ϕ^4 KINK COLLISIONS

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Introduction

The ϕ^4-model is a 1+1D model, with solitons called kinks. Here we are looking at collisions between kinks and antikinks in this model.

Model

Field $\phi(x,t)$ ($\mathbb{R}^2 \to \mathbb{R}$).

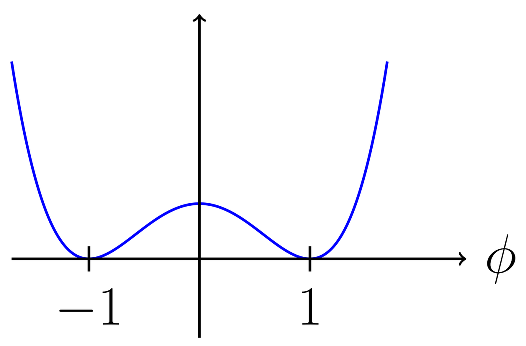
Lagrangian

$$L = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - U(\phi).$$

Potential

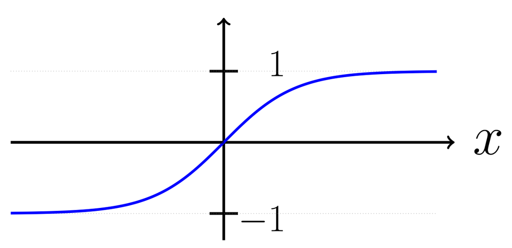
$$U(\phi) = \tfrac{1}{2}(1-\phi^2)^2$$

has two vacua, at $\phi = \pm 1$ (plotted on right).



A minimal energy solution interpolates between the vacua, giving a kink from -1 to 1 (below), or an aintikink from 1 to -1.

$$\phi_{\text{kink}} = \tanh(x)$$



Since there are only two vacua, a kink cannot folow another kink without an antikink in between, so the only two soliton collisions are kink-antikink collisions.

Shape/wobble mode

There is an oscillating wobble / shape mode, approximated by linearisation about the kink for small amplitudes by:

$$\phi(x,t) \approx \tanh(x) + A\sin\left(\sqrt{3}t\right)\operatorname{sech}(x)\tanh(x).$$

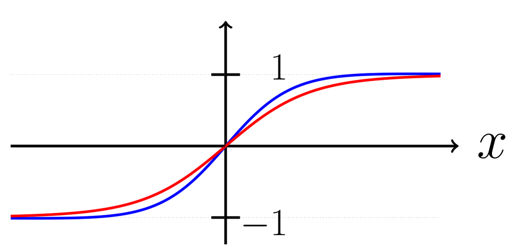


Figure 1: Wobbling kink approx for $A = 0.15$ with maximum positive (blue) and negative (red) coefficient of $\operatorname{sech}(x)\tanh(x)$.

This oscillation decays slowly over time, storing and gradually radiating away energy. This mode can be excited in collisions.

Collisions (setup)

The initial condition is a kink and an antikink (initial kink/antikink velocity v_0 / $-v_0$), symmetric about the origin with the kink on the left and antikink on the right as plotted below.

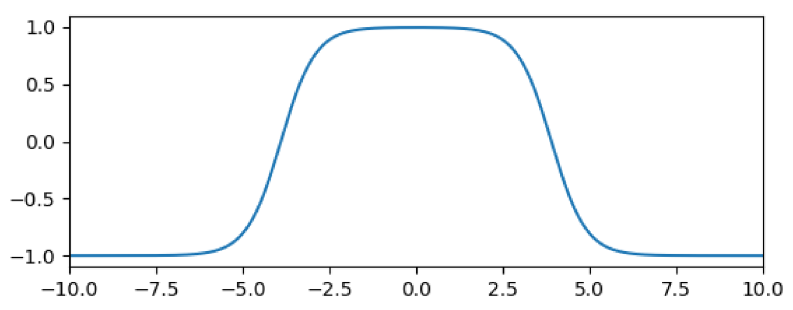


Figure 2: An example initial condition

They collide and bounce, losing kinetic energy to radiation and excited shape modes.

With sufficient kinetic energy after collision, the kink and antikink may escape after one bounce. Otherwise, they attract and collide again.

In each collision, energy is transferred between kinetic energy and the shape mode, which allows escape after multiple bounces.

They may also never escape, resulting in an oscillating bion which slowly decays to the vacuum.

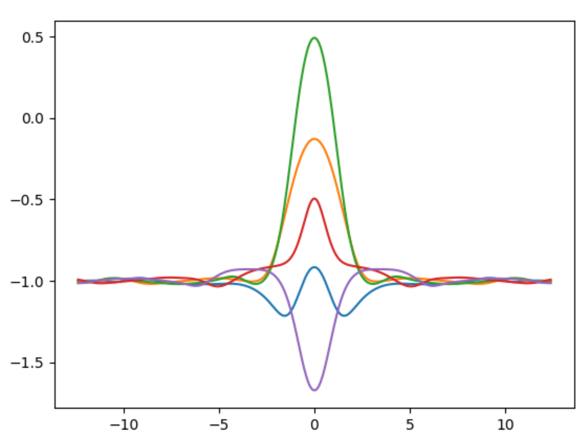


Figure 3: A plot of a bion at a few times

The behaviour depends upon the initial condition.

In the image plots that follow, each vertical slice is a separate field, with a different initial condition as indicated. As an example, the initial condition for the no wobble case is plotted below (not all exactly the same positions, but they are adjusted so that they collide at roughly the same time).

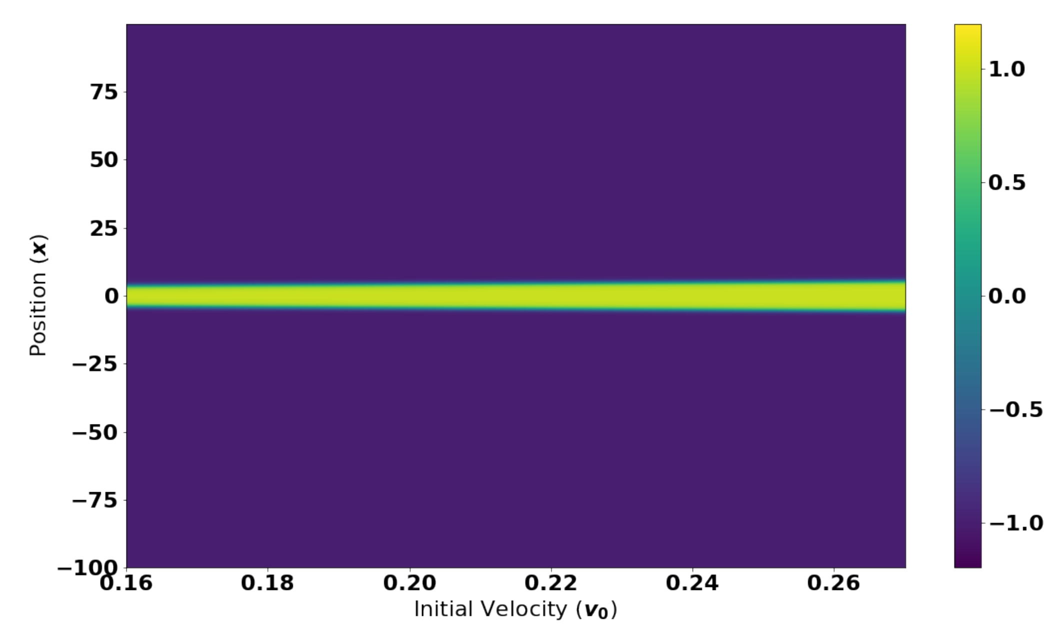


Figure 4: Initial condition (no wobble)

wobble: none

The result after collision is a pattern of escape windows. The n-bounce windows are surrounded (in a fractal pattern) by $(n+1)$-bounce windows.

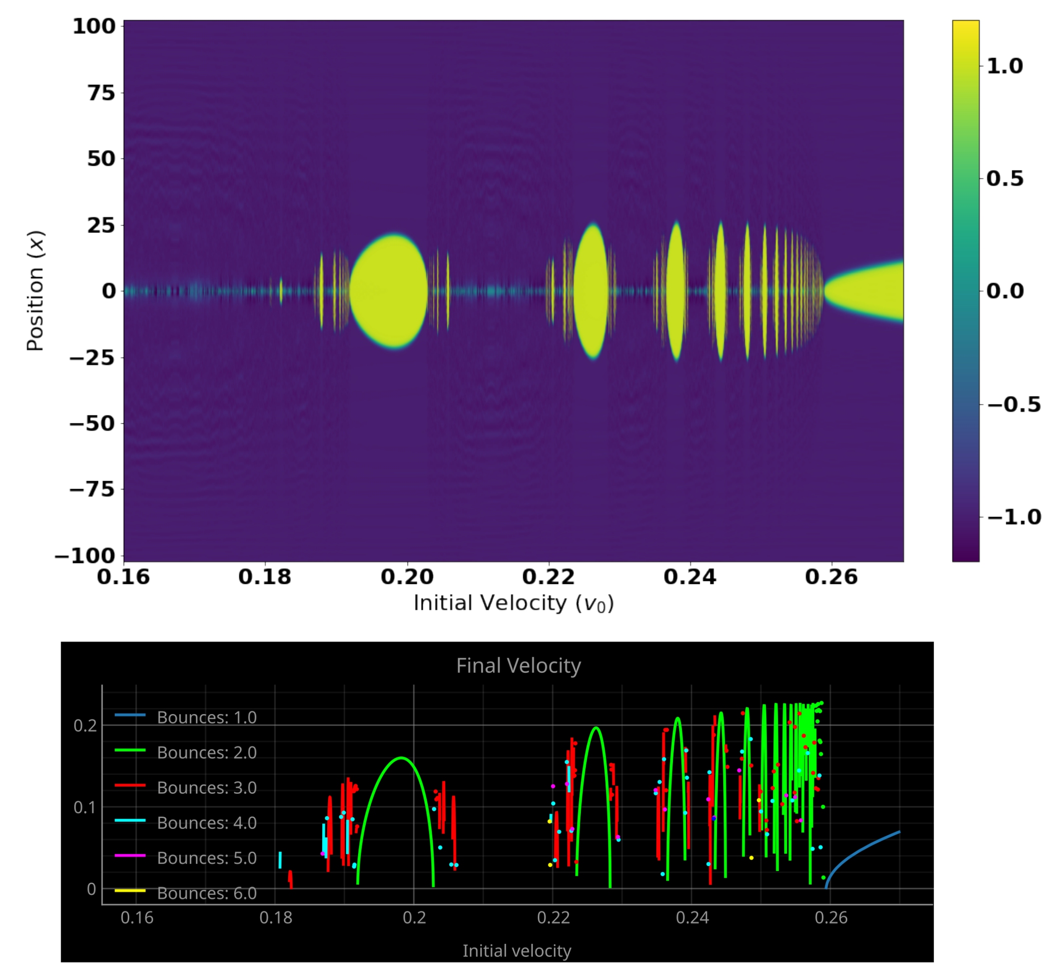


Figure 5: field values at $t = 160$ (top), final velocities (bottom)

wobble: 0.02, sync at start

Adding a wobble symmetrically to the kink and antikink, where the phase is aligned at the start of the simulation, the pattern changes compared to no wobble.

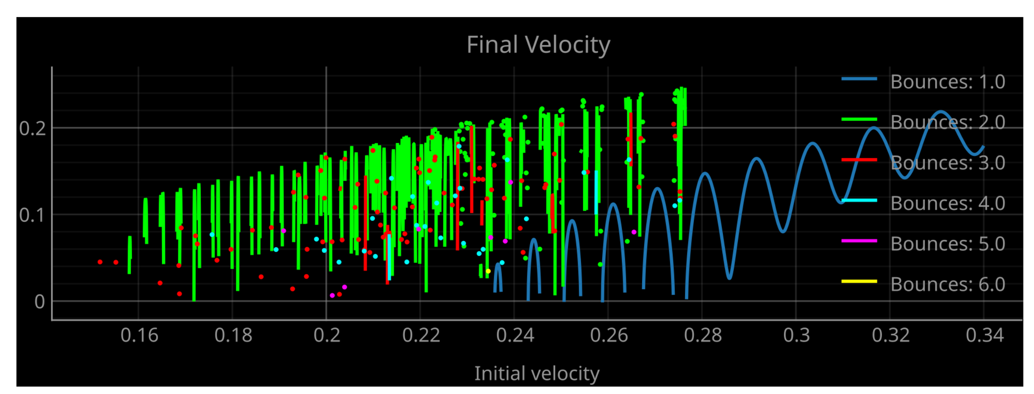


Figure 6: final velocities

wobble: 0.02, sync at collision (approx)

When we set the phases of the initial wobble so they align at the time of collision, the pattern is much closer to the original fractal, but changed and shifted.

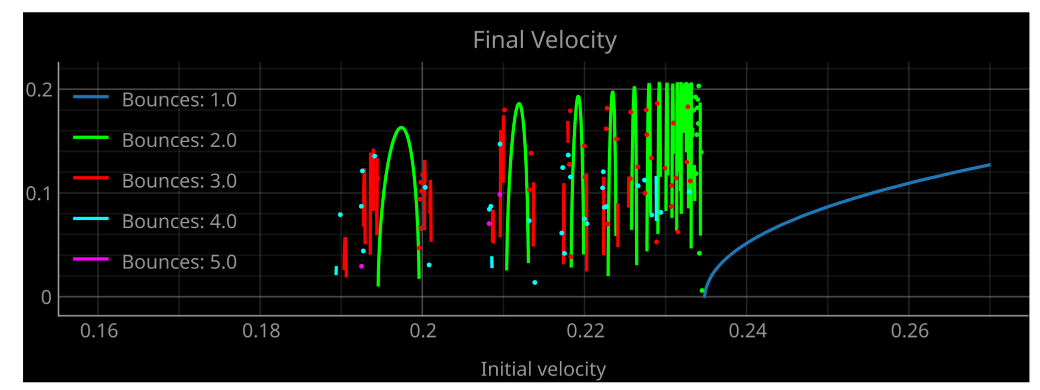


Figure 7: final velocities

wobble: 0.02, v_0=0.25 fixed phase plot

When we fix the initial velocity and vary the initial phase, a distorted version of the fractal pattern emerges.

The shape varies depending on the choice of initial velocity and amplitude. For example, the one bounce window, and the others, can cover the entire range or can disappear completely.

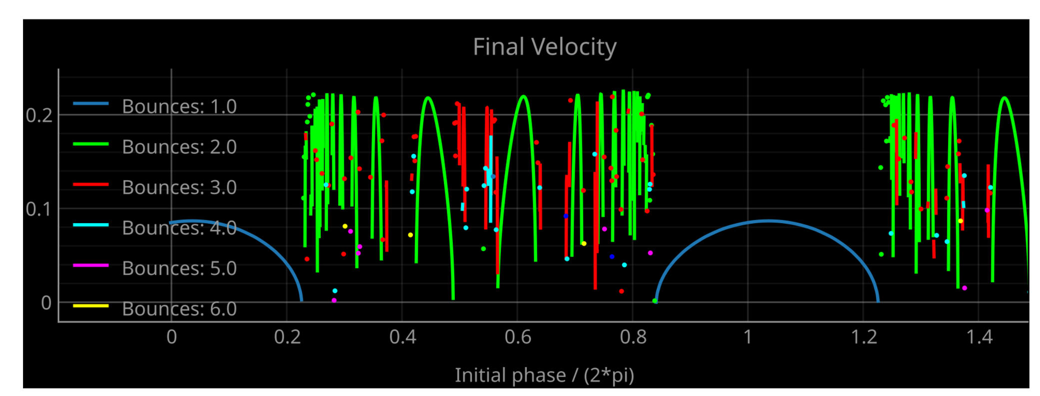


Figure 8: final velocities

References

A. Alonso Izquierdo, J. Queiroga-Nunes and L. M. Nieto. 'Scattering between wobbling kinks'. In: *Phys. Rev. D* 103.4 (2021), p. 045003. DOI: 10.1103/PhysRevD.103.045003. arXiv: 2007.15517

A. Alonso-Izquierdo, L. M. Nieto and J. Queiroga-Nunes. 'Asymmetric scattering between kinks and wobblers'. In: *Communications in Nonlinear Science and Numerical Simulation* 107 (2022), p. 106183. DOI: 10.1016/j.cnsns.2021.106183. arXiv: 2109.13904

Conclusion

We have looked at symmetric collisions between wobbling kinks and antikinks in the ϕ^4-model, and have shown that initial phase is important in such collisions. There are also interesting patterns in the plots, taking initial phase as a function of initial velocity.