MCV 4U - Unit 7 Test
Applications of Vectors

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Date: 2016

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Fill in the blanks with the most appropriate answer:

1. A force vector has a magnitude of 23 N and makes an angle of $35°$ with the x-axis. What is the magnitude of its vertical component? 1. 13.2 N

$\vec{f_v} = 23 \sin 35°$

2. An airplane has an airspeed of 400 km/h and is heading due west. If it encounters a wind blowing north at 120 km/h, how many degrees north from west is the resultant velocity of the plane? 2. W16.7°N or N73.3°W

3. Is the following set of forces acting on an object, at equilibrium? 6 N, 7 N, 10 N 3. YES

$6+7>10$

4. Suppose that $|\vec{a}|=4$. What is $\vec{a}\cdot\vec{a}$? 4. 16

5. Suppose $\vec{x}\cdot\vec{y}=2$, $|\vec{x}|=2$ *and* $|\vec{y}|=2$. What is the angle between the vectors $\vec{x}$ *and* $\vec{y}$? $\cos\theta = \frac{\vec{x}\cdot\vec{y}}{|\vec{x}||\vec{y}|} = \frac{2}{(2)(2)} = \frac{1}{2}$ 5. $60°$

6. For what value of s is the line segment connecting the origin with (3, s) and the line segment connecting the origin with (18, 6) perpendicular? 6. -9

7. Suppose $\vec{a}$ *and* $\vec{b}$ are vectors such that $\vec{a}\times\vec{b}=(1,0,2)$. What is $\vec{b}\times(-2\vec{a})$?

$= -2(\vec{b}\times\vec{a})$
$= -2(-1,0,-2)$

7. $(2,0,4)$

8. How much work is done sliding a desk 5 m across the floor against a frictional force of 120 N? $\vec{W} = |\vec{F}||\vec{s}|\cos\theta$ 8. 600 J

$= (120)(5)\cos 0°$

9. Which of the following pairs of vectors are perpendicular to each other?

a. (1, 3, 2) and (–2, –6, –4)
b. (13, 4, 2) and (2, –5, –3)
c. (4, 14, –18) and (6, 21, –27)
d. (5, –4, 3) and (–3, 4, –5)

10. Suppose $\vec{a}=(4,-6,10)$ and $\vec{b}=(-6,9,-15)$. What is $\vec{a}\times\vec{b}$?

a. (–24, –54, –150)
b. (0, 0, 0)
c. (1, –1, –1)
d. (–3, –2, 0)

$$\begin{matrix} -6 & & 9 \\ 10 & \times & -15 \\ 4 & \times & -6 \\ -6 & \times & 9 \end{matrix}$$

$\vec{a}\times\vec{b} = (0,0,0)$

TO

11. Determine the value of k such that the vectors $(1,2,1)$ and $(k,2k+1,8)$ are perpendicular.

[3]

$(1,2,1)\cdot(K,2K+1,8)=0$

$K+2(2K+1)+8=0$

$K+4K+2+8=0$

$5K+10=0$

$5K=-10$

$K=-2$

12. Determine the angle (nearest degree), for the vector $\vec{a}=(6,-2,-3)$ with the x-axis.

[3]

$\cos\alpha=\dfrac{6}{\sqrt{6^2+(-2)^2+(-3)^2}}$

$\cos\alpha=\dfrac{6}{\sqrt{36+4+9}}$

$\cos\alpha=\dfrac{6}{\sqrt{49}}$

$\cos\alpha=\dfrac{6}{7}$

$\alpha=31°$

13. Determine the value of k such that the scalar projection of $\vec{a}=(k,0)$ on $\vec{b}=(1,\sqrt{3})$ is 4.

[3]

$|\text{proj}_{\vec{b}}\vec{a}|=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$4=\dfrac{(K,0)\cdot(1,\sqrt{3})}{\sqrt{1^2+\sqrt{3}^2}}$

$4=\dfrac{K+0}{\sqrt{4}}$

$4=\dfrac{K}{2}$

$K=8$

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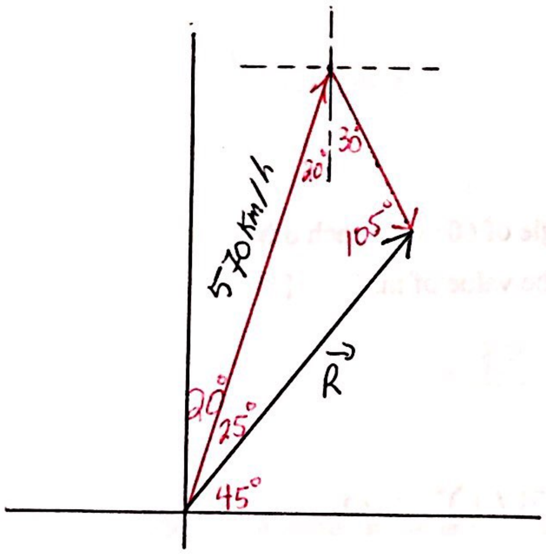
14. A 80 N force is applied at the end of a 40 cm wrench. If the force makes a 70° angle with the wrench, calculate the magnitude of the torque. [2]

$$|\vec{\tau}| = |\vec{r}||\vec{f}| \sin\theta$$

$$= (0.4)(80) \sin 70^\circ$$

$$= 30.1 \text{ N·m}$$

15. An airplane has an airspeed of 570 km/h and is heading in a direction of $N\,20^\circ\,E$ when it encounters a wind from $N30^\circ W$. The resultant ground velocity has a direction of $N\,45^\circ\,E$. How long does it take for the plane to travel 1180 km. [5]



$$\frac{\sin 105^\circ}{570} = \frac{\sin 50^\circ}{\vec{R}}$$

$$\vec{R} = 452 \text{ Km/h}$$

$$\text{time} = \frac{\text{distance}}{\text{Speed}}$$

$$= \frac{1180 \text{Km}}{452 \text{ Km/h}}$$

$$= 2.61 \text{ hours}$$

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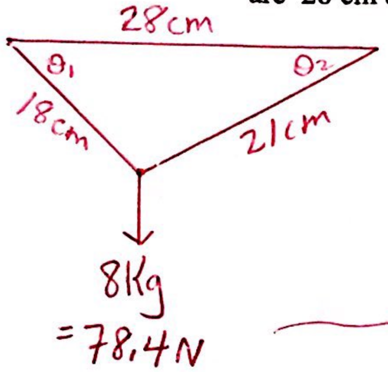
16. A mass of 8 kg is suspended by two strings, 18 cm and 21 cm long, from two points that are 28 cm apart and at the same level. Determine the tension in each of the strings. [5]



$21^2 = 28^2 + 18^2 - 2(28)(18)\cos\theta_1$

$441 = 784 + 324 - 1008\cos\theta_1$

$-667 = -1008\cos\theta_1$

$\cos\theta_1 = 0.6617$

$\theta_1 = 48.6°$

$\dfrac{\sin 48.6}{21} = \dfrac{\sin\theta_2}{18}$

$\sin\theta_2 = 0.6430$

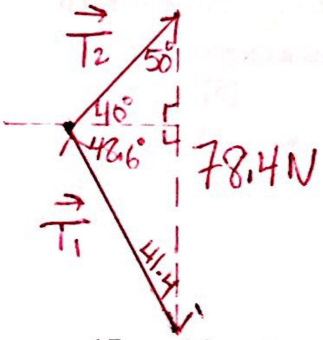
$\theta_2 = 40°$

$\dfrac{\sin 88.6°}{78.4} = \dfrac{\sin 50°}{|\vec{T_1}|}$

$|\vec{T_1}| = 60.1\text{ N}$

$\dfrac{\sin 88.6°}{78.4} = \dfrac{\sin 41.4°}{|\vec{T_2}|}$

$|\vec{T_2}| = 51.9\text{ N}$



17. The vectors $\vec{a}$ and $\vec{b}$ are unit vectors that make an angle of 60° with each other. If $\vec{a} - 3\vec{b}$ and $m\vec{a} + \vec{b}$ are perpendicular, determine the value of m. [4]

$(\vec{a} - 3\vec{b}) \cdot (m\vec{a} + \vec{b}) = 0$

$m|\vec{a}|^2 + \vec{a}\cdot\vec{b} - 3m\,\vec{a}\cdot\vec{b} - 3|\vec{b}|^2 = 0$

$m(1)^2 + |\vec{a}||\vec{b}|\cos 60° - 3m|\vec{a}||\vec{b}|\cos 60° - 3(1)^2 = 0$

$m + \frac{1}{2} - 3m\left(\frac{1}{2}\right) - 3 = 0$

$-\frac{1}{2}m - \frac{5}{2} = 0$

$-\frac{1}{2}m = \frac{5}{2}$

$-m = 5$

$m = -5$

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18. Determine the area of the triangle formed by the points A (3, -1, 2), B (-4, 0, 5) and C (1, -2, 3). [4]

$\vec{AB} = (-7, 1, 3)$

$\vec{AC} = (-2, -1, 1)$

$$\begin{array}{ccc} 1 & \times & -1 \\ 3 & & 1 \\ -7 & \times & -2 \\ 1 & \times & -1 \end{array}$$

$\vec{AB} \times \vec{AC} = (1+3, -6+7, 7+2)$

$= (4, 1, 9)$

Area of triangle $= \dfrac{|(4,1,9)|}{2}$

$= \dfrac{\sqrt{4^2+1^2+9^2}}{2}$

$= \dfrac{\sqrt{98}}{2} = \dfrac{7\sqrt{2}}{2} = 4.95 \doteq 5 \text{ units}^2$

19. Without doing any calculation, explain why one might conjecture that two vectors of the form $(a, b, 0)$ and $(c, d, 0)$ would have a cross product of the form $(0, 0, e)$? [2]

(a,b,0) and (c,d,0) are vectors on the x-y plane. The cross product of these vectors would produce a vector that is perpendicular to the x-y plane. Vectors of the form (0,0,e) are perpendicular to the x-y plane as they run along the z-axis.

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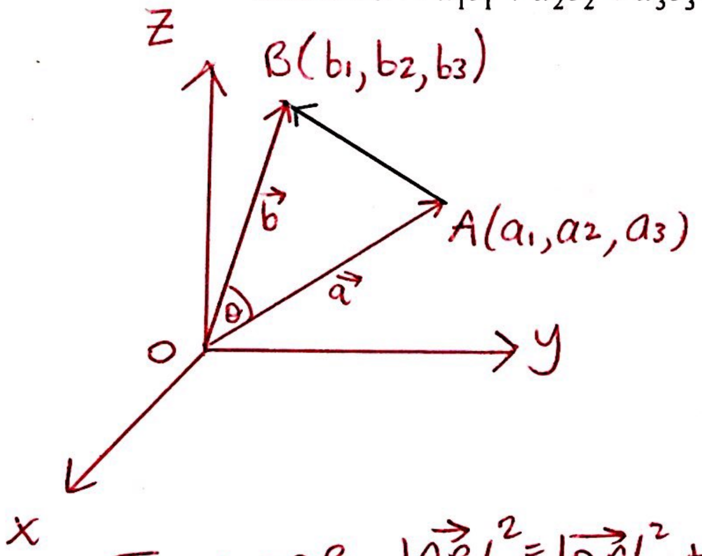
20. Prove the following theorem for dot product of algebraic vectors: in R^3, if $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$, then $\vec{a} \bullet \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$. [6]



In ΔOAB, $|\vec{AB}|^2 = |\vec{OA}|^2 + |\vec{OB}|^2 - 2|\vec{OA}||\vec{OB}| \cos\theta$

$\vec{AB} = (b_1 - a_1, b_2 - a_2, b_3 - a_3)$

$|\vec{AB}|^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2$

$|\vec{OA}|^2 = a_1^2 + a_2^2 + a_3^2$, $|\vec{OB}|^2 = b_1^2 + b_2^2 + b_3^2$

and $\vec{a} \cdot \vec{b} = |\vec{OA}||\vec{OB}| \cos\theta$

$\therefore (b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2 = a_1^2 + a_2^2 + a_3^3 + b_1^2 + b_2^2 + b_3^2 - 2\vec{a} \cdot \vec{b}$

$\cancel{b_1^2} - 2a_1b_1 + \cancel{a_1^2} + \cancel{b_2^2} - 2a_2b_2 + \cancel{a_2^2} + \cancel{b_3^2} - 2a_3b_3 + \cancel{a_3^2} = \cancel{a_1^2} + \cancel{a_2^2} + \cancel{a_3^3} + \cancel{b_1^2} + \cancel{b_2^2} + \cancel{b_3^2} - 2\vec{a} \cdot \vec{b}$

$-2a_1b_1 - 2a_2b_2 - 2a_3b_3 = -2\vec{a} \cdot \vec{b}$

$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$