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TOPICAL ISSUE

Control of a Free-Flying Space Manipulation Robot with a Payload

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Abstract—The control modes of a free-flying space manipulation robot during the transportation and installation of a building element on a large space structure are considered. It is proposed to save the working fluid of the gas-jet engines of the robot body when moving along the trajectory by using the mobility of a manipulator with electromechanical drives for the angular stabilization of the mechanical "robot—transported element" system. Conditions ensuring the stable motion of the manipulator in the working area when installing the element on the assembled structure are obtained. A stability domain is determined to select the initial configuration of the manipulator before installing the element and its admissible change during installation. The control algorithms are designed based on the principle of dynamic feedback systems.

Keywords: free-flying space manipulation robot, working area, technical controllability, control algorithm, motion stability

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1. INTRODUCTION

In space technology, space manipulation robots (SMRs) are used for servicing and assembling various-purpose spacecraft in orbit. Such robots fly freely in space due to their movement system independently of the spacecraft that delivered the robot to the destination point [1]. The feasibility of developing this type of space robotic devices was declared at the 6th IFAC Symposium on Space Control (1974), which was held under the leadership of Academician B.N. Petrov [2]. Currently, there are two ways of connecting spacecraft and modules in space: direct docking and berthing (docking by means of a manipulator) [3]. The latter term defines several operations, such as soft docking, payload stowage in the receiving compartment of a cargo spacecraft, etc. This paper considers the problem of attaching a building element to a large space structure (LSS) being assembled in orbit by means of an SMR, another operation of the same type. As in [3], the mass of the LSS element may significantly exceed that of the manipulator, whose gripper with the held payload may be located at a significant distance from the center of gravity of the SMR body and the entire mechanical system. The kinematic algorithm used to control the manipulator converts control signals into the required rotational velocities of the actuators; this algorithm considers the geometric and kinematic constraints determined by the current configuration of the manipulator.

Structurally, a freely-flying space manipulation robot (FSMR) is designed as a platform with one or several manipulators attached. The platform is equipped with control devices and a set of actuators that provide the required orientation and desired trajectory of the platform in outer space. Such SMRs are called free-flying robots in [4, 5]. One of the first domestic publications [6] presented a methodology for analyzing the dynamics of a manipulator on a moving base and one solution to capture a payload in inertial space by means of an FSMR in the cases of its stabilizable and non-stabilizable body.

An assembly operation performed in space includes two stages as follows. In the first stage, an FSMR approaches the installation zone of a building element; at the end of this stage, the robot hovers in the vicinity of the docking point of the element with an LSS. The boundary of the working area is determined, on the one hand, by safety conditions (no possible contact between the hovering robot and the LSS when installing the element) and, on the other, by goal attainability conditions (the successful installation of the element with a given orientation in the required point of the LSS) [7, 8]. The latter is the content of the

† Deceased.

second stage of the assembly operation. The first stage of the assembly operation is implemented by means of a control system of the translational and angular movements of the FSMR using reaction forces and torque applied by the actuators to the robot body. The manipulator with the transported element is stationary in this stage, and its configuration should be as close to optimal as possible [7].

The list of problems arising in the design of control systems for FSMRs was described in [1]. This paper considers those of manipulator control during the FSMR movement to the working area and in it. When controlling an FSMR in its working area in the free-floating mode (i.e., the angular position control system of the robot body is disabled), the challenges include the narrowing of the working area [4, 9] and the presence of dynamic singularities [10, 11]. The dynamics and kinematics of the mechanical structure of an FSMR in this mode are significantly complicated due to the disturbing effect of the manipulator motions on the body position [12, 13]. Therefore, we consider manipulator control based on the feedback principle using information about the angular position of the robot body and estimates of the deviation of the manipulator's endpoint from the target point [14, 15].

We present solutions of two problems as follows. The first problem arises if it is necessary to save the working fluid of the gas-jet engines of the robot body when moving along the trajectory. This problem is solved using the manipulator mobility. The second problem is related to the stabilization of the manipulator movement in the working area when installing the transported element on the LSS.

2. THE MECHANICAL STRUCTURE OF A FREE-FLYING SPACE MANIPULATION ROBOT: KEY FEATURES

The mechanical structure of an FSMR is a set of elements connected through joints. The main element is the body equipped with a control system and jet engines. Multilink manipulators are attached to the body. A gripper, a device for capturing and holding a payload during manipulation operations of an FSMR, is rigidly fixed on the end link of each manipulator. This mechanical structure is characterized by many degrees of freedom and the mutual influence of the movements of its elements. The FSMR body responds to dynamic reaction forces arising from the movements of the manipulator's links. When controlling the configuration and angular movements in such a mechanical system, it is necessary to consider the dynamic coupling between the body and manipulators [15].

To illustrate the key features of its mechanical structure, we consider the plane motion of an FSMR with one three-link manipulator [16] as one possible setup. The coordinates X_0 and Y_0 of the FSMR body's center of gravity and its angle of rotation ϑ are the generalized coordinates describing the position of the FSMR body in the inertial frame CXY whose axes are associated with an LSS. They form the vector $q_K = (X_0, Y_0, \vartheta)^{\mathrm{T}}$. The vector $q_\alpha = (\alpha_1, \alpha_2, \alpha_3)^{\mathrm{T}}$ consists of the generalized coordinates of the inter-link angles that specify the manipulator's configuration. The vector $\rho_{BA} = (X_\varepsilon, Y_\varepsilon)^{\mathrm{T}}$ represents the controlled coordinates, i.e., the deviations of the endpoint of the transported payload $B = (X_B, Y_B)$ from the target point $A = (X_A, Y_A)$ in the inertial frame: $X_\varepsilon = X_A - X_B$ and $Y_\varepsilon = Y_A - Y_B$.

The plane motion of this setup is described by the equation

$$A(q)\ddot{q} = M(q,u) + F(q,\dot{q}) \tag{1}$$

for the vector $q = (q_K, q_\alpha)^{\mathrm{T}}$ with the following notations [15, 16]: the matrix $A(q) \in R^{6\times 6}$ contains blocks of symmetric matrices specifying the mass-inertia parameters of the body and manipulator ($A_{11}(q) \in R^{3\times 3}$ and $A_{22}(q) \in R^{3\times 3}$, respectively) as well as the dynamic interaction coefficients of the body and manipulator's links ($A_{12}(q) \in R^{3\times 3}$ and $A_{21}(q) \in R^{3\times 3}$, respectively, where $A_{12}(q) = A_{21}(q)$); $M(q, u) = (M_K, M_\alpha)^{\mathrm{T}}$, where $M_K \in R^3$ is the vector of control actions applied to the robot body and $M_\alpha \in R^3$ is the vector of control actions applied by actuators to the manipulator's links when feeding control voltages $u(t)$ to the former's inputs; finally, $F(q, \dot{q}) = (f_K(q, \dot{q}), f_\alpha(q, \dot{q}))^{\mathrm{T}}$ is the vector of nonlinear disturbance functions from Coriolis and centrifugal forces. The expressions for calculating the elements of these matrices and vectors were given in [16].

In this paper, the actuators of manipulator's links are assumed to have DC motors with independent excitation [16]. In the first approximation with the time constant of the motor and mechanical nonlinearities being neglected [16, 17], the dynamics of each jth actuator ($j = \overline{1, 3}$) are described by the equations

$$J_j i_{pj}\ddot{\alpha}_j = (k_{bj}k_{aj})^{-1}u_j(t) - k_{aj}^{-1}i_{pj}\dot{\alpha}_j - M_{Rj}(t), \quad j = \overline{1, 3}, \tag{2}$$

where $\alpha_j \in q_\alpha$, J_j is the moment of inertia of the jth actuator reduced to the motor shaft, i_{pj} is the gearing ratio, M_{Rj} is the moment of dynamic load on the motor shaft from the manipulator, and k_{bj} and k_{aj} are constants.

Self-braking mechanical gears [15, 16] are often used in the link actuators to reduce the energy cost of controlling the FSMR manipulator. The self-braking property is provided by imposing an impulse coupling on the moving link of the manipulator; as a result, $\dot{\alpha}_j = 0$ and $u_j = 0$. Due to self-braking, the equation for α_j disappears from (1), which is mathematically expressed as a decrease (or increase) in the order of system (1) by $2 \times r$, where r denotes the number of simultaneously braked (or unbraked) links of the manipulator. The FSMR model (1), (2) serves to design robot motion control algorithms in different operation modes of the manipulator [15].

When designing angular motion control algorithms for an FSMR, it is necessary to consider the property of technical controllability, a necessary condition for the performance of the robot [18]. For an FSMR, this property means that the angular motion of the robot body and the movements of the manipulator's links must be controllable. In other words, when control signals are supplied to change their positions, these changes must be implemented in a required direction and with a given speed. It is reasonable to analyze the controllability of FSMRs based on a simplified angular motion model of the robot mechanical system under the following assumptions [18]: for each q_i, there exists M_i with the constraint $|M_i| \leqslant M_i^{\max} > 0$, $i = \overline{1, 6}$; given $M_j = 0$, $i, j = \overline{1, 6}, j \neq i$, at a time instant $t = t_0$ and $q_i(t) = \dot{q}_i(t) = \ddot{q}_i(t) = 0$ $(t < t^*)$, the desired response to $M_i^{\max} > 0$ is $q_i(t) \geqslant 0$ at time instants $t > t^*$; the velocities $\dot{q}$ are small enough to nullify the terms of the full motion model that depend on the products of $\dot{q}$; the motion equations of the model can be linearized with respect to the position $q = q^*$, where $q_i^* = \text{const}$, $i = \overline{1, 6}$.

The angular motion model linearized in the position q^* has the form

$$A(q^*)\Delta\ddot{q} = P(q^*)M(q), \tag{3}$$

where $\Delta q = q - q^*$, $A(q^*)$ is a positive definite matrix, and the matrix $P(q^*)$ relates the generalized forces to the vector of control forces and moments [18].

The FSMR with model (3) is controllable in Δq_i, $i = \overline{1, 6}$ in the position $q = q^*$ if under the zero initial conditions $\Delta q_i(t) = \Delta\dot{q}_i(t) = \Delta\ddot{q}_i(t) = 0\ \forall t < t_0$, supplying the maximum control $|M_i(t)| = M_i^{\max}\ \forall t \geqslant t_0$ at the time instant t_0 generates an acceleration $\Delta\ddot{q}_i(t) \geqslant \eta_i \neq 0$ of the same sign as $M_i(t)$ irrespective of the other control actions $M_i(t)$ $(j = \overline{1, 6}; j \neq i)$, where η_i are known characteristic values of the mechanical system. According to the theorem proved in [18], the controllability of the FSMR in the neighborhood of the point $q = q^*$ is determined only by the design parameters of the robot's mechanical system and not by the vector of control constraints $M^{\max}$.

3. TRAJECTORY MOTION CONTROL OF A FREE-FLYING SPACE MANIPULATION ROBOT

Consider a section of the FSMR trajectory that starts when turning the cruise engine off and ends when reaching the boundary of the manipulator's working area. On this section, the FSMR motion control system must eliminate the residual lateral velocity and the lateral deviation of the robot from the line of sight as well as stabilize the angular position of its body. If gas-jet engines are used as actuators, the problem is to reduce the consumption of the onboard working fluid of the engines. This problem will be solved for control design by the joint use of gas-jet nozzles and torque actuators of the manipulator. For brevity, such control will be referred to as cost-efficient control.

When the FSMR with the transported element of the LSS moves along the trajectory, its manipulators must be fixed in a position that aligns the center of gravity of the robot's mechanical system with the center of application of the control forces [7]. The manipulator with the transported element is stationary, and the trajectory and angular motion of the FSMR are controlled using basic algorithms that form the control actions M_ϑ applied to the robot body from gas-jet nozzles. Under cost-efficient control on the considered trajectory section, we propose to provide the limited mobility of the manipulator. In this case, the required angular stabilization of the body is implemented through motion exchange between the robot body and the manipulator's links by applying control torques from the electromechanical actuators of the manipulator, the electrical energy of which can be recovered. Due to restrictions on the admissible movements of the manipulator's links to control the angular position of the FSMR body, the angles of rotation of the links may reach the limit values, making further control by the electromechanical method impossible. When restoring the initial configuration of the manipulator, the required angular orientation of the body is provided by means of gas-jet nozzles. For brevity, this restoration process will be called the manipulator's unloading mode.

The following features must be considered when forming cost-efficient control algorithms: there are bounded domains of varying the coordinates of the manipulator's links ($|\alpha_i(t)| \leqslant \alpha_{i\,\max}$, $|\dot{\alpha}_i(t)| \leqslant \dot{\alpha}_{i\,\max}$); the deviation of the manipulator's links from the initial position displaces the FSMR's center of gravity relative to the center of application of the forces and, therefore, is a parametric disturbance in the robot orientation system; gas-jet actuators are relay elements, and the torques of electromechanical actuators are bounded; the conditions of technical controllability by the vector q_α hold in the entire domain of varying the coordinates of system (1).

Let $u_\vartheta(\vartheta, \dot{\vartheta}, t)$ be the basic orientation control algorithms for the FSMR and $u_\alpha(\alpha, t)$ be the manipulator's configuration control algorithms. In the case of cost-efficient control, initially implemented by the control action $M_{\alpha 1}$ from the arm link actuator only, the FSMR motion equations of motion of the SCMR have the form

$$A_1(q)\ddot{q}_1 = F_q + F_q^d, \tag{4}$$

where $q_1 = (\vartheta, \alpha_1, X_0, Y_0)^{\mathrm{T}}$, $F_q = (0, M_{\alpha 1}, 0, F_y)^{\mathrm{T}}$ is the vector of control actions used, $F_q^d = (M_\vartheta^d, 0, 0, 0)^{\mathrm{T}}$ is the vector of disturbances considered, $A_1(q) = [a_{ij}(\alpha_1, \lambda)]$ is a symmetric matrix, and λ is the vector of the parameters of the FSMR and LSS element.

The coordinate ϑ varies according to the solution of Eq. (4) of the form

$$\ddot{\vartheta} = k_0(k_\alpha M_{\alpha 1} + k_d M_\vartheta^d + k_y F_y), \tag{5}$$

where $k_0 = (\det[A_1(q)])^{-1}$; $k_\alpha(\alpha_1, \lambda) = -D_{21}q$ is the efficiency coefficient of $M_{\alpha 1}$ when applied to ϑ, representing the algebraic complement of the element $a_{21}(q)$ for $\det[A_1(q)]$; $k_d(\alpha_1, \lambda) = D_{11}(q)$ is the efficiency coefficient of the exogenous disturbance M_ϑ^d on ϑ, representing the algebraic complement of the element $a_{11}(q)$ for $\det[A_1(q)]$; finally, $k_y(\alpha_1, \lambda) = -D_{41}(q)$ is the efficiency coefficient of the control channel F_y, representing the algebraic complement of the element $a_{41}(q)$ for $\det[A_1(q)]$.

We construct the stabilizing control action $M_{\alpha 1}$ for the coordinate ϑ in the form

$$M_{\alpha 1}[u_{\alpha 1}(t)] = -\tilde{k}_0 k_A(\vartheta + k_{\dot{\vartheta}}\dot{\vartheta}), \tag{6}$$

where $k_A = (k_m k_b)^{-1}$ is the static gain of the actuator; $\tilde{k}_0$ is a tunable parameter of the control algorithm $u_{\alpha 1}(t)$ (if necessary); $k_{\dot{\vartheta}}$ is a constant.

If the control action (6) is implementable, then the linear part of the basic algorithm is designed to ensure stability and the desired quality of the motion (5). Considering (6), let us write (5) as

$$\ddot{\vartheta} + k_A k_{\dot{\vartheta}} \tilde{k}_0 \bar{k}_\alpha(\alpha_1, \lambda)\dot{\vartheta} + k_A \tilde{k}_0 \bar{k}_\alpha(\alpha_1, \lambda)\vartheta = \bar{M}_\Sigma^d(\alpha_1, \lambda, t), \tag{7}$$

where $\bar{k}_\alpha(\alpha_1, \lambda) = k_0 k_\alpha(\alpha_1, \lambda)$ is the reduced efficiency coefficient of the control action $M_{\alpha 1}$ (according to the technical implementability theorem and [18], this coefficient satisfies the condition $\bar{k}_\alpha(\alpha_1, \lambda) > 0$ $\forall(\alpha_1, \alpha_2) \in (0, \pm\pi)$); $\bar{M}_\Sigma^d(\alpha_1, \lambda, t) = \bar{k}_d(\alpha_1, \lambda) M_\vartheta^d + \bar{k}_y(\alpha_1, \lambda) F_y$ is the resulting reduced disturbing torque; finally, $\bar{k}_y(\alpha_1, \lambda) = k_0 k_y(\alpha_1, \lambda)$ and $\bar{k}_d(\alpha_1, \lambda) = k_0 k_d(\alpha_1, \lambda)$.

If the parameters λ are known and $\alpha_1(t)$ is measured, we propose an algorithm to change $k_0(t)$ in (6) based on the stationarity condition

$$\tilde{k}_0(t)\bar{k}_\alpha(\alpha_1, \lambda) = K, \tag{8}$$

where K is a constant satisfying the desired quality of motion for ϑ.

Under (8), the coefficients in (7) are constant, which ensures the stability of motion for ϑ. For the motion (7) with (8), the required static accuracy $|\vartheta(t)| \leqslant \vartheta_{\min}$ of FSMR orientation, where $\vartheta_{\min}$ is a given value, is achieved by fulfilling the condition $k_A \geqslant (K\vartheta_{\min})^{-1}[M_\Sigma^d(\alpha_1, \lambda, t)]_{\max}$.

The control action (5) is implemented by supplying the voltage $u_{\alpha 1}(t)$ to the input of the electrical actuator (2) of the manipulator's shoulder link according to the algorithm [16]

$$u_{\alpha 1}(t) = -\frac{\tilde{k}_0}{i_g}\left[\left(1 + \frac{k_{\dot{\vartheta}}}{k_m J_m}\right)\vartheta + (k_m J_m)^{-1}\int \vartheta dt + k_{\dot{\vartheta}}\dot{\vartheta} + \frac{k_b J_L}{\tilde{k}_0 J_m}\dot{\alpha}_1\right], \tag{9}$$

where $M_L \approx -J_L\ddot{\alpha}_1$ and J_L is the moment of inertia of the load reduced to the shoulder joint.

The algorithm (9) is used until reaching the domain $|\vartheta(t)| < \vartheta_{\min}$ by ϑ. The system then switches to a nonlinear algorithm containing nonlinearities (dead zone and hysteresis) to organize highly cost-efficient unilateral auto oscillations in this coordinate domain.

Generally, the residual nonzero initial conditions ϑ_0, $\dot{\vartheta}_0$ and the forced motions generated by exogenous disturbances are damped using the algorithm (9) by changing the coordinates $\alpha_i(t)$. The damping process ends either with steady-state small oscillations in the domain $|\vartheta(t)| \leqslant \vartheta_{\min}$ or with $\alpha_i(t)$ reaching the constraints. In the latter case, it becomes necessary to return the manipulator to the initial position (the unloading mode) in order to implement the orientation control method for the FSMR using its mobility again. In the unloading mode, the manipulator's links are transferred to the initial state, $\alpha_i(t) \to \alpha_i^*$, under the action of its control $M_\alpha(u_\alpha)$ while keeping ϑ in the domain $|\vartheta(t)| \leqslant \vartheta_{\min}$. The angular stabilization of the body is implemented by the torque $M_\vartheta(u_\vartheta) \leqslant M_\vartheta^{\max}$, where $M_\vartheta^{\max}$ is the existing constraint. In this mode, robot control is a multilink control problem in an essentially nonlinear system with control constraints.

When describing the unloading mode in (4), it is necessary to assume

$$F_q = (M_\vartheta, M_{\alpha 1}, 0, F_y)^{\mathrm{T}}.$$

Then the behavior of the coordinate ϑ is described by the equation

$$\ddot{\vartheta} = \bar{k}_{M\vartheta}(\alpha_1, \lambda)M_\vartheta(u_\vartheta) + f_p(\alpha_1, \lambda), \tag{10}$$

where $\bar{k}_{M\vartheta}(\alpha_1, \lambda)$ is the efficiency coefficient of $M_\vartheta(u_\vartheta)$, calculated by analogy with (5); $f_p(\alpha_1, \lambda) = \bar{M}_\Sigma^d(\alpha_1, \lambda, t) + \bar{k}_\alpha(\lambda_1, \lambda)M_{\alpha 1}$ are disturbances for $|\vartheta(t)| \leqslant \vartheta_{\min}$.

When designing control algorithms for the coordinates ϑ and α, it is necessary to consider the contradictory requirements for the operation of each subsystem. Minimizing the roll time of the manipulator's link that has reached the constraint implies using the maximum achievable speeds $\dot{\alpha}_{1\max}$ of the output shaft of the actuator (under the existing constraints). However, the disturbances $f_p(\alpha_1, \lambda)$ arising in the control subsystem ϑ under the constraint $M_\vartheta^{\max}$ may violate the orientation accuracy requirements. In this case, the roll rate of the manipulator's link should be bounded by a value smaller than $\dot{\alpha}_{1\max}$. It is reasonable to use the phase plane method based on (10) to determine the optimal controller parameters ensuring the desired dynamics in the unloading mode.

Let the basic nonlinear algorithm for stabilizing the angular position of the FSMR $u_\vartheta(\vartheta, \dot{\vartheta}, t)$ generate unilateral auto oscillations represented on the phase plane $(\vartheta, \dot{\vartheta})$ as a limit cycle Γ_0 (Fig. 1). In the unloading mode of the manipulator, under the action of $f_p(\alpha_1, \lambda)$, the undisturbed cycle Γ_0 is transformed into another stable cycle Γ_1. The cycle Γ_1 is formed so that for the maximum possible value $f_{p,\max}(\alpha_1, \lambda)$, its phase trajectory would not cross the limits of the admissible deviations of the controlled coordinates ($|\vartheta| = \vartheta_p$, $|\dot{\vartheta}| = \dot{\vartheta}_p$); see the dashed box in Fig. 1. When the unloading process is complete, the original cycle is restored ($\Gamma_1 \to \Gamma_0$), and a return to the control action $M_\alpha(u_\alpha)$ follows.

Simultaneously with FSMR orientation control, the correction system continues to work for the transverse displacements of the body: when the deviation exceeds $Y_0 = Y_{0\min}$, it generates the control action F_y in (4). Since the mechanical structure of the FSMR has an unbalanced configuration, the disturbing torque $M_{Fy}^d = F_y x_c$ arises in the orientation control channel; its compensation by the action of $M_\alpha(u_\alpha)$ may be insufficient. Therefore, when F_y acts in the orientation control system, it is necessary to provide an automatic transition to the efficient nonlinear control $M_\vartheta(u_\vartheta)$.

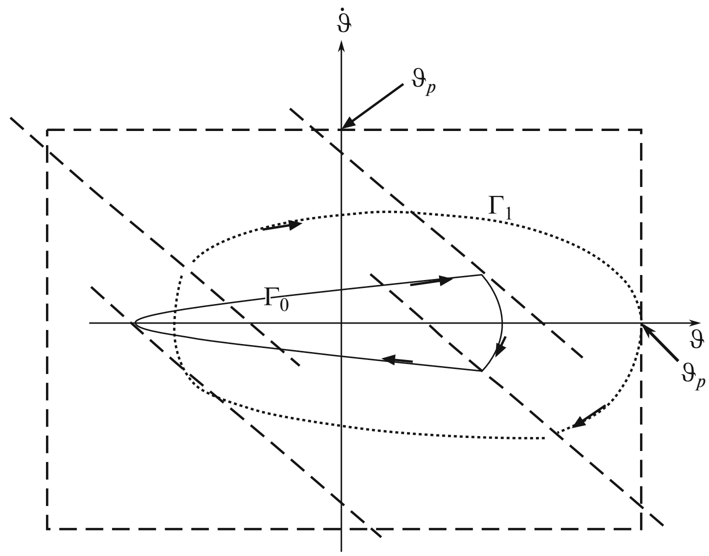

Fig. 1. Limit cycles in the manipulator's unloading mode.

4. MANIPULATOR CONTROL WHEN INSTALLING AN ELEMENT ON AN OBJECT

Consider FSMR manipulator control in the soft installation mode of a building element on an LSS in the working area. Here, the motion of the robot body when using manipulator's self-braking actuators does not change q_α. The controlled motion of each link must not change the values of other interlink angles. This property, characteristic of the class of mechanical systems under consideration, holds under the conditions of technical controllability (if satisfied during the design process). These conditions allow neglecting the mutual influence of joints and, consequently, treating the matrix A_{22} as a diagonal one. For $t \geqslant t_0$, where t_0 is the time of entering the working area, the coordinates of the FSMR mechanical system with the transported element of the LSS change with sufficiently small rates. Hence, linear mathematical models can be used to design algorithms [15]. The terms of the functions $f_K(q, \dot{q})$ and $f_\alpha(q, \dot{q})$ contain products of small values $(\dot{q}_i \dot{q}_j)$, $i, j = \overline{1, 6}$; hence, their contribution to FSMR dynamics can be neglected in the first approximation. In the presence of all these features, the motion of (1) can be approximately described by

$$A_r(q)\ddot{q} = M(q,u), \tag{11}$$

where the matrix $A_r(q)$ consists of the blocks $A_{r,11} = A_{11}$, $A_{r,12} = A_{12}$, $A_{r21} = 0$, and $\dot{q}(t_0) = 0$.

When the coordinates X_ε and Y_ε are selected as the controlled ones, manipulator control by the coordinates q_α becomes open-loop and it is possible to reach the domain $|X_\varepsilon| \leqslant X_{\varepsilon,\min}$, $|Y_\varepsilon| \leqslant Y_{\varepsilon,\min}$ by purposefully varying $q_\alpha(t)$, where $X_{\varepsilon,\min}$ and $Y_{\varepsilon,\min}$ are given values. Using only the rotating degrees of freedom of the mechanical FSMR system allows neglecting the displacement of the body's center of gravity and treating q_1 and q_2 as constants. If the control actions by α_1 and α_2 are formed in the soft installation mode of the element and the manipulator's end link is fixed (α_3 is a constant), then the motion for X_ε and Y_ε based on (11) is described by

$$\begin{aligned} \ddot{X}_\varepsilon &= d_{11}(q)M_{\alpha 1} + d_{12}(q)M_{\alpha 2}, \\ \ddot{Y}_\varepsilon &= d_{21}(q)M_{\alpha 1} + d_{22}(q)M_{\alpha 2}, \end{aligned} \tag{12}$$

where

$$d_{11}(q) = b_\Delta a_{44}^{-1}\Big[(b_m - a_{23}^2)(a_{14}a_{33} - a_{13}a_{34}) + b_3(a_{24}a_{33} - a_{23}a_{34})\Big],$$

$$d_{12}(q) = b_\Delta a_{55}^{-1}\Big[(b_m - a_{23}^2)(a_{15}a_{33} - a_{13}a_{35}) + b_3(a_{25}a_{33} - a_{23}a_{35})\Big],$$

$$d_{21}(q) = b_\Delta a_{44}^{-1}\Big[(b_m - a_{13}^2)(a_{24}a_{33} - a_{23}a_{34}) + b_3(a_{14}a_{33} - a_{13}a_{34})\Big],$$

$$d_{22}(q) = b_\Delta a_{55}^{-1}\Big[(b_m - a_{13}^2)(a_{25}a_{33} - a_{23}a_{35}) + b_3(a_{15}a_{33} - a_{13}a_{35})\Big],$$

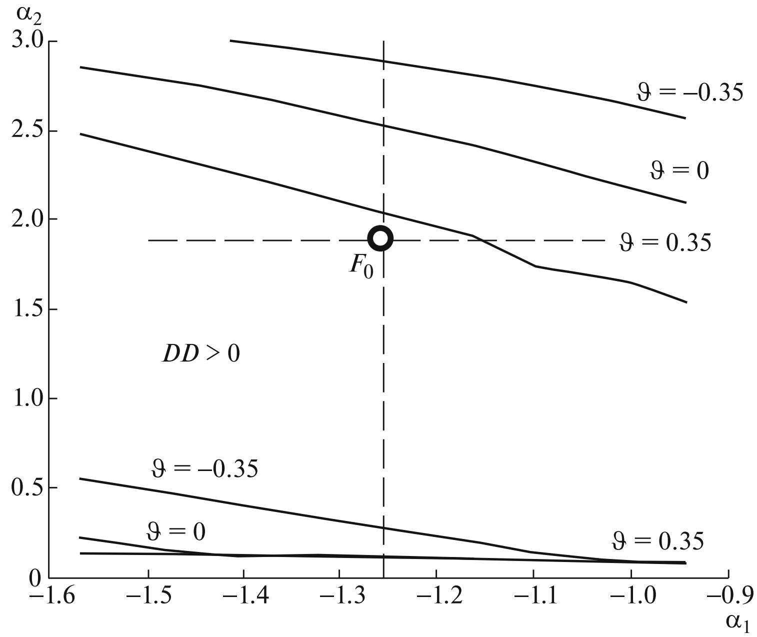

Fig. 2. The effect of the angle ϑ on the boundaries of the stability domain.

$b_\Delta = [a_{33}m_S(b_m - a_{13}^2 - a_{23}^2)]^{-1}$, $b_m = a_{33}m_S^2$, $b_3 = a_{13}a_{23}$, and m_S denotes the FSMR mass.

The coefficients $d_{ij}(q)$, $i, j = \overline{1, 2}$, vary due to their dependence on the angular position of the FSMR body through ϑ and on the joint angles α_1 and α_2. During manipulator control in the working area, its links may take positions in which $d_{ij}(q) < 0$, causing instability for X_ε and Y_ε.

If the information about X_ε, Y_ε, $\dot{X}_\varepsilon$, and $\dot{Y}_\varepsilon$ is available, then stable control by X_ε and Y_ε is ensured by the PD algorithms

$$\begin{aligned} M_{\alpha 1} &= k_{0x}(k_{1x}X_\varepsilon + k_{2x}\dot{X}_\varepsilon), \\ M_{\alpha 2} &= k_{0y}(k_{1y}Y_\varepsilon + k_{2y}\dot{Y}_\varepsilon), \end{aligned} \tag{13}$$

where the gains k_{jx}, k_{jy} ($j = \overline{0, 2}$) must be appropriately chosen to stabilize the trivial solution of system (12), (13). These stability requirements are defined when analyzing the characteristic equation

$$\sum_{j=0}^{4} c_j \lambda^j = 0,$$

where

$$\begin{gathered} c_0 = \Delta d k_{1x}k_{1y}; \quad c_1 = \Delta d(k_{1y}k_{2x} + k_{1x}k_{2y}); \\ c_2 = \Delta d k_{2x}k_{2y} - (k_{1x}k_{0x}d_{11} + k_{2x}k_{0y}d_{22}); \\ c_3 = -(k_{1y}k_{0x}d_{11} + k_{2y}k_{0y}d_{22}); \quad c_4 = 1; \\ \Delta d = k_{0x}k_{0y}(d_{11}d_{22} - d_{12}d_{21}). \end{gathered}$$

The necessary stability condition $c_j > 0$ $\forall j = \overline{0, 4}$ holds for $\Delta d > 0$ and $\operatorname{sgn} d_{11} \neq \operatorname{sgn} k_{0x}$, $\operatorname{sgn} d_{22} \neq \operatorname{sgn} k_{0y}$. The condition $\Delta d > 0$ does not depend on the gains k_{jx}, k_{jy}, $j = \overline{0, 2}$, and is satisfied under the relations

$$\begin{gathered} (\operatorname{sgn} d_{11} \neq \operatorname{sgn} d_{22} \wedge \operatorname{sgn} d_{12} = \operatorname{sgn} d_{21}) \vee (\operatorname{sgn} d_{11} = \operatorname{sgn} d_{22} \wedge \operatorname{sgn} d_{12} \neq \operatorname{sgn} d_{21}); \\ (\operatorname{sgn} d_{11} = \operatorname{sgn} d_{22} \wedge \operatorname{sgn} d_{12} = \operatorname{sgn} d_{21} \wedge |d_{11}d_{22}| > |d_{12}d_{21}|) \\ \vee (\operatorname{sgn} d_{11} \neq \operatorname{sgn} d_{22} \wedge \operatorname{sgn} d_{12} \neq \operatorname{sgn} d_{21} \wedge |d_{11}d_{22}| < |d_{12}d_{21}|). \end{gathered} \tag{14}$$

If the variations of $d_{ij}(q)$ do not violate the condition $\Delta d > 0$, then the stability conditions are satisfied by varying the gains in (13). If α_1 and α_2 are measurable during FSMR manipulator control, then $d_{ij}(q)$ can be calculated and the stability conditions can be maintained by tuning the gains in (13) at appropriate time instants.

Based on (14), it is reasonable to form the stability domain in the coordinates α_1 and α_2. Information about this domain serves to choose the initial configuration of the manipulator before the element installation and the admissible variation of α_1 and α_2 during the installation process. The topology of the stability domain depends on the values ϑ and α_3, which determine the relative position of the body and the element to be installed. As one example with the data from [15], Fig. 2 shows a segment of the stability domain for $\alpha_3 = -0.2\pi$ and three initial positions of the FSMR body ($\vartheta = [-0.35; 0; 0.35]$). Here, the point F_0 indicates the initial position of the links: $\alpha_1(t_0) = -1.26$ and $\alpha_2(t_0) = 1.58$. According to Fig. 2, increasing the positive value of the angle ϑ reduces the stability domain where $\Delta d > 0$. (In this figure, the stability domain is indicated by $DD > 0$.) This fact decreases the range of varying the angles α_1 and α_2 when the element is installed by the manipulator.

Note that the variation of the angle α_3 (the gripper's position) has a smaller effect on the boundaries of the stability domain compared to the variation of the angular position of the FSMR body.

5. CONCLUSIONS

The features of the mechanical structure of the FSMR have been analyzed, and a solution has been proposed to reduce the consumption of the onboard working fluid of gas-jet engines during transportation of the LSS element and during its assembly in orbit. This solution involves the mobility of the manipulator to stabilize the angular position of the FSMR body. On separate sections of the FSMR motion trajectory, the control is jointly implemented by two types of actuators: gas-jet nozzles and torque electromechanical actuators of the manipulator. The mathematical models of FSMR motion used in this paper are convenient for designing control algorithms based on the feedback principle and studying the manipulation processes of the FSMR. The control algorithms of the FSMR satisfy the conditions of technical controllability and maintain the required configuration of the mechanical structure of the robot during the transportation and installation of the LSS element. Under sufficiently small velocities of the manipulator's joints, the algorithms presented above provide in the working area a soft installation of the element at a given point of the LSS. The stability domain in the space of the angles of the manipulator's joints has to be determined in advance in order to choose the initial configuration of the robot's mechanical system before the manipulation operation and the range of these angles during the operation that ensures stable motion.

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