

CQF 2009
Module 4.2
Live Lecture: April 1, 2009
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Stochastic Interest Rate Modeling

In this lecture:

- Stochastic models for interest rates
- How to derive the pricing equation for many fixed-income products
- The structure of many popular one-factor interest rate models
- The theoretical framework for multi-factor interest rate modeling
- Popular two-factor models

By the end of this lecture, you will:

- Be able to derive the pricing equation for fixed-income instruments with one and two random factors
- Appreciate the meaning of the market price of interest rate risk
- Know the names of many popular interest rate models

Introduction.....	3
One-factor interest rate modeling	4
The spot interest rate.....	6
Stochastic interest rates.....	7
The pricing equation for the general model.....	9
Hedging one bond with another.....	11
Both sides must be equal to a universal constant.....	17
The fixed-income pricing equation.....	20
The market price of risk.....	23
Interpreting the market price of risk and risk neutrality	25
Risk-neutral spot rate	27
Risk-adjusted drift rate.....	28
The relationship between prices and expectations.....	30
Rule of Thumb	32
Tractable models and solutions of the pricing equation	33
Named models	36
Vasicek.....	37
Cox, Ingersoll & Ross.....	47
Ho & Lee.....	50
Yield curve fitting.....	51
Hull & White.....	53
A more general model.....	54
Positive interest rates	57
Mean reversion.....	58
Solution for constant parameters	65
Multi-factor interest rate modeling	71

Theoretical framework.....	73
Popular models.....	82
Brennan and Schwartz (1982).....	83
Fong and Vasicek (1991).....	86
Longstaff and Schwartz (1992).....	88
General affine model.....	89

Summary

- Interest rates can be modeled as stochastic variables
- Whenever a modeled quantity is not traded the pricing equation contains a market price of risk term
- The pricing equation is another partial differential equation, similar in form to the Black-Scholes equation