HotBits Statistical Testing

Introduction

While it's essential that the theory behind a random number generator be well understood, and that its hardware and software realisation be carefully verified to implement the theoretical design, there is no substitute for detailed statistical testing of the actual output of the generator against the expectation for genuinely random data. Although a generator's design may be simple, its implementation on a complex computer and software environment makes it vulnerable to a multitude of potential problems ranging from simple programming errors to subtle bias introduced by the behaviour of instruction and data caches in the microprocessor, interference from interrupts (if not prevented), and metastability in the logic gates which receive events from the generator.

A large data set produced by the HotBits generator has been subjected to the scrutiny of three different randomness test suites, whose results are presented below. You can download this large data set (which took almost two days to create with the HotBits generator running continuously) and subject it to your own analyses, if you are so inclined.

There are many different ways to test for randomness, but all of them, in essence, boil down to computing a mathematical metric from the data stream being tested and comparing the result with the expectation value for an infinite sequence of genuinely random data. For a truly random sequence, *any* value is equally probable. The sequence of bits “`0000000000000000`” is just as likely to occur in a random data stream as “`1100100100001111`”, and is no less “random”. (The latter sequence is, in fact, the first sixteen bits of the mathematical constant π in binary, whose algorithmic complexity is only modestly greater than the all zero sequence!)

Randomness can be defined only statistically over a long sequence, which is why it is essential to test a large data set. Data can fail to be random in many ways. For example, one of the most obvious tests one can apply to a sequence of binary data is to count the number of ones and zeroes: as the length of the sequence increases, the difference in these values can be used to calculate the probability the sequence is random. But this test, used in isolation, would consider a sequence of alternating zero and one bits (“`0101010101010101…`”) perfectly random. Hence, it must be used as part of a test battery, including other measures which are sensitive to repeating patterns, improbably long runs of zeroes and ones, and other, more subtle, deviations from randomness.

Speaking as a programmer and not a mathematician or statistician, the two widely-used randomness test batteries: Diehard and the NIST SP 800-22 Statistical Test Suite, whose results are reported below, are quite messy and fragile programs. When using them, it is wise to use data sets of the same size as those employed in the examples supplied with the programs, and to select test parameters identical to those of the examples. In my experience, deviating from the domain in which the programs are known to have been tested may yield surprising and dismaying results. Also, before testing your own data with one of these test batteries, be sure to re-run the examples in the documentation and verify that you're able to reproduce the published results; changes in compilers and libraries, file formats, and operating system compatibility issues may have to be resolved before you can obtain reliable results from these tests.

Tests with the Fourmilab ENT Utility

The Fourmilab ENT program is a public domain utility which tests binary data sequences, either as a series of 8 bit bytes, or as a bit stream, with five standard tests for randomness which are described in the document linked to above. These are all straightforward mathematical metrics, and while they identify major departures from randomness, may miss subtle forms of bias identified by the more comprehensive test suites. The following are the results of an ENT test of the same 11,468,800 data set used for the Diehard test battery in the next section.

```
Entropy = 7.999975 bits per byte.

Optimum compression would reduce the size
of this 11468800 byte file by 0 percent.

Chi square distribution for 11468800 samples is 402.53, and randomly
would exceed this value 0.01 percent of the times.

Arithmetic mean value of data bytes is 127.5423 (127.5 = random).
Monte Carlo value for Pi is 3.141486168 (error 0.00 percent).
Serial correlation coefficient is -0.000053 (totally uncorrelated = 0.
```

- Download ENT test results (`ent_results.zip` 448 bytes)
- Download data set for ENT and Diehard tests (`FourmilabHotBits_Diehard.zip` 11 Mb)

Tests with the Marsaglia Diehard Battery of Tests of Randomness

Professor George Marsaglia of Florida State University published the "Diehard Battery of Tests of Randomness" in 1995, as part of the Marsaglia Random Number CDROM; these programs may now be downloaded from the link above. The Diehard tests are rather "quirky" measurements of randomness compared to the mathematical properties tested by ENT. Diehard tests include items such as a spacings between birthdays in a random population, monkeys pounding on keyboards, and games of craps. These tests, however, can be exquisitely sensitive to subtle departures from randomness, and their results can all be expressed as the probability the results obtained would be observed in a genuinely random sequence. Probability values close to zero or one indicate potential problems, while probabilities in the middle of the range are expected for random sequences. Please read the “NOTE” at the top of the results presented below about interpreting the reported probability values: with hundreds of probability values computed, some may be expected, purely by chance, to be close to one or zero.

The Diehard test suite was run on a 11,468,800 byte data set extracted from the beginning of the 16,779,776 HotBits test data set. I limited the data set length to that used by other Diehard examples to avoid possible problems in the code dependent upon the size of the data set.

```
                    NOTE

Most of the tests in DIEHARD return a p-value, which
should be uniform on [0,1) if the input file contains truly
independent random bits.   Those p-values are obtained by
p=1-F(X), where F is the assumed distribution of the sample
random variable X---often normal. But that assumed F is often
an asymptotic approximation, for which the fit will be worst
in the tails. Thus you should not be surprised with  occasion-
al p-values near 0 or 1, such as .0012 or .9983. When a bit
stream really FAILS BIG, you will get p`s of 0 or 1 to six
or more places.  By all means, do not, as a Statistician
might, think that a p < .025 or p> .975 means that the RNG
has "failed the test at the .05 level".  Such p`s happen
among the hundreds that DIEHARD produces, even with good RNGs.
 So keep in mind that "p happens"

Enter the name of the file to be tested.
This must be a form="unformatted",access="direct" binary
file of about 10-12 million bytes. Enter file name:

FourmilabHotBits.32


        HERE ARE YOUR CHOICES:

        1   Birthday Spacings
        2   Overlapping Permutations
        3   Ranks of 31x31 and 32x32 matrices
        4   Ranks of 6x8 Matrices
        5   Monkey Tests on 20-bit Words
        6   Monkey Tests OPSO,OQSO,DNA
        7   Count the 1`s in a Stream of Bytes
        8   Count the 1`s in Specific Bytes
        9   Parking Lot Test
        10  Minimum Distance Test
        11  Random Spheres Test
        12  The Sqeeze Test
        13  Overlapping Sums Test
        14  Runs Test
        15  The Craps Test
        16  All of the above

To choose any particular tests, enter corresponding numbers.
Enter 16 for all tests. If you want to perform all but a few
```

```
tests, enter corresponding numbers preceded by "-" sign.
Tests are executed in the order they are entered.

Enter your choices.

16

|-------------------------------------------------------------
|           This is the BIRTHDAY SPACINGS TEST
|Choose m birthdays in a "year" of n days.  List the spacings
|between the birthdays.  Let j be the number of values that
|occur more than once in that list, then j is asymptotically
|Poisson distributed with mean m^3/(4n).  Experience shows n
|must be quite large, say n>=2^18, for comparing the results
|to the Poisson distribution with that mean.  This test uses
|n=2^24 and m=2^10, so that the underlying distribution for j
|is taken to be Poisson with lambda=2^30/(2^26)=16. A sample
|of 200 j''s is taken, and a chi-square goodness of fit test
|provides a p value.  The first test uses bits 1-24 (counting
|from the left) from integers in the specified file.  Then the
|file is closed and reopened, then bits 2-25 of the same inte-
|gers are used to provide birthdays, and so on to bits 9-32.
|Each set of bits provides a p-value, and the nine p-values
|provide a sample for a KSTEST.
|------------------------------------------------------------

        RESULTS OF BIRTHDAY SPACINGS TEST FOR FourmilabHotBits
(no_bdays=1024, no_days/yr=2^24, lambda=16.00, sample size=500

Bits used       mean            chisqr          p-value
 1 to 24        15.86           23.2107         0.142517
 2 to 25        15.81           13.7843         0.682308
 3 to 26        15.76           13.6039         0.694917
 4 to 27        15.55           18.6599         0.348401
 5 to 28        15.63           25.9381         0.075596
 6 to 29        15.83           15.1730         0.583021
 7 to 30        16.18           20.5640         0.246401
 8 to 31        15.95           13.3762         0.710659
 9 to 32        15.92           11.0928         0.851700

                degree of freedoms is: 17
-------------------------------------------------------------
        p-value for KStest on those 9 p-values: 0.915681


|-------------------------------------------------------------
|           THE OVERLAPPING 5-PERMUTATION TEST
|This is the OPERM5 test.  It looks at a sequence of one mill-
|ion 32-bit random integers.  Each set of five consecutive
|integers can be in one of 120 states, for the 5! possible or-
|derings of five numbers.  Thus the 5th, 6th, 7th,...numbers
|each provide a state. As many thousands of state transitions
|are observed,  cumulative counts are made of the number of
|occurences of each state.  Then the quadratic form in the
|weak inverse of the 120x120 covariance matrix yields a test
|equivalent to the likelihood ratio test that the 120 cell
|counts came from the specified (asymptotically) normal dis-
|tribution with the specified 120x120 covariance matrix (with
|rank 99).  This version uses 1,000,000 integers, twice.
|-------------------------------------------------------------
```

```
                  OPERM5 test for file
            (For samples of 1,000,000 consecutive 5-tuples)

                    sample 1
chisquare=94.907067 with df=99; p-value= 0.597706
```

```
                    sample 2
chisquare=60.297852 with df=99; p-value= 0.999246
```

```
|-------------------------------------------------------------
|This is the BINARY RANK TEST for 31x31 matrices. The leftmost
|31 bits of 31 random integers from the test sequence are used
|to form a 31x31 binary matrix over the field {0,1}. The rank
|is determined. That rank can be from 0 to 31, but ranks< 28
|are rare, and their counts are pooled with those for rank 28.
|Ranks are found for 40,000 such random matrices and a chisqu-
|are test is performed on counts for ranks 31,30,28 and <=28.
|-------------------------------------------------------------
        Rank test for binary matrices (31x31) from FourmilabHo

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=28   205             211.4           0.195           0.195
r=29    5155            5134.0          0.086           0.281
r=30    22974           23103.0         0.721           1.001
r=31    11666           11551.5         1.134           2.136

        chi-square = 2.136 with df = 3;  p-value = 0.545
--------------------------------------------------------------

|-------------------------------------------------------------
|This is the BINARY RANK TEST for 32x32 matrices. A random 32x
|32 binary matrix is formed, each row a 32-bit random integer.
|The rank is determined. That rank can be from 0 to 32, ranks
|less than 29 are rare, and their counts are pooled with those
|for rank 29.  Ranks are found for 40,000 such random matrices
|and a chisquare test is performed on counts for ranks  32,31,
|30 and <=29.
|-------------------------------------------------------------
        Rank test for binary matrices (32x32) from FourmilabHo

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=29   206             211.4           0.139           0.139
r=30    5042            5134.0          1.649           1.788
r=31    23133           23103.0         0.039           1.827
r=32    11619           11551.5         0.394           2.221

        chi-square = 2.221 with df = 3;  p-value = 0.528
--------------------------------------------------------------

|-------------------------------------------------------------
|This is the BINARY RANK TEST for 6x8 matrices.  From each of
|six random 32-bit integers from the generator under test, a
|specified byte is chosen, and the resulting six bytes form a
|6x8 binary matrix whose rank is determined.  That rank can be
```

```
|from 0 to 6, but ranks 0,1,2,3 are rare; their counts are
|pooled with those for rank 4. Ranks are found for 100,000
|random matrices, and a chi-square test is performed on
|counts for ranks 6,5 and (0,...,4) (pooled together).
|-------------------------------------------------------------

        Rank test for binary matrices (6x8) from FourmilabHotB

                      bits  1 to  8

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    924             944.3           0.436           0.436
r=5     21622           21743.9         0.683           1.120
r=6     77454           77311.8         0.262           1.381

        chi-square = 1.381 with df = 2;  p-value = 0.501
--------------------------------------------------------------

                      bits  2 to  9

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    936             944.3           0.073           0.073
r=5     21877           21743.9         0.815           0.888
r=6     77187           77311.8         0.201           1.089

        chi-square = 1.089 with df = 2;  p-value = 0.580
--------------------------------------------------------------

                      bits  3 to 10

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    955             944.3           0.121           0.121
r=5     21949           21743.9         1.935           2.056
r=6     77096           77311.8         0.602           2.658

        chi-square = 2.658 with df = 2;  p-value = 0.265
--------------------------------------------------------------

                      bits  4 to 11

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    964             944.3           0.411           0.411
r=5     21689           21743.9         0.139           0.550
r=6     77347           77311.8         0.016           0.566

        chi-square = 0.566 with df = 2;  p-value = 0.754
--------------------------------------------------------------

                      bits  5 to 12

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    945             944.3           0.001           0.001
r=5     21697           21743.9         0.101           0.102
r=6     77358           77311.8         0.028           0.129
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```
        chi-square = 0.129 with df = 2;  p-value = 0.937
--------------------------------------------------------------

                      bits  6 to 13

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    999             944.3           3.169           3.169
r=5     21929           21743.9         1.576           4.744
r=6     77072           77311.8         0.744           5.488

        chi-square = 5.488 with df = 2;  p-value = 0.064
--------------------------------------------------------------

                      bits  7 to 14

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    956             944.3           0.145           0.145
r=5     21790           21743.9         0.098           0.243
r=6     77254           77311.8         0.043           0.286

        chi-square = 0.286 with df = 2;  p-value = 0.867
--------------------------------------------------------------

                      bits  8 to 15

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    950             944.3           0.034           0.034
r=5     21435           21743.9         4.388           4.423
r=6     77615           77311.8         1.189           5.612

        chi-square = 5.612 with df = 2;  p-value = 0.060
--------------------------------------------------------------

                      bits  9 to 16

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    950             944.3           0.034           0.034
r=5     21739           21743.9         0.001           0.036
r=6     77311           77311.8         0.000           0.036

        chi-square = 0.036 with df = 2;  p-value = 0.982
--------------------------------------------------------------

                      bits 10 to 17

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    936             944.3           0.073           0.073
r=5     21681           21743.9         0.182           0.255
r=6     77383           77311.8         0.066           0.320

        chi-square = 0.320 with df = 2;  p-value = 0.852
--------------------------------------------------------------

                      bits 11 to 18
```

RANK	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	892	944.3	2.897	2.897
r=5	21752	21743.9	0.003	2.900
r=6	77356	77311.8	0.025	2.925

chi-square = 2.925 with df = 2; p-value = 0.232

---

bits 12 to 19

RANK	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	950	944.3	0.034	0.034
r=5	21614	21743.9	0.776	0.810
r=6	77436	77311.8	0.200	1.010

chi-square = 1.010 with df = 2; p-value = 0.604

---

bits 13 to 20

RANK	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	945	944.3	0.001	0.001
r=5	21658	21743.9	0.339	0.340
r=6	77397	77311.8	0.094	0.434

chi-square = 0.434 with df = 2; p-value = 0.805

---

bits 14 to 21

RANK	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	955	944.3	0.121	0.121
r=5	21683	21743.9	0.171	0.292
r=6	77362	77311.8	0.033	0.324

chi-square = 0.324 with df = 2; p-value = 0.850

---

bits 15 to 22

RANK	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	922	944.3	0.527	0.527
r=5	21719	21743.9	0.029	0.555
r=6	77359	77311.8	0.029	0.584

chi-square = 0.584 with df = 2; p-value = 0.747

---

bits 16 to 23

RANK	OBSERVED	EXPECTED	(O-E)^2/E	SUM
r<=4	904	944.3	1.720	1.720
r=5	21900	21743.9	1.121	2.841
r=6	77196	77311.8	0.173	3.014

```
        chi-square = 3.014 with df = 2;  p-value = 0.222
--------------------------------------------------------------

                      bits 17 to 24

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    924             944.3           0.436           0.436
r=5     21714           21743.9         0.041           0.478
r=6     77362           77311.8         0.033           0.510

        chi-square = 0.510 with df = 2;  p-value = 0.775
--------------------------------------------------------------

                      bits 18 to 25

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    935             944.3           0.092           0.092
r=5     21747           21743.9         0.000           0.092
r=6     77318           77311.8         0.000           0.093

        chi-square = 0.093 with df = 2;  p-value = 0.955
--------------------------------------------------------------

                      bits 19 to 26

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    950             944.3           0.034           0.034
r=5     21716           21743.9         0.036           0.070
r=6     77334           77311.8         0.006           0.077

        chi-square = 0.077 with df = 2;  p-value = 0.962
--------------------------------------------------------------

                      bits 20 to 27

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    923             944.3           0.480           0.480
r=5     21745           21743.9         0.000           0.481
r=6     77332           77311.8         0.005           0.486

        chi-square = 0.486 with df = 2;  p-value = 0.784
--------------------------------------------------------------

                      bits 21 to 28

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    889             944.3           3.238           3.238
r=5     21903           21743.9         1.164           4.403
r=6     77208           77311.8         0.139           4.542

        chi-square = 4.542 with df = 2;  p-value = 0.103
--------------------------------------------------------------

                      bits 22 to 29
```

```
RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    952             944.3           0.063           0.063
r=5     21753           21743.9         0.004           0.067
r=6     77295           77311.8         0.004           0.070

        chi-square = 0.070 with df = 2;  p-value = 0.965
--------------------------------------------------------------

                      bits 23 to 30

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    986             944.3           1.841           1.841
r=5     22062           21743.9         4.654           6.495
r=6     76952           77311.8         1.674           8.170

        chi-square = 8.170 with df = 2;  p-value = 0.017
--------------------------------------------------------------

                      bits 24 to 31

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    977             944.3           1.132           1.132
r=5     21823           21743.9         0.288           1.420
r=6     77200           77311.8         0.162           1.582

        chi-square = 1.582 with df = 2;  p-value = 0.453
--------------------------------------------------------------

                      bits 25 to 32

RANK    OBSERVED        EXPECTED        (O-E)^2/E       SUM

r<=4    968             944.3           0.595           0.595
r=5     21885           21743.9         0.916           1.510
r=6     77147           77311.8         0.351           1.862

        chi-square = 1.862 with df = 2;  p-value = 0.394
--------------------------------------------------------------
    TEST SUMMARY, 25 tests on 100,000 random 6x8 matrices
    These should be 25 uniform [0,1] random variates:

0.501240        0.580088        0.264713        0.753662
0.064310        0.866791        0.060452        0.982397
0.231666        0.603516        0.805025        0.850268
0.221576        0.774874        0.954789        0.962434
0.103210        0.965486        0.016827        0.453440
        The KS test for those 25 supposed UNI's yields
                KS p-value = 0.088531

|-------------------------------------------------------------
|                 THE BITSTREAM TEST
|The file under test is viewed as a stream of bits. Call them
|b1,b2,... .  Consider an alphabet with two "letters", 0 and 1
|and think of the stream of bits as a succession of 20-letter
|"words", overlapping.  Thus the first word is b1b2...b20, the
|second is b2b3...b21, and so on.  The bitstream test counts
```

```
|the number of missing 20-letter (20-bit) words in a string of
|2^21 overlapping 20-letter words.  There are 2^20 possible 20
|letter words.  For a truly random string of 2^21+19 bits, the
|number of missing words j should be (very close to) normally
|distributed with mean 141,909 and sigma 428.  Thus
| (j-141909)/428 should be a standard normal variate (z score)
|that leads to a uniform [0,1) p value.  The test is repeated
|twenty times.
|---------------------------------------------------------------

       THE OVERLAPPING 20-TUPLES BITSTREAM  TEST for Fourmila
 (20 bits/word, 2097152 words 20 bitstreams. No. missing words
  should average 141909.33 with sigma=428.00.)
---------------------------------------------------------------
          BITSTREAM test results for FourmilabHotBits.32.

Bitstream       No. missing words       z-score          p-valu
   1            142235                   0.76            0.2233
   2            141920                   0.02            0.4900
   3            142064                   0.36            0.3589
   4            142338                   1.00            0.1582
   5            141548                  -0.84            0.8007
   6            141988                   0.18            0.4270
   7            141888                  -0.05            0.5198
   8            141930                   0.05            0.4807
   9            141610                  -0.70            0.7578
   10           142059                   0.35            0.3632
   11           142442                   1.24            0.1066
   12           142265                   0.83            0.2029
   13           142704                   1.86            0.0316
   14           141658                  -0.59            0.7214
   15           141818                  -0.21            0.5844
   16           141810                  -0.23            0.5917
   17           142139                   0.54            0.2957
   18           142066                   0.37            0.3571
   19           141959                   0.12            0.4538
   20           141860                  -0.12            0.5458
---------------------------------------------------------------

|---------------------------------------------------------------
|        OPSO means Overlapping-Pairs-Sparse-Occupancy
|The OPSO test considers 2-letter words from an alphabet of
|1024 letters.  Each letter is determined by a specified ten
|bits from a 32-bit integer in the sequence to be tested. OPSO
|generates  2^21 (overlapping) 2-letter words  (from 2^21+1
|"keystrokes")  and counts the number of missing words---that
|is 2-letter words which do not appear in the entire sequence.
|That count should be very close to normally distributed with
|mean 141,909, sigma 290. Thus (missingwrds-141909)/290 should
|be a standard normal variable. The OPSO test takes 32 bits at
|a time from the test file and uses a designated set of ten
|consecutive bits. It then restarts the file for the next de-
|signated 10 bits, and so on.
|--------------------------------------------------------------

                   OPSO test for file FourmilabHotBits.32

Bits used       No. missing words       z-score          p-valu
23 to 32                141580          -1.1356          0.8719
22 to 31                141827          -0.2839          0.6117
```

21 to 30	141592	-1.0942	0.8630
20 to 29	141567	-1.1804	0.8810
19 to 28	141530	-1.3080	0.9045
18 to 27	141664	-0.8460	0.8012
17 to 26	141445	-1.6011	0.9453
16 to 25	141851	-0.2011	0.5797
15 to 24	141327	-2.0080	0.9776
14 to 23	141867	-0.1460	0.5580
13 to 22	141517	-1.3529	0.9119
12 to 21	141589	-1.1046	0.8653
11 to 20	141630	-0.9632	0.8322
10 to 19	141897	-0.0425	0.5169
9 to 18	141473	-1.5046	0.9337
8 to 17	141741	-0.5804	0.7191
7 to 16	141697	-0.7322	0.7679
6 to 15	142348	1.5127	0.0651
5 to 14	141311	-2.0632	0.9804
4 to 13	141515	-1.3598	0.9130
3 to 12	141648	-0.9011	0.8162
2 to 11	141756	-0.5287	0.7015
1 to 10	141559	-1.2080	0.8864

```
-----------------------------------------------------------------

|-------------------------------------------------------------
|    OQSO means Overlapping-Quadruples-Sparse-Occupancy
|  The test OQSO is similar, except that it considers 4-letter
|words from an alphabet of 32 letters, each letter determined
|by a designated string of 5 consecutive bits from the test
|file, elements of which are assumed 32-bit random integers.
|The mean number of missing words in a sequence of 2^21 four-
|letter words,  (2^21+3 "keystrokes"), is again 141909, with
|sigma = 295.  The mean is based on theory; sigma comes from
|extensive simulation.
|-------------------------------------------------------------
```

OQSO test for file FourmilabHotBits.32

Bits used	No. missing words	z-score	p-valu
28 to 32	141900	-0.0316	0.5126
27 to 31	141966	0.1921	0.4238
26 to 30	141500	-1.3876	0.9173
25 to 29	141357	-1.8723	0.9694
24 to 28	142099	0.6429	0.2601
23 to 27	141732	-0.6011	0.7261
22 to 26	141330	-1.9638	0.9752
21 to 25	141966	0.1921	0.4238
20 to 24	141817	-0.3130	0.6228
19 to 23	142012	0.3480	0.3639
18 to 22	142448	1.8260	0.0339
17 to 21	142369	1.5582	0.0595
16 to 20	141741	-0.5706	0.7158
15 to 19	141902	-0.0248	0.5099
14 to 18	142162	0.8565	0.1958
13 to 17	141921	0.0396	0.4842
12 to 16	142013	0.3514	0.3626
11 to 15	141955	0.1548	0.4384
10 to 14	141800	-0.3706	0.6445
9 to 13	141915	0.0192	0.4923
8 to 12	142330	1.4260	0.0769
7 to 11	142155	0.8328	0.2024

```
6 to 10                  142132           0.7548         0.2251
5 to 9                   142252           1.1616         0.1227
4 to 8                   142121           0.7175         0.2365
3 to 7                   142450           1.8328         0.0334
2 to 6                   141697          -0.7198         0.7641
1 to 5                   142027           0.3989         0.3449
-----------------------------------------------------------------

|-------------------------------------------------------------
|    The DNA test considers an alphabet of 4 letters: C,G,A,T,
|determined by two designated bits in the sequence of random
|integers being tested.  It considers 10-letter words, so that
|as in OPSO and OQSO, there are 2^20 possible words, and the
|mean number of missing words from a string of 2^21  (over-
|lapping)  10-letter  words (2^21+9 "keystrokes") is 141909.
|The standard deviation sigma=339 was determined as for OQSO
|by simulation.  (Sigma for OPSO, 290, is the true value (to
|three places), not determined by simulation.
|-------------------------------------------------------------

                   DNA test for file FourmilabHotBits.32

Bits used       No. missing words       z-score          p-valu
31 to 32                 142299           1.1495         0.1251
30 to 31                 141702          -0.6116         0.7295
29 to 30                 141705          -0.6027         0.7266
28 to 29                 141910           0.0020         0.4992
27 to 28                 141889          -0.0600         0.5239
26 to 27                 141104          -2.3756         0.9912
25 to 26                 142020           0.3265         0.3720
24 to 25                 141725          -0.5437         0.7066
23 to 24                 141376          -1.5732         0.9421
22 to 23                 141444          -1.3727         0.9150
21 to 22                 142121           0.6244         0.2661
20 to 21                 141715          -0.5732         0.7167
19 to 20                 141282          -1.8505         0.9678
18 to 19                 141857          -0.1544         0.5613
17 to 18                 141921           0.0344         0.4862
16 to 17                 142055           0.4297         0.3337
15 to 16                 142639           2.1524         0.0156
14 to 15                 141865          -0.1308         0.5520
13 to 14                 142382           1.3943         0.0816
12 to 13                 141725          -0.5437         0.7066
11 to 12                 141746          -0.4818         0.6850
10 to 11                 141594          -0.9302         0.8238
9 to 10                  142113           0.6008         0.2739
8 to 9                   142013           0.3058         0.3798
7 to 8                   141896          -0.0393         0.5156
6 to 7                   141759          -0.4435         0.6712
5 to 6                   141432          -1.4081         0.9204
4 to 5                   141645          -0.7797         0.7822
3 to 4                   142413           1.4858         0.0686
2 to 3                   142099           0.5595         0.2879
1 to 2                   141658          -0.7414         0.7707
-----------------------------------------------------------------

|-------------------------------------------------------------
|    This is the COUNT-THE-1''s TEST on a stream of bytes.
|Consider the file under test as a stream of bytes (four per
|32 bit integer).  Each byte can contain from 0 to 8 1''s,
```

```
|with probabilities 1,8,28,56,70,56,28,8,1 over 256.  Now let
|the stream of bytes provide a string of overlapping  5-letter
|words, each "letter" taking values A,B,C,D,E. The letters are
|determined by the number of 1''s in a byte: 0,1,or 2 yield A,
|3 yields B, 4 yields C, 5 yields D and 6,7 or 8 yield E. Thus
|we have a monkey at a typewriter hitting five keys with vari-
|ous probabilities (37,56,70,56,37 over 256).  There are 5^5
|possible 5-letter words, and from a string of 256,000 (over-
|lapping) 5-letter words, counts are made on the frequencies
|for each word.   The quadratic form in the weak inverse of
|the covariance matrix of the cell counts provides a chisquare
|test: Q5-Q4, the difference of the naive Pearson sums of
|(OBS-EXP)^2/EXP on counts for 5- and 4-letter cell counts.
|-------------------------------------------------------------

       Test result for the byte stream from FourmilabHotBits.
  (Degrees of freedom: 5^4-5^3=2500; sample size: 2560000)

               chisquare       z-score         p-value
               2493.54         -0.091          0.536401

|-------------------------------------------------------------
|    This is the COUNT-THE-1''s TEST for specific bytes.
|Consider the file under test as a stream of 32-bit integers.
|From each integer, a specific byte is chosen , say the left-
|most: bits 1 to 8. Each byte can contain from 0 to 8 1''s,
|with probabilitie 1,8,28,56,70,56,28,8,1 over 256.  Now let
|the specified bytes from successive integers provide a string
|of (overlapping) 5-letter words, each "letter" taking values
|A,B,C,D,E. The letters are determined  by the number of 1''s,
|in that byte: 0,1,or 2 ---> A, 3 ---> B, 4 ---> C, 5 ---> D,
|and  6,7 or 8 ---> E.  Thus we have a monkey at a typewriter
|hitting five keys with with various probabilities: 37,56,70,
|56,37 over 256. There are 5^5 possible 5-letter words, and
|from a string of 256,000 (overlapping) 5-letter words, counts
|are made on the frequencies for each word. The quadratic form
|in the weak inverse of the covariance matrix of the cell
|counts provides a chisquare test: Q5-Q4, the difference of
|the naive Pearson  sums of (OBS-EXP)^2/EXP on counts for 5-
|and 4-letter cell  counts.
|-------------------------------------------------------------

       Test results for specific bytes from FourmilabHotBits.
  (Degrees of freedom: 5^4-5^3=2500; sample size: 256000)

bits used      chisquare       z-score         p-value
1 to 8         2370.17         -1.836          0.966830
2 to 9         2590.32          1.277          0.100746
3 to 10        2513.16          0.186          0.426206
4 to 11        2452.96         -0.665          0.747062
5 to 12        2598.68          1.396          0.081421
6 to 13        2462.08         -0.536          0.704108
7 to 14        2423.88         -1.076          0.859147
8 to 15        2596.54          1.365          0.086087
9 to 16        2501.68          0.024          0.490549
10 to 17       2475.61         -0.345          0.634949
11 to 18       2535.65          0.504          0.307060
12 to 19       2467.15         -0.465          0.678858
13 to 20       2484.22         -0.223          0.588298
14 to 21       2606.47          1.506          0.066070
```

```
15 to 22        2458.25        -0.590          0.722563
16 to 23        2546.29         0.655          0.256356
17 to 24        2531.40         0.444          0.328502
18 to 25        2519.09         0.270          0.393567
19 to 26        2526.07         0.369          0.356168
20 to 27        2595.14         1.346          0.089231
21 to 28        2443.93        -0.793          0.786095
22 to 29        2469.71        -0.428          0.665820
23 to 30        2485.01        -0.212          0.583966
24 to 31        2518.65         0.264          0.395996
25 to 32        2530.07         0.425          0.335318
|--------------------------------------------------------------
|              THIS IS A PARKING LOT TEST
|In a square of side 100, randomly "park" a car---a circle of
|radius 1.   Then try to park a 2nd, a 3rd, and so on, each
|time parking "by ear".  That is, if an attempt to park a car
|causes a crash with one already parked, try again at a new
|random location. (To avoid path problems, consider parking
|helicopters rather than cars.)   Each attempt leads to either
|a crash or a success, the latter followed by an increment to
|the list of cars already parked. If we plot n: the number of
|attempts, versus k: the number successfully parked, we get a
|curve that should be similar to those provided by a perfect
|random number generator.  Theory for the behavior of such a
|random curve seems beyond reach, and as graphics displays are
|not available for this battery of tests, a simple characteriz
|ation of the random experiment is used: k, the number of cars
|successfully parked after n=12,000 attempts. Simulation shows
|that k should average 3523 with sigma 21.9 and is very close
|to normally distributed.  Thus (k-3523)/21.9 should be a st-
|andard normal variable, which, converted to a uniform varia-
|ble, provides input to a KSTEST based on a sample of 10.
|--------------------------------------------------------------

        CDPARK: result of 10 tests on file FourmilabHotBits.32
  (Of 12000 tries, the average no. of successes should be
   3523.0 with sigma=21.9)

   No. succeses         z-score         p-value
        3543             0.9132         0.180558
        3506            -0.7763         0.781201
        3533             0.4566         0.323972
        3524             0.0457         0.481790
        3526             0.1370         0.445521
        3510            -0.5936         0.723613
        3543             0.9132         0.180558
        3532             0.4110         0.340551
        3533             0.4566         0.323972
        3533             0.4566         0.323972
  Square side=100, avg. no. parked=3528.30 sample std.=11.70
     p-value of the KSTEST for those 10 p-values: 0.002300


|--------------------------------------------------------------
|              THE MINIMUM DISTANCE TEST
|It does this 100 times:  choose n=8000 random points in a
|square of side 10000.  Find d, the minimum distance between
|the (n^2-n)/2 pairs of points.  If the points are truly inde-
|pendent uniform, then d^2, the square of the minimum distance
|should be (very close to) exponentially distributed with mean
```

```
|.995 .  Thus 1-exp(-d^2/.995) should be uniform on [0,1) and
|a KSTEST on the resulting 100 values serves as a test of uni-
|formity for random points in the square. Test numbers=0 mod 5
|are printed but the KSTEST is based on the full set of 100
|random choices of 8000 points in the 10000x10000 square.
|---------------------------------------------------------------

        This is the MINIMUM DISTANCE test for file FourmilabHo

Sample no.       d^2              mean             equiv uni
   5            0.0066           0.7395            0.006640
   10           3.0735           1.2046            0.954449
   15           0.1273           0.9023            0.120123
   20           5.4762           1.1602            0.995928
   25           0.9729           1.2054            0.623852
   30           0.7707           1.1989            0.539084
   35           0.0274           1.2205            0.027205
   40           0.0148           1.1269            0.014762
   45           1.8536           1.1399            0.844779
   50           0.3695           1.1442            0.310181
   55           0.4188           1.1592            0.343554
   60           0.2675           1.1129            0.235763
   65           3.1837           1.1337            0.959224
   70           2.3059           1.1826            0.901478
   75           1.0426           1.1678            0.649299
   80           0.0274           1.1610            0.027178
   85           3.4117           1.1430            0.967576
   90           3.3879           1.1558            0.966792
   95           0.4765           1.1227            0.380508
   100          1.1551           1.1143            0.686808
---------------------------------------------------------------
Result of KS test on 100 transformed mindist^2's: p-value=0.29


|---------------------------------------------------------------
|             THE 3DSPHERES TEST
|Choose  4000 random points in a cube of edge 1000.  At each
|point, center a sphere large enough to reach the next closest
|point. Then the volume of the smallest such sphere is (very
|close to) exponentially distributed with mean 120pi/3.  Thus
|the radius cubed is exponential with mean 30. (The mean is
|obtained by extensive simulation).  The 3DSPHERES test gener-
|ates 4000 such spheres 20 times.  Each min radius cubed leads
|to a uniform variable by means of 1-exp(-r^3/30.), then a
| KSTEST is done on the 20 p-values.
|---------------------------------------------------------------

            The 3DSPHERES test for file FourmilabHotBits.32

        sample no       r^3             equiv. uni.
           1            95.178          0.958106
           2            102.500         0.967178
           3            0.896           0.029438
           4            47.776          0.796592
           5            31.541          0.650536
           6            20.136          0.488897
           7            20.419          0.493707
           8            7.454           0.220000
           9            11.429          0.316806
           10           32.907          0.666098
```

```
     11           10.626          0.298265
     12           38.004          0.718272
     13           6.657           0.199018
     14           11.230          0.312259
     15           47.004          0.791291
     16           1.492           0.048505
     17           0.666           0.021962
     18           20.268          0.491147
     19           0.664           0.021891
     20           42.920          0.760853
--------------------------------------------------------------
        p-value for KS test on those 20 p-values: 0.539573


|-------------------------------------------------------------
|                 This is the SQUEEZE test
| Random integers are floated to get uniforms on [0,1). Start-
| ing with k=2^31=2147483647, the test finds j, the number of
| iterations necessary to reduce k to 1, using the reduction
| k=ceiling(k*U), with U provided by floating integers from
| the file being tested.  Such j''s are found 100,000 times,
| then counts for the number of times j was <=6,7,...,47,>=48
| are used to provide a chi-square test for cell frequencies.
|-------------------------------------------------------------

                RESULTS OF SQUEEZE TEST FOR FourmilabHotBits.3

            Table of standardized frequency counts
        (obs-exp)^2/exp  for j=(1,..,6), 7,...,47,(48,...)
        -0.1     0.5    -1.1     0.5    -1.3     0.4
        -1.4    -1.2     0.6    -0.9    -1.2    -1.9
         0.7    -1.8     1.7    -2.2     0.6     1.3
         0.8    -0.8     0.1     0.1     0.1     0.6
         0.6     0.4     3.1    -1.7     1.0     0.0
         1.4     1.0     0.2    -0.2     0.9    -1.6
         1.2    -0.1     0.1     0.4     1.6     0.0
         1.8
        Chi-square with 42 degrees of freedom:55.429856
        z-score=1.465317, p-value=0.080144
```

```
|-------------------------------------------------------------
|            The  OVERLAPPING SUMS test
|Integers are floated to get a sequence U(1),U(2),... of uni-
|form [0,1) variables.  Then overlapping sums,
|  S(1)=U(1)+...+U(100), S2=U(2)+...+U(101),... are formed.
|The S''s are virtually normal with a certain covariance mat-
|rix.  A linear transformation of the S''s converts them to a
|sequence of independent standard normals, which are converted
|to uniform variables for a KSTEST.
|-------------------------------------------------------------

                Results of the OSUM test for FourmilabHotBits.

                Test no                 p-value
                  1                     0.222186
                  2                     0.841877
                  3                     0.064437
                  4                     0.936267
```

```
                      5                     0.676322
                      6                     0.305676
                      7                     0.679858
                      8                     0.181487
                      9                     0.992939
                      10                    0.653249
```

```
        p-value for 10 kstests on 100 kstests:0.642506

|-------------------------------------------------------------
|    This is the RUNS test.  It counts runs up, and runs down,
|in a sequence of uniform [0,1) variables, obtained by float-
|ing the 32-bit integers in the specified file. This example
|shows how runs are counted: .123,.357,.789,.425,.224,.416,.95
|contains an up-run of length 3, a down-run of length 2 and an
|up-run of (at least) 2, depending on the next values.  The
|covariance matrices for the runs-up and runs-down are well
|known, leading to chisquare tests for quadratic forms in the
|weak inverses of the covariance matrices.  Runs are counted
|for sequences of length 10,000.  This is done ten times. Then
|another three sets of ten.
|-------------------------------------------------------------

                The RUNS test for file FourmilabHotBits.32
        (Up and down runs in a sequence of 10000 numbers)
                        Set 1
         runs up; ks test for 10 p's: 0.005535
         runs down; ks test for 10 p's: 0.144368
                        Set 2
         runs up; ks test for 10 p's: 0.157631
         runs down; ks test for 10 p's: 0.668033

|-------------------------------------------------------------
|This the CRAPS TEST.  It plays 200,000 games of craps, counts
|the number of wins and the number of throws necessary to end
|each game.  The number of wins should be (very close to) a
|normal with mean 200000p and variance 200000p(1-p), and
|p=244/495.  Throws necessary to complete the game can vary
|from 1 to infinity, but counts for all>21 are lumped with 21.
|A chi-square test is made on the no.-of-throws cell counts.
|Each 32-bit integer from the test file provides the value for
|the throw of a die, by floating to [0,1), multiplying by 6
|and taking 1 plus the integer part of the result.
|-------------------------------------------------------------

        RESULTS OF CRAPS TEST FOR FourmilabHotBits.32
No. of wins:  Observed  Expected
                 98599       98585.858586
        z-score= 0.059, pvalue=0.47657

Analysis of Throws-per-Game:

Throws  Observed        Expected        Chisq    Sum of (O-E)^
1       66719           66666.7         0.041           0.041
2       37257           37654.3         4.192           4.234
3       27201           26954.7         2.250           6.484
4       19333           19313.5         0.020           6.503
5       13682           13851.4         2.072           8.575
6       10159           9943.5          4.669          13.244
```

7	7134	7145.0	0.017	13.261
8	5036	5139.1	2.067	15.328
9	3669	3699.9	0.257	15.586
10	2741	2666.3	2.093	17.679
11	1948	1923.3	0.316	17.995
12	1423	1388.7	0.845	18.840
13	1048	1003.7	1.954	20.794
14	720	726.1	0.052	20.846
15	518	525.8	0.117	20.963
16	386	381.2	0.062	21.025
17	281	276.5	0.072	21.097
18	201	200.8	0.000	21.097
19	151	146.0	0.172	21.269
20	104	106.2	0.046	21.315
21	289	287.1	0.012	21.328

```
Chisq=  21.33 for 20 degrees of freedom, p= 0.37808

        SUMMARY of craptest on FourmilabHotBits.32
 p-value for no. of wins: 0.476565
 p-value for throws/game: 0.378076
```

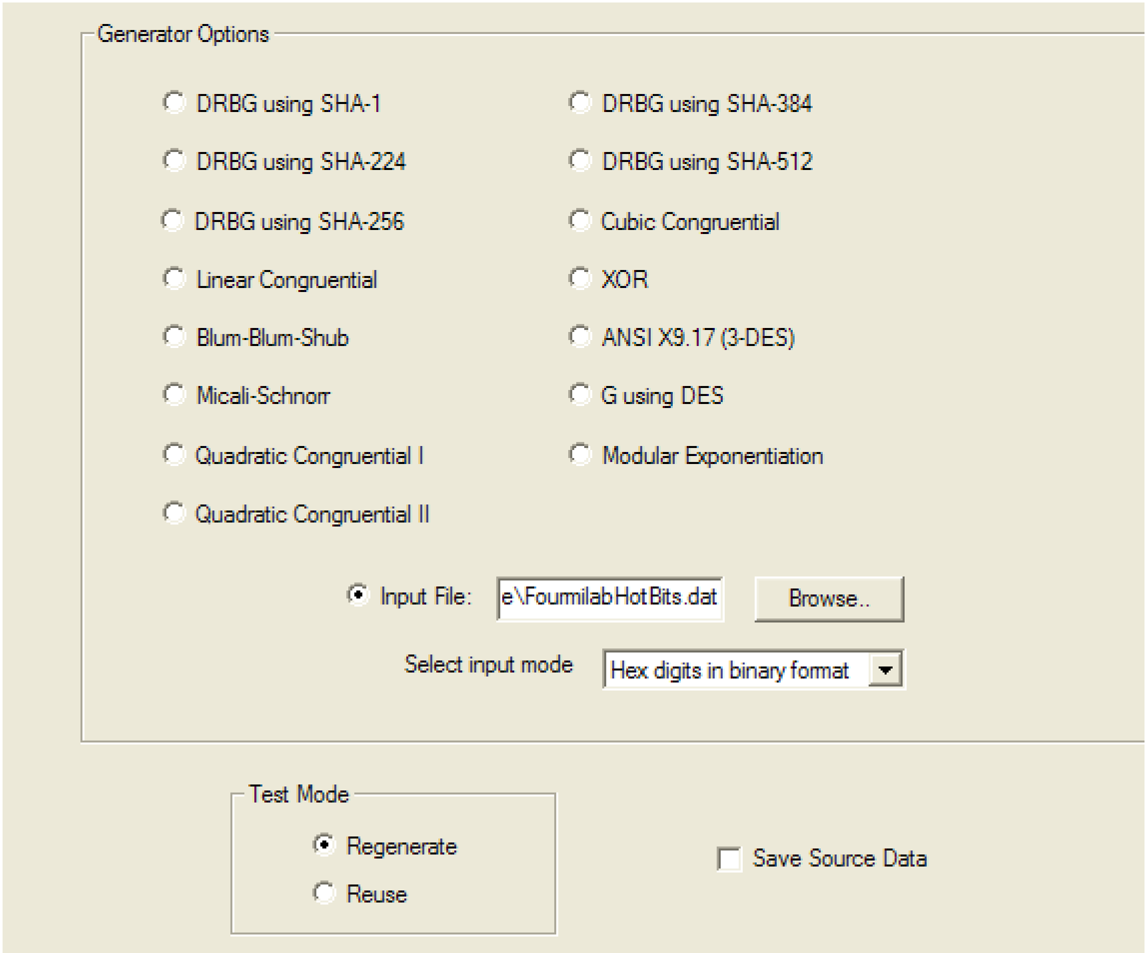
- Download Diehard test results (`diehard_results.zip` 13 Kb)
- Download data set for ENT and Diehard tests (`FourmilabHotBits_Diehard.zip` 11 Mb)

Tests with the NIST Statistical Test Suite (SP 800-22)

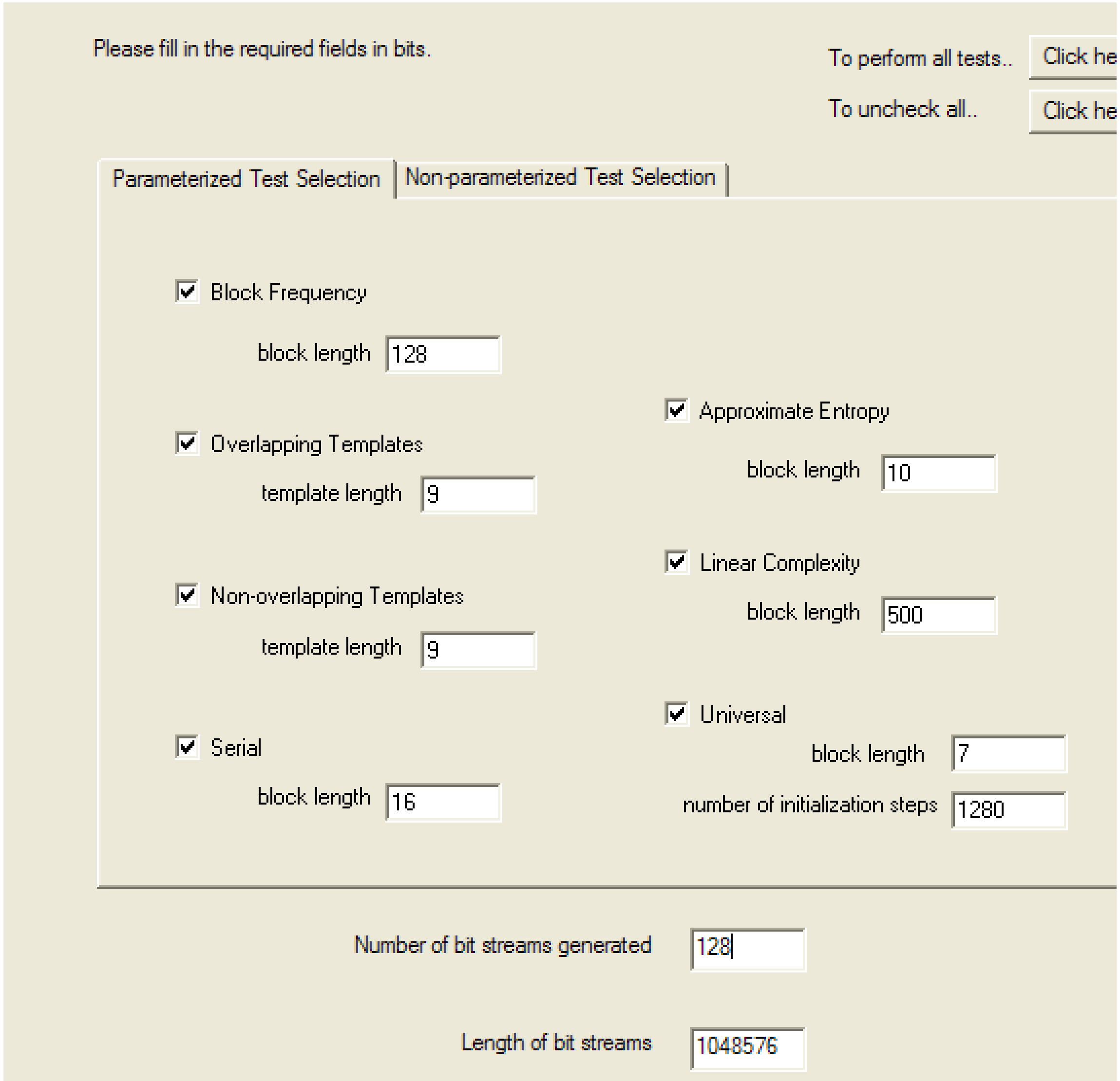
The following results were produced by testing a sequence of 16,779,776 bytes from the HotBits generator with version 1.8 of the U.S. National Institute of Standards and Technology Statistical Test Suite for Random and Pseudorandom Number Generators for Cryptographic Applications as described in NIST Special Publication 800-22 [PDF]. This test suite is supplied as source and executable binaries compiled with Microsoft Visual C++. In testing the program, I encountered numerous crashes due to buffer overflows in editing messages. I corrected these in the source code, rebuilt with Visual C++ .NET, and used my patched version for the tests. Since the executable supplied by NIST crashed on my machine, even running the examples, I had no option but to rebuild it. If you're interested in building your own version that doesn't crash, you can download my patches to the source code.

The data set used in the following tests is identical in the first 11,468,800 bytes to that used in the ENT and Diehard tests above. This is the complete data set I generated for testing; due to the sensitivity of the Diehard tests to the data set size, I limited the data I tested to be the same as that used in the Diehard examples and tested that data set with ENT. The NIST tests seem to have no problems with large data sets, so I used the complete data set in these tests.

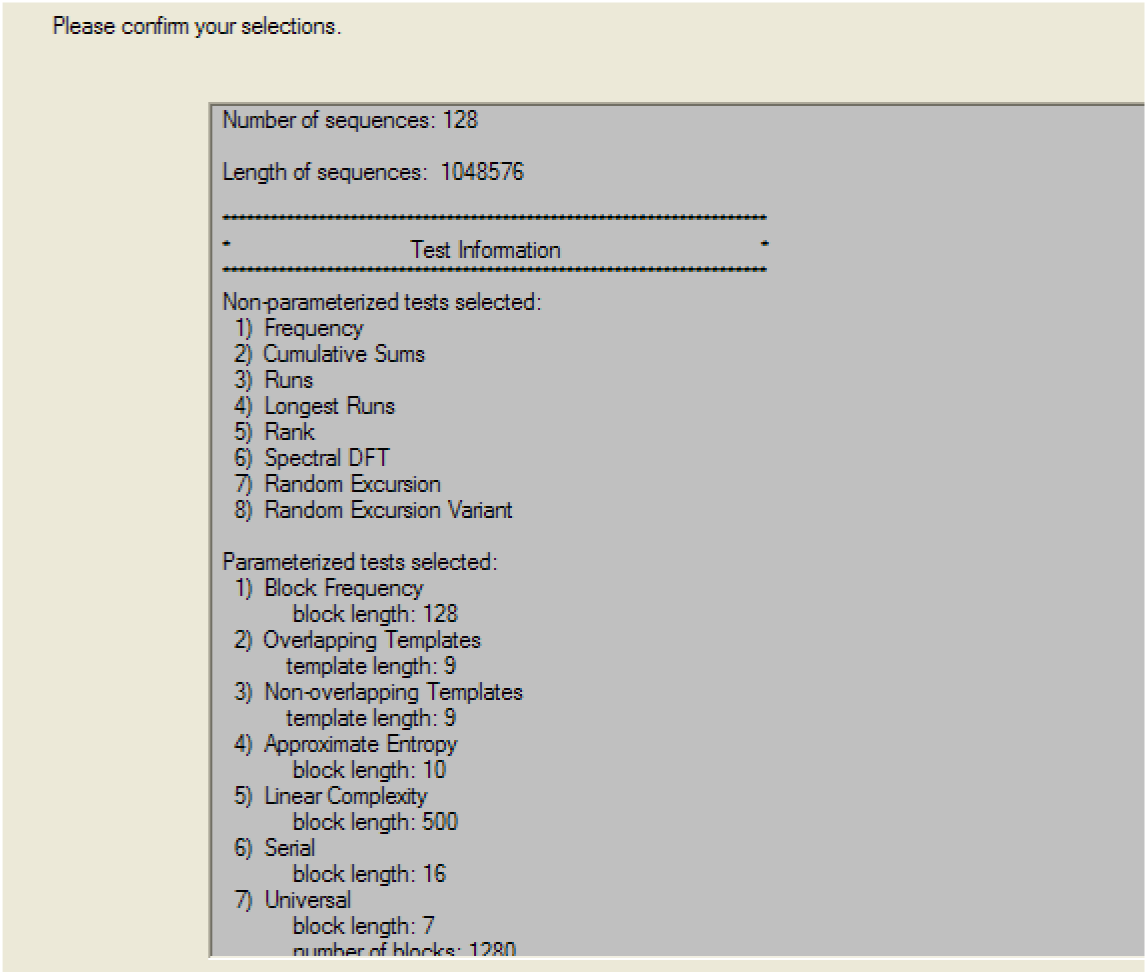
Version 1.8 of the NIST test suite has a rudimentary graphical user interface which runs only under Microsoft Windows. I performed these tests on my development machine, on which I booted Windows XP SP2. The test suite was compiled on a Windows 2000 machine using Microsoft Visual C++ .NET. The first configuration page of the test suite selects the data to be tested. You can choose from a variety of pseudorandom generators, data from a binary file, or a text file containing a sequence of bits represented as ASCII `0` and `1` characters.



The next page selects the tests to be performed and specifies the parameters to those tests which require them. I enabled all tests and used the parameters from the examples in the paper documenting the tests. Unless you really understand what you're doing, have studied the math underlying the test, and have read the code that implements it, it's a bad idea to get too creative changing these parameters. The LavaRnd people, for example, chose a block length of 14 for the Approximate Entropy test and discovered that with that setting even the “gold standard” Blum-Blum-Shub pseudorandom generator failed this test.



The tests selected and parameters are then displayed on confirmation page, and if accepted the tests begin. On my 3.4 GHz Pentium 4 machine, they run for a couple of hours.



Please confirm your selections.

Number of sequences: 128

Length of sequences: 1048576

****
* Test Information *
****

Non-parameterized tests selected:
1) Frequency
2) Cumulative Sums
3) Runs
4) Longest Runs
5) Rank
6) Spectral DFT
7) Random Excursion
8) Random Excursion Variant

Parameterized tests selected:
1) Block Frequency
block length: 128
2) Overlapping Templates
template length: 9
3) Non-overlapping Templates
template length: 9
4) Approximate Entropy
block length: 10
5) Linear Complexity
block length: 500
6) Serial
block length: 16
7) Universal
block length: 7
number of blocks: 1280

After all the tests have completed, a summary report is written, which is reproduced below. The key values to look at for each test are the "P-value": the probability the results obtained were due to chance, and the "Proportion" of sequences generated which were deemed to pass the test. Extremal P-values (very close to zero or one), and Proportions which are significantly less than one are indicative of potential problems

RESULTS FOR THE UNIFORMITY OF P-VALUES AND THE PROPORTION OF PASSING S

C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	P-VALUE	PROPORTION	STATISTI
15	15	14	16	9	10	8	10	9	22	0.090936	0.9766	frequenc
15	12	12	15	19	7	8	14	10	16	0.275709	1.0000	block-fr
14	16	17	14	8	11	17	6	10	15	0.232760	0.9766	cumulati
15	17	15	17	11	6	20	7	8	12	0.041438	0.9766	cumulati
28	17	8	16	9	8	11	12	10	9	0.000569	0.9609	runs
14	13	12	13	6	14	16	17	8	15	0.407091	1.0000	longest-
13	8	11	14	14	12	18	15	12	11	0.739918	0.9844	rank
10	17	18	10	13	12	14	10	12	12	0.689019	0.9922	fft
15	24	14	13	15	10	10	8	9	10	0.048716	0.9844	nonperio
15	12	17	6	12	13	14	15	12	12	0.637119	0.9766	nonperio

15	11	15	12	15	15	9	11	11	14	0.888137	0.9922	nonperio
4	7	14	14	10	15	13	12	23	16	0.014216	1.0000	nonperio
17	13	13	7	10	11	17	15	11	14	0.534146	0.9922	nonperio
14	14	10	17	16	9	13	11	9	15	0.654467	0.9922	nonperio
8	11	21	9	7	10	13	15	18	16	0.057146	1.0000	nonperio
12	12	16	14	15	14	14	10	8	13	0.848588	0.9922	nonperio
10	15	10	10	15	18	16	9	15	10	0.468595	0.9844	nonperio
14	16	13	10	18	11	7	16	7	16	0.213309	0.9844	nonperio
17	16	15	12	8	9	14	11	11	15	0.585209	0.9922	nonperio
9	15	9	8	7	13	19	11	17	20	0.043745	1.0000	nonperio
13	11	11	12	13	13	10	17	14	14	0.941144	1.0000	nonperio
14	15	11	13	14	11	10	14	13	13	0.980883	0.9922	nonperio
10	15	12	6	10	18	12	17	16	12	0.287306	1.0000	nonperio
20	14	10	8	15	10	11	11	11	18	0.232760	0.9766	nonperio
15	10	17	9	14	15	13	15	9	11	0.671779	0.9844	nonperio
12	13	12	11	14	14	13	15	15	9	0.957319	0.9844	nonperio
10	15	15	12	14	8	13	15	13	13	0.875539	0.9766	nonperio
14	10	17	15	7	16	8	11	14	16	0.350485	0.9844	nonperio
20	12	10	9	14	14	18	11	12	8	0.242986	0.9609	nonperio
14	11	15	9	14	17	9	14	9	16	0.585209	0.9766	nonperio
17	18	11	9	10	12	11	6	13	21	0.063482	0.9922	nonperio
13	10	9	15	15	21	8	7	18	12	0.070445	1.0000	nonperio
11	8	18	15	10	6	11	18	15	16	0.134686	1.0000	nonperio
15	14	17	12	9	12	9	10	14	16	0.671779	0.9766	nonperio
17	10	18	11	16	10	10	11	9	16	0.378138	0.9844	nonperio
15	12	15	14	17	12	8	9	12	14	0.706149	1.0000	nonperio
12	10	14	12	14	8	16	13	12	17	0.756476	0.9922	nonperio
13	11	15	13	8	15	10	12	12	19	0.585209	1.0000	nonperio
13	14	14	16	11	12	16	10	11	11	0.911413	0.9766	nonperio
14	12	14	11	7	13	18	11	16	12	0.602458	0.9922	nonperio
13	11	12	15	7	11	11	16	17	15	0.602458	0.9688	nonperio
14	6	12	16	16	14	15	13	12	10	0.585209	1.0000	nonperio
12	9	10	10	14	14	15	10	15	19	0.534146	1.0000	nonperio
6	15	11	17	19	14	11	10	13	12	0.287306	0.9922	nonperio
12	16	9	13	13	14	11	11	13	16	0.900104	1.0000	nonperio
8	10	14	10	15	8	17	25	13	8	0.008879	0.9922	nonperio
9	6	25	8	19	8	14	12	15	12	0.002316	0.9844	nonperio
16	11	13	14	14	14	9	7	17	13	0.585209	0.9844	nonperio
16	10	13	15	7	14	14	13	8	18	0.378138	0.9844	nonperio
10	19	14	13	12	11	12	13	7	17	0.422034	0.9922	nonperio
17	10	9	13	11	10	13	14	16	15	0.723129	1.0000	nonperio
13	10	12	19	13	20	9	9	12	11	0.242986	0.9922	nonperio
6	18	11	11	11	11	17	19	12	12	0.186566	1.0000	nonperio
12	12	18	14	9	11	15	15	8	14	0.602458	0.9922	nonperio
23	10	12	10	12	12	11	17	8	13	0.110952	0.9766	nonperio
13	15	11	15	7	15	9	12	24	7	0.022503	0.9766	nonperio
8	8	13	17	16	10	15	9	18	14	0.253551	0.9922	nonperio
8	10	10	13	11	18	10	18	17	13	0.299251	1.0000	nonperio
13	10	13	15	13	10	14	17	9	14	0.819544	0.9922	nonperio
18	11	7	12	12	14	14	17	10	13	0.500934	0.9766	nonperio
16	16	14	11	16	8	12	14	11	10	0.689019	0.9844	nonperio
11	19	14	14	14	15	9	7	15	10	0.364146	0.9922	nonperio
18	15	13	14	6	10	16	16	12	8	0.242986	0.9922	nonperio
11	15	13	14	17	17	7	12	12	10	0.551026	1.0000	nonperio
12	17	11	14	17	12	7	12	14	12	0.637119	0.9844	nonperio
9	13	13	5	10	12	15	20	12	19	0.078086	0.9922	nonperio
11	11	11	17	18	16	15	6	9	14	0.242986	1.0000	nonperio
14	13	8	15	10	15	9	14	13	17	0.654467	0.9922	nonperio
18	7	12	15	7	16	15	13	10	15	0.264458	0.9844	nonperio
3	13	15	17	13	16	20	5	17	9	0.005490	1.0000	nonperio

10	16	15	13	10	19	13	8	12	12	0.500934	0.9844	nonperio
16	16	10	12	7	12	7	16	14	18	0.222869	0.9844	nonperio
12	6	15	20	17	12	7	16	11	12	0.100508	0.9844	nonperio
12	15	10	14	12	15	14	14	13	9	0.931952	0.9766	nonperio
6	10	11	17	14	12	13	11	20	14	0.232760	0.9922	nonperio
11	20	13	9	16	8	13	11	10	17	0.242986	1.0000	nonperio
18	8	5	12	17	16	18	9	15	10	0.054199	1.0000	nonperio
11	13	10	17	14	7	15	14	15	12	0.654467	0.9922	nonperio
14	10	9	13	9	8	20	17	14	14	0.232760	0.9766	nonperio
12	10	16	10	11	12	14	12	12	19	0.689019	0.9844	nonperio
10	13	13	12	16	12	15	10	11	16	0.888137	0.9844	nonperio
13	10	14	14	18	17	12	12	6	12	0.422034	0.9922	nonperio
15	24	14	13	15	10	10	8	9	10	0.048716	0.9844	nonperio
14	10	13	16	11	15	11	6	15	17	0.452799	0.9766	nonperio
8	14	9	13	8	14	21	13	13	15	0.222869	1.0000	nonperio
17	16	9	14	11	11	14	10	12	14	0.772760	0.9688	nonperio
11	12	11	10	14	11	15	19	15	10	0.654467	1.0000	nonperio
19	7	8	12	11	14	20	11	13	13	0.141256	0.9844	nonperio
12	15	19	15	8	8	13	12	18	8	0.178278	0.9844	nonperio
8	16	11	13	13	9	15	20	10	13	0.337162	0.9922	nonperio
9	8	15	15	8	17	16	17	9	14	0.242986	0.9922	nonperio
18	12	6	12	12	9	13	24	12	10	0.023812	0.9922	nonperio
6	11	11	15	10	17	11	21	18	8	0.041438	0.9922	nonperio
7	13	14	14	12	14	4	16	18	16	0.116519	1.0000	nonperio
11	9	17	11	14	13	13	14	13	13	0.911413	1.0000	nonperio
15	9	11	16	15	5	19	13	12	13	0.213309	0.9688	nonperio
13	18	8	12	11	8	14	18	14	12	0.392456	0.9766	nonperio
17	11	16	10	13	14	9	14	12	12	0.804337	0.9844	nonperio
15	16	11	8	12	14	13	18	6	15	0.299251	0.9766	nonperio
12	10	9	13	17	10	14	12	18	13	0.637119	0.9844	nonperio
11	12	17	12	11	11	14	16	9	15	0.788728	0.9844	nonperio
7	14	17	13	10	14	14	13	14	12	0.739918	0.9922	nonperio
19	13	14	10	14	12	18	12	9	7	0.275709	0.9766	nonperio
16	8	8	13	9	15	18	14	14	13	0.407091	0.9844	nonperio
10	14	15	12	9	12	12	12	20	12	0.585209	0.9844	nonperio
11	10	11	22	8	15	10	16	14	11	0.162606	0.9922	nonperio
13	18	11	9	16	16	15	8	12	10	0.437274	0.9922	nonperio
17	10	10	11	8	18	11	13	13	17	0.392456	0.9922	nonperio
11	15	14	12	17	8	13	13	17	8	0.517442	1.0000	nonperio
15	8	13	10	9	13	12	15	21	12	0.287306	1.0000	nonperio
9	8	9	20	17	12	7	15	17	14	0.078086	0.9922	nonperio
16	15	13	10	12	15	10	8	17	12	0.637119	0.9766	nonperio
6	11	10	16	12	15	17	11	15	15	0.422034	0.9844	nonperio
11	16	6	13	15	5	15	17	16	14	0.128379	1.0000	nonperio
14	10	16	12	9	9	10	17	18	13	0.437274	0.9766	nonperio
12	12	11	9	18	18	12	10	12	14	0.585209	0.9844	nonperio
12	12	14	16	12	12	11	21	7	11	0.299251	0.9922	nonperio
15	16	12	8	15	14	10	11	18	9	0.468595	1.0000	nonperio
11	17	18	12	17	7	9	14	12	11	0.311542	1.0000	nonperio
20	16	12	11	17	10	13	12	8	9	0.253551	0.9844	nonperio
10	14	17	10	13	7	15	15	8	19	0.204076	1.0000	nonperio
14	17	14	8	15	13	11	9	14	13	0.723129	0.9922	nonperio
11	15	16	16	11	11	19	10	11	8	0.392456	0.9766	nonperio
16	10	16	7	9	17	10	13	13	17	0.311542	0.9844	nonperio
12	17	14	10	17	9	12	11	11	15	0.689019	0.9844	nonperio
9	11	14	18	9	17	14	17	10	9	0.311542	0.9766	nonperio
8	16	15	7	11	10	11	15	15	20	0.170294	1.0000	nonperio
16	11	13	14	3	16	15	12	12	16	0.213309	0.9766	nonperio
12	22	11	15	12	9	19	11	7	10	0.057146	1.0000	nonperio
14	17	7	13	7	13	17	18	9	13	0.178278	0.9922	nonperio

14	12	10	15	12	12	14	10	13	16	0.941144	1.0000	nonperio
12	14	10	16	9	10	17	12	12	16	0.689019	1.0000	nonperio
14	10	11	9	20	10	10	19	12	13	0.232760	1.0000	nonperio
12	15	17	9	11	11	12	12	17	12	0.756476	0.9844	nonperio
12	16	19	13	11	14	9	15	12	7	0.392456	0.9922	nonperio
16	10	11	13	11	11	16	20	10	10	0.407091	1.0000	nonperio
17	8	19	12	13	16	8	13	12	10	0.299251	0.9922	nonperio
15	15	12	12	16	9	19	14	4	12	0.148094	0.9922	nonperio
15	14	11	10	10	21	7	13	13	14	0.264458	0.9922	nonperio
16	16	13	14	13	10	10	9	18	9	0.500934	0.9844	nonperio
19	5	16	13	14	6	13	14	16	12	0.100508	0.9922	nonperio
13	13	17	11	13	12	11	14	12	12	0.970538	1.0000	nonperio
16	13	8	12	9	15	18	13	15	9	0.452799	1.0000	nonperio
14	13	11	9	19	14	10	12	10	16	0.568055	1.0000	nonperio
10	13	18	12	20	13	13	10	10	9	0.324180	1.0000	nonperio
9	18	9	12	13	12	13	14	12	16	0.706149	1.0000	nonperio
11	15	12	13	14	11	18	11	14	9	0.788728	0.9922	nonperio
11	19	12	14	10	10	17	14	10	11	0.534146	1.0000	nonperio
9	14	12	9	18	8	13	13	15	17	0.422034	1.0000	nonperio
11	13	8	13	17	8	12	18	18	10	0.253551	0.9922	nonperio
10	15	17	16	8	18	12	5	11	16	0.110952	0.9922	nonperio
7	14	11	10	10	25	16	11	14	10	0.022503	0.9922	nonperio
7	18	12	14	8	10	15	18	11	15	0.232760	1.0000	nonperio
16	17	11	16	11	14	15	12	8	8	0.468595	0.9844	nonperio
12	11	14	13	12	13	5	20	14	14	0.299251	0.9688	nonperio
13	10	15	13	18	17	12	12	6	12	0.407091	0.9922	nonperio
14	16	17	10	14	10	14	8	14	11	0.654467	0.9766	overlapp
17	10	16	11	16	9	13	11	9	16	0.517442	0.9609	universa
17	12	10	18	11	12	13	11	11	13	0.756476	1.0000	apen
7	5	10	9	11	9	7	8	7	8	0.947557	1.0000	random-e
7	13	5	5	6	12	4	10	11	8	0.235285	0.9877	random-e
3	4	7	14	8	7	9	9	7	13	0.146359	1.0000	random-e
11	6	6	9	8	9	7	10	6	9	0.934318	1.0000	random-e
6	6	9	9	5	10	8	8	8	12	0.845066	1.0000	random-e
7	16	4	6	5	8	8	11	12	4	0.050710	0.9877	random-e
10	6	6	6	8	12	11	7	6	9	0.752361	1.0000	random-e
6	10	7	6	10	7	9	11	10	5	0.823278	1.0000	random-e
7	10	7	7	12	7	7	9	6	9	0.919445	0.9877	random-e
8	5	8	10	7	11	4	13	7	8	0.521600	0.9877	random-e
8	7	7	8	8	11	4	6	9	13	0.624107	0.9877	random-e
8	7	8	9	8	10	4	10	10	7	0.919445	1.0000	random-e
7	10	7	6	7	5	11	13	7	8	0.650132	1.0000	random-e
7	5	12	9	11	5	3	7	7	15	0.087559	1.0000	random-e
7	8	5	13	8	7	10	11	5	7	0.598138	1.0000	random-e
5	8	7	8	6	14	11	5	8	9	0.472584	0.9753	random-e
5	6	9	10	6	9	11	9	8	8	0.902994	0.9753	random-e
6	7	11	9	9	8	7	6	10	8	0.959132	0.9877	random-e
5	8	14	10	9	7	5	12	4	7	0.235285	1.0000	random-e
9	10	7	10	6	6	9	8	8	8	0.984058	0.9877	random-e
12	6	3	17	7	3	6	9	7	11	0.013217	0.9877	random-e
11	5	6	7	14	7	11	7	5	8	0.360699	0.9753	random-e
5	9	8	10	9	8	3	8	13	8	0.521600	0.9630	random-e
5	8	8	10	13	3	7	11	8	8	0.425817	0.9753	random-e
5	7	13	9	11	6	10	8	9	3	0.360699	0.9877	random-e
5	10	10	7	13	9	8	2	10	7	0.302291	0.9877	random-e
11	19	13	16	12	12	10	14	10	11	0.671779	0.9922	serial
17	10	9	14	12	21	17	11	7	10	0.095617	0.9922	serial
15	10	13	12	12	14	11	18	8	15	0.671779	0.9922	linear-c

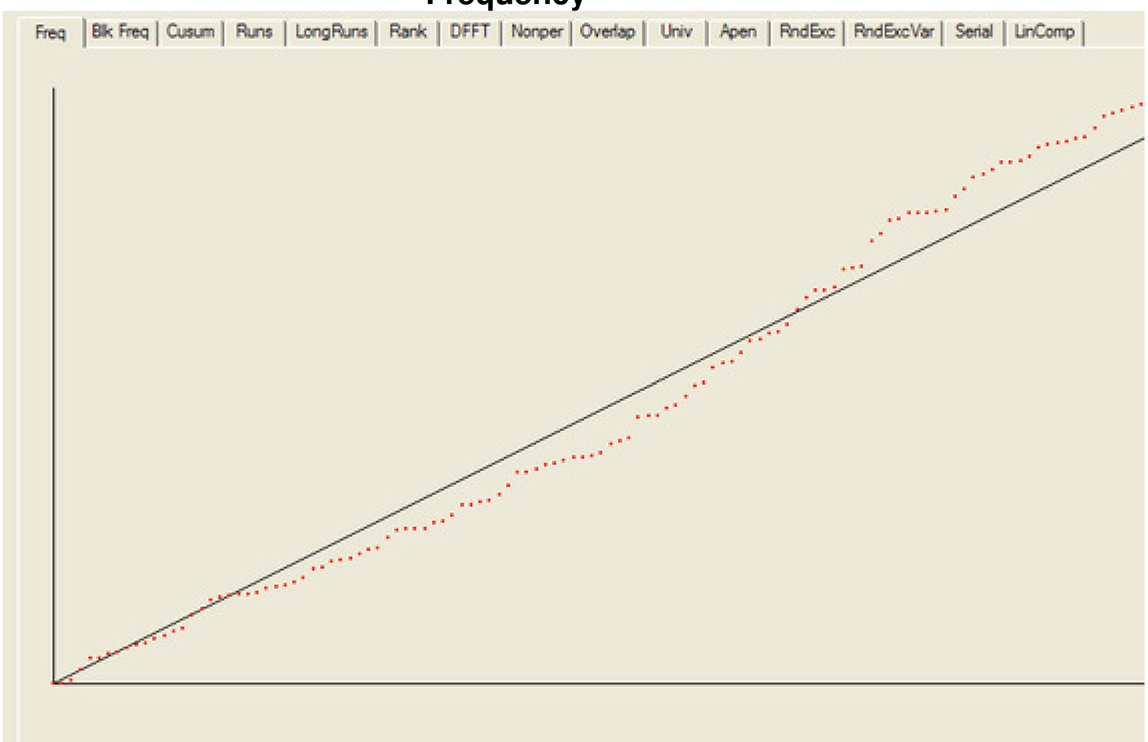
```
- - - - - - - - - - - - - - - - - - - - - - - - - -- - - - - - - -
The minimum pass rate for each statistical test with the exception of
excursion (variant) test is approximately = 0.963616 for a sample size
binary sequences.

The minimum pass rate for the random excursion (variant) test is appro
0.956834 for a sample size = 81 binary sequences.

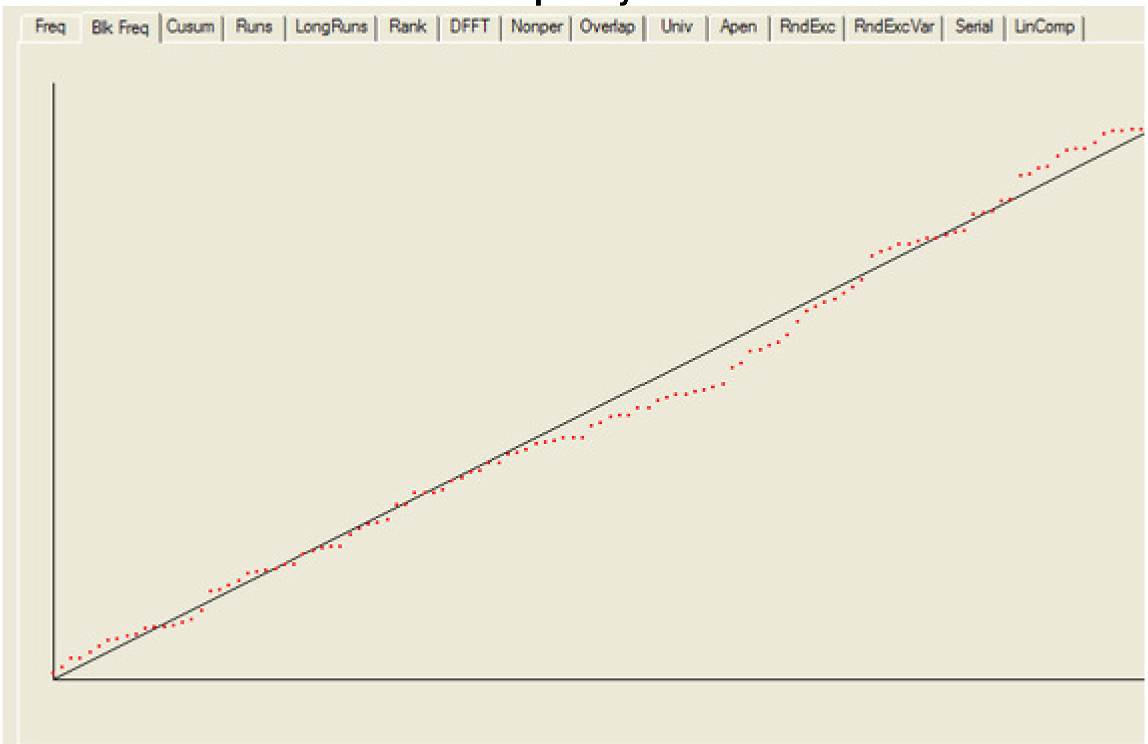
For further guidelines construct a probability table using the MAPLE p
provided in the addendum section of the documentation.
- - - - - - - - - - - - - - - - - - - - - - - - - -- - - - - - - -
```

For each test a chart is generated which shows the expectation for a random sequence as a solid line with the results from the data being tested plotted as a series of red dots for each sub-sequence tested. The following are the charts for the HotBits data.

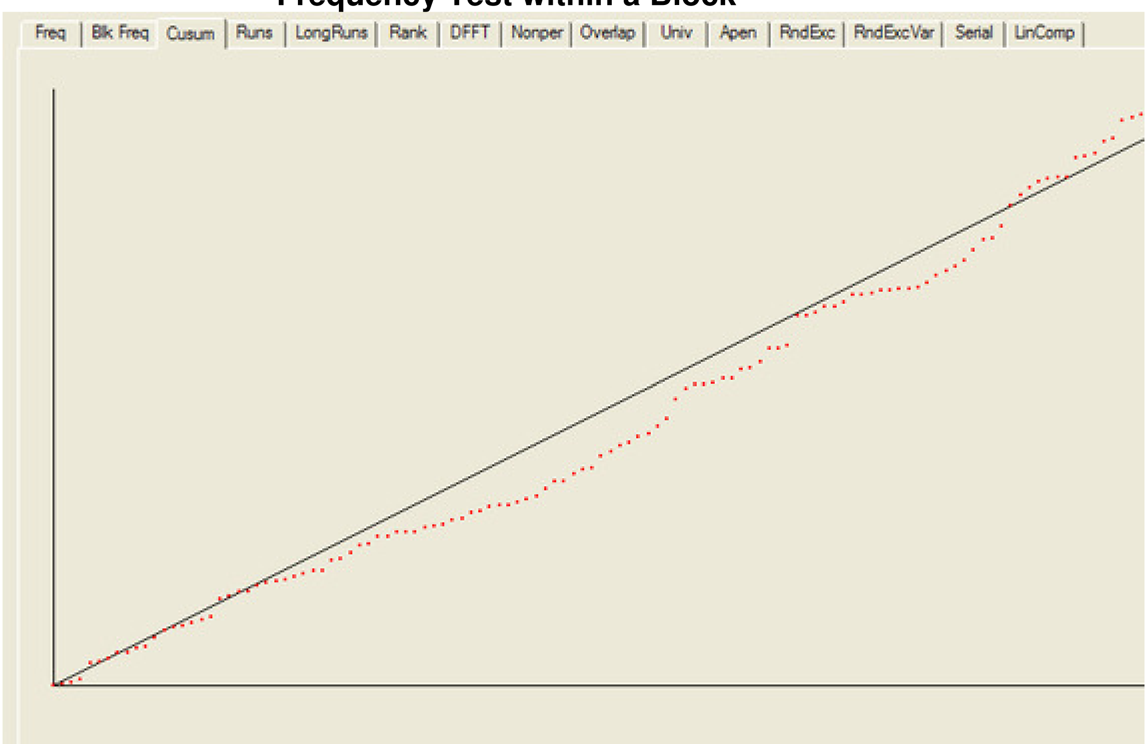
Frequency



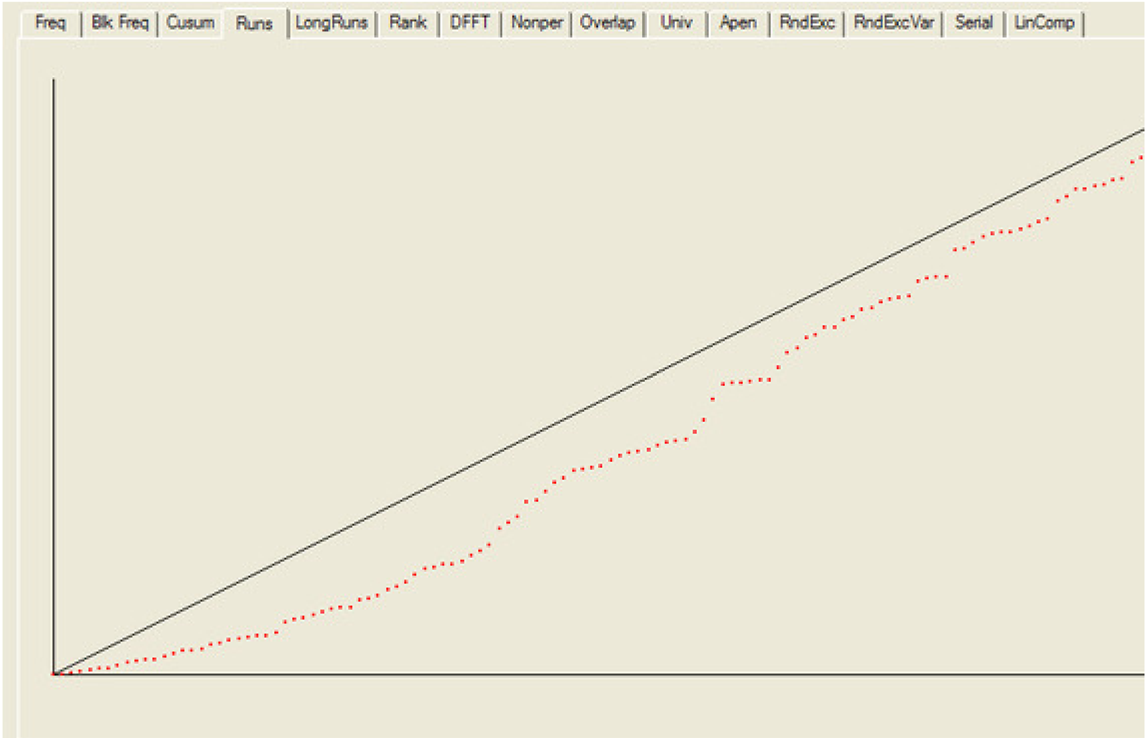
Block Frequency



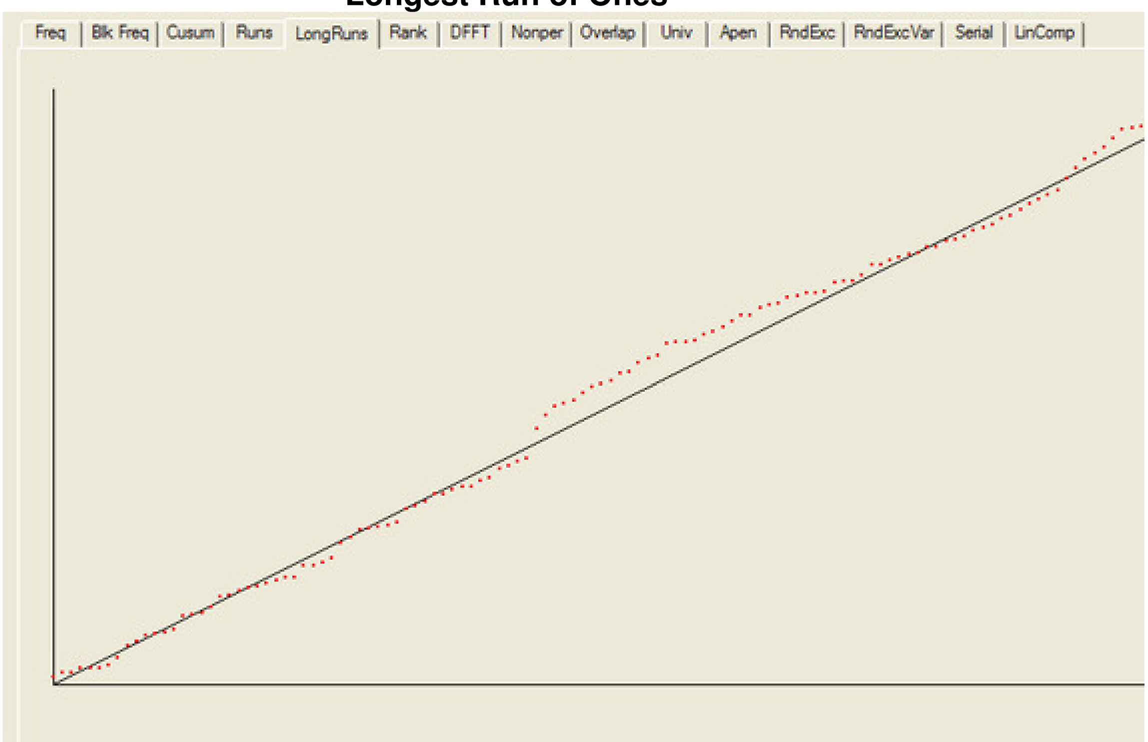
Frequency Test within a Block



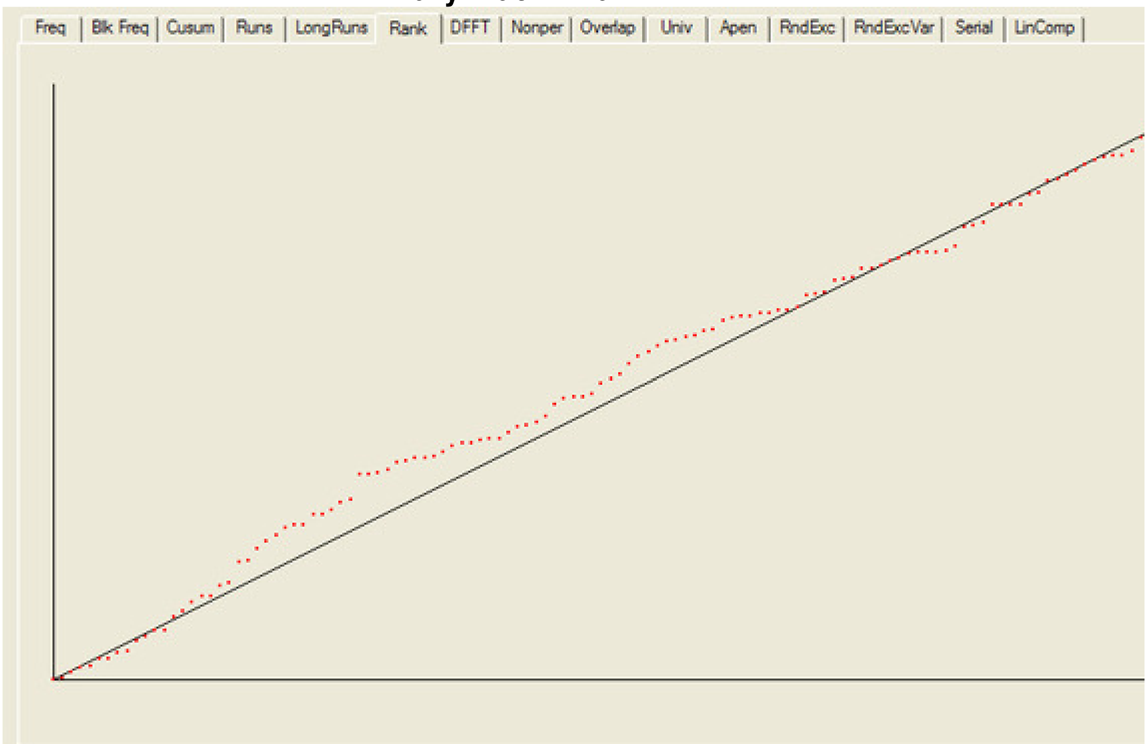
Runs



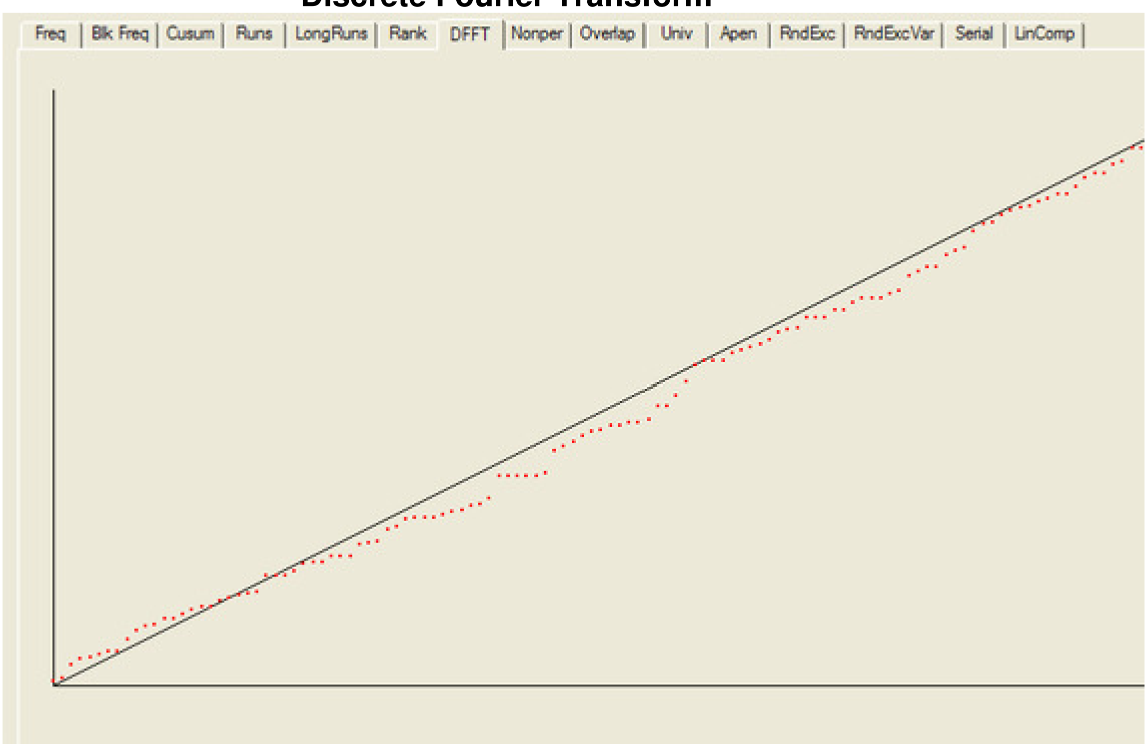
Longest Run of Ones



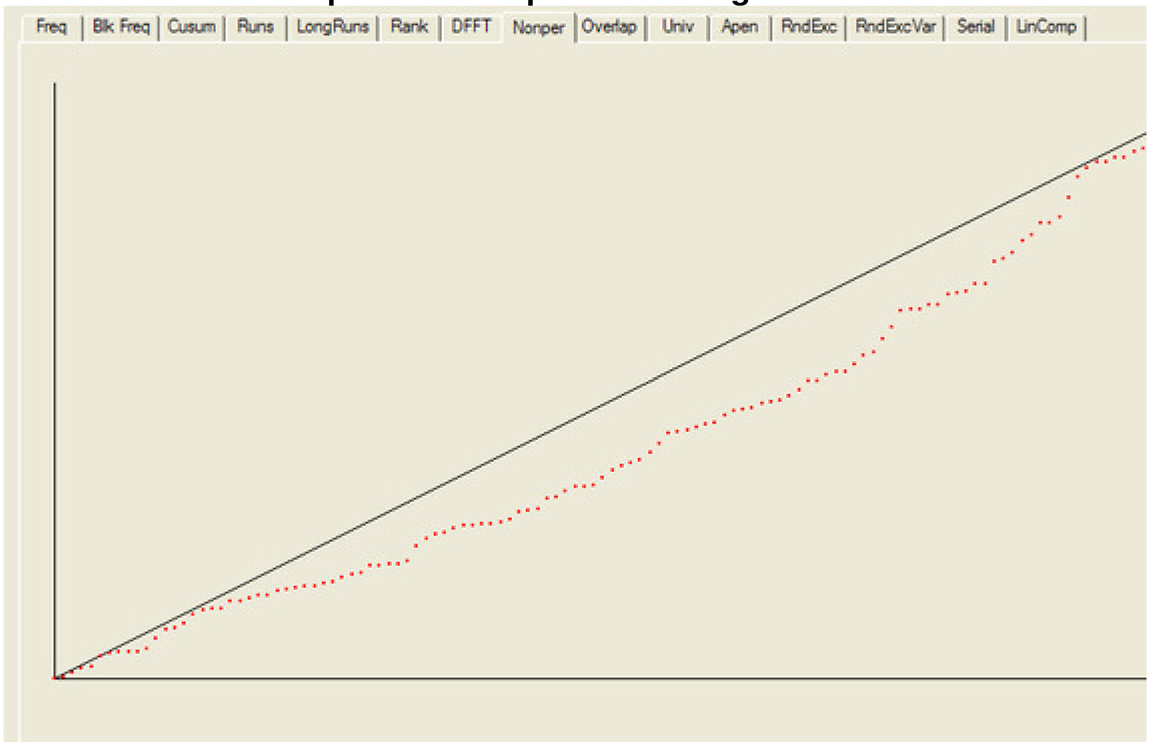
Binary Matrix Rank



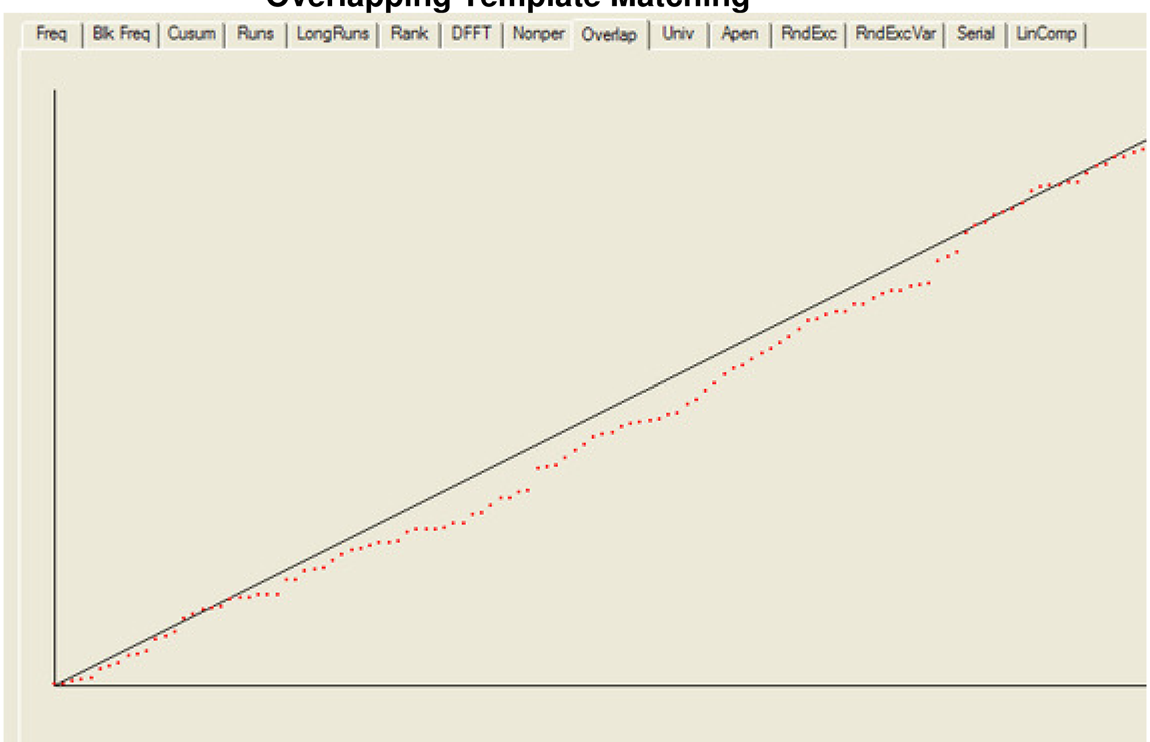
Discrete Fourier Transform



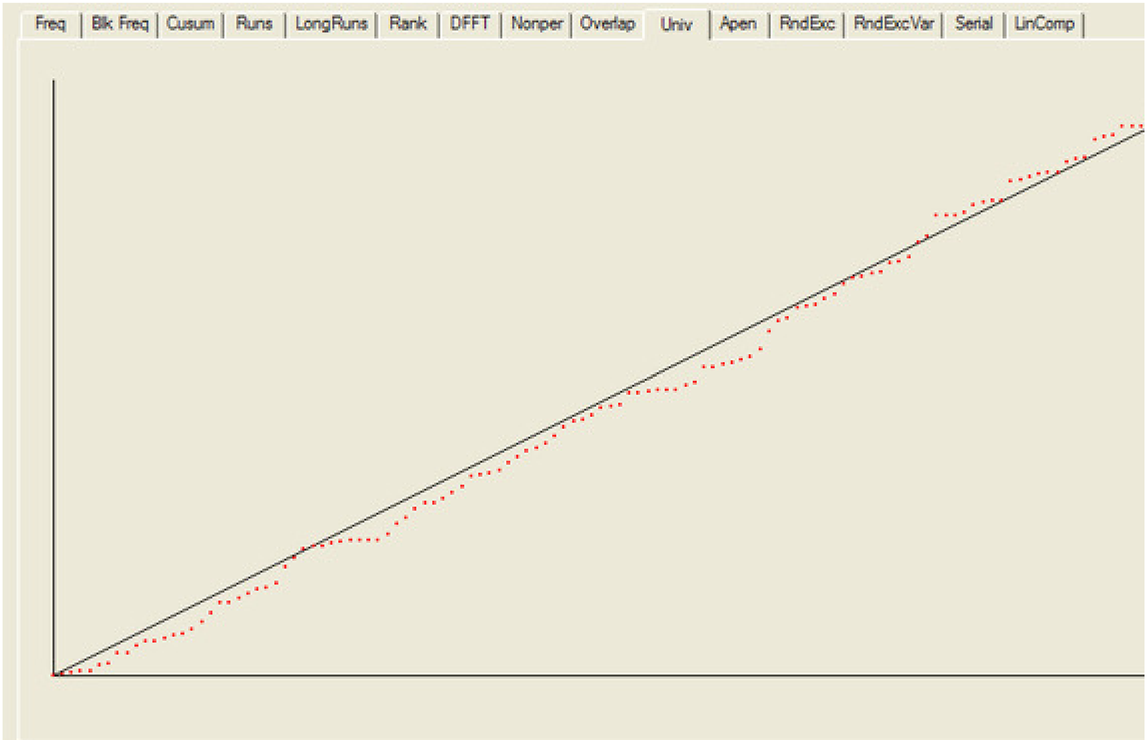
Nonperiodic Template Matching



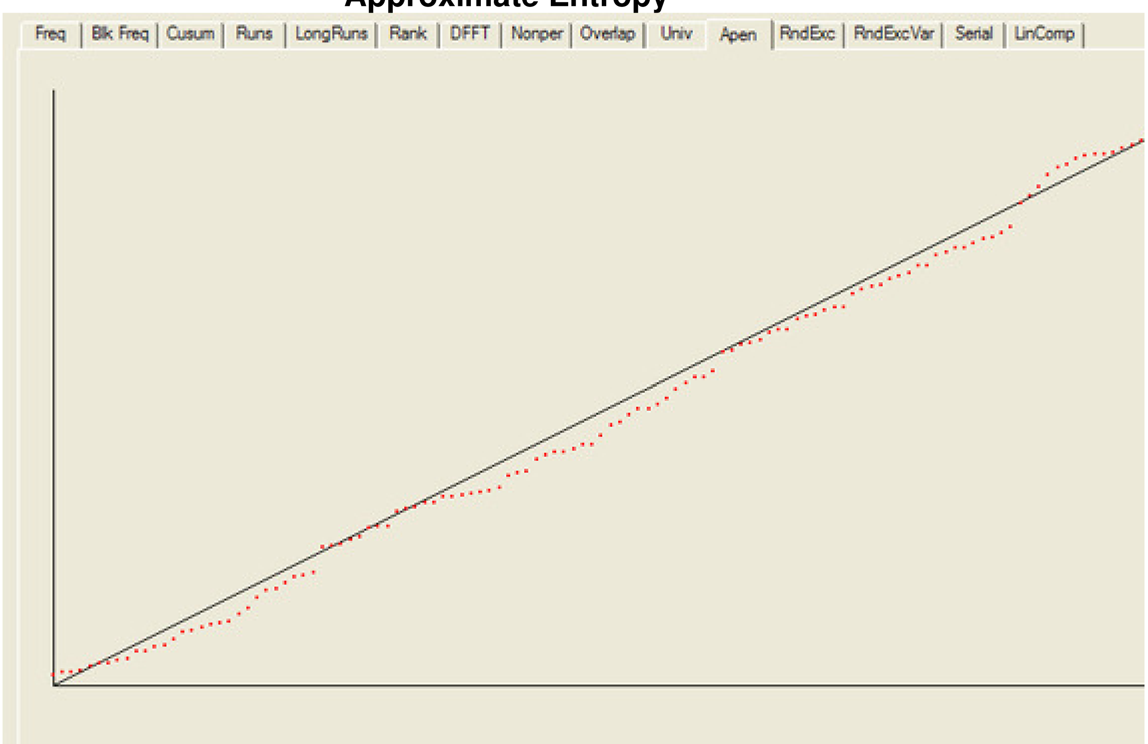
Overlapping Template Matching



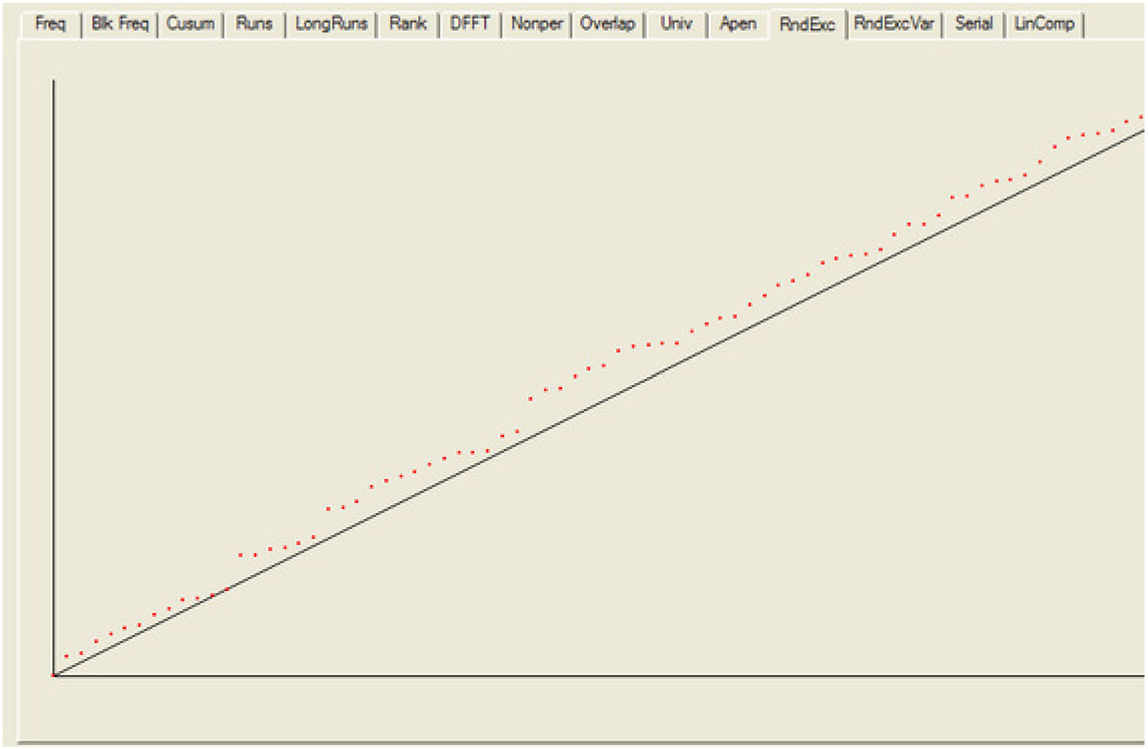
Maurer's "Universal Statistical" Test



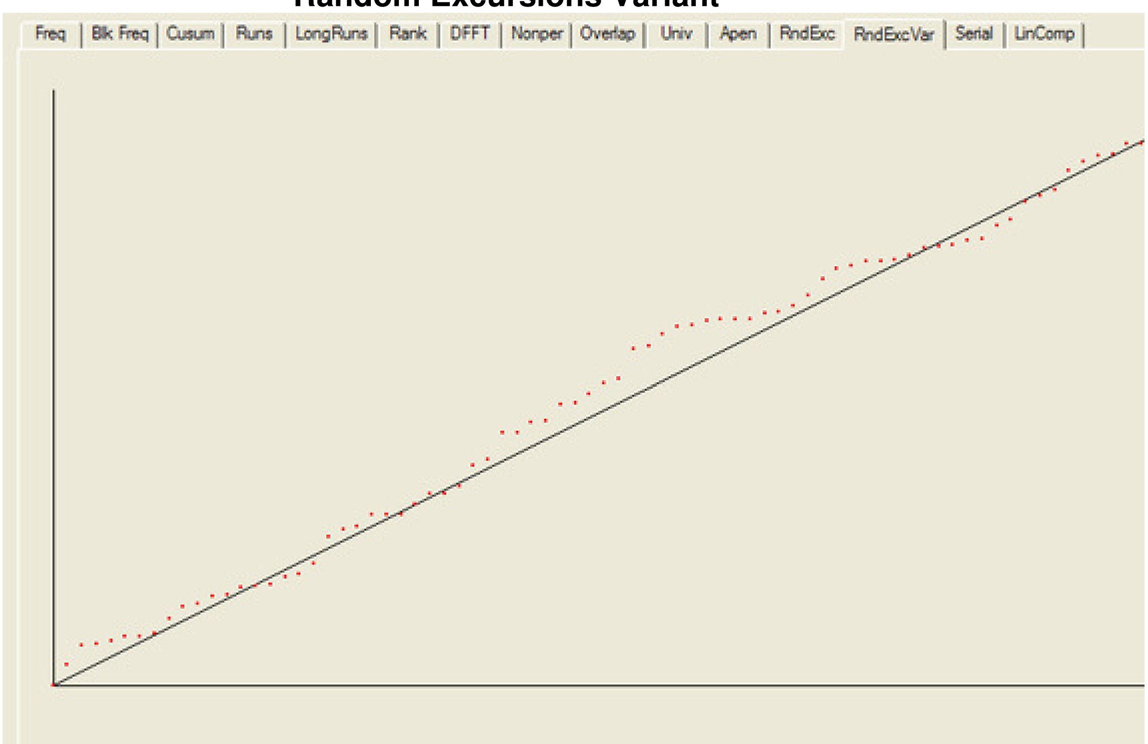
Approximate Entropy



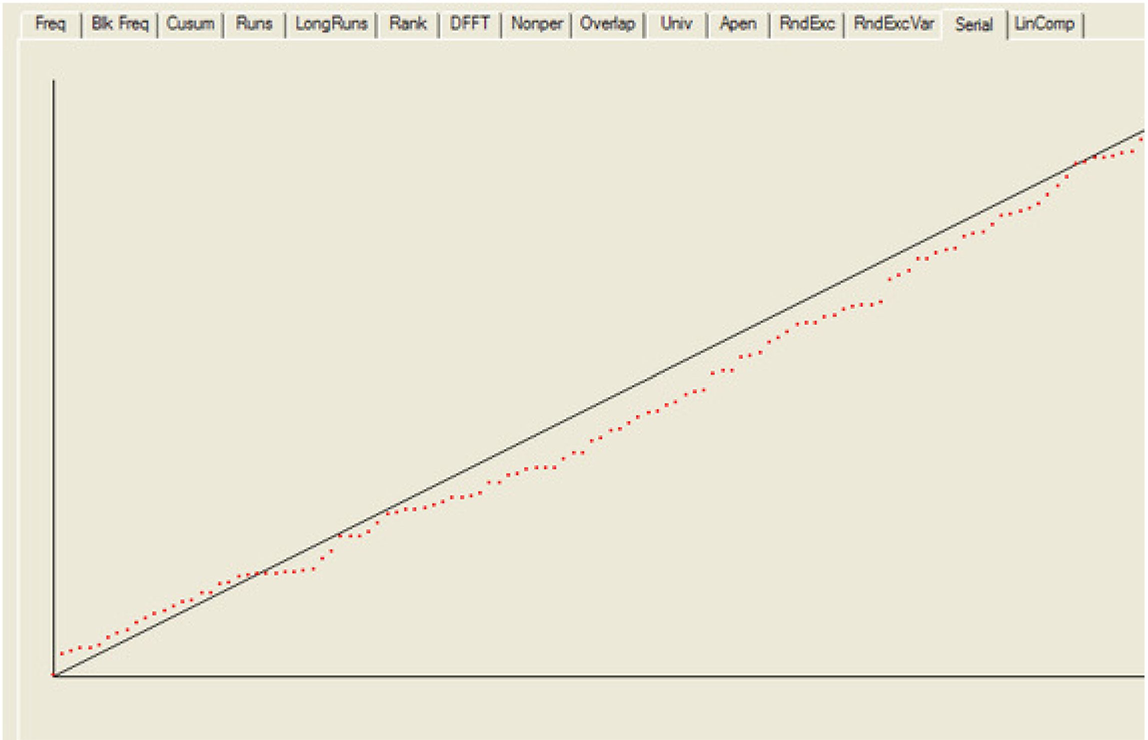
Random Excursions



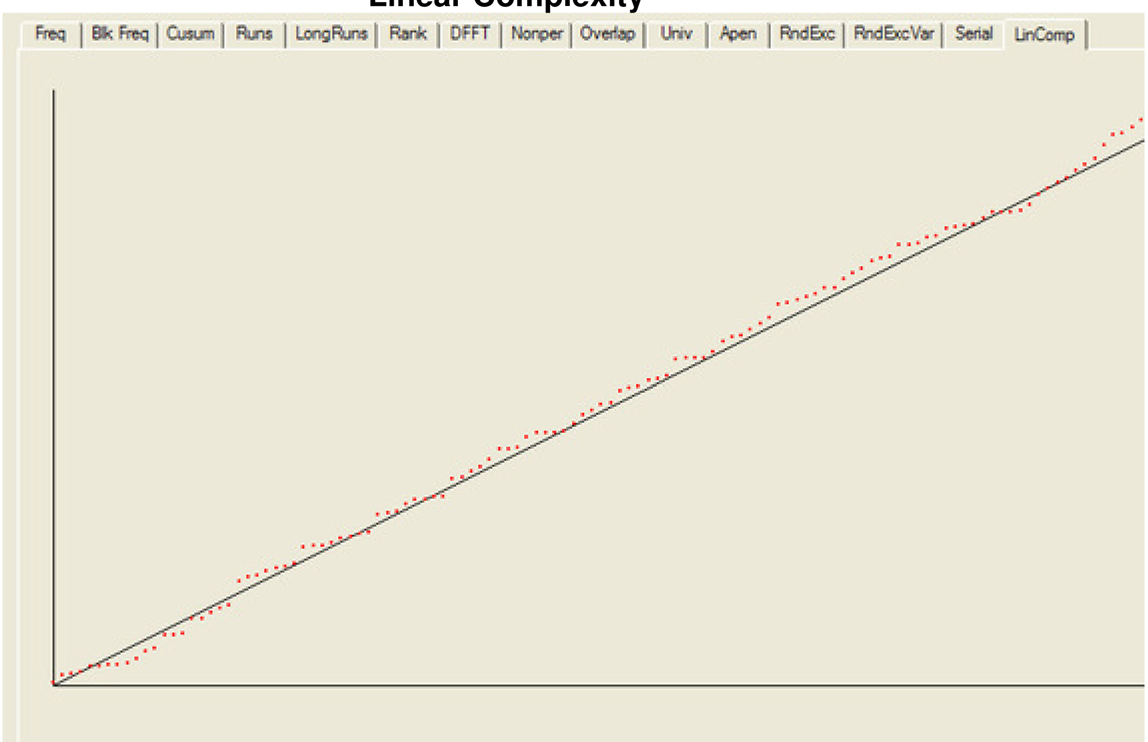
Random Excursions Variant



Serial



Linear Complexity



In addition to the test summary and graphs, detailed results from the various tests are written to a number of data files. You can download a ZIPped archive containing all of these results, and the HotBits data stream which was tested to produce them from the links below.

- Download NIST test results (`nist_results.zip` 102 Kb)
- Download data set for NIST tests (`FourmilabHotBits_NIST_SP_800-22.zip` 17 Mb)

Generator Quality Monitoring

Each HotBits generator machine performs a continuous quality control check on the random data it produces. For each block of random data generated, a specified percentage (10% for the Fourmilab generators) of additional data is generated and subjected to the ENT tests described above. The results of these tests, both for all bytes produced since the HotBits generator was started and those generated since the last status report was requested, are reported in the server status report as shown below. This status report is available only on the Fourmilab local network—it is not accessible over the Internet.

Quality control testing is performed on bytes generated expressly for that purpose and discarded immediately after being tested. The random bytes produced for inventory and returned to requesters have not been examined by any program or human before being delivered.

HotBits Server Status Report

Server **hotbits0.dmz.fourmilab.ch** up since Monday, 25 September 2006 15:12:14 CEST
Inventory loaded from file: 7647232
Requests processed: 10006, 10001 OK, 5 rejected
Unauthorised access attempts: 0
Bytes returned: 10239757
Inventory length: 7103488
Inventory file (/server/var/hbproxy/inventory.dat) length: 7103488, valid.
Bytes in current buffer: 243
Bytes built for inventory: 9696256
Seconds spent building inventory: 92926, 104 bytes/second
Quota queue length: 0

Quality measurements on a sample of 965838 bytes generated since the server was started:

> Entropy: 7.999825 bits per byte.
> Chi-square distribution: 233.74 (random exceeds this 75.00% of the time).
> Mean value: 127.5374 (expectation: 127.5).
> Monte Carlo π value: 3.141470930 (error 0.00 percent).

Serial correlation coefficient: 0.000885 (expectation 0).

Quality measurements on a sample of 159018 bytes generated since Friday, 29 September 2006 13:39:55 CEST:

Entropy: 7.998742 bits per byte.
Chi-square distribution: 277.54 (random exceeds this 25.00% of the time).
Mean value: 127.8514 (expectation: 127.5).
Monte Carlo π value: 3.138663548 (error 0.09 percent).
Serial correlation coefficient: 0.002713 (expectation 0).

Release 3.0, September 2006
Built on Sep 25 2006 at 15:12:09

HotBits Main Page
How HotBits Works
HotBits Hardware Description

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by John Walker
September, 2006

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