Convertible Bonds

In this lecture. . .

- the basic Convertible Bond (CB)
- market conventions for the pricing and analysis of CBs
- converts as options
- CB arbitrage
- pricing convertibles

By the end of this lecture you will

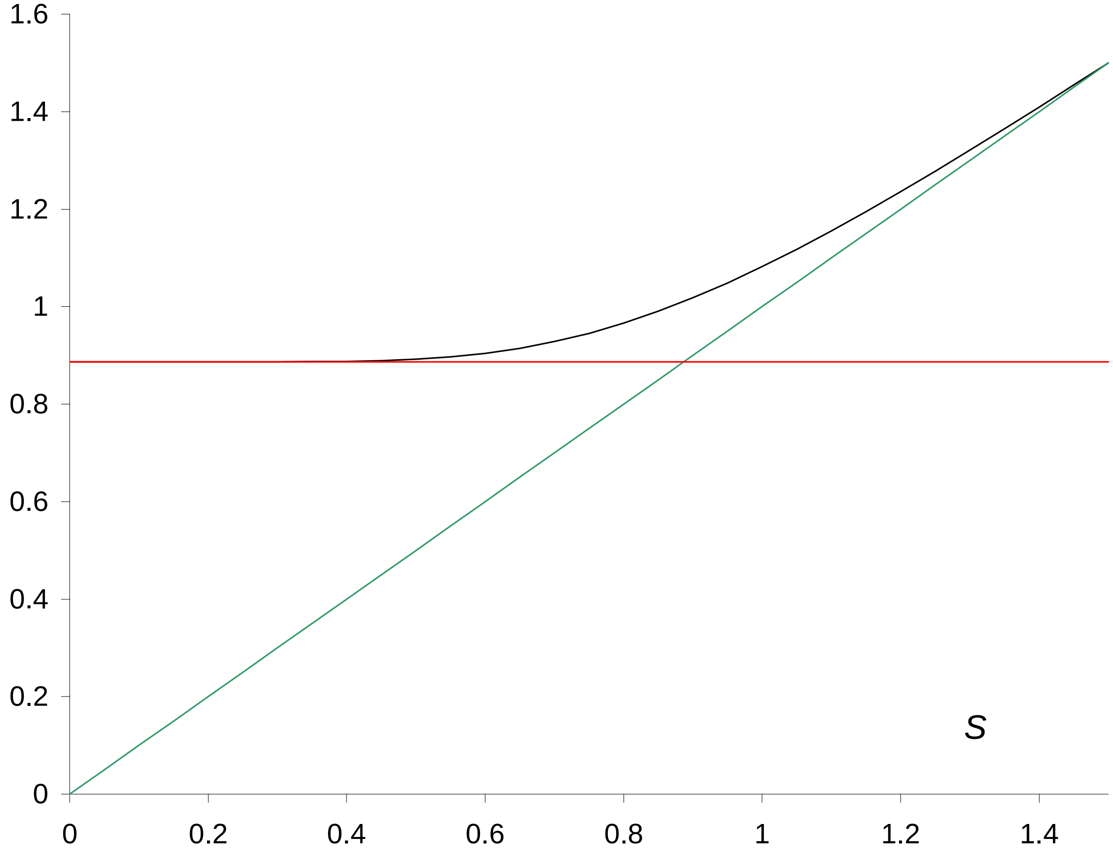
- know what convertible bonds are used for
- know most of the common features to be found in convertible bonds
- be able to find their value and greeks

Introduction

The **convertible bond** or **CB** or **convert** is a contract that may at a time of the holder's choosing be exchanged for a specified number of the issuer's stock. In the meantime it pays coupons.

A convertible bond thus has the characteristics of an ordinary bond but with the extra feature that the bond may be exchanged for a specified asset. This exchange is called **conversion**.

Converts sometimes behave like a bond and sometimes like a stock. Their price is greater than both.



Convertible bond basics

The convertible bond has characteristics that make it sometimes behave like stock, and sometimes like a bond.

The conversion feature of convertible bonds also makes these contracts similar to American options. The question of when to exercise an American option is very similar to the question of when to convert a convertible bond.

It is this 'optionality' that adds value to the convertible bond.

On conversion, new shares are issued.

The CB is a **hybrid** instrument since it has features of both equity and debt.

The issuers of CBs

- CBs are issued by corporations

- These companies are often not of the highest quality, in terms of credit risk

By selling bonds which can be later converted to equity the issuer can get away with a lower coupon than might otherwise be expected.

Why issue a convertible?

Corporations in need of capital have many choices available to them.

These choices have two common themes: Issue equity; Issue debt.

Issue equity: Dilutes earnings per share but has low initial financing costs.

Issue debt: Does not dilute earnings per share but may have high initial financing costs.

The other possibility is to issue a hybrid instrument with both of these features, the Convertible Bond.

- The bond can be sold with a lower coupon that a plain bond with the same maturity and price. (Or equivalently, it can be sold for a higher price with the same coupon.)

- It does not dilute earnings per share, until the bond is converted and new shares are issued.

- If the bond is converted, the principal does not have to be repaid.

The typical issuer of CBs has high cash requirements, perhaps with a very rapid use of cash and low credit quality.

They may be startups.

In the US only 30% of the CB market is Investment Grade (but rising). In Europe and Japan this figure is 85%.

Examples: Technology/Media/Telecommunications (TMT) and Biotechnology companies, high burn rate (use of cash), high volatility, high risk of default. (High volatility increases price of CBs, high risk of default lowers the price.)

Why buy a convertible?

- Upside participation with downside protection. (Unless there is a default.)

- Coupon is typically greater than the dividend yield of the equity.

- Some investors may be barred from participating directly in the equity market. The debt nature of the instrument may make it appealing.

- Legally rank, being debt securities, above equity in case of default.

Some statistics

- Globally, capitalization of CBs is \$500 billion

- 400 hedge funds focused on CB arbitrage (out of 7000 hedge funds in total)

- CB arbitrage is the third best (since 1993) hedge fund strategy after Equity Market Neutral and Event Driven, with a Sharpe ratio of 1.04. (NB S&P500 Sharpe ratio was 1.18 over same period.)

- Hedge funds hold 70% of CBs

Some important definitions

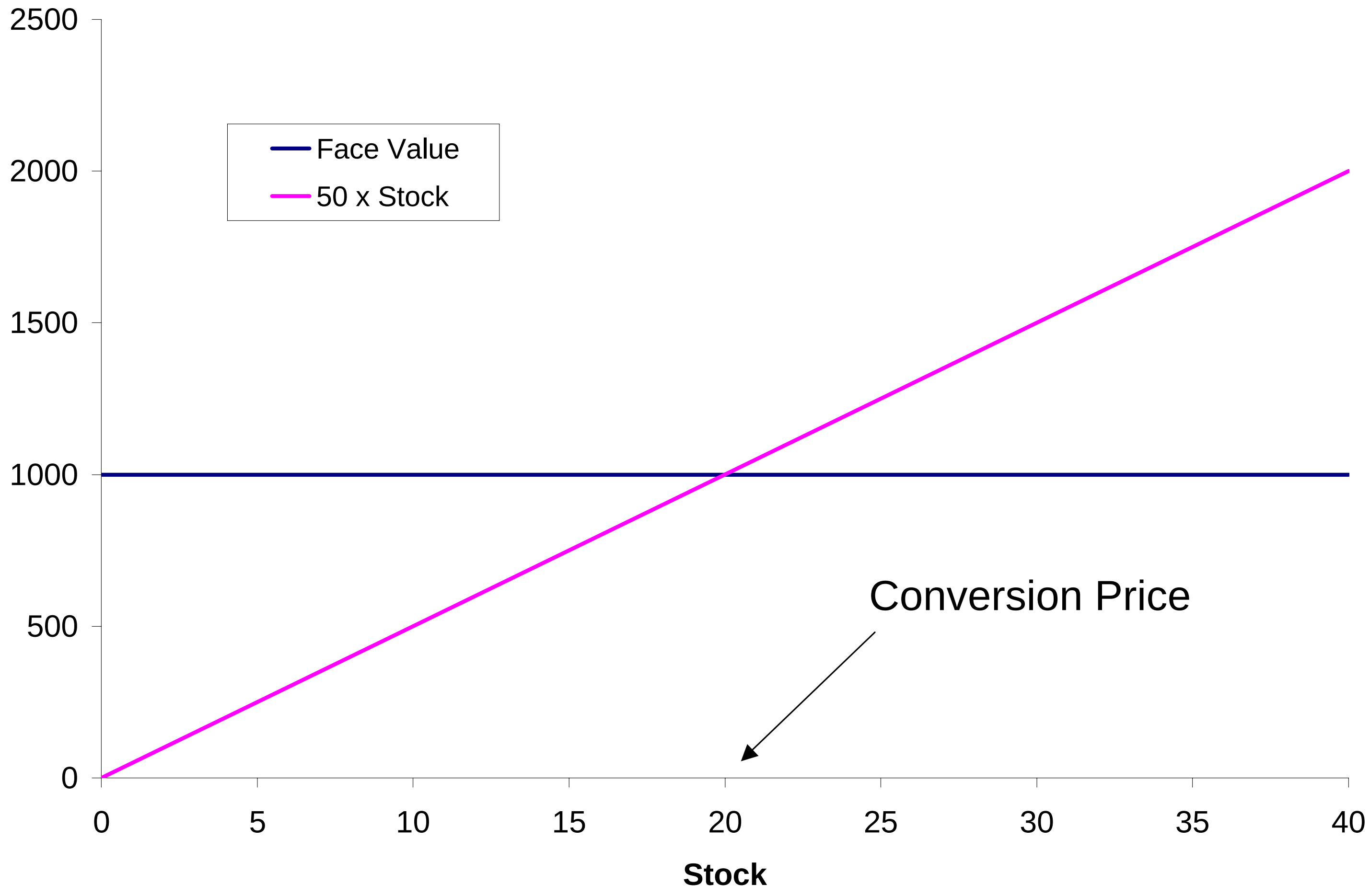
Face value: The bond principal.

Conversion ratio: The number of stock into which the bond may be converted.

Conversion Price: Ratio of face value to conversion ratio.

Example: The face value is \$1000 and the conversion ratio is 50. The conversion price is therefore

$$= \frac{1000}{50} = 20.$$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Face Value
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Conversion Price
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Stock

The value of a CB is clearly bounded below by both

- its **Gross Parity**, which is the amount received if the bond is converted immediately (regardless of whether this is optimal)

  $$\text{Gross Parity} = \text{market price of stock} \times \text{conversion ratio}.$$

  Example: Stock is at \$22, conversion ratio is 50, Gross Parity is

  $$22 \times 50 = 1100.$$

- its value as a corporate bond, with a final principal and coupons during its life. This is called its **straight value**.

The latter point shows how there are credit risk issues in the pricing of CBs. Here we assume that there is no risk of default.

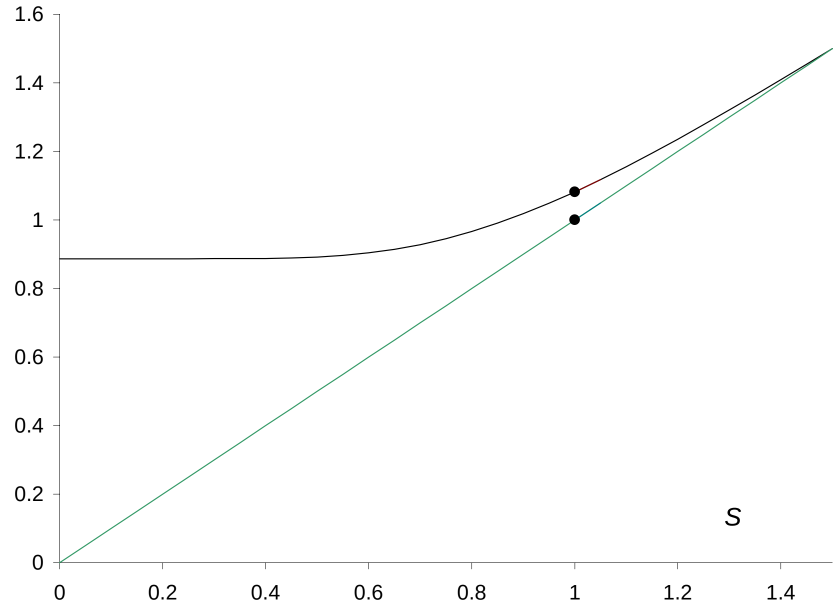
Conversion premium

$$\text{Conversion premium} = \frac{\text{Market value of CB} - \text{Gross parity}}{\text{Gross parity}}.$$

This is measured as a percentage.

It is the extra value in the CB above what you would get by converting now.

It is a measure of how much the bond component adds to the contract above converting immediately.



(Here the conversion ratio has been chosen as 1, and face value as 1.)

$$\text{Conversion premium} = \frac{1.0819 - 1}{1} = 8.19\%.$$

Break-even Time

Hold the CB and you will receive coupons. Hold the underlying stock and you will receive dividends.

Ignoring conversion, how long before holding the CB becomes more profitable than holding the equity?

This is the **Break-even Time**.

$$\text{B.E.T.} = \frac{\text{Conversion premium}}{\text{Coupon rate (CB)} - \text{Dividend yield (equity)}}.$$

Example:

A CB has a conversion ratio of 10, it has a coupon of 6% per annum. Its market value is \$260.

The equity is trading at \$23. It has a dividend yield of 2% per annum.

What is the Break-even Time?

The Gross parity is $10 \times 23 = 230$. Therefore the Conversion premium is

$$\frac{260 - 230}{230} = 13\%.$$

Thus

$$\text{B.E.T.} = \frac{13}{6 - 2} = 3.25 \text{ years.}$$

Important considerations

1. Contract terms, conversion, callability, putability etc.

2. Behavior of underlying, e.g. volatility

3. Credit risk, especially for lower stock prices

4. Interest rate exposure

Converts as options

The ‘payoff’ for a convertible is similar to that for a vanilla call option. In some special cases the convertible can be decomposed into a pure bond and a vanilla call on the stock.

For this to be valid the CB cannot be callable or putable. To make things simple let’s also assume that there are no dividends on the stock.

Step 1: Calculate the value of the straight bond component

Step 2: Calculate the strike of the vanilla call

Step 3: Use Black–Scholes to calculate the value of the option component

Example:

- Five-year CB, no call or put, principal \$1000
- 2% coupon, paid in two semi-annual instalments
- Conversion ratio of 20
- Stock at \$47
- Five-year continuously compounded interest rate 5% (equivalent to 5.06% semi-annual)
- Stock volatility 22%

Step 1: Straight bond component

The present value (at a rate of 5% continuously compounded) per annum of the \$1000 is \$786.59. The present value of all the coupons adds up to \$79.59.

The value of the CB as a straight bond is therefore \$866.18.

Step 2: The strike

The effective strike is simply the principal divided by the conversion ratio:

$$\frac{1000}{20} = 50.$$

Step 3: The option

What is the value of a call option with a strike of \$50 expiring in five years when the underlying is \$47, the volatility is 22% and the risk-free rate is 5%?

Plug these numbers into a Black–Scholes calculator and you will get \$12.97 per share, or \$259.40 for the 20 shares.

Therefore the value of the CB is the sum of the straight bond and option components:

$$866.18 + 259.40 = 1125.58.$$

Note that the Delta of the option is 0.74 so the CB is behaving more like a stock than a bond.

Optimal conversion

Just as American options can be exercised before expiry, so can convertible bonds be converted to stock before maturity.

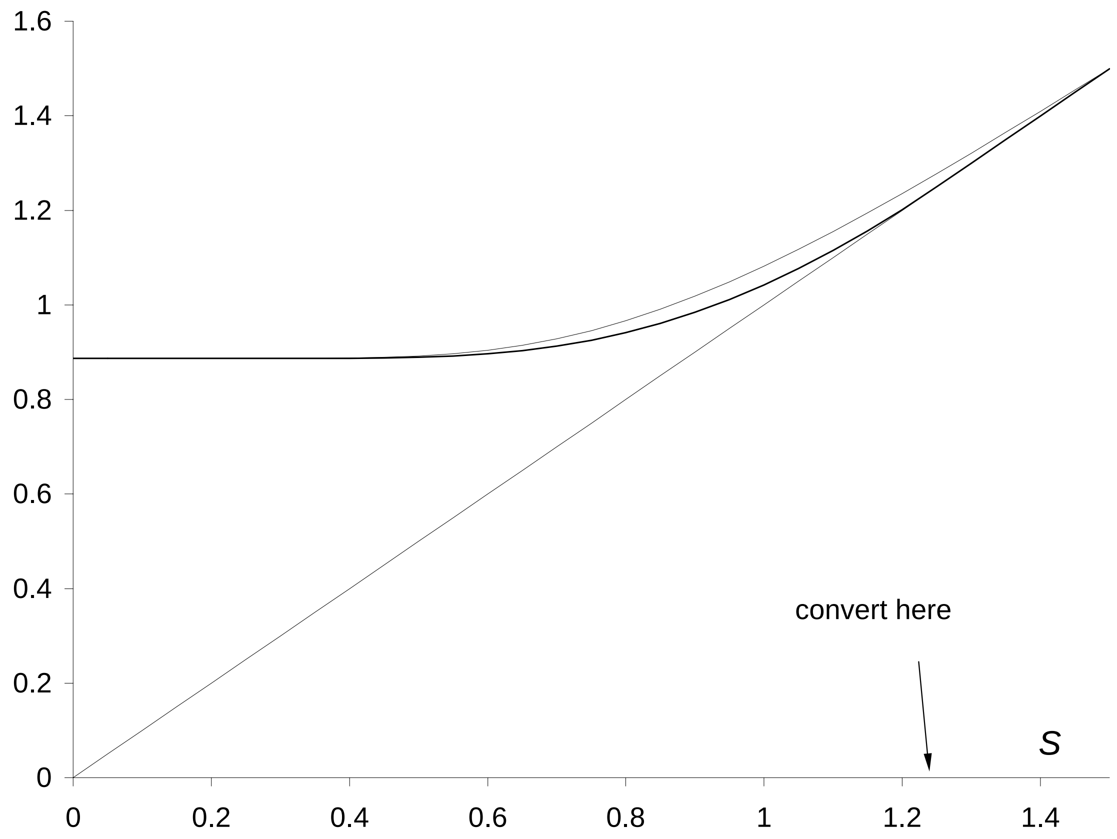
This conversion is usually associated with an underlying stock having a dividend and the stock value being high enough.

The reason for conversion is that you would rather take the stock's dividend than the bond's coupon. However, the relevant calculation is not that simple. Again, think in terms of American options, it is not that easy to determine *when* it is best to exercise.

The calculation is usually performed by finite-difference methods or via trees.

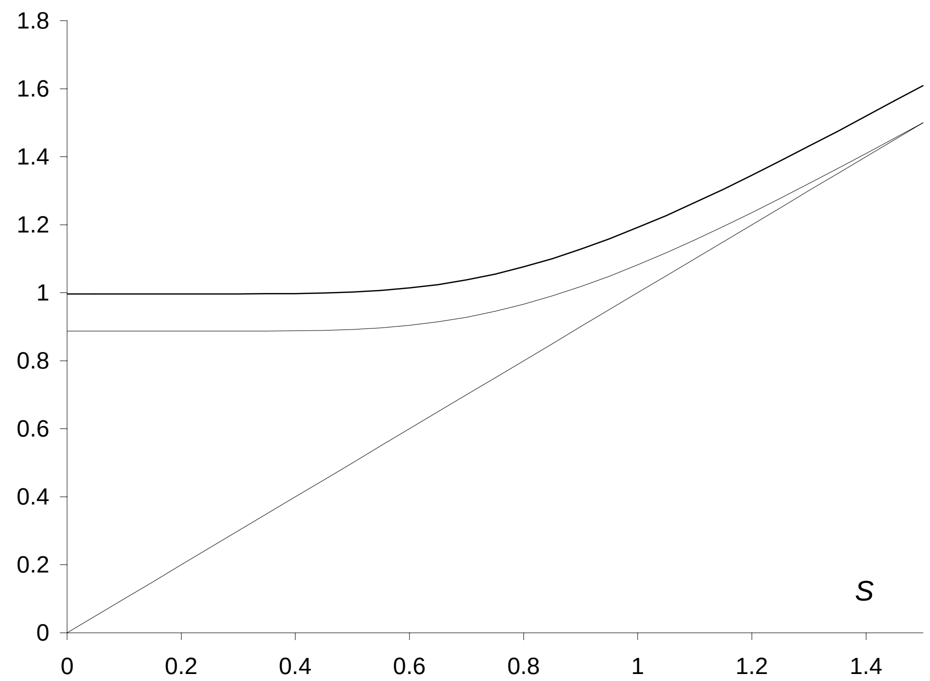
Note also that if the bond is both callable and convertible the bond issuer can force conversion by announcing its intention of calling the bond.

All things being equal, a CB on a stock paying dividends has a lower value than a CB on one not paying dividends. Conversion is therefore more likely.



The effect of coupons

It is clear that the greater the coupon the higher the value of the bond.

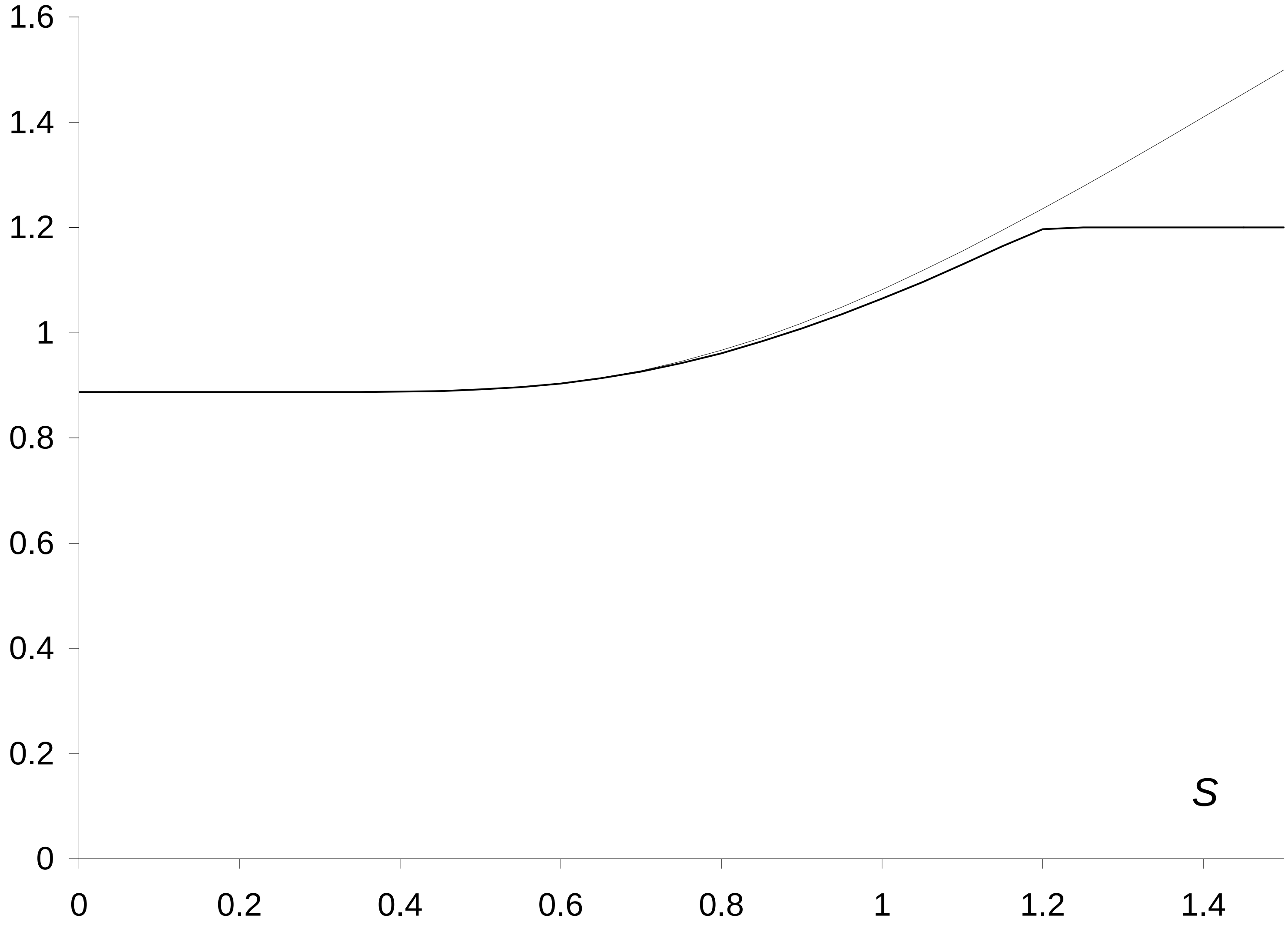


Callable converts

The **Call** feature allows the issuer to call back or redeem the convertible before maturity. The issuer pays the holder the **Call price**. The dates on which the bond may be called back will be specified in its term sheet.

The value of the CB is decreased by the call feature.

As with conversion, the question of when to call back the bond is not a simple one to answer.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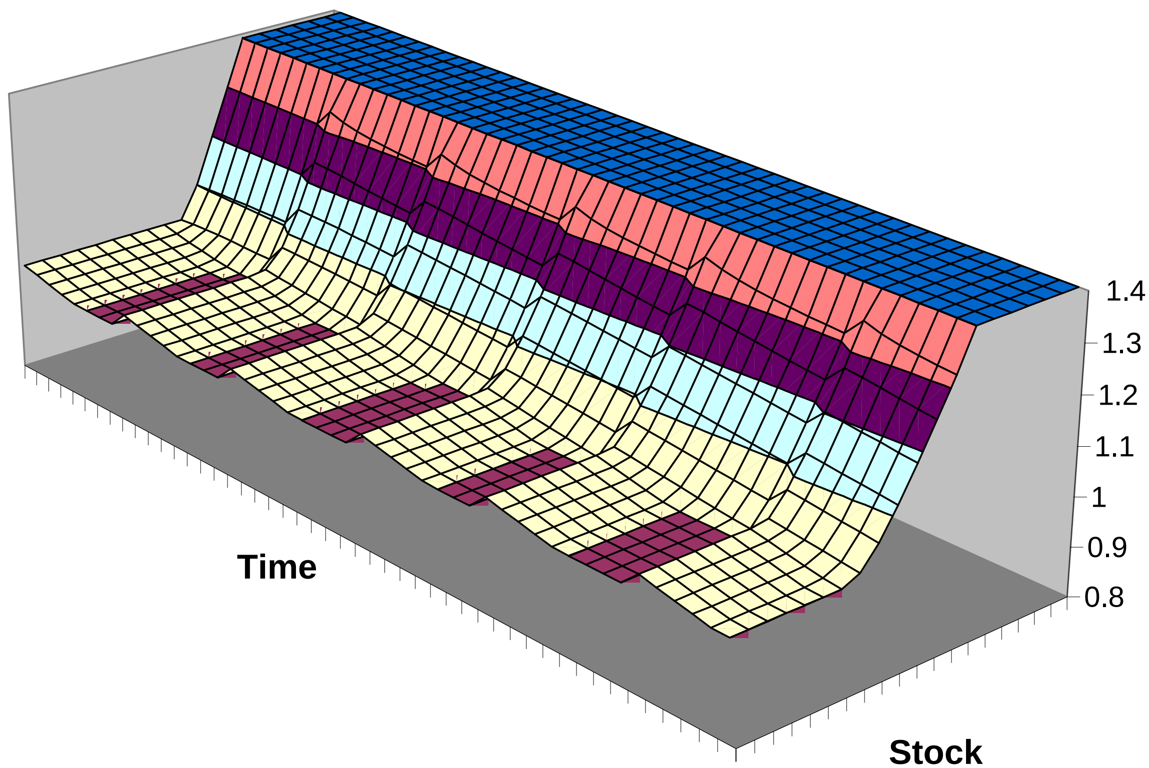
Interpret the following plot of value of CB against underlying asset and time.



Solution: That CB has the following features:

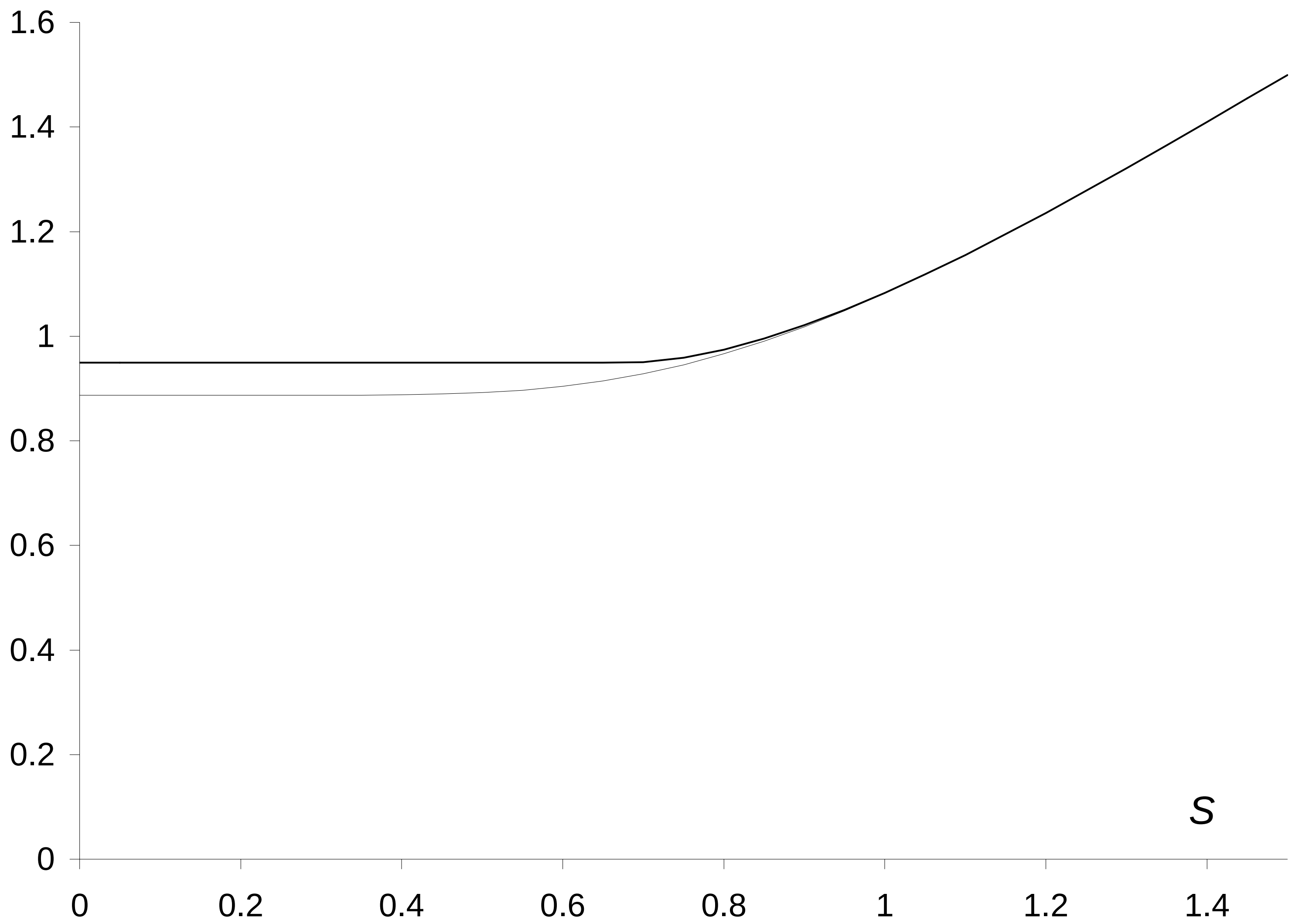
- Conversion at any time
- Call at any time
- Discretely paid coupons

Putable converts

The call feature is a right/option held by the issue, as such it *decreases* the CB's value.

The **Put** feature is a right/option of the bond holder. It allows them to put the bond back to the issuer. It *increases* the CB's value.

The dates on which the bond may be put back will be specified in its term sheet.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Hedging and the Greeks

CBs are nonlinear in the underlying stock. Therefore it is important to be able to measure and understand the various Greeks.

The Greeks are sensitivities to various quantities.

They are useful in risk management, in hedging and in the exploitation of arbitrage opportunities.

The most important Greeks are Delta, Gamma, Theta and Vega.

Delta Δ: This is the sensitivity of the CB to the stock price.

By how much does the CB change in value if the underlying move \$1? This is the Delta.

Buy one CB and sell Delta of the underlying asset to eliminate (most) exposure to the movement in the stock, market risk.

Subtleties of delta hedging:

- Cost of borrowing

- Feedback effects

- Correlation between stock price and volatility (and credit)

Gamma Γ: This is the convexity or curvature in the CB. It is also the sensitivity of the CB's Delta to movement in the underlying.

It tells you how often you will have to rehedge to eliminate market risk. When Gamma is large you will have to rehedge frequently.

If you are delta hedged then your exposure to the market is via Gamma. Positive Gamma means that you will benefit from large stock moves. Negative Gamma means that you will suffer if the market makes a large move.

Long CB positions are usually Gamma positive. However, there are exceptions due to the existence of call features or credit risk.

Theta Θ: This is the sensitivity of the CB to time. If the underlying stock does not move it tells you by how much your CB will change in value with the passing of time.

Vega: This is the sensitivity of the CB value to the stock volatility.

This is important to know because CB arbitrage strategies often involve estimating volatility.

Credit risk

CBs are issued by corporations as a way of raising capital. There is always the question of how likely the company is to pay the coupons on the bond or the principal at maturity. CBs have an important element of **credit risk**.

Risk of default lowers the value of the bond.

There are several ways of modeling and interpreting this risk.

We will look at

- Credit spread

Credit spread: A simple default model

Earlier we saw how to value a simple CB (without coupons, call or put features) as a straight bond plus a call option.

In that example we concluded that the fair value of the CB was \$1188.98.

Suppose the market's value for this bond was in fact \$1079.76. How can we reconcile these two valuations?

One way is to assume that the market has priced in the possibility of default.

The simplest way to model default is via a **probability of default** or **hazard rate**.

This says that if at time t the company has not defaulted then the probability of default between times t and $t+dt$ is $p\,dt$ where p is the **instantaneous risk of default**.

The result of this model is that when present valuing risky cash-flows (such as coupons, principal or even the stock after conversion) you should add p to the risk-free interest rate used for discounting:

$$\text{Risky interest rate } = \text{Risk-free rate } + p.$$

We can go back to our example and ask the question "What p must be used so that our theoretical value and the market's value are the same?"

This is the implied risk of default. In the above example it turns out that $p = 2\%$.

We can use this implied risk of default to measure the **relative value** of bonds issued by this company, or across companies (perhaps with the same credit rating).

This is done by converting all bond prices to implied risks of default and seeing which ps are large, which small.

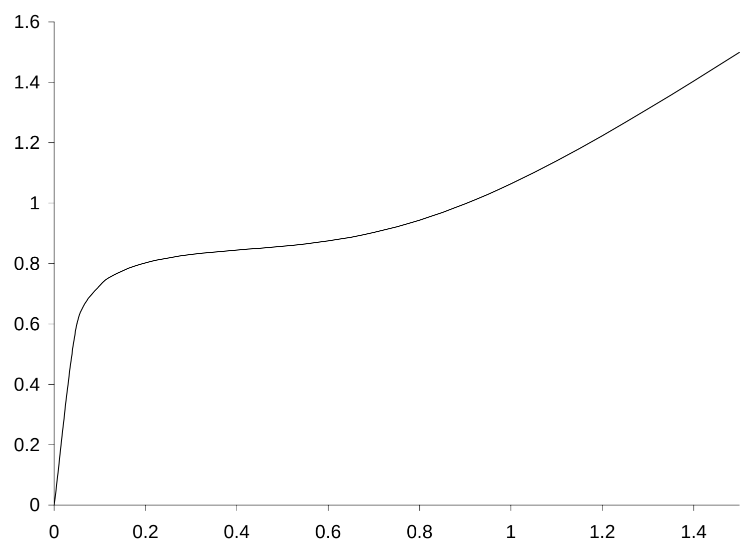
This model also introduces another 'Greek,' the sensitivity of the bond price to the risk of default. This Greek is used for determining how sensitive the CB price is to the risk of default.

A long/short strategy might involve a convergence of one or more risks of default.

Default depending on stock price

Modern models allow for risk of default to depend on the level of the stock price. For obvious reasons, the lower the stock price the more likely the risk of default.

This can be modeled via a risk of default function $p(S)$. The analysis is quite complicated but result in pictures such as the following.



Dilution

In reality, the conversion of the bond into the underlying stock requires the company to issue new shares in the company. This contrasts with options for which exercise leaves the number of shares unchanged.

We've seen a simple example of the use of the Black–Scholes model for valuing a CB. But Black–Scholes is only valid when exercise does not affect the underlying stock.

- On conversion the number of shares increases. This is called **dilution**.

Notation:

- N = number of old stock
- V = value of CB
- Q = quantity of CBs issued
- n = conversion ratio for each CB
- A = assets of the company
- σ_A = volatility of the assets
- S = stock price
- σ_S = volatility of the stock price

Before CBs are converted into stock:

Owning the stock gives you a share in the company's assets less its liabilities. The CBs count as liabilities.

$$\text{OldCo} = A - QV = NS.$$

After CBs are converted into stock:

Owning the stock gives you a share in the company's assets. There are no longer any liabilities. But there are more stocks. More people share a bigger pot.

$$\text{NewCo} = A = (N + Qn)S.$$

Volatility of the stock before conversion:

$$\sigma_A A = (N + Q\Delta)\sigma_S S,$$

where Δ is the Delta of the CB.

Volatility of the stock after conversion:

$$\sigma_A A = (N + Qn)\sigma_S S.$$

At this point we have to decide which volatility we are measuring, which is the most stable?

It is reasonable to assume that the volatility of the assets is not affected by conversion.

Therefore the volatility of the stock does change at conversion.

- Because $\Delta < n$ before conversion the actual volatility of the stock is *higher* than an equivalent company with no outstanding CBs.

Interest rate risk

CBs usually have a long lifespan, much longer than options for example. Therefore there is some sensitivity to interest rates.

It is common to price CBs using a term structure of interest rates, and to examine sensitivity of the price to parallel shifts in the yield curve.

Sometimes a stochastic interest rate model is used, but this requires solving by tree or finite-difference methods.

Aside: It is possible to hedge interest rate risk using risk-free government bonds.

Under 'normal' market conditions this may be successful.

However, in times of market turmoil the government bonds are seen as a safe haven. The flight to quality and away from lower quality investments will result in the CB hedger losing on *both* their CB (as investors sell and as credit spreads widen) and on their short (hedging) bond position as investors buy.

Other features in converts

Step up coupons: The coupon rate varies deterministically with time in step up/down coupon bonds. The coupon structure is specified in the term sheet.

Conversion into shares and cash: Some bonds are convertible into a combination of shares and cash. The conversion ratio and the cash amount may vary as a function of time.

Protected call and unprotected call: A bond may become callable by the issuer from a specified date. This is known as an **unprotected call**. In the **protected call** the bond may only be called if the share price (or the average share price over the past specified number of days) is above a certain level. Many bonds have some unprotected calls and some protected calls.

Path dependence in convertible bonds: A bond callable when the underlying has been above a specified level for a specified number of days is path dependent. Another form of path dependency is to set the conversion to be a specified function of the underlying asset value on some specified date.

Convertibles arbitrage

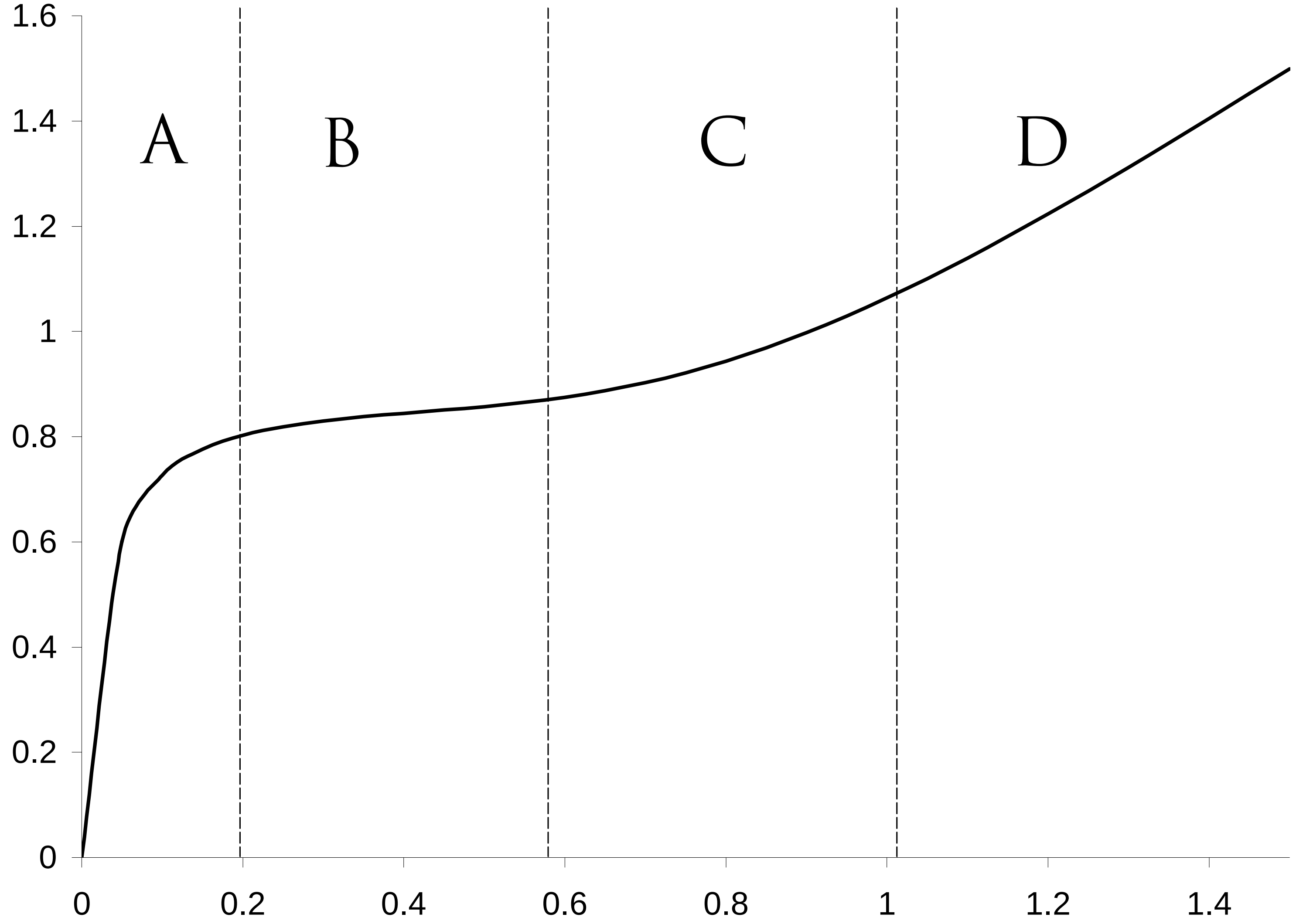
Hedge funds hold 70% of CBs. CB arbitrage is a very popular hedge fund strategy (400 out of 7000 hedge funds).

Hedge funds provide increased demand and liquidity for CBs, both new issues and the secondary market.

Aside: There is a problem with so many convertibles being in the hands of hedge funds. Hedge funds typically sell the underlying short in order to hedge the market risk in the CB. If the ability to borrow the issuer's stock becomes hampered then many hedge funds might simultaneously have to sell their, now unhedgeable, CBs.

Basic principles

- Market neutral
- Produce ‘standstill income’ from the coupon
- Exploit pricing discrepancies between the CB and the company’s stock/options
- Capture profits from stock movements (regardless of direction), volatility strategies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Region A: Benefit from an improvement in credit quality of the issuer. Or from a rise in stock value.

Region B: Bond behaving like a (risky) bond.

Region C: The domain of volatility strategies. Exploit actual volatility or (relative/absolute) movements in implied volatility.

Region D: Bond behaving like the underlying stock.

CB arbitrage strategy

Convertible bond arbitrage will involve one or both of the following factors:

- A view on the volatility implied by the CB price, in comparison to actual volatility and/or implied volatility of exchanged-traded options on the same stock

- A view on the credit-worthiness implied in the bond price, in comparison with historical, fundamental or similar companies

Question: How can you deduce an implied volatility and an implied risk of default?

Deal structure 1: Delta hedging the CB

Buy the CB and hedge with the underlying

This trade extracts the difference between actual volatility and implied volatility. (In fact, the difference between the squares of these quantities.)

Details:

- Buy the CB

- Delta hedge with the underlying. This Delta hedge must be maintained and constantly revised.

- The strategy produces standstill income: Bond coupon less dividends on the underlying (the short stock position)

- For the short sale of the underlying stock, the CB holder must borrow stock. (The cost of stock borrowing is similar to the effect of dividends on the underlying and reduces the value of the CB.)

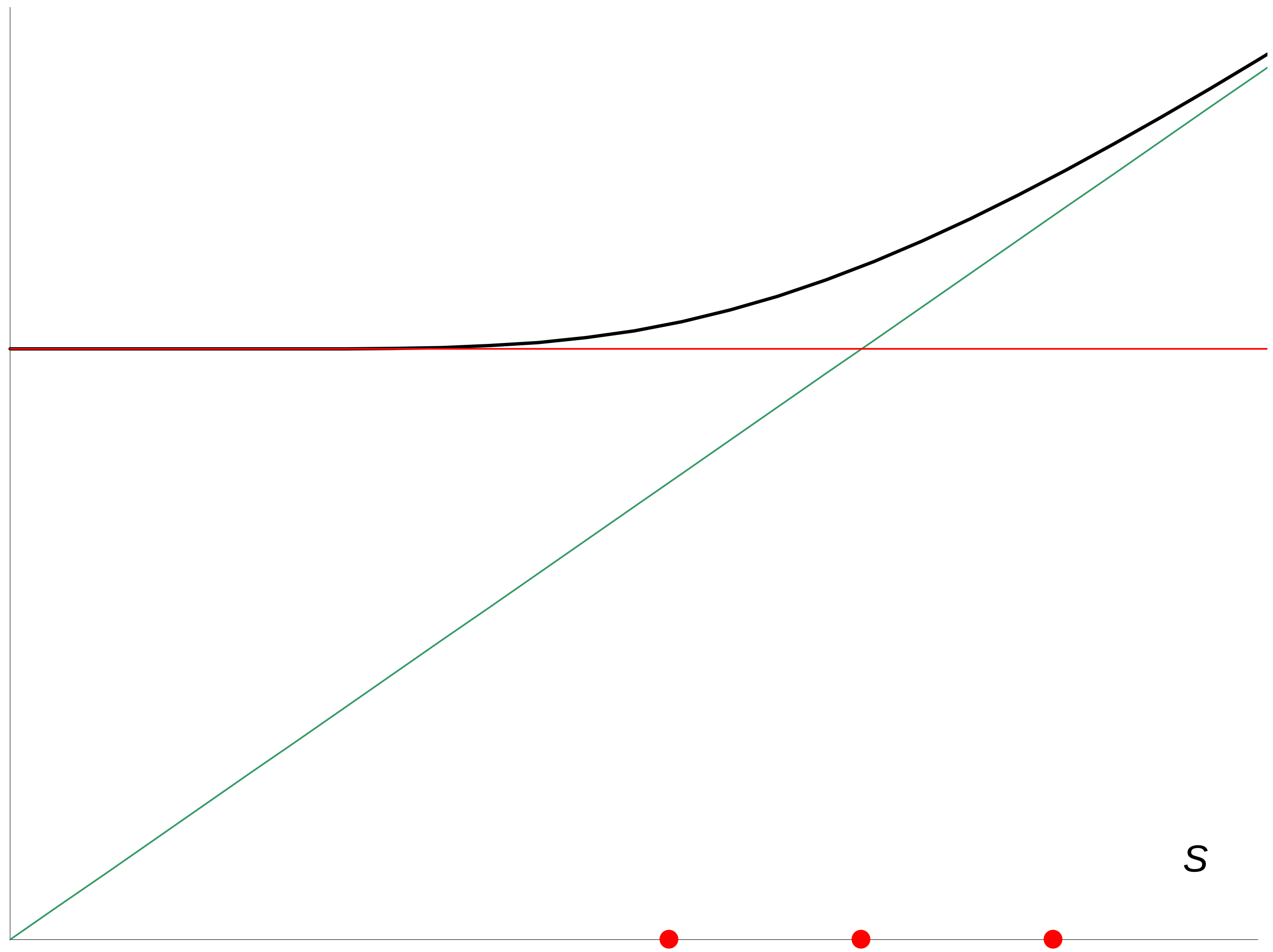
Risks:

- Costly
- Delta hedging is never perfect
- Incorrect estimate for actual volatility
- Large fall in stock will result in profit due to positive Gamma. But this may result in the sudden fall in credit quality and a fall in the value of the straight bond component of the CB.

Example:

- Stock = \$10
- Conversion ratio = 100
- Face value =\$1000
- Coupon = 2%
- Risk-free rate = 5%
- CB = \$1100
- $\Delta = 50$
- Maturity in one year

What happens if we Delta hedge today, and come back one year later? Meanwhile either stock has not moved, has risen to \$12.5 or fallen to \$7.5.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Cashflows today:

Buy bond = \$1100.

Sell stock = \$500

Net cost \$600. To be borrowed at 5%p.a.

At maturity:

Underlying	**CB**	**Stock**	**Coupon**	**Cash + Int.**	**Net**
10	+1000	−500	+20	−630	−10
12.5	+1250	−625	+20	−630	+15
7.5	+1000	−375	+20	−630	+15

Deal structure 2: Extracting value from volatility

Example: The implied volatility for a certain CB is 25%. This is low compared with the 40% implied volatility of exchange-traded options on the same stock. How can you extract value from the CB (assuming no credit risk problems)?

Buy the CB and hedge with a call

Hedging with an exchange-traded option, a call, say, can eliminate exposure to movements in the stock price. Sell the number of calls given by the ratio of the Deltas of the CB and the vanilla call.

This trade assumes that the implied volatilities of the CB and the vanilla call with converge to each other.

Risks:

- Convergence does not materialize

- Large move in underlying. Need to examine Gamma

Deal structure 3: Extracting value from credit

Example: Implied risk of default appears too high, compared with historical, fundamental assessment or similar companies. How can you extract value from this CB?

Buy the CB and hedge the market risk

Wait for the market to reassess the risk of default and for the CB price to rise.

Risk:

- Market stubbornly refuses to change its default estimate

Long position in CB and short position in a contract on similar company with lower risk of default

This is a relative value/convergence trade. It does not matter whether the CB risk of default falls, only whether the two risks of default move towards each other.

Risks:

- Market stubbornly refuses to change its default estimates
- Default occurs in the CB issuer. You are hedging with a *similar* company, not the same one

To hedge against actual default you could buy a default swap.

Pricing CBs with known interest rate

We will use S to mean the underlying asset price, the maturity date is T and the CB can be converted into n of the underlying. There is a continuous dividend yield of $D(S,t)$.

To introduce the ideas behind pricing convertibles we will start by assuming that interest rates are deterministic for the life of the bond.

Since the bond value depends on the price of that asset we have

$$V = V(S, t);$$

the contract value depends on an asset price and on the time to maturity.

Repeating the Black–Scholes analysis, with a portfolio consisting of one convertible bond and $-\Delta$ assets, we find that the change in the value of the portfolio is

$$d\Pi = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} dt - \Delta\, dS.$$

As before, choose

$$\Delta = \frac{\partial V}{\partial S}$$

to eliminate risk from this portfolio.

The return on this risk-free portfolio is at most that from a bank deposit and so

$$\frac{\partial V}{\partial t} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (rS - D(S,t))\frac{\partial V}{\partial S} - rV \le 0. \qquad (1)$$

This inequality is the basic Black–Scholes inequality.

Scaling the principal to \$1, the final condition is

$$V(S,T) = 1.$$

Coupons are paid discretely every quarter or half year and so we have the jump condition across each coupon date

$$V(S,t_c^-) = V(S,t_c^+) + K,$$

where K is the amount of the discrete coupon paid on date t_c.

The early conversion feature makes the convertible bond similar to an American option problem; mathematically, we have another free boundary problem.

Since the bond may be converted into n assets we have the constraint

$$V \geq nS.$$

In addition to this constraint, we require the continuity of V and $\partial V/\partial S$.

Call and put features

The convertible bond permits the holder to exchange the bond for a certain number of the underlying asset at any time of their choosing.

CBs often also have a **call feature** which gives the issuing company the right to purchase back the bond during specified periods for a specified amount. Sometimes this amount varies with time.

The bond with a call feature is clearly worth less than the bond without. This is modeled exactly like US-style exercise again.

If the bond can be repurchased by the company for an amount M_C then elimination of arbitrage opportunities leads to

$$V(S,t) \leq M_C.$$

Now we must solve a constrained problem in which our bond price is bounded below by nS and above by M_C.

To eliminate arbitrage and to optimize the bond's value, V and $\partial V/\partial S$ must be continuous.

Some convertible bonds incorporate a **put feature**. This right permits the holder of the bond to return it to the issuing company for an amount M_P, say. The value M_P can be time dependent.

Now we must impose the constraint

$$V(S,t) \geq M_P.$$

This feature increases the value of the bond to the holder.

Two-factor modeling: Convertible bonds with stochastic interest rate

The lifespan of a typical convertible is much longer than that for a traded option. It is therefore safer to price CBs using a stochastic interest rate model.

When interest rates are stochastic, the convertible bond has a value of the form

$$V = V(S, r, t).$$

Before, r was just a parameter, now it is an independent variable.

We continue to assume that the asset price is governed by the lognormal model

$$dS = \mu S \, dt + \sigma S \, dX_1, \tag{2}$$

and the interest rate by

$$dr = u(r,t) \, dt + w(r,t) \, dX_2. \tag{3}$$

Observe that in (2) and (3) there are two Wiener processes. This is because S and r are governed by two different random variables; this is a **two-factor model**.

dX_1 and dX_2 are both still drawn from Normal distributions with zero mean and variance dt, but they are not the same random variable. They may, however, be correlated and we assume that

$$E[dX_1\,dX_2] = \rho\,dt,$$

with $-1 \leq \rho(r, S, t) \leq 1$.

To handle correlated random walks we need some theory. We require a multi-dimensional Itô's lemma.

The usual Taylor series expansion together with a few rules of thumb results in the correct expression for the small change in any function of both S and r.

These rules of thumb are

- $dX_1^2 = dt$;
- $dX_2^2 = dt$;
- $dX_1\, dX_2 = \rho\, dt$.

The equation for dV is

$$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{\partial V}{\partial r} dr + \frac{1}{2} \left(\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + 2\rho\sigma S w \frac{\partial^2 V}{\partial S \partial r} + w^2 \frac{\partial^2 V}{\partial r^2} \right) dt.$$

Now we come to the pricing of the convertible bond.

Construct a portfolio consisting of the convertible bond with maturity T_1, $-\Delta_2$ zero-coupon bonds with maturity date T_2 and $-\Delta_1$ of the underlying asset.

We are therefore going to hedge both the interest rate risk and the underlying asset risk.

Thus

$$\Pi = V - \Delta_2 Z - \Delta_1 S.$$

The analysis is much as before; the choice

$$\Delta_2 = \frac{\partial V}{\partial r} \bigg/ \frac{\partial Z}{\partial r}$$

and

$$\Delta_1 = \frac{\partial V}{\partial S}$$

eliminates risk from the portfolio.

The usual analysis leads to

$$\frac{\partial V}{\partial t}+\tfrac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2}+\rho\sigma Sw\frac{\partial^2V}{\partial S\partial r}+\tfrac{1}{2}w^2\frac{\partial^2V}{\partial r^2}$$

$$+rS\frac{\partial V}{\partial S}+(u-\lambda w)\frac{\partial V}{\partial r}-rV=0. \tag{4}$$

where again $\lambda(r,S,t)$ is the market price of interest rate risk.

This is exactly the same market price of risk as for ordinary bonds with no asset dependence and so we would expect it not to be a function of S, only of r and t.

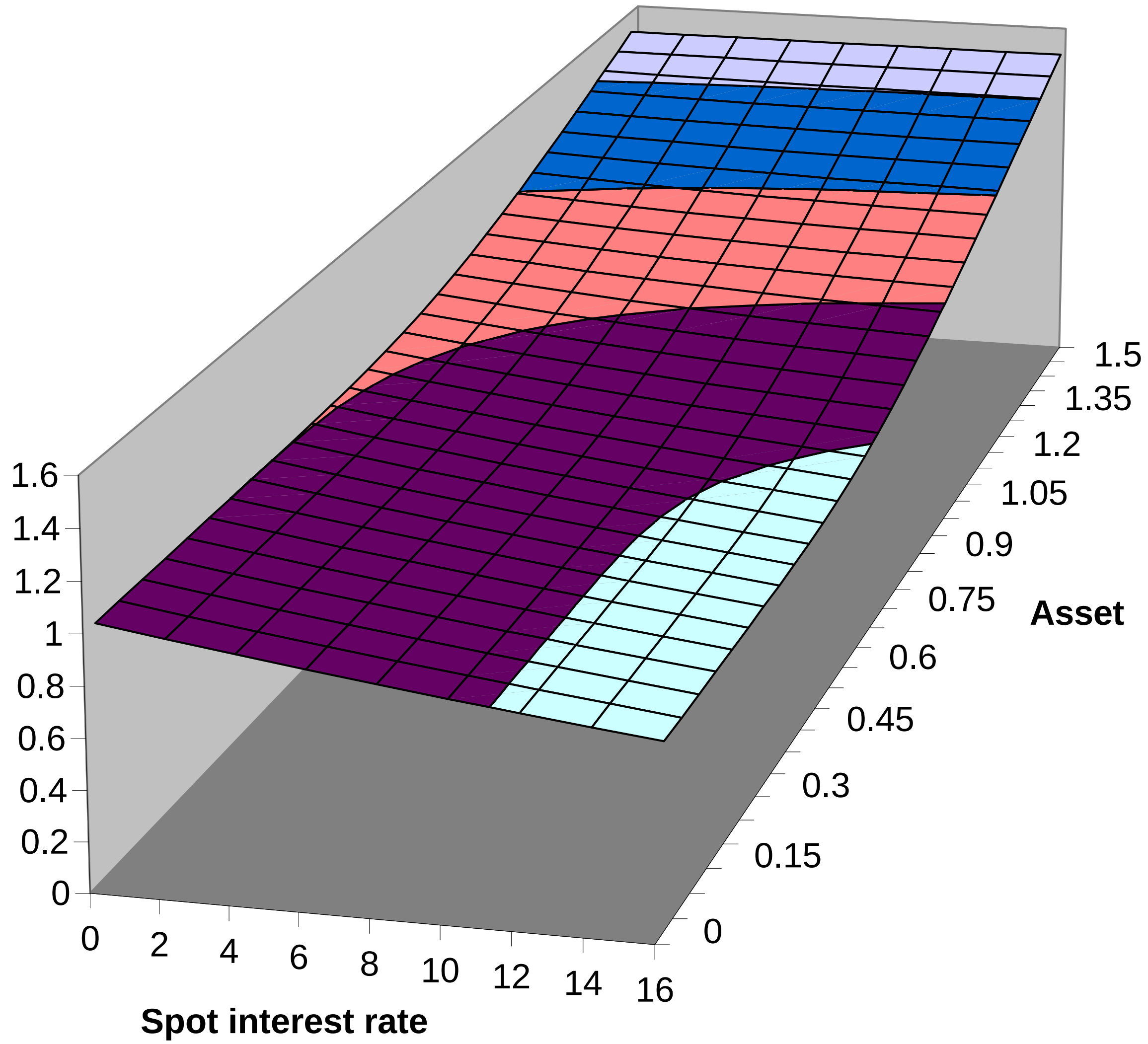
This is the convertible bond pricing equation.

There are two special cases of this equation that we have seen before.

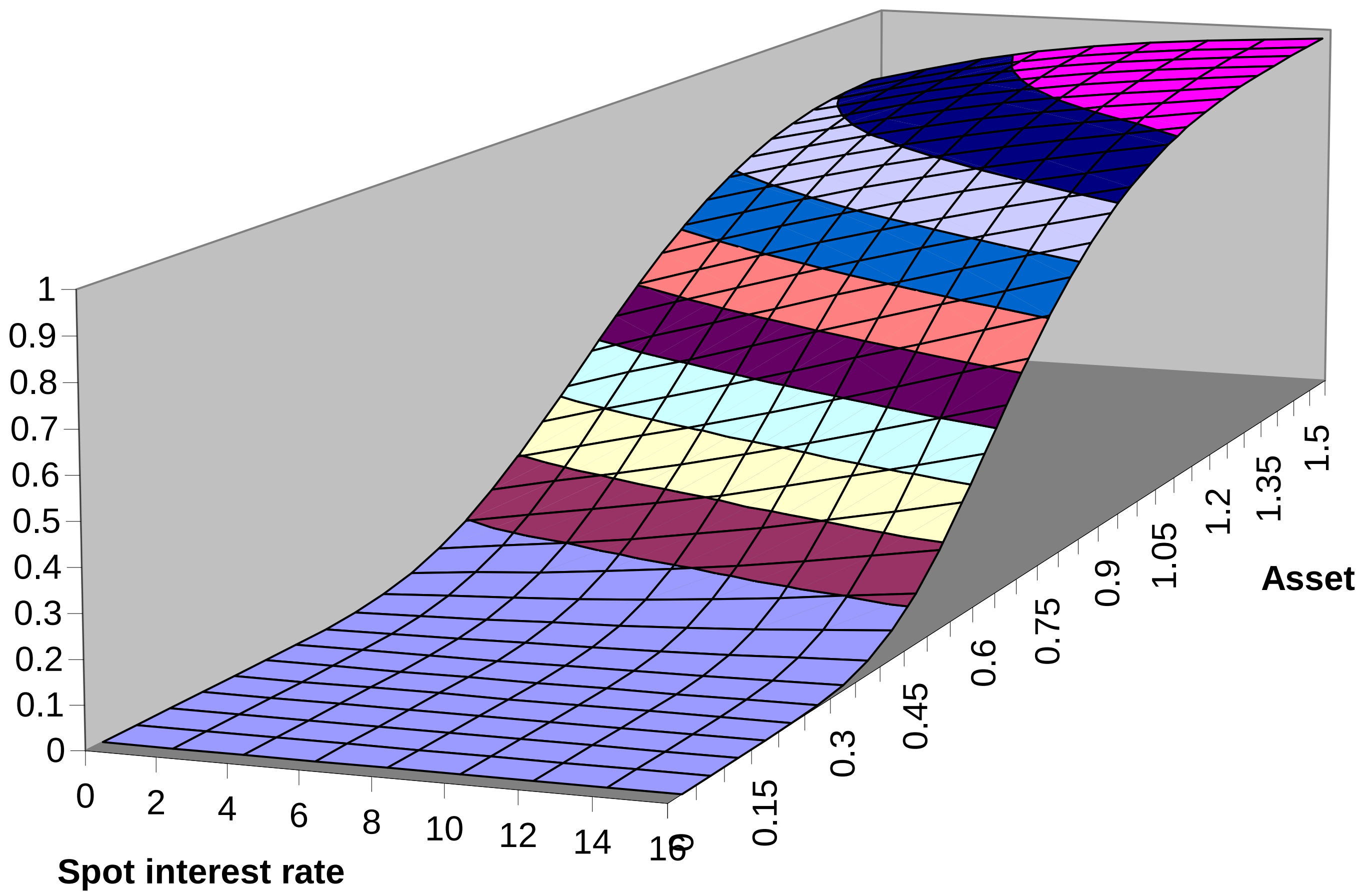
- When $u = 0 = w$ we have constant interest rate r, Equation (4) collapses to the Black–Scholes equation.

- When there is no dependence on an asset $\partial/\partial S = 0$ we return to the basic bond pricing equation.

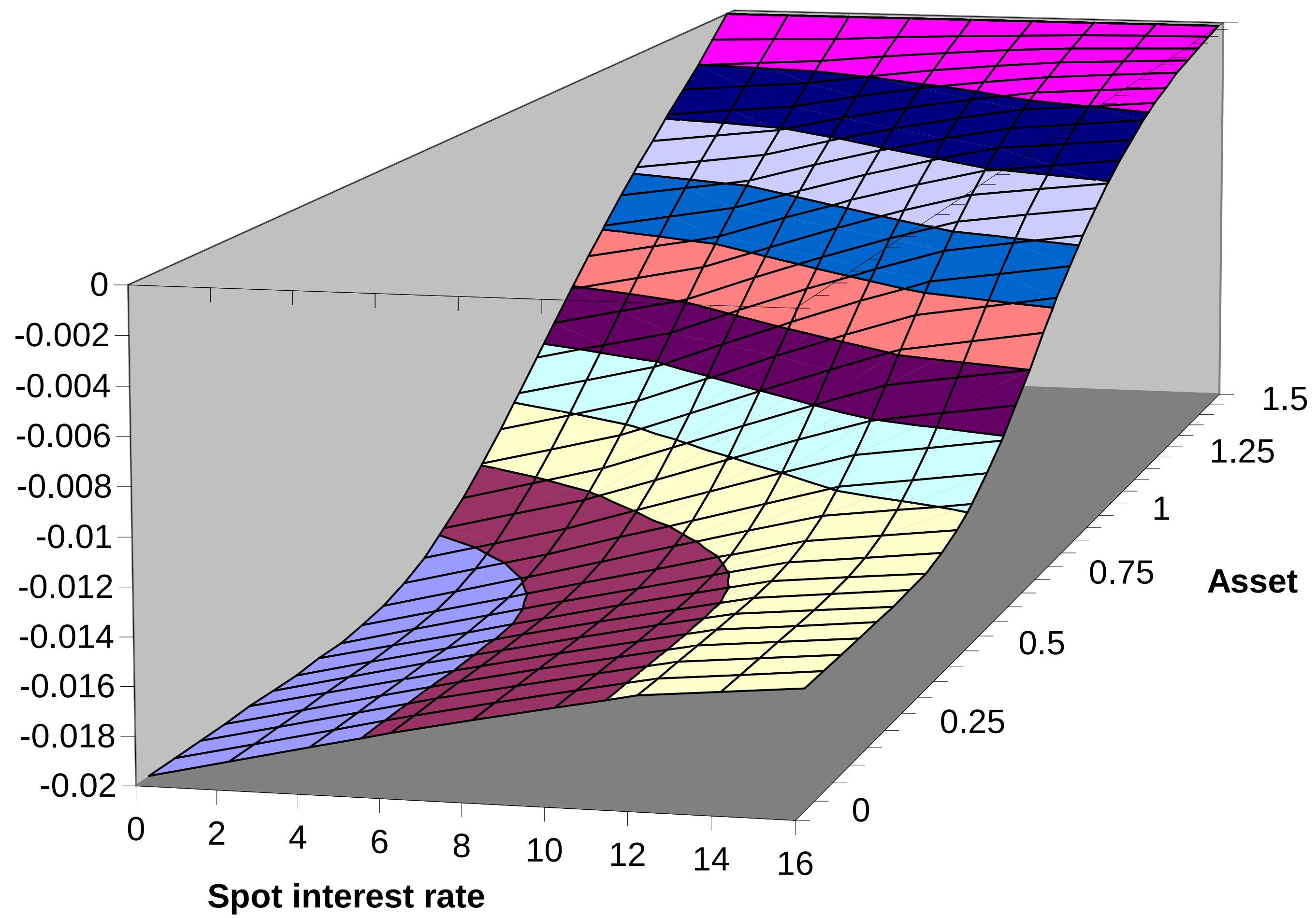
In the next figures are shown the value and sensitivities of a CB when the underlying asset is lognormal and interest rates evolve according to the Vasicek model fitted to a flat 7% yield curve.



The value of a CB with stochastic asset and interest rates.



$\frac{\partial V}{\partial S}$ for a CB with stochastic asset and interest rates.



$\frac{\partial V}{\partial r}$ for a CB with stochastic asset and interest rates.

Summary

Please take away the following important ideas

- Convertible bonds are a hybrid equity/debt instrument
- They can be priced in the familiar Black–Scholes environment
- Credit risk is an important factor in pricing CBs