Section 8.4 – Vector & Parametric Equations of a Plane

Vector Equation → $\vec{r} = \vec{r_0} + s\vec{a} + t\vec{b}$, $s,t \in \mathbb{R}$

$$(x,y,z) = (x_0,y_0,z_0) + s(a_1,a_2,a_3) + t(b_1,b_2,b_3)$$

Parametric Equations:

$$x = x_0 + sa_1 + tb_1$$
$$y = y_0 + sa_2 + tb_2$$
$$z = z_0 + sa_3 + tb_3$$

Things to remember:

- determining the equation of a plane requires 2 direction vectors.
- any pair of non-collinear vectors are coplanar. ∴ can use these as direction vectors.
- vector equation of a plane always requires 2 parameters, s and t. ∴ a plane is described as 2-dimensional.
- vector equation of a line, $\vec{r} = \vec{r_0} + t\vec{m}$ requires only one parameter ∴ a line is described as one-dimensional.
- not possible to derive a corresponding symmetric eqn. of a plane.

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ex1: Consider the plane with direction vectors $\vec{a}=(8,-5,4)$ & $\vec{b}=(1,-3,-2)$ through Po (3,7, 0).

a) Write vector and parametric equations of the plane.

b) Determine if the point Q(-10,8,-6) is on the plane.

c) Determine the point of intersection between π and the z-axis.

Solution:

a) $(x,y,z)=(3,7,0)+t(8,-5,4)+s(1,-3,-2)$

Parametric: $x=3+8t+s$

$y=7-5t-3s$

$z=0+4t-2s$

b) $(-10,8,-6)=(3,7,0)+t(8,-5,4)+s(1,-3,-2)$

① $-10=3+8t+s$ $\boxed{③\ -6=4t-2s}$

② $8=7-5t-3s$

① $-13=8t+s$

② $1=-5t-3s$

$\boxed{s=-8t-13}$

$1=-5t-3(-8t-13)$

$1=-5t+24t+39$

$-38=19t$

$\boxed{-2=t}$

$\therefore s=-8(-2)-13$

$s=16-13$

$\boxed{s=3}$

Sub $s=3$ & $t=-2$ into equation ③

$-6=4t-2s$

$-6=4(-2)-2(3)$

$-6=-8-6$

$-6\neq-14$

Since LS $\neq$ RS, the point Q(-10,8,-6) does not lie on the plane.

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c) $0 = 3 + 8t + s$

$0 = 7 - 5t - 3s$

$\times 3 \quad 0 = 9 + 24t + 3s$

$+ \quad 0 = 7 - 5t - 3s$

$0 = 16 + 19t$

$19t = -16$

$\boxed{t = \frac{-16}{19}}$

$\therefore 0 = 3 + 8\left(\frac{-16}{19}\right) + s$

$0 = 3 - \frac{128}{19} + s$

$0 = \frac{57}{19} - \frac{128}{19} + s$

$0 = -\frac{71}{19} + s$

$\boxed{s = \frac{71}{19}}$

$z = 0 + 4\left(\frac{-16}{19}\right) - 2\left(\frac{71}{19}\right)$

$z = \frac{-64}{19} - \frac{142}{19}$

$\boxed{z = -\frac{206}{19}}$

$\therefore$ POI is $\left(0, 0, \frac{-206}{19}\right)$

ex: Determine the vector and parametric equations of the plane containing the point $P(2,1,3)$ and the line $L: \vec{r} = (1,2,4) + s(-1,1,0),\ s \in \mathbb{R}$

Solution:

need another direction vector

$\vec{m} = (2-1, 1-2, 3-4)$

$\vec{m} = (1, -1, -1)$

$\therefore \vec{r} = (1,2,4) + s(-1,1,0) + t(1,-1,-1)$

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