CQF 2009
Module 3.1
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The Black-Scholes Model

In this lecture:

- The assumptions that go into the Black-Scholes model
- Foundations of options theory: delta hedging and no arbitrage
- The Black-Scholes partial differential equation
- The Black-Scholes formulae for calls, puts and simple digitals
- The meaning and importance of the 'greeks,' delta, gamma, theta, vega, and rho
- American options and early exercise
- The relationship between option values and expectations

By the end of this lecture, you will be able to:

- Derive the Black-Scholes partial differential equation
- Quote formulae for simple contracts
- Understand the meaning of the common greeks
- Interpret the early-exercise feature mathematically and graphically

Introduction....3
What determines the value of an option?....4
The Black-Scholes assumptions....6
A very special portfolio....8
Elimination of risk: delta hedging....11
No arbitrage....13
The Black-Scholes equation....15
Observations: The BS equation is a linear parabolic PDE....16
Replication....17
Final conditions....18
Options on dividend-paying equities....19
Currency options....21
Commodity options....22
Solving the equation and the greeks....23
Derivation of the formulae for calls, puts, and simple digitals....24
The Dirac delta function....35
Observations....38
Formula for a call....40
Call Option Value – box....47
Formula for a put....48
Put Option Value – box....52
Formula for a binary call....53
Binary Call Option Value - box....54
Formula for a Binary Put and box....56

Delta 58
Dynamic hedging and rehedging 60
Deltas of common contracts 61
Gamma 63
Gammas of common contracts – box 66
Theta 68
Thetas of common contracts - box 69
Vega 71
Vegas of common contracts – box 72
Rho 74
Rhos of common contracts 75
Sensitivity to dividend for common contracts 76
Early exercise and American options 77
High contact or smooth-pasting condition 81
Bermudan options 84
The relationship between option prices and expectations 85
Example 1 87
Example 2 88
Example 3 89
Backward Kolmogorov and Black-Scholes 90
Options as expectations 92
Summary 93

Takeaways:

- Using tools from stochastic calculus, we can build up an option pricing model from our lognormal asset price random walk model
- There are some ‘simple’ formulae for the prices of simple contracts
- The greeks are important measures of the sensitivities of the option value to variables and parameters
- American options must always have a value greater than the payoff
- Option values can be interpreted in terms of expectations