

## Section 6.7 - Operations w vectors in $\mathbb{R}^3$

Unit vectors - As in 2-D, we define the 3-D unit vectors along the axes.

The unit vector along the x-axis is

$\vec{i} = [1, 0, 0]$ , along the y-axis is  $\vec{j} = [0, 1, 0]$ , and along the z-axis is  $\vec{k} = [0, 0, 1]$ . Again, unit vectors have length one.

<sup>"Algebraic or Position"</sup>  
Magnitude of a Cartesian Vector -

- Similar to 2-D,

$$\text{if } \vec{u} = [a, b, c], |\vec{u}| = \sqrt{a^2 + b^2 + c^2}$$

ex: Find the magnitude of  $\vec{v} = [-2, 0, 1]$

$$\begin{aligned} |\vec{v}| &= \sqrt{(-2)^2 + (0)^2 + (1)^2} \\ &= \sqrt{4 + 1} \\ &= \sqrt{5} \end{aligned}$$

Scalar Multiplication in 3-D

For any vector  $\vec{u} = [u_1, u_2, u_3]$  and any scalar  $k \in \mathbb{R}$ ,  $k\vec{u} = [ku_1, ku_2, ku_3]$

ex: Find  $b$  and  $c$  such that  $[-2, b, 7] + [c, 6, 21]$  are collinear.

Let  $k$  represent the scalar. Then  $[c, 6, 21] = k[-2, b, 7]$ .

$$[c, 6, 21] = [-2k, bk, 7k]$$

$$\therefore c = -2k; 6 = bk; 21 = 7k$$

$$\therefore c = -2(3) \quad \begin{cases} 6 = b(3) \\ 21 = 7(3) \end{cases} \quad \boxed{3 = k}$$

$$c = -6 \quad \begin{cases} \therefore 2 = b \end{cases}$$

Vector Addition and Subtraction

If  $\vec{u} = [u_1, u_2, u_3]$  and  $\vec{v} = [v_1, v_2, v_3]$ ,

$$\vec{u} + \vec{v} = [u_1 + v_1, u_2 + v_2, u_3 + v_3]$$

and

$$\vec{u} - \vec{v} = [u_1 - v_1, u_2 - v_2, u_3 - v_3]$$

Vector Between Two Points

The vector  $\vec{P_1P_2}$  from  $P_1(x_1, y_1, z_1)$  to point  $P_2(x_2, y_2, z_2)$  is  $\vec{P_1P_2} = [x_2 - x_1, y_2 - y_1, z_2 - z_1]$

Magnitude of a Vector Between Two Points

The magnitude of the vector  $\vec{P_1P_2}$  between the points  $P_1(x_1, y_1, z_1)$  &  $P_2(x_2, y_2, z_2)$  is

$$|\vec{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

ex: Given  $A(3, 6, -1)$  and  $B(-1, 0, 5)$  express  $\vec{AB}$  as an ordered triple.

b) Determine the magnitude of  $\vec{AB}$

c) Determine a unit vector,  $\vec{u}$ , in the direction of  $\vec{AB}$

$$\begin{aligned} \text{a) } \vec{AB} &= [-1 - 3, 0 - 6, 5 - (-1)] \\ &= [-4, -6, 6] \end{aligned}$$

$$\begin{aligned} \text{b) } |\vec{AB}| &= \sqrt{(-4)^2 + (-6)^2 + (6)^2} \\ &= \sqrt{16 + 36 + 36} \\ &= \sqrt{88} \\ &= 2\sqrt{22} \end{aligned}$$

c) Let  $\vec{u}$  represent the unit vector in the direction of  $\vec{AB}$ .

$$\begin{aligned} \text{Then } \vec{u} &= \frac{1}{|\vec{AB}|} \vec{AB} = \frac{1}{2\sqrt{22}} [-4, -6, 6] \\ &= \left[ \frac{-2}{\sqrt{22}}, \frac{-3}{\sqrt{22}}, \frac{3}{\sqrt{22}} \right] \end{aligned}$$

