4.5 An Algorithm for Curve Sketching

Example 1: Without using a graphing calculator, sketch the graph of:

$$f(x) = \frac{x-4}{x^2}$$

Domain: $\{x \mid x \neq 0, x \in \Re\}$

Intercepts: The x-intercept is 4 and there is no y-intercept since $x = 0$ is not in the domain.

Asymptotes: $f(x) = \dfrac{x-4}{x^2}$

The equation of the *vertical* asymptote is $x = 0$.

$$\lim_{x\to 0^+} \frac{x-4}{x^2} = \lim_{x\to 0^-} \frac{x-4}{x^2}$$

$$= -\infty$$

The equation of the *horizontal* asymptote is $y = 0$.

$$\lim_{x\to \infty^+} \frac{x-4}{x^2} = \lim_{x\to \infty^-} \frac{x-4}{x^2}$$

$$= 0$$

Asymptotes: $f(x) = \dfrac{x-4}{x^2}$

There are no *oblique* asymptotes.

Symmetry: none

Critical numbers: $f'(x) = \dfrac{8-x}{x^3}, x \neq 0$

The only zero of the derivative is $f'(8) = 0$.

The derivative is defined for all values of x except where $x = 0$.

The critical numbers are 0 and 8.

$$f(x) = \frac{x-4}{x^2} \qquad f'(x) = \frac{8-x}{x^3}, x \neq 0$$

Local Maximum and Minimum Values:

$$0 = \frac{8-x}{x^3} \qquad x = 8$$

Intervals of Increase and Decrease:

	$x < 0$	$0 < x < 8$	$x > 8$
$f'(x)$	$-$	$+$	$-$
$f(x)$	decreasing	increasing	decreasing

$\left(8, \frac{1}{16}\right)$ is a local maximum

Concavity and point of inflection:

After simplifying $f''(x) = \dfrac{2(x-12)}{x^3}, x \neq 0$

$f''(x) = 0$ when $x = 12$ so there might be a point of inflection at $x = 12$.

$f''(x)$ is defined for all values of x except $x = 0$.

	$x < 0$	$0 < x < 12$	$x > 12$
$f''(x)$	$-$	$-$	$+$
$f(x)$	concave down	concave down	concave up

There is a point of inflection at $x = 12$

$$f(x) = \frac{x-4}{x^2}$$