

Name:

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Date:

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MCV4U - Unit 6 Test - Intro to Vectors

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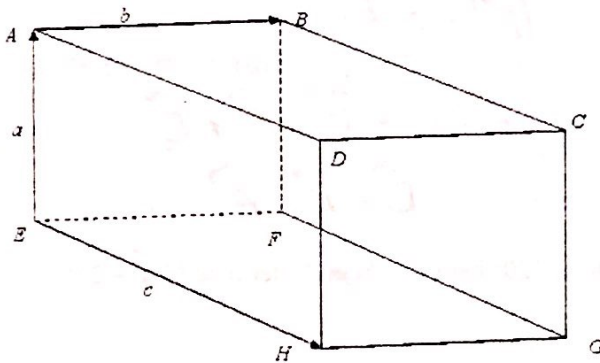
Problem

1. Vector \overrightarrow{AB} goes from (1, 0) to (2, 1). Vector \overrightarrow{CD} starts at (4, 2).
- Calculate the magnitude of \overrightarrow{AB} . [2]
 - If $\overrightarrow{AB} = \overrightarrow{CD}$, determine the endpoint of \overrightarrow{CD} . [1]

a) $\overrightarrow{AB} = (2-1, 1-0)$
 $\overrightarrow{AB} = (1, 1)$
 $|\overrightarrow{AB}| = \sqrt{1^2 + 1^2}$
 $= \sqrt{2}$

b) go over 1 and up 1 from (4, 2)
 $\therefore (5, 3)$

2. Using the prism, answer the following questions.



- a. Write \overrightarrow{EC} in terms of \vec{a} , \vec{b} , and \vec{c} . [1]

$$= \vec{a} + \vec{b} + \vec{c}$$

- b. Write \overrightarrow{CE} in terms of \vec{a} , \vec{b} , and \vec{c} . [1]

$$= -\vec{a} - \vec{b} - \vec{c}$$

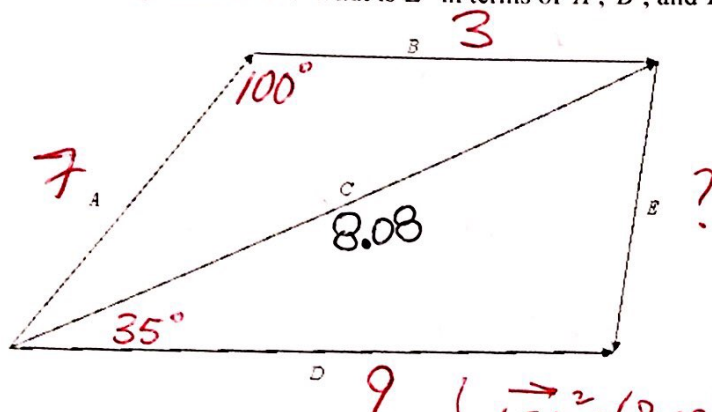
- c. What type of vectors are \overrightarrow{EC} and \overrightarrow{CE} in relation to each other? Explain. [2]

\overrightarrow{EC} and \overrightarrow{CE} are opposite vectors.

They have the same magnitude but opposite directions.

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3. If $|\vec{A}| = 7$, $|\vec{B}| = 3$, $|\vec{D}| = 9$ the angle between \vec{A} and \vec{B} is 100° and the angle between \vec{C} and \vec{D} is 35° , what is the magnitude of \vec{E} ? What is \vec{E} in terms of \vec{A} , \vec{B} , and \vec{D} ? [5]



$$|\vec{C}|^2 = 7^2 + 3^2 - 2(7)(3)\cos 100^\circ$$

$$|\vec{C}|^2 = 49 + 9 + 7.29$$

$$|\vec{C}|^2 = 65.29$$

$$|\vec{C}| = 8.08$$

$$|\vec{E}|^2 = (8.08)^2 + 9^2 - 2(8.08)(9)\cos 35^\circ$$

$$|\vec{E}|^2 = 65.29 + 81 - 119.14$$

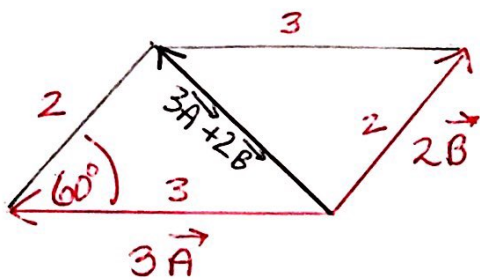
$$|\vec{E}|^2 = 27.15$$

$$|\vec{E}| = 5.21$$

$$\vec{E} = -\vec{B} - \vec{A} + \vec{D}$$

$$= \vec{D} - \vec{B} - \vec{A}$$

4. \vec{A} and \vec{B} are unit vectors with an angle of 120° between them. Determine $|3\vec{A} + 2\vec{B}|$. [5]



$$|3\vec{A} + 2\vec{B}|^2 = 2^2 + 3^2 - 2(2)(3)\cos 60^\circ$$

$$= 4 + 9 - 12\left(\frac{1}{2}\right)$$

$$= 7$$

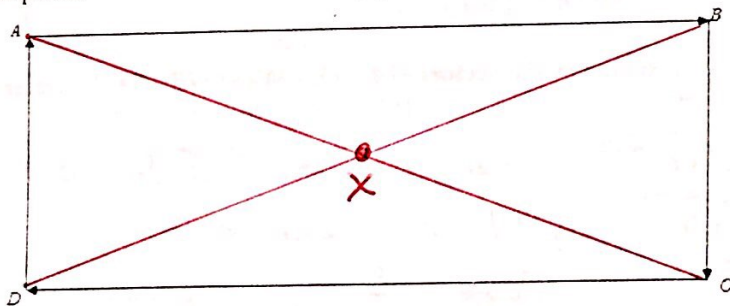
$$|3\vec{A} + 2\vec{B}| = \sqrt{7}$$

$$\approx 2.65$$

5. If $\vec{x} = \vec{i} - \vec{j} + 2\vec{k}$, $\vec{y} = 3\vec{i} + \vec{j} - 7\vec{k}$ and $\vec{z} = 3\vec{j} + \vec{k}$, simplify $2(\vec{x} - \vec{y}) + \vec{z}$? [3]

$$\begin{aligned}
 &= 2(\vec{i} - \vec{j} + 2\vec{k} - (3\vec{i} + \vec{j} - 7\vec{k})) + \vec{z} \\
 &= 2(-2\vec{i} - 2\vec{j} + 9\vec{k}) + 3\vec{j} + \vec{k} \\
 &= -4\vec{i} - 4\vec{j} + 18\vec{k} + 3\vec{j} + \vec{k} \\
 &= -4\vec{i} - \vec{j} + 19\vec{k}
 \end{aligned}$$

6. In rectangle $ABCD$, the diagonals meet at X . Determine an expression for \vec{AX} in terms of \vec{AB} and \vec{BC} . Explain. [2]



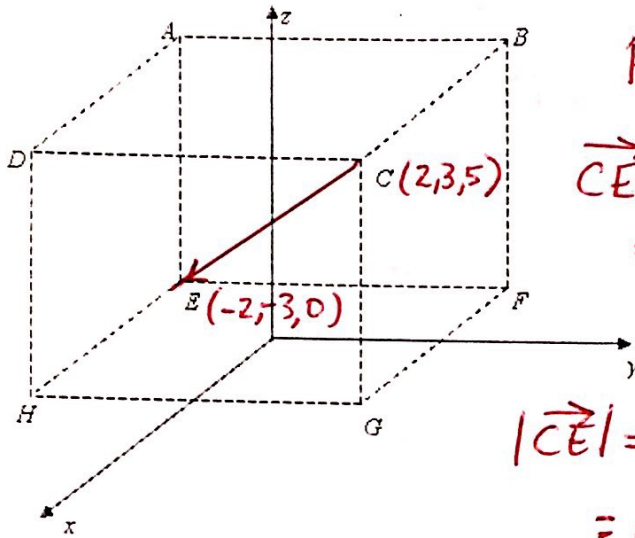
$$\vec{AC} = \vec{AB} + \vec{BC}$$

Since the diagonals in a rectangle bisect each other,

$$\vec{AX} = \frac{1}{2} \vec{AC}$$

$$= \frac{1}{2} (\vec{AB} + \vec{BC})$$

7. The prism is bisected by the xz -plane and the yz -plane. The point $C = (2, 3, 5)$. Determine $|\overrightarrow{CE}|$. [4]

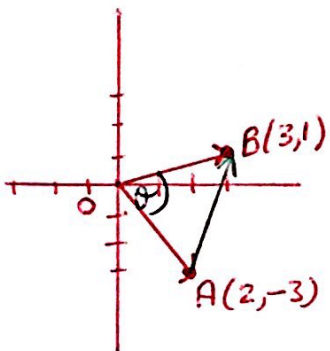


$$\text{Point } E = (-2, -3, 0)$$

$$\begin{aligned}\overrightarrow{CE} &= (-2-2, -3-3, 0-5) \\ &= (-4, -6, -5)\end{aligned}$$

$$\begin{aligned}|\overrightarrow{CE}| &= \sqrt{(-4)^2 + (-6)^2 + (-5)^2} \\ &= \sqrt{16 + 36 + 25} \\ &= \sqrt{77} \\ &= 8.77\end{aligned}$$

8. A triangle in R^2 has two sides represented by the vectors $\overrightarrow{OA} = (2, -3)$ and $\overrightarrow{OB} = (3, 1)$. Determine the measures of the angles of the triangle. [5]



$$\begin{aligned}|\overrightarrow{OA}| &= \sqrt{2^2 + (-3)^2} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13}\end{aligned}$$

$$\begin{aligned}|\overrightarrow{OB}| &= \sqrt{3^2 + 1^2} \\ &= \sqrt{10}\end{aligned}$$

$$\begin{aligned}\overrightarrow{AB} &= (3-2, 1-(-3)) \\ &= (1, 4)\end{aligned}$$

$$\begin{aligned}|\overrightarrow{AB}| &= \sqrt{1^2 + 4^2} \\ &= \sqrt{17}\end{aligned}$$

$$\begin{aligned}17 &= 13 + 10 - 2\sqrt{13}\sqrt{10} \cos \theta \\ 17 &= 23 - 22.8 \cos \theta \\ \cos \theta &= \frac{6}{22.8}\end{aligned}$$

$$\cos \theta = 0.2632$$

$$\theta = 74.7^\circ$$

$$\begin{aligned}13 &= 17 + 10 - 2\sqrt{17}\sqrt{10} \cos \beta \\ 13 &= 27 - 26.1 \cos \beta\end{aligned}$$

$$\cos \beta = \frac{14}{26.1}$$

$$\cos \beta = 0.5364$$

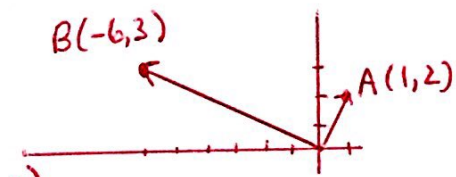
$$\beta = 57.6^\circ$$

$$\alpha = 180 - (74.7 + 57.6)$$

$$\alpha = 47.7^\circ$$

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9. A rectangle is formed in R^2 by the vectors $\vec{OA} = (1, 2)$ and $\vec{OB} = (-6, 3)$. [5]
- Determine its perimeter.
 - Determine its area.
 - Determine the length of its diagonals.



$$|\vec{OA}| = \sqrt{1^2 + 2^2}$$

$$= \sqrt{5}$$

$$|\vec{OB}| = \sqrt{(-6)^2 + (3)^2}$$

$$= \sqrt{36 + 9}$$

$$= \sqrt{45} \text{ or } 3\sqrt{5}$$

$$\text{Perimeter} = 2\sqrt{5} + 2\sqrt{45}$$

$$= 2\sqrt{5} + 6\sqrt{5}$$

$$= 8\sqrt{5} = 17.9$$

$$\text{b) Area: } Lw$$

$$= \sqrt{45} \cdot \sqrt{5}$$

$$= 3\sqrt{5} \cdot \sqrt{5}$$

$$= \boxed{15 \text{ units}^2}$$

c) using pythagorean,

$$d^2 = (\sqrt{45})^2 + (\sqrt{5})^2$$

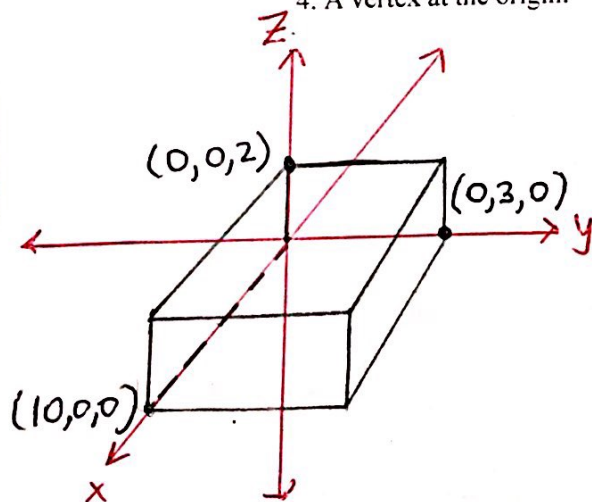
$$d^2 = 45 + 5$$

$$d^2 = 50$$

$$d = \sqrt{50}$$

$$= \boxed{5\sqrt{2}}$$

10. Determine 3 vectors that form the rectangular prism that satisfies the following conditions. [3]
- Volume of 60 cubic units
 - Length in the x -direction of 10 units
 - Height in the y -direction of 3 units
 - A vertex at the origin.



$$V = lwh$$

$$60 = (10)(3)(h)$$

$$60 = 30h$$

$$\boxed{h = 2}$$

11. If $a(0, -1, 3) + b(1, 1, 1) - c(1, 2, 5) = (-2, 3, -8)$, determine a , b and c .

[5]

$$(1) \quad b - c = -2$$

$$(2) \quad -a + b - 2c = 3$$

$$(3) \quad 3a + b - 5c = -8$$

$$\boxed{b = -2 + c}$$

$$\therefore -a + (-2 + c) - 2c = 3$$

$$-a - 2 + c - 2c = 3$$

$$-a - c = 5$$

$$-a = c + 5$$

$$\boxed{a = -c - 5}$$

$$\therefore 3a + b - 5c = -8$$

$$3(-c - 5) + (-2 + c) - 5c = -8$$

$$-3c - 15 - 2 + c - 5c = -8$$

$$-7c = 9$$

$$\boxed{c = -\frac{9}{7}}$$

$$b = -2 - \frac{9}{7}$$

$$\boxed{b = -\frac{23}{7}}$$

$$a = -\left(-\frac{9}{7}\right) - 5$$

$$a = \frac{9}{7} - 5 = \boxed{\frac{-26}{7} = a}$$