

CQF January 2009
Module 2.4
Live Class: February 17
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Martingales I: Advanced Stochastic Calculus and Martingales

In this lecture:

We expand on the stochastic calculus lecture (Lecture 1.3)

- to introduce further probabilistic methods:
- the probabilistic universe;
- sample space,
- Filtration and probability measures;
- conditional and unconditional expectation;
- change of measure and the Radon Nicodým derivative;
- definition and properties of martingales.

Introduction.....	3
Section 1: The Probabilistic Universe.....	4
1.1 The Sample Space Ω	5
1.2 The Filtration \mathcal{F}	23
1.3 The Probability Measure P	33
Section 2. Martingales.....	65
2.1 Definitions.....	70
2.2 Markov vs. Martingales.....	78
2.3 The Brownian Motion as a Martingale.....	80
2.4 Itô Integral and Martingales.....	84

Summary:

- The probability space $(\Omega; \mathcal{F}; P)$ is the space where stochastic processes live;
- The expectation is an integration with respect to the probability measure P ;
- The conditional expectation has a number of important properties, i.e. linearity, Tower, “taking out what is known,” independence, positivity and Jensen’s inequality;
- The Radon Nicodým Theorem is a useful result to help us change our setting from a measure P to a measure Q ;
- Martingales are driftless processes;
- Brownian motion can be defined in terms of martingales;
- Itô integrals are martingales and that martingales can be represented as Itô integrals.