Consumer Preferences

1. Principles of consumer choice
2. Features of consumer preferences
3. Substitution between goods
4. Utility

School of Business, Nanjing University

Zhen-Hua Wu

PRINCIPLES OF CONSUMER CHOICE

Economists refer to likes and dislikes as **preferences**.

EX: Three students order dinner at a restaurant. One picks a salad, another chooses a steak, and the third selects pasta. Why do the three make different choices? They don't pick their meals at random; their decisions reflect their likes and dislikes.

- **The Ranking Principle**:
  A consumer can rank, in order of preference (though possibly with ties), all potentially available alternatives.

  The consumer has a clear idea of what's good (something with a high rank) and what's bad (something with a low rank).
  => the consumer is <u>never uncertain in making comparisons</u>—at least not after some reflection.

  While the Ranking Principle may not hold in all circumstances, it's a reasonable starting point for thinking about most economic decisions.

  Notice: the Ranking Principle allows for ties. This doesn't mean that the consumer is uncertain; it simply means that he likes two (or more) alternatives equally. Economists say that the consumer is **indifferent** between such alternatives.

  Two more implications:
  1) preferences are **complete**: between any pair of alternatives, a consumer either prefers one to the other or is indifferent between them.
  2) preferences are **transitive**: if the consumer prefers one alternative to a second, and prefers the second alternative to a third, then he also prefers the first alternative to the third.

  *The Ranking Principle is in fact equivalent to the assumption that preferences involving pairs of objects are **complete** and **transitive**.*

- **The Choice Principle**:
  Among the available alternatives, the consumer selects the one that he ranks the highest.

  The consumers always try to achieve the highest possible level of well-being/satisfication.

FEATURES OF CONSUMER PREFERENCES

In practice, our decisions tend to be interrelated in two ways. First, the enjoyment of one activity often depends on other activities. For example, many people enjoy jogging and drinking beer, but usually not at the same time. Second, when an individual spends money to purchase one good, less money is available for other goods. A decision to consume more of one good is therefore also a decision to consume less of another.

To make better decisions, consumers need to consider these interrelationships. They must keep an eye on the big picture— allocating their limited funds to competing needs and desires over some fixed period, such as an hour, a day, a month, a year, or even a lifetime. By following such a plan, the consumer ends up with a collection of goods, known as a **consumption bundle/basket**, for the period in question.

A consumer's choices should reflect how he feels about various consumption bundles, rather than how he feels about any one good in isolation. Comparisons between consumption bundles involve **trade-offs**. For example, when comparing three restaurant meals and two movies versus one meal and eight movies, you must decide whether it is worth giving up two meals to get six more movies.

How Do People Rank Consumption Bundles?

- **The More-Is-Better Principle**:
  When one consumption bundle contains more of every good than a second bundle, a consumer prefers the first bundle to the second.

  * **Monotonicity of preferences**, a special case of a more technical property called **local non-satiation**, i.e., there is always some small change that would make the consumer better off.
  **No doubt you can think of situations in which someone might have too much of a good thing. Consumer theory can accommodate this relatively rare possibility. But for the typical decision, we can reasonably assume that consumers prefer more to less.

EX: Bill's Preferences for Meals
The preference ranking shown in right Table satisfies the More-Is-Better Principle. In any single column (e.g., the one highlighted in yellow), the numbers at the top are smaller than the numbers at the bottom. This means that, given a fixed amount of yogurt, Bill prefers more pizza. Similarly, in any row, the numbers at the right-hand side are smaller than the numbers at the left-hand side. This means that, given a fixed amount of pizza, Bill prefers more yogurt.

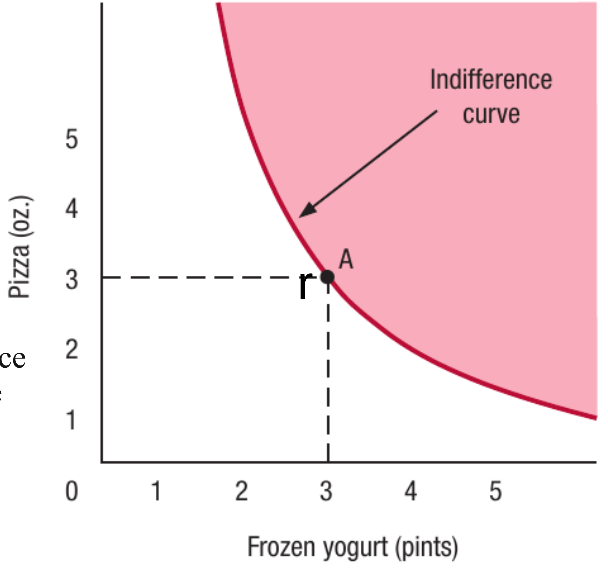
One scoop of yogurt, two slices of pizza

Cheese pizza (slices) \ Frozen yogurt (scoops)	0	1	2	3
3	11	7	3	1
2	13	8	4	2
1	15	9	6	5
0	16	14	12	10

Question: If Bill starts with three scoops of yogurt and no pizza, is she willing to trade one scoop of yogurt for two slices of pizza?

FEATURES OF CONSUMER PREFERENCES

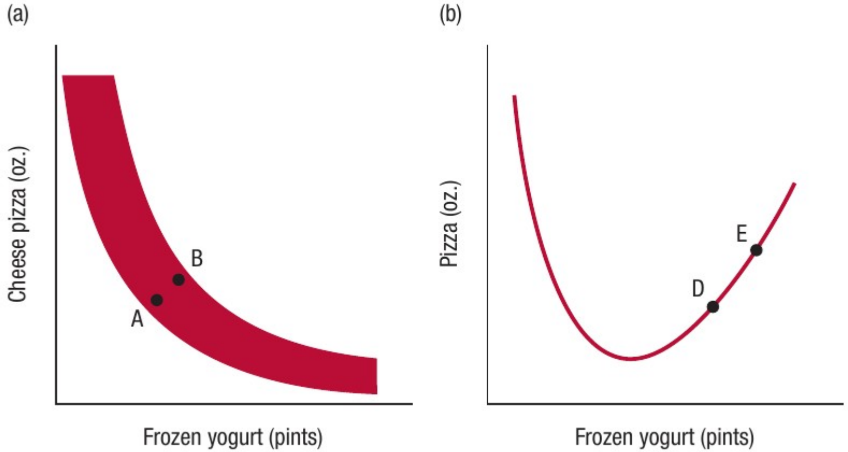
Consumer Indifference Curves



To depict preference of an individual, we introduce the **indifference curve**.

Economists say that <u>an individual is indifferent between two alternatives if he or she likes (or dislikes) them equally</u>. When goods are finely divisible, we can start with any alternative and find others that the consumer likes equally well. An **indifference curve** shows all these alternatives. When we draw an indifference curve, we declare a “tie” between all the points on the curve. The right Figure shows one of Bill’s indifference curves. The curve identifies all the consumption bundles Bill finds just as attractive as bundle A, which consists of 3 pints of yogurt and 3 ounces of pizza.

Some Properties of Indifference Curves



- <u>Indifference curves are thin</u>
  See Figure (a) on the right. Since the red curve is thick, we can start at a bundle like A and move to the northeast, reaching a bundle like B, while staying on the curve. Since B contains more yogurt and more pizza than A, the consumer must like B better than A. But this means the thick red curve can’t be an indifference curve.

- <u>Indifference curves do not slope upward</u>
  See Figure (b) on the right. Since part of the red curve slopes upward, we can start at a bundle like D and move to the northeast, reaching a bundle like E, while staying on the curve. Since E contains more yogurt and more pizza than D, the consumer must like E better than D. But this means the red curve can’t be an indifference curve.

- <u>Separates all the better-than-A bundles from the worse-than-A bundles</u>
  Since more is better, the better-than-A bundles lie to the northeast of the indifference curve, while the worse-than-A bundles lie to the southwest. In the Figure on the top-right, we’ve shaded the better-than-A bundles light red.

FEATURES OF CONSUMER PREFERENCES

Consumer Indifference Curves (continued...)

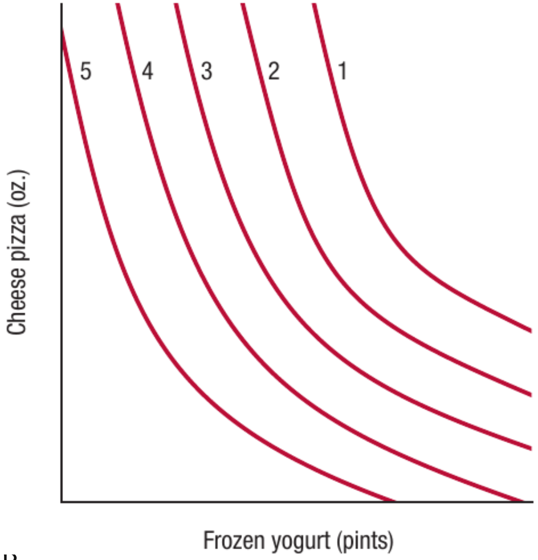
Families of Indifference Curves (indifference map)

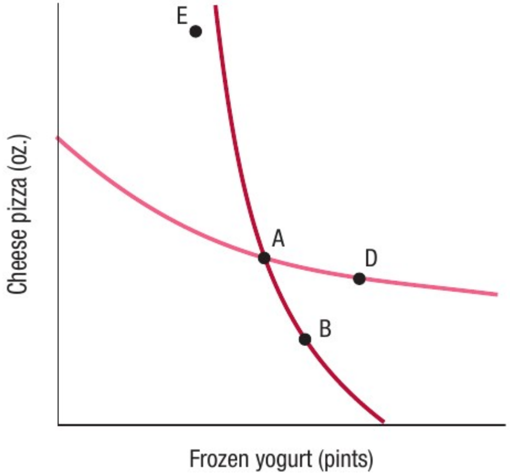
A family of indifference curves is a collection of indifference curves that represent one individual's preferences. Within a family, each indifference curve corresponds to a different level of well-being. The right Figure shows 5 indifference curves belonging to the family representing Bill's preferences.

When the More-Is-Better Principle holds, families of indifference curves have two important properties.

- Indifference curves from the same family do not cross
  Proof: See the right Figure, which shows two red curves crossing at bundle A. If the dark red curve is an indifference curve, then the consumer is indifferent between bundles A and B. Since bundle D contains more yogurt and more pizza than bundle B, the consumer prefers D to B, so he also prefers D to A. But that means the light red curve isn't one of his indifference curves..

- In comparing any 2 bundles, the consumer prefers the one located on the indifference curve that is furthest from the origin
  For any bundle A, the better-than-A bundles lie to the northeast of the indifference curve running through A, and the worse-than-A bundles lie to the southwest. For example, Bill ranks the five indifference curves shown in top-right Figure according to the numbers (1 through 5) that appear next to them in the figure.





Let's summarize:

Assuming the following 2 principles hold:

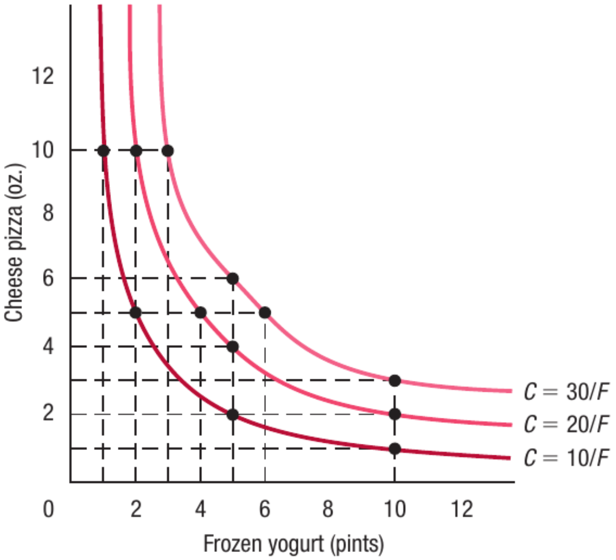
Ranking Principle
The More-Is-Better Principle $\Longrightarrow$

Properties of Indifference Curves and Families of Indifference Curves

1. Indifference curves are thin.
2. Indifference curves do not slope upward.
3. The indifference curve that runs through any consumption bundle—call it A—separates all the better-than-A bundles from all the worse-than-A bundles.
4. Indifference curves from the same family never cross.
5. In comparing any two bundles, the consumer prefers the one located on the indifference curve that is furthest from the origin.

FEATURES OF CONSUMER PREFERENCES

Consumer Indifference Curves (continued...)



Formulas of Indifference Curves

So far, we've been studying consumer preferences using graphs. Though the graphical approach helps to build understanding and intuition, it doesn't allow us to make precise numerical statements about consumer behavior. Instead, economists usually describe consumer preferences using mathematical formulas.

One way to describe consumer preferences mathematically is to write down the formulas for their indifference curves.

For example, the formula for the dark red indifference curve in the right Figure is $C = 10/F$, where C stands for ounces of cheese pizza and F for pints of frozen yogurt. We've graphed this formula by plotting a few points and connecting the dots.

The right Figure also shows two other indifference curves, the formulas for which are $C = 20/F$ and $C = 30/F$. All three curves belong to the same family. The general formula for this family of indifference curves is $C = U/F$, where U is a constant.

To obtain a particular indifference curve, we simply plug in a value for the constant U (such as 10, 20, or 30), and plot the relationship between C and F. Higher values of U lead to indifference curves that are further from the origin. Therefore, the value of U for the indifference curve that runs through any bundle provides a measure of the consumer's well-being, or "**utility**" (hence the letter U), when consuming that bundle.

FEATURES OF CONSUMER PREFERENCES

Consumer Indifference Curves (continued...)

Formulas of Indifference Curves (Applicaiton)

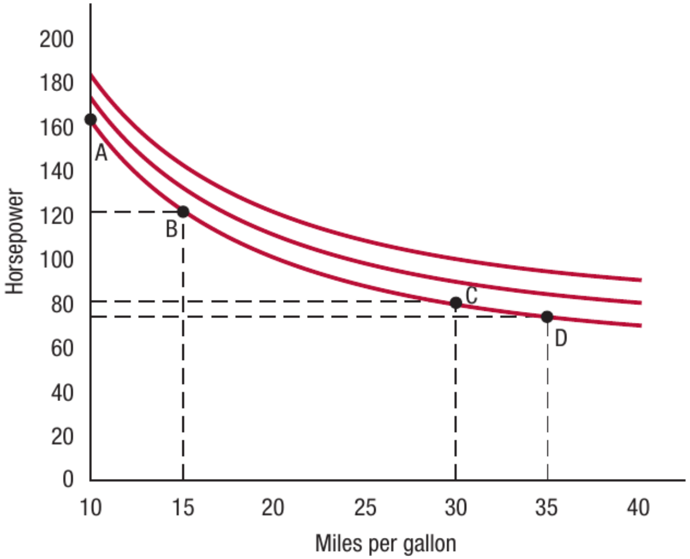
<u>Preferences for Automobile Characteristics*</u>

Why does a consumer choose one type of automobile over another?

An automobile is a bundle of characteristics and features: style, comfort, power, handling, fuel efficiency, reliability, and so forth. To comprehend the consumer's choice, we must therefore study his preference for bundles of these characteristics. As with bundles of goods, we can understand the customer's preferences by examining his indifference curves.

In one study, economist Pinelopi Goldberg examined data on purchases of large passenger cars in the United States between 1984 and 1987. The right Figure, which is based on her results, shows the preferences of the typical new car buyer for two characteristics, horsepower and fuel economy. Since the curves of buyer is slope downward, to sacrifice the power and acceleration in return for greater fuel efficiency. For example, consumers are willing to give up roughly 40 horsepower to increase fuel efficiency from 10 to 15 miles per gallon (compare points A and B).

Understanding consumers' willingness to trade horsepower for fuel efficiency is important for both automobile manufacturers and public policymakers. Automobile manufacturers can use information of this type to determine whether a particular design change will improve a car's appeal to consumers. Policymakers can use it to evaluate the likely success of policies that encourage consumers to purchase fuel-efficient automobiles.



Indifference Curves for Horsepower and Fuel Economy.

The typical new car buyer's preferences for horsepower and fuel economy correspond to the family of indifference curves shown in this figure. Consumers are willing to give up roughly 40 horsepower to increase fuel efficiency from 10 to 15 miles per gallon (compare points A and B), but they are willing to give up only 6 horsepower to increase fuel efficiency from 30 to 35 miles per gallon (compare points C and D).

*Pinelopi Koujianou-Goldberg, "Product Differentiation and Oligopoly in International Markets: The Case of the U.S. Automobile Industry," Econometrica 63, July 1995, pp. 891–951.

FEATURES OF CONSUMER PREFERENCES

Goods versus Bads

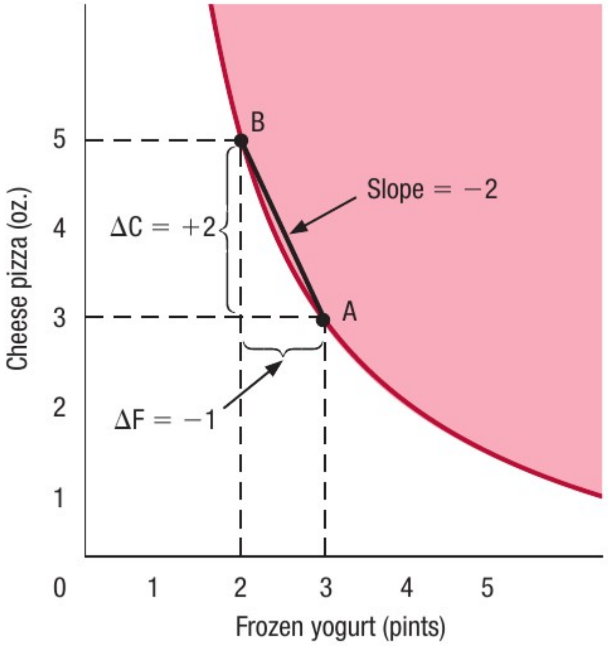
So far, we have focused on decisions involving things that people desire (**goods**). But people also often make decisions involving objects, conditions, or activities that make them worse off, and that they wish to avoid (**bads**). Think, for example, about studying for your final exam in this course. Everyone likes to get good grades, and most people like to learn, but few people *enjoy* studying.

Do we need separate theories for goods and bads? Fortunately, we don't, because we can always think of a bad as the absence of a good.
In our example, studying is a bad because it crowds out leisure time. So we can think of the student as choosing leisure time (a good) instead of study time (a bad).

We'll attack this question later by studying the choice of leisure hours (a good) rather than the choice of work hours (a bad).

Can you give some examples of bads?

SUBSTITUTION BETWEEN GOODS



To determine whether a consumer is better off with one bundle or another, we need to know the rate at which he is willing to make trade-offs. Indifference curves provide us with that information.

Rates of Substitution

When we move from one bundle to another along a consumer's indifference curve, we learn how many units of one good must be added to compensate him for receiving fewer units of the other good, so that he is neither better nor worse off.

> The right Figure illustrates this point using Bill's preferences. Since bundles A and B lie on the same indifference curve, he is equally happy with either. In moving from bundle A to bundle B, the change in frozen yogurt, ΔF, is -1 pint, and the change in cheese pizza, ΔC, is +2 ounces. So starting from bundle A, two additional ounces of pizza exactly compensate Bill for the loss of a pint of yogurt. The rate at which she substitutes for yogurt with pizza in moving from bundle A to bundle B is $-\Delta C/\Delta F = 2$ ounces per pint, which equals the slope of the straight line connecting bundles A and B.

Economists usually measure rates of substitution in terms of very small changes. Let's refer to the goods in question as X and Y.

The marginal rate of substitution for X with Y, written MRS_{XY}, is the rate at which a consumer must adjust Y to maintain the same level of well-being when X changes by a tiny amount, from a given starting point. The phrase "for X with Y" means that we measure the rate of substitution compensating for a given change in X with an adjustment to Y.

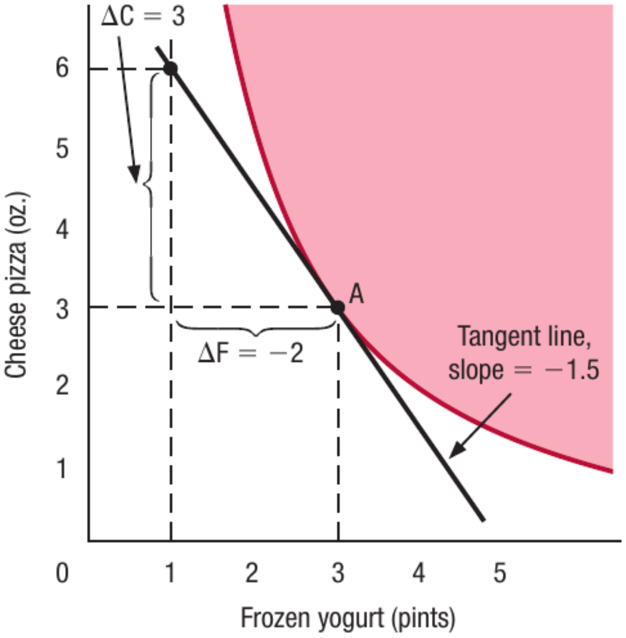
Mathematically, if ΔX is the tiny change (either + or -) in X and ΔY is the compensating adjustment to Y, then $MRS_{XY} = -\Delta Y/\Delta X$.

> Because ΔX and ΔY always have opposite signs, including the negative sign converts the ratio into a positive number, making it easier to interpret (a larger positive value then indicates that the adjustment to Y must be larger to compensate for a change in X).

SUBSTITUTION BETWEEN GOODS

Rates of Substitution (continued...)

The right Figure illustrates Bill's marginal rate of substitution for frozen yogurt with pizza at bundle A. Notice that the figure includes a line that lies tangent to her indifference curve at bundle A. Starting from bundle A, for very small movements along either the indifference curve or the tangent line, the value of $\Delta C/\Delta F$ is nearly the same. Therefore, the marginal rate of substitution for frozen yogurt with cheese pizza, MRS_{FC}, at bundle A is simply the slope of this tangent line times negative one. In the figure, the slope of the tangent line is -1.5 ounces per pint (because the line runs through the bundle containing 6 ounces of pizza and 1 pint of yogurt, as well as point A which contains 3 ounces of pizza and 3 pints of yogurt), so Bill's marginal rate of substitution for frozen yogurt with pizza is 1.5 ounces per pint.



Note that MRS_{XY}, is not the same as MRS_{YX},. For MRS_{XY} , we compensate for a given change in X with an adjustment to Y, and divide this adjustment by the change in X (that is, $-\Delta Y/\Delta X$). For MRS_{YX}, we compensate for a given change in Y with an adjustment to X and divide this adjustment by the change in Y (that is, $-\Delta X/\Delta Y$).

However, Since $\Delta X/\Delta Y = 1/(\Delta Y/\Delta X)$, it follows that $MRS_{XY} = 1/MRS_{YX}$.

For example, if the marginal rate of substitution for yogurt with pizza is 1.5 ounces per pint, then the marginal rate of substitution for pizza with yogurt is 0.667 pints per ounce.

SUBSTITUTION BETWEEN GOODS

Rates of Substitution (continued...)

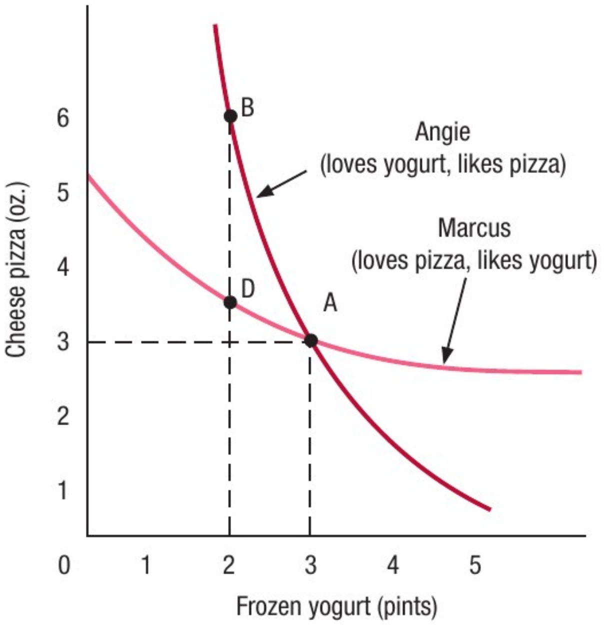
What Determines Rates of Substitution?

- <u>Consumers' tastes</u> (How do these differences in taste affect their rates of substitution?)

  Right Figure illustrates this point by showing the indifference curves for two consumers. Angie loves yogurt and likes pizza, while Marcus loves pizza and likes yogurt.

  Starting at bundle A in right Figure, imagine reducing the amount of yogurt by one pint. Angie needs a large amoun of pizza, which she likes, to compensate for the lost yogurt, which she loves. So at A, Angie's marginal rate of substitution for yogurt with pizza is high and her indifference curve (dark red), is relatively steep (it runs through bundle B). In contrast, Marcus needs only a small amount of pizza, which he loves, to compensate him for the lost yogurt, which he likes. So at A, Marcus's marginal rate of substitution for yogurt with pizza is low and his indiffere curve, shown in light red, is relatively flat (it runs through D).

  <u>More generally, the marginal rate of substitution for X with Y is large when the consumer values X highly compared to Y (so that a lot of Y is needed to compensate for lost X), and small when the consumer values Y highly compared to X (so that only a little Y is needed to compensate for lost X).</u>

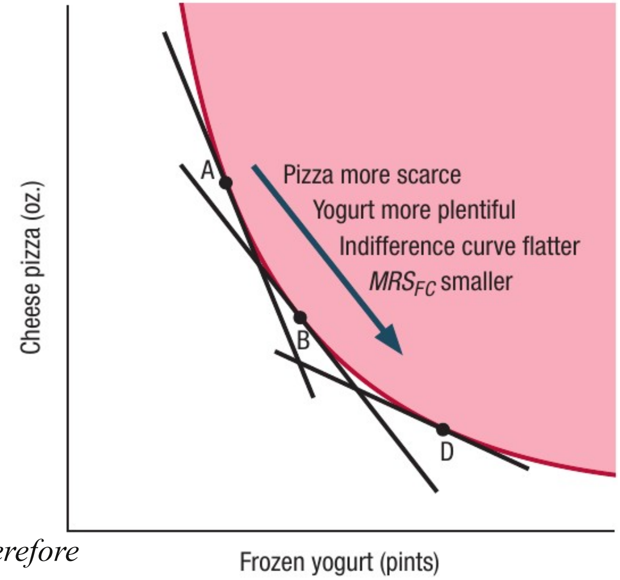


SUBSTITUTION BETWEEN GOODS

Rates of Substitution (continued...)

What Determines Rates of Substitution? (continued...)

- <u>Consumers' starting point</u>



In the Right Figure, Bill's marginal rate of substitution for yogurt with pizza is different at bundles A, B, and D.

Notice that the indifference curve in the right Figure becomes flatter as we move in the direction of the blue arrow, from the northwest (top left) to the southeast (bottom right). Thus, MRS_{FC} declines as we progress toward bundles offering more yogurt and less pizza (for example, from A to B to D).

*This pattern is typical of consumers' preferences. One important reason is that people like **variety**, and therefore attach more value to an additional unit of a good when it is relatively scarce.*

When yogurt is scarce and pizza is plentiful (at a bundle like A), it takes a great deal of pizza to compensate Bill for a lost pint of yogurt, so her MRS_{FC} is high. Alternatively, when pizza is scarce and yogurt is plentiful (at a bundle like D), a small amount of pizza compensates her for a lost pint of yogurt, so her MRS_{FC} is low.

*Any indifference curve that becomes flatter as we move along the curve from the northwest to the southeast is said to have a **declining MRS**.*

† The notion of a declining MRS is associated with a mathematical concept called **convexity**.

Notice that, in above Figure, the set of better-than-A alternatives (shaded light red) is shaped like a convex lens that bulges in the direction of the origin. Economists and mathematicians refer to **this type of set as convex**. The indifference curve illustrated in above Figure is also called a **convex function**, in the sense that the slope of the line drawn tangent to it increases (becomes less negative) as we move from left to right. These characteristics of preferences are both mathematically equivalent to a declining MRS.

SUBSTITUTION BETWEEN GOODS

Rates of Substitution (continued...)

Formulas for Rates of Substitution

As we've seen, one way to describe consumers' preferences mathematically is to write formulas for their indifference curves. Another way is to write formulas for their marginal rates of substitution. An MRS formula tells us the rate at which the consumer is willing to exchange one good for another, given the amounts consumed.

To illustrate, suppose the rate at which a particular consumer is willing to substitute for yogurt with pizza is given by the formula $MRS_{FC} = C/F$, where C stands for ounces of cheese pizza and F stands for pints of frozen yogurt. Then, if the consumer starts out with C ounces of pizza and F ounces of yogurt, tiny changes in the amounts of pizza and yogurt, ΔC and ΔF, will leave him (roughly) on the same indifference curve as long as $\Delta C/\Delta F = - C/F$.

For example, with $F = 12$ and $C = 2$, the MRS for yogurt with pizza is 1/6 ounce per pint. Therefore, starting with 12 pints of yogurt and 2 ounces of pizza, the consumer must receive $(1/6) \times \Delta F$ ounces of pizza to compensate for the loss of ΔF pints of yogurt (where ΔF is tiny).

Checking whether a consumer's indifference curves have declining MRSs using a formula for the MRS is usually easy.

For example, when $MRS_{FC} = C/F$, the MRS for yogurt with pizza increases with the amount of pizza and decreases with the amount of yogurt. Every indifference curve must therefore become flatter as we move along the curve from the northwest to the southeast, toward bundles with less pizza and more yogurt. Therefore, those indifference curves have declining MRSs.

Because an MRS formula tells us how a consumer makes trade-offs, it completely describes his preferences. The same is true of an indifference curve formula. Accordingly, for every indifference curve formula, there is a corresponding MRS formula associated with the same preferences, and vice versa.

For example, the marginal rate of substitution formula examined above, $MRS_{FC} = C/F$, describes the same preferences as the indifference curve formula discussed in previous page where $C = U/F$.

- <u>Mutually beneficial ←→ Trade ←→ MRS</u>

SUBSTITUTION BETWEEN GOODS

Special Cases: Perfect Substitutes and Complements

Two products are **perfect substitutes** if their functions are identical, so that a consumer is willing to swap one for the other at a fixed rate.

- In practice, substitutability is a matter of degree. For example: Coke and Pepsi, Nintendo Wii and Microsoft Xbox …

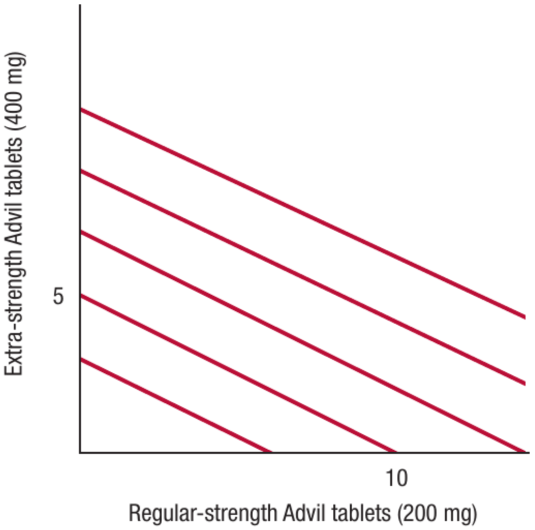
If two goods are valuable only when used together in fixed proportions, we call them **perfect complements**.

- In practice, complementarity is also a matter of degree. For example: bicycle tires and frames, left and right gloves, and left and right shoes … However, some view a single glove as a fashion statement.

We study the case of perfect substitutes/complements because it is one end of the theoretical spectrum.

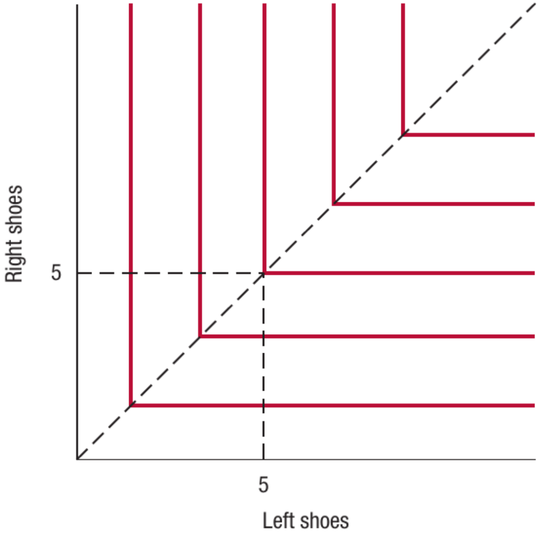
Indifference Curves for Perfect Substitutes.

The indifference curves for perfect substitutes are straight lines. Because the consumer only cares about the total amount of Advil purchased, two 200-milligram regular-strength tablets are a perfect substitute for one 400-milligram extra-strength tablet.



Indifference Curves for Perfect Complements.

Indifference curves for perfect complements are L-shaped (*Leontief preferences*). Assuming that a left shoe is of no value without a right shoe and vice versa, a consumer's indifference curves for left and right shoes are vertical above the 45-degree line and horizontal below it, with a **kink** where they meet.



UTILITY

When discussing consumers' preferences, economists often use a concept called **utility**. This is simply a numeric value indicating the consumer's relative well-being—higher utility indicates greater satisfaction than lower utility.

To describe a consumer's preferences over consumption bundles, we assign a utility value to each bundle; the better the bundle in the eyes of the consumer, the higher the value. Usually, we describe these values using mathematical formulas called **utility functions**.

Given a utility function, we can determine which of any two bundles the consumer likes better simply by comparing their utility values: he prefers the one with the higher value and is indifferent between bundles whose values are identical.

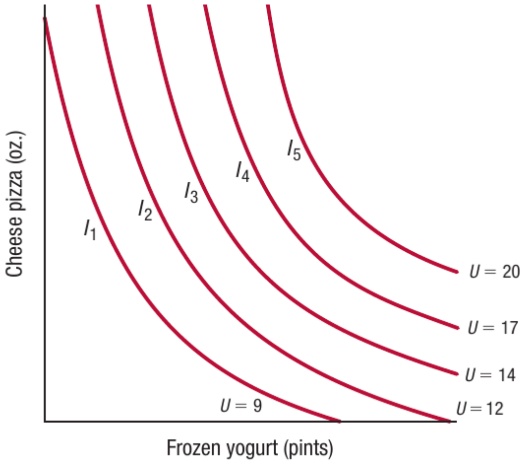
For example, the formula $U(F, C) = 2F + 5(F \times C)$ assigns utility values to consumption bundles based on pints of frozen yogurt, F, and ounces of cheese pizza, C. Then the utility value:

1. associated with12 pints of yogurt and 3 ounces of pizza is $204 = (2 \times 12) + (5 \times 12 \times 3)$.
2. associated with 9 pints of yogurt and 4 ounces of pizza is $198 = (2 \times 9) + (5 \times 9 \times 4)$.
3. associated with 17 pints of yogurt and 2 ounces of pizza, is $204 = (2 \times 17) + (5 \times 17 \times 2)$.

In this case, the utilities associated with the first and third bundles are the same, and both are higher than the utility associated with the second bundle. Therefore, the consumer is indifferent between the first and third bundles, and prefers both to the second bundle.

UTILITY

From Indifference Curves to Utility Functions and Back

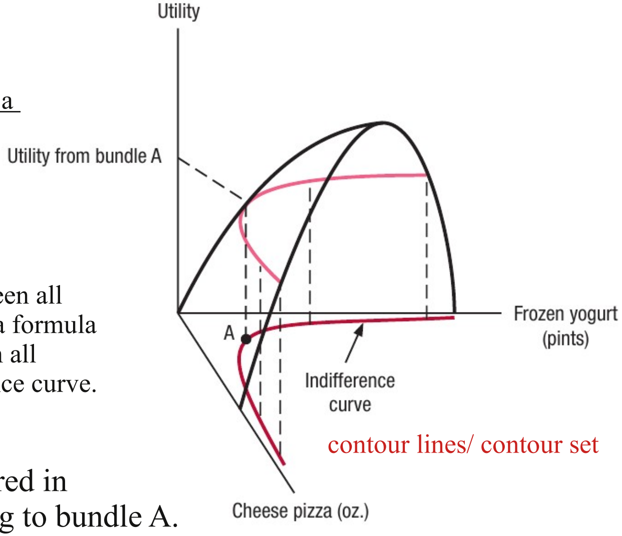


Consumers don't actually have utility functions; they have preferences. A utility function is a formula that an economist develops to summarize consumer preferences. Starting with information about preferences, then, *how do we derive an appropriate utility function*?

In general, a utility function must assign the same value to all the bundles on a single indifference curve. So all we need to do is choose a utility value for each indifference curve, picking higher values for indifference curves that correspond to higher levels of well-being.

When the More-Is-Better Principle holds, indifference curves that are further from the origin have larger utility values.

> For example, look at the right Figure, which shows five indifference curves (labeled I_1 through I_5) for someone who consumes yogurt and pizza. As shown in the figure, we've assigned utility values of 9 to I_1, 12 to I_2, 14 to I_3, 17 to I_4, and 20 to I_5. This utility function represents the consumer's preferences: bundles with a higher utility value are always preferred to bundles with a lower value, and the consumer is indifferent between any bundles with the same utility.



We can also start with a utility function and construct the associated indifference curves. To find an indifference curve, all we need to do is fix a level of utility and identify all the bundles that will deliver it.

> For example, take the utility function $U(F, C) = F \times C$. Choose any utility value, say 10. The consumer will be indifferent between all combinations of frozen yogurt and cheese pizza that satisfy the equation $10 = F \times C$. We can rewrite this equation as $C = 10/F$, a formula that describes a single indifference curve. If we select any other utility value, call it U, the consumer will be indifferent between all combinations of yogurt and pizza that satisfy the formula $U = F \times C$, so the formula $C = U/F$ describes the associated indifference curve. In other words, the utility function $U(F, C) = F \times C$ and the indifference curve formula $C = U/F$ summarize the same preference

Another way: For any consumption bundle, like A, the level of utility corresponds to the height of the hill pictured in the right figure. The light red curve shows all the points on the hill that are just as high as the point corresponding to bundle A. The dark red curve directly below it (at "ground level") shows the combinations of yogurt and pizza that are associated with the points on the light red curve. The dark red curve is the indifference curve passing through bundle A.

Ordinal versus Cardinal Utility

Information about preferences can be either **ordinal** or **cardinal**.

- **Ordinal information** allows us to determine only whether one alternative is better or worse than another.
- **Cardinal information** tells us something about the intensity of those preferences—it answers the question "How much worse?" or "How much better?"

During the 19th century and for much of the 20th century, many prominent scholars, including the influential moral philosopher Jeremy Bentham (1748–1832), thought that utility functions should provide cardinal information about preferences. According to this view, people are "pleasure machines"—they use consumption goods as inputs to produce utility as an output. Bentham and others argued that the aim of public policy should be to maximize the total utility generated through economic activity.

In modern microeconomic theory, utility functions are only intended to summarize ordinal information. If one consumption bundle has a utility value of 10 and a second has a utility value of 5, we know the consumer prefers the first to the second, but it doesn't necessarily make him twice as happy.

Most economists believe that there's no meaningful way to measure human well-being on an absolute scale. From the modern "ordinalist" perspective, the scale used to measure utility is completely arbitrary. So they reject cardinal interpretations of utility.

You can probably say whether you're generally happier today than you were yesterday (ordinal). But you can't measure the difference in your happiness.

When we change the scale used to measure utility, the consumer's family of indifference curves, and therefore his preferences, remain unchanged.

For example, let's examine the utility function $U(F, C) = 2 \times F \times C$, which assigns exactly twice as many "utils" (units of utility) to each consumption bundle as the utility function $U(F, C) = F \times C$, considered above. With this new function, the consumer's indifference curve formula is $C = 0.5U/F$ instead of $C = U/F$. For any given value of U, these two formulas generate different indifference curves. But if we plug any value of U into the formula $C = 0.5U/F$, and plug a value twice as large into the formula $C = U/F$, we generate the same indifference curve. Therefore, the two formulas generate the same family of indifference curves.

Monotonic transformation cannot change preference.

UTILITY

Utility Functions and the Marginal Rate of Substitution

Knowing a consumer's utility function helps us analyze his behavior because it allows us to determine which trade-offs he is willing to make.

Marginal utility: the change in the consumer's utility resulting from the addition of a very small amount of some good, divided by the amount added.

- Mathematically, if ΔX is the tiny change in the amount of a good X and ΔU is the resulting change in the utility value, then the **marginal utility of X**, written MU_X, is: $MU_X = \frac{\Delta U}{\Delta X}$. If U(X,Y) is differentiable, we define the marginal utility as the partial derivative of the utility function with respect to the quantity of that good: $MU_X = \frac{\partial U(X,Y)}{\partial X}$
- The **marginal rate of substitution** for any good, call it X, with any other good, call it Y, equals the ratio of the marginal utility of X to the marginal utility of Y. Mathematically, it is $MRS_{XY} = \frac{MU_X}{MU_Y}$, why?

  Proof: A small change in X, call it ΔX, causes utility to change by approximately $MU_X = \Delta X$. Similarly, a small change in Y, ΔY, causes utility to change by approximately $MU_Y = \Delta Y$. If the combination of these changes leaves us on the same indifference curve, then utility is unaffected, so the changes offset: $MU_X \times \Delta X = MU_Y \times \Delta Y$. Since along an indifference curve, we have $-\Delta Y/\Delta X = MU_X / MU_Y$, and because $MRS_{XY} = -\Delta Y/\Delta X$, then we have above expression.

- *The marginal utility associated with a particular good is completely meaningless*.
- However, the ratio of marginal utilities give us the marginal rate of substitution, which is meaningful.
  That is because when we change the units used to measure utility, we don't change the ratio of marginal utilities.

The Problem Bobby enjoys reading books and watching movies. His utility function is $U(M, B) = M + 2B$. Find a formula for his indifference curves. What do these curves look like? What is Bobby's marginal utility of movies? Of books? What is his MRS for movies with books? From his perspective, are movies and books perfect substitutes, perfect complements, or something else?

UTILITY

Some Special Utility Functions

Utility functions for **perfect substitutes** have the form U(X, Y) = AX + BY for some positive numbers A and B.
Utility functions for **perfect complements** corresponds to utility functions of the form U(X, Y) = min{AX, BY} for some positive numbers A and B.

Cobb-Douglas Utility Functions [mathematician Charles Cobb and economist Paul Douglas (also a former U.S. Senator)]

$$U(X,Y) = X^a Y^b \qquad \mathrm{MU}_X = aX^{a-1}Y^b, \quad \mathrm{MU}_Y = bX^aY^{b-1}, \text{ and } \mathrm{MRS}_{XY} = \frac{MU_X}{MU_Y} = \frac{a}{b}\frac{Y}{X}$$

Here X and Y measure the quantities of goods such as pizza and frozen yogurt, while a and b are constants that may differ from one consumer to another. For example, he function U(X, Y) = XY is a special case of a Cobb-Douglas utility function (a = b = 1).

Quasi-Linear Utility Functions

$$U(X,Y) = f(X) + Y$$

Here, f(X) is a function, the value of which is assumed to increase along with X. For example, the utility function $U(X,Y) = 2\sqrt{X} + Y$ belongs to this class.

The constants a and b affect the relative attractiveness of the two goods. An increase in a enhances the marginal value of X to the consumer relative to that of Y. Because more Y is then required to compensate for any loss of X, MRS_{XY} rises.

Indifference Curves for Special Utility Functions.

Part (a) shows two families of indifference curves for Cobb-Douglas utility functions: one with a = b = 1 (the light red curves) and the other with a = 3 and b = 1 (the dark red curves). At points where the curves from the two families intersect, the ones corresponding to the Cobb-Douglas utility function with the higher value of a are steeper.

Part (b) shows a family of indifference curves for a quasi-linear utility function. In that case, if we shift any indifference curve either upward or downward, it will lie right on top of another indifference curve from the same family.