5.5 The Derivative of Tan(x)

The derivative of $y = \tan x$ is $y' = \sec^2 x$

› Proof:

$$y = \tan x$$

$$y = \frac{\sin x}{\cos x}$$

$$y' = \frac{\cos x \cos x \ - (-\sin x) sinx}{(\cos x)^2}$$

$$= \frac{(\cos x)^2 + (\sin x)^2}{(cosx)^2}$$

$$= \frac{1}{(cosx)^2}$$

$$= \sec^2 x$$

› Ex 1:

determine the derivative for $y = \tan(x^2 + 3x)$

$$y^1 = \sec^2(x^2 + 3x)$$

$$y^1 = (2x + 3) \cdot \sec^2(x^2 + 3x)$$

› Ex 2:

determine the derivative for $y = (\sin x + \tan x)^4$

$$y' = 4(\sin x + \tan x)^3(\cos x + \sec^2 x)$$

Ex 3: $determine\ the\ derivative\ for\ y = (sin^3\ x)(tan\ x)$

- $y = (\sin x)^3(tanx)$
- $y' = 3(\sin x)^2(\cos x)(tanx) + \sec^2 x(\sin x)^3$
- $y' = \sin^2 x(3\cos x \tan x + \sec^2 x \sin x)$

EX 4: $determine\ the\ derivative\ for\ y = e^{tan\sqrt{x}}$

- $y = e^{\tan x^{\frac{1}{2}}}$
- $y' = e^{\tan x^{\frac{1}{2}}}\left(\sec^2 x^{\frac{1}{2}}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$