Value and Variance

Now that Dr. Gomes has introduced the concepts of expected value and variance, read below about calculating these values.

Expected value

The term expected value means exactly what it sounds like: the result you can expect from some action. It can be calculated by summing the values of a random variable with each value multiplied by its probability of occurrence. Formulaically, expected value is expressed as:

$$E\left[X\right] = \sum_{i=1}^{n} X_i P_i$$

Where:

- $\sum$ - sum
- i - index
- n - total number of possible outcomes
- Xi - value of outcome i
- Pi - probability of observing outcome

An example

Consider the scenario where a fair, three-sided die is rolled to obtain a number based upon a random variable. The total number of possible outcomes is three, so *n*=3. The value of each of the outcomes can only be one, two, or three, and the probability of seeing any specific outcome would be one-third (a three-sided fair die).

To calculate the expected value, you take the first possibility from the die (one) and multiply it by the probability of that result coming up (one-third). Then, you do the same for the other possible results (two multiplied by one-third, and three multiplied by one-third) until all possibilities are exhausted. Once the multiplication is complete, you add the results together to get the expected value. Therefore, your calculation looks like the following:

$$(1 * \tfrac{1}{3}) + (2 * \tfrac{1}{3}) + (3 * \tfrac{1}{3}) = 2$$

Therefore, the expected value would be 2 (one-third + two-thirds + 1).

Expected variance

Utilizing the expected value, you can calculate the long-term average, and since random variables change in each turn, roll, draw, or trial, you understand that these variables will have some type of variation. The formula for calculating expected variance is as follows:

$$Var[X] = E[X^2] - (E[X])^2$$

As with the first example, you now want to calculate the variance based upon a three-sided die. In order to calculate this, you have already found E[X]. You can calculate $(E[X])^2$ as $2^2 = 4$. In order to complete the formula, you now need to solve for $E[X^2]$. This can be calculated in the following manner:

$$E[X^2] = (1^2 * \tfrac{1}{3}) + (2^2 * \tfrac{1}{3}) + (3^2 * \tfrac{1}{3}) = 4.66$$

The final step in the process would be to subtract the final equation as shown below:

$$Var[X] = E[X^2] - (E[X])^2 = 4.66 - 4 = .66$$