CQF Module 5 Examination

2009 May

Instruction

All questions must be attempted. Books and lecture notes may be referred to. Spreadsheets and VBA may be used. Help from other people is not permitted.

Part A

There is only one correct answer for each question.

1. Which of the following are possible reason(s) to run a short or long position in a credit default swap

   (A) hedging a portfolio of syndicated loans

   (B) speculating on the future credit rating of a corporate entity

   (C) investing in structured finance securities

   (D) all of the above

2. By which set of the following terms is a credit default swap classified?

   (A) Reference entity, settlement mechanism, term and premium

   (B) reference entity, settlement mechanism, term, premium and credit event definition

   (C) reference entity, settlement mechanism, term, premium, credit event definition and deliverable obligation

   (D) all of the above

3. The spread to Libor paid or received in a total return swap is a function of which of the following?

   (A) credit rating of counterparty, credit quality of reference asset, funding cost of beneficiary bank and the capital charge associated with the swap

   (B) maturity term of swap, credit quality of reference asset, current level of Libor

   (C) credit quality of reference asset, credit rating of counterparty

   (D) insufficient information to answer

4. What is "jump-to-default risk"?

   (A) the credit risk exposure associated with a particular reference name

   (B) sudden default of the reference name in the market in the very near future, as opposed to a gradual credit deterioration

   (C) default probability distribution of any reference name

   (D) the risk of extreme market volatility impacting the reference name asset swap spread

Part B

1. Find hazard rate(intensity) of the following function.

   (a) X has exponential distribution with parameter λ.

   (b) X has Weibull distribution, i.e. the CDF of X is

   $$F(x) = 1 - \exp\left(-\alpha x^{\beta}\right).$$

2. The **Probability Generating Function** of a discrete random variable X is defined to be the generating function $G(s) = \mathbb{E}(s^X)$ of its probability mass function.

   (a) Show that

   $$\mathbb{E}(X) = G'(1).$$

   (b) Consider a Poisson random variable N with probability distribution function

   $$\Pr(N = i) = \frac{\lambda^i}{i!} e^{-\lambda}.$$

   Show that the probability generating function can be written as

   $$G(s) = e^{\lambda(s-1)}.$$

   (c) Suppose the number of default $N(t)$ in a large portfolio follows a Poisson process with intensity $\lambda = 1$, what is the expected number of default in one year?(**Hint**:use results from (a) and (b)).

   (d) Denote $\mathcal{F}_t$ the filtration adapted to Poisson process $N(t)$, show that the compensated Poisson process defined as

   $$M(t) = N(t) - \lambda t$$

   is a martingale.

3. In the context of the Merton (1974) model, at any time t the firm assets V_t are assumed to be sum of its debt D_t and its equity E_t,

$$V_t = E_t + D_t.$$

   (a) Based on the two possible credit scenarios that the company can face at maturity, i.e. it can either go on default or not, explain why the company's equity can be interpreted as a call option on the value of the assets with a strike price equal to the repayment required by the debt.

   (b) Consider a privately-held company which is about to go public and start selling equity shares to the general public. The total value of the company's assets is 100 million USD. These assets are divided in two parts or tranches. One tranche is a zero-coupon bond with a one-year maturity and a face value of 60 million USD. The other tranche corresponds to the equity. Assume that the average asset price volatility is 30 percent and that the risk-free interest rate is 5 percent. Based on the assumptions of the Merton model compute the values of the company's equity (E_0) and debt (D_0) today.

4. Suppose that the probability of company A defaulting in one year is 10% and the probability of company B defaulting in one year is 15%. Assuming default correlation is 30%, calculate the probability that both company default in one year by using bivariate Gaussian copula.

5. Suppose there is a risky bond $V(r,t;p)$, where interest rate is stochastic with SDE

$$dr = u(r,t)dt + w(r,t)dx,$$

and the risk of default is governed by Poisson Process with intensity p. Now consider the risky bond is hedged by a risk-free bond $Z(r,t)$, in particular the hedging portfolio can be written as

$$\Pi = V - \Delta Z.$$

By assuming fractional recovery on market value θ, show that pricing PDE of of the risky bond is

$$\frac{\partial V}{\partial t} + \tfrac{1}{2}w^2 \frac{\partial^2 V}{\partial r^2} + (u - \lambda w)\frac{\partial V}{\partial r} - (r + (1-\theta)p)V = 0.$$

What is the fundamental risky bond pricing formula?

6. The following table shows the term structure of CDS spreads and the current discount factors $Z(t,T)$ for a company:

Maturity	CDS Spreads	$Z(t,T)$
1Y	21	0.9801
2Y	36	0.9513
3Y	42	0.9151

Table 1: Company data, spreads in bps.

Assume that the CDS premium is paid annually (once a year) and a recovery rate of 50% . Compute the term-structure of the implied survival probabilities for this company.

Hint:Use the standard CDS bootstrapping methodology (JP Morgan) without accruals to determine directly $P(T_1)$, $P(T_2)$ and $P(T_3)$. There is no need to compute hazard rates.

7. A CDO is comprised of the following underlying portfolio:

Assets:	125 single-name CDS
Principal:	0.8 million for each name
Maturity:	5 years
1 year PD:	3% for each name
Recovery:	40% for each name

The CDO is structured with the following capital structure:

Tranche	Attachment point	Fair Spread	Rating
Senior	7%-10%	0.002%	AAA
Mezzanine	3%-7%	2.3%	A
Equity	0%-3%	26.27%	NR

(a) How many defaults must there be before the Mezzanine tranche experiences capital loss?

(b) Consider one factor Gaussian copula model with constant pairwise correlation

$$A_i = wZ + \sqrt{1-w^2}\,\varepsilon_i,$$

show that conditional default probability

$$F(t=1|Z) = \Phi\left(\frac{-1.88 - wZ}{\sqrt{1-w^2}}\right),$$

where ϕ is CDF of standard normal.

(c) Denote K to be the number of default by year 1 conditional on Z, explain why K follows binomial distribution $\mathcal{B}\left(F(1|Z), 125\right)$.

(d) Conditional on the first percentile of Z and $w = 0.3$, calculate one year default probability for Mezzanine tranche.