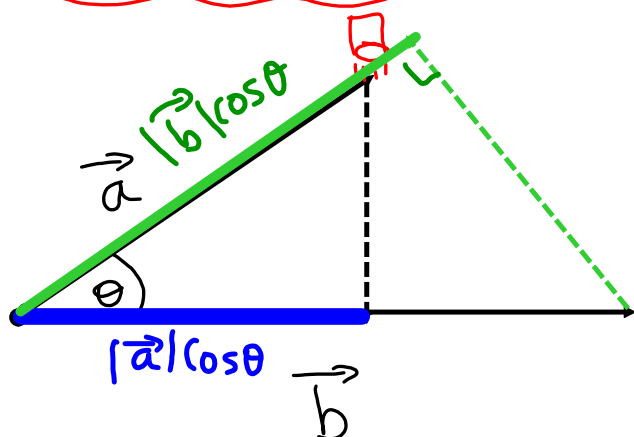# Section 7.5 - Scalar & Vector Projections

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$\vec{a} \cdot \vec{b} = |\vec{a}|\cos\theta |\vec{b}|$$

$$|\vec{a}|\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

∴ Scalar projection of $\vec{a}$ on $\vec{b}$: $\left| \text{proj}_{\vec{b}}\,\vec{a} \right|$

$$= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

what about projection of $\vec{b}$ on $\vec{a}$?

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$|\vec{b}|\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$$

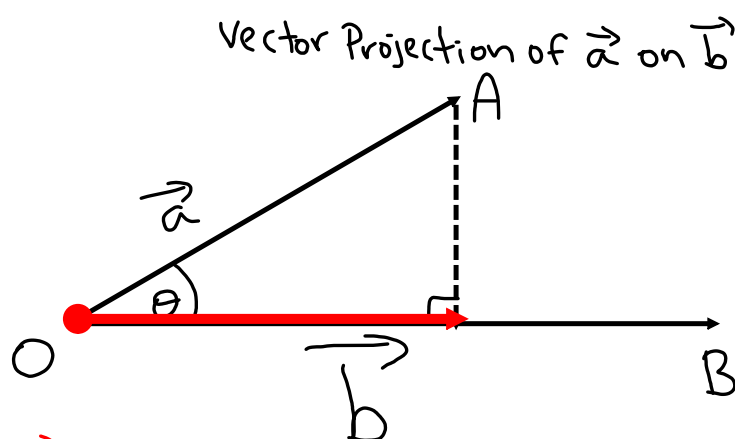$$\left| \text{proj}_{\vec{a}}\,\vec{b} \right| = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$$

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Vector projection of $\vec{a}$ on $\vec{b}$

$= ($scalar projection of $\vec{a}$ on $\vec{b}$ $)($unit vector in the direction of $\vec{b}$ $)$

$= \left(\dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right) \left(\dfrac{\vec{b}}{|\vec{b}|} \right)$

$= \left(\dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right) \vec{b}$

$= \left(\dfrac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}} \right) \vec{b} \quad , \quad \vec{b} \neq \vec{0}$

Vector Projection of $\vec{a}$ on $\vec{b}$

Direction Cosines for $\overrightarrow{OP} = (a, b, c)$

If α, β, γ are the angles that $\overrightarrow{OP}$ makes with the positive x-axis, y-axis and z-axis respectively, then

$\cos \alpha = \dfrac{(a,b,c) \cdot (1,0,0)}{|\overrightarrow{OP}|} = \dfrac{a}{\sqrt{a^2 + b^2 + c^2}}$

$\cos \beta = \dfrac{b}{\sqrt{a^2 + b^2 + c^2}}$

$\cos \gamma = \dfrac{c}{\sqrt{a^2 + b^2 + c^2}}$

May 8-7:56 AM

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ex: If $\vec{p} = (3, 6, -22)$ and $\vec{q} = (-4, 5, -20)$, what are the scalar and vector projections of $\vec{p}$ on $\vec{q}$?

Solution:

Scalar proj of $\vec{p}$ on $\vec{q}$: $|proj_{\vec{q}}\vec{p}|$

$$= \frac{\vec{p} \cdot \vec{q}}{|\vec{q}|}$$

$$= \frac{(3, 6, -22) \cdot (-4, 5, -20)}{\sqrt{(-4)^2 + (5)^2 + (-20)^2}}$$

$$= \frac{(3)(-4) + (6)(5) + (-22)(-20)}{\sqrt{16 + 25 + 400}}$$

$$= \frac{-12 + 30 + 440}{\sqrt{441}}$$

$$= \frac{458}{21} \approx 21.8$$

Vector projection of $\vec{p}$ on $\vec{q}$: $proj_{\vec{q}}\vec{p}$

$$= (\text{Scalar proj of } \vec{p} \text{ on } \vec{q})(\text{unit vector in the direction of } \vec{q})$$

$$= \left(\frac{458}{21}\right)\left(\frac{(-4, 5, -20)}{21}\right)$$

$$= \left(\frac{458}{441}\right)(-4, 5, -20)$$

$$= \left(\frac{-1832}{441}, \frac{2290}{441}, \frac{-9160}{441}\right)$$