MCV4U - Unit 3 Test
Derivatives and Their Applications

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Date: March 11, 2016

Mark: ________
43

Answer all questions on this paper. Be sure to show all applicable work and express all answers in simplest form. Marks are awarded for presentation and technical correctness.

1. A paint company estimates that the cost in dollars, C, of producing x litres of paint per day is $C(x) = 0.0006x^2 + 8x + 3000$.

A) Find the cost of producing 125 litres. [2]

$C(125) = 0.0006(125)^2 + 8(125) + 3000$
$= 9.375 + 1000 + 3000$
$= \$4009.38$

costs \$4009.38 to produce 125 L

B) Find the marginal cost of producing 125 litres. [2]

$C'(x) = 0.0012x + 8$
$C'(125) = 0.0012(125) + 8$
$= \$8.15$

∴ marginal cost is \$8.15/L for the 125th L

2. Let the function $s(t) = -\frac{5}{2}t^2 + 10t + 15$ represent the motion of a toy car as it travels near a sensor. At what time t will the car's distance from the sensor be the greatest? What is its velocity when the car is at that point? [4]

$S'(t) = -5t + 10$
Set $S'(t) = 0$
$-5t + 10 = 0$
$-5t = -10$
$t = 2$

At the 2 second mark, the car's distance from the sensor will be the greatest. It's velocity at this point is 0

$\overline{8}$

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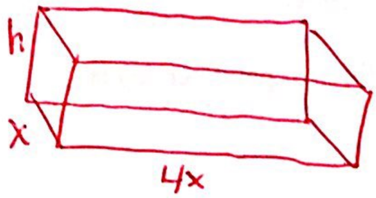
CHOICE: **Do any <u>five</u> questions from #3 - #8.**
Be sure to label the five you want marked.

3. A rectangular wooden bedding chest will be built so that its length is 4 times its width. The top, front, and two sides of the chest will be oak. The back and bottom of the chest will be cedar. The volume of the chest must be 2.4 m^3. Oak costs 2 times as much as cedar. Find the dimensions that will minimize the cost of the chest. [7]

*otherway

$0.475 \times 1.9 \times 2.66m$



$V = 4x^2h$

$2.4 = 4x^2h$

$h = \frac{2.4}{4x^2}$

$\boxed{h = \frac{0.6}{x^2}}$

$$\begin{array}{ccccc} \text{top} & \text{front} & \text{sides} & \text{bottom} & \text{back} \end{array}$$

$C = 2(4x^2) + 2(4xh) + 2(2xh) + 4x^2 + 4xh$

$C = 8x^2 + 8xh + 4xh + 4x^2 + 4xh$

$C = 12x^2 + 16xh$

$C = 12x^2 + 16x\left(\frac{0.6}{x^2}\right)$

$C = 12x^2 + \frac{9.6}{x}$

$C = 12x^2 + 9.6x^{-1}$

$C' = 24x - 9.6x^{-2}$

Set $C' = 0$

$24x = \frac{9.6}{x^2}$

$24x^3 = 9.6$

$x^3 = \frac{9.6}{24}$

$x^3 = 0.4$

$x = 0.7368$ m

$4x = 2.96$ m

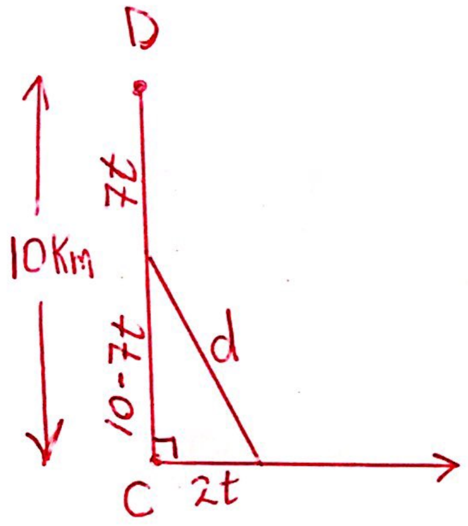
$h = \frac{0.6}{(0.7368)^2}$

$h = 1.11$ m

$\therefore$ the dimensions that will minimize the cost of the chest are

$2.96m \times 0.74m \times 1.11m$

4. Deeg and Coutu are both training for a marathon. Deeg's house is located 10 km north of Coutu's house. At 8 AM one Saturday morning, Deeg leaves his house and jogs south at 7 km/h. At the same time, Coutu leaves his house and jogs east at 2 km/h. When are Deeg and Coutu closest together, given that they both run for 3 hours? [7]



$$d^2 = (2t)^2 + (10-7t)^2$$

$$d^2 = 4t^2 + 100 - 140t + 49t^2$$

$$d^2 = 53t^2 - 140t + 100$$

$$(d^2)' = 106t - 140$$

$$106t = 140$$

$$t = 1.32 \text{ hours}$$

$\therefore$ Deeg and Coutu are closest together after 1.32 hours or approximately 9:19 AM

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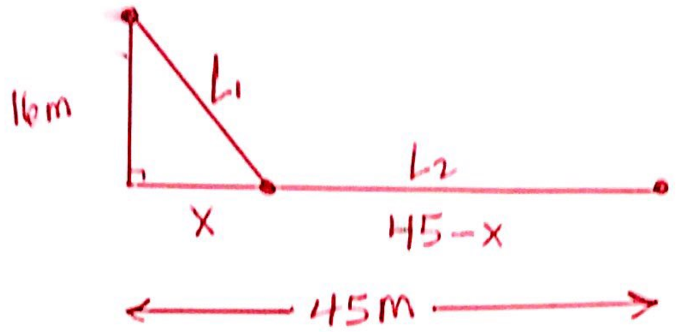
Cable service is being installed in a new subdivision. The cable must cross a 16 metre wide river and reach a point which is 45 metres downstream from its starting point on the other bank. Laying cable under water costs three times as much as laying it over ground. How should the cable be routed to minimize the cost? [7]

$(L_1)^2 = x^2 + 16^2$

$(L_1)^2 = x^2 + 256$

$L_1 = \sqrt{x^2 + 256}$



$C = 3(x^2 + 256)^{1/2} + (45 - x)$

$C' = 1.5(x^2 + 256)^{-1/2}(2x) - 1$

$1 = \dfrac{1.5(2x)}{\sqrt{x^2 + 256}}$

$\sqrt{x^2 + 256} = 3x$

$x^2 + 256 = 9x^2$

$256 = 8x^2$

$32 = x^2$

$x = 5.66$

$\therefore$ the cable should be routed 5.66m from its starting point on the other bank.

or 39.34m

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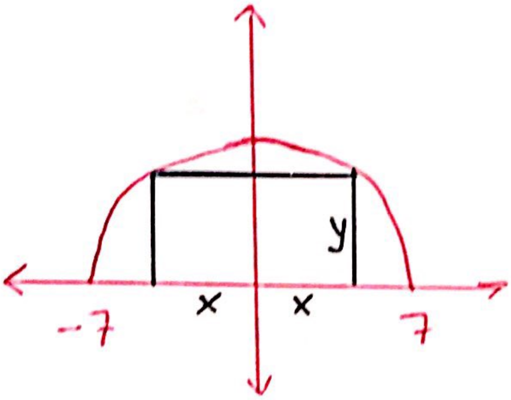
6. A rectangle is bounded by a semi-circle with equation $y=\sqrt{49-x^2}$, $-7 \le x \le 7$ and the x-axis. Find the dimensions of the rectangle having the largest area. [7]



$y=\sqrt{49-x^2}$

$A=2xy$

$A=2x\sqrt{49-x^2}$

$A=2x(49-x^2)^{1/2}$

$A'=2(49-x^2)^{1/2}+2x(\frac{1}{2})(49-x^2)^{-1/2}(-2x)$

$A'=2(49-x^2)^{1/2}-2x^2(49-x^2)^{-1/2}$

$\frac{2x^2}{\sqrt{49-x^2}}=2\sqrt{49-x^2}$

$x^2=49-x^2$

$2x^2=49$

$x^2=\frac{49}{2}$

$x=\frac{7}{\sqrt{2}}$ or $\frac{7\sqrt{2}}{2}$

$y=\sqrt{49-\frac{49}{2}}$

$y=\sqrt{\frac{49}{2}}$

$y=\frac{7}{\sqrt{2}}$ or $\frac{7\sqrt{2}}{2}$

$\therefore$ the dimensions of the rectangle having the largest area is

$\frac{7}{\sqrt{2}} \times \frac{7}{\sqrt{2}}$ units

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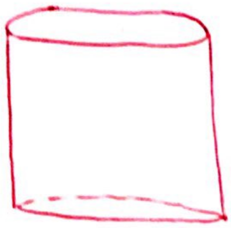
7. A cylindrical pot, without a top, is to have a volume of 2000 cm^3. The bottom will be made of copper and the rest of aluminum. Copper is three times as expensive as aluminum. Determine the dimensions that will minimize the cost of the pot. [7]



$V = \pi r^2 h$

$2000 = \pi r^2 h$

$\boxed{h = \frac{2000}{\pi r^2}}$

$$C = \underbrace{3\pi r^2}_{\text{bottom}} + \underbrace{2\pi r h}_{\text{side}}$$

$C = 3\pi r^2 + 2\pi r\left(\frac{2000}{\pi r^2}\right)$

$C = 3\pi r^2 + 4000 r^{-1}$

$c' = 6\pi r - 4000 r^{-2}$

$\frac{4000}{r^2} = 6\pi r$

$6\pi r^3 = 4000$

$r^3 = \frac{4000}{6\pi}$

$r^3 = 212.2$

$\boxed{r = 5.96}$

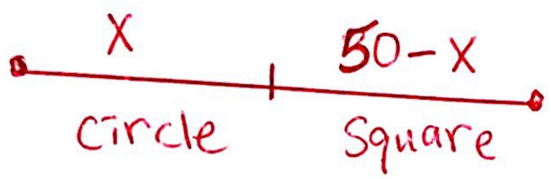
$h = \frac{2000}{3.14(5.96)^2}$

$h = \frac{2000}{111.7}$

$\boxed{h = 17.9}$

$\therefore$ the dimensions that will minimize the cost of the pots are radius of 5.96 cm and a height of 17.9 cm

8. A 50 cm piece of wire is cut into two pieces. One piece is bent to form a square. The other piece is bent to form a circle. What are the lengths of the two pieces such that the sum of the areas of the square and circle is a minimum? [7]



$C = 2\pi r$

$x = 2\pi r$

$r = \dfrac{x}{2\pi}$

$P = 4s$

$50 - x = 4s$

$s = \dfrac{50-x}{4}$

$A = \pi r^2$

$A = \pi\left(\dfrac{x}{2\pi}\right)^2$

$\cancel{s=}$

$A = s^2$

$A = \left(\dfrac{50-x}{4}\right)^2$

$A = \dfrac{x^2}{4\pi} + \left(\dfrac{50-x}{4}\right)^2$

$A' = \dfrac{2x}{4\pi} + 2\left(\dfrac{50-x}{4}\right)\left(\dfrac{-1}{4}\right)$

$A' = \dfrac{x}{2\pi} - \dfrac{50-x}{8}$

$\dfrac{50-x}{8} = \dfrac{x}{2\pi}$

$8x = 2\pi(50-x)$

$8\cancel{0}x = 100\pi - 2\pi x$

$8x + 2\pi x = 100\pi$

$x(8+2\pi) = 100\pi$

$x = \dfrac{100\pi}{8+2\pi}$

$x = 21.99$

$x \doteq 22\text{ cm}$

$50 - x = 28\text{ cm}$

$\therefore$ Square is 28 cm
Circle is 22 cm