Name: ________________________ **Date:** ________________

MCV4U - Unit 1: Intro to Calculus - Practice Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Determine the conjugate radical of the expression $-\sqrt{8}+3\sqrt{5}$.

a. $-\sqrt{8}+3\sqrt{5}$
b. $-\sqrt{8}+5\sqrt{3}$
c. $-\sqrt{8}-3\sqrt{5}$
d. $\sqrt{8}+3\sqrt{5}$

____ 2. Determine which expression is the correct rationalization of the denominator of $\dfrac{\sqrt{5}+3\sqrt{3}}{\sqrt{6}}$.

a. $\dfrac{\sqrt{30}+9\sqrt{2}}{6}$
b. $\dfrac{3\sqrt{10}+3\sqrt{2}}{6}$
c. $\dfrac{\sqrt{30}-6\sqrt{2}}{3}$
d. $\dfrac{3\sqrt{10}-6\sqrt{2}}{6}$

____ 3. Determine which expression is the correct rationalization of the denominator of $\dfrac{\sqrt{5}+\sqrt{10}}{\sqrt{5}-\sqrt{10}}$.

a. -1
b. $-3-2\sqrt{2}$
c. 1
d. $-3+2\sqrt{2}$

____ 4. Determine which expression is the correct rationalization of the numerator of $\dfrac{\sqrt{x}+3}{x-6}$.

a. $\dfrac{x+3}{\sqrt{x}-6}$
b. $\dfrac{x+9}{(x-6)(\sqrt{x}+3)}$
c. $\dfrac{x-9}{(x+6)(\sqrt{x}-3)}$
d. $\dfrac{x-9}{(x-6)(\sqrt{x}-3)}$

____ 5. Determine an equation of the line tangent to the curve $y=\sqrt{x-8}$ at the point with x-coordinate 9.

a. $-x+2y+7=0$
b. $-x+2y+8=0$
c. $x-2y+7=0$
d. $x-2y+8=0$

____ 6. Determine an equation of the line tangent to the curve $y=\dfrac{1}{x+3}$ at the point with x-coordinate 2.

a. $-7x+25y+1=0$
b. $7x-25y+1=0$
c. $-x-25y-7=0$
d. $x+25y-7=0$

____ 7. If s is a position function, determine what $s(5)-s(4)$ represents.

a. instantaneous velocity at $t=5$ s
b. instantaneous velocity at $t=4$ s
c. average velocity between $t=4$ s and $t=5$ s
d. position between $t=4$ s and $t=5$ s

____ 8. Suppose the motion of an avalanche is described by the function $s(t) = 4t^2$, where s is the distance in metres travelled by the leading edge of the snow at t minutes. Determine the rate at which the avalanche is moving at 3 minutes.

a. 16 m/min.
b. 24 m/min.
c. 30 m/min.
d. 40 m/min.

____ 9. An oil tank is being drained for cleaning. After t minutes there are V litres of oil left in the tank, where $V(t) = 10(10 - t)^2$, $0 \le t \le 10$. Determine the rate of change of volume at the time t = 5 minutes.

a. -200 litres/min.
b. -150 litres/min.
c. -100 litres/min.
d. -50 litres/min.

____ 10. For $f(x) = -x^2 + 1$, which value has the largest magnitude?

a. average rate of change from $x = -1$ to $x = 1$
b. instantaneous rate of change at $x = 3$
c. instantaneous rate of change at $x = 0$
d. average rate of change from $x = -4$ to $x = 0$

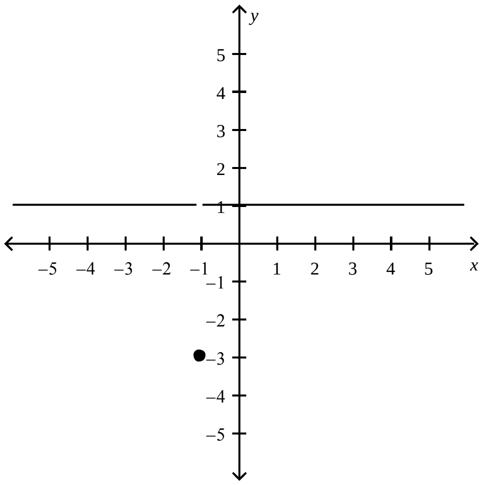
____ 11. Which of the following must be equal to $\lim_{x \to 3^-} f(x)$ so that the limit exists at the point with x-coordinate 3?

a. $\lim_{x \to 3^+} f(x)$
b. $f(3)$
c. 0
d. Nothing else. The one limit is sufficient.

____ 12. Which of the following must be equal to $\lim_{x \to 25^+} f(x)$ so that the limit exists at the point with x-coordinate 25?

a. $\lim_{x \to 25^-} f(x)$
b. $f(25)$
c. 0
d. Nothing else. The one limit is sufficient.

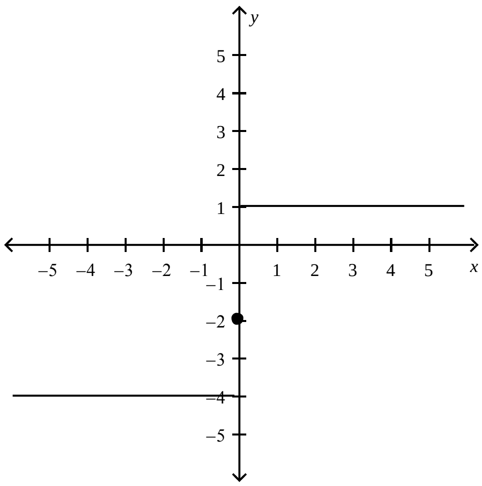
____ 13. Does the following graph represent a function that satisfies the following conditions?
$f(-1) = -3$, $\lim_{x \to -1} f(x) = 1$



a. Yes

b. No
c. Only if $f(0) = 0$.
d. There is not enough given information.

____ 14. Does the following graph represent a function that satisfies the following conditions: $f(0) = -2,\ \lim_{x \to 0} f(x) = -4$?



a. Yes
b. No
c. Only if $f(1) = 1$.
d. There is not enough given information.

____ 15. Determine $\lim_{x \to 5} \dfrac{\sqrt{x^2 - 9}}{8x}$.

a. 10
b. $\dfrac{1}{10}$
c. 4
d. $\dfrac{2}{5}$

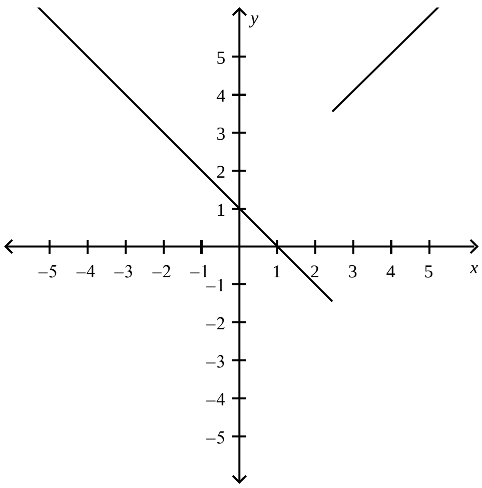
____ 16. Determine, using the properties of limits, $\lim_{x \to 2} \dfrac{3f(x) + x^2}{x}$ given $\lim_{x \to 2} f(x) = 6$.

a. 14
b. 18
c. 8
d. 11

____ 17. Determine, using the properties of limits, $\lim_{x \to -1} \dfrac{-f(x)^2 + x}{2}$ given $\lim_{x \to -1} f(x) = 3$.

a. -5
b. -10
c. 5
d. 10

____ 18. Determine which of the following limits is represented by the following graph.



a. $\lim_{x \to 2.5} \dfrac{|2x - 5|(x + 1)}{2x - 5}$

b. $\lim_{x \to -2.5} \dfrac{|2x - 5|(x + 1)}{2x - 5}$

c. $\lim_{x \to 2.5} \dfrac{|5x - 2|(x + 1)}{5x - 2}$

d. $\lim_{x \to -2.5} \dfrac{|5x - 2|(x + 1)}{5x - 2}$

____ 19. Graph the function $z(x) = \begin{cases} 6x - 10, \text{ if } x < 1 \\ 8 - x, \text{ if } x \geq 1 \end{cases}$. Determine the x-coordinate for which the function is discontinuous.

a. 1
b. -1
c. -16
d. There do not exist any x-coordinates for which the function is discontinuous.

____ 20. Determine the value of x for which the function $f(x) = \begin{cases} 5x, \text{ if } x < 0 \\ 1, \text{ if } x = 0 \\ -5x, \text{ if } x > 0 \end{cases}$ is discontinuous.

a. 0
b. 1
c. 5
d. -5

Short Answer

21. Explain the steps for rationalizing the denominator of $\dfrac{\sqrt{7}}{5\sqrt{5} - 8}$.

22. Determine the conjugate radical of the expression $6\sqrt{22} + 5\sqrt{13}$.

23. Rationalize the denominator of $\dfrac{\sqrt{10}}{3\sqrt{3} + \sqrt{15}}$.

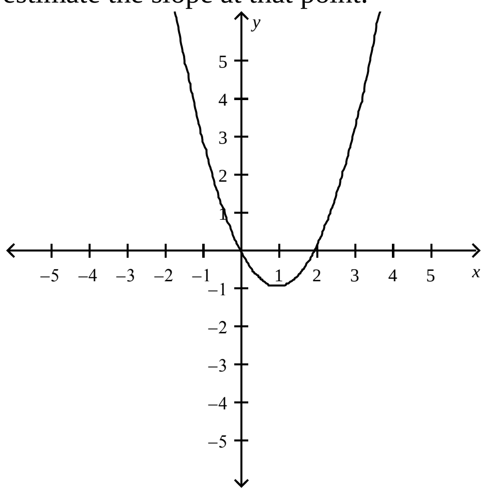
24. Rationalize the denominator of $\dfrac{\sqrt{14}}{4\sqrt{6}+\sqrt{7}}$.

25. Rationalize the numerator of $\dfrac{\sqrt{x+144}-12}{x}$.

26. Explain the steps for rationalizing the denominator of $\dfrac{\sqrt{19}+3\sqrt{2}}{\sqrt{5}+\sqrt{6}}$.

27. Determine the slope of the tangent to the curve $y = x^2 - x$ at the point with *x*-coordinate 8.

28. The graph of the function $f(x) = x^2 - 2x$ is shown below. Draw the line tangent to the point (0, 0). Then, estimate the slope at that point.



29. Describe what can be inferred about the line tangent to a curve if the slope at a point is found to be 0.

30. Determine the average velocity of the function $f(t) = \sqrt{t-2}$ between the time intervals $t = 3$ and $t = 5$.

31. A manufacturer of tennis balls determines that the profit from the sale of *x* cans of tennis balls per week measured in hundreds is given by the function $P(x) = 180x - 2x^2$, where *P* is measured in dollars. Determine the rate of cans of tennis balls being sold for $x = 3$.

32. What does the rate of change of the position function represent?

33. Let $f(x) = mx + b$ where *m* and *b* are constants. If $\lim\limits_{x \to -1} f(x) = 3$ and $\lim\limits_{x \to 0} f(x) = 5$, determine *m* and *b*.

34. Determine $\lim\limits_{x \to -5} (6x - x^2)$, if it exists.

35. Describe what happens if $\lim\limits_{x \to a} f(x) = f(a)$ for a function $f(x)$.

36. $j(x) = \begin{cases} x - 2, \text{ if } x \neq -2 \\ 3kx + 5, \text{ if } x = -2 \end{cases}$. Determine k so that $j(x)$ is continuous.

37. Examine the continuity of the function $h(x) = \begin{cases} -x, \text{ if } x < 0 \\ 0, \text{ if } x = 0 \\ x, \text{ if } x > 0 \end{cases}$.

38. Graph the function $g(x) = \begin{cases} x^2 + 2, \text{ if } x < 1 \\ x + 2, \text{ if } x \geq 1 \end{cases}$. Is the function continuous? Explain.

39. Determine the values of x for which the function $f(x) = \dfrac{\sqrt{3x - 6}}{x - 5}$ is continuous.

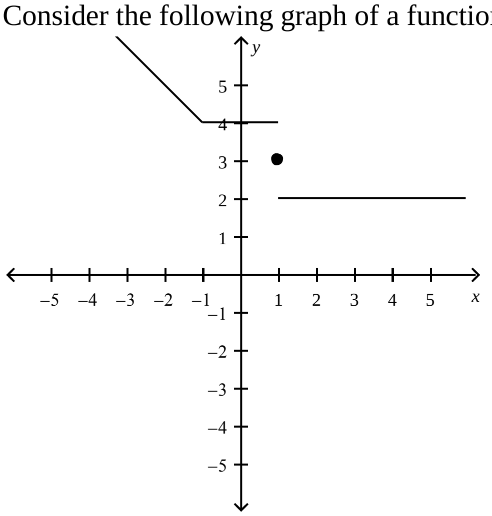
40. Determine the values of x for which the function $f(x) = \dfrac{x + 2}{x - 2}$ is discontinuous.

Problem

41. Rationalize the numerator of $\dfrac{4\sqrt{x + 25} - 20}{4x}$. Describe each step of the process.

42. A man drops a penny from the top of a 500 m tall building. After t seconds, the penny has fallen a distance of s metres, where $s(t) = 500 - 5t^2,\ 0 \leq t \leq 10$.
a. Determine the average velocity between 1 s and 5 s.
b. Determine the average velocity between 5 s and 9 s.
c. Determine the velocity at $t = 5$.

43. Consider the following graph of a function $f(x)$.



Determine the following and explain each answer.

a. $\lim\limits_{x \to 0} f(x)$

b. $\lim\limits_{x \to 1^-} f(x)$

c. $\lim\limits_{x \to 1^+} f(x)$

d. $\lim\limits_{x \to 1} f(x)$

e. $f(1)$

44. Sometimes, substituting $x = a$ into $\lim\limits_{x \to a} f(x)$ can yield the indeterminate form $\frac{0}{0}$. Consider the limit $\lim\limits_{x \to 2} \dfrac{x^2 - x - 2}{x - 2}$.

a. Explain why direct substitution does not work for this limit.

b. What is the equivalent function that is the same as $\dfrac{x^2 - x - 2}{x - 2}$ for all values except at $x = 2$?

c. Determine the limit and explain why this method is acceptable.

d. Explain what the graph will look like.

45. a. Determine $\lim\limits_{x \to 4} \sqrt{3 - \sqrt{x}}$.

b. What restriction is placed on the expression $\sqrt{x}$ for this limit?

c. What restriction is placed on *x* for this limit?

46. a. Graph the function $f(x) = \begin{cases} \dfrac{x^3 + x^2 - 12x}{x - 3}, & \text{if } x \neq 3 \\ 15, & \text{if } x = 3 \end{cases}$.

b. Determine $f(3)$.

c. Determine $\lim\limits_{x \to 3^-} f(x)$, $\lim\limits_{x \to 3^+} f(x)$, and $\lim\limits_{x \to 3} f(x)$.

d. Is $f(x)$ continuous? Explain.

47. Determine the values of *x* for which the following functions are continuous. Explain your steps.

a. $f(x) = \dfrac{275}{x + 8}$

b. $g(x) = \sqrt{x^2 + 1}$

c. $h(x) = \dfrac{1}{\sqrt{x^2 + 1}}$

d. $z(x) = \dfrac{2x^2 - x - 15}{4x - 12}$

MCV4U Chapter 1 - Practice Test
Answer Section

MULTIPLE CHOICE

1. ANS: D PTS: 1 REF: Knowledge and Understanding
   OBJ: 1.1 - Radical Expressions: Rationalizing Denominators
2. ANS: A PTS: 1 REF: Knowledge and Understanding
   OBJ: 1.1 - Radical Expressions: Rationalizing Denominators
3. ANS: B PTS: 1 REF: Knowledge and Understanding
   OBJ: 1.1 - Radical Expressions: Rationalizing Denominators
4. ANS: D PTS: 1 REF: Application
   OBJ: 1.1 - Radical Expressions: Rationalizing Denominators
5. ANS: A PTS: 1 REF: Application OBJ: 1.2 - The Slope of a Tangent
6. ANS: D PTS: 1 REF: Application OBJ: 1.2 - The Slope of a Tangent
7. ANS: C PTS: 1 REF: Thinking OBJ: 1.3 - Rates of Change
8. ANS: B PTS: 1 REF: Application OBJ: 1.3 - Rates of Change
9. ANS: C PTS: 1 REF: Application OBJ: 1.3 - Rates of Change
10. ANS: B PTS: 1 REF: Thinking OBJ: 1.3 - Rates of Change
11. ANS: A PTS: 1 REF: Thinking OBJ: 1.4 - Limit of a Function
12. ANS: A PTS: 1 REF: Thinking OBJ: 1.4 - Limit of a Function
13. ANS: A PTS: 1 REF: Application OBJ: 1.4 - Limit of a Function
14. ANS: B PTS: 1 REF: Application OBJ: 1.4 - Limit of a Function
15. ANS: B PTS: 1 REF: Knowledge and Understanding
    OBJ: 1.5 - Properties of Limits
16. ANS: D PTS: 1 REF: Thinking OBJ: 1.5 - Properties of Limits
17. ANS: A PTS: 1 REF: Thinking OBJ: 1.5 - Properties of Limits
18. ANS: A PTS: 1 REF: Application OBJ: 1.5 - Properties of Limits
19. ANS: A PTS: 1 REF: Application OBJ: 1.6 - Continuity
20. ANS: A PTS: 1 REF: Knowledge and Understanding
    OBJ: 1.6 - Continuity

SHORT ANSWER

21. ANS:

    First, determine the conjugate radical of the denominator. It is $5\sqrt{5}+8$. Then, multiply numerator and denominator by this value. Finally, simplify the expression. The denominator will be an integer, while the numerator will contain a radical expression.

    PTS: 1 REF: Communication
    OBJ: 1.1 - Radical Expressions: Rationalizing Denominators
22. ANS:
    $6\sqrt{22}-5\sqrt{13}$

    PTS: 1 REF: Knowledge and Understanding
    OBJ: 1.1 - Radical Expressions: Rationalizing Denominators

23. ANS:
$$\frac{30\sqrt{30}-5\sqrt{6}}{12}$$

PTS: 1 REF: Knowledge and Understanding
OBJ: 1.1 - Radical Expressions: Rationalizing Denominators

24. ANS:
$$\frac{8\sqrt{21}-7\sqrt{2}}{89}$$

PTS: 1 REF: Knowledge and Understanding
OBJ: 1.1 - Radical Expressions: Rationalizing Denominators

25. ANS:
$$\frac{\sqrt{x+144}-12}{x}\times\frac{\sqrt{x+144}+12}{\sqrt{x+144}+12}=\frac{x+144-144}{x(\sqrt{x+144}+12)}=\frac{1}{\sqrt{x+144}+12}$$

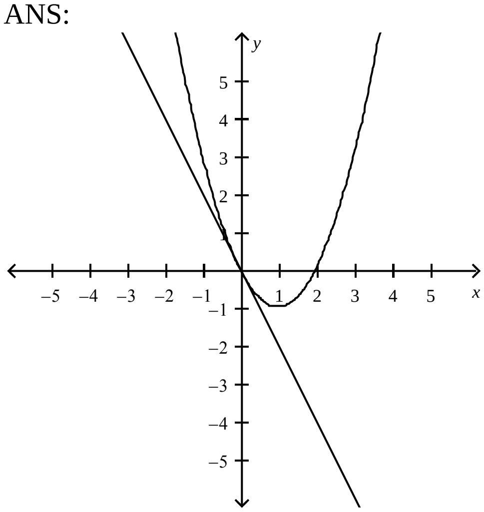
PTS: 1 REF: Application OBJ: 1.1 - Radical Expressions: Rationalizing Denominators

26. ANS:
First, determine the conjugate radical of the denominator. It is $\sqrt{5}-\sqrt{6}$. Then, multiply numerator and denominator by this value. Finally, simplify the expression. The denominator will be an integer, while the numerator will contain a radical expression.

PTS: 1 REF: Communication
OBJ: 1.1 - Radical Expressions: Rationalizing Denominators

27. ANS:
15

PTS: 1 REF: Knowledge and Understanding OBJ: 1.2 - The Slope of a Tangent

28. ANS:


The slope at the point (0, 0) is -2.

PTS: 1 REF: Thinking OBJ: 1.2 - The Slope of a Tangent

29. ANS:
If the slope is zero, the line at that point is horizontal. It is of the form $y = b$ where *b* is a constant.

PTS: 1 REF: Thinking OBJ: 1.2 - The Slope of a Tangent

30. ANS:

$\dfrac{\sqrt{3}-1}{2} \doteq 0.37$

PTS: 1 REF: Knowledge and Understanding OBJ: 1.3 - Rates of Change

31. ANS:

16 800 cans per week

PTS: 1 REF: Application OBJ: 1.3 - Rates of Change

32. ANS:

It represents the velocity function at a point *x* = *a*. It can be found by determining the limiting value of the average velocity as *h* approaches 0.

PTS: 1 REF: Thinking OBJ: 1.3 - Rates of Change

33. ANS:

$m = 2$ and $b = 5$

PTS: 1 REF: Thinking OBJ: 1.4 - Limit of a Function

34. ANS:

-55

PTS: 1 REF: Knowledge and Understanding OBJ: 1.4 - Limit of a Function

35. ANS:

In this scenario, the graph of $f(x)$ passes through the point (*a*, *f*(*a*)). This means the function is continuous at the point (*a*, *f*(*a*)).

PTS: 1 REF: Communication OBJ: 1.4 - Limit of a Function

36. ANS:

$k = \dfrac{3}{2}$

PTS: 1 REF: Thinking OBJ: 1.6 - Continuity

37. ANS:

Since x and $-x$ are polynomial functions, they are continuous on the entire real line. Furthermore, $y = 0$ is a line that is always continuous. At the point $x = 0$, $x = (0) = 0$ and $-x = -(0) = 0$. So, the function is continuous at $x = 0$. The limit at this point is 0 and $f(0) = 0$. So, the function is continuous on the entire real line.

PTS: 1 REF: Communication OBJ: 1.6 - Continuity

38. ANS:

Yes, the function is continuous. The limit from the left and right of 1 is 3. Also, $f(1) = 3$. Furthermore, the functions $x^2 + 2$ and $x + 2$ are polynomial functions, which are continuous on the entire real line. So, this fact along with the knowledge that $\lim_{x \to 1} f(x) = f(3)$ shows that the function is continuous.

PTS: 1 REF: Communication OBJ: 1.6 - Continuity

39. ANS:

$x \geq 2$ but $x \neq 5$

PTS: 1 REF: Knowledge and Understanding OBJ: 1.6 - Continuity

40. ANS:
$x = 2$

PTS: 1 REF: Knowledge and Understanding OBJ: 1.6 - Continuity

PROBLEM

41. ANS:
First, the given expression can be simplified slightly by taking a 4 out of each term.
$$\frac{4\sqrt{x+25}-20}{4x} = \frac{\sqrt{x+25}-5}{x}$$
Next, determine the conjugate radical. It is $\sqrt{x+25}+5$.
Now, rationalize the numerator by multiplying and dividing by the conjugate radical.
$$\frac{\sqrt{x+25}-5}{x} = \frac{\sqrt{x+25}-5}{x} \times \frac{\sqrt{x+25}+5}{\sqrt{x+25}+5} = \frac{x+25-25}{x(\sqrt{x+25}+5)}$$
Finally, simplify the expression as much as possible. Note that the *x* in the numerator and denominator cancel.
$$\frac{x+25-25}{x(\sqrt{x+25}+5)} = \frac{1}{\sqrt{x+25}+5}$$

PTS: 1 REF: Application OBJ: 1.1 - Radical Expressions: Rationalizing Denominators

42. ANS:
a. $\frac{s(5)-s(1)}{5-1} = \frac{500-5(5)^2-(500-5(1)^2)}{4} = \frac{-125+5}{4} = \frac{-120}{4} = -30$
So, the average velocity between 1 s and 5 s is -30 m/s.
b. $\frac{s(9)-s(5)}{9-5} = \frac{500-5(9)^2-(500-5(5)^2)}{4} = \frac{-405+125}{4} = \frac{-280}{4} = -70$
So, the average velocity between 5 s and 9 s is -70 m/s.

c. $v(5) = \lim_{h \to 0} \frac{s(5+h) - s(5)}{h}$

$= \lim_{h \to 0} \frac{(500 - 5(5+h)^2) - 375}{h}$

$= \lim_{h \to 0} \frac{500 - 5(25 + 10h + h^2) - 375}{h}$

$= \lim_{h \to 0} \frac{500 - 125 - 50h - 5h^2 - 375}{h}$

$= \lim_{h \to 0} \frac{-50h - 5h^2}{h}$

$= \lim_{h \to 0} -50 - 5h$

$= -50 - 5(0)$

$= -50$

So, the velocity at $t = 5$ is -50 m/s.

PTS: 1 REF: Application OBJ: 1.3 - Rates of Change

43. ANS:

a. $\lim_{x \to 0} f(x) = 4$. There is no discontinuity at (0, 4). It is a straight line. The limit can easily be seen to be 4.

b. $\lim_{x \to 1^-} f(x) = 4$. This limit is the value x is approaching from the left of 1. This is on the line at the value

4. The limit is 4.

c. $\lim_{x \to 1^+} f(x) = 2$ This limit is the value x is approaching from the right of 1. This is on the line at the value

2. The limit is 2.

d. $\lim_{x \to 1^-} f(x) = 4$ does not equal $\lim_{x \to 1^+} f(x) = 2$. Therefore, $\lim_{x \to 1} f(x)$ does not exist.

e. $f(1) = 3$. This is a point on the graph.

PTS: 1 REF: Communication OBJ: 1.4 - Limit of a Function

44. ANS:

a. Substitution produces the indeterminate form $\frac{0}{0}$, so the limit cannot be found using this method.

b. $\frac{x^2 - x - 2}{x - 2} = \frac{(x-2)(x+1)}{x-2} = x + 1$

So, an equivalent function is $\lim_{x \to 2} (x + 1)$.

c. By direct substitution, $\lim_{x \to 2} (x + 1) = 2 + 1 = 3$ for $x \neq 2$. This method is acceptable because the limit is all values around a point. It is what the values approach around $x = 2$.

d. The graph will be the line $y = x + 1$ with a whole at the point with x-coordinate 2.

PTS: 1 REF: Communication OBJ: 1.5 - Properties of Limits

45. ANS:
a. 1
b. $\sqrt{x}$ needs to be greater than or equal to zero, because there cannot be a square root of a negative number. However, the expression also needs to be less than 3 so the bigger radical is always positive. So, the expression $\sqrt{x}$ needs to be greater than or equal to zero and less than or equal to 3.
c. Essentially, this part of the problem is asking for the domain of the function $f(x) = \sqrt{3 - \sqrt{x}}$. The inside radical is $\sqrt{x}$. $x \geq 0$ for this radical. The big radical is $\sqrt{3 - \sqrt{x}}$. In order for this to be valid, $3 - \sqrt{x} \geq 0$. So,

$3 - \sqrt{x} \geq 0$

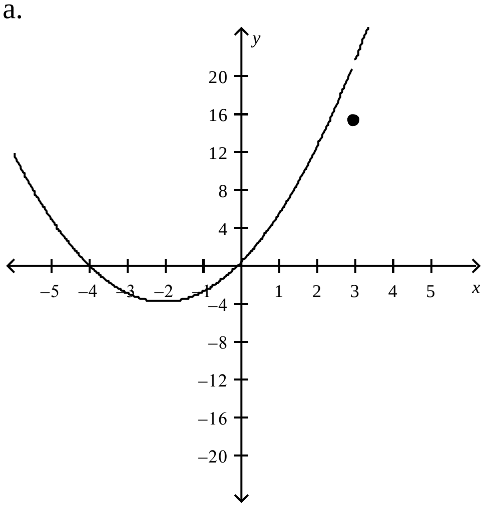
$3 \geq \sqrt{x}$

$9 \geq x$

Therefore, $0 \leq x \leq 9$.

PTS: 1 REF: Thinking OBJ: 1.5 - Properties of Limits

46. ANS:
a.



b. $f(3) = 15$
c. By observing the graph, $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 21$. So, $\lim_{x \to 3} f(x) = 21$.
d. No, $f(x)$ is not continuous because $\lim_{x \to 3} f(x) \neq f(3)$.

PTS: 1 REF: Application OBJ: 1.6 - Continuity

47. ANS:
a. The denominator cannot be zero. So, the function is continuous for all real numbers except $x = -8$.
b. The values inside the radical cannot be negative. The x^2 guarantees a positive number and then adding one will keep that positive. So, this function is continuous on the entire real line.
c. This is the same as b., except now the denominator cannot be zero. As in b. however, there is no real number such that the denominator will equal zero. So, this function is continuous on the entire real line.

d. The denominator cannot be zero. So, the function is continuous for all real numbers except $x = 3$. Notice that cancelling an $x - 3$ out of numerator and denominator after factoring will not change the answer to this problem. If determining the limit, factoring and cancelling would be the strategy. Either way, whether a hole or a vertical asymptote, the function would still have a discontinuity at $x = 3$.

PTS: 1 REF: Knowledge and Understanding OBJ: 1.6 - Continuity