MCV4U - Unit 5 Test
Exponential & Trig Functions

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Date: April 18, 2016

Mark: ______
48

Answer all questions on this paper. Be sure to show all applicable work and express all answers in simplest form. Marks are awarded for presentation and technical correctness.

For questions 1 - 4, fill in the blanks with the correct answer. (1 mark each)

1. What famous Swiss mathematician introduced the number e? Leonard Euler

2. Evaluate: $\ln e^{3x-1}$ $3x-1$

3. The value of 'e' to three decimal places is 2.718

4. Differentiate the following:

a) $y = e^{4x^2-7x}$ $(8x-7)e^{4x^2-7x}$

b) $f(x)=4x^3\cos x$ $12x^2\cos x + 4x^3(-\sin x)$ $12x^2\cos x - 4x^3\sin x$ or $4x^2(3\cos x - x\sin x)$

c) $f(x)=(\tan 5x)^2$ $2(\tan 5x)\sec^2 5x\,(5)$ $10\tan(5x)\sec^2(5x)$

d) $y=6^{3x+2}$ $6^{3x+2}\ln 6\,(3)$ $3(6^{3x+2})\ln 6$

e) $y=\cos(2^x)$ $-\sin(2^x)\,2^x\ln 2$ $-\sin(2^x)\,2^x\ln 2$

f) $f(x)=\dfrac{e^{\cos x}}{x}$ $\dfrac{e^{\cos x}(-\sin x)x - e^{\cos x}}{x^2}$ or $\dfrac{e^{\cos x}(-x\sin x - 1)}{x^2}$

g) $y = x\ln 2x$ $(1)\ln 2x + x\left(\frac{1}{2x}\right)(2)$ $\ln(2x)+1$

h) $y=\sin^3(5x^2-4x)$ $= \sin(5x^2-4x)^3$ $= 3(\sin 5x^2-4x)^2\cos(5x^2-4x)(10x-4$

i) $f(x)=\log_7(x^2+x+1)$ $= \dfrac{1}{(x^2+x+1)\ln 7}(2x+1)$ $= \dfrac{(2x+1)}{(x^2+x+1)\ln 7}$

j) $y = 3^x\log_3 x$ $3^x\ln 3\log_3 x + 3^x\left(\frac{1}{x\ln 3}\right)$ or $3^x\left(\ln 3\log_3 x + \frac{1}{x\ln 3}\right)$

$\overline{13}$

2016

6. Determine the equation of the line tangent to the graph of $y = xe^x$ at the point where x = 2. Use "e" in your answer. ie, no decimals.

$y = 2e^2 \therefore (2, 2e^2)$

[4]

$y' = (1)(e^x) + xe^x$
$y' = e^x + xe^x$
at $x = 2$
$y' = e^2 + 2e^2$
$y' = 3e^2$

$y = mx + b$
$2e^2 = 3e^2(2) + b$
$2e^2 = 6e^2 + b$
$-4e^2 = b$
$\therefore y = 3e^2 x - 4e^2$

7. If $f(t) = 10^{3t-5} \cdot e^{2t^2}$, then find the value(s) of t so that $f'(t) = 0$.

$f'(t) = 10^{3t-5}(\ln 10)(3)e^{2t^2} + 10^{3t-5}e^{2t^2}(4t)$

[4]

$= 10^{3t-5}e^{2t^2}[3\ln 10 + 4t]$

$\therefore 10^{3t-5} \neq 0$
$e^{2t^2} \neq 0$

$3\ln 10 + 4t = 0$
$4t = -3\ln 10$
$t = \frac{-3\ln 10}{4}$

or $t = -1.72$

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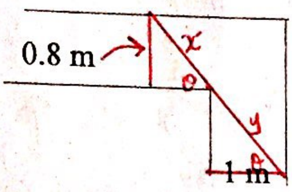
Thinking/Inquiry/Problem Solving:

10. A ladder needs to be carried horizontally around a corner joining two corridors, which are 1 m and 0.8 m wide. Calculate the length of the longest ladder that can be carried around this corner. See diagram below.

[6]



$\sin\theta = \frac{0.8}{x}$

$x = \frac{0.8}{\sin\theta}$

$\cos\theta = \frac{1}{y}$

$y = \frac{1}{\cos\theta}$

$L = x + y$

$L = \frac{0.8}{\sin\theta} + \frac{1}{\cos\theta}$

$L = 0.8(\sin\theta)^{-1} + (\cos\theta)^{-1}$

$L' = -0.8(\sin\theta)^{-2}(\cos\theta) - (\cos\theta)^{-2}(-\sin\theta)$

$L' = \frac{-0.8\cos\theta}{(\sin\theta)^2} + \frac{\sin\theta}{(\cos\theta)^2}$

$\frac{0.8\cos\theta}{\sin^2\theta} = \frac{\sin\theta}{\cos^2\theta}$

$0.8\cos^3\theta = \sin^3\theta$

$\boxed{0.8 = \tan^3\theta}$

$0.9283 = \tan\theta$

$\theta = 42.87°$ or 0.75 rad

$L = \frac{0.8}{\sin 42.87°} + \frac{1}{\cos 42.87°}$

$L = 1.18 + 1.36$

$L = 2.54$ m

$L \approx 2.5$ m

11. Prove that $\frac{d}{dx}(\cot x) = -\csc^2 x$

[3]

$\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$

$\frac{-\sin x\sin x - \cos x\cos x}{\sin^2 x}$

$\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$\frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$\frac{-1}{\sin^2 x}$

$= -\csc^2 x$

9

8. Sketch the function $f(x) = x^3e^x$. Be sure to state and explain the following: Domain, intercepts, equations of asymptotes, critical numbers, intervals of increase and decrease, interval(s) of concavity, maximum and minimum points and point(s) of inflection(s). Include interval chart(s) in this question. [10]

Domain: $x \in \mathbb{R}$

as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to 0$

x-int; let $y=0$

$\therefore x^3 = 0 \quad e^x \neq 0$

$\boxed{x=0}$

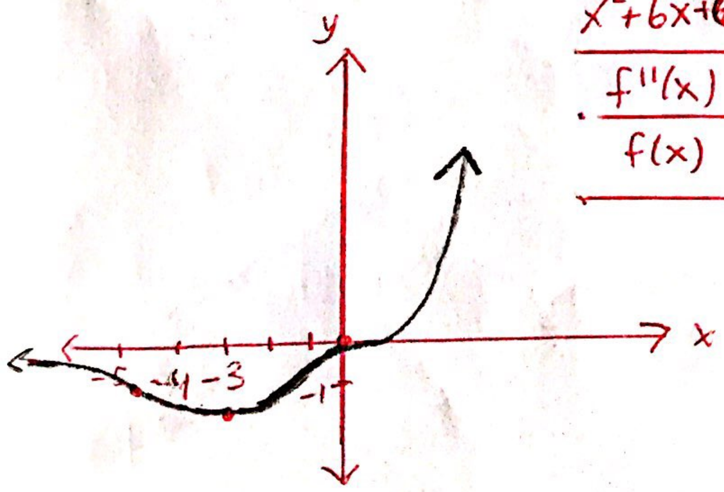
$\therefore$ y-int is 0 also

$f'(x) = 3x^2e^x + x^3e^x$

$0 = x^2e^x(3+x)$

$\therefore x=0$ & $x=-3$

	$x<-3$	$-3<x<0$	$x>0$
e^x	+	+	+
x^2	+	+	+
$x+3$	−	+	+
$f'(x)$	−	+	+
$f(x)$	↘	↗	↗
		min @ $x=-3$	Plateau @ $x=0$



$f''(x) = 6xe^x + 3x^2e^x + 3x^2e^x + x^3e^x$

$= 6xe^x + 6x^2e^x + x^3e^x$

$= xe^x(6 + 6x + x^2)$

$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(6)}}{2(1)}$

$x = \dfrac{-6 \pm \sqrt{36-24}}{2}$

$x = \dfrac{-6 \pm \sqrt{12}}{2}$ or $\dfrac{-6 \pm 2\sqrt{3}}{2} = -3 \pm \sqrt{3}$

$x = \dfrac{-6 \pm 3.5}{2}$

$x = -4.75$ & -1.25

as well as $x = 0$

	$x<-4.75$	$-4.75<x<-1.25$	$-1.25<x<0$	$x>0$
x	−	−	−	+
e^x	+	+	+	+
x^2+6x+6	+	−	+	+
$f''(x)$	−	+	−	+
$f(x)$	∩	∪	∩	∪
	POI	POI	POI	

POI $(-4.75, -0.9)$

POI $(-1.25, -0.6)$

Min $(-3, -1.3)$

9. Suppose that a particle moves along so that at time t measured in seconds, its position in meters is given by $s(t)=5\sin(2t)$. $t\in[0,\pi]$ When is the particle changing direction.

$V(t) = 5\cos(2t)(2)$

$V(t) = 10\cos(2t)$

[3]

$10\cos(2t) = 0$

$\cos(2t) = 0$

$2t = \cos^{-1} 0$

$2t = \pi/2$ | $2t = 3\pi/2$

$t = \pi/4$ | $t = 3\pi/4$

$\doteq 0.79$ sec | $\doteq 2.36$ Sec

10. Prove that $\frac{d}{dx}(\cot x) = -\csc^2 x$

$\cot x = \frac{\cos x}{\sin x}$

[3]

$$\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= -\csc^2 x$$

6

11. Prove that the derivative of $y = 2^x$ is $y' = 2^x \ln 2$ two different ways; one using first principles and the other using logarithmic differentiation.

[5]

1

$y = 2^x$

$y' = \frac{2^{x+h} - 2^x}{h}$

$y' = \lim_{h \to 0} \frac{2^x(2^h - 1)}{h}$

$y' = 2^x \lim_{h \to 0} \left(\frac{2^h - 1}{h}\right)$

$y' = 2^x (0.6931)$

$y' = 2^x \ln 2$

2

$y = 2^x$

$\ln y = \ln 2^x$

$\ln y = x \ln 2$

$\frac{1}{y} \frac{dy}{dx} = \ln 2$

$\frac{dy}{dx} = y \ln 2$

$\frac{dy}{dx} = 2^x \ln 2$