MCV4UR
Unit 1 Test, Introduction to Calculus
Mark: /47

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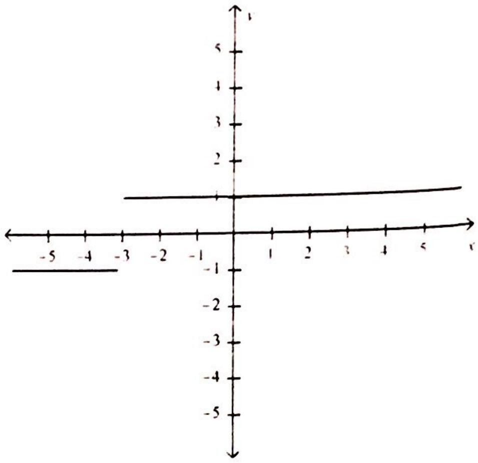
Show **all** work **neatly** in the space provided using methods taught in this course.

1. Determine which of the following limits is represented by the graph on the right.



a. $\lim_{x \to 3} \frac{|x-3|}{x-3}$ c. $\lim_{x \to 3} \frac{|x+3|}{x+3}$

b. $\lim_{x \to -3} \frac{|x-3|}{x-3}$ d. $\lim_{x \to -3} \frac{|x+3|}{x+3}$

2. For $f(x) = x^2$, which value has the largest magnitude?

$f'(x) = 2x$

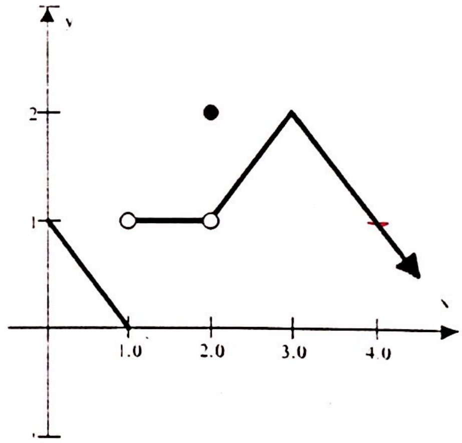
a. average rate of change from $x = -1$ to $x = 1$ $= 0$

b. instantaneous rate of change at $x = 1$ $= 2$

c. instantaneous rate of change at $x = -2$ $= |-4| = 4$

d. average rate of change from $x = -3$ to $x = 0$ $\frac{f(0) - f(-3)}{3} = \frac{0-9}{3} = |-3| = 3$

3. For the function in the diagram on the right, state the following:



a) $\lim_{x \to 1} f(x)$ DNE

b) $\lim_{x \to 2} f(x)$ 1

c) $\lim_{x \to 4^-} f(x)$ 1

d) value(s) of *x* for which *f* is discontinuous 1 & 2

e) Type of discontinuity at the point identified in question "*d*" above.
$x = 1$ jump discontinuity
$x = 2$ hole or removable

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4. Determine where *f*(*x*) is discontinuous and explain why the function is discontinuous.

$$f(x) = \begin{cases} \frac{x^2 - 5x + 6}{x - 3} & \text{if } x < 2 \\ 3x - 1 & \text{if } x \geq 2 \end{cases}$$

$$f(x) = \begin{cases} \frac{(x-3)(x-2)}{(x-2)}, & \text{if } x < 2 \\ 3x - 1, & \text{if } x \geq 2 \end{cases}$$

T2

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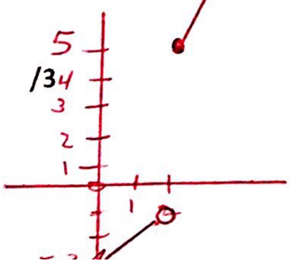

$f(2) = 2 - 3$
$= \boxed{-1}$

$f(2) = 3(2) - 1$
$= \boxed{5}$

$\therefore$ the function is discontinuous at $x = 2$. There is a jump discontinuity there.

5. Determine each of the following limits. If the limit does not exist show why.

a) $\lim_{x \to -2} -7$

$= -7$

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b) $\lim_{x \to \frac{\pi}{2}} \tan x$

$= \tan\frac{\pi}{2}$

$=$ undefined

V.A. at $\pi/2$

c) $\lim_{x \to -1} \frac{5x^2-2x+3}{2x^2+4x-1}$

$= \frac{5(-1)^2-2(-1)+3}{2(-1)^2+4(-1)-1}$

$= \frac{5+2+3}{2-4-1}$

$= -10/3$

d) $\lim_{x \to 4} \frac{2x}{x-4}$ /2

$= \frac{2(4)}{4-4}$

$= \frac{8}{0}$

undefined

$\lim_{x \to 4} \frac{2x}{x-4} =$ DNE

V.A.

e) $\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$ /3

Substitution yields 0/0

$= \lim_{x\to5} \frac{\sqrt{x-1}-2}{x-5} \cdot \frac{\sqrt{x-1}+2}{\sqrt{x-1}+2}$

$= \lim_{x\to5} \frac{(x-1)-4}{(x-5)(\sqrt{x-1}+2)}$

$= \lim_{x\to5} \frac{(x-5)}{(x-5)(\sqrt{x-1}+2)}$

$= \lim_{x\to5} \frac{1}{\sqrt{x-1}+2} = \boxed{\frac{1}{4}}$

f) $\lim_{x \to -1} \frac{5|x+1|}{x+1}$ /2 3

= when $x+1>0$

$\lim_{x\to-1} \frac{5(x+1)}{(x+1)}$

$= \lim_{x\to-1} 5$

$= 5$

when $x+1<0$

$\lim_{x\to-1} \frac{-5(x+1)}{(x+1)}$

$= \lim_{x\to-1} -5$

$= -5$

$\therefore \lim_{x\to-1} \frac{5|x+1|}{x+1}$

DNE

6. Given $f(x) = \frac{x}{5x-1}$, find $f'(x)$ using ***the definition of a derivative (first principles)***. /4

$= \lim_{h\to0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to0} \frac{\frac{x+h}{5(x+h)-1} - \frac{x}{5x-1}}{h}$

$= \lim_{h\to0} \frac{\frac{(x+h)(5x-1)-x[5(x+h)-1]}{(5(x+h)-1)(5x-1)}}{h}$

$= \lim_{h\to0} \frac{\frac{5x^2-x+5xh-h-5x^2-5xh+x}{(5(x+h)-1)(5x-1)}}{h}$

$= \lim_{h\to0} \frac{\frac{-h}{(5(x+h)-1)(5x-1)}}{h}$

$= \lim_{h\to0} \frac{-1}{(5(x+h)-1)(5x-1)}$

$= \frac{-1}{(5x-1)(5x-1)}$

$= \frac{-1}{(5x-1)^2}$.

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7. Determine **the equation** of the tangent line to the function $f(x) = \sqrt{2x-1}$ if the tangent line is ***perpendicular*** to $4x + 2y = 7$.

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$$f'(x) = \lim_{h \to 0} \frac{\sqrt{2(x+h)-1} - \sqrt{2x-1}}{h} \cdot \frac{\sqrt{2(x+h)-1} + \sqrt{2x-1}}{\sqrt{2(x+h)-1} + \sqrt{2x-1}}$$

$$= \lim_{h \to 0} \frac{2(x+h)-1-(2x-1)}{h(\sqrt{2(x+h)-1} + \sqrt{2x-1})}$$

$$= \lim_{h \to 0} \frac{2x+2h-1-2x+1}{h(\sqrt{2(x+h)-1} + \sqrt{2x-1})}$$

$$= \lim_{h \to 0} \frac{2}{\sqrt{2(x+h)-1} + \sqrt{2x-1}}$$

$$= \frac{2}{\sqrt{2x-1} + \sqrt{2x-1}}$$

$$= \frac{2}{2\sqrt{2x-1}} \quad \boxed{= \frac{1}{\sqrt{2x-1}}}$$

$4x + 2y = 7$

$2y = 7 - 4x$

$y = \frac{7}{2} - 2x$

$m = -2$

$\boxed{m_{\perp} = \frac{1}{2}}$

$\frac{1}{2} = \frac{1}{\sqrt{2x-1}}$

$\sqrt{2x-1} = 2$

$2x - 1 = 4$

$2x = 5$

$\boxed{x = 5/2}$

$y = \sqrt{2x-1}$

$y = \sqrt{5-1}$

$y = 2$

$\therefore y = mx + b$

$2 = \frac{1}{2}\left(\frac{5}{2}\right) + b$

$2 = \frac{5}{4} + b$

$b = 3/4$

$\boxed{y = \frac{1}{2}x + \frac{3}{4}}$

8. Evaluate $\lim_{x \to 256} \frac{x^{\frac{1}{4}} - 4}{x - 256}$ Use change of variable.

Let $u = x^{1/4}$

$u^4 = x$

as $x \to 256$, $u \to 4$

$$\therefore \lim_{u \to 4} \frac{u-4}{u^4 - 256}$$

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$$= \lim_{u \to 4} \frac{u-4}{(u^2-16)(u^2+16)}$$

$$= \lim_{u \to 4} \frac{u-4}{(u-4)(u+4)(u^2+16)}$$

$$= \lim_{u \to 4} \frac{1}{(u+4)(u^2+16)}$$

$$= \frac{1}{(4+4)(16+16)}$$

$$= \frac{1}{(8)(32)}$$

$$= \frac{1}{256}$$

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9. If $\lim_{x \to 1} f(x) = 1$ what is the value of $\lim_{x \to 1} \frac{2f(x) - x^2}{f(x)+1}$? Demonstrate use of the laws of limits. /3

$$\lim_{x \to 1} \frac{2f(x) - x^2}{f(x)+1}$$

$$= \frac{2\lim_{x \to 1} f(x) - \lim_{x \to 1} x^2}{\lim_{x \to 1} f(x) + 1}$$

$$= \frac{2(1) - (1)^2}{1+1}$$

$$= \frac{1}{2}$$

10. An oil tank is being drained for cleaning. After t minutes, there are V litres of oil left in the tank, where $V(t) = 55(32-t)^2$, $0 \leq t \leq 32$. Calculate the average rate of change in volume during the first 22 minutes.

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$$\frac{V(22) - V(0)}{22 - 0}$$

$$= \frac{55(32-22)^2 - 55(32-0)^2}{22}$$

$$= \frac{55(100) - 55(1024)}{22}$$

$$= \frac{5500 - 56320}{22}$$

$$= \frac{-50820}{22}$$

$$= -2310$$

∴ The average rate of change in the first 22 minutes is -2310 litres/minute

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11. Suppose the motion of an avalanche is described by the function $s(t) = 5t^2$, where s is the distance in metres travelled by the leading edge of the snow at t minutes. Determine the rate at which the avalanche is moving at 4 minutes.

/3 (3)

/4

$$\lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

$$= \lim_{h \to 0} \frac{5(t+h)^2 - 5t^2}{h}$$

$$= \frac{5(t^2 + 2th + h^2) - 5t^2}{h}$$

$$= \lim_{h \to 0} \frac{5t^2 + 10th + 5h^2 - 5t^2}{h}$$

$$= \lim_{h \to 0} \frac{10th + 5h^2}{h}$$

$$= \lim_{h \to 0} 10t + 5h$$

$$= 10t$$

at 4 minutes, rate is $10(4) = 40$

40 m/minute

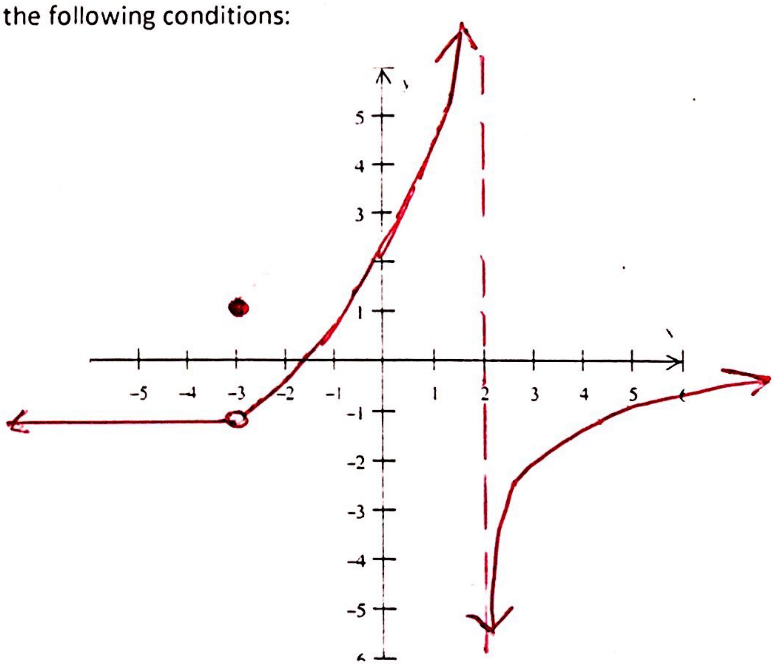
12. Sketch the graph of any function that satisfies the following conditions:

$$\lim_{x \to 2^-} f(x) = \infty$$

$$\lim_{x \to 2^+} f(x) = -\infty$$

$$\lim_{x \to -3} f(x) = -1$$

$$f(-3) = 1$$



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