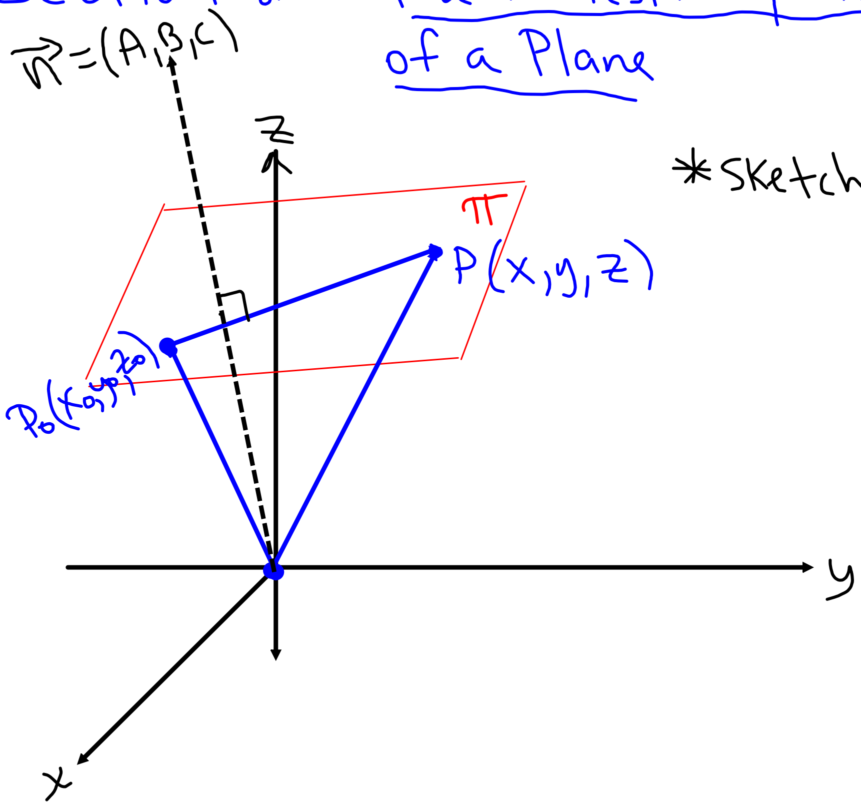
Section 8.5 – The Cartesian Equation of a Plane



* sketch is on p.461

$\overrightarrow{P_0P} = (x-x_0, y-y_0, z-z_0)$

$\therefore \vec{n}\cdot\overrightarrow{P_0P}=0$

$(A,B,C)\cdot(x-x_0, y-y_0, z-z_0)=0$

$Ax-Ax_0+By-By_0+Cz-Cz_0=0$

$Ax+By+Cz+\underbrace{(-Ax_0-By_0-Cz_0)}_{D}=0$

$Ax+By+Cz+D=0$

* Cartesian Equation with normal $\vec{n}=(A,B,C)$

* normal is a non-zero vector $\perp$ to all vectors in the plane.

ex: The point $P_0(1,2,-3)$ (x_0 y_0 z_0) is a point on the plane with normal $\vec{n}=(3,-2,5)$. Determine the cartesian equation of this plane.

Solution:

Method 1: Use the dot product

use arbitrary pt. $P(x,y,z)$

$\overrightarrow{P_0P}=(x-1,y-2,z+3)$ is a vector on the plane.

$\therefore \overrightarrow{P_0P}\cdot\vec{n}=0$

$(x-1,y-2,z+3)\cdot(3,-2,5)=0$

$3x-3-2y+4+5z+15=0$

$3x-2y+5z+16=0$ (Cartesian Equation) (Scalar Eqn)

Method 2:

$Ax+By+Cz+D=0$

$(3)(1)-2(2)+5(-3)+D=0$

$3-4-15+D=0$

$D=16$

$\therefore 3x-2y+5z+16=0$

b) Is vector $\vec{a}=(4,1,-2)$ parallel to the plane?

Is $\vec{a}\cdot\vec{n}=0$

$(4,1,-2)\cdot(3,-2,5)$

$=12-2-10$

$=0$

$\therefore$ vector is parallel

c) Is vector $\vec{b}=(15,-10,25)$ normal to the plane.

Solution:

Vector $\vec{b}$ is $\perp$ to the plane only if it is parallel to the normal vector, $\vec{n}$, that is only if $\vec{b}=k\vec{n}$, where k is a scalar.

$\therefore$ Does $(15,-10,25)=k(3,-2,5)$

$15=3k$	$-10=-2k$	$25=5k$
$5=k$	$5=k$	$5=k$

$\therefore$ yes

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ex: Find the Scalar equation of the plane containing the points $A(-3,-1,-2)$ $B(4,6,2)$ and $C(5,-4,1)$.

[redacted]

Solution: $3x+y-7z-4=0$

$\vec{AB}=(7,7,4)$

$\vec{BC}=(1,-10,-1)$

$\vec{AC}=(8,-3,3)$

$\vec{AB}\times\vec{BC}=(33,11,-77)=(3,1,-7)$

$$\begin{array}{ccc} 7 & \times & -10 \\ 4 & \times & -1 \\ 7 & \times & 1 \\ 7 & \times & -10 \end{array}$$

$$\therefore \vec{n}=(\overset{A}{3},\overset{B}{1},\overset{C}{-7})$$

$\therefore Ax+By+Cz+D=0$ (use $(\overset{x}{4},\overset{y}{6},\overset{z}{2})$)

$(3)(4)+(1)(6)+(-7)(2)+D=0$

$12+6-14+D=0$

$4+D=0$

$D=-4$

$\therefore 3x+y-7z-4=0$

ex: Determine the Cartesian form of the plane whose equation in vector form is

$\vec{r} = (2,3,1) + s(1,1,2) + t(3,4,1)$

Solution:

2 direction vectors are needed.

$\vec{m}_1 = (1,1,2)$
$\vec{m}_2 = (3,4,1)$

The normal vector is $\vec{m}_1 \times \vec{m}_2$

$\begin{array}{cc} 1 & 4 \\ 2 & 1 \\ 1 & 3 \\ 1 & 4 \end{array}$ $\quad \vec{n}_1 = (-7,5,1)$

$\therefore Ax + By + Cz + D = 0$
$-7x + 5y + z + D = 0$

Sub in point (2,3,1) to get

$-7(2) + 5(3) + (1)(1) + D = 0$
$-14 + 15 + 1 + D = 0$
$D = -2$

$\therefore -7x + 5y + z - 2 = 0$

$7x - 5y - z + 2 = 0$

ex: Determine the angle between the planes
$\pi_1: x+y-z+1=0$ and
$\pi_2: 2x-y+3z+4=0$.

Solution:

$\vec{n_1} = (1,1,-1) \quad \vec{n_2} = (2,-1,3)$

$\vec{n_1} \cdot \vec{n_2} = |\vec{n_1}||\vec{n_2}| \cos\theta$

$(1,1,-1)\cdot(2,-1,3) = |(1,1,-1)|\,|(2,-1,3)| \cos\theta$

$2-1-3 = \sqrt{3}\sqrt{14} \cos\theta$

$-2 = \sqrt{42} \cos\theta$

$\cos\theta = \frac{-2}{\sqrt{42}}$

$\boxed{\theta = 108^\circ \text{ or } 72^\circ}$

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Ex: Determine the vector and parametric equations of the plane with Cartesian equation

$$x - y + 3z + 2 = 0$$