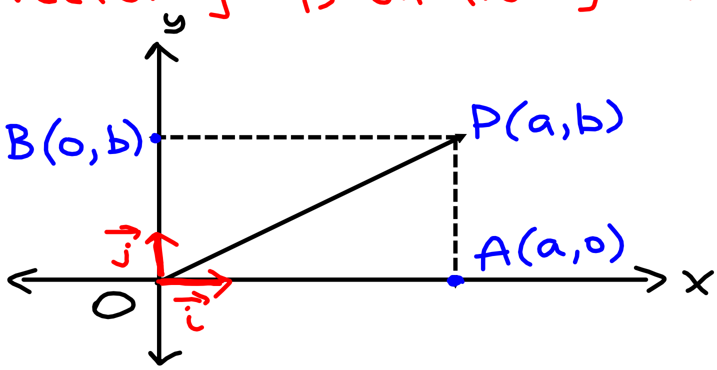
Section 6.6: Operations with Vectors in R^2

Recall that $\overrightarrow{OP} = (a,b)$ is the vector formed when we joined the origin, $(0,0)$ to the point $P(a,b)$. $\overrightarrow{OP}$ is a special Cartesian vector called a "position Vector". $\overrightarrow{OP} = (a,b)$ can also be written as $[a,b]$.

A second way of writing $\overrightarrow{OP} = (a,b)$ is with the use of the unit vectors $\vec{i}$ and $\vec{j}$. $\vec{i}$ and $\vec{j}$ are special unit vectors that have their tails on the origin. The head of vector $\vec{i}$ is on the x-axis at $(1,0)$ and the head of vector $\vec{j}$ is on the y-axis at $(0,1)$.



Since $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are scalar multiples of $\vec{i}$ and $\vec{j}$, we can write
$\overrightarrow{OA} = a\vec{i}$ and $\overrightarrow{OB} = b\vec{j}$

Since $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$
then $\overrightarrow{OP} = a\vec{i} + b\vec{j}$

And since $\overrightarrow{OP} = (a,b)$
the $(a,b) = a\vec{i} + b\vec{j}$

(ie) $\overrightarrow{OP} = (3,4)$ can be written as $3\vec{i} + 4\vec{j}$

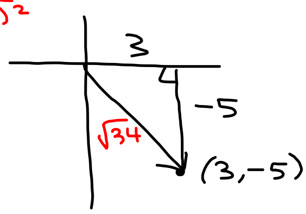
Magnitude of a Vector:

To find the magnitude of a position vector, use the formula for finding the distance between two points.

ex: Find the magnitude of vector $\vec{OA} = (3,-5)$.

If $O = (0,0)$ and $A = (3,-5)$, then

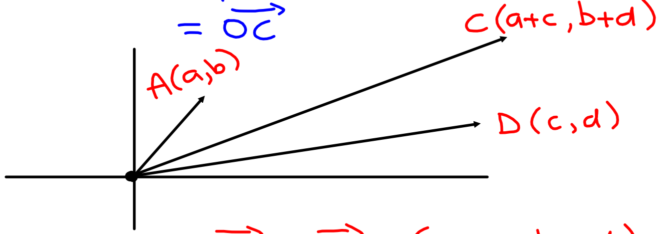
$$|\vec{OA}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(3-0)^2+(-5-0)^2}$$
$$= \sqrt{9+25}$$
$$= \sqrt{34}$$



Adding Vectors

To add two position vectors, $\vec{OA} = (a,b)$ and $\vec{OD} = (c,d)$, first rewrite them in the form $(a,b) = a\vec{i}+b\vec{j}$ and $(c,d) = c\vec{i}+d\vec{j}$.

$$\therefore \vec{OA}+\vec{OD} = a\vec{i}+b\vec{j}+c\vec{i}+d\vec{j}$$
$$= a\vec{i}+c\vec{i}+b\vec{j}+d\vec{j}$$
$$= (a+c)\vec{i}+(b+d)\vec{j}$$
$$= (a+c,\ b+d)$$
$$= \vec{OC}$$

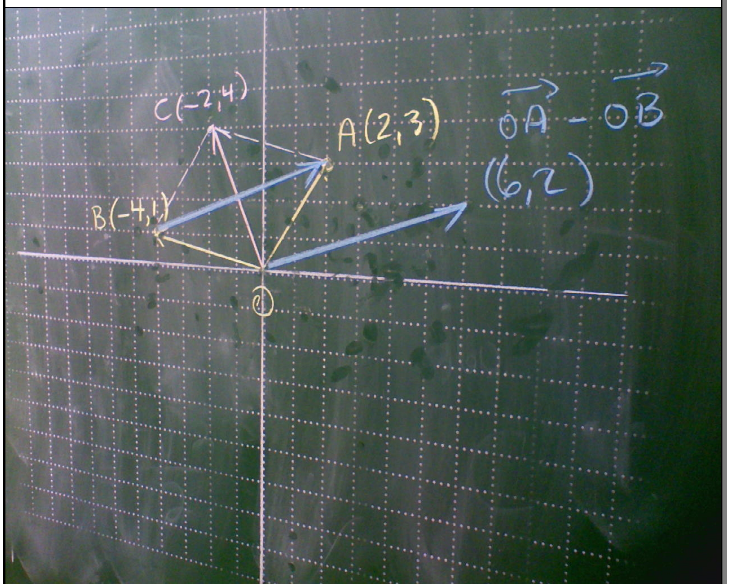


Similarly $\vec{OA} - \vec{OD} = (a-c, b-d)$

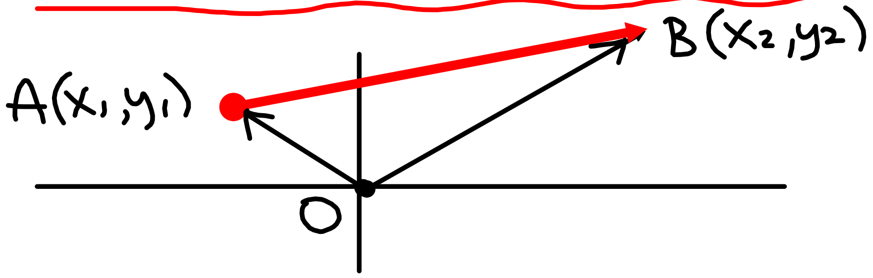
ex: If $\vec{a} = \vec{OA} = (2,3)$ and $\vec{b} = \vec{OB} = (-4,1)$ find $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.

Solution: $\vec{a}+\vec{b} = (2,3)+(-4,1) = (-2,4)$

$\vec{a}-\vec{b} = (2,3)-(-4,1) = (6,2)$



Vectors in R^2 defined by Two Points



In considering the Cartesian Vector $\overrightarrow{AB}$ with points $A(x_1, y_1)$ & $B(x_2, y_2)$, it is important to be able to find the related position vector.

Since $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$,
then $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$$= (x_2, y_2) - (x_1, y_1)$$
$$= (x_2 - x_1, y_2 - y_1) \ *$$

Also $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ex: what is the position vector for A(3,5) and B(4,7)?

$$\overrightarrow{AB} = (4-3, 7-5)$$
$$= (1, 2)$$

ex: A parallelogram is formed by the vectors $\overrightarrow{OA} = (2,3)$ and $\overrightarrow{OB} = (1,1)$.

a) Determine the lengths of the diagonals.

b) Determine the perimeter of the parallelogram

Solution:

1st diagonal: $\overrightarrow{OA} + \overrightarrow{OB}$

$$= (2,3) + (1,1)$$
$$= (3,4)$$

$$|(3,4)| = \sqrt{3^2+4^2}$$
$$= \sqrt{25}$$
$$= 5$$

2nd diagonal: $\overrightarrow{OA} - \overrightarrow{OB}$

$$= (2,3) - (1,1)$$
$$= (1,2)$$

$$|(1,2)| = \sqrt{1^2+2^2}$$
$$= \sqrt{1+4}$$
$$= \sqrt{5}$$

Perimeter?

$$\overrightarrow{OA} = (2,3)$$
$$|\overrightarrow{OA}| = \sqrt{2^2+3^2}$$
$$= \sqrt{4+9}$$
$$= \sqrt{13}$$

$$\overrightarrow{OB} = (1,1)$$
$$|\overrightarrow{OB}| = \sqrt{1^2+1^2}$$
$$= \sqrt{2}$$

$\therefore$ perimeter is

$$2|\overrightarrow{OA}| + 2|\overrightarrow{OB}|$$
$$= 2\sqrt{13} + 2\sqrt{2}$$
$$\doteq 10 \text{ units}$$

Apr 30-9:00 AM