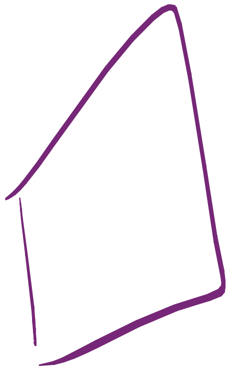
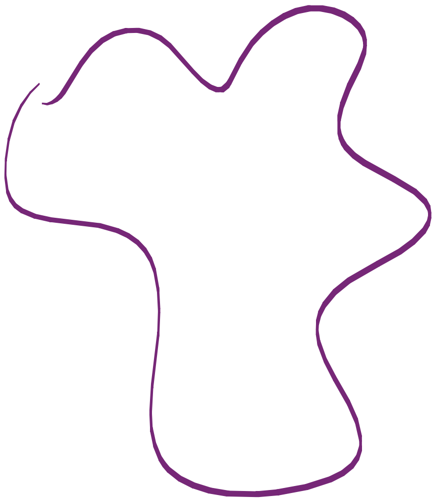
$$\int_a^b f(x)\,dx$$





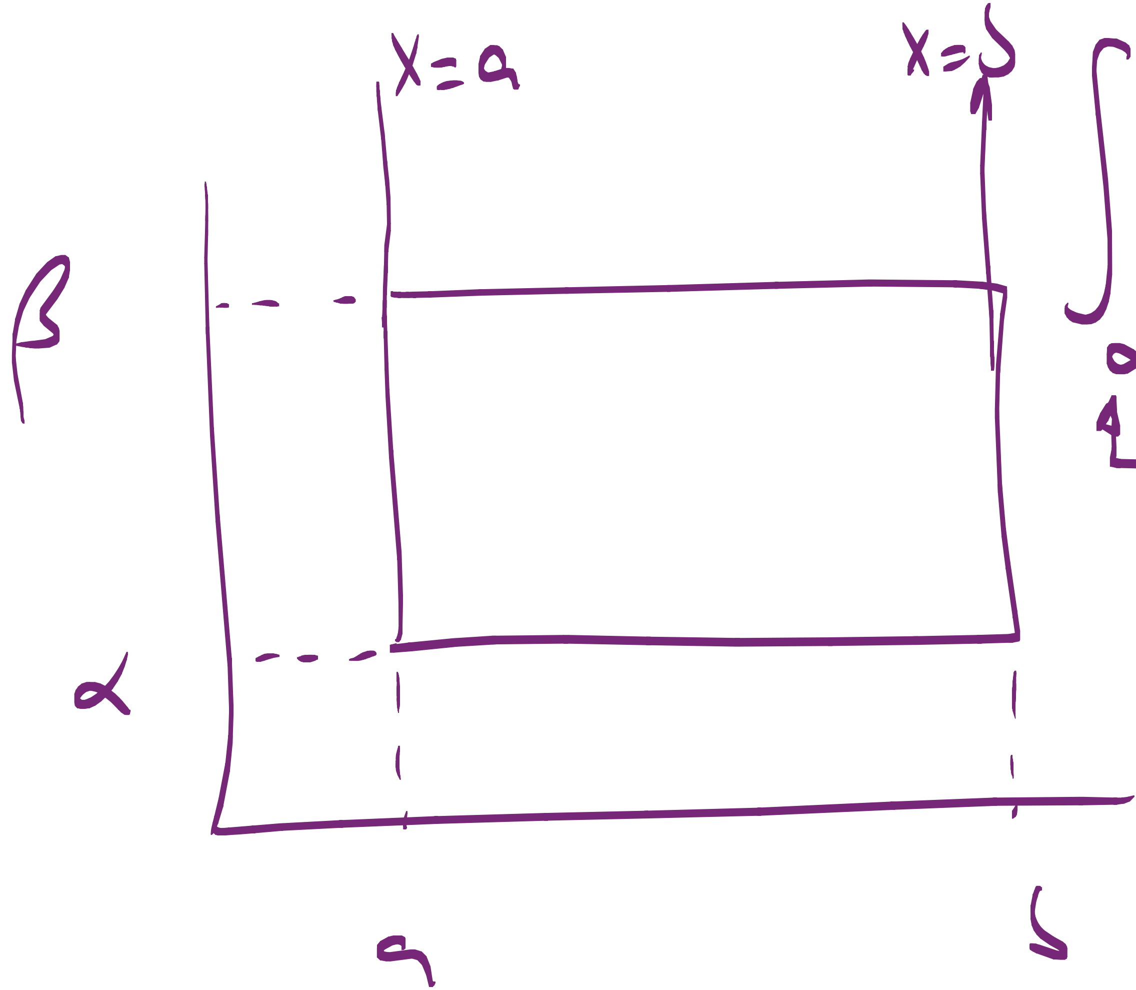
$$\int\int \overbrace{f(x,y)}^{1}\,dy\,dx$$

$$\int \int_{x=g(y)} f(x,y)\,dx\,dy$$

$x = g(y)$ a fn. of y

$y = x \rightarrow x = y$

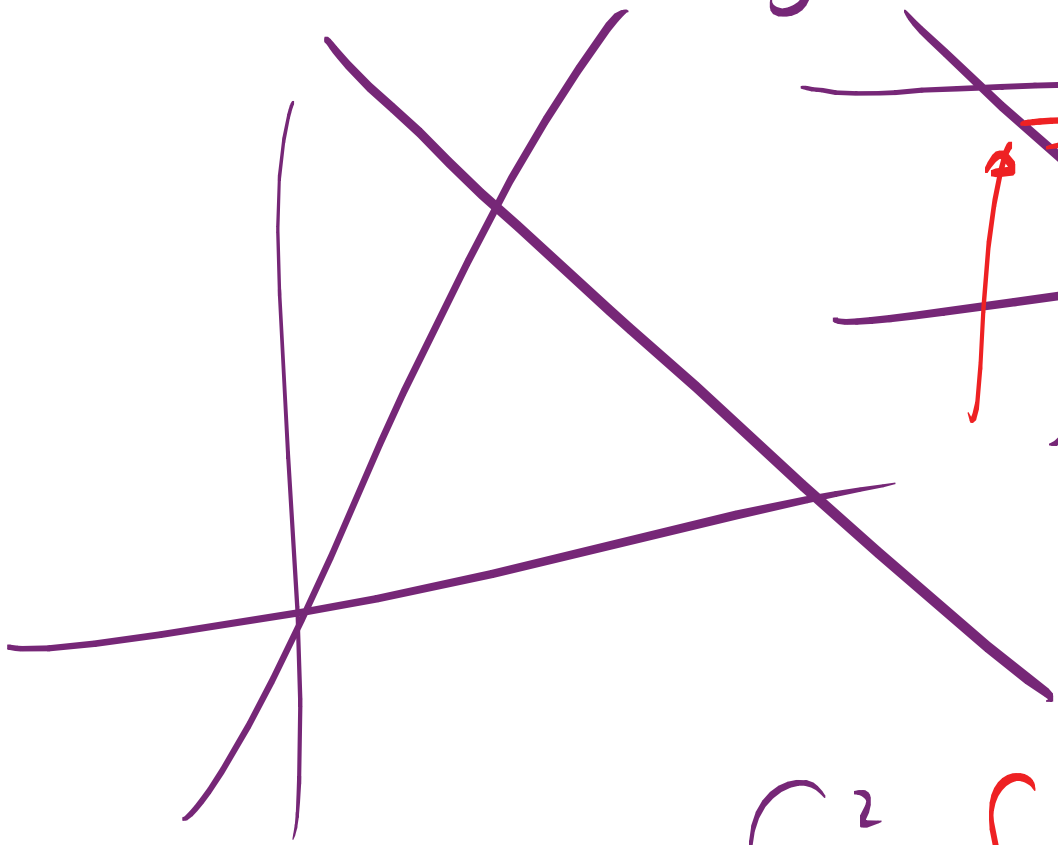
$y = x^2 \quad x = \sqrt{y}$

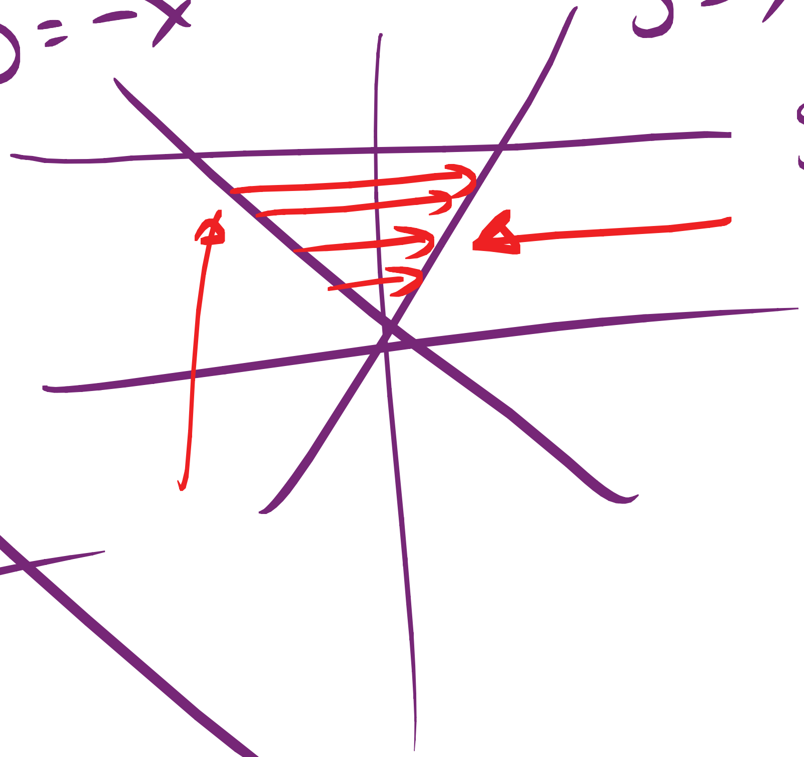


$$\int_{\alpha}^{\beta}\int_{x=a}^{x=b} dx\,dy$$

x as a fn. of y

$$x = f(y)$$



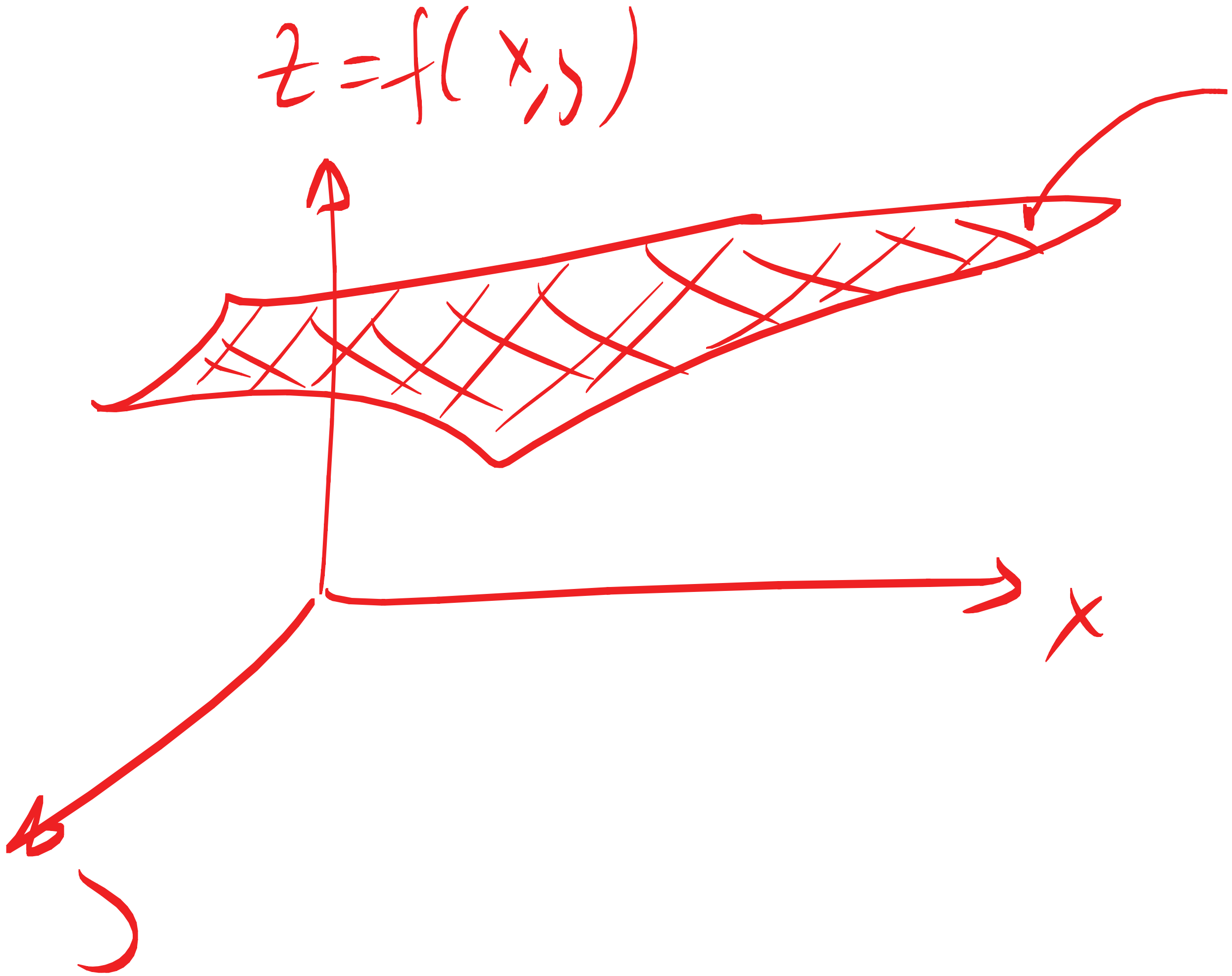


$$\int_0^2 \int_{x=-y}^{x=+y}$$

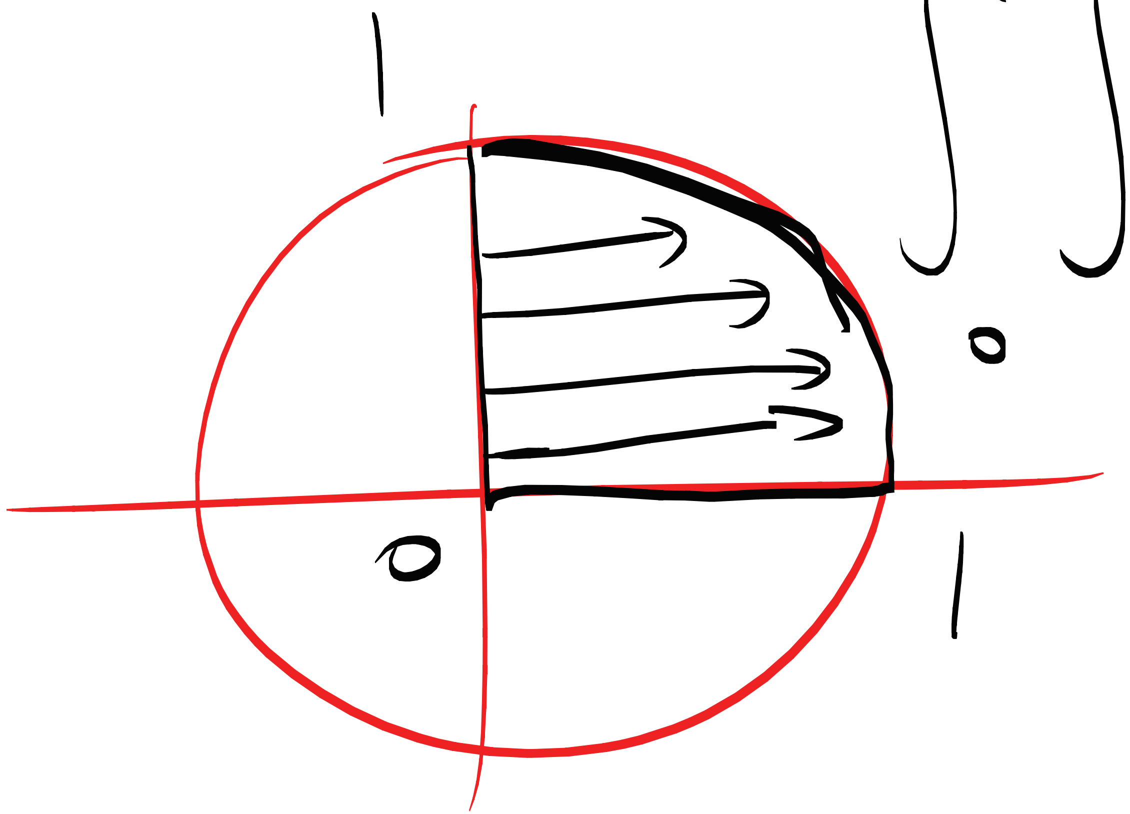
If $f(x,y)$ is separable

$$f(x,y) = g(x)\,h(y)$$

$$\iint f(x,y)\,dx\,dy = \int g(x)\,dx \int h(y)\,dy$$



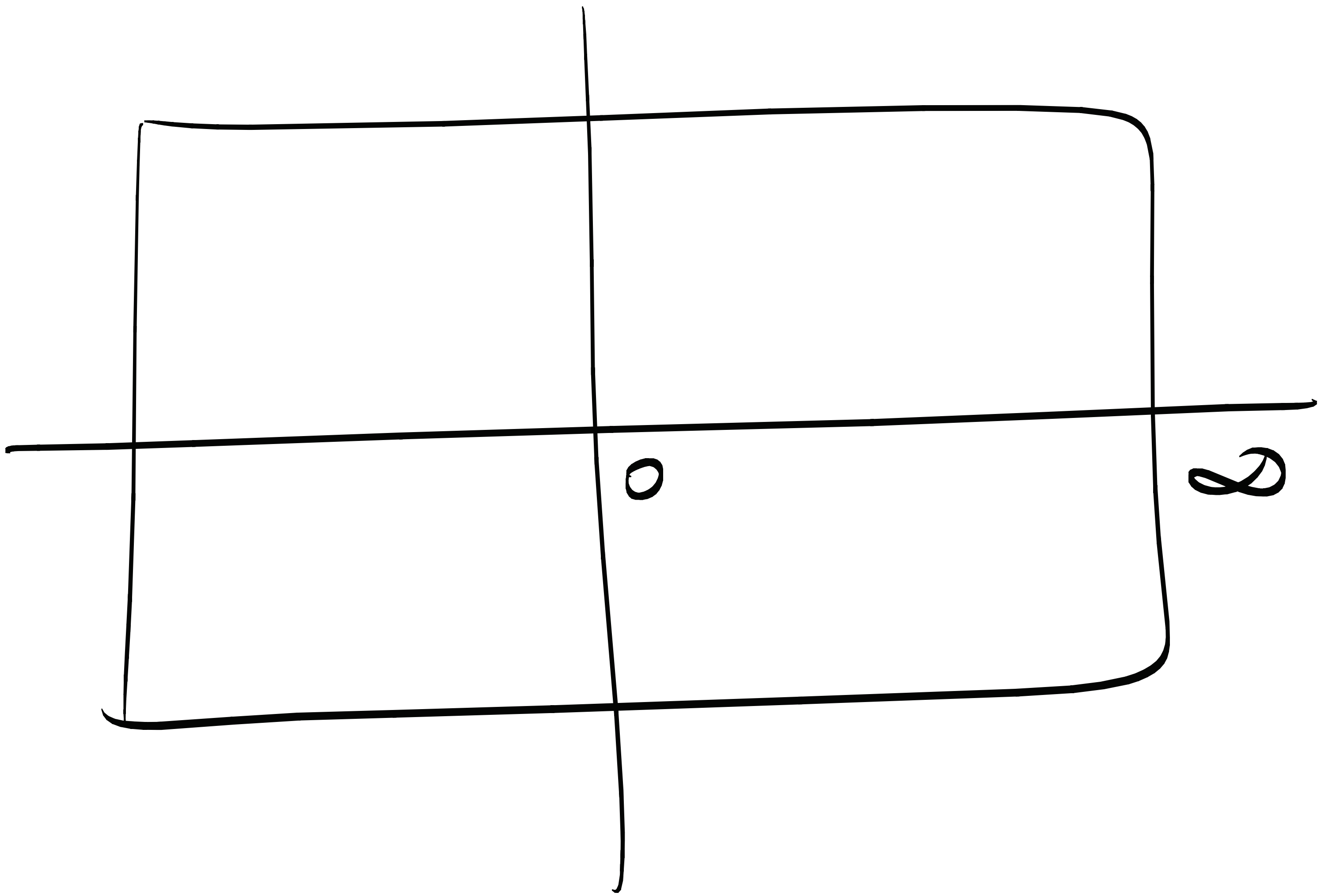
z = f(x, y)
x
y

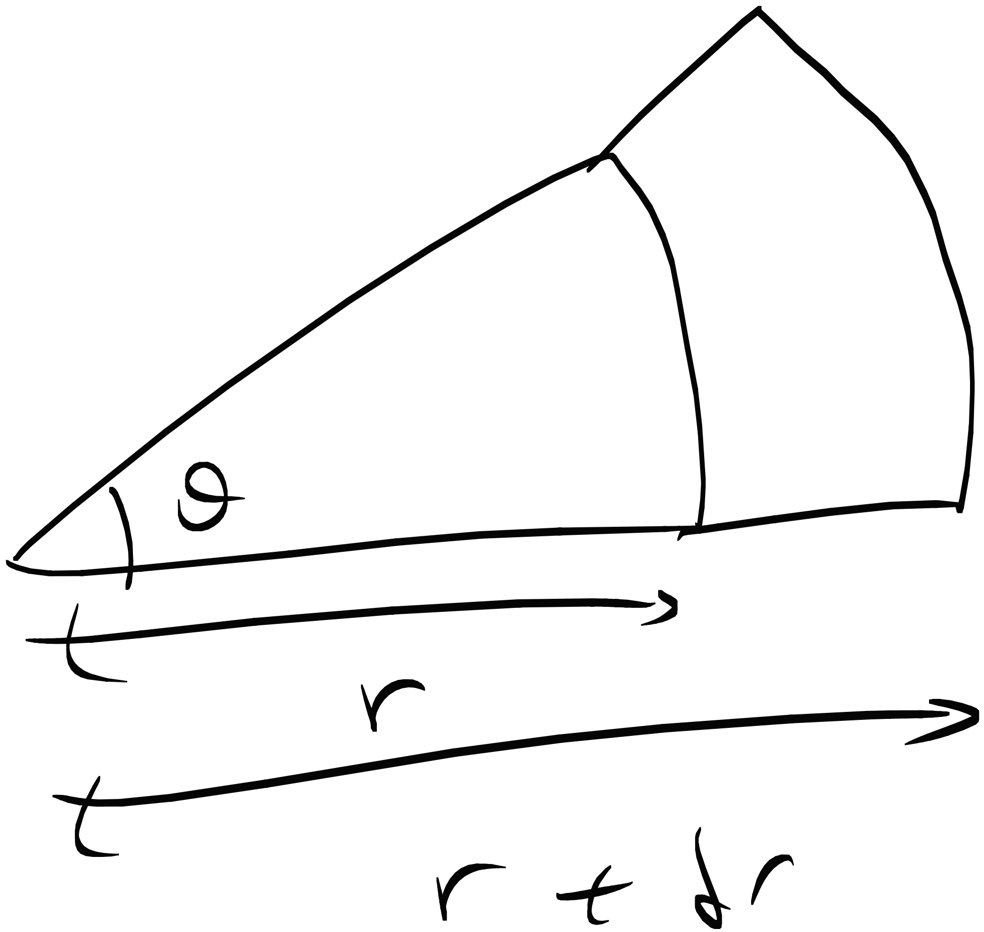


$$\int_0^1 \int_0^{\sqrt{1-y^2}} dx\, dy$$

$$x^2 + y^2 = 1$$

$$x = \sqrt{1-y^2}$$


0
∞



θ
r
r + dr

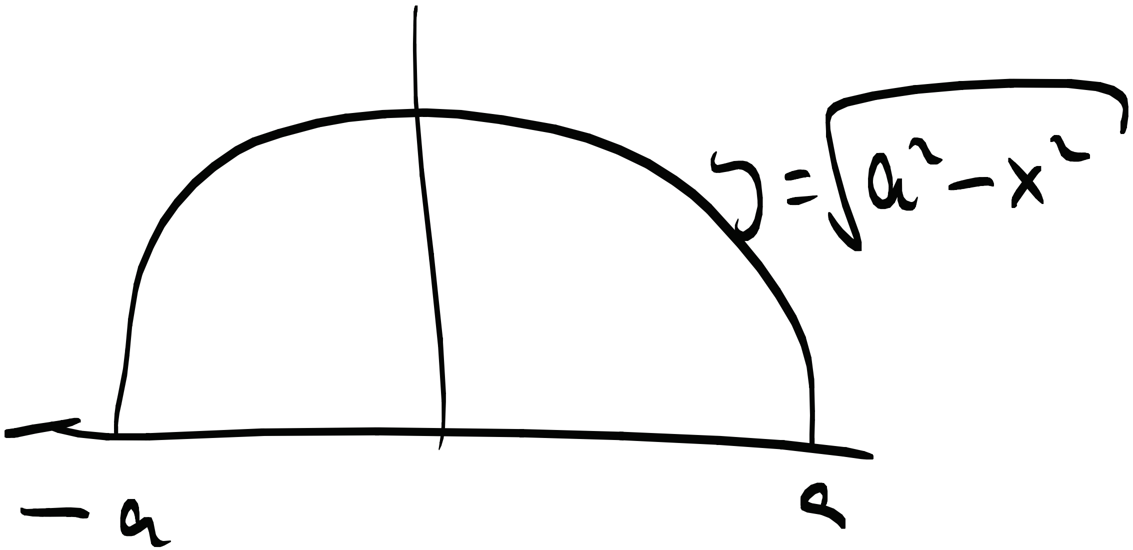
Ex: Use polar co-ords to evaluate

$$(r^2)^{3/2}$$

$$\int_{-a}^{a} \int_{0}^{\sqrt{a^2-x^2}} \boxed{(x^2+y^2)^{3/2}} \, dy \, dx$$

$x = r\cos\theta$

$y = r\sin\theta$



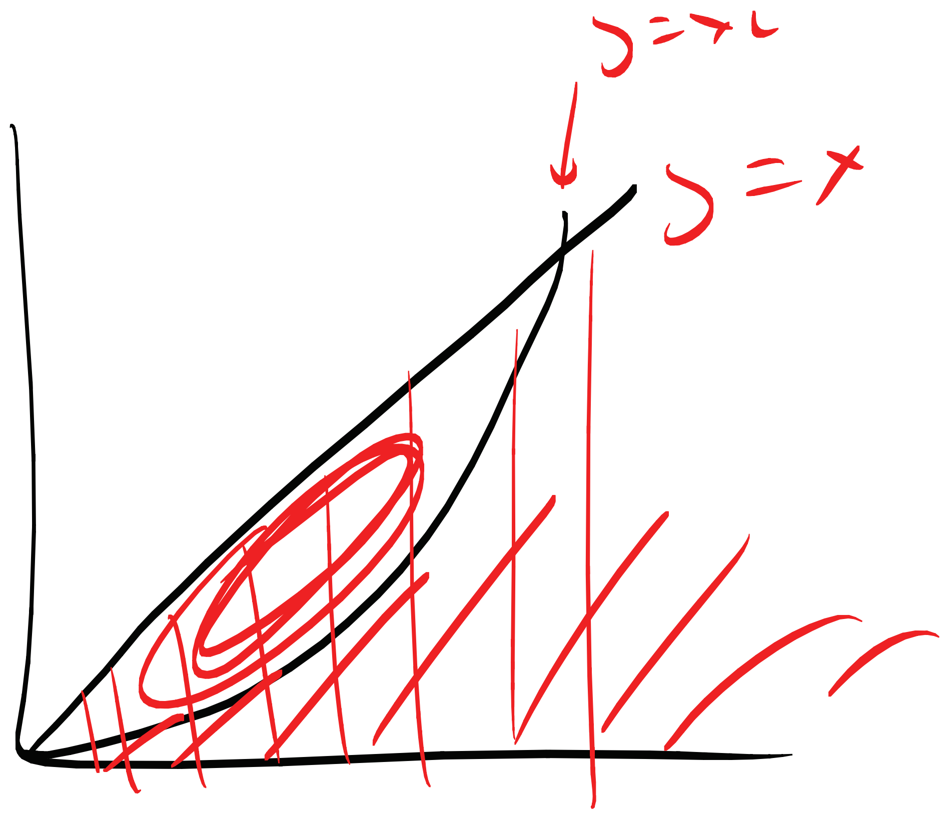
$x^2+y^2 = r^2$

$dy\,dx = r\,dr\,d\theta$

$$\int_{\theta=0}^{\theta=\pi} \int_0^a \left(r^2\right)^{3/2} r \, dr \, d\theta$$

$$\int_{\theta=0}^{\pi} \int_0^a r^4 \, dr \, d\theta$$

$$\int_{\theta=0}^{\pi} r^5/5 \Big|_0^a d\theta = \frac{a^5}{5} \int_0^{\pi} d\theta = \frac{\pi a^5}{5}$$



y = x²
y = x

$$a y'' + b y' + c y = 0$$ const. coeff problem

$$y = e^{\lambda x}$$

$$a x^2 y'' + b x y' + c x^0 y^{(0)} = 0$$ Cauchy-Euler.

Ex: $(x^2-9)^2 y'' + (x+3)y' + 2y = 0$

$$y'' + \frac{\cancel{(x+3)}}{(x+3)^{\cancel{2}}(x-3)^2} y' + \frac{2}{(x+3)^2(x-3)^2} y = 0$$

$x = \pm 3$ singular pts.

$$\left.\begin{aligned} p(x) &= \frac{1}{(x+3)(x-3)^2} \\ q(x) &= \frac{2}{(x+3)^2(x-3)^2} \end{aligned}\right\} \text{s.p. at } x = \pm 3$$

Check

$x_0 = 0$

$(x \pm 3)\, p(x)$

$(x \pm 3)^2 q(x)$

$\left.\begin{array}{l} +3 \\ -3 \end{array}\right\}$ have to be done separately.

$(x - 0)$

$(x - 0)^2$

Application of Gamma Func^n

Beta fn. $$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

$$(m, n > 0)$$

$$B(m,n) = B(n,m) \quad \text{(symmetric)}$$

$$B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

Calculate $I = \int_0^1 x^4 (1-x)^5 \, dx$

Comparing with $\int_0^1 x^{m-1} (1-x)^{n-1} \, dx$

$m-1 = 4 \longrightarrow m = 5$

$n-1 = 5 \longrightarrow n = 6$

$\int_0^1 x^{m-1} (1-x)^{n-1} \, dx$

$$= \boxed{B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}} = \frac{\Gamma(5)\,\Gamma(6)}{\Gamma(11)}$$

$$= \frac{4! \cdot 5!}{10!} = \boxed{\frac{1}{1260}}$$

$$\int_{1 \longleftarrow 0}^{4 \longleftarrow 1} x^{n-1}(1-x)^{m-1}$$

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$$\int e^{-x^2} x \, dx$$

$u = x^2$

$du = 2x \, dx$