Section 7.7 – Applications of Dot & Cross Product

Dot Product:

- finding work done
- determining angle between 2 vectors
- finding the projection of one vector onto another.

Find the work done:

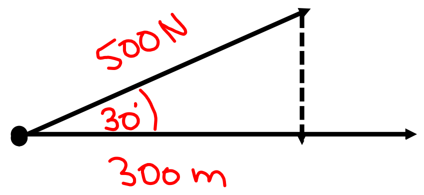
work is defined as the product of the distance an object has been displaced and the component of force along the line of displacement.

$W = \vec{f} \cdot \vec{s}$

$\vec{f}$ = force acting on object (N)
$\vec{s}$ = displacement of object (m)
W = work done, (J) Joules

ex: Angela has entered the wheelchair division of a marathon race. She races her wheelchair up a 300m hill with a constant force of 500 N applied at an angle of 30° to the surface of the hill. Find the work done by Angela, to the nearest 100 J.

Solution:



$$W = \vec{f} \cdot \vec{s}$$
$$= |\vec{f}||\vec{s}|\cos\theta$$
$$= (500)(300)(\cos 30°)$$
$$= 129\,904 \text{ N}\cdot\text{m}$$
$$= 129\,900 \text{ J}$$

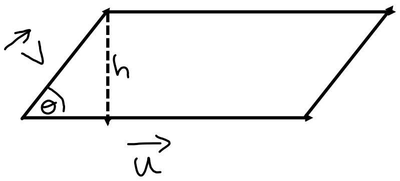
or 129.9 KJ

Area of a Parallelogram

a) Determine the area of a parallelogram defined by the vectors $\vec{u} = (4,5,2)$ and $\vec{v} = (3,2,7)$.

b) Determine the angle between $\vec{u}$ and $\vec{v}$

Solution:



$A = bh$

Since $\sin\theta = \dfrac{h}{|\vec{v}|}$

$h = |\vec{v}|\sin\theta$

$\therefore A = |\vec{u}||\vec{v}|\sin\theta$

See proof on pg. 411 which shows

$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta$

$\therefore$ Area $= |\vec{u}||\vec{v}|\sin\theta$

This is the cross product of $\vec{u}$ and $\vec{v}$

$\therefore$ Area of the parallelogram is $|\vec{u} \times \vec{v}|$

$\vec{u} \times \vec{v} = (35 - 4, 6 - 28, 8 - 15)$

$= (31, -22, -7)$

$$\begin{array}{ccc} 5 & & 2 \\ & \times & \\ 2 & & 7 \\ & \times & \\ 4 & & 3 \\ & \times & \\ 5 & & 2 \end{array}$$

$\therefore |\vec{u} \times \vec{v}| = \sqrt{(31)^2 + (-22)^2 + (-7)^2}$

$= \sqrt{1494}$

$\doteq 38.65$

b) $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta$

$\sin\theta = \dfrac{|\vec{u} \times \vec{v}|}{|\vec{u}||\vec{v}|}$

$\sin\theta = \dfrac{\sqrt{1494}}{\sqrt{45}\sqrt{62}}$

$\sin\theta = 0.7318$

$\theta = 47^\circ$

$\therefore$ angle could be 47° or 133°

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ex: Calculate the area of the triangle formed by
$A(-2,4,2)$, $B(-1,0,0)$ $C(6,-2,8)$

Solution:

$\vec{AB} = (-1-(-2), 0-4, 0-2)$
$= (1,-4,-2)$

$\vec{AC} = (6-(-2), -2-4, 8-2)$
$= (8,-6,6)$

$\vec{AB} \times \vec{AC} = (-24-12, -16-6 \quad 6+32)$
$= (-36,-22,26)$

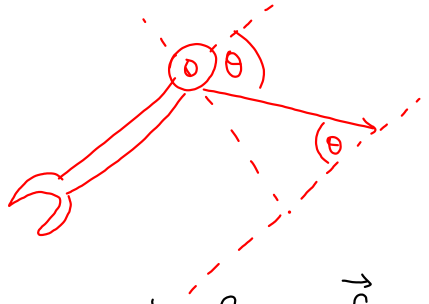
$$\begin{matrix} -4 & \times & -6 \\ -2 & \times & 6 \\ 1 & \times & 8 \\ -4 & & -6 \end{matrix}$$

So $|\vec{AB} \times \vec{AC}| = \sqrt{(-36)^2+(-22)^2+(26)^2}$
$= \sqrt{2456}$
$= 49.55$

Since
$A = \frac{1}{2}bh$
$= \frac{1}{2}(49.55)$
$\doteq 24.8$ units^2

Physical Application of the Cross Product

Consider using a wrench to tighten a bolt.



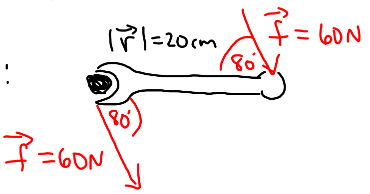
Suppose the force, $\vec{f}$, on the wrench turns it clockwise. Length of the wrench is $\vec{r}$.
The effect of turning the wrench is called the moment, $\vec{M}$ or torque $\vec{\tau}$ (tau) of the force about the centre of the bolt.

$\therefore$ Torque $= \vec{r} \times \vec{f}$
$= |\vec{r}||\vec{f}| \sin\theta$

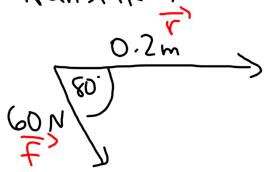
where $\vec{f}$ = force in Newtons
$\vec{r}$ = distance in metres
Torque is in newton metres → Joules

Ex: A wrench is used to tighten a bolt. A force of 60 N is applied in a clockwise direction of 80° to the handle, 20 cm from the centre of the bolt.
a) Calculate the magnitude of the torque
b) In what direction does the vector point?

Solution:



20 cm = 0.2 m
Translate $\vec{f}$ so that it is tail-to-tail.



$\therefore |\vec{\tau}| = |\vec{r}||\vec{f}| \sin\theta$
$= (0.2)(60)\sin 80°$
$= 11.8\ N\cdot m$

$\therefore$ The torque has ~~a magnitude~~ of 11.8 N·m or $\boxed{11.8\,J}$