Equities

$$dS = \mu S\, dt + \sigma S\, dX$$

$$\Pi = V - \Delta S$$

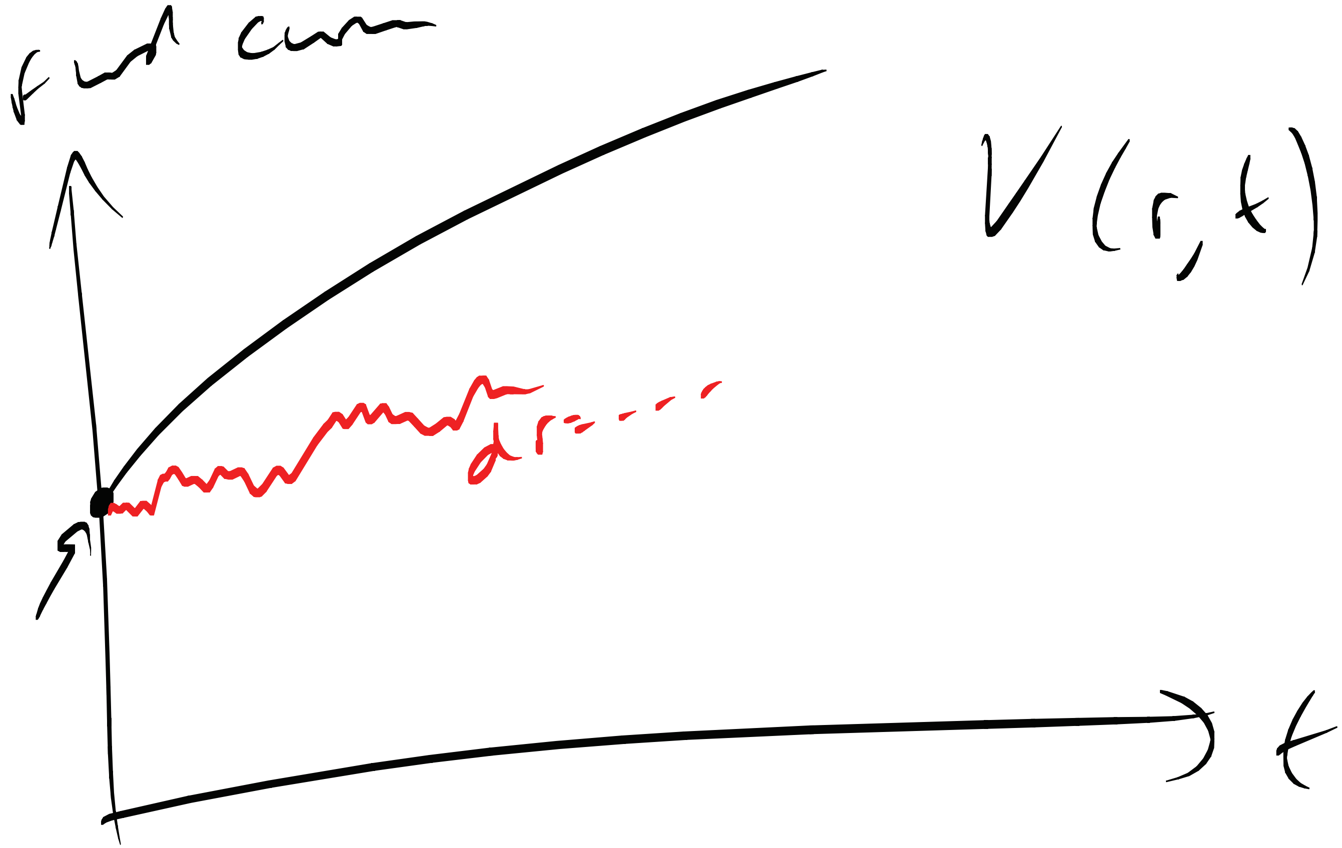
Fixed Income

$$dr = u\, dt + w\, dX$$

$$\Pi = V - \Delta r$$

not traded

(Aside: Bond = fixed income derivative)



fwd curve
V(r, t)
dr
t

← risk-neutral drift

$$\frac{\partial V}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 V}{\partial r^2} + (u - \lambda w)\frac{\partial V}{\partial r} - rV = 0$$

r is random

$$V = E^{+}\left[PV \text{ of Payoff} \right]$$

$$dr = (u - \lambda w)\,dt + w\,dX$$

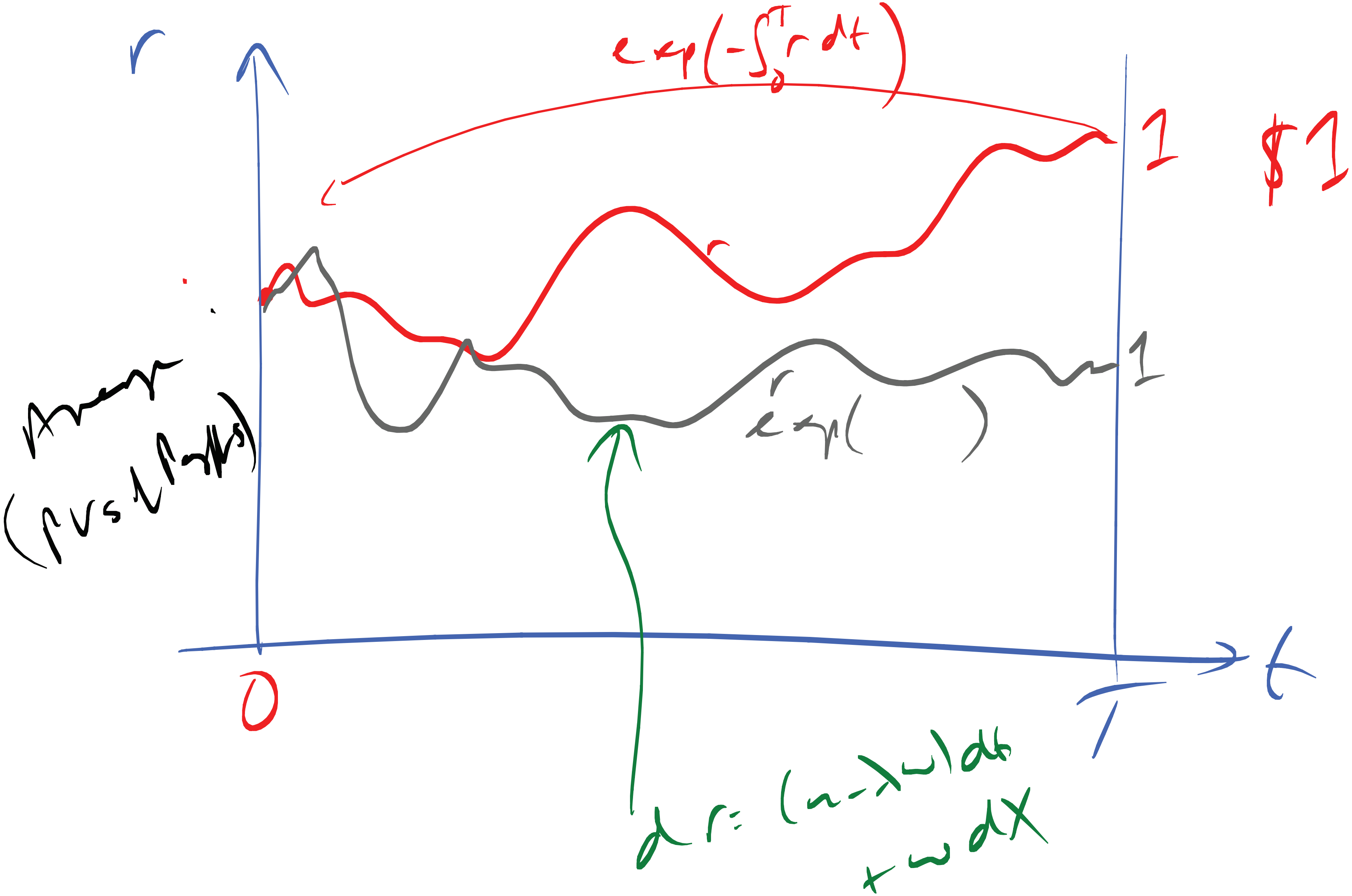
$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

r known

$$V = E^{+}\left[PV \text{ of Payoff} \right]$$

$$dS = r S\,dt + \sigma S\,dX$$

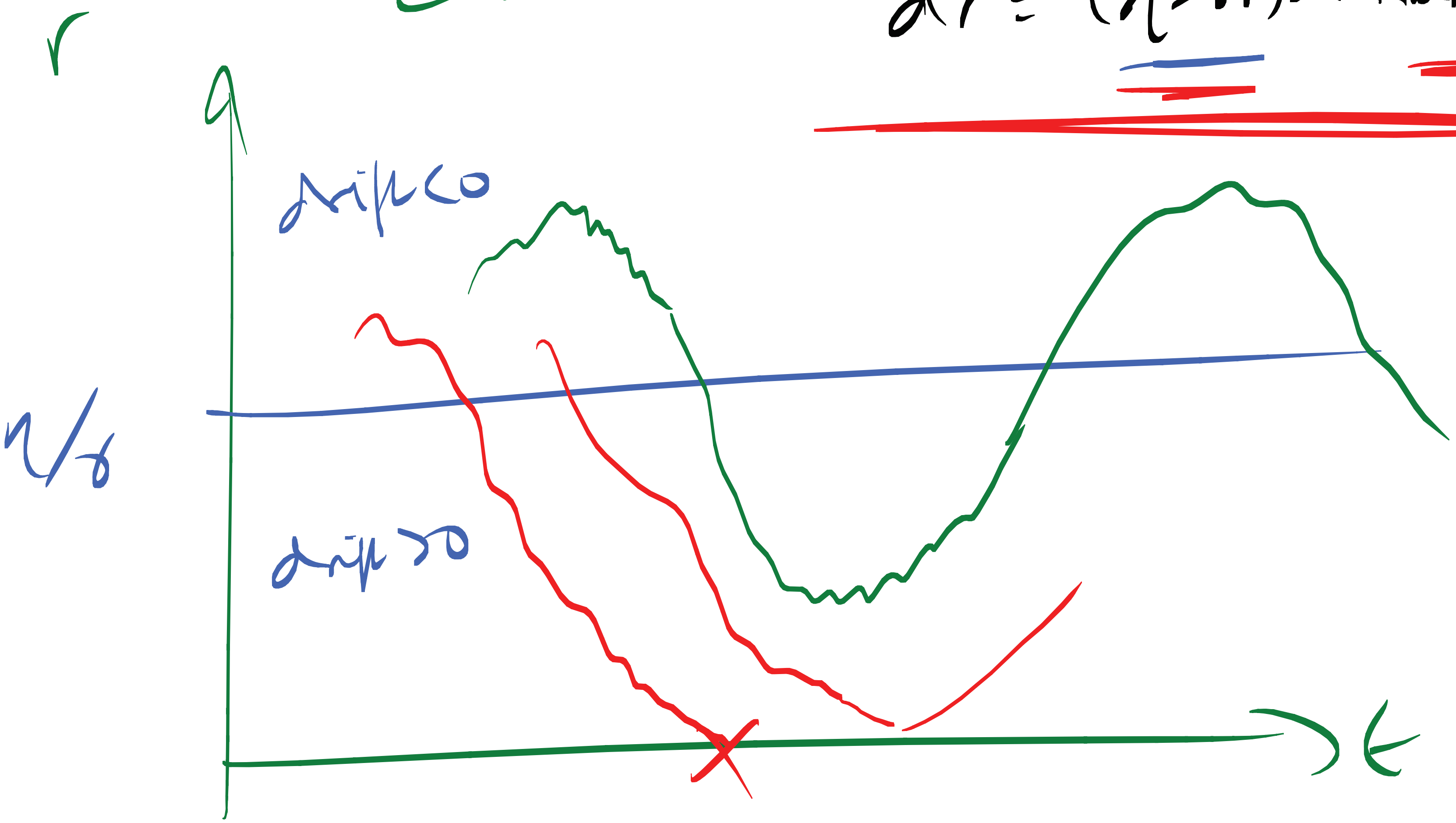
$$\cancel{\mu}$$



r
t
0
T
1
$1
r
$\exp(-\int_0^T r\,dt)$
exp(
1
Average (over all paths)
$dr = (u - \lambda w)dt + w\,dX$

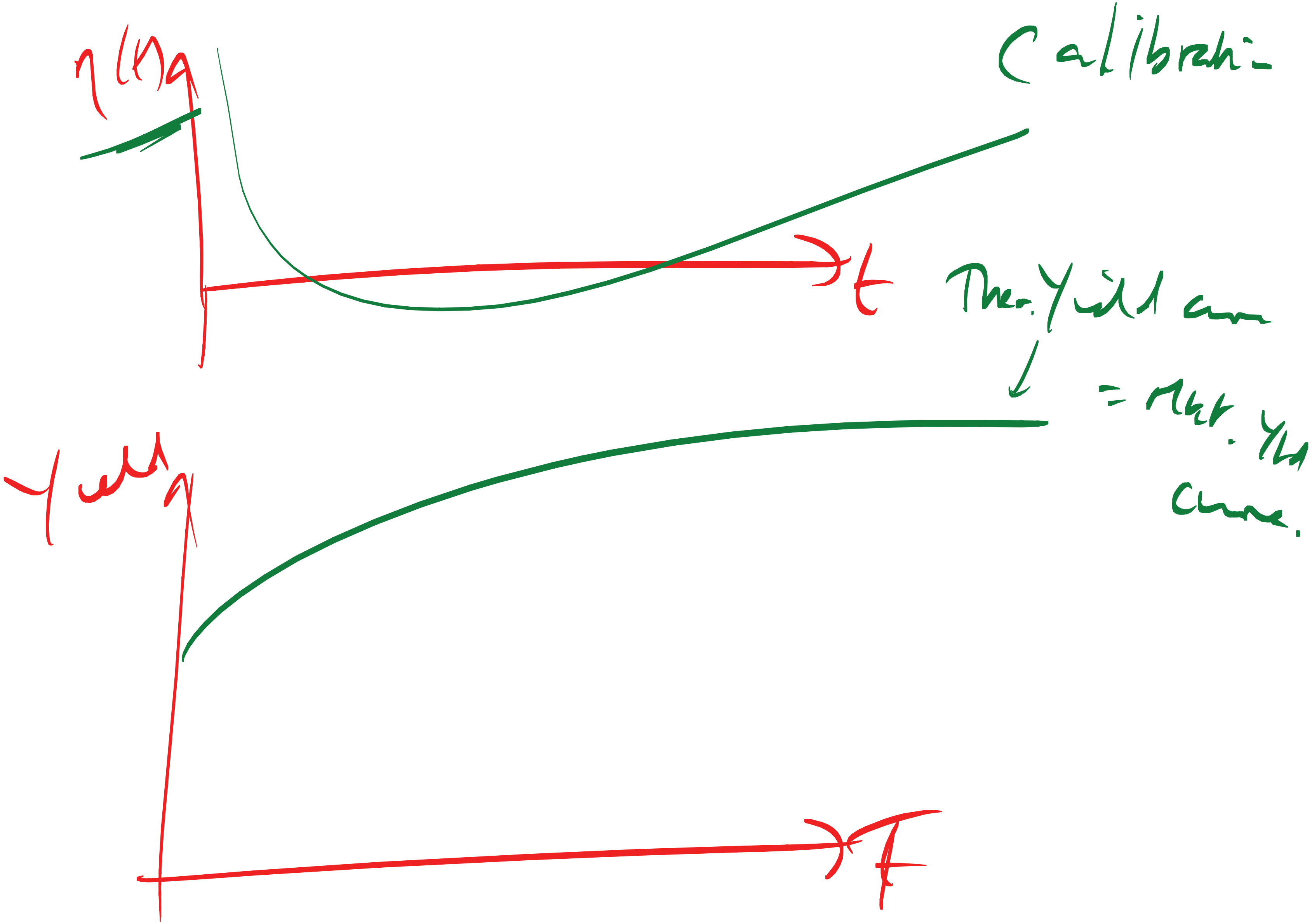
CIR

$$dr = (\eta - \gamma r)\,dt + \sqrt{\alpha r}\,dX$$



→ mean reverting

$r > 0$


r(t)
t
Calibration
Theo. Yield curve
= Mkt. Yld curve
Yield
T

$$V = e^{A(t) - rB(t)} \leftarrow \qquad \dot{} = \frac{d}{dt}$$

$$\frac{\partial V}{\partial t} + \frac{1}{2}\omega^2 \frac{\partial^2 V}{\partial r^2} + (u - \lambda\omega)\frac{\partial V}{\partial r} - rV = 0$$

$$(\dot{A} - r\dot{B})\cancel{e^{A-rB}} + \frac{1}{2}\omega^2 B^2 \cancel{e^{A-rB}} - (u-\lambda\omega)B\cancel{e^{A-rB}} - r\cancel{e^{A-rB}} = 0$$

$$\dot{A} - r\dot{B} + \frac{1}{2}\omega^2 B^2 - (u-\lambda\omega)B - r = 0$$

$$\frac{\partial}{\partial r}: \quad 0 - \dot{B} + \frac{1}{2}B^2\frac{\partial(\omega^2)}{\partial r} - B\frac{\partial}{\partial r}(u - \lambda\omega) - 1 = 0$$

$$\frac{\partial}{\partial r}:$$

$$\frac{1}{2}\beta\,\frac{\partial}{\partial r}(\omega^2) - \frac{\partial^2}{\partial r^2}\underbrace{(u-\lambda\omega)}_{XT!} > 0$$

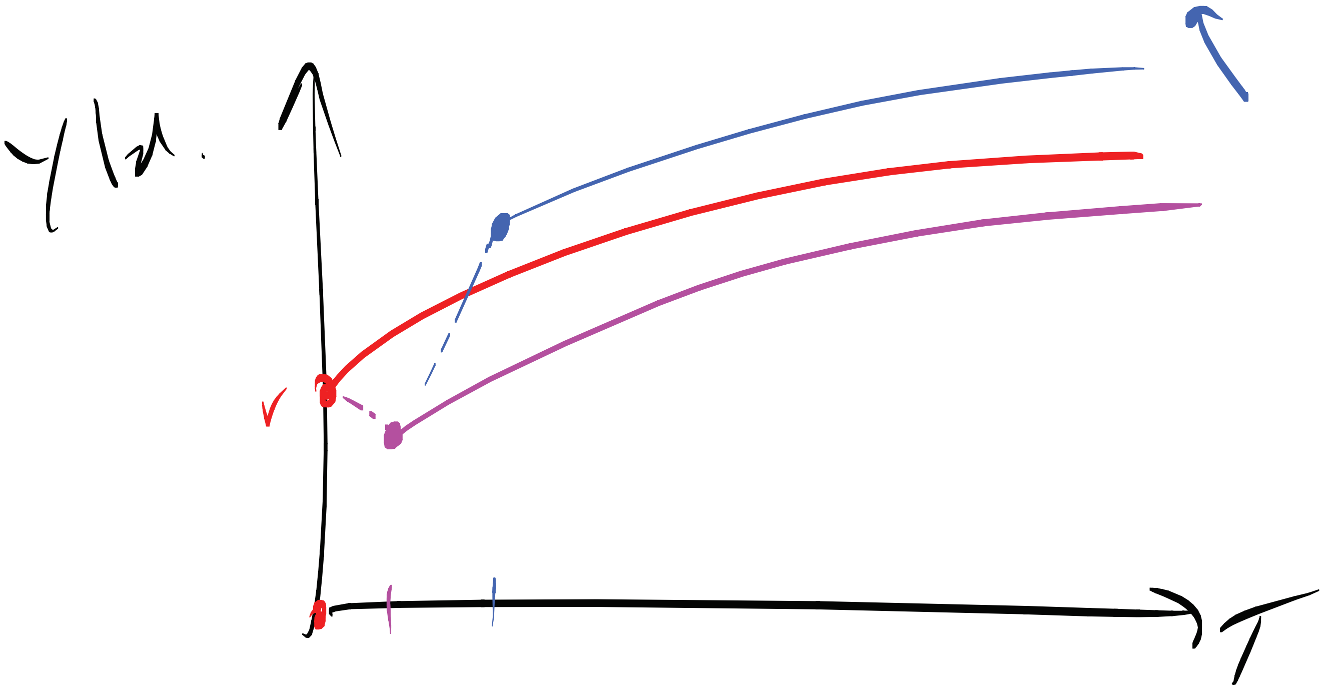
$4 \quad 0 \qquad = 0$

$dr = \underbrace{- - -}$

$\omega^2 = \text{lin. in } r$

$u - \lambda\omega = \text{lin. in } r$

$dr = \ldots\ldots dx$ $\qquad V(r, t)$



Don't panic!

Normal service will be resumed asap!

♡

$V(r, l, t)$

$\longrightarrow \quad \dfrac{\partial V}{\partial t} \quad - \qquad \dfrac{\partial V}{\partial l}\, dl \quad -\dfrac{\partial V}{\partial r}\, dr \quad - \dfrac{\partial^2 V}{\partial r^2} \qquad \dfrac{\partial^2 V}{\partial l^2} \qquad \dfrac{\partial^2 V}{\partial r \partial l}$

$\times 2 \qquad \times \dfrac{3}{2} \qquad\qquad 6$

$$L(V) - rV$$

$$= \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial r^2} + \frac{1}{2}\xi^2 \frac{\partial^2 V}{\partial \ell^2} + \rho\sigma\xi \frac{\partial^2 V}{\partial r \partial \ell} - rV$$

$$- \Delta_1 \left(\quad V_1 \quad \right)$$

$$- \Delta_2 \left(\quad V_2 \quad \right) = 0$$

$$\boxed{\begin{aligned} 3x - 4y + 7 &= 0 \\ 2x + 17y - 6 &= 0 \end{aligned}} \quad x, y$$

$$-4x + 2y + 1 = 0$$

$$\begin{pmatrix} 3 & -4 \\ 2 & 17 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ 6 \end{pmatrix}$$

Phase plane analysis

$\boxed{\text{Jordan \& Smith}}$

$$dr = (a_1 + b_1(l - r))\,dt + \cancel{\sigma_1 r\, dX_1}$$

$$dl = l(a_2 - b_2 r + c_2 l)\,dt + \cancel{\sigma_2 l\, dX_2}$$

$$dr = (a_1 + b_1(l - r))\,dt$$

$$dl = l(a_2 - b_2 r + c_2 l)\,dt$$

$$\frac{dl}{dr} = \frac{\overbrace{l(a_2 - b_2 r + c_2 l)}}{a_1 + b_1(l - r)}$$

$$0 = a_1 + b_1(l - r)$$