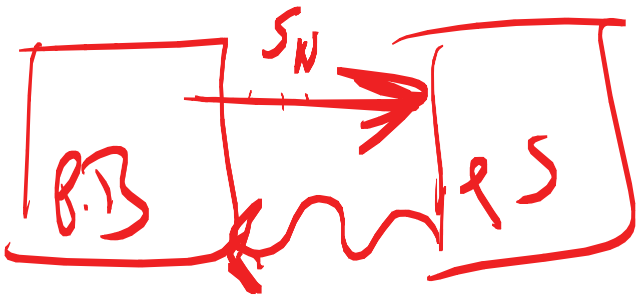
$$DL_N = \sum_{n=1}^{N} P(T_{n-1})(1 - e^{-\lambda_n \Delta t}) D(0, T_n)(1 - R).$$



CDS Pricing: Fair Spread

The fair pricing of the $N-$period CDS, i.e. the fair quote of the spread S_N, must be such that the expected present value of payments made by buyer and seller are equal, i.e. $\boxed{PL_N = DL_N}$. Thus we obtain

$$S_N = \frac{\sum_{n=1}^{N} P(T_{n-1})(1 - e^{-\lambda_n \Delta t})D(0, T_n)(1 - R)}{\Delta t \sum_{n=1}^{N} P(T_{n-1})D(0, T_n)}.$$



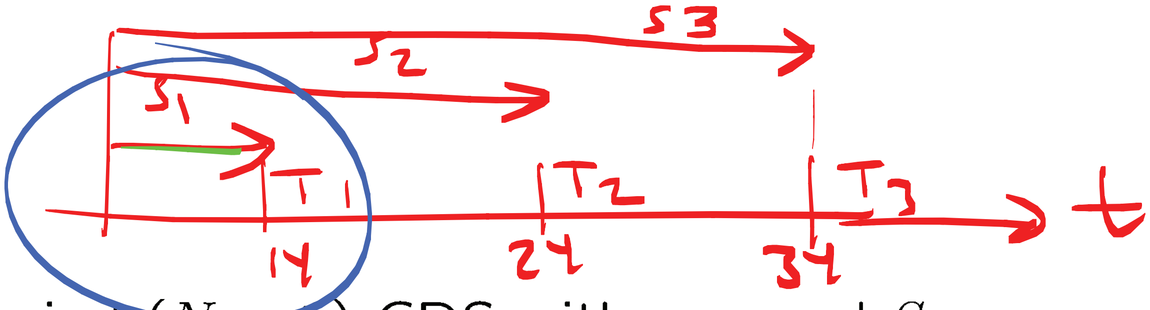
Bootstrapping Hazard Rates

In the previous equations the spread S_N and the discount factors $D(0, T_n)$ are observable in the default risk and government bond markets, respectively. However, the default hazard rates λ_n are not directly observed and need to be inferred from the observable variables.

Bootstrapping Hazard Rates

Since there are N periods, we may use N CDSs of increasing maturity, each with spread S_n, and impose $PL_n = DL_n$, $n = 1, \ldots, N$. Thus we have N equations with as many unknowns, which can be identified in a recursive manner using bootstrapping. Let us detail some of this procedure.

STEP 1



Starting with the one-period ($N = 1$) CDS with a spread S_1 per annum, we equate payments on the swap as follows:

N = 1

$$
\begin{aligned}
PL_1 &= DL_1 \\
\underbrace{S_1 \Delta t \overset{1}{\cancel{P(T_0)}} D(0,T_1)} &= \underbrace{\left(1 - e^{-\lambda_1 \Delta t}\right) D(0,T_1)(1-R)} \\
S_1 \Delta t &= \left(1 - e^{-\lambda_1 \Delta t}\right)(1-R).
\end{aligned}
$$

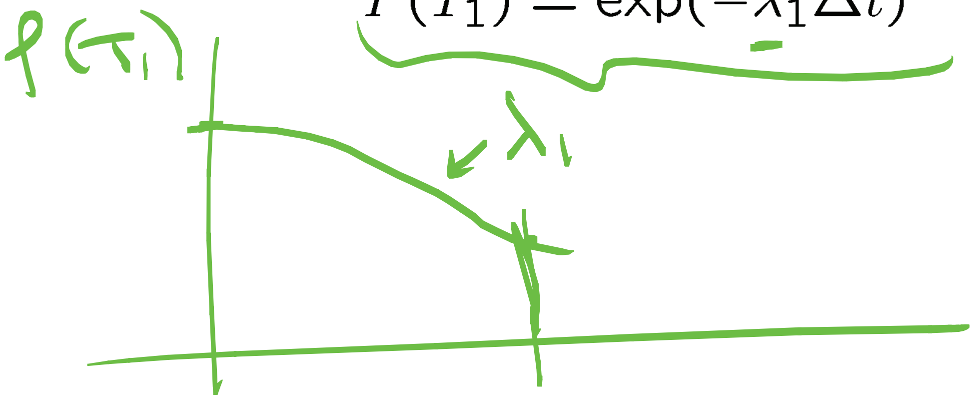
This results in an identification of λ_1, which is:

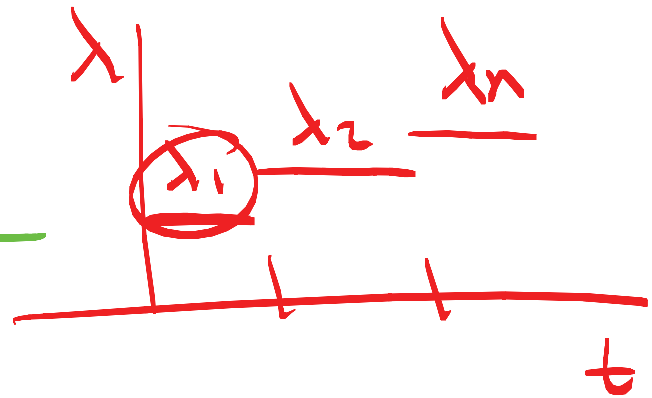
$$\lambda_1 = -\frac{1}{\Delta t} \ln \left(\frac{1 - R - S_1 \Delta t}{1 - R} \right),$$

which also provides the survival function for the first period, i.e.

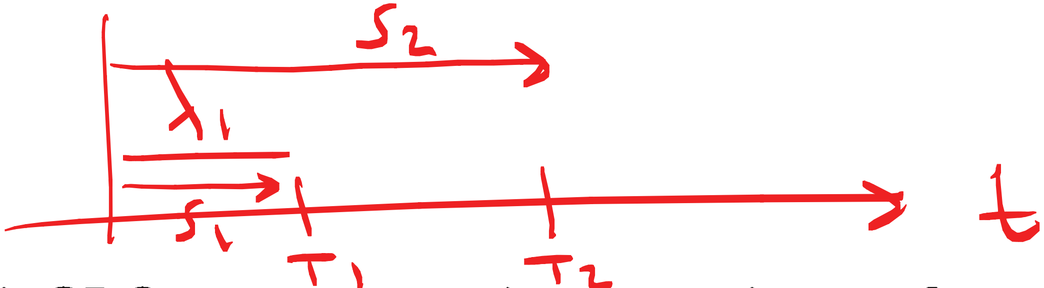
$N = 1$

$$\underbrace{P(T_1) = \exp(-\lambda_1 \Delta t)}$$





STEP 2



We now use the 2−period CDS to extract the hazard rate for the second period, whose spread is denoted as S_2. We set

$PL_2 = DL_2$ and obtain the following equation which can be solved for λ_2:

$$S_2 \Delta t \sum_{n=1}^{2} P(T_{n-1}) D(0, T_n) =$$

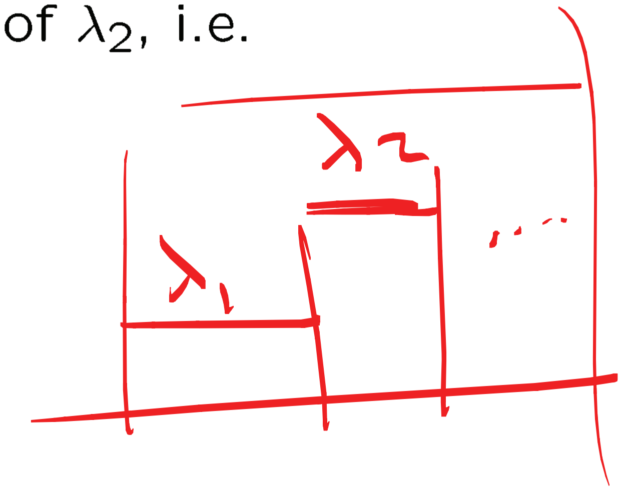
$$\sum_{n=1}^{2} P(T_{n-1}) \left(1 - e^{-\lambda_n \Delta t}\right) D(0, T_n)(1 - R)$$

Expanding this equation, we have

$$
\begin{aligned}
S_2 \Delta t [P(T_0) D(0, T_1) + P(T_1) D(0, T_2)] = \\
P(T_0) \left(1 - e^{-\lambda_1 \Delta t}\right) D(0, T_1)(1 - R) \\
+ P(T_1) \left(1 - e^{-\lambda_2 \Delta t}\right) D(0, T_2)(1 - R).
\end{aligned}
$$

Rearranging this equation delivers the value of λ_2, i.e.

$$\lambda_2 = -\frac{1}{\Delta t} \ln \left(\frac{L_1}{L_2} \right)$$



where

$$L_1 = P(T_0)\left(1 - e^{-\lambda_1 \Delta t}\right) D(0,T_1)(1-R)$$

$$+P(T_1)D(0,T_2)(1-R) - S_2\Delta t[D(0,T_1) + P(T_1)D(0,T_2)]$$

$$L_2 = P(T_1)D(0,T_2)(1-R)$$

and $P(T_0) = 1$.

STEP 3

In general, we can write down the expression for the k^{th} default hazard rate:

$$\lambda_k = -\frac{1}{\Delta t} \ln \left(\frac{P(T_{k-1})D(0,T_k)(1-R) + \sum_{n=1}^{k-1} G_n - S_k \Delta t \sum_{n=1}^{k} H_n}{P(T_{k-1})D(0,T_k)(1-R)} \right),$$

where

$$G_n = P(T_{n-1}) \left(1 - e^{-\lambda_n \Delta t}\right) D(0,T_n)(1-R),$$

$H_n = P(T_{n-1})D(0,T_n).$

Thus, we begin with λ_1 and, through a process of bootstrapping, we arrive at all λ_n, $n = 1, \ldots, N$.

Part 4

CDS Examples

Example 1: Hedging

A pension fund owns \$10 million of a five-year bond issued by General Motors. In order to manage the risk of losing money if General Motors defaults on its debt, the pension fund buys a CDS from Bank of America in a notional amount of \$10 million. The CDS trades at 200 basis points. In return for this credit protection, the pension fund pays 2% of 10 million (\$200,000) pa in quarterly installments of \$50,000 to Bank of America.

Example 2: Speculation

A hedge fund believes that General Motors will soon default on its debt. Therefore it buys \$10 million worth of CDS protection for 2 years from Bank of America, with General Motors as the reference entity, at a spread of 500 bps pa.

Scenario 1: No Default if General Motors does not default, then the CDS contract will run for 2 years, and the hedge fund will have ended up paying \$1 million, without any return, thereby making a loss. **Scenario 2: Default** if General Motors does indeed default after, say, one year, then the hedge fund will have paid \$500,000 to Bank of America, but will then receive \$10 million (assuming zero recovery rate, and that Bank of America has the liquidity to cover the loss), thereby making a tidy profit. Bank of America, and its investors, will incur a \$9.5 million loss unless the bank has somehow offset the position before the default.

Example 3: Arbitrage

Consider a company which has announced some bad news and its share price has dropped by 25%, but its CDS spread has remained unchanged, then an investor might expect the CDS spread to increase relative to the share price.

Therefore a basic strategy would be to go long on the CDS spread (by buying CDS protection) while simultaneously hedging oneself by buying the underlying stock.

This technique would benefit in the event of the CDS spread widening relative to the equity price, but would lose money if the company's CDS spread tightened relative to its equity.

Excel Workshop

Implementing CDS Pricing and Bootstrapping

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