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Module 5.3
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Valuing CDOs Using Copulas

In this lecture:

- We will learn about pricing credit instruments;
- Formulate a pricing model for static CDOs;
- Define copulas;
- Apply copulas to price and manage the risk of credit derivatives;

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Summary:

On the upside, Copula-based model is simple and take advantage of some interesting properties of copulas, such as

- the ability to de-couple the choice of marginal probability functions and of a dependence structure;
- the fact that the multivariate ($n \geq 3$) theory is a straight extension of bivariate theory, making high dimensionality problems not significantly more difficult to crack than low dimensionality problems;
- the very wide choice of dependency structures.

On the downside, this type of models is far from satisfactory:

- we have assumed that interest rates and default risks are independent while we know that they are not;
- because it is based on copulas, this model is limited to a one-period time frame and cannot accommodate embedded decision. Actively managed CDOs cannot be priced;
- how do we choose the copula function? So far we have limited ourselves to a Normal copula, which is just a multivariate normal distribution. However, if we wanted to choose another function for our copula, how would we proceed? The copula literature does not provide a clear answer, and the conventional wisdom is to try several copula functions and see which one presents the best to your data.
- data! Calibration and parameter estimation are difficult since the closest that can be used to approximate actual default probabilities are CDS rates. We would not only need default probabilities for all the securities held in the collateral... but also default correlations...

However, it is still very much considered as a “standard” model because

- no other model has proved that it can really do better!
- the lack of data is the major limitation when it comes to CDO pricing and they will affect every model. So beware!

- empirical analysis have shown that most models gives prices that are quite close to market prices for the equity tranche but that the pricing accuracy drops rapidly as we get into mezzanine and senior tranches.

The search for a good CDO pricing model is still on!