Section 6.8: Linear Combinations and Spanning Sets

$\vec{a} = (2,3)$, $\vec{b} = (9,1)$, $\vec{c} = (5,4)$

Express vector $\vec{c}$ as a linear combination of $\vec{a}$ and $\vec{b}$.

$m(2,3) + n(9,1) = (5,4)$

$(2m, 3m) + (9n, n) = (5,4)$

① $2m + 9n = 5$

② $3m + n = 4$

re-arranging eqn ②,

$\boxed{n = 4 - 3m}$

$\therefore 2m + 9(4 - 3m) = 5$

$2m + 36 - 27m = 5$

$\dfrac{-25m}{-25} = \dfrac{-31}{-25}$

$\boxed{m = \dfrac{31}{25}}$

$\therefore n = 4 - 3\left(\dfrac{31}{25}\right)$

$n = 4 - \dfrac{93}{25}$

$n = \dfrac{100}{25} - \dfrac{93}{25}$

$\boxed{n = \dfrac{7}{25}}$

$\therefore \dfrac{31}{25}(2,3) + \dfrac{7}{25}(9,1) = (5,4)$

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ex: Write $(6,-1)$ as a linear combination of $(2,4)$ and $(-3,-6)$

Solution:

$a(2,4)+b(-3,-6)=(6,-1)$

① $2a-3b=6$
② $4a-6b=-1$

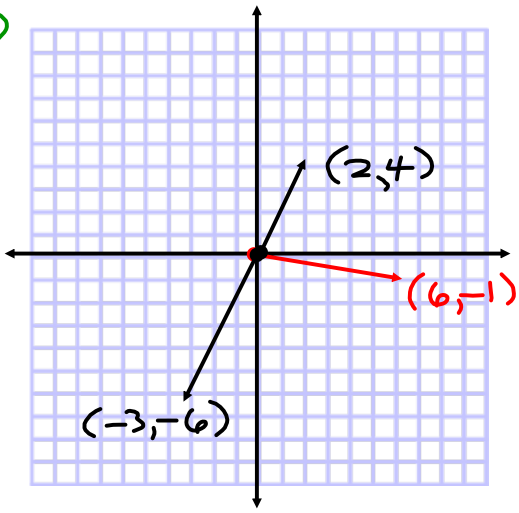
mult ① $\times 2$

$$\begin{array}{r} 4a-6b=12 \\ -\;4a-6b=-1 \\ \hline 0a+0b=13 \end{array}$$

$\therefore$ there are no solutions for "a" & "b" in this linear combination.

(ie) $(6,-1)$ cannot be written as a L.C. of $(2,4)$ and $(-3,-6)$

Why?



Since $(2,4)$ and $(-3,-6)$ are collinear, they can only form other collinear vectors.
Since $(6,-1)$ is not collinear with $(2,4)$ and $(-3,-6)$, no linear combination can be written.

p. 340 #1.

$2(1,0)+4(-1,0)=(-2,0)$

$(1,0)$ and $(-1,0)$ can only span the x-axis. (ie) $(1,0)$ + $(-1,0)$ are collinear.

p.341 # 13. Show that vectors $(-1,2,3)$, $(4,1,-2)$ & $(-14,-1,16)$ do not lie on the same plane.
(ie) not coplanar.

Solution: Can these vectors be written as a linear combination of each other?

$$a(-1,2,3)+b(4,1,-2)=(-14,-1,16)$$

① $-a+4b=-14$
② $2a+b=-1$
③ $3a-2b=16$

① $\boxed{a=4b+14}$

$\therefore 2(4b+14)+b=-1$
$8b+28+b=-1$
$9b=-29$
$\boxed{b=-\frac{29}{9}}$

$\therefore a=4\left(-\frac{29}{9}\right)+14$
$a=-\frac{116}{9}+\frac{126}{9}$
$\boxed{a=\frac{10}{9}}$

Sub these values into the 3^{rd} eqn to check for consistency.

$3a-2b=16$
$3\left(\frac{10}{9}\right)-2\left(-\frac{29}{9}\right)=16$
$\frac{30}{9}+\frac{58}{9}=16$
$\frac{88}{9}\neq 16$

$\therefore$ these vectors are not coplanar.