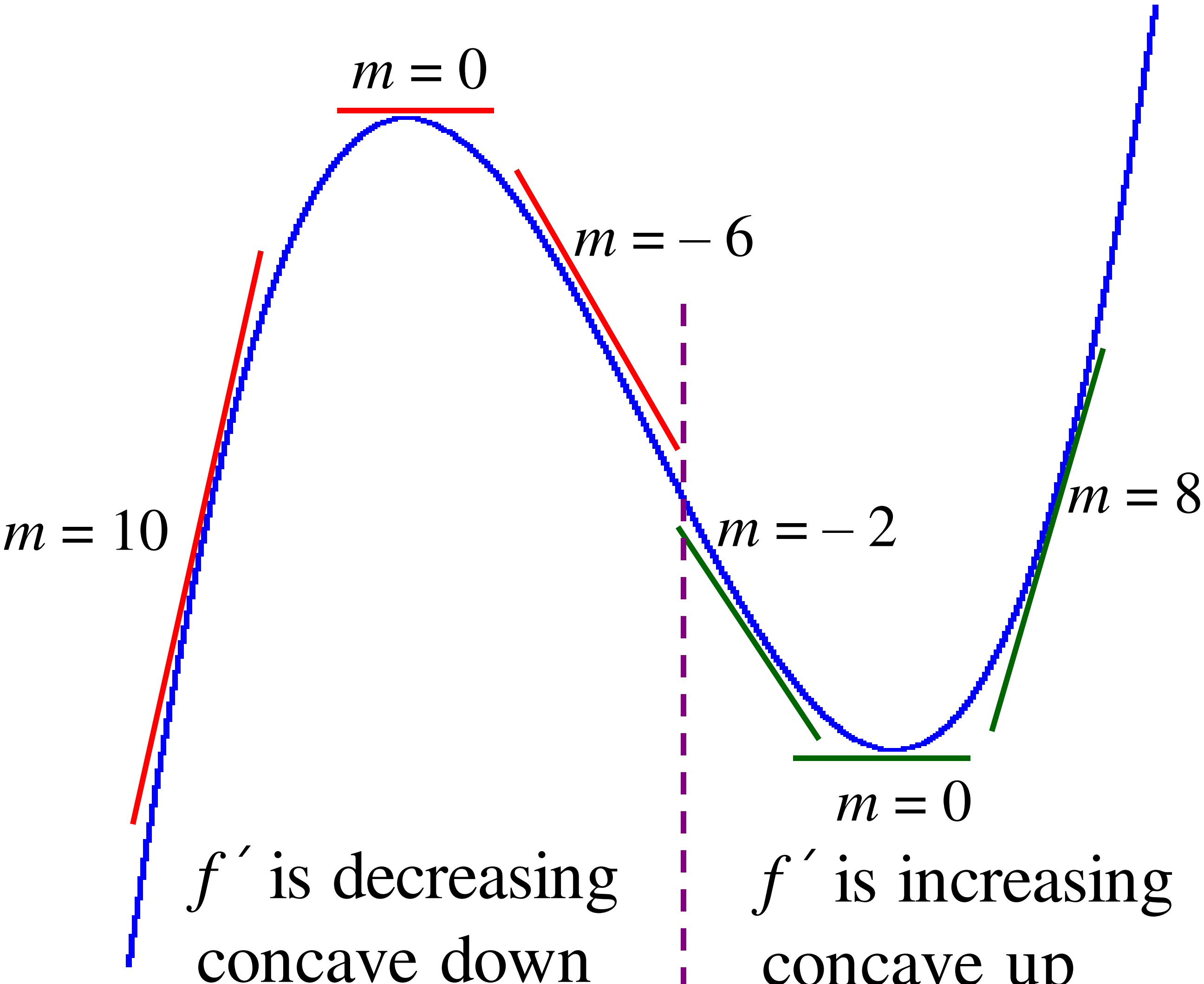
4.4 Concavity and Points of Inflection



Definition of Concavity

Let f be differentiable on an open interval l. The graph of f is concave up on l if f' is increasing on the interval. The graph of f is concave down on f if f' is decreasing.

The graph of f is <u>concave up</u>	Graph of f is <u>concave down</u>
f' is increasing	f' is decreasing
$f'' > 0$	$f'' < 0$

Test for Concavity

Let $f(x)$ be a differentiable function whose second derivative exists on an open interval I.

The graph of f is concave up if $f''(x) > 0$ for all x in I.

The graph of f is concave down if $f''(x) < 0$ for all x in I.

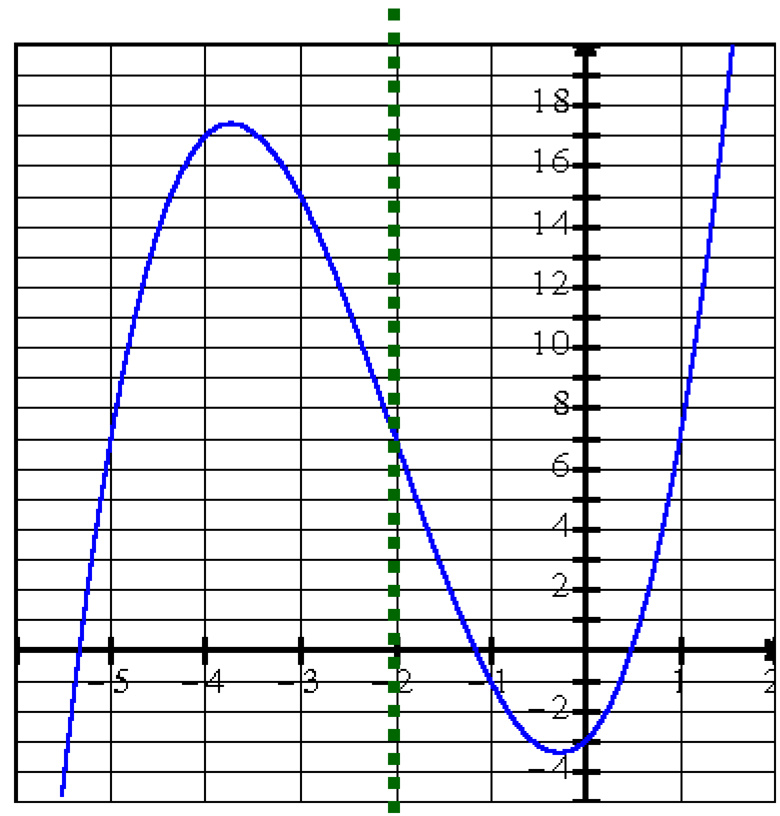
Example:

$$f(x) = x^3 + 6x^2 + 3x - 4$$

$$f'(x) = 3x^2 + 12x + 3$$

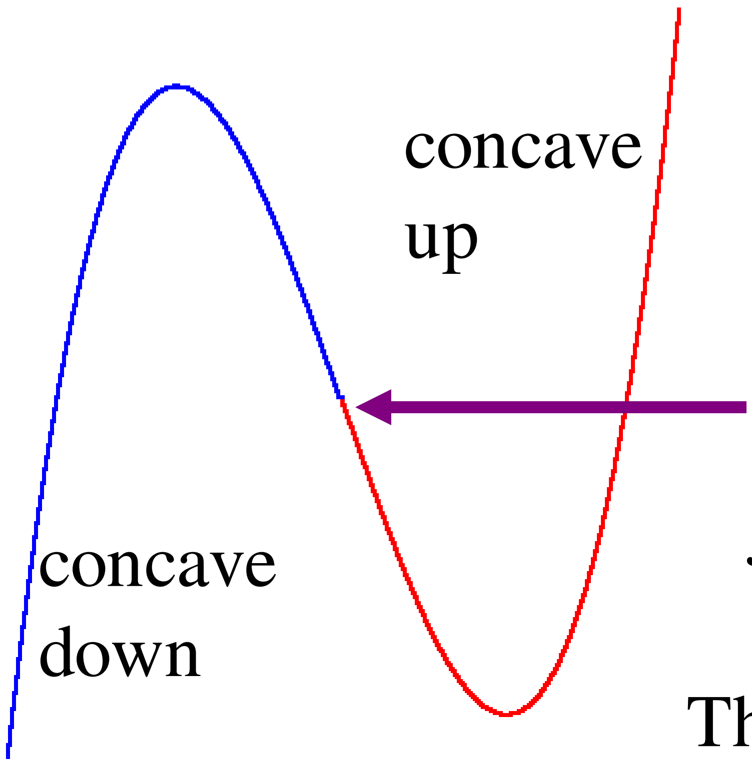
$$f''(x) = 6x + 12$$

concave down	concave up
$6x + 12 < 0$	$6x + 12 > 0$
$x < -2$	$x > -2$



Point of Inflection

The point where the concavity changes from concave up to concave down or vice versa.



point of inflection

$$f''(x) = 0$$

The fact that $f''(x) = 0$ does not necessarily mean that it is a point of inflection.

The Second Derivative Test

For $f(x)$ where $f'(x) = 0$ and the second derivative exists on an interval containing c:

if $f''(c) > 0$, then $f(c)$ is a local minimum value

if $f''(c) < 0$, then $f(c)$ is a local maximum value

if $f''(c) = 0$, then *the test fails*. Use the first derivative test.