Unit 7 - Applications of Vectors

Section 7.1 - Vectors as Forces

Force: defined as that which changes, or tends to change, the state of rest, or uniform motion of a body.

- force is described by both magnitude and direction.
- ∴ it is a vector
- on earth, force is defined as the product between the mass of an object and the acceleration due to gravity ($9.8\ m/sec^2$)

eg: $1\ Kg \times 9.8\frac{m}{s^2} = 9.8\ Kg \cdot \frac{m}{s^2} = 9.8$ Newtons

- Since earth's gravitational field acts downward, we say that a 1 Kg mass exerts a force of 9.8 N.
- weight, expressed in Newtons, is a force acting with a downward direction

Resultant & Composition of Forces

- the resultant of several forces is the single force that can be used to represent the combined effect of all the forces. The individual forces that make up the resultant are referred to as the components of the resultant.

Equilibrant of Several Forces

- The equilibrant of a number of forces is the single force that opposes the resultant of the forces acting on the object. When the equilibrant is applied to the object, this force maintains the object in a state of equilibrium. ($\vec{0}$)

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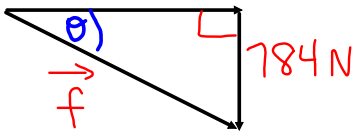
ex1: A clown with a mass of 80 Kg is shot out of a canon with a horizontal force of 2000 N. The vertical force is the acceleration due to gravity, which is 9.8 m/s², times the mass of the clown.

a) Find the magnitude and direction of the resultant force on the clown.

b) Find the magnitude and direction of the equilibrant.

Solution:



* vertical force
$80\text{Kg} \times 9.8\ \text{m/s}^2$
$= 784\ \text{N}$

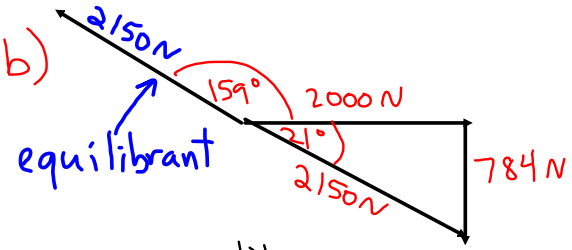
$|\vec{f}|^2 = 2000^2 + 784^2$
$|\vec{f}| = 2148.2\ \text{N}$ ← resultant vector
$|\vec{f}| \approx 2150\ \text{N}$

$\tan\theta = \frac{784}{2000}$

$\theta = 21.4°$

∴ the resultant has a magnitude of 2150 N and a direction of 21.4° below the horizontal.

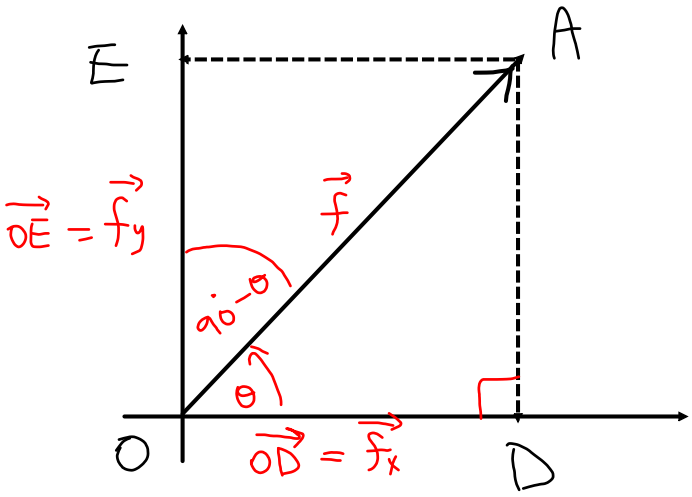
b)



The equilibrant has a magnitude of 2150N and a direction of 159° counterclockwise from the horizontal

Resolving a vector into its components

Resolving vectors: opposite of composition
- taking a single force & decomposing it into 2 components.
- these 2 components are at right angles to each other; referred to as horizontal and vertical components.



Horizontal Component: $\vec{OD}$

$\cos\theta = \frac{|\vec{OD}|}{|\vec{OA}|}$ $\therefore |\vec{OD}| = |\vec{OA}|\cos\theta$

$\therefore |\vec{OD}|$, the horizontal component of $\vec{OA}$ has magnitude $|\vec{OA}|\cos\theta$ or $|\vec{f}|\cos\theta = |\vec{f_x}|$

Vertical Component: $\vec{OE}$

Using $\triangle OEA$

$\cos(90-\theta) = \frac{|\vec{OE}|}{|\vec{OA}|}$, remember $\cos(90-\theta) = \sin\theta$

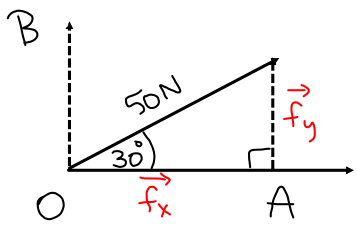
$\therefore |\vec{OA}|\sin\theta = |\vec{OE}|$ or $|\vec{f}|\sin\theta = |\vec{f_y}|$

ex 2: A girl pulls on the rope of her sled with a 50 N force at an angle of 30° to the horizontal.

a) Determine the force that pulls the **sled forward**.

b) the force that tends to lift the **sled**

Solution:



$|\vec{OA}| = |\vec{f}_x| = |\vec{f}|\cos 30$
$= 50\cos 30°$
$= 43.3N$

$|\vec{OB}| = |\vec{f}_y| = |\vec{f}|\sin 30°$
$= 50\sin 30$
$= 25N$

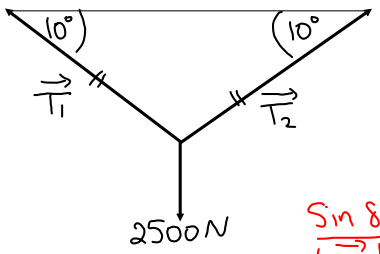
The sled is pulled forward with a force of 43.3N and the force that tends to lift it is 25N.

ex: Solving a Tension Problem

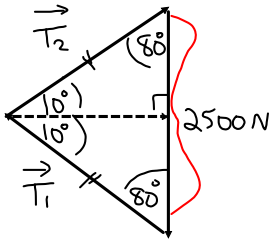
A traffic light at an intersection is hanging from two wires of equal length making angles of 10° below the horizontal. The traffic light weighs 2500N. What are the tensions in the wires?

Solution:

Position Diagram:



Vector diagram



$\frac{\sin 80°}{|\vec{T}_2|} = \frac{\sin 80°}{|\vec{T}_1|} = \frac{\sin 20°}{2500}$

$|\vec{T}_1| = 7198N$

∴ The wires each have a tension of 7198N at 10° below the horizontal.

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