MCV4U - Unit 5 Test
Exponential & Trig Functions

Name: _____
Date: _____

Mark: _____
48

Answer all questions on this paper. Be sure to show all applicable work and express all answers in simplest form. Marks are awarded for presentation and technical correctness.

For questions 1 - 4, fill in the blanks with the correct answer. (1 mark each)

1. What famous Swiss mathematician introduced the number *e*? _____

2. Evaluate: $\ln e^{3x-1}$ _____

3. The value of 'e' to three decimal places is _____

4. Differentiate the following:

a) $y = e^{4x^2-7x}$ _____

b) $f(x)=4x^3\cos x$ _____

c) $f(x)=(\tan 5x)^2$ _____

d) $y=6^{3x+2}$ _____

e) $y=\cos(2^x)$ _____

f) $f(x)=\dfrac{e^{\cos x}}{x}$ _____

g) $y = x\ln 2x$ _____

h) $y=\sin^3\left(5x^2-4x\right)$ _____

i) $f(x)=\log_7\left(x^2+x+1\right)$ _____

j) $y = 3^x \log_3 x$ _____

5. Determine the equation of the line tangent to the graph of $y = x\,e^x$ at the point where x = 2. Use "e" in your answer. ie, no decimals.

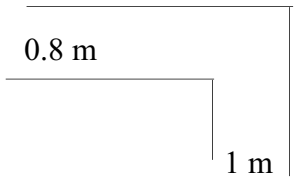
[4]

6. If $f(t) = 10^{3t-5} \cdot e^{2t^2}$, then find the value(s) of *t* so that $f'(t) = 0$.

[4]

7. A ladder needs to be carried horizontally around a corner joining two corridors, which are 1 m and 0.8 m wide. Calculate the length of the longest ladder that can be carried around this corner. See diagram below.

[6]



8. Sketch the function $f(x) = x^3 e^x$. Be sure to state and explain the following:

Domain, intercepts, equations of asymptotes, critical numbers, intervals of increase and decrease, interval(s) of concavity, maximum and minimum points and point(s) of inflection(s). Include interval chart(s) in this question. [10]

9. Suppose that a particle moves along so that at time *t* measured in seconds, its position in meters is given by $s(t)=5\sin(2t)$. $t\in[0,\pi]$ When is the particle changing direction.

[3]

10. Prove that $\frac{d}{dx}(\cot x) = -\csc^2 x$

[3]

11. Prove that the derivative of $y = 2^x$ is $y' = 2^x \ln 2$ two different ways; one using first principles and the other using logarithmic differentiation.

[5]