Section 7.6 - The Cross Product of Two Vectors

Cross Product: $\vec{a} \times \vec{b}$

- referred to as the "vector product" because the result is a vector, not a scalar.
- exists only in R^3
- trying to find a particular vector that is perpendicular to each of the given vectors.
- many answers but only need to show one.

ex: Calculate the cross product
Given Vectors $\vec{a} = (5,1,6)$ and $\vec{b} = (-1,2,4)$

Solution: Let $\vec{v} = (x,y,z)$

$\therefore \vec{v} \cdot \vec{a} = 0$ and $\vec{v} \cdot \vec{b} = 0$

$(x,y,z)\cdot(5,1,6) = 0$ $\quad$ $(x,y,z)\cdot(-1,2,4) = 0$

① $5x + y + 6z = 0$ $\quad$ ② $-x + 2y + 4z = 0$ mult by 5

$-5x + 10y + 20z = 0$

$11y + 26z = 0$

$11y = -26z$

$\boxed{z = \frac{-11}{26}y}$

$5x + y + 6\left(\frac{-11}{26}y\right) = 0$

$5x + y - \frac{66}{26}y = 0$

$5x - \frac{40}{26}y = 0$

$5x = \frac{20}{13}y$

$x = \frac{20}{65}y$

$\boxed{x = \frac{4}{13}y}$

$\therefore \vec{v} = \left(\frac{4}{13}y, y, \frac{-11}{26}y\right)$

$\therefore y\left(\frac{4}{13}, 1, \frac{-11}{26}\right)$

usually write $k\left(\frac{4}{13}, 1, \frac{-11}{26}\right)$

$\therefore$ infinite # of Solutions $\quad$ "k" is a parameter representing real #'s.

* $\vec{a} \times \vec{b}$ is the opposite of $\vec{b} \times \vec{a}$

$\therefore$ not commutative

More efficient method:

If $\vec{u} = (a_1, a_2, a_3)$ and $\vec{v} = (b_1, b_2, b_3)$

$$\begin{matrix} a_2 & & b_2 \\ a_3 & \times & b_3 \\ a_1 & \times & b_1 \\ a_2 & \times & b_2 \end{matrix}$$

$\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1)$

ex: Consider the vectors $\vec{a} = (7,1,-2)$, $\vec{b} = (4,3,6)$ and $\vec{c} = (-1,2,4)$

a) Find $\vec{a} \times \vec{b}$

$$\begin{matrix} 1 & & 3 \\ -2 & \times & 6 \\ 7 & \times & 4 \\ 1 & \times & 3 \end{matrix}$$

$\therefore \vec{a} \times \vec{b} = (6+6,\ -8-42,\ 21-4)$

$= (12, -50, 17)$

b) Confirm that $\vec{a} \times \vec{b}$ is orthogonal $(\perp)$ to $\vec{a} = (7,1,-2)$ and $\vec{b} = (4,3,6)$

$\therefore (12,-50,17) \cdot (7,1,-2) = 0$

c) Determine $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

d) Determine $(\vec{a} + \vec{b}) \times \vec{c}$ and $\vec{a} \times \vec{c} + \vec{b} \times \vec{c}$

c) $\vec{b} + \vec{c} = (3,5,10)$

$\vec{a} \times (\vec{b} + \vec{c}) = (10+10, -6-70, 35-3)$

$= (20, -76, 32)$

$$\begin{matrix} 1 & & 5 \\ -2 & \times & 10 \\ 7 & \times & 3 \\ 1 & \times & 5 \end{matrix}$$

Right Side:

$\vec{a} \times \vec{b} = (12,-50,17)$

$\vec{a} \times \vec{c} = (7,1,-2) \times (-1,2,4)$

$= (8,-26,15)$

Add

$$\begin{matrix} 1 & & 2 \\ -2 & \times & 4 \\ 7 & \times & -1 \\ 1 & \times & 2 \end{matrix}$$

$\therefore (12,-50,17) + (8,-26,15)$

$= (20,-76,32)$

$\therefore LS = RS$

May 9-9:15 AM

Properties of Cross Product

Let $\vec{p}, \vec{q}$, and $\vec{r}$ be 3 vectors in R^3, $K \in R$

Vector multiplication is not commutative:

ie) $\vec{p} \times \vec{q} = -(\vec{q} \times \vec{p})$

Distributive Law for Vector Multiplication:

$\vec{p} \times (\vec{q} + \vec{r}) = \vec{p} \times \vec{q} + \vec{p} \times \vec{r}$

Scalar Law for Vector Multiplication:

$K(\vec{p} \times \vec{q}) = (K\vec{p}) \times \vec{q} = \vec{p} \times (K\vec{q})$

May 9-9:29 AM

ex: Suppose $\vec{a}$, $\vec{b}$ and $\vec{c}$ are vectors such that $\vec{a} \times \vec{b} = (2,-1,7)$ and $\vec{a} \times \vec{c} = (10,8,-3)$.
Determine $(3\vec{b} - \vec{c}) \times (2\vec{a})$

Solution:

$$(3\vec{b} - \vec{c}) \times (2\vec{a})$$
$$= 6\vec{b} \times \vec{a} - 2\vec{c} \times \vec{a}$$
$$= -6\vec{a} \times \vec{b} + 2\vec{a} \times \vec{c}$$
$$= -6(2,-1,7) + 2(10,8,-3)$$
$$= (-12,6,-42) + (20,16,-6)$$
$$= (8,22,-48)$$
$$= (4,11,-24)$$