CQF Module 3.4 Volatility

1. Consider two European call options A and B with the following information:

Option	Strike price	Expiration (months)	Market price of option
A	25	6	3.29
B	30	6	0.64

a) The underlying is 27, there are no dividends and the risk-free interest rate r is 5%. Estimate the implied volatility σ_{imp} for both options.

b) Calculate delta , gamma and vega for each option.

c) Now set up a portfolio that is delta and gamma neutral and will profit if both implied volatilities fall by the same amount. How much return will be made?

Solution: σ_{imp} for option A = 23%, σ_{imp} for option B = 19%.

1. Calculate delta , gamma and vega for each option.

   Now set up a portfolio that is delta and gamma neutral and will profit if both implied volatilities fall by the same amount.

   How much return will be made?

$$\Delta_{\text{call}} = N(d_1) \qquad \Gamma_{\text{call}} = \frac{N'(d_1)}{S\sigma\sqrt{T-t}} \qquad \upsilon\text{ega}_{\text{call}} = SN'(d_1)\sqrt{T-t}$$

$$\Delta_{\text{A}} = 0.761 \qquad \Delta_{\text{B}} = 0.298$$

$$\Gamma_{\text{A}} = 0.071 \qquad \Gamma_{\text{B}} = 0.096$$

$$\text{vega}_{\text{A}} = 5.93 \qquad \text{vega}_{\text{B}} = 6.62$$

For delta neutrality we want

$$0.761\ A + 0.298\ B + 1 = 0$$

where A is the quantity of option A, etc. and 1 represents the quantity of stock (arbitrarily chosen to be one, but can be scaled later) and for gamma neutrality we want

$$0.071\ A + 0.096\ B = 0$$

so that $A = -1.85$ and $B = 1.37$. The net vega position is

$$-1.85 \times 5.93 + 1.37 \times 6.62 = -1.91.$$

The cost of the position is

$$-1.85 \times 3.29 + 1.37 \times 0.64 + 1 \times 27 = 21.79.$$

If implied volatilties both fall by 1% (from 23 to 22 and 19 to 18) then the portfolio will make a profit of

$$1.91 \times \frac{1}{100} = 0.0191$$

a return of

$$\frac{0.0191}{21.79} = 0.00087.$$

That is, approximately 8 basis points.

2. Using market data for European options with the same expiry but different exercise prices, find and plot the implied volatilities. What shape does the plotted curve have?
   See attached PDF