2.3 The Product Rule

If we are given two functions $f(x)$ and $g(x)$, what is the first derivative of $f(x)g(x)$?

Example: $f(x) = x^2 + 4$ and $g(x) = x^3 - 3x^2$

If $h(x) = f(x)g(x)$ what is $h'(x)$

Let $f(x) = u$ and $g(x) = v$

then $h(x) = uv$

$$h'(x) = u'v + uv'$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

Example 1: Determine the first derivative of:

$$y = (x^2 + 4)\,(\,x^3 - 3x\,)$$

let $u = x^2 + 4$ and $v = x^3 - 3x$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = (x^2+4)\left[\frac{d}{dx}(x^3-3x)\right] + \left[\frac{d}{dx}(x^2+4)\right](x^3-3x)$$

$$\frac{dy}{dx} = (x^2+4)(3x^2-3) + (x^3-3x)(2x)$$

$$\frac{dy}{dx} = 3x^4 - 3x^2 + 12x^2 - 12 + 2x^4 - 6x^2$$

$$\frac{dy}{dx} = 5x^4 + 3x^2 - 12$$

Example 2: Determine the slope of the tangent at $x = 2$. $f(x) = (3x + 4)(x^2 - 4x)$

$$\frac{d}{dx} f(x) = (3x+4)\left[\frac{d}{dx}(x^2-4x)\right] + \left[\frac{d}{dx}(3x+4)\right](x^2-4x)$$

$$f'(x) = (3x + 4)(2x - 4) + (3)(x^2 - 4x)$$

$$f'(x) = 6x^2 - 12x + 8x - 16 + 3x^2 - 12x$$

$$f'(x) = 9x^2 - 16x - 16$$

$$f'(2) = 9(2)^2 - 16(2) - 16$$

$$f'(2) = 36 - 32 - 16$$

$$f'(2) = -12$$

the slope of the tangent is -12