MCV4U **Unit #2 Derivatives** **Name: _________________**
March 1, 2016

Mark: /47

Answer all questions on this paper. Be sure to show all applicable work and express all answers in simplest form. Marks are awarded for presentation and technical correctness.

Knowledge & Understanding:

Multiple Choice:

1. Which of the following is not a way for a derivative to fail to exist?
   a. cusp c. vertical tangent
   b. horizontal tangent d. discontinuity

2. Determine the derivative $\frac{dy}{dx}$ for $y = 2x^3 - 3x + 1$.
   a. $6x^2 - 3$ c. $3x^2 - 3$
   b. $6x^2 - 3x$ d. $x^2 - 3$

3. What is the slope of the tangent to $f(x) = \sqrt{x-1}$ at (5, 2)?
   a. $\frac{1}{4}$ c. 2
   b. $\frac{1}{2}$ d. 4

4. Under what condition is the tangent to $f(x)$ at $(a, f(a))$ horizontal?
   a. $f'(a) > 0$ c. $f'(a) = 0$
   b. $f'(a) < 0$ d. $f'(a)$ is undefined

5. Which function has a derivative that is equal to 5 when $x = 2$?
   a. $f(x) = x^2 + 1$ c. $h(x) = \frac{1}{x+1}$
   b. $g(x) = x^3 - 1$ d. $j(x) = 5x - 3$

6. All but one of the functions is differentiable for all real values of x. Which function is not differentiable for at least one real value of x?
   a. $f(x) = x^2 + 1$ c. $h(x) = |x|$
   b. $g(x) = \frac{1}{x^2+1}$ d. $j(x) = x^3 - 3x$

7. Determine the value of k for which $f(x) = 4x^2 - kx + 6$ has a horizontal tangent at $x = \frac{1}{2}$.
   a. 1 c. 4
   b. 2 d. 8

8. Which function has the most horizontal tangents?
   a. $f(x) = 3x^3$ c. $h(x) = x^3 + 3x^2 - 9x - 1$
   b. $g(x) = x^2 - 2x + 1$ d. $j(x) = x^4 - 1$

9. The position *s*, in metres, of an object moving in a straight line is given by $s(t) = 5t(t-2)^2$, where t is the time in seconds. Determine the velocity of the object at time $t = 1$.
   a. 15 m/s c. 0 m/s
   b. 5 m/s d. –5 m/s

10. What is the degree of the derivative of $h(x) = (x+3)^4(x-2)^5$?
   a. 4 c. 8
   b. 5 d. 9

Full Solution:

11. Differentiate and simplify the following functions:

a) $y = 2x^6 + x^4 - 2\sqrt[3]{x}$ [2]

b) $f(x) = \frac{(3x-1)^3}{(4x+3)^4}$ [4]

12. Rewrite $h(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$ as a product and use the product rule to derive the quotient rule.

[4]

13. Calculate the derivative of $y = \sqrt{5 - x}$ from first principles. [4]

14. Find the value of *p* and *q* so that *f(x)* is continuous **and** differentiable (has a derivative) at x = -1.

$$\boldsymbol{f(x)} = \begin{cases} \boldsymbol{x^2 + p}, & \boldsymbol{if\ x <} -1 \\ \boldsymbol{qx + 5}, & \boldsymbol{if\ x \geq -1} \end{cases}$$

[4]

15. If *f(4)* = 3 and $f'(4) = 5,\ \ find\ \ g'(4)\ where\ g(x) = \sqrt{x}f(x).$ [4]

16. Determine the value(s) of *k* such that $g'\,(-1) = -\frac{1}{2}$ if $(x) = \frac{x-k}{1+x^2}$. [3]

17. Determine $\frac{dy}{dx}$ $at\;\; x = -2\; for\; y = 3u^2 +\; 2u\; and\; u = \sqrt{x^2 + 5}.$ [4]

18. The tangent to the curve $y = x^3 + 3x^2 - 1$ at x = 0 intersects the curve at another point. Determine the coordinates of the other point. [4]

19. Determine the slope of the normal to $x^2 - 4x + 4 + (y-1)^2 = 49$ at (2, -3).

[4]