3.4 Optimization in Economics & Science

This section analyzes the unit rates of change for profit, revenue and cost.

Basic Business Model

cost $\qquad C(x) =$ total cost of producing x units.

revenue $\qquad R(x) =$ total revenue from the sale of x units

$$= (\text{price per unit}) \times x$$

profit $\qquad P(x) =$ total profit from the sale of x units

$$= R(x) - C(x) \qquad (\text{revenue} - \text{cost})$$

The **demand function** p is the price at which units can be sold.

$$R(x) = px$$

Example 1: Marginal Cost

The owner of a hat manufacturer uses regression to estimate that the cost, in dollars, of producing a certain hat is $C(x) = 4500 + 6.2x - 0.0004x^2 \qquad 0 < x < 3000$

a) Find the marginal cost at a production level of 500 hats.

b) Find the actual cost of producing the 501st hat.

The **marginal** cost is instantaneous rate of change of cost

marginal cost $= C'(x)$

$$C'(x) = 6.2 - 0.0008x$$

$$C'(500) = 6.2 - 0.0008(500)$$

$$C'(500) = 5.80$$

The marginal cost when $x = 500$ is \$5.80

b) Find the actual cost of producing the 501st hat.

$$C(x) = 4500 + 6.2x - 0.0004x^2$$

$$\begin{aligned} C(501) - C(500) &= [4500 + 6.2(501) - 0.0004(501)^2] - \\ &\quad [4500 + 6.2(500) - 0.0004(500)^2] \\ &= 7505.7996 - 7500 \\ &= 5.7996 \end{aligned}$$

The actual cost of producing the 501st hat is \$5.7996

Example 2:

A commuter train carries 2000 passengers daily. The cost to ride the train is \$7. Market surveys indicate that 40 fewer passengers would ride the train for each increase in fare of \$0.10. If they need at least 1600 passengers to ride the train and the train holds 2600 people, what fare will give the largest possible revenue?

revenue = (number of passengers) $\times$ (fare per passenger)

let x = number of \$0.10 increases

fare per passenger is $7 + 0.10x$

number of passengers is $2000 - 40x$

Domain of x $\quad 1600 < 2000 - 40x < 2600$

$$-15 < x < 10$$

$$R(x) = (7 + 0.10x)(2000 - 40x)$$

$$R(x) = -4x^2 - 80x + 14000$$

$R'(x) = -8x - 80$ **(marginal revenue)**

$$0 = -8x - 80$$

$$8x = -80$$

$x = -10$ $\qquad -15 < x < 10$

Evaluate $R(x)$ for $x = -15$, $x = 10$ and $x = -10$

Example 3: A theatre company determines that the demand function, p, based on the weekly sales of x number of tickets is $p = \frac{2000 - x}{1000}$

What is the marginal revenue for the weekly sale of 400 tickets?

revenue = price × number sold

$$R(x) = px$$

$$= \left(\frac{2000 - x}{1000}\right)x$$

$$= 2x - \frac{x^2}{1000}$$

$$R'(x) = \frac{d}{dx}\left(2x - \frac{x^2}{1000}\right)$$

$$R'(x) = 2 - \frac{x}{500}$$

$$R'(400) = 2 - \frac{400}{500} = 1.20$$

For weekly sales of 400 tickets, revenue increases by $1.20 per ticket.

Break Even Point: Profit changes from negative to positive or vice versa.

$P(x) = 0$ and $C(x) = R(x)$

Marginal revenue: $R'(x)$ Change in revenue from selling one more unit

Marginal cost: $C'(x)$ Change cost of producing one more unit

Marginal profit: $P'(x)$ Change in profit from selling one more unit