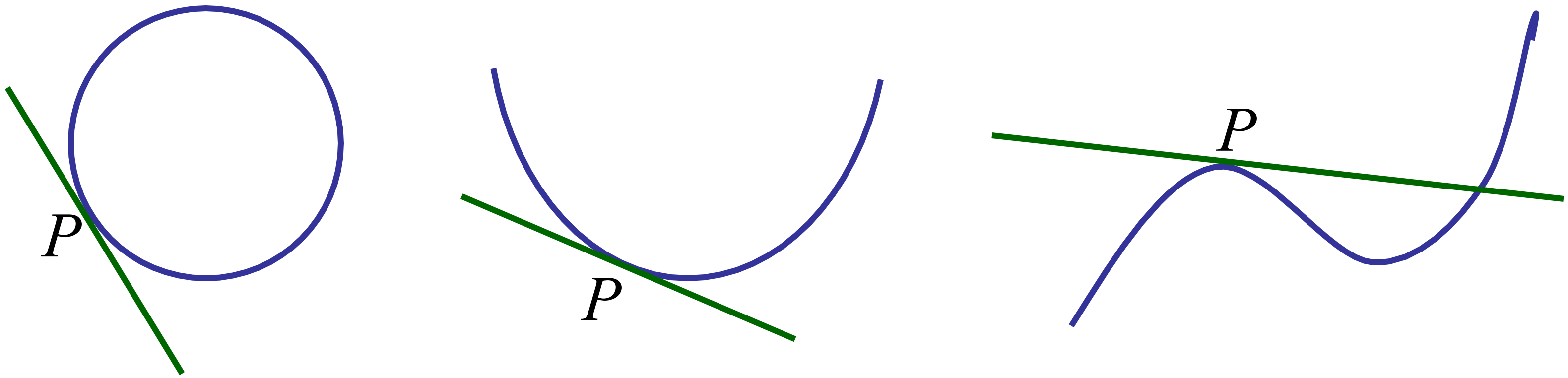
1.2-1.3 The Slope of a Tangent & Rates of Change

The slope of the line that is tangent to the graph at a single point represents the instantaneous rate of change.

The ***tangent problem*** is fundamental to calculus.

The tangent line touches the circle at one point.



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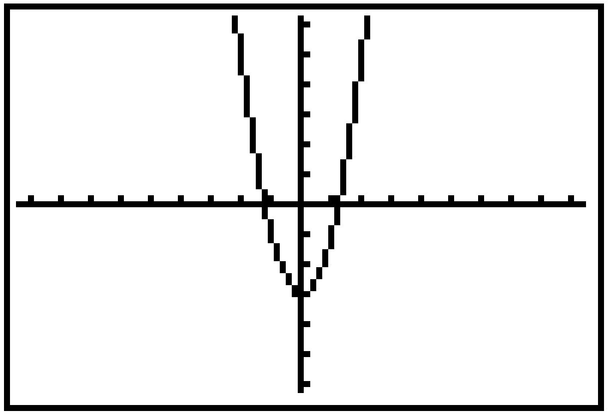
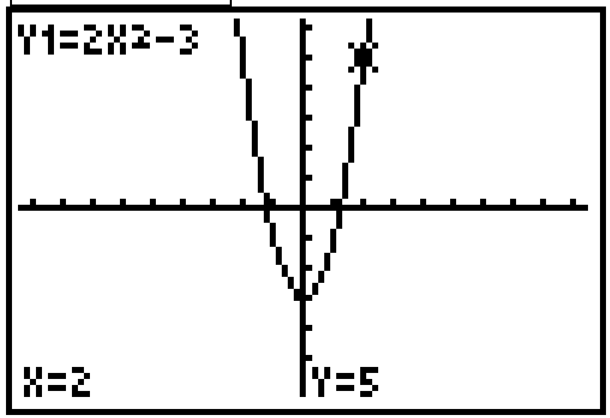
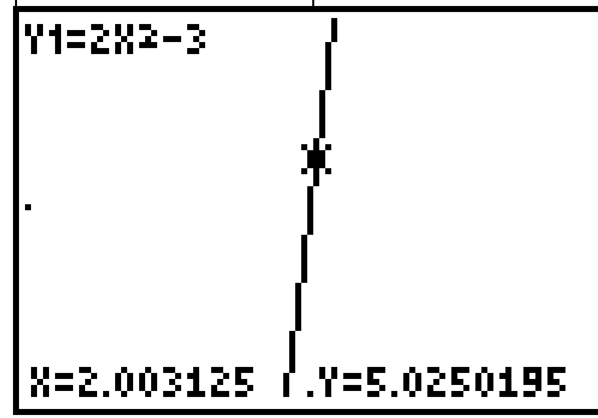
Example1: Determine equation of the tangent line of $f(x) = 2x^2 - 3$ at point (2, 5)



trace



zoom in



$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{5.0250195 - 5}{2.003125 - 2}$$

$$m = \frac{0.0250195}{0.003125}$$

$$m = 8.00624$$

$$y = mx + b$$

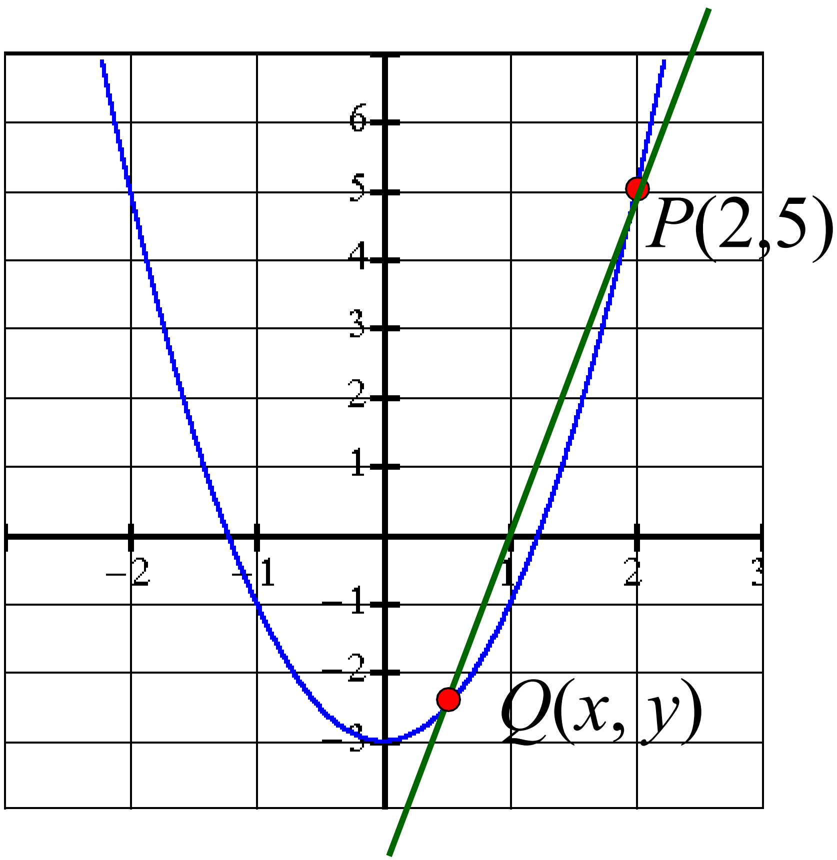
$$5 = 8.00624(2) + b$$

$$b = -11.01248$$

$$\boxed{y = 8.00624x - 11.01248}$$

A ***secant*** is a line that passes through two points on the graph of a relation.

$f(x) = 2x^2 - 3$ at point (2, 5)

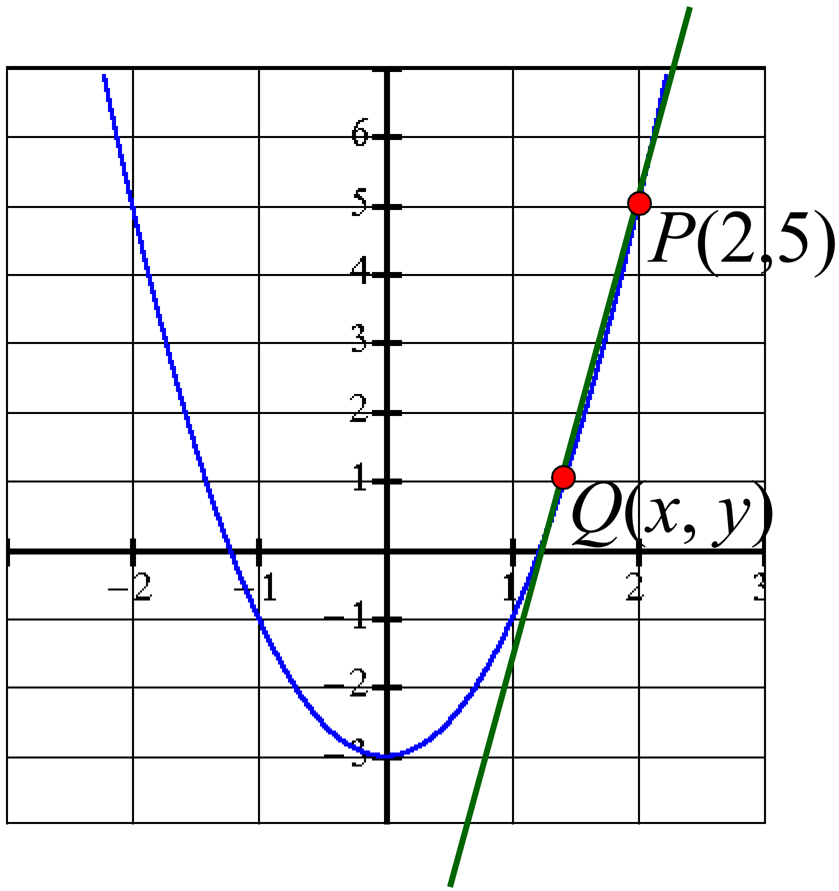


In the previous example we estimated the slope of the tangent line by choosing a point close to point *P*.

By choosing a point to the right of point P, we found the slope to be equal to 8.00624.

$f(x) = 2x^2 - 3$ at point (2, 5)

If we had chosen a point to the left of point P, i.e. Q(1.99, 4.9202) the slope would have been …



$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{5 - 4.9202}{2 - 1.99}$$

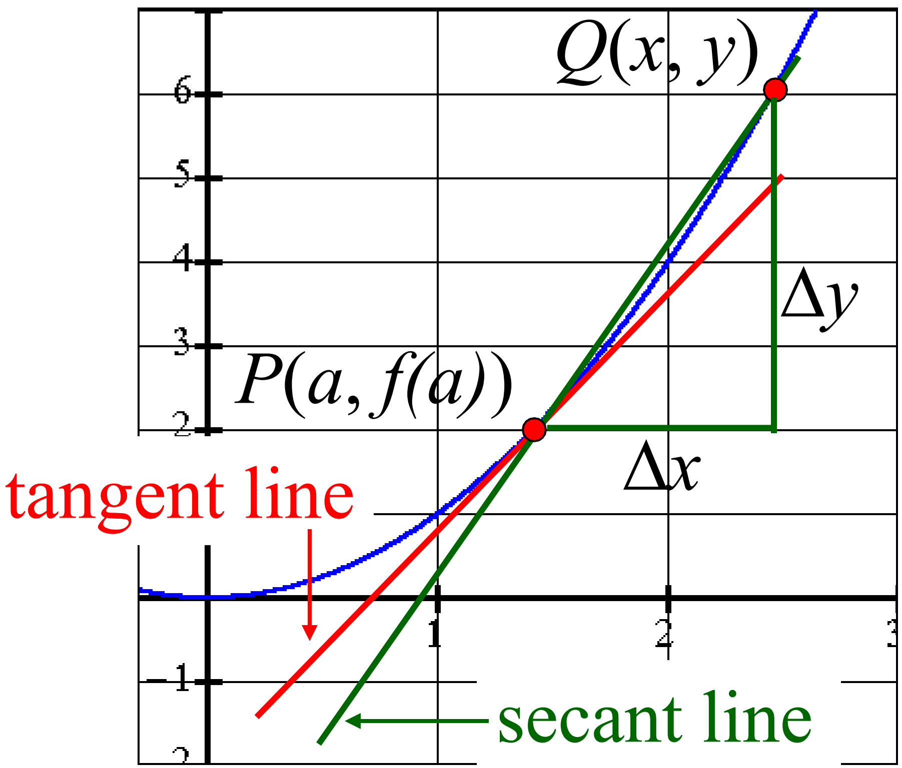
$$m = 7.98$$

As point Q gets closer to point P the slope gets closer to 8.

Definition of Limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

is called a limit. It represents the value that the ratio approaches as Δx gets close to zero.



Slope of *PQ* is

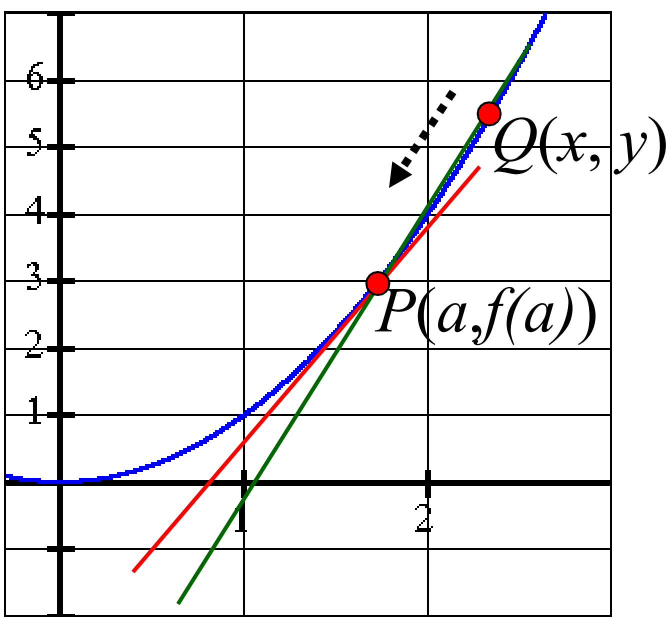
$$m_{PQ} = \frac{\Delta y}{\Delta x}$$

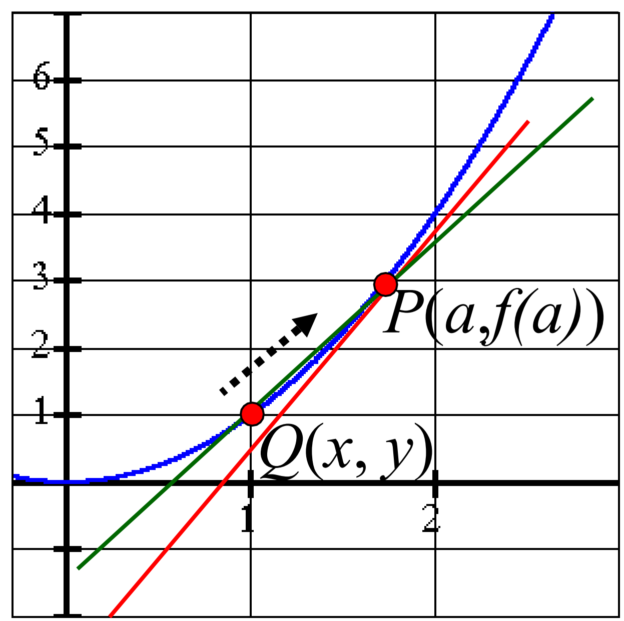
$$= \frac{f(x) - f(a)}{x - a}$$

Slope of a Secant line: represents the average rate of change between two points on the graph of a relation.

Slope of a Tangent: at *P* is the limiting value of the slopes of the secants as *Q* approaches point *P*.

When using a series of secant lines to approximate the slope of the tangent you must approach the point of tangency from *both* left and right sides.





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As $\Delta x = x - a$ decreases to 0, the limiting value of the secant slopes is the *slope of the tangent line*.

Instantaneous Rate of Change

slope of tangent = instantaneous rate of change

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Using Secants to Approximate Tangents (*instantaneous rates of change*).

Example: The amount of algae, A, in kg, formed in a garden pond during the summer can be modelled by $A(t) = 10t - 0.6t^2$, where t is the time in weeks after June 1. Estimate the rate at which the amount of algae is changing on July 6.

June 1 to July 6 is five weeks so $t = 5$.

instantaneous rate of change $= \lim\limits_{\Delta t \to 0} \dfrac{\Delta A}{\Delta t}$

$$= \lim_{t \to 5} \frac{A(t) - A(5)}{t - 5}$$

$$m_{secant} = \frac{\Delta A}{\Delta t}$$

$$= \frac{A(t) - A(5)}{t - 5}$$

$$= \frac{10t - 0.6t^2 - [10(5) - 0.6(5^2)]}{t - 5}$$

$$= \frac{10t - 0.6t^2 - 35}{t - 5}$$

from the right

t	$m_{secant} = \frac{\Delta A}{\Delta t}$
6	3.4
5.5	3.7
5.1	3.94
5.01	3.994
5.001	3.9994

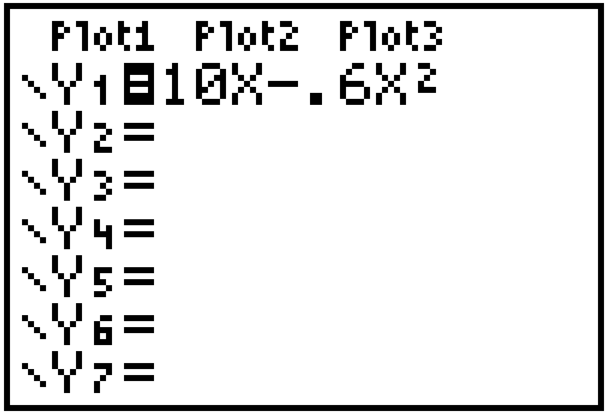
from the left

t	$m_{secant} = \frac{\Delta A}{\Delta t}$
4	4.6
4.5	4.3
4.9	4.06
4.99	4.006
4.999	4.0006

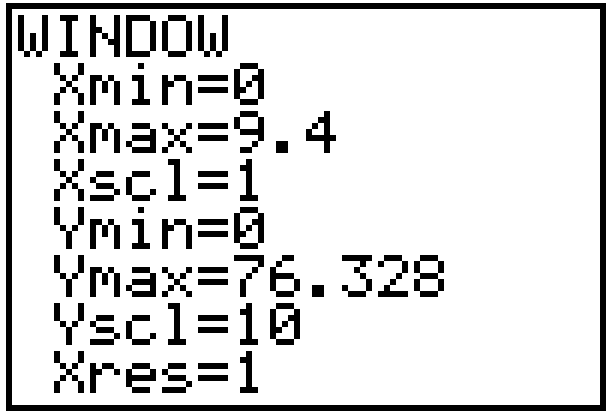
The value of m_{secant} approaches 4. $\therefore$ The instantaneous rate of change at 5 weeks is 4 kg/week.

Using a Graphing Calculator

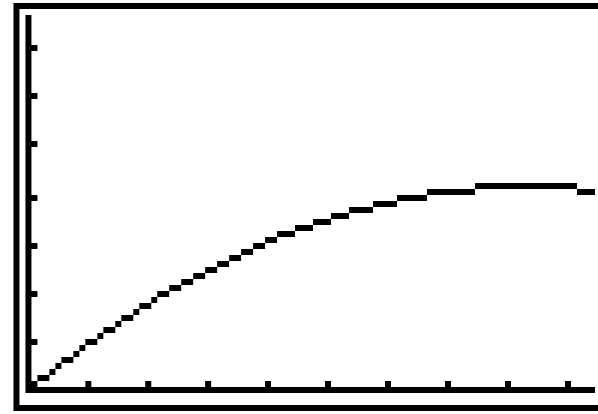
Y=



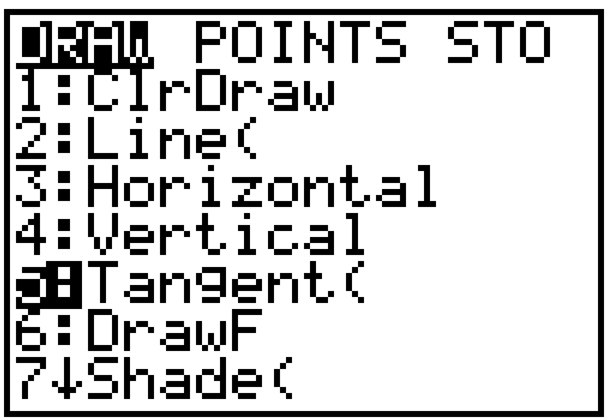
Window



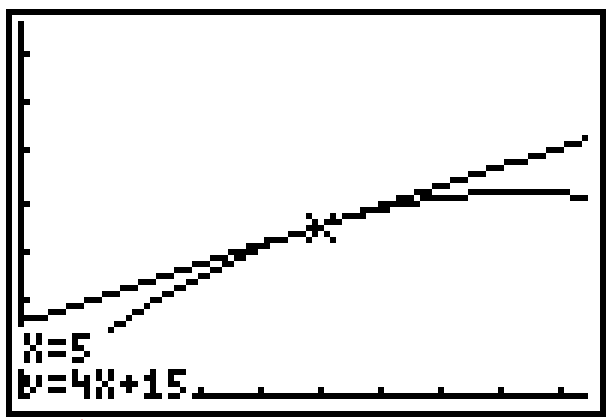
Graph



Draw



Trace and Enter



We see that the slope of the tangent line at $x = 5$ equals 4.