1.6 Continuity

Continuity at a Point:

For function to be ***continuous*** at a point, the function must exist at the point and any small change in x produces a small change in $f(x)$.

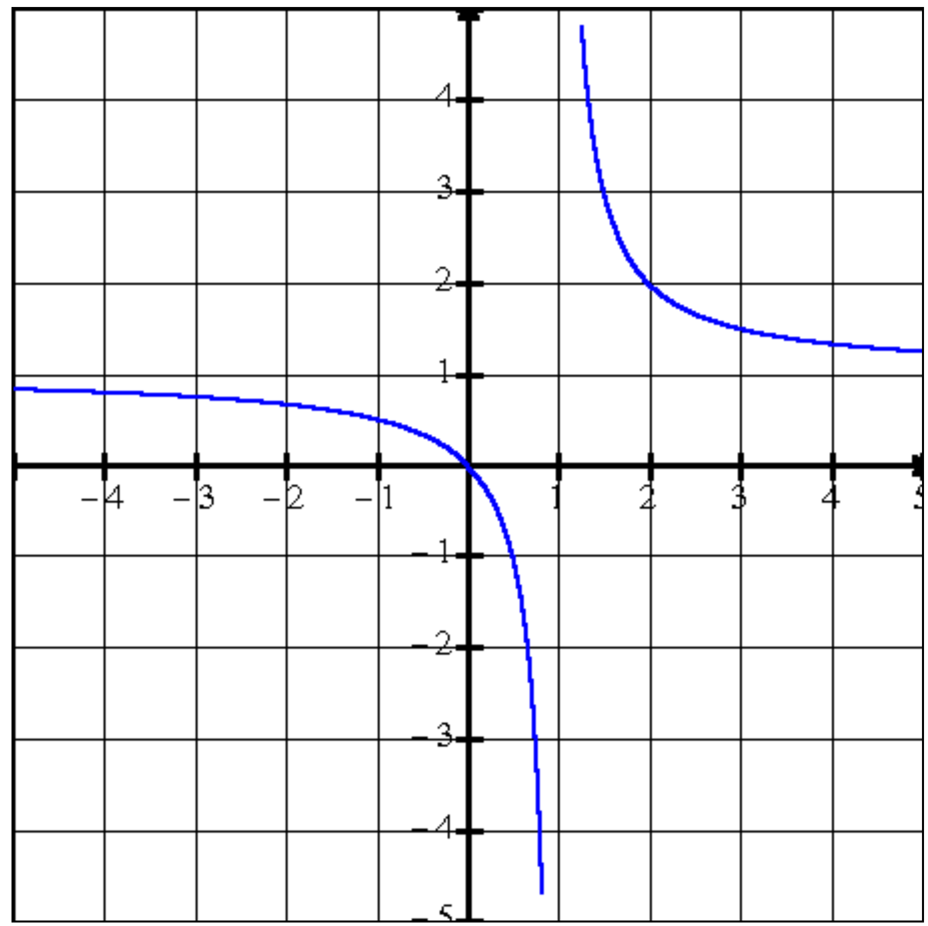
The following conditions must be true.

1. $\lim_{x \to a} f(x)$ exists
2. $f(a)$ exists (or is defined).
3. $\lim_{x \to a} f(x) = f(a)$ (from both sides of a)

A rational function is ***discontinuous*** at $x = a$ if the denominator equals zero.

Example: find all numbers, $x = a$, for which each function is discontinuous.

$a)\ \ f(x) = \dfrac{x}{x-1}$



The function is discontinuous at $x = 1$.

$f(1)$ does not exist.

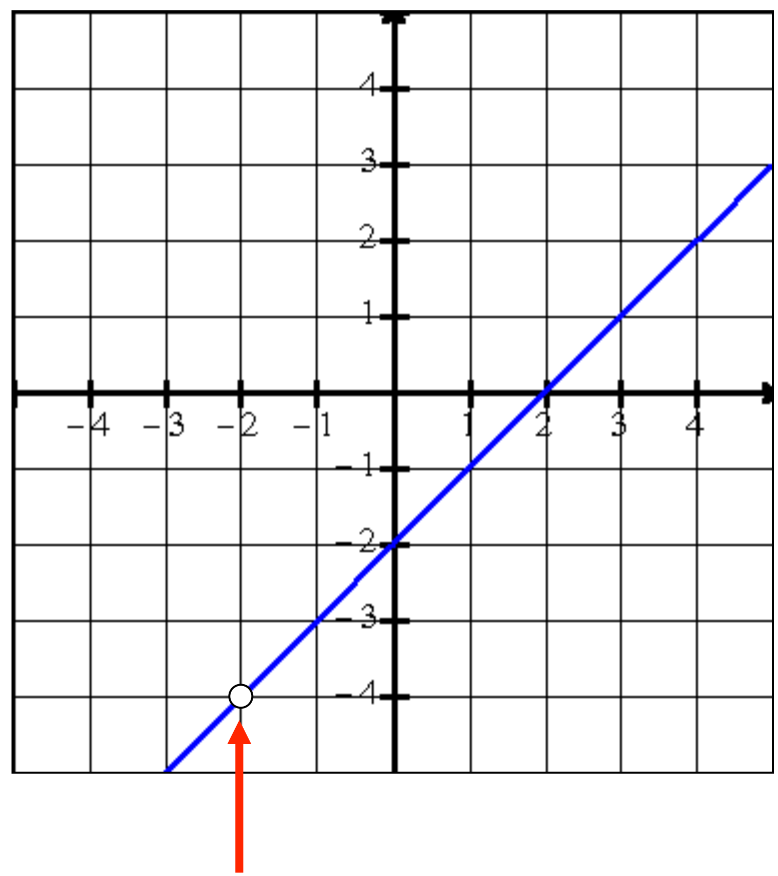
$\lim_{x\to1} f(x)$ does not exist

$$b)\ g(x)=\frac{x^2-4}{x+2}$$ factor

$$=\frac{(x-2)(x+2)}{x+2}$$

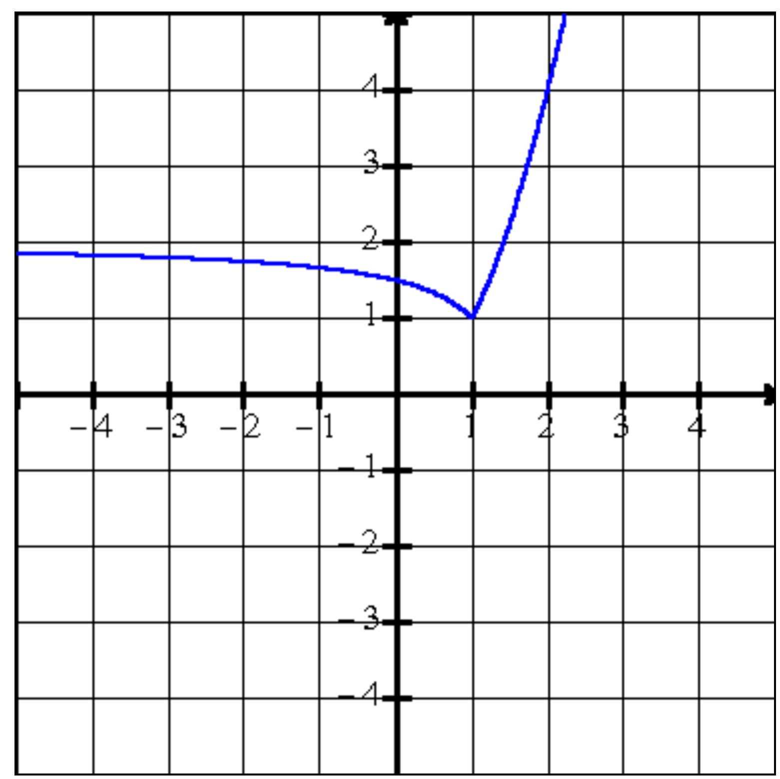
$$= x - 2, \text{ if } x \neq -2$$

The graph of the original function has a hole at $x = -2$. $g(-2)$ does not exist.



c) $h(x) = \begin{cases} \dfrac{2x-3}{x-2} & \text{for } x < 1 \\ x^2 & \text{for } x \geq 1 \end{cases}$

$h(x)$ is undefined for $x = 2$, however, $x = 2$ is not part of the domain.

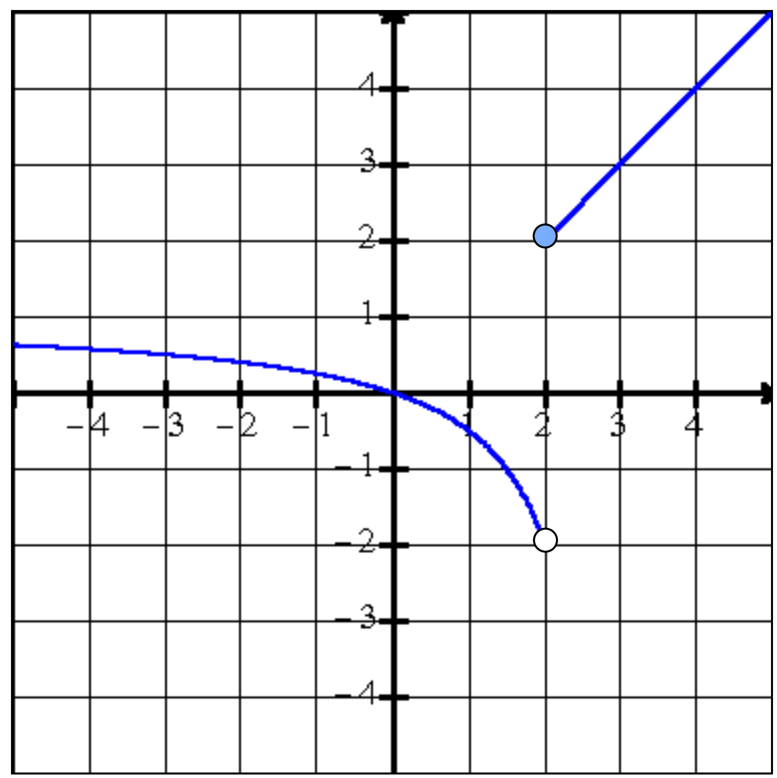


The function might be discontinuous at $x = 1$.

We can see from the graph that the function is continuous.

d) $k(x) = \begin{cases} \dfrac{x}{x-3} & \text{for } x < 2 \\ x & \text{for } x \geq 2 \end{cases}$

$k(x)$ is undefined for $x = 2$, however, $x = 2$ is not part of the domain.



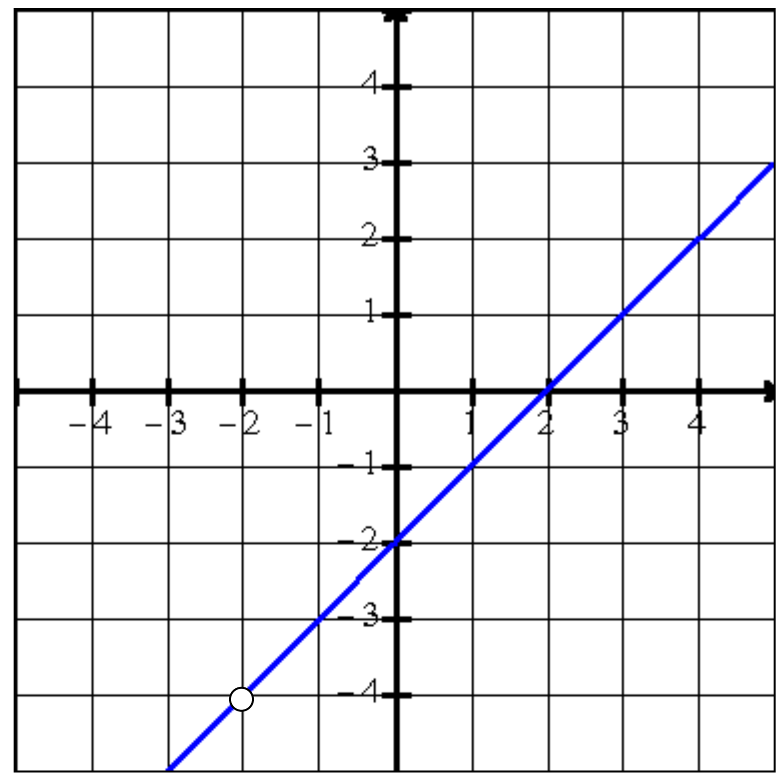
The function might be discontinuous at $x = 2$.

We can see from the graph that there is a jump at $x = 2$, therefore it is discontinuous.

Removing a Discontinuity

We saw in example b that the function was discontinuous at $x = -2$.

$$b)\ g(x) = \frac{x^2 - 4}{x + 2}$$



We can remove the discontinuity by defining it as follows:

$$g(x) = \begin{cases} \dfrac{x^2 - 4}{x + 2} & \text{if } x \neq -2 \\ -4 & \text{if } x = -2 \end{cases}$$