Asymmetric nuclear matter in the Skyrme model

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The Skyrme model

- Effective Lagrangian of meson fields: $\mathrm{SU}(2) \ni U = e^{i\pi^a \tau^a} = \sigma \mathbb{I}_2 + i\pi_a \tau^a$ (2 flavors)
  Quartic (and sextic) terms allow the stability of topological solitons (Skyrmions):

$$\mathcal{L} = -\frac{f_\pi^2}{16}\,\mathrm{Tr}\,L_\mu L^\mu + \frac{1}{32e^2}\,\mathrm{Tr}\,[L_\mu, L_\nu]^2 - \lambda^2\pi^4 \mathcal{B}_\mu \mathcal{B}^\mu + \frac{m_\pi^2 f_\pi^2}{8}\,\mathrm{Tr}\,(U - I) \quad L_\mu = U^\dagger \partial_\mu U$$

- Topological degree of finite energy configurations is conserved on each topological sector.

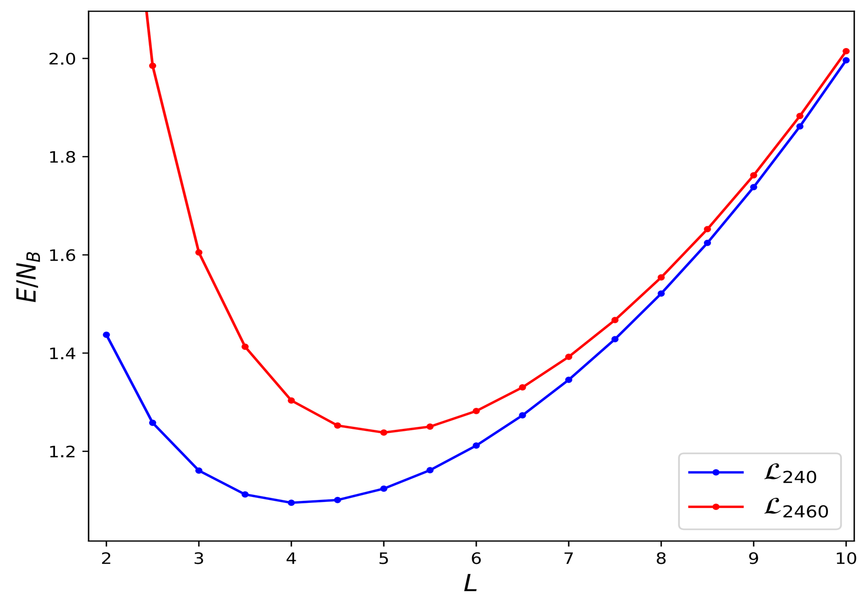
$$\pi_3(\mathrm{SU}(2) \sim \mathrm{S}^3) = \mathbb{Z} \;\ni\; B = \int d^3x \mathcal{B}^0, \quad \mathcal{B}^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\,\{L_\nu L_\alpha L_\beta\}$$

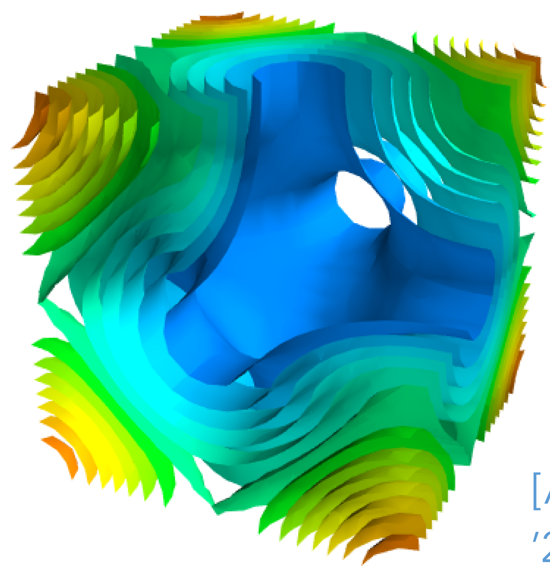
  … It can be identified with the Baryon number.

- Unified description of baryons and mesons, in the low energy phase, non-perturbatively.

Classical Skyrmion crystals

- Minimize static energy imposing some symmetries over a unit cell of size L to obtain classical crystal configurations.





[Adam et al. '22]

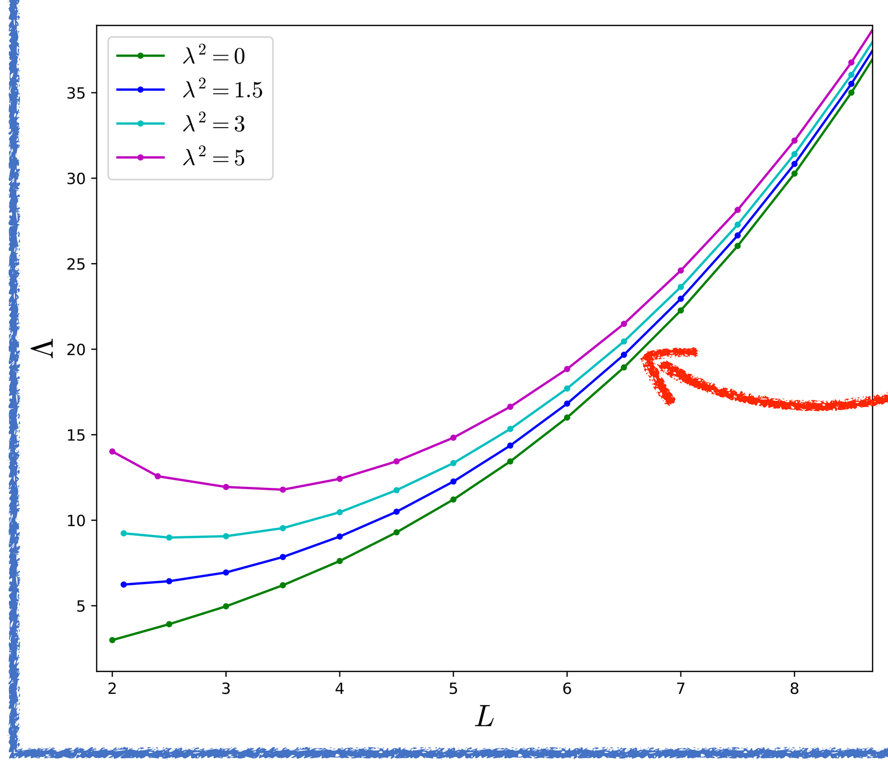
…a good model for symmetric nuclear matter.

Isospin quantization: symmetry energy

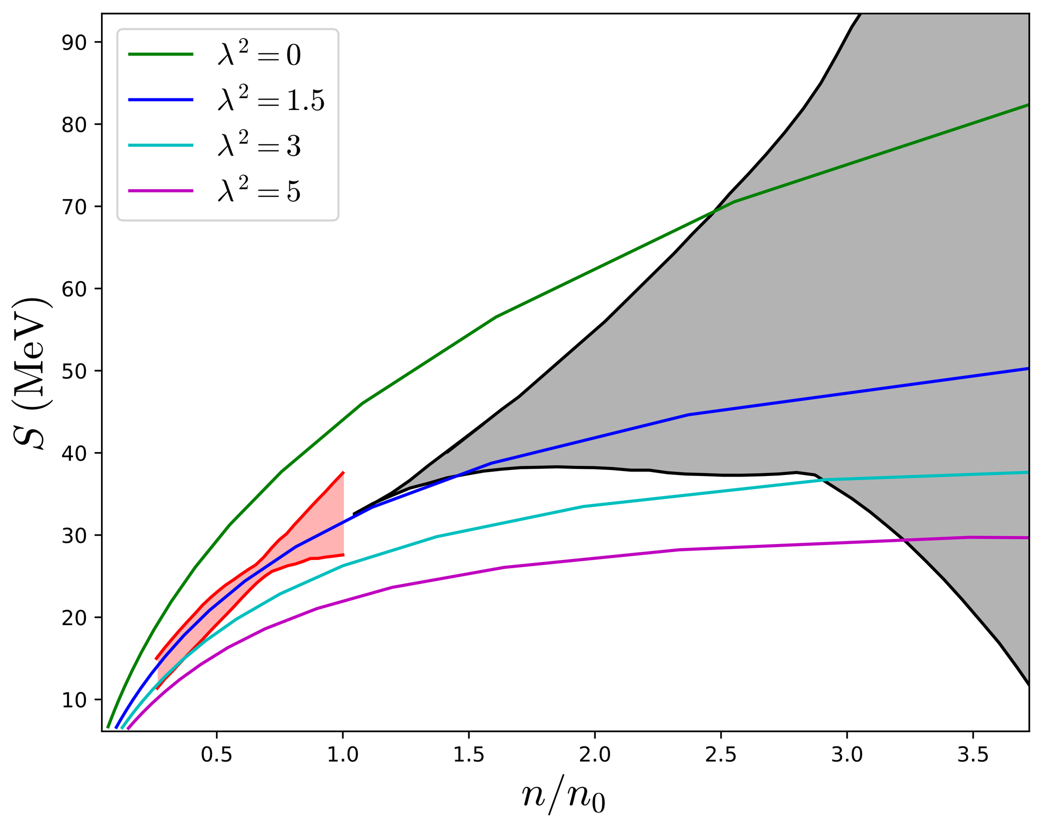
- In nuclear matter, the energy per baryon is a function of density n_B and the asymmetry parameter, $\delta = (1 - 2\gamma)$

$$\frac{E(n_B, \delta)}{N_B} = E_0(n_B) + S_N(n_B)\delta^2 + \mathcal{O}(\delta^3)$$

- Isospin asymmetry is included by quantizing isospin collective coordinates.



The isospin moment of inertia Λ is a function of L



- In a mean field approximation, we obtain a contribution to the energy (symmetry energy)

$$E_{\rm iso}^{\rm cell} = \frac{2\delta^2}{\Lambda} \longrightarrow S_N(n_B) = \frac{1}{2\Lambda}$$

[Adam et al. 2202.00953]

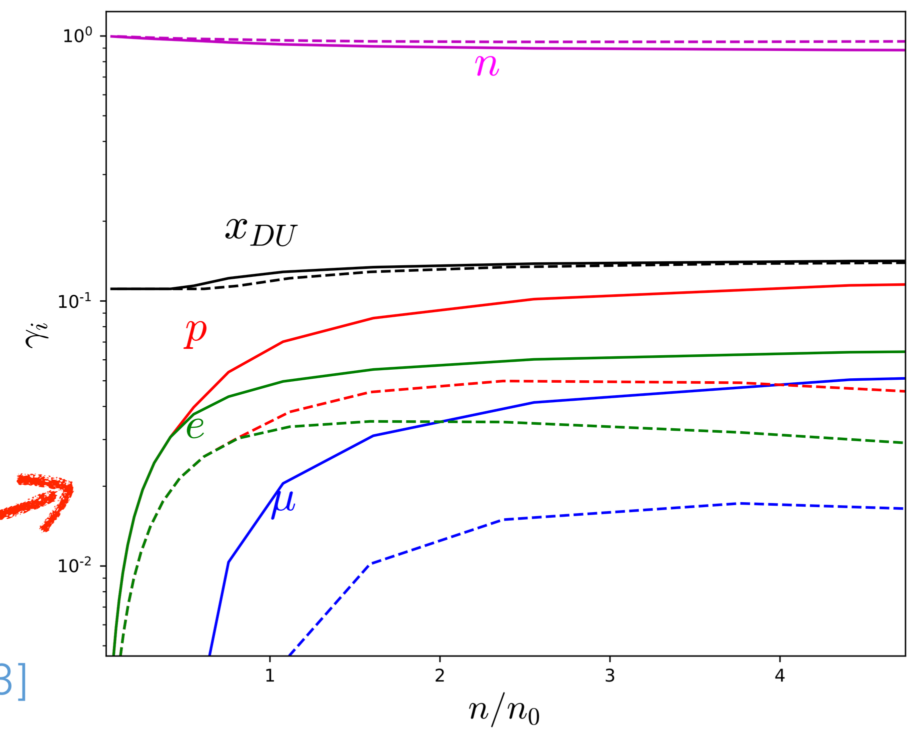
- We define an Isospin chemical potential: $\mu_I = -\frac{\partial E}{\partial N_I} = \frac{2(1-2\gamma)^2}{\Lambda}$
- Global neutrality achieved by including a leptonic background.

$$n_e = n_p = \gamma n_B \quad , \quad \gamma = n_p / n_B$$

- β-equilibrium:

$$n \leftrightarrow p + e + \bar{\nu}_e \longrightarrow \mu_n - \mu_p \equiv \mu_I = \mu_e = (3\pi^2 n_e)^{1/3}$$

Solve for $\gamma(L)$ to obtain particle fractions as function of density



Adding strangeness

In QCD with 3 flavors (u, d, s), there are 8 Goldstone mesons:

$$U = e^{i\theta_a \lambda_a / f_\pi}, \qquad \theta_a\lambda_a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

… but we expect charged Kaons (K^-) to condensate first.

- Modified Lagrangian (2 additional terms):
  1. Explicit flavor symmetry breaking ($\mathrm{SU}(3)_\mathrm{F} \to \mathrm{SU}(2)_\mathrm{I} \times U(1)_\mathrm{Y}$) potential:

$$\mathcal{L}_0^{\rm new} = \frac{f_\pi^2}{48}\left(m_\pi^2 + 2m_K^2\right)\mathrm{Tr}\{U + U^\dagger - 2\} + \frac{\sqrt{3}}{24} f_\pi^2 \left(m_\pi^2 - m_K^2\right)\mathrm{Tr}\{\lambda_8 \left(U + U^\dagger\right)\}, \quad m_s \gg m_{u,d}$$

  2. A (topological) WZW term implements the effect of the axial anomaly :

$$S_{WZ} = -i\frac{N_c}{240\pi^2}\int d^5x\, \epsilon^{\mu\nu\alpha\beta\gamma}\,\mathrm{Tr}\{L_\mu L_\nu L_\alpha L_\beta L_\gamma\} \quad \neq 0 \quad \text{for } N_f \geq 3$$

- Field parametrization via the Callan-Klebanov approach : kaons are small fluctuations along the strange directions over an $\mathrm{SU}(2)$ solitonic background.

$$\mathrm{SU}(3) \ni U = \Sigma U_\pi \Sigma$$

[Callan, Klebanov, 85]

$$U_\pi = \begin{pmatrix} u & 0 \\ 0 & 1 \end{pmatrix}, \quad \Sigma = e^{i\frac{2\sqrt{2}}{f_\pi}\mathcal{D}}, \quad u = \sigma + i\pi_k\tau^k, \quad \mathcal{D} = \begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix}, \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \quad K^\dagger = (K^-, \bar{K}^0).$$

At the onset of condensation, charged kaons develop non-zero vacuum expectation values, i.e. the vacuum "rotates" in flavor space:

$$\langle K^\mp \rangle = \phi e^{\mp i\mu_K t} \implies \tilde{\mathcal{D}} = \begin{pmatrix} 0 & 0 & \phi e^{i\mu_K t} \\ 0 & 0 & 0 \\ \phi e^{-i\mu_k t} & 0 & 0 \end{pmatrix} \implies \Sigma = e^{i\frac{\sqrt{2}}{f_\pi}\tilde{\mathcal{D}}} = \begin{pmatrix} \cos\tilde{\phi} & 0 & ie^{i\mu_K t}\sin\tilde{\phi} \\ 0 & 1 & 0 \\ ie^{-i\mu_K t}\sin\tilde{\phi} & 0 & \cos\tilde{\phi} \end{pmatrix}$$

time dependence of the vev given by the kaon chemical potential

Kaon condensation

- At a critical density n_B^*, a K^- condensate becomes energetically more favorable than electrons, due to the lack of a Fermi surface.
  Processes involving K^- start to take place along with standard β - equilibrium:

$$n \to p + l + \bar{\nu}_l \quad , \quad p + l \to n + \nu_l \qquad n \leftrightarrow p + K^-, \qquad l \leftrightarrow K^- + \nu_l \qquad (l = e, \mu)$$

$$\mu_n - \mu_p = \mu_I = \mu_l = \mu_K$$

- Total energy of the system (Kaons+Skyrmion+electrons):

$$E_{\rm Tot} = E_{Clas}(L) + E_{\rm Iso}(\gamma, \tilde{\phi}) + E_K(\mu_e, \tilde{\phi}) + E_e(\mu_e)$$

e^- energy (degenerate Fermi gas)

Classical crystal energy

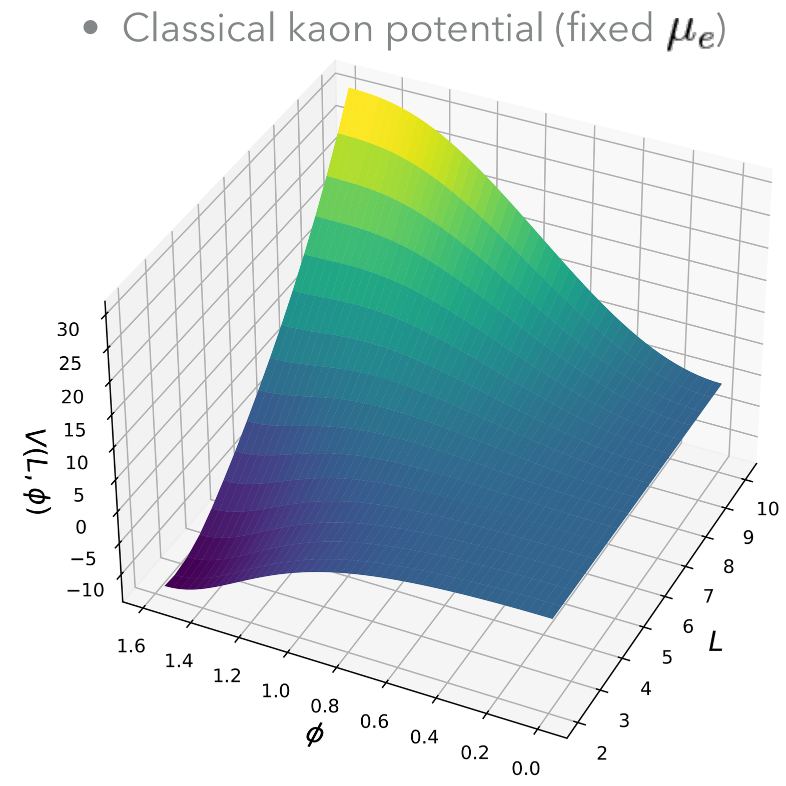
Quantum isospin correction

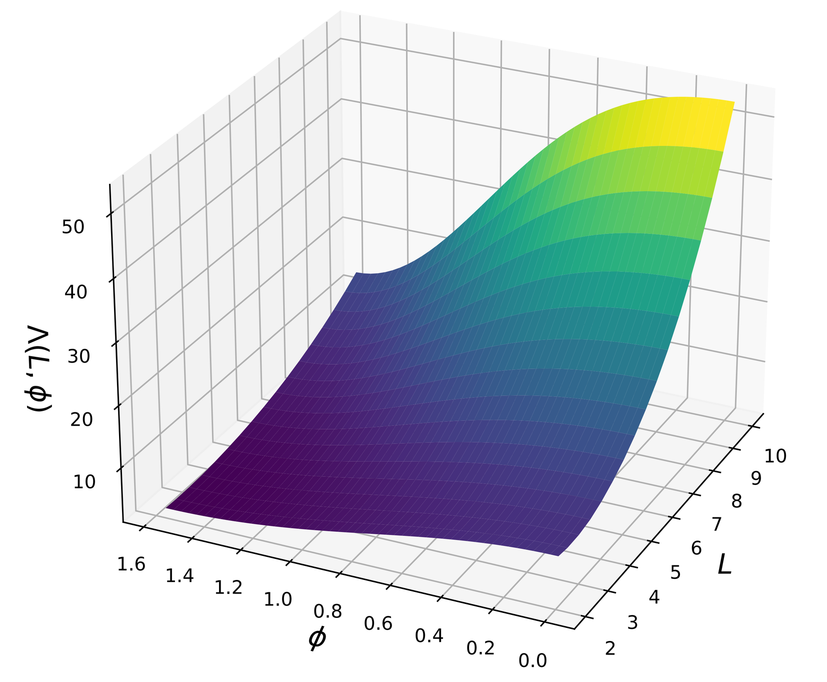
Classical K^- potential

- At a given density, the energy of the system depends on 3 parameters: $(\gamma, \mu_e, \tilde{\phi})$
  Their values can be obtained fixing L and minimizing the Grand Canonical Potential:

$$\Omega = E_{\rm Tot} - \mu_e(N_e - \gamma N_B) \longrightarrow \left.\frac{\partial \Omega}{\partial \gamma}\right|_{n_B}(\gamma, \tilde{\phi}, \mu_e) = \left.\frac{\partial \Omega}{\partial \tilde{\phi}}\right|_{n_B}(\gamma, \tilde{\phi}, \mu_e) = \left.\frac{\partial \Omega}{\partial \mu_e}\right|_{n_B}(\gamma, \tilde{\phi}, \mu_e) = 0.$$

- Classical kaon potential (fixed μ_e)





- Due to isospin of Kaons, Λ is now a function of both L and the condensate angle $\tilde{\phi}$.

Conclusions

The Skyrme model is a useful approach for the non-perturbative description of strongly interacting matter in the low energy regime. Although it is usually employed to model finite nuclei, we have extended its validity to the treatment of dense nuclear matter such as that of the interior of neutron stars. We have analyzed the effects of a non-trivial isospin asymmetry on the energy density, which allows us to describe beta-equilibrated matter within the model. Furthermore, strange degrees of freedom, such as kaons and hyperons, are expected to appear at sufficiently high densities. We have studied the kaon condensed phase of isospin asymmetric matter by modeling kaons as fluctuations along the strange directions, and how these affect the energy functional, including isospin quantum corrections.

References & Acknowledgments

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