Chapter 8: Equations of Lines and Planes

Section 8.1 – Vector + Parametric Equations of a Line in $\mathbb{R}^2$

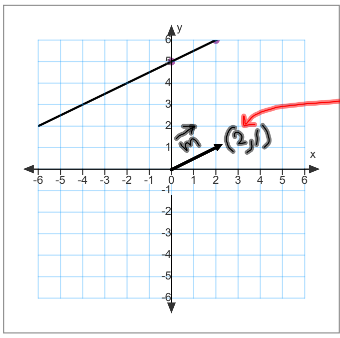
Review: In $\mathbb{R}^2$,

Equation in Standard Form: $Ax + By + C = 0$
(also called scalar equation)

Slope y-intercept form: $y = mx + b$

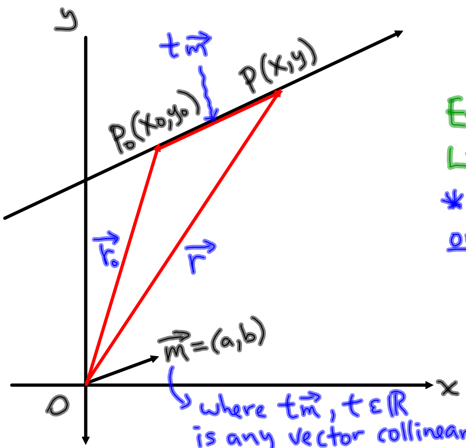
*Vectors can be used to define a line in $\mathbb{R}^2$.

ex: consider $-x + 2y - 10 = 0 \rightarrow y = \frac{1}{2}x + 5$



Slope: x-component right 2
y-component up 1

Direction Vector: a nonzero vector $\vec{m} = (a,b)$ parallel (collinear) to the given line. Vector always has its tail at the origin and head points at the point (a,b)
direction numbers



where $t\vec{m}$, $t \in \mathbb{R}$ is any vector collinear to $\vec{m} = (a,b)$

Expressing Equations of Lines Using Vectors:
* Need either 2 points or one point and a direction vector.

Examining $\triangle OP_0P$; $\vec{OP} = \vec{OP_0} + \vec{P_0P}$

but $\vec{P_0P} = t\vec{m}$, $\vec{OP_0} = \vec{r}_0$, $\vec{OP} = \vec{r}$

$\boxed{\therefore \vec{r} = \vec{r}_0 + t\vec{m},\ t \in \mathbb{R}}$ ← vector form

In component form,
$(x,y) = (x_0,y_0) + t(a,b),\ t \in \mathbb{R}$

or $(x,y) = (x_0,y_0) + (ta,tb)$

$\therefore \left.\begin{aligned} x &= x_0 + ta \\ y &= y_0 + tb \end{aligned}\right\}$ Parametric Equations a)

ex: Write a vector equation for the line passing through the points $A(1,4)$ & $B(3,1)$

Solution:

$\vec{AB} = (3-1, 1-4)$

$\vec{AB} = (2,-3)$ ← direction vector

* choose one of the points, A or B, to be the position vector.

$\therefore (x,y) = (x_0,y_0) + t\vec{m}$
$= (3,1) + t(2,-3)$

b) Determine 2 more points on the line.

Since $(x,y) = (3,1) + t(2,-3)$

let parameter t equal 1.

$\therefore (x,y) = (3,1) + 1(2,-3)$
$= (3,1) + (2,-3)$
$= (5,-2)$

or let $t=2$

$\therefore (x,y) = (3,1) + 2(2,-3)$
$= (3,1) + (4,-6)$
$= (7,-5)$

c) Determine if the point $(2,3)$ lies on the line.

Since $(x,y) = (3,1) + t(2,-3)$
$(2,3) = (3,1) + t(2,-3)$

x-coordinate	y-coordinate
$2 = 3 + 2t$	$3 = 1 - 3t$
$-1 = 2t$	$2 = -3t$
$-\frac{1}{2} = t$	$\frac{-2}{3} = t$

$\therefore$ Since t values are not the same, the point $(2,3)$ does not lie on the line.

ex2: Consider line ℓ_1

$\ell_1: \begin{cases} x = 3+2t \\ y = -5+4t \end{cases}$ } parametric equations

a) Find the coordinates of 2 points on the line.

b) Write a vector equation of the line

c) Write a scalar equation of the line ie) standard form (grade 9)

d) Determine if ℓ_1 is parallel to ℓ_2 if

$\ell_2: \begin{cases} x = 1+3t \\ y = 8+12t \end{cases}$

a) let $t=0$ $\therefore x=3, y=-5$, $P_1(3,-5)$

let $t=1$ $\therefore x=5, y=-1$, $P_2(5,-1)$

b) $(x,y) = (3,-5)+t(2,4)$

c) $x = 3+2t$ $\qquad$ $y = -5+4t$

$x-3 = 2t$ $\qquad$ $y+5 = 4t$

$\frac{x-3}{2} = t$ $\qquad$ $\frac{y+5}{4} = t$

$\therefore \frac{x-3}{2} = \frac{y+5}{4}$

$4(x-3) = 2(y+5)$

$4x-12 = 2y+10$

$4x-2y-22=0$

$\boxed{2x-y-11=0}$

d) direction vectors of parallel lines are scalar multiples of each other.

$\therefore \ell_1, \vec{m} = (2,4)$ $\qquad$ $\ell_2: \vec{m} = (3,12)$

If they are $\parallel$,

$(2,4) = k(3,12)$

$2 = 3k$ and $4 = 12k$

$\frac{2}{3} = k$ $\qquad$ $\frac{1}{3} = k$

Since k values are not equal, the lines ℓ_1 and ℓ_2 are not parallel.

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ex 3: Determine the vector equation for the line that is perpendicular to points $A(-1,5)$ and $B(6,11)$ and passes through $E(7,8)$

Solution: