Section 8.3: Vector, Parametric and Symmetric Equations of Lines in R^3

In R^3, vector equation is

$\vec{r} = \vec{r_0} + t\vec{m}$ ← direction vector is $\vec{m} = (a,b,c)$

In component form: $(x,y,z) = (x_0,y_0,z_0) + t(a,b,c)$

Parametric Equation: $x = x_0 + ta$
$y = y_0 + tb$
$z = z_0 + tc$

Ex 1: A line passes through points $A(2,-1,5)$ and $B(3,6,-4)$

a) Write a vector eqn. of the line
b) Write parametric equations
c) Determine if the point $C(0,-15,9)$ lies on the line.

Solution: a) $\vec{AB} = (1,7,-9)$

$\therefore$ Vector eqn: $(x,y,z) = (2,-1,5) + t(1,7,-9)$

b) $x = 2 + t$
$y = -1 + 7t$
$z = 5 - 9t$

c) Sub $C(\overset{x}{0}, \overset{y}{-15}, \overset{z}{9})$ into the parametric equations and then solve.

$0 = 2 + t$
$\boxed{-2 = t}$

$-15 = -1 + 7t$
$-14 = 7t$
$\boxed{-2 = t}$

$9 = 5 - 9t$
$4 = -9t$
$\boxed{\frac{-4}{9} = t}$

$\therefore$ Since t values are not equal, the point does not lie on the line.

Symmetric Equations of a Line in R^3

- derived from its Parametric Equations
- a new form for a line in 3 Space

parametric equations
$$\begin{cases} x = x_0 + ta \\ y = y_0 + tb \\ z = z_0 + tc \end{cases}$$

$x = x_0 + ta$, $\frac{x - x_0}{a} = t$

$y = y_0 + tb$, $\frac{y - y_0}{b} = t$

$z = z_0 + tc$, $\frac{z - z_0}{c} = t$

$$\therefore \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} = t$$

$a, b, c \neq 0$

ex2: Write the symmetric equations of the line passing through the points $X(1,2,5)$ & $Y(-1,3,9)$

Solution:

$\vec{XY} = (-1-1, 3-2, 9-5)$

$\vec{XY} = (-2, 1, 4) = \vec{m}$

$\therefore (x,y,z) = (1,2,5) + t(-2,1,4)$

$x = 1 - 2t$

$y = 2 + t$

$z = 5 + 4t$

$x - 1 = -2t$, $\frac{x-1}{-2} = t$

$\frac{y-2}{1} = t$

$z - 5 = 4t$, $\frac{z-5}{4} = t$

$\therefore$ Symmetric equations are

$$\frac{x-1}{-2} = \frac{y-2}{1} = \frac{z-5}{4}$$

ex 1: Determine a Cartesian equation for the line that passes through the point $(4,-3)$ and is $\perp$ to the line $\vec{r}=(2,-3)+t(5,-7), t \in \mathbb{R}$.

Solution:

direction vector $\perp$ to $(5,-7)$ is $(7,5)$.

$\therefore \vec{r}=(4,-3)+t(7,5)$

$x=4+7t$ | $y=-3+5t$

$\frac{x-4}{7}=t$ | $\frac{y+3}{5}=t$

$\therefore \frac{x-4}{7}=\frac{y+3}{5}$

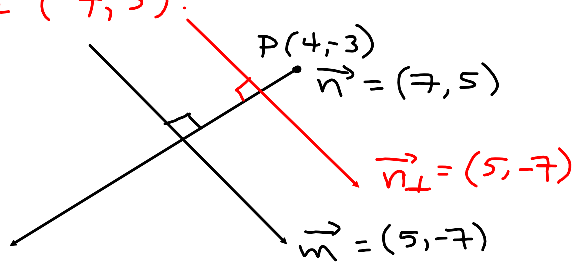
$5(x-4)=7(y+3)$

$5x-20=7y+21$

$\boxed{5x-7y-41=0}$

OR

A normal to the desired line would be $(5,-7)$, since its direction vector would be $(7,5)$.



$\therefore$ Cartesian equation would be

$Ax+By+C=0$

$5x+(-7)y+C=0$

Sub in $(4,-3)$

$5(4)-7(-3)+C=0$

$20+21+C=0$

$C=-41$

$\therefore \boxed{5x-7y-41=0}$