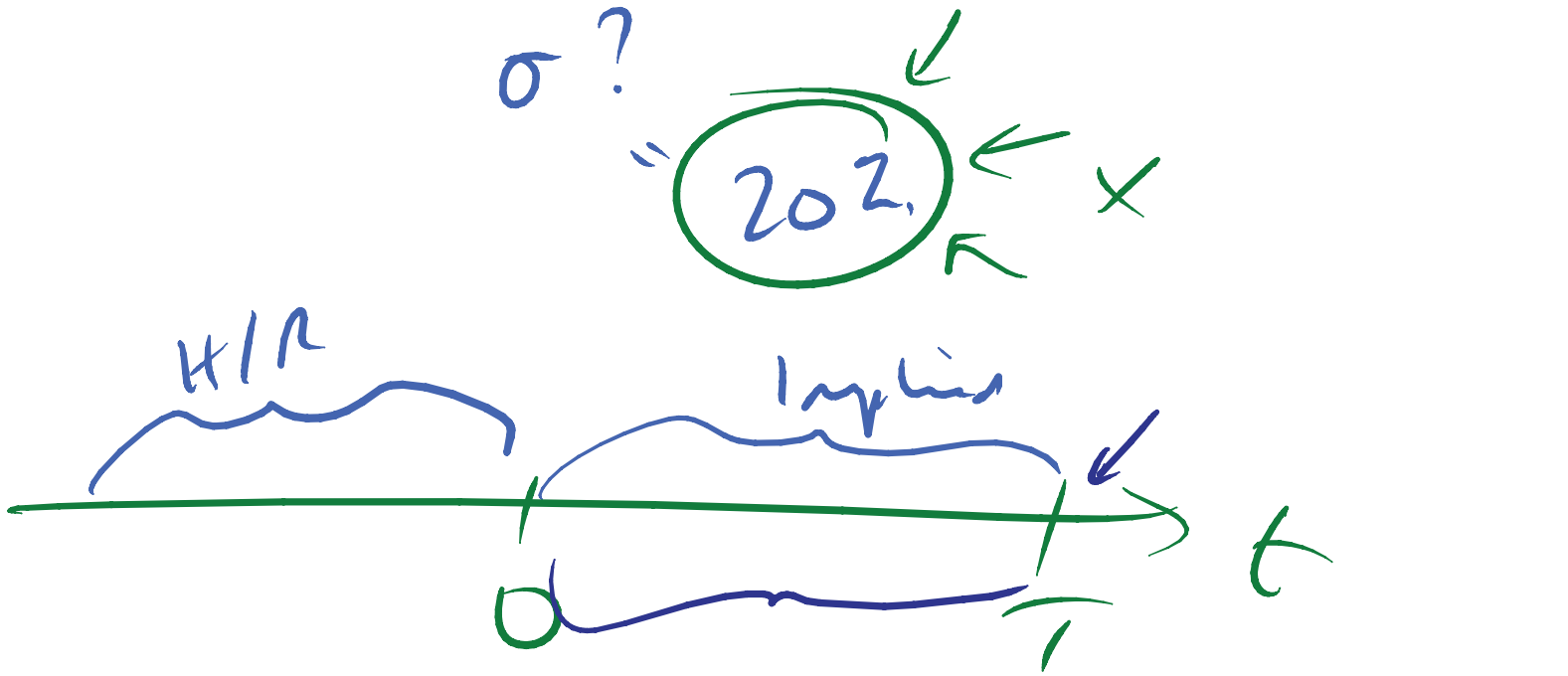
$(\mu, S=100=K, r=5\%, T=1, V=10.45$



Barrier

σ^2

$20

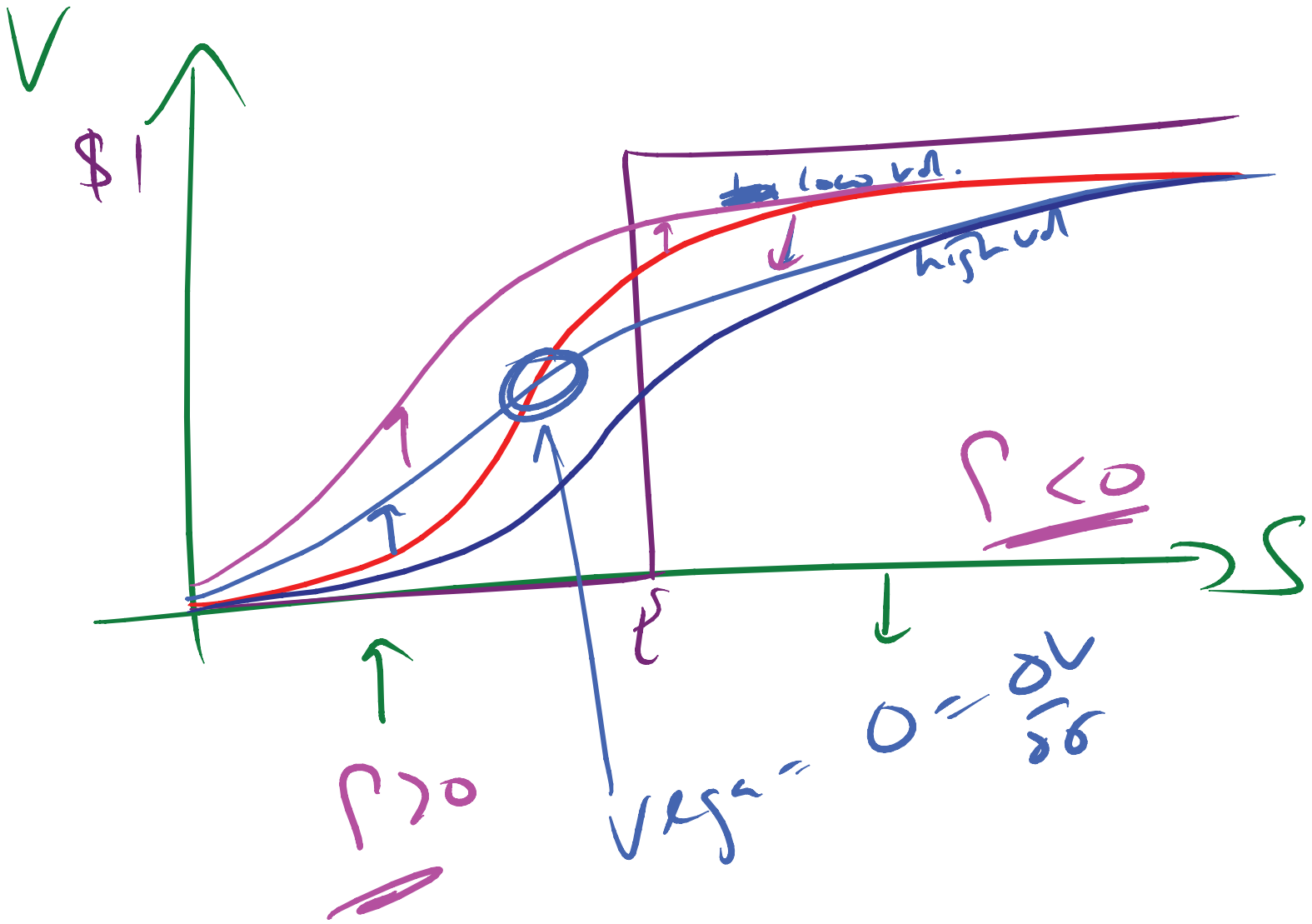
Vanilla Call imp. vol = 20%

$10

1) Inc. van. call by $1 imp. vol → 24%
2) Barrier using 24%, Barrier $22
3) Portfolio --- Barrier − 2 Vanilla

20 − 2·10 = 0 (coincident)

" 22 − 2·11 = 0


V
$1
low vol.
high vol
Γ < 0
S
K
Γ > 0
Vega = 0 = ∂V/∂σ

Historical Buy

Past

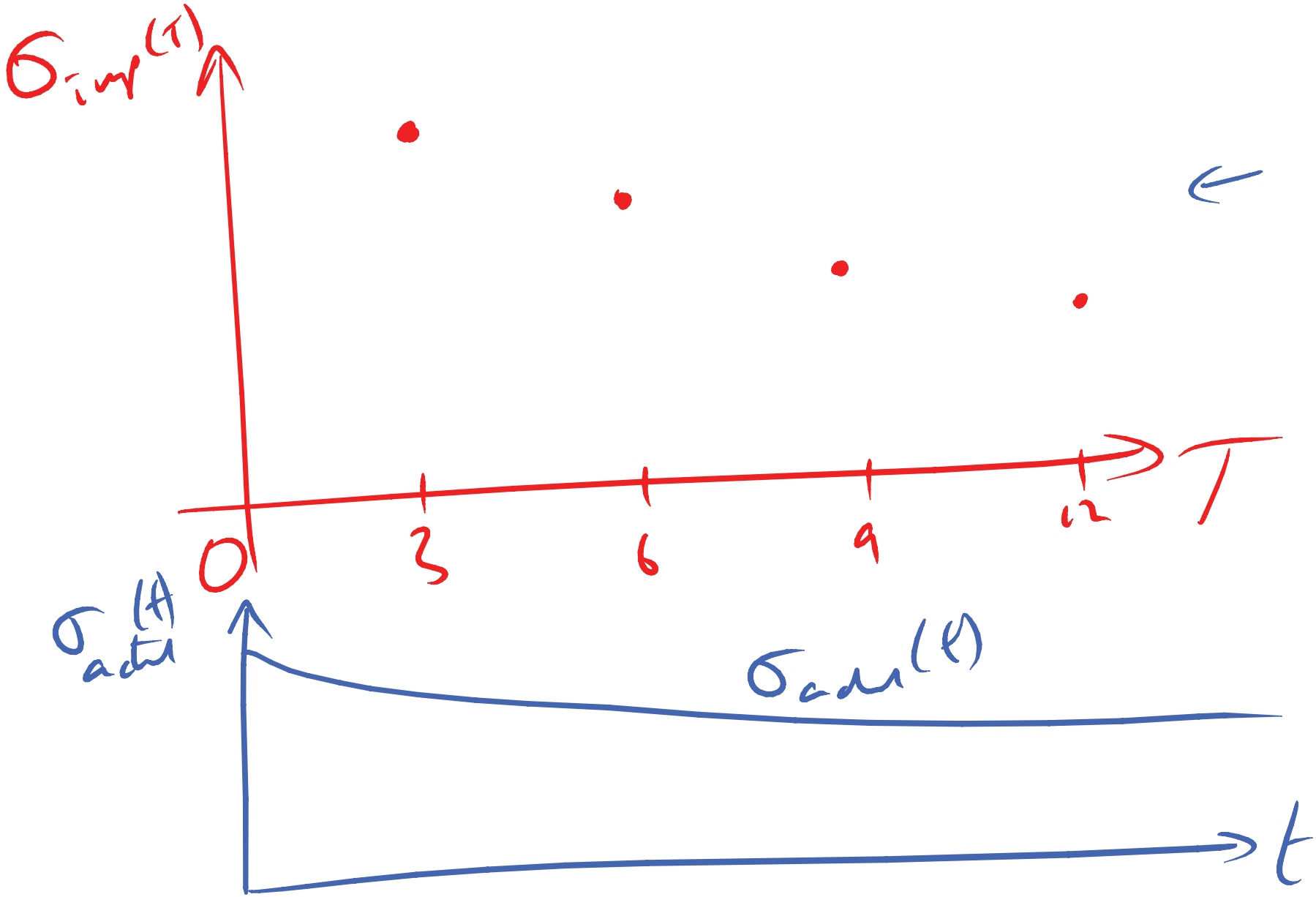
Future ?!!

Snapshot Sell

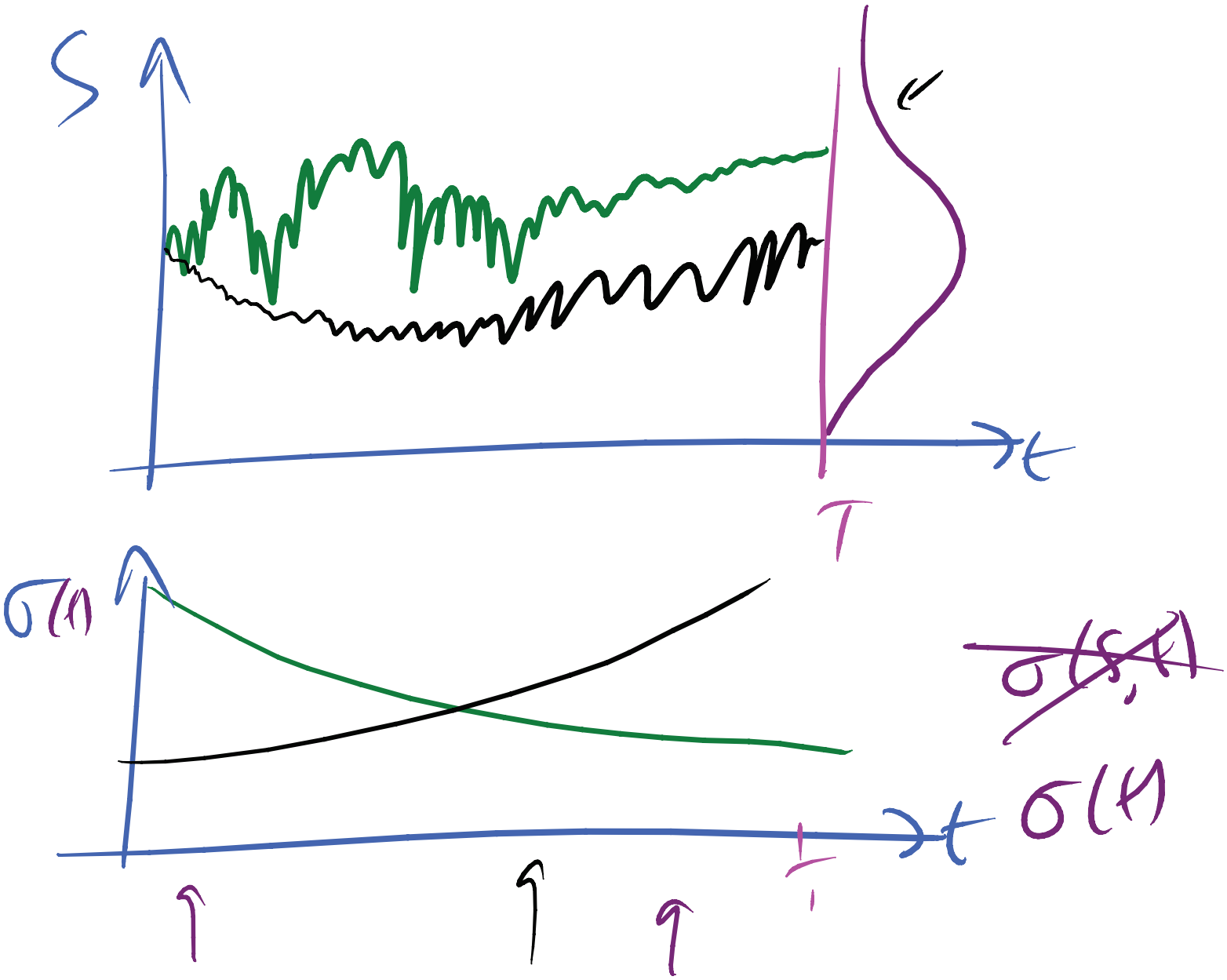
of

Options.

Calibration


$\sigma_{imp}(T)$
0
3
6
9
12
T
$\sigma_{actu}(t)$
$\sigma_{actu}(t)$
t

actual


S
t
T
σ(t)
σ(S,t)
σ(t)
t

"average"

$$\sqrt{\frac{1}{T}\int_0^T \sigma(\tau)^2 \, d\tau}$$

$$BSE \quad \ldots \quad + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \ldots$$

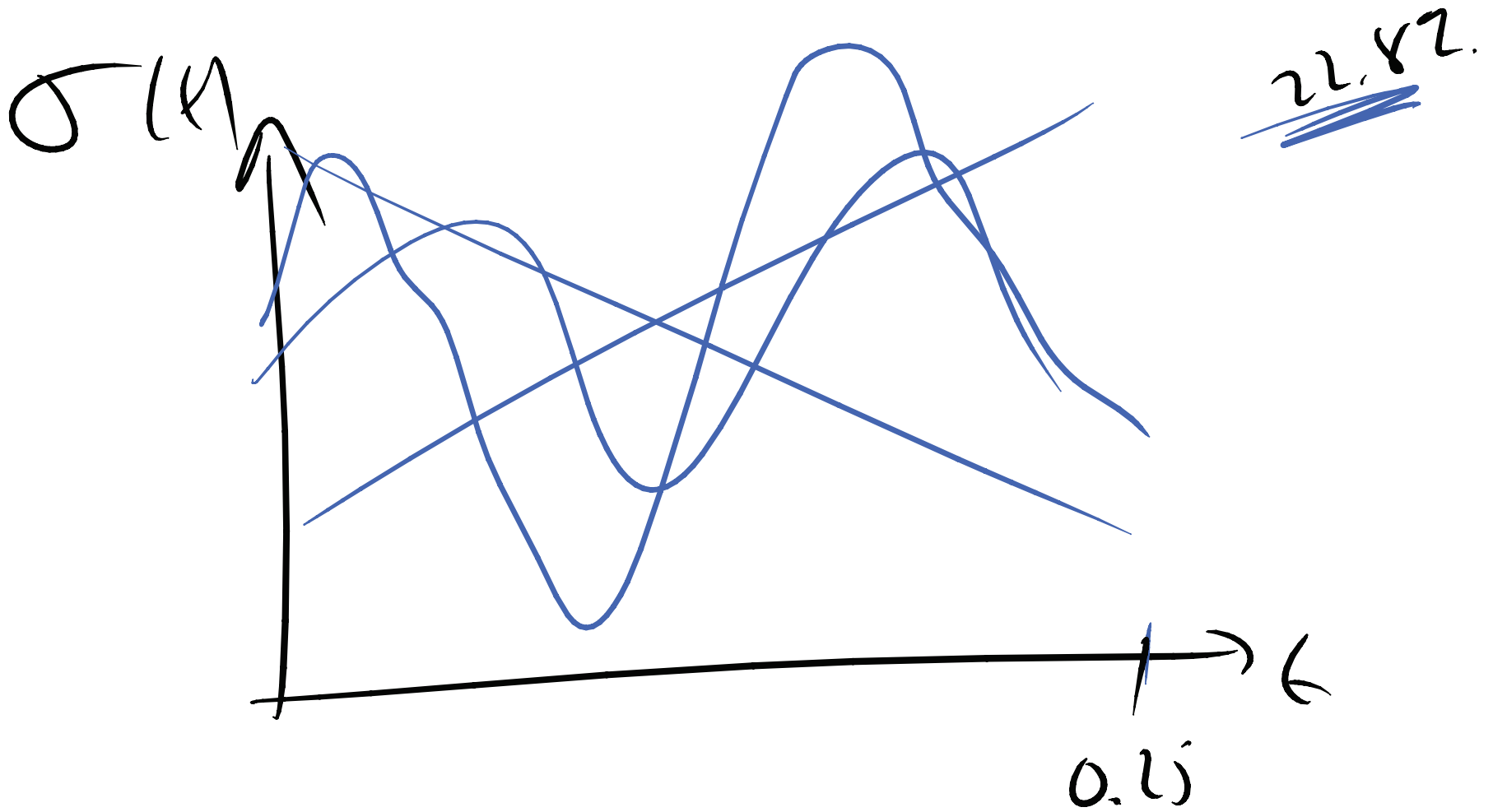
1) BSE valid $\sigma(S,t)$

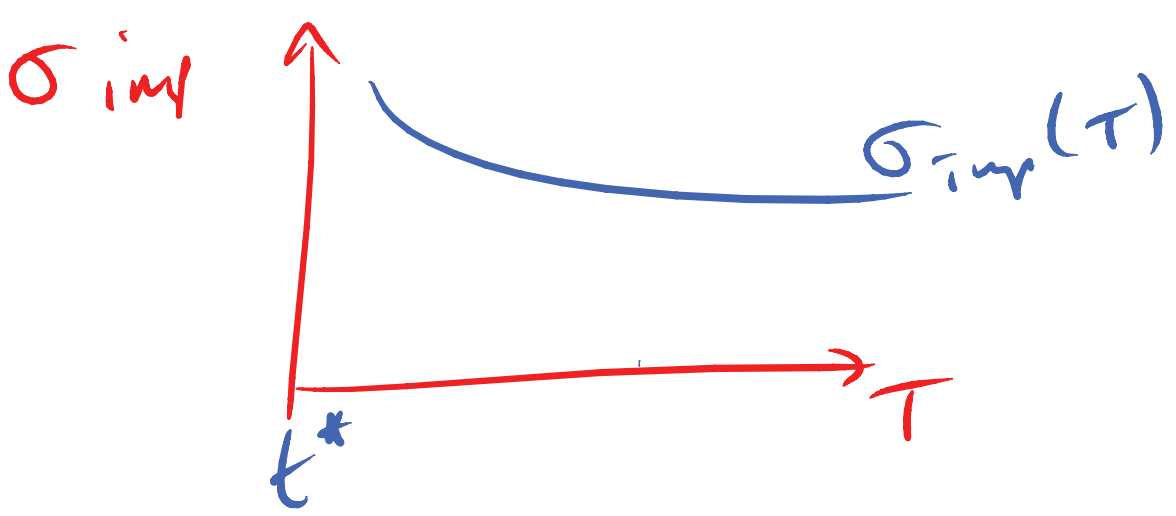
2) BSF valid when σ = constant

3) BSF valid when $\sigma(t)$

same formulae! ... provided

$$\sigma \longrightarrow \sqrt{\frac{1}{T-t}\int_t^T \sigma^2(\tau)\, d\tau}$$


σ(k)
22.82.
k
0.05



t = calendar

T = expiry

τ = int

t^* = time at which we calibrate

$$\sigma_{imp}(T) = \sqrt{\frac{1}{T-t^*}\int_{t^*}^{T} \sigma^2(\tau)\, d\tau}$$

$\sigma_{imp}(T)$ ✓ for all T

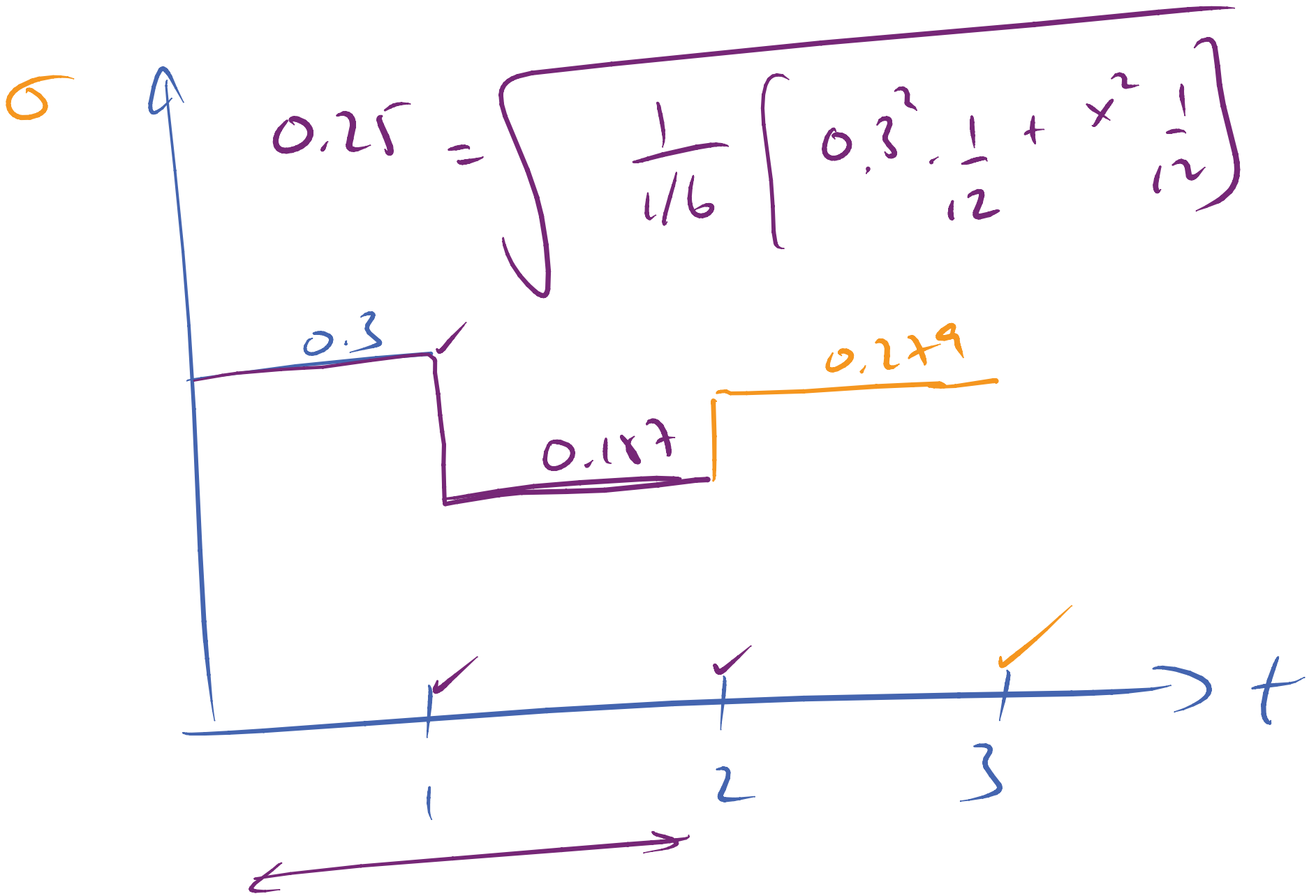
t^* ?

$$(T-t^*)\,\sigma_{imp}^2(T) = \int_{t^*}^{T} \sigma^2(\tau)\,d\tau$$

$$\frac{d}{dT}$$

$$\sigma_{imp}^2(T) + 2(T-t^*)\,\sigma_{imp}(T)\,\frac{\partial}{\partial T}\sigma_{imp}(T) = \sigma^2(T)$$

$$\sigma_{imp}^2(t) + 2(t-t^*)\,\sigma_{imp}(t)\,\frac{\partial}{\partial t}(\sigma_{imp}(t)) = \sigma(t)$$


σ
$0.25 = \sqrt{\frac{1}{1/6}\left[0.3^2 \cdot \frac{1}{12} + x^2 \frac{1}{12}\right]}$
0.3
0.187
0.279
t
1
2
3

$$\frac{1}{4} \cdot .26^2 = \frac{1}{12} 0.3^2 + \frac{1}{12} \cdot 0.187^2 + \frac{1}{12} \sigma_3^2$$