#2.

If 3000 cm^2 of material is available to make a box with a square base and open top, find the la
possible volume of the box. [7]

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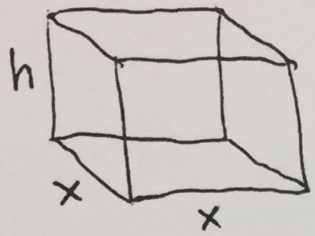


$$SA = x^2 + 4xh$$

$$3000 = x^2 + 4xh$$

$$3000 - x^2 = 4xh$$

$$\boxed{\frac{3000 - x^2}{4x} = h}$$

$$V = x^2 h$$

$$V = x^2\left(\frac{3000 - x^2}{4x}\right)$$

$$V = \frac{3000x^2}{4x} - \frac{x^4}{4x}$$

$$V = 750x - \frac{x^3}{4}$$

$$V' = 750 - \frac{3}{4}x^2$$

$$\frac{3}{4}x^2 = 750$$

$$x^2 = 1000$$

$$x = 31.62 \text{ cm}$$

$\therefore$ Length $= 31.62$cm
Width $= 31.62$cm

$$h = \frac{3000 - (31.6)^2}{4(31.6)}$$

$$h = 15.82$$

$$V = lwh$$

$$V = (31.62)(31.62)(15.82)$$

$$V = 15820 \text{ cm}^3$$

or

15811.4 cm^3

3. Ottawa Travel advertises a package plan for a Florida vacation. The fare for the flight is $400/person plus $8/person for each unsold seat on the plane. The plane holds 120 passengers and the flight will be cancelled if there are fewer than 50 passengers. What number of passengers will maximize revenue? Be sure to use derivatives to solve this equation. [4]

#3.

Let x = # of empty seats

$R(x) = (400 + 8x)(120 - x)$

$R(x) = 48000 - 400x + 960x - 8x^2$

$R(x) = -8x^2 + 560x + 48000$

$R'(x) = -16x + 560$

$16x = 560$

$x = 35$

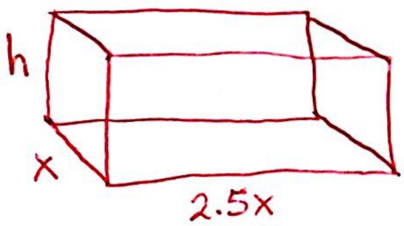
$\therefore$ passengers $= 120 - x$

$= 120 - 35$

$= 85$ passengers will maximize revenue

4. A rectangular wooden bedding chest will be built so that its length is 2.5 times its width. The top, front, and two sides of the chest will be oak. The back and bottom of the chest will be cedar. The volume of the chest must be 0.5 m^3. Oak costs 1.5 times as much as cedar. Find the dimensions that will minimize the cost of the chest. [7]

#4



$V = 2.5x^2h$

$0.5 = 2.5x^2h$

$\boxed{h = \frac{1}{5x^2}}$

$$C = \underbrace{1.5(2.5x^2)}_{\text{top}} + \underbrace{1.5(2.5xh)}_{\text{Front}} + \underbrace{1.5(2xh)}_{\text{sides}} + \underbrace{2.5x^2}_{\text{bottom}} + \underbrace{2.5xh}_{\text{back}}$$

$C = 3.75x^2 + 3.75xh + 3xh + 2.5x^2 + 2.5xh$

$C = 6.25x^2 + 9.25xh$

$C = 6.25x^2 + 9.25x\left(\frac{1}{5x^2}\right)$

$C = 6.25x^2 + 1.85x^{-1}$

$C' = 12.5x - \frac{1.85}{x^2}$

$\frac{1.85}{x^2} = 12.5x$

$12.5x^3 = 1.85$

$x^3 = 0.148$

$\boxed{x = 0.53 \text{ m}}$

$2.5x = 1.32$ m

$h = \frac{1}{5(0.53)^2}$

$= 0.71$

$\boxed{1.32\text{m} \times 0.53\text{m} \times 0.71\text{ m}}$

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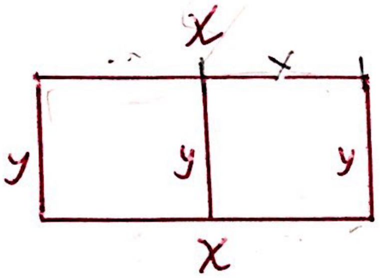
#5. A rectangular dog kennel will be surrounded by a fence and then divided into two sections by a block wall. The area of the kennel must be 72 m². Fencing costs $10/m and the block wall costs $20/m. What should be the dimensions of the kennel to minimize the costs? [6]



$A = xy$

$72 = xy$

$\boxed{\frac{72}{x} = y}$

Fence / Block wall

$C = 10(2x + 2y) + 20(y)$

$C = 20x + 20y + 20y$

$C = 20x + 40y$

$C = 20x + 40\left(\frac{72}{x}\right)$

$C = 20x + 2880x^{-1}$

$C' = 20 - 2880x^{-2}$

Set $C' = 0$

$\frac{2880}{x^2} = 20$

$20x^2 = 2880$

$x^2 = 144$

$x = 12$

∴ front/back is 12m

Sides are 6 m

12m X 6m

CHOICE: Do any four questions from #5 - #9.

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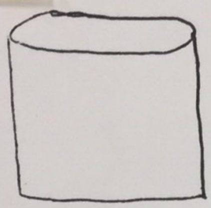
#7

A cylindrical pot, without a top, is to have a volume of 1000 cm^3. The bottom will be made of copper and the rest of aluminum. Copper is five times as expensive as aluminum. Determine the dimensions that will minimize the cost of the pot. [7]



$V = \pi r^2 h$

$1000 = \pi r^2 h$

$h = \frac{1000}{\pi r^2}$

$A = \pi r^2 + 2\pi r h$

$C = 5(\pi r^2) + 1(2\pi r h)$

$C = 5\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$C = 5\pi r^2 + 2000 r^{-1}$

$C' = 10\pi r - \frac{2000}{r^2}$

$\frac{2000}{r^2} = 10\pi r$

$10\pi r^3 = 2000$

$r^3 = \frac{2000}{10\pi}$

$r^3 = \frac{2000}{63.66}$

$r^3 = 63.66$

$r = 4$ cm

$h = \frac{1000}{\pi(4)^2}$

$= 19.9$ or 20 cm

height $= 20$ cm

and

radius $= 4$ cm

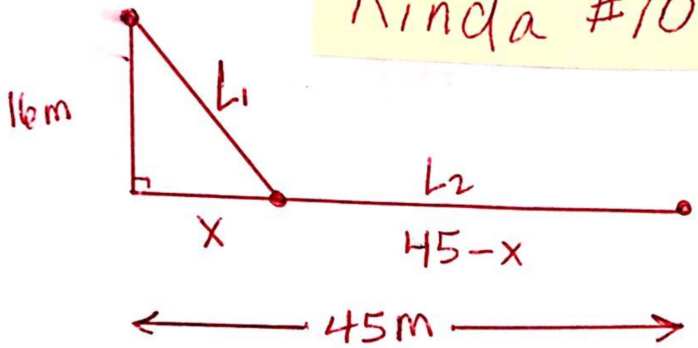
Cable service is being installed in a new subdivision. The cable must cross a 16 metre wide river and reach a point which is 45 metres downstream from its starting point on the other bank. Laying cable under water costs three times as much as laying it over ground. How should the cable be routed to minimize the cost? [7]

Kinda #10

$(L_1)^2 = x^2 + 16^2$

$(L_1)^2 = x^2 + 256$

$L_1 = \sqrt{x^2 + 256}$



$$C = 3(x^2+256)^{1/2} + (45-x)$$

$$C' = 1.5(x^2+256)^{-1/2}(2x) - 1$$

$$1 = \frac{1.5(2x)}{\sqrt{x^2+256}}$$

$$\sqrt{x^2+256} = 3x$$

$$x^2 + 256 = 9x^2$$

$$256 = 8x^2$$

$$32 = x^2$$

$$x = 5.66$$

∴ the cable should be routed 5.66 m from its starting point on the other bank.

or 39.34m

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