Notion of Control-Law Module and Modular Framework of Cooperative Transportation Using Multiple Nonholonomic Robotic Agents With Physical Rigid-Formation-Motion Constraints

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***Abstract*—Consider cooperative manipulation and transportation of a rigid body by multiple two-wheeled nonholonomic robotic agents that attached to it, the agents are then physically constrained to maintain rigid-formation-motion (RFM); thus the system has *two physical motion-constraints at two levels*: 1) the nonholonomic constraint at the individual level and 2) the RFM constraint at the system level. First, we provide a novel notion: the encapsulation of a category of control with certain constraints for one motion-mode as a *control-law module* (CLM), any concrete control law with such constraints is called an *instance* of the CLM; here two CLMs are provided as the examples. Then we provide an RFM control framework by decomposing a feasible RFM configuration-path as a concatenation of partitions, with one type of CLMs for each partition; thus any instance for each partition can be designed separately and incorporated easily with the interchangeable property, which makes the framework modular, flexible, and adaptive, to satisfy different kinematics requirements. As a result, the transportation is achieved by RFM control of agents. Also, the RFM framework implies a valuable *rigid-closure-method* for accurate rigid body manipulation even when agents are not attached to the body.**

***Index Terms*—Control-law module (CLM), cooperative control, cooperative manipulation, cooperative transportation, encapsulation design methodology, flocking, formation, modular framework, multiagent system, nonholonomic constraint, rigid-closure-method, rigid-formation-motion (RFM).**

I. Introduction

Multiagent systems have many applications in cooperative control [1]–[3], [44]–[47], cooperative manipulation, and transportation [8], [31]–[33]. For example, a single robot may manipulate or transport a not-too-large object; however, for a large object, a single robot is insufficient to perform such operation, in this case, it is feasible and efficient to use multiple robots, which may include wheeled robots [10], quadrotors [31], and vessels [24], etc. In many scenarios, using a system of multiple cooperative wheeled robots (typically with physically nonholonomic constraints) is an appropriate choice.

Cooperative control of multiple physical robotic agents can be roughly divided into two categories: one category is pure motion coordination, e.g., flocking of inertial agents with general nonequal velocity coupling and position coupling [2], [3], swarming with general directed and weighted topology [21], consensus [5]–[7], [9], [43] (including high-order consensus [49]), formation (most work considering formation of agents in the Euclidean space [11], [20], [22], [34], [45]–[48], fewer on non-Euclidean manifold, e.g., a sphere manifold [4]), with/without trajectory following, etc.; another category is cooperative manipulation or transportation of a rigid body by multiple agents, such as prehensile manipulation [23], nonprehensile manipulation [8], [10], [24]–[32], caging [26], [29], [32], transportation [8], [10], [29]–[31] and grasp [32], manipulation with multiple cooperative unilateral thrusters [8], etc., some of which may also include motion coordination in the manipulation.

For motion coordination of multiple physical robotic agents, there are also roughly two categories: one is motion coordination without physical rigid-formation constraints (most researches on formation are in this category; note that in [42], the rigid means topologically rigid); another is with physical rigid-formation constraints, e.g., multiple agents attached to a rigid body [8]. With such constraints, the agents must maintain a rigid formation during their collective motion, or simply rigid-formation-motion (RFM), which is an important consideration for control design of such system.

Generally, there are two motion control paradigms for cooperative agents: 1) centralized control and 2) decentralized control. From the perspective of whether formation is rigid during the evolution, one can conclude that: 1) in motion coordination, the agents with decentralized control may converge to a desired formation, but generally the agents cannot maintain RFM in the whole process, the agents have RFM only in the steady state and 2) the control for RFM cannot be decentralized (or at least fully decentralized), since RFM does not allow deformation.

This paper considers transportation of a planar rigid body by multiple two-wheeled nonholonomic robots that attached to it. In this case, since the body is rigid, the agents must have RFM. As a result, the manipulation and transportation can be achieved by RFM control of the agents, generally without considering the design of the forces applied to the body (if the inertia of the body can be neglected).

Compared with relative researches on formation, here one difficulty is that: the system has two physical motion-constraints at two levels, i.e., the nonholonomic constraint at the individual level and the RFM constraint at the system level, which are important considerations.

Also, it is impossible to ensure RFM of nonholonomic agents with arbitrary initial orientations, which will violate the nonholonomic constraints. If the RFM of the agents is successfully maintained, then the agents, as a whole, can be viewed as a virtually large rigid body which is generally not small-time locally controllable (STLC) [25]. Thus the smooth trajectory tracking or path following of the RFM is not an easy task or even not feasible in many cases.

Here we first provide a novel notion, i.e., the encapsulation of a category of control with certain constraints (including initial and terminal conditions for control of agents) for one motion-mode as a control-law module (abbreviated as CLM); any concrete control law with such constraints is called an instance of the CLM. As the examples, two CLMs are presented for RFM control of the agents, respectively aimed at performing a rigid-translation mode and a rigid-rotation mode, based on two sets of the formation-parameters. Then we provide a control framework by essentially decomposing a feasible configuration path (or trajectory) of the agents with RFM constraint as a concatenation of some single-mode partitions,

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and in each partition, control the agents with an instance of the corresponding CLM, thus provide a noncomplex solution for RFM control of nonholonomic agents.

The proposed framework is modular and flexible, with an instance of the CLM for each partition designed separately and incorporated easily. Each partition has its own continuous dynamics, with the common characteristics described by the CLM and the specific characteristics determined by the instance of the CLM. Instances for each partition are interchangeable: one can simply replace an instance for a partition to suit different kinematic requirements for this partition, without influence on other partitions. As a result, the algorithmic architecture is also modular and flexible. Such methodology is similar to an encapsulation design of components in software engineering.

This paper has four primary contributions.

1) From the perspective of methodology, this paper provides a novel notion of CLM and an encapsulation design method, which is fundamental to build a control framework that is modular and flexible.
2) From the perspective of motion control, this paper extends control for a simple two-wheeled robot to a system of such agents attached to a rigid body, and presents a modular framework for RFM control of nonholonomic agents with different motion modes, which satisfies the two physical constraints at two levels. As compared with relative work, in [1]–[3], [20], [22], [35], and [39]–[41], the nonrigid formation of the agents during their transient evolving process allows certain deformation; compared with the undirected formations of neighboring agents [36], [38] designated via graphs, the formation of all the agents in this paper can be viewed strictly as a rigid body; as for [37] which considers only translation mode with fixed formation-parameters, our framework allows different motion modes (for the reason, refer to Section VI-A) with considerations of mode-switches and formation-parameters that mode-depended.
3) From the perspective of manipulation, the RFM control framework provides a modular and flexible solution on nonprehensile manipulations [8], [10], [26]–[29] of a rigid-body without explicitly considering the pull or push forces applied to the body.
4) Also, the RFM control framework implies the *rigid-closure-method* for accurate rigid body manipulation, as compared with the object closure for caging [26], [29], [32]. To be specific, the rigid-closure-method for manipulation is that: put the rigid body (possibly with not-too-large inertia) in the closure formed by the initial positions of agents, with perfect size such that the body has no degree of freedom in it; then control the agents for RFM; as a result, the initial closure maintains rigid during motion, thus the accurate manipulation of the body is achieved by RFM control of agents. In this case, more perfect size and more rigidity of the closure, more precise control on the body. Thus the RFM control framework is useful for manipulation (the modular and flexible properties preserve) even when the agents are not attached on the body. As a comparison, the object closure [26], [29], [32] is not rigid, thus requires a relatively low degree of precision; e.g., in [29], the agents move toward a goal position for caging a convex object in translation motion while maintaining the closure such that the object in the closure has some degrees of freedom but only cannot be removed from it.

As a less rigorous case, for cooperative transportation of a deformable body [12], [13], the distances between agents may allow certain (e.g., 5%) variations, as compared with the RFM control, the quasi-rigid motion control of agents, which can be possibly more or even fully decentralized, will be investigated in future.

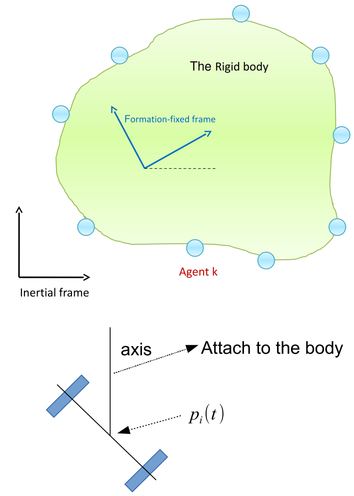

Fig. 1. Top: collective motion of multiple robots (represented as circles) attached to a rigid body. Bottom: two-wheeled robot.

This rest of this paper is arranged as follows. Sections II and III describe the problem. Section IV is the notion of CLM. Sections V and VI provide the framework, with examples in Section VII. Section VIII is the conclusion.

II. Scenarios and Motivation

A. Scenario I: RFM of Attached Two-Wheeled Robots

Consider transportation of a rigid body using n two-wheeled differentially-driven robotic agents in the 2-D Euclidean space $\mathcal{W} = \mathbb{R}^2$, each agent is attached to the body (Fig. 1) by the axis that passes through the middle of the two wheels, and the agent can turn in place freely about the axis. Define the position $p_i(t) = [x_i(t), y_i(t)]^T \in \mathbb{R}^2$ of agent i as the middle position of the two wheels in the inertial frame, with orientation $\theta_i(t) \in \mathbb{R}$, where $x_i(t), y_i(t) \in \mathbb{R}$ are the X–Y coordinates, respectively. The differentially-driven agent can be described as [17], [22]

$$\begin{aligned} \dot{x}_i(t) &= v_i(t)\cos\theta_i(t) \\ \dot{y}_i(t) &= v_i(t)\sin\theta_i(t), \quad i = 1, 2, \ldots, n \\ \dot{\theta}_i(t) &= u_i(t) \end{aligned} \tag{1}$$

where $v_i(t)$ and $u_i(t) \in \mathbb{R}$ are the linear and angular velocity controllers, respectively, and assume $v_i(t) \geq 0$, $i = 1, 2, \ldots, n$, i.e., the agents can only go forward, not backward. The extension of agents that can go both forward and backward is not difficult, and thus omitted here.

B. Scenario II: Rigid-Closure-Method Using Two-Wheeled Agents

Consider cooperative manipulation of a rigid body (possibly with not-too-large inertia) by nonattached agents with the kinematics described as (1). First, put the body in the closure formed by the initial positions of the agents, with perfect size such that the body has no degree of freedom in it (also refer to Fig. 1 except the attachment). Then, to manipulate the body accurately, just ensure the initial closure of the agents rigid during collective motion. As a result, the manipulation is achieved by RFM control of agents.

III. Problem Description: RFM Constraint of Nonholonomic Agents

A. Two Physical Constraints at Two Levels

The system for each scenario has two physical constraints: 1) the nonholonomic (two-wheeled) constraint at the individual level and 2) the RFM constraint at the system level by either the physical attachment (scenario I) or physical requirement (scenario II). In control design, the two levels of constraints are important considerations.

B. Configuration of RFM

Denote $\mathcal{A}$ as the initial formation of agents, denote $d_{ij}(t) = p_i(t) - p_j(t)$ as the relative position of agents i, j.

Definition 1: The agents are said to have RFM, or formation $\mathcal{A}$ is said rigid, if and only if

$$||d_{ij}(t)|| \equiv ||d_{ij}(0)||, \ \forall t \geq 0, \ i, j = 1, 2, \ldots, n \tag{2}$$

where $|| \cdot ||$ represents the Euclidean norm; the agents have rigid translation if and only if $d_{ij}(t) \equiv d_{ij}(0), \ \forall t \geq 0, \ i, j = 1, 2, \ldots, n$.

Remark 1: The rigid translation of agents does not necessarily mean that the agents move in a straight line trajectory.

When formation $\mathcal{A}$ is ensured rigid, define $\bar{p}(t) \in \mathbb{R}^2$ and $\psi(t) \in \mathbb{R}$ as the position and orientation of the formation-fixed frame $\mathcal{F}$ in the inertial frame, respectively. Then the configuration $q(t)$ of formation $\mathcal{A}$ can be described as $q(t) = (\bar{p}(t), \psi(t))$, which is a function of the agents' positions. Without loss of generality, define the initial position $\bar{p}(0)$ and orientation $\psi(0)$ of frame $\mathcal{F}$ as

$$\bar{p}(0) \ := \bar{p}_0 = \frac{1}{n}\sum_{i=1}^{n} p_i(0), \ \ \psi(0) := \psi_0 = 0. \tag{3}$$

Denote the initial configuration and desired configuration as $q_0 = (\bar{p}_0, \psi_0)$ and $q_d = (\bar{p}_d, \psi_d)$, respectively. The agents are expected to have RFM and transport, as a whole, from initial configuration q_0 to desired configuration q_d in the free space $\mathcal{C}_{\text{free}} = \{q \in \mathcal{C} | \mathcal{A}(q) \cap \mathcal{O} = \emptyset\}$, where $\mathcal{C}$ is the configuration space, $\mathcal{A}(q) \subset \mathcal{W}$ is formation $\mathcal{A}$ at configuration q, and $\mathcal{O} \subset \mathcal{W}$ is the obstacle region.

IV. Notion of CLM in General Background

First, we describe the notion of the CLM in general background, then provide a solution for RFM control described in Section II.

Consider n cooperative agents with the concatenation state x

$$\dot{x} = f(x, u) \tag{4}$$

where u is the control input. We want to control the system from the initial state α_0 to a desired state α_d, under some state-constraints, particularly the physical RFM constraint of agents with other constants [e.g., the hard (physical) nonholonomic constraint at the individual level of each agent and obstacle-avoidance constraint].

There are some kinematic *modes* (e.g., rigid-translation mode, rigid-rotation mode in Section V, and some complex modes, e.g., rigid-translation while rotation) of the system, denote $c_m > 1$ as the number of modes used in the system control.

Definition 2: Define the CLM $\mathcal{M}_c := \mathcal{M}_c(x_{\text{ini}}, x_{\text{ter}})$ as the set of all (or some types of) admissible control laws that control the state of the system from initial state x_{ini} to terminal state x_{ter} in a single mode, with the state-constraints ensured during the process, provided that $x_{\text{ini}}, x_{\text{ter}}$ are compatible with the mode. The subscript $c \in \{1, 2, \ldots, c_m\}$ denotes the mode for this module.

Definition 3: Any concrete control law that belongs to a CLM is called an instance of the CLM.

Remark 2: The common characteristics of the control are described by the CLM (e.g., see Section V), and the specific characteristics of the control are determined by its instance.

Assume a path of the system will reach the $m-1$ ($m > 1$) sequential states (denoted as $\alpha_1, \alpha_2, \ldots, \alpha_{m-1}$) from α_0 to α_d, e.g., a feasible path $\xi^*(\alpha_0 \to \alpha_d)$ can be divided as m partitions

$$\xi^*(\alpha_0 \to \alpha_d) = \xi^*(\alpha_0 \to \alpha_1) \cup \xi^*(\alpha_1 \to \alpha_2) \ldots \cup \xi^*(\alpha_{m-1} \to \alpha_d).$$

Denote all such feasible paths as the sets $\Omega(\alpha_0 \to \alpha_d) = \Omega(\alpha_0 \to \alpha_1) \cup \Omega(\alpha_1 \to \alpha_2) \ldots \cup \Omega(\alpha_{m-1} \to \alpha_d)$, and assume each partition has a single mode. For convenience, denote $\alpha_m = \alpha_d$, we call α_i, $i = 0, 1, \ldots, m-1$, the partition-states.

As a result, we may use the CLMs to build a framework which provides a feasible and noncomplex solution that is modular and flexible in the sense that, different control-law instances for each partition can be incorporated or interchangeable easily within this framework, according to different kinematic requirements of the system, e.g., maximum velocities of agents. Thus the framework has flexibility and convenience for implementation in both software architecture and algorithms.

For implementation of the framework, there are more considerations: 1) what are the formation-parameters of agents for different CLMs? 2) conditions of partition-states and partition-switches; and 3) error handling. For example, to ensure the RFM, for each partition, the control laws of the system will unavoidably require the formation-parameters that are mode depended and partition-state depended.

Remark 3: As a comparison, for a general system with state x, different operation modes results in a switched system: $\dot{x} = f_\sigma(x)$, where $\sigma : [0, \infty) \to \mathcal{P}$ is a piecewise constant function of time, as a switching signal, $\mathcal{P}$ is some index set [18]. Generally, $\dot{x} = \mathbf{0}$ is not necessarily required during switches if without special constraints. However, in RFM control of nonholonomic agents, a complete stop ($\dot{x} = \mathbf{0}$) may be required for the partition-switches to avoid violation of the two physical constraints (Section III-A).

V. CLMs for RFM Control

For the RFM of nonholonomic agents, the system is highly nonlinear, and generally, it is not STLC [25] for nonholonomic agents with the RFM constraint. Here, simply consider two examples of modes of RFM, i.e., pure rigid-rotation mode and pure rigid-translation mode, to reduce the complexity, which are sufficient to illustrate the framework. One may also consider additional complex modes, which are not the focus of this paper and omitted here.

A. Partition-Switches

The system requires to satisfy the two physical constraints (Section III-A) with an appropriate initial condition for each mode. To avoid violation of the two physical constraints, the complete stop (particularly $\dot{p}_i = \mathbf{0}$, $i = 1, 2, \ldots, n$) of agents is preferred during the partition-switch. Else, in scenario I, the wheels of agents may slip, which reflects the violation of the physical constraints, although the formation is rigid since the agents are attached to the rigid body; while for scenario II, the wheels of agents may not slip since no hard (i.e., attachment) physical constraints at the system level, but the result is that the formation of agents is not accurately rigid.

Proposition 1: Consider two adjacent partitions P_k and P_{k+1} of the configuration path of formation $\mathcal{A}$, $k \in \{1, 2, \ldots\}$, denote q_k as the configuration of $\mathcal{A}$ at the partition-switch. For partition-switch

1) From P_k with rotation mode to P_{k+1} with rotation mode, the complete stop of formation $\mathcal{A}$ (either an asymptotic stop or a hard stop) is required at configuration q_k.
2) From P_k with rotation mode to P_{k+1} with translation mode, the complete stop is required.
3) From P_k with translation mode to P_{k+1} with rotation mode, the complete stop is required.
4) From P_k with translation mode to P_{k+1} with translation mode, the complete stop is not required.

B. Encapsulation of CLM for the Two Modes

Lemma 1: Assume the consensus orientation at $t = 0$

$$\theta_i(0) = \theta_0, \ \ i = 1, 2, \ldots, n \tag{5}$$

where $\theta_0 \in \mathbb{R}$. If

$$v_i(t) = v_j(t),\ \ u_i(t) = u_j(t),\ \ i, j = 1, 2, \dots, n \tag{6}$$

then the agents maintain the rigid formation in translation.

Remark 4: The control laws are same for every agent, thus the result is straightforward with initial condition (5). As for agent i, assume the solution $p_i(t) := \bar{p}_i(t)$, compare with agent j, we have $p_j(t) = \bar{p}_i(t) - d_i + d_j$, so $p_i(t) - p_j(t) \equiv d_i - d_j = d_{ij}(0)$.

The following is the CLM for rigid-translation.

CLM for Rigid-Translation

Inputs: initial configuration q_{ini} and terminal configuration q_{ter}.
Compatibility check:
- i) if the orientations of q_{ini} and q_{ter} are equal,
- ii) check or reset the orientations of agents to satisfy (5)
- iii) condition for partition-switches: Proposition 1.

Control-law structure and restrictions: (6).
Configuration transformation: from q_{ini} to q_{ter}.

Definition 4: Define a set of formation-parameters of agents as

$$\gamma_i := p_i(0) - p_r \in \mathbb{R}^2,\ \ i = 1, 2, \dots, n \tag{7}$$

for rotation control mode, where $p_r \in \mathbb{R}^2$ is an arbitrary position in the inertial frame $\mathcal{F}$ that serves as the rotation point of the RFM.

Remark 5: The formation-parameters remain same in rigid rotation in the sense that: $||\gamma_i|| \equiv ||p_i(t) - p_r||$, $i = 1, 2, \dots, n$.

Lemma 2: Represent the formation parameter γ_i defined in (7) as $\gamma_i = ||\gamma_i||(\cos\phi_i, \sin\phi_i)^T$, where $\phi_i \in \mathbb{R}$, $i = 1, 2, \dots, n$. Assume the agents have the initial orientations: $\theta_i(0) = \phi_i - \pi/2$, or

$$\theta_i(0) = \phi_i + \frac{\pi}{2},\ \ i = 1, 2, \dots, n. \tag{8}$$

Then if the control-law structure of the agents

$$v_i(t) = ||\gamma_i||u(t),\ \ u_i(t) = u(t),\ \ i = 1, 2, \dots, n \tag{9}$$

satisfies, where $u(t) \in \mathbb{R}$ is an arbitrary function, then the agents have RFM and rotate about p_r in counter-clockwise ($u(t) > 0$) or clockwise ($u(t) < 0$) direction.

Proof: Refer to Section VII. ■

Remark 6: Define the turning radius r of the RFM as $r := ||\bar{p}(0) - p_r|| = 1/n||\sum_{i=1}^{n} \gamma_i||$, which is the average turning radius of the agents. If $p_r = \bar{p}(0)$, then $r = 0$, the rigid formation rotates about its center $\bar{p}(t) \equiv \bar{p}(0)$ to adjust its orientation. In practice, the selection of p_r is crucial, an improper p_r may lead to collision with possible obstacles. Selection of suitable p_r and thus the turning radius r to for obstacle-avoidance is a pure geometry problem and omitted here.

The following is the CLM for rigid-rotation.

CLM for Rigid-Rotation

Inputs: initial configuration q_{ini} and terminal configuration q_{ter}, and possibly the rotation point p_r.
Compatibility check:
- i) check or determine p_r,
- ii) recalculate the formation-parameters according to Definition 4,
- iii) check or reset the initial orientations according to Lemma 2,
- iv) condition for partition-switches: Proposition 1.

Control-law structure and restrictions: (9).
Configuration transformation: from q_{ini} to q_{ter}.

Remark 7: To avoid violation of the two physical constraints when the system switches from one mode to another, the formation-parameters need to be recalculated, and the states of agents (particularly the orientations) need to be adjusted.

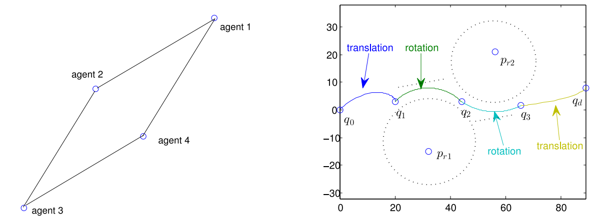

Fig. 2. Left: illustration of a formation shape of agents. Right: example of the trajectory of RFM. The dotted circles and lines represent obstacles, the details of partitions 1–4 are illustrated in Fig. 3.

Remark 8: For the two-wheeled differentially-driven robot, it can turn in place freely, so it is easy to adjust its orientation before the implementation of the control laws of each partition.

C. Control-Law Instances

The CLMs provide two kinds of abstract control-law restrictions on $v_i(t)$ and $u_i(t)$ for rigid rotation and rigid translation, respectively, any concrete instance can be then designed provided that the restrictions are satisfied. Certainly, an instance for each module can be designed with different complexity to satisfy different requirements of kinematic properties of RFM which is not the focus of this paper.

VI. Modular Framework for RFM Control

A. Why Hybrid Mode-Switched Framework

Since the RFM control is designed in a centralized paradigm (Section I), one may wonder "why not directly plan paths of agents individually so that the RFM constraint is satisfied?" It is indeed true when $\mathcal{C}_{\text{free}}$ contains a feasible path of formation $\mathcal{A}$ with pure translation motion, in this case, control of agents is straightforward and trivial (Lemma 1) when considering vast work on path planning of a single nonholonomic robot [14]–[16].

However, when the RFM with pure rigid-translation mode is not feasible, for example, the rigid formation (as illustrated in Fig. 2) is expected to pass through a narrow environment with avoidance of obstacles, then rigid rotation control is unavoidable, and as a result, the control framework is hybrid (refer to Section III-A) and we need to consider different modes with proper mode-switches and flexibility.

B. General Description of Framework

The framework has three hierarchical phases.

Phase I: Generation of feasible reference path $\xi(q_0 \to q_d)$. Consider the agents as a virtually rigid body (the advantage is the state-dimension reduction from $3n$ to 3-D) with configuration q and formation $\mathcal{A}$, and generate a feasible path $\xi(q_0 \to q_d)$ in $\mathcal{C}_{\text{free}}$. Path planning for a rigid body has been widely investigated, e.g., in [14]–[16], the details are not the focus in this paper and thus omitted.

Then check the question: "is there a feasible path with translation motion?" If yes, use Lemma 1 plus path planning techniques for a single agent, the RFM control is trivial. If not, consider Phase II.

Phase II: Generate a feasible approximation path $\xi^*(q_0 \to q_d) \subset \mathcal{C}_{\text{free}}$, which can be divided into m feasible partitions, $m > 1$

$$\xi^*(q_0 \to q_d) = \xi^*(q_0 \to q_1) \cup \xi^*(q_1 \to q_2) \dots \cup \xi^*(q_{m-1} \to q_d)$$

where each partition $\xi^*(q_{k-1} \to q_k)$, $k \in 1, 2, \dots, m$, is in either the rigid-rotation mode or rigid-translation mode of formation $\mathcal{A}$. Then we get m feasible partition-states q_k, $k = 1, 2, \dots, m - 1$.

Phase III: RFM control for each partition. For each partition, design an appropriate instance, and according to different kinematic requirements of the system (e.g., maximum velocities of agents), different instances can be designed and easily interchangeable at this partition, without influence on other partitions. As a result, the framework is modular and flexible in software algorithmic architecture.

C. Error Handling

In scenario I, the attached rigid body itself serves as a very good stabilizer for RFM of agents, thus no additional control for robustness needed. If, in some cases, one of the agents does not follow the specified trajectory duo to modeling errors or sensor noise, etc., then the consequence is that: one or some wheels of one or more robots may slip due to the physical constraint of the rigid body to which the robots are attached. If this case occurs, the solution is also easy: simply set a threshold for the error tolerance of the wheels, if one of the robots detects the error that exceeds this threshold, then stop all robots, and recalculate the trajectory with current configuration of the system as the new initial configuration. Thus in implementation of real robots, this error handling is robust and can be rather easily incorporated into the proposed framework. The robustness for scenario II can be handled similarly.

VII. Example of RFM Control

Although design of control-law instances is not the focus in this paper, some examples would provide more understanding of the framework. This section provides some examples and an algorithm of the RFM control framework. Although these examples are rather simple, they are sufficiently enough to illustrate the framework.

A. Examples of Control-Law Instances

Denote $\psi_d \in \mathbb{R}$ as the desired orientation of the rigid formation, generally $\psi_d \in [-2\pi, 0)$ or $\psi_d \in (0, 2\pi)$. Define the rotation matrix

$$R(\theta) := \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

which rotates points in the X–Y plane counter-clockwise through an angle θ about the origin of the Cartesian coordinates.

Proposition 2: Assume $0 \leq \psi_d < 2\pi$, and the agents have the initial orientation (8). In Lemma 2, let $u(t) = k_1(\phi_i + \pi/2 + \psi_d - \theta_i(t))$, $k_1 > 0$. Then the agents maintain rigid formation and rotate about p_r in counter-clockwise direction, with orientation $\psi(t) \to \psi_d$ and position $\bar{p}(t) \to p_r + R(\psi_d)(\bar{p}(0) - p_r)$.

Proof: Refer to Section VII-C. ■

Remark 9: $u(t)$ has the same value for any agent i. The coefficient k_1 only affects the convergence rate of the agents from the initial configuration to the desired configuration. If the RFM is achieved, $\bar{p}(t)$ is always the current average position: $\bar{p}(t) = 1/n \sum_{i=1}^{n} p_i(t)$.

Remark 10: If $-2\pi < \psi_d < 0$, use the initial condition $\theta_i(0) = \phi_i - \pi/2$, $i = 1, 2, \ldots, n$, and $u(t) = k_1[\phi_i - \pi/2 + \psi_d - \theta_i(t)]$, $i = 1, 2, \ldots, n$, then the rigid formation rotates in clockwise direction.

Define a set of formation-parameters of agents as

$$d_i := p_i(0) - \bar{p}(0) \in \mathbb{R}^2, \; i = 1, 2, \ldots, n \tag{10}$$

for translation control mode, where $\bar{p}(0)$ is defined in (3).

Remark 11: $\sum_{i=1}^{N} d_i = \mathbf{0}$. For rigid translation, $p_i(t) - \bar{p}(t) \equiv d_i$.

Represent $d_i = a_i + ib_i \in \mathbb{C}$, $a_i, b_i \in \mathbb{R}$, $i = 1, 2, \ldots, n$. $\bar{p}_d = x_d + iy_d \in \mathbb{C}$ is the desired position of the formation. Then:

Proposition 3: In Lemma 1, consider the control law

$$\begin{aligned} v_i(t) &= k_1\sqrt{e_{xi}^2 + e_{yi}^2}\cos(e_{\theta i}) \\ u_i(t) &= k_2 e_{\theta i} \end{aligned}, \; i = 1, 2, \ldots, n \tag{11}$$

where $e_{xi} = x_d + a_i - x_i(t)$, $e_{yi} = y_d + b_i - y_i(t)$, $e_{\theta i} = \vartheta_i(t) - \theta_i(t)$, $\vartheta_i(t) = \operatorname{atan2}(e_{yi}, e_{xi})$, $k_1, k_2 > 0$. Then the agents maintain rigid shape and move in translation with $\bar{p}(t) \to \bar{p}_d$.

Proof: From Lemma 1, the agents have rigid translation, from Lemma 3 in Section VII-C, $e_{xi} \to 0$, $e_{yi} \to 0$, i.e., $p_i(t) \to \bar{p}_d + d_i$, $i = 1, 2, \ldots, n$, so $\bar{p}(t) = 1/n \sum_{i=1}^{n} p_i(t) \to \bar{p}_d$, the result holds. ■

Remark 12: The coefficients k_1 and k_2 only affect the convergence rate the agents moving to the desired configuration.

Remark 13: The orientation ψ_0 influences the curve of the trajectory. If $e_{\theta i}(0) = 0$, i.e., $\theta_i(0) = \vartheta_i(0)$ for all i, then the control law reduces to be the straightforward form: $v_i(t) = k_1\sqrt{e_{xi}^2 + e_{yi}^2}$, $u_i(t) = 0$, for all i, the trajectory of $\bar{p}(t)$ is a straight line from $\bar{p}(0)$ to $\bar{p}_d$. In this case, the rigid body can move to the desired position in a straight line rather than a curved line as in Fig. 3 (partition 1).

In Propositions 2 and 3, although the convergence of the control is asymptotic and requires infinite time, the rigid motion property always holds whenever the evolution interrupts.

One may use other control-law instances that can be easily incorporated in the algorithm, and omitted here.

B. Example of Transportation

Consider $\xi^*(q_0 \to q_d)$ as in Fig. 2 with $m = 4$ partitions.

1) *Partition 1:* Rigid translation from configuration q_0 to q_1.
2) *Partition 2:* Rigid rotation from q_1 to q_2 about p_{r1}.
3) *Partition 3:* Rigid rotation from q_2 to q_3 about p_{r2}.
4) *Partition 4:* Rigid translation from q_3 to q_d.

The algorithm for tracking $\xi^*(q_0 \to q_d)$ is provided as follows with a sequential execution.

Algorithm for Tracking $\xi^*(q_0 \to q_d)$

i) translation from q_0 to q_1:
   (1) set $\bar{p}(0) = \bar{p}_0$, calculate the formation-parameters d_i, $i = 1, 2, \ldots, n$, for translation,
   (2) select an orientation θ_0, adjust $\theta_i(0) = \theta_0$, $i = 1, 2, \ldots, n$,
   (3) move the formation to q_1 using control law (11),
ii) rotation from q_1 to q_2 about p_{r1}:
   (1) reset $t = 0$,
   (2) calculate the formation-parameters γ_i, $i = 1, 2, \ldots, n$,
   (3) adjust $\theta_i(0)$, $i = 1, 2, \ldots, n$, to satisfy condition (8),
   (4) control the orientation using control law in Proposition 2,
iii) rotation from q_2 to q_3 about p_{r2}:
   (1) reset $t = 0$,
   (2) recalculate new formation-parameters γ_i, $i = 1, 2, \ldots, n$
   (3) adjust $\theta_i(0)$, $i = 1, 2, \ldots, n$, to satisfy condition (8),
   (4) move the formation using control law in Proposition 2,
iv) translation from q_2 to q_d:
   (1) reset $t = 0$,
   (2) recalculate new values of d_i, $i = 1, 2, \ldots, n$,
   (3) select an orientation θ_0, set $\theta_i(0) = \theta_0$, $i = 1, 2, \ldots, n$,
   (4) move the formation to q_d using control law (11).

Remark 14: The values of the formation-parameters (7), (10) are generally different on different partitions, to be more specific for the following.

1) The formation-parameters (7) for rotation should be always recalculated in every partition for rotation.
2) The formation-parameters (10) for translation should be recalculated if the agents have previously moved in rotation.

Remark 15: As the convergence of the control is asymptotic and requires infinite time, so just interrupt the evolution when the

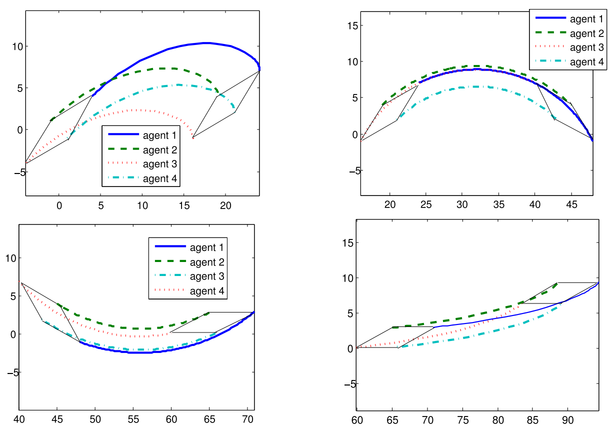

Fig. 3. Illustration of RFM of four agents, with partitions 1–4, respectively, from the top left to the bottom right. Partition 1: rigid translation (NOT rotation, refer to Remark 13). Partition 2: rigid rotation. Partition 3: rigid rotation. Partition 4: rigid translation.

formation $\mathcal{A}$ approaches the desired state sufficiently enough, the implementation of the algorithm does not affect the performance.

In the example, consider $n = 4$ agents with initial positions: $p_1(0) = [4, 4]^T$, $p_2(0) = [-1, 1]^T$, $p_3(0) = [-4, -4]^T$, and $p_4(0) = [1, -1]^T$. Then $q_0 = (\bar{p}_0, \psi_0)$, where $\bar{p}_0 = [0, 0]^T$, $\psi_0 = 0$. The configurations $q_i = [\bar{p}_i, \psi_i]$, $i = 1, 2, 3, d$, and p_{r1}, p_{r1} are that: $\bar{p}_1 = [20, 3]^T, \bar{p}_2 = [44, 3]^T, \bar{p}_3 = [65.4, 1.54]^T, \bar{p}_d = [89, 8]^T$; $\psi_1 = 0$, $\psi_2 = -\pi/2$, $\psi_3 = -\pi/6$, $\psi_d = -\pi/6$; $p_{r1} = [32, -9]^T$, and $p_{r2} = [56, 21]^T$. Then we have (refer to Fig. 3) the following.

1) *Partition 1:* The formation-parameters in partition 1 are that: $d_1 = [4, 4]^T, d_2 = [-1, 1]^T, d_3 = [-4, -4]^T, d_4 = [1, -1]^T$. $\theta_0 = \pi/4$. The motion of the agents is illustrated in Fig. 3.
2) *Partition 2:* $\gamma_1 = [-8, 16]^T$, $\gamma_2 = [-13, 13]^T$, $\gamma_3 = [-16, 8]^T$, and $\gamma_4 = [-11, 11]^T$.
3) *Partition 3:* $\gamma_1 = [-8, -22]^T$, $\gamma_2 = [-11, -17]^T$, $\gamma_3 = [-16, -14]^T$, and $\gamma_4 = [-13, -19]^T$.
4) *Partition 4:* $d_1 = [5.5, 1.4]^T$, $d_2 = [-0.3, 1.4]^T$, $d_3 = [-5.4, -1.4]^T$, and $d_4 = [0.4, -1.4]^T$. $\theta_0 = 0.1$.

C. Proofs

Proof of Lemma 2: From (1)

$$\dot{x}_i(t) = ||\gamma_i|| u_i(t)\cos\theta_i(t) = ||\gamma_i||\dot{\theta}_i(t)\cos\theta_i(t)$$

$$\int_0^t \dot{x}_i(t)dt = x_i(t) - x_i(0) = ||\gamma_i||[\sin\theta_i(t) - \sin\theta_i(0)]$$

similarly

$$\int_0^t \dot{y}_i(t)dt = y_i(t) - y_i(0) = ||\gamma_i||[-\cos\theta_i(t) + \cos\theta_i(0)]$$

$i = 1, 2, \ldots, n$, then

$$p_i(t) = ||\gamma_i||\begin{pmatrix} \cos\left(\theta_i(t) - \frac{\pi}{2}\right) + \cos\left(\theta_i(0) + \frac{\pi}{2}\right) \\ \sin\left(\theta_i(t) - \frac{\pi}{2}\right) + \sin\left(\theta_i(0) + \frac{\pi}{2}\right) \end{pmatrix} + p_i(0).$$

Note that $p_i(0) - p_r = ||\gamma_i||(\cos\phi_i, \sin\phi_i)^T$, we have

$$\begin{aligned} p_i(t) - p_r & \\ &= ||\gamma_i||\begin{pmatrix} \cos\left(\theta_i(t) - \frac{\pi}{2}\right) + \cos\left(\theta_i(0) + \frac{\pi}{2}\right) + \cos\phi_i \\ \sin\left(\theta_i(t) - \frac{\pi}{2}\right) + \sin\left(\theta_i(0) + \frac{\pi}{2}\right) + \sin\phi_i \end{pmatrix} \\ &= ||\gamma_i||\begin{pmatrix} \cos\left(\theta_i(t) - \frac{\pi}{2}\right) \\ \sin\left(\theta_i(t) - \frac{\pi}{2}\right) \end{pmatrix} \end{aligned}$$

i.e., $p_i(t) - p_r = R(\theta_i(t) - \pi/2 - \phi_i)(p_i(0) - p_r)$. And $\dot{\theta}_i(t) = u_i(t) = u(t)$, then $\theta_i(t) = \int_0^t u(t)dt + \theta_i(0)$, that is

$$\theta_i(t) - \frac{\pi}{2} - \phi_i = \int_0^t u(t)dt + \theta_i(0) - \frac{\pi}{2} - \phi_i$$

which is same for any i (note the initial condition). That is, the result holds. ■

Proof of Proposition 2: Note that $\dot{\theta}_i(t) = u(t)$, then $\theta_i(t) = \phi_i + \pi/2 + \psi_d - \psi_d e^{-k_1 t}$, so from the proof of Lemma 2, $p_i(t) - p_r = R(\psi_d - \psi_d e^{-k_1 t})(p_i(0) - p_r)$, $i = 1, 2, \ldots, n$, and since the initial orientation is zero, the result holds. ■

Inspired by the tracking of a single agent in [19], we have the following lemma.

Lemma 3: Consider a single agent i, assume its desired position: $p_{di} = (x_{di}, y_{di})^T \in \mathbb{R}^T$, p_{di} is constant. Define $e_{xi} = x_{di} - x_i(t)$, $e_{yi} = y_{di} - y_i(t)$. Consider the control law

$$\begin{aligned} v_i(t) &= k_1\sqrt{e_{xi}^2 + e_{yi}^2}\ \cos(e_{\theta i}) \\ u_i(t) &= k_2 e_{\theta i} \end{aligned} \tag{12}$$

where $e_{\theta i} = \vartheta_i(t) - \theta_i(t)$, $\vartheta_i(t) = \mathrm{atan2}(e_{yi}, e_{xi})$, $k_1 > 0$, $k_2 > 0$. Then $p_i(t) \to p_{di}$.

Proof: Define the function $V_i(t) := (1/2)(e_{xi}^2 + e_{yi}^2)$. From the definition of $\vartheta_i(t)$, $\cos(\vartheta_i) = (e_{xi})/(\sqrt{e_{xi}^2 + e_{yi}^2})$, $\sin(\vartheta_i) = (e_{yi})/(\sqrt{e_{xi}^2 + e_{yi}^2})$; and

$$\begin{aligned} \dot{e}_{xi} &= -v_i(t)\cos(\vartheta_i - e_{\theta i}) \\ &= -k_1\cos(e_{\theta i})[e_{xi}\cos(e_{\theta i}) + e_{yi}\sin(e_{\theta i})] \\ \dot{e}_{yi} &= -v_i(t)\sin(\vartheta_i - e_{\theta i}) \\ &= -k_1\cos(e_{\theta i})[e_{yi}\cos(e_{\theta i}) - e_{xi}\sin(e_{\theta i})]. \end{aligned}$$

Then $\dot{V}_i(t) = e_{xi}\dot{e}_{xi} + e_{yi}\dot{e}_{yi} = -k_1\cos^2(e_{\theta i})(e_{xi}^2 + e_{yi}^2)$, so $e_{xi} \to 0$, $e_{yi} \to 0$, i.e., $p_i(t) \to p_{di}$. ■

Remark 16: $e_{\theta i} = \pi/2$ is not a singularity (although for this value $v_i = 0$), since $u_i(t)$ will adjust the orientation and make $v_i(t) \neq 0$.

VIII. Conclusion

This paper provides a notion of CLM and a hierarchical and modular framework for RFM control of multiple nonholonomic agents. The RFM control also implies a rigid-closure-method for accurate manipulation of a rigid body. Although the framework is not optimal from the perspective of trajectory tracking, it provides a feasible and noncomplex solution, which is modular and flexible, with different control-law instances easily incorporated and interchangeable, which also provides further adaption for software algorithmic architecture and physical implementation. The framework for RFM control of agents may possibly serve as a unit (e.g., a large virtual vehicle) that may be incorporated into the transportation systems [50].

References

[1] W. Dong, "Flocking of multiple mobile robots based on backstepping," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 41, no. 2, pp. 414–424, Apr. 2011.
[2] W. Li and M. W. Spong, "Stability of general coupled inertial agents," *IEEE Trans. Autom. Control*, vol. 55, no. 6, pp. 1411–1416, Jun. 2010.
[3] W. Li and M. W. Spong, "Analysis of flocking of cooperative multiple inertial agents via a geometric decomposition technique," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 44, no. 12, pp. 1611–1623, Dec. 2014.
[4] W. Li and M. W. Spong, "Unified cooperative control of multiple agents on a sphere for different spherical patterns," *IEEE Trans. Autom. Control*, vol. 59, no. 5, pp. 1283–1289, May 2014.

[5] Z. Meng, W. Ren, Y. Cao, and Z. You, "Leaderless and leader-follower consensus with communication and input delays under a directed network topology," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 41, no. 1, pp. 75–88, Feb. 2011.

[6] H. Li, X. Liao, and T. Huang, "Second-order locally dynamical consensus of multiagent systems with arbitrarily fast switching directed topologies," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 43, no. 6, pp. 1343–1353, Nov. 2013.

[7] J. Qin, W. Zheng, and H. Gao, "Coordination of multiple agents with double-integrator dynamics under generalized interaction topologies," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 42, no. 1, pp. 44–57, Feb. 2012.

[8] W. Li and M. W. Spong, "Decomposition frameworks for cooperative manipulation of a planar rigid body with multiple unilateral thrusters," *Nonlin. Dyn.*, vol. 79, no. 1, pp. 31–46, 2015.

[9] D. Meng and K. L. Moore, "Studies on resilient control through multiagent consensus networks subject to disturbances," *IEEE Trans. Cybern.*, vol. 44, no. 11, pp. 2050–2064, Nov. 2014.

[10] G. Lionis and K. J. Kyriakopoulos, "Centralized motion planning for a group of micro agents manipulating a rigid object," in *Proc. Mediterr. Conf. Control Autom.*, Limassol, Cyprus, 2005, pp. 662–667.

[11] Z. Kan, A. P. Dani, J. M. Shea, and W. E. Dixon, "Network connectivity preserving formation stabilization and obstacle avoidance via a decentralized controller," *IEEE Trans. Autom. Control*, vol. 57, no. 7, pp. 1827–1832, Jul. 2012.

[12] A.-M. Cretu, P. Payeur, and E. M. Petriu, "Soft object deformation monitoring and learning for model-based robotic hand manipulation," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 42, no. 3, pp. 740–753, Jun. 2012.

[13] G. L. Foresti and F. A. Pellegrino, "Automatic visual recognition of deformable objects for grasping and manipulation," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 34, no. 3, pp. 325–333, Aug. 2004.

[14] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. New York, NY, USA: Wiley, 2005.

[15] S. M. LaValle, *Planning Algorithms*. New York, NY, USA: Cambridge Univ. Press, 2006.

[16] H. Choset *et al.*, *Principles of Robot Motion*. Cambridge, MA, USA: MIT Press, 2005.

[17] J. P. Laumond, *Robot Motion Planning and Control*. Berlin, Germany: Springer, 1998.

[18] D. Liberzon, *Switching in Systems and Control* (Systems & Control: Foundations & Applications). Boston, MA, USA: Birkhaüser, 2003.

[19] S. Mastellone, D. M. Stipanovic, and M. W. Spong, "Remote formation control and collision avoidance for multi-agent nonholonomic systems," in *Proc. IEEE Int. Conf. Robot. Autom.*, Rome, Italy, 2007, pp. 1062–1067.

[20] S. Mastellone, J. S. Mejia, D. M. Stipanovic, and M. W. Spong, "Formation control and coordinated tracking via asymptotic decoupling for Lagrangian multi-agent systems," *Automatica*, vol. 47, no. 11, pp. 2355–2363, 2011.

[21] W. Li, "Stability analysis of swarms with general topology," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 38, no. 4, pp. 1084–1097, Aug. 2008.

[22] A. Ailon and I. Zohar, "Control strategies for driving a group of nonholonomic kinematic mobile robots in formation along a time-parameterized path," *IEEE/ASME Trans. Mechatronics*, vol. 17, no. 2, pp. 326–336, Apr. 2012.

[23] J. M. Esposito, "Decentralized cooperative manipulation with a swarm of mobile robots: The approach problem," in *Proc. Amer. Control Conf.*, Baltimore, MD, USA, 2010, pp. 4762–4767.

[24] M. G. Feemster and J. M. Esposito, "Comprehensive framework for tracking control and thrust allocation for a highly overactuated autonomous surface vessel," *J. Field Robot.*, vol. 28, no. 1, pp. 80–100, 2011.

[25] K. M. Lynch, "Controllability of a planar body with unilateral thrusters," *IEEE Trans. Autom. Control*, vol. 44, no. 6, pp. 1206–1211, Jun. 1999.

[26] K. M. Lynch and M. T. Mason, "Dynamic nonprehensile manipulation: Controllability, planning, and experiments," *Int. J. Robot. Res.*, vol. 18, no. 1, pp. 64–92, 1999.

[27] M. W. Spong, "Impact controllability of an air hockey puck," *Syst. Control Lett.*, vol. 42, no. 5, pp. 333–345, 2001.

[28] C. B. Partridge and M. W. Spong, "Control of planar rigid body sliding with impacts and friction," *Int. J. Robot. Res.*, vol. 19, no. 4, pp. 336–348, 2000.

[29] G. A. S. Pereira, M. F. M. Campos, and V. Kumar, "Decentralized algorithms for multi-robot manipulation via caging," *Int. J. Robot. Res.*, vol. 23, nos. 7–8, pp. 783–795, 2004.

[30] N. Michael, J. Fink, and V. Kumar, "Cooperative manipulation and transportation with aerial robots," *Auton. Robot.*, vol. 30, no. 1, pp. 73–86, 2011.

[31] J. Fink, N. Michael, S. Kim, and V. Kumar, "Planning and control for cooperative manipulation and transportation with aerial robots," *Int. J. Robot. Res.*, vol. 30, no. 3, pp. 324–334, 2011.

[32] A. Rodriguez, M. T. Mason, and S. Ferry, "From caging to grasping," *Int. J. Robot. Res.*, vol. 31, no. 7, pp. 886–900, 2012.

[33] N. E. Du Toit and J. W. Burdick, "Robot motion planning in dynamic, uncertain environments," *IEEE Trans. Robot.*, vol. 28, no. 1, pp. 101–115, Feb. 2012.

[34] M. M. Zavlanos, H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Hybrid control for connectivity preserving flocking," *IEEE Trans. Autom. Control*, vol. 54, no. 12, pp. 2869–2875, Dec. 2009.

[35] M. Egerstedt and X. Hu, "Formation constrained multi-agent control," *IEEE Trans. Robot. Autom.*, vol. 17, no. 6, pp. 947–951, Dec. 2001.

[36] P. Tabuada, G. J. Pappas, and P. Lima, "Motion feasibility of multiagent formations," *IEEE Trans. Robot.*, vol. 21, no. 3, pp. 387–392, Jun. 2005.

[37] C. Belta and V. Kumar, "Motion generation for formations of robots: A geometric approach," in *Proc. Int. Conf. Robot. Autom.*, Seoul, Korea, 2001, pp. 1245–1250.

[38] T. Eren, P. N. Belhumeur, B. D. O. Anderson, and A. S. Morse, "A framework for maintaining formations based on rigidity," in *Proc. 15th IFAC World Congr. Autom. Control*, Barcelona, Spain, 2002, pp. 2752–2757.

[39] P. Ogren, M. Egerstedt, and X. Hu, "A control Lyapunov function approach to multi-agent coordination," *IEEE Trans. Robot. Autom.*, vol. 18, no. 5, pp. 847–851, Oct. 2002.

[40] R. Olfati-Saber and R. M. Murray, "Distributed structural stabilization and tracking for formations of dynamics multi-agents," in *Proc. 41st IEEE Conf. Decis. Control*, Las Vegas, NV, USA, Dec. 2002, pp. 209–215.

[41] N. Roy, P. Newman, and S. Srinivasa, "Rigidity maintenance control for multi-robot systems," in *Robotics: Science and Systems VIII*. Cambridge, MA, USA: MIT Press, 2013.

[42] D. Carboni, R. K. Williams, A. Gasparri, G. Ulivi, and G. S. Sukhatme, "Rigidity-preserving team partitions in multiagent networks," *IEEE Trans. Cybern.*, to be published.

[43] A. V. Proskurnikov, "Average consensus for nonlinearly coupled agents: Quadratic criteria," in *Proc. Eur. Control Conf.*, Strasbourg, France, Jun. 2014, pp. 2839–2844.

[44] S. Das, U. Halder, and D. Maity, "Chaotic dynamics in social foraging swarms—An analysis," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 42, no. 4, pp. 1288–1293, Aug. 2012.

[45] A. Franchi *et al.*, "Modeling and control of UAV bearing formations with bilateral high-level steering," *Int. J. Robot. Res.*, vol. 31, no. 12, pp. 1504–1525, 2012.

[46] D. Zelazo, A. Franchi, H. H. Bülthoff, and P. R. Giordano, "Decentralized rigidity maintenance control with range measurements for multi-robot systems," *Int. J. Robot. Res.*, vol. 34, no. 1, pp. 105–128, 2015.

[47] A. Franchi, G. Oriolo, and P. Stegagno, "Mutual localization in multirobot systems using anonymous relative measurements," *Int. J. Robot. Res.*, vol. 32, no. 11, pp. 1302–1322, 2013.

[48] D. Zelazo and F. Allgower, "Growing optimally rigid formations," in *Proc. Amer. Control Conf.*, Montreal, QC, Canada, 2012, pp. 3901–3906.

[49] H. Rezaee and F. Abdollahi, "Average consensus over high-order multiagent systems," *IEEE Trans. Autom. Control*, to appear.

[50] K.-D. Kim and P. R. Kumar, "An MPC-based approach to provable system-wide safety and liveness of autonomous ground traffic," *IEEE Trans. Autom. Control*, vol. 59, no. 12, pp. 3341–3356, Dec. 2014.