$\sigma_i(T) \Longleftarrow \sigma(t)$

$\Longrightarrow$ ✓

$$\sigma_i(T) = \sqrt{\frac{1}{T-t}\int_t^T \sigma^2(\tau)\,d\tau} \Leftarrow$$

✓ ?

$\Rightarrow \sigma(t)$

$\sigma(t)$

1994 :

Dupire CB

Derman + Kani

Rubinstein

Deterministic Volatility

$$\int_{\xi}^{\infty} (s-\xi)\, p(\quad)\, ds$$

$$\frac{\partial}{\partial \xi}: \quad -\int_{\xi}^{\infty} p(\quad)\, ds - \cancel{(\xi-\xi)\, p(\quad)}$$

$$? \int_{\xi}^{\infty} \frac{\partial^2}{\partial S^2}\left(\sigma^2 S^2 p\right)(S-\xi)\, dS$$

$$= \left[ \frac{\partial}{\partial S}\left(\sigma^2 S^2 p\right)(S-\xi)\right]_{\xi}^{\infty} - \int_{\xi}^{\infty} \frac{\partial}{\partial S}\left(\sigma^2 S^2 p\right) dS$$

$$= 0 - \left[\sigma^2 S^2 p\right]_{\xi}^{\infty} = \sigma^2 \xi^2 p = \sigma^2 \xi^2 e^{\dots} \frac{\partial^2 V}{\partial \xi^2}$$

evaluated at $\xi$

$\sigma(\xi, \;)$

$$\int_\xi^\infty \frac{\partial (rSp)}{\partial S}\,(S-\xi)\,dS$$

$$= \Big[\, rSp\,(S-\xi) \Big]_\xi^\infty - \int_\xi^\infty rSp\,dS$$

$$= -r\int_\xi^\infty Sp\,dS = \qquad = r\xi\frac{\partial V}{\partial \xi} - rV$$

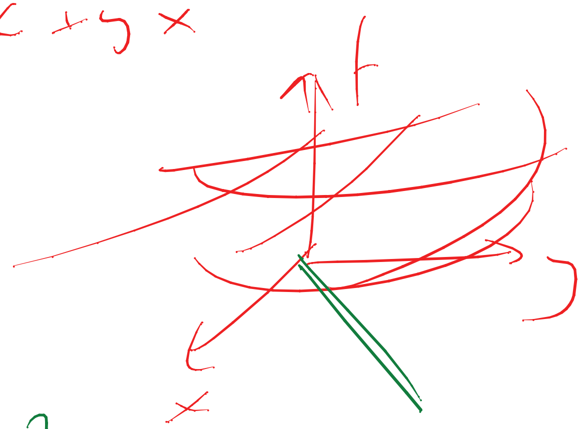
Know: $V = \dots \int_\xi^\infty (S-\xi)\,p\,dS$ $\qquad V - \xi\dfrac{\partial V}{\partial \xi} = \int_\xi^\infty S\,p\,dS$

$$\xi\frac{\partial V}{\partial \xi} = \dots \int_\xi^\infty \xi\,p\,dS$$

$$V(S^*, t^*, E, T, r) \leftarrow$$

$$\left.\frac{\partial V}{\partial E}\right|_{E=S}$$

$$f(x,y) = x^2 + y^3 x$$




$$\left.\frac{\partial f}{\partial x}\right|_{y=x} = 2x + y^3$$

$$= 2x + x^3$$

$f(x, y)$

NS EW

walk along a winding road $y = g(x)$


$$\left.\frac{\partial f(x,y)}{\partial x}\right|_{y=g(x)}$$

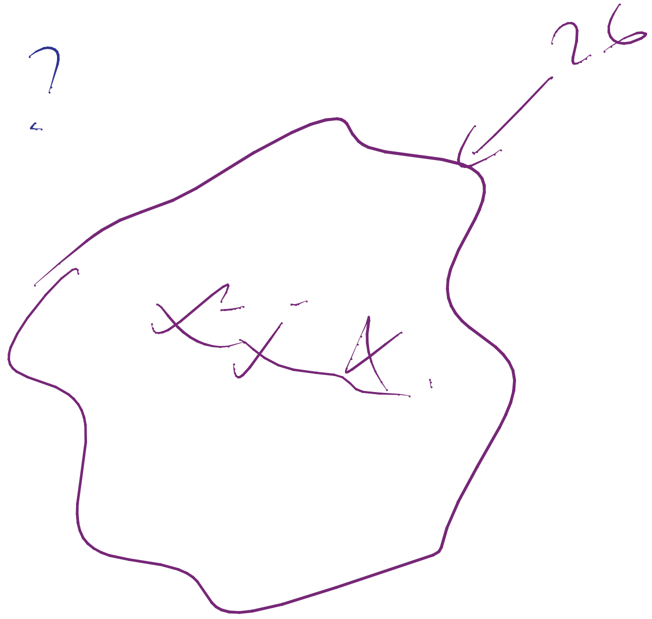
Turn to face north, what is the slope?

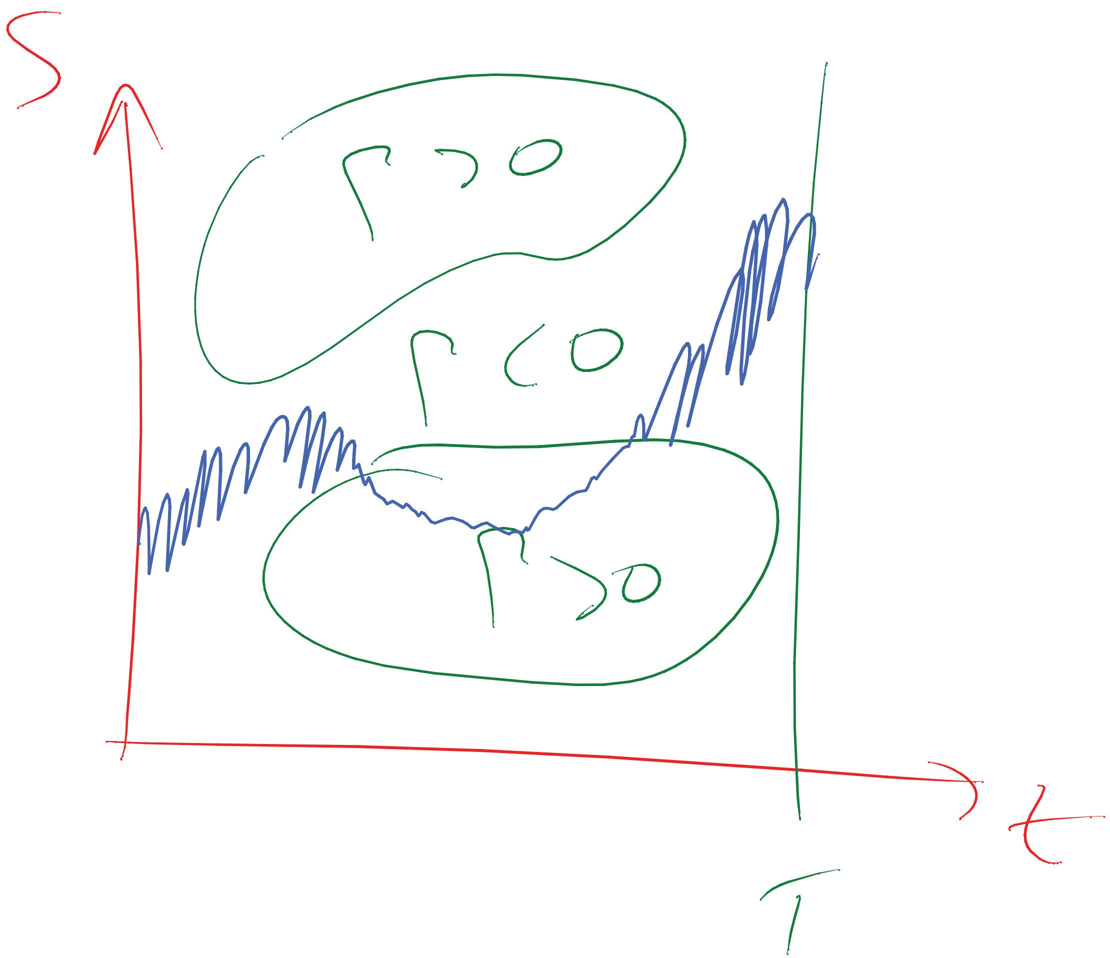
Numerical $\frac{\partial^2}{\partial S^2}$ !

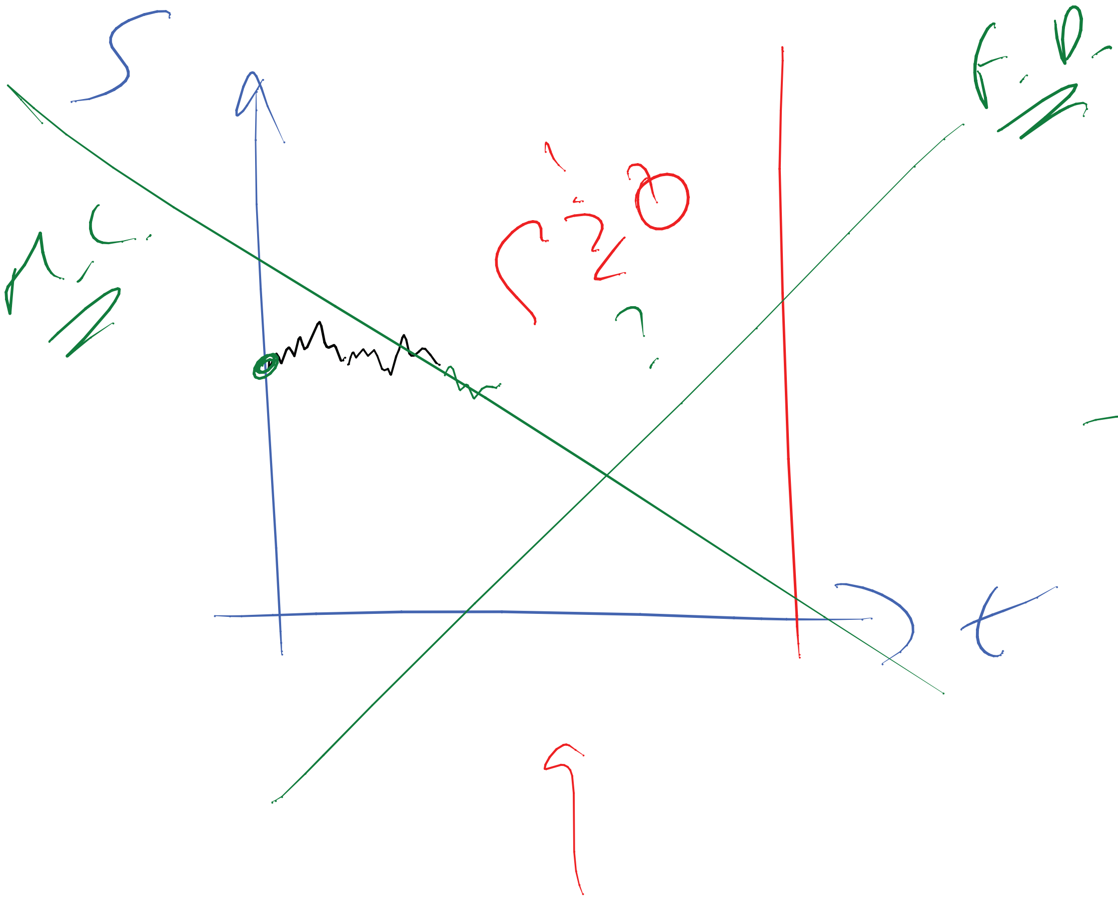
Model.

$\sigma(S,t)$ ?

Inverse




S
t
T



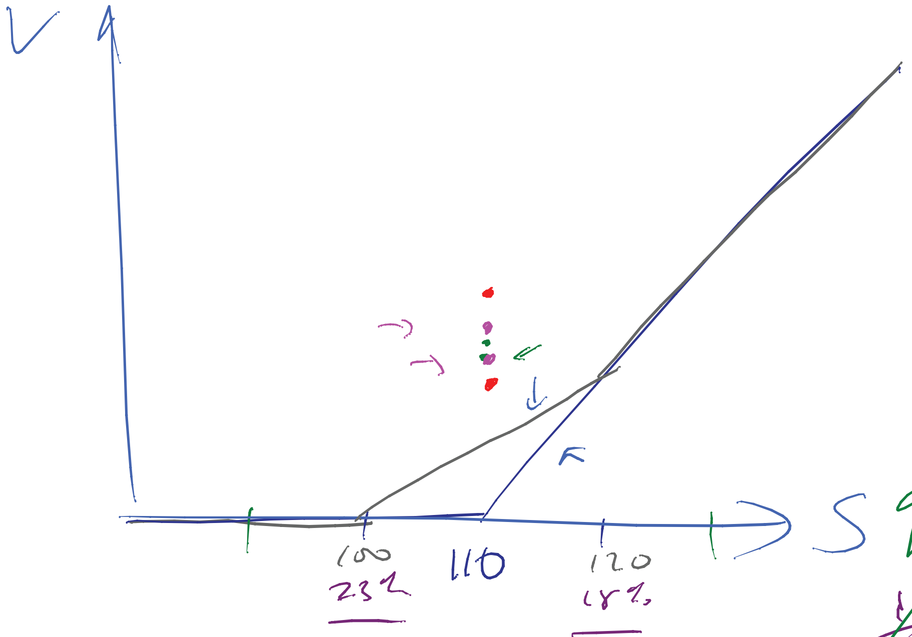

Delta =

Gamma =

Vol = Volmin

If Gamma < 0 then Vol = Volmax

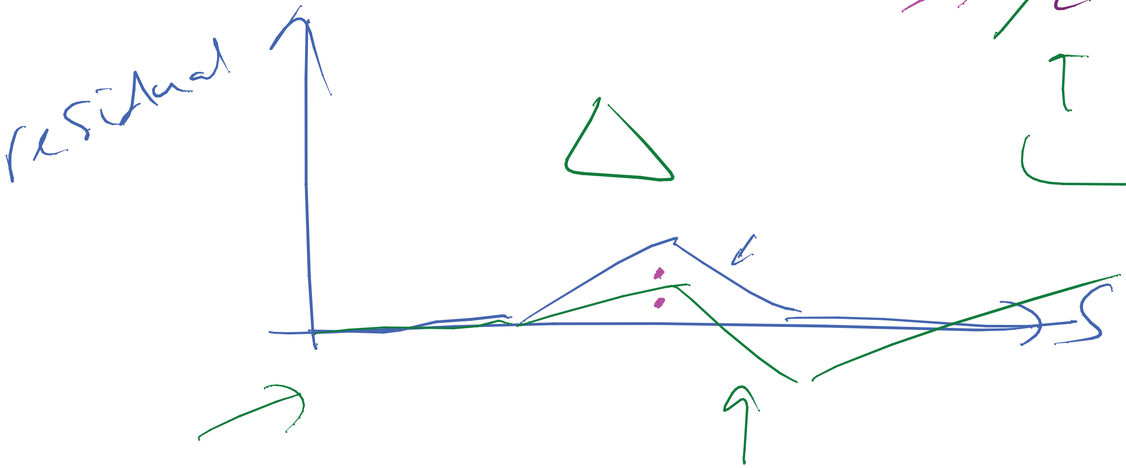
Theta =




15-25%

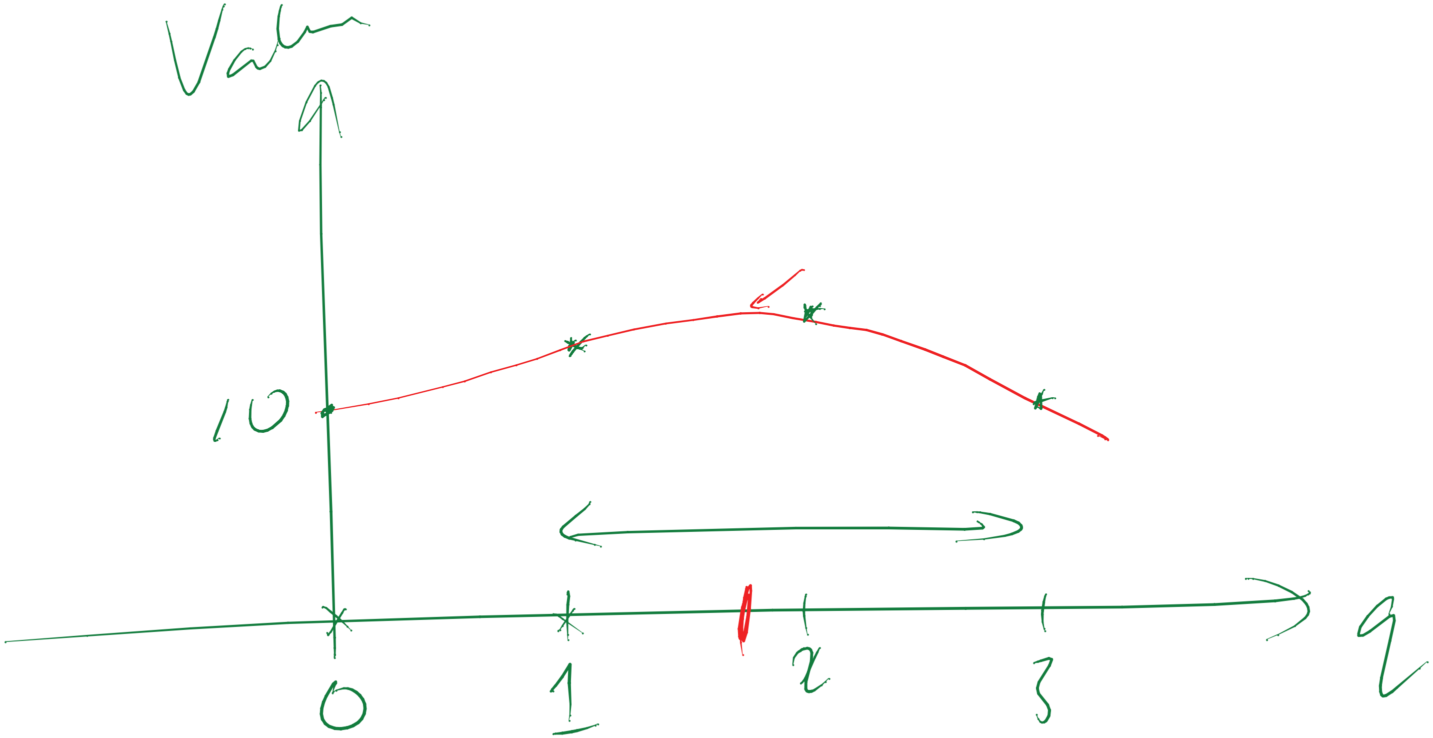
$q_1 \cancel{\tfrac{1}{2}}$ Cost 100 + $q_2 \cancel{\tfrac{1}{2}}$ Cost 120 = Cost Static Hedge.

optimize



$$\mathrm{Var}\left(-C^5\right) = \ldots$$

$$\mathrm{Var}\left(-C^5 + \frac{1}{2}V_1 + \frac{1}{2}V_2\right) - \frac{1}{2}\mathrm{Cov}\ldots - \frac{1}{2}\mathrm{Cov}\ldots =$$




→ Value(Barrier + 1 × valve) = 16 − 5 = 11

Value( .. + 2 × .. ) = 21½ − 10 = 11½

Value( .. + 3 × .. ) = 24½ − 15 = 9½

$$\mathrm{Value}(E) = \max_{q's}\left[\mathrm{SolveNLPDE}\left(E + \sum_{i=1}^{N} q_i V_i\right) - \sum_{i=1}^{N} q_i \cos t V_i(q_i)\right]$$

$q_3 = 1, \quad q_i = 0$

$$\text{Value}(V_3) = \max_{q\text{s}} \left[ \text{SOLVENLPDE}\left( V_3 + \cancel{\sum_{i=1}^{N} q_i V_i} \; 0 \right) - \cancel{\sum_{i=1}^{N} q_i \,\text{Cost}V_i(q_i)} \; \text{Cost}V_3 \right]$$

7.84 – 11.52

↓ 9.08

↓ 11.27