MCV4U - Unit 4Test
Curve Sketching

Name: Key
Date: 2014

K/U:	APP:	TIPS:	COMM:
11	15	12	3 + 3

*Answer all questions on this paper. Be sure to show all **applicable** work and express all answers in simplest form. Marks are awarded for presentation and technical correctness.*

Knowledge and Understanding:

1. State the x-intercept(s) of the function $y = \dfrac{x^2 - 3x}{(x-3)}$ $\quad \dfrac{x(x-3)}{x-3}$ 1. $x = 0$

2. State the vertical asymptote(s) of the function $f(x) = \dfrac{3x-5}{4x^2+4x+1}$ 2. $x = -1/2$

$$\begin{array}{r} 4x+2 \\ x+2\,\overline{)\,4x^2+10x-6} \\ 4x^2+8x \\ 2x \end{array}$$

3. State the equation of the oblique asymptote of $y = \dfrac{4x^2+10x-6}{x+2}$ 3. $y = 4x+2$

4. Evaluate: $\lim\limits_{x \to \infty} \dfrac{-3x^2+5x^3+7}{6x^3+x^2-2}$ 4. $5/6$

5. Sketch a graph of a function *f* that satisfies these conditions. [2]
- points (-1,10) and (3,1) are local extrema on the graph
- (1,3) is an inflection point
- the graph is concave down only when x < 1
- the x-intercept is - 4 and the y-intercept is 8

6. Let $f(x) = \dfrac{3x^3+2x^2+x+1}{x^2+x-2}$. What types of asymptotes does $f(x)$ have?

a. horizontal and vertical asymptotes
b. oblique and vertical asymptotes
c. horizontal and oblique asymptotes
d. None

7. Which function has an oblique asymptote?

a. $f(x) = \dfrac{2x^2 - x}{3x^2+1}$

b. $f(x) = \dfrac{x^3+x+2}{x^2-3x+7}$

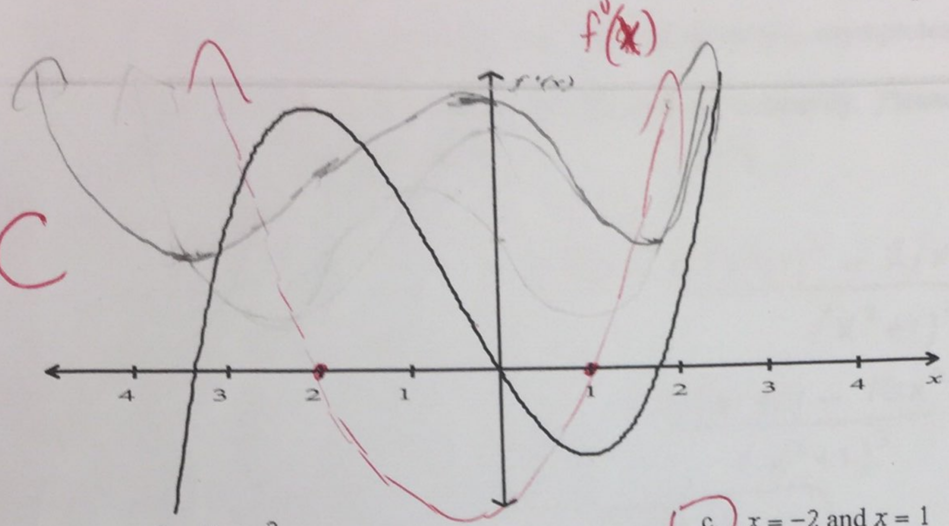
c. $f(x) = \dfrac{x-3}{x^2-4x+3}$

d. $f(x) = \dfrac{x^2-2x+1}{x-1}$ $\quad (x-1)(x-1)$

$\overline{8}$

8. Below is the graph of $f'(x)$. For what value(s) of x does $f(x)$ have a point of inflection?

C



a. $x = -2$

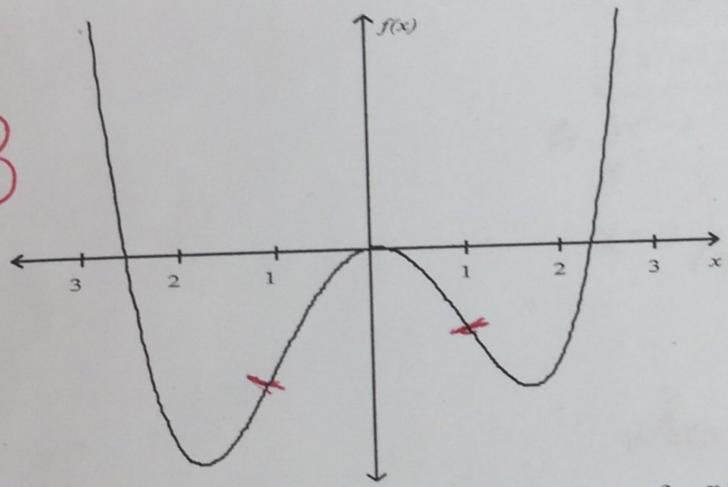
b. $x = 1$

c. $x = -2$ and $x = 1$

d. $x = 0$

9. Below is the graph of $f(x)$. For what values of x is $f(x)$ concave down?

B



a. $x < -1$

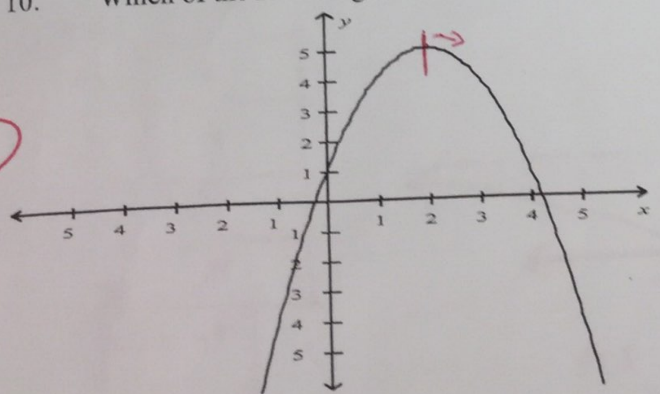
b. $-1 < x < 1$

c. $x > 1$

d. $x < -1$ and $x > 1$

10. Which of the following is true for the interval $(2, \infty)$ for the graph of $f(x)$ shown below?

D



a. $f'(x) > 0, f''(x) > 0$

b. $f'(x) > 0, f''(x) < 0$

c. $f'(x) < 0, f''(x) > 0$

d. $f'(x) < 0, f''(x) < 0$

Application:

11. If $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$, determine the domain, intercepts, asymptotes, intervals of increase and decrease, local extrema, points of inflection and concavity. Please use interval charts in your solution. Graph the function. [15]

$f(x) = \dfrac{(x-1)(x+1)}{x^2+1}$

Domain : $x \in \mathbb{R}$
x-int : $x = \pm 1$
y-int : $y = -1$
No vertical asymptote
Horizontal asymptote @ $y = 1$
No oblique Asymptote

$f'(x) = \dfrac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2}$

$= \dfrac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2}$

$= \boxed{\dfrac{4x}{(x^2+1)^2}}$

$4x = 0$
$x = 0$
critical pt.

	$x<0$	$x>0$
$4x$	−	+
$(x^2+1)^2$	+	+
$f'(x)$	−	+
$f(x)$	↘	↗
	Min at $x=0$	

$(0,-1)$ is a local min

$f''(x) = \dfrac{4(x^2+1)^2 - 2(x^2+1)(2x)(4x)}{(x^2+1)^4}$

$= \dfrac{4(x^2+1) - 16x^2}{(x^2+1)^3}$

$\boxed{= \dfrac{-12x^2 + 4}{(x^2+1)^3}}$

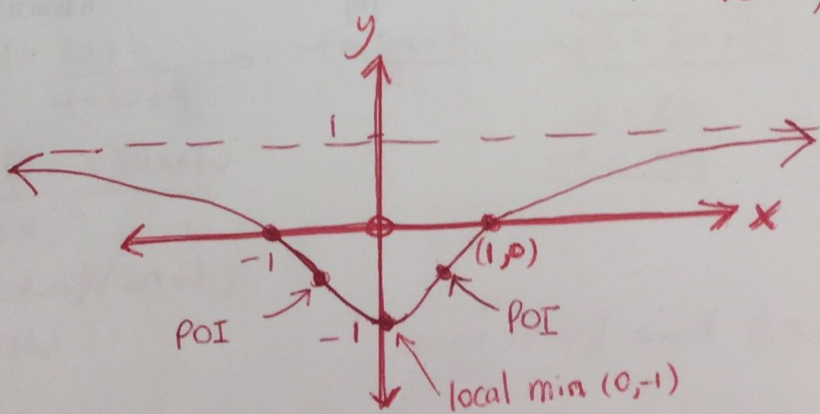
$= \boxed{\dfrac{-4(3x^2-1)}{(x^2+1)^3}}$

$\therefore 3x^2 - 1 = 0$
$x^2 = \frac{1}{3}$
$x = \pm \frac{1}{\sqrt{3}}$

	$x < -\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$	$x > \frac{1}{\sqrt{3}}$
$-12x^2+4$	−	+	−
$(x^2+1)^3$	+	+	+
$f''(x)$	−	+	−
$f(x)$	∩	∪	∩
	POI at $x = -1/\sqrt{3}$	POI at $x = 1/\sqrt{3}$	

when $x = \frac{-1}{\sqrt{3}}$ $f(\frac{-1}{\sqrt{3}}) = \dfrac{\frac{1}{3} - 1}{\frac{1}{3} + 1} = \dfrac{\frac{-2}{3}}{\frac{4}{3}} = -\frac{1}{2}$

$(\frac{-1}{\sqrt{3}}, \frac{-1}{2}) + (\frac{1}{\sqrt{3}}, \frac{-1}{2})$ are POI's



T/I/PS:

12. The point (-1, 5) is a point of inflection on the graph of $f(x) = 2x^3 + mx^2 - 3x + n$. Determine the values of m and n. [4]

$5 = 2(-1)^3 + m(-1)^2 - 3(-1) + n$
$5 = -2 + m + 3 + n$
$\boxed{4 = m + n}$

$f'(x) = 6x^2 + 2mx - 3$
$f''(x) = 12x + 2m$
$0 = 12(-1) + 2m$
$12 = 2m$
$\boxed{6 = m}$

$\therefore 4 = 6 + n$
$-2 = n$

$\therefore m = 6$ and $n = -2$

13. Determine the conditions on the parameter k, such that the function $f(x) = \dfrac{2x+4}{x^2 - k^2}$ will have critical points. [4]

$f'(x) = \dfrac{2(x^2 - k^2) - 2x(2x+4)}{(x^2 - k^2)^2}$

$= \dfrac{2x^2 - 2k^2 - 4x^2 - 8x}{(x^2 - k^2)^2}$

$= \boxed{\dfrac{-2x^2 - 8x - 2k^2}{(x^2 - k^2)^2}}$

using quadratic formula,

$x = \dfrac{8 \pm \sqrt{(-8)^2 - 4(-2)(-2k^2)}}{-4}$

$= \dfrac{8 \pm \sqrt{64 - 16k^2}}{-4}$

$64 - 16k^2 \geq 0$
$64 \geq 16k^2$
$4 \geq k^2$
$2 \geq k \geq -2$

$-2 \leq k \leq 2$
$x \neq \pm k$

14. If the graph of the function $g(x) = \dfrac{ax + b}{(x-1)(x-4)}$ has a horizontal tangent at point (2,-1), determine the values of a and b. [4]

$g(x) = \dfrac{ax+b}{x^2 - 5x + 4} \rightarrow -1 = \dfrac{2a+b}{4-10+4} \rightarrow -1 = \dfrac{2a+b}{-2} \rightarrow \boxed{2 = 2a + b}$

$2 = 2a$
$\boxed{1 = a}$

$g'(x) = \dfrac{a(x^2 - 5x + 4) - (2x-5)(ax+b)}{(x^2 - 5x + 4)^2}$

$0 = a(4 - 10 + 4) - (4-5)(2a+b)$
$0 = a(-2) - (-1)(2a+b)$
$0 = -2a + 2a + b$
$\boxed{b = 0}$

$\therefore a = 1$ and $b = 0$

12

Communication:

15. What is an inflection point? How do you identify points of inflection? /3

An inflection point is a point on the graph where the graph changes from Concave up to concave down or vice versa.

Points of inflection are found by Setting the second derivative to zero and Solving

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16. Use the ***second derivative test*** to show that $f(x) = x^3 - 3x^2$ has a local maximum at the origin.

[4]

$f'(x) = 3x^2 - 6x$
$0 = 3x(x-2)$
$x = 0 ; x = 2$

$f''(x) = 6x - 6$

$f''(0) = -6$
$\therefore$ concave down
$\therefore$ local max

$f(0) = (0)^3 - 3(0)^2$
$= 0$
$(0,0)$

17. Below is the graph of $f'(x)$. Sketch a possible graph of $f(x)$. [3]