Section 8.2 – Cartesian Equation of a line

Review: Direction vectors & slope

Given line segment AB.
If Slope $m = \frac{b}{a}$ ← rise / ← run
then $\vec{m} = (a, b)$ ⟵ direction vector

ex 1: Determine the equivalent vector and parametric eqns of the line $y = \frac{2}{3}x - 4$.

Solution:

y-int: $(0, -4)$

If $m = \frac{2}{3}$, $\vec{m} = (3, 2)$

vector eqn.

$\therefore (x, y) = (0, -4) + t(3, 2)$

parametric equation:

$x = 0 + 3t$ or $x = 3t$
$y = -4 + 2t$ $\quad y = -4 + 2t$

ex 2: Given the vector equation
$\vec{r} = (2, -2) + t(-1, 3)$
determine the equivalent slope y-intercept form.

Solution:

$\vec{m} = (-1, 3)$

$\therefore m = \frac{3}{-1} = -3$

line contains the point $(2, -2)$

$\therefore y = mx + b$

$-2 = -3(2) + b$

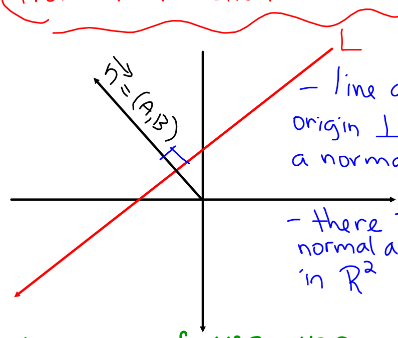
$-2 = -6 + b$

$b = 4$

$\therefore \boxed{y = -3x + 4}$

May 18-7:54 AM

Developing the Cartesian Equation from a direction vector:



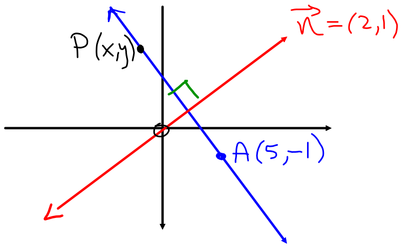
- line drawn from the origin $\perp$ to L is called a normal axis.
- there is only one normal axis that exists in R^2

∗ see proof 438–439

Using Cartesian Equation (Scalar eqn.)

$Ax+By+C=0$, where a normal to this line is $\vec{n}=(A,B)$.

ex: Determine the Cartesian equation of the line passing through $A(5,-1)$ which has $\vec{n}=(2,1)$ as its normal.



Since $\vec{n}\perp\vec{AP}$, then $\vec{n}\cdot\vec{AP}=0$

$\vec{PA}=(5-x,-1-y)$

$\therefore (2,1)\cdot(5-x,-1-y)=0$

$(2)(5-x)+(1)(-1-y)=0$

$10-2x-1-y=0$

$-2x-y+9=0$

$2x+y-9=0$

2nd method:

Since $\vec{n}=(2,1)$

$Ax+By+C=0$

$(2)(5)+(1)(-1)+C=0$

$10-1+C=0$

$9+C=0$

$C=-9$

$\therefore \boxed{2x+y-9=0}$

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ex: Determine the acute $\angle$ formed at the point of intersection created by the lines:

$L_1: (x,y) = (3,1) + t(1,3)$

$L_2: (x,y) = (-1,-2) + t(2,-3)$

Solution: dot product the direction vectors

$(1,3)\cdot(2,-3) = |(1,3)|\,|(2,-3)| \cos\theta$

$(1)(2) + (3)(-3) = \sqrt{1^2+3^2}\sqrt{2^2+(-3)^2}\cos\theta$

$2 - 9 = \sqrt{10}\sqrt{13}\cos\theta$

$\frac{-7}{\sqrt{130}} = \cos\theta$

$\theta = 127.9°$

$\therefore$ the acute angle is $52.1°$

ex: For the pair of lines $x = 3$ and $5x - 10y + 20 = 0$, determine the size of the acute angle created by the intersection of the lines.

Solution:

For $x = 3$, $\vec{m_1} = (0,1)$

and $5x - 10y + 20 = 0$, $\vec{n} = (5,-10)$

$\therefore \vec{m_2} = (10,5)$ or $(2,1)$

So $\cos\theta = \dfrac{\vec{m_1} \cdot \vec{m_2}}{|\vec{m_1}|\,|\vec{m_2}|}$

$\cos\theta = \dfrac{(0,1)\cdot(2,1)}{\sqrt{0^2+1^2}\,\sqrt{2^2+1^2}}$

$\cos\theta = \dfrac{(0)(2)+(1)(1)}{\sqrt{1}\,\sqrt{5}}$

$\cos\theta = \dfrac{1}{\sqrt{5}}$

$\theta = 63°$