Basel II capital requirements for structured credit products and economic capital: a comparative analysis

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Abstract

The huge development experienced by the market for structured credit products has occurred under a banks' capital adequacy regime inadequate to deal with the complexity of these instruments. The new prudential framework (Basel II) contains a specific section devoted to the treatment of tranched products, that put much store on public ratings. In this paper we try to evaluate the reliability of the new rules, comparing the forthcoming charges for structured products with an estimate of their risk. The analysis has been performed on the tranches of the iTraxx index, chosen as a proxy of CDOs built on corporate exposures. The results obtained cast some doubts about the actual degree of conservatism of the choice to map from ratings to prudential charges.

Keywords: CDO, economic capital, regulatory capital, Value at Risk.

1 Introduction

For a long time the great success obtained by tranched securities has been regarded as a remarkable example of the capital arbitrage strategies flourished under the previous version of the Basel Accord: a system of risk weights based on notional amount of investments has proven to be a strong incentive for banks to reduce the capital requirements without shedding a proportional amount of risk. In order to curb this incentive the new regulatory treatment of structured credit products has been designed along the principles of:

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- Capital *neutrality*
- Economic capital *centrality*

Capital neutrality implies that the amount of regulatory charges should be invariant to the decision to hold assets on the balance sheet or to securitize it; in addition, the prudential rules should align more closely regulatory and economic capital.

Under the new regulatory treatment of structured credit products internal risk estimates will not play a big role for prudential purposes, as regulators preferred to link capital charges to public ratings, even for banks allowed to determine on their own the prudential charges of traditional credit exposures (so-called *IRB* banks).

For IRB banks two alternative approaches have been introduced, depending on the existence of a public rating for the structured credit product: should a rating exist, the tranche capital charge comes from a coefficient to be applied to the notional exposure (*Rating Based Approach*, RBA); for unrated products, instead, IRB banks must provide their internal estimate of probability of default (PD) and recovery rate (LGD) for the underlying assets, as an input of a regulatory formula set by regulators (*Supervisory Formula Approach*, SFA).

A number of studies has compared RBA and SFA approaches over a range of structured credit products [2]. It turned out that there is a clear capital advantage in using the RBA; this evidence could create an incentive for IRB banks to obtain an external assessment for transactions that otherwise wouldn't be rated. The magnitude of such a phenomenon should increase if the RBA charges were found inadequate to fully cover the risk of these products.

This paper investigates the coherence between the economic capital absorbed by structured products and their regulatory treatment, focusing on *iTraxx* tranches, a stylized and actively traded version of "corporate CDOs", that can represent a useful *boot camp* for the forthcoming prudential regulation.

The paper is organized as follows: section 2 introduces *synthetic CDOs*, section 3 provides an overview of the new prudential rules for tranched products, sections 4 and 5 describe the methodologies used to obtain an estimate of the tranches' riskiness, section 6 illustrates a comparison between economic capital and prudential requirements whose results are sketched in section 7; the final section reports our conclusions.

2 *Single-Tranche* CDOs

Collateralised Debt Obligations (CDOs) could be defined as fixed income securities originated by engineering the risk of a portfolio of credit instruments. Mixing securitisation and credit derivative techniques, these products are used to transfer the risks arising from a portfolio of corporate exposures (bonds, loans or the like) or a pool of securitized retail credits conveyed by a collection of Asset Backed Securities. The former transactions are usually termed as "*corporate CDOs*", while the latter are known as "*CDO of ABS*"[1].

The original scheme of corporate CDOs has experienced continuous innovation from its appearance at the beginning of '90s; along this path of evolution the use of credit derivatives to transfer the risk of the underlying portfolio has been a structural break-trough, which gave life to a class of product on its own: "*synthetic CDOs*".

In the traditional version of synthetic CDOs, a *special-purpose vehicle* ("SPV") sells protection on an underlying pool by synthetically referencing a portfolio of single-name credit default swaps ("CDS"), each referred to a specific company. The SPV offsets the risk by issuing notes that provide investors with exposure to different "*tranches*" of the underlying reference portfolio.

Investors pay up-front the tranche's notional amount and receive a recurring premium for selling protection on the reference pool via the SPV. If one or more defaults occur in the portfolio, the SPV will refund the swap counterparts for the losses incurred on the defaulted assets using investors' money.

A priority scheme governs the loss absorbtion: each tranche starts to suffer a reduction of its notional amount when losses in the portfolio breach the tranche's *attachment point* (L_b), expressed as a percentage of the total pool notional. The investment will be wiped out should portfolio losses reach the tranche's *detachment point* (H_b). The *thickness* of each tranche is defined by the difference between H_b and L_b.

The first tranche to suffer losses is usually termed as the "*equity*", whose $L_b = 0$; next comes the *"mezzanine"*, and then one or more "*senior*" tranches. Usually the *credit enhacement* enjoyed by a tranche does not depend only on its position in the capital structure, as the contractual rules that govern the allocation of portfolio's cash-flows ("*cash-flow waterfall*") tend to relax the strict prioritization scheme given by the sequence of L_b and H_b values.

Nevertheless, the distinctive feature of all CDOs is that the sequential allocation of portfolio losses along the tranches concentrates much of the pool credit risk in the equity and mezzanine tranches, often referenced as *"leveraged"* products; senior tranches, instead, allow investors to get a *"deleveraged"* exposure to the pool's credit risk.

The appearance of CDS indexes (basically portfolios of single-name credit default swaps) and the rapid development of default swaps referred to these portfolios, so-called *index swaps*, set the ground for the quotation by major investment banks of a bilateral derivative agreement whose mechanics mimics that of a real CDO: *"Single-Tranche CDOs"*.

The great success achieved by this product, basically a standardized version of CDOs contracts, has contributed to complete the market for credit risk, by allowing the emergence of a market for default correlation between the most important

[1] Although corporate exposures and ABS can be viewed as the most popular form of CDOs' collateral, many other assets have been used as underlying of these transactions comprising project financing loans, private equity exposures, as well as other CDOs.

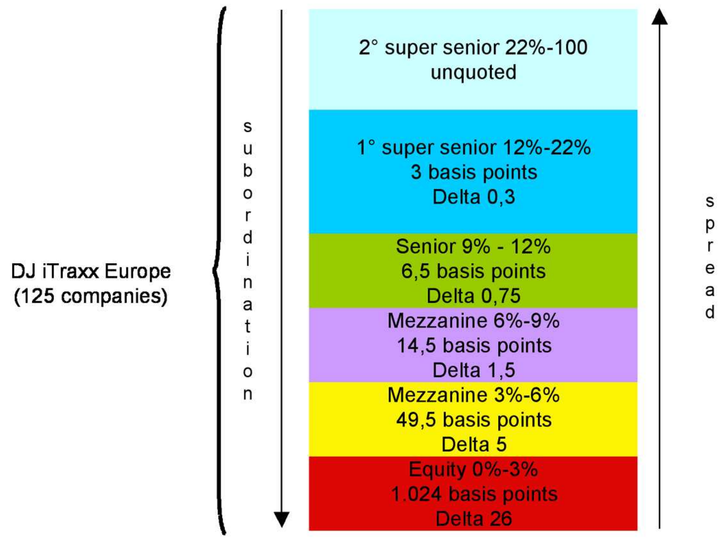

Figure 1: Single tranche CDOs.

corporations worldwide.

Nowdays many investment banks operate as licensed delears of this product quoting the four standard tranches of the Dow Jones indexes that equally weight the 125 most liquid CDS referred to top European investment grade companies (*iTraxx*) and their American counterparties (*CDX.IG*)[2].

In both cases most trades regard the index series that comprise CDS whose maturity is of 3, 5 and 7 years, with very tight bid-ask spreads for the "on the run" series (lower than 1 bp per annum).

While in traditional CDOs there is a clear distinction between the arranger and the investor, in this contract the difference is blurred, as it is possible to assume either short or long positions. Investment banks acting as market makers stand ready to buy protection from the "*protection sellers*" and sell it to the "*protection buyers*".

Who enters into the deal as a protection seller is entitled to receive a quarterly agreed spread from the counterpart that, as the buyer of protection, will pay the recovery-adjusted amount upon default of a name in the pool, once losses have breached the tranche's L_b.

If we look to the investor position, i.e. the seller of protection, we can isolate three different sources of risk: one arising from changes of credit spreads on the underlying portfolio ("*market risk*"); another linked to the eventual default of one or more firms in the pool ("*default risk*"); the last one emerging from changes in

[2] A complete description of the rules governing the composition of both the iTraxx and CDX.IG indexes can be found at www.indexco.com, while the templates of the documentation used for trading both indexes and their tranches are available at www.markit.com

the level of expected correlation between defaults (”*correlation risk*”). The same risks are faced by the buyer of protection, unless she had a long position on the underlying portfolio and is able to place the entire capital structure (i.e. all CDO’s tranches).

Both the seller and the buyer of protection can immunize themselves, by managing a hedge portfolio which will leave them exposed just to model and liquidity risks. The simplest risk to hedge away is the market risk, which requires buying protection (for the protection seller) in CDS referenced to each company in the pool, or in the swap referenced to the index as a whole.

In both cases, the amount of protection traded is a multiple (the ”delta”) of the tranches notional value and is calculated using the CDO’s pricing model. Being a hedge ratio, the delta represents the sensitivity of the tranche value to a shift of the reference portfolio credit spreads and therefore can be interpreted as a measure of the *leverage / deleverage* of the transaction[3].

3 Regulatory treatment

One of the most important objective of the new prudential framework was to address the capital arbitrage strategies that flourished under the 1988 Basel I Capital Accord, often based on the use of structured credit products.

A comprehensive policy regarding this class of products is provided in a specific section (*Securitisation Framework*) of the new *Basel Accord*. The Securitisation Framework contemplates a range of methods for banks to determine the regulatory capital charges of structured credit products, depending on whether the bank will implement the Internal Rating Based approach or the standardized one for the underlying exposures[4].

In both cases the regulators put great store on the evaluations released by rating agencies, recognising the crucial role played by these institutions in the market for structured credit products, mainly due to the complex characteristic of many products.

Under the standardized approach the capital charges are linked to the security’s external credit rating by a set of weights to be applied to the notional exposure (table 2). For unrated positions banks must hold one unit of capital for each unit of exposure (known as a ”dollar-by-dollar” capital charge), being a strong capital disincentive against holding unrated products.

Capital charges for IRB banks depend on whether the position held has a credit rating from a recognized credit rating agency:

[3]These hedging positions are frequently reported as one of the most important source of the booming growth recently experienced by credit derivatives’ volume. It is worth to notice that the actual performance of these hedging strategies relies mainly on the stability of the parameters that govern the hedge ratio quantification; as example, the extreme movements experienced by default correlation during market turmoils have been disruptive for these strategies.

[4]Under the Internal Rating Based (IRB) approach, banks are allowed to calculate the prudential charges for credit exposures, according to the results of their rating system.

- For all tranches with a credit rating, IRB banks must use what is known as the *Rating Based Approach* (RBA). Primarily targeted to banks that invest in a securitization they did not originate, RBA maps from the tranche's external rating to its capital charge via a specific look-up table (table 3). The weights for tranches of a given rating vary according to the tranche seniority and the pool granularity[5]. Further, if a tranche is the most senior in its structure and enjoys an investment grade rating, it attracts a lower risk weight than the base case, as long its underlying pool is perceived to be sufficiently granular ($N > 6$).

- For unrated exposures, IRB banks must use a bottom up approach in which a set of parameters related to the pool credit quality and to the contractual terms governing the cash flow waterfall are plugged into the *Supervisory Formula*, SFA. The formula, conceived for banks that act as originators of a securitization, depends on five inputs of which only one reflects banks' internal estimates (the *Kirb*, which is the capital charge that the bank would face if the exposures had been retained on balance sheet).

Decisions about the levels of structured credit product capital charges generated by the RBA and SFA approaches have been strongly influenced by studies performed by analysts at the Federal Reserve Board and the Bank of England.

Peretyatkin and Perraudin [11] examined the robustness of RBA weights on a portfolio of stilyzed tranched products, checking the impact of different maturities, granularity and default correlation; for this purpose they developed a Monte Carlo model aimed to calculate the marginal Value at Risk of tranches of stilyzed deals held into a well diversified portfolio.

Gordy and Jones [6] provided the financial engineering background to the SFA, devising a model (*Uncertainty in Loss Prioritization*) that smooths the step function for capital charges that comes out when one employs a strict prioritization rule to allocate the underlying's capital charge along the tranches of a structured credit product.

Supervisory formula approach

The supervisory formula has been designed in order to allocate the capital requirement of a given portfolio (Kirb) along the tranches of a structured credit product built on this pool of assets. As it is not practical for regulators to take into account the myriad possible deal-specific features of each product, the formula was developed on a limited set of information regarding the cash-flow waterfall.

[5]A pool is said to be highly granular if it contains a large number of exposures, none of which contributes a large part of the total risk. As a measure of granularity it is used the statistic:

$$N = \frac{(\sum EAD_i)^2}{\sum EAD_i}$$

where EAD denotes the exposure at default of the ith exposure in the pool

Pykthin and Dev [8], [9] tried to summarize the waterfall for a particular tranche in terms of its thickness T and credit enhancement level ζ[6]. They assumed the allocation of the pool's economic losses to be governed deterministically, according to a strict loss prioritization rule (SLP).

Let ζ denote the tranche's credit enhancement level and T its thickness (both values easily observable for a market participant), under the SLP rule the tranche absorbs pool losses only in excess of ζ, up to a maximum of T; that is, if L denotes pool losses, then the loss for the tranche can be defined as $min[L, \zeta + T] - min[L, \zeta]$.

Notwithstanding its simplicity, the SLP approach brings an unsatisfactory knife-edge property when used to determine capital requirements: for an infinitely grained pool, the capital requirement for a tranche is dollar by dollar if $\zeta + T$ is less or equal to $Kirb$ and zero thereafter[7].

Gordy and Jones [6] generalized the SLP approach by adding a stochastic element to the distribution of losses across tranches in order to address the uncertainty about the true characteristics of the cash-flow waterfall. In doing this they assumed that the credit enhancement ζ_i represents the ex-ante expected protection against losses in the pool, while the actual protection provided by the waterfall is randomly determined (they indicated with $Z(\zeta_i)$ - where Z is a random variable - the unknown level of credit enhancement) and known only at the horizon.

Based on this intuition they derived a model, called *Uncertainty in Loss Prioritisation* (ULP), which has been proven suitable for regulatory purposes.

The supervisory formula is indeed based on an approximation of the ULP along with some "supervisory overrides", like a dollar for dollar capital requirements for tranches whose credit enhancement fall short of $Kirb$, and an imposed floor level for most senior positions[8].

4 The conditional loss distribution

From the protection seller's perspective the value of a single tranche CDO is simply the difference between the discounted value at time t_0 of the spread payments she expects to

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[6]The thickness T_i is defined as the ratio between the i-th tranche notional C_i and that of the underlying portfolio, $T_i = C_i/\Sigma C_i$; the credit enhancement ζ is defined as the sum of the notional values of all more-junior tranches.

[7]To have an idea about the unsatisfactoriness of this "twice" allocation, one can just consider that the subordinated tranches are tipically entitled to some cash payouts prior to more senior investors being paid out in full and hence do not deserve such a harsh treatment.

[8]To compute the capital charge via SFA we need to solve a function on the following seven parameters:

- $Kirb$
- n: number of names on the pool
- LGD: exposure-weighted loss given default of the pool
- ζ: tranche's credit enhancement
- T: tranche's thickness
- τ: parameter that sets the level of uncertainty capital allocation between the tranches (fixed by regulators);
- γ: parameter that governs the LGD distribution.

receive (*fee leg*), and the present value at time t_0 of the sum due when defaults affect the tranche (*contingent leg*):

$$Value(t_0) = MtM(t_0) = fee(t_0) - contingent(t_0) \tag{1}$$

The key input to evaluate both the fee and the contingent leg is the *expected loss* of the underlying pool, which reflects the probability distribution of losses in the pool.

The market standard approach to generate the portfolio's loss distribution relies on the so-called *One Factor Gaussian Copula Model* based upon Vasicek's model [13]. To derive the portfolio loss distribution the normal copula is typically implemented using either Monte Carlo simulation or quasi-analytical methods, as that described by Gibson [4].

Under the One Factor Gaussian Copula Model[9] we assume that a company defaults when the value of its assets (A_i) decreases below a threshold (α); in this conceptual framework α could be thought to represent the level of the firm's debt.

In the simplest version of the Merton approach [10] the evolution of the firm's assets is driven by a systematic factor *X*, that can be interpreted as a proxy of the general state of economy, and a idiosyncratic component ϵ_i

$$A_i = w_i X + \sqrt{1 - w_i^2}\epsilon_i \tag{2}$$

where X and ϵ_i are supposed to be uncorrelated standard normal variables ($X \sim N(0,1)$, $\epsilon_i \sim N(0,1)$ and $E[X\epsilon_i] = 0$).

If we further assume uncorrelation ($E[\epsilon_i\epsilon_j = 0]$) between the idiosyncratic components and common dependence of all firms in the portfolio to the state of economy ($w_i = w$), the pairwise "*default time correlation*" between company i and j is equal to the square of the factor loading:

$$\rho_{ij} = E[A_i A_j] = w^2 \tag{3}$$

Given the default probability of the company i-*th* from (2) we can then recognize the threshold α as:

$$A_i = wX + \sqrt{1 - w^2}\epsilon_i < \alpha \equiv \Phi(PD_i(t))$$

As for every other credit-related security, a critical choice regards the probability measure to use: *risk neutral* or *objective* probabilities of default. If one is interested in pricing, modelling under the risk neutral measure is a natural choice; when estimating the capital absorbed by an investment, one could instead adopt an actuarial-like approach choosing historical probabilities.

Starting from both risk neutral and real world default probabilities, we can get the conditional default probability for the i-*th* exposure, given X, as:

$$q_i(t|X = x) = \Phi\left(\frac{\Phi^{-1}(PD_i(t)) - \sqrt{\rho}x}{\sqrt{1-\rho}}\right) \tag{4}$$

Next we derive the entire default distribution of the pool from the conditional default probabilities of its components. The computation of the distribution appears simpler if we recall that under the common dependence hypothesis (3) the defaults are independent, conditional to the realizations of the systematic factor X.

[9]The One Factor Gaussian Copula Model could be considered as an extension at a portfolio level of the "Merton approach" for evaluating the debt of a firm based on the option pricing theory.

An easy way to compute the entire default distribution $p(l,t)$ is to implement the iterative algorithm proposed by Andersen, Basu and Sidenius [12].

Let $p^k(l,t|X)$ to denote the conditional probability to observe exactly l defaults at time t in a portfolio containing k credit exposures. If we know the entire distribution $p^k(l,t|X)$ (for $l = 0,\ldots,k$), using the following iteration we get the default distribution for a $k+1$ portfolio:

$$\begin{aligned} p^{k+1}(0,t|X) &= p^k(0,t|X)(1-q_{k+1}(t|X)) \\ p^{k+1}(l,t|X) &= p^k(l,t|X)(1-q_{k+1}(t|X)) + p^k(l-1,t|X)q_{k+1}(t|X) \\ p^{k+1}(k+1,t|X) &= p^k(l,t|X)q_{k+1}(t|X) \end{aligned} \tag{5}$$

where $q_{k+1}(t|X)$ is the default probability of the *k+1* exposure added to the k-portfolio.

Starting from the degenerate default distribution for $k = 0$, $p^0(0,t|X) = 1$, we can then use the recursion (5) to solve for the default distribution of the reference portfolio of N credits:

$$p^N(l,t|X)l = 0,...,N \tag{6}$$

Once we have the conditional default distribution, the unconditional default distribution $p(l,T)$ can be solved as[10]:

$$p(l,t) = \int_{-\infty}^{\infty} p^N(l,t|X)g(X)dX \tag{7}$$

where *g* is the probability density of *X*. The integral is solved numerically.

Obtained the distribution of defaults we need to move on to the distribution of portfolio losses. To reduce the computational burden of this step some simplificative assumptions about the risk drivers are usually accepted.

A popular approach, so-called *Large Homogeneous Pool* (LHP), assumes that the portfolio exposures are homogeneous in terms of probability of default ($PD_i = PD$) and recovery rate ($R_i = R$); in addition it is assumed that the pool is composed by an infinite number of counterparties, all sharing the same weight ($A_i = A$). A more realistic version of LHP, known as *Homogeneous pool*, is broadly used by market participants to get a quick estimate of tranches prices. The *Homogenous pool* diverges from the LHP with respect to the number of counterparties, assumed equal to the true number of companies in the portfolio, while shares the assumptions about the common value of default probability (equal to the one obtained by the quoted index spread) and recovery rate, conventionally fixed at 40%[11].

From a risk management perspective, like that adopted in this paper, the so-called *Heterogenous Pool* [1] approach looks more desirable as it relaxes the strong assumption regarding the common value of firms' default probability, allowing us to take into account the dispersion between the CDS spreads of the 125 portfolio companies.

It is worth noting that the shape of the portfolio loss distribution depends largely on the strong assumption about defaults interdependence. In the framework so far described the uniform pairwise correlation impacts on the individual conditional default probability (4) and, by (5), affects the entire distribution, influencing the allocation of the risk along the capital structures (see section 7).

[10] To simplify the notation there on we omitted the suffix N

[11] The 40% recovery rate is thought to be coherent with the historical rate observed on the US unsecured bond market.

As the risk of different tranches varies with default time correlation so does their fair values: default correlation is indeed the most important parameter for Single-Tranche pricing, as highlighted by the market practice to quote the tranche's *implied correlation* rather than its fair spread[12].

4.1 The pricing model

Let $f^L(c,t)$ be the density of the cumulative portfolio loss[13] at time t. Given $f^L(.)$ we can compute the expected loss for a given tranche in a risk neutral world as:

$$E_t^Q[C^L(t)] = \int_{L_b}^{H_b} (c - L_b) f^L(c,t) dc \tag{8}$$

Provided that the index components share the same notional amount and hold for true the assumption of constant recovery rate across the pool, it is easy to see that the loss distribution will assume discrete values: let N denote the exposures comprised in the reference portfolio, the loss distribution will assume N+1 values (from 0 to N defaults).

The (8) will then reduce to[14]:

$$E_t[L] = \sum_{l=0}^{N} p(l,t) max(min(lA(1-R), H_b) - L_b, 0) \tag{9}$$

where $p(l,t)$ is the probability to get l defaults by time t computed by (7) and N is the number of single name CDS composing the underlying portfolio.

The settlement convention to pay the contingent at the payment dates following the default further simplifies the pricing, as it allows us to determine the expected loss only on a quarterly basis.

From (9) the expected value of the payments due in case of defaults will be equal to

$$contingent = \sum_{t=1}^{n} D_t(E_t[L] - L_{t-1}[L]) \tag{10}$$

while the fee leg could be expressed as

$$Fee = s \sum_{t=1}^{n} D_t \Delta_t (H_b - L_b) - E_t[L] \tag{11}$$

where $\Delta_t = t - (t-1)$ and s is the yearly spread due by the protection buyer.

In (11) the amount $(H_b - L_b) - E_t[L]$ indicates the notional reduction caused by the defaults and reflects the right of the investor (protection seller) to get payments only on the residual amount of the tranche.

Like interest rate swaps, at inception synthetic CDO tranches will have their spreads set to constrain the mark-to-market value to zero; setting MTM = 0 in (1) and using (10) and (11) to solve for s, the "par spread" can be obtained as

$$s_{par} = \frac{contingent}{\sum_{t=1}^{n} D_t \Delta_t (H_b - L_b) - E_t[L]} \tag{12}$$

[12]In the market jargon the implied correlation is the number that makes the fair or theoretical value of a tranche equal to its market quote under the *One Factor Gaussian Copula Model*

[13]With c we refer to the cumulative loss of the tranche.

[14]We omitted the Q since this quantity will be computed under both the risk neutral and real world measures

5 Risk measures and economic capital

The risk of a tranched product can be evaluated by different metrics. From now on we describe those we believe could be eligible as proxies of the economic capital absorbed by a corporate CDO, adopting the one year time horizon chosen by Basel II for all credit exposures.

We are aware that this holding period is clearly too extended for index tranches that, mostly held in the trading book, need a much shorter period to be closed or hedged. When adopting this "regulatory constrained" time horizon we are therefore implementing the idea of using the index tranches as a proxy of corporate CDOs, whose scarce liquidity can well justify the regulators' choice.

In line with the *Value at Risk* conceptual framework on which Basel II relies, we computed a first measure of the tranches economic capital using a *VaR* measure derived from the Gaussian Copula model, taking the expected loss on the one year time horizon conditioned at the $99.9 - th$ percentile of the systematic factor[15].

In our framework that's equivalent to solve the (9) considering the probability distribution conditioned to $X = x_{99.9}$ (6). We indicate this measure as

$$E_t[L|X = x_{99.9}] = \sum_{l=0}^{N} p(l, t|X = x_{99.9}) max(min(lA(1 - R), H_b) - L_b, 0) \qquad (13)$$

This approach fails to consider the Mark to Market effect, which arises every time the maturity of the product is longer than the chosen holding period horizon (t_1): if we compute the VaR at t_1 for two different tenor but otherwise identical tranches (i.e. one with one-year maturity and the other with five-years maturity) we end up with the same value. Further it doesn't consider the spreads cashed in by the protection seller.

To obtain a more comprehensive risk measure we calculated the total loss suffered by an hypothetical investor, which comprises both the *cash flows* expected during the holding period and an estimate of the MtM effect suffered at the end of the holding period. We call this measure *Modified Unexpected Loss*:

$$UL_{mod} = E[CashFlow(t_0, t_1)|X = x_{99.9}] + |MtM(t_1)| \qquad (14)$$

The first term is equivalent to the mark to market of a contract with maturity of 1 year, having observed a negative state of the economy during this time horizon ($X = x_{99.9}$).

The second one accounts for the mismatch between the time horizon of the *VaR* measure and the product maturity. Since its evaluation is equivalent to that of a 4-year CDO written on the original pool of names that starts at t_1 and matures at t_5, it resembles the pricing of a *forward starting CDO*, an issue recently studied by Hull & White [15].

Forward starting CDOs represent contracts that compel two counterparties to enter into a CDO transaction at a specified future time on a given portfolio at an agreed spread; the contract is canceled out if during the period between the *deal date* and the *start date* the portfolio experiences an amount of losses exceeding the tranche detachment point. In line with standard CDOs, we define the *forward CDO spread* as the specific spread value that determines a zero Mark to Market at the deal date.

[15] Gordy[5] shows that when dependence across exposures is driven by a single systematic risk factor, and no exposure accounts for more than an arbitrarily small share of total portfolio (*perfect granularity*), the loss distribution is completely described by the systematic factor distribution: this allows to compute the VaR without generating the entire loss distribution.

Market participants distinguish the case where the underlying portfolio currently exists (*specific forward start CDOs*), and may have suffered defaults before the start date occurs, from the *de novo* portfolio which will come into existence at the start date (*general forward start CDOs*).

Hull & White have showed that *specific forward CDOs* can be priced in a consistent way by the One Factor Gaussian Copula Model, as it produces coherent results with those obtained by a more general dynamic model [14]. Accordingly, to compute the $MtM(t_1)$ in (14) we used the One Factor Gaussian Copula Model, considering the reduction of the notional portfolio caused by losses incurred from t_0 to t_1.

In this respect we need to calculate the number of defaults (l) associated to the losses incurred during the 1 year time horizon. To get l we compute the portfolio unexpected loss at time t_1 by (13); hence, considering the equivalence

$$E_1[L|X = x_{99.9}] = lA(1 - R)$$

we obtain

$$l = \frac{E_1}{A(1 - R)} \tag{15}$$

Once we known l from (15) we can easily compute $MtM(t_1, T_5)$ just evaluating (13) starting from $l = l$.

6 Comparative analysis

Both the regulatory approaches allowed to IRB banks (RBA and SFA) have been applied to iTraxx tranches and compared with the two described risk measures.

The analysis has been performed with respect to the standard tranches quoted on the fifth series of the iTraxx index, using the market data as of September 15, 2006 for the five years maturity.

As first we checked the soundness of the pricing methodology calibrating the model (4-12) to correlation values and single-name CDS spreads quoted by dealers (see figure 2) so to obtain exactly the market spread of each tranche.

Then we computed economic capital for each position under the two proposed measures using both risk neutral and real world probabilities.

Risk neutral probabilities have been derived from the CDS spreads. In doing so for each name we derived the annual default intensity λ from the 5-years CDS spread quoted at the September 15; to keep the computation simple we assumed a flat term structure for the default intensity and a constant recovery rate of 40%. To get the default probability we assumed a Poisson process for default, implying $PD_i = 1 - exp(-\lambda t)$.

The actual measures have been derived from the ratings that Fitch Ratings assigned to each of the 125 companies ("issuer ratings") comprised in the fifth series of the iTraxx index.

As a proxy of the probability of default of different rating classes we considered the 5-years cumulative default rate (CRD) computed over a portfolio encompassing all the corporate finance long-term debt issuer ratings assigned from 1990 to 2006 [3]. From those figures we derived the constant intensity of default as $\lambda = -log(1 - CRD)/5$.

Table (1) shows data about rating and probabilities used in the exercise. As highlighted by many studies real probabilities are on average lower that those backed out from CDS spreads, mainly due to scarcity of defaults occurred on the best rating classes. For the Itraxx constituencies the average default probability on a five years time horizon reduces

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Losses	Bid	Offer	Base Corr	Comp Corr	Delta
0-3%	500+15.50%	500+15.75%	12.7%	12.7%	26x
3-6%	49	50	22.5%	6.4%	5x
6-9%	14	15	29.7%	13.4%	1.5x
9-12%	5.9	6.9	35.4%	17.5%	0.75x
12-22%	2.7	3.2	51.3%	25.0%	0.3x

10Y Reference 48

Losses	Bid	Offer	Base Corr	Comp Corr	Delta
0-3%	500+46.88%	500+47.13%	9.9%	9.9%	10.5x
3-6%	397	400	14.9%	31.9%	12x
6-9%	98	100	22.8%	2.4%	5x
9-12%	46	47	29.6%	8.6%	2.5x
12-22%	16.75	17.75	48.8%	16.1%	1x

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2006 Bloomberg L.P.
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Figure 2: iTraxx trading screen.

of half a percentage point (from 2.45% to 1.91%), if one chooses not to take into account the risk premia asked by investors and to adopt an objective measure based on historical rate of default.

In order to evaluate the sensitivity of the two proposed risk measures to the correlation parameter we considered values of ρ ranging from 5% to 30%. We choose to use the same correlation measure for all the tranche rather than the different tranche's correlations quoted in the market[16], as the same idea of multiple correlation for the same pool lacks of logical background and should be viewed as a signal of the limited adequacy of the standard pricing model to explain the equilibrium tranches' prices.

The regulatory charges have been determined using both the approaches reserved to IRB banks: RBA and SFA. In implementing the Supervisory Formula we obtained the K_{irb} parameter using the risk neutral PDs described before[17] and a fixed LGD value of 60%.

The RBA charges are instead based on the informal "credit assessments" expressed by Fitch on the tranches of the fifth series of the iTraxx. Even if these credit opinions should be viewed as general assessments of the creditworthiness of different tranches and there is no assurance that, if requested, an actual rating would have been the same as the assessment, we think that these evaluations represent a reliable proxy of ratings as they are based on the results produced by the proprietary model used by Fitch for CDOs evaluations (VECTOR)[18]. Moreover, in our opinion the straightforward nature of the iTraxx tranches do not allow much room to modify the model results based on the qualitative characteristics

[16]On September 15 the implied compound correlation spanned from 13% of the 0-3% tranche to 25% of the 12-22%.

[17]At this stage of the study we didn't compute the SFA capital charge using the real world probabilities.

[18]The centerpiece of Fitch's CDO rating methodology, VECTOR is a multirisk factor model that takes as input rating based default probabilities, recovery rate assumptions and correlation to produce default and loss distribution using a Monte Carlo simulation.

of the transaction.

7 Results

Previous studies reported the emergence of material differences between the charges generated by the two approaches introduced to measure banks' capital requirements for structured credit products, with the Supervisory Formula being the tougher one.

Applying the new prudential regime to iTraxx tranches we found the gap between SFA and RBA to be quite conspicuous, as for mezzanine tranches the former approach brings to capital requirements that are 27 times higher than those based on ratings (table 4)[19].

Moreover, our exercise shows that under certain circumstances RBA charges do not fully cover the economic risk of tranches, particularly for mezzanine investments. We obtained a better alignment between regulatory charges and risks with the Supervisory Formula.

A key role in explaining differences between RBA charges and our risk estimates is played by the value of correlation between defaults of firms in the pool. Using *risk neutral* probabilities of default and correlation of 10% or greater, our estimates of economic capital absorbtion (UL_{mod}) tend to exceed the RBA charges for all tranches, but the equity [20].

The gap appear really wide for mezzanine tranches: considering a default correlation of 15% the regulatory charges becomes almost 8 times lower (table 6) than the capital absorbtion computed taking into account the Mark to Market effect.

It is worth noting that with a default correlation of 15% there is a large probability that under extreme scenarios losses will wipe out the equity tranche (for $\rho = 30\%$ also the mezzanine will be canceled out) and inflict some minor losses even to the senior tranche.

As one could expect the differences between regulatory and economic capital reduces moving to *actual* probabilities of default. However, even under these measures the economic capital goes beyond the regulatory measure should default correlation assume values greater than 15% (table 7).

The gap tends also to reduce a little bit if we discard the maturity effect, applying a measure of risk that doesn't take into account the cost of disposing the investment at the end of the one year holding period. As reported in table 9, if we adopt such a measure of risk using actual default probabilities and a default correlation of 15% it emerges a substantial coherence between RBA weights and risk estimates. Even in this case, however, moving to a risk neutral enviroment causes the emergence of a substantial gap between regulatory and economic capital (table 8) and the same happens if we allow the default correlation to increase.

8 Conclusion

We found a gap between RBA charges and those produced by SFA that for same kind of investments looks can be defined too large, especially if one thinks of the possible reactions of market participants.

Moreover, the magnitude of the gaps between RBA charges and economic capital seems to be larger than those previously computed by Peretyatkin and Perraudin for stylized transactions [11].

[19] Probably the magnitude of the difference could reduce if the SFA charges were computed using risk neutral probabilities rather than real ones.

[20] Being unrated, the equity tranche is entitled of a dollar by dollar (100%) capital charge.

A major role in explaining these differences should be played the *mono-dimensional* nature of ratings, mainly linked to a PD measure rather than to extreme values of the loss distributions.

Moreover, bias can arise because the PDs underlying Fitch's ratings are generated by a process which diverges from the One Factor Gaussian Copula model, especially with respect to the assumption concerning the interdependence of defaults.

As highlighted, part of the gap depends on the use of risk neutral probabilities of default; in this regard we are aware of the favour toward employing *actual* measures for risk management purposes, but we think that failing to consider the information conveyed by the a deep and liquid market, like that of single-name CDS, could bring to unsatisfactory results, especially in the case of corporate CDOs.

Our analysis shows a sensible underestimation of risk when default correlation increases. This finding is quite relevant from a regulator's perspective: many studies on asset returns (e.g. [7]) show that correlation levels tend to increase during economic downturns, exactly the situations regulatory capital has to cope with.

We think that it could be useful to explore the robustness of RBA charges with respect to other structured credit products, as CDOs of ABS or less stylized corporate CDOs; should the same findings emerge even in these cases, it could be really questioned whether the Basel II choice of mapping from ratings to capital is the right one, or it is encouraging new forms of regulatory arbitrage.

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Names	Rating (Fitch)	Real PD (%)	Risk Neutral PD (%)
Abn Amro	AA-	0.03	0.93
Accor	BBB	3.74	3.15
Adecco S.A.	BBB-	4.76	3.58
Aegon N.V.	AA	0.15	1.21
Akzo	A-	1.42	2.00
Allianz	A+	0.37	0.91
Altadis	BBB+	2.24	2.56
Arcelor	BBB	3.74	3.31
Auchan	A	0.5	1.15
Aviva	A+	0.37	1.19
AXA	AA-	0.03	1.24
BAA	A-	1.42	4.09
BAE system	BBB	3.74	2.36
Banca BPI	BBB	3.74	2.32
Banca Intesa	AA-	0.03	1.09
Banca MPS	A+	0.37	1.32
Banco Bilbao	AA-	0.03	1.07
Banco Comercial Portugues	A+	0.37	1.14
Banco Espirito Santo	A+	0.37	1.24
Banco Santander	AA	0.15	1.28
Barclays	AA+	0.03	0.79
Bayer Aktiengesellschaft	BBB+	2.24	2.56
Bayerische Motoren Werke	A+	0.37	1.11
Bertelsmann	BBB+	2.24	2.76
Boots	BBB	3.74	2.64
British American Tobacco	A-	1.42	2.16
British Telecommunications	BBB+	2.24	3.39
Cadbury Schweppes	BBB	3.74	2.84
Capitalia	A-	1.42	1.40
Carrefour	A	0.5	1.13
Centrica	A	0.5	1.96
Ciba Specialty Chemicals	BBB	3.74	3.89
Commerzbank	A	0.5	1.72
Compagnie de Saint-Gobain	A-	1.42	2.24
Compagnie Financiere Michelin	BBB+	2.24	3.04
Compass	BBB+	2.24	3.08
Continental	BBB+	2.24	2.92
DaimlerChrysler	BBB+	2.24	4.39
Degussa	BBB-	4.76	7.26
Deutsche Lufthansa	BBB-	4.76	3.93
Deutsche Telekom	A-	1.42	3.47
Deutshe bank	AA-	0.03	1.11
Diageo	A+	0.37	1.24

Names	Rating (Fitch)	Real PD (%)	Risk Neutral PD (%)
Dsg International	BBB+	2.24	2.96
E.ON	AA-	0.03	0.99
EADS	A	0.5	1.76
Edison	BBB	3.74	1.44
EDP - Energias de Portugal	A	0.5	1.09
Electrcite de France	AA-	0.03	0.66
Electrolux	BBB+	2.24	3.08
EnBW Energie Baden-Wuerttemberg	A-	1.42	0.99
Endesa	A-	1.42	1.17
Enel	A+	0.37	0.99
Finmeccanica	BBB	3.74	1.72
Fortum	A+	0.37	1.07
France Telecom	A-	1.42	2.88
Gallaher Group	BBB	3.74	2.52
Gas Natural	A+	0.37	1.44
Generali	AA-	0.03	0.95
GKN Holdings	BBB-	4.76	4.59
Glencore International	BBB-	4.76	7.15
GUS	BBB+	2.24	3.06
Hannover Rueckversicherung	A+	0.37	2.08
Hellenic Telecommunications	BBB	3.74	3.35
Henkel	A-	1.42	1.36
HVP	A	0.5	1.07
Iberdrola	A+	0.37	1.42
Imperial Chemical Industries	BBB	3.74	3.11
Imperial Tobacco	BBB	3.74	3.15
ITV	BBB-	4.76	2.41
Kingfisher	BBB	3.74	3.78
Koninklijke	A-	1.42	10.17
Koninklijke Philips Electronics	BBB+	2.24	1.64
Lafarge	BBB	3.74	2.92
Linde Aktiengesellschaft	BBB-	4.76	3.94
Louis Vuitton	BBB	3.74	1.88
Marks and Spencer	BBB+	2.24	2.60
Metro	BBB	3.74	2.52
mmO2	BBB+	2.24	2.08
Muenchener Ruck	AA-	0.03	1.58
National Grid	BBB+	2.24	2.12
Nestle	AAA	0.03	0.33
Nokia	A+	0.37	0.91
Pearson	BBB+	2.24	3.11
Peugeot	A-	1.42	2.30

Names	Rating (Fitch)	Real PD (%)	Risk Neutral PD (%)
Portugal Telecom Internat. Finance	BBB	3.74	3.94
PPR	BBB-	4.76	3.78
Reed Elsevier	A-	1.42	1.80
Renault	BBB+	2.24	3.00
Rentokil Initial 1972	BBB	3.74	1.88
Repsol	BBB+	2.24	2.32
Reuters	A-	1.42	1.80
RWE	A+	0.37	1.01
Safeway	BBB	3.74	3.55
Sanofi Aventis	AA-	0.03	1.60
Sanpaolo Imi	AA-	0.03	1.09
Siemens	AA-	0.03	0.91
Sodexho Alliance	BBB	3.74	2.12
Stora Enso	BBB	3.74	3.19
Suez	A	0.5	0.99
Svenska Cellulosa Aktiebolaget	BBB+	2.24	2.12
Swiss Reinsurance Company	AA-	0.03	1.64
Tate & Lyle	BBB	3.74	2.44
Technip	BBB	3.74	2.00
Telecom Italia	BBB+	2.24	5.64
Telefonica	BBB+	2.24	3.47
TeliaSonera	A-	1.42	3.55
Tesco	A+	0.37	1.07
The Royal Bank o Scotland	AA+	0.03	0.75
Thomsom	BBB	3.74	5.23
Unicredir	A+	0.37	1.30
Unilever	A+	0.37	1.24
Union Fenosa	BBB+	2.24	2.28
United Utilities	BBB+	2.24	1.56
UPM-Kymmene	BBB	3.74	3.11
Valleo	BBB	3.74	7.80
Vattenfall Aktiebolag	A+	0.37	1.07
Veolia Environnement	A-	1.42	2.12
Vivendi Universal	BBB	3.74	3.51
Vodafone Group	A-	1.42	2.56
Volkswagen	A-	1.42	1.96
Volvo	A-	1.42	2.82
Wolters Kluwer	BBB	3.74	3.78
WPP 2005	BBB+	2.24	2.44
Zurich Insurance Company	A	0.5	1.68

Table 1: *Risk neutral* vs *Real world* 5-years probabilities for the 125 names.

Counterpart	*Rating classes*						
	AAA to AA-	A+ to A-	BBB+ to BBB-	BB+ to BB-	B+ to B-	below B-	*unrated*
Sovereign	0%	20 %	50%	100%	100%	150%	100%
Banks (options 1)	20%	50 %	100%	100%	100%	150%	100%
Banks (options 2)	20%	50 %	50%	100%	100%	150%	100%
Corporates	20%	50 %	100%	100%	150%	150%	100%
Securitisation products	20%	50 %	100%	350%	Capital deduction	Capital deduction	Capital deduction

Table 2: Standard approach. Risk weights.

<table>
<tr><td rowspan="2">Long Term Rating</td><td colspan="3">Tranche type</td></tr>
<tr><td>Senior</td><td>Base case</td><td>Backed by non granular pools</td></tr>
<tr><td>Aaa/AAA</td><td>7%</td><td>12%</td><td>20%</td></tr>
<tr><td>Aa/AA</td><td>8%</td><td>15%</td><td>25%</td></tr>
<tr><td>A1/A+</td><td>10%</td><td>18%</td><td rowspan="3">35%</td></tr>
<tr><td>A2/A</td><td>12%</td><td>20%</td></tr>
<tr><td>A3/A</td><td>20%</td><td>35%</td></tr>
<tr><td>Baa1/BBB+</td><td>35%</td><td colspan="2">50%</td></tr>
<tr><td>Baa2/BBB</td><td>60%</td><td colspan="2">50%</td></tr>
<tr><td>Baa3/BBB-</td><td colspan="3">100%</td></tr>
<tr><td>Ba1/BB+</td><td colspan="3">250%</td></tr>
<tr><td>Ba2/BB</td><td colspan="3">425%</td></tr>
<tr><td>Ba3/BB-</td><td colspan="3">650%</td></tr>
<tr><td>Ba3/BB-Unrated</td><td colspan="3">Deduction</td></tr>
</table>

Table 3: RBA risk weights.

Tranche	Fitch Ratings	Delta*	RBA cc**	FSA cc	FSA/RBA
$0-3$	NA	26x	100.00	100.00	1.00
$3-6$	BBB-	5x	8.00	92.61	11.58
$6-9$	AAA	1.5x	0.56	15.51	27.70
$9-12$	AAA	0.75x	0.56	1.30	2.32
$12-22$	AAA	0.3x	0.56	0.56	1.00
$22-100$	NA	-	0.56	0.56	1

*Delta measures the sensitivity of the tranche's MtM to a change in the index spread.
** *cc* indicates *capital charge*. A 100% capital charge corresponds to 1250% RWA.

Table 4: Regulatory capital charge as percentage of tranche's notional amount.

Tranche	RBA	UL_{mod} - *Risk Neutral Probabilities* ρ					
		5%	10%	15%	20%	25%	30%
$0-3$	100	48.7	71.8	100	100	100	100
$3-6$	8.00	6	20.8	43.5	63.4	78.4	100
$6-9$	0.56	0	1	4.4	11	29	55
$9-12$	0.56	0	0	0.6	2	4.5	9
$12-22$	0.56	0	0	0	0	0.5	1.2
$22-100$	-	0	0	0	0	0	0

Table 5: Economic capital charge as percentage of tranche's notional amount.

Tranche	UL_{mod} vs RBA - *Risk Neutral Probabilities* ρ					
	5%	10%	15%	20%	25%	30%
$0-3$	0.5	0.7	1	1	1	1
$3-6$	0.7	2.6	5.4	7.9	9.8	12.5
$6-9$	0	1.9	7.8	19.4	51.8	98.5
$9-12$	-	-	1	3.2	8	16.3
$12-22$	-	-	-	0.2	1	2
$22-100$	-	-	-	-	-	-

Table 6: Economic capital vs RBA. Capital charge ratio.

Tranche	RBA	UL_{mod} - *Actual Probabilities* ρ					
		5%	10%	15%	20%	25%	30%
$0-3$	100	31	57.3	76.6	100	100	100
$3-6$	8.00	0	8	20	41.3	60	78
$6-9$	0.56	0	0	0.8	3.2	8.4	20.4
$9-12$	0.56	0	0	0	0.4	1.3	2.6
$12-22$	0.56	0	0	0	0	0	0.3
$22-100$	-	0	0	0	0	0	0

Table 7: Economic capital charge as percentage of tranche's notional amount.

Tranche	RBA	$E_t(L\|x_{99.9})$ - *Risk Neutral Probabilities* ρ					
		5%	10%	15%	20%	25%	30%
$0-3$	100	45	71.5	89.4	97.3	99.5	100
$3-6$	8.00	0.4	5	20.6	46.8	73.6	91
$6-9$	0.56	0	0	0	3	13.8	37.1
$9-12$	0.56	0	0	0	0	0	3
$12-22$	0.56	0	0	0	0	0	0
$22-100$	-	0	0	0	0	0	0

Table 8: Economic capital charge as percentage of tranche's notional amount.

Tranche	RBA	$E_t(L\|x_{99.9})$ - *Actual Probabilities*					
		ρ					
		5%	10%	15%	20%	25%	30%
$0-3$	100	35.3	58.4	78.3	91.3	97.4	99.5
$3-6$	8.00	0	1.5	8.4	24.2	47.4	71.1
$6-9$	0.56	0	0	0	0.4	3	11.7
$9-12$	0.56	0	0	0	0	0	0
$12-22$	0.56	0	0	0	0	0	0
$22-100$	-	0	0	0	0	0	0

Table 9: Economic capital charge as percentage of tranche's notional amount.