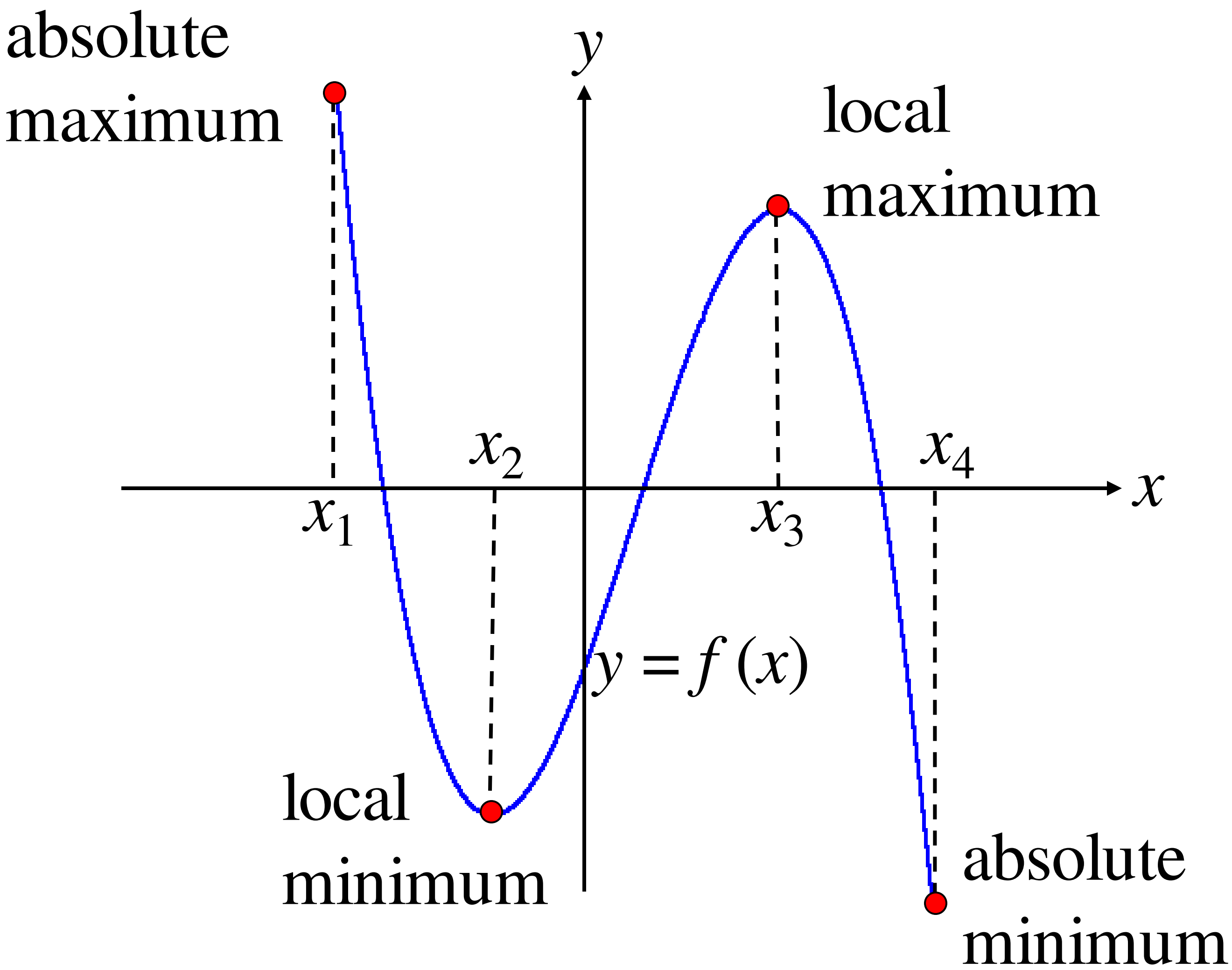
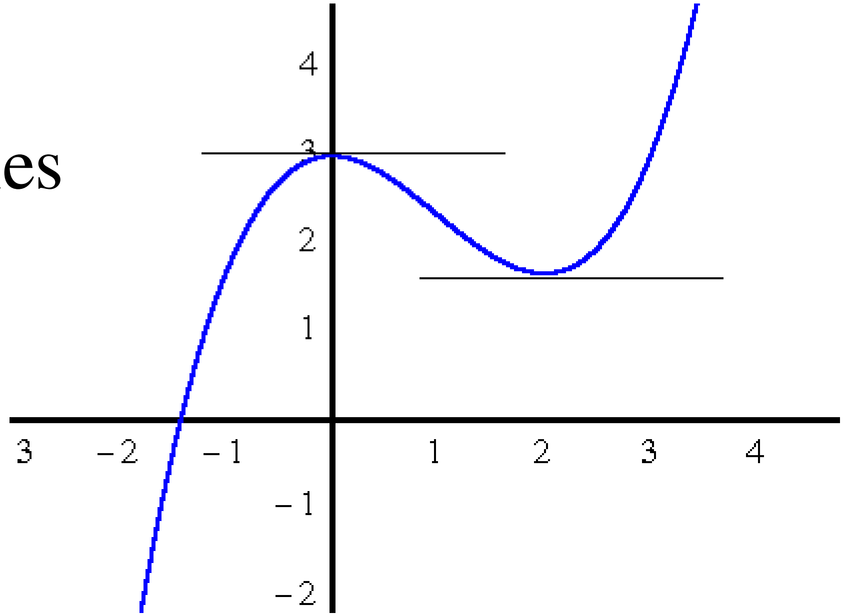
4.2 Critical Points, Local Maxima & Minima



Recall: Critical Numbers

Points on the graph where the slope of the tangent lines are zero.

Points where $f'(x) = 0$.



Example:

$$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 2$$

$$f'(x) = x^2 - 2x - 3$$

$$f'(x) = (x + 1)(x - 3)$$

$f'(x) = 0$ where $x = -1$ and $x = 3$

The First Derivative Test

We saw that when $f'(x) = 0$, we had a critical point.
However, is this point a max, min or neither?

Example: $y = 2x^3$

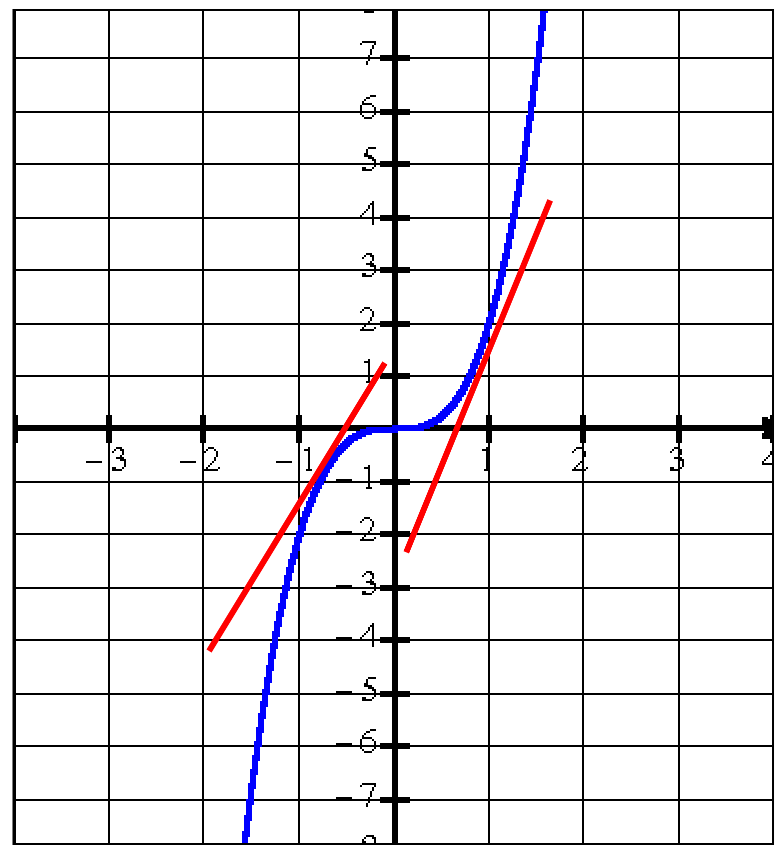
$$y' = 6x^2$$

If $6x^2 = 0$ then

$$x = 0$$

There is a horizontal tangent at (0, 0).

The slope is zero at (0, 0) but is positive on either side of (0, 0), so it is not a max or min.



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The first derivative test for Local Extrema.

Let c be a critical number of a polynomial function.

If $f\,'(x)$ changes from negative to positive at c, then $(c, f(c))$ is a local **minimum** of f.

If $f\,'(x)$ changes from positive negative to at c, then $(c, f(c))$ is a local **maximum** of f.

If $f\,'(x)$ does not change, then $(c, f(c))$ is neither a maximum nor a minimum.

The first derivative test for Absolute Extrema.

Let c be a critical number of a polynomial function.

If $f\,'(x)$ is negative for all $x < c$, and $f\,'(x)$ is positive for all $x > c$ then $(c, f(c))$ is the absolute **minimum** of f.

If $f\,'(x)$ is positive for all $x < c$, and $f\,'(x)$ is negative for all $x > c$ then $(c, f(c))$ is the absolute **maximum** of f.

Example 1: Determine the local and absolute extrema for the function: $y = 2x^3 - 3x^2 - 12x + 1, \ -6 \le x \le 2$

$$y = 2x^3 - 3x^2 - 12x + 1$$

$$y' = 6x^2 - 6x - 12$$

$$y' = 6(x^2 - x - 2)$$

$$y' = 6(x - 2)(x + 1)$$

There are critical points at $x = 2$ and $x = -1$

Sub at $x = 2$ and $x = -1$ into the original equation.

$$y = 2(2)^3 - 3(2)^2 - 12(2) + 1$$

$$y = -19$$

$$y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$$

$$y = 8$$

$\boxed{(2, -19) \text{ and } (-1, 8)}$