

## 5.2 The derivative of the exponential function $y = a^x$

$$\text{let } y = a^x, a > 0$$

$$\begin{aligned}\ln y &= \ln a^x \\ &= x \ln a\end{aligned}$$

$$\frac{d(\ln y)}{dy} \frac{dy}{dx} = \ln a$$

$$\frac{1}{y} \frac{dy}{dx} = \ln a$$

$$\frac{dy}{dx} = y \ln a$$

$$\frac{dy}{dx} = a^x \ln a$$

$$\text{If } y = a^x, a > 0$$

$$\text{then } \frac{dy}{dx} = a^x \ln a$$

**Example:** Find the derivative of

a)  $y = 4^x$

$$\frac{dy}{dx} = 4^x \ln 4$$

b)  $y = 6^{3x+5}$

$$y = 6^u$$

$$\frac{dy}{dx} = 6^u \ln 6 \frac{du}{dx}$$

$$= (3)6^{3x+5} \ln 6$$

# The derivative of a composite function involving $y = a^x$

If  $y = a^{f(x)}$ ,  $a > 0$  then

$$\frac{dy}{dx} = a^{f(x)} \ln a \cdot f'(x)$$

**Example:**  $y = 600(2)^{\frac{x}{3}}$

Determine the rate of change when  $x = 4$ .

$$\frac{dy}{dx} = 600(2)^{\frac{x}{3}} \ln 2 \cdot \frac{1}{3}$$

$$\left. \frac{dy}{dx} \right|_{x=4} = 600(2)^{\frac{4}{3}} \ln 2 \cdot \frac{1}{3} = 349.3$$