2. **Using market data for European options with the same expiry but different exercise prices, find and plot the implied volatilities. What shape does the plotted curve have?**

For European call options,

$$C_{BS}(S,t;E,T) = SN(d_1) - Ee^{-r(T-t)}N(d_2),$$

where

$$d_1 = \frac{\log(S/E) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$

and

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

If we know the value of an option, then we can work out its implied volatility (as this is the only unknown left in the equation). It is easy to do this using an iterative procedure (or for instance, using Solver in Excel). We find the value of the volatility such that our Black–Scholes formula gives the observed price. Figure 22.4 shows the implied volatilities for call options with various strike prices. The shape of this curve is called the smile.
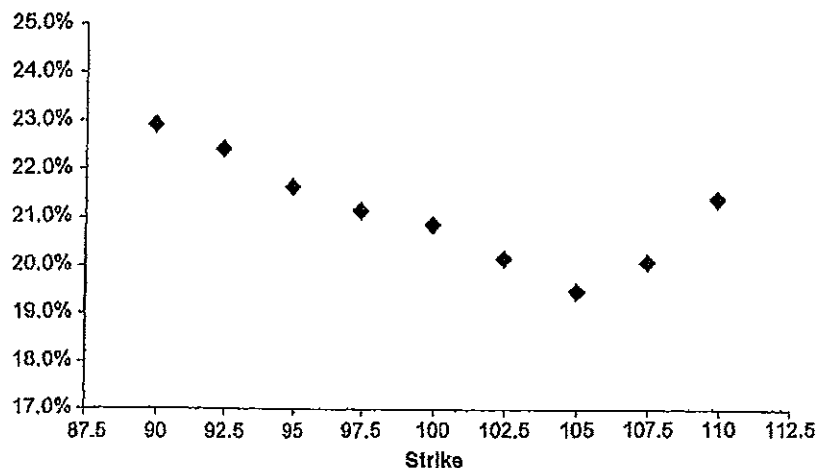

Figure 22.4: Implied volatilities against strike price