## CQF Exercises 4.1 Fixed Income

- 1. A coupon bond pays out 3% every year, with a principal of \$1 and a maturity of five years. Decompose the coupon bond into a set of zero coupon bonds.
- 2. Construct a spreadsheet to examine how \$1 grows when it is invested at a continuously-compounded rate of 7%. Redo the calculation for a discretely compounded rate of 7%, paid once per annum. Which rate is more profitable?
- 3. A zero-coupon bond (ZCB) has a principal of \$100 and matures in 4 years. The market price for the bond is \$72. Calculate the yield to maturity, duration and convexity for the bond.
- 4. A coupon bond pays out 2% every year on a principal of \$100. The bond matures in 6 years and has a market value \$92. Calculate the yield to maturity, duration and convexity for the bond.
- 5. Zero-coupon bonds are available with principal of \$1 and the following maturities:
- 1 year (market price \$0.93)
- 2 years (market price \$0.82)
- 3 years (market price \$0.74)

Calculate the yield to maturities for the three bonds. Use a bootstrapping method to obtain the forward rates that apply between 1-2 years and 2-3 years.

6. Consider the following problem

$$\begin{cases} \frac{dV}{dt} + K(t) = r(t)V\\ V(T) = 1 \end{cases}$$

where  $V=V\left(t\right)$  is the value of a coupon bond and the interest rate  $r\left(t\right)$  is known.  $K\left(t\right)$  represents a coupon payment, and T is maturity. By assuming a solution of the form

$$V = f(t) e^{-\int_t^T r(\tau)d\tau},$$

for the non-homogeneous part of the equation, obtain a particular solution.

What must we do if the interest rates are not known in advance?