Determine the derivative of the function $f(x) = (7^x)(x^7)$ in factored form. [2]

$f'(x) = 7^x \ln 7 (x^7) + 7^x (7x^6)$

$= 7^x x^6 (x \ln 7 + 7)$

2. Determine the first and second derivative of the function $f(x) = e^{0.5x^2}$ in factored form. [3]

$f'(x) = e^{0.5x^2}(x)$

$f''(x) = e^{0.5x^2}(x)(x) + (1)e^{0.5x^2}$

$= e^{0.5x^2}(x^2 + 1)$

Application:

3. The number, N, of bacteria in a culture at time t in hours is $N = 1000\left[30 + e^{\frac{-t}{30}}\right]$.

A) What is the initial number of bacteria? [1]

$N = 1000[30 + e^0]$

$= 1000[31]$

$= 31000$

B) How fast is the number of bacteria changing when t = 20? [3]

$N' = 1000\left[0 + e^{-t/30}(-1/30)\right]$

$= 1000\left(-\frac{1}{30}\right)e^{-t/30}$

$= -\frac{1000}{30}e^{-t/30}$

$N'(20) = -\frac{1000}{30}e^{-20/30}$

$= -17.11$

decreasing by about 17 bacteria per hour

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T/I/PS:

4. A certain radioactive substance decays exponentially over time. The amount of a sample of the substance that remains, P, after t years is given by $P(t) = 10e^{-4t}$, where P is expressed as a percentage. [5]

A) At what time has 50% of the substance decayed?

$50 = 10e^{-4t}$

$0.5 = e^{-4t}$

$\ln .5 = \ln e^{-4t}$

$\ln .5 = -4t \ln e$

$\ln .5 = -4t$

$\frac{\ln .5}{-4} = t$

$0.17 = t$

50% of the substance has decayed at 0.17 years or about 62 days

B) What is the rate of decay when 50% of the original sample has decayed?

$P'(t) = 100e^{-4t}(-4)$

$P'(t) = -400e^{-4t}$

$P'(0.17) = -400e^{-4(0.17)}$

$= -400e^{-0.68}$

$= -400(0.5)$

$= -200$

decreasing at $\approx$ 200 %/year

-202%

Communication:

5. Why can you not use the power rule for derivatives to differentiate $y = 2^x$? [2]

The power rule is used to take the derivatives of polynomial & rational functions the exponent has to be a number, not a variable.