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MCV4U - Unit 8 Test - Equations of Lines & Planes

Multiple Choice

Identify the choice that best completes the statement or answers the question.

A 1. What is the vector equation of the line with parametric equations $x = 2t + 7$, $y = -3t + 2$, $t \in \mathbf{R}$?

a. $\vec{r} = (7,2) + s(2,-3)$, $s \in \mathbf{R}$

b. $\vec{r} = (2,-3) + s(7,2)$, $s \in \mathbf{R}$

c. $\vec{r} = (2,7) + s(-3,2)$, $s \in \mathbf{R}$

d. none of the above

D 2. Which three points are on the line L: $\vec{r} = (2,-5) + s(2,1)$, $s \in \mathbf{R}$?

a. $P(2,-5)$, $Q(2,1)$, $R(0,-6)$

b. $Q(2,1)$, $Q(6,-3)$, $R(2,-5)$

c. $P(1,7)$, $Q(2,-5)$, $R(6,-3)$

d. $P(2,-5)$, $Q(6,-3)$, $R(0,-6)$

D 3. Which of the following is a direction vector for the line $x = 2t - 1$, $y = -3t + 2$, $t \in \mathbf{R}$?

a. $\vec{m} = (4,-6)$

b. $\vec{m} = \left(\frac{2}{3},-1\right)$

c. $\vec{m} = (-2,3)$

d. all of the above

$\vec{m} = (2,-3)$

C 4. Determine which line is perpendicular to the line $2x - 3y + 17 = 0$.

$\vec{n} = (2,-3)$

a. $\vec{r} = (2,-3) + s(3,-2)$, $s \in \mathbf{R}$

b. $\vec{r} = (1,2) + s(3,2)$, $s \in \mathbf{R}$

c. $\vec{r} = (1,7) + s(2,-3)$, $s \in \mathbf{R}$

d. $\vec{r} = s(3,2)$, $s \in \mathbf{R}$

B 5. Which value of k will make the lines $\vec{r} = (1,2) + s(2,3)$, $s \in \mathbf{R}$ and $12x + ky = 0$ parallel?

$\vec{m} = (2,3)$ $\vec{n} = (12,k)$

a. 18

b. -8

c. 8

d. -18

$(2,3) \cdot (12,k) = 0$

$24 + 3k = 0$

A 6. Which of the following equations determines a line with normal vector $\vec{n} = (4,3)$ going through the point $P(1,-1)$?

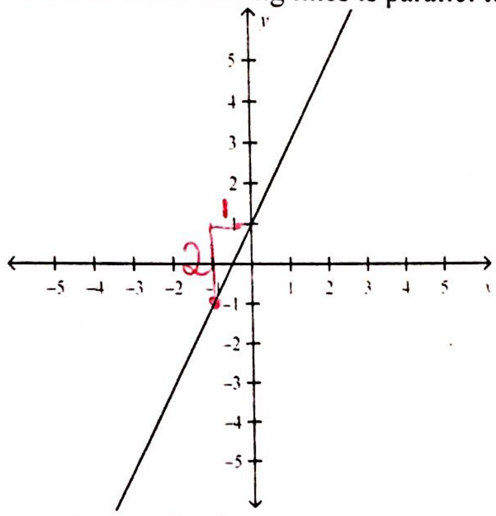
a. $4x + 3y - 1 = 0$

b. $4x + 3y + 1 = 0$

c. $3x + 4y - 1 = 0$

d. $3x + 4y + 1 = 0$

B 7. Which of the following lines is parallel to the line shown below?



$m = \frac{2}{1}$

$\vec{m} = (1,2)$

a. $2x + y + 3 = 0$
b. $2x - y - 2 = 0$
c. $x + 2y - 4 = 0$
d. $x - 2y - 4 = 0$

A 8. Which of the following is the parametric equation of the line with symmetric equation $\frac{x+3}{3} = \frac{y+2}{2} = \frac{z-1}{2}$?

a. $x = 3t - 3,\ y = 2t - 2,\ z = 2t + 1,\ t \in \mathbf{R}$
b. $x = 3t + 3,\ y = 2t + 2,\ z = 2t - 1,\ t \in \mathbf{R}$
c. $x = 3t - 3,\ y = 2t - 2,\ z = t + 2,\ t \in \mathbf{R}$
d. none of the above

A 9. What value of k will place the point $P(k, 2k, k-2)$ on the line $\vec{r} = (-3,2,9) + s(3,2,-4),\ s \in \mathbf{R}$?

a. 3
b. 4
c. 2
d. 1

$K = -3 + 3S$
$2K = 2 + 2S$
$2K = -6 + 6S$

$8 = -4S$
$-2 = S$

C 10. Which of the following is not an equation for the line passing through the points $P(1,4,-3)$ and $Q(3,2,1)$?

$\vec{PQ} = (2,-2,4)$

a. $\vec{r} = (1,4,-3) + s(2,-2,4),\ s \in \mathbf{R}$
b. $x = -t + 3,\ y = t + 2,\ z = -2t + 1,\ t \in \mathbf{R}$
c. $\frac{x-3}{2} = \frac{y-2}{-2} = \frac{z+1}{4}$
d. $\vec{r} = (3,2,1) + s(1,-1,2),\ s \in \mathbf{R}$

D 11. Determine the value of k that makes the lines $\frac{x+2}{4} = \frac{y+1}{5} = \frac{z-3}{3}$ and $\vec{r} = (1,3,6) + s(-2k, 2, k),\ s \in \mathbf{R}$, perpendicular.

$\boxed{K = 2}$

$\vec{m_1} = (4,5,3)$ $\vec{m_2} = (-2K, 2, K)$

$\vec{m_1} \cdot \vec{m_2}$

$-8K + 10 + 3K = 0$
$-5K + 10 = 0$
$-5K = -10$
$\boxed{K = 2}$

a. 1
b. 3
c. -3
d. none of the above

D 12. Which of the following determines a plane?

a. a line and a point not on the line
b. two intersecting lines
c. two parallel, non-coincident lines
d. all of the above

C 13. Which of the following is not a plane?

a. $\vec{r} = (1,3,4) + s(2,-1,2) + t(1,1,1),\ s,t \in \mathbf{R}$

b. $\vec{r} = (2,4,2) + s(1,-2,3) + t(3,2,2),\ s,t \in \mathbf{R}$

c. $\vec{r} = (3,2,3) + s(4,-4,2) + t(-2,2,-1),\ s,t \in \mathbf{R}$

d. $\vec{r} = (-2,1,4) + s(2,2,-1) + t(2,2,1),\ s,t \in \mathbf{R}$

D 14. A plane is defined by the equation $3x - 2z = 4y + 1$. Which of the following is the normal vector of this plane?

a. $\vec{n} = (3,-2,4)$

b. $\vec{n} = (3,4,-2)$

c. $\vec{n} = (3,2,4)$

d. $\vec{n} = (3,-4,-2)$

~~$3x - 2z - 4y = 1$~~
~~$\vec{n} = (3,-2,-4)$~~
$3x - 4y - 2z = 1$
$\vec{n} = (3,-4,-2)$

B 15. On which of the following planes could the point $P(2,3,-4)$ lie?

a. $x = 3$

b. $y = 3$

c. $z = 3$

d. none of the above

C 16. Which of the following is the x-intercept of the plane $2x - 4z + 1 = 0$?

a. $P(2,0,-4)$

b. $P(0,0,0)$

c. $P\left(-\frac{1}{2},0,0\right)$

d. $P\left(0,0,\frac{1}{4}\right)$

$2x = -1$
$x = -1/2$

B 17. Which of the following best describes the intersection of the three planes π_1: $x = 2$, π_2: $y = 6$, and π_3: $z = -3$?

a. $\vec{v} = (2,0,-3) + s(0,6,0),\ s \in \mathbf{R}$

b. $P(2,6,-3)$

c. There is no intersection.

d. none of the above

A 18. Which of the following would describe the sketch of the expression $xy + 2y - 3x - 6 = 0$?

a. the planes $x = -2$ and $y = 3$

b. the planes $x = 3$ and $y = -2$

c. the planes $x = 6$ and $y = 2$

d. the planes $x = 3$ and $y = 6$

$y(x+2) - 3(x+2) = 0$
$(x+2)(y-3) = 0$

A 19. Which of the following planes is the equation for the plane with an x-intercept at $P(2,0,0)$, a y-intercept at $Q(0,-3,0)$, and is parallel to the z-axis?

a. $3x - 2y - 6 = 0$

b. $2x - 3y + z - 6 = 0$

c. $3y - 2z - 6 = 0$

d. $z = -6$

A 20. Which three points are on the plane $2x - 7y + 3z - 5 = 0$?

a. $P(1,0,1)$, $Q(3,1,2)$, and $R(4,3,6)$

b. $P(1,0,1)$, $Q(2,2,3)$, and $R(3,1,2)$

c. $P(3,1,2)$, $Q(4,3,6)$, and $R(5,0,-2)$

d. $P(4,3,6)$, $Q(0,0,0)$, and $R(3,1,2)$

Full Solution: Show all applicable work for the following questions.

1. Calculate the acute angle that is formed by the intersection of the planes with equations $2x+3y-z+9=0$ and $x+2y+4=0$.

good wrong answer is 76.5°

$\vec{n_1}=(2,3,-1)$ $\vec{n_2}=(1,2,0)$

[3]

$\cos\theta = \dfrac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$

$\cos\theta = \dfrac{(2,3,-1)\cdot(1,2,0)}{\sqrt{2^2+3^2+(-1)^2}\sqrt{1^2+2^2}}$

$\cos\theta = \dfrac{2+6+0}{\sqrt{14}\sqrt{5}}$

$\cos\theta = \dfrac{8}{\sqrt{70}}$

$\cos\theta = 0.9562$

$\theta = 17°$

2. Determine the x-intercept of the plane with equation $[x,y,z]=[1,8,6]+s[1,-12,-12]+t[2,4,-3]$.

[4]

$x=1+s+2t$
$y=8-12s+4t$
$z=6-12s-3t$

Let $y=0, z=0$

$0=8-12s+4t$
$0=6-12s-3t$

$0=2+7t$
$7t=-2$
$\boxed{t=-\frac{2}{7}}$

$0=8-12s+4\left(-\frac{2}{7}\right)$
$0=8-12s-\frac{8}{7}$
$12s=-\frac{8}{7}+\frac{56}{7}$
$12s=\frac{48}{7}$
$s=\frac{48}{84}$
$\boxed{s=\frac{4}{7}}$

$\therefore x=1+\frac{4}{7}+2\left(-\frac{2}{7}\right)$
$x=\frac{7}{7}+\frac{4}{7}-\frac{4}{7}$
$x=1$

$\therefore$ the x-intercept is $(1,0,0)$

$\frac{}{7}$

3. Determine the parametric equations of the line whose direction vector is perpendicular to the direction vectors of the two lines

$$\frac{x-5}{3}=\frac{y-5}{2}=\frac{z+5}{4} \text{ and } \frac{x}{-4}=\frac{y+10}{-7}=\frac{z+2}{3}$$

and passes through the point $(2,-5,0)$. [4]

$\vec{m}_1=(3,2,4)$

$\vec{m}_2=(-4,-7,3)$

$\vec{m}_1 \times \vec{m}_2=(34,-25,-13)$

2 -7
4 × 3
3 × -4
2 × -7

$\therefore \vec{r}=(2,-5,0)+t(34,-25,-13)$

$x=2+34t$

$y=-5-25t$

$z=-13t$

4. A plane is determined by a normal $\vec{n}=(2,5,3)$ and contains the point $P_1(-1,-4,7)$. Determine a Cartesian equation for this plane using two different methods. [5]

Method 1:

Select an arbitrary pt. $P(x,y,z)$

$\overrightarrow{PP_1}=(x+1,y+4,z-7)$

$\overrightarrow{PP_1}\cdot\vec{n}=(x+1,y+4,z-7)\cdot(2,5,3)$

$2(x+1)+5(y+4)+3(z-7)=0$

$2x+2+5y+20+3z-21=0$

$\boxed{2x+5y+3z+1=0}$

Method 2:

$Ax+By+Cz+D=0$

$2x+5y+3z+D=0$

$2(-1)+5(-4)+3(7)+D=0$

$-2-20+21+D=0$

$-1+D=0$

$D=1$

$\therefore \boxed{2x+5y+3z+1=0}$

5. A line segment with endpoints P (1,2) and Q (3,6) is the hypotenuse of a right triangle. The third vertex, R, lies on the line with Cartesian equation $-x+2y-1=0$. Determine the coordinates of R. [4]

R(x, y) arbitrary point

$\vec{PR} = (x-1, y-2)$

$\vec{QR} = (x-3, y-6)$

$\vec{PR} \cdot \vec{QR} = 0$

$(x-1, y-2) \cdot (x-3, y-6) = 0$

$x^2 - 4x + 3 + y^2 - 8y + 12 = 0$

Since $\boxed{x = 2y-1}$,

$(2y-1)^2 - 4(2y-1) + y^2 - 8y + 15 = 0$

$4y^2 - 4y + 1 - 8y + 4 + y^2 - 8y + 15 = 0$

$5y^2 - 20y + 20 = 0$

$y^2 - 4y + 4 = 0$

$(y-2)(y-2) = 0$

$\boxed{y = 2}$

$\therefore x = 2(2) - 1$

$\boxed{x = 3}$

$\therefore R(3, 2)$

6. Explain why the line $\vec{r} = (4, 9, -3) + t(1, -4, 2)$ and the point $(8, -7, 5)$ do not determine a plane? **[2]**

$8 = 4 + t$, $t = 4$

$-7 = 9 - 4t$, $4t = 16$, $t = 4$

$5 = -3 + 2t$, $8 = 2t$, $4 = t$

$\therefore$ point (8, −7, 5) is on the actual line.

$\therefore$ it cannot define a plane. need a line and a point not on the line.

7. Explain why the plane with Cartesian equation $2x + 5z - 3 = 0$ never intersects the y-axis. [2]

$2x + 0y + 5z - 3 = 0$

Since there is no value for y, we cannot Solve for y. $\therefore$ no y values, no y-intercept. This is a plane parallel to the y-axis.

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