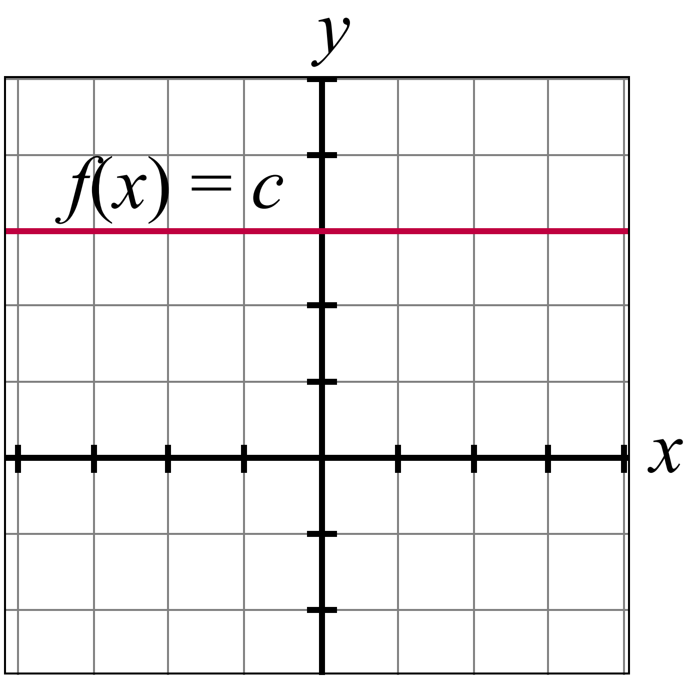
2.2 The Derivatives of Polynomial Functions

Differentiation Rules:

1) The derivative of a constant: (Constant Function Rule)



The function $f(x) = c$, where c is a constant is the same for all values of x so the rate of change is zero.

If $f(x) = c$, c is a constant

$$f'(x) = 0$$

$$\frac{d}{dx}(c) = 0$$ (Liebnitz notation)

Example: determine the derivative of the following:

a) $f(x) = 3$

$f'(x) = 0$

b) $y = -3$

$$\frac{dy}{dx} = 0$$

c) $y = -\frac{2}{5}$

$y' = 0$

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2) The Derivative of a Power Function: (Power Rule)

If n is a positive integer and $f(x) = x^n$ then $f'(x) = nx^{n-1}$

Example: determine the derivative of the following:

a) $f(x) = x^5$

$f'(x) = 5x^4$

b) $y = t^7$

$$\frac{dy}{dt} = 7t^6$$

c) $y = t^{13}$

$y' = 13t^{12}$

3) The General Power Rule

If n is a real number and $f(x) = x^n$ then $f'(x) = nx^{n-1}$

Liebnitz notation: $\dfrac{d}{dx}(x^n) = nx^{n-1}$

Example: differentiate each function.

a) $f(x) = x^{-7}$

$$f'(x) = -7x^{-7-1}$$

$$= -7x^{-8}$$

$$= \frac{-7}{x^8}$$

b) $y = \sqrt[4]{x^3}$

$$y = x^{\frac{3}{4}}$$

$$\frac{dy}{dx} = \frac{3}{4}x^{\frac{3}{4}-1}$$

$$= \frac{3}{4}x^{-\frac{1}{4}}$$

$$= \frac{3}{4}\left(\frac{1}{x^{\frac{1}{4}}}\right)$$

$$= \frac{3}{4\sqrt[4]{x}}$$

4) The Constant Multiple Rule

If $g(x) = cf(x)$ then $g'(x) = cf'(x)$

or $\frac{d}{dx}[cf(x)] = c\left[\frac{d}{dx}f(x)\right]$

Example: differentiate each function.

a) $f(x) = 5x^4$

$f'(x) = 5(4)x^3$

$= 20x^3$

b) $y = 8\sqrt[2]{x^3}$

$= 8x^{\frac{3}{2}}$

$\frac{dy}{dx} = 8\left(\frac{3}{2}\right)x^{\frac{3}{2}-1}$

$= 12x^{\frac{1}{2}}$

$= 12\sqrt{x}$

Example: Determine the equation of the tangent to the graph $y = 2x^2 + 5x$ that has a slope -3.

The slope of the tangent line is the derivative.

$$y = 2x^2 + 5x$$

$$y' = 4x + 5$$

$$4x + 5 = -3$$

$$4x = -3 - 5$$

$$4x = -8$$

$$x = -\frac{8}{4}$$

$$x = -2$$

$$y = 2x^2 + 5x$$

$$y = 2(-2)^2 + 5(-2)$$

$$y = -2$$

The point on the graph is $(-2, -2)$

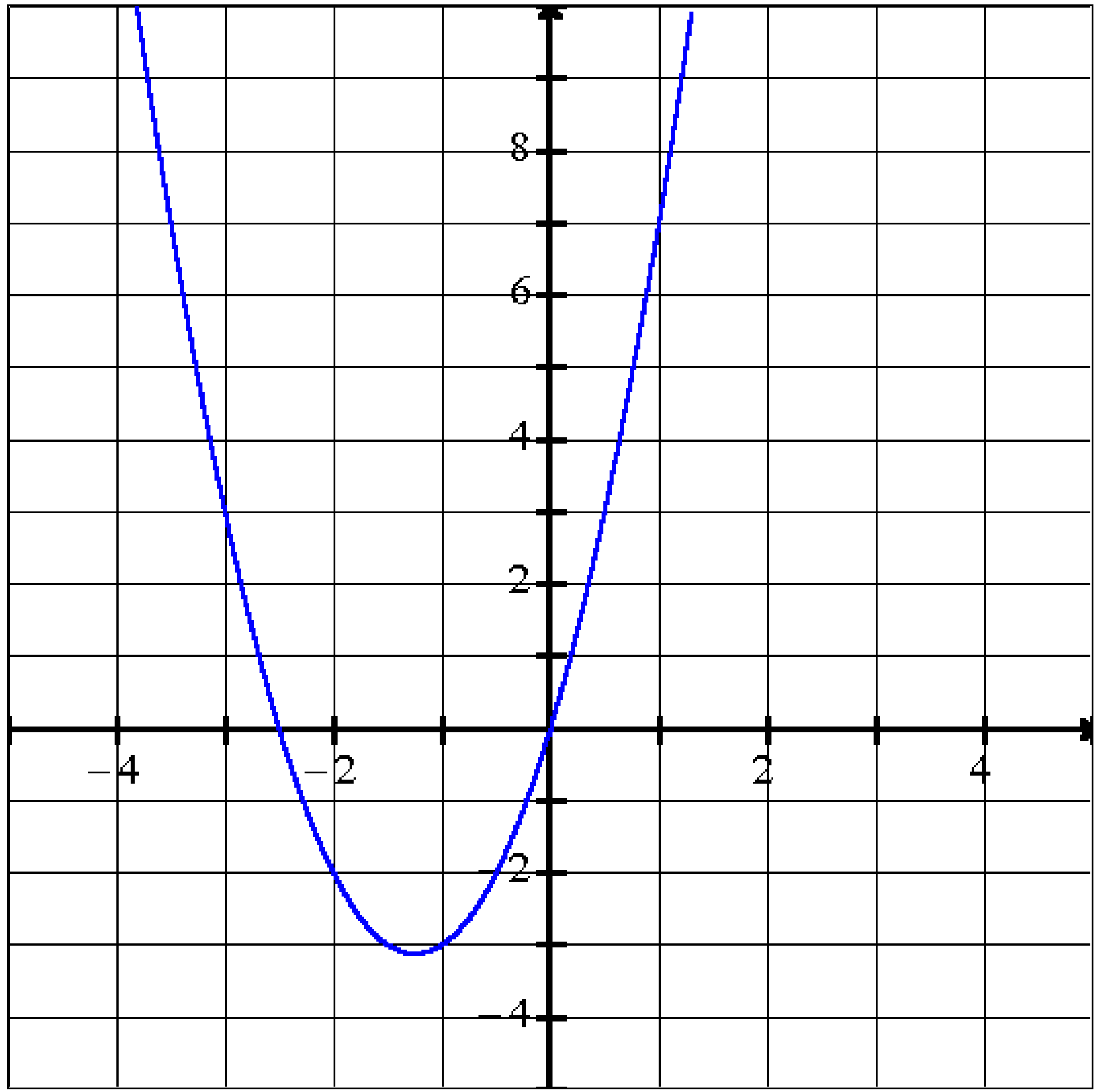
$$(y - y_1) = m(x - x_1)$$

$$(y + 2) = -3(x + 2)$$

$$y + 2 = -3x - 6$$

$$\boxed{3x + y + 8 = 0}$$

$$y = 2x^2 + 5x$$



The Sum and Difference Rules

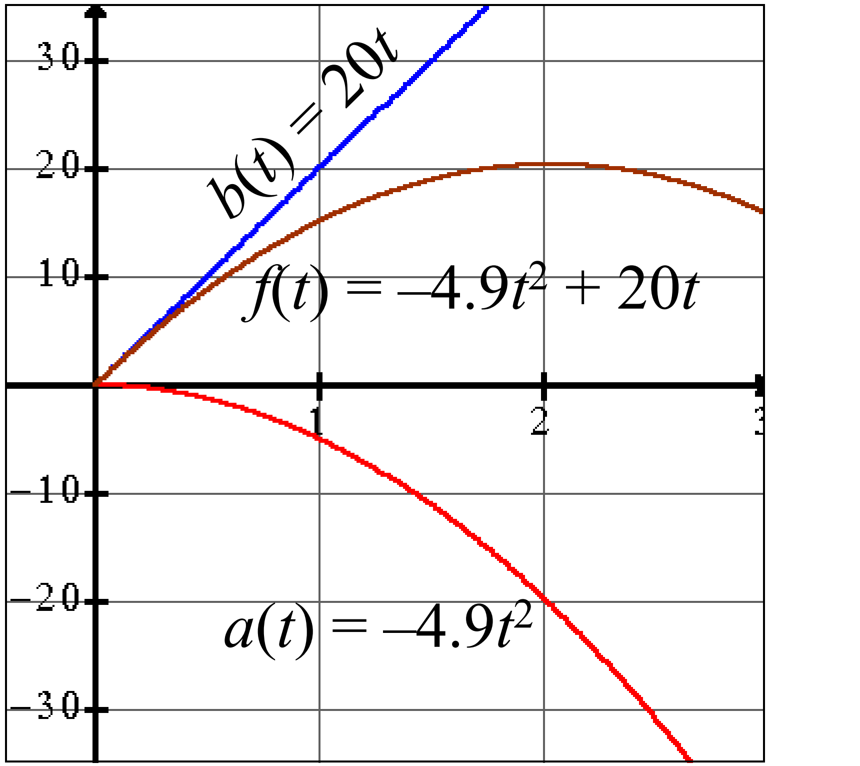
1) A golf ball is hit with an initial velocity of 50 m/s. The function $f(t) = -4.9t^2 + 20t$ models the height of the ball. Determine the instantaneous rate of change of the height of the ball at 0.5 s.

The function $f(t) = -4.9t^2 + 20t$ is the sum of two different functions.

$$f(t) = a(t) + b(t)$$

Is $f'(t) = a'(t) + b'(t)$?

Is $f'(t) = -9.8t + 20$?



Determine the derivative from first principles.

$f(t) = -4.9t^2 + 20t$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[-4.9(t+h)^2 + 20(t+h)] - [-4.9t^2 + 20t]}{h}$$

$$= \lim_{h \to 0} \frac{-4.9(t^2 + 2th + h^2) + 20t + 20h + 4.9t^2 - 20t}{h}$$

$$= \lim_{h \to 0} \frac{-4.9t^2 - 9.8th - 4.9h^2 + 20h + 4.9t^2}{h}$$

$$= \lim_{h \to 0} \frac{-9.8th - 4.9h^2 + 20h}{h}$$

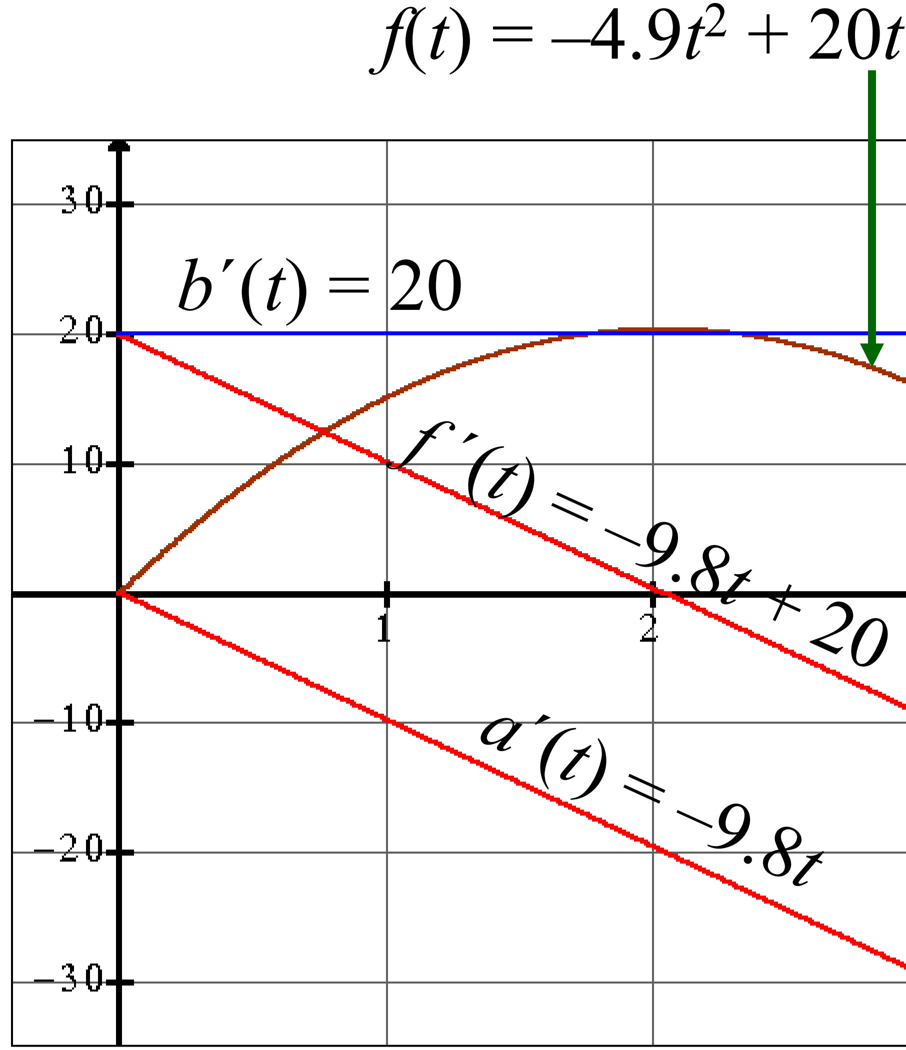
$$= \lim_{h \to 0} (-9.8t - 4.9h + 20)$$

$$= -9.8t - 4.9(0) + 20$$

$$\boxed{= -9.8t + 20}$$

$$\therefore f'(t) = a'(t) + b'(t)$$

We can see graphically that $f'(t) = a'(t) + b'(t)$



The Sum Rule: if $h(t) = f(t) + g(t)$

then: $h'(t) = f'(t) + g'(t)$

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

The Difference Rule: if $h(t) = f(t) - g(t)$

then: $h'(t) = f'(t) - g'(t)$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]$$

The Derivative of any Polynomial Function

For any polynomial function:

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$$

$$P'(x) = na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + 2a_2x^2 + a_1$$

Example: Determine the derivative of

$$y = 5x^3 + 3x^2 - 2x - 4$$

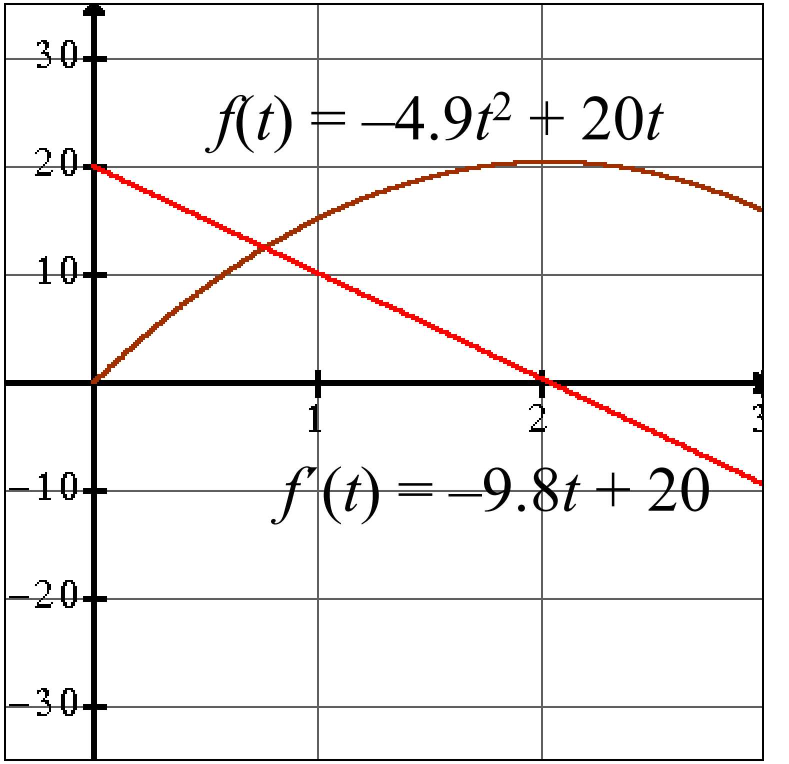
$$\frac{dy}{dx} = \frac{d}{dx}\left[5x^3 + 3x^2 - 2x - 4\right]$$

$$= \frac{d}{dx}\left[5x^3\right] + \frac{d}{dx}\left[3x^2\right] + \frac{d}{dx}\left[-2x\right] + \frac{d}{dx}\left[-4\right]$$

$$= 15x^2 + 6x - 2 + 0$$

Returning to the initial problem:

1) A golf ball is hit with an initial velocity of 50 m/s. The function $f(t) = -4.9t^2 + 20t$ models the height of the ball. Determine the instantaneous rate of change of the height of the ball at 0.5 s.



$$f(t) = -4.9t^2 + 20t$$

$$f'(t) = -9.8t + 20 \qquad \text{sub } t = 0.5$$

$$\begin{aligned} f'(0.5) &= -9.8(0.5) + 20 \\ &= -4.9 + 20 \\ &= 15.1 \end{aligned}$$

After 0.5 seconds the rate of change of the golf ball is 15.1 m/s.