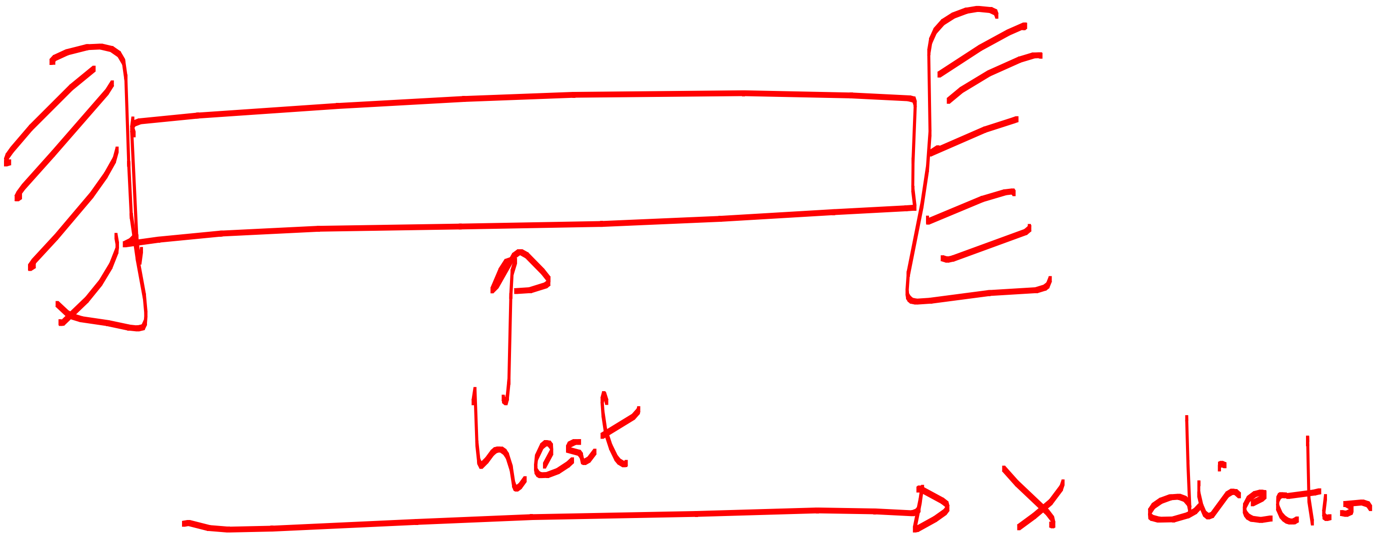
$$u_t = u_{xx}$$

first order in time

2nd order in spatial direction

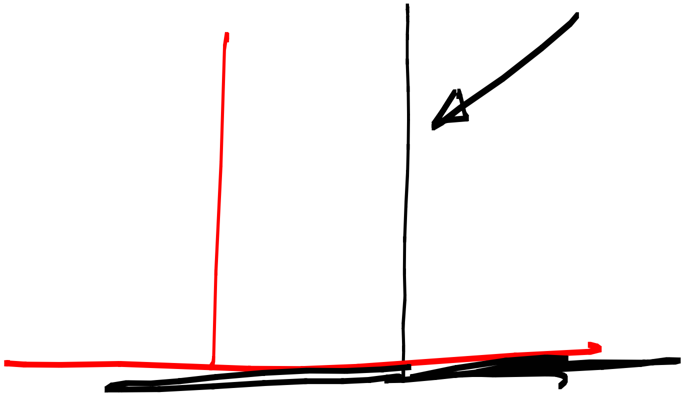
$$\boxed{u_t = u_{xx} + u_{yy}}$$

$$\frac{\partial u}{\partial t} = \nabla^2 u$$



B.S.E $V_t + \frac{1}{2}\sigma^2 S^2 V_{SS} + rSV_S = rV$

$$\downarrow$$



Dirac delta function

$$\frac{\partial u}{\partial t} = \sigma^2 \frac{\partial^2 u}{\partial x^2} \longrightarrow u \text{ F.S.}$$

$$\int u \cdot f \longrightarrow$$

Payoff

$$\boxed{\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}}$$

$$\begin{tikzcd}
N(d_1) & & N(d_2) \\
& \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}\phi^2}\,d\phi \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

$$\begin{tikzcd}
t^{\alpha} f\left(y/t^{\beta}\right) \arrow[d] \\
\boxed{C.D.F}
\end{tikzcd}$$

$$Y(s) = \frac{F(s) + a y'(0) + (as + b) y(0)}{as^2 + bs + c}$$

Now Take $\mathcal{L}^{-1}$

$$\mathcal{L}^{-1}[Y(s)] = y(t) = \mathcal{L}^{-1}\left\{ \qquad \right\}$$

$$I = \int_{x_0}^{x_n} f(x)\, dx$$

$$= I_1 + I_2 + \cdots + I_n$$

$$= \int_{x_0}^{x_1} + \int_{x_1}^{x_2} + \cdots + \int_{x_{n-1}}^{x_n}$$

<u>Ex:</u>

$$\int_1^2 x^2 \qquad \int_0^6 e^{x^2}$$

5

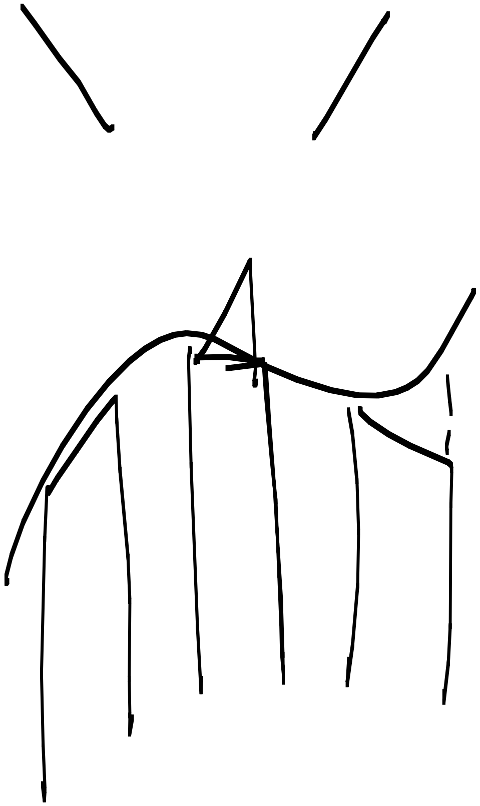
10

100

$$\int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$$

$\uparrow$

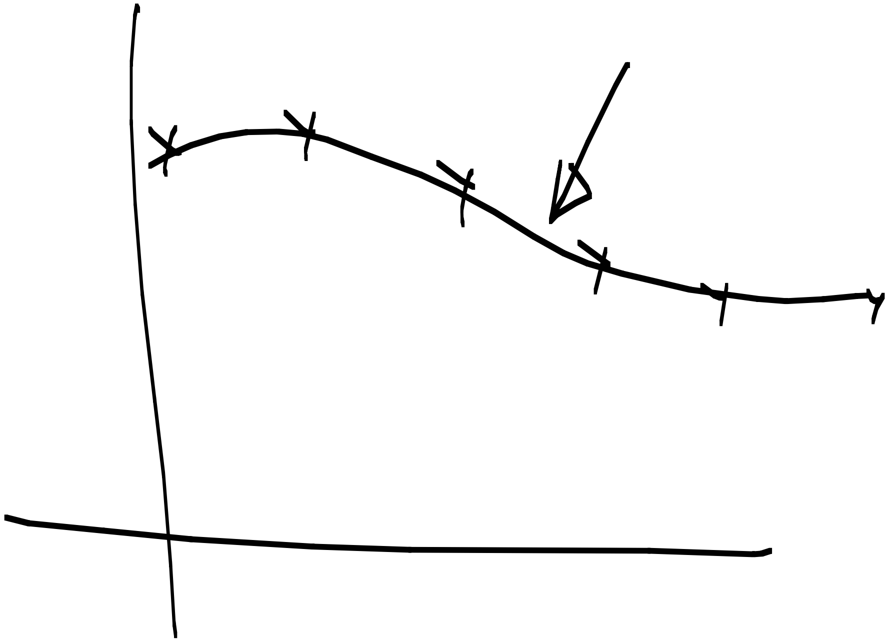
5



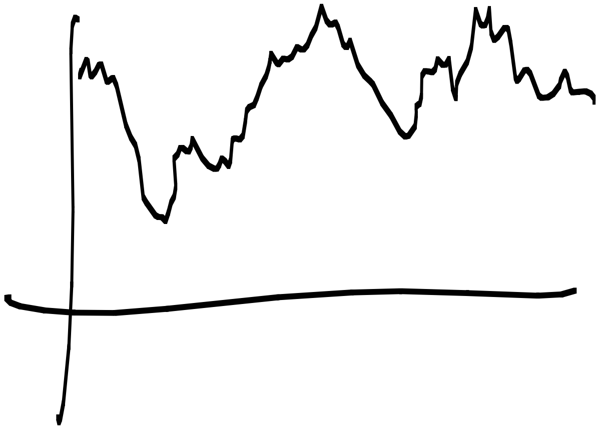
$$\boxed{e^{-rt}}$$

$$\boxed{e^{-\int_{t=0}^{T} r_t \, dt}}$$

$$dS = rS\,dt + \sigma S\,dX$$



Stochastic

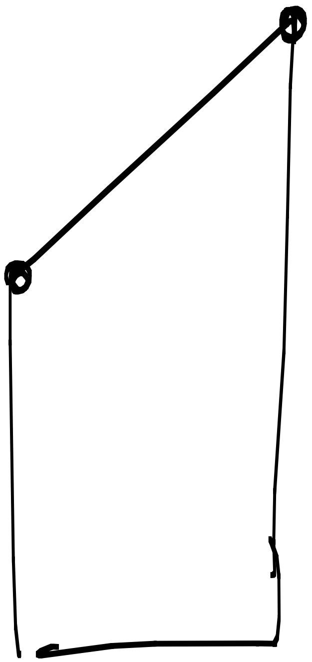


1

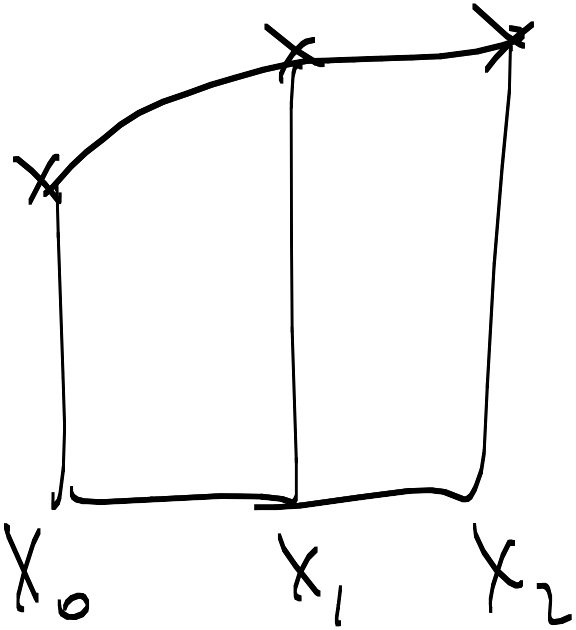
$$e^{rt} \equiv 1+rt$$

$$\frac{1}{e^{rt}} \qquad \frac{1}{(1+rt)^{-1}} \equiv \boxed{1-rt}$$

Trapezium.



Simpson



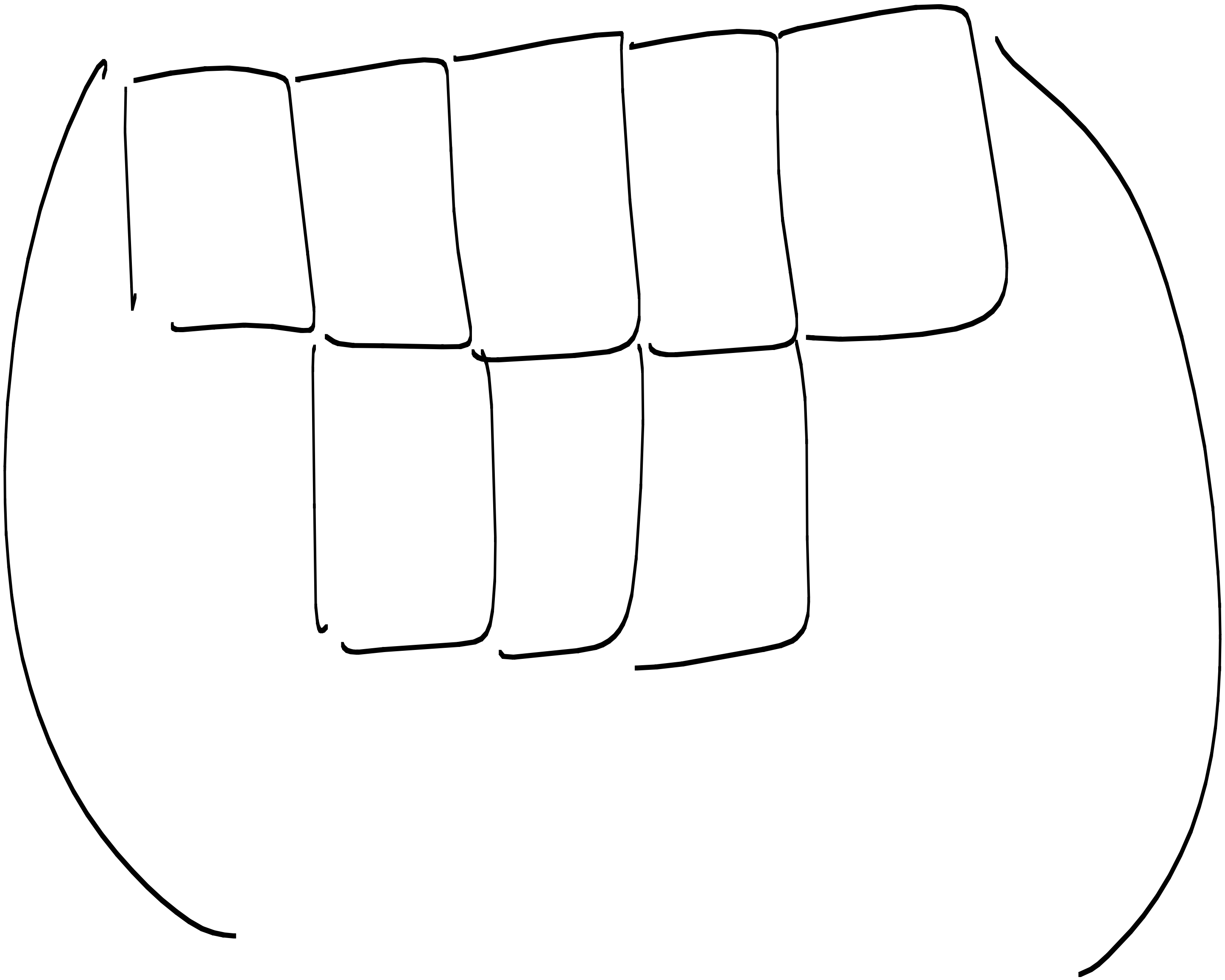
Application :

$$x = A^{-1} \underline{b}$$

$$\begin{array}{c} \begin{array}{ccc|ccc} & A & & & & \end{array} \\ \left(\begin{array}{ccc|ccc} a & b & c & 1 & 0 & 0 \\ d & e & f & 0 & 1 & 0 \\ g & h & i & 0 & 0 & 1 \end{array}\right) \end{array} \xrightarrow{\text{Row ops}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & A^{-1} \\ 0 & 0 & 1 & \end{array}\right)$$

$$A\underline{x} = \underline{b}$$

$$\left[\begin{array}{c|c} A & b \end{array}\right]$$



Direct	Iterative Techniqu
Done once	Repeated
Gaussian Elim$^{\underline{n}}$	Jacobi
LUD	G/S
	Gauss - Siedel

We are off to

Burger Burger

$$\lim_{k \to \infty} \left\{ \underline{x}^{(k)} \right\} \longrightarrow \underline{x}$$

$$A\underline{x} = b$$

$$\underline{x}^{(k+1)} = T\underline{x}^{(k)} + \underline{c}$$

$$K = 0 \longrightarrow \cancel{x_1} \; \underline{x}^{(1)}$$

$$\begin{array}{cc} X_1^{(n)} & X_1^{(n+1)} \\ \vdots & \\ X_N^{(n)} & X_N^{(n+1)} \end{array} \qquad \left\{ \begin{array}{l} 100 \to 110 \\ 0.1 \to \\ 0.6 \to \\ \vdots \\ 5 \to \end{array} \right.$$

$K \neq 0$:

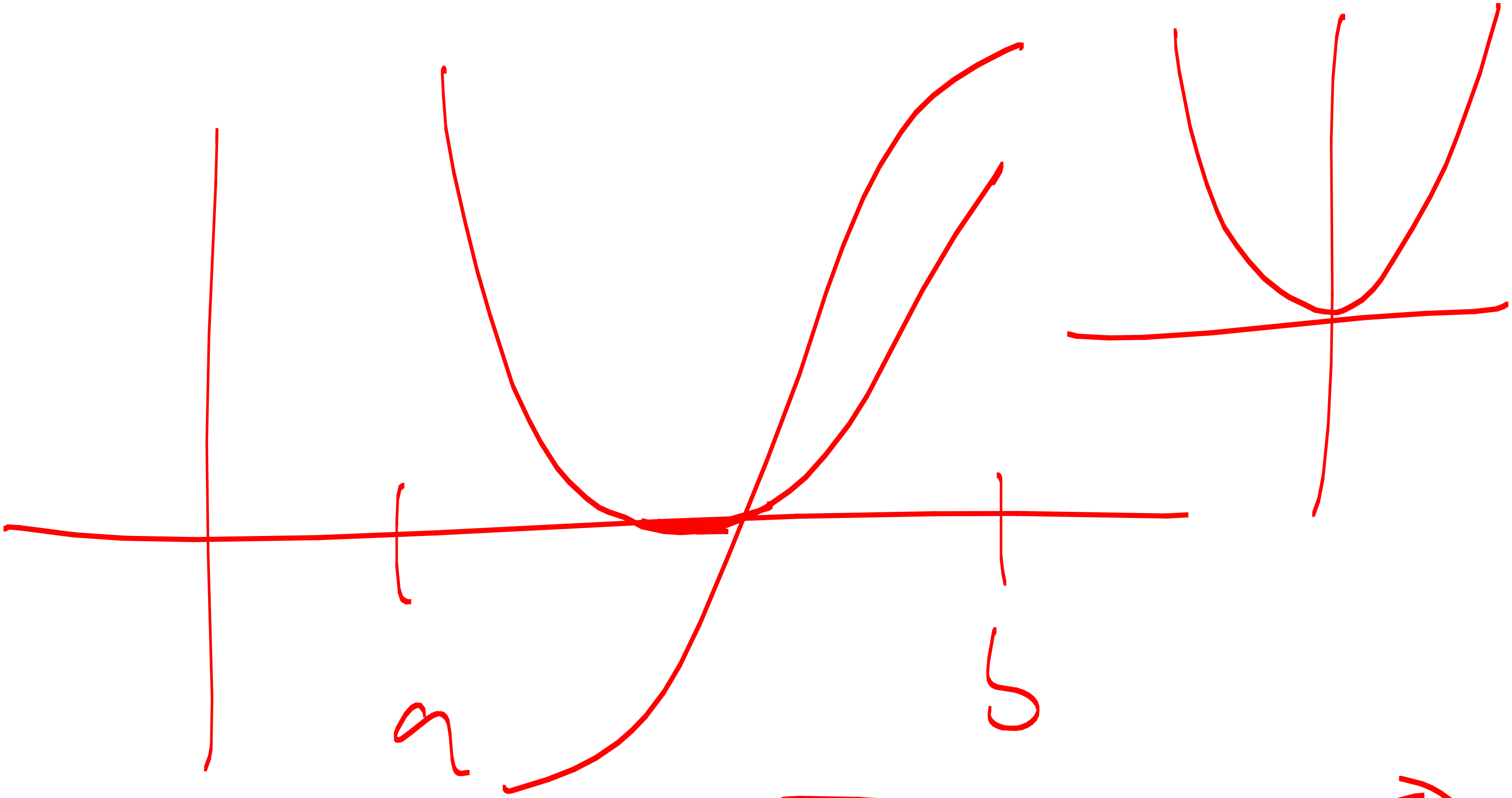
$K = 2$:

To calc. $\boxed{x_1^{(2)}}$: $x_2^{(1)}, x_3^{(1)}, x_4^{(1)}$

" " $x_2^{(2)}$: $x_1^{(2)}, x_3^{(1)}, x_4^{(1)}$

" " $x_3^{(2)}$: $x_1^{(2)}, x_2^{(2)}, x_4^{(1)}$

" " $x_4^{(2)}$: $x_1^{(2)}, x_2^{(2)}, x_3^{(2)}$



$$\boxed{f(a)f(b) \leq 0}$$

B.S option price = Market Price of option

$f(x) = y$ Explicit

$f(x,y) = 0$ Implicit

$$\boxed{\alpha_i y_{i-1} + \beta_i y_i + \gamma_i y_{i+1} = 0}$$

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

2D T.S.E $\quad u(x,t) \quad x \to x + \delta x$

$t \to t + \delta t$

$$u(\cancel{x}\,t + \delta t, x) = u(x,t) + \frac{\partial u}{\partial t}\,\delta t + O(\delta t^2)$$

$$\frac{\partial u}{\partial t} = \frac{u(t+\delta t) - u(x,t)}{\delta t} + O(\delta t)$$

t : 0 to T $\quad$ M steps

$$\delta t = \frac{T-0}{M} \qquad t_i = i\,\delta t \qquad 0 \leq i \leq M$$

x : a to b $\quad N$ steps

$$\delta x = \frac{b-a}{N} \qquad x_i = a + j\,\delta x \quad 0 \leq j \leq N$$

$$u(t, x+\delta x) = u(x,t) + \frac{\partial u}{\partial x}\delta x + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}\delta x^2 + O(\delta x^3)$$

$$u(t, x-\delta x) = u(x,t) - \frac{\partial u}{\partial x}\delta x + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}\delta x^2 + O(\delta x^3)$$

Add :

$$u(t, x+\delta x) + u(t, x-\delta x) = 2u + \frac{\partial^2 u}{\partial x^2}\delta x^2 + O(\delta x^4)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u(t, x-\delta x) - 2u(x,t) + u(t, x+\delta x)}{\delta x^2} + O(\delta x^2)$$

$$u(x,t) = u(x_j, t_i) = u_j^i$$

$$\frac{\partial u}{\partial t} \sim \frac{u_j^{i+1} - u_j^i}{\delta x}$$

fwd
deriv.

①

$$\frac{\partial^2 u}{\partial x^2} \sim \frac{u^i_{j-1} - 2u^i_j + u^i_{j+1}}{\delta x^2}$$

② centred deriv.

Subst. into heat eq^n

$$\frac{u^{i+1}_j - u^i_j}{\delta t} = c^2 \, \frac{u^i_{j-1} - 2u^i_j + u^i_{j+1}}{\delta x^2}$$

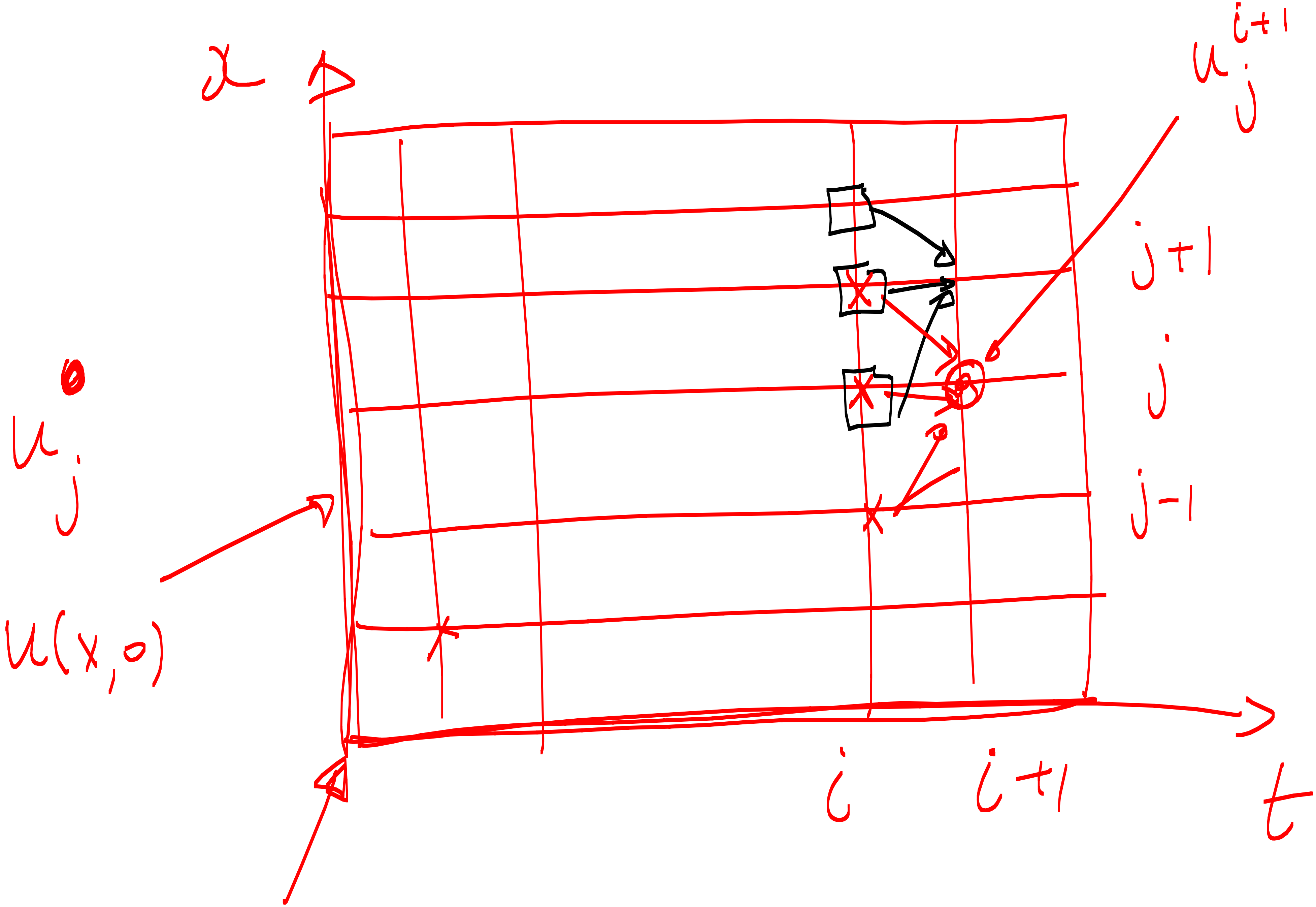
$$r = c^2 \delta t / \delta x^2$$

$$u_j^{i+1} - u_j^i = r\left[u_{j-1}^i - 2u_j^i + u_{j+1}^i\right]$$

$$u_j^{i+1} = r\,u_{j-1}^i + \left[1 - 2r\right)u_j^i + r\,u_{j+1}^i$$

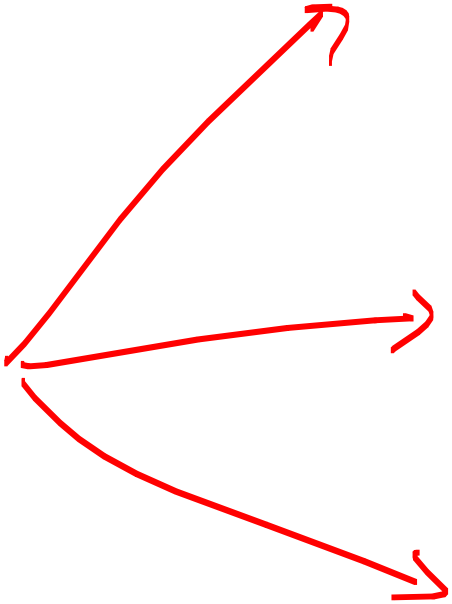
difference eq^n (explicit)

$$u_j^{i+1} = F\left(u_{j-1}^i,\ u_j^i,\ u_{j+1}^i\right)$$



x
u_j^{i+1}
$j+1$
j
$j-1$
u_j^0
$u(x,0)$
i
$i+1$
t

trinomial [illegible]



Explicit scheme is 1st order accurate in t $O(dt)$

2nd order accurate in x

$O(dx^2)$

ξ