

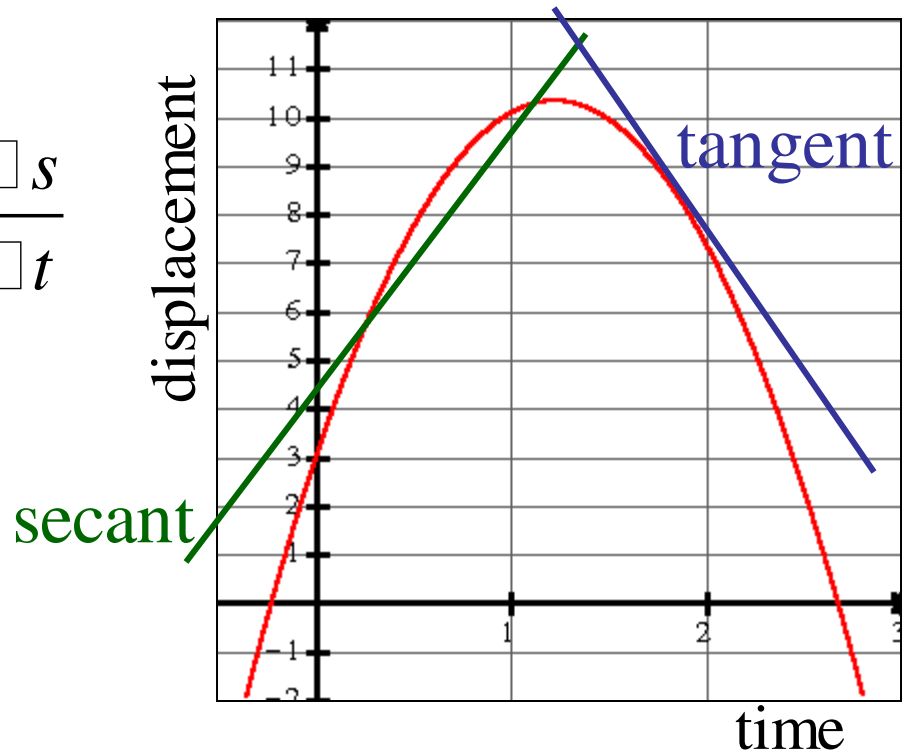
# 3.1 Higher-Order Derivatives, Velocity and Acceleration

**Average Velocity:** rate of change of displacement over an interval of time.

**Instantaneous Velocity:** rate of change of displacement at a specific point in time.

**Average Velocity:** is the slope of the secant line.  $= \frac{\Delta s}{\Delta t}$

**Instantaneous Velocity:** is the slope of the tangent line.  $v(t) = f'(t)$



# Velocity and Speed

The **velocity** of an object measures how fast an object is moving and the direction of the movement.

**Speed** is the magnitude or absolute value of the velocity without regard to direction.

*Vertical motion:*

positive velocity  $\rightarrow$  object is moving up.

negative velocity  $\rightarrow$  object is moving down.

*Horizontal motion:*

positive velocity  $\rightarrow$  object is moving right.

negative velocity  $\rightarrow$  object is moving left.

If  $v(t) = 0$  then the object is stationary or at its maximum height.

## Example:

The motion of a lion moving across level ground is given by the equation:  $s(t) = t^3 - 12t^2 + 36t$

a) What is the lion's velocity after 1 s, 5 s?

$$\begin{aligned} v &= s'(t) \\ &= 3t^2 - 24t + 36 \end{aligned}$$

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After 1 s:

$$\begin{aligned} s'(1) &= 3(1)^2 - 24(1) + 36 \\ &= 15 \text{ m/s} \end{aligned}$$

After 5 s:

$$\begin{aligned} s'(5) &= 3(5)^2 - 24(5) + 36 \\ &= -9 \text{ m/s} \end{aligned}$$

b) When is the lion momentarily stopped? When  $s'(t) = 0$

b) When is the lion momentarily stopped?  $s'(t) = 0$

$$3t^2 - 24t + 36 = 0$$

$$3(t^2 - 8t + 12) = 0$$

$$3(t - 2)(t - 6) = 0$$

The lion is stopped at  $t = 2$  sec and  $t = 6$  sec.

c) What are the positions of the lion when it is stopped?

$$s(t) = t^3 - 12t^2 + 36t$$

$$s(2) = 2^3 - 12(2)^2 + 36(2) \quad s(6) = 6^3 - 12(6)^2 + 36(6)$$

$$s(2) = 32$$

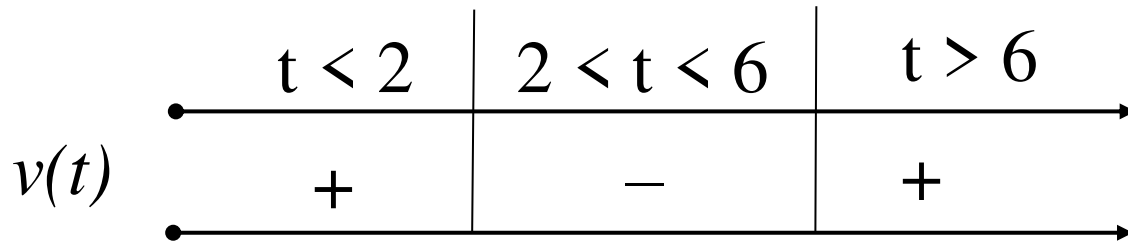
$$s(6) = 0$$

After 2 s the lion is 32 m from the starting point and after 6 s it is back at starting point.

d) When is the lion moving in a positive direction?

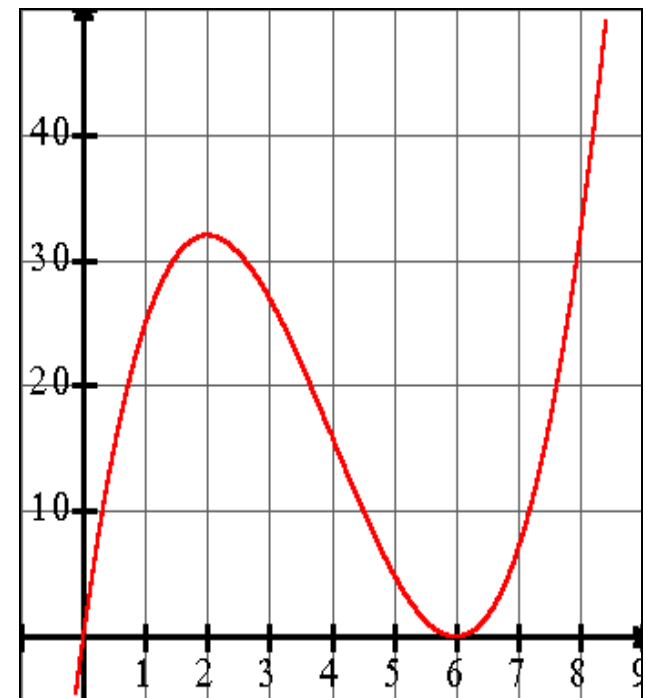
When is it moving in a negative direction?

The lion is moving in a positive direction when  $v(t) > 0$ ,  
and moving in a negative direction when  $v(t) < 0$



The lion moves in a positive  
direction for  $t \in (0, 2)$   
and  $t \in (6, \infty)$

The lion moves in a negative  
direction for  $t \in (2, 6)$



$$s(t) = t^3 - 12t^2 + 36t$$

e) Determine the position of the lion after 10 sec.

$$s(t) = t^3 - 12t^2 + 36t$$

$$s(10) = 10^3 - 12(10)^2 + 36(10)$$

$$s(10) = 160$$

The lion is 160 m from where it started after 10 sec.

f) Find the total distance travelled during the first 10 s.

$$t = 0, s = 0$$

$$t = 2, s = 32$$

$$t = 6, s = 0$$

$$t = 10, s = 160$$

The total distance travelled is  $32 + 32 + 160 = 224$  m.