Differentiating Other Logarithmic and Exponential Functions.

Example: What is the derivative of $y = \log_2 x$?

Suppose: $y = \log_a x$, $a > 0$.

$$y = \frac{\ln x}{\ln a} \quad \text{(change of base)}$$

$$\frac{dy}{dx} = \frac{d\left(\frac{\ln x}{\ln a}\right)}{dx} \quad (\ln a \text{ is a constant})$$

$$\frac{dy}{dx} = \frac{1}{\ln a}\frac{d(\ln x)}{dx}$$

The derivative of $y = \log_a x$:

$$\frac{dy}{dx} = \frac{1}{x \ln a}$$

Example: find the slope of the tangent line of $y = \log_3 x$ at the point (2, 9).

$$y = \frac{\ln x}{\ln 3}$$

$$\frac{dy}{dx} = \frac{1}{x \ln 3}$$

$$\left.\frac{dy}{dx}\right|_{(2,9)} = \frac{1}{2 \ln 3}$$

$$= 0.455$$

Finding derivatives of functions involving $\log_a x$.

Example: find the derivative of

$$y = \log_2(4x + 1)^3 \qquad \text{let } u = (4x + 1)^3$$

$$y = \frac{\ln u}{\ln 2} \qquad \text{(use the chain rule)}$$

$$\frac{dy}{dx} = \frac{1}{u \ln 2}\frac{du}{dx}$$

$$= \frac{1}{(4x+1)^3 \ln 2} 3(4x+1)^2(4)$$

$$= \frac{12}{(4x+1)\ln 2}$$

The Derivative of a Composite Function involving $\log_a x$

If $y = \log_a f(x)$, $a > 0$

$$\frac{dy}{dx} = \frac{1}{f(x)\ln a} f'(x)$$

$$\boxed{\frac{dy}{dx} = \frac{f'(x)}{f(x)\ln a}}$$

Example:

find the derivative of

$y = \log_4 (2x-1)^5$

$$\frac{dy}{dx} = \frac{5(2x-1)^4(2)}{(2x-1)^5 \ln 4}$$

$$\boxed{\frac{dy}{dx} = \frac{10}{(2x-1)\ln 4}}$$

Summary

$$y = e^{f(x)}$$

$$y' = e^{f(x)} \cdot f'(x)$$

$$y = \ln f(x)$$

$$y' = \frac{1}{f(x)} \cdot f'(x)$$

$$y = a^{f(x)}$$

$$y' = a^{f(x)} In \cdot f'(x)$$

$$y = \log_a f(x)$$

$$y' = \frac{1}{f(x) In\, a} \cdot f'(x)$$