MCV4U
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Unit #2 Derivatives

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Mark: ***/47***

Answer all questions on this paper. Be sure to show all <u>applicable</u> work and express all answers in simplest form. Marks are awarded for presentation and technical correctness.

Knowledge & Understanding:

Multiple Choice:

1. Which of the following is not a way for a derivative to fail to exist?

B
a. cusp — c. vertical tangent
b. horizontal tangent — d. discontinuity

2. Determine the derivative $\frac{dy}{dx}$ for $y = 2x^3 - 3x + 1$.

A
a. $6x^2 - 3$ — c. $3x^2 - 3$
b. $6x^2 - 3x$ — d. $x^2 - 3$

3. What is the slope of the tangent to $f(x) = \sqrt{x-1}$ at (5, 2)?

A
a. $\frac{1}{4}$ — c. 2
b. $\frac{1}{2}$ — d. 4

4. Under what condition is the tangent to $f(x)$ at $(a, f(a))$ horizontal?

C
a. $f'(a) > 0$ — c. $f'(a) = 0$
b. $f'(a) < 0$ — d. $f'(a)$ is undefined

5. Which function has a derivative that is equal to 5 when $x = 2$?

D
a. $f(x) = x^2 + 1$ — c. $h(x) = \frac{1}{x+1}$
b. $g(x) = x^3 - 1$ — d. $j(x) = 5x - 3$

6. All but one of the functions is differentiable for all real values of x. Which function is not differentiable for at least one real value of x?

C
a. $f(x) = x^2 + 1$ — c. $h(x) = |x|$
b. $g(x) = \frac{1}{x^2+1}$ — d. $j(x) = x^3 - 3x$

7. Determine the value of k for which $f(x) = 4x^2 - kx + 6$ has a horizontal tangent at $x = \frac{1}{2}$.

C
a. 1 — c. 4
b. 2 — d. 8

8. Which function has the most horizontal tangents?

C
a. $f(x) = 3x^3$ — c. $h(x) = x^3 + 3x^2 - 9x - 1$
b. $g(x) = x^2 - 2x + 1$ — d. $j(x) = x^4 - 1$

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9. The position s, in metres, of an object moving in a straight line is given by $s(t) = 5t(t-2)^2$, where t is the time in seconds. Determine the velocity of the object at time $t = 1$.

D

a. 15 m/s
b. 5 m/s
c. 0 m/s
d. –5 m/s

10. What is the degree of the derivative of $h(x) = (x+3)^4(x-2)^5$?

C

a. 4
b. 5
c. 8
d. 9

Full Solution:

11. Differentiate and simplify the following functions:

a) $y = 2x^6 + x^4 - 2\sqrt[3]{x}$ [2]

$y = 2x^6 + x^4 - 2x^{1/3}$

$y' = 12x^5 + 4x^3 - \frac{2}{3}x^{-2/3}$

$y' = 12x^5 + 4x^3 - \frac{2}{3\sqrt[3]{x^2}}$

b) $f(x) = \frac{(3x-1)^3}{(4x+3)^4}$ [4]

$$f'(x) = \frac{3(3x-1)^2(3)(4x+3)^4 - (3x-1)^3(4)(4x+3)^3(4)}{(4x+3)^8}$$

$$= \frac{9(3x-1)^2(4x+3)^4 - 16(3x-1)^3(4x+3)^3}{(4x+3)^8}$$

$$= \frac{9(3x-1)^2(4x+3) - 16(3x-1)^3}{(4x+3)^5}$$

$$= \frac{(3x-1)^2[9(4x+3) - 16(3x-1)]}{(4x+3)^5}$$

$$= \frac{(3x-1)^2(-12x+43)}{(4x+3)^5}$$

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12. Rewrite $h(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$ as a product and use the product rule to derive the quotient rule. [4]

$$h(x)g(x) = f(x)$$

$$h'(x)g(x) + h(x)g'(x) = f'(x)$$

$$h'(x) = \frac{f'(x) - h(x)g'(x)}{g(x)}$$

$$h'(x) = \frac{f'(x) - \frac{f(x)}{g(x)}g'(x)}{g(x)} \cdot \frac{g(x)}{g(x)}$$

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

13. Calculate the derivative of $y = \sqrt{5-x}$ from first principles. [4]

$$y' = \lim_{h \to 0} \frac{\sqrt{5-(x+h)} - \sqrt{5-x}}{h} \cdot \frac{\sqrt{5-(x+h)} + \sqrt{5-x}}{\sqrt{5-(x+h)} + \sqrt{5-x}}$$

$$= \lim_{h \to 0} \frac{5-(x+h) - (5-x)}{h(\sqrt{5-(x+h)} + \sqrt{5-x}}$$

$$= \lim_{h \to 0} \frac{-h}{h\sqrt{5-(x+h)} + \sqrt{5-x}}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{5-(x+h)} + \sqrt{5-x}}$$

$$= \frac{-1}{\sqrt{5-x} + \sqrt{5-x}}$$

$$= \frac{-1}{2\sqrt{5-x}}$$

$\overline{8}$

14. Find the value of p and q so that $f(x)$ is continuous **and** differentiable (has a derivative) at ~~x = 2~~.
$x = -1$

$$f(x) = \begin{cases} x^2 + p, & if\ x < -1 \\ qx + 5, & if\ x \geq -1 \end{cases}$$

[4]

$x^2 + p = qx + 5$
$(-1)^2 + p = q(-1) + 5$
$1 + p = -q + 5$
$p + q = 4$
$\therefore p - 2 = 4$
$\boxed{p = 6}$

$f'(x) = 2x \quad + \quad f'(x) = q$
$2x = q$
$2(-1) = q$
$\boxed{q = -2}$

15. If $f(4) = 3$ and $f'(4) = 5$, $find\ \ g'(4)\ where\ g(x) = \sqrt{x}f(x)$. [4]

$g(x) = \sqrt{x}\, f(x)$

$g'(x) = \frac{1}{2\sqrt{x}} f(x) + \sqrt{x}\, f'(x)$

$g'(4) = \frac{1}{2\sqrt{4}} f(4) + \sqrt{4}\, f'(4)$

$= \frac{1}{4}(3) + 2(5)$

$= \frac{3}{4} + 10$

$= 10\frac{3}{4}$

$= \boxed{\frac{43}{4}}$

$\overline{8}$

16. Determine the value(s) of k such that $g'(-1) = -\frac{1}{2}$ if $g(x) = \frac{x-k}{1+x^2}$. [3]

$$g'(x) = \frac{(1)(1+x^2) - 2x(x-k)}{(1+x^2)^2}$$

$$= \frac{1+x^2-2x^2+2kx}{(1+x^2)^2}$$

$$= \frac{-x^2+2kx+1}{(1+x^2)^2}$$

Since $g'(-1) = -\frac{1}{2}$,

$$-\frac{1}{2} = \frac{-(-1)^2-2k+1}{(1+(-1)^2)^2}$$

$$-\frac{1}{2} = \frac{-2k}{4}$$

$$-4k = -4$$

$$\boxed{k=1}$$

17. Determine $\frac{dy}{dx}$ at $x = -2$ for $y = 3u^2 + 2u$ and $u = \sqrt{x^2+5}$. [4]

When $x=-2$

$u = \sqrt{(-2)^2+5}$

$u = 3$

$u = (x^2+5)^{1/2}$

$\frac{du}{dx} = \frac{1}{2}(x^2+5)^{-1/2}(2x)$

$= \frac{x}{\sqrt{x^2+5}}$

$\frac{dy}{du} = 6u+2$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$=(6u+2)\left(\frac{x}{\sqrt{x^2+5}}\right)$$

$$=(6(3)+2)\left(\frac{-2}{\sqrt{(-2)^2+5}}\right)$$

$$=(20)(-2/3)$$

$$=-\frac{40}{3}$$

$\frac{}{7}$

18. The tangent to the curve $y = x^3 + 3x^2 - 1$ at x = 0 intersects the curve at another point. Determine the coordinates of the other point. [4]

If $x=0$,
$y=-1$
$(0,-1)$
$y' = 3x^2 + 6x$
at $x=0$
$y'=0$ (Slope)
$\therefore$ tangent is $y=-1$

Set equal to each other

$-1 = \cancel{x}^3 + 3x^2 - 1$
$0 = x^3 + 3x^2$
$0 = x^2(x+3)$
$\therefore x=0$; $x=-3$
$y = (-3)^3 + 3(-3)^2 - 1$
$= -27 + 27 - 1$
$= -1$
$\therefore (-3,-1)$

19. Determine the slope of the normal to $x^2 - 4x + 4 + (y-1)^2 = 49$ at (2, -3). [4]

$2x - 4 + 2(y-1)\frac{dy}{dx} = 0$

$2(y-1)\frac{dy}{dx} = -2x + 4$

$\frac{dy}{dx} = \frac{2(-x+2)}{2(y-1)}$

$\frac{dy}{dx} = \frac{-x+2}{y-1}$

@ $(2,-3)$

$\frac{dy}{dx} = \frac{-2+2}{-3-1}$

$= \frac{0}{-4}$

$= 0$

$m_\perp$ is undefined.
The normal would be a vertical line.

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