Implied Correlations: Smiles or Smirks?

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This version: Aug 15, 2007

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Abstract

With standardized collateralized debt obligation (CDO) tranches on credit default swap (CDS) indices trading very actively, the concept of implied correlation and correlation trading have gained popularity in recent times. In this paper, we examine the commonly observed 'implied correlation smile' pattern that is oftentimes interpreted to signal relative value opportunities across tranches. We investigate whether implied correlation smile can arise as a result of model mis-specifications in the industry standard one factor Gaussian copula default time model rather than relative mis-pricing across tranches. Our evidence shows that empirical features like fat tails in return distributions, heterogeneous pair-wise correlations, heterogeneous spreads, and correlation between default probabilities and recovery rates can give rise to smile patterns in implied correlations even when all the tranches are fairly priced. The standard Gaussian copula model assumes away these features and is thus mis-specified. Our results suggest that implied correlations computed from this model are not very useful to determine relative mis-pricing across tranches. On the other hand, we also show that one factor Gaussian copula model and implied correlations obtained from this model perform well for pricing a given tranche across time, even in periods of market turmoil related to credit risk events.

JEL Classification: G 12, G 13
Key words: Collateralized Debt Obligations, Gaussian Copula, Implied Correlation Smile

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1. Introduction

Recent developments in credit markets have led to a renewed focus on correlations among credits. A number of securities whose values depend on correlations among a set of credits have started trading actively. The most popular ones include collateralized debt obligation (CDO) tranches and basket default swaps. Correlation affects the values of these securities because of its pronounced impact on the shape of the default loss distribution of credit portfolios that underlie these securities. One of the most important drivers of active trading of such correlation dependent securities has been the rapid standardization in credit derivatives market. CDS indices like the 'Dow Jones CDX North America Investment Grade (DJ.CDX.NA.IG)' and 'iTraxx Europe' are standardized, equally-weighted, tradable portfolios of credit default swaps that have high liquidity. A further development has been the standardization of CDO tranches on standardized indices, which trade very actively[1]. With active trading of these tranches, market participants have the opportunity to trade correlations, just like option traders talk about trading volatility when they trade options. Borrowing further from the analogy of 'implied volatility' in the options market, it has become commonplace to talk about 'implied correlation' to refer to correlation extracted from the observed tranche prices using a simple one-factor Gaussian copula model. Furthermore, market participants rely on these implied correlations to propose relative value strategies among tranches.[2]

To infer implied correlations from observed CDO tranche prices, one needs a standard model for valuing CDOs similar to the Black and Scholes (1973) model (Black-Scholes model hereafter) that is used to obtain implied volatilities in the options market. It has become an industry practice to use a simple, one-factor Gaussian copula model first introduced by Li (2000)[3] to obtain implied correlations from CDO prices. This model further assumes that all pair-wise correlations among the credits underlying a CDO tranche are homogeneous.

[1] Amato and Gyntelberg (2005) provide an overview of how standardization has been one of the main drivers of active trading in CDS indices and index tranches markets.

[2] For example, an article in December 2004 issue of *Credit* magazine suggests that one can infer relative mis-pricing across tranches by looking at the implied correlation chart. It says "investors are not being properly compensated for the risk of buying equity and junior mezzanine tranches…. The implied correlation smile shows the current strength of demand for equity and mezzanine tranches." The article goes on to "…advising clients to take advantage of the richness of the 3-6% tranche in particular, either to express a bearish view on the market or as a cheap hedge against other tranches…".

[3] See Laurent and Gregory (2005) and Burtschell, Gregory and Laurent (2005) for a comparative analysis of alternative CDO pricing models. See Hull and White (2004) for details of CDO pricing.

Therefore, one can back-solve the standard Gaussian copula model to compute one implied correlation number from the observed price of each tranche.

When implied correlations are computed from a set of CDO tranches on a given reference portfolio, the resulting correlations should theoretically be identical because they all refer to correlations among credits in the same reference portfolio. In practice, however, implied correlations computed from the observed prices of standardized tranches on CDS indices show a pronounced and persistent U-shaped pattern (with reference to tranche attachment points). This pattern is often called the 'correlation smile' and is analogous to the 'volatility smile' seen in the equity options markets.

It is not uncommon to observe that market participants rely on these implied correlations as measures of true underlying correlations or to propose relative value strategies among tranches based on correlation smile. In this paper, we show that correlation smile does not always suggest relative mis-pricing of tranches. Since the computation of implied correlations is based on the industry standard Gaussian copula model with its strong simplifying assumptions, various model mis-specifications can cause implied correlations to be different from the true underlying correlations, and lead to smile shaped patterns even when all tranches are fairly priced. Thus, one has to be cautious in interpreting observed correlation smile patterns as relative value opportunities.

We show that correlation smile can arise from the following assumptions of the industry standard Gaussian copula model: (1) a Gaussian copula instead of a fat-tailed copula, (2) homogenous correlations among underlying credits in the reference portfolio instead of heterogeneous ones, (3) homogeneous spreads for underlying credits rather than heterogeneous ones, and (4) uncorrelated default probabilities and recovery rates rather than correlated ones. Thus, analyzing implied correlation smiles for relative value opportunities across tranches can be misleading unless one can take into account the impact of certain model mis-specifications.

Given these results, can the implied correlations obtained from the standard Gaussian copula model still be useful for pricing purposes? We examine the performance of implied correlations computed from the standard Gaussian copula model as for pricing a given tranche on a future date. We find that this model and the computed implied correlations work well for pricing tranches across time, even in periods of credit market stress. Overall evidence suggests that the industry standard Gaussian copula default time model and implied correlations obtained from this model are useful for pricing tranches across time, but should be used with caution for relative value strategies across tranches.

The rest of the paper is organized as follows: Section 2 discusses implied correlation and implied volatilities. Section 3 examines the standard Gaussian copula default time model in terms of the impact of model specifications on implied correlation smiles and the performance of this model for pricing tranches across time. Section 4 concludes.

2. Implied Volatility and Implied Correlation

Since the idea of implied correlations developed as a direct analog of implied volatility idea in the option pricing literature, in this section, we begin with a brief discussion of implied volatility and volatility smile in equity options market. We then examine implied correlation and correlation smile patterns commonly seen in the liquid market of standardized tranches on CDS indices.

2.1. Implied Volatility and Volatility Smile

Implied volatility is a commonly used construct in equity options market. It refers to the volatility parameter in the Black-Scholes option pricing model that represents the volatility of returns on the underlying stock. It is computed by equating the observed option price to the model price obtained from the Black-Scholes model and then solving for the volatility parameter. The assumption is that all other inputs to the model are known. Implicitly, this calculation presumes that observed market prices are adequately described by the Black-Scholes model. If this assumption were indeed true, then implied volatilities computed from a set of options on the same underlying stock but with different strike prices would all be identical because they all represent the volatility of returns on the same underlying stock. However, in practice, the implied volatilities from such a set of options are not all equal. They exhibit a skew, very often a U-shaped pattern, commonly known as the volatility smile. As an illustration, Figure 1 shows the skew observed in implied volatilities computed from a set of call options on S&P 500 index on a randomly selected day.

Insert Figure 1

The volatility smile curve suggests that the implied volatility is higher for out of the money (OTM) options than for at the money (ATM) options. Prior research shows that the reason of this smile is mainly the discrepancy between the assumptions of the Black-Scholes model and the real world[4]. The construct of implied volatility is based on the Black-Scholes model, but many of the assumptions of this model are not empirically valid. In particular, (1) return distributions are known to be fat-tailed while the Black-Scholes model uses a Geometric Brownian motion to model asset returns, thus assuming a Normal distribution, and (2) return volatility is known to be stochastic but the Black-Scholes model assumes it to be constant. The impact of any of these two violations is that large changes in the price of the underlying stock may be observed empirically with a higher frequency than what is assumed in the Black-Scholes model. OTM options, therefore, have a higher chance of becoming in the money than what is implied by the model. So, the market price of an OTM option is more than the model price. In computing implied volatility, one can match the market price with the Black-Scholes model price only by increasing the volatility parameter in the model. As a result, the implied volatility of OTM options is higher than that of ATM options. There are a number of studies that examine deviations from Black-Scholes model assumptions to fit the observed implied volatilities. For example, Das and Sundaram (1999) consider the impact of jump in stock returns as well as stochastic volatility to explain the term structure of implied volatilities.

2.2. Implied Correlation and Correlation Smile

Implied correlation is a construct that is analogous to implied volatility. As implied volatility is based on Black-Scholes model, implied correlation also has to be based on a standard and widely accepted model used to price CDO tranches. In the CDO market, a single factor Gaussian copula default time model with homogeneous correlation assumption is commonly used as the standard model for this purpose. The implied correlation parameter is

[4] Chriss (1996) provides a detailed discussion of implied volatility.

extracted from a given tranche price, assuming that all other inputs to the model are known. The standard Gaussian copula default time model[5] makes the following assumptions:[6]

(i) The dependence structure of asset returns of the names in the reference portfolio is given by a Gaussian copula.

(ii) The correlations among asset returns are driven by a single common factor.

(iii) All pair-wise correlations among asset returns are identical (homogeneous correlation assumption). This assumption ensures that there is only one correlation parameter in the model and thus one can solve for a single implied correlation number from a given tranche price.

(iv) All the credits in the reference portfolio have identical spreads (homogeneous spread assumption).

(v) Recovery rates on underlying credits are homogeneous across credits and are constant through time.

If implied correlation is indeed a measure of correlation among underlying credits, then one could compute it from different CDO tranches with the same underlying reference portfolio and get identical numbers. However, in practice, this is not the case. The implied correlations computed from observed prices of tranches on a given reference portfolio typically exhibit a U-shaped pattern, which is commonly referred to as the 'correlation smile', analogous to the 'volatility smile'. Figure 2.A shows a typical correlation smile pattern obtained from the market prices of standardized tranches on the CDX.NA.IG index.

Insert Figure 2

The smile pattern is quite pervasive. Figure 2.B shows that the correlation smile is consistently observed using the prices of standardized tranches on various CDS indices across time. In Figure 2.B, implied correlations are the lowest for the [3%,7%] mezzanine tranche and the highest for the [15%,30%] senior tranche. These results confirm that the correlation smile is a fairly persistent phenomenon.

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[5] A default time model works as follows: Given a default probability term structure, $DP_{t1}, DP_{t2}, \ldots, DP_T$, the default time model samples a variate u uniformly distributed between 0 and 1. Assuming that T is the maturity or horizon of interest, if $u > DP_T$, then the exposure does not default. If $DP_{t(i)} < u < DP_{t(i+1)}$, then the exposure defaults between time $t(i)$ and $t(i+1)$.

[6] For example, see Hull and White (2004) who state "the standard market model has become a one-factor Gaussian copula model with constant pairwise correlations, constant CDS spreads, and constant default intensities for all companies in the reference portfolio."

2.3. Correlation smile as an indicator of relative values among tranches

The wide range of implied correlations observed suggests that the correlation smile is unlikely to arise from minor technical factors or microstructure effects in the market. The observed correlation smile is many times perceived by the market participants to be an indicator of relative richness or cheapness of tranches on a given reference portfolio.

The spreads on the senior tranches increase as correlations among credits increase. When correlations among credits increase, the likelihood of many defaults occurring together increases, thereby increasing the likelihood of an extreme loss on the portfolio. Since senior tranches suffer losses only when the underlying portfolio has an extreme loss, the likelihood of senior tranches incurring a loss increases with a rise in correlations. This leads to a decrease in their values. Correlation smile suggests a higher than average implied correlation for senior-most tranches, which is oftentimes interpreted as their spreads being too high i.e. they are considered to be cheap. This cheapness is frequently attributed to a lack of demand for senior-most tranches, which may arise because they pay a tiny spread.

The spreads on equity tranches, on the other hand, decrease with a rise in correlation across assets. When correlations increase, the likelihoods of both very large losses and very small losses increase. Equity, being the first loss tranche, gains from a higher likelihood of very small losses. Since the loss of equity tranche is capped, it does not suffer in the same proportion from a higher likelihood of very large losses. As a result, the equity tranche overall becomes safer as correlations rise and its spread declines.[7] Correlation smile may suggest that equity tranche has a higher than average implied correlation and is thus overpriced or rich. This is again commonly attributed to excess demand for equity tranche, because it yields attractively high spreads in a low yield environment.

Since implied correlations are based on the standard one-factor Gaussian copula default time model with its strong simplifying assumptions, one has to exercise great caution before interpreting correlation smile patterns as indicating relative richness or cheapness of various tranches on a given reference portfolio. The observed smile patterns can, in substantial part, if not completely, arise from the fact that many of the assumptions in the

[7] Equity tranches are usually quoted on an upfront fee basis. A fall in spread is equivalent to a fall in the upfront fee.

standard Gaussian copula model do not reflect empirical reality. In the next section, we explore specific assumptions of this model and examine if the commonly observed correlation smile is an artifact of model mis-specifications.

3. Standard Gaussian Copula Model and Implied Correlations

We first examine the impact of certain assumptions of the standard Gaussian copula model on implied correlations. Then, we examine the performance of this model for pricing a given CDO tranche through time.

3.1. Model Mis-specifications and Implied Correlation Smile

In this section we outline our methodology to explore if model mis-specifications of the standard Gaussian copula model can cause smile patterns in implied correlations. We start with assuming a reference portfolio that resembles an actively traded CDS index and focus on pricing of CDO tranches on this index. The tranche structure that we choose is similar to the standardized tranche structure that is commonly seen in the marketplace.

We compute the fair values of these tranches using a 'price generating model'. The price generating model is assumed to be the 'true model' that drives the fair tranche values. In other words, it is the model that takes the reference portfolio and tranche characteristics as given and generates the fair values of tranches. Next, we back solve for the implied correlation parameter from the fair values of tranches, but this time using the standard Gaussian copula model, as is the common industry practice.

When the price generating model is the same as the model used to extract correlations (i.e. both are standard Gaussian copula models), we should not observe any smile pattern in the computed implied correlations i.e. we should recover the same correlation parameter value for all tranches. We allow the 'price generating model' to deviate from the standard Gaussian copula model to reflect various empirical regularities, and yet continue to use the standard Gaussian copula model to compute implied correlations, consistent with the market practice. We then examine the resulting implied correlation curves for correlation smile patterns. We consider the following deviations from the standard Gaussian copula model, each motivated by empirical evidence, (1) a fat-tailed return distribution rather than a Gaussian distribution,

(2) heterogeneous spreads on underlying credits rather than homogenous spreads, (3) heterogeneous pair-wise correlations among underlying credits rather than homogenous correlations, and (4) recovery rates correlated with default probabilities rather than constant and homogeneous recovery rates.[8] We study each deviation separately to isolate the marginal impact of each on implied correlations curve. We mainly adopt Monte-Carlo simulation technique to compute tranche prices under each 'price generating model'.

We assume a reference portfolio of 125 credit default swaps, each with a tenor of five years and equal weight (similar to the CDX.NA.IG index). The tranches on this portfolio are assumed to have break-points at 3%, 7%, 10%, 15% and 30%, again similar to the standardized tranches on the CDX.NA.IG index. In our baseline standard Gaussian copula model, we assume that all credits in the reference portfolio have identical spread of 49 basis points, which is the spread on the CDX.NA.IG Series 5 index on September 22, 2005, just after it became an on-the-run series[9]. We also assume the loss given default (LGD) to be constant at 50 percent.

We follow market convention for quoting the fair values of tranches. The fair values of all tranches except the equity tranche are expressed in terms of the fixed annual spread earned on the notional balance of the tranche. Fair spread on the equity tranche is decomposed into two components – a fixed annual spread of five percent that is earned on the notional balance of the equity tranche, called the running spread, and the remainder that is paid in advance as an upfront fee.

In the following sub-sections, we consider various deviations from the standard Gaussian copula model, one at a time, in generating the fair tranche values and then investigate the resulting implied correlation patterns.

3.1.1. Fat tailed return distributions

It has been well documented that the observed returns on financial assets have fat tailed empirical distributions (e.g. see Fama (1965)). The standard Gaussian copula model, however, assumes that asset returns are normally distributed. To examine the impact of this

[8] Given that one factor models can capture most of the variation in asset returns, in this paper, we do not explore whether the assumption of a one-factor model can also lead to correlation smile pattern.

[9] We obtain Index spread data from Mark-It Group. CDX.NA.IG Series 5 is initiated in September 20, 2005. To avoid beginning of the week effects, we use the index spread as of September 22, 2005 which is a Wednesday.

assumption on implied correlations when the true asset return distribution has fat tails, we assume a 'price generating model' that incorporates a double t-copula model for asset returns, with a degree of freedom parameter set to four.[10] As shown in Figure 3, a t-copula is a convenient way to generate fat-tailed return distribution. The remaining assumptions in our 'price generating model' are the same as the standard Gaussian copula model.

Insert Figure 3

In our price generating model, the asset return X_i on i^{th} asset is given by

$$X_i = a_i Z + \sqrt{1 - a_i^2}\,\varepsilon_i \tag{1}$$

Here, Z is the common risk factor and ε_i's are the identically distributed shocks independent of Z. In the double t-copula model, we assume that both Z and ε_i's have standard t-distributions with ν degrees of freedom (we assume $\nu=4$). By contrast, a Gaussian copula model assumes that all these distributions are standard Normal. By construction, the correlation between X_i and X_j is $a_i a_j$. In this section, we assume homogenous correlations across assets, i.e. $a_i = a$, for all i.

We generate fair values of standard CDO tranches using above price generating model. These values are given in Table 1, Panel (A). For comparison, Panel (B) of the same table gives the fair values obtained using the standard Gaussian copula model. Panel (C) and Figure 4 show implied correlations computed treating the fair values in Panel (A) as market prices and using the standard Gaussian copula model to solve for implied correlations. As presented in Panel (C) and Figure 4, there is a clear U-shaped pattern in these implied correlations, similar to that observed in Figure 2 using actual market prices.[11] Thus the evidence suggests that using standard Gaussian copula model when in reality asset returns are fat-tailed can lead to a pattern similar to the observed implied correlation smile.

Insert Table 1, Figure 4

Another important point to note from Figure 4 is that model mis-specification like the one discussed in this sub-section can drive a substantial wedge between true correlations and

[10] Hull and White (2004) find a good fit between model prices and market quotes for the iTraxx EUR index tranches using a degree of freedom equal to four.

[11] We repeat the analysis using different degrees of freedom parameters for the t-copula model and obtain a similar correlation smile. As degrees of freedom increase and the t-distribution approaches normal distribution, correlation smile flattens out as expected.

the implied correlations. Each curve here represents a particular level of true correlation and the corresponding implied correlations can be read off the vertical axis. This illustration brings out the pitfalls of interpreting implied correlation as true correlation for the purpose of relative value assessments. As an example, suppose that historically observed average level of correlation for investment grade names is 20% and an investor expects these levels to remain the same in the future. Under this assumption, when we examine the implied correlation on the equity tranche in Table 1, Panel (C), it appears that equity tranche is priced at an implied correlation level of 13.1% instead of 20%. This may suggest that equity tranche spread is higher relative to historical levels (or equivalently equity tranche value is lower relative to historical levels). If so, one may infer that equity is cheap, i.e. selling protection (or going long on the equity tranche) would be a profitable strategy since equity would become less risky and its spread would decrease as correlations move up to historical levels. The investor using this relative value strategy would be making an erroneous correlation trade because in our example, equity is, in fact, fairly priced by the double t-copula price generating model.

The evidence, therefore, suggests that assuming a Gaussian copula in an environment in which asset return distributions are fat-tailed can cause correlation smiles similar to those observed with market prices. Furthermore, a relative value strategy based on these implied correlations would be misleading.

3.1.2 Heterogeneous Correlations

A critical assumption of the standard Gaussian copula model is that of homogeneous correlations i.e. all pair-wise correlations across assets in the reference portfolio are considered to be identical. This assumption is needed if one has to back out a single correlation number from an observed tranche price. In reality, however, one can expect a significant heterogeneity in pair-wise correlations for the credits in various CDS indices. For example, Figure 5 shows the distribution of KMV estimates of pair-wise asset correlations among the names in CDX.NA.IG index on September 22, 2005. These pair-wise correlations have a wide range from 0.12 to 0.63, thus confirming that the assumption of homogenous correlations is a strong one.

Insert Figure 5

Can the assumption of a homogenous dependence structure in an environment with heterogeneous correlations across assets lead to an implied correlation smile? To examine this question, we first carry out the following analysis. We assign random pair-wise correlations to each asset pair in our reference portfolio of 125 names. These correlations are generated by assigning a random uniform variate between 0 and 1 to each of the a_i coefficients in the model represented in equation (1), with Z, and ε_i following standard normal distributions.

Each name is assigned a different random sensitivity to the systematic factor Z, which results in a well-behaved heterogeneous dependence structure. We compute the fair values for our standardized tranches, keeping all other assumptions of the standard Gaussian copula model except that of homogenous correlation. Next we invert these fair values and compute the implied correlations using the standard Gaussian copula model. [12]

We repeat above analysis using different heterogeneous correlation scenarios. The results are presented in Table 2. Panel (A) gives fair values of tranches obtained in five different random heterogeneous correlations scenarios. In Panel (B), for comparison purposes, we show fair values of the same tranches as generated by the standard Gaussian copula model with a homogenous correlation value equal to the average of all pair-wise correlations in each scenario. Finally, we back out implied correlations from the fair values given in Panel (A). These results are reported in Panel (C) and plotted in Figure 6.A. The results show that these implied correlations have a U-shaped pattern similar to that is observed empirically. Thus an implied correlation smile can arise due to the homogeneous correlation assumption of the standard Gaussian copula model.

Insert Table 2, Figure 6

The evidence presented so far is based on an analysis where the correlation structures are generated in a random fashion. We next examine whether a correlation structure that is empirically estimated for the reference portfolio can also produce an implied correlation curve similar to that is observed in practice. In this case, we repeat above analysis using pair-wise asset correlations for our reference portfolio of 125 names as estimated by the Global

[12] Mashal, Naldi and Tejwani (2004) and Hager and Schobel (2005) also discuss the impact of homogenous correlations assumption on implied correlations.

Correlation model of KMV.[13] The results are reported in Table 3. The first row gives the fair values of tranches obtained from the heterogeneous correlation model. The second row shows the fair values under the standard Gaussian copula model with homogenous correlation equal to the average of pair-wise KMV correlations for our reference portfolio. The third row reports the implied correlations that are backed out from tranche fair values given in the first row. The implied correlations obtained are also plotted in Figure 6.B.

Insert Table 3

As shown in Table 3 and Figure 6.B, the shape of the implied correlation smile using KMV estimated correlations is similar to the ones obtained from a random heterogeneous correlation model given in Figure 6.A. Thus, both analyses confirm that the homogenous correlation assumption of the standard Gaussian copula model is another likely explanation for the observance of a correlation smile in CDO tranche prices.

Another important finding in above analyses is that the deviation of the implied correlation from 'true' correlation is at its maximum for mezzanine tranche ([3%,7%] tranche). Hence the homogenous correlation assumption appears to impact the mezzanine tranche considerably more than other tranches. We explain this finding as follows. For the investment grade index with an average spread of 49 basis points and recovery of 50 percent, we find that the risk-neutral expected number of defaults in the collateral portfolio over a horizon of five years is about five percent.[14] Since the mezzanine tranche covers portfolio losses in the range of three to seven percent, the expected loss on the portfolio lies in the mezzanine tranche interval. Changes in correlation structure have no impact on the first moment of the default distribution, which is the expected number of defaults, but only affect the second moment or the spread of the default distribution. Therefore, the mezzanine tranche, due to its location in the capital structure becomes relatively correlation insensitive once the correlation increases beyond a certain threshold. The insensitivity of mezzanine tranche to correlation values

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[13] KMV uses a factor model to measure asset return correlations between firms. More specifically, KMV estimates an R^2 value for each firm, which is the proportion of a firm's risk that can be explained by systematic factors. Thus R^2 captures the systematic component of a firm's risk and in a single-factor world, the product R_iR_j is a measure of the correlation between firm i and j. Using KMV estimates of R^2 for the 125 names in our reference portfolio, we construct a heterogeneous correlation structure that is more reflective of real-world correlations and re-estimate fair tranche prices and implied correlations.

[14] In the standard Gaussian copula model, default intensity, λ, is equal to credit spread divided by loss given default. Therefore, for 49 basis points spread and recovery rate of 50 percent, default intensity is 0.0098. As a result, default probability over a horizon of five years is 4.8 percent (=1-exp($-\lambda t$) = 1-exp(-0.0098*5)).

implies that different levels of correlation inputs to the standard Gaussian copula model tend to produce similar prices for the mezzanine tranche. Applying the same argument in reverse, fairly similar tranche prices when inverted can produce substantially different implied correlation numbers for this particular tranche, thus sometimes causing large deviations between implied correlation and input correlation.

Overall, the presented evidence in this section suggests that the assumption of a homogenous correlation structure in an environment with heterogeneous correlations is another source of implied correlation smile similar to that observed in the market.

3.1.3 Heterogeneous Spreads (Default Probabilities)

The standard Gaussian copula model assumes that all underlying credits in the reference portfolio have the same spread (often set equal to the average spread on the portfolio). This is a strong assumption. In Figure 7, we show the distribution of spreads in CDX.NA.IG index as on September 22, 2005 (the date which was used as our reference date earlier). The spreads are rather heterogeneous and range from a few basis points to more than 300 basis points, even in a benign credit environment. We examine below whether this assumption also contributes to a correlation smile. We first estimate fair values of tranches using the standard Gaussian copula model except that we allow each name in our reference portfolio to have a distinct CDS spread. Observed CDS spreads for the 125 CDX.NA.IG index components on September 22, 2005 are used for this purpose. The average spread on these credits (or the index spread) is 49 basis points on Sep. 22, 2005 - which is what we use for CDS spread in the homogenous spread model of previous sections. All other assumptions of the standard Gaussian copula model are retained so as to isolate the impact of heterogeneity in spreads on implied correlations.

Insert Figure 7

Table 4 Panel (A) shows the fair values of tranches with the assumption of heterogeneous spreads. The corresponding fair values with homogenous spread of 49 basis points is the same as shown previously in Table 1, Panel (B). We translate the fair values computed in Panel (A) to implied correlations using standard Gaussian copula model with homogenous spread of 49 basis points. The results are given in Table 4, Panel (B) as well as in Figure 8. We can see that the homogenous spread assumption in the standard Gaussian copula model also induces a correlation smile. This smile appears to be driven mainly by the mezzanine tranche. Moreover, the smile becomes more pronounced as the actual levels of 'true' or input correlations increase. As explained earlier, this is consistent with the general correlation insensitivity of the mezzanine tranche at medium to high levels of input correlation.

Insert Table 4, Figure 8

The evidence presented here suggests that assuming homogenous spreads in an environment where spreads are heterogeneous can also lead to a correlation smile. Therefore, the use of implied correlations for relative value strategies can be misleading. These strategies can be especially problematic when implied correlations are obtained from mezzanine tranches which are not correlation sensitive beyond a certain correlation threshold.

3.1.4. Correlated recovery rate and probability of default

Another major assumption in the standard Gaussian copula model is that recovery rate in default is constant. There is growing evidence in the literature that recovery rates are stochastic and correlated with the state of the economy.[15] For example, Altman, Brady, Resti and Sironi (2004) find a strong negative correlation between default rates and recovery rates in the US corporate bond market. Hull, Predescu, and White (2005) develop a model for CDO pricing that incorporates the relation between recovery rates and default rates.

In this section, we investigate implied correlation patterns when recoveries are stochastic and correlated with the systematic factor – our proxy for the state of the economy. High default rates correspond to low realizations of our systematic factor (bad state of the economy). Therefore, the empirically observed negative correlation between default rates and recovery rates translates into a positive correlation between our systematic factor and recovery rates.

We assume the following recovery model, wherein recovery on instrument i is decomposed into systematic and idiosyncratic components,

[15] See Acharya, Bharath, and Srinavasan (2005), Altman, Brooks, Resti, and Sironi (2005), Carey and Gordy (2005), and Frye (2000) for empirical evidence. Bakshi, Madan, and Zhang (2006), and Das and Hanouna (2006) suggest methods to extract implied recoveries from market data considering the stochastic nature of the recovery levels. Altman, Resti, and Sironi (2005) provide a detailed review on the theoretical and empirical developments related to recovery rates.

$$R_i = N\left(\rho_{RR} Z + \sqrt{1-\rho_{RR}^2}\, Y_i\right) \qquad (2)$$

where Z ,the common systematic factor, has a standard normal distribution and drives both the default and recovery processes; Y_i, the idiosyncratic recovery factor, also has a standard normal distribution; and ρ_{RR} is the correlation between the systematic factor and the recovery rate. This specification ensures that the average recovery is 50 percent, which is consistent with our fixed recovery of 50 percent assumption in the previous sections.

We first compute fair tranche prices for different values of ρ_{RR} keeping all other assumptions of the standard Gaussian copula model. We consider four different values of ρ_{RR} i.e. $\rho_{RR} = 0$, $\rho_{RR} = 0.1$, $\rho_{RR}=0.2$, and $\rho_{RR} = 0.3$. We utilize only non-negative values of ρ_{RR} to be consistent with the negative correlation between recovery rate and default probability that is documented in the literature. The fair values of the tranches corresponding to different ρ_{RR} are given in Table 5.

Insert Table 5

Using the Gaussian copula model with a fixed recovery rate of 50 percent, we extract implied correlations from fair tranche values given in Table 5. In Table 6 and Figure 9, we show these implied correlations for four different values of ρ_{RR} considered above. Each figure corresponds to a particular assumed value of ρ_{RR} and different curves in a figure correspond to a range of assumed homogeneous correlations of individual asset returns with the systematic factor in the standard Gaussian copula model. For example, in Figure 9.A, the correlation between recovery rate and the systematic factor is zero, i.e. $\rho_{RR} = 0$. In this figure, the curve labeled 0% shows implied correlations backed out when the correlation between individual asset returns and systematic factor is also zero, whereas the curve labeled 10% corresponds to the case when this correlation is 10%. These results show some interesting patterns of implied correlations. The correlation smile gets inverted when recovery is stochastic and positively correlated with the systematic factor. The smile effect is insignificant when ρ_{RR} is equal to zero, but becomes substantial for higher values of ρ_{RR}. Moreover, as the correlation between recovery rate and systematic factor, ρ_{RR}, increases, the standard model fails to converge to an implied correlation solution for the mezzanine tranche for most of the cases. In these cases, mezzanine tranche spreads are substantially high in the 'true' model where systematic factor and recovery rates are correlated. Therefore, the standard model with constant recovery rate of 50% is not able to obtain these prices for any level of constant pair-wise asset correlation. Additionally, as shown in Table 6, Panel (D) and Figure 9.D, the standard Gaussian copula model fails to back out implied correlations for all tranches except the equity tranche when ρ_{RR} is equal to 30% and when the correlation among credits is also 30%. In these cases, the increase in the 'true' prices of all tranches (except equity tranche) could not be matched with any correlation by the standard Gaussian copula model. When the correlation among credits increases, the number of joint defaults in poor states of the world increases. Since recovery is positively related to systematic factor, higher values of ρ_{RR} lead to substantially lower recoveries in these states. The resulting increase in expected losses conditional on bad states of the world gets reflected in higher prices for more senior tranches. This higher price cannot be matched at any level of correlation by assuming constant pair-wise correlations in the standard Gaussian copula model.

Insert Table 6, Figure 9

Why are the results of this section so different than those of the previous sections in terms of the shape of the implied correlation curve? The fair value of a tranche is a function of the risk-neutral loss distribution of underlying collateral assets. When recoveries are fixed, as is the case in previous sections, the loss distribution for the collateral has the same shape as the (number of) default distribution since the recovery (or LGD, the loss given default) simply scales the default distribution to a loss distribution when recovery rate is constant. Moreover, in this case, the mean of the loss distribution is simply the average number of defaults multiplied by the LGD. When recovery rate is positively correlated with the systematic factor, however, good states of the world that correspond to a small number of defaults have high recovery values. Therefore, levels of loss are low in good states of the world. Conversely, bad states of the world that correspond to a large number of defaults have low recovery values. In these cases, the losses are higher. The relation between recovery rates and default severity observed when ρ_{RR} is 30 percent is given in Figure 10. As shown in the figure, the positive correlation between recovery rate and the systematic factor leads to an increase in the expected loss.[16] Therefore, tranches become riskier as ρ_{RR} increases, and thus their prices increase. The standard Gaussian copula model that assumes 50 percent fixed recovery can match the corresponding rise in the prices only by increasing the correlation. Since mezzanine tranche is the least correlation sensitive among others, a substantial rise in correlation is required to match the corresponding prices. This leads to the inverted implied correlation curves observed in this section.

Insert Figure 10

3.2. Implied correlations for pricing of tranches through time

In the previous sections, we show that implied correlations obtained using the standard Gaussian copula model are rather inaccurate measures of true correlations among underlying credits. They can deviate substantially from true correlations because of the many simplifying assumptions in the standard Gaussian copula model that are violated in practice. We also emphasize the perils of executing correlation trades across tranches on the basis of implied correlations.

Despite the poor performance of the standard Gaussian copula model for relative value assessments across different tranches using implied correlations, the model is still quite appealing for its simplicity and efficiency in implementation. In this respect, a natural question is whether the model is useful for pricing a given CDO tranche through time. To address this question, we conduct an empirical analysis using actual market data on CDX.NA.IG index and its standardized tranches i.e. we price [0%-3%], [3%-7%], [7%-10%], and [10%-15%] tranches through time. We do not include the [15%-30%] tranche due to a large number of missing market quotes for this tranche in our dataset. Our analysis is based on the following methodology. We use observed CDX tranche prices (from GFINet), recovery estimates (from GFINet), and CDS spreads on names in the index (from Mark-It) to back out an implied correlation for each tranche on each date t. Next, we compute the model price of each tranche on the following date, i.e. $t+1$, using a Gaussian copula model. The inputs to the

[16] The increase in the mean of loss distribution is mainly driven with higher recovery rates in low default environments and lower recovery rates in high default environments due to the positive correlation between recovery rates and the systematic factor. Note that default distributions do not change whether recovery is stochastically linked to the systematic factor or it is constant. This is because default distributions are functions of the collateral asset default probabilities and their pair-wise asset return correlations only.

model are the CDS spreads and recovery estimates of date $t+1$ as well as the implied correlation for this tranche estimated at the previous date t. We then compare our model tranche prices with actual tranche prices on date $t+1$ to gauge the relative pricing performance of the standard Gaussian copula model. We repeat this process weekly until the end of the analysis period.

Our analysis covers the period from March 3, 2005 to August 17, 2005. We choose this particular period since it includes a period of stress in credit derivatives market that was triggered by rating downgrades of GM and Ford, both components of the CDX.NA.IG index. Both credits were simultaneously downgraded by S&P on May 5, 2005. Thus, our sample period allows us to test the performance of Gaussian copula model in normal times as well as in periods of market stress.

The time series of model and actual tranche prices are presented in Figure 11 for various standardized tranches. The results suggest that implied correlations computed from the standard Gaussian copula model perform well in pricing a given tranche through time. This is valid across different tranches and through both stable and turbulent periods in the markets. The model prices of mezzanine tranche occasionally have a higher deviation from actual prices. This occurs at points where implied correlations could not be backed out for this tranche and zero correlation is used instead (see Figure 2.B).[17] In particular, the standard Gaussian copula model is able to capture the considerable widening of tranche premiums around the downgrades of GM and Ford. Thus this model is valuable for pricing a given tranche through time.

Insert Figure 11

Overall, the presented evidence suggests that although a Gaussian copula model is useful for pricing tranches through time, the implied correlations obtained by this model should be used with caution for relative value strategies across tranches.

4. Conclusion

Given the increasing importance of correlation trading and the pervasiveness of a smile pattern in implied correlations, it is important to understand why such patterns arise. It

[17] In these cases, we use zero as implied correlation since zero correlation produces model prices closest to market prices.

is a standard practice in the marketplace to back out implied correlations using a simple Gaussian copula default time model with homogenous pair-wise correlations, homogenous spreads, and constant recoveries for underlying assets.

While the simplicity of this model is appealing, the evidence presented in this study shows that its assumptions can give rise to correlation smile patterns even with fairly priced tranches. Specifically, we show that standard Gaussian copula model results in correlation smiles due to the assumptions of (1) Gaussian dependence structure, (2) homogenous pair-wise correlations across assets, (3) homogenous spreads for underlying credits of a portfolio, and (4) independence of default probabilities and recovery rates. The presented evidence suggests that one should be cautious while interpreting correlation smiles obtained from the standard Gaussian copula model as indicators of relative value opportunities. On the other hand, we show that, although this model is not reliable for valuations across different tranches, it works reasonably well for pricing a given tranche across time - through periods of market stability as well as turbulence.

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Table 1

Fair Tranche Values and Implied Correlations: A Double t-Copula Model

This table shows fair values and implied correlations considering fat-tailed nature of asset return distribution, a feature that is not incorporated in the standard Gaussian copula model. For this analysis, we use standardized tranches on a reference portfolio of 125 credit default swaps. All credits are assumed identical with a spread of 49 basis points and a recovery value of 50 percent. Pair-wise correlations are assumed to be homogenous. Their assumed values are shown in the first column. Panel (A) shows tranche fair-values generated with a double t-copula model (with degrees of freedom parameter, *v*, set equal to four). Panel (B) shows fair-values of the same tranches using standard Gaussian copula model for comparison purposes. Panel (C) shows the implied correlations computed using a Gaussian copula model (as is the common practice) from the fair tranche values shown in Panel (A). Fair values are given in basis points, except the equity [0%-3%] tranche.

Panel (A): Fair values of tranches using a double t-copula model ($v = 4$)

Correlation	0%-3% Tranche (Upfront)	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.00	53.3%	77	0	0	0
0.05	49.7%	133	10	3	1
0.10	46.1%	167	26	11	4
0.15	42.8%	193	43	19	6
0.20	39.2%	206	59	28	11
0.25	36.4%	222	75	39	16
0.30	33.1%	229	90	52	22

Panel (B): Fair values of tranches using standard Gaussian copula model

Correlation	0%-3% Tranche (Upfront)	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.00	53.3%	77	0	0	0
0.05	47.3%	170	5	0	0
0.10	42.0%	231	26	3	0
0.15	37.7%	272	54	10	0
0.20	33.7%	295	79	22	2
0.25	30.2%	314	103	35	4
0.30	26.9%	324	122	48	8

Panel (C): Implied correlations computed from the fair values in Panel (A) above

Correlation	0%-3% Tranche	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.00	0	0	0	0	0
0.05	0.029	0.028	0.063	0.1	0.177
0.10	0.061	0.048	0.1	0.152	0.249
0.15	0.093	0.066	0.13	0.186	0.282
0.20	0.131	0.075	0.157	0.224	0.346
0.25	0.167	0.088	0.188	0.262	0.389
0.30	0.209	0.097	0.222	0.31	0.439

Table 2
Fair Tranche Values and Implied Correlations:
A Gaussian Copula Model with Heterogeneous Input Correlations

This table shows fair values and implied correlations considering heterogeneous nature of pair-wise asset correlations, a feature that is not incorporated in the standard Gaussian copula model. For this analysis, we use standardized tranches on a reference portfolio of 125 credit default swaps. All credits are assumed identical with a spread of 49 basis points and a recovery value of 50 percent. Heterogeneous correlations are created by assigning a random value between zero and one to the correlation between each credit and the systematic factor. Panel (A) shows tranche fair-values generated by considering heterogeneous pair-wise correlations in a Gaussian copula model. The average values of pair-wise correlations are shown in the first column. For comparison purposes, Panel (B) shows fair-values of the same tranches using standard Gaussian copula model with homogenous correlation equal to the average of pair-wise heterogeneous correlations. Panel (C) shows the implied correlations computed from the fair tranche values shown in Panel (A) using a standard Gaussian copula model with homogenous correlation equal to the average pair-wise heterogeneous correlations. Fair values are given in basis points, except the equity [0%-3%] tranche.

Panel (A): Fair values of tranches for a heterogeneous correlation structure in a Gaussian copula model

Average Correlation	0%-3% Tranche (Upfront)	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.223	31.2%	241	108	57	8
0.239	30.1%	240	112	62	9
0.246	29.4%	241	116	65	11
0.269	28.3%	258	123	68	13
0.300	26.1%	258	126	73	16

Panel (B): Fair values of tranches for the standard Gaussian copula model

Constant Correlation	0%-3% Tranche (Upfront)	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.223	31.6%	297	84	26	3
0.239	30.7%	306	95	31	3
0.246	30.5%	318	102	36	4
0.269	28.8%	317	110	41	5
0.300	27.3%	336	126	52	7

Panel (C): Implied correlations computed from the fair values in Panel (A) above

Input Average Correlation	0%-3% Tranche	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.223	0.236	0.107	0.262	0.327	0.302
0.239	0.251	0.106	0.267	0.349	0.318
0.246	0.260	0.107	0.280	0.359	0.347
0.269	0.277	0.130	0.301	0.374	0.363
0.300	0.310	0.130	0.310	0.395	0.389

Table 3
Fair Tranche Values and Implied Correlations:
A Heterogeneous Correlation Model with Historical Correlations

This table shows fair values and implied correlations considering the heterogeneous nature of pair-wise asset correlations, a feature that is not incorporated in the standard Gaussian copula model. For this analysis, we use standardized tranches on a reference portfolio of 125 credit default swaps. All credits are assumed identical with a spread of 49 basis points and a recovery value of 50 percent. The heterogeneous correlations are obtained from the Global Correlation model of KMV. Average of these correlations is 0.347. First row shows tranche fair-values generated by assuming heterogeneous pair-wise correlations in a Gaussian copula model. For comparison purposes, second row shows fair-values of the same tranches using standard Gaussian copula model with homogenous correlation equal to average correlation of 0.347. Third row shows the implied correlations computed from the fair tranche values shown in Panel (A) using a standard Gaussian copula model with homogenous correlation of 0.347. Fair values are given in basis points, except the equity [0%-3%] tranche.

	0%-3% Tranche	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
(a) Fair values with heterogeneous correlations	23.4% upfront	315	137	63	13
(b) Fair values with constant correlation	23.7% upfront	330	138	61	11
(c) Implied Correlations computed from fair values in (a)	0.347	0.257	0.347	0.354	0.356

Table 4
Fair Tranche Values and Implied Correlations: A Heterogeneous Spread Model

This table shows fair values and implied correlations considering heterogeneous nature of spreads on credits in the reference portfolio of a CDO, a feature that is not incorporated in the standard Gaussian copula model. For this analysis, we use standardized tranches on a reference portfolio of 125 credit default swaps. All credits are assumed to have a recovery value of 50 percent. We use CDS spreads on 125 CDX.NA.IG index components on September 22, 2005 for heterogeneous spreads. Panel (A) shows tranche fair-values generated by considering heterogeneous spreads in a Gaussian copula model. The pair-wise correlations are assumed homogeneous and their values are shown in the first column. Panel (B) shows the implied correlations computed) from the fair tranche values shown in Panel (A) using a standard Gaussian copula model with homogenous spread equal to the average spread on credits, which is 49 basis points. Fair values are given in basis points, except the equity [0%-3%] tranche.

Panel (A): Model prices for a heterogeneous spread model

Correlation	0%-3% Tranche (Upfront)	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.00	53.1%	71	0	0	0
0.05	47.3%	155	4	0	0
0.10	42.9%	215	21	2	0
0.15	38.8%	257	44	8	0
0.20	35.2%	284	68	17	1
0.25	31.8%	301	89	28	3
0.30	28.6%	313	108	40	6

Panel (B): Implied Correlations for a heterogeneous spread model

Input Correlation	0%-3% Tranche	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.00	0.002	0	0	0	0
0.05	0.049	0.039	0.044	0.04	0.05
0.10	0.092	0.08	0.088	0.089	0.092
0.15	0.137	0.128	0.132	0.134	0.134
0.20	0.18	0.172	0.178	0.181	0.181
0.25	0.228	0.219	0.221	0.224	0.231
0.30	0.274	0.244	0.263	0.265	0.278

Table 5
Fair Tranche Values:
A Model with Recovery Rates Correlated with Systematic Factor

This table shows the fair values of standardized tranches on a CDO when recovery rates on credits in the reference portfolio are stochastic and correlated with the systematic factor, a feature that is not incorporated in the standard Gaussian copula model. Systematic factor drives the asset returns. The correlation between the systematic factor and the recovery rate is ρ_{RR}. For this analysis, we use standardized tranches on a reference portfolio of 125 credit default swaps. Panel (A),(B),(C) and (D) show the tranche fair-values generated by assuming $\rho_{RR} = 0$, $\rho_{RR} = 0.1$, $\rho_{RR} = 0.2$ and $\rho_{RR} = 0.3$ respectively. Fair values are given in basis points, except the equity [0%-3%] tranche.

Panel (A): $\rho_{RR} = 0$

Correlation	0%-3% Tranche (Upfront)	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.00	51.63%	104	0	0	0
0.05	46.02%	189	8	0	0
0.10	41.46%	247	32	4	0
0.15	36.73%	279	57	12	1
0.20	33.14%	303	83	24	2
0.25	29.61%	318	105	38	4
0.30	26.68%	373	162	77	18

Panel (B): $\rho_{RR} = 0.1$

Correlation	0%-3% Tranche (Upfront)	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.00	51.63%	104	0	0	0
0.05	46.02%	189	8	0	0
0.10	41.46%	247	32	4	0
0.15	36.73%	279	57	12	1
0.20	33.14%	303	83	24	2
0.25	29.61%	318	105	38	4
0.30	26.41%	350	146	65	13

Panel (C): $\rho_{RR} = 0.2$

Correlation	0%-3% Tranche (Upfront)	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.00	51.26%	110	0	0	0
0.05	45.73%	249	25	2	0
0.10	41.24%	308	61	14	1
0.15	37.13%	340	96	30	3
0.20	33.76%	362	124	47	7
0.25	29.73%	365	143	62	12
0.30	26.76%	372	162	79	18

Panel (D): $\rho_{RR} = 0.3$

Correlation	0%-3% Tranche (Upfront)	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0.00	50.80%	117	0	0	0
0.05	45.47%	277	36	5	0
0.10	41.15%	336	78	20	2
0.15	37.16%	367	115	40	5
0.20	33.90%	388	144	59	10
0.25	29.95%	389	162	75	16
0.30	27.03%	394	180	92	24

Table 6
Implied Correlations:
A Model with Recovery Rates Correlated with Systematic Factor

This table shows the implied correlations obtained from the fair values of the standardized tranches on a CDO given in Table 5 using the standard Gaussian copula model with recovery rate of 50 percent. The model that drives the fair values in Table 5 consider that recovery rates on credits in the reference portfolio are correlated with the systematic factor, a feature that is not incorporated in the standard Gaussian copula model. The correlation between the systematic factor and the recovery rate is ρ_{RR}. For this analysis, we use standardized tranches on a reference portfolio of 125 credit default swaps. Panel (A),(B),(C) and (D) show the implied correlations where $\rho_{RR} = 0$, $\rho_{RR} = 0.\ 1$, $\rho_{RR} = 0.2$ and $\rho_{RR} = 0.3$ respectively. When an implied correlation could not be backed out using the standard Gaussian copula model, it is represented as not available, N/A.

Panel (A): $\rho_{RR} = 0$

Correlation	0%-3% Tranche	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0	0.0139	0.0119	0.0052	0	0
0.05	0.0615	0.0627	0.0573	0.0556	0.05
0.1	0.1082	0.112	0.1083	0.1073	0.0988
0.15	0.1609	0.1584	0.1537	0.1547	0.1623
0.2	0.208	0.2113	0.2063	0.2052	0.2047
0.25	0.2568	0.2917	0.2548	0.2576	0.2548
0.3	0.3026	N/A	0.4423	0.4156	0.4095

Panel (B): $\rho_{RR} = 0.1$

Correlation	0%-3% Tranche	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0	0.0133	0.0129	0.0098	0	0
0.05	0.0622	0.0837	0.0765	0.0757	0.0736
0.1	0.1117	0.1538	0.1342	0.1334	0.1331
0.15	0.1551	0.2394	0.1876	0.1877	0.1885
0.2	0.2007	N/A	0.26	0.2534	0.2563
0.25	0.2532	N/A	0.3113	0.3046	0.3014
0.3	0.3061	N/A	0.3763	0.3575	0.3591

Panel (C): $\rho_{RR} = 0.2$

Correlation	0%-3% Tranche	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0	0.017	0.0147	0.0102	0	0
0.05	0.0639	0.1145	0.0967	0.0942	0.0938
0.1	0.1109	0.23	0.16	0.1653	0.1724
0.15	0.1562	N/A	0.2401	0.2324	0.2347
0.2	0.1992	N/A	0.3053	0.2937	0.2944
0.25	0.2553	N/A	0.37	0.3502	0.35
0.3	0.3015	N/A	0.4432	0.4206	0.4126

Panel (D): $\rho_{RR} = 0.3$

Correlation	0%-3% Tranche	3%-7% Tranche	7%-10% Tranche	10%-15% Tranche	15%-30% Tranche
0	0.0207	0.0181	0.016	0	0
0.05	0.0661	0.1566	0.1144	0.1137	0.1178
0.1	0.1118	N/A	0.1944	0.1887	0.1933
0.15	0.1558	N/A	0.2738	0.2638	0.2671
0.2	0.1977	N/A	0.3708	0.3428	0.3411
0.25	0.2527	N/A	0.4421	0.4068	0.3913
0.3	0.2974	N/A	N/A	N/A	N/A

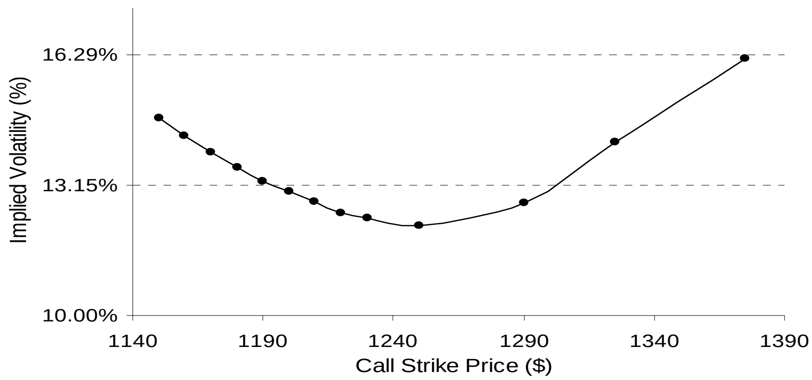

Figure 1 : Implied volatility smile on S&P 500 index options
This figure shows implied volatilities obtained using call prices as of April 7, 2004 for options expiring on June 17, 2004.

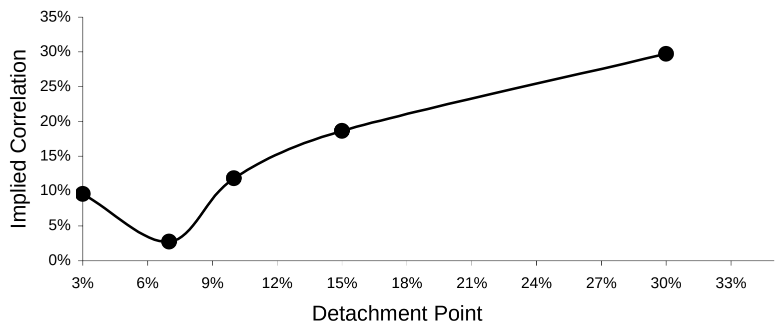

Figure 2.A

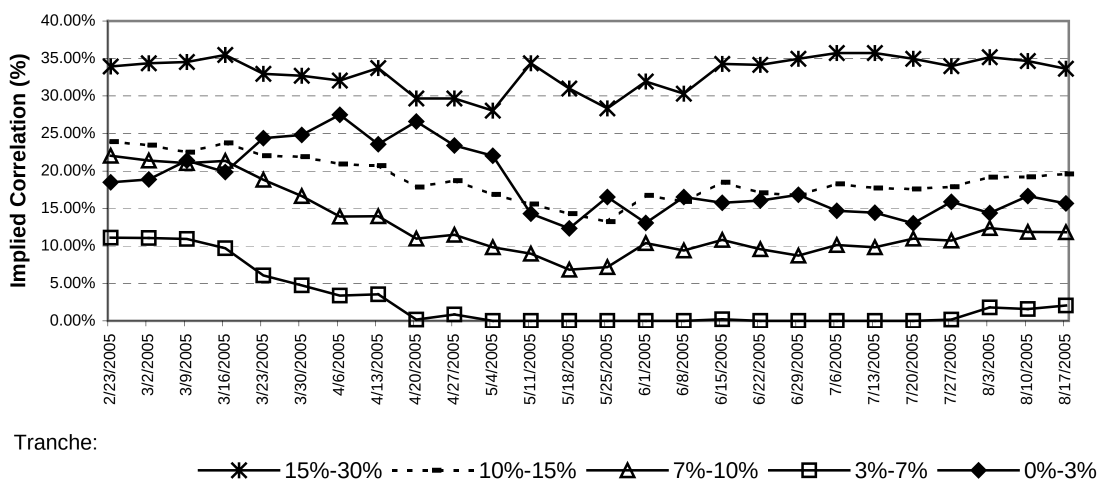

Figure 2.B

Figure 2: Implied Correlation Smiles from Market Prices of Standardized Tranches on the CDX.NA.IG Index

Figure 2.A. shows the implied correlations as of September 22, 2005. Figure 2.B shows implied correlations weekly from February 23, 2005 to August 17, 2005.

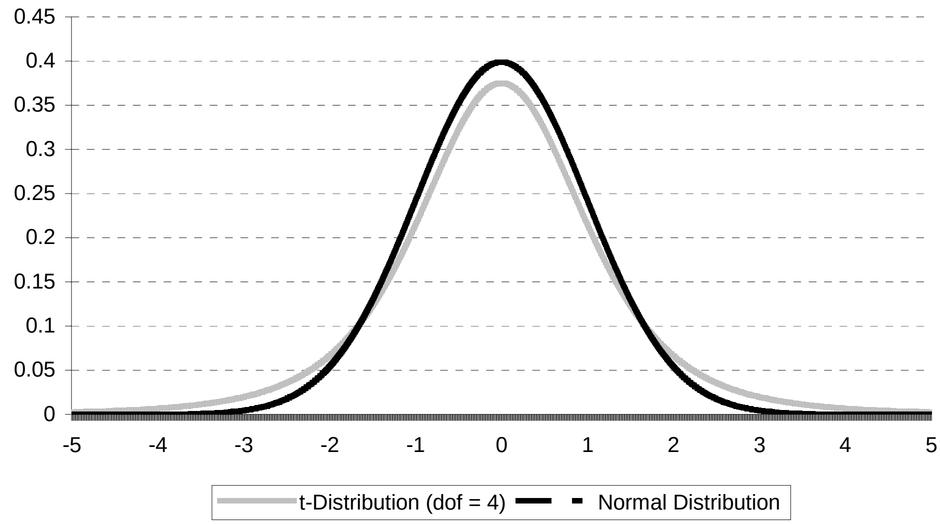

Figure 3: Probability Densities of Standard Normal Distribution and t-Distributions with Degrees of Freedom of Four

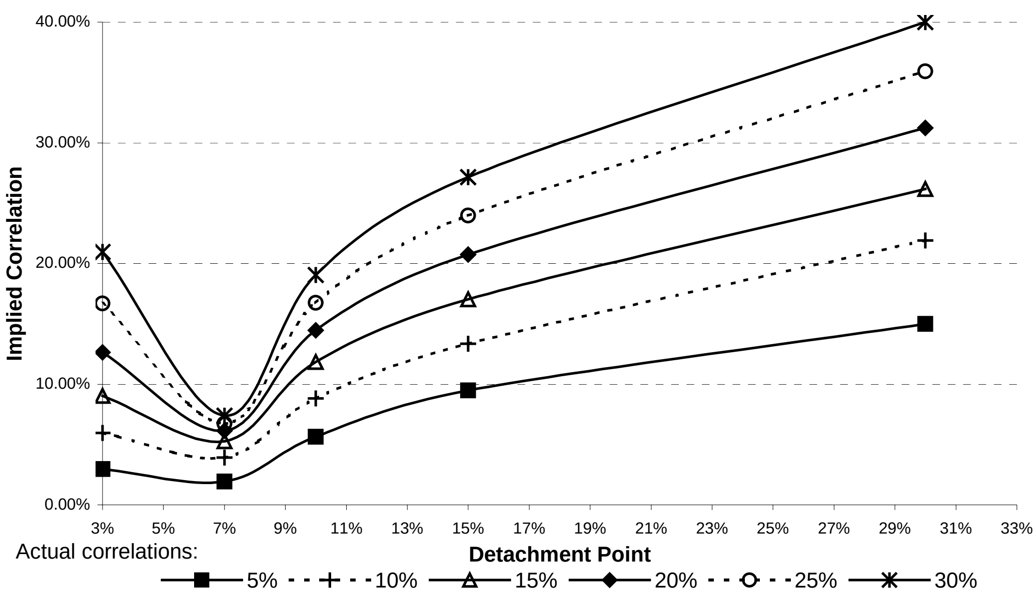

Figure 4: Implied Correlations Computed from Fair Tranche Prices where Asset Returns Follow a Double t-Copula Model

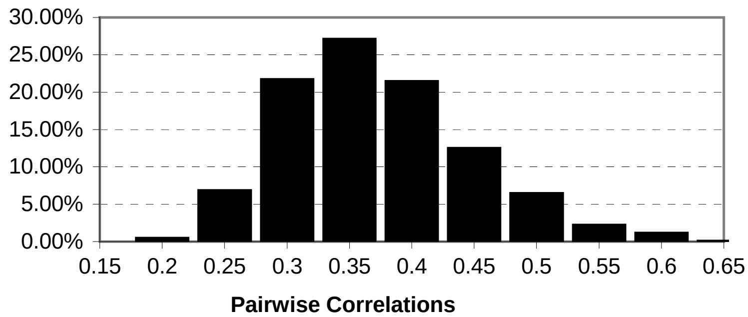

Figure 5: Pair-wise Asset Correlation Estimates
This figure shows the distribution of KMV estimates of pair-wise asset correlations among the names in CDX.NA.IG index on September 22, 2005.

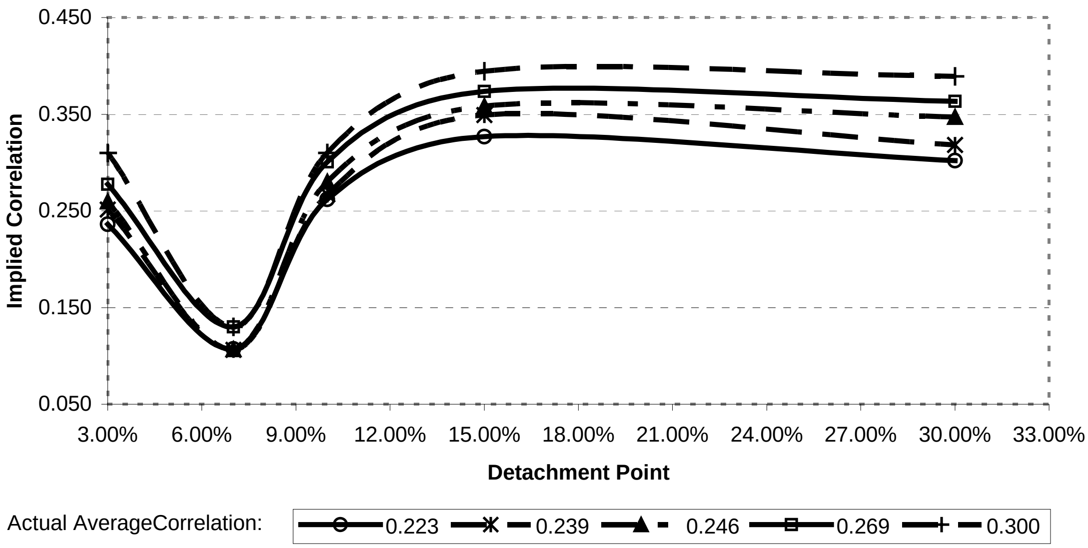

Figure 6.A

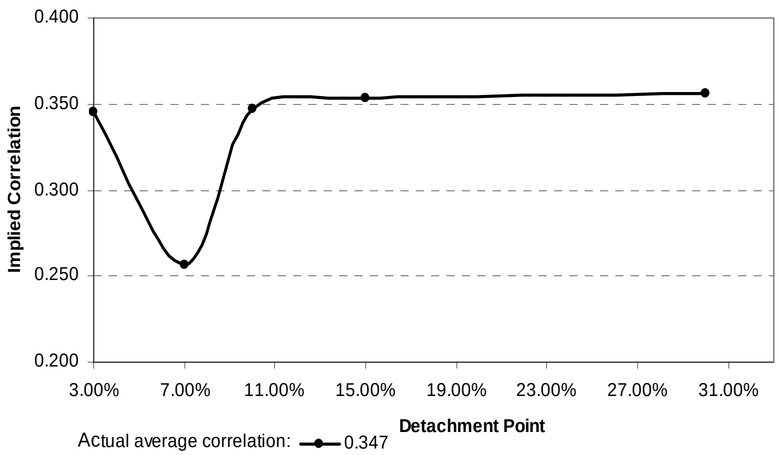

Figure 6.B

Figure 6: Implied Correlations Computed from Fair Tranche Prices with Heterogeneous Pair-Wise Correlations

This figure shows implied correlations backed out using Gaussian copula model where homogenous correlation is equal to the average of heterogeneous pair-wise correlations. In Figure 6.A, heterogeneous pair-wise correlations are random. In Figure 6.B, heterogeneous pair-wise correlations are from the Global Correlation model estimated by KMV.

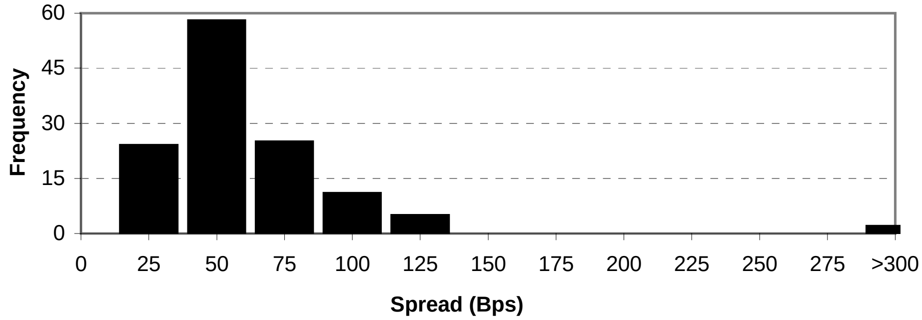

Figure 7: Distribution of CDS spreads for CDX.NA.IG index
This figure shows the distribution of five year CDS spreads for the constituents of the CDX.NA.IG index as on September 22, 2005. The spreads have a substantial heterogeneity.

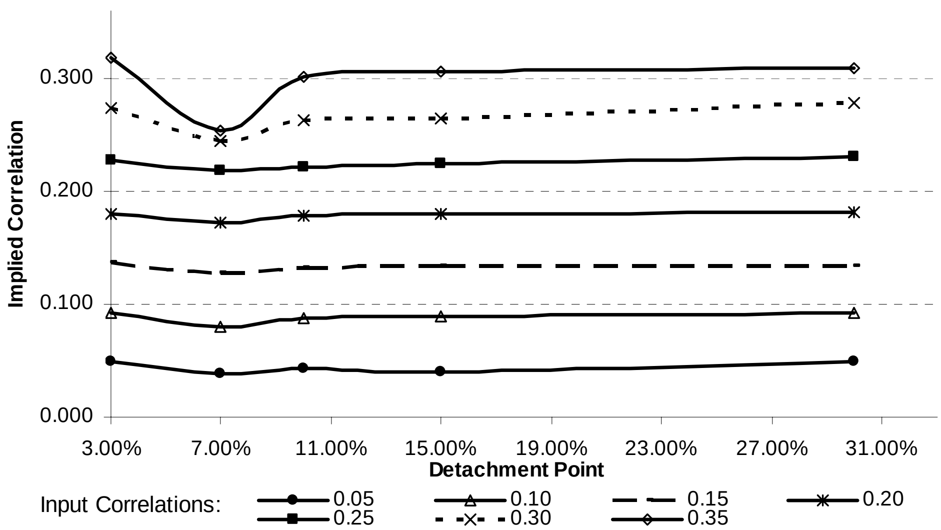

Figure 8: Implied Correlations Computed from Fair Tranche Prices with Heterogeneous Spreads

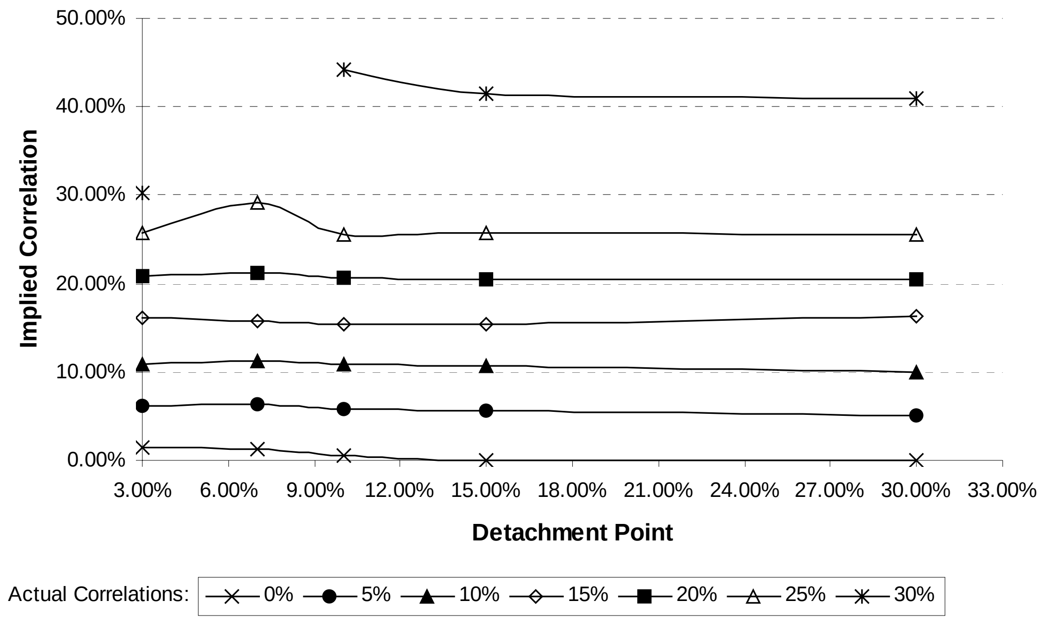

Figure 9.A

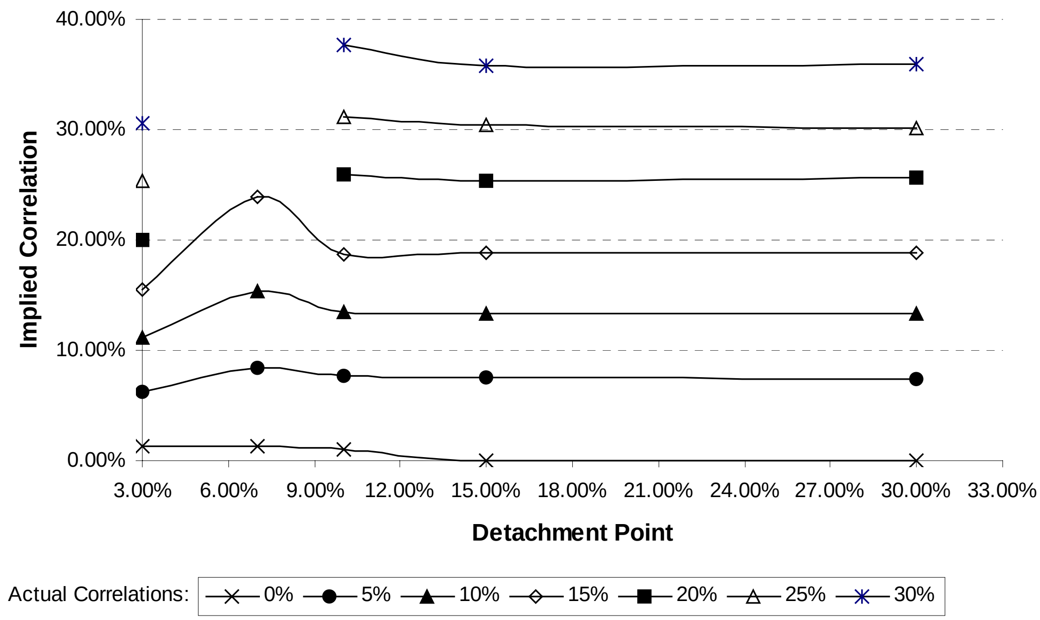

Figure 9.B

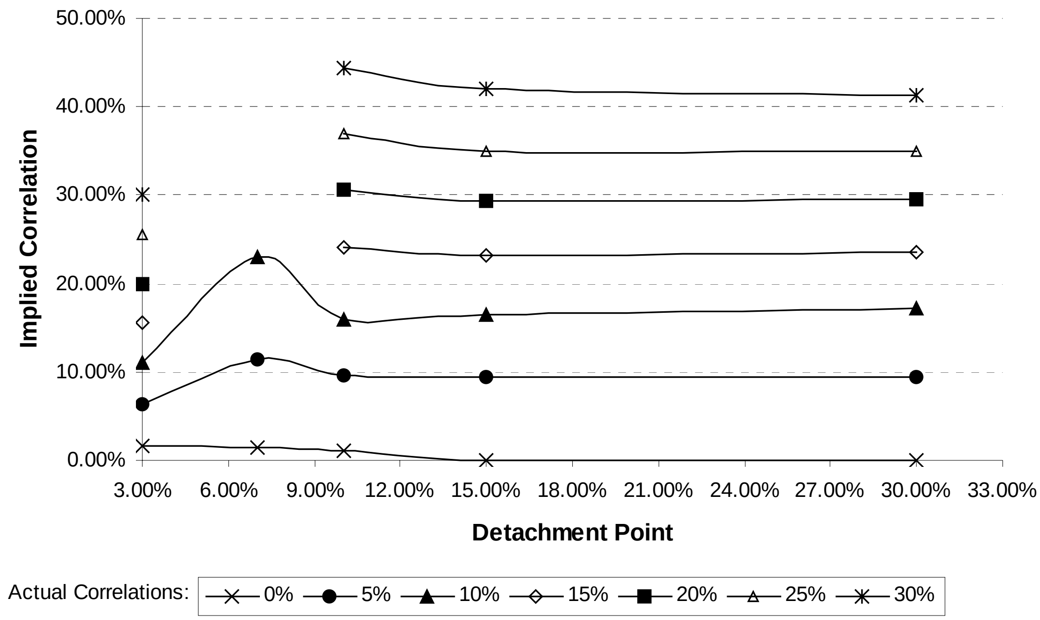

Figure 9.C

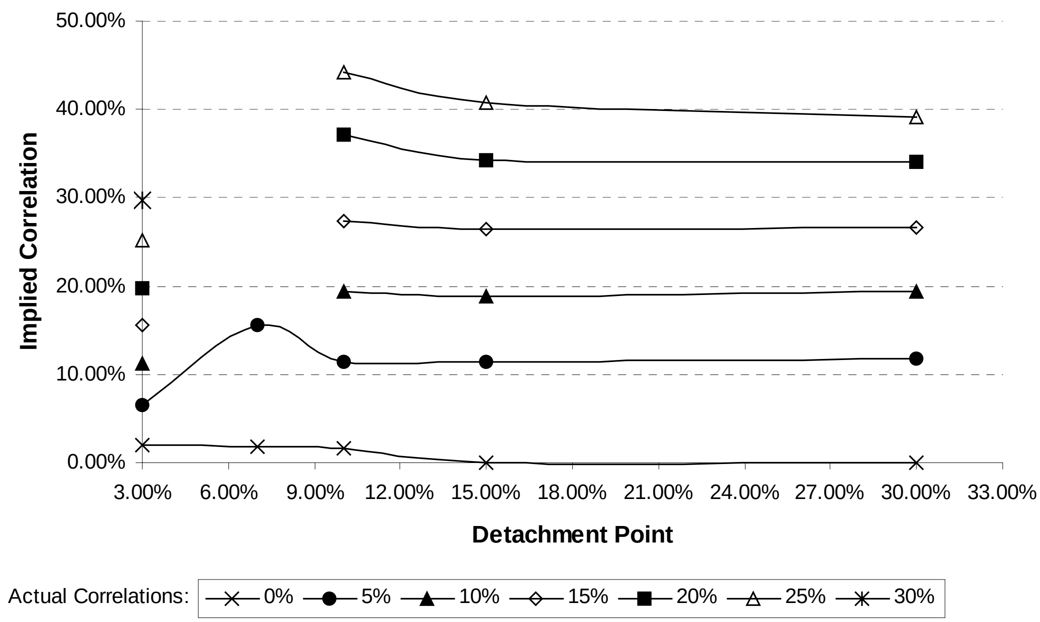

Figure 9.D

Figure 9: Implied Correlations with Stochastic Recovery

This figure shows implied correlation curves with stochastic recovery. In figures 9.A, 9.B, 9.C and 9.D, the correlation between recovery rate and systematic factor, ρ_{RR}, is 0, 0.1, 0.2 and 0.3, respectively. Different curves correspond to different homogeneous correlations between asset returns and systematic factor.

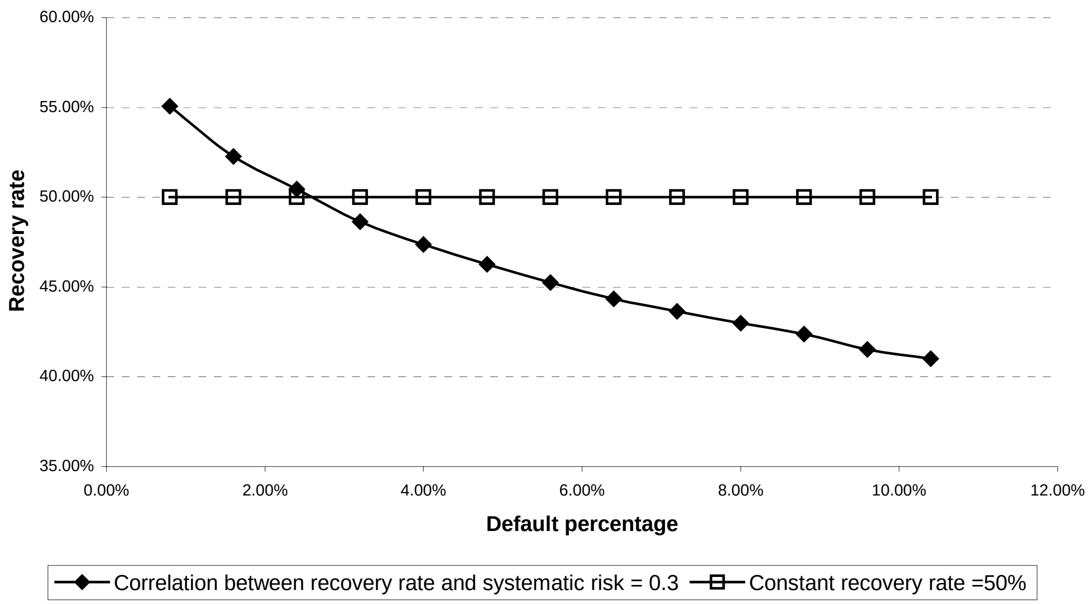

Figure 10: Recovery Rates for Different Default Percentages.
This figure shows the expected recovery rate (i) when there is positive correlation of 30 percent between recovery rates and systematic risk, (ii) when recovery rate is constant and equal to 50 percent.

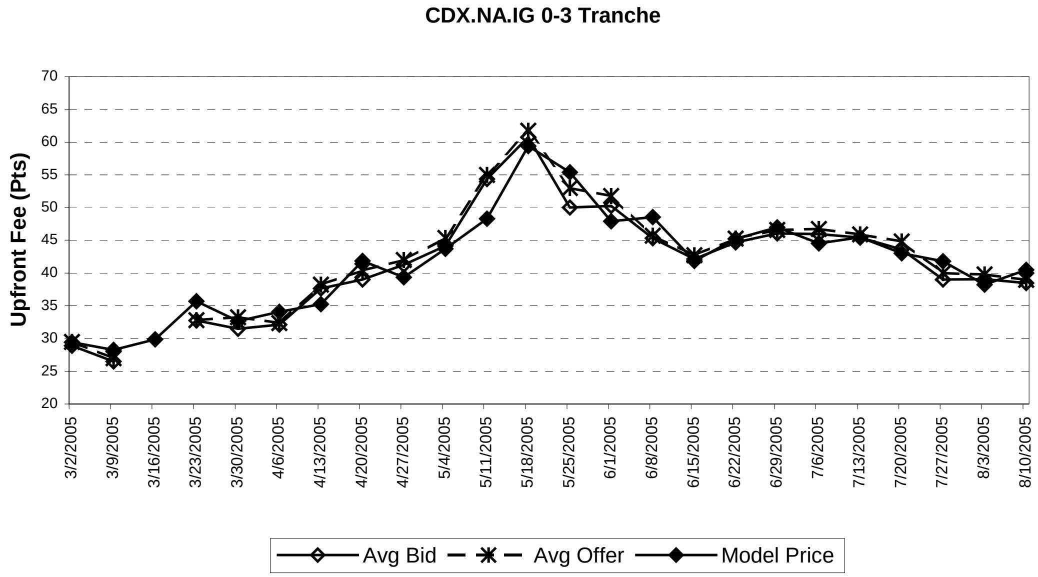

Figure 11.A

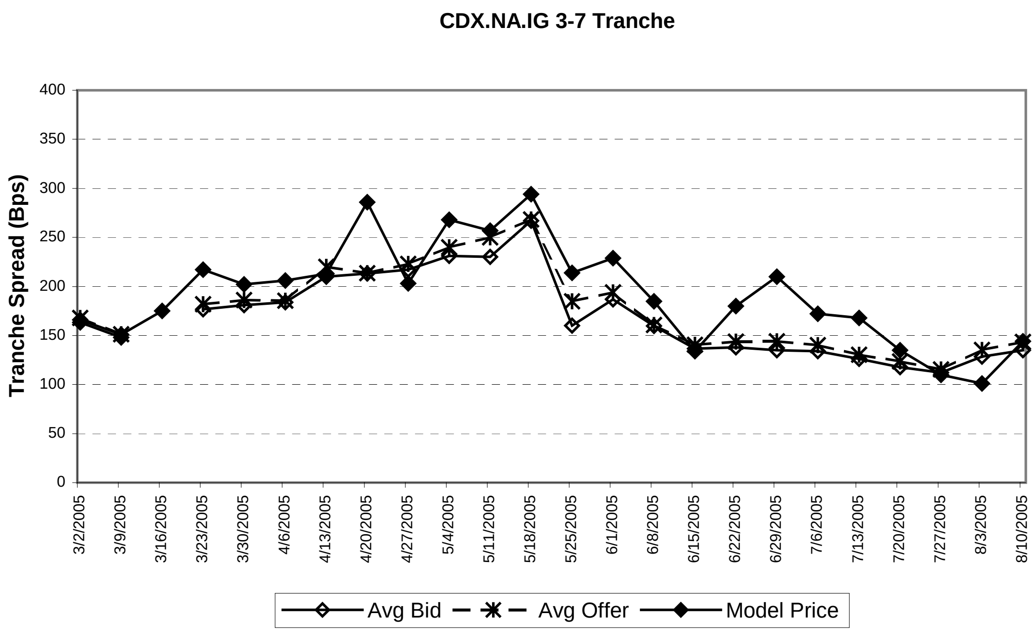

Figure 11.B

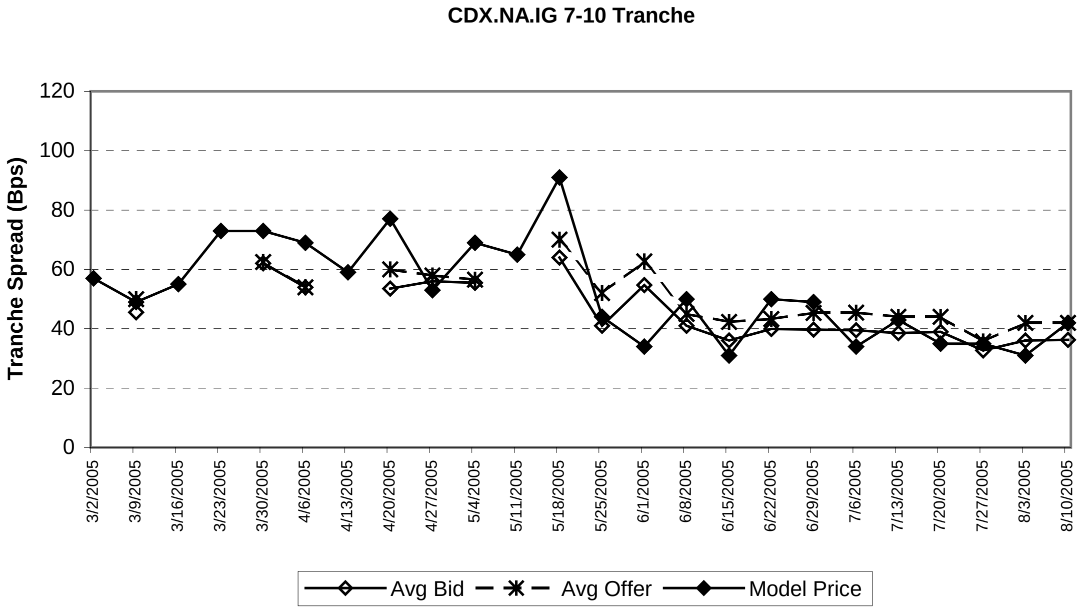

Figure 11.C

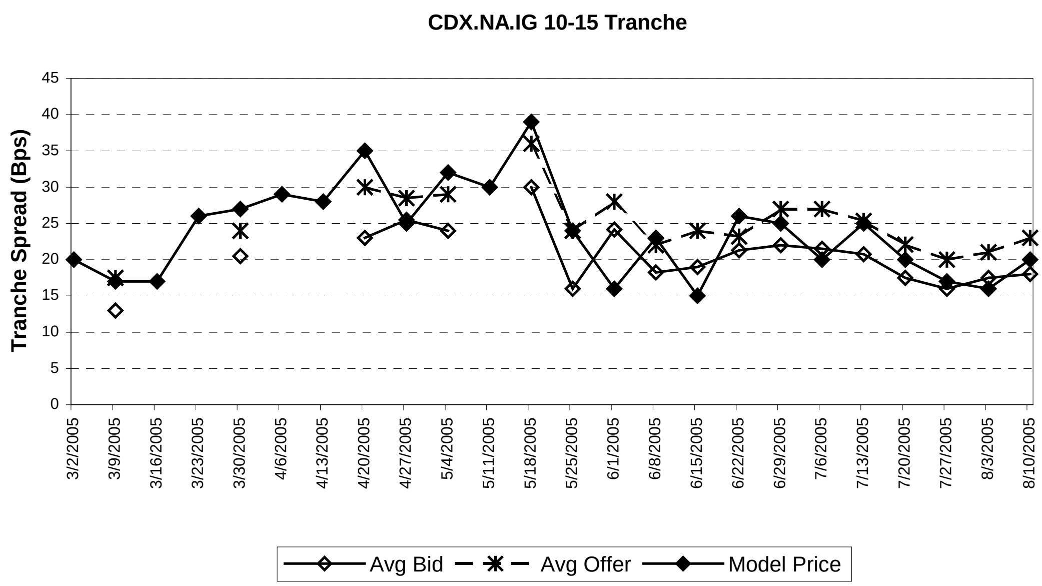

Figure 11.D

Figure 11: Actual and Model Prices of CDX.NA.IG Index Tranches.
Figures 11.A, 11.B, 11.C and 11.D show the actual and model prices of equity, [3%-7%] mezzanine tranche, [7%-10%] tranche and [10%-15%] tranche, respectively. Model prices are computed using the standard Gaussian copula model where lagged implied correlation of the tranche is used as input correlation.