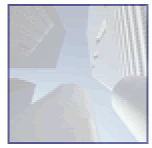



:: The Greek Letters - Gamma ::

The Greek Letters or simply the "Greeks" are quantities representing the market sensitivities of the options or other derivatives. Each Greek measures a different aspect of the risk in an option position. Through understanding and managing these Greeks, market makers, traders, financial institutions and portfolio managers can manage their risks appropriately, whether they deal in OTC or exchange-traded options. This article looks at the Greek that measures the curvature between option price and underlying price - Gamma.

Gamma

The gamma of an option is defined as the rate of change of the option's delta w.r.t. the price of the underlying, when all else remains the same. It's the second partial derivative of the option price w.r.t. the underlying price and is mathematically expressed as:

$$\Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{\partial \Delta}{\partial S}$$

If gamma is small, delta only changes slowly and in order to keep a portfolio (a basket of shares and options) delta-neutral, adjustments to the portfolio can be made less frequently. If gamma is large, i.e. delta is very sensitive to the underlying price, the portfolio will need to be adjusted frequently to maintain delta-neutrality.

Derivation of Gamma

Recalling from the **delta** article for a European call option on a non-dividend-paying underlying:

$$\Delta = N(d_1)$$

where

$$d_1 = \frac{\log(S/E) + \left(r - q + \frac{1}{2}\sigma^2\right)(T - t)}{\sigma\sqrt{T - t}}$$

Gamma is simply:

$$\begin{aligned}\Gamma &= \frac{\partial \Delta}{\partial S} \\ &= N'(d_1)\frac{\partial d_1}{\partial S} \\ &= \frac{N'(d_1)}{S\sigma\sqrt{T - t}}\end{aligned}$$

with

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

It can be shown that gamma for a put option on the same underlying is the same as the gamma for a call option, so one less formula to remember.

For a European call or put option on a dividend-paying underlying at continuous rate q, gamma is therefore:

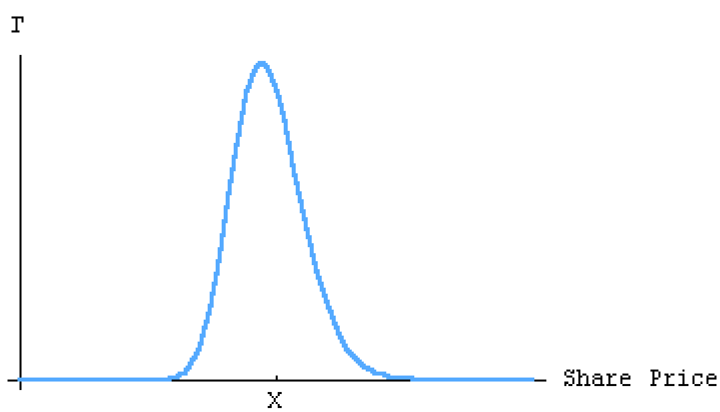
$$\Gamma = \frac{e^{-q(T-t)} N'(d_1)}{S\sigma\sqrt{T-t}}$$

with

$$d_1 = \frac{\log(S/E) + \left(r - q + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

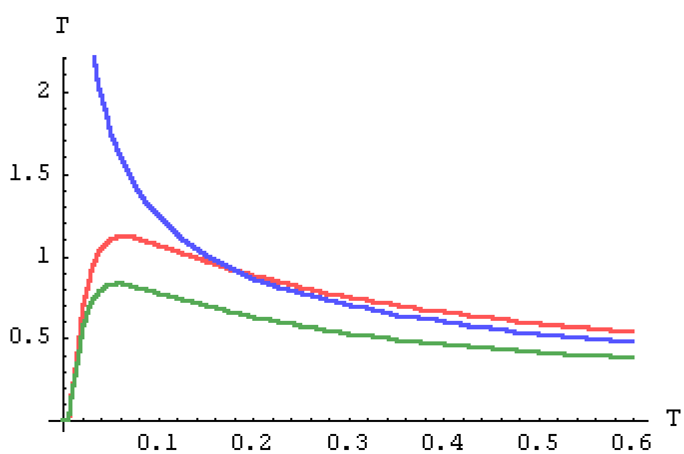
Variation of Gamma with Share Price

Variation of gamma with share price (S) for a European option on a dividend-paying share with exercise price of X. Gamma is always positive and increases to a maximum with share price close to the exercise price.



Variation of Gamma with Time to Expiry

Variation of gamma with Time to Expiry (T) for European option on a dividend-paying share with strike price of X. **Red**, **Blue** and **Green** lines denote out-of-the-money, at-the-money and in-the-money options respectively. (Values used: S = £4.75, £5.00, £5.25, X = £50, vol = 20%, r = 8% and q = 3%).



Gamma increases as time to expiry decreases for at-the-money options, i.e. value of option is highly sensitive to the underlying share price. Gammas for out-of-the-money and in-the-money options decreases to zero because either the option will not be exercised or the option are certain to be exercised, in both cases, adjustments to make portfolio delta-neutral are increasingly unnecessary.

Making a Portfolio Gamma Neutral

For a portfolio containing a basket of shares and options with a value of ?. Assuming the volatility and risk-free interest rate of the underlying is constant for now, Taylor series expansion of the change in the portfolio value d? with the underlying share price and time demonstrates the roles played by various Greek letters:

$$\delta\Pi = \frac{\partial\Pi}{\partial S}\delta S + \frac{\partial\Pi}{\partial t}\delta t + \frac{1}{2}\frac{\partial^2\Pi}{\partial S^2}\delta S^2 + \frac{1}{2}\frac{\partial^2\Pi}{\partial t^2}\delta t^2 + \ldots$$

Ignoring all terms higher order than dt, the change in the portfolio value is therefore

$$\delta\Pi \simeq \Delta\,\delta S + \Theta\,\delta t + \frac{1}{2}\Gamma\,\delta S^2$$

For a delta neutral portfolio, delta of the portfolio is zero, thus:

$$\delta\Pi = \Theta\,\delta t + \frac{1}{2}\Gamma\,\delta S^2$$

When gamma is positive, theta tends to be negative and vice versa if there is no change in portfolio value. Since there is no uncertainty about the passage of time, a portfolio therefore cannot be hedge against changes in time. From the equation above, if gamma is positive, any changes in share price in a short time (first term on r.h.s. ≃ 0) will cause the portfolio to increase in value. The reverse is also true.

In order to protect the value of portfolio against (wide) fluctuation in share price, gamma of the portfolio can be made neutral. Delta neutrality protects against small share price moves between rebalancing, whereas gamma neutrality protects against large share price movements between delta-hedge rebalancing.

If a delta-neutral portfolio has a gamma Γ_Π and to hedge against this gamma, traded options with delta Δ_T and gamma Γ_T are introduced:

$$N_T\Gamma_T + \Gamma_\Pi = 0$$
$$N_T = -\frac{\Gamma_\Pi}{\Gamma_T}$$

N_T is number of traded options required to make the portfolio neutral. The portfolio, after adjusting for gamma neutrality will not be delta-neutral anymore. Therefore to adjust for this change in delta ($= N_T\Delta_T$), a quantity of the underlying asset equal to this amount must be either bought or sold off to maintain delta neutrality (Positive delta = long call short underlying; negative delta = long put long underlying).

Written by Henry Tang.

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