t calendar time

T maturity

s integration variable.

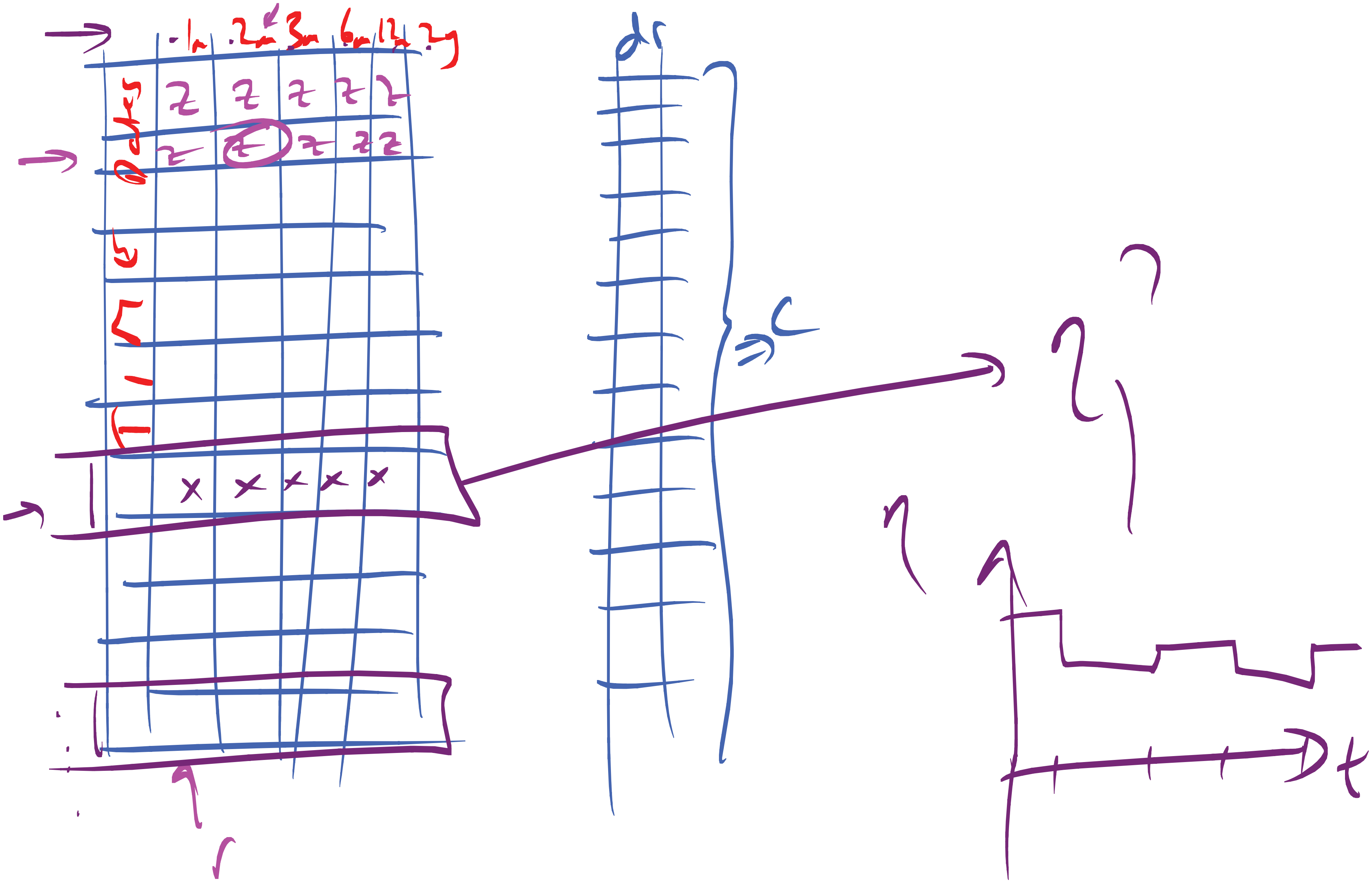
→ t^* time at which we perform the calibration.

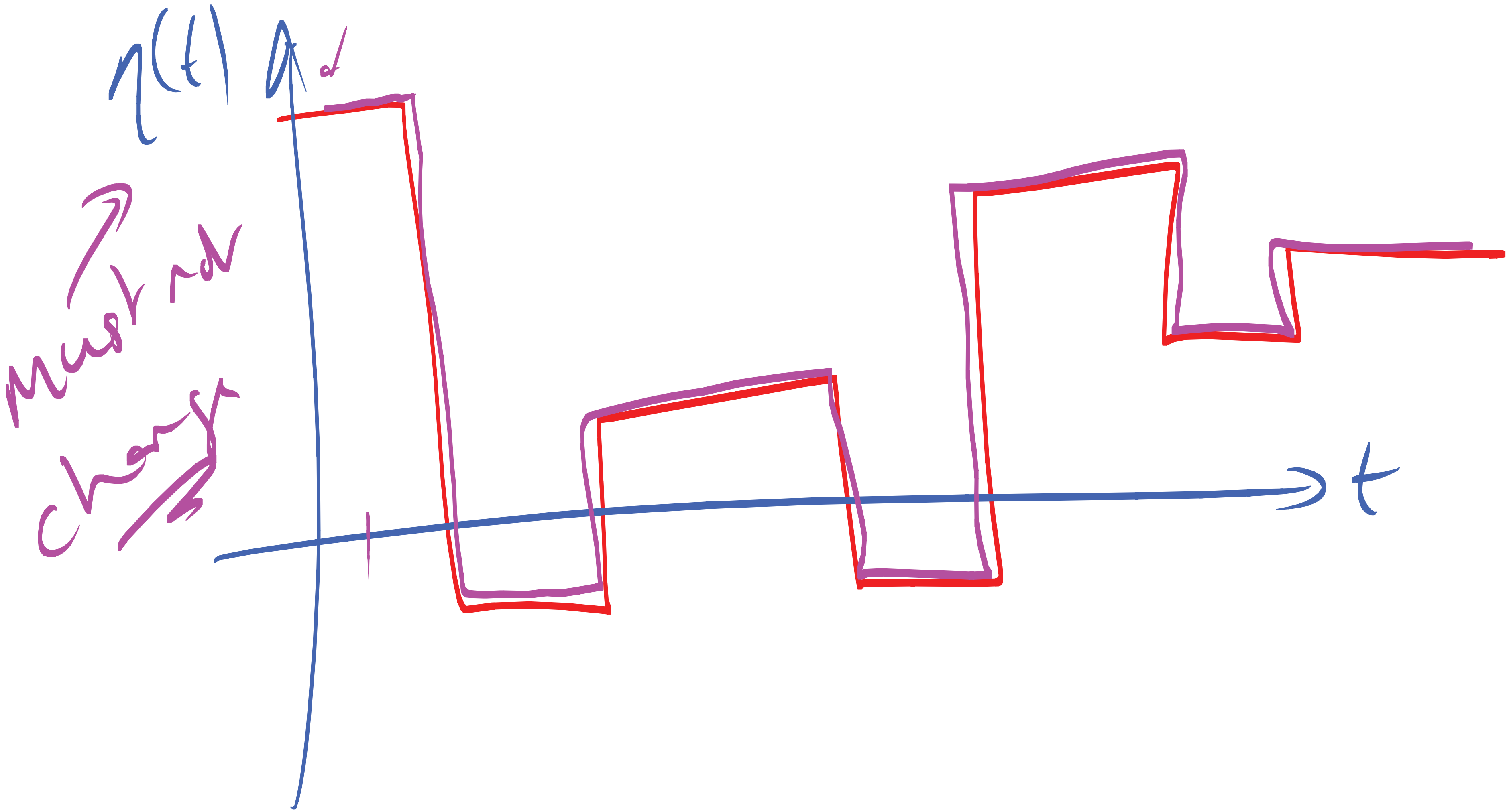
$$\int_{t^*}^{T} \eta^*(s)\,(T-s)\,ds = F(T) \qquad \text{I.C.}$$

$$\frac{\partial}{\partial T}: \int_{t^*}^{T} \eta^*(s)\,ds + \cancel{\eta^*(T)(T-T)} = \frac{dF}{dT}$$

$$\frac{\partial^2}{\partial T^2}: \qquad \eta^*(T) = \frac{d^2F}{dT^2}$$

$$\cancel{dr = \left(f(t)\sin r + e^{r^3 \ln t}\right)dt + e^{1/t\,r}\,dx}$$


.1m .2m 3m 6m 12m 2y
Dates
dr
t



η(t)
t
must not change

→ 1. "Oh, it's hard to calibrate"

→ 2. "It's unstable"

Sophisticated Math.

Sell side

Exotic, vanillas

CALIBRATE

QUANTS

Buyside

Don't Do

Hedge funds

Less sophisticated

Buy / Sell

Stats.

- r = constant → Pretend all underlyings are lognormal. Black model.
- $r(t)$ deterministic

Calib →

- $dr = \ldots dt + \ldots dX.$

SABR →

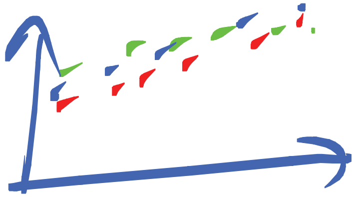
- $\begin{cases} dr = \ldots dt + \ldots dX_1 \\ dl = \ldots dt + \ldots dX_2 \\ d\ldots \end{cases}$

~~No Arb.~~ Arb.

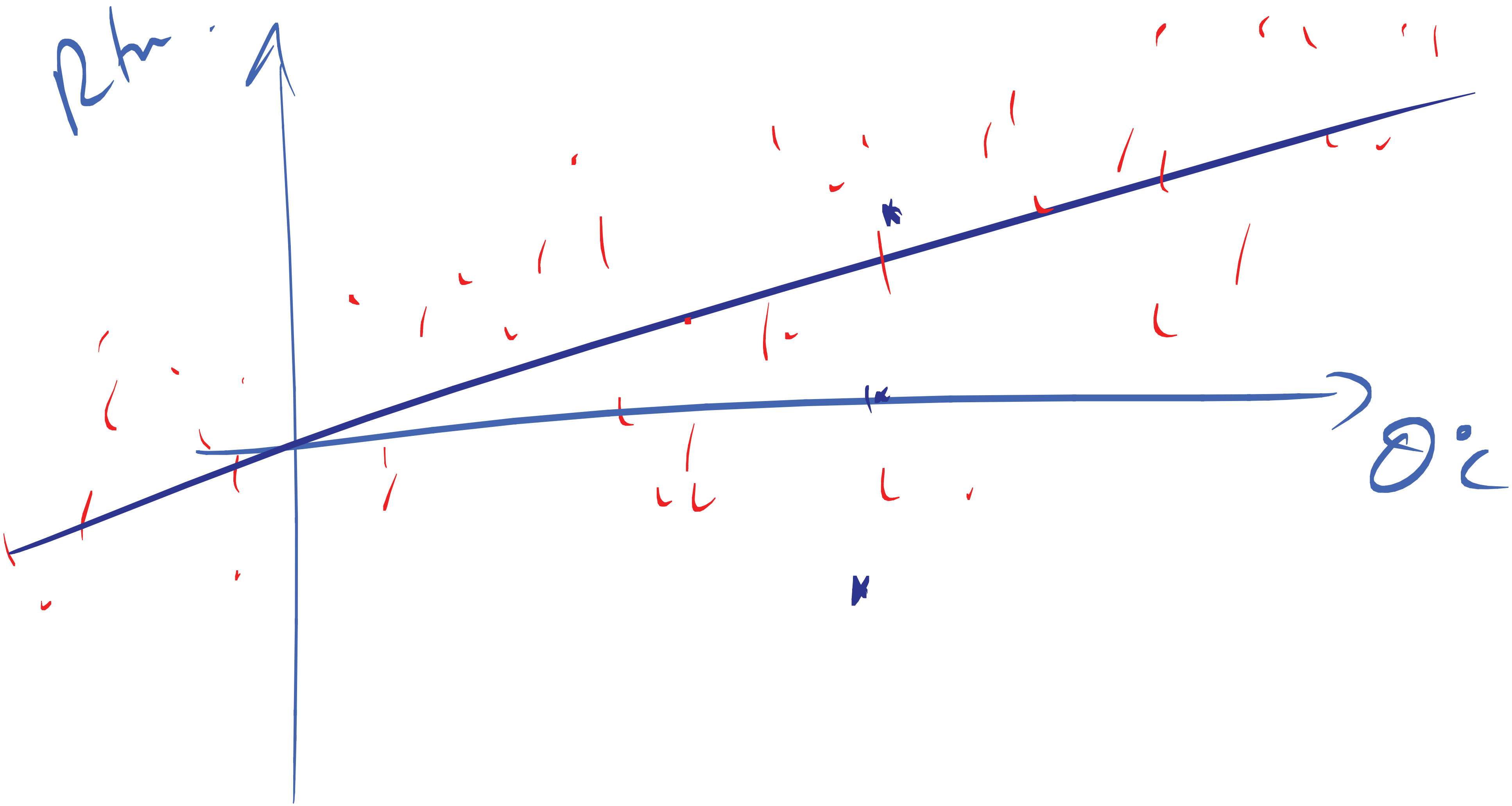
Arb

✓ Calib • HJM

✓ Calib • BGM



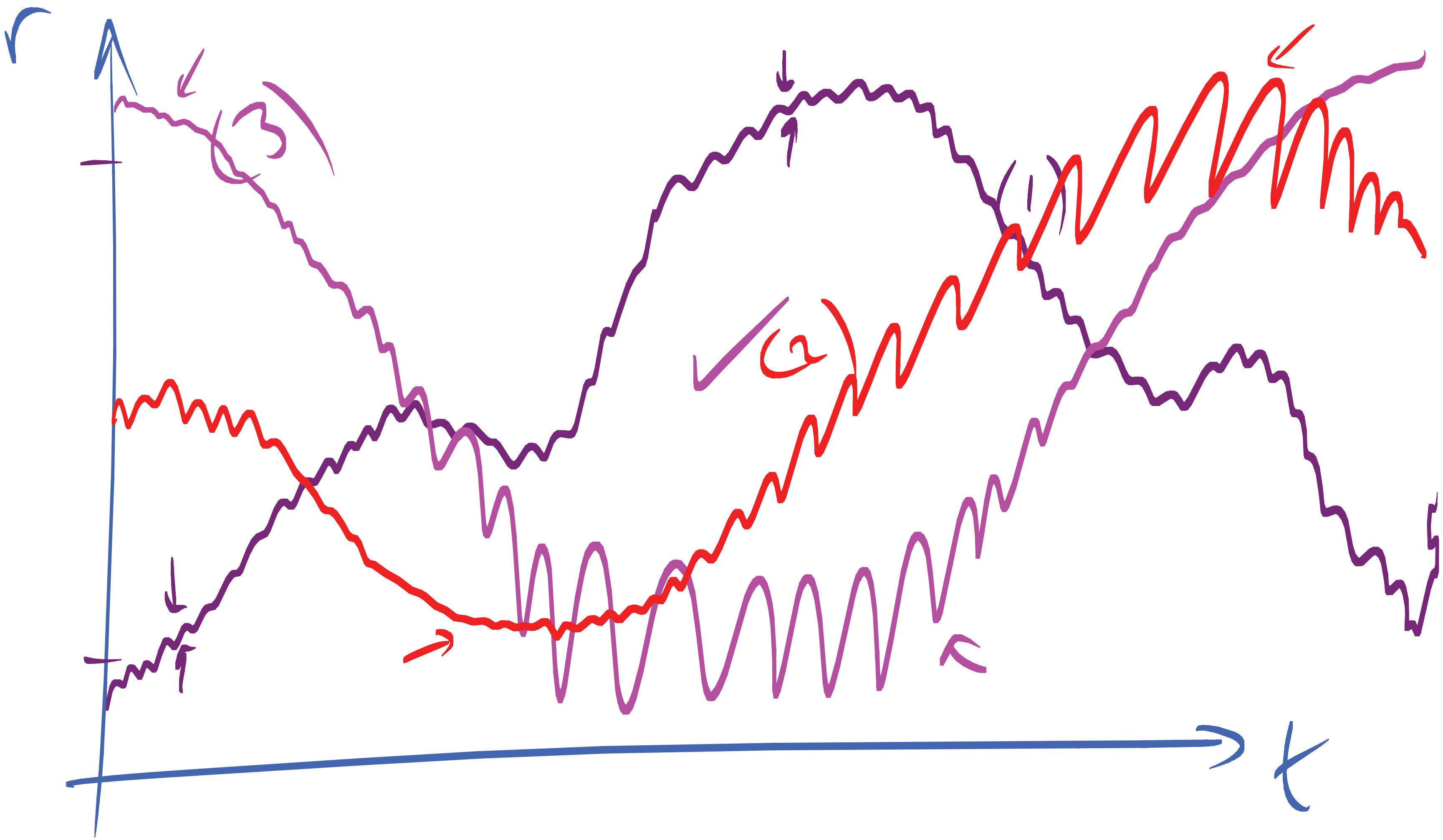
Quantic. No Arb.

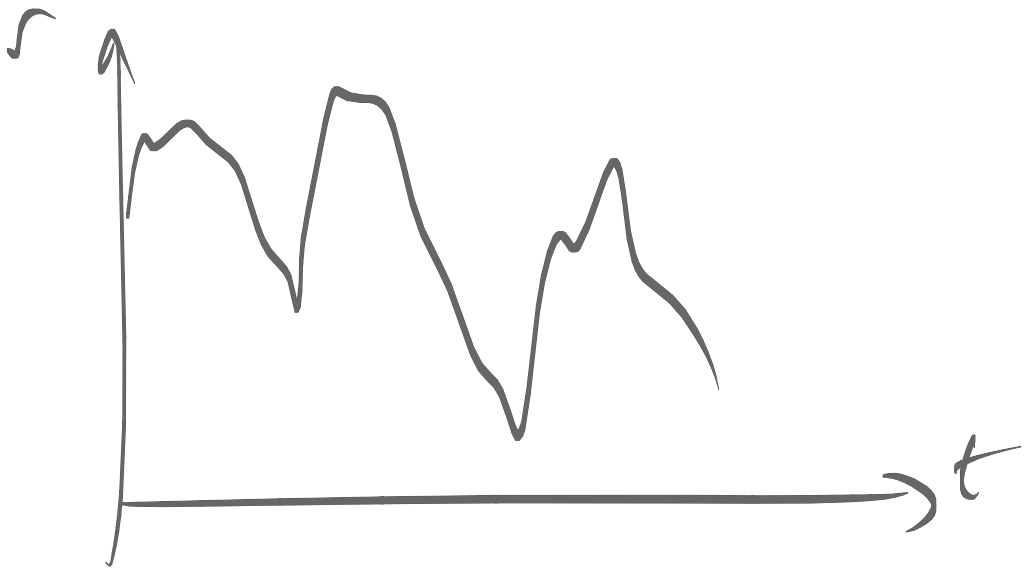

Rh
θ°c

r	dr		
5			
6.7	1.7		
7.1	x		
7.6	x		
8.1			
8.5			
7.9	x		
7.3	x		
6.2			

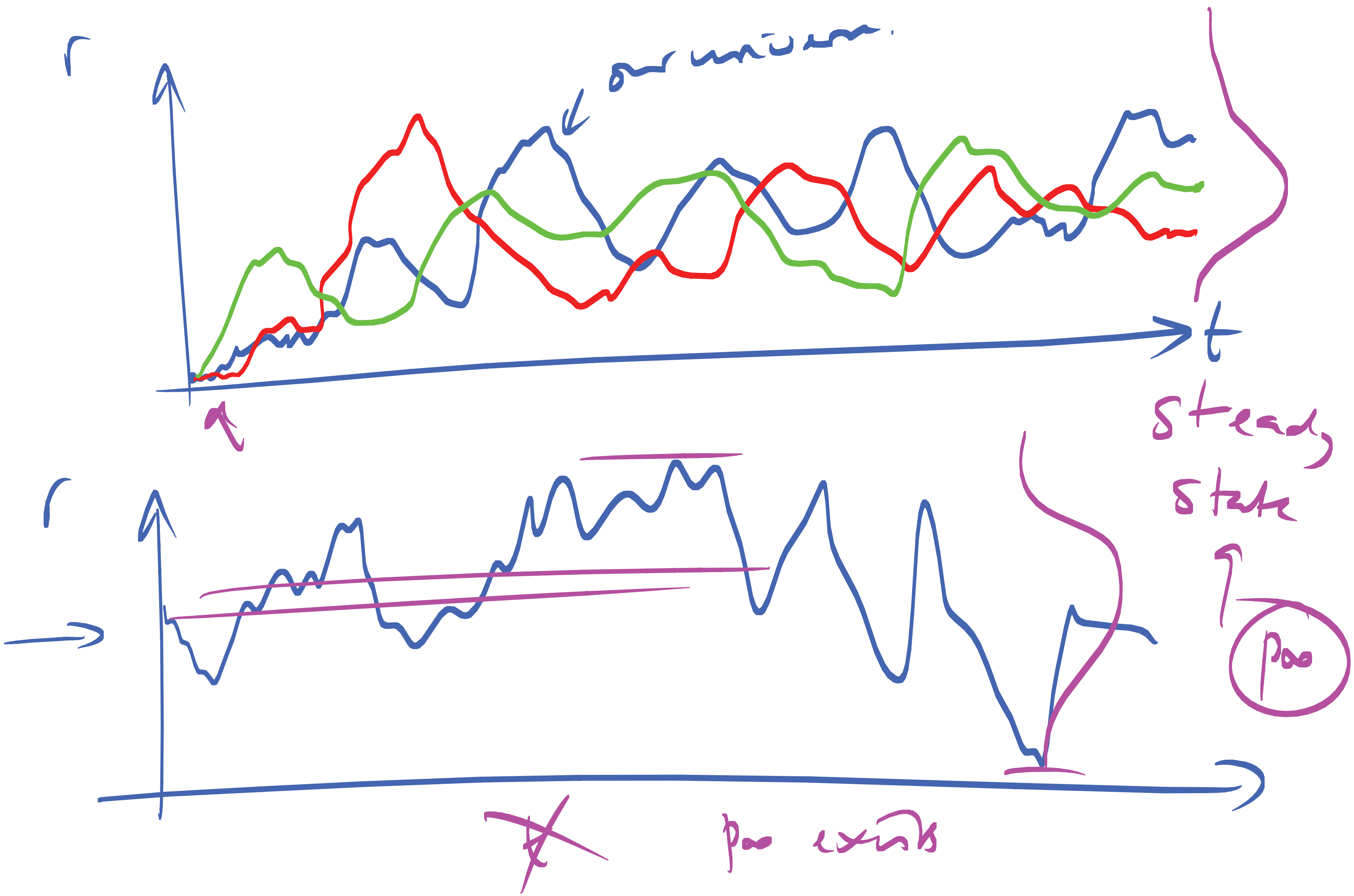
T

δr


r
t
(1)
(2)
(3)



Ergodic Theorem


r
t
steady
state
p_∞
r
p_∞ exists

$$\frac{\partial Z}{\partial t} + \frac{1}{2}\omega^2 \frac{\partial^2 Z}{\partial r^2} + (u - \lambda\omega)\frac{\partial Z}{\partial r} - rZ = 0 \qquad ' = \frac{d}{dr}$$

Given: $Z = 1 + a(r)(T-t) + b(r)(T-t)^2 + c(r)(\quad)^3 + \dots$

$$-a(r) - 2b(r)(T-t) - 3c(r)(T-t)^2 - \dots$$

$$+ \frac{1}{2}\omega^2(r)\left(a''(r)(T-t) + b''(r)(T-t)^2 + \dots \right)$$

$$+ (u-\lambda\omega)\left(a'(r)(T-t) + b'(r)(T-t)^2 + \dots \right)$$

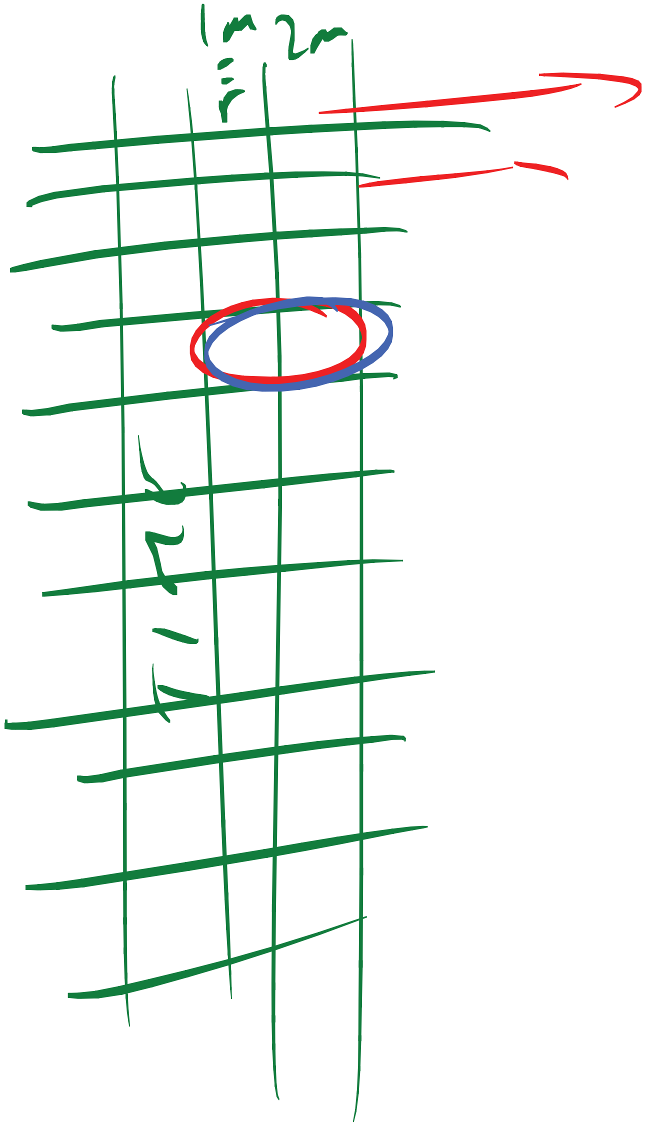
$$- r\left(1 + a(r)(T-t) + b(r)(T-t)^2 + \dots \right) = 0$$

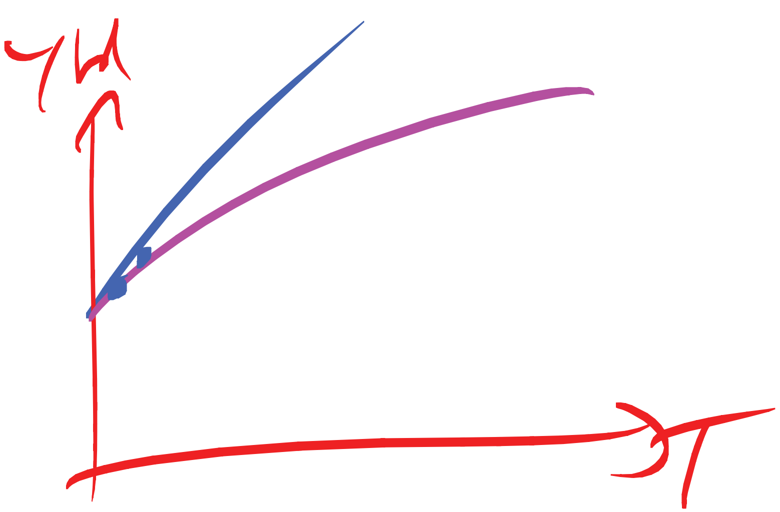
$$\boxed{a(r) = -r}$$

$$2b = \frac{1}{2}\omega^2 a'' + (u-\lambda\omega)a' - ra$$

$$= 0 - (u-\lambda\omega) + r^2$$

$$\boxed{b = \frac{r^2 - (u-\lambda\omega)}{2}}$$


1m
2m


T

$$\Pi = V$$

$$d\Pi - r\Pi\, dt = \quad \cdot \qquad - \; - \left(\lambda\, dt + dX \right)$$