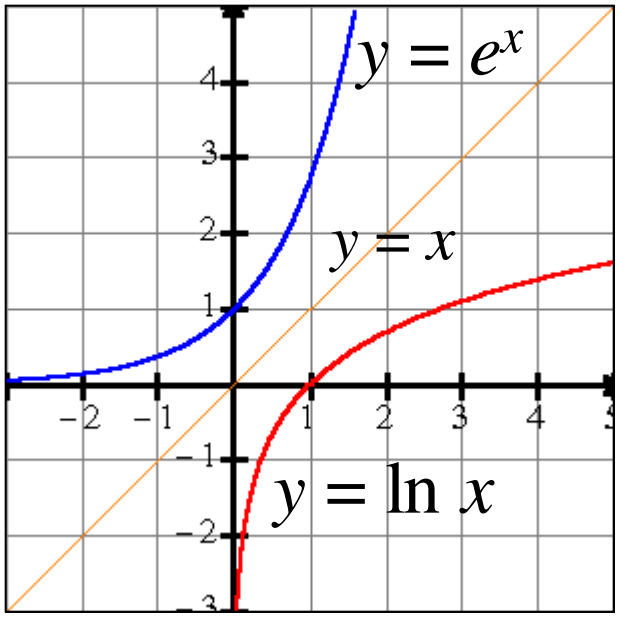
The Natural Logarithm and Its Derivative

Recall that the inverse of $y = 2^x$ is $y = \log_2 x$

The inverse of $y = e^x$ is $y = \log_e x$.

The function $y = \log_e x$ can be written as $y = \ln x$

Natural Logarithm: $y = \ln x$ is the logarithm function with base e.



$y = \ln x$ is equivalent to $x = e^y$

$y = \ln x$ is the inverse of $y = e^x$

Laws of Logarithms

$$\ln ab = \ln a + \ln b \qquad \ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

$$\ln a^p = p \ln a$$

Solve for x. $e^{x+3} = 56.1$

Change it to logarithmic form

$$e^{x+3} = 56.1 \quad \rightarrow \quad \ln 56.1 = x + 3$$

$$x = \ln 56.1 - 3$$

$$x = 1.027$$

Finding the Derivative of $y = \ln x$

Change $y = \ln x$ to exponential form.

$$y = \log_e x$$

$$x = e^y$$

$$\frac{d(x)}{dx} = \frac{d(e^y)}{dx}$$

$$\frac{d(x)}{dx} = \frac{d(e^y)}{dy}\frac{dy}{dx}$$

$$1 = e^y \frac{dy}{dx}$$

$$\frac{1}{e^y} = \frac{dy}{dx}$$

$$\frac{1}{x} = \frac{dy}{dx}$$

If $y = \ln x$ then $\dfrac{dy}{dx} = \dfrac{1}{x}$

or $\dfrac{d(\ln x)}{dx} = \dfrac{1}{x}$

Example: If $y = \ln x^3$ find $\dfrac{dy}{dx}$ (use the chain rule)

$$\frac{dy}{dx} = \frac{d(\ln x^3)}{d(x^3)} \frac{d(x^3)}{dx}$$

$$\frac{dy}{dx} = \frac{1}{x^3}(3x^2)$$

$$\boxed{\frac{dy}{dx} = \frac{3}{x}}$$

Derivative of a composite function involving $y = \ln x$

If $y = \ln f(x)$

$$\frac{dy}{dx} = \frac{1}{f(x)} = f'(x)$$

Find $\frac{dy}{dx}$

(ii) $y = (\ln x)^3$

$$\frac{dy}{dx} = 3(\ln x)^2 \frac{1}{x}$$

$$= \frac{3(\ln x)^2}{x}$$

(iii) $y = \ln (3x)$

$$y = \ln 3 + \ln x$$

$$\frac{dy}{dx} = \frac{d}{dx}(\ln 3 + \ln x)$$

$$= 0 + \frac{1}{x}$$

$$= \frac{1}{x}$$

Finding the Derivative Without Using the Quotient or Product Rule.

Given: $y = \ln \dfrac{2x-5}{x^3}$ find $\dfrac{dy}{dx}$ (use the laws of logs)

$$y = \ln(2x-5) - \ln x^3$$

$$\frac{dy}{dx} = \frac{1}{2x-5}(2) - \frac{1}{x^3}(3x^2) = \frac{2x-3(2x-5)}{x(2x-5)}$$

$$\frac{dy}{dx} = \frac{2}{2x-5} - \frac{3}{x} = \frac{-4x+15}{x(2x-5)}$$

Using Natural Logarithms to Find Logarithms with Other Bases

Example: Find $\log_5 30$ using natural logarithms

Let: $\log_5 30 = y$

$$5^y = 30$$

$$\ln 5^y = \ln 30$$

$$y \ln 5 = \ln 30$$

$$y = \frac{\ln 30}{\ln 5}$$

$$y = 2.11$$

In general:

$$\log_b x = \frac{\ln x}{\ln b}$$

or

$$\log_b x = \frac{\log x}{\log b}$$