

Analysis

April 8, 2018

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1 Preamble

```
In [1]: import pandas as pd
       import numpy as np
       import scipy.stats as stats
       import scipy.optimize as optim
       import seaborn as sb
       from matplotlib import pyplot as plt

       import statsmodels.api as sm

       import warnings
       warnings.filterwarnings("ignore")

%matplotlib inline

pd.set_option('display.max_columns', None)

plot_style = 'bmh'

plot_color_dict = {
    'data': '#348ABD',
    'fitted_pred': '#A60628',
    'loo_pred': '#E24A33',
    'kde': '#7A68A6',
}

measure_label_dict = {
    'total_root_length':'$RL$',
    'total_root_surface_area':'$RSA$',
    'total_root_volume':'$RV$',
    'maximal_rooting_depth':'$z_{max}$',
    'maximal_horizontal_spread':'$r_{max}$',
    'convex_hull_volume':'$conv$',
    'number_of_root_branches':'$NB$',
    'rhizosphere_volume_phosphate':'$V_{rhizo}$',
    'root_tip_density':'$RND$',
    'total_root_length_per_convex_hull_volume':'$RLD$',
    'total_root_surface_area_per_convex_hull_volume':'$RSAD$',
    'total_root_volume_per_convex_hull_volume':'$RVD$',
    'rhizosphere_volume_phosphate_per_convex_hull_volume':'$VD_{rhizo}$',
    'root_system_total_conductance':'$K_{rs}$',
    'half_mean_distance_between_roots':'$HMD$',
    'root_system_conductance_per_unit_of_root_area':'$K_{rs,A}$',
    'root_system_conductance_per_unit_of_root_length':'$K_{rs,L}$',
    'mean_depth_of_standard_root_water_uptake':'$z_{SUF}$',
}
```

```

input_param_label_dict = {
    'N': '$N$',
    'ln': '$l_n$',
    'maxB': '$max_B$',
    'r': '$r$',
    'sigma': r'$\sigma$',
    'theta': r'$\theta$',
}

```

1.1 Import data

In [2]: `root_data = pd.read_hdf('root_data.h5', '/data')`

In [3]: `root_data.describe()`

```

Out[3]:      variable_value  maximal_rooting_depth  maximal_horizontal_spread \
count      53000.000000          53000.000000          53000.000000
mean       5.073742           96.360813          140.675772
std        9.593408          18.432926          40.466511
min        0.011111          17.556674          31.591177
25%        0.785398          89.052064          113.039483
50%        1.333333          101.731758          141.611663
75%        2.333333          109.150982          170.431956
max        40.000000          133.102523          246.381642

      convex_hull_volume  total_root_length  total_root_surface_area \
count      5.300000e+04          53000.000000          53000.000000
mean       5.068756e+05         12327.403791         4376.284805
std        3.113330e+05         13000.233771         4556.386593
min        2.037828e+04         928.977099          386.926864
25%        2.902084e+05         6480.670294         2310.025457
50%        4.430118e+05         8751.795049         3082.694361
75%        6.386111e+05         9183.583233         3232.672234
max        1.787740e+06         63098.115898        22217.951110

      total_root_volume  number_of_root_branches \
count      53000.000000          53000.000000
mean       143.558379          1555.515453
std        146.169334          1648.684200
min        16.361829          139.000000
25%        77.564691          828.000000
50%        99.248805          1110.000000
75%        105.408625          1159.000000
max        724.107142          7956.000000

      rhizosphere_volume_phosphate  root_system_total_conductance \
count      53000.000000          53000.000000

```

mean	2424.364206	0.040605
std	2544.861981	0.009128
min	194.122440	0.008537
25%	1276.985390	0.036379
50%	1716.621970	0.044755
75%	1800.116067	0.046622
max	12372.691625	0.053383
mean_depth_of_standard_root_water_uptake \		
count	53000.000000	53000.000000
mean	-34.478992	0.003286
std	11.486981	0.002252
min	-62.782556	0.000345
25%	-43.842738	0.001821
50%	-33.646199	0.002680
75%	-26.704811	0.004176
max	-2.781809	0.024725
total_root_length_per_convex_hull_volume \		
count	53000.000000	
mean	0.025915	
std	0.017492	
min	0.003346	
25%	0.014535	
50%	0.021064	
75%	0.032514	
max	0.197229	
total_root_surface_area_per_convex_hull_volume \		
count	53000.000000	
mean	0.009240	
std	0.006119	
min	0.001452	
25%	0.005205	
50%	0.007459	
75%	0.011538	
max	0.068808	
total_root_volume_per_convex_hull_volume \		
count	53000.000000	
mean	0.000305	
std	0.000196	
min	0.000061	
25%	0.000175	
50%	0.000245	
75%	0.000377	
max	0.002165	

```

        rhizosphere_volume_phosphate_per_convex_hull_volume \
count                      53000.000000
mean                       0.005105
std                        0.003421
min                        0.000713
25%                         0.002865
50%                         0.004137
75%                         0.006399
max                        0.038546

        half_mean_distance_between_roots \
count                      53000.000000
mean                       4.013829
std                        1.208540
min                        1.270399
25%                         3.128878
50%                         3.887375
75%                         4.679627
max                        9.754158

        root_system_conductance_per_unit_of_root_area \
count                      53000.000000
mean                       0.000013
std                        0.000004
min                        0.000002
25%                         0.000014
50%                         0.000015
75%                         0.000016
max                        0.000029

        root_system_conductance_per_unit_of_root_length
count                      5.300000e+04
mean                       4.908577e-06
std                        1.671638e-06
min                        7.325552e-07
25%                         4.976969e-06
50%                         5.187365e-06
75%                         5.695776e-06
max                        1.127802e-05

```

```

In [4]: root_means = root_data \
    .groupby(['variable_name', 'variable_value']) \
    .mean() \
    .dropna() \
    .reset_index()
root_covariances = root_data \
    .groupby(['variable_name', 'variable_value']) \
    .cov() \

```

```
.unstack() \  
.reset_index()
```

1.2 Code for probability distributions

```
In [5]: param_dists = {  
    'normal': stats.norm,  
    'inverse-gamma': stats.invgamma,  
}  
  
def plot_dist(x_name, dist_name='normal'):   
    '''Plot KDE, histogram and fitted distribution'''  
    dist = param_dists[dist_name]  
    var_names = np.unique(root_data.variable_name)  
  
    for var_name in var_names:  
        var_values = np.unique(  
            root_data.variable_value.loc[root_data.variable_name == var_name])  
        for var_value in var_values:  
            plot_data = root_data.loc[  
                (root_data.variable_name == var_name) &  
                (root_data.variable_value == var_value),  
                x_name]  
  
            dist_params = dist.fit(plot_data)  
  
            print(f"variable name: {str(var_name)}")  
            print(f"variable value: {var_value}")  
            print(f"distribution: {dist_name}(shape={dist_params[:-2]}, " +  
                  f"loc={dist_params[-2]}, scale={dist_params[-1]})")  
            print(stats.describe(plot_data))  
  
            with plt.style.context(plot_style):  
                fig, ax = plt.subplots()  
                sb.distplot(plot_data, kde=True, fit=dist, ax=ax,  
                            hist_kws={  
                                'color': plot_color_dict['data'],  
                                'label': 'histogram'  
                            },  
                            kde_kws={  
                                'color': plot_color_dict['kde'],  
                                'label': 'KDE'  
                            },  
                            fit_kws={  
                                'color': plot_color_dict['fitted_pred'],  
                                'label': 'fitted PDF'  
                            })  
                ax.set_xlabel(measure_label_dict[x_name])
```

```

        ax.legend()
        fig.tight_layout()
        plt.show(fig)
        plt.close(fig)

        fig, ax = plt.subplots()
        probplot = sm.ProbPlot(plot_data, dist, fit=True)
        probplot.qqplot(ax=ax, color=plot_color_dict['data'])
        sm.qqline(line='45', ax=ax, fmt=plot_color_dict['fitted_pred'])
        fig.tight_layout()
        plt.show(fig)
        plt.close(fig)

```

1.3 Code for parameter regression

```

In [6]: from sklearn.linear_model import LinearRegression
        from sklearn.pipeline import Pipeline
        from sklearn.preprocessing import PolynomialFeatures

        regression_poly1 = Pipeline([
            ('ols', LinearRegression())
        ])
        regression_poly2 = Pipeline([
            ('poly', PolynomialFeatures(degree=2, include_bias=False)),
            ('ols', LinearRegression())
        ])

        reg_models = {
            'poly1': regression_poly1,
            'poly2': regression_poly2,
        }

def eval_reg(data, var_name, y_name, model_name, y_label=''):
    '''Plot parameter regression models'''
    from sklearn.model_selection import LeaveOneOut, cross_val_predict
    from sklearn.metrics import r2_score, explained_variance_score

    reg_data = data.loc[data.variable_name == var_name]

    X = reg_data.loc[:, ['variable_value']]
    y = reg_data.loc[:, [y_name]]
    model = reg_models[model_name]

    y_loo_pred = cross_val_predict(reg_models[model_name], X, y, cv=LeaveOneOut())
    y_fitted_pred = model.fit(X,y).predict(X)

```

```

print(f"variable name: {var_name}")
print(f"model: {model_name}")
print(f"r-squared: {r2_score(y, y_loo_pred)}")
print(f"explained variance: {explained_variance_score(y, y_loo_pred)}")

with plt.style.context(plot_style):
    X_plot = X.iloc[:,0]
    fig, ax = plt.subplots()
    ax.plot(X, y, color=plot_color_dict['data'],
             linestyle=' ', marker='o', label="data")
    ax.plot(X, y_fitted_pred, color=plot_color_dict['fitted_pred'],
             linestyle='-', marker='|', label="fully fitted OLS")
    ax.legend(loc='best')
    ax.set_xlabel(input_param_label_dict[var_name])
    ax.set_ylabel(y_label)
    plt.show(fig)
    plt.close(fig)

def mean_reg(x_name, y_name, model_name):
    y_label = r'$\overline{{{:s}}}$'.format(measure_label_dict[y_name].strip('$'))
    eval_reg(root_means, x_name, y_name, model_name, y_label)

```

1.4 Code for correlations

```

In [7]: def compute_correlation(x_name: str, y_name: str):
    var_x = np.sqrt(root_covariances.loc[:, (x_name, x_name)])
    var_y = np.sqrt(root_covariances.loc[:, (y_name, y_name)])
    corr = root_covariances.loc[:, (x_name, y_name)] / (var_x * var_y)

    col_name = f"{x_name}X{y_name}"
    data = pd.DataFrame({
        'variable_name': root_covariances.variable_name,
        'variable_value': root_covariances.variable_value,
        col_name: corr,
    })

    return col_name, data

def corr_reg(x_name, y1_name, y2_name, model_name):
    var_name, corr_data = compute_correlation(y1_name, y2_name)
    y_label = r'$\bar{{r}}({{:s}}, {{{:s}}})$'.format(measure_label_dict[y1_name].strip('$'),
                                                    measure_label_dict[y2_name].strip('$'))
    eval_reg(corr_data, x_name, var_name, model_name, y_label)

```

```

In [8]: def plot_corr_matrix():
    var_names = np.unique(root_data.variable_name)
    for var_name in var_names:
        var_values = np.unique(

```

```

root_data.variable_value.loc[root_data.variable_name == var_name])
for var_value in var_values:

    print(f"variable name: {str(var_name)}")
    print(f"variable value: {var_value}")

plot_data = root_data.loc[(root_data.variable_name == var_name) &
                           (root_data.variable_value == var_value)]
plot_data = plot_data.drop(['variable_name', 'variable_value'], axis=1)
plot_data.rename(columns=measure_label_dict,
                  index=measure_label_dict, inplace=True)
corr_data = plot_data.corr()

mask = np.zeros_like(corr_data, dtype=np.bool)
mask[np.triu_indices_from(mask)] = True

cmap = sb.diverging_palette(220, 10, as_cmap=True)
with sb.axes_style("white"):
    fig, ax = plt.subplots()
    sb.heatmap(corr_data, cmap=cmap, mask=mask, center=0, square=True,
                linewidths=.5, cbar_kws={"shrink": .5},
                vmin=-1, vmax=1, ax=ax)
    plt.show(fig)
    plt.close(fig)

```

2 Maximal rooting depth

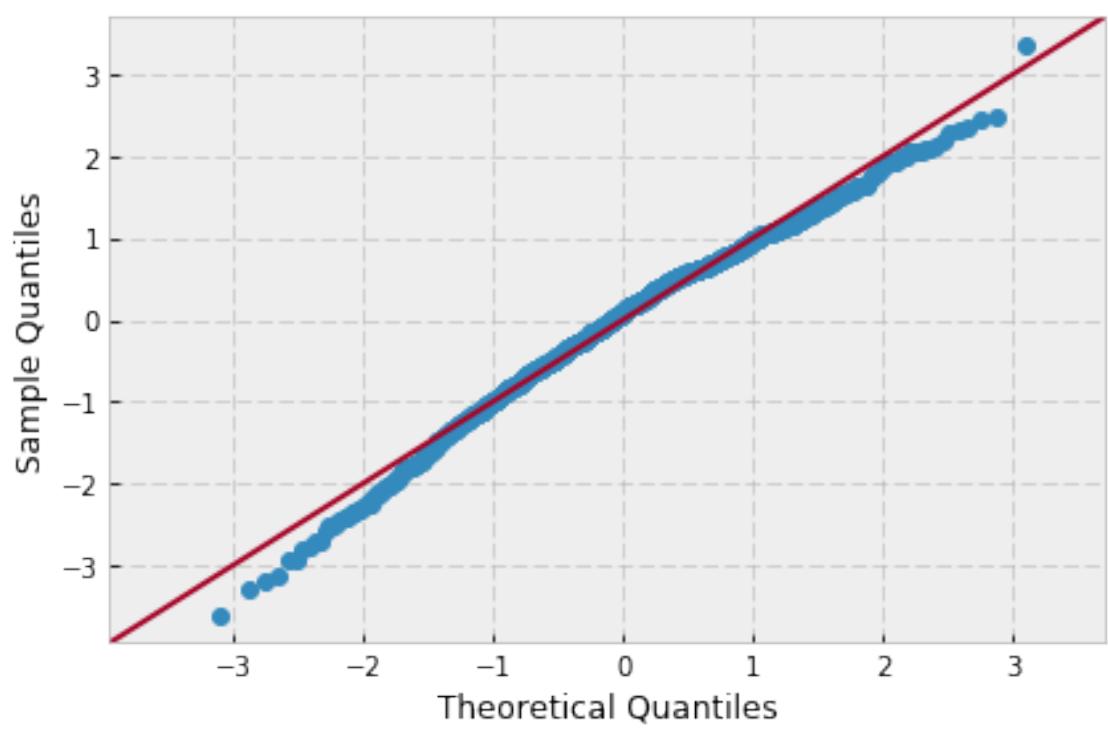
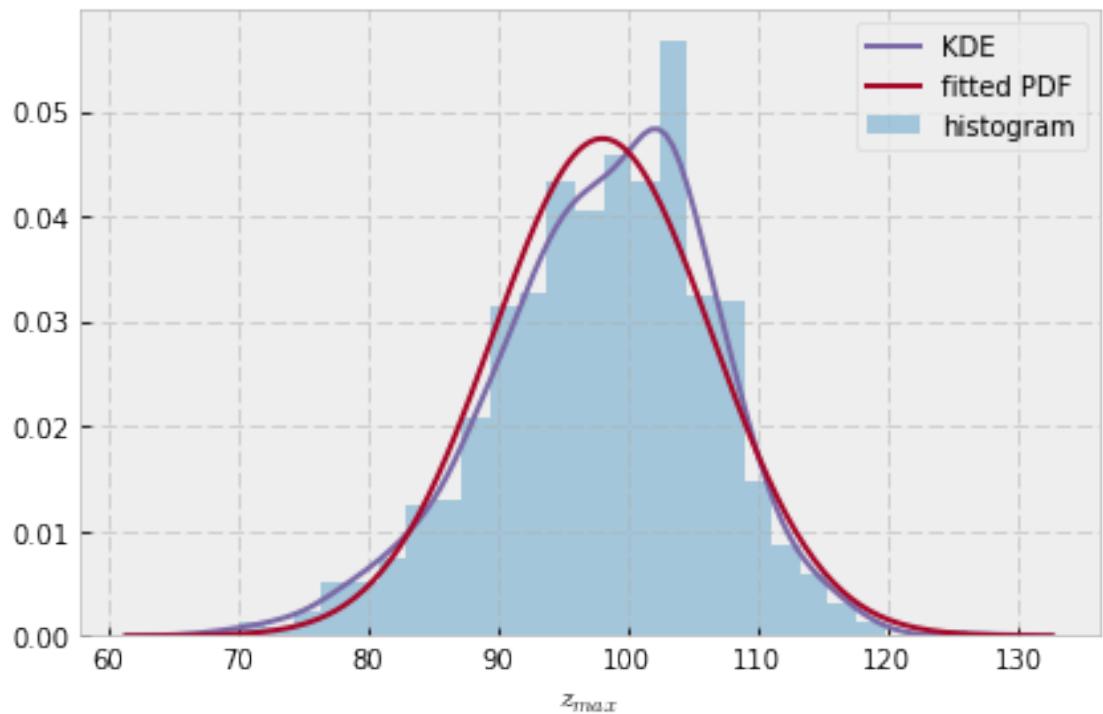
2.1 Probability distributions

In [9]: `plot_dist('maximal_rooting_depth')`

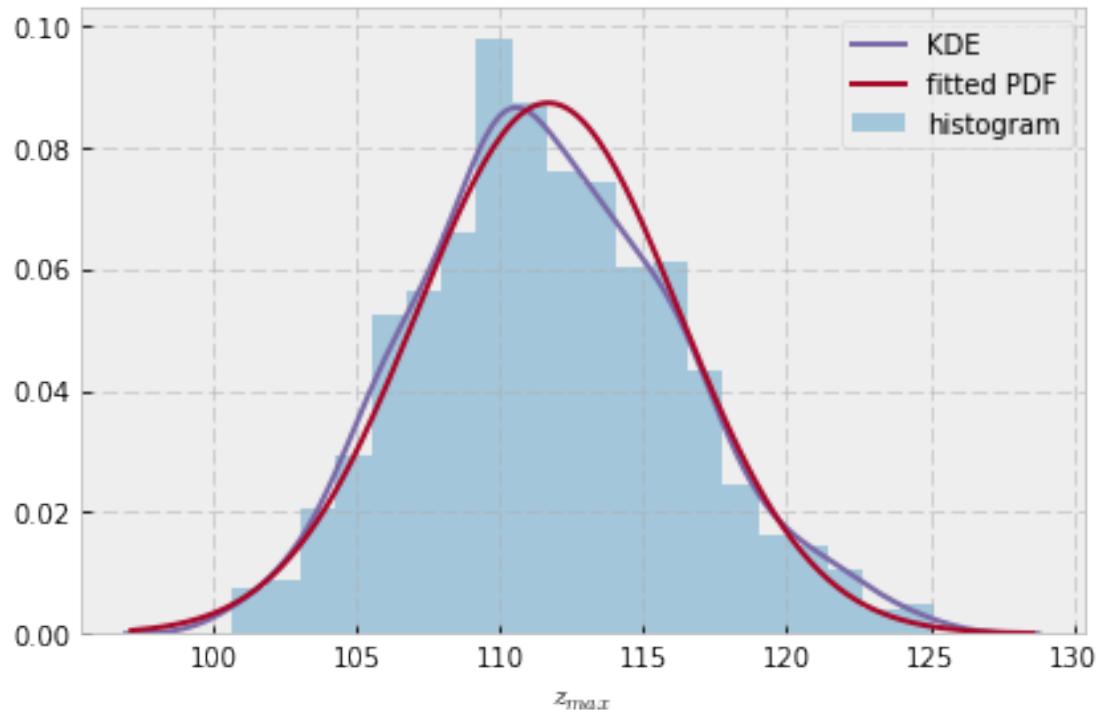
```

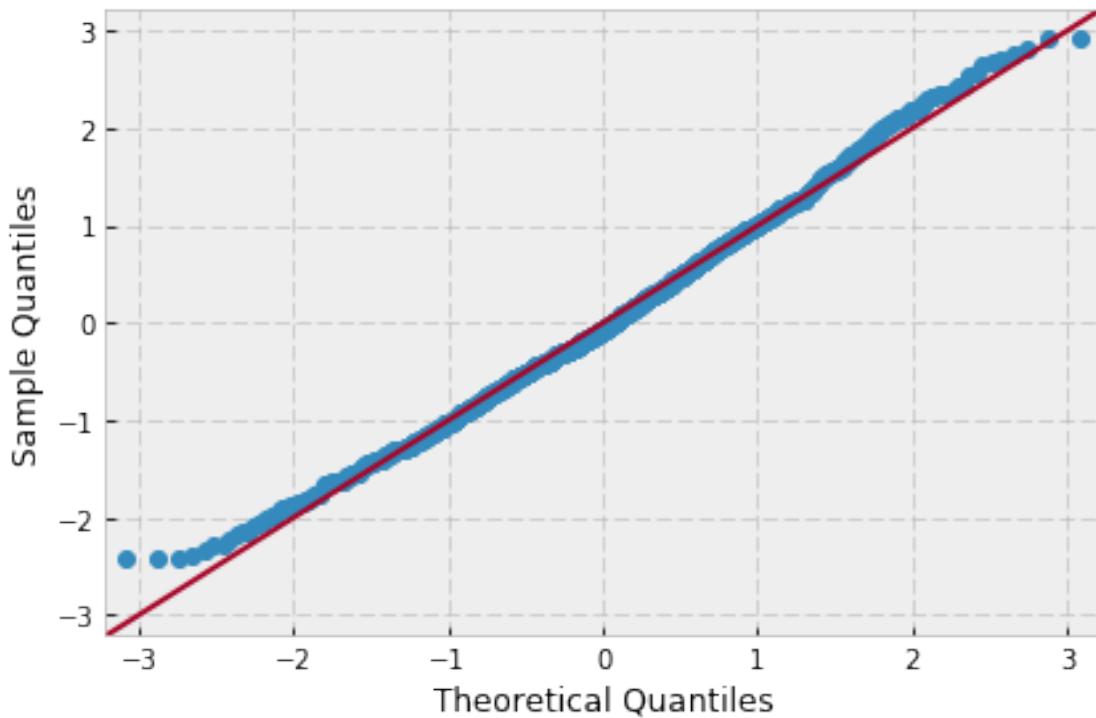
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=98.06661357456376, scale=8.41378708881183)
DescribeResult(nobs=1000, minmax=(67.811421795526883, 126.24922860625531)
               mean=98.06661357456376, variance=70.862675851708374,
               skewness=-0.4037788075684776, kurtosis=0.24472839692745607)

```

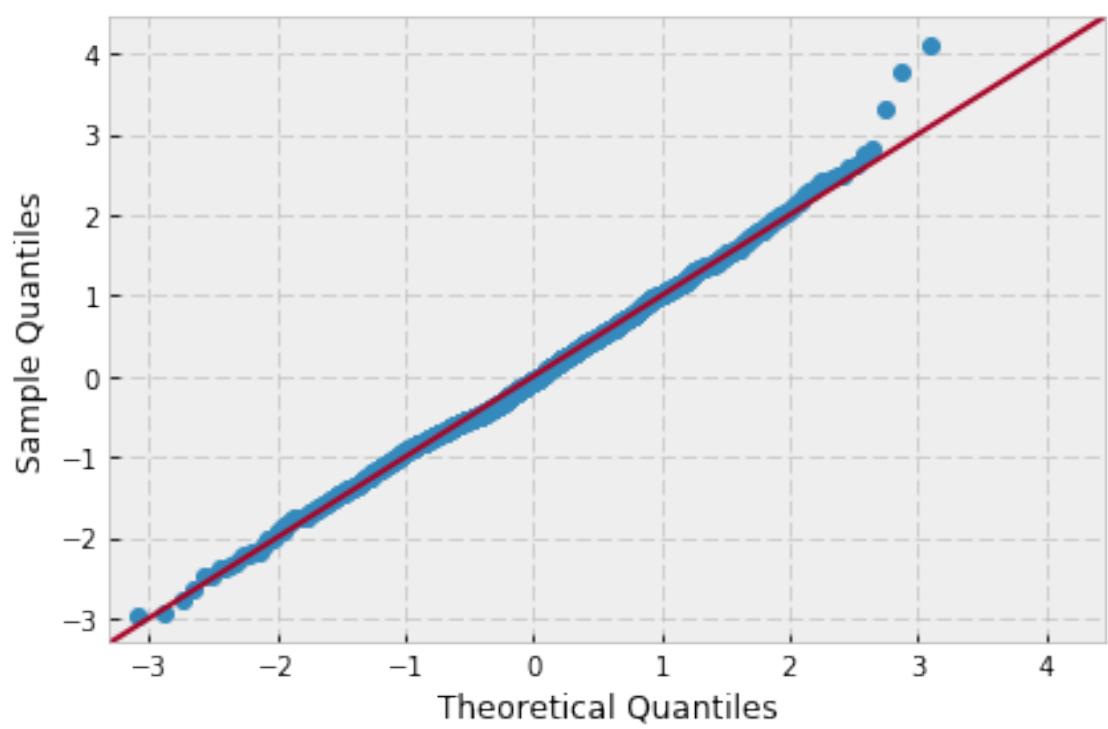
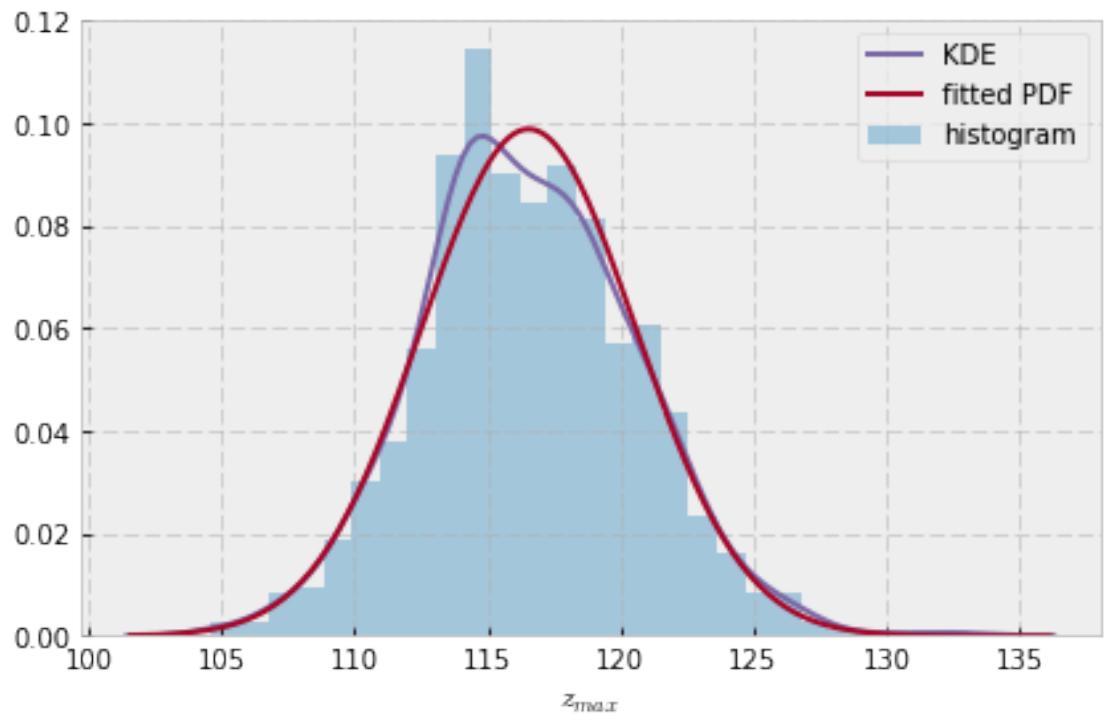


```
variable name: N
variable value: 1.0
distribution: normal(shape=(),loc=111.6824343109389, scale=4.5735361508299635)
DescribeResult(nobs=1000, minmax=(100.61035374317909, 125.11799583794614)
               mean=111.6824343109389, variance=20.938171094042605,
               skewness=0.23179243643468922, kurtosis=-0.1766191864831259)
```

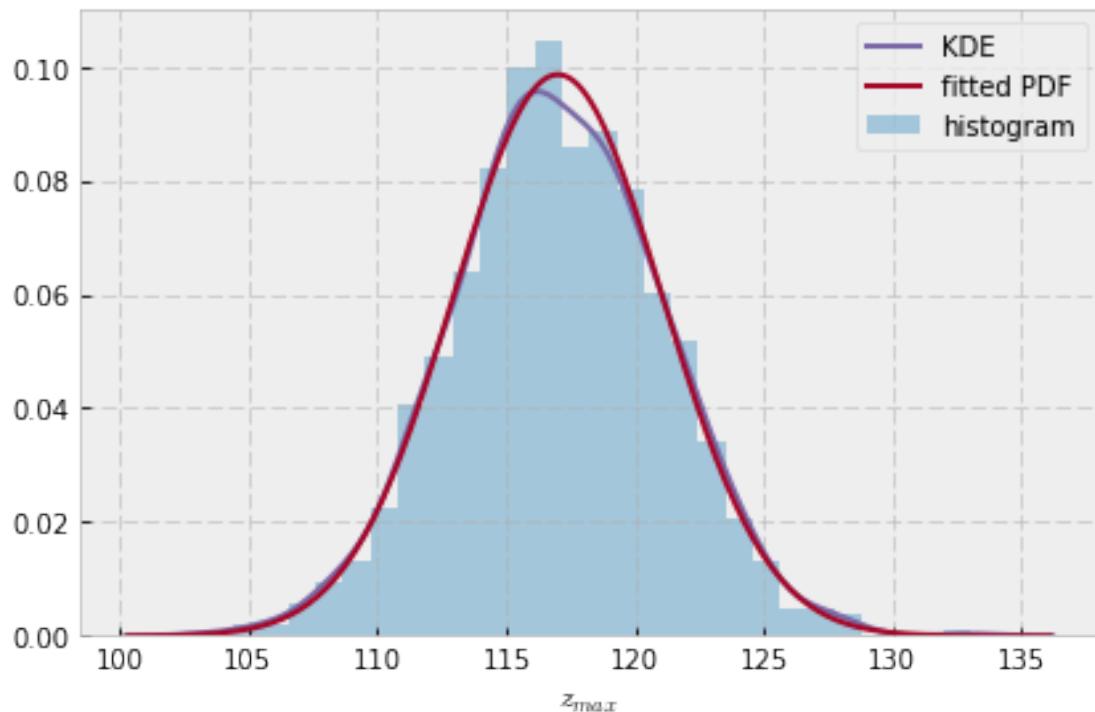


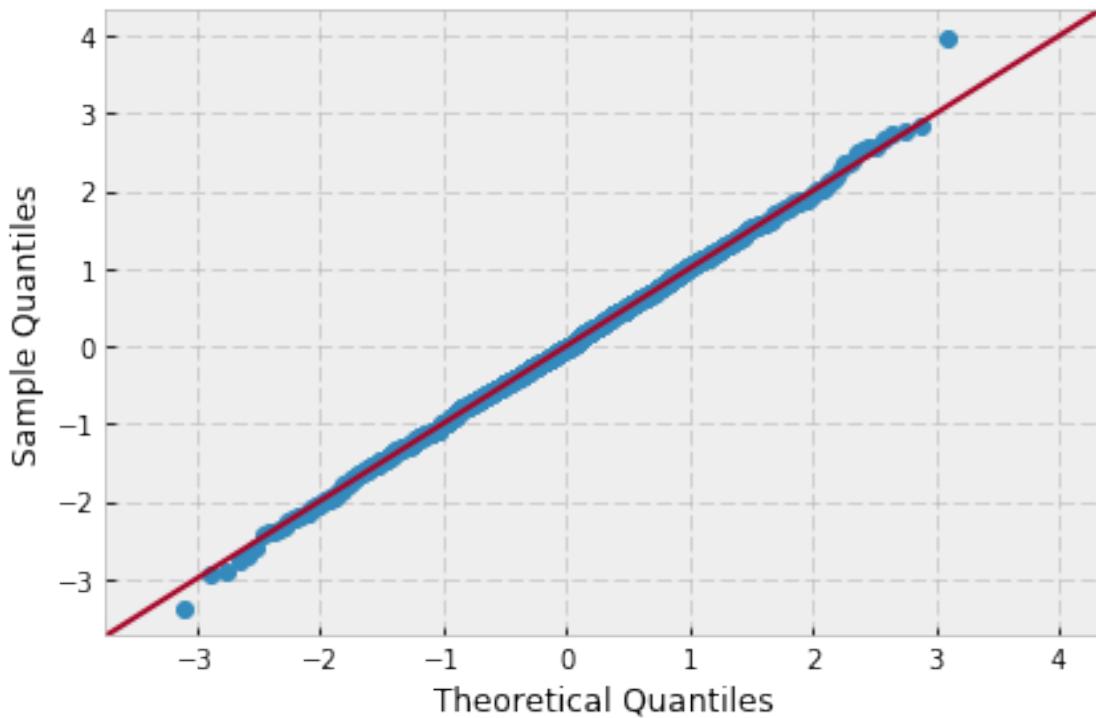


```
variable name: N
variable value: 1.5
distribution: normal(shape=(),loc=116.54189435753531, scale=4.034838584000744)
DescribeResult(nobs=1000, minmax=(104.61675284833174, 133.1025230840801)
               mean=116.54189435753531, variance=16.296218617558694,
               skewness=0.1977161007700751, kurtosis=0.26571479848197077)
```

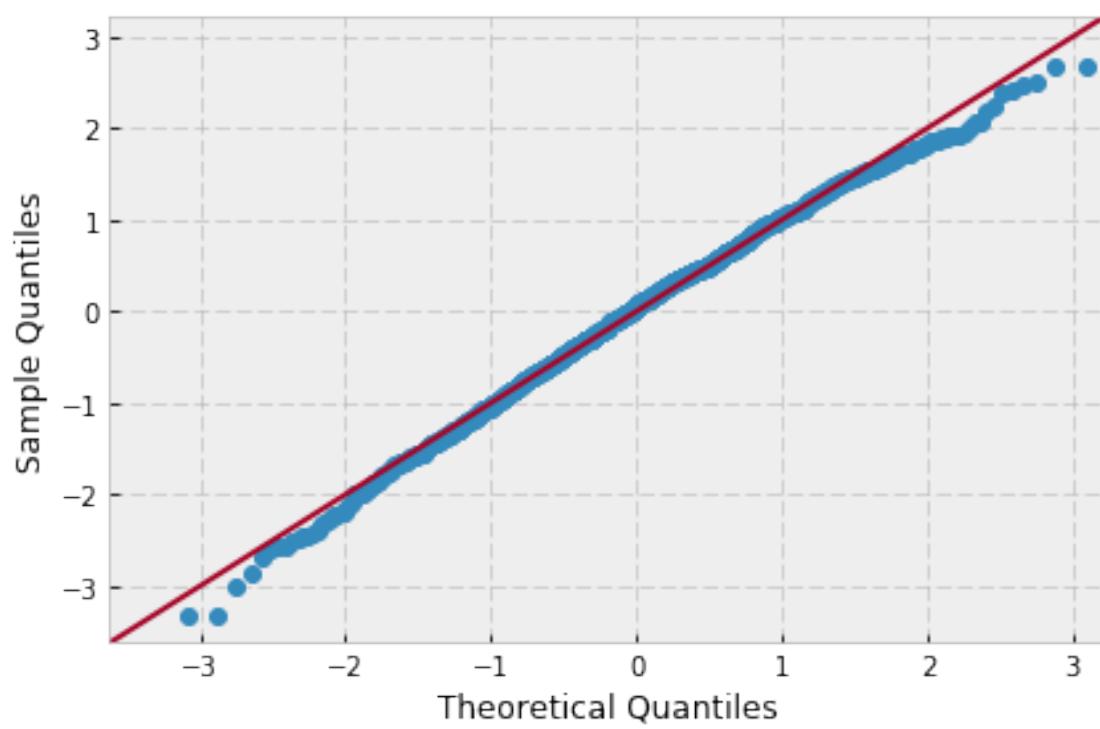
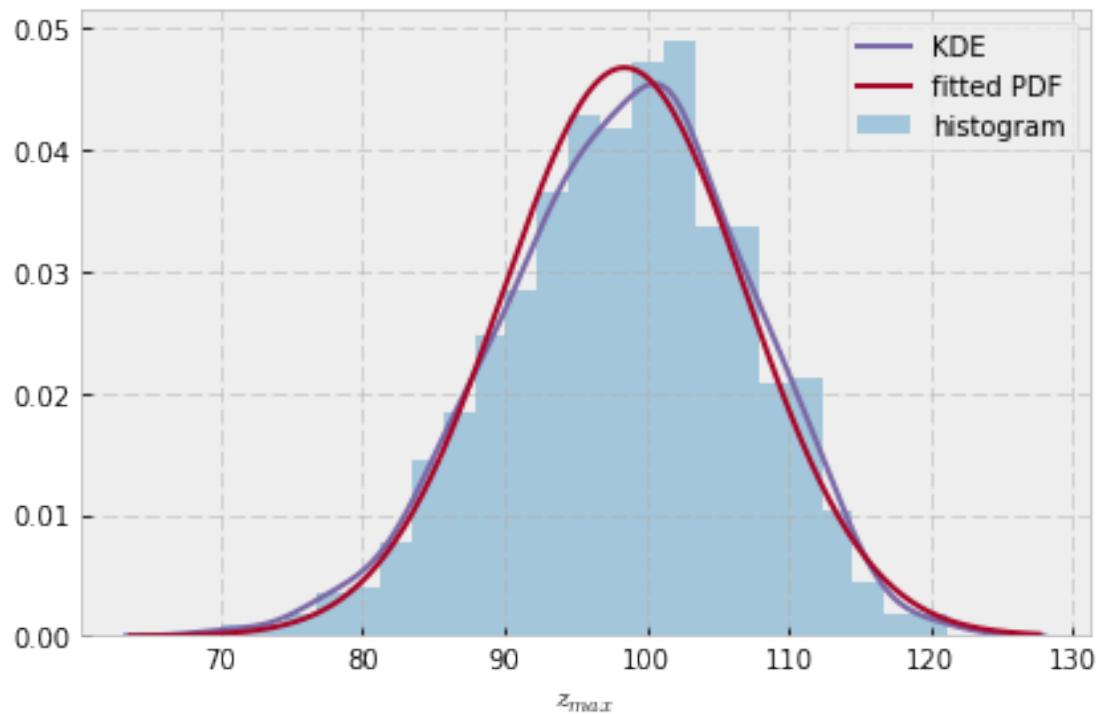


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=117.00625048326003, scale=4.045286398861881)
DescribeResult(nobs=1000, minmax=(103.37558044223572, 133.05445251997489)
               mean=117.00625048326003, variance=16.380722771588516,
               skewness=0.025668814080644642, kurtosis=0.1152173615807679)
```

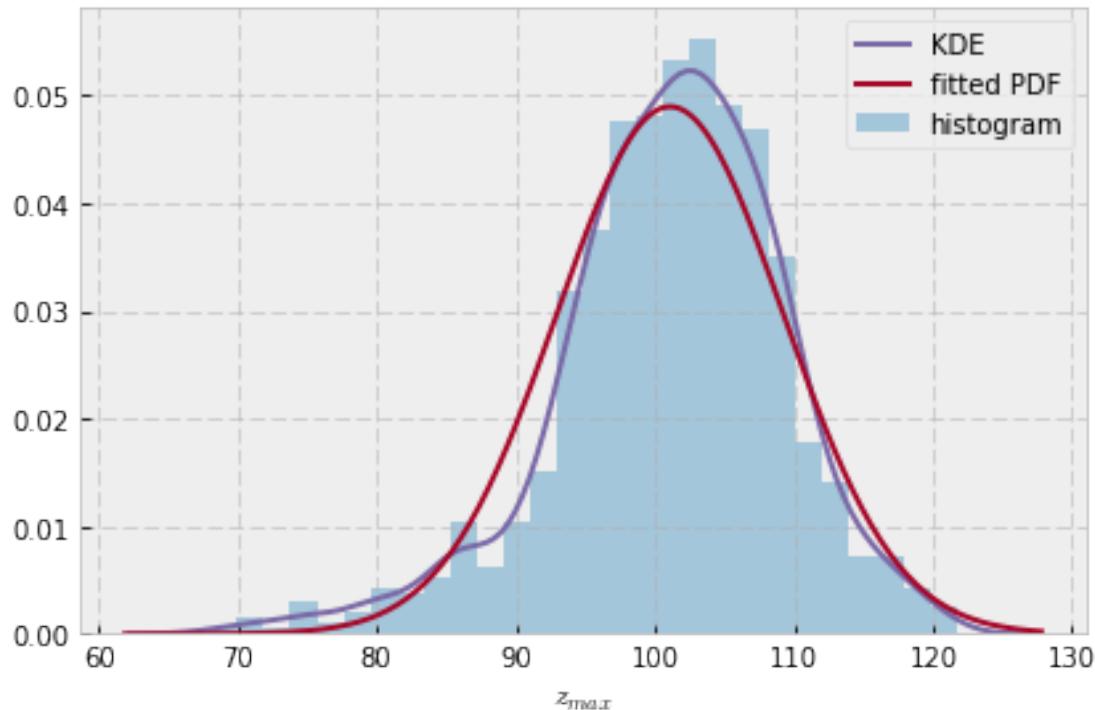


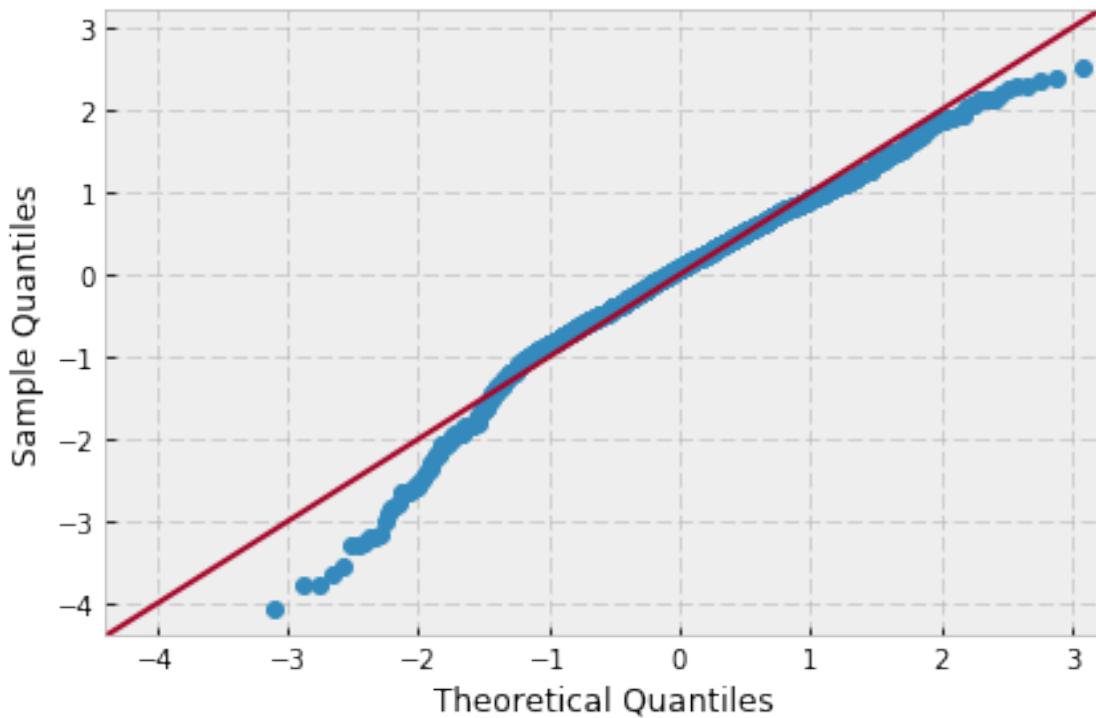


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(),loc=98.4236915370448, scale=8.528023124097365)
DescribeResult(nobs=1000, minmax=(70.09408222777455, 121.18070579475602)
               mean=98.423691537044803, variance=72.799978383522884,
               skewness=-0.2298389624751914, kurtosis=-0.12429895900797483)
```

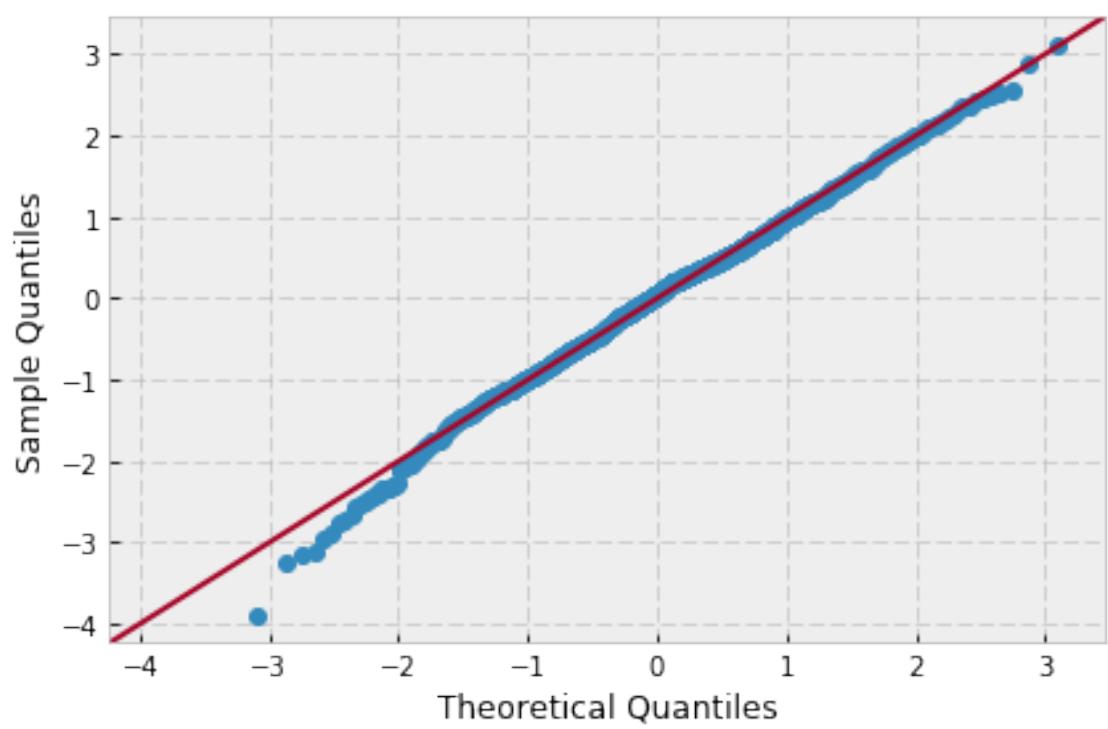
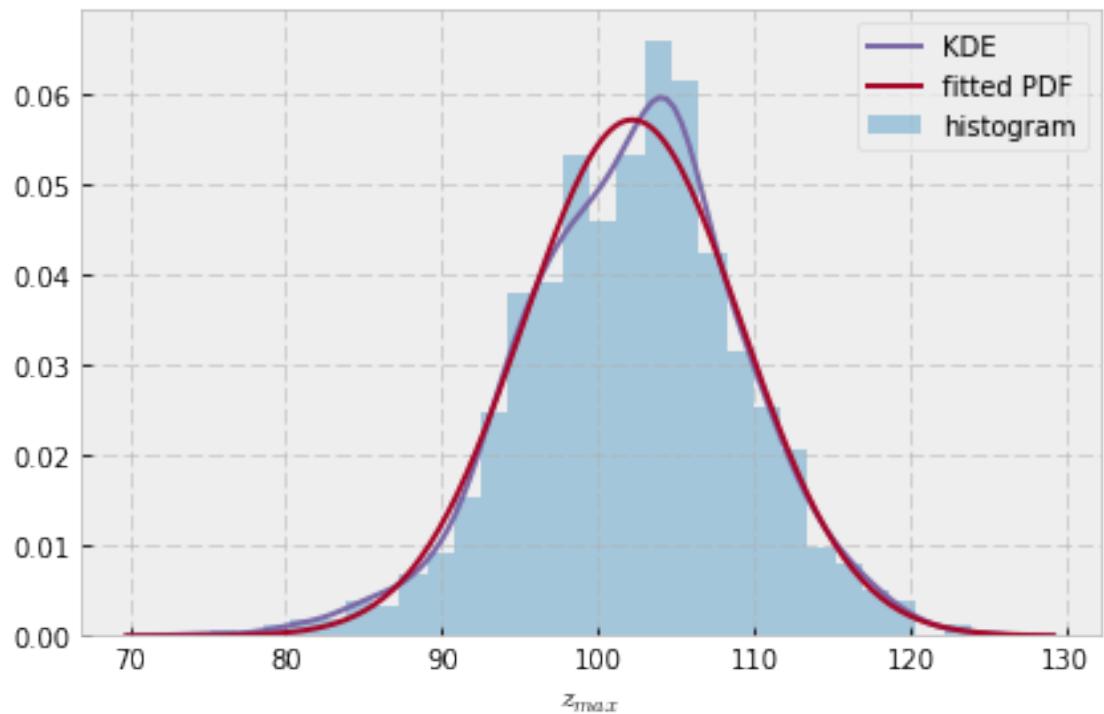


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=101.08709539269374, scale=8.145232724298712)
DescribeResult(nobs=1000, minmax=(68.039121574816008, 121.64257942547901)
               mean=101.08709539269374, variance=66.411227360346956,
               skewness=-0.6620871704156552, kurtosis=1.161046370735698)
```

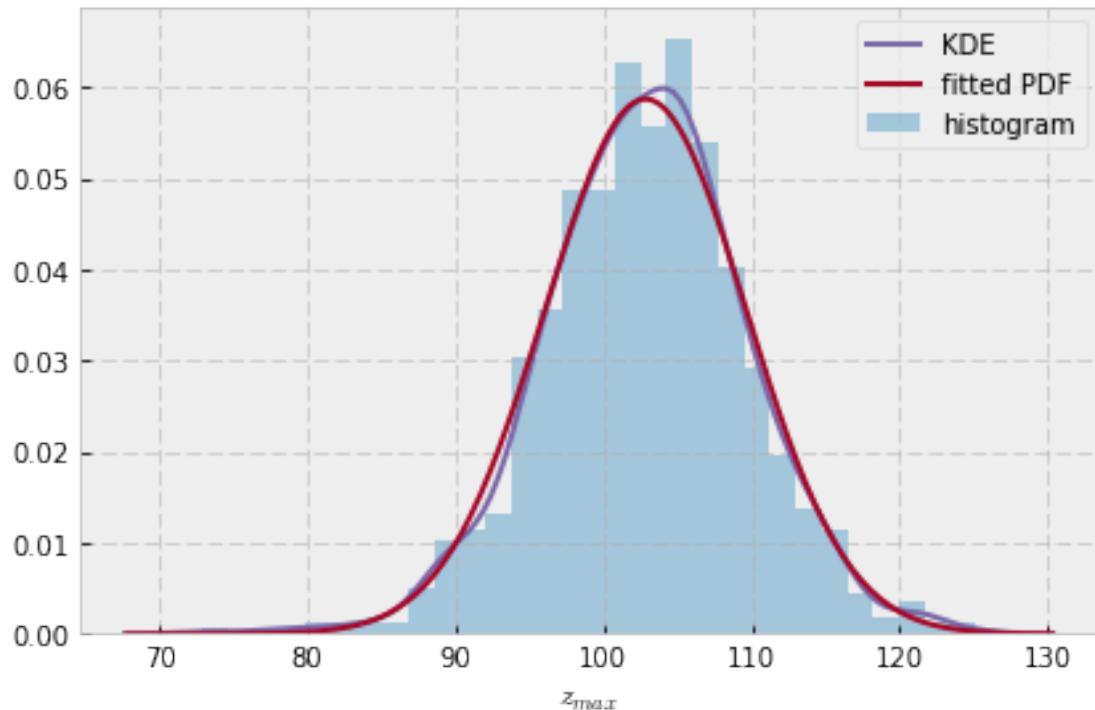


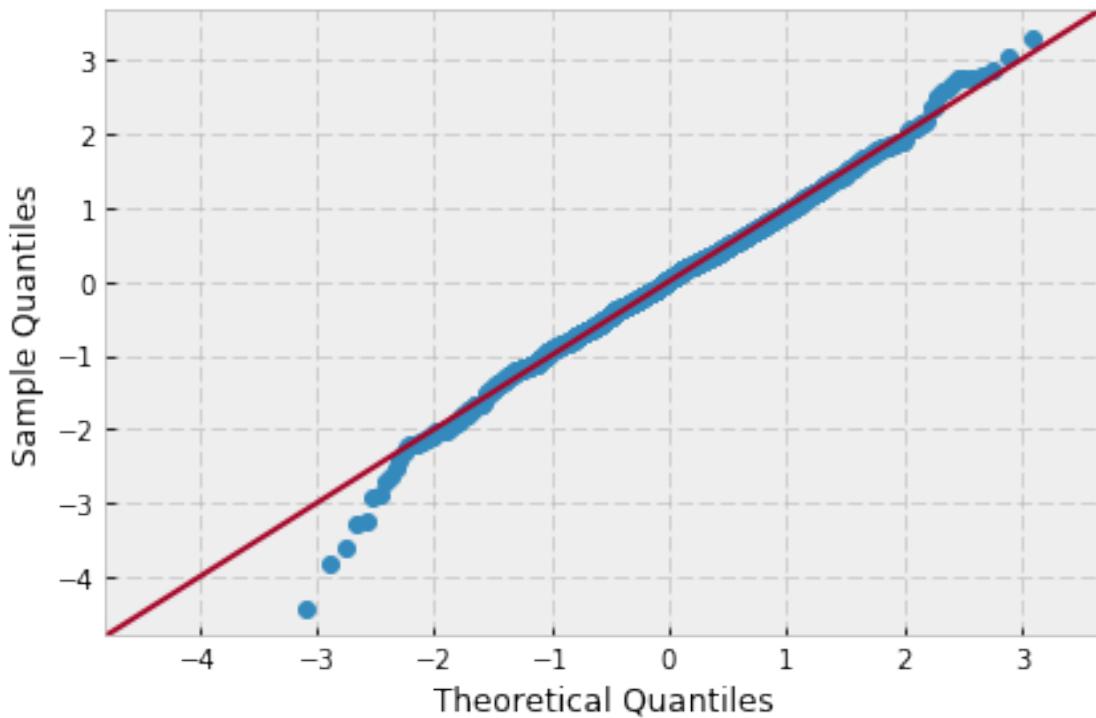


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=102.2083509387657, scale=6.982024993765703)
DescribeResult(nobs=1000, minmax=(75.100300909797767, 123.89422842115083)
               mean=102.2083509387657, variance=48.797470484053008,
               skewness=-0.18042755603725819, kurtosis=0.3049628948890639)
```

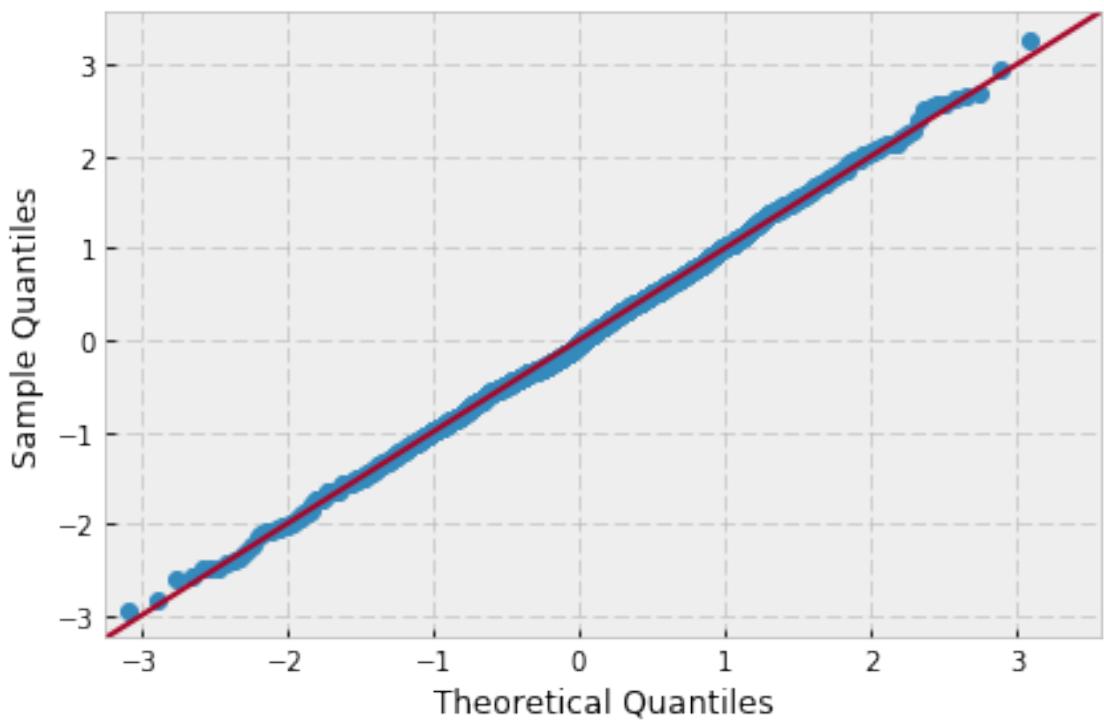
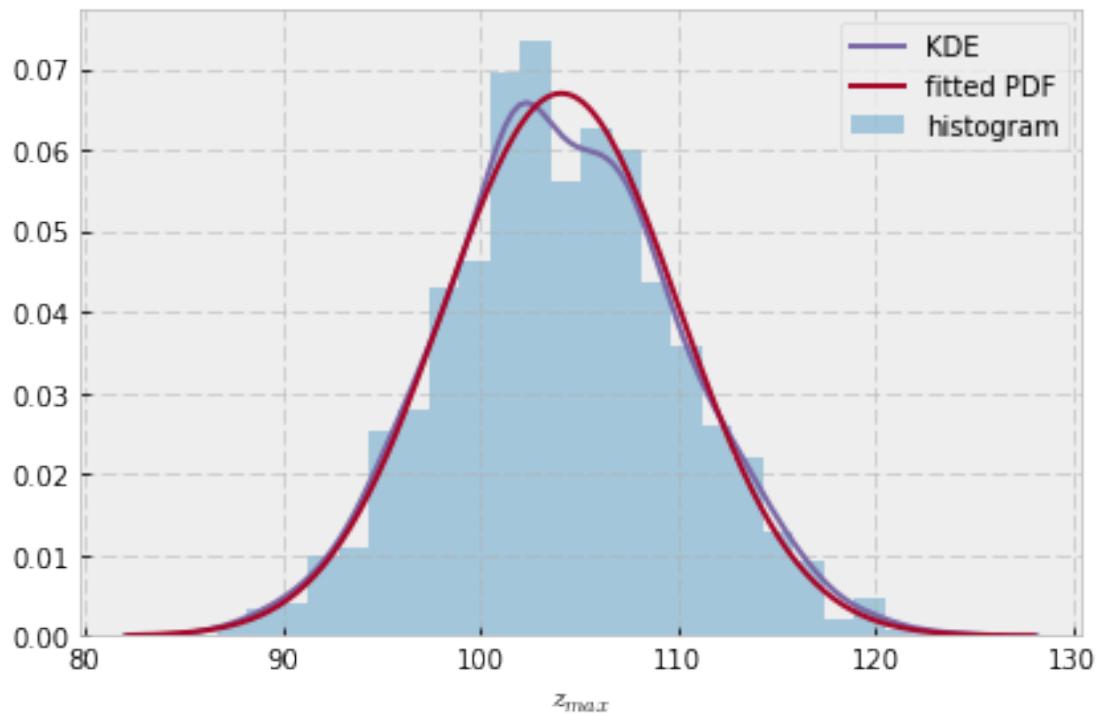


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=102.8204842914613, scale=6.798791620660571)
DescribeResult(nobs=1000, minmax=(72.858706874798344, 125.15416026039316)
               mean=102.8204842914613, variance=46.269837338502903,
               skewness=-0.15998265665302436, kurtosis=0.7517261444040466)
```

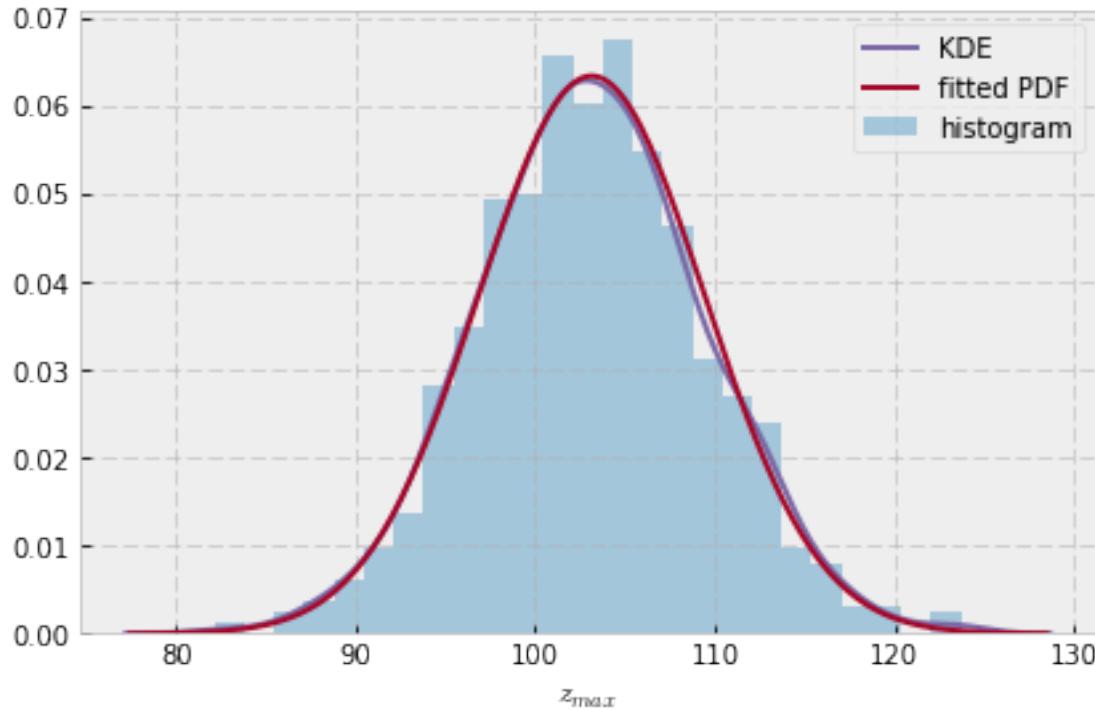


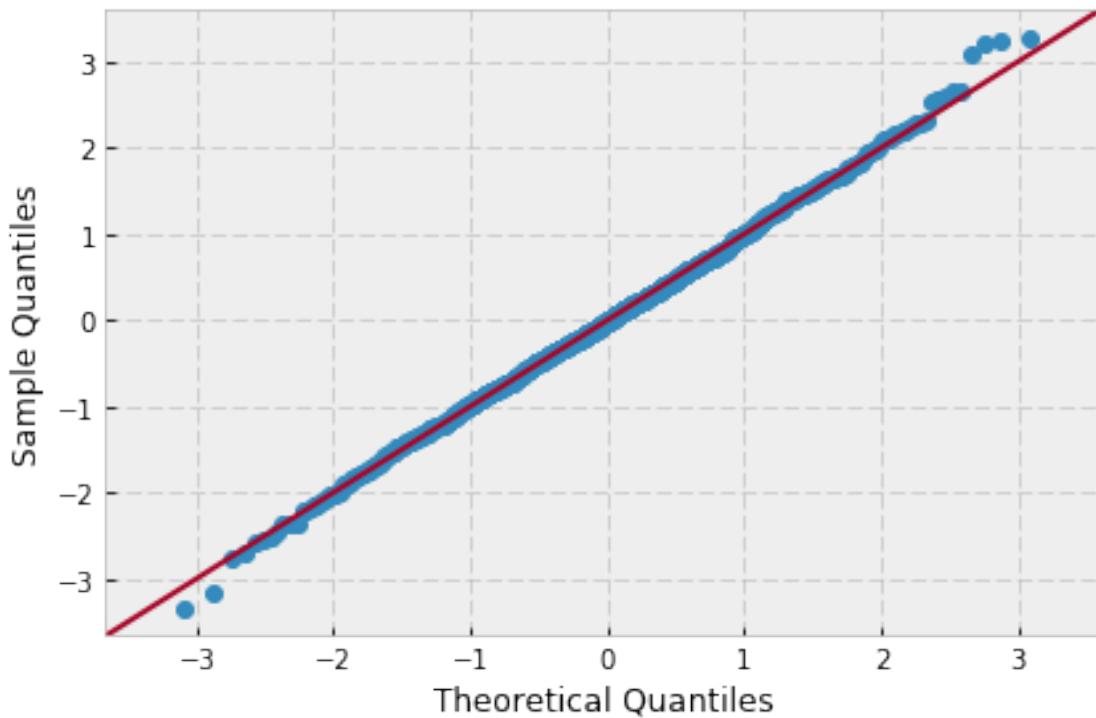


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=104.10638908666206, scale=5.9562156551704994)
DescribeResult(nobs=1000, minmax=(86.660471061730931, 123.51387453580392)
               mean=104.10638908666206, variance=35.512016947845986,
               skewness=0.07735478337023846, kurtosis=-0.10855563399660806)
```

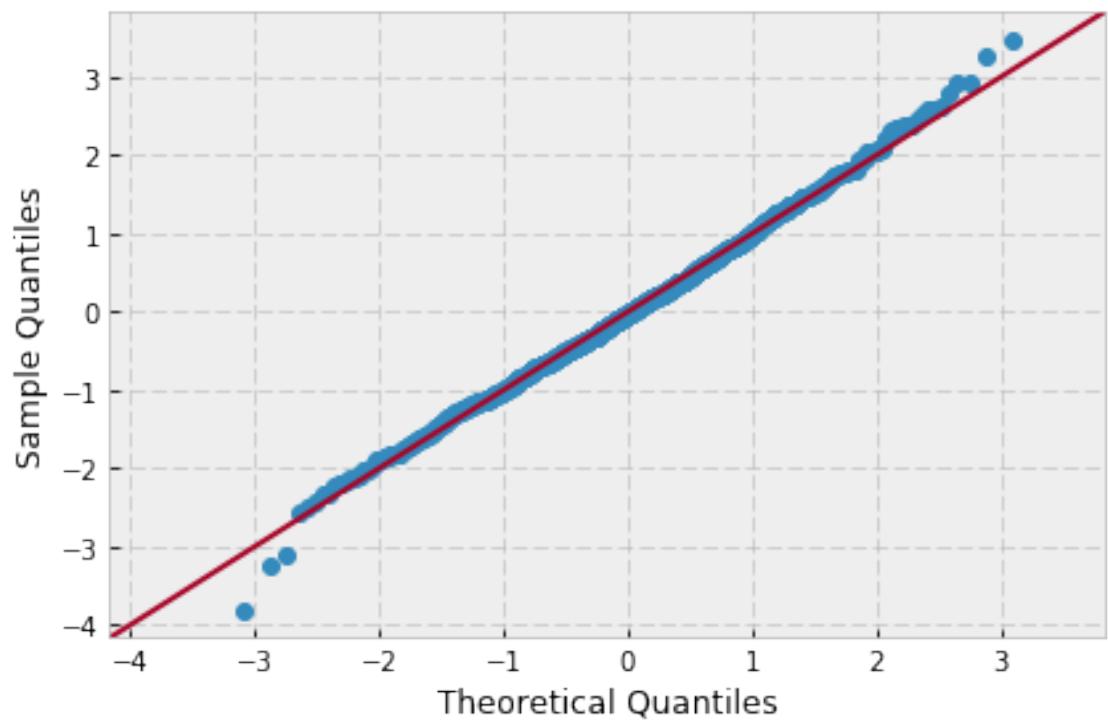
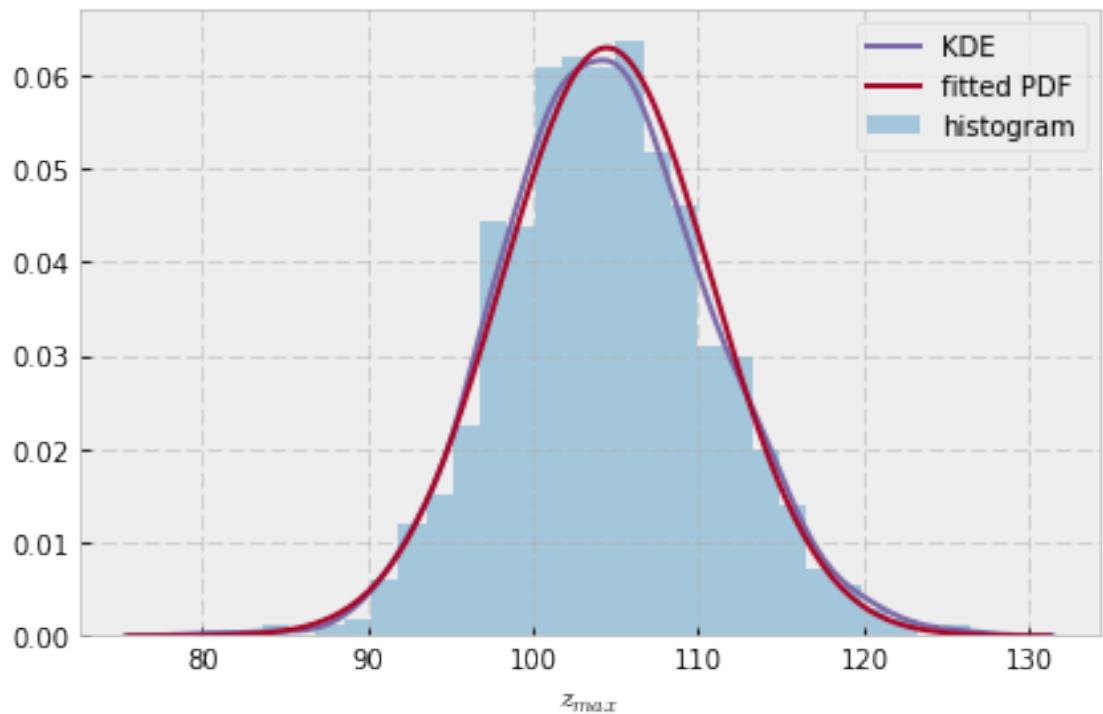


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=103.13484990889526, scale=6.30015739743069)
DescribeResult(nobs=1000, minmax=(82.132745973266353, 123.72029145742846)
               mean=103.13484990889526, variance=39.731714947347989,
               skewness=0.06167579792066425, kurtosis=0.14005412212456836)
```

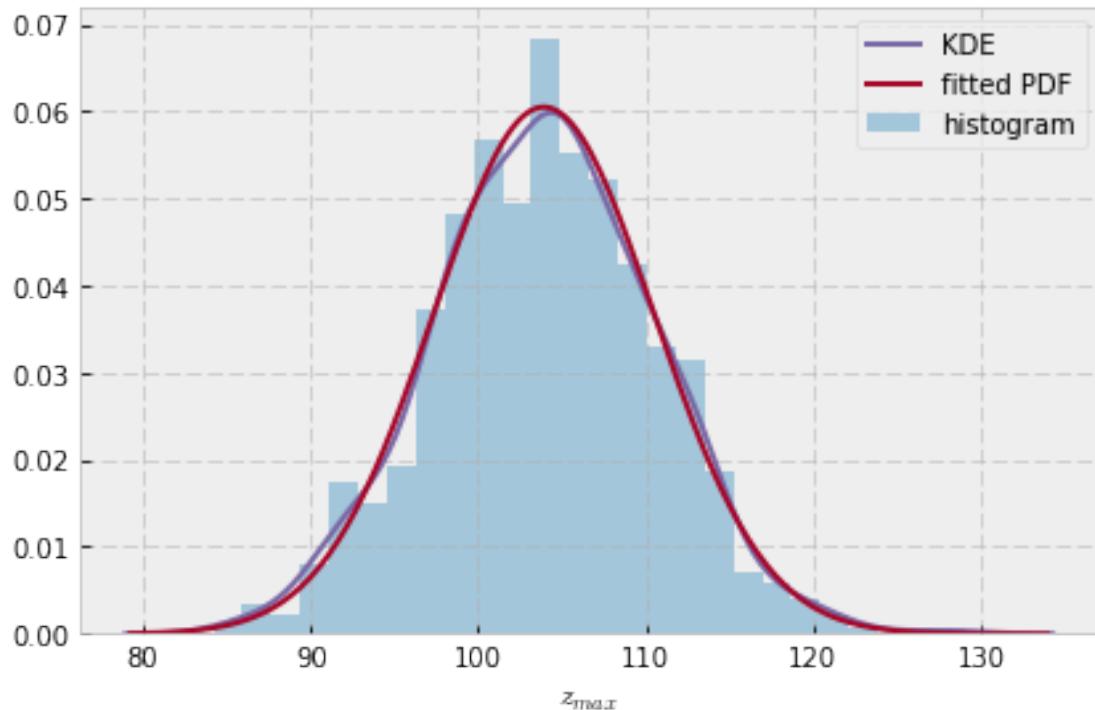


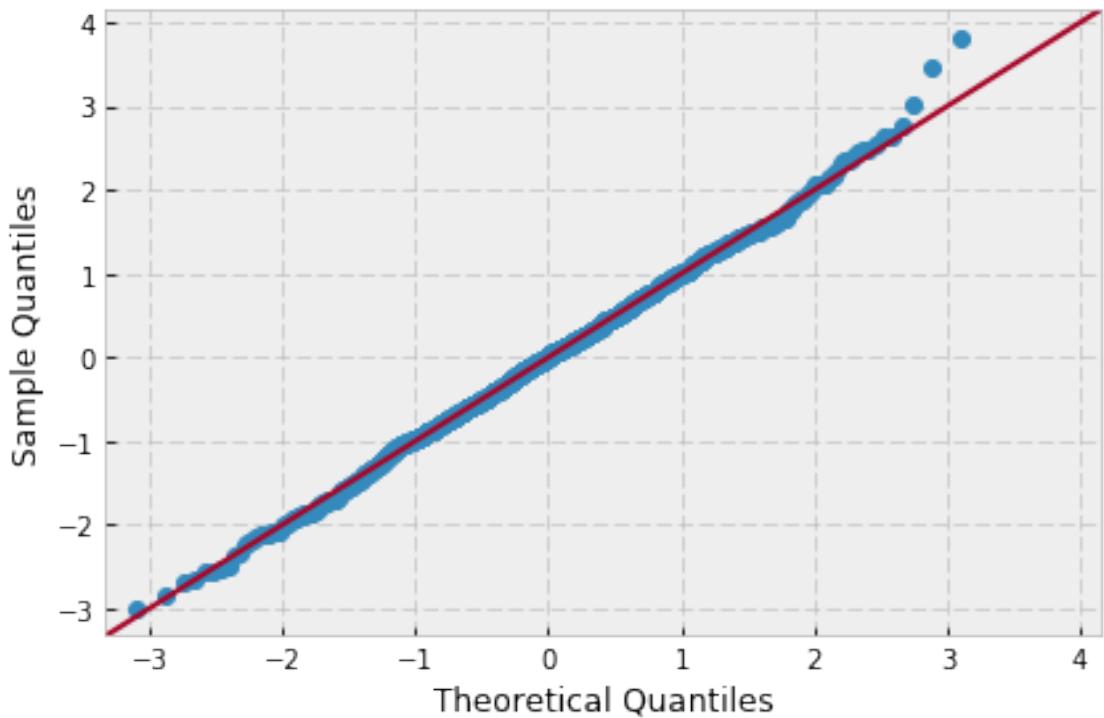


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(),loc=104.45294921425058, scale=6.3409783187082756)
DescribeResult(nobs=1000, minmax=(80.330215700741959, 126.44056072868223)
               mean=104.45294921425058, variance=40.248254292621048,
               skewness=0.1272666678465081, kurtosis=0.1987289332606399)
```

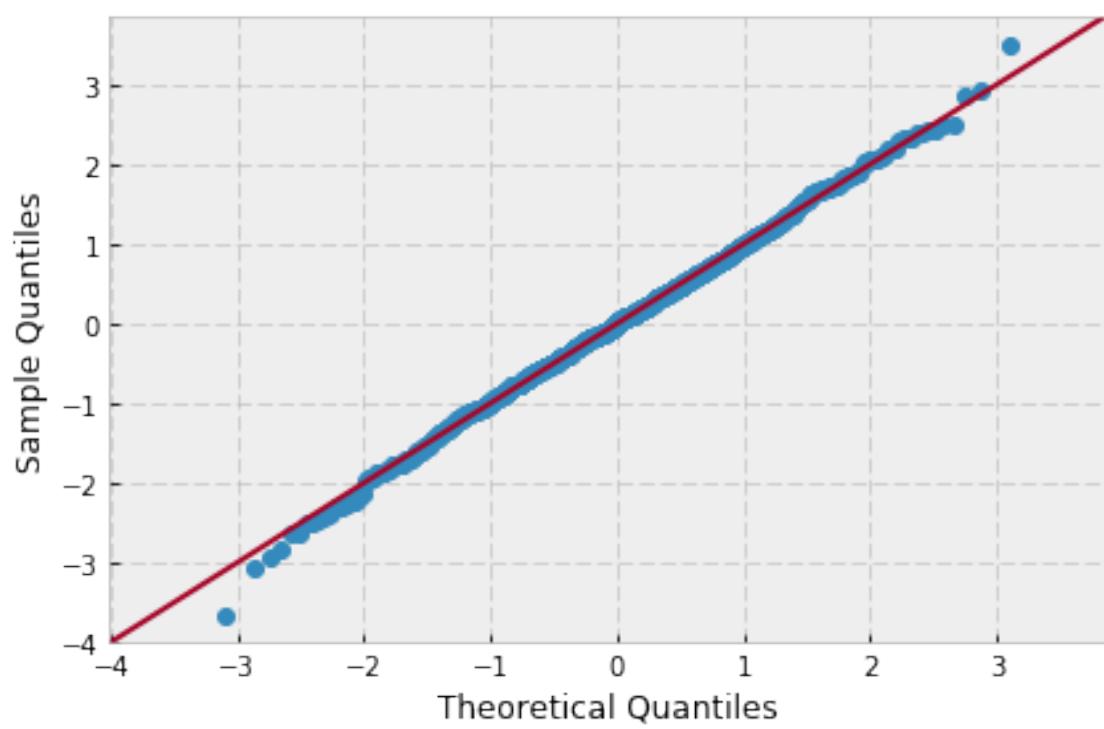
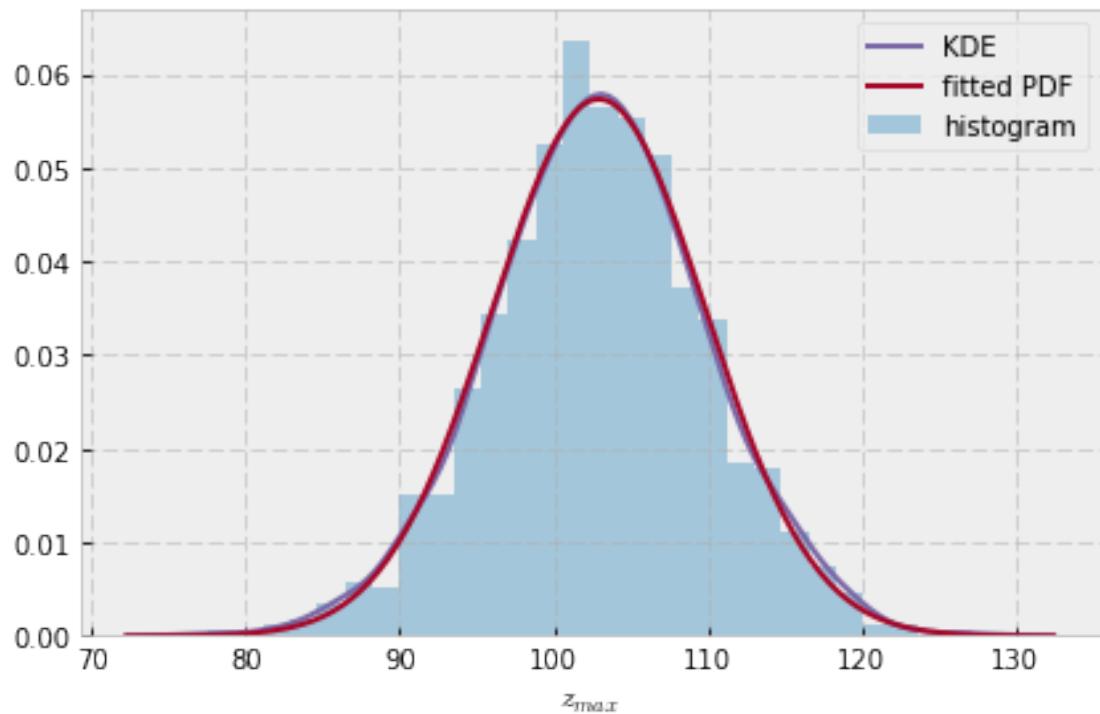


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=103.9117614445074, scale=6.594152948501469)
DescribeResult(nobs=1000, minmax=(84.200666262724098, 129.02345453489252)
               mean=103.91176144450741, variance=43.52637948771833,
               skewness=0.041902939947205, kurtosis=0.11852497021606645)
```

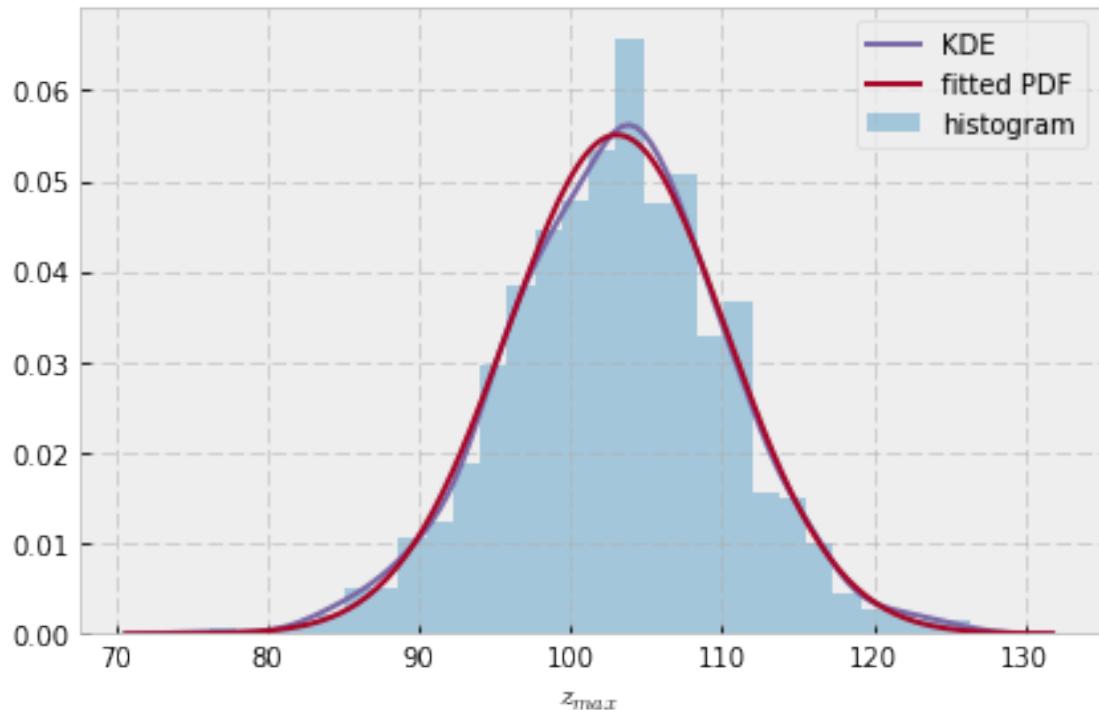


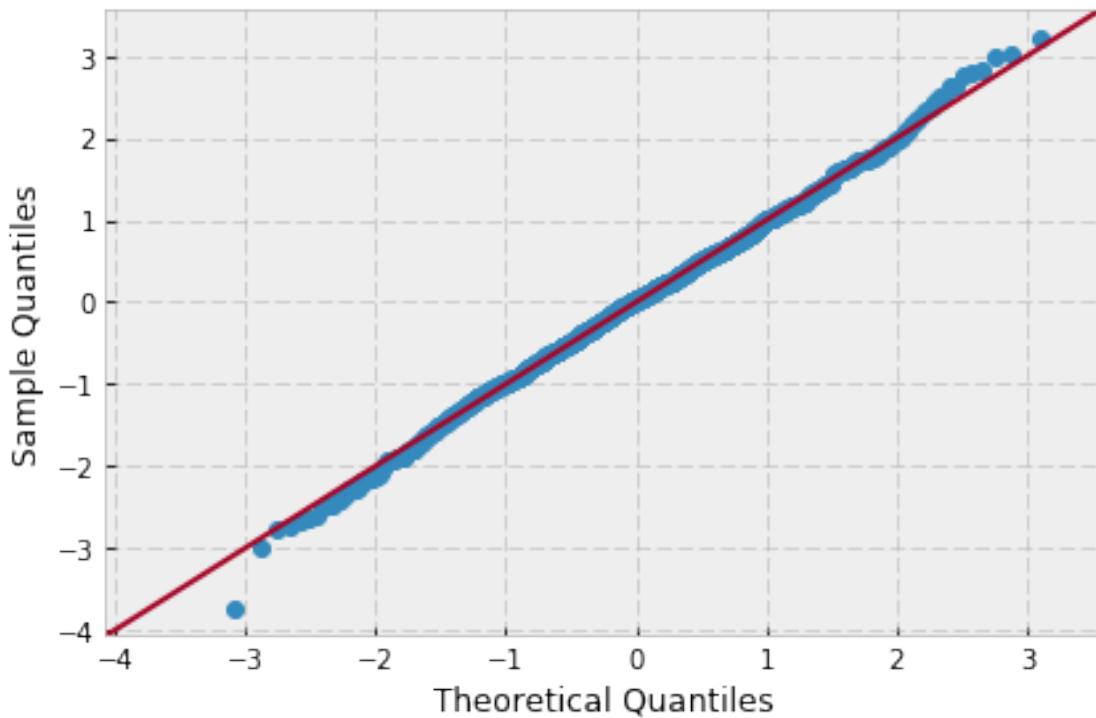


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=102.93562838632057, scale=6.9588822453592645)
DescribeResult(nobs=1000, minmax=(77.512672894604208, 127.21263818356731)
               mean=102.93562838632057, variance=48.474516621397797,
               skewness=-0.03489237457361211, kurtosis=0.1682974668437267)
```

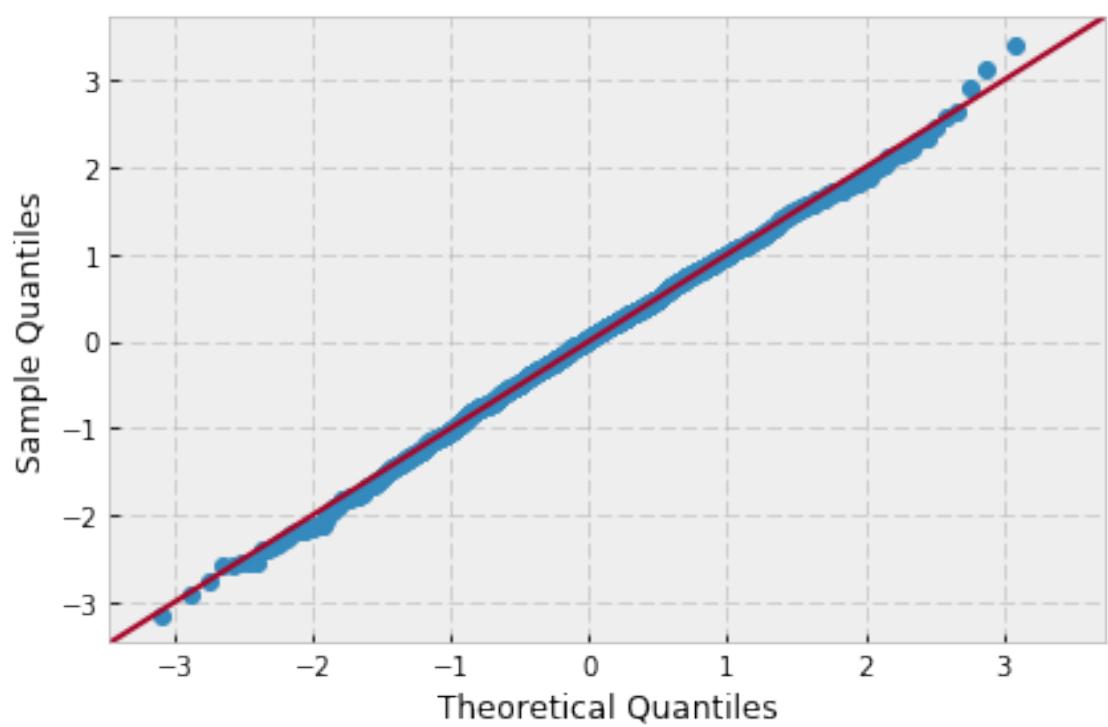
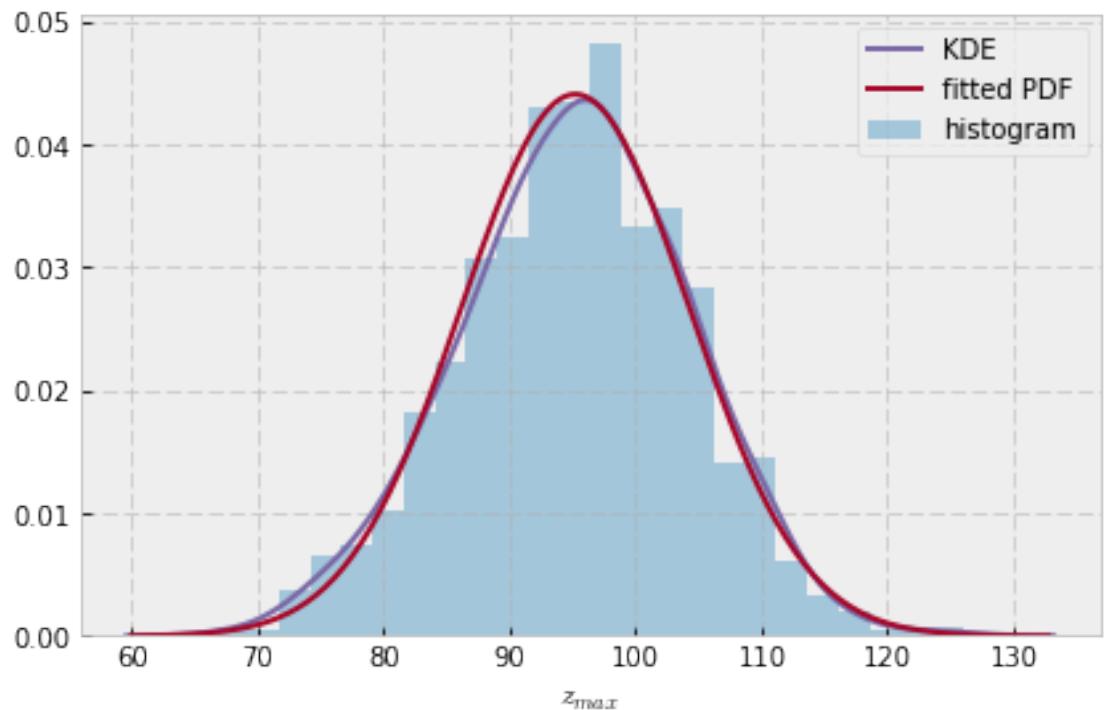


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=103.0551998594697, scale=7.244605932846012)
DescribeResult(nobs=1000, minmax=(76.097407121320217, 126.32986159809586)
               mean=103.0551998594697, variance=52.536851974201838,
               skewness=-0.03854570962836135, kurtosis=0.23603443273151337)
```

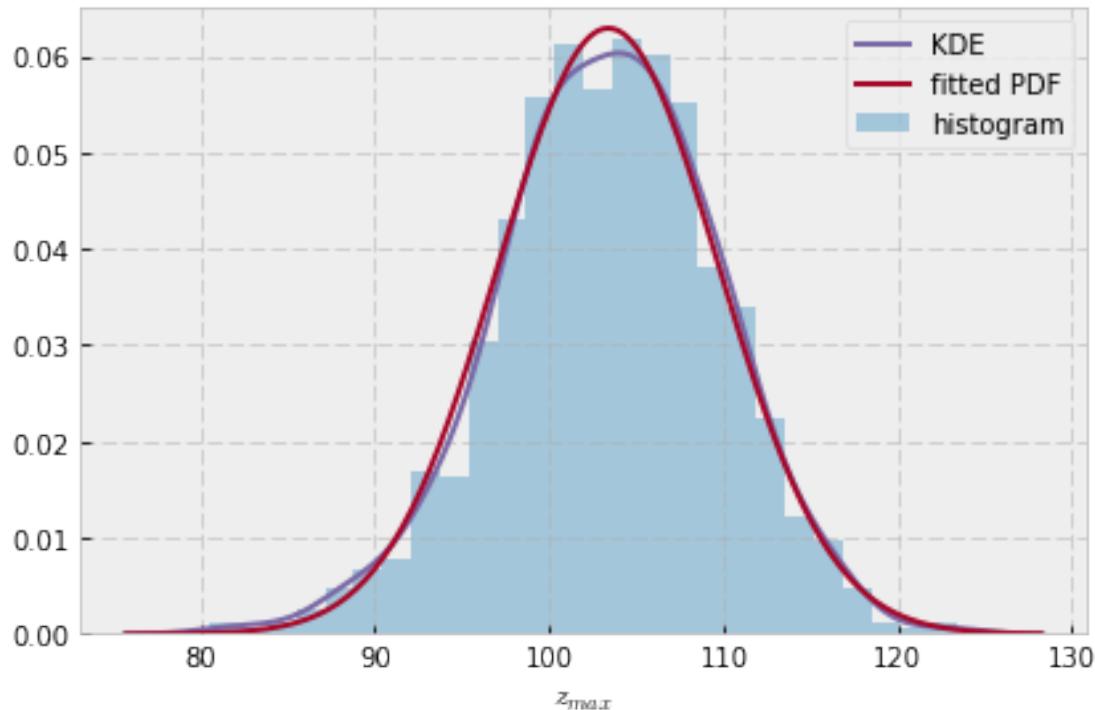


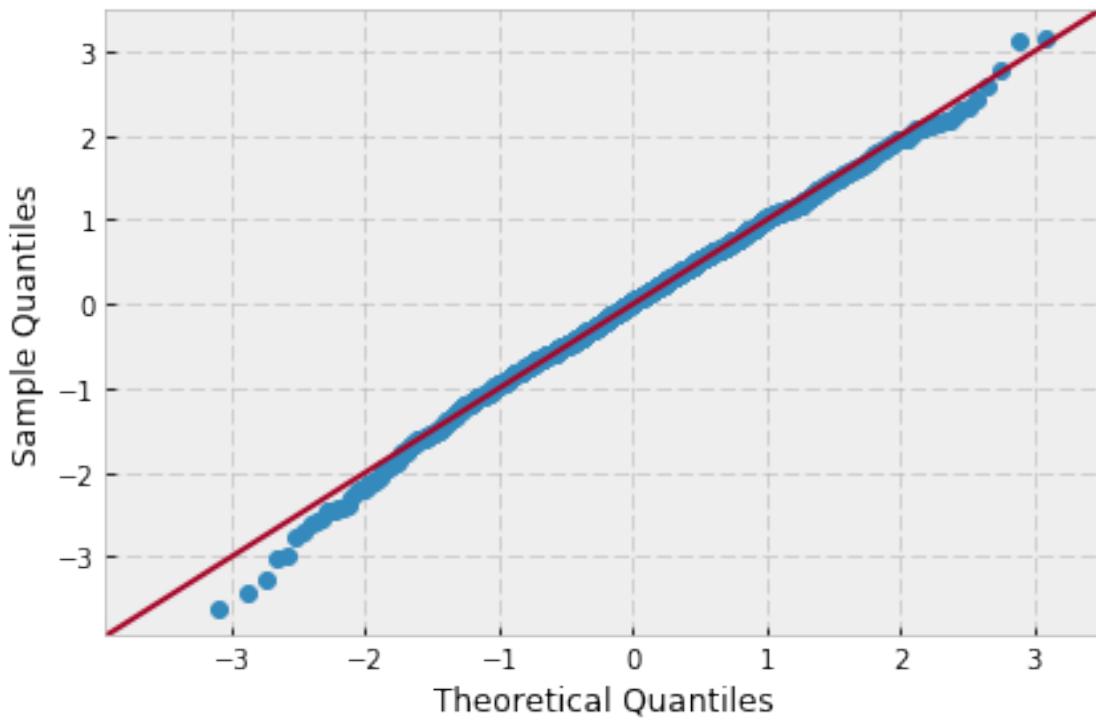


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(),loc=95.206438040821, scale=9.054750143151974)
DescribeResult(nobs=1000, minmax=(66.776645922882508, 125.98658663894506)
               mean=95.206438040820998, variance=82.070570725636316,
               skewness=-0.09390555385536832, kurtosis=-0.03157057437860811)
```

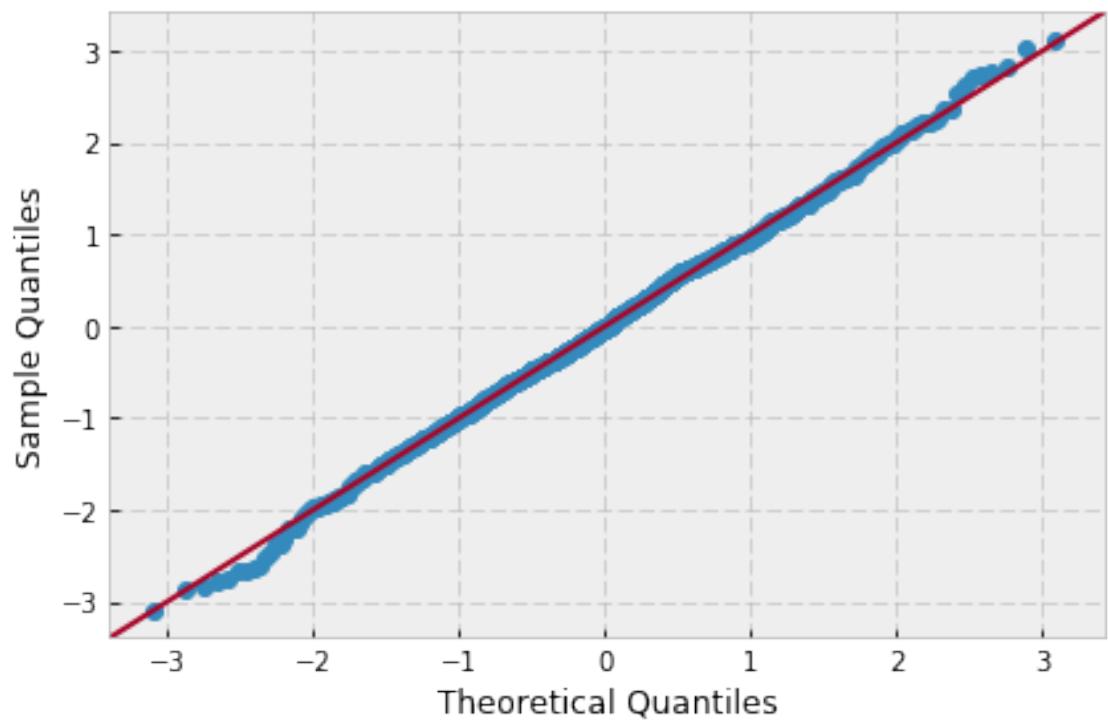
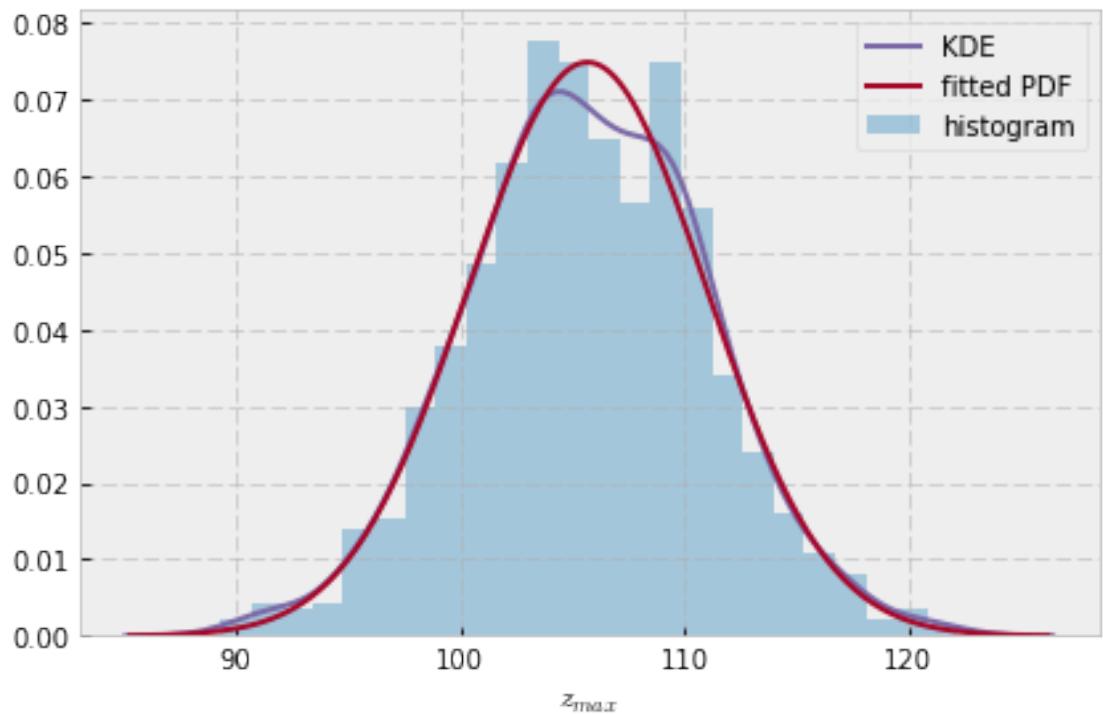


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(), loc=103.39298842529463, scale=6.34302424350122)
DescribeResult(nobs=1000, minmax=(80.551740679899922, 123.39654391986384)
               mean=103.39298842529463, variance=40.274230784428653,
               skewness=-0.1840813343761588, kurtosis=0.2312380070886837)
```

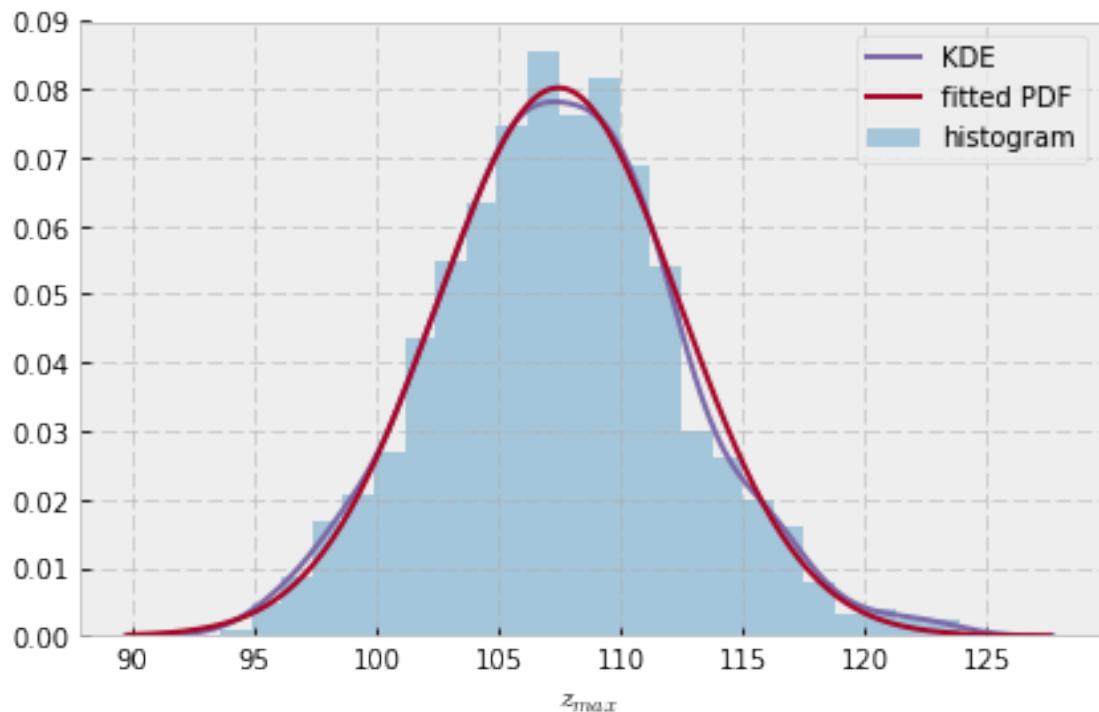


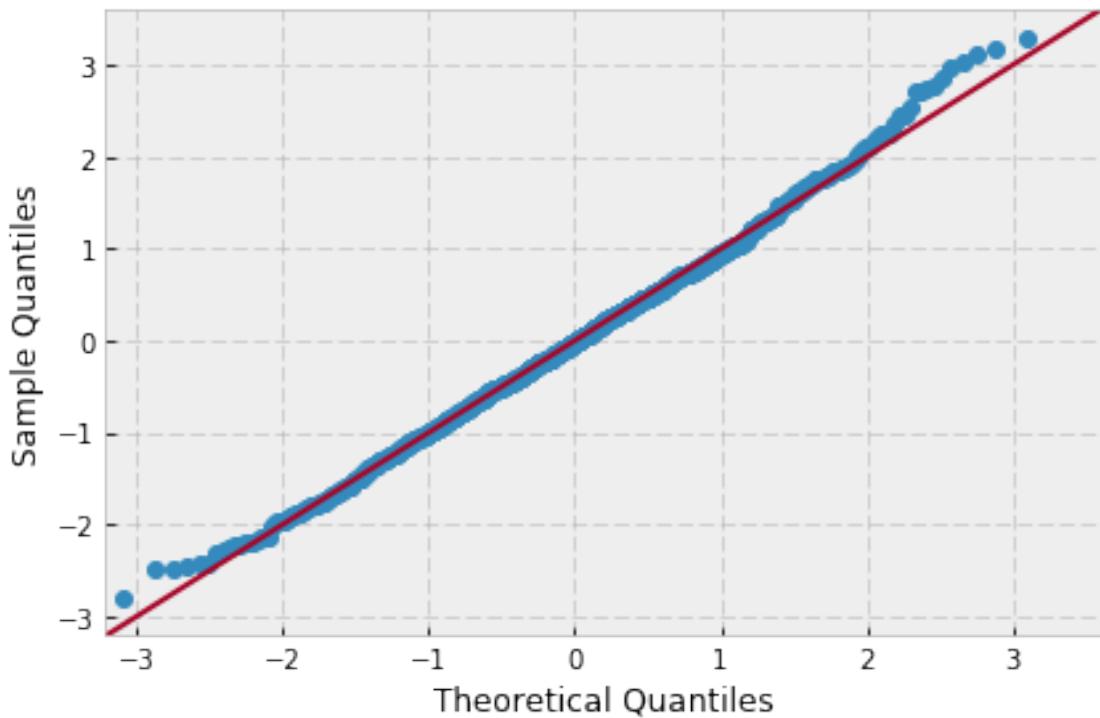


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(),loc=105.68183585527572, scale=5.324592553022331)
DescribeResult(nobs=1000, minmax=(89.262521951028191, 122.24514834321495)
               mean=105.68183585527572, variance=28.379665521222083,
               skewness=-0.020347359245835434, kurtosis=0.03334617716783406)
```

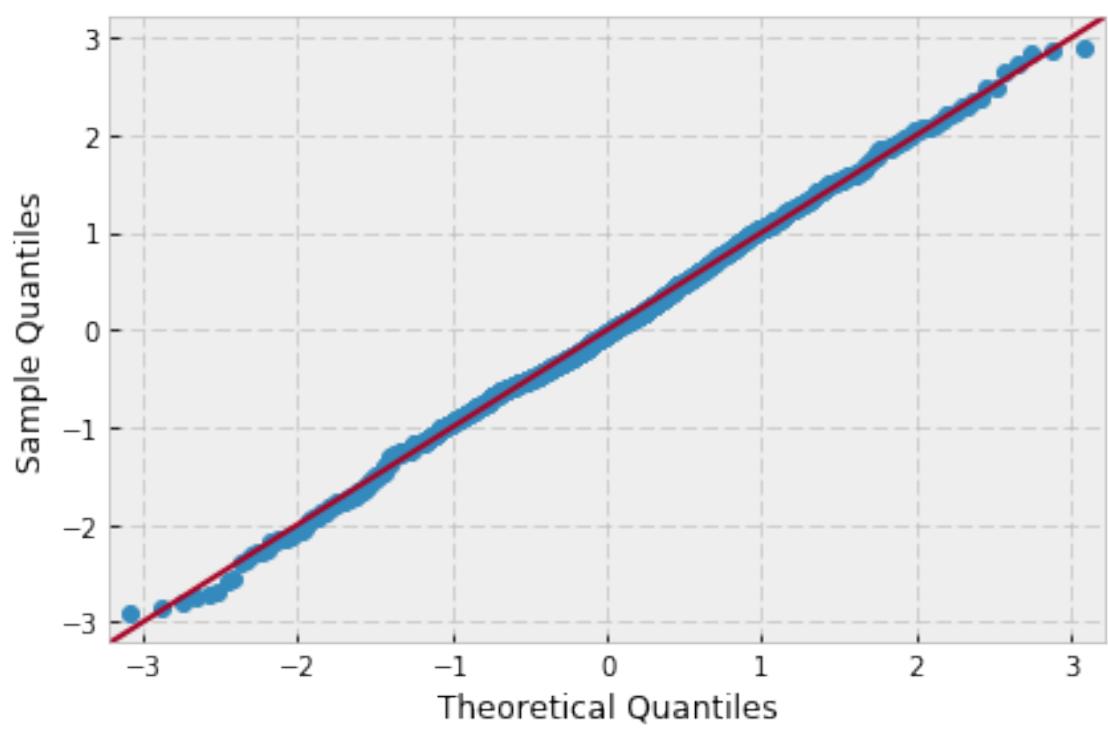
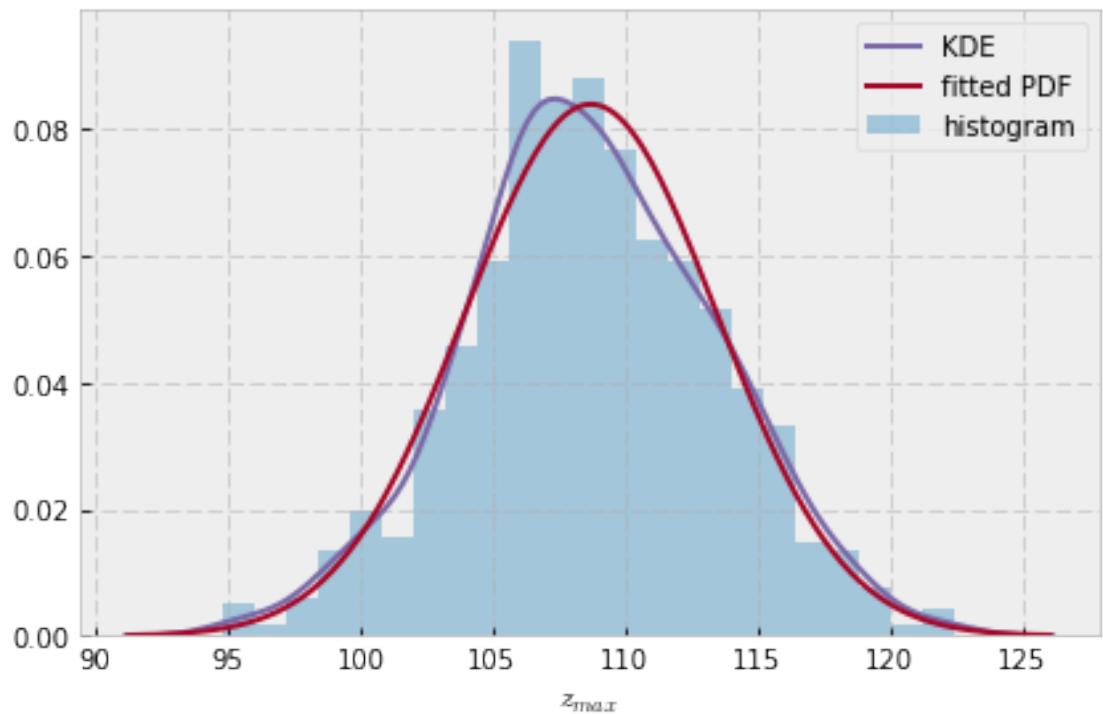


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(), loc=107.45602735059164, scale=4.976427983060977)
DescribeResult(nobs=1000, minmax=(93.570670530785875, 123.84931162099116)
               mean=107.45602735059164, variance=24.789625095688031,
               skewness=0.1652875779076341, kurtosis=0.10839432172309582)
```

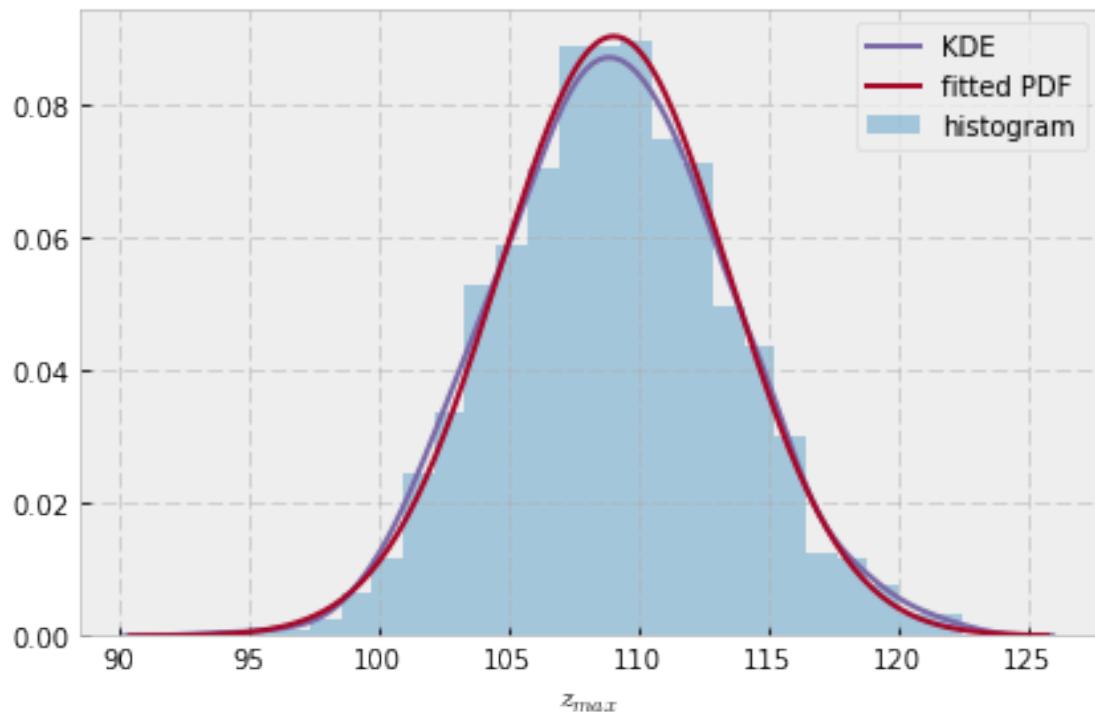


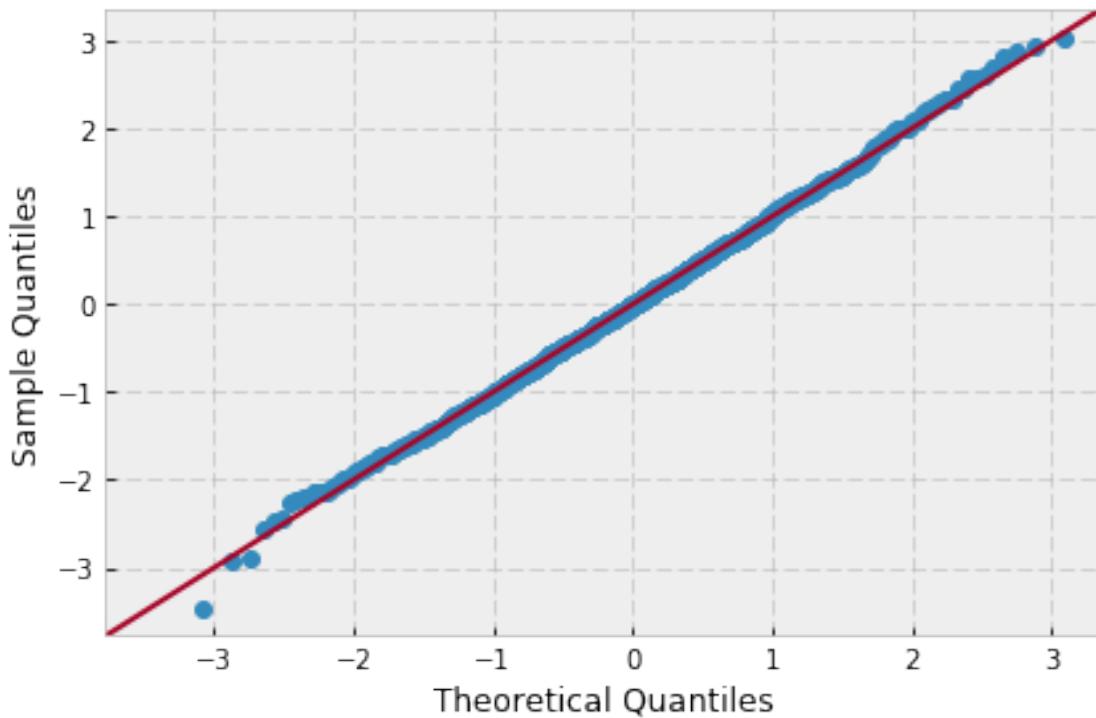


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(),loc=108.70573735279149, scale=4.757006801628118)
DescribeResult(nobs=1000, minmax=(94.81809009611726, 122.46056498903984)
               mean=108.70573735279149, variance=22.651765476212393,
               skewness=0.02547152522431889, kurtosis=-0.04595438126928153)
```

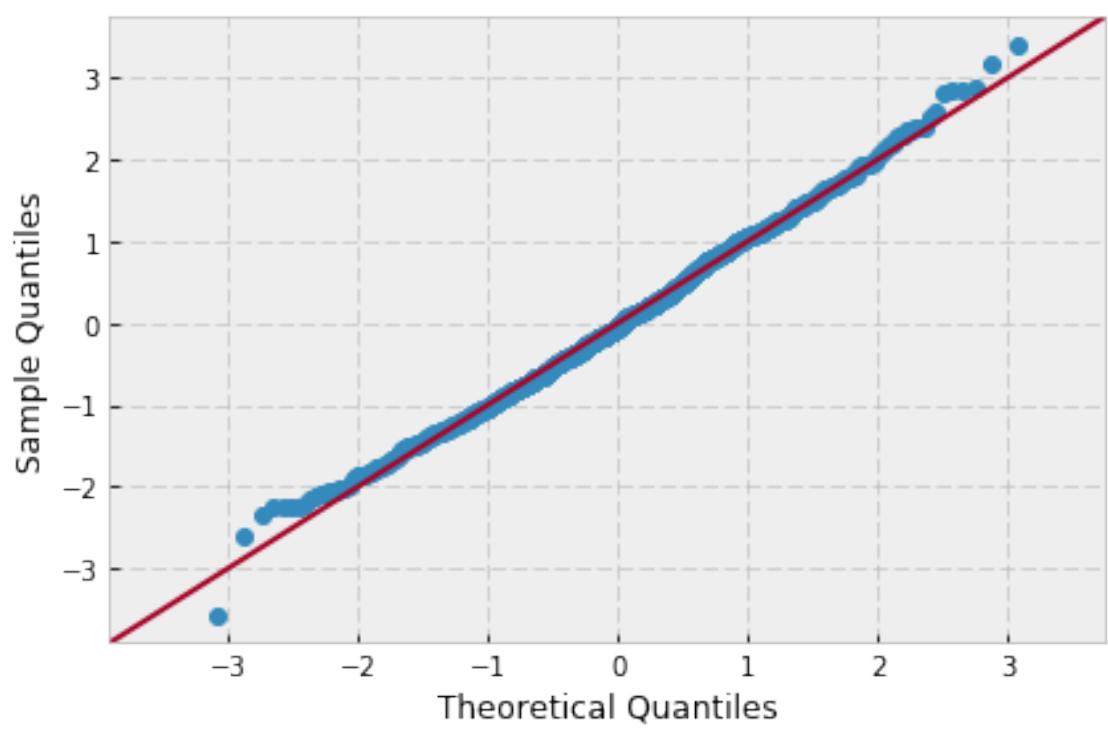
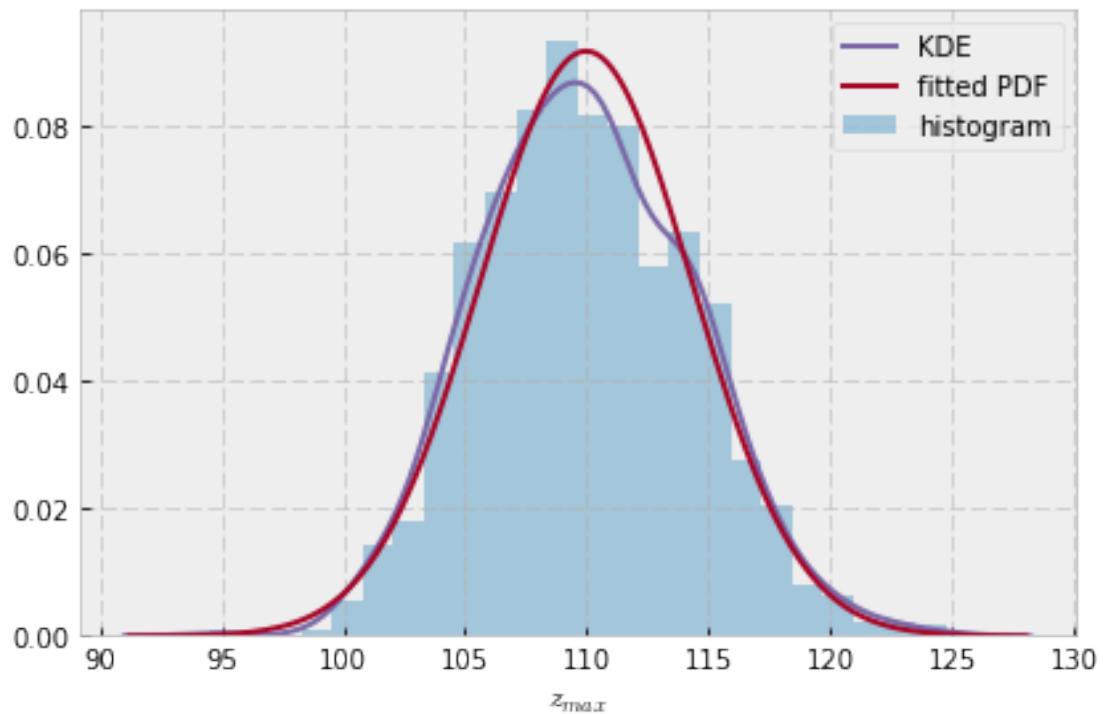


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(), loc=109.05680922380455, scale=4.424041252219403)
DescribeResult(nobs=1000, minmax=(93.785495860383392, 122.42535185226852)
               mean=109.05680922380455, variance=19.591732734073091,
               skewness=0.06627686934457923, kurtosis=-0.06266588954495145)
```

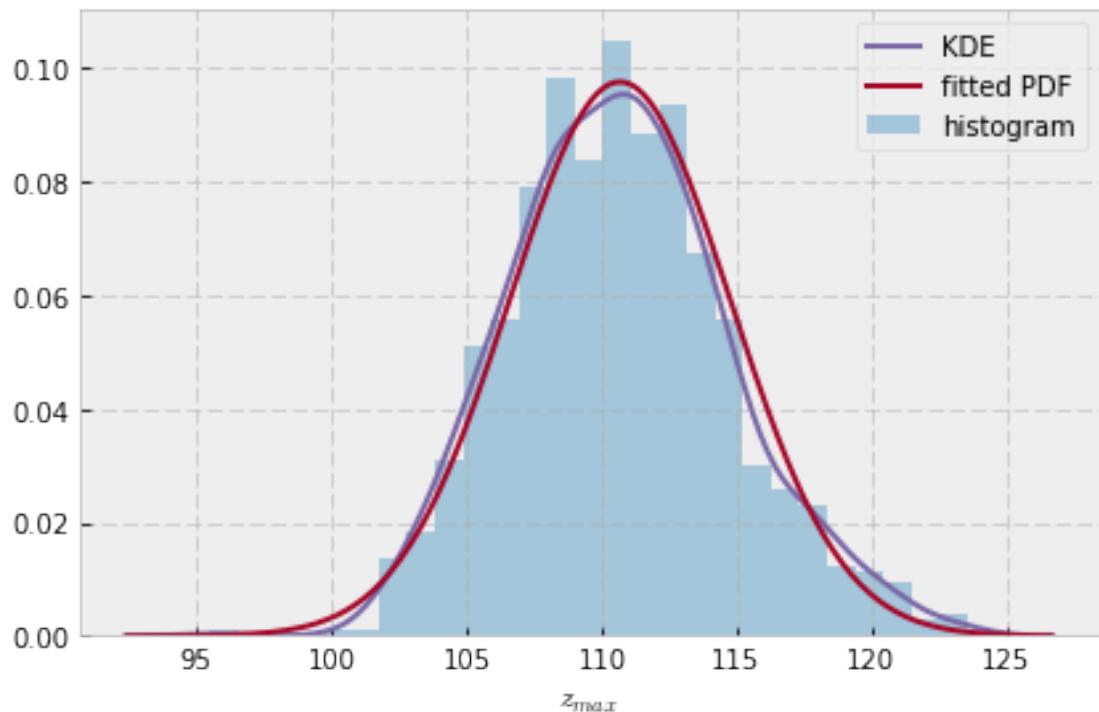


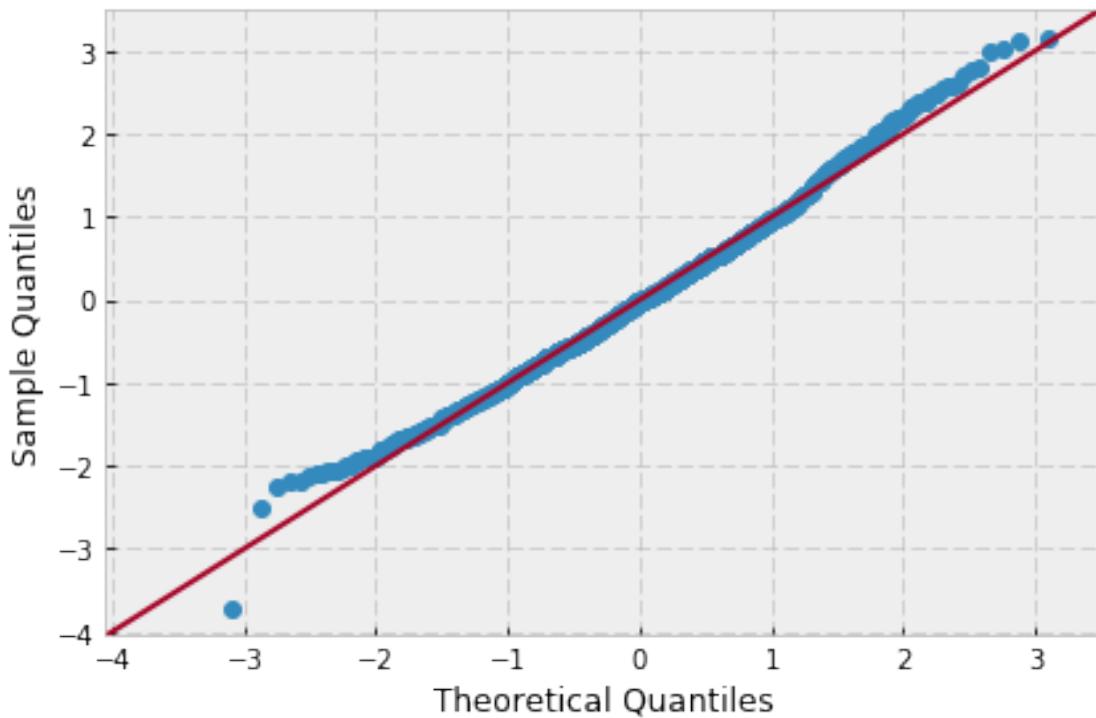


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(),loc=109.968309494804, scale=4.343796241353436)
DescribeResult(nobs=1000, minmax=(94.477441495342291, 124.72923731883311)
               mean=109.968309494804, variance=18.887453239635878,
               skewness=0.160659917899985, kurtosis=-0.11123910267374137)
```

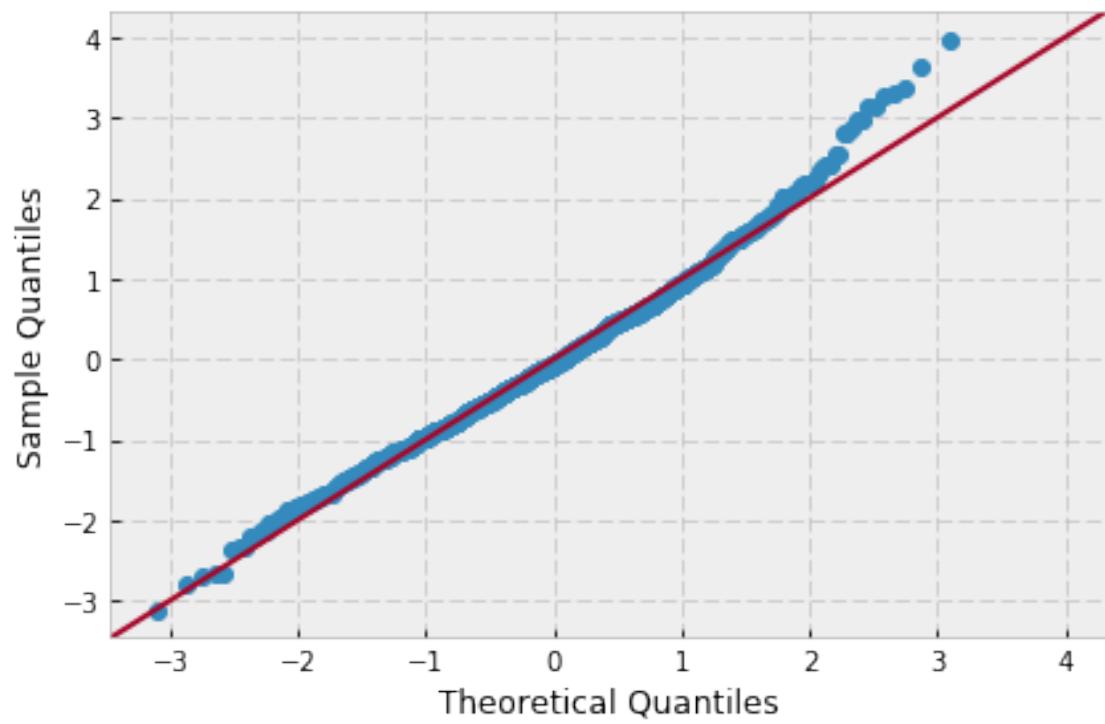
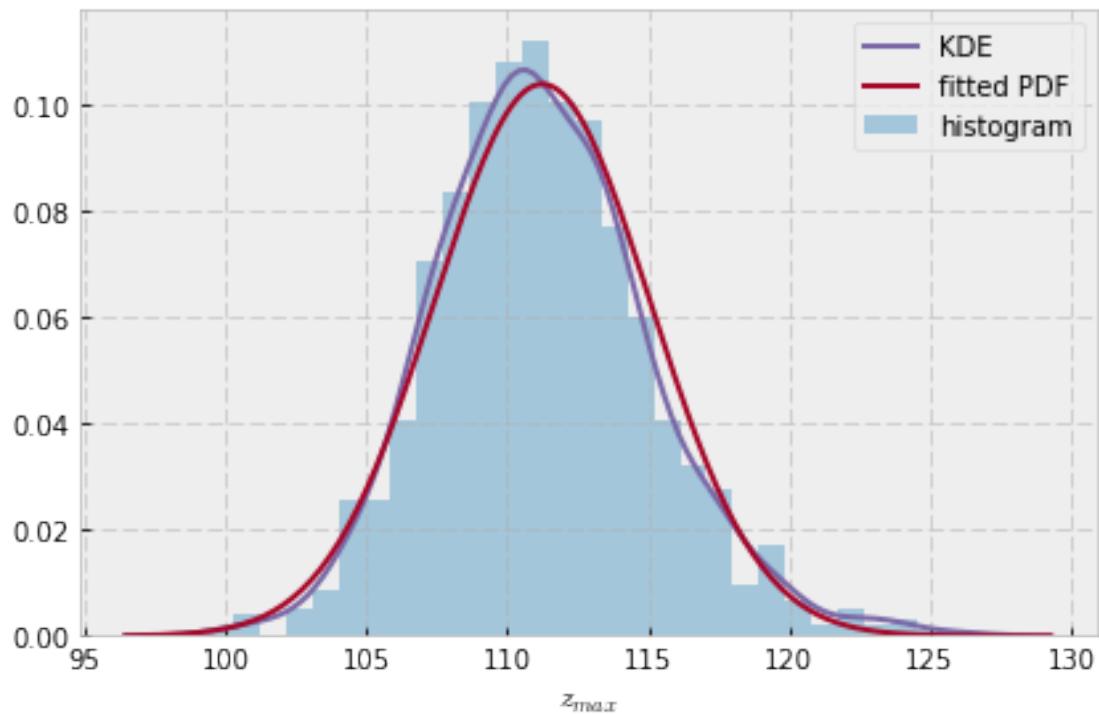


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(), loc=110.68294486149487, scale=4.0880241262735355)
DescribeResult(nobs=1000, minmax=(95.523275268892974, 123.5539518929744)
               mean=110.68294486149487, variance=16.728669926921427,
               skewness=0.27018391968920885, kurtosis=0.0862430519709636)
```

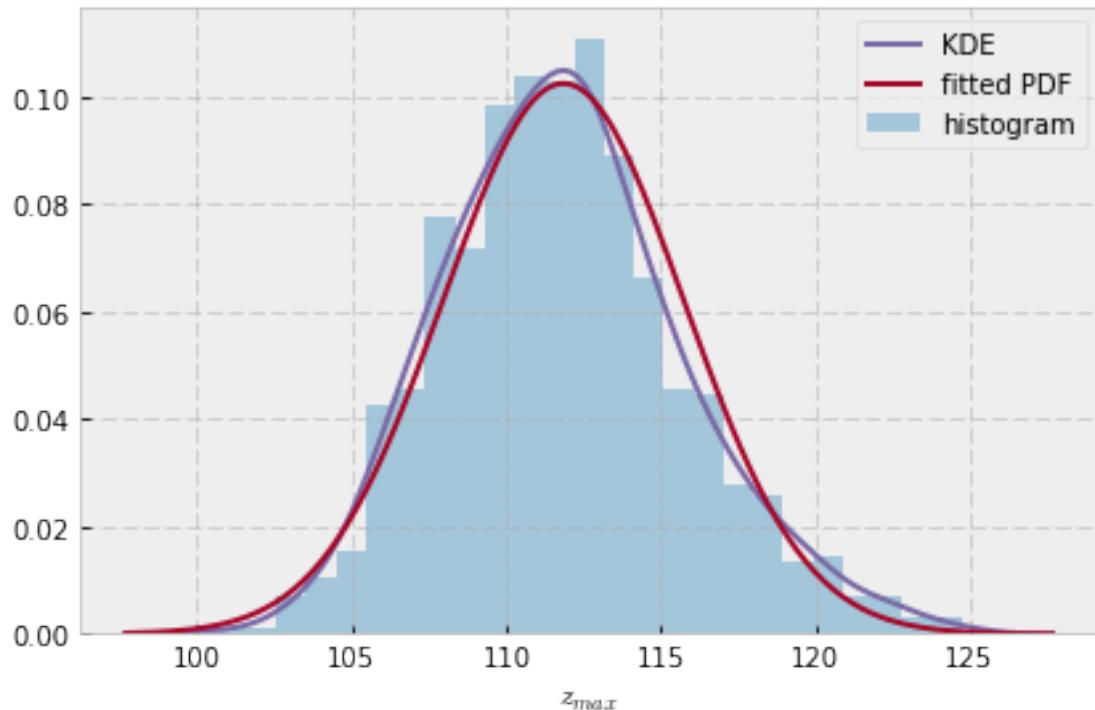


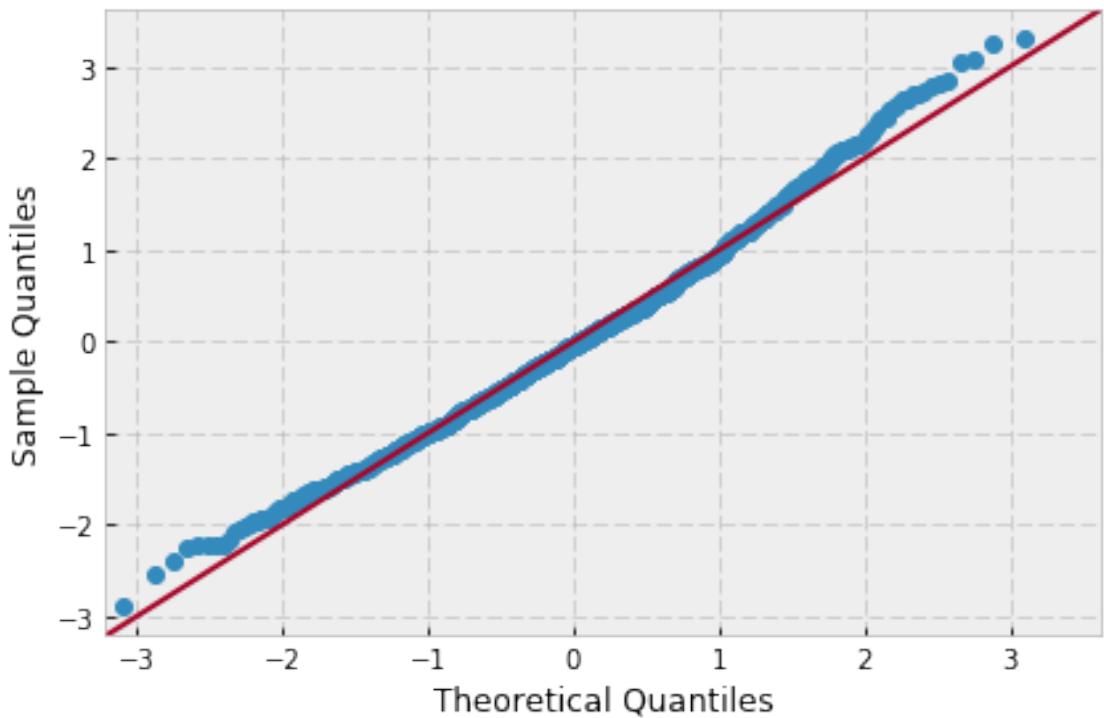


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(),loc=111.23261398397614, scale=3.8330112278812467)
DescribeResult(nobs=1000, minmax=(99.318421937405148, 126.40027758925274)
               mean=111.23261398397614, variance=14.70668175481852,
               skewness=0.3834501711923076, kurtosis=0.6059753359286639)
```

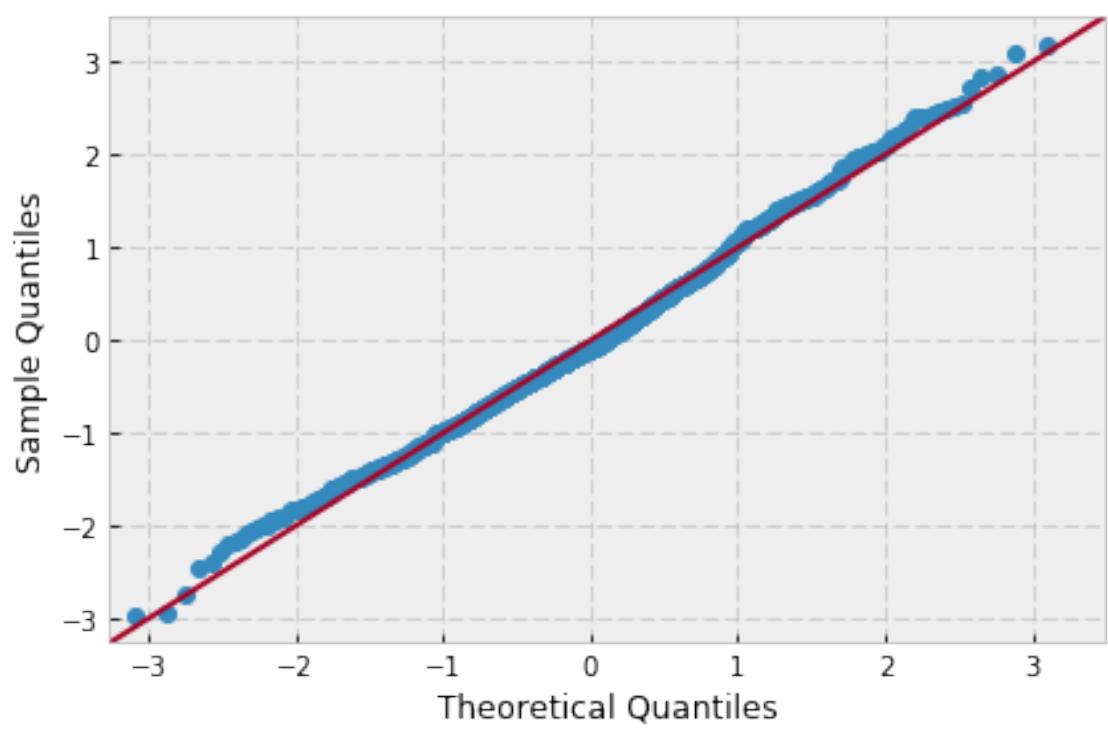
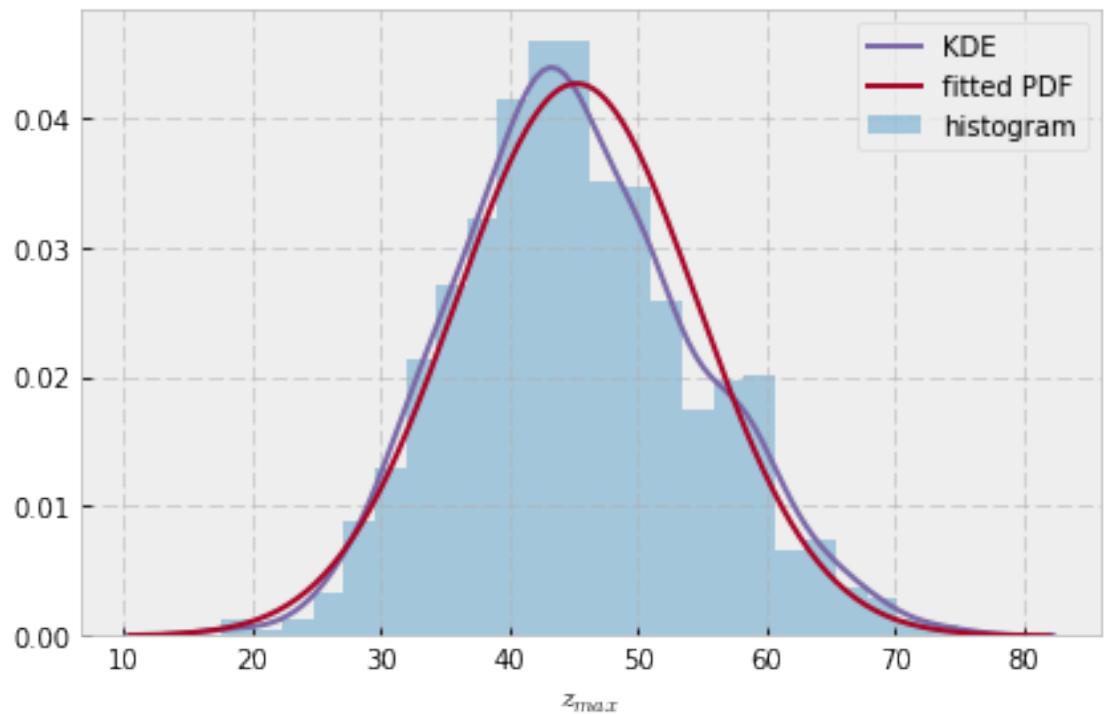


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(), loc=111.8509104248681, scale=3.8921714236231866)
DescribeResult(nobs=1000, minmax=(100.6307411543829, 124.7295389013818)
               mean=111.8509104248681, variance=15.164162553422365,
               skewness=0.36104973635143695, kurtosis=0.13531156326625915)
```

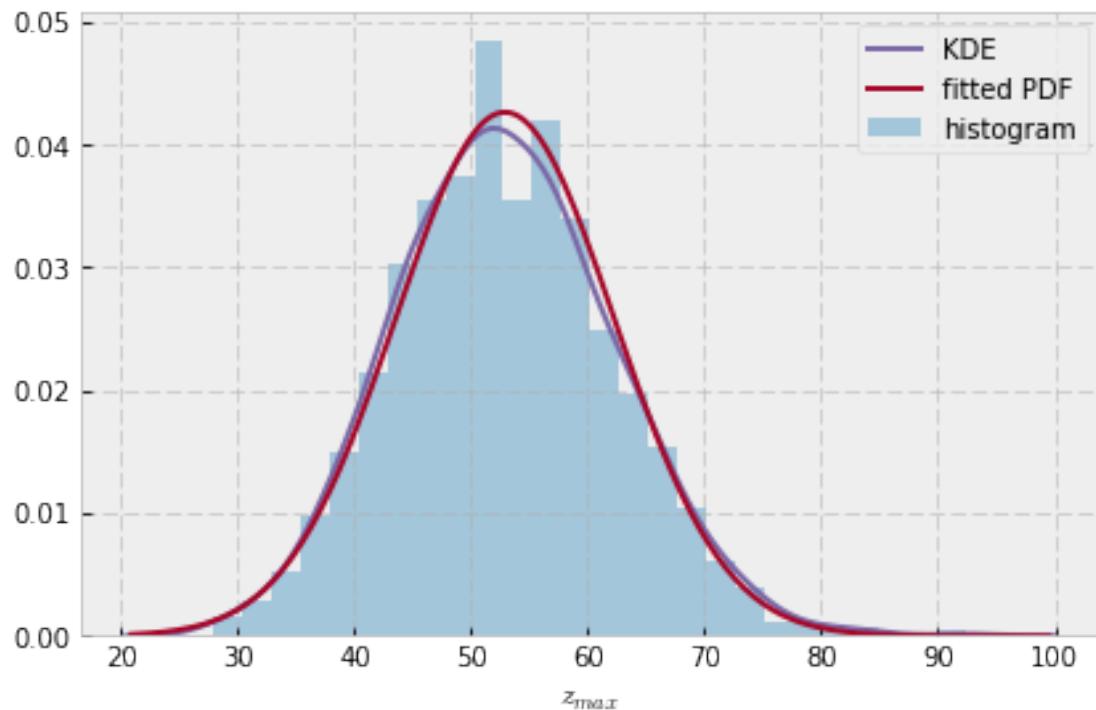


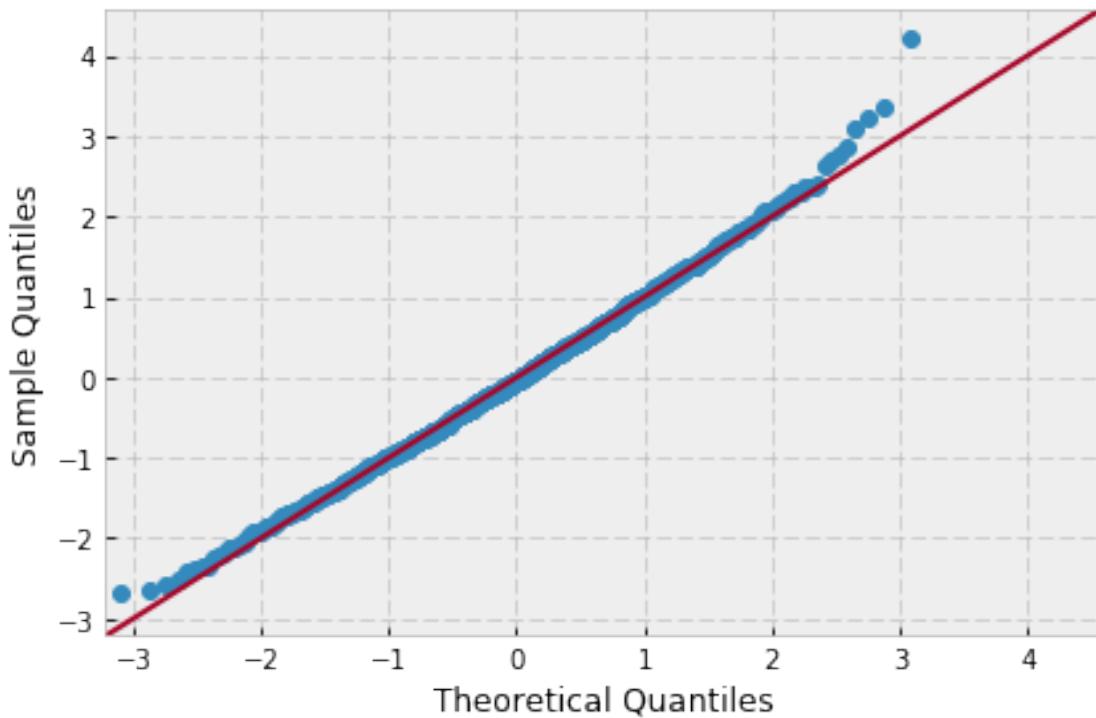


```
variable name: r
variable value: 1.0
distribution: normal(shape=(),loc=45.205285166459504, scale=9.360406526335002)
DescribeResult(nobs=1000, minmax=(17.556673588177649, 74.947419994626387)
               mean=45.205285166459504, variance=87.704915253508403,
               skewness=0.24973157484222006, kurtosis=-0.10575855713740667)
```

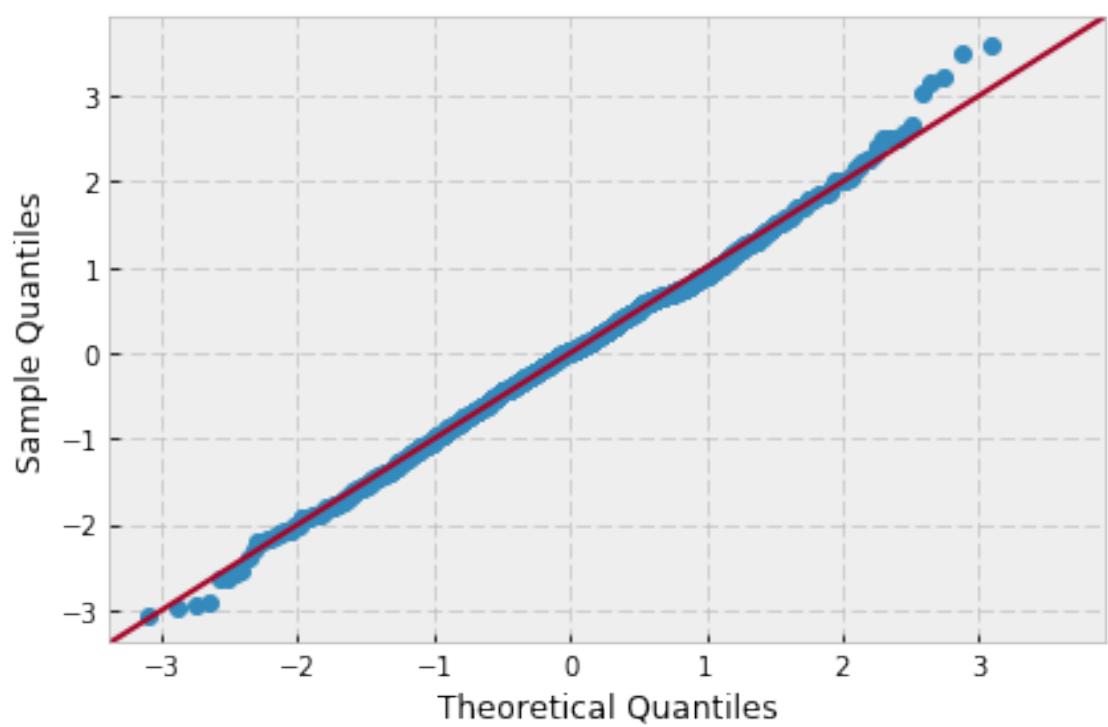
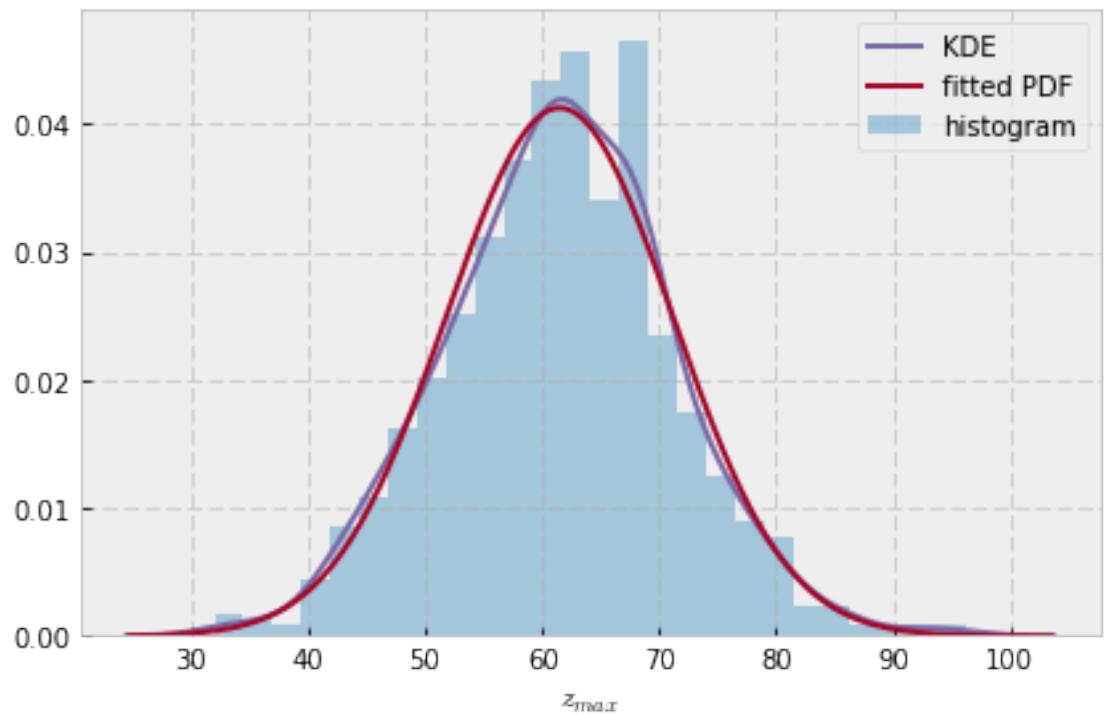


```
variable name: r
variable value: 1.22222222222223
distribution: normal(shape=(), loc=52.88795893855268, scale=9.363198545472523)
DescribeResult(nobs=1000, minmax=(27.928625415728927, 92.431641181066141)
               mean=52.887958938552678, variance=87.757244246184953,
               skewness=0.23129303304236967, kurtosis=0.15886171075258204)
```

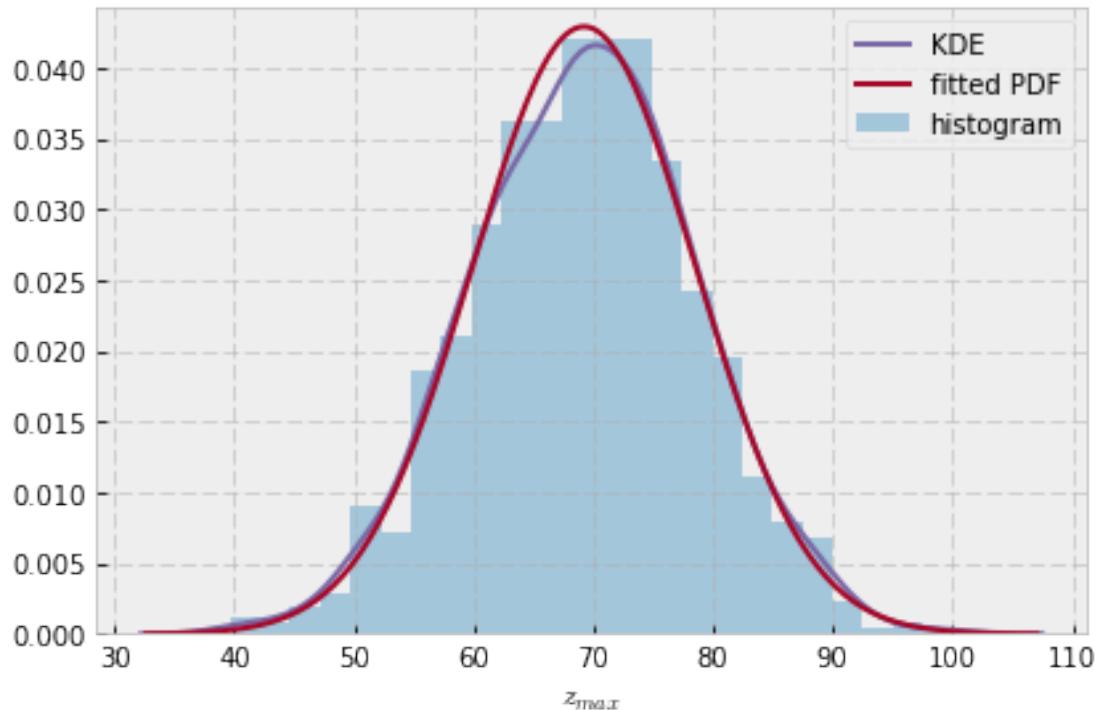


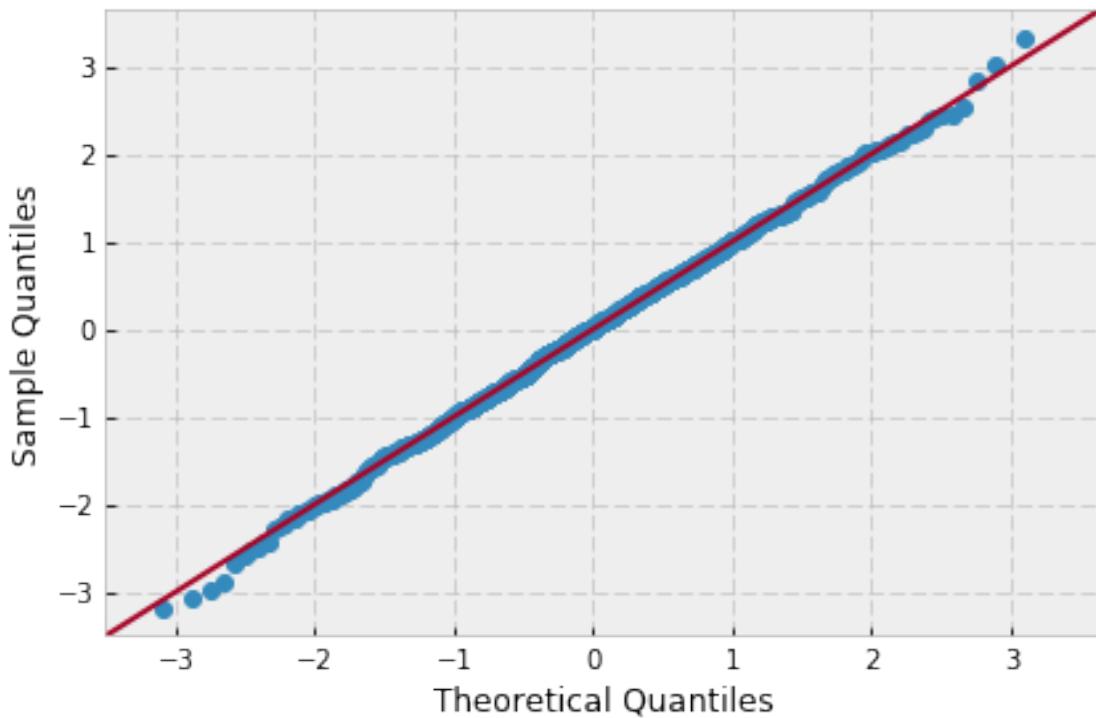


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=61.501683008721045, scale=9.66663395214342)
DescribeResult(nobs=1000, minmax=(32.0246927982827, 96.19057480013791)
               mean=61.501683008721045, variance=93.53734931404594,
               skewness=0.04231019971385085, kurtosis=0.2775895511256352)
```

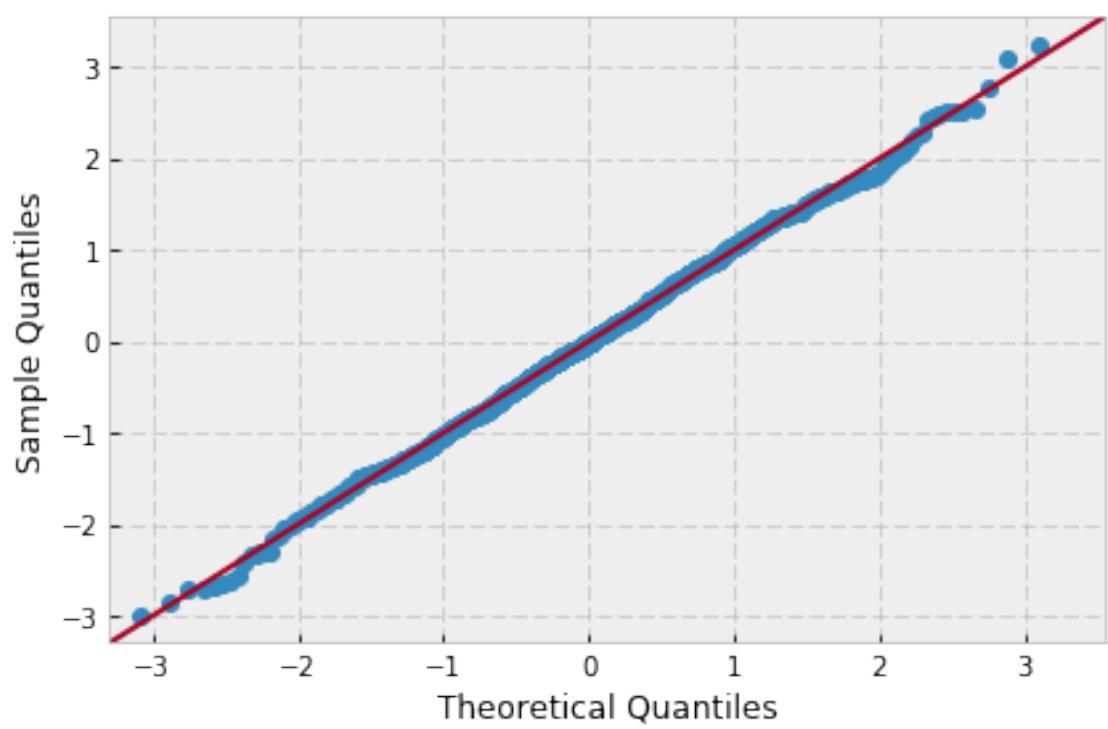
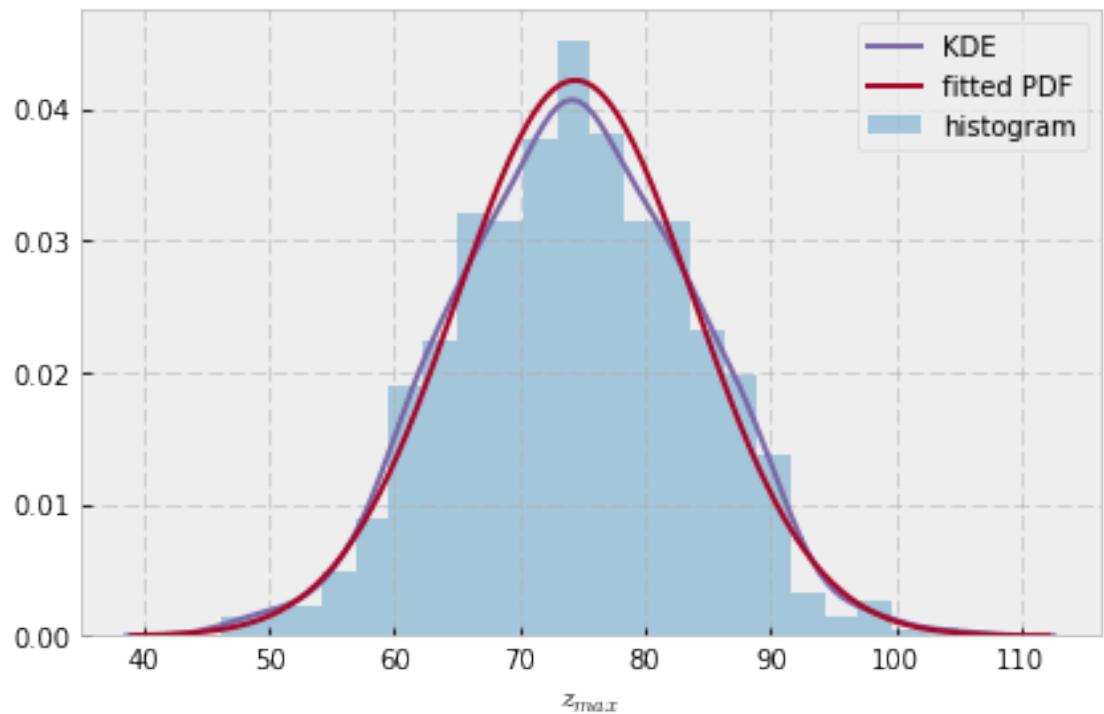


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=69.21249252966805, scale=9.299893201120295)
DescribeResult(nobs=1000, minmax=(39.63679603766851, 100.05602341445064)
               mean=69.212492529668054, variance=86.574588140383852,
               skewness=-0.048738357756973806, kurtosis=-0.023720104287471067)
```

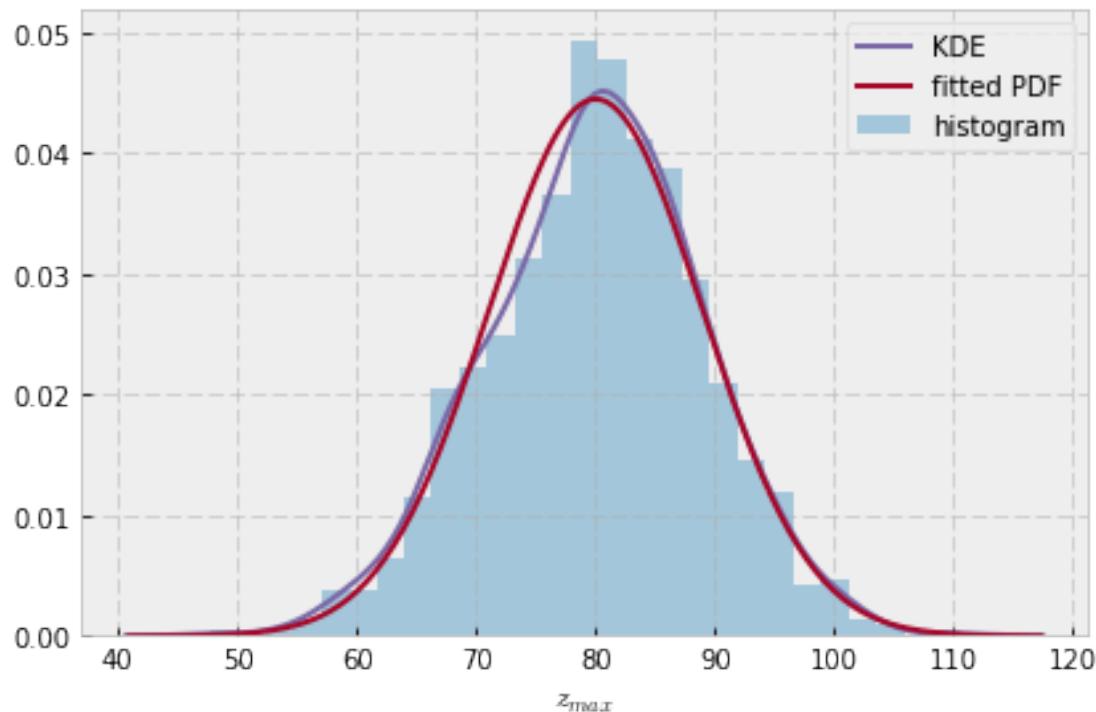


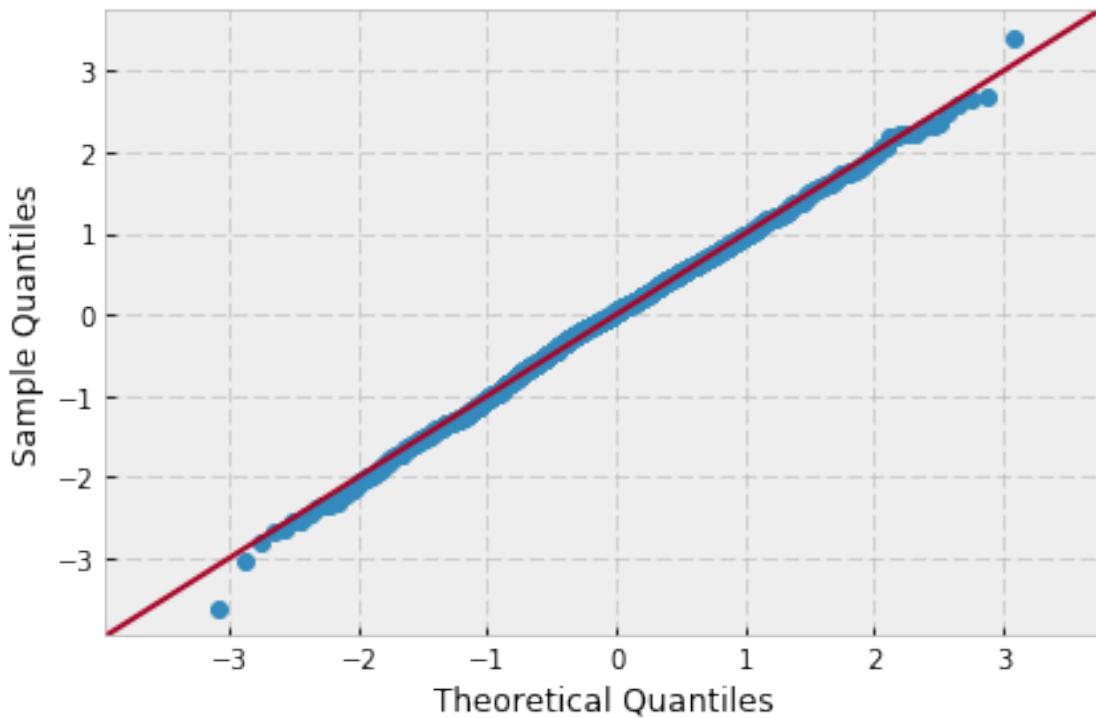


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(),loc=74.42791690784797, scale=9.468511709709516)
DescribeResult(nobs=1000, minmax=(46.168046723299909, 105.05020204305494)
               mean=74.42791690784793, variance=89.742456453359566,
               skewness=-0.008624385637929118, kurtosis=-0.1656982866322383)
```

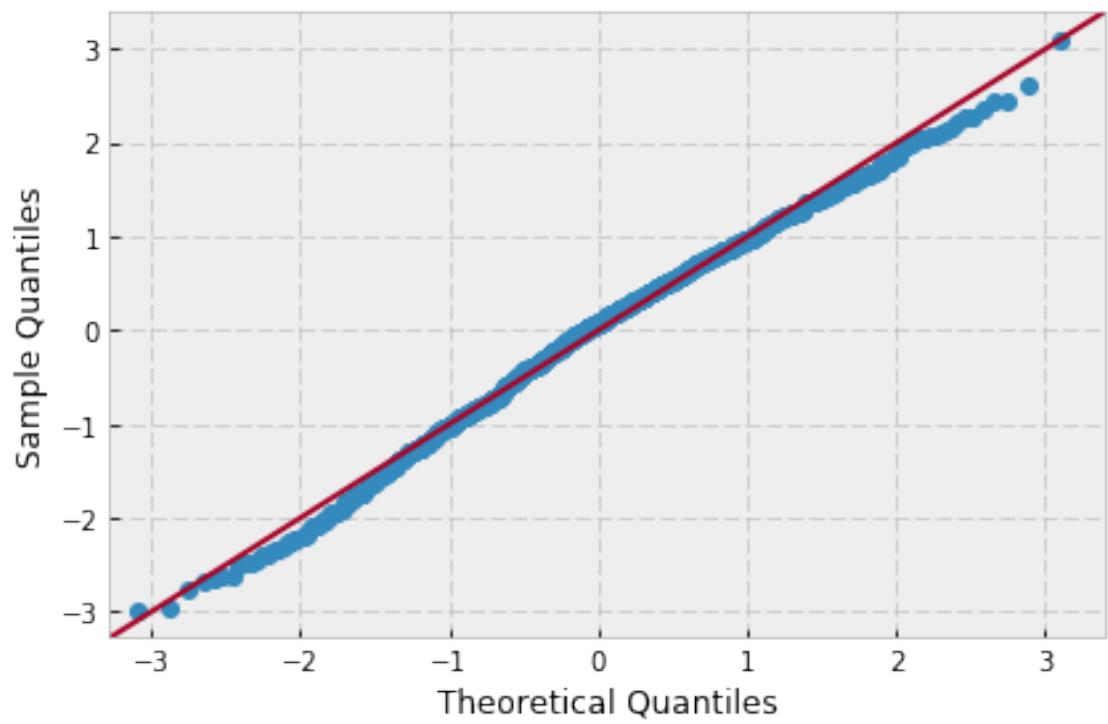
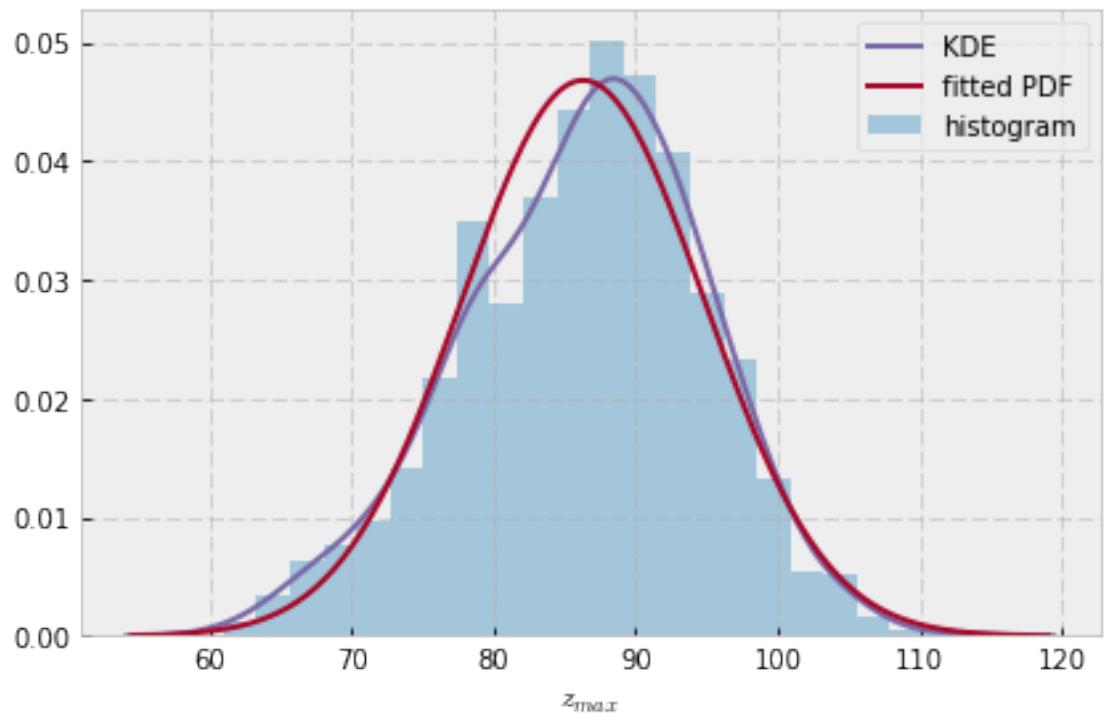


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=80.02078601959352, scale=8.962672524951222)
DescribeResult(nobs=1000, minmax=(47.669684283026825, 110.49258859696749)
               mean=80.020786019593515, variance=80.409908698213712,
               skewness=-0.12161739385737447, kurtosis=0.017119662632304244)
```

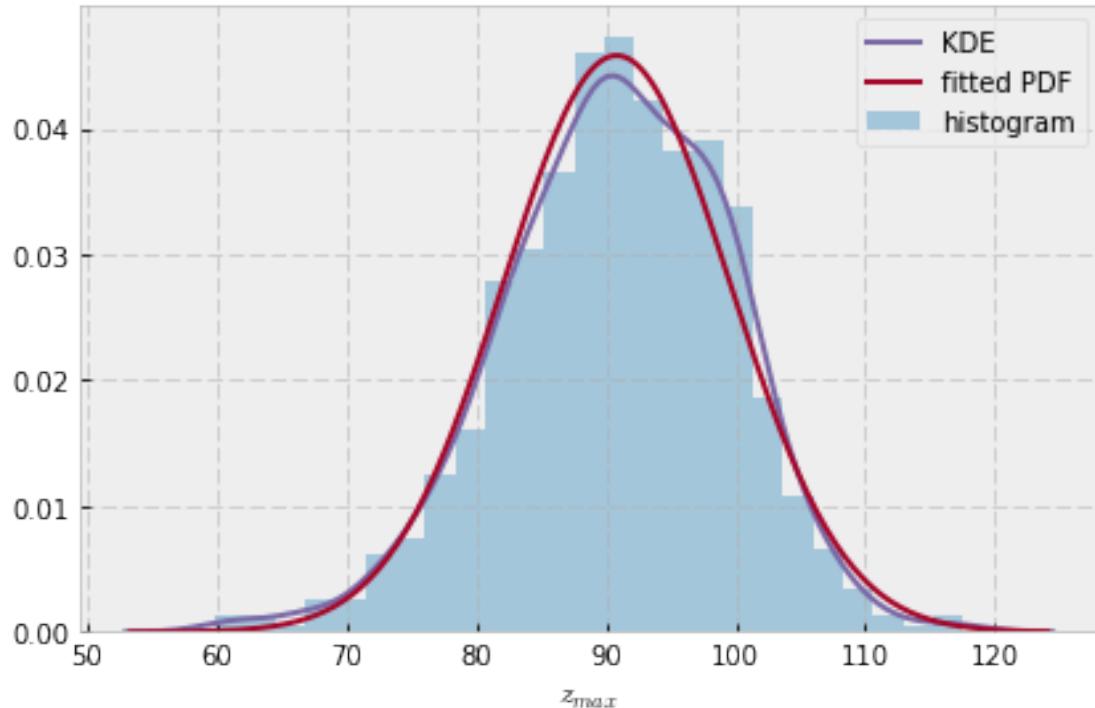


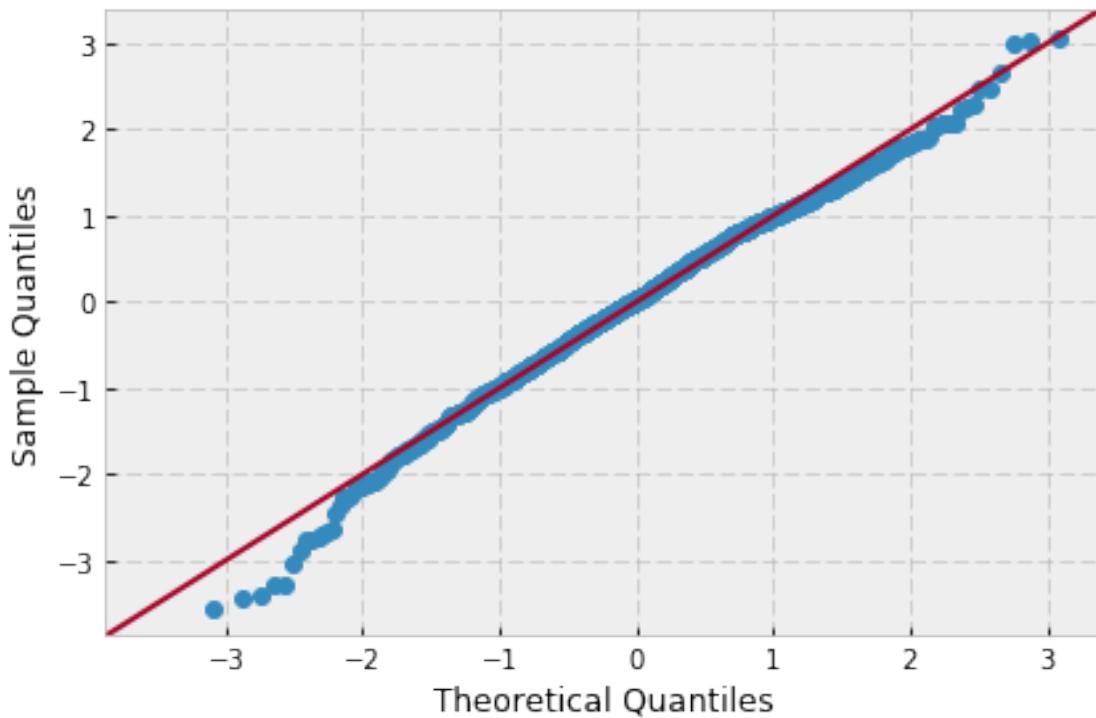


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(),loc=86.29576113872088, scale=8.522301980386318)
DescribeResult(nobs=1000, minmax=(60.947885935078268, 112.63299760682918)
               mean=86.295761138720877, variance=72.702333378274844,
               skewness=-0.25876063359601054, kurtosis=-0.12267861318030171)
```

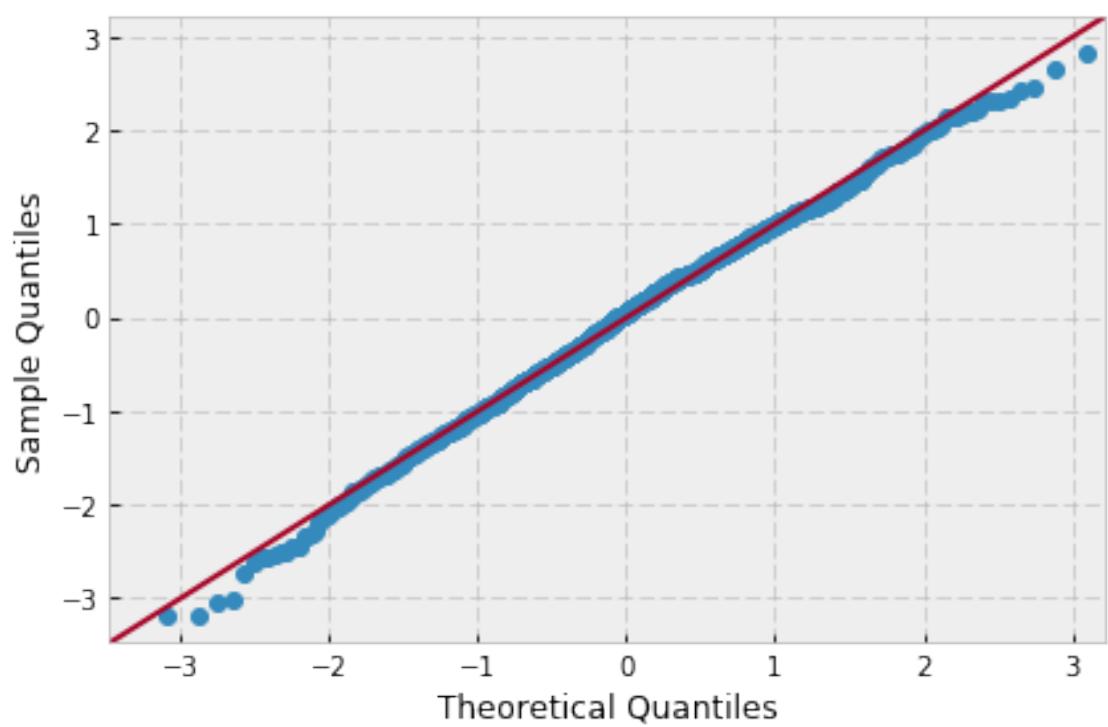
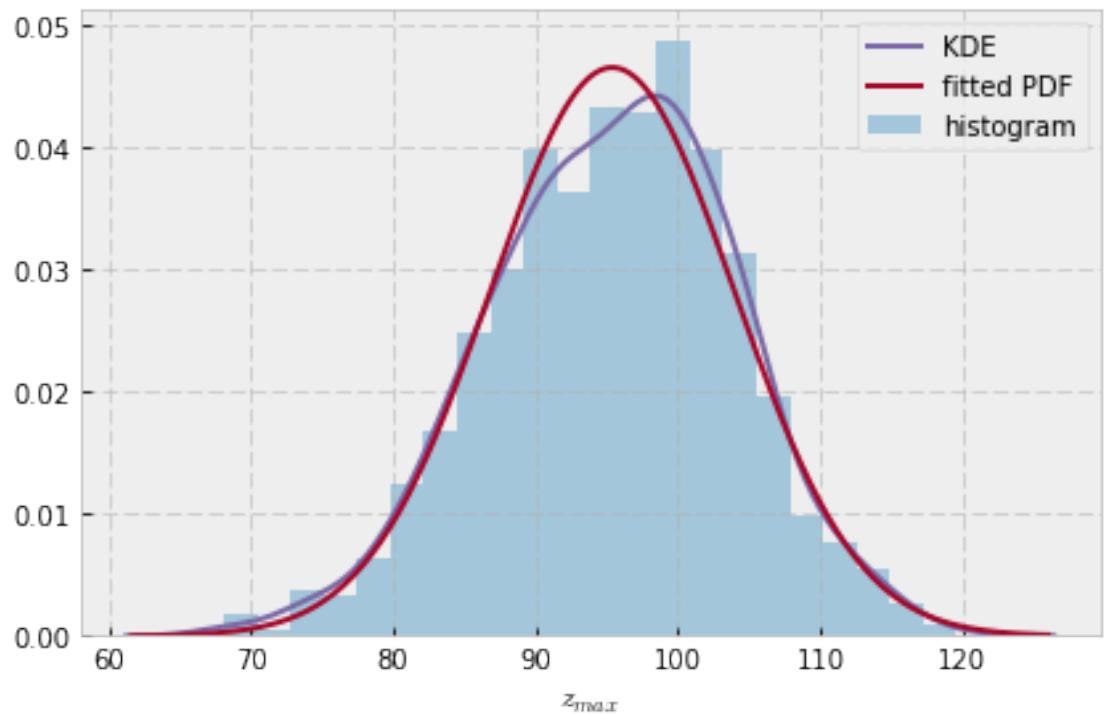


```
variable name: r
variable value: 2.555555555555554
distribution: normal(shape=(),loc=90.83499151541906, scale=8.710627833386317)
DescribeResult(nobs=1000, minmax=(59.93836635455235, 117.47312310192567)
               mean=90.834991515419063, variance=75.950988240004421,
               skewness=-0.2930929132723749, kurtosis=0.31830796321898935)
```

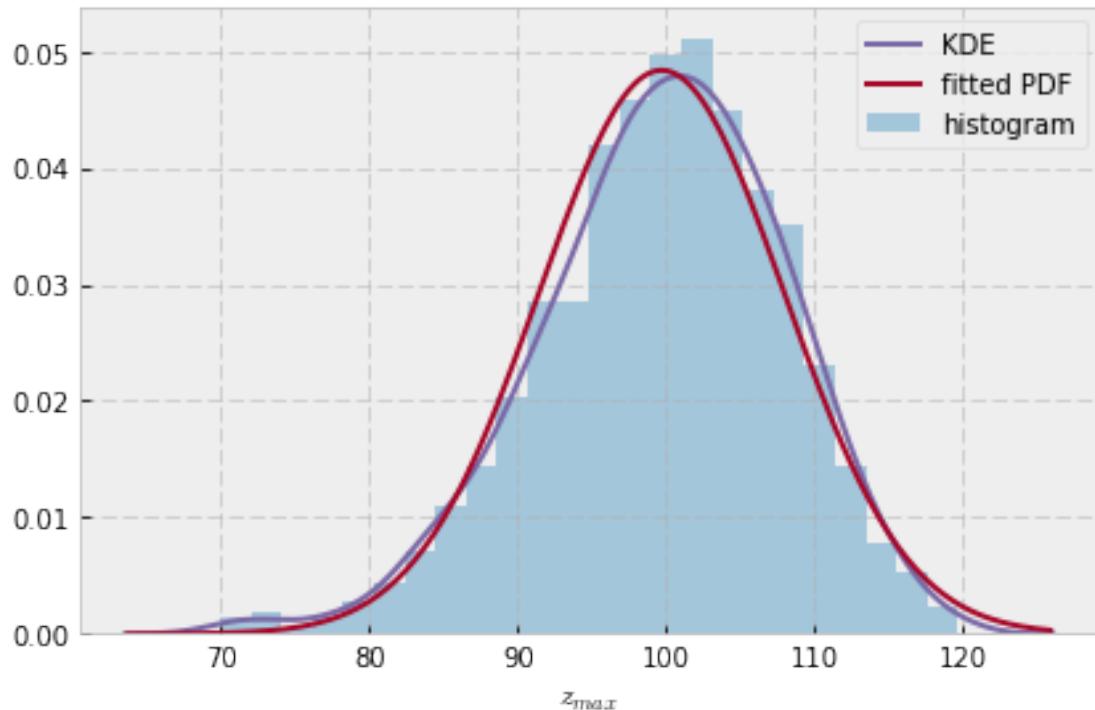


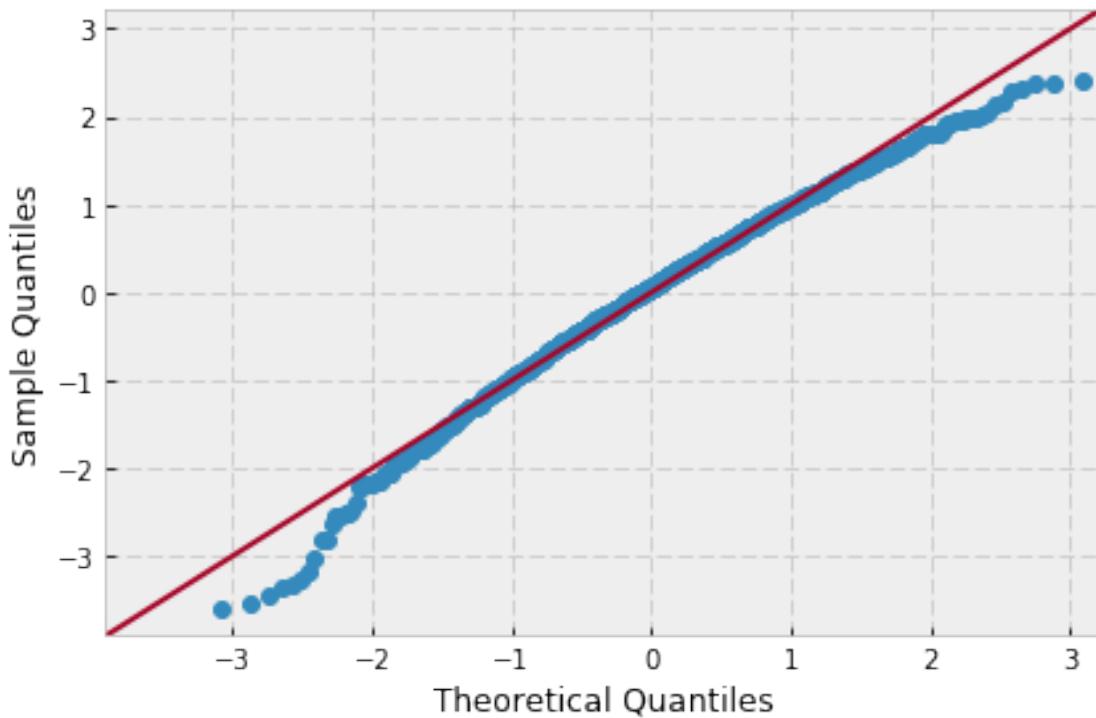


```
variable name: r
variable value: 2.7777777777777777
distribution: normal(shape=(),loc=95.38613846331187, scale=8.588411253169193)
DescribeResult(nobs=1000, minmax=(68.080635877401491, 119.56377372342365)
               mean=95.386138463311866, variance=73.834642496059303,
               skewness=-0.19066909042504146, kurtosis=-0.07786581080024346)
```

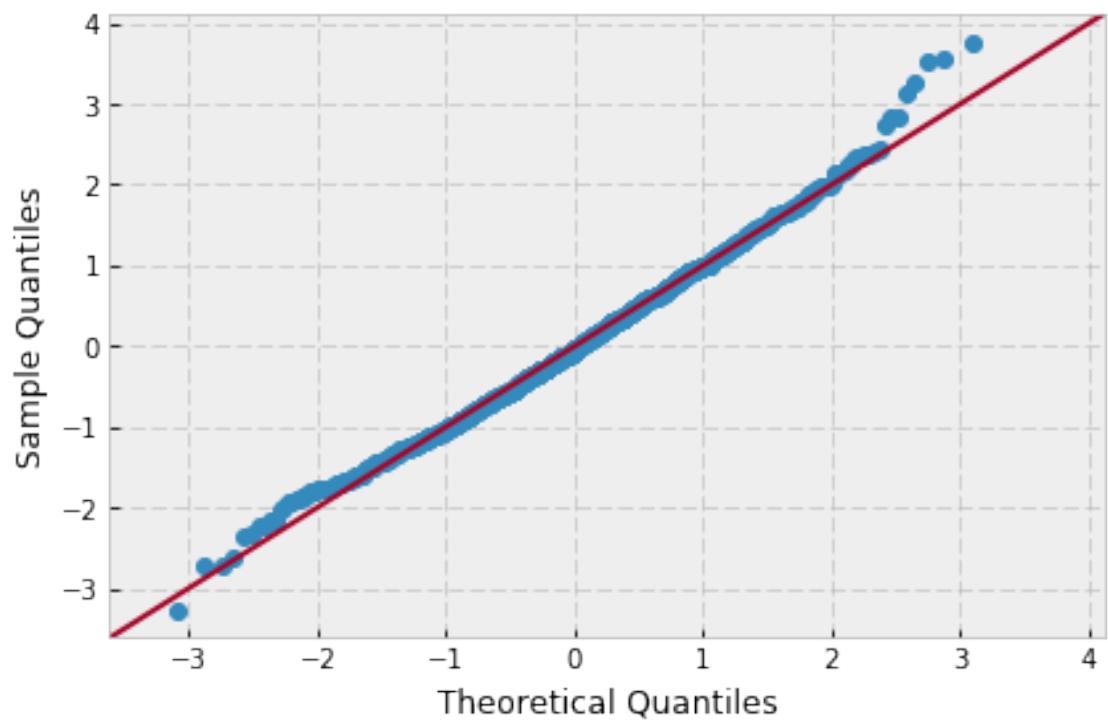
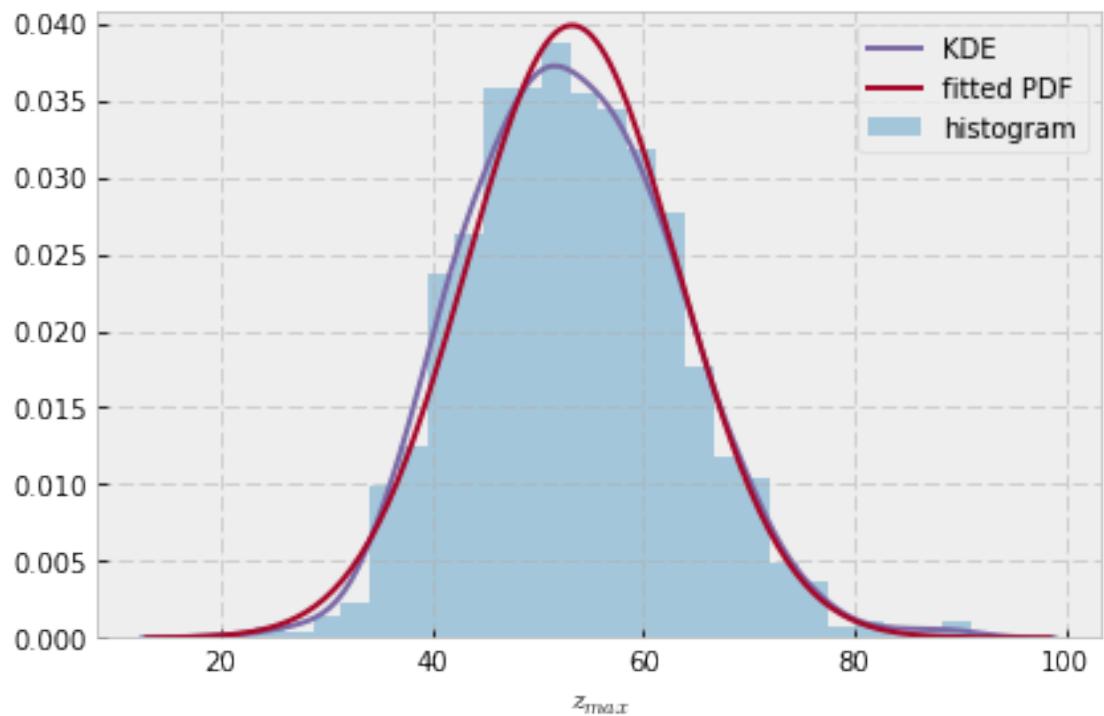


```
variable name: r
variable value: 3.0
distribution: normal(shape=(),loc=99.6946059357553, scale=8.247815757622336)
DescribeResult(nobs=1000, minmax=(69.940457934179065, 119.67036036898186)
               mean=99.694605935755305, variance=68.094559331014324,
               skewness=-0.43501632855243383, kurtosis=0.3101412779718955)
```

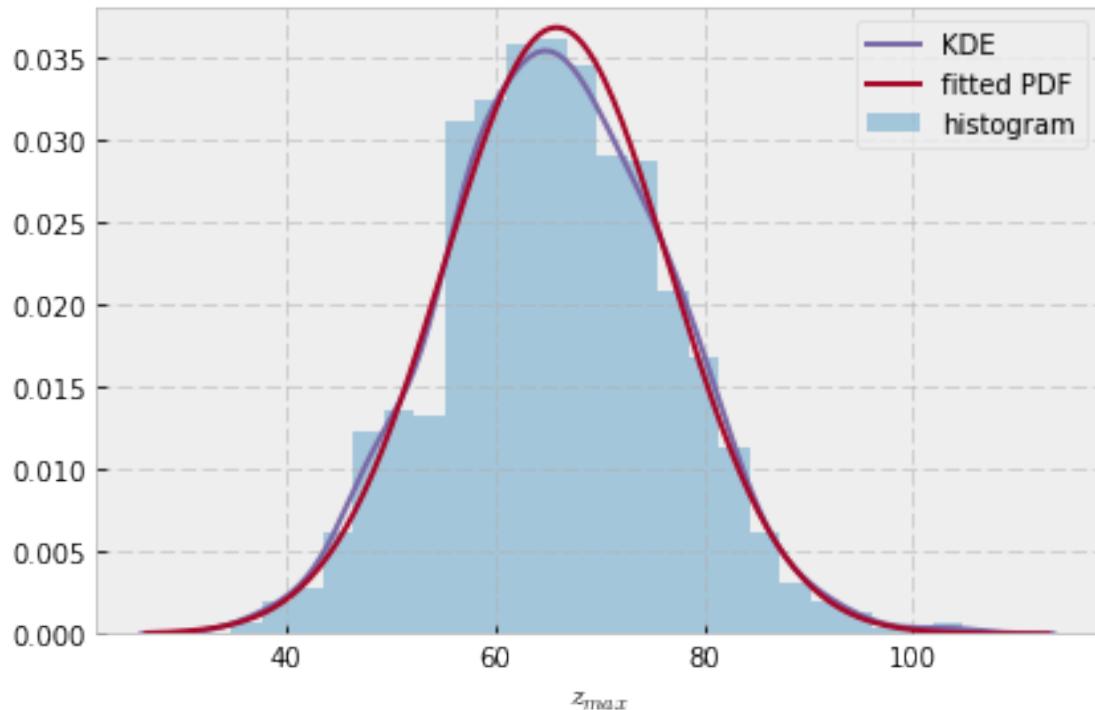


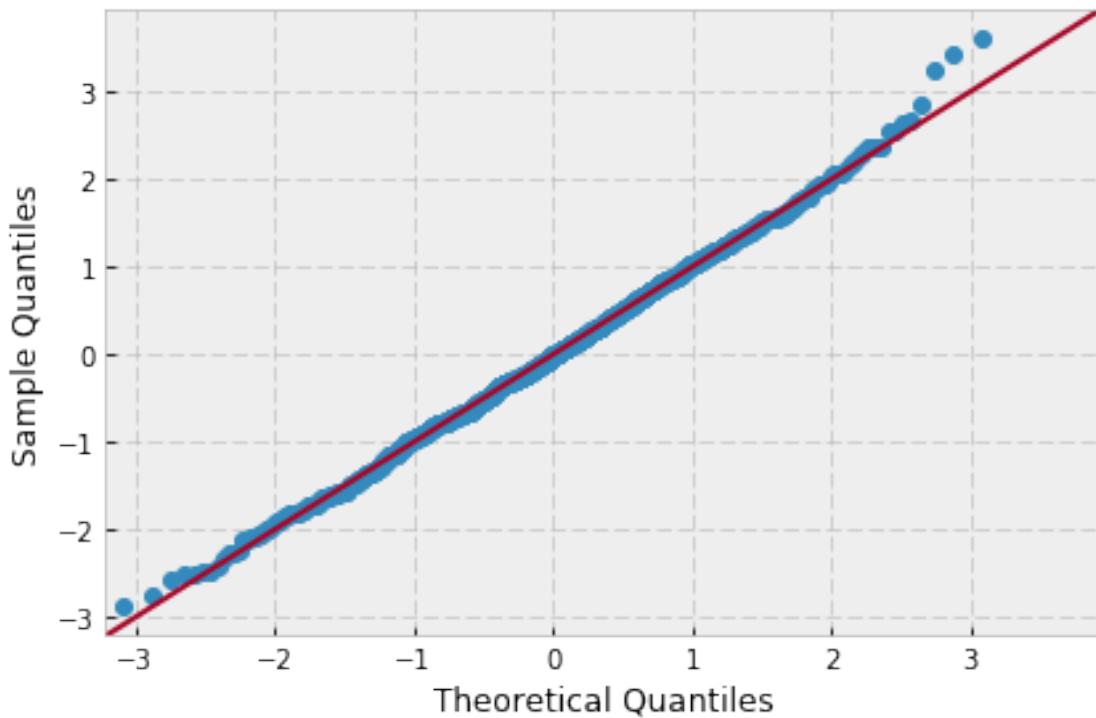


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(), loc=53.25820727057305, scale=10.003563733286029)
DescribeResult(nobs=1000, minmax=(20.58574701345233, 90.947315232995948)
               mean=53.258207270573052, variance=100.17145882474026,
               skewness=0.2523089023038684, kurtosis=0.1643538088733143)
```

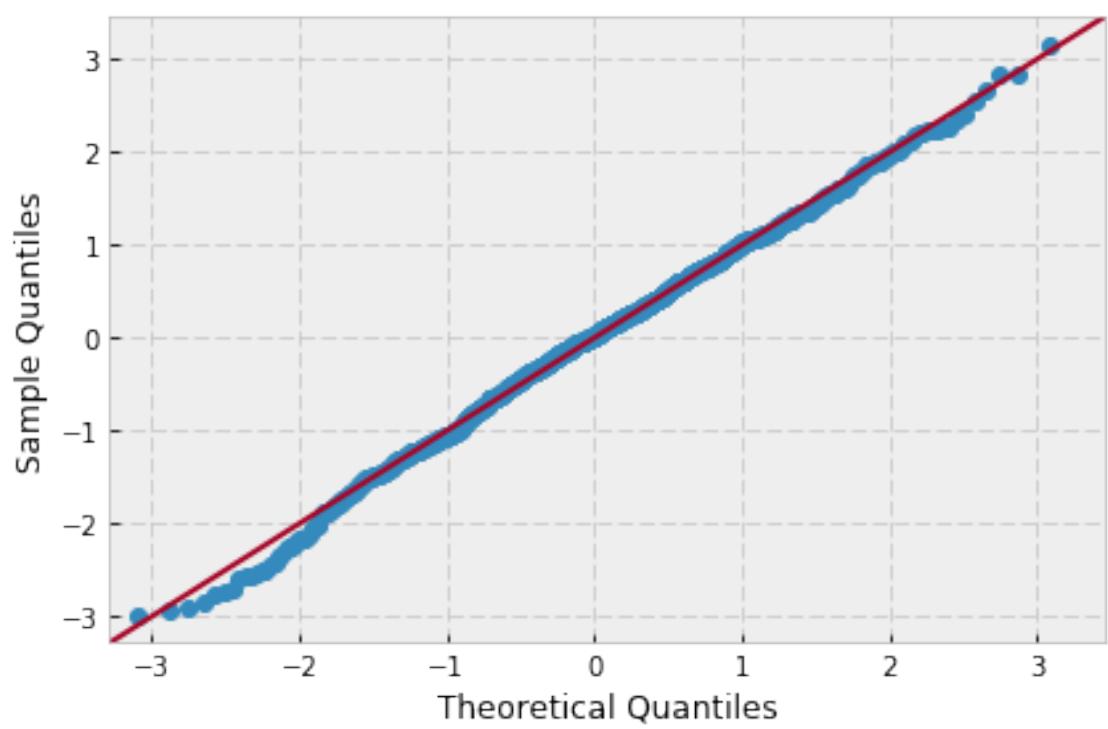
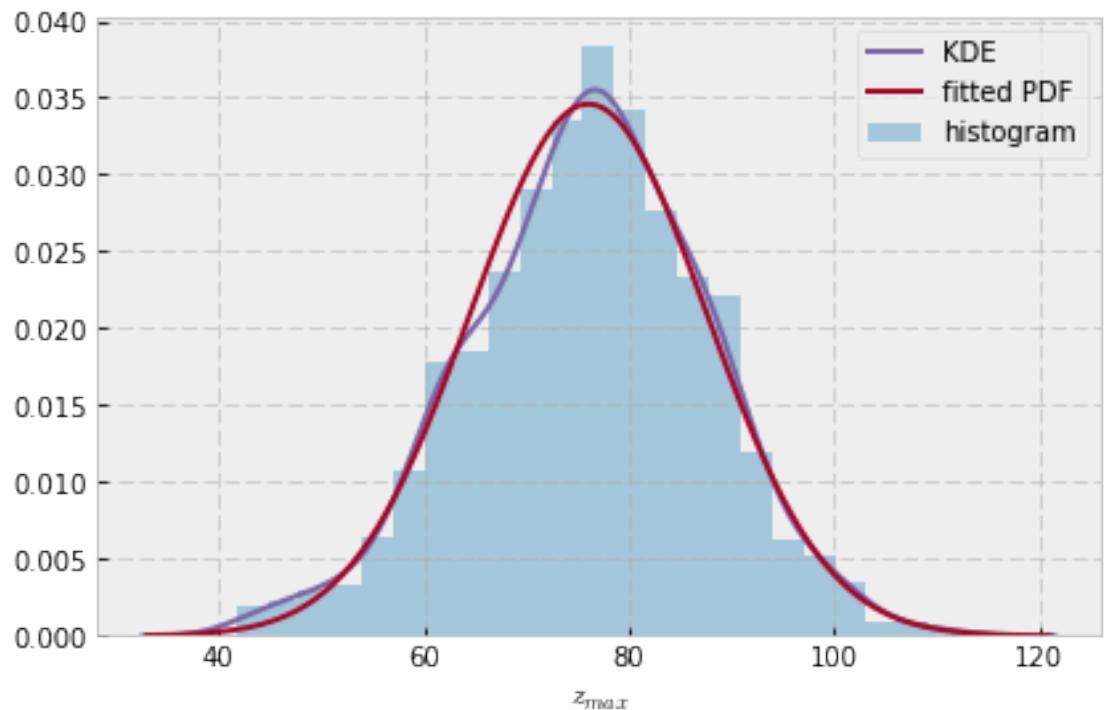


```
variable name: sigma
variable value: 0.0222222222222223
distribution: normal(shape=(),loc=65.82811425988608, scale=10.813440799213247)
DescribeResult(nobs=1000, minmax=(34.685781715317383, 104.8351075017038)
               mean=65.828114259886078, variance=117.04754946755716,
               skewness=0.09670946461300595, kurtosis=0.00501826228983937)
```

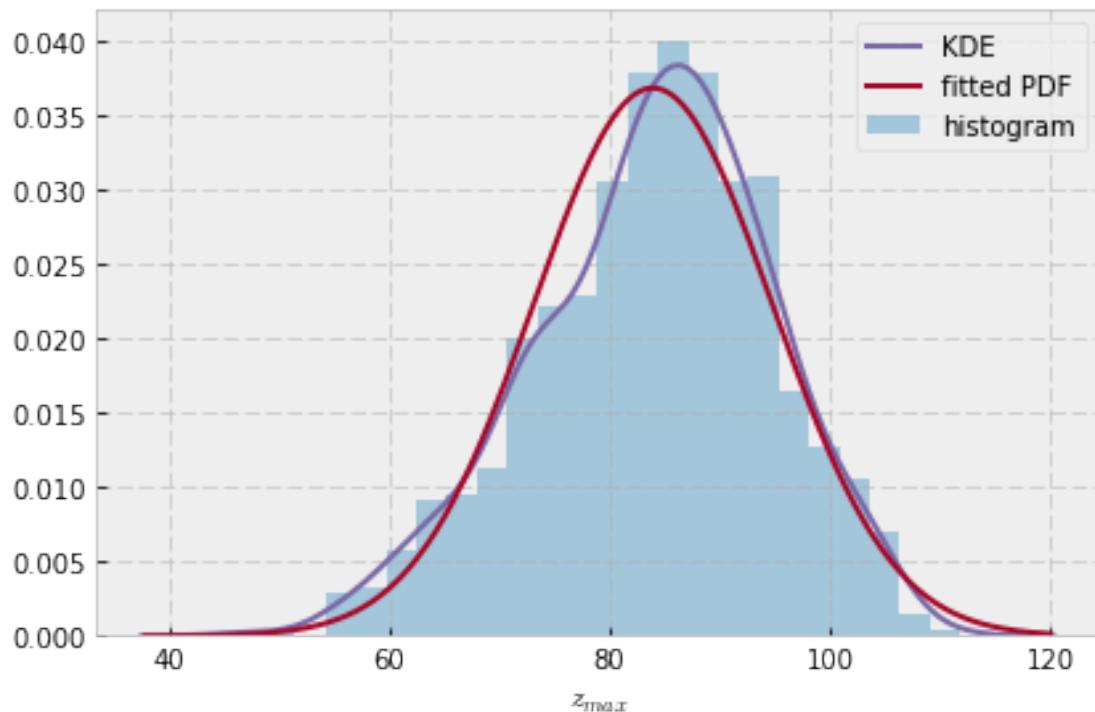


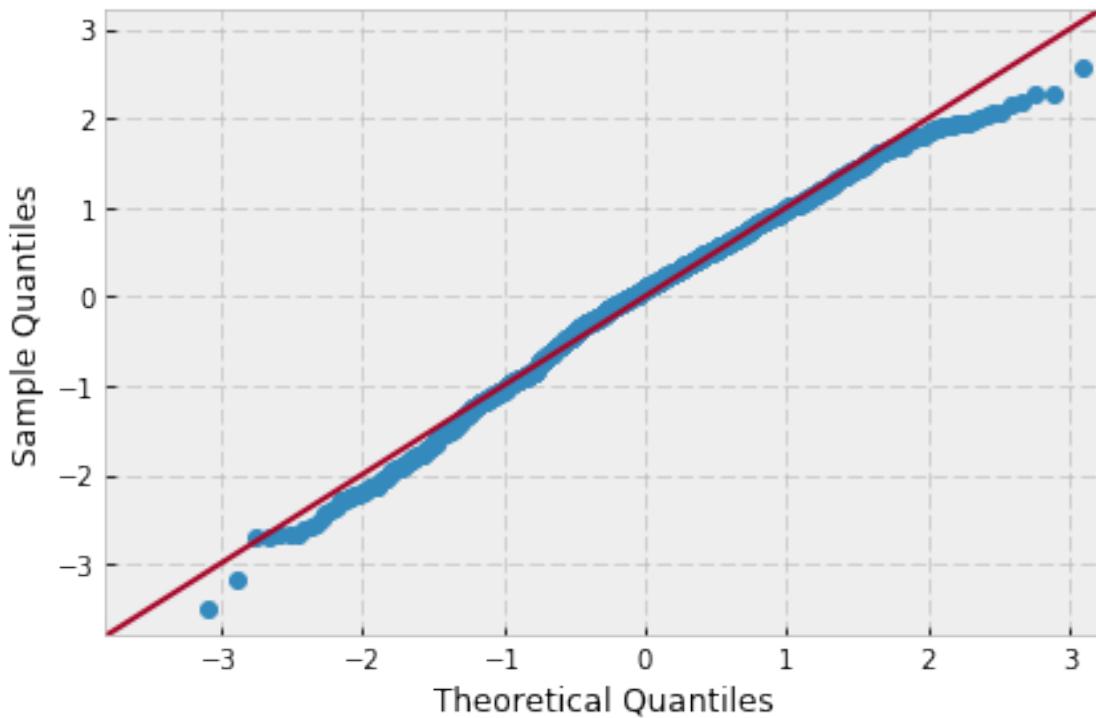


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=76.05183429297685, scale=11.530845153545284)
DescribeResult(nobs=1000, minmax=(41.691474355196426, 112.37481818395885)
               mean=76.051834292976849, variance=133.09348343847725,
               skewness=-0.14147139367396772, kurtosis=0.05053770451815787)
```

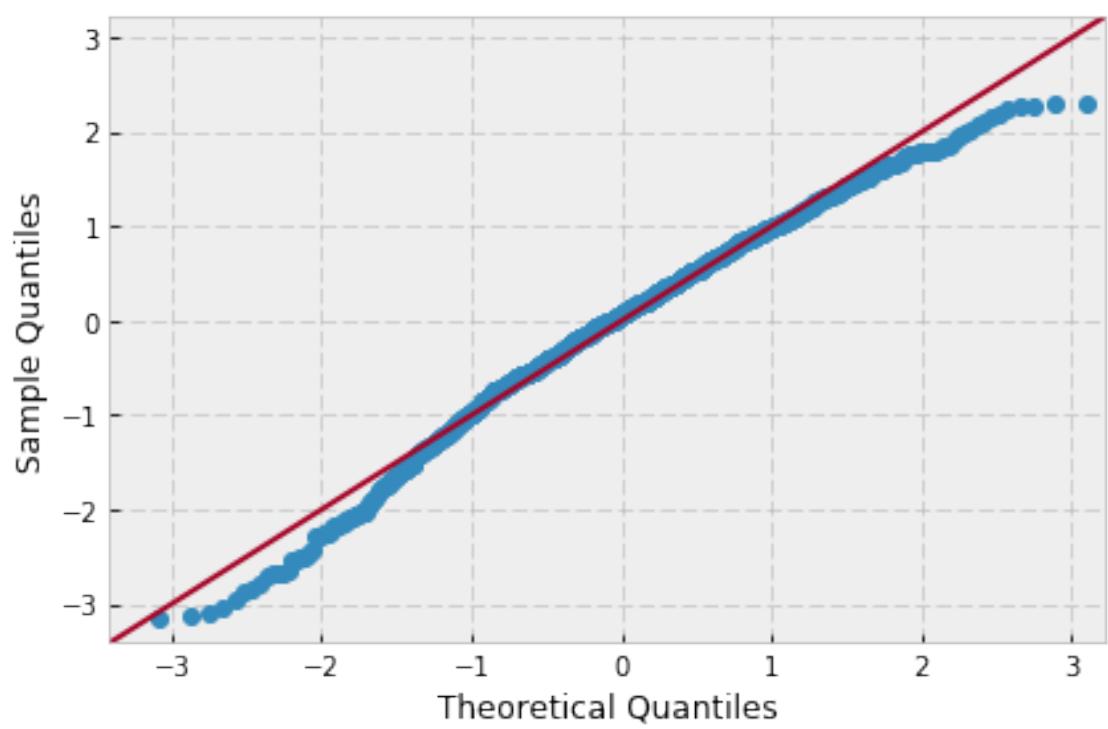
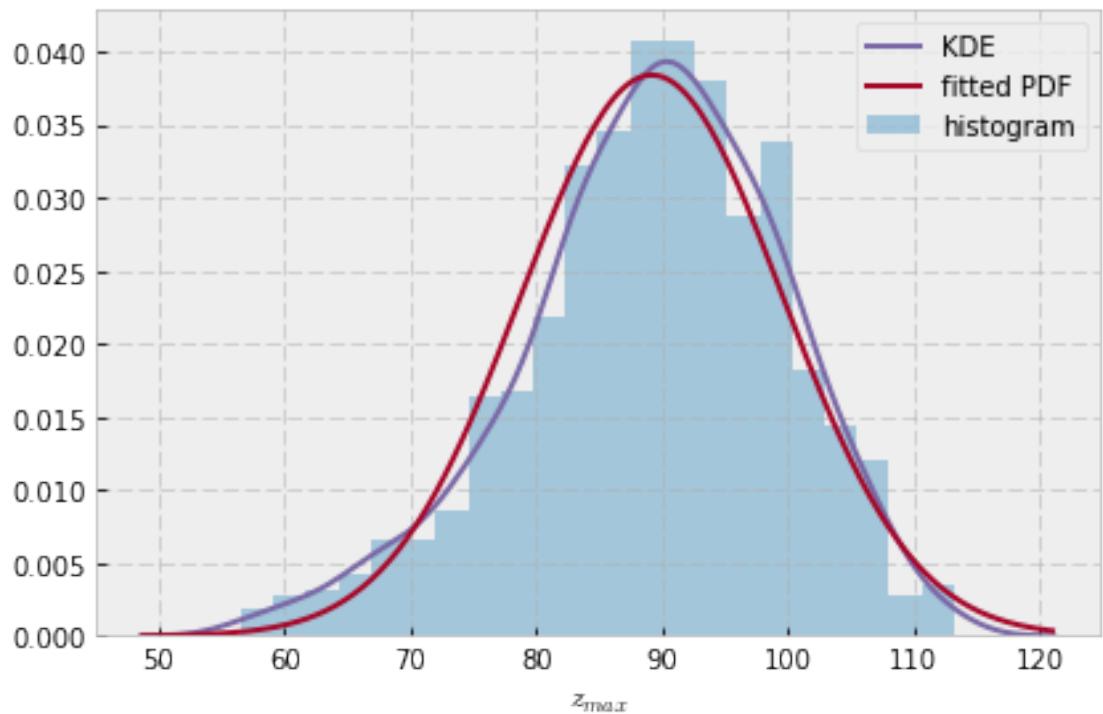


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(),loc=83.94823062944394, scale=10.83682393913526)
DescribeResult(nobs=1000, minmax=(45.962462236411952, 111.87485720735332)
               mean=83.948230629443941, variance=117.55430739521023,
               skewness=-0.35003014377849867, kurtosis=-0.09268331770222638)
```

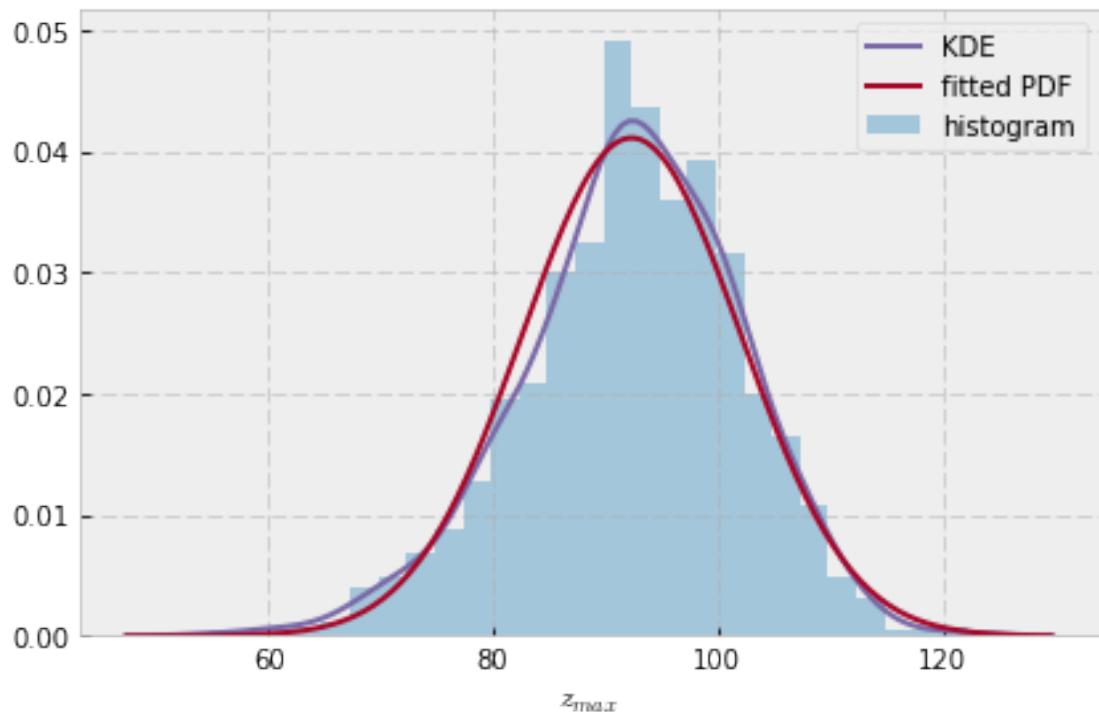


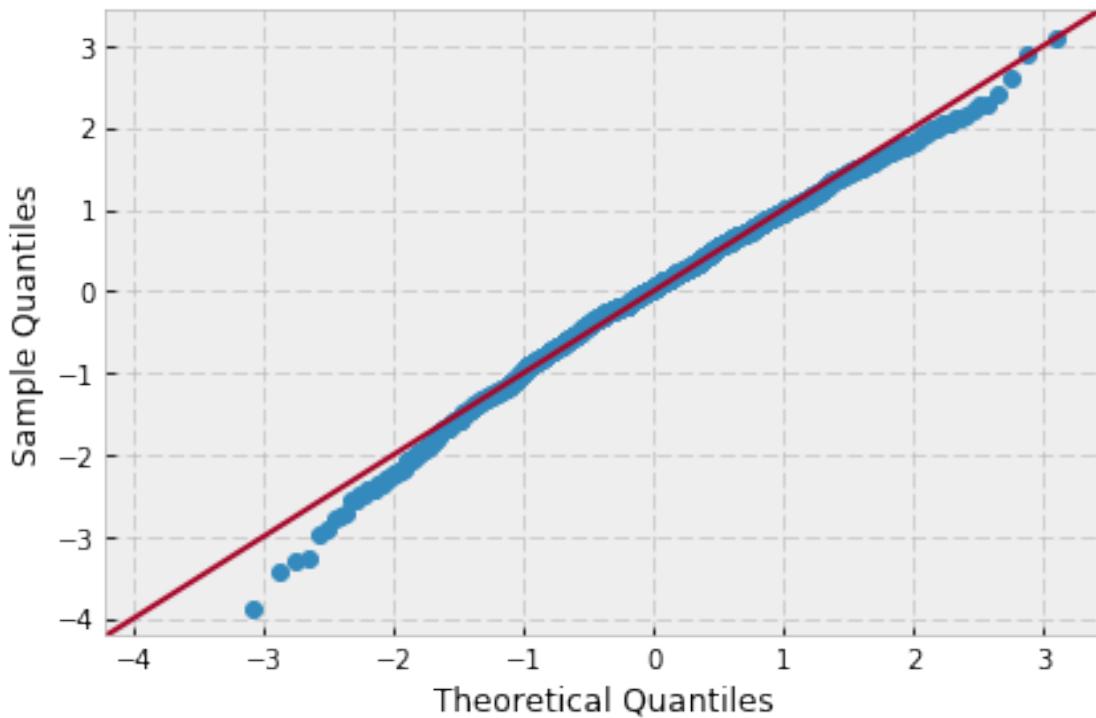


```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(),loc=89.18925484603722, scale=10.386577110070284)
DescribeResult(nobs=1000, minmax=(56.592457361096471, 113.1781357597005)
               mean=89.189254846037215, variance=107.98897303647244,
               skewness=-0.4338974288274305, kurtosis=0.12149538359127465)
```

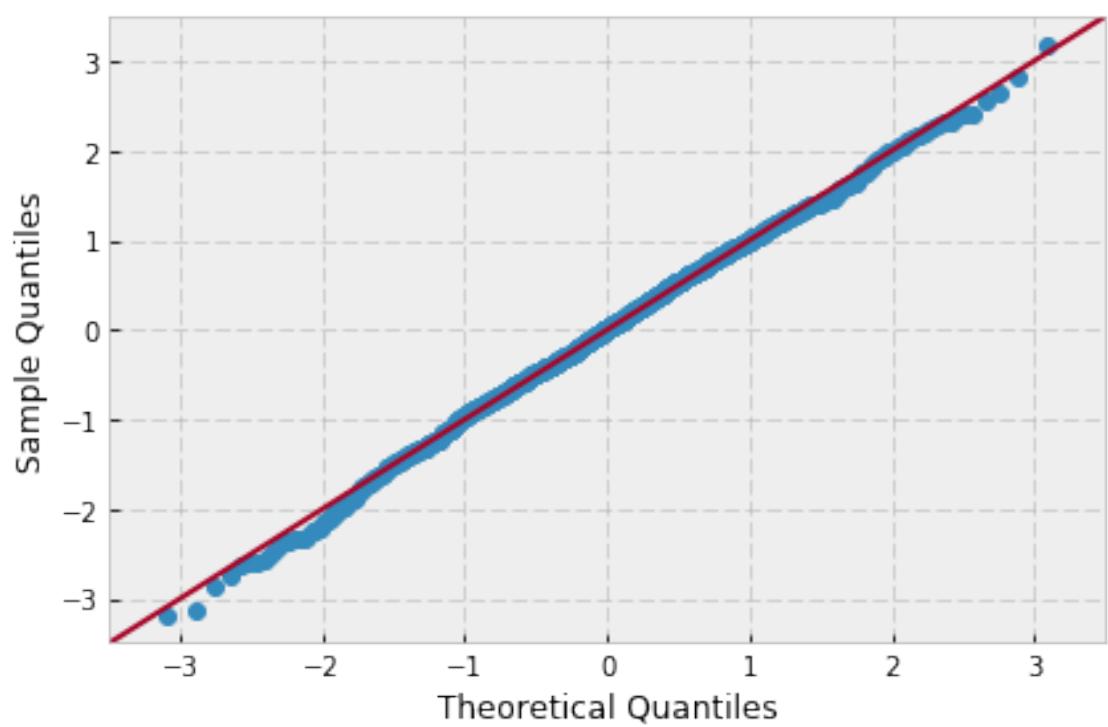
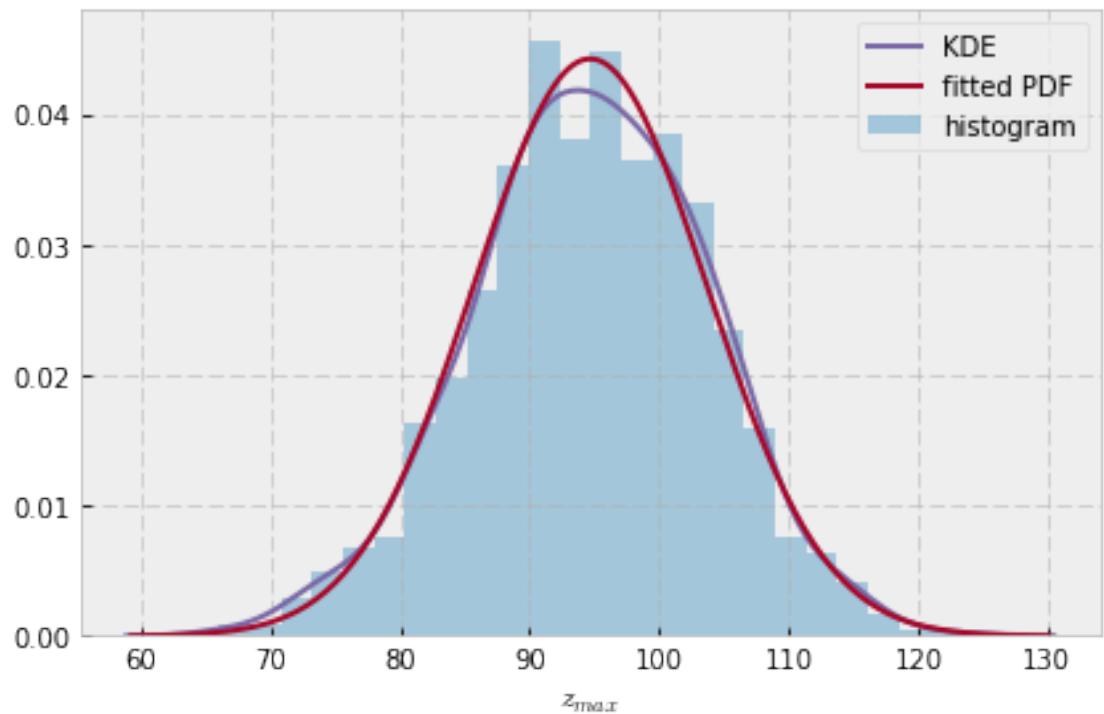


```
variable name: sigma
variable value: 0.06666666666666667
distribution: normal(shape=(),loc=92.3363376953257, scale=9.70902925389129)
DescribeResult(nobs=1000, minmax=(54.791630793994074, 122.31737433228407)
               mean=92.336337695325696, variance=94.359608661578434,
               skewness=-0.3412048573989796, kurtosis=0.32437900150778054)
```

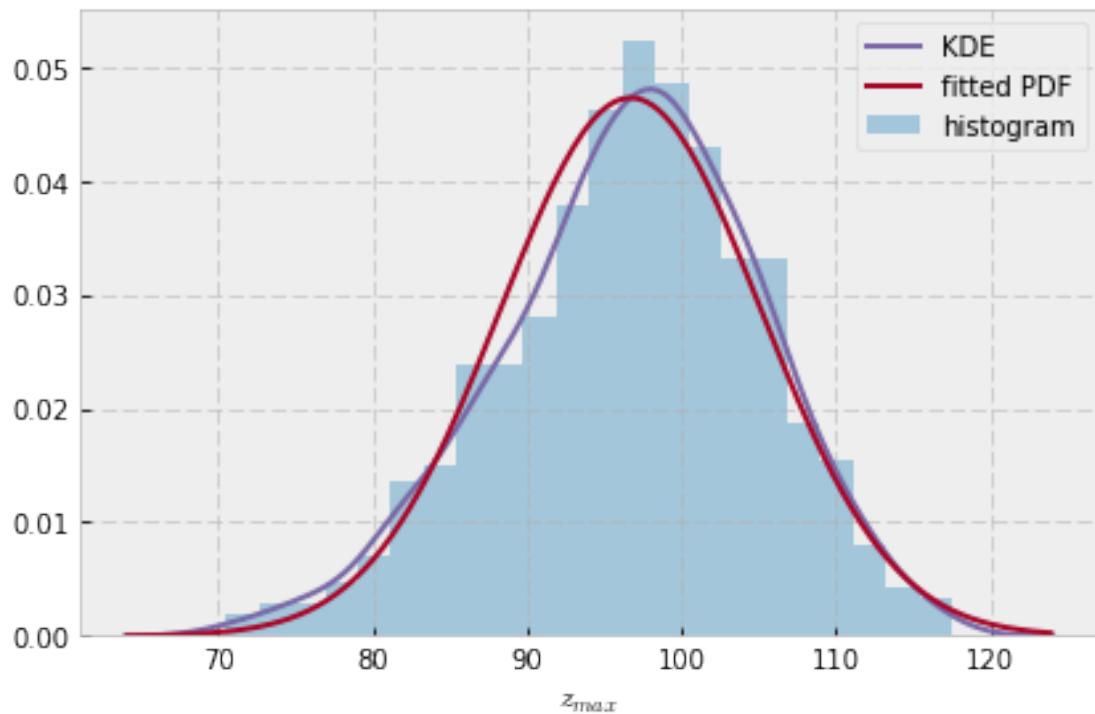


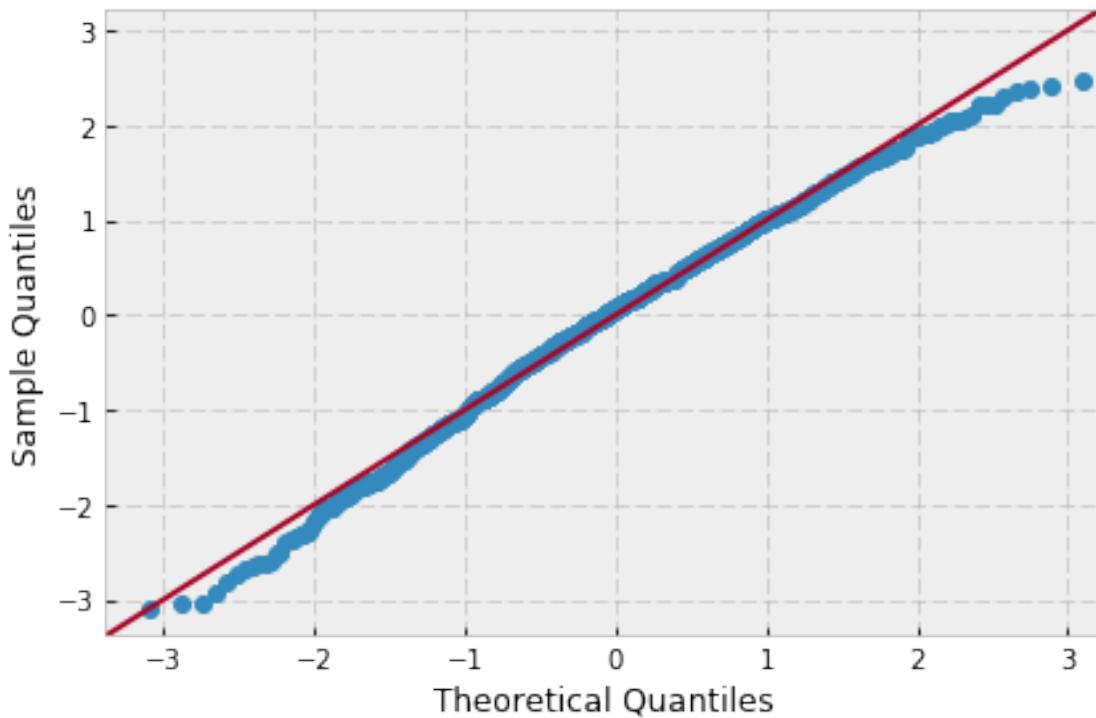


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=94.68581574791783, scale=9.018698823092159)
DescribeResult(nobs=1000, minmax=(66.045526114269066, 123.30089761104692)
               mean=94.685815747917829, variance=81.418346808452355,
               skewness=-0.1199383142900634, kurtosis=-0.014342960396485527)
```

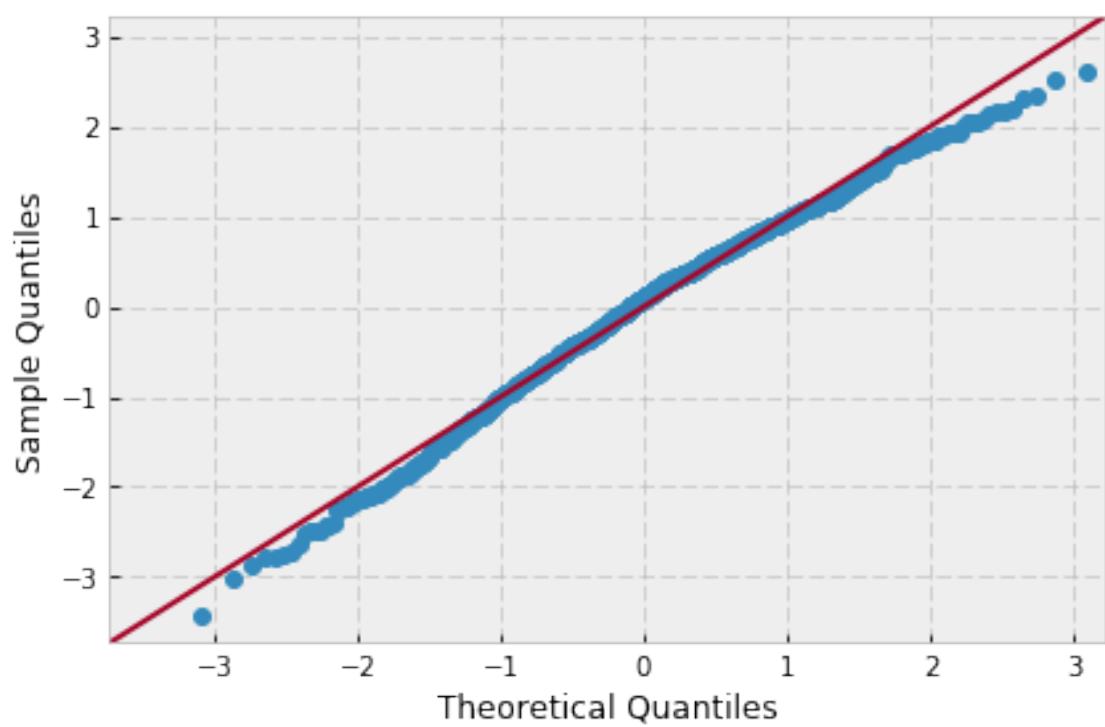
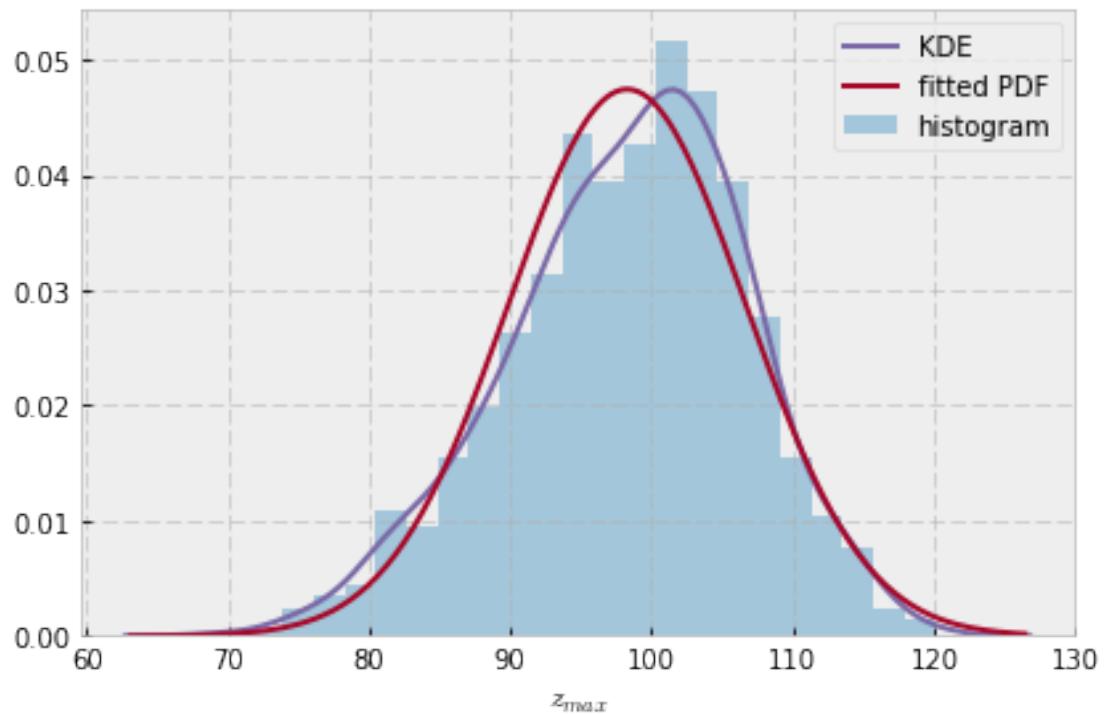


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(),loc=96.68031540470055, scale=8.438940073106293)
DescribeResult(nobs=1000, minmax=(70.462503882156398, 117.51155212176558)
               mean=96.680315404700551, variance=71.28699655403328,
               skewness=-0.30462660357898436, kurtosis=-0.05508815040866111)
```

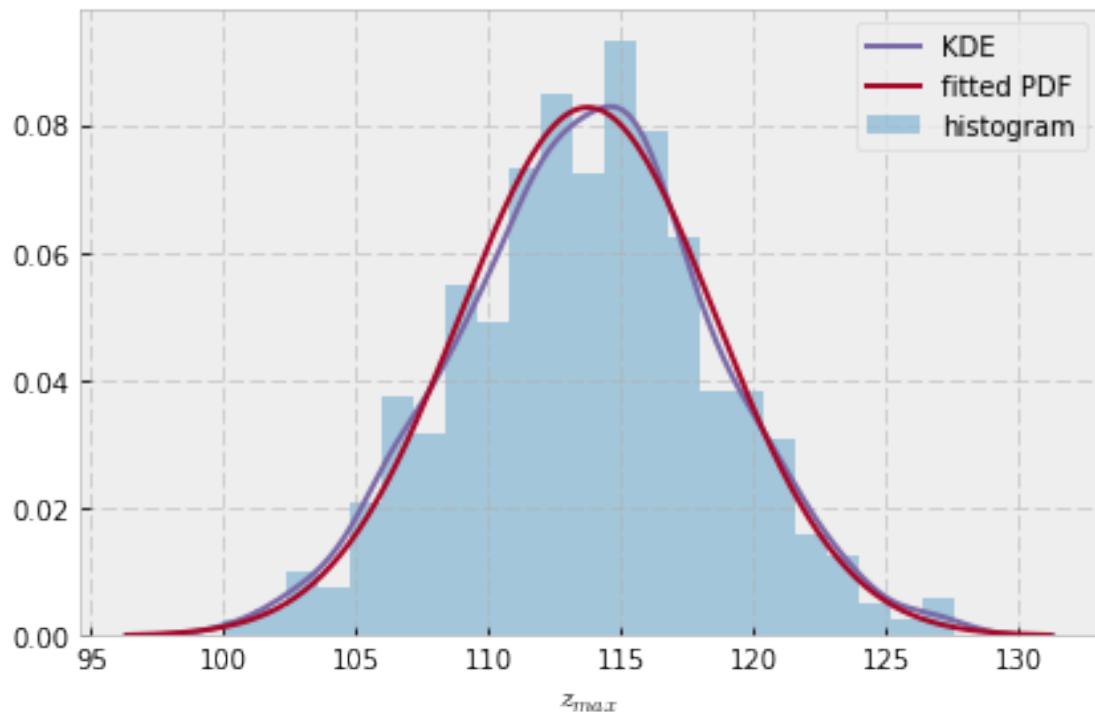


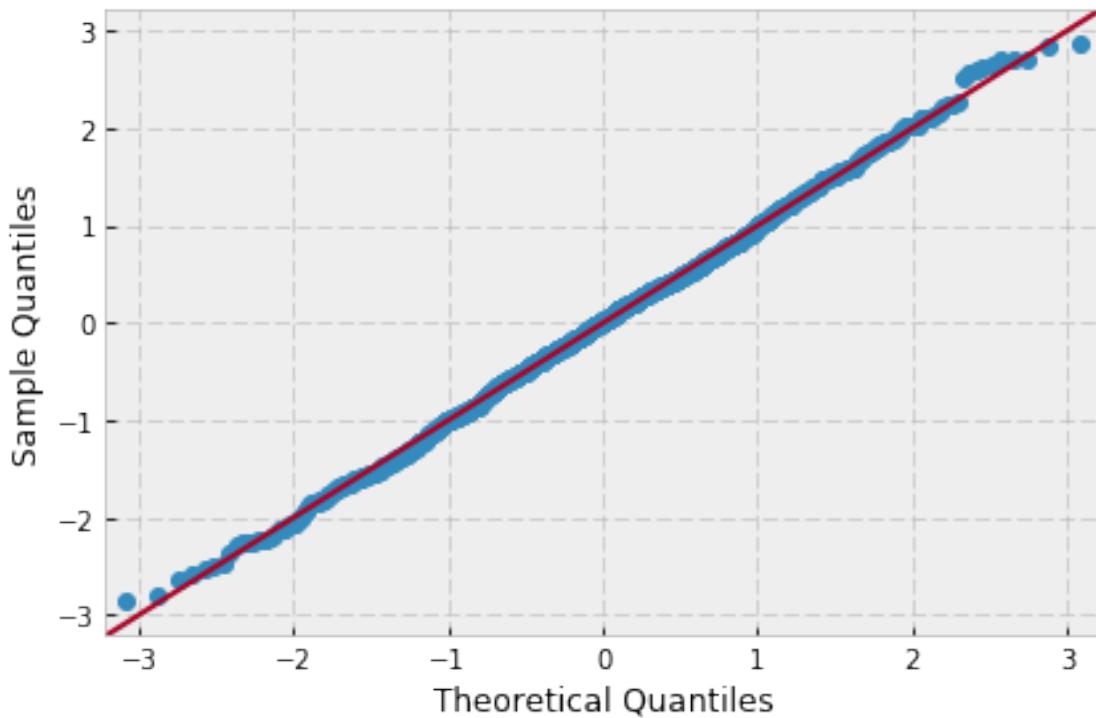


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(),loc=98.27628994146184, scale=8.408310282713316)
DescribeResult(nobs=1000, minmax=(69.434516026966577, 120.11294504698445)
               mean=98.27628994146184, variance=70.770452262645136,
               skewness=-0.34643730176474963, kurtosis=-0.0813633532822573)
```

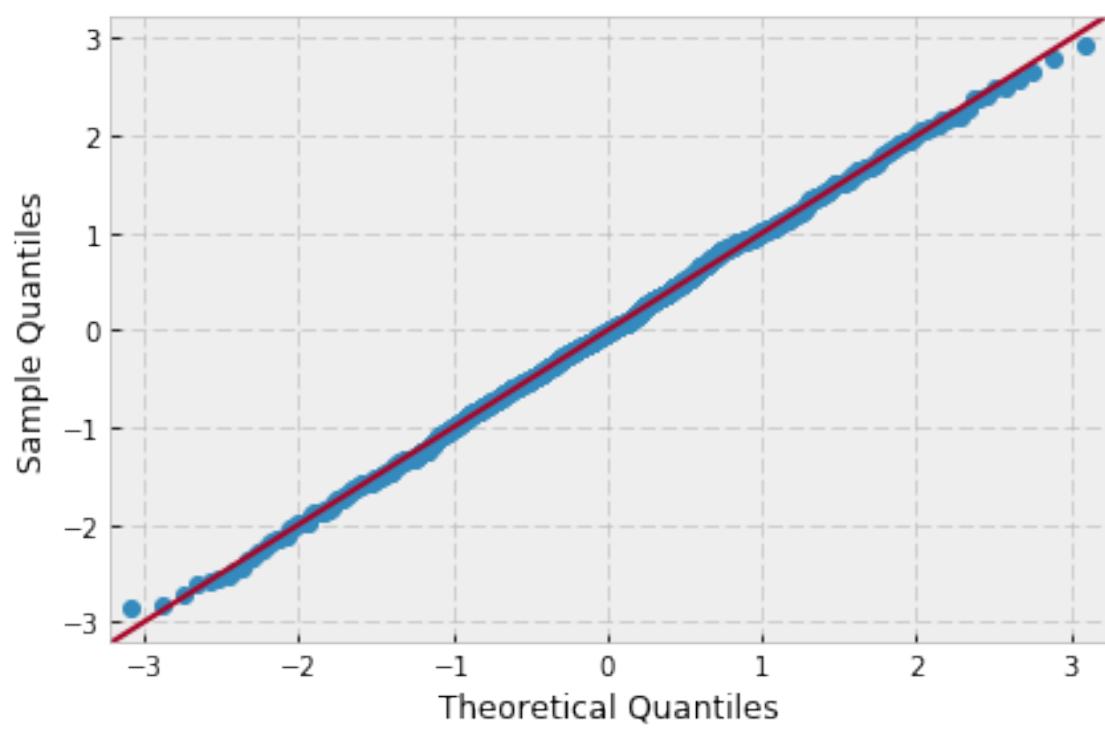
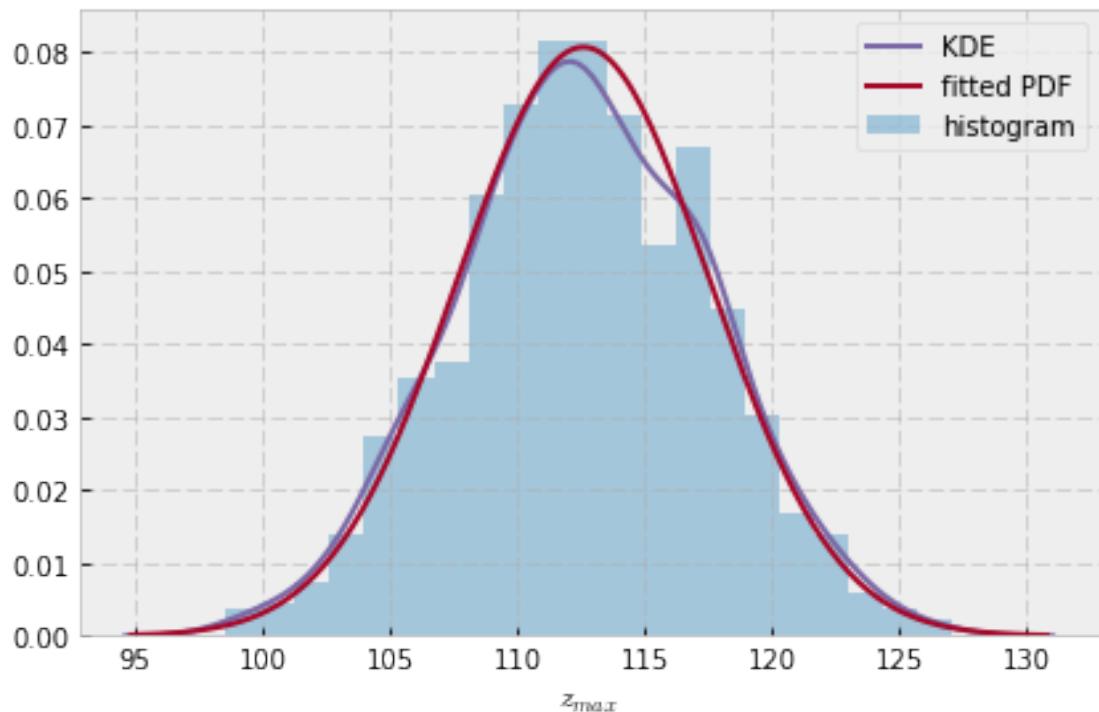


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=113.75613846789926, scale=4.824611361253967)
DescribeResult(nobs=1000, minmax=(99.997773847745933, 127.61737017492753)
               mean=113.75613846789926, variance=23.300174962102957,
               skewness=0.012851131180975597, kurtosis=-0.10413580982941983)
```

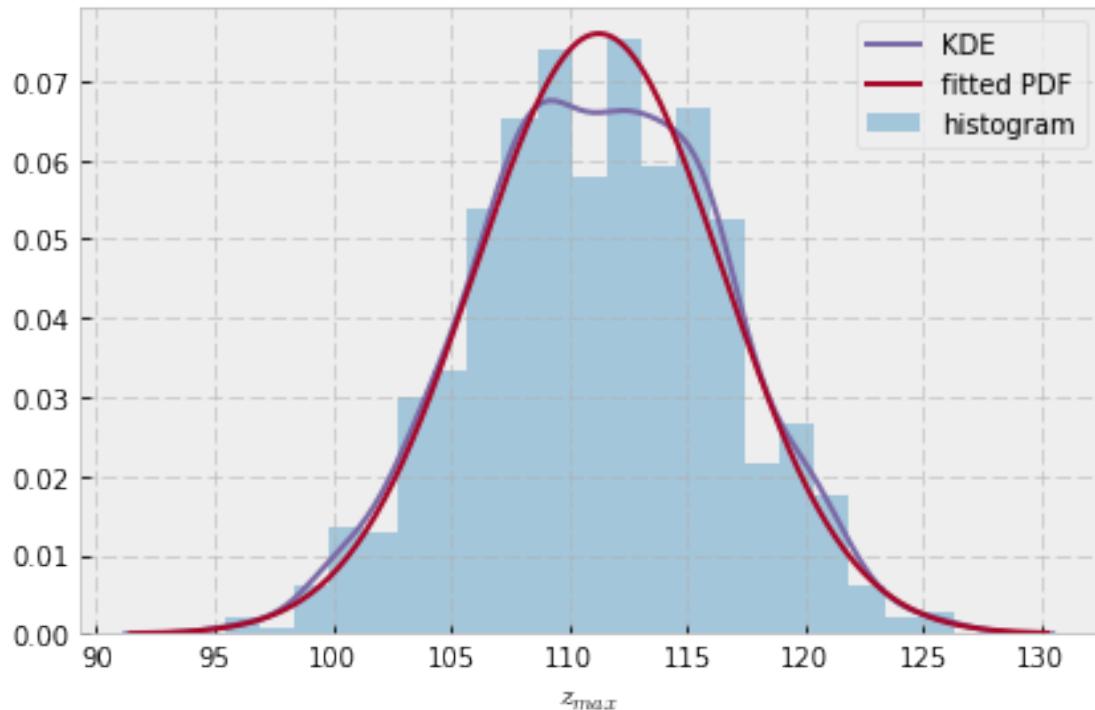


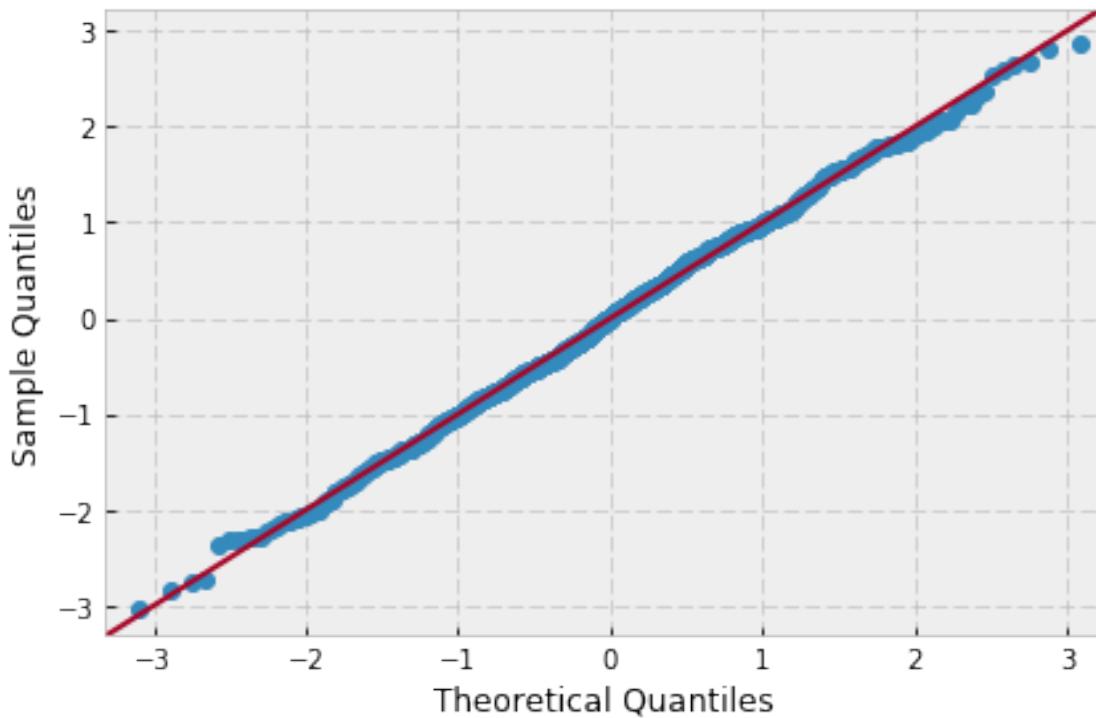


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(),loc=112.61517499968441, scale=4.943762098225543)
DescribeResult(nobs=1000, minmax=(98.571356626562363, 127.10558824425472)
               mean=112.61517499968441, variance=24.465248932784203,
               skewness=-0.016192117636339053, kurtosis=-0.1978695286948824)
```

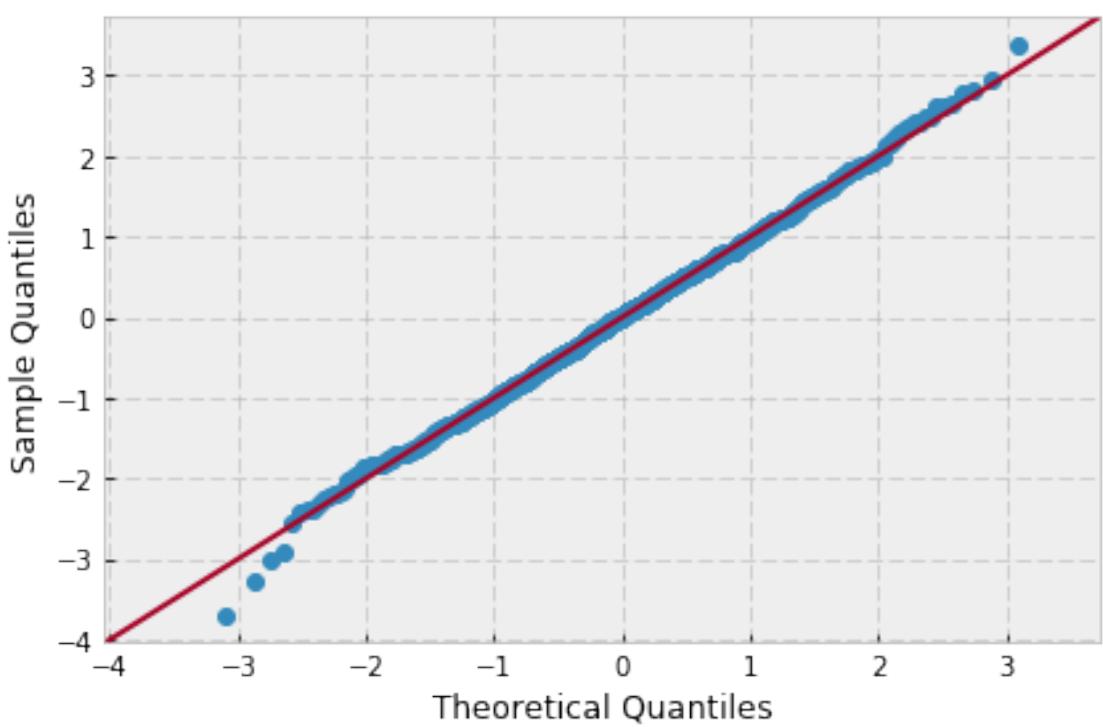
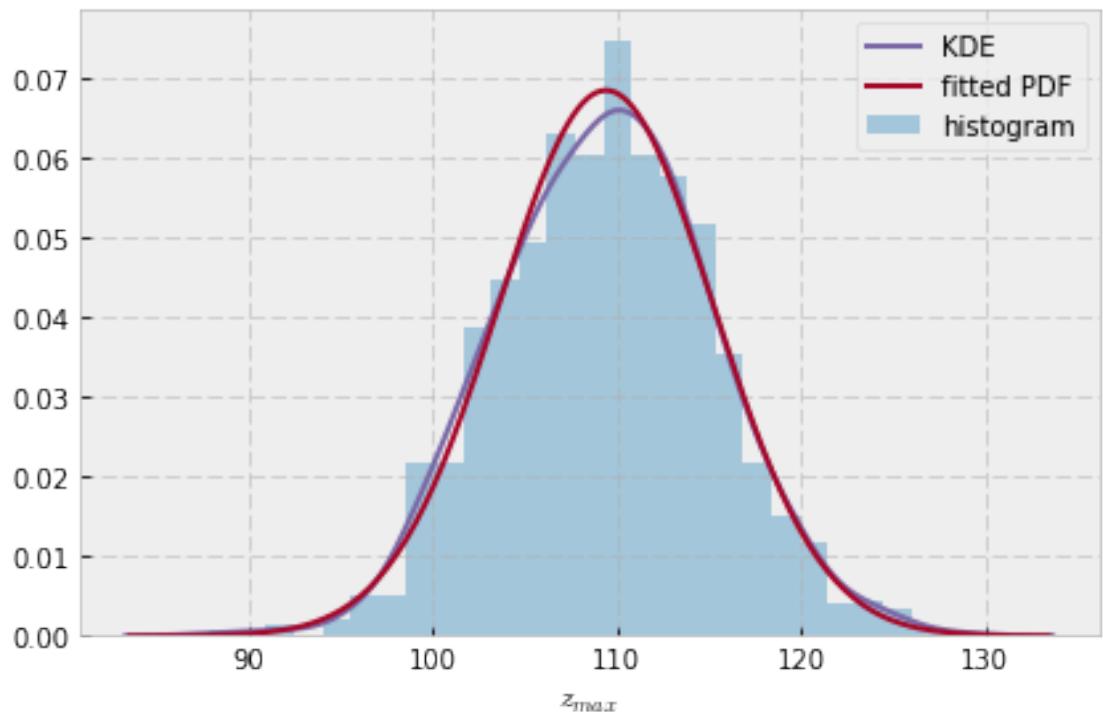


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=111.26135547213534, scale=5.248360006096953)
DescribeResult(nobs=1000, minmax=(95.410623431763284, 126.2935477884815)
               mean=111.26135547213534, variance=27.57285560920722,
               skewness=-0.031022284909504344, kurtosis=-0.2754189295893874)
```

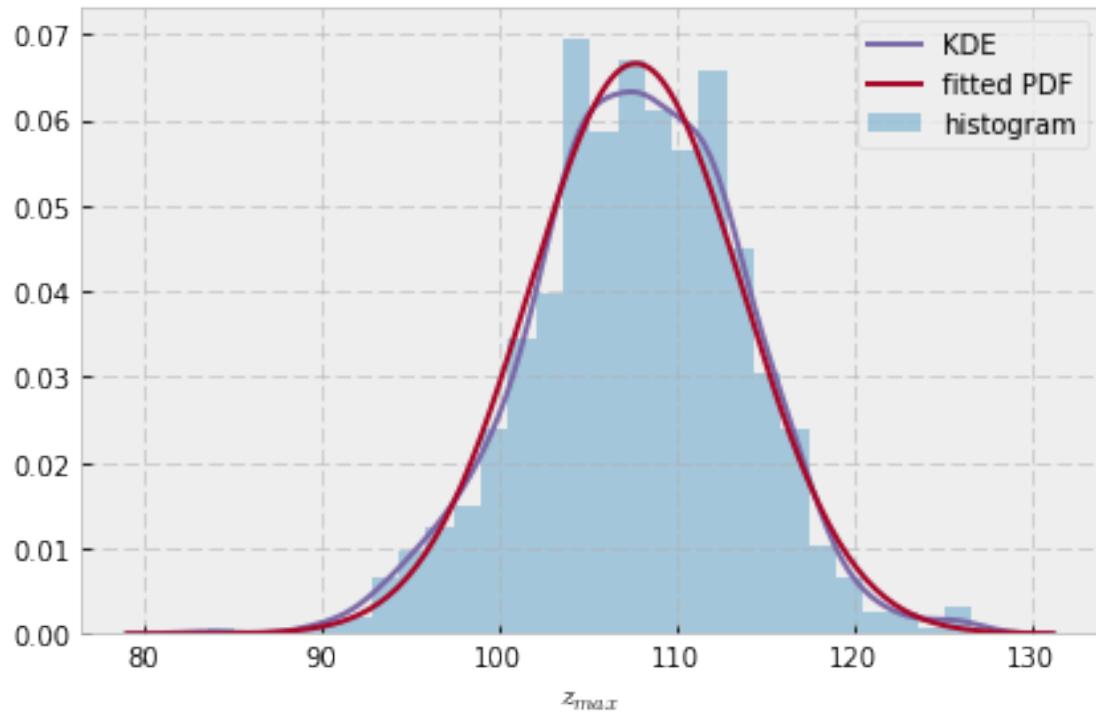


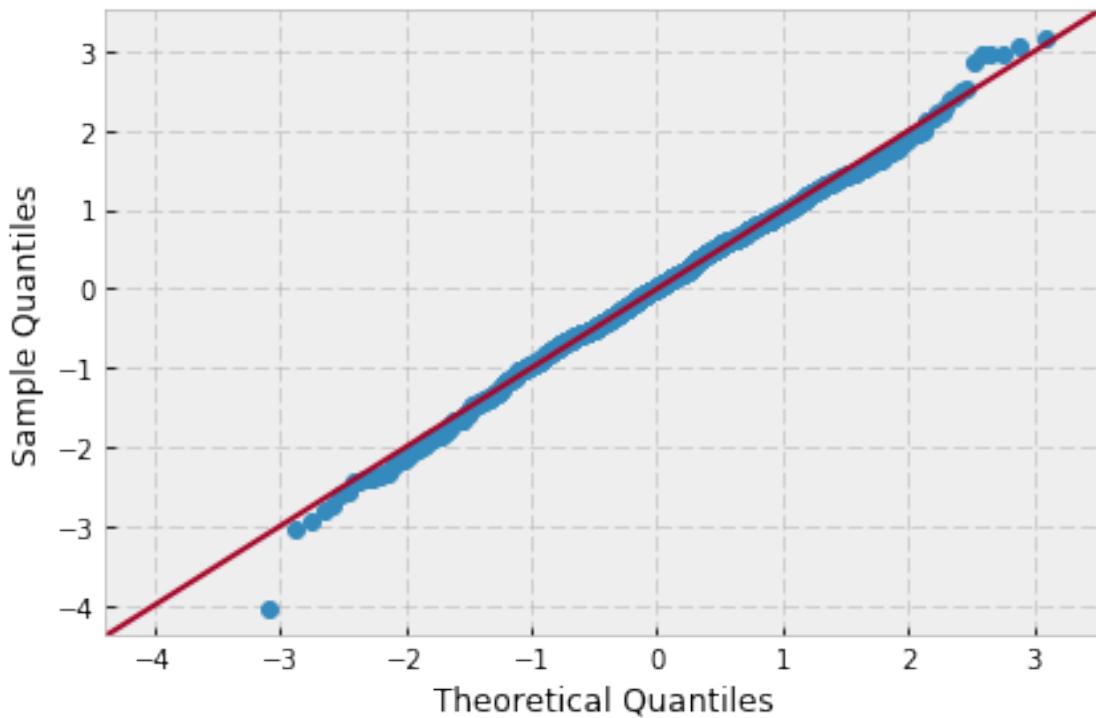


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(),loc=109.41224444222787, scale=5.822222664925973)
DescribeResult(nobs=1000, minmax=(87.936901192111392, 129.05011937586397)
               mean=109.41224444222787, variance=33.932208968946647,
               skewness=0.014930706934920558, kurtosis=0.08112124748901639)
```

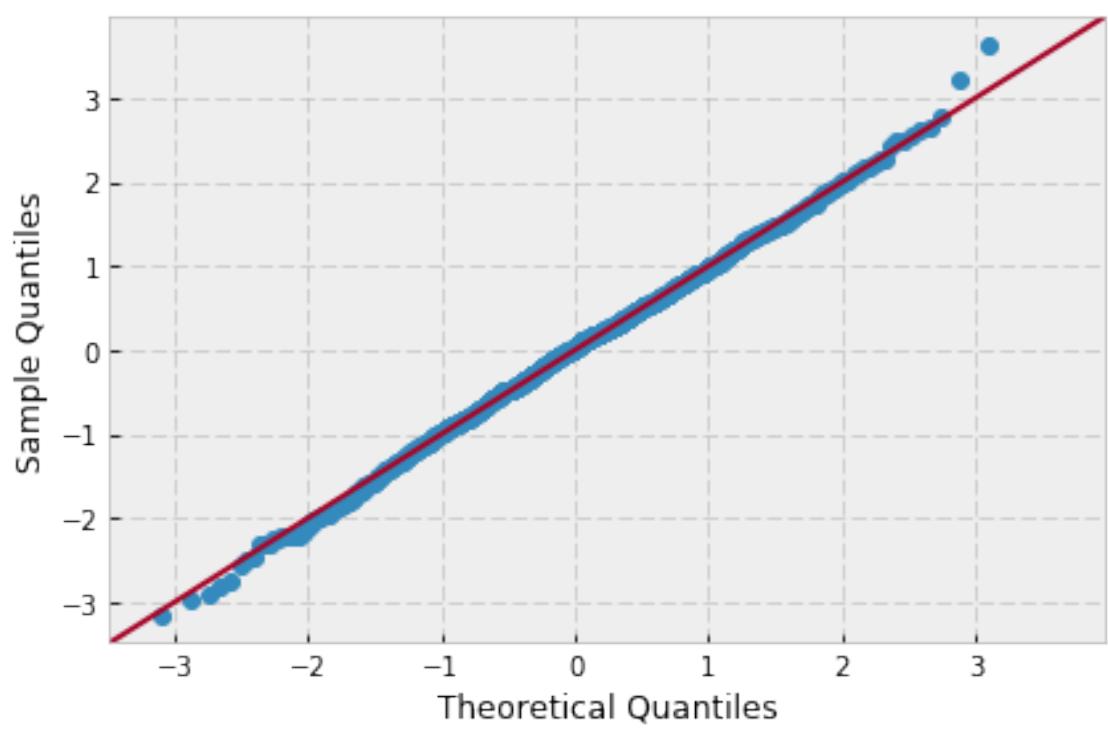
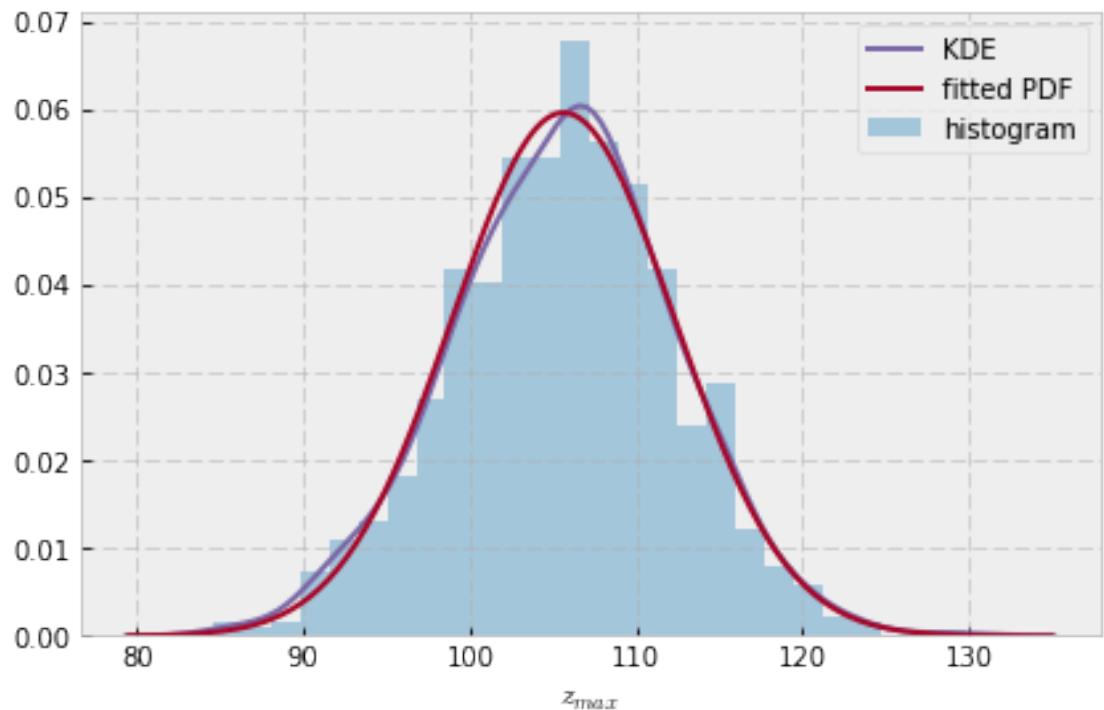


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(),loc=107.73206779251451, scale=5.9923332276339645)
DescribeResult(nobs=1000, minmax=(83.619660779368004, 126.68198941577324)
               mean=107.73206779251451, variance=35.944001512518604,
               skewness=-0.13077213553878544, kurtosis=0.2561801900542191)
```

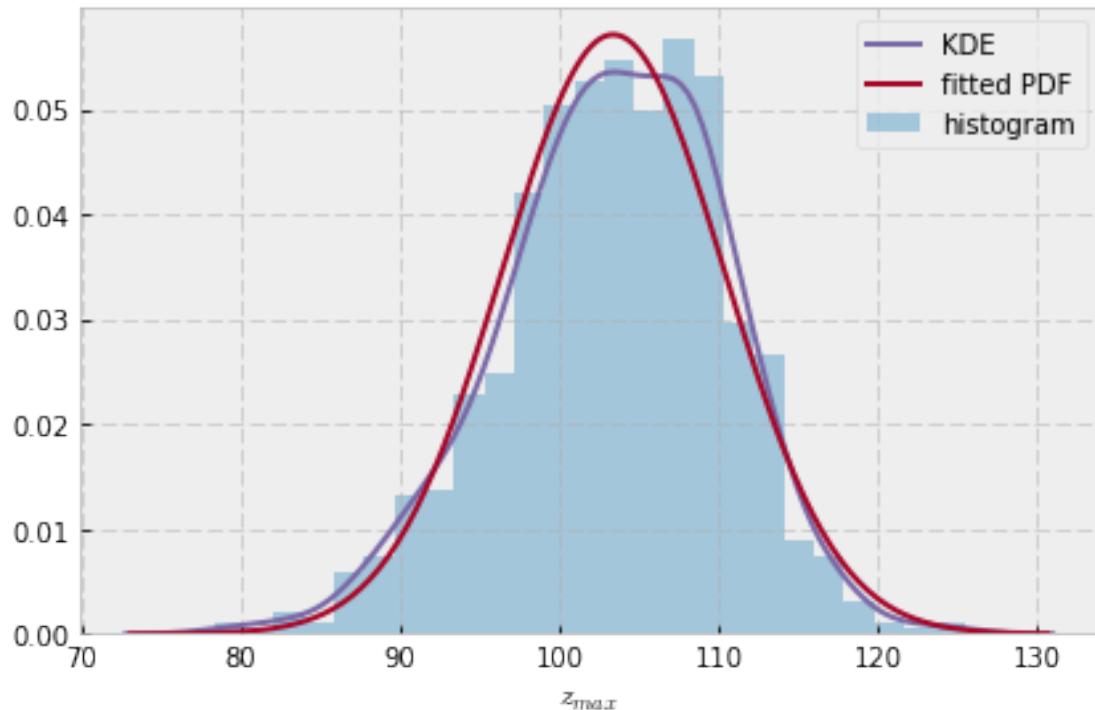


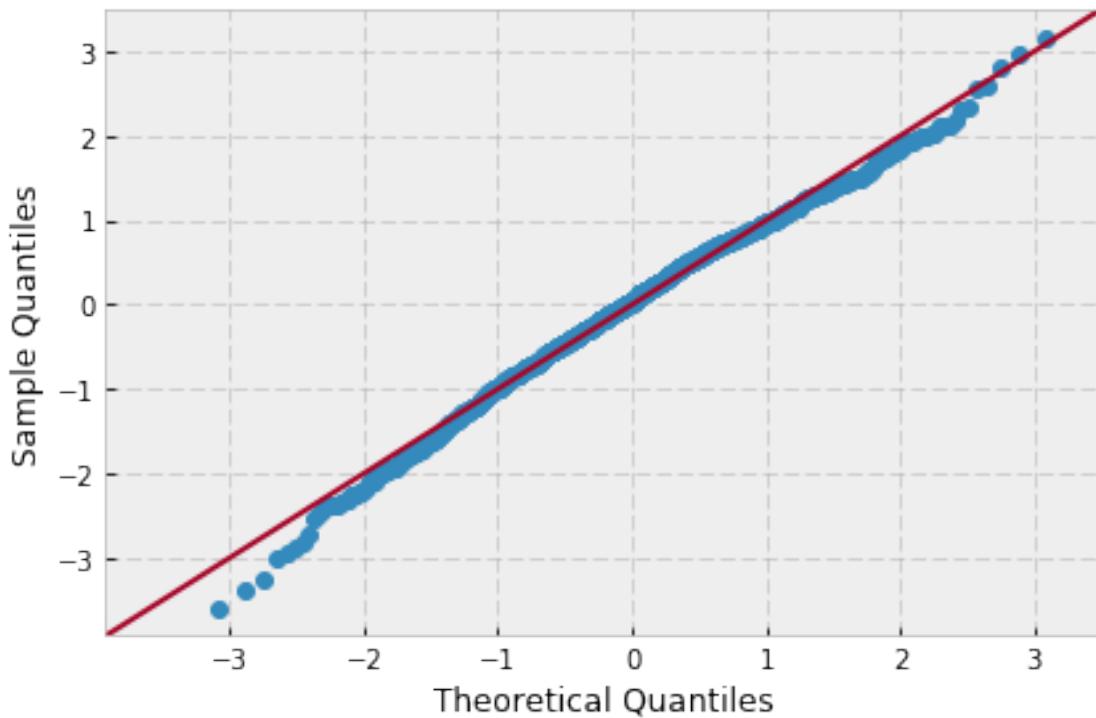


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(),loc=105.62295535944867, scale=6.694946121232342)
DescribeResult(nobs=1000, minmax=(84.543148955944005, 129.89500674636938)
               mean=105.62295535944867, variance=44.867170736940928,
               skewness=-0.05152944294715397, kurtosis=0.09977455145523484)
```

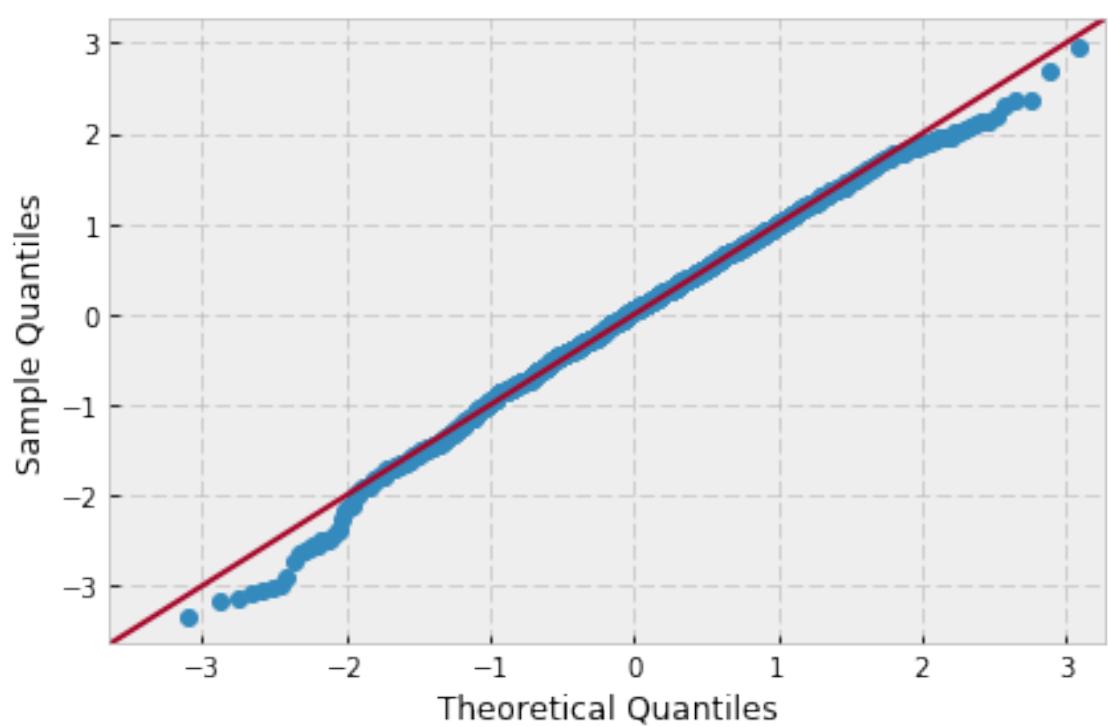
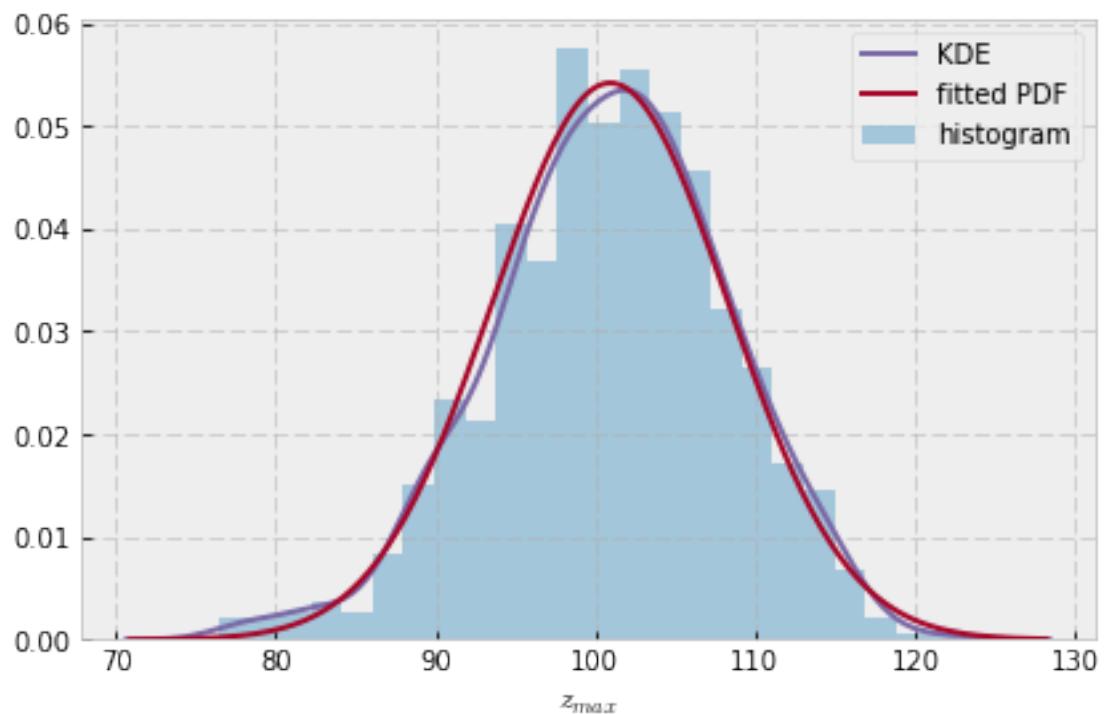


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=103.38930087823523, scale=6.985086050939321)
DescribeResult(nobs=1000, minmax=(78.32854403557964, 125.41789456451492)
               mean=103.38930087823523, variance=48.840267406433519,
               skewness=-0.2952671524217359, kurtosis=0.17177853296810808)
```

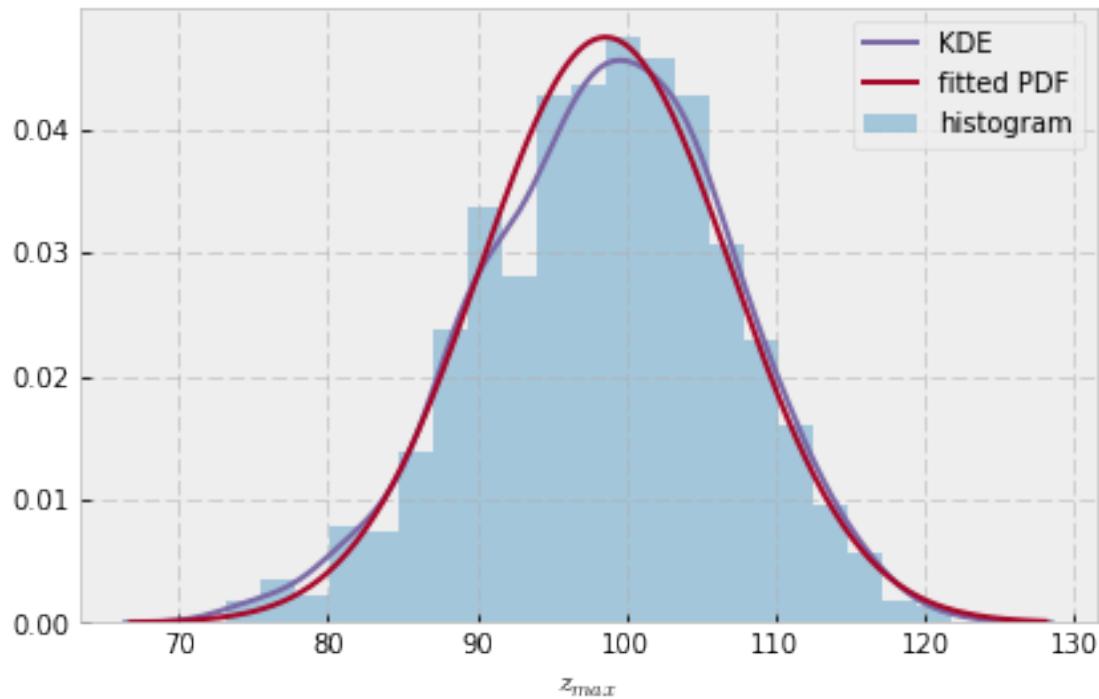


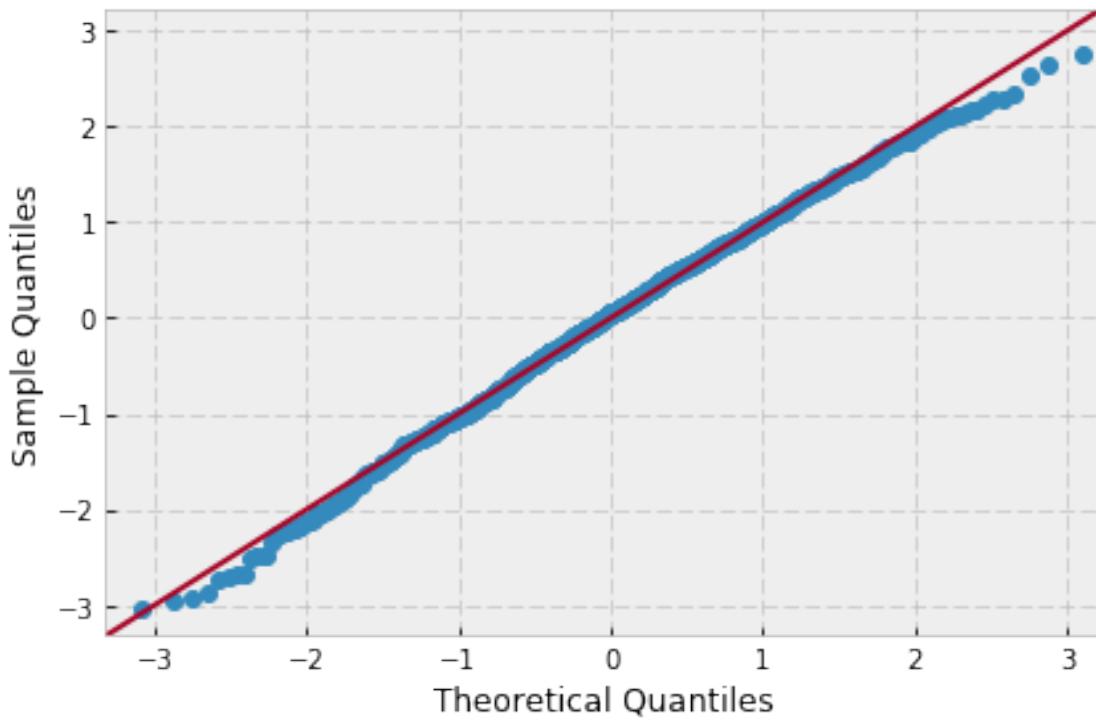


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(),loc=100.9254592090742, scale=7.364918299262674)
DescribeResult(nobs=1000, minmax=(76.419007032369265, 122.66190166680514)
               mean=100.92545920907421, variance=54.29631787268687,
               skewness=-0.26884274903822325, kurtosis=0.13654815060043912)
```

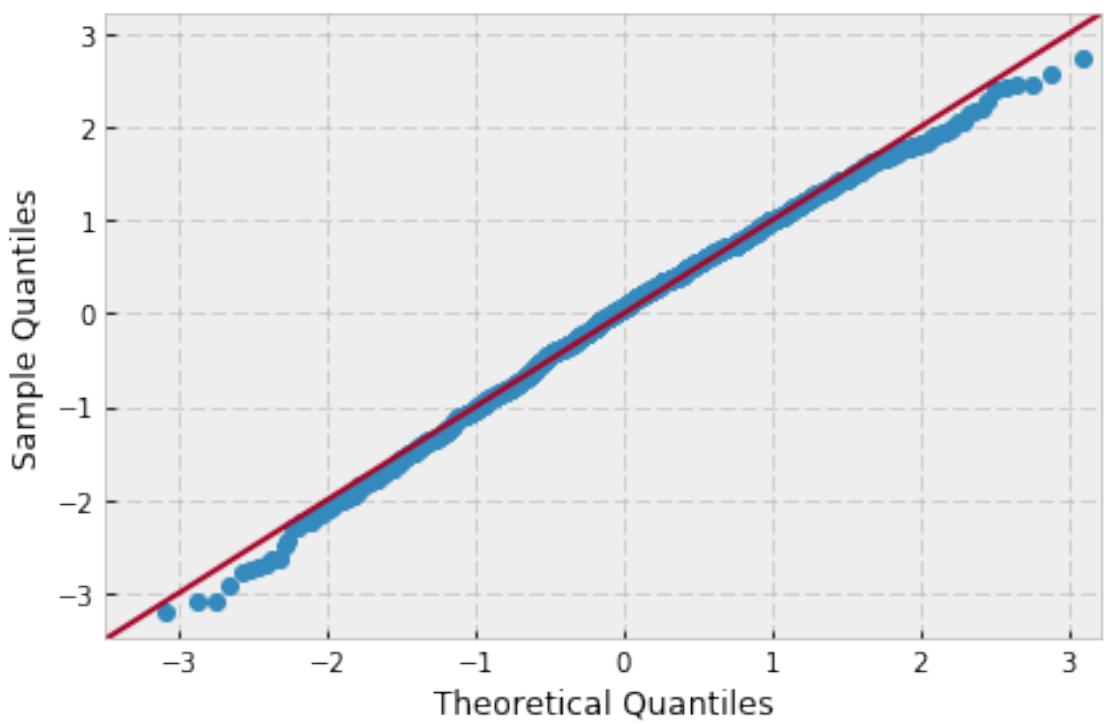
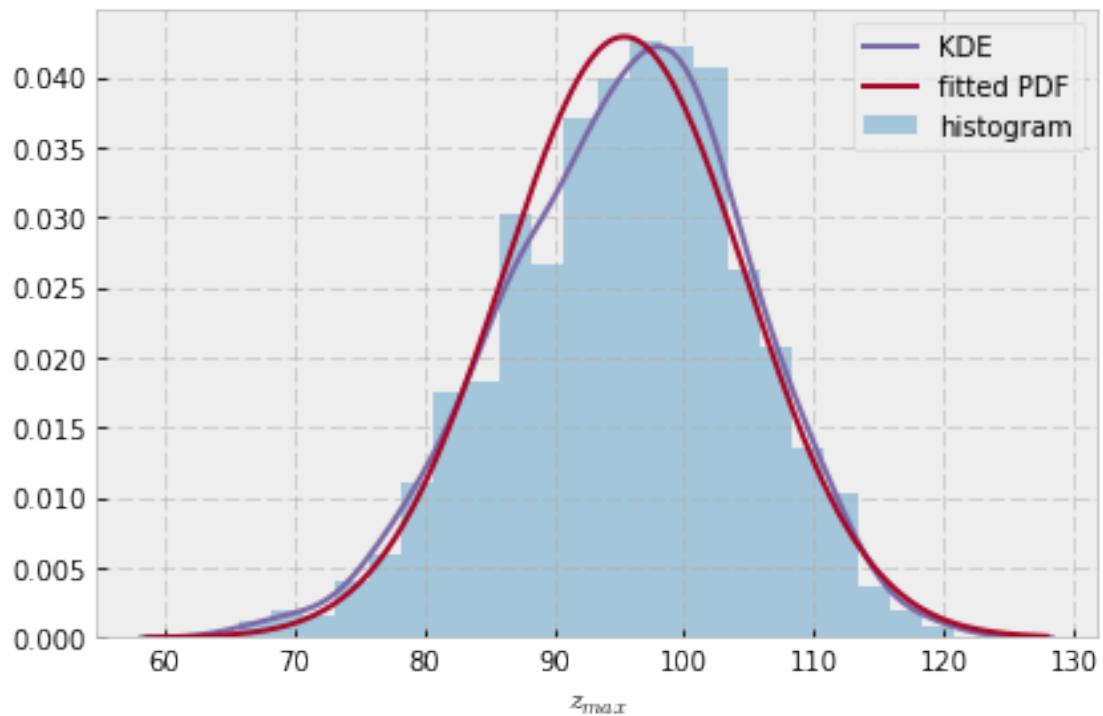


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(),loc=98.62311877353746, scale=8.40534983906537)
DescribeResult(nobs=1000, minmax=(73.145891239168904, 121.76698799378813)
               mean=98.623118773537456, variance=70.720626543619844,
               skewness=-0.19169823044996068, kurtosis=-0.14325041321901466)
```





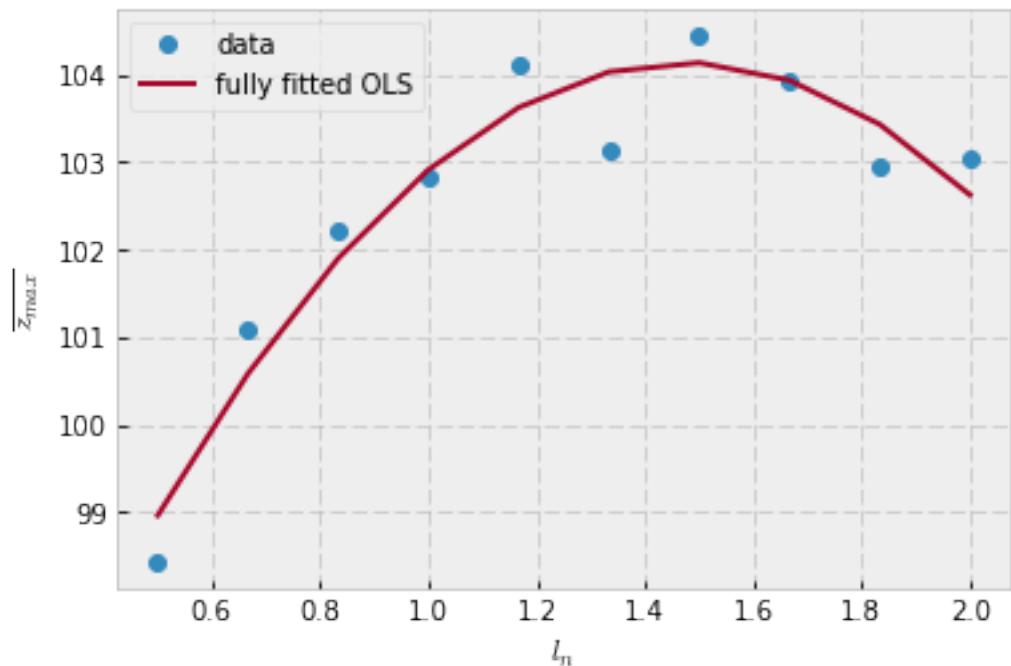
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(),loc=95.39502366609055, scale=9.299356995380673)
DescribeResult(nobs=1000, minmax=(65.693974642697455, 120.85966567410048)
               mean=95.395023666090552, variance=86.564605132668135,
               skewness=-0.2473434943618572, kurtosis=-0.10269905935028945)
```



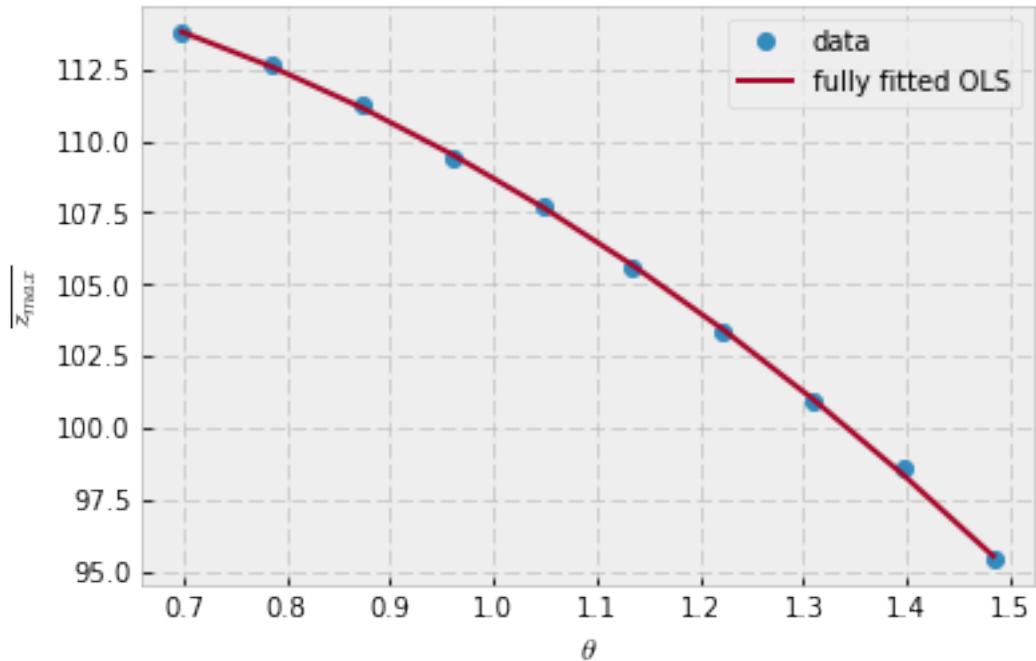
2.2 Parameter regression

```
In [10]: var_name = 'maximal_rooting_depth'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly2')
mean_reg('maxB', var_name, 'poly2')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly2')
mean_reg('N', var_name, 'poly2')
```

variable name: ln
model: poly2
r-squared: 0.7753816325517857
explained variance: 0.7754962177441524



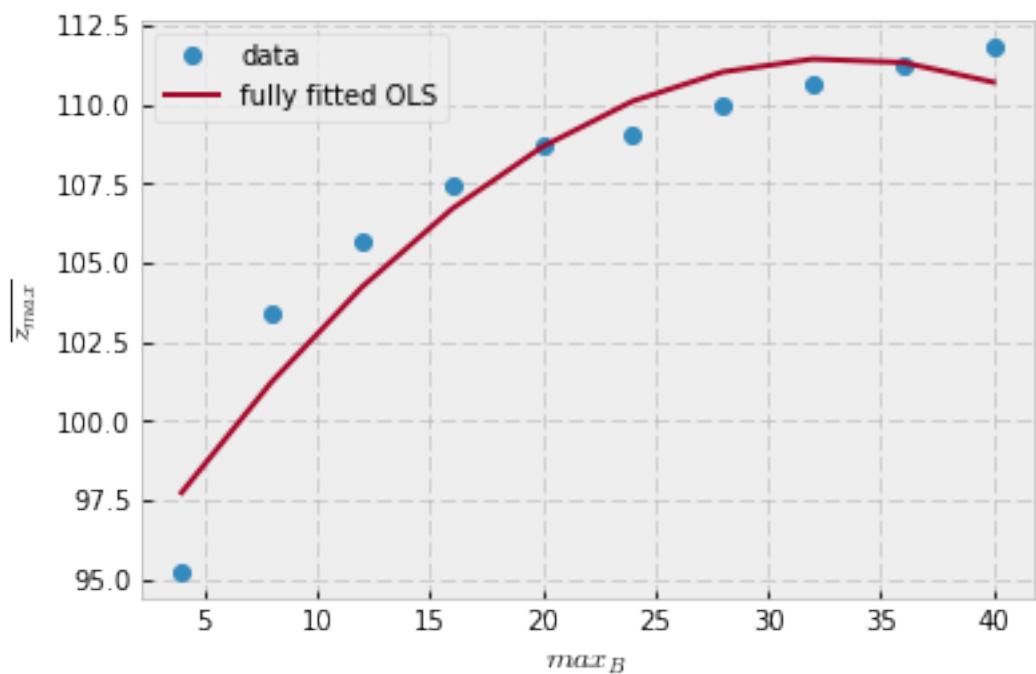
```
variable name: theta
model: poly2
r-squared: 0.9990843892116165
explained variance: 0.9990957781741907
```



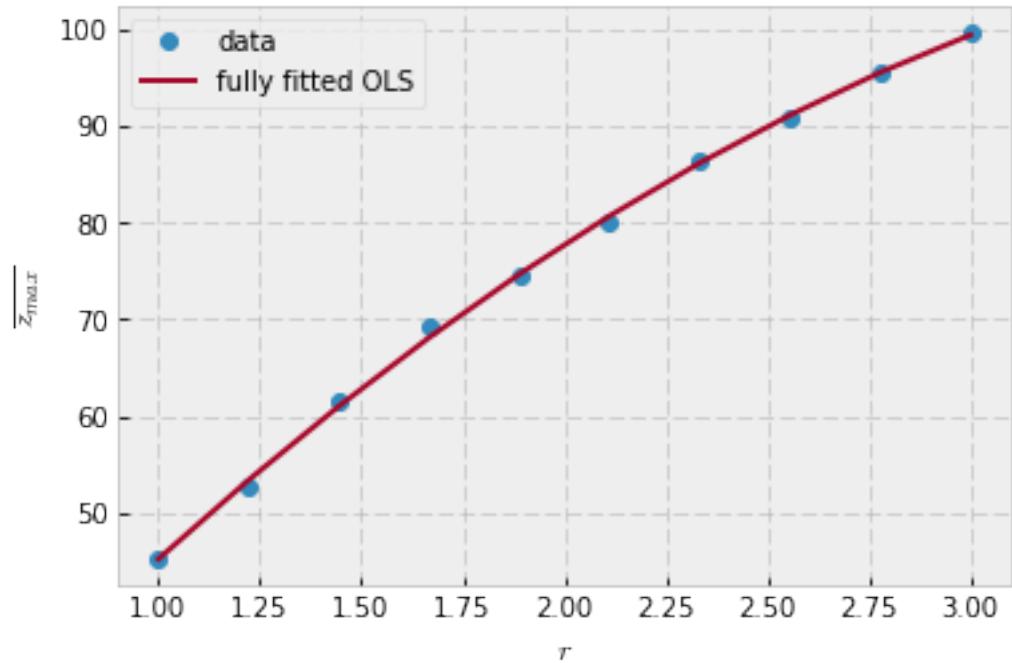
```

variable name: maxB
model: poly2
r-squared: 0.6849723517504815
explained variance: 0.6862501252820524

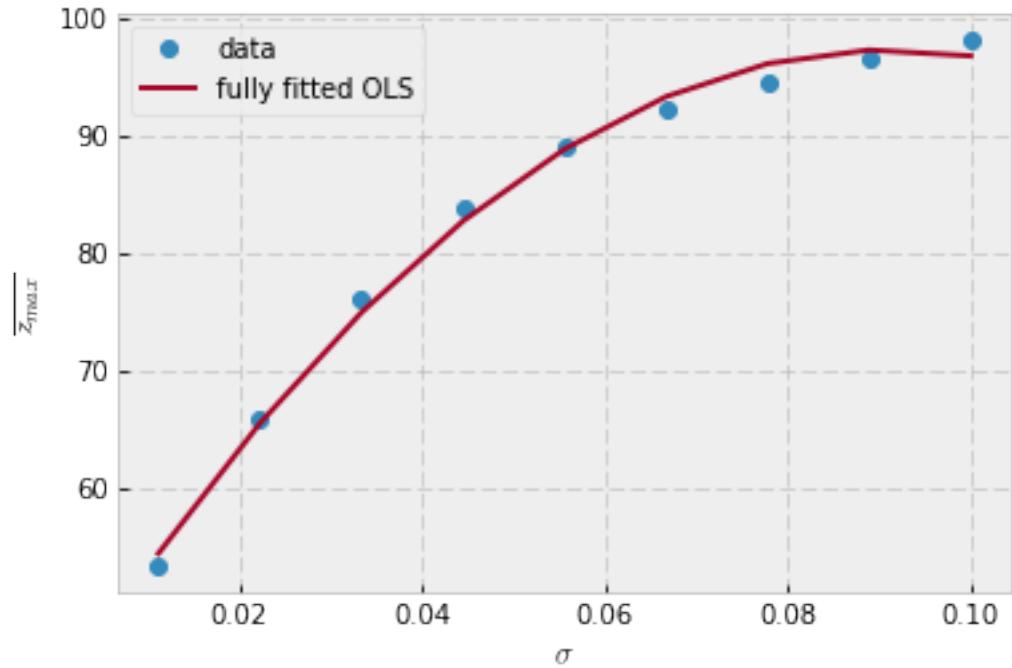
```



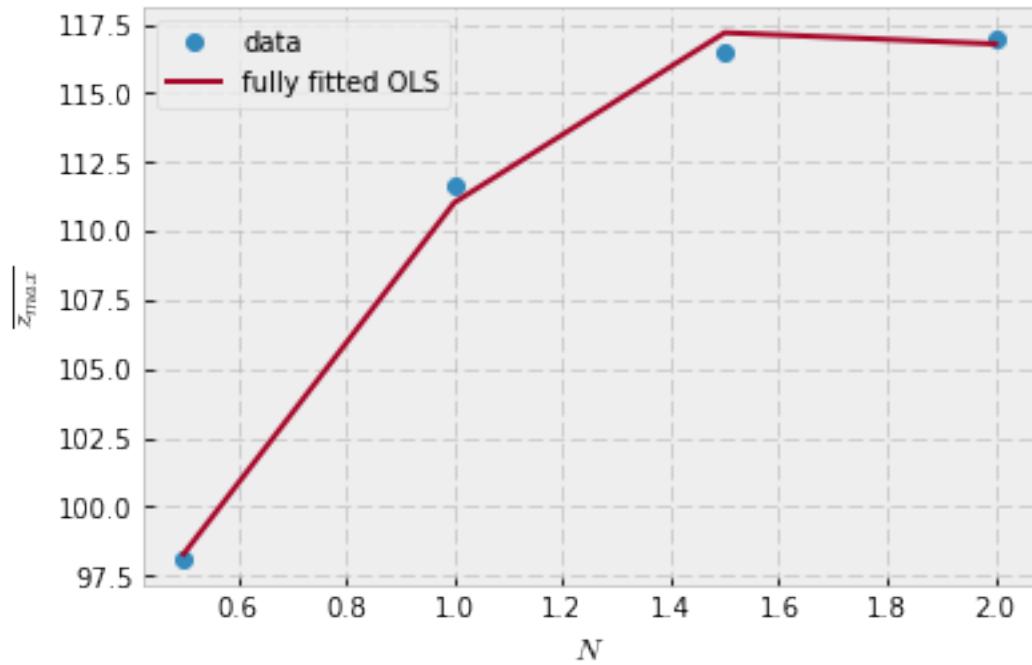
```
variable name: r
model: poly2
r-squared: 0.998522699383428
explained variance: 0.998523764819633
```



```
variable name: sigma
model: poly2
r-squared: 0.9790504019281303
explained variance: 0.9790634755552131
```



```
variable name: N
model: poly2
r-squared: 0.8196779526594422
explained variance: 0.8196779526594422
```

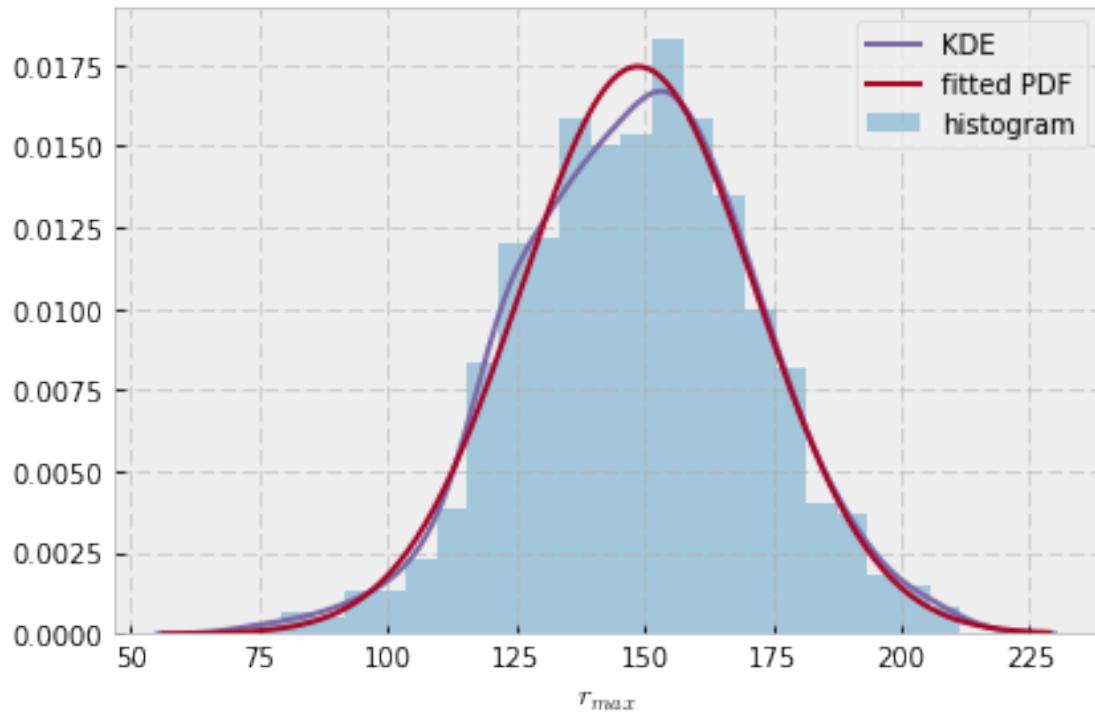


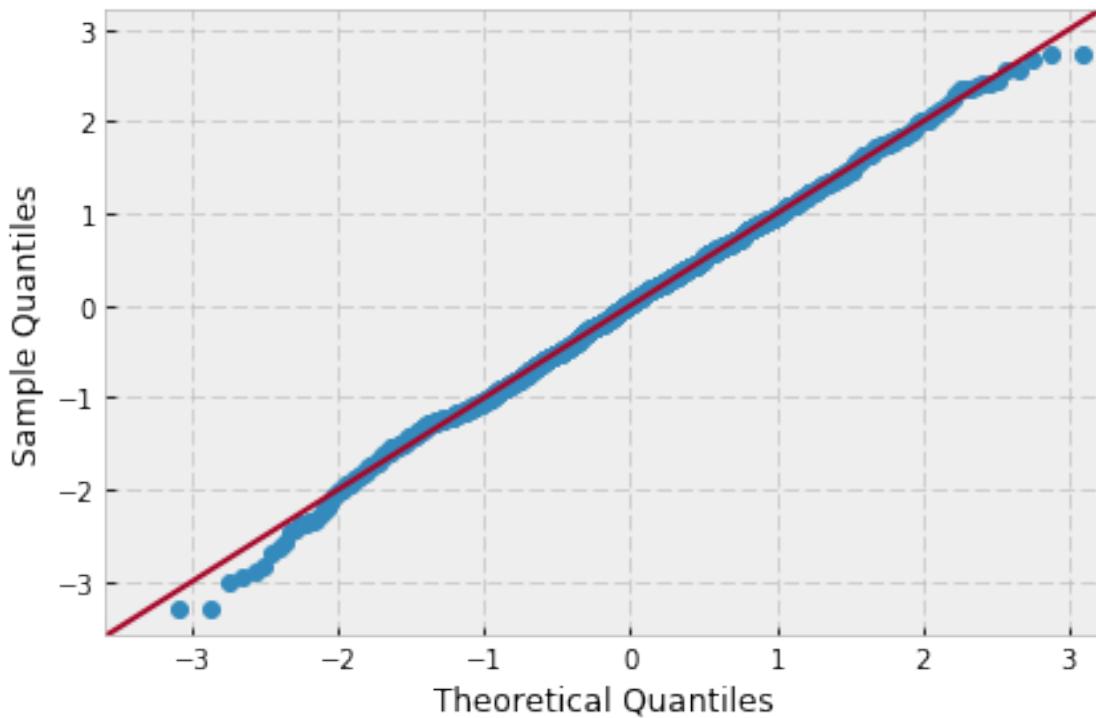
3 Maximal horizontal spread

3.1 Probability distributions

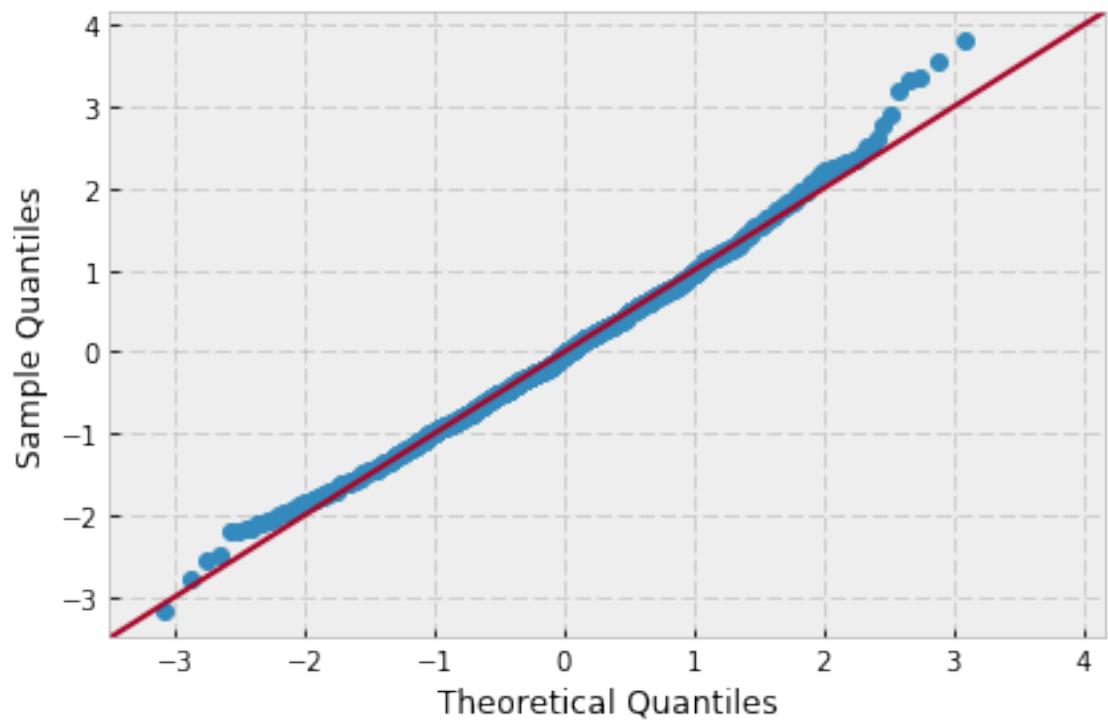
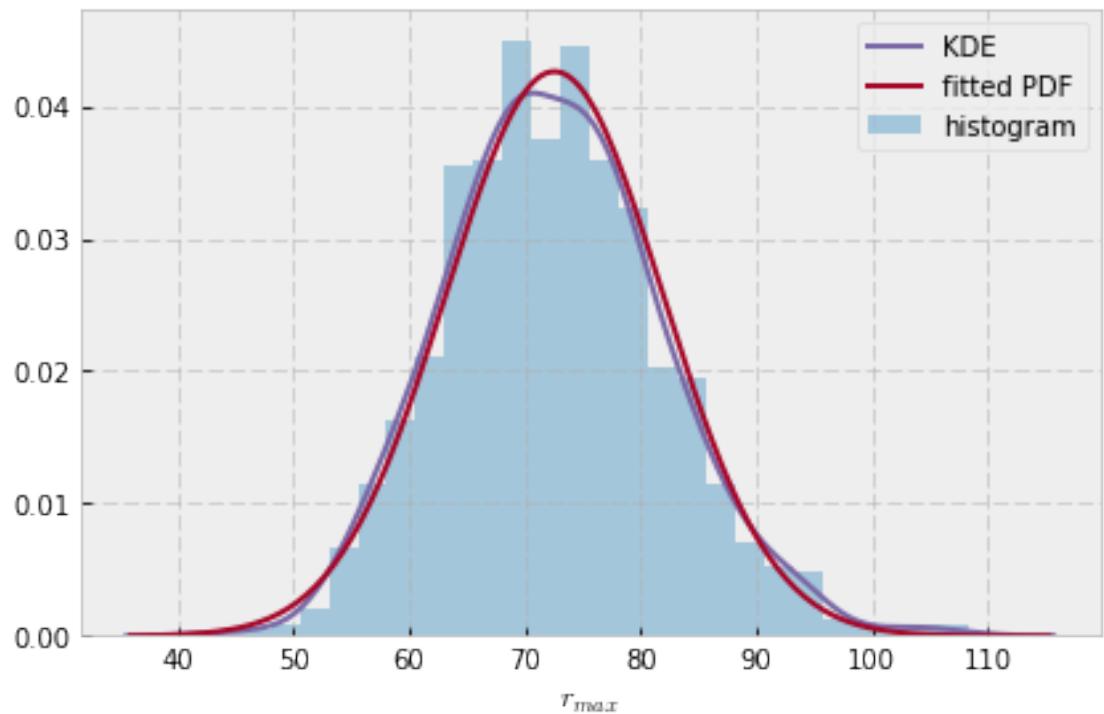
```
In [11]: plot_dist('maximal_horizontal_spread')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=148.7670340811633, scale=22.868543671466135)
DescribeResult(nobs=1000, minmax=(73.49364497532612, 211.53158736979407)
    mean=148.7670340811633, variance=523.49378343719115,
    skewness=-0.076536902793879, kurtosis=0.01832299861350517)
```

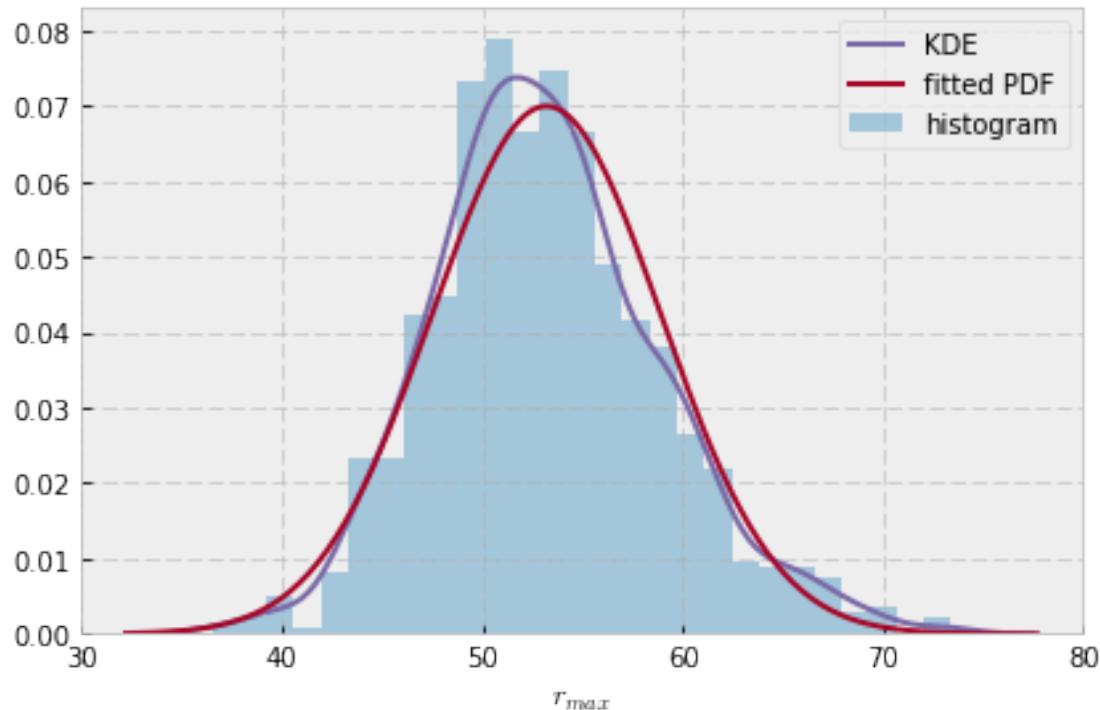


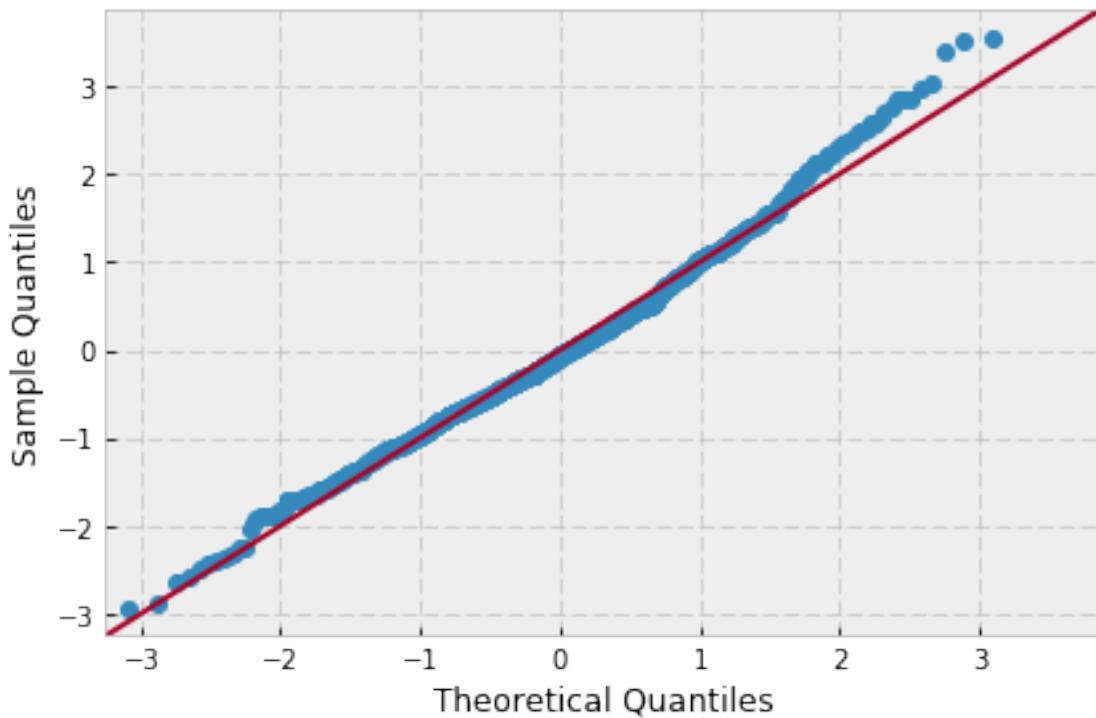


```
variable name: N
variable value: 1.0
distribution: normal(shape=(),loc=72.5526522785993, scale=9.359055594877024)
DescribeResult(nobs=1000, minmax=(43.006608147751479, 108.26387995255632)
               mean=72.552652278599297, variance=87.679601229228169,
               skewness=0.28418739234318724, kurtosis=0.25375593502875216)
```

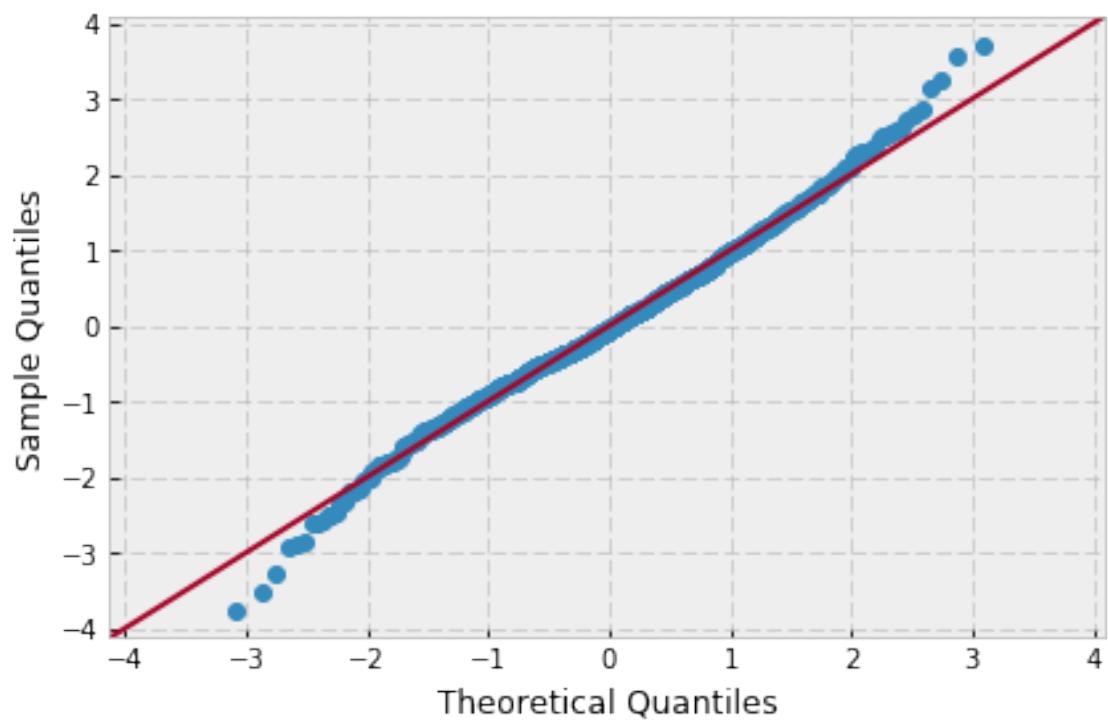
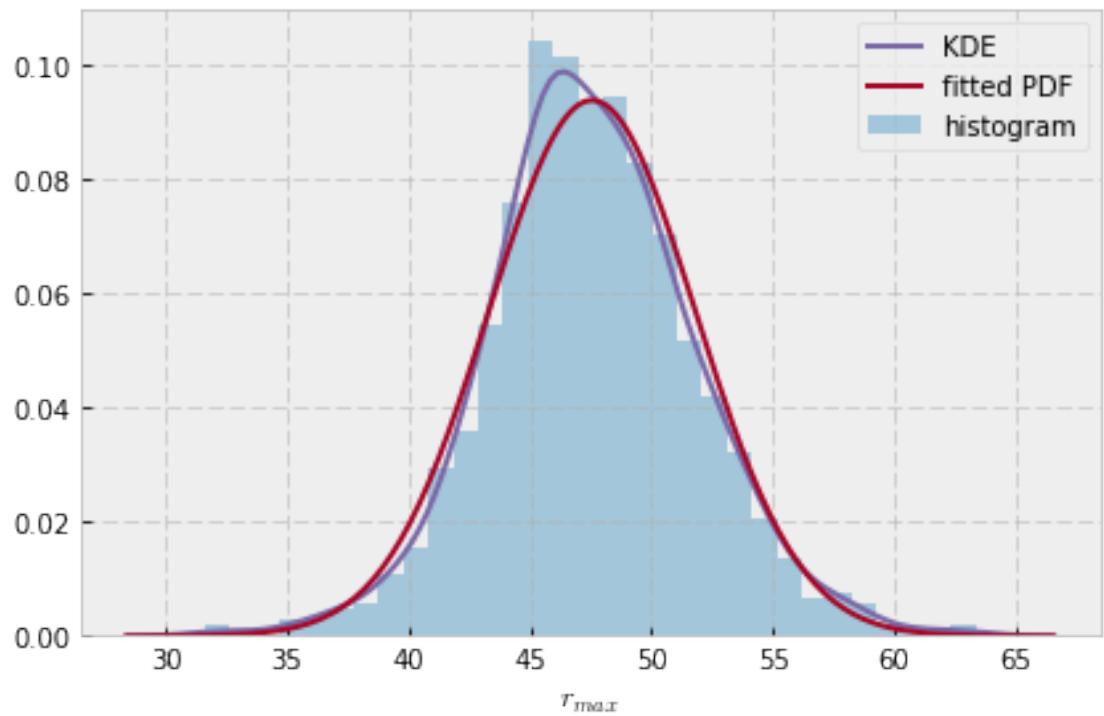


```
variable name: N
variable value: 1.5
distribution: normal(shape=(),loc=53.20058703721677, scale=5.6984750082593845)
DescribeResult(nobs=1000, minmax=(36.508482005477305, 73.386107246960222)
               mean=53.200587037216771, variance=32.505122542299098,
               skewness=0.38098171980612555, kurtosis=0.40224268574793376)
```

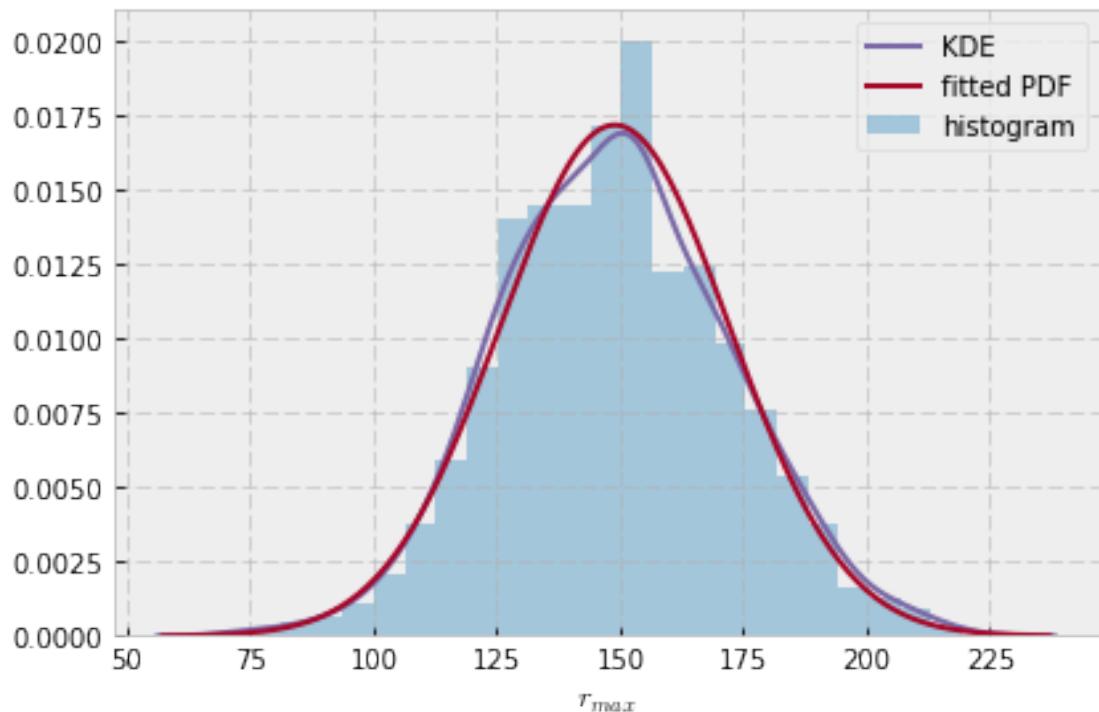


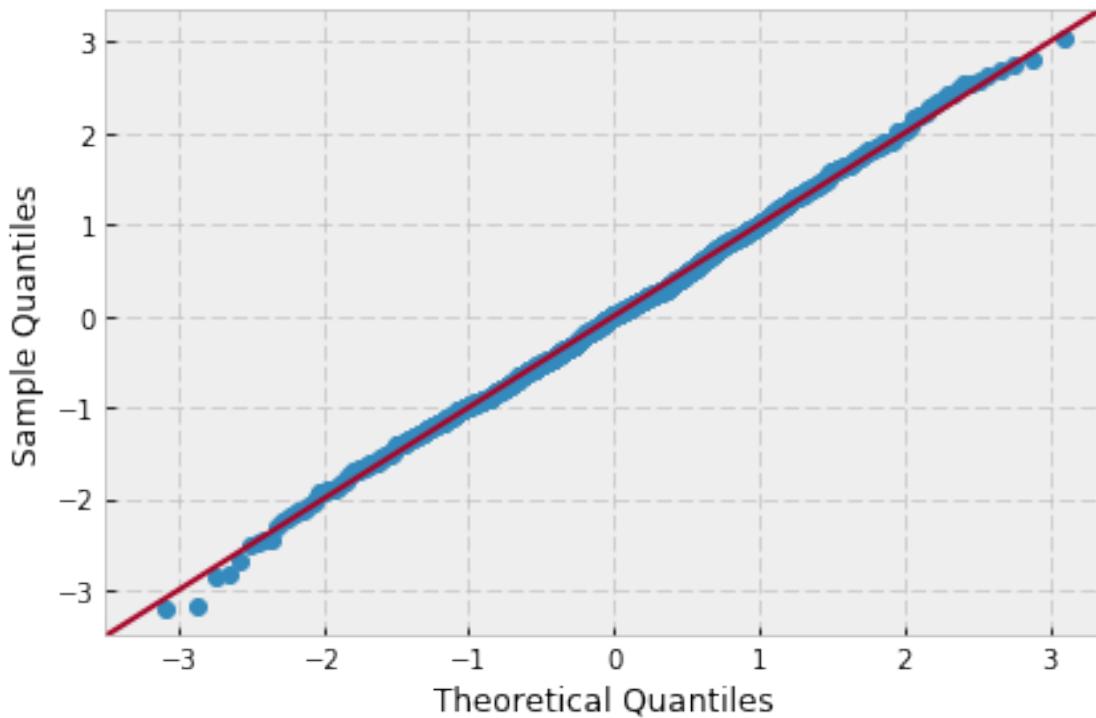


```
variable name: N
variable value: 2.0
distribution: normal(shape=(),loc=47.55665739148845, scale=4.251142903429944)
DescribeResult(nobs=1000, minmax=(31.59117740483428, 63.35385033970006)
               mean=47.55665739148845, variance=18.090306291674452,
               skewness=0.10320251173641505, kurtosis=0.70189997929121)
```

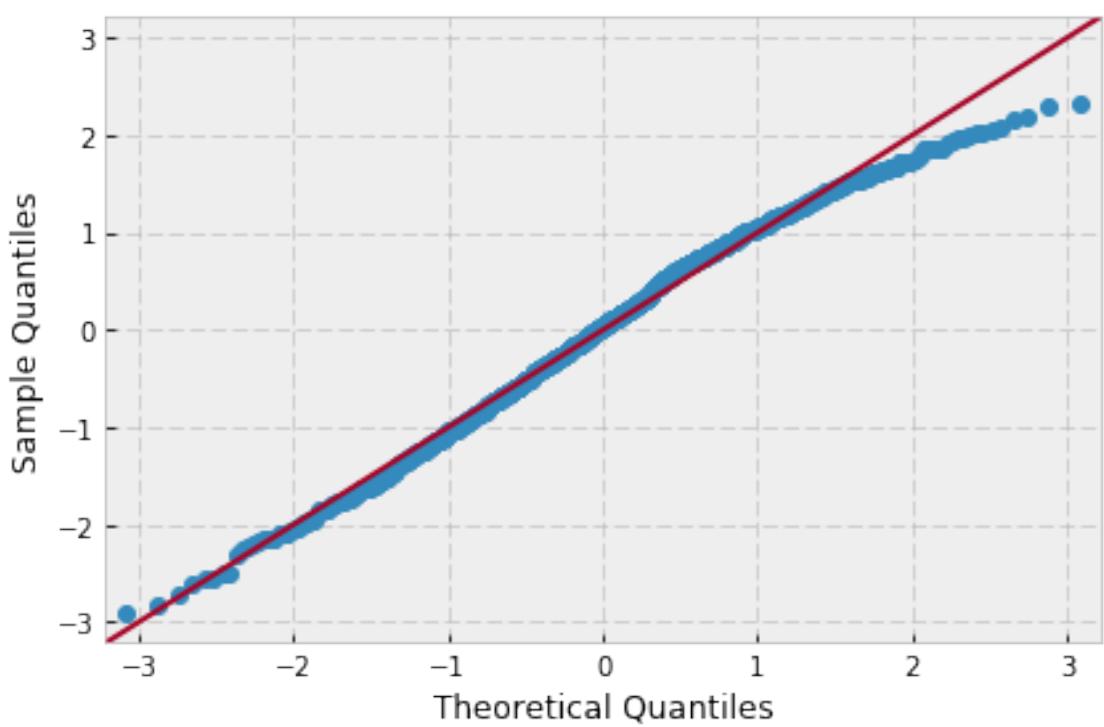
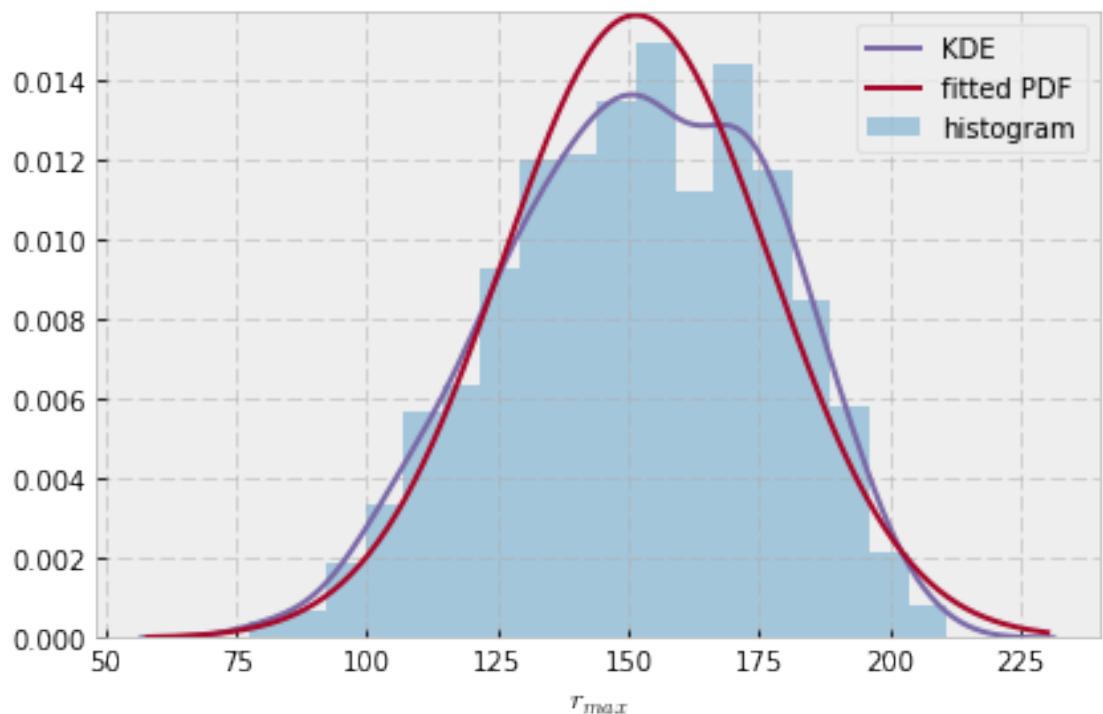


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=148.90296673943087, scale=23.239785323289333)
DescribeResult(nobs=1000, minmax=(74.766450162712147, 219.46380253224368)
               mean=148.90296673943087, variance=540.62825012269707,
               skewness=0.07574756874372819, kurtosis=-0.034949240933307646)
```

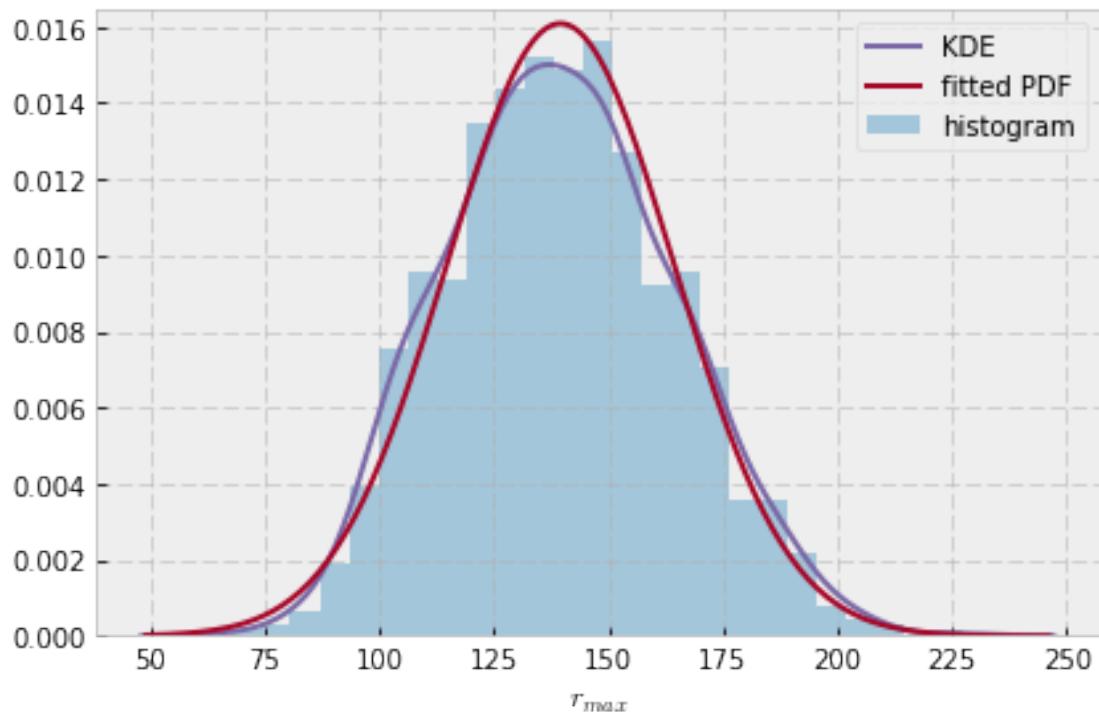


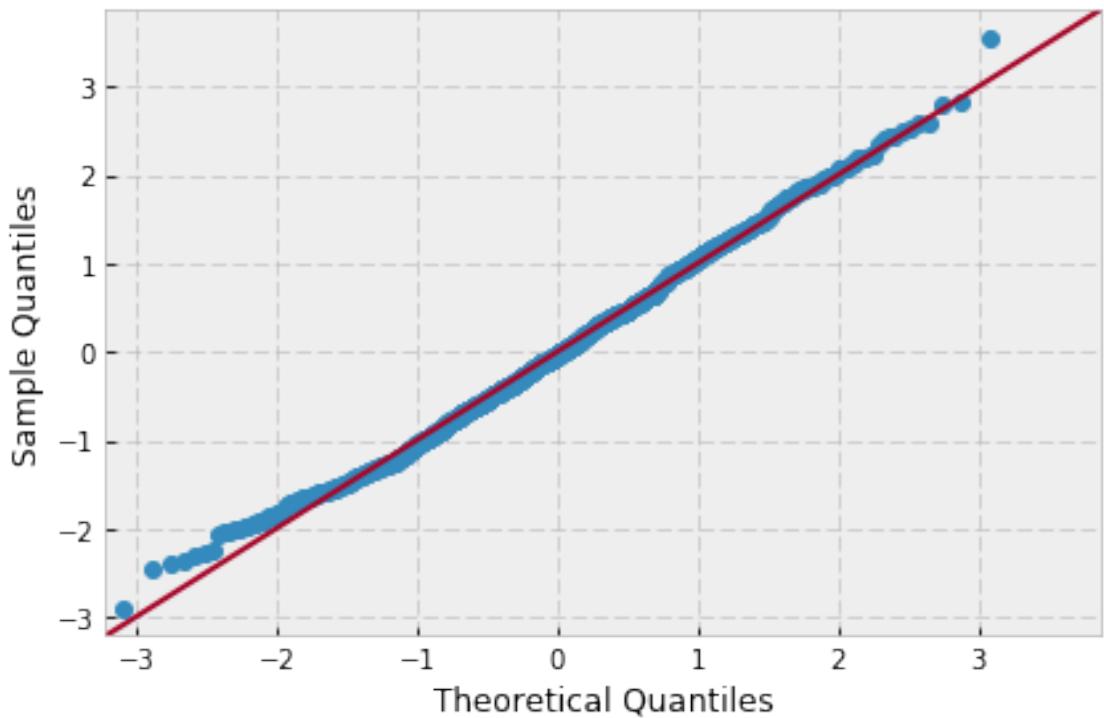


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(),loc=151.44659026586172, scale=25.526977121218657)
DescribeResult(nobs=1000, minmax=(77.261252758788117, 210.84727231243824)
               mean=151.44659026586172, variance=652.27883978700777,
               skewness=-0.21274152489448162, kurtosis=-0.5353679563091762)
```

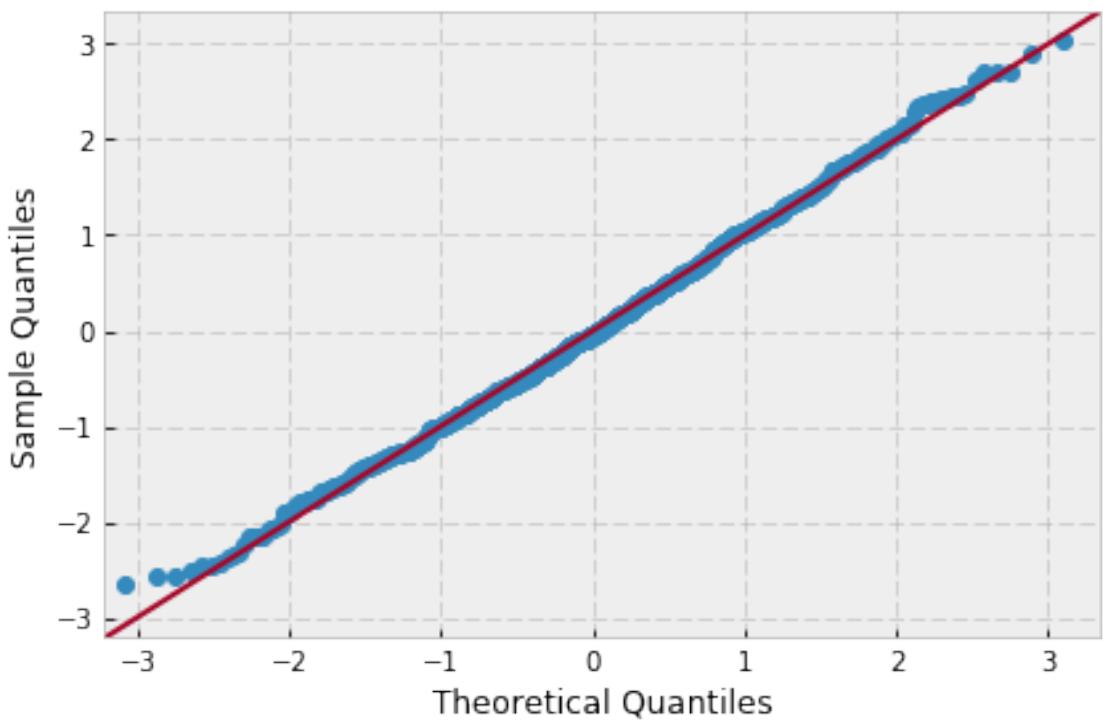
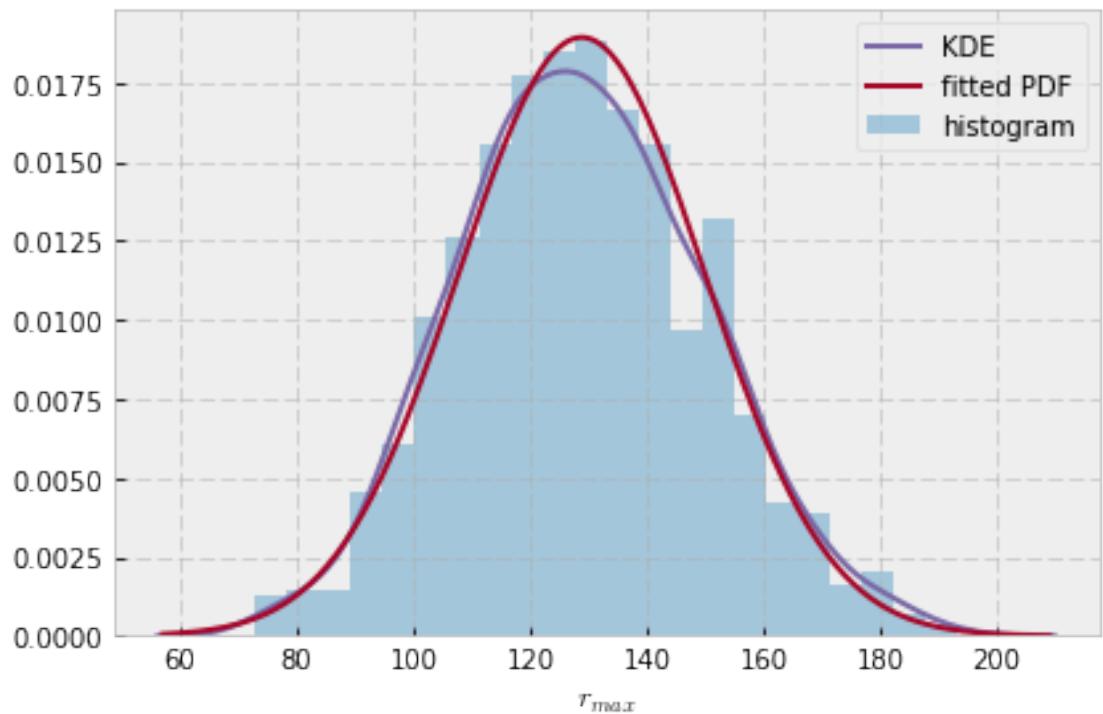


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=139.48395803187313, scale=24.79959005529943)
DescribeResult(nobs=1000, minmax=(67.743594021528878, 227.40073952783086)
               mean=139.48395803187313, variance=615.63530221311942,
               skewness=0.17161379436290974, kurtosis=-0.2889590670121662)
```

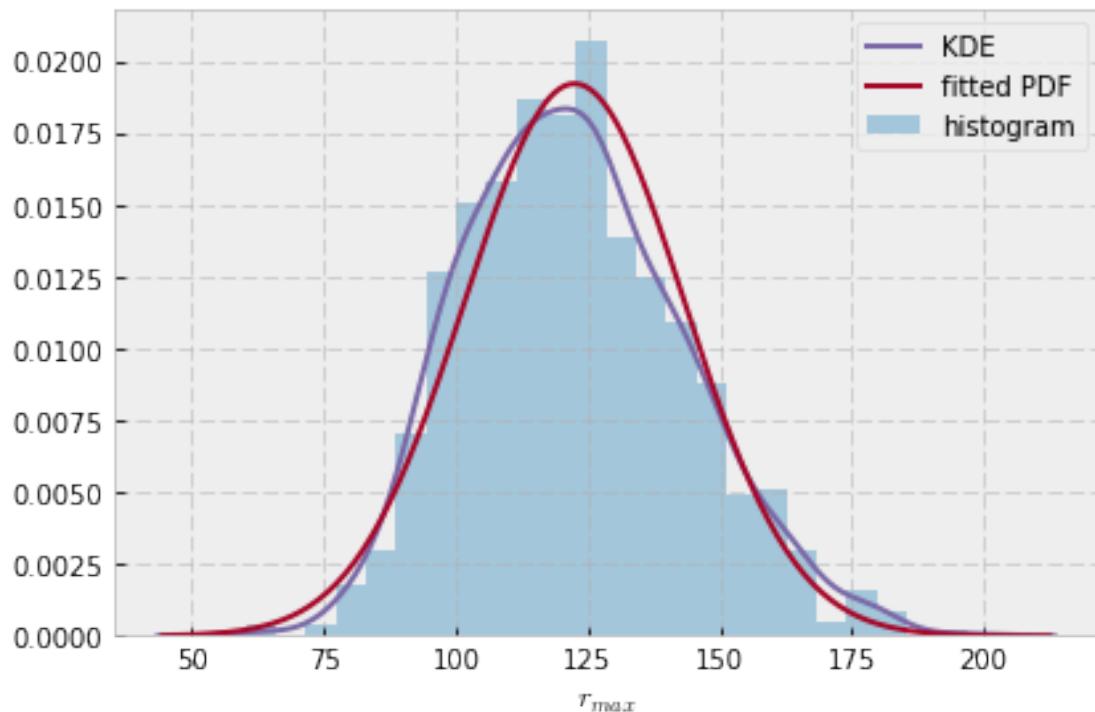


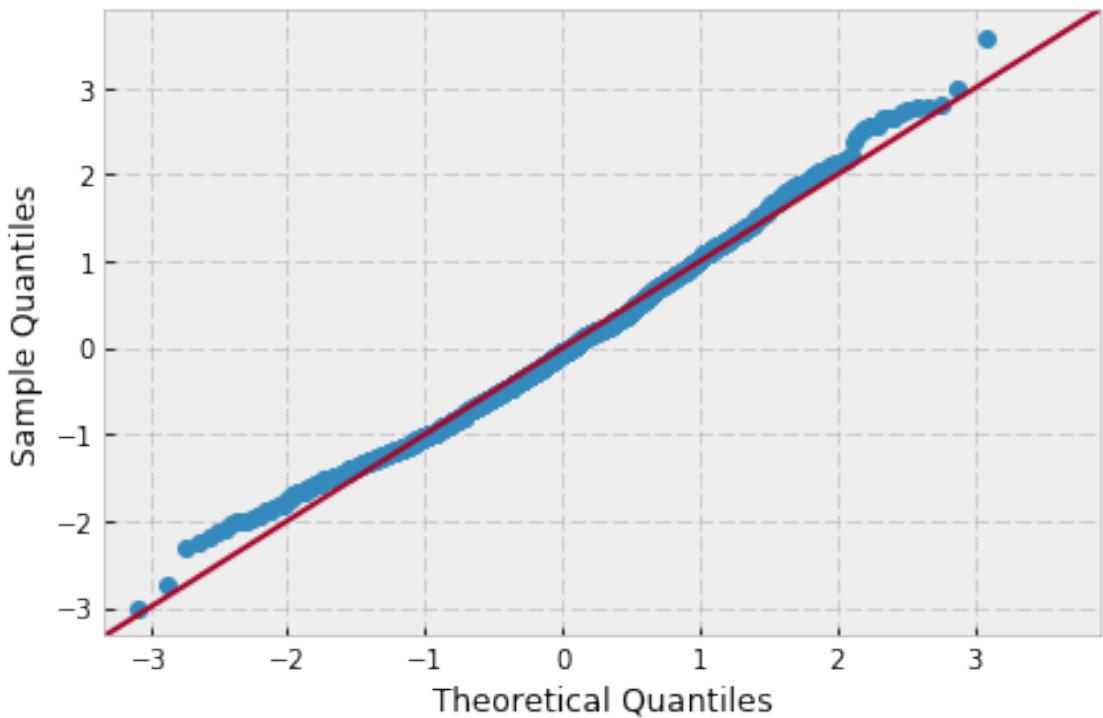


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(),loc=128.86320307741187, scale=21.062512218903645)
DescribeResult(nobs=1000, minmax=(72.948648134499948, 193.07022386438365)
               mean=128.86320307741187, variance=444.07349446593122,
               skewness=0.13736131209561597, kurtosis=-0.18562919333712635)
```

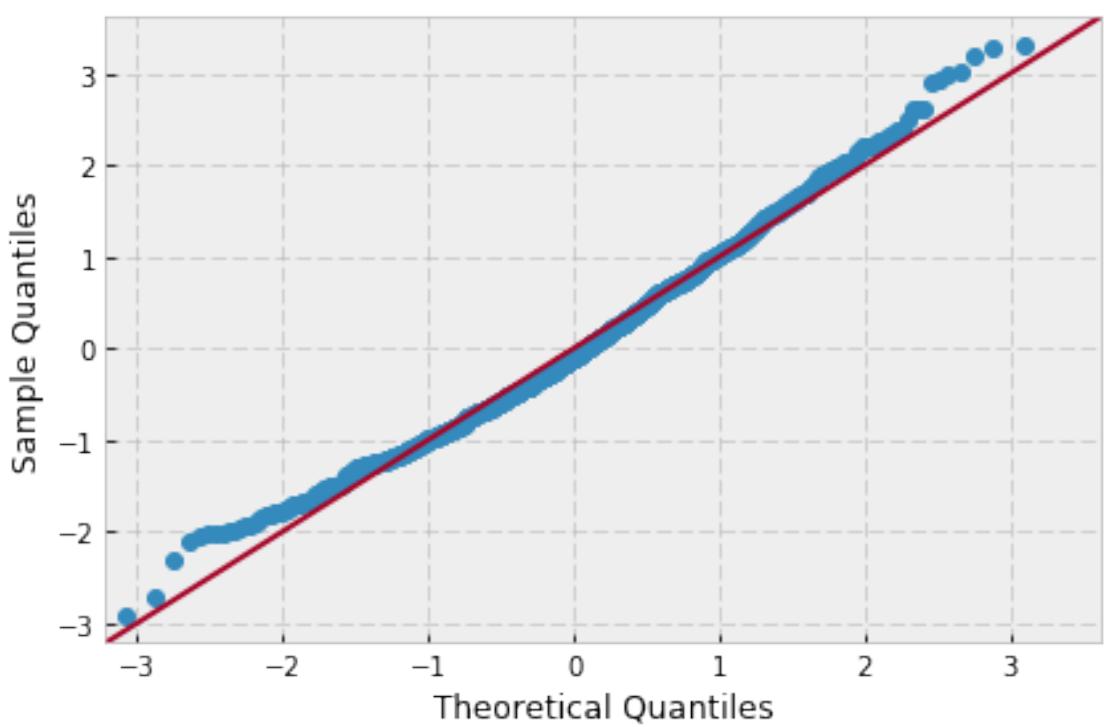
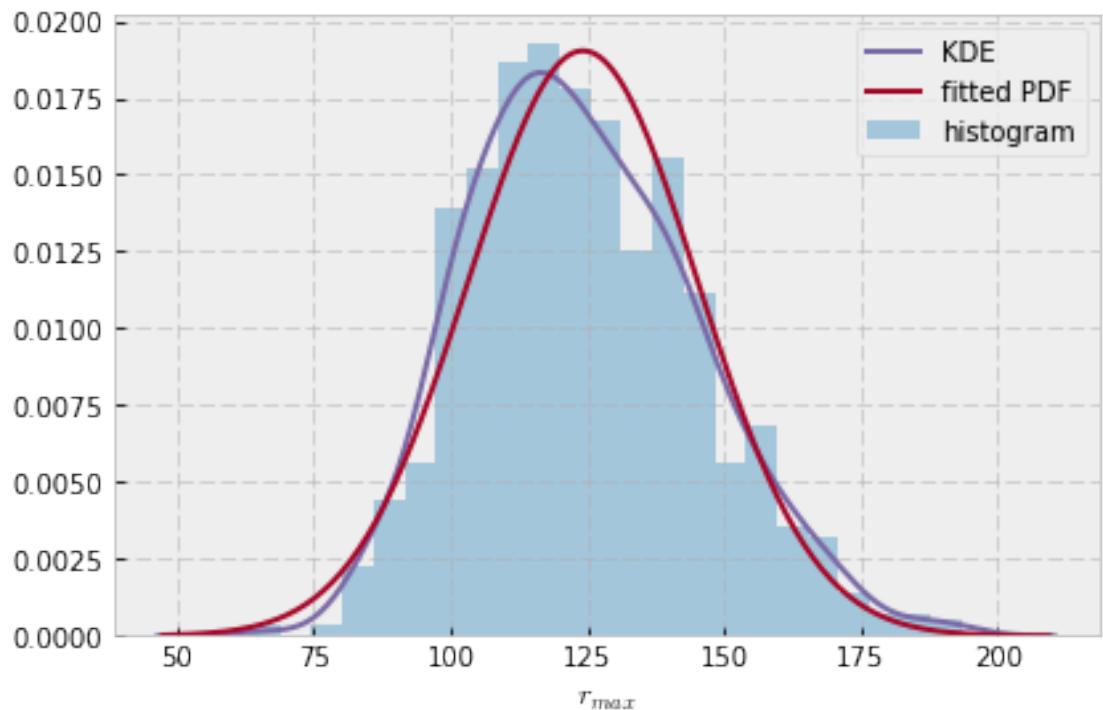


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=122.47198732664005, scale=20.775041701721)
DescribeResult(nobs=1000, minmax=(60.118334755838234, 196.73815101324948)
               mean=122.47198732664005, variance=432.03439210034702,
               skewness=0.3477712374816198, kurtosis=-0.06646915419758903)
```

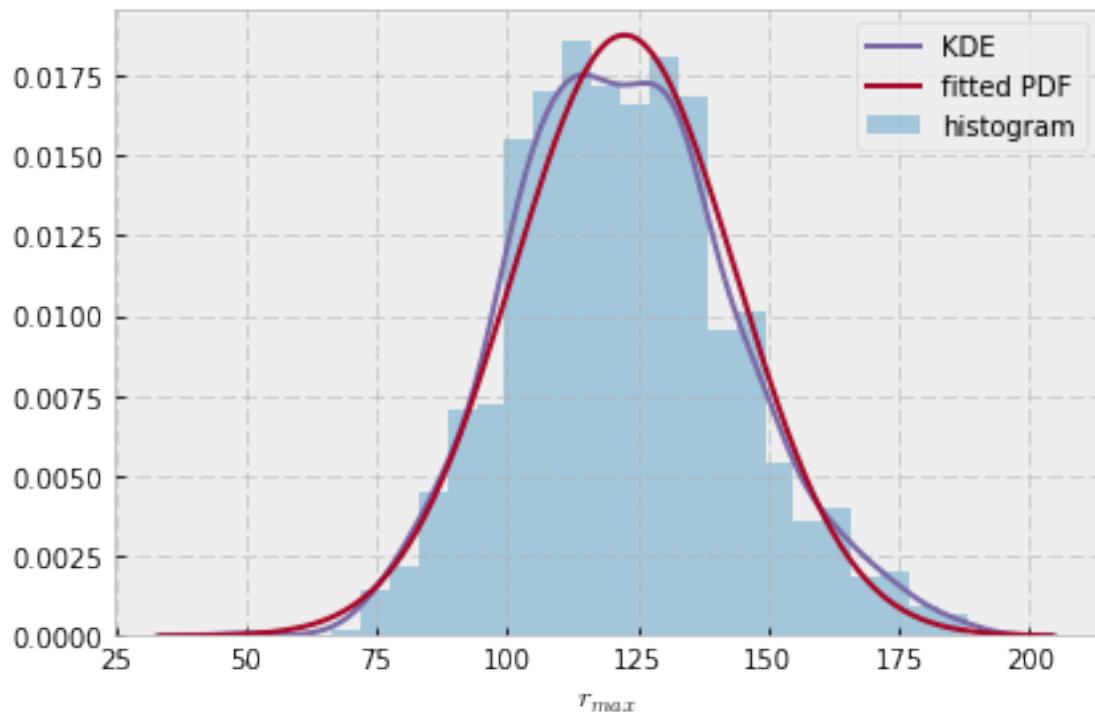


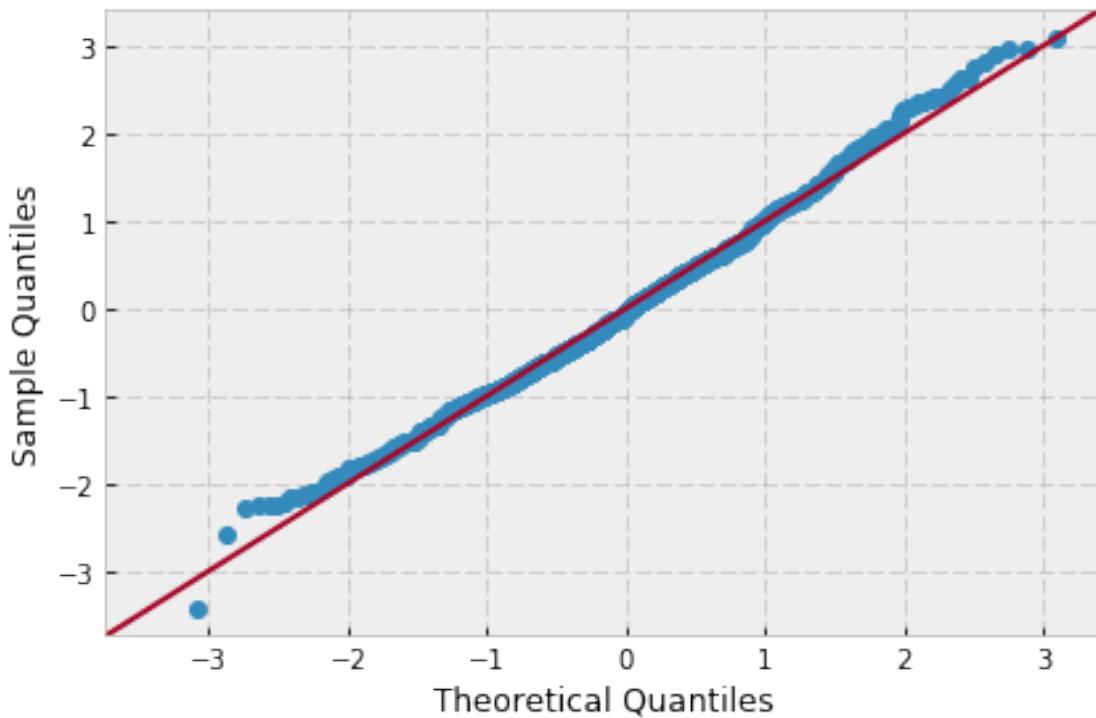


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=124.19589766478032, scale=20.959319971701934)
DescribeResult(nobs=1000, minmax=(63.230887299876436, 193.56052202848676)
               mean=124.19589766478032, variance=439.73282650268629,
               skewness=0.3820344649694134, kurtosis=-0.04581117173943827)
```

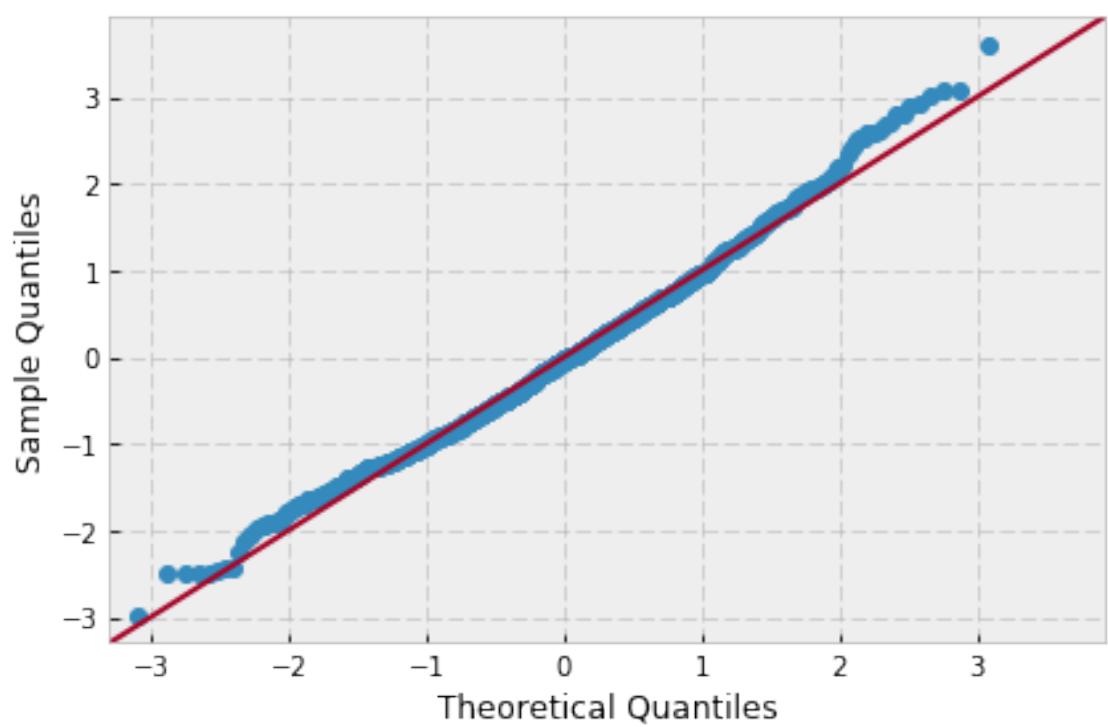
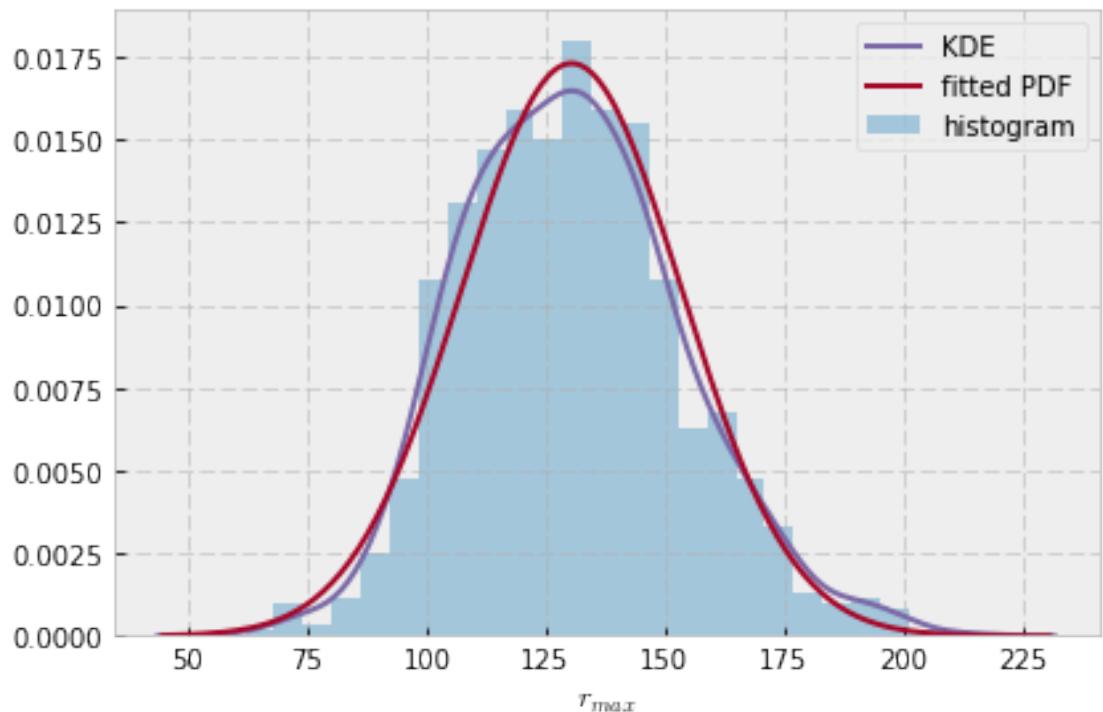


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=122.47812753547664, scale=21.277207624850245)
DescribeResult(nobs=1000, minmax=(49.722915373416171, 188.15142238921288)
               mean=122.47812753547664, variance=453.17273704803347,
               skewness=0.25049113033841386, kurtosis=-0.004921000140198917)
```

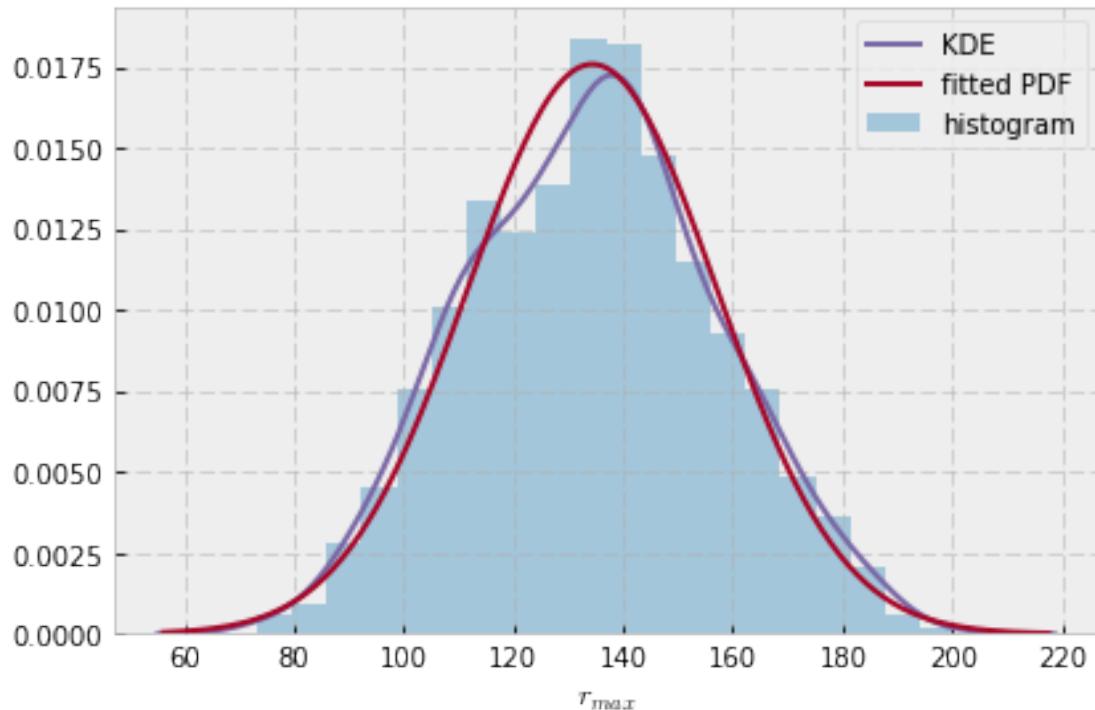


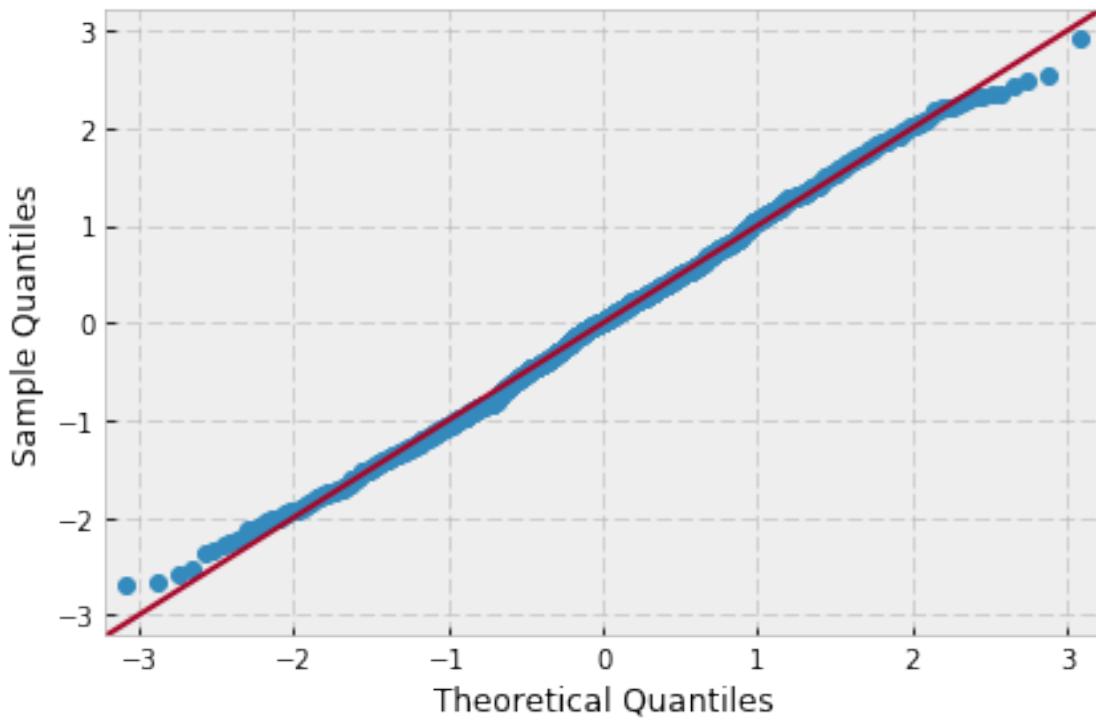


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=130.4412528354438, scale=23.0524547099134)
DescribeResult(nobs=1000, minmax=(62.037973703180533, 213.28019544436609)
               mean=130.44125283544381, variance=531.94761576837686,
               skewness=0.33594149279257324, kurtosis=0.13939549305427024)
```

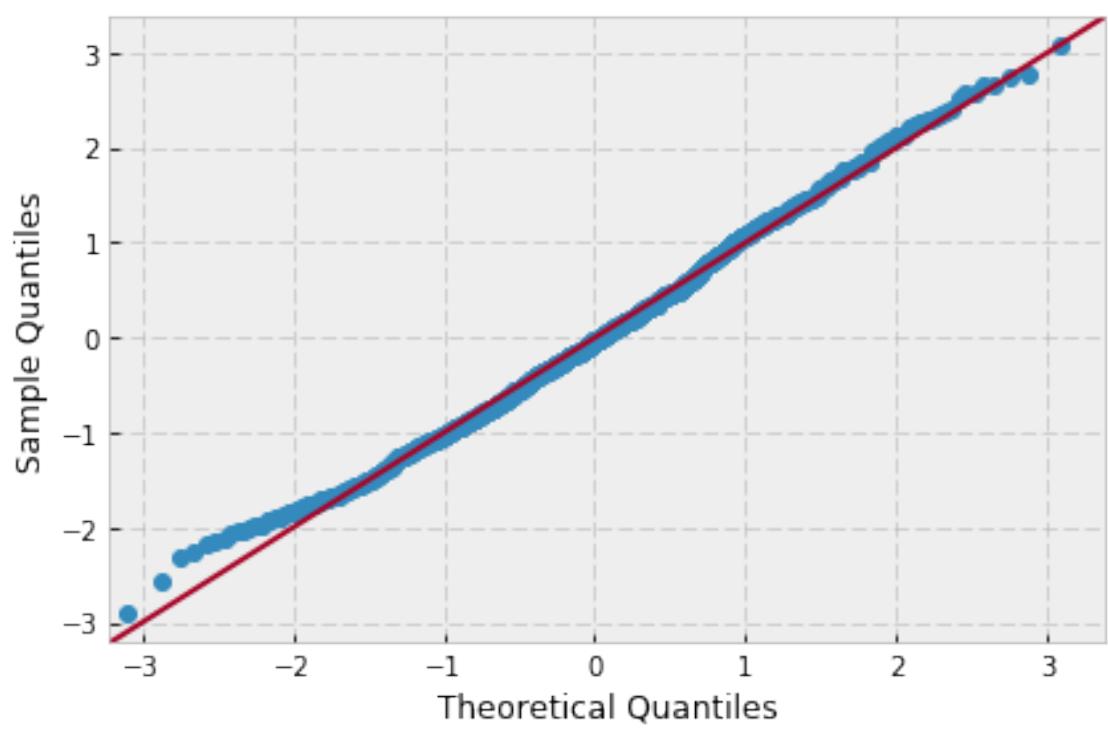
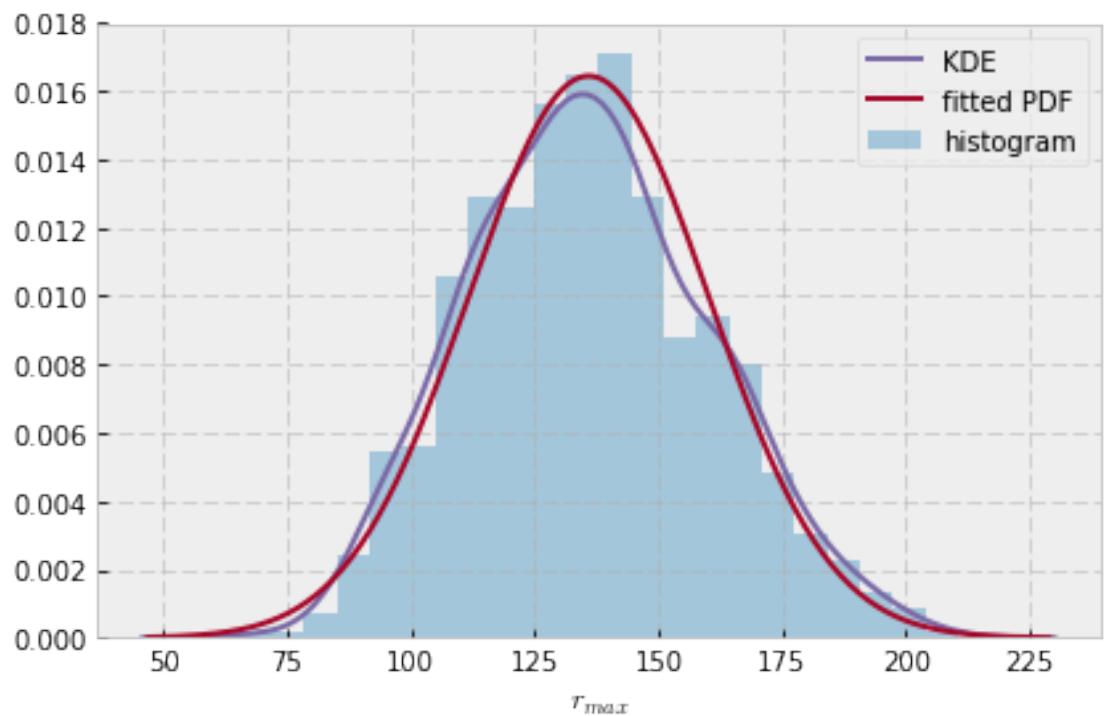


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=134.38406973468983, scale=22.699088131238156)
DescribeResult(nobs=1000, minmax=(73.243829866960226, 200.48783848871031)
               mean=134.38406973468983, variance=515.76436635607297,
               skewness=0.04455250226264711, kurtosis=-0.3783570402579728)
```

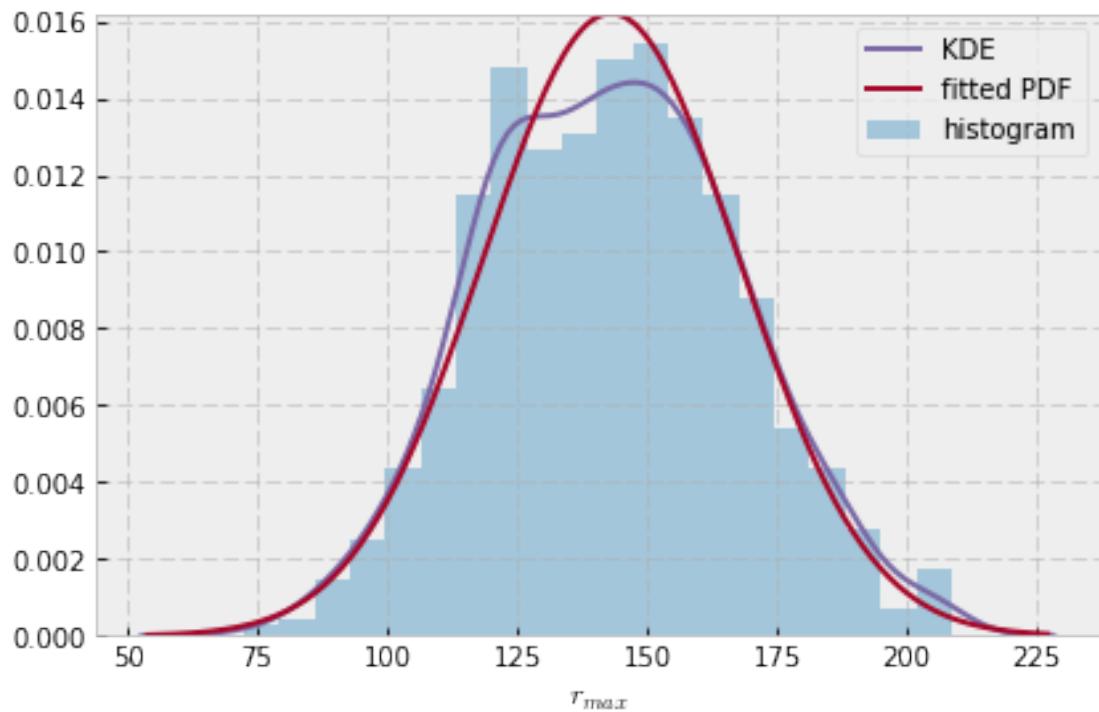


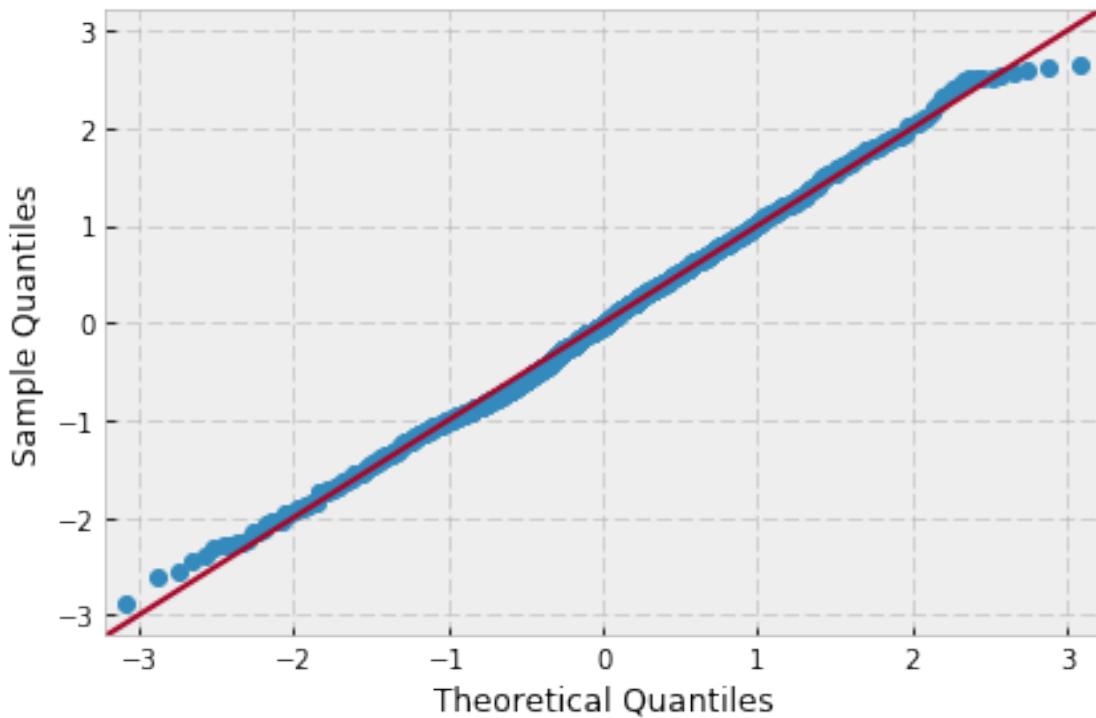


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(),loc=135.92808521935845, scale=24.268542886686955)
DescribeResult(nobs=1000, minmax=(65.268714607813237, 210.55450727388578)
               mean=135.92808521935845, variance=589.55172556853256,
               skewness=0.19640785357324397, kurtosis=-0.3014706413684656)
```

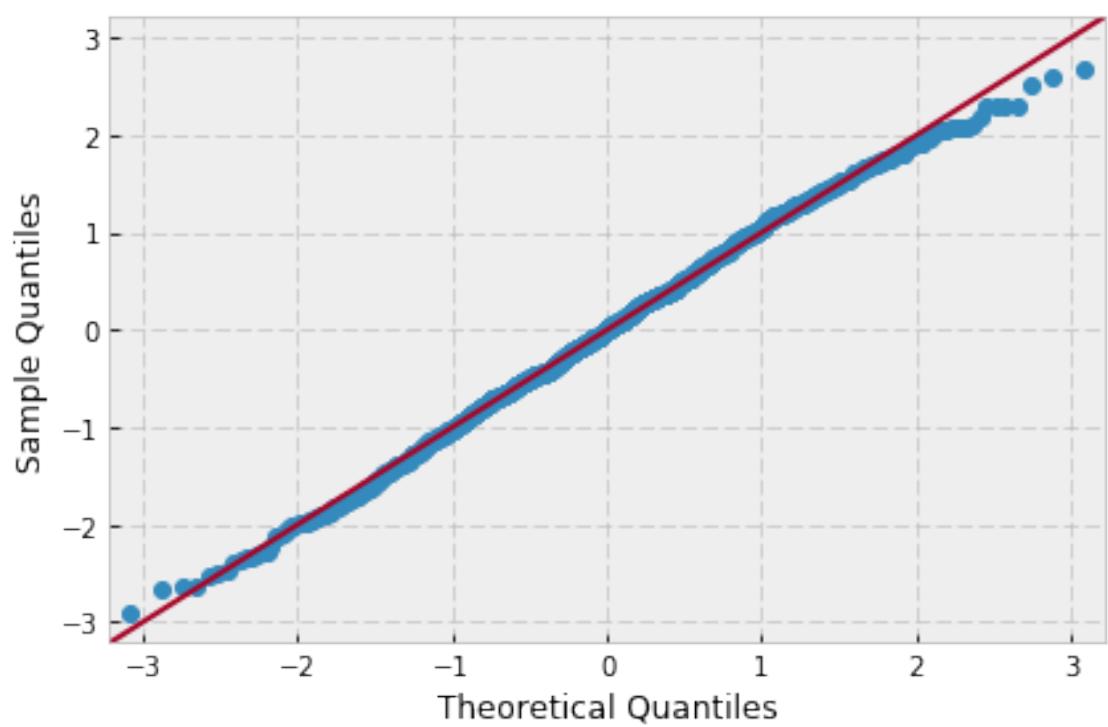
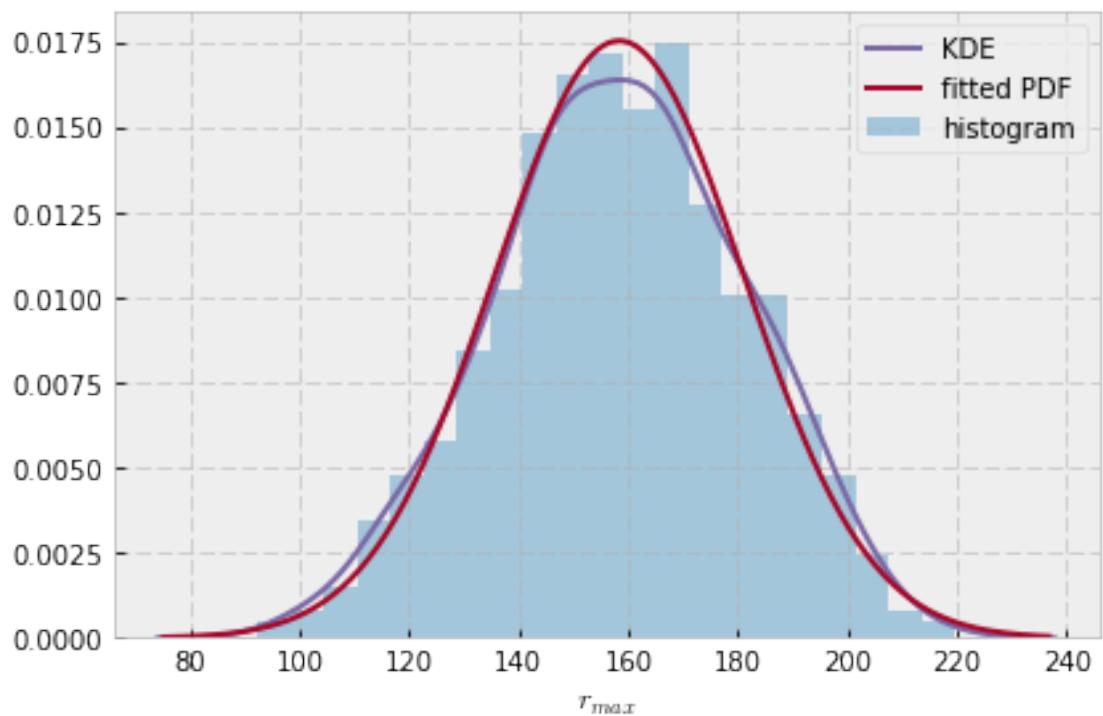


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(), loc=143.096839234708, scale=24.62769492667748)
DescribeResult(nobs=1000, minmax=(72.382540440009521, 208.60283614481276)
               mean=143.096839234708, variance=607.13048788938522,
               skewness=0.09546727600565144, kurtosis=-0.3248290683376567)
```

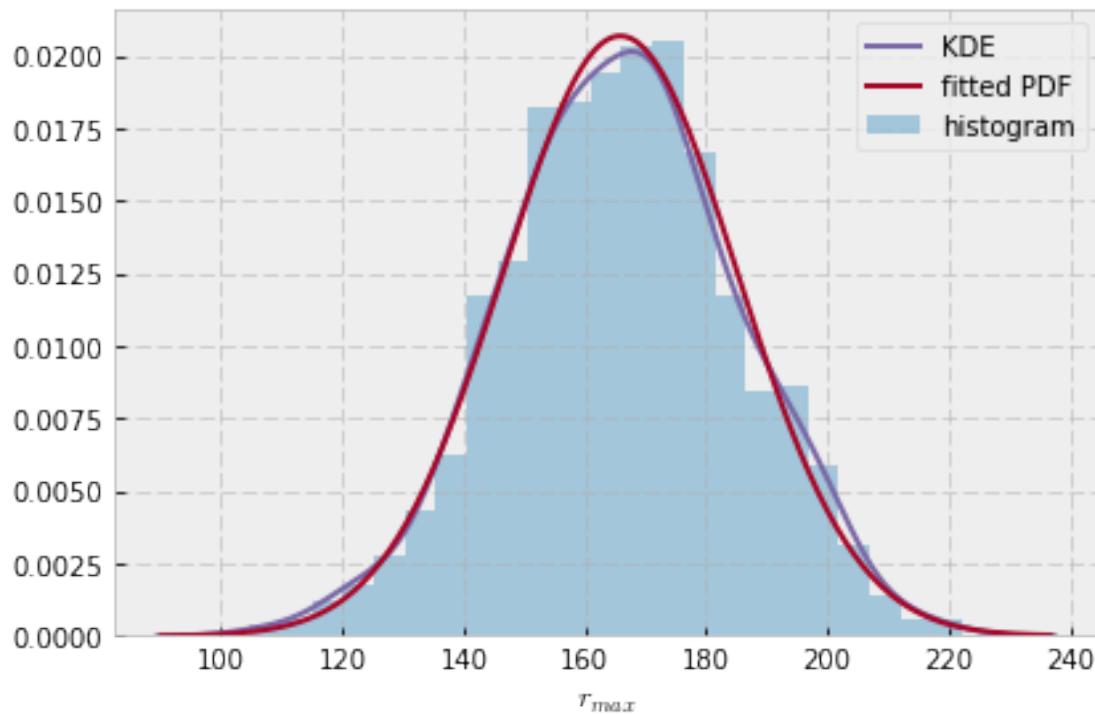


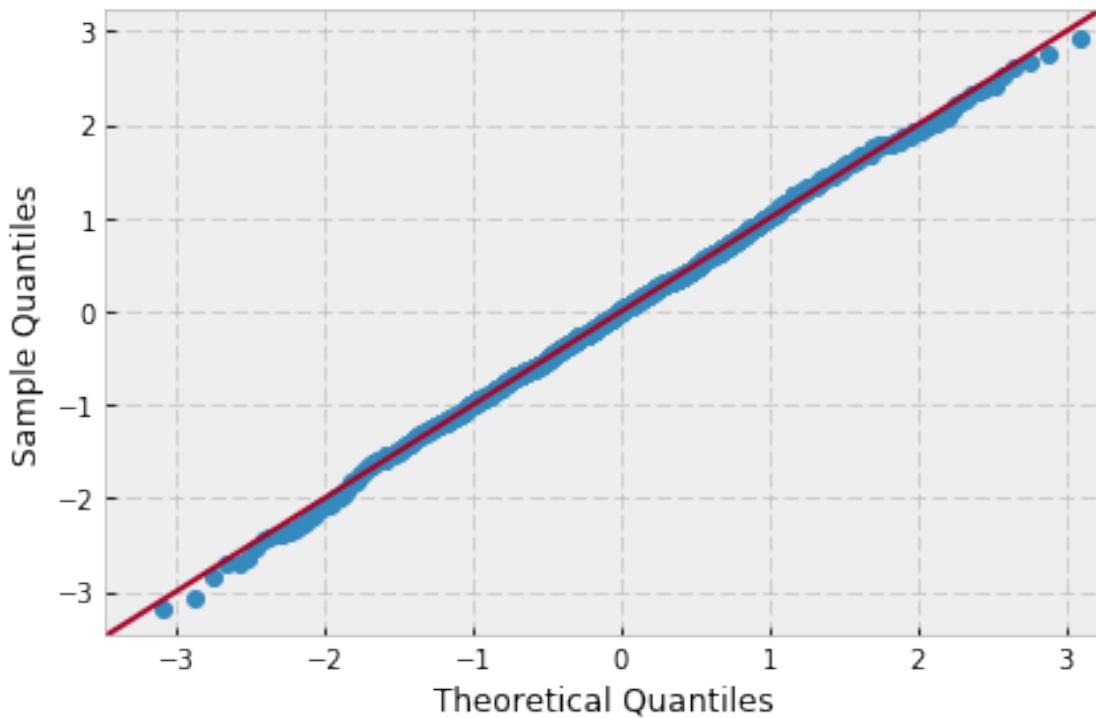


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(),loc=158.35921551415035, scale=22.76209826982621)
DescribeResult(nobs=1000, minmax=(92.326699586234668, 219.55000757930097)
               mean=158.35921551415035, variance=518.63174939461999,
               skewness=-0.08193869230673463, kurtosis=-0.33815498132559085)
```

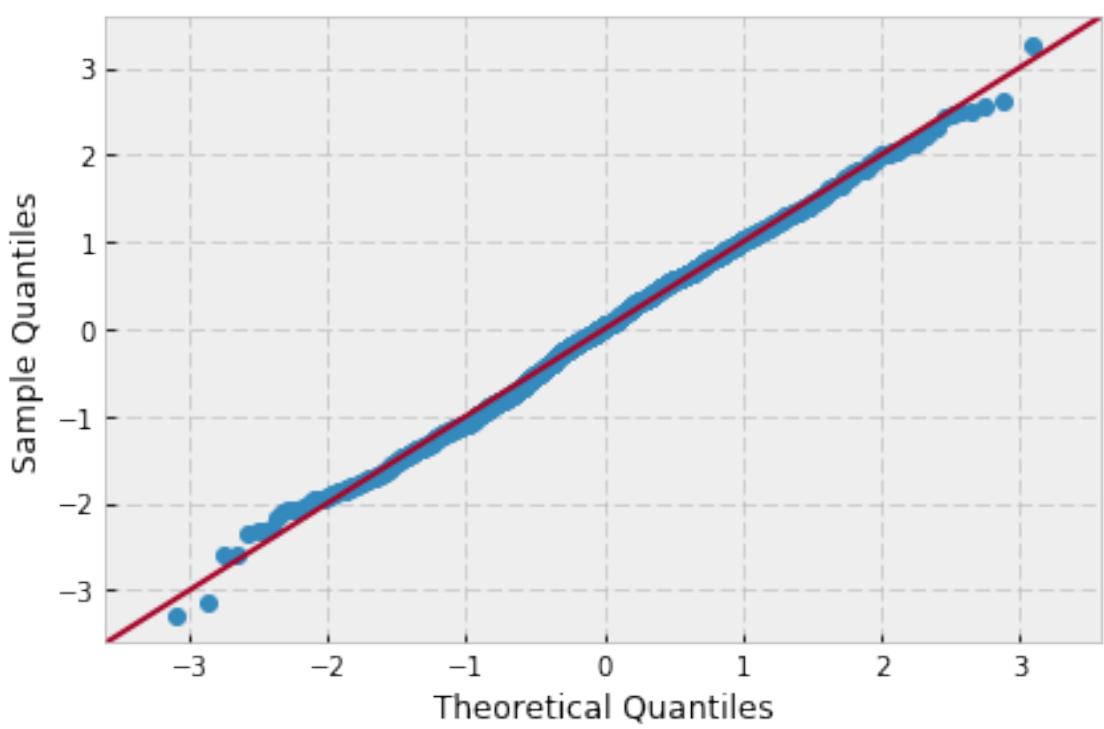
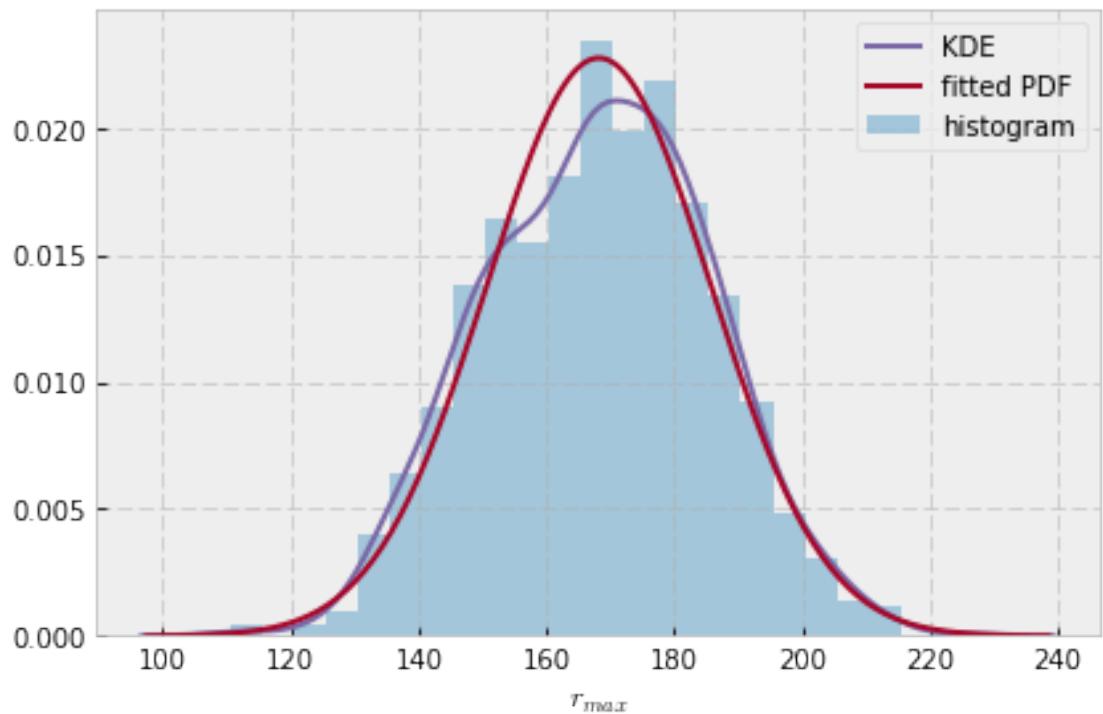


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(), loc=165.85033481333045, scale=19.301123588640476)
DescribeResult(nobs=1000, minmax=(104.69061004813781, 222.28537197736881)
               mean=165.85033481333045, variance=372.90627806203588,
               skewness=-0.04144107618787465, kurtosis=-0.08574129405267739)
```

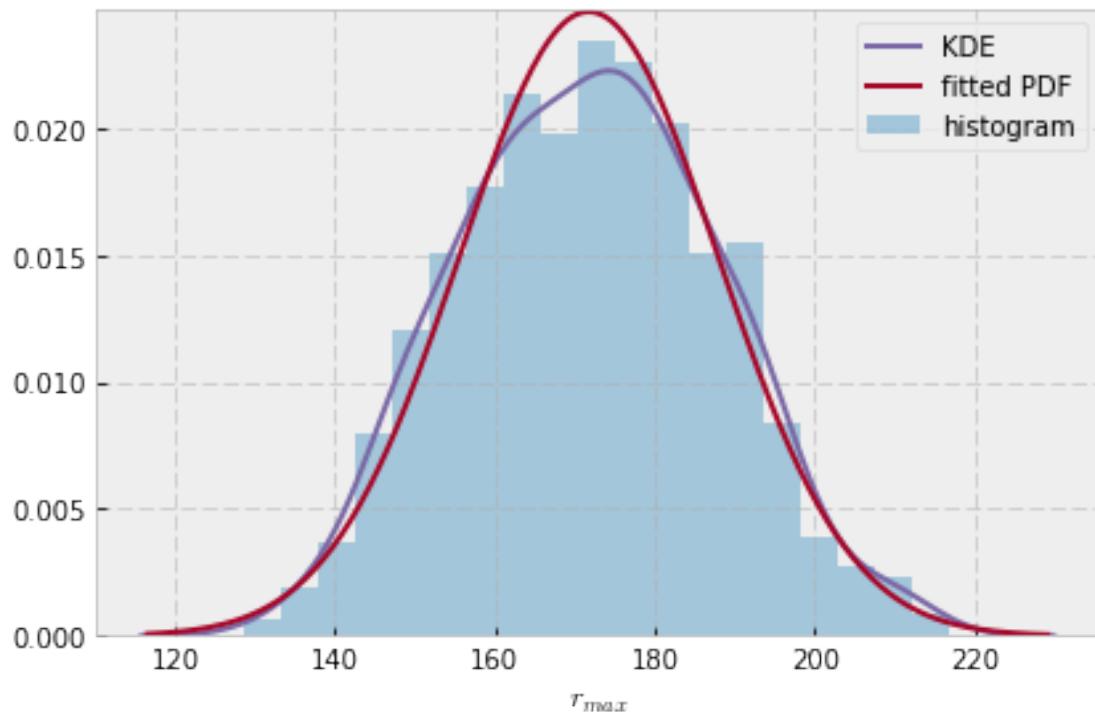


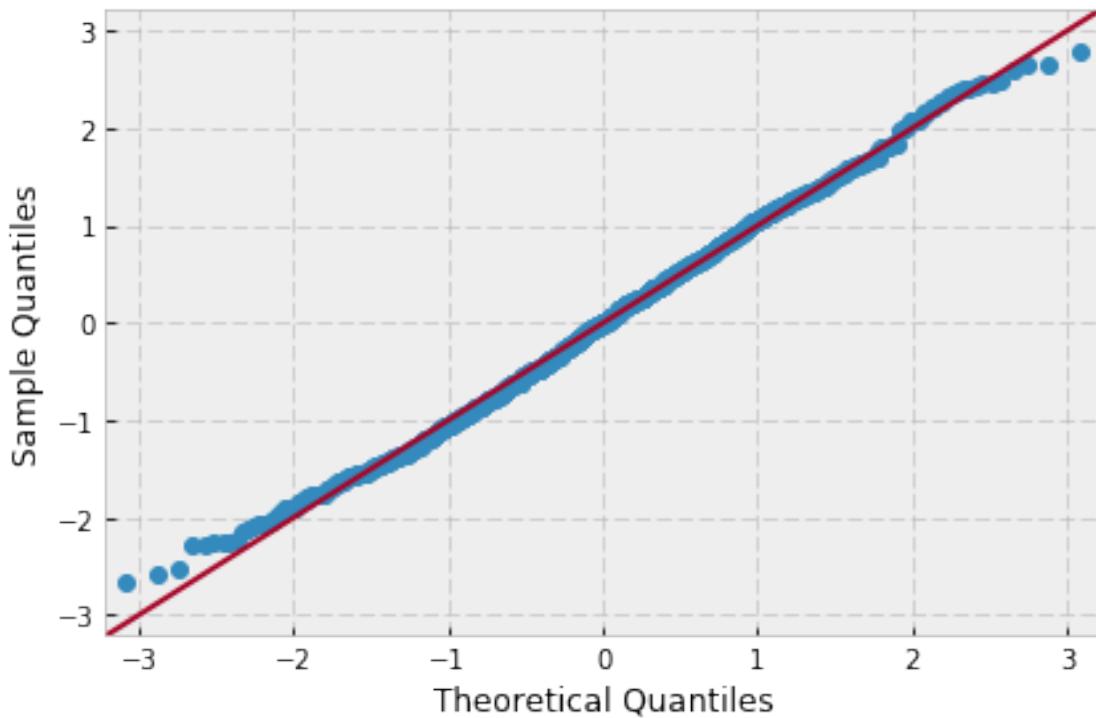


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(),loc=168.20172370878421, scale=17.536419545073496)
DescribeResult(nobs=1000, minmax=(110.65114352328639, 225.33414439630224)
               mean=168.20172370878421, variance=307.83384430514081,
               skewness=-0.045974547227893105, kurtosis=-0.3013072201052682)
```

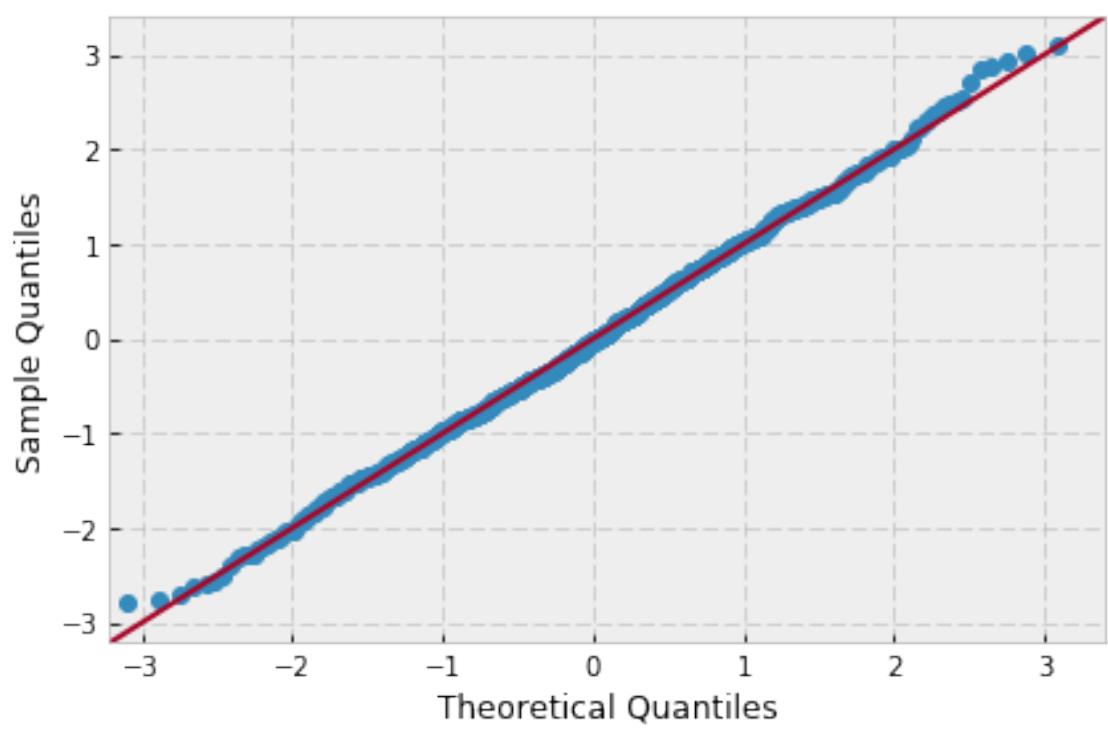
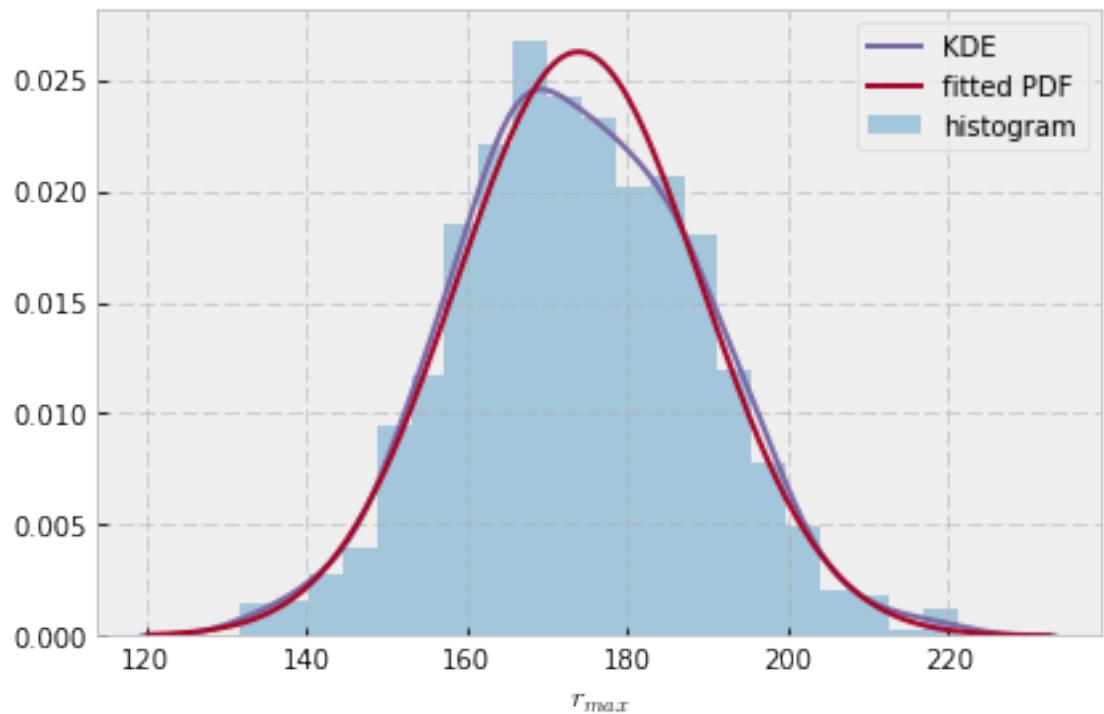


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(), loc=171.73792594513904, scale=16.220855319245523)
DescribeResult(nobs=1000, minmax=(128.73544334285515, 216.91617221426043)
               mean=171.73792594513904, variance=263.37952681471052,
               skewness=0.0707239462713022, kurtosis=-0.41180942540334664)
```

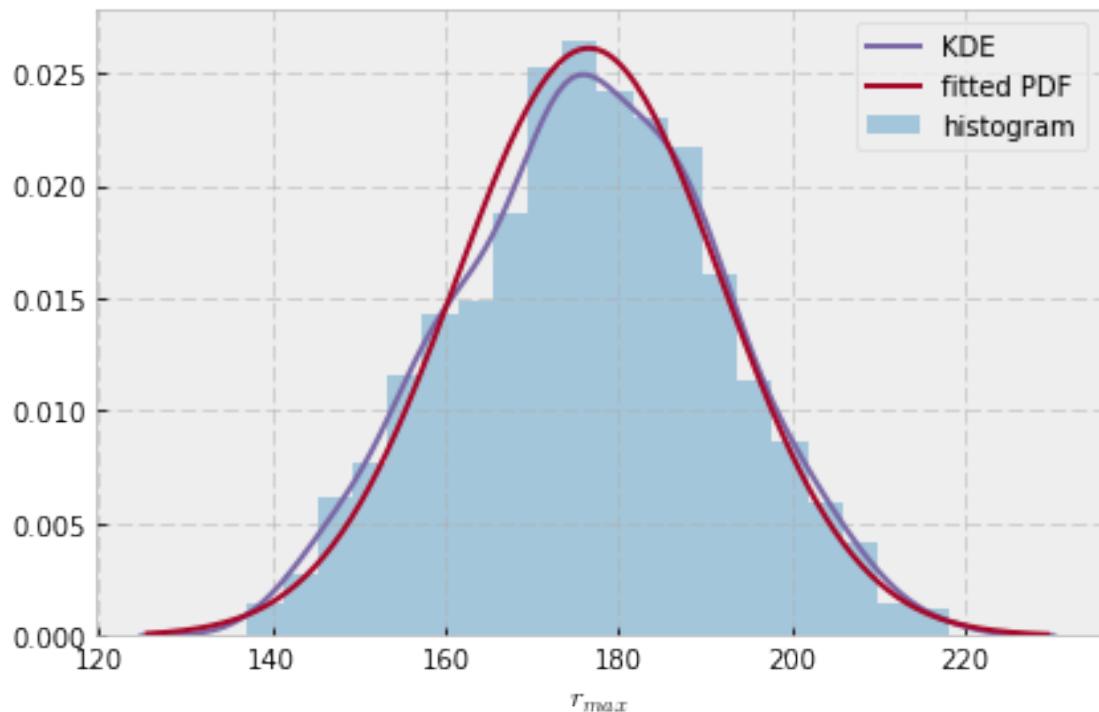


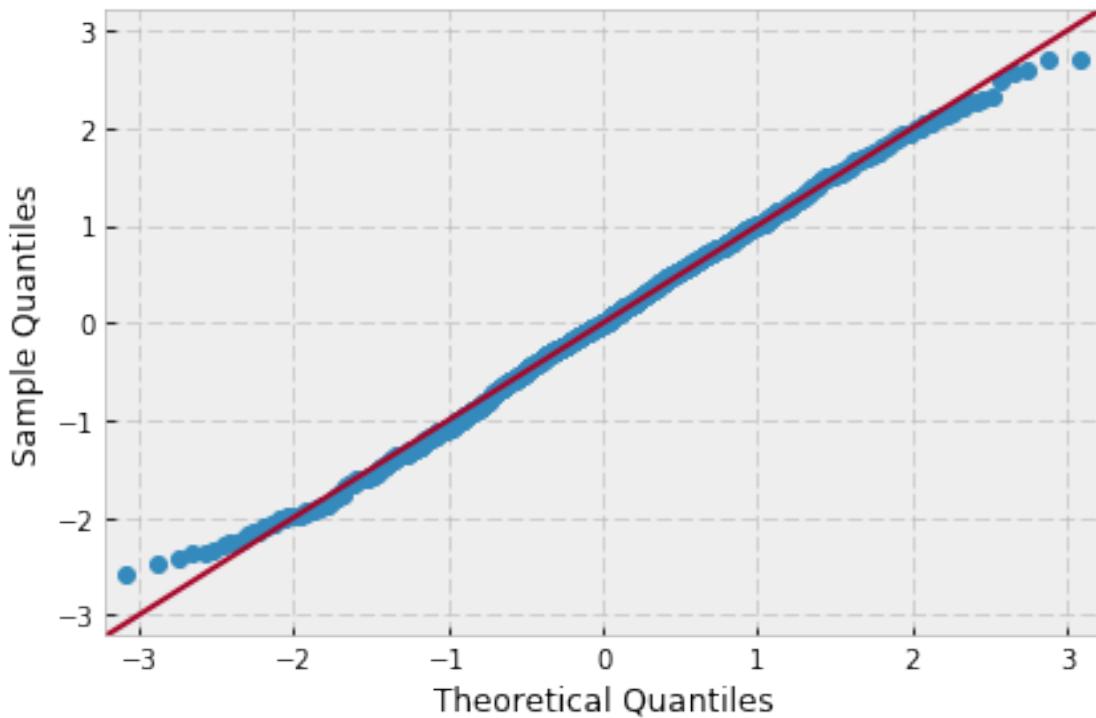


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(),loc=173.896039710697, scale=15.198563134085)
DescribeResult(nobs=1000, minmax=(131.69123738296818, 221.04983353714553)
               mean=173.89603971069701, variance=231.2275488896573,
               skewness=0.07910436829414269, kurtosis=-0.12247183297206332)
```

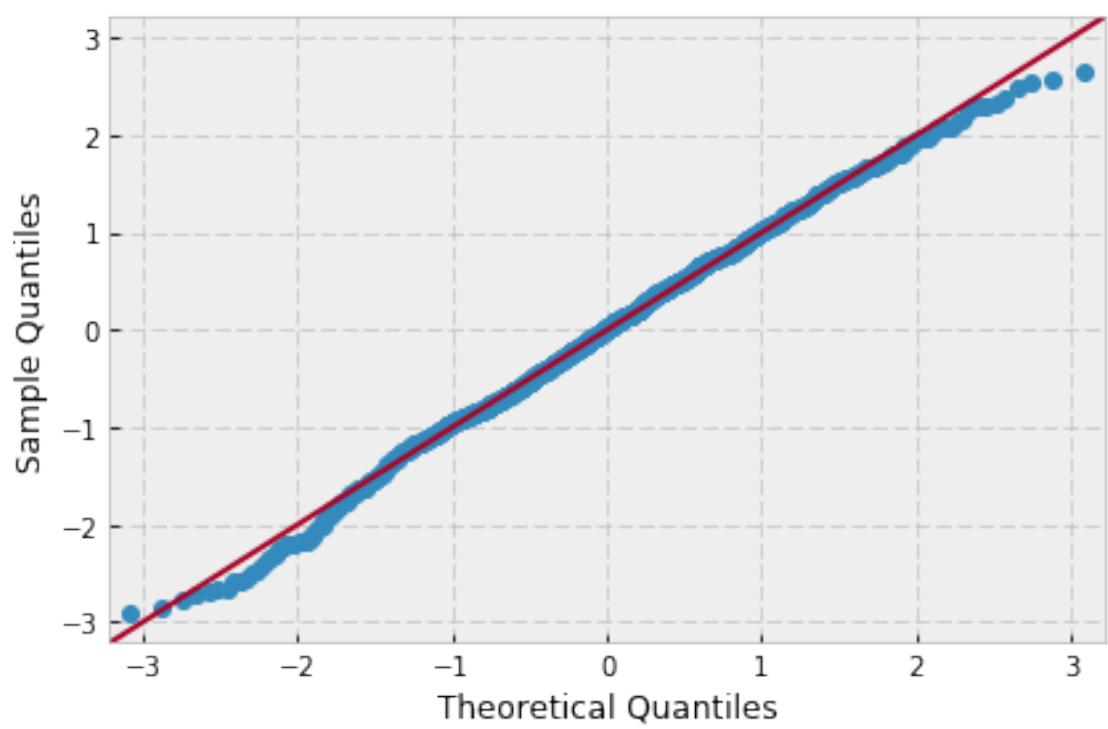
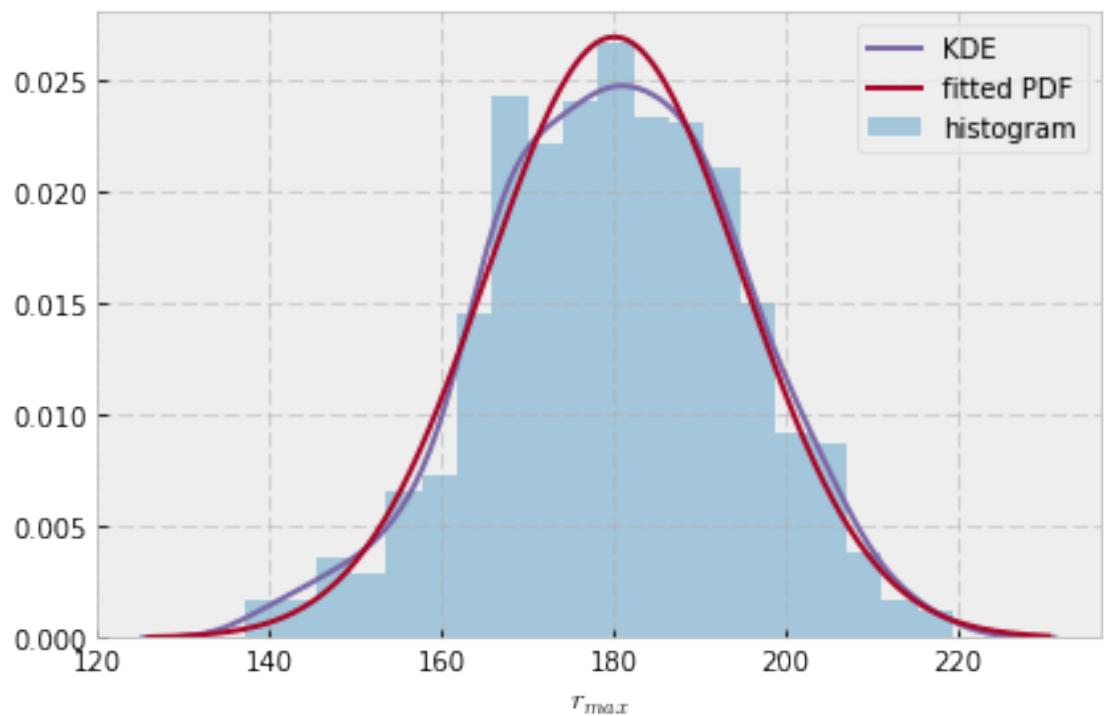


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(), loc=176.52010616457824, scale=15.291244750143395)
DescribeResult(nobs=1000, minmax=(137.09891616397442, 217.96043589313044)
               mean=176.52010616457824, variance=234.05622223101898,
               skewness=-0.021925573310730643, kurtosis=-0.3941728228820818)
```

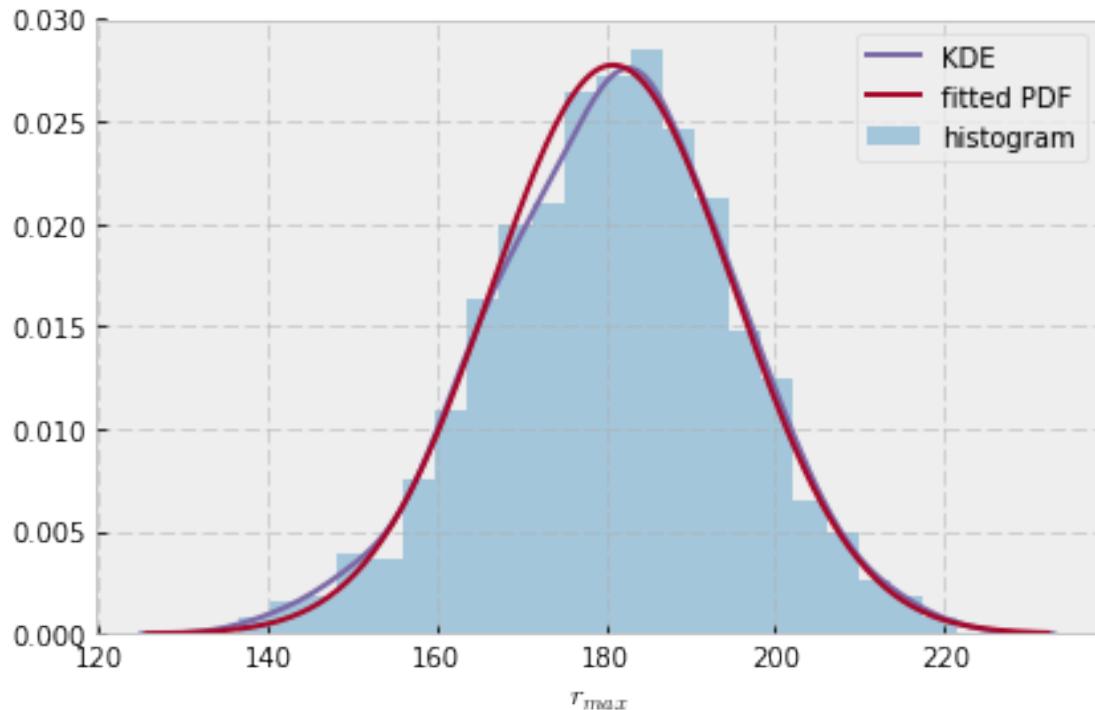


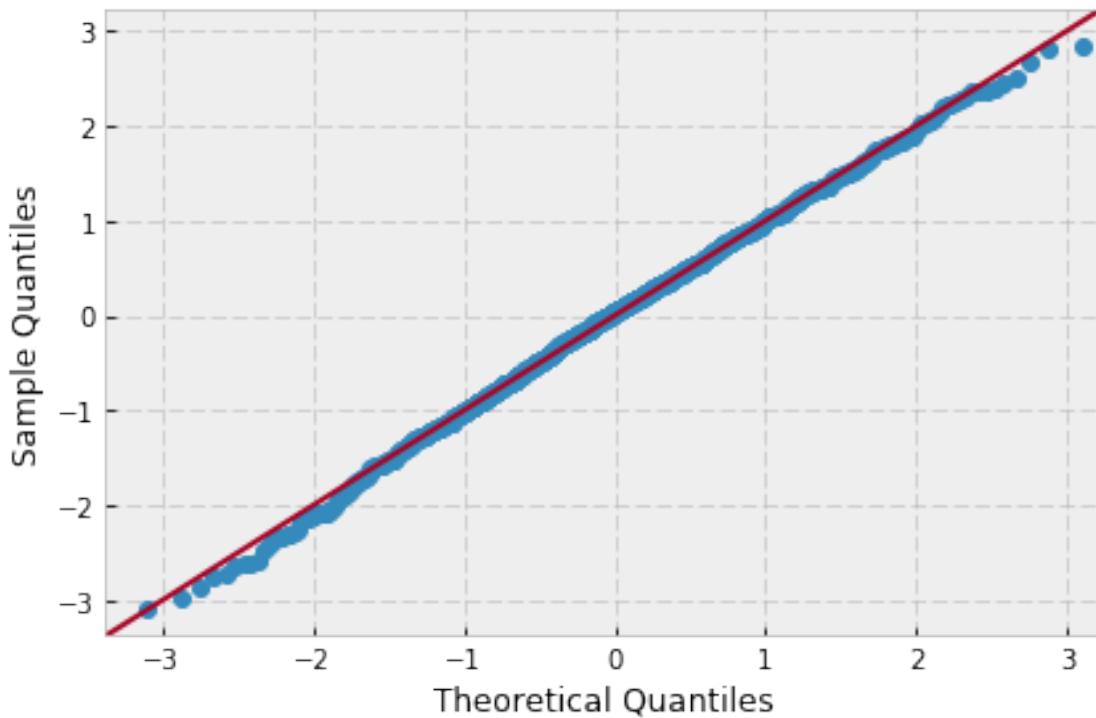


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(),loc=180.07906213313203, scale=14.816885436213513)
DescribeResult(nobs=1000, minmax=(137.05614314776574, 219.42212602755725)
               mean=180.07906213313203, variance=219.75985388375983,
               skewness=-0.12859925357460236, kurtosis=-0.1438821700406585)
```

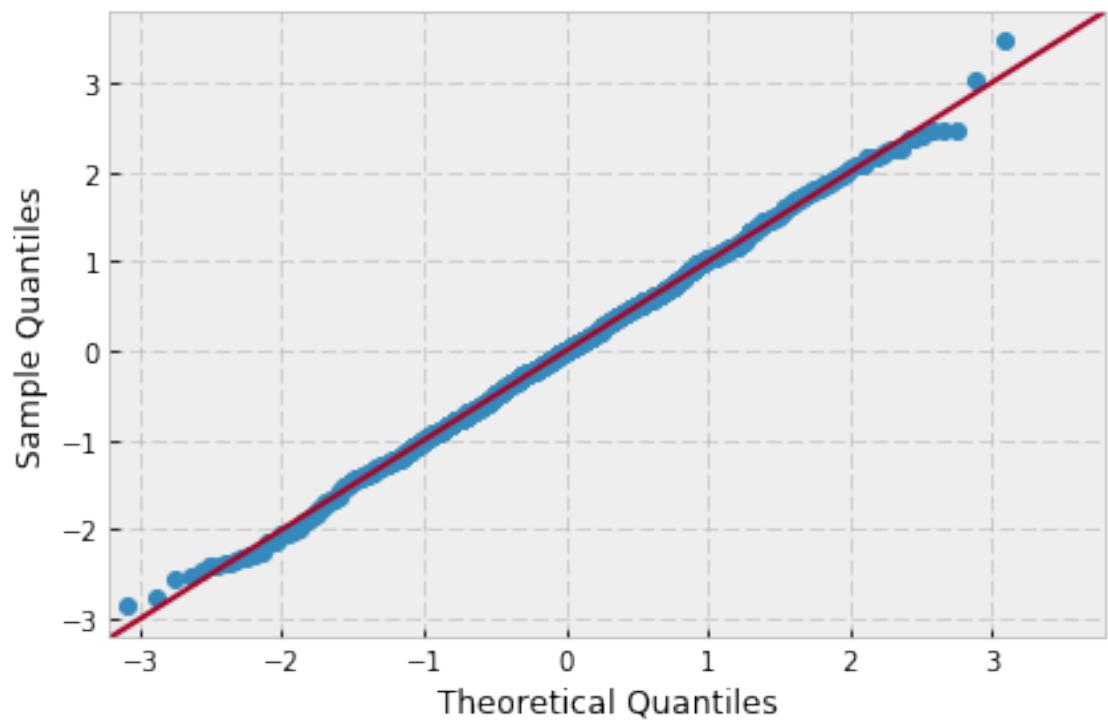
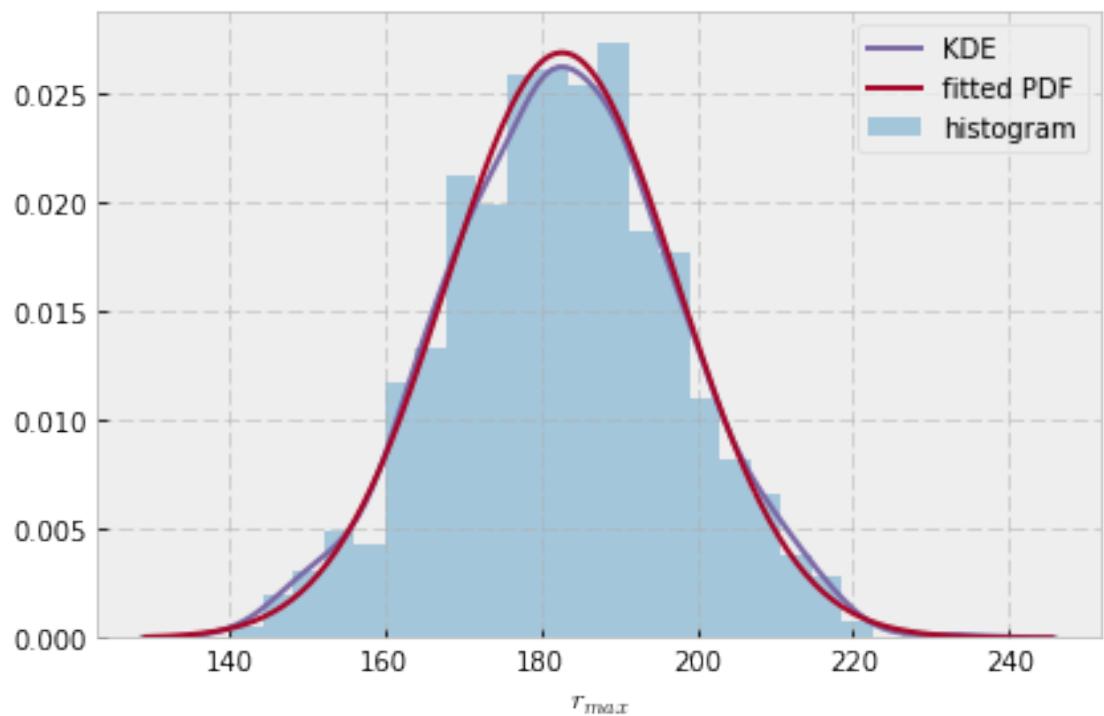


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(), loc=180.79751697228116, scale=14.3850408711608)
DescribeResult(nobs=1000, minmax=(136.53995035941253, 221.41153698654293)
               mean=180.79751697228116, variance=207.13653740236904,
               skewness=-0.12201875080490893, kurtosis=-0.06349968230064773)
```

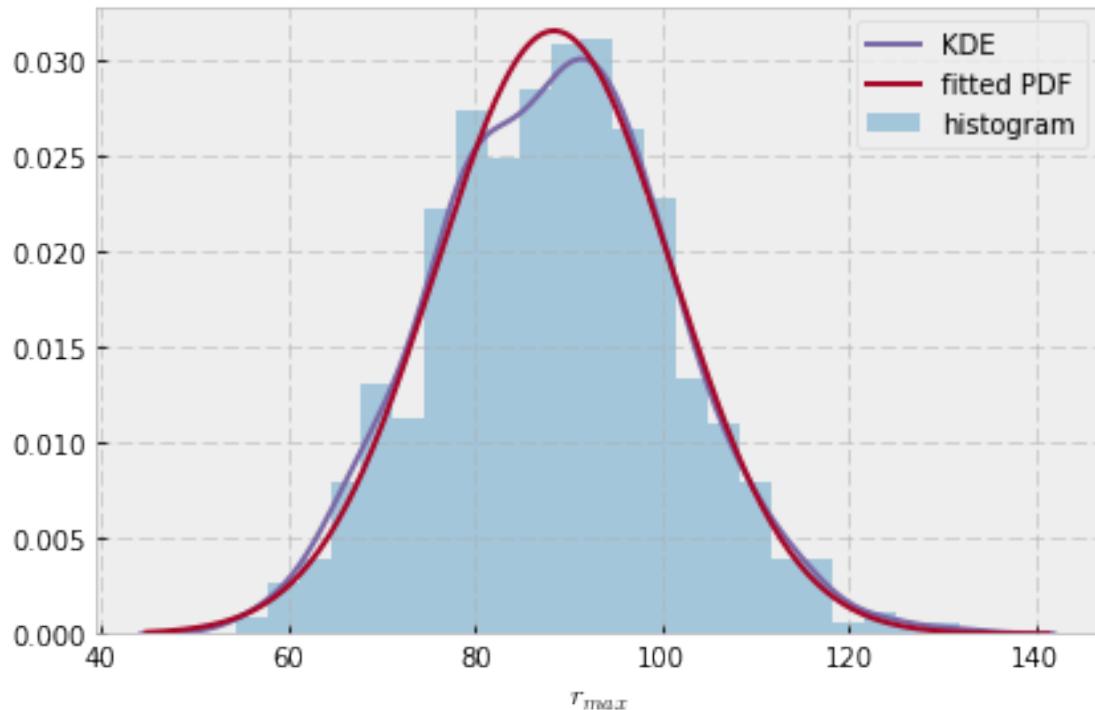


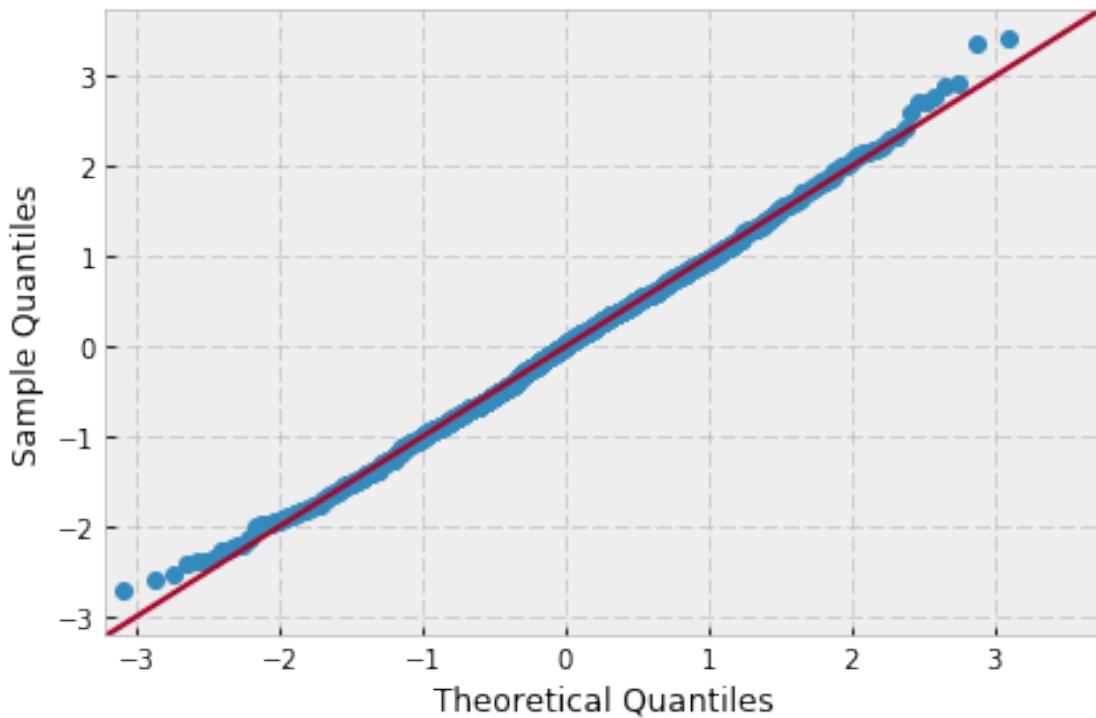


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(),loc=182.7122688141931, scale=14.817809980281883)
DescribeResult(nobs=1000, minmax=(140.55036368634666, 234.20369977619674)
               mean=182.7122688141931, variance=219.78727989163298,
               skewness=0.01606110483132167, kurtosis=-0.11626360728297414)
```

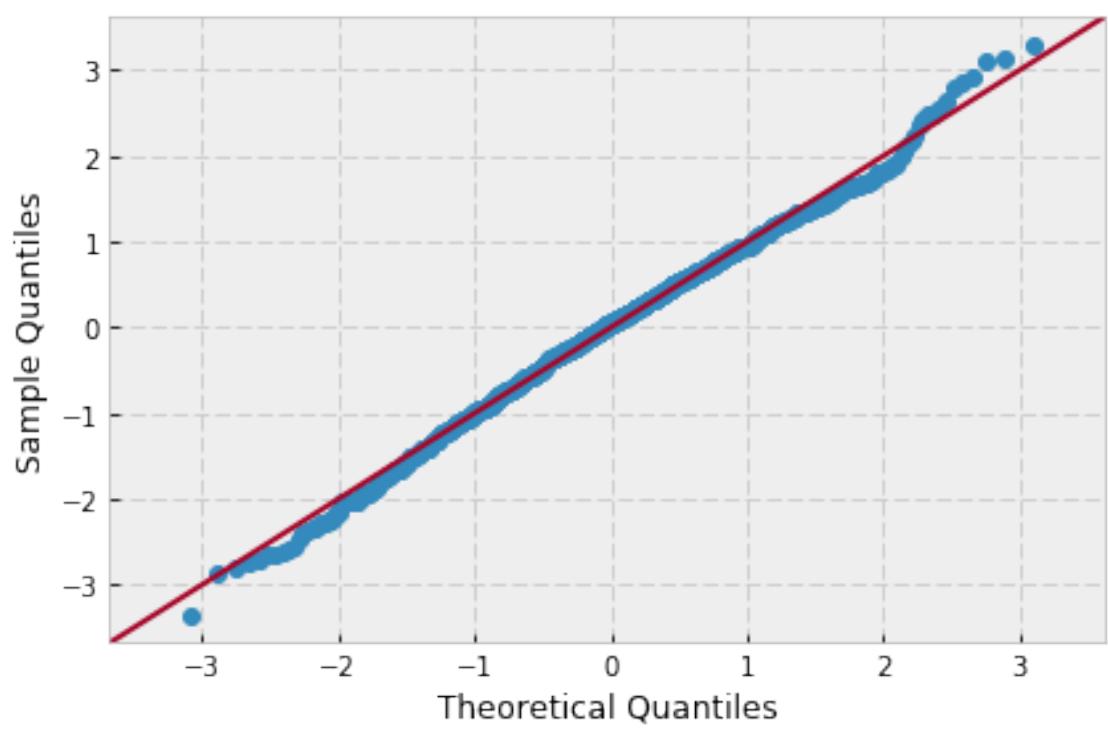
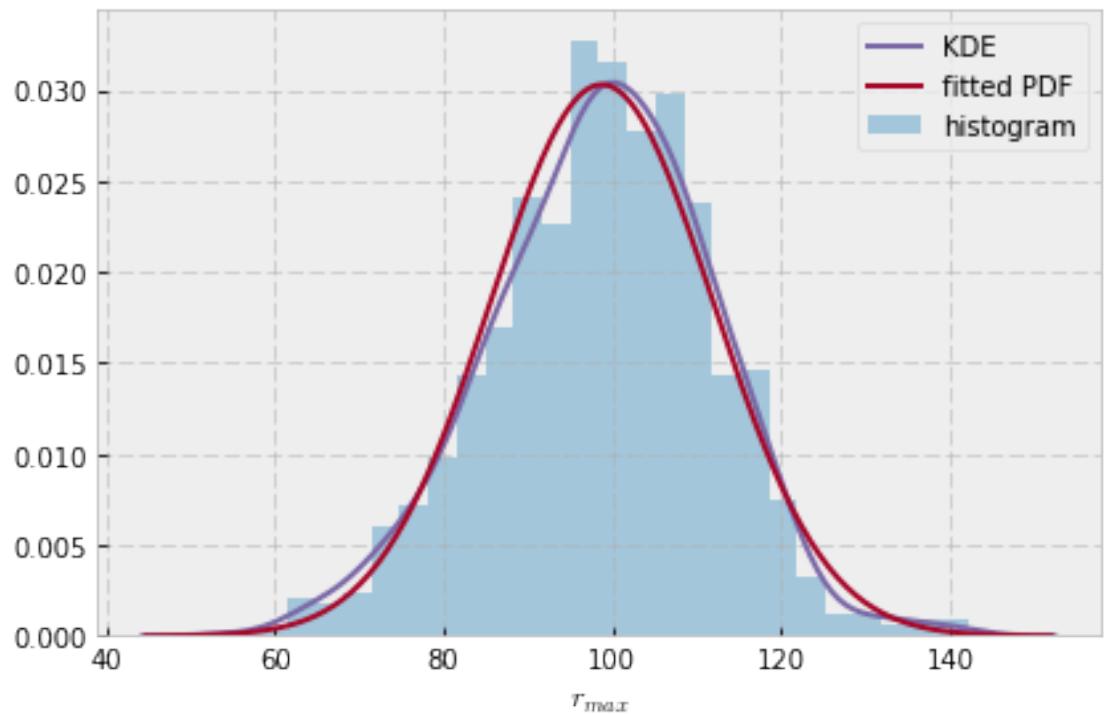


```
variable name: r
variable value: 1.0
distribution: normal(shape=(),loc=88.45013404746486, scale=12.634414118917265)
DescribeResult(nobs=1000, minmax=(54.412895439480025, 131.79424698378924)
               mean=88.450134047464857, variance=159.78820833663258,
               skewness=0.11064125049702866, kurtosis=-0.07273639665424803)
```

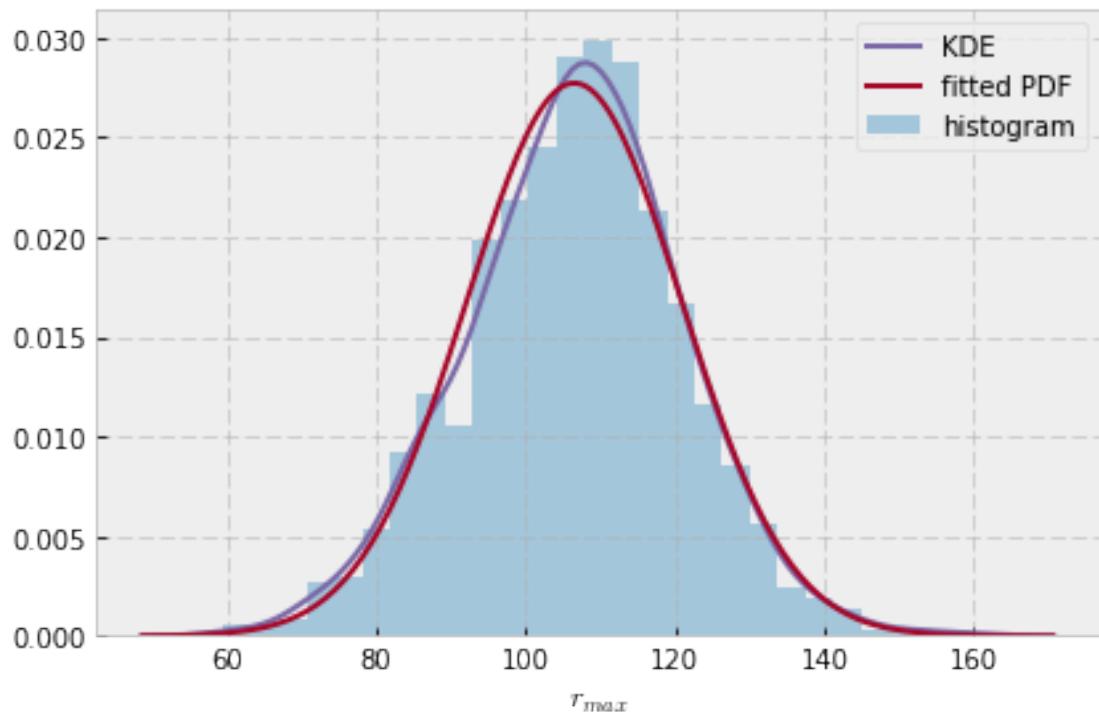


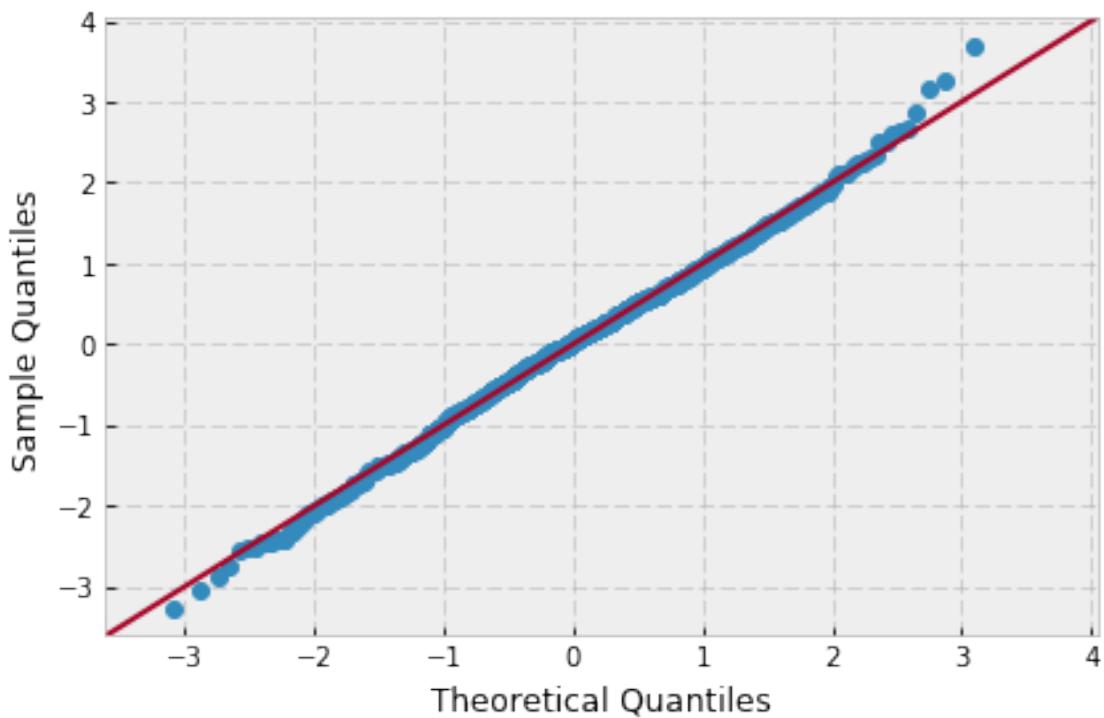


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(),loc=98.72812475358317, scale=13.169909893790619)
DescribeResult(nobs=1000, minmax=(54.675365624739634, 141.99855721806688)
               mean=98.728124753583174, variance=173.62014675732135,
               skewness=-0.13926691401210062, kurtosis=0.17641981657564276)
```

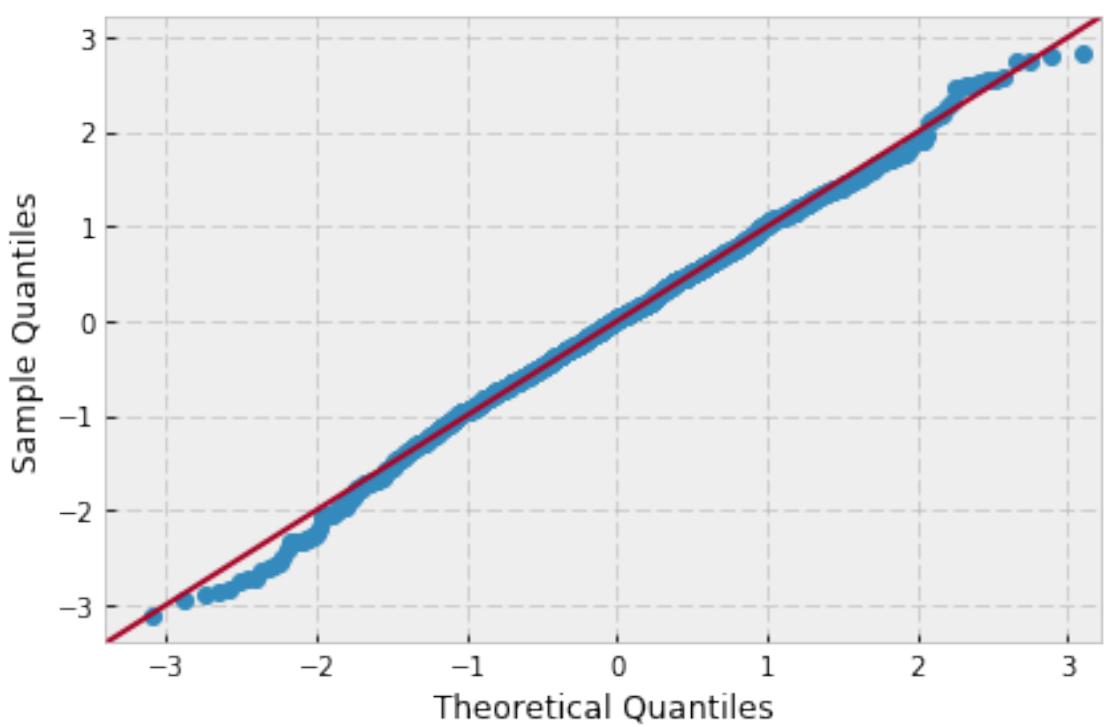
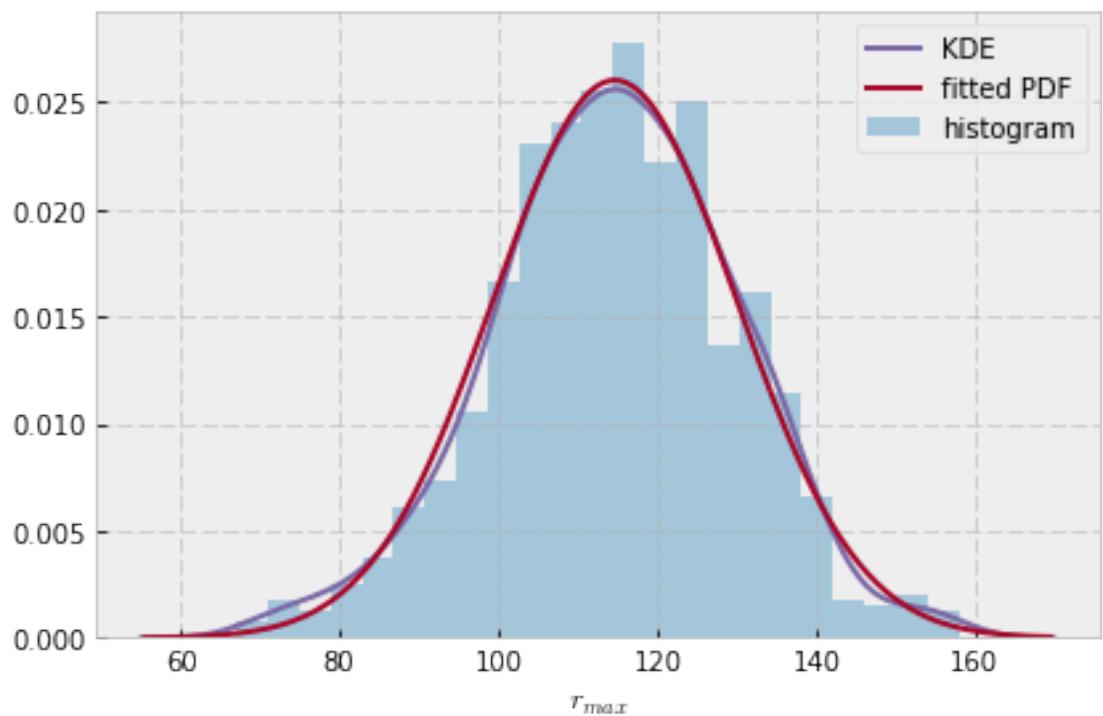


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=106.47812226117752, scale=14.392227816427084)
DescribeResult(nobs=1000, minmax=(59.465894381812689, 159.73678753488289)
               mean=106.47812226117752, variance=207.34356508502253,
               skewness=-0.05984384369323599, kurtosis=0.2328935393355165)
```

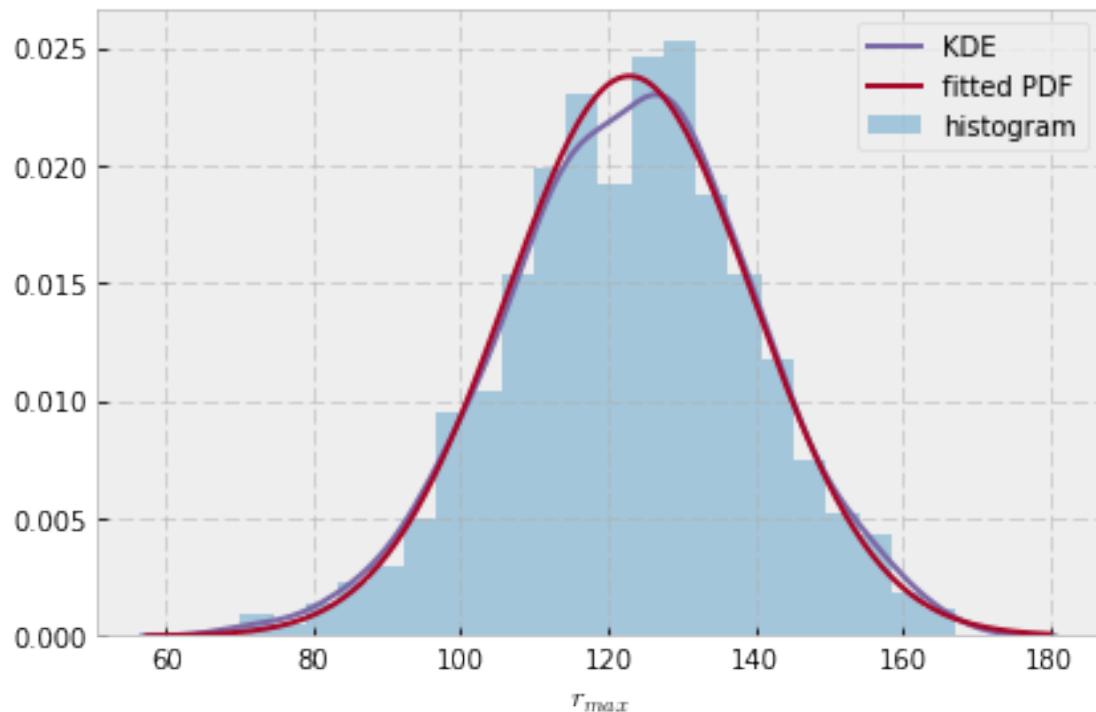


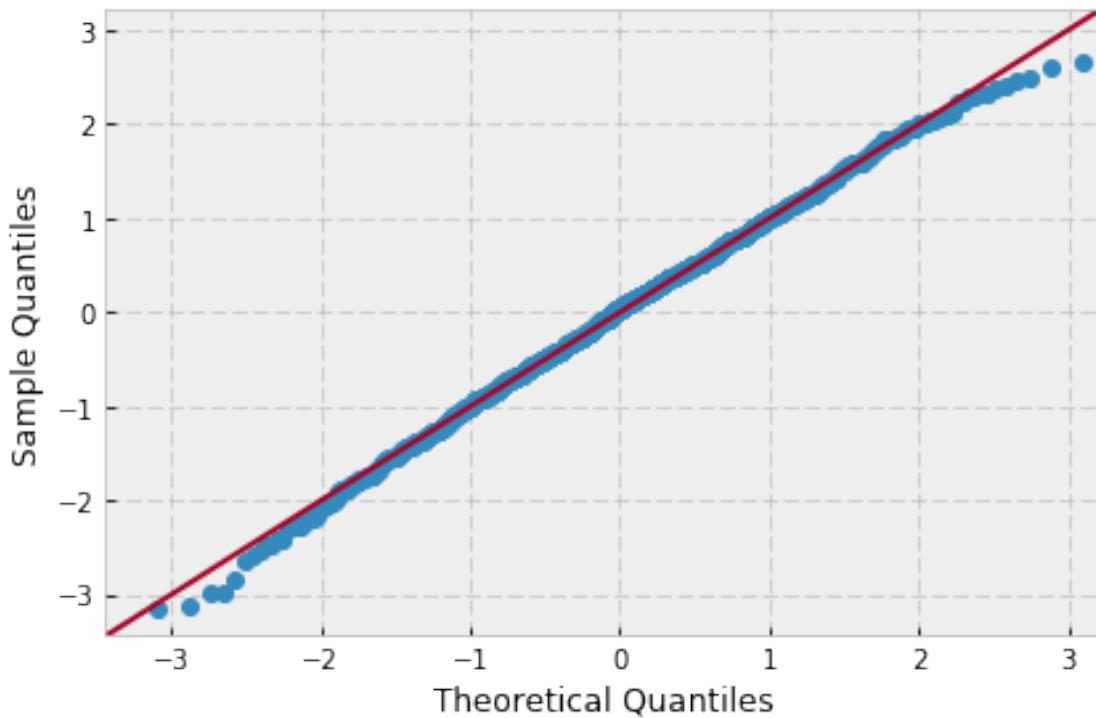


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=114.66443725344944, scale=15.330812442108211)
DescribeResult(nobs=1000, minmax=(67.003195889599709, 157.96535225434997)
               mean=114.66443725344944, variance=235.26907921431425,
               skewness=-0.1373038767239501, kurtosis=0.13947072464252708)
```

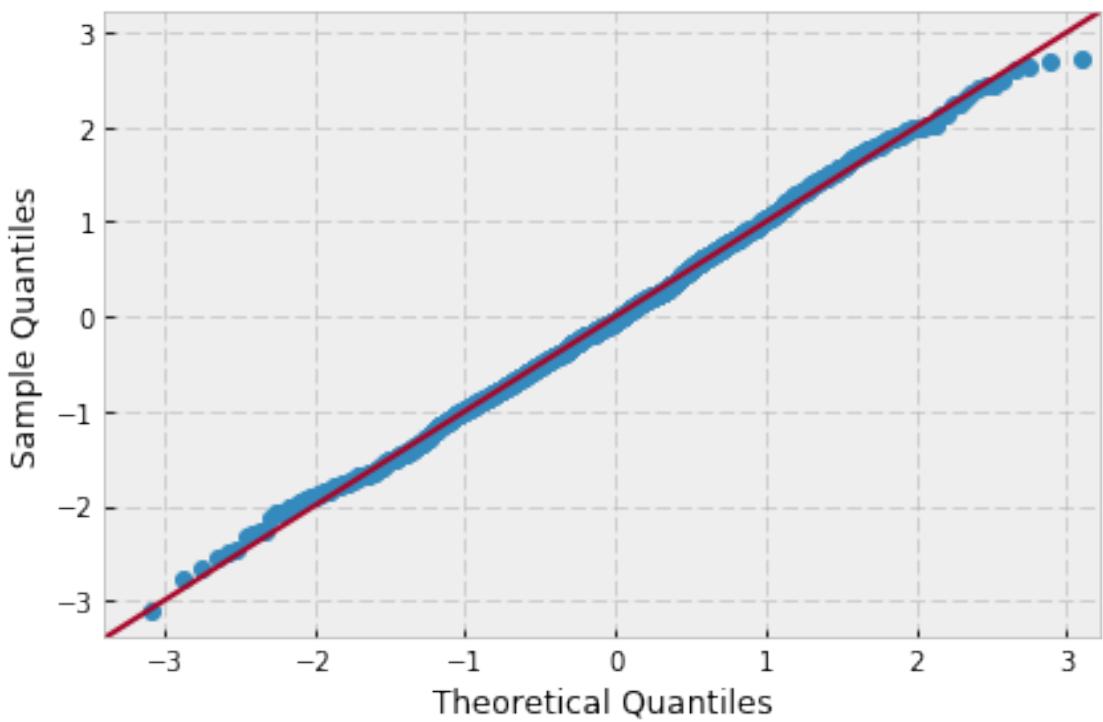
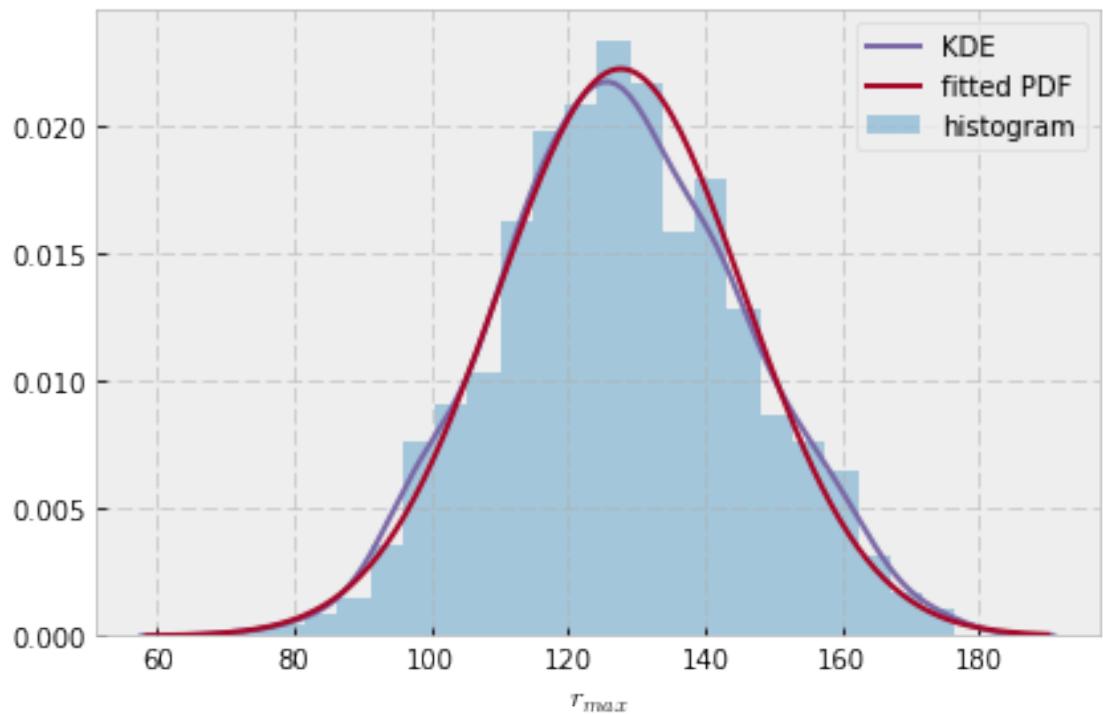


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(),loc=122.92601659093837, scale=16.76295910991927)
DescribeResult(nobs=1000, minmax=(70.088367062293372, 167.31642852554907)
               mean=122.92601659093837, variance=281.27807619702241,
               skewness=-0.12715037149858957, kurtosis=-0.06384209180502998)
```

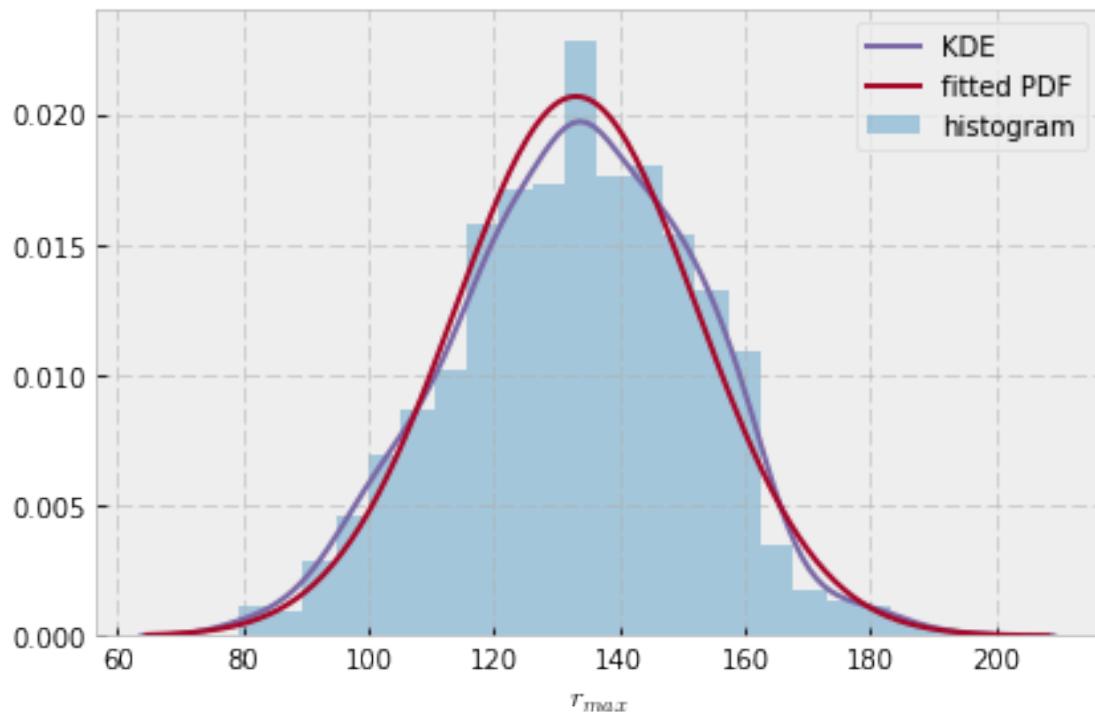


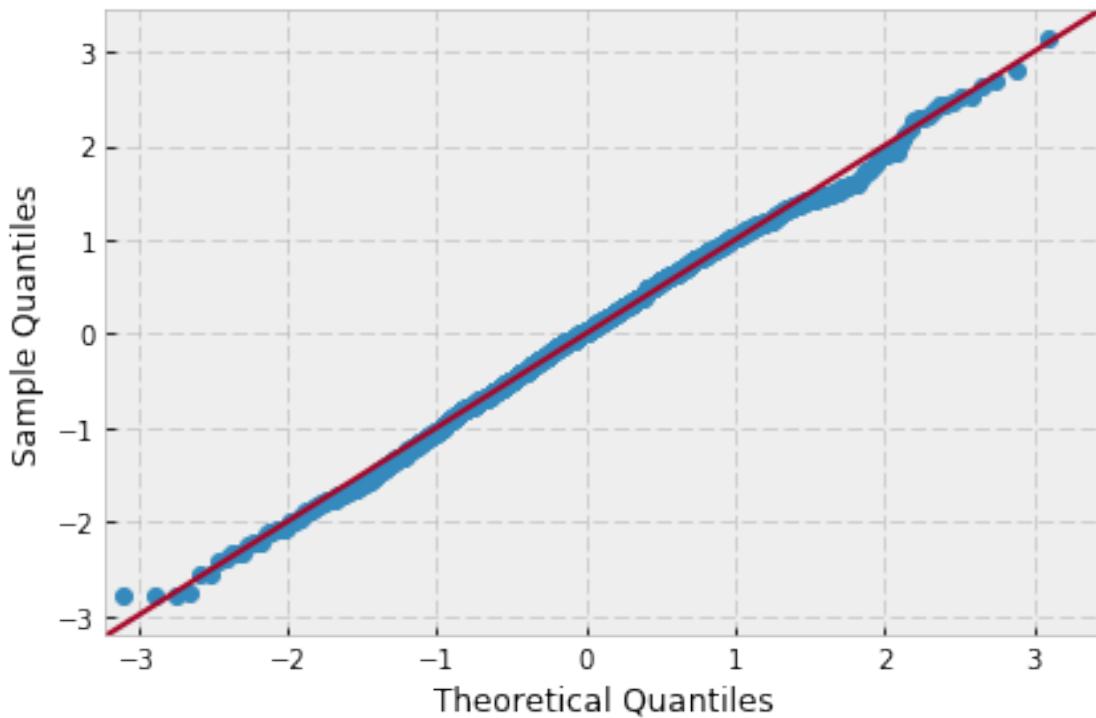


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(),loc=127.76104503355424, scale=17.949857767341893)
DescribeResult(nobs=1000, minmax=(72.072845915542857, 176.59308566534582)
               mean=127.76104503355424, variance=322.51991378158573,
               skewness=0.06533587829104441, kurtosis=-0.2715229717671712)
```

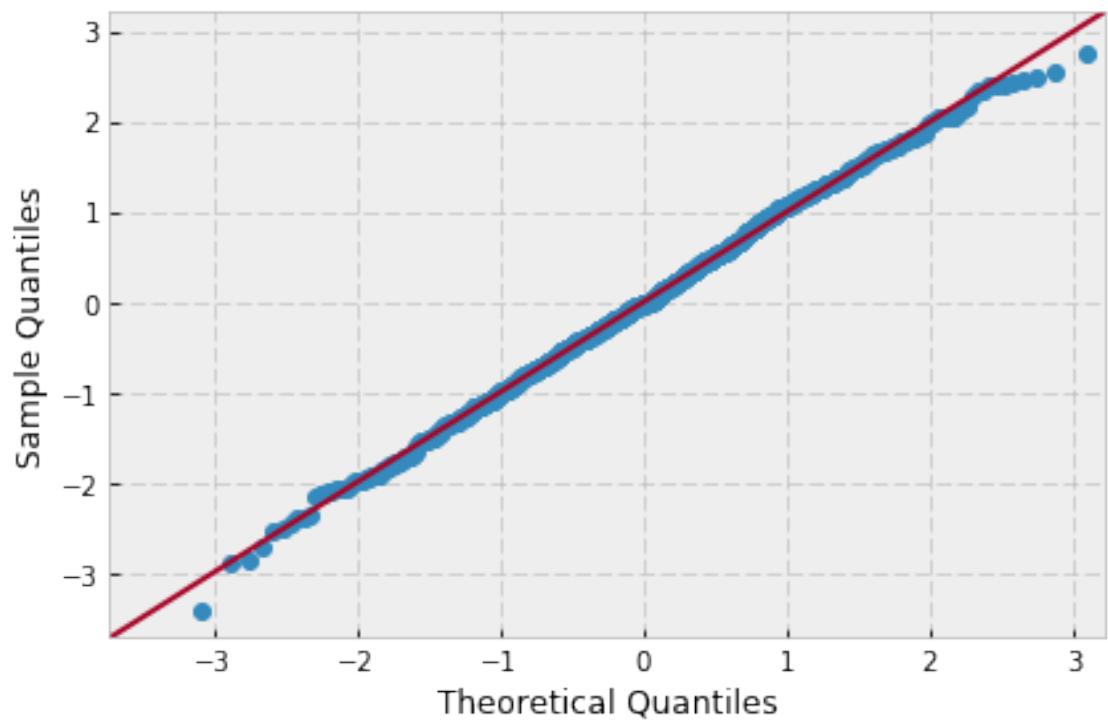
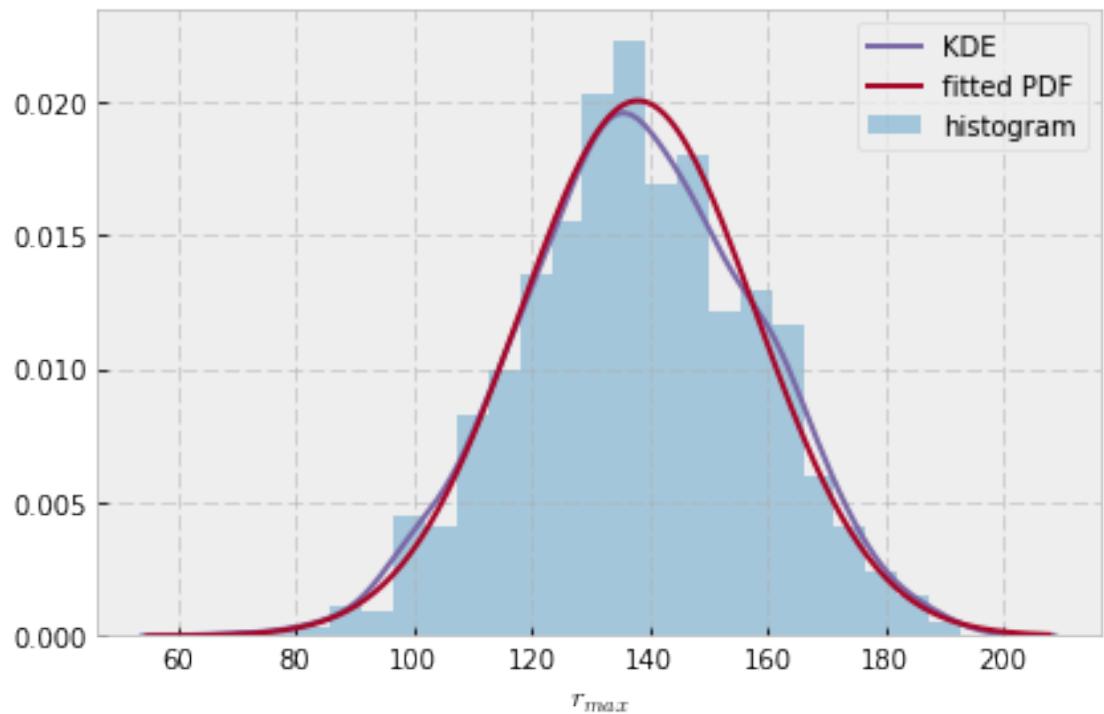


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(),loc=133.06869304796285, scale=19.29560781723754)
DescribeResult(nobs=1000, minmax=(79.185155068106269, 193.78039131438365)
               mean=133.06869304796285, variance=372.69317421084924,
               skewness=-0.09992785862432038, kurtosis=-0.20651980078089327)
```

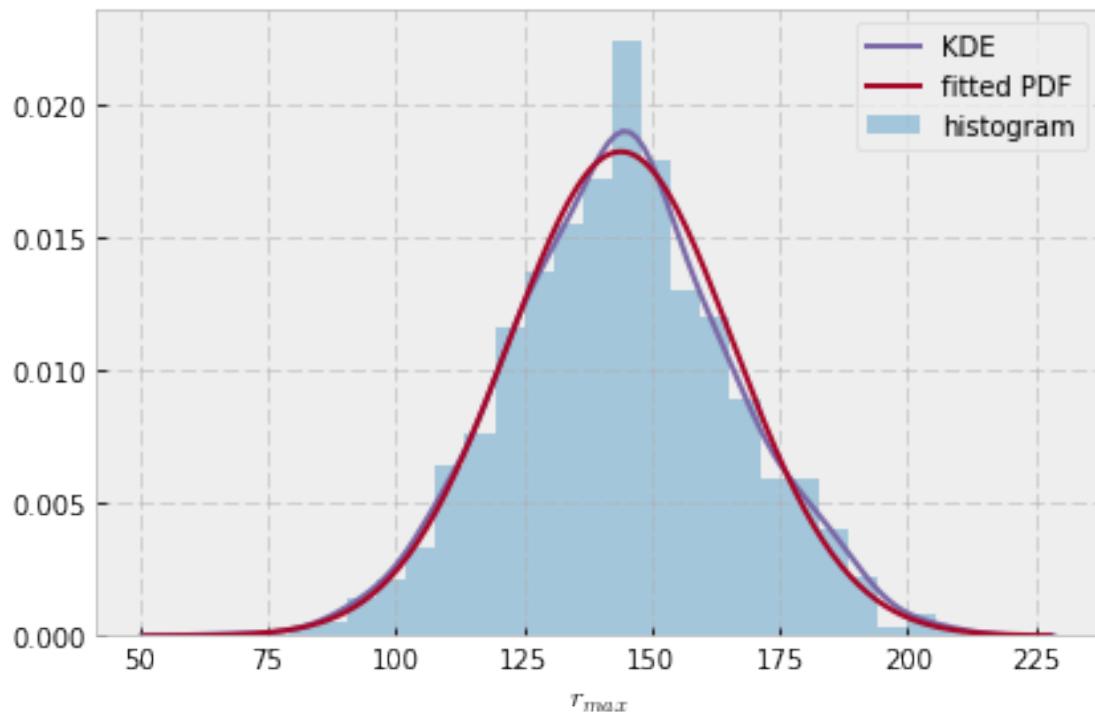


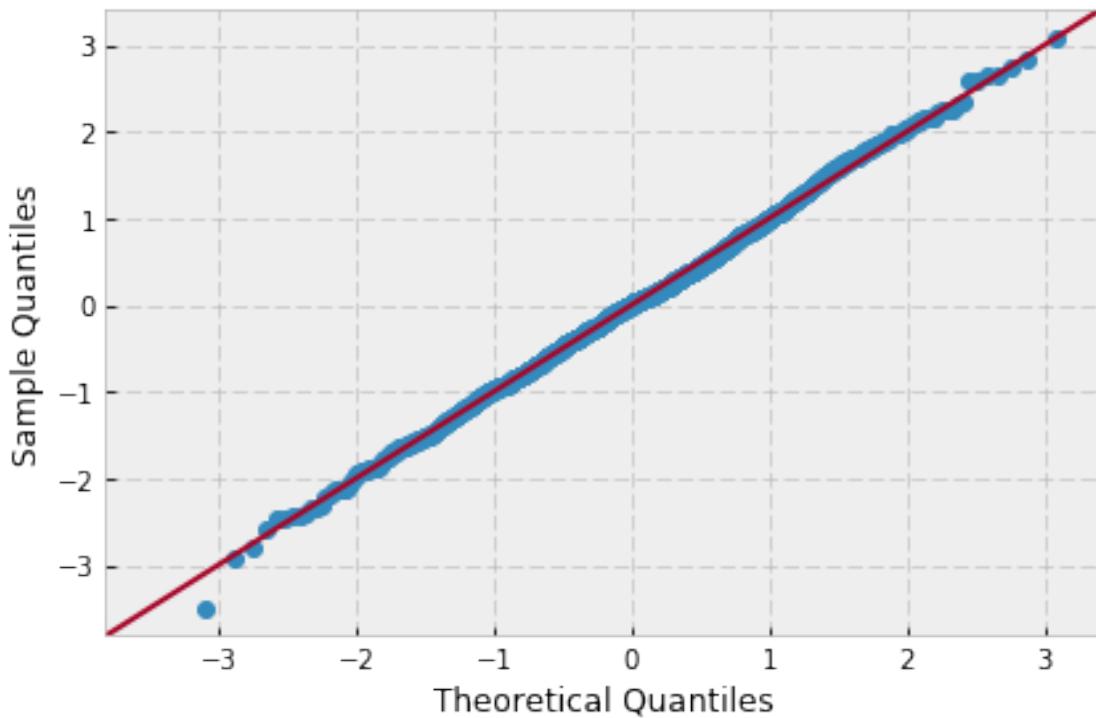


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(),loc=138.0739922987366, scale=19.930980529494565)
DescribeResult(nobs=1000, minmax=(70.016350481144528, 192.76726166582748)
               mean=138.07399229873661, variance=397.64162649358497,
               skewness=-0.04986544486799727, kurtosis=-0.21425343218380455)
```

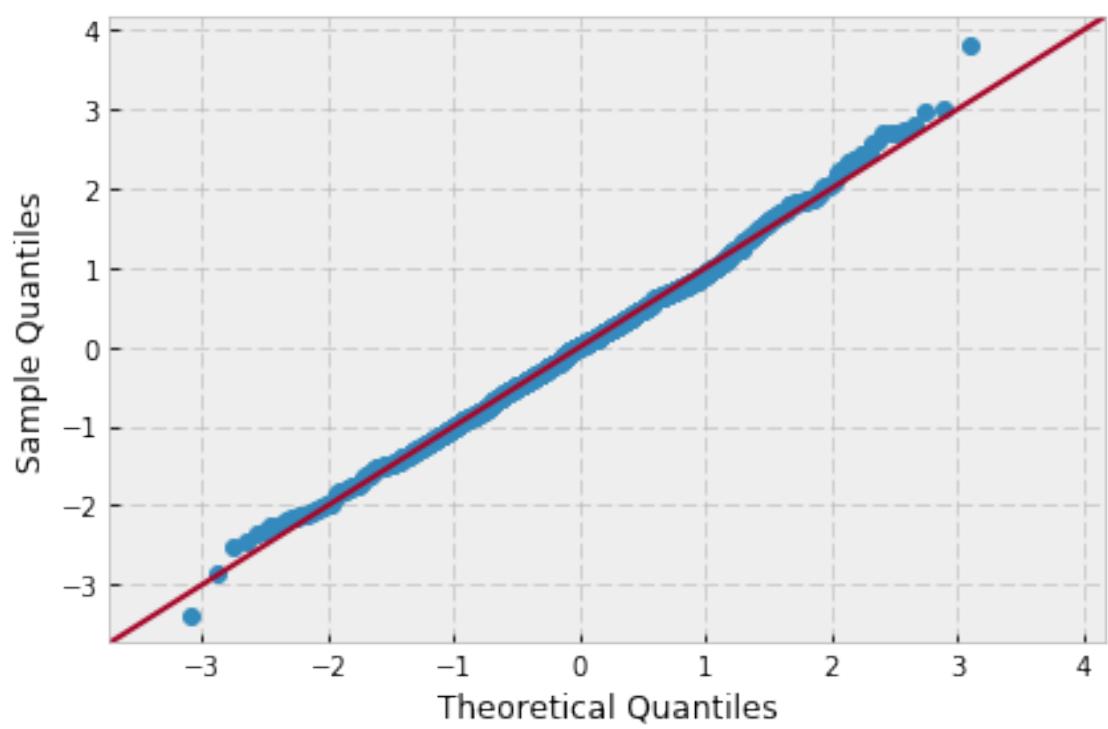
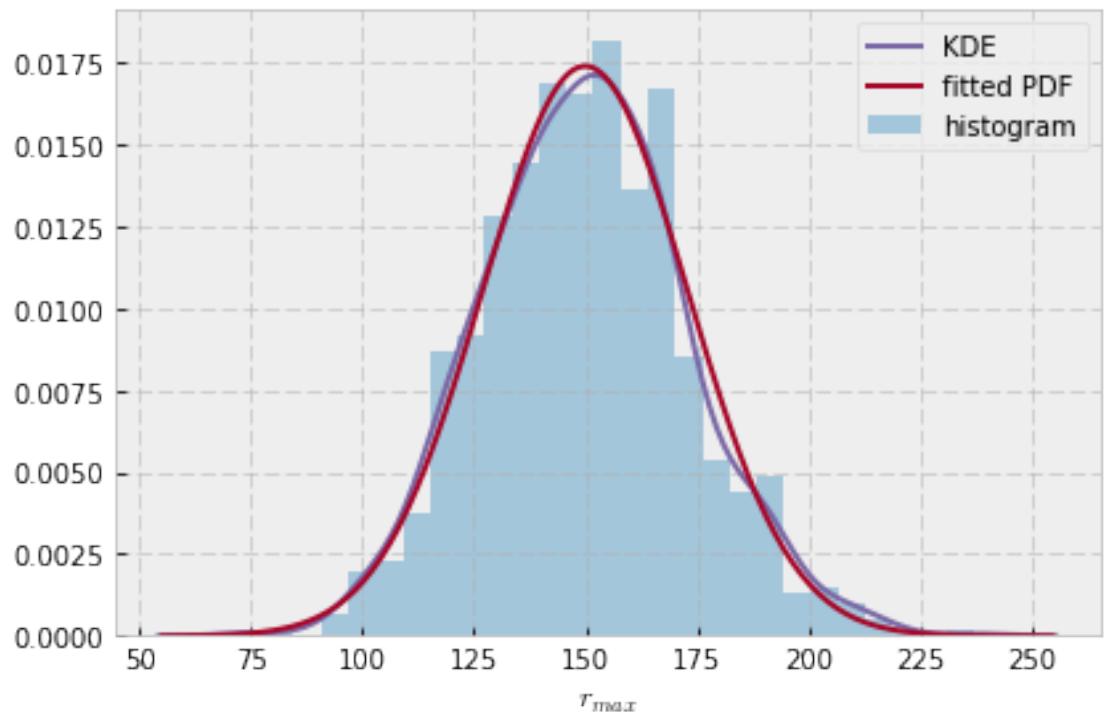


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(),loc=143.97732612075882, scale=21.884844065306282)
DescribeResult(nobs=1000, minmax=(67.566600175302526, 211.28328720464529)
               mean=143.97732612075882, variance=479.42582558835994,
               skewness=0.03544201763149416, kurtosis=-0.015230560927653958)
```

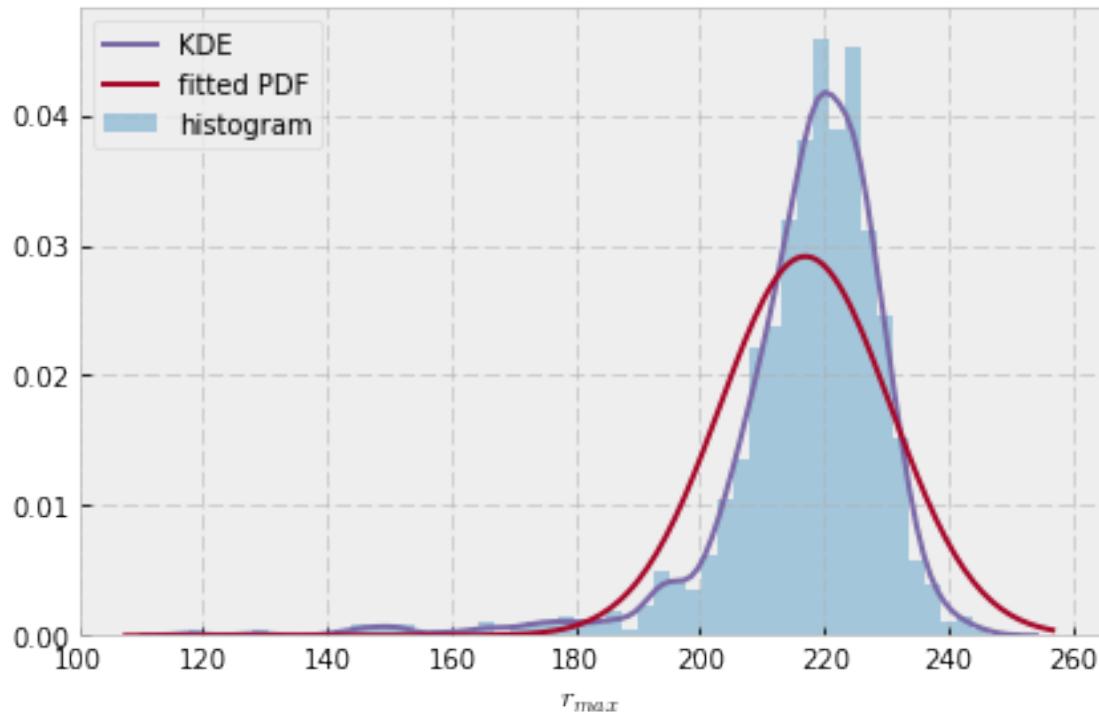


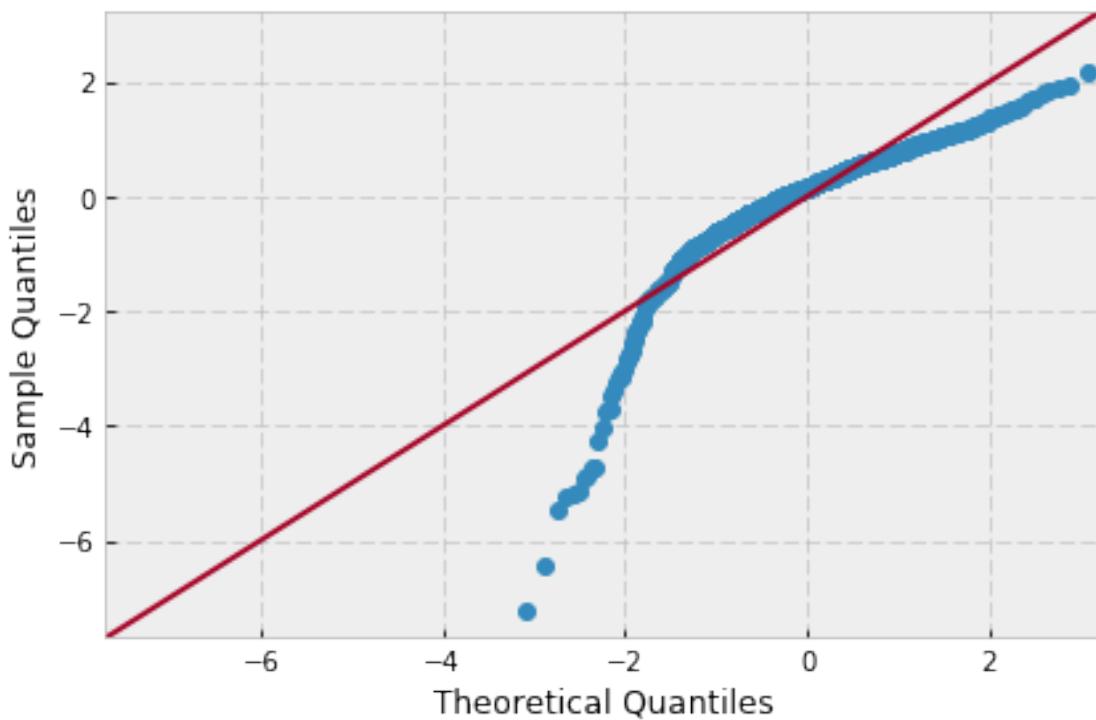


```
variable name: r
variable value: 3.0
distribution: normal(shape=(),loc=149.8509684564523, scale=22.908956603679382)
DescribeResult(nobs=1000, minmax=(72.466055248371163, 236.986802788695)
               mean=149.85096845645231, variance=525.3456383075727,
               skewness=0.1609327492946264, kurtosis=0.1390139516471991)
```

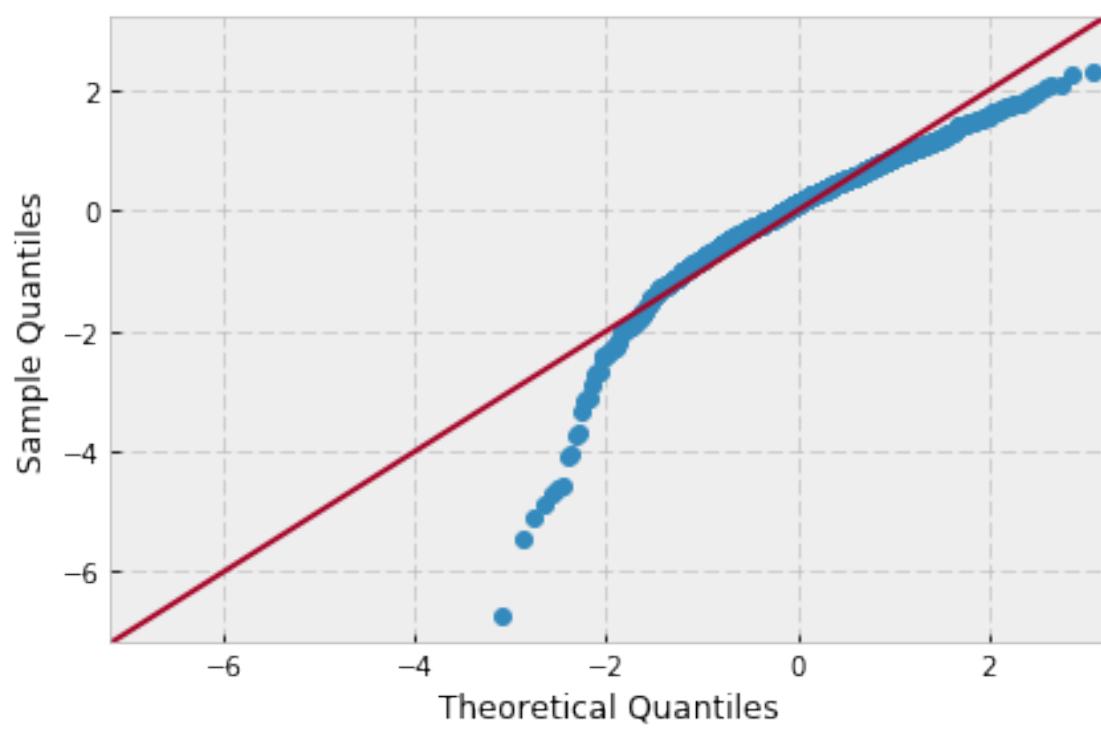
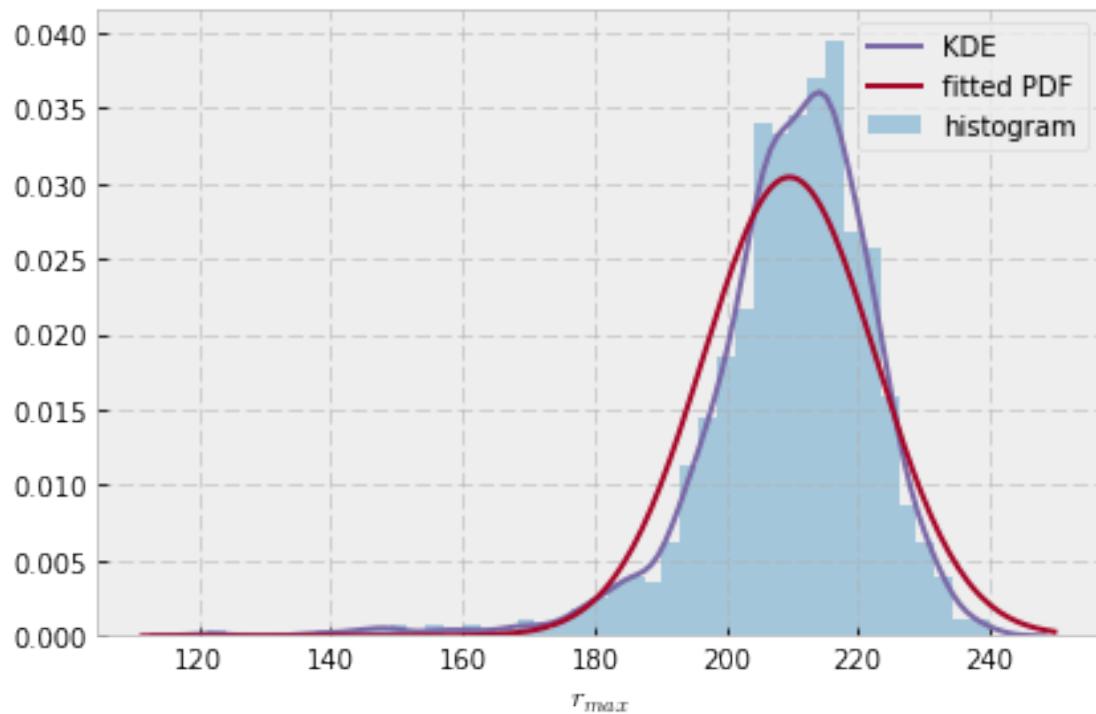


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(), loc=216.92030551530544, scale=13.69112952493616)
DescribeResult(nobs=1000, minmax=(117.9165848806994, 246.38164168464965)
               mean=216.92030551530544, variance=187.63466233090955,
               skewness=-2.37397193760475, kurtosis=9.81373206296626)
```

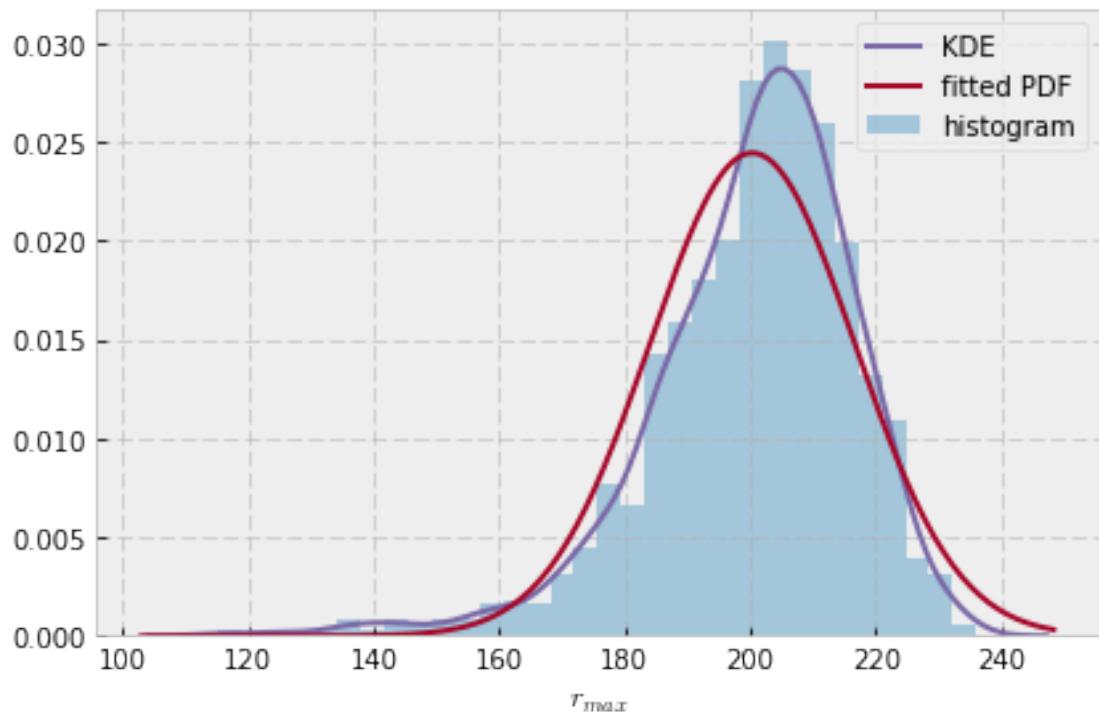


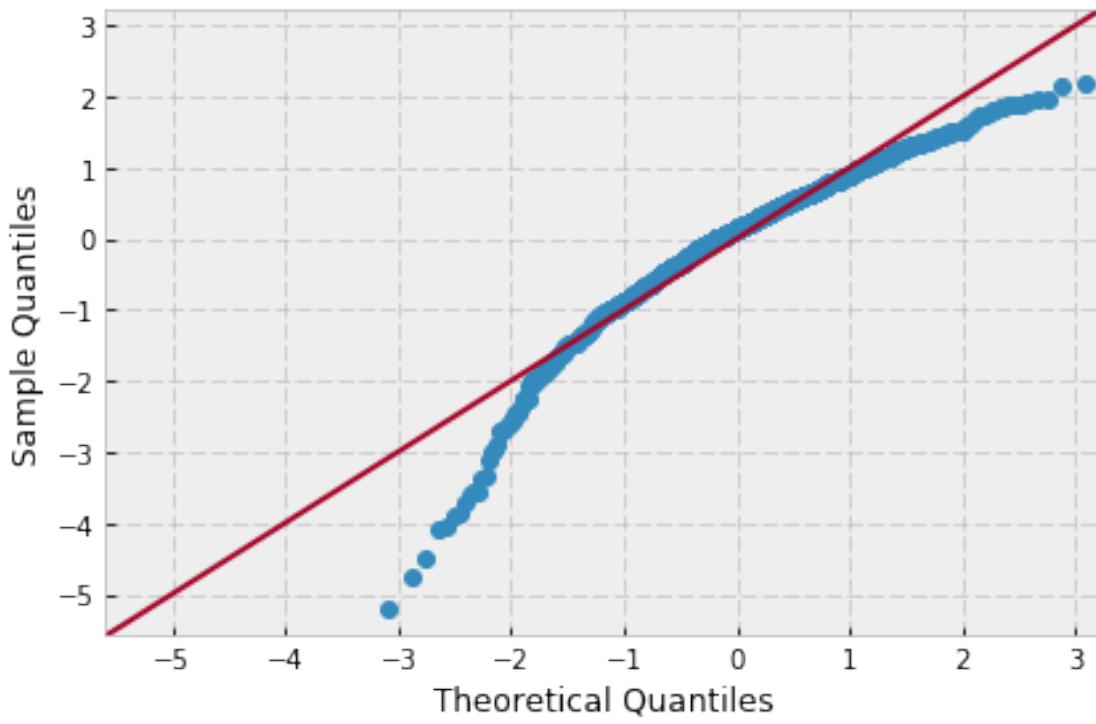


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(),loc=209.62501579396073, scale=13.118023180542133)
DescribeResult(nobs=1000, minmax=(121.32844660230974, 239.94634862656957)
               mean=209.62501579396073, variance=172.25478695219292,
               skewness=-1.52521879143638, kurtosis=5.559423841669057)
```

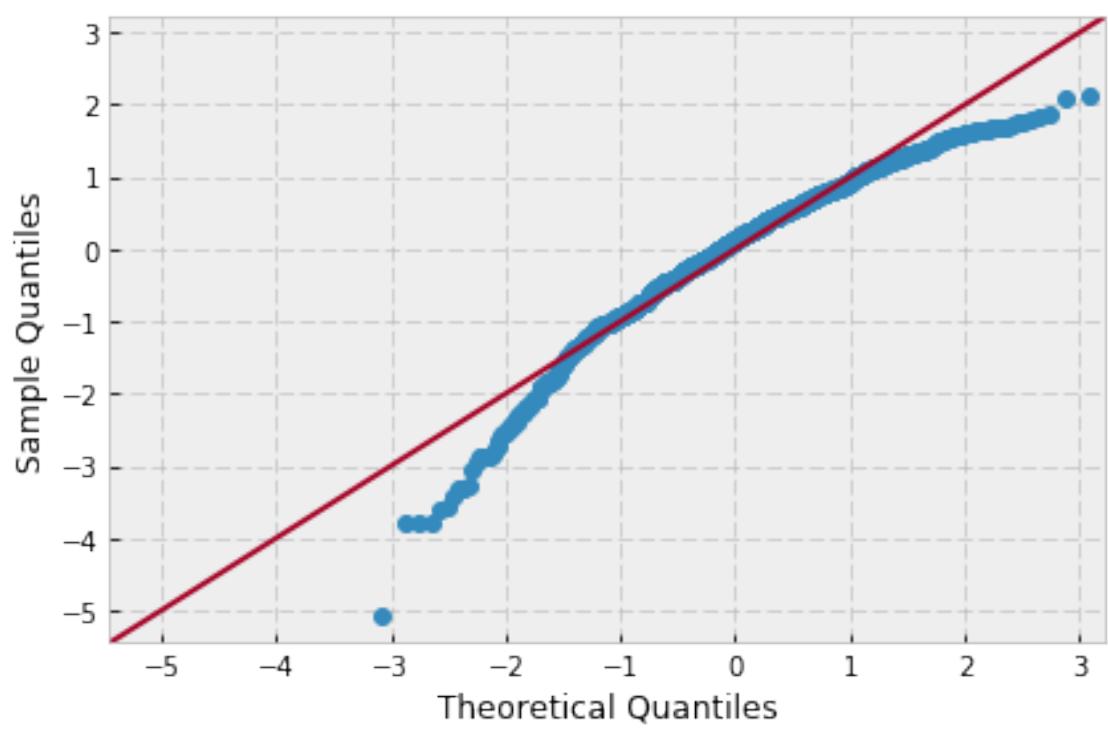
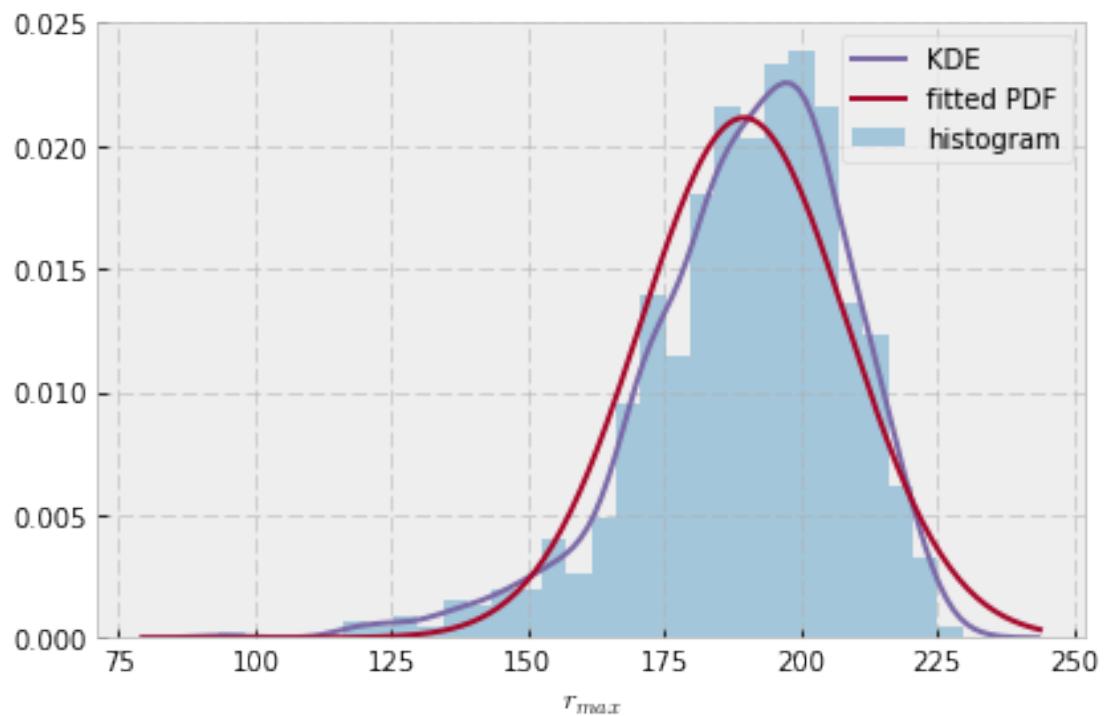


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=200.32299877169407, scale=16.30906444237312)
DescribeResult(nobs=1000, minmax=(115.22546382455286, 236.05255786715892)
               mean=200.32299877169407, variance=266.25183482029956,
               skewness=-1.1435268559540674, kurtosis=2.66291351248608)
```





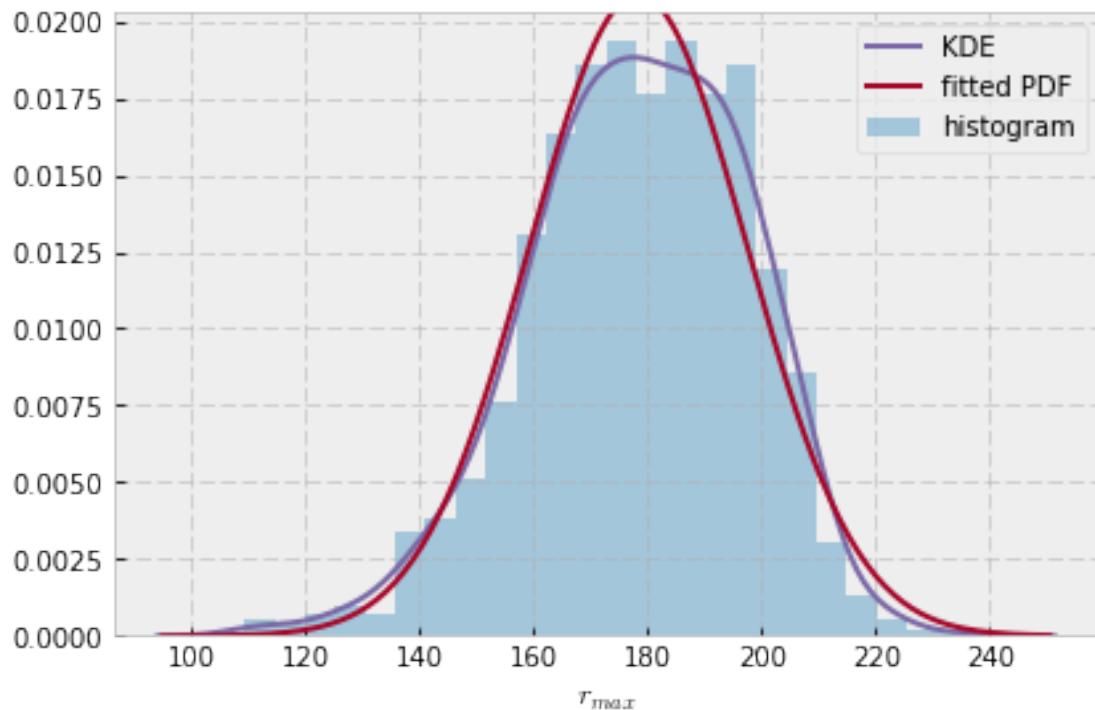
```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=189.5703129117257, scale=18.87918396210126)
DescribeResult(nobs=1000, minmax=(93.495399697815429, 229.58722454843149)
               mean=189.57031291172569, variance=356.78036744230366,
               skewness=-0.9199188645873662, kurtosis=1.4814986275394766)
```

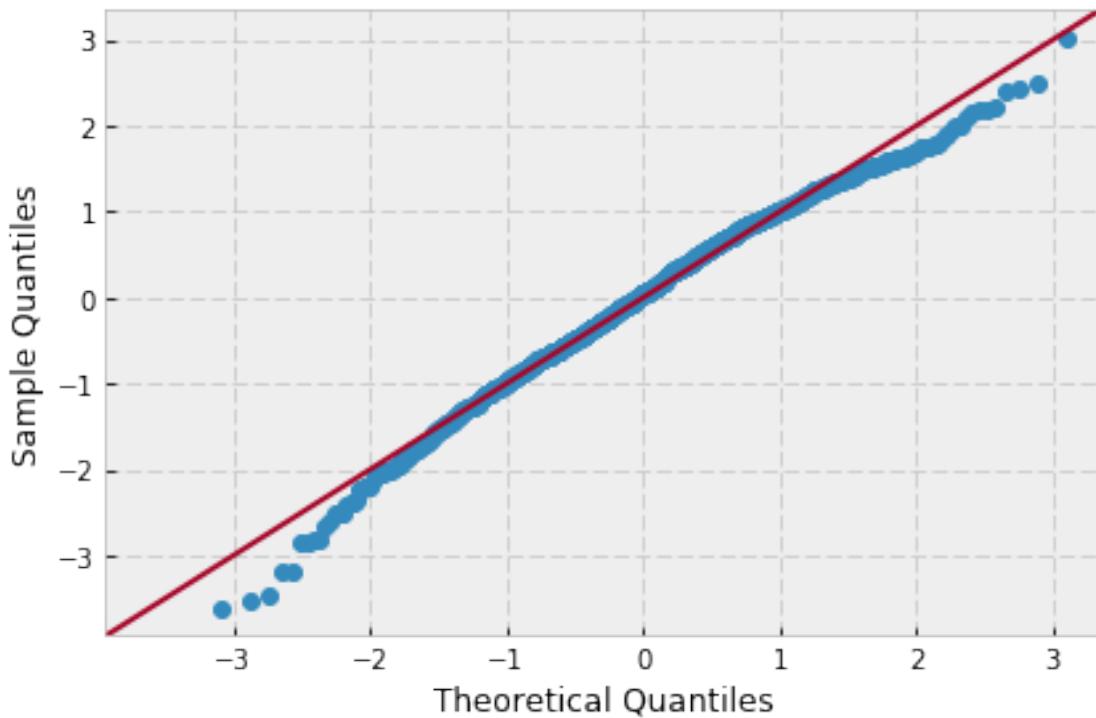


```

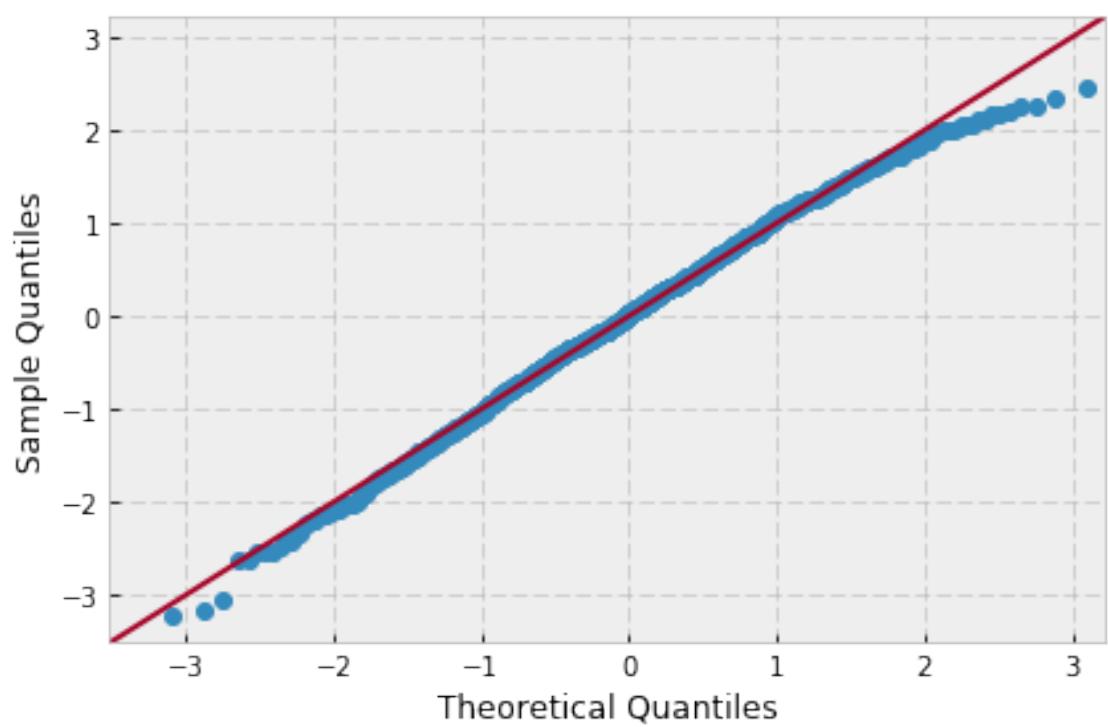
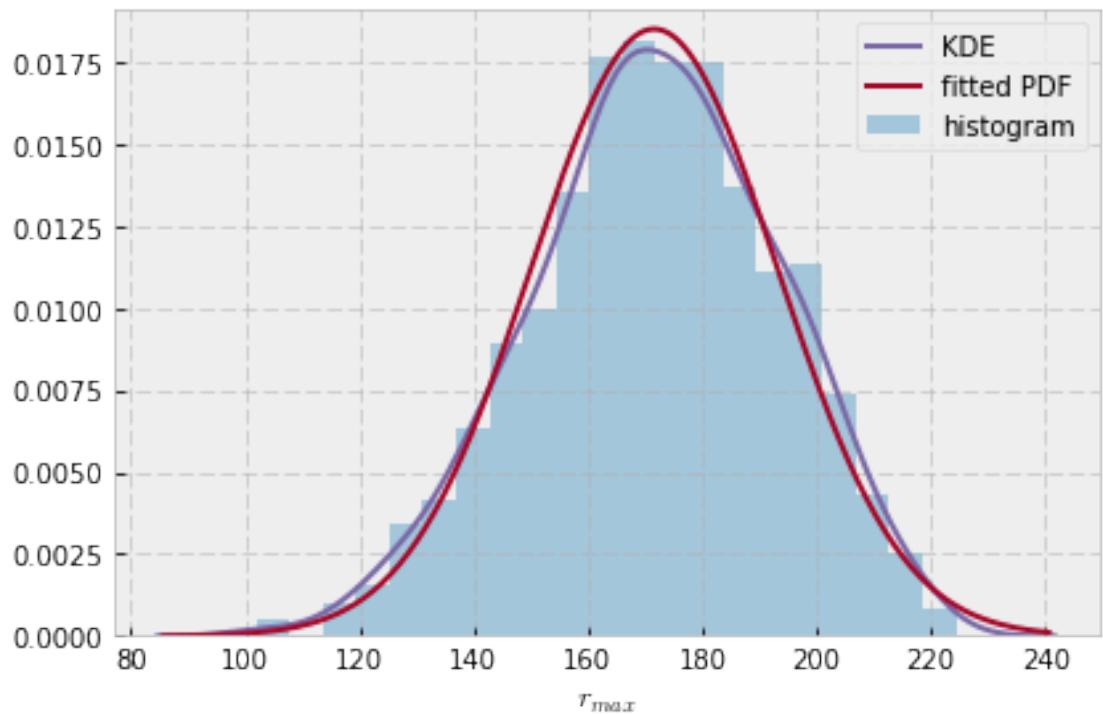
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(),loc=178.40672496823242, scale=19.113233678519073)
DescribeResult(nobs=1000, minmax=(109.49991857300593, 235.99505905191623)
               mean=178.40672496823242, variance=365.68138303270842,
               skewness=-0.372124384001116, kurtosis=0.137660928813093)

```

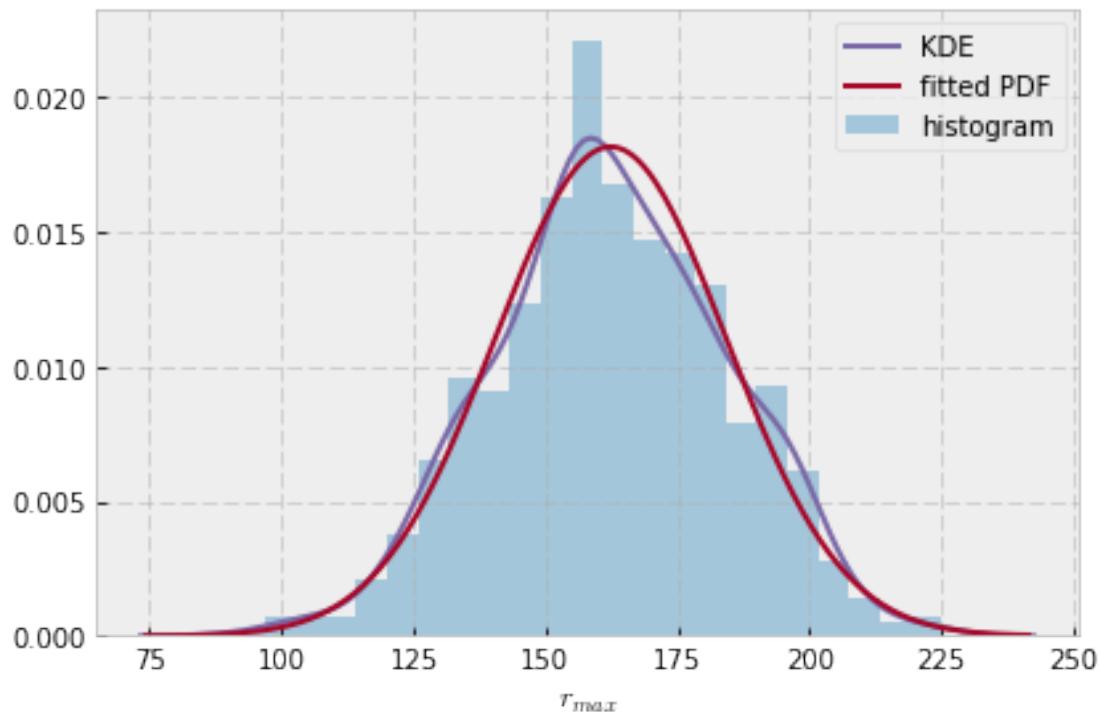


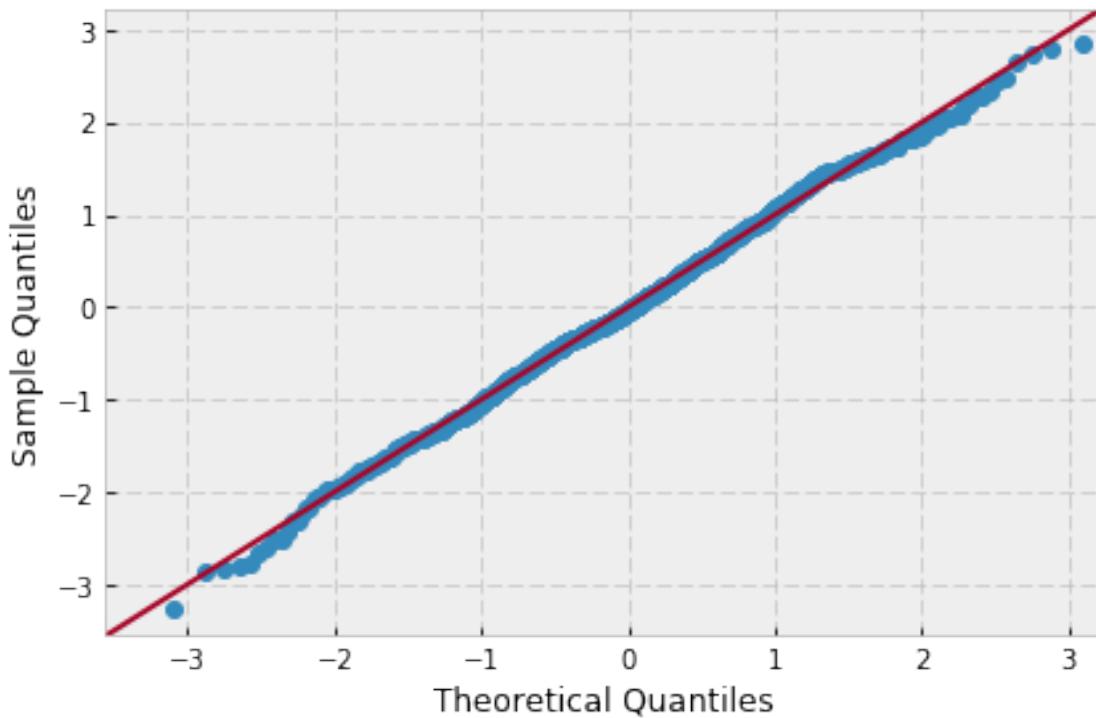


```
variable name: sigma
variable value: 0.0666666666666667
distribution: normal(shape=(),loc=171.56029008385835, scale=21.557343505579194)
DescribeResult(nobs=1000, minmax=(101.89700133471638, 224.35923132561763)
               mean=171.56029008385835, variance=465.18424326079821,
               skewness=-0.1957723507844222, kurtosis=-0.23004491256159687)
```

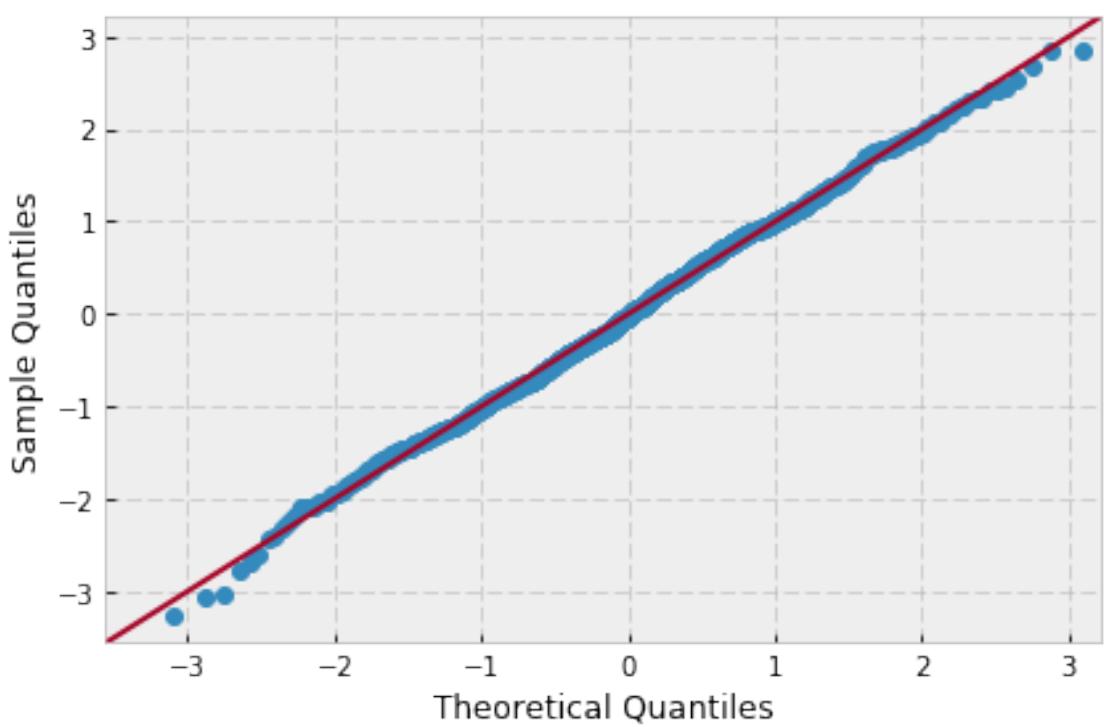
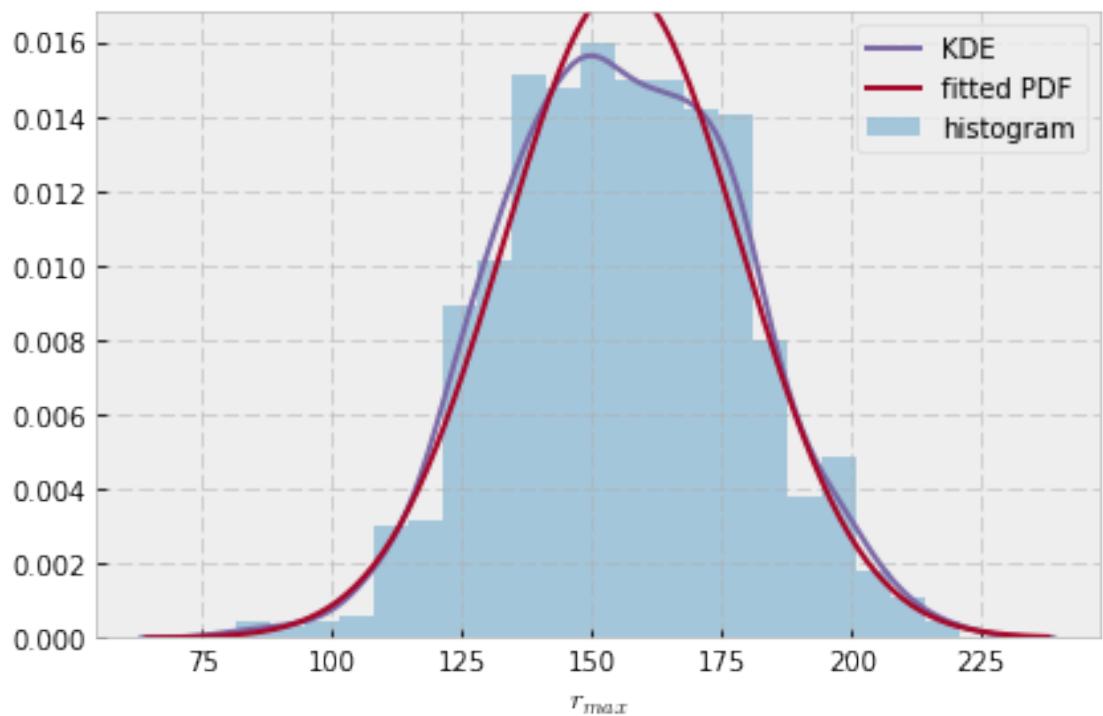


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=162.32963699765298, scale=21.974303765895975)
DescribeResult(nobs=1000, minmax=(90.831720946641468, 224.92479773713458)
               mean=162.32963699765298, variance=483.35337937524537,
               skewness=-0.03902629039064222, kurtosis=-0.18029408913063039)
```

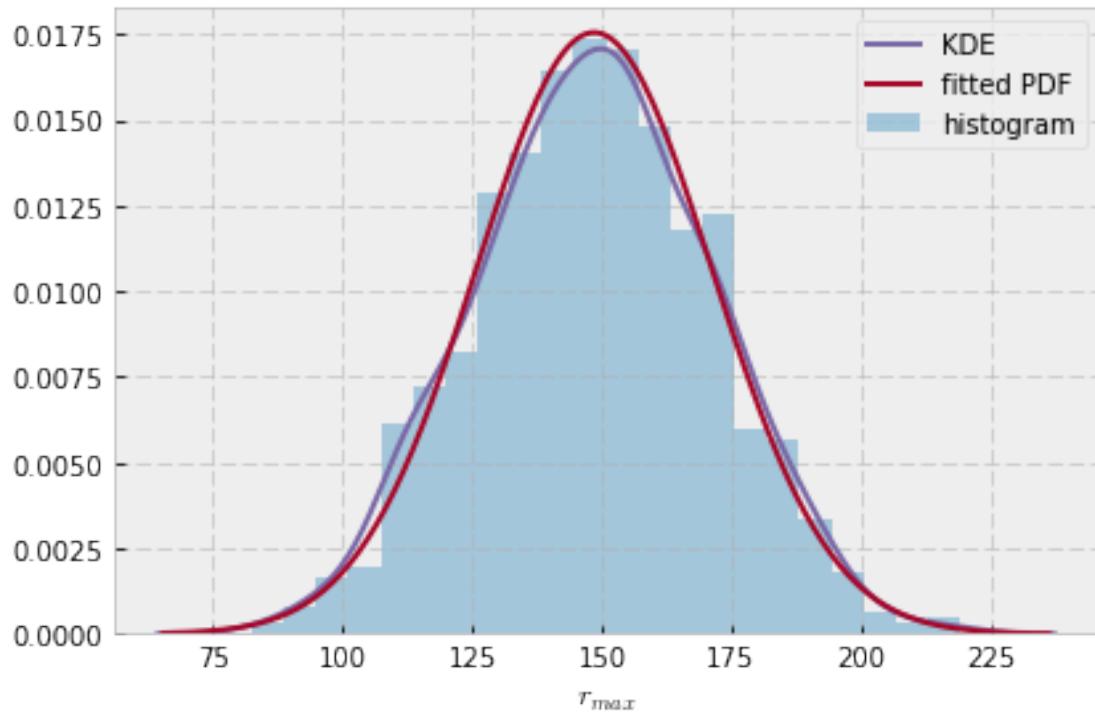


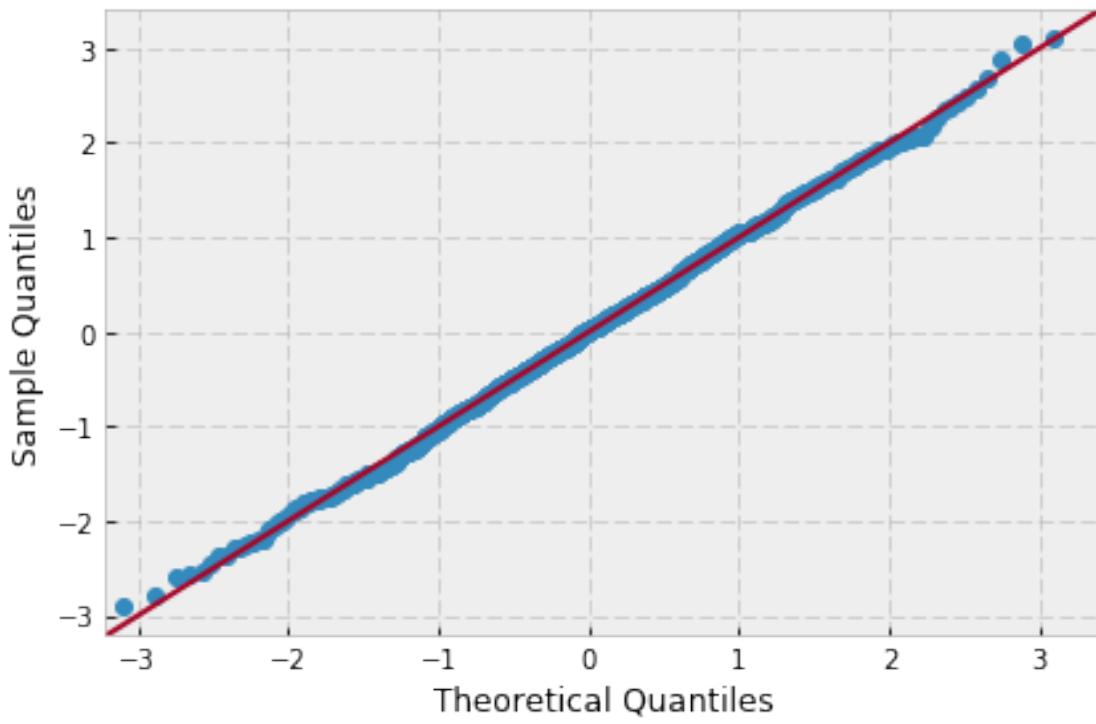


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(),loc=155.79673281226843, scale=22.809837779121732)
DescribeResult(nobs=1000, minmax=(81.537291166236585, 220.67130263844427)
               mean=155.79673281226843, variance=520.80950901886797,
               skewness=0.0029767192063026634, kurtosis=-0.2191217194629469)
```

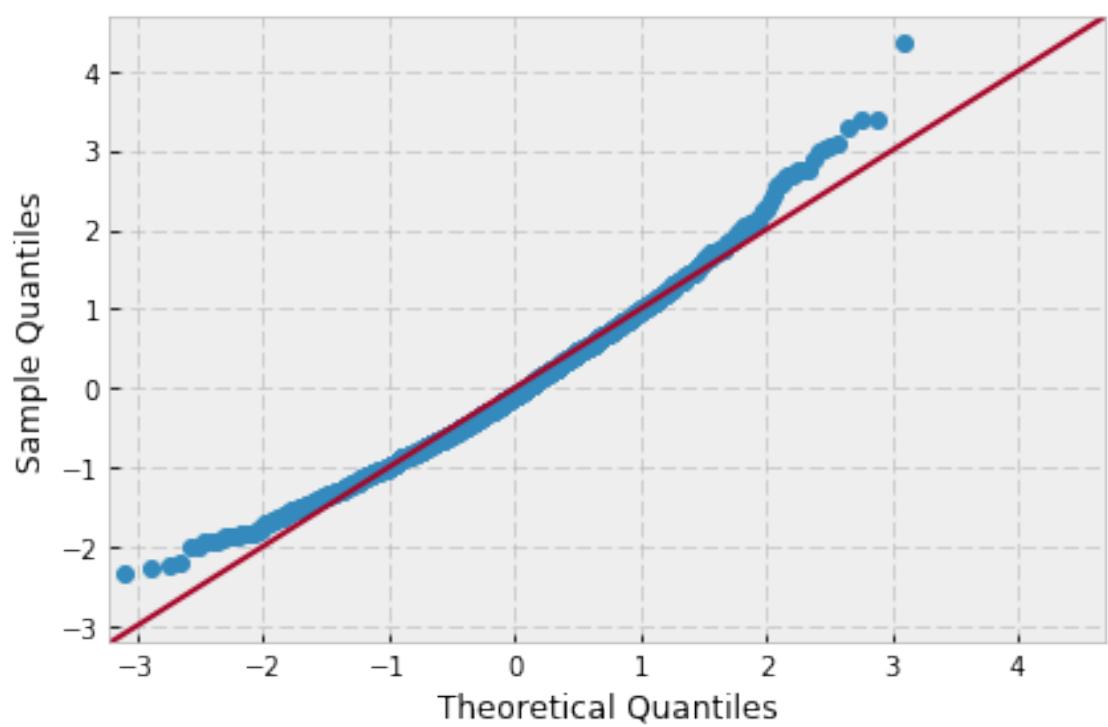
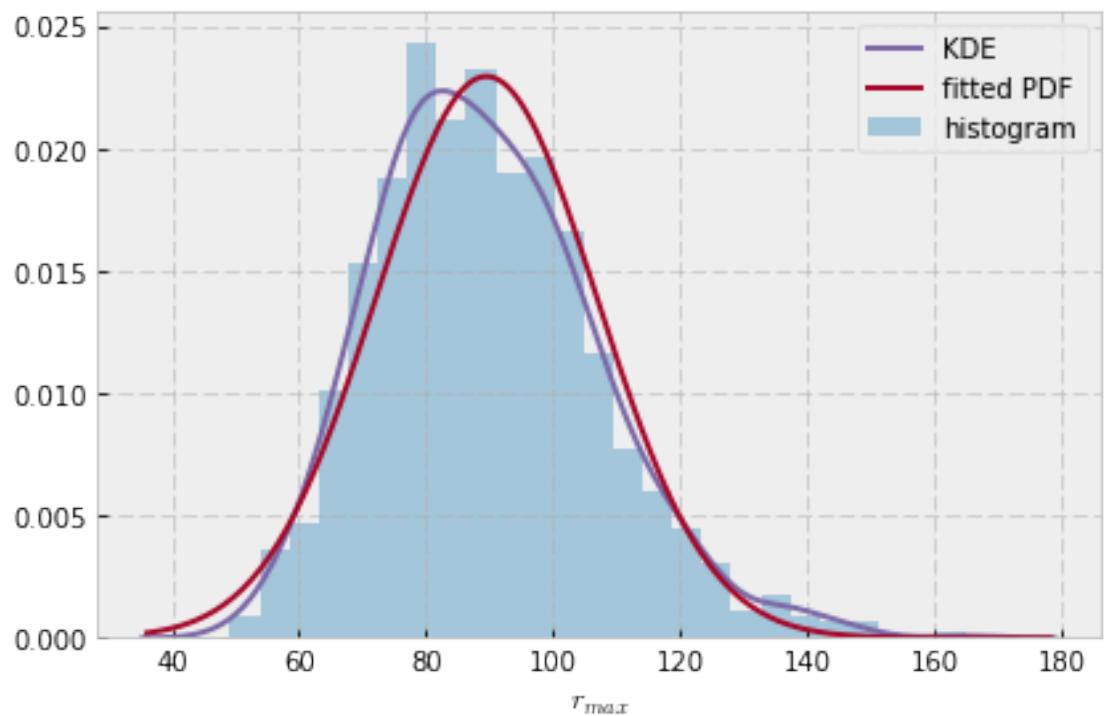


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=148.50357707872163, scale=22.734809401150915)
DescribeResult(nobs=1000, minmax=(82.484830702125493, 219.10079463733251)
               mean=148.50357707872163, variance=517.38894745411415,
               skewness=0.0060766259955762365, kurtosis=-0.20852446721850226)
```

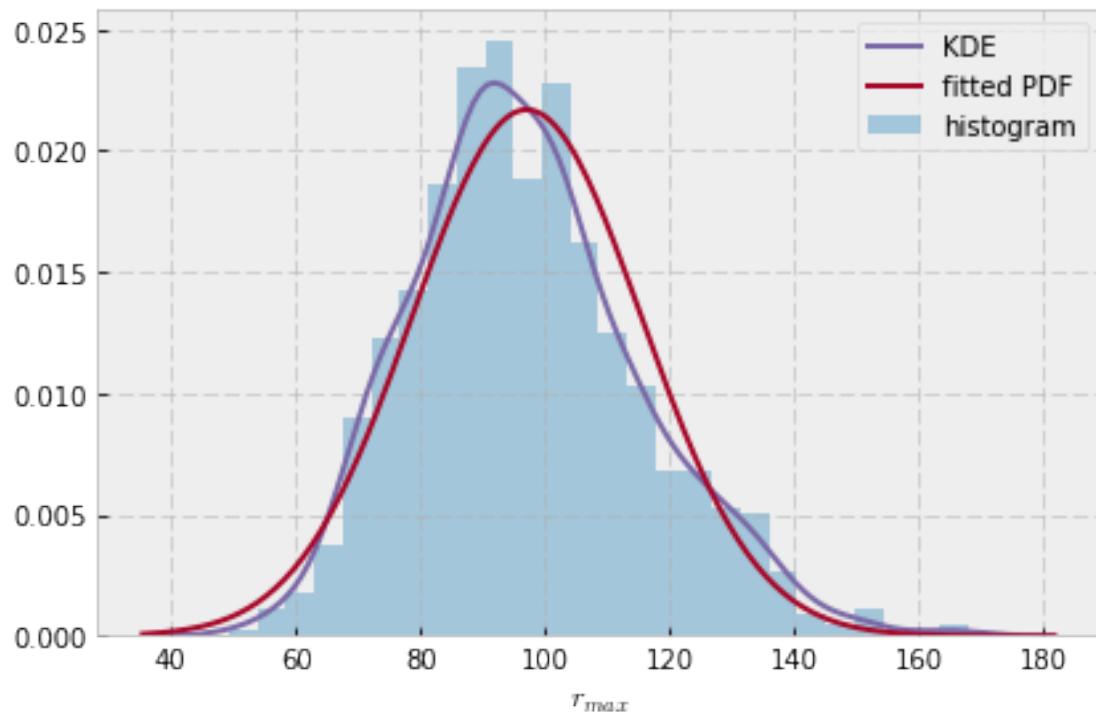


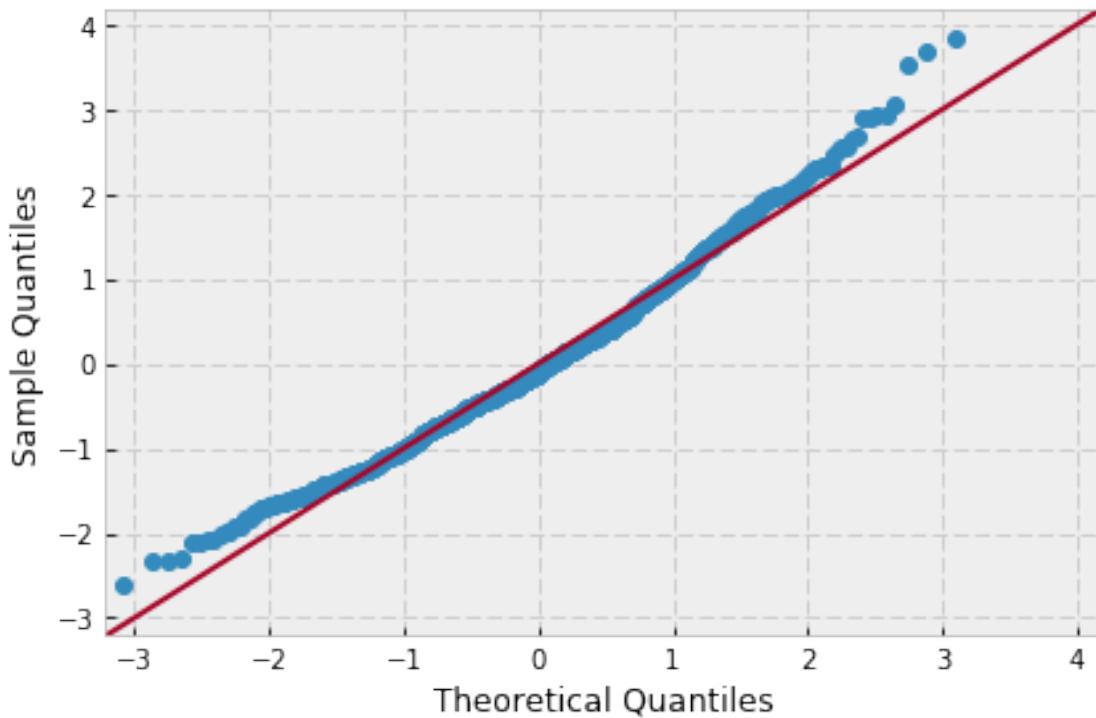


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(),loc=89.50500897834814, scale=17.390592275741632)
DescribeResult(nobs=1000, minmax=(49.157876901299176, 165.16032175104638)
               mean=89.50500897834814, variance=302.73543513622081,
               skewness=0.5575036328013092, kurtosis=0.43910038556231257)
```

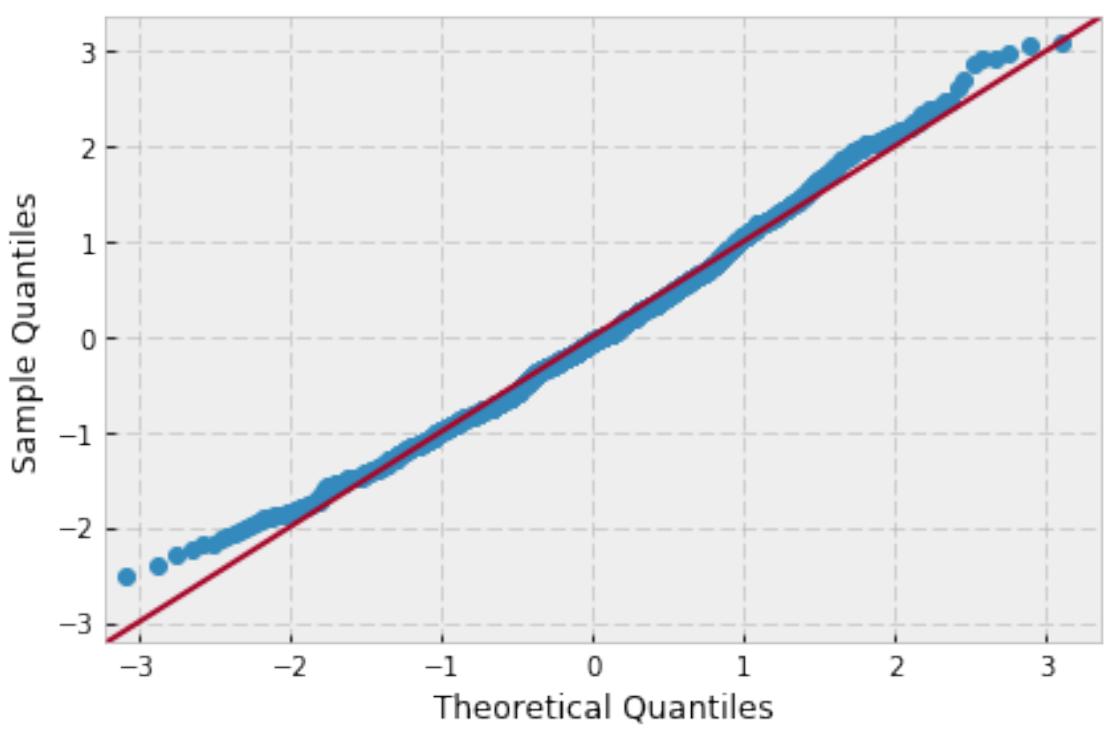
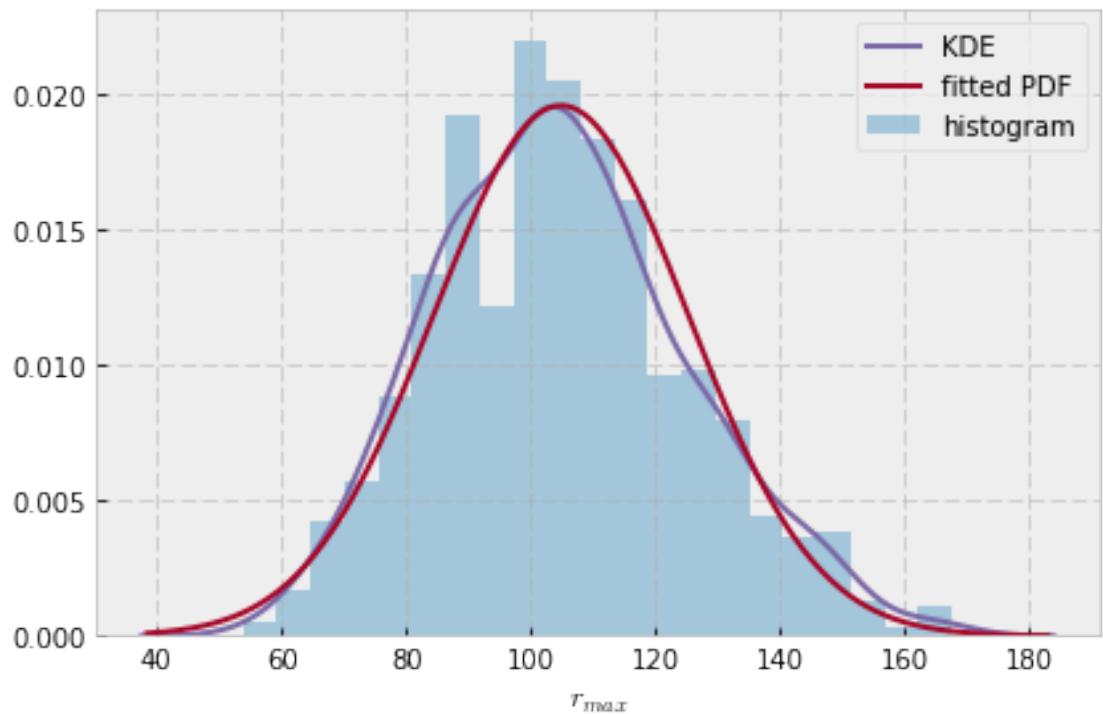


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(),loc=97.0769900936782, scale=18.391228087010845)
DescribeResult(nobs=1000, minmax=(49.345639862903639, 167.85148114986728)
               mean=97.076990093678205, variance=338.57584639485145,
               skewness=0.49364835712446986, kurtosis=0.2551187827231085)
```

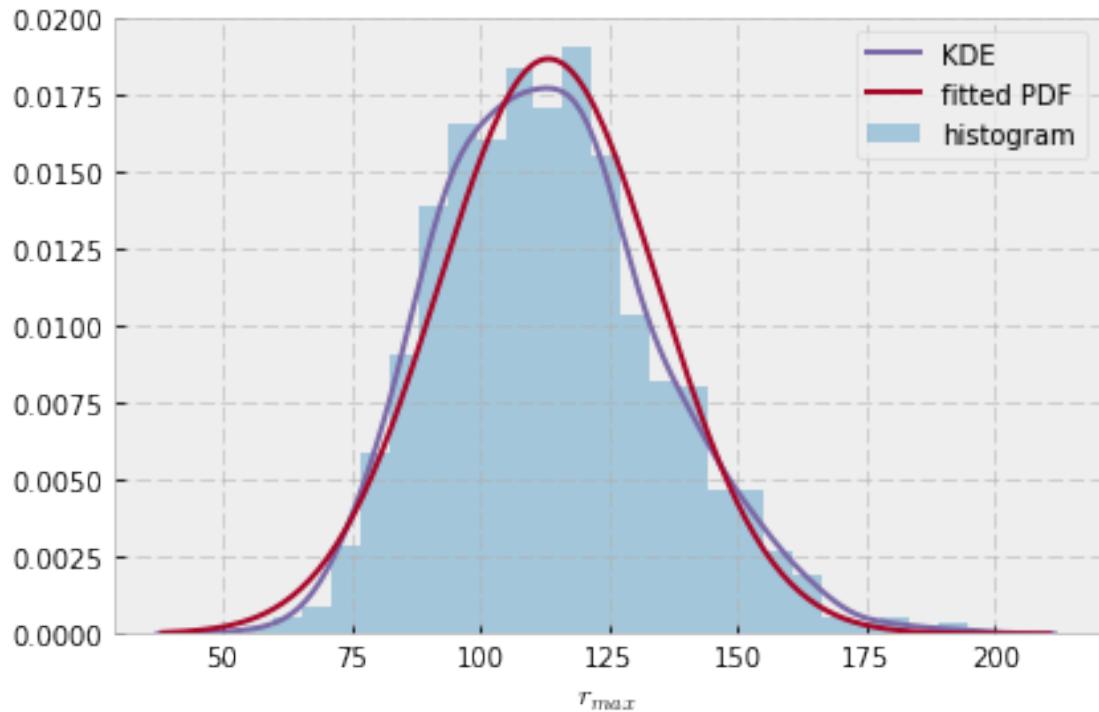


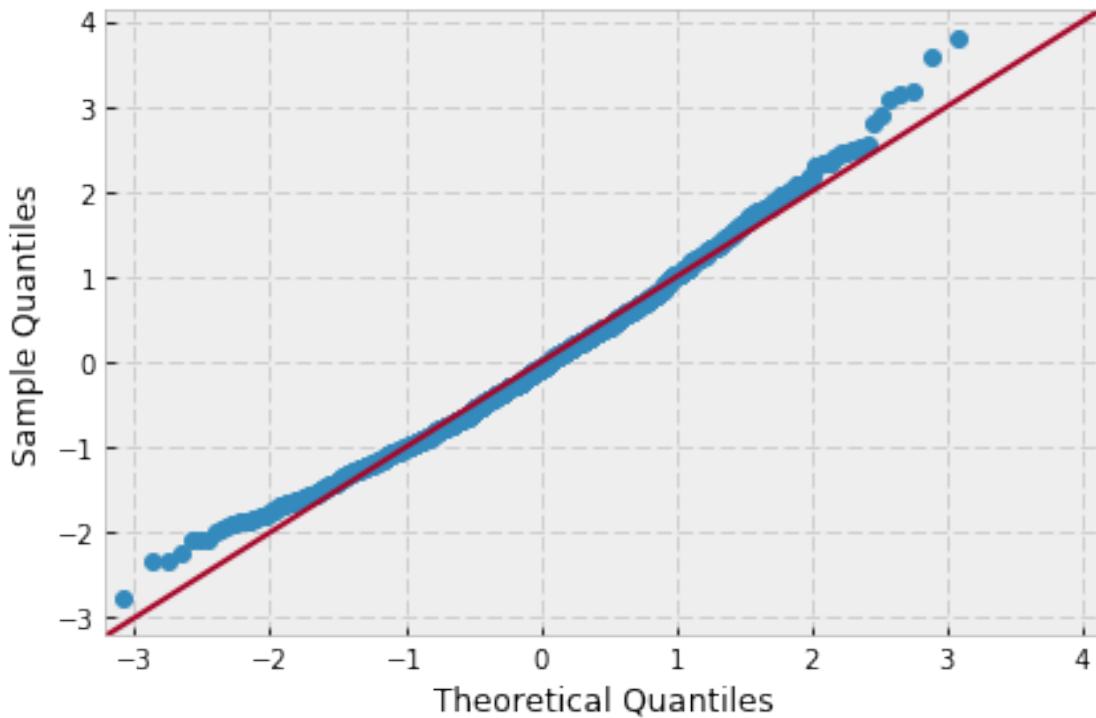


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=104.84655174527103, scale=20.402980397593414)
DescribeResult(nobs=1000, minmax=(53.846687968938959, 167.56669563734263)
               mean=104.84655174527103, variance=416.69830741199303,
               skewness=0.3240125222597742, kurtosis=-0.1320580695257294)
```

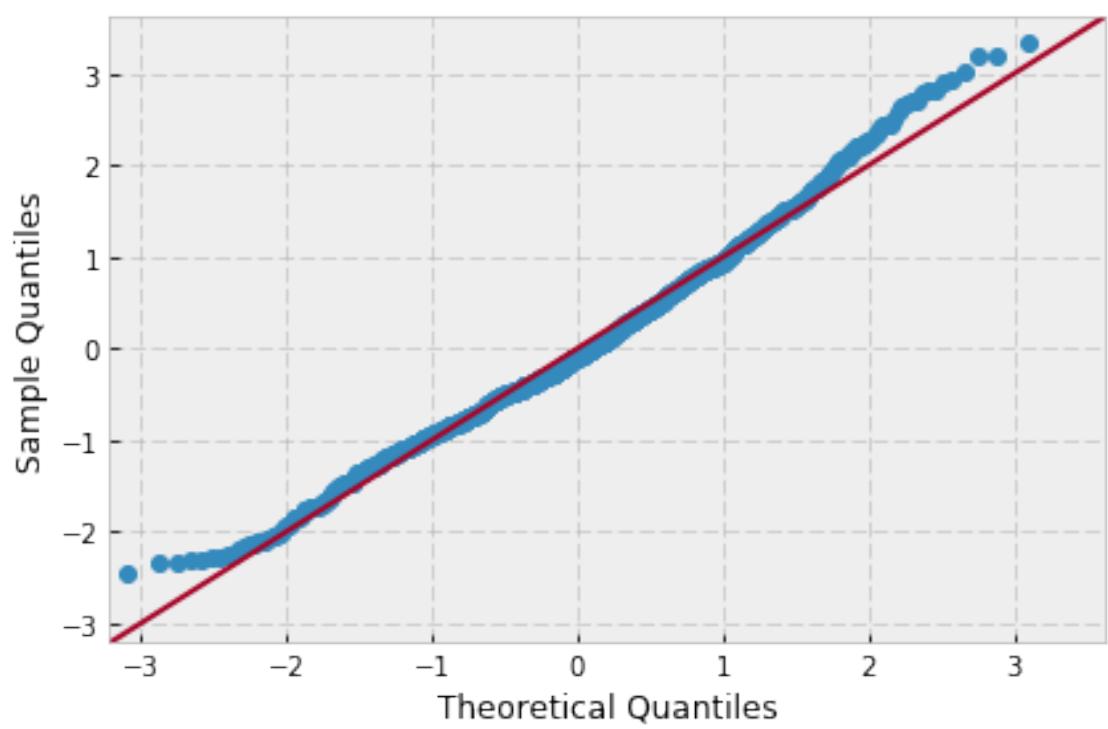
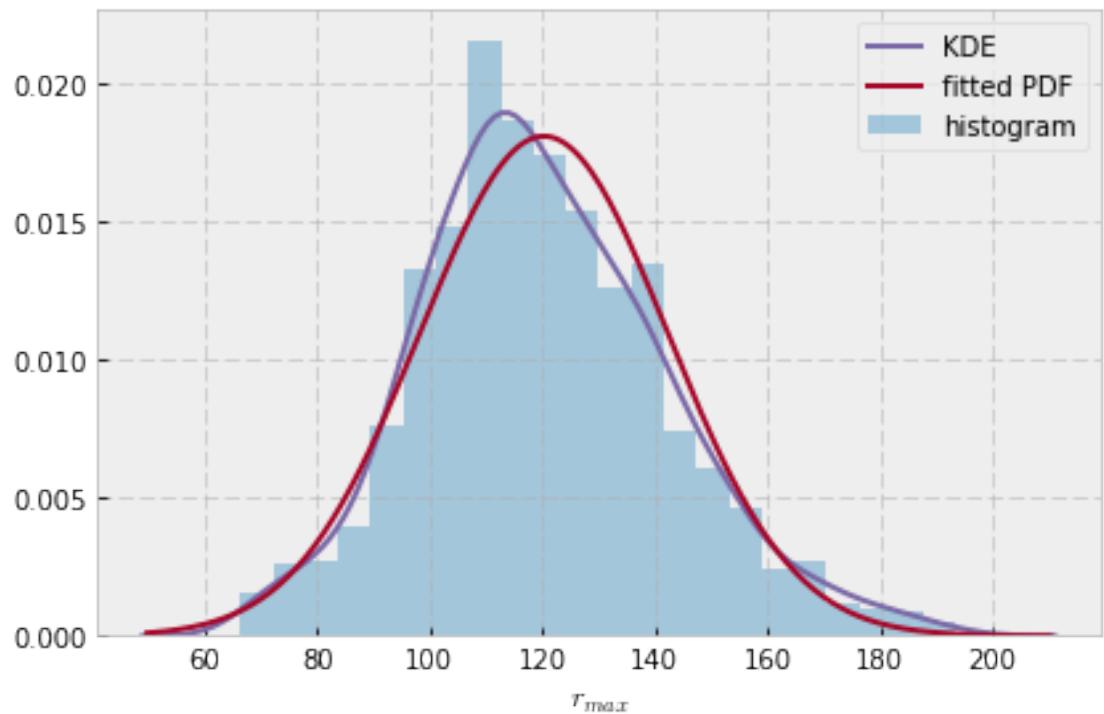


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(),loc=113.29507808494814, scale=21.336769236046536)
DescribeResult(nobs=1000, minmax=(54.425689234355282, 194.48093596538408)
               mean=113.29507808494814, variance=455.71343486716904,
               skewness=0.42379780174048287, kurtosis=0.10554195061152072)
```

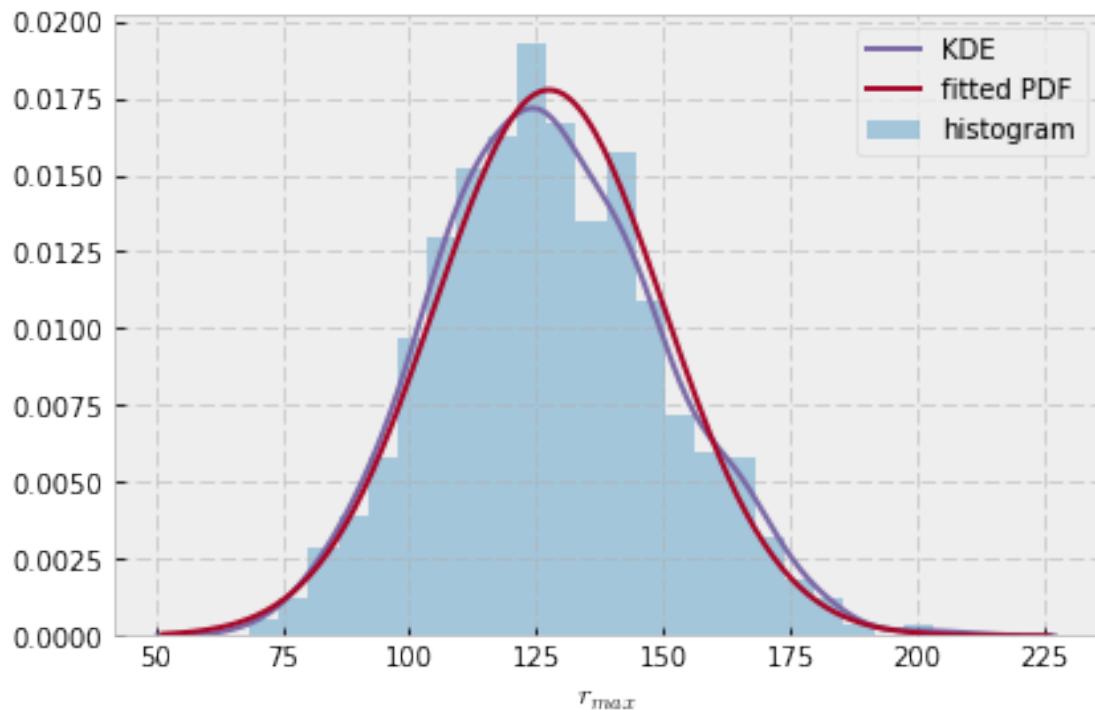


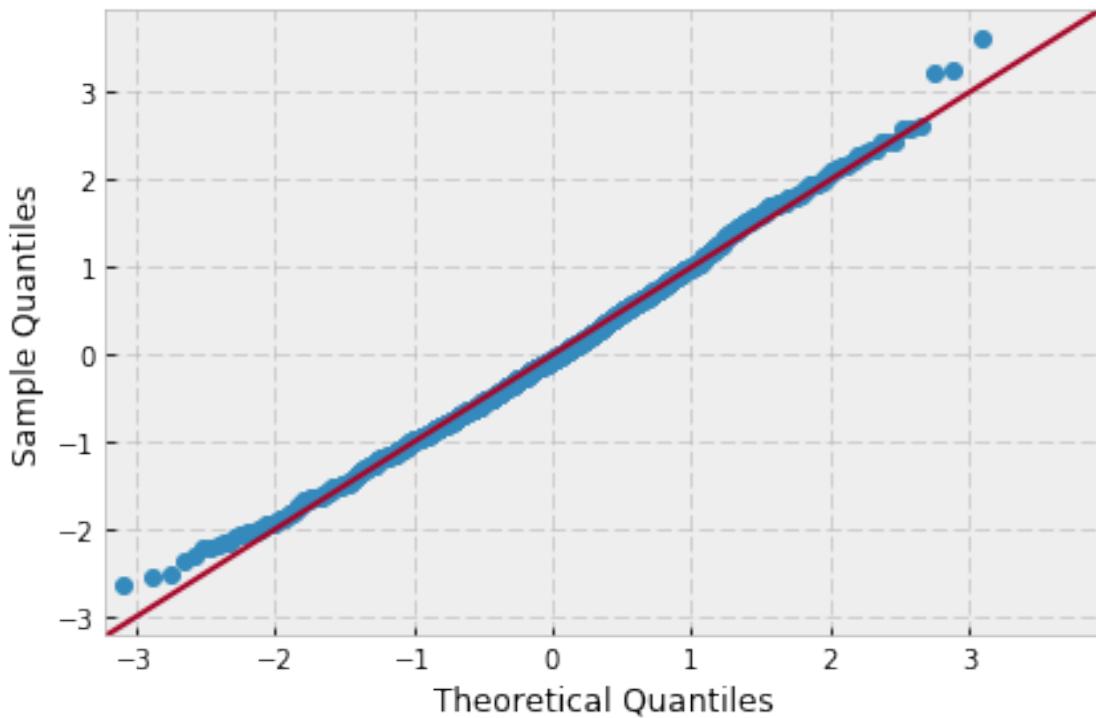


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=120.25416435587485, scale=22.013790762121534)
DescribeResult(nobs=1000, minmax=(66.360570805998478, 193.56934527366829)
               mean=120.25416435587485, variance=485.09207579426169,
               skewness=0.3814690311858938, kurtosis=0.2119930897079234)
```

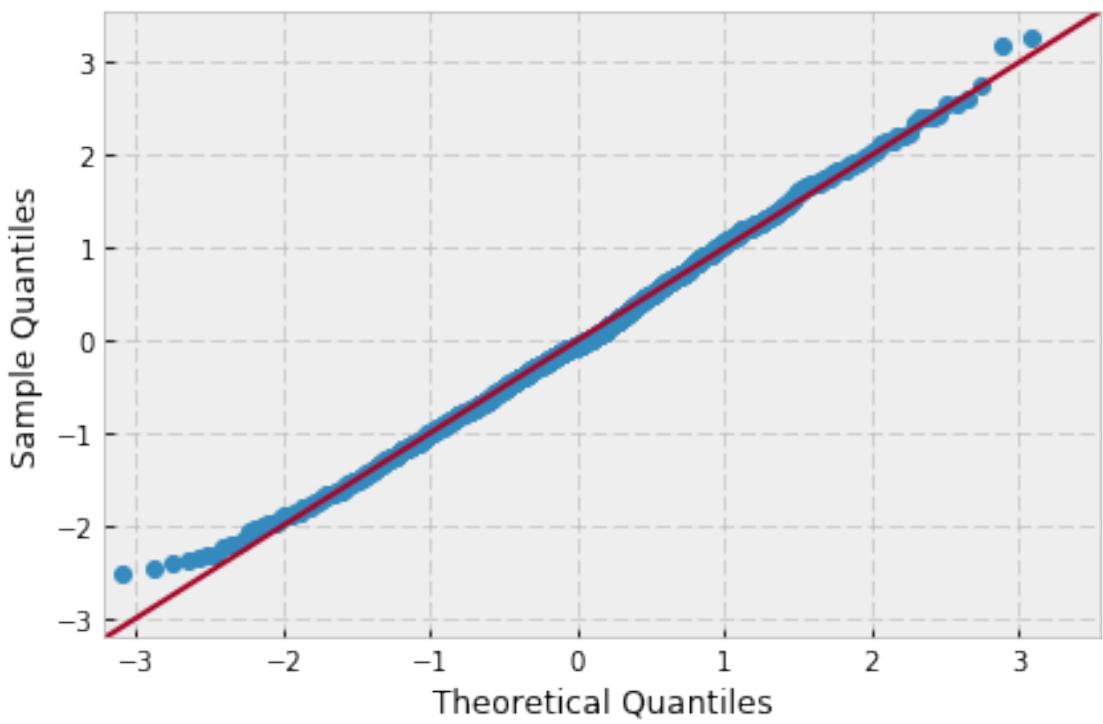
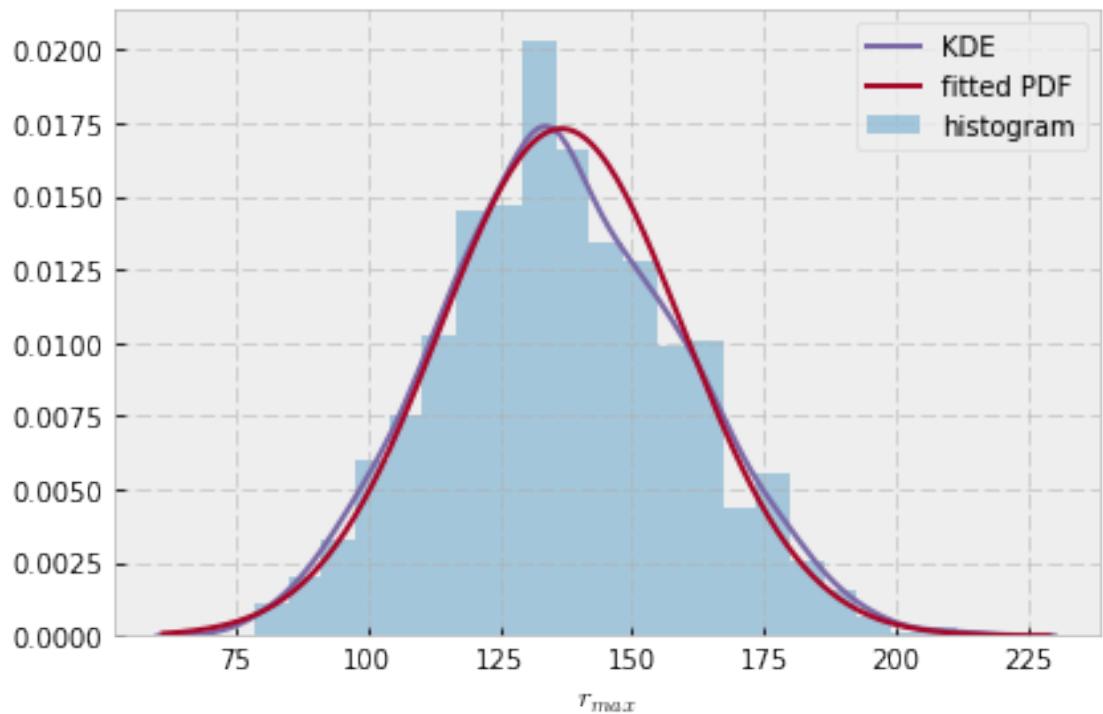


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=127.48200844442529, scale=22.45955636175725)
DescribeResult(nobs=1000, minmax=(68.360738204347072, 208.98386955068534)
               mean=127.48200844442529, variance=504.93660857552618,
               skewness=0.23131302224287373, kurtosis=-0.11865049635915392)
```

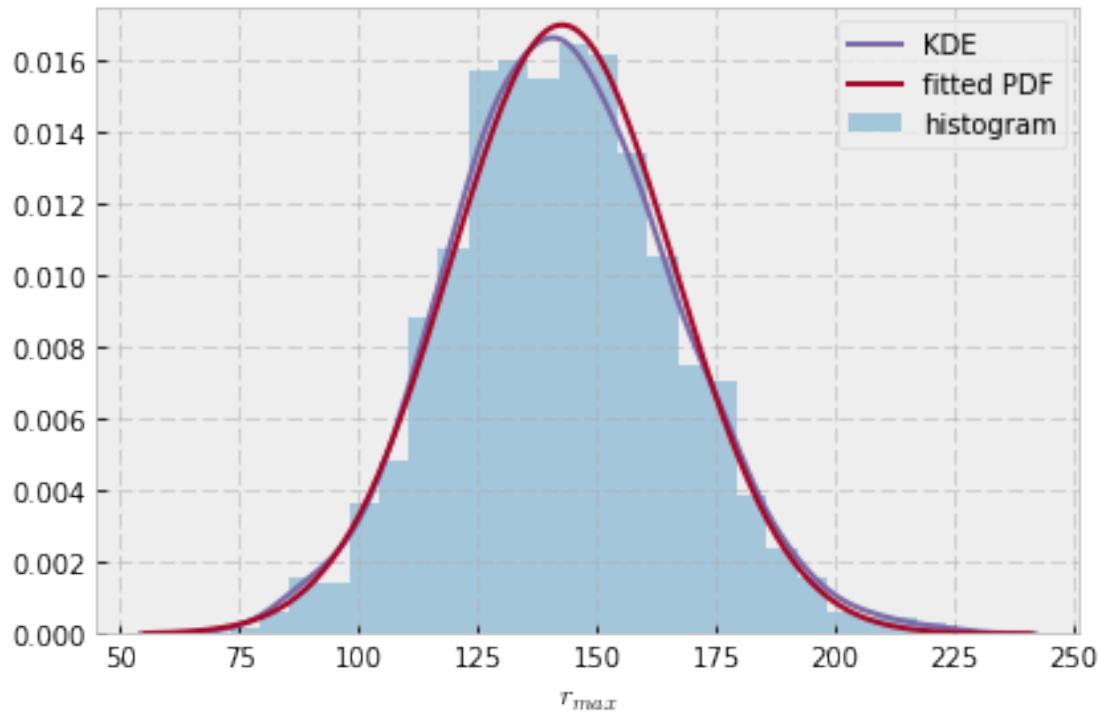


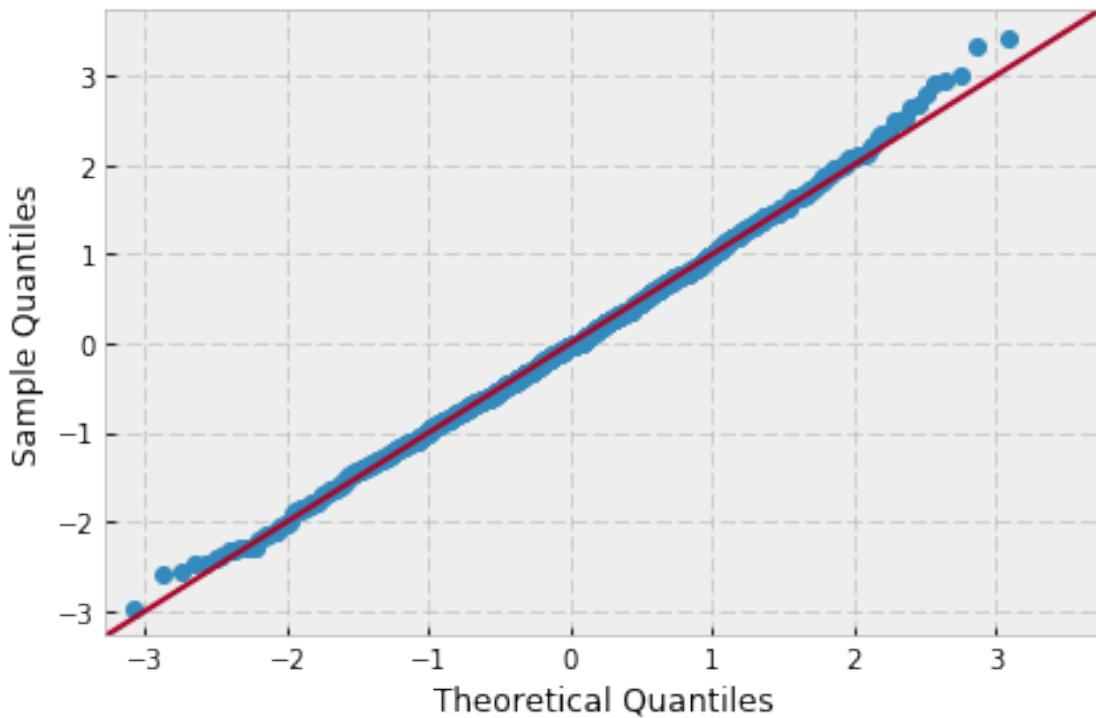


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(),loc=136.69013495230598, scale=23.0460404695473)
DescribeResult(nobs=1000, minmax=(78.545777154705462, 211.69275209322763)
               mean=136.69013495230598, variance=531.65163295696891,
               skewness=0.1465037839420444, kurtosis=-0.24230417370819346)
```

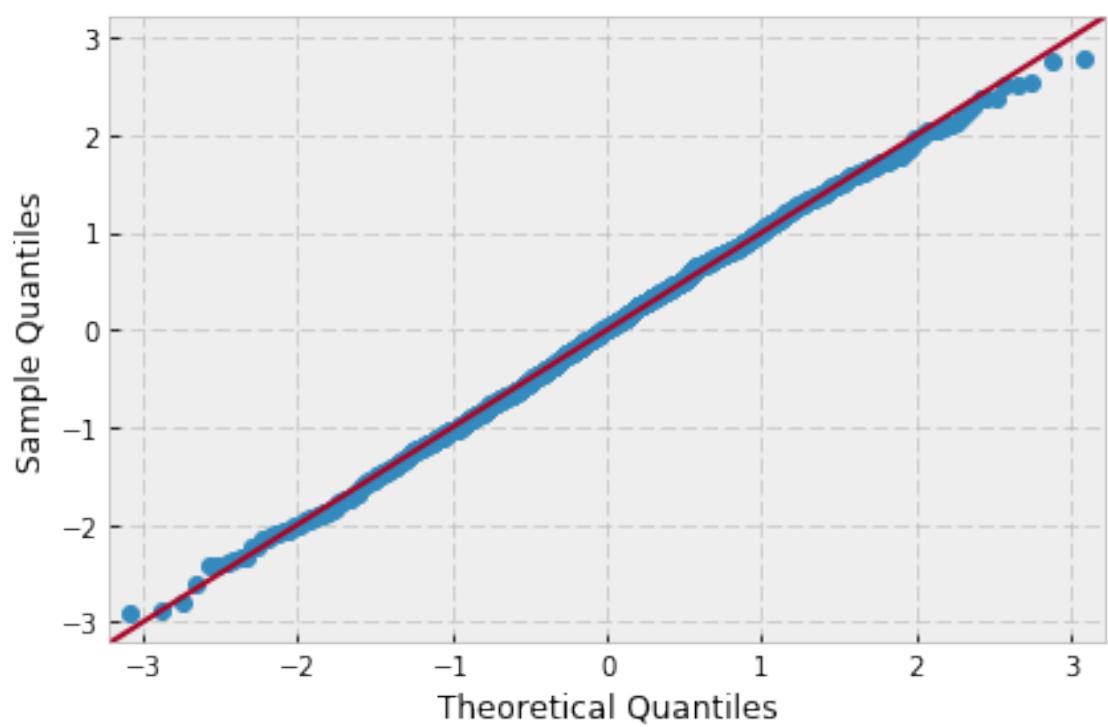
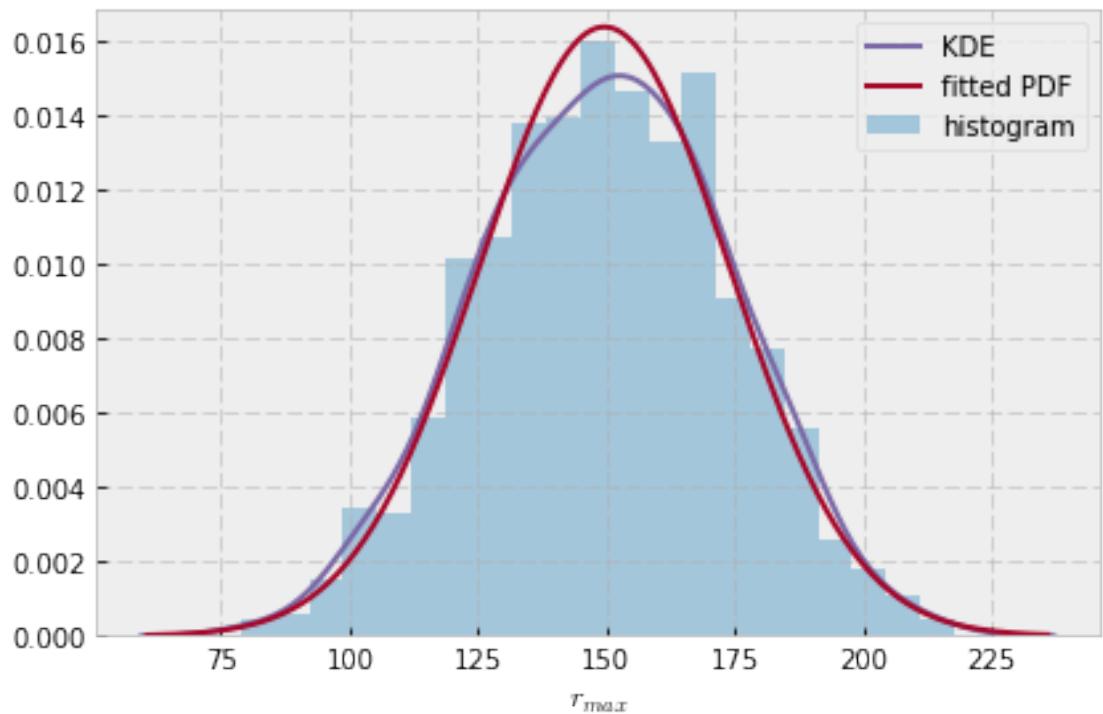


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(),loc=142.7353507742947, scale=23.527996587436594)
DescribeResult(nobs=1000, minmax=(73.035932491125763, 223.26710490710201)
               mean=142.73535077429469, variance=554.12074416259054,
               skewness=0.16507227211302966, kurtosis=0.07252355837766622)
```

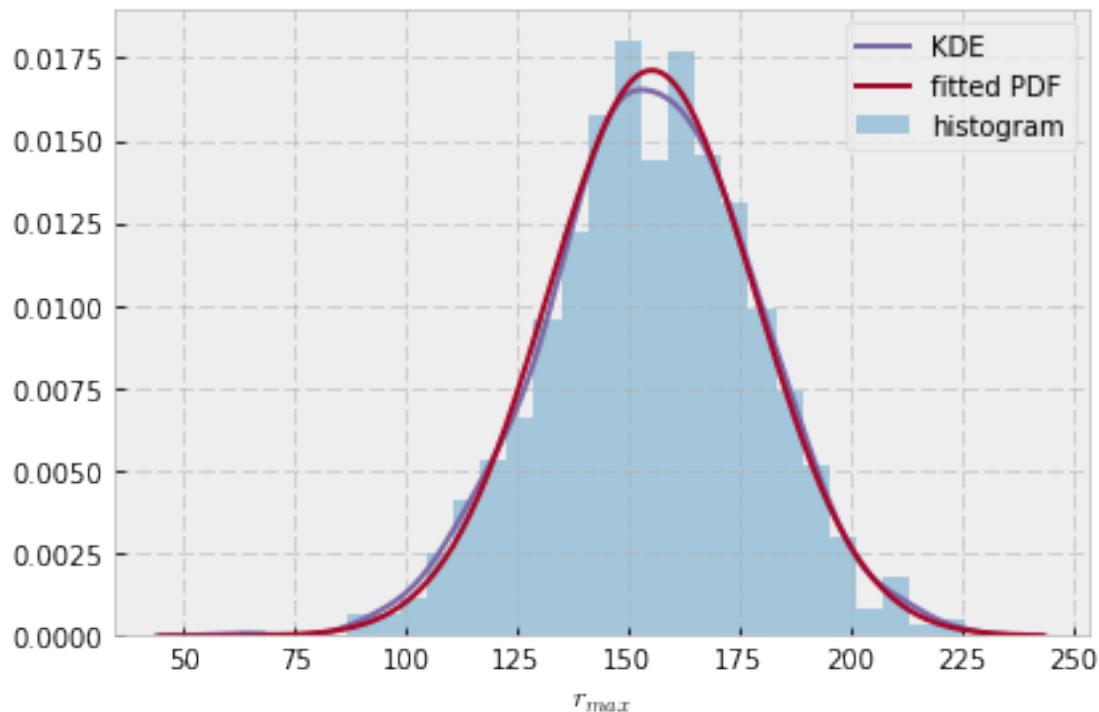


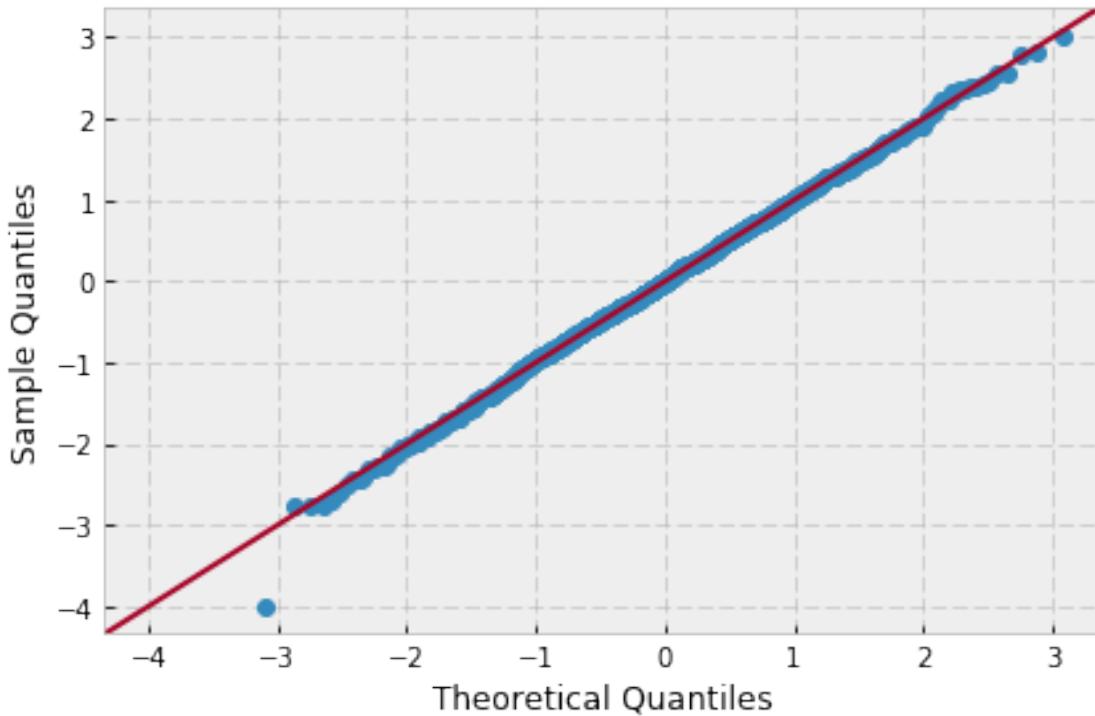


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(),loc=149.62154650913376, scale=24.320632691486722)
DescribeResult(nobs=1000, minmax=(78.961385011195659, 217.60255718055666)
               mean=149.62154650913376, variance=592.08525977398665,
               skewness=-0.05203836733032086, kurtosis=-0.3095679010941108)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(),loc=155.37052239221526, scale=23.302150684049227)
DescribeResult(nobs=1000, minmax=(62.447561454192353, 225.54396874804226)
               mean=155.37052239221526, variance=543.53376026239823,
               skewness=-0.08501699580941136, kurtosis=0.06445531527982151)
```

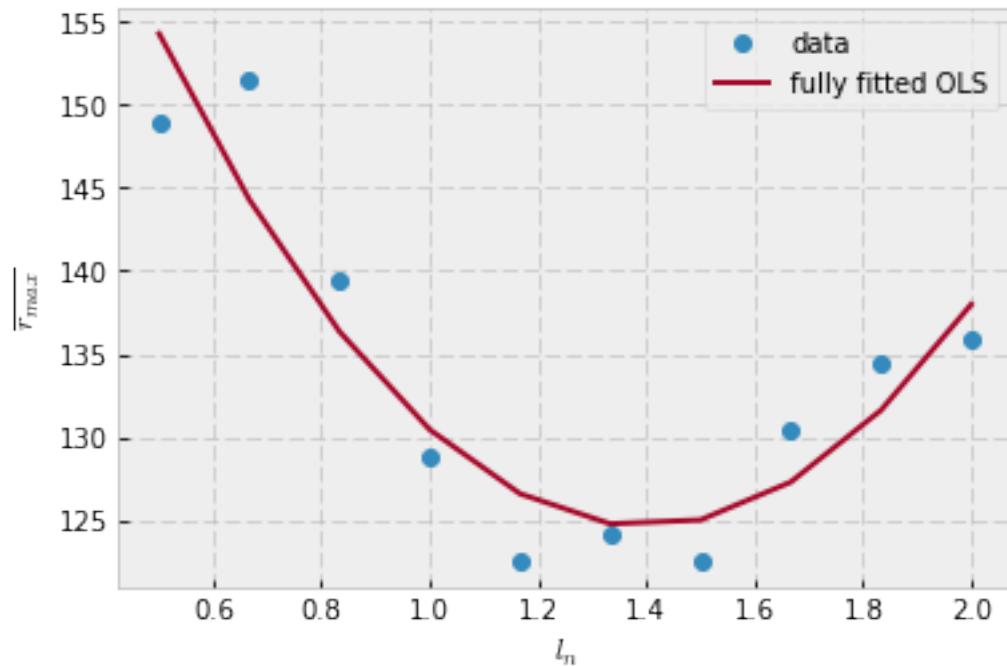




3.2 Parameter regression

```
In [12]: var_name = 'maximal_horizontal_spread'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly1')
mean_reg('maxB', var_name, 'poly2')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly2')
mean_reg('N', var_name, 'poly2')
```

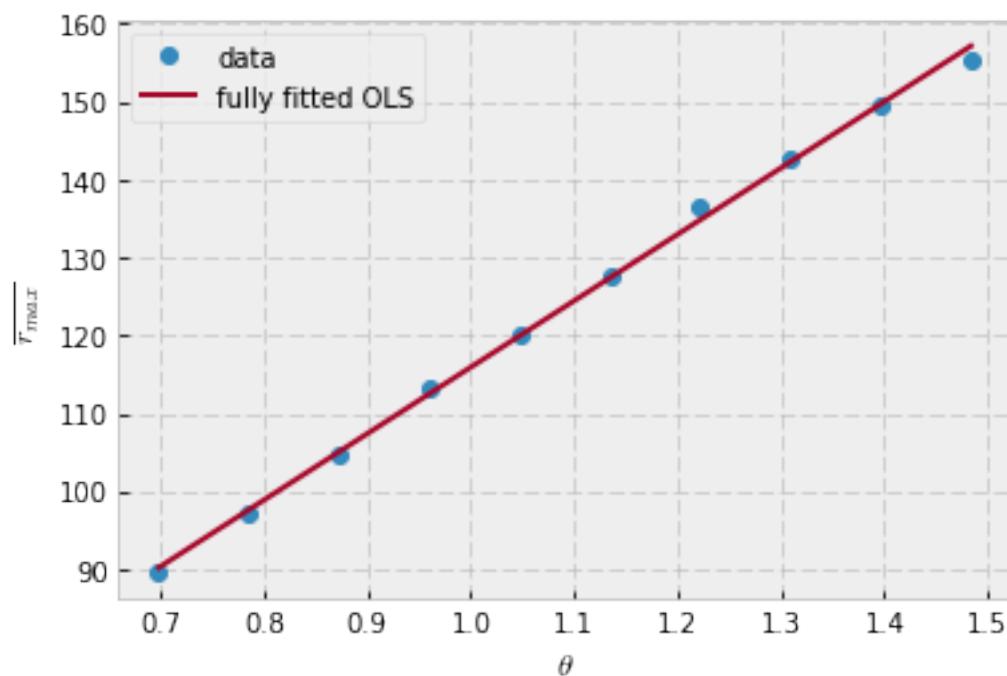
```
variable name: ln
model: poly2
r-squared: 0.5703655389383372
explained variance: 0.5790566613786263
```



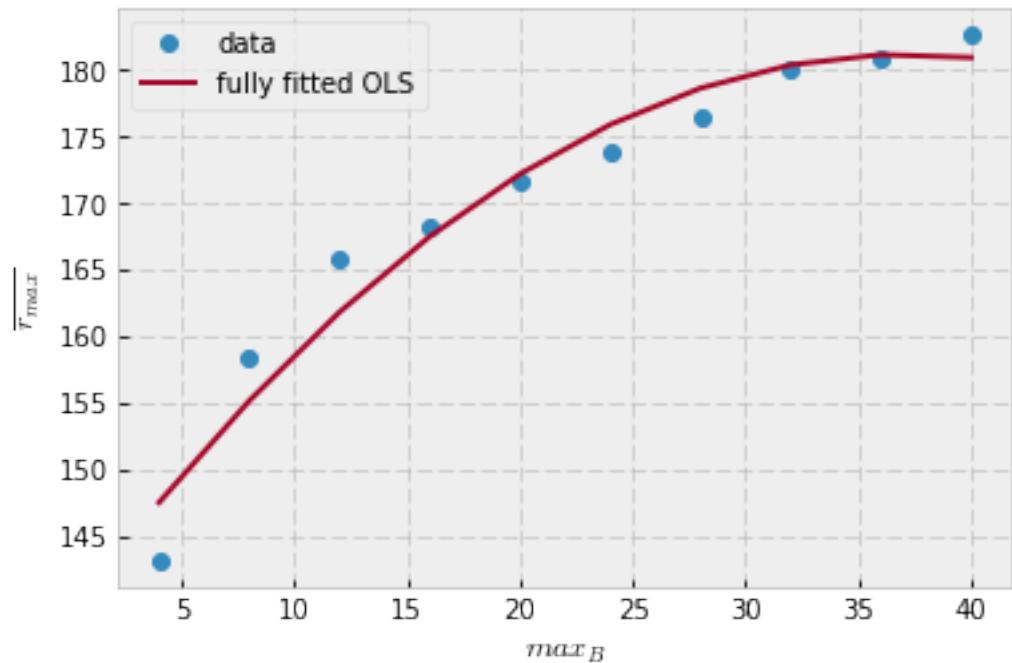
```

variable name: theta
model: poly1
r-squared: 0.9967204506218775
explained variance: 0.99674581917928

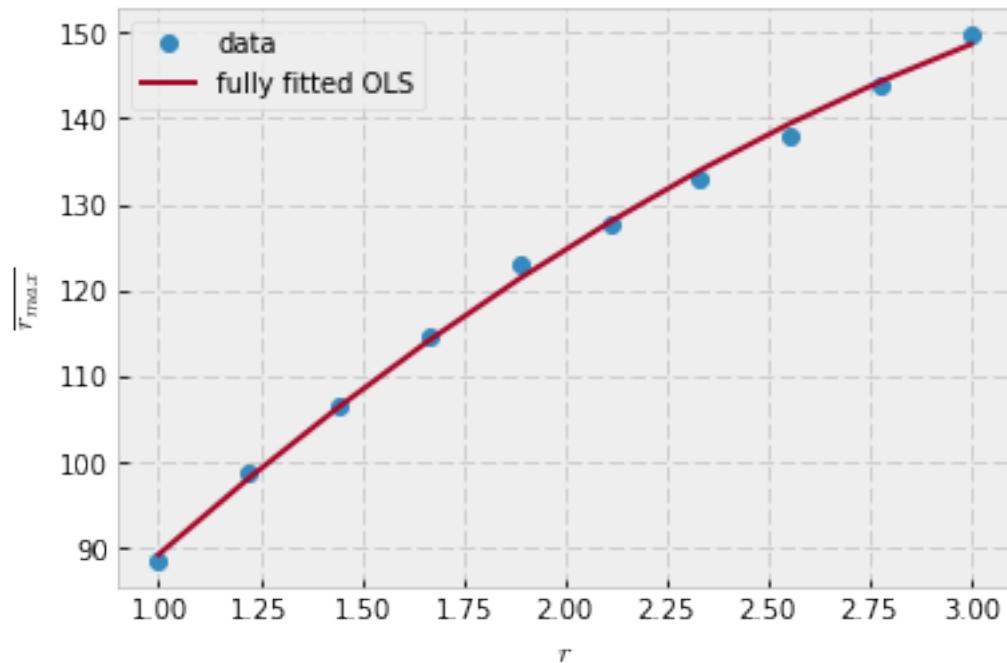
```



```
variable name: maxB
model: poly2
r-squared: 0.8358873850289893
explained variance: 0.8367903090840739
```



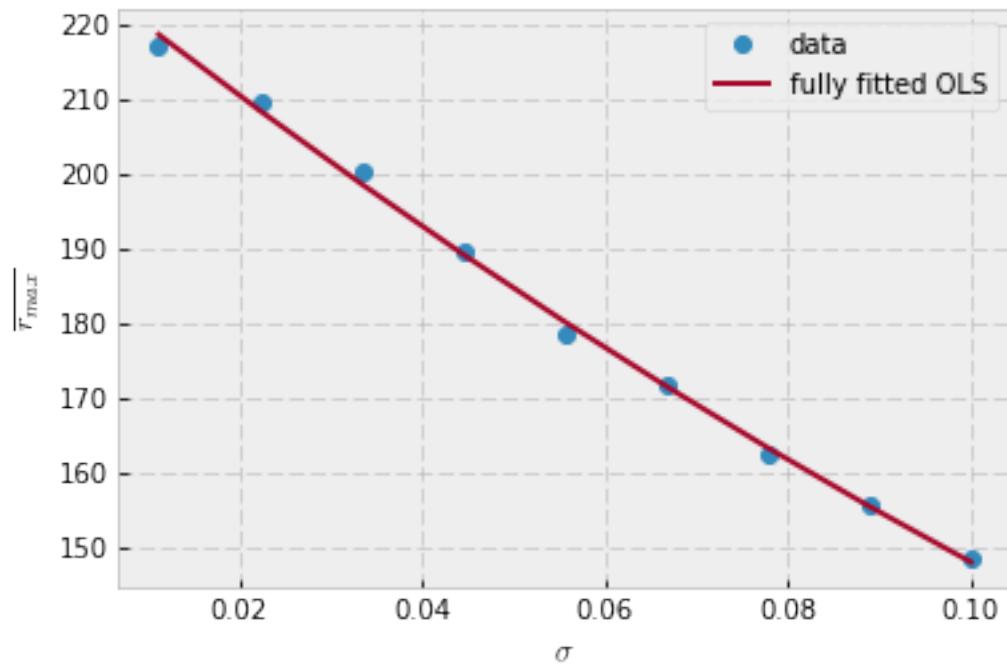
```
variable name: r
model: poly2
r-squared: 0.9932210684801768
explained variance: 0.9932308023322289
```



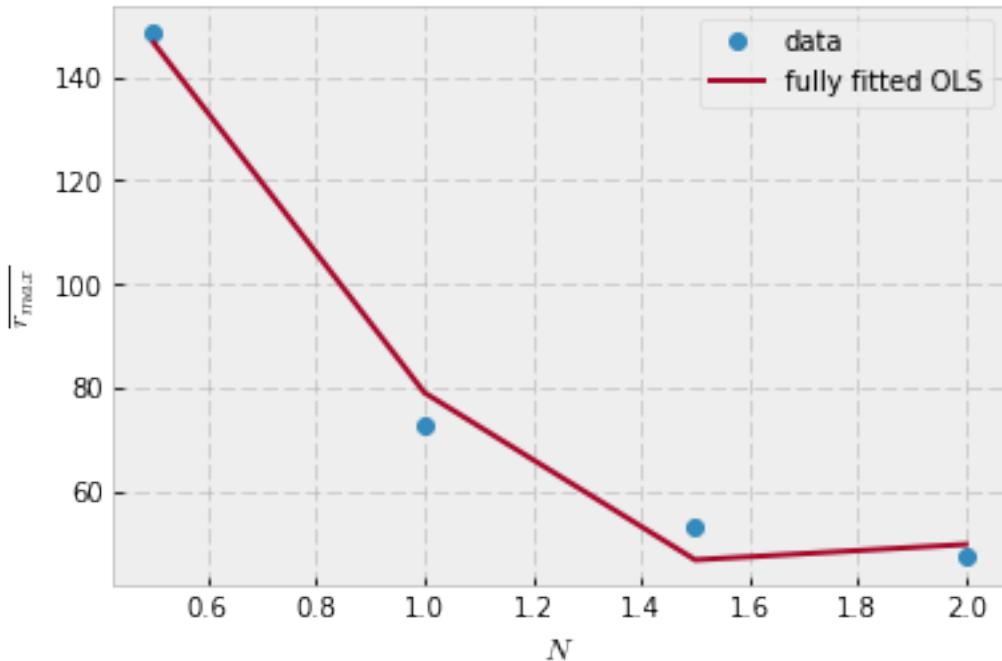
```

variable name: sigma
model: poly2
r-squared: 0.9913245648220792
explained variance: 0.9914048624864077

```



```
variable name: N
model: poly2
r-squared: 0.3685742882848678
explained variance: 0.3685742882848678
```

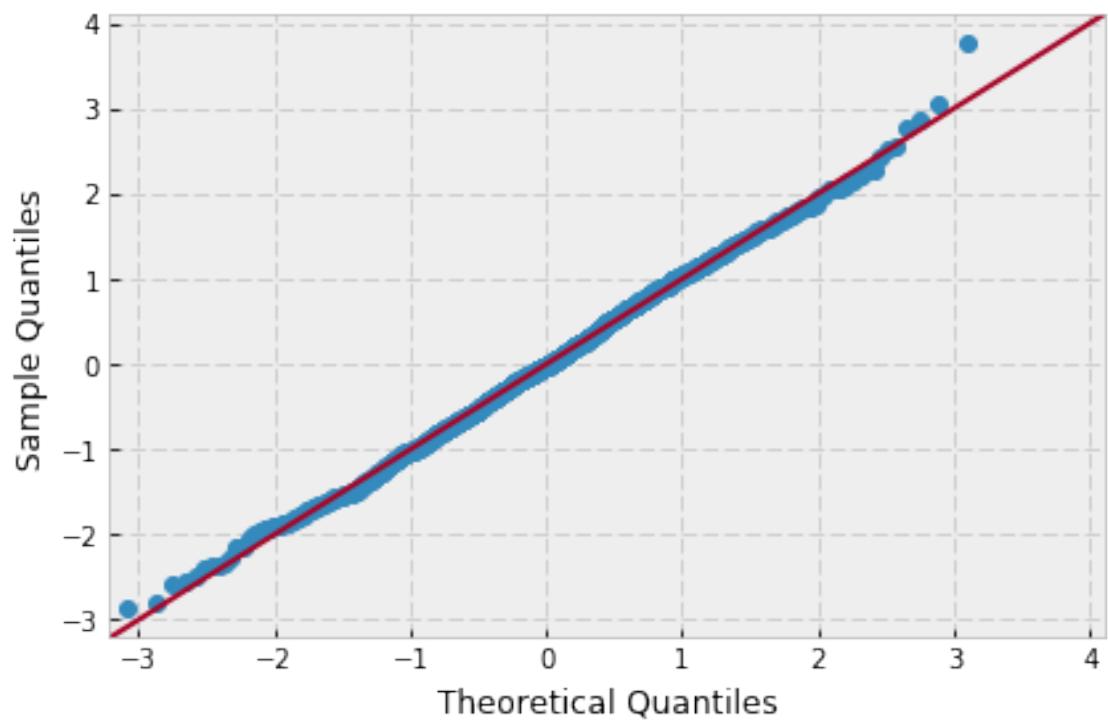
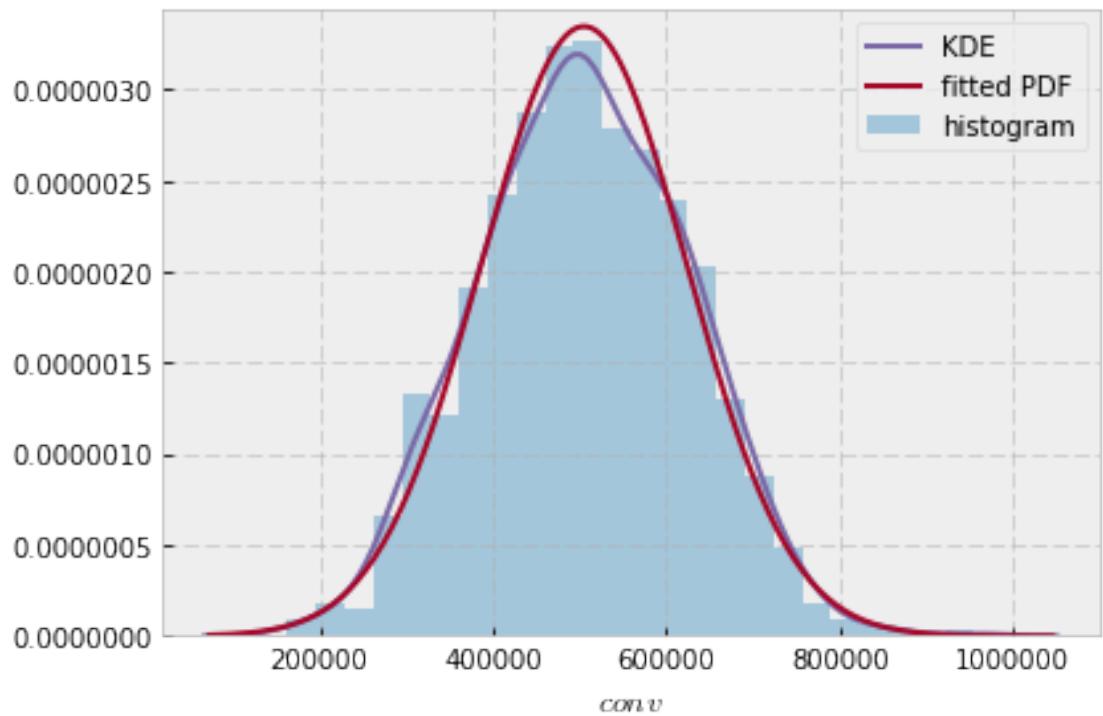


4 Volume of the convex hull

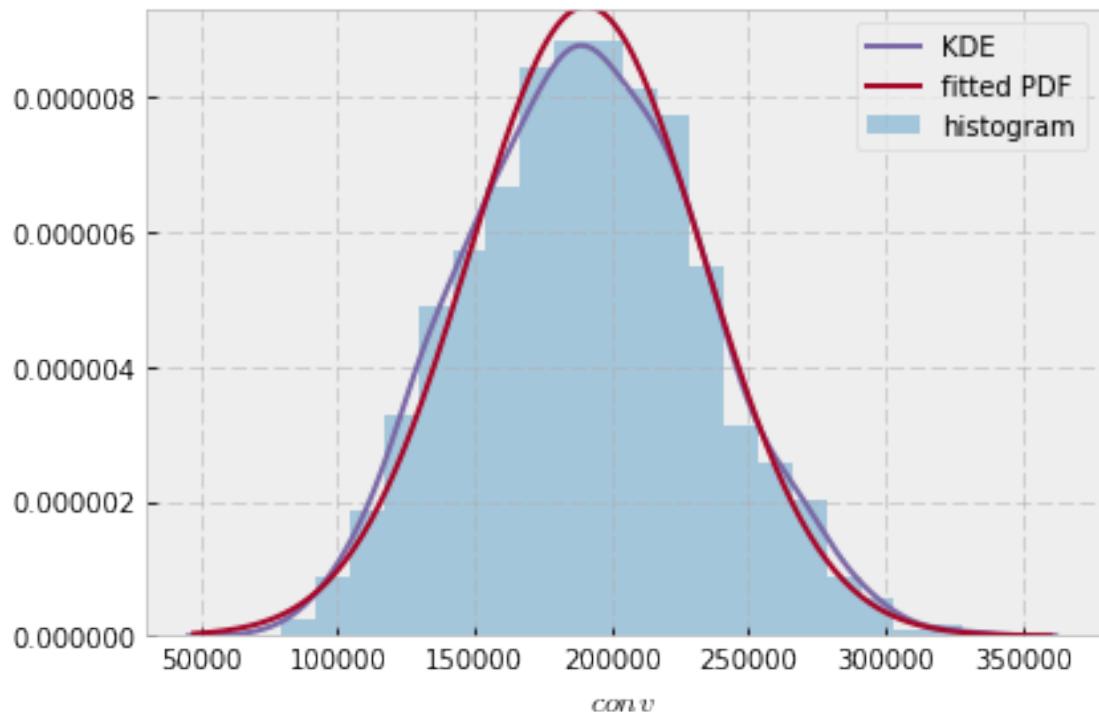
4.1 Probability distributions

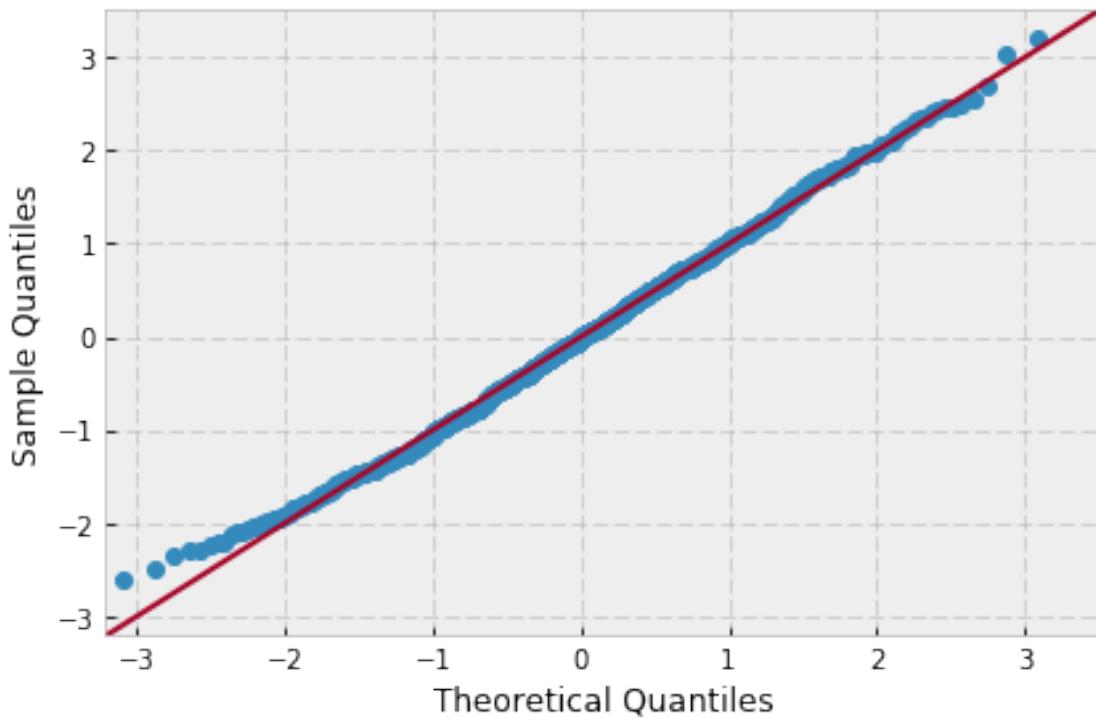
```
In [13]: plot_dist('convex_hull_volume')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=504890.9797993, scale=119154.77130099032)
DescribeResult(nobs=1000, minmax=(162948.9911999999, 954513.8253999997)
               mean=504890.9797993002, variance=14212071595.386694,
               skewness=0.04025480510545997, kurtosis=-0.19149133717461853)
```

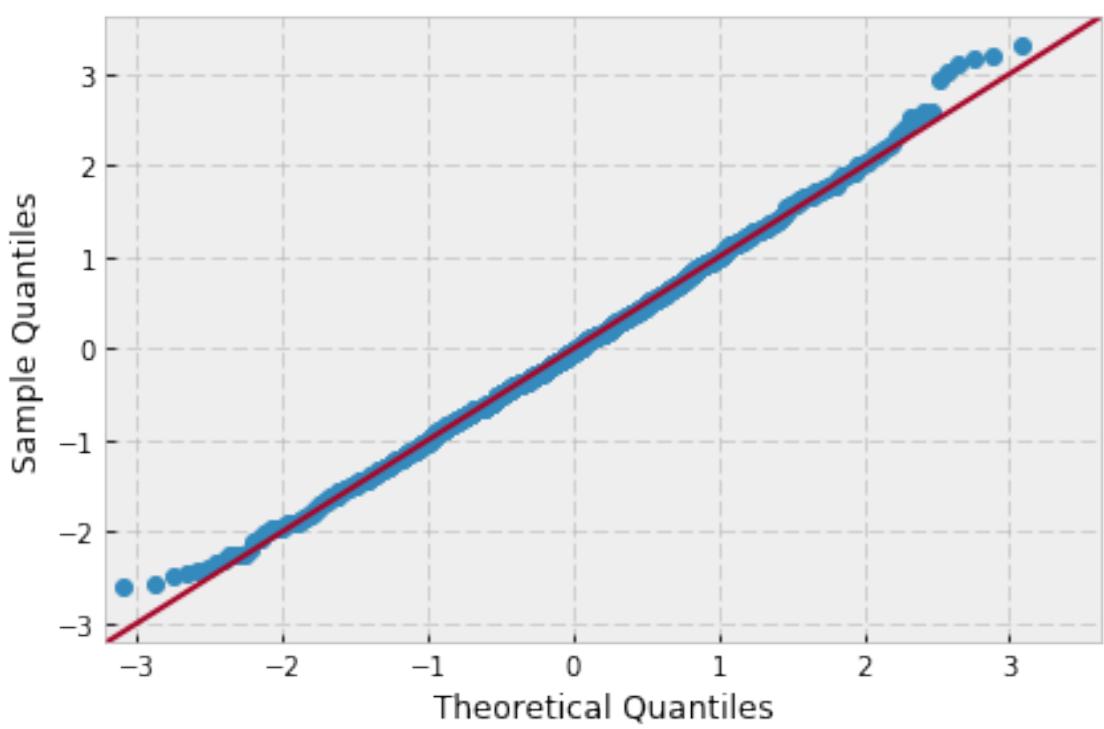
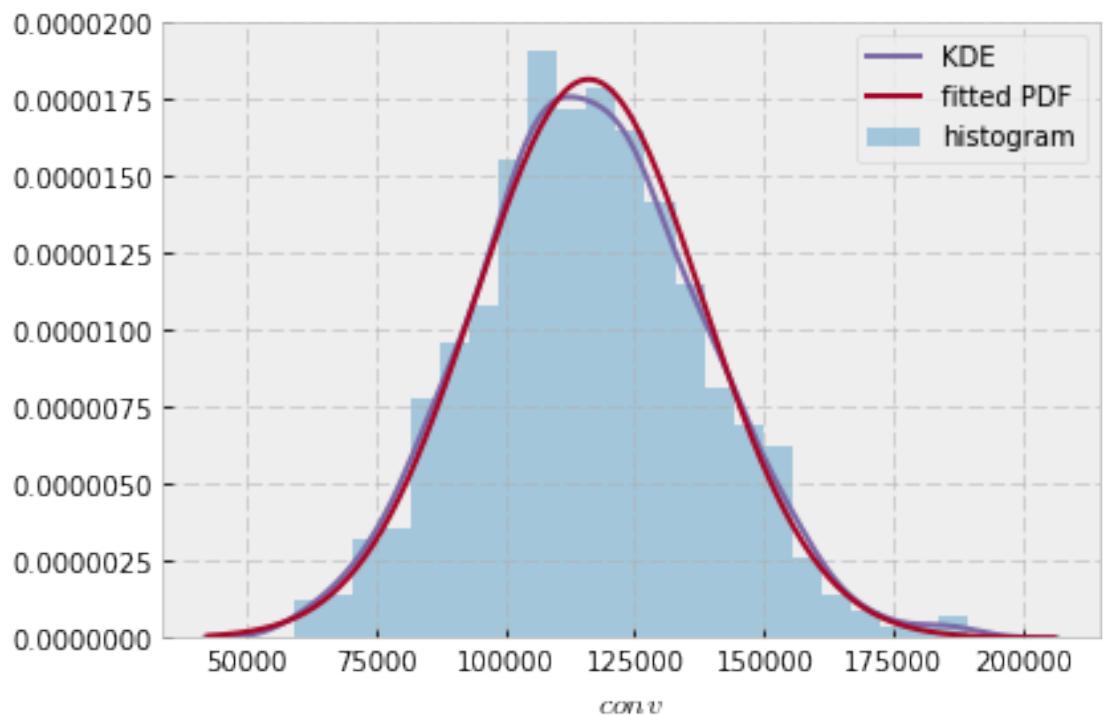


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=190796.34017243, scale=42651.64594323637)
DescribeResult(nobs=1000, minmax=(79754.78372000007, 328133.67469999997)
               mean=190796.34017243001, variance=1820983885.5527439,
               skewness=0.12717073592123052, kurtosis=-0.3176677340881753)
```

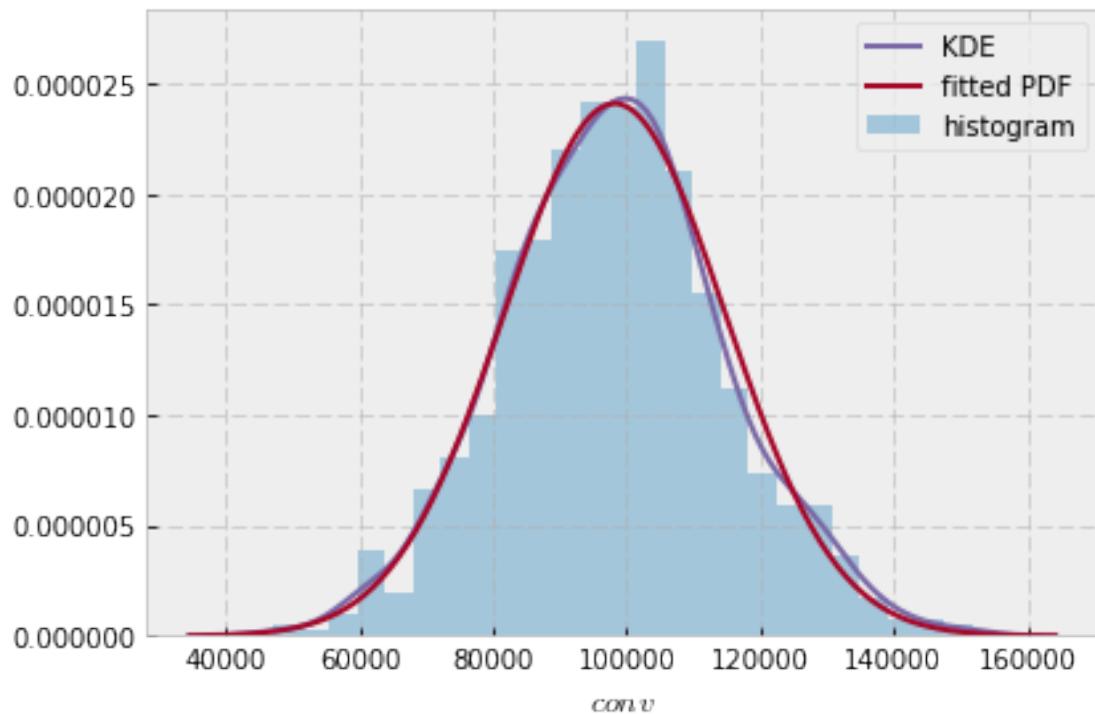


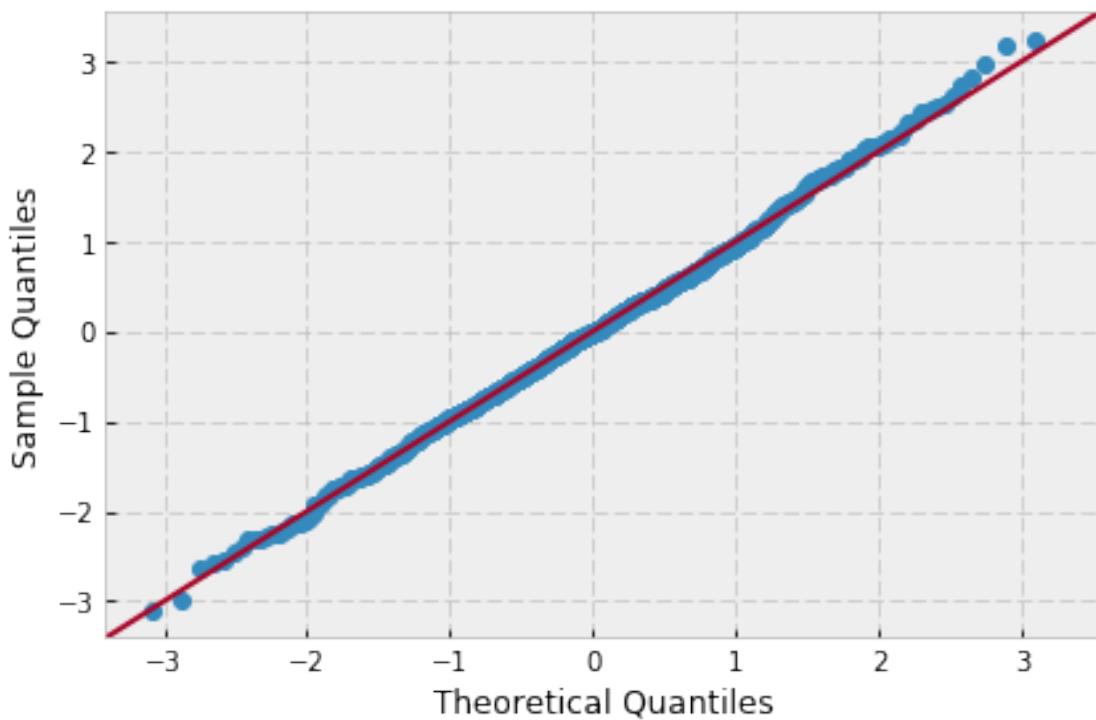


```
variable name: N
variable value: 1.5
distribution: normal(shape=(),loc=116116.57567993, scale=21966.93052871548)
DescribeResult(nobs=1000, minmax=(59282.98060999999, 189205.0229999999)
               mean=116116.57567993, variance=483029065.91933149,
               skewness=0.16411676973676737, kurtosis=-0.012440850202235954)
```

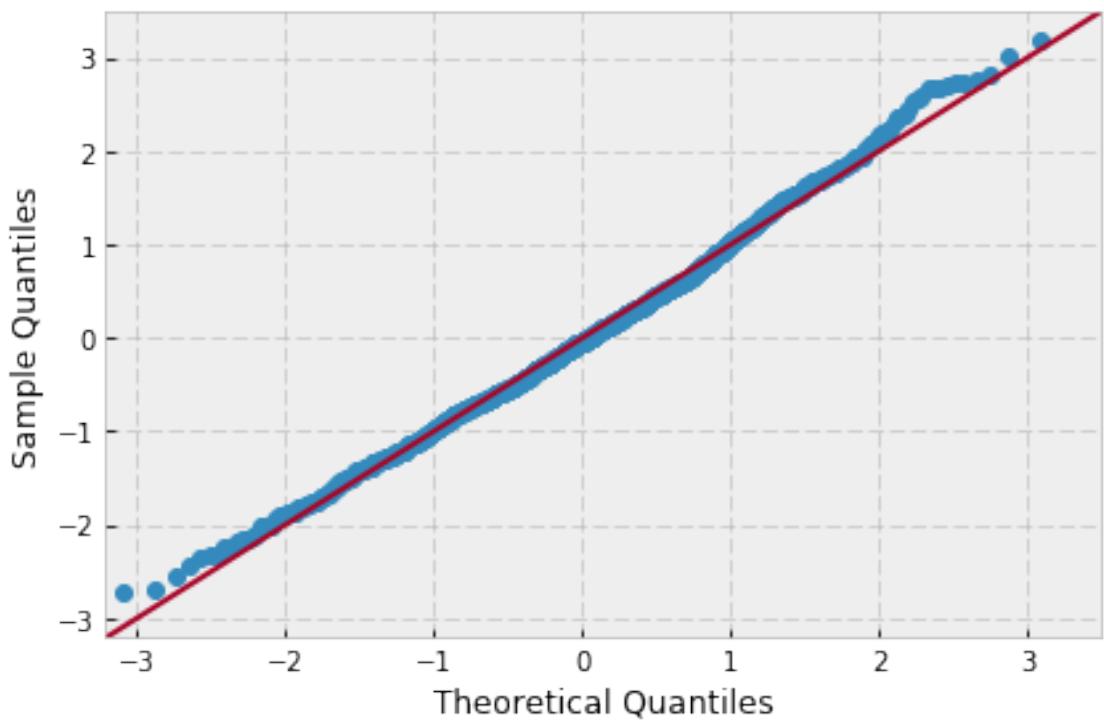
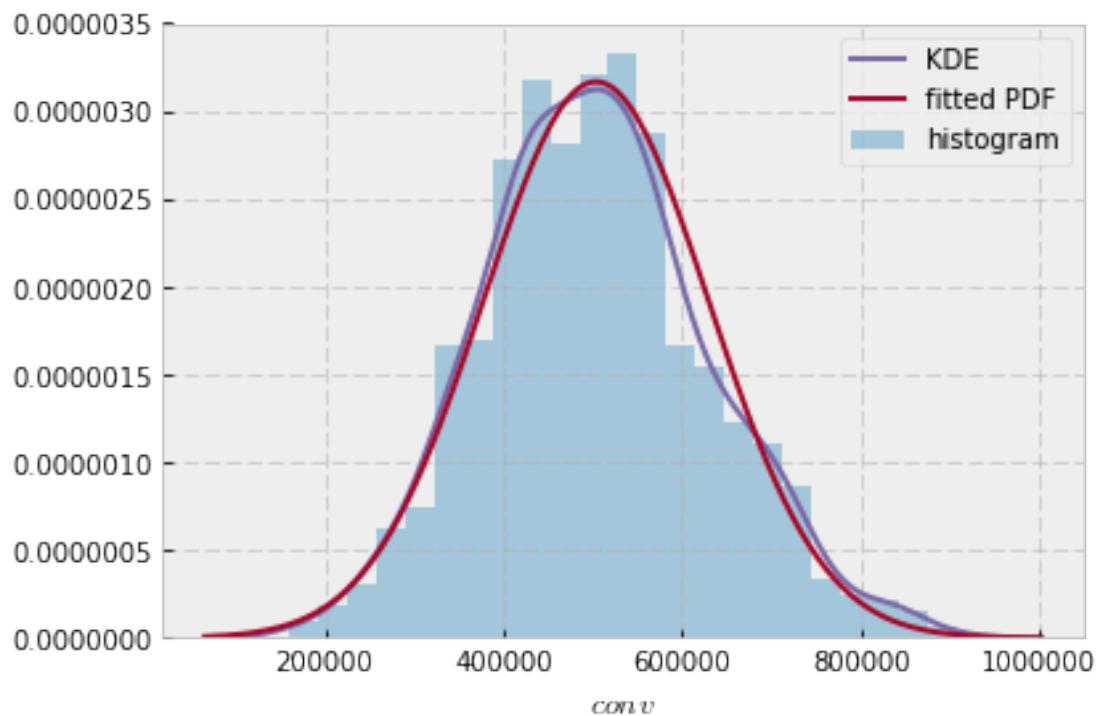


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=98174.67485847, scale=16549.48459837525)
DescribeResult(nobs=1000, minmax=(46989.559849999998, 151651.8168)
               mean=98174.674858469996, variance=274159600.07193148,
               skewness=0.09662930695731461, kurtosis=0.08135984899798654)
```

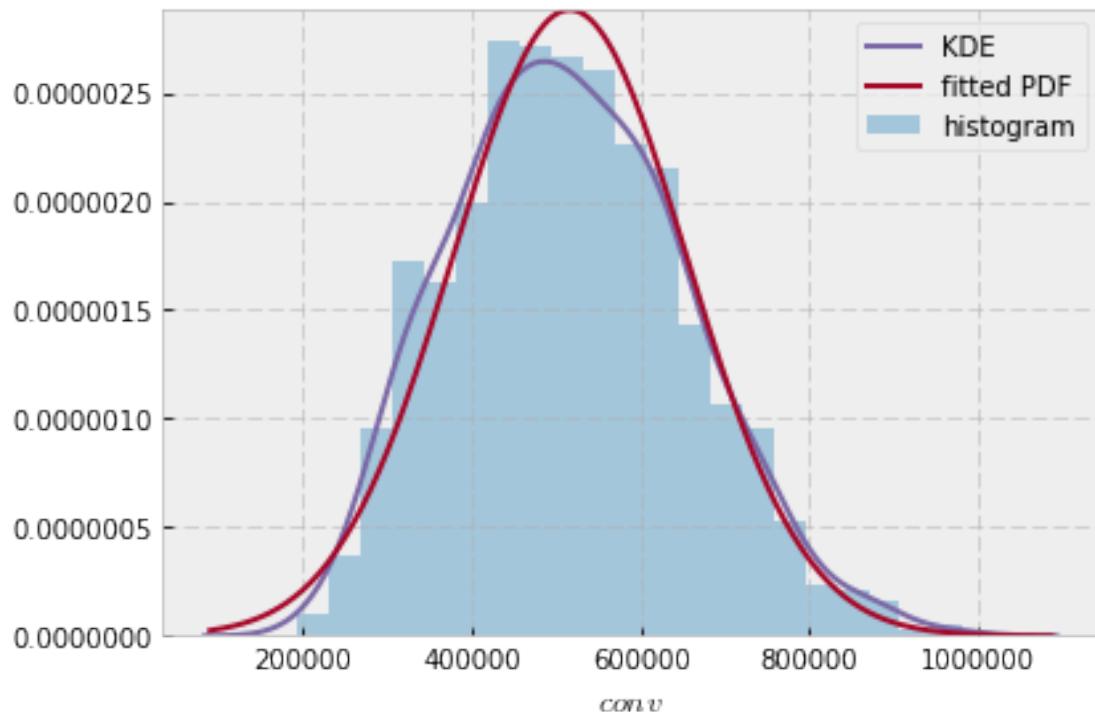


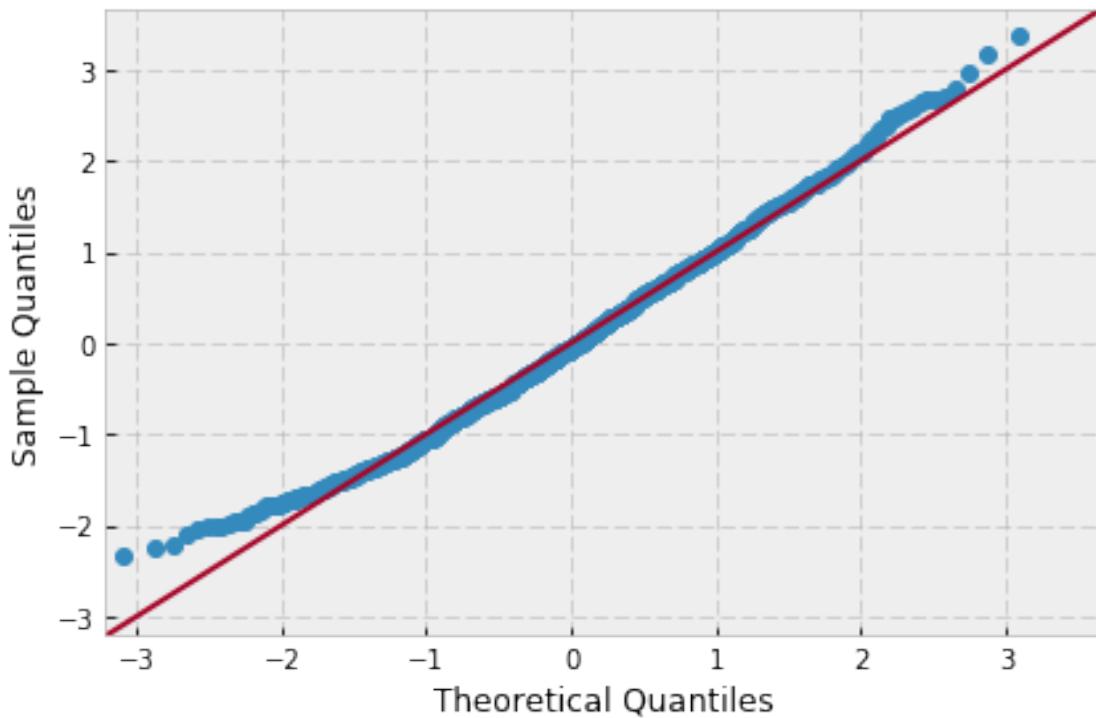


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(),loc=503673.4695341, scale=126089.99686070206)
DescribeResult(nobs=1000, minmax=(160880.73860000001, 906451.18229999999)
               mean=503673.46953409998, variance=15914601910.242096,
               skewness=0.2425691971486958, kurtosis=-0.02108312386823563)
```

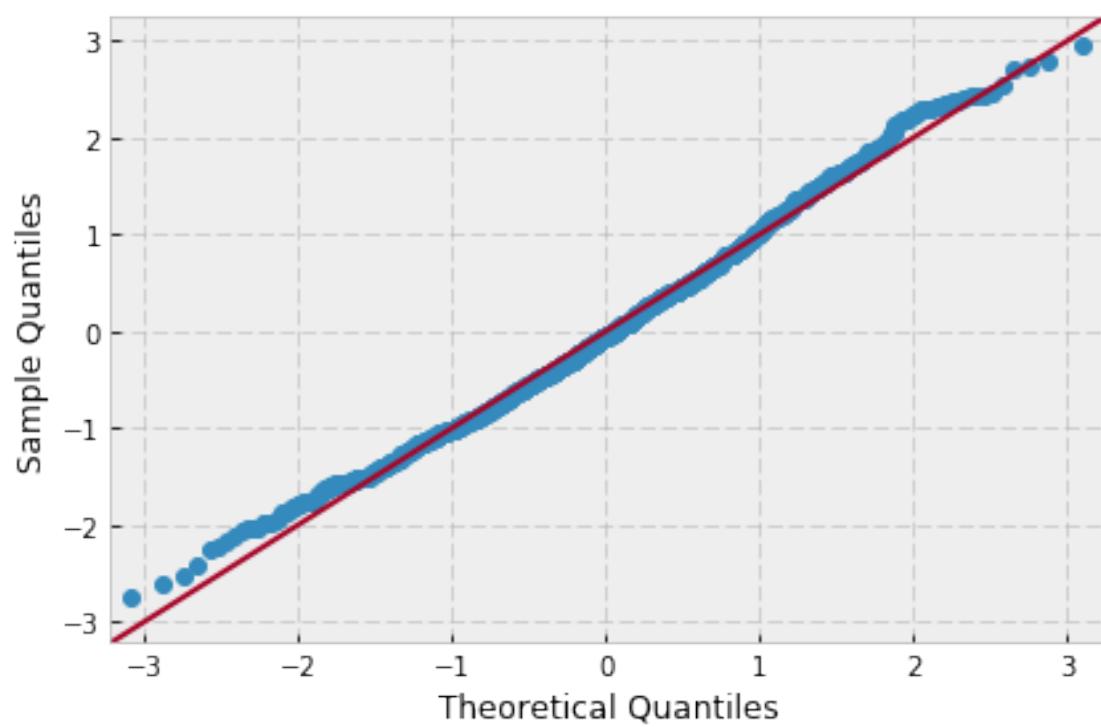
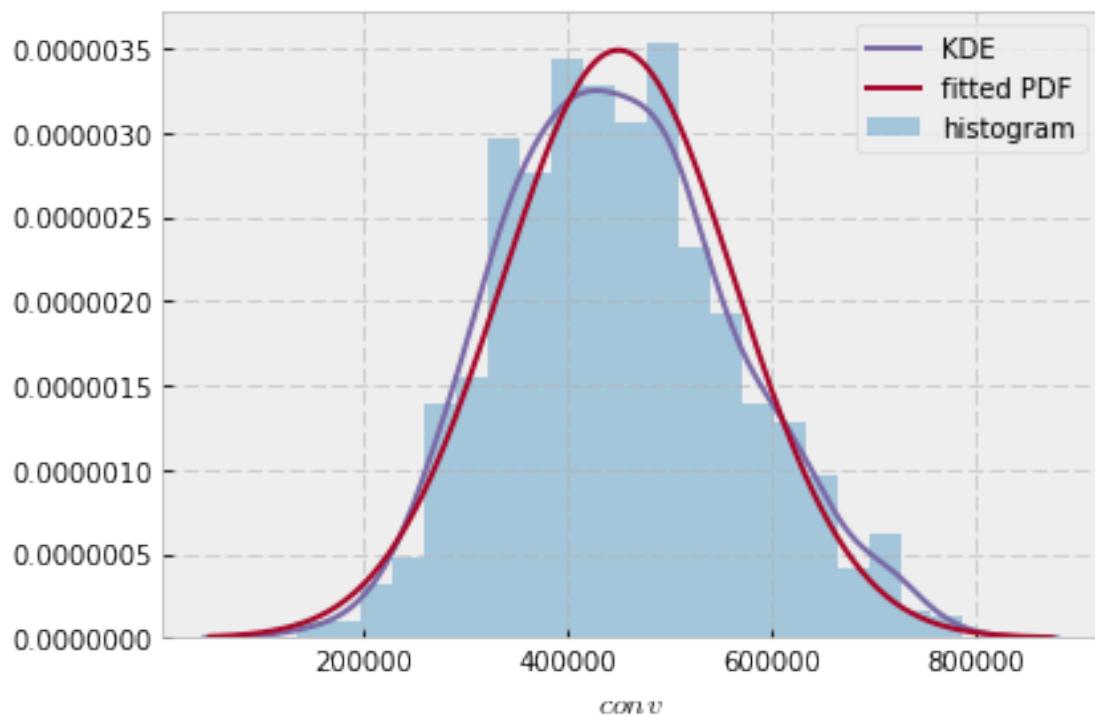


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=515898.30276329996, scale=138420.63455963708)
DescribeResult(nobs=1000, minmax=(193882.8063, 982139.9277)
               mean=515898.30276329996, variance=19179451523.416008,
               skewness=0.288200380759445, kurtosis=-0.2502412728473815)
```

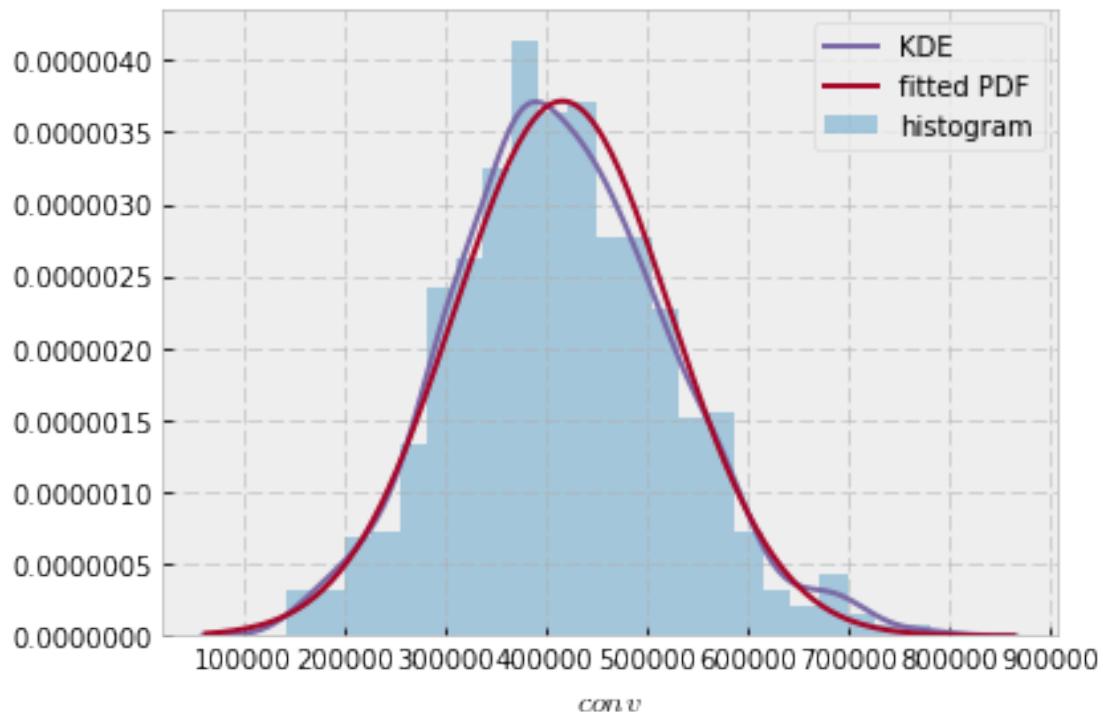


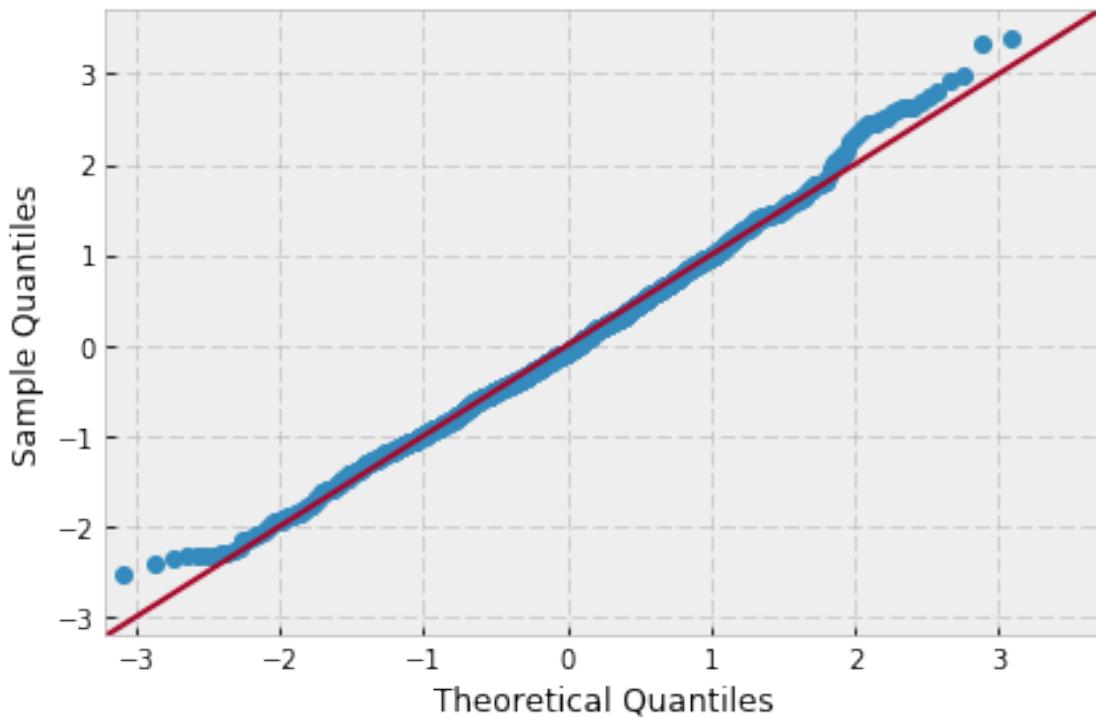


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=450548.74619140005, scale=114276.44329359241)
DescribeResult(nobs=1000, minmax=(136238.0100000001, 788679.19889999996)
               mean=450548.74619140005, variance=13072177669.503143,
               skewness=0.2636808281207275, kurtosis=-0.24830182547694735)
```

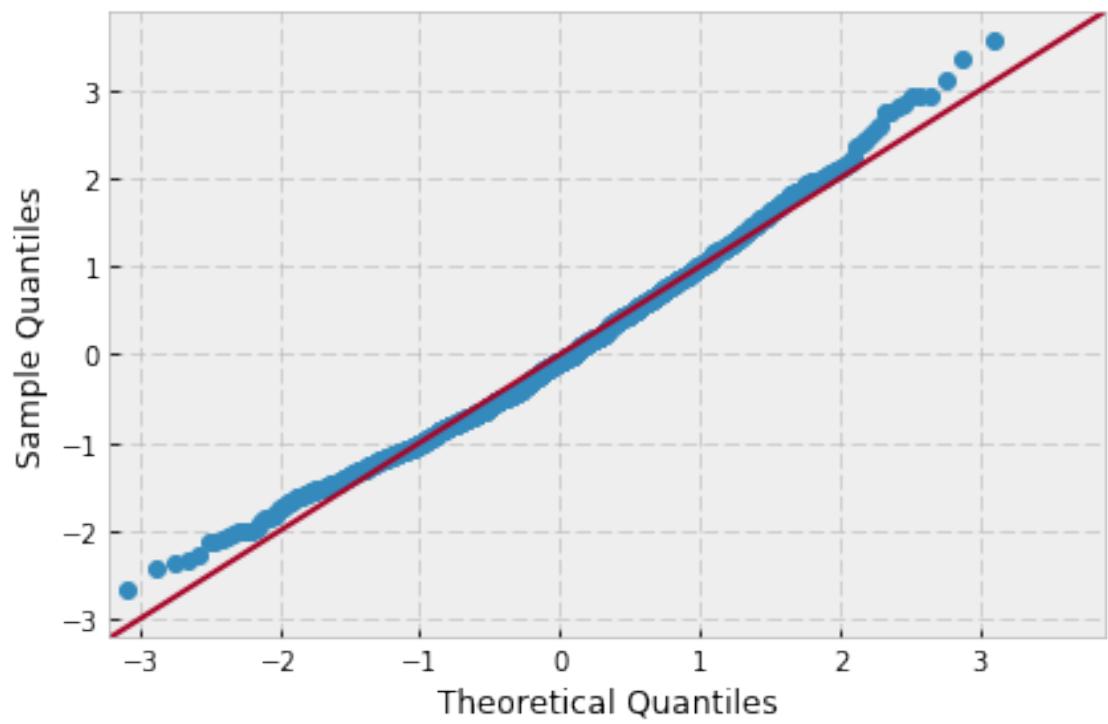
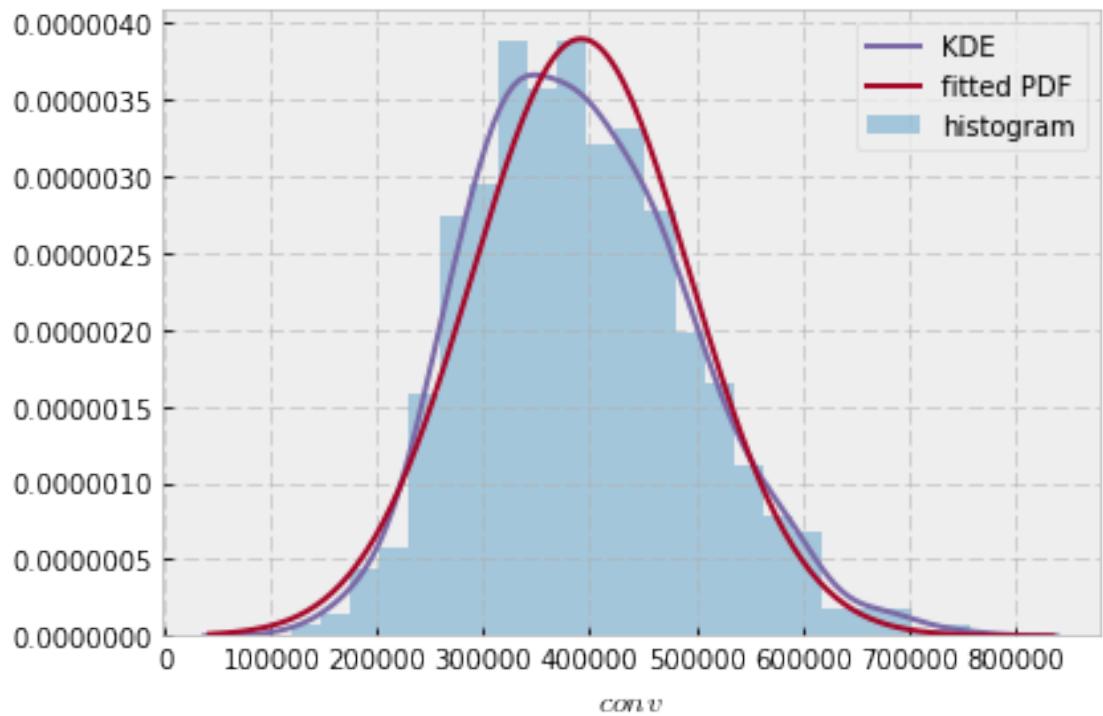


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=417104.8582146, scale=107373.10018362661)
DescribeResult(nobs=1000, minmax=(144323.296, 783000.0860000001)
               mean=417104.85821460001, variance=11540523166.209324,
               skewness=0.2684073482256936, kurtosis=0.08931849008600246)
```

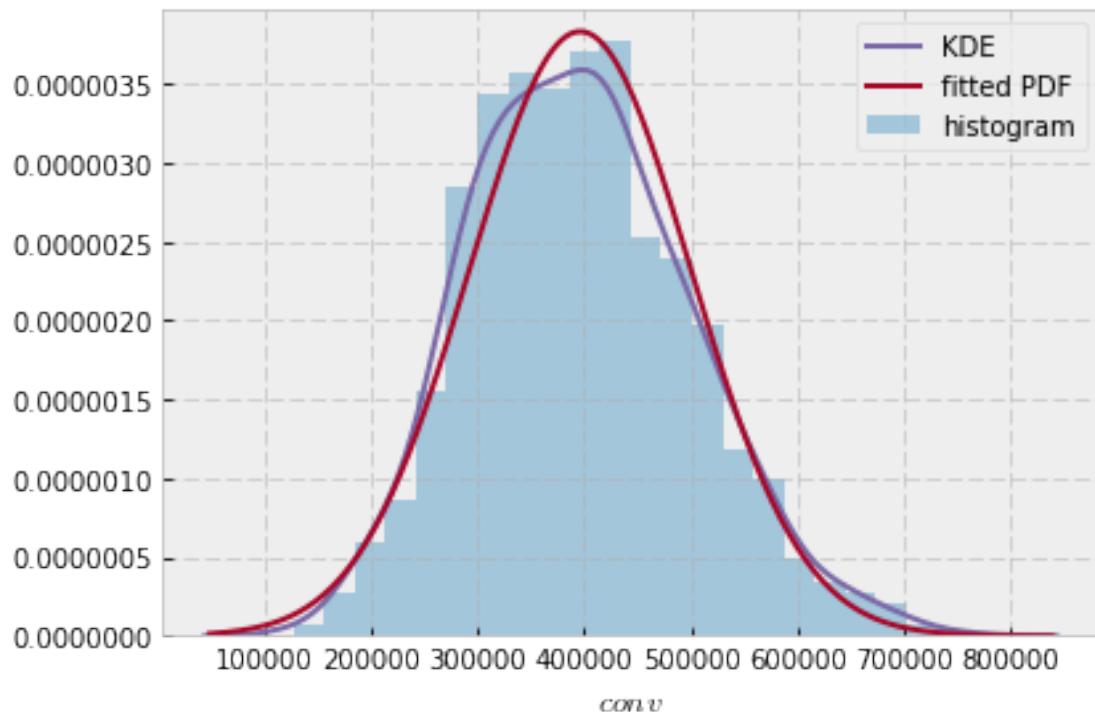


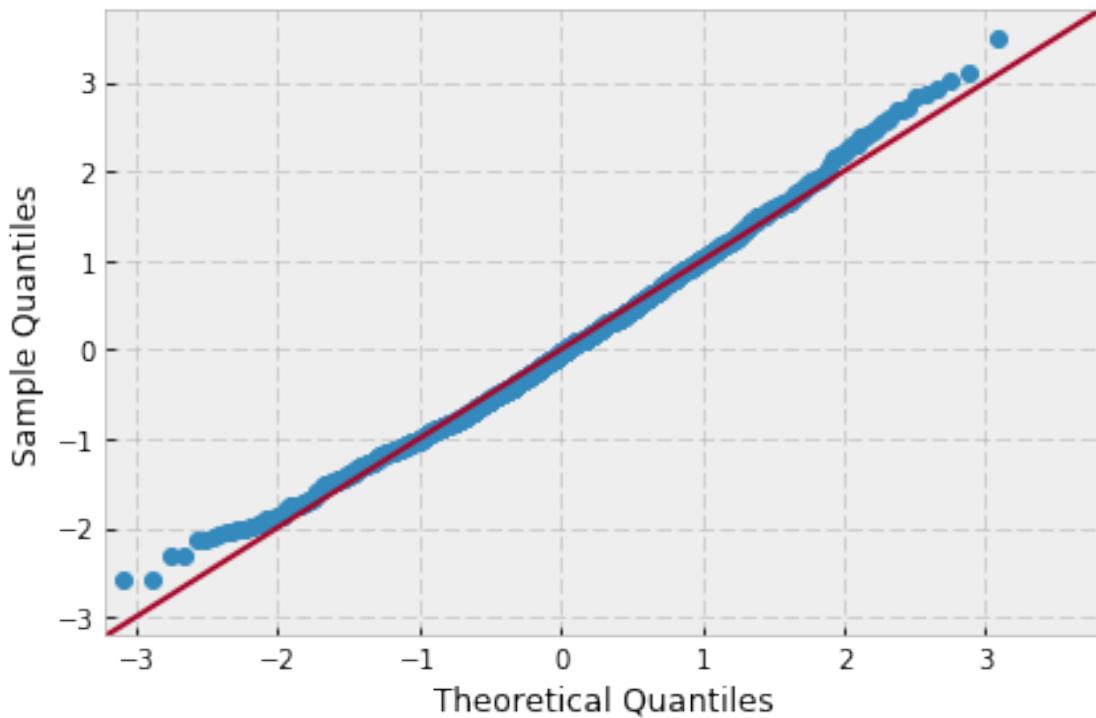


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=392001.6336975, scale=102313.01051772699)
DescribeResult(nobs=1000, minmax=(119508.0107, 757578.2249999998)
               mean=392001.63369749999, variance=10478430551.752266,
               skewness=0.3916979367958242, kurtosis=0.011273116407439776)
```

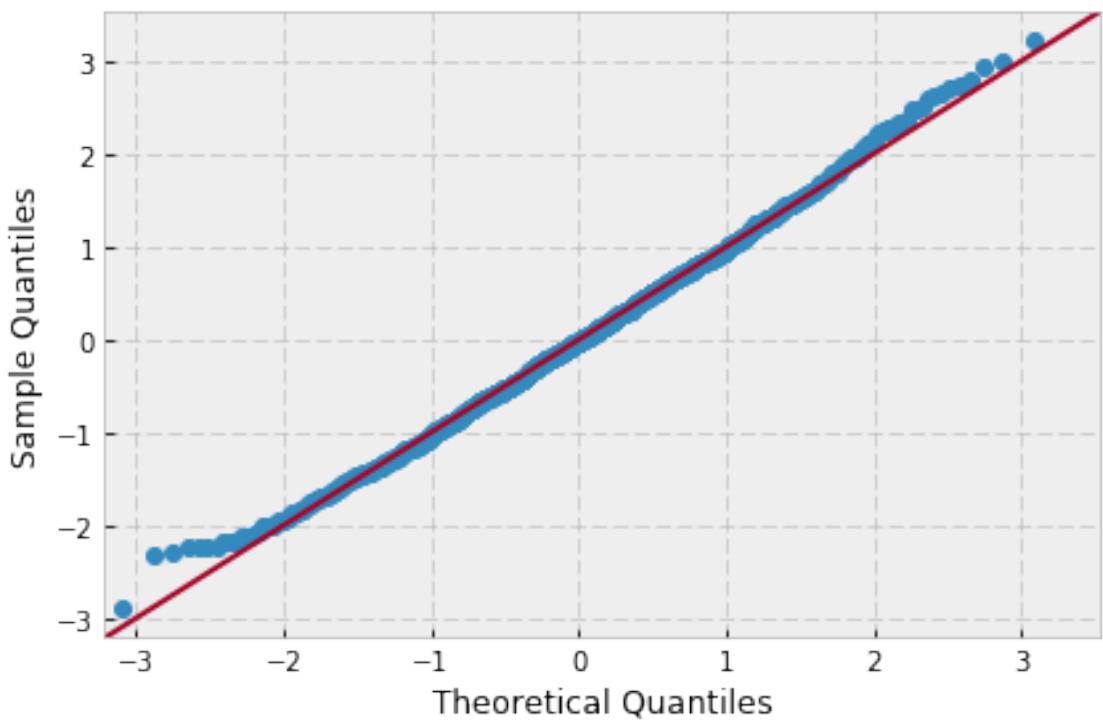
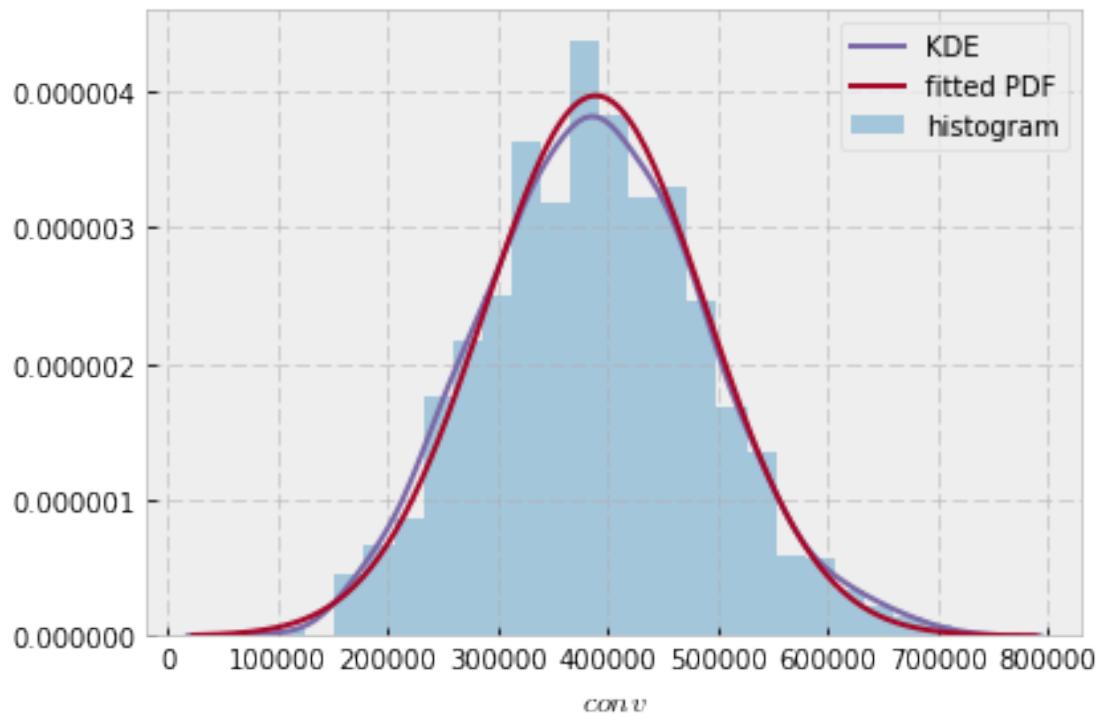


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=396205.54994389997, scale=104061.5064887984)
DescribeResult(nobs=1000, minmax=(126500.8039, 760721.6352000002)
               mean=396205.54994389997, variance=10839636769.487719,
               skewness=0.3241096915345769, kurtosis=-0.0407524575733114)
```

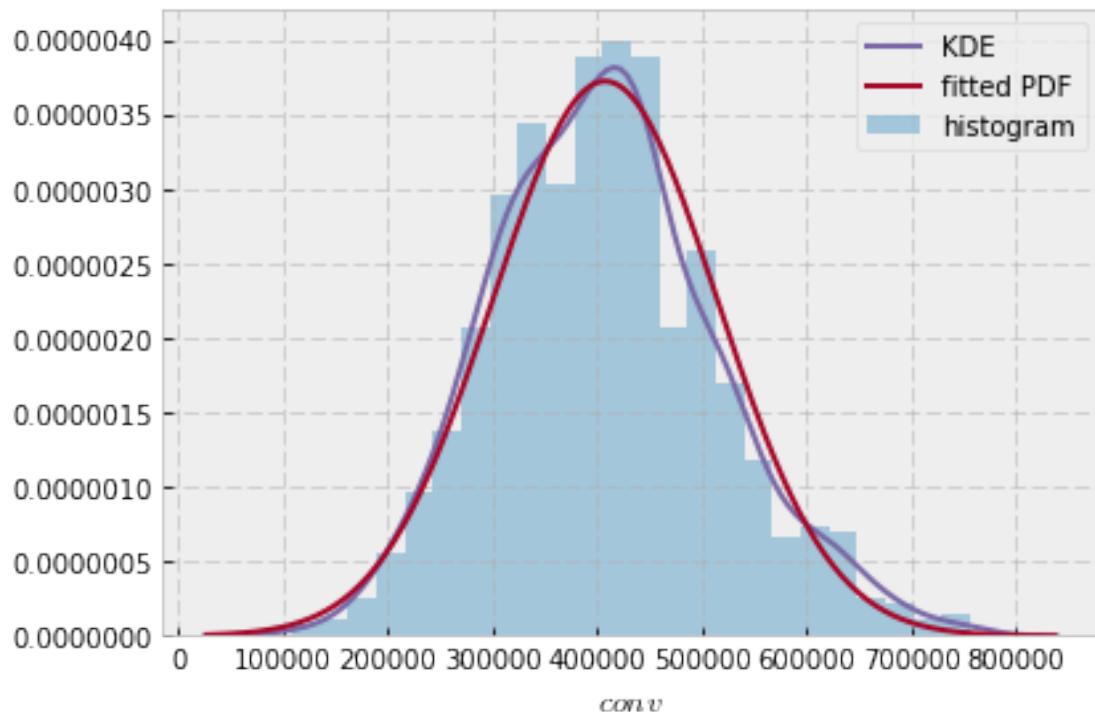


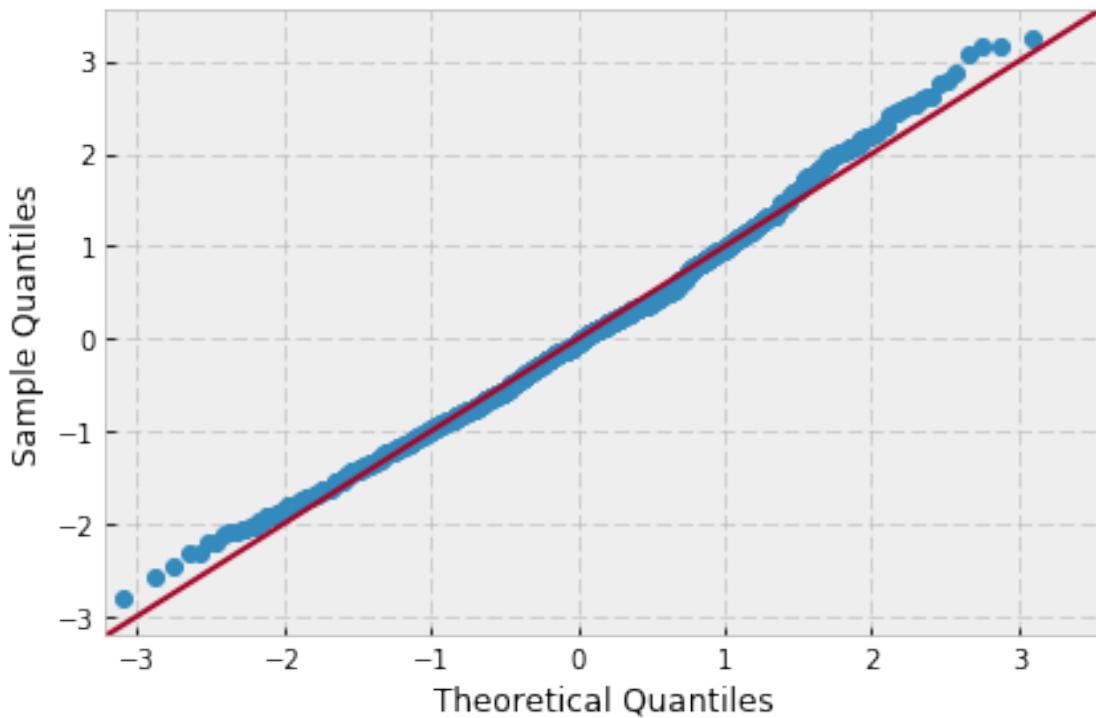


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(),loc=389371.82484827, scale=100491.01449238499)
DescribeResult(nobs=1000, minmax=(98916.171069999997, 713210.14980000001)
               mean=389371.82484826999, variance=10108552546.254984,
               skewness=0.1716632561568022, kurtosis=-0.12113763945525147)
```

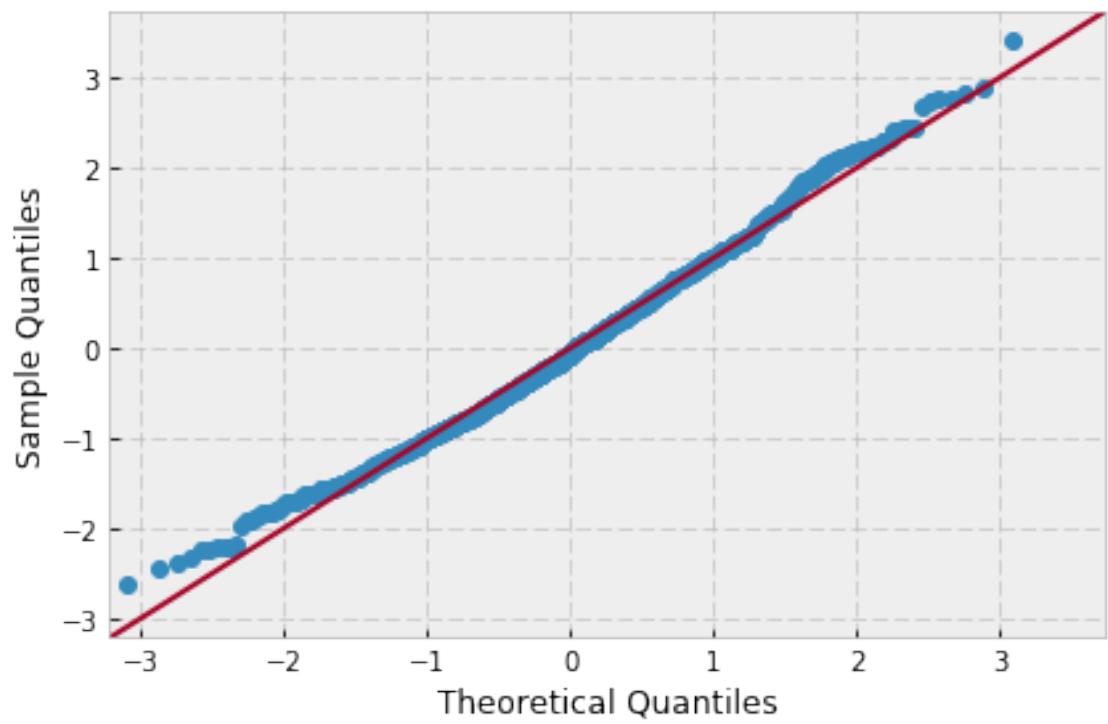
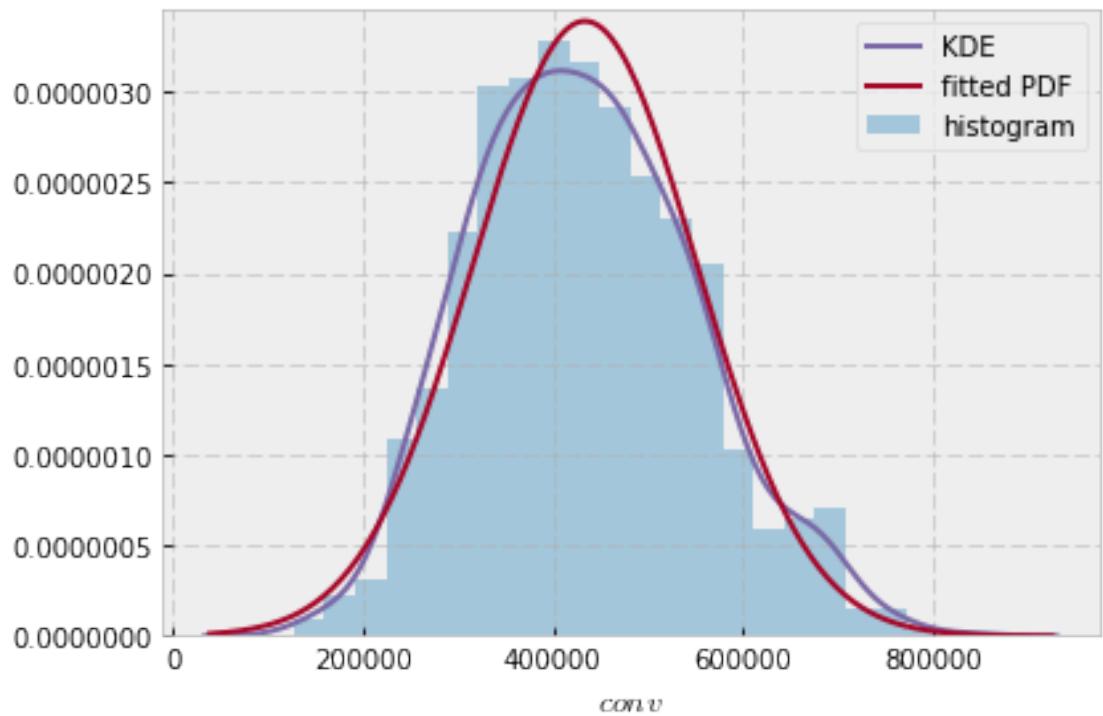


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=407289.73211240006, scale=107192.65716517481)
DescribeResult(nobs=1000, minmax=(107686.5505, 755768.6770000003)
               mean=407289.73211240006, variance=11501767517.648352,
               skewness=0.33106515981685425, kurtosis=0.0857063192236911)
```

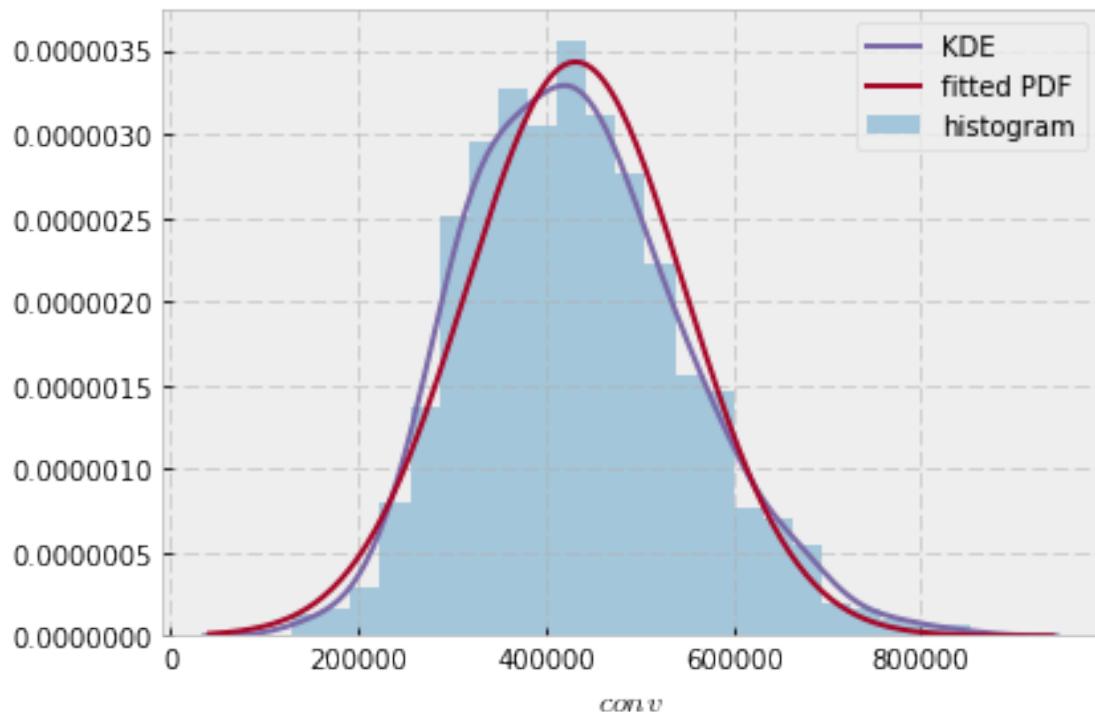


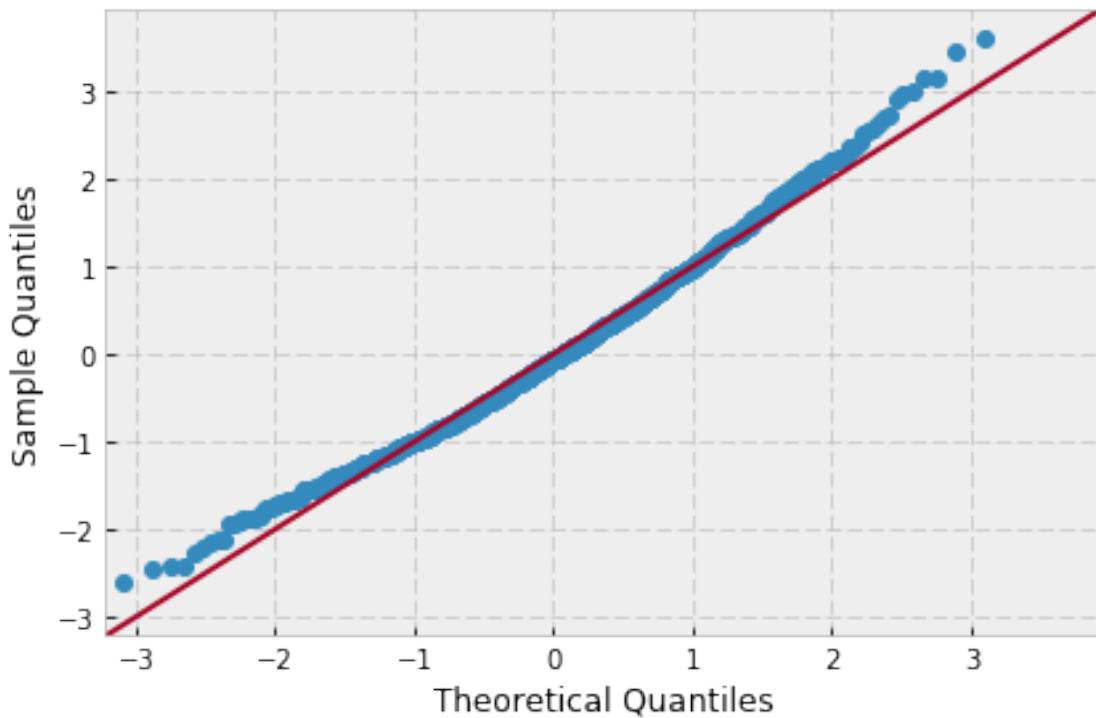


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(),loc=433713.60826209997, scale=117722.31849987003)
DescribeResult(nobs=1000, minmax=(127050.341, 836748.8177999996)
               mean=433713.60826209997, variance=13872416689.674515,
               skewness=0.3056543992784221, kurtosis=-0.1780757939979445)
```

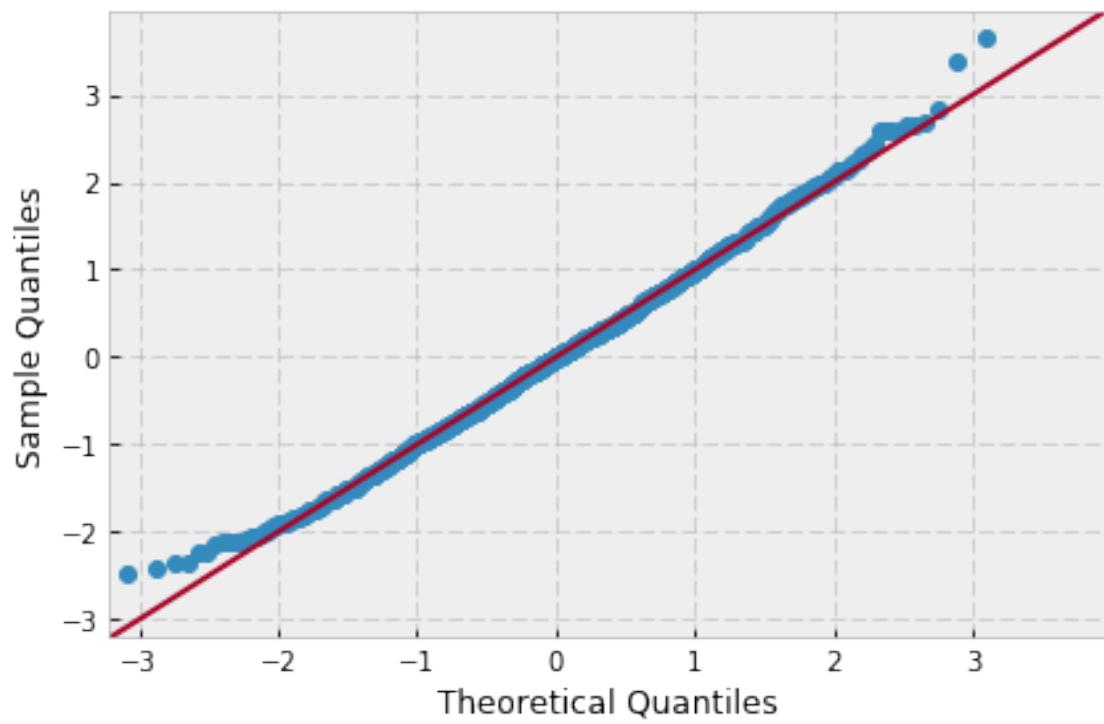
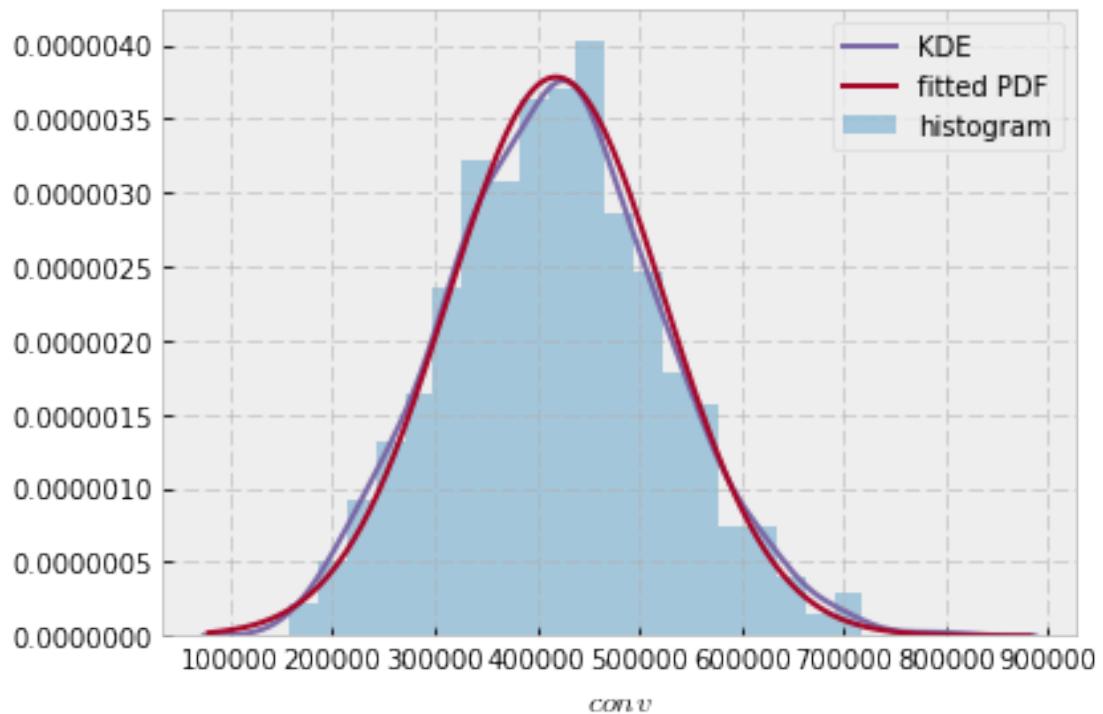


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=431883.4697952, scale=116244.52652265171)
DescribeResult(nobs=1000, minmax=(129213.7752999999, 851959.4647000001)
               mean=431883.46979519998, variance=13526316262.738214,
               skewness=0.42444180182667923, kurtosis=0.093509690887974)
```

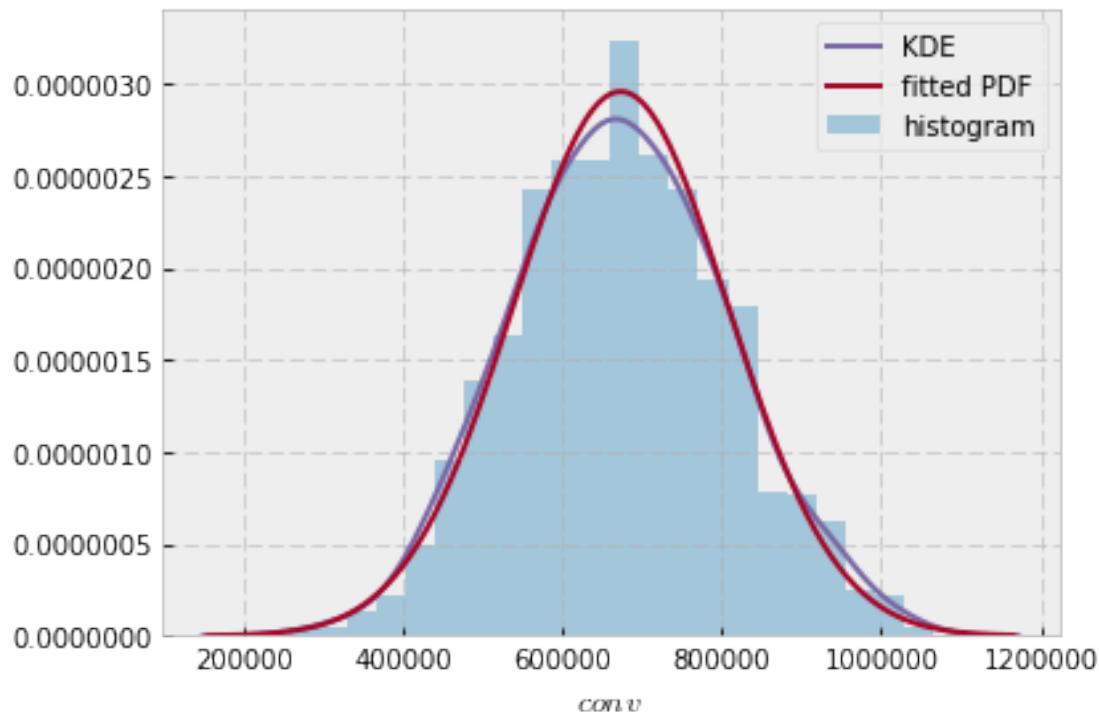


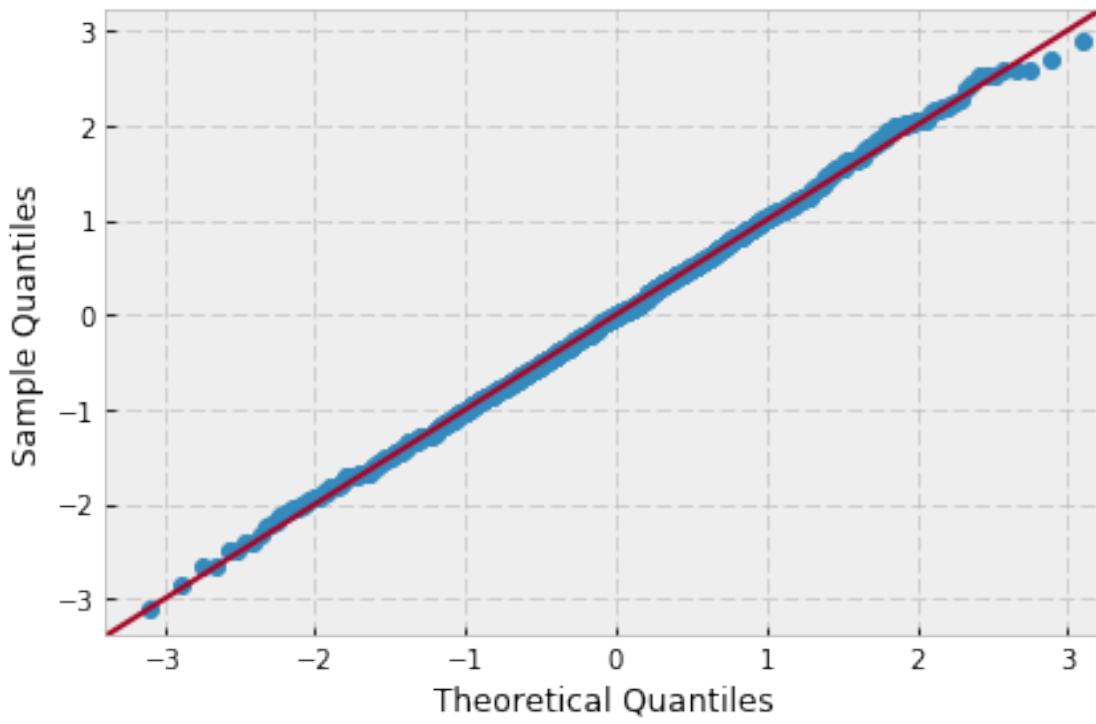


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(),loc=418634.4643035001, scale=105475.59351796337)
DescribeResult(nobs=1000, minmax=(158426.14069999999, 802599.42319999996)
               mean=418634.4643035008, variance=11136237065.031668,
               skewness=0.18451584950856026, kurtosis=-0.07006918421029784)
```

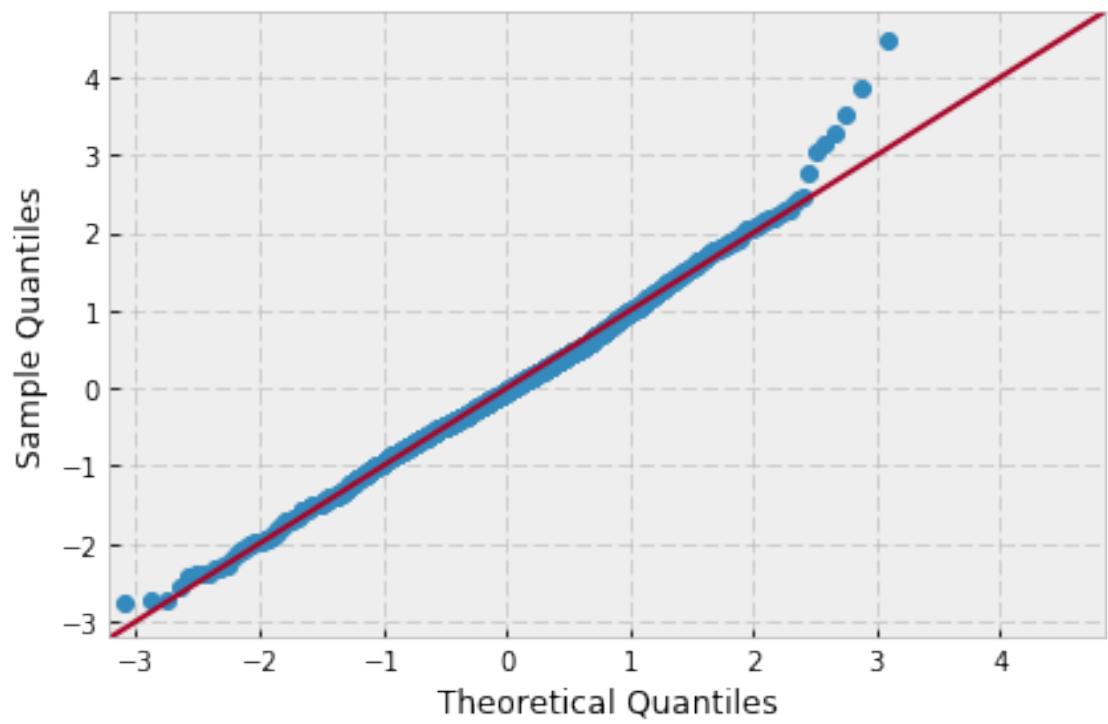
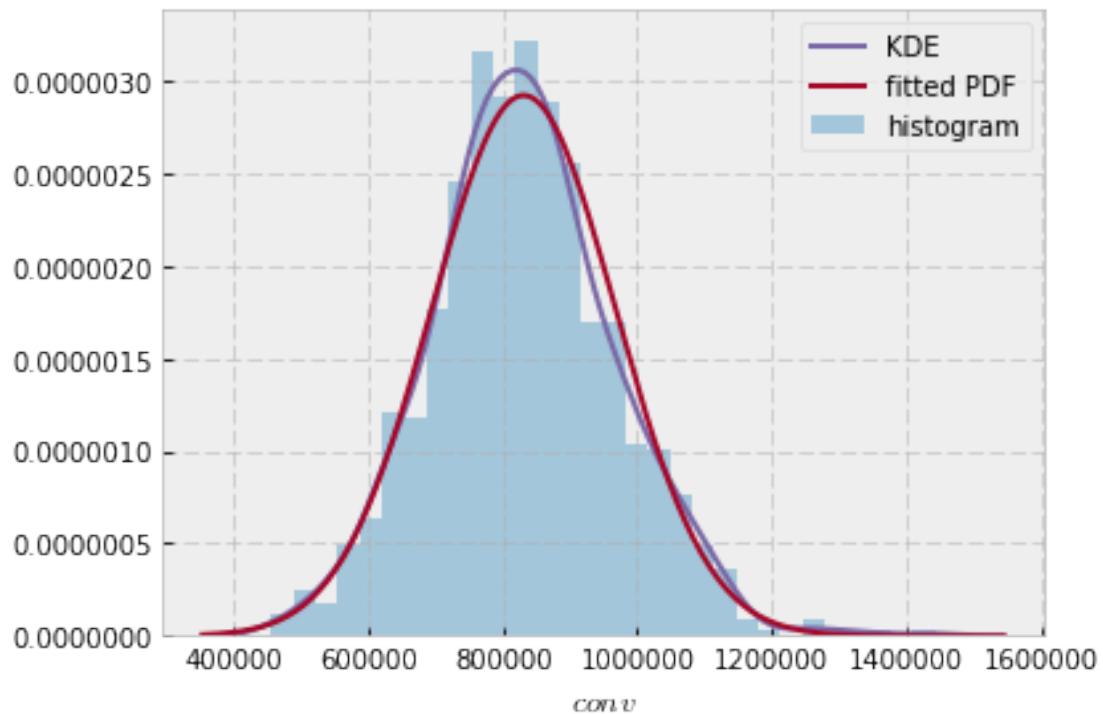


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(), loc=673217.2827369001, scale=134787.10983327928)
DescribeResult(nobs=1000, minmax=(256850.1686, 1064546.773)
               mean=673217.28273690015, variance=18185750727.936428,
               skewness=0.06305142906665993, kurtosis=-0.1944775395930698)
```

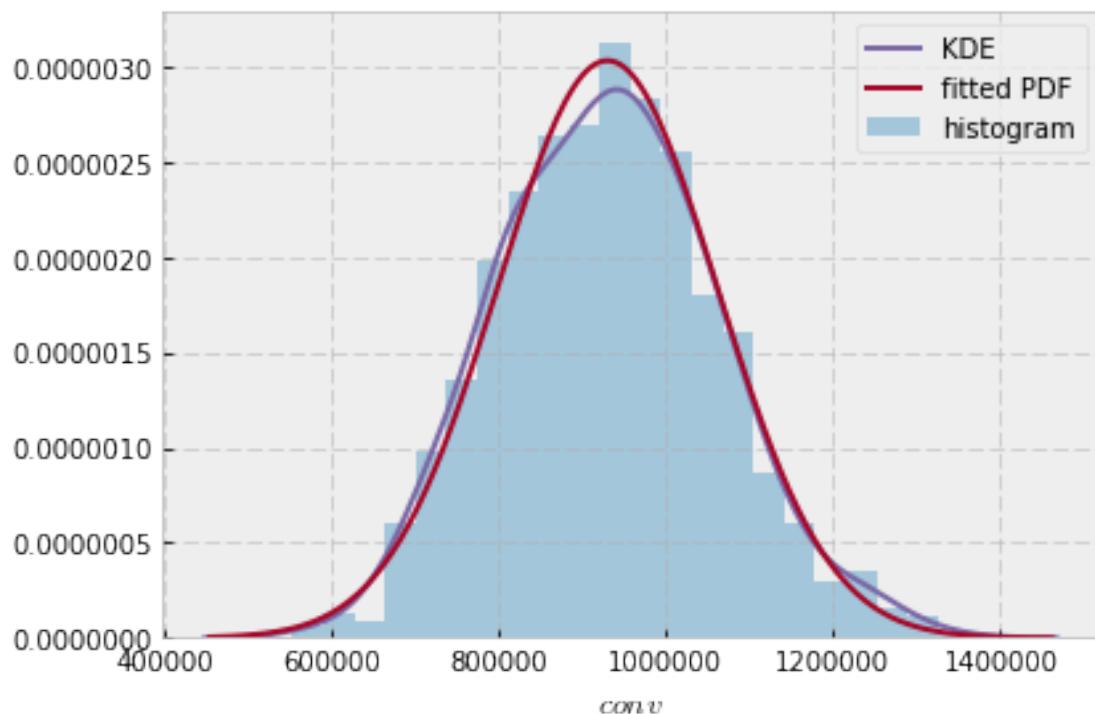


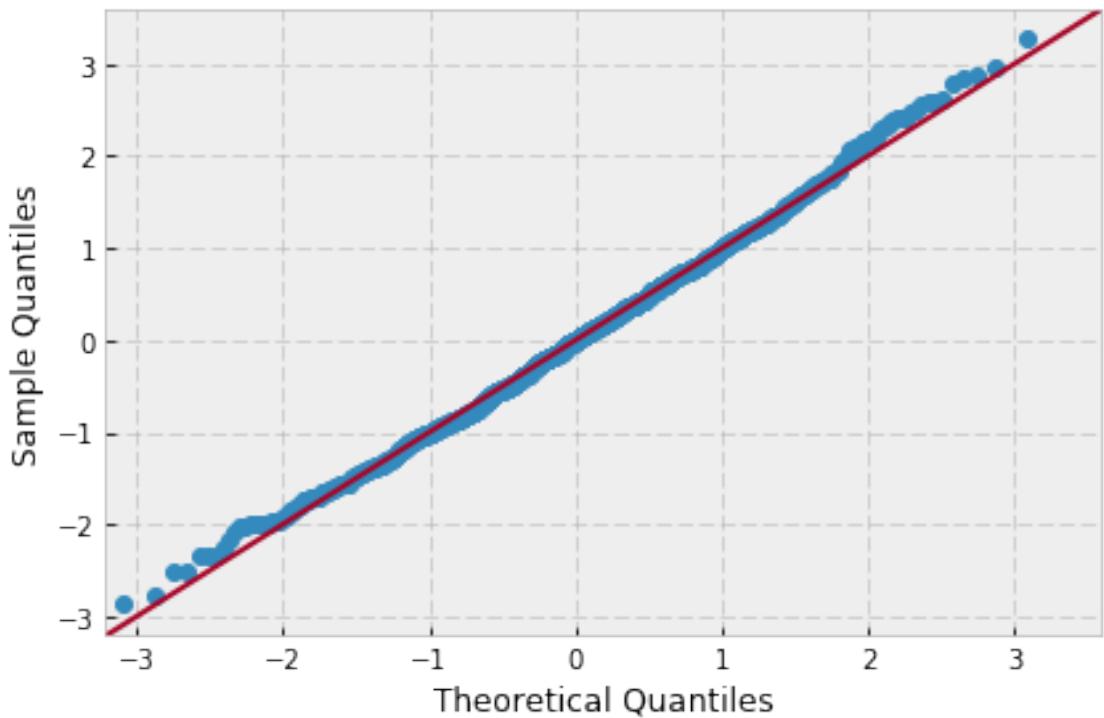


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(),loc=830972.0054825001, scale=136449.3689947833)
DescribeResult(nobs=1000, minmax=(456283.99070000002, 1442641.7549999999)
               mean=830972.00548250007, variance=18637067366.440964,
               skewness=0.27027871328610165, kurtosis=0.5432195137097606)
```

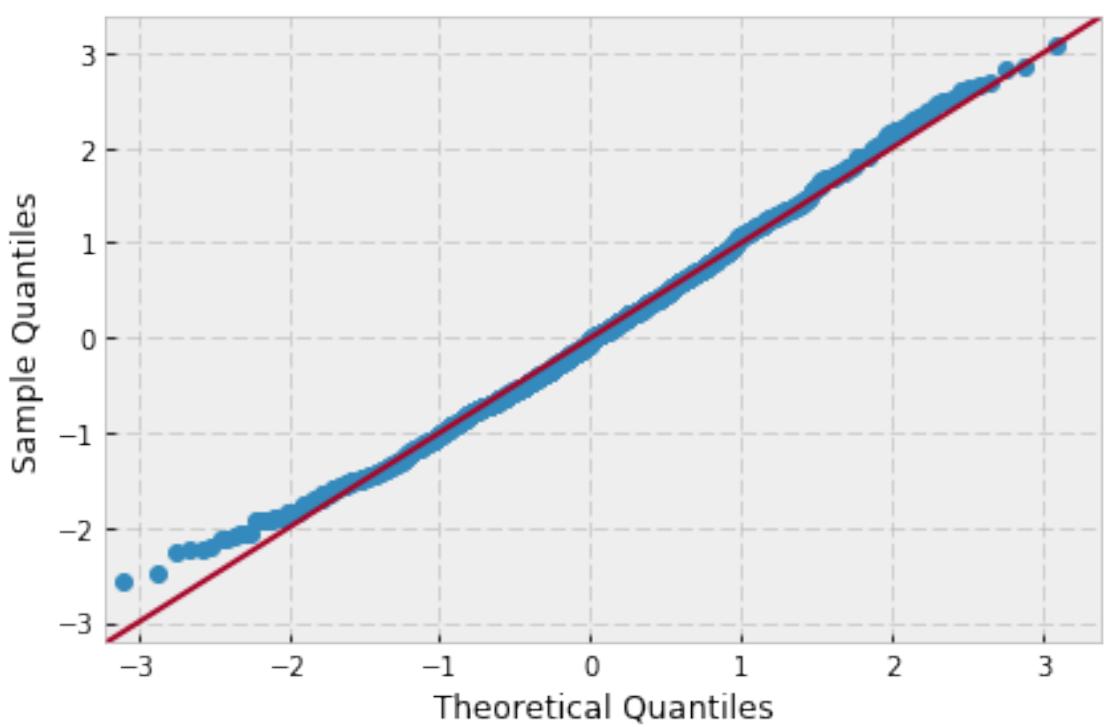
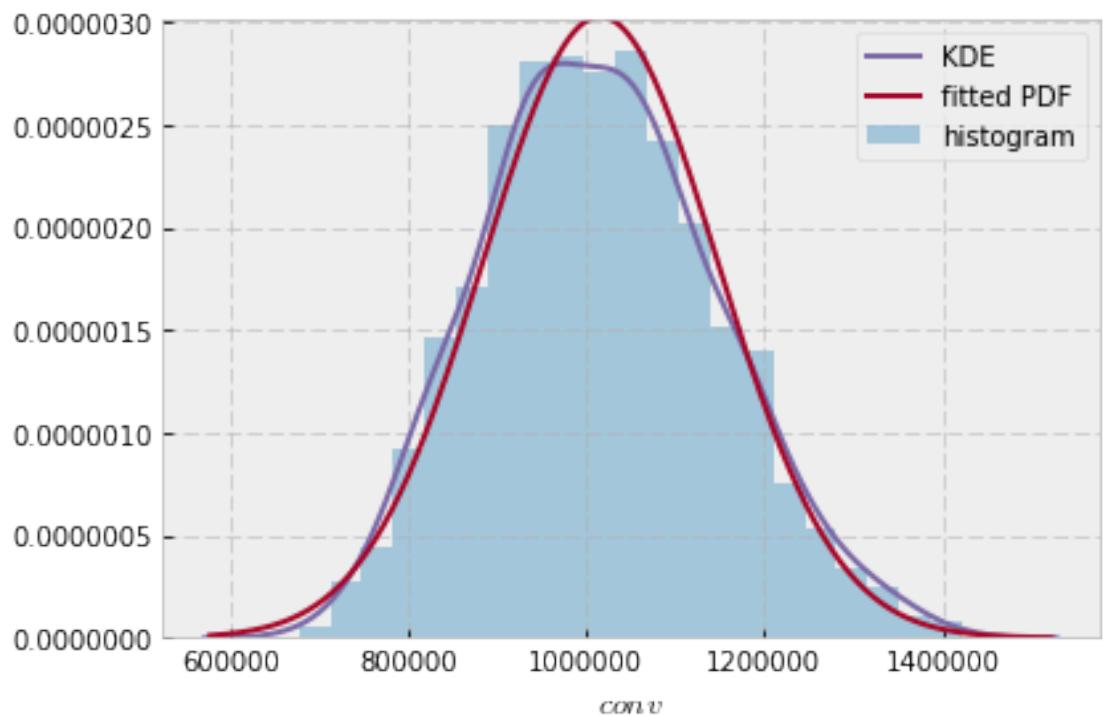


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(), loc=929958.864188, scale=131573.10656357638)
DescribeResult(nobs=1000, minmax=(553605.52339999995, 1361742.4269999999)
               mean=929958.86418799998, variance=17328811181.972202,
               skewness=0.16626964136508887, kurtosis=-0.0909816971214572)
```

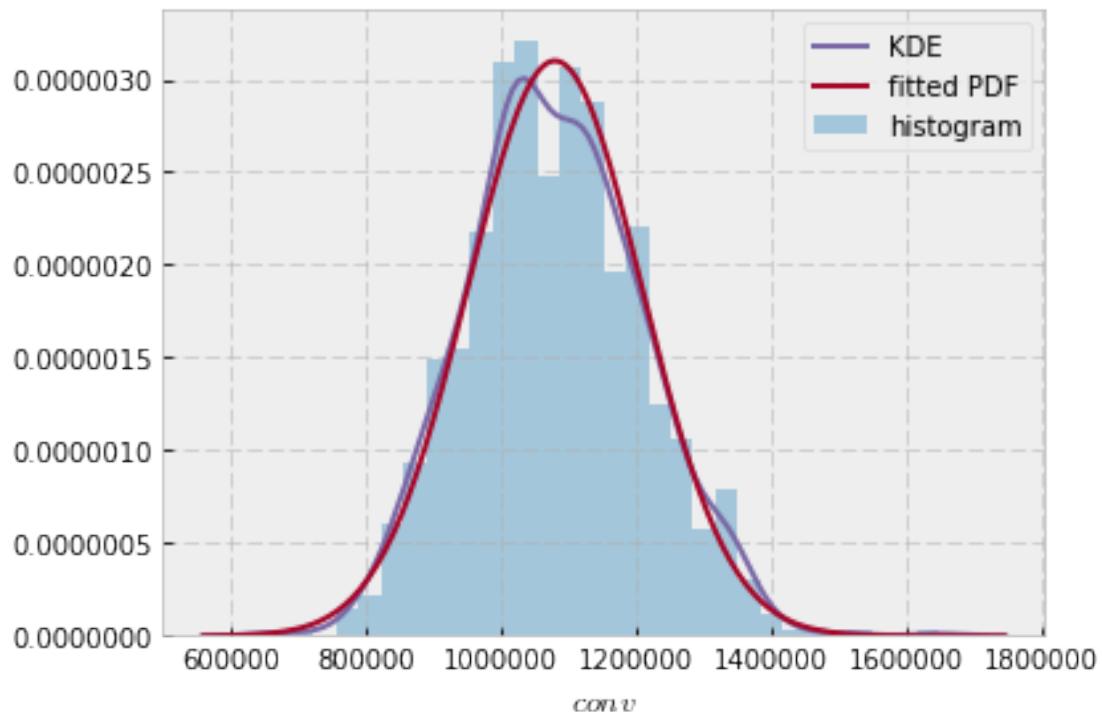


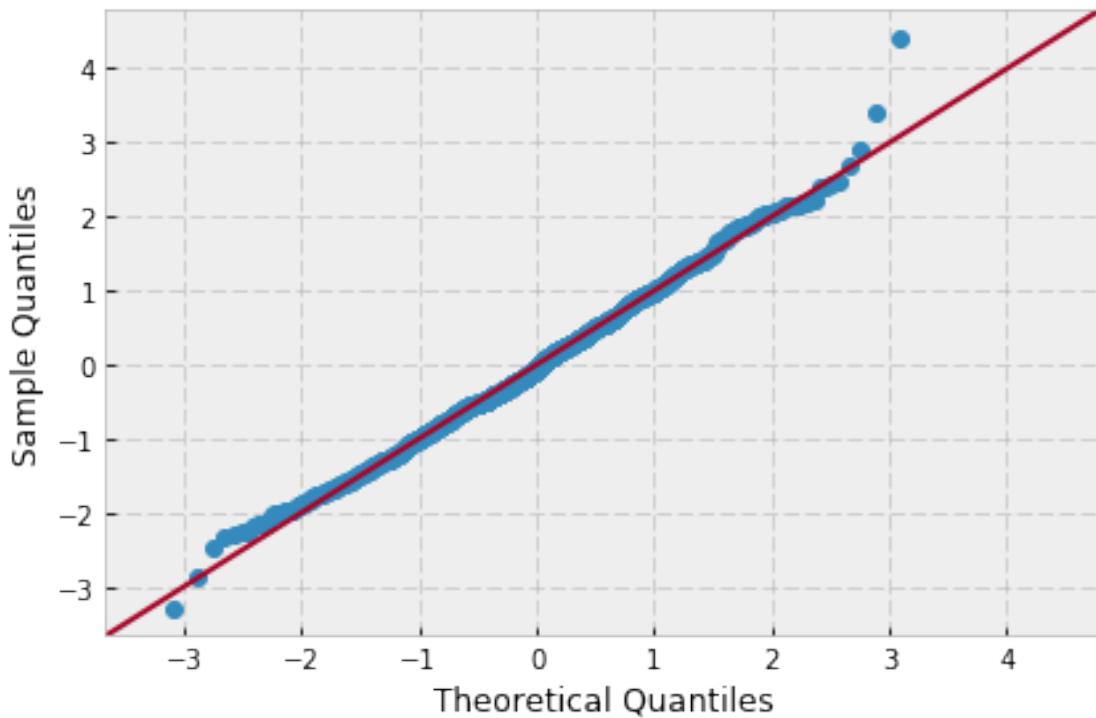


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(),loc=1015523.4947795001, scale=131665.62652654198)
DescribeResult(nobs=1000, minmax=(676500.5145999999, 1422924.818)
               mean=1015523.4947795001, variance=17353190399.025856,
               skewness=0.2307113192709296, kurtosis=-0.2608331314635377)
```

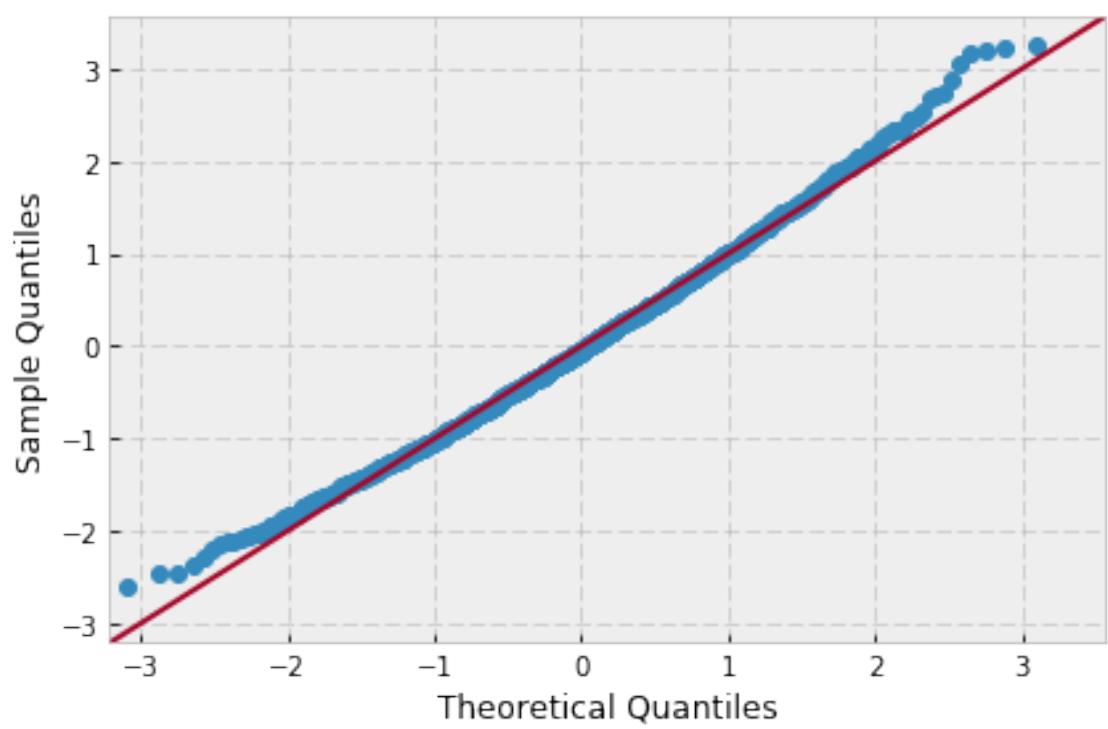
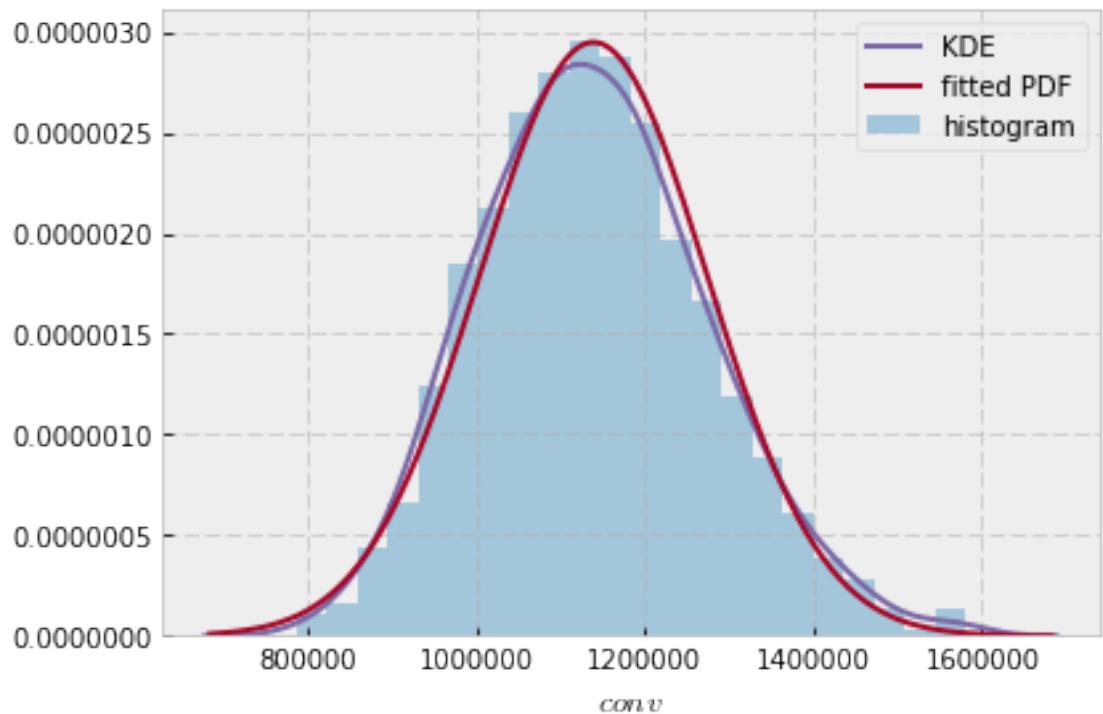


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(), loc=1080650.7421215, scale=128696.45735385464)
DescribeResult(nobs=1000, minmax=(658217.4264, 1647913.000999999)
               mean=1080650.7421215, variance=16579357492.925453,
               skewness=0.1924786002638292, kurtosis=0.07846495742118265)
```

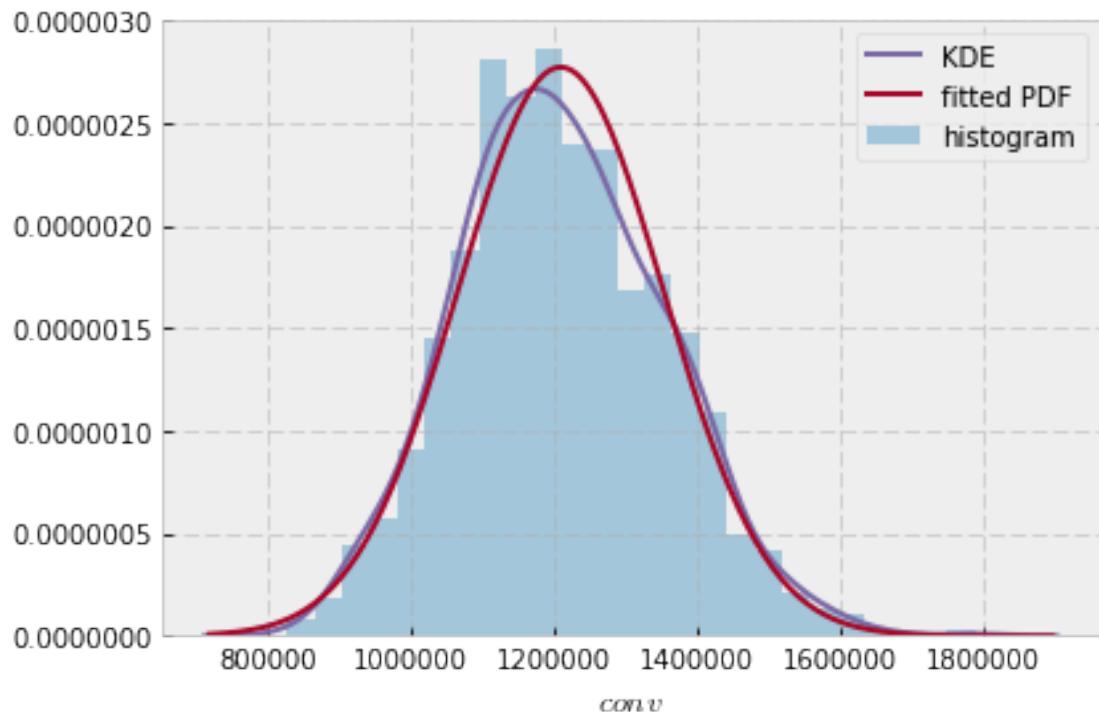


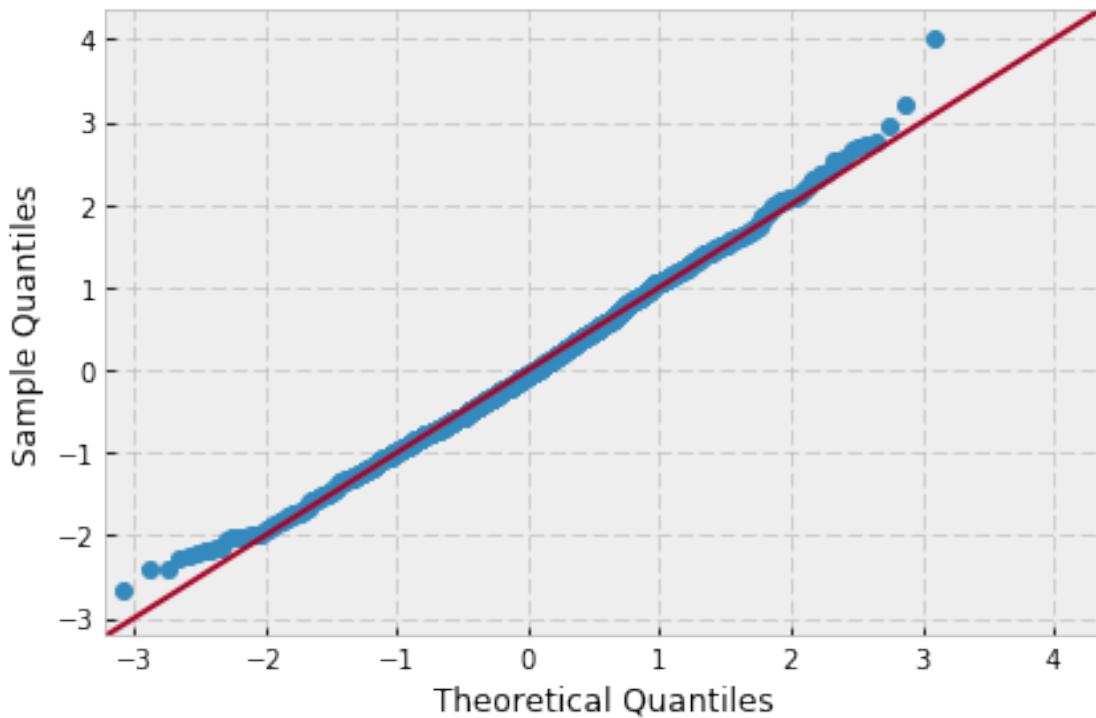


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(),loc=1138581.4527681002, scale=135275.2992819908)
DescribeResult(nobs=1000, minmax=(785340.38280000002, 1580333.548)
               mean=1138581.4527681002, variance=18317724320.152328,
               skewness=0.2979683177505183, kurtosis=0.007297104140776334)
```

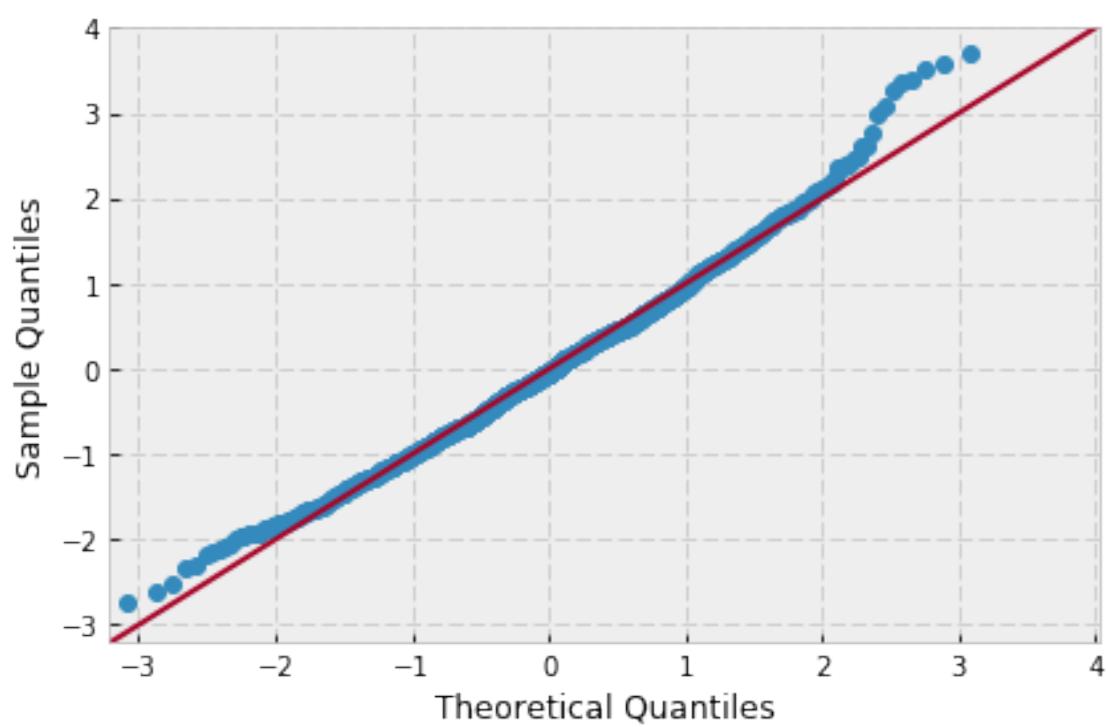
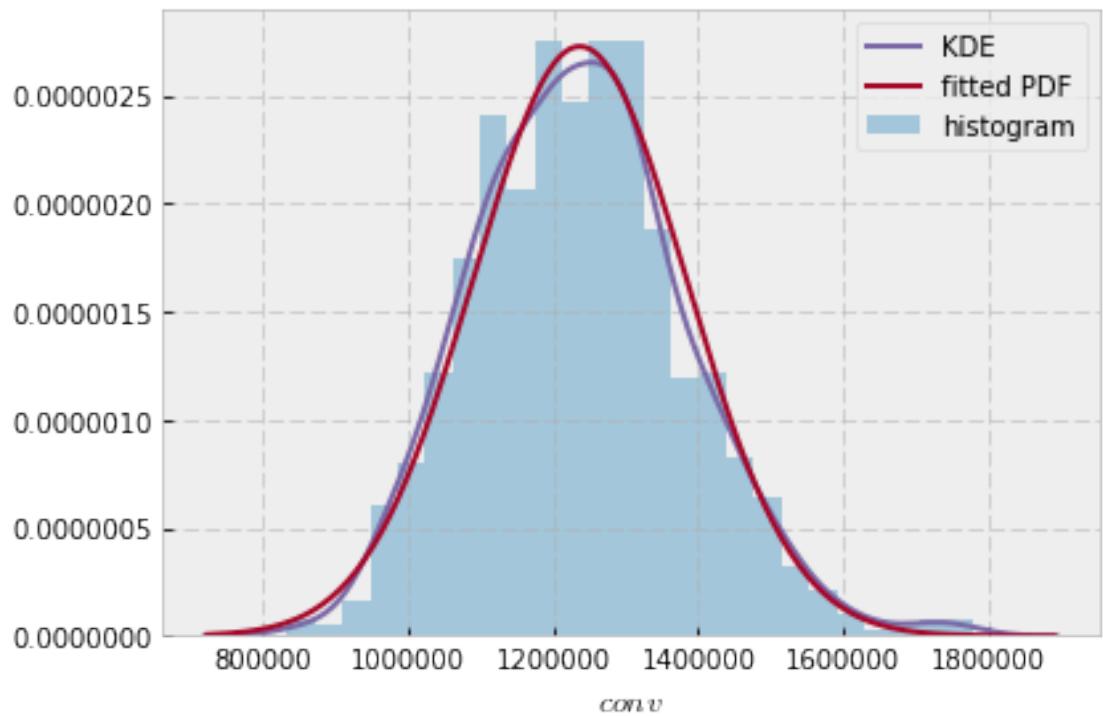


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(), loc=1209545.9841411, scale=143850.3852602835)
DescribeResult(nobs=1000, minmax=(827041.8993999999, 1787739.993)
               mean=1209545.9841411, variance=20713646986.518509,
               skewness=0.24314803845305522, kurtosis=-0.04346023012953193)
```

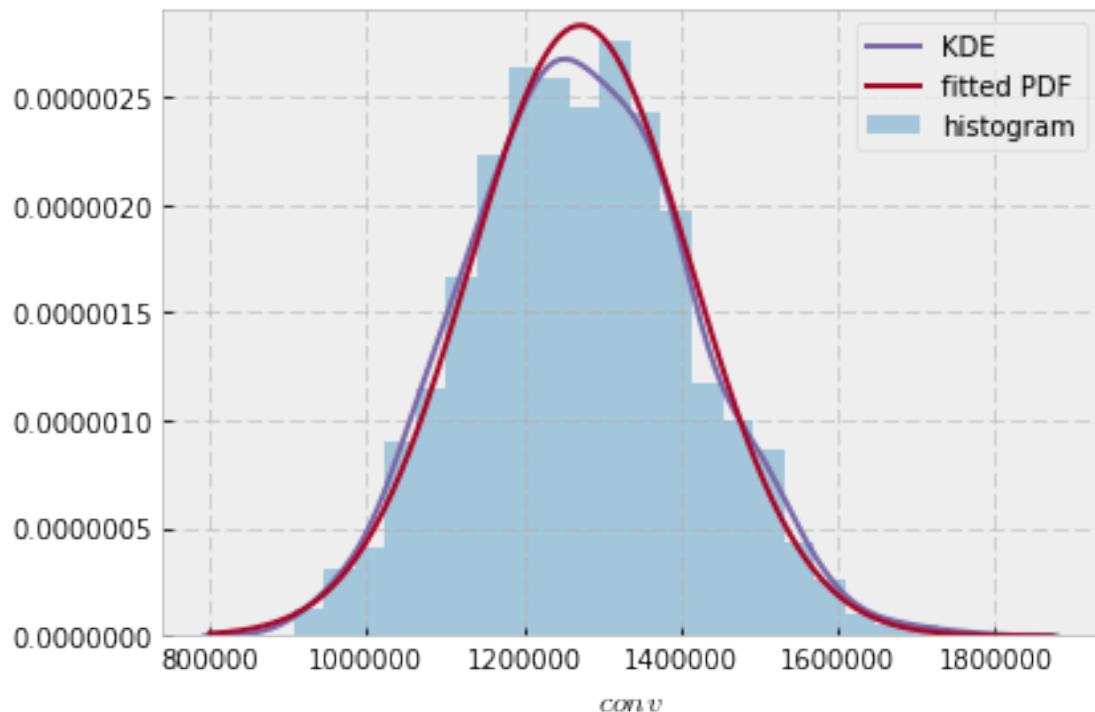


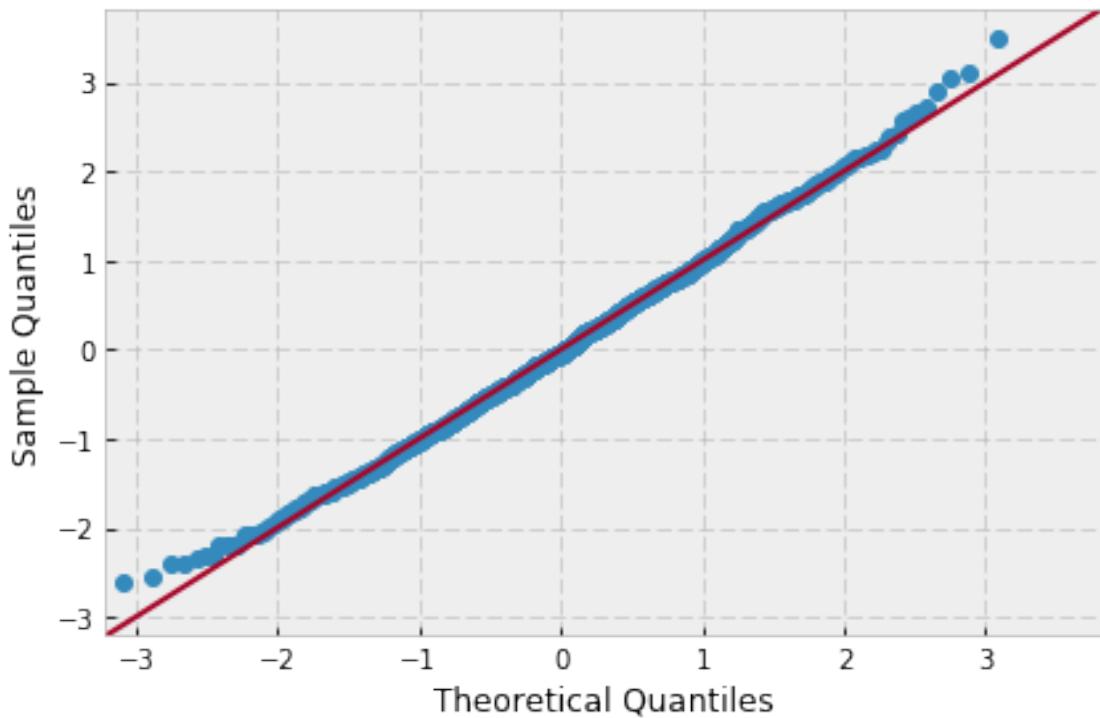


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(),loc=1236887.0064407, scale=146355.89116826386)
DescribeResult(nobs=1000, minmax=(835198.5808999999, 1778444.686999999)
               mean=1236887.0064407, variance=21441488368.024719,
               skewness=0.34914806358093503, kurtosis=0.3192321014646149)
```

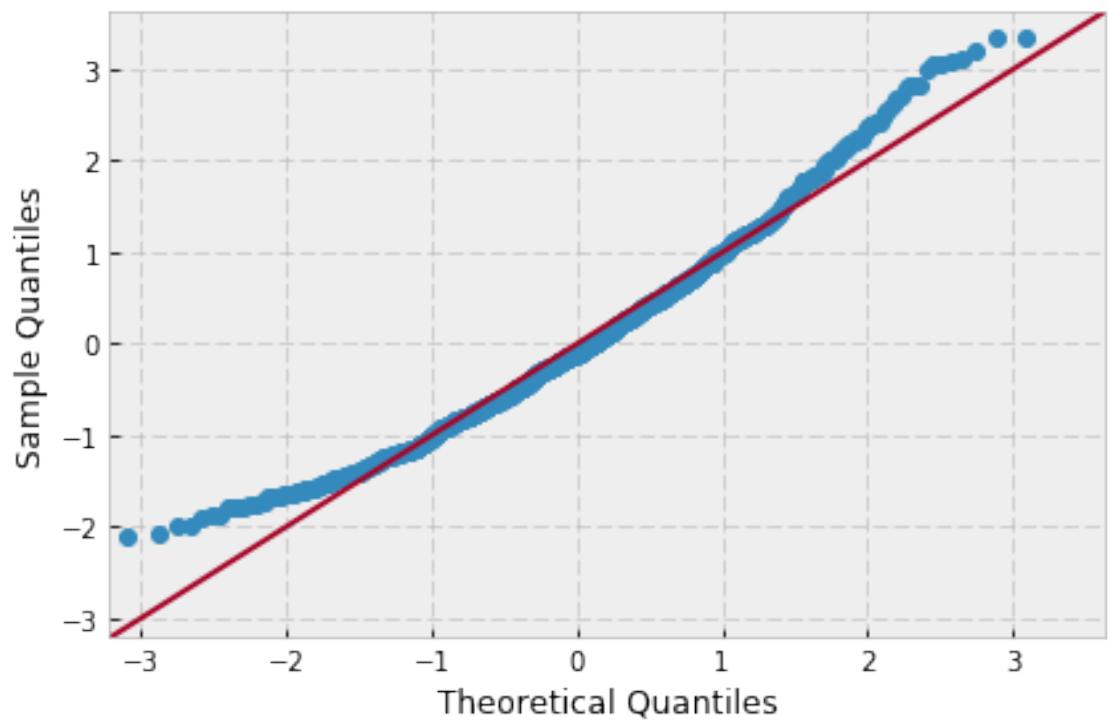
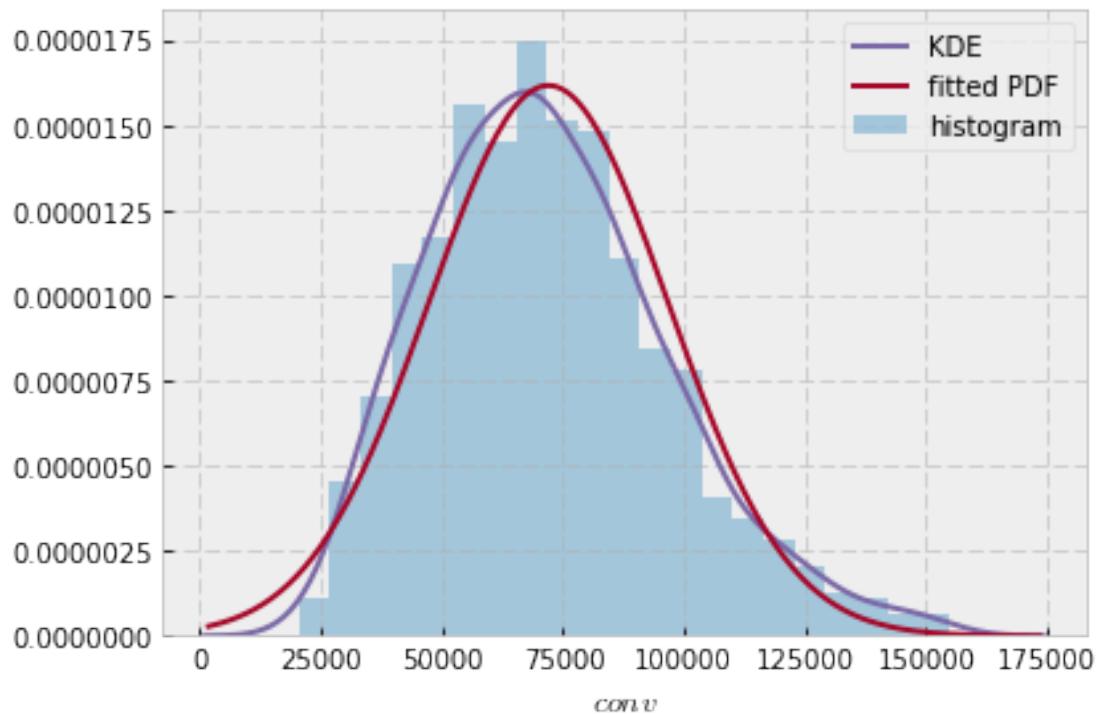


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(), loc=1272471.2107255, scale=140851.03513770364)
DescribeResult(nobs=1000, minmax=(906773.96990000003, 1765829.29)
               mean=1272471.2107255, variance=19858872972.334965,
               skewness=0.16374302251022493, kurtosis=-0.14422378805198832)
```

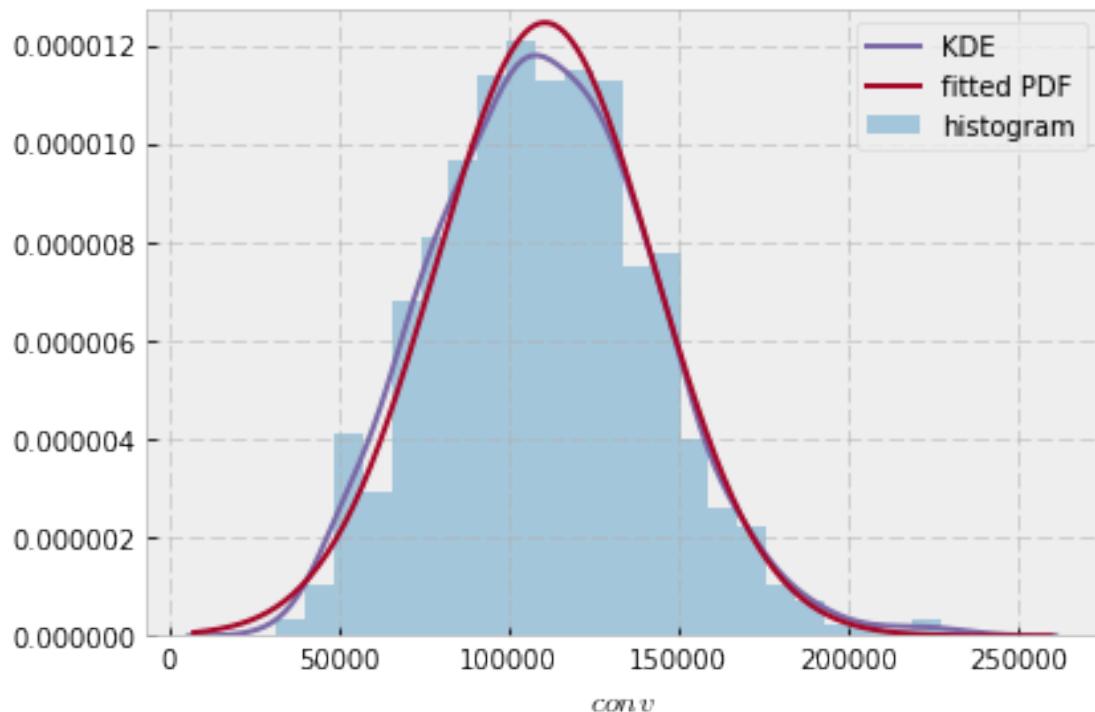


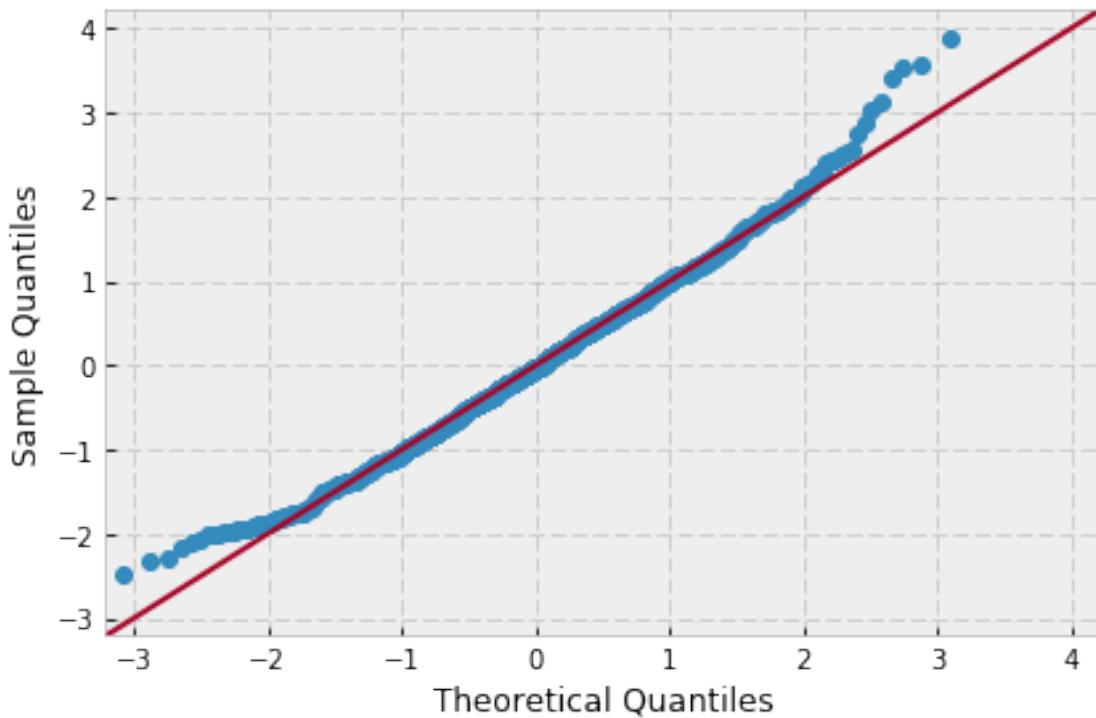


```
variable name: r
variable value: 1.0
distribution: normal(shape=(),loc=71953.96866771001, scale=24686.739728622724)
DescribeResult(nobs=1000, minmax=(20378.284810000001, 154768.9191)
               mean=71953.968667710011, variance=610045163.59235191,
               skewness=0.5643227169879185, kurtosis=0.19009478671950264)
```

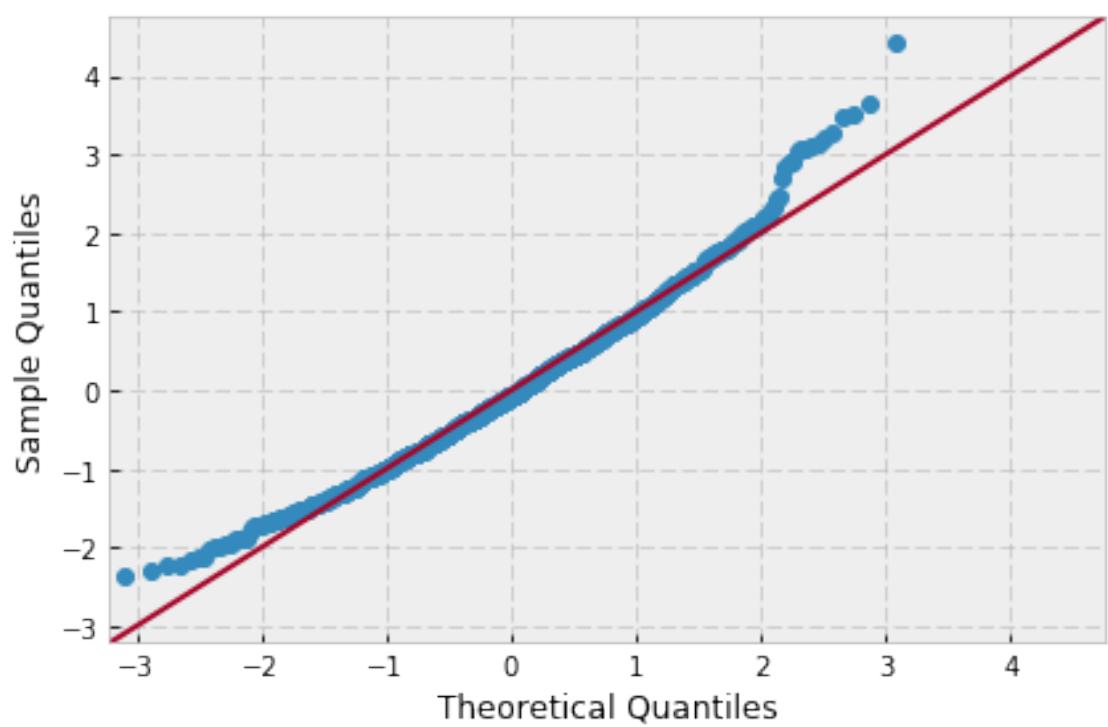
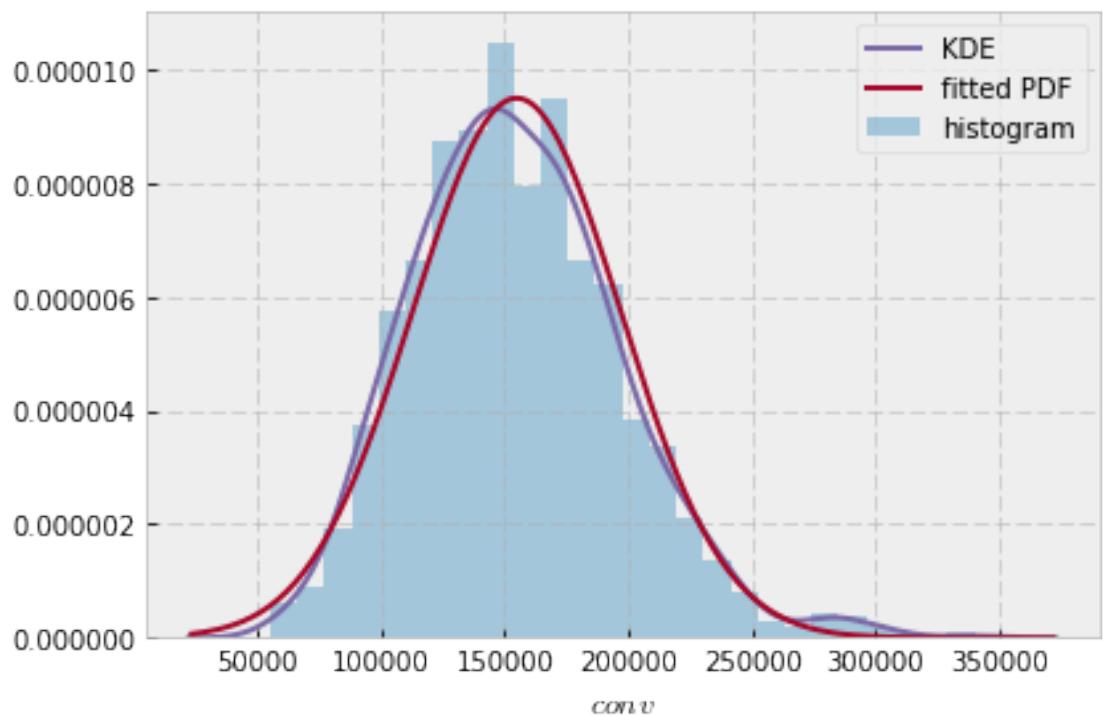


```
variable name: r
variable value: 1.22222222222223
distribution: normal(shape=(), loc=110677.27159736001, scale=32034.84393583683)
DescribeResult(nobs=1000, minmax=(31382.269090000002, 235601.8665)
               mean=110677.27159736001, variance=1027258484.4778996,
               skewness=0.30245145451960903, kurtosis=0.15870271906577793)
```

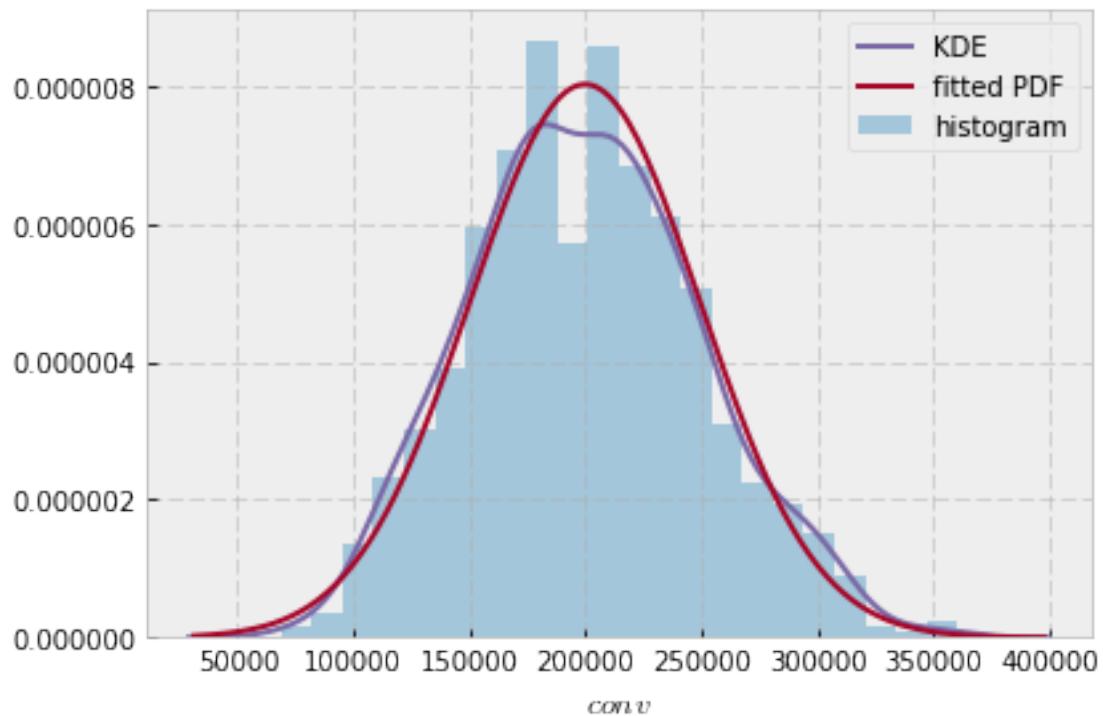


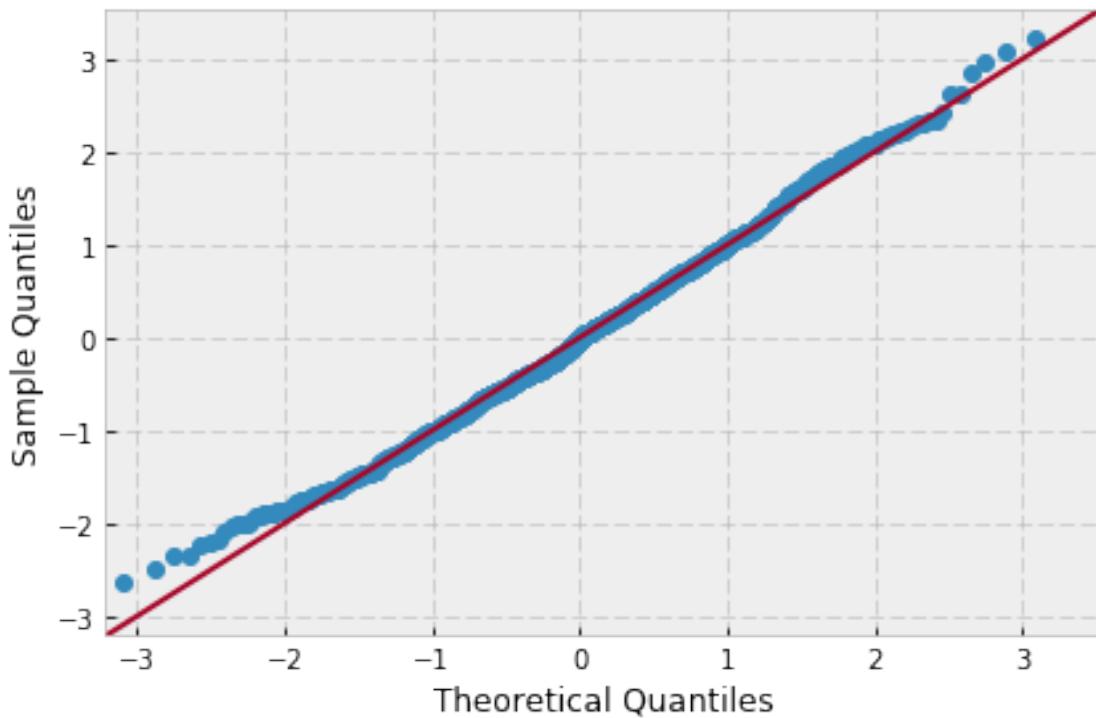


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(),loc=154740.98993698, scale=42025.20351681824)
DescribeResult(nobs=1000, minmax=(54858.680639999999, 340075.59590000001)
               mean=154740.98993698001, variance=1767885616.2462385,
               skewness=0.5329125666339823, kurtosis=0.633425463759755)
```

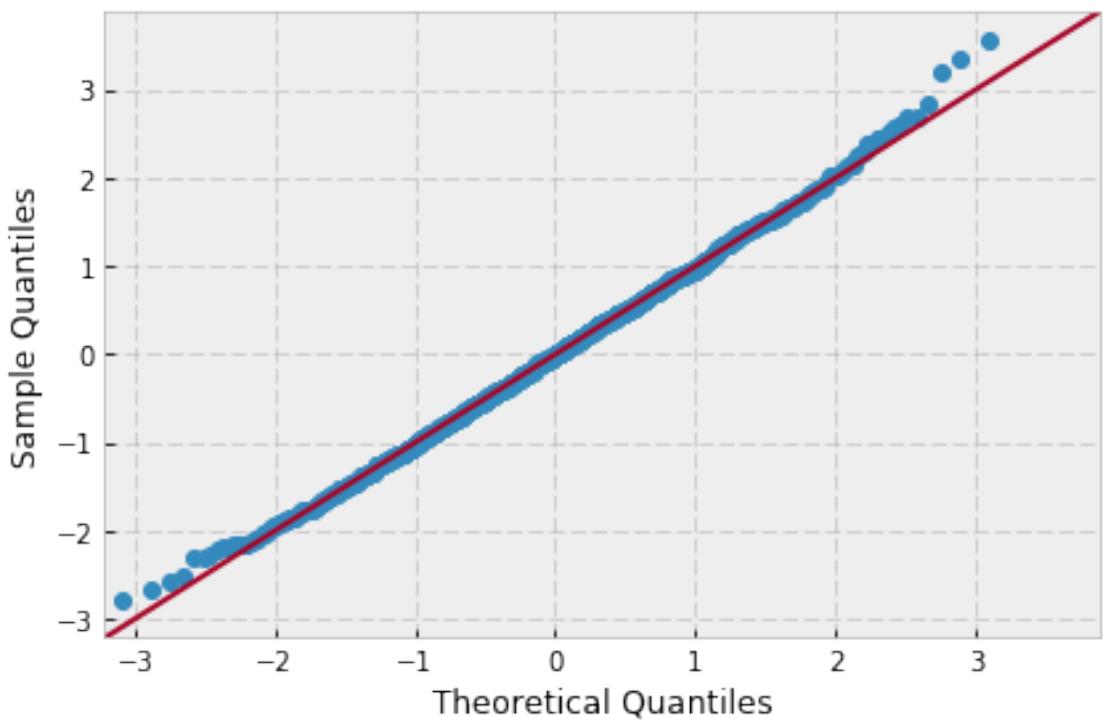
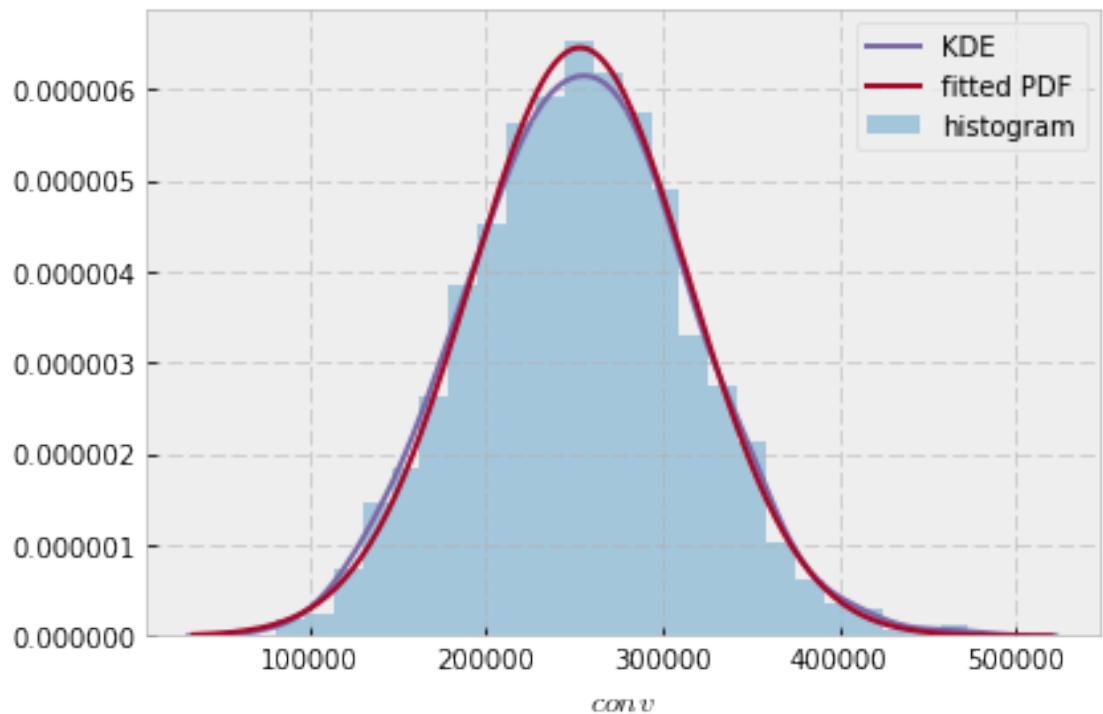


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=199799.08903824002, scale=49696.35358252225)
DescribeResult(nobs=1000, minmax=(68585.520990000005, 360333.6060999998)
               mean=199799.08903824002, variance=2472199759.1582298,
               skewness=0.2084562765772152, kurtosis=-0.2291917116856279)
```

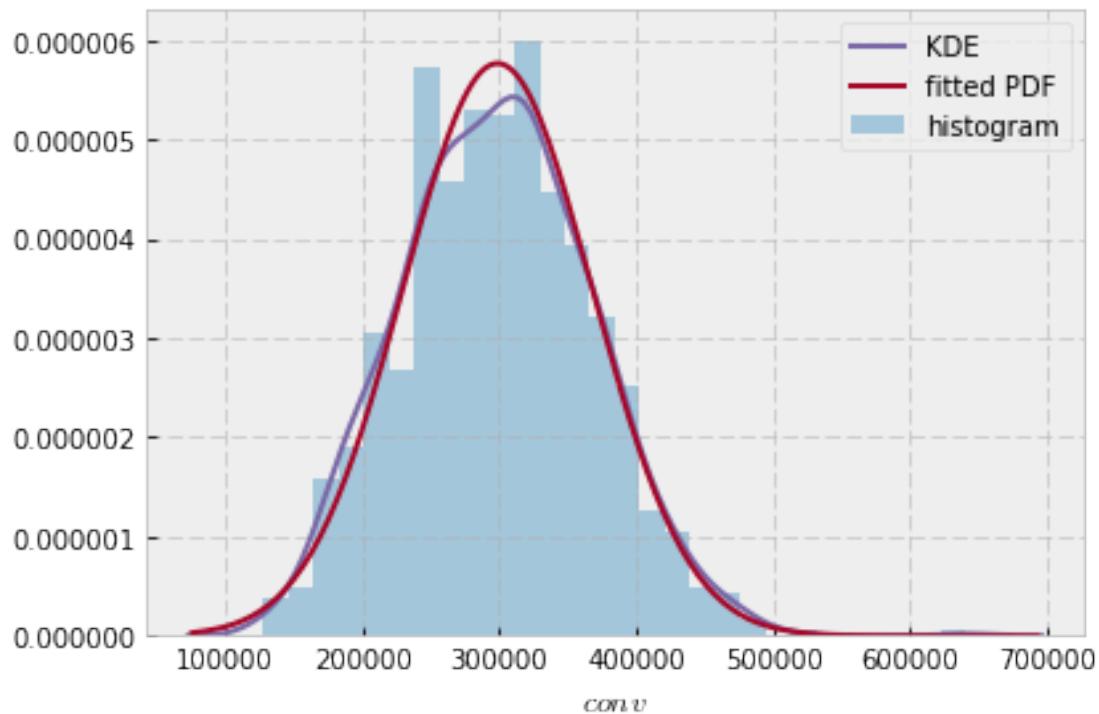


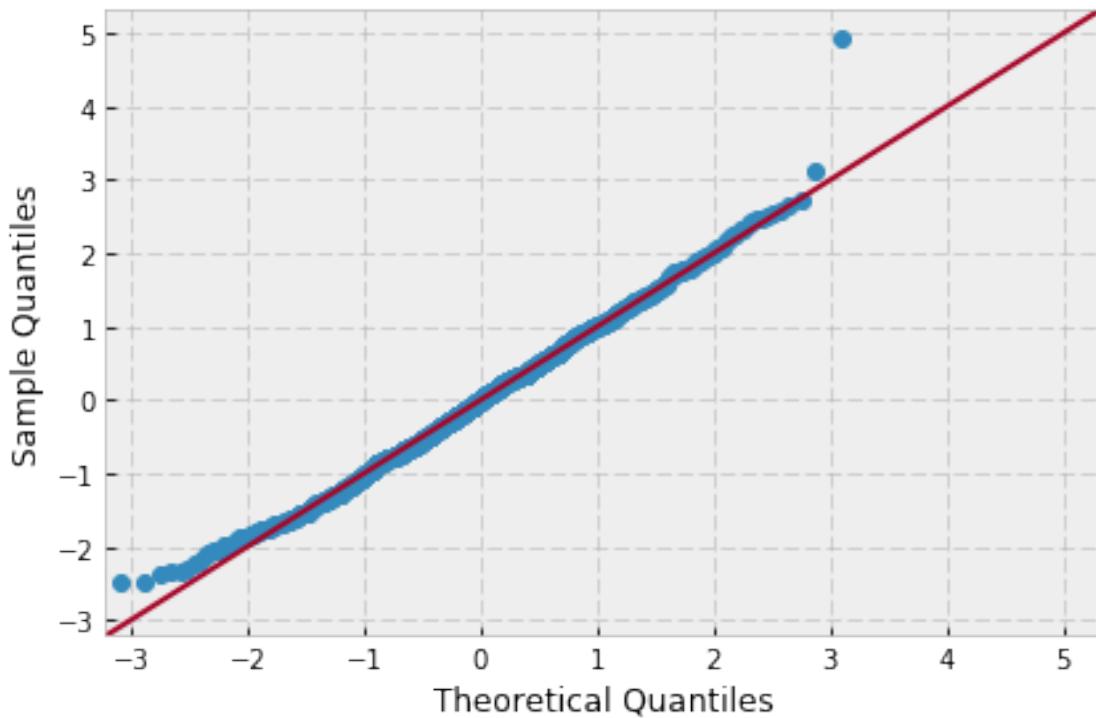


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(),loc=253147.82558320998, scale=61779.66280428897)
DescribeResult(nobs=1000, minmax=(80766.888260000007, 473191.98700000002)
               mean=253147.82558320998, variance=3820547283.495141,
               skewness=0.11914893259991571, kurtosis=-0.03768702976584937)
```

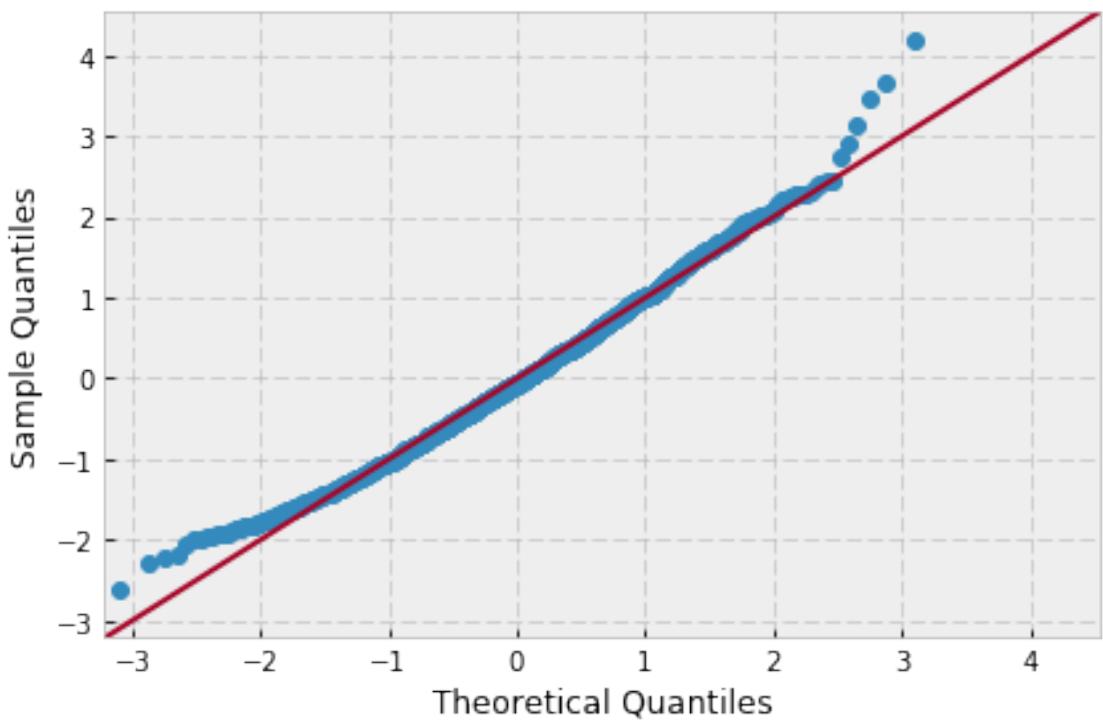
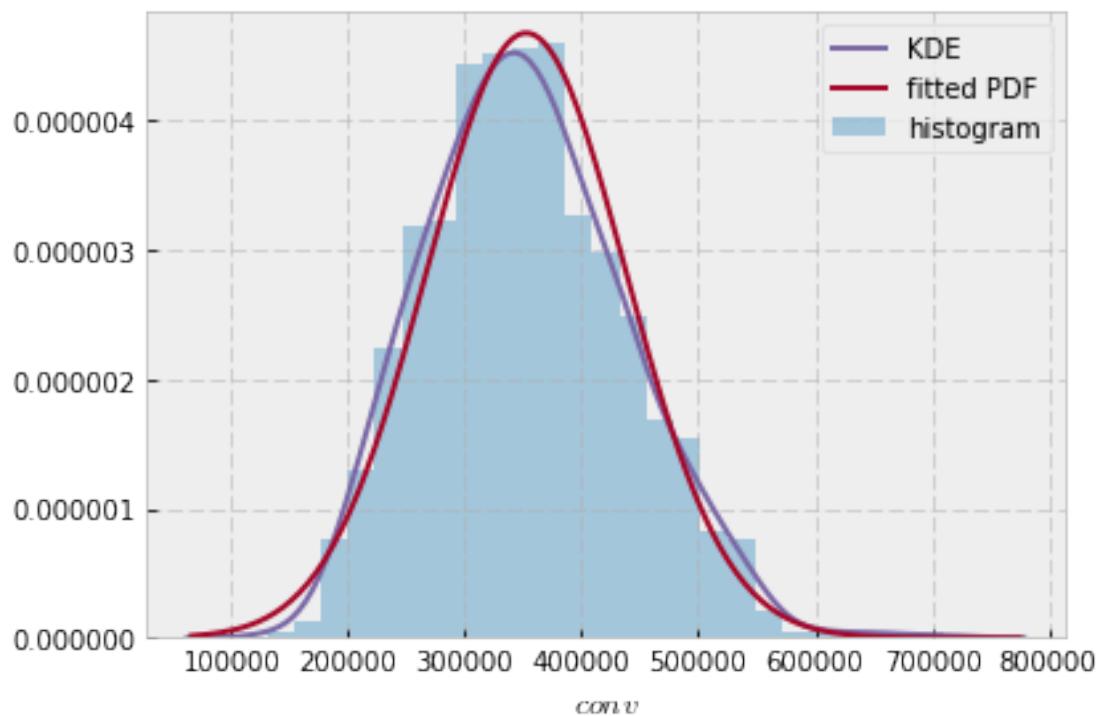


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=298528.80682490004, scale=69188.92658019639)
DescribeResult(nobs=1000, minmax=(127736.73910000001, 640770.04550000001)
               mean=298528.80682490004, variance=4791899460.7805862,
               skewness=0.2089285125102461, kurtosis=0.15026728963883418)
```

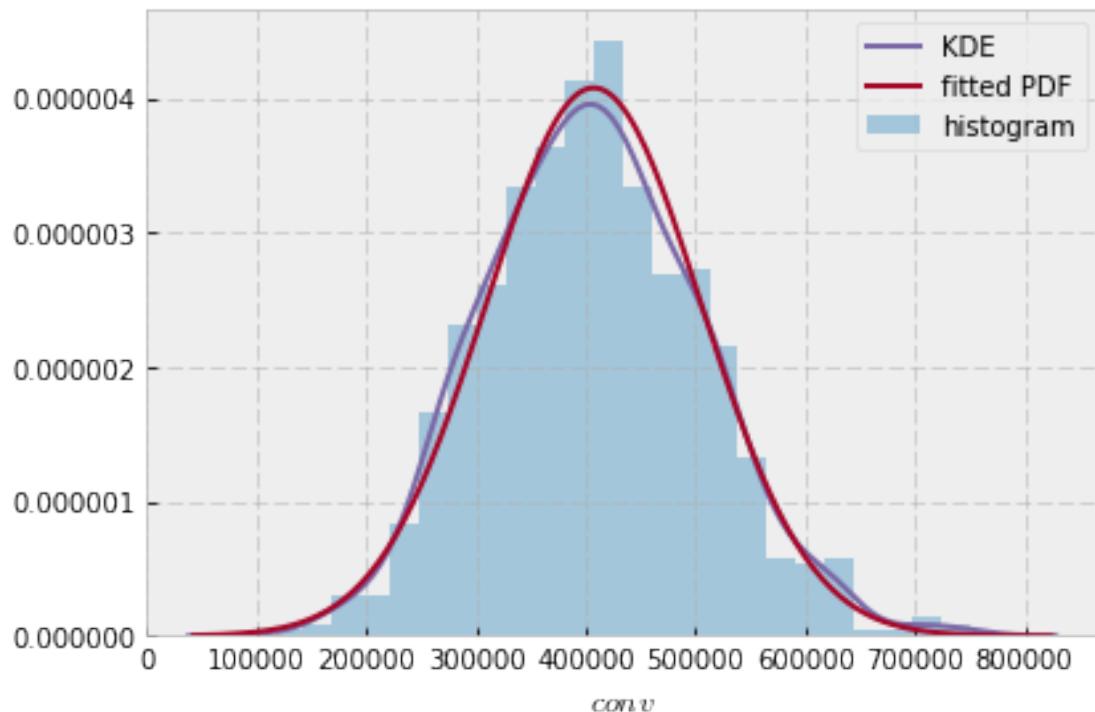


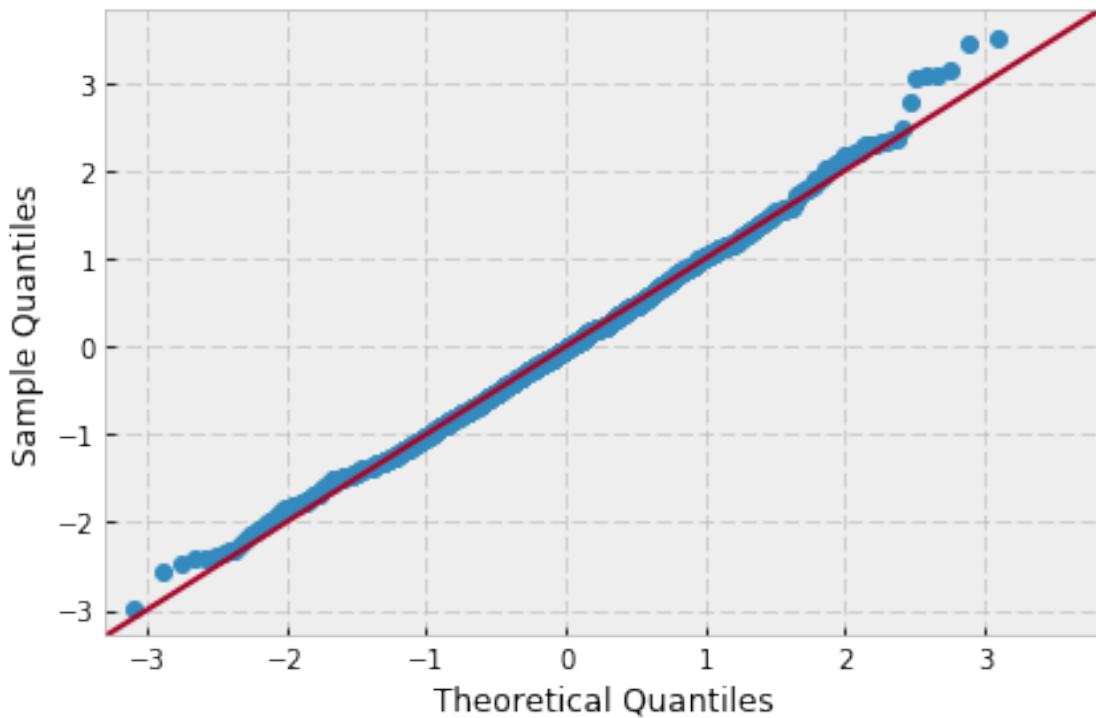


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(),loc=352424.3462058, scale=85297.97206474436)
DescribeResult(nobs=1000, minmax=(130148.24649999999, 710537.18240000005)
               mean=352424.34620580002, variance=7283027065.4233313,
               skewness=0.36760204451717987, kurtosis=0.07146622035924421)
```

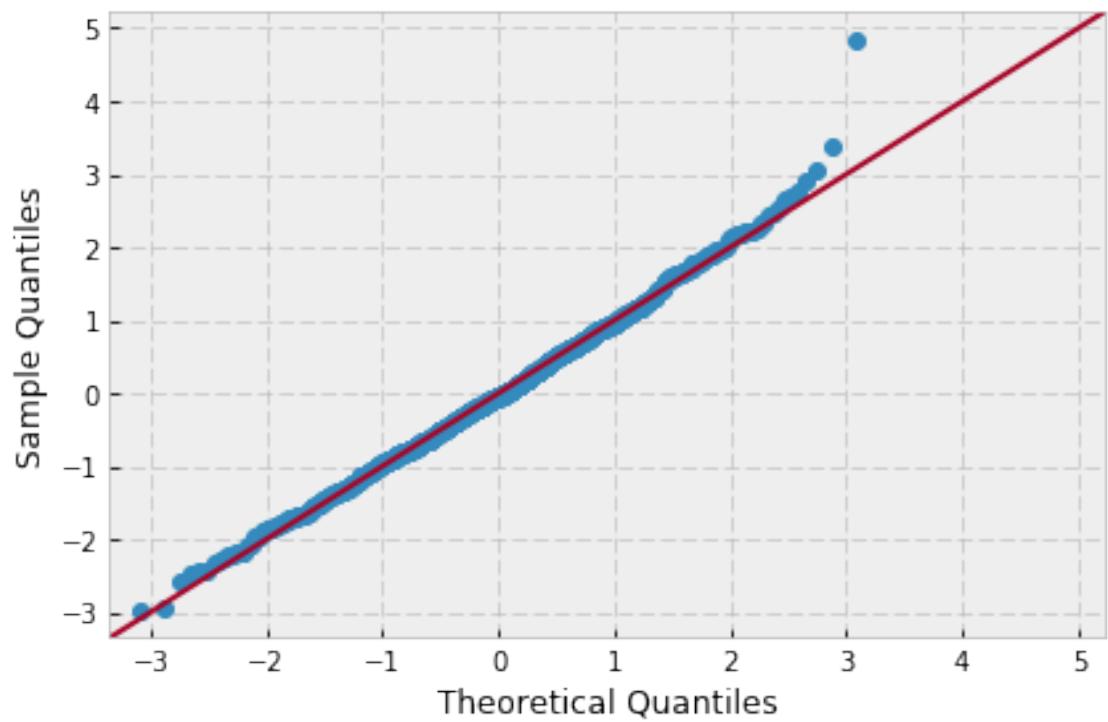
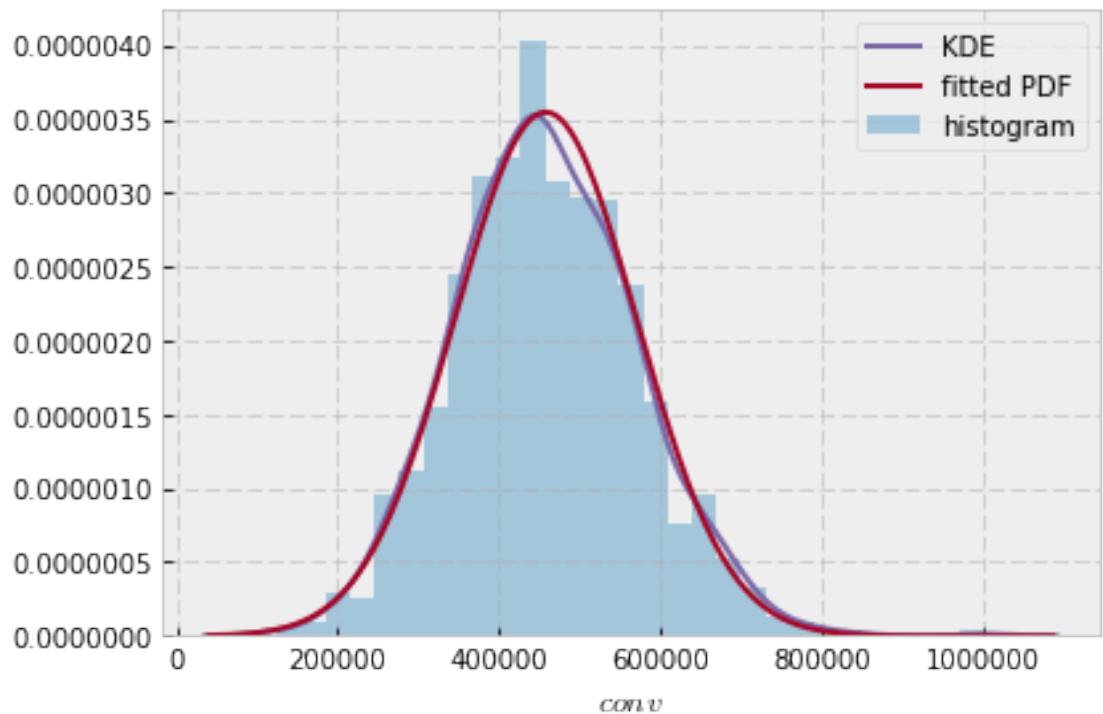


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(),loc=407294.7136827, scale=97887.05788278453)
DescribeResult(nobs=1000, minmax=(116504.94, 750024.1001999999)
               mean=407294.71368270001, variance=9591467568.5161247,
               skewness=0.21878148510466197, kurtosis=0.04746503754658038)
```

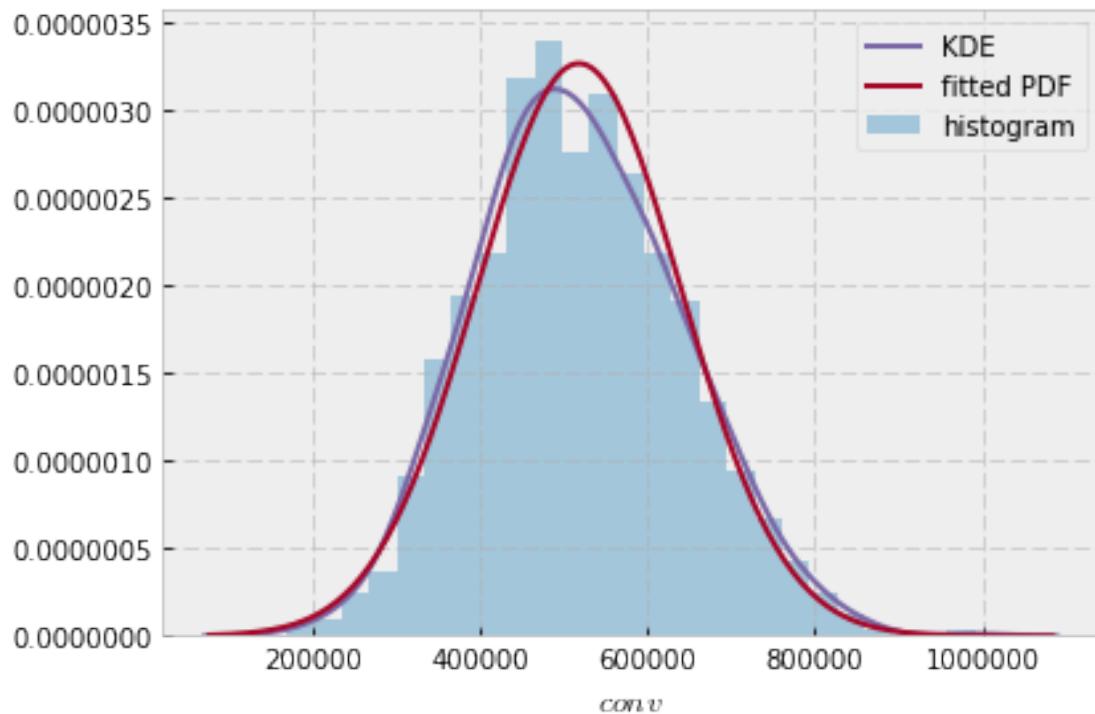


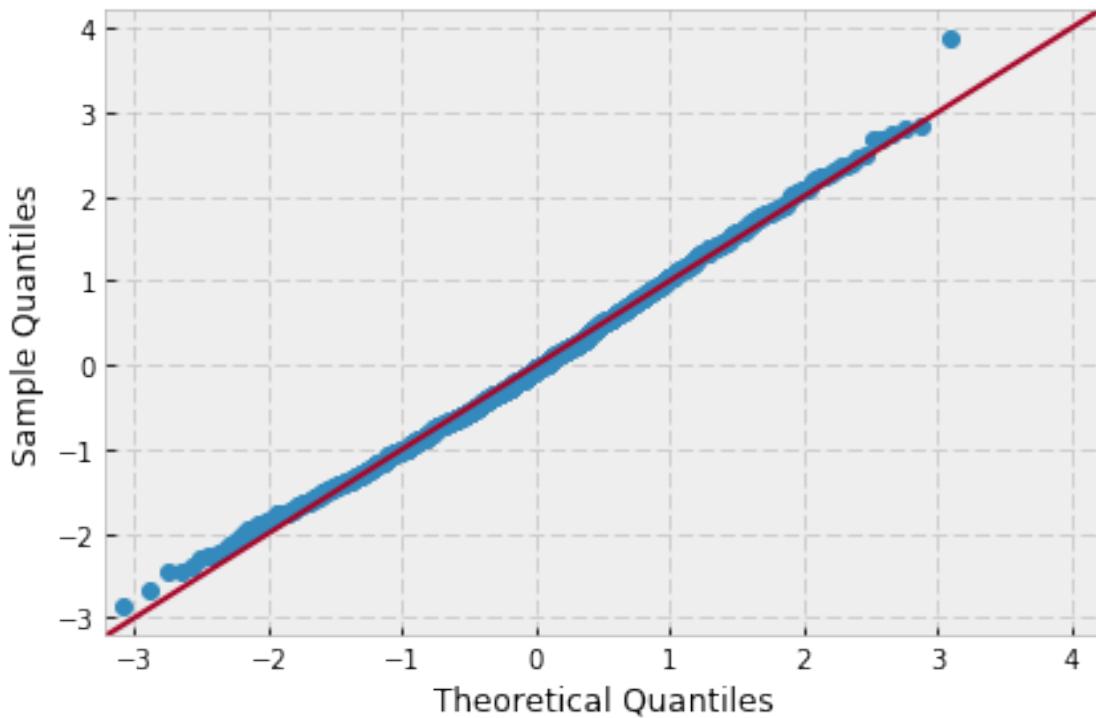


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(),loc=457472.8951913, scale=112473.37744279904)
DescribeResult(nobs=1000, minmax=(124498.3317, 1000879.329)
               mean=457472.89519130002, variance=12662923556.947285,
               skewness=0.23557302978873548, kurtosis=0.39077027106980866)
```

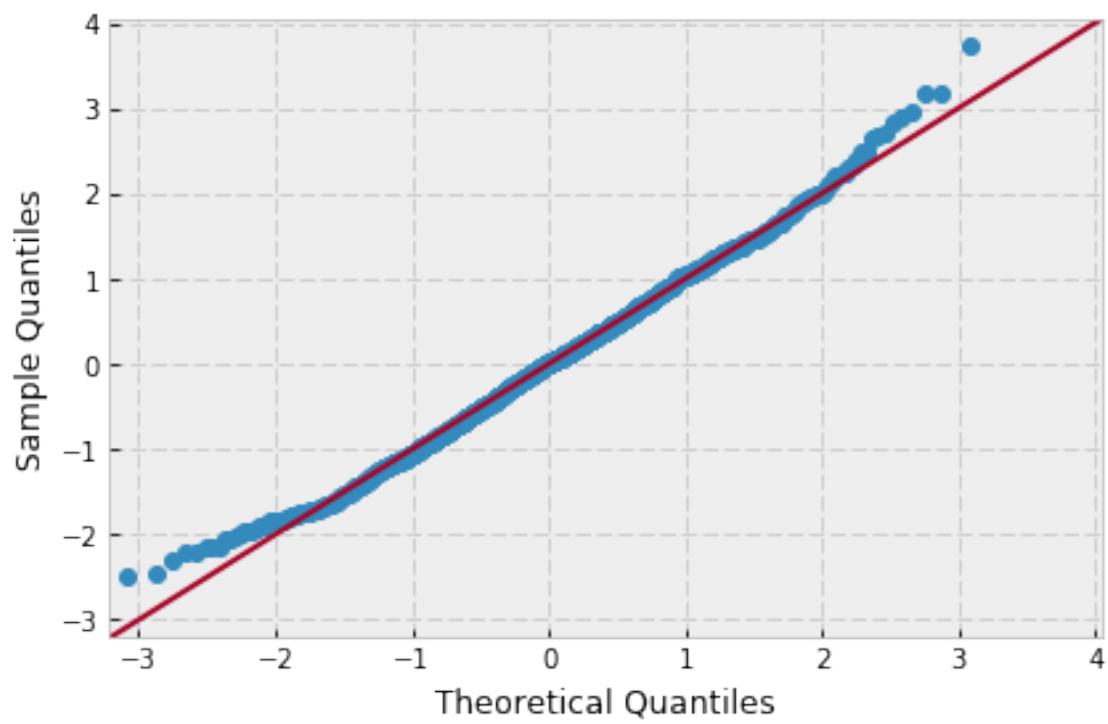
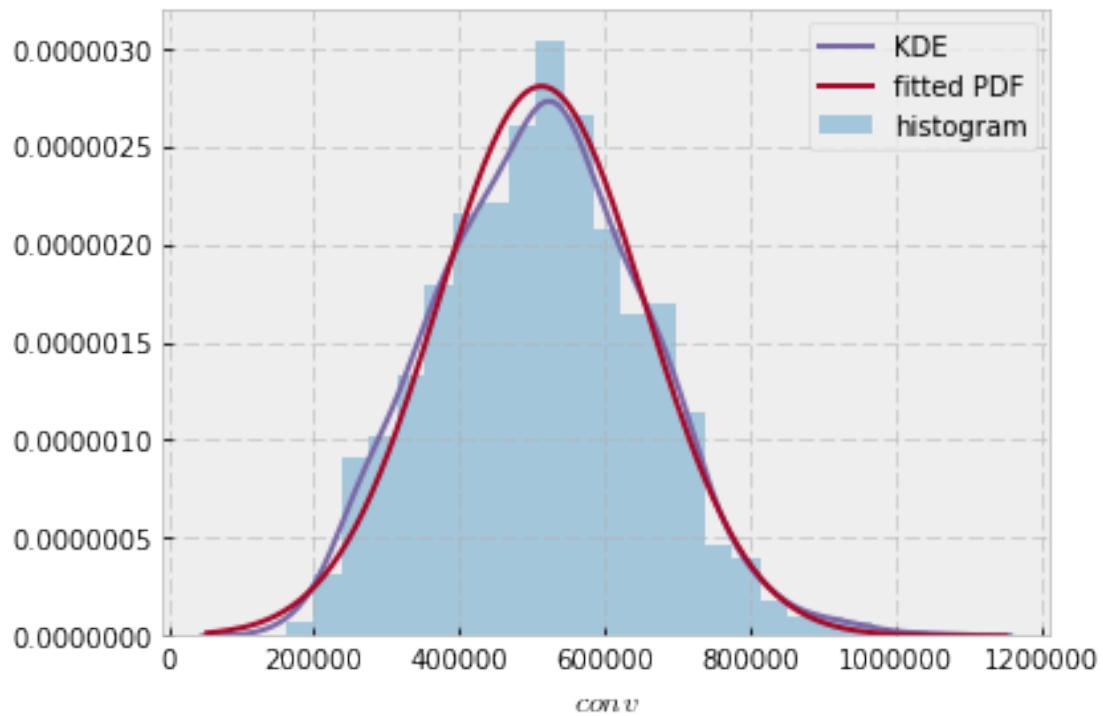


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=518031.91771280003, scale=122092.07069640115)
DescribeResult(nobs=1000, minmax=(168411.99189999999, 991325.055599999996)
               mean=518031.91771280003, variance=14921395122.057074,
               skewness=0.2122082586362381, kurtosis=-0.12248451890562384)
```

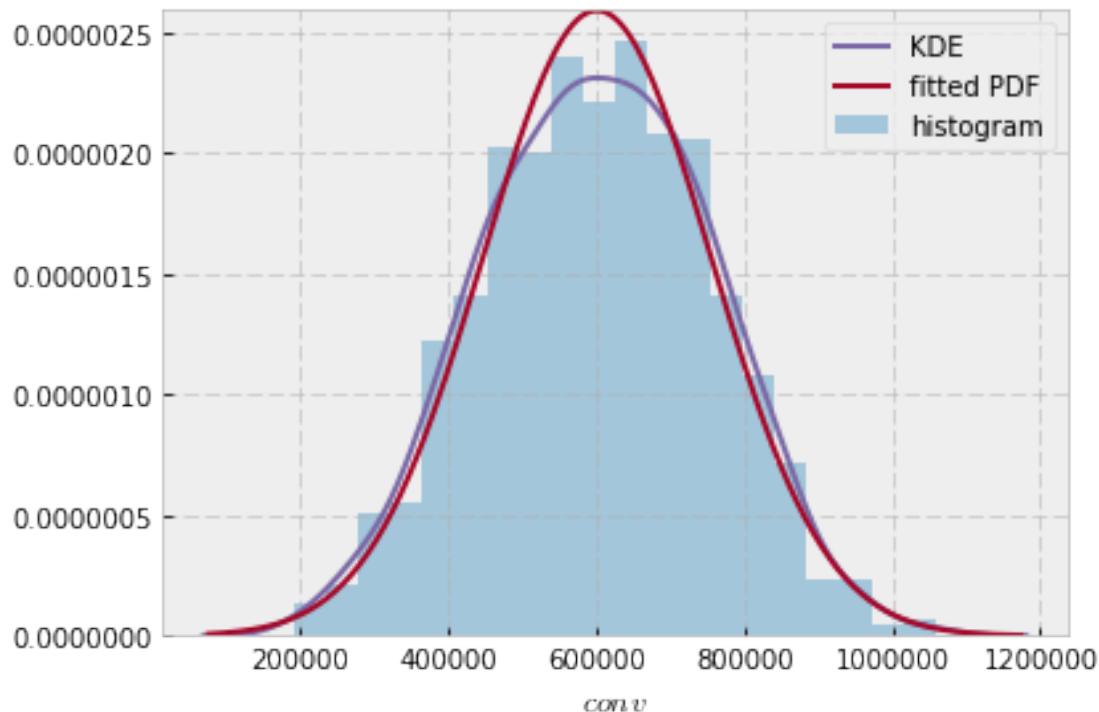


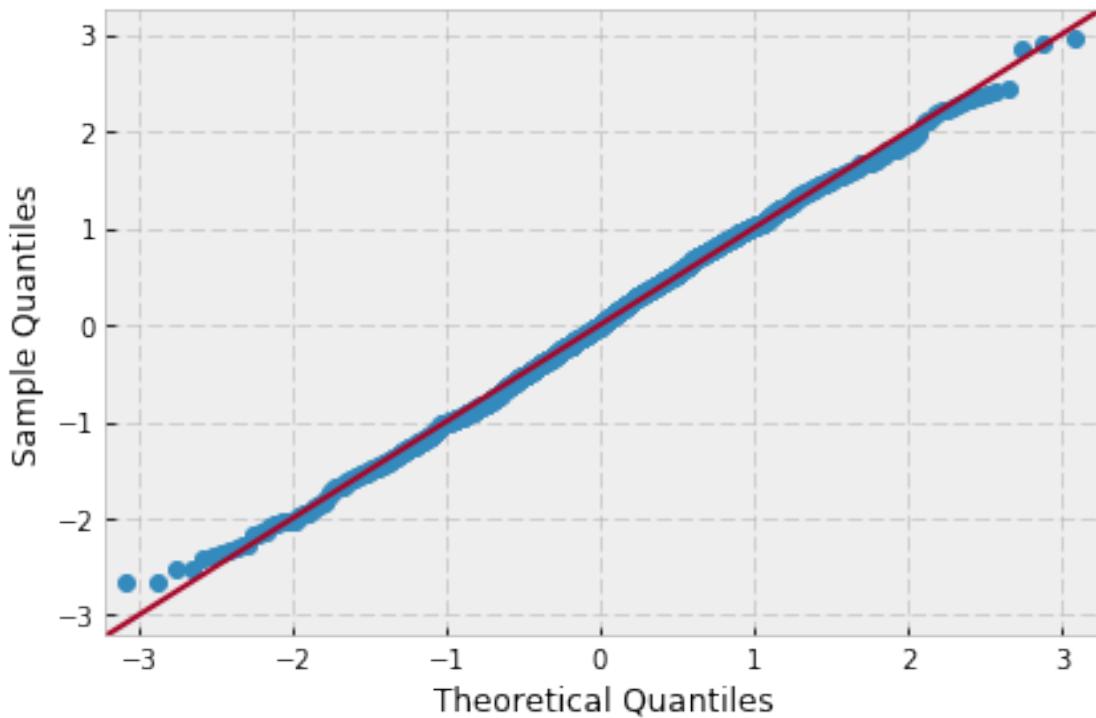


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(),loc=512766.83339419996, scale=142215.66931879084)
DescribeResult(nobs=1000, minmax=(160078.61859999999, 1044569.102)
               mean=512766.83339419996, variance=20245542141.933598,
               skewness=0.18458266585157482, kurtosis=-0.10055771947382519)
```

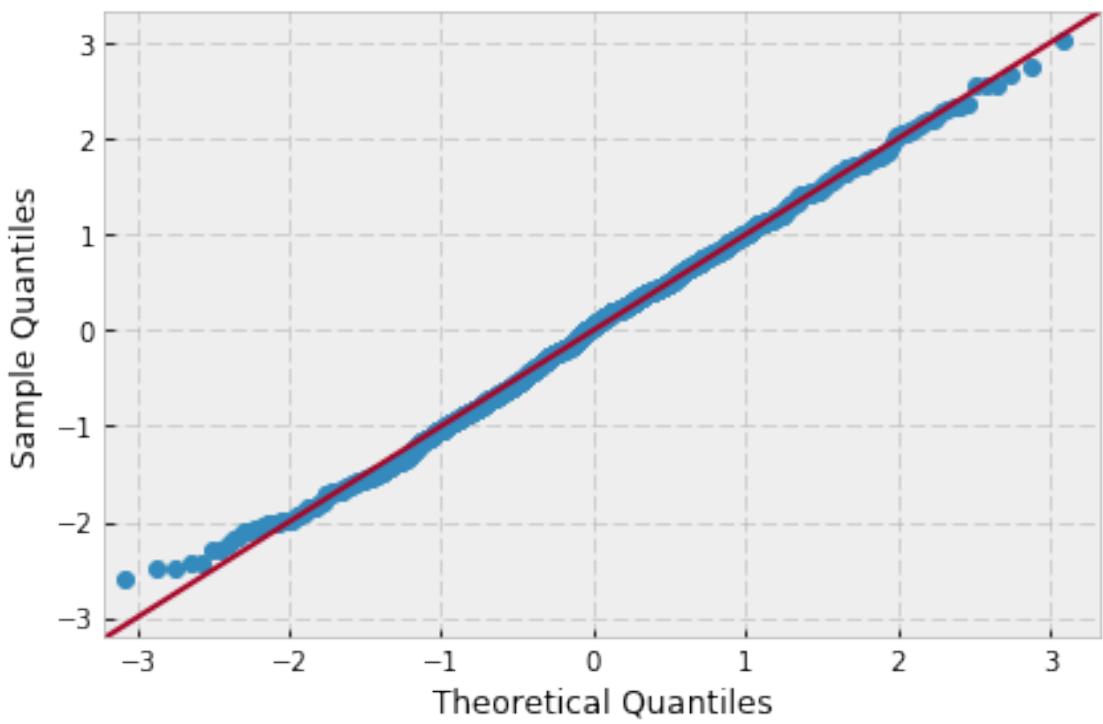
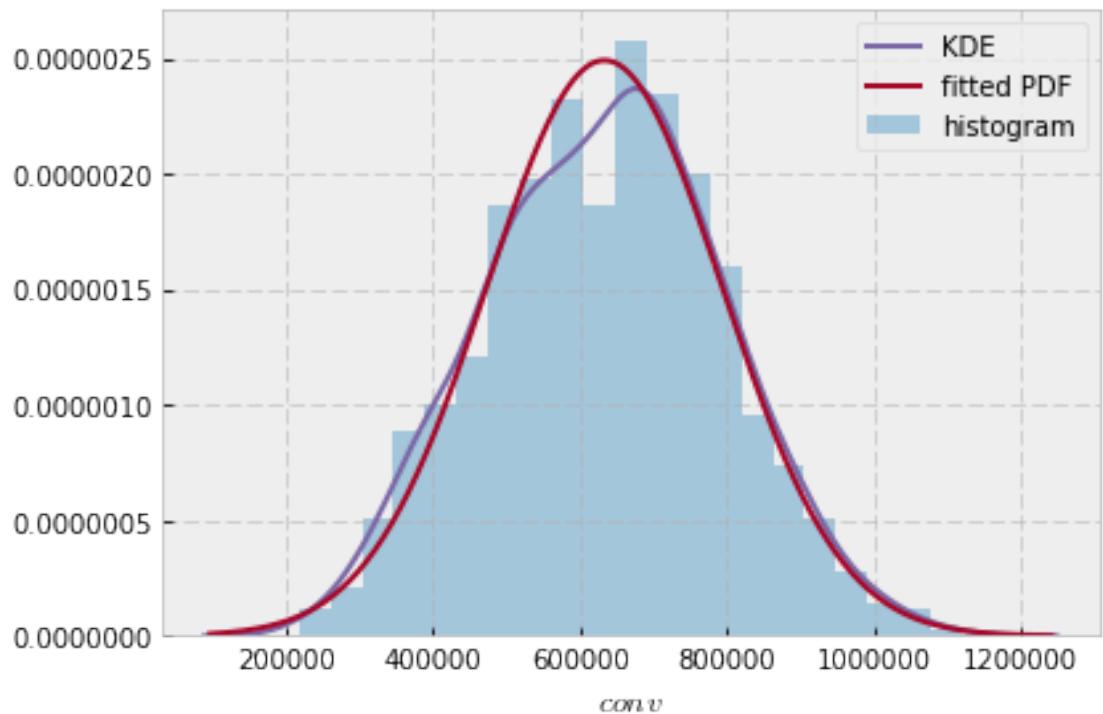


```
variable name: sigma
variable value: 0.0222222222222223
distribution: normal(shape=(), loc=601297.1546831, scale=153860.41245579987)
DescribeResult(nobs=1000, minmax=(192433.2677, 1058373.013)
               mean=601297.15468309994, variance=23696723244.313171,
               skewness=-0.010546926145286305, kurtosis=-0.36187442953224735)
```

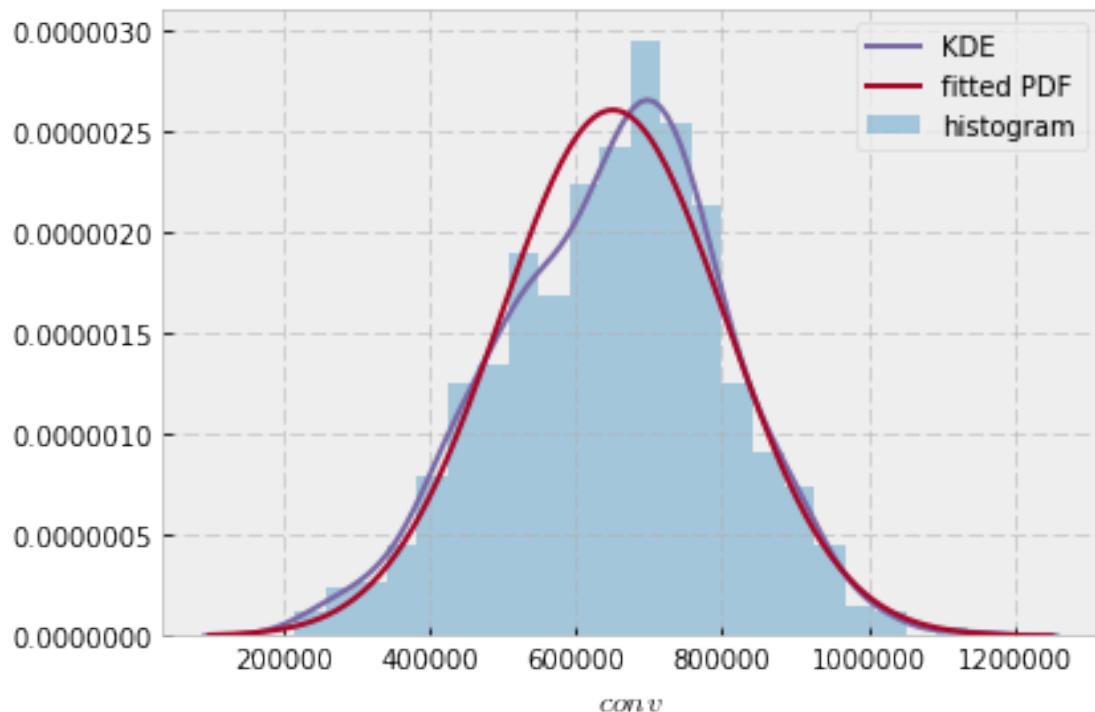


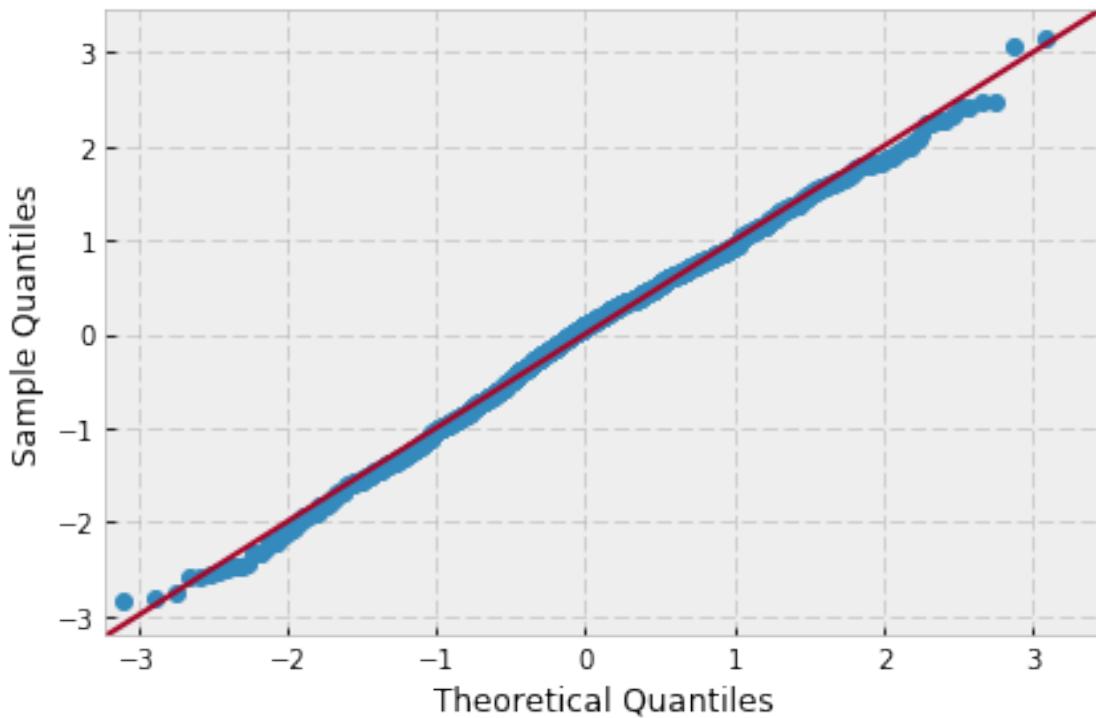


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=633580.6953345999, scale=160125.32064901068)
DescribeResult(nobs=1000, minmax=(217638.155, 1120990.199)
               mean=633580.6953345994, variance=25665784097.045528,
               skewness=0.014226381327634994, kurtosis=-0.35734135573376147)
```

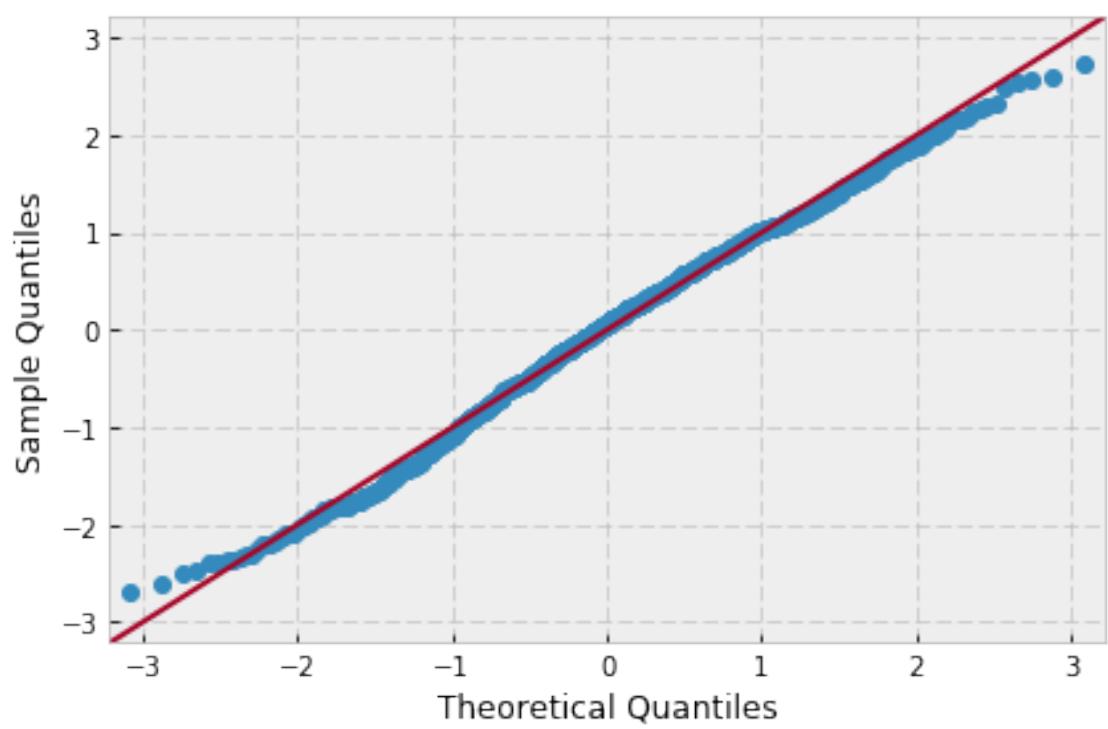
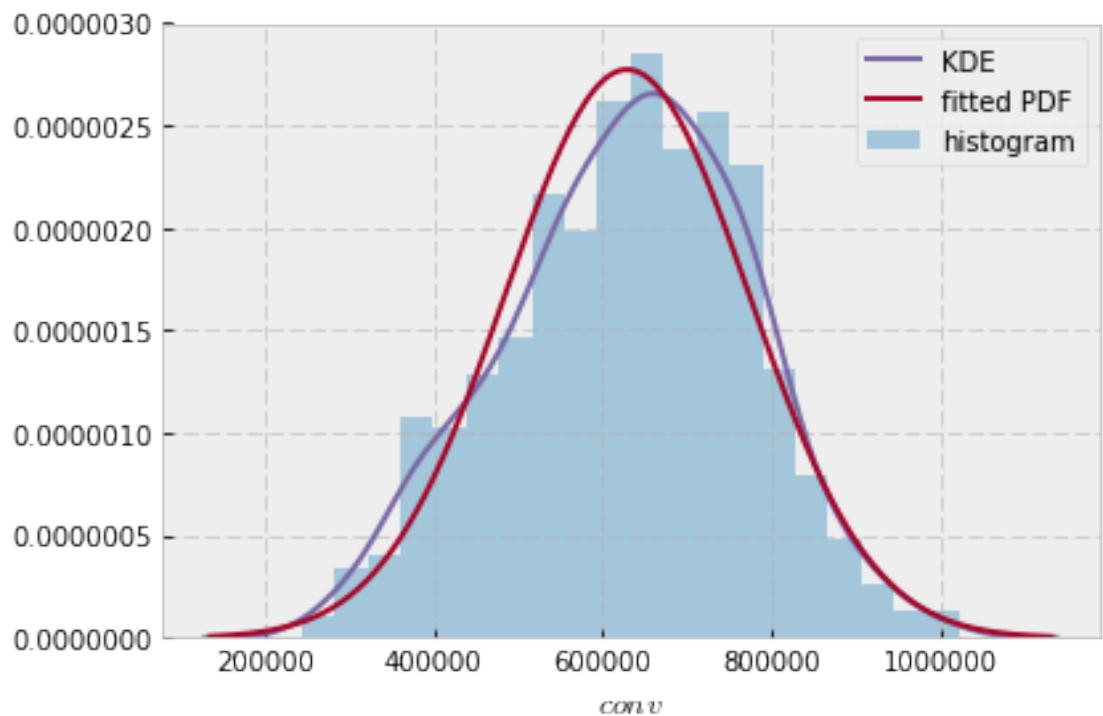


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=650616.7296519999, scale=153359.53469997967)
DescribeResult(nobs=1000, minmax=(216123.0013, 1133795.733)
               mean=650616.72965199989, variance=23542689572.967239,
               skewness=-0.15738114054966287, kurtosis=-0.16555456814853864)
```

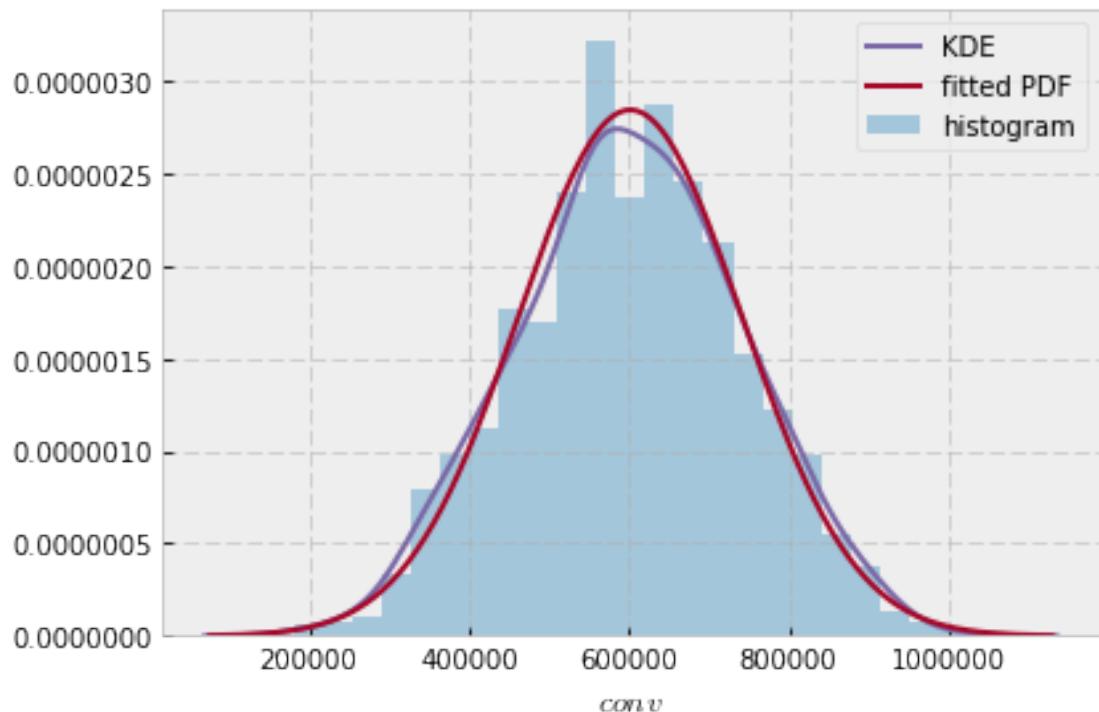


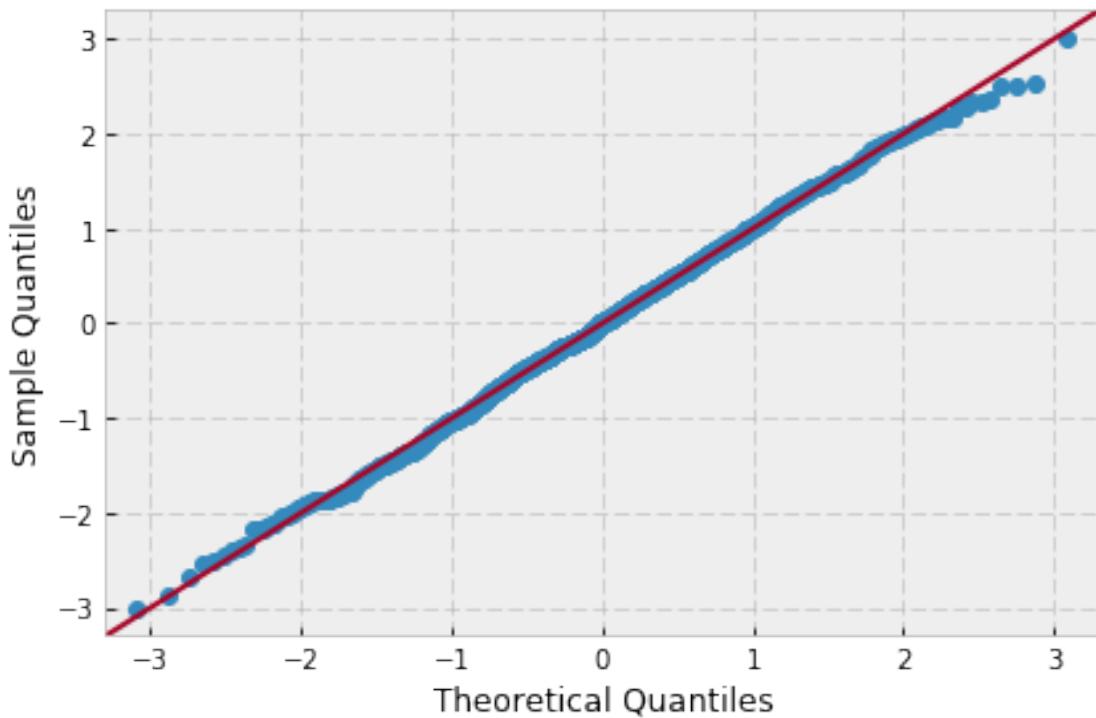


```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(),loc=628690.1118337, scale=143979.4436223398)
DescribeResult(nobs=1000, minmax=(243577.2874, 1022190.179)
               mean=628690.11183369998, variance=20750831016.815342,
               skewness=-0.16498634021035208, kurtosis=-0.3575422414484546)
```

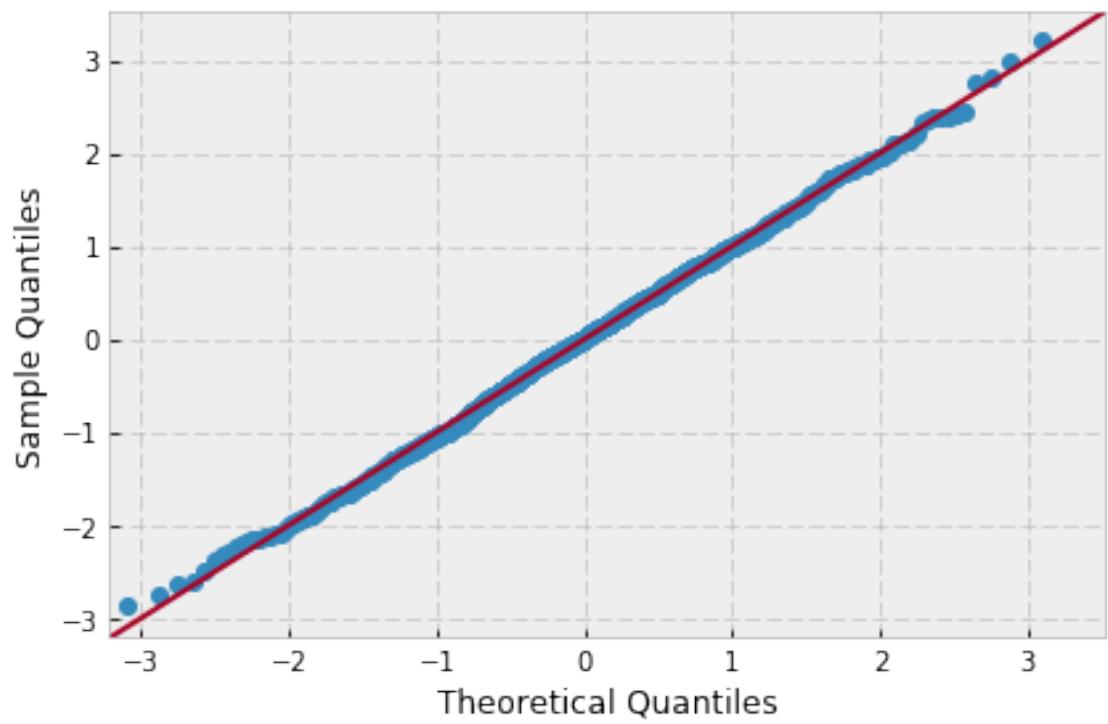
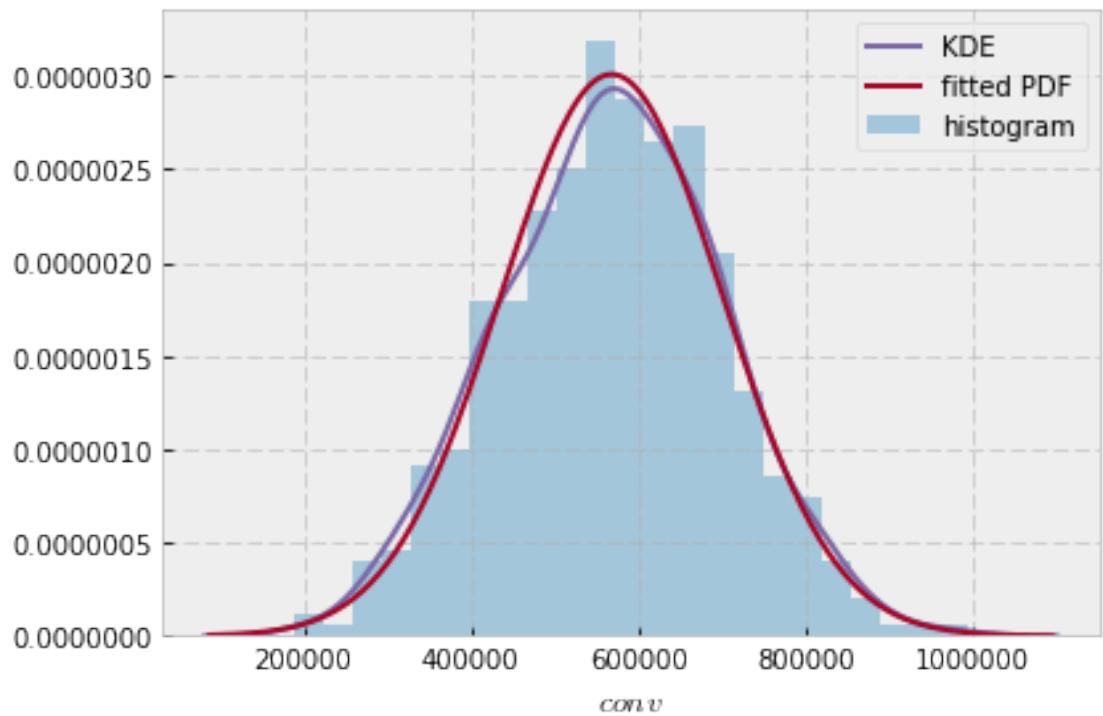


```
variable name: sigma
variable value: 0.06666666666666667
distribution: normal(shape=(), loc=601718.151852, scale=140098.06036857425)
DescribeResult(nobs=1000, minmax=(181756.03690000001, 1022764.11)
               mean=601718.15185200004, variance=19647113632.66935,
               skewness=-0.03289885587670868, kurtosis=-0.2861596011809855)
```

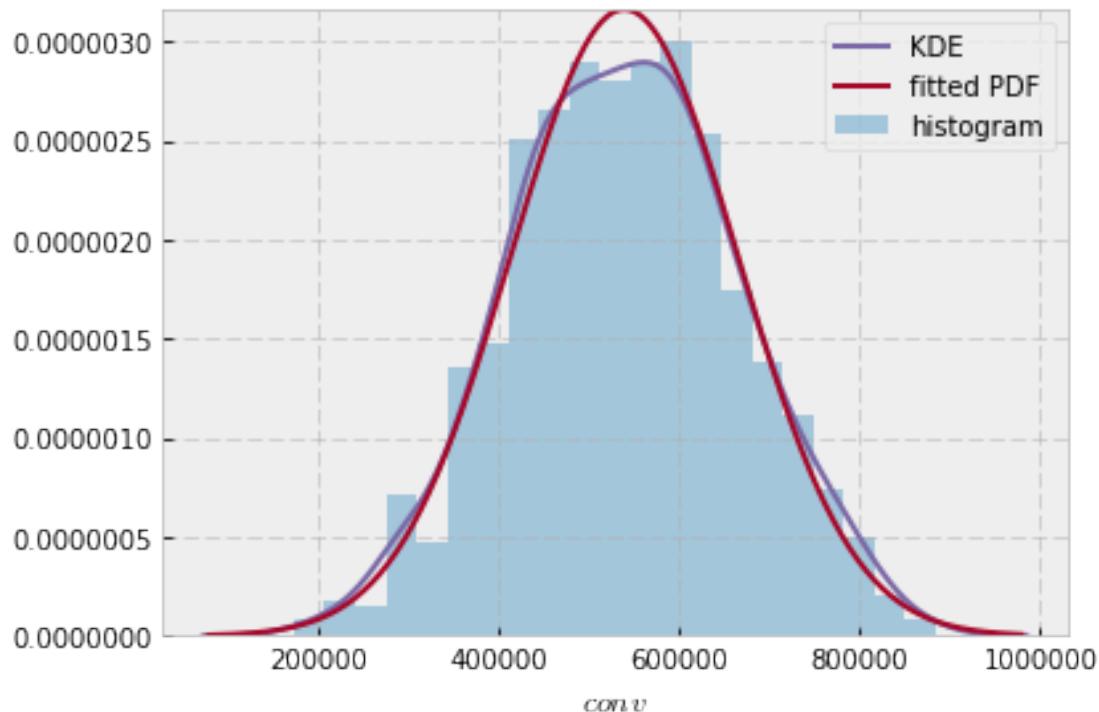


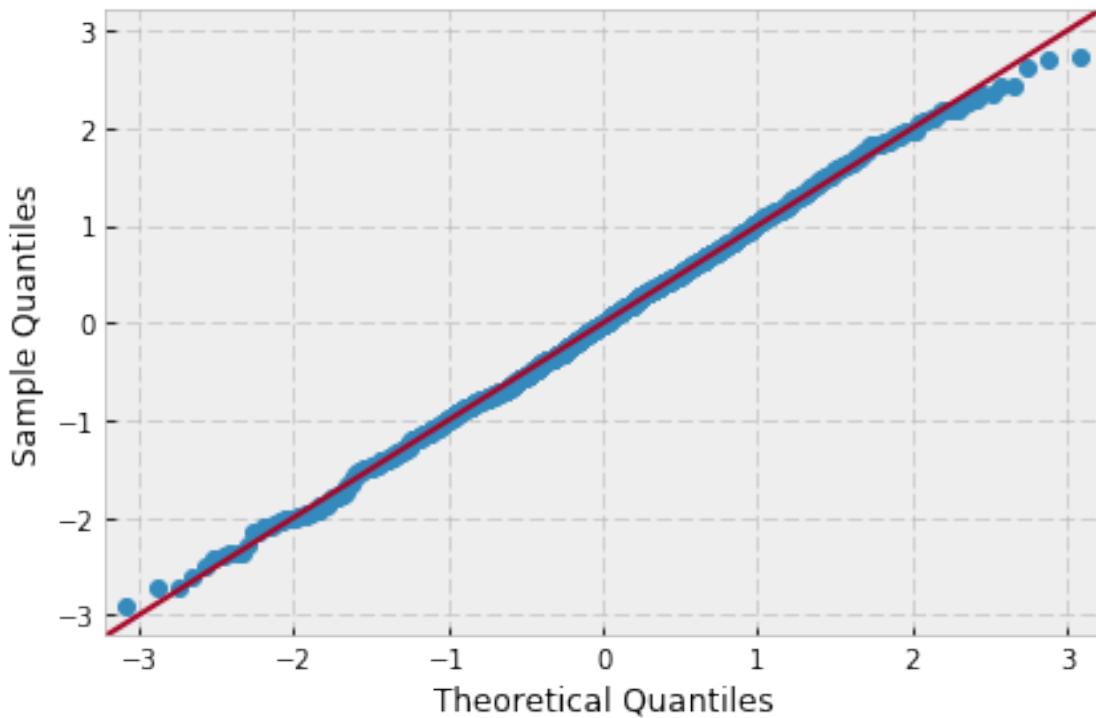


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(),loc=568411.1205014999, scale=132629.99025931334)
DescribeResult(nobs=1000, minmax=(187699.0333000001, 994481.2582000004)
               mean=568411.12050149986, variance=17608322638.824371,
               skewness=-0.005543336584638611, kurtosis=-0.18682856941844062)
```

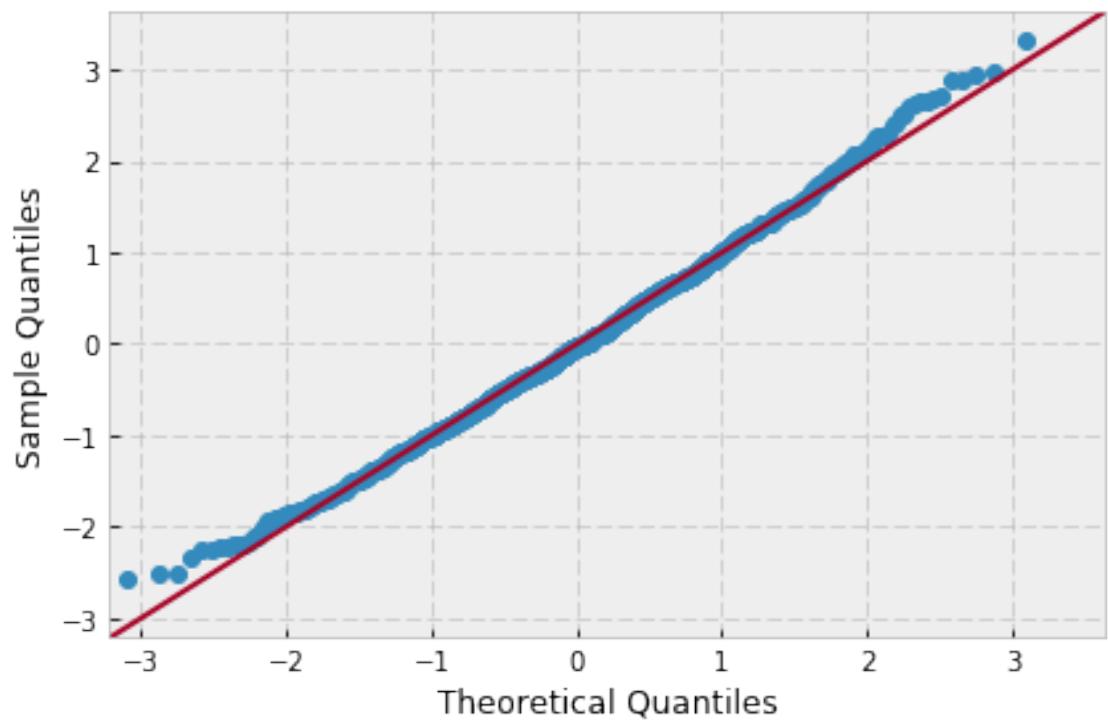
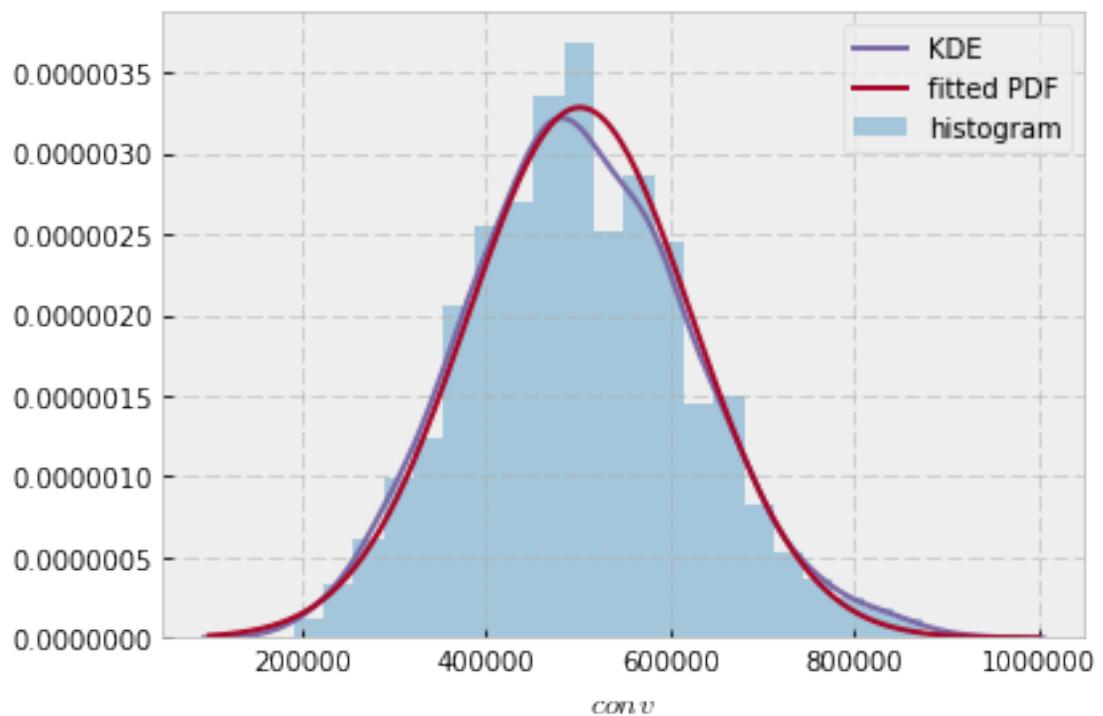


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(), loc=538497.6524252, scale=126118.46823446287)
DescribeResult(nobs=1000, minmax=(172191.1951999999, 883964.7798000002)
               mean=538497.65242519998, variance=15921789819.626846,
               skewness=0.012015047164622403, kurtosis=-0.27701168992351466)
```

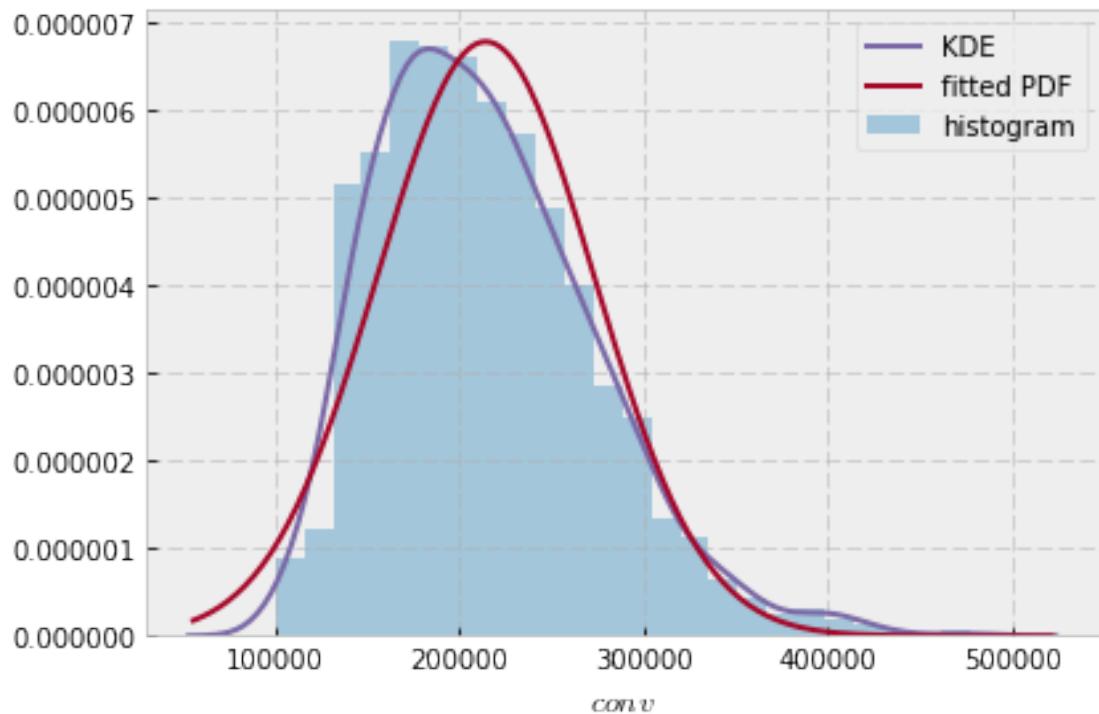


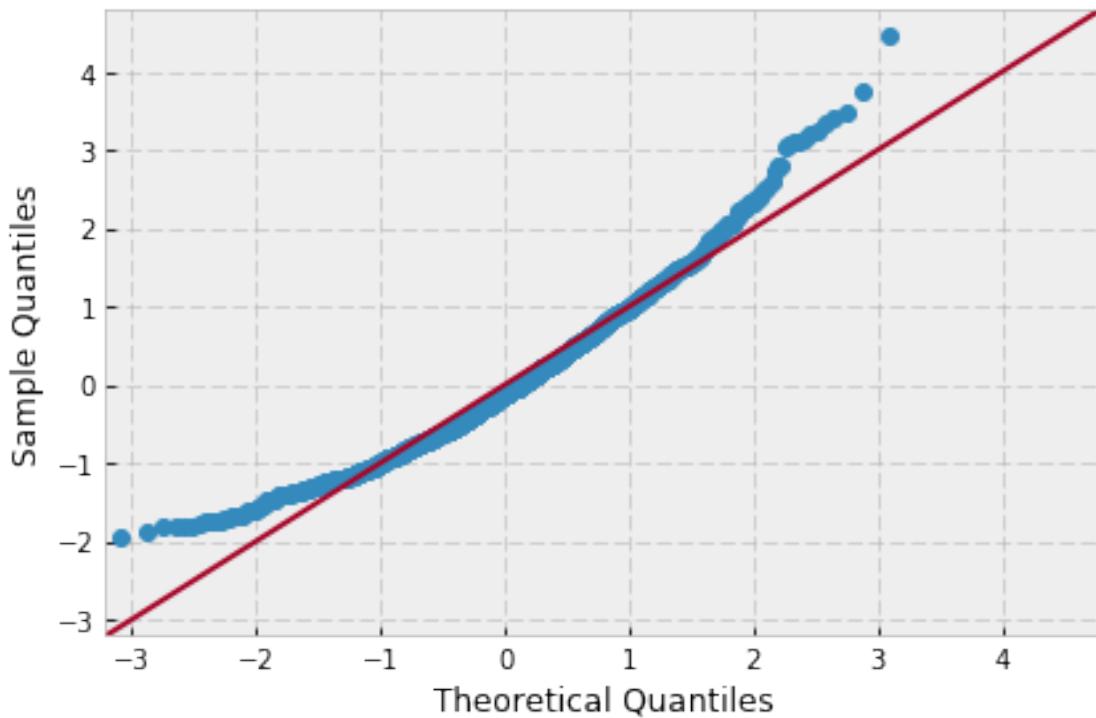


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(),loc=502052.04192870005, scale=121504.74227330228)
DescribeResult(nobs=1000, minmax=(191700.4602, 907213.3769999998)
               mean=502052.04192870005, variance=14778180575.477089,
               skewness=0.21871445767274342, kurtosis=-0.05468264478629514)
```

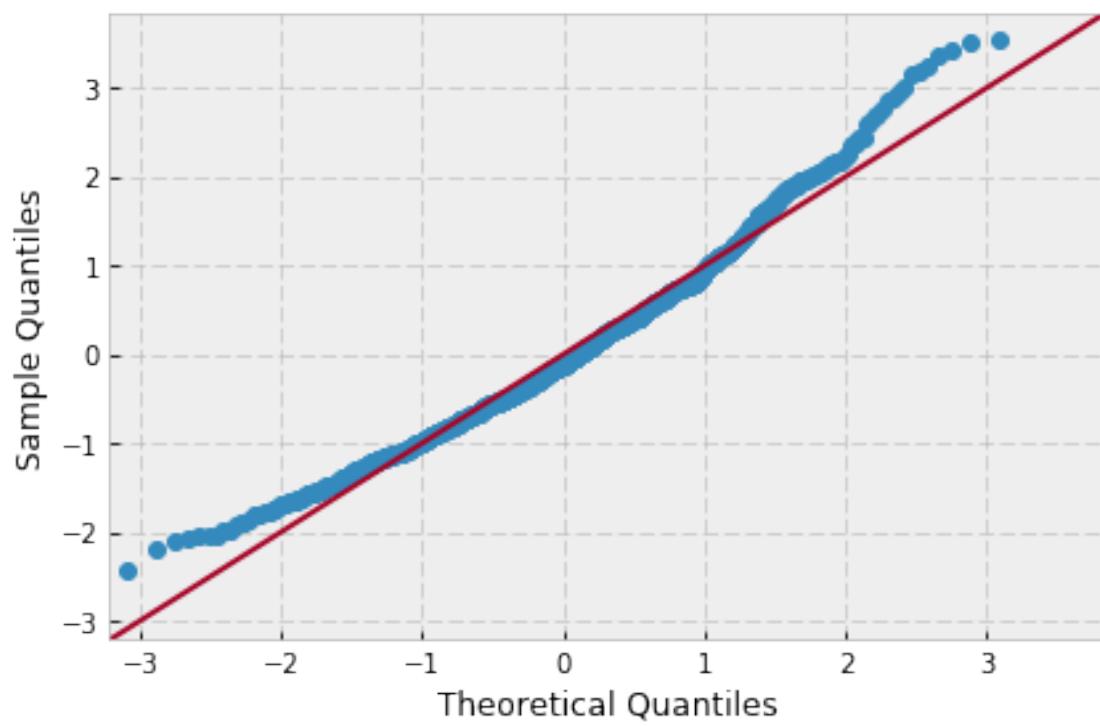
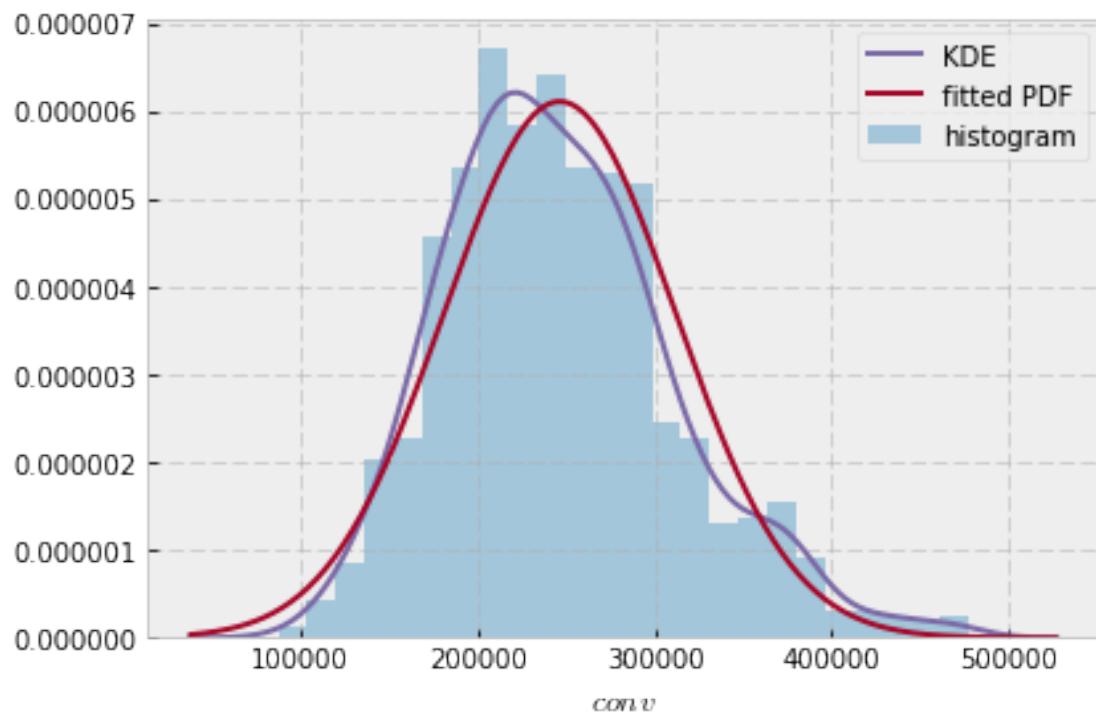


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=214142.67314539, scale=58760.23283084854)
DescribeResult(nobs=1000, minmax=(99718.223790000004, 476803.50599999999)
               mean=214142.67314539, variance=3456221183.5190496,
               skewness=0.759025883693774, kurtosis=0.6808478674189975)
```

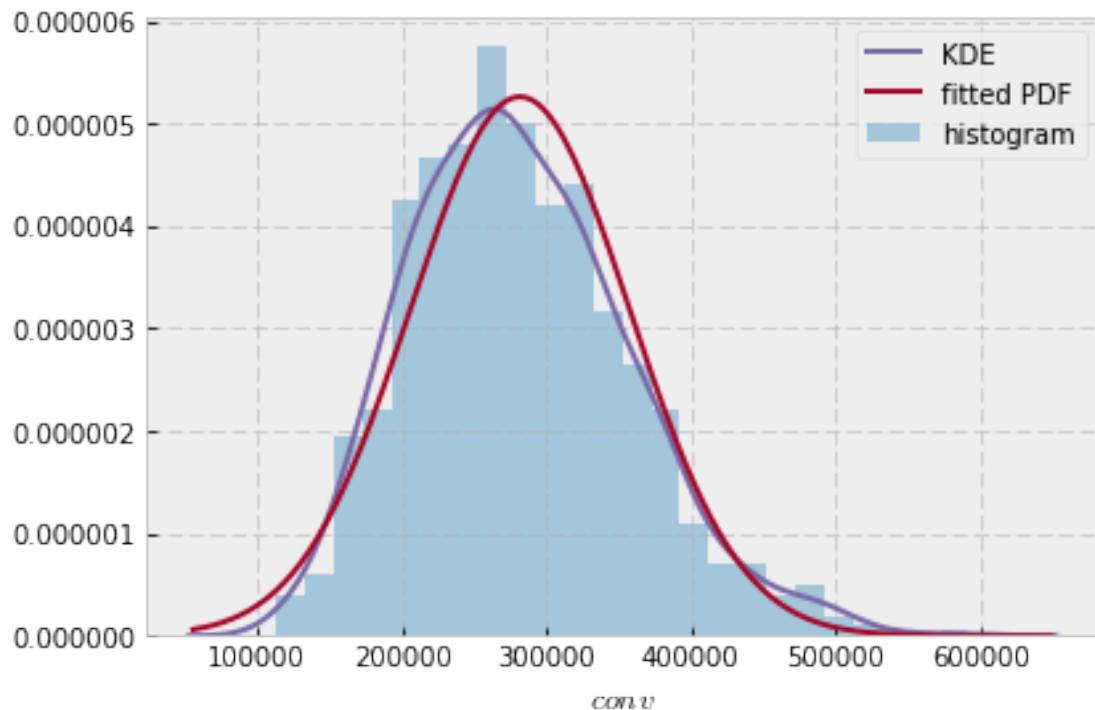


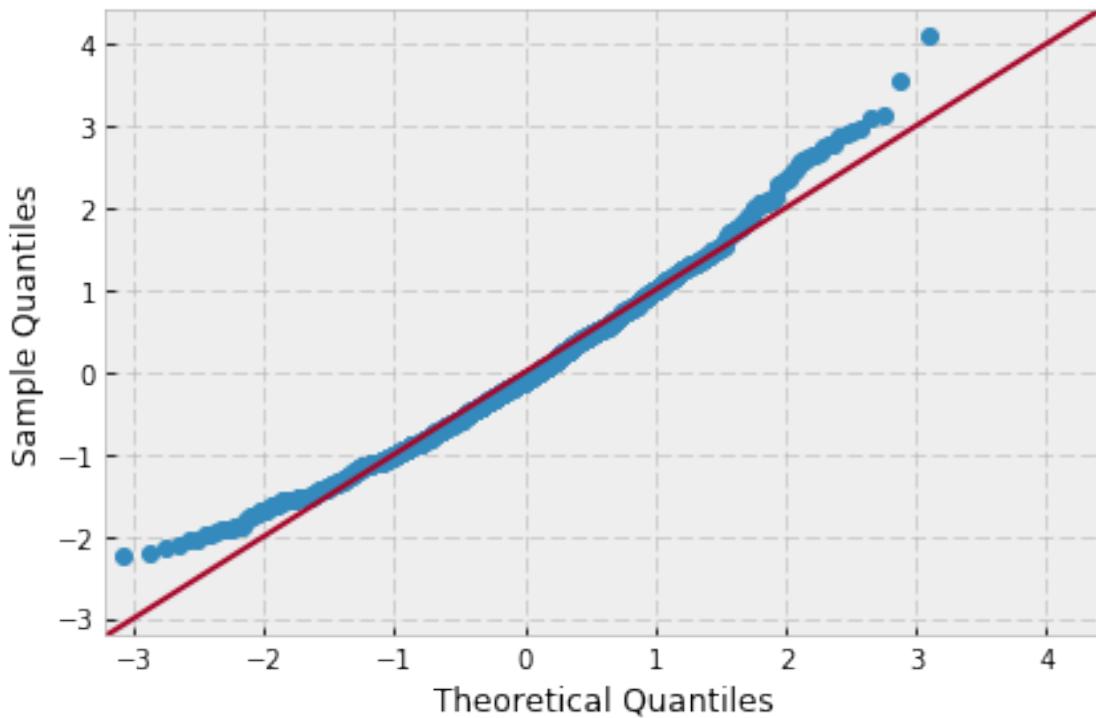


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=246251.6763355, scale=65345.67301162494)
DescribeResult(nobs=1000, minmax=(87226.03010000004, 476918.6423000001)
               mean=246251.6763355, variance=4274331312.6548629,
               skewness=0.6008531628820079, kurtosis=0.40649637779731407)
```

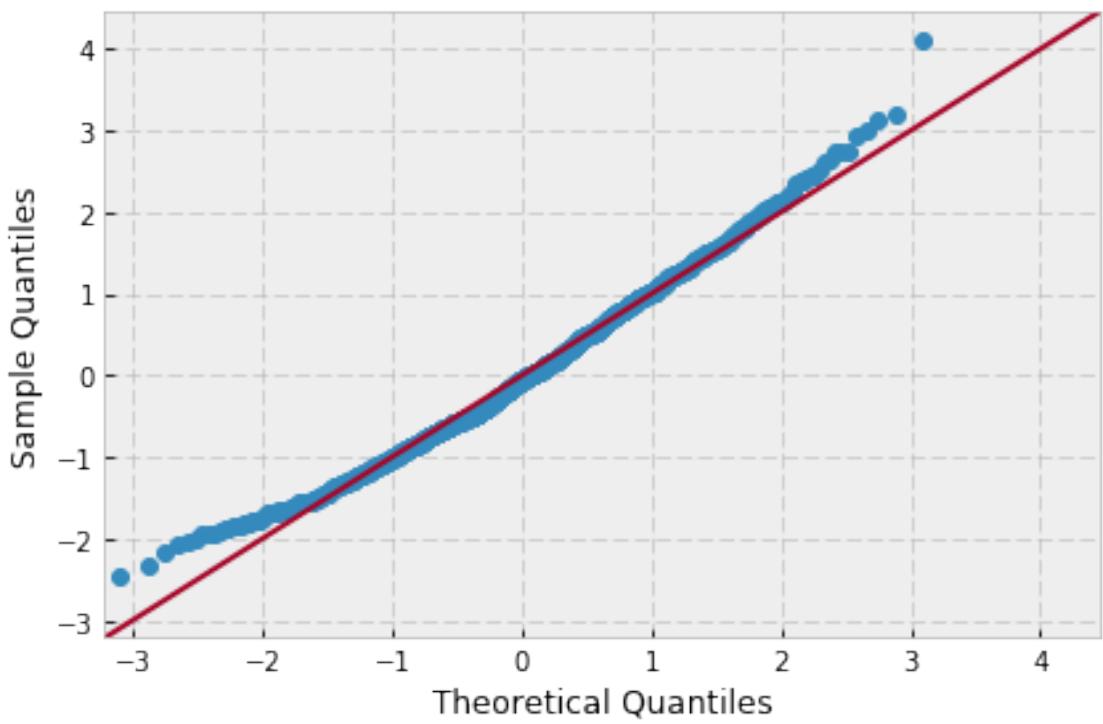
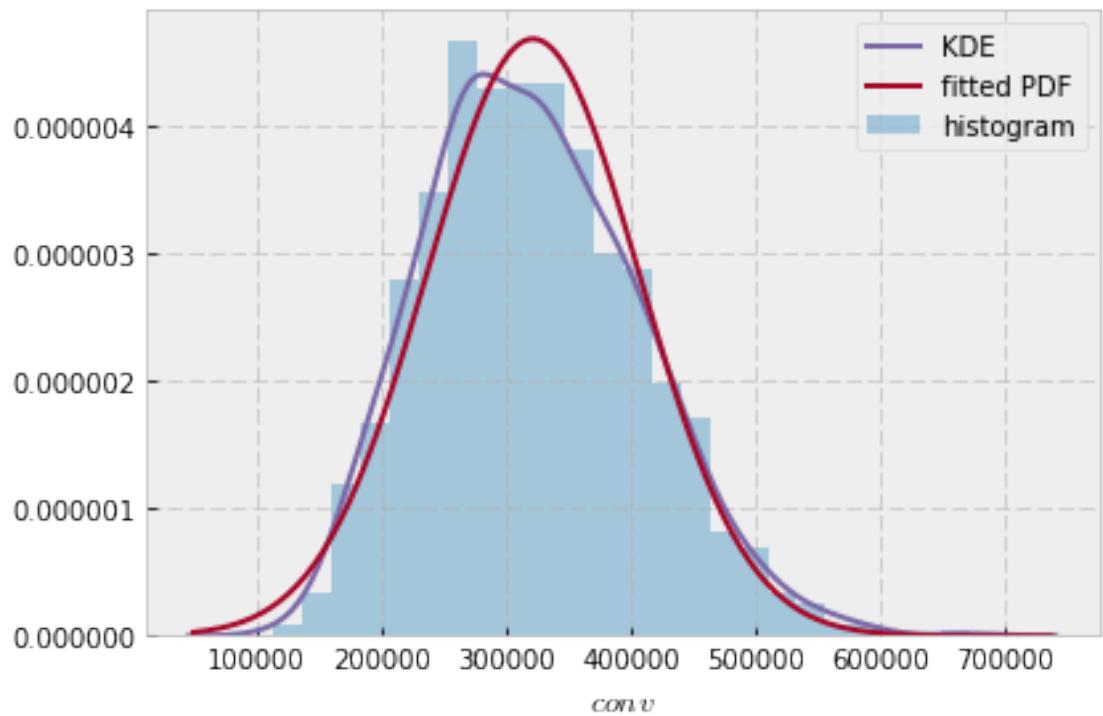


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=281708.87613530003, scale=75846.79110012914)
DescribeResult(nobs=1000, minmax=(112724.5307, 592217.05599999998)
               mean=281708.87613530003, variance=5758494214.4010277,
               skewness=0.5236058637063428, kurtosis=0.27164025214962306)
```

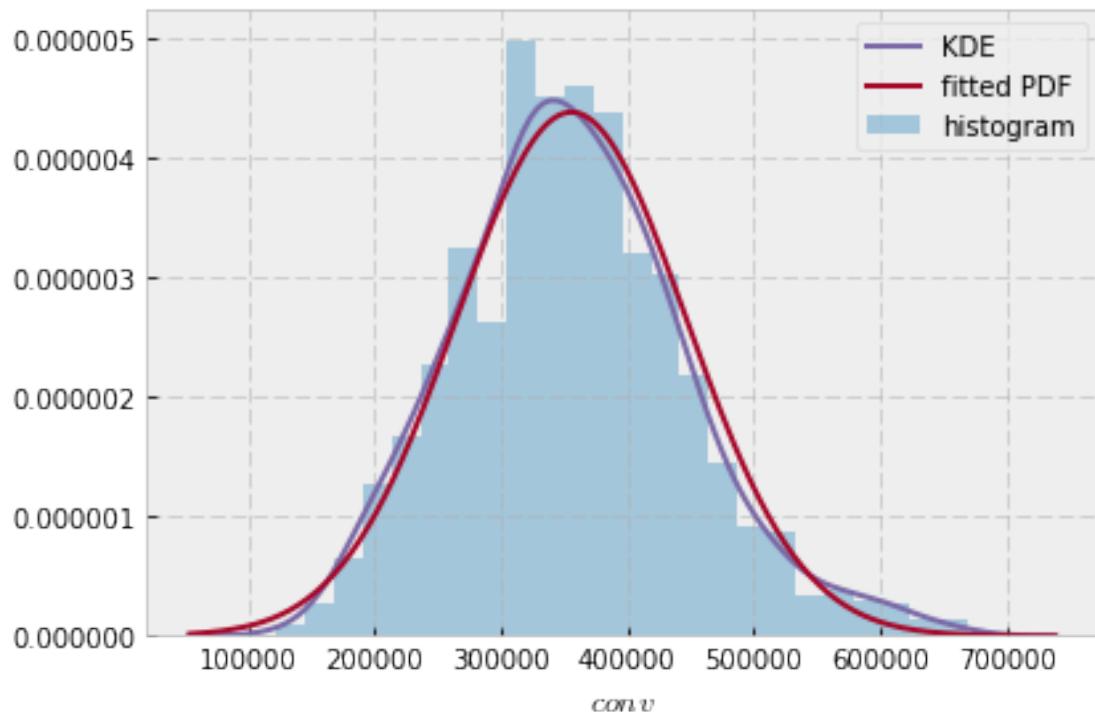


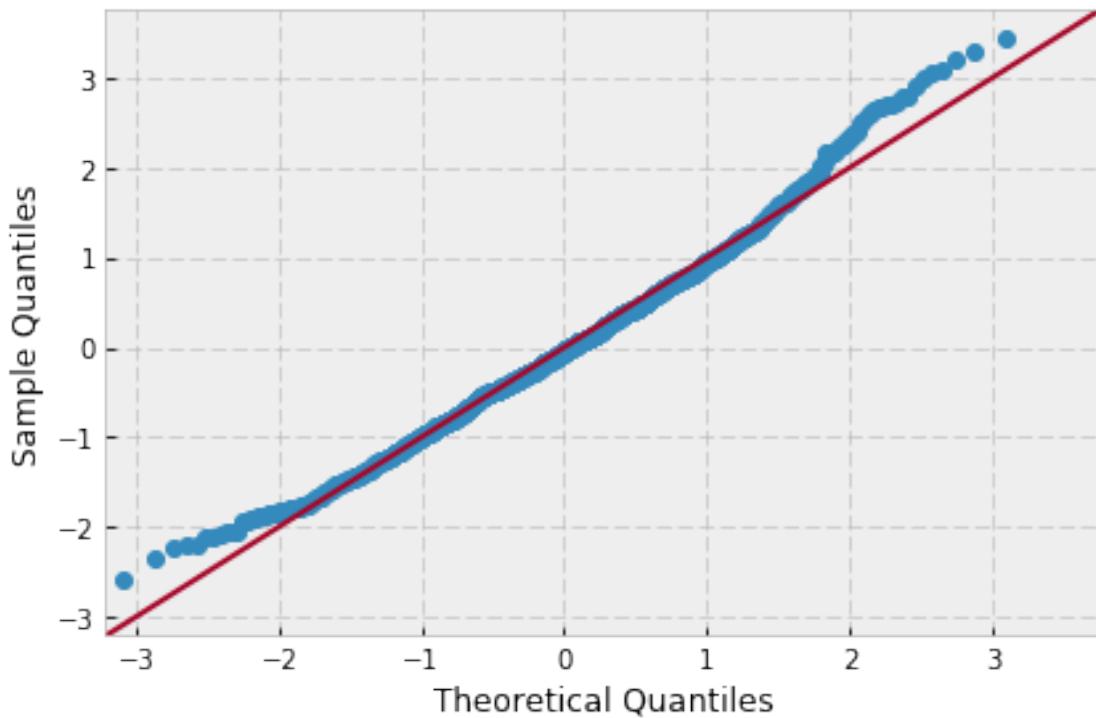


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(),loc=321351.8899207, scale=85057.62919188368)
DescribeResult(nobs=1000, minmax=(113544.219, 672129.8438999998)
               mean=321351.88992069999, variance=7242042326.0700521,
               skewness=0.3928924956665979, kurtosis=-0.02329470689271007)
```

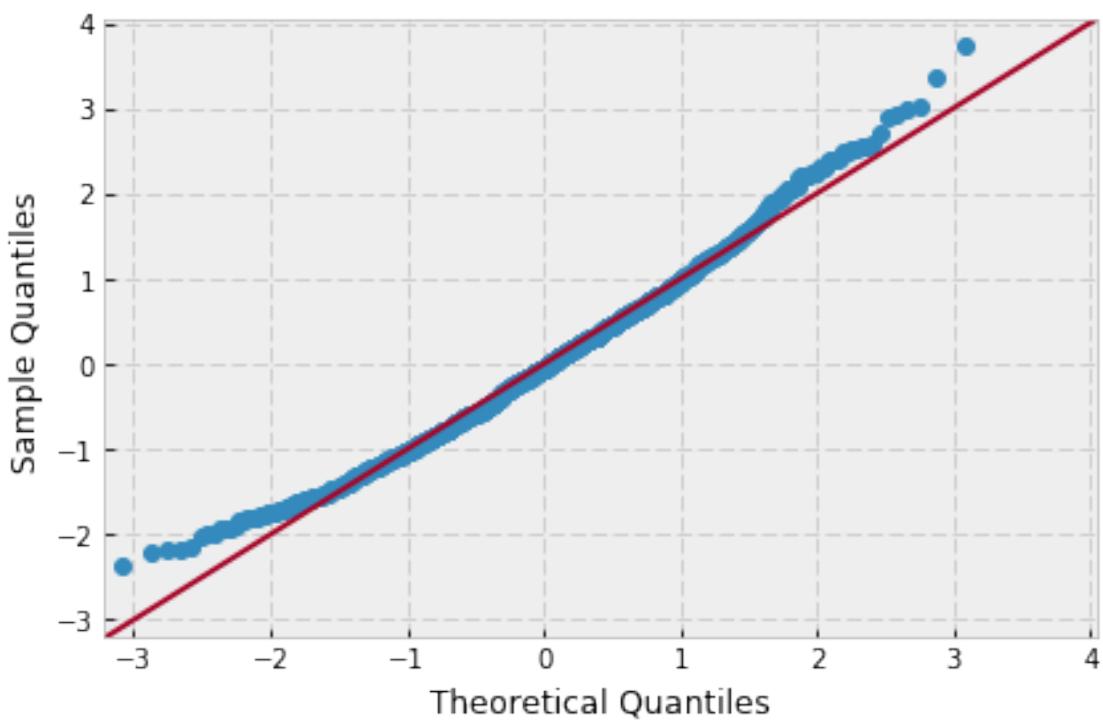
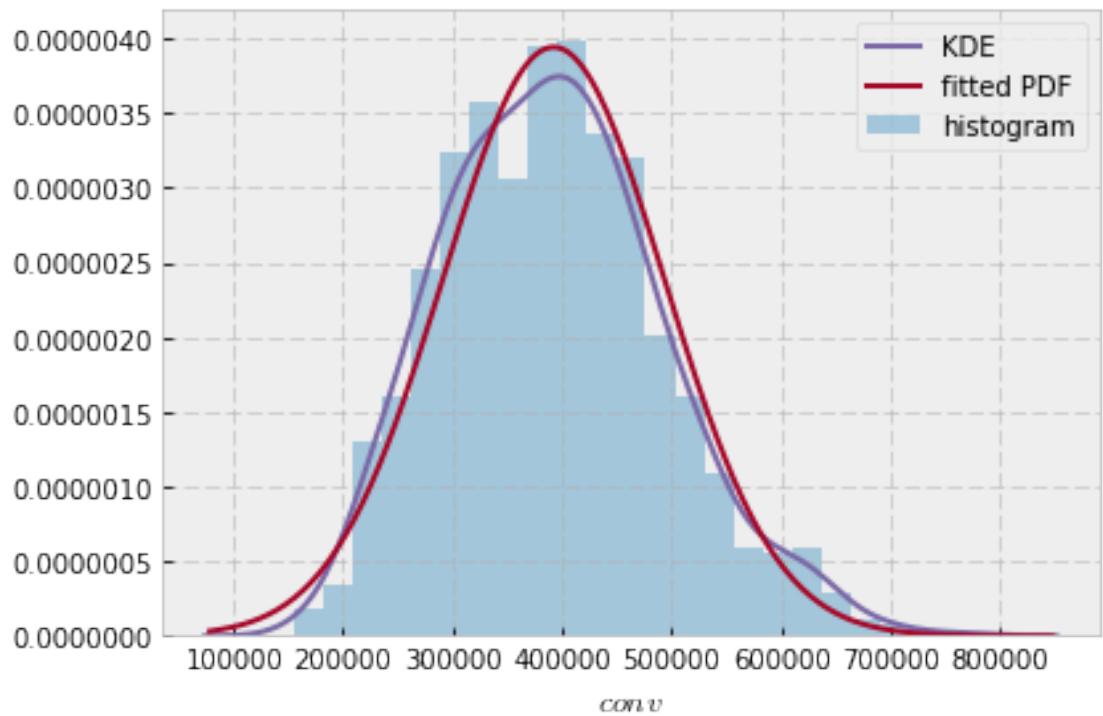


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=355634.3666453, scale=90889.52680538228)
DescribeResult(nobs=1000, minmax=(122019.4255, 669656.1299000006)
               mean=355634.3666453, variance=8269175258.1644678,
               skewness=0.3871333081514963, kurtosis=0.27171959553035574)
```

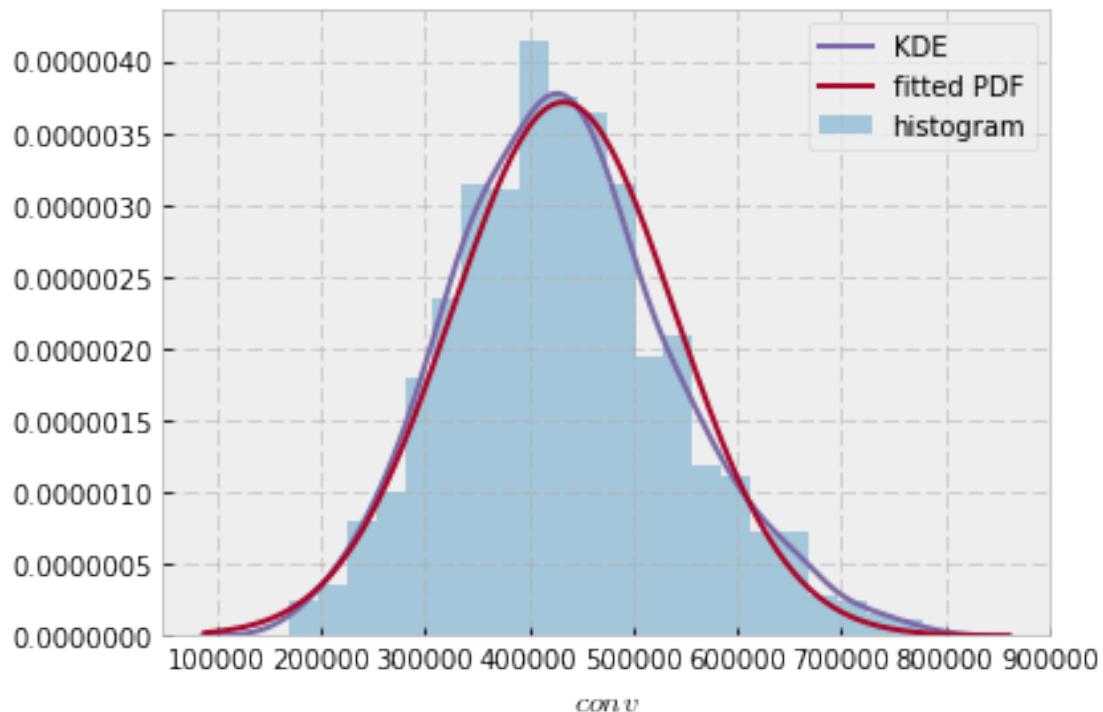


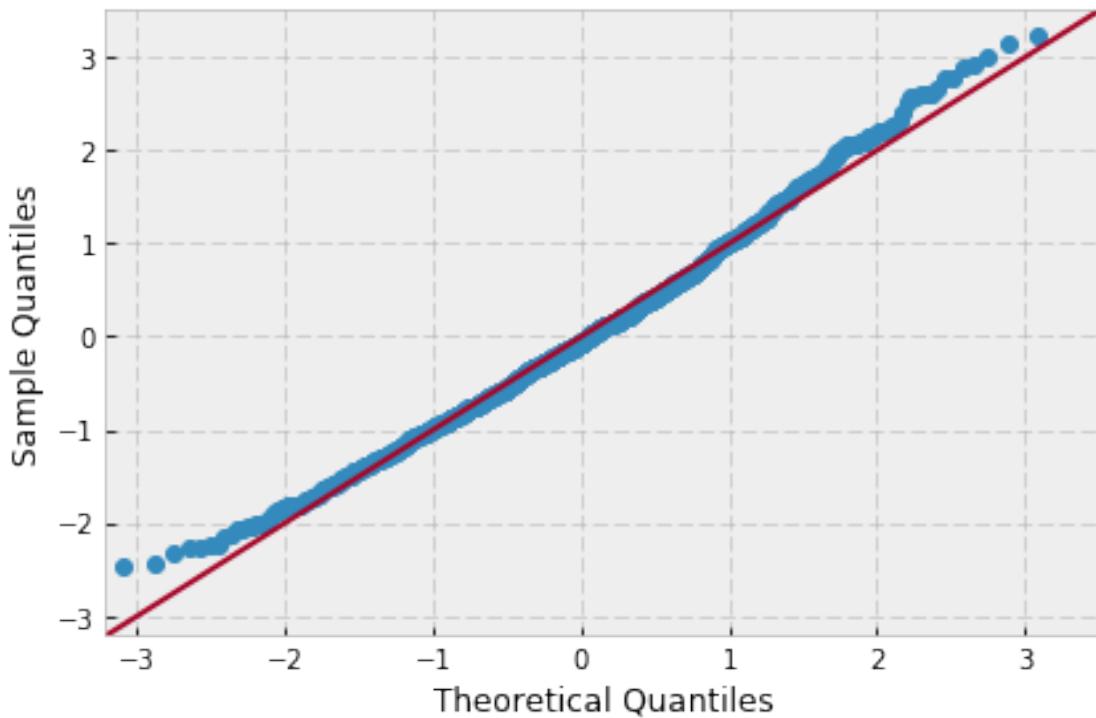


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(),loc=392101.0012248, scale=101159.20084219502)
DescribeResult(nobs=1000, minmax=(154287.77849999999, 770681.17870000005)
               mean=392101.00122480001, variance=10243427342.373922,
               skewness=0.40066269827409334, kurtosis=0.030575682832897577)
```

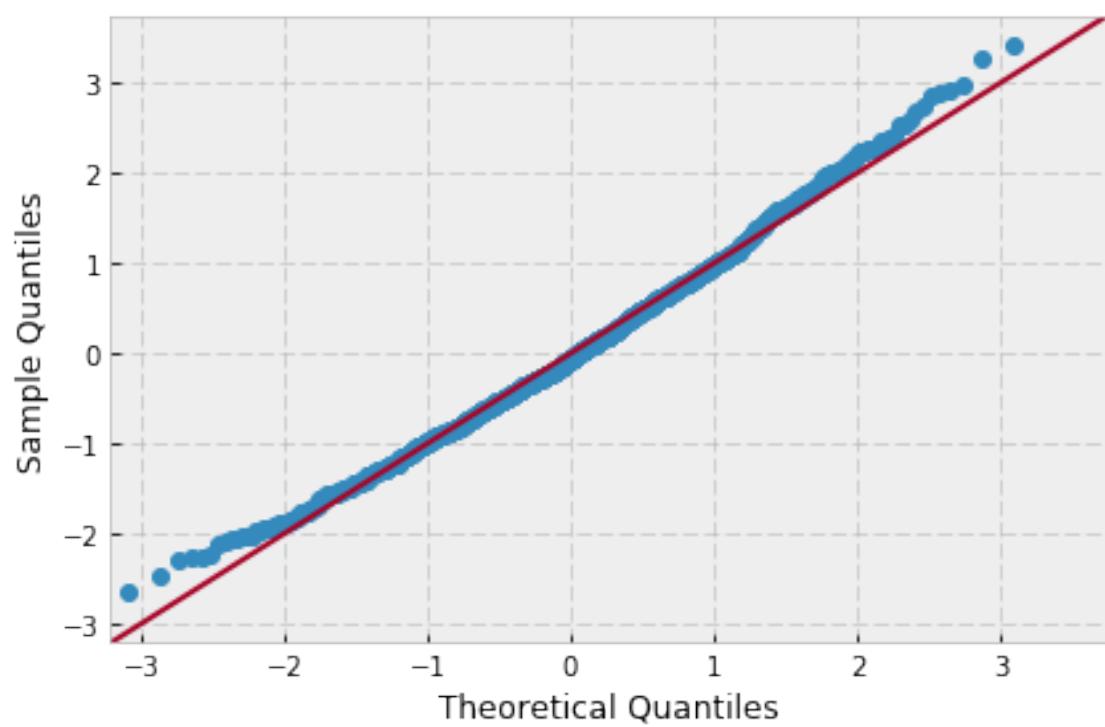
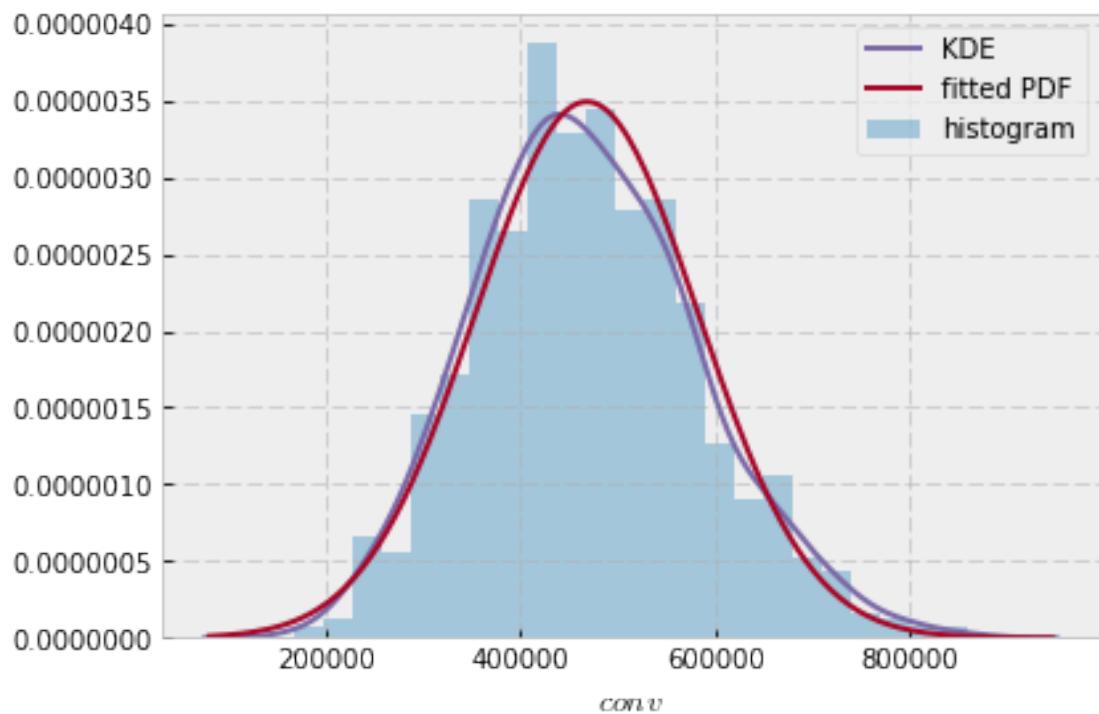


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=432650.6826091, scale=107439.02060220322)
DescribeResult(nobs=1000, minmax=(168988.19959999999, 778974.1925)
               mean=432650.68260910001, variance=11554697845.806456,
               skewness=0.3330155717450263, kurtosis=0.014940025898865006)
```

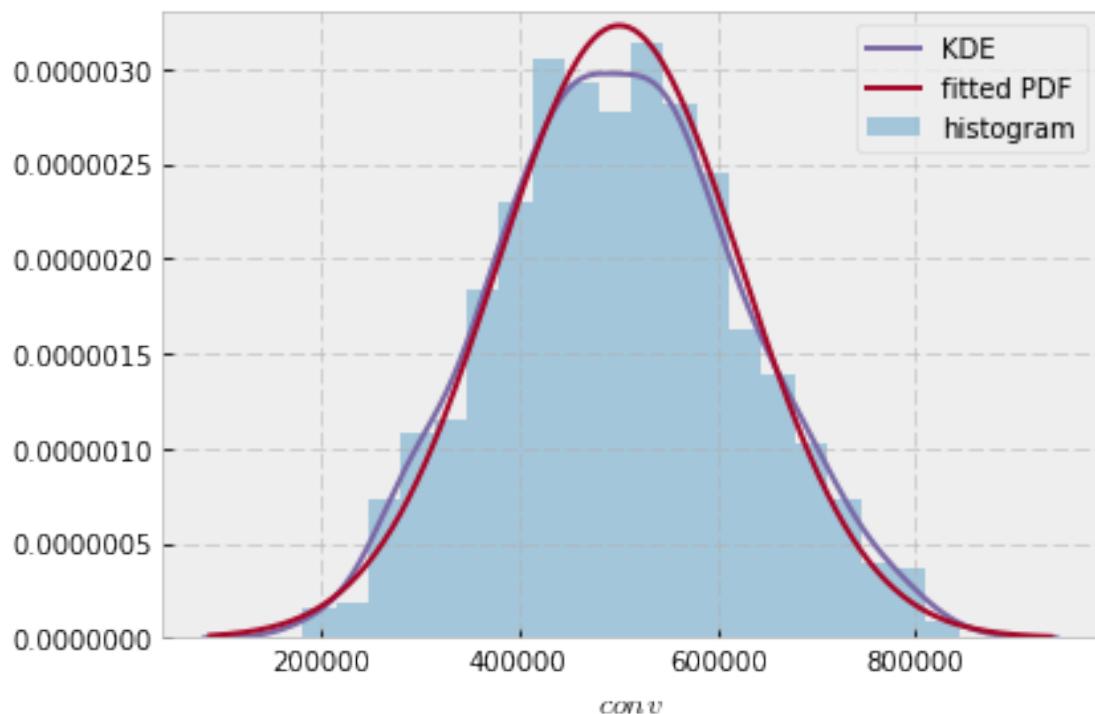


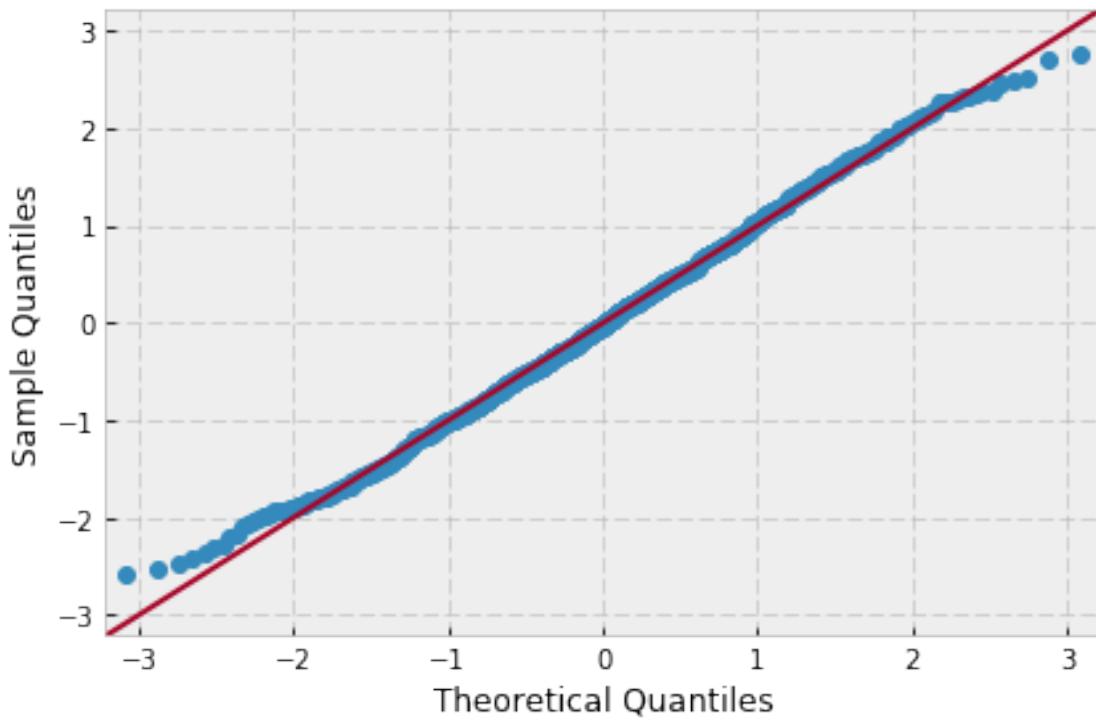


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(),loc=468398.275182, scale=114283.83046188799)
DescribeResult(nobs=1000, minmax=(166503.7758, 860508.5365000005)
               mean=468398.27518200001, variance=13073867772.814371,
               skewness=0.3108572595807076, kurtosis=-0.0161516244254849)
```

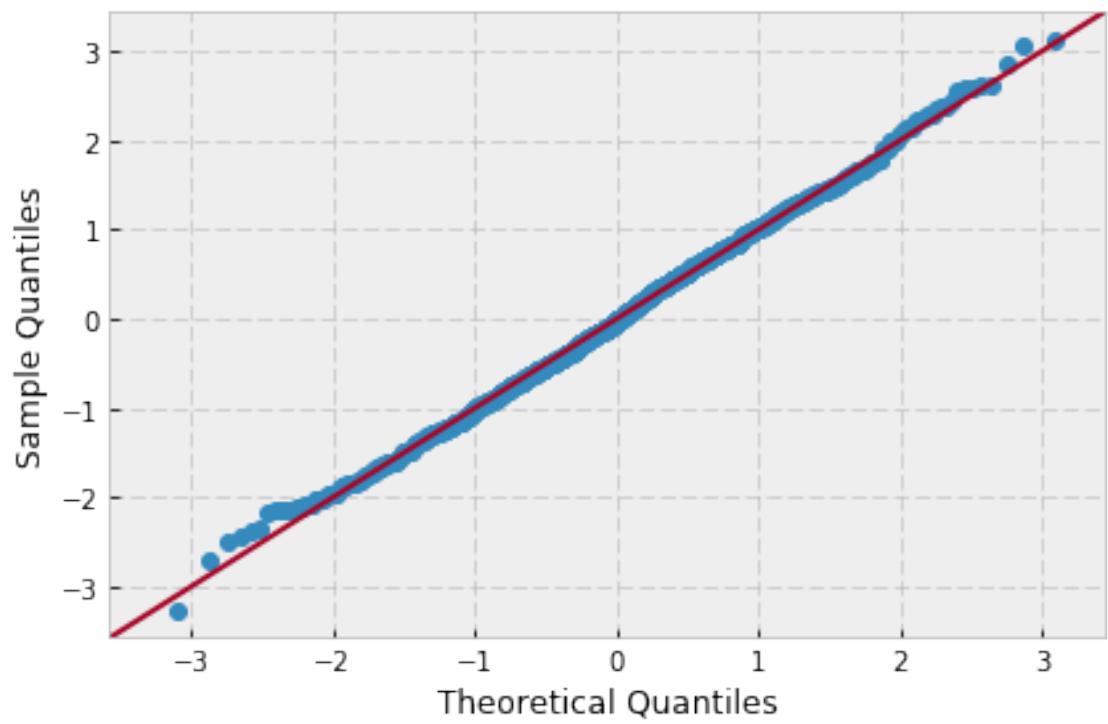
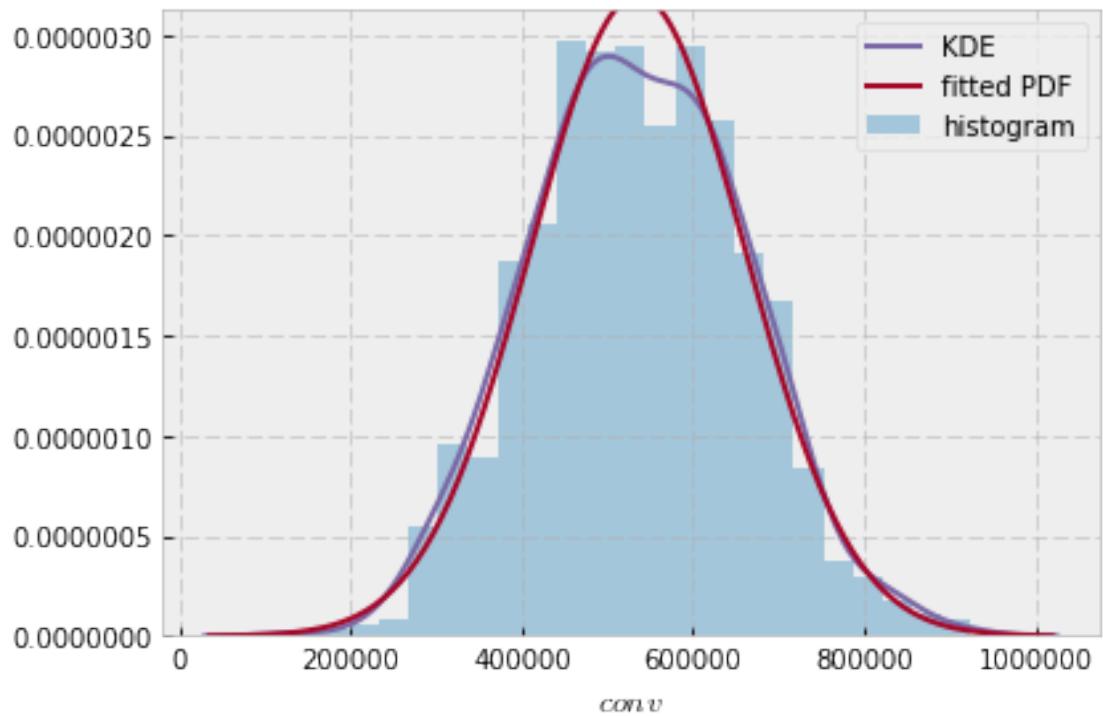


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=500954.6194148, scale=123478.3705690747)
DescribeResult(nobs=1000, minmax=(181782.4604999999, 843188.1448000001)
               mean=500954.6194148002, variance=15262170168.562296,
               skewness=0.09784035236321317, kurtosis=-0.36924626797699966)
```





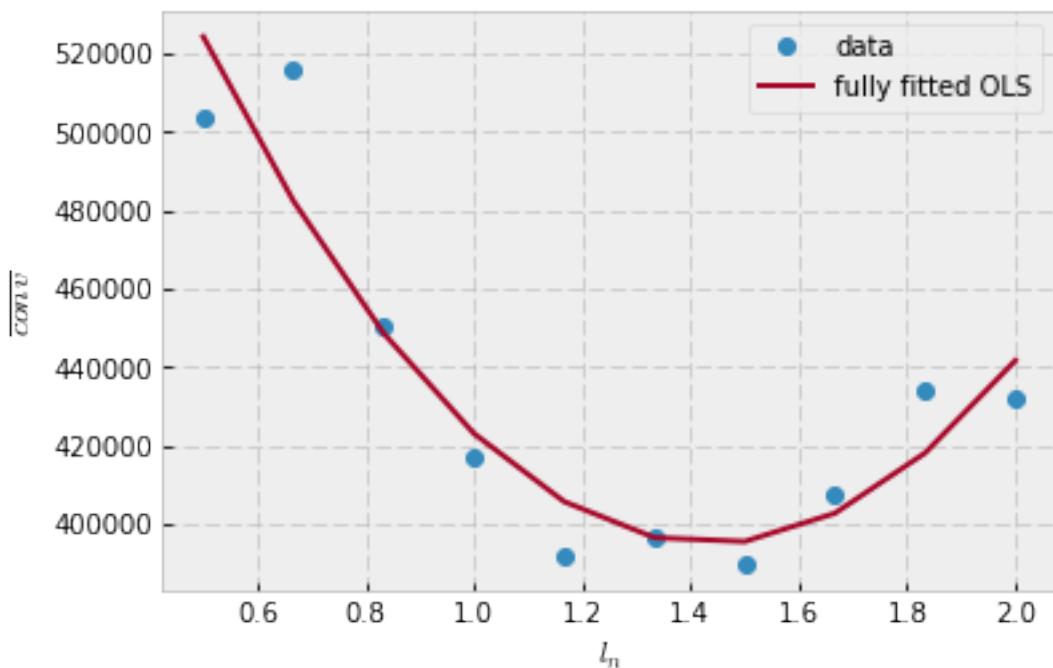
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(),loc=535398.1395644001, scale=124733.44338422752)
DescribeResult(nobs=1000, minmax=(129205.51240000001, 924951.08140000002)
               mean=535398.13956440007, variance=15574005904.39068,
               skewness=0.06756287513672024, kurtosis=-0.20984422330416308)
```



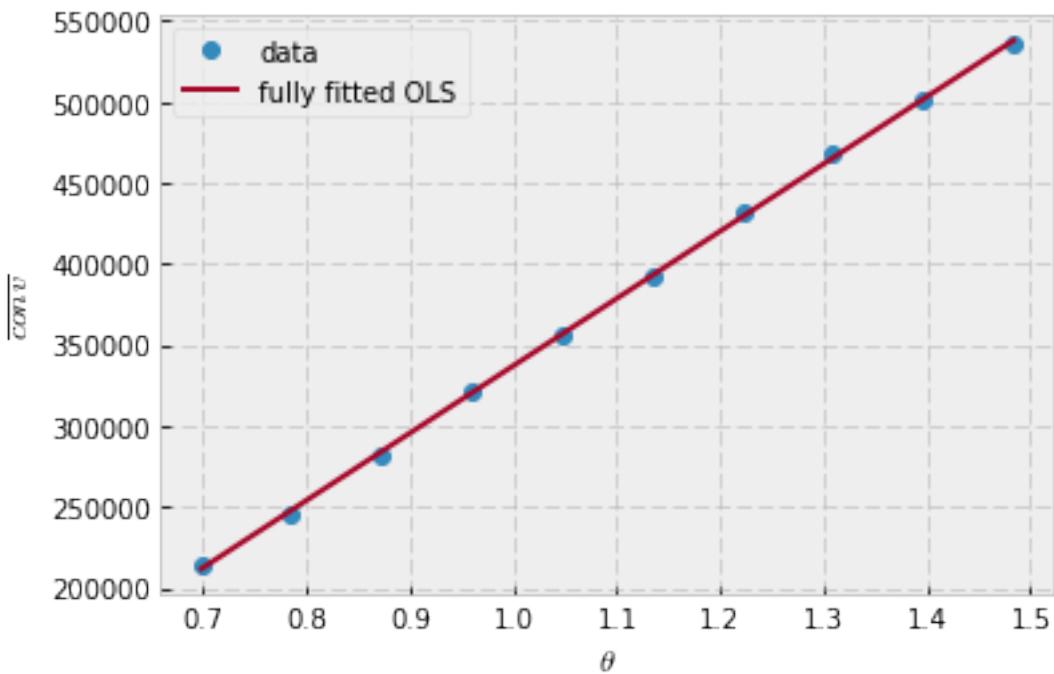
4.2 Parameter regression

```
In [14]: var_name = 'convex_hull_volume'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly1')  
        mean_reg('maxB', var_name, 'poly2')  
        mean_reg('r', var_name, 'poly1')  
        mean_reg('sigma', var_name, 'poly1')  
        mean_reg('N', var_name, 'poly1')
```

variable name: ln
model: poly2
r-squared: 0.6298836281717848
explained variance: 0.6368751435413271



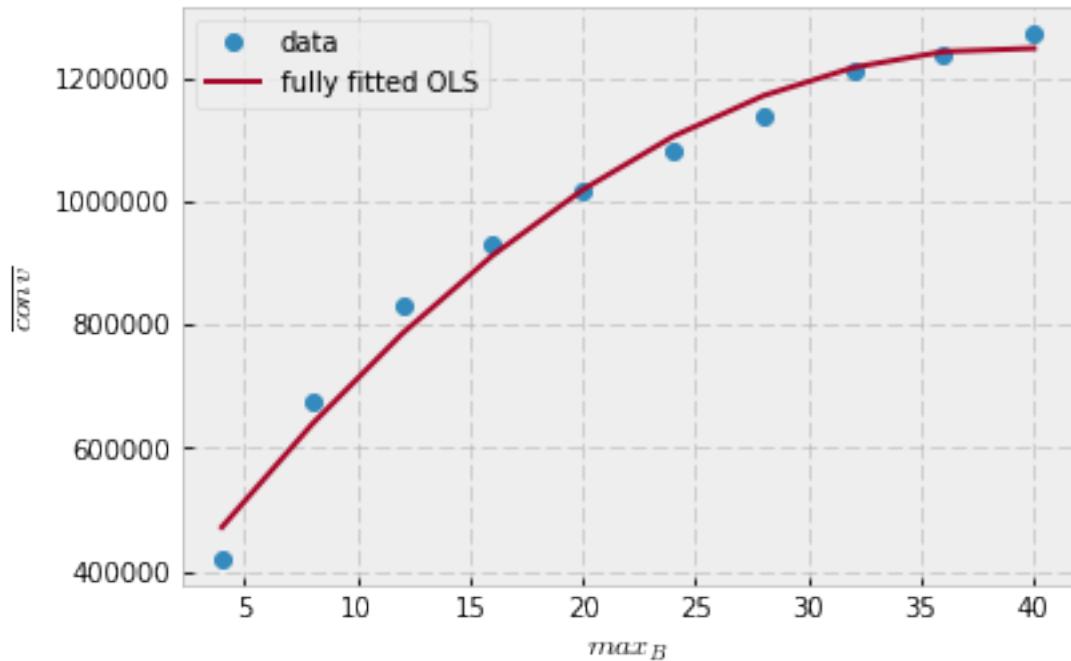
```
variable name: theta  
model: poly1  
r-squared: 0.9993036610422747  
explained variance: 0.9993038434190464
```



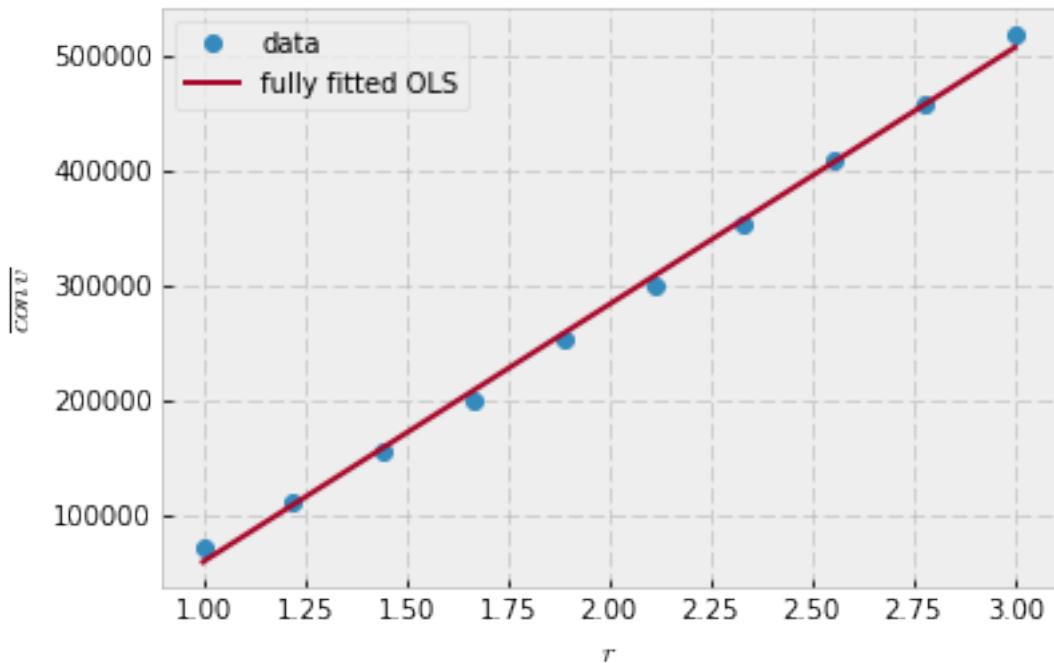
```

variable name: maxB
model: poly2
r-squared: 0.9520481161903153
explained variance: 0.9522426897960711

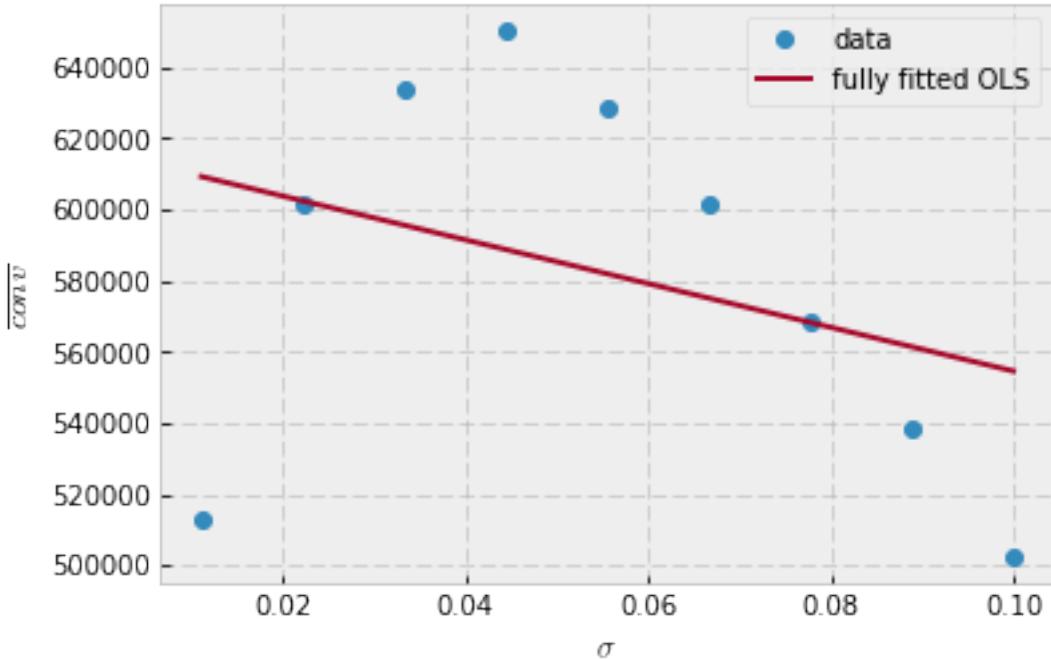
```



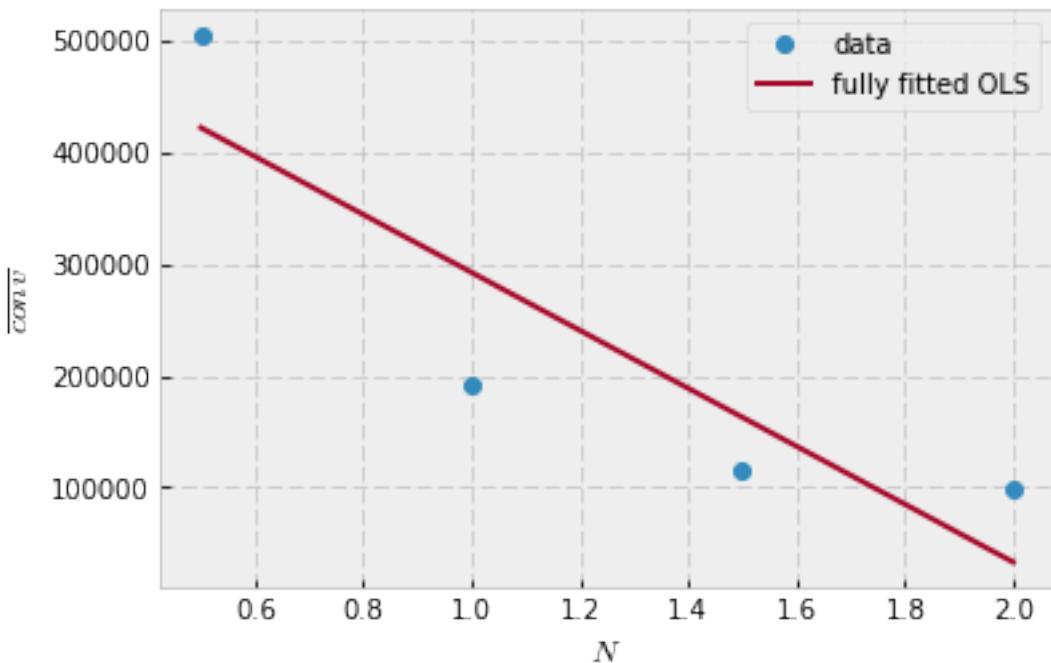
```
variable name: r
model: poly1
r-squared: 0.9951076557668824
explained variance: 0.9951610086934837
```



```
variable name: sigma
model: poly1
r-squared: -0.8252206212184423
explained variance: -0.8007694238703627
```



```
variable name: N
model: poly1
r-squared: -0.3880222724920994
explained variance: -0.20288634608959488
```

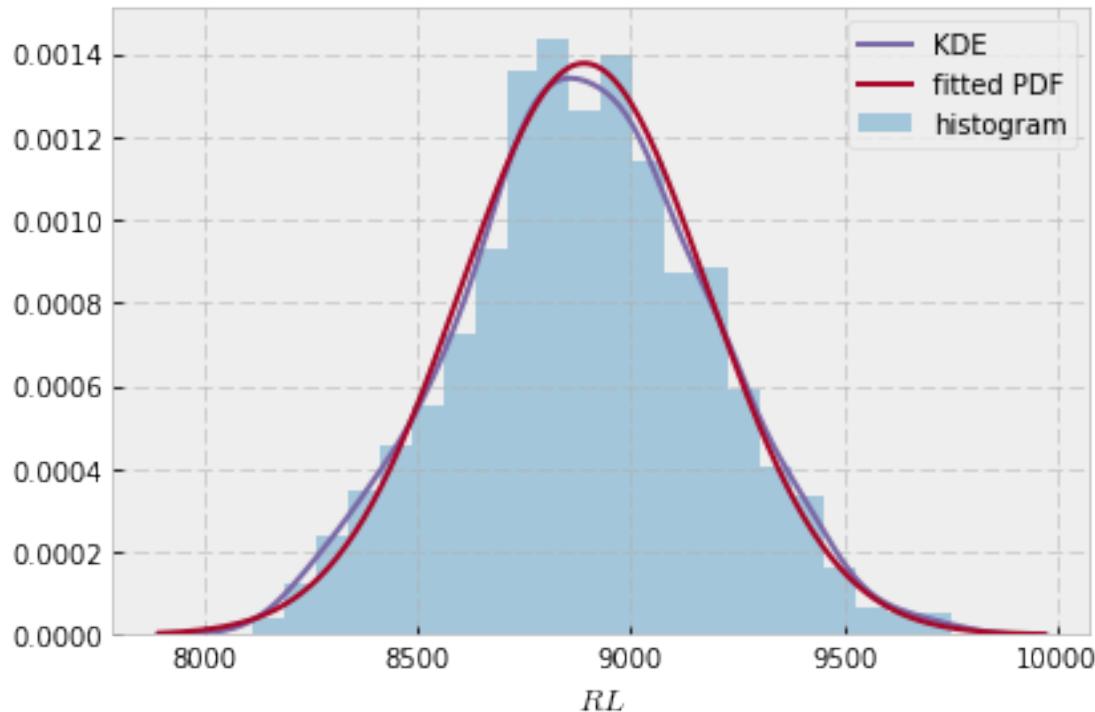


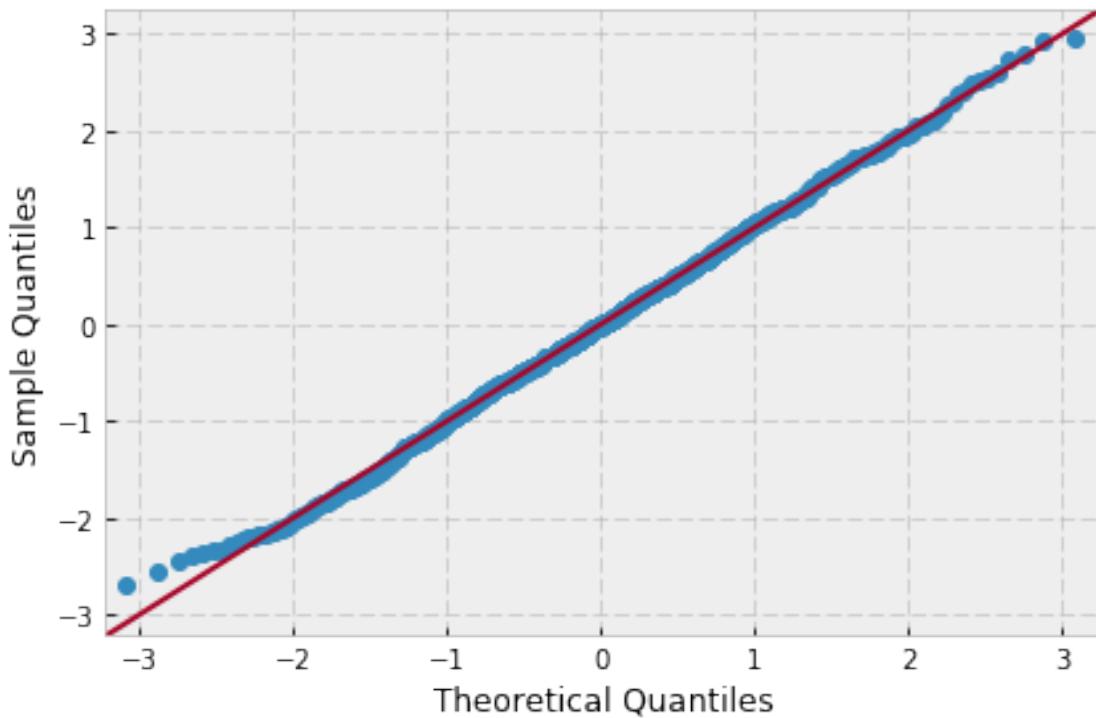
5 Total root length

5.1 Probability distributions

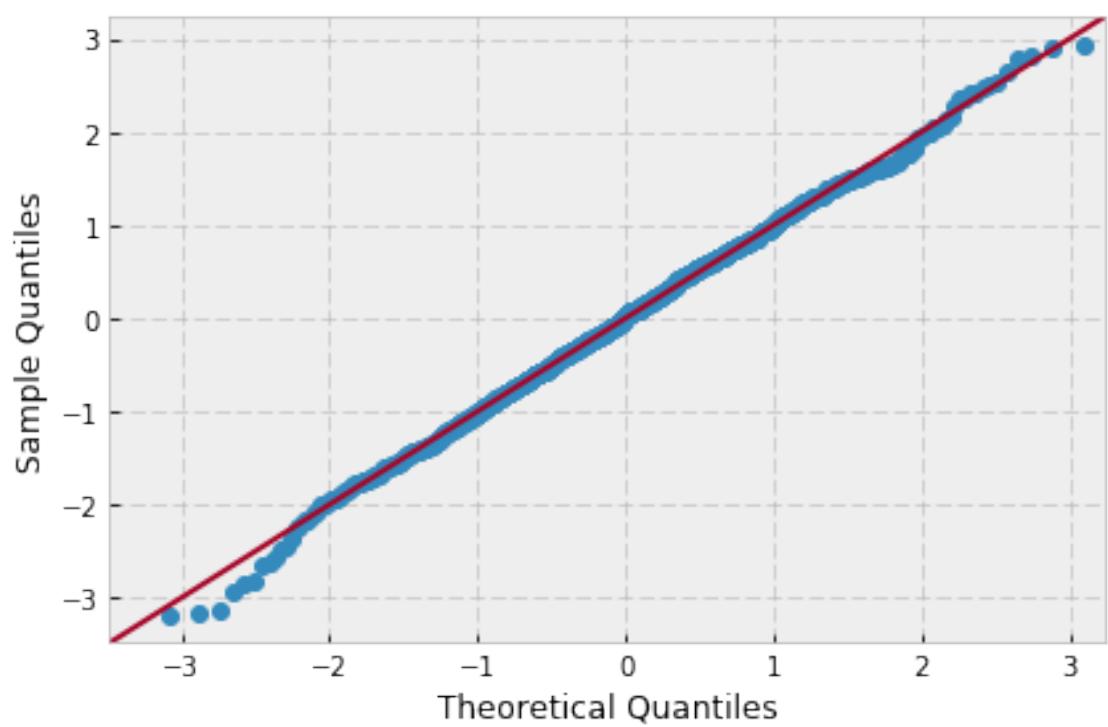
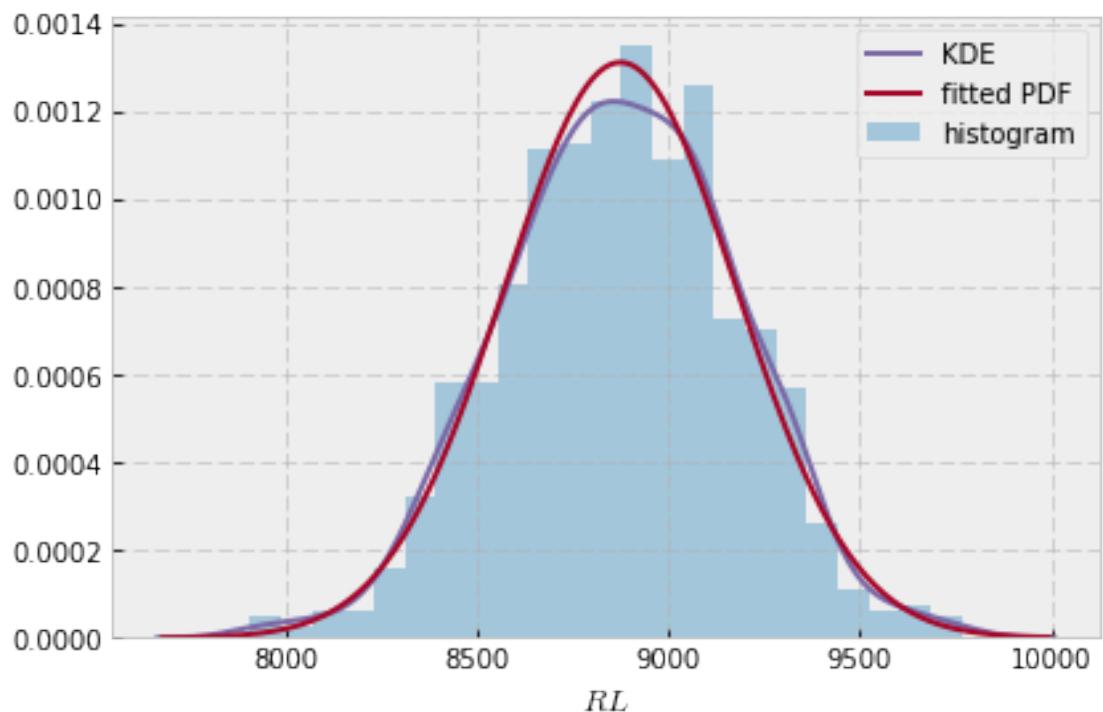
```
In [15]: plot_dist('total_root_length')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=8890.529435828374, scale=289.84949503462246)
DescribeResult(nobs=1000, minmax=(8112.5984027331015, 9750.0770228082965)
    mean=8890.5294358283736, variance=84096.826598424072,
    skewness=0.031793353772924894, kurtosis=-0.20400781064700801)
```

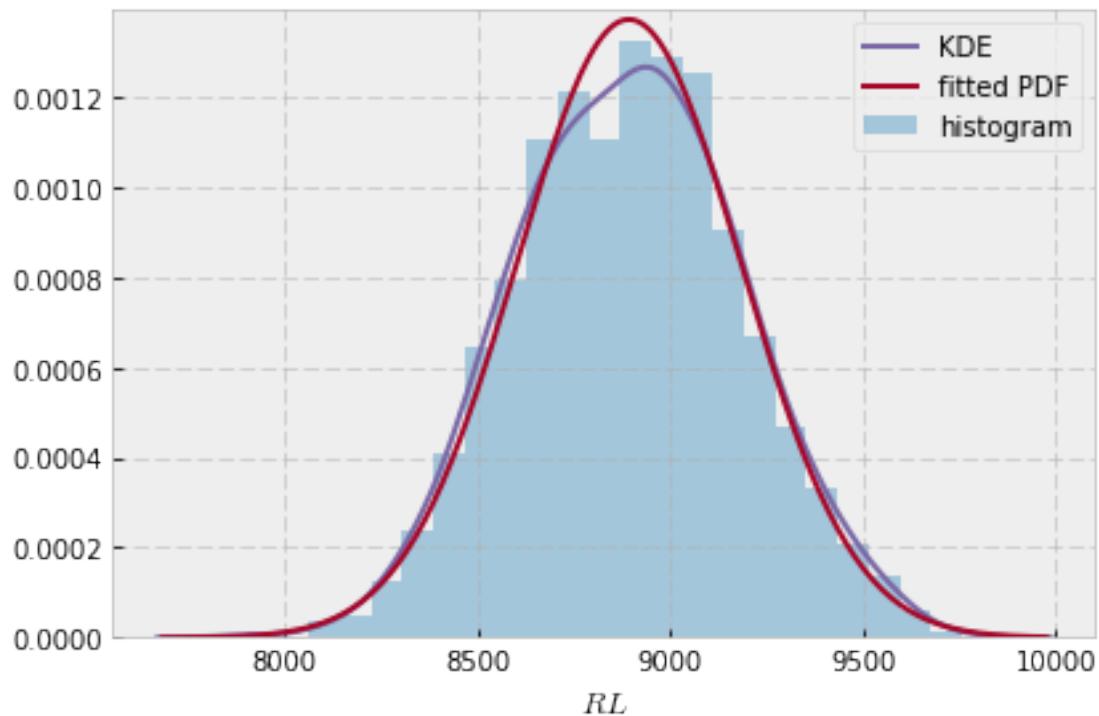


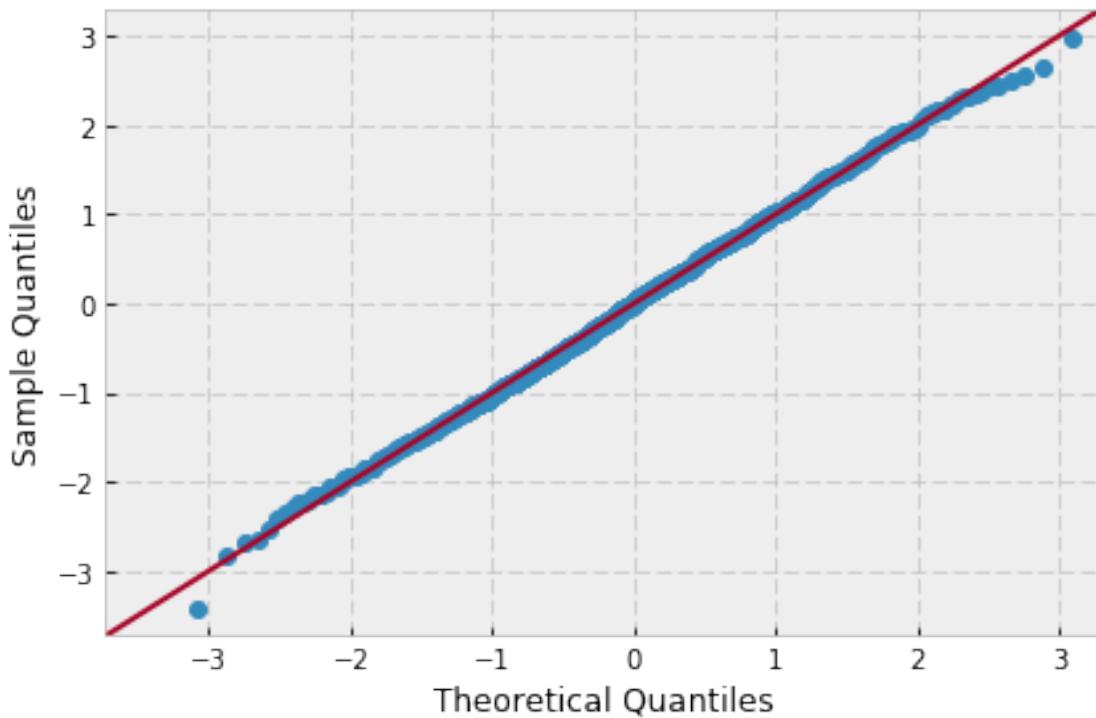


```
variable name: N
variable value: 1.0
distribution: normal(shape=(),loc=8873.204074936704, scale=304.4019958243945)
DescribeResult(nobs=1000, minmax=(7905.0238376431189, 9764.8300570286683)
               mean=8873.2040749367043, variance=92753.328390264956,
               skewness=-0.07983531310620573, kurtosis=-0.011499039756156293)
```

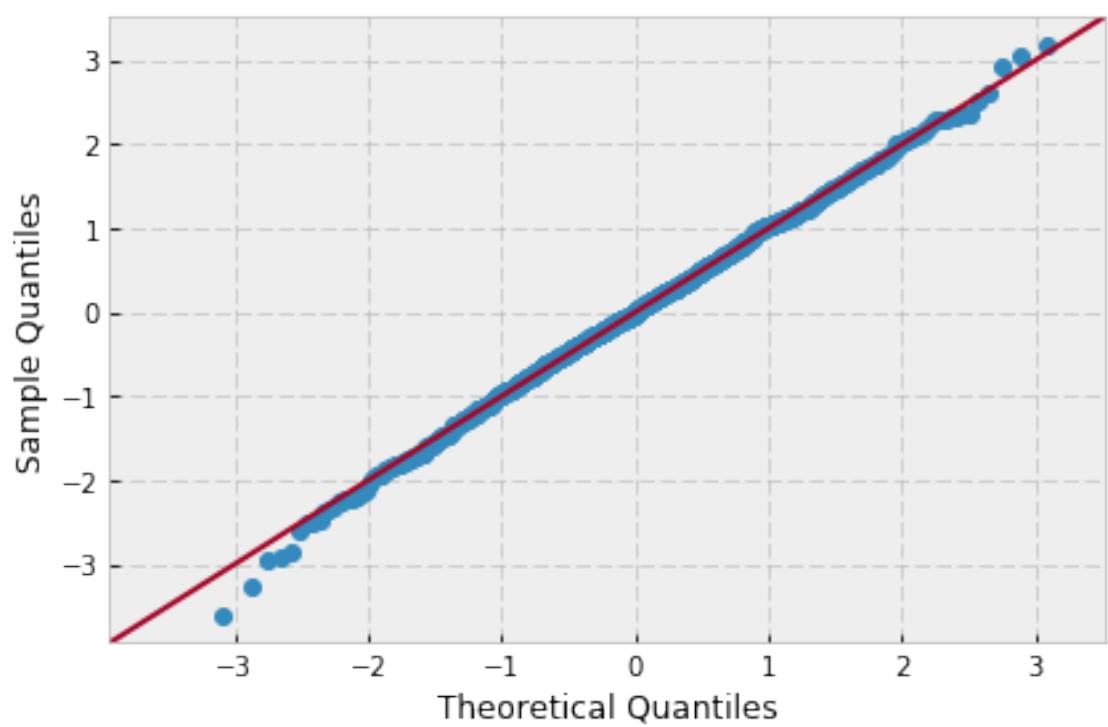
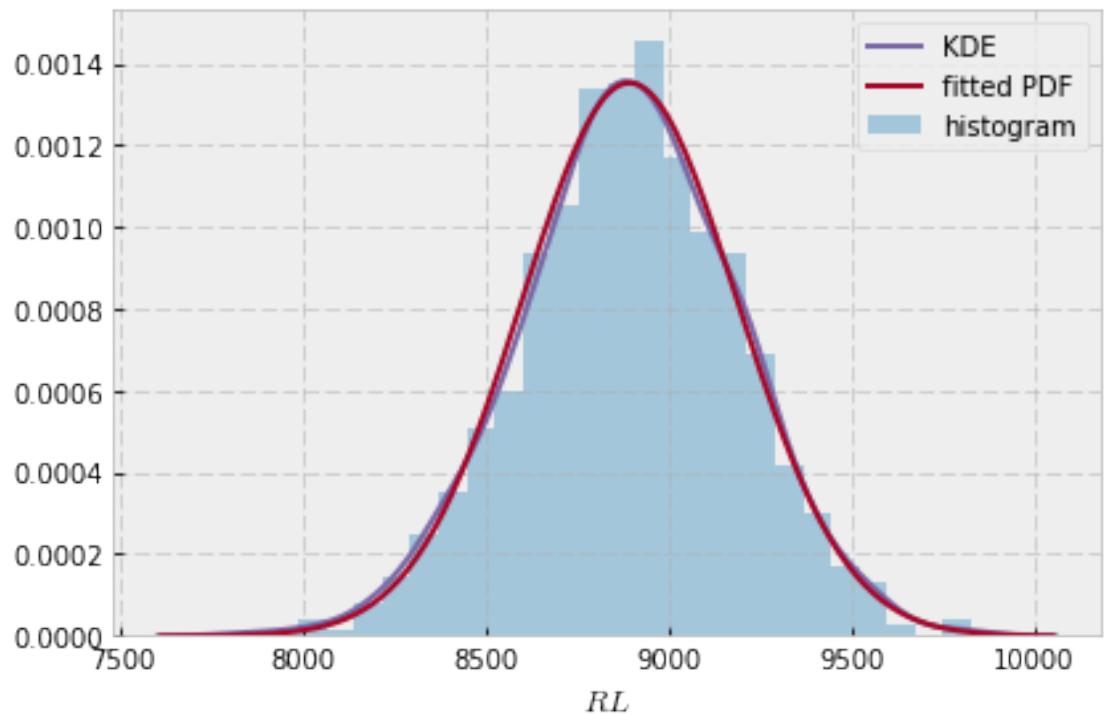


```
variable name: N
variable value: 1.5
distribution: normal(shape=(),loc=8892.405371161714, scale=290.35831355187025)
DescribeResult(nobs=1000, minmax=(7902.4172017548881, 9754.9720688726557)
               mean=8892.4053711617144, variance=84392.342591277469,
               skewness=0.012046332878024873, kurtosis=-0.2456007996134537)
```

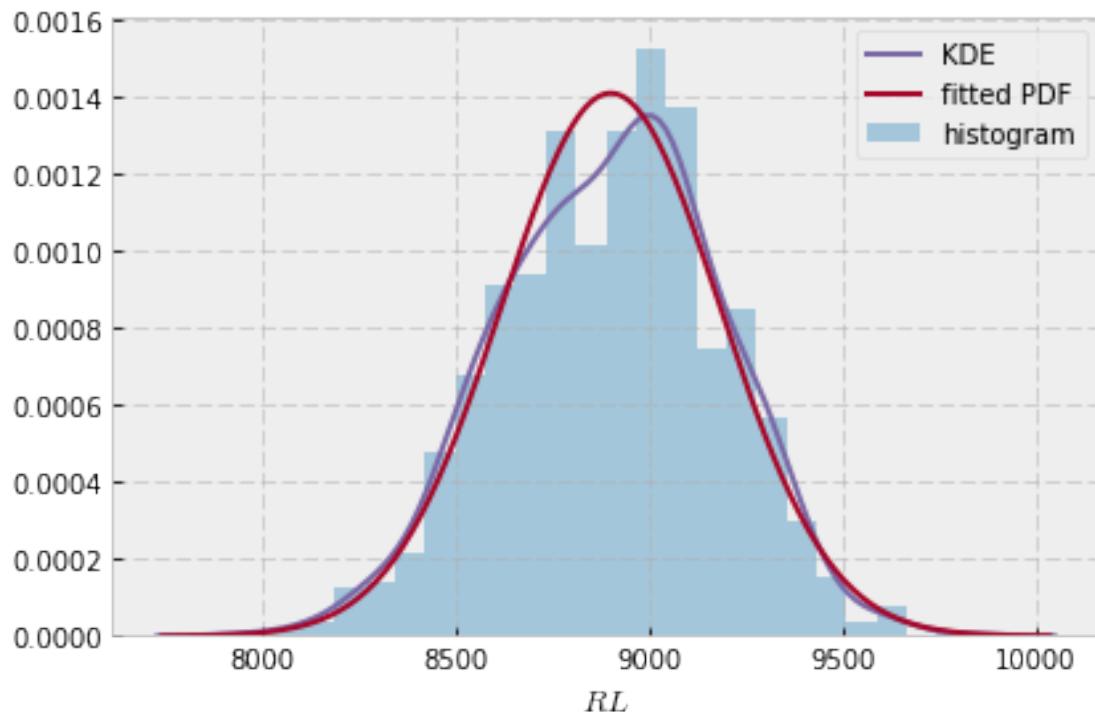


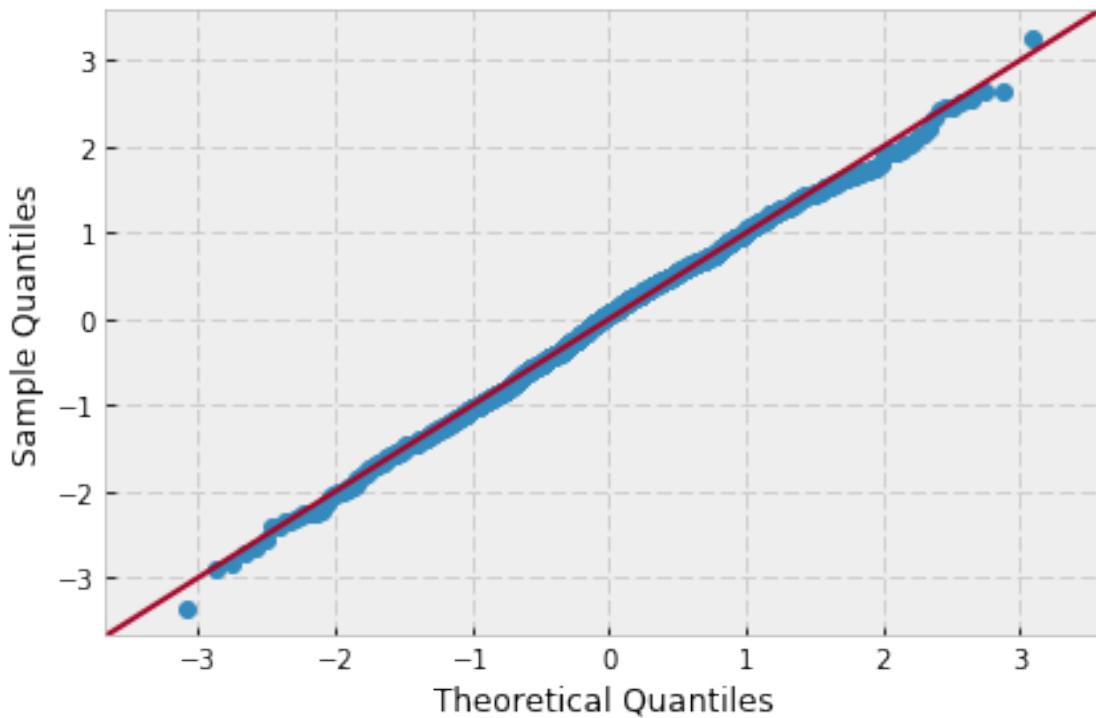


```
variable name: N
variable value: 2.0
distribution: normal(shape=(),loc=8890.936801042883, scale=294.8469442908724)
DescribeResult(nobs=1000, minmax=(7830.2964079502053, 9826.9353065293544)
               mean=8890.9368010428825, variance=87021.742299964768,
               skewness=-0.09043646275551268, kurtosis=0.11323556761954556)
```

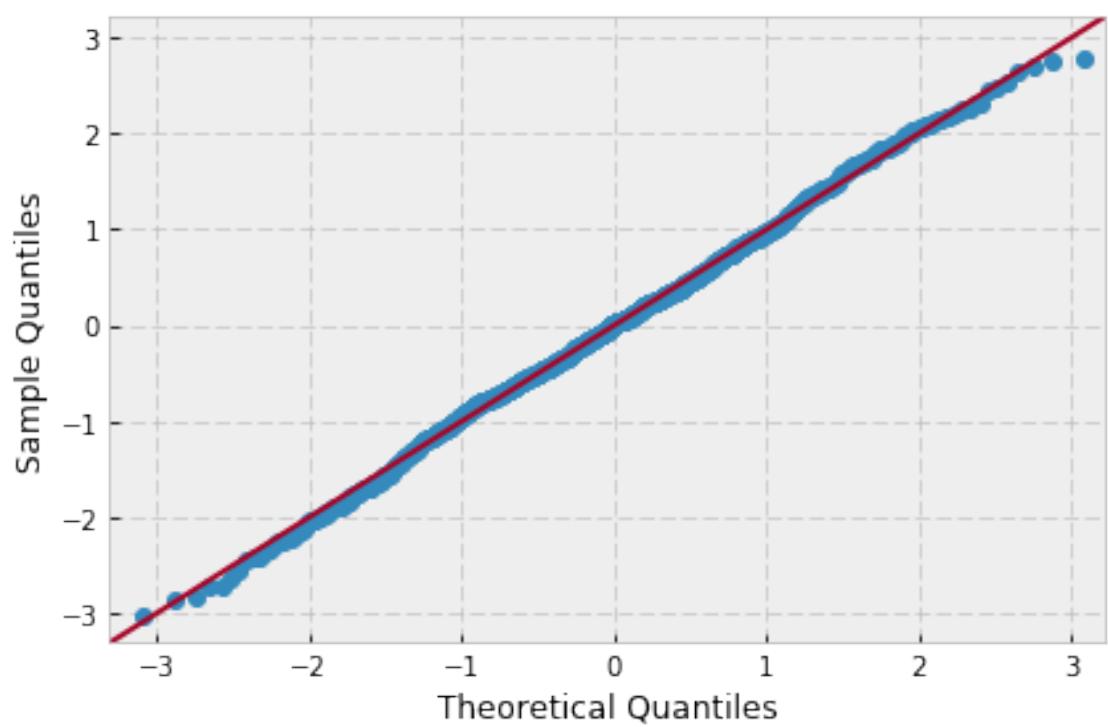
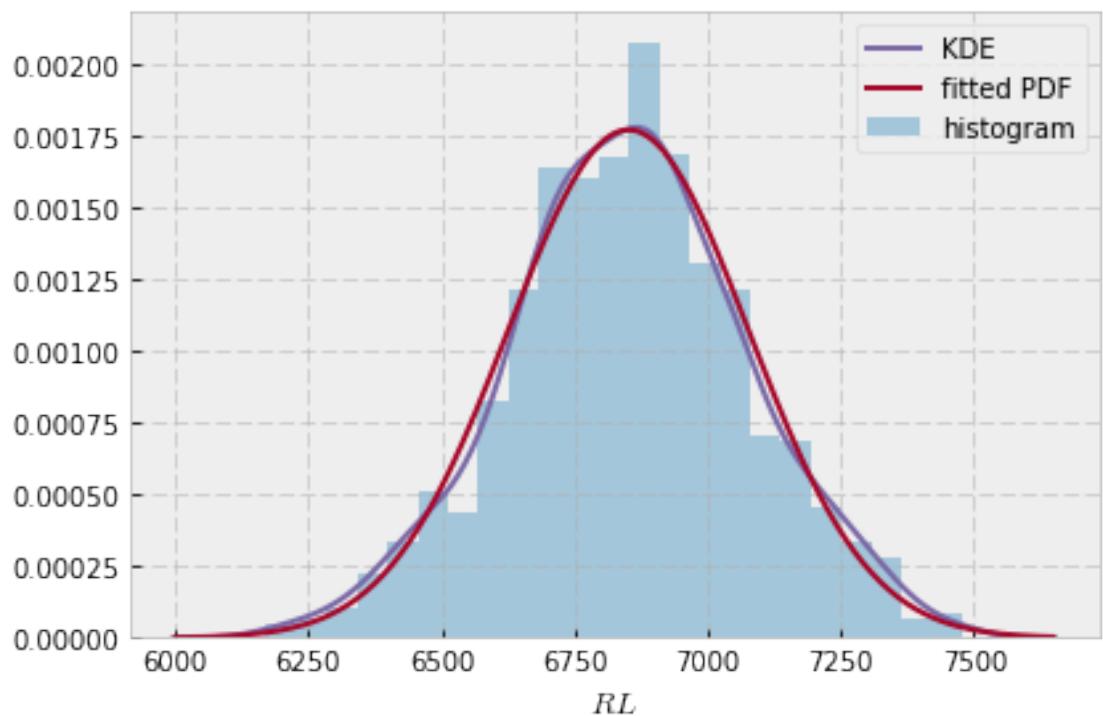


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=8900.776052858922, scale=282.6044073466456)
DescribeResult(nobs=1000, minmax=(7955.1925778830719, 9820.9575209055874)
               mean=8900.7760528589224, variance=79945.196247996777,
               skewness=-0.11533371946890708, kurtosis=-0.2035079698058544)
```

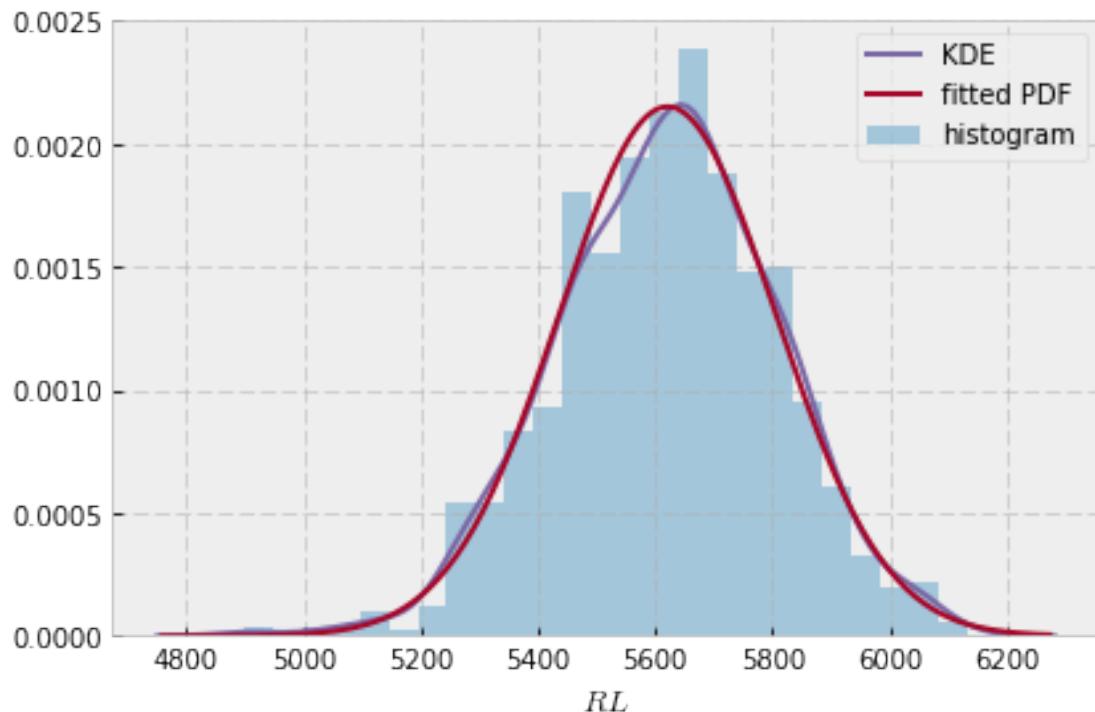


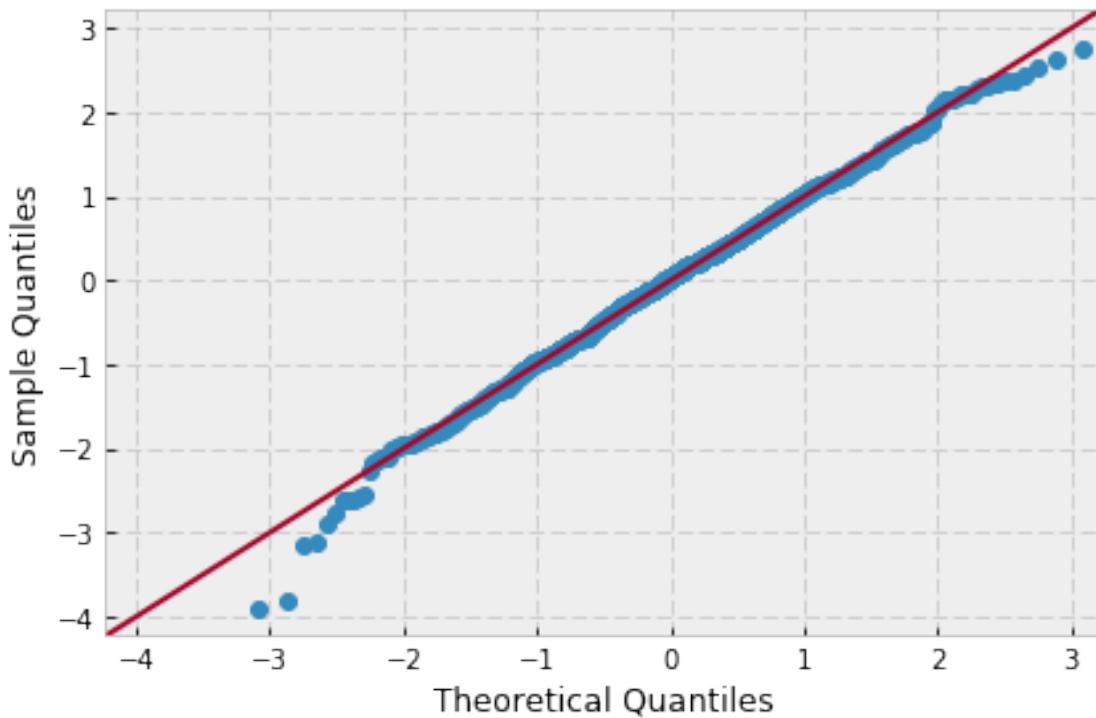


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=6850.932396277722, scale=225.2918586774509)
DescribeResult(nobs=1000, minmax=(6171.1624871924187, 7477.4676692632702)
               mean=6850.9323962777216, variance=50807.228815155657,
               skewness=-0.020611569816929326, kurtosis=-0.014867059102519065)
```

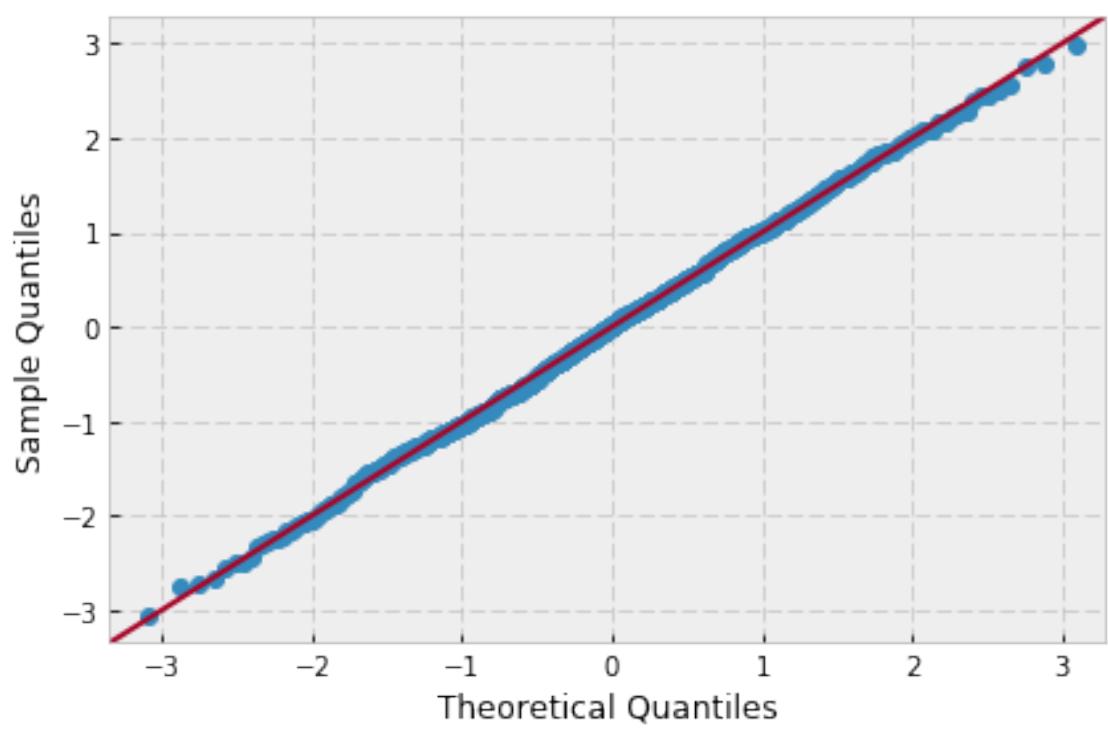
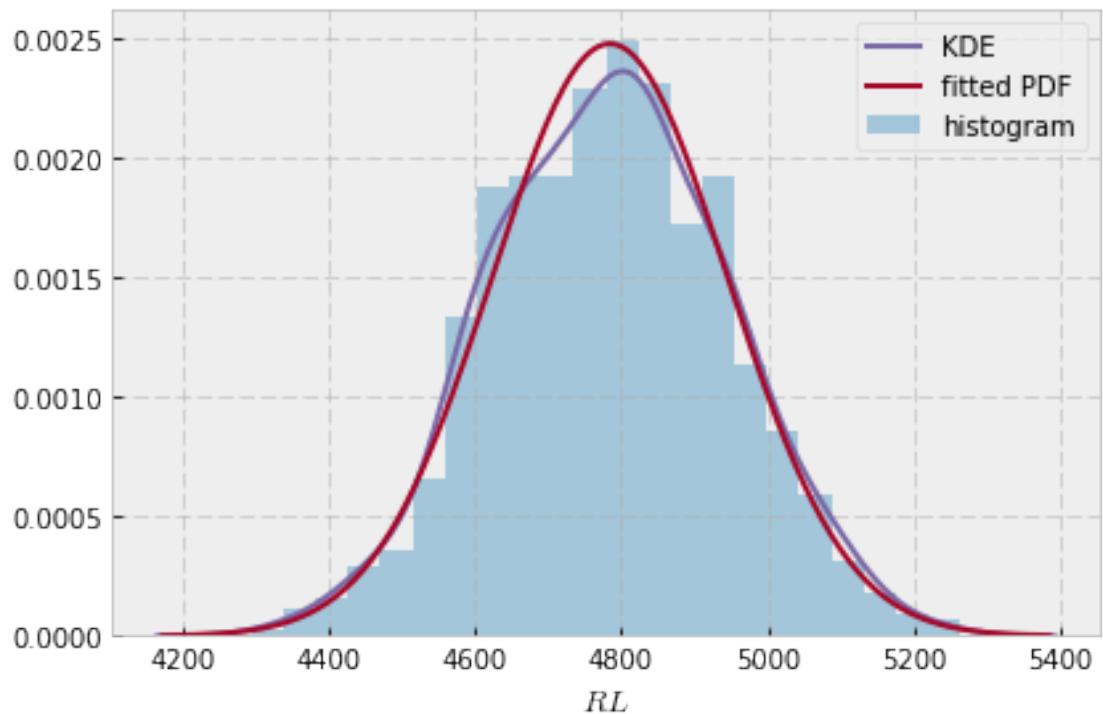


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=5620.805627685591, scale=185.50625961667004)
DescribeResult(nobs=1000, minmax=(4897.7482895044286, 6133.0176440698897)
               mean=5620.8056276855914, variance=34447.019376343735,
               skewness=-0.17651174512214668, kurtosis=0.17163382480163758)
```

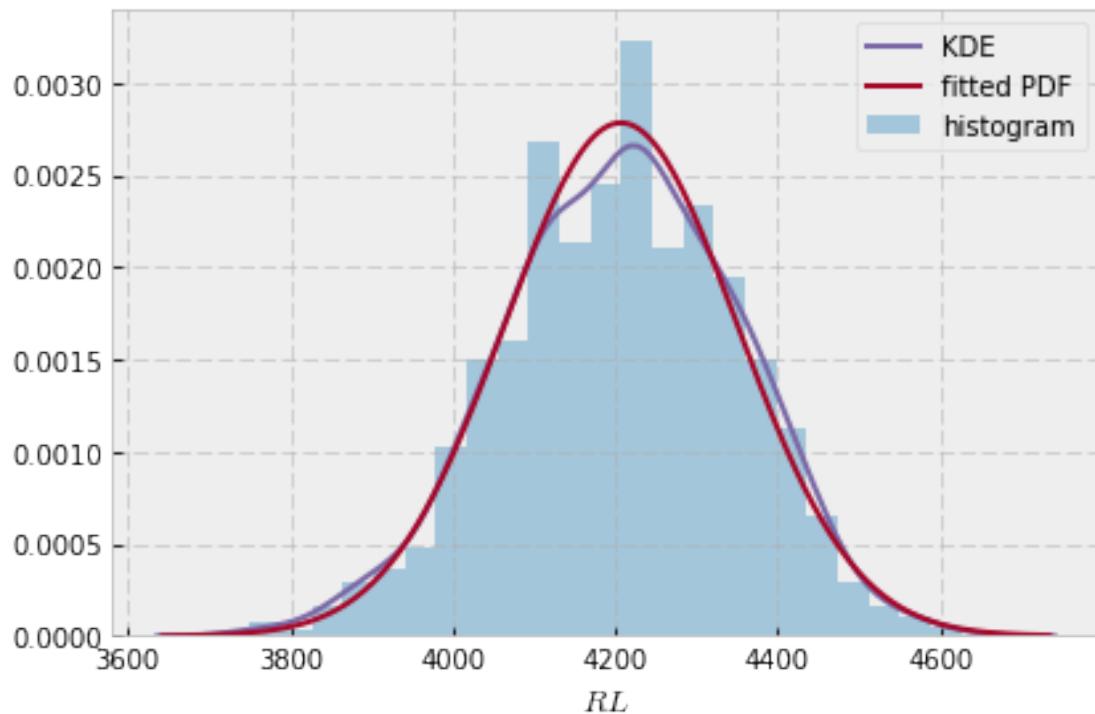


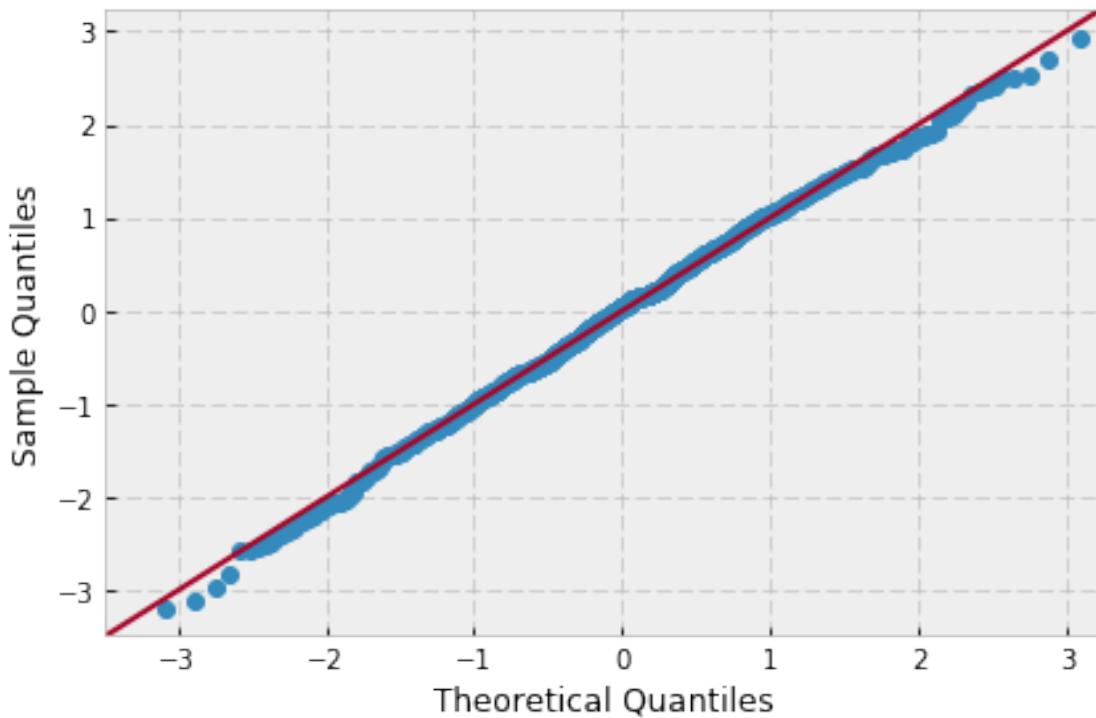


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(),loc=4784.389372724816, scale=160.7516435428016)
DescribeResult(nobs=1000, minmax=(4294.5114110423237, 5263.1649763019759)
               mean=4784.389372724816, variance=25866.957859571514,
               skewness=0.012379054961225256, kurtosis=-0.2097827580149758)
```

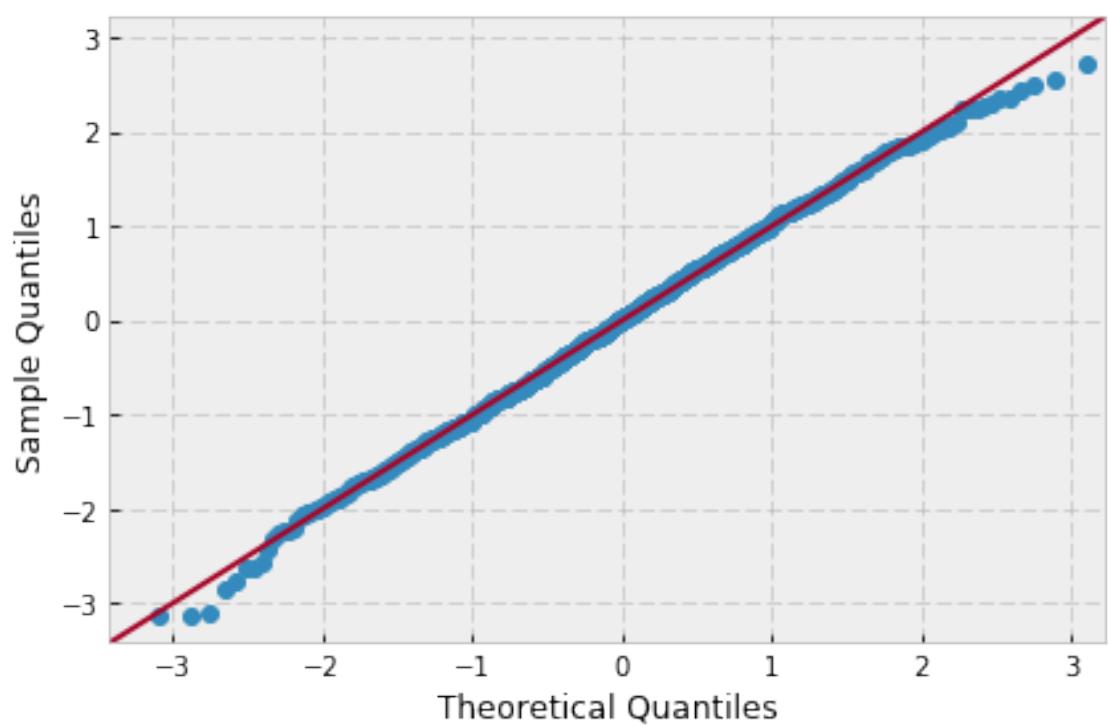
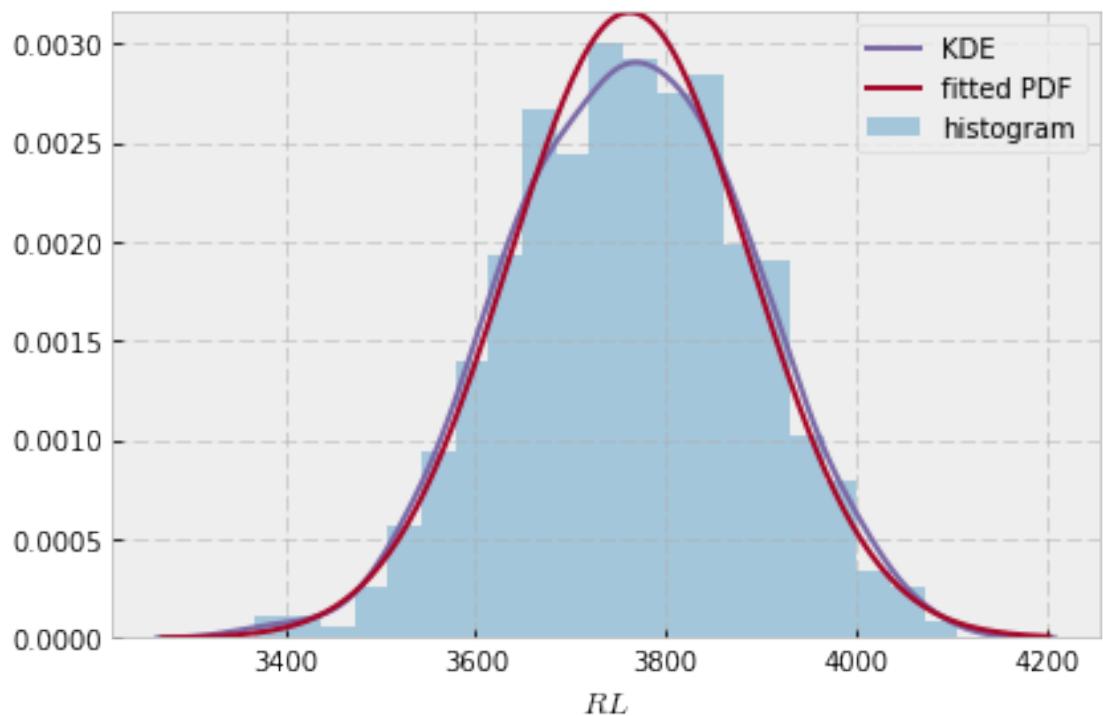


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=4206.746806533822, scale=143.3159677812821)
DescribeResult(nobs=1000, minmax=(3750.8179848574632, 4625.5541288561899)
               mean=4206.7468065338217, variance=20560.026647733222,
               skewness=-0.13398904242060464, kurtosis=-0.14836545976938575)
```

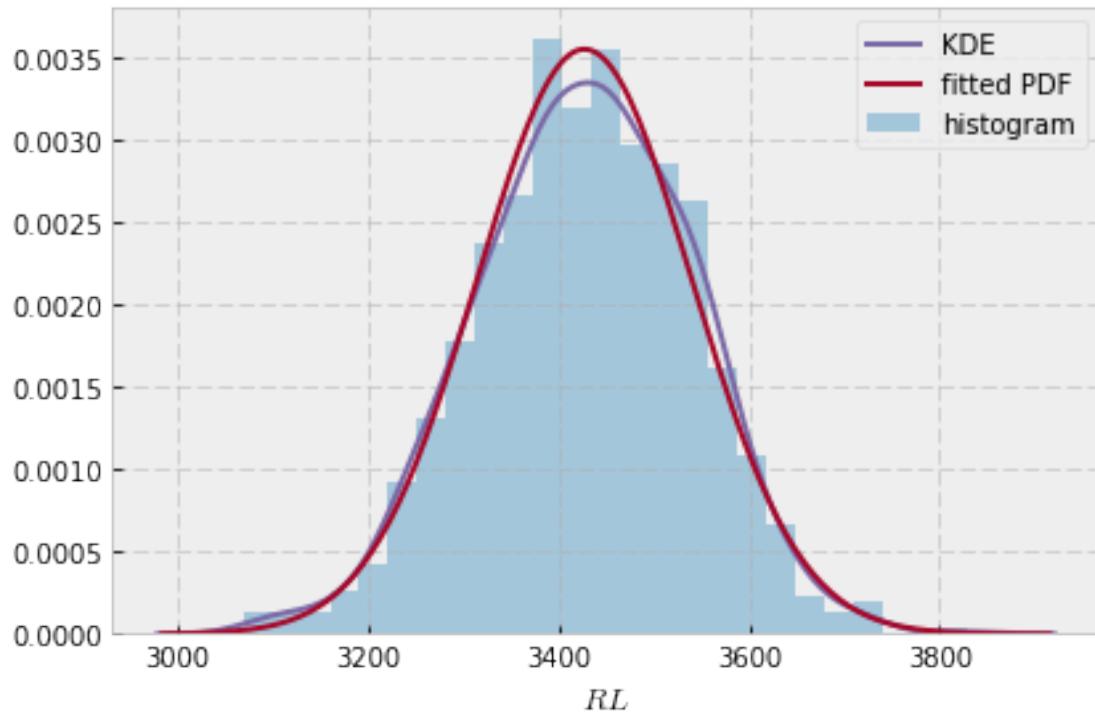


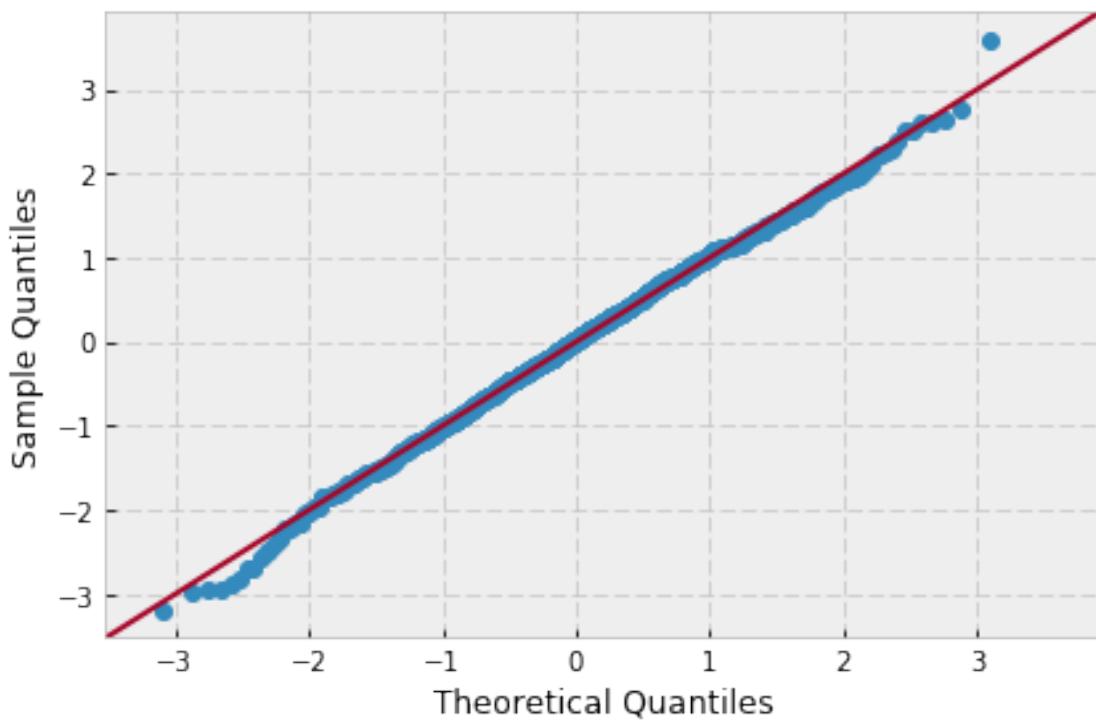


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=3761.9597386000996, scale=126.35956674860543)
DescribeResult(nobs=1000, minmax=(3366.8678093598078, 4106.9079484135336)
               mean=3761.9597386000996, variance=15982.722831726998,
               skewness=-0.0827710348967881, kurtosis=-0.22525111461016722)
```

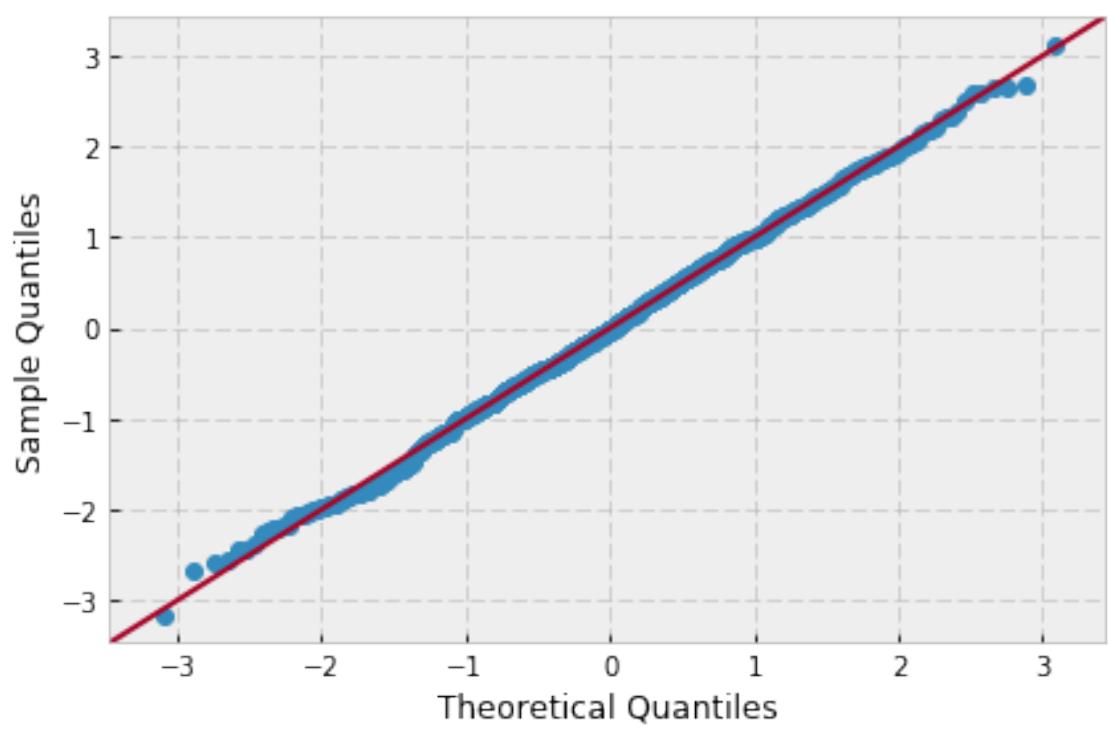
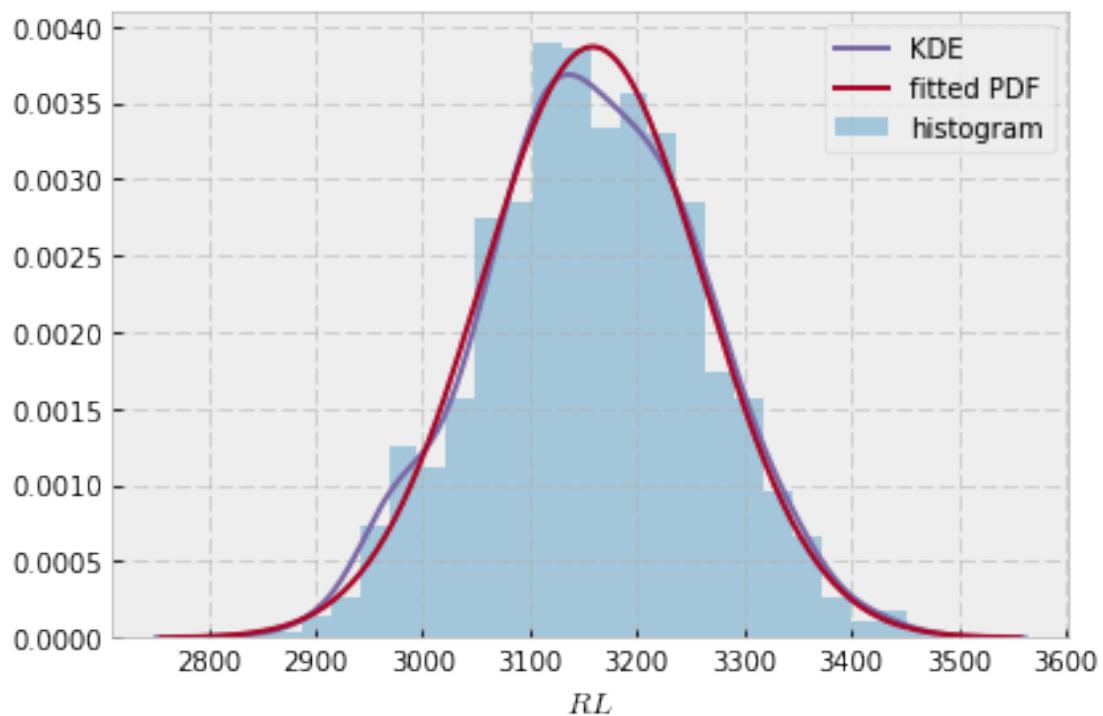


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=3427.360485073408, scale=112.52590688544345)
DescribeResult(nobs=1000, minmax=(3069.3605578012694, 3830.2740888522012)
               mean=3427.3604850734082, variance=12674.754474866357,
               skewness=-0.11342266240425428, kurtosis=-0.00517966282550697)
```

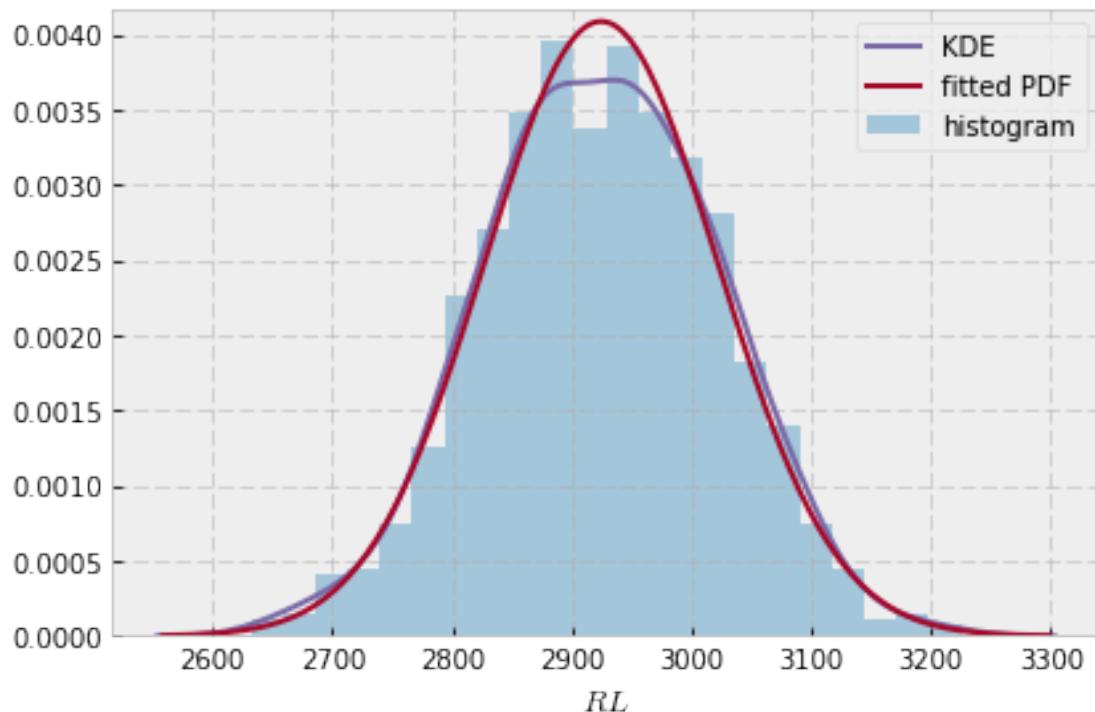


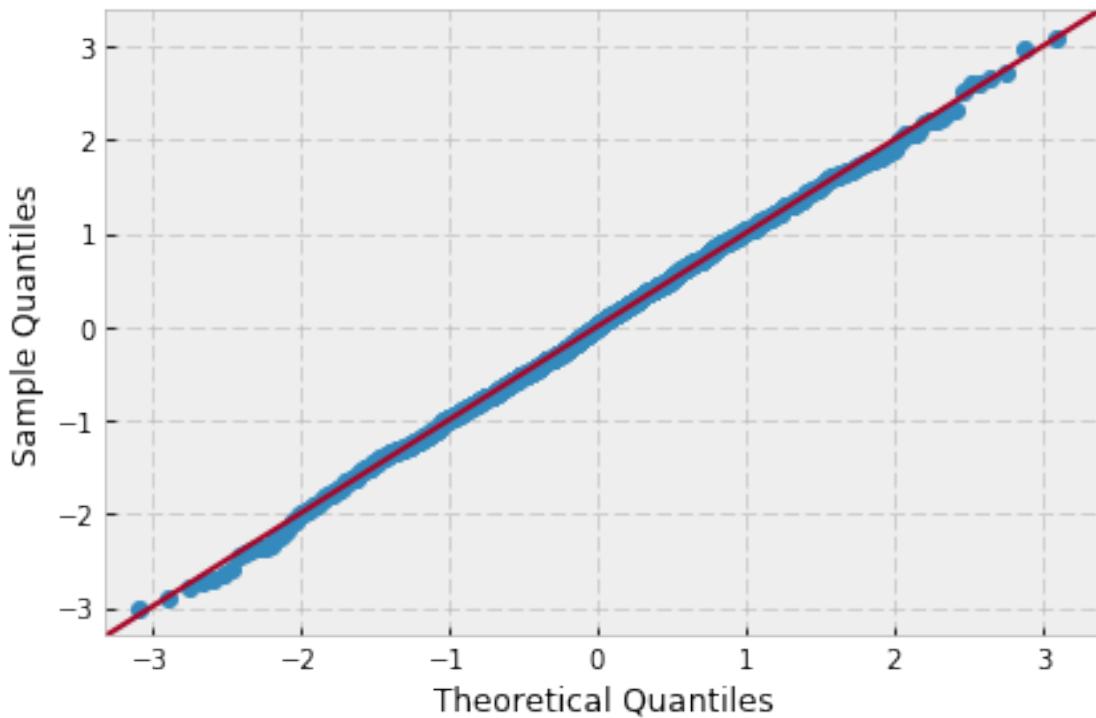


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=3158.666475600519, scale=103.15864875983623)
DescribeResult(nobs=1000, minmax=(2833.4478495650496, 3479.6269142749657)
               mean=3158.6664756005189, variance=10652.359173128387,
               skewness=-0.010504683125537847, kurtosis=-0.20147794697938215)
```

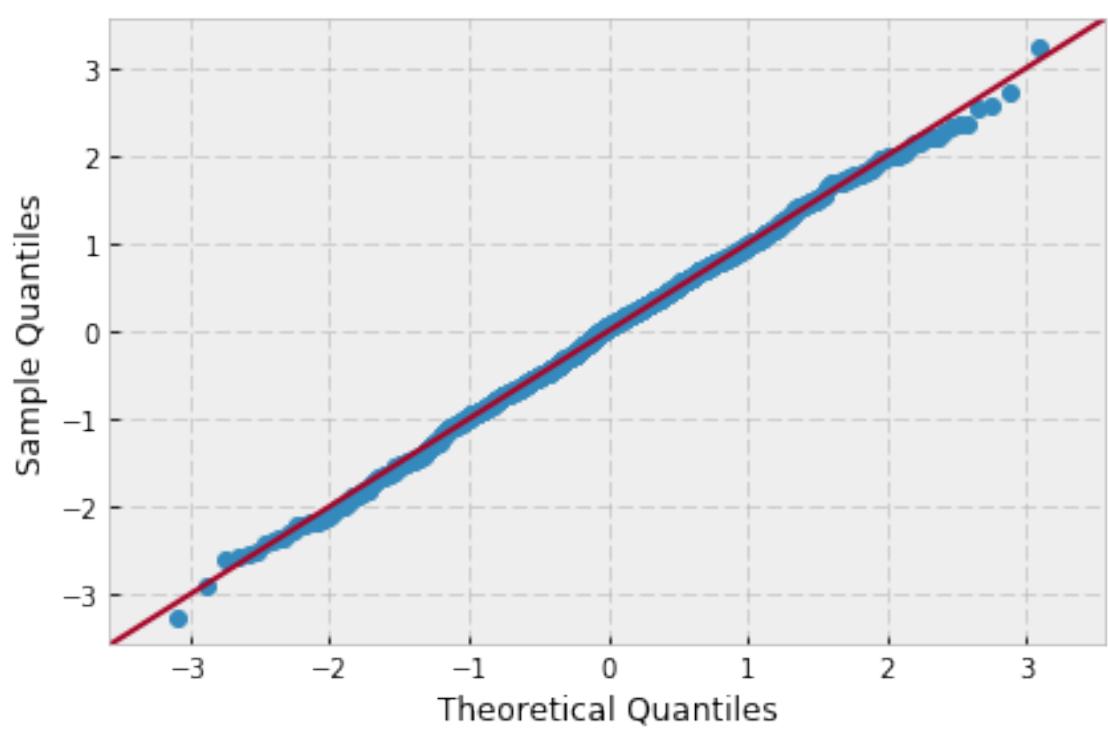
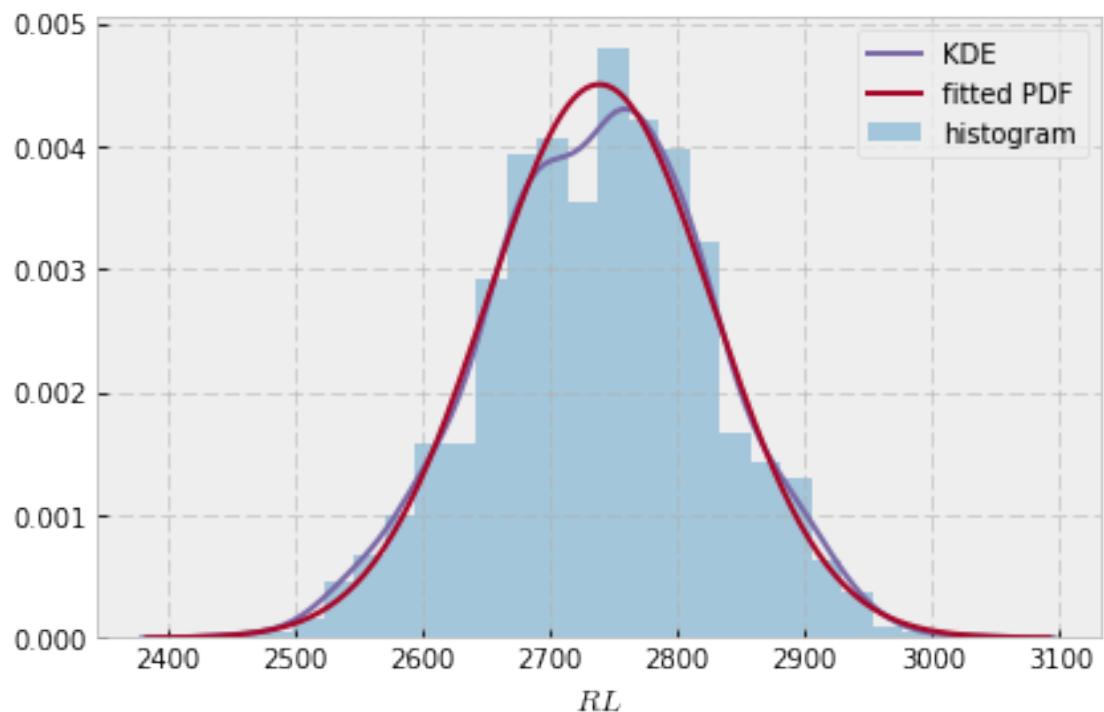


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=2924.502294457333, scale=97.59089803025766)
DescribeResult(nobs=1000, minmax=(2631.5967404984822, 3225.2199607539574)
               mean=2924.5022944573329, variance=9533.5168952473978,
               skewness=-0.02870972391914144, kurtosis=-0.1379215911565077)
```

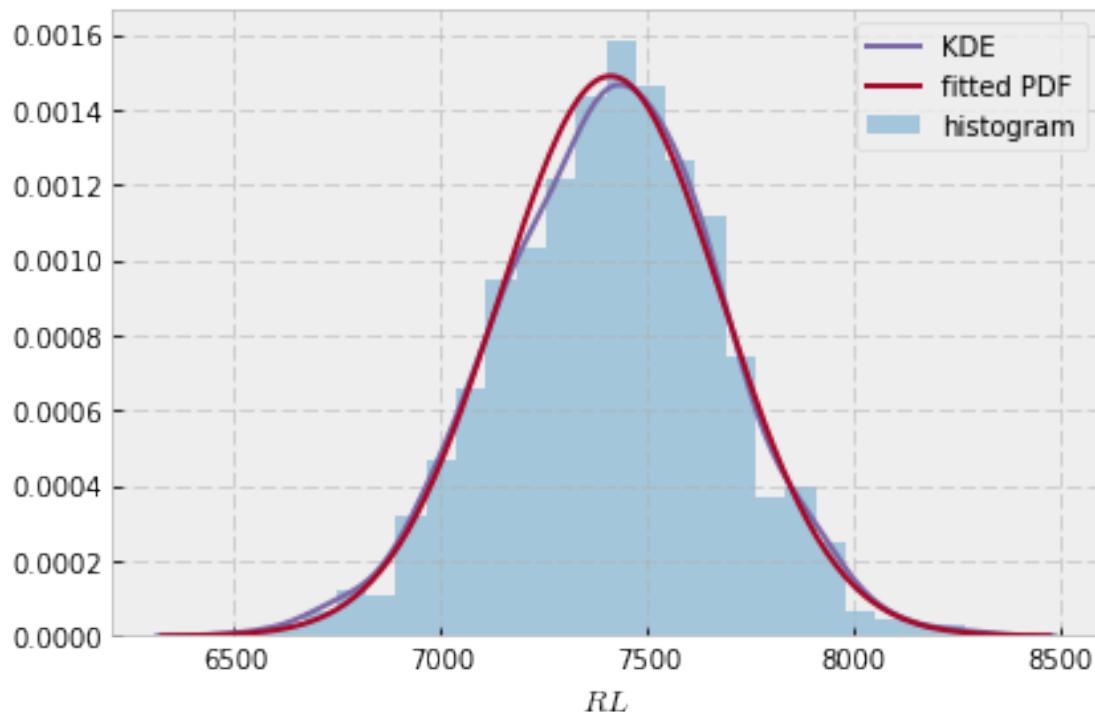


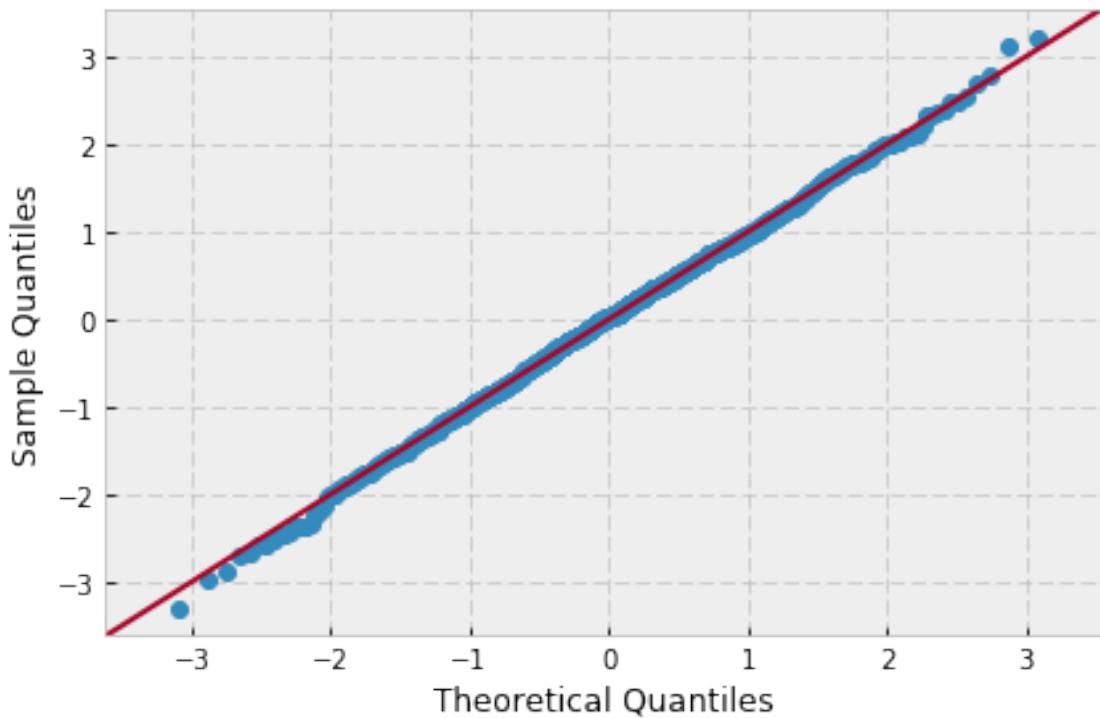


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=2738.7636936870203, scale=88.63310823777364)
DescribeResult(nobs=1000, minmax=(2450.7794475778883, 3025.0895344693217)
               mean=2738.7636936870203, variance=7863.6915674563534,
               skewness=-0.0672496364558661, kurtosis=-0.14962943412561058)
```

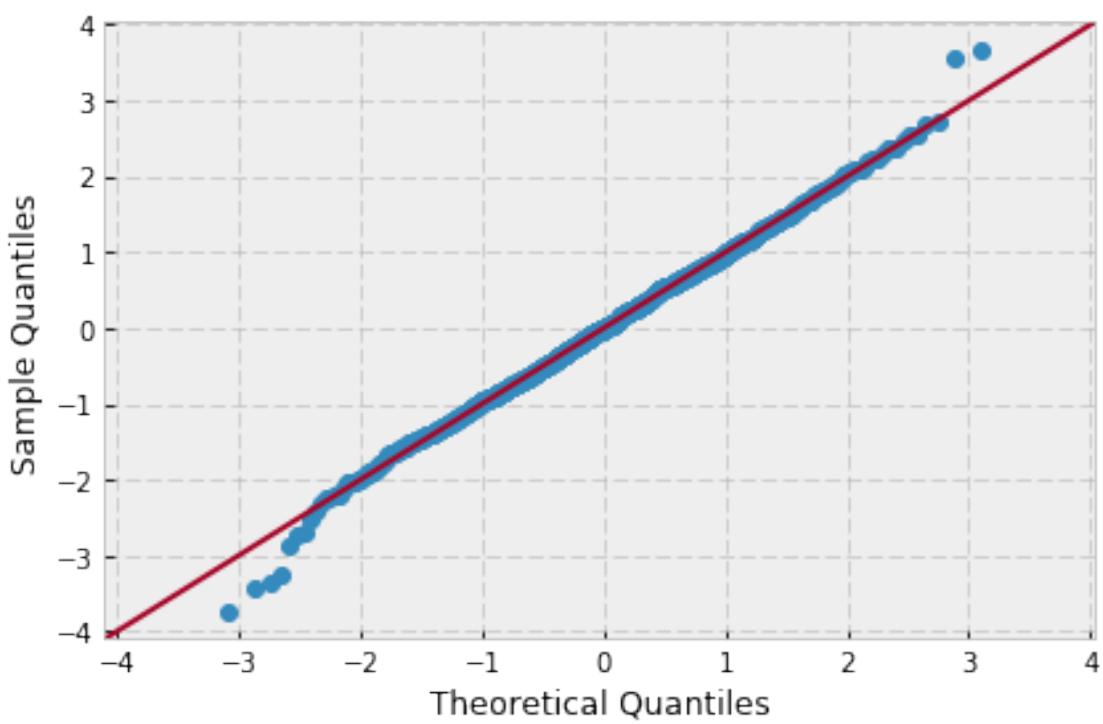
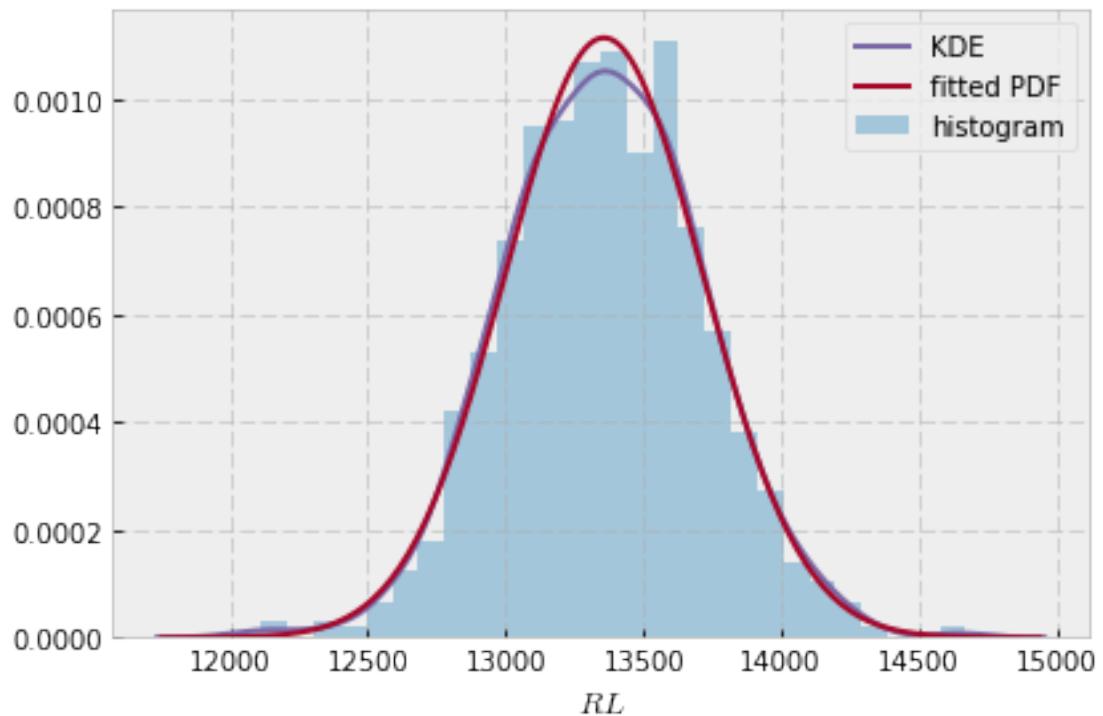


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(), loc=7409.690513553469, scale=267.9966521765638)
DescribeResult(nobs=1000, minmax=(6527.7502679954532, 8269.0018235244679)
               mean=7409.6905135534689, variance=71894.099677523656,
               skewness=-0.06277769497652826, kurtosis=0.02547766388910766)
```

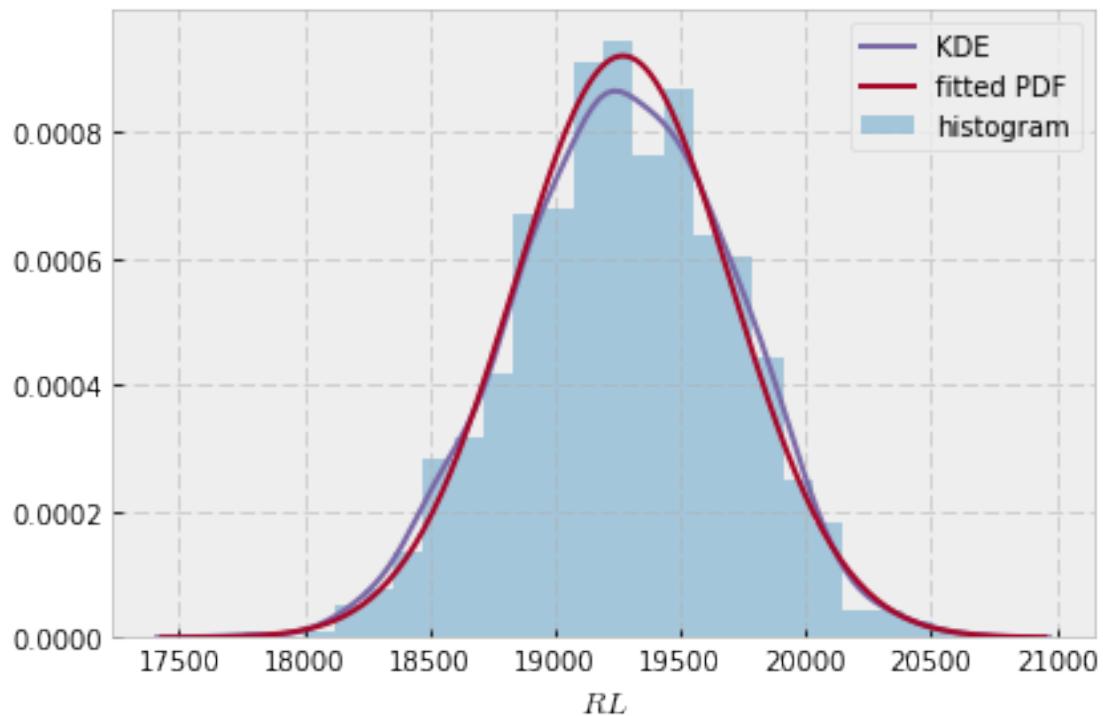


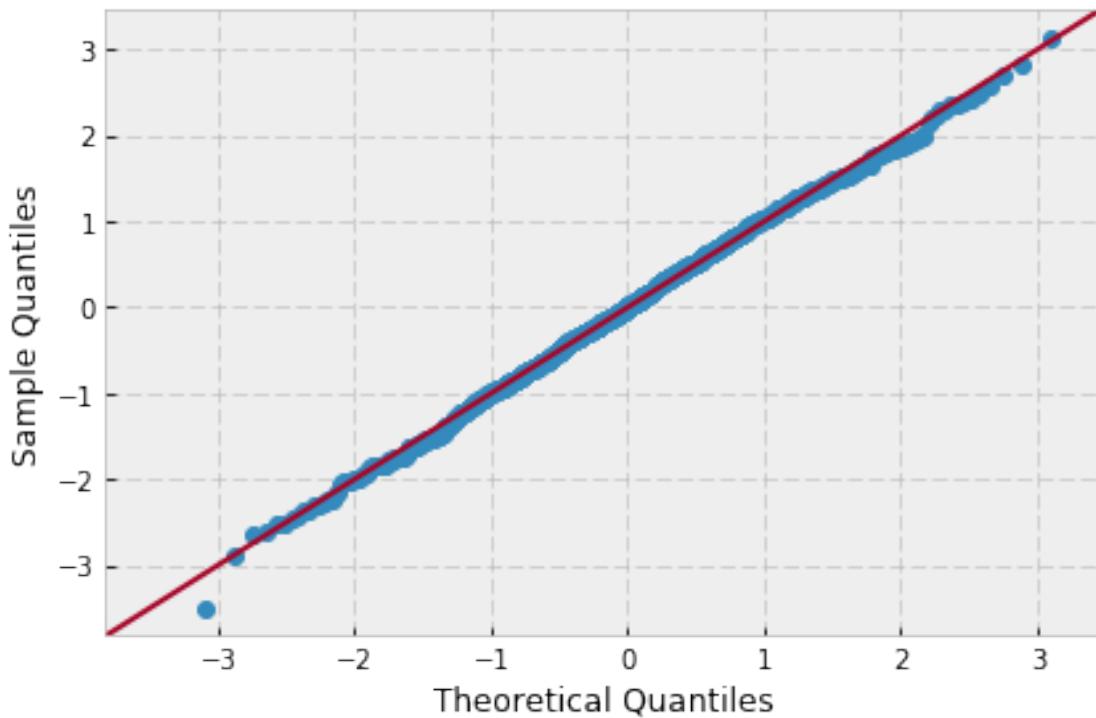


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(),loc=13356.337228294986, scale=358.13075222657824)
DescribeResult(nobs=1000, minmax=(12019.876944371916, 14668.869846186924)
               mean=13356.337228294986, variance=128386.02171208685,
               skewness=-0.022808967433929173, kurtosis=0.3117518132110906)
```

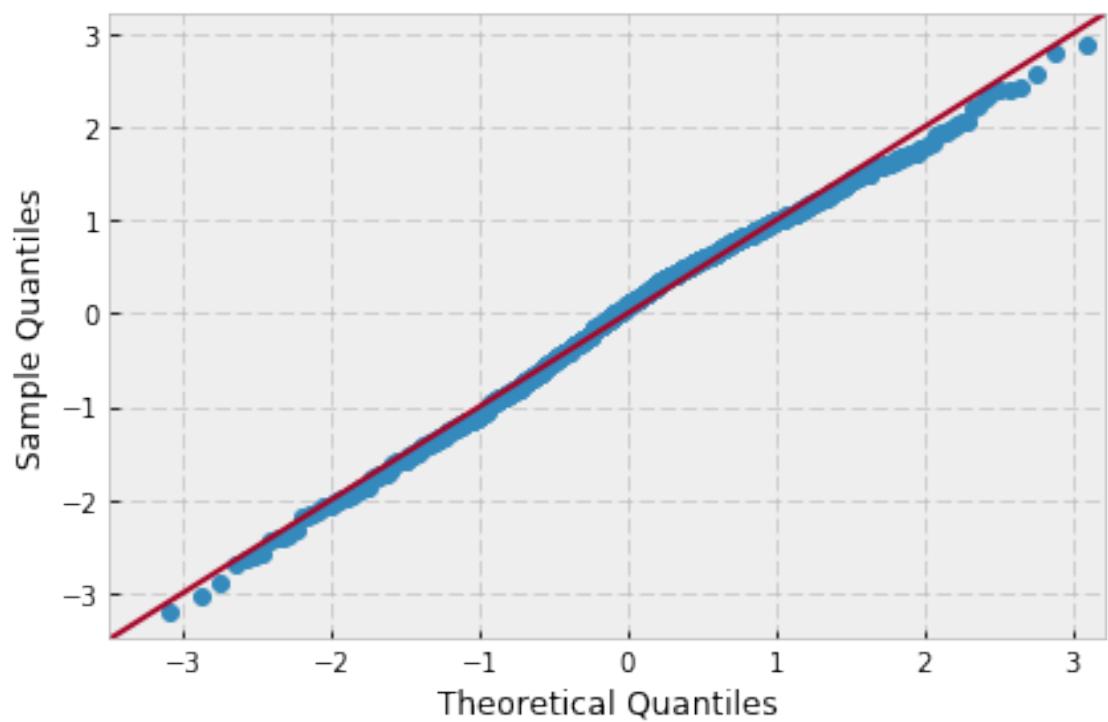
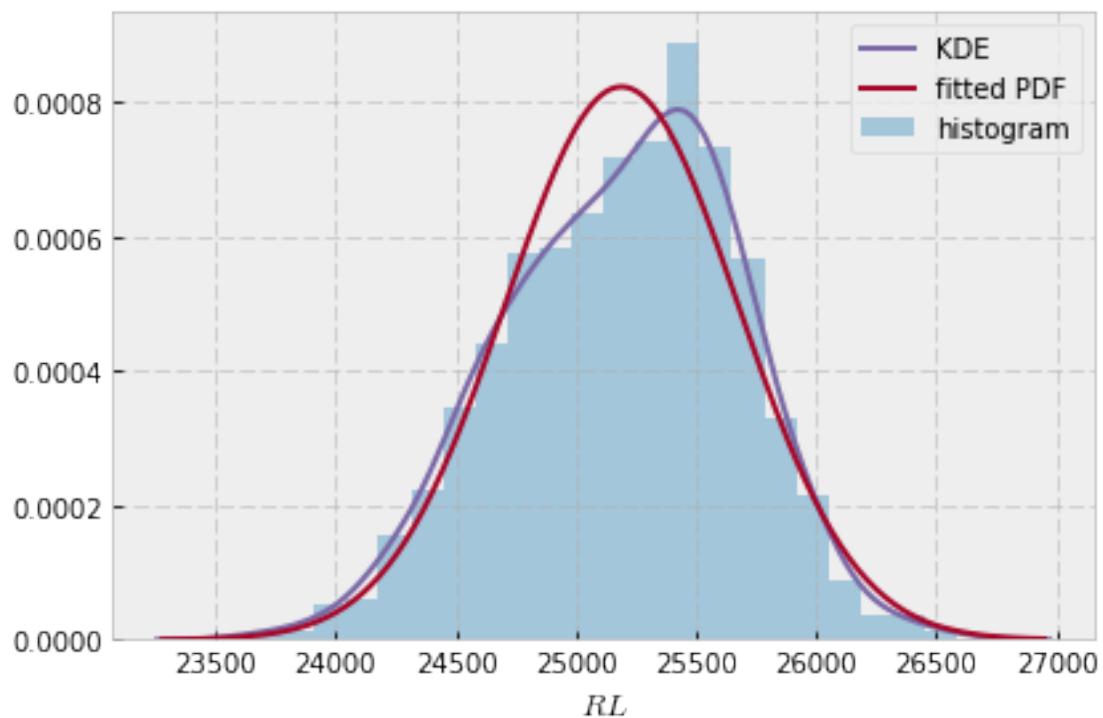


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(), loc=19270.670531146, scale=433.5323001902279)
DescribeResult(nobs=1000, minmax=(17757.663620377603, 20626.321432418146)
               mean=19270.670531145999, variance=188138.39370193181,
               skewness=-0.08725808785972644, kurtosis=-0.17795475201211497)
```

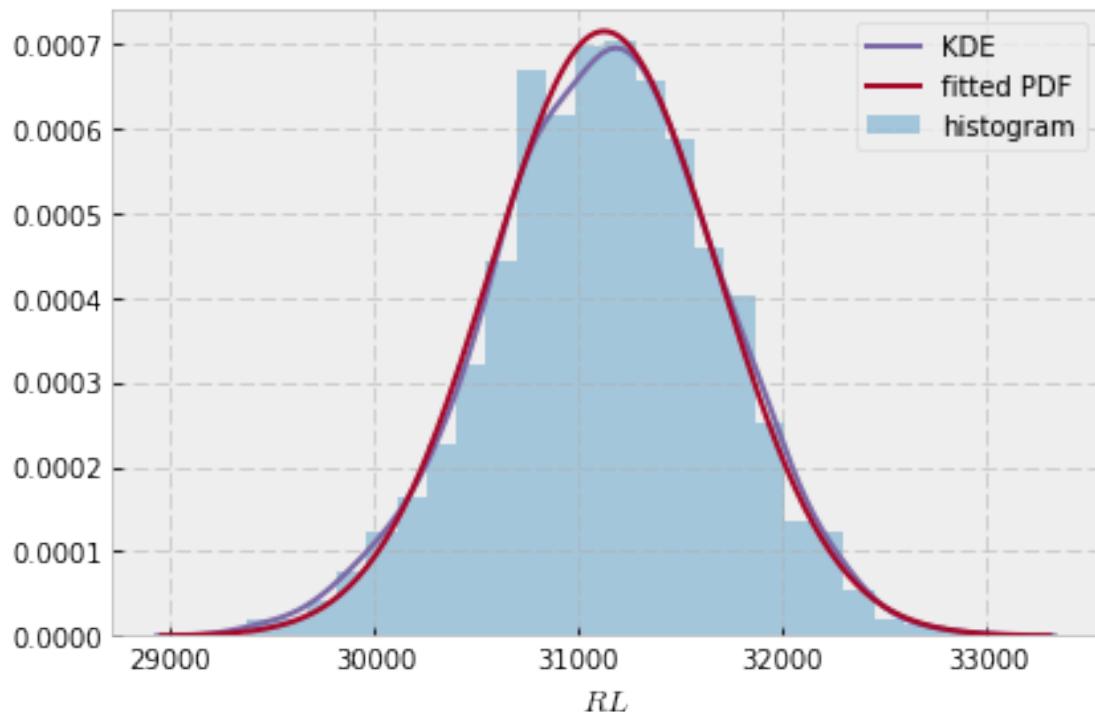


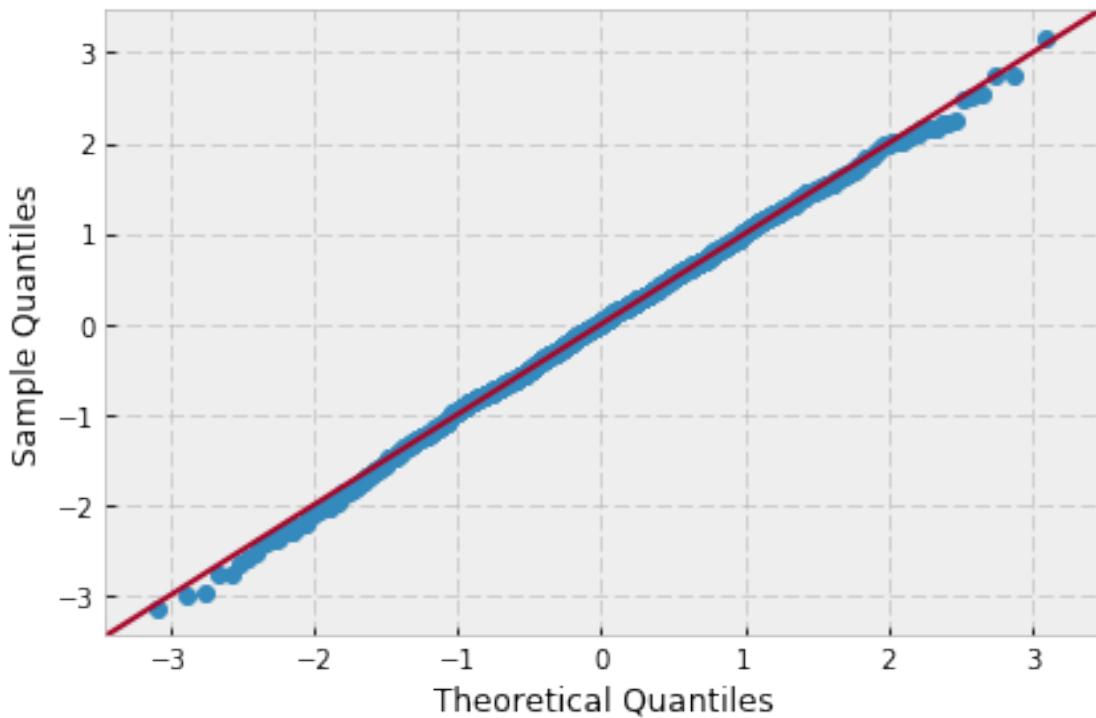


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(),loc=25189.69380010707, scale=484.8663667831329)
DescribeResult(nobs=1000, minmax=(23642.123697203271, 26585.403340460733)
               mean=25189.69380010707, variance=235330.72436183738,
               skewness=-0.21723937374568827, kurtosis=-0.2751994383044343)
```

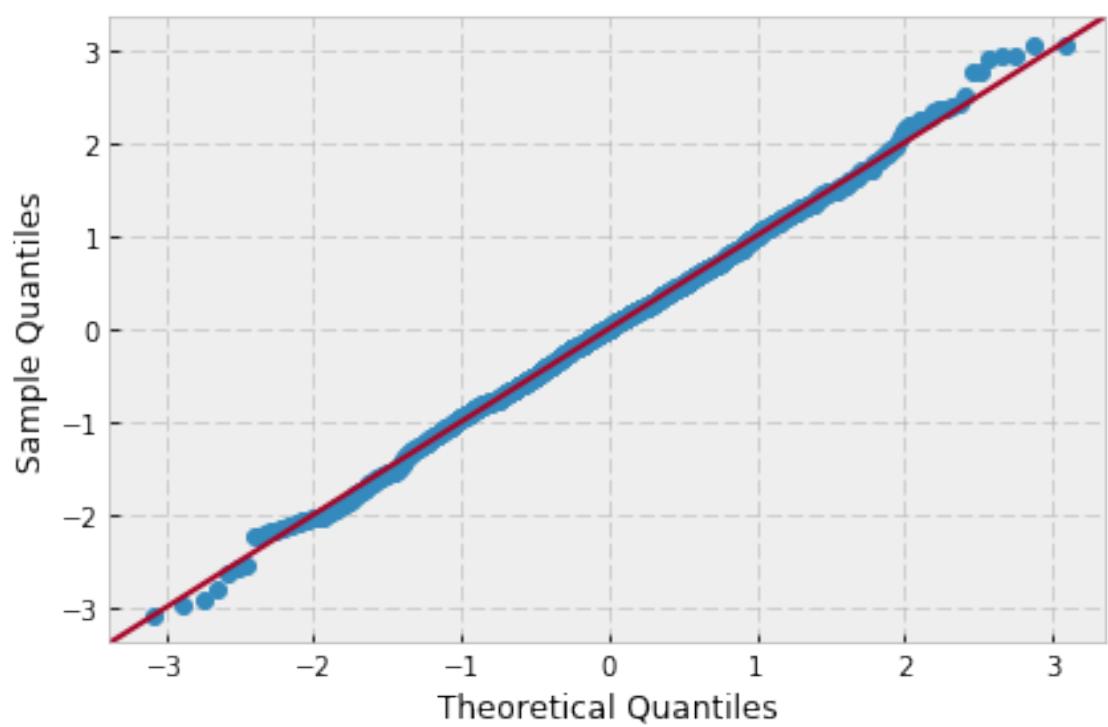
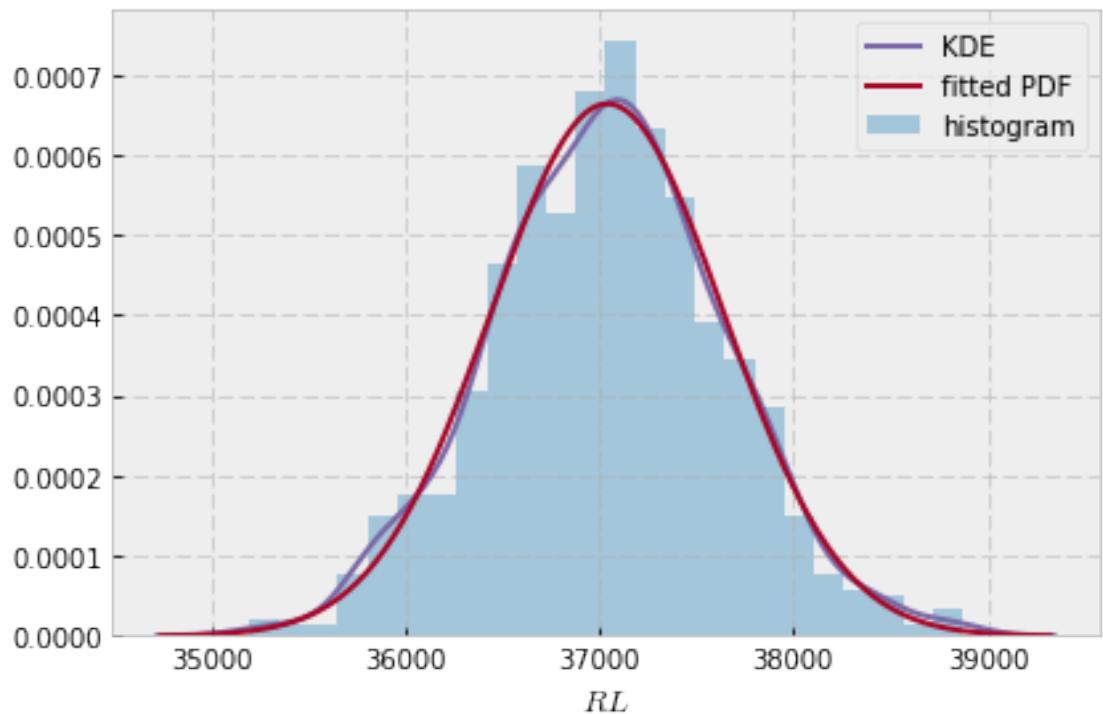


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(), loc=31122.74460758081, scale=557.9087603374098)
DescribeResult(nobs=1000, minmax=(29376.923701765532, 32883.471664190751)
               mean=31122.744607580811, variance=311573.7586198452,
               skewness=-0.10826706326997004, kurtosis=-0.03600778490692402)
```

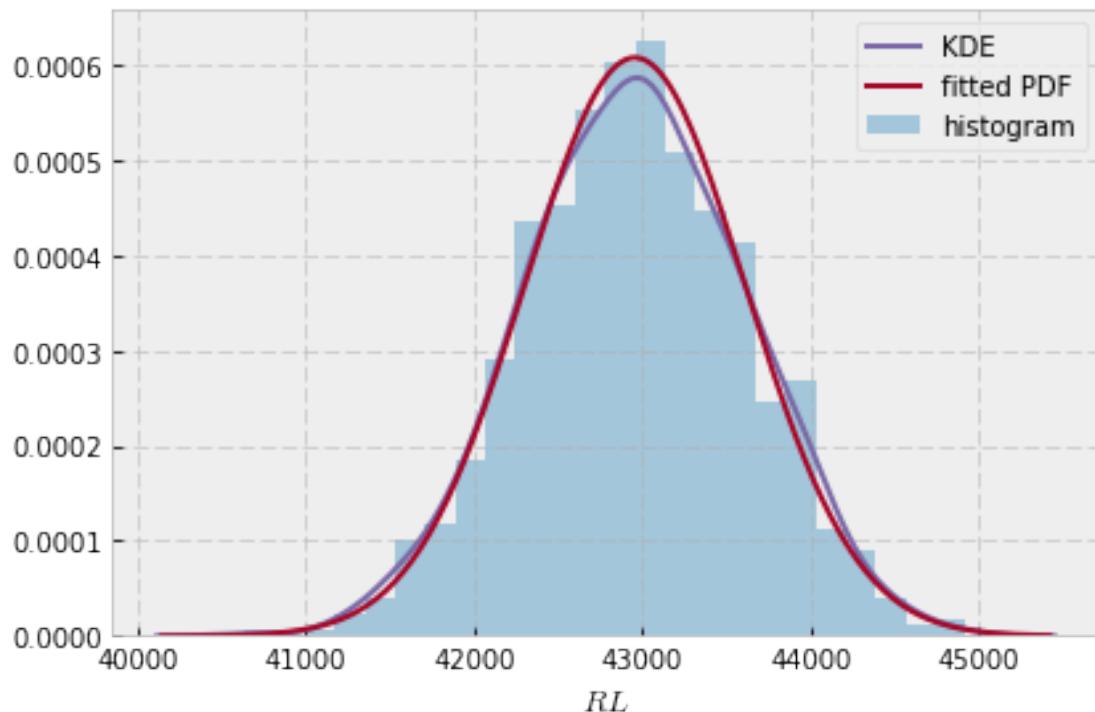


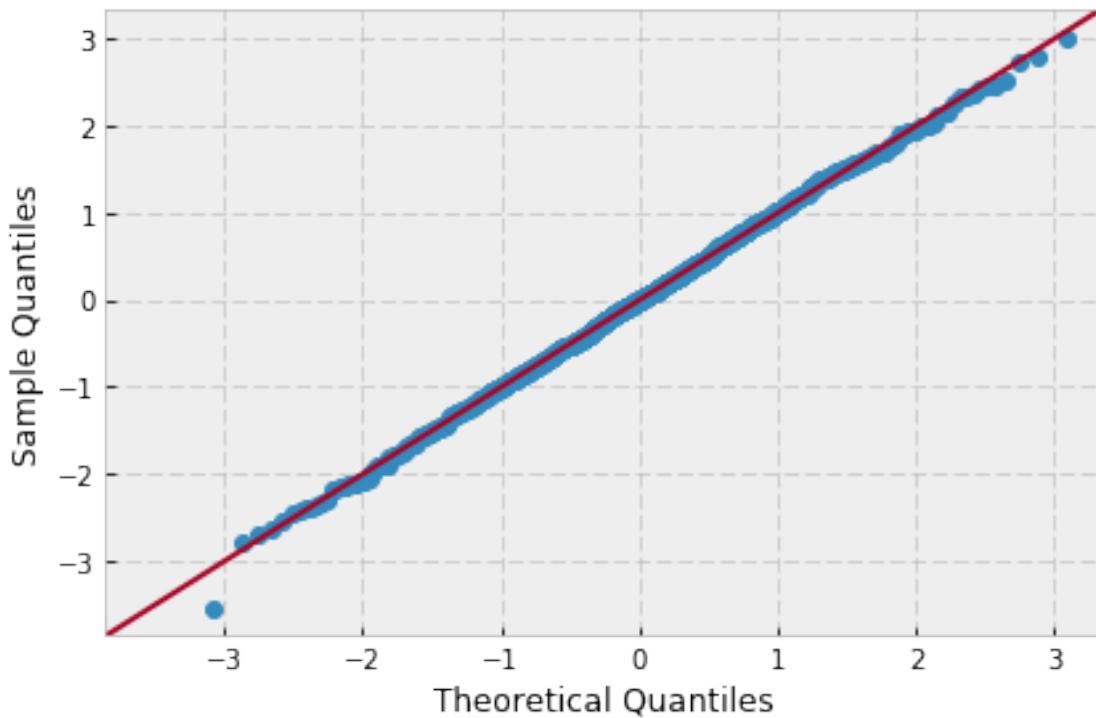


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(),loc=37041.66625173225, scale=600.8707504969034)
DescribeResult(nobs=1000, minmax=(35194.048422962645, 38872.299519513501)
               mean=37041.666251732247, variance=361407.06586858042,
               skewness=0.024907311890289788, kurtosis=0.08505813625470404)
```

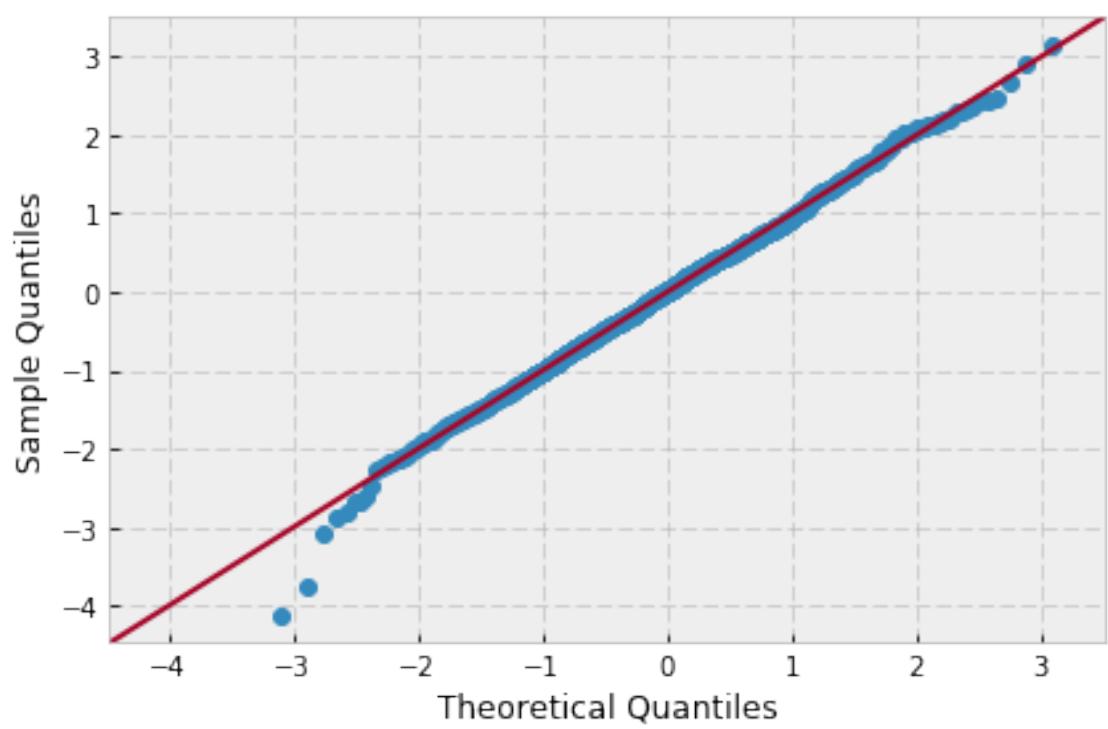
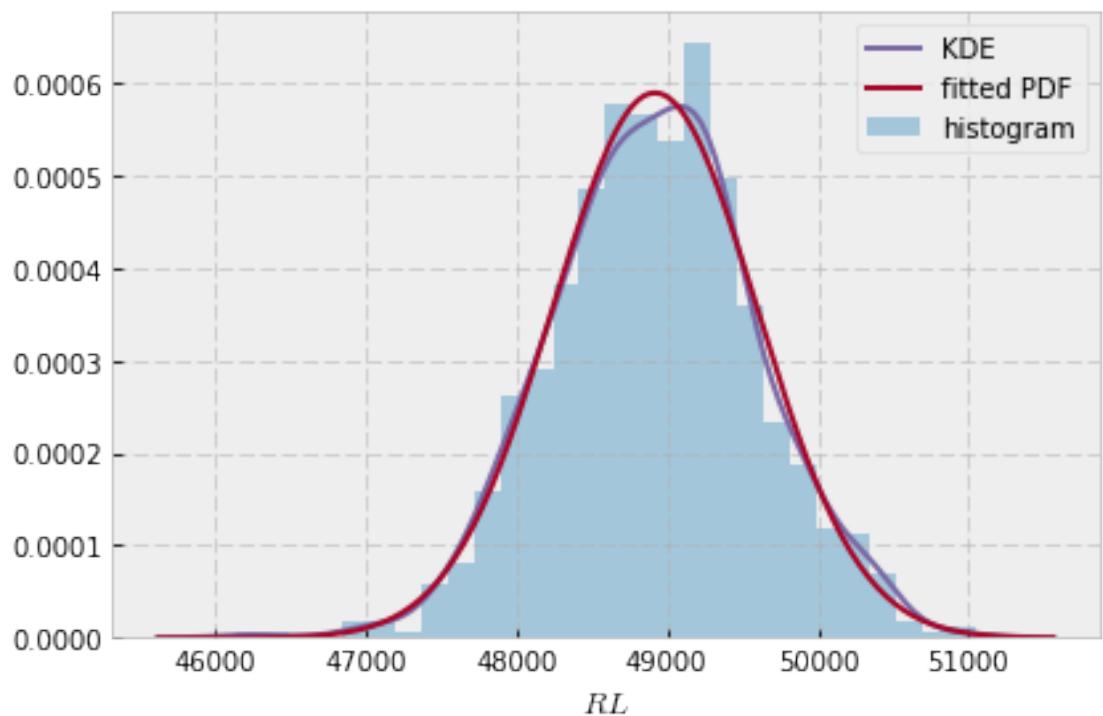


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(), loc=42955.26548529134, scale=655.4102831588602)
DescribeResult(nobs=1000, minmax=(40639.661102610131, 44923.415747877159)
               mean=42955.26548529134, variance=429992.63190227957,
               skewness=-0.03161101473936712, kurtosis=-0.1409510396537983)
```

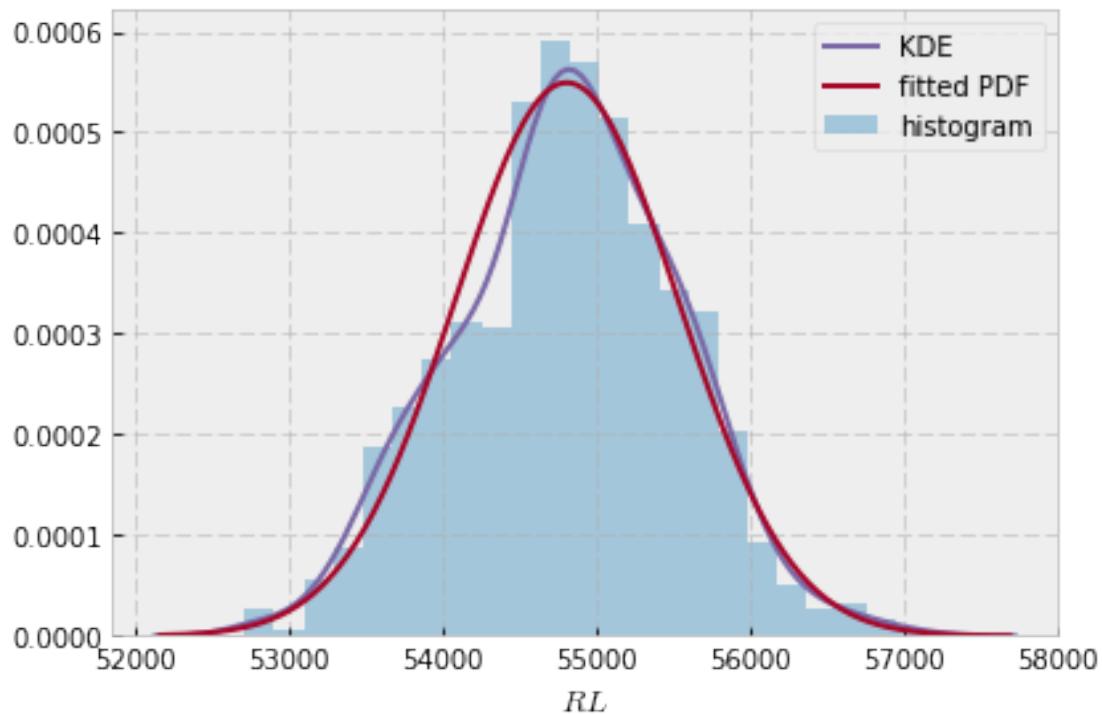


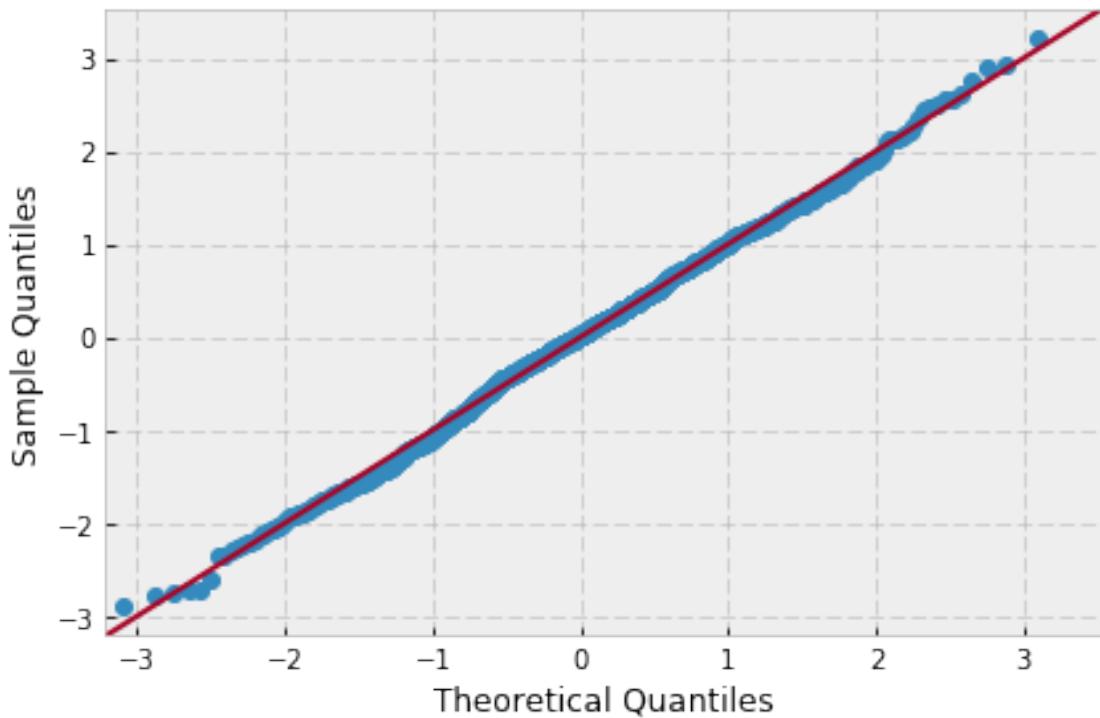


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(),loc=48919.24073746399, scale=676.4539293676997)
DescribeResult(nobs=1000, minmax=(46139.778203215894, 51045.402847824676)
               mean=48919.240737463988, variance=458047.96652352437,
               skewness=-0.0735254144546601, kurtosis=0.23913725706287492)
```

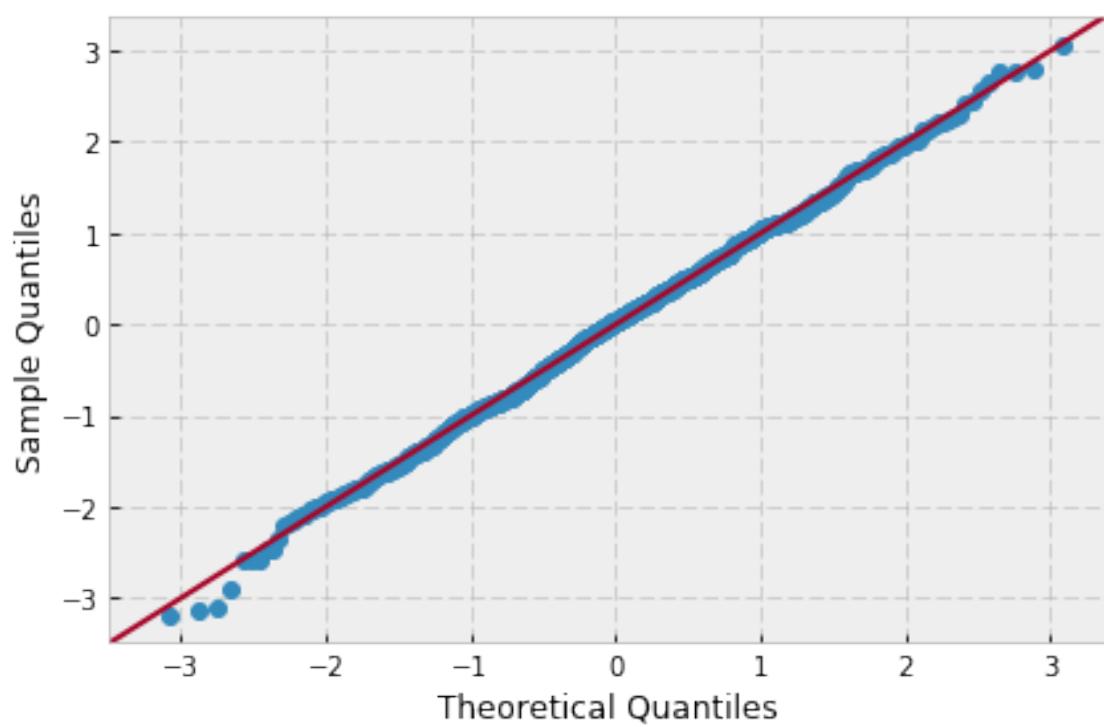
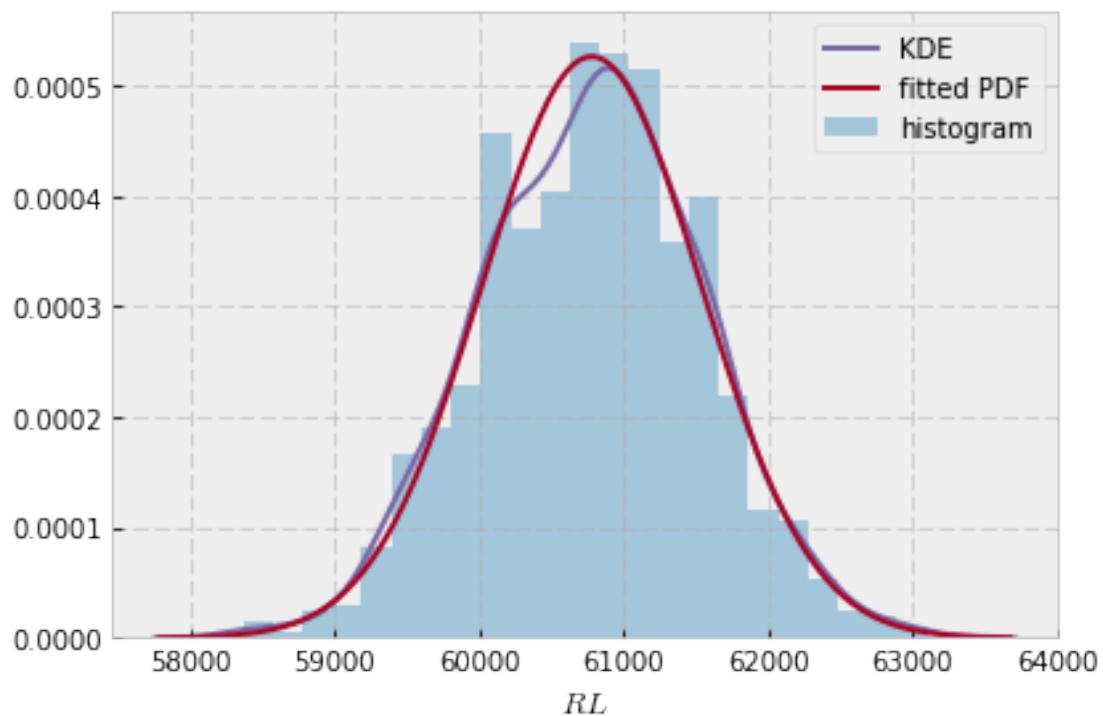


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(), loc=54807.84717718455, scale=726.3313436316798)
DescribeResult(nobs=1000, minmax=(52710.247649438039, 57140.913107282635)
               mean=54807.847177184551, variance=528085.30604784912,
               skewness=-0.05654143450283809, kurtosis=-0.1134884808851182)
```

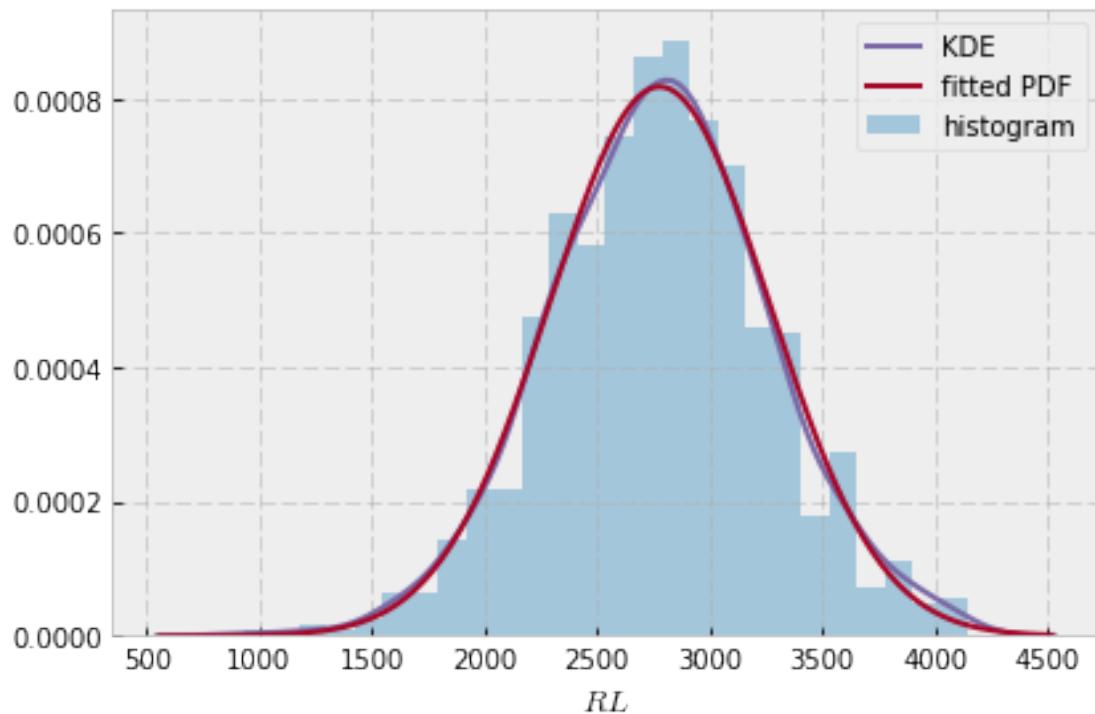


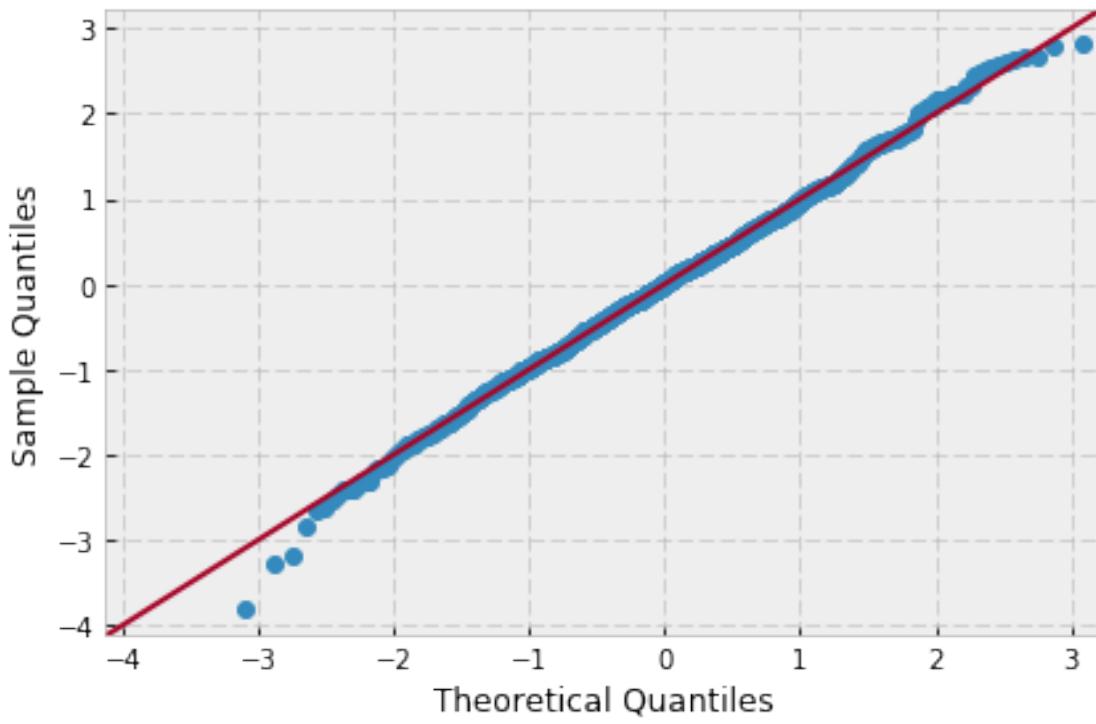


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(),loc=60778.814121159005, scale=758.3118108488464)
DescribeResult(nobs=1000, minmax=(58365.73295189596, 63098.115897691336)
               mean=60778.814121159005, variance=575612.41488774447,
               skewness=-0.05240603149469387, kurtosis=-0.07820523650411237)
```

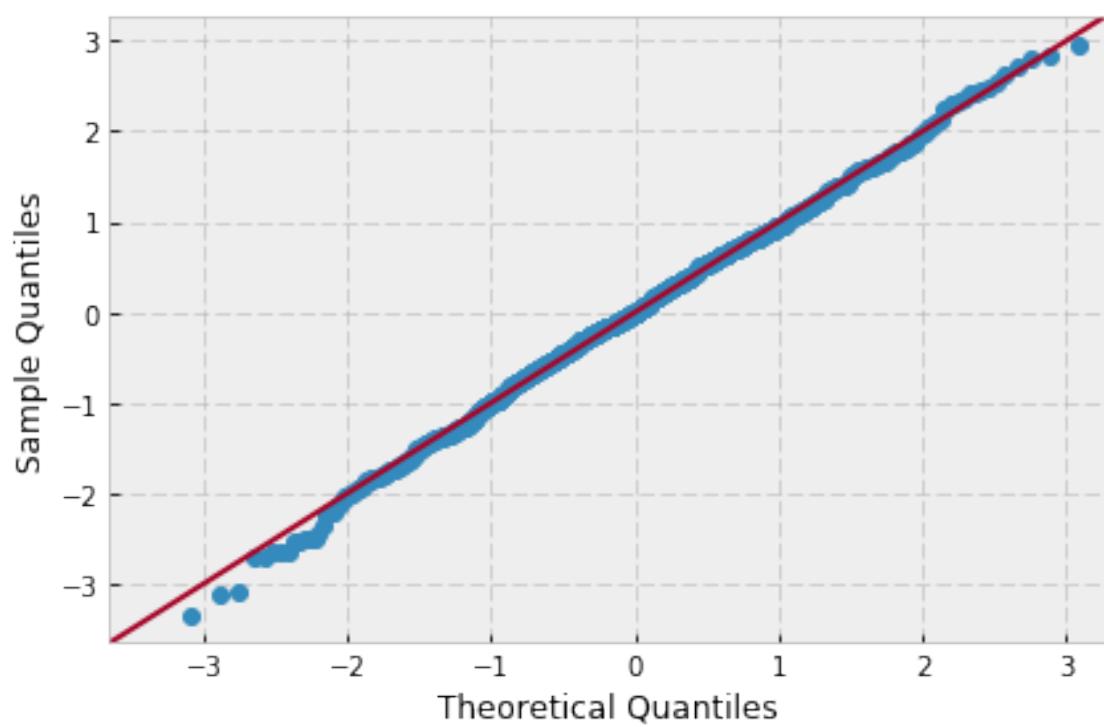
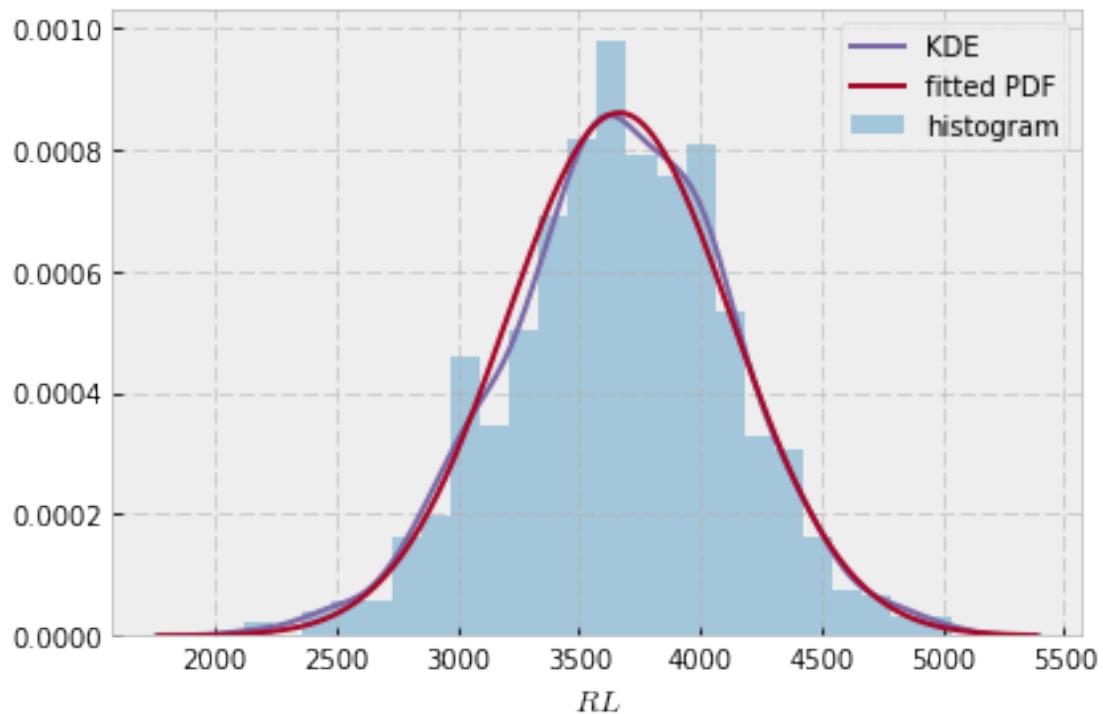


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=2778.8305555319703, scale=487.6168625823738)
DescribeResult(nobs=1000, minmax=(928.97709919566614, 4149.963508197432)
               mean=2778.8305555319703, variance=238008.2128875652,
               skewness=-0.034434738782295006, kurtosis=0.20391834649899332)
```

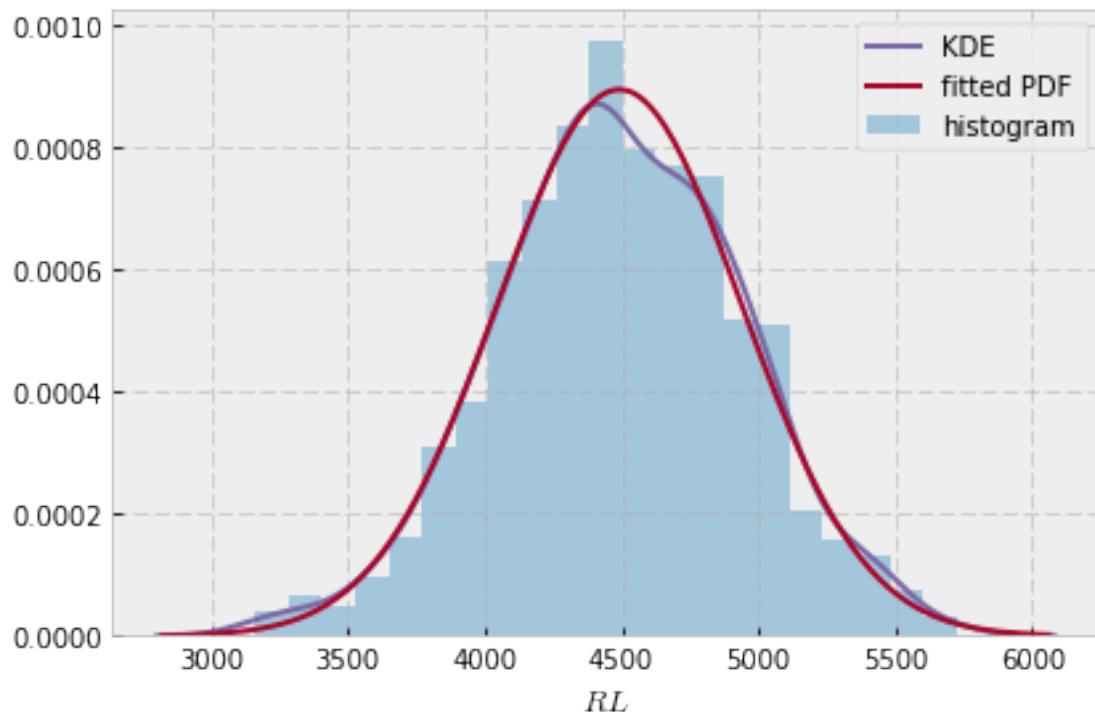


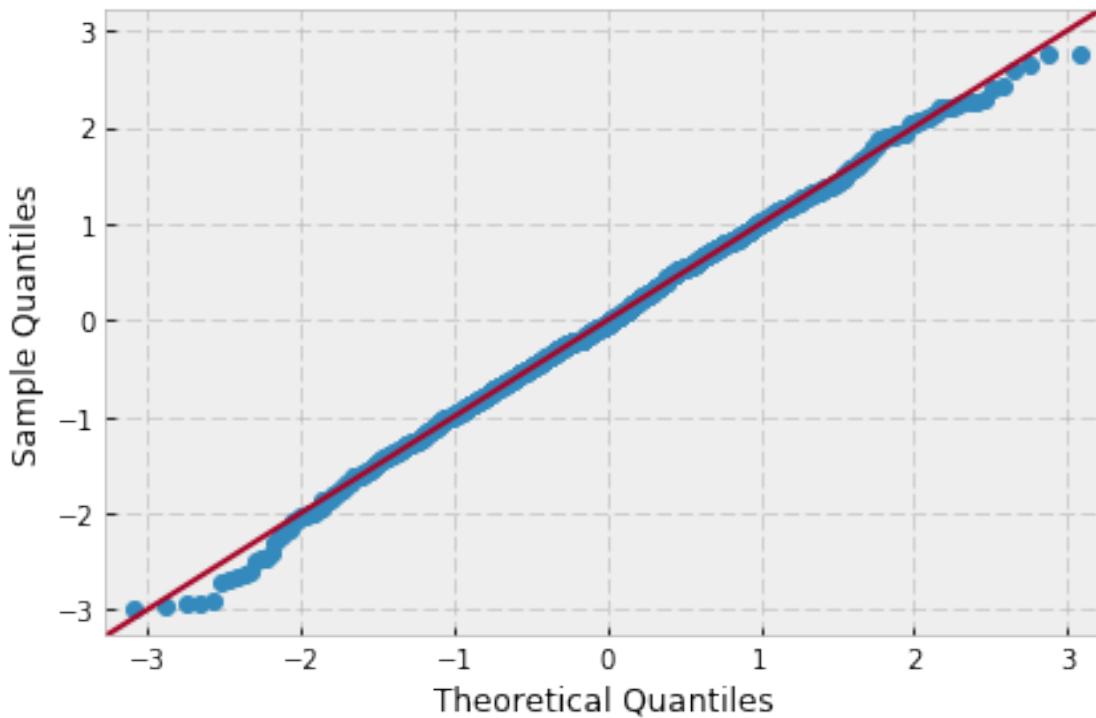


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(),loc=3666.6413680982487, scale=463.8199694028768)
DescribeResult(nobs=1000, minmax=(2119.3972371087125, 5033.3713420090271)
               mean=3666.6413680982487, variance=215344.30832521073,
               skewness=-0.1125076691042469, kurtosis=0.09181807008577048)
```

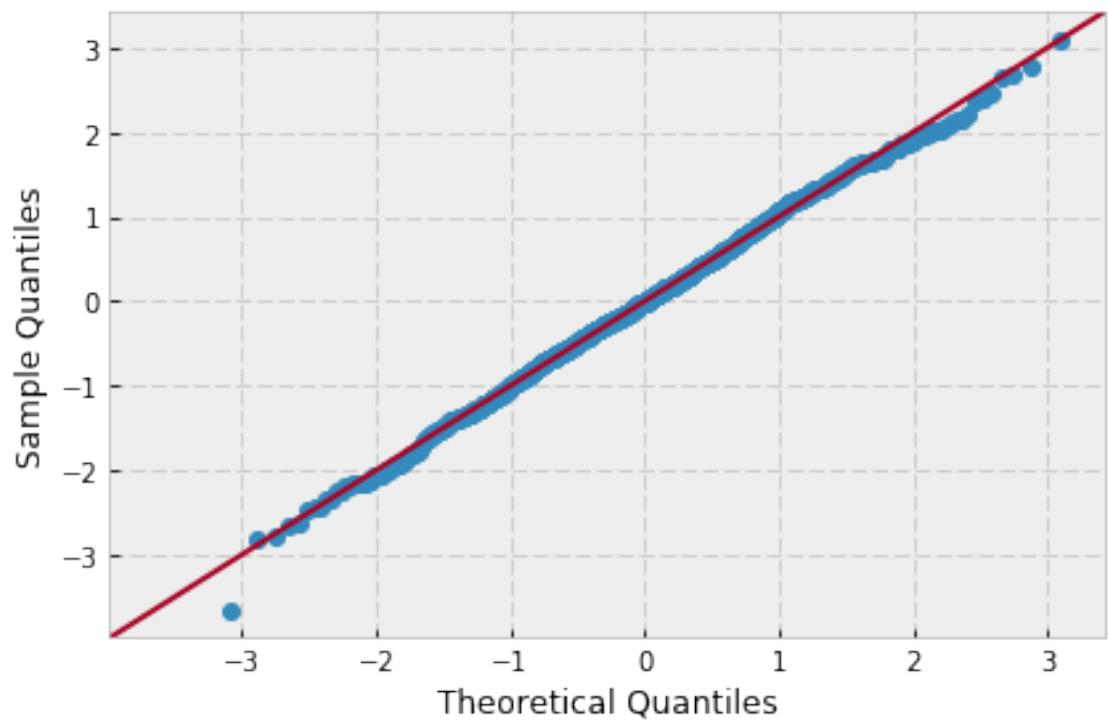
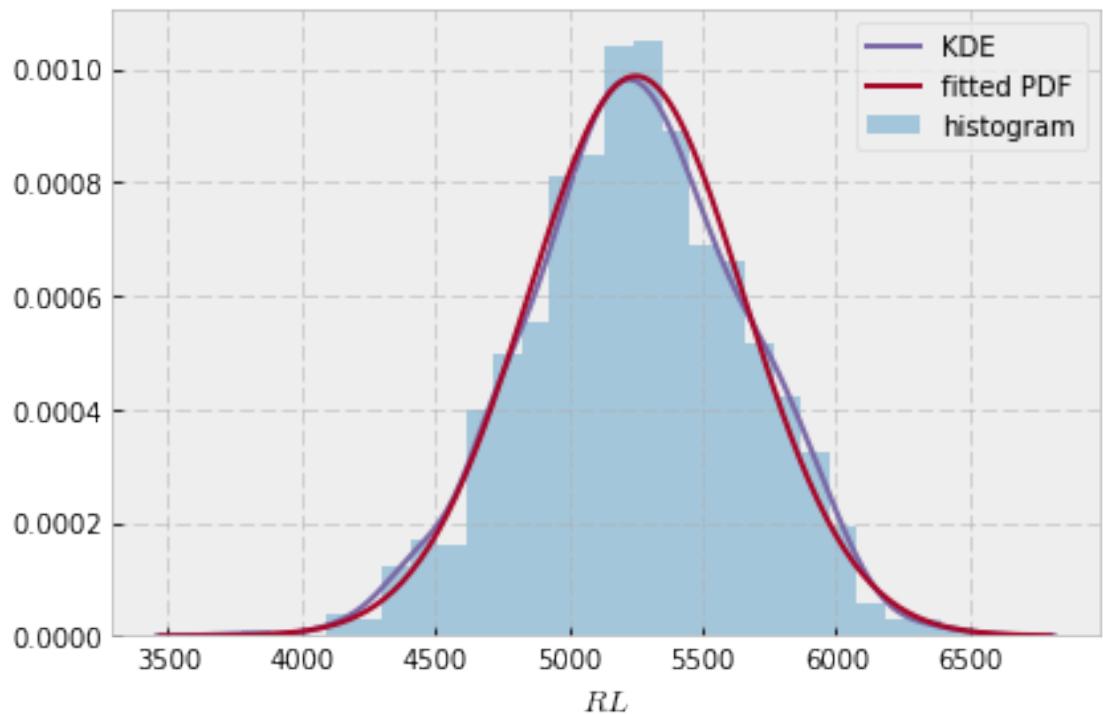


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=4491.64579534022, scale=446.73503950449265)
DescribeResult(nobs=1000, minmax=(3158.8515030722101, 5723.8451121044909)
               mean=4491.6457953402196, variance=199771.96748856918,
               skewness=-0.08378288559119029, kurtosis=-0.03482076741237616)
```

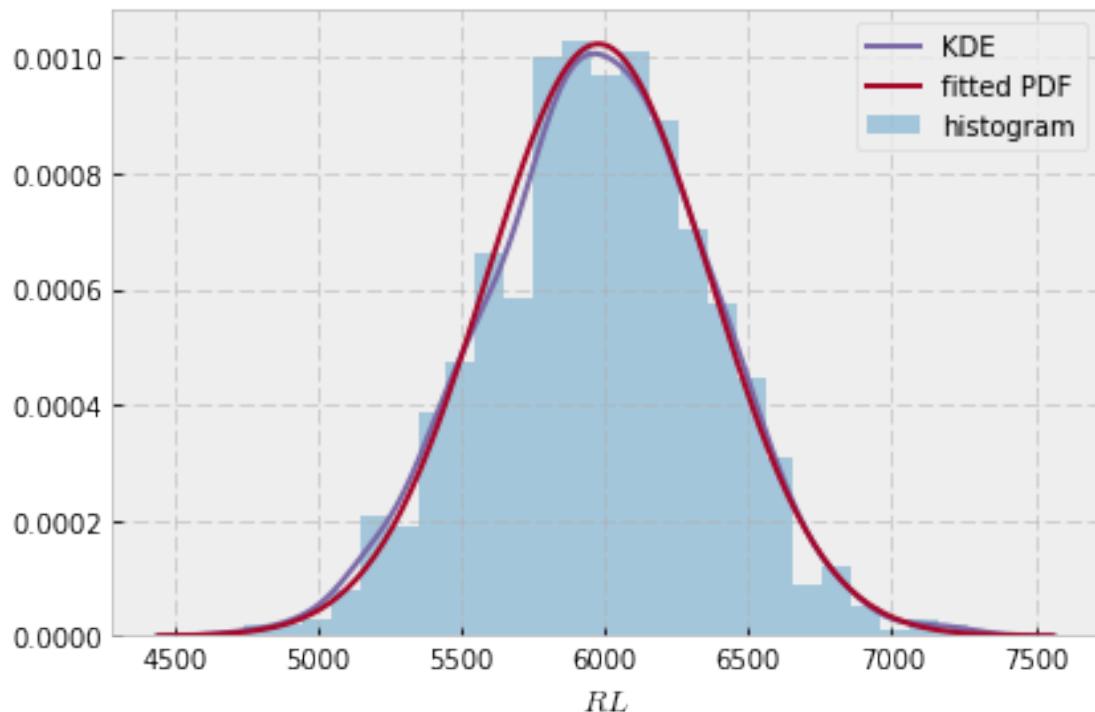


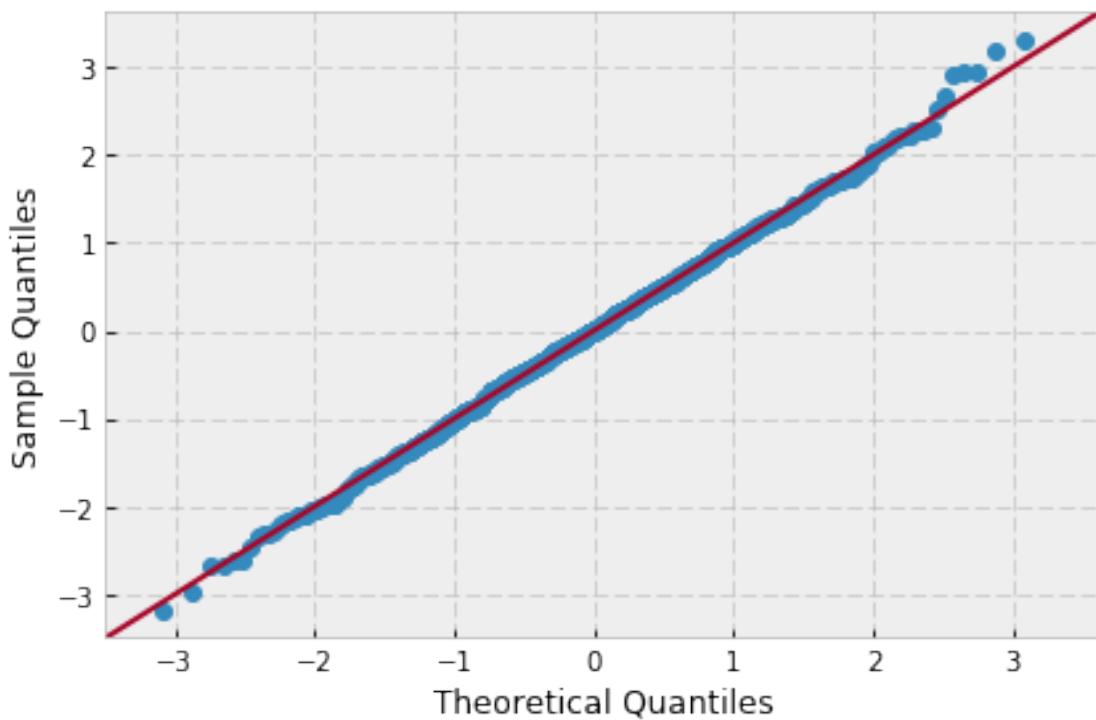


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=5250.218678257399, scale=404.11051375494884)
DescribeResult(nobs=1000, minmax=(3776.7481691455723, 6497.0921181139975)
               mean=5250.218678257399, variance=163468.77610339207,
               skewness=-0.07038202238047254, kurtosis=-0.12379853671581698)
```

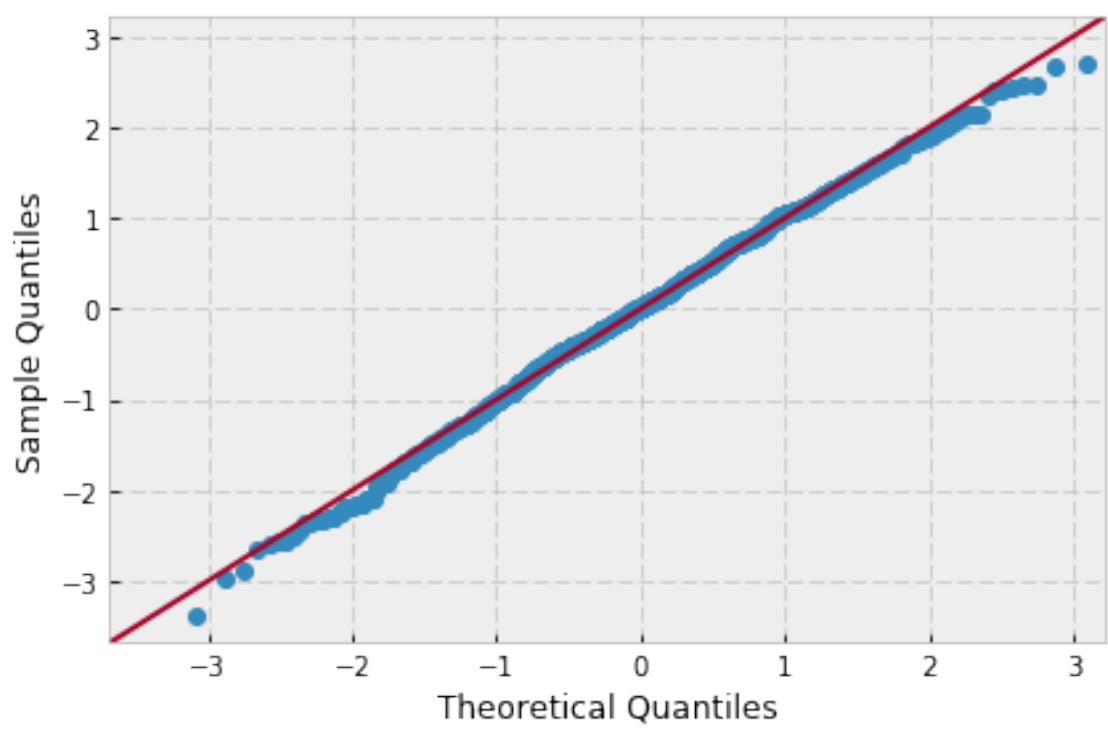
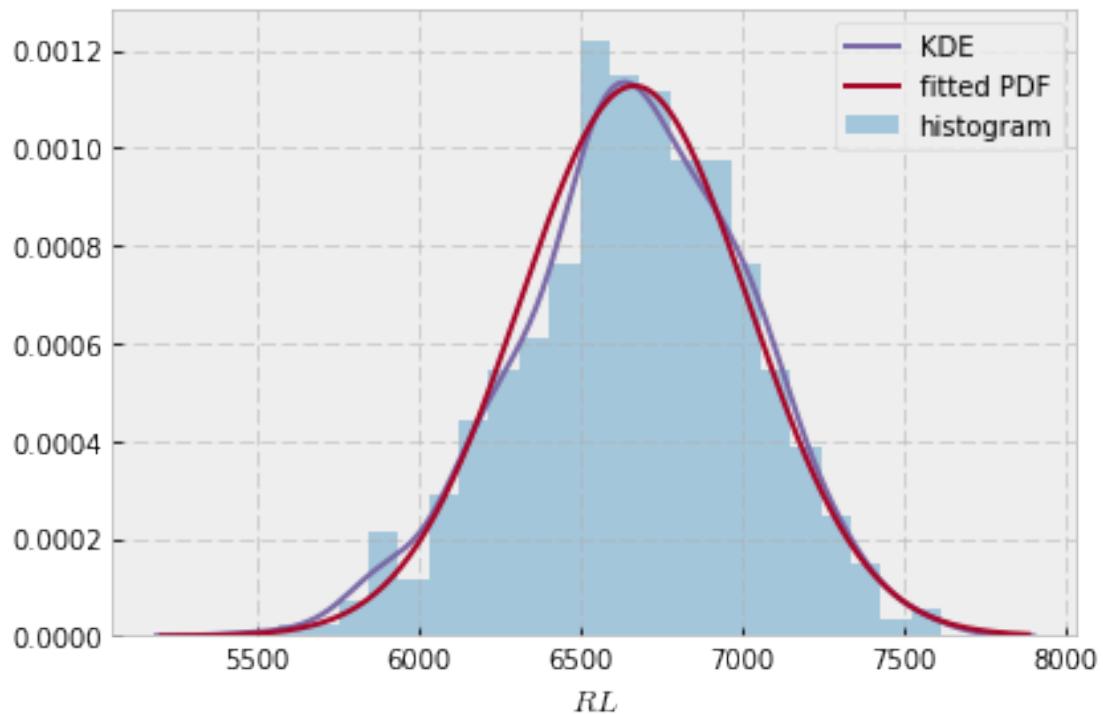


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=5976.549011430559, scale=389.7891706211633)
DescribeResult(nobs=1000, minmax=(4740.4744459980029, 7263.2498175016035)
               mean=5976.5490114305594, variance=152087.68521875309,
               skewness=-0.021929565544741465, kurtosis=0.005469709109163112)
```

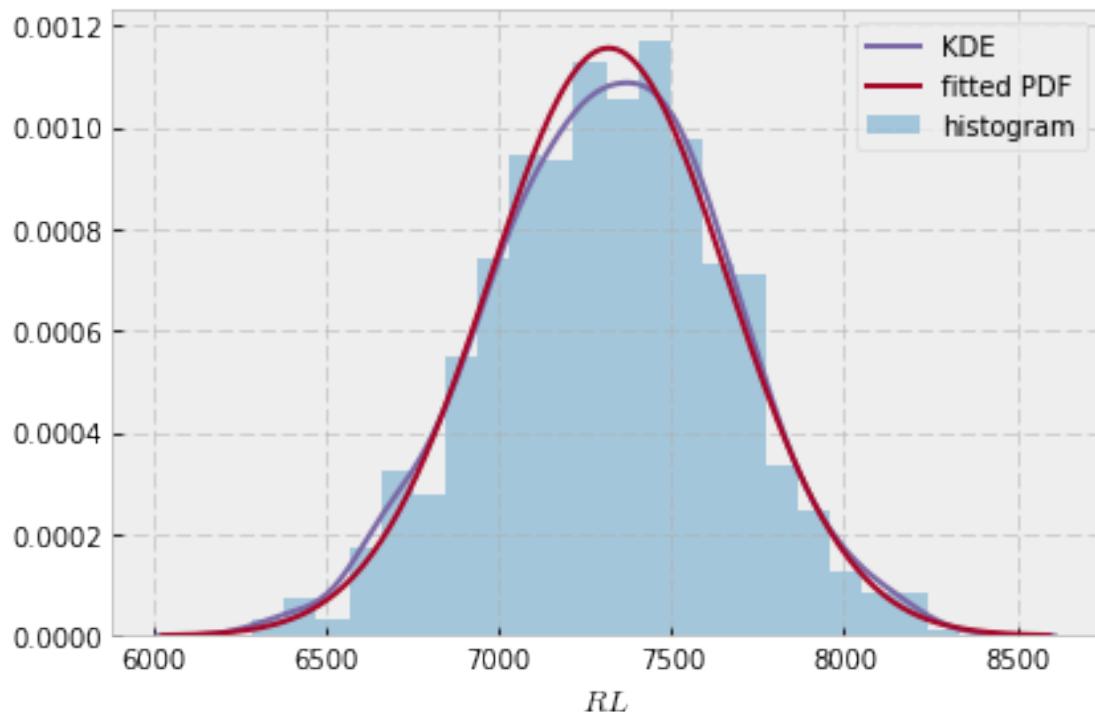


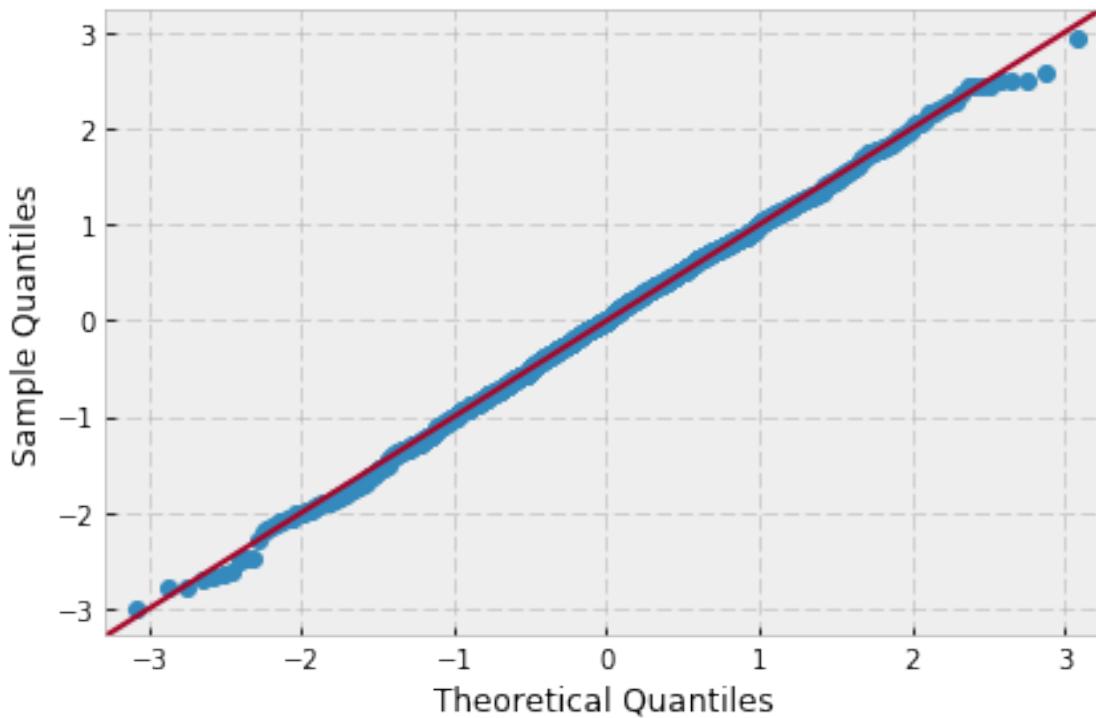


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(),loc=6668.262734627992, scale=353.9170371959584)
DescribeResult(nobs=1000, minmax=(5472.5546506290393, 7618.8417434054209)
               mean=6668.2627346279924, variance=125382.65186943483,
               skewness=-0.18346564276431093, kurtosis=-0.09314313218330961)
```

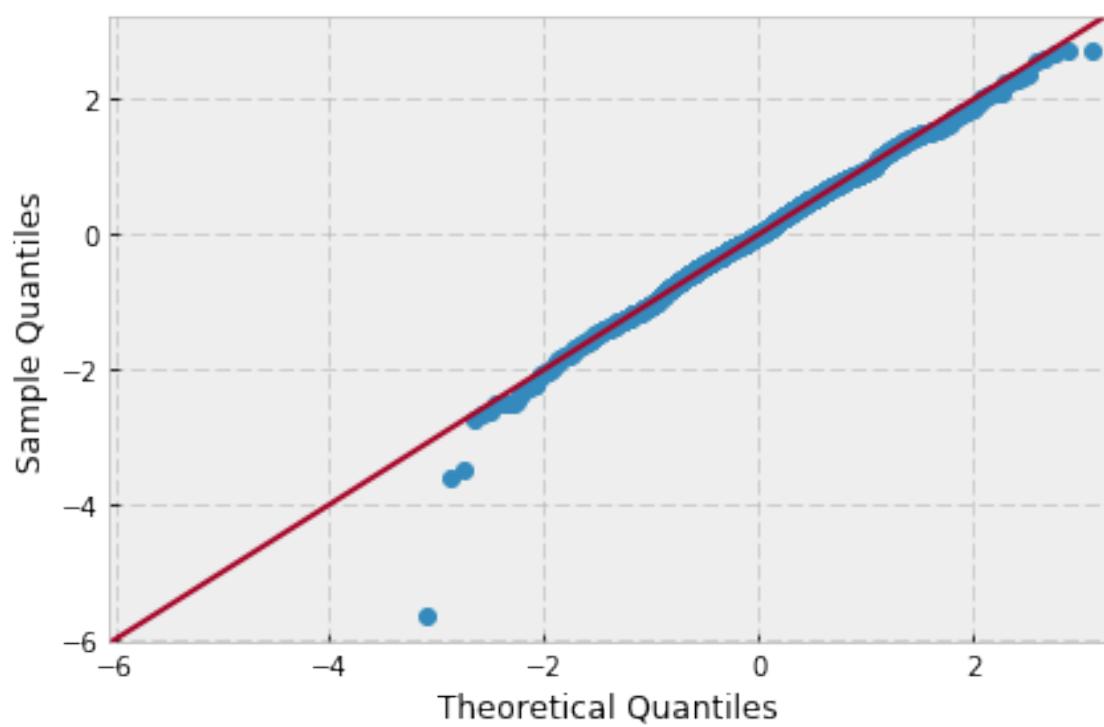
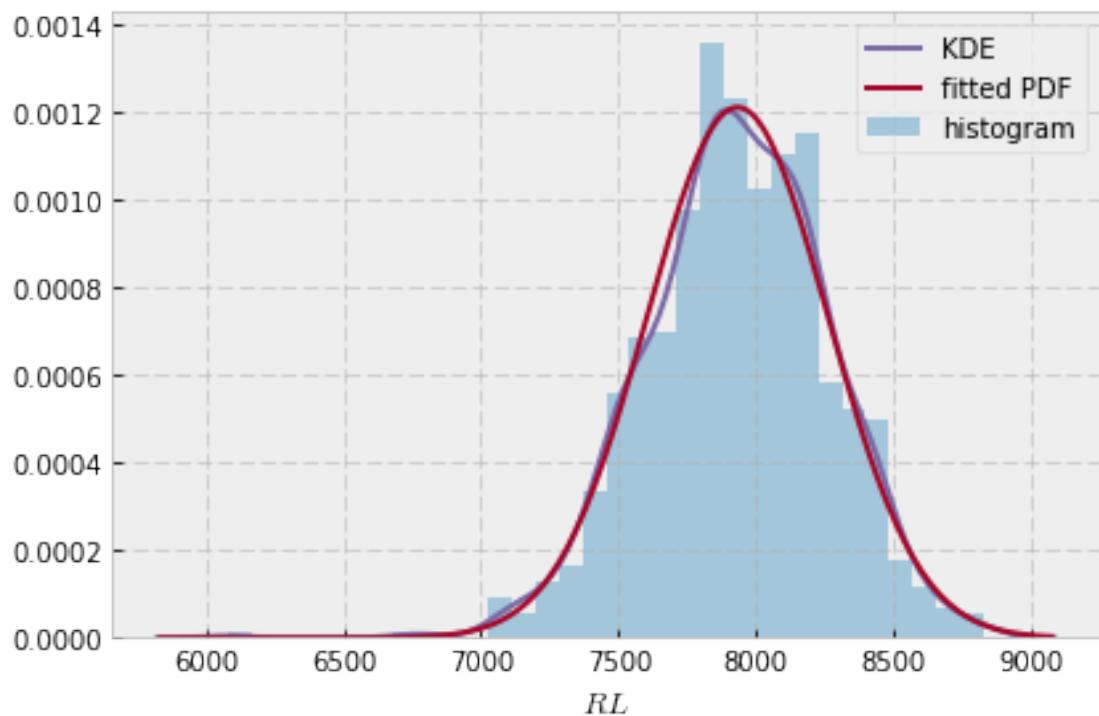


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=7318.090957984401, scale=345.1540074769156)
DescribeResult(nobs=1000, minmax=(6286.8219968220983, 8332.6005903332189)
               mean=7318.090957984401, variance=119250.53941679146,
               skewness=-0.07211706638044871, kurtosis=-0.16118421762596213)
```

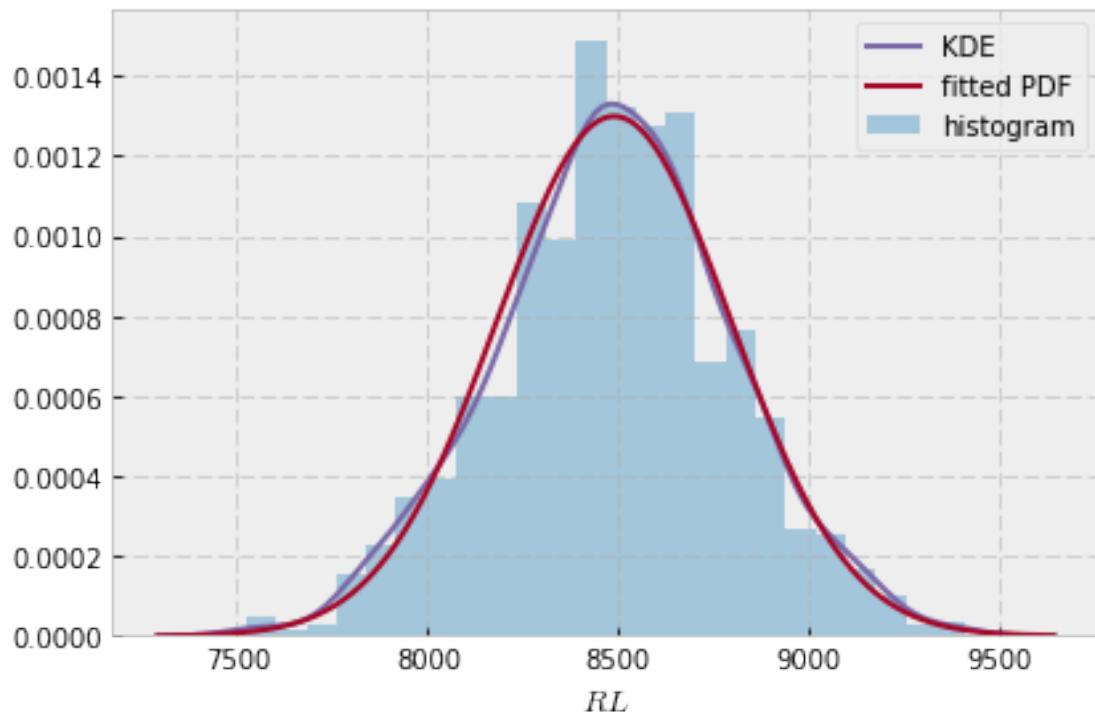


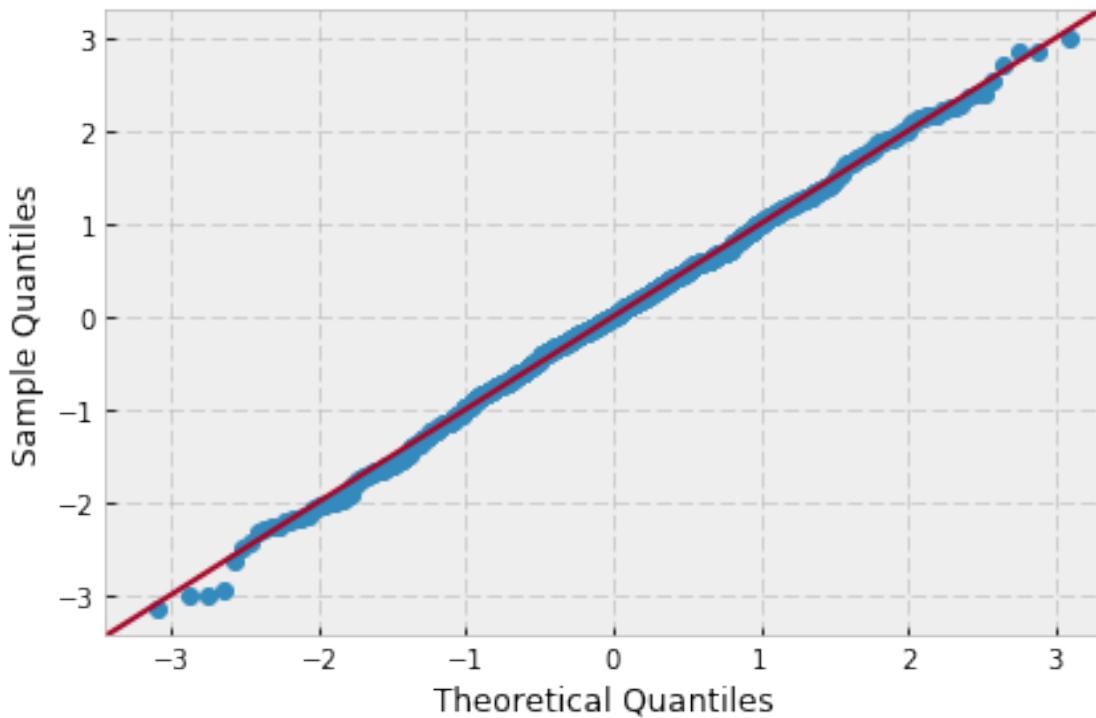


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(),loc=7933.120002365586, scale=329.16994540378676)
DescribeResult(nobs=1000, minmax=(6078.8521077481319, 8828.7090118494998)
               mean=7933.1200023655856, variance=108461.31427140336,
               skewness=-0.30507684617641323, kurtosis=0.8414012375527364)
```

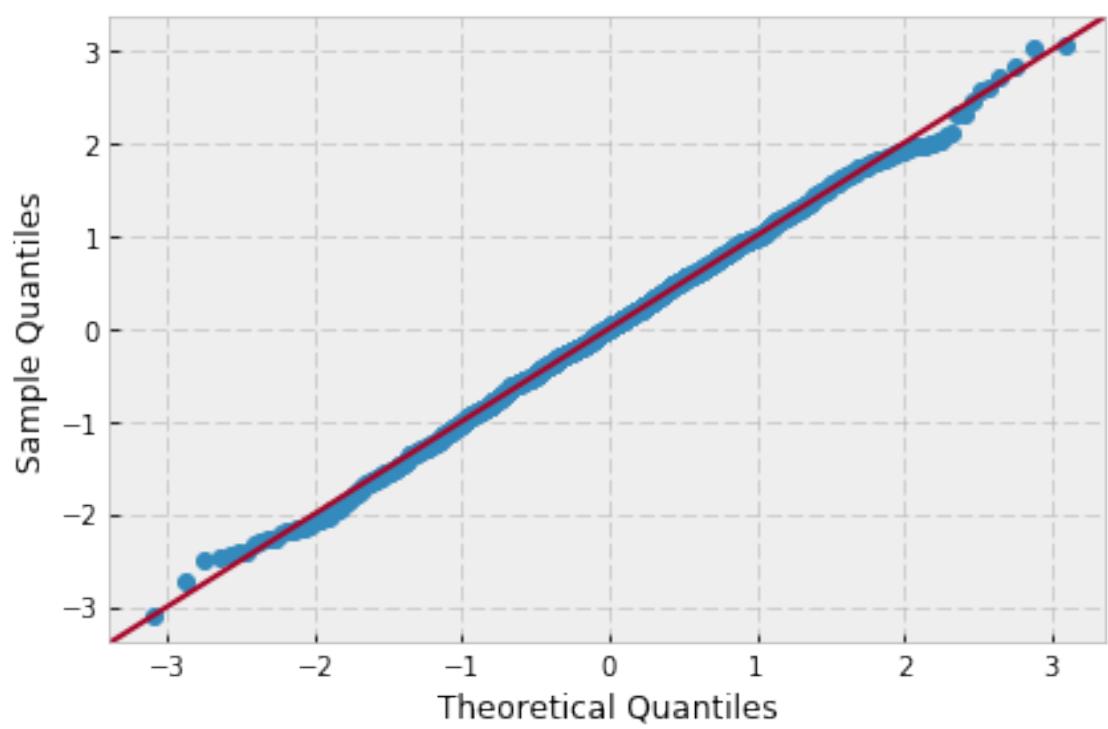
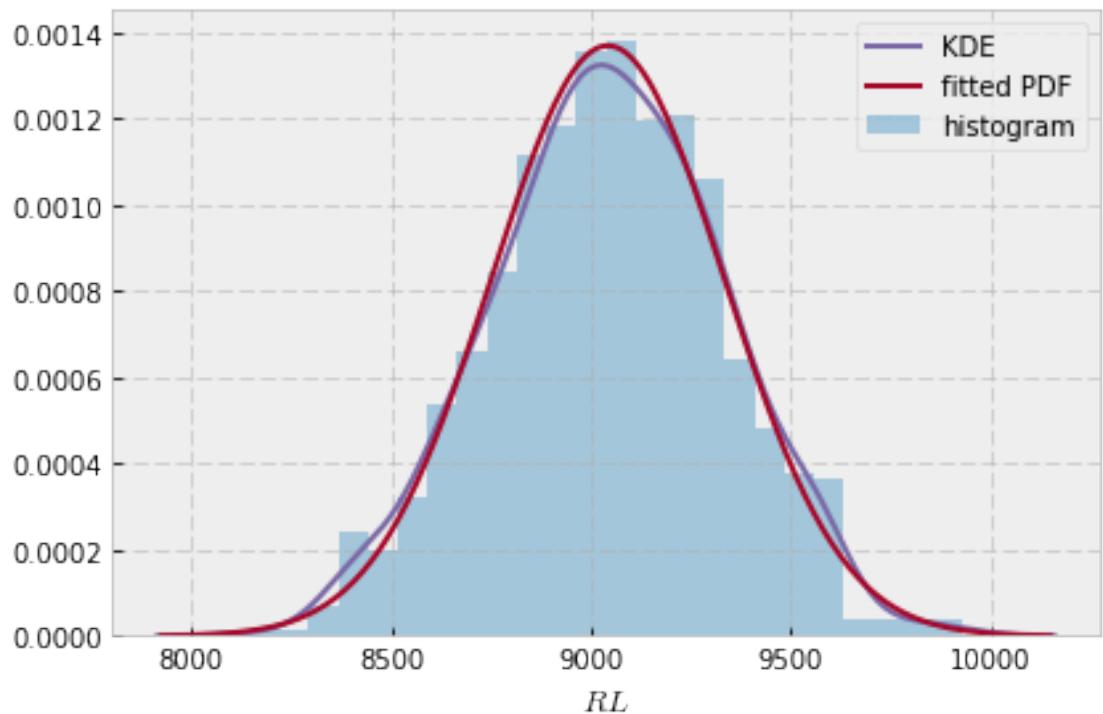


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=8487.809292537817, scale=307.0370953340848)
DescribeResult(nobs=1000, minmax=(7524.1072107198643, 9408.117144972146)
               mean=8487.8092925378169, variance=94366.144055247132,
               skewness=-0.059315394191519114, kurtosis=0.013568290924173887)
```

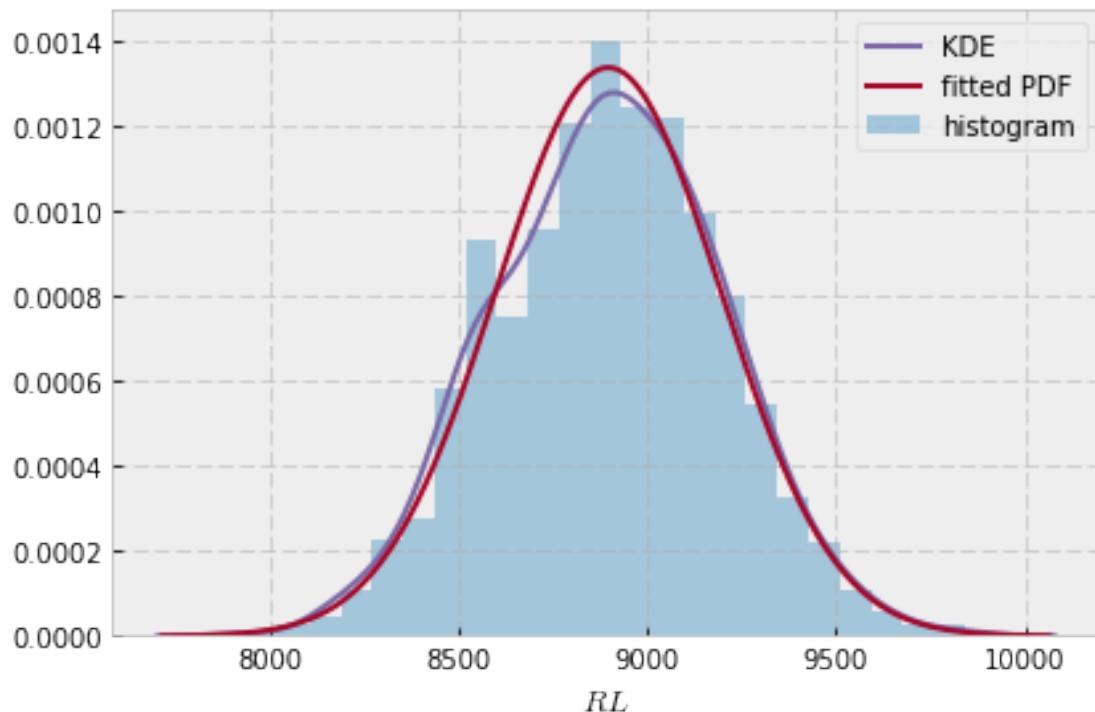


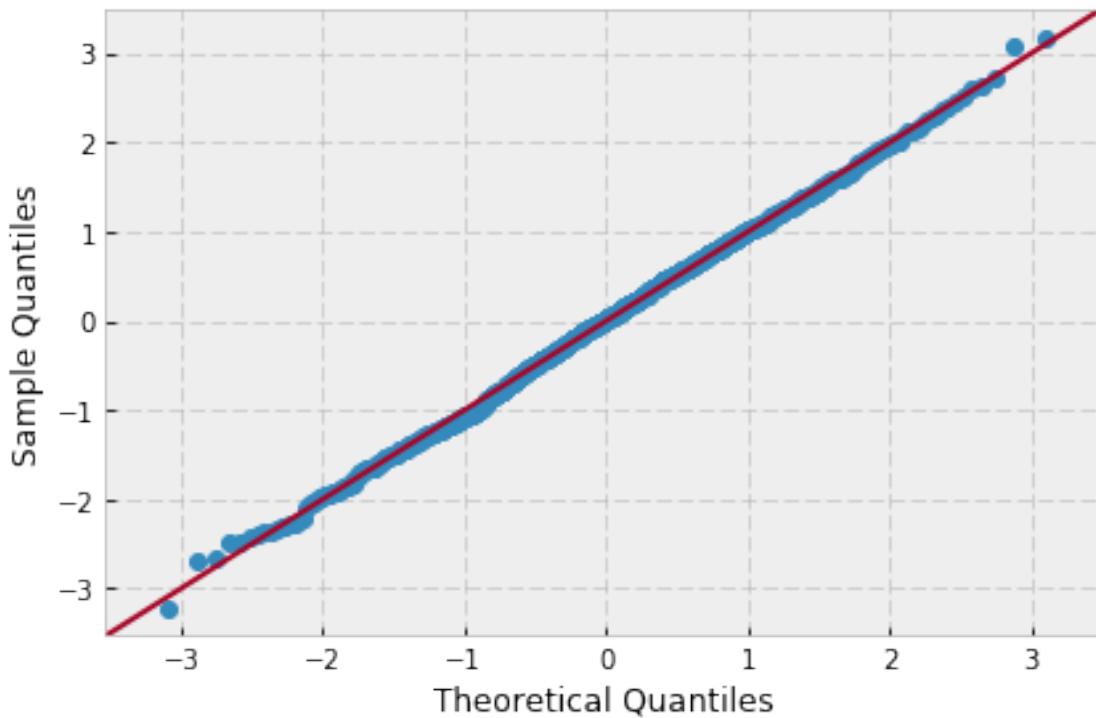


```
variable name: r
variable value: 3.0
distribution: normal(shape=(),loc=9041.529155753151, scale=291.2178935897016)
DescribeResult(nobs=1000, minmax=(8143.1705465589612, 9929.9588385041916)
               mean=9041.5291557531509, variance=84892.754301123889,
               skewness=-0.04039113819656151, kurtosis=-0.17886253192973012)
```

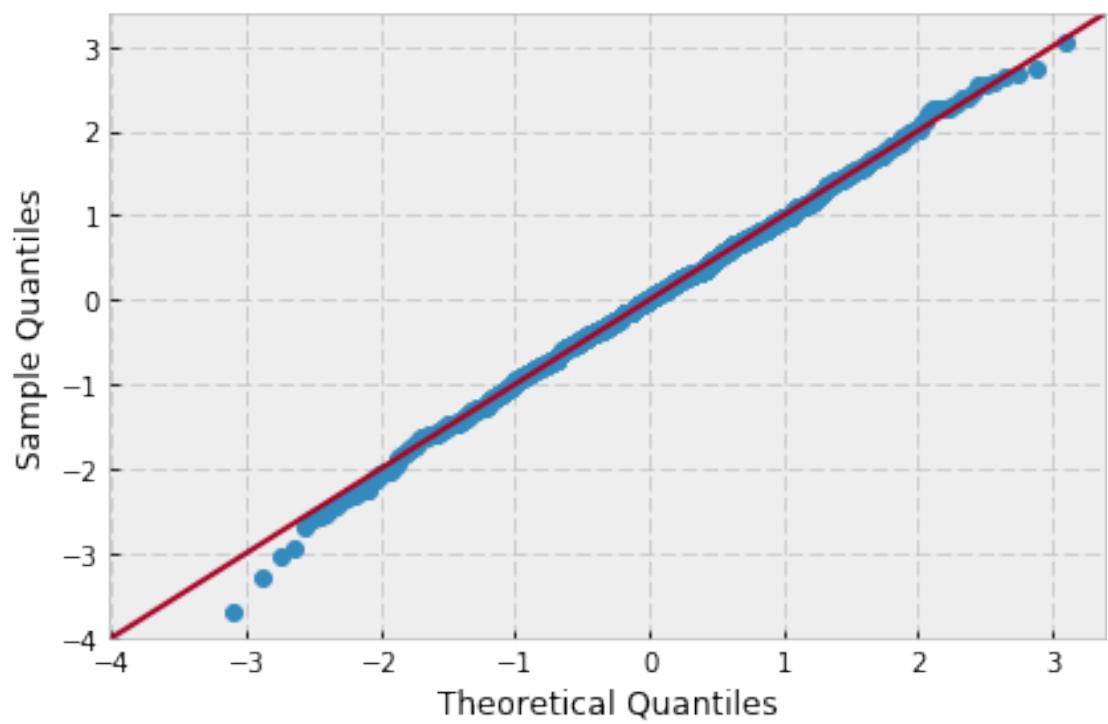
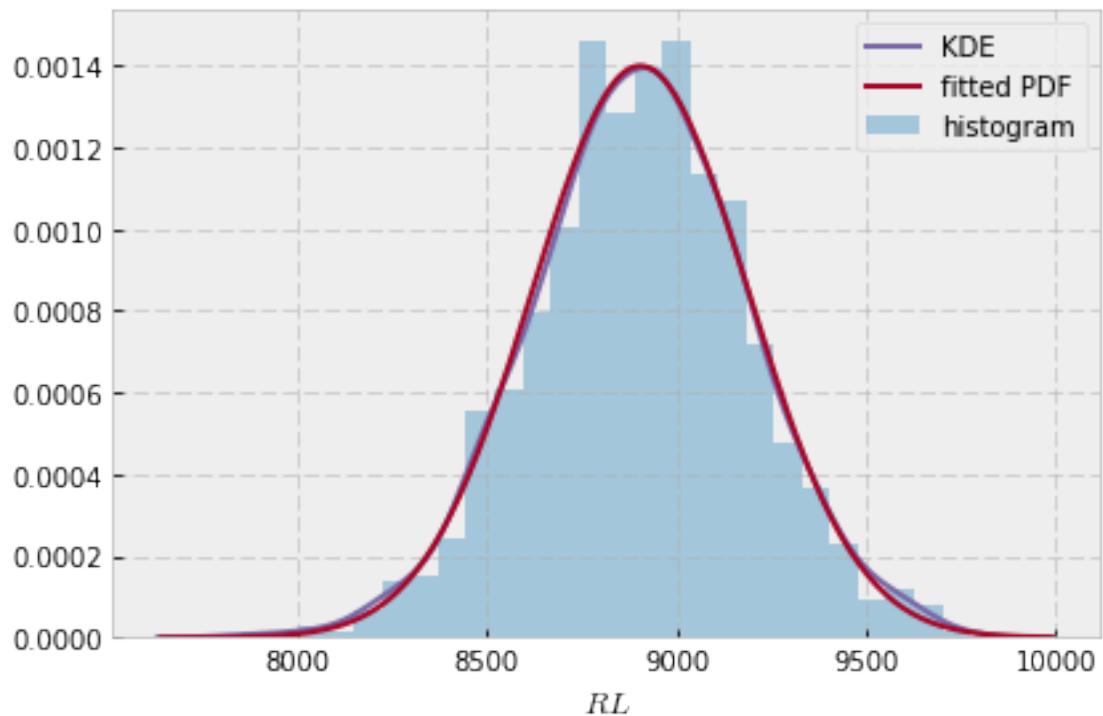


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(), loc=8895.769522462751, scale=298.2551213003227)
DescribeResult(nobs=1000, minmax=(7938.1826643519853, 9839.9339456579582)
               mean=8895.7695224627514, variance=89045.162544414619,
               skewness=-0.023366441845583726, kurtosis=-0.18422205431911776)
```

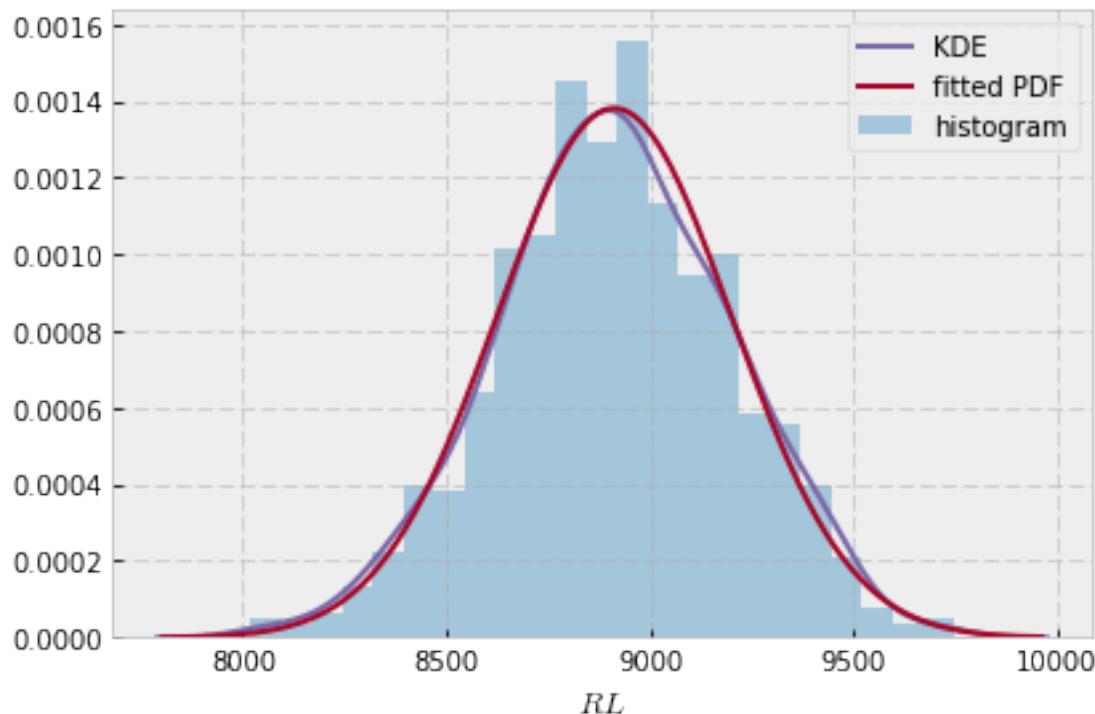


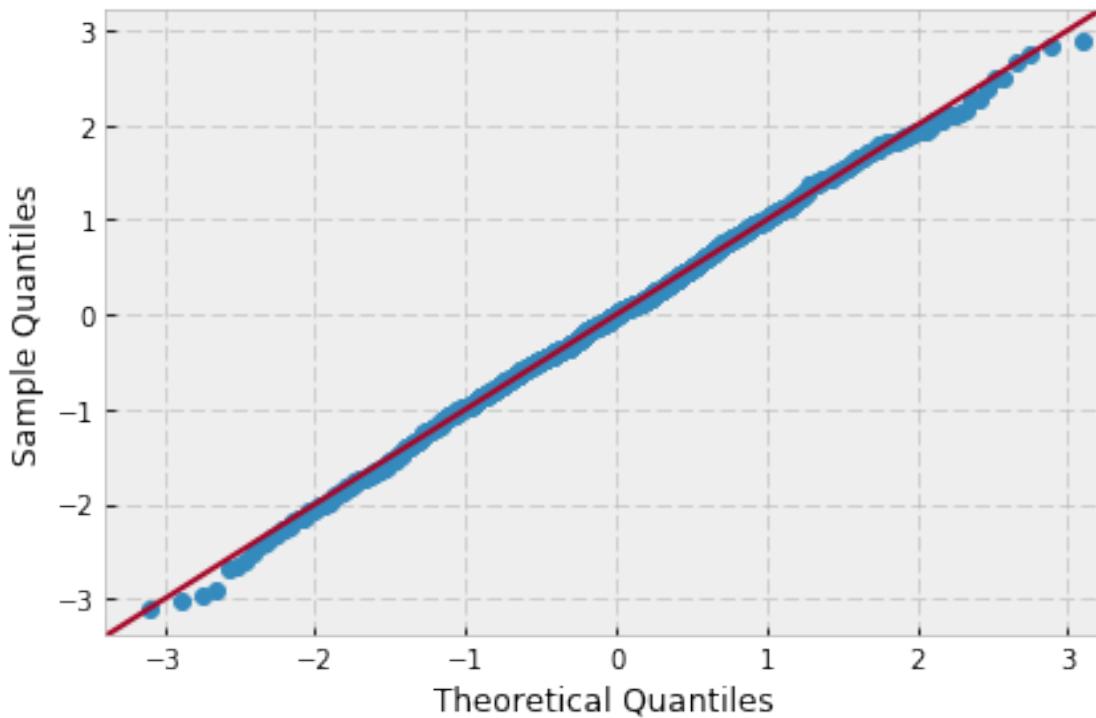


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(),loc=8904.767236624006, scale=284.9716443735661)
DescribeResult(nobs=1000, minmax=(7855.5230875669258, 9774.2197959192035)
               mean=8904.7672366240058, variance=81290.128225199427,
               skewness=-0.06356555924718664, kurtosis=0.15351952388324452)
```

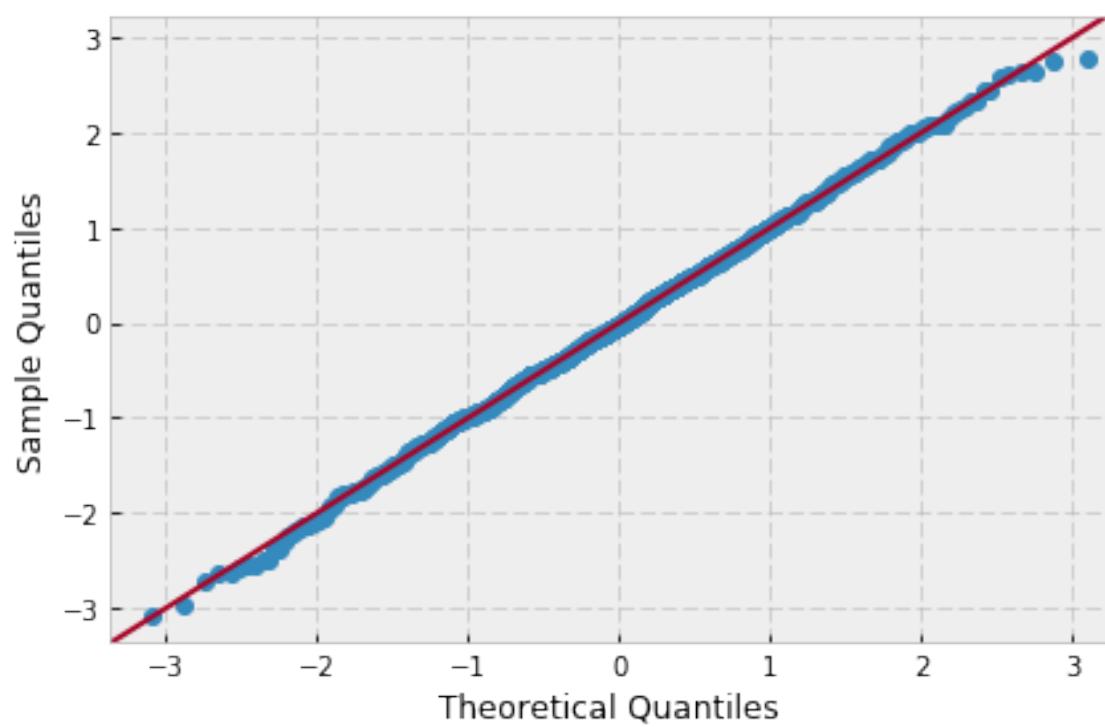
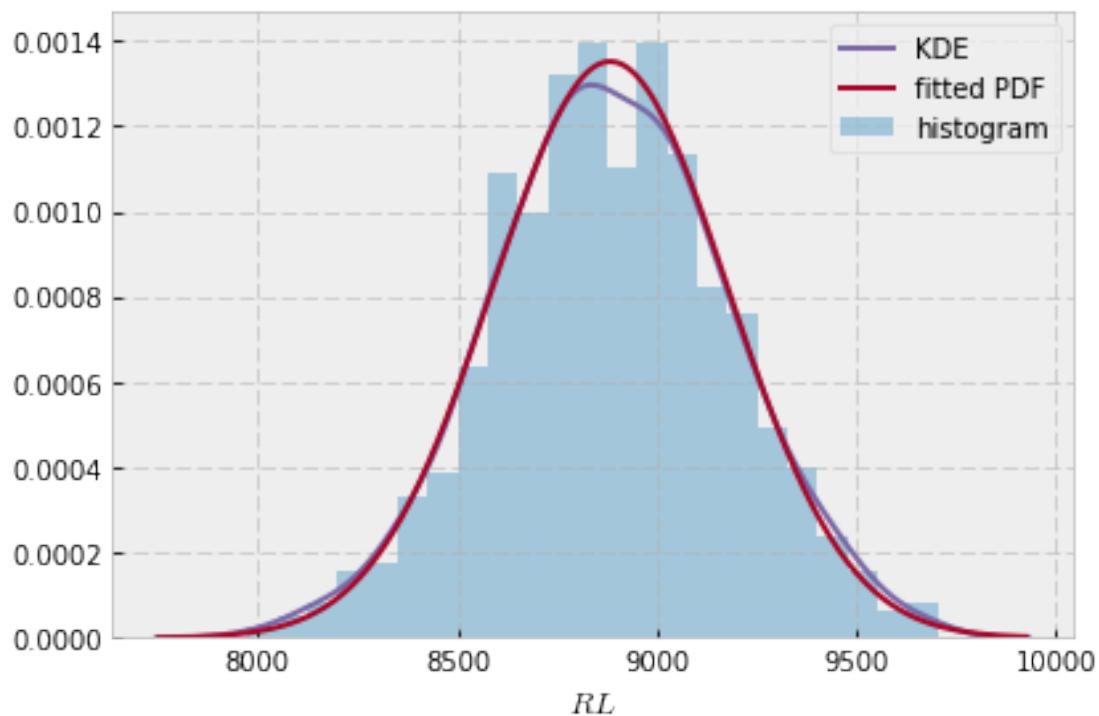


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(), loc=8912.05678466684, scale=288.70975385299636)
DescribeResult(nobs=1000, minmax=(8019.6749037870786, 9744.5332035191204)
               mean=8912.05678466684, variance=83436.758728586341,
               skewness=-0.06184953107086665, kurtosis=-0.07463624118297885)
```

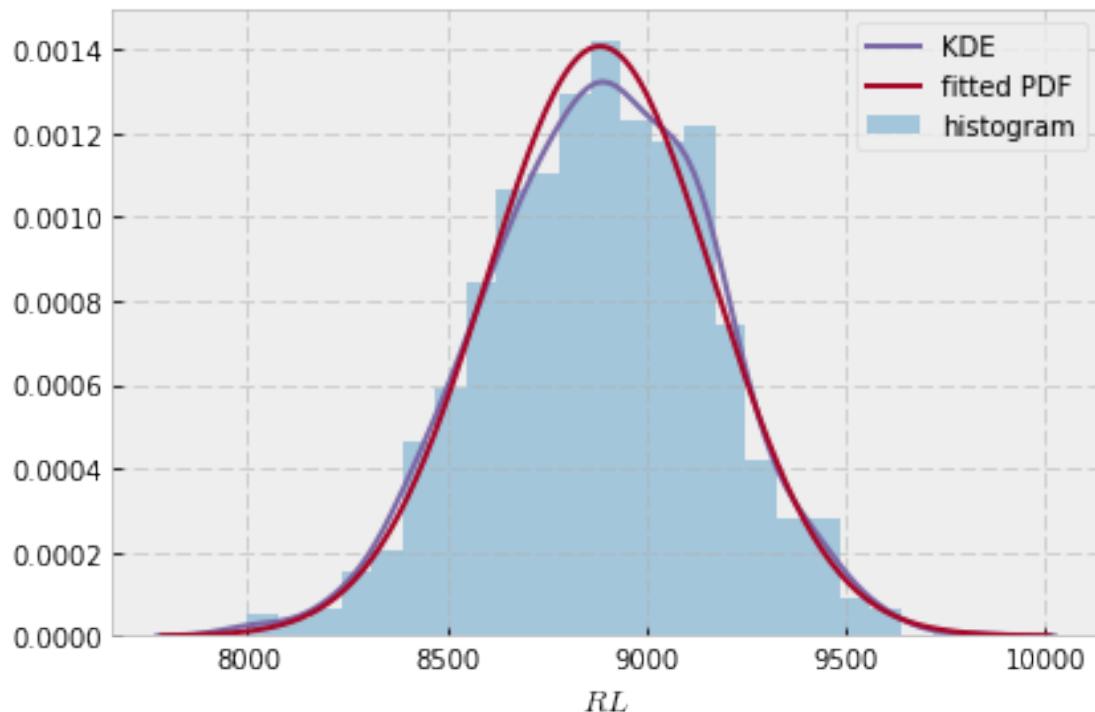


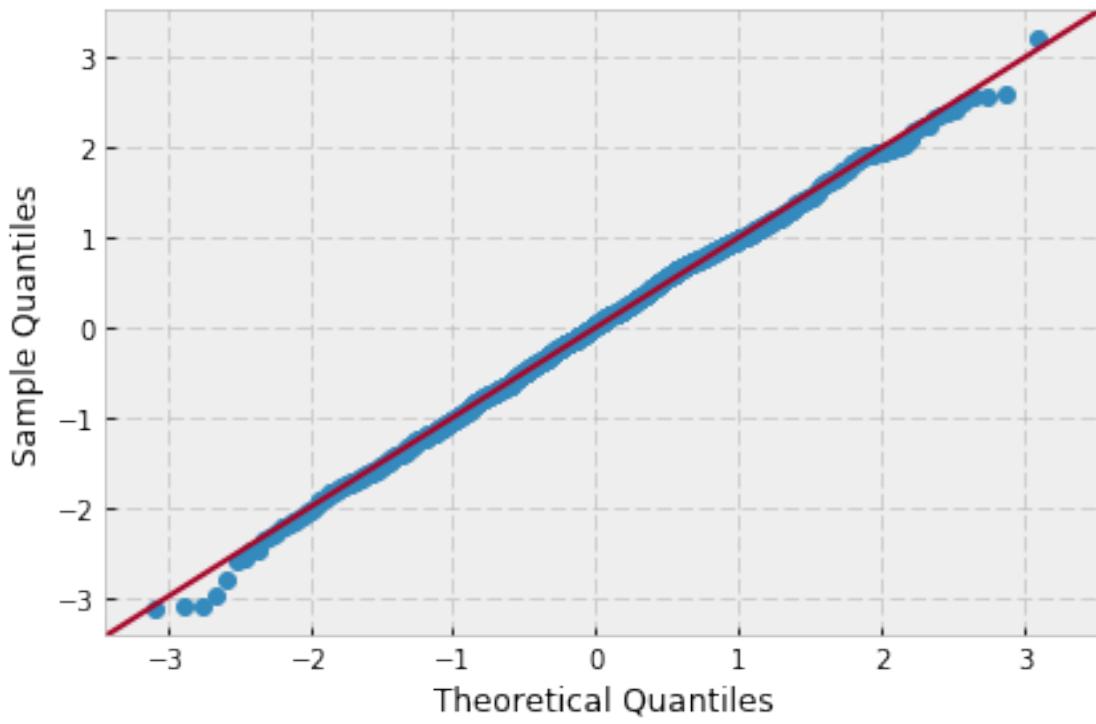


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=8882.388591785026, scale=295.2185321624677)
DescribeResult(nobs=1000, minmax=(7974.1056391051961, 9702.1130222822503)
               mean=8882.3885917850257, variance=87241.222955117075,
               skewness=-0.019151181455849705, kurtosis=-0.08671630676229825)
```

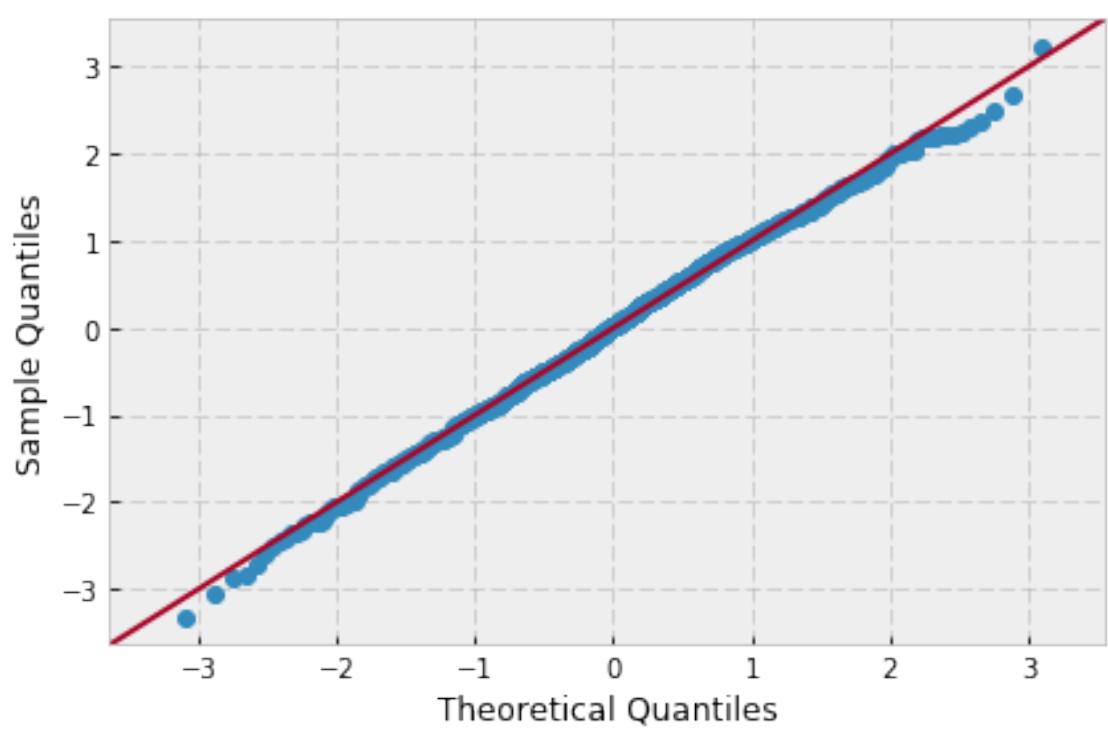
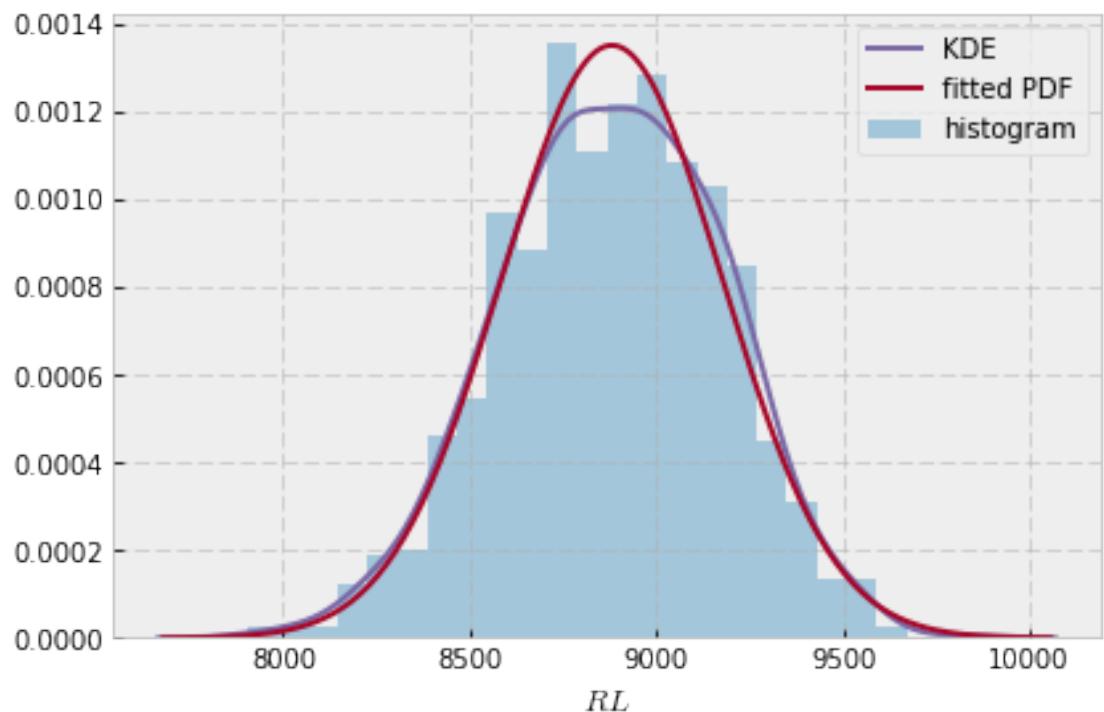


```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(), loc=8883.078759286605, scale=283.60051139482044)
DescribeResult(nobs=1000, minmax=(7999.9007281057811, 9795.0109674953801)
               mean=8883.0787592866054, variance=80509.759823226908,
               skewness=-0.10452390330558342, kurtosis=-0.08960350482126511)
```

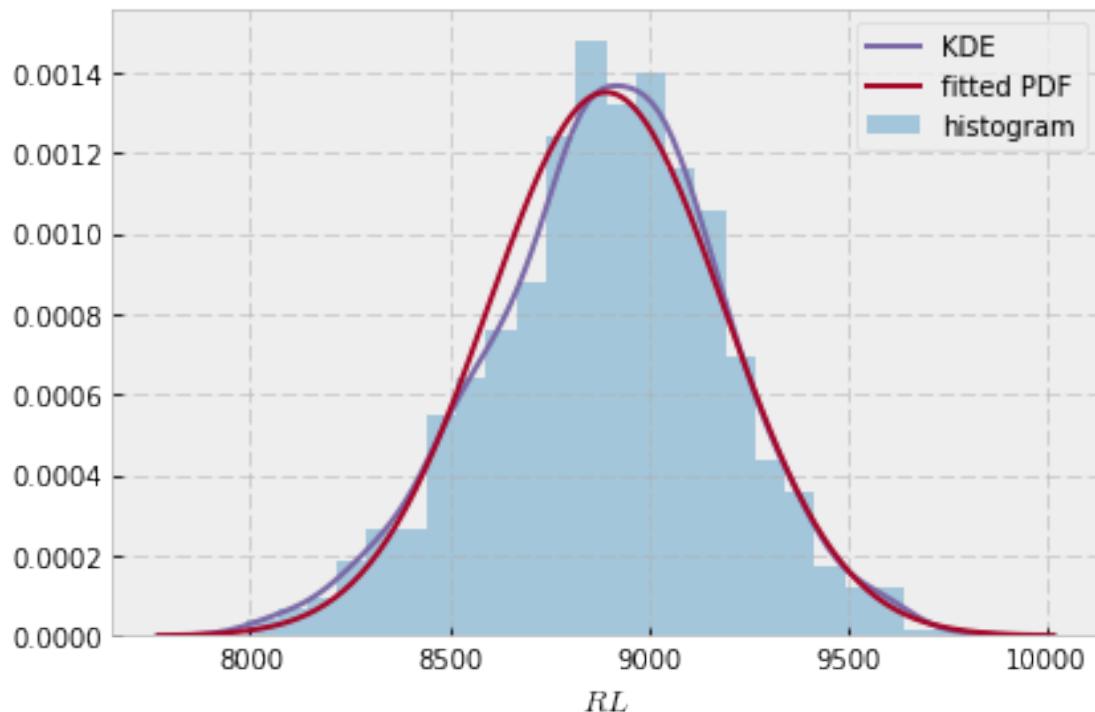


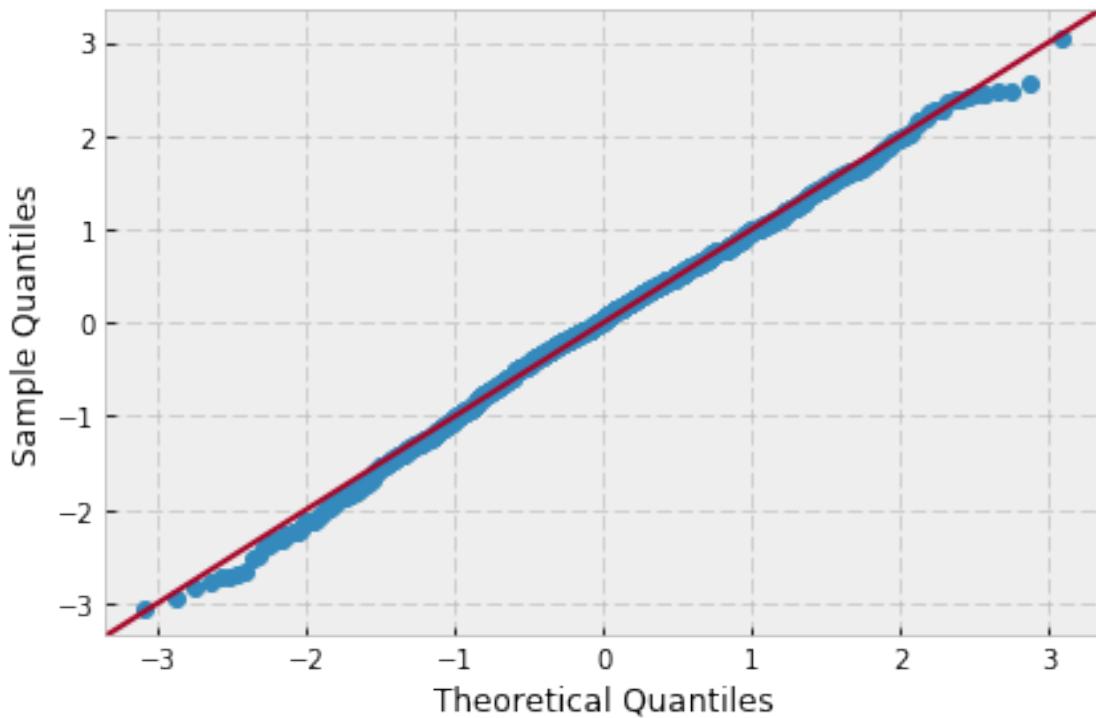


```
variable name: sigma
variable value: 0.0666666666666667
distribution: normal(shape=(),loc=8880.946242795184, scale=295.4858738721719)
DescribeResult(nobs=1000, minmax=(7902.0075517346449, 9832.492235231879)
               mean=8880.9462427951839, variance=87399.300958960055,
               skewness=-0.12394269070725283, kurtosis=-0.1844533895835032)
```

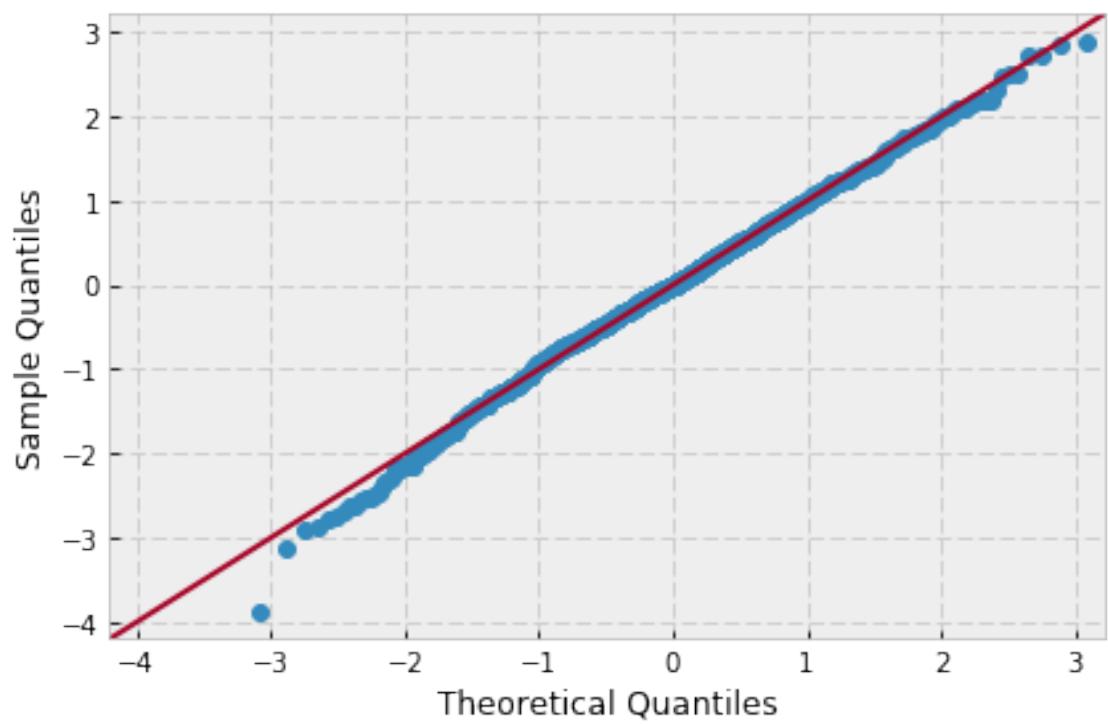
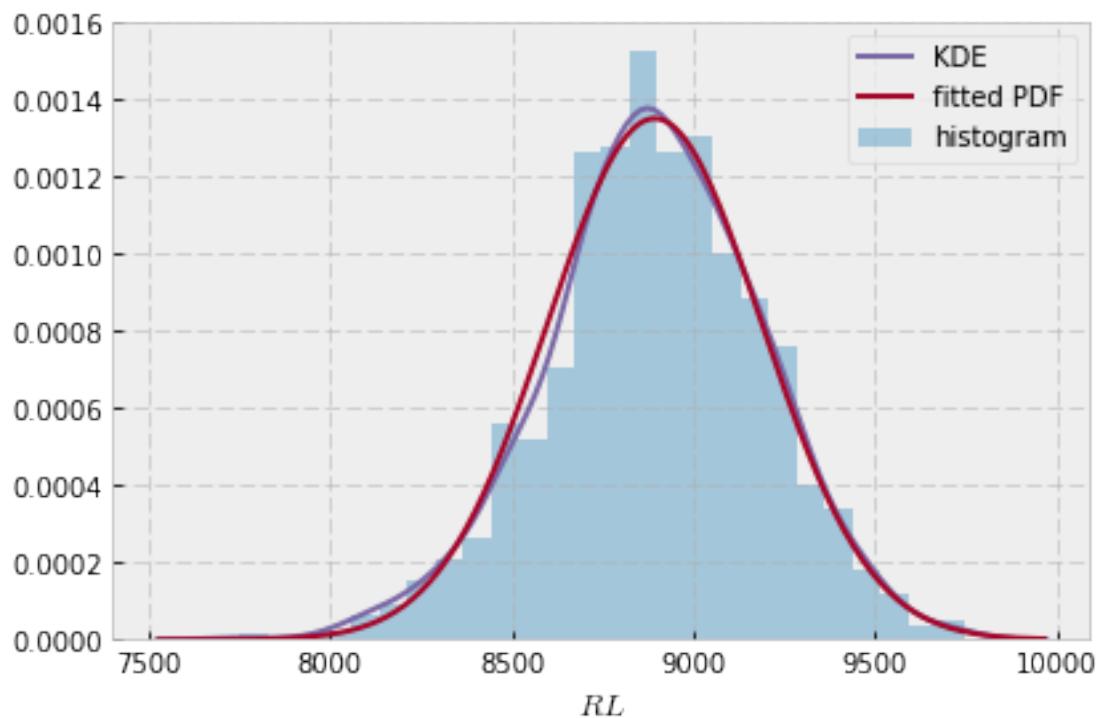


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=8892.846635460086, scale=295.3353972893993)
DescribeResult(nobs=1000, minmax=(7993.0336899914473, 9791.5603214926687)
               mean=8892.846635460086, variance=87310.307199286603,
               skewness=-0.17274084783082305, kurtosis=0.01072024969397889)
```

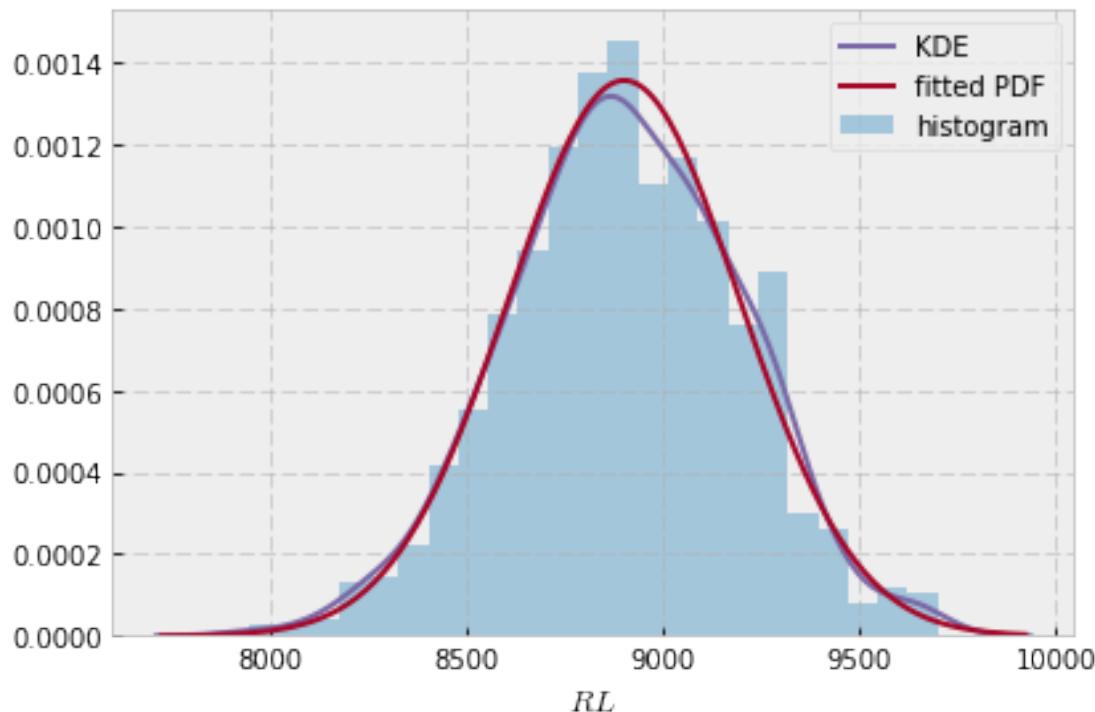


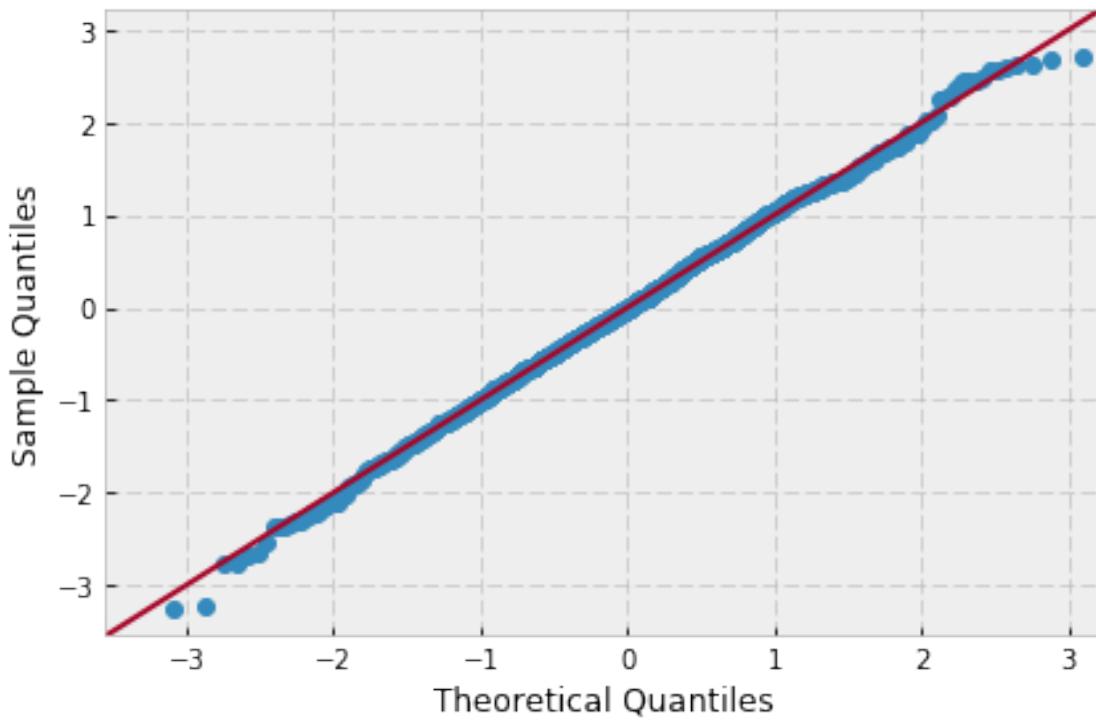


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(),loc=8892.68520866906, scale=295.0987883293162)
DescribeResult(nobs=1000, minmax=(7752.1433839712427, 9742.1728012617223)
               mean=8892.6852086690596, variance=87170.465338769354,
               skewness=-0.17399448884873608, kurtosis=0.20011292625464927)
```

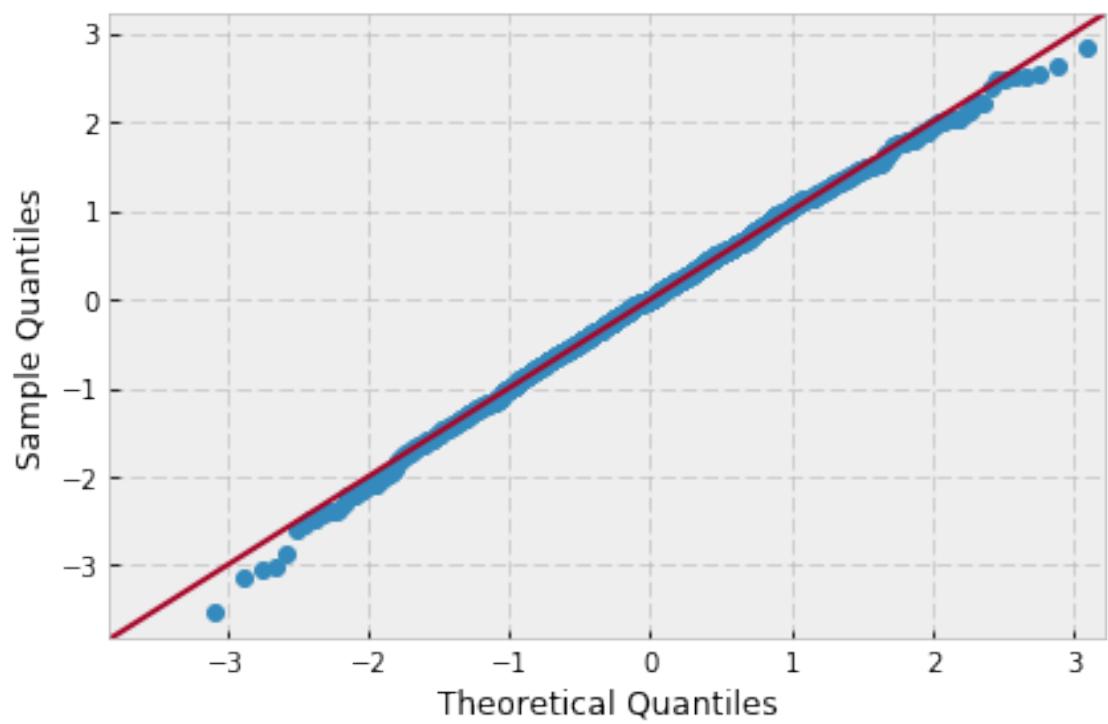
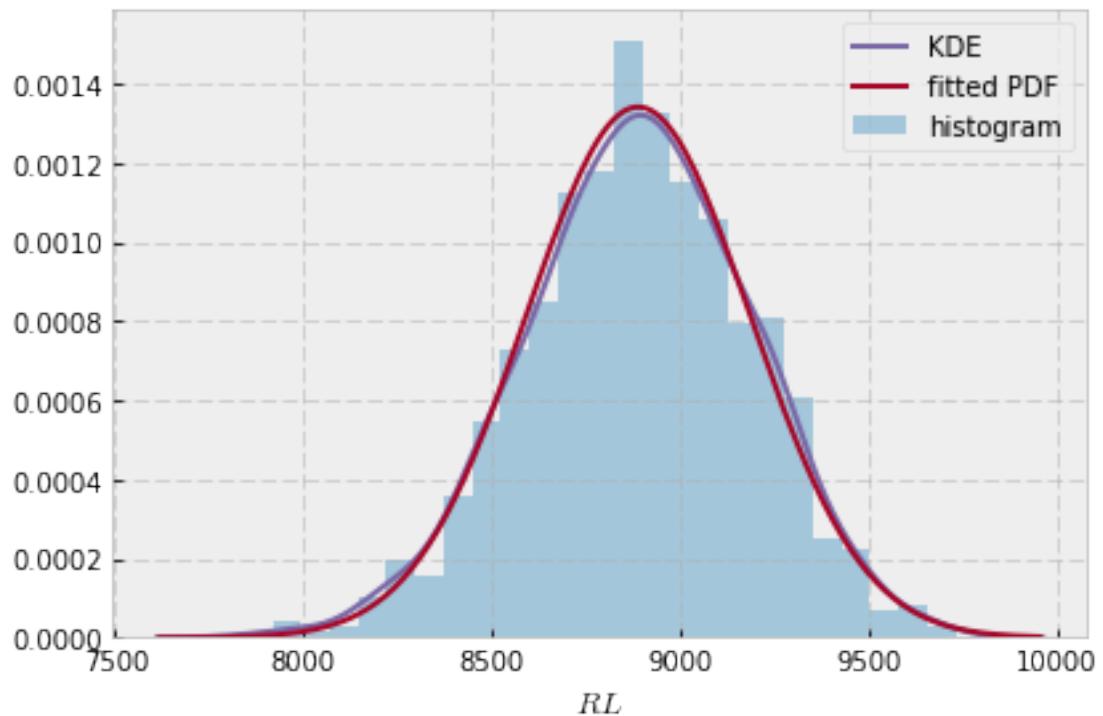


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=8902.022643208613, scale=293.780166286381)
DescribeResult(nobs=1000, minmax=(7947.4223047121595, 9699.2606528527358)
               mean=8902.0226432086129, variance=86393.179282536221,
               skewness=-0.06843267545565648, kurtosis=-0.0600662055564154)
```

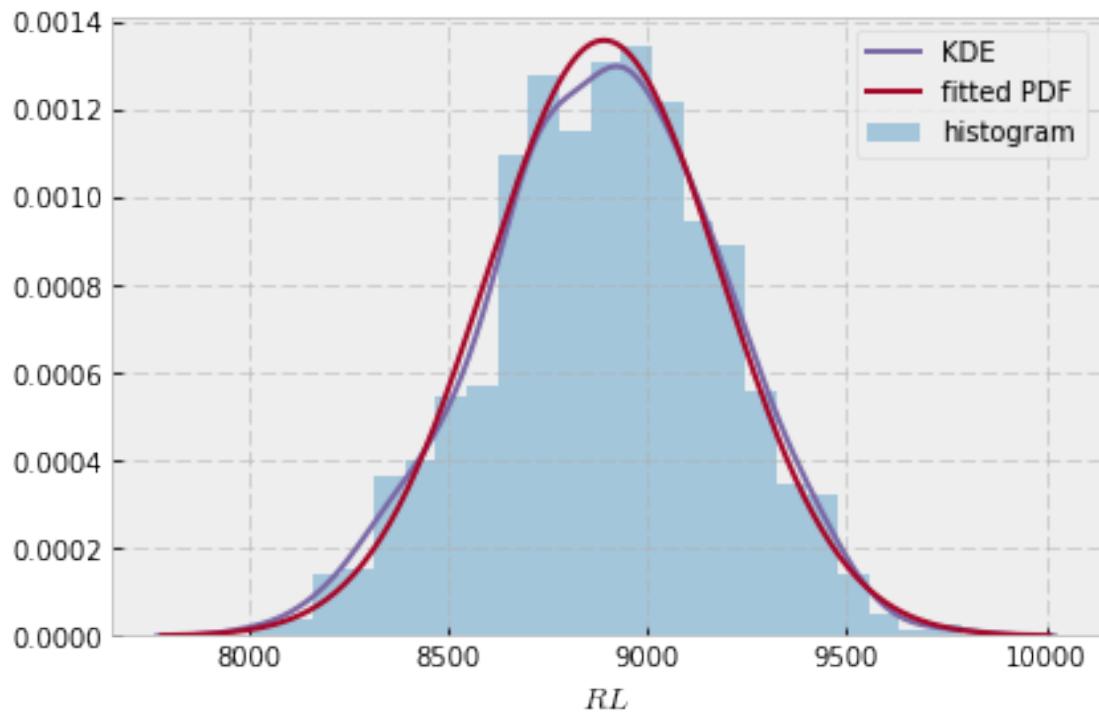


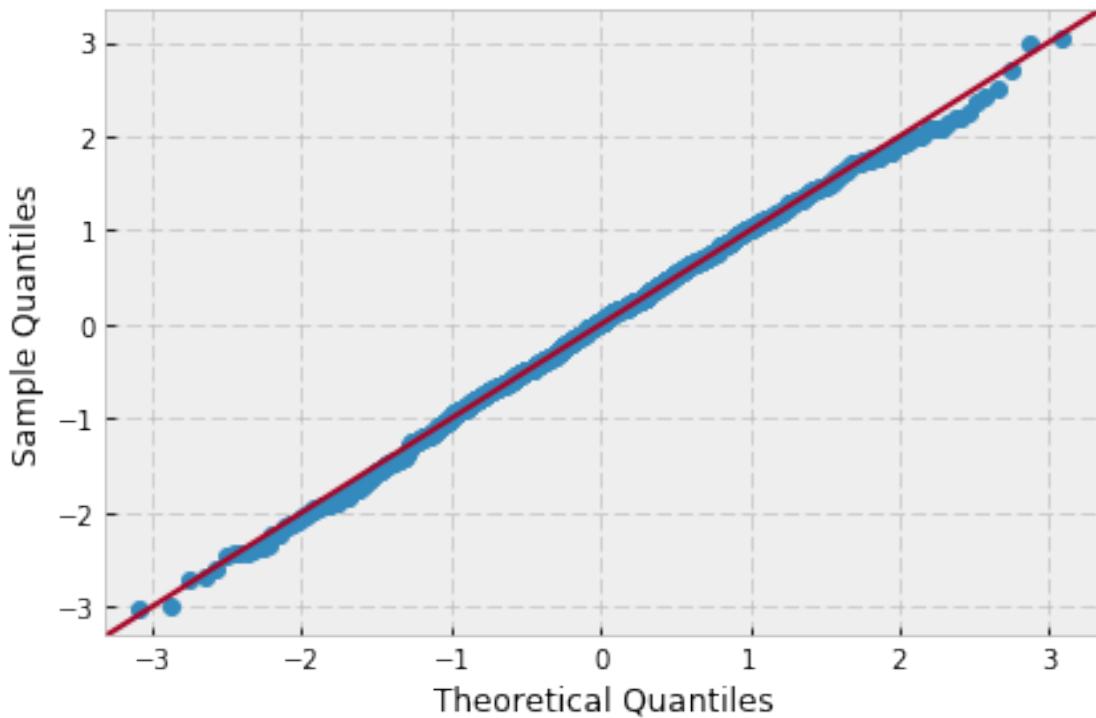


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(),loc=8887.539035040167, scale=297.4444911597158)
DescribeResult(nobs=1000, minmax=(7842.4921710504022, 9729.0927400968085)
               mean=8887.5390350401667, variance=88561.787108370627,
               skewness=-0.14674352216049869, kurtosis=-0.01448045150698185)
```

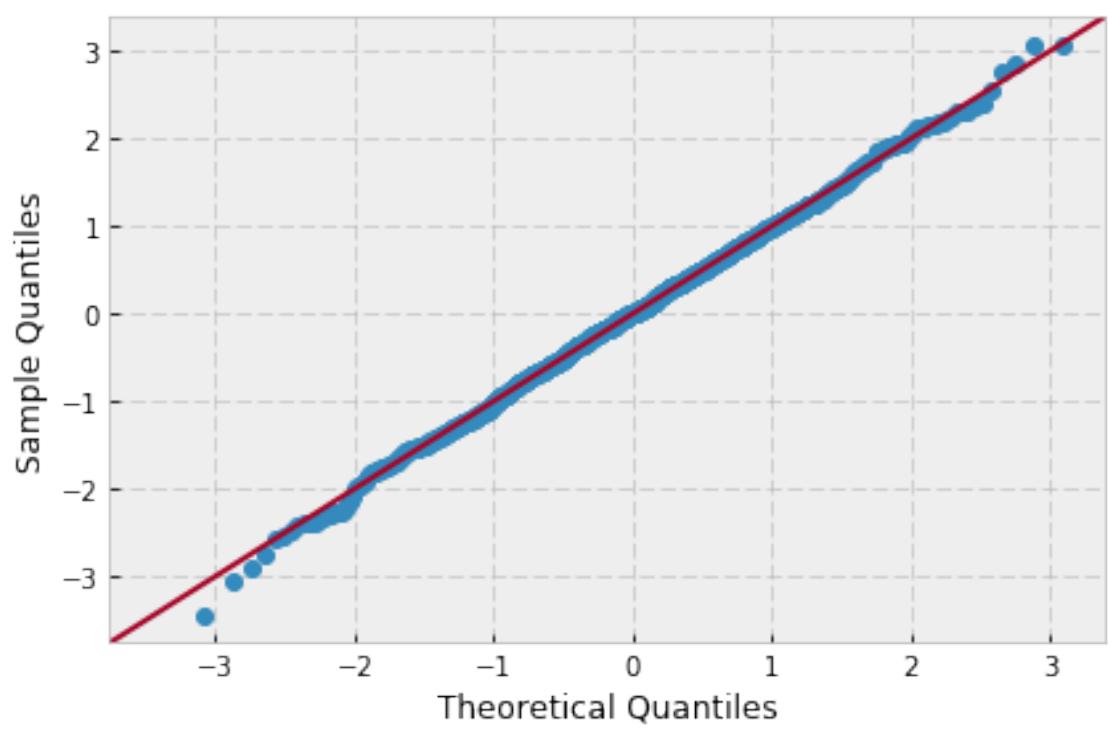
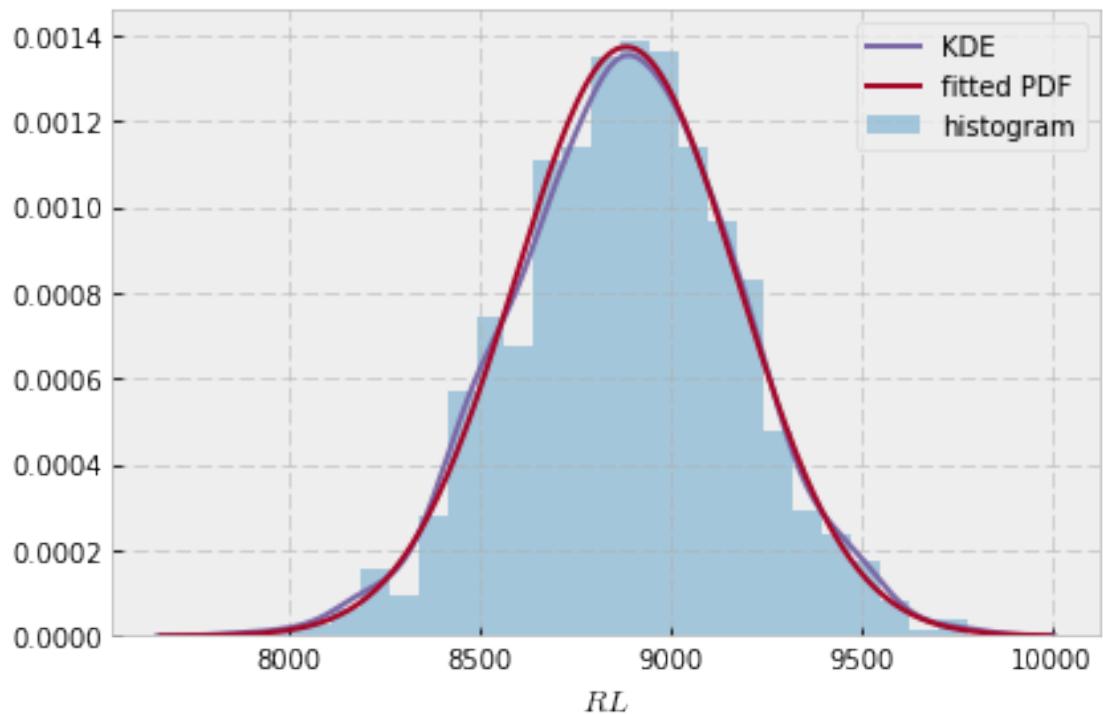


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=8892.267021434629, scale=294.2293252162429)
DescribeResult(nobs=1000, minmax=(8006.5779603882538, 9787.0355191459639)
               mean=8892.267021434629, variance=86657.553370576192,
               skewness=-0.10718694159862996, kurtosis=-0.15514707727498944)
```

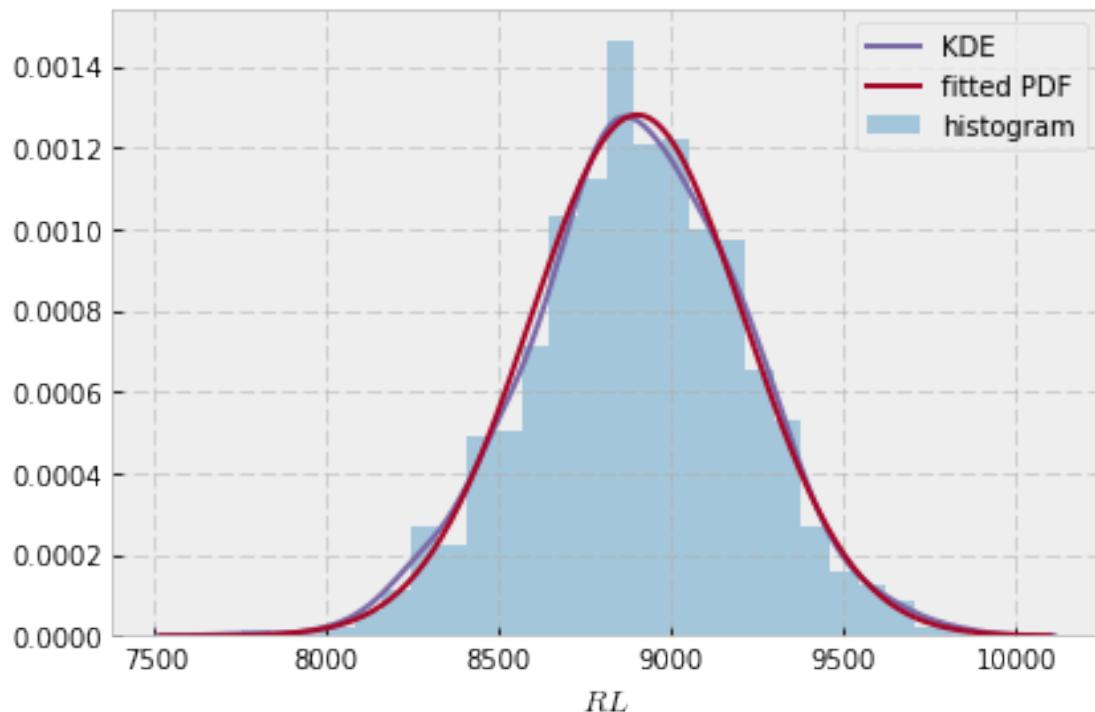


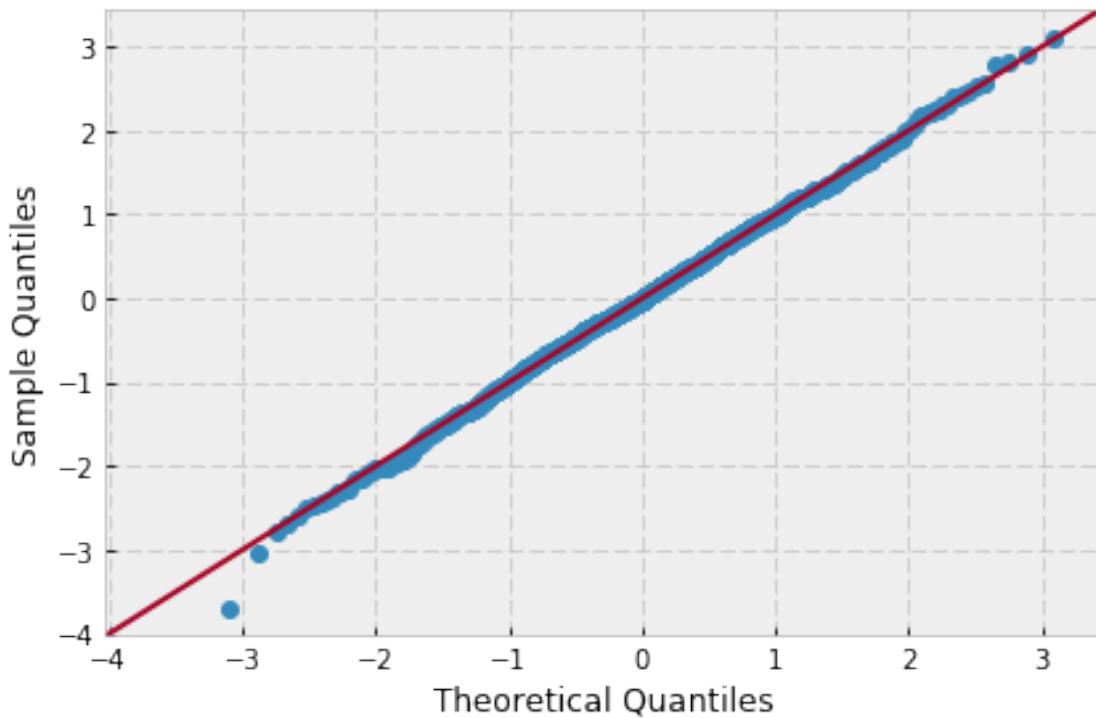


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(),loc=8884.07524885335, scale=290.71440392177175)
DescribeResult(nobs=1000, minmax=(7885.2456133858568, 9775.3011578971727)
               mean=8884.0752488533508, variance=84599.464111702764,
               skewness=-0.03902436714050272, kurtosis=0.014378776910580537)
```

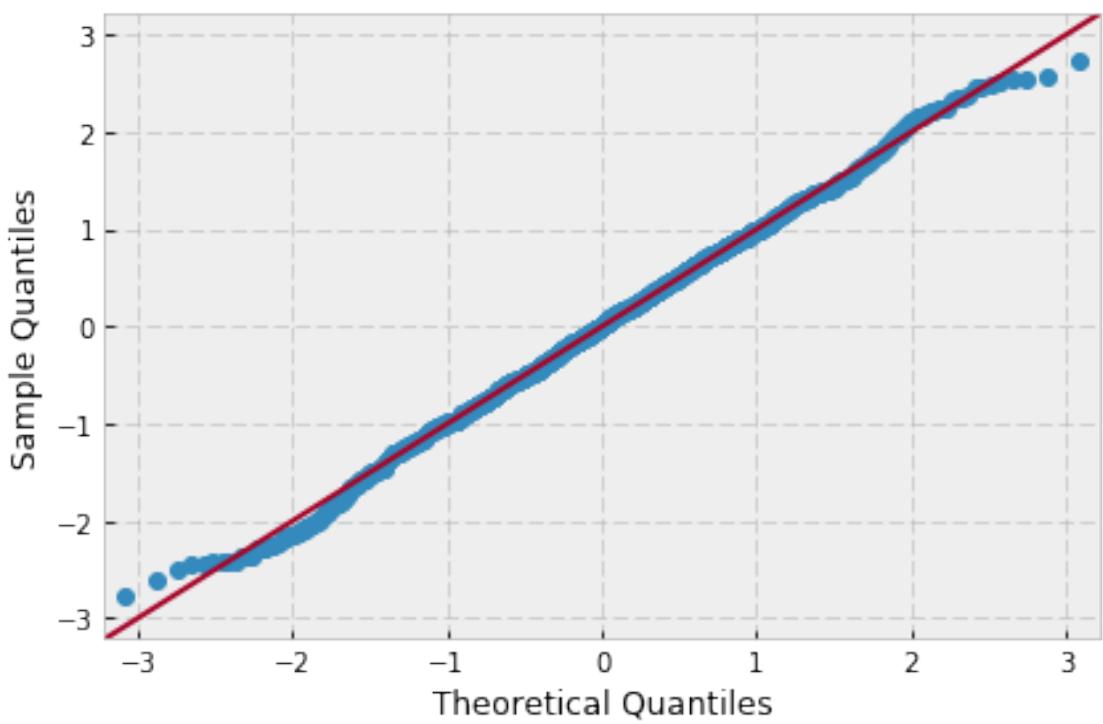
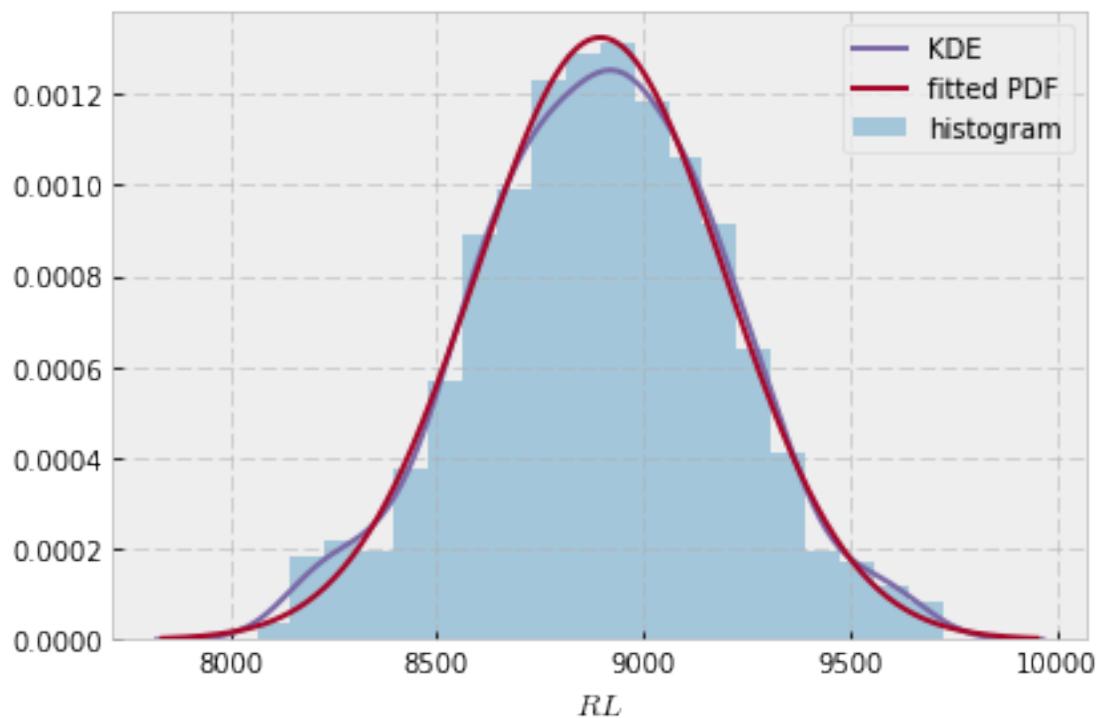


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(),loc=8901.436779045735, scale=311.6398215719581)
DescribeResult(nobs=1000, minmax=(7753.4187457500011, 9866.1348001363749)
               mean=8901.4367790457345, variance=97216.594984386291,
               skewness=-0.06784939812456103, kurtosis=0.05491467644058057)
```

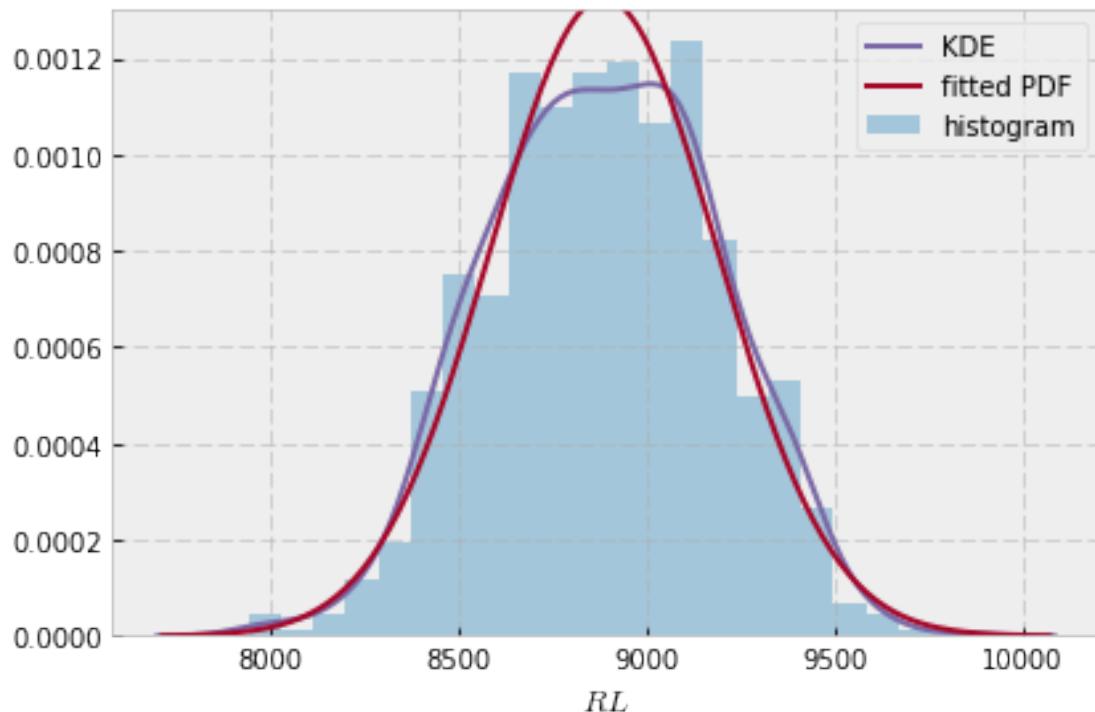


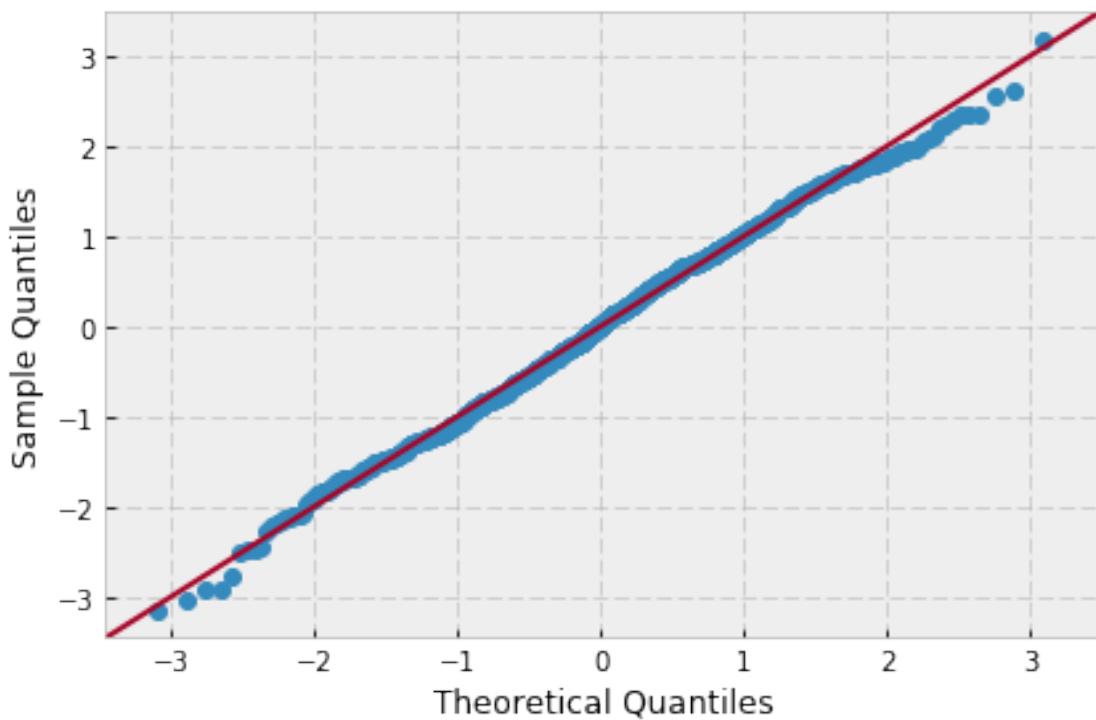


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=8895.568954602593, scale=301.3678006006078)
DescribeResult(nobs=1000, minmax=(8064.5719637805933, 9723.6400576445521)
               mean=8895.5689546025933, variance=90913.464703551261,
               skewness=-0.04328960005626475, kurtosis=-0.16322488515177502)
```

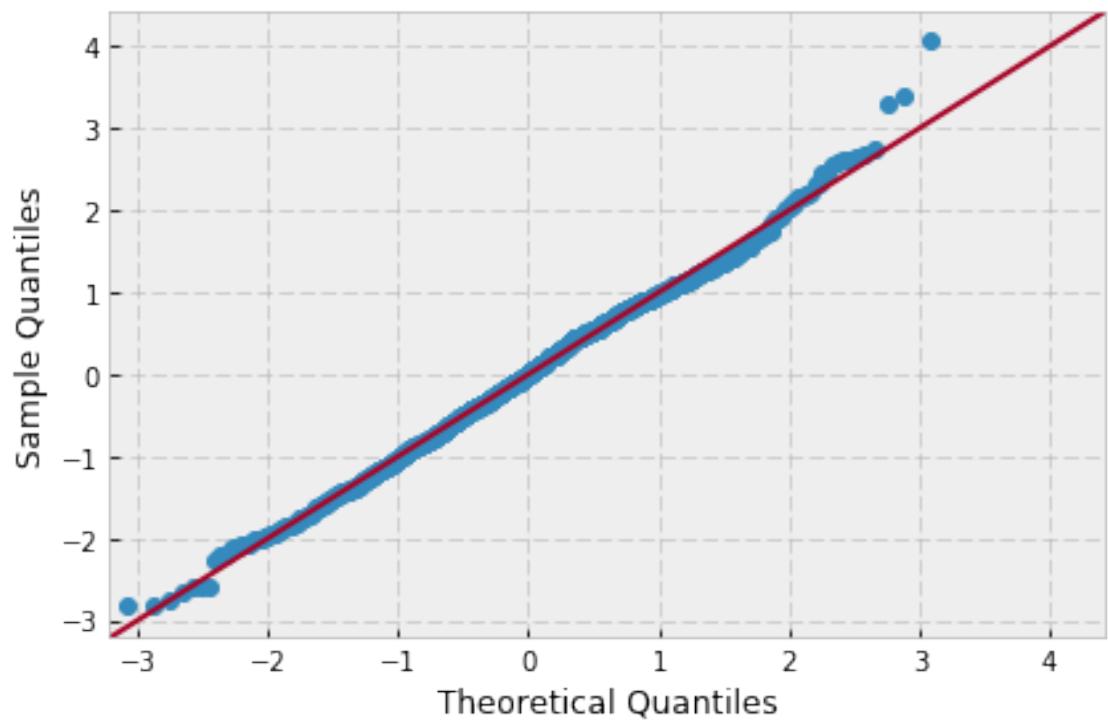
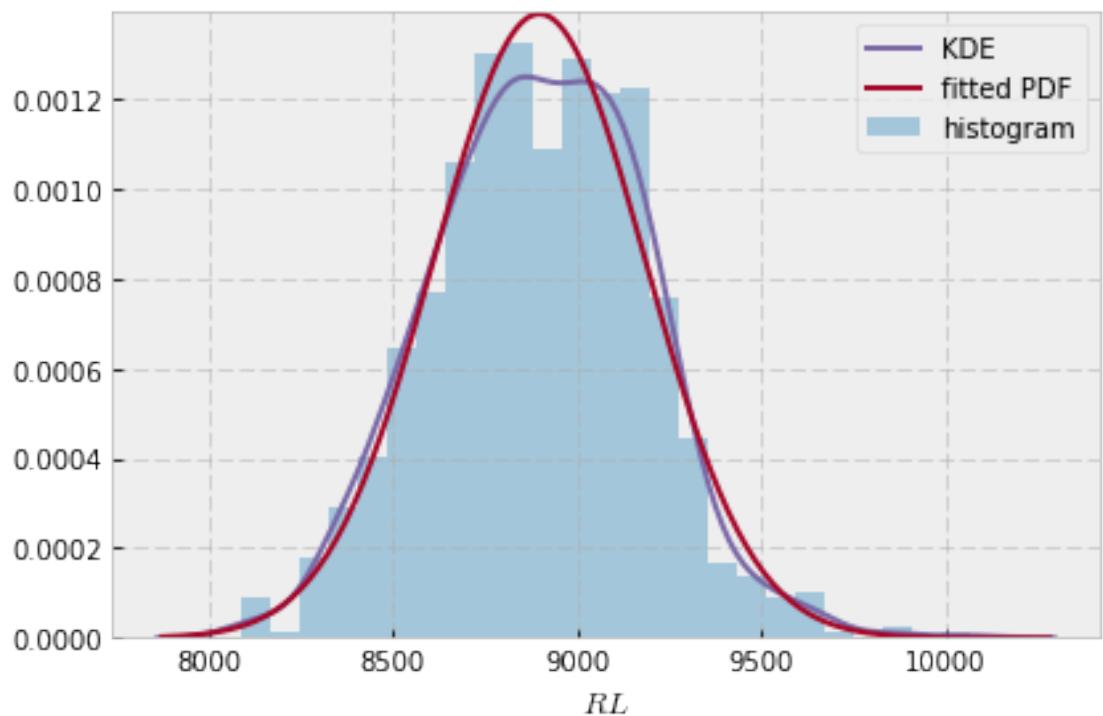


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(),loc=8885.237795545188, scale=301.3774584913905)
DescribeResult(nobs=1000, minmax=(7940.0224167996748, 9841.3184400439131)
               mean=8885.2377955451884, variance=90919.291778508312,
               skewness=-0.05827576953576213, kurtosis=-0.32636682108884907)
```

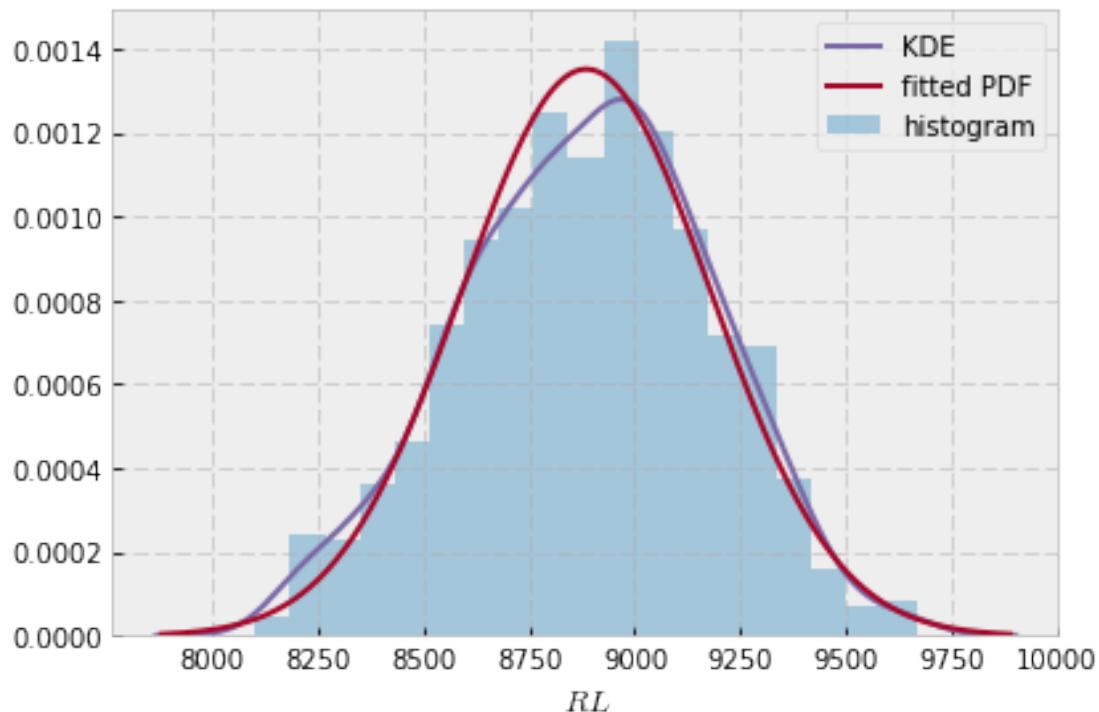


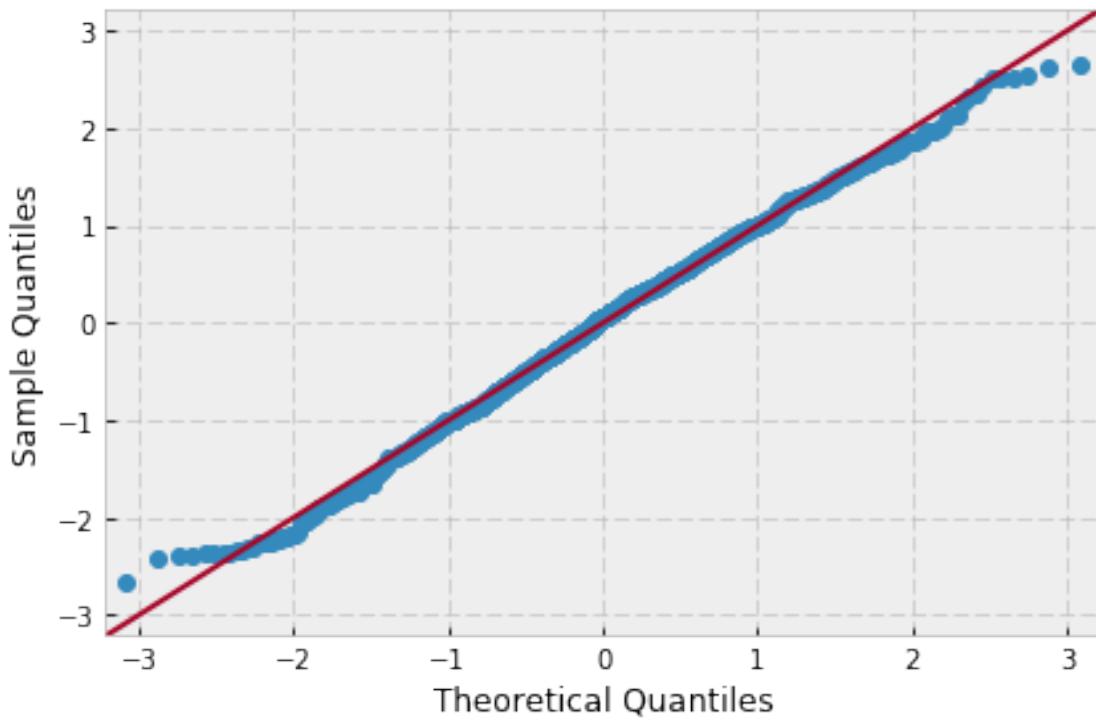


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(),loc=8896.608017686714, scale=287.14739898191385)
DescribeResult(nobs=1000, minmax=(8088.0252981169169, 10065.960188501833)
               mean=8896.6080176867144, variance=82536.164906985388,
               skewness=0.060182073482801954, kurtosis=0.10510345267401933)
```

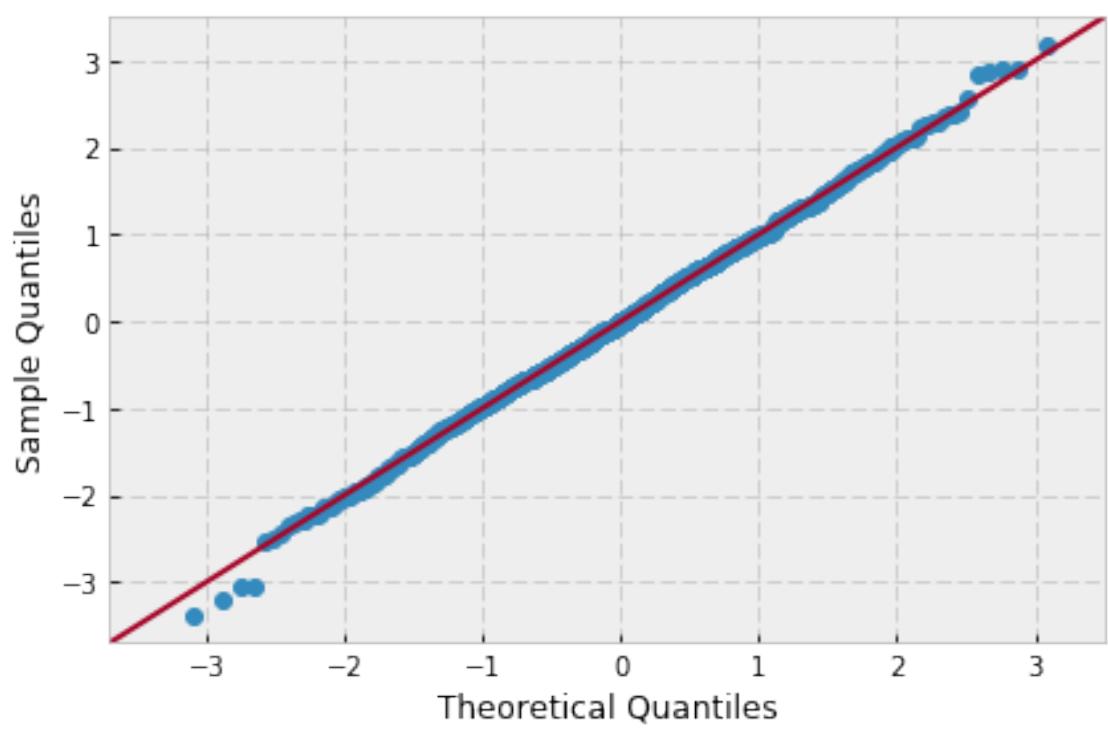
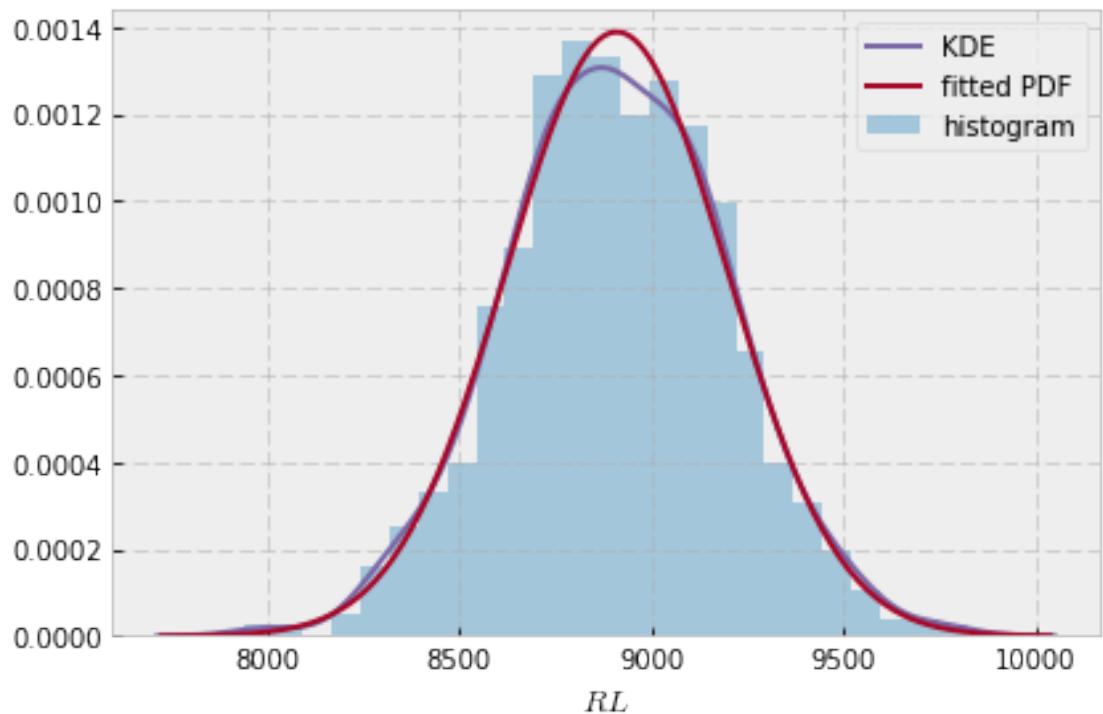


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(),loc=8884.981863815598, scale=295.3469140469768)
DescribeResult(nobs=1000, minmax=(8101.3833447968173, 9665.9848721160561)
               mean=8884.9818638155975, variance=87317.116753826136,
               skewness=-0.11497487302266225, kurtosis=-0.3432571250917271)
```

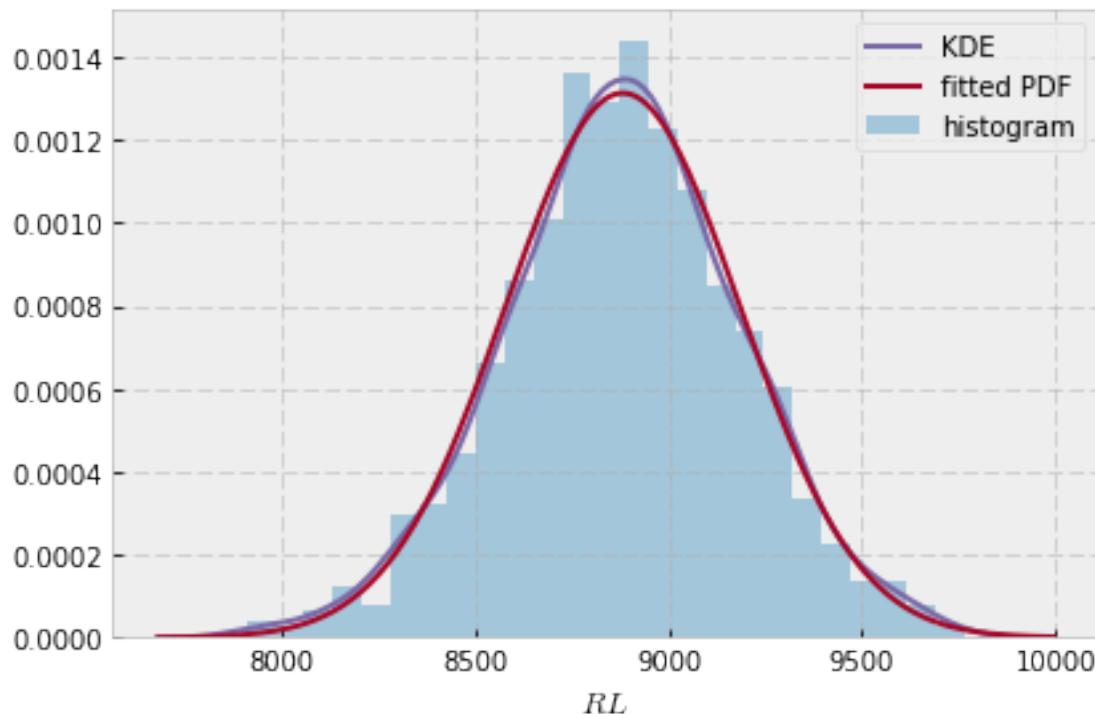


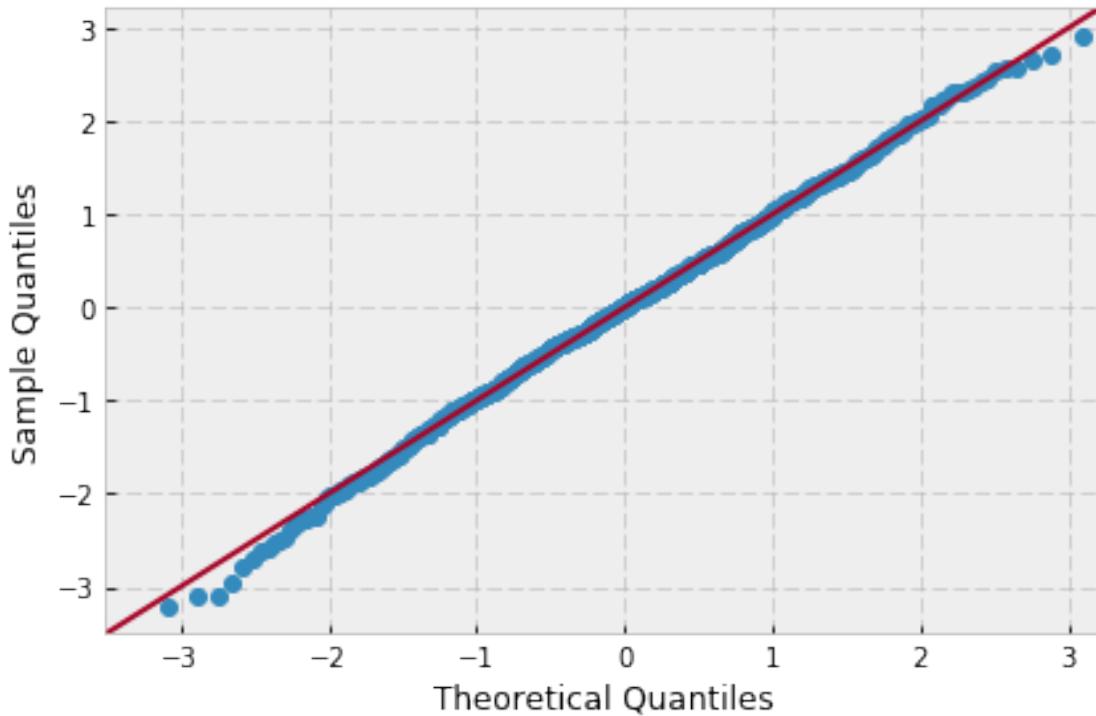


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(),loc=8909.889963416197, scale=287.3499877560367)
DescribeResult(nobs=1000, minmax=(7941.7428632239989, 9821.0408548768064)
               mean=8909.8899634161971, variance=82652.668131525948,
               skewness=-0.014273990063063373, kurtosis=0.056166833053407306)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=8881.587970969511, scale=303.7754744405431)
DescribeResult(nobs=1000, minmax=(7909.30493754894, 9765.0451107403551)
               mean=8881.5879709695109, variance=92371.910782359424,
               skewness=-0.08187450849267093, kurtosis=0.09133626571536091)
```

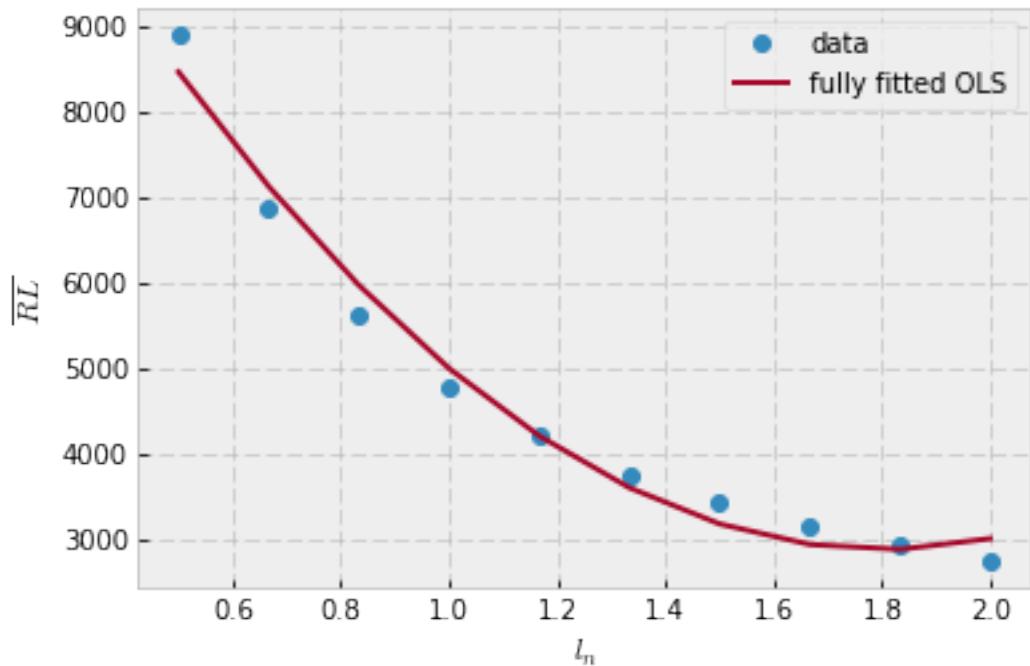




5.2 Parameter regression

```
In [16]: var_name = 'total_root_length'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly1')
mean_reg('maxB', var_name, 'poly1')
mean_reg('r', var_name, 'poly1')
mean_reg('sigma', var_name, 'poly1')
mean_reg('N', var_name, 'poly1')

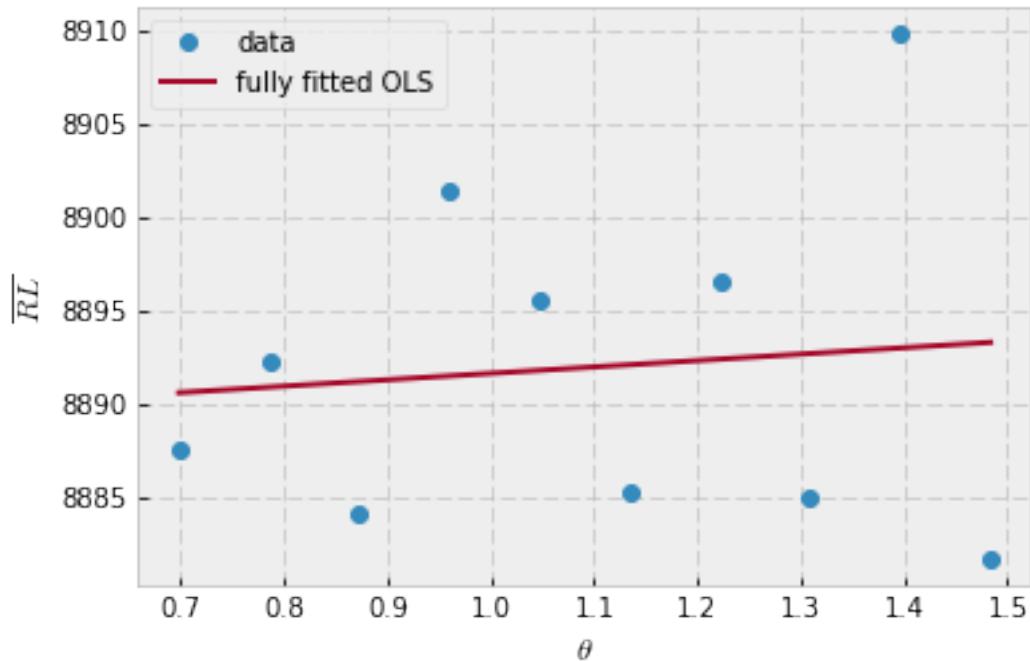
variable name: ln
model: poly2
r-squared: 0.9297134976359078
explained variance: 0.9298455993260001
```



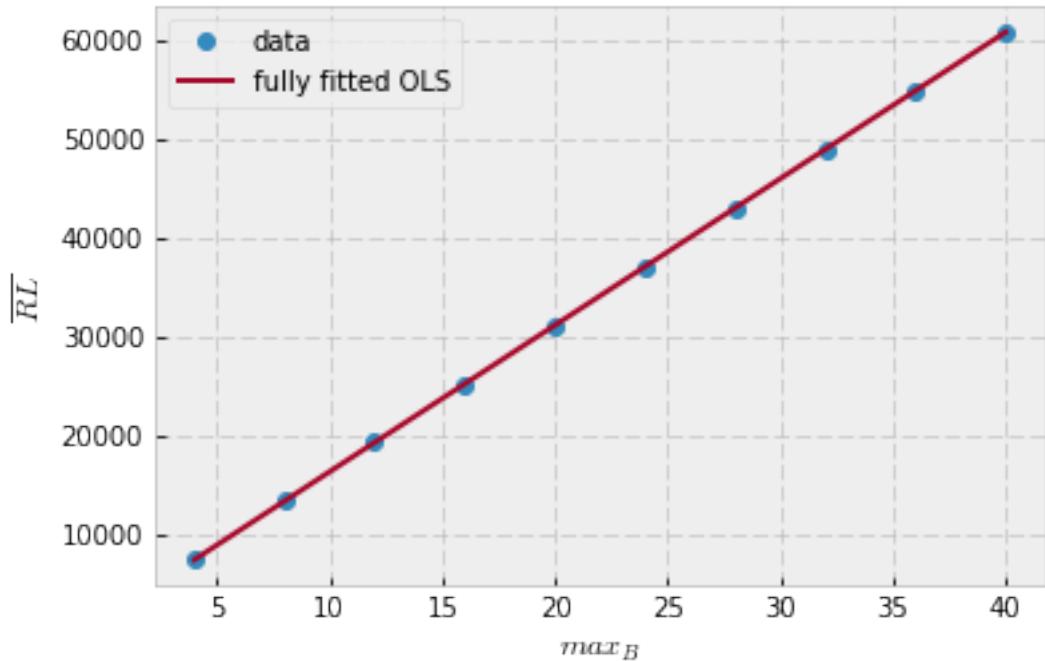
```

variable name: theta
model: poly1
r-squared: -0.7024425628746567
explained variance: -0.7010939118978465

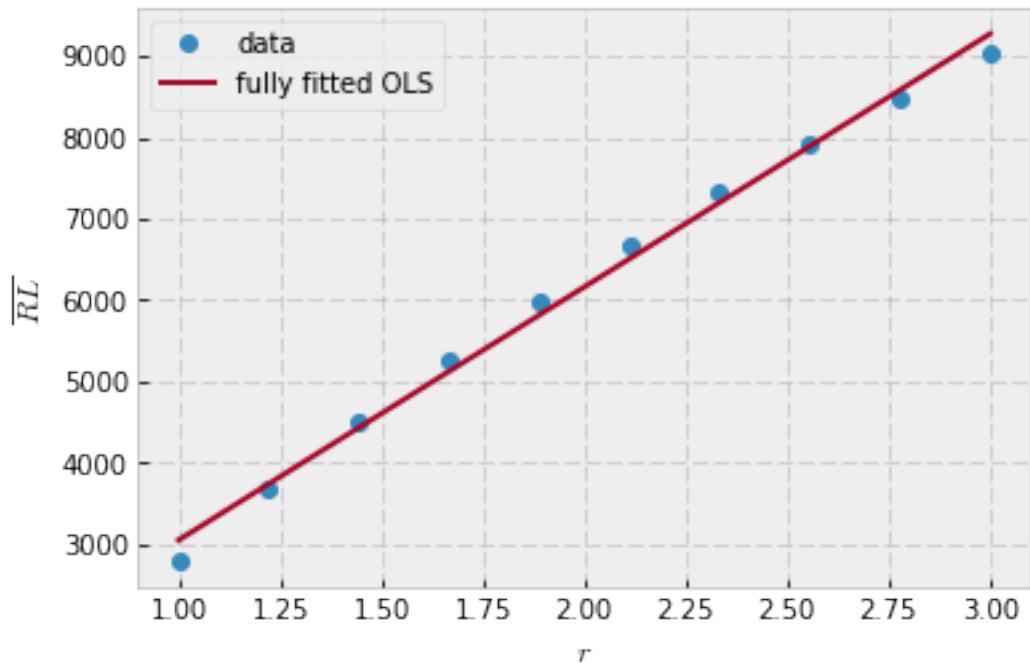
```



```
variable name: maxB  
model: poly1  
r-squared: 0.9999987826703696  
explained variance: 0.9999987844942784
```



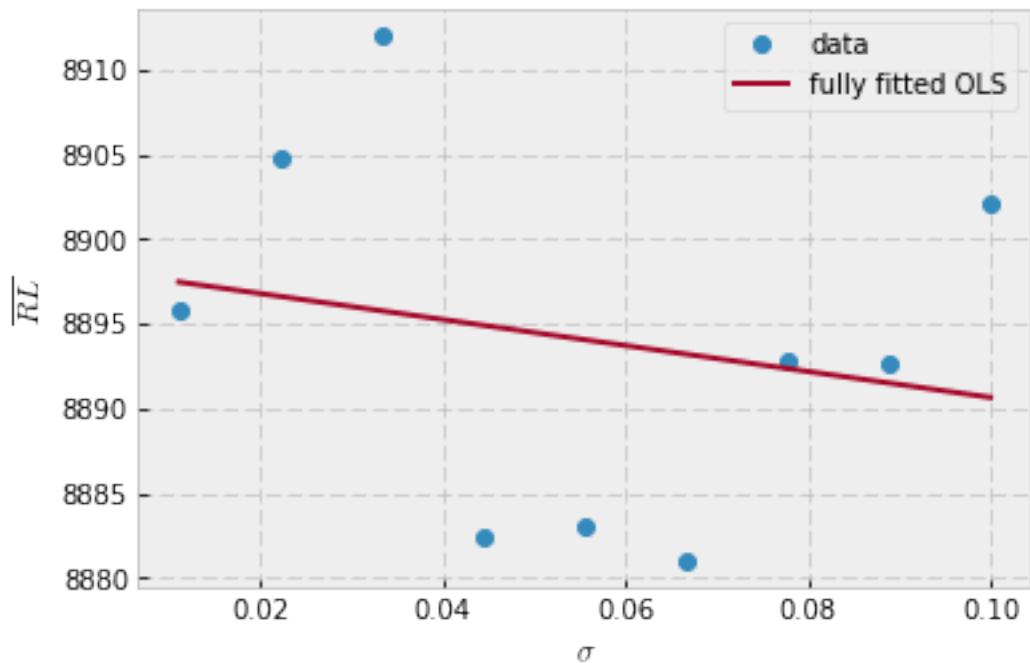
```
variable name: r  
model: poly1  
r-squared: 0.9891012756025228  
explained variance: 0.989229692864304
```



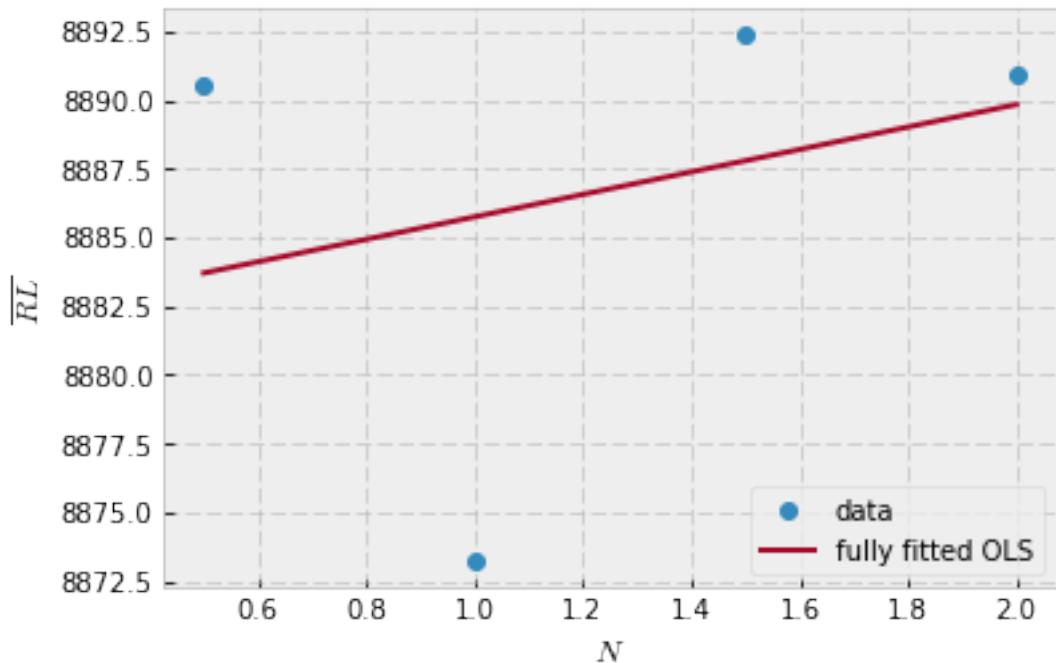
```

variable name: sigma
model: poly1
r-squared: -0.5100679957527106
explained variance: -0.5028739755407698

```



```
variable name: N
model: poly1
r-squared: -2.621483202587284
explained variance: -2.3909199112887927
```

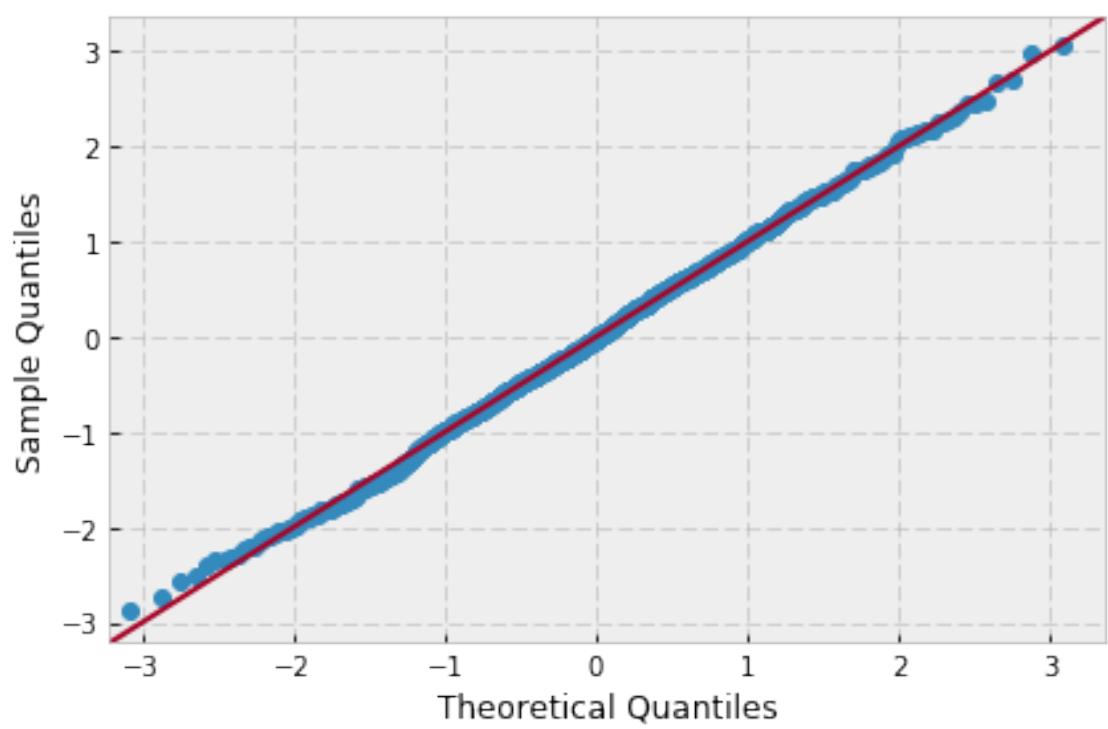
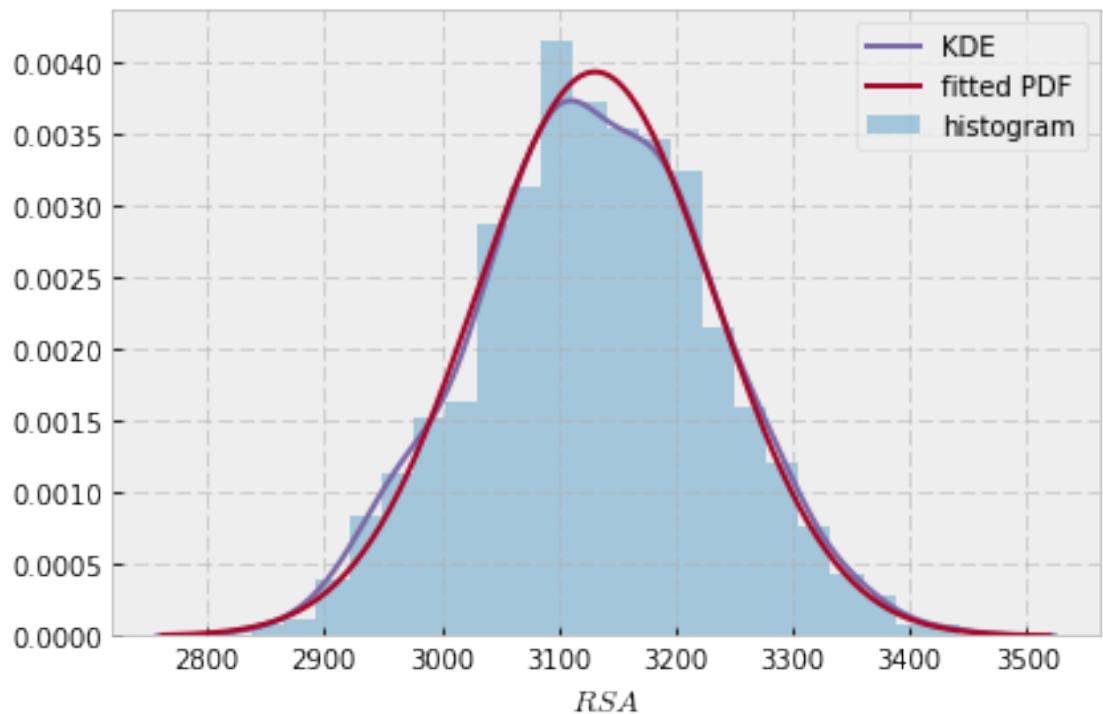


6 Total root surface area

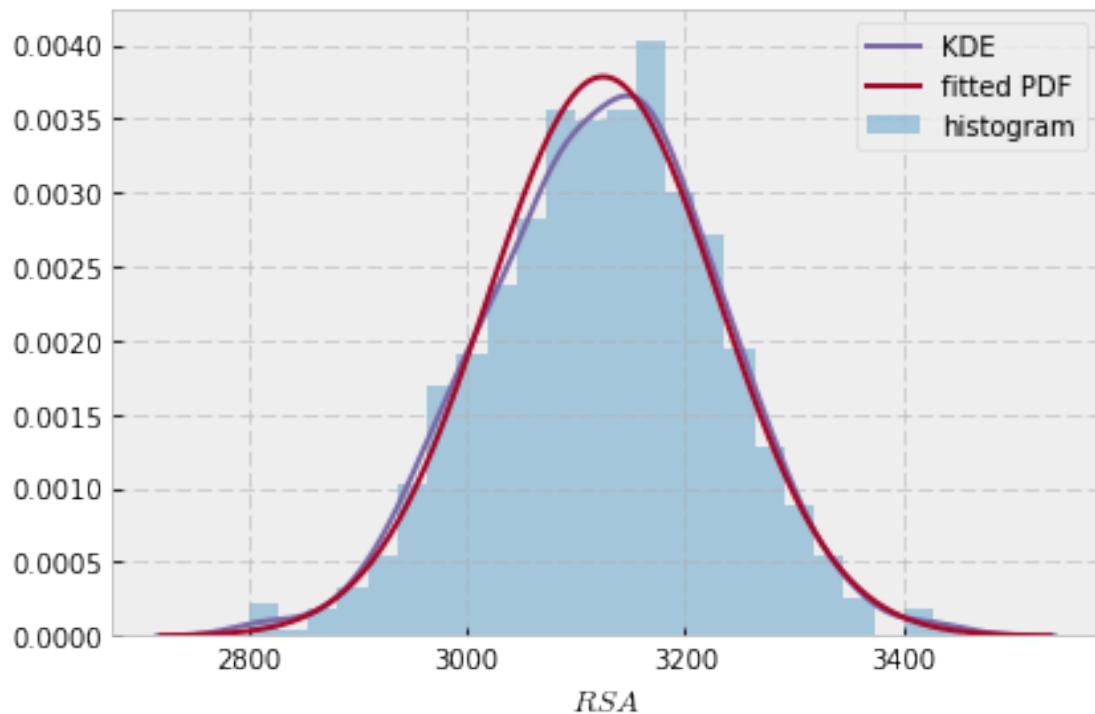
6.1 Probability distributions

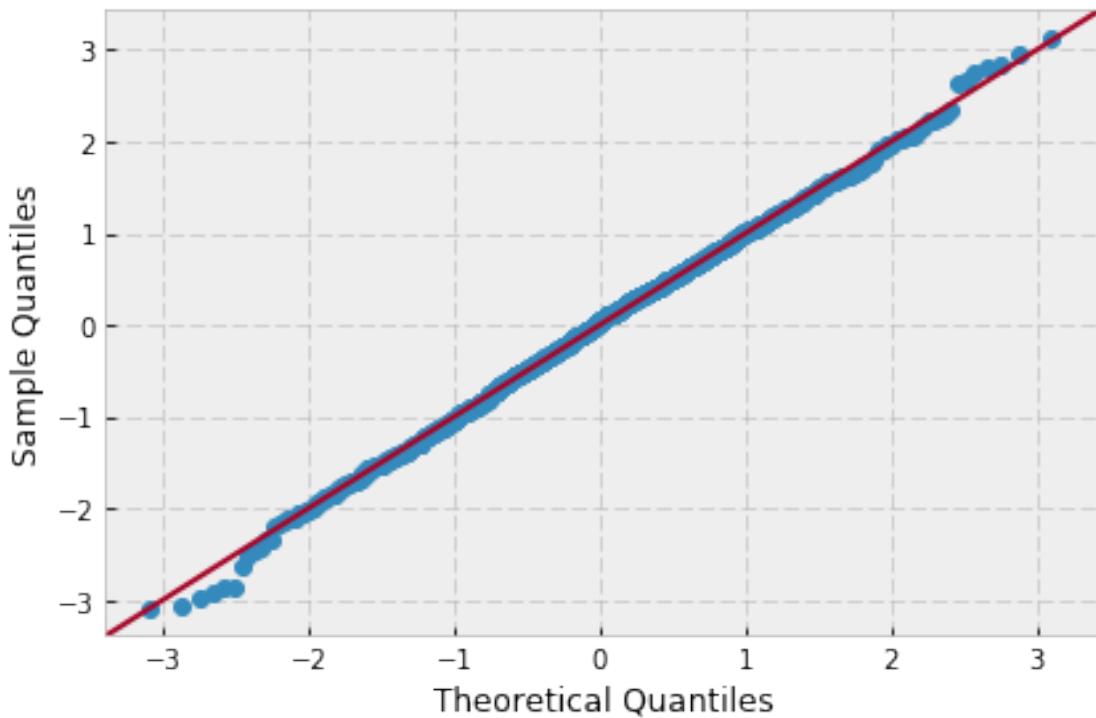
```
In [17]: plot_dist('total_root_surface_area')

variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=3131.2876212139995, scale=101.50440008313855)
DescribeResult(nobs=1000, minmax=(2838.4246830000002, 3442.2302709999999)
               mean=3131.2876212139995, variance=10313.456692930788,
               skewness=0.01080016267018221, kurtosis=-0.22351992728685843)
```

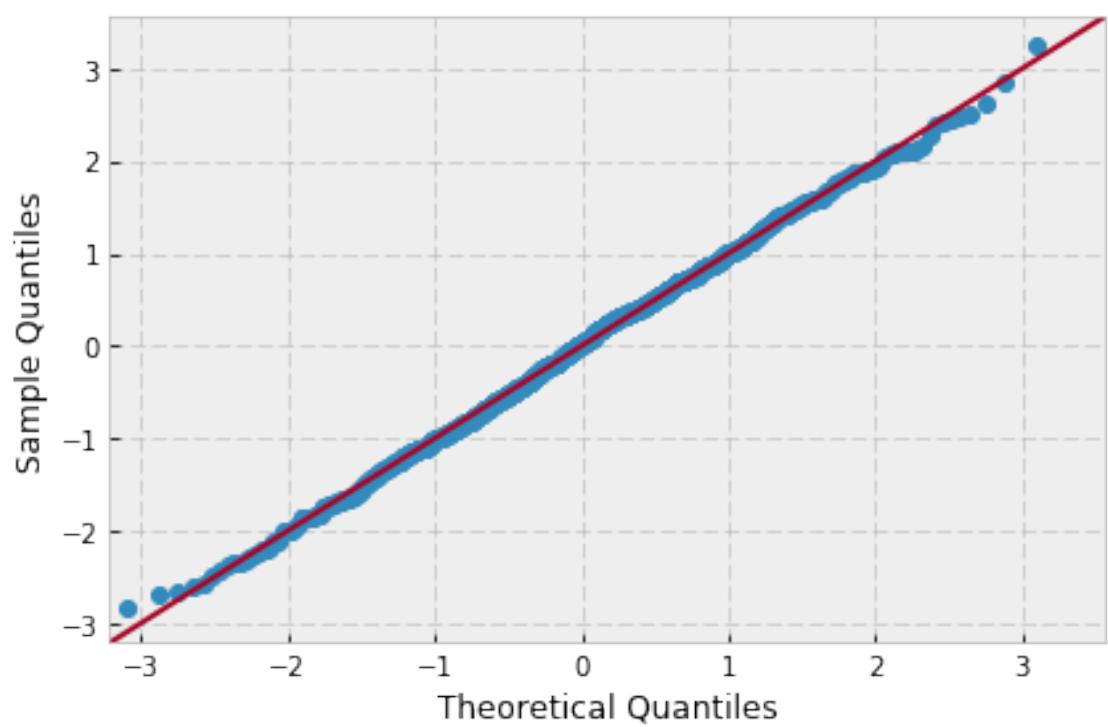
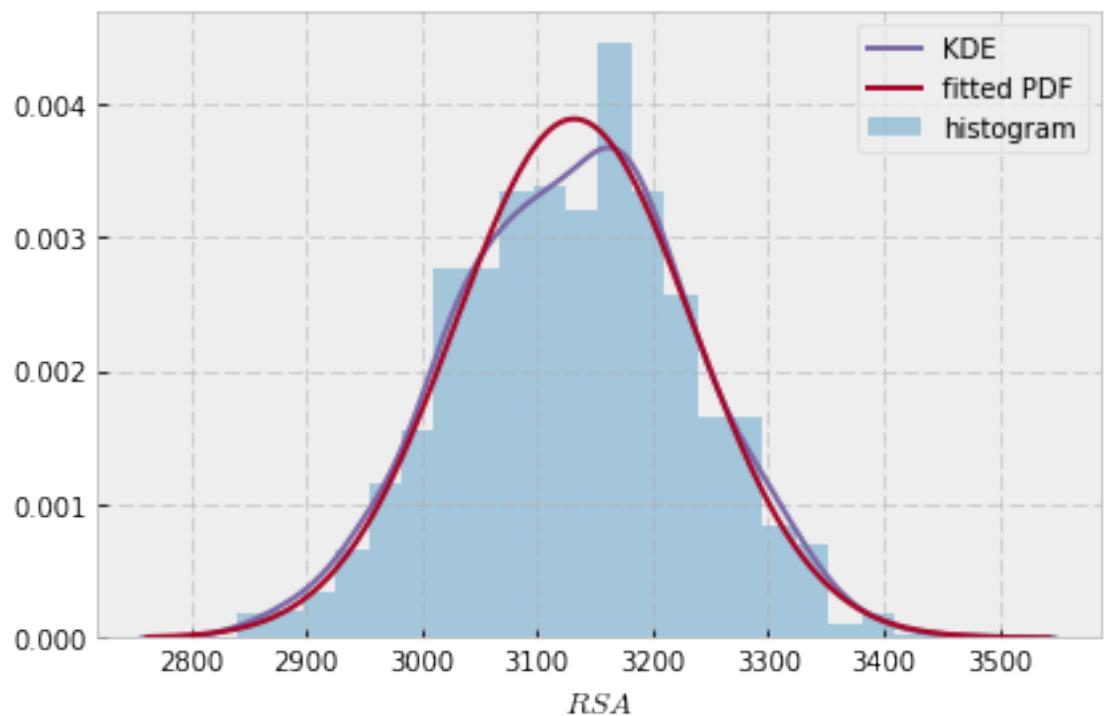


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=3125.196250888, scale=105.40504298983605)
DescribeResult(nobs=1000, minmax=(2800.373325999999, 3454.705645)
    mean=3125.1962508880001, variance=11121.344432121305,
    skewness=-0.076617523185813, kurtosis=-0.01339880264033022)
```

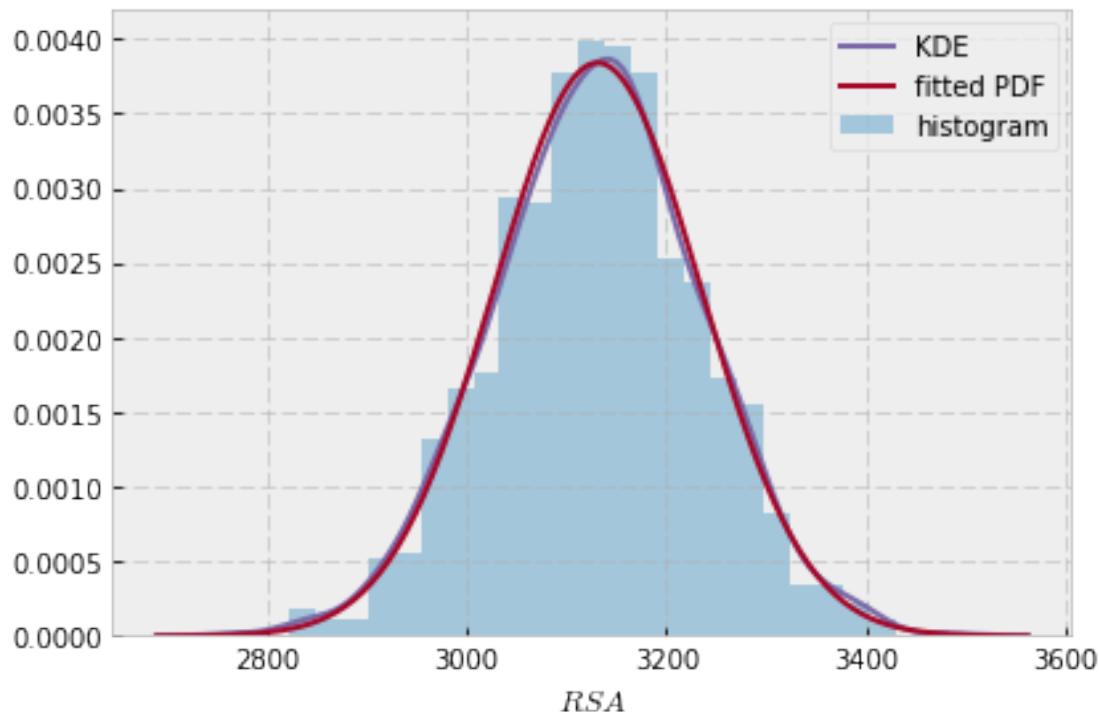


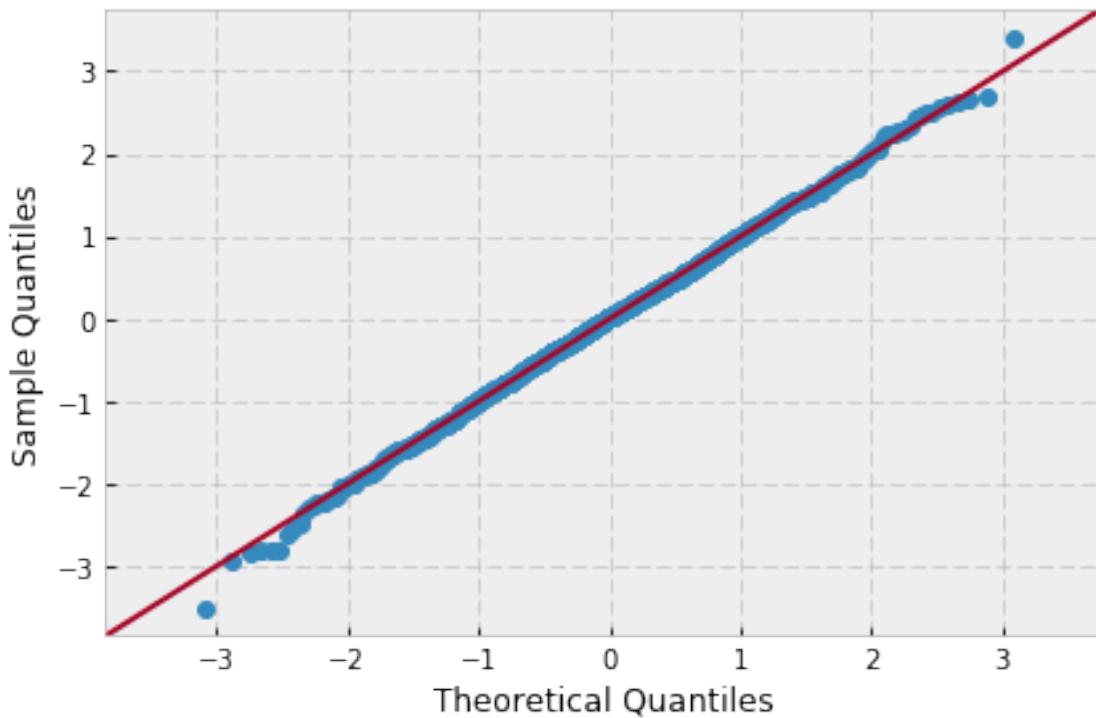


```
variable name: N
variable value: 1.5
distribution: normal(shape=(),loc=3131.853244065, scale=102.60712761733893)
DescribeResult(nobs=1000, minmax=(2839.986269, 3465.6309110000002)
               mean=3131.8532440650001, variance=10538.761399280156,
               skewness=-0.025240995162718834, kurtosis=-0.234375106316715)
```

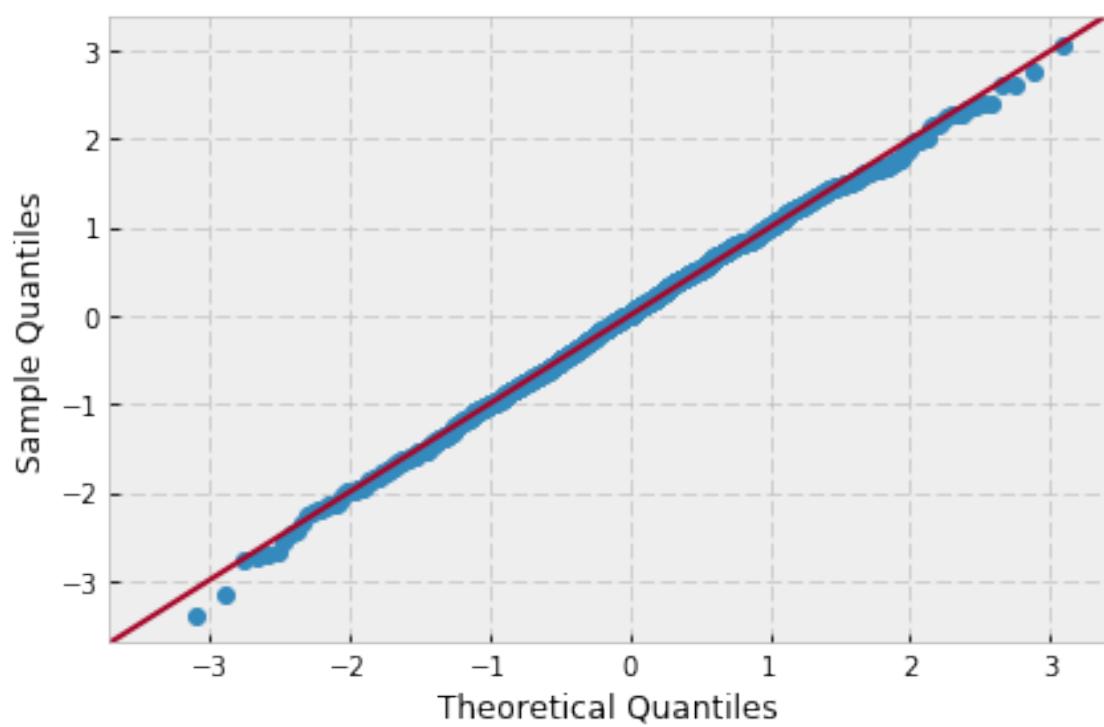
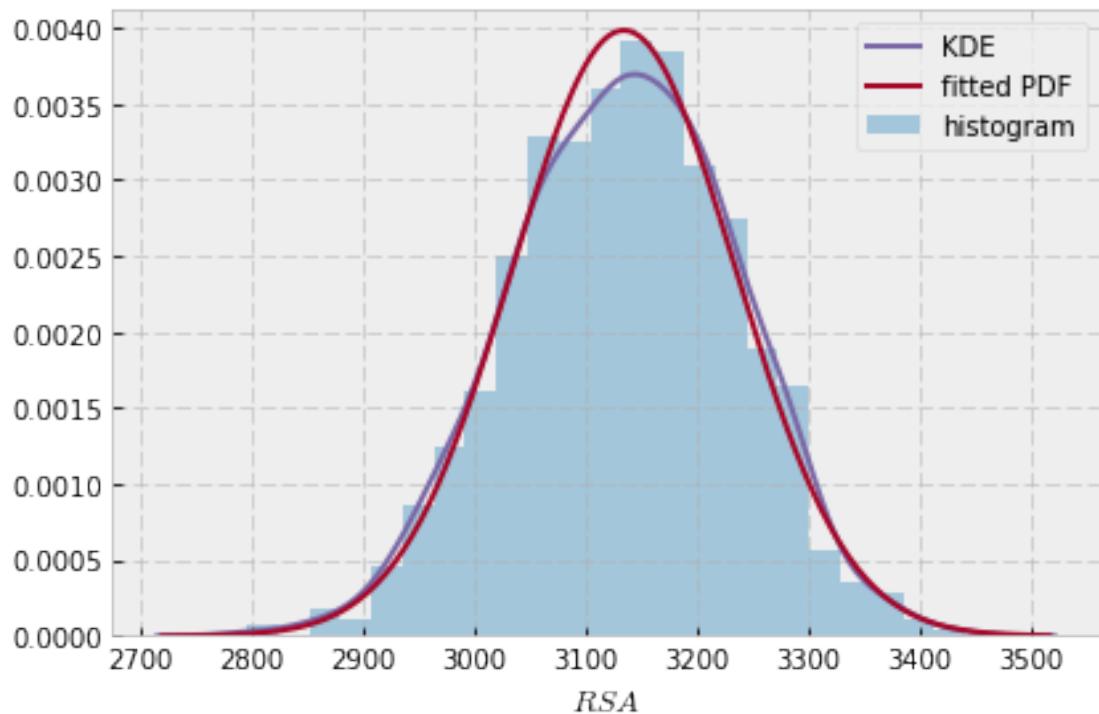


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=3130.7434038300003, scale=103.89043137847723)
DescribeResult(nobs=1000, minmax=(2767.162245, 3483.72487)
               mean=3130.7434038300003, variance=10804.025757763849,
               skewness=-0.03994483074551858, kurtosis=0.07698597154542997)
```

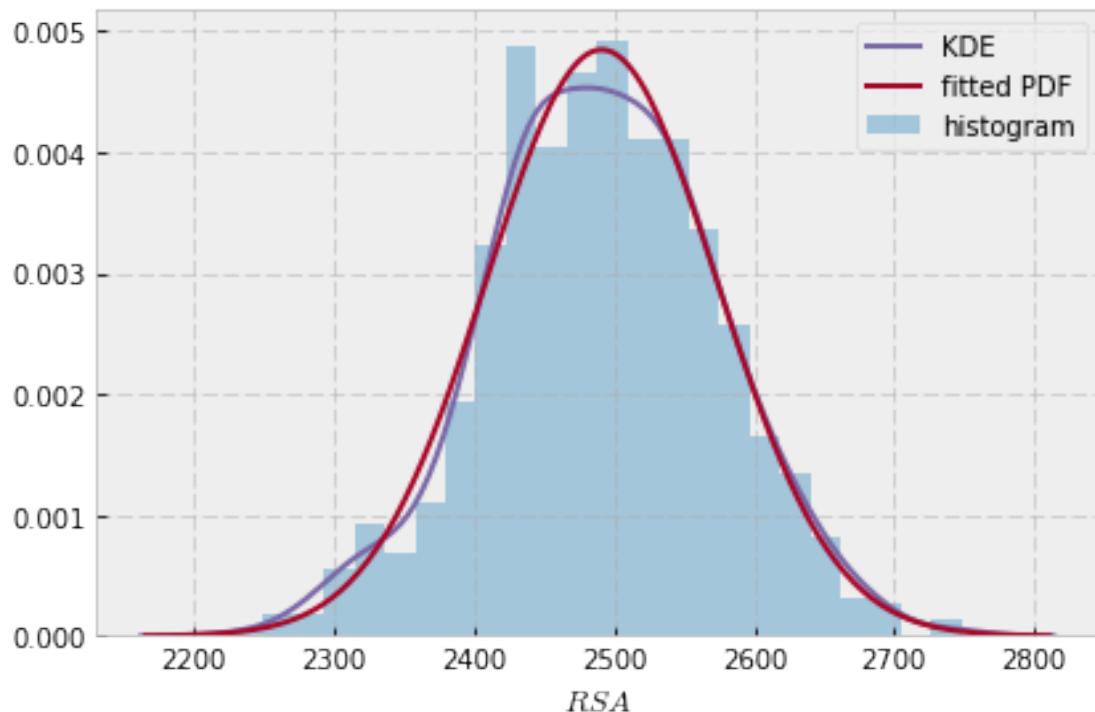


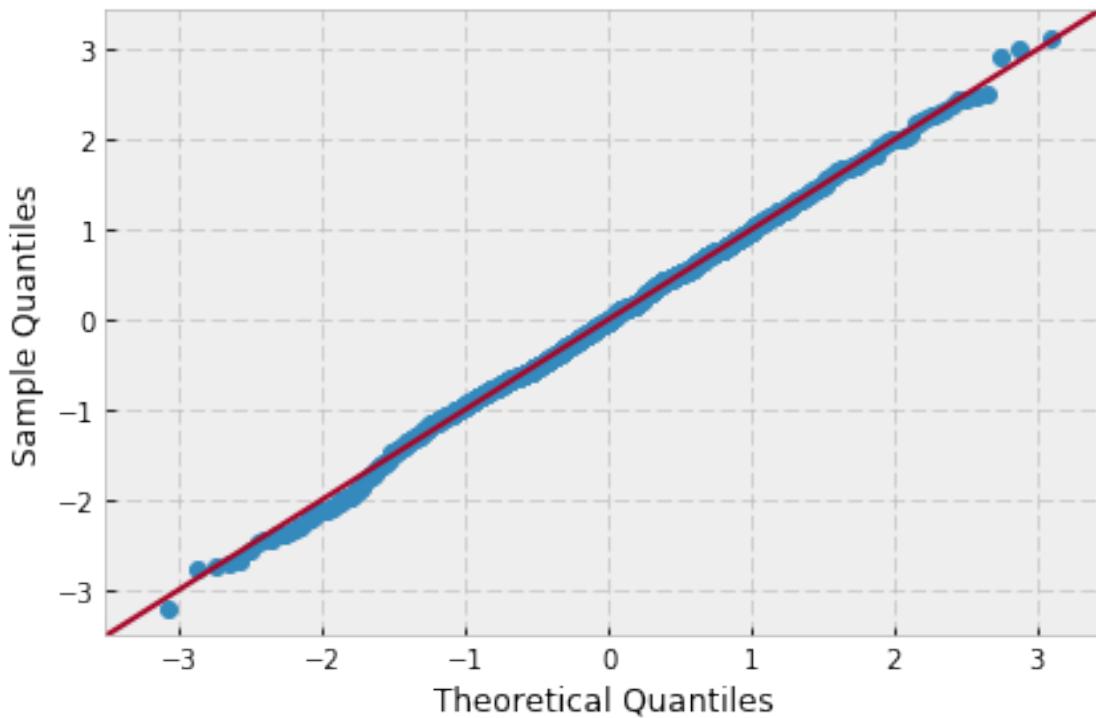


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(),loc=3133.9899223879997, scale=100.13462029867279)
DescribeResult(nobs=1000, minmax=(2795.214829999999, 3440.3626840000002)
               mean=3133.9899223879997, variance=10036.979161520892,
               skewness=-0.10388945608652751, kurtosis=-0.155107707478094)
```

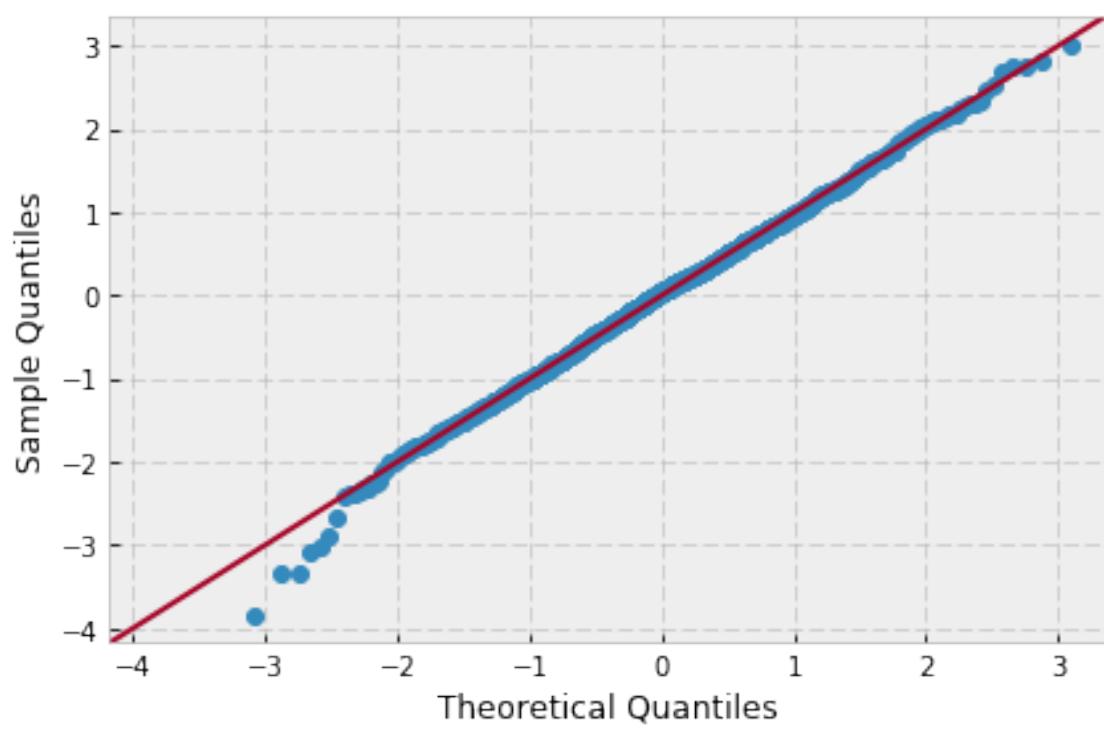
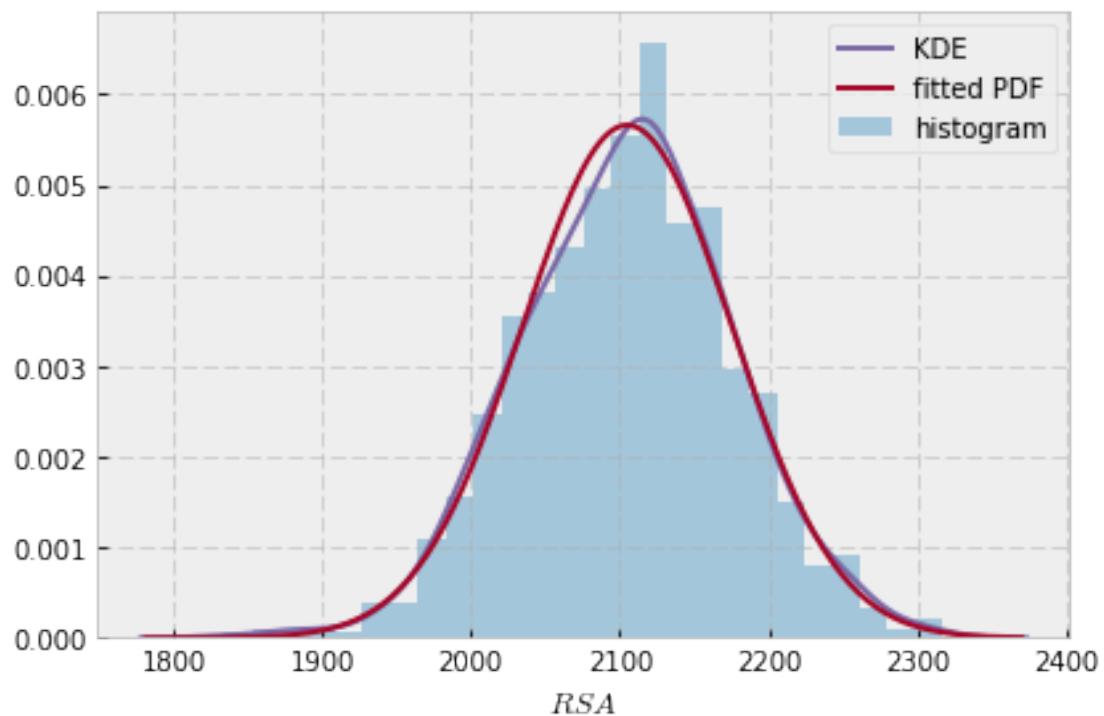


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=2490.9434045989997, scale=82.31769768753115)
DescribeResult(nobs=1000, minmax=(2227.36409, 2748.452072999999)
               mean=2490.9434045989997, variance=6782.9863389146858,
               skewness=-0.042516860382876435, kurtosis=0.010071239513129271)
```

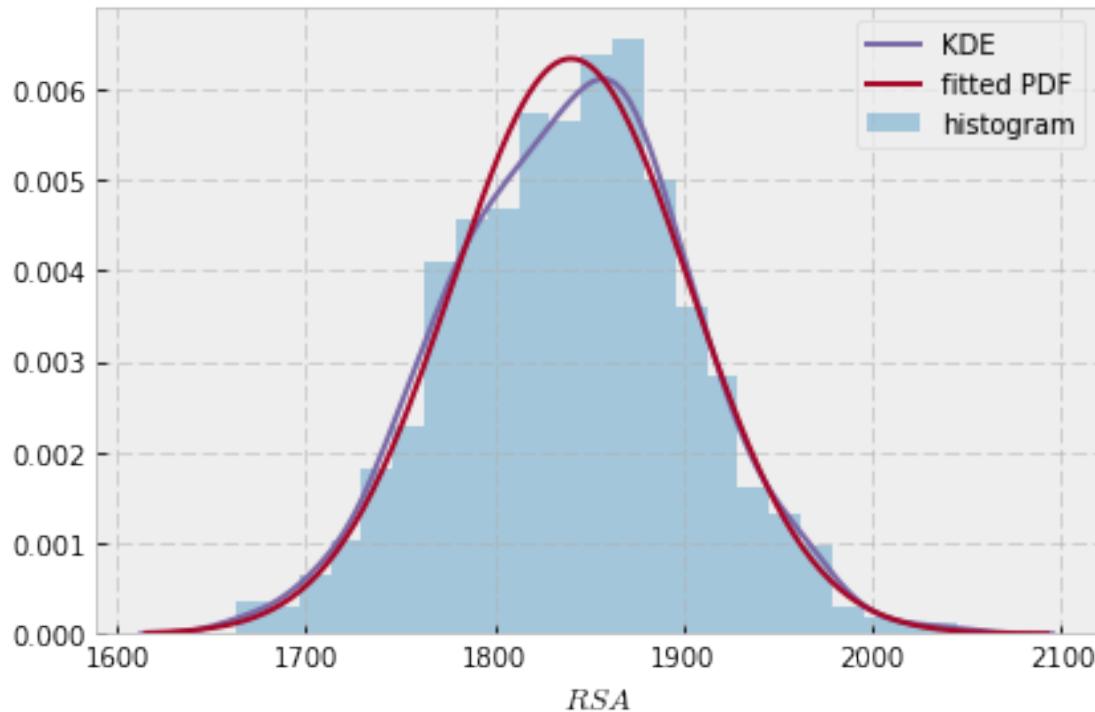


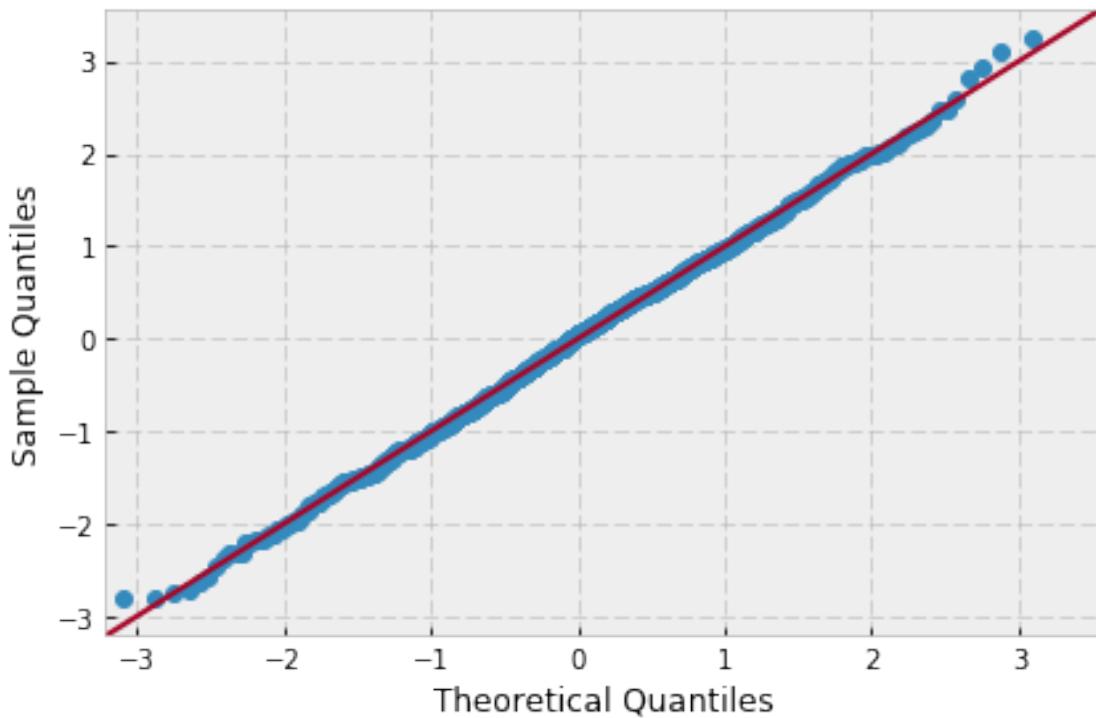


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(),loc=2104.969479507, scale=70.49775362464702)
DescribeResult(nobs=1000, minmax=(1834.5886499999999, 2316.7585039999999)
               mean=2104.969479507, variance=4974.9081742957278,
               skewness=-0.10426518367370975, kurtosis=0.19657049335478005)
```

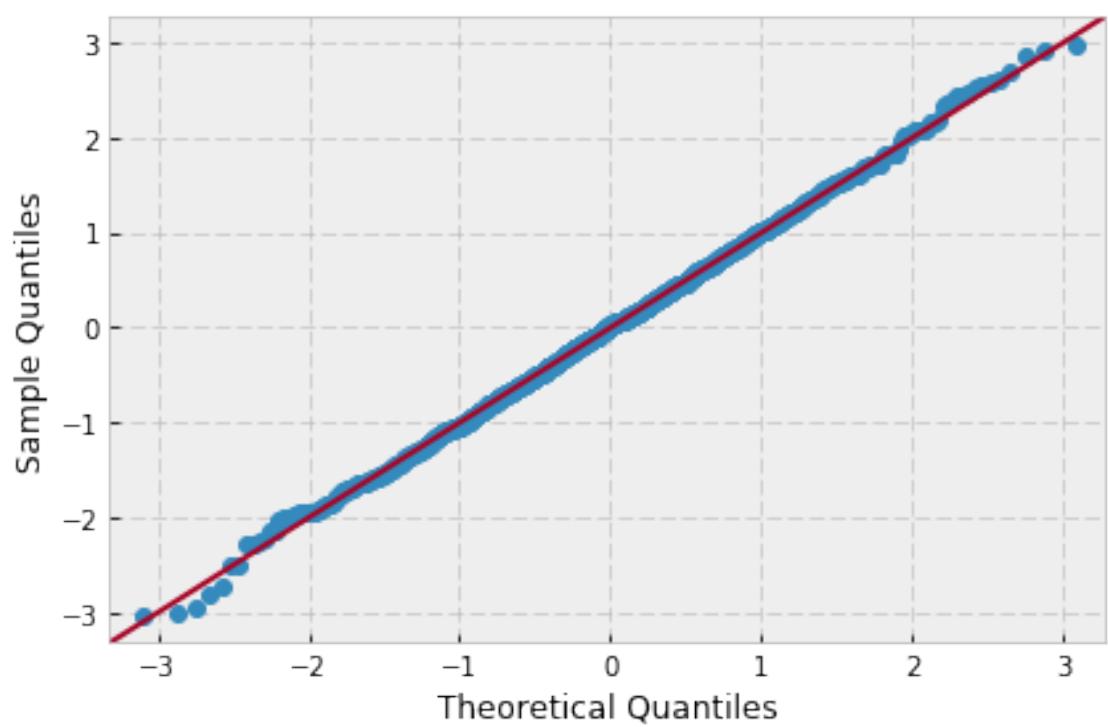
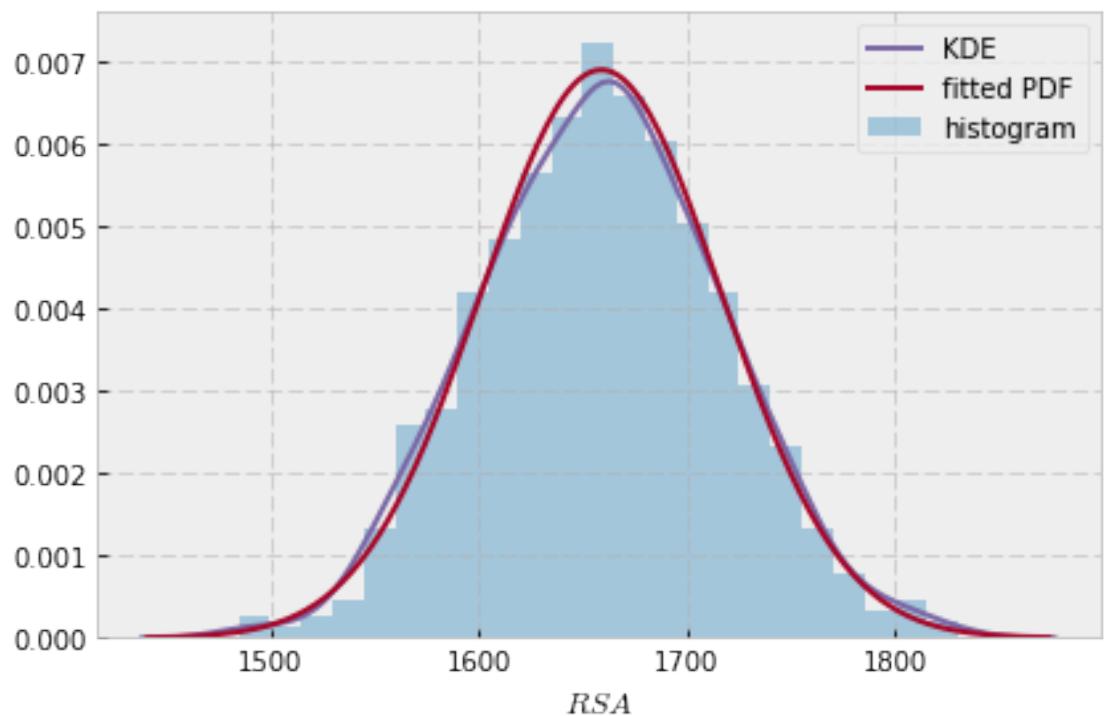


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=1840.3293523240002, scale=62.999218684689936)
DescribeResult(nobs=1000, minmax=(1663.0208459999999, 2045.096994)
               mean=1840.3293523240002, variance=3972.8744293106965,
               skewness=-0.006961758432932275, kurtosis=-0.11511598872258588)
```

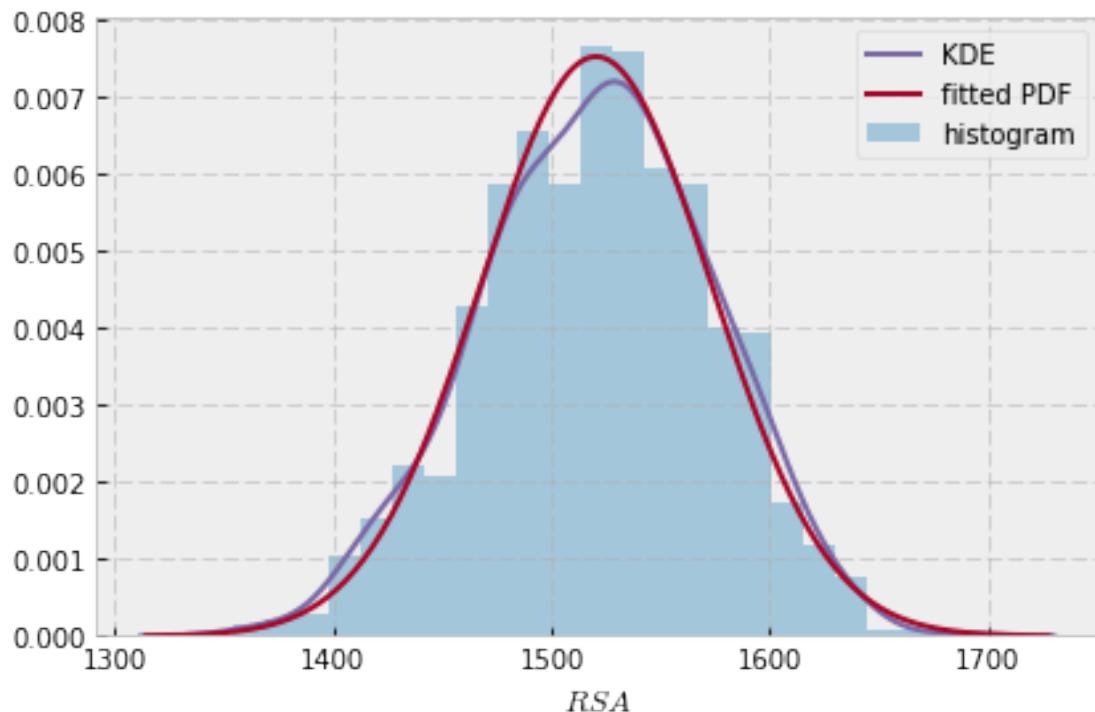


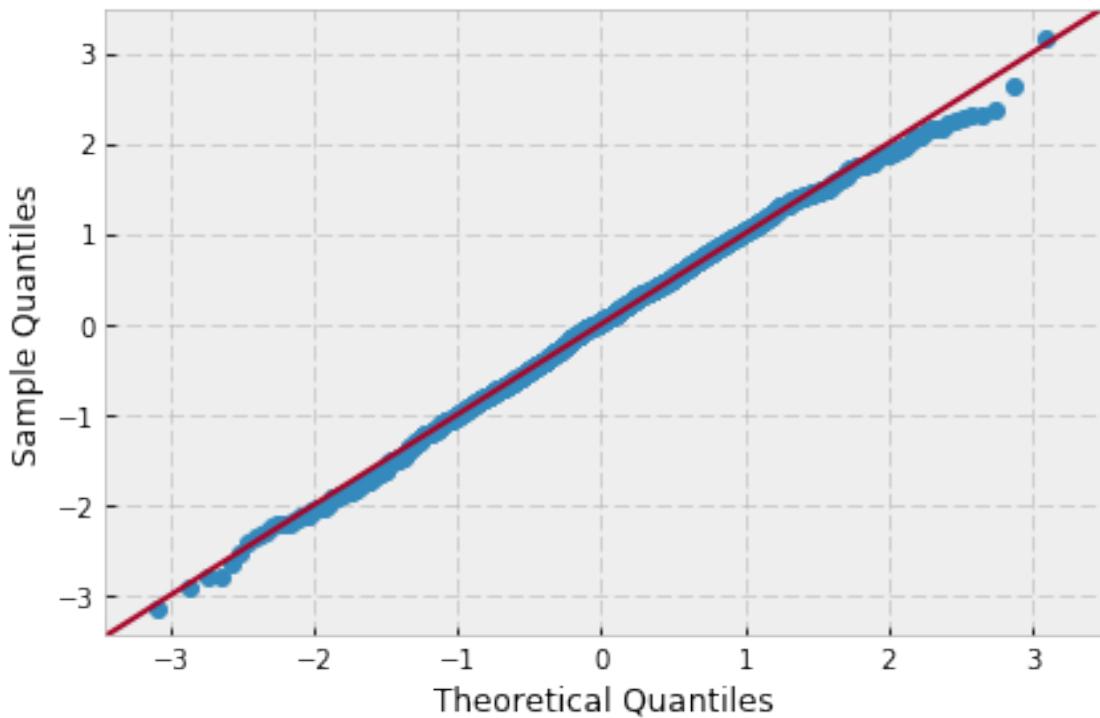


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(),loc=1658.671955247, scale=57.815129710446165)
DescribeResult(nobs=1000, minmax=(1484.1245730000001, 1830.353959)
               mean=1658.6719552469999, variance=3345.9351585943095,
               skewness=0.02831859064367777, kurtosis=-0.09470756332564934)
```

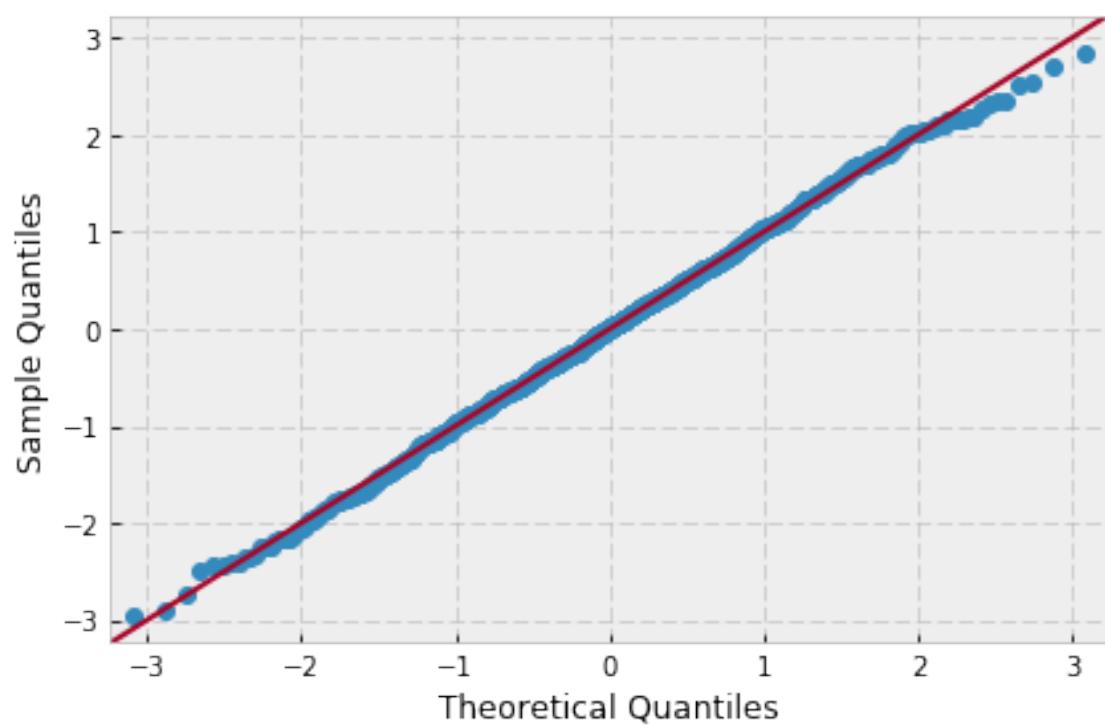
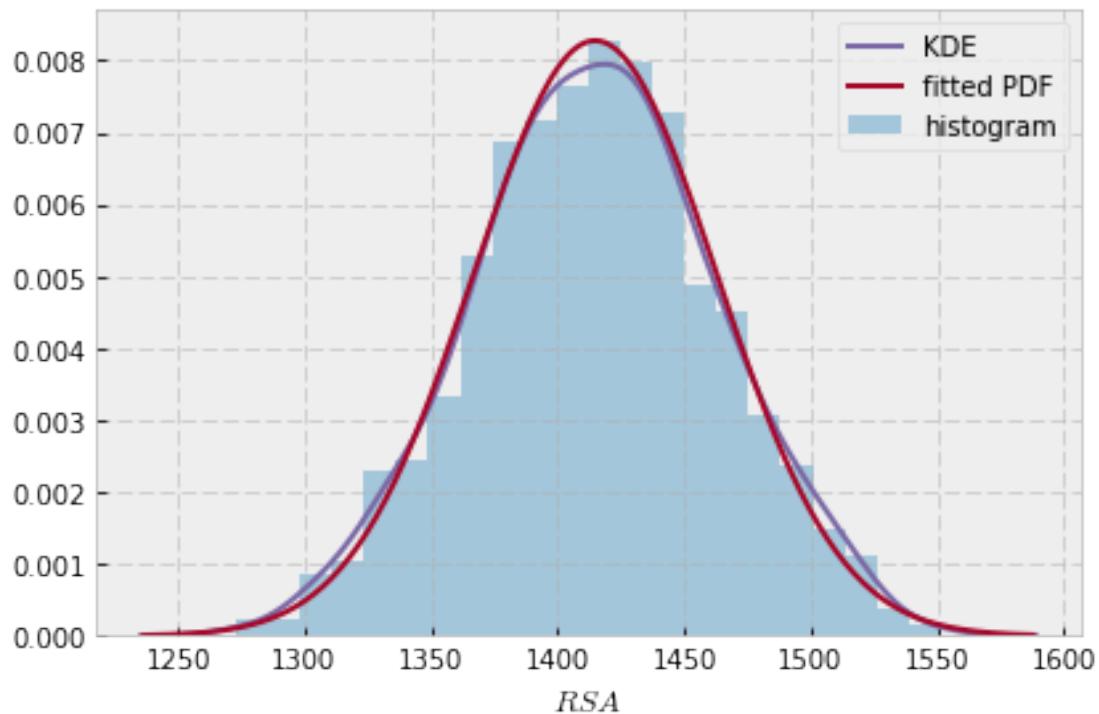


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=1520.334157062, scale=52.9501413775849)
DescribeResult(nobs=1000, minmax=(1354.175606, 1687.626906)
    mean=1520.334157062, variance=2806.5239959021301,
    skewness=-0.12884564501093296, kurtosis=-0.22822779407861704)
```

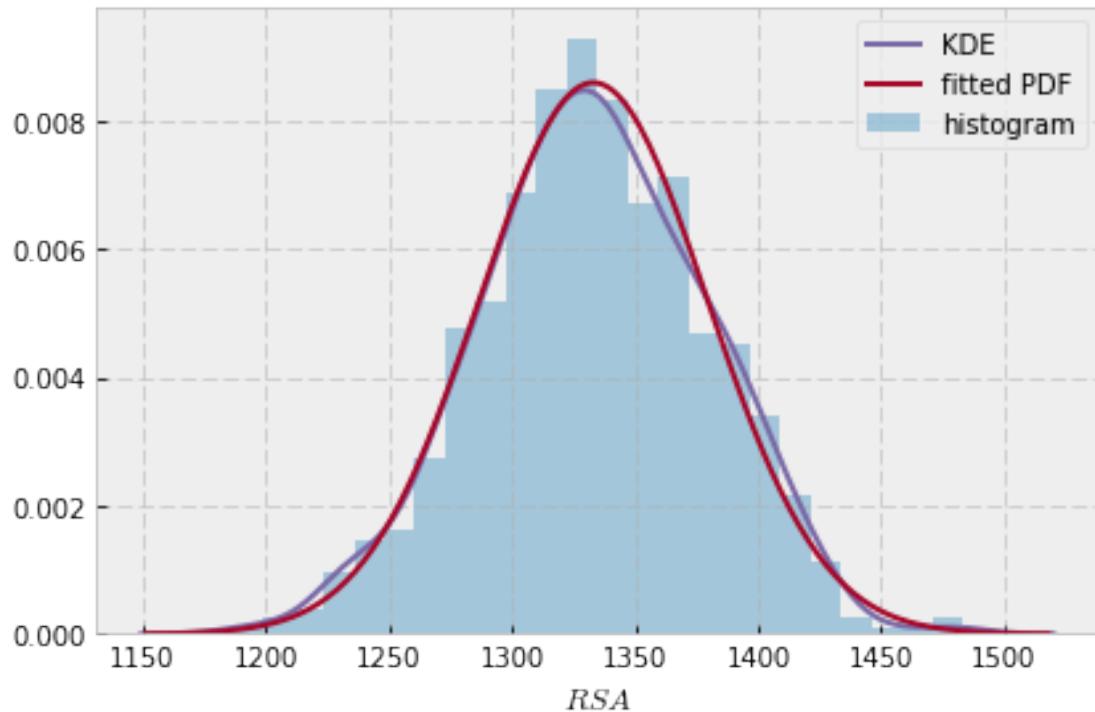


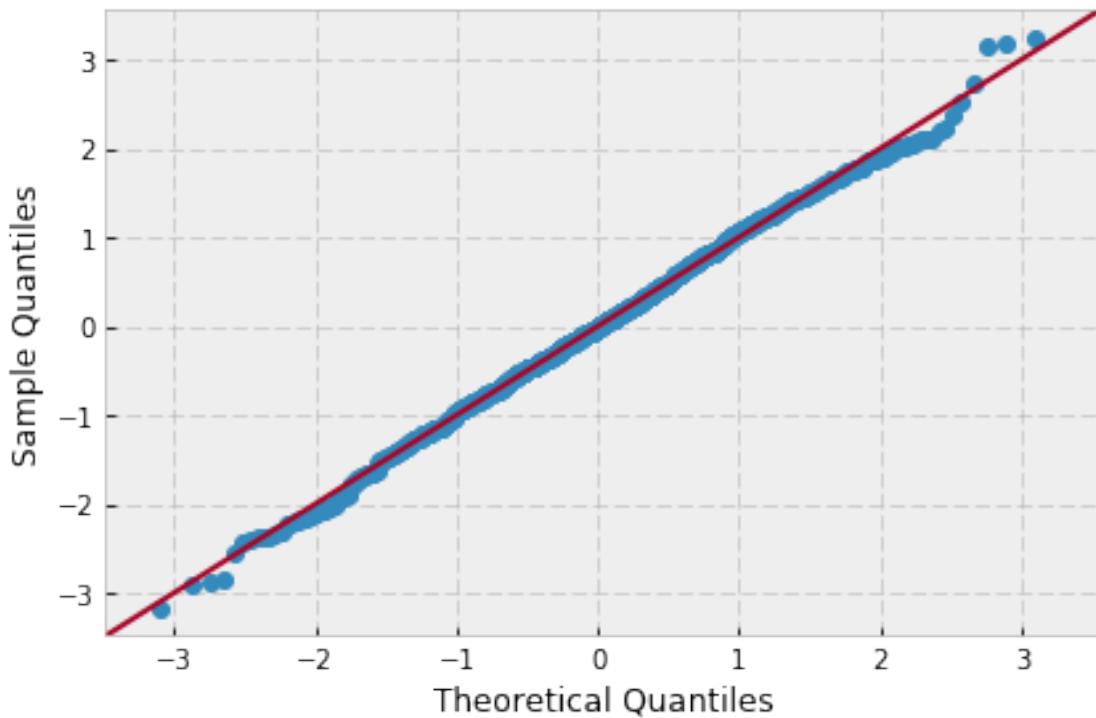


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(),loc=1414.7847791610002, scale=48.199346781365016)
DescribeResult(nobs=1000, minmax=(1272.7391990000001, 1551.432076999999)
               mean=1414.7847791610002, variance=2325.502532682965,
               skewness=-0.0242390478690617, kurtosis=-0.2066254739906559)
```

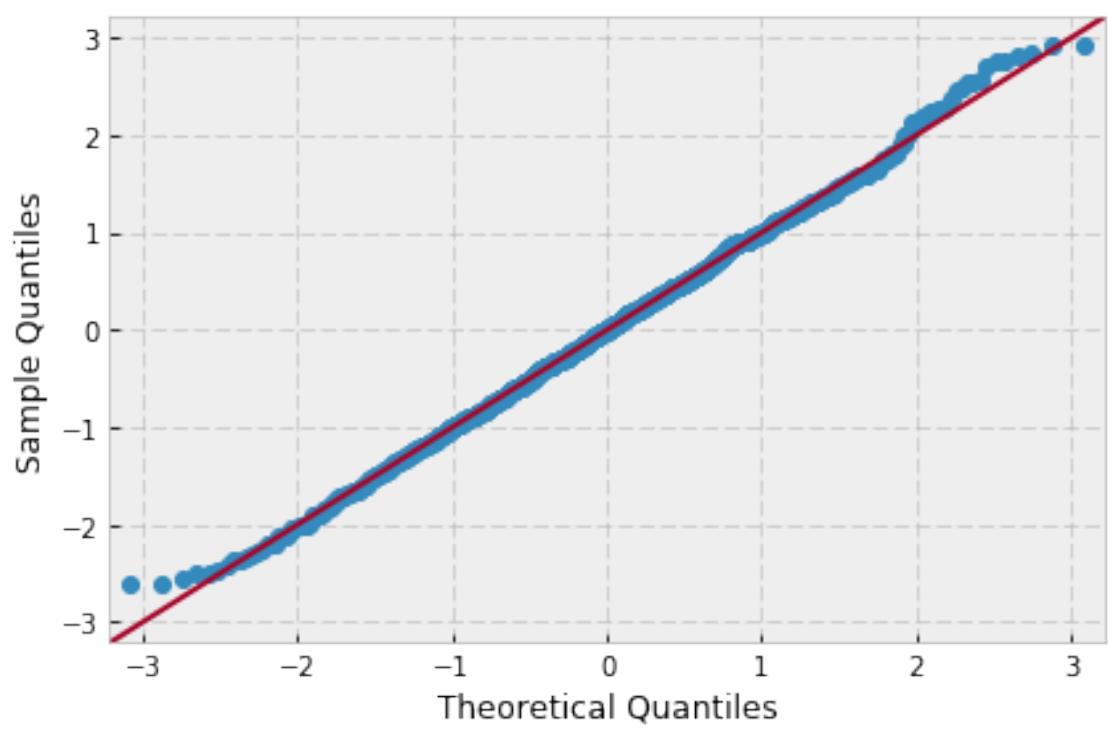
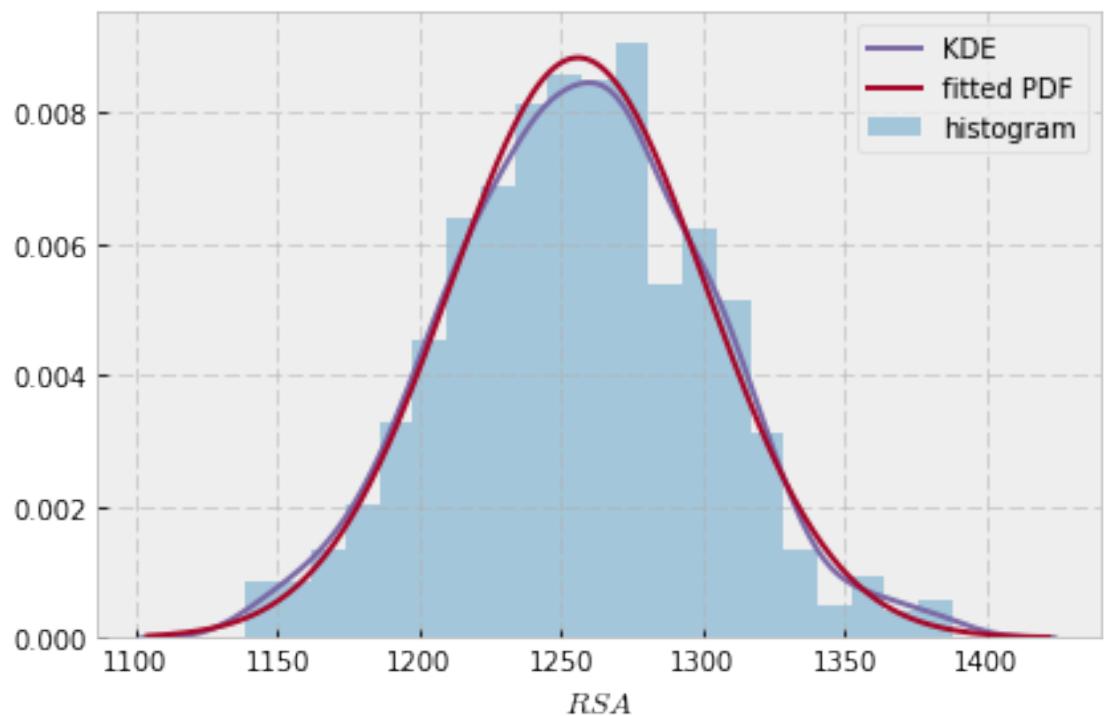


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=1332.9750984749999, scale=46.39189454210657)
DescribeResult(nobs=1000, minmax=(1186.203759, 1483.031291)
               mean=1332.9750984749999, variance=2154.3622414473848,
               skewness=-0.0471367329783479, kurtosis=-0.0645531273429314)
```

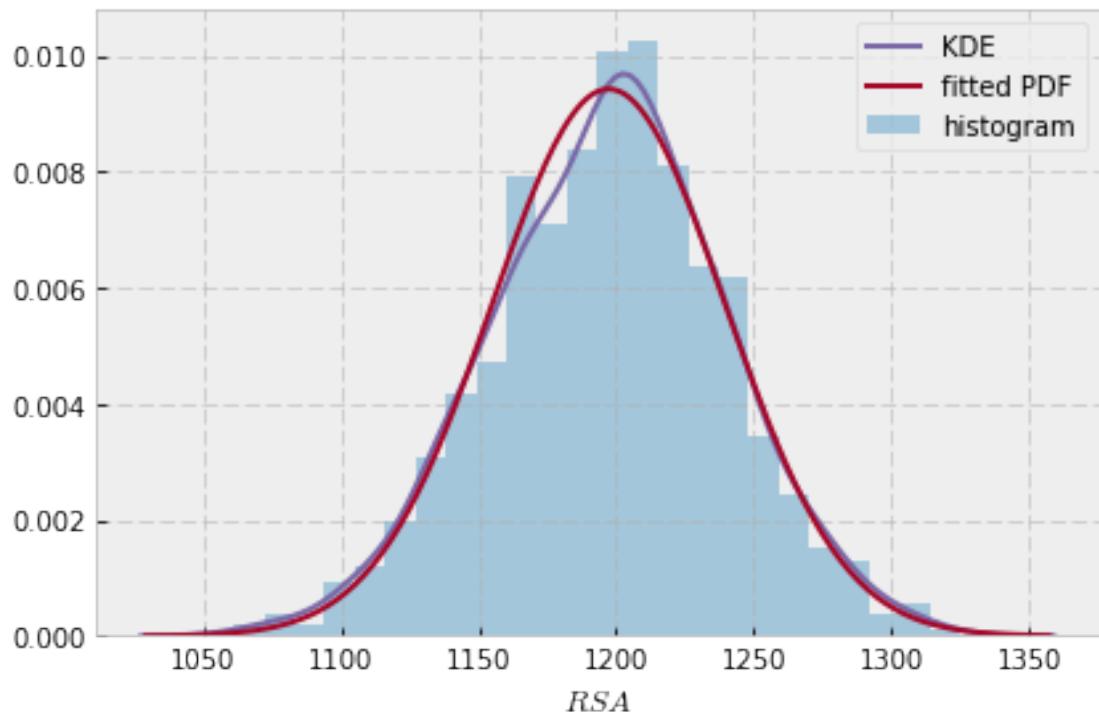


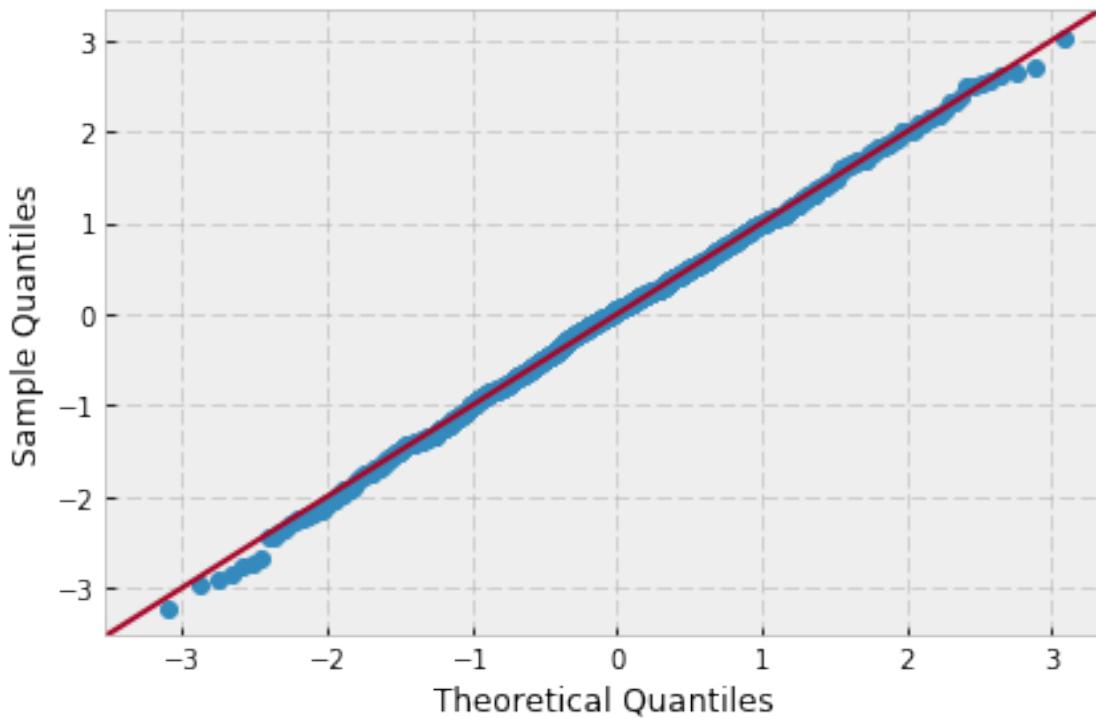


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(),loc=1255.9900557659998, scale=45.12357134418236)
DescribeResult(nobs=1000, minmax=(1138.2948610000001, 1387.9921629999999)
               mean=1255.9900557659998, variance=2038.1748657192345,
               skewness=0.048980941342156116, kurtosis=-0.098616451124105)
```

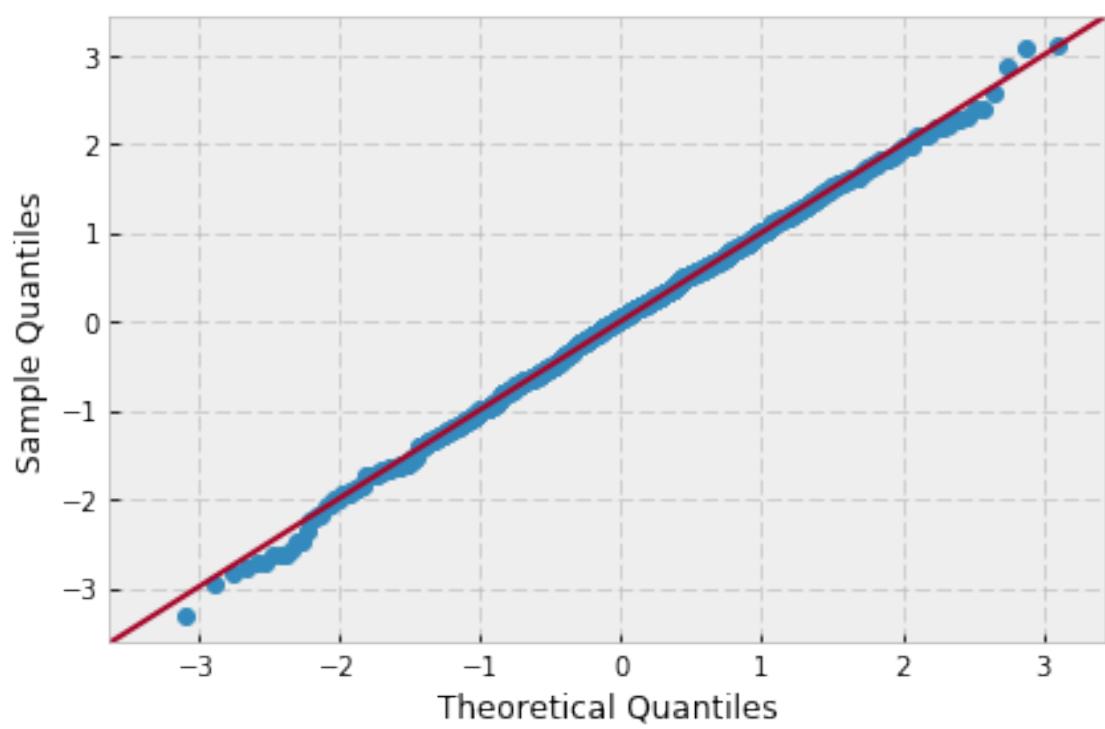
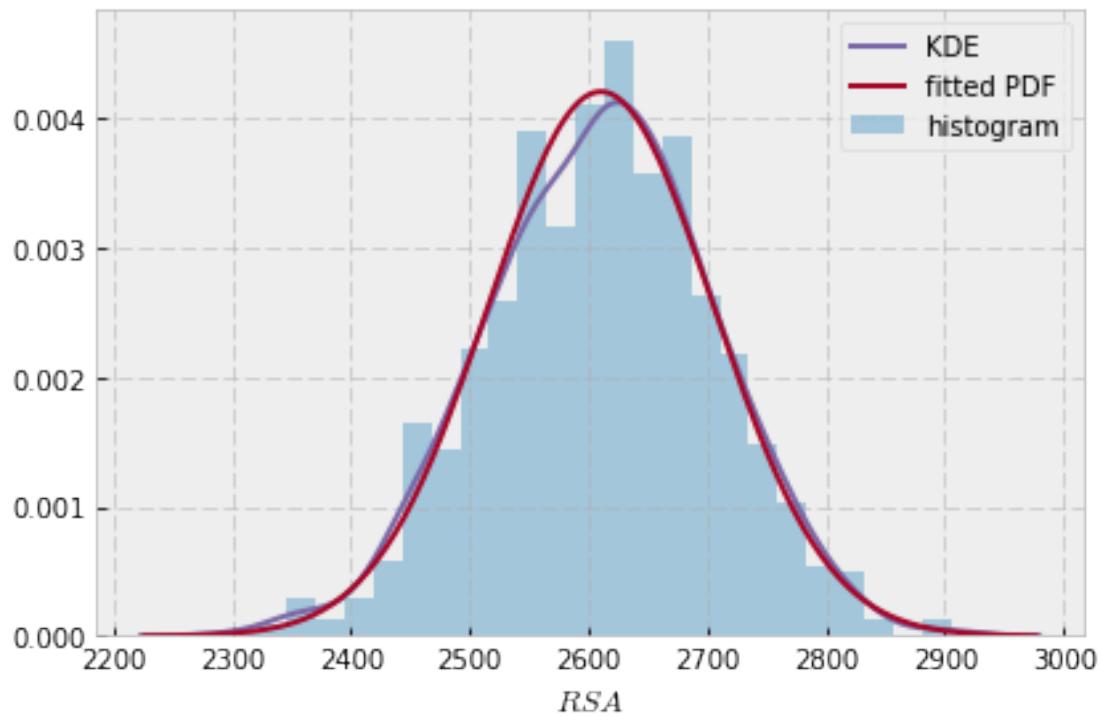


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=1197.226307884, scale=42.3453275523533)
DescribeResult(nobs=1000, minmax=(1061.0753380000001, 1325.1668219999999)
               mean=1197.2263078840001, variance=1794.9216872032944,
               skewness=-0.08458251958564857, kurtosis=-0.006273407190194202)
```

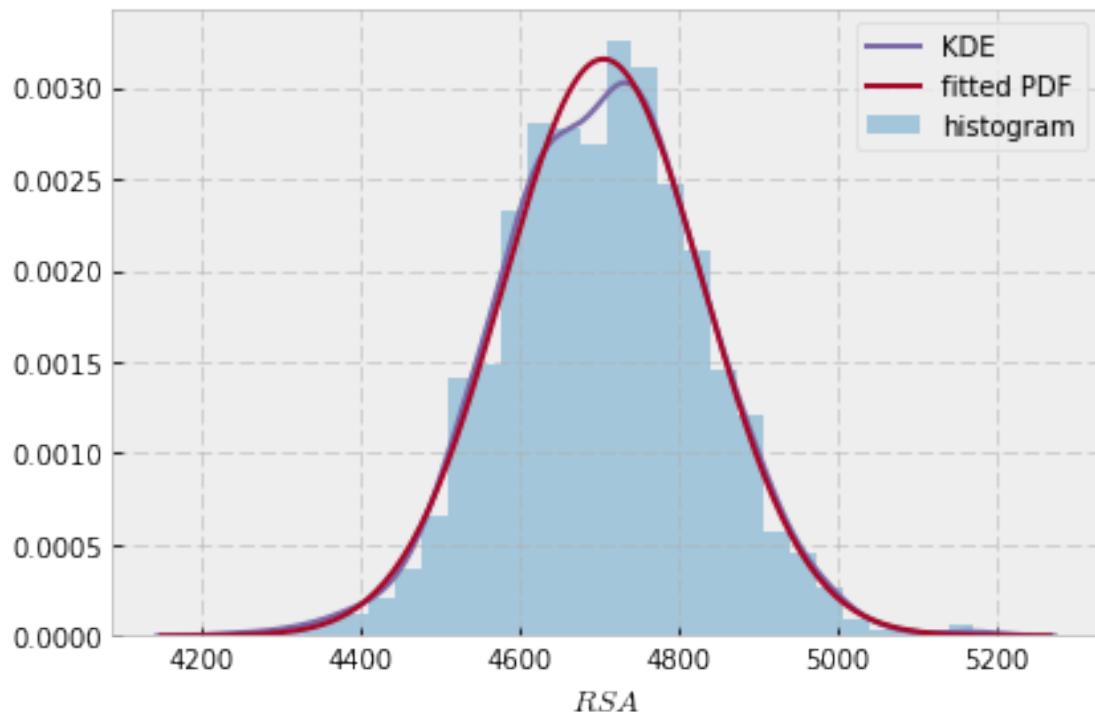


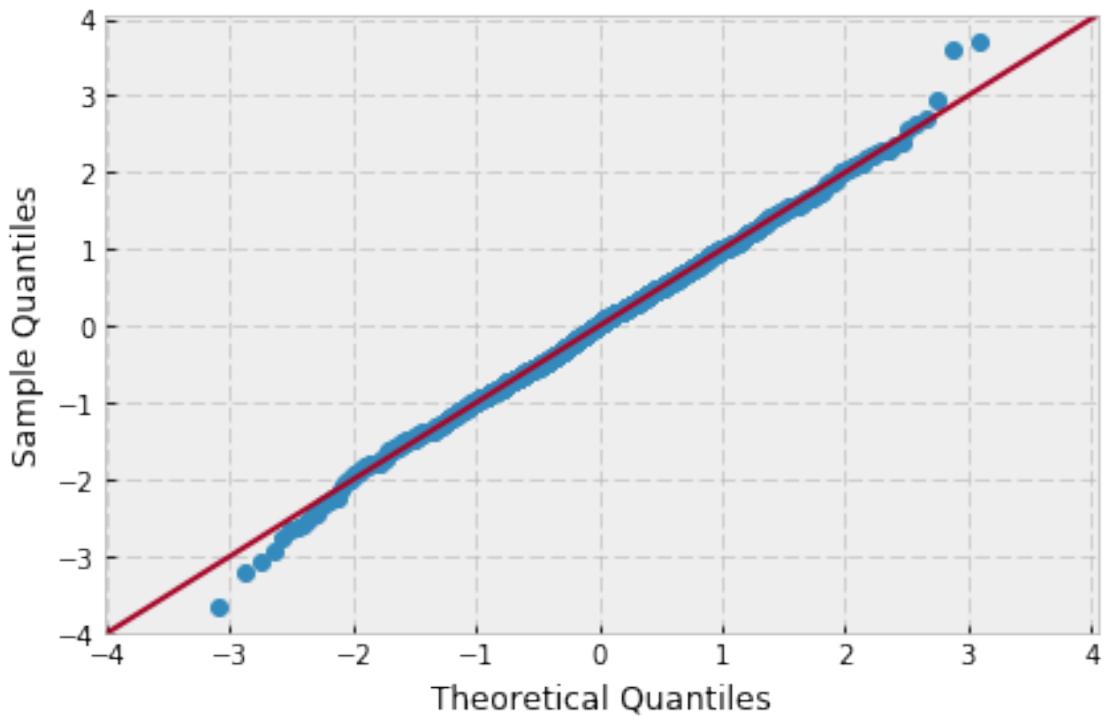


```
variable name:  maxB
variable value: 4.0
distribution: normal(shape=(),loc=2610.3565048230003, scale=94.79205914802228)
DescribeResult(nobs=1000, minmax=(2297.462156, 2904.900277000002)
               mean=2610.3565048230003, variance=8994.5290065286827,
               skewness=-0.09082572839061173, kurtosis=-0.039911396007745736)
```

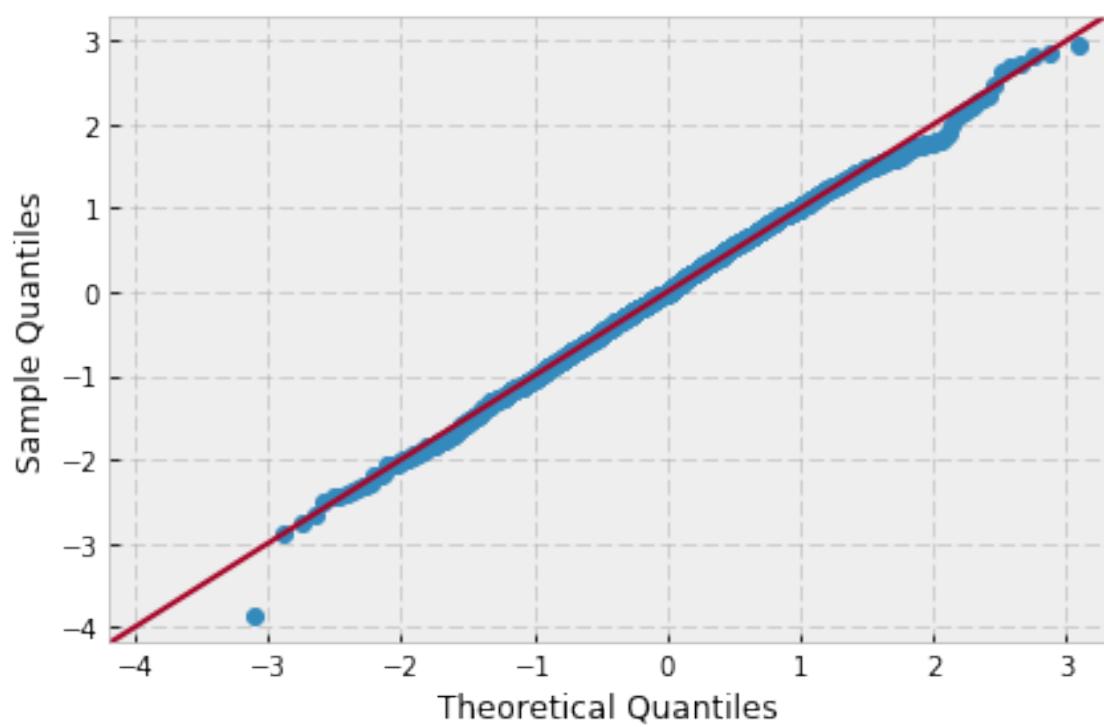
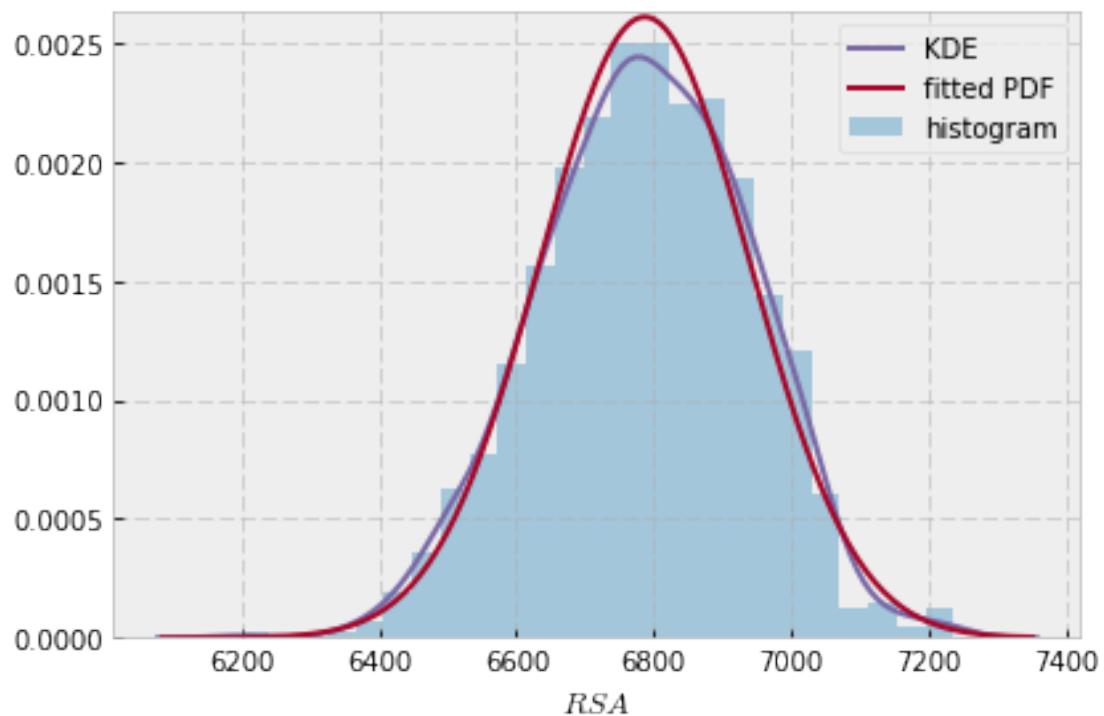


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(), loc=4704.548169934, scale=126.40174688358954)
DescribeResult(nobs=1000, minmax=(4243.886731999999, 5171.118051000004)
               mean=4704.5481699339998, variance=15993.395010233271,
               skewness=-0.006814727099776592, kurtosis=0.23352207800067726)
```

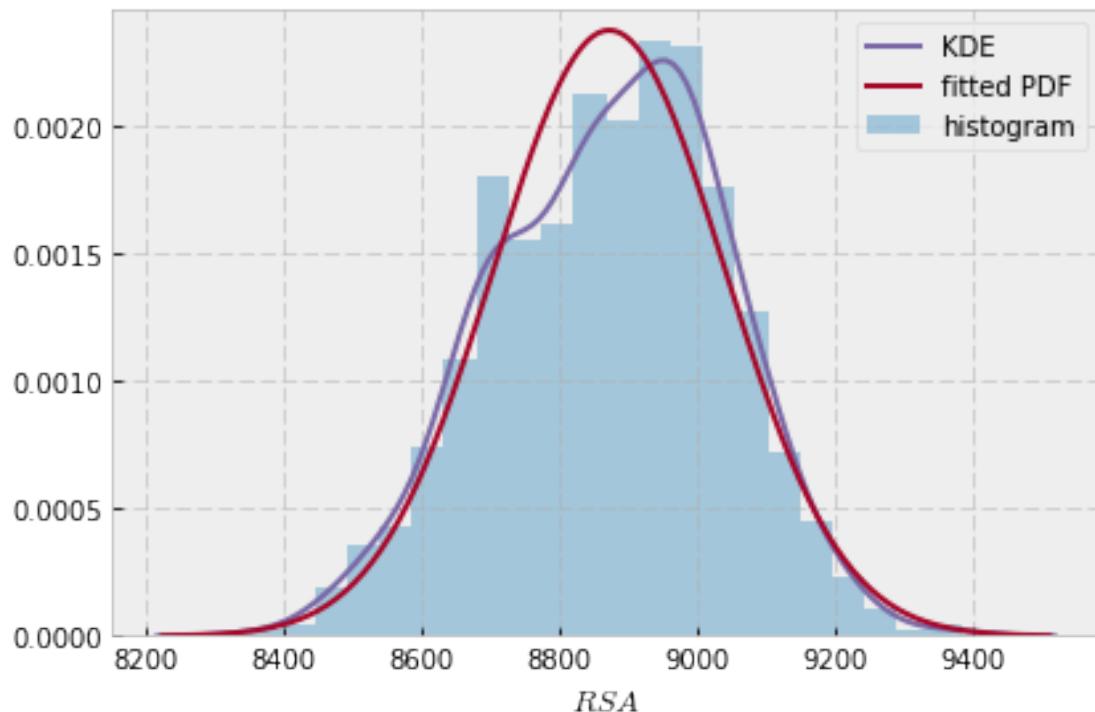


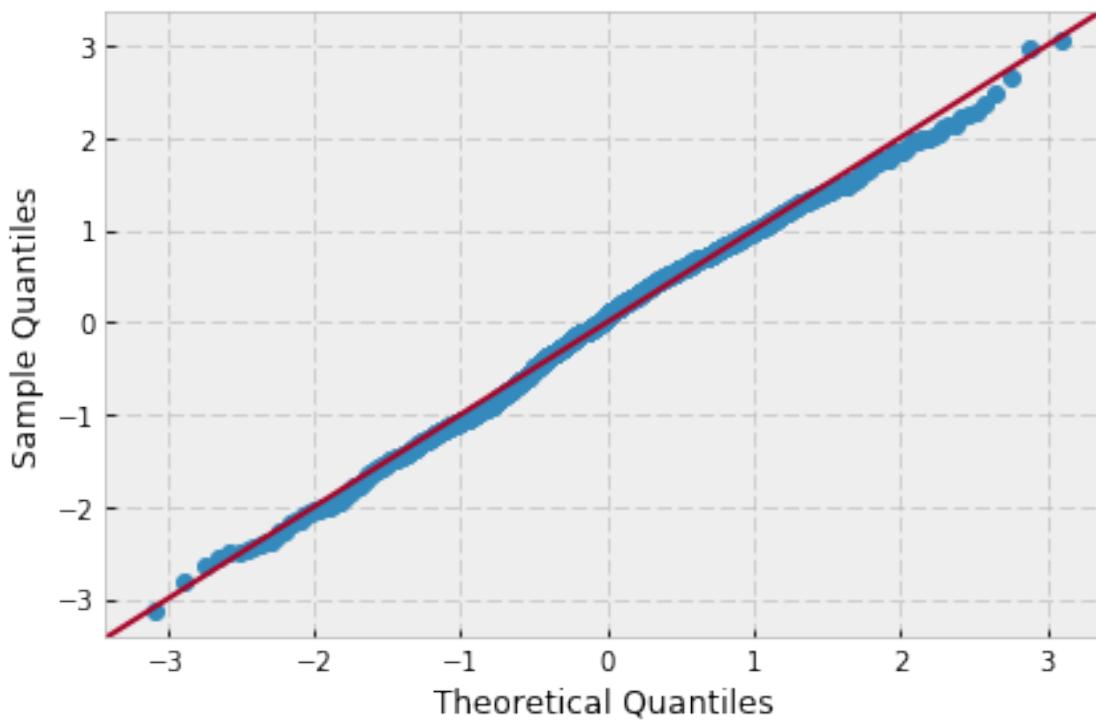


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(),loc=6787.010499125, scale=152.8268676558993)
DescribeResult(nobs=1000, minmax=(6199.105507999999, 7236.660941000001)
               mean=6787.010499125, variance=23379.430908422182,
               skewness=-0.11511507212845526, kurtosis=-0.10294388576845703)
```

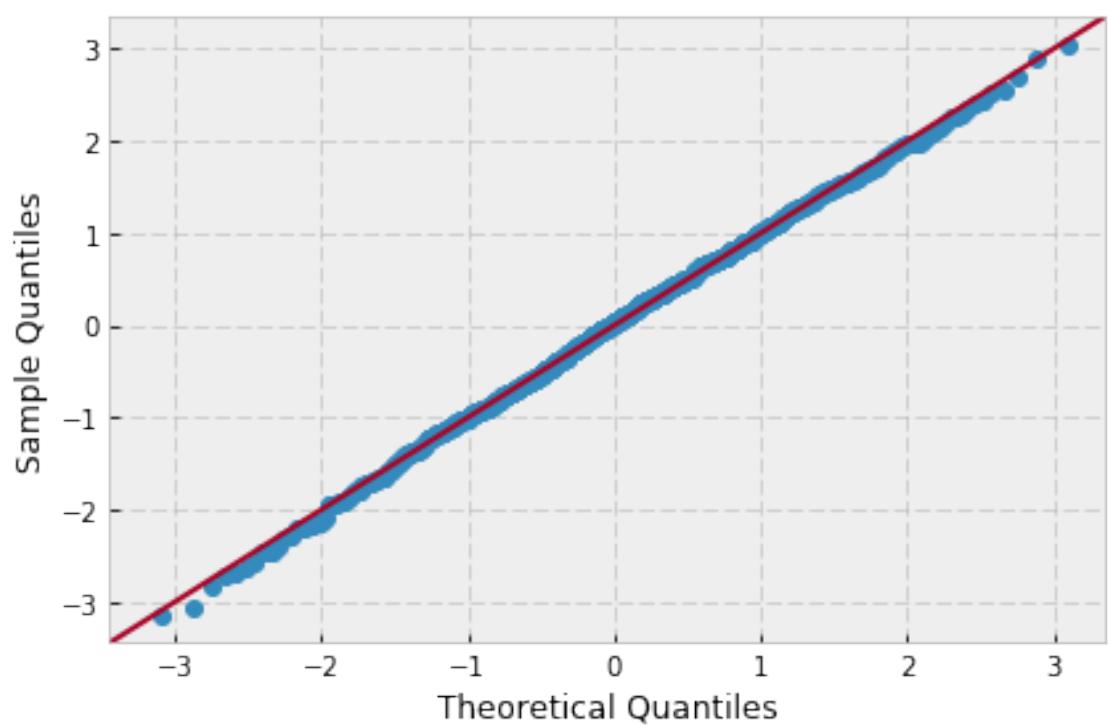
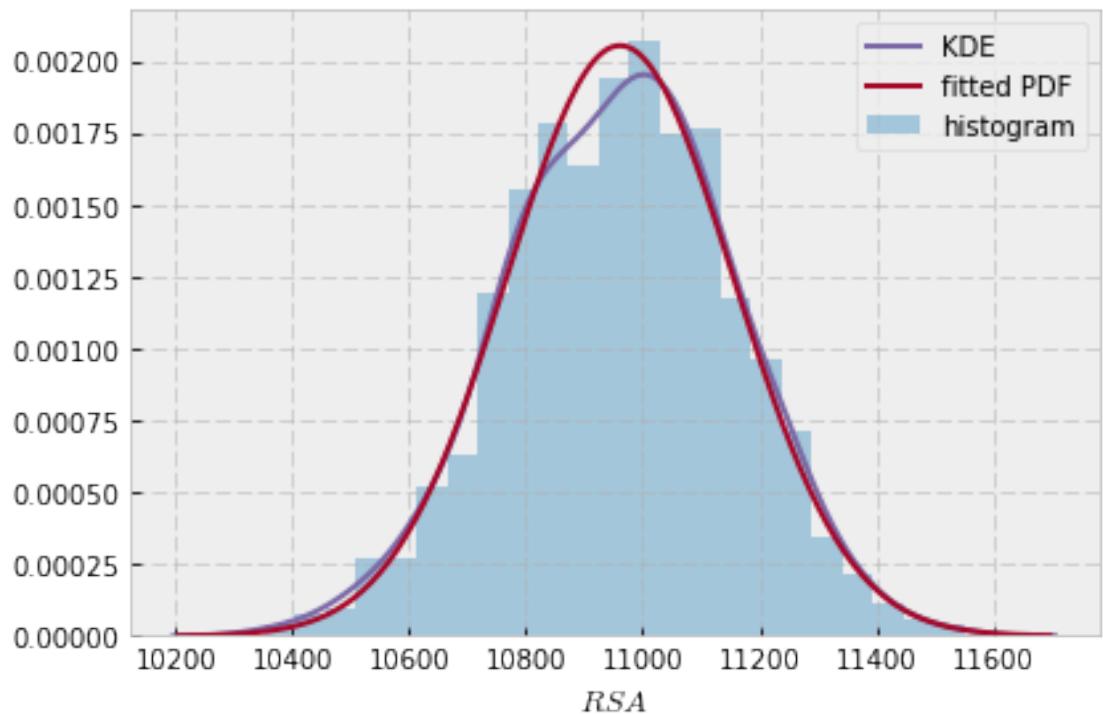


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(), loc=8872.185561866001, scale=167.7405939446958)
DescribeResult(nobs=1000, minmax=(8350.2240770000008, 9384.0257220000003)
               mean=8872.1855618660011, variance=28165.071928848167,
               skewness=-0.1642882175820818, kurtosis=-0.3139346832917336)
```

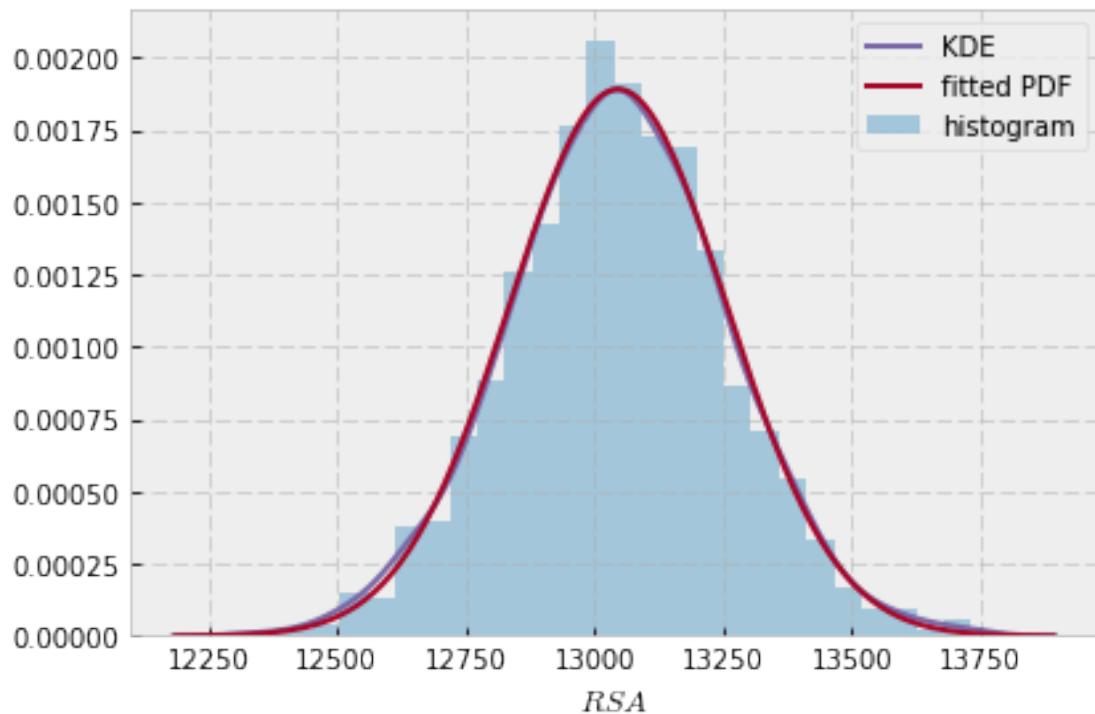


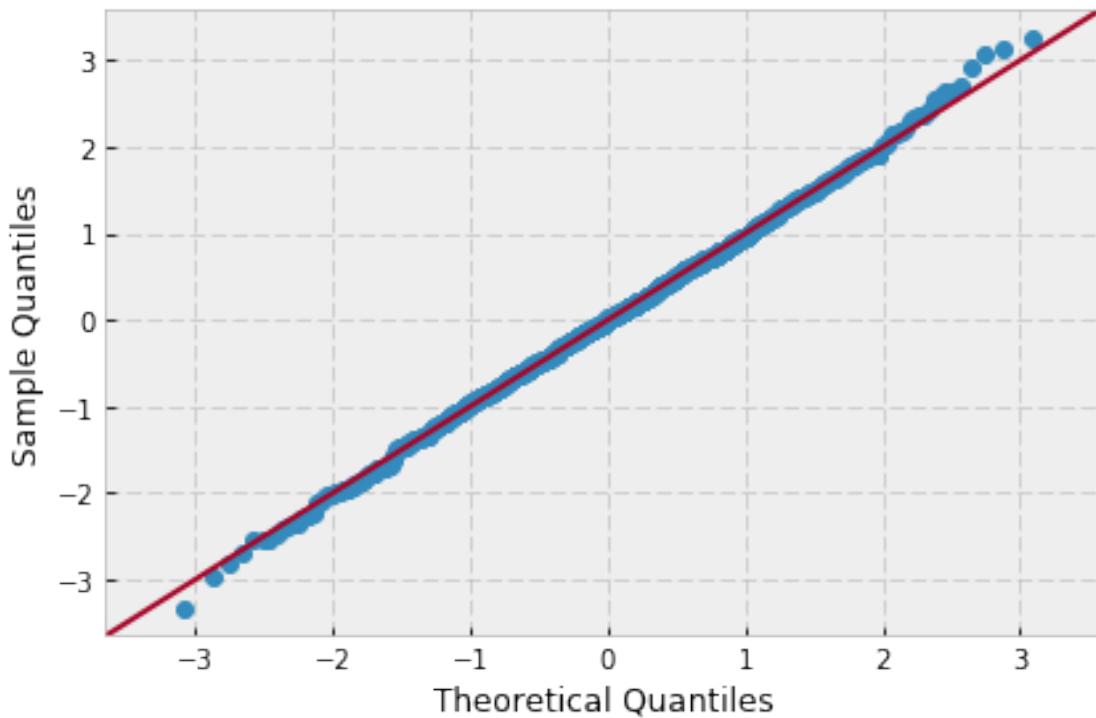


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(),loc=10961.02237656, scale=194.32737134396638)
DescribeResult(nobs=1000, minmax=(10351.98674, 11550.25304999999)
               mean=10961.02237656, variance=37800.928181637442,
               skewness=-0.08941247285642467, kurtosis=-0.1040135914175222)
```

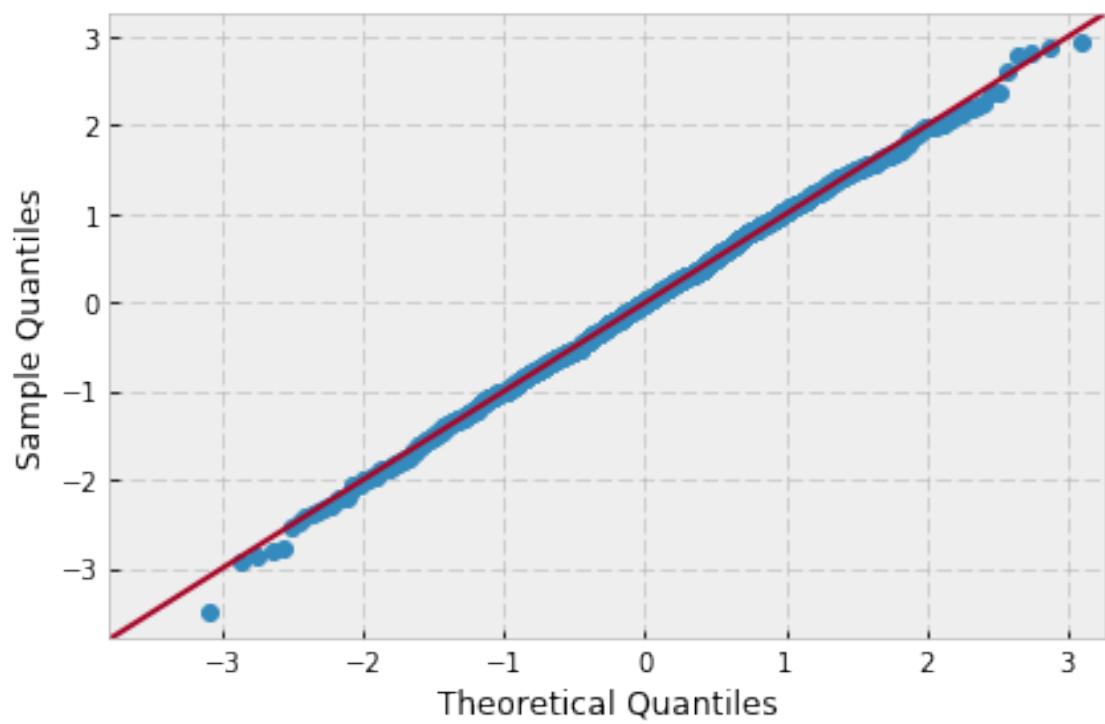
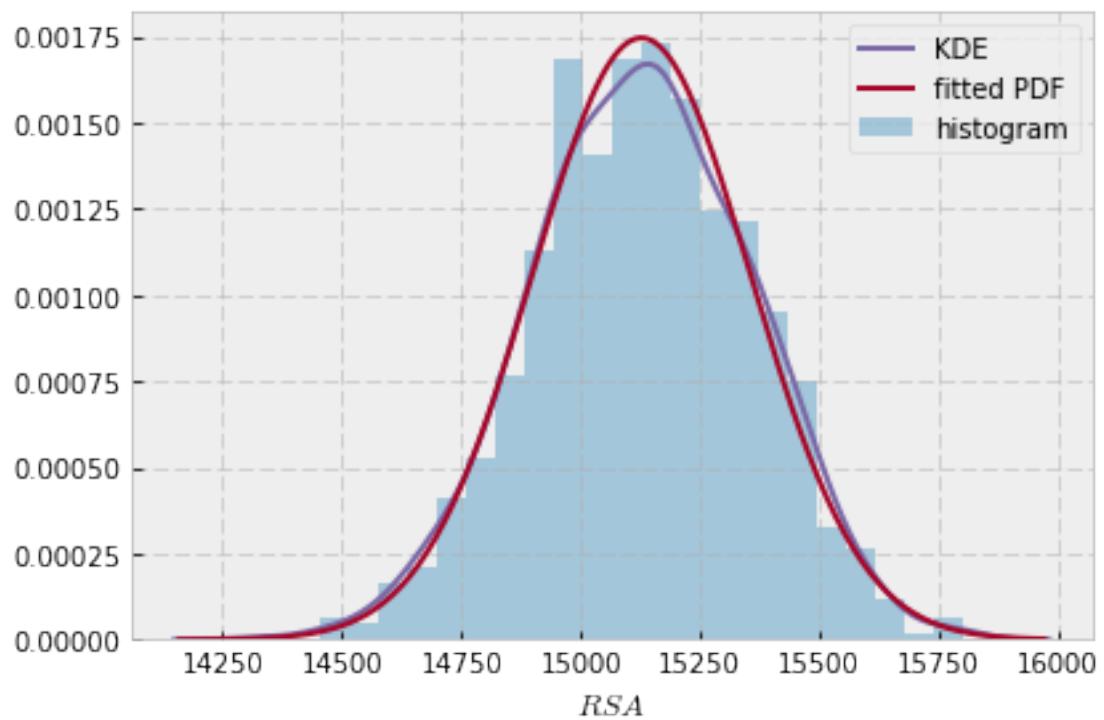


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(), loc=13045.92600808, scale=210.89529635550429)
DescribeResult(nobs=1000, minmax=(12345.145210000001, 13731.93764)
               mean=13045.92600808, variance=44521.347372248223,
               skewness=0.013902165220700917, kurtosis=0.12788028446997002)
```

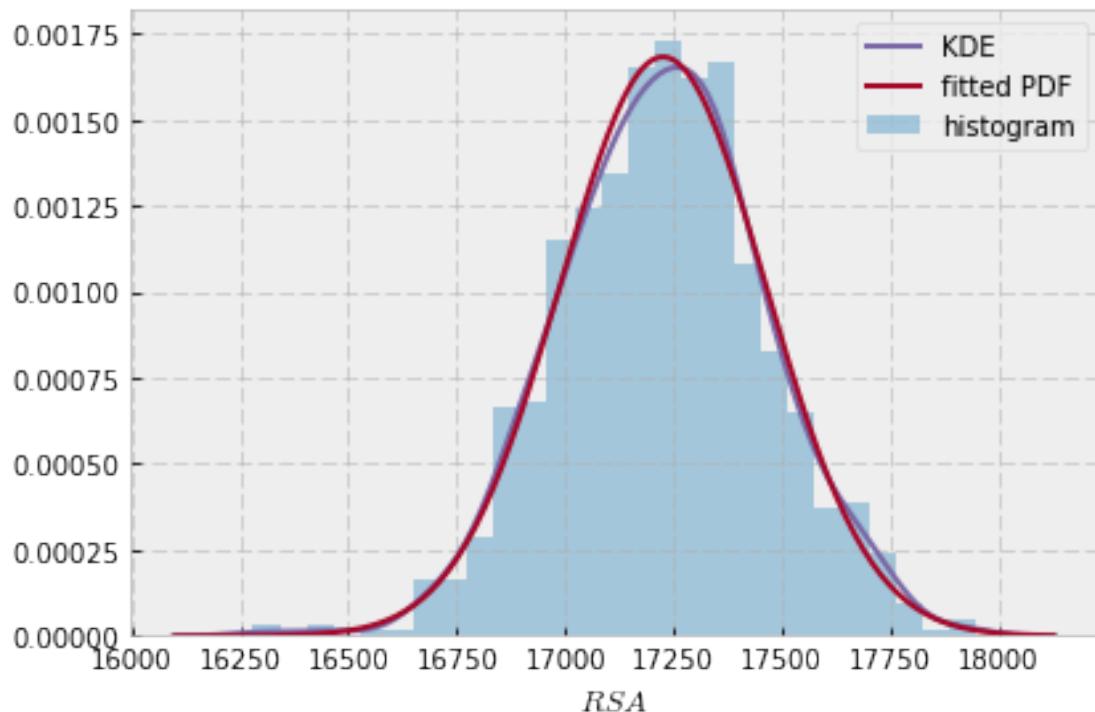


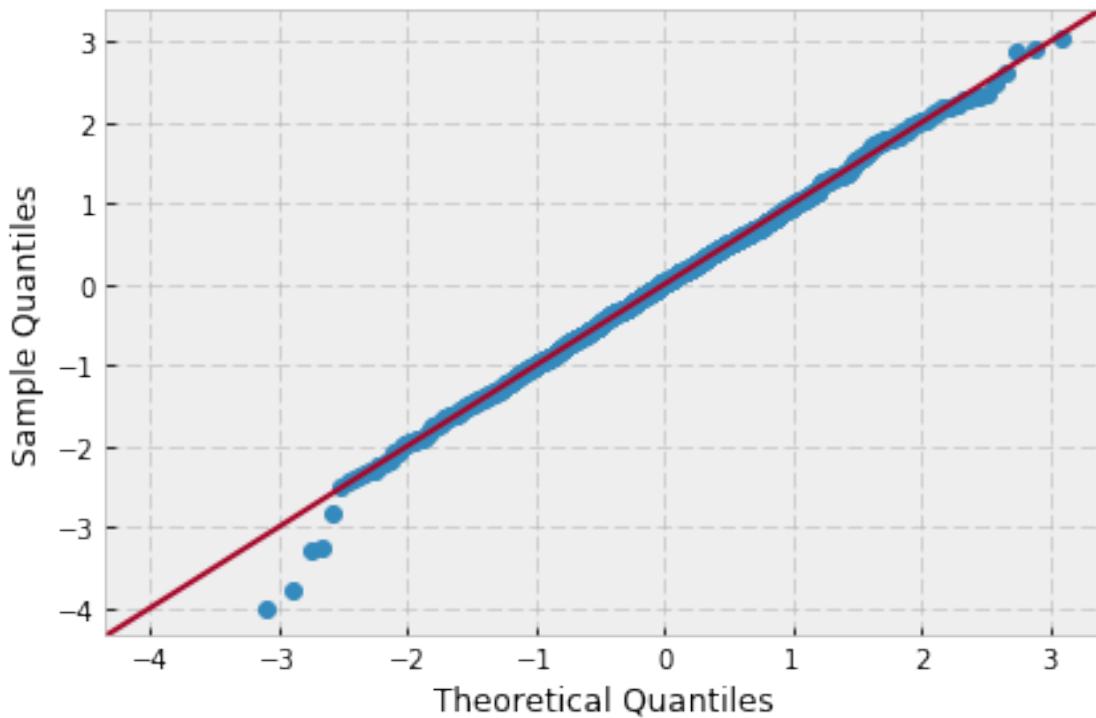


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(),loc=15127.00496831, scale=228.4676219682814)
DescribeResult(nobs=1000, minmax=(14331.670249999999, 15798.459049999999)
               mean=15127.004968310001, variance=52249.703991833376,
               skewness=-0.06596048896386075, kurtosis=-0.09664912273809989)
```

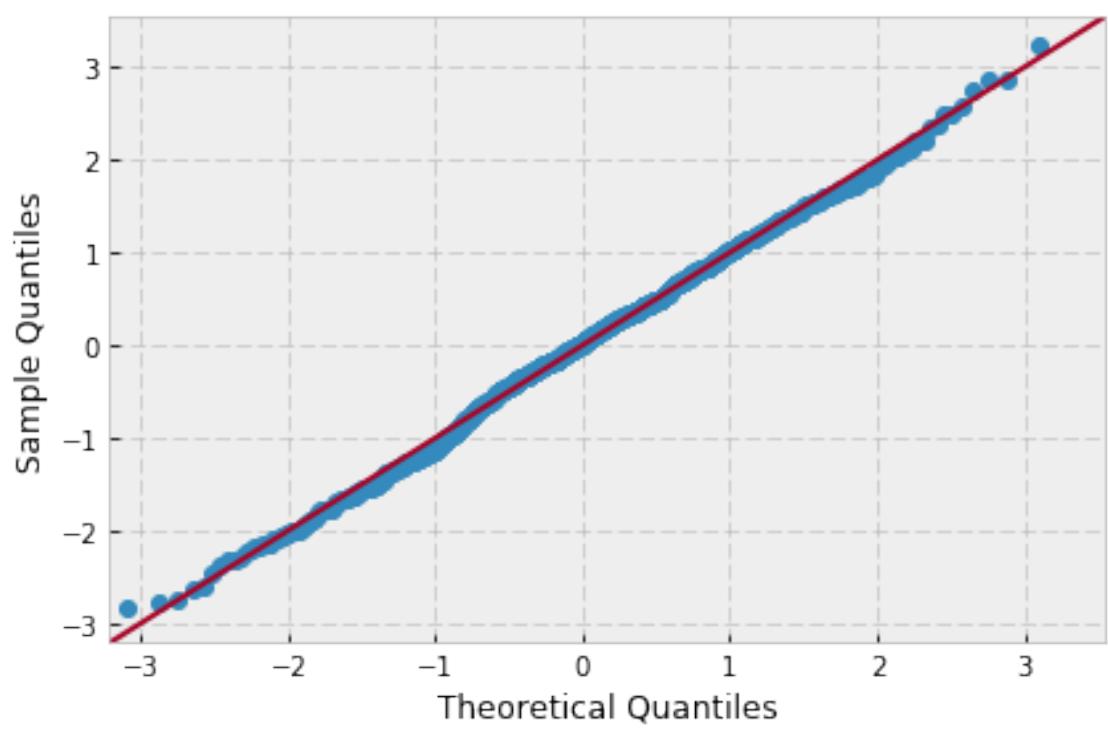
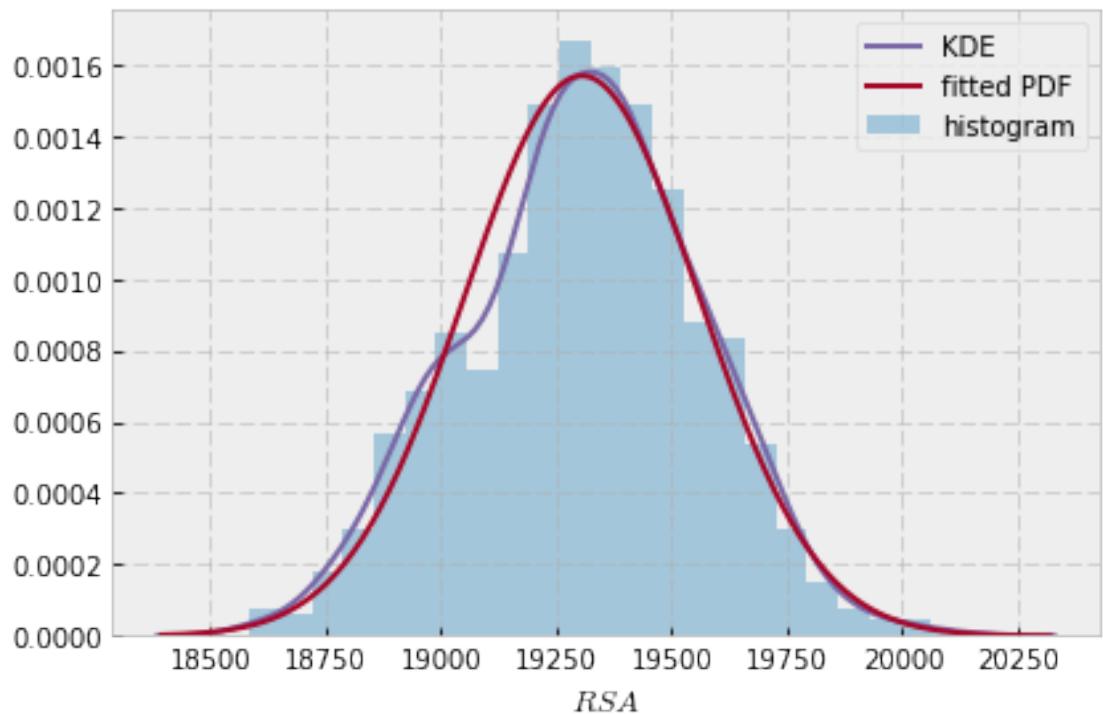


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(), loc=17225.55199468, scale=237.02449013147708)
DescribeResult(nobs=1000, minmax=(16279.35621, 17946.61204)
               mean=17225.551994680001, variance=56236.845767854531,
               skewness=-0.08959536573074788, kurtosis=0.2507542087755521)
```

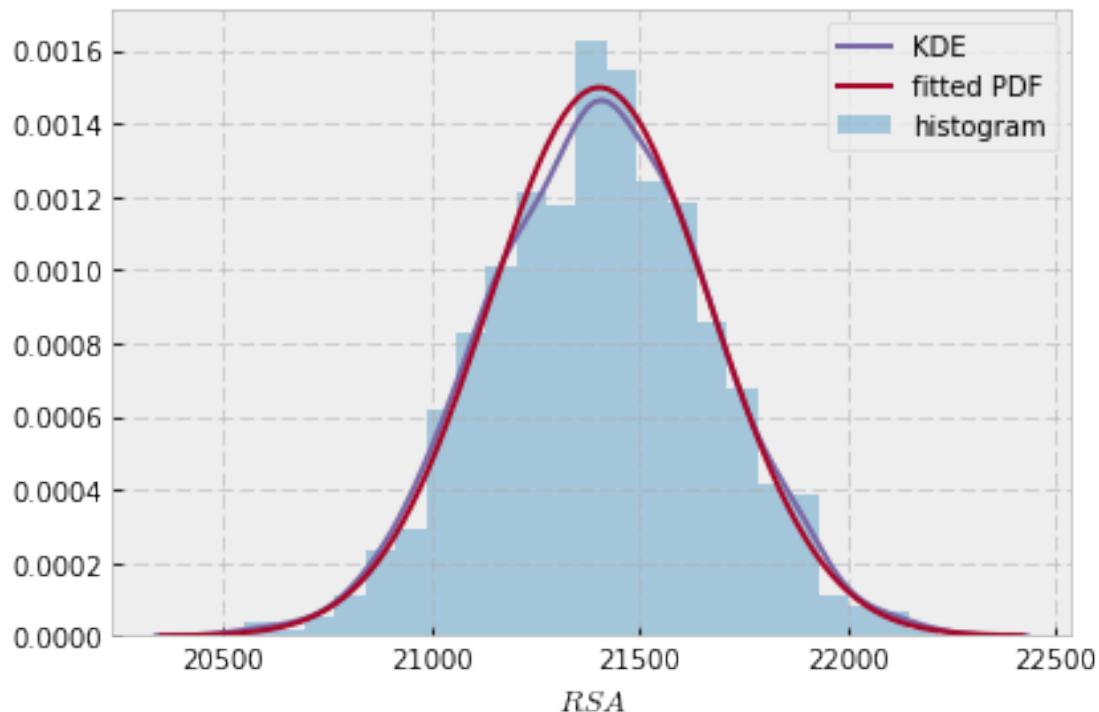


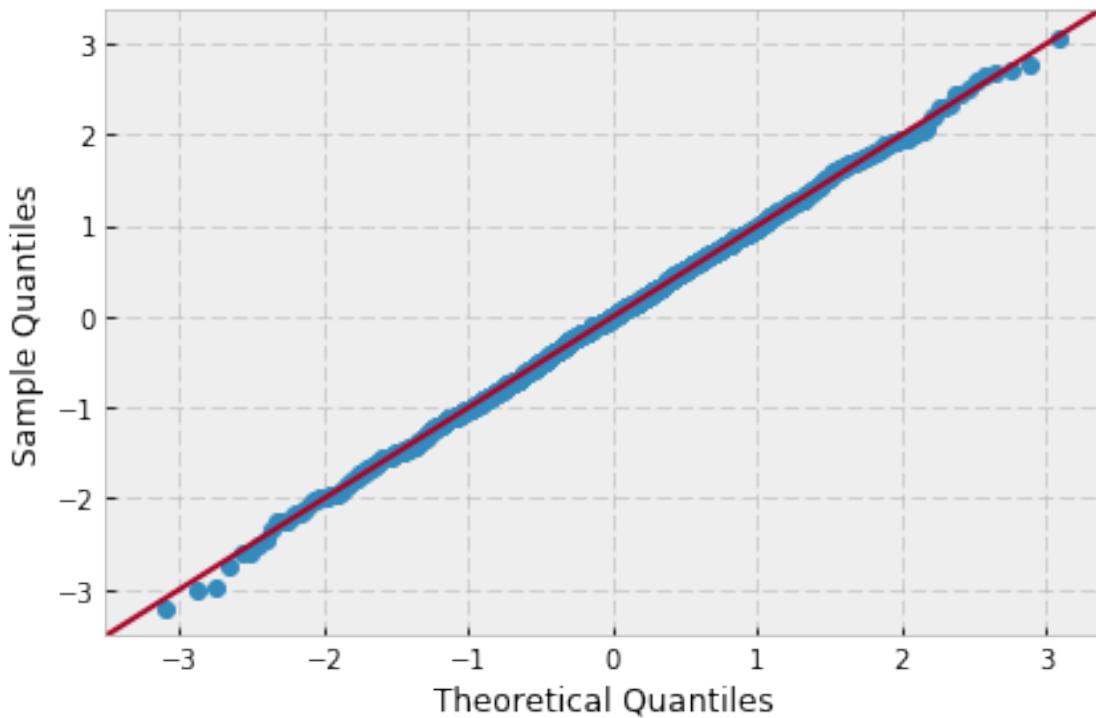


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(),loc=19305.08306065, scale=254.194602882026)
DescribeResult(nobs=1000, minmax=(18585.701789999999, 20128.191309999998)
               mean=19305.083060649999, variance=64679.57571006097,
               skewness=-0.07892359868384538, kurtosis=-0.187384196335588)
```

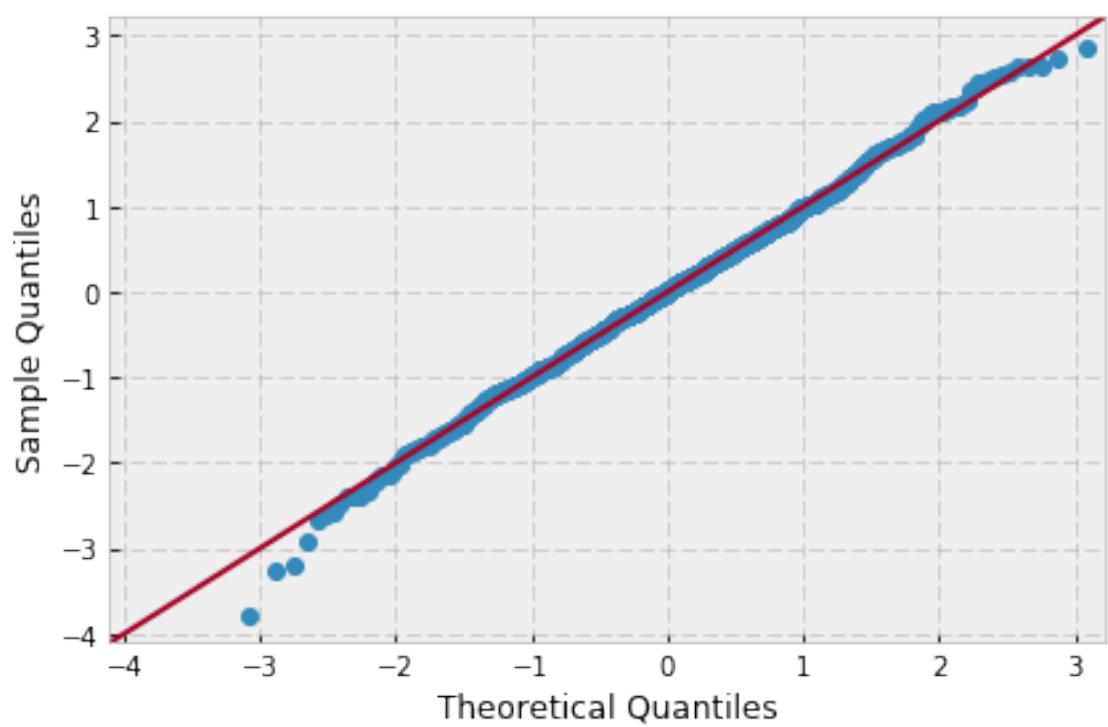
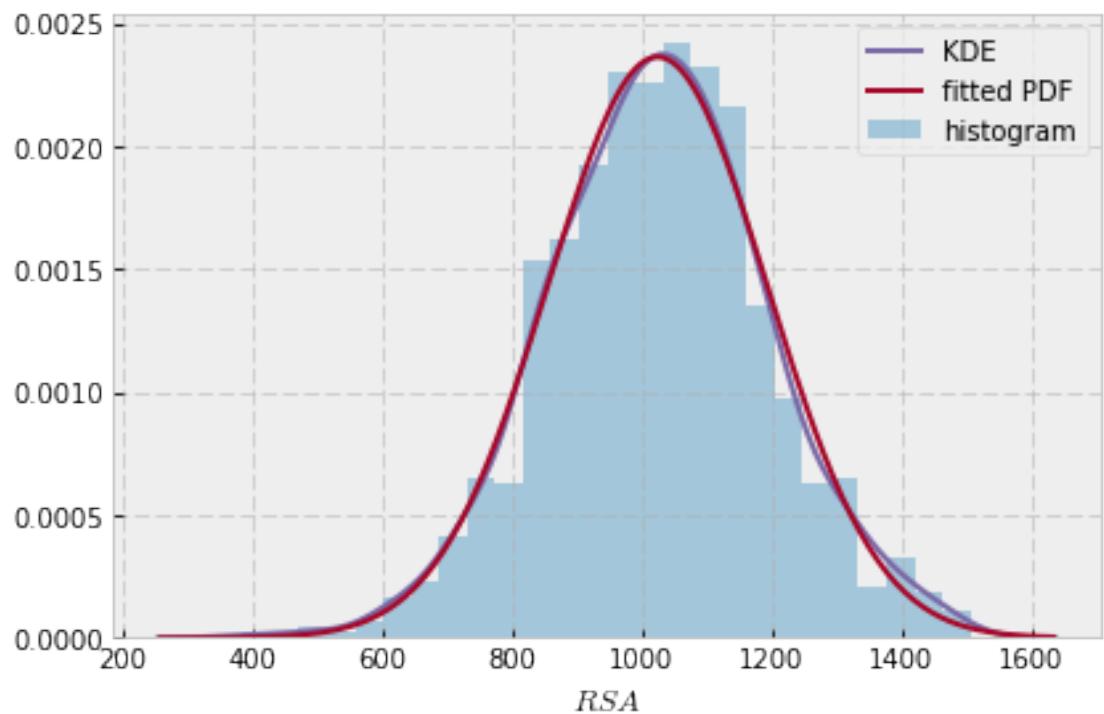


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(), loc=21404.03708287, scale=266.15666264964904)
DescribeResult(nobs=1000, minmax=(20552.2081, 22217.951110000002)
               mean=21404.03708287, variance=70910.279352151236,
               skewness=-0.01671377513545639, kurtosis=-0.09943647266825506)
```

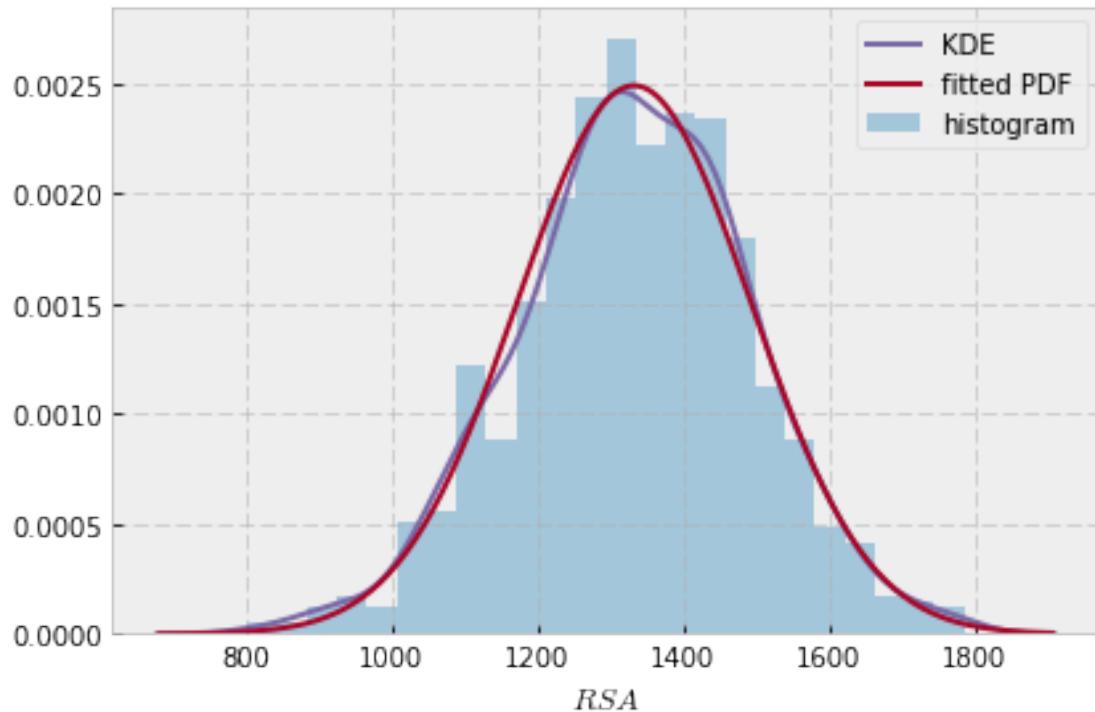


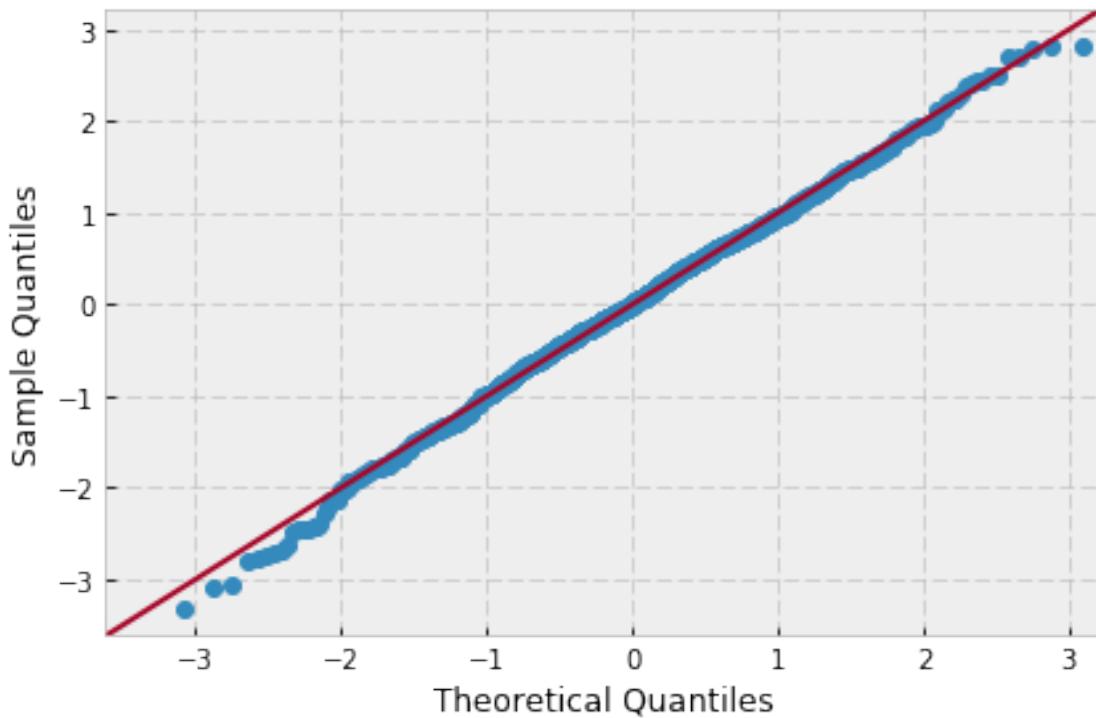


```
variable name: r
variable value: 1.0
distribution: normal(shape=(),loc=1024.3574376793, scale=168.81592523306577)
DescribeResult(nobs=1000, minmax=(386.9268644, 1505.707535)
               mean=1024.3574376792999, variance=28527.343956252305,
               skewness=-0.040293074667804595, kurtosis=0.1950175771307059)
```

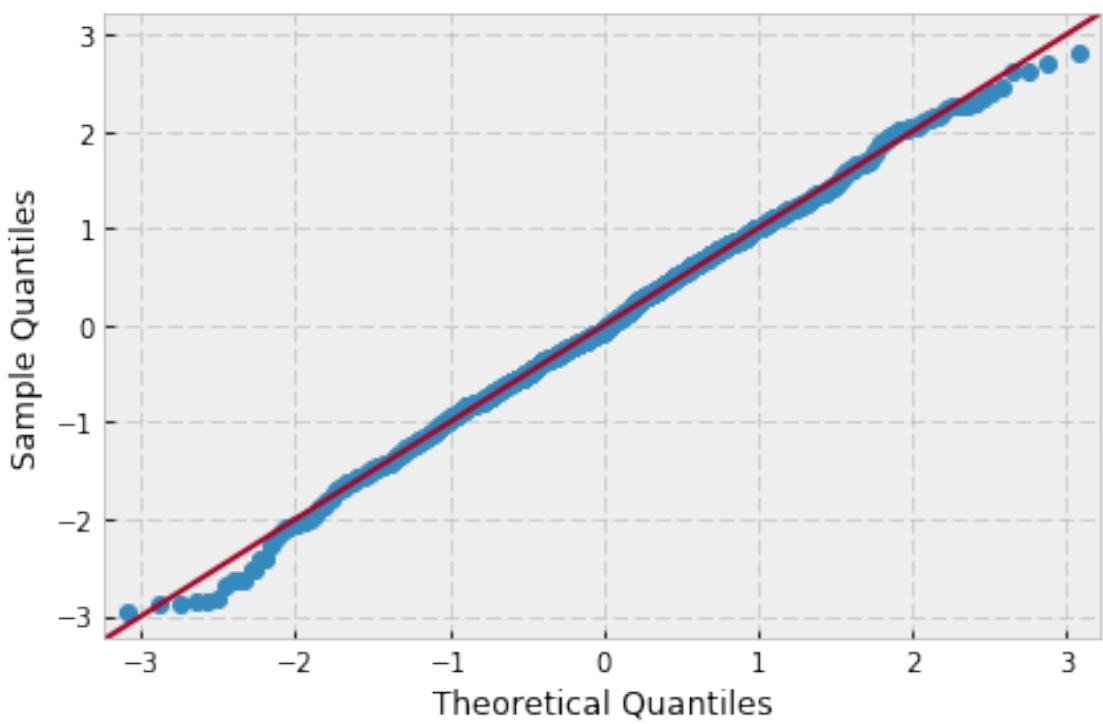
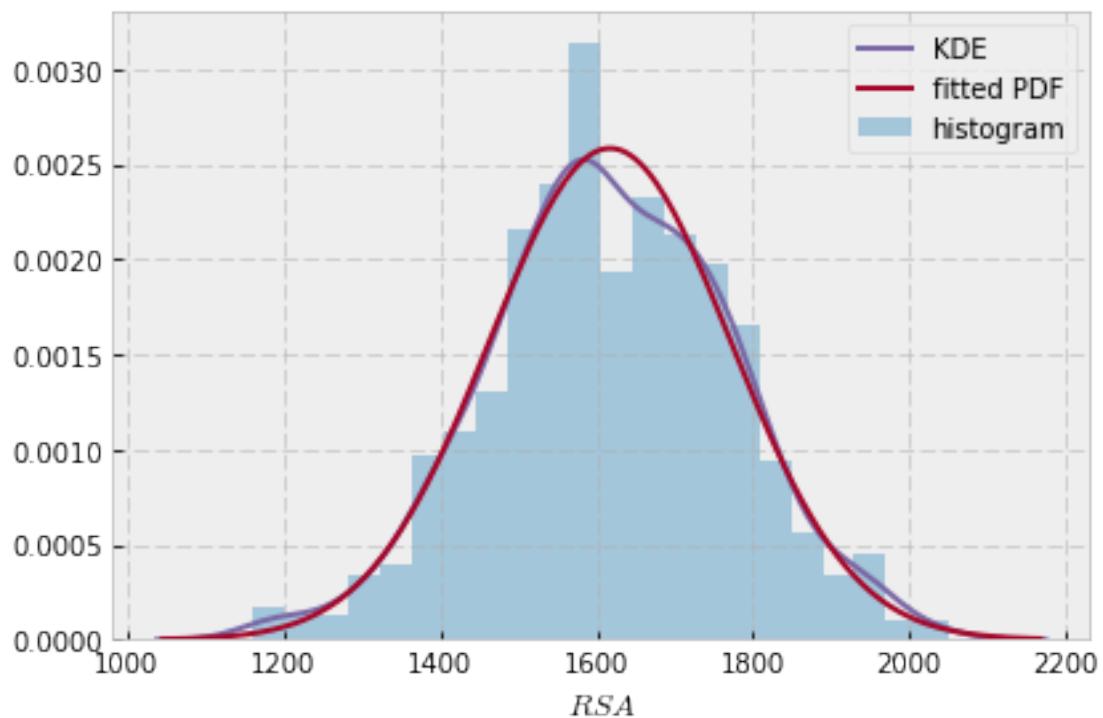


```
variable name: r
variable value: 1.22222222222223
distribution: normal(shape=(), loc=1332.0712591418, scale=160.05047655724445)
DescribeResult(nobs=1000, minmax=(801.83507899999995, 1785.017061)
    mean=1332.0712591418001, variance=25641.796843044103,
    skewness=-0.1234857026577768, kurtosis=0.09774267107312662)
```

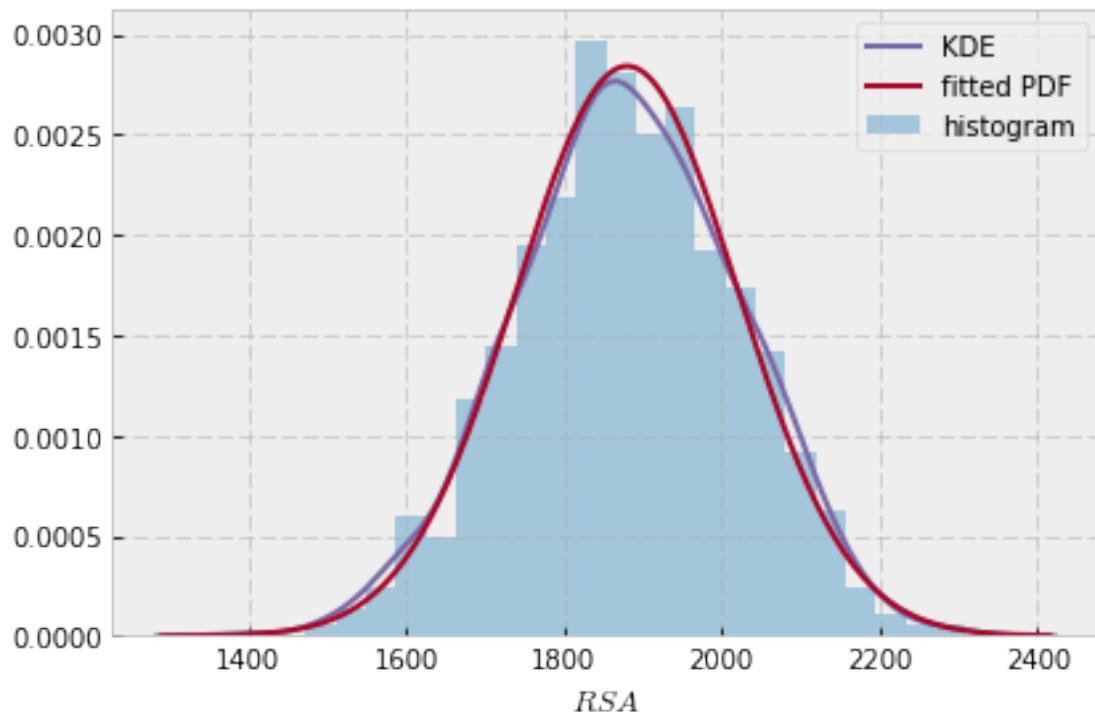


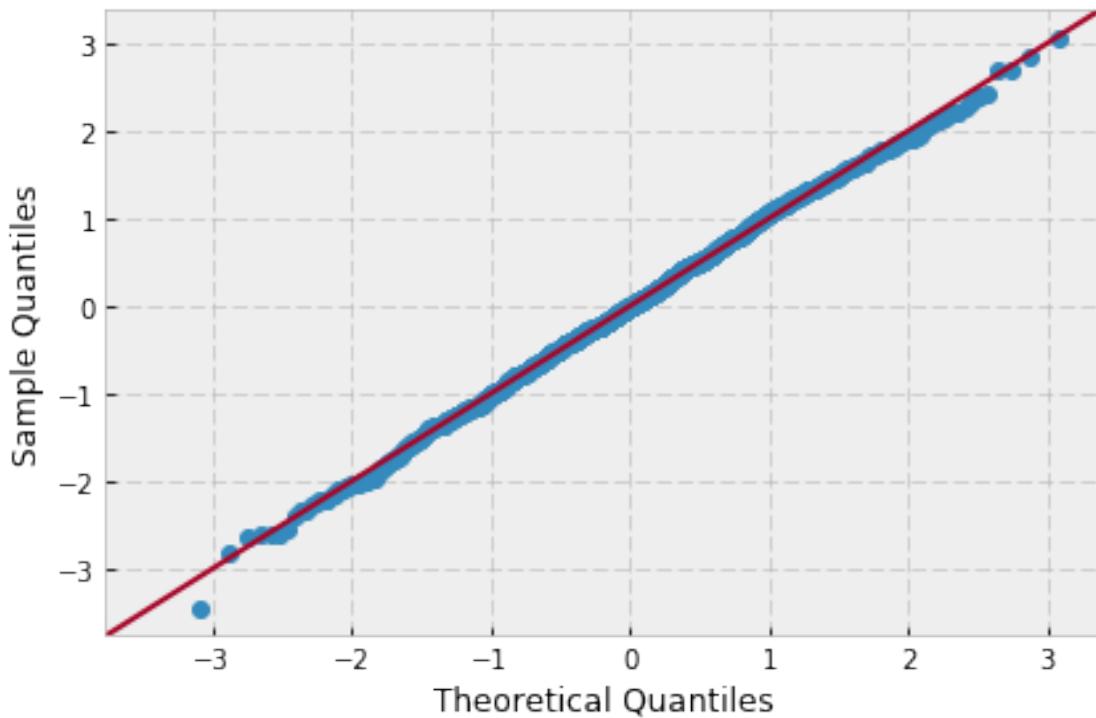


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(),loc=1617.132068707, scale=154.42690014158183)
DescribeResult(nobs=1000, minmax=(1161.5365400000001, 2050.594137999999)
               mean=1617.1320687069999, variance=23871.539026364451,
               skewness=-0.06901387998540189, kurtosis=-0.03489512925374472)
```

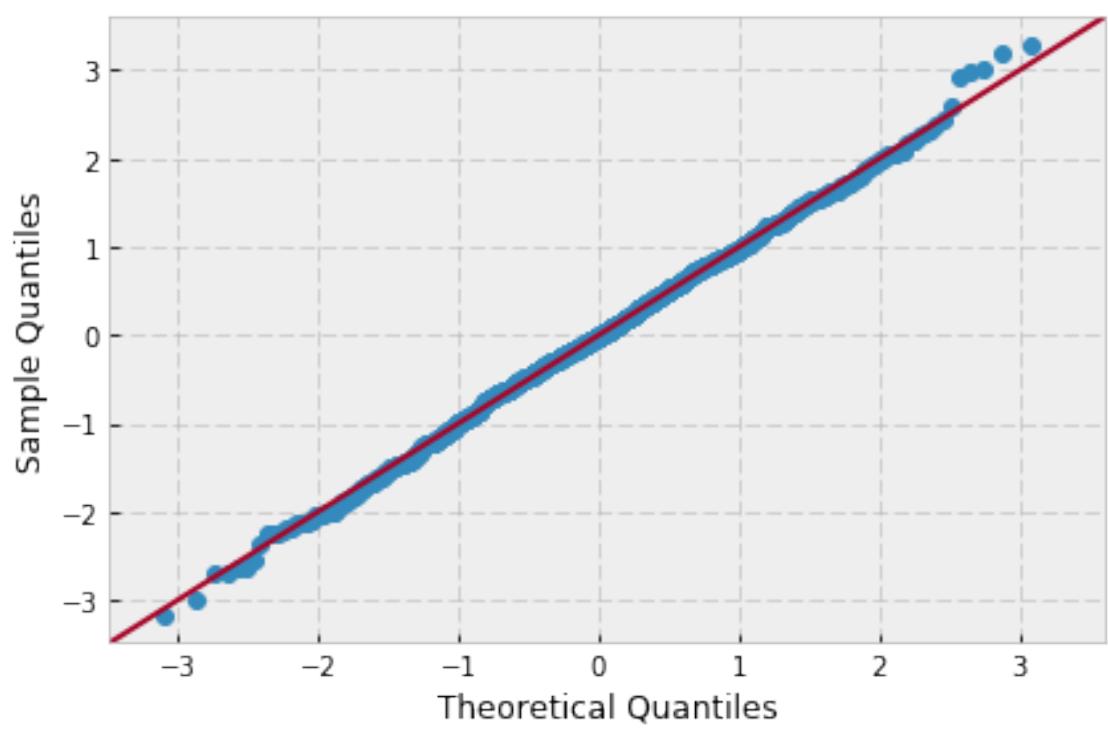
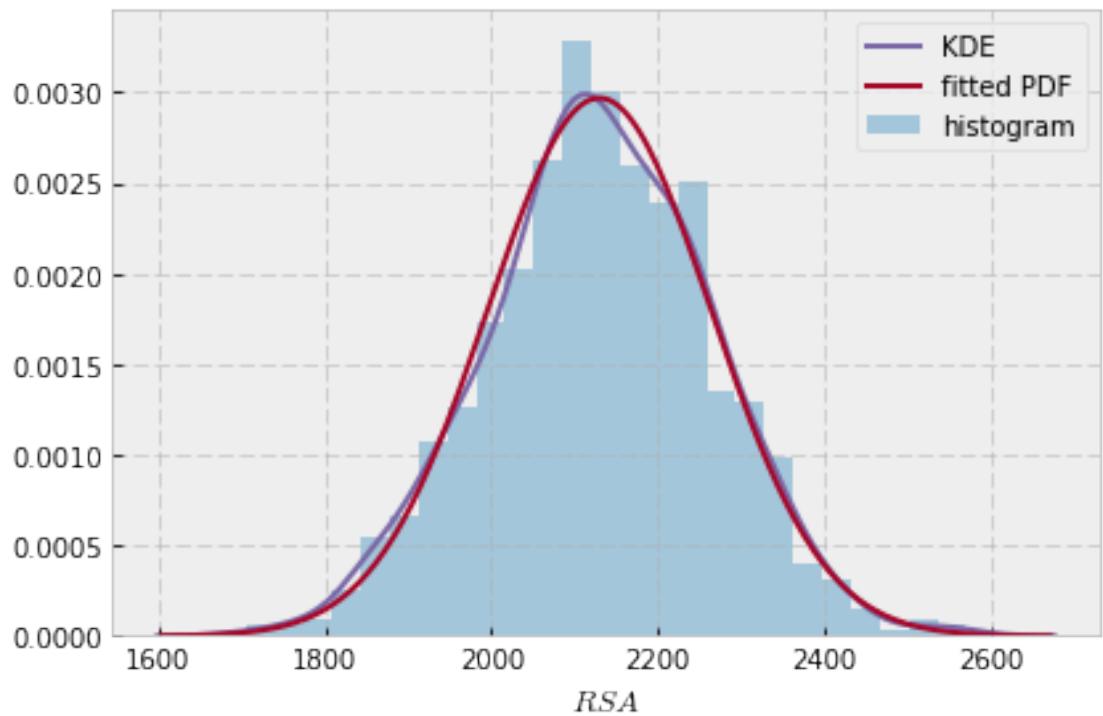


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=1879.8836284400002, scale=140.44224066147925)
DescribeResult(nobs=1000, minmax=(1396.595378, 2309.177610000002)
               mean=1879.8836284400002, variance=19743.766728745602,
               skewness=-0.061215389164794055, kurtosis=-0.16225347202693952)
```

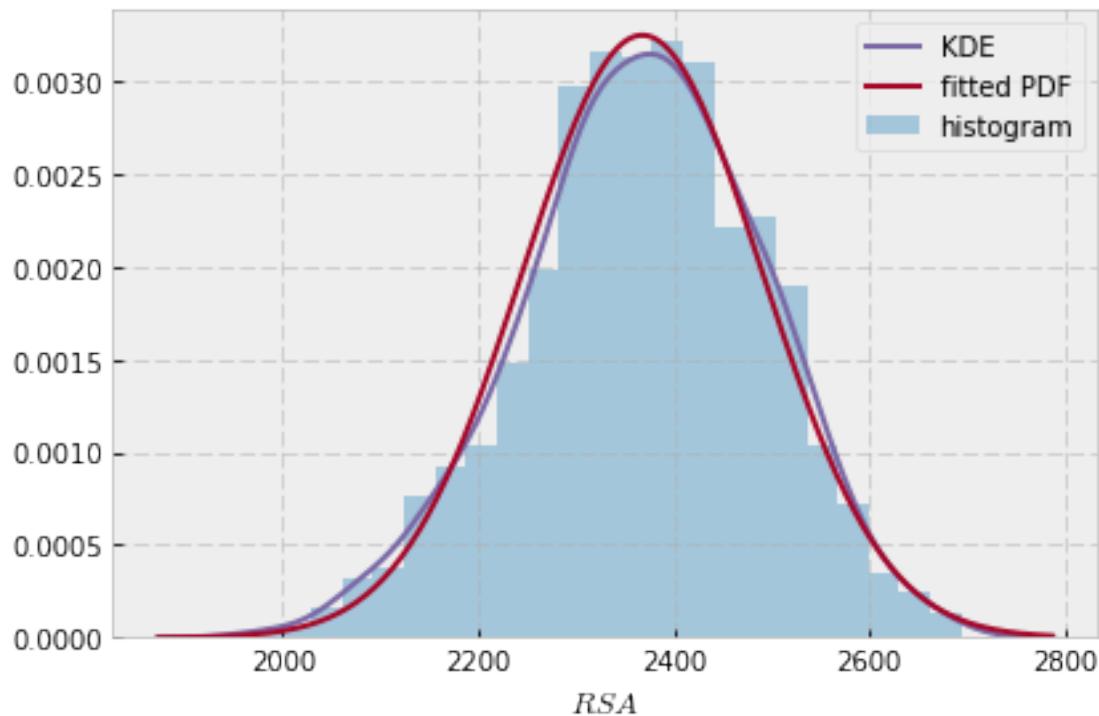


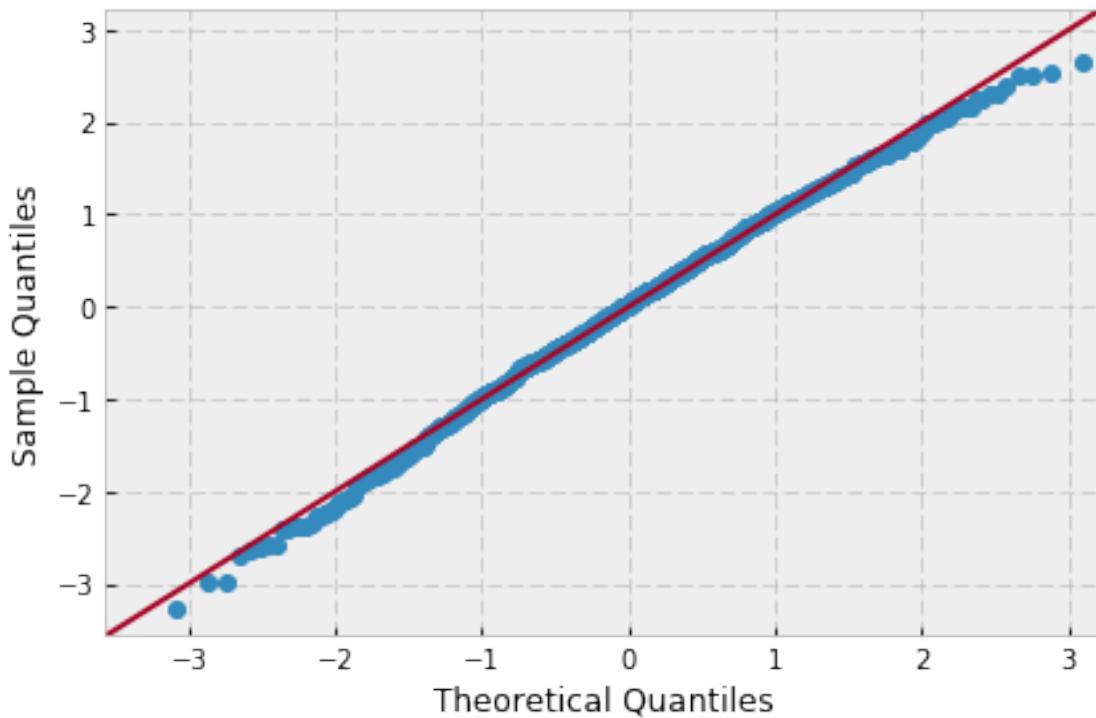


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(),loc=2129.551411247, scale=134.37568053059414)
DescribeResult(nobs=1000, minmax=(1704.2691580000001, 2570.5182030000001)
               mean=2129.551411247, variance=18074.898416476775,
               skewness=-0.026068149235202825, kurtosis=0.03321642863989327)
```

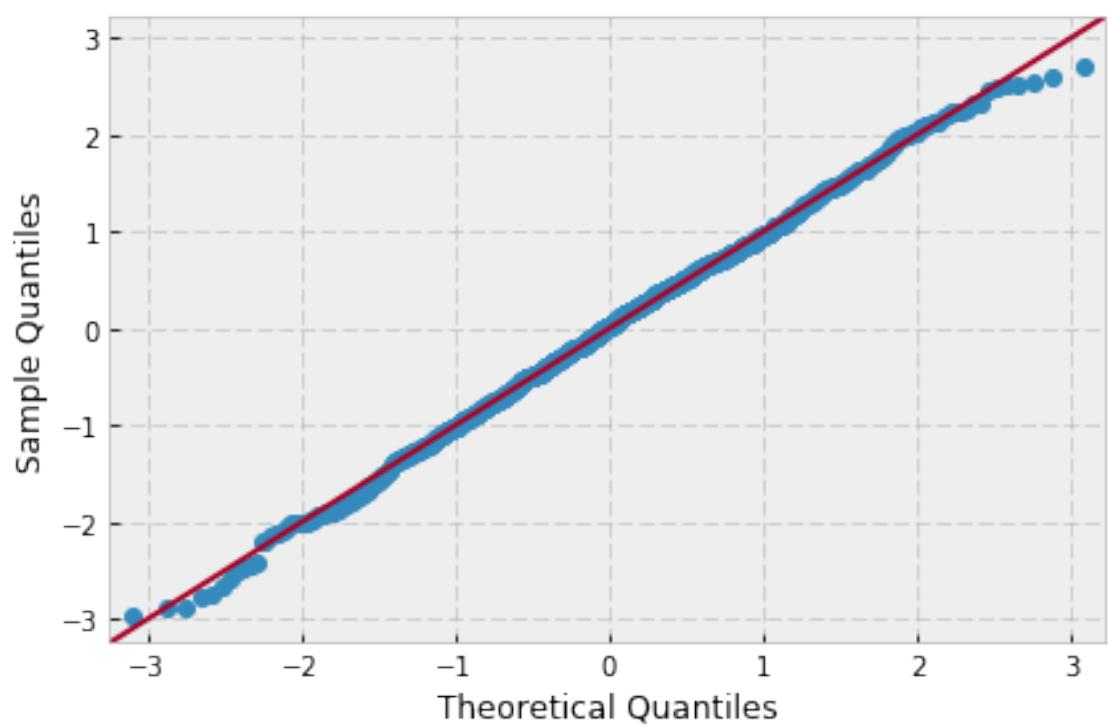
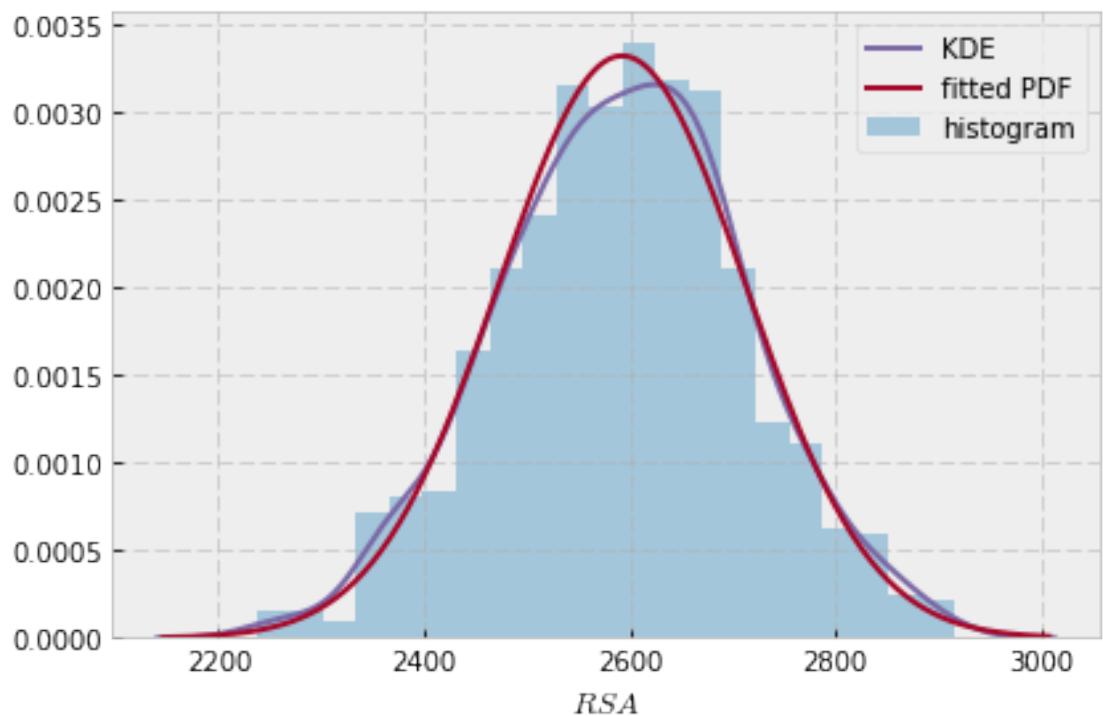


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=2367.786662375, scale=122.76800805361428)
DescribeResult(nobs=1000, minmax=(1966.2054439999999, 2693.8730919999998)
               mean=2367.7866623750001, variance=15087.070872324626,
               skewness=-0.19626632746709033, kurtosis=-0.10694694572270746)
```

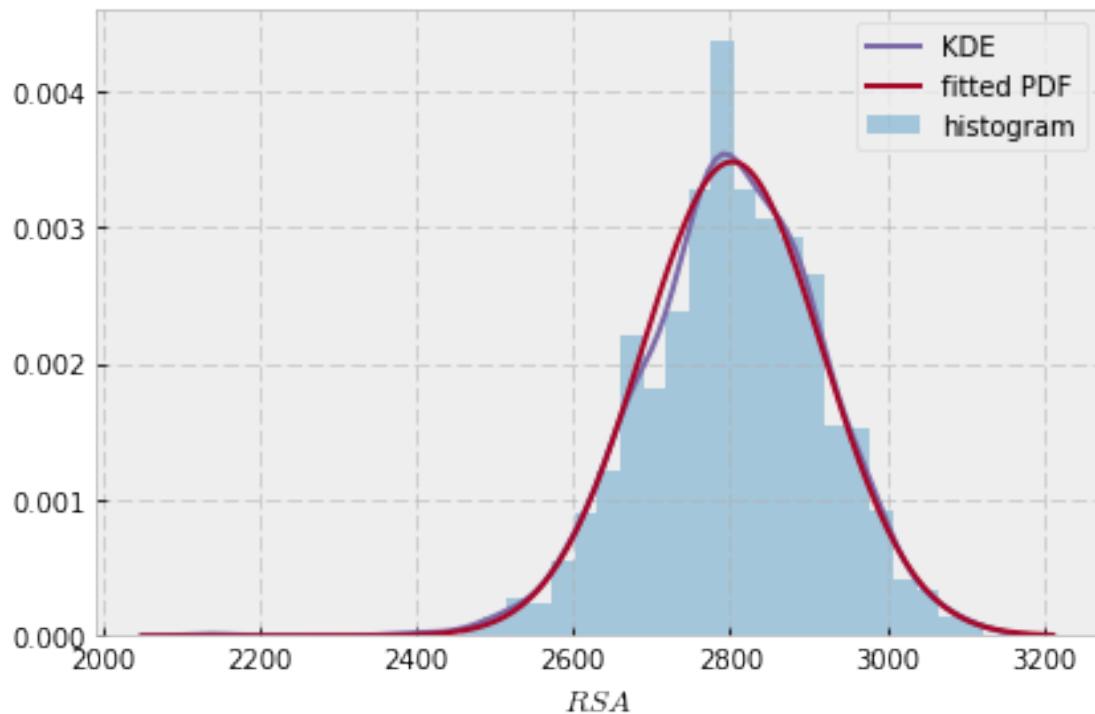


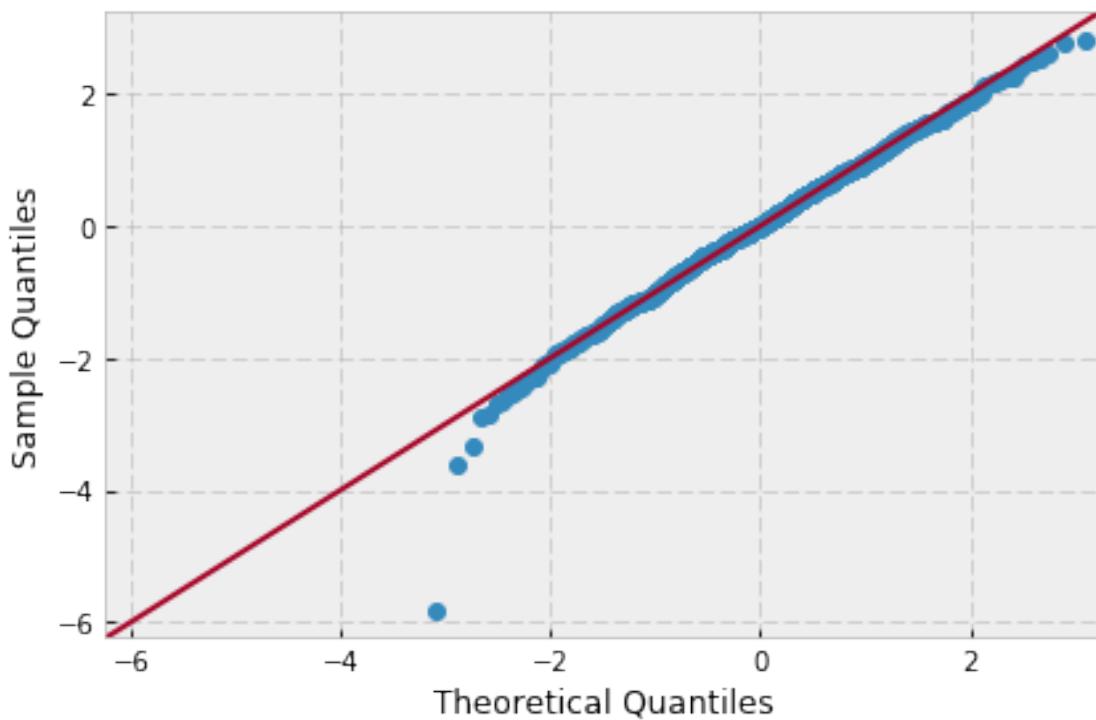


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(),loc=2592.352942994, scale=120.20420414047769)
DescribeResult(nobs=1000, minmax=(2236.6155520000002, 2916.280921999999)
               mean=2592.3529429939999, variance=14463.514207252885,
               skewness=-0.07268717108422972, kurtosis=-0.12652192317465616)
```

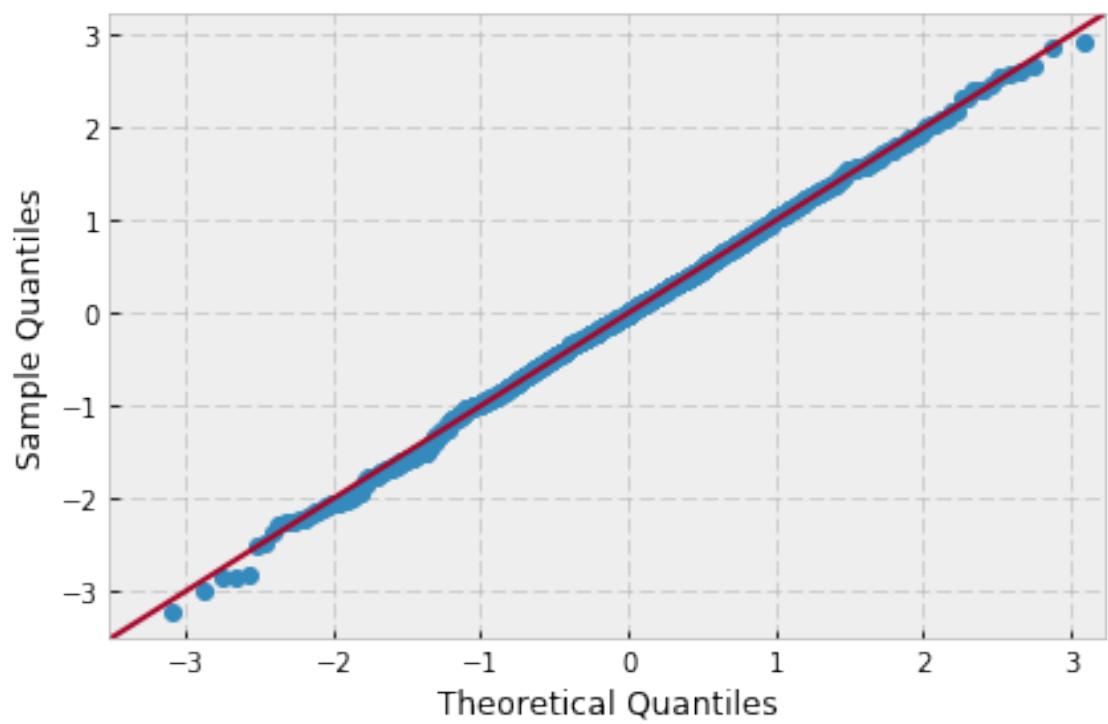
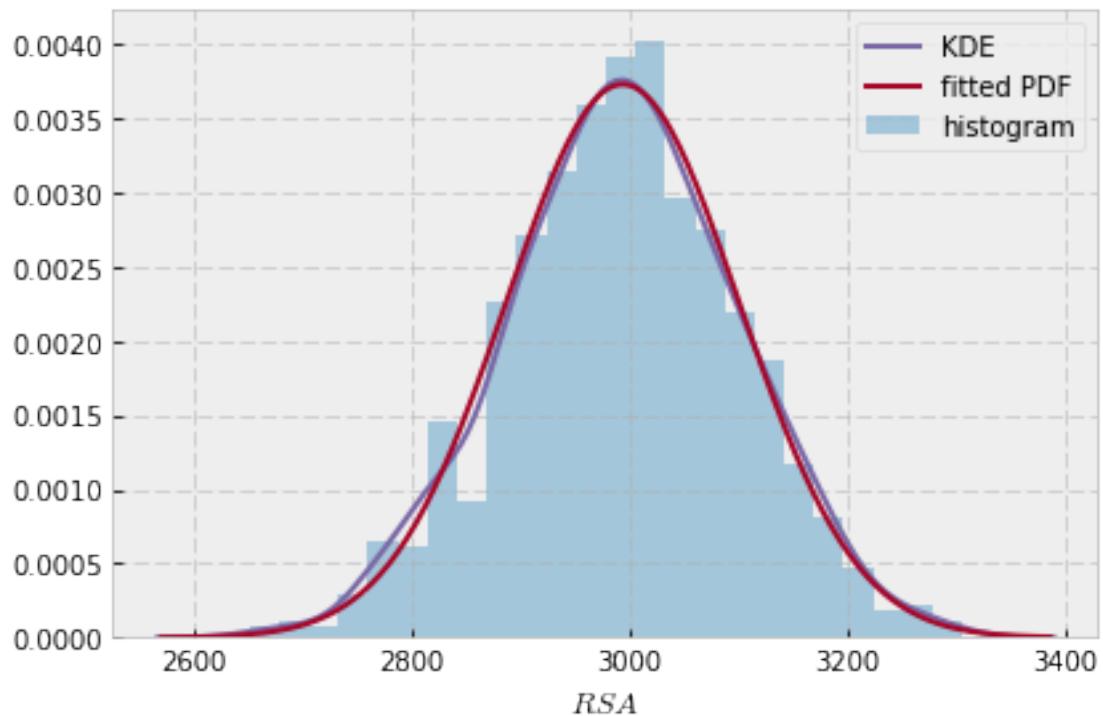


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=2802.68245541, scale=114.74032984894401)
DescribeResult(nobs=1000, minmax=(2136.6289550000001, 3124.406183999999)
               mean=2802.6824554099999, variance=13178.521815660131,
               skewness=-0.3212252875771477, kurtosis=1.0105798118627094)
```

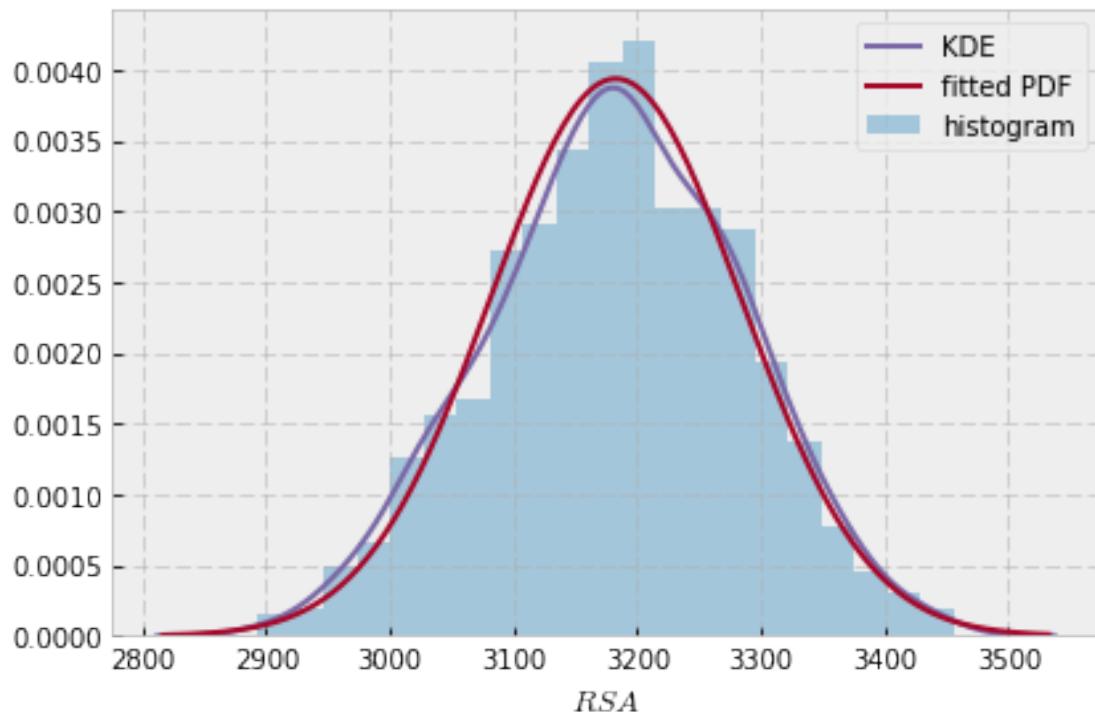


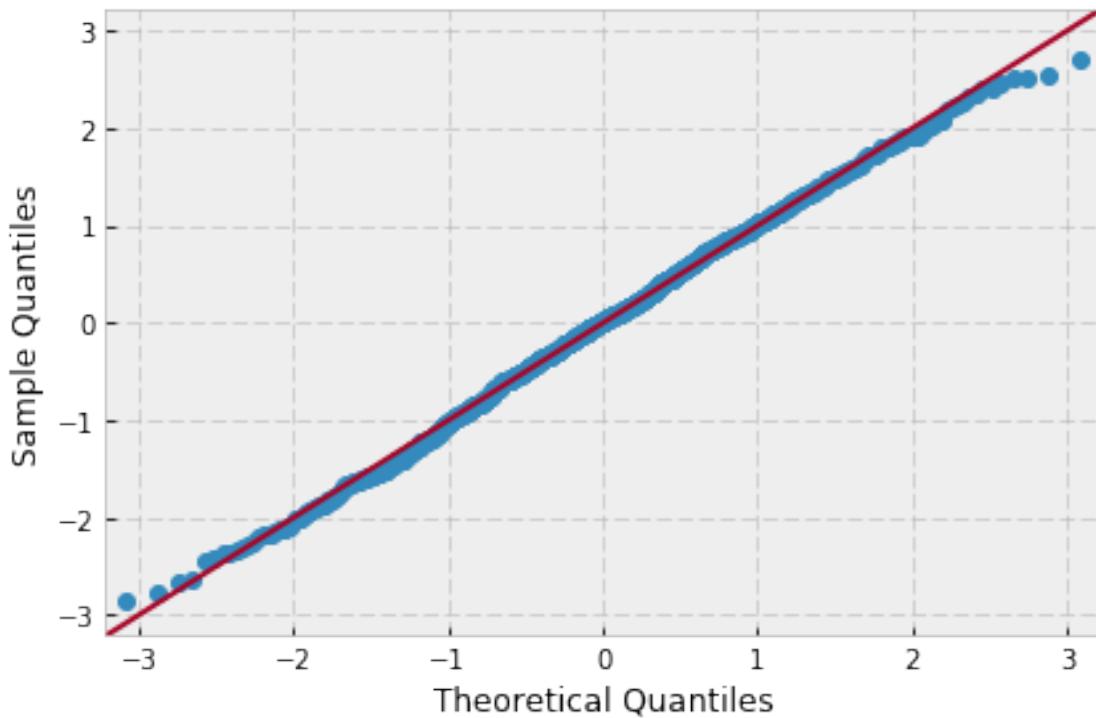


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=2993.39682701, scale=106.97190904660643)
DescribeResult(nobs=1000, minmax=(2650.163395, 3305.489986)
               mean=2993.3968270099999, variance=11454.443768844285,
               skewness=-0.05740855710702652, kurtosis=-0.044039418865050806)
```

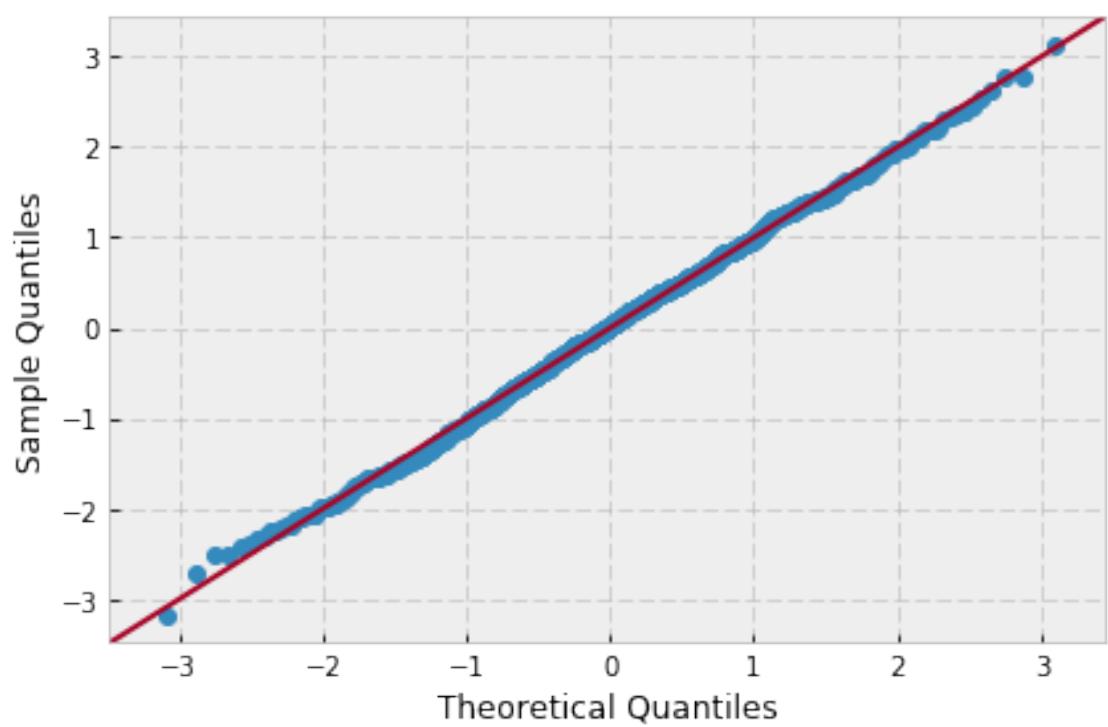
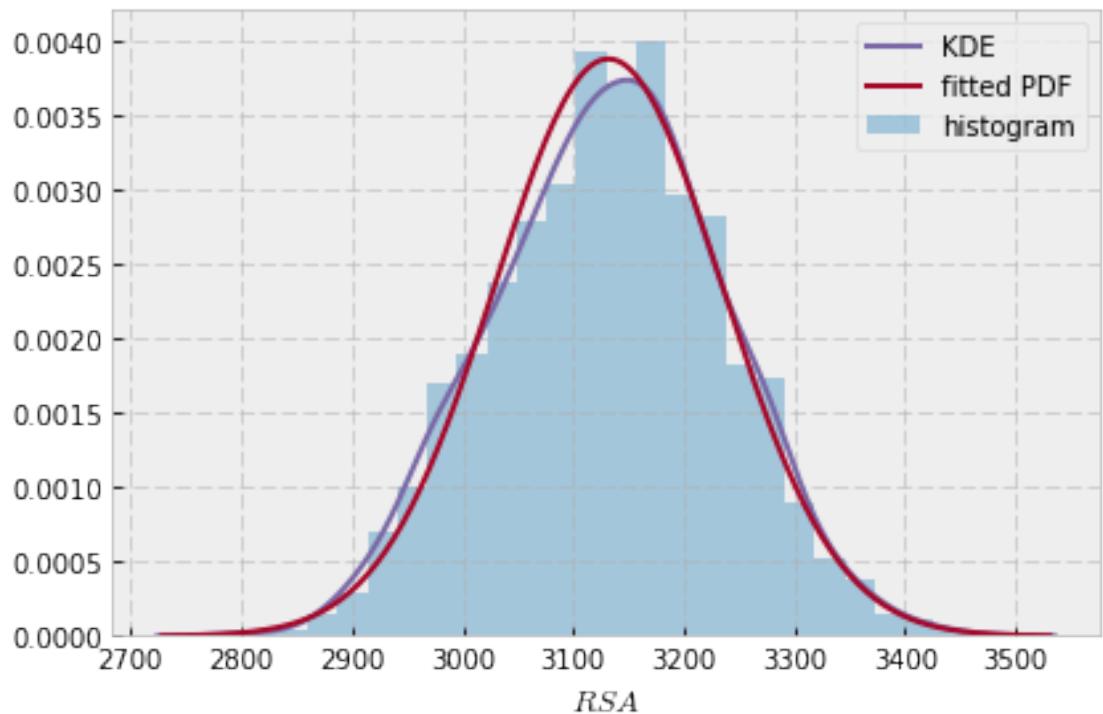


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=3182.486474712, scale=101.34504517217748)
DescribeResult(nobs=1000, minmax=(2892.7434250000001, 3456.2147810000001)
               mean=3182.486474712, variance=10281.099280230925,
               skewness=-0.08071190992711148, kurtosis=-0.2919981758034922)
```

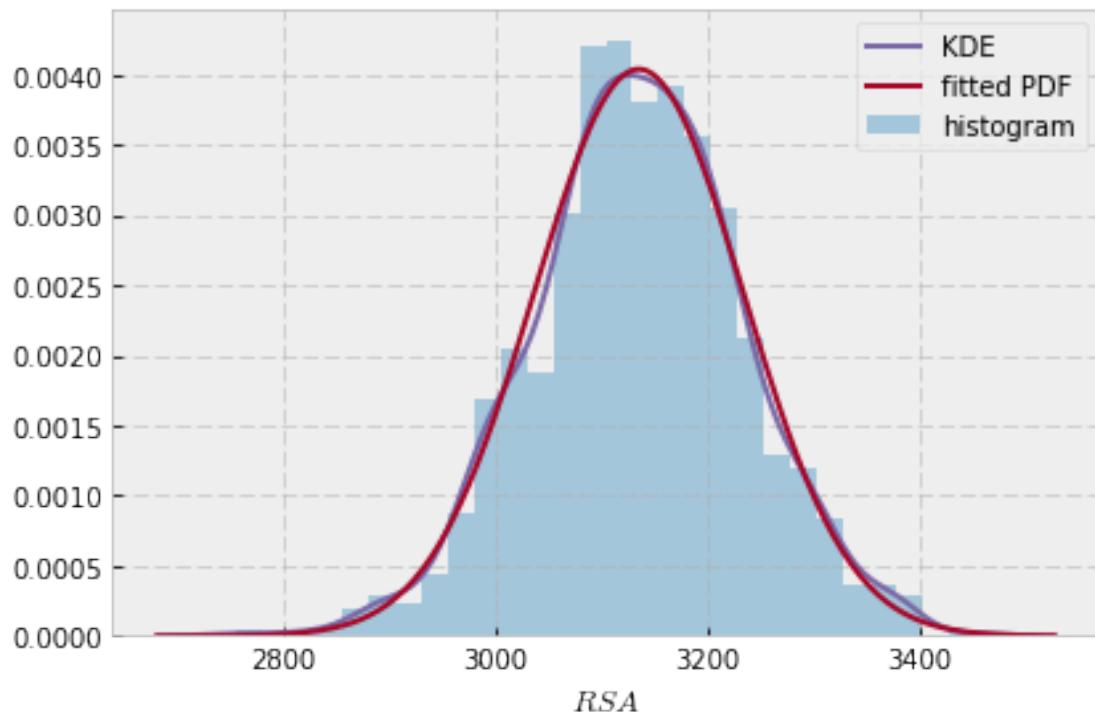


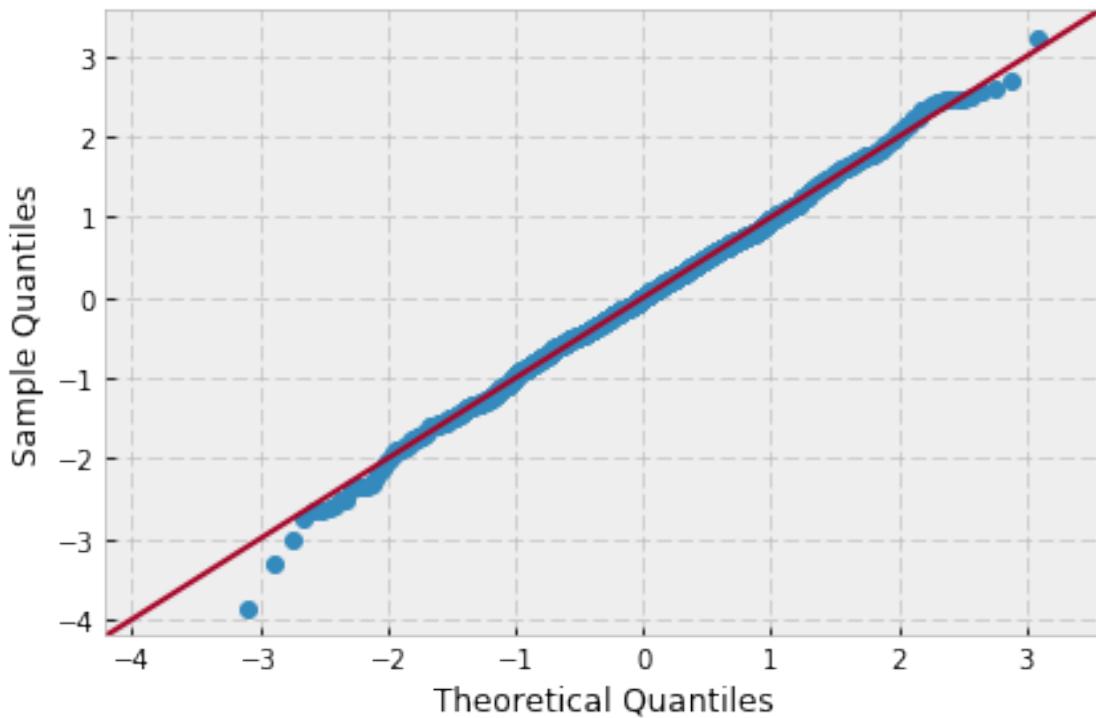


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(),loc=3131.6742278399997, scale=102.8848516584566)
DescribeResult(nobs=1000, minmax=(2805.783715, 3452.838463999999)
               mean=3131.674227839997, variance=10595.888589371989,
               skewness=-0.03637774804533017, kurtosis=-0.24899669593619533)
```

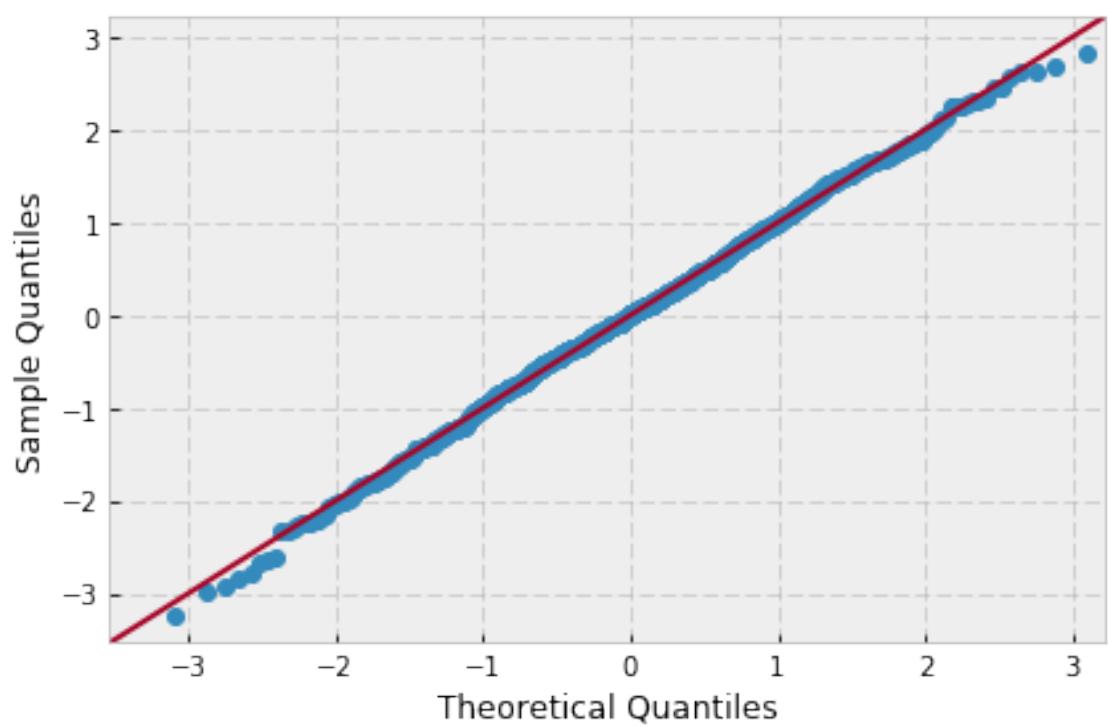
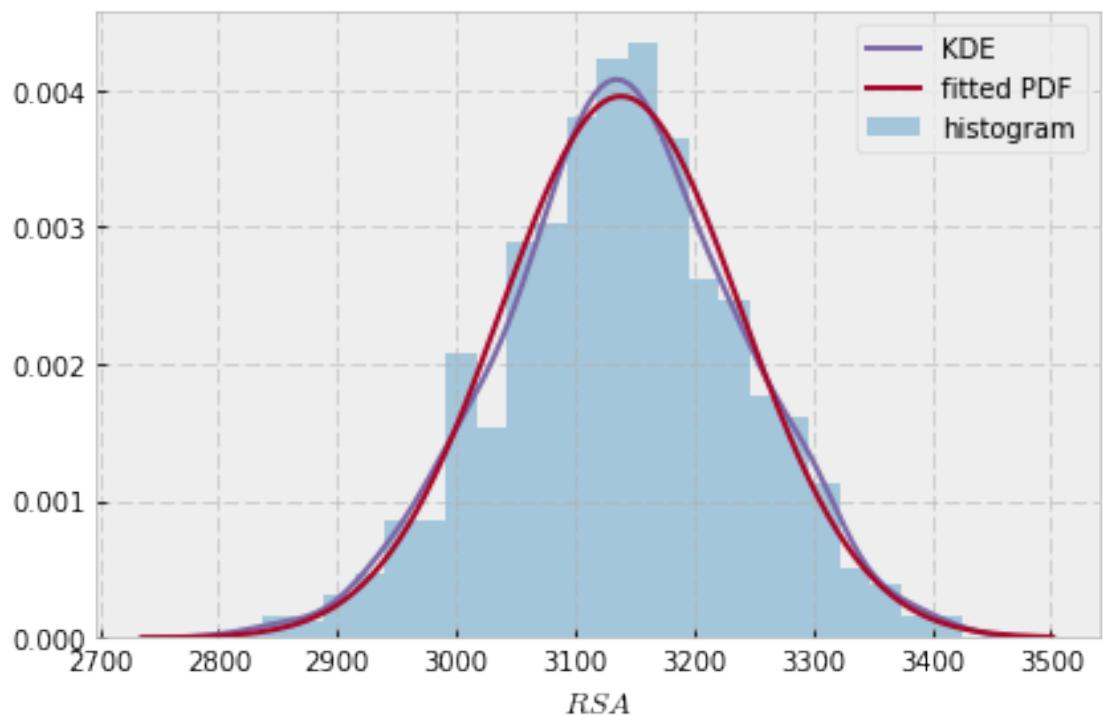


```
variable name: sigma
variable value: 0.0222222222222223
distribution: normal(shape=(), loc=3135.180332738, scale=98.73353826641761)
DescribeResult(nobs=1000, minmax=(2754.8087270000001, 3453.2319859999998)
               mean=3135.1803327379998, variance=9758.0696482544045,
               skewness=-0.06222954670862398, kurtosis=0.20072995217901424)
```

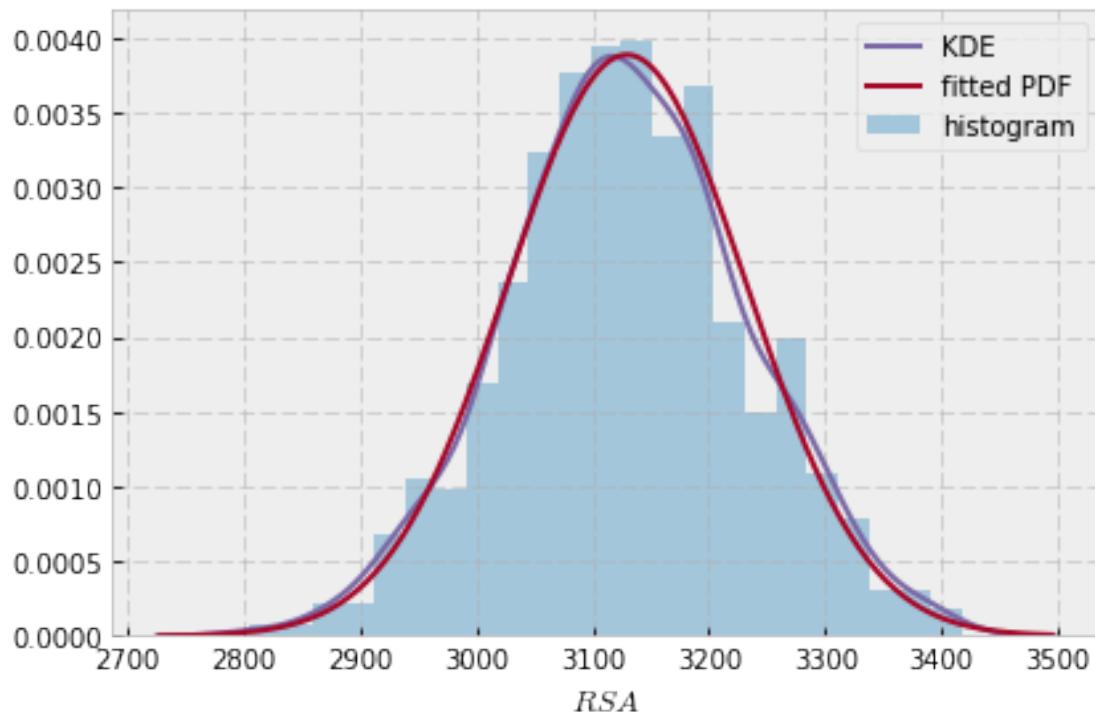


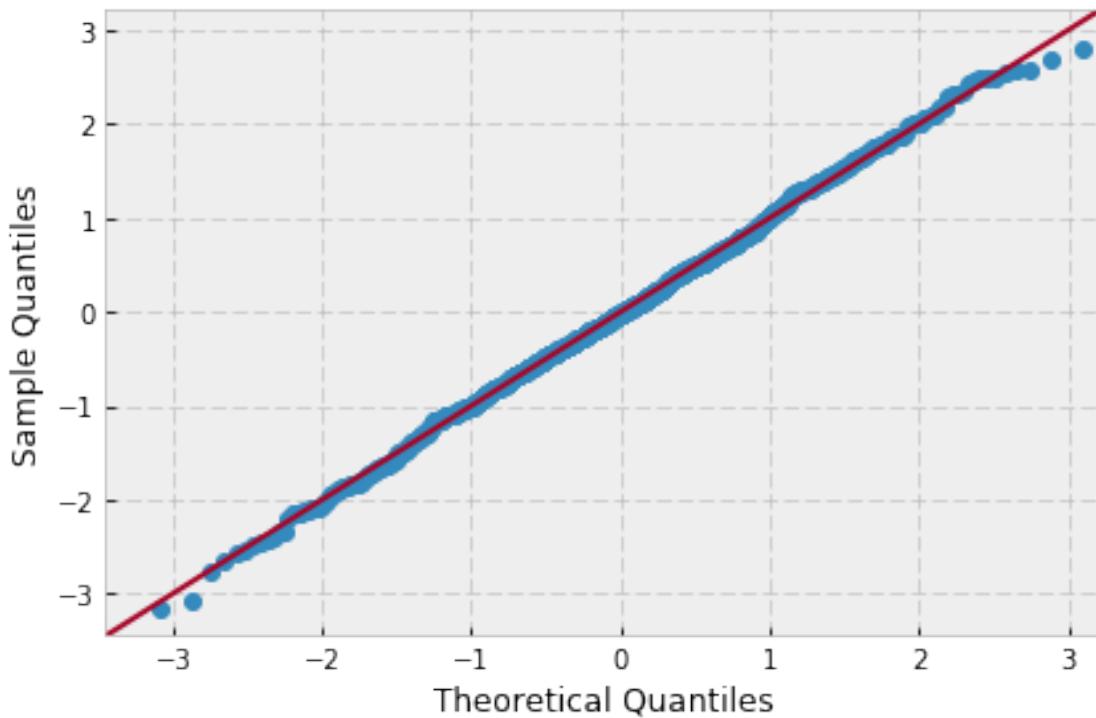


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=3138.4312522640002, scale=100.90761640080488)
DescribeResult(nobs=1000, minmax=(2812.3734060000002, 3424.6216639999998)
               mean=3138.4312522640002, variance=10192.539587279265,
               skewness=-0.06944095027188492, kurtosis=-0.04591695347800551)
```

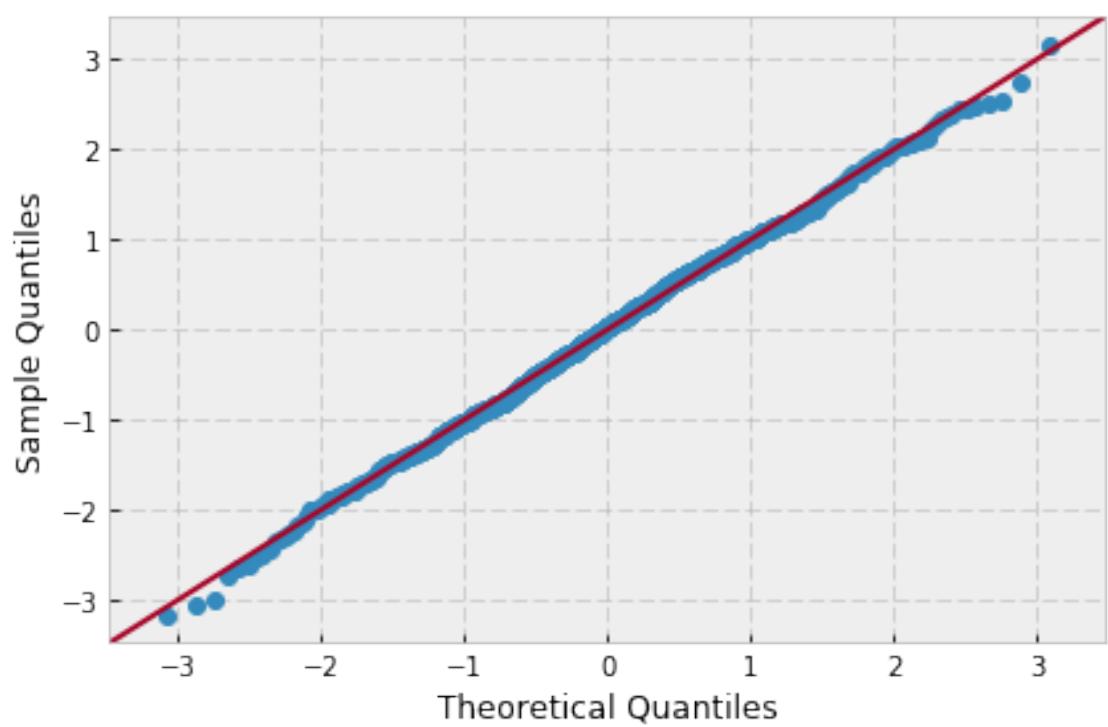
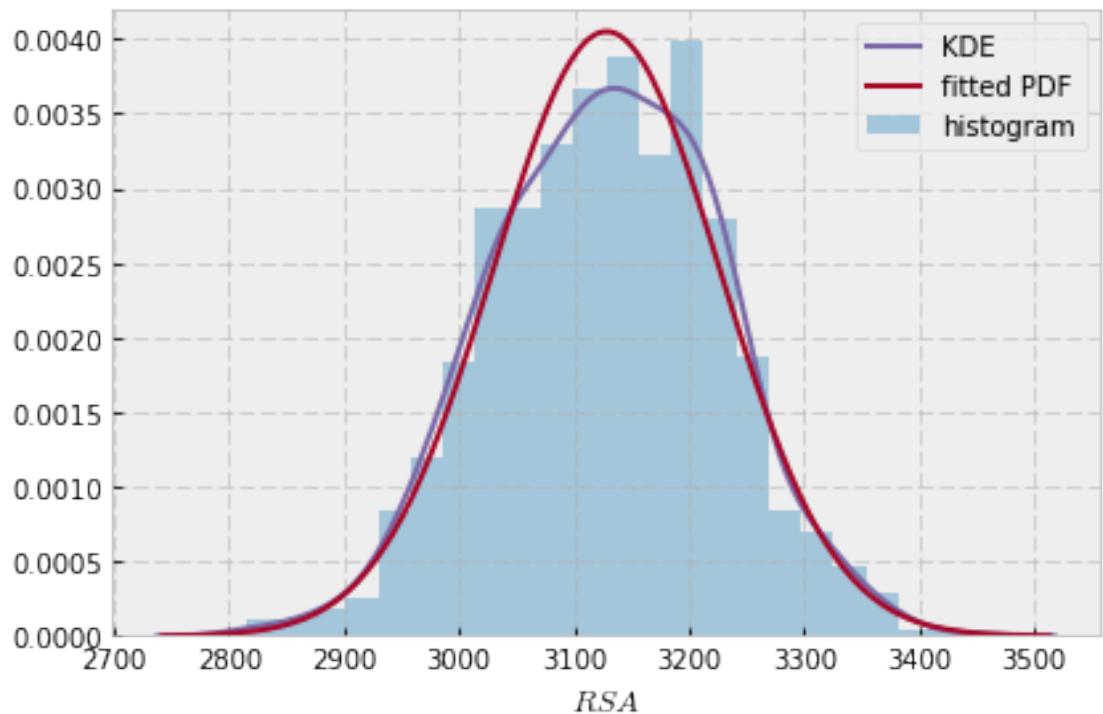


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=3129.4717979650004, scale=102.70841872548829)
DescribeResult(nobs=1000, minmax=(2805.444555, 3417.3452200000002)
               mean=3129.4717979650004, variance=10559.57885594618,
               skewness=0.003885077446856966, kurtosis=-0.06123539107783538)
```

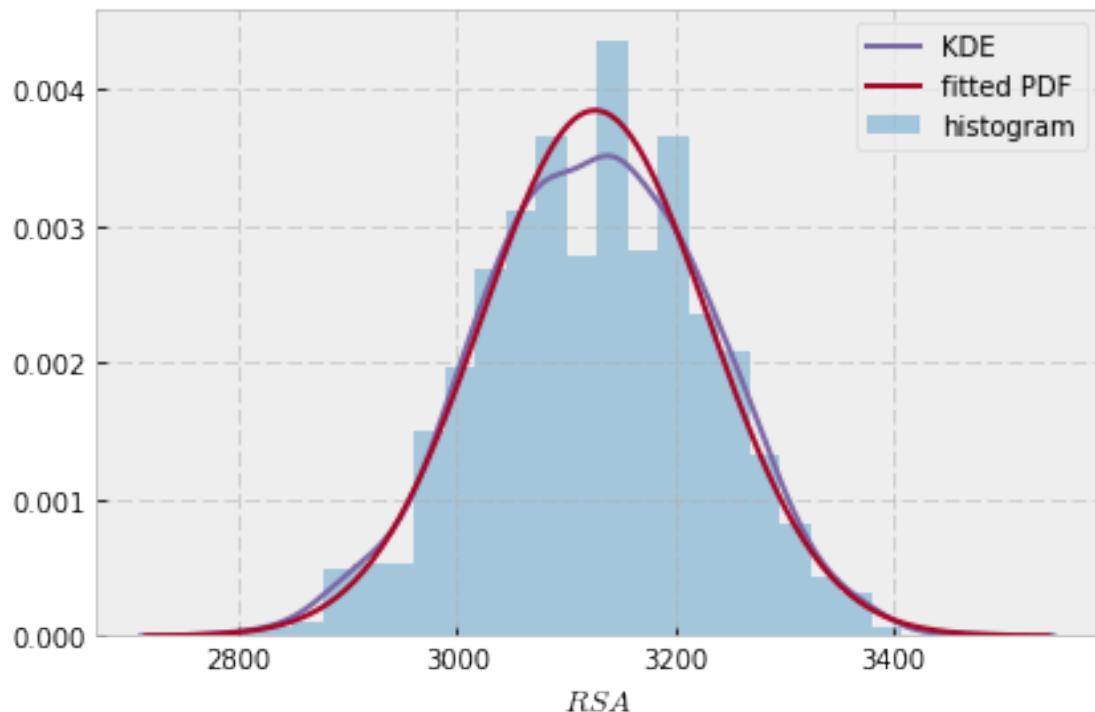


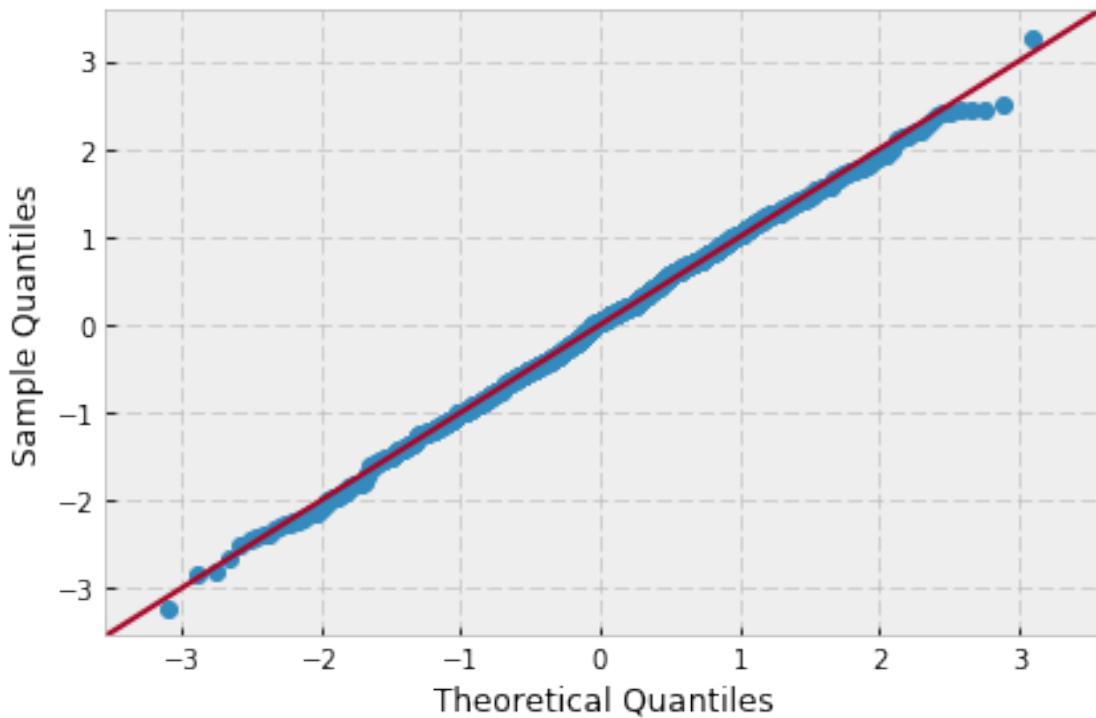


```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(),loc=3128.2603472779997, scale=98.48809012727699)
DescribeResult(nobs=1000, minmax=(2816.4959699999999, 3438.7202160000002)
               mean=3128.2603472779997, variance=9709.6135104290643,
               skewness=-0.07102087682309616, kurtosis=-0.16965625172688892)
```

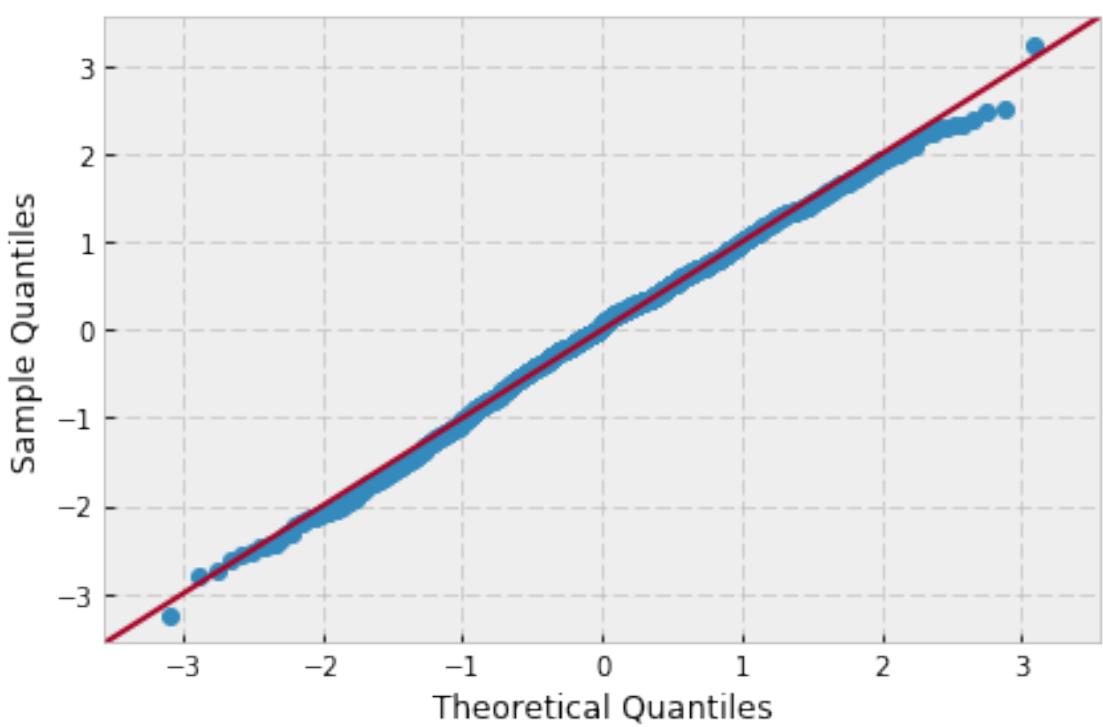
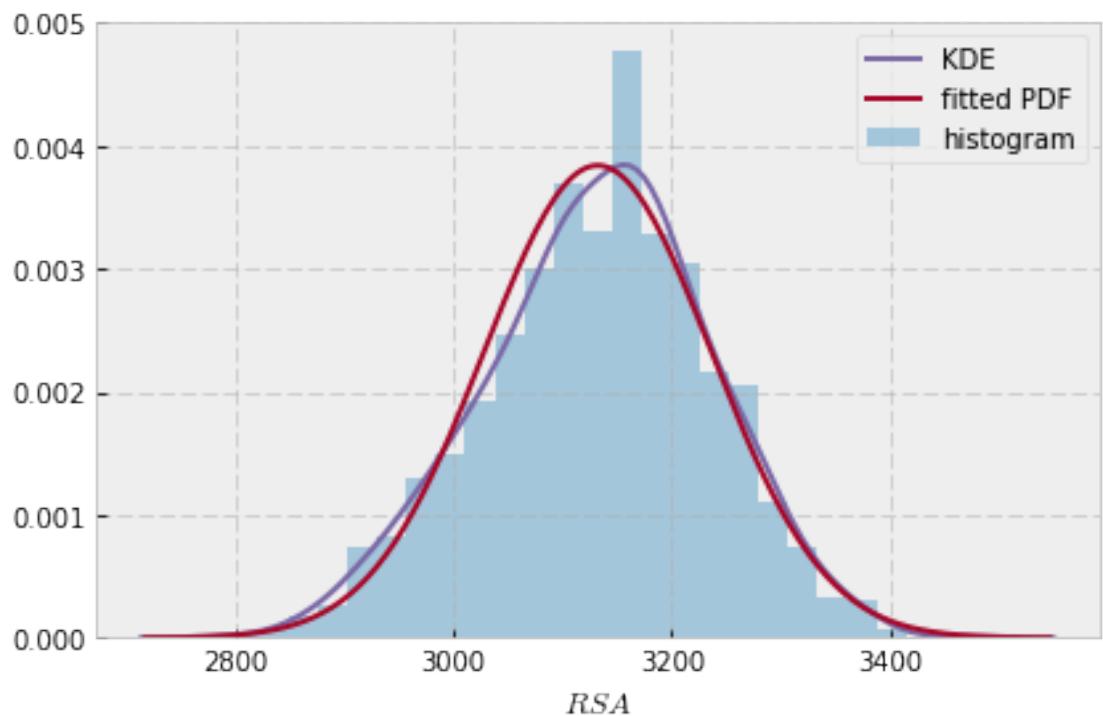


```
variable name: sigma
variable value: 0.06666666666666667
distribution: normal(shape=(), loc=3126.866145847, scale=103.69282646154554)
DescribeResult(nobs=1000, minmax=(2793.532193, 3464.476735000002)
               mean=3126.8661458470001, variance=10762.965224809006,
               skewness=-0.062031733294812534, kurtosis=-0.21739579066584414)
```

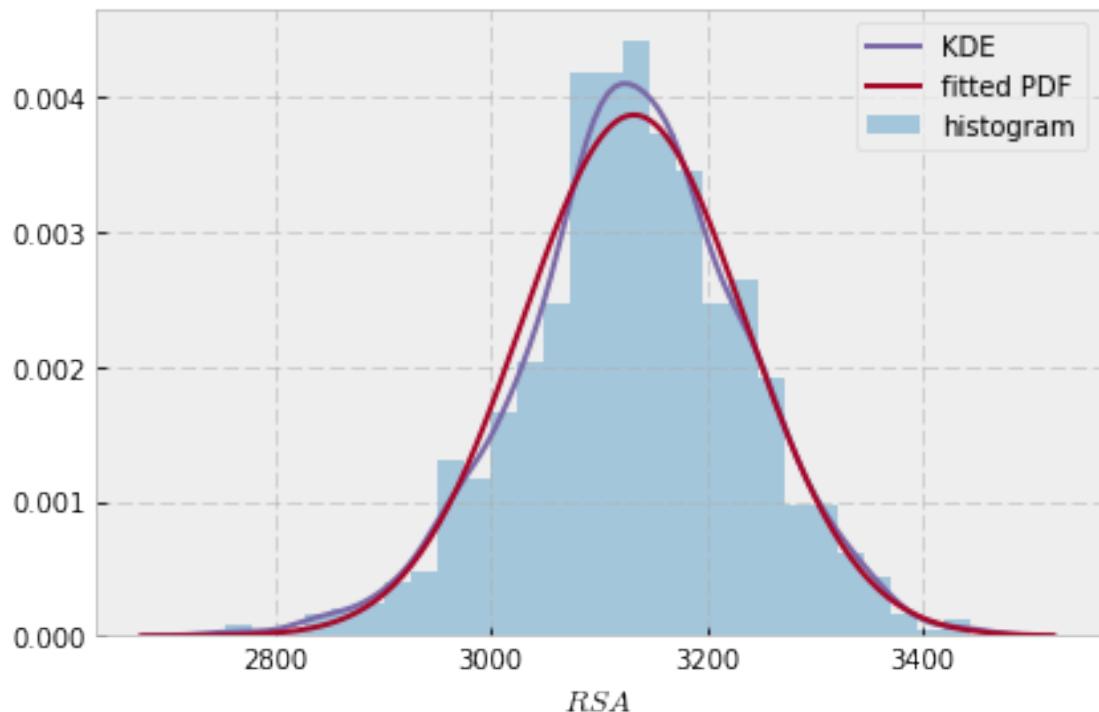


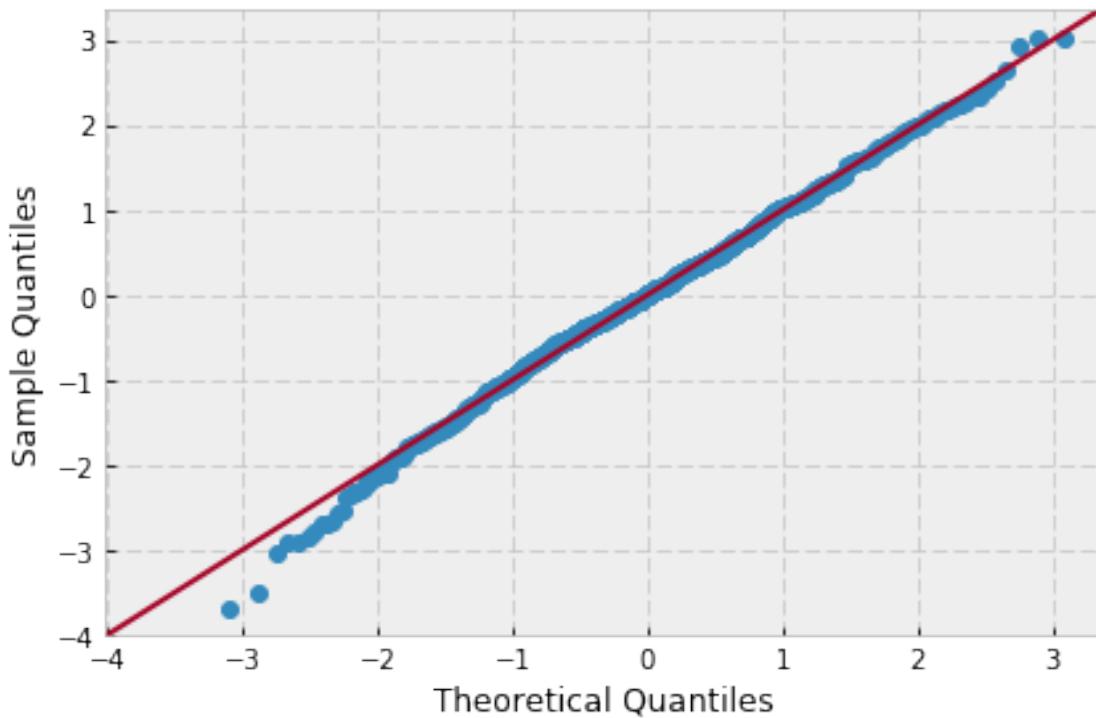


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=3132.2611929940003, scale=103.90601629996502)
DescribeResult(nobs=1000, minmax=(2796.0598450000002, 3467.87698)
               mean=3132.2611929940003, variance=10807.267490819413,
               skewness=-0.16775403640233016, kurtosis=-0.1544603597203018)
```

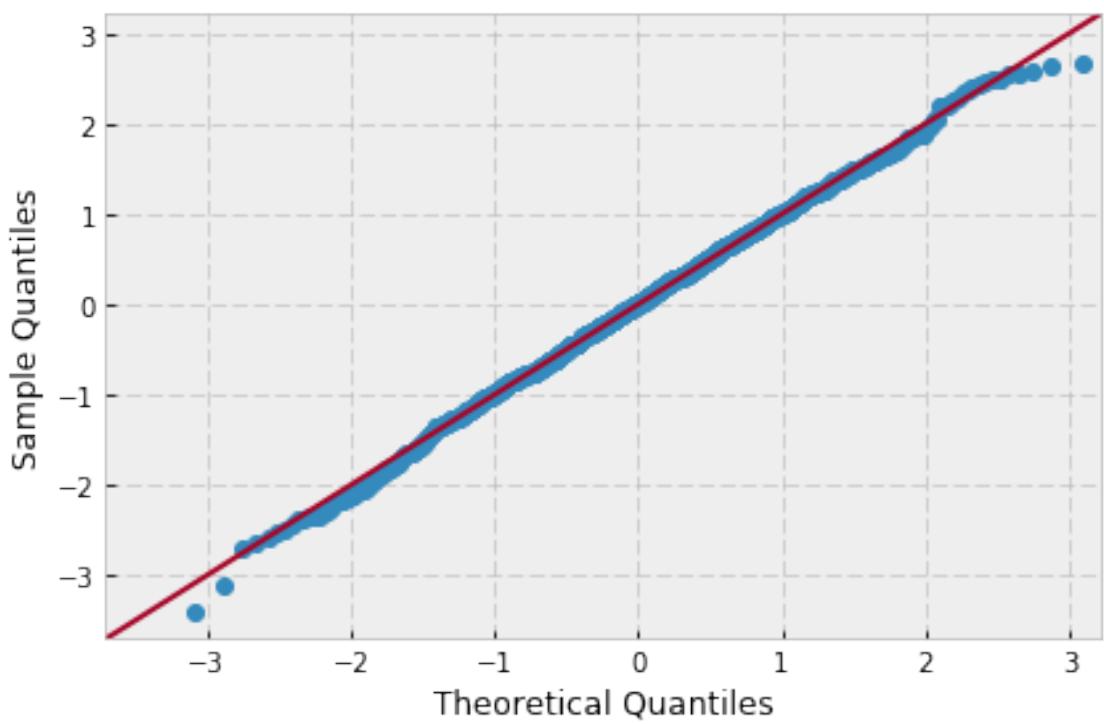
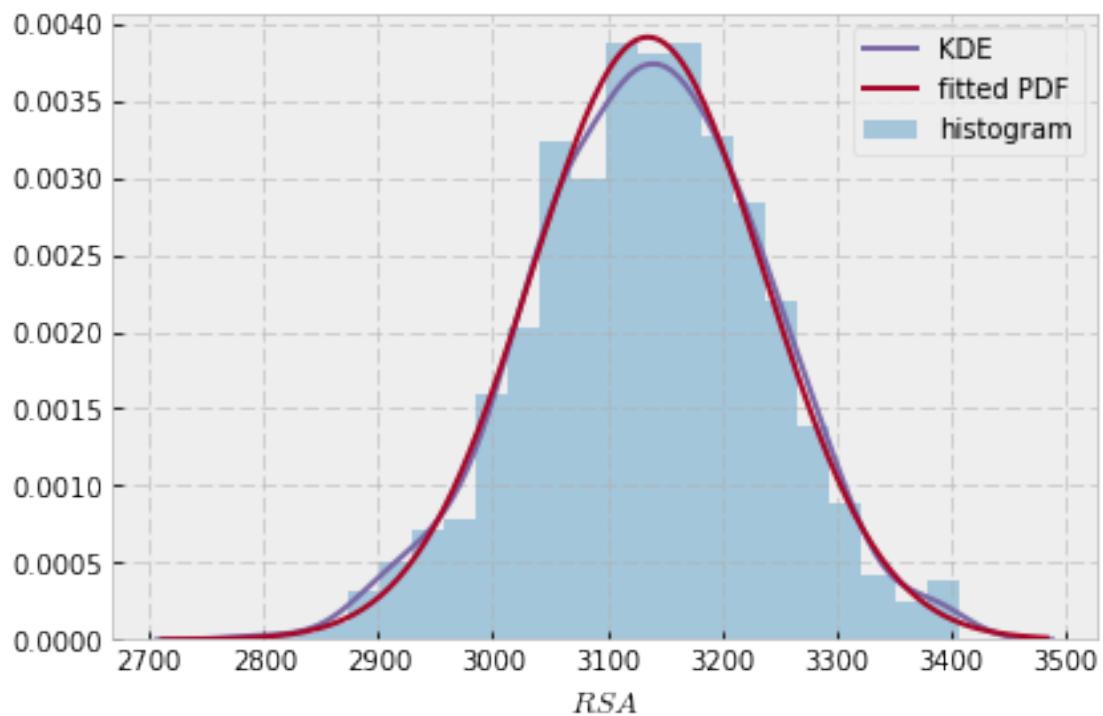


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(), loc=3132.474594838, scale=103.20292174979019)
DescribeResult(nobs=1000, minmax=(2753.552737, 3443.590436999998)
               mean=3132.474594838, variance=10661.504562255574,
               skewness=-0.15332866392599062, kurtosis=0.30317313502586574)
```

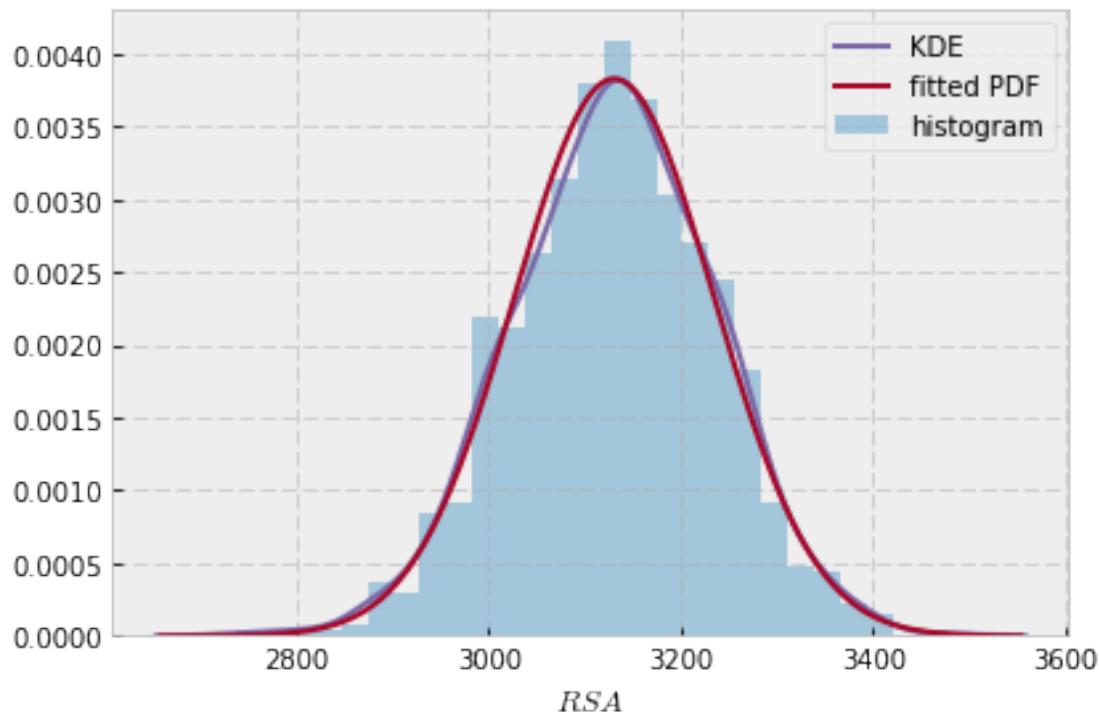


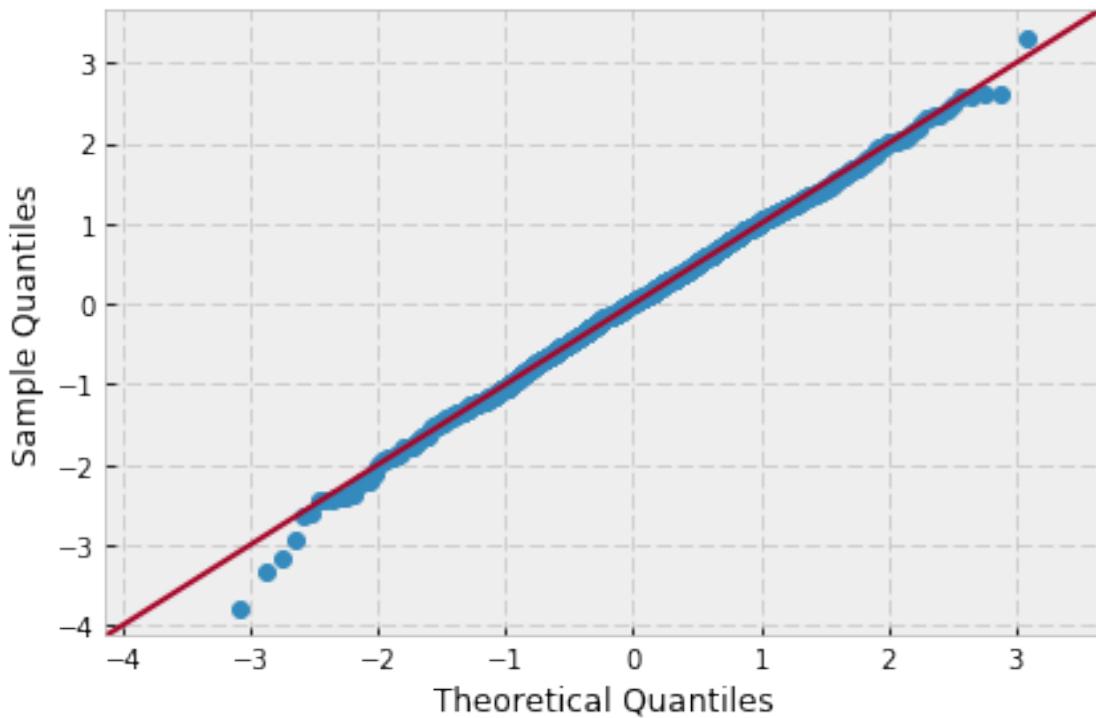


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(),loc=3135.335721614, scale=101.98304423576943)
DescribeResult(nobs=1000, minmax=(2788.4941269999999, 3406.8294179999998)
               mean=3135.3357216139998, variance=10410.952263858764,
               skewness=-0.09346452339326895, kurtosis=-0.061822405569606165)
```

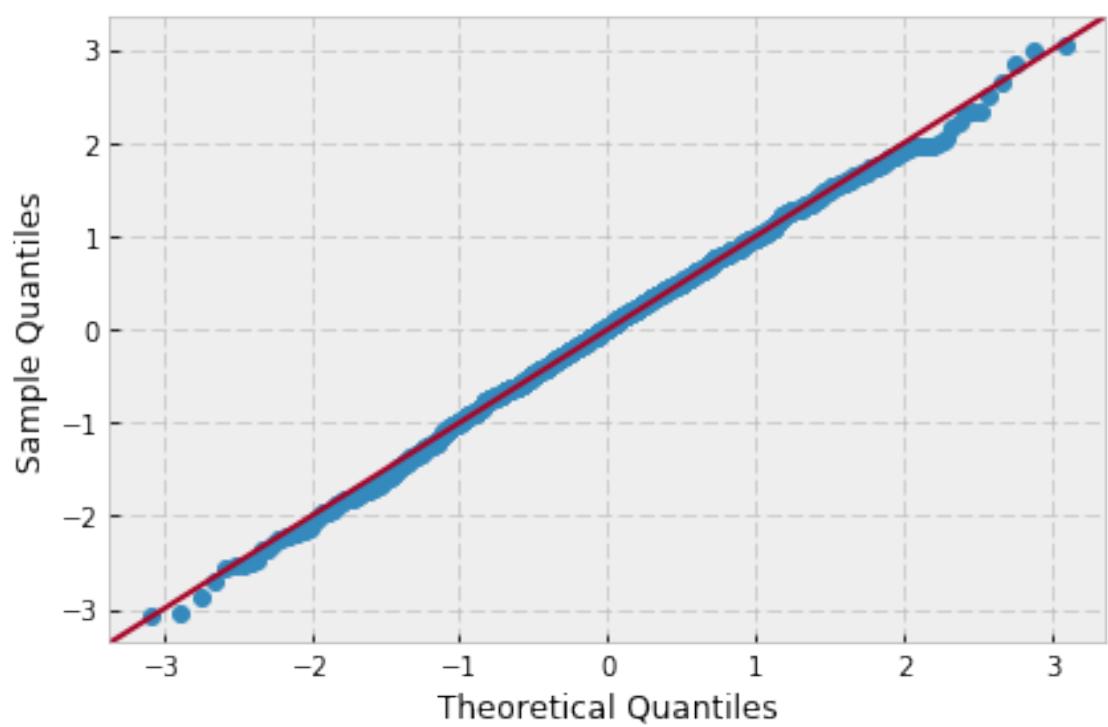
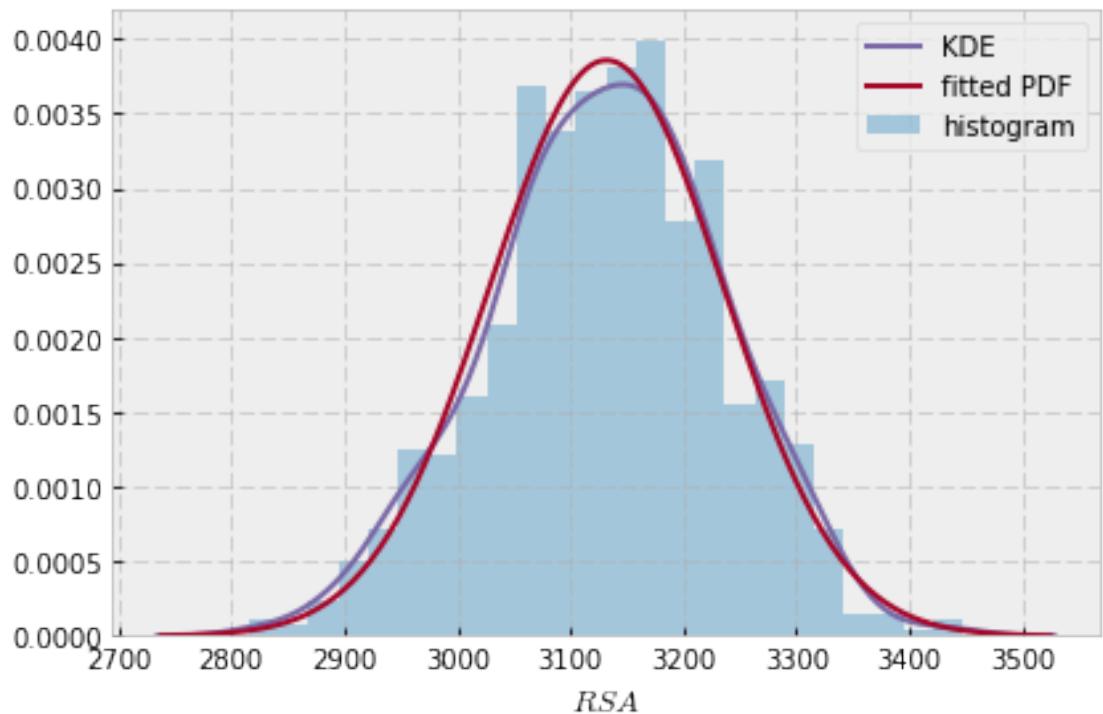


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(),loc=3130.6240909289995, scale=104.10676482653479)
DescribeResult(nobs=1000, minmax=(2737.131617, 3474.753212000001)
               mean=3130.6240909289995, variance=10849.067550197618,
               skewness=-0.11598763273079538, kurtosis=0.08916224845270992)
```

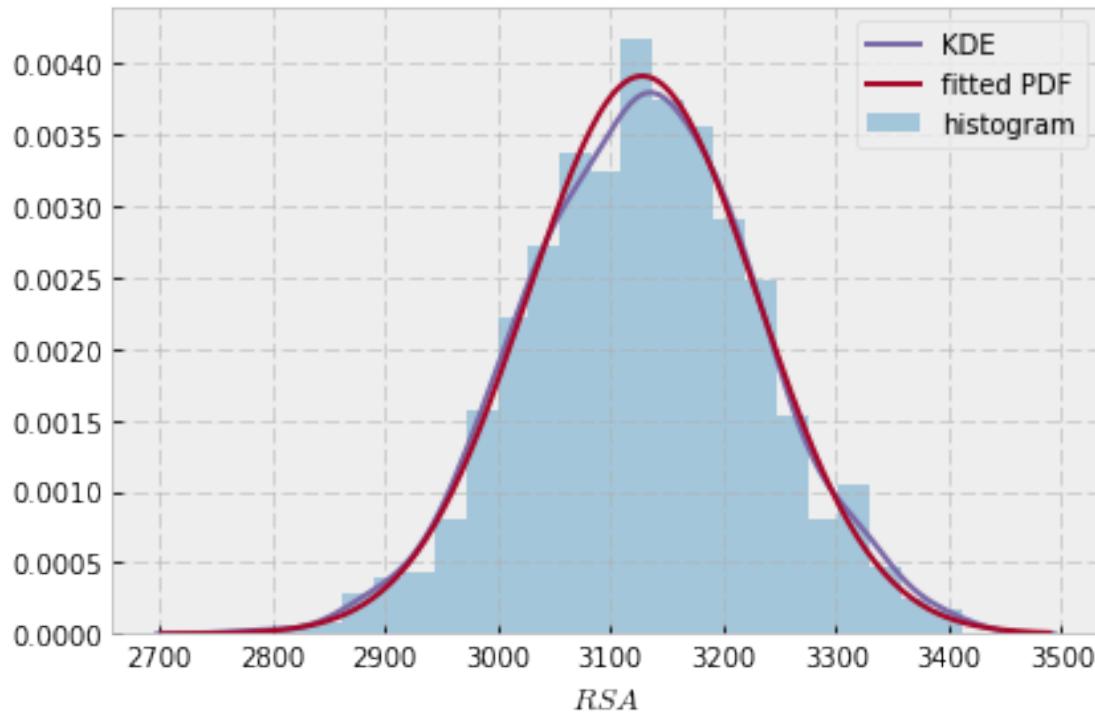


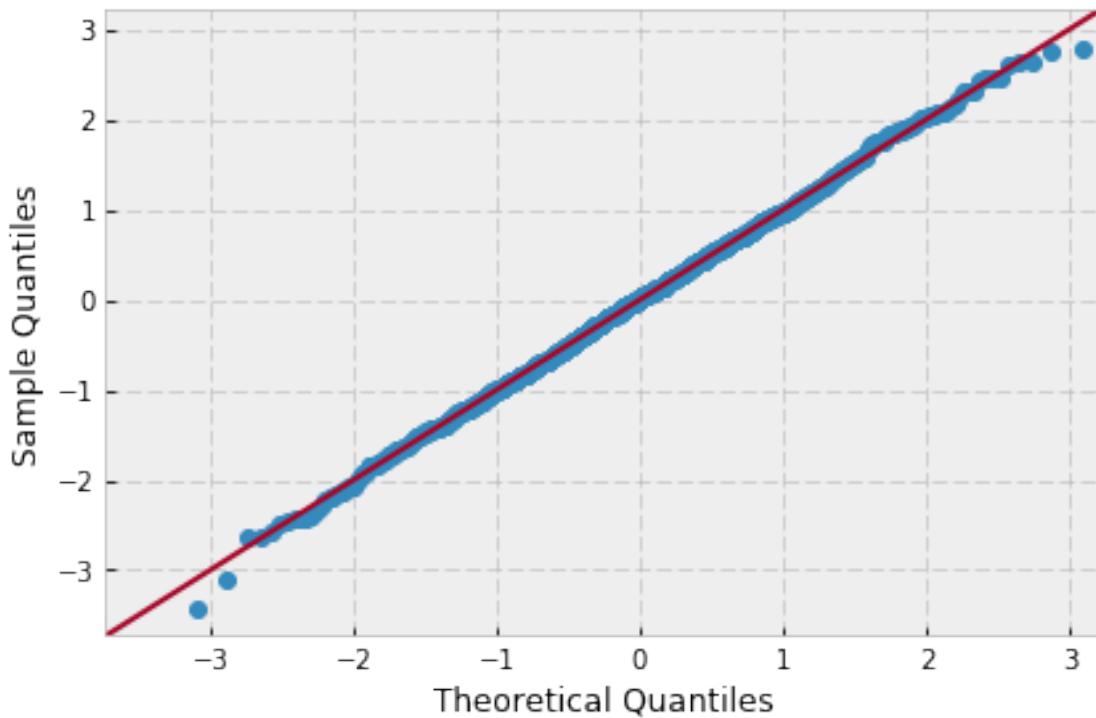


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=3131.941092627, scale=103.43043559279894)
DescribeResult(nobs=1000, minmax=(2815.853119999998, 3446.803296)
               mean=3131.9410926270002, variance=10708.563570486616,
               skewness=-0.12002354255325746, kurtosis=-0.11017003273435932)
```

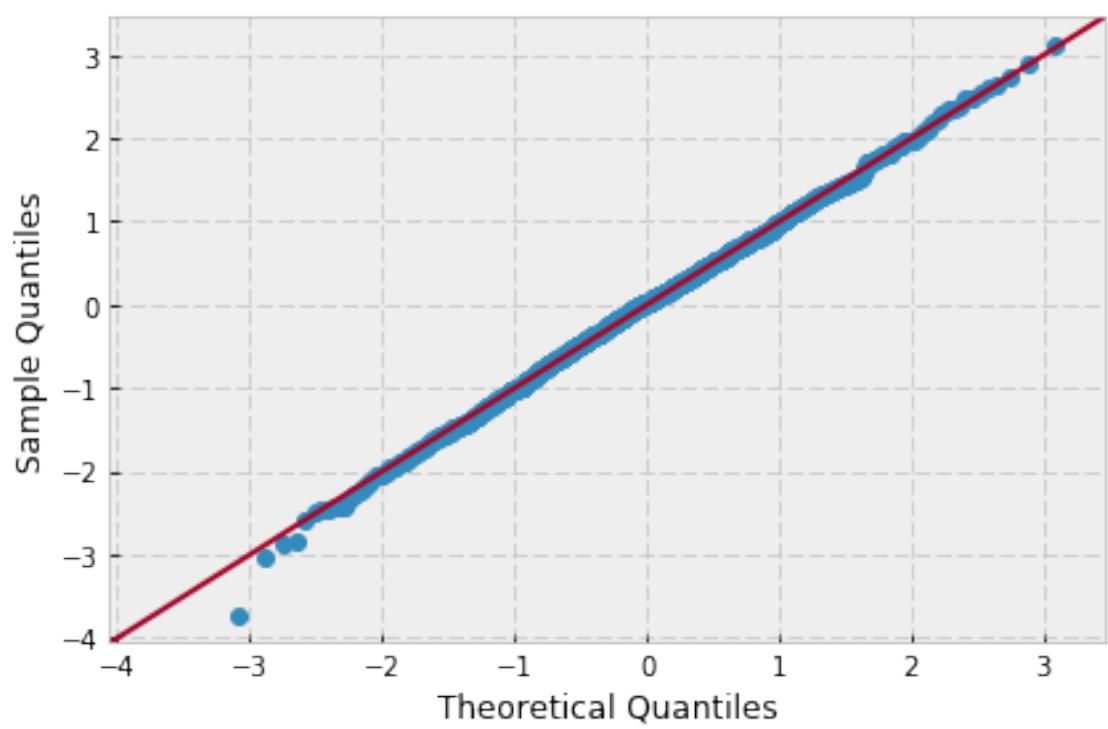
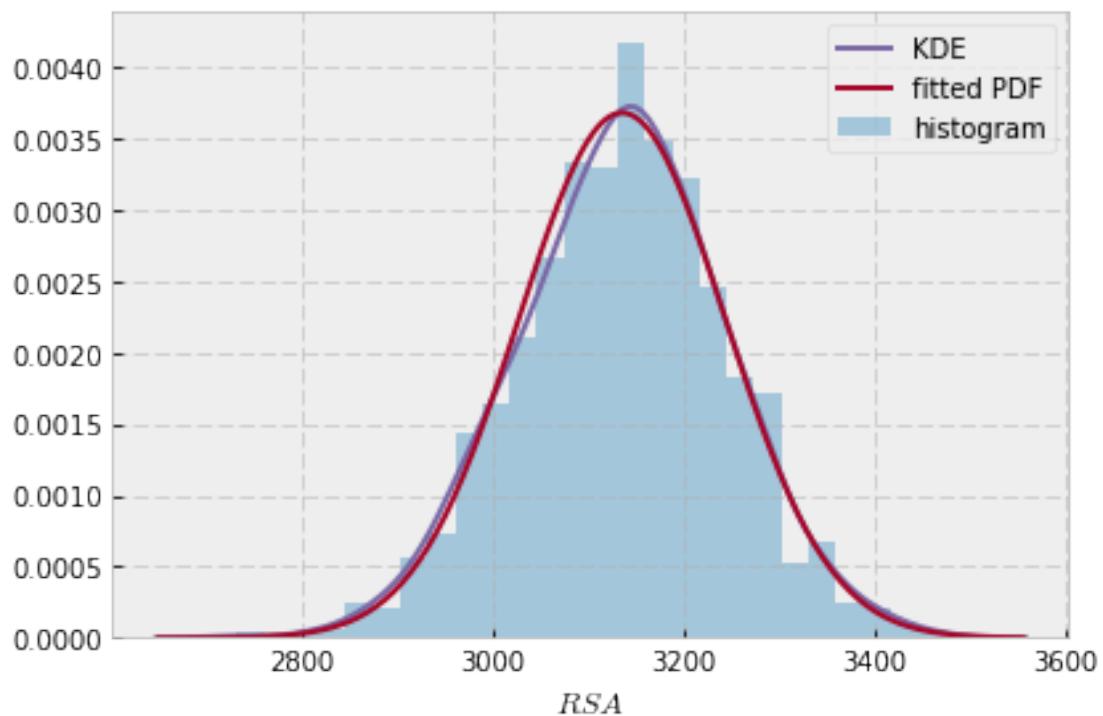


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=3128.374348625, scale=101.9010432695996)
DescribeResult(nobs=1000, minmax=(2779.8397679999998, 3412.170098999999)
               mean=3128.374348625, variance=10394.216836269079,
               skewness=-0.01824470972938809, kurtosis=-0.07458069159333602)
```

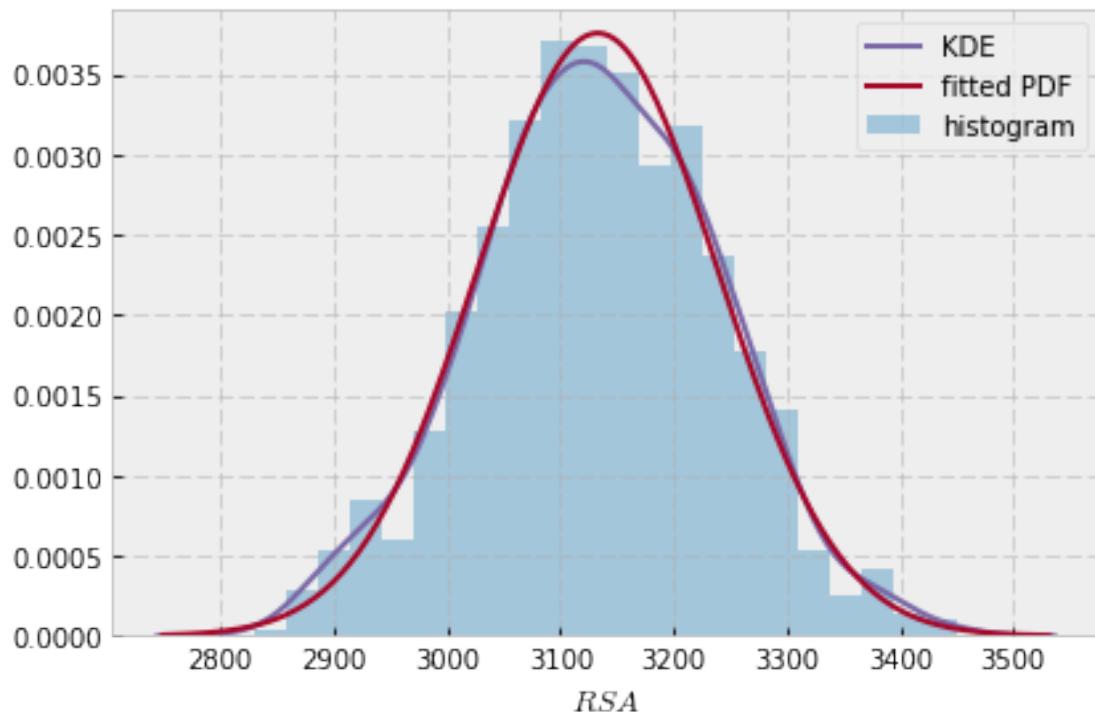


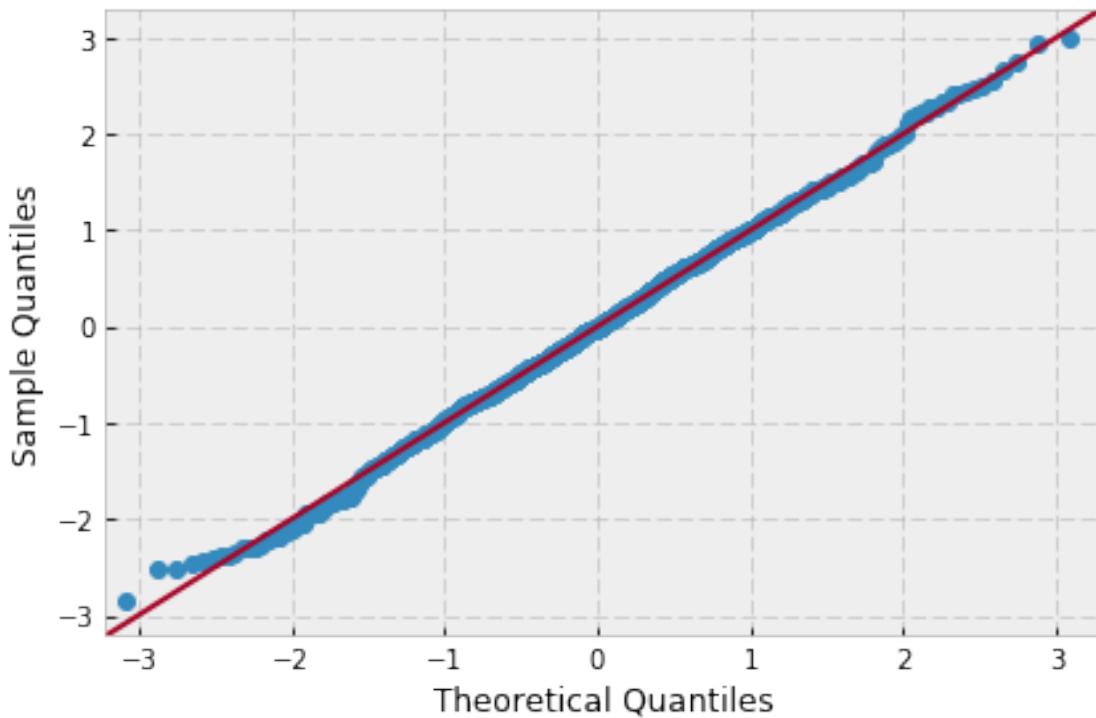


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(),loc=3135.771148513, scale=108.41537302195535)
DescribeResult(nobs=1000, minmax=(2732.7028100000002, 3473.413826999999)
               mean=3135.7711485129998, variance=11765.65876625598,
               skewness=-0.07790252261790732, kurtosis=0.061350853811289685)
```

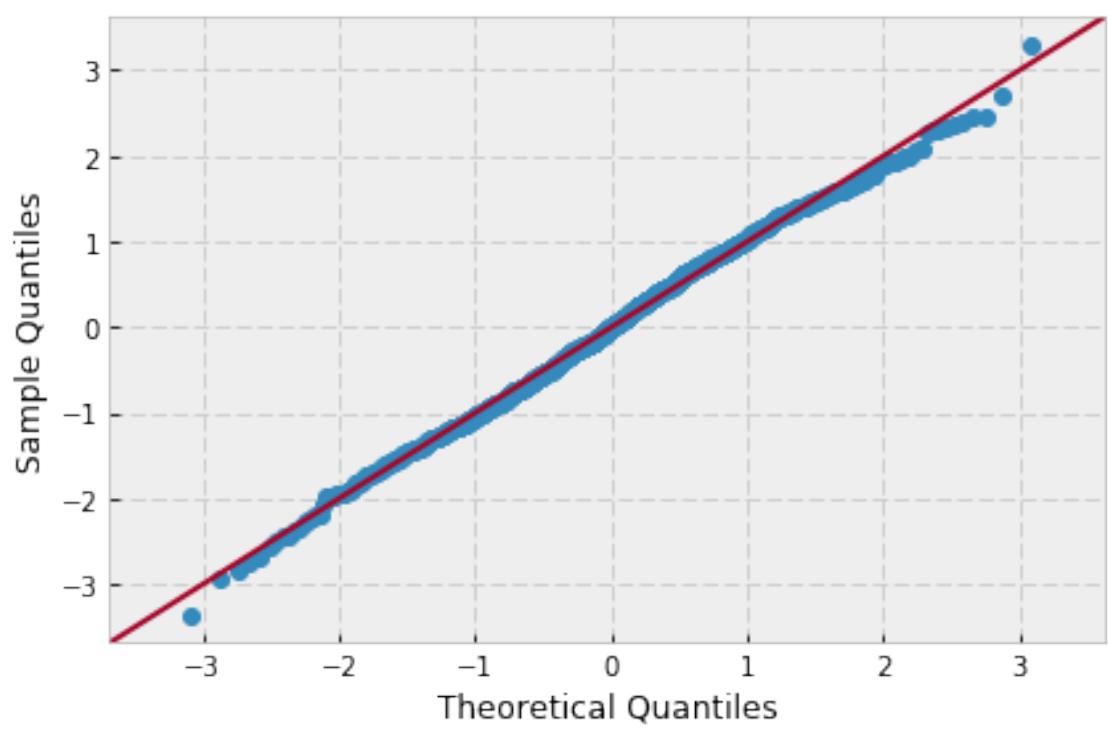
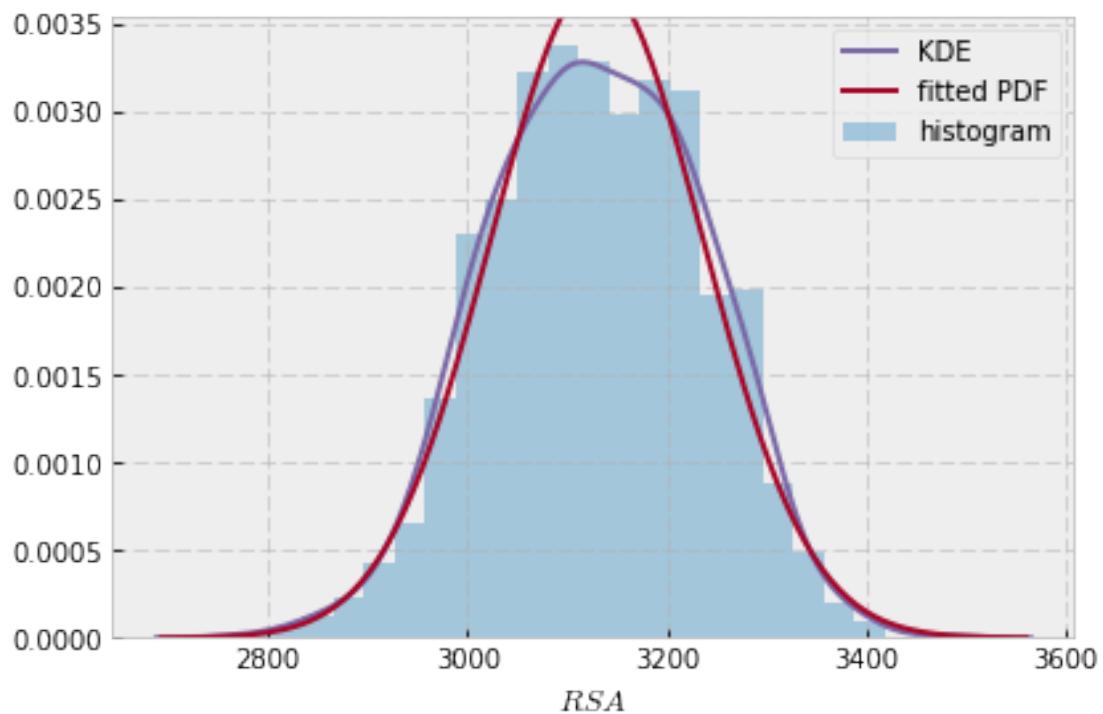


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=3132.9198656630006, scale=106.23586816174215)
DescribeResult(nobs=1000, minmax=(2829.1171639999998, 3451.1239770000002)
               mean=3132.9198656630006, variance=11297.357041120178,
               skewness=-0.014891754496721398, kurtosis=-0.13728432423407444)
```

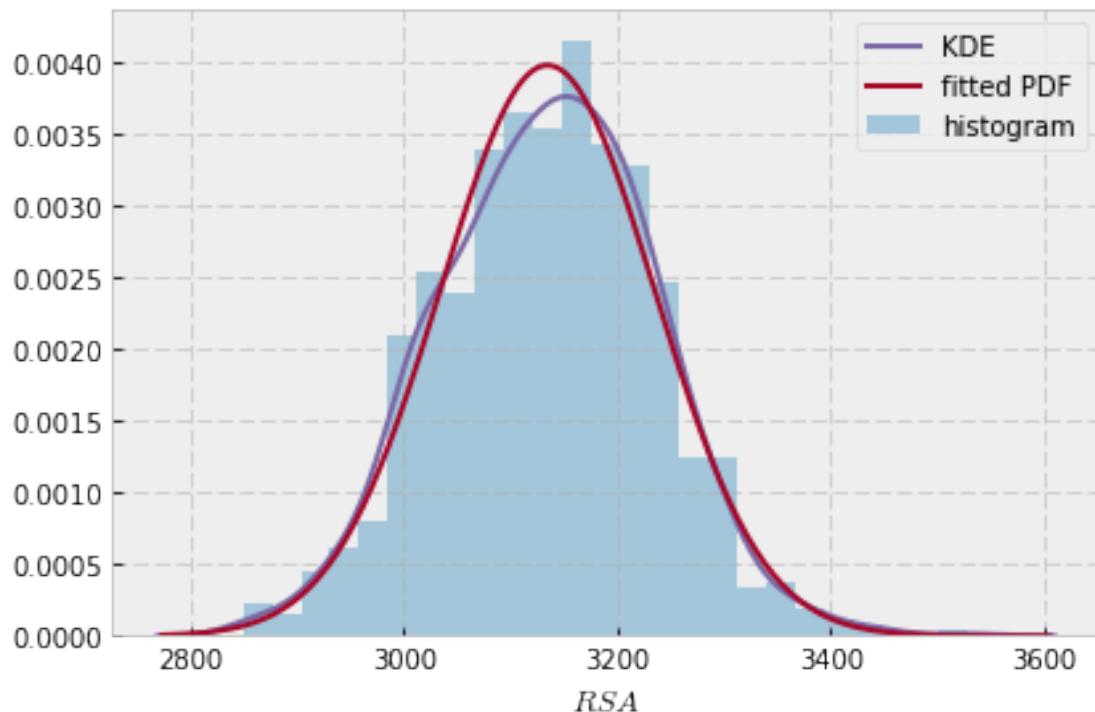


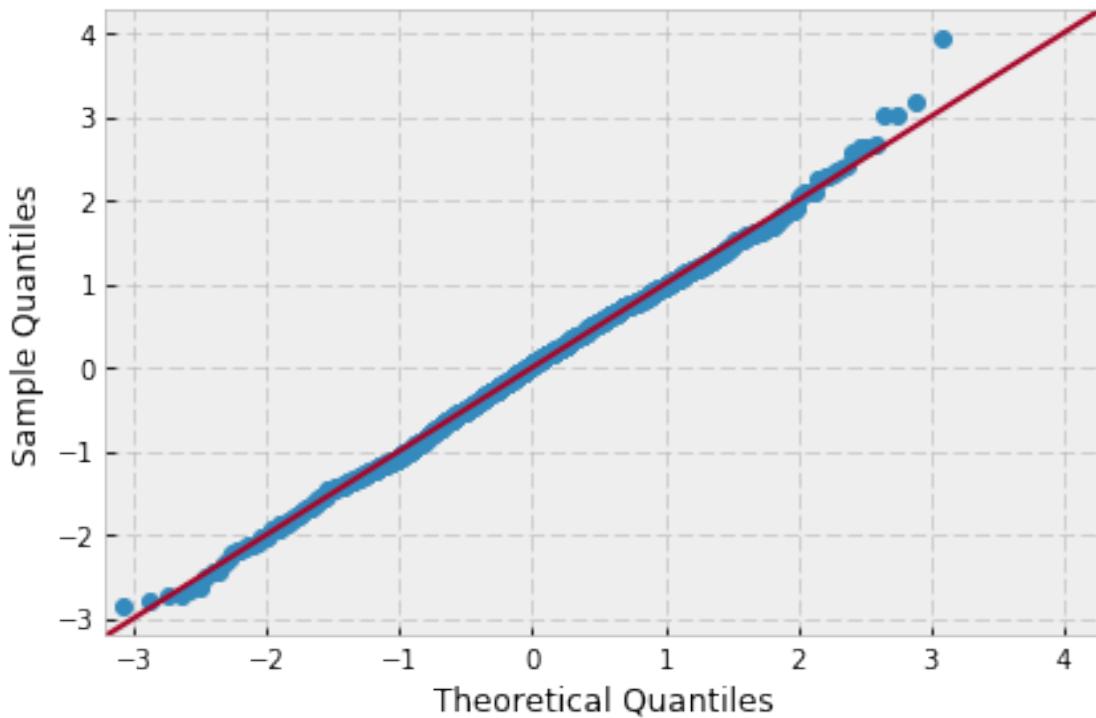


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(),loc=3129.806037346, scale=106.62148683207414)
DescribeResult(nobs=1000, minmax=(2772.3326480000001, 3480.5504550000001)
               mean=3129.8060373459998, variance=11379.520975257416,
               skewness=-0.06651873952300266, kurtosis=-0.28730683229675735)
```

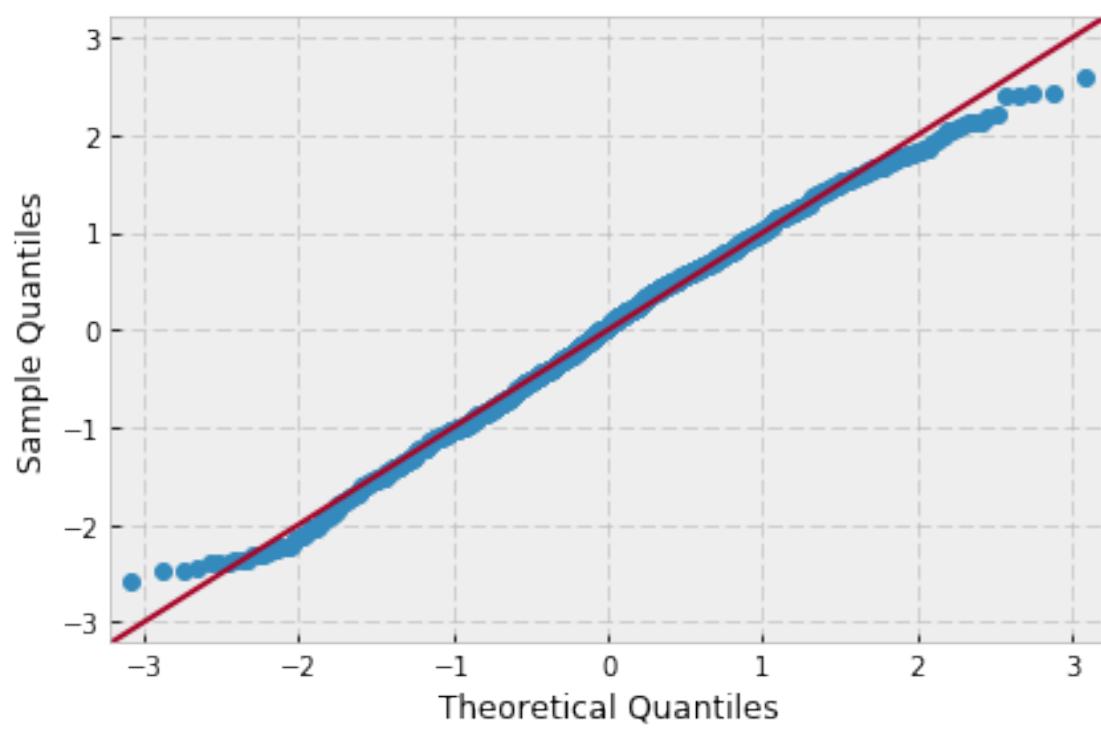
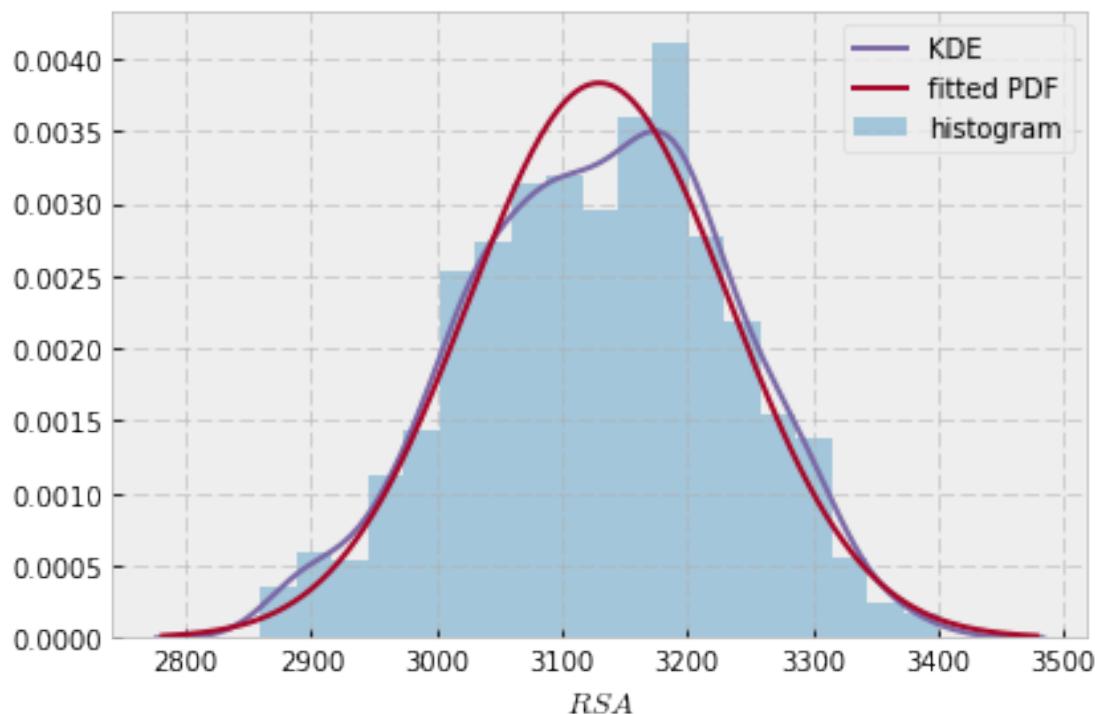


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=3134.4767507969996, scale=100.31516207703177)
DescribeResult(nobs=1000, minmax=(2849.116935, 3529.180042)
               mean=3134.4767507969996, variance=10073.204947488641,
               skewness=0.018713248503409278, kurtosis=0.05257816959205375)
```

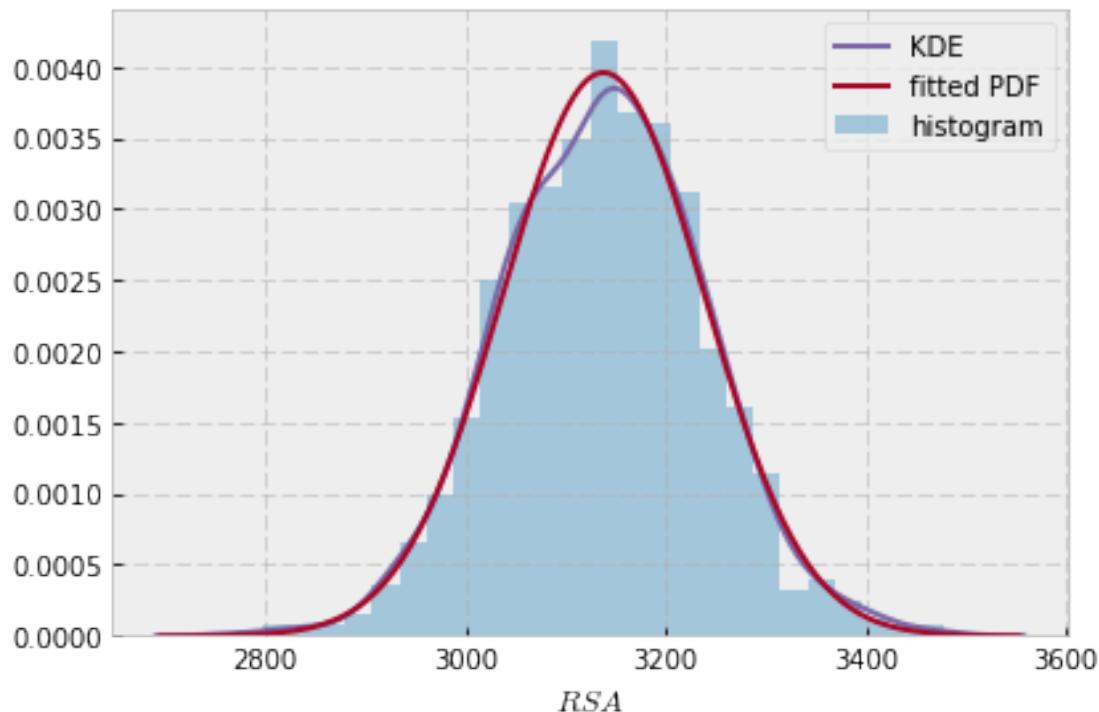


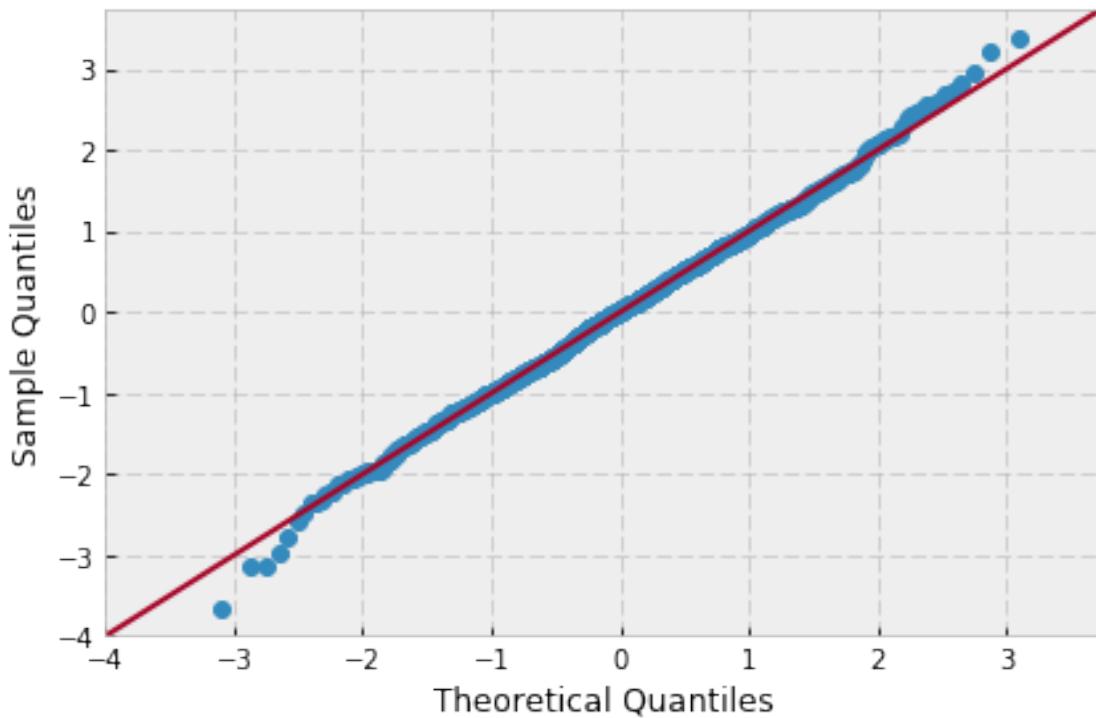


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(),loc=3129.619333926, scale=104.13567843400001)
DescribeResult(nobs=1000, minmax=(2860.0776040000001, 3400.2378440000002)
               mean=3129.6193339259999, variance=10855.094617526982,
               skewness=-0.12636398754521208, kurtosis=-0.4371318854534154)
```

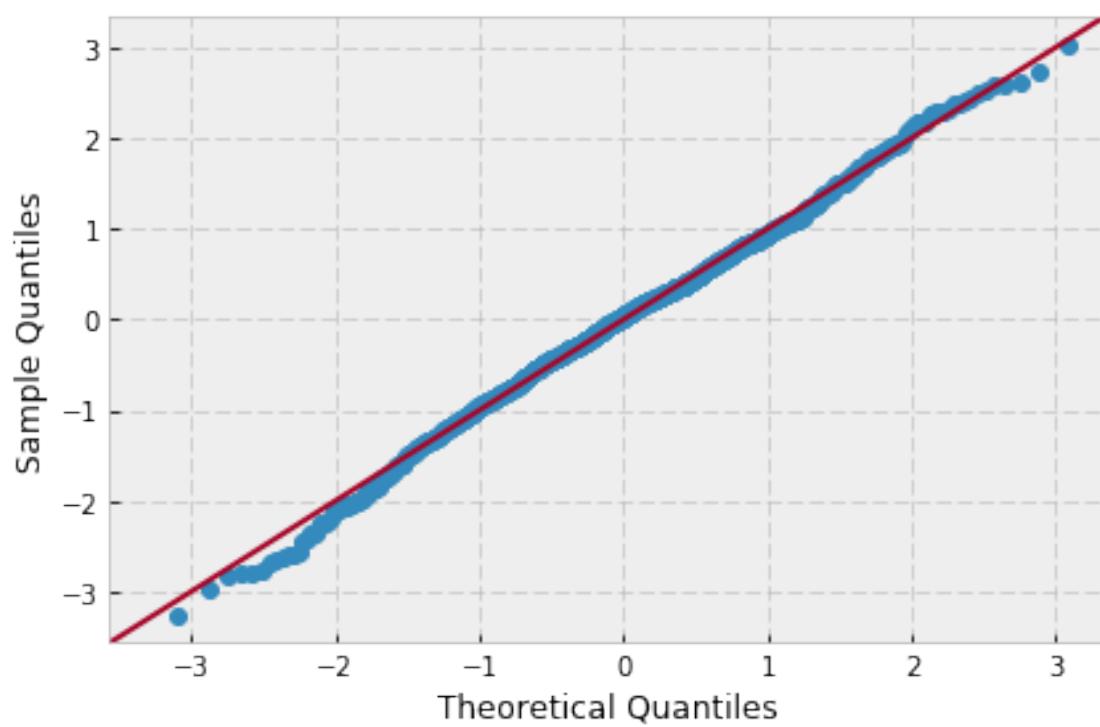
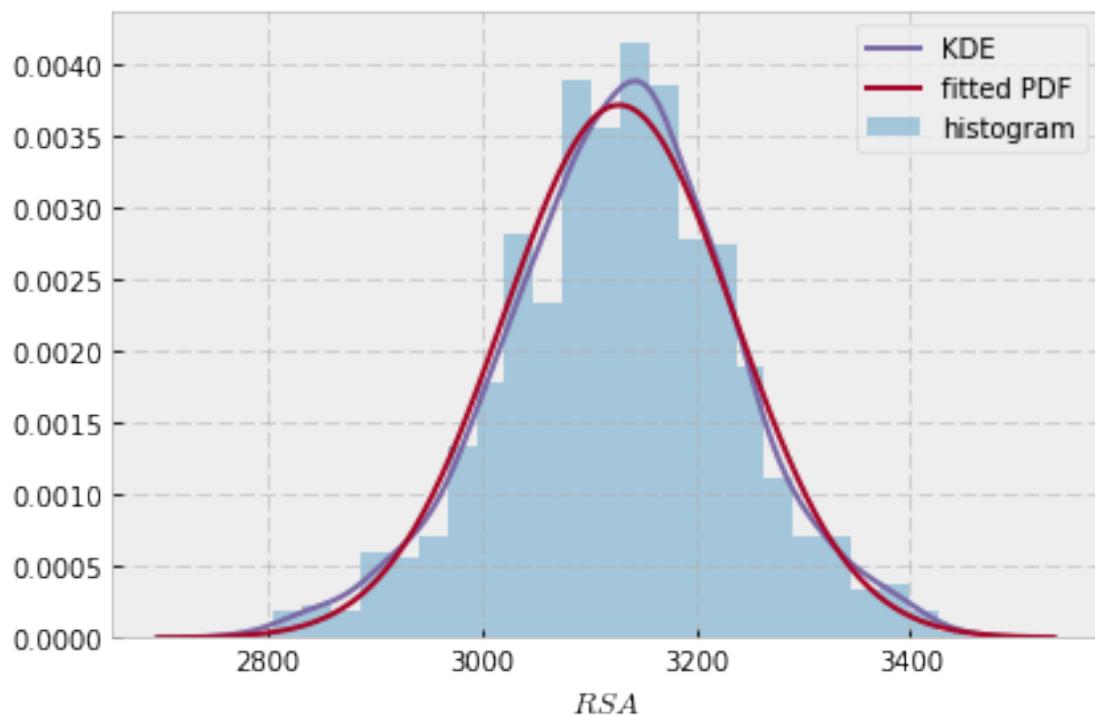


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=3137.7558778820003, scale=100.75007694386433)
DescribeResult(nobs=1000, minmax=(2770.4740190000002, 3477.819105)
               mean=3137.7558778820003, variance=10160.738742937521,
               skewness=0.0025182411502170728, kurtosis=0.1939114435931364)
```





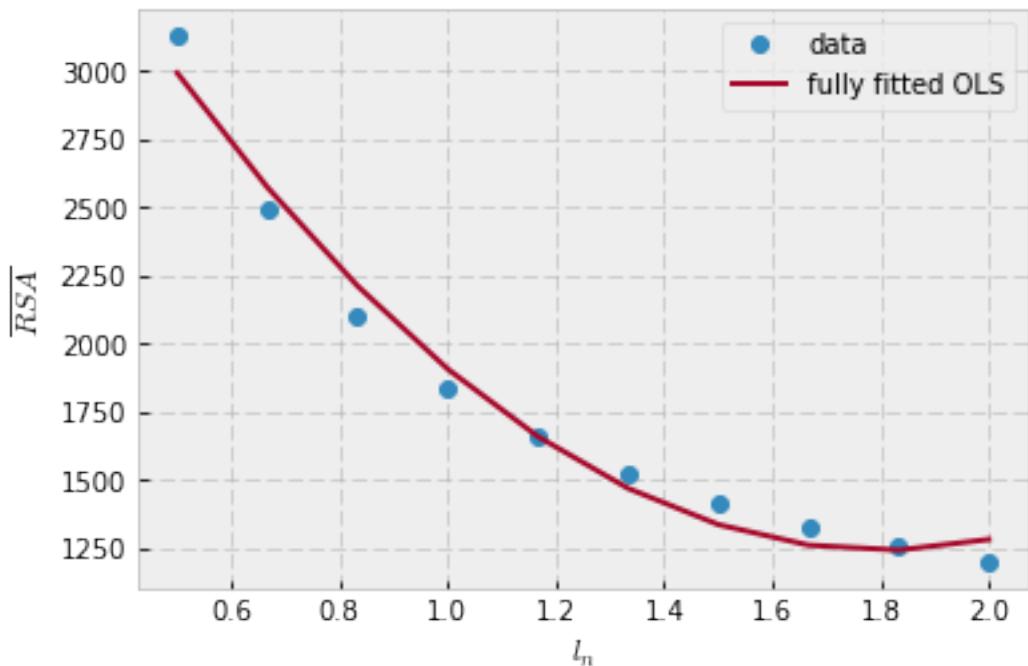
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=3128.128093959, scale=107.51595255469948)
DescribeResult(nobs=1000, minmax=(2778.378048999999, 3452.965385999999)
               mean=3128.1280939590001, variance=11571.25130504944,
               skewness=-0.10673511722115682, kurtosis=0.20392335432762598)
```



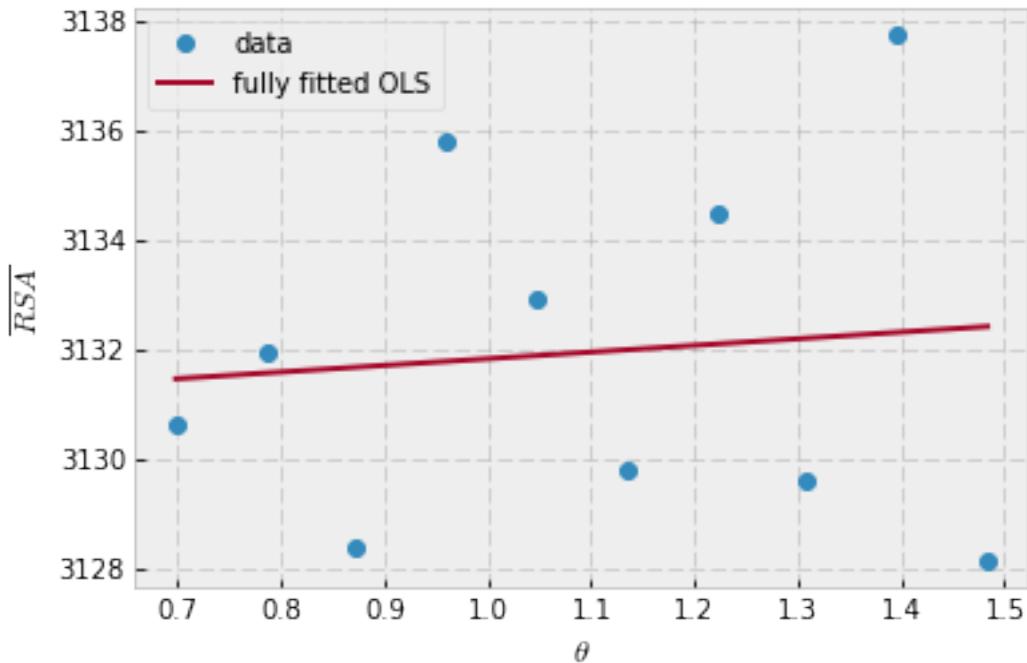
6.2 Parameter regression

```
In [18]: var_name = 'total_root_surface_area'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly1')  
        mean_reg('maxB', var_name, 'poly1')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly1')  
        mean_reg('N', var_name, 'poly1')
```

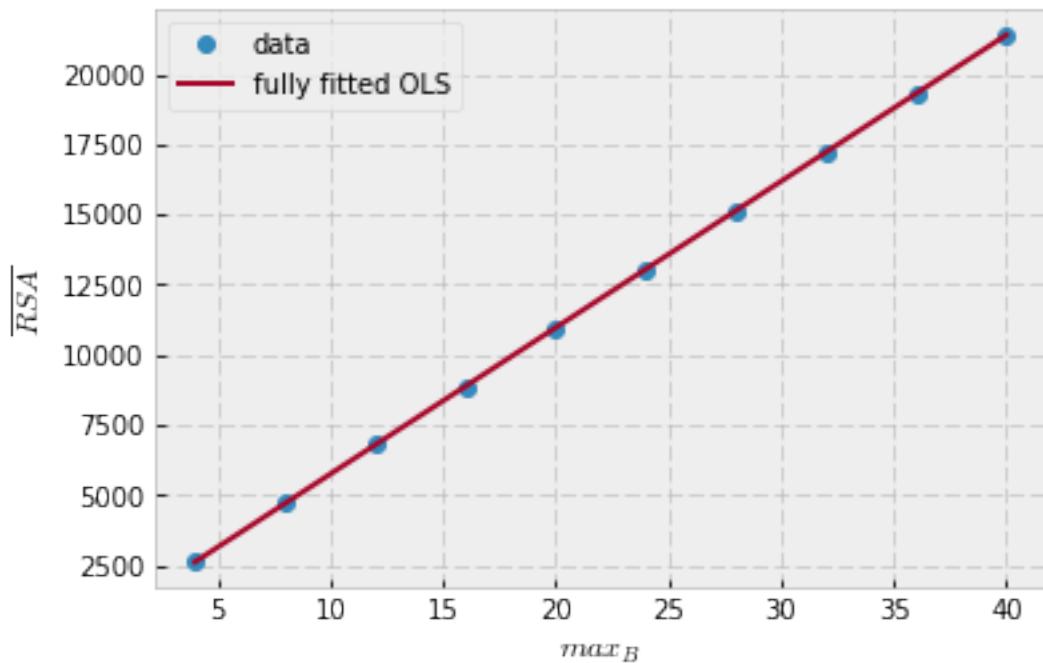
variable name: ln
model: poly2
r-squared: 0.9295032719231
explained variance: 0.9296302758409631



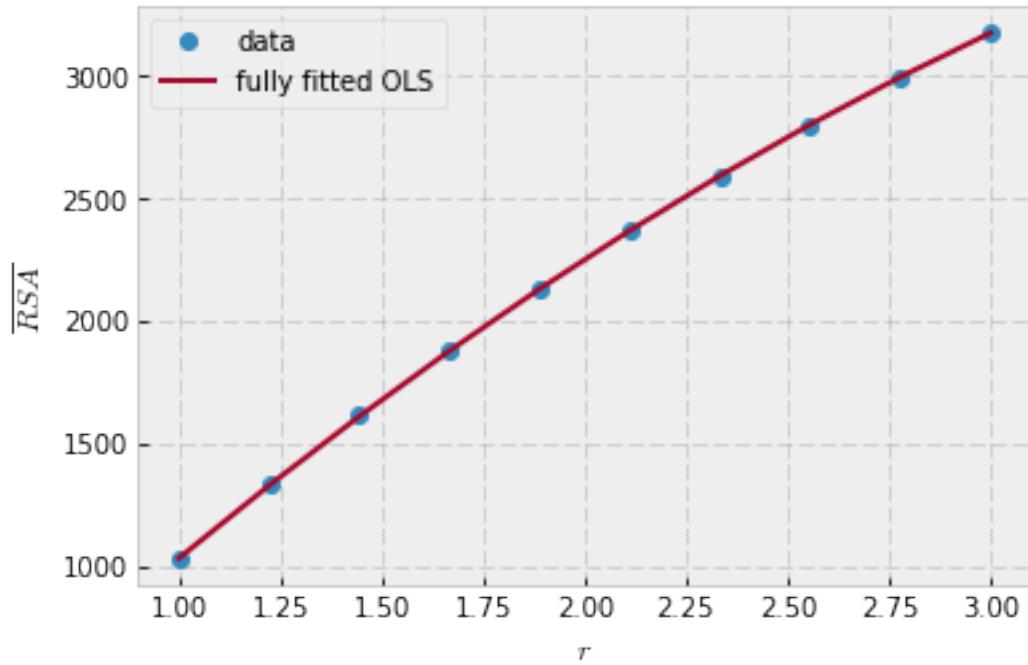
```
variable name: theta  
model: poly1  
r-squared: -0.6748710714199542  
explained variance: -0.673232271334455
```



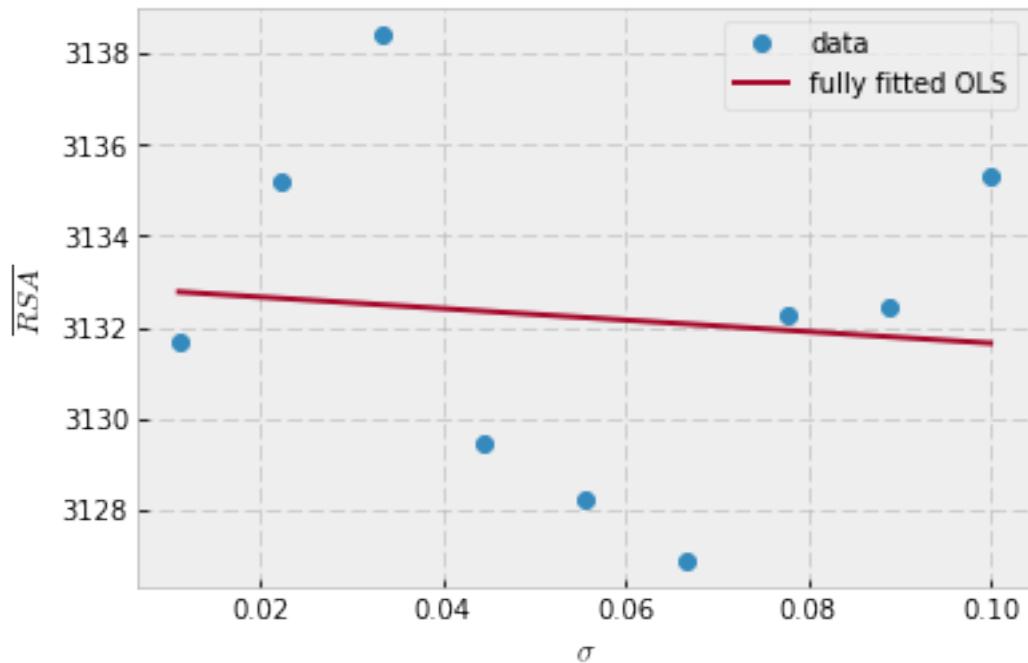
```
variable name: maxB
model: poly1
r-squared: 0.9999990874295286
explained variance: 0.9999990895408468
```



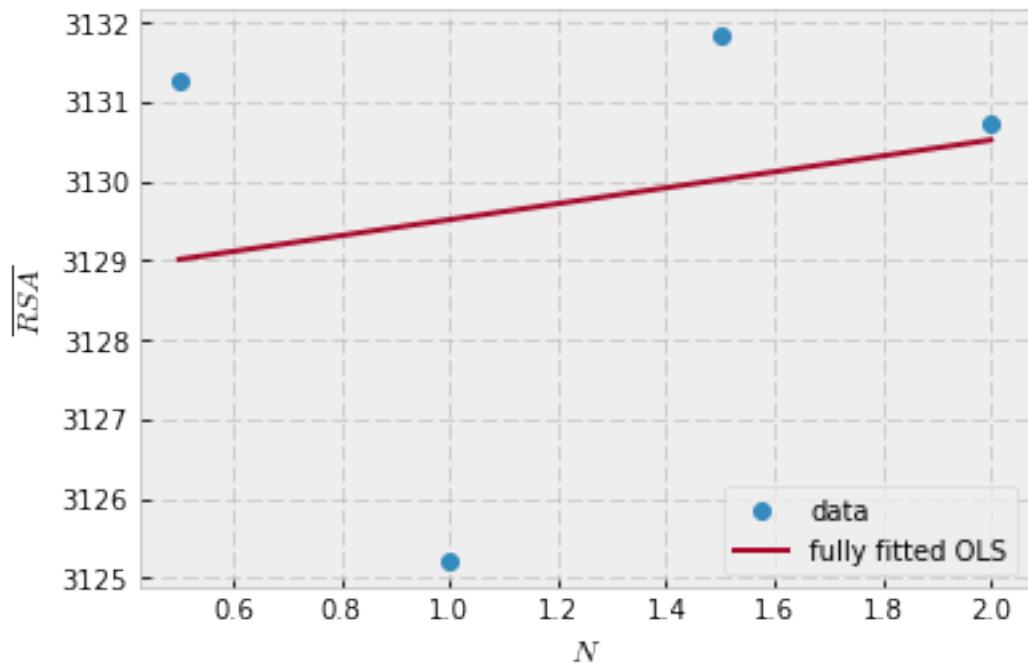
```
variable name: r
model: poly2
r-squared: 0.9998700274611575
explained variance: 0.9998701785865394
```



```
variable name: sigma
model: poly1
r-squared: -0.5504862995175284
explained variance: -0.5447578928753471
```



```
variable name: N
model: poly1
r-squared: -2.6068389717609137
explained variance: -2.409463526183814
```

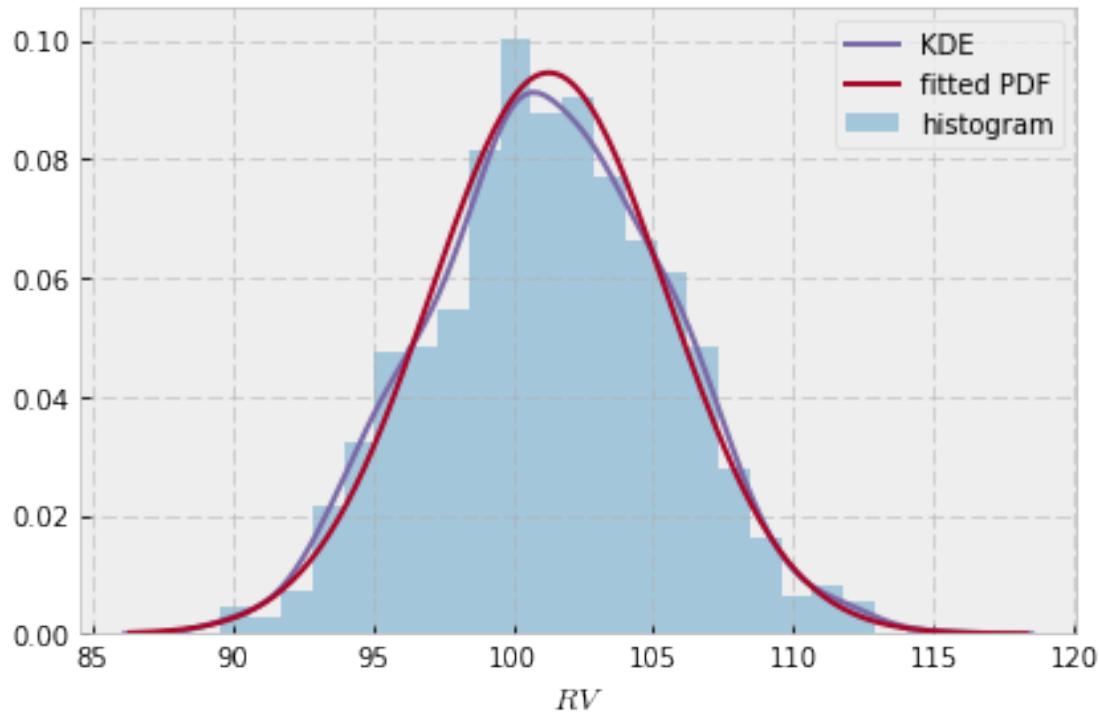


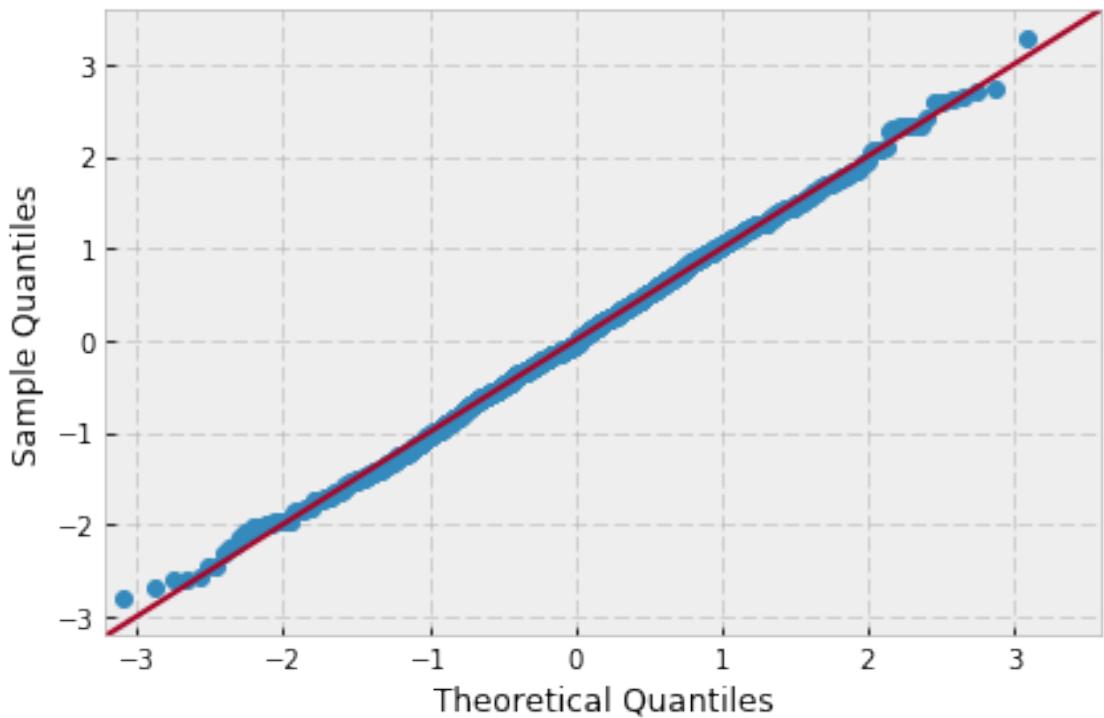
7 Total root volume

7.1 Probability distributions

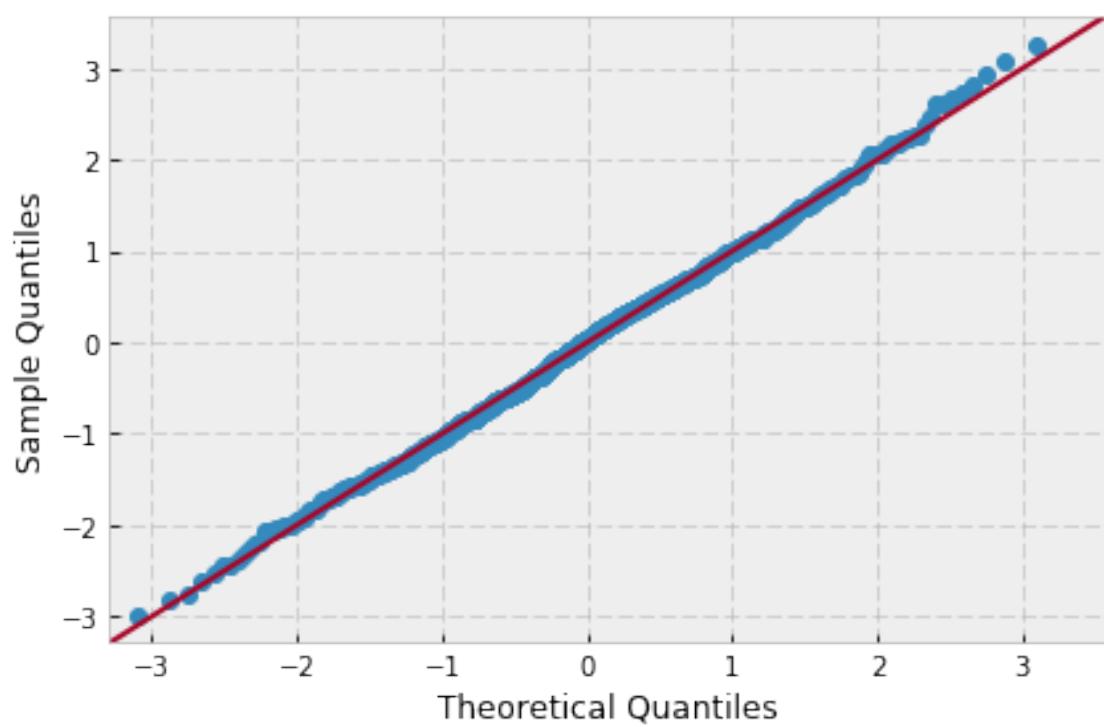
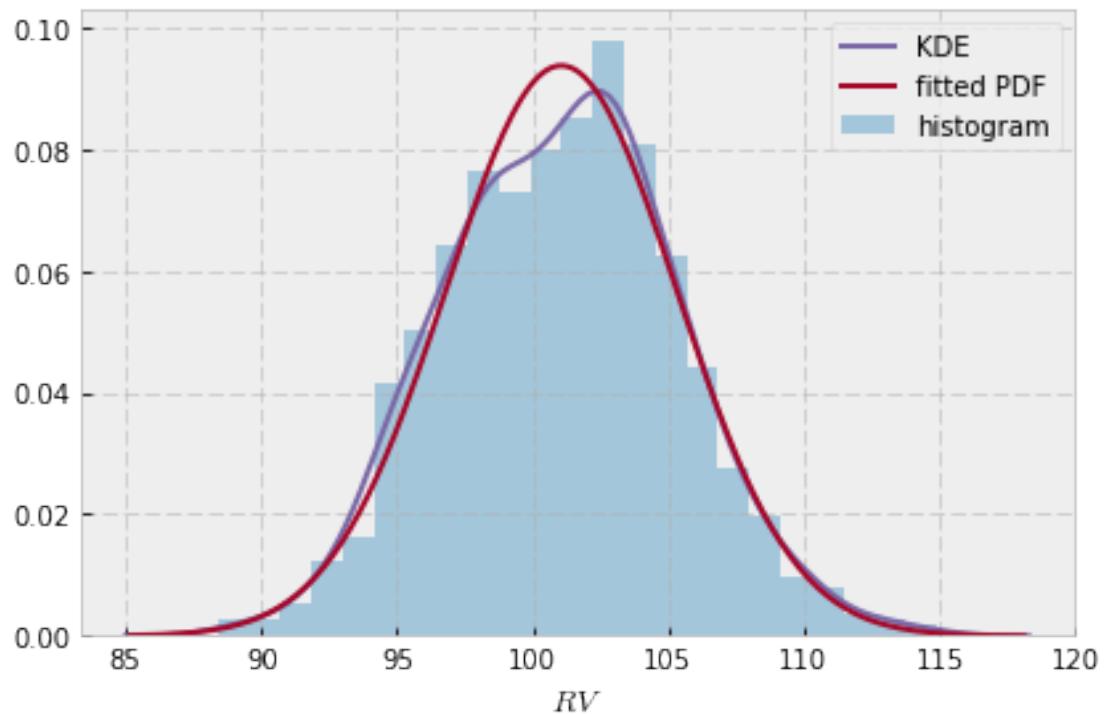
```
In [19]: plot_dist('total_root_volume')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=101.26882389756608, scale=4.219858325785858)
DescribeResult(nobs=1000, minmax=(89.487767541940372, 115.14035880159976)
               mean=101.26882389756608, variance=17.825029319023251,
               skewness=0.02672889812458017, kurtosis=-0.22150820637437807)
```

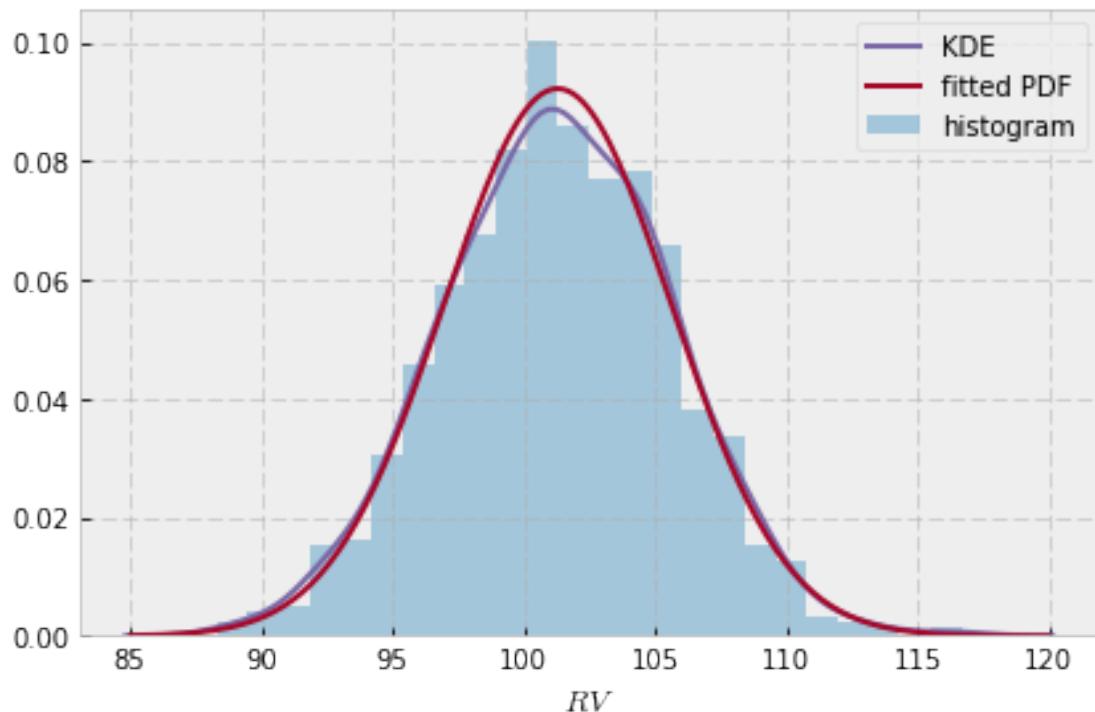


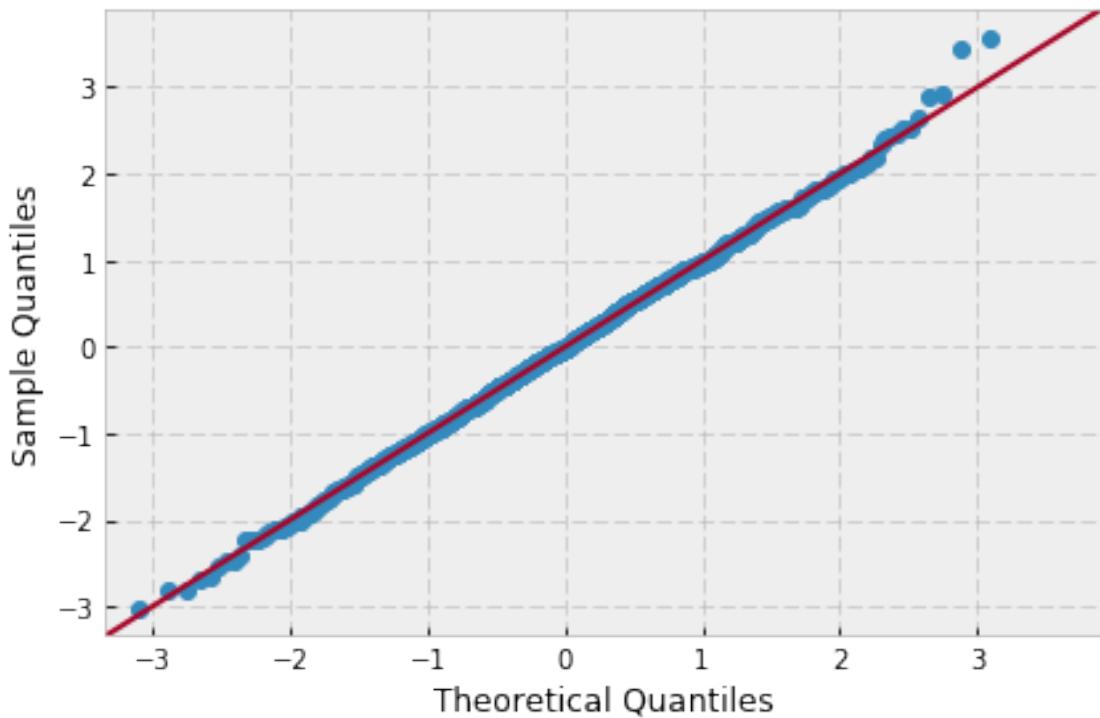


```
variable name: N
variable value: 1.0
distribution: normal(shape=(),loc=101.05618038869707, scale=4.2559158573617175)
DescribeResult(nobs=1000, minmax=(88.373667758270031, 114.89968798675888)
               mean=101.05618038869707, variance=18.130950735678603,
               skewness=0.05693329832595995, kurtosis=-0.11121314979498109)
```

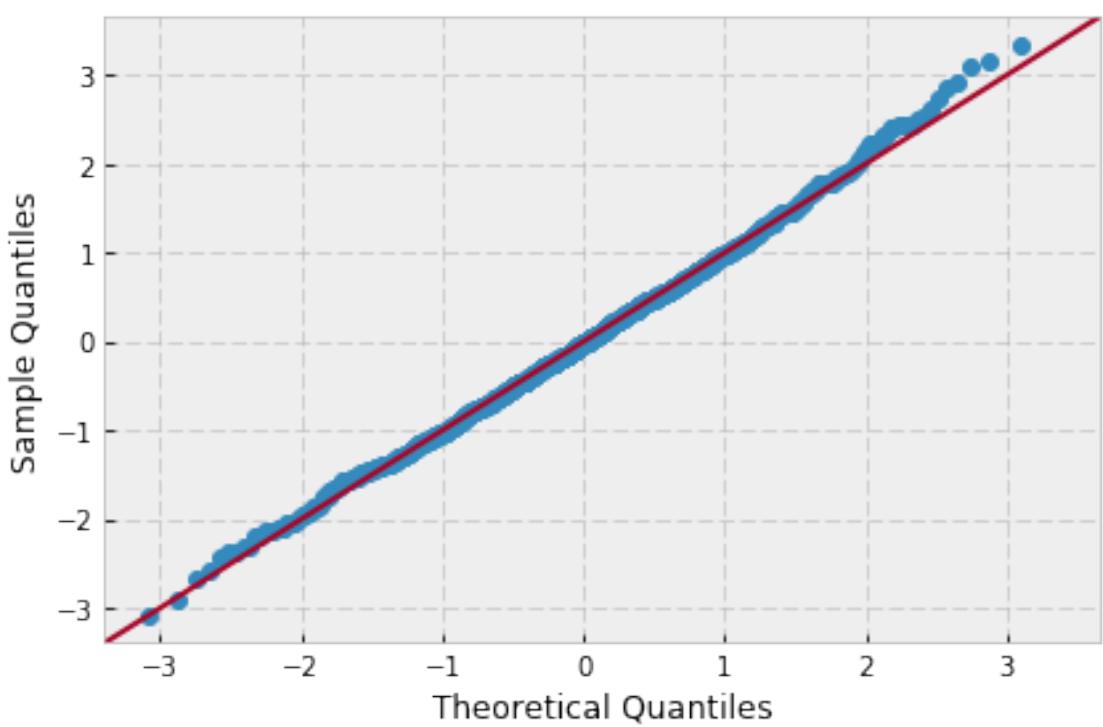
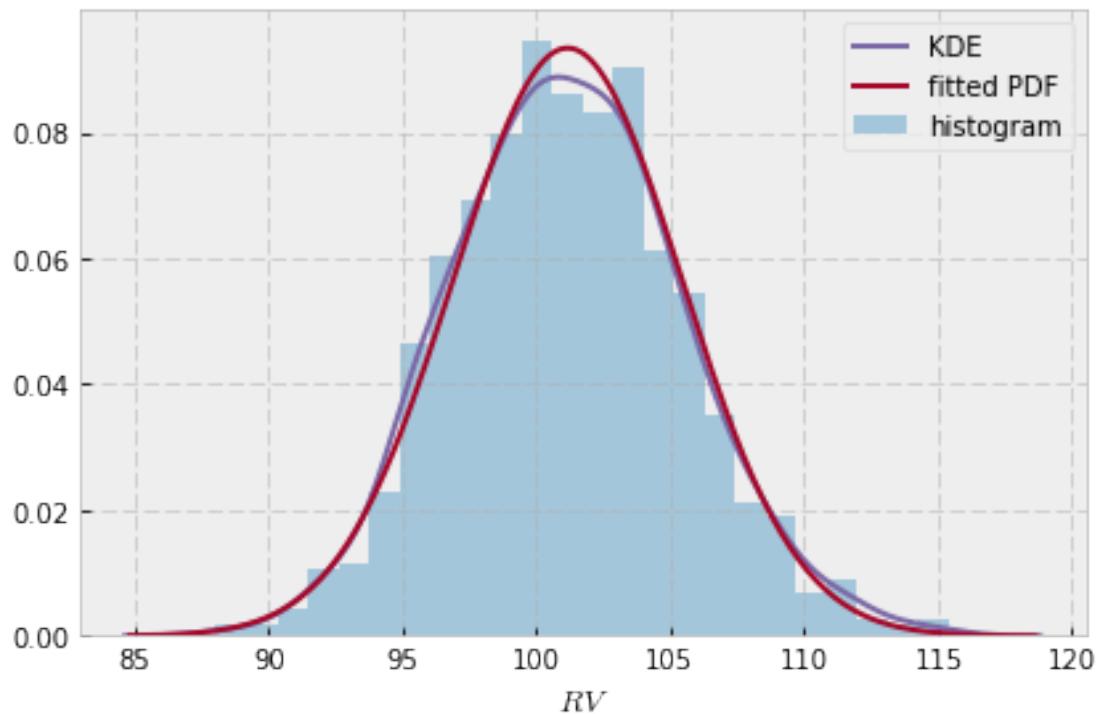


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=101.29189455885032, scale=4.325639741071396)
DescribeResult(nobs=1000, minmax=(88.274251596622463, 116.70402397771248)
               mean=101.29189455885032, variance=18.729889058594811,
               skewness=0.010518147178201457, kurtosis=0.016520041257692242)
```

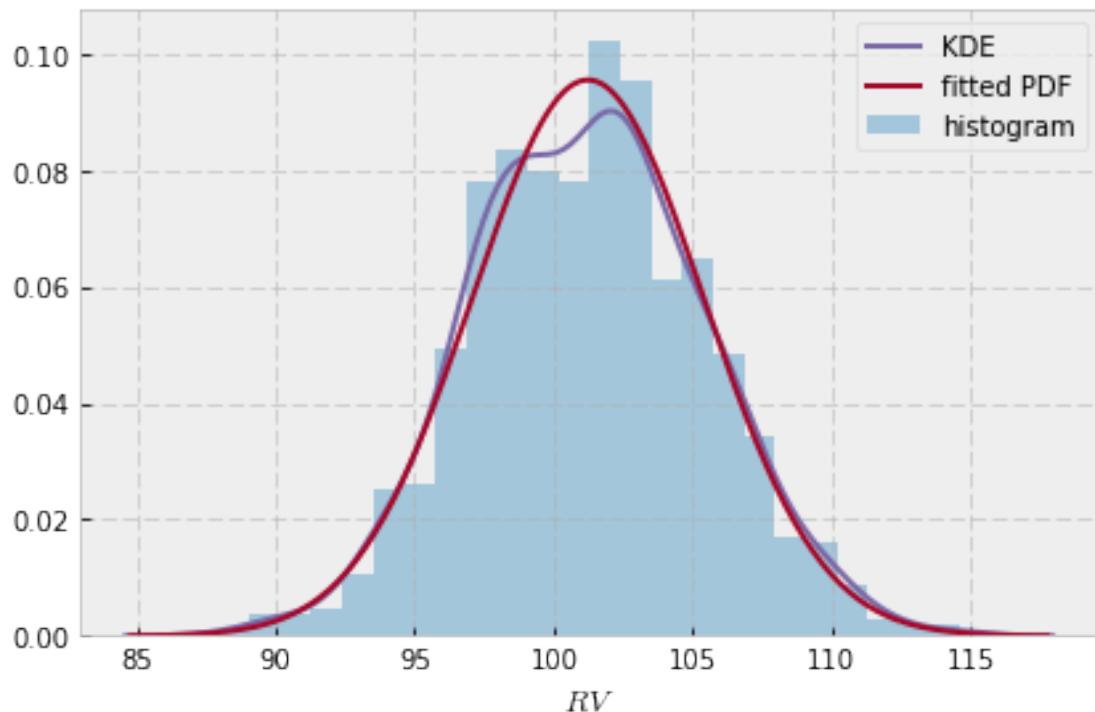


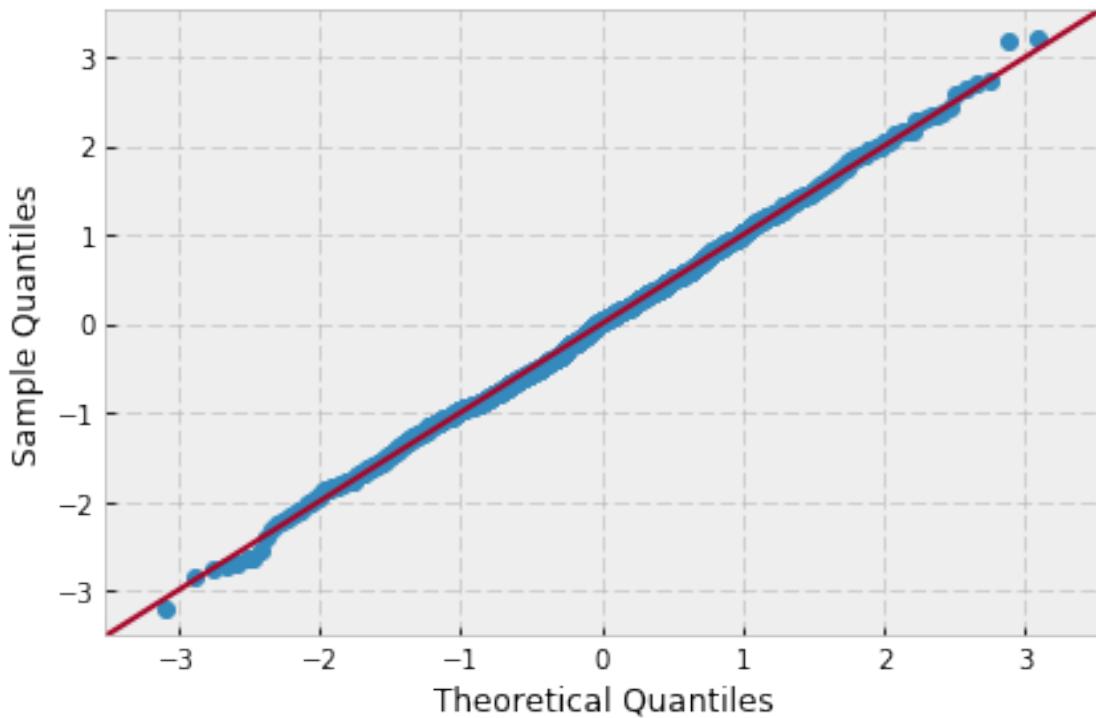


```
variable name: N
variable value: 2.0
distribution: normal(shape=(),loc=101.17013187249015, scale=4.268051129516685)
DescribeResult(nobs=1000, minmax=(88.051216980926327, 115.39809198977892)
               mean=101.17013187249015, variance=18.234494939107762,
               skewness=0.15397499680099977, kurtosis=0.03924819750492148)
```

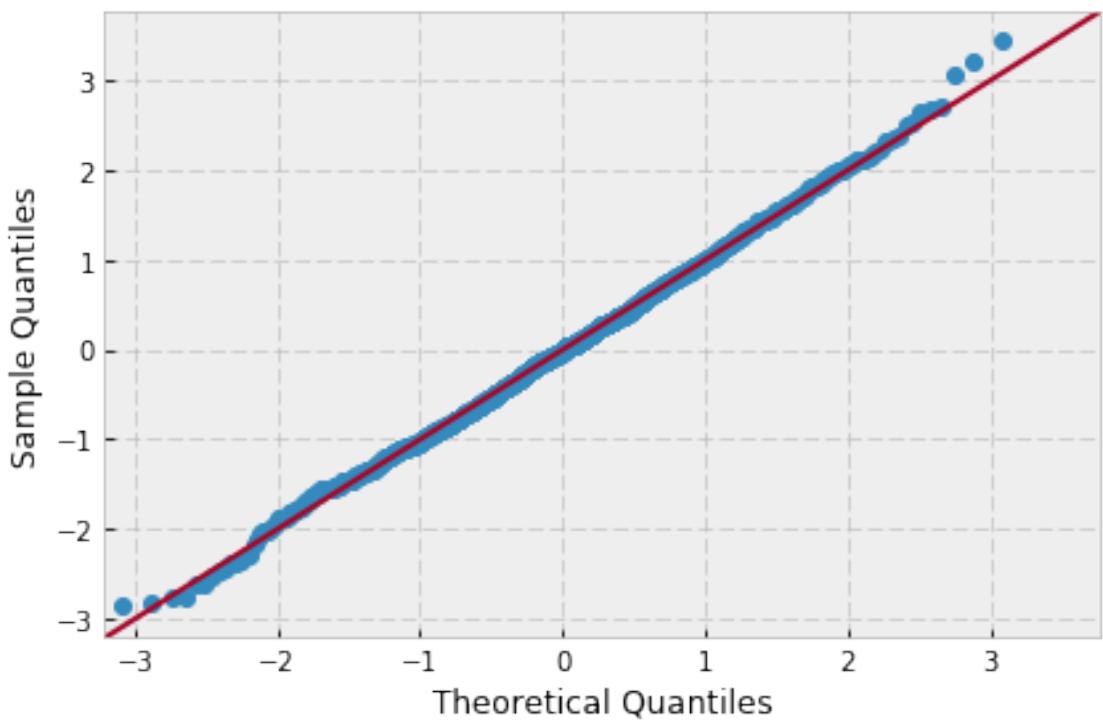
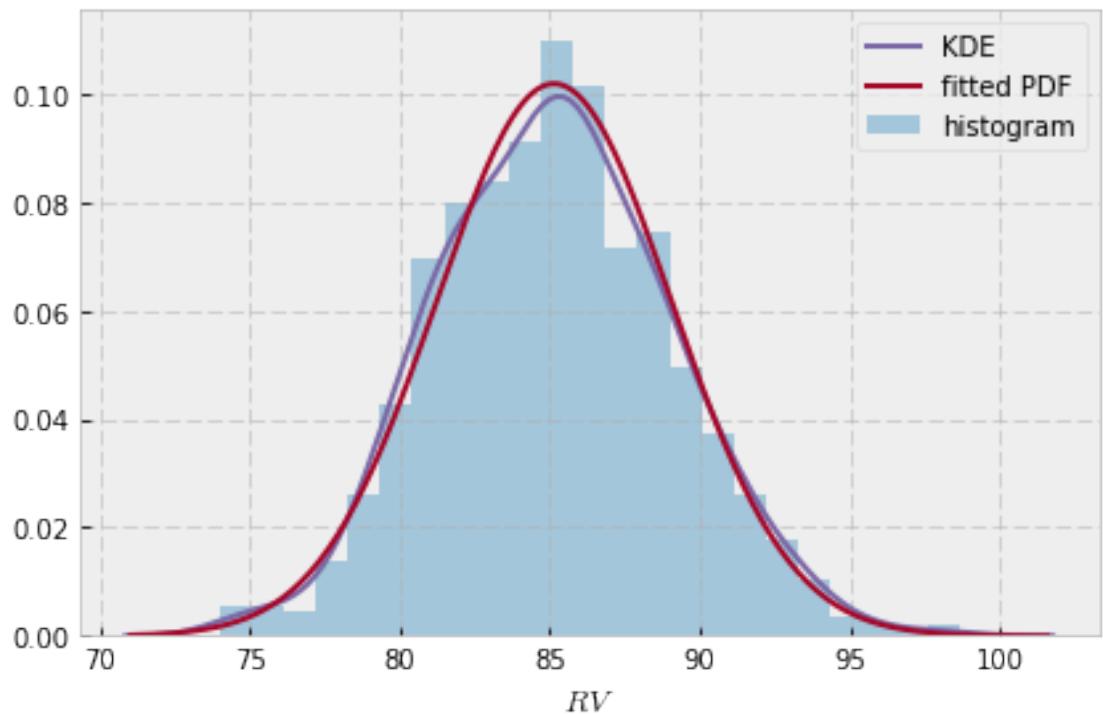


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=101.23454911831331, scale=4.1769410656327866)
DescribeResult(nobs=1000, minmax=(87.91588336932972, 114.63439649906908)
               mean=101.23454911831331, variance=17.464300966736296,
               skewness=0.05570545389390994, kurtosis=-0.06999844753951168)
```

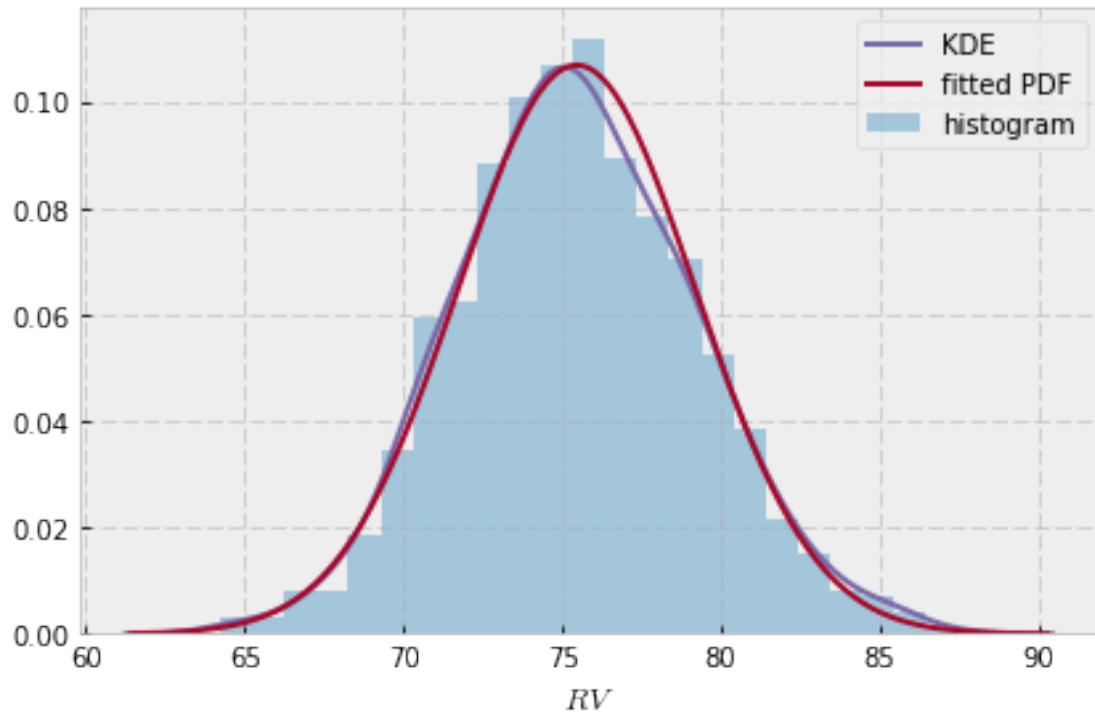


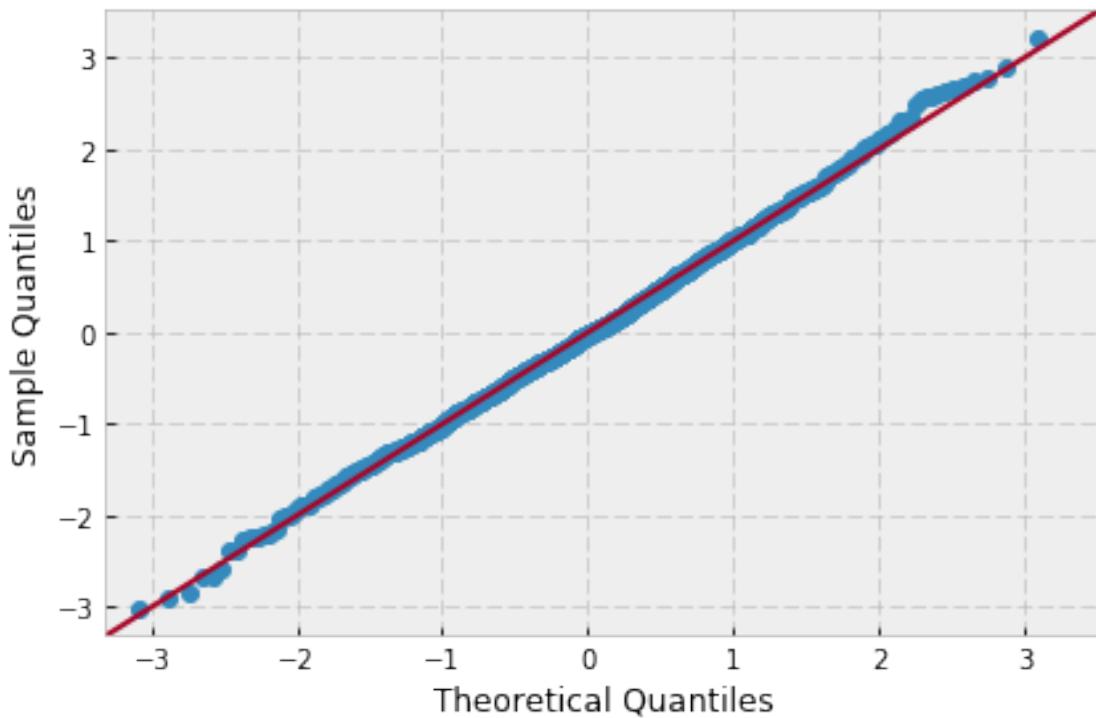


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(),loc=85.14206008992265, scale=3.9128237410223004)
DescribeResult(nobs=1000, minmax=(73.98026765863753, 98.638283161269428)
               mean=85.142060089922651, variance=15.3255151434512,
               skewness=0.10033925911010178, kurtosis=-0.015549373193721738)
```

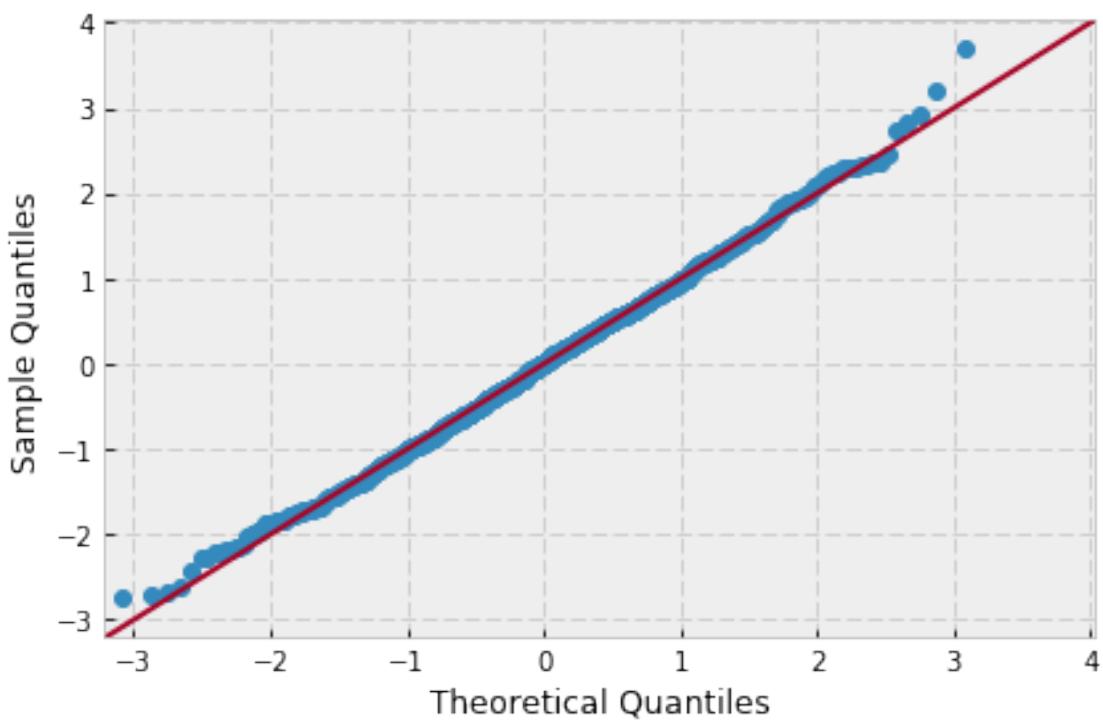
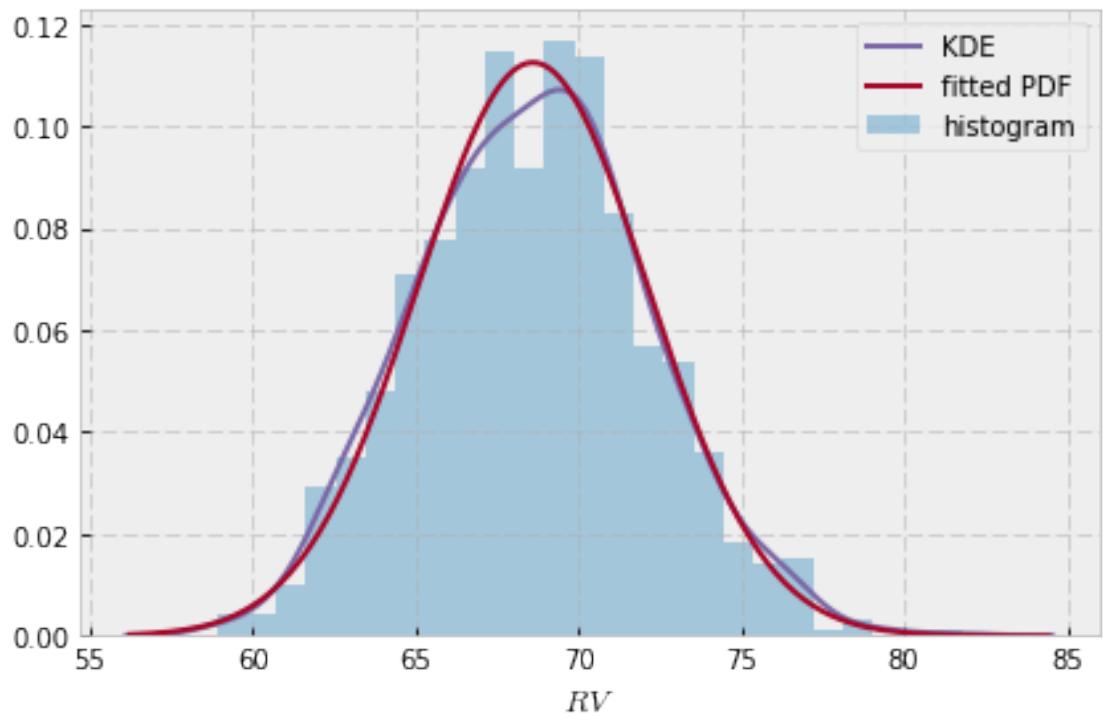


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(),loc=75.45082095817237, scale=3.7344118296274273)
DescribeResult(nobs=1000, minmax=(64.233386042354653, 87.44660102604422)
               mean=75.450820958172372, variance=13.959791504766036,
               skewness=0.12206324654615237, kurtosis=0.0007907445530834778)
```

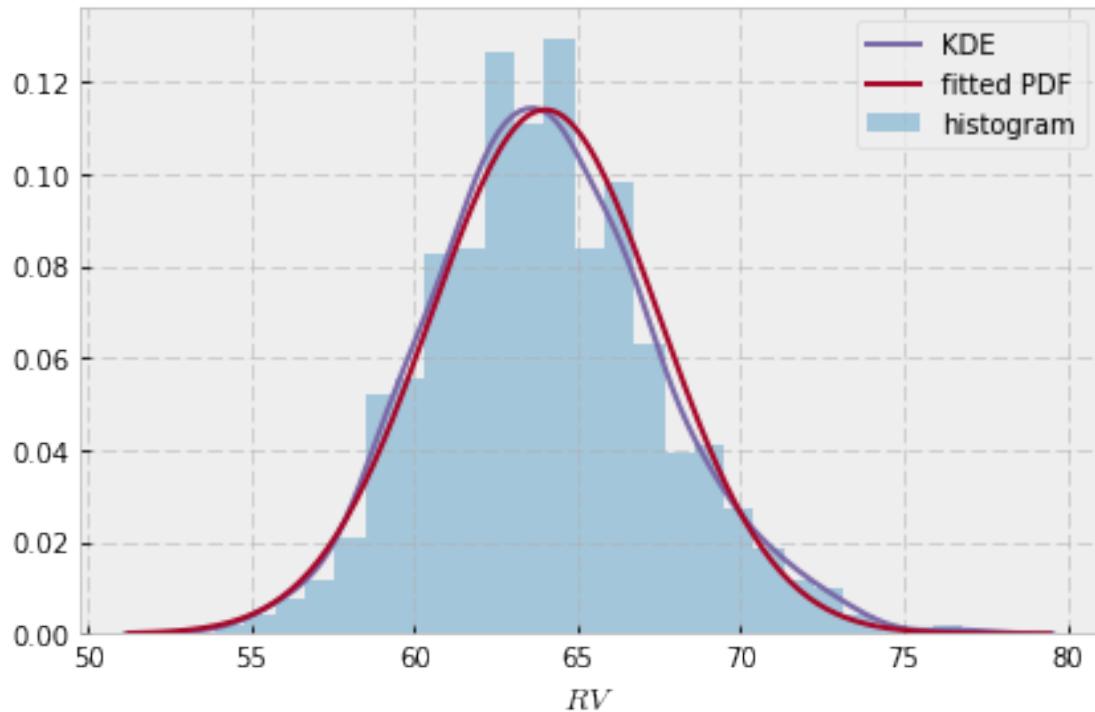


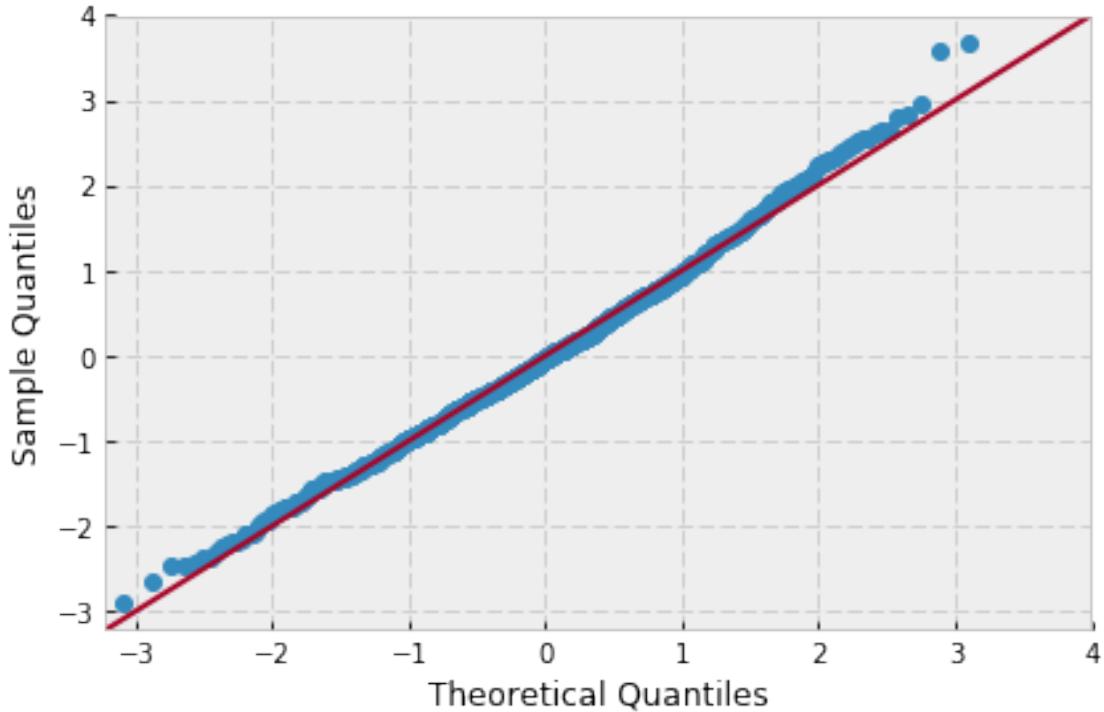


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(),loc=68.59772086519894, scale=3.5475194562700856)
DescribeResult(nobs=1000, minmax=(58.888116177346966, 81.767687540593315)
               mean=68.597720865198937, variance=12.597491784399201,
               skewness=0.11676249684898082, kurtosis=-0.07856581270998264)
```

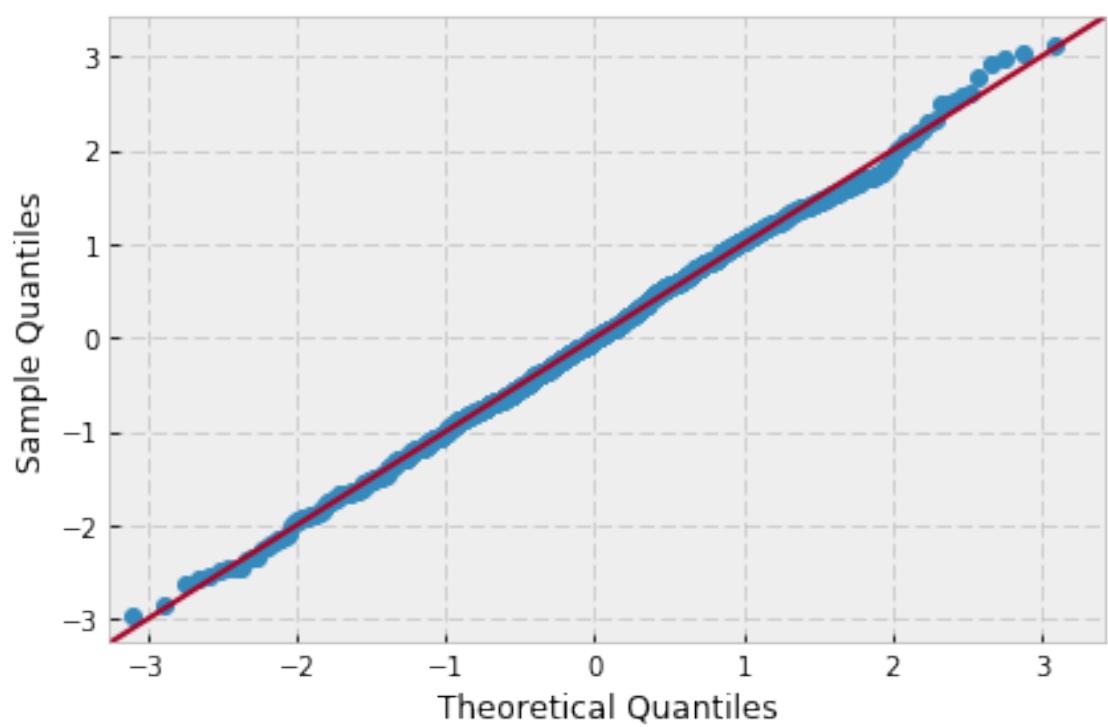
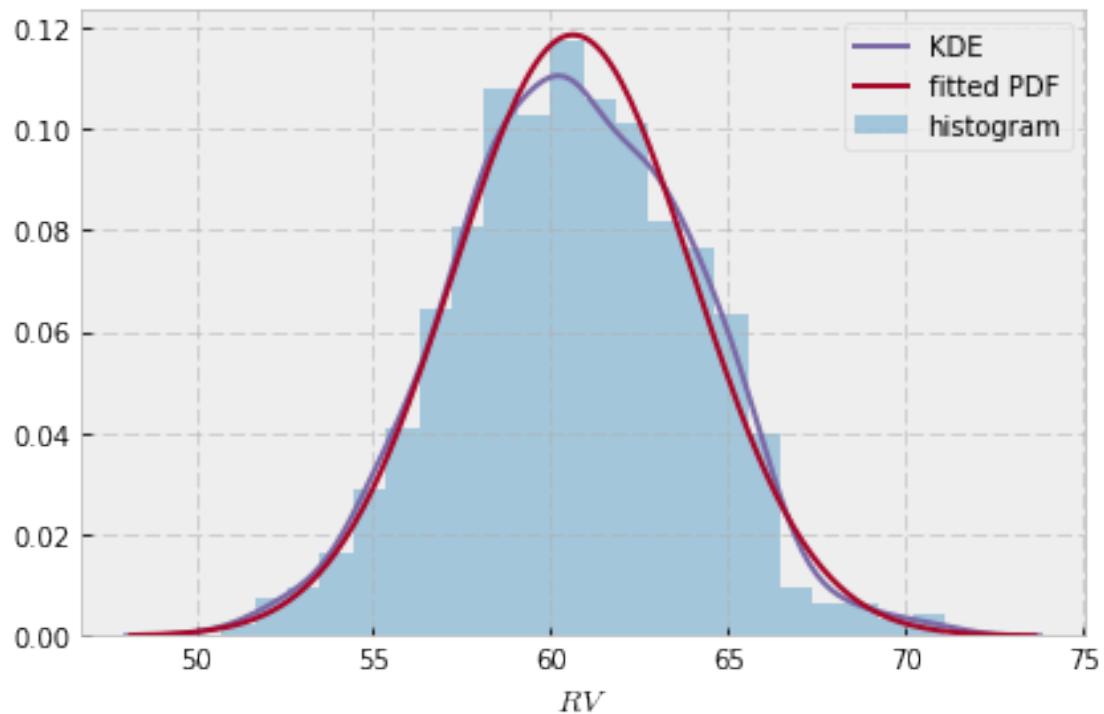


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=63.99753941769221, scale=3.499202205019487)
DescribeResult(nobs=1000, minmax=(53.880148755922797, 76.823733528455037)
               mean=63.997539417692209, variance=12.256672744357596,
               skewness=0.26792509563517464, kurtosis=0.14502141984922012)
```

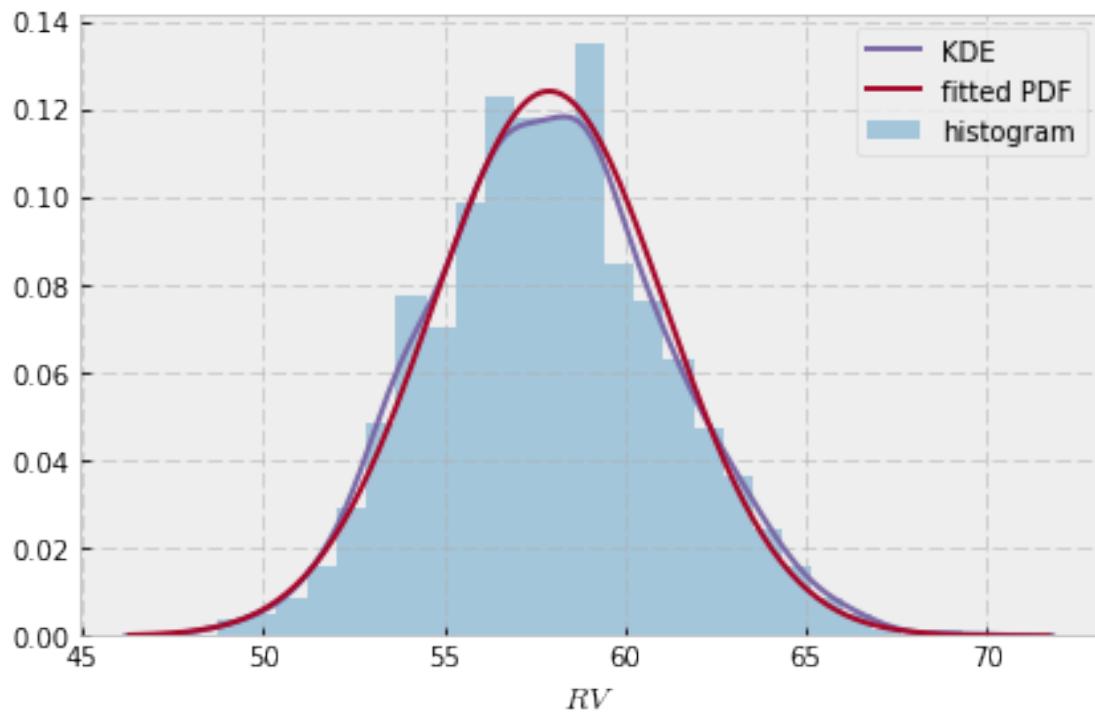


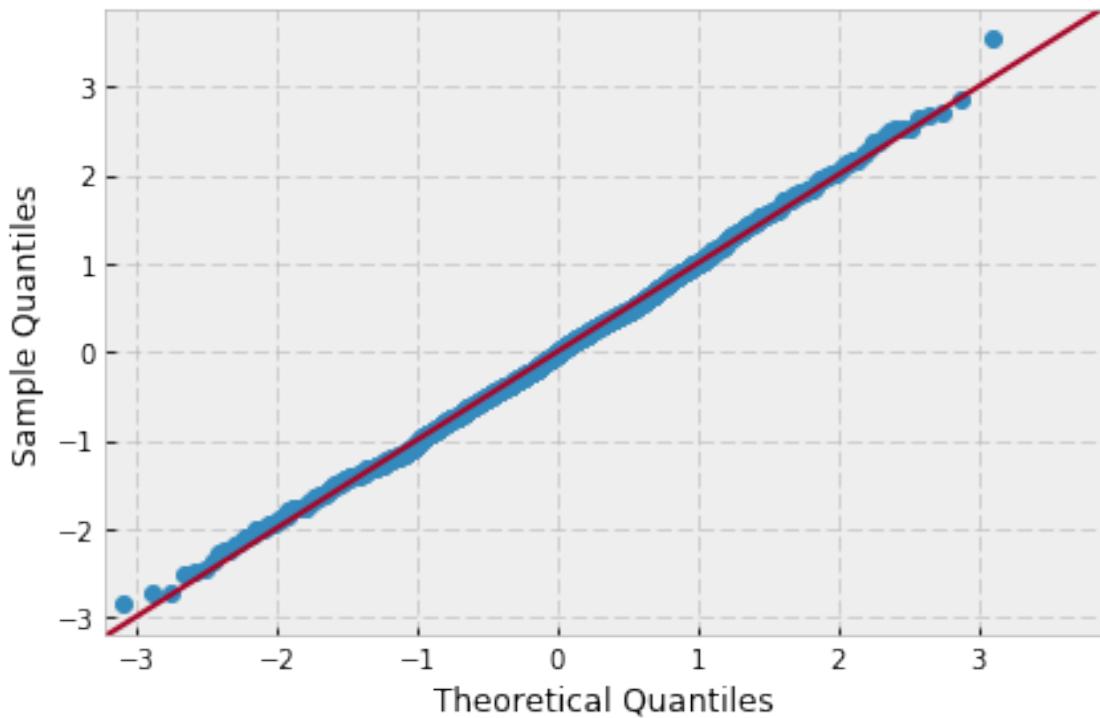


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=60.62771026498317, scale=3.3637252710235814)
DescribeResult(nobs=1000, minmax=(50.713748598381407, 71.102543164625246)
               mean=60.627710264983172, variance=11.325973672595262,
               skewness=0.011427520962545975, kurtosis=-0.13545922377180997)
```

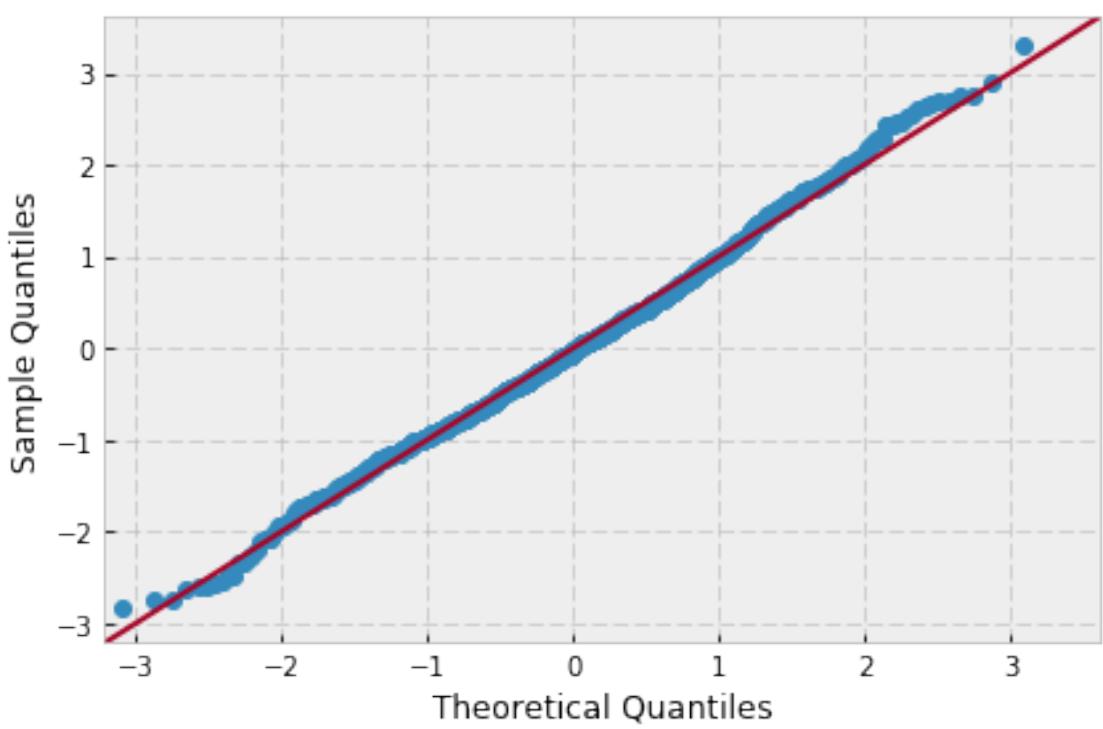
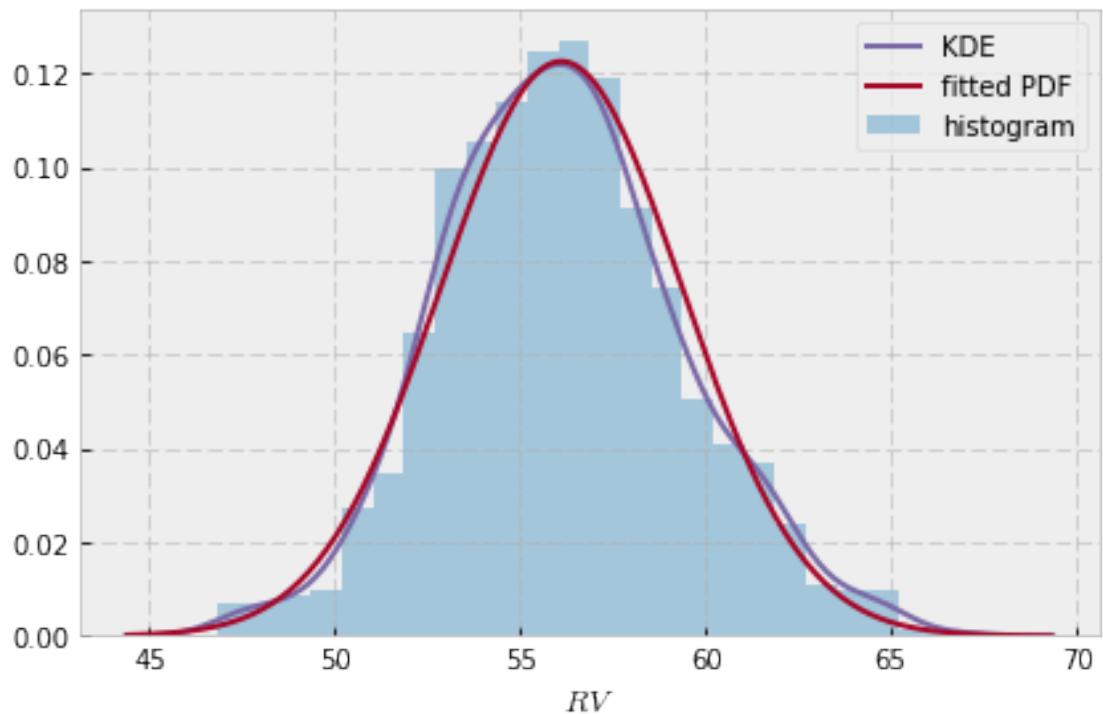


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=57.90760661129035, scale=3.2161963809467786)
DescribeResult(nobs=1000, minmax=(48.724930509218495, 69.307489337033374)
               mean=57.907606611290348, variance=10.354273434249405,
               skewness=0.13497036706076487, kurtosis=-0.12503692806395827)
```

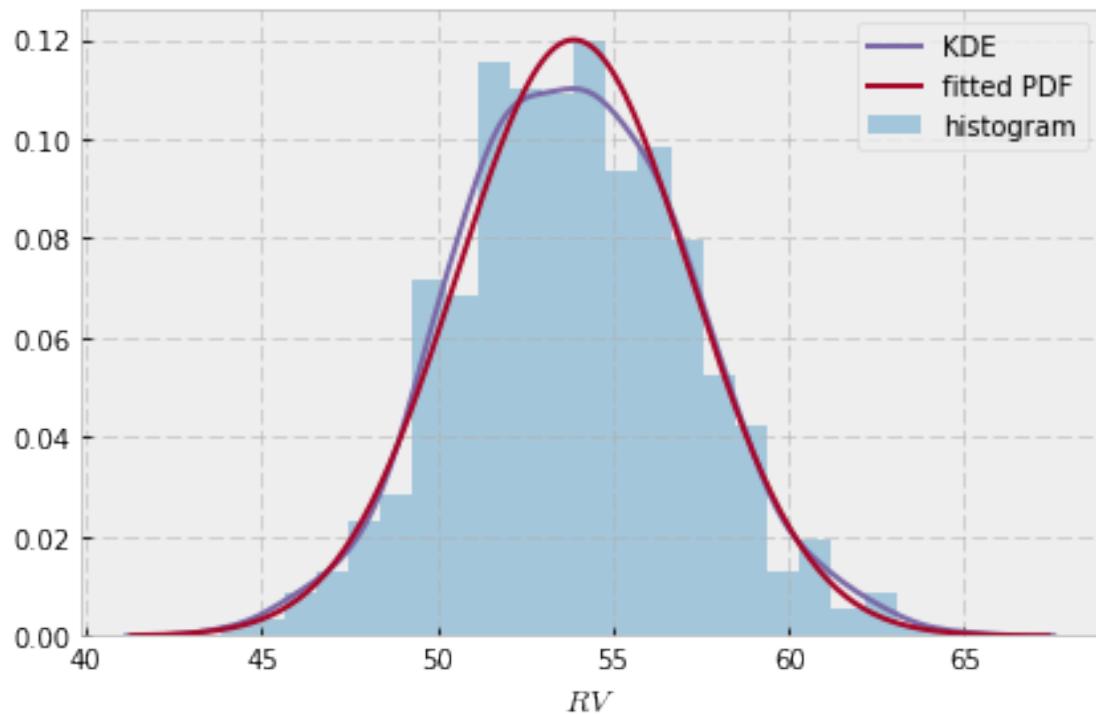


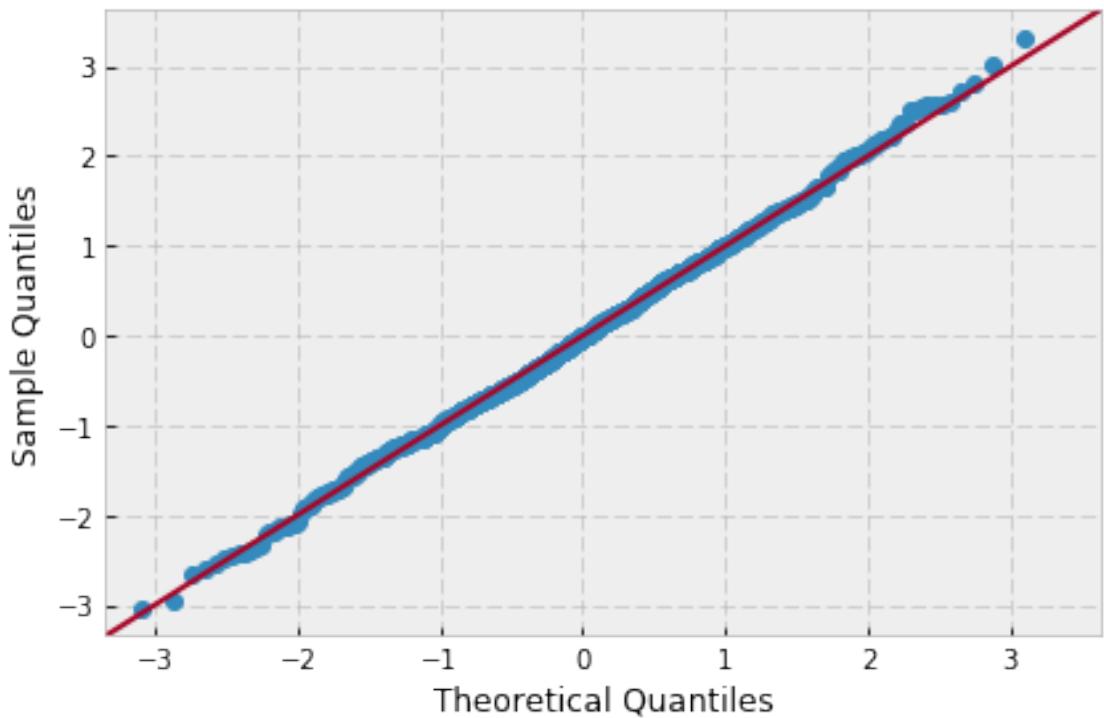


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=56.10954999199473, scale=3.2546764575848512)
DescribeResult(nobs=1000, minmax=(46.857209903028995, 66.863021393210062)
               mean=56.109549991994733, variance=10.603522365922998,
               skewness=0.18941176166957863, kurtosis=0.09371484993653789)
```

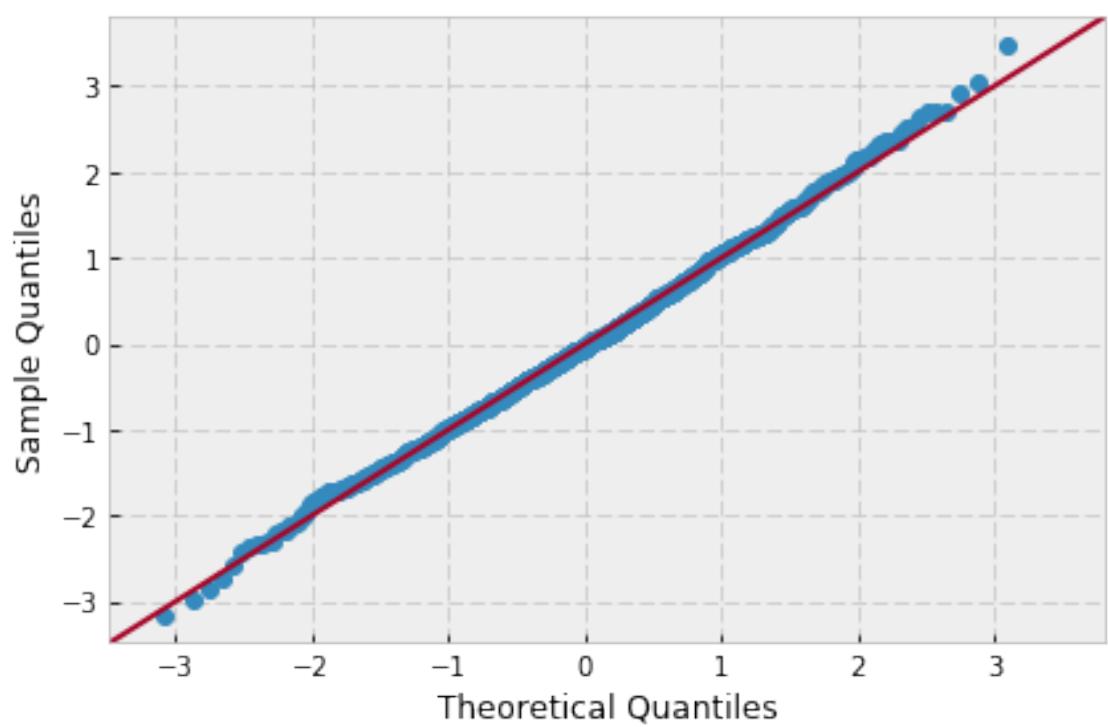
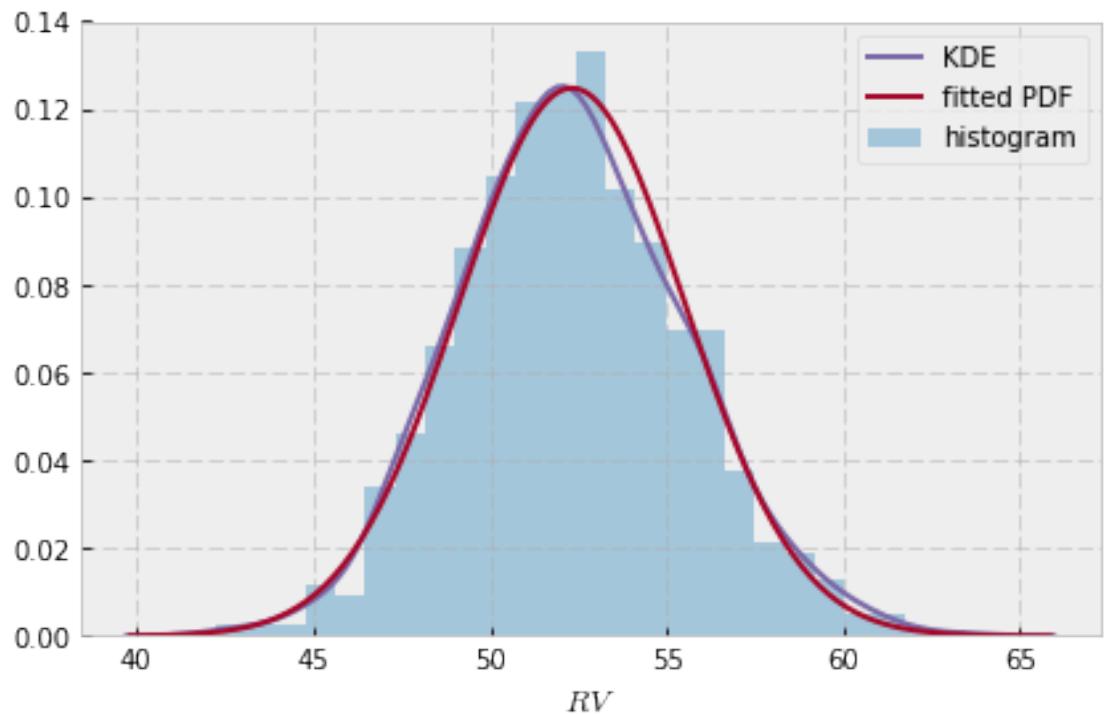


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=53.878520087658416, scale=3.325124535223977)
DescribeResult(nobs=1000, minmax=(43.802498520536041, 64.889751474520935)
               mean=53.878520087658416, variance=11.06752069544391,
               skewness=0.08634570806433597, kurtosis=-0.028637478127238936)
```

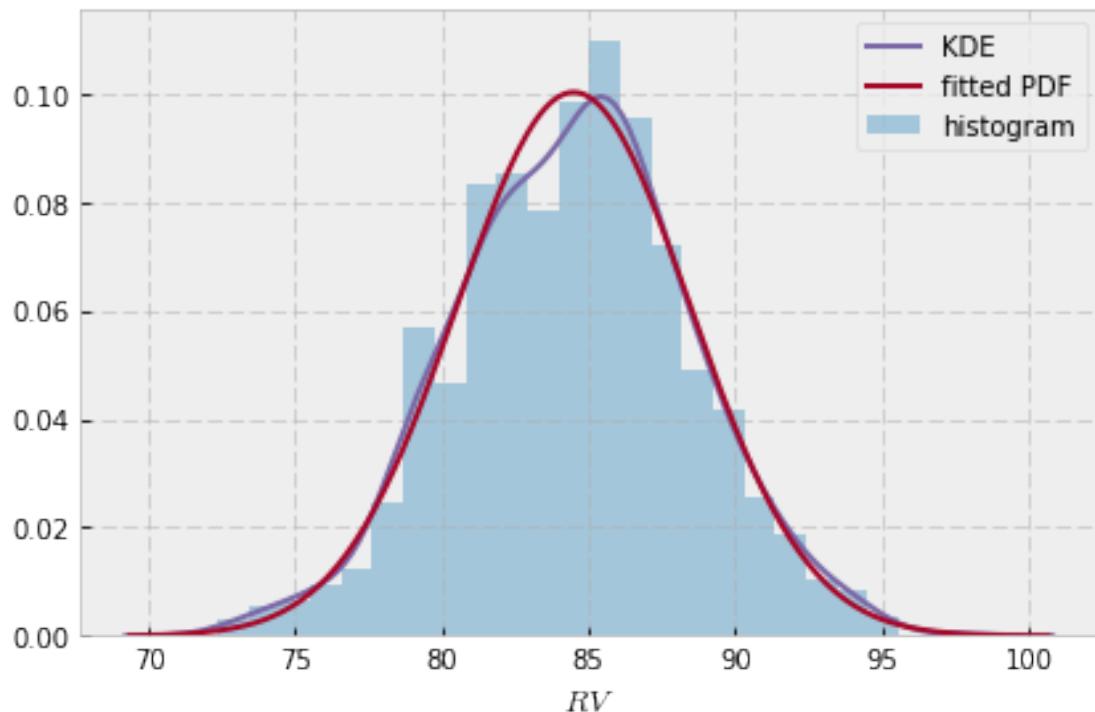


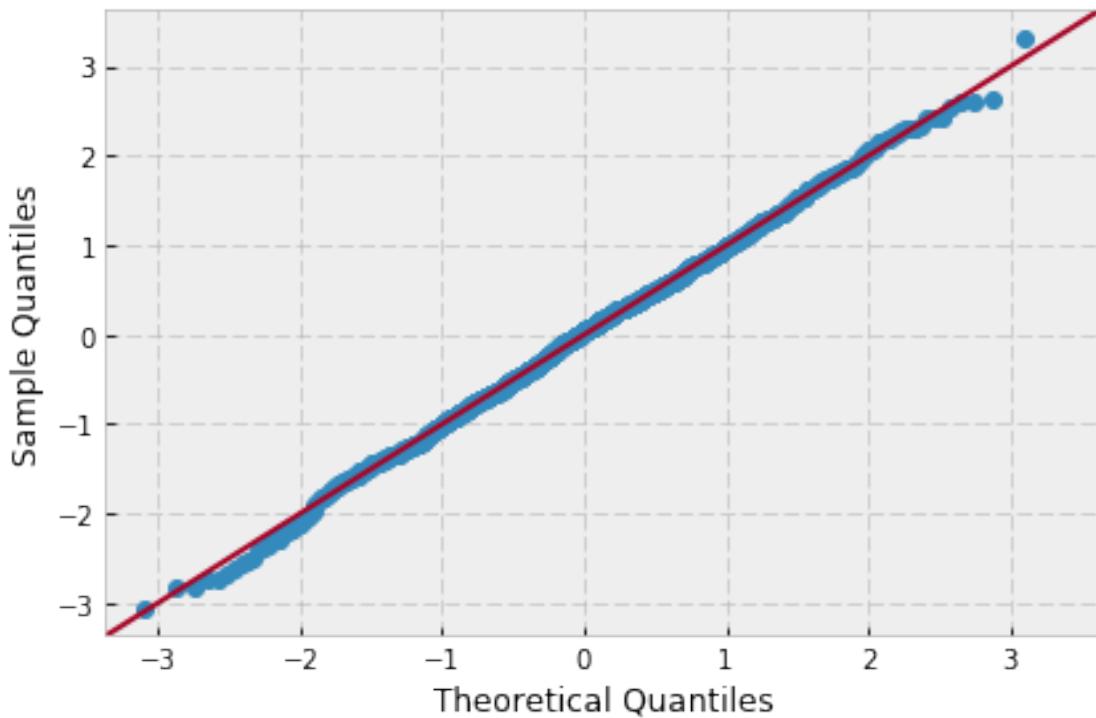


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(),loc=52.314897302048685, scale=3.2007856693261445)
DescribeResult(nobs=1000, minmax=(42.228940064152155, 63.436839060894627)
               mean=52.314897302048685, variance=10.255284185148763,
               skewness=0.14596570621139093, kurtosis=0.0367653421704639)
```

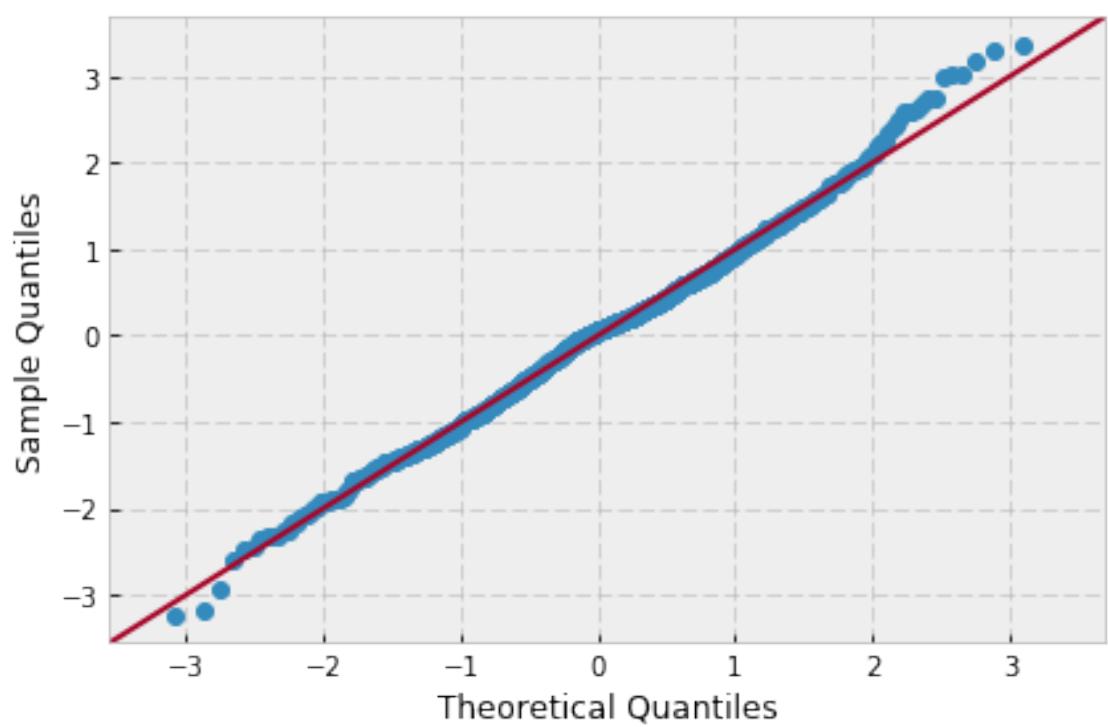
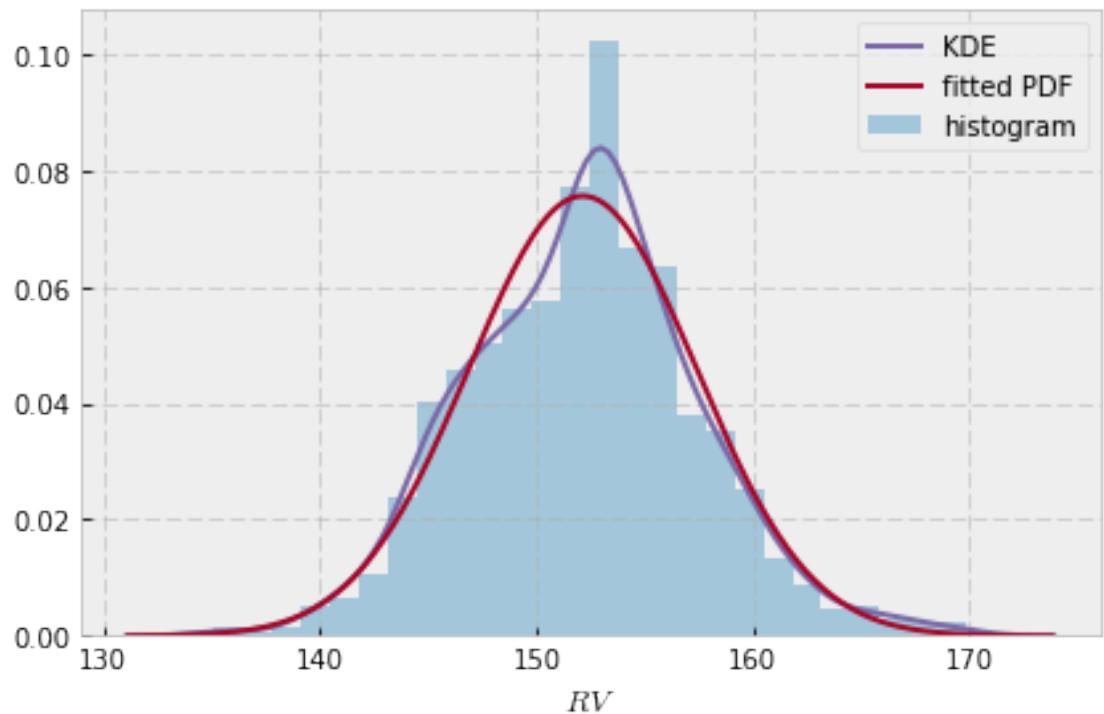


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(), loc=84.48483078826804, scale=3.9810279994424045)
DescribeResult(nobs=1000, minmax=(72.342342547688801, 97.67668488101036)
               mean=84.484830788268042, variance=15.864448380725118,
               skewness=-0.03567716418897212, kurtosis=-0.026116667443468966)
```

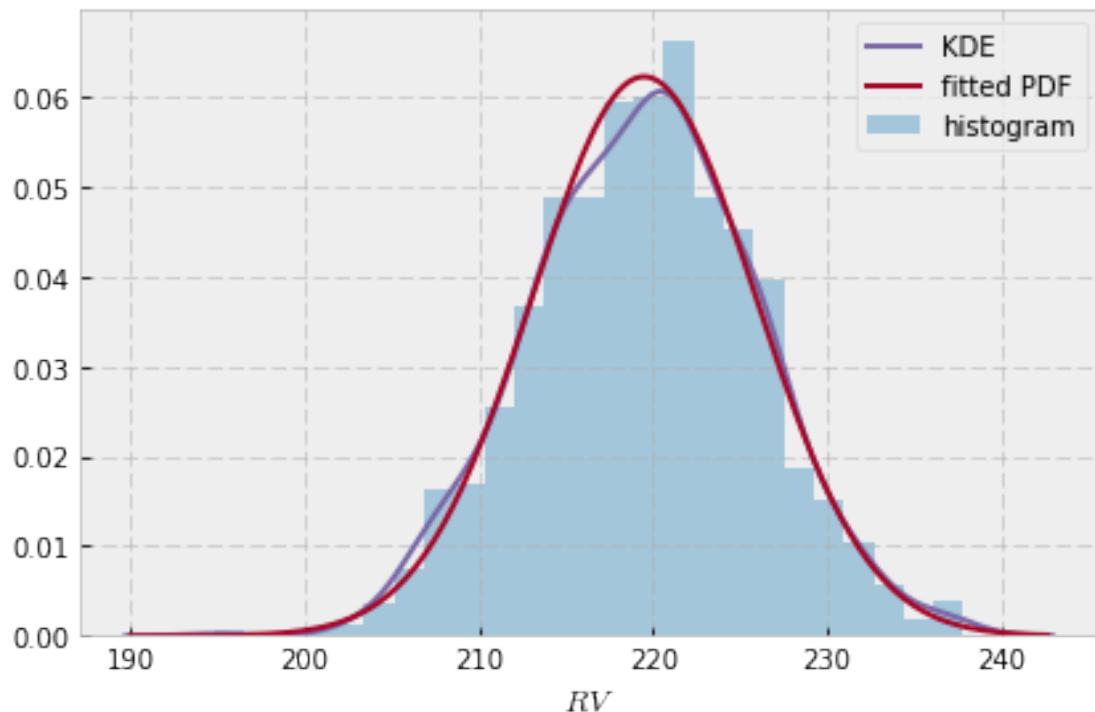


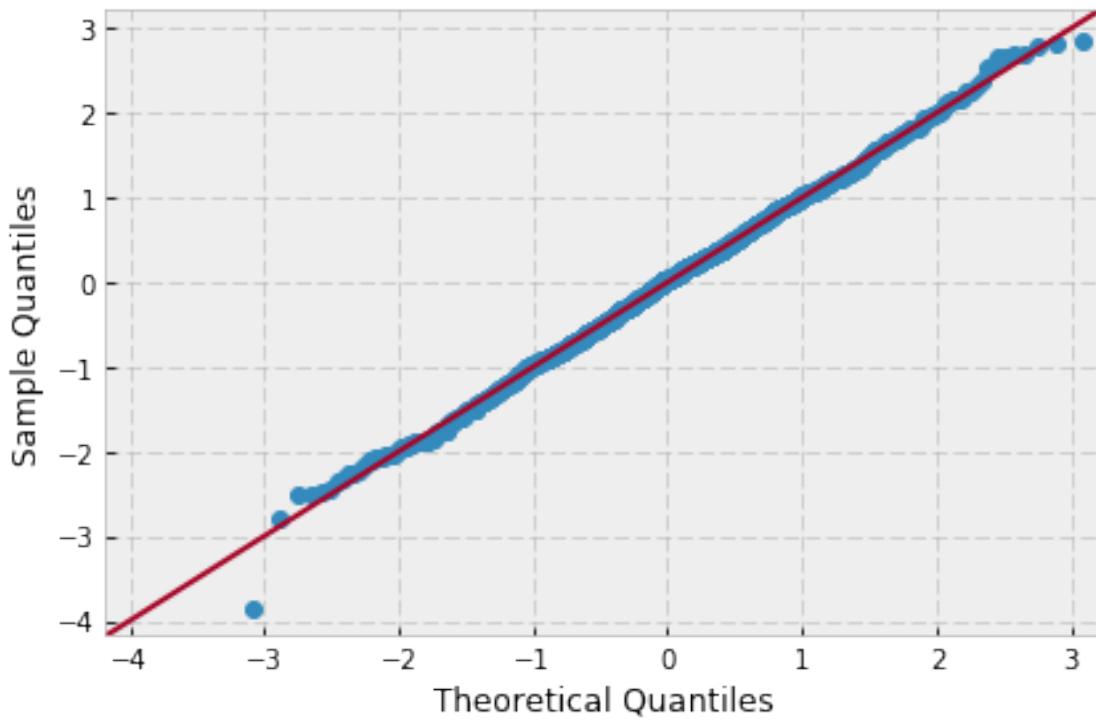


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(),loc=152.14408166747202, scale=5.2715157906555445)
DescribeResult(nobs=1000, minmax=(135.11486908677719, 169.85388209466294)
               mean=152.14408166747202, variance=27.816695426557303,
               skewness=0.11981062033707697, kurtosis=0.2763904874710037)
```

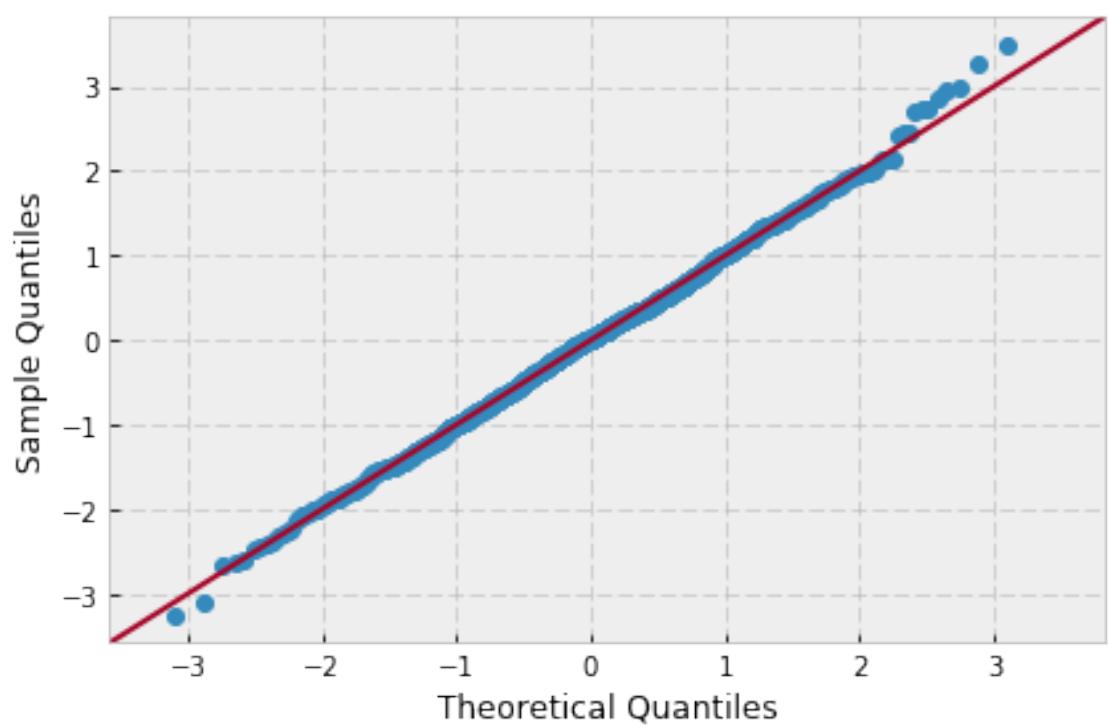
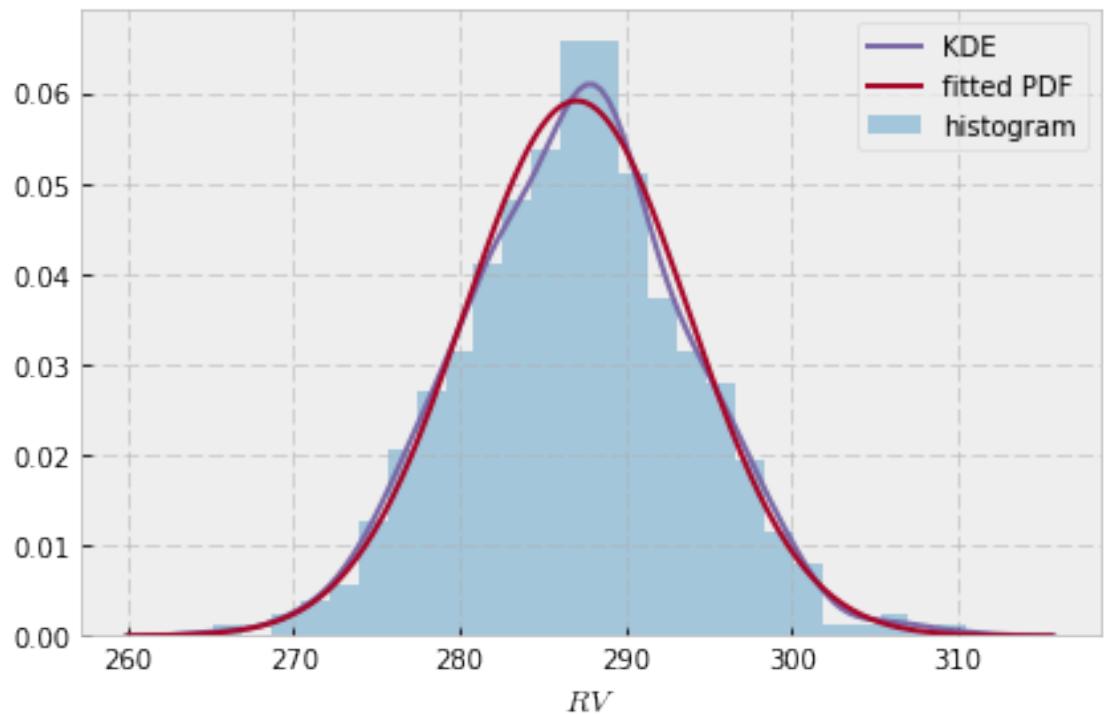


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(),loc=219.49760066171962, scale=6.409232328978432)
DescribeResult(nobs=1000, minmax=(194.86413709431392, 237.81933196379939)
               mean=219.49760066171962, variance=41.119378425247554,
               skewness=-0.020719827584291976, kurtosis=-0.05350857438202583)
```

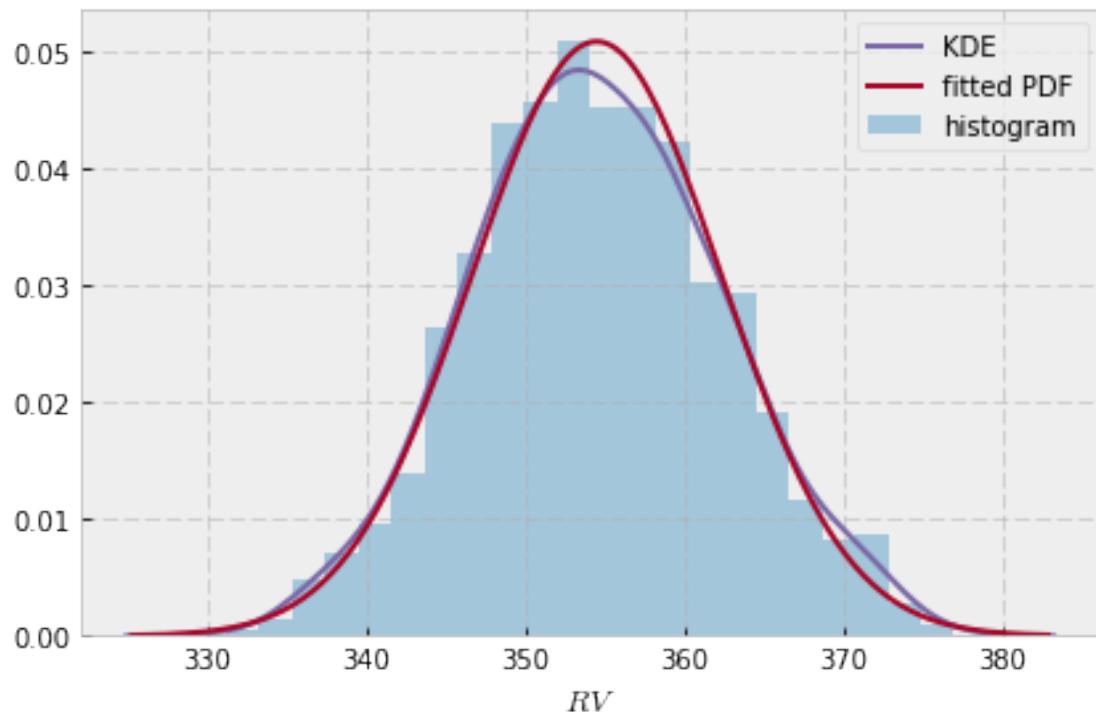


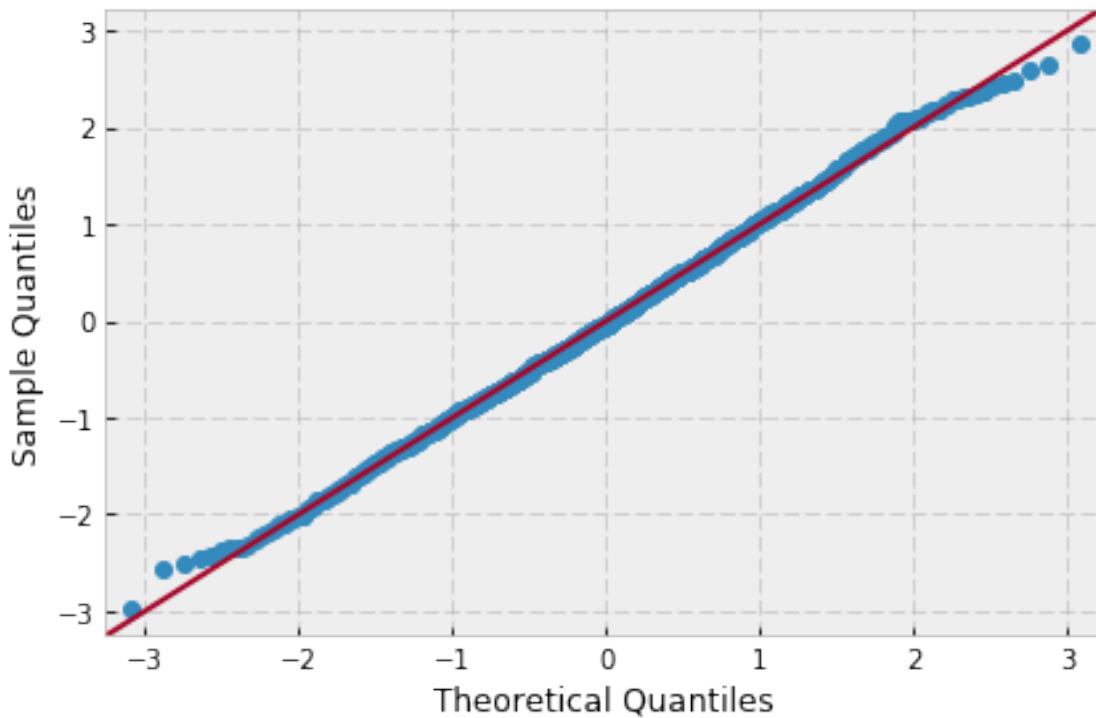


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(),loc=287.05617268542807, scale=6.746134812581202)
DescribeResult(nobs=1000, minmax=(265.16972365508201, 310.55484582539015)
               mean=287.05617268542807, variance=45.555890800320327,
               skewness=0.04765019063548426, kurtosis=0.07090046933749639)
```

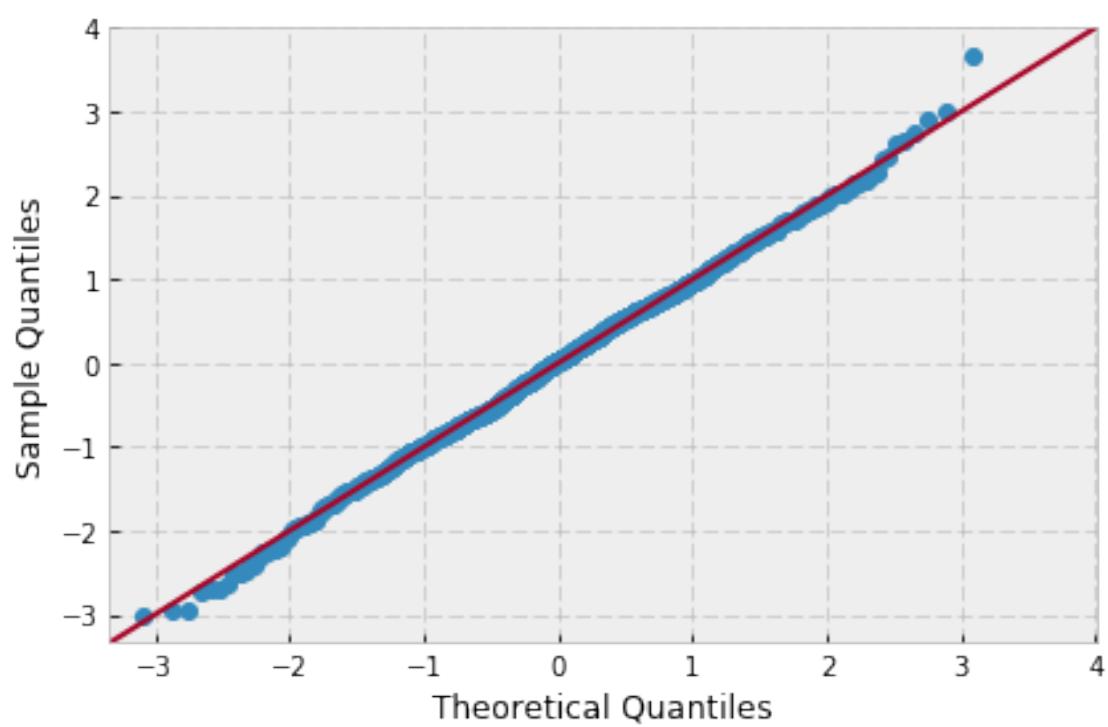
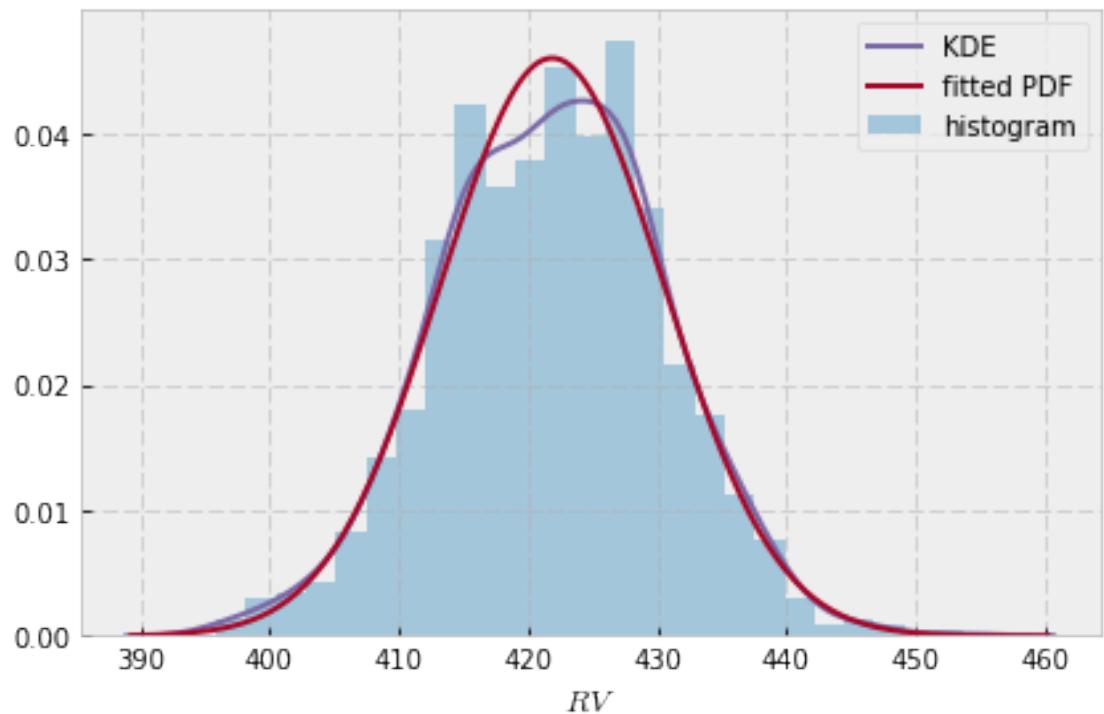


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(), loc=354.473927899195, scale=7.852935119847973)
DescribeResult(nobs=1000, minmax=(331.17403106105837, 376.94793039691353)
               mean=354.47392789919502, variance=61.730320316858545,
               skewness=0.06700782786716879, kurtosis=-0.23328490161528315)
```

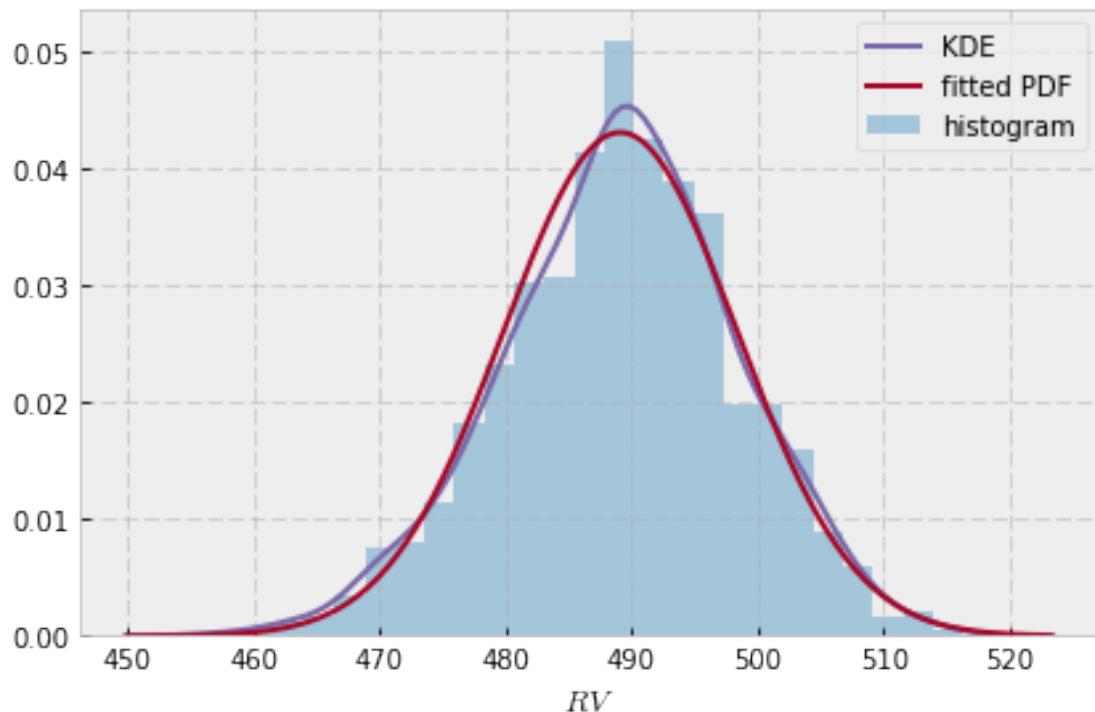


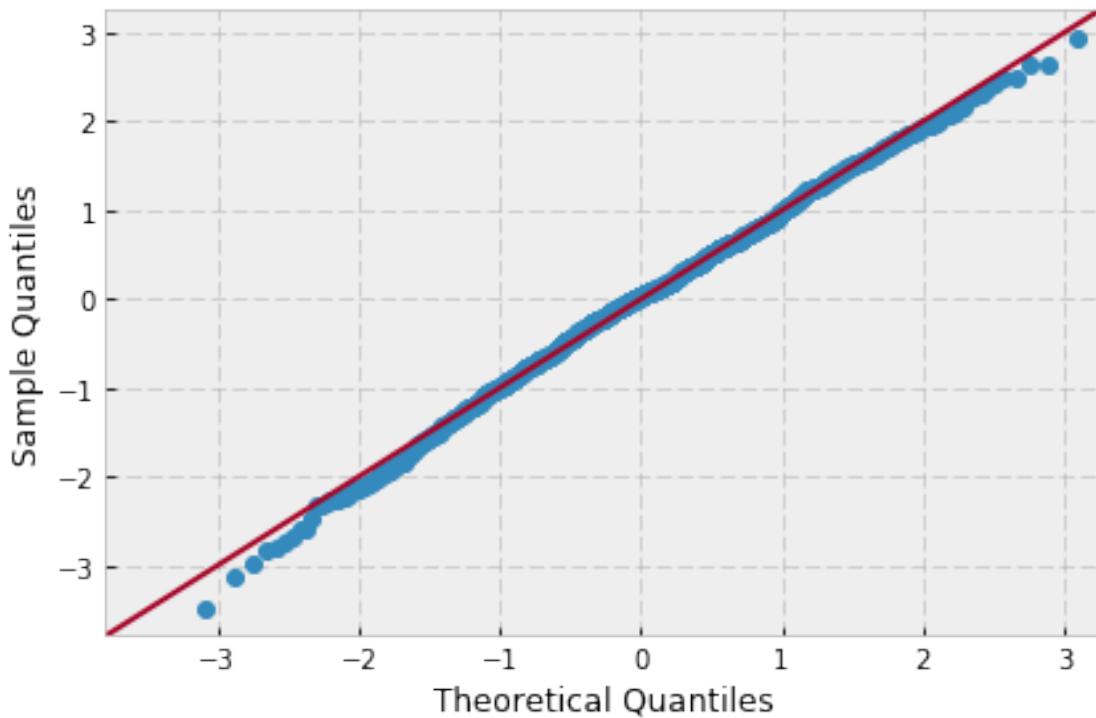


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(),loc=421.8606937951904, scale=8.669687374686895)
DescribeResult(nobs=1000, minmax=(395.82613124252867, 453.75793895391382)
               mean=421.8606937951904, variance=75.238717892698034,
               skewness=-0.03377201852960518, kurtosis=0.019386529388161655)
```

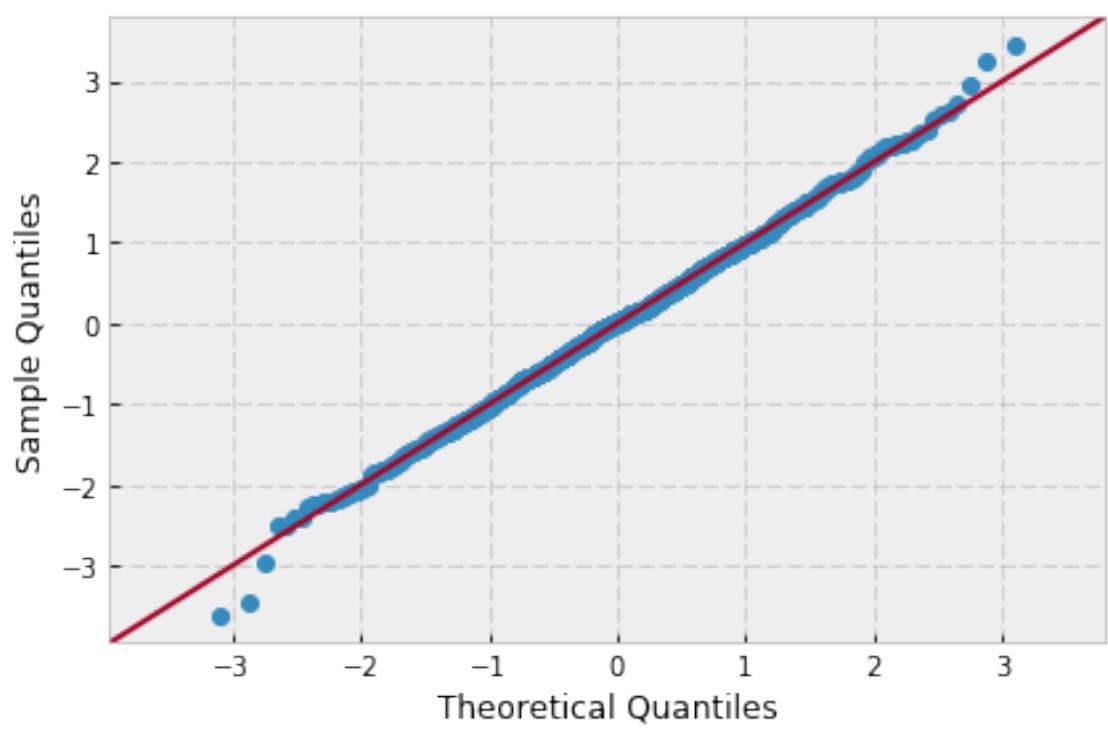
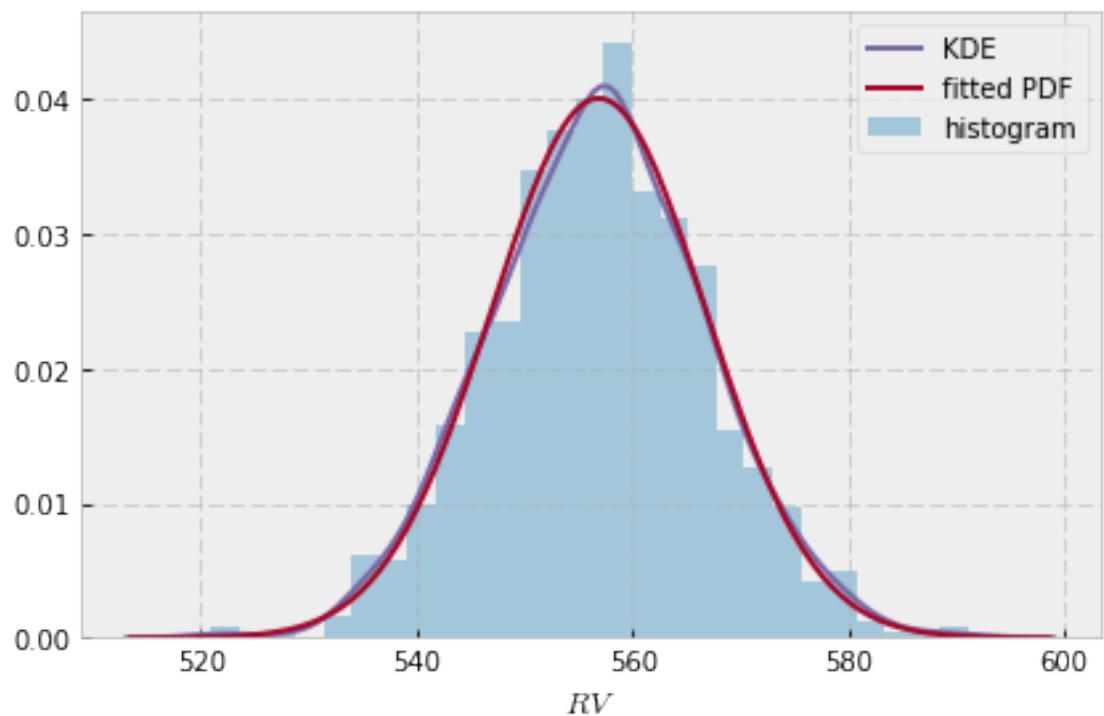


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(), loc=489.0782015028997, scale=9.25366841083901)
DescribeResult(nobs=1000, minmax=(456.96696144554312, 516.23227578162744)
               mean=489.07820150289967, variance=85.7160951529127,
               skewness=-0.17143712196111155, kurtosis=0.056592692519986)
```

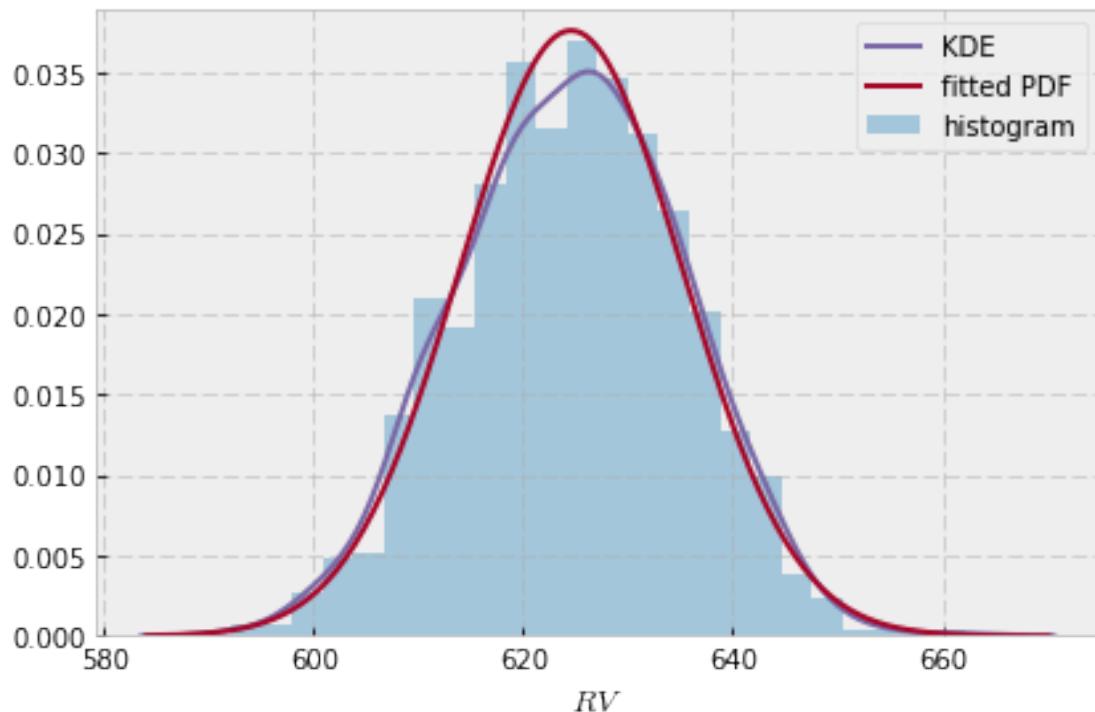


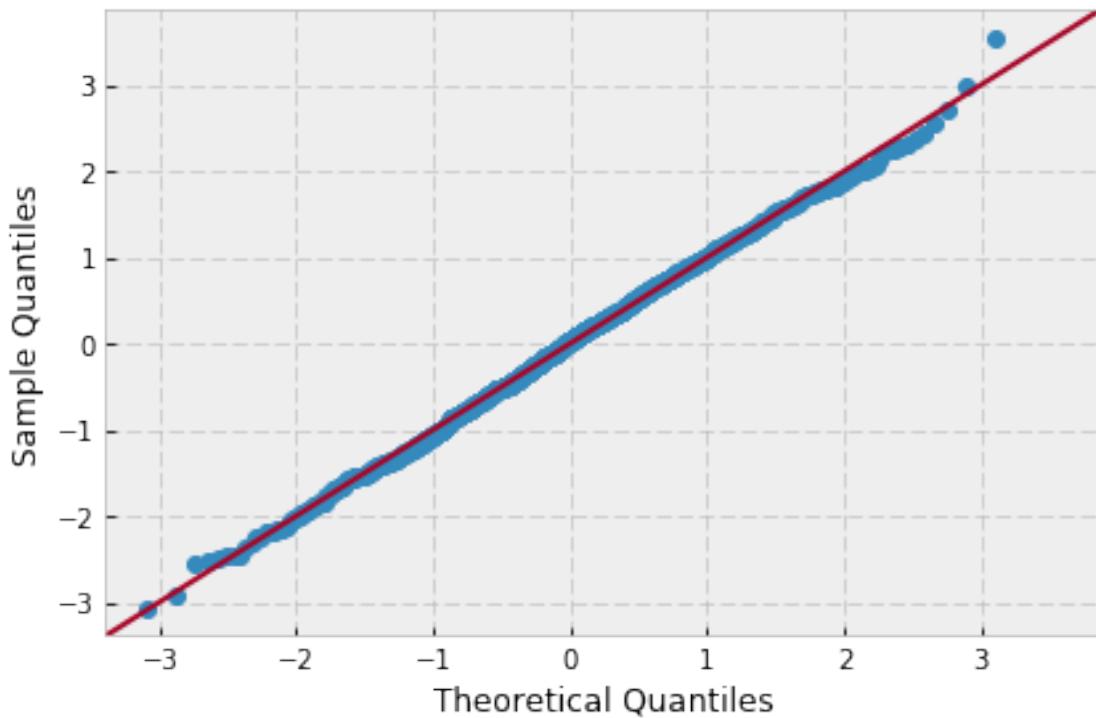


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(),loc=556.8661373654461, scale=9.956978204150074)
DescribeResult(nobs=1000, minmax=(520.98916369840629, 591.13974314677273)
               mean=556.86613736544609, variance=99.240655613533178,
               skewness=0.01303834268092583, kurtosis=0.13248871018993214)
```

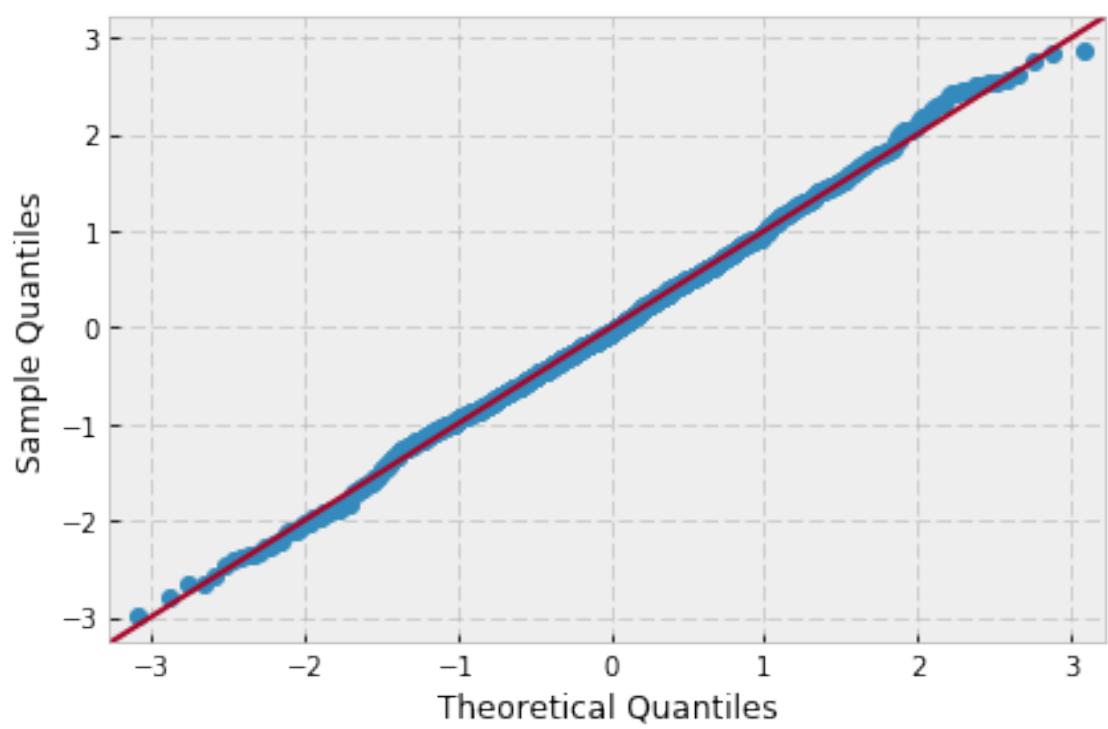
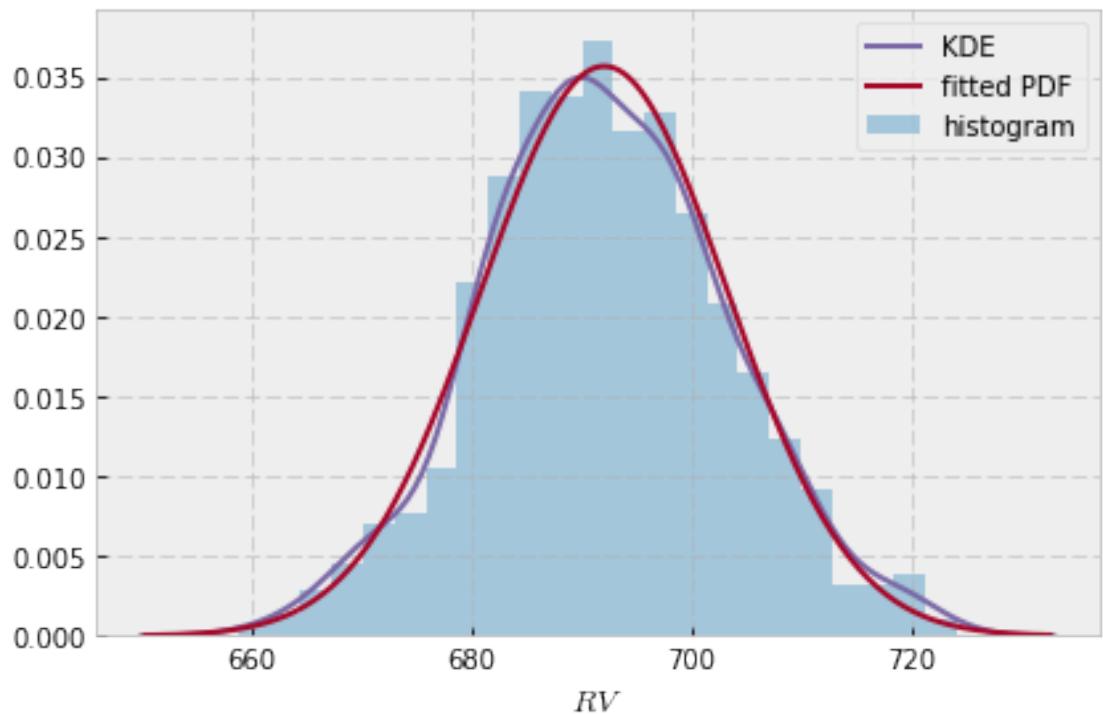


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(), loc=624.5594660596901, scale=10.616667637878615)
DescribeResult(nobs=1000, minmax=(592.05193818114356, 662.10162502629441)
               mean=624.55946605969007, variance=112.82645819137046,
               skewness=-0.03923896621861167, kurtosis=-0.2159071285701053)
```

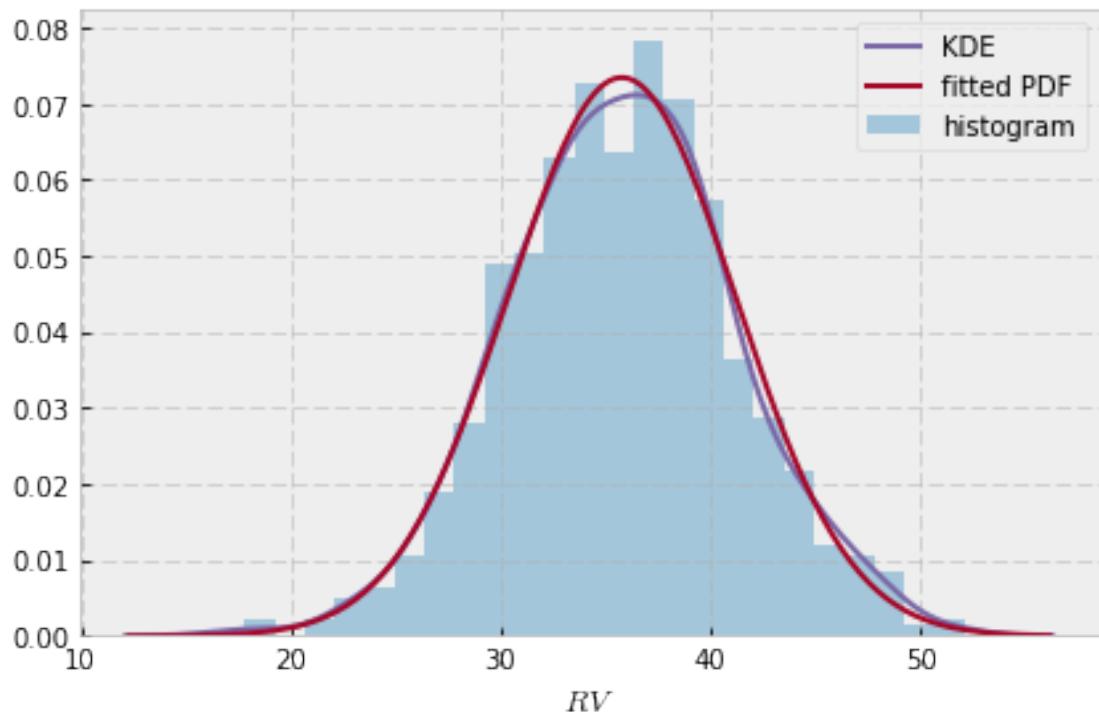


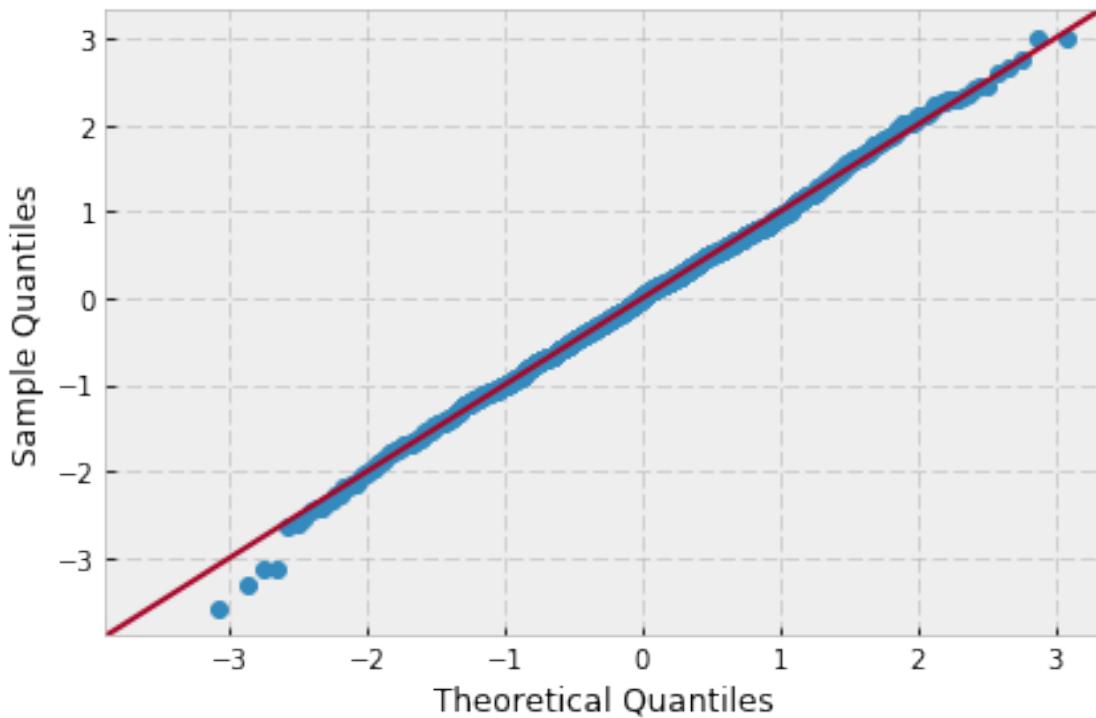


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(),loc=692.0501399017521, scale=11.185537061208304)
DescribeResult(nobs=1000, minmax=(658.77847156892597, 724.10714153511503)
               mean=692.05013990175212, variance=125.241480828493,
               skewness=0.06843054369092691, kurtosis=-0.02546065468532177)
```

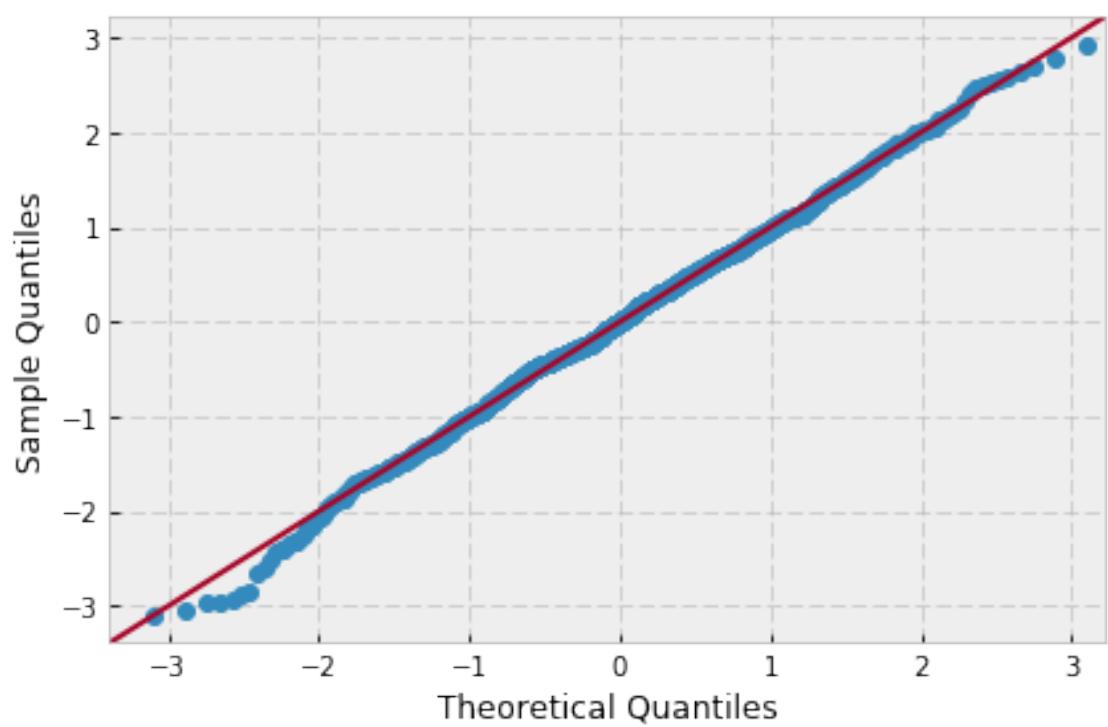
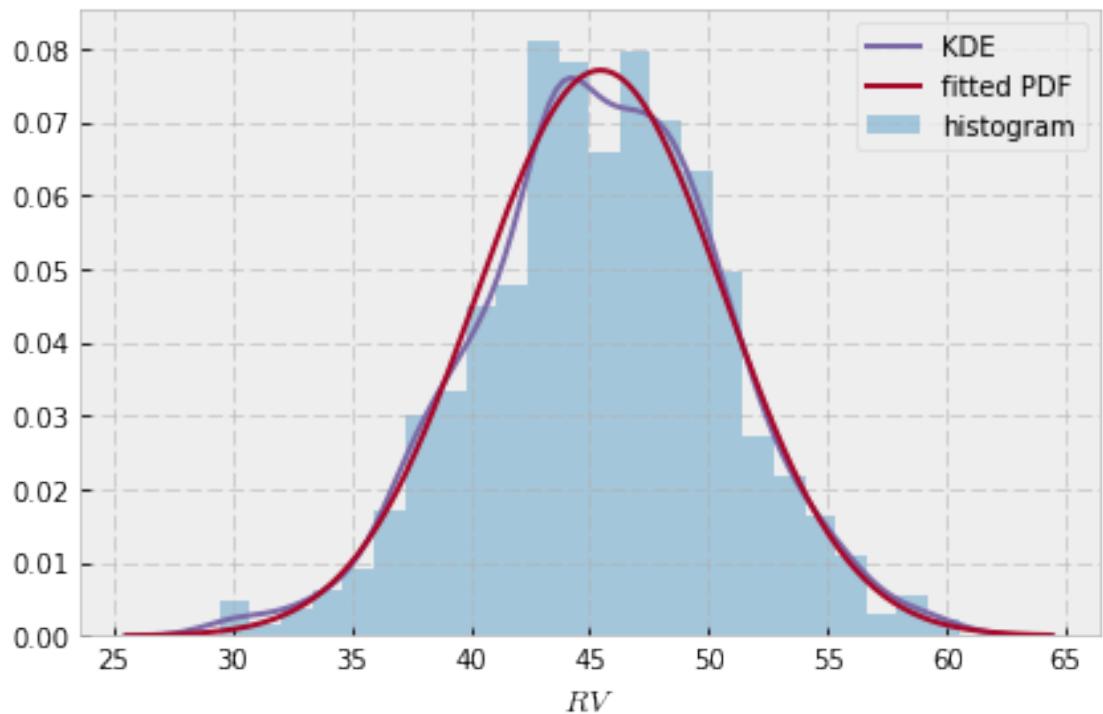


```
variable name: r
variable value: 1.0
distribution: normal(shape=(),loc=35.7814677128654, scale=5.432232152230148)
DescribeResult(nobs=1000, minmax=(16.361828550315362, 52.07559786853664)
               mean=35.7814677128654, variance=29.538684840563551,
               skewness=-0.014759006877380547, kurtosis=0.15199633482234676)
```

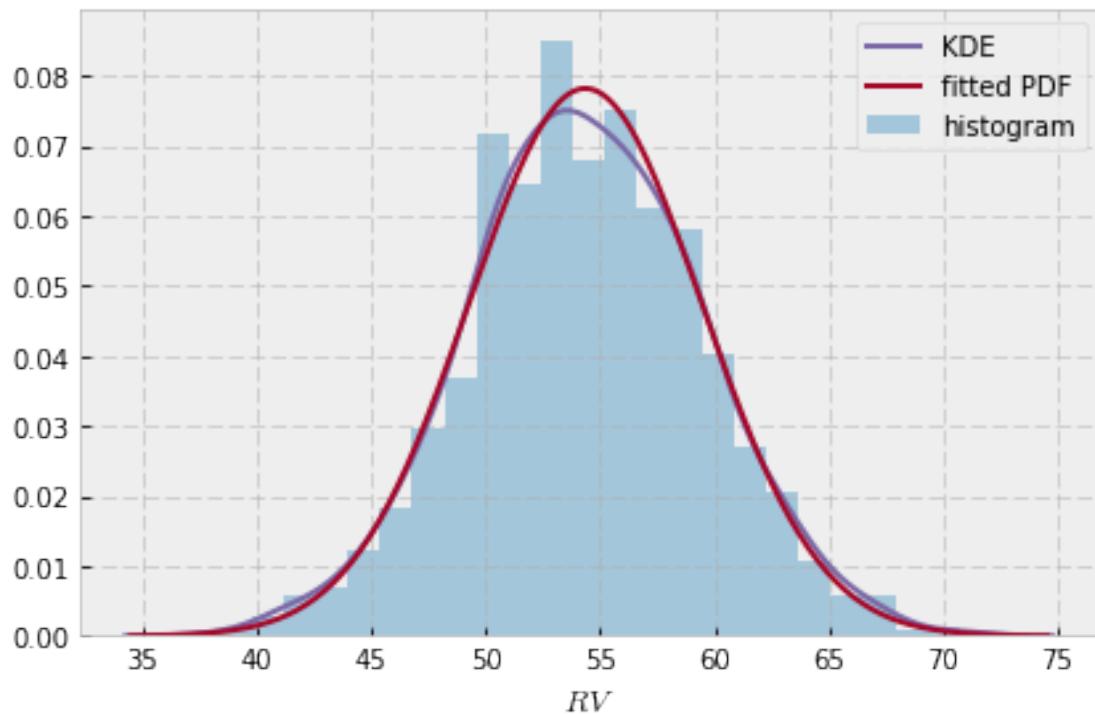


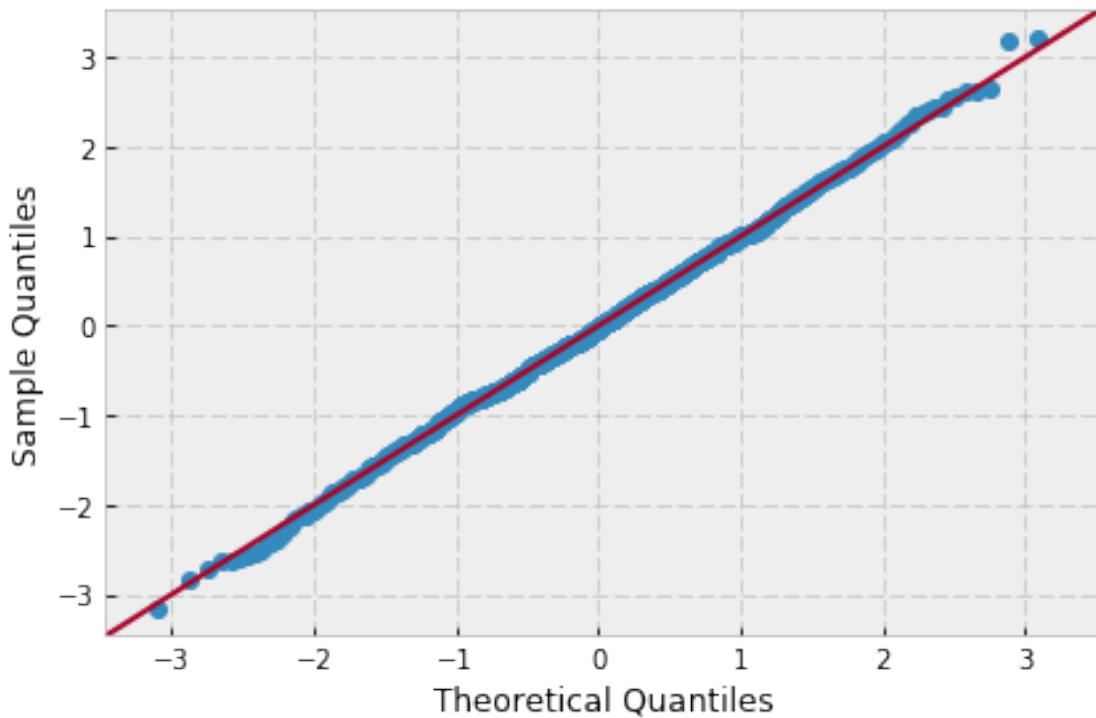


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(),loc=45.45380934715949, scale=5.177241552141578)
DescribeResult(nobs=1000, minmax=(29.461517124278895, 60.494128592071839)
               mean=45.453809347159492, variance=26.830660749971308,
               skewness=-0.09330966607748555, kurtosis=0.07646068878738355)
```

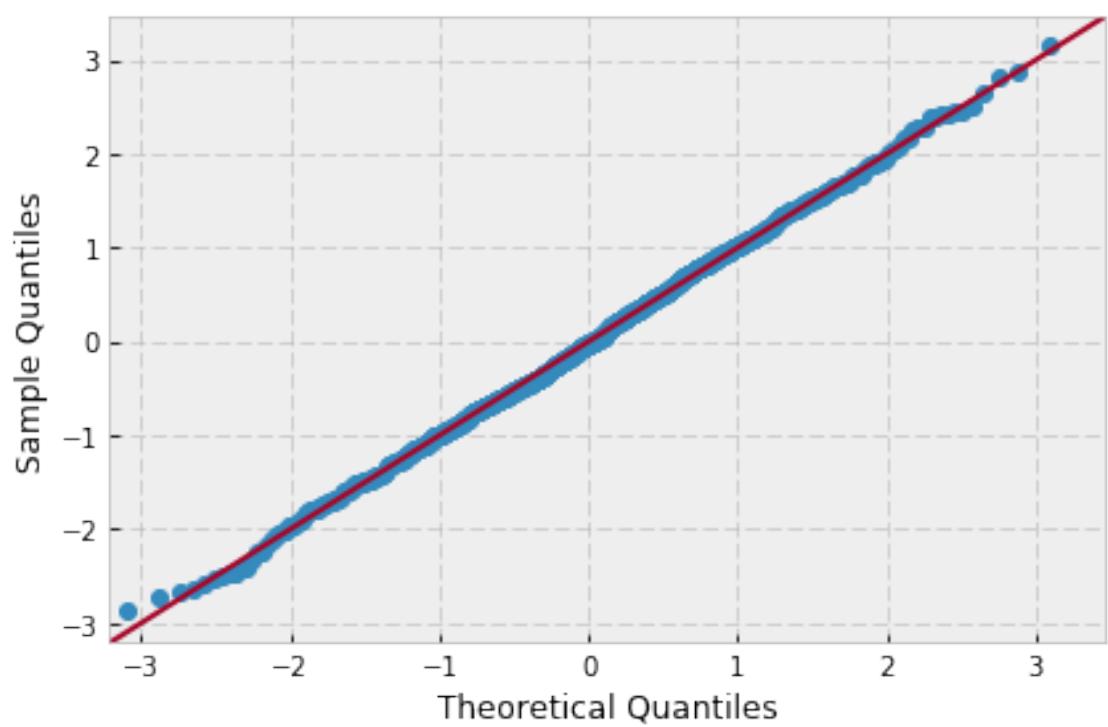
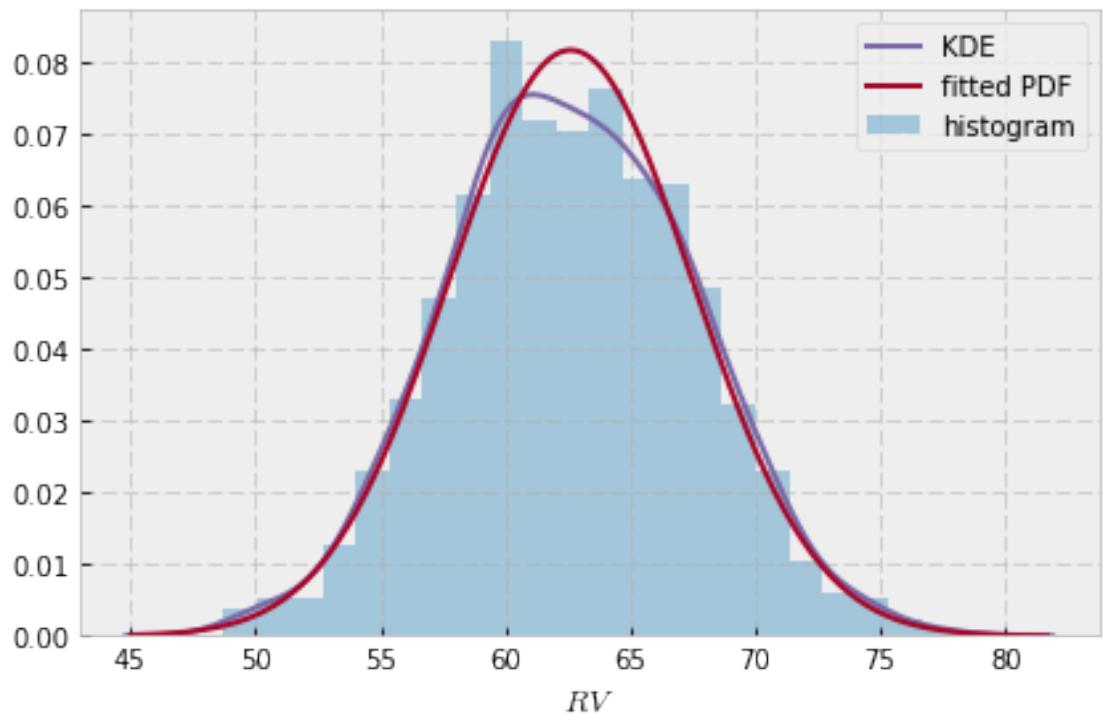


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(),loc=54.355700001195636, scale=5.099001011046041)
DescribeResult(nobs=1000, minmax=(38.320642199259296, 70.71815527442935)
               mean=54.355700001195636, variance=26.025837147796341,
               skewness=0.024630935324187194, kurtosis=0.011135811712001509)
```

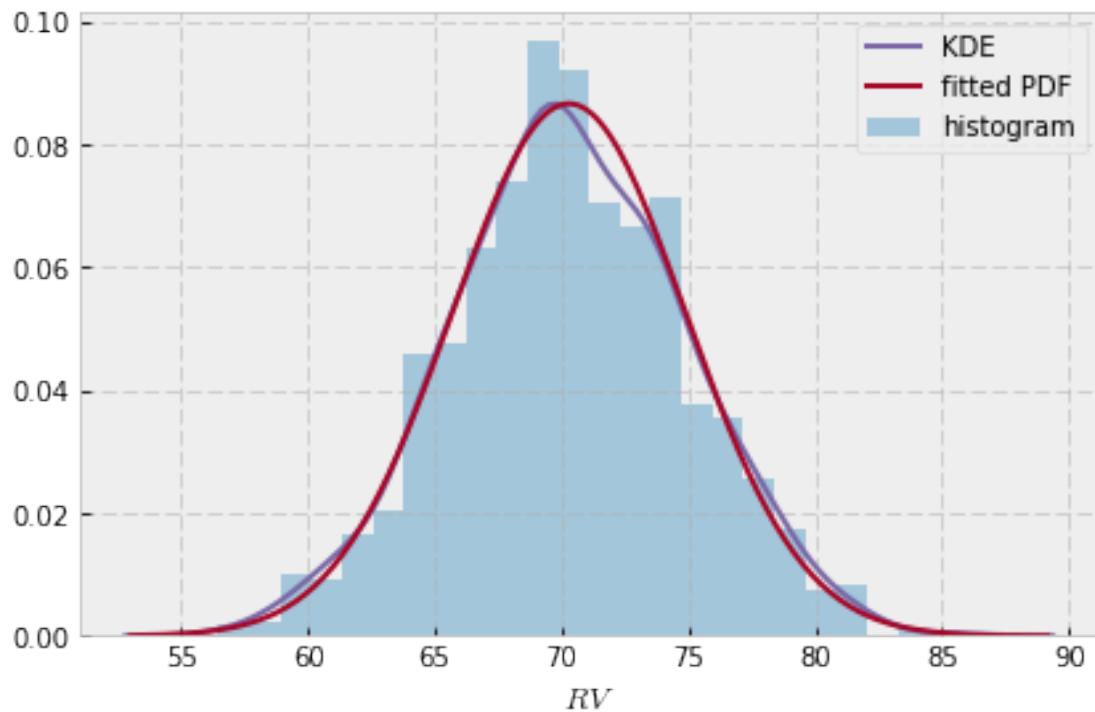


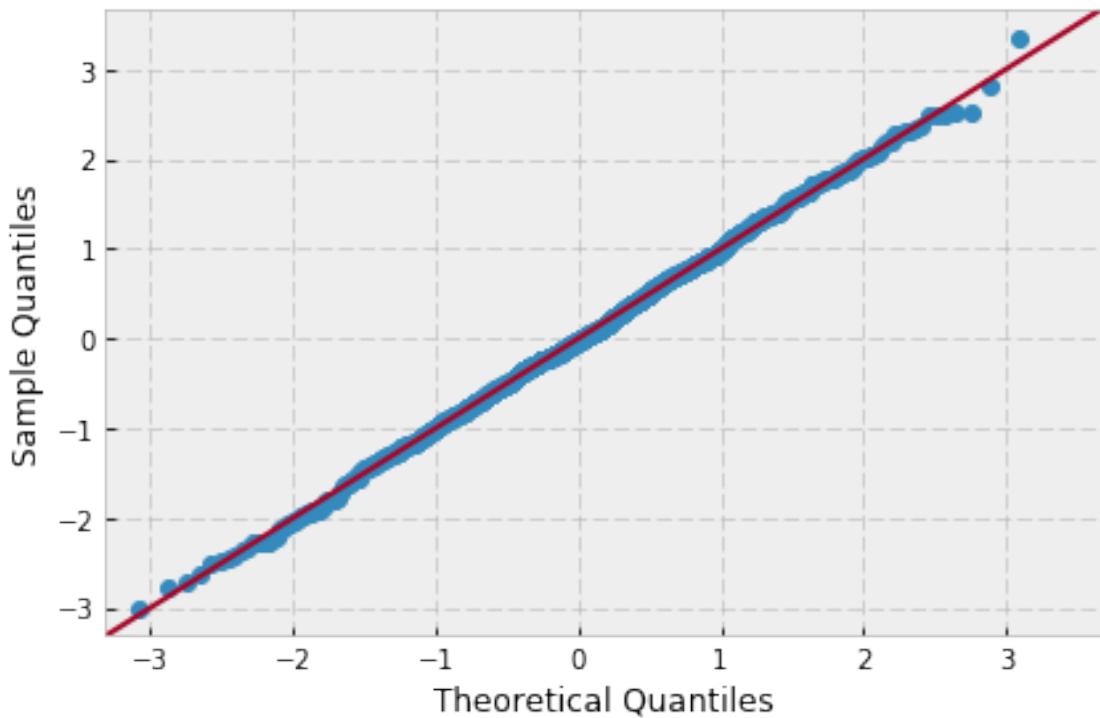


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=62.613439923823535, scale=4.873966167590694)
DescribeResult(nobs=1000, minmax=(48.688686409957022, 78.00989549153671)
               mean=62.613439923823535, variance=23.779325528347059,
               skewness=0.03645723479838222, kurtosis=-0.1769424524492189)
```

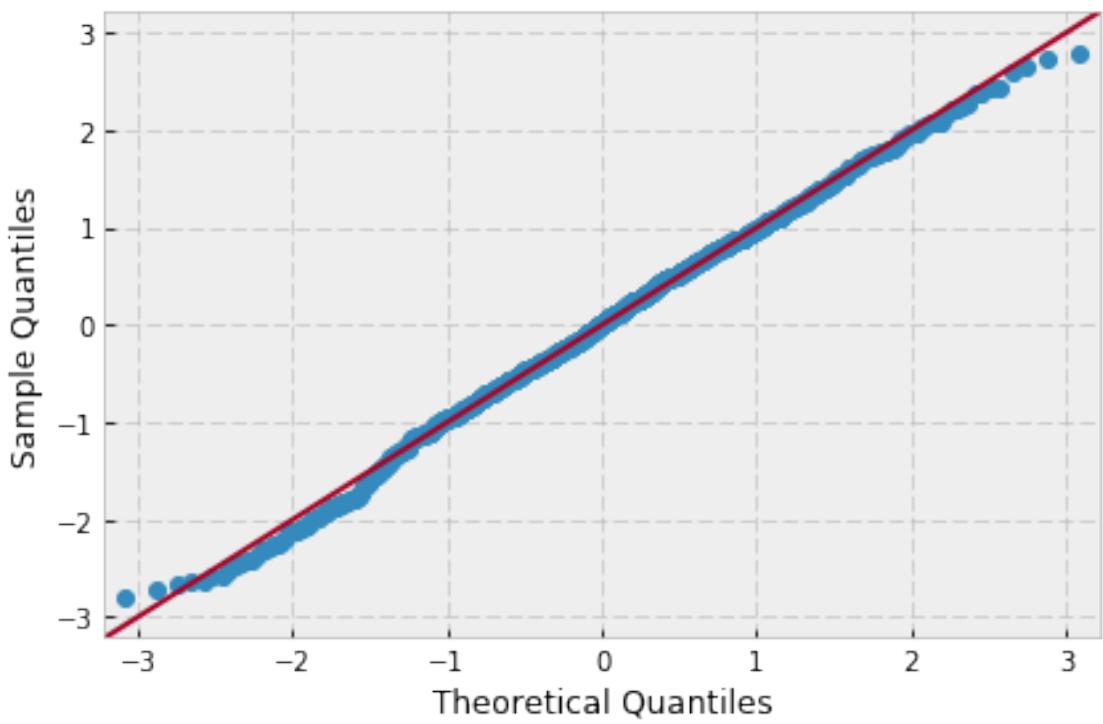
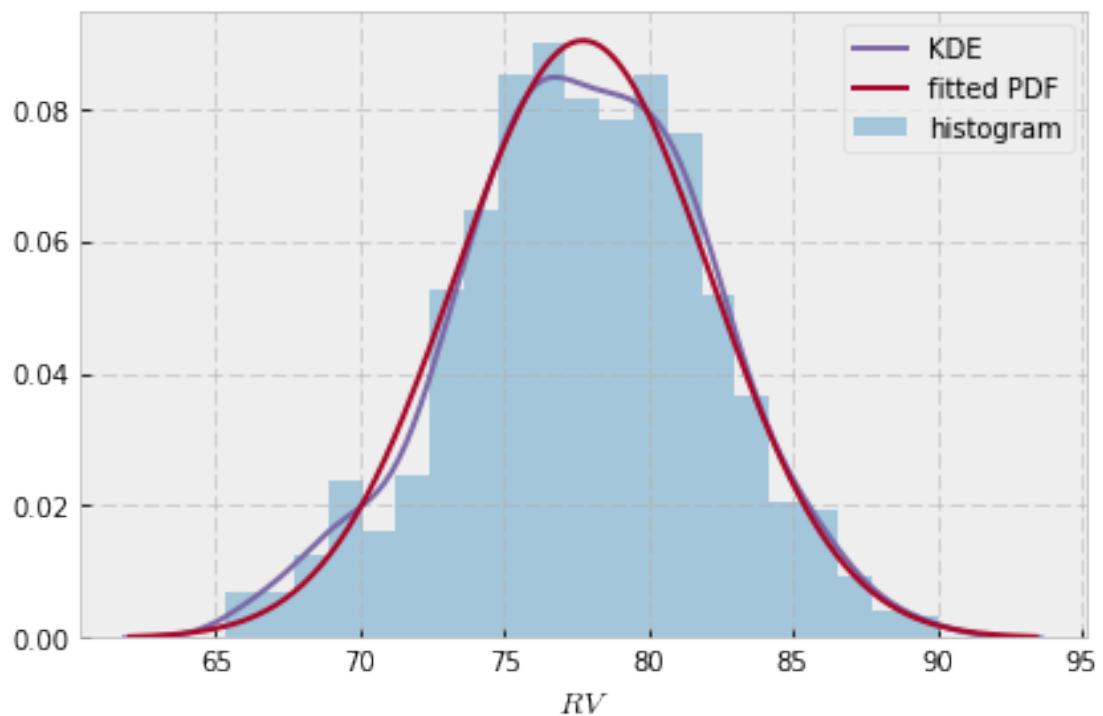


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(),loc=70.28158918342174, scale=4.605318935512461)
DescribeResult(nobs=1000, minmax=(56.472150131445055, 85.680418878658685)
               mean=70.281589183421744, variance=21.230192690480102,
               skewness=0.015555349127356084, kurtosis=-0.10806168329142496)
```

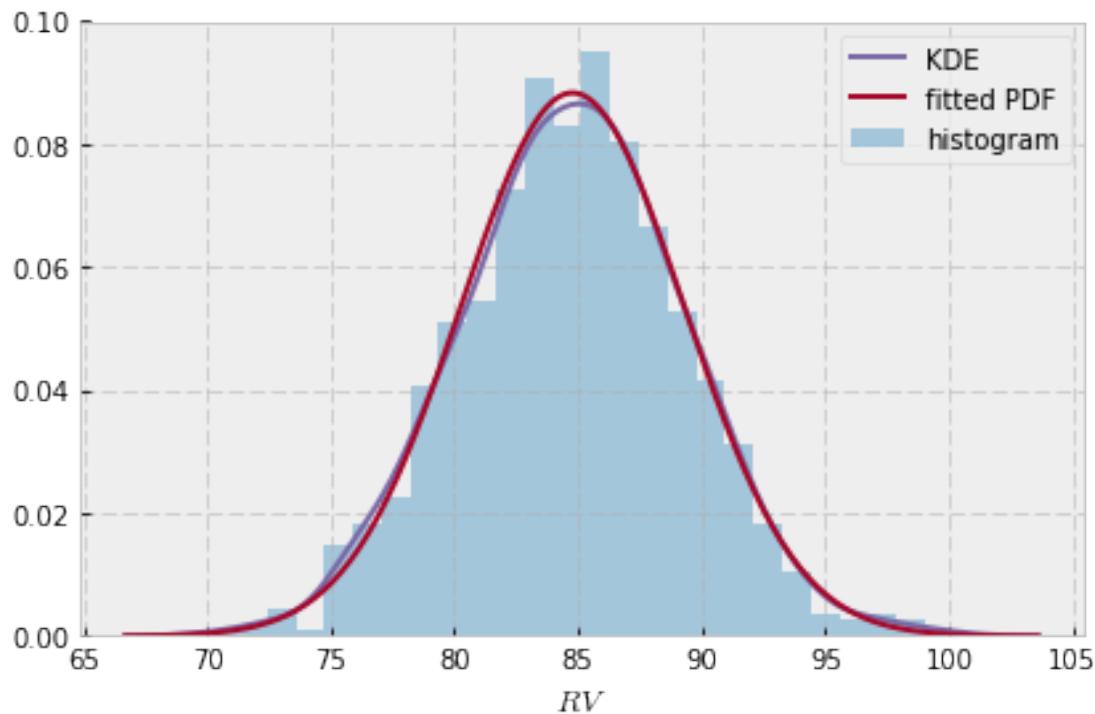


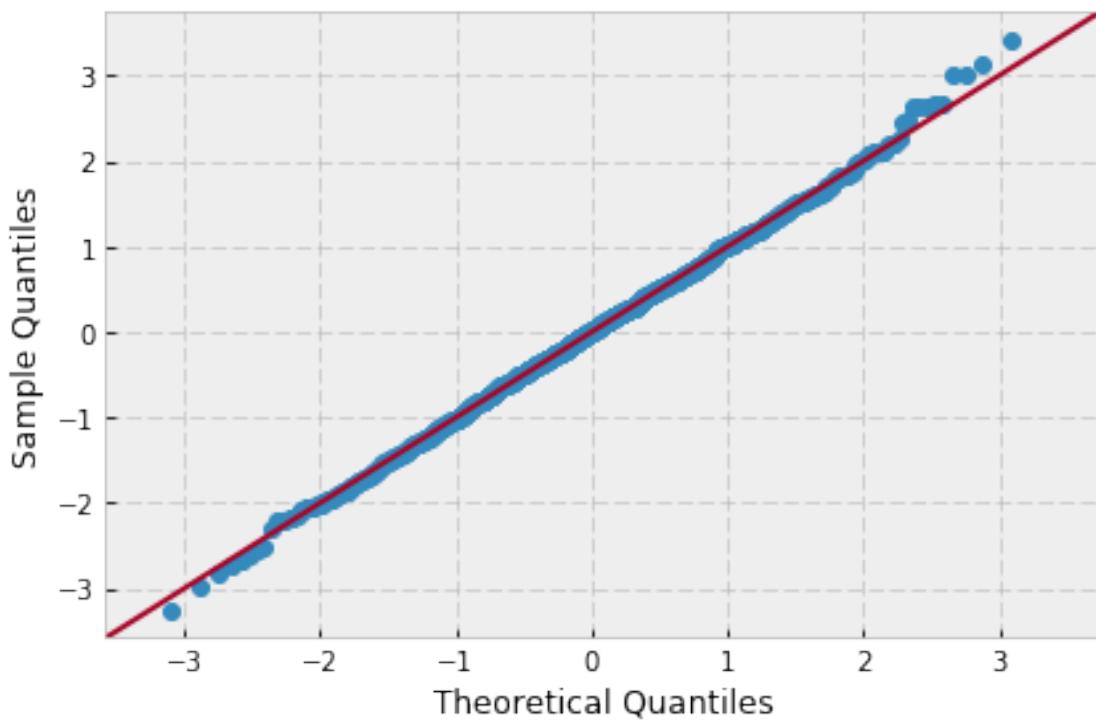


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(),loc=77.72022936241277, scale=4.421312541232051)
DescribeResult(nobs=1000, minmax=(65.35550597927714, 90.072358520659634)
               mean=77.720229362412766, variance=19.567572159415231,
               skewness=-0.1214190666418257, kurtosis=-0.11258691412347321)
```

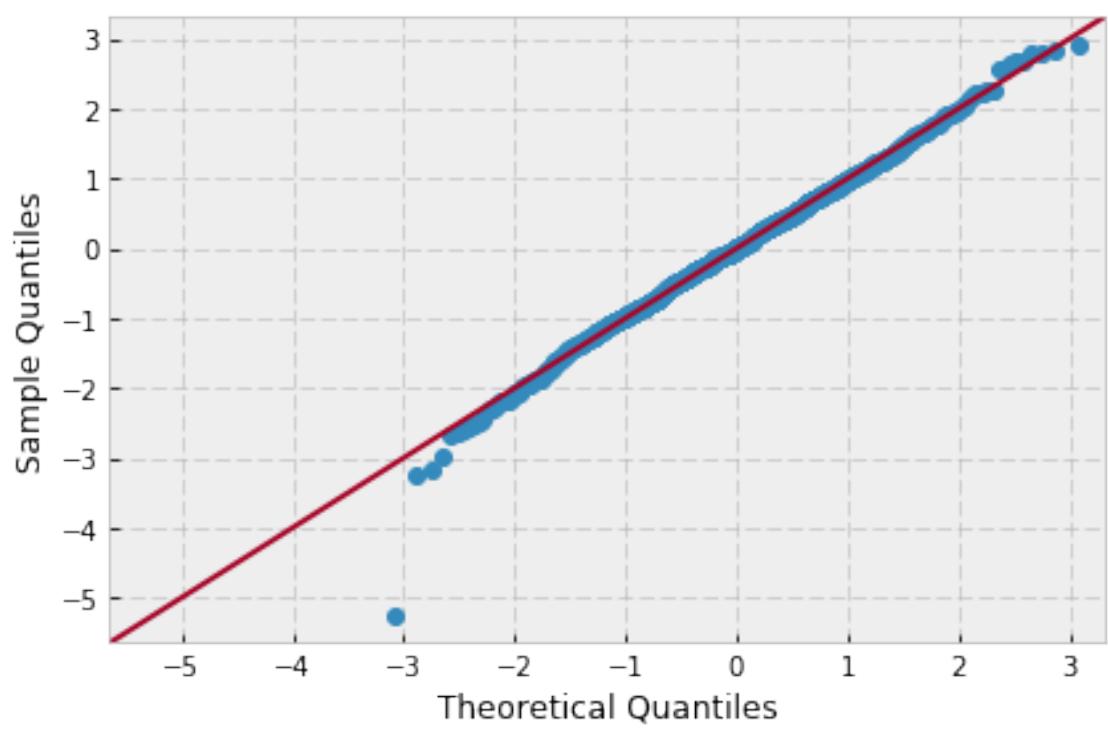
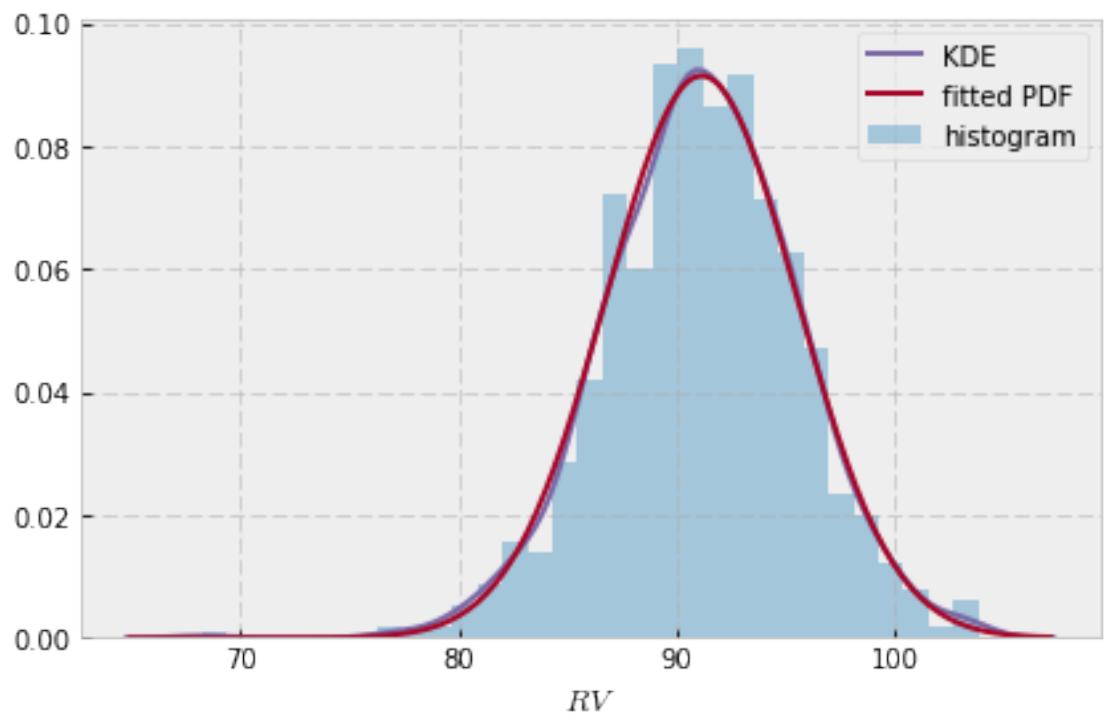


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(),loc=84.77397328399014, scale=4.52005557904689)
DescribeResult(nobs=1000, minmax=(70.10808535482316, 100.18209128685776)
               mean=84.773973283990145, variance=20.451353791464381,
               skewness=0.01426528060830885, kurtosis=0.08292079462216151)
```

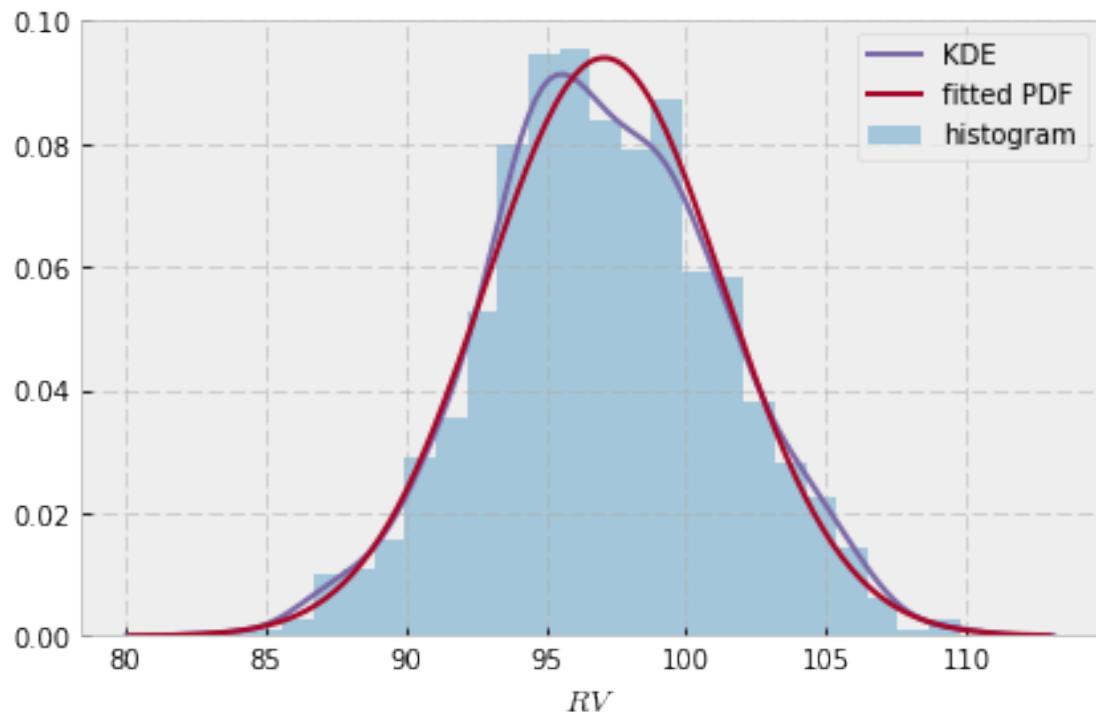


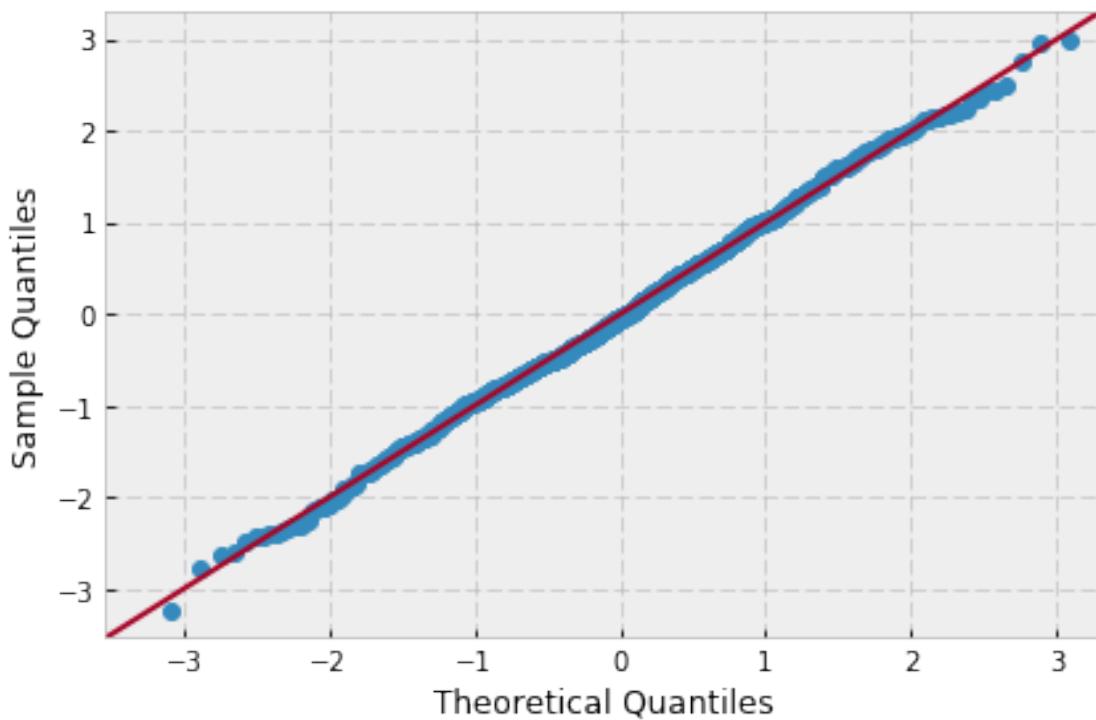


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(),loc=91.12692732433489, scale=4.365276620164248)
DescribeResult(nobs=1000, minmax=(68.239621267233574, 103.80859752080968)
               mean=91.126927324334886, variance=19.07471468523784,
               skewness=-0.17891452797984672, kurtosis=0.7279076491405982)
```

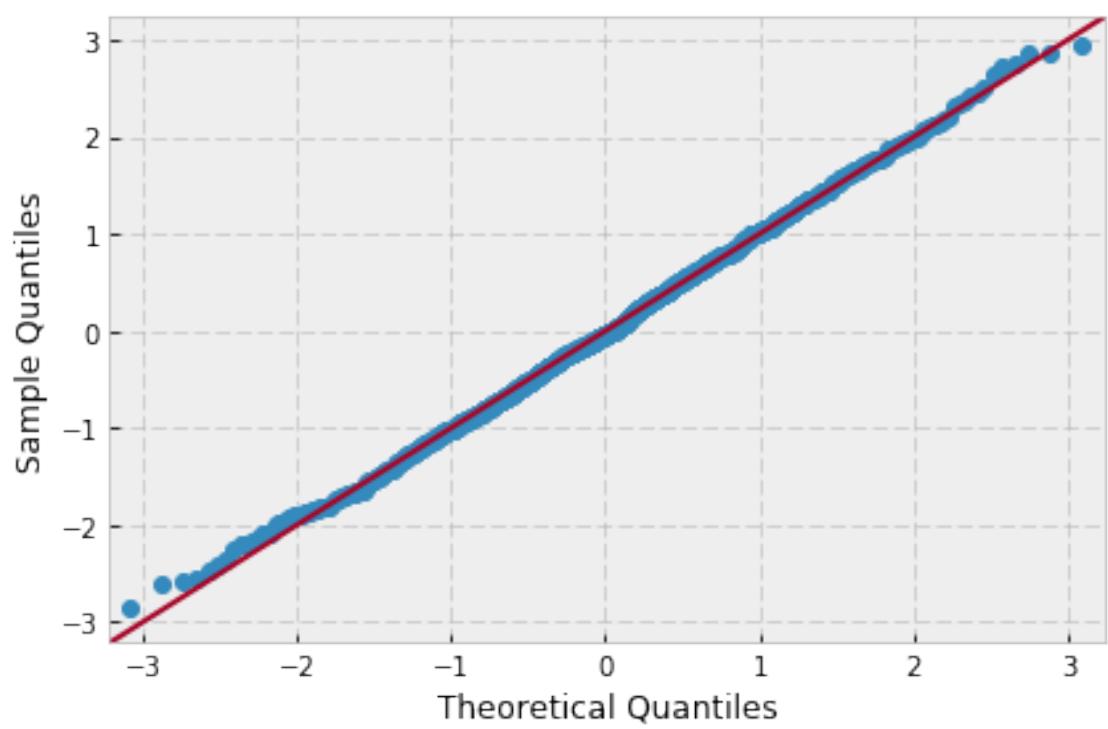
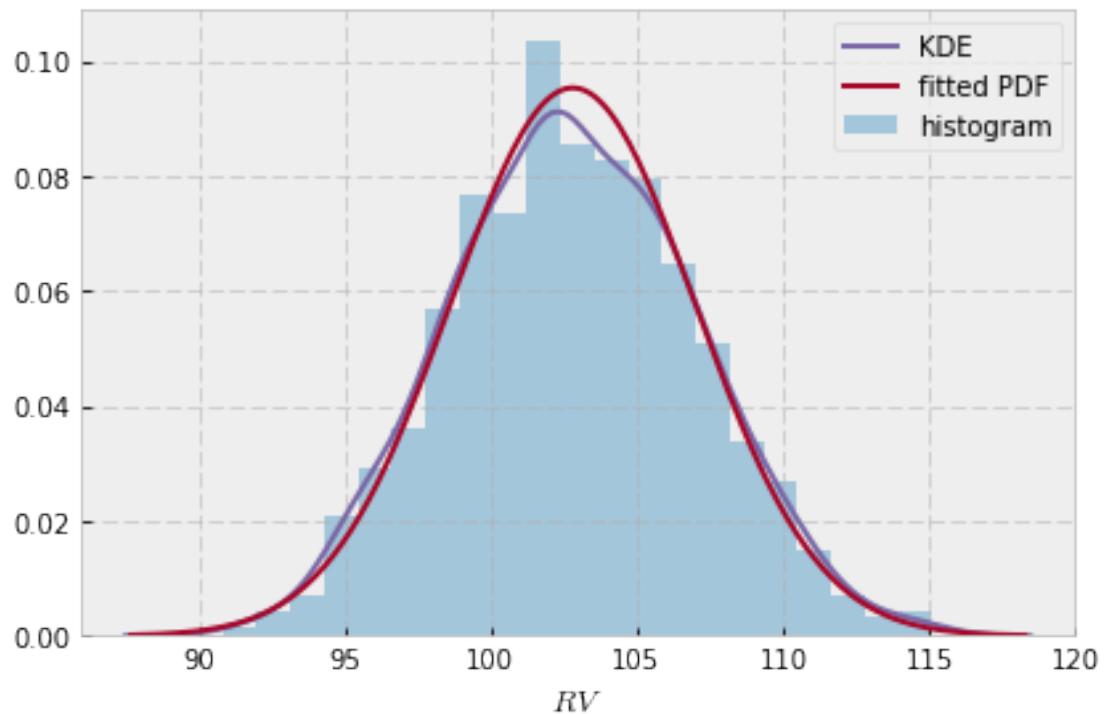


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(),loc=97.07809915915477, scale=4.245575589699727)
DescribeResult(nobs=1000, minmax=(83.394140256351321, 109.75180155509851)
               mean=97.078099159154775, variance=18.042955042897081,
               skewness=0.040131009370359516, kurtosis=-0.1331137587744493)
```

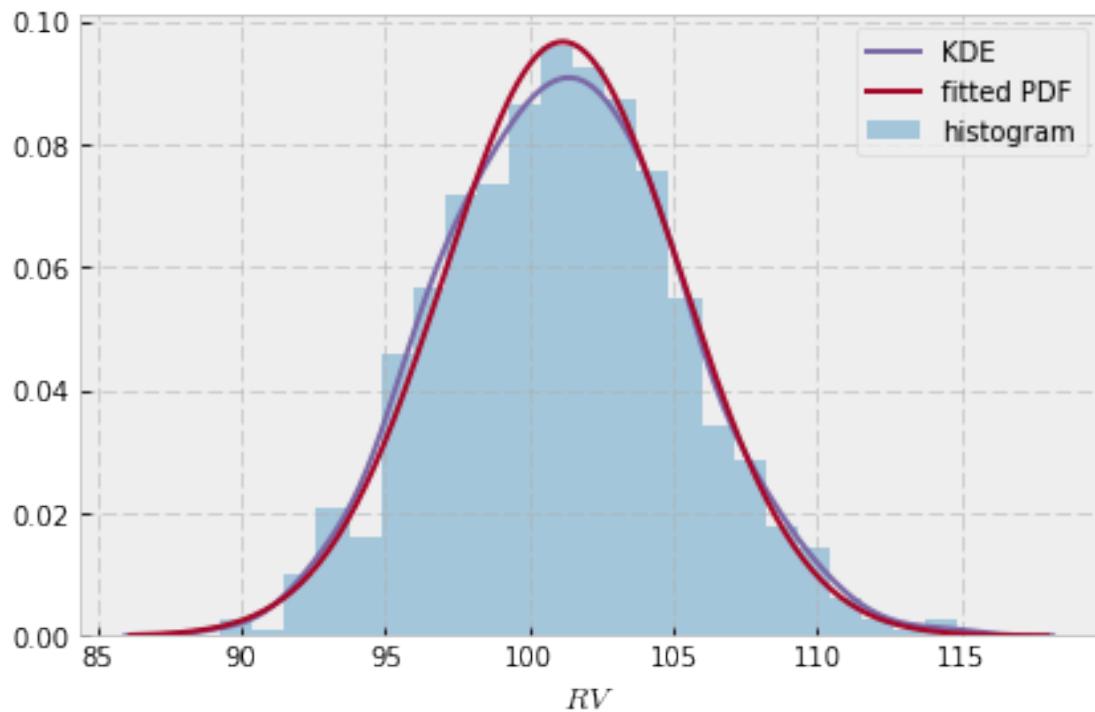


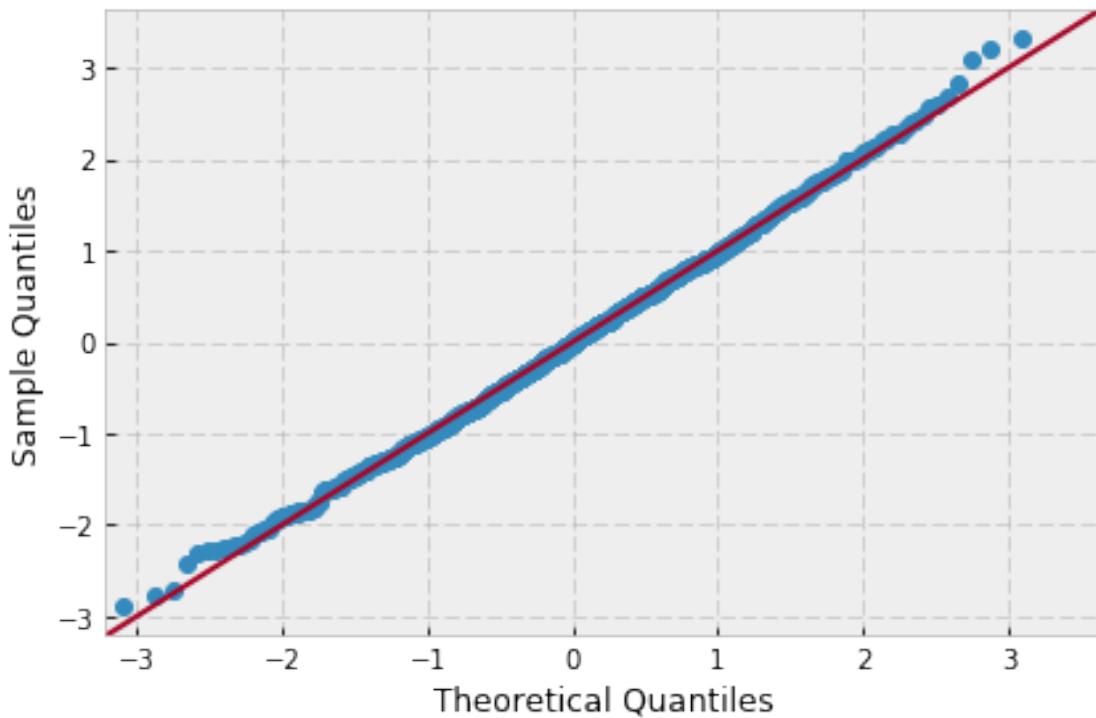


```
variable name: r
variable value: 3.0
distribution: normal(shape=(),loc=102.80632920641915, scale=4.186404137277915)
DescribeResult(nobs=1000, minmax=(90.822516020447438, 115.13578344714658)
               mean=102.80632920641915, variance=17.54352312374138,
               skewness=0.07095836949651567, kurtosis=-0.2393216862098848)
```

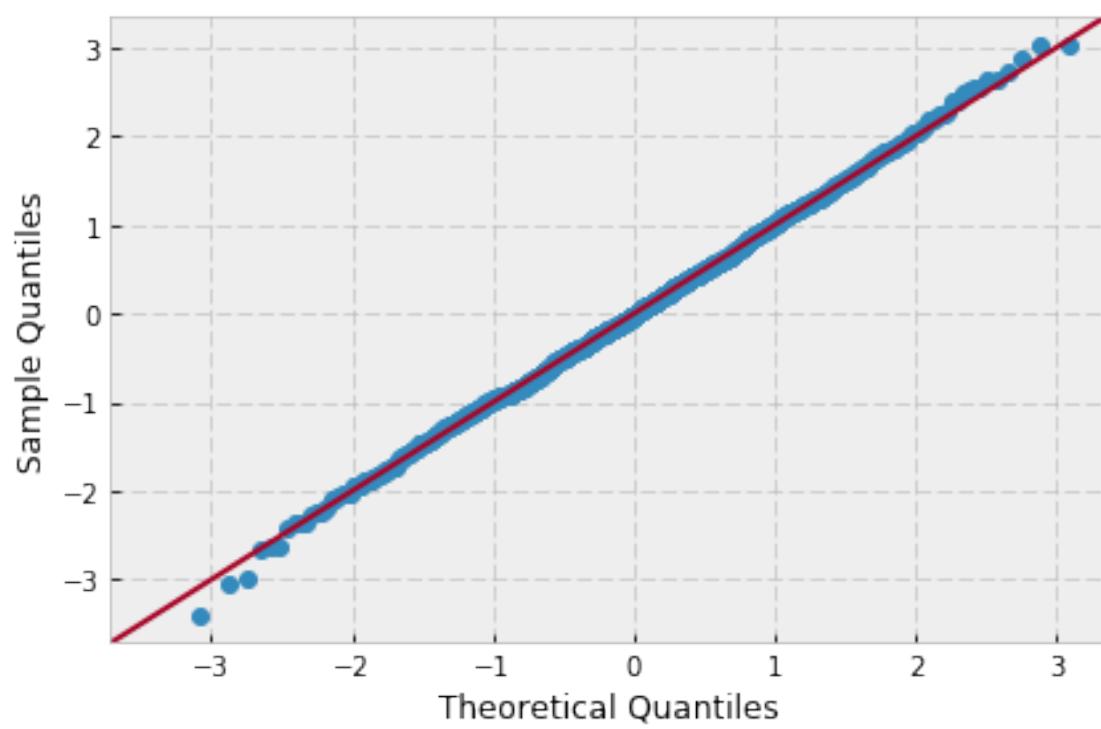
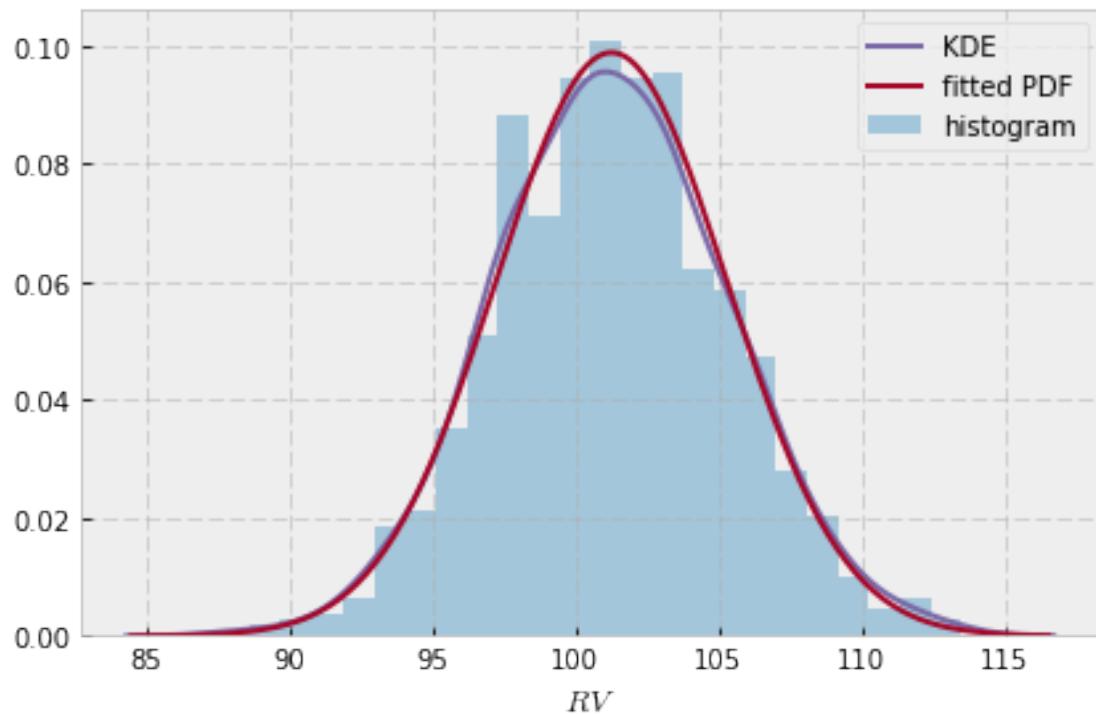


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(), loc=101.16707411529069, scale=4.129002823051369)
DescribeResult(nobs=1000, minmax=(89.276193464270492, 114.88518797794389)
               mean=101.16707411529069, variance=17.065730042808983,
               skewness=0.11886525372948281, kurtosis=-0.1057825455327821)
```

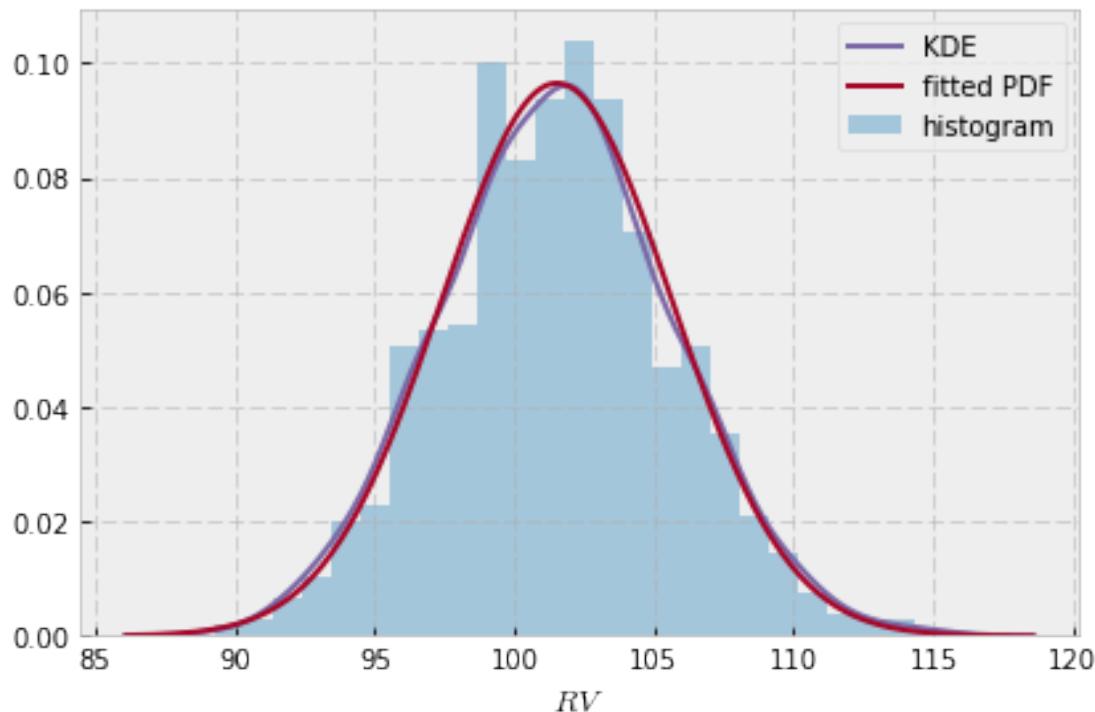


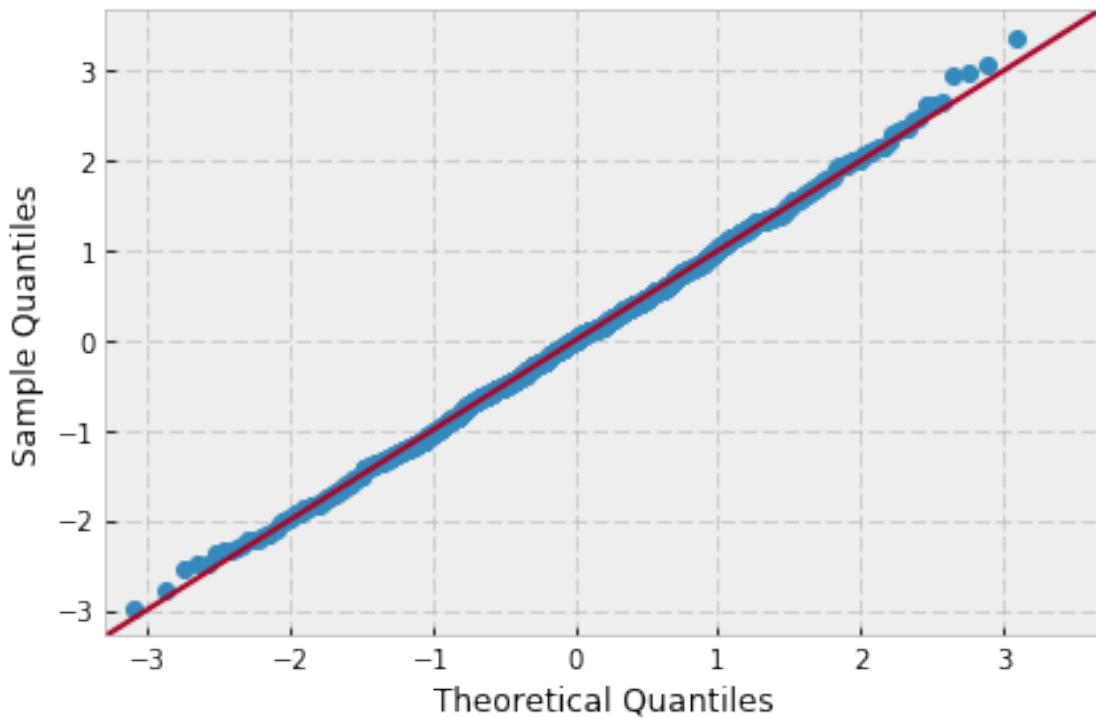


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(),loc=101.25382650648379, scale=4.03899753302194)
DescribeResult(nobs=1000, minmax=(87.548071989906518, 113.46675036822299)
               mean=101.25382650648379, variance=16.329830902659971,
               skewness=0.04001655257963342, kurtosis=0.020255162540871563)
```

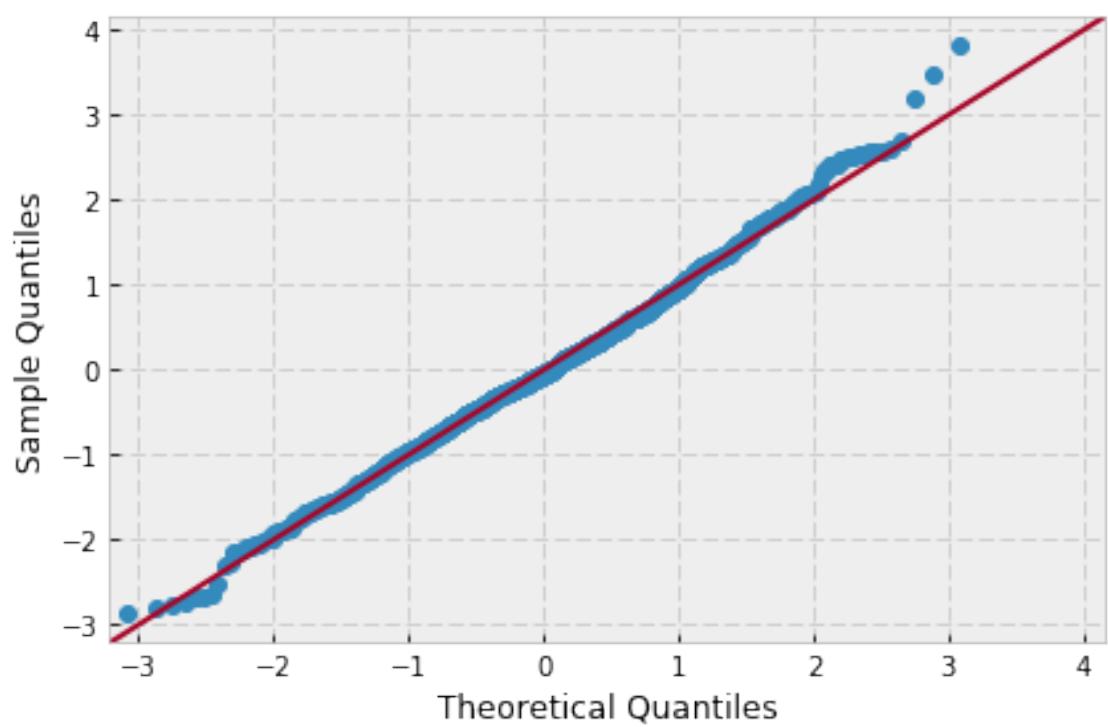
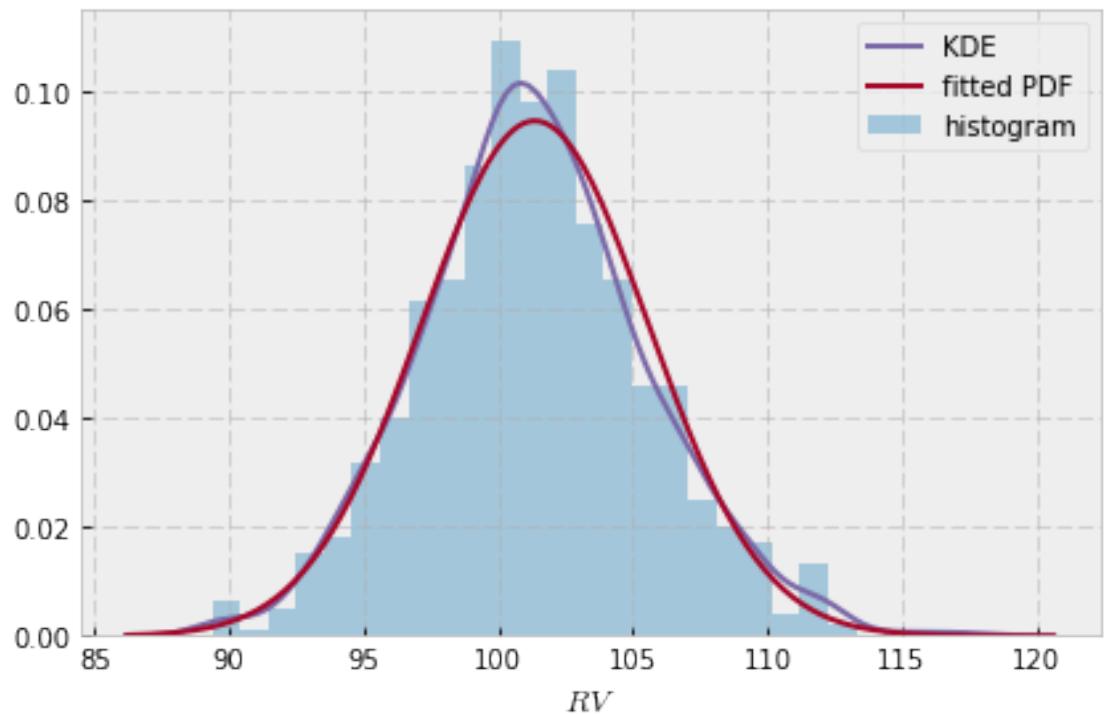


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=101.50937664401867, scale=4.135014813406515)
DescribeResult(nobs=1000, minmax=(89.220189382315198, 115.40909541573504)
               mean=101.50937664401867, variance=17.115462970061383,
               skewness=0.07998571874613579, kurtosis=-0.047882623631088084)
```

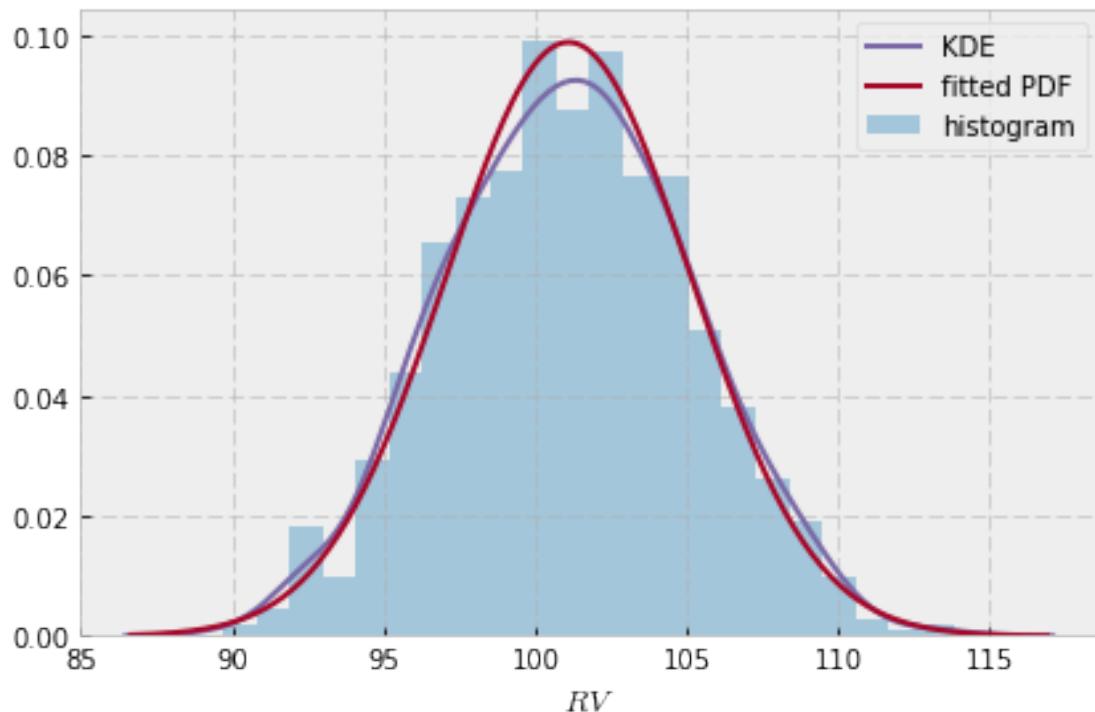


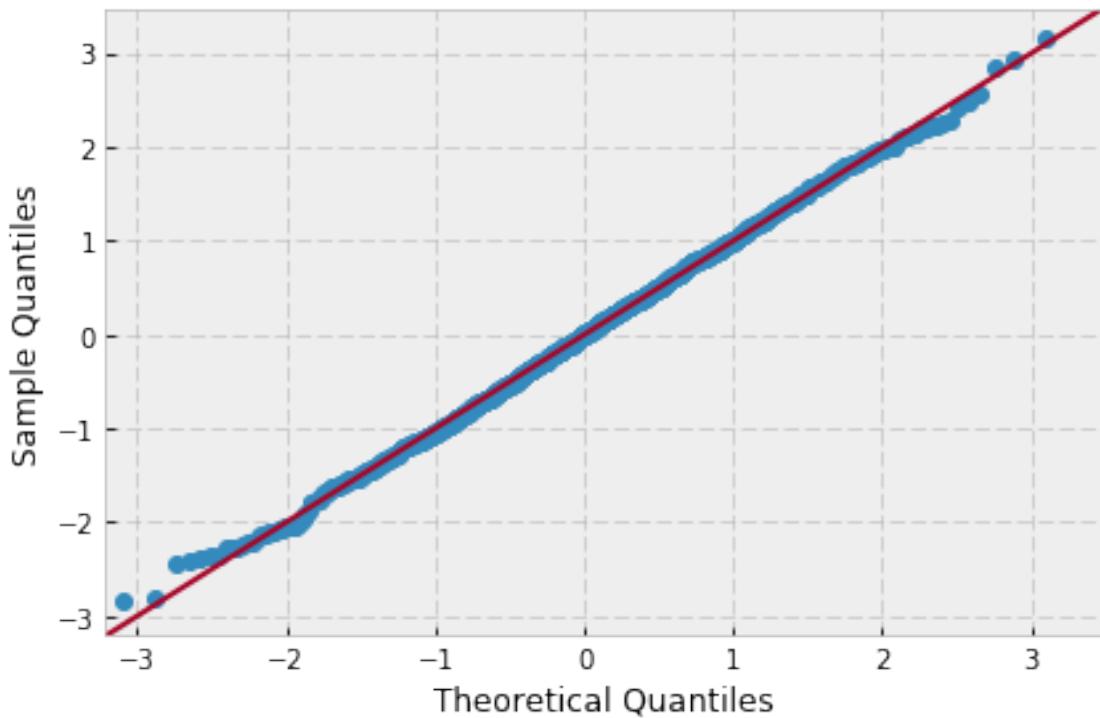


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=101.36042499860415, scale=4.216248093195603)
DescribeResult(nobs=1000, minmax=(89.362093876270606, 117.46755642666183)
               mean=101.36042499860415, variance=17.794542525901456,
               skewness=0.19486114752527411, kurtosis=0.2864522672647576)
```

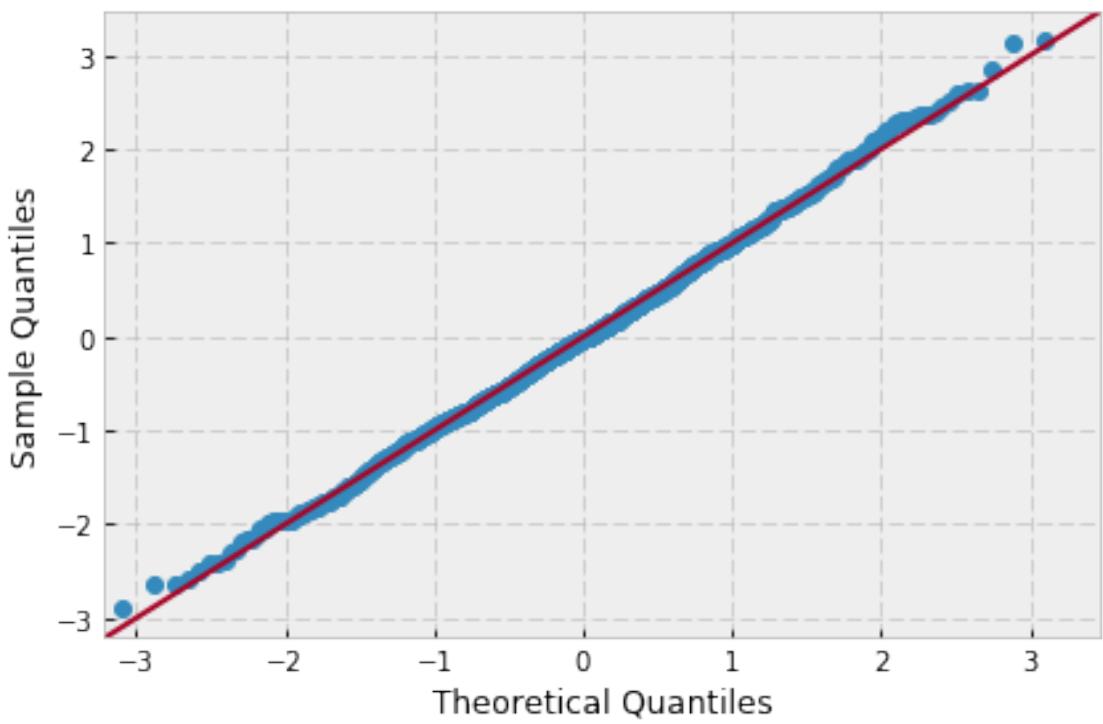
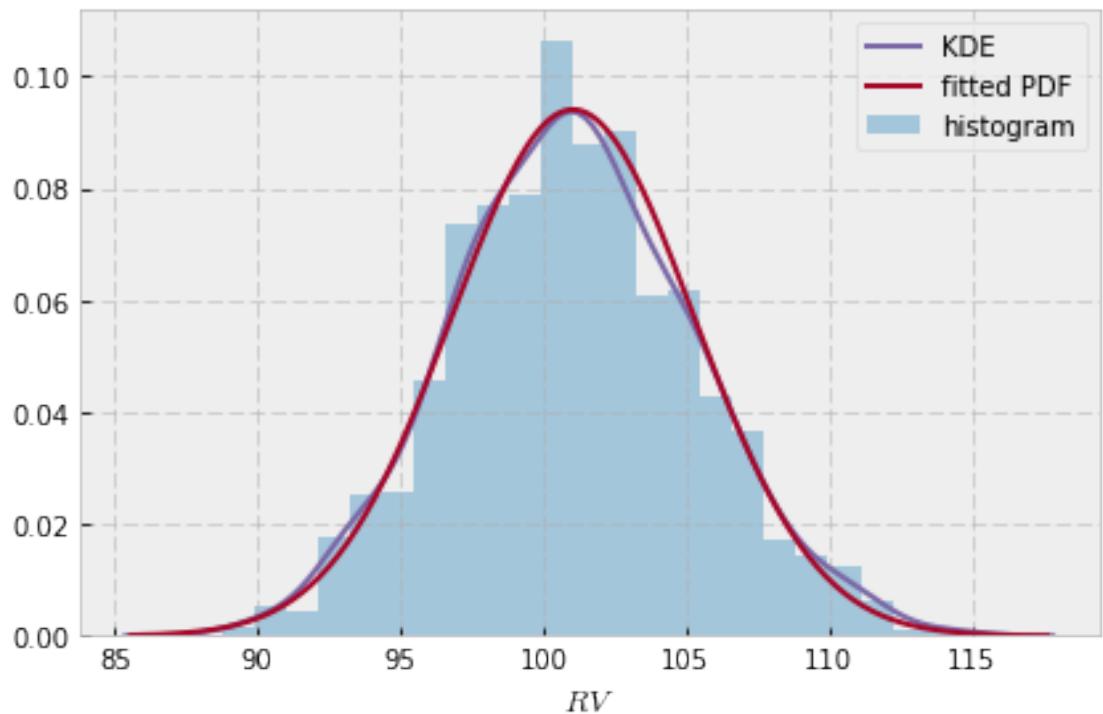


```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(),loc=101.10475492847465, scale=4.036458862612048)
DescribeResult(nobs=1000, minmax=(89.700345427878787, 113.86671651106936)
               mean=101.10475492847465, variance=16.309309459018369,
               skewness=0.025815977181383388, kurtosis=-0.26617151692700336)
```

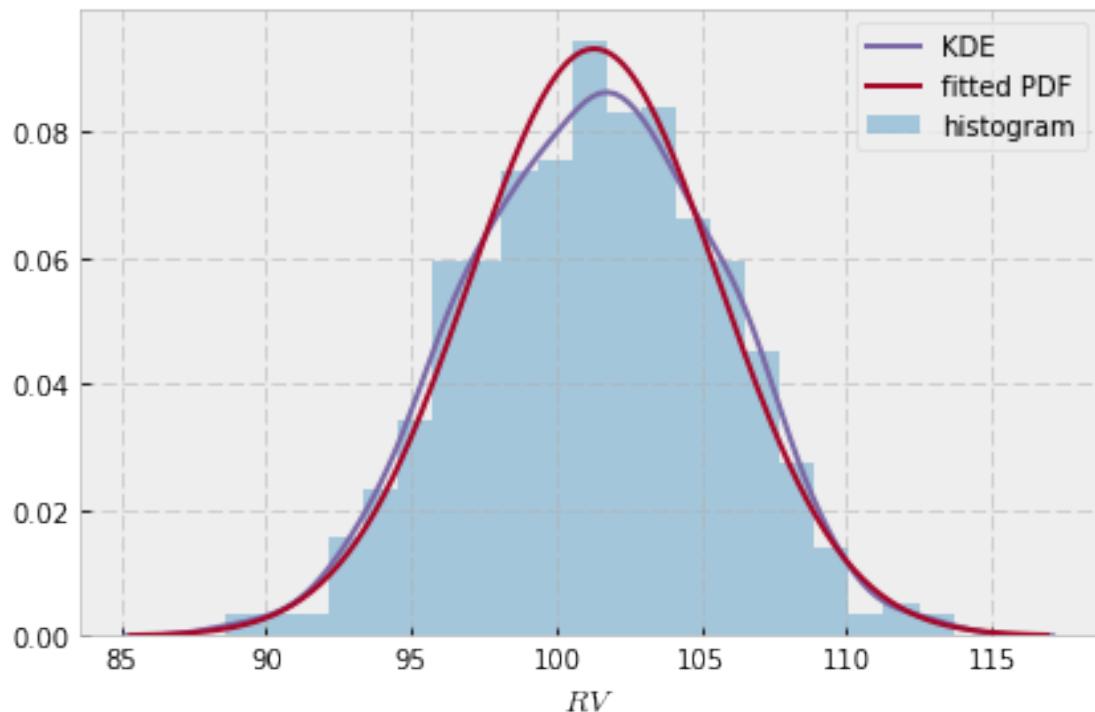


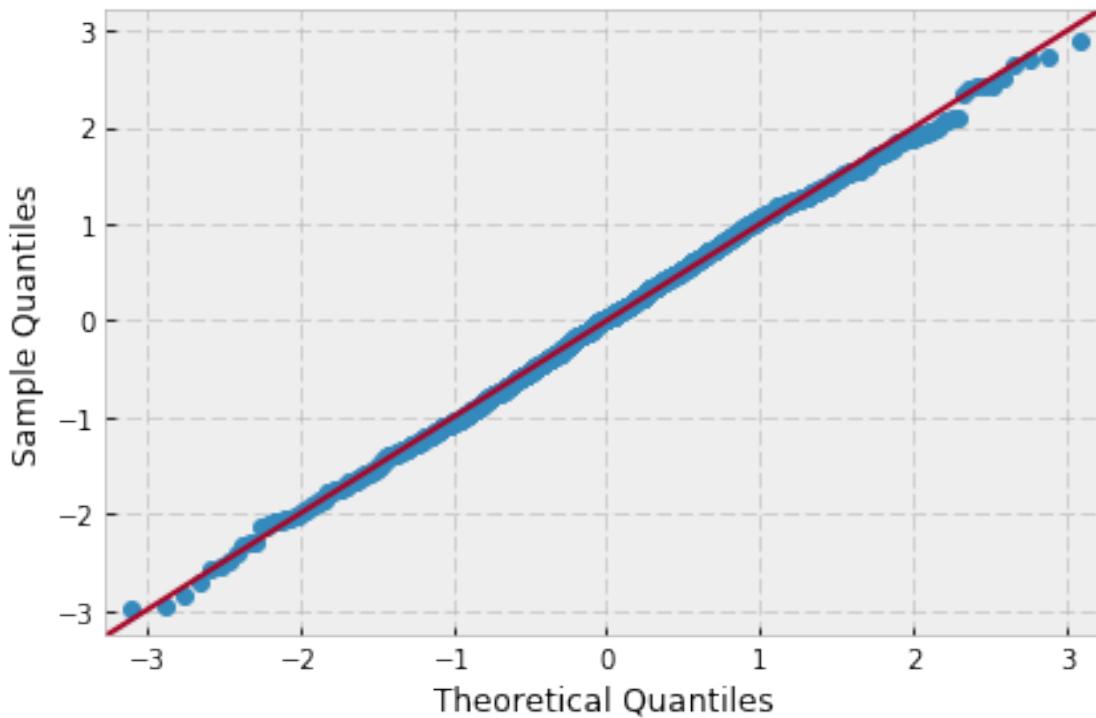


```
variable name: sigma
variable value: 0.0666666666666667
distribution: normal(shape=(),loc=101.04374162580568, scale=4.240211480936028)
DescribeResult(nobs=1000, minmax=(88.764859185756364, 114.45022652565935)
               mean=101.04374162580568, variance=17.997390793855558,
               skewness=0.11229308556448422, kurtosis=-0.06549810375534326)
```

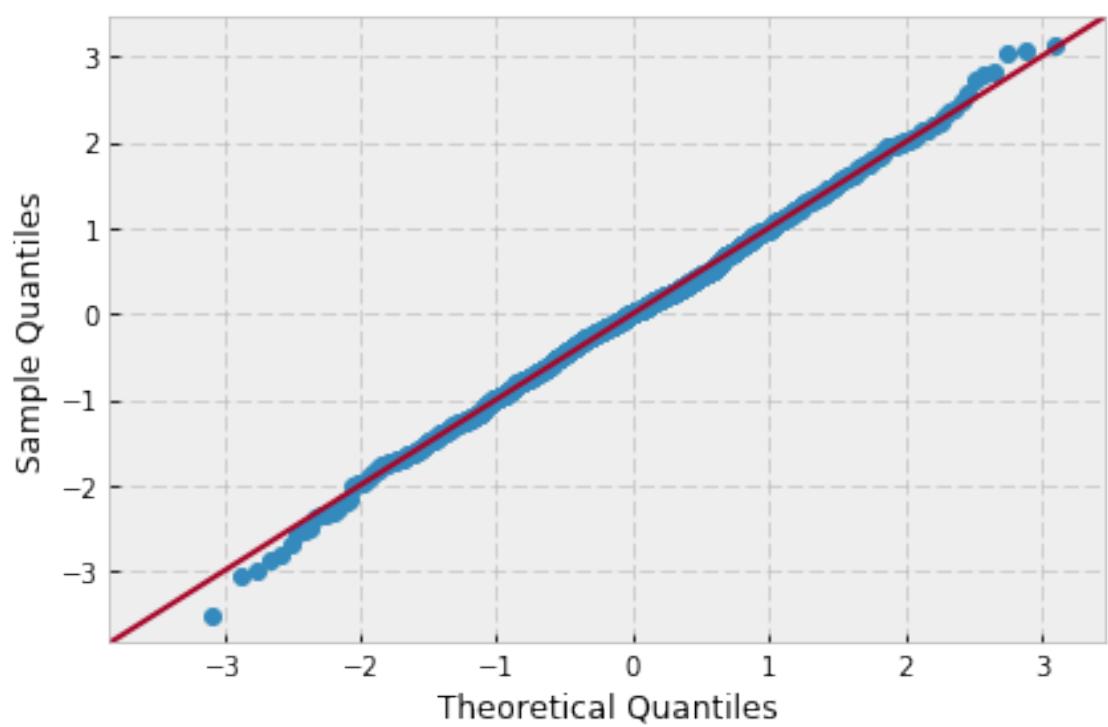
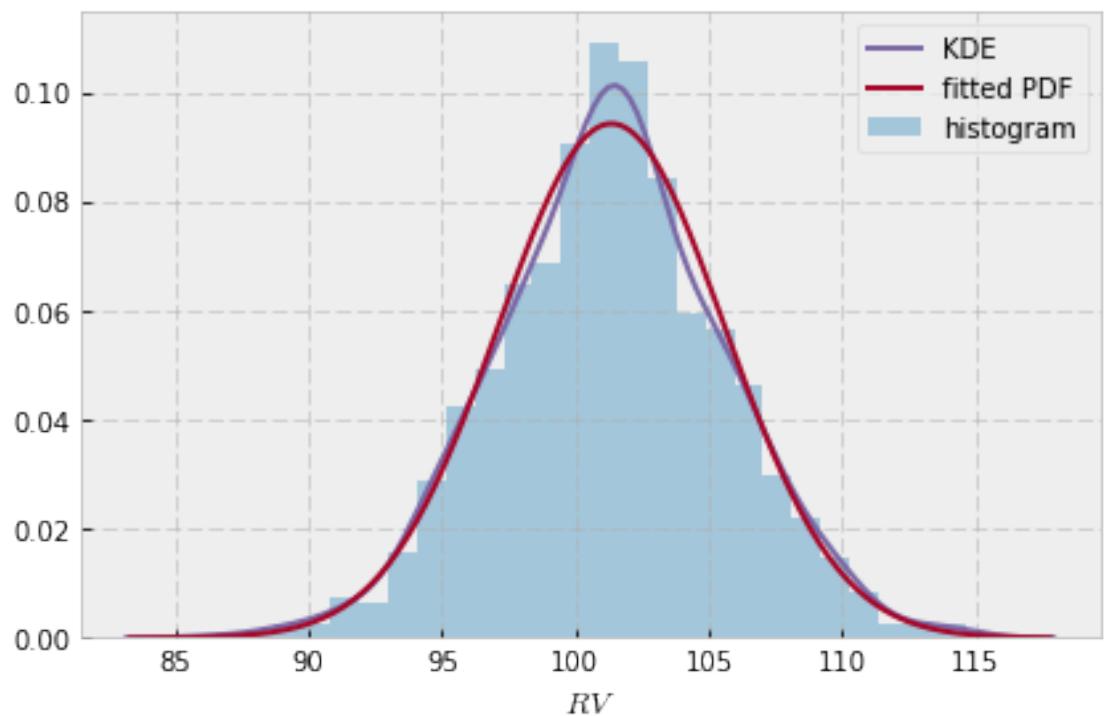


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(),loc=101.30625903436065, scale=4.283434899322953)
DescribeResult(nobs=1000, minmax=(88.581668779665293, 113.67605820933456)
               mean=101.30625903436065, variance=18.366180717455286,
               skewness=-0.05666330864672879, kurtosis=-0.29819600908548693)
```

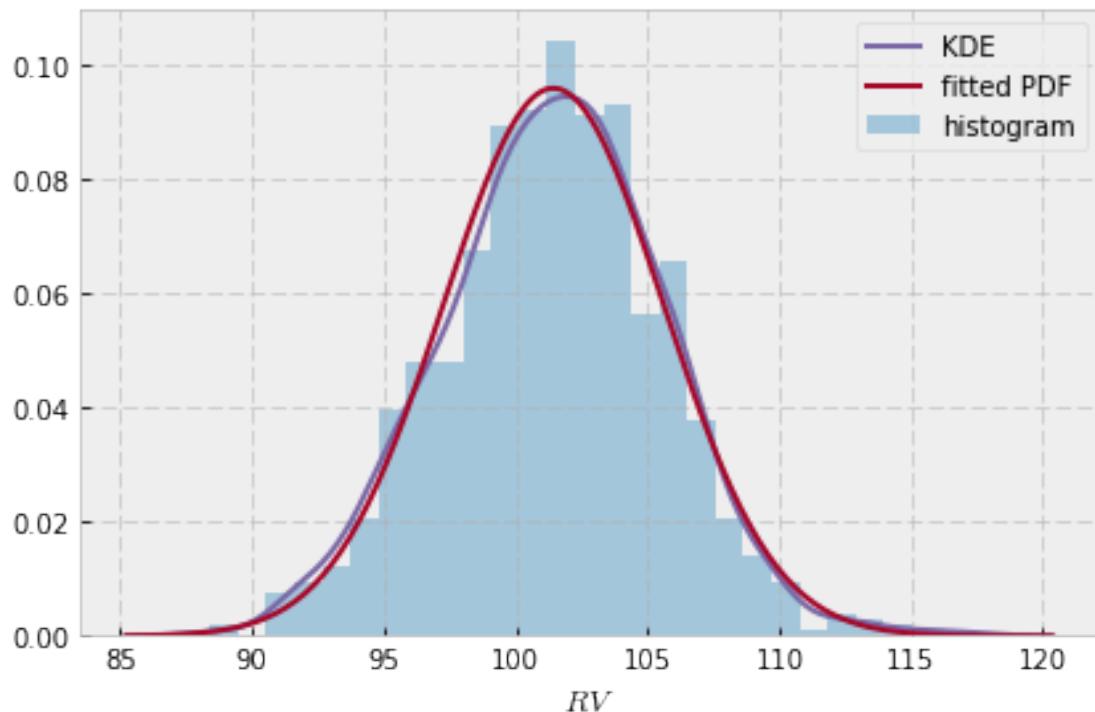


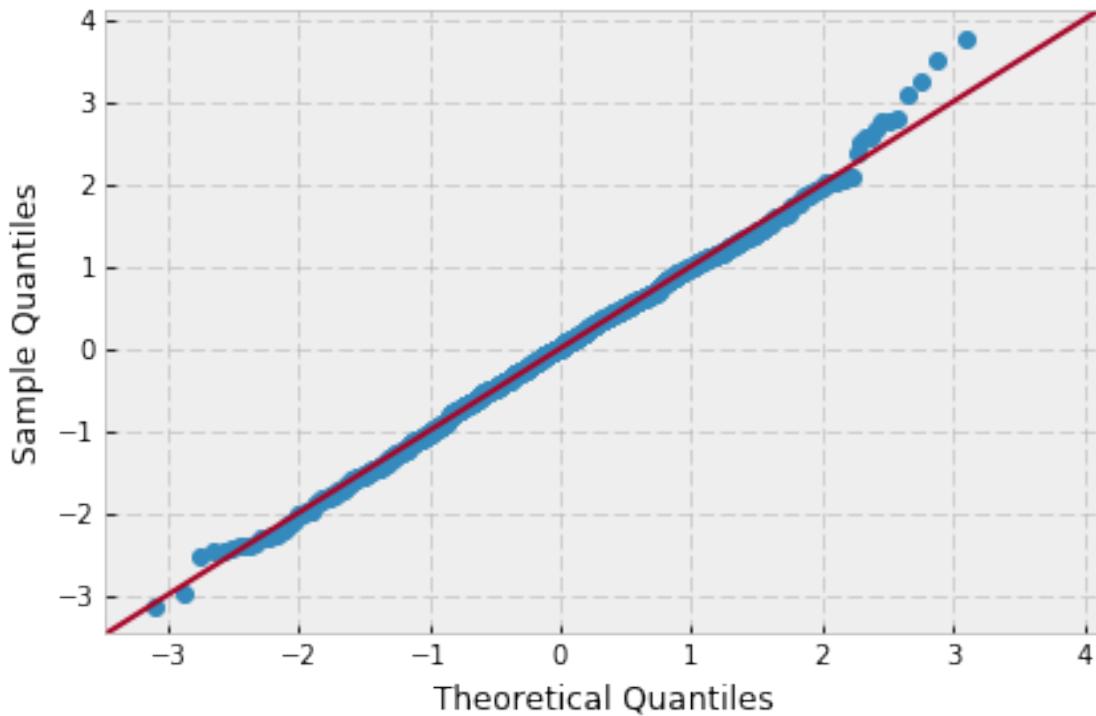


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(),loc=101.3469278489566, scale=4.235837715108321)
DescribeResult(nobs=1000, minmax=(86.496020630764406, 114.59832602558187)
               mean=101.3469278489566, variance=17.960281430164244,
               skewness=-0.0022627737350483287, kurtosis=0.16610093657567937)
```

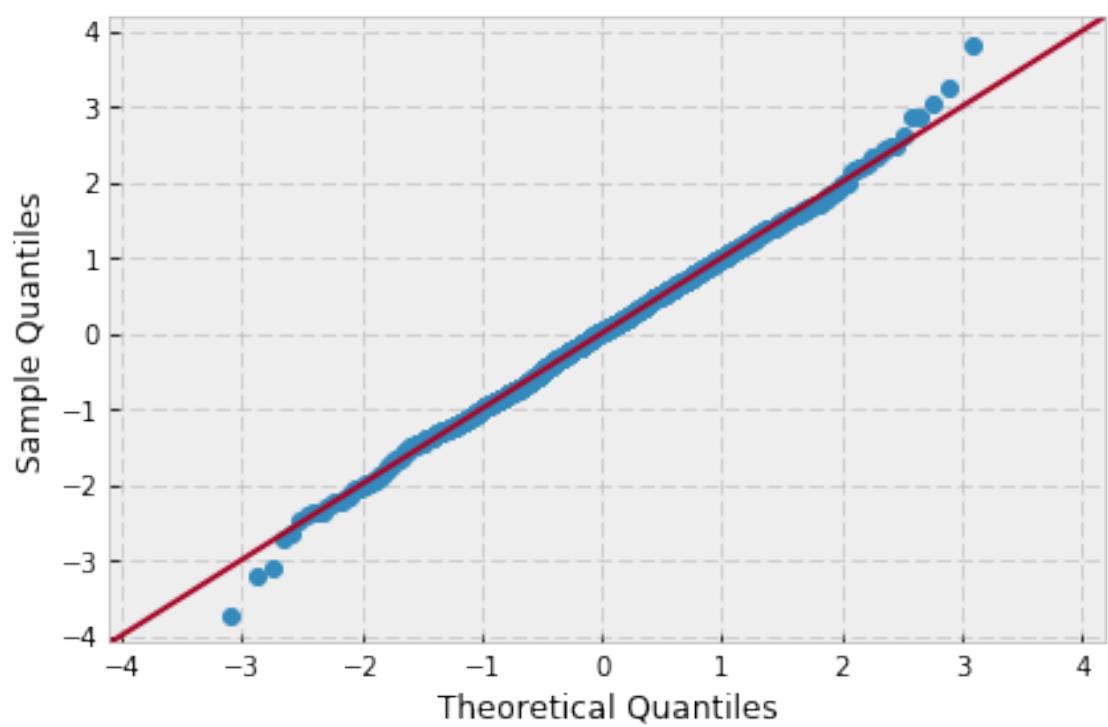
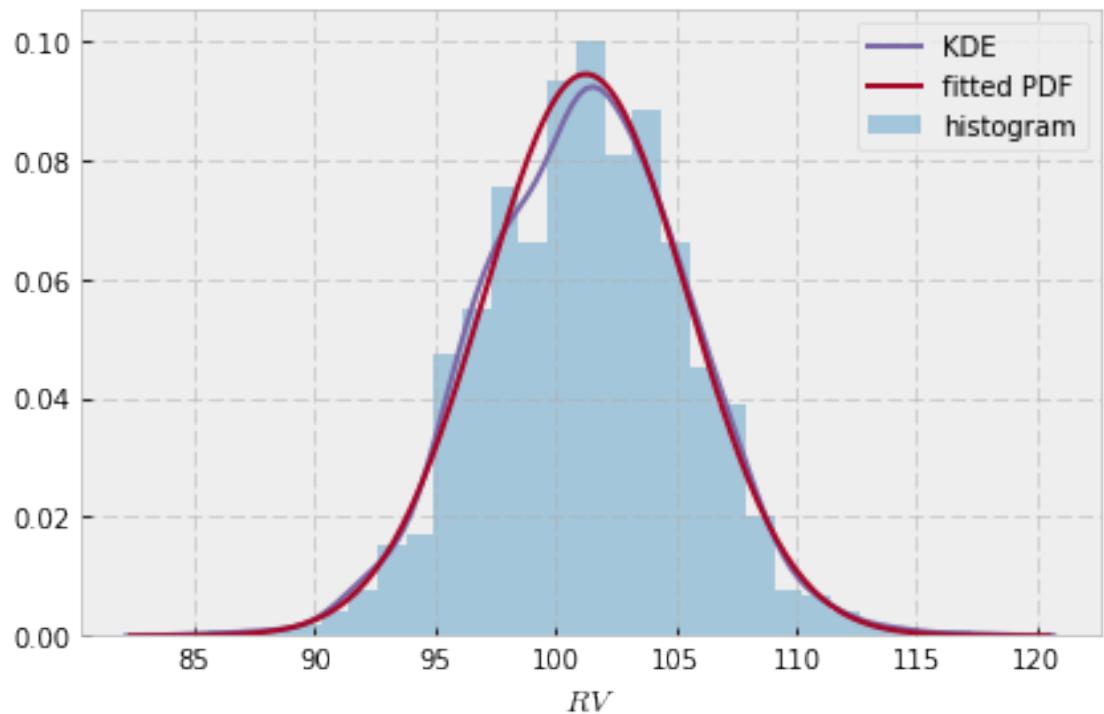


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=101.4306885448615, scale=4.166512139702129)
DescribeResult(nobs=1000, minmax=(88.404103473282419, 117.16442534304852)
               mean=101.4306885448615, variance=17.377200610896107,
               skewness=0.018898887632125792, kurtosis=0.20353049672607648)
```

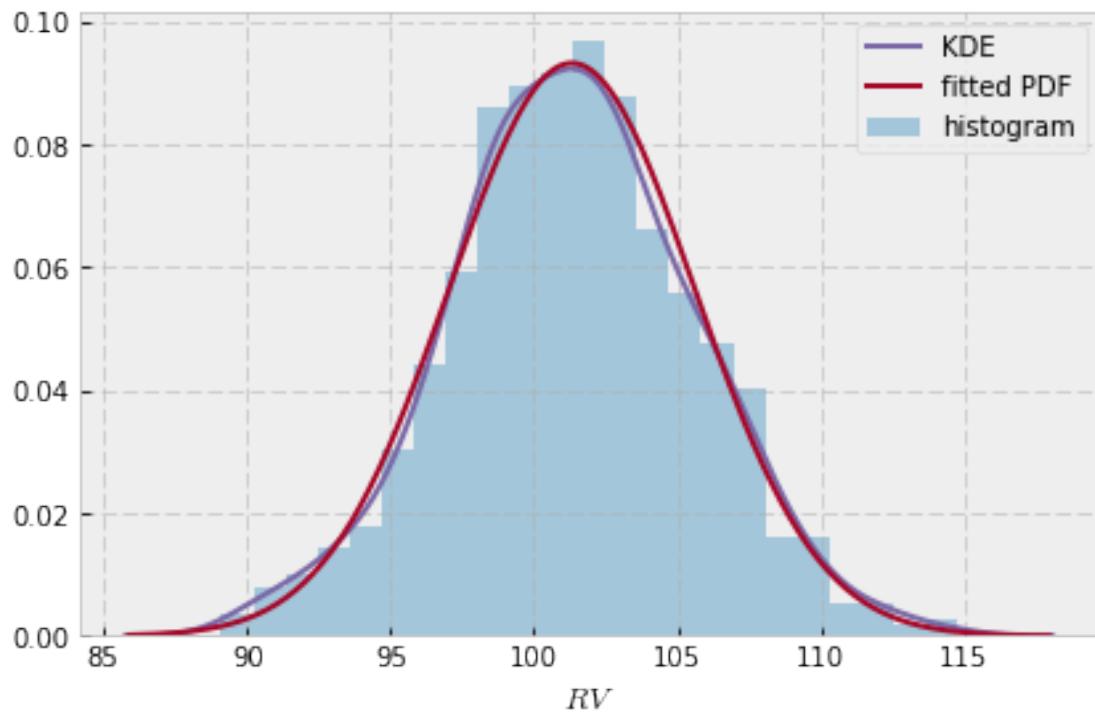


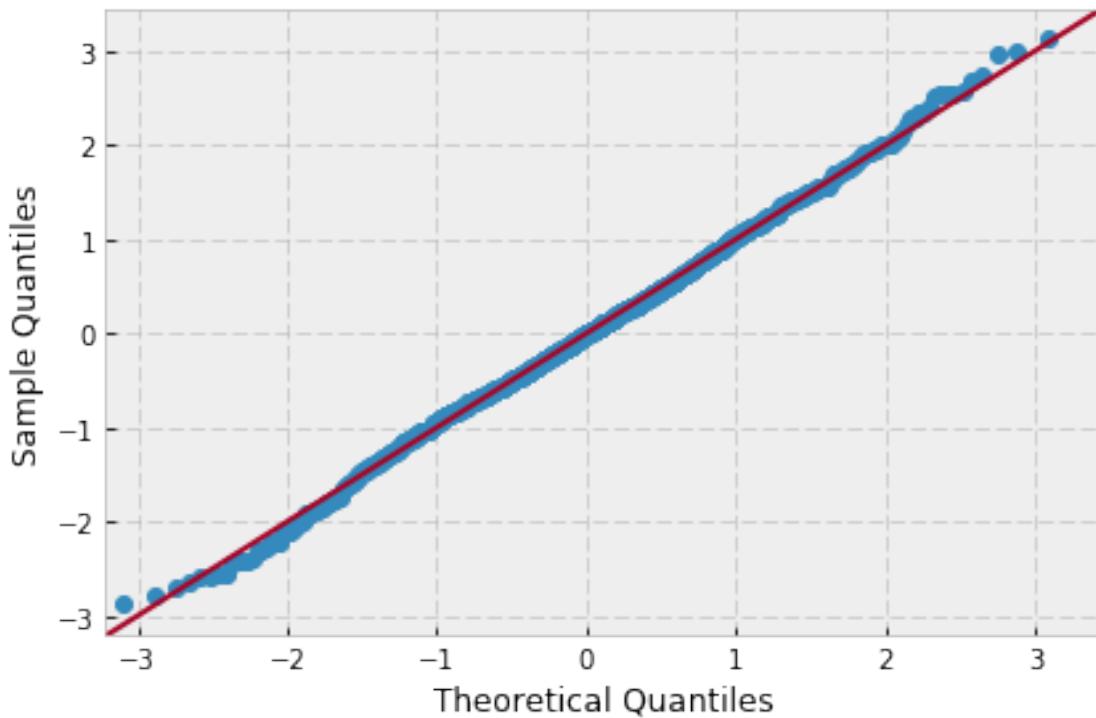


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(),loc=101.26700483058498, scale=4.220533390856076)
DescribeResult(nobs=1000, minmax=(85.559027579456242, 117.32641967356341)
               mean=101.26700483058498, variance=17.830732836167254,
               skewness=-0.0004824218991789193, kurtosis=0.19975426033022048)
```

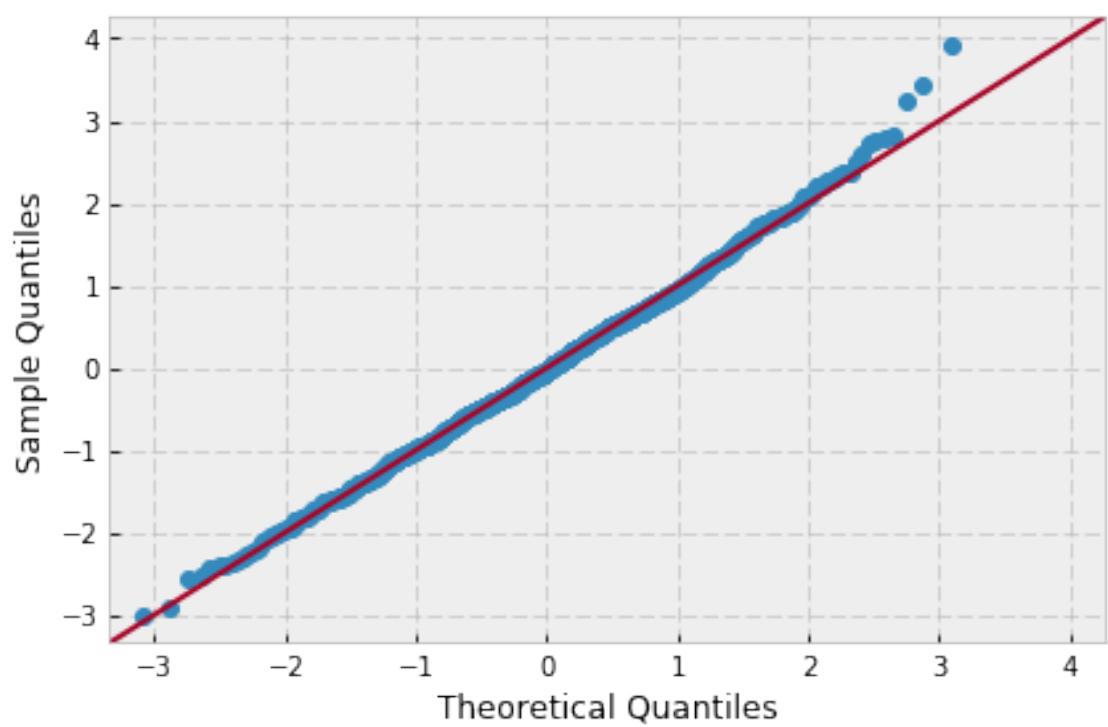
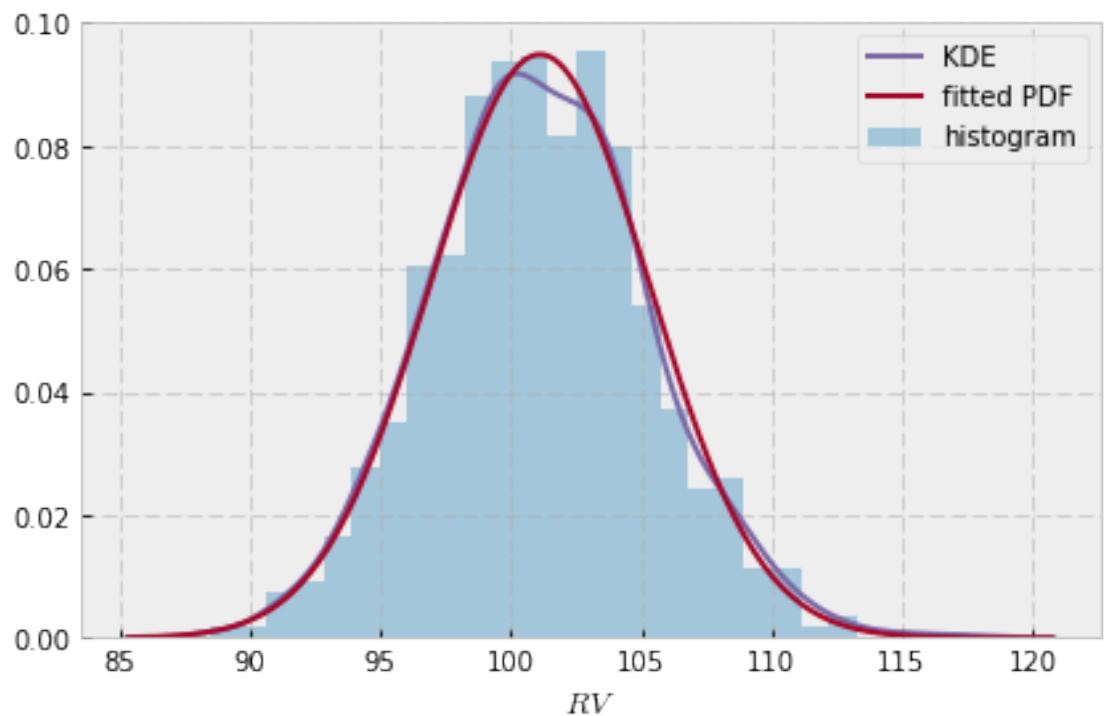


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=101.31719607488004, scale=4.279947104668575)
DescribeResult(nobs=1000, minmax=(89.099129471082122, 114.73472667518823)
               mean=101.31719607488004, variance=18.336283502263186,
               skewness=0.023578016683336506, kurtosis=0.05961327422201146)
```

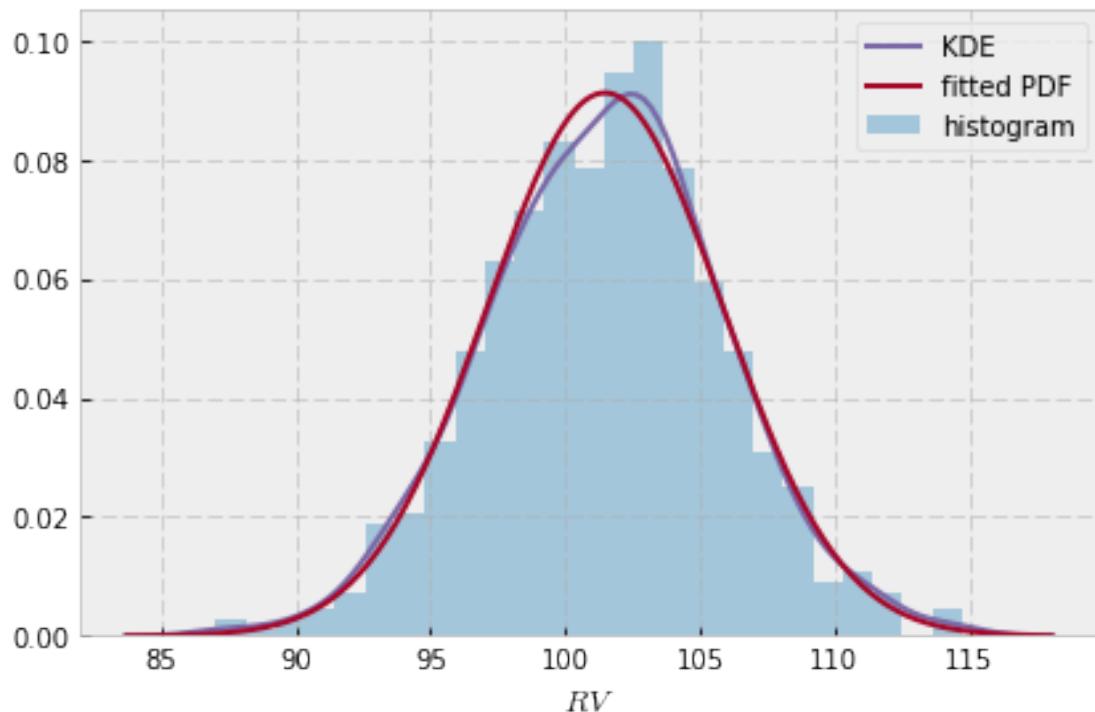


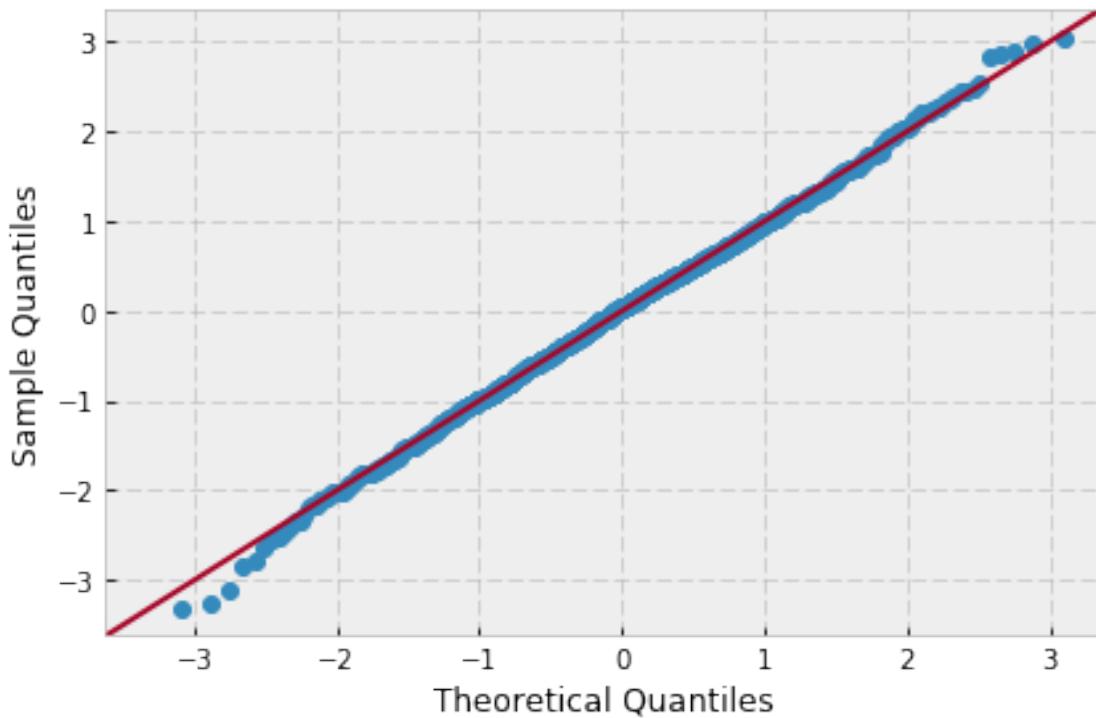


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(),loc=101.11109817830997, scale=4.20587812064204)
DescribeResult(nobs=1000, minmax=(88.494915815230939, 117.57777636392383)
               mean=101.11109817830997, variance=17.707117883578995,
               skewness=0.16360724552260164, kurtosis=0.18969079403273303)
```

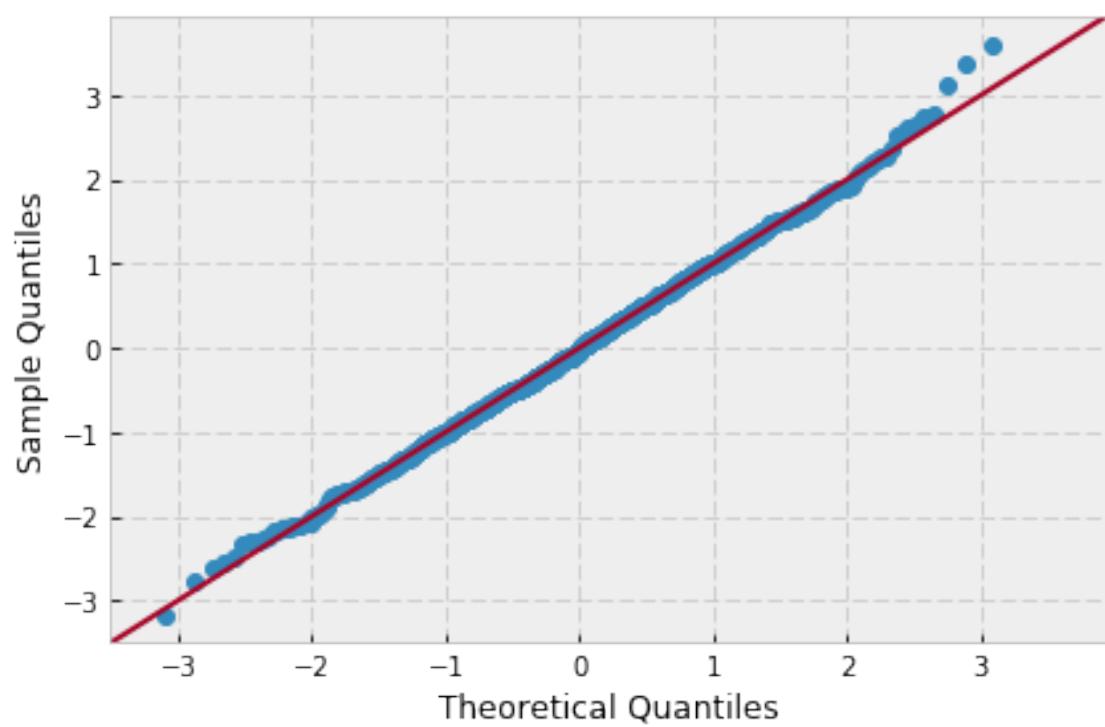
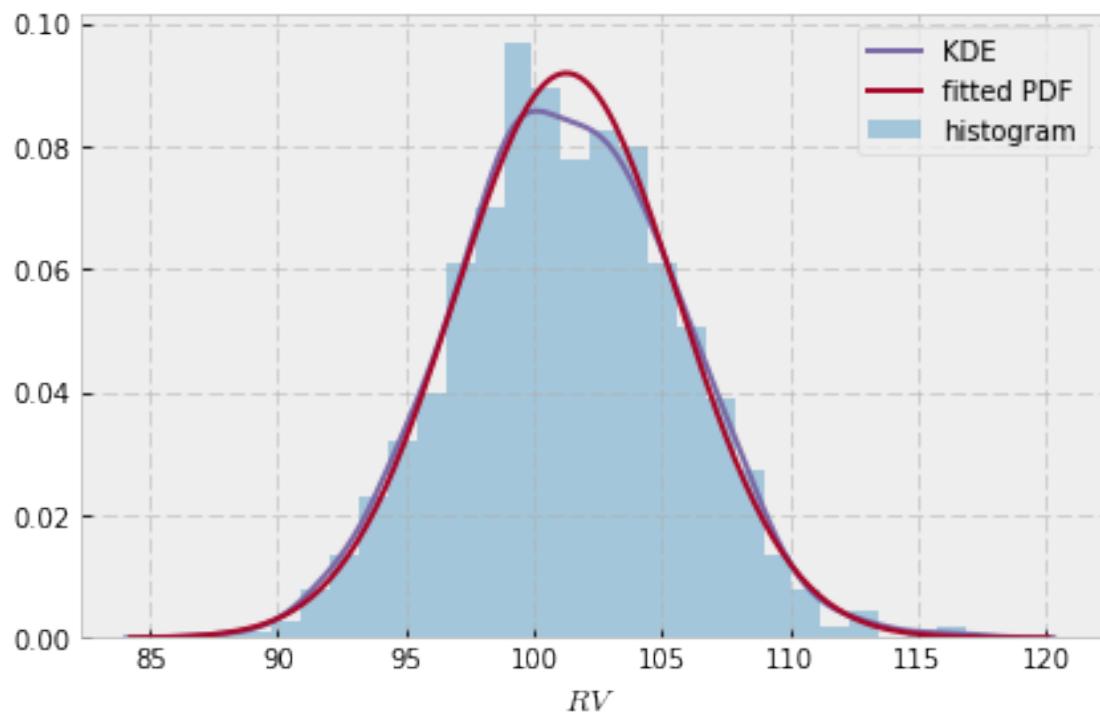


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(),loc=101.46877491199076, scale=4.373403981438741)
DescribeResult(nobs=1000, minmax=(87.025177575444474, 114.73930717480692)
               mean=101.46877491199076, variance=19.145808193057288,
               skewness=-0.04895075429293916, kurtosis=0.12665858615513503)
```

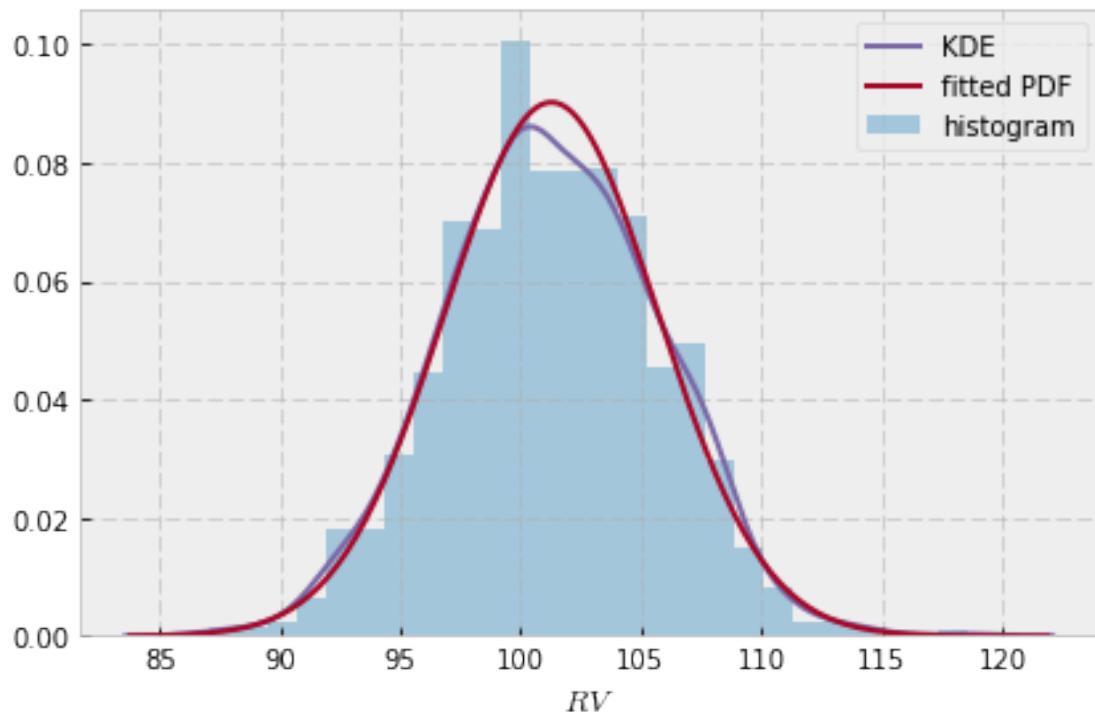


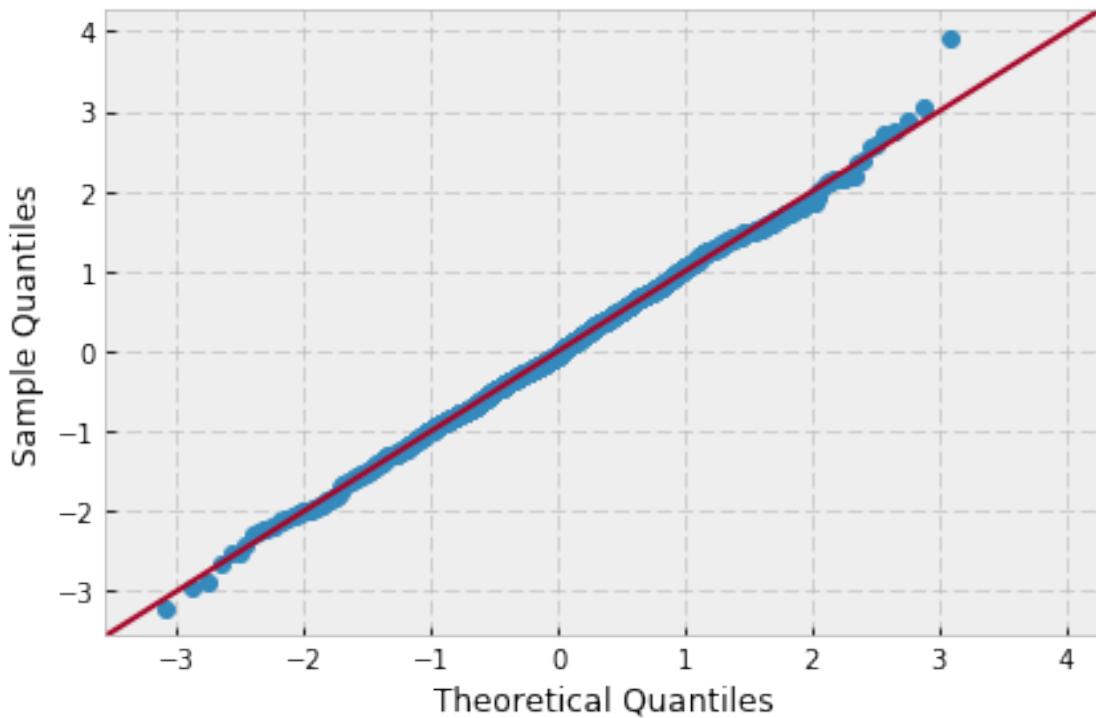


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(),loc=101.26713668490784, scale=4.341668139744876)
DescribeResult(nobs=1000, minmax=(87.529079044703707, 116.85791013392898)
               mean=101.26713668490784, variance=18.868951186862599,
               skewness=0.07734547651421207, kurtosis=-0.03297634785440229)
```

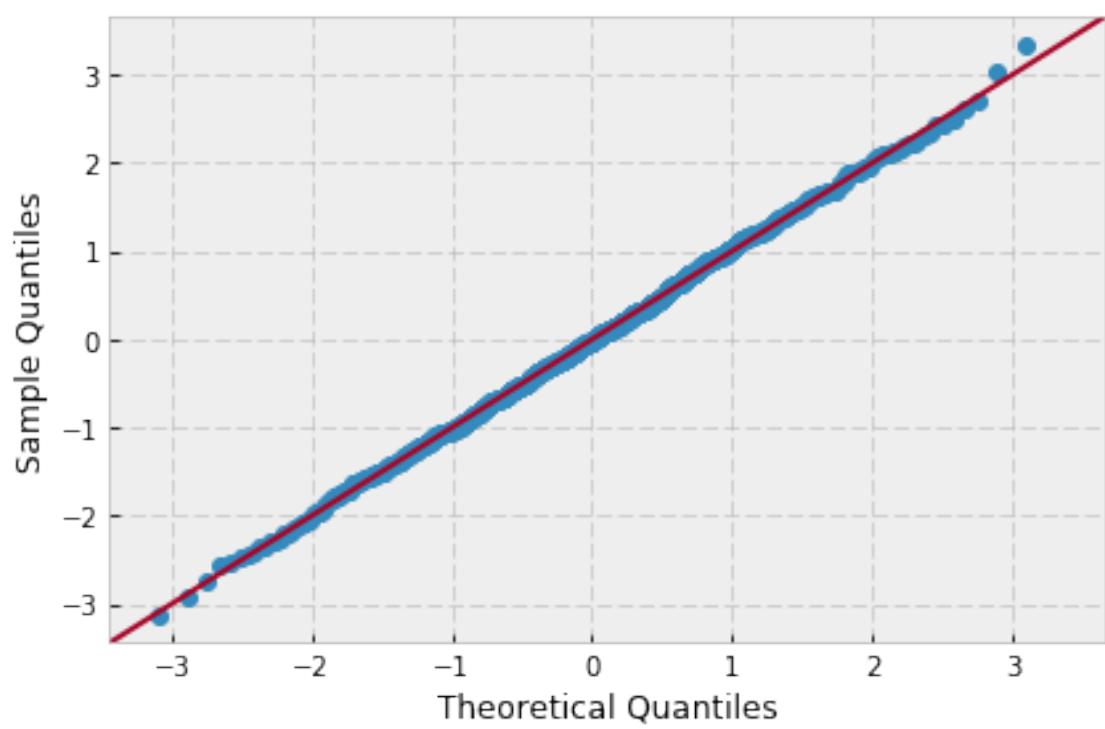
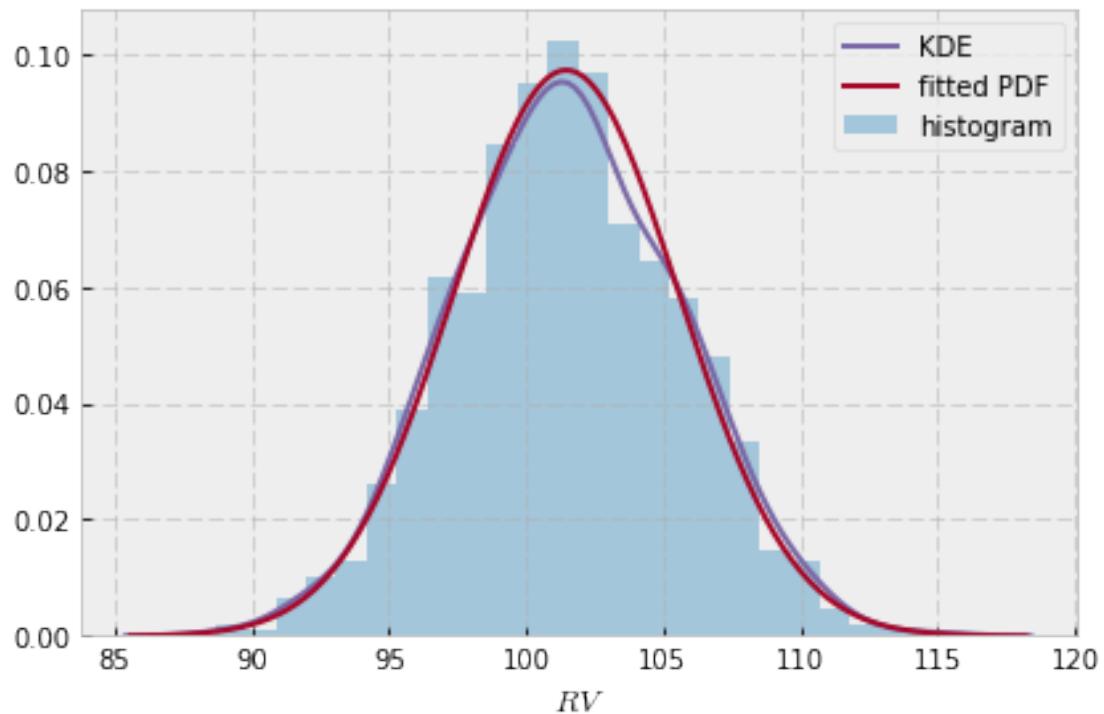


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(),loc=101.26882378436302, scale=4.421768385919707)
DescribeResult(nobs=1000, minmax=(87.099568889349541, 118.57088716470518)
               mean=101.26882378436302, variance=19.571607265984952,
               skewness=0.029202262480166837, kurtosis=-0.019401716852288864)
```

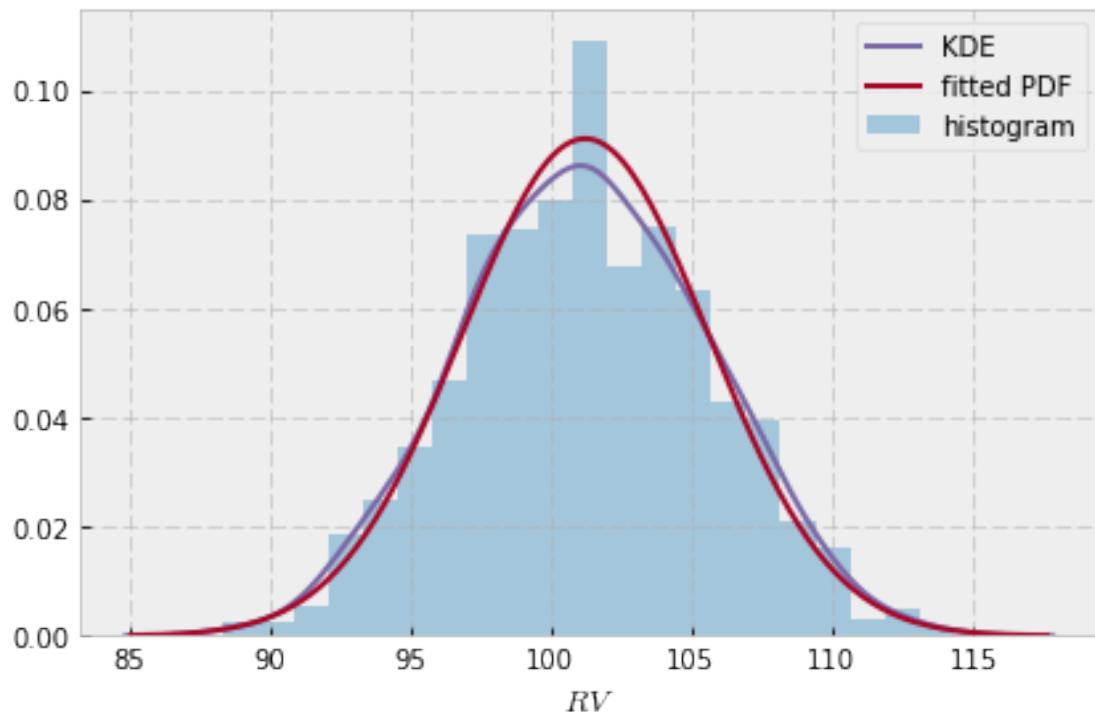


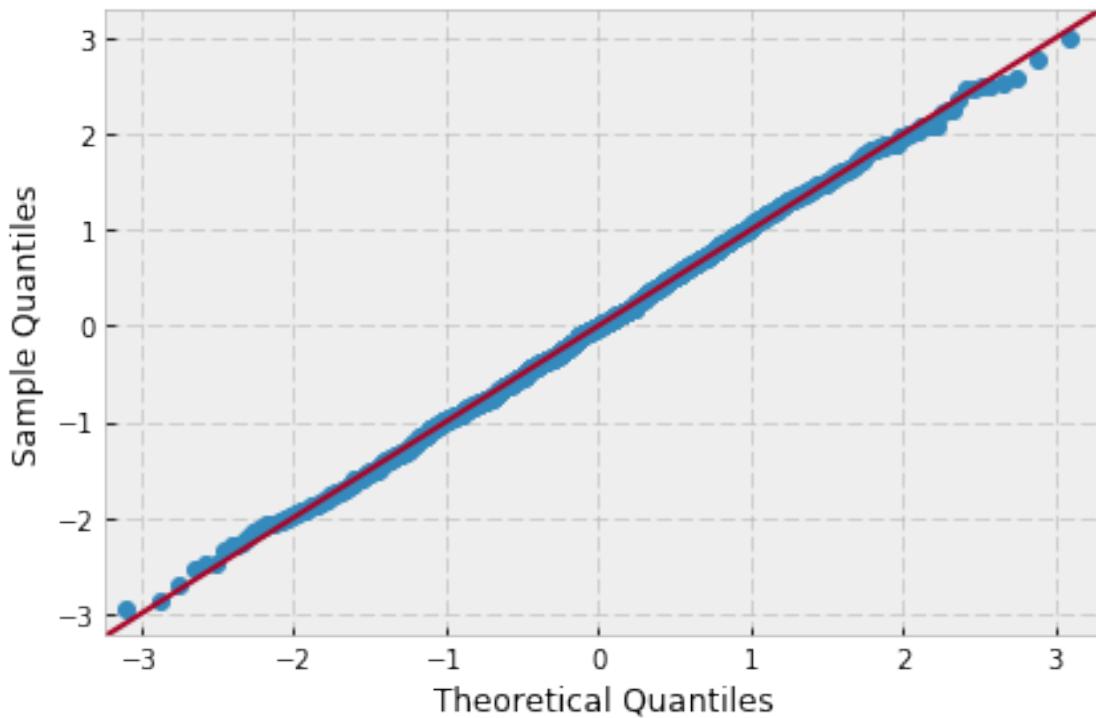


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(),loc=101.45986107843298, scale=4.101883603390274)
DescribeResult(nobs=1000, minmax=(88.658835936885609, 115.1229900260665)
               mean=101.45986107843298, variance=16.842291387149128,
               skewness=0.02739713245525337, kurtosis=-0.1427706011975216)
```

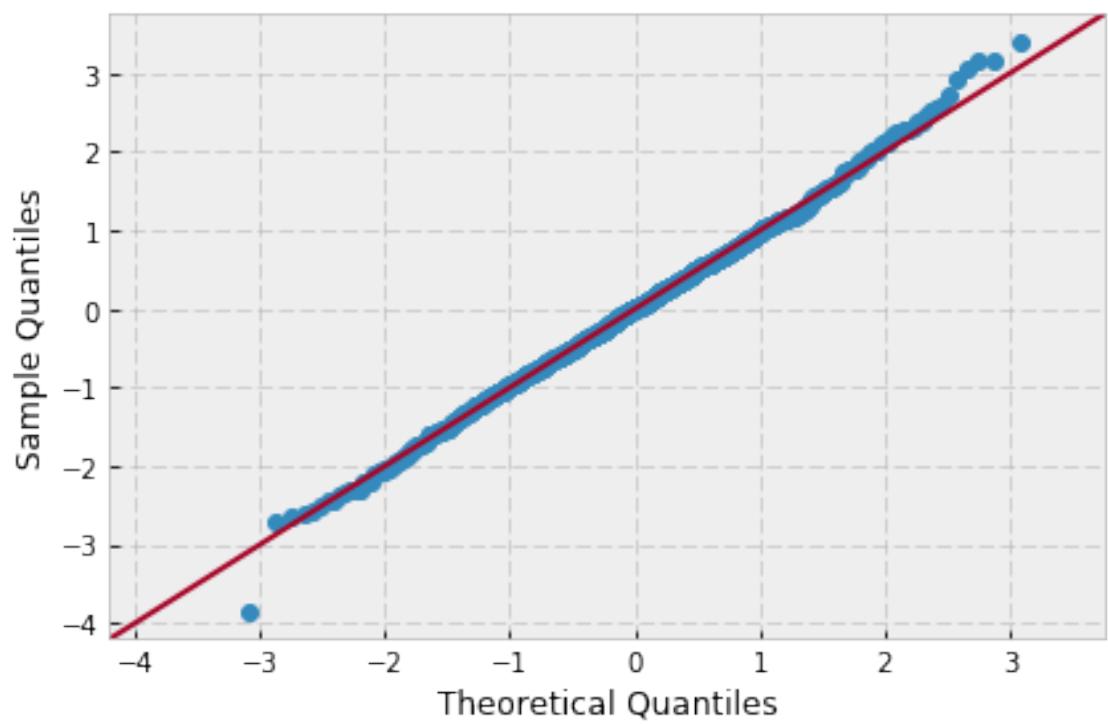
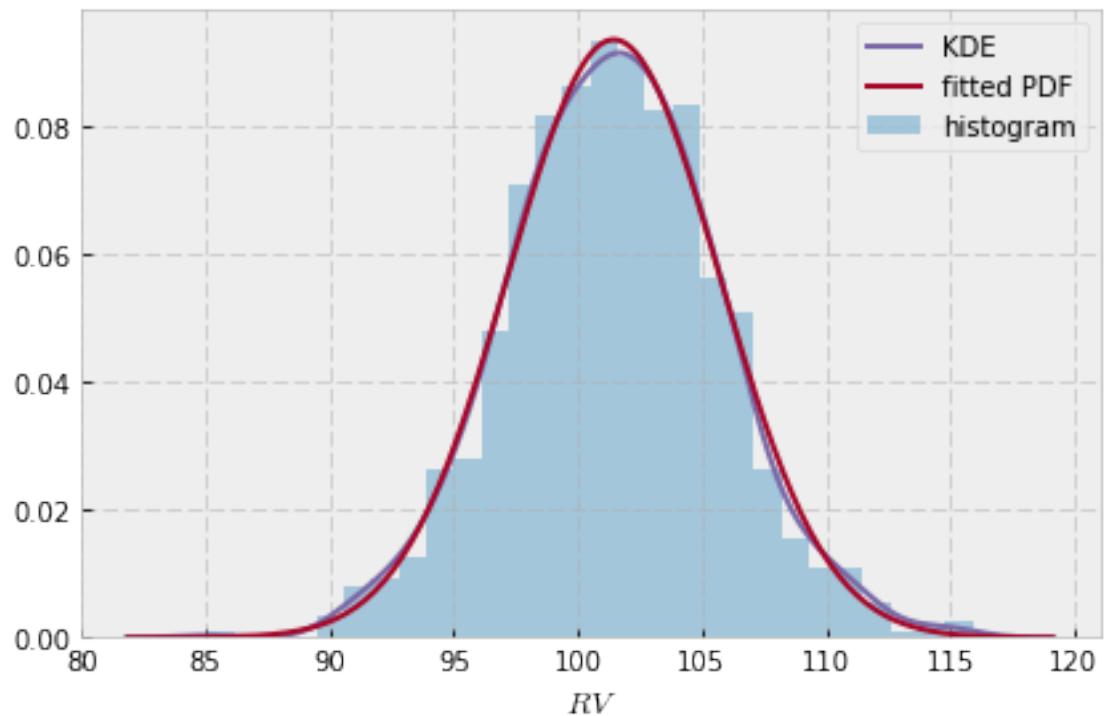


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(),loc=101.22468672827206, scale=4.3798707509707455)
DescribeResult(nobs=1000, minmax=(88.381106613273914, 114.3456593868941)
               mean=101.22468672827206, variance=19.202470265474521,
               skewness=0.017228907875422847, kurtosis=-0.27317982269604935)
```

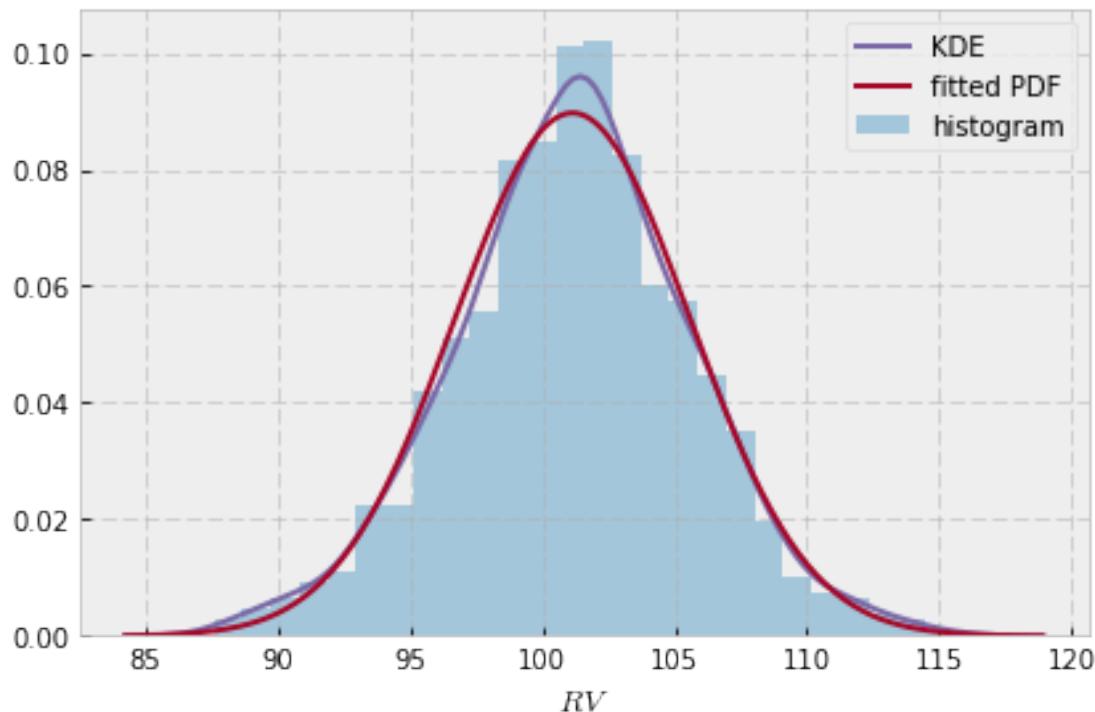


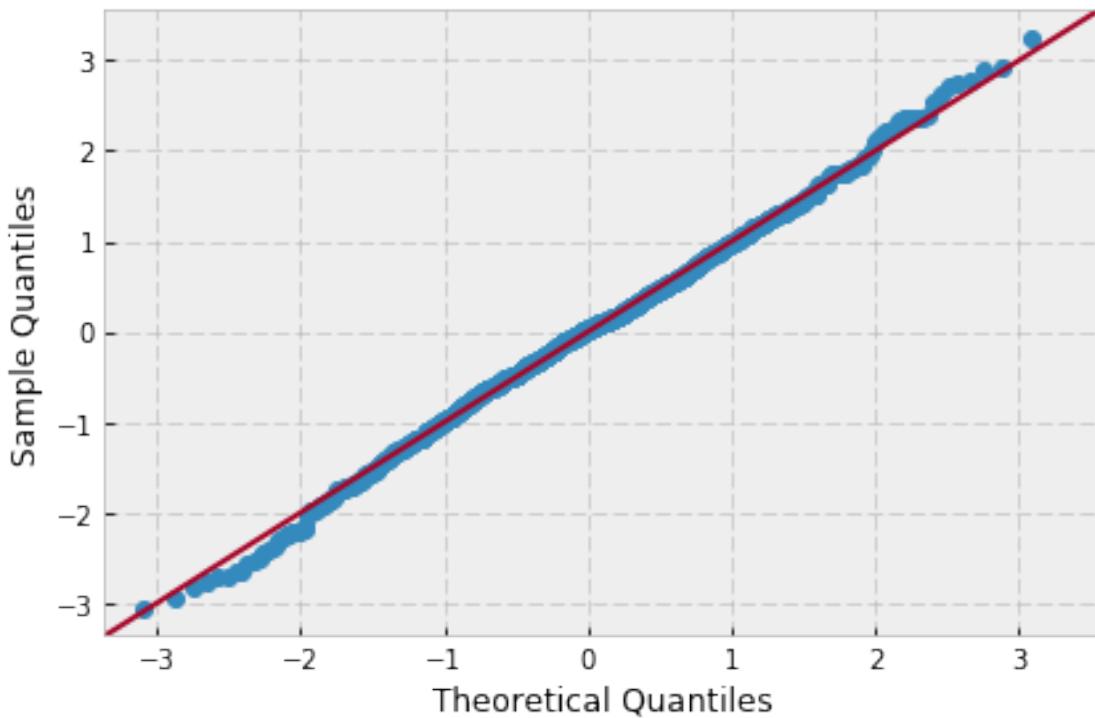


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(),loc=101.44034016581446, scale=4.261105842786701)
DescribeResult(nobs=1000, minmax=(85.076870460427472, 115.89494082527349)
               mean=101.44034016581446, variance=18.175198201632593,
               skewness=0.05543401983927169, kurtosis=0.2529277754504835)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(),loc=101.13529450620068, scale=4.447145818722859)
DescribeResult(nobs=1000, minmax=(87.581090903341618, 115.55236257119448)
               mean=101.13529450620068, variance=19.796902835820028,
               skewness=-0.04768215305289694, kurtosis=0.2032617143713189)
```

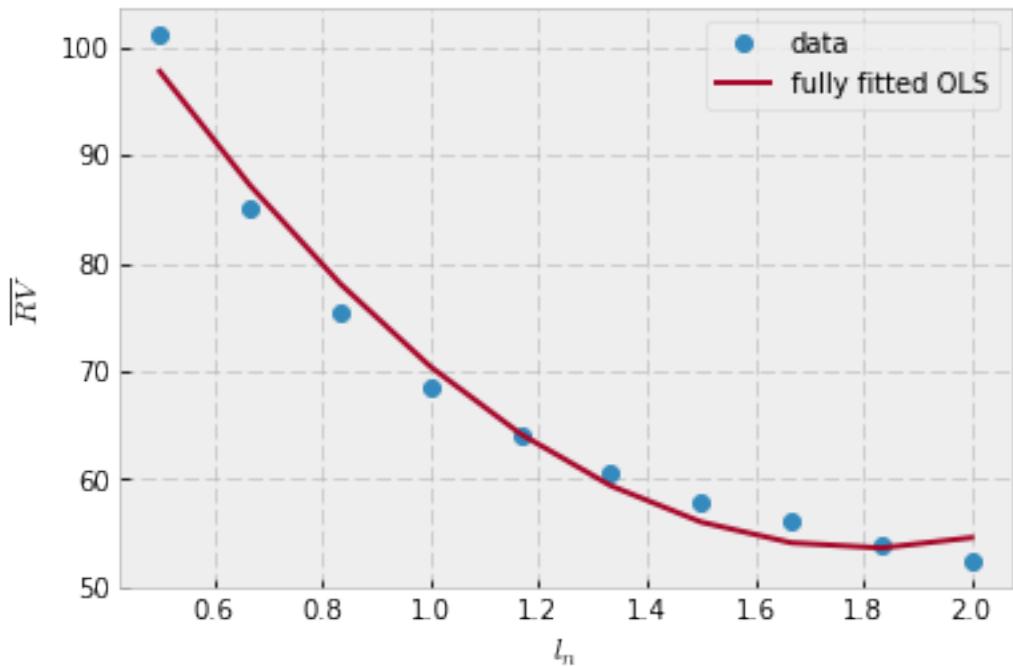




7.2 Parameter regression

```
In [20]: var_name = 'total_root_volume'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly1')
mean_reg('maxB', var_name, 'poly1')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly1')
mean_reg('N', var_name, 'poly1')
```

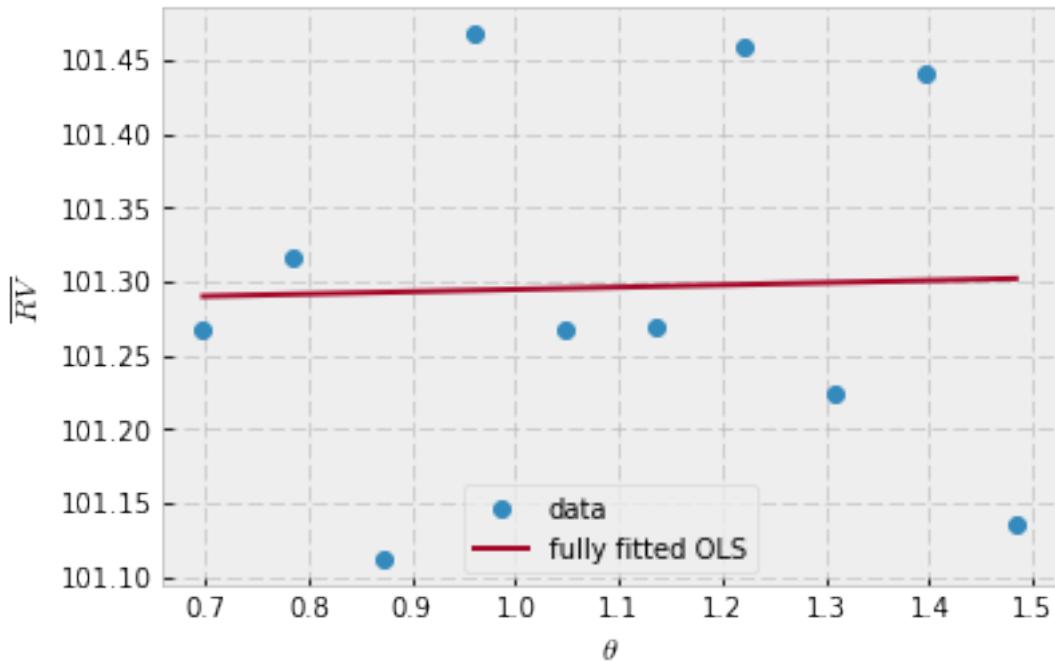
```
variable name: ln
model: poly2
r-squared: 0.9290524592320429
explained variance: 0.929156955888541
```



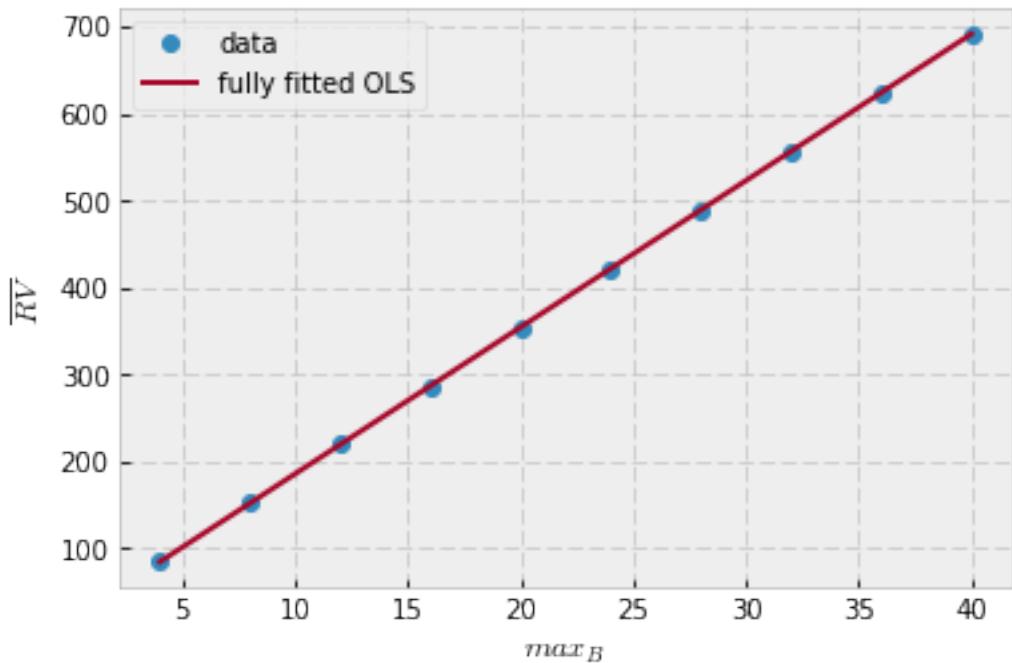
```

variable name: theta
model: poly1
r-squared: -0.6152367594290664
explained variance: -0.6129705364384297

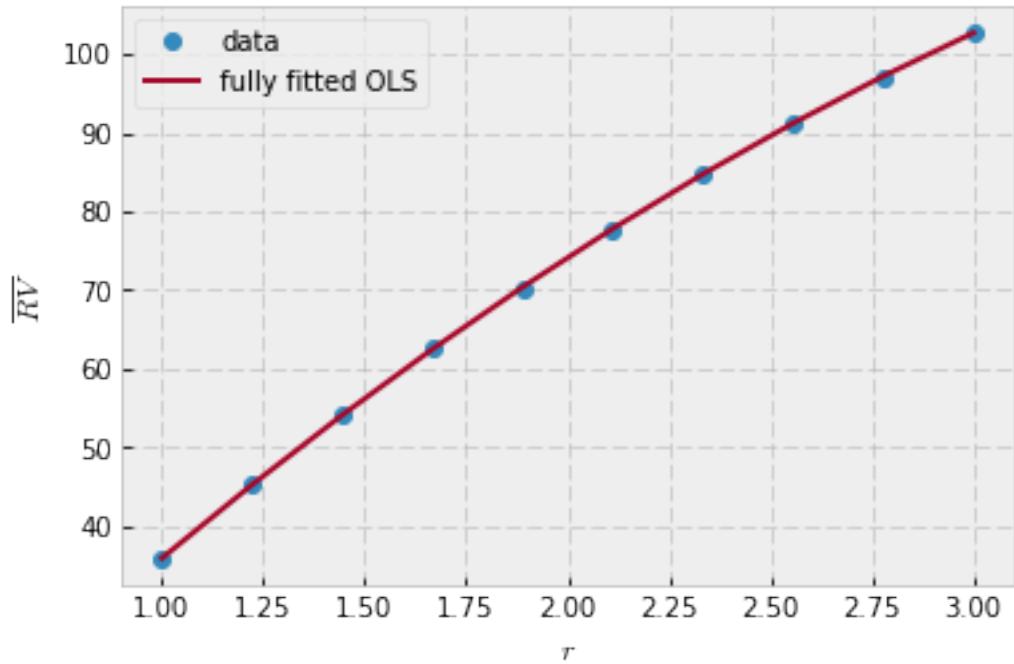
```



```
variable name: maxB  
model: poly1  
r-squared: 0.9999991382596531  
explained variance: 0.9999991408489118
```



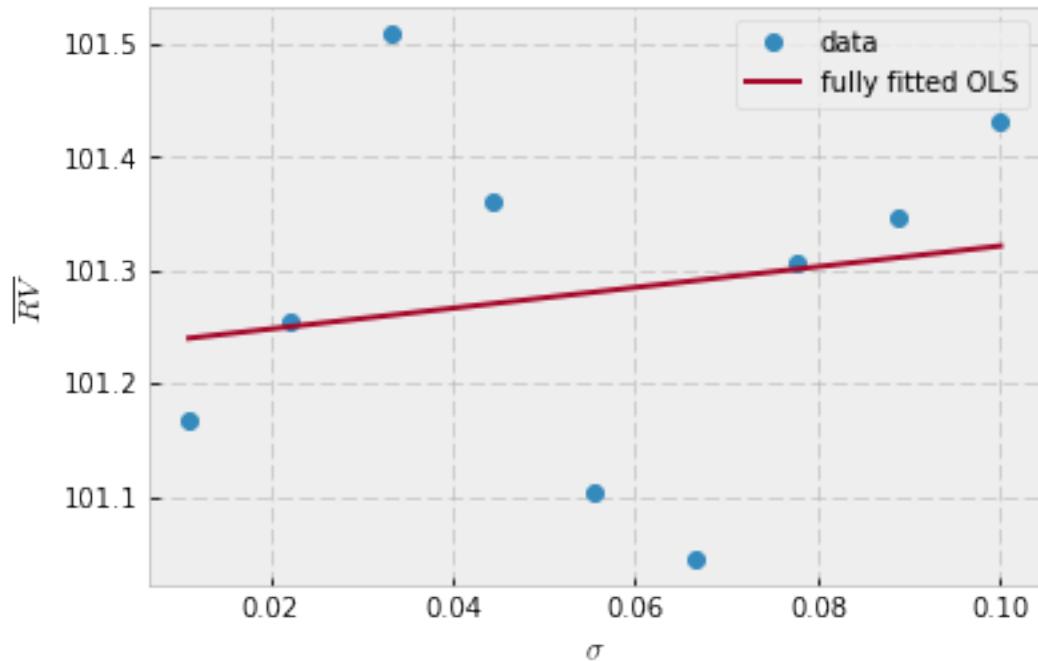
```
variable name: r  
model: poly2  
r-squared: 0.9998644129208107  
explained variance: 0.99986469258683
```



```

variable name: sigma
model: poly1
r-squared: -0.4384121481657073
explained variance: -0.43710351286484106

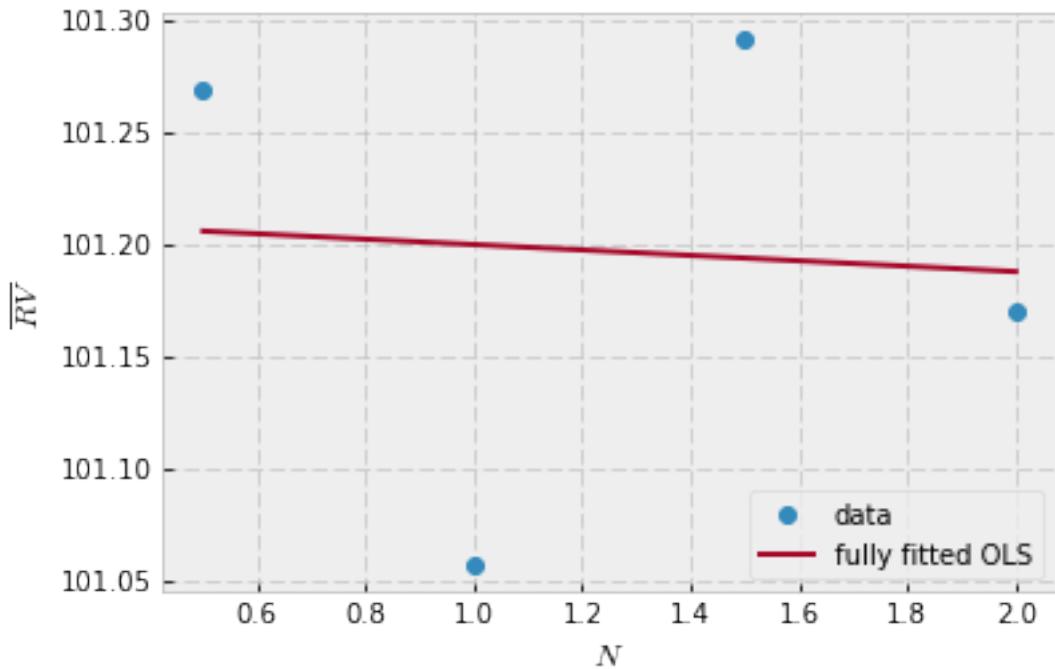
```



```

variable name: N
model: poly1
r-squared: -2.148171967324859
explained variance: -2.094223040445508

```



8 Number of root branches

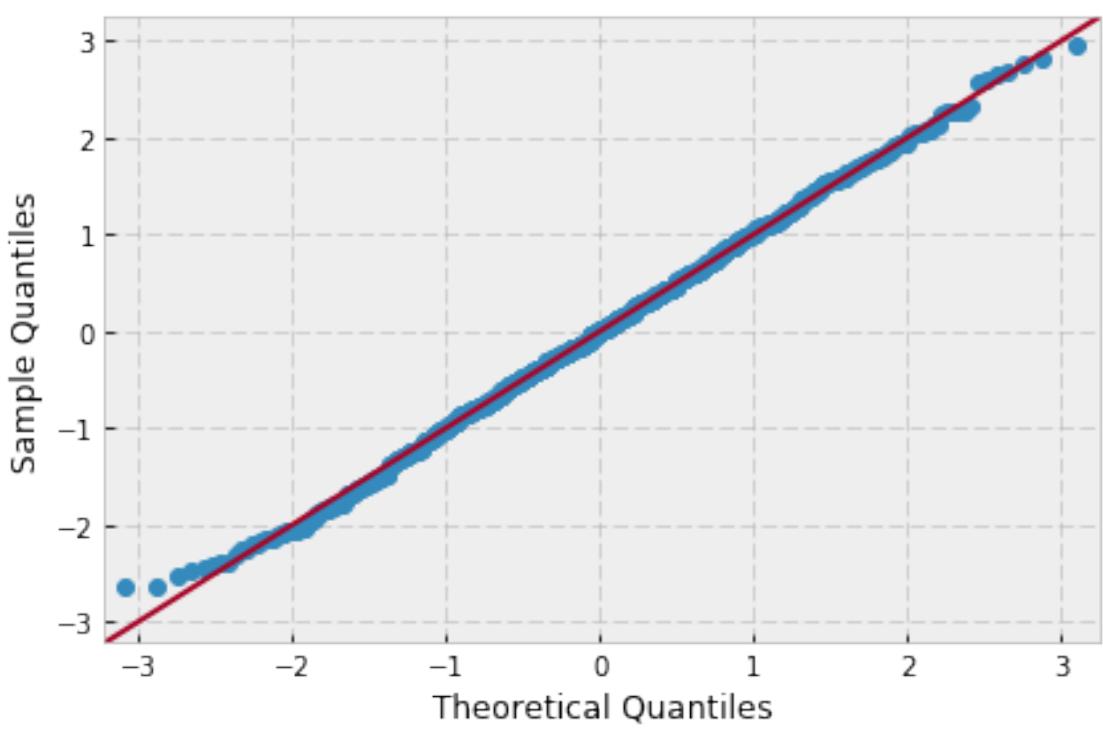
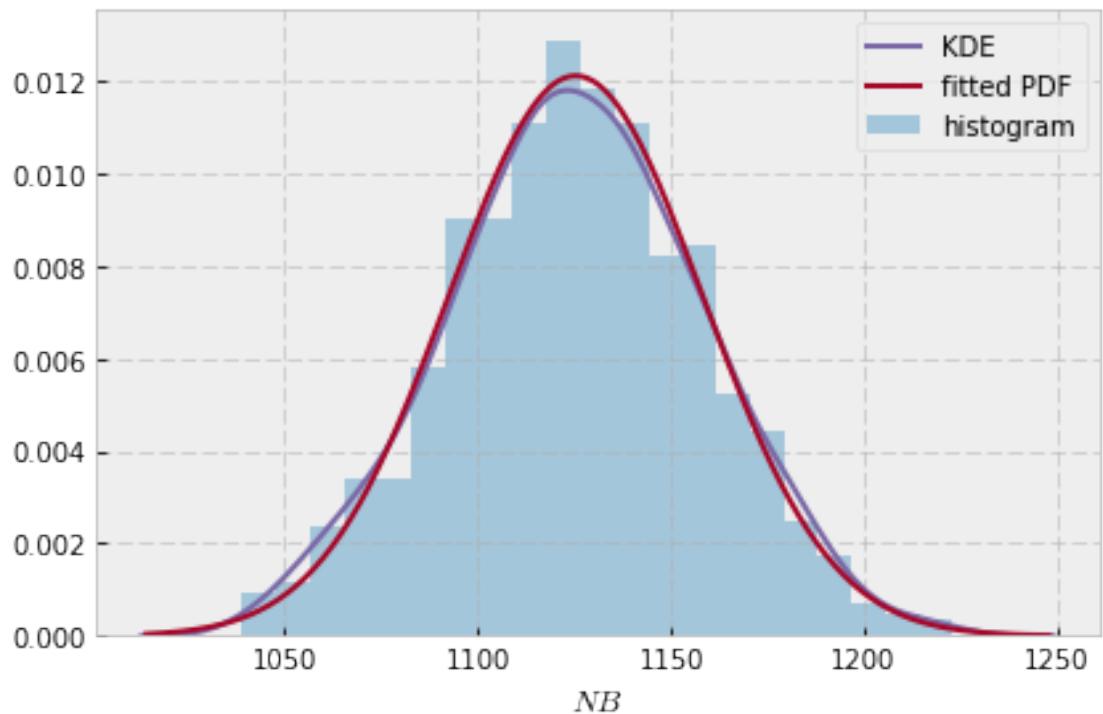
8.1 Probability distributions

```

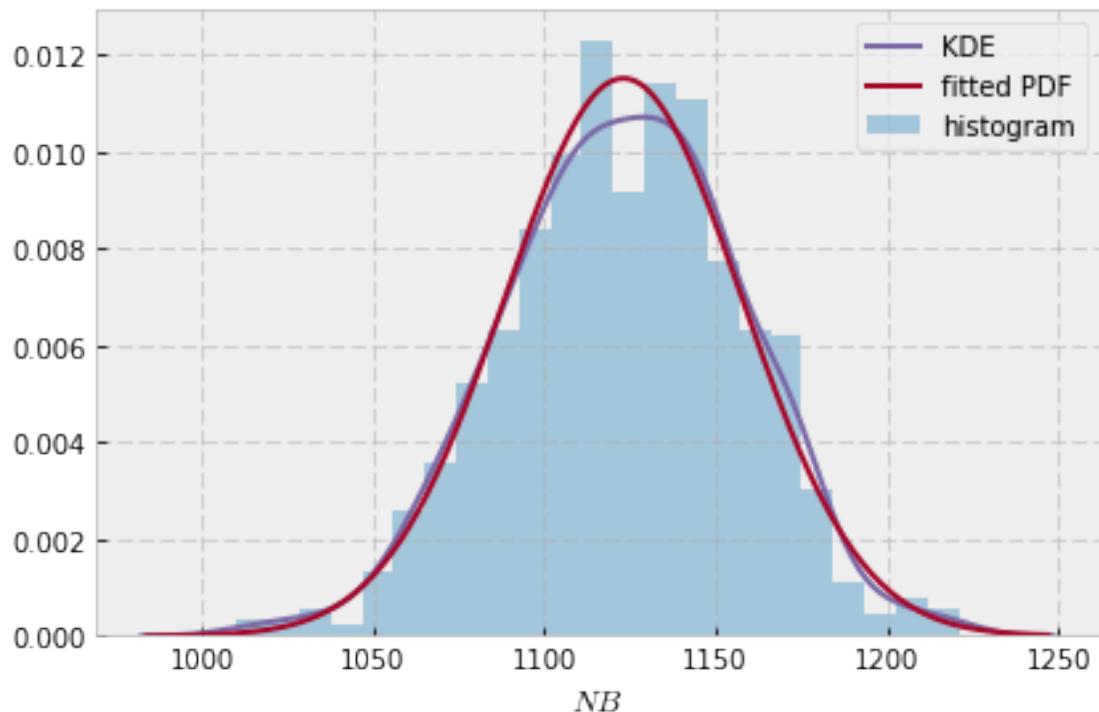
In [21]: plot_dist('number_of_root_branches')

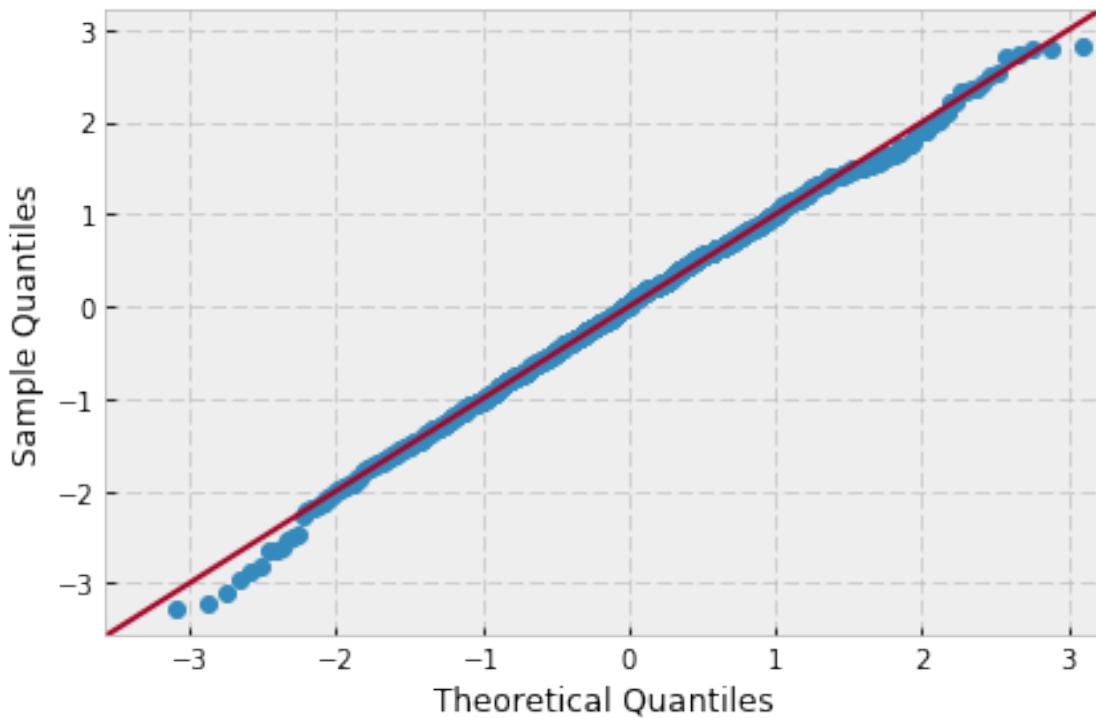
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=1125.35, scale=32.922294877483864)
DescribeResult(nobs=1000, minmax=(1039.0, 1223.0)
               mean=1125.349999999999, variance=1084.9624624624626,
               skewness=0.004017565743661127, kurtosis=-0.20101022395219248)

```

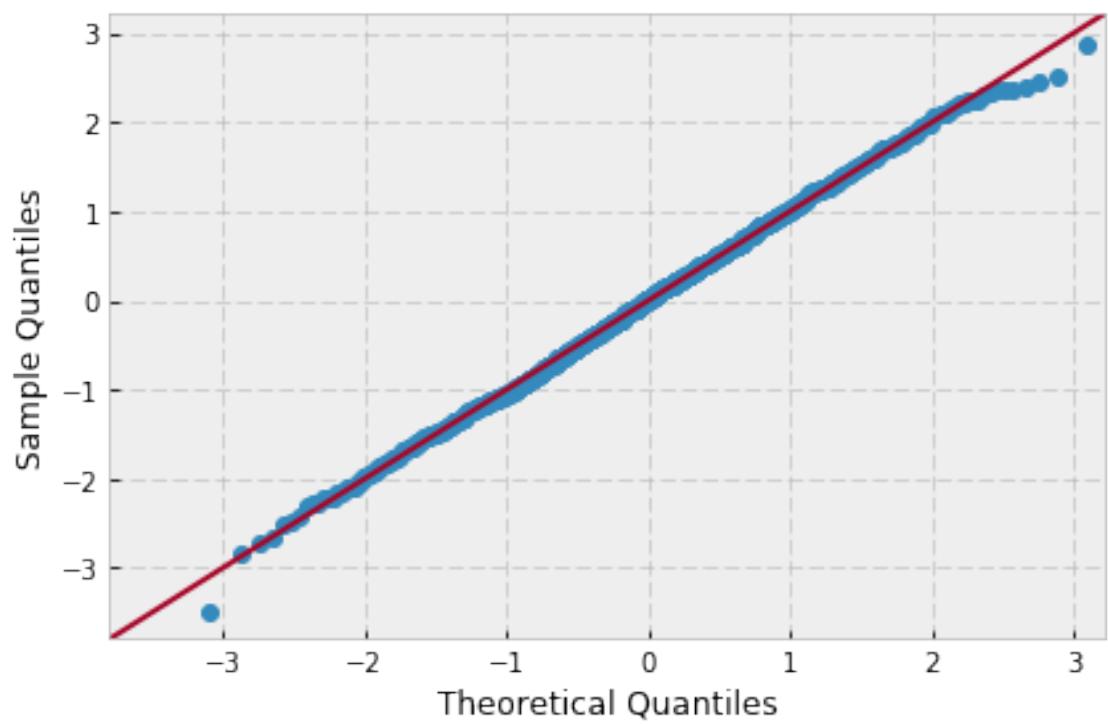
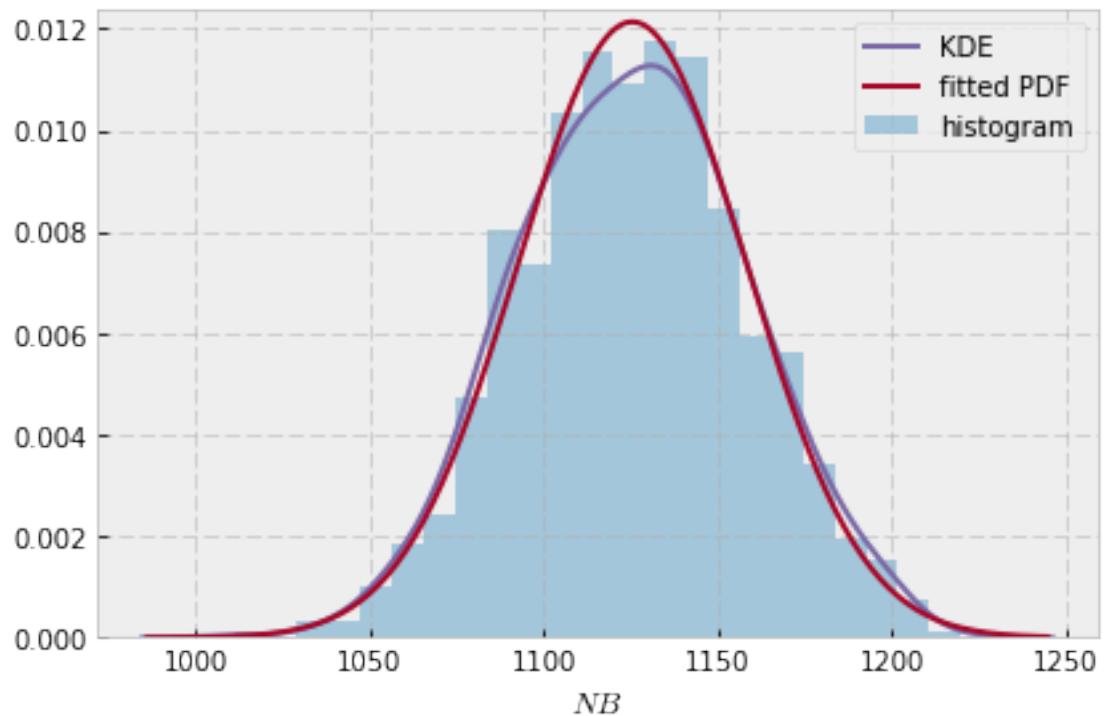


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=1123.051, scale=34.62560322940237)
DescribeResult(nobs=1000, minmax=(1010.0, 1221.0)
    mean=1123.050999999999, variance=1200.1325315315312,
    skewness=-0.11720220180575858, kurtosis=-0.011838820362719993)
```

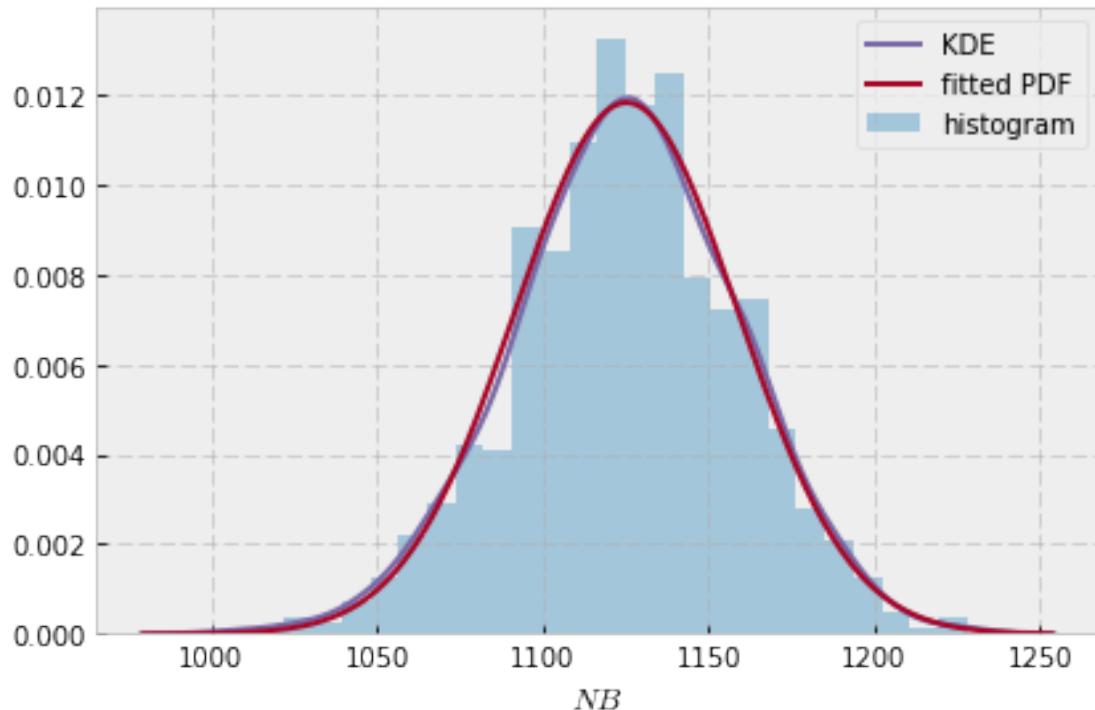


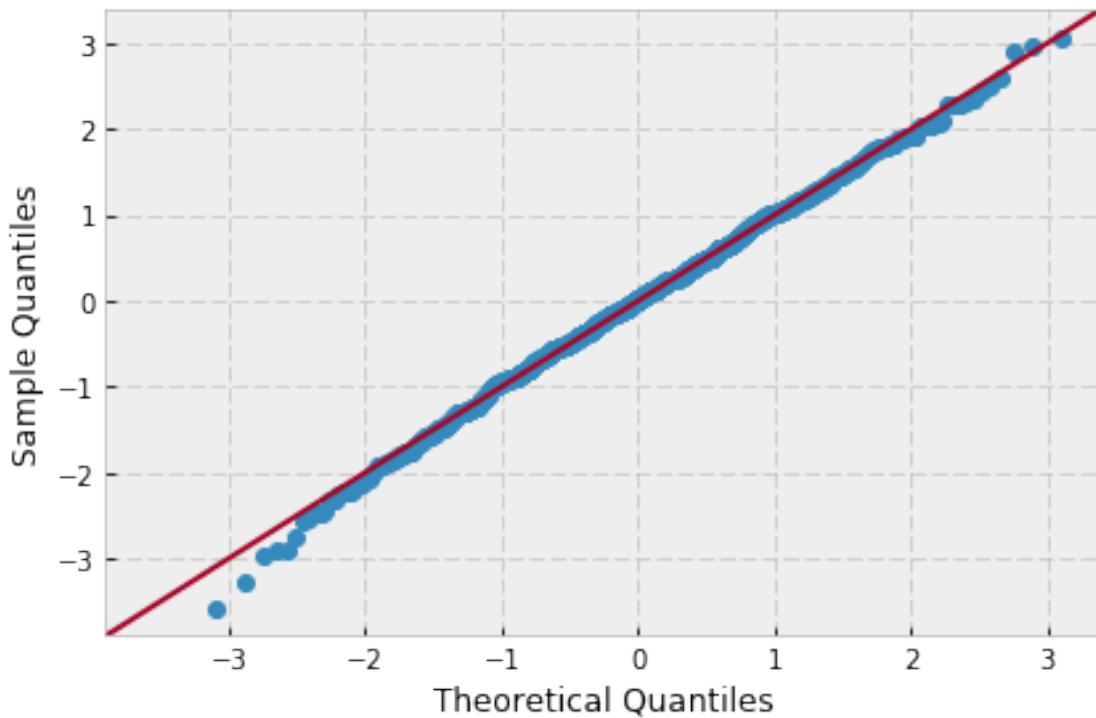


```
variable name: N
variable value: 1.5
distribution: normal(shape=(),loc=1125.494, scale=32.883977314187526)
DescribeResult(nobs=1000, minmax=(1011.0, 1220.0)
               mean=1125.493999999999, variance=1082.4384024024023,
               skewness=-0.01881562277316979, kurtosis=-0.24260505717075365)
```

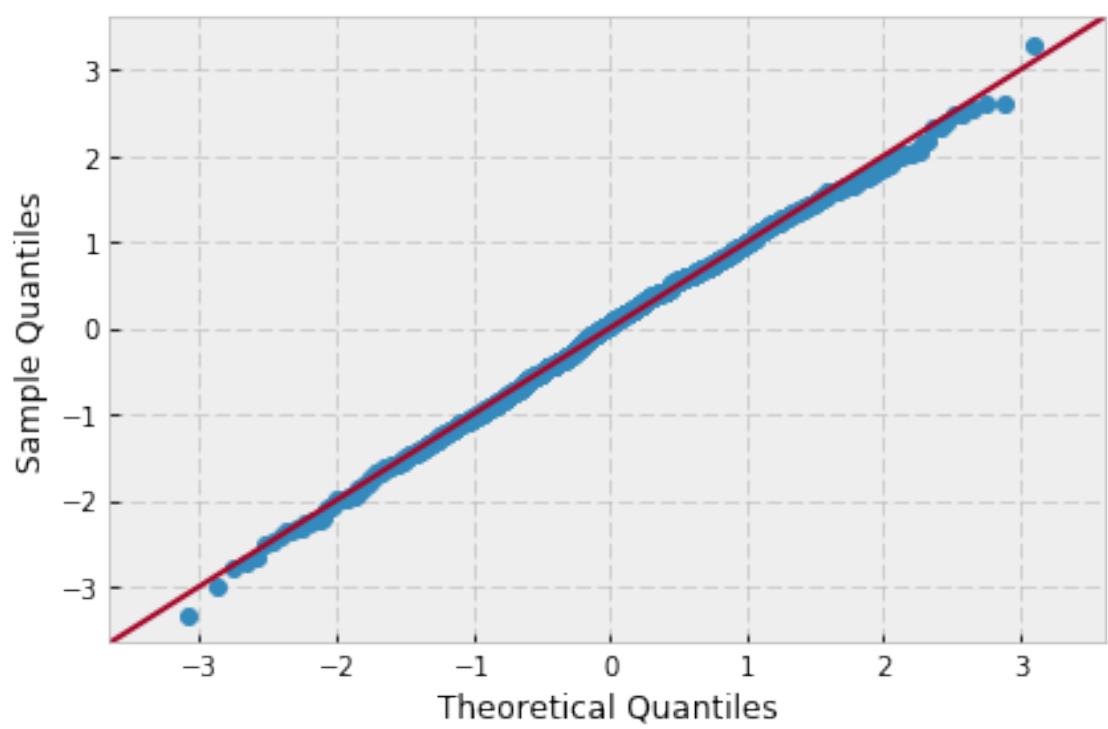
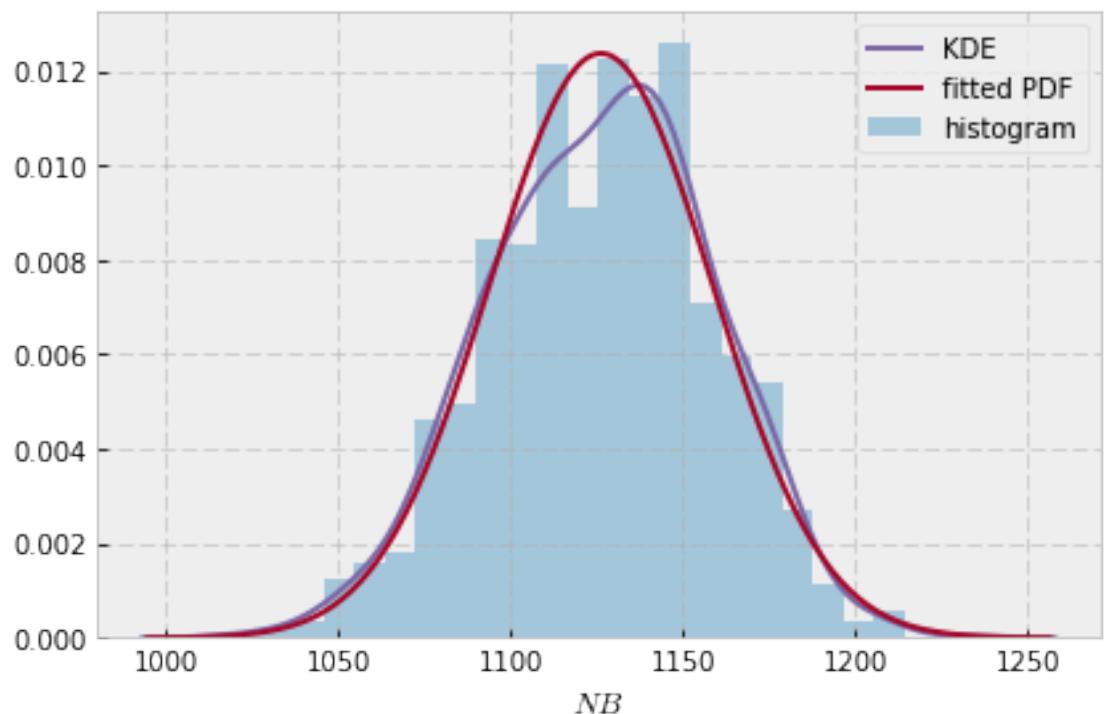


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=1125.217, scale=33.674350936580794)
DescribeResult(nobs=1000, minmax=(1005.0, 1228.0)
    mean=1125.2170000000001, variance=1135.0970080080078,
    skewness=-0.11923946548790126, kurtosis=0.10179567372187925)
```

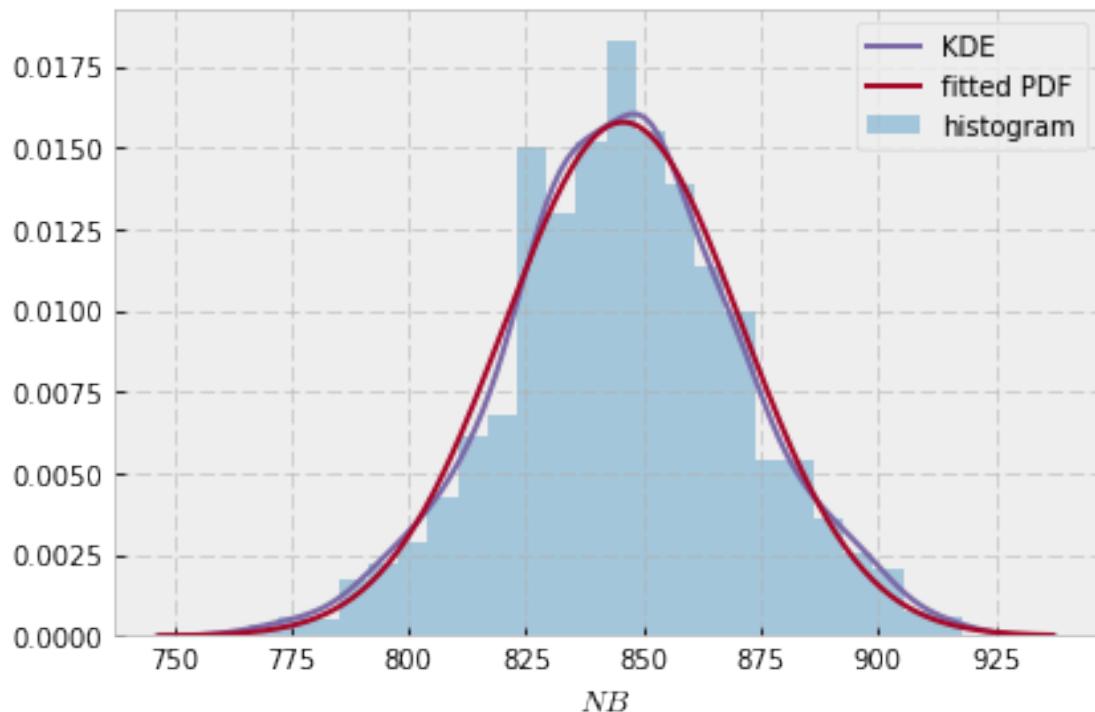


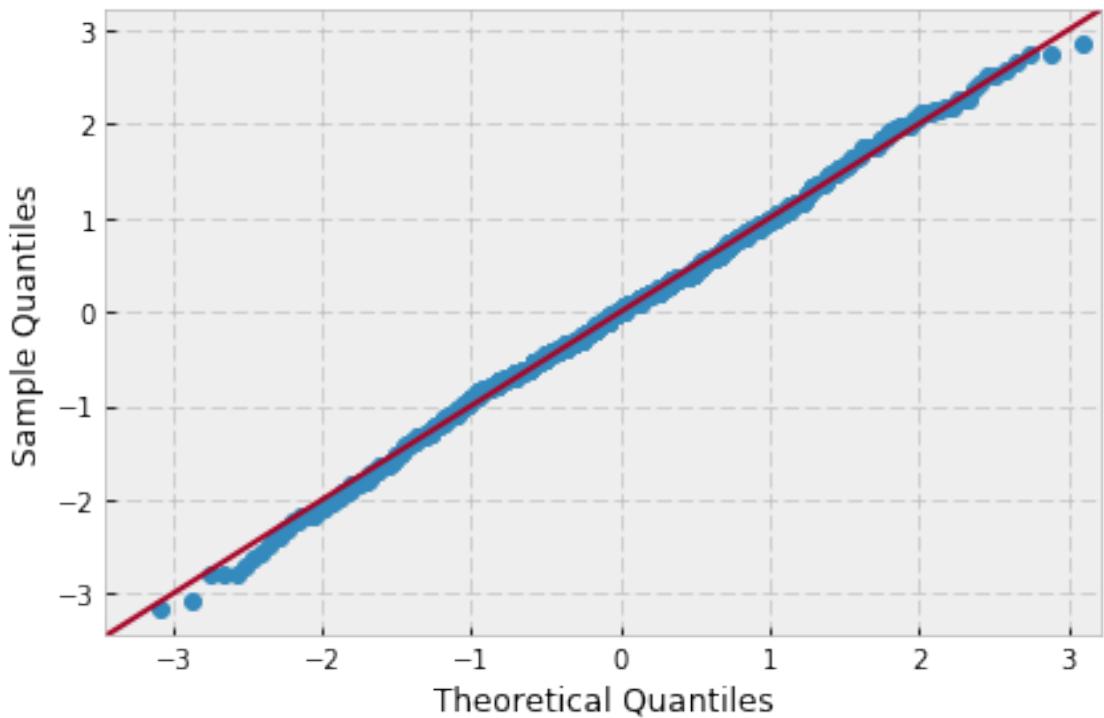


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=1126.213, scale=32.23668145141494)
DescribeResult(nobs=1000, minmax=(1019.0, 1232.0)
               mean=1126.213, variance=1040.243874874875,
               skewness=-0.12385709780118952, kurtosis=-0.1956372245266862)
```

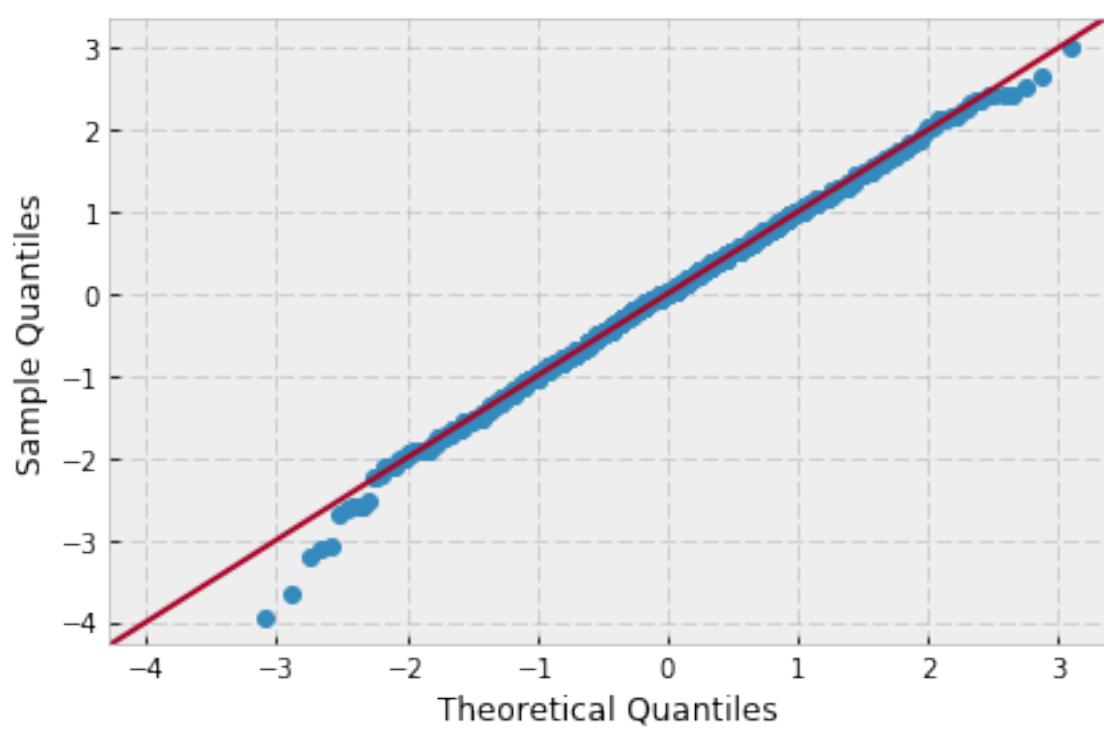
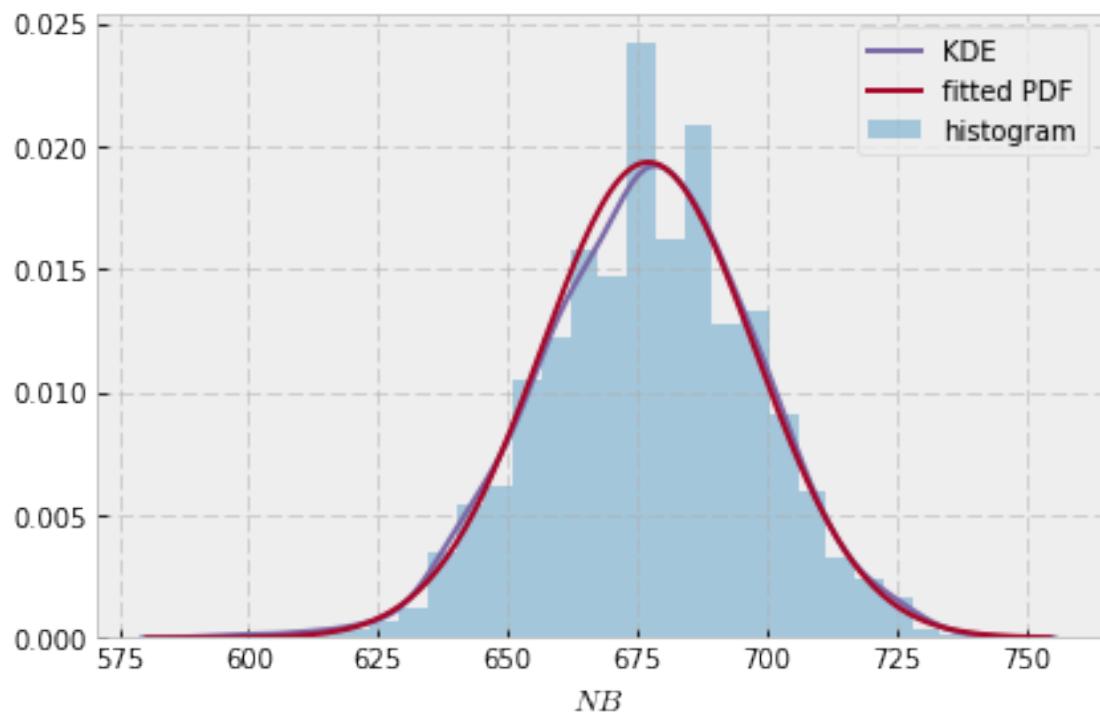


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=845.649, scale=25.273143828973872)
DescribeResult(nobs=1000, minmax=(766.0, 918.0)
    mean=845.649, variance=639.37117017017022,
    skewness=-0.026153096234814068, kurtosis=0.08519074217099876)
```

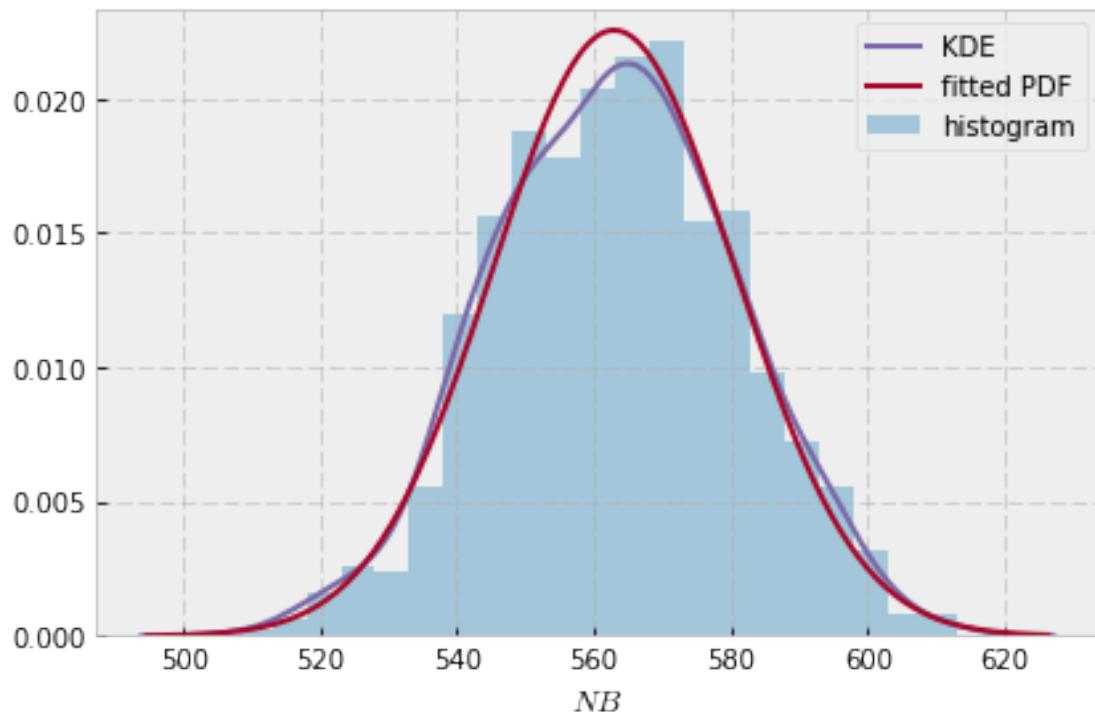


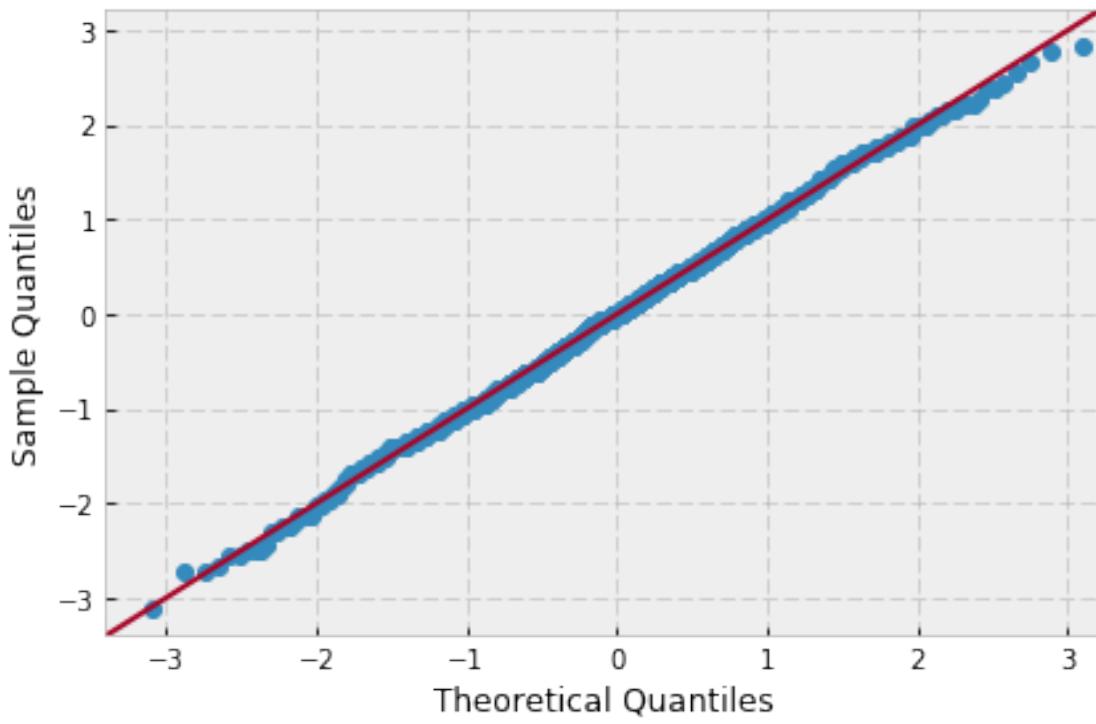


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=677.086, scale=20.61777398265875)
DescribeResult(nobs=1000, minmax=(596.0, 739.0)
               mean=677.0860000000001, variance=425.51812212212218,
               skewness=-0.16457009287128668, kurtosis=0.185175006453556)
```

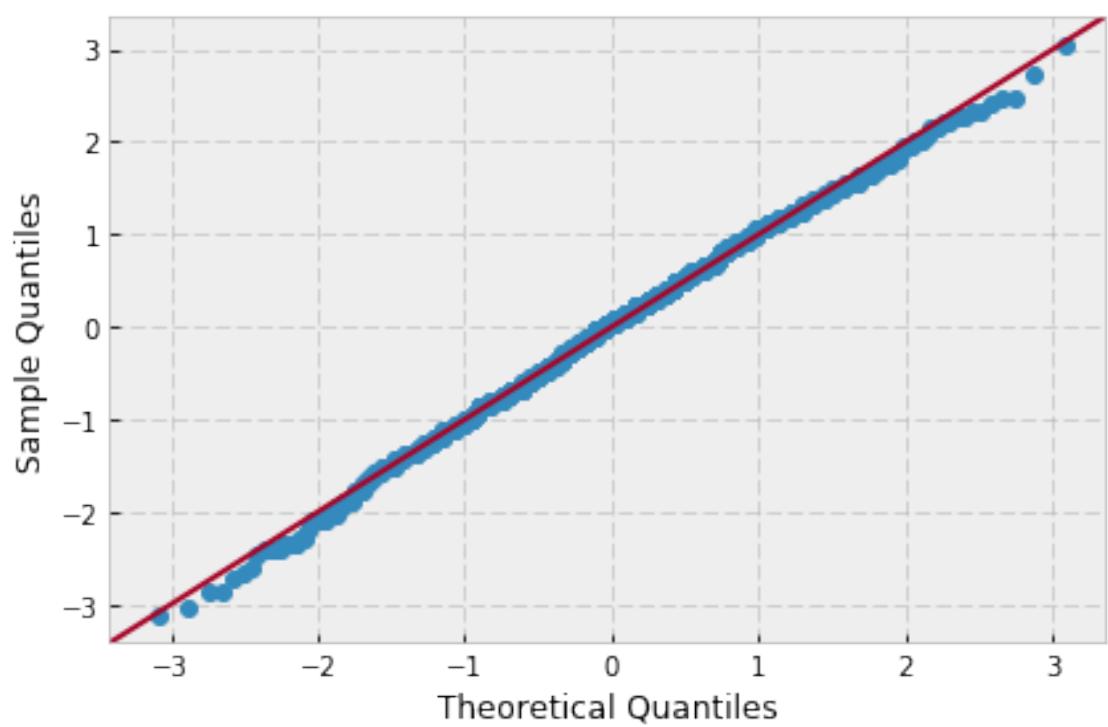
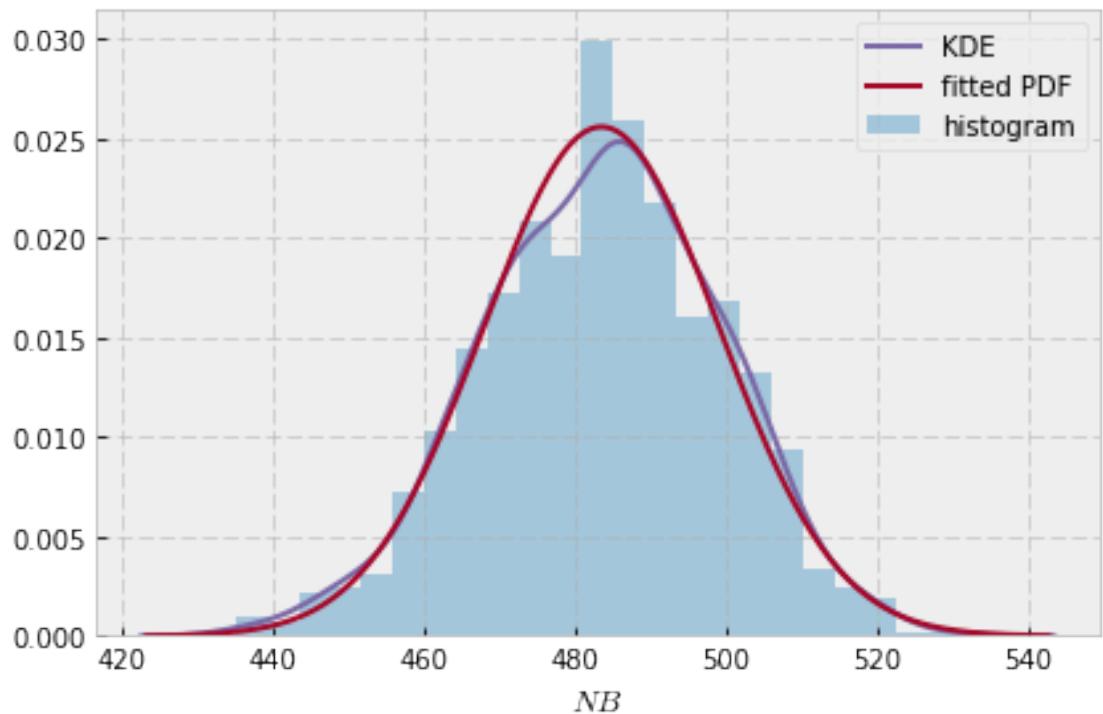


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=562.914, scale=17.68515207737836)
DescribeResult(nobs=1000, minmax=(508.0, 613.0)
    mean=562.9139999999999, variance=313.07768168168172,
    skewness=-0.026932254005413655, kurtosis=-0.23322213920994495)
```

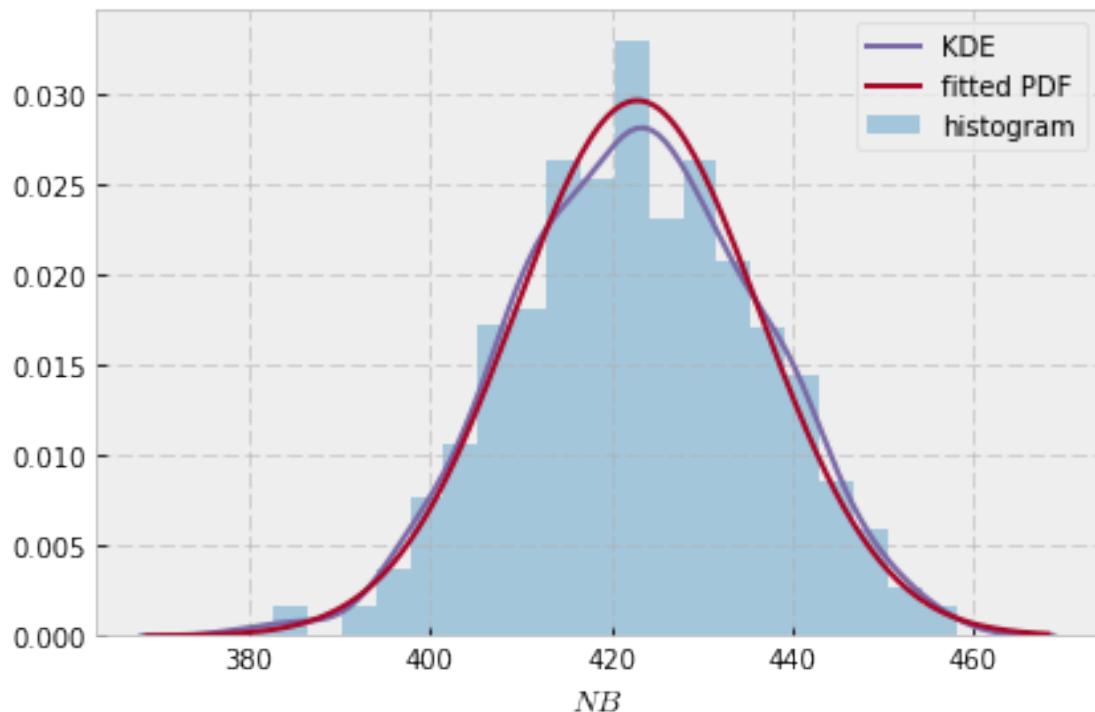


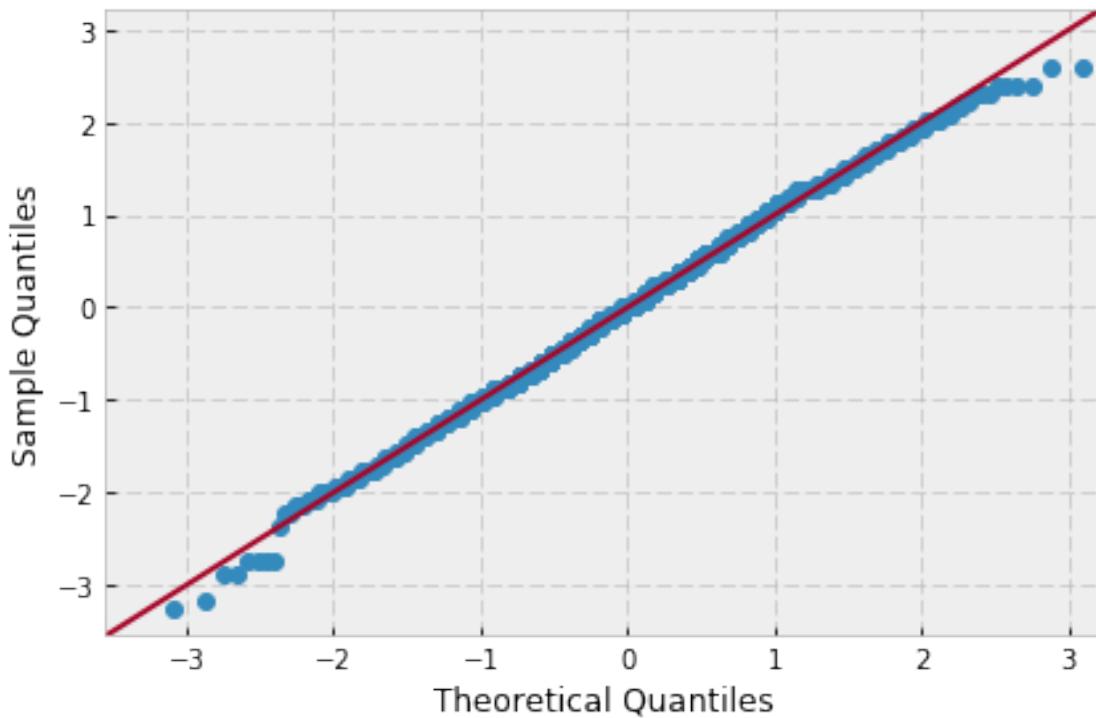


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=483.521, scale=15.598575543939901)
DescribeResult(nobs=1000, minmax=(435.0, 531.0)
               mean=483.5210000000002, variance=243.55911811811814,
               skewness=-0.13335032481599696, kurtosis=-0.13101918862707107)
```

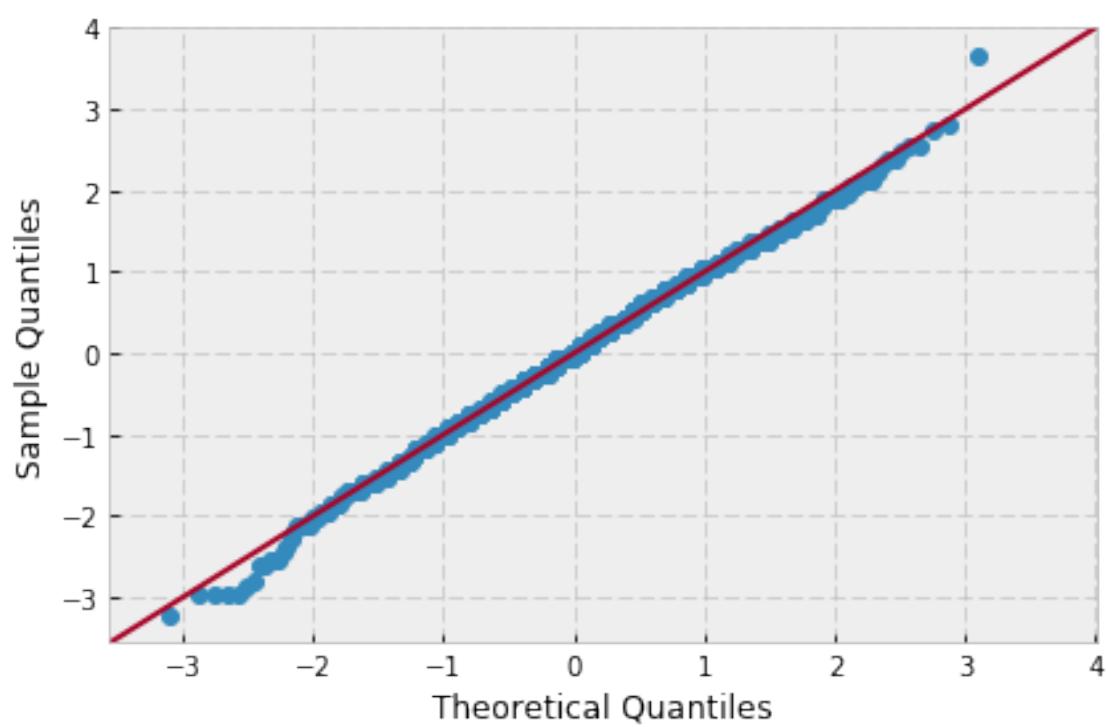
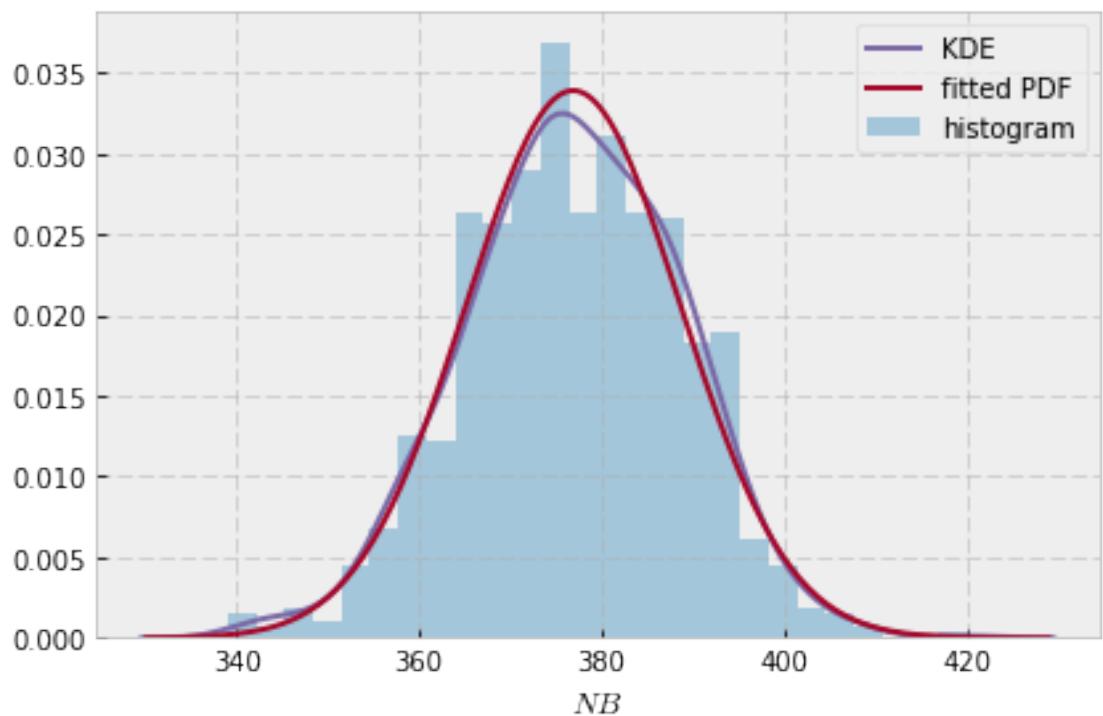


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=422.898, scale=13.477299284352188)
DescribeResult(nobs=1000, minmax=(379.0, 458.0)
               mean=422.8980000000002, variance=181.81941541541545,
               skewness=-0.07306004315916424, kurtosis=-0.2138575404554275)
```

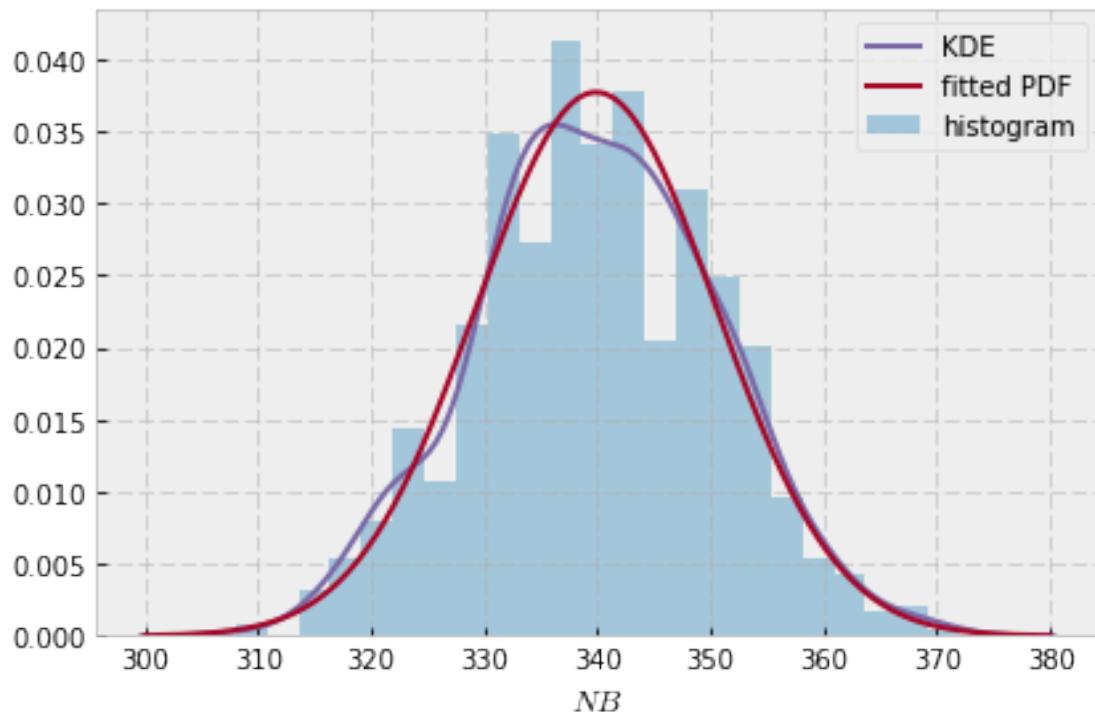


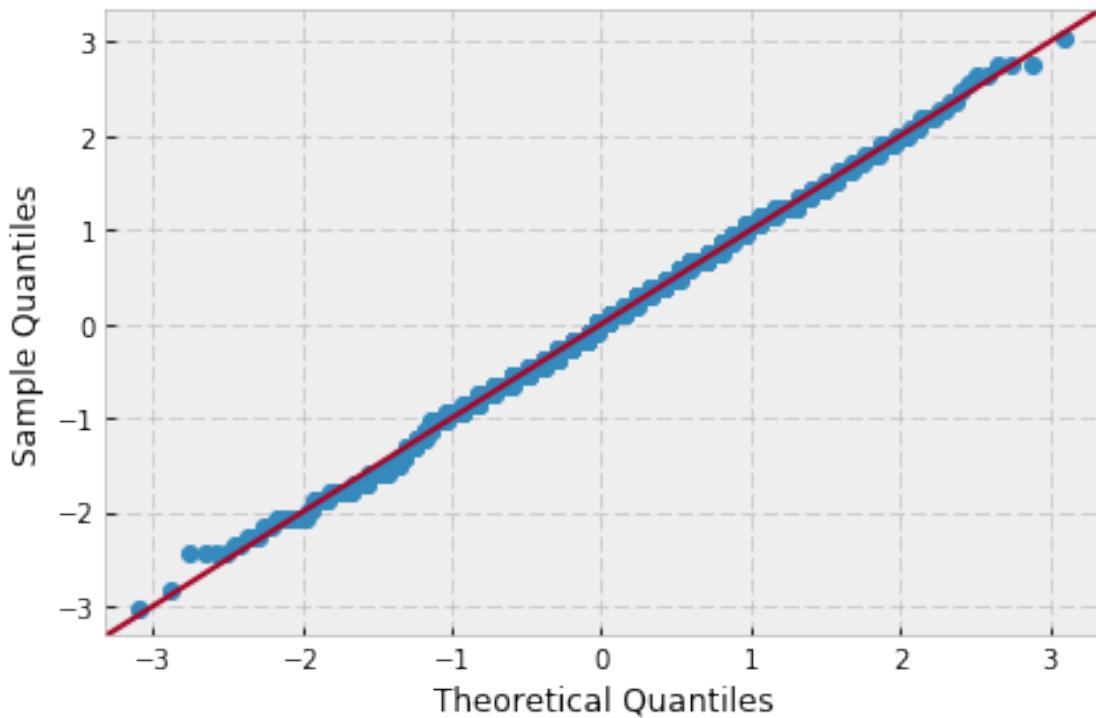


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(),loc=376.834, scale=11.770150551288628)
DescribeResult(nobs=1000, minmax=(339.0, 420.0)
               mean=376.834, variance=138.6751191191191,
               skewness=-0.1276480682047909, kurtosis=0.05875491930131593)
```

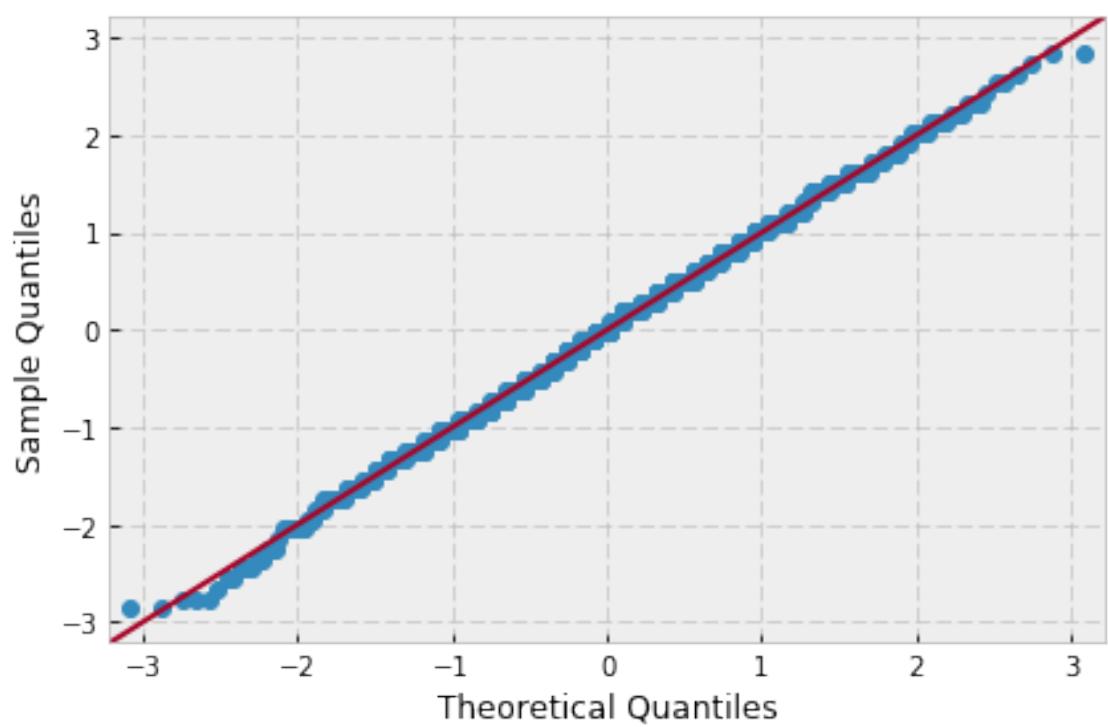
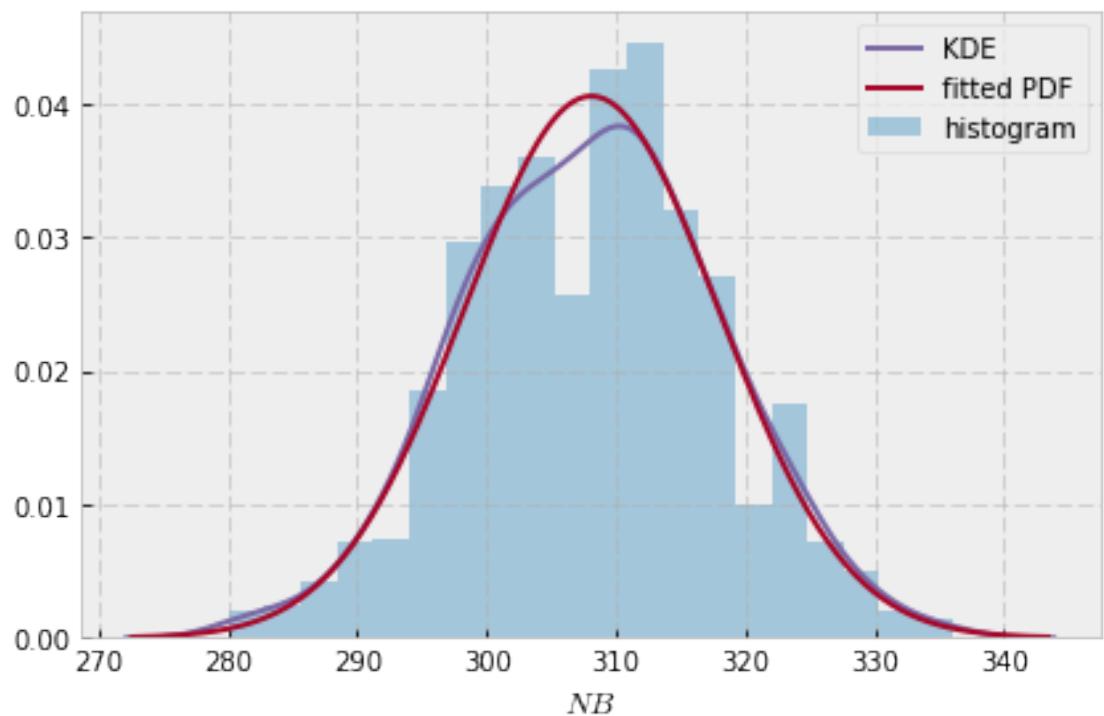


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=339.878, scale=10.58419179720398)
DescribeResult(nobs=1000, minmax=(308.0, 372.0)
    mean=339.8779999999999, variance=112.13725325325326,
    skewness=0.007775191650277858, kurtosis=-0.17709829535942578)
```

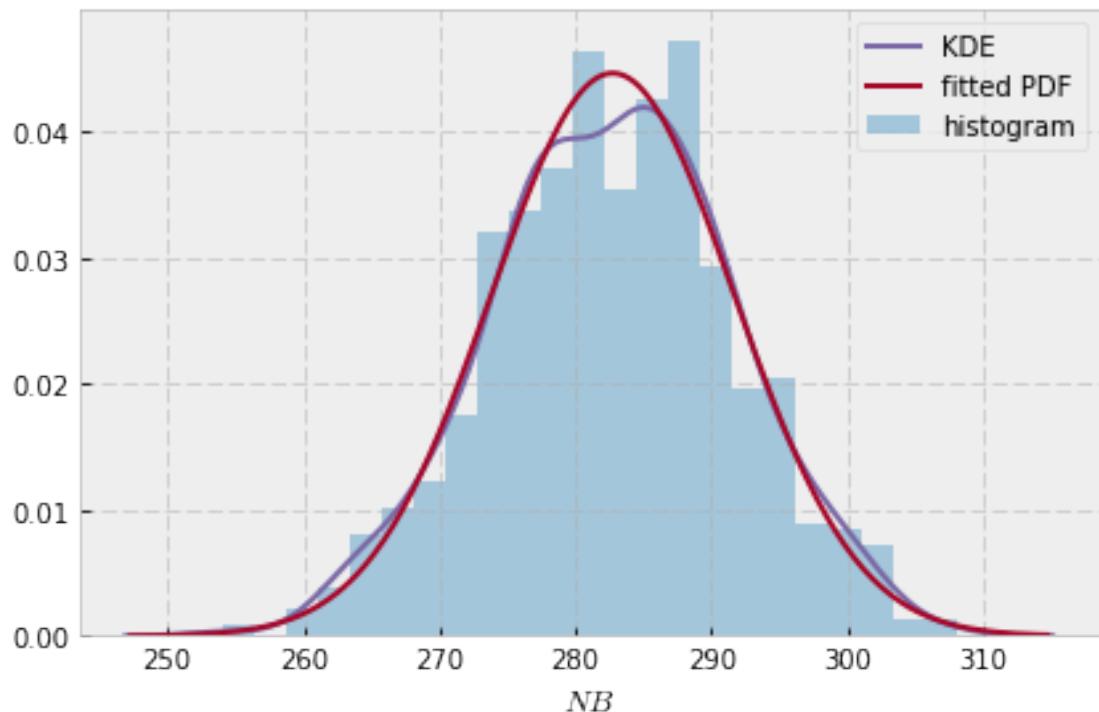


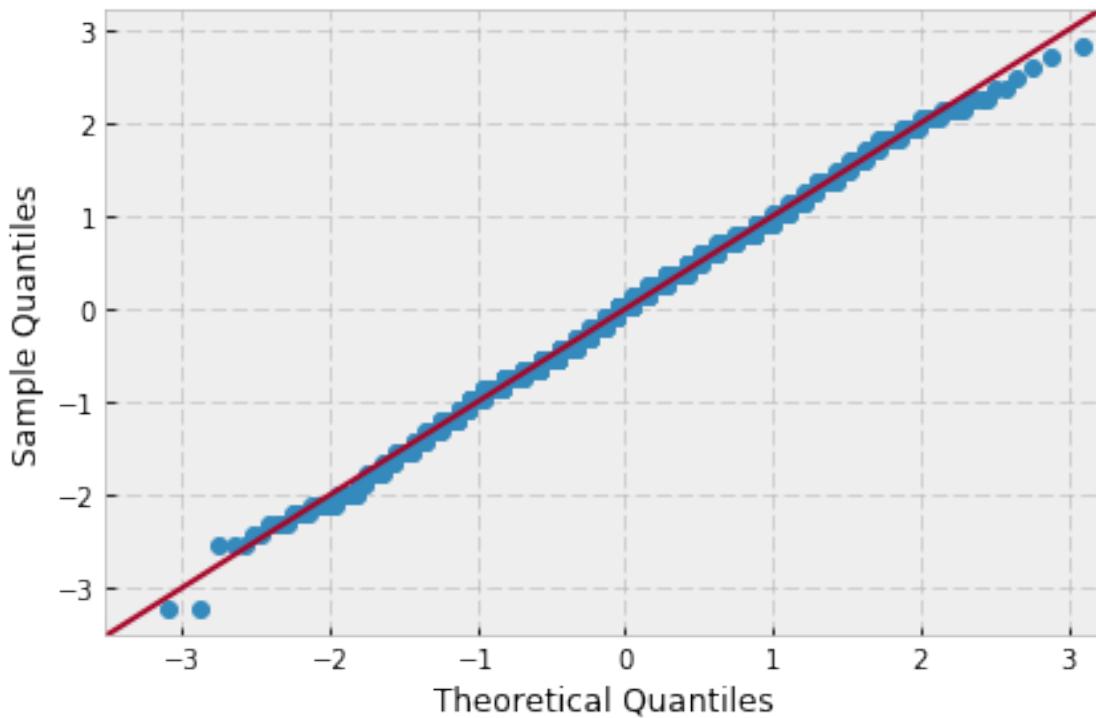


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=308.104, scale=9.833167546625045)
DescribeResult(nobs=1000, minmax=(280.0, 336.0)
               mean=308.1039999999998, variance=96.787971971971984,
               skewness=-0.03143814027898271, kurtosis=-0.15737602860204358)
```

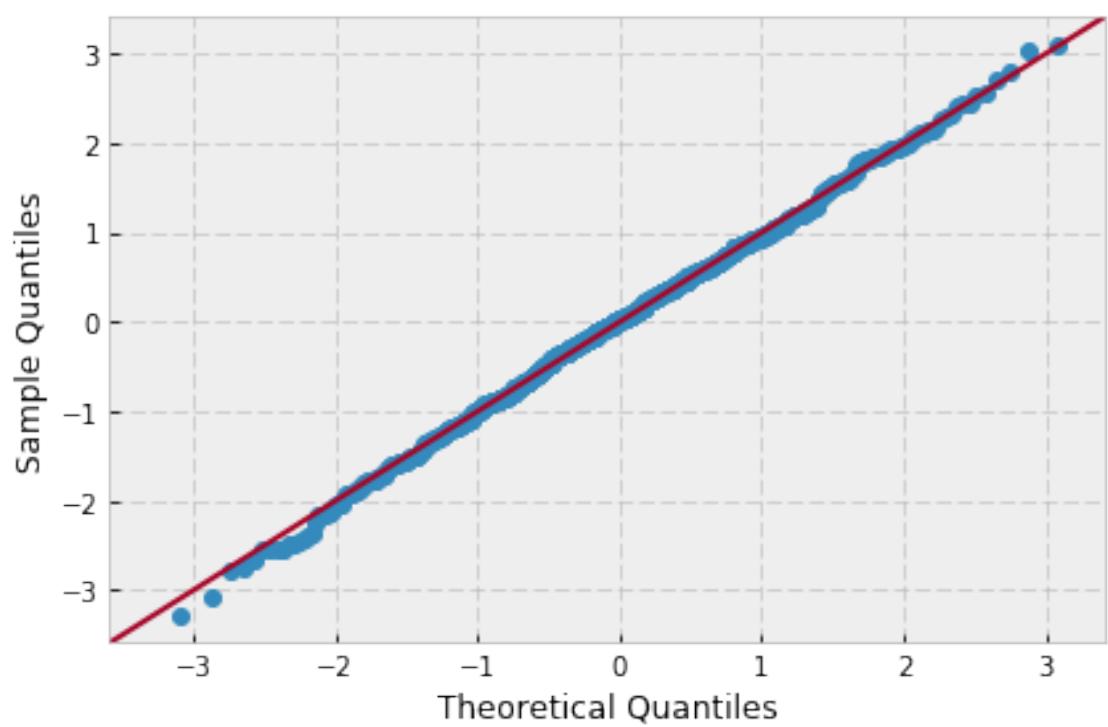
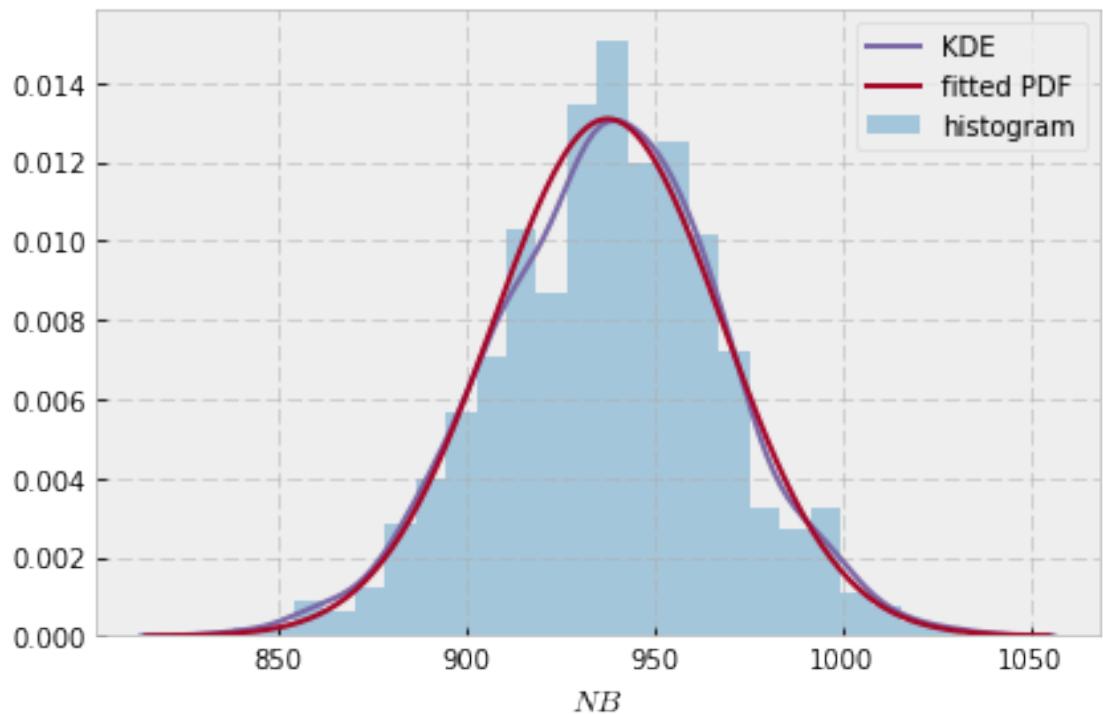


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=282.737, scale=8.938894282851768)
DescribeResult(nobs=1000, minmax=(254.0, 308.0)
               mean=282.73700000000002, variance=79.983814814814821,
               skewness=-0.06645930661160593, kurtosis=-0.16105646384122174)
```

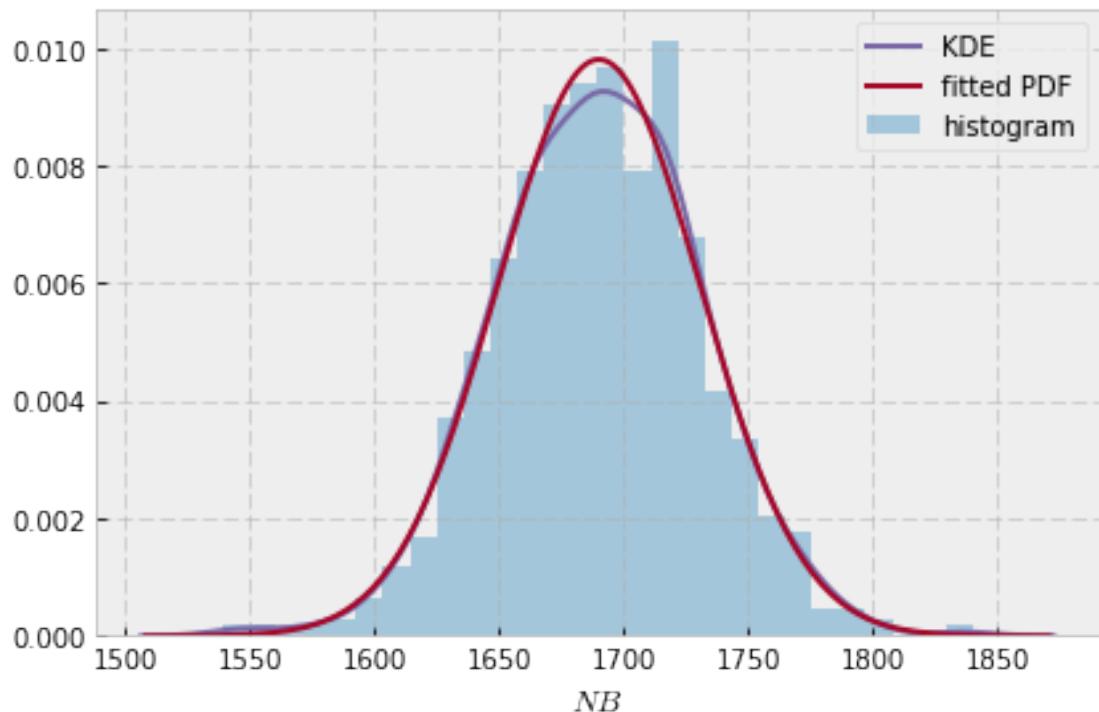


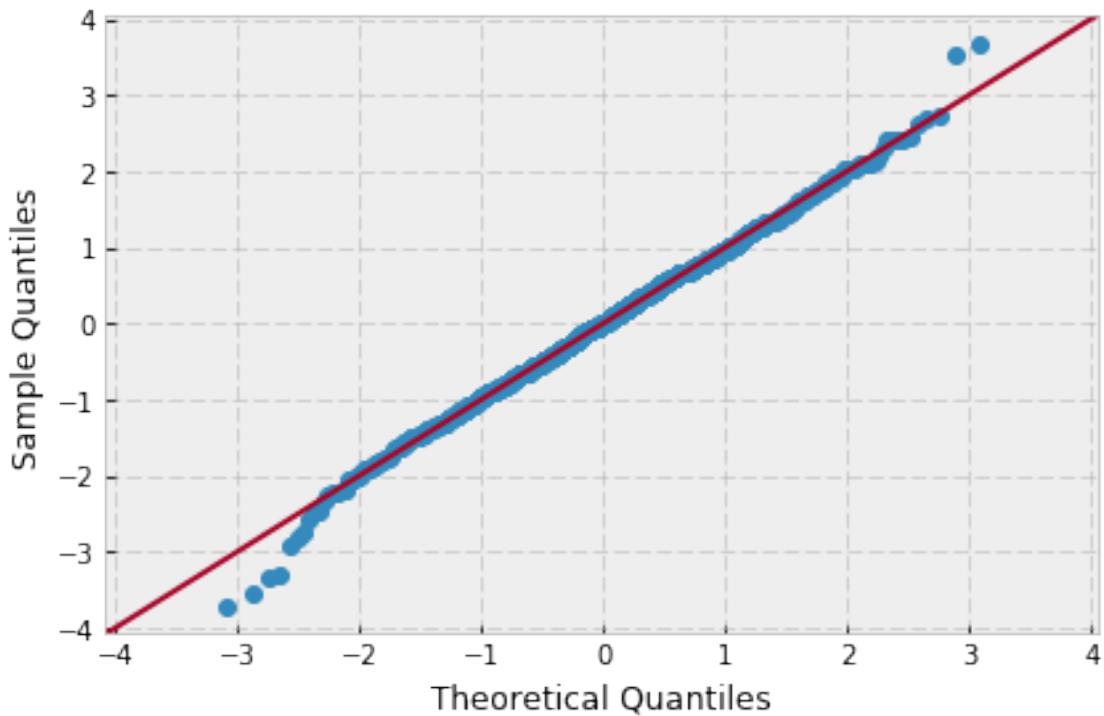


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(),loc=937.742, scale=30.478507771870984)
DescribeResult(nobs=1000, minmax=(838.0, 1032.0)
               mean=937.7419999999996, variance=929.86930530530526,
               skewness=-0.07192809513497425, kurtosis=0.047342989496574894)
```

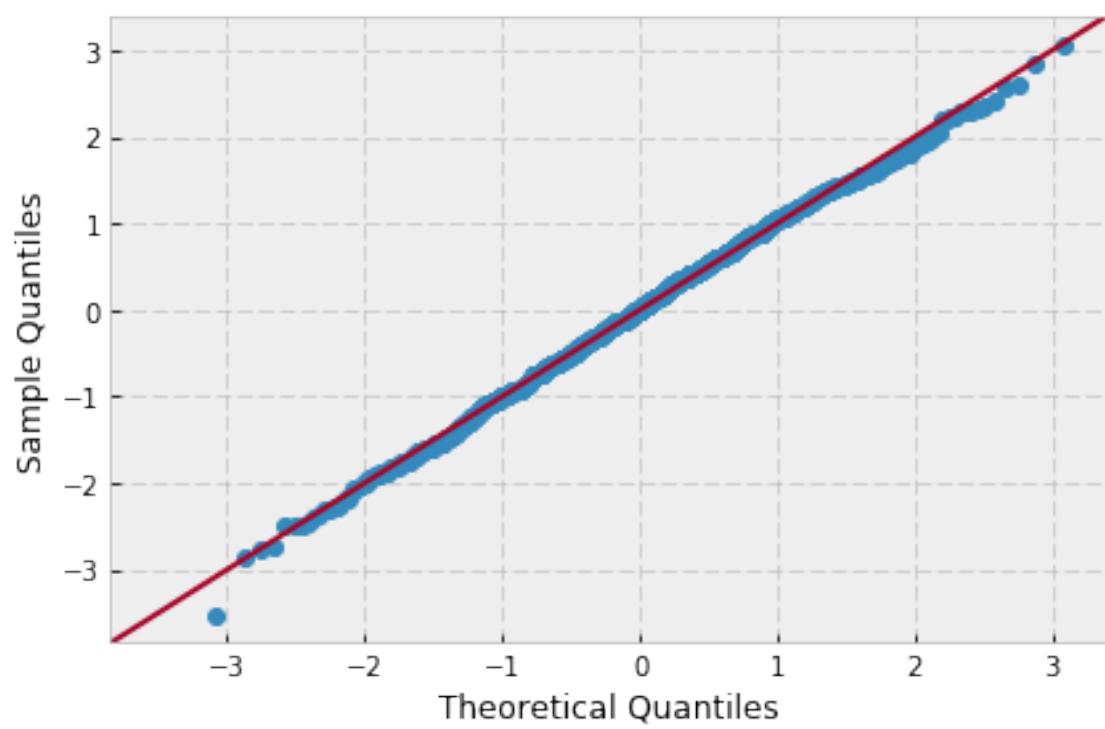
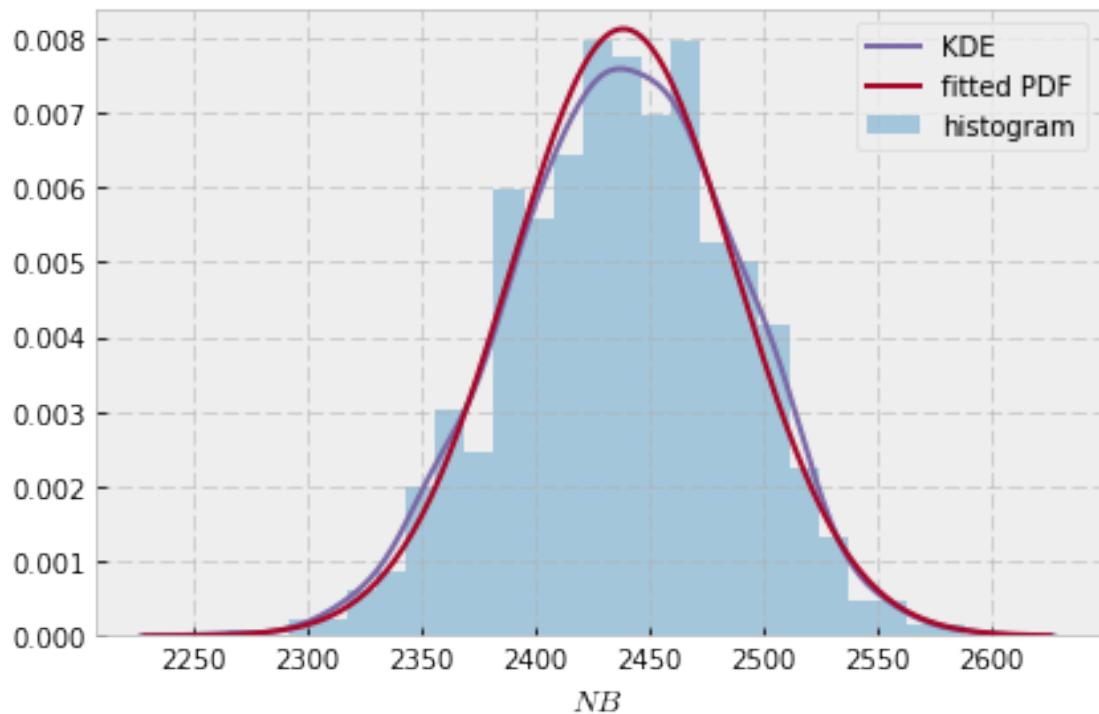


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(), loc=1690.117, scale=40.68485358213791)
DescribeResult(nobs=1000, minmax=(1539.0, 1840.0)
    mean=1690.117, variance=1656.9142252252252,
    skewness=-0.06781641435944913, kurtosis=0.3563007867942418)
```

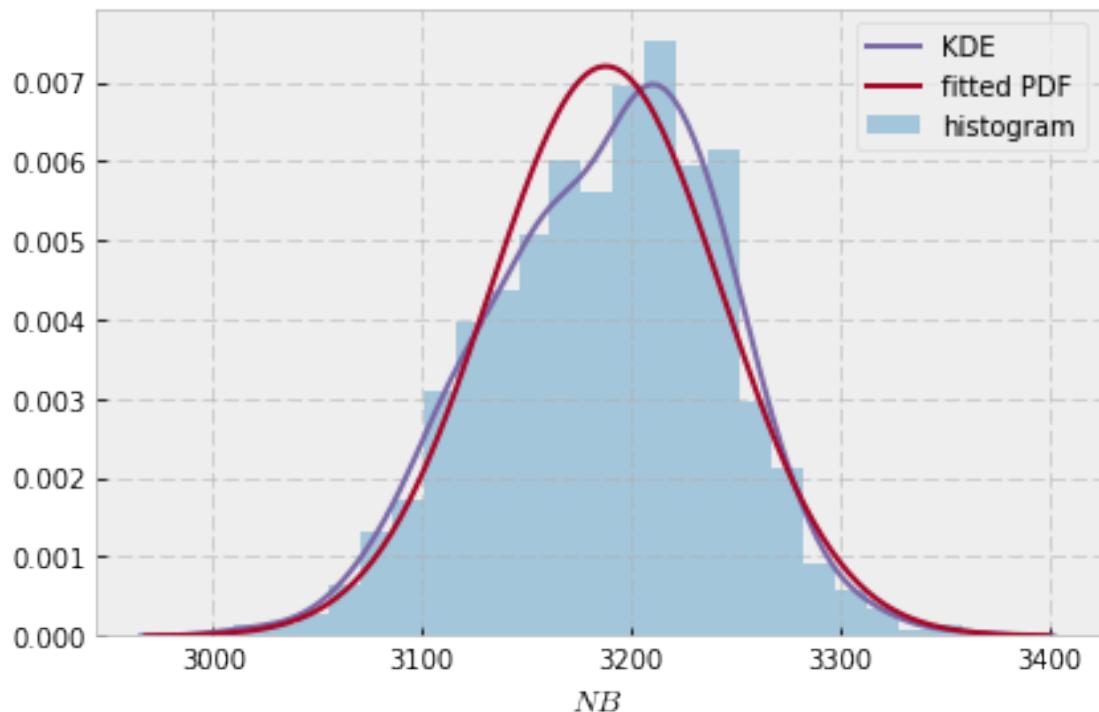


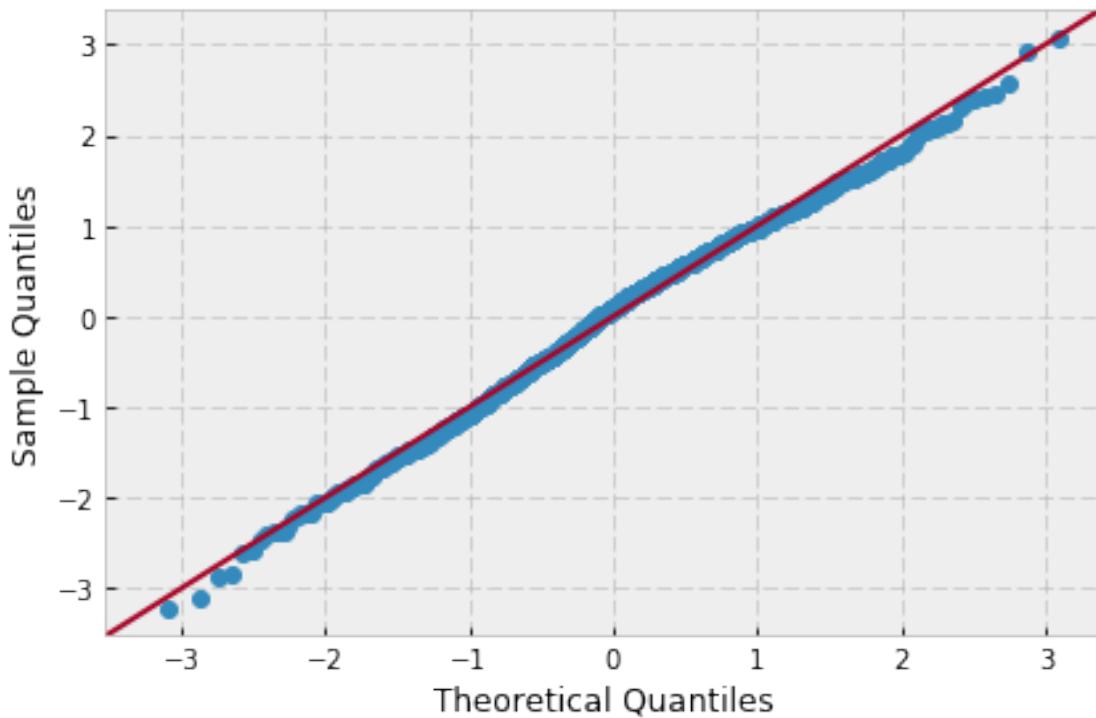


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(),loc=2438.902, scale=49.14275527481136)
DescribeResult(nobs=1000, minmax=(2266.0, 2589.0)
               mean=2438.902, variance=2417.4278238238235,
               skewness=-0.10688761114207036, kurtosis=-0.17412822892679847)
```

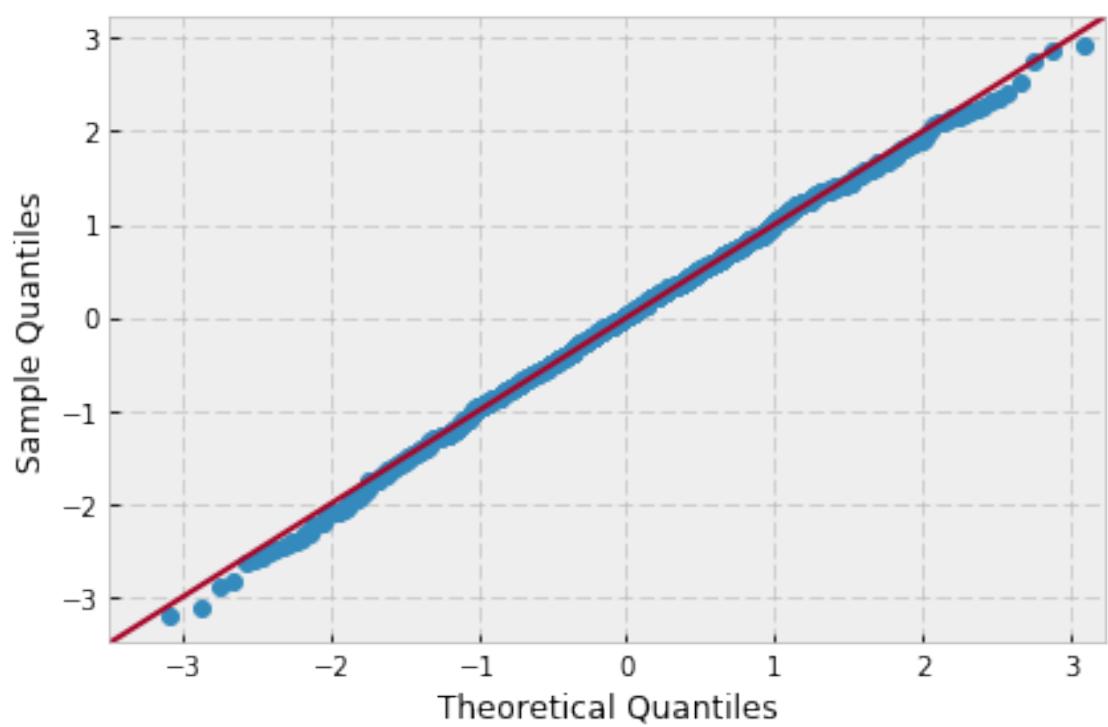
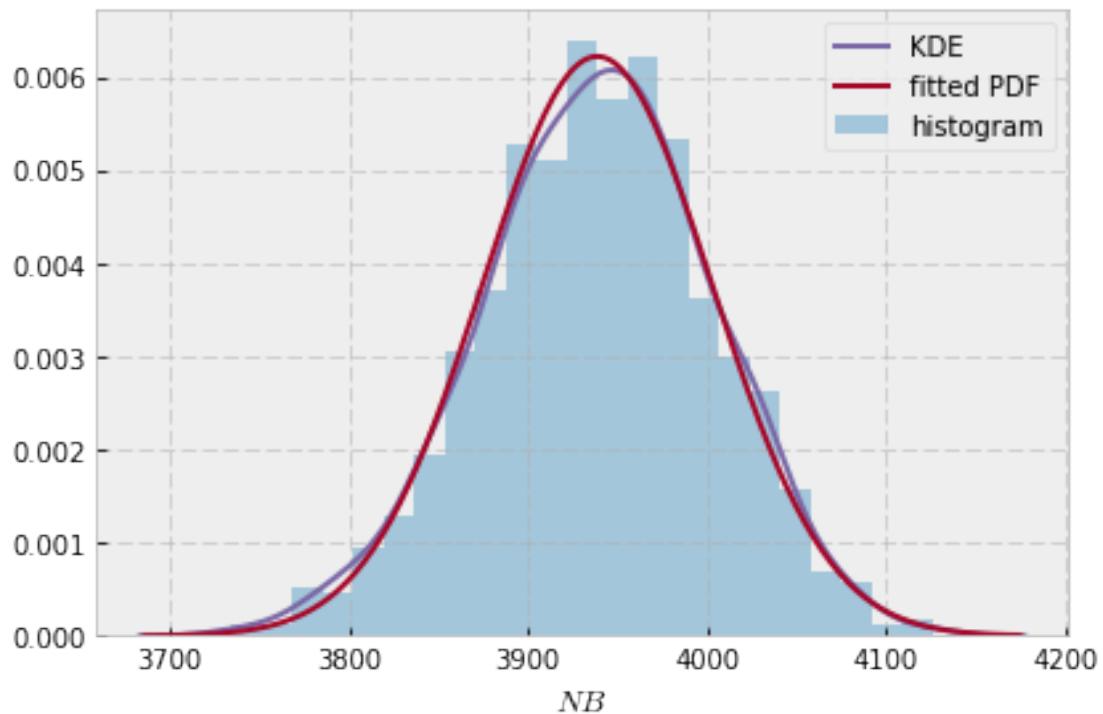


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(), loc=3188.05, scale=55.411709051427025)
DescribeResult(nobs=1000, minmax=(3010.0, 3358.0)
    mean=3188.0500000000002, variance=3073.5310310310306,
    skewness=-0.20771568496649212, kurtosis=-0.2243180124111177)
```

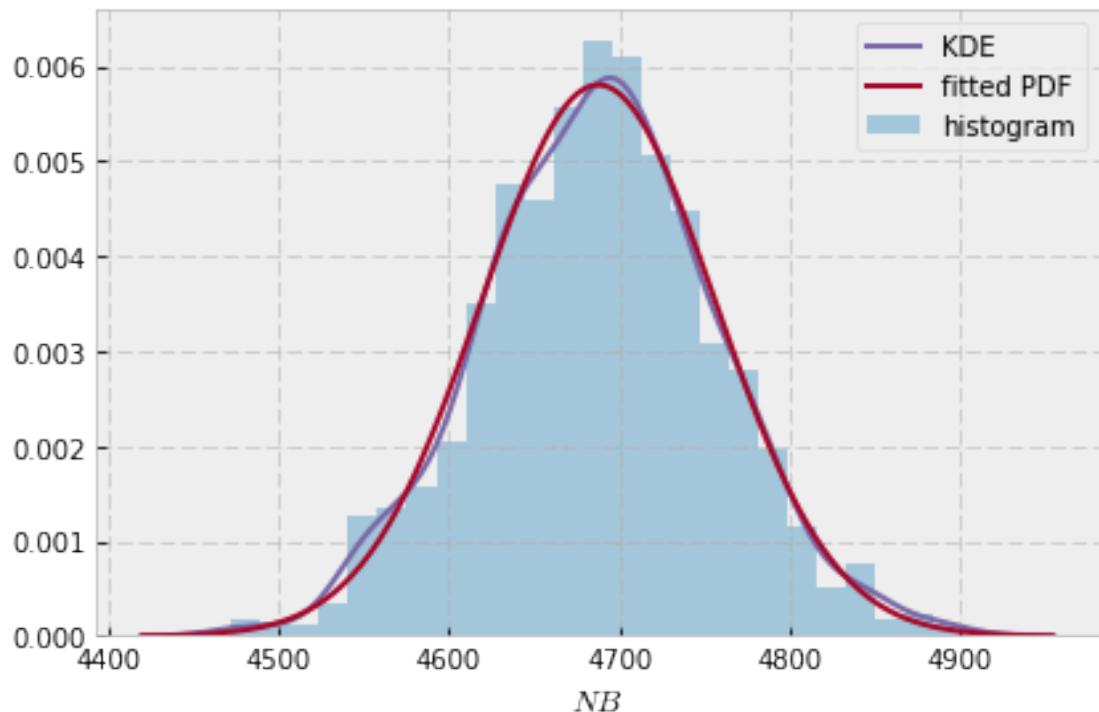


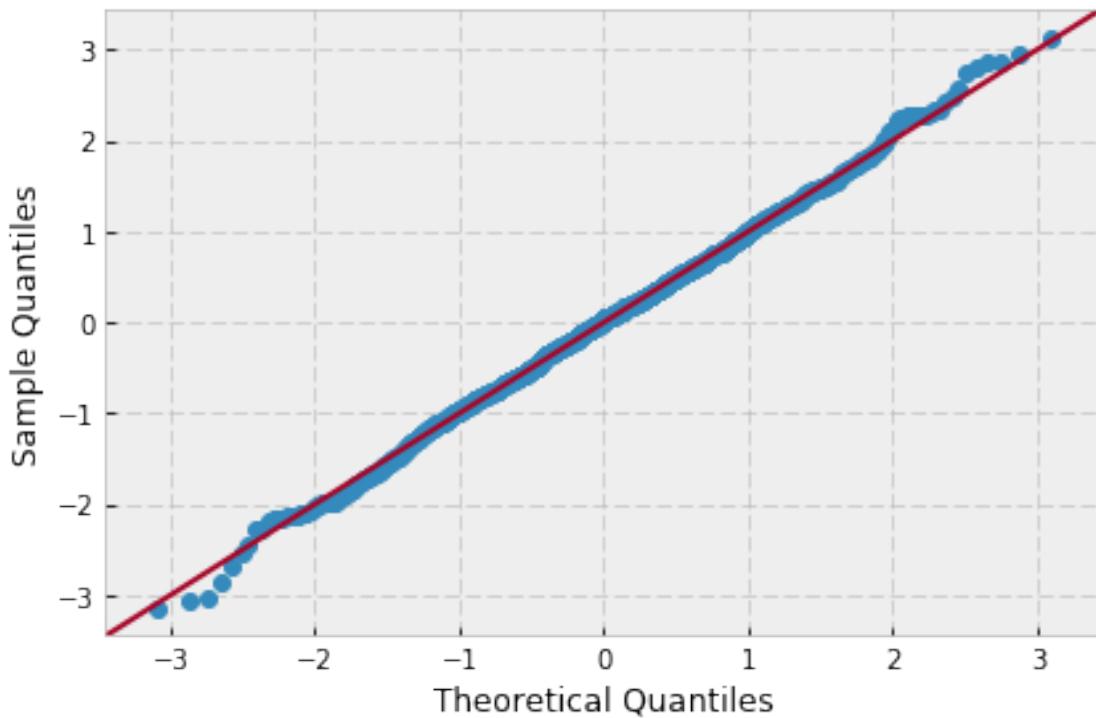


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(),loc=3938.519, scale=64.1689460642763)
DescribeResult(nobs=1000, minmax=(3734.0, 4126.0)
               mean=3938.518999999998, variance=4121.7754144144146,
               skewness=-0.12990029447688103, kurtosis=-0.04942373928773325)
```

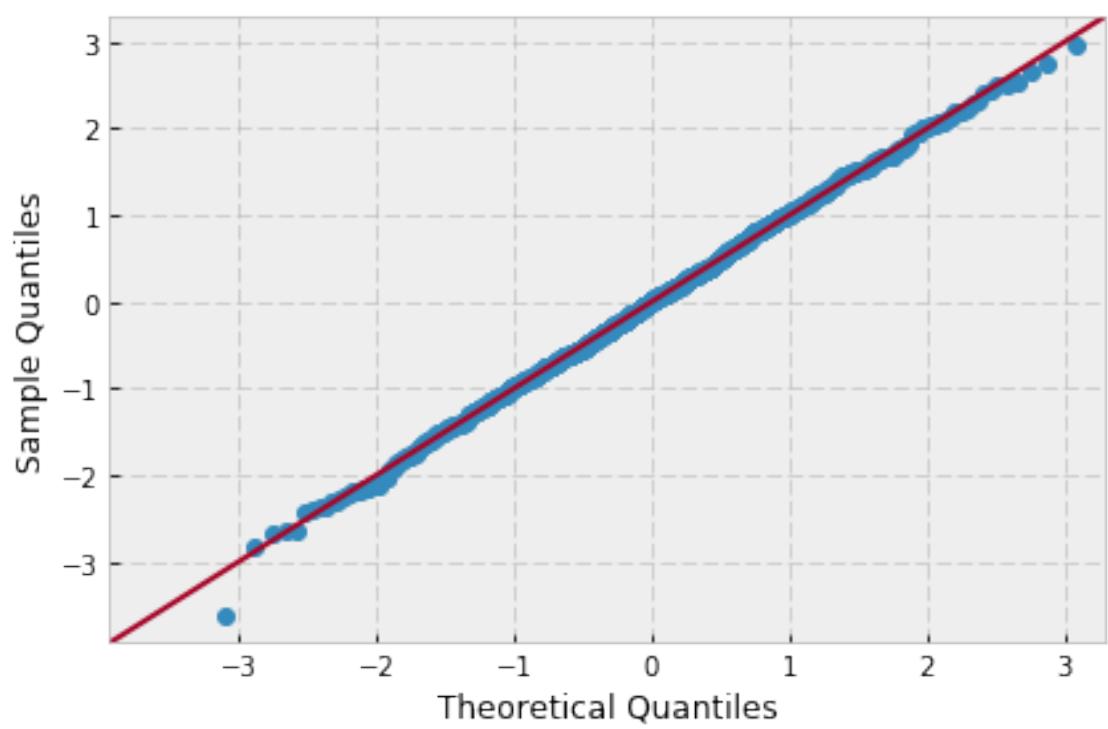
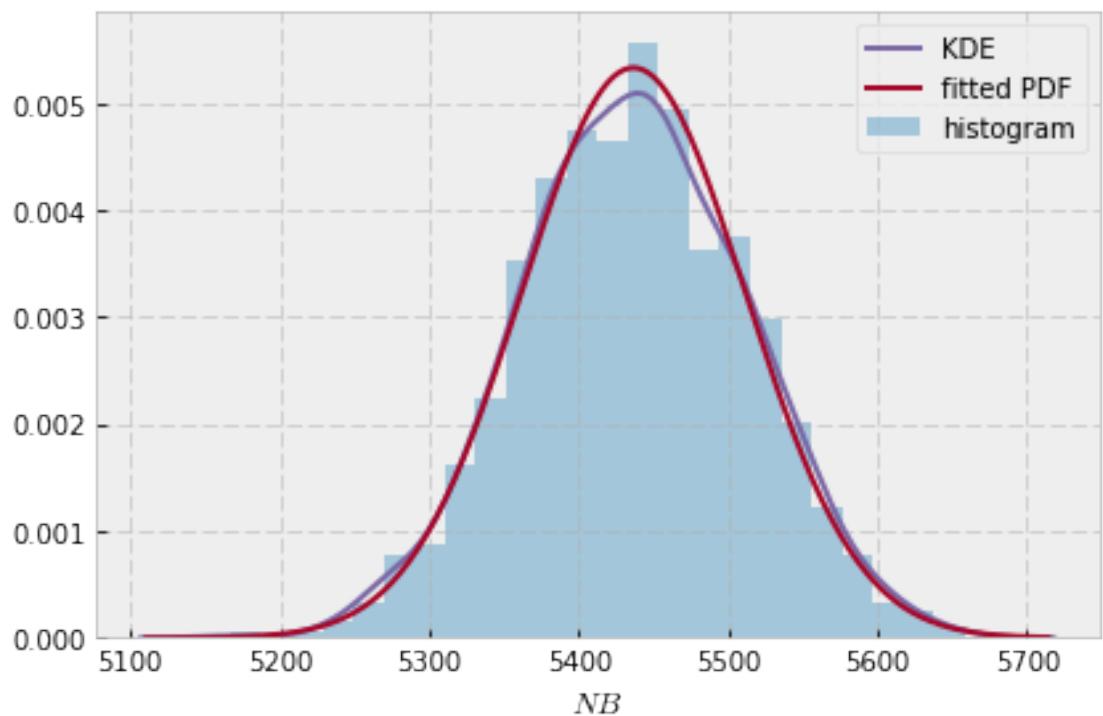


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(), loc=4687.547, scale=68.72833324764976)
DescribeResult(nobs=1000, minmax=(4472.0, 4902.0)
               mean=4687.546999999999, variance=4728.3121031031033,
               skewness=0.006059228212327086, kurtosis=0.088866771895181)
```

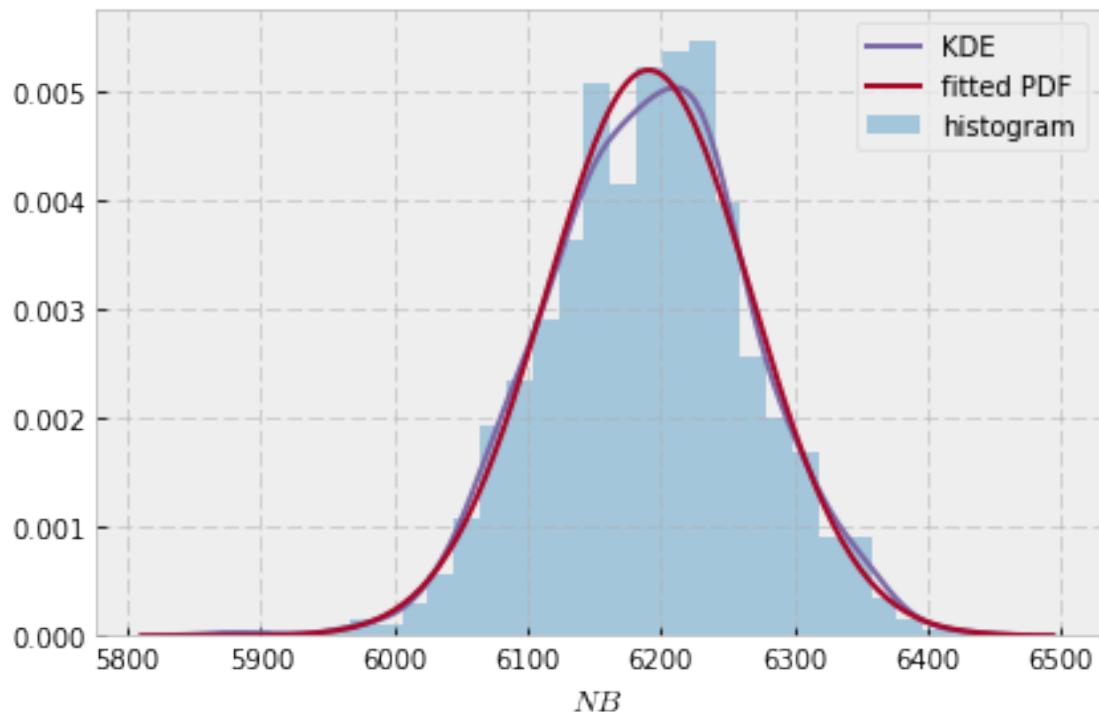


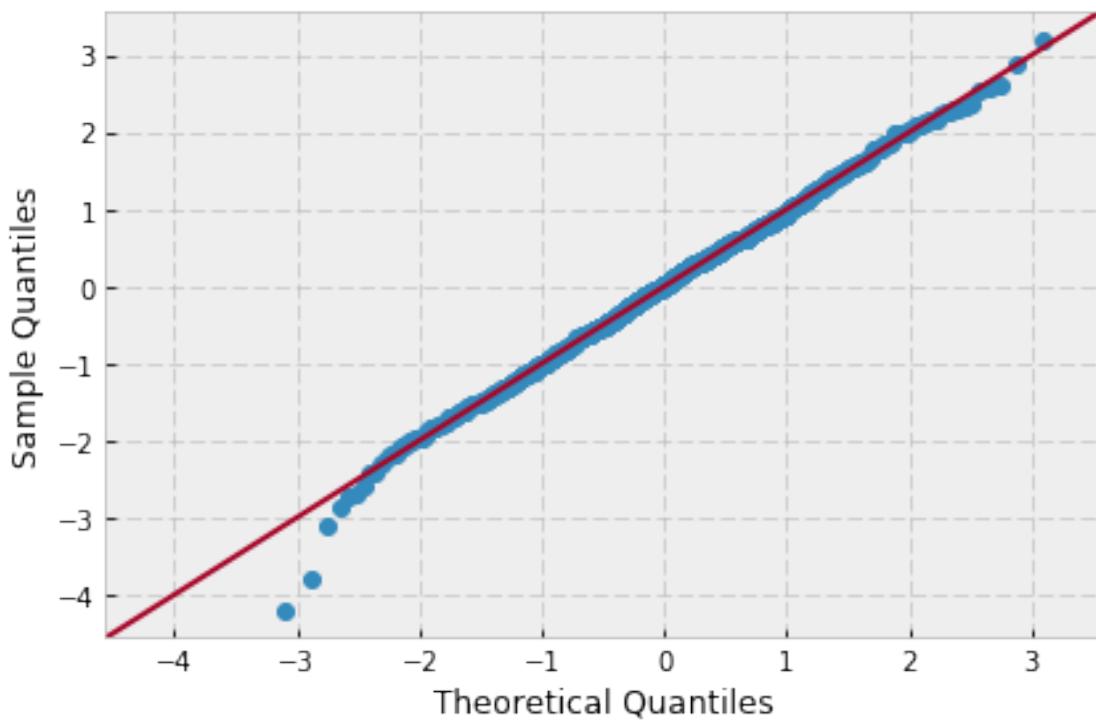


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(),loc=5436.656, scale=74.85241254629005)
DescribeResult(nobs=1000, minmax=(5167.0, 5658.0)
               mean=5436.655999999999, variance=5608.4921561561569,
               skewness=-0.027567631581954754, kurtosis=-0.11874901949210148)
```

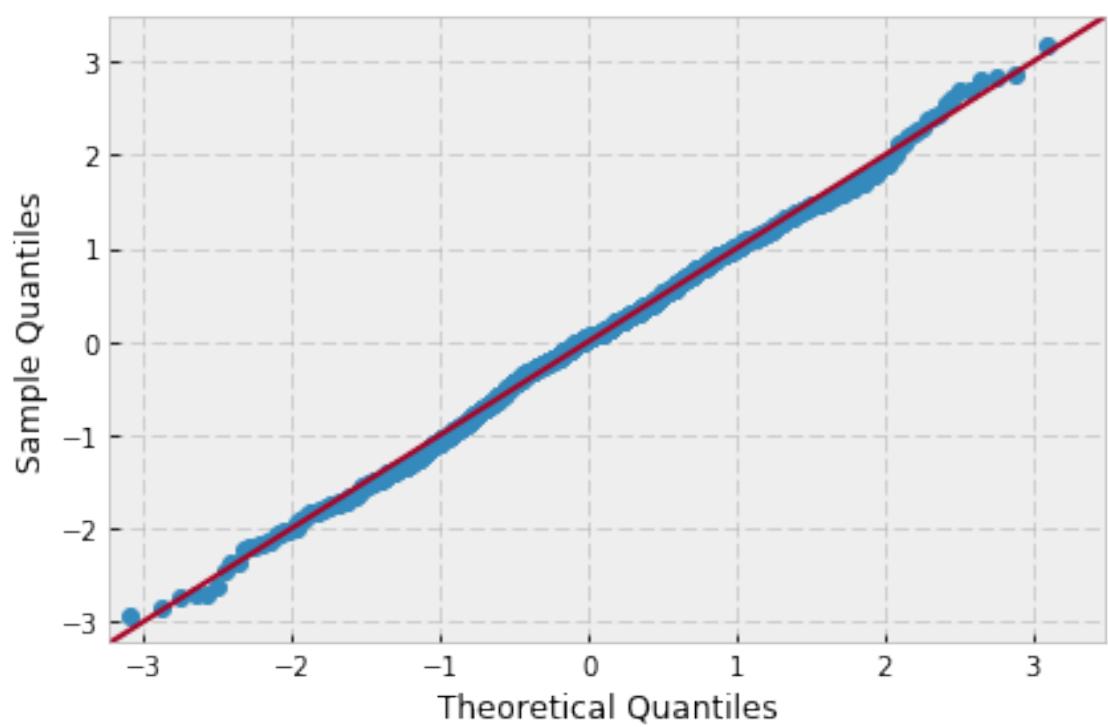
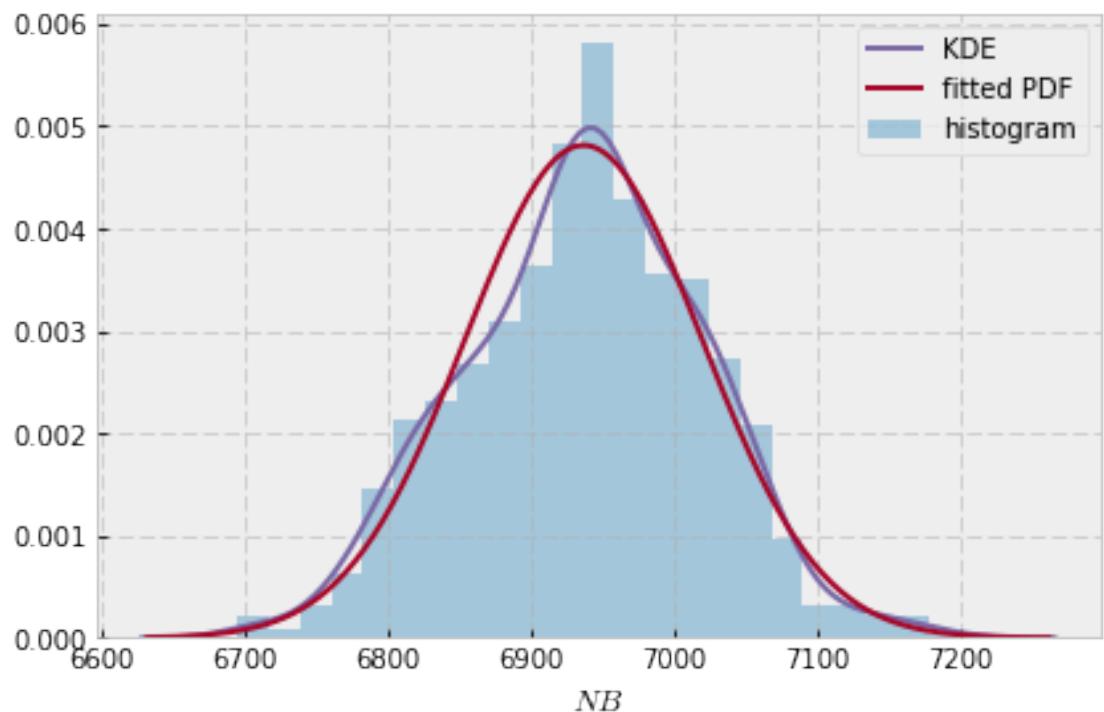


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(), loc=6190.992, scale=76.79893186757222)
DescribeResult(nobs=1000, minmax=(5869.0, 6436.0)
    mean=6190.992000000002, variance=5903.9799159159165,
    skewness=-0.0919954222535947, kurtosis=0.21438002338780526)
```

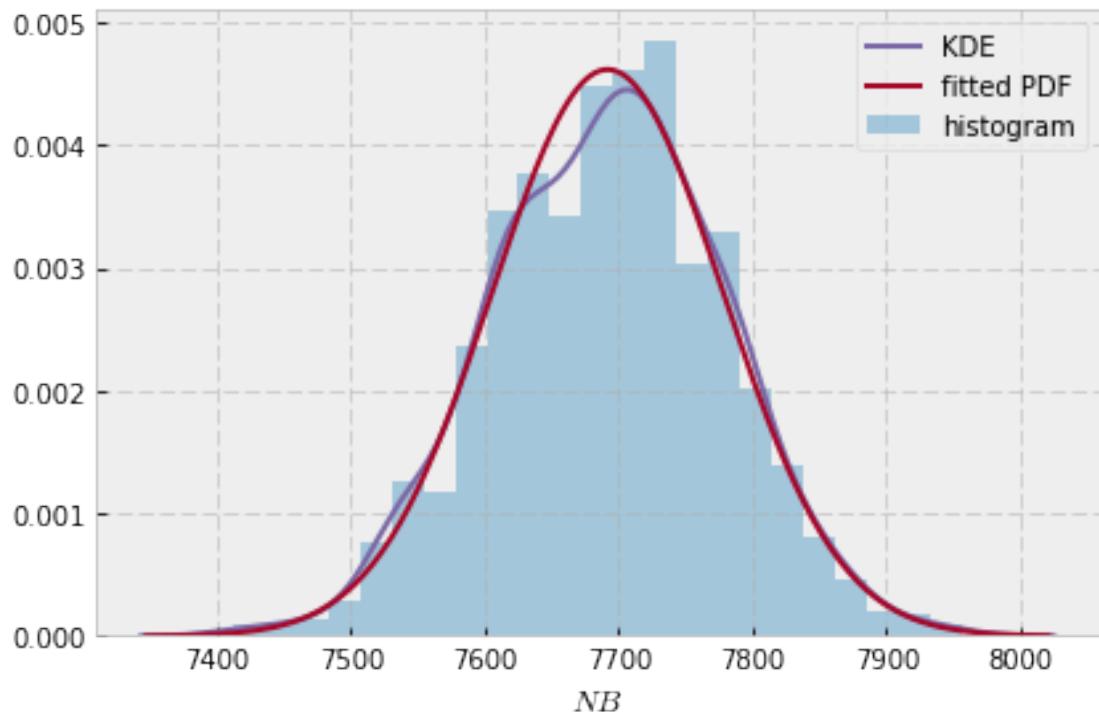


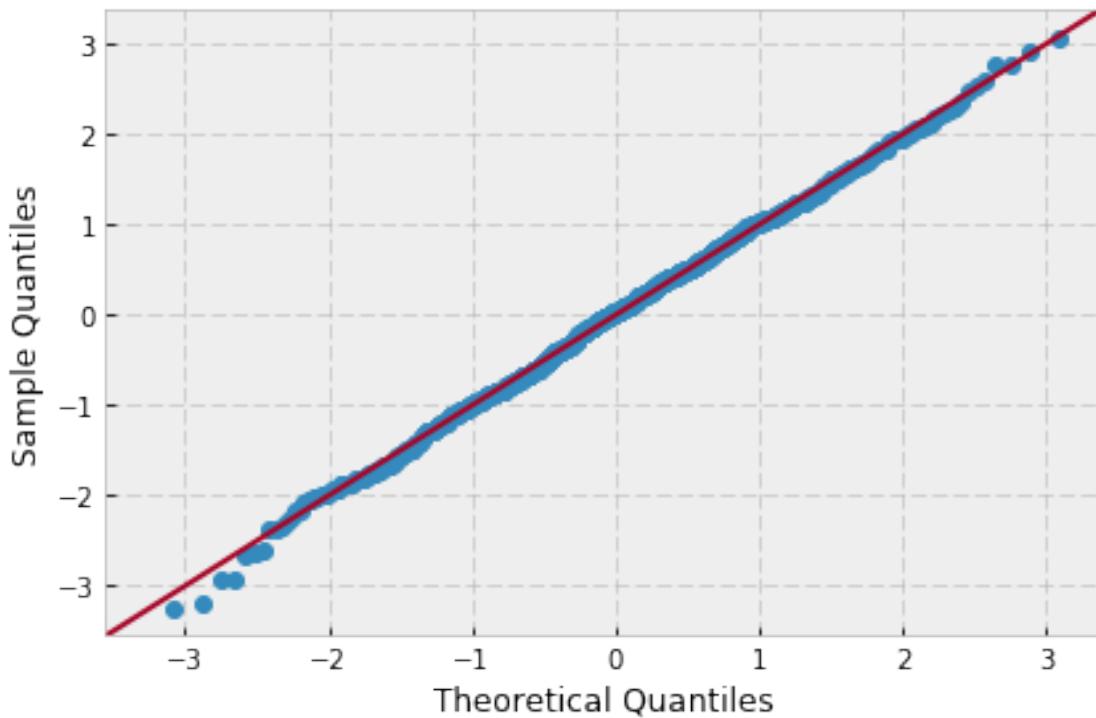


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(),loc=6936.651, scale=82.9174722178625)
DescribeResult(nobs=1000, minmax=(6694.0, 7200.0)
               mean=6936.650999999998, variance=6882.1893883883886,
               skewness=-0.05876604345319681, kurtosis=-0.11185173642756352)
```

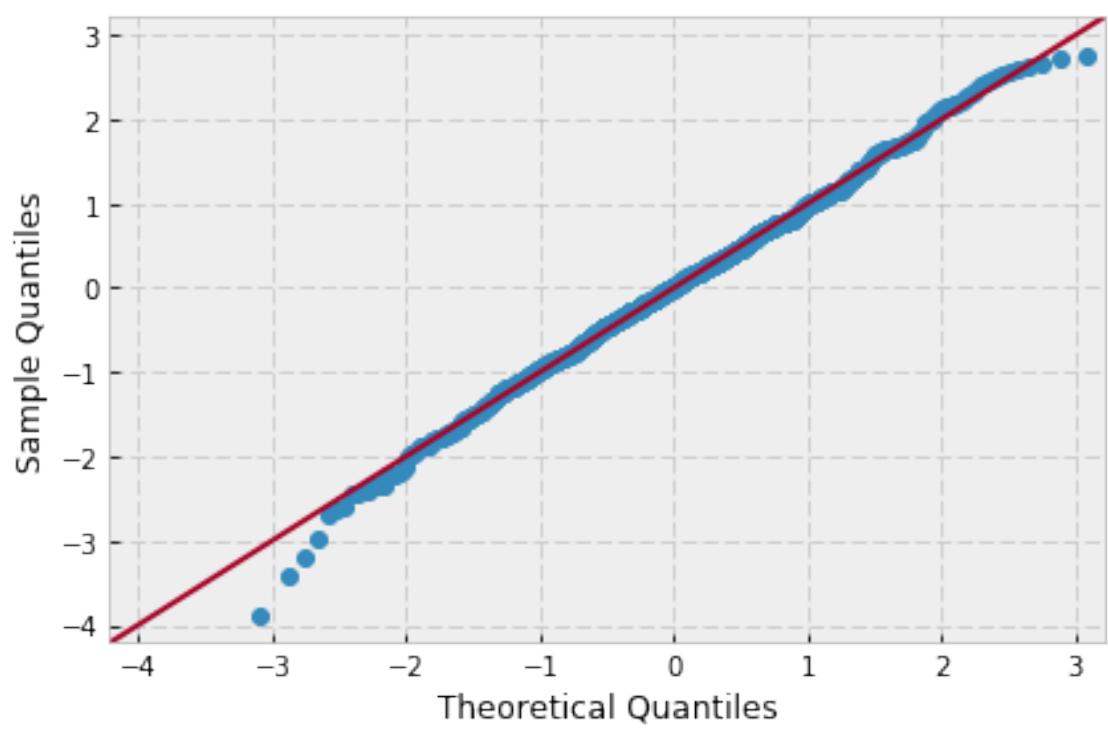
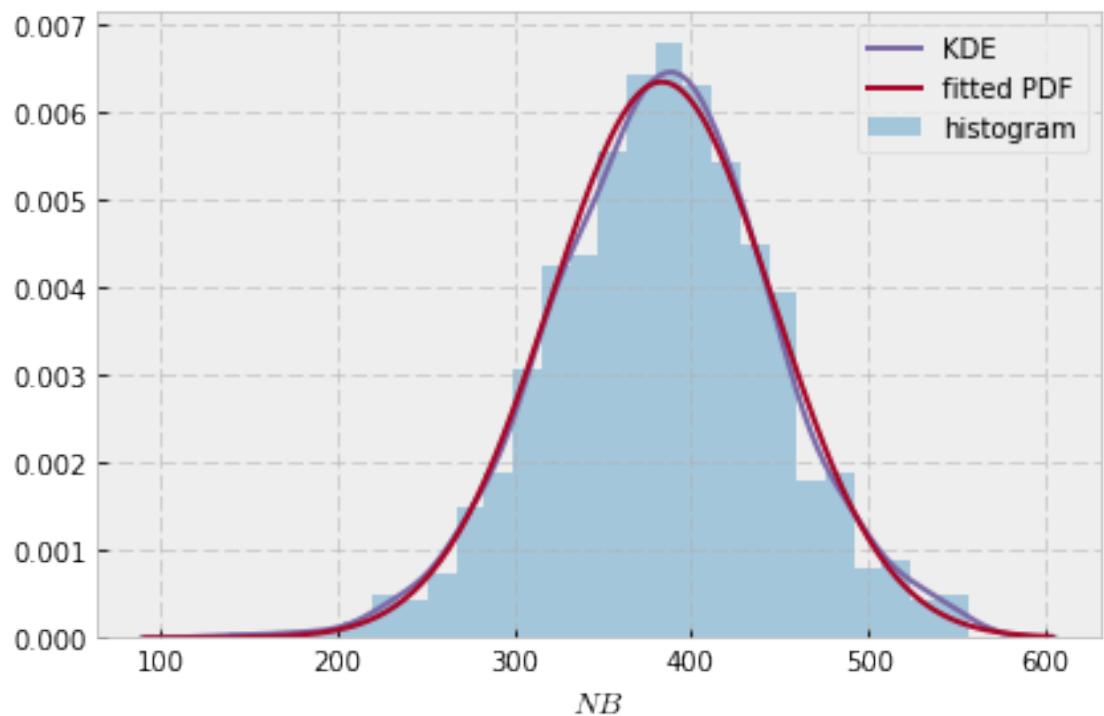


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(), loc=7691.652, scale=86.352086807442)
DescribeResult(nobs=1000, minmax=(7412.0, 7956.0)
    mean=7691.652, variance=7464.1470430430427,
    skewness=-0.06574630912998834, kurtosis=-0.08442355811346003)
```

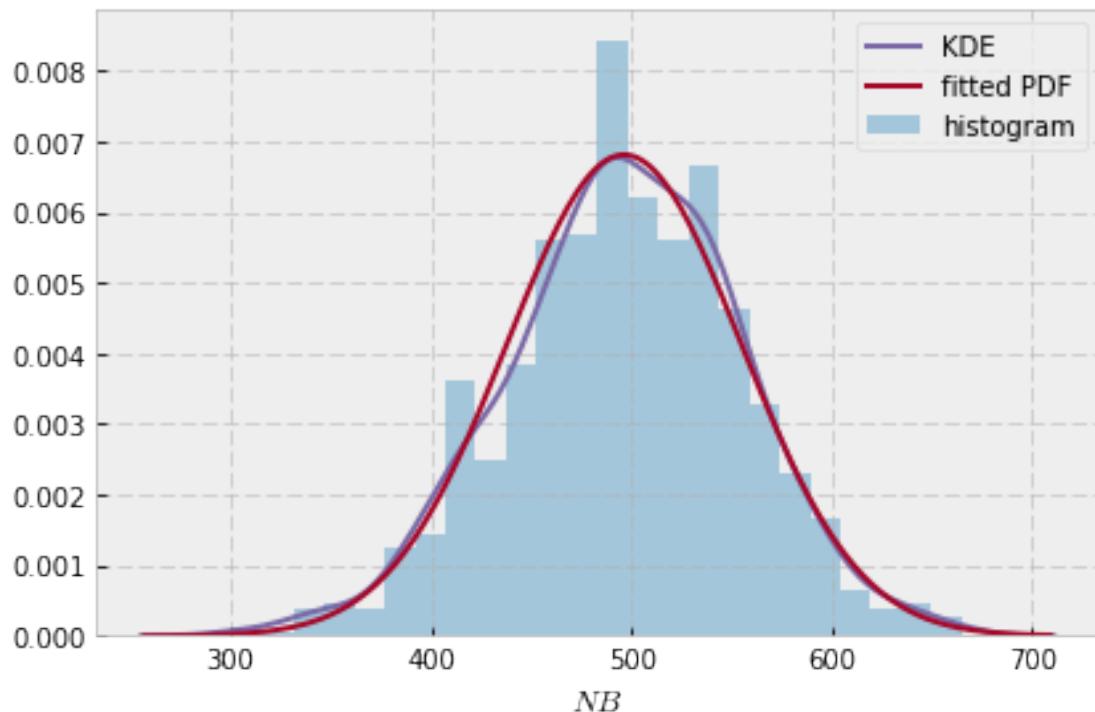


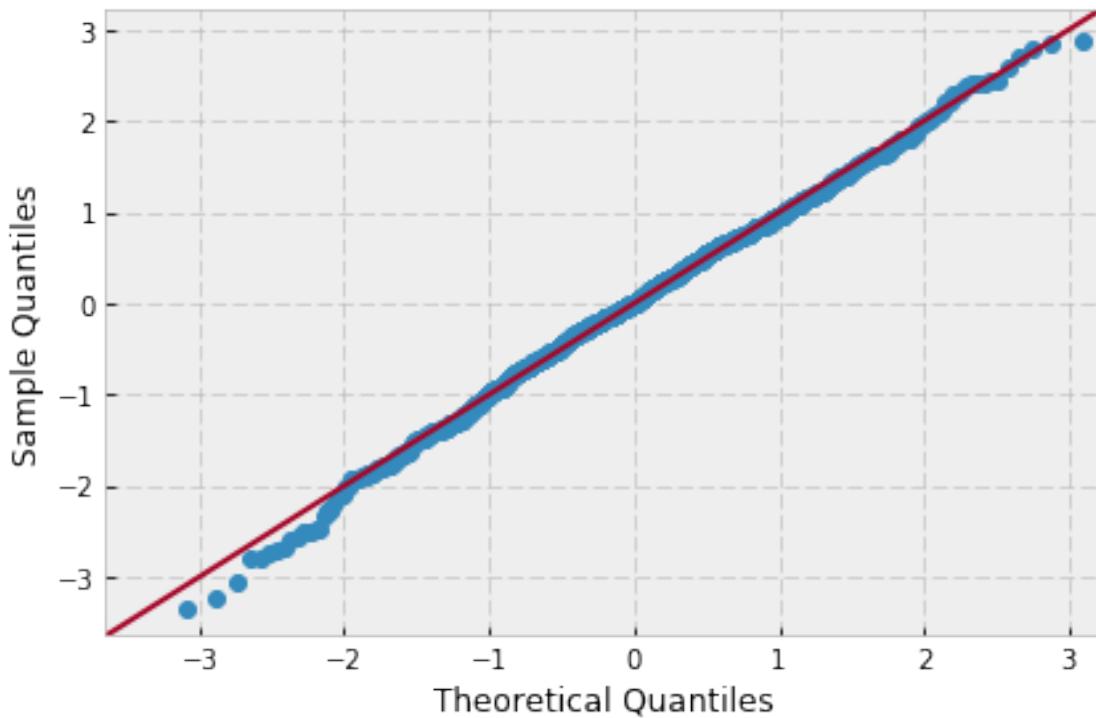


```
variable name: r
variable value: 1.0
distribution: normal(shape=(),loc=383.343, scale=62.93259370946028)
DescribeResult(nobs=1000, minmax=(139.0, 556.0)
               mean=383.3430000000002, variance=3964.4758268268265,
               skewness=-0.08783031725367633, kurtosis=0.25029828426630907)
```

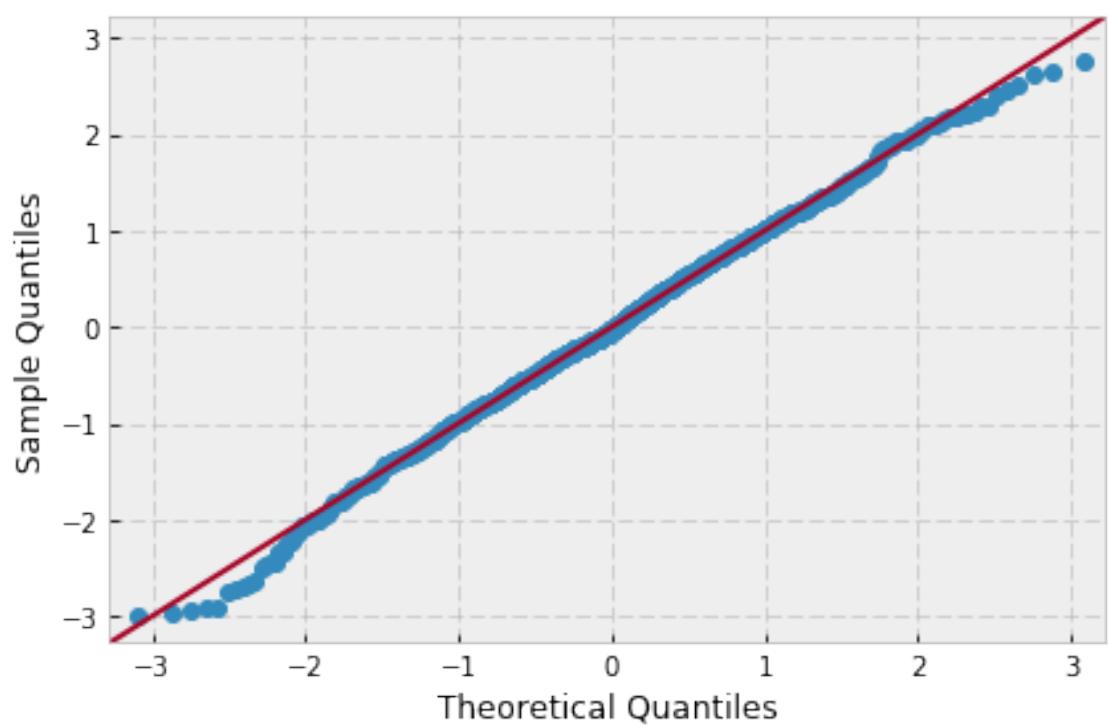
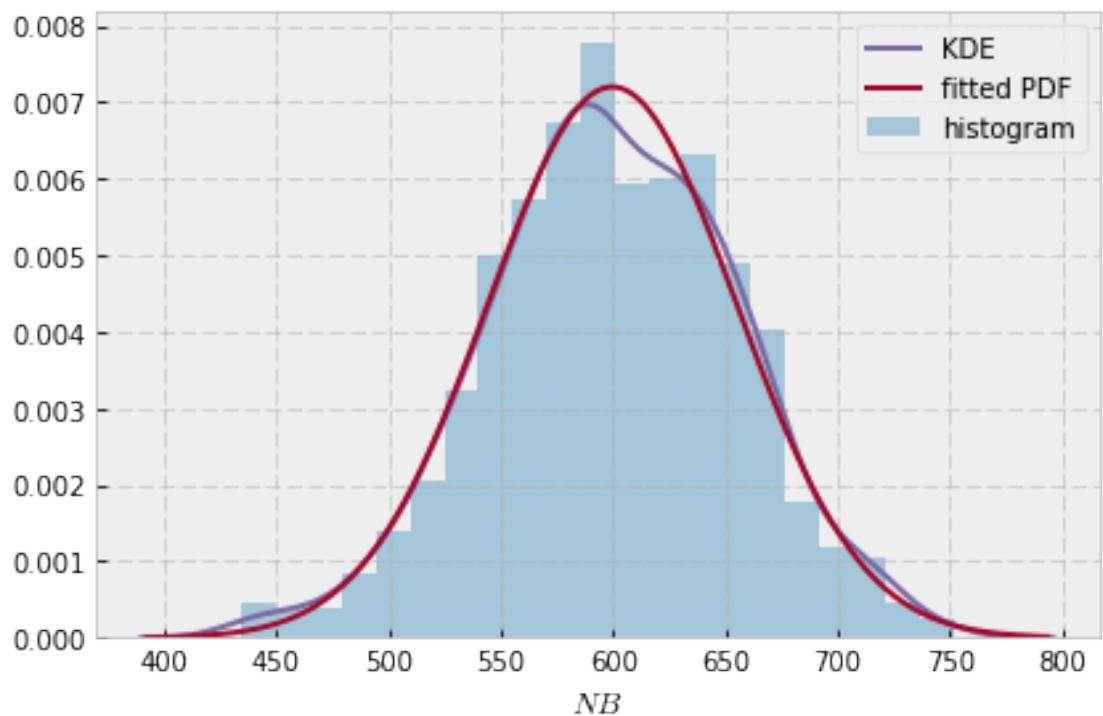


```
variable name: r
variable value: 1.22222222222223
distribution: normal(shape=(), loc=496.363, scale=58.56400969025259)
DescribeResult(nobs=1000, minmax=(301.0, 665.0)
    mean=496.363, variance=3433.1764074074072,
    skewness=-0.1531167148634229, kurtosis=0.1199001046742163)
```

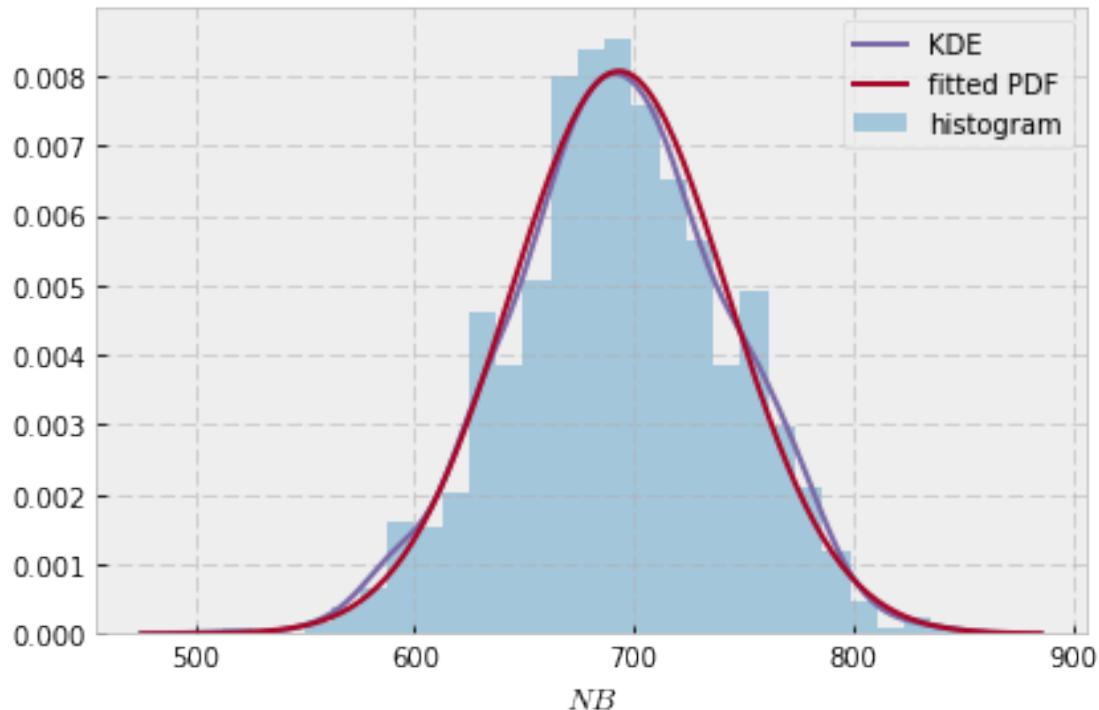


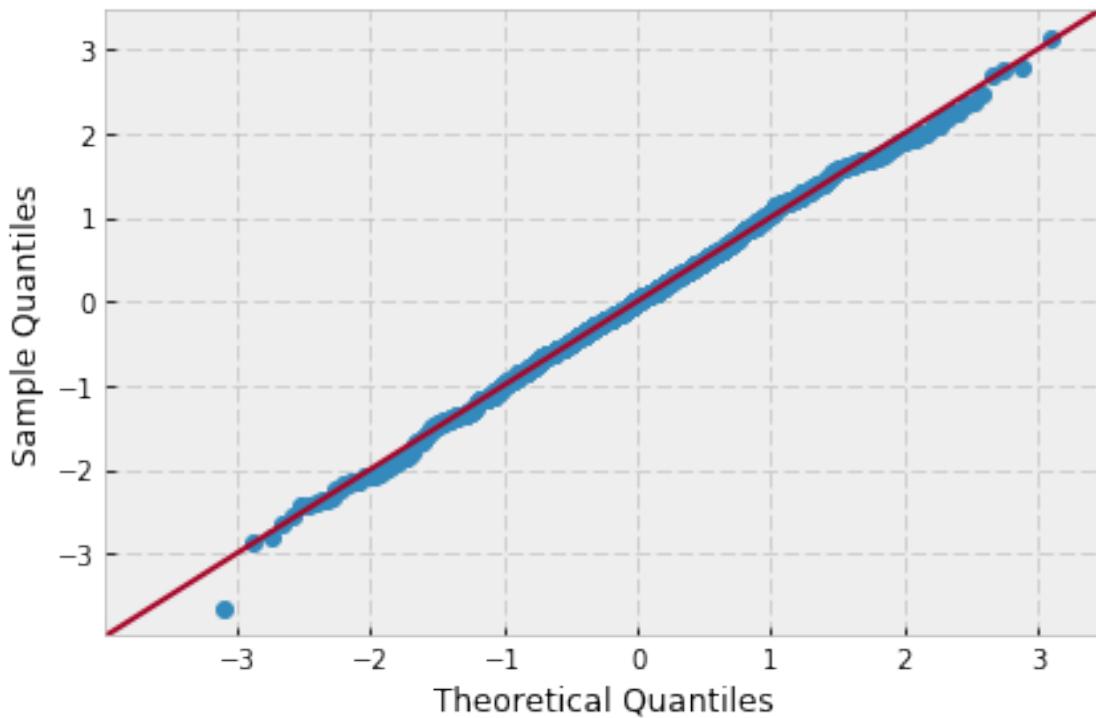


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=599.644, scale=55.37753392848042)
DescribeResult(nobs=1000, minmax=(434.0, 752.0)
               mean=599.6440000000001, variance=3069.7410050050048,
               skewness=-0.11206276161203048, kurtosis=-0.034758066601593196)
```

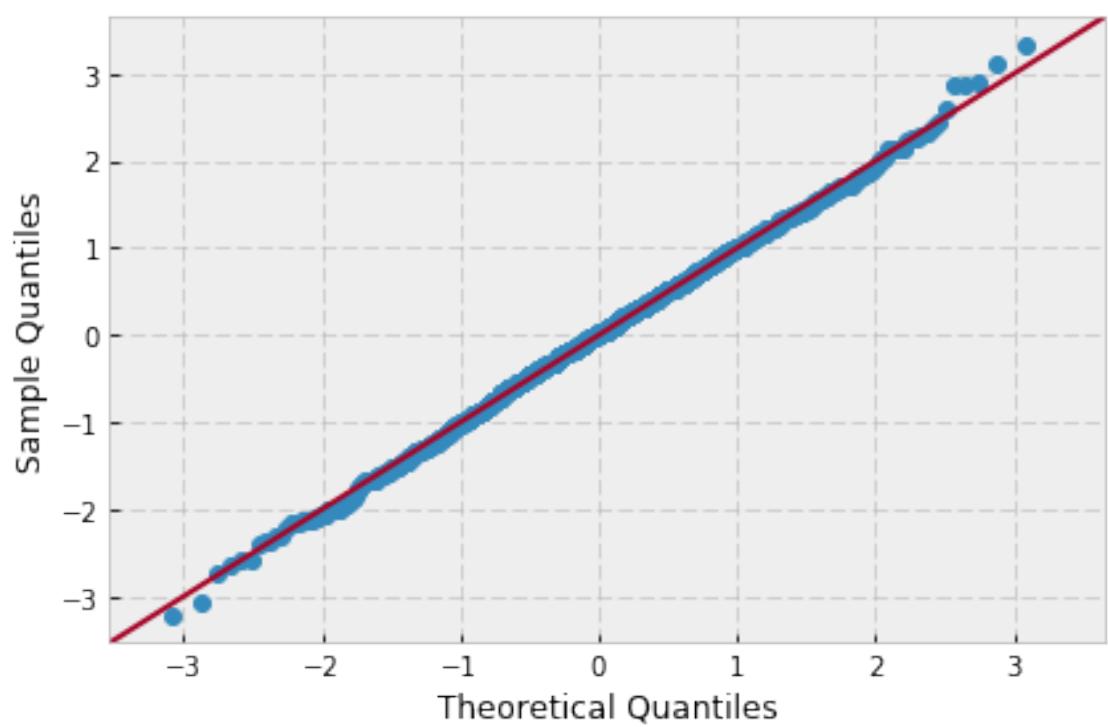
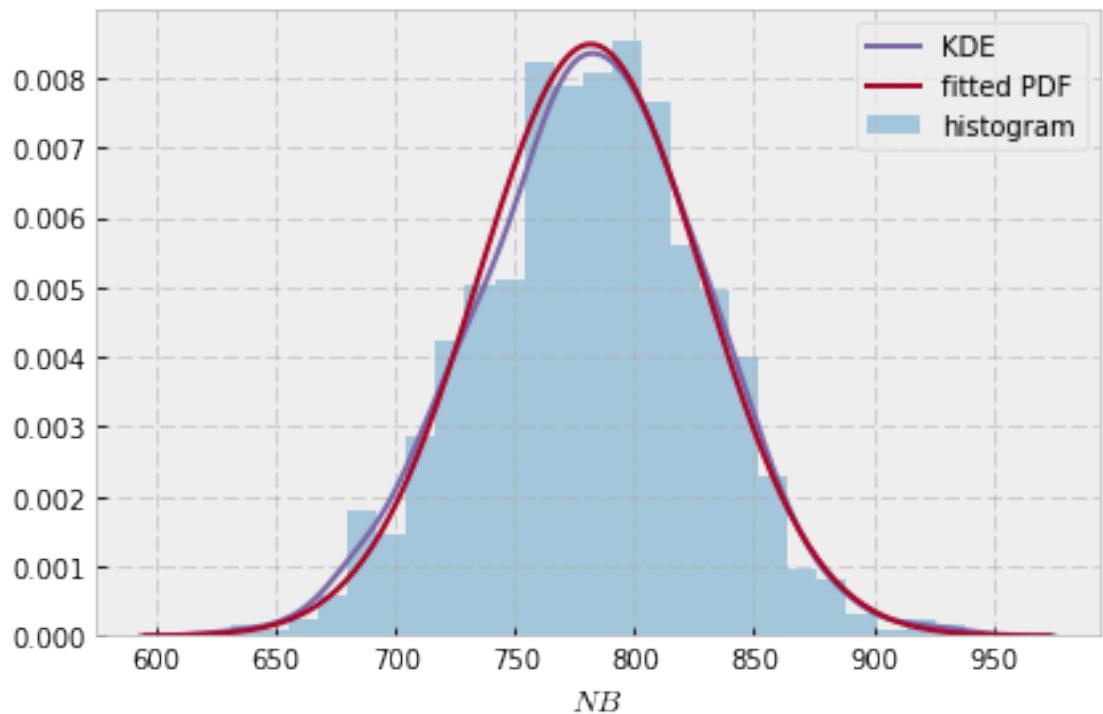


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=693.246, scale=49.46525532128587)
DescribeResult(nobs=1000, minmax=(513.0, 848.0)
    mean=693.2459999999998, variance=2449.2607447447449,
    skewness=-0.07285452182703331, kurtosis=-0.09355322698702917)
```

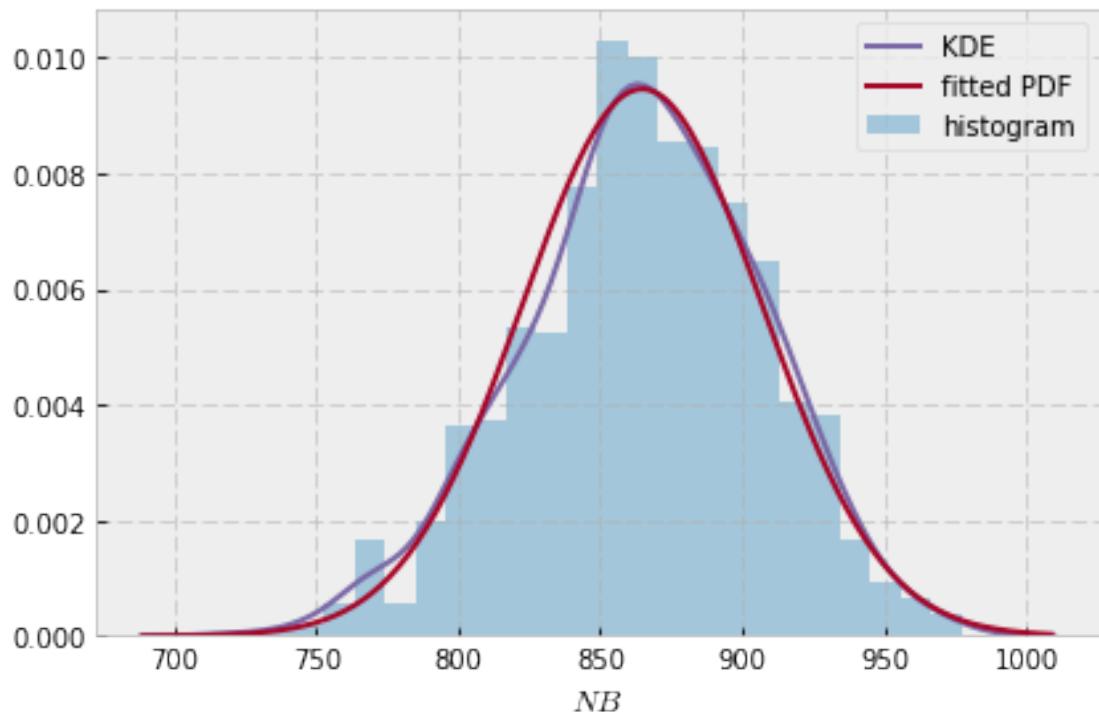


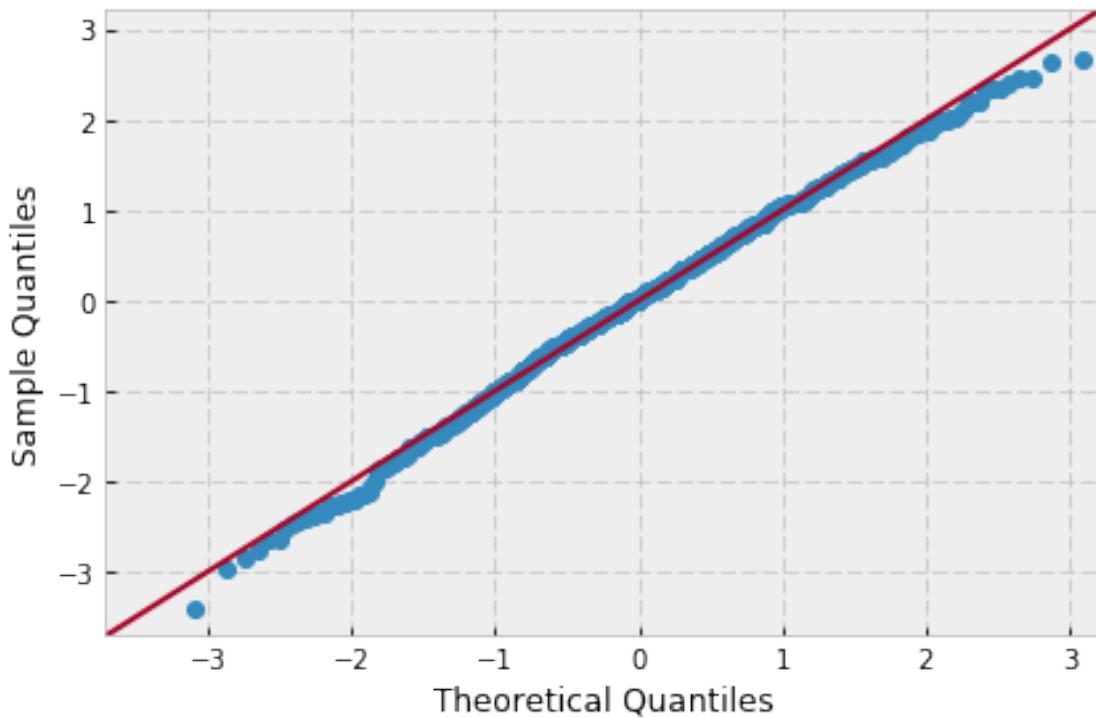


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(),loc=781.738, scale=47.00418445202512)
DescribeResult(nobs=1000, minmax=(631.0, 938.0)
               mean=781.7380000000006, variance=2211.6049609609613,
               skewness=-0.04671110838746178, kurtosis=-0.008189824907279153)
```

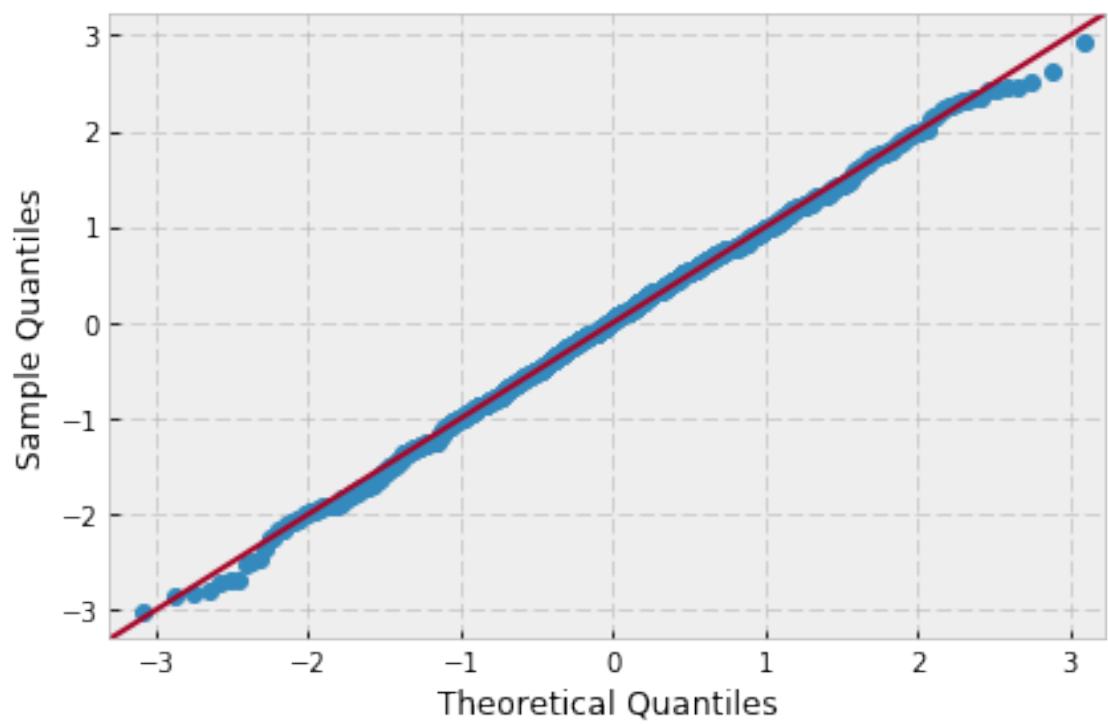
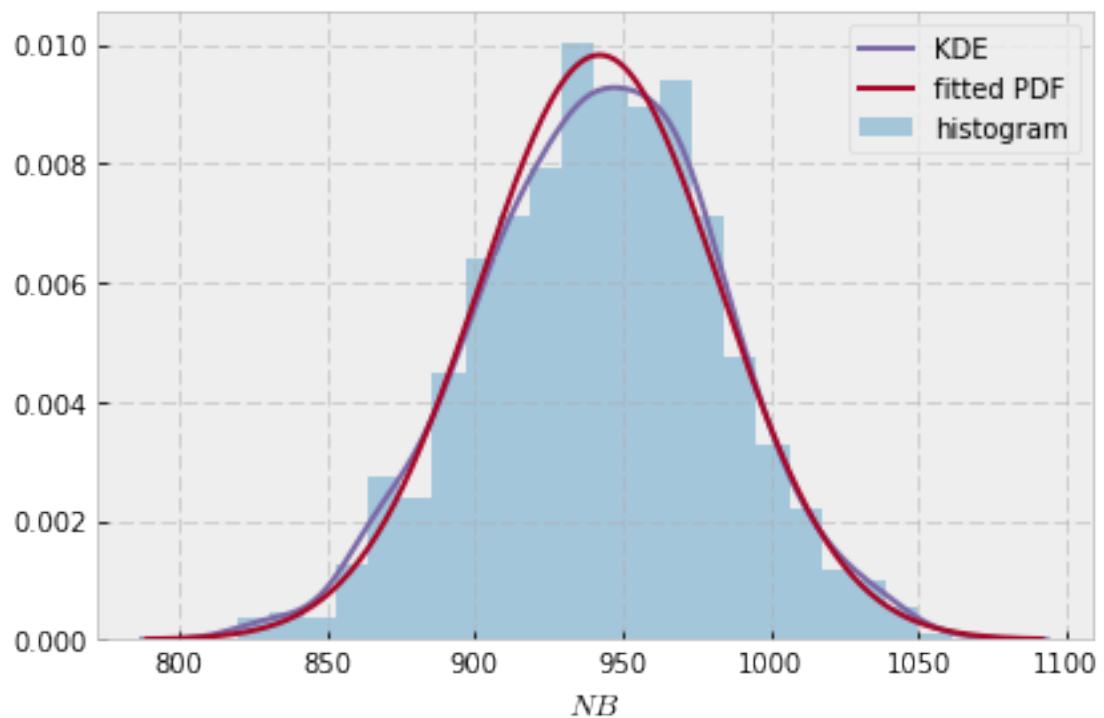


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=864.792, scale=42.15911213486356)
DescribeResult(nobs=1000, minmax=(721.0, 977.0)
    mean=864.7920000000003, variance=1779.169905905906,
    skewness=-0.2044947772622384, kurtosis=-0.08557391822357419)
```

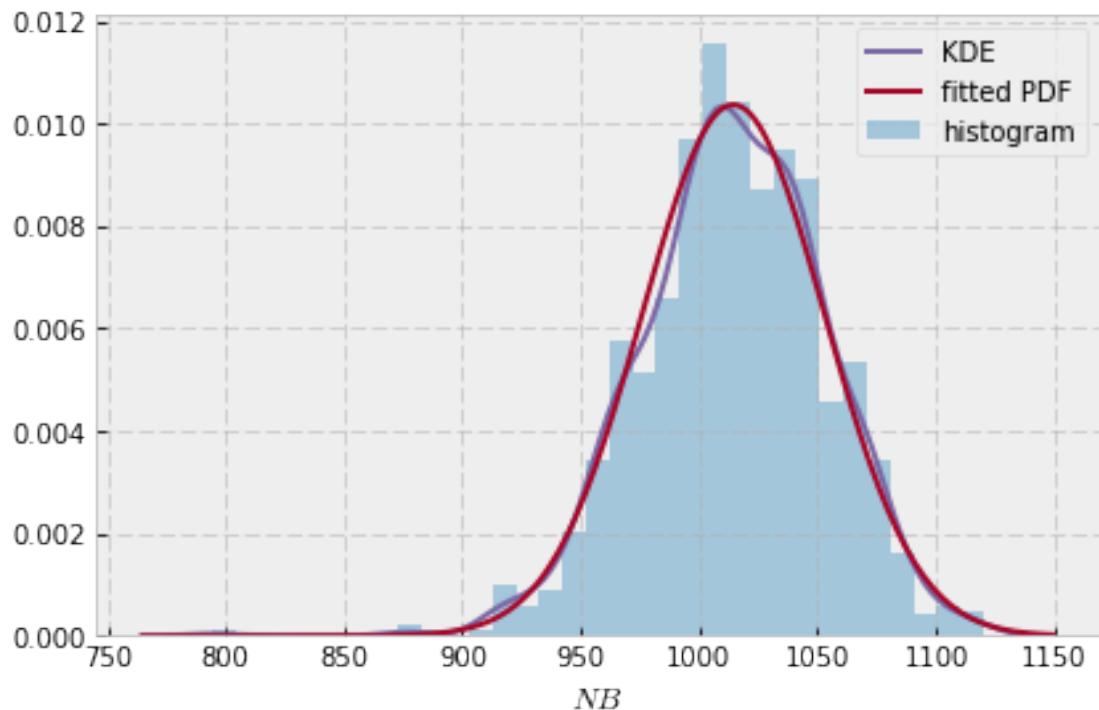


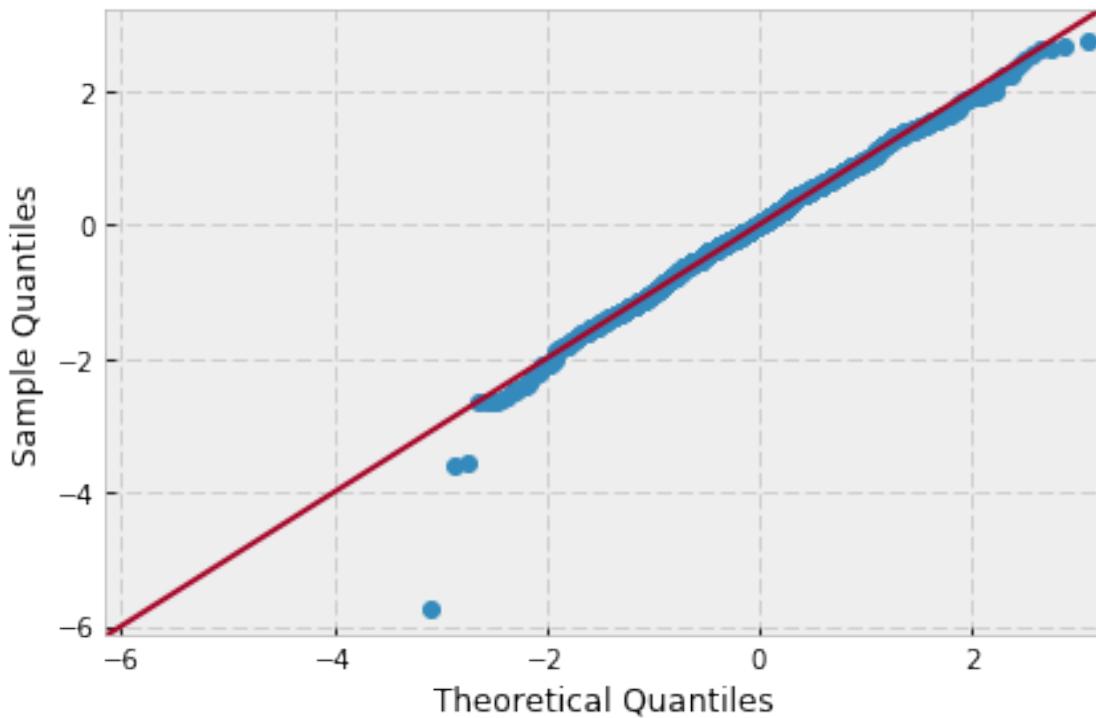


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(),loc=942.21, scale=40.61612856981817)
DescribeResult(nobs=1000, minmax=(820.0, 1061.0)
               mean=942.2100000000004, variance=1651.3212212212211,
               skewness=-0.09269976444565865, kurtosis=-0.12501356055381008)
```

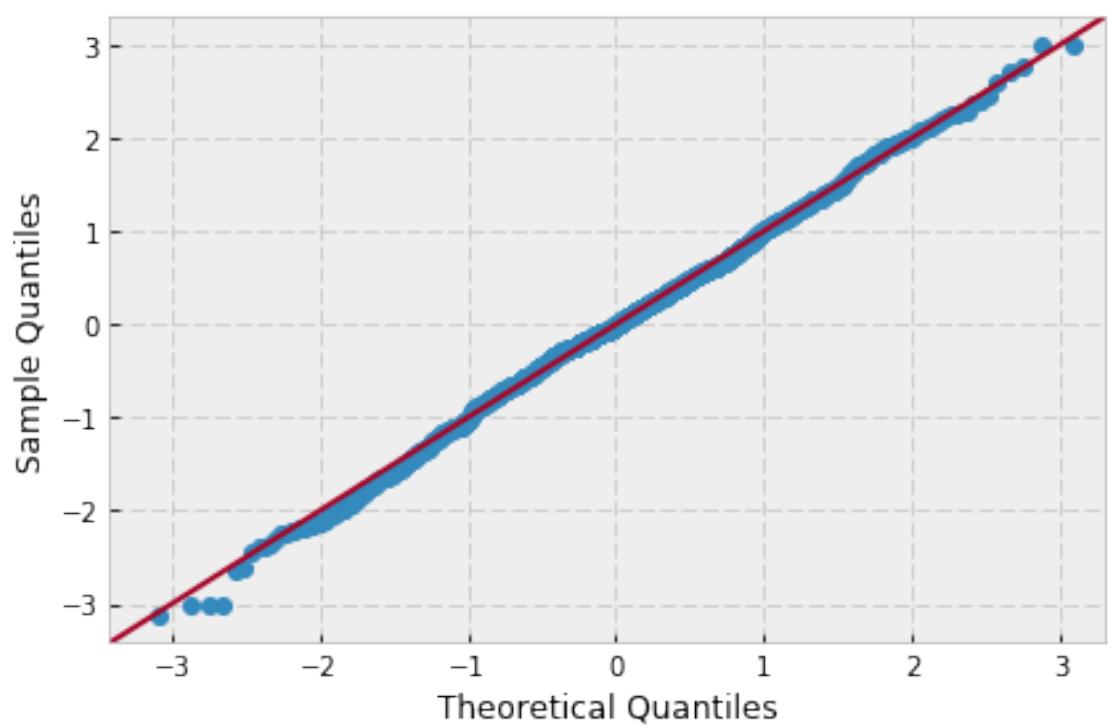
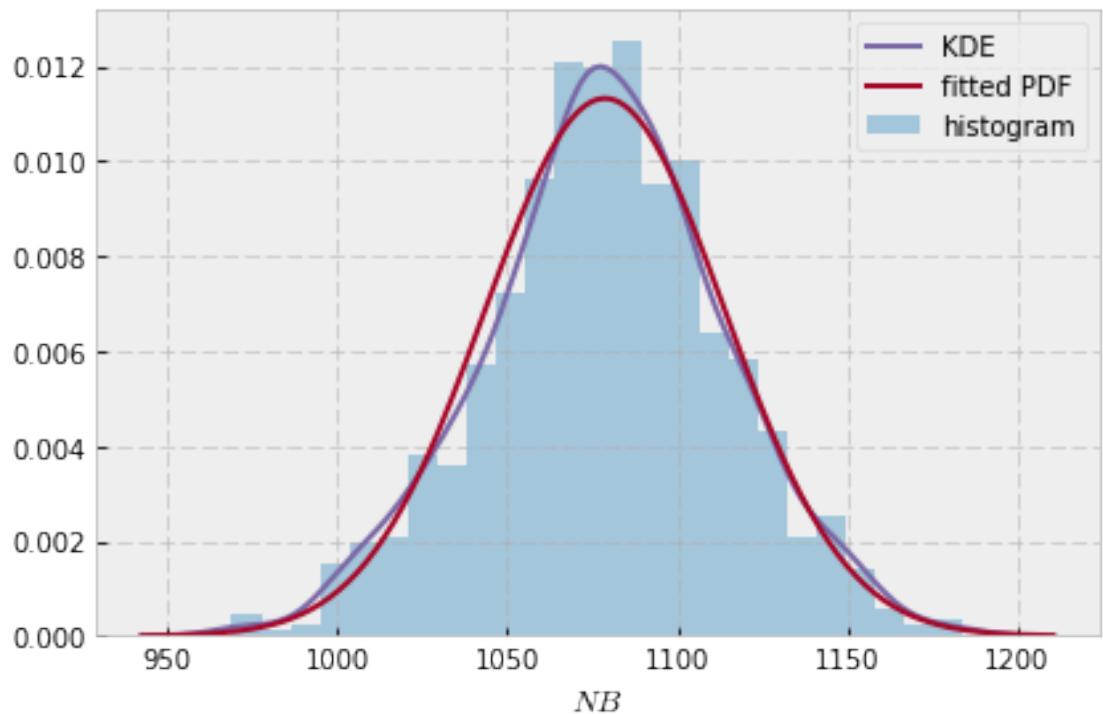


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=1014.355, scale=38.479851545971435)
DescribeResult(nobs=1000, minmax=(794.0, 1120.0)
    mean=1014.355, variance=1482.1811561561562,
    skewness=-0.32732736064385143, kurtosis=0.9188522852271306)
```

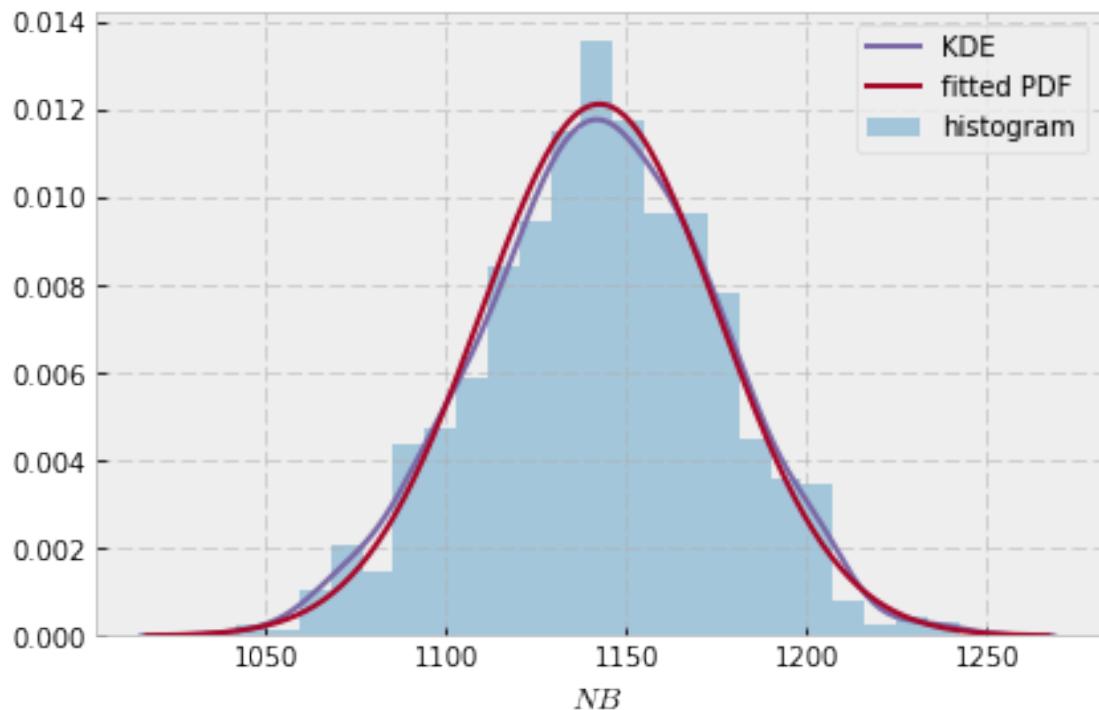


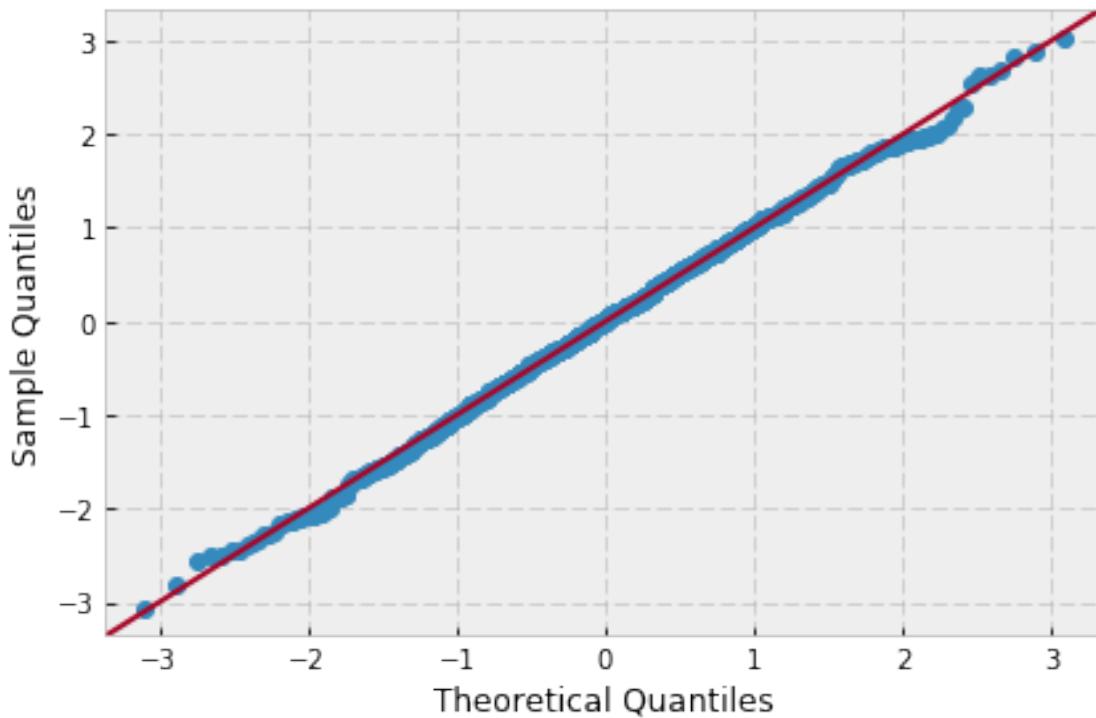


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(),loc=1078.689, scale=35.23055888004049)
DescribeResult(nobs=1000, minmax=(969.0, 1184.0)
               mean=1078.6890000000001, variance=1242.4347137137138,
               skewness=-0.06904495035345062, kurtosis=0.06237211002773835)
```

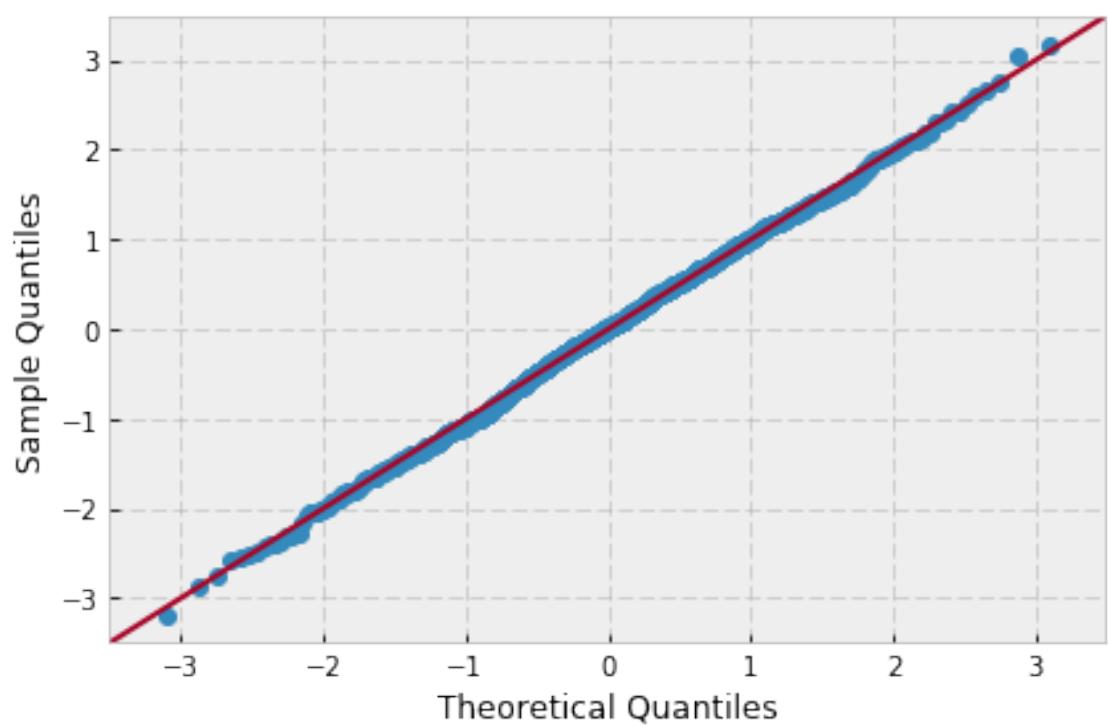
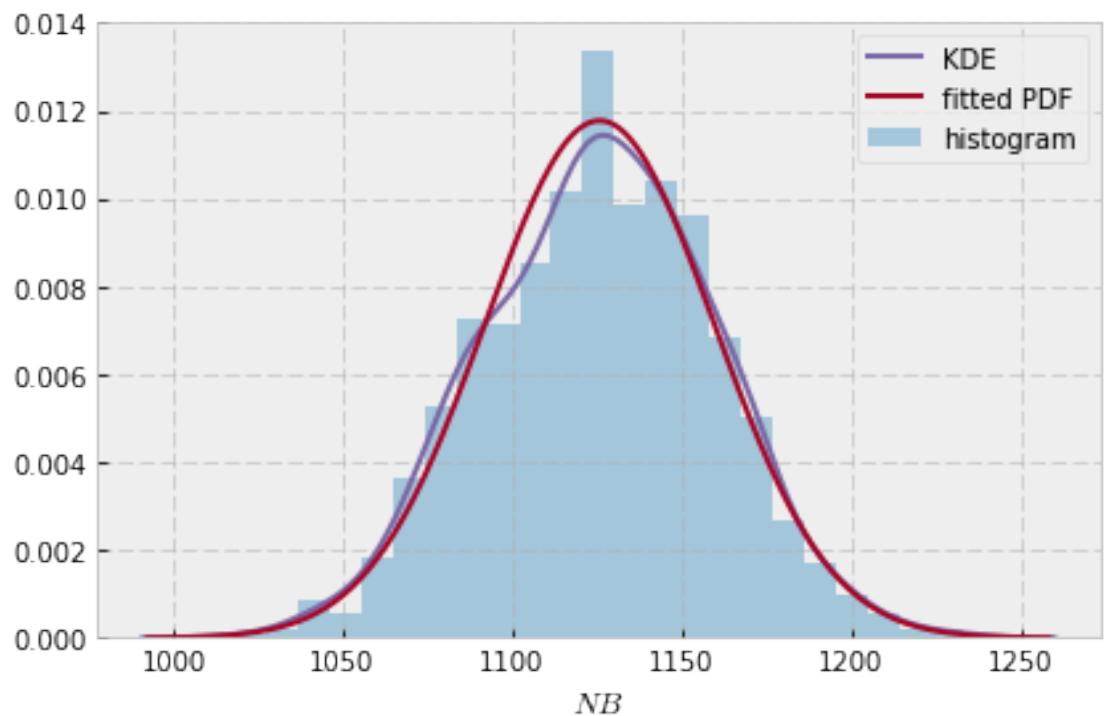


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=1142.544, scale=32.92521927034048)
DescribeResult(nobs=1000, minmax=(1042.0, 1242.0)
    mean=1142.544000000001, variance=1085.1552192192191,
    skewness=-0.05579009020026107, kurtosis=-0.17150475882089466)
```

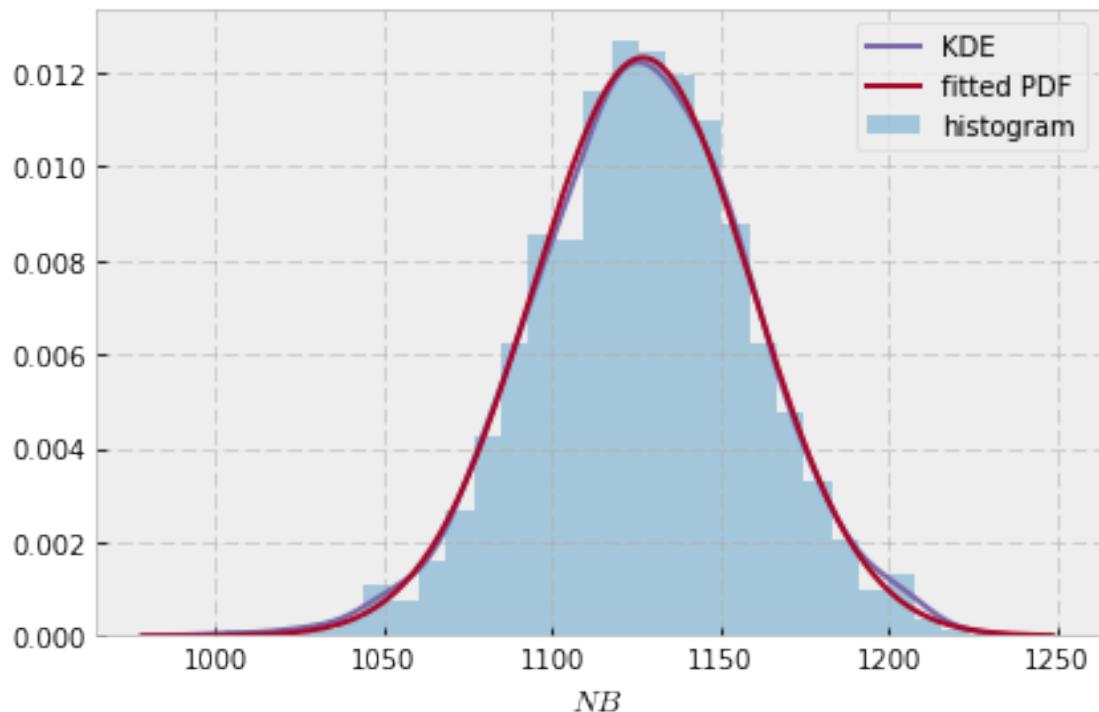


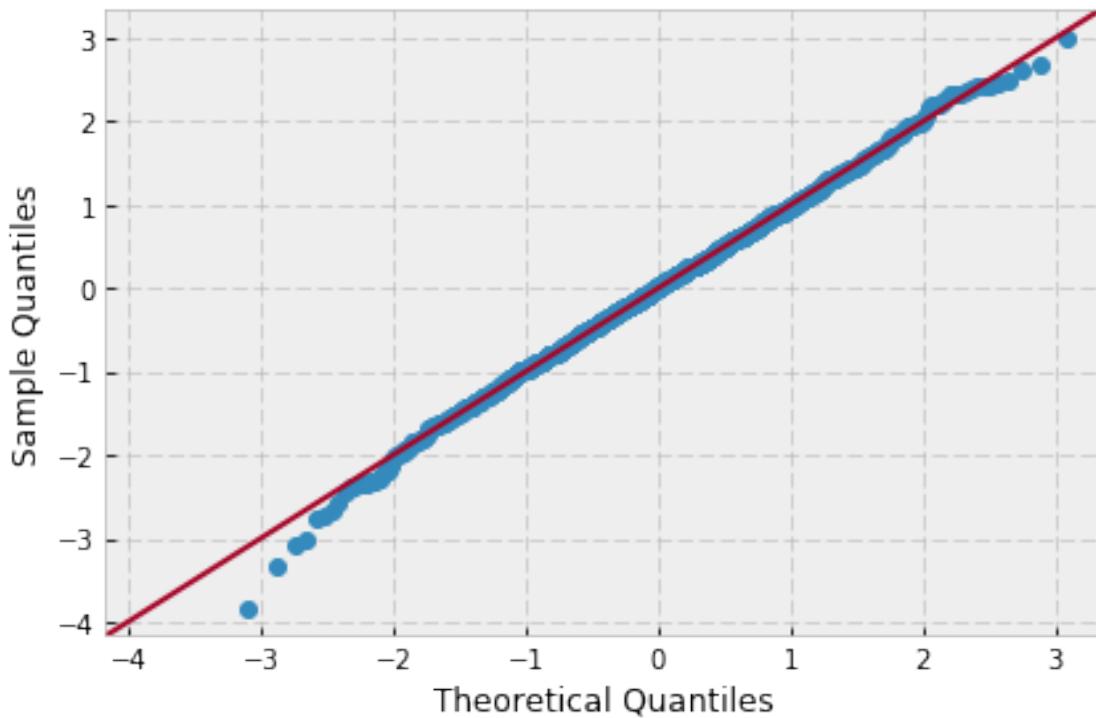


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(), loc=1125.81, scale=33.887282275213515)
DescribeResult(nobs=1000, minmax=(1018.0, 1233.0)
               mean=1125.809999999999, variance=1149.4973973973974,
               skewness=-0.040734876247970744, kurtosis=-0.1545700909627823)
```

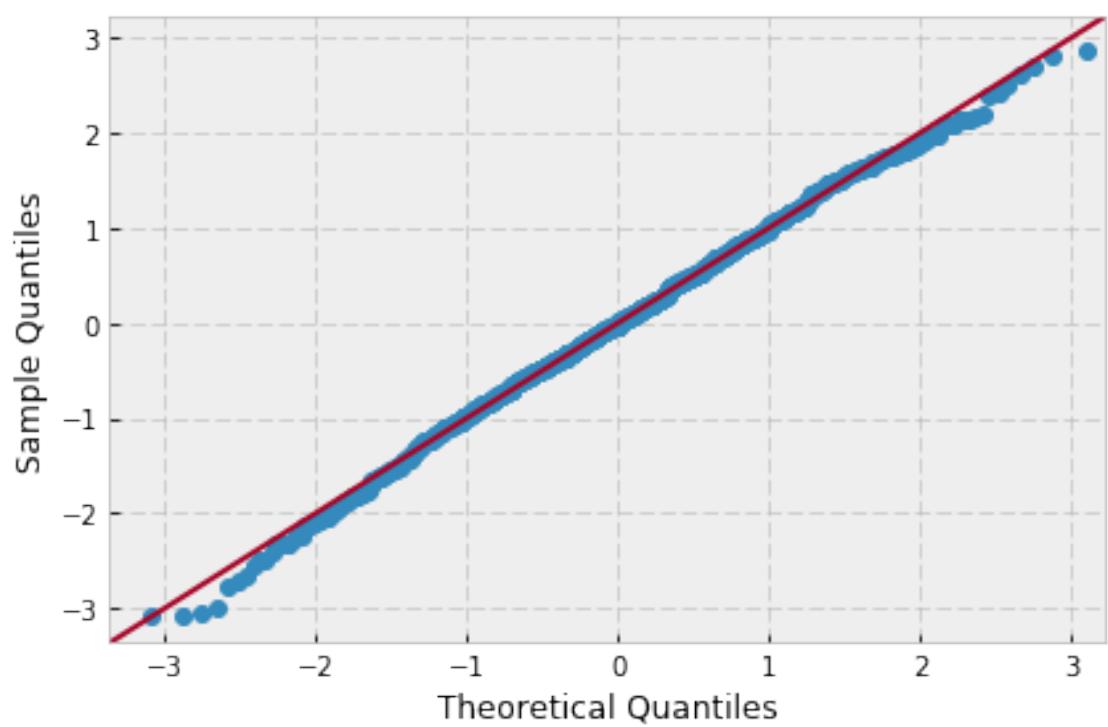
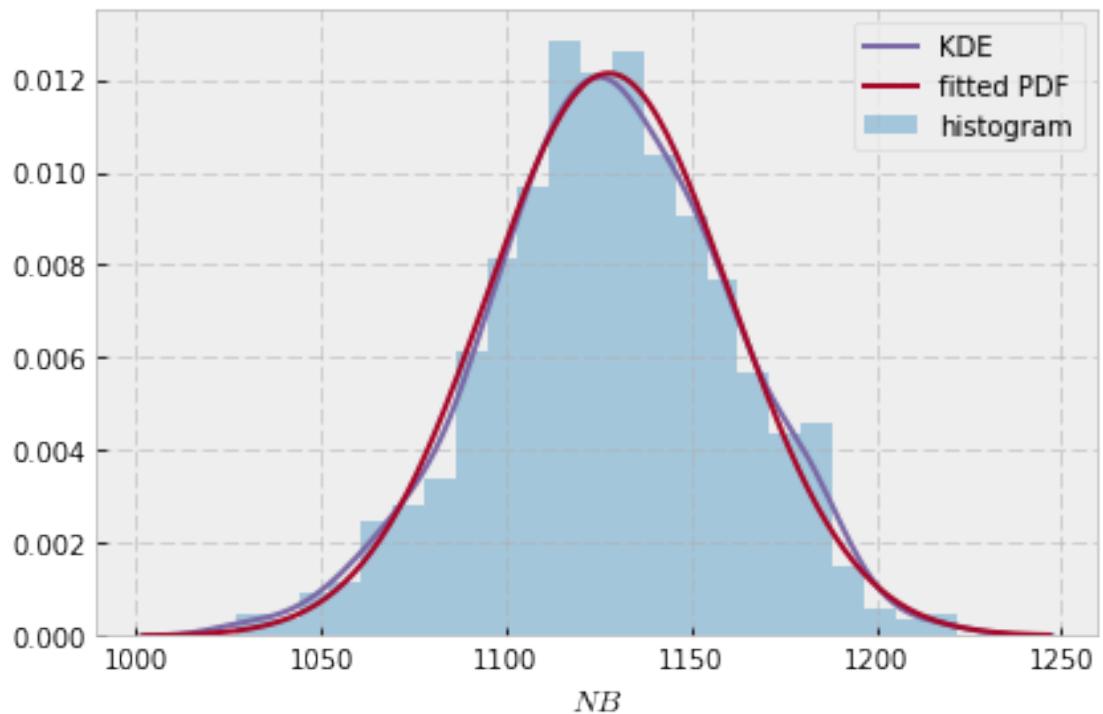


```
variable name: sigma
variable value: 0.0222222222222223
distribution: normal(shape=(), loc=1126.968, scale=32.369877602487165)
DescribeResult(nobs=1000, minmax=(1003.0, 1224.0)
    mean=1126.968000000001, variance=1048.8578338338339,
    skewness=-0.09439785318895536, kurtosis=0.1829447817911256)
```

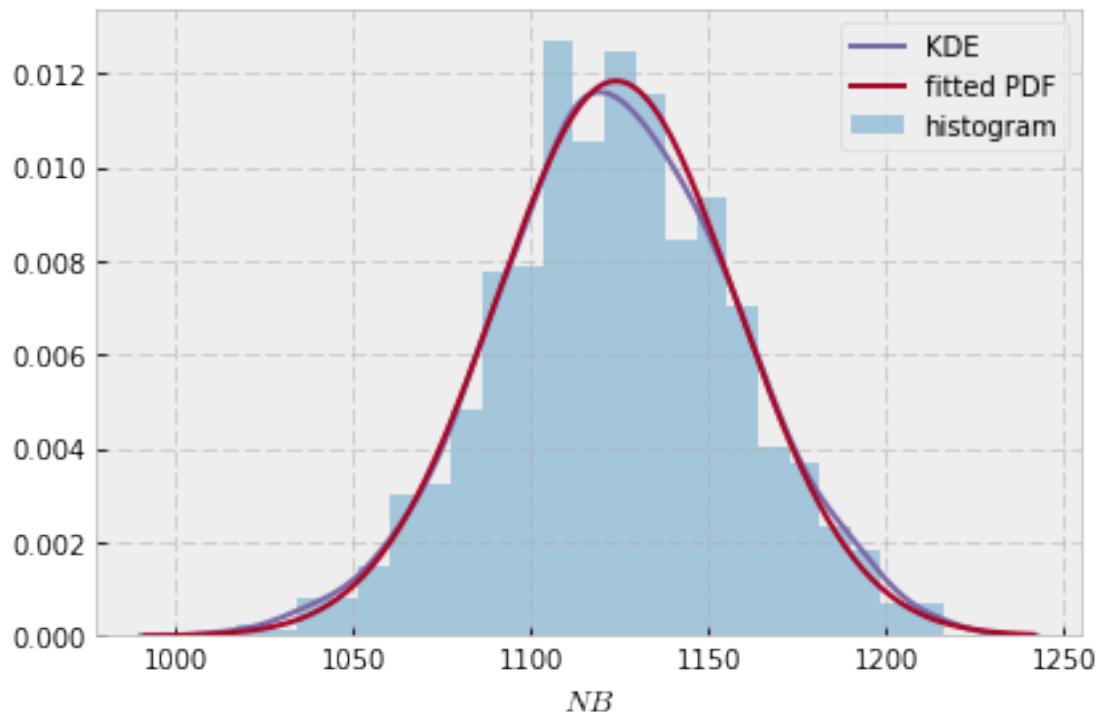


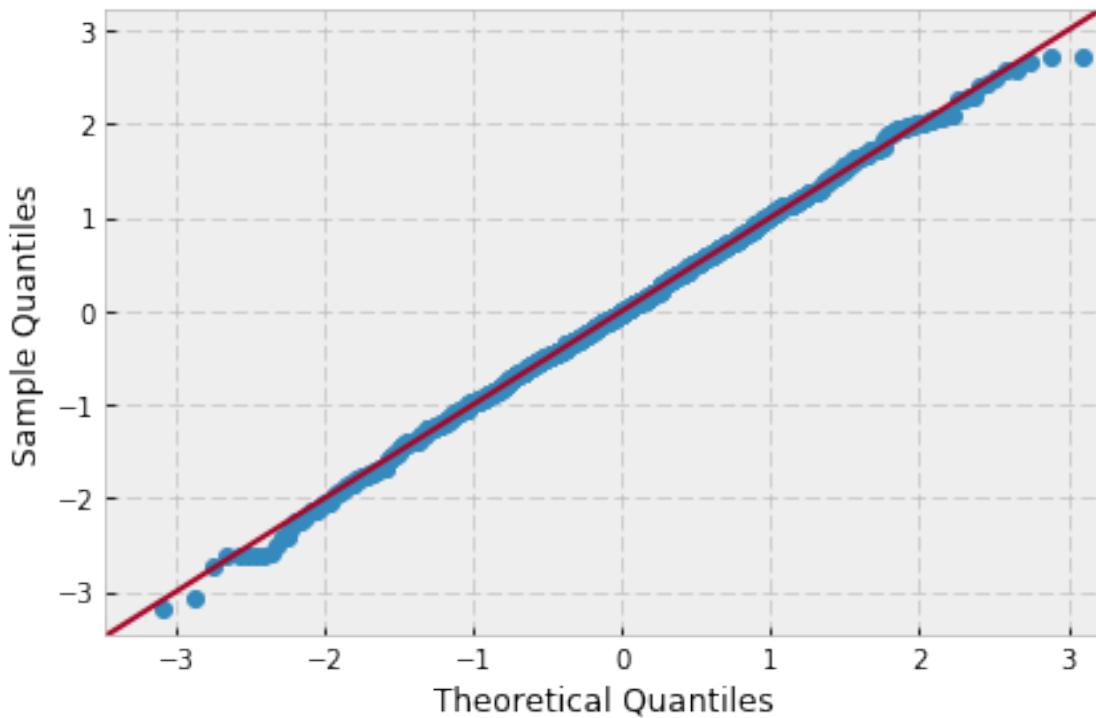


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=1127.744, scale=32.87607738158553)
DescribeResult(nobs=1000, minmax=(1027.0, 1222.0)
               mean=1127.743999999999, variance=1081.9183823823823,
               skewness=-0.11655800281377422, kurtosis=-0.031236712500033903)
```

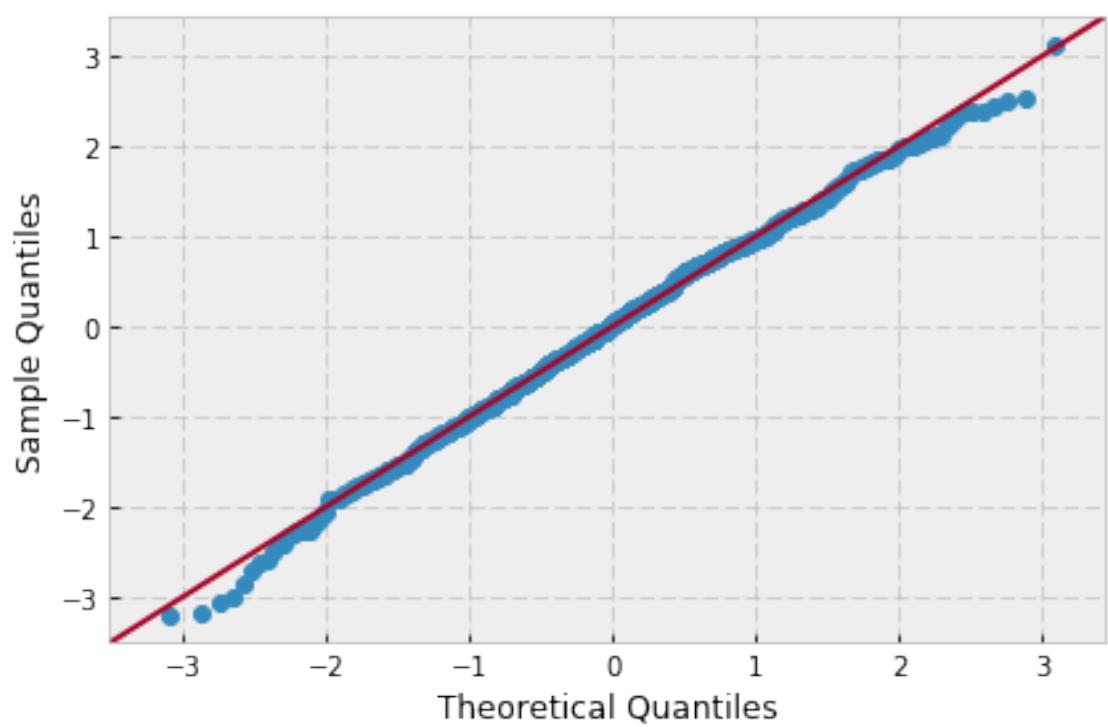
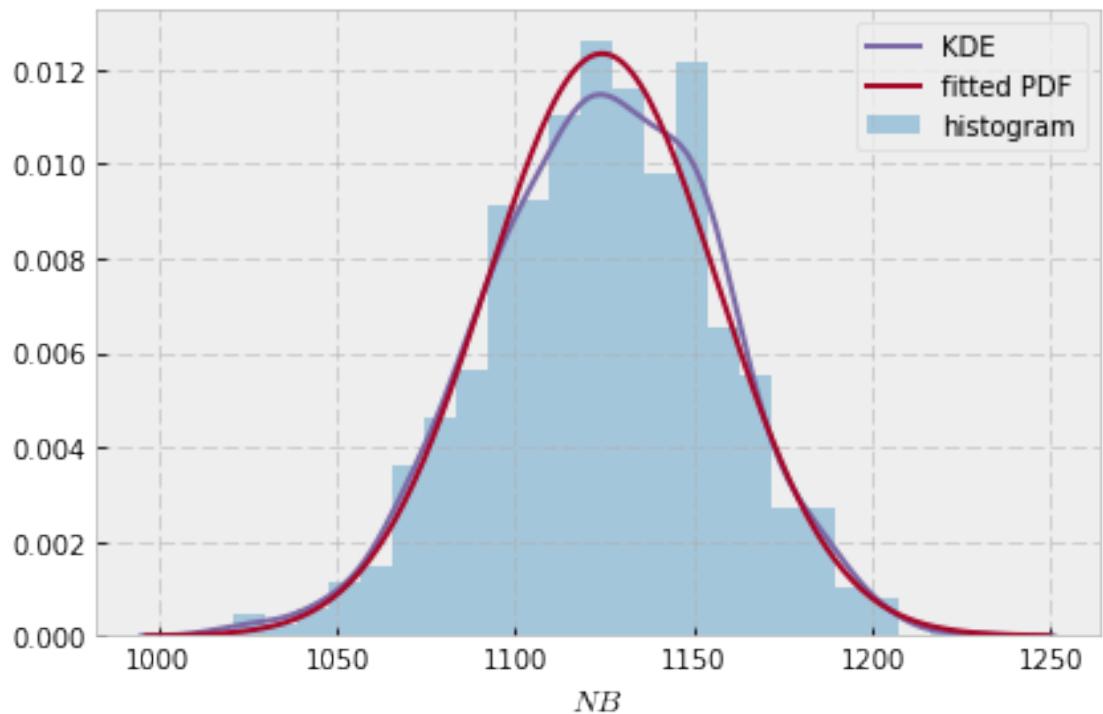


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=1124.12, scale=33.71079352373658)
DescribeResult(nobs=1000, minmax=(1017.0, 1216.0)
    mean=1124.119999999999, variance=1137.5551551551553,
    skewness=-0.04018597198919402, kurtosis=-0.06693167662379951)
```

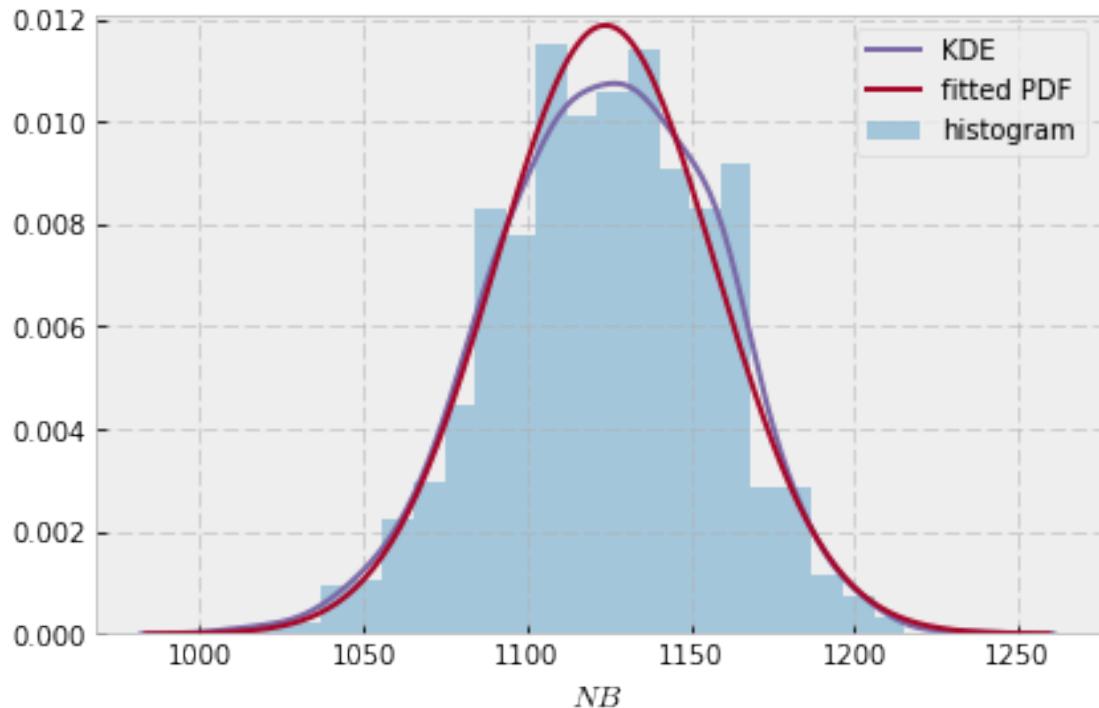


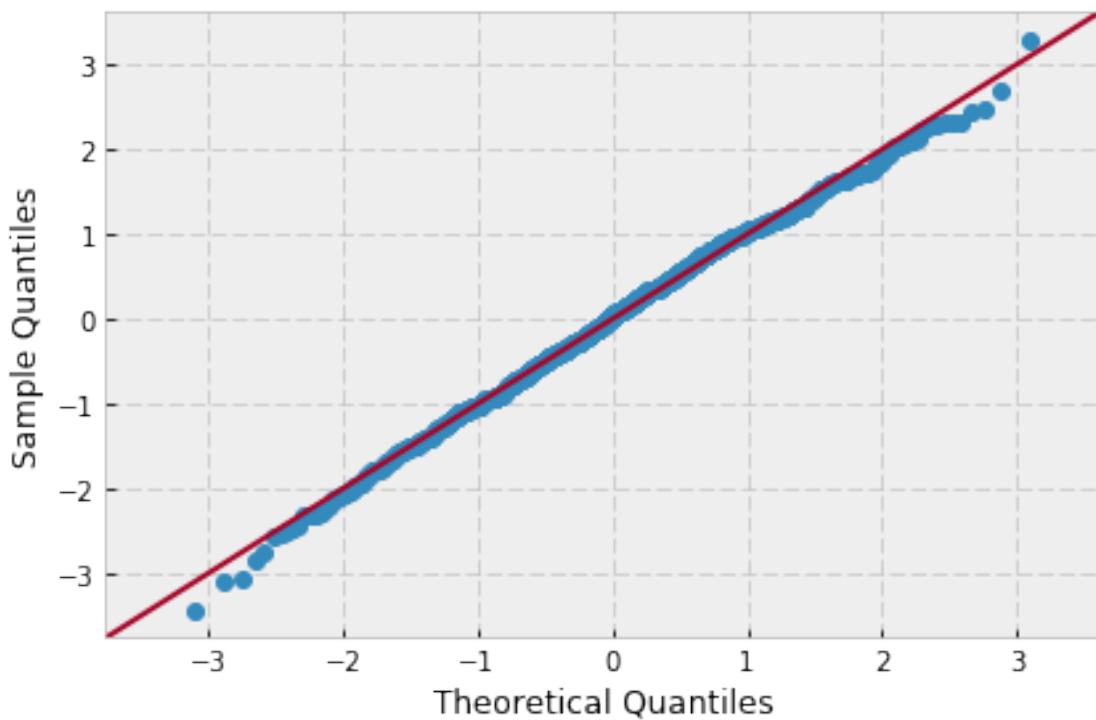


```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(),loc=1124.321, scale=32.34130422540192)
DescribeResult(nobs=1000, minmax=(1021.0, 1225.0)
               mean=1124.320999999999, variance=1047.0069659659659,
               skewness=-0.1446382751410086, kurtosis=-0.09347261337961354)
```

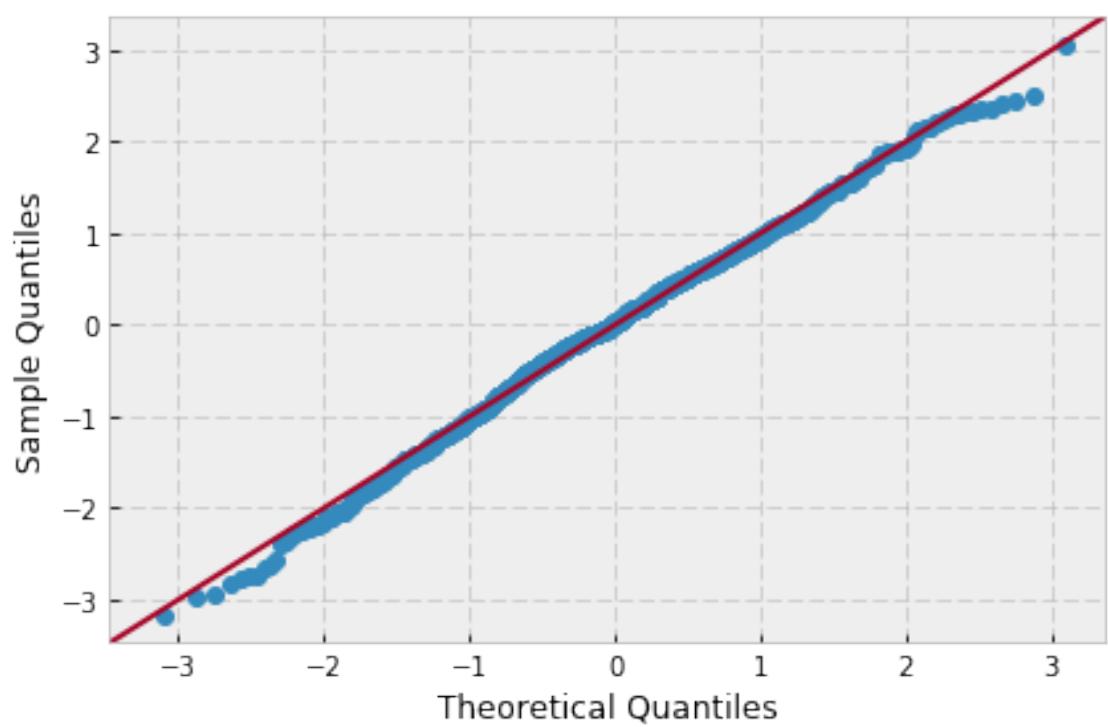
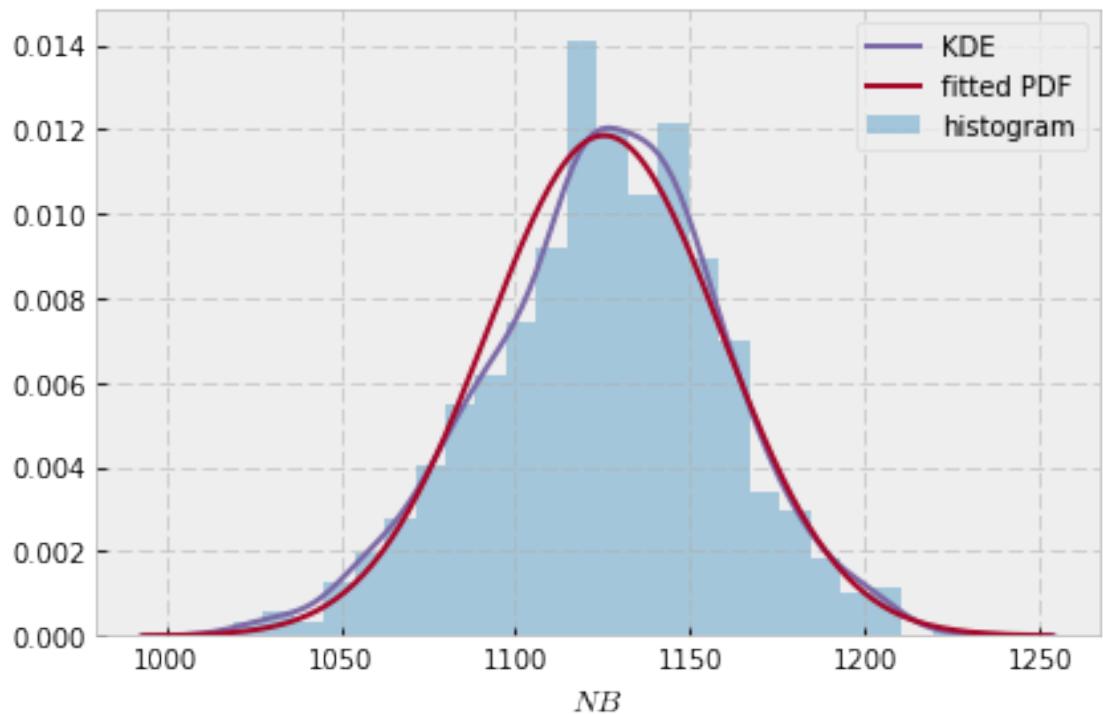


```
variable name: sigma
variable value: 0.06666666666666667
distribution: normal(shape=(), loc=1123.887, scale=33.59265144343328)
DescribeResult(nobs=1000, minmax=(1009.0, 1234.0)
    mean=1123.886999999999, variance=1129.5958268268271,
    skewness=-0.14428918900665513, kurtosis=-0.13127712639115652)
```

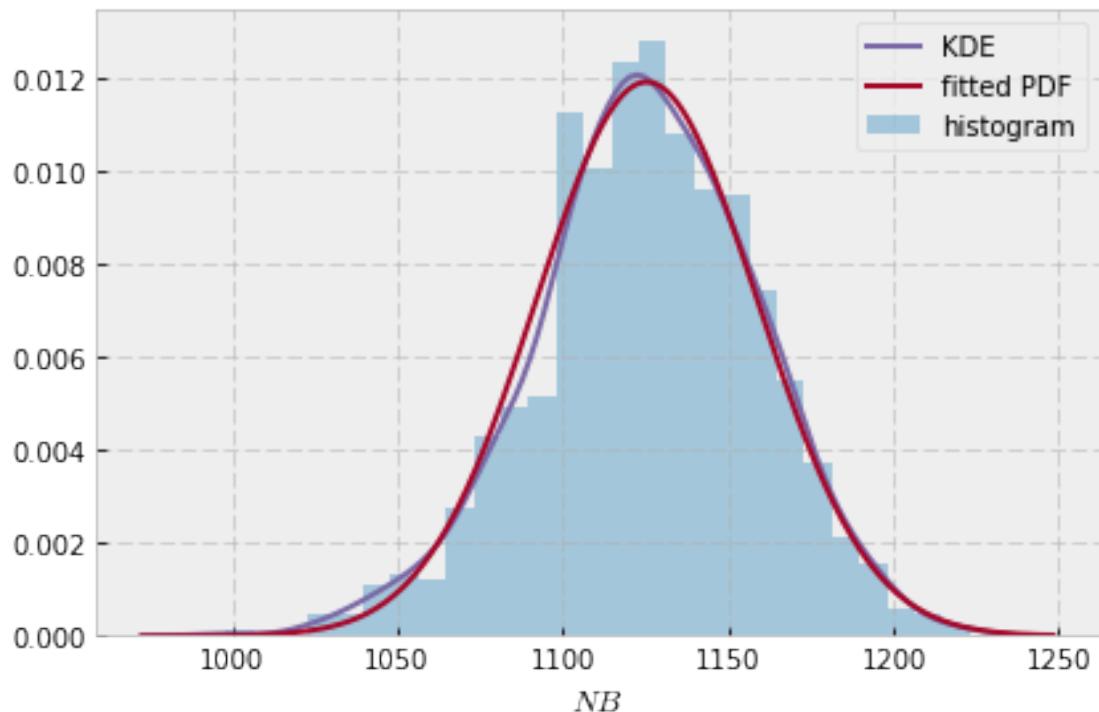


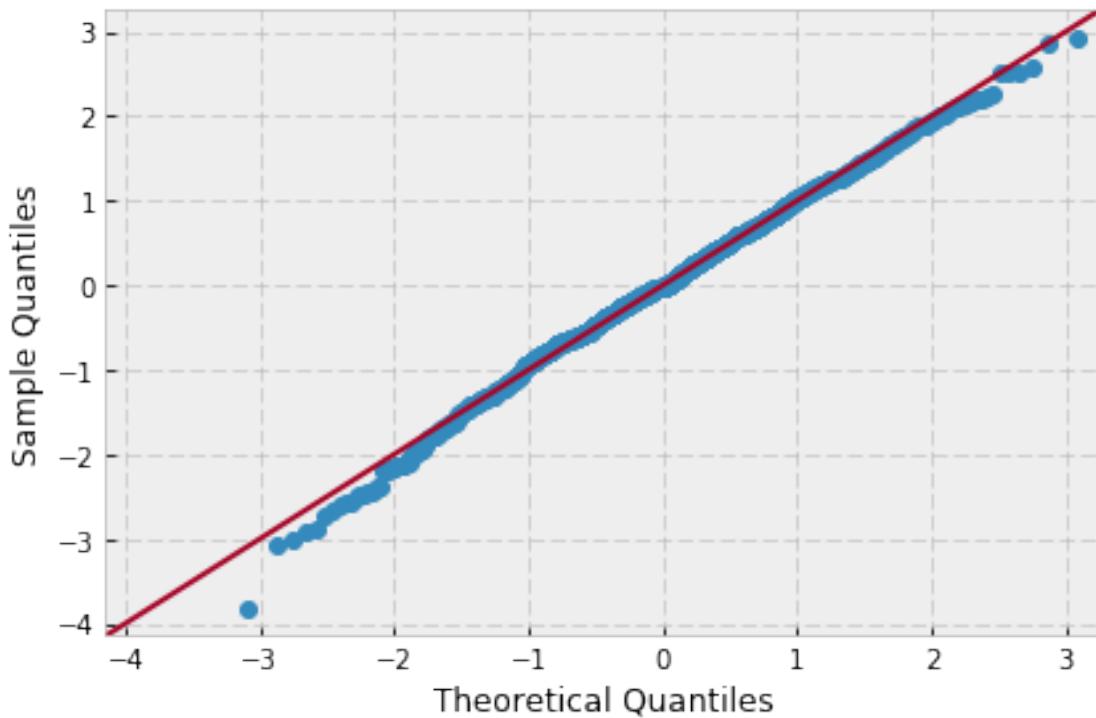


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(),loc=1125.403, scale=33.642244143338594)
DescribeResult(nobs=1000, minmax=(1019.0, 1228.0)
               mean=1125.403, variance=1132.9335245245245,
               skewness=-0.19099742125188365, kurtosis=0.0259460957522597)
```

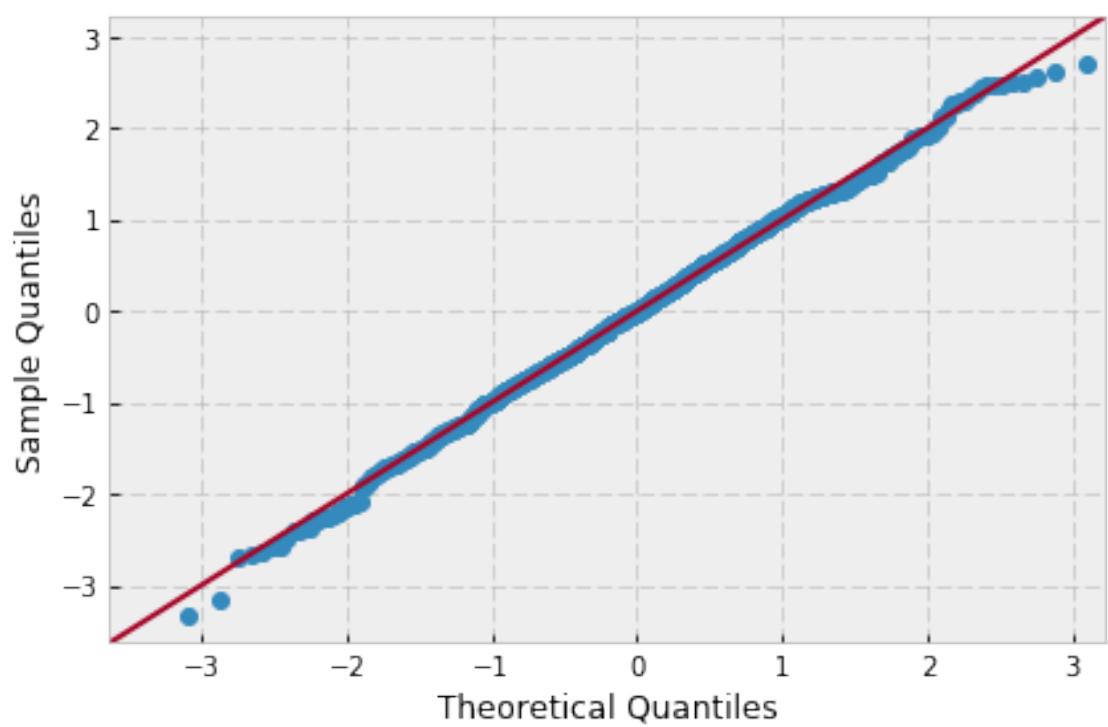
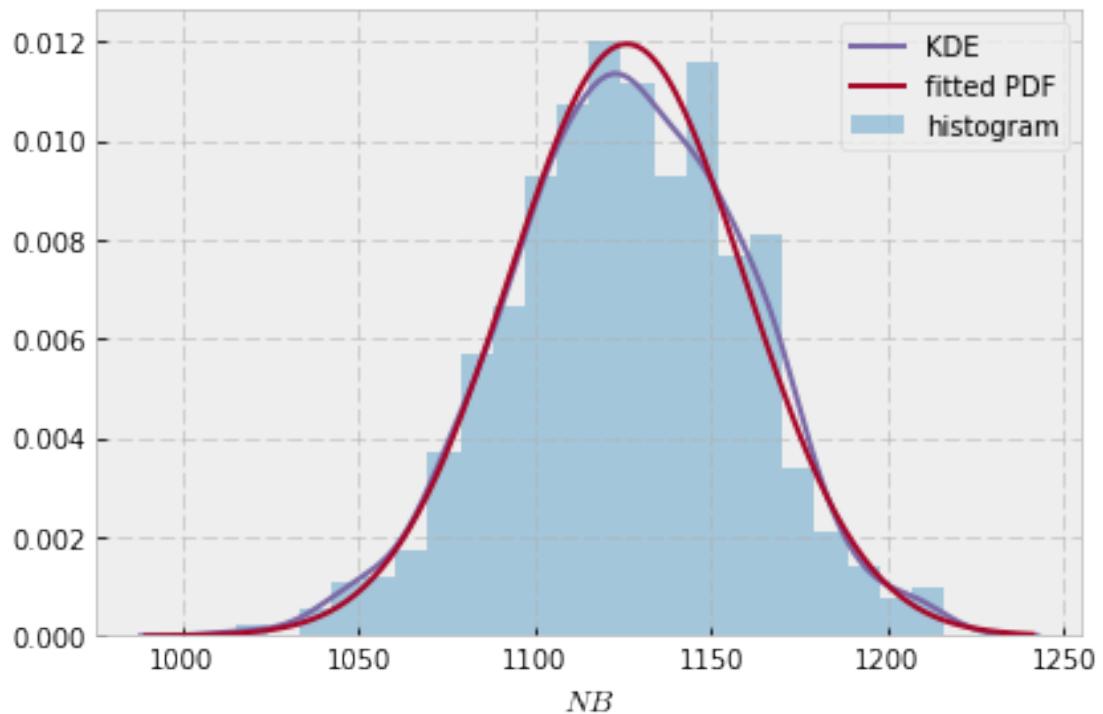


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(), loc=1125.407, scale=33.43222025232545)
DescribeResult(nobs=1000, minmax=(998.0, 1223.0)
    mean=1125.406999999999, variance=1118.8321831831831,
    skewness=-0.1947796711059768, kurtosis=0.1687175173532509)
```

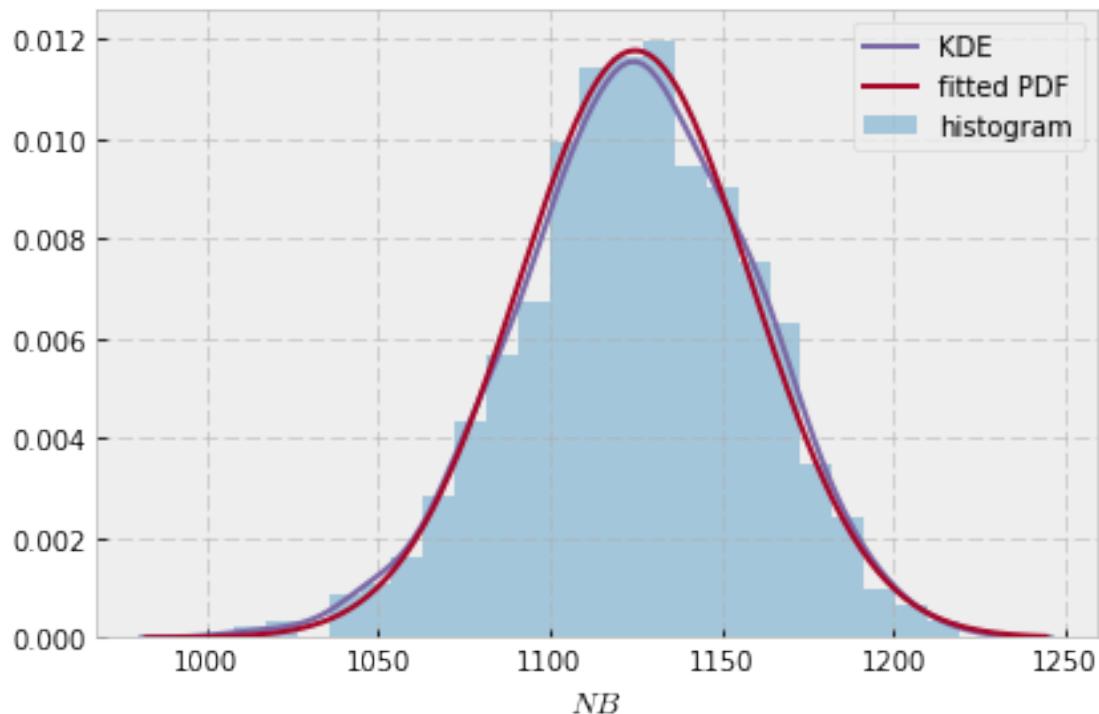


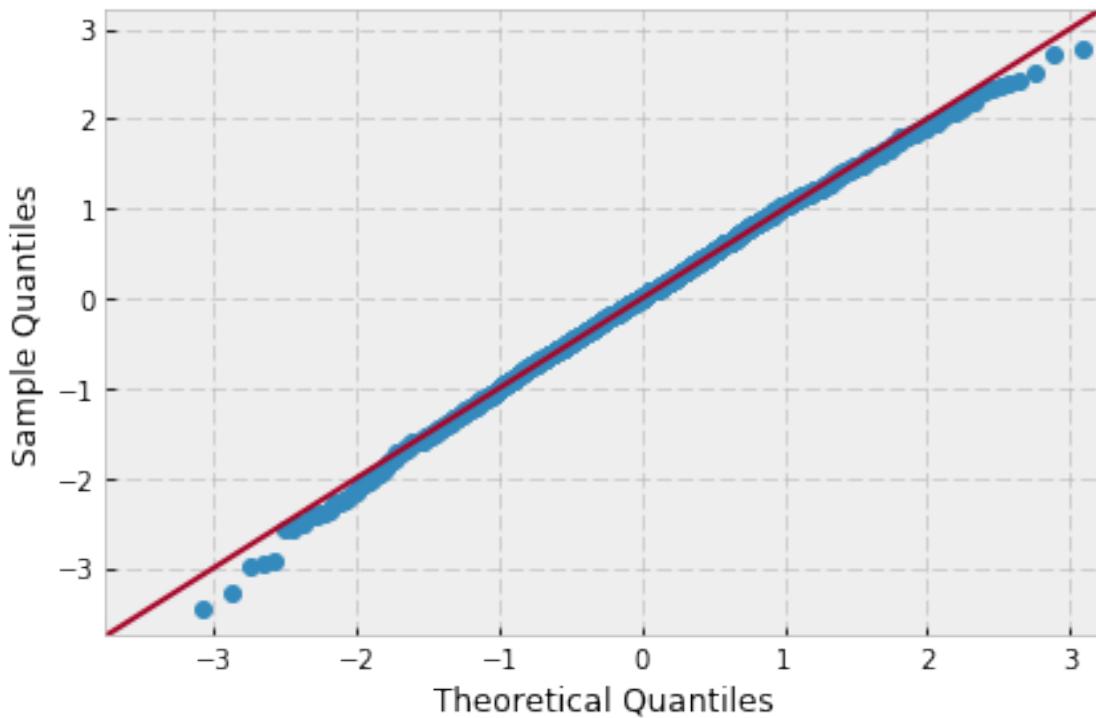


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(),loc=1126.156, scale=33.367254367118676)
DescribeResult(nobs=1000, minmax=(1015.0, 1216.0)
               mean=1126.155999999999, variance=1114.4881521521525,
               skewness=-0.09512224845291055, kurtosis=-0.10404193744004253)
```

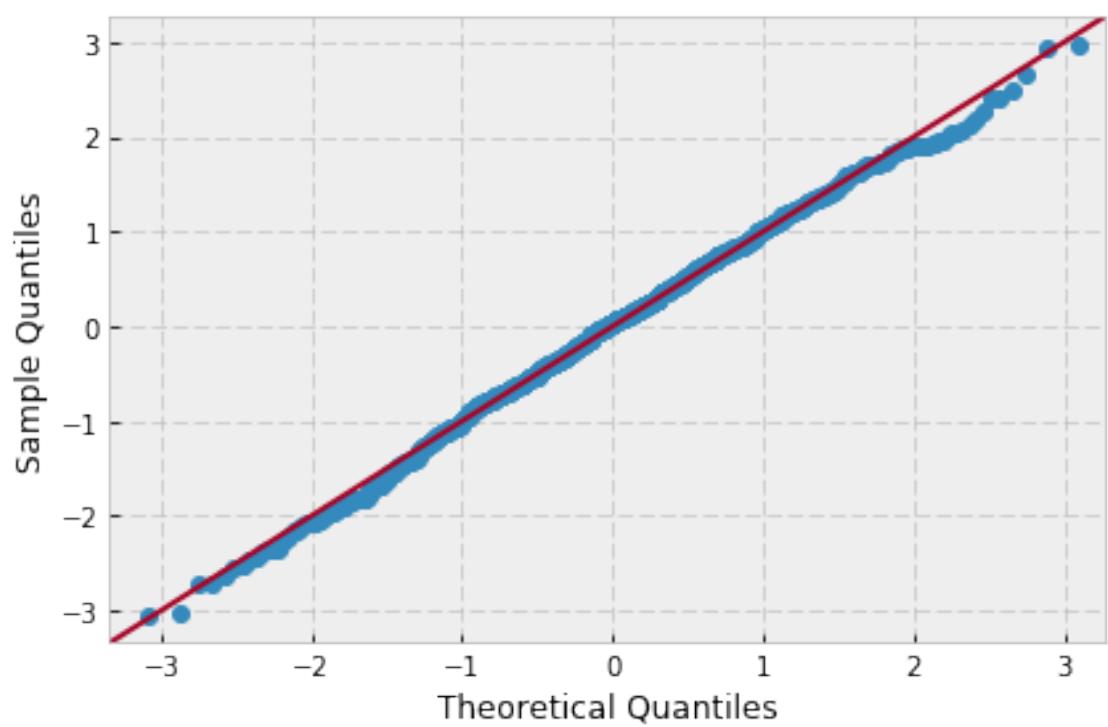
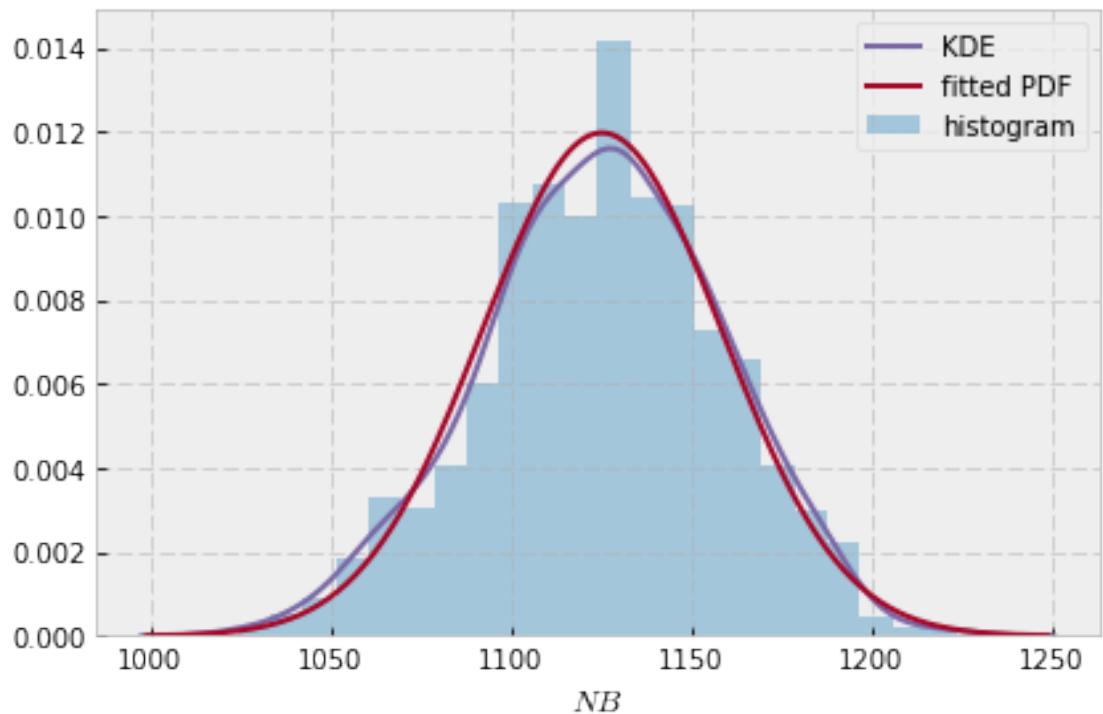


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=1124.84, scale=33.87078977526211)
DescribeResult(nobs=1000, minmax=(1008.0, 1219.0)
    mean=1124.839999999999, variance=1148.378778778779,
    skewness=-0.1646716565367034, kurtosis=-0.030850689612061277)
```

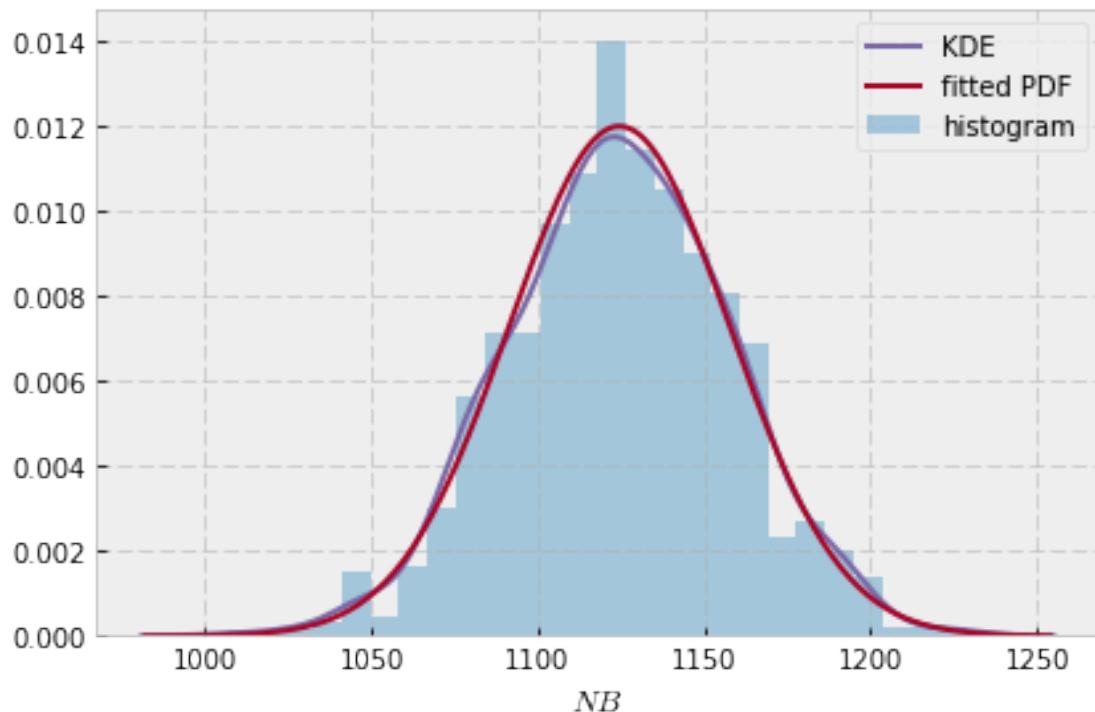


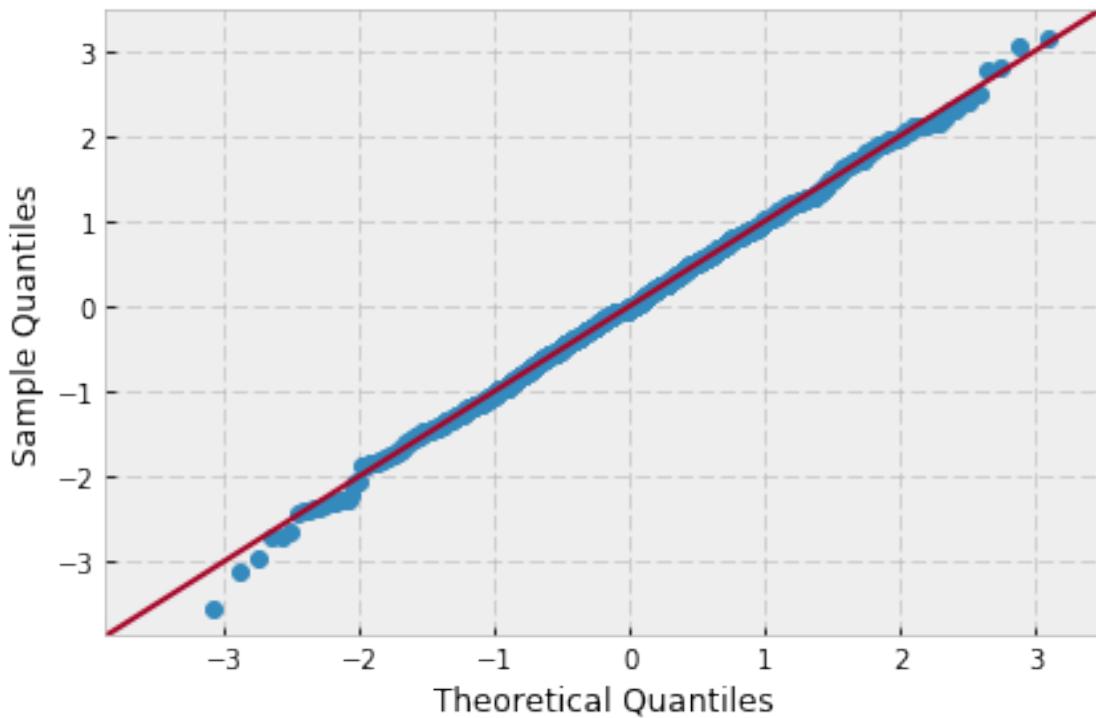


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=1125.233, scale=33.312650915230385)
DescribeResult(nobs=1000, minmax=(1024.0, 1224.0)
               mean=1125.232999999999, variance=1110.8435545545544,
               skewness=-0.1330176477832919, kurtosis=-0.15418822339204485)
```

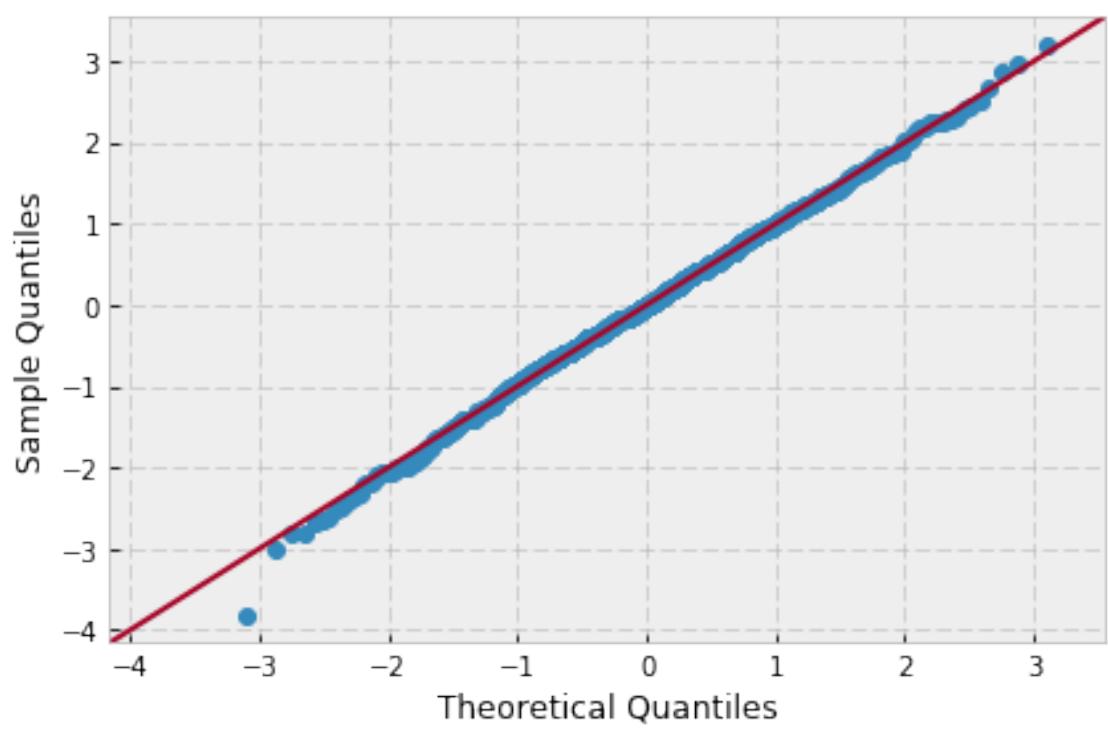
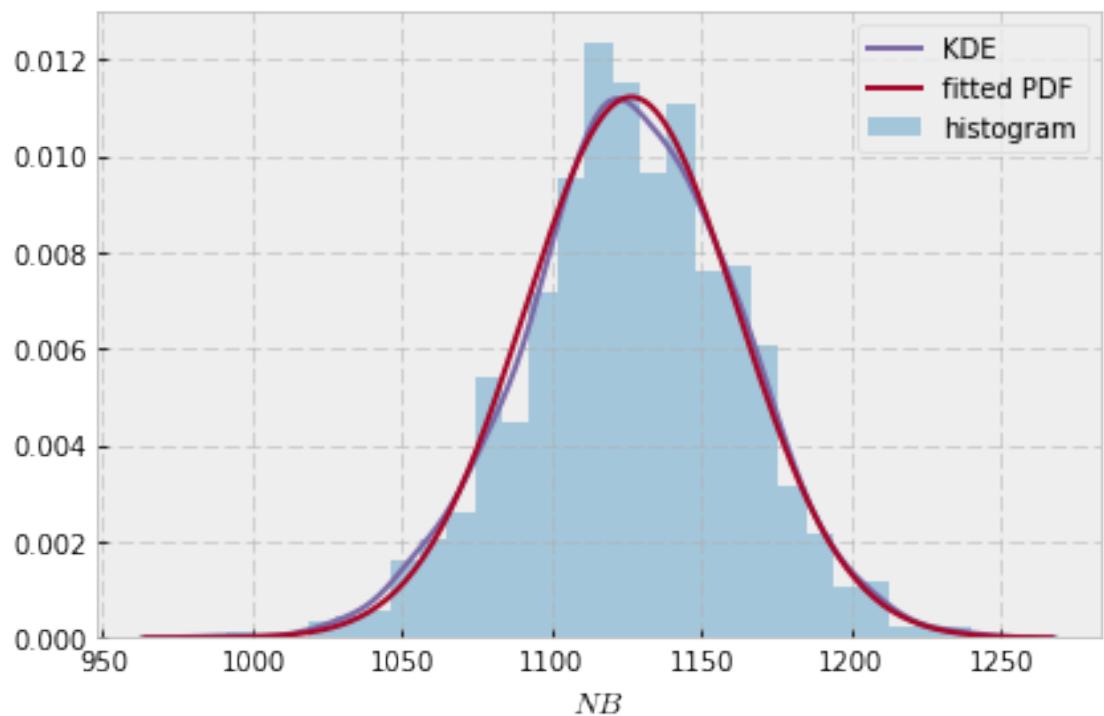


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=1124.592, scale=33.199962891545525)
DescribeResult(nobs=1000, minmax=(1007.0, 1229.0)
    mean=1124.5920000000001, variance=1103.3408768768768,
    skewness=-0.049291557297831363, kurtosis=0.014989542627259933)
```

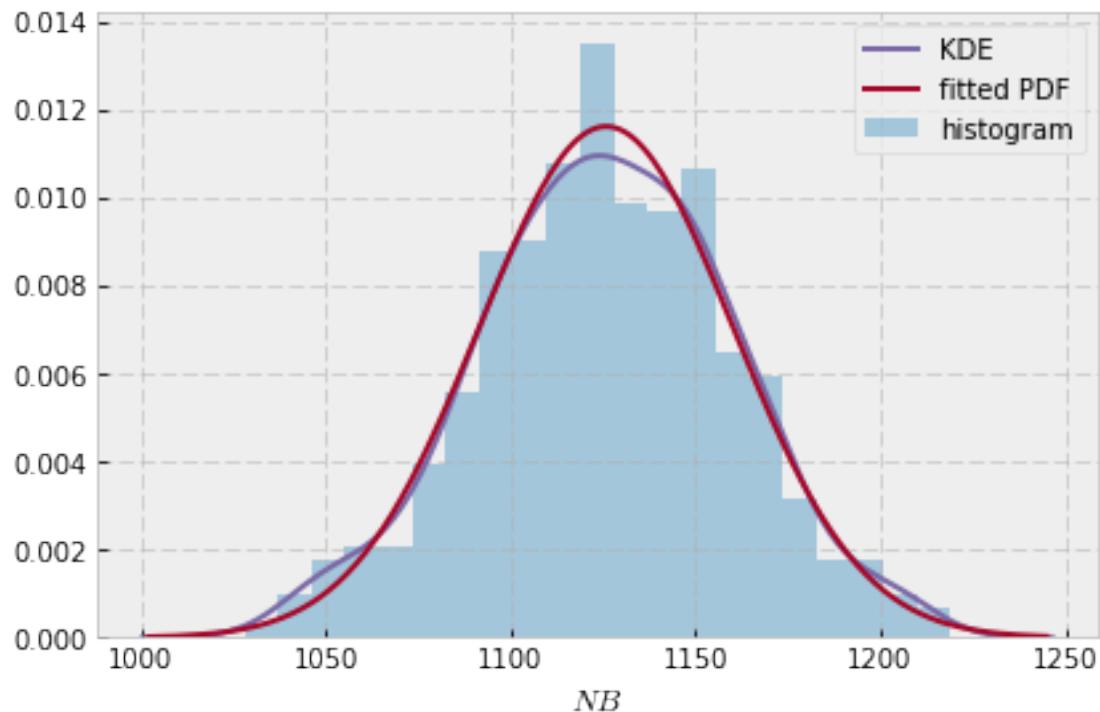


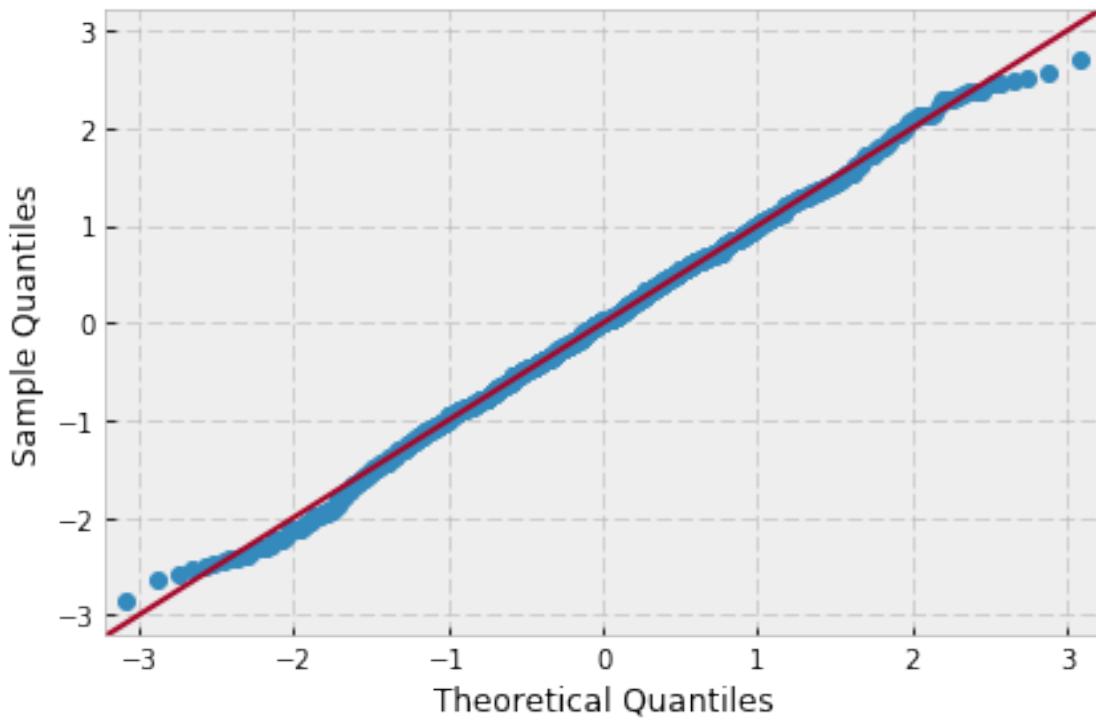


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(),loc=1126.455, scale=35.56501054407266)
DescribeResult(nobs=1000, minmax=(991.0, 1240.0)
               mean=1126.454999999999, variance=1266.136111111109,
               skewness=-0.09400306760277793, kurtosis=0.11482109663821705)
```

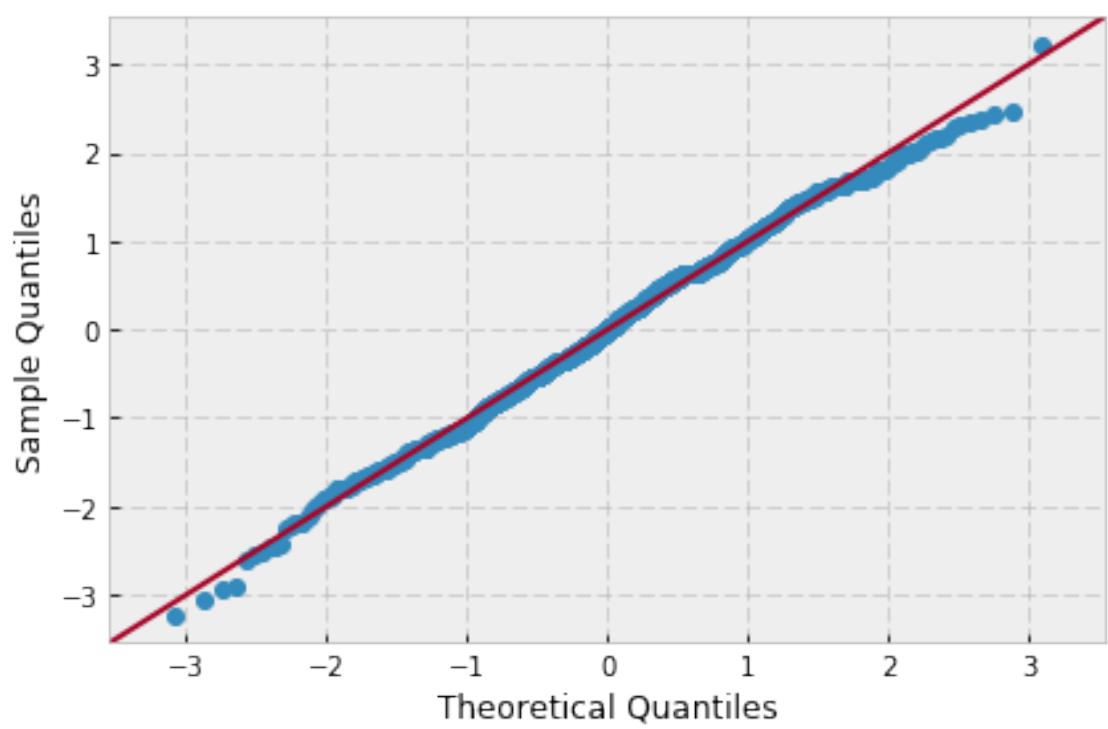
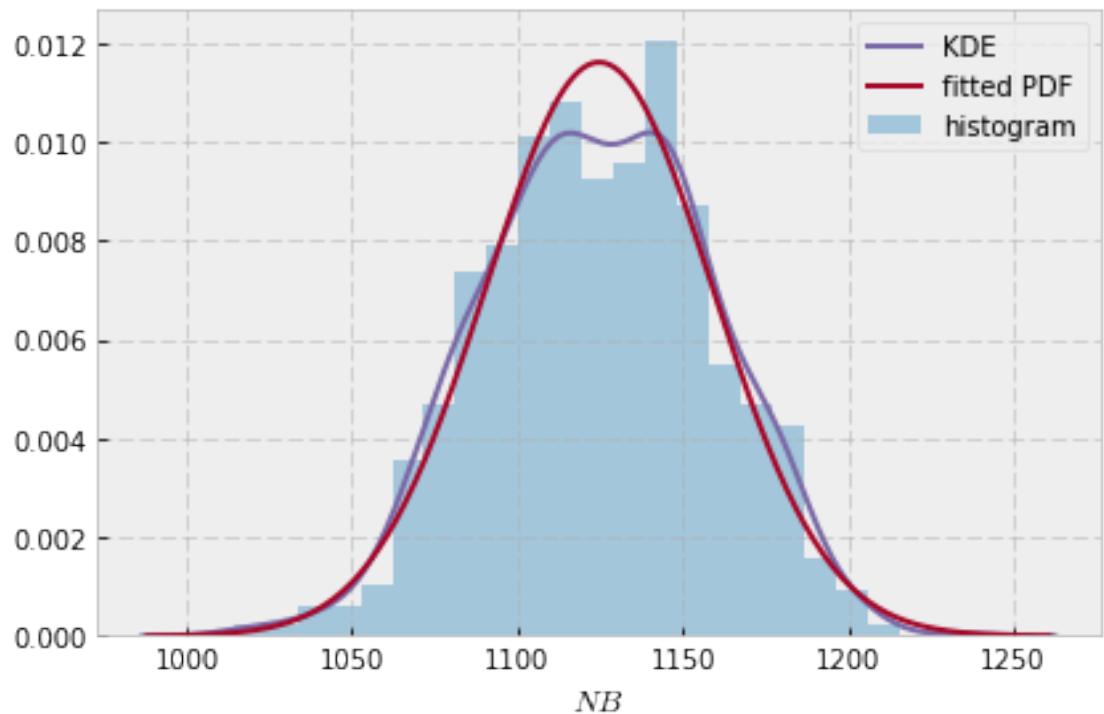


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=1125.864, scale=34.356360459163895)
DescribeResult(nobs=1000, minmax=(1028.0, 1219.0)
    mean=1125.864, variance=1181.5410450450452,
    skewness=-0.06482603268710212, kurtosis=-0.15223939461831426)
```

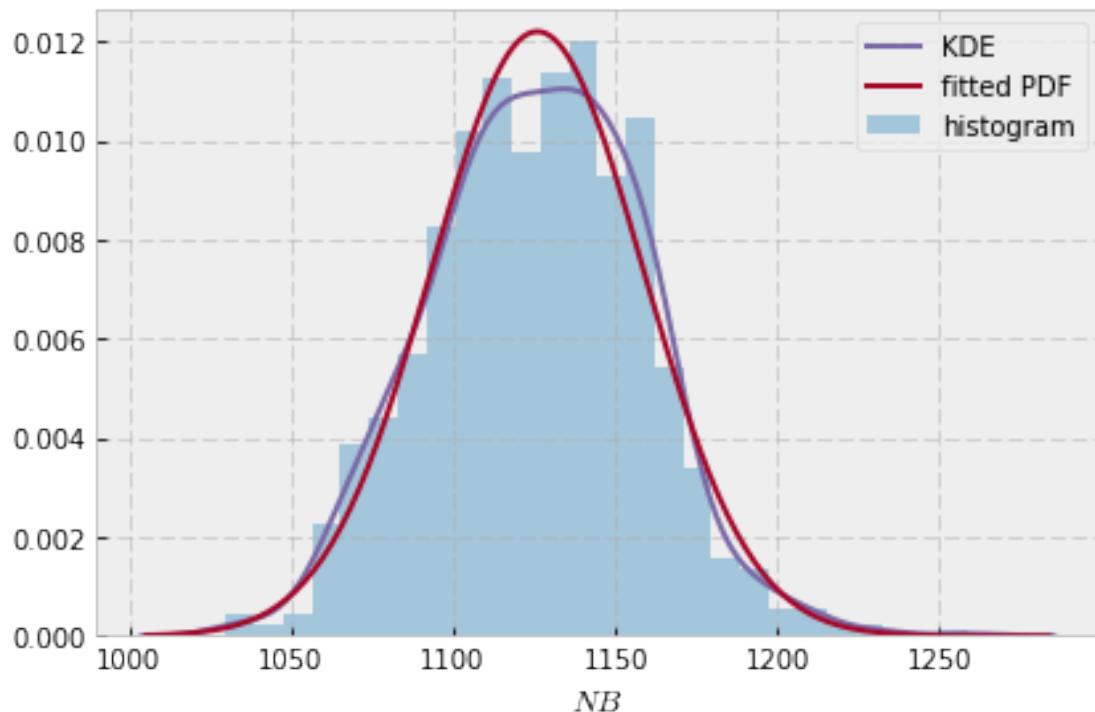


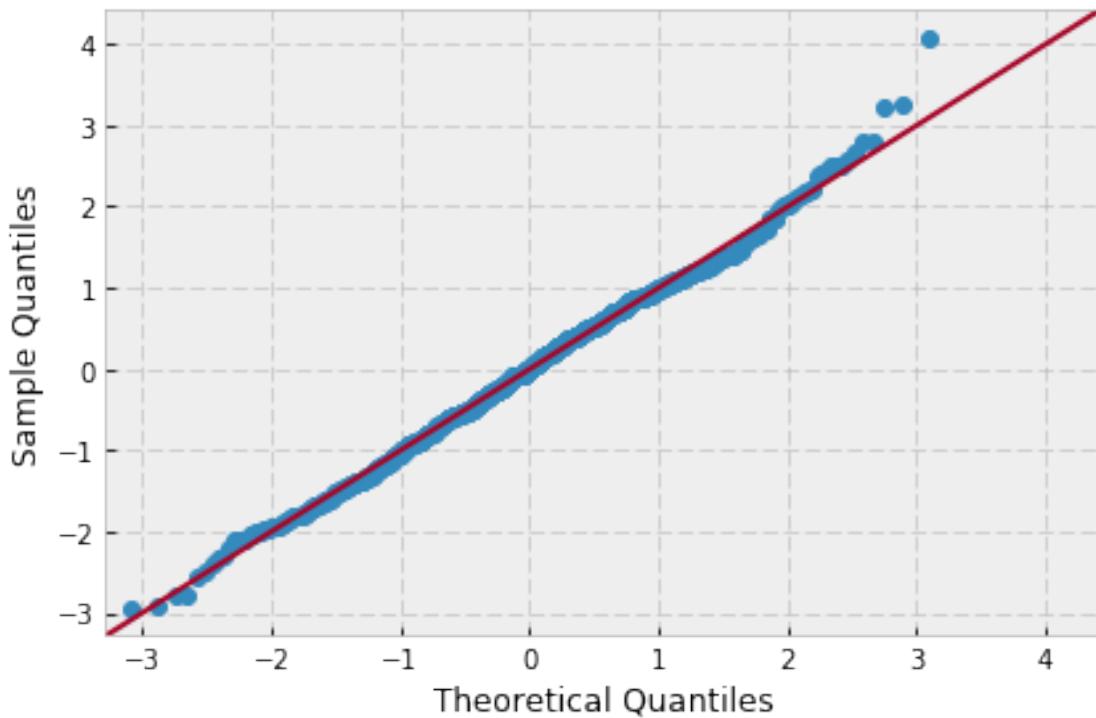


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(),loc=1124.705, scale=34.34009282165673)
DescribeResult(nobs=1000, minmax=(1014.0, 1235.0)
               mean=1124.704999999999, variance=1180.4223973973974,
               skewness=-0.0745899834064309, kurtosis=-0.30335655444578125)
```

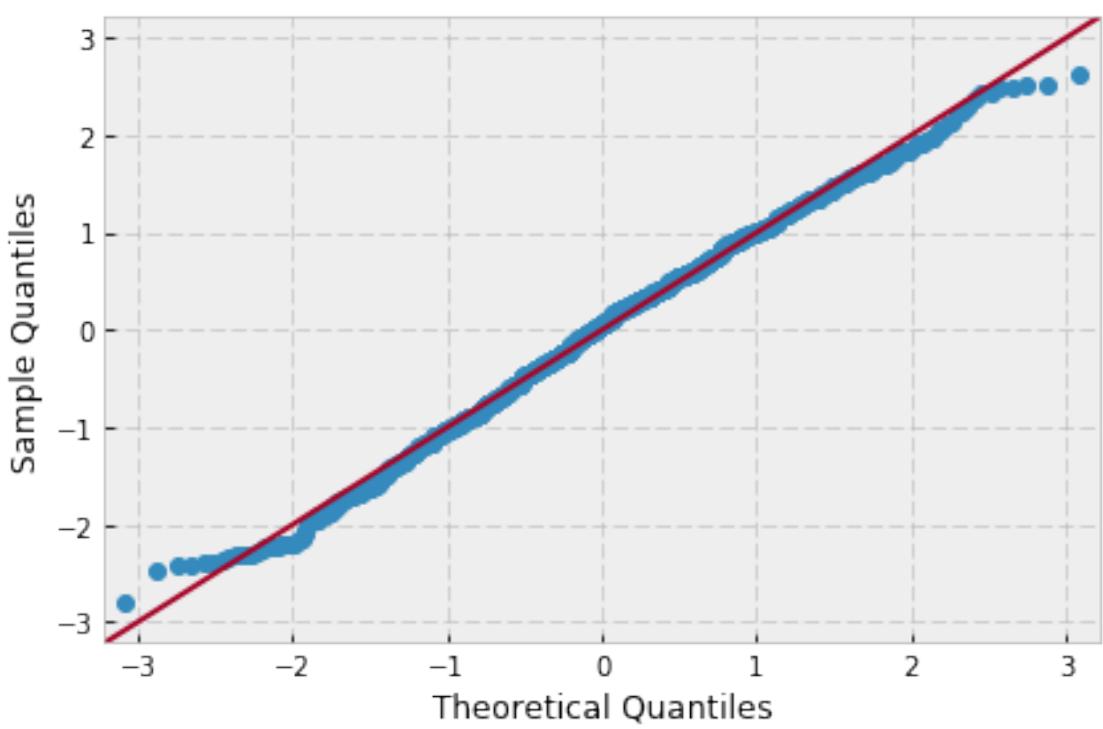
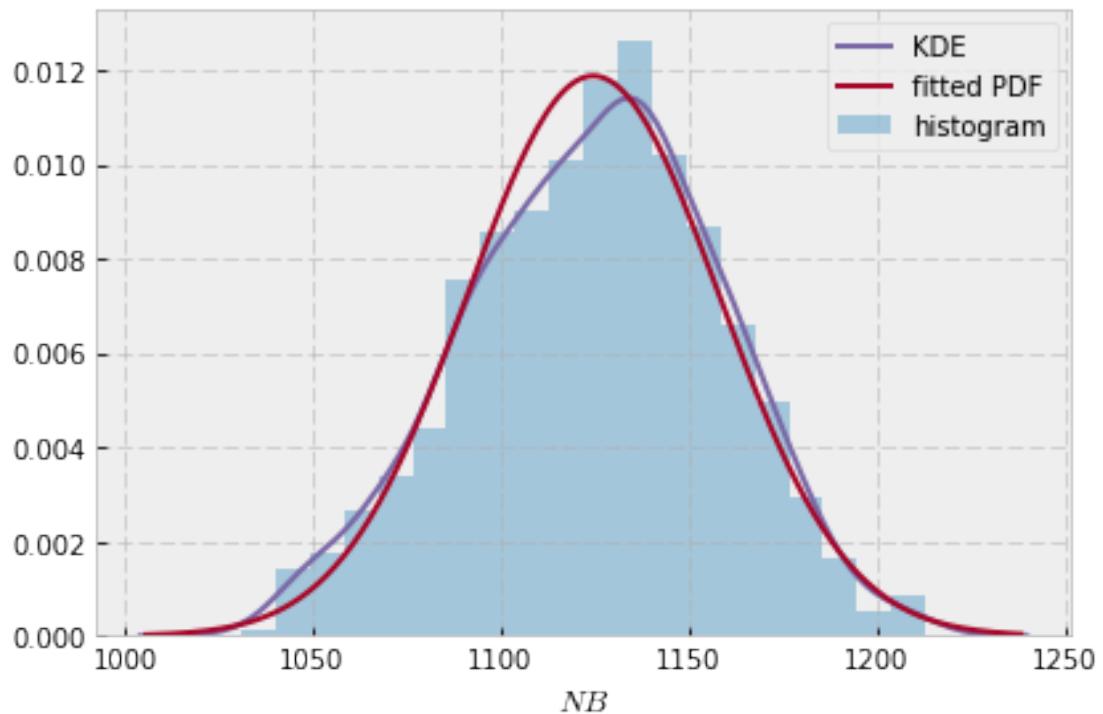


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=1125.945, scale=32.69278169565875)
DescribeResult(nobs=1000, minmax=(1030.0, 1259.0)
    mean=1125.944999999999, variance=1069.8878628628629,
    skewness=0.03609539504448593, kurtosis=0.08337080499981164)
```

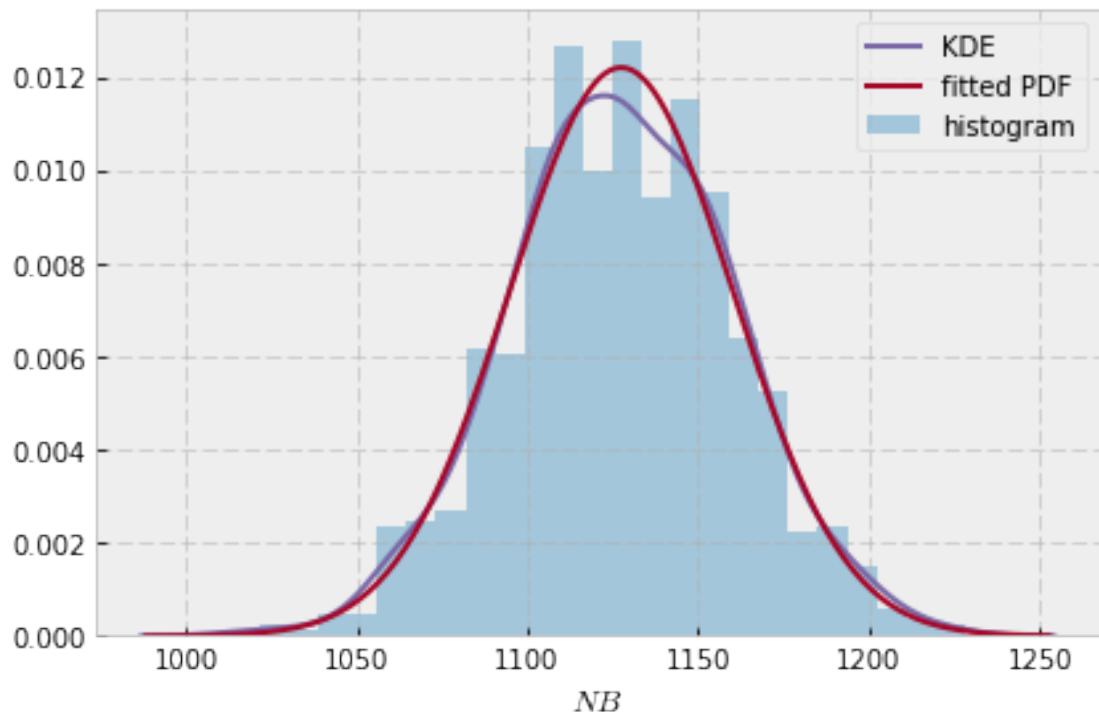


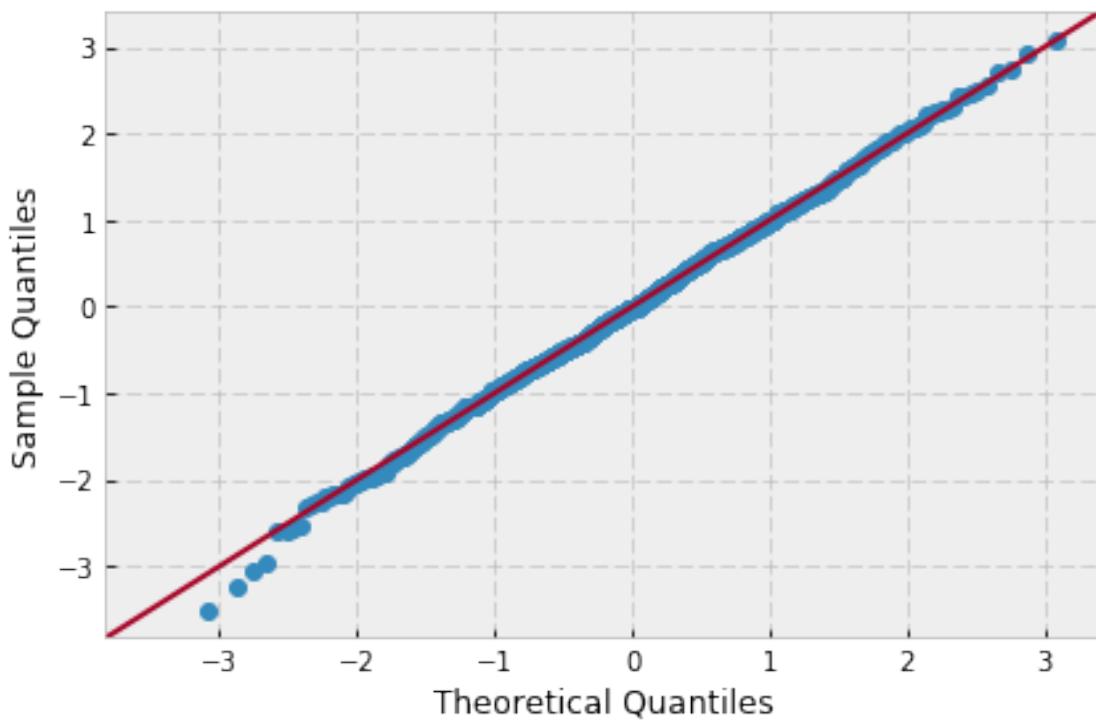


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(),loc=1124.604, scale=33.606326547244045)
DescribeResult(nobs=1000, minmax=(1031.0, 1213.0)
               mean=1124.604, variance=1130.5156996996996,
               skewness=-0.13722826137320196, kurtosis=-0.34163839340394064)
```

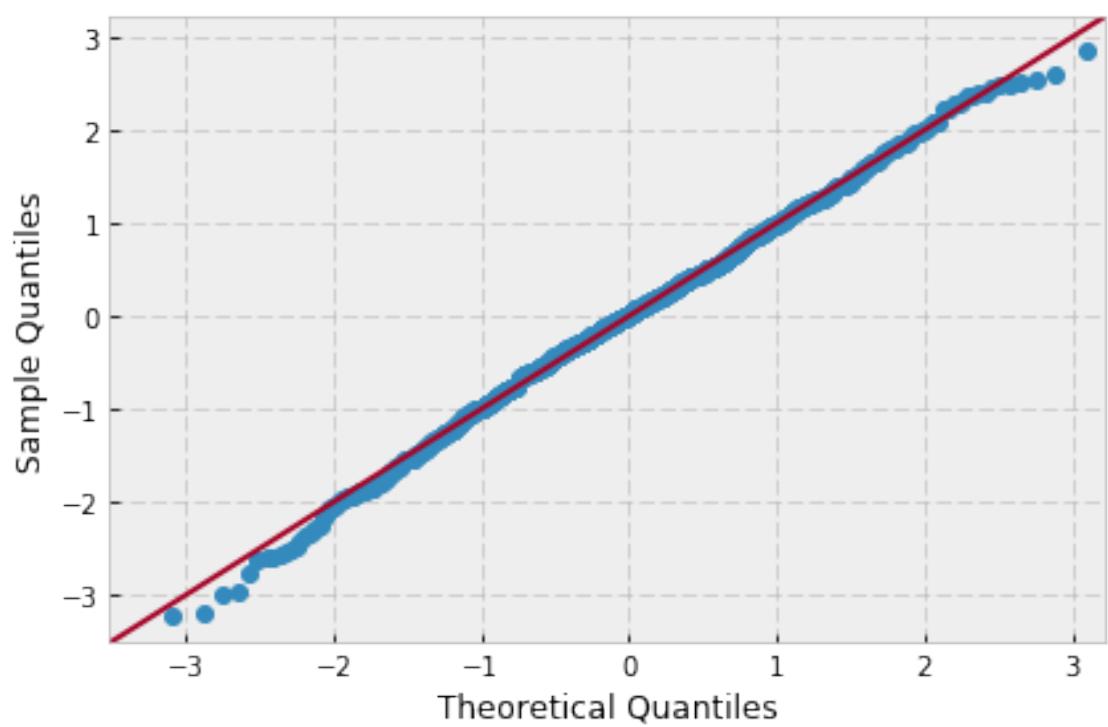
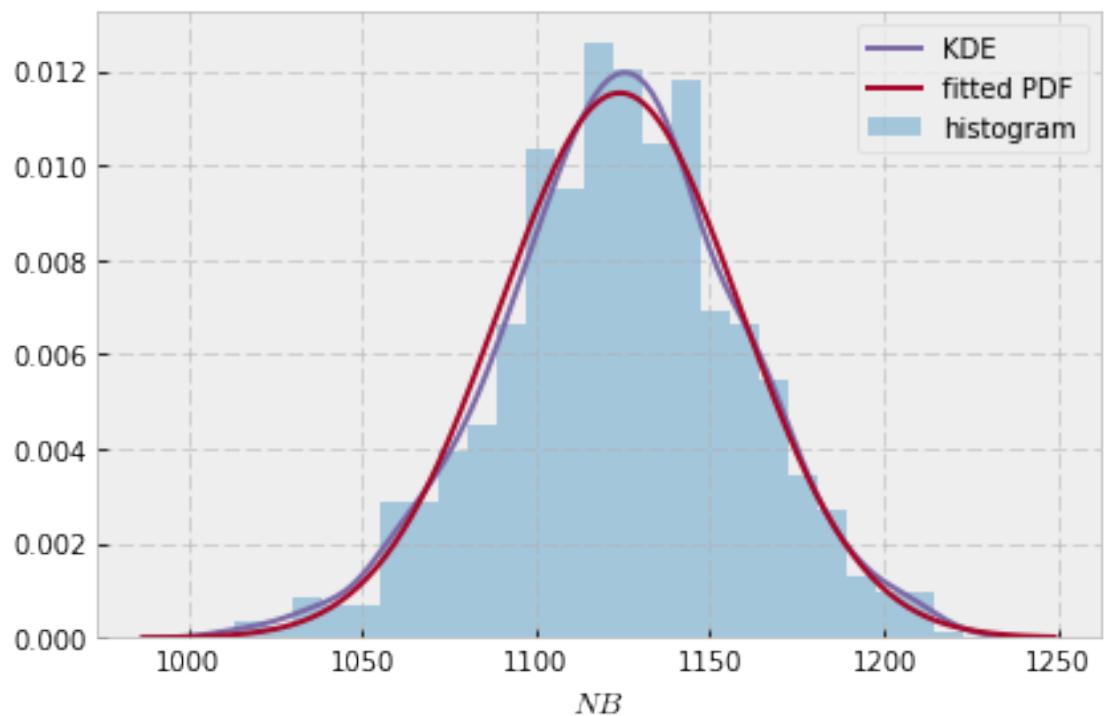


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=1127.431, scale=32.71380807854689)
DescribeResult(nobs=1000, minmax=(1013.0, 1228.0)
    mean=1127.431, variance=1071.2645035035036,
    skewness=-0.037223917139381696, kurtosis=0.04656044739166498)
```





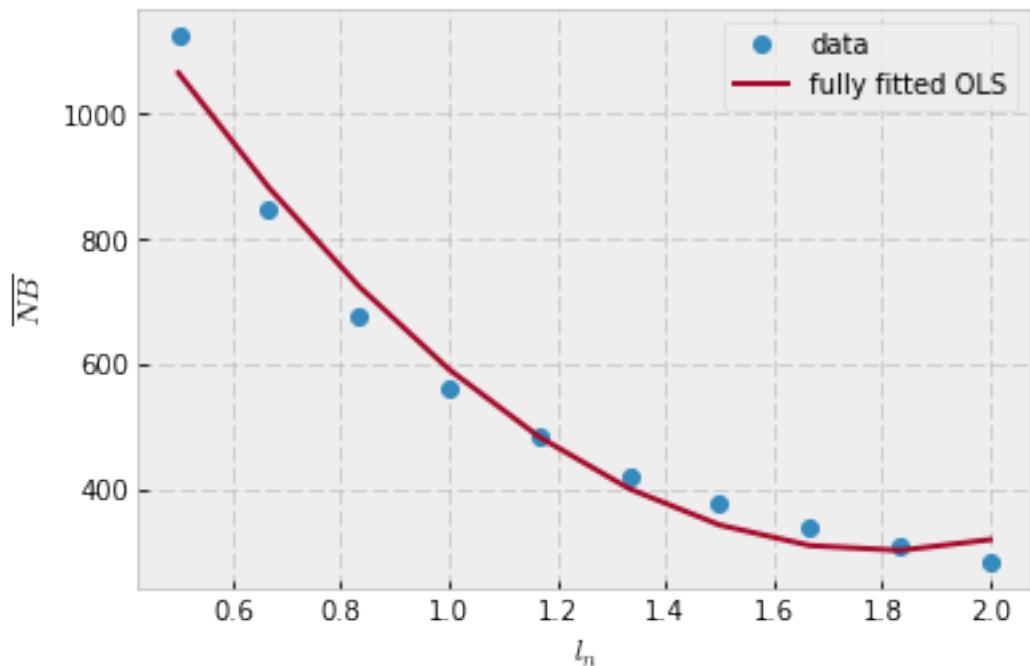
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(),loc=1124.136, scale=34.57969785871473)
DescribeResult(nobs=1000, minmax=(1013.0, 1223.0)
               mean=1124.136, variance=1196.9524564564565,
               skewness=-0.11867892564585361, kurtosis=0.0887417851092911)
```



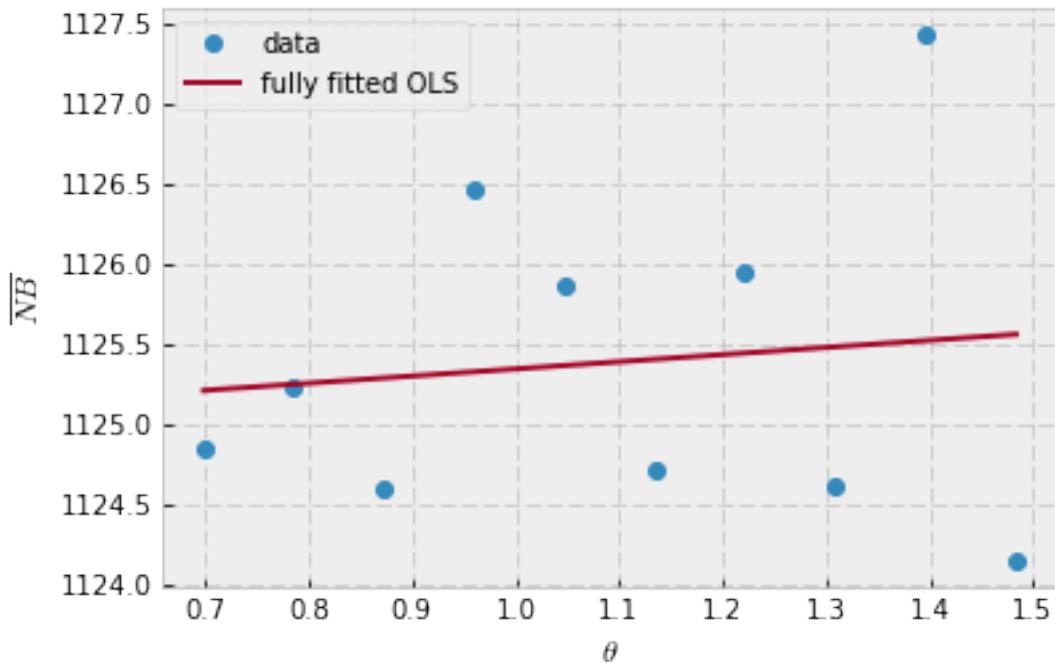
8.2 Parameter regression

```
In [22]: var_name = 'number_of_root_branches'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly1')  
        mean_reg('maxB', var_name, 'poly1')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly1')  
        mean_reg('N', var_name, 'poly1')
```

```
variable name: ln  
model: poly2  
r-squared: 0.9298722998961004  
explained variance: 0.9300052503727689
```



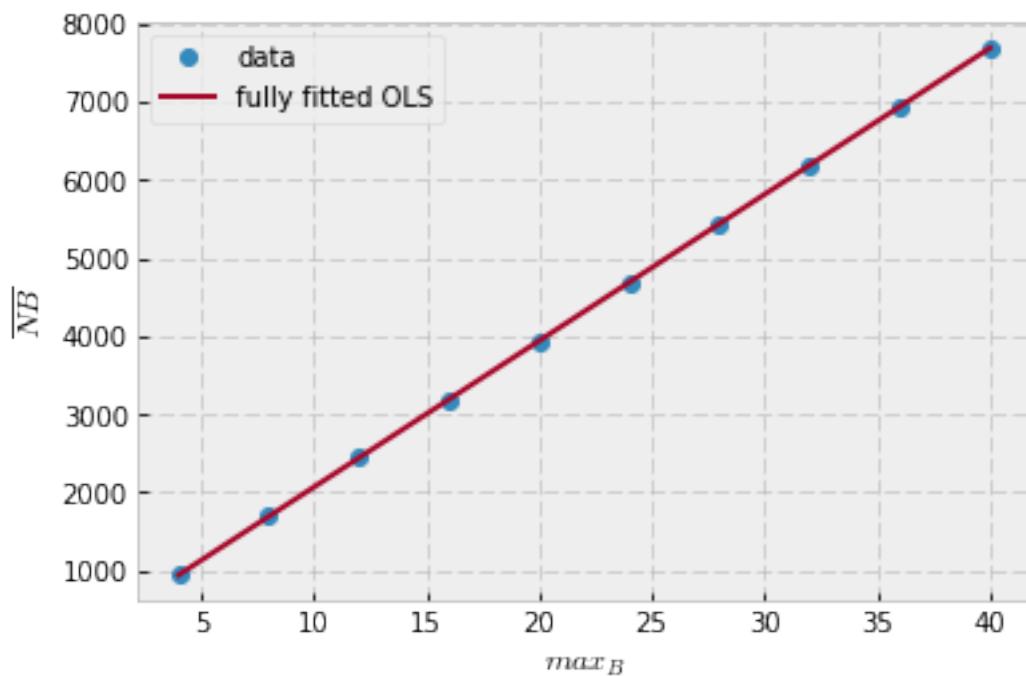
```
variable name: theta  
model: poly1  
r-squared: -0.7204747521799859  
explained variance: -0.7185227403913084
```



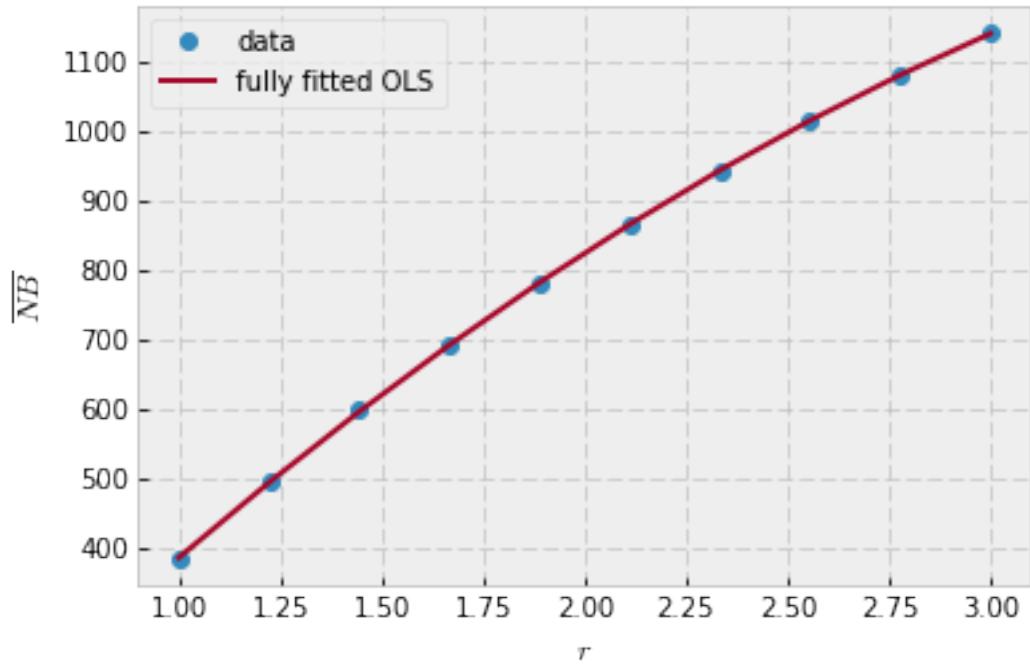
```

variable name: maxB
model: poly1
r-squared: 0.9999990103244277
explained variance: 0.9999990119736687

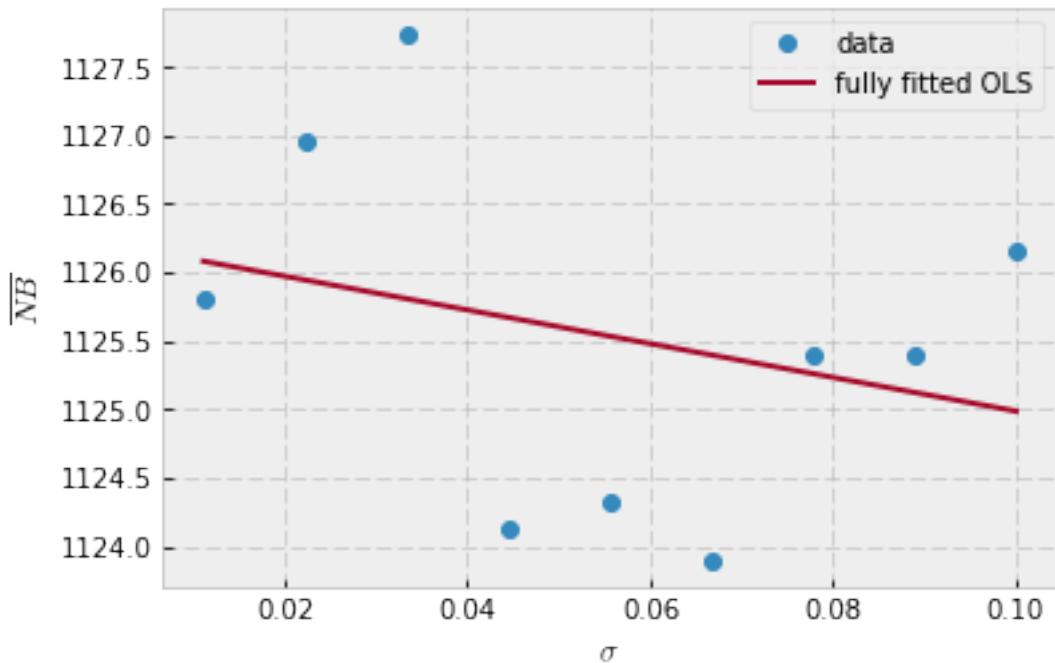
```



```
variable name: r
model: poly2
r-squared: 0.9997413980506127
explained variance: 0.9997415858540416
```



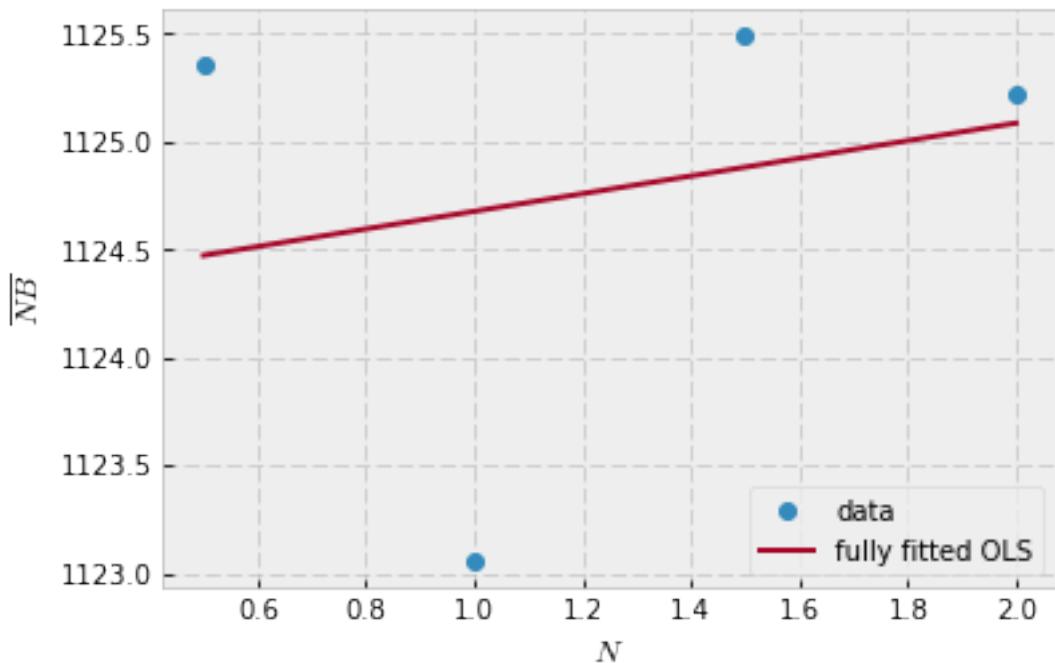
```
variable name: sigma
model: poly1
r-squared: -0.4229704136433907
explained variance: -0.41702817206359466
```



```

variable name: N
model: poly1
r-squared: -2.7180850660386775
explained variance: -2.4871826759176514

```

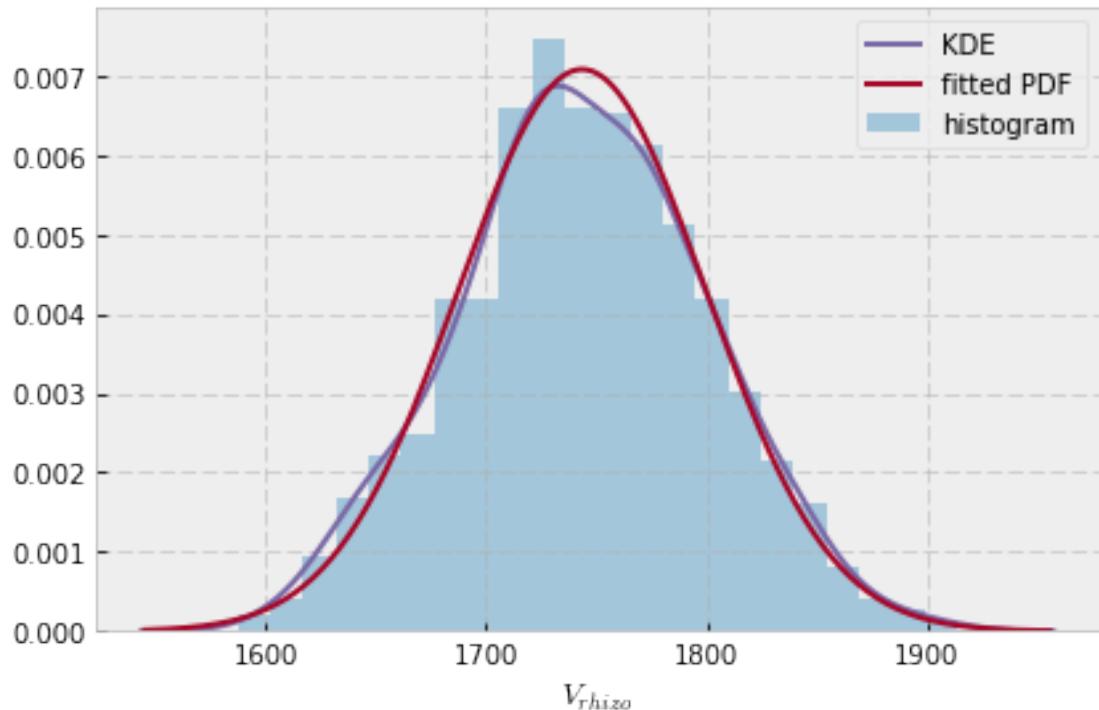


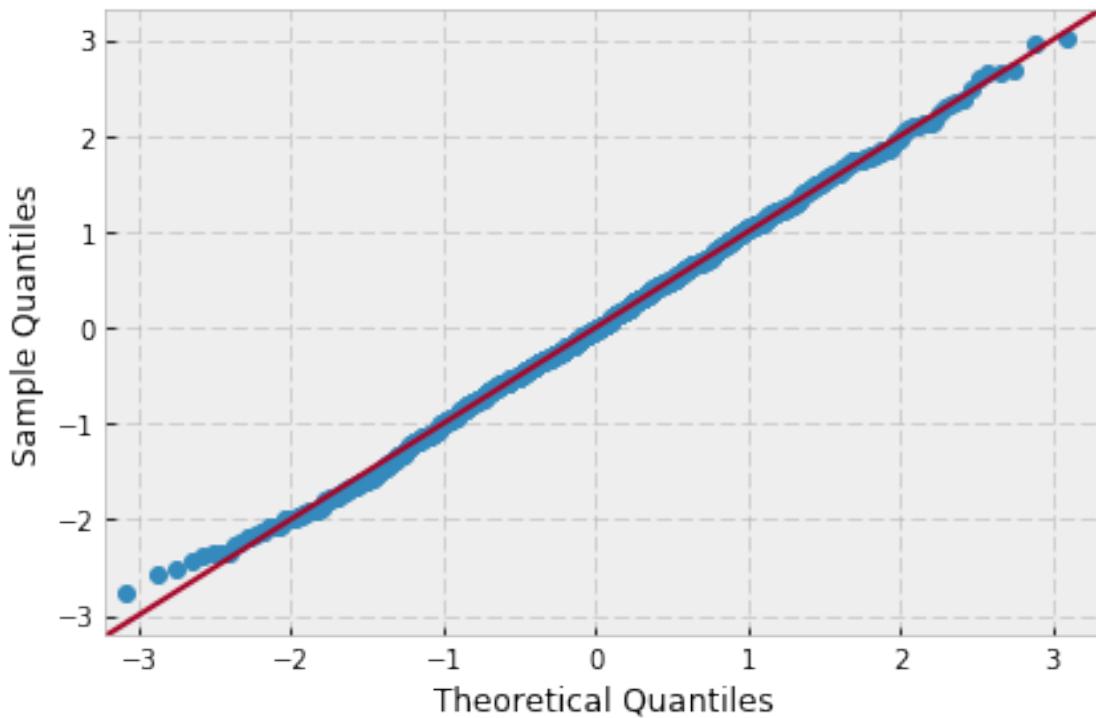
9 Volume of the rhizosphere: phosphate (0.2cm)

9.1 Probability distributions

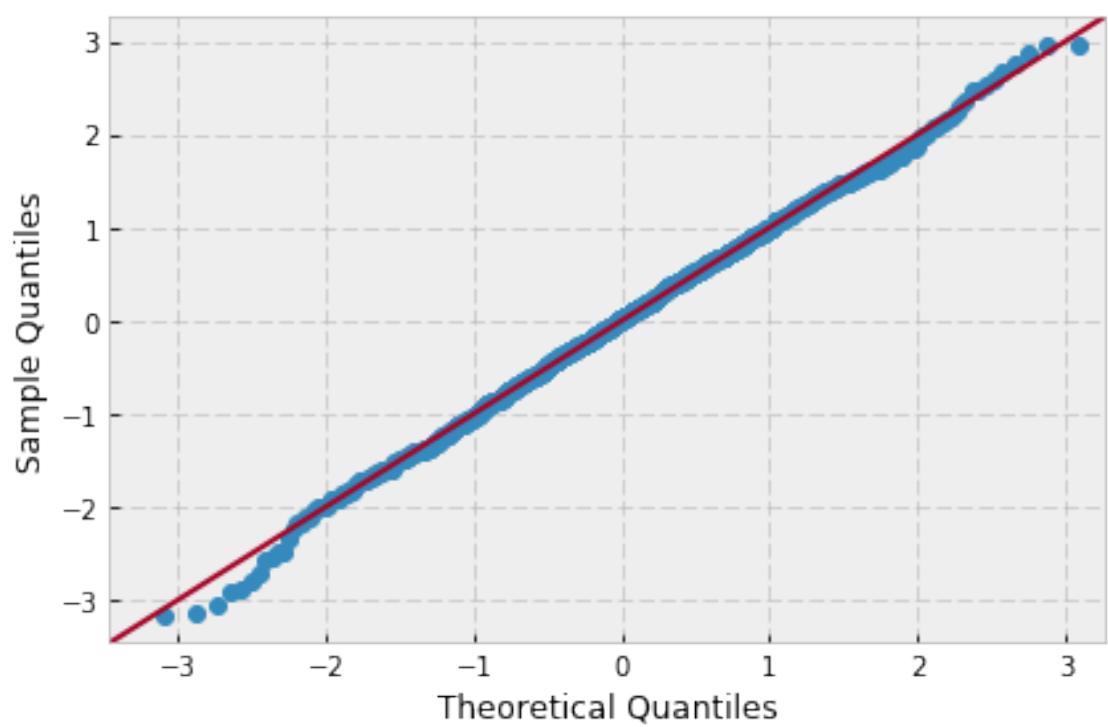
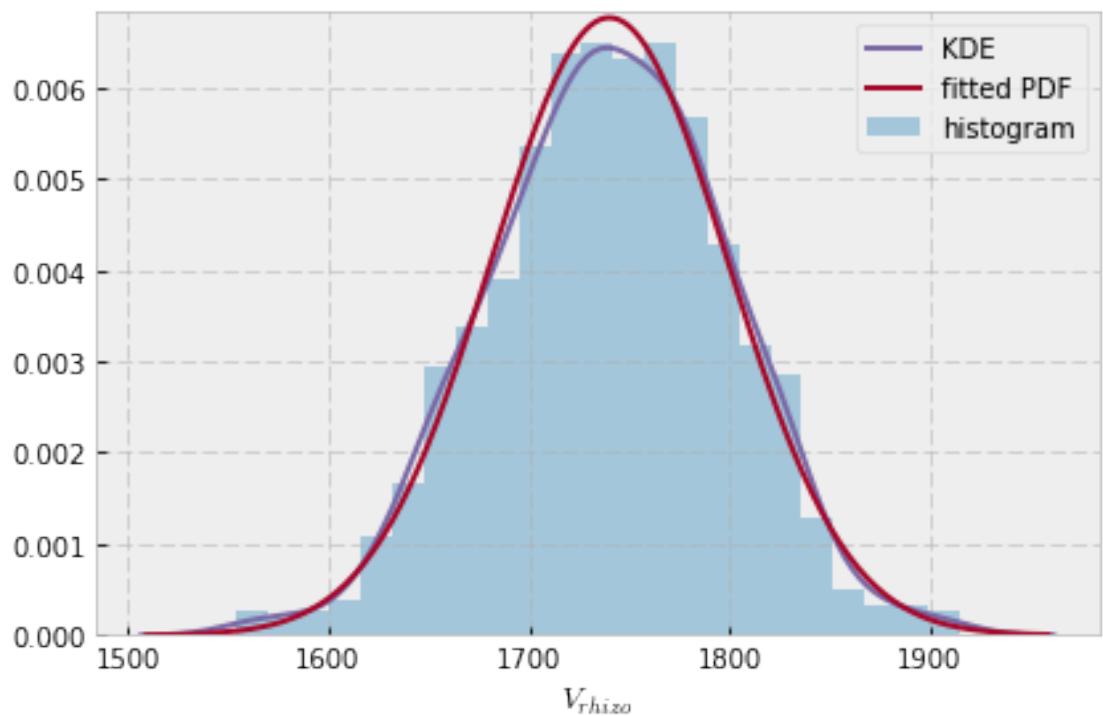
```
In [23]: plot_dist('rhizosphere_volume_phosphate')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=1743.4741617982802, scale=56.34885530654683)
DescribeResult(nobs=1000, minmax=(1587.1453283196986, 1913.6607466333917)
    mean=1743.4741617982802, variance=3178.371866224375,
    skewness=0.022062274213444245, kurtosis=-0.21362384647677146)
```

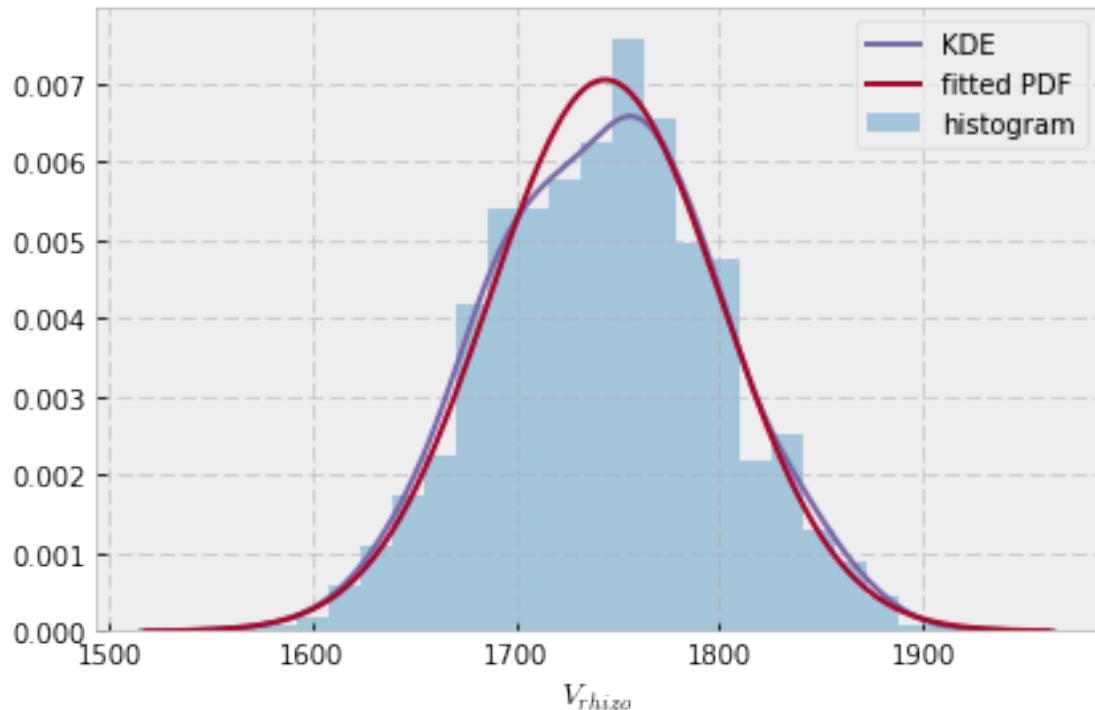


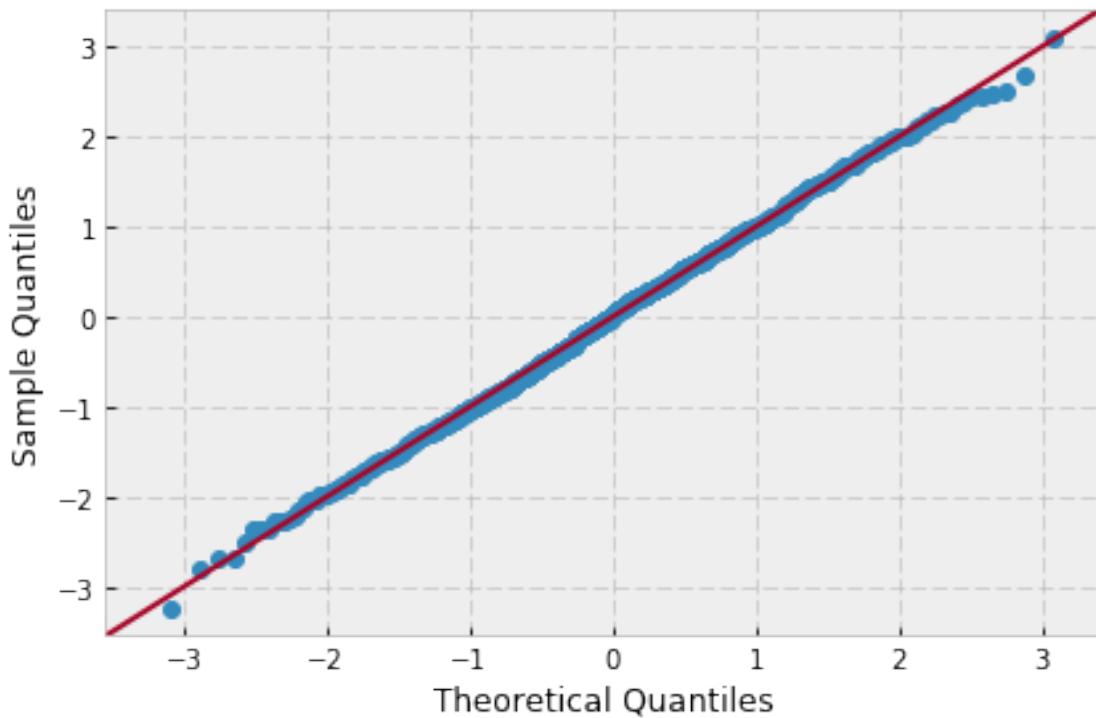


```
variable name: N
variable value: 1.0
distribution: normal(shape=(),loc=1740.0786446364023, scale=58.95035475191497)
DescribeResult(nobs=1000, minmax=(1553.730716150292, 1914.4069569206029)
               mean=1740.0786446364023, variance=3478.622948324949,
               skewness=-0.08497535908975812, kurtosis=-0.0083629574555113)
```

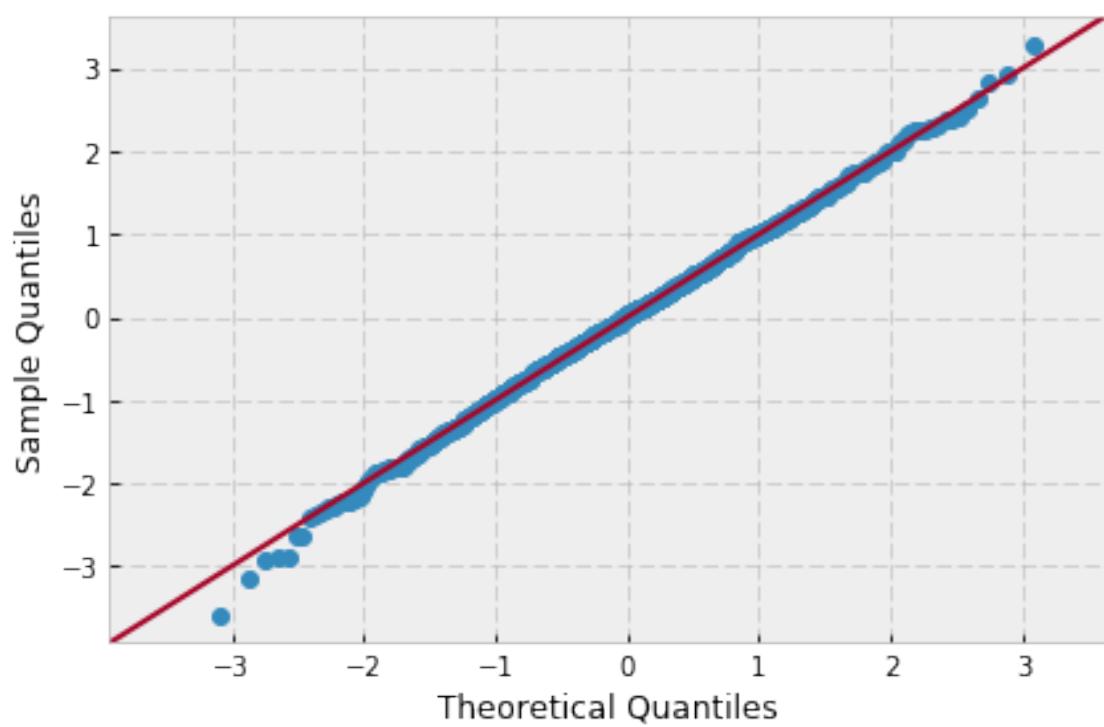
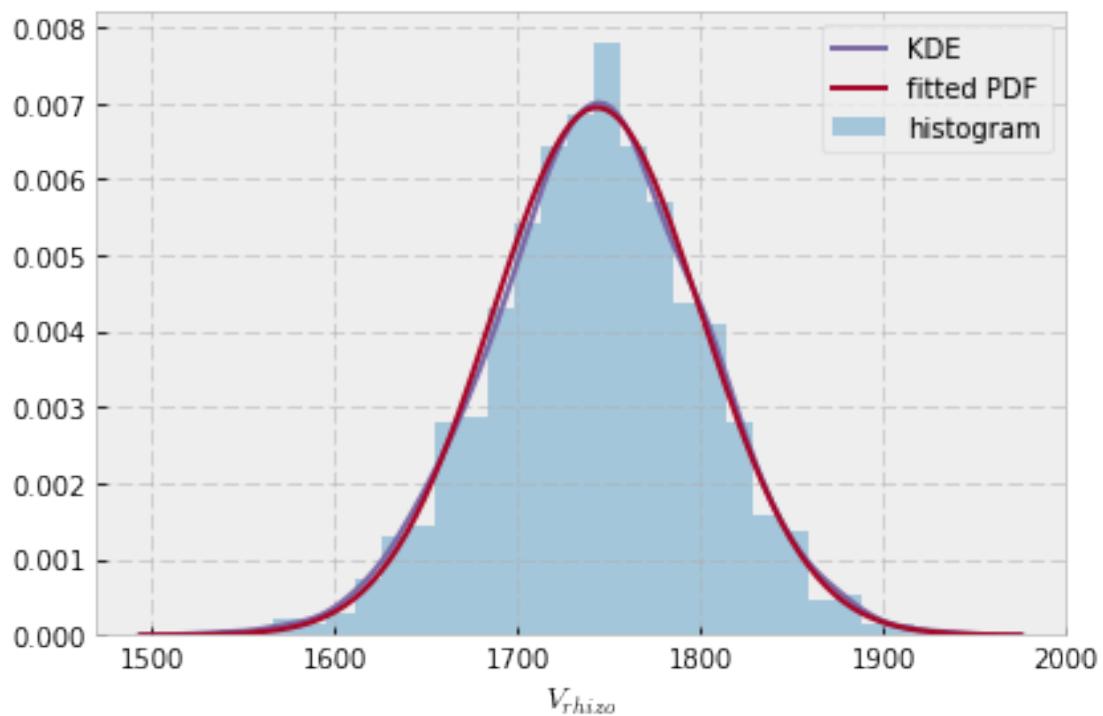


```
variable name: N
variable value: 1.5
distribution: normal(shape=(),loc=1743.8235139448395, scale=56.62980892592853)
DescribeResult(nobs=1000, minmax=(1561.0375285437965, 1918.9619370335054)
               mean=1743.8235139448395, variance=3210.145404391566,
               skewness=-0.0027353403151204126, kurtosis=-0.2579884682537892)
```

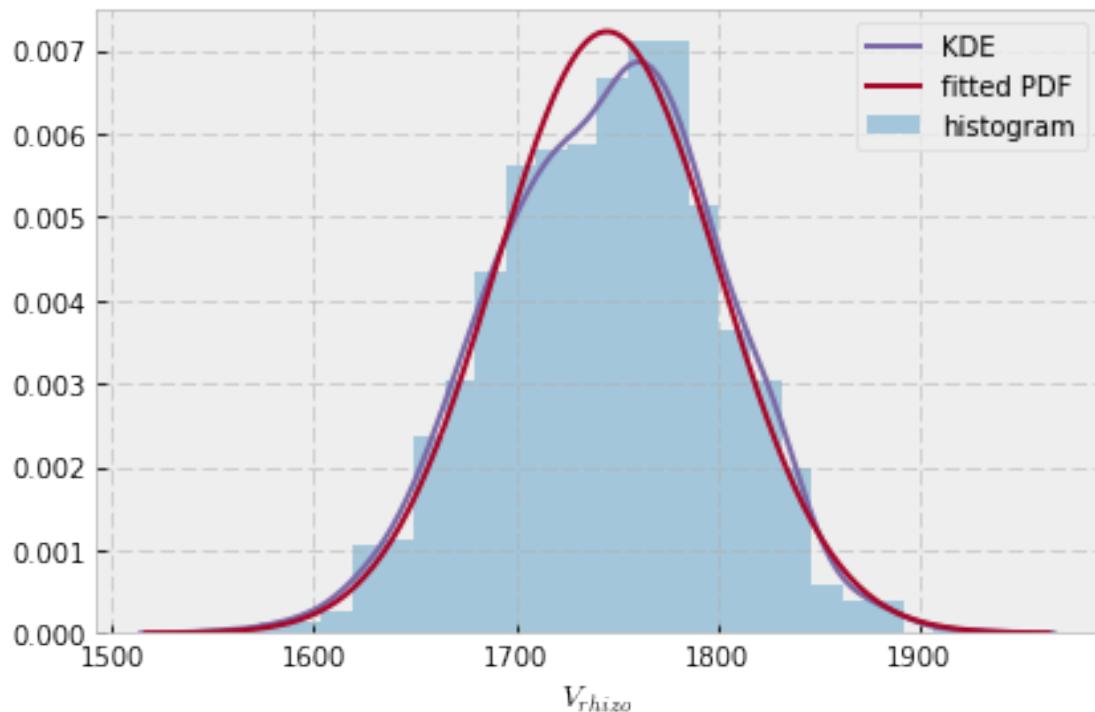


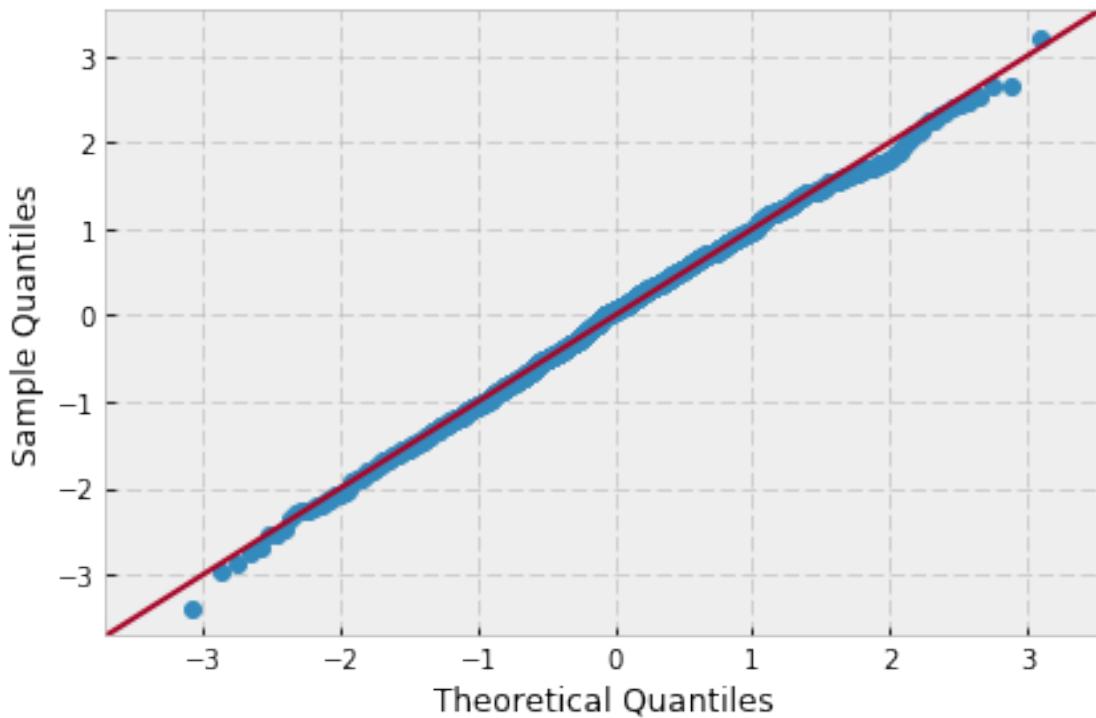


```
variable name: N
variable value: 2.0
distribution: normal(shape=(),loc=1743.4163974878481, scale=57.48835017818454)
DescribeResult(nobs=1000, minmax=(1537.4109722920443, 1931.6269148734352)
               mean=1743.4163974878481, variance=3308.2186248344046,
               skewness=-0.07657739239300747, kurtosis=0.10053855352414143)
```

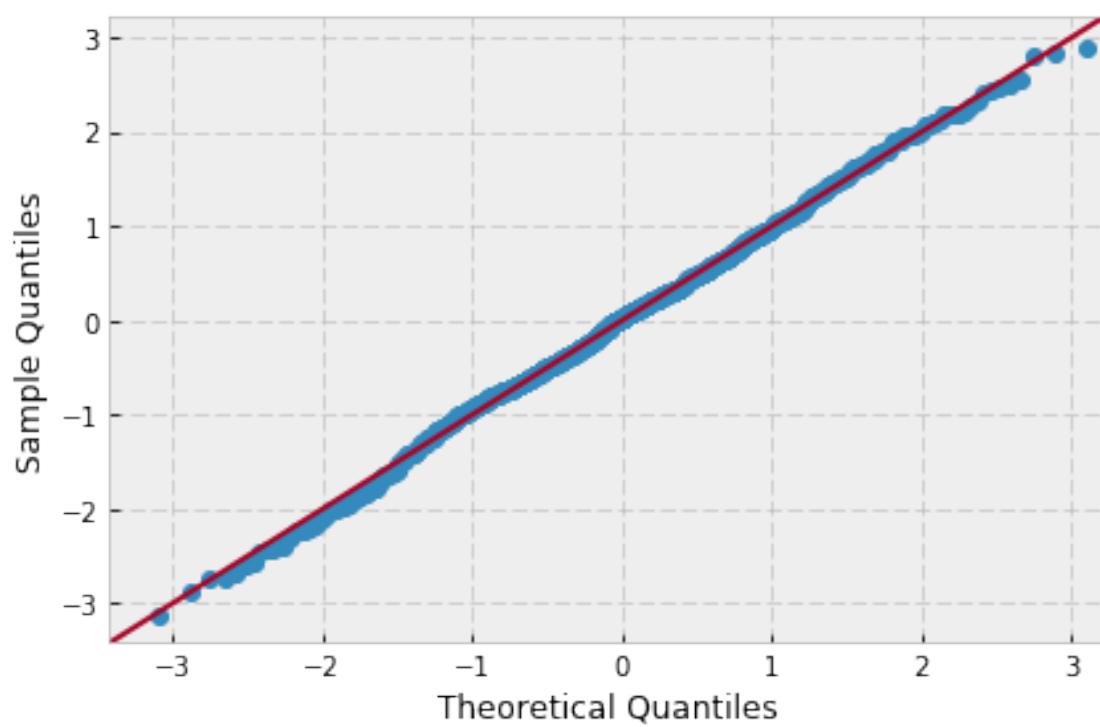
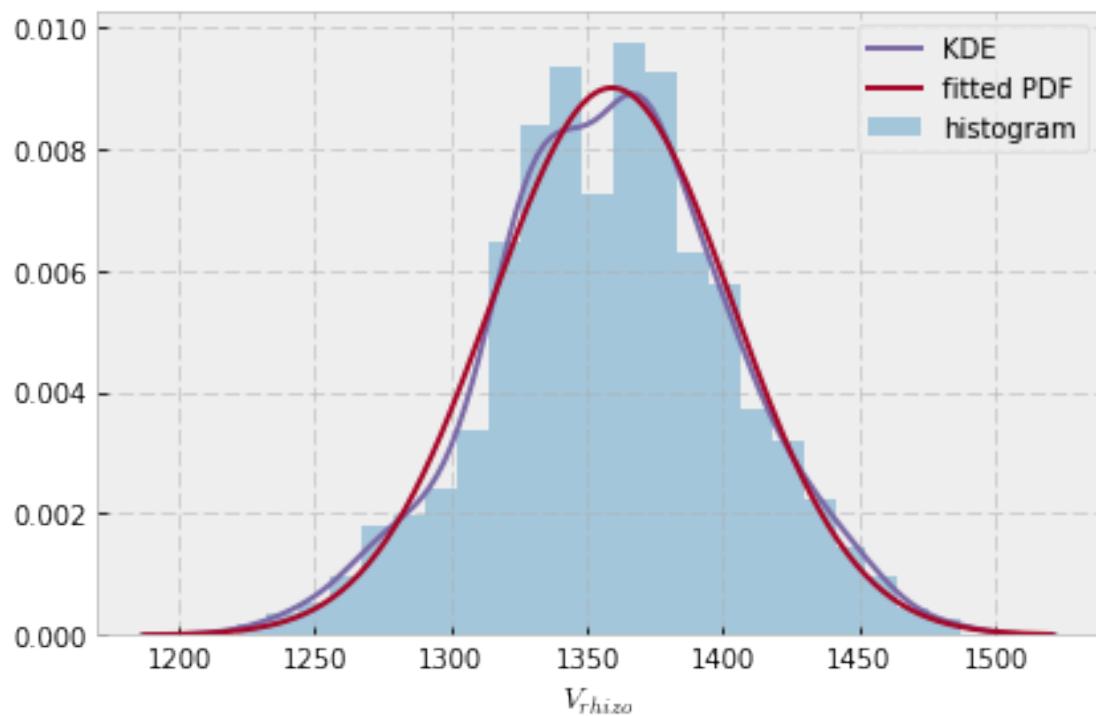


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=1745.3025263915774, scale=55.18759184786652)
DescribeResult(nobs=1000, minmax=(1558.7245484534405, 1922.214826422746)
               mean=1745.3025263915774, variance=3048.7190129796832,
               skewness=-0.11731129396596043, kurtosis=-0.18910121418171277)
```

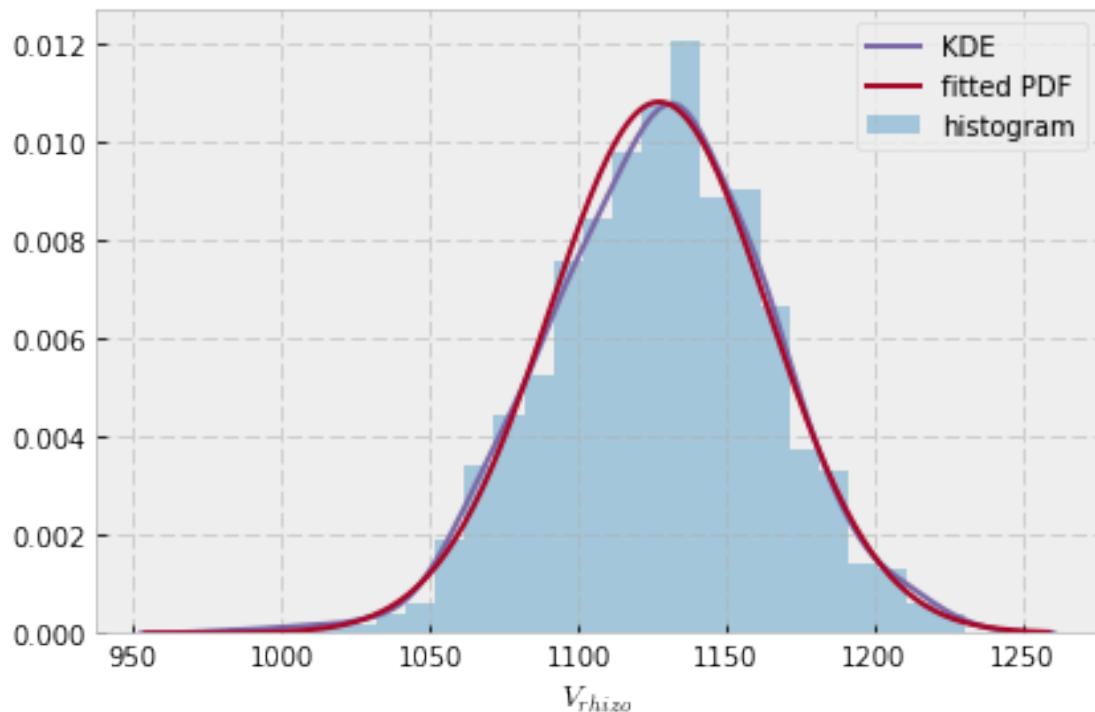


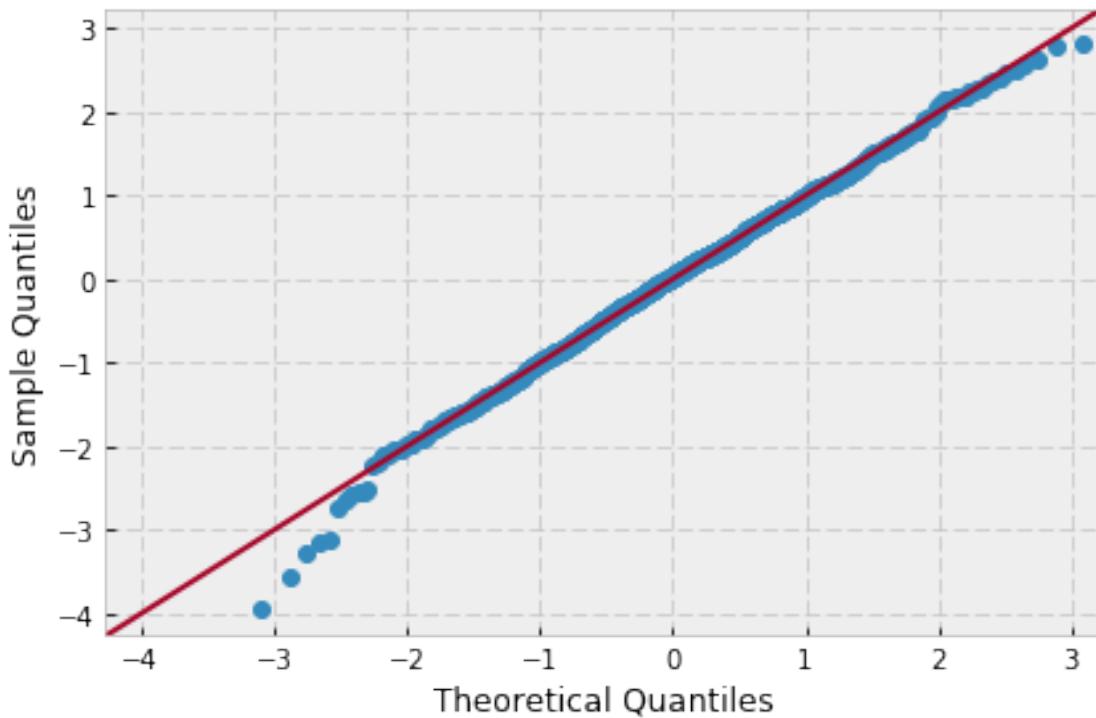


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(),loc=1359.1023075777214, scale=44.3142541782257)
DescribeResult(nobs=1000, minmax=(1220.9659697147761, 1487.2281065014752)
               mean=1359.1023075777214, variance=1965.7188422146087,
               skewness=-0.03595848369353028, kurtosis=0.0027399819645710366)
```

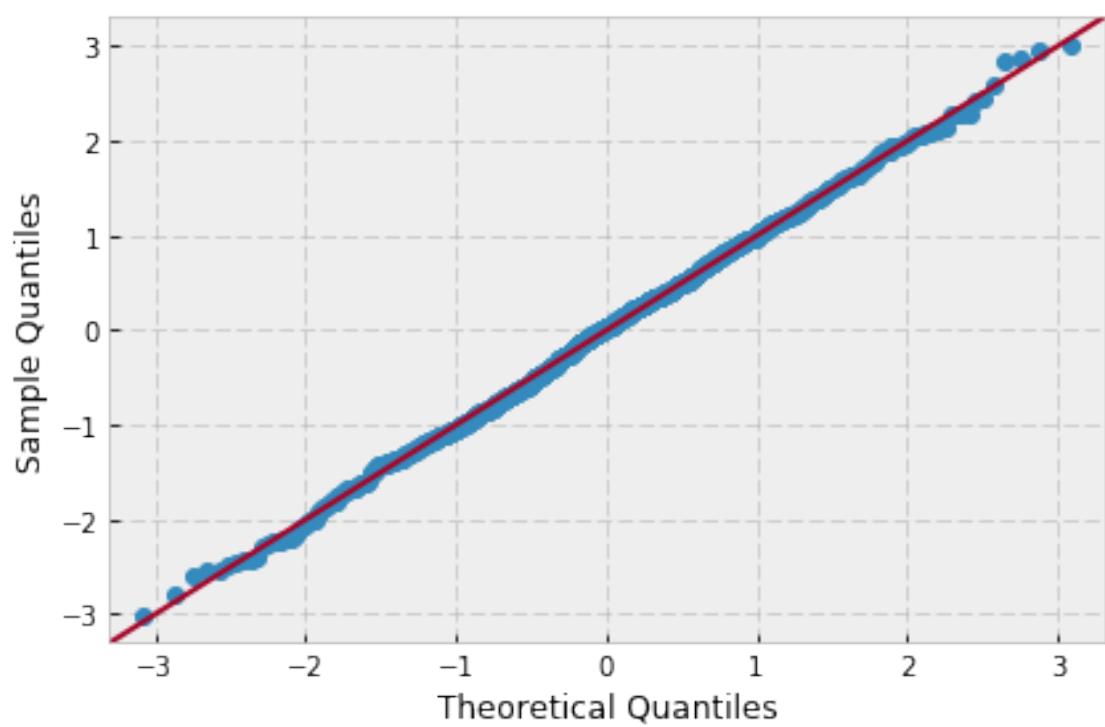
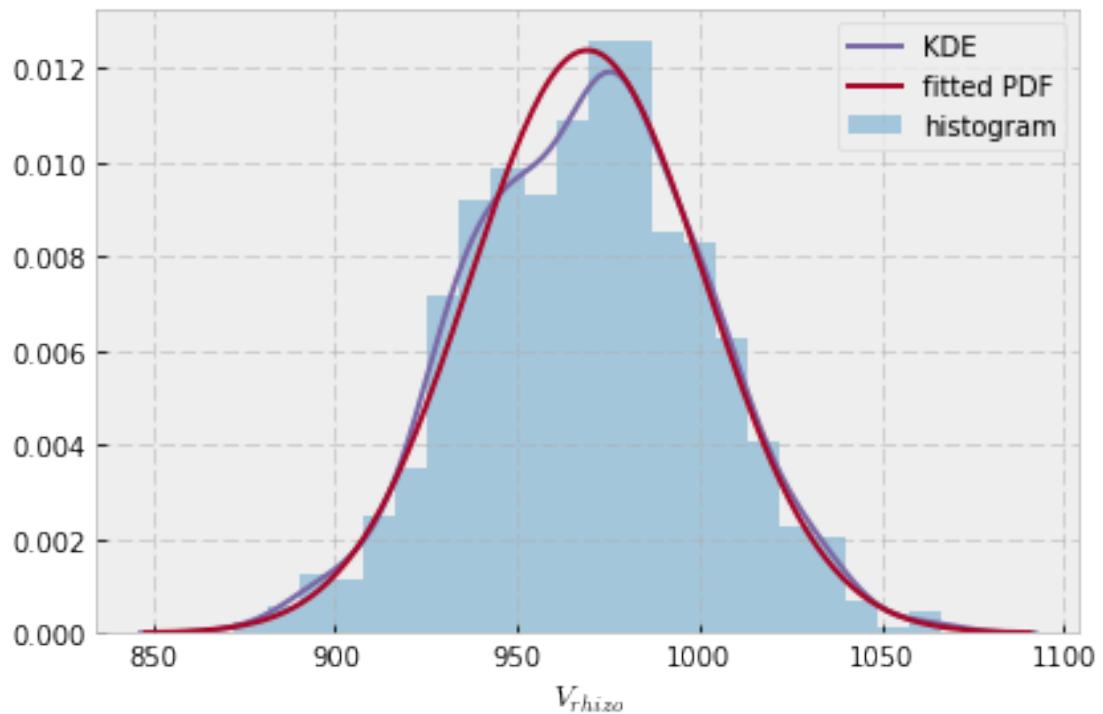


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=1127.325765668571, scale=36.912233380766175)
DescribeResult(nobs=1000, minmax=(982.38200330585539, 1230.829527818905)
               mean=1127.325765668571, variance=1363.8768500061547,
               skewness=-0.1556825439518013, kurtosis=0.19739135553949172)
```

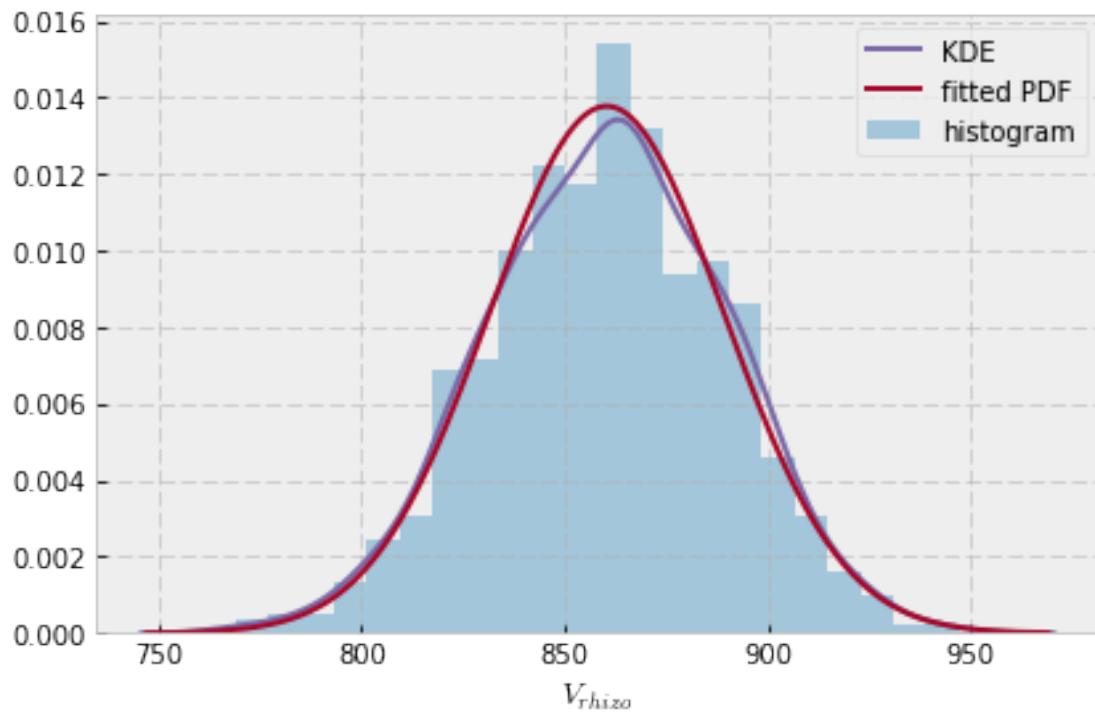


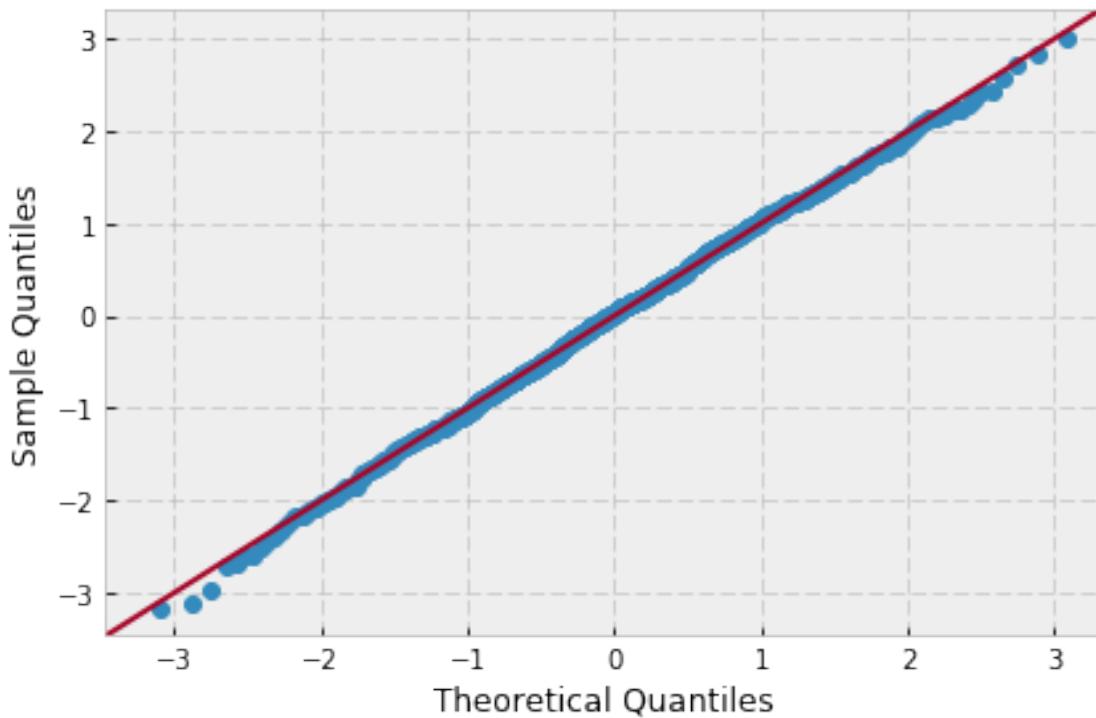


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(),loc=969.2904029374874, scale=32.24133049953211)
DescribeResult(nobs=1000, minmax=(872.48456839252538, 1066.3152246489285)
               mean=969.29040293748744, variance=1040.5439363163759,
               skewness=-0.0006985976129427895, kurtosis=-0.1809348631726002)
```

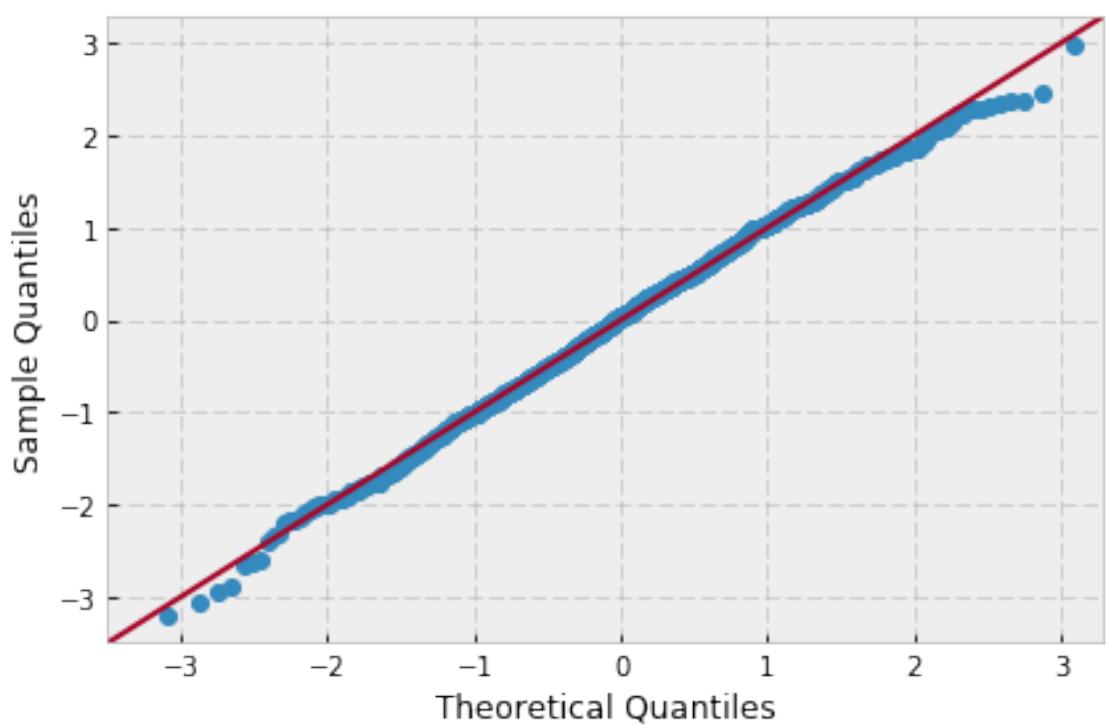
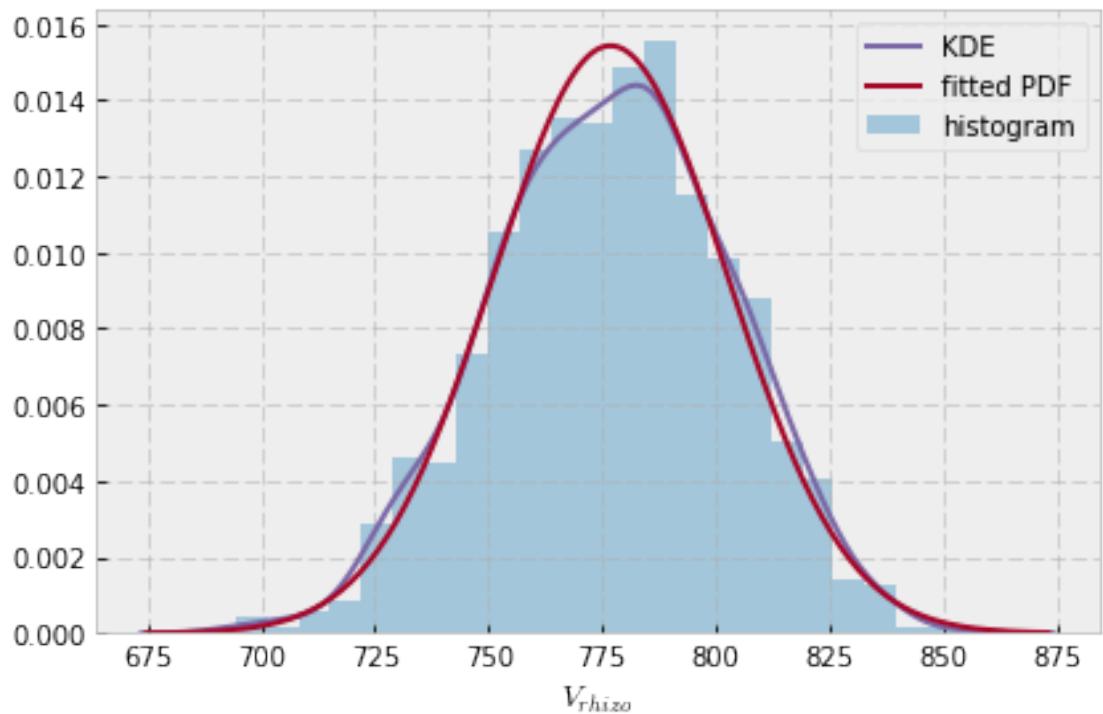


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=860.3694050202039, scale=28.95991888285074)
DescribeResult(nobs=1000, minmax=(768.89766672913163, 947.33966452144489)
               mean=860.36940502020389, variance=839.5164181194142,
               skewness=-0.08366165321630135, kurtosis=-0.1346401936921775)
```

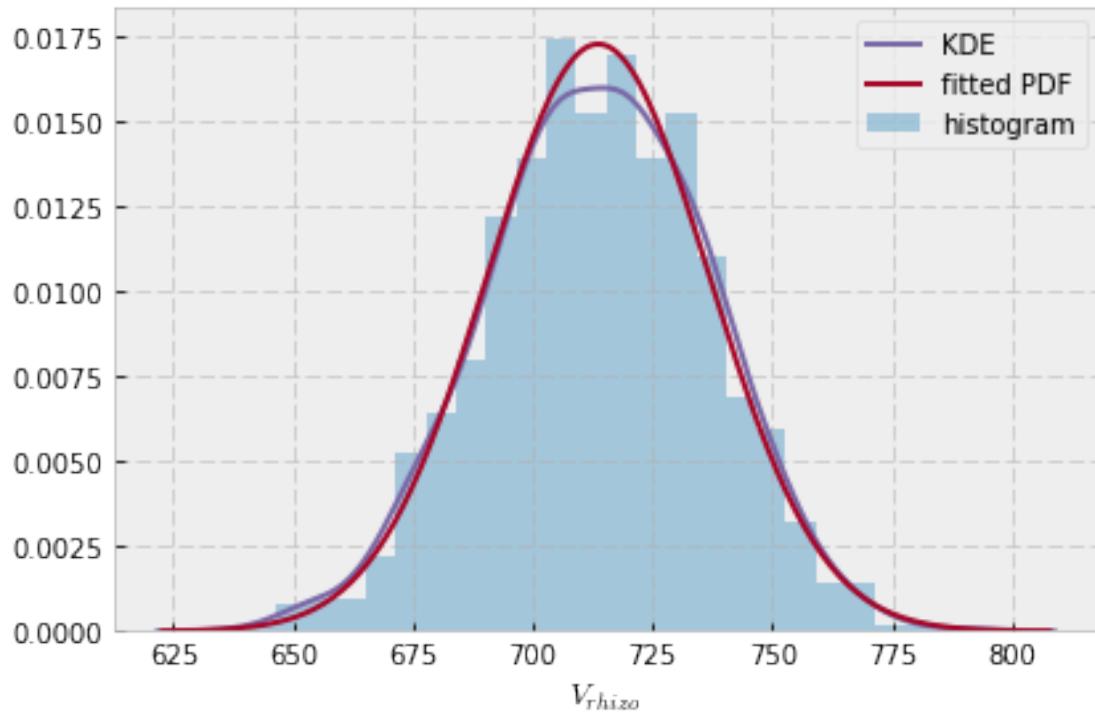


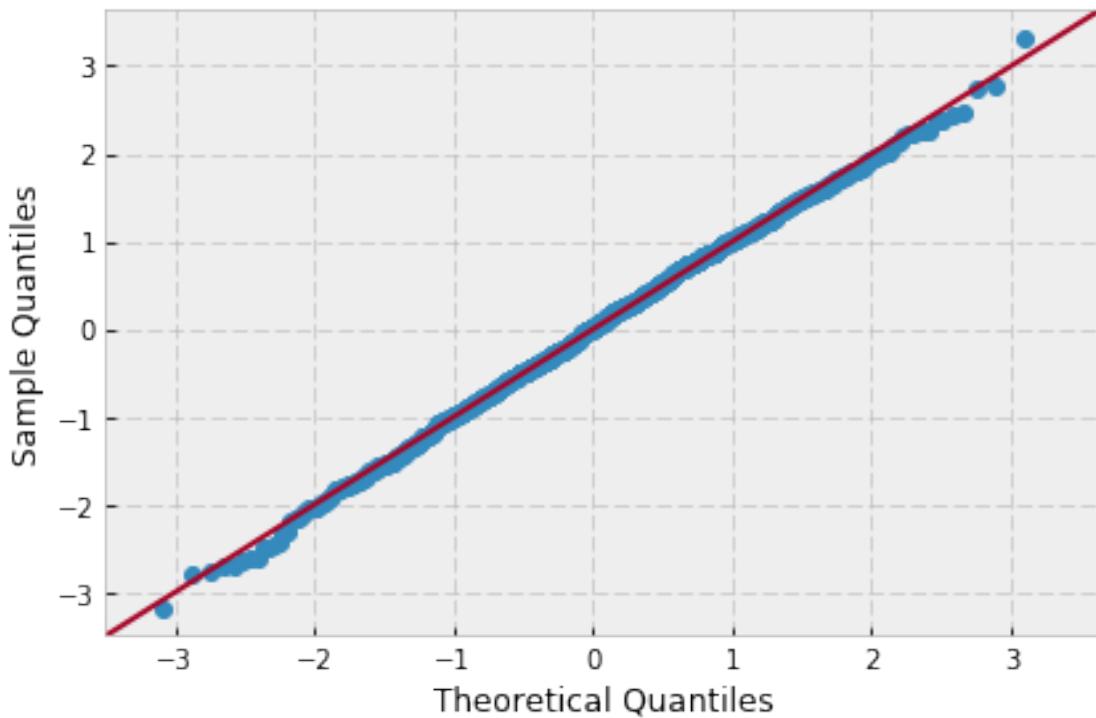


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=776.8083050213021, scale=25.851368458406093)
DescribeResult(nobs=1000, minmax=(694.29776658748312, 853.62564735869512)
               mean=776.80830502130209, variance=668.96221338565908,
               skewness=-0.11181693791090777, kurtosis=-0.24227672837564818)
```

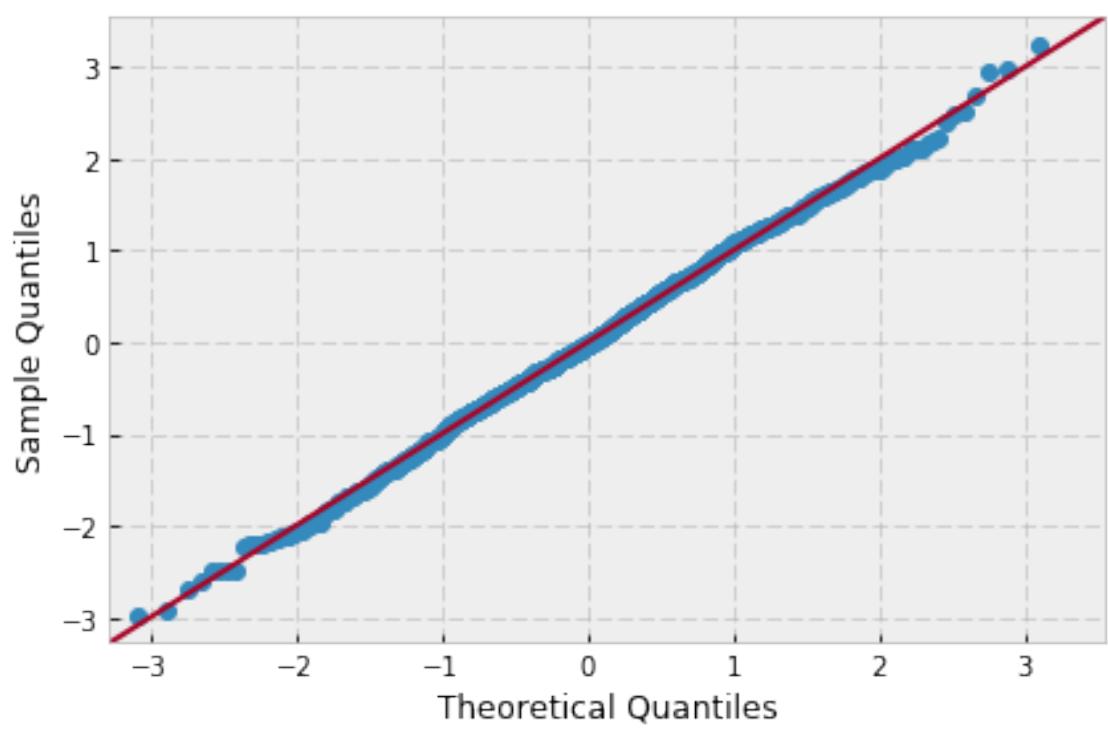
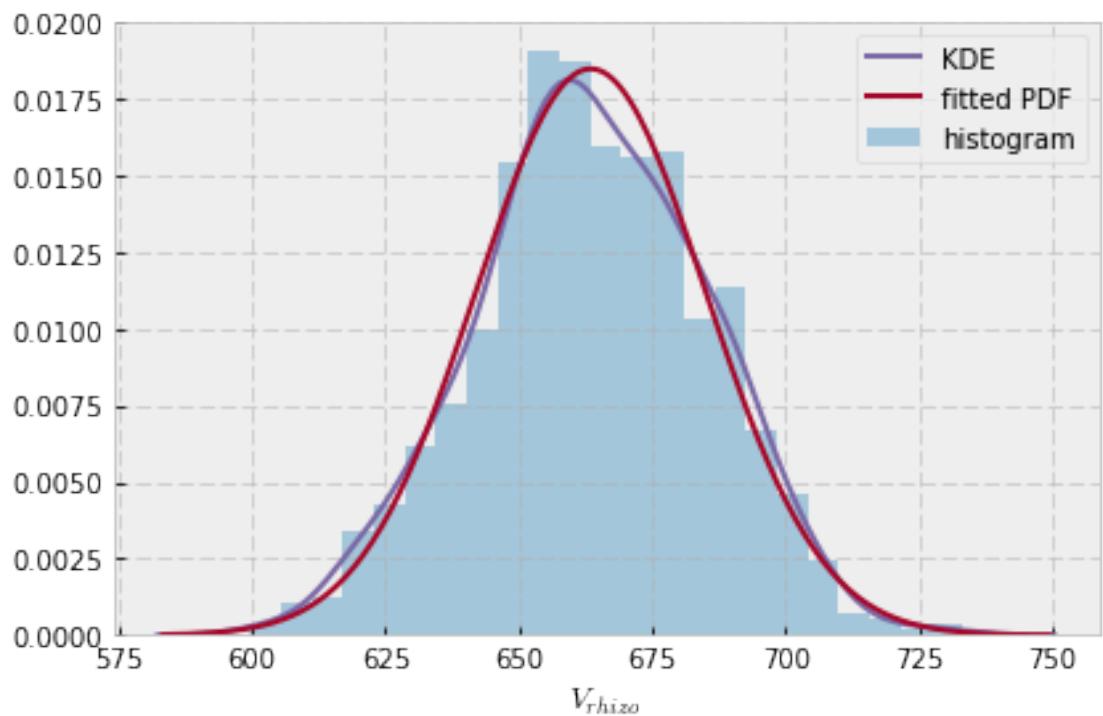


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=713.6518044950167, scale=23.13310737042945)
DescribeResult(nobs=1000, minmax=(640.24533335280034, 790.30783354688288)
               mean=713.65180449501668, variance=535.67633294476207,
               skewness=-0.08706452415170406, kurtosis=-0.11827624565716821)
```

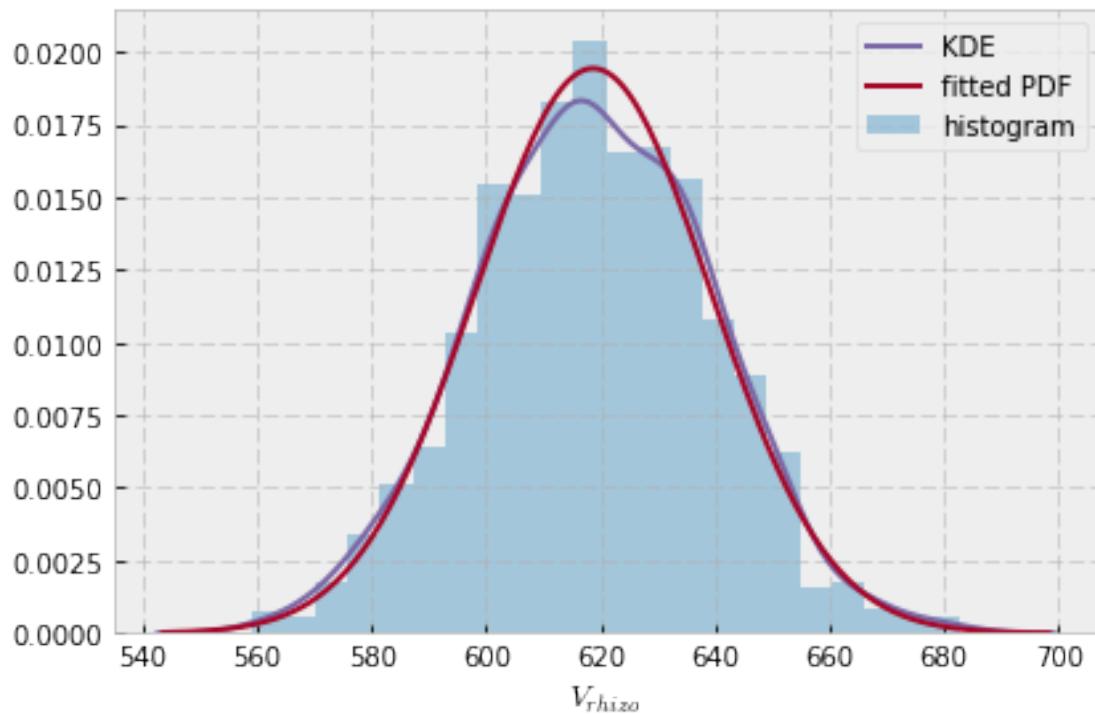


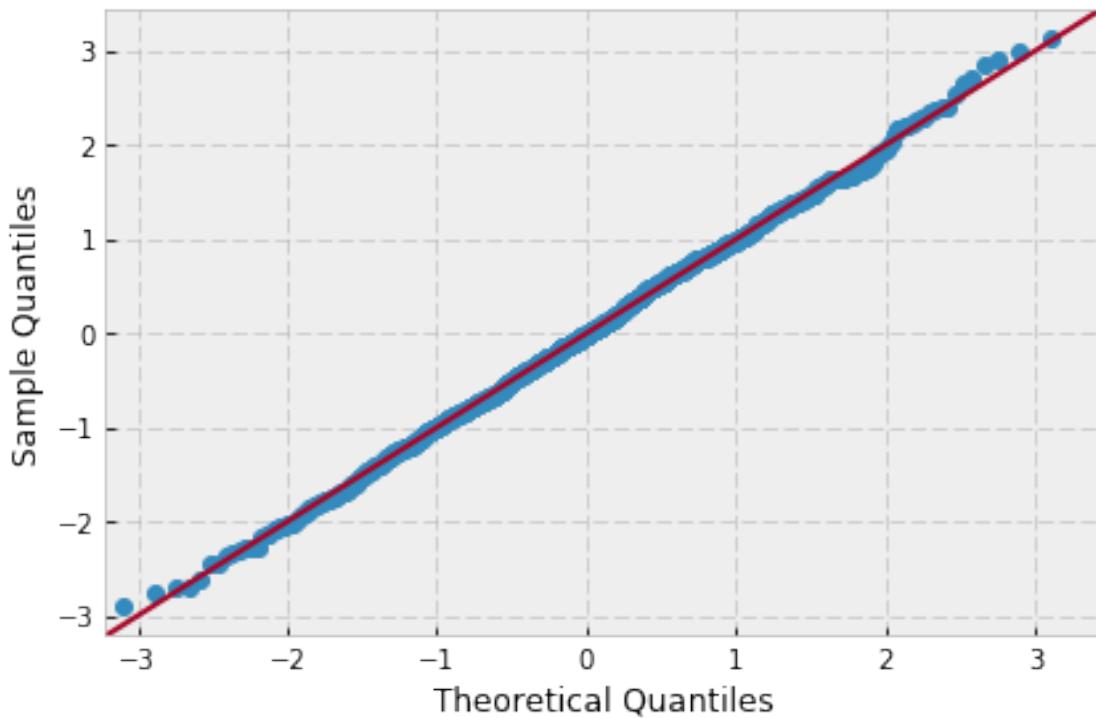


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=663.5243731438107, scale=21.569489612170766)
DescribeResult(nobs=1000, minmax=(599.54382555309894, 733.21312480490565)
               mean=663.52437314381075, variance=465.70859072026292,
               skewness=-0.044737882403939576, kurtosis=-0.16553682249537482)
```

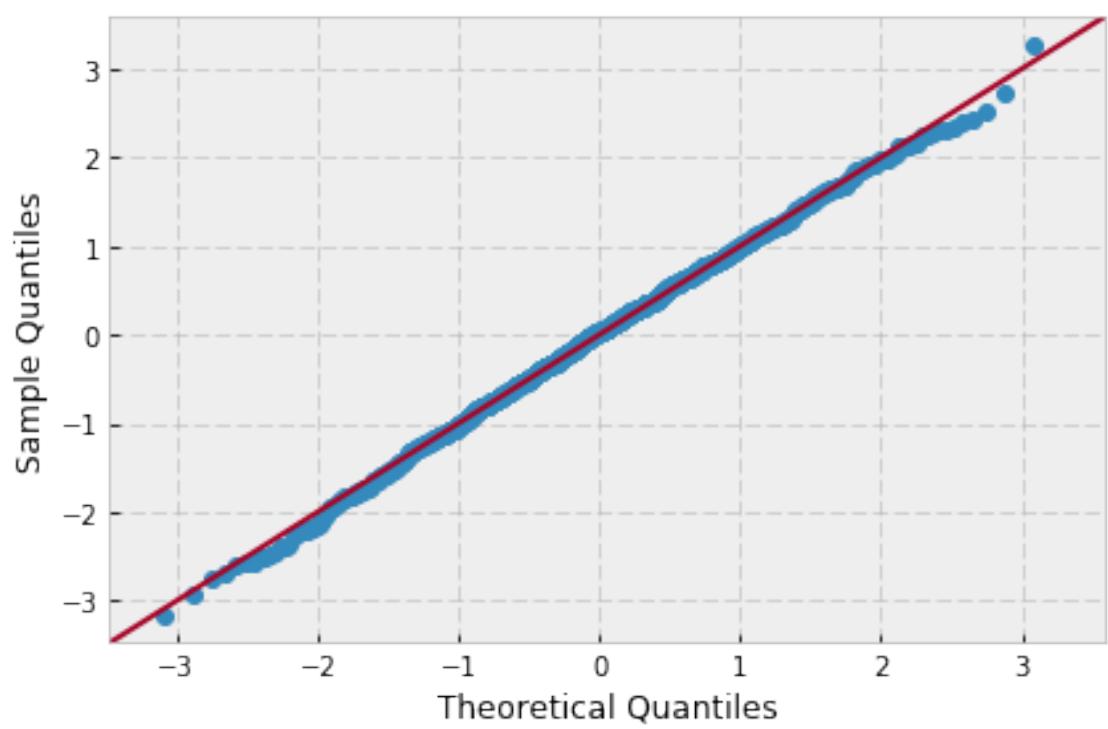
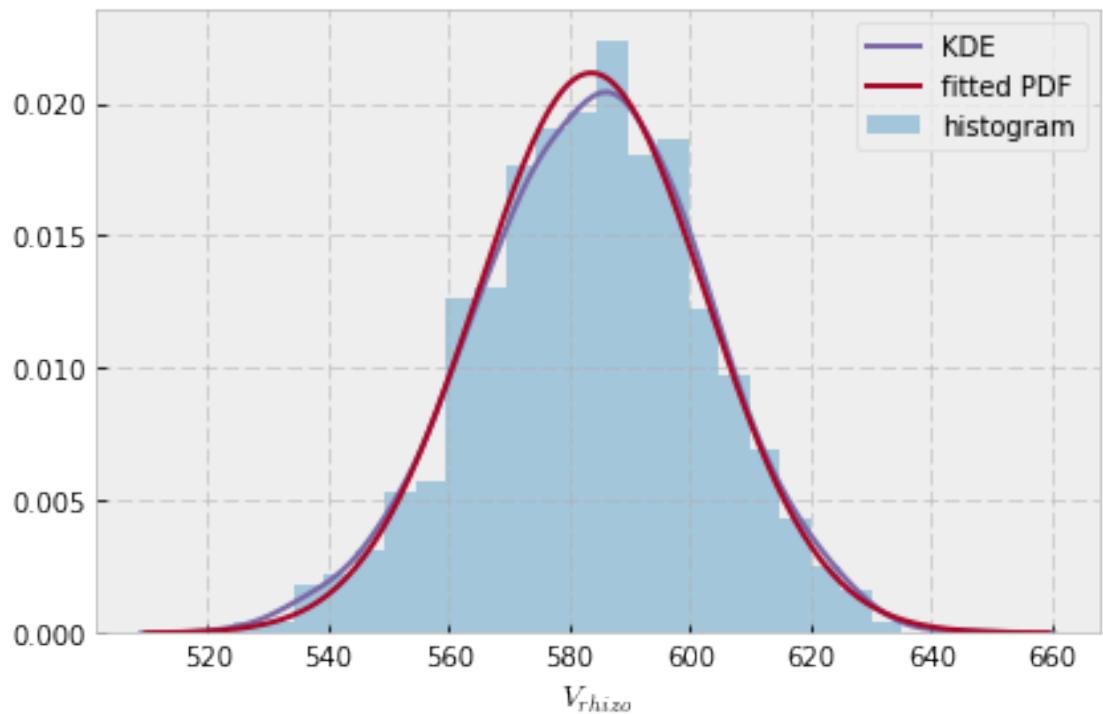


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=618.7014398691354, scale=20.52914960483281)
DescribeResult(nobs=1000, minmax=(558.99778236435952, 682.89014232025681)
               mean=618.70143986913536, variance=421.86785134895615,
               skewness=0.004077681134218601, kurtosis=-0.11604790909890594)
```

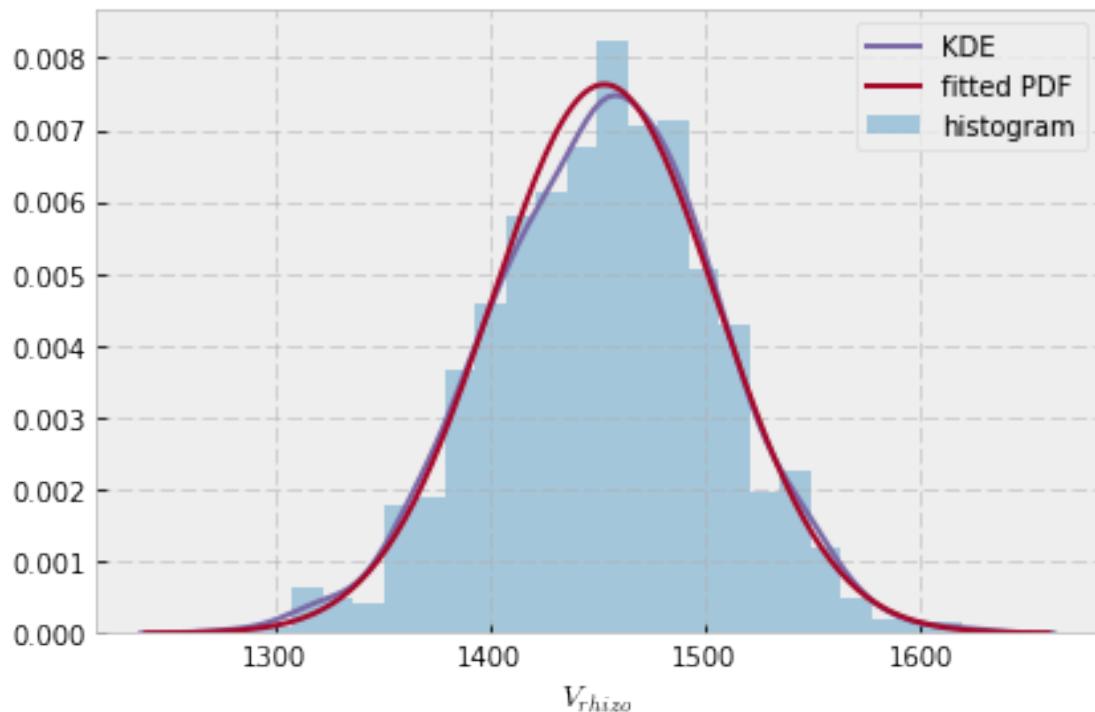


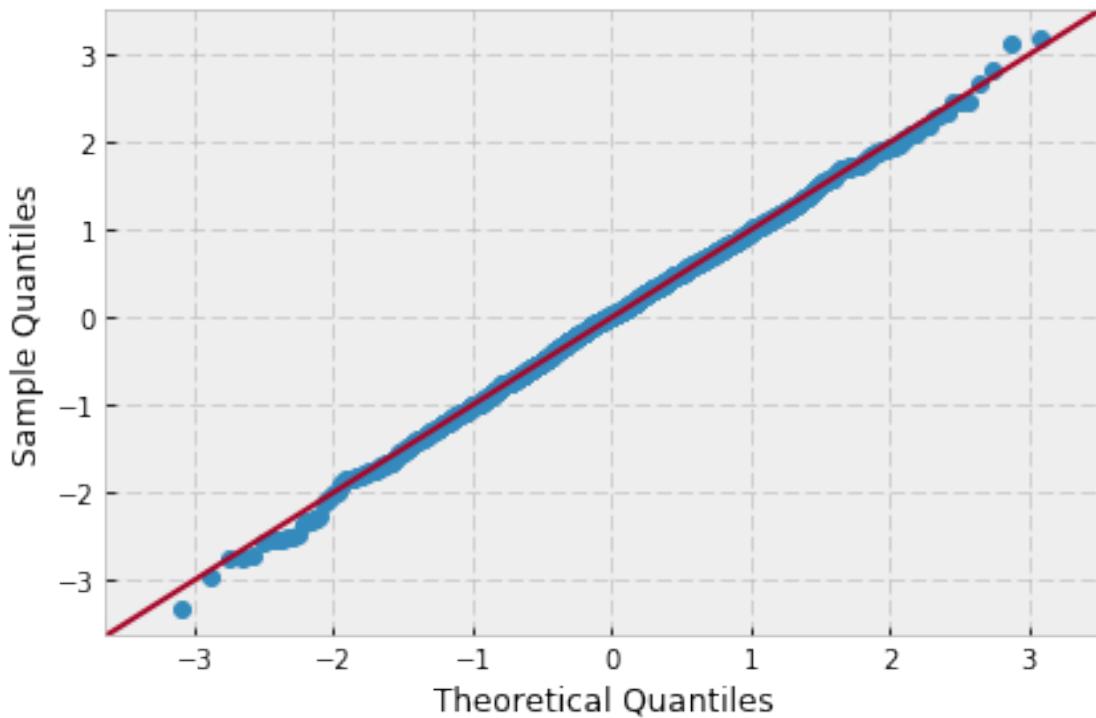


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=583.6085688254342, scale=18.868468503559978)
DescribeResult(nobs=1000, minmax=(524.0977992400974, 645.17288604673274)
               mean=583.60856882543419, variance=356.37547914898391,
               skewness=-0.10448371797412752, kurtosis=-0.08800487880415675)
```

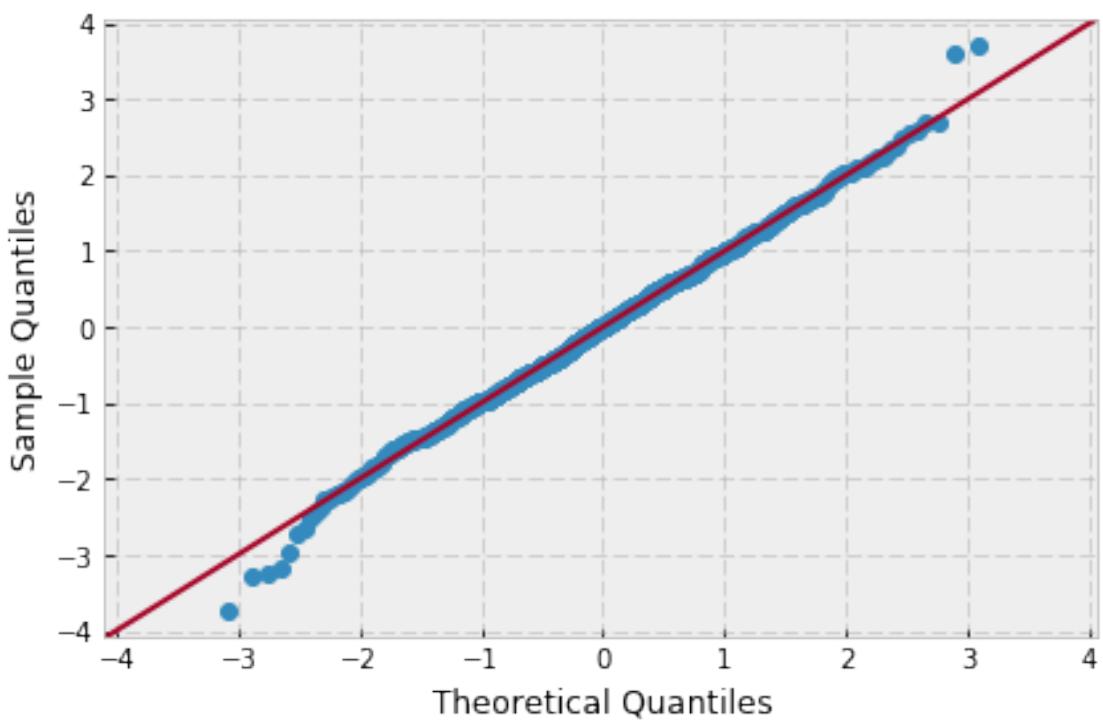
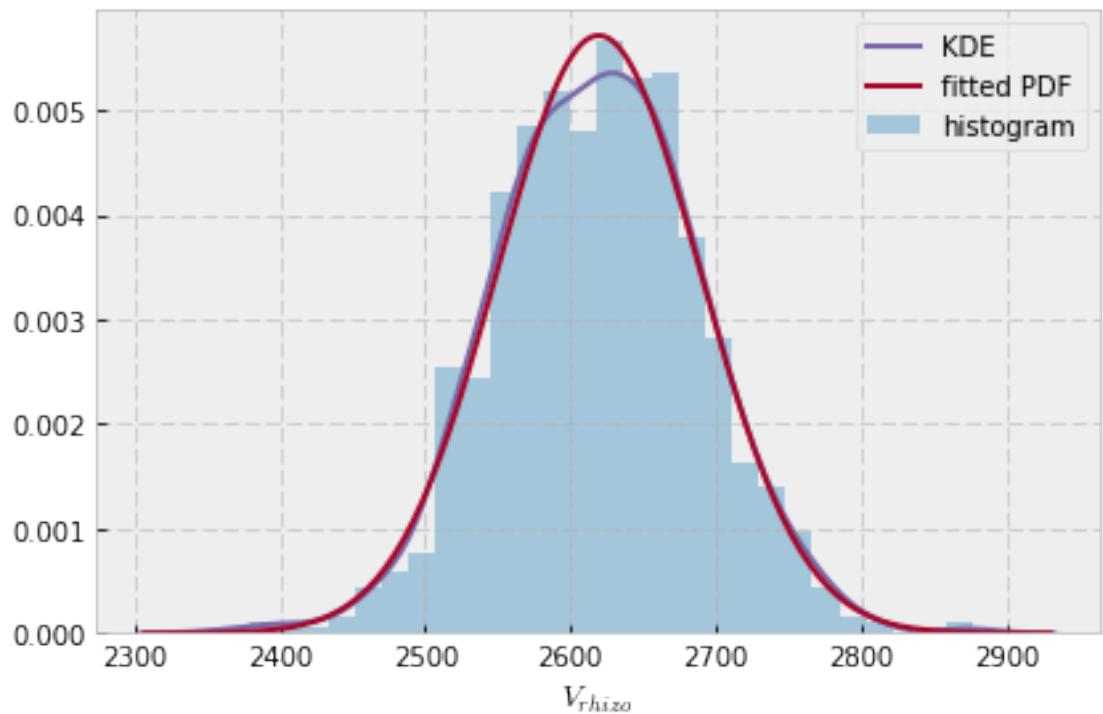


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(), loc=1453.2000252399907, scale=52.291292775562155)
DescribeResult(nobs=1000, minmax=(1279.7861901139552, 1620.0927712190121)
               mean=1453.2000252399907, variance=2737.1164165561149,
               skewness=-0.07494477495340478, kurtosis=-0.0020462324231185214)
```

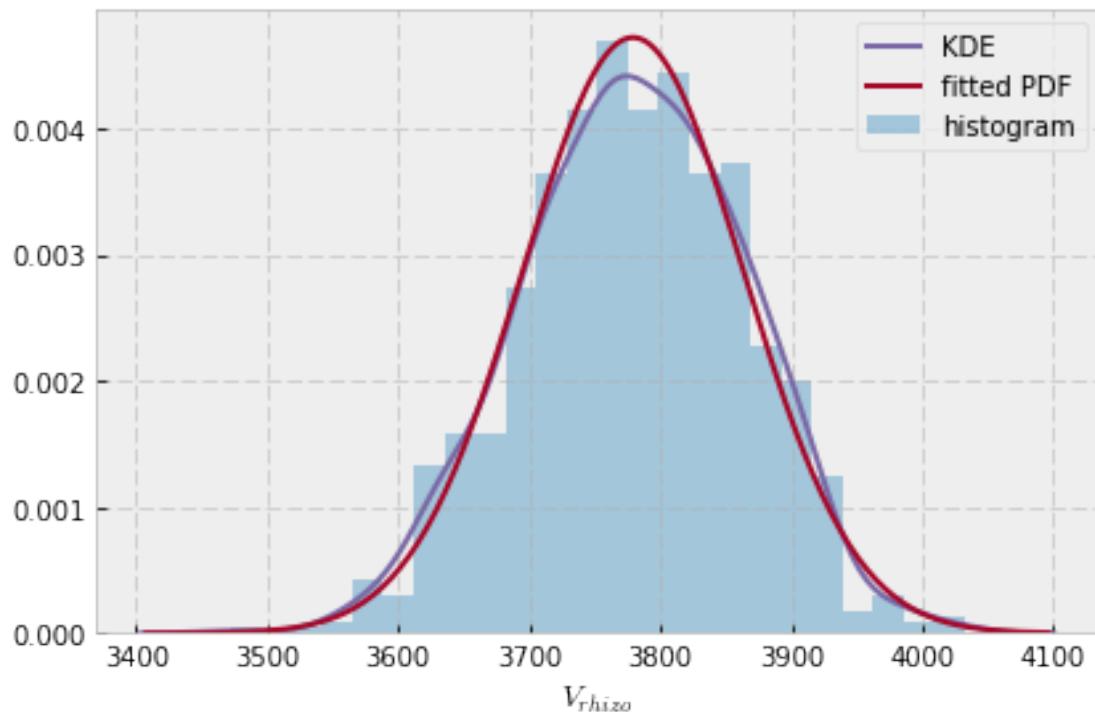


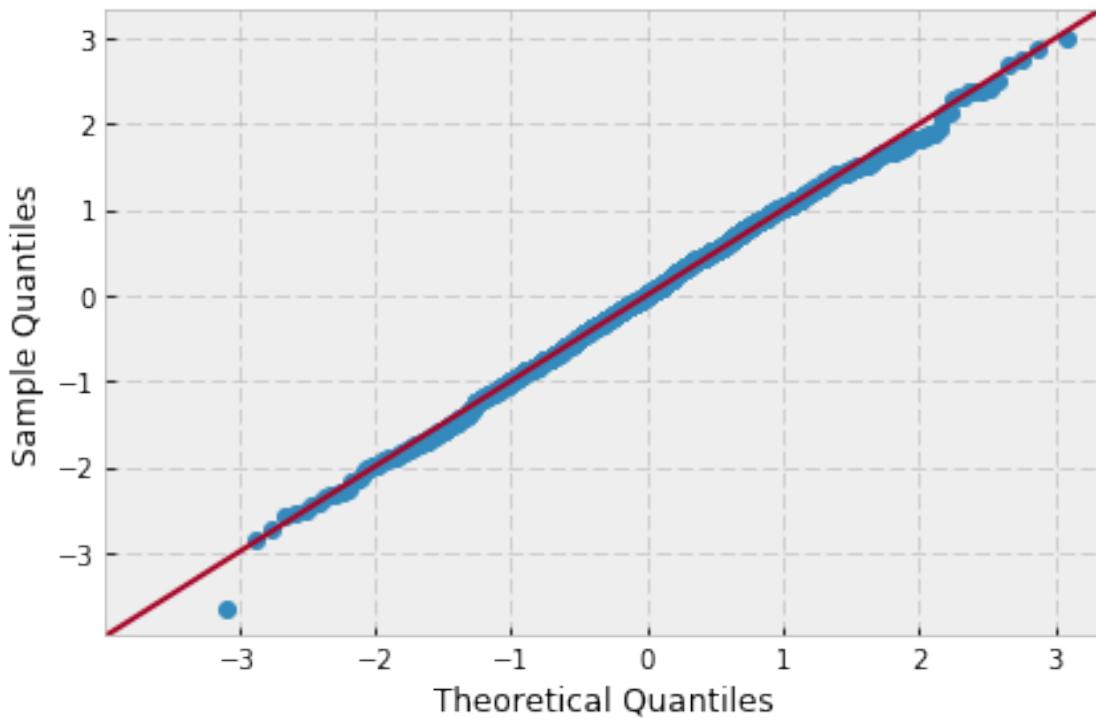


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(),loc=2619.3161444390225, scale=69.83402074958904)
DescribeResult(nobs=1000, minmax=(2359.2462103718108, 2877.5493371573675)
               mean=2619.3161444390225, variance=4881.6721261802149,
               skewness=-0.01806337145342376, kurtosis=0.27764515788961663)
```

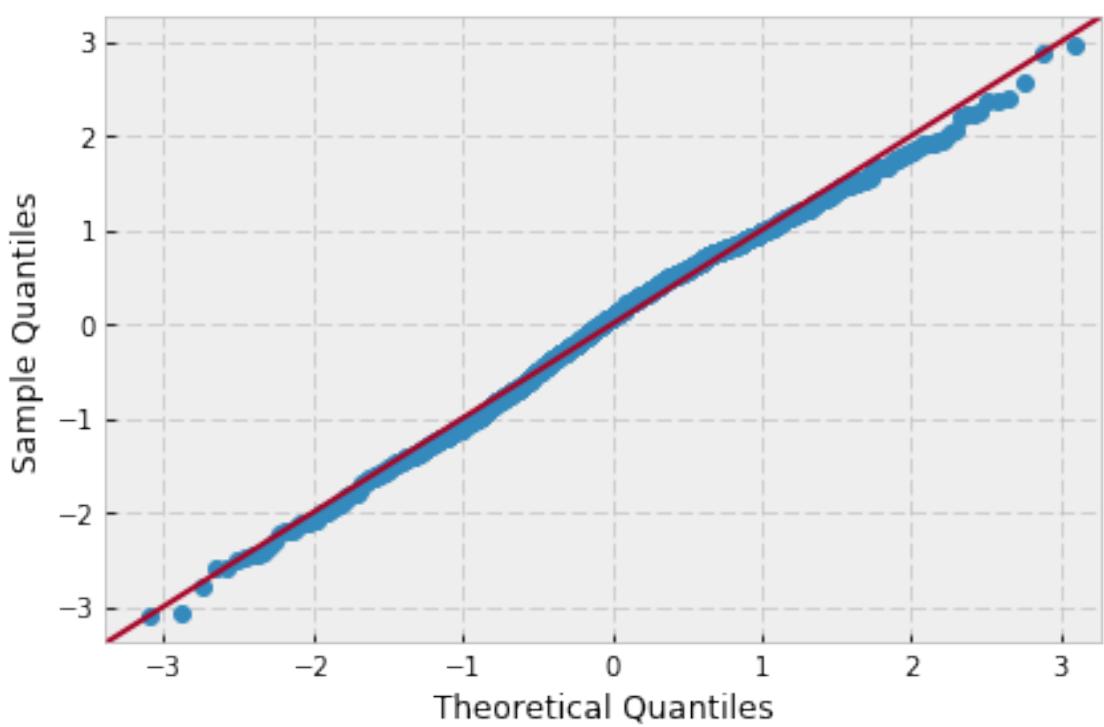
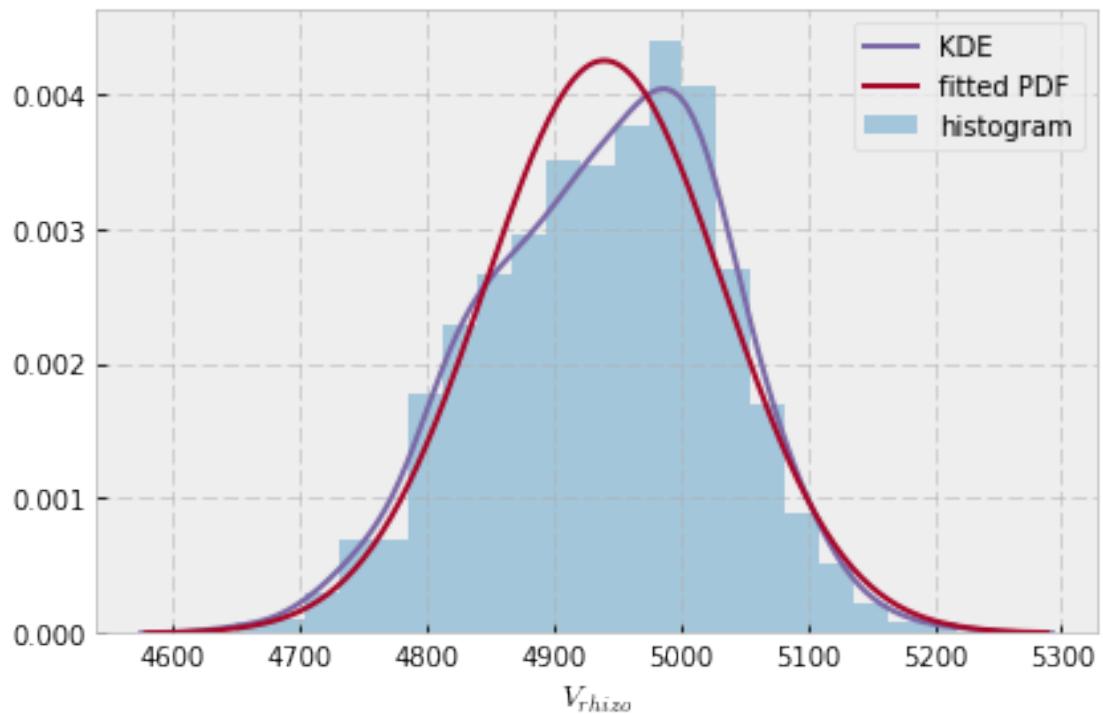


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(), loc=3779.025635818468, scale=84.47973176434652)
DescribeResult(nobs=1000, minmax=(3471.2889210383782, 4032.4431490596708)
               mean=3779.0256358184679, variance=7143.9690480239624,
               skewness=-0.10144830352750757, kurtosis=-0.1530162622386988)
```

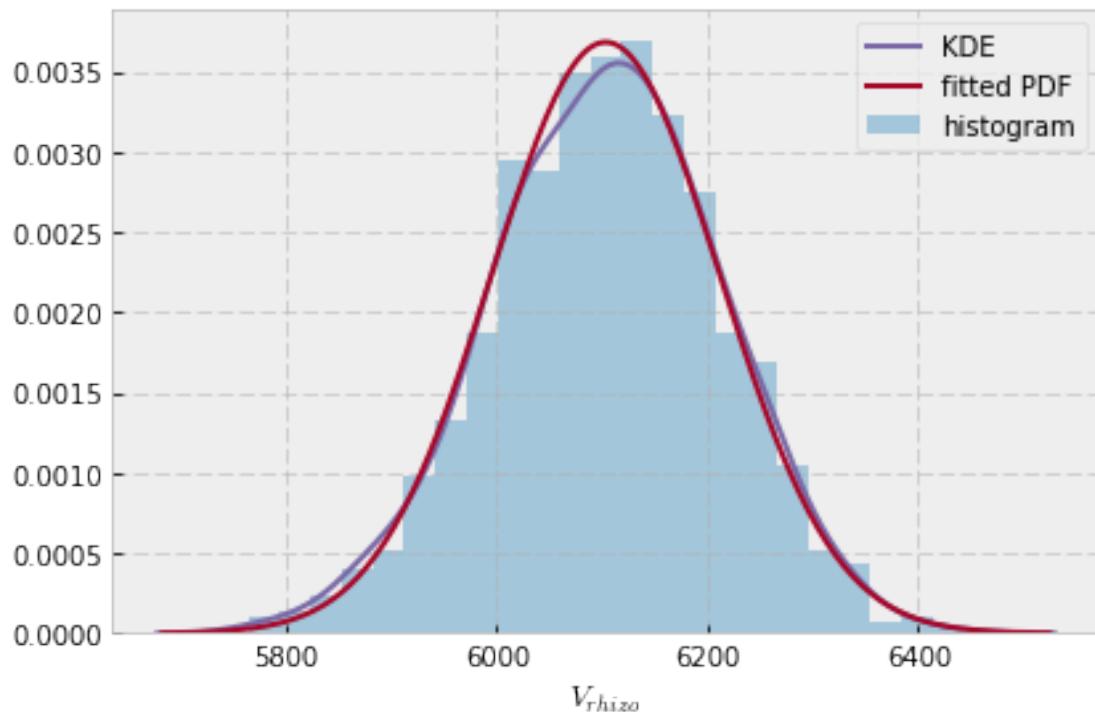


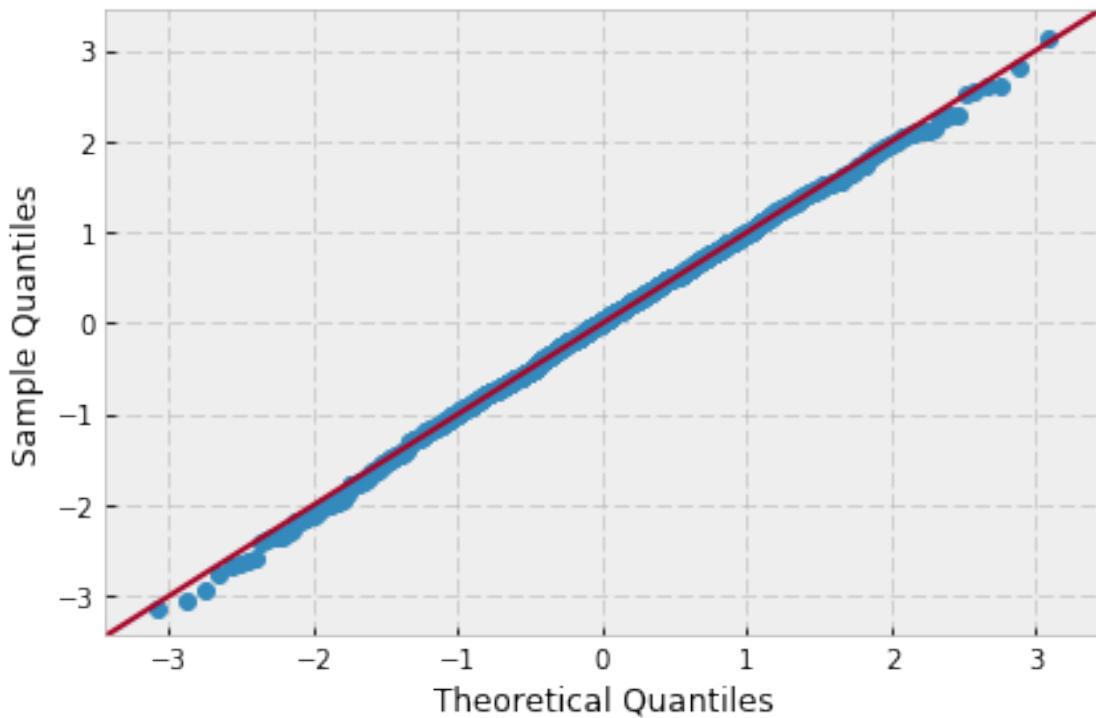


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(),loc=4939.86810783088, scale=93.8759518209005)
DescribeResult(nobs=1000, minmax=(4650.2534185531113, 5217.6119452653566)
               mean=4939.86810783088, variance=8821.5158461261562,
               skewness=-0.20408375480087557, kurtosis=-0.3066819118551116)
```

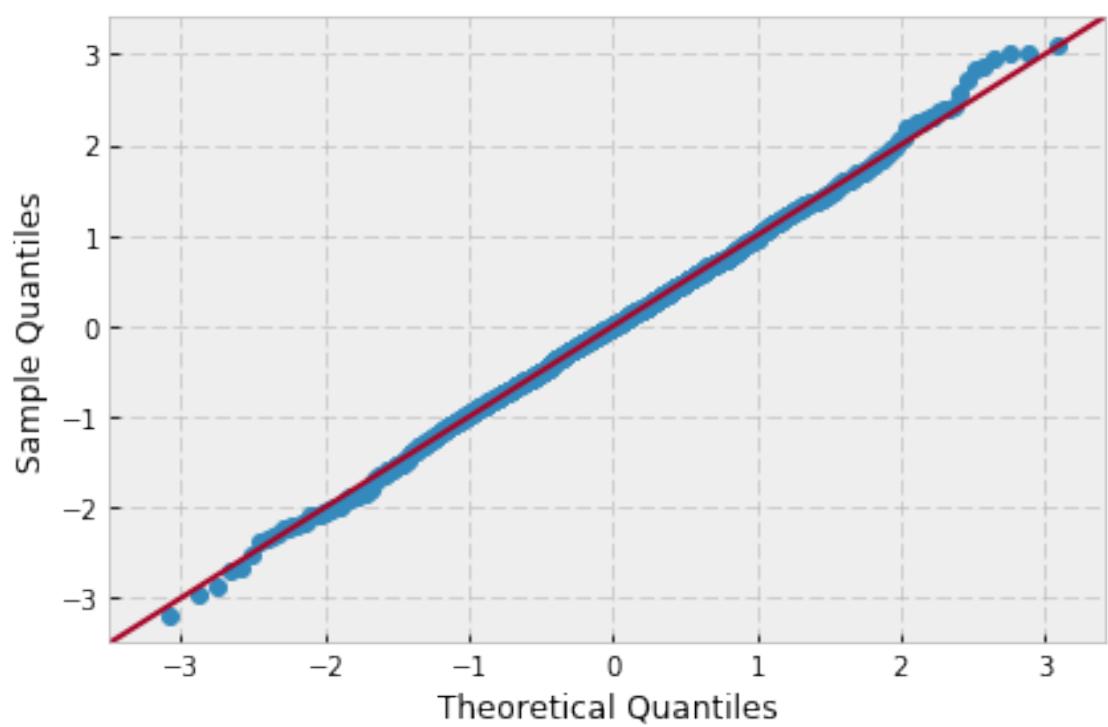
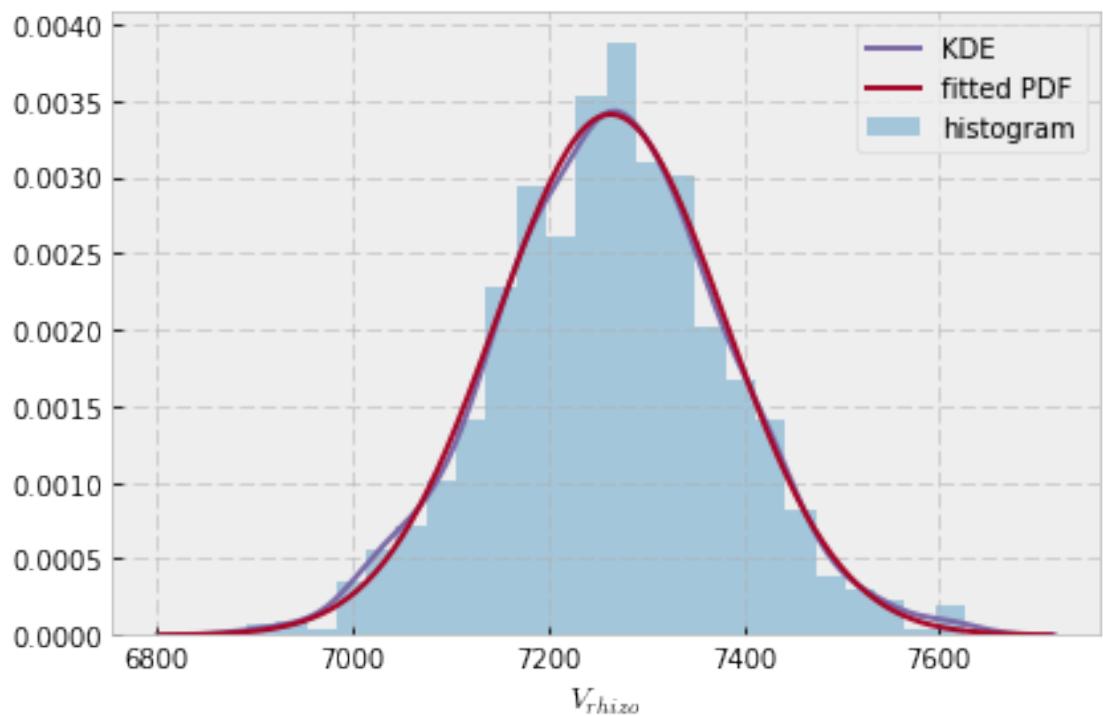


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(), loc=6103.204632546824, scale=108.33474421442249)
DescribeResult(nobs=1000, minmax=(5764.3417117346589, 6442.2836692424999)
               mean=6103.2046325468236, variance=11748.164968973319,
               skewness=-0.10615545566631834, kurtosis=-0.05271192226347354)
```

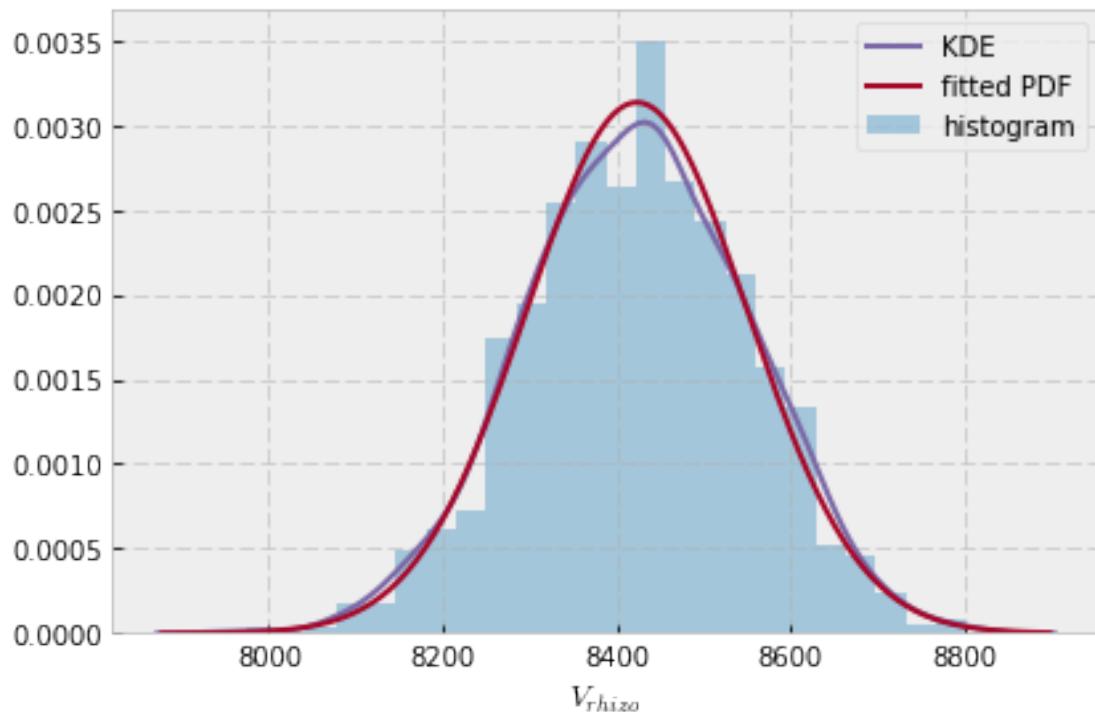


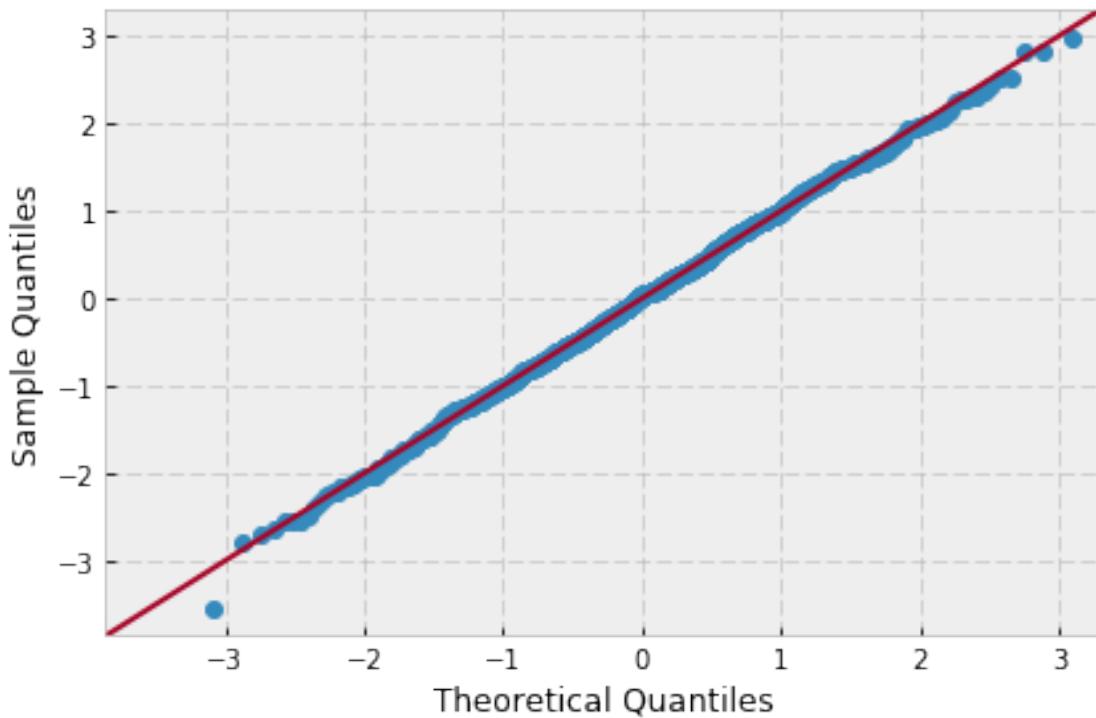


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(),loc=7263.977198564112, scale=116.9420867009029)
DescribeResult(nobs=1000, minmax=(6891.6594080339719, 7626.3200068976248)
               mean=7263.9771985641119, variance=13689.140782744236,
               skewness=0.022920329721936857, kurtosis=0.10608726887044417)
```

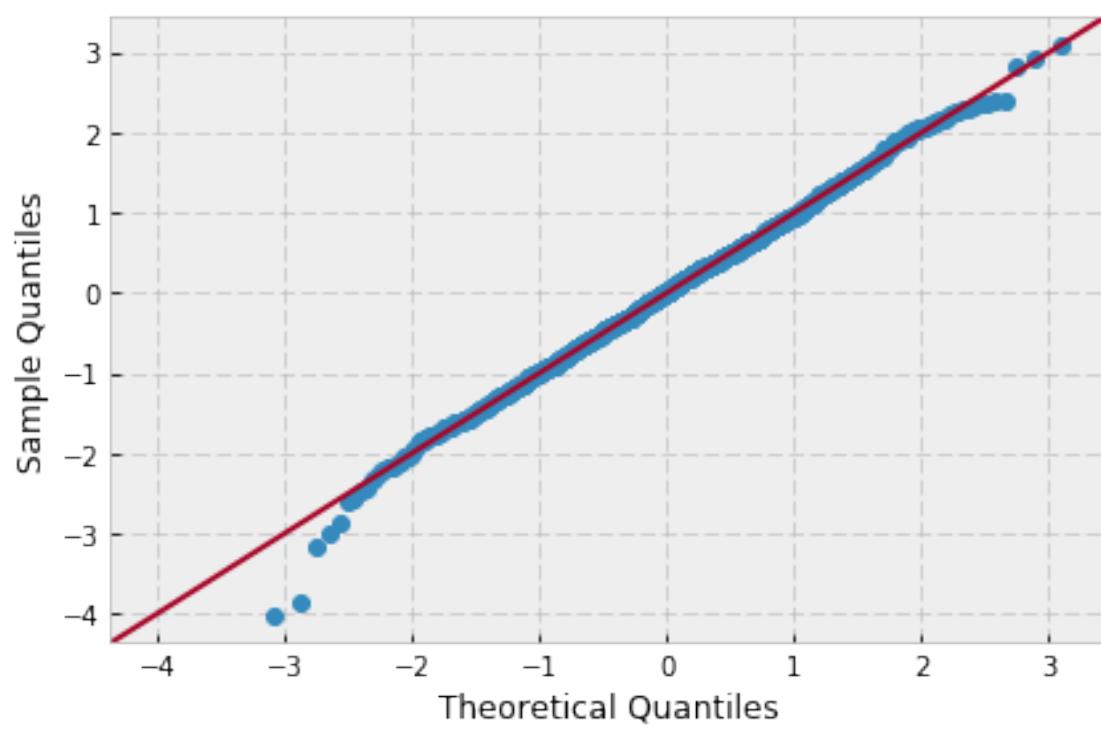
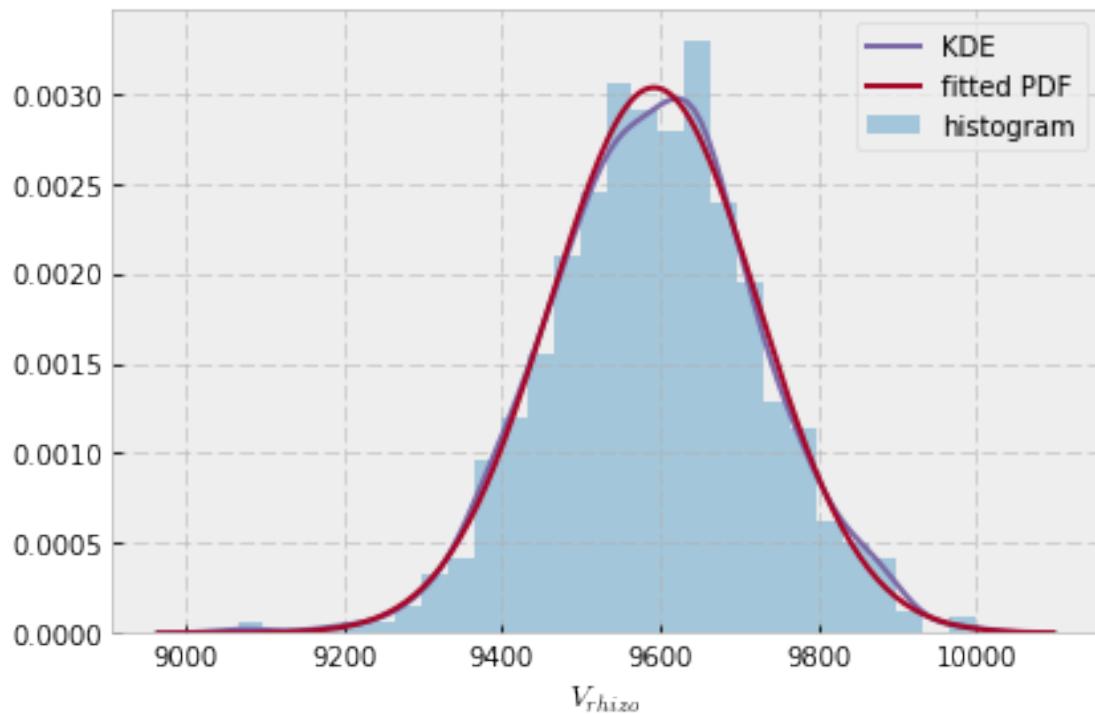


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(), loc=8423.317702499939, scale=127.276689179703)
DescribeResult(nobs=1000, minmax=(7973.2233053348828, 8801.7943204923358)
               mean=8423.3177024999386, variance=16215.571179726452,
               skewness=-0.044137073625663686, kurtosis=-0.1318161752535727)
```

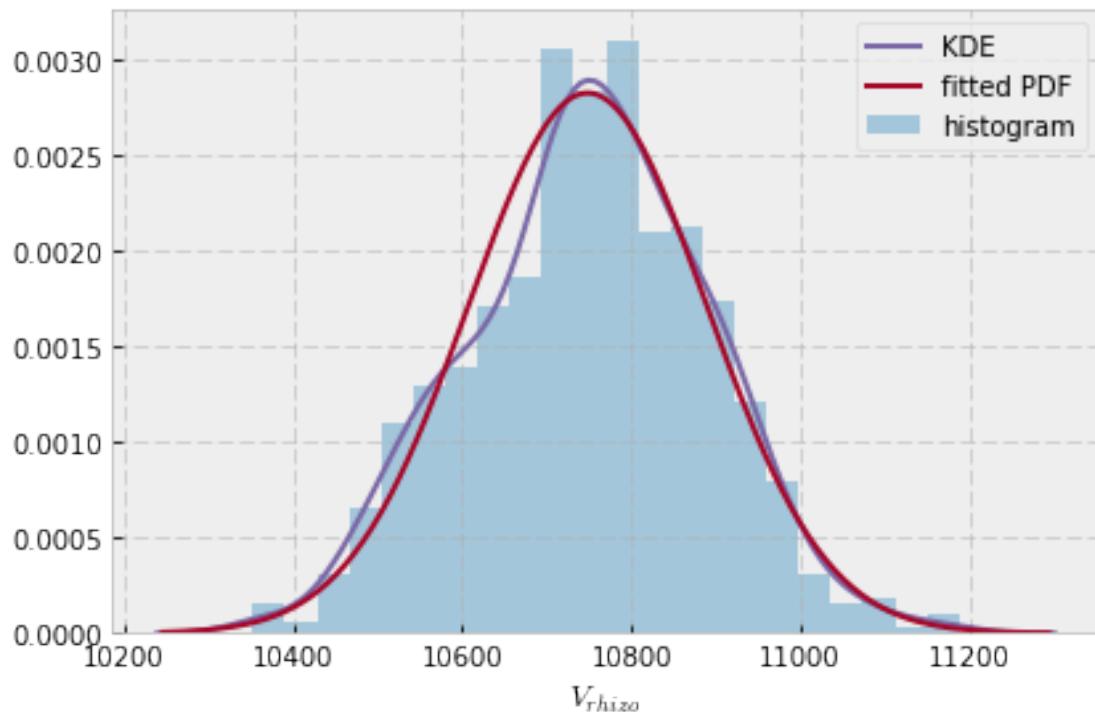


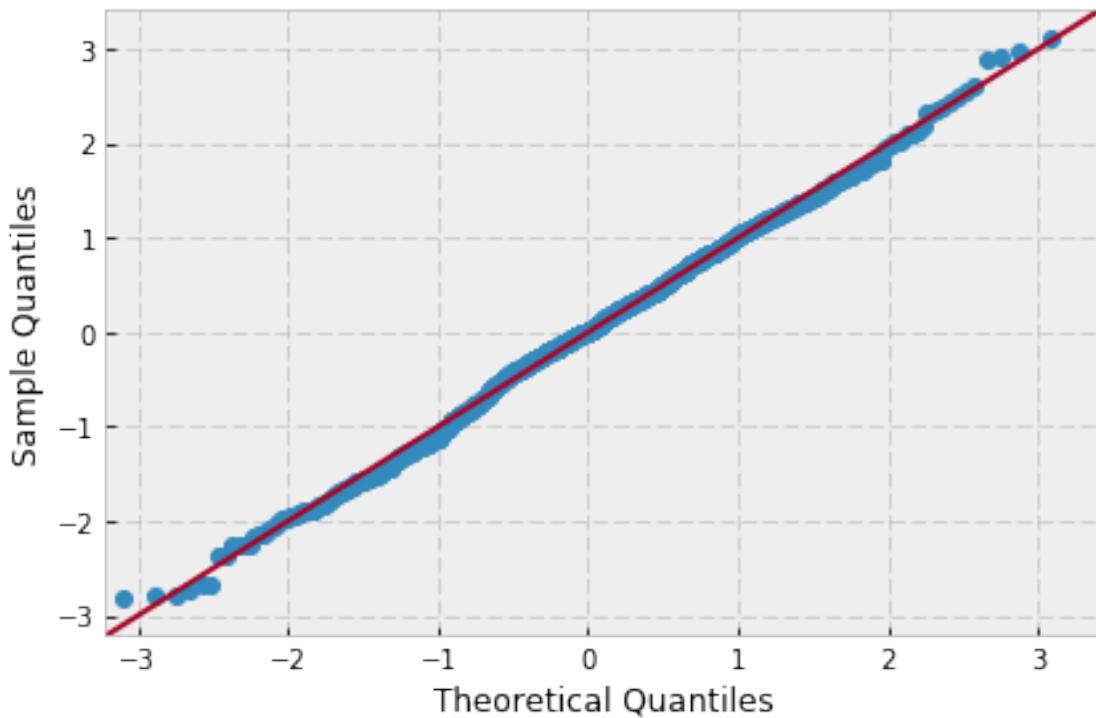


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(),loc=9592.481543127351, scale=131.51341247791228)
DescribeResult(nobs=1000, minmax=(9064.6013404512687, 9998.103298443013)
               mean=9592.481543127351, variance=17313.090752337834,
               skewness=-0.08190263385696116, kurtosis=0.24929818807648418)
```

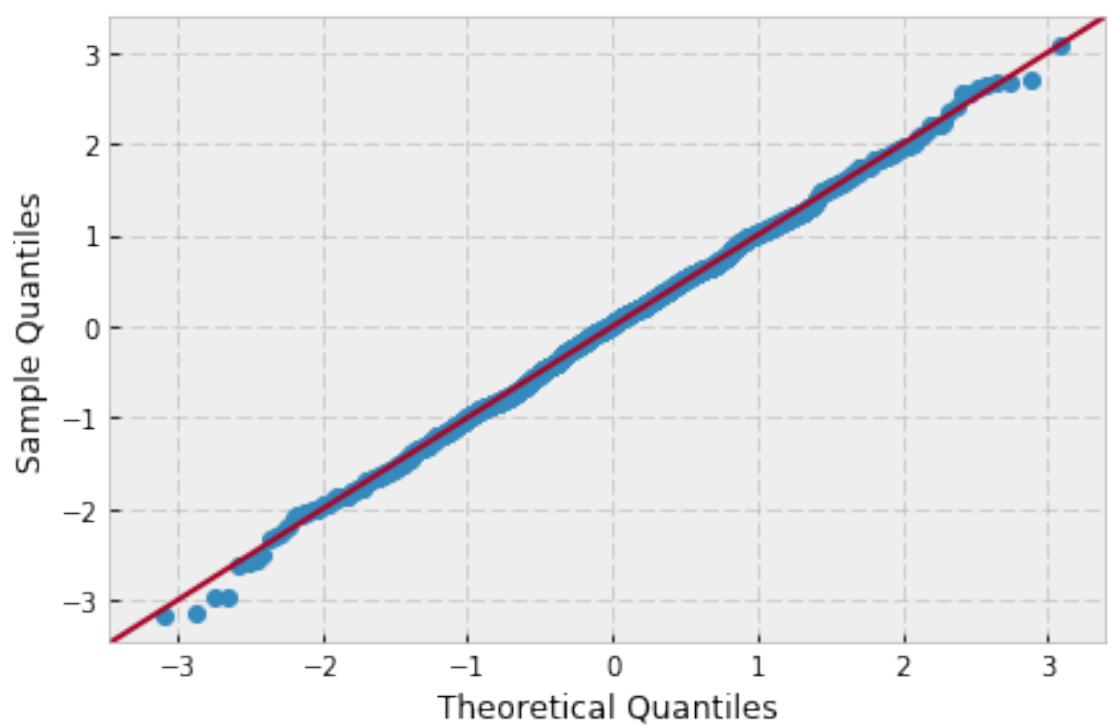
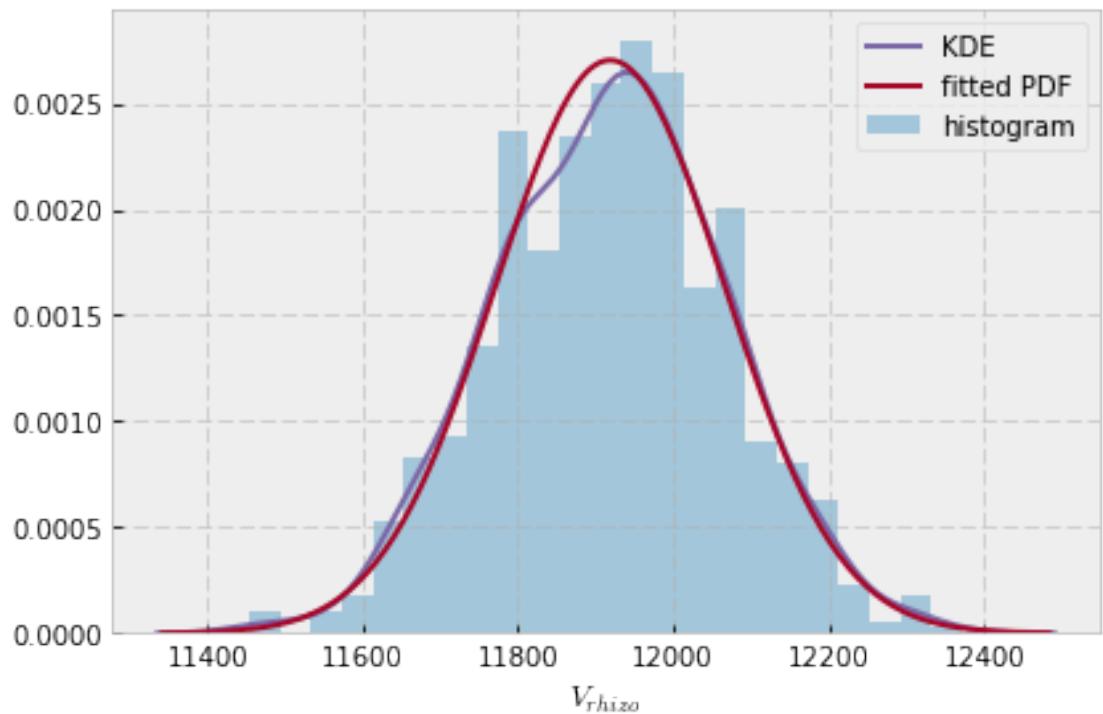


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(), loc=10748.373674957764, scale=141.12787398039495)
DescribeResult(nobs=1000, minmax=(10351.995344607487, 11187.862268502406)
               mean=10748.373674957764, variance=19937.0138280543,
               skewness=-0.06626703510759054, kurtosis=-0.1394440396658574)
```

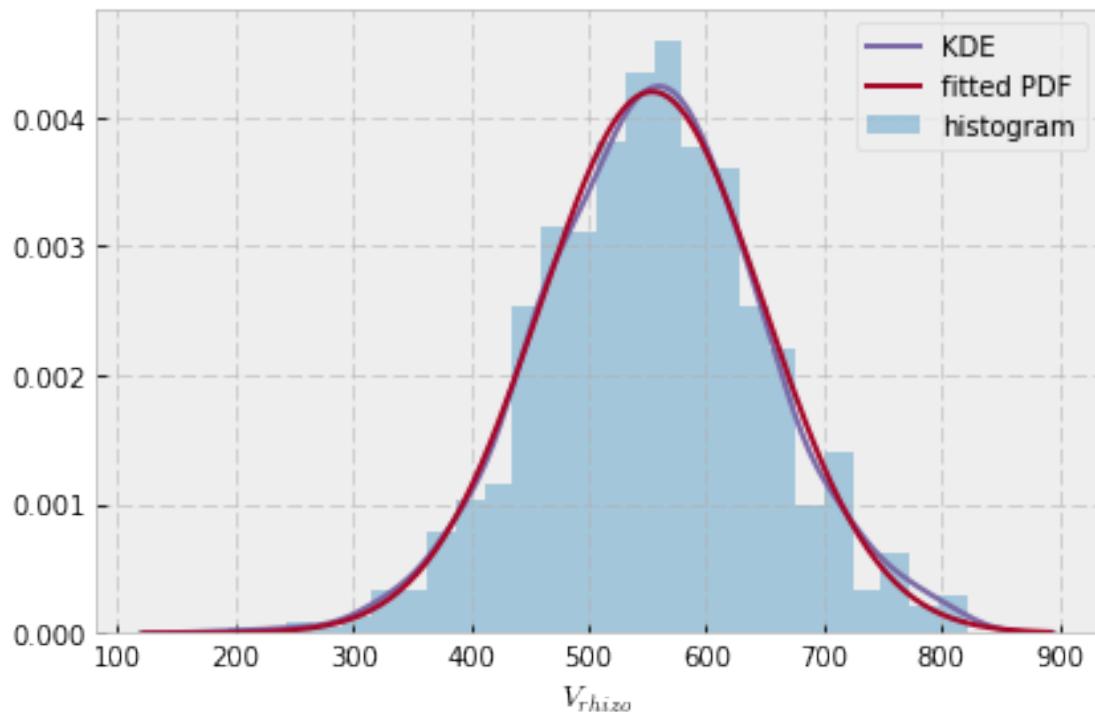


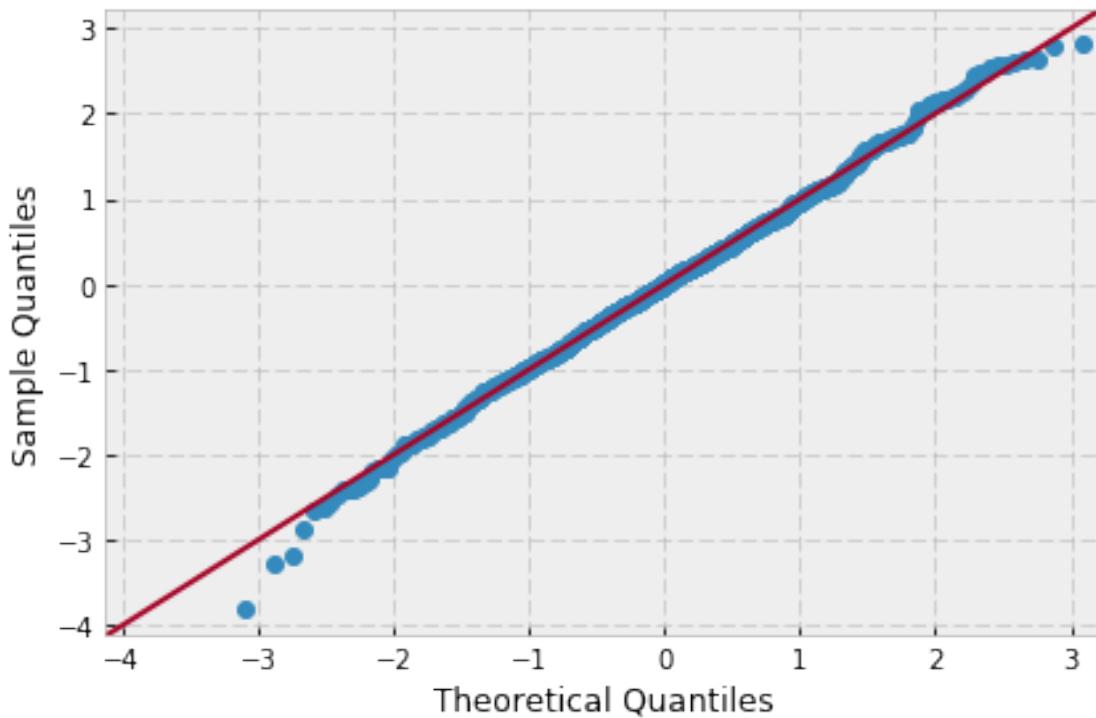


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(),loc=11918.500080630009, scale=147.51511086240794)
DescribeResult(nobs=1000, minmax=(11453.104213308425, 12372.691625170364)
               mean=11918.500080630009, variance=21782.490423171676,
               skewness=-0.039993092148023296, kurtosis=-0.08976101321260277)
```

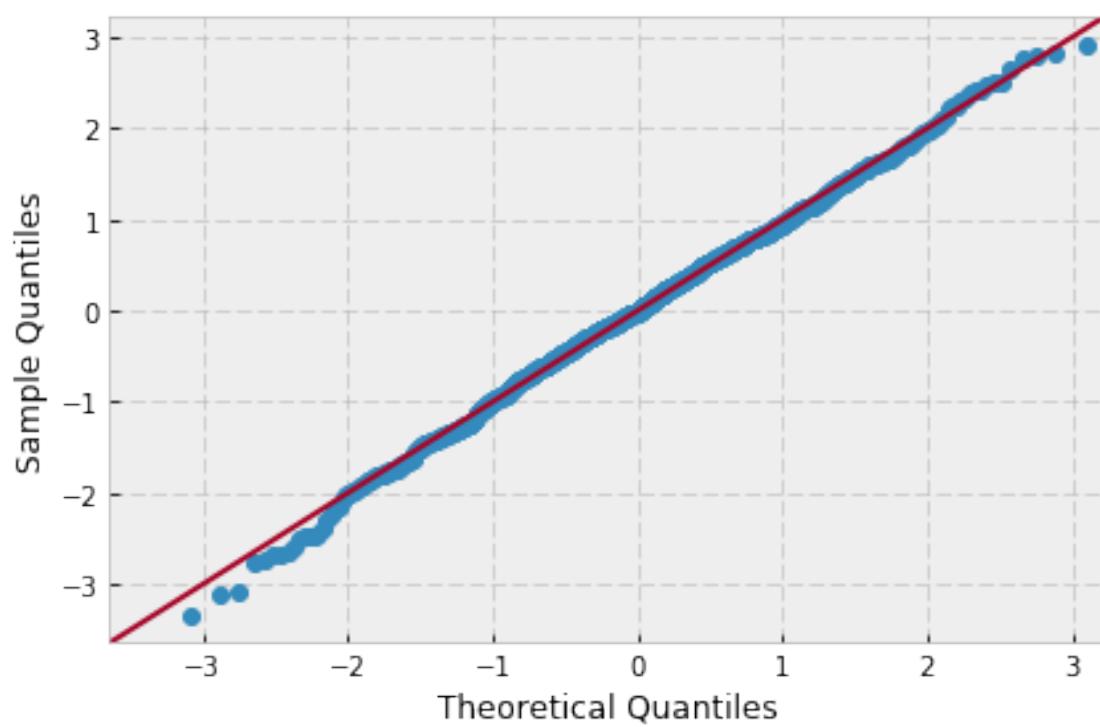
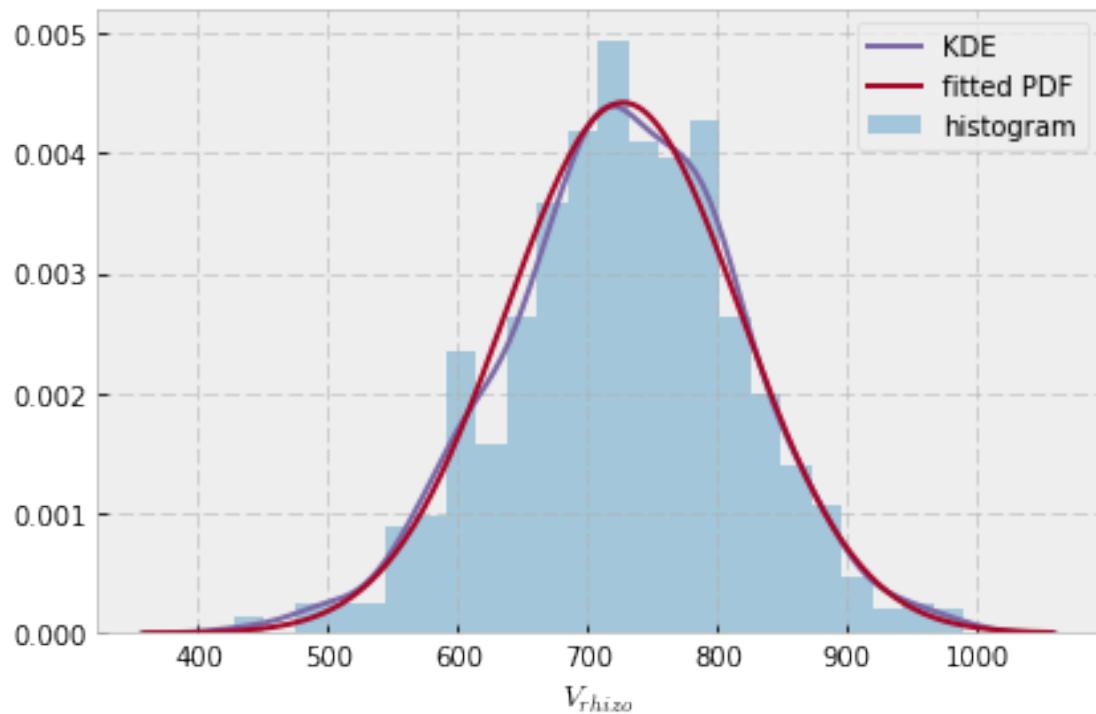


```
variable name: r
variable value: 1.0
distribution: normal(shape=(),loc=554.0695184939487, scale=94.98980447552621)
DescribeResult(nobs=1000, minmax=(194.12243994791757, 821.08722486193494)
               mean=554.06951849394875, variance=9032.0950493480486,
               skewness=-0.037257818690103646, kurtosis=0.20132820932186002)
```

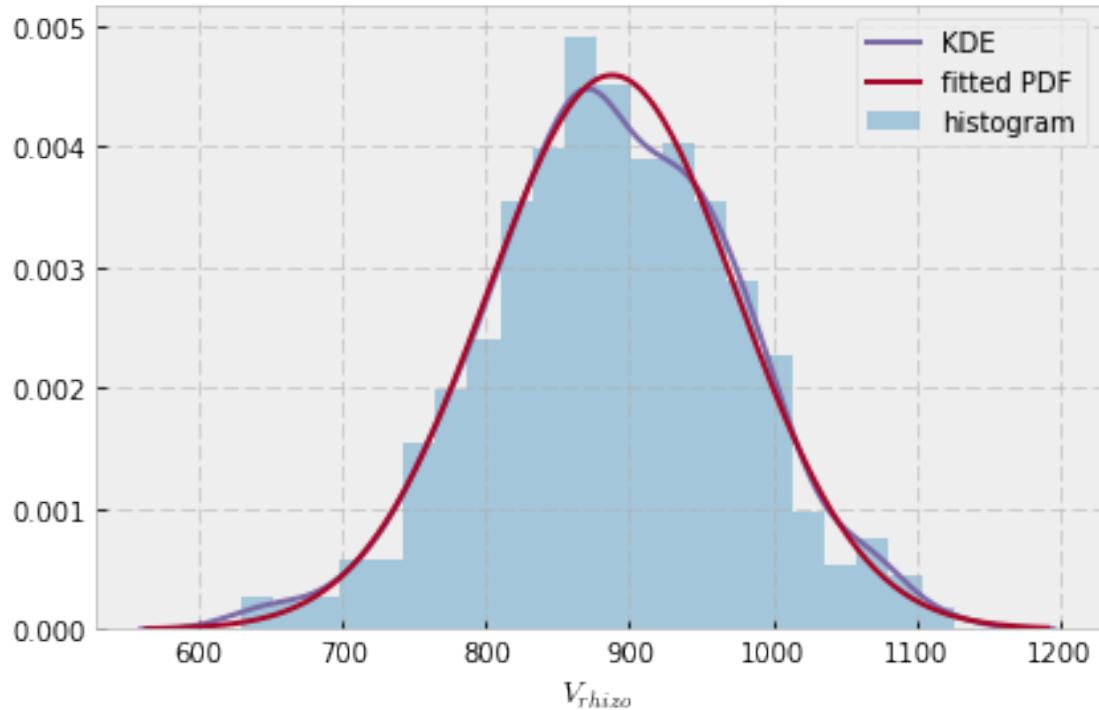


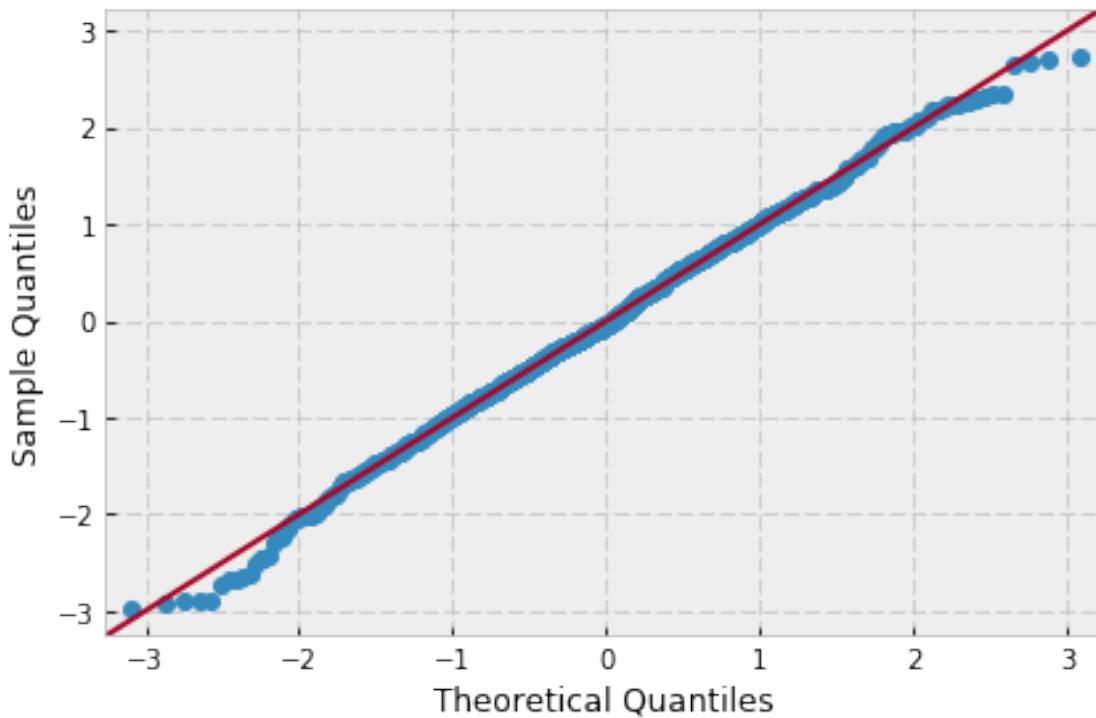


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(),loc=727.1782391383954, scale=90.22201733567485)
DescribeResult(nobs=1000, minmax=(426.70468346696657, 989.52769108779614)
               mean=727.17823913839538, variance=8148.160572691505,
               skewness=-0.11763545118187402, kurtosis=0.09436765762173183)
```

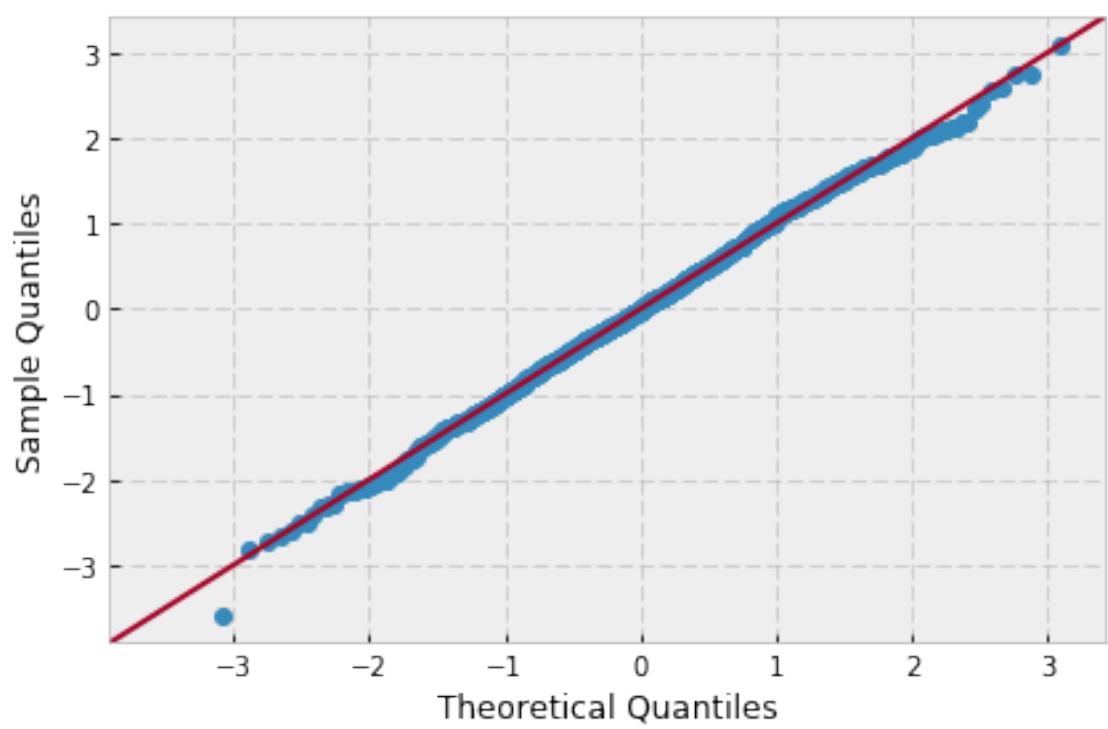
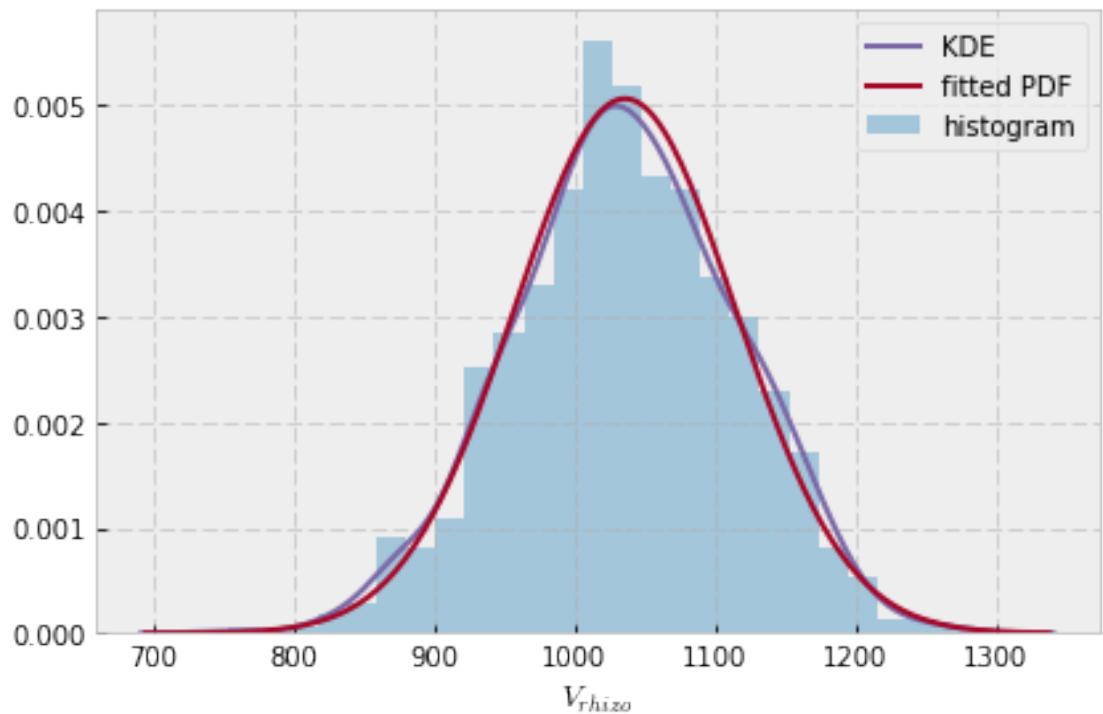


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(),loc=887.8635223277263, scale=86.92725355544732)
DescribeResult(nobs=1000, minmax=(629.26180817424824, 1125.9216764474302)
               mean=887.86352232772629, variance=7563.9113220150439,
               skewness=-0.08002316912995791, kurtosis=-0.035673105615332545)
```

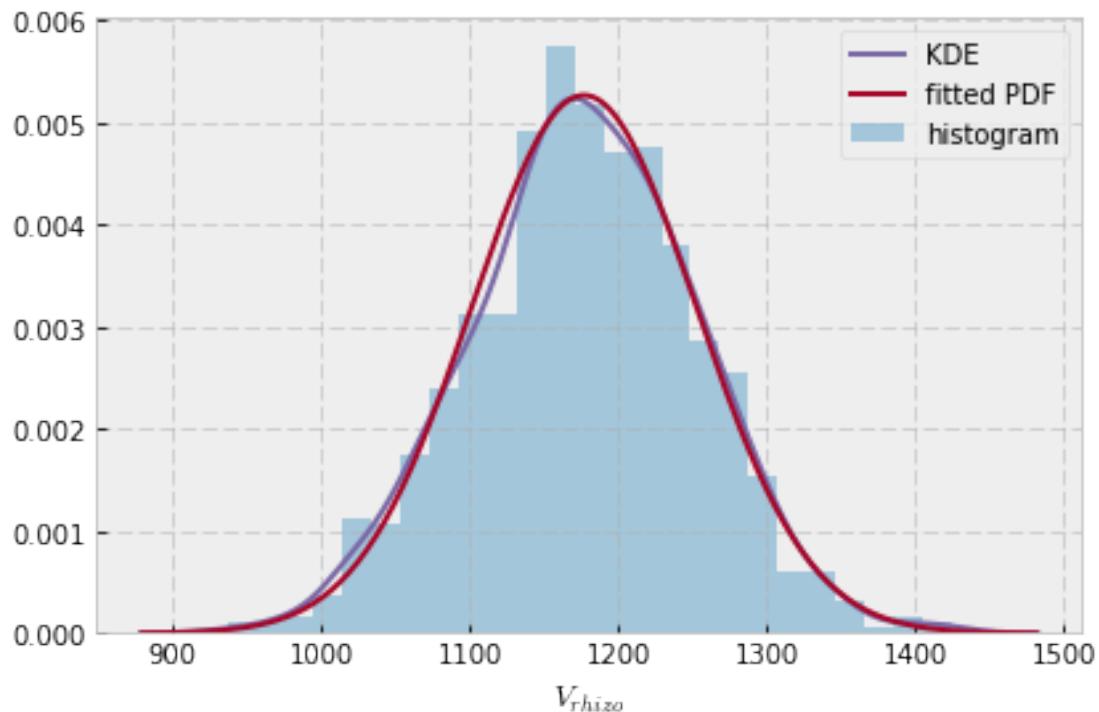


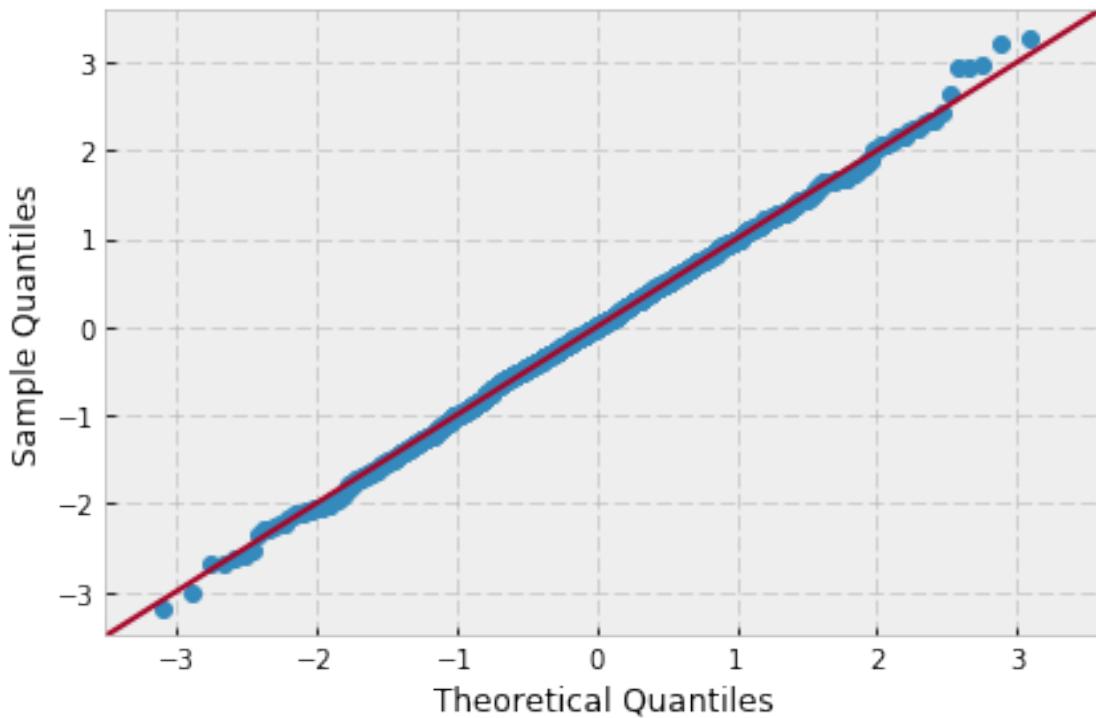


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=1035.7385124384205, scale=78.73259094504611)
DescribeResult(nobs=1000, minmax=(753.92546358301684, 1278.2784393481159)
               mean=1035.7385124384205, variance=6205.0259028227792,
               skewness=-0.06943449567652406, kurtosis=-0.13960000116426663)
```

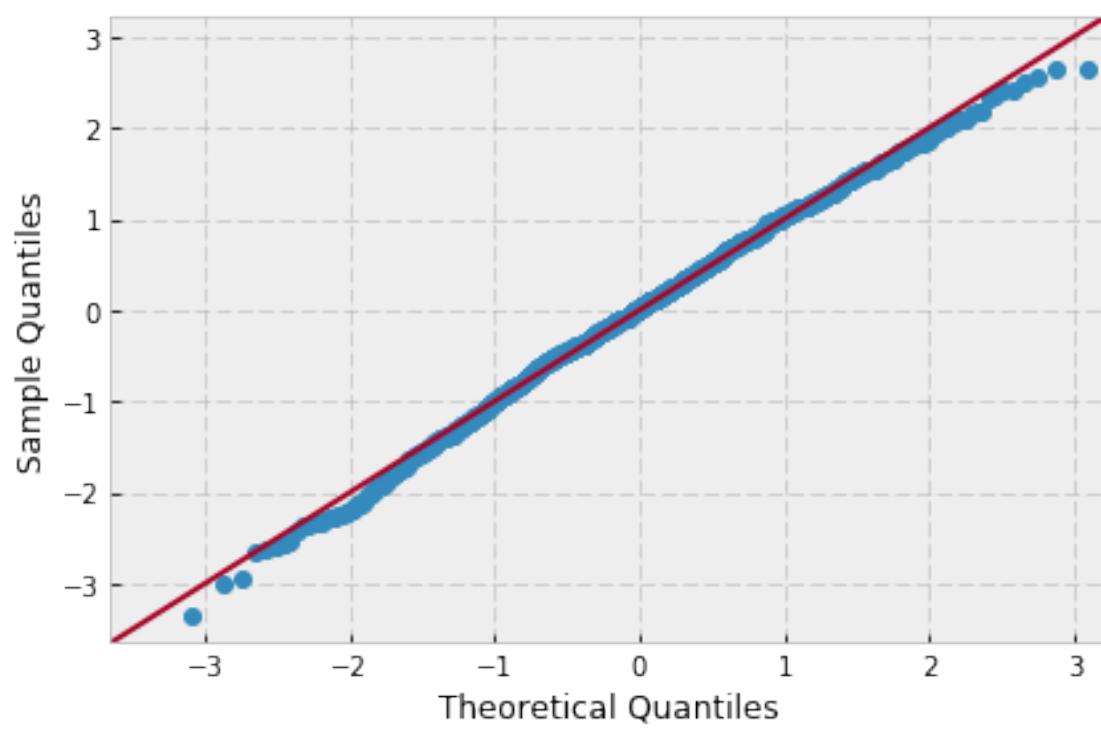
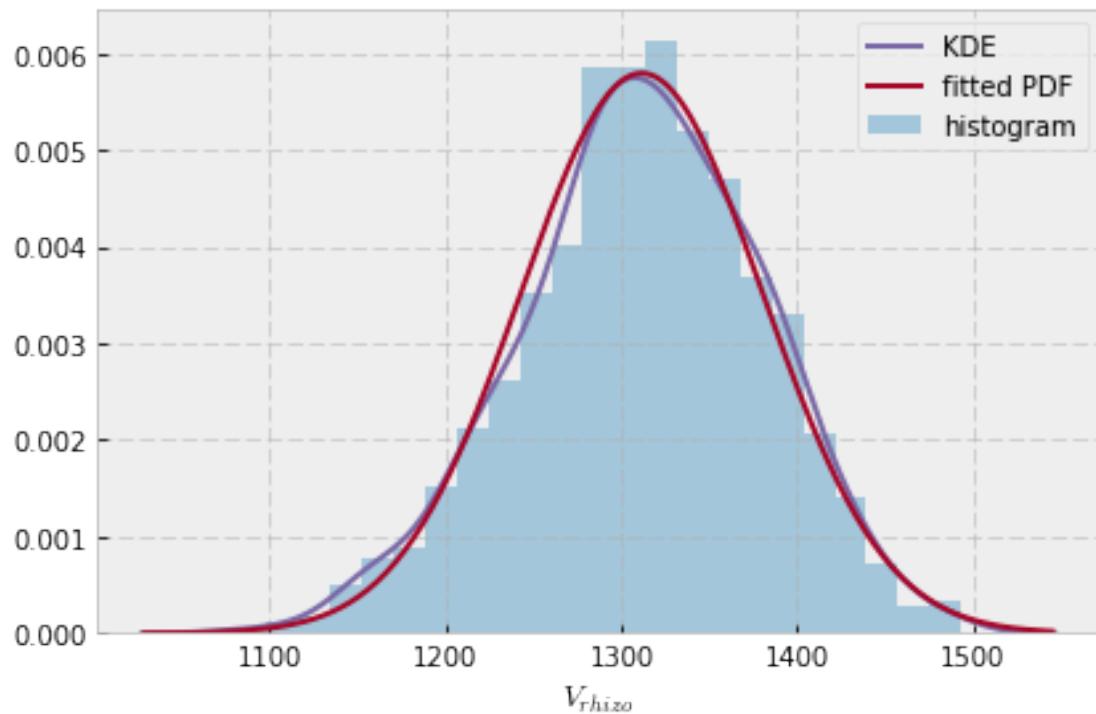


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=1176.945623231736, scale=75.69637548997625)
DescribeResult(nobs=1000, minmax=(936.5618226403418, 1424.2304460941016)
               mean=1176.9456232317359, variance=5735.6769392587339,
               skewness=-0.02425823983324145, kurtosis=0.020030255057090773)
```

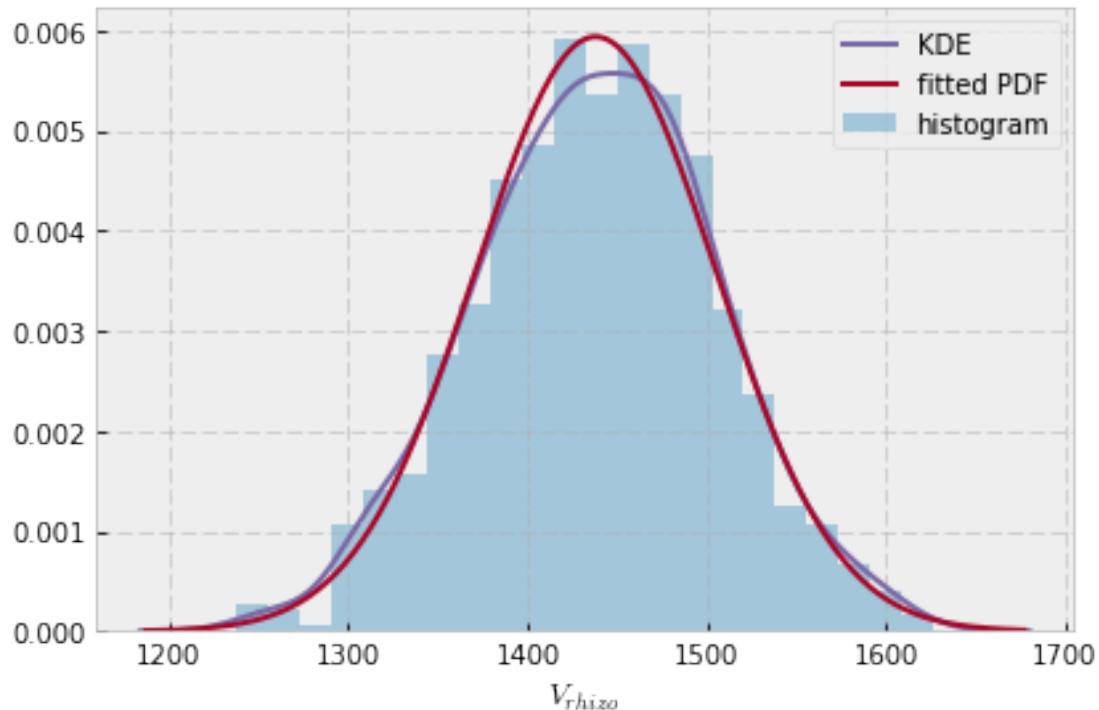


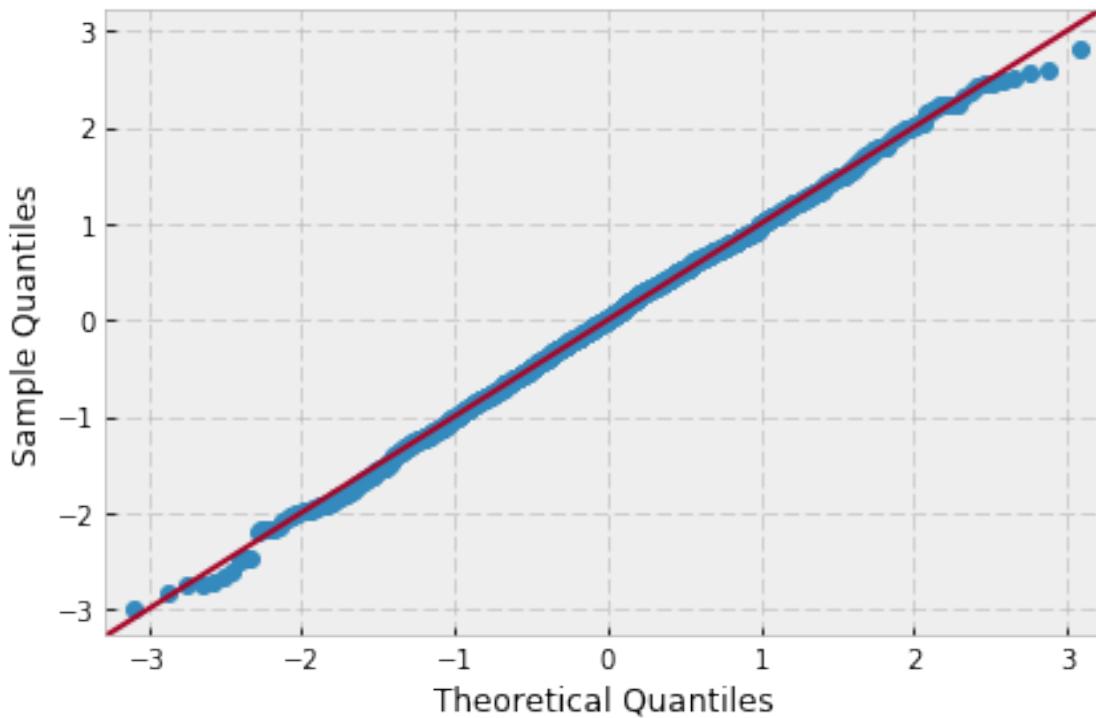


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(),loc=1311.5159194490443, scale=68.83404745560766)
DescribeResult(nobs=1000, minmax=(1080.9210140874004, 1492.8067560118589)
               mean=1311.5159194490443, variance=4742.868958078926,
               skewness=-0.1899052283283912, kurtosis=-0.09678024166003052)
```

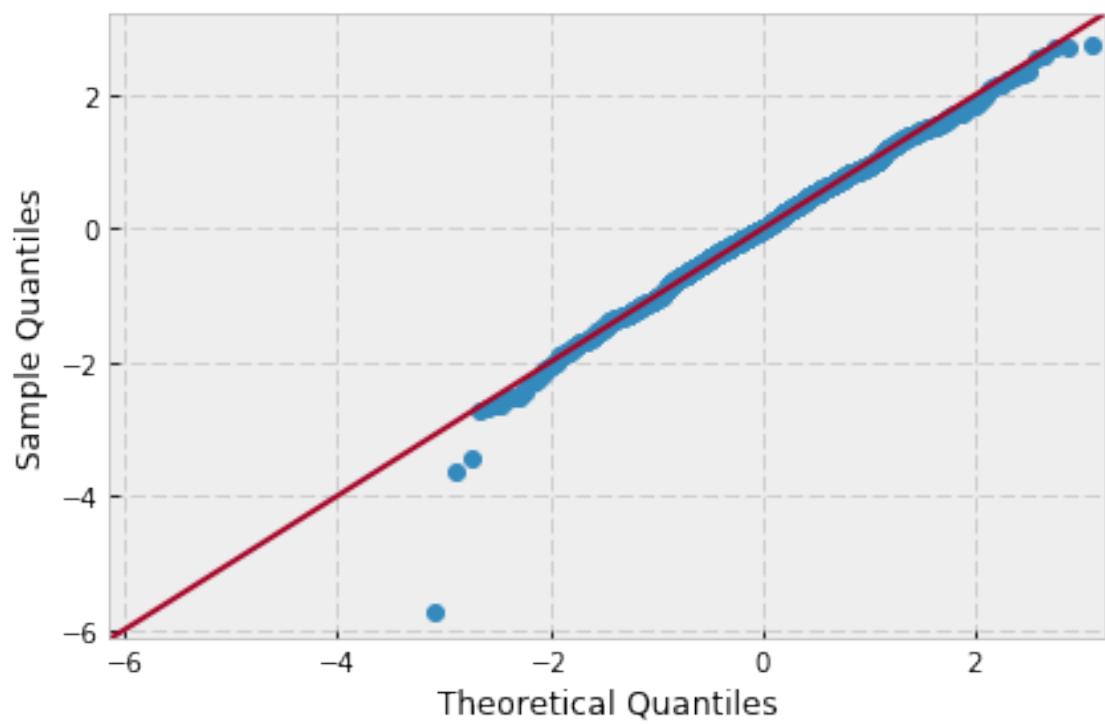
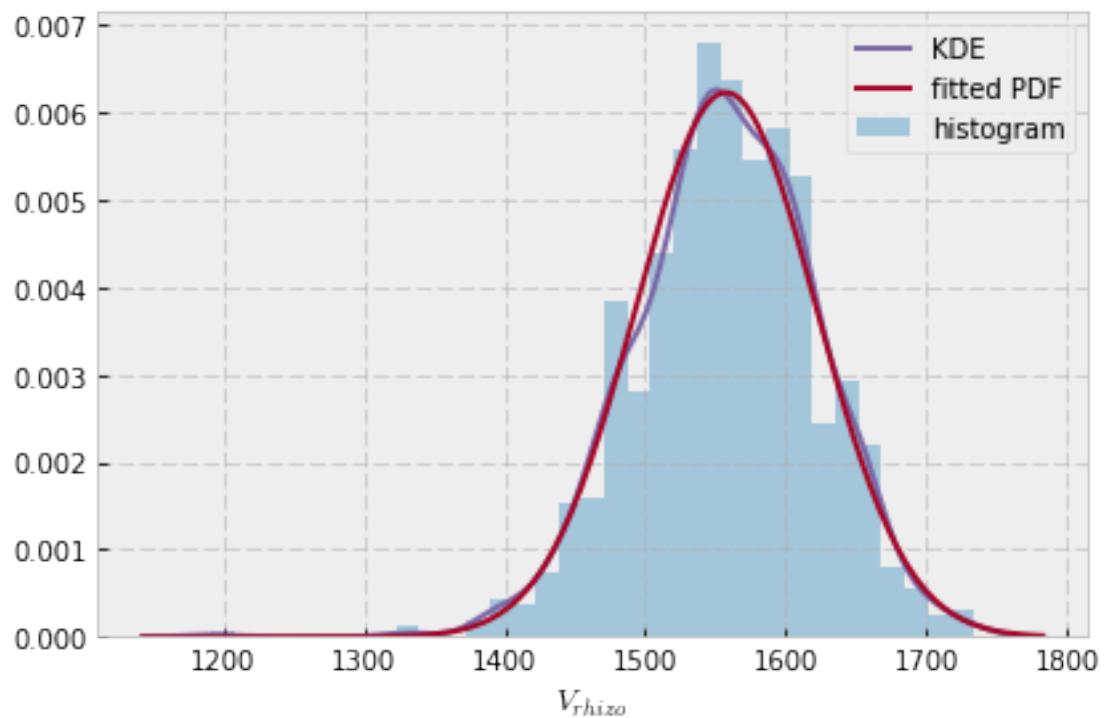


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(),loc=1438.089150227518, scale=67.16611768823748)
DescribeResult(nobs=1000, minmax=(1237.3394537521651, 1626.8638261592102)
               mean=1438.0891502275181, variance=4515.8031684786465,
               skewness=-0.07439554285302573, kurtosis=-0.14928771747911895)
```

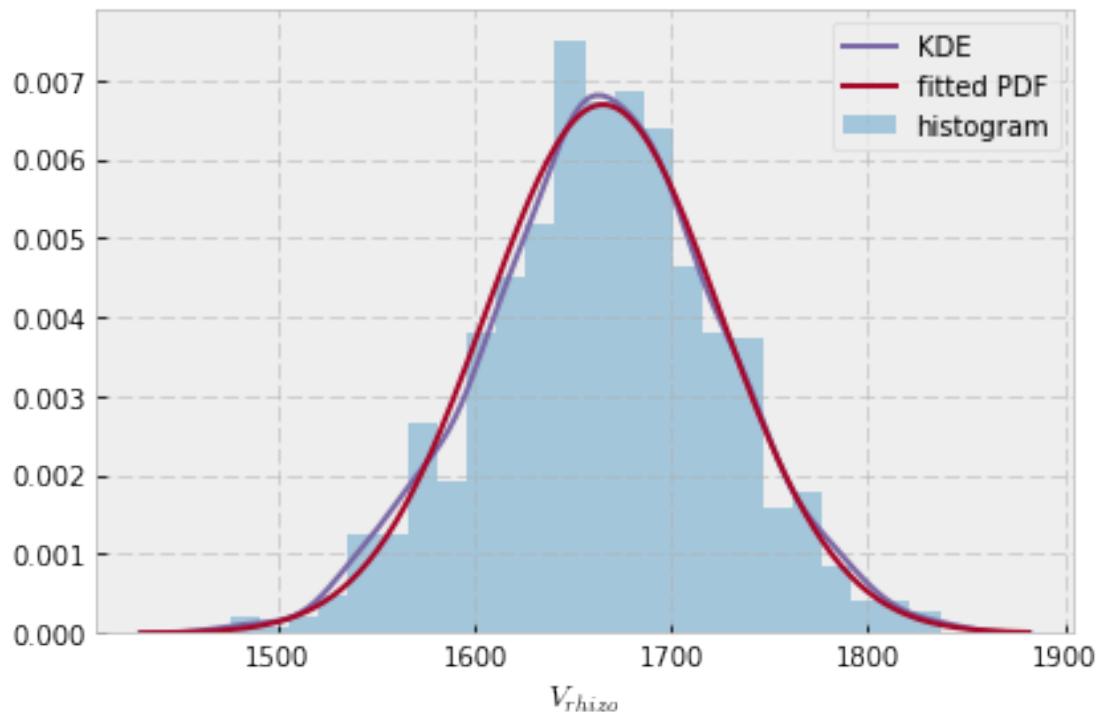


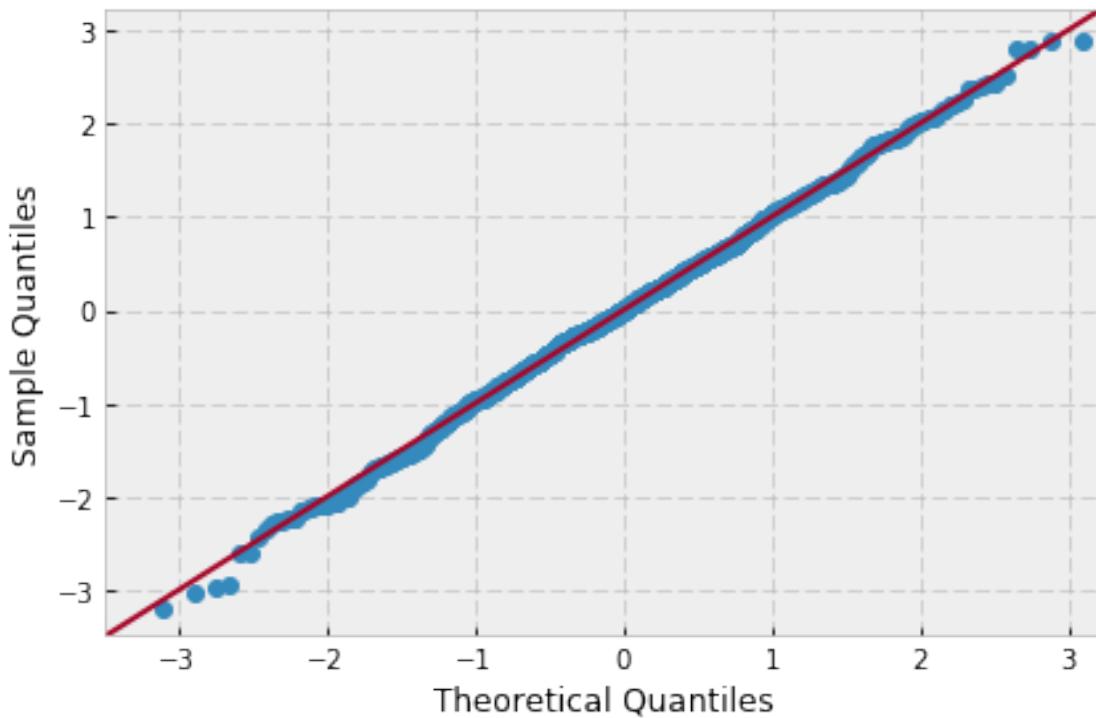


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(),loc=1557.4419973547194, scale=64.04623605704194)
DescribeResult(nobs=1000, minmax=(1191.2110900659579, 1734.0632746113849)
               mean=1557.4419973547194, variance=4106.0263794537923,
               skewness=-0.3157220973298956, kurtosis=0.9181155786926878)
```

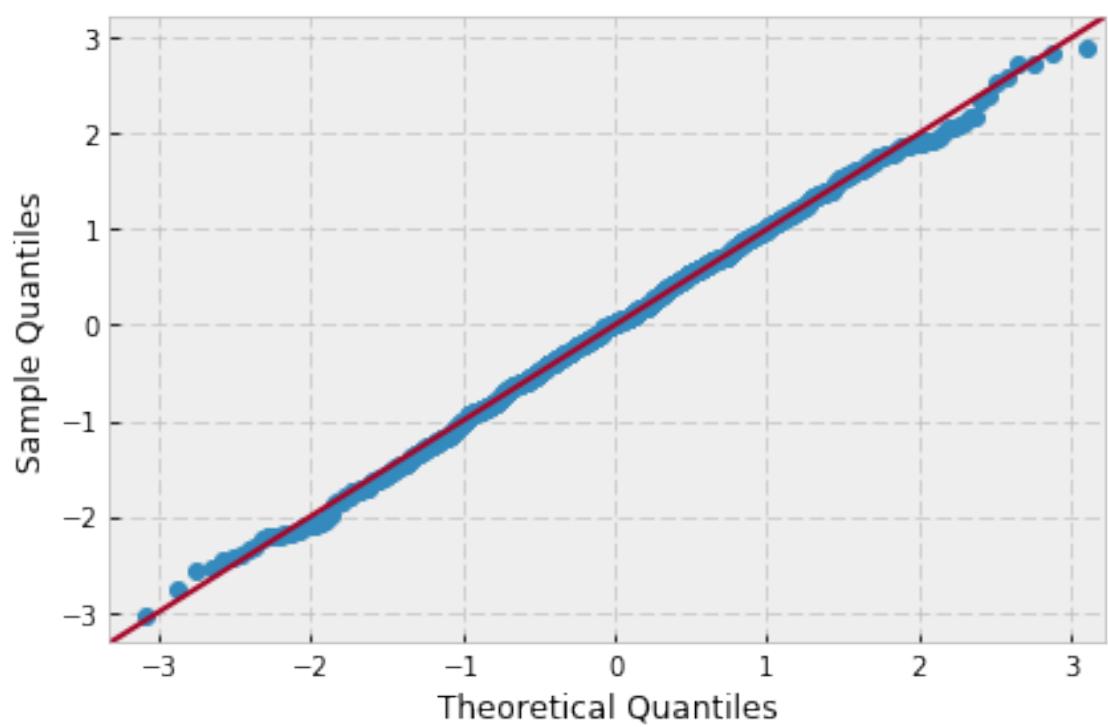
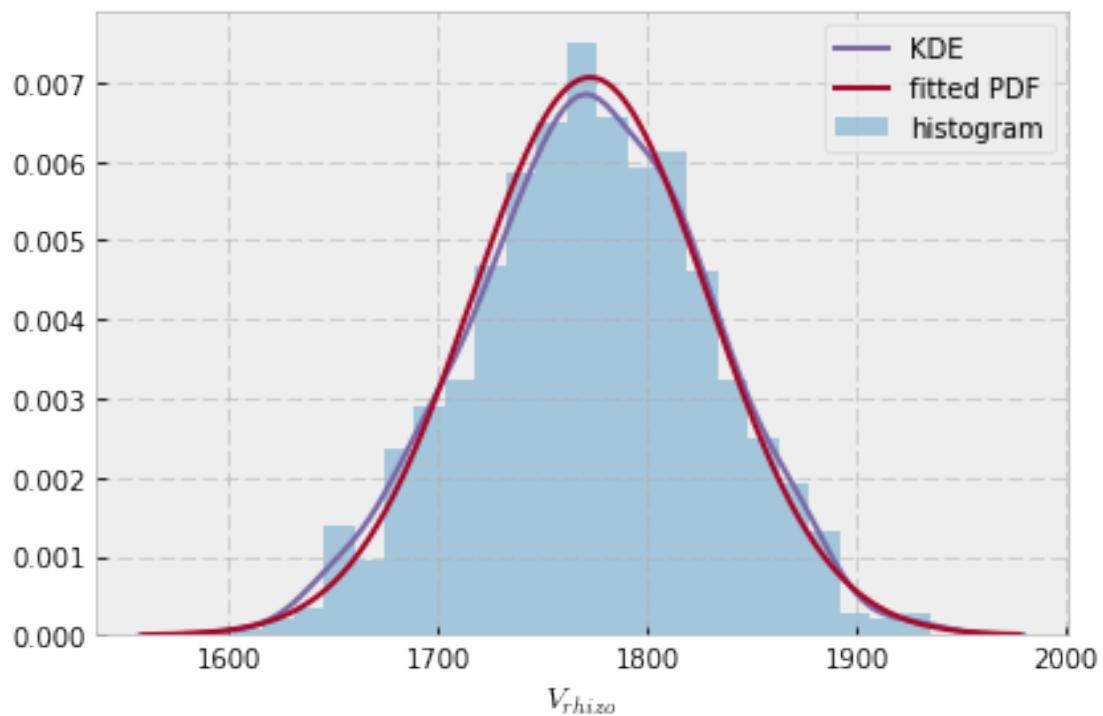


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=1665.2889584676113, scale=59.653828534556204)
DescribeResult(nobs=1000, minmax=(1475.5193323468764, 1836.6618343016617)
               mean=1665.2889584676113, variance=3562.1414002304632,
               skewness=-0.0617862553401327, kurtosis=-0.00720709554466481)
```

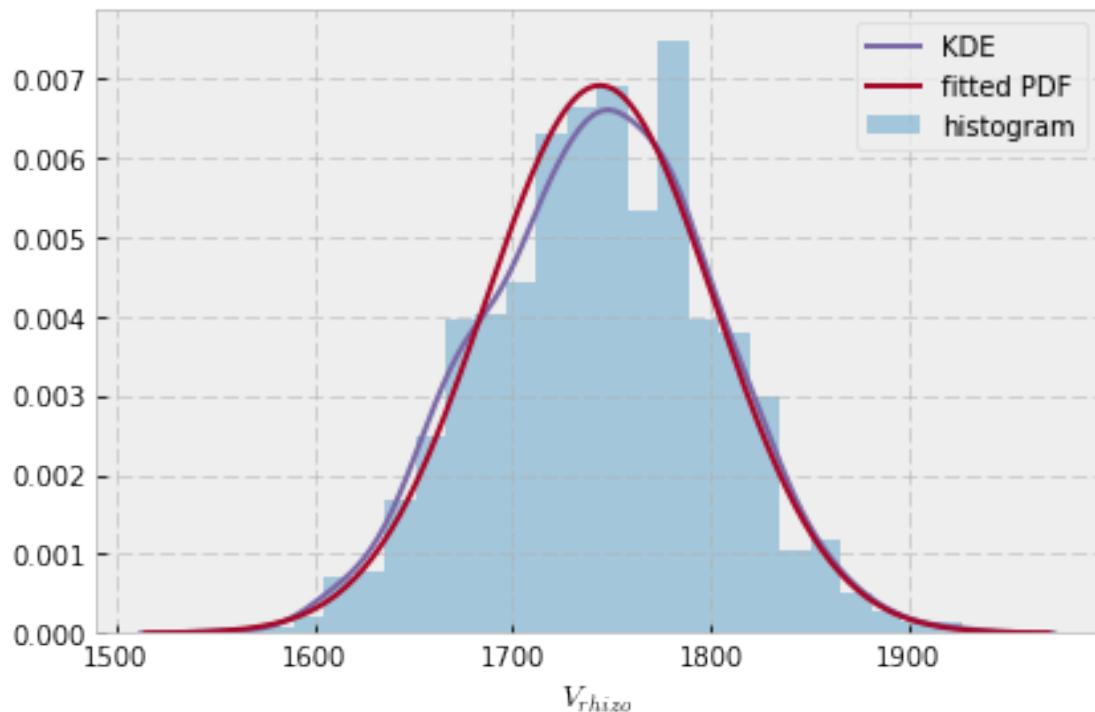


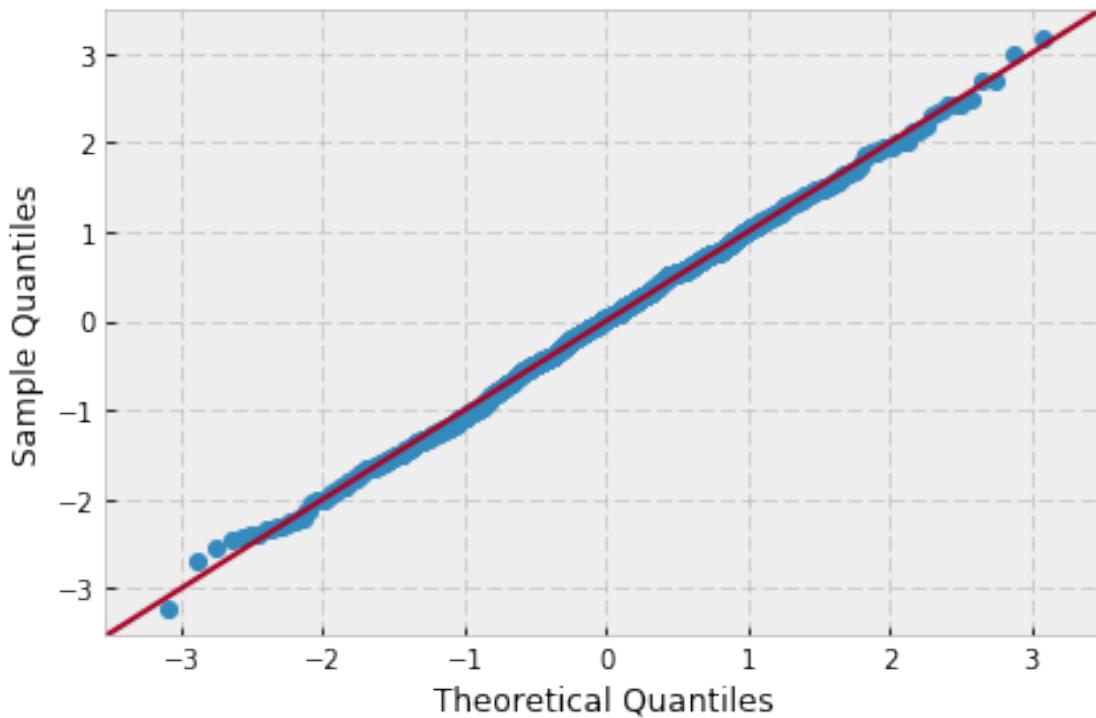


```
variable name: r
variable value: 3.0
distribution: normal(shape=(),loc=1772.68950017063, scale=56.47637714789638)
DescribeResult(nobs=1000, minmax=(1601.8244768790551, 1935.3467862183136)
               mean=1772.68950017063, variance=3192.773949701133,
               skewness=-0.062006993297589595, kurtosis=-0.22945001613591076)
```

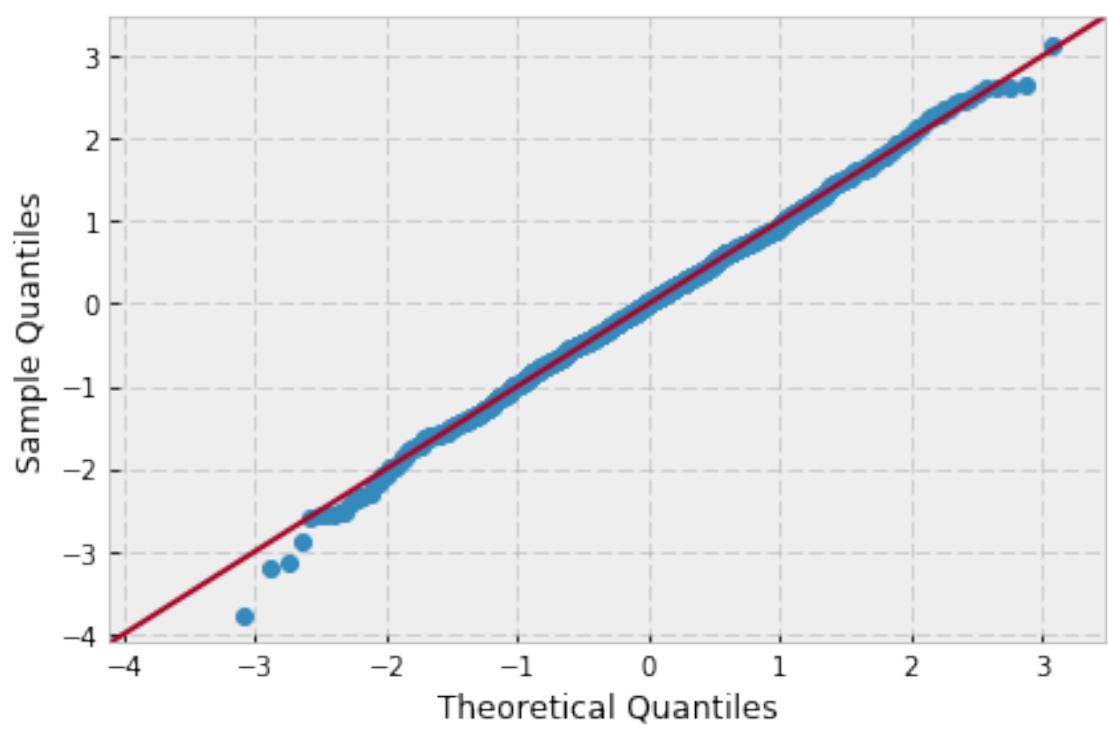
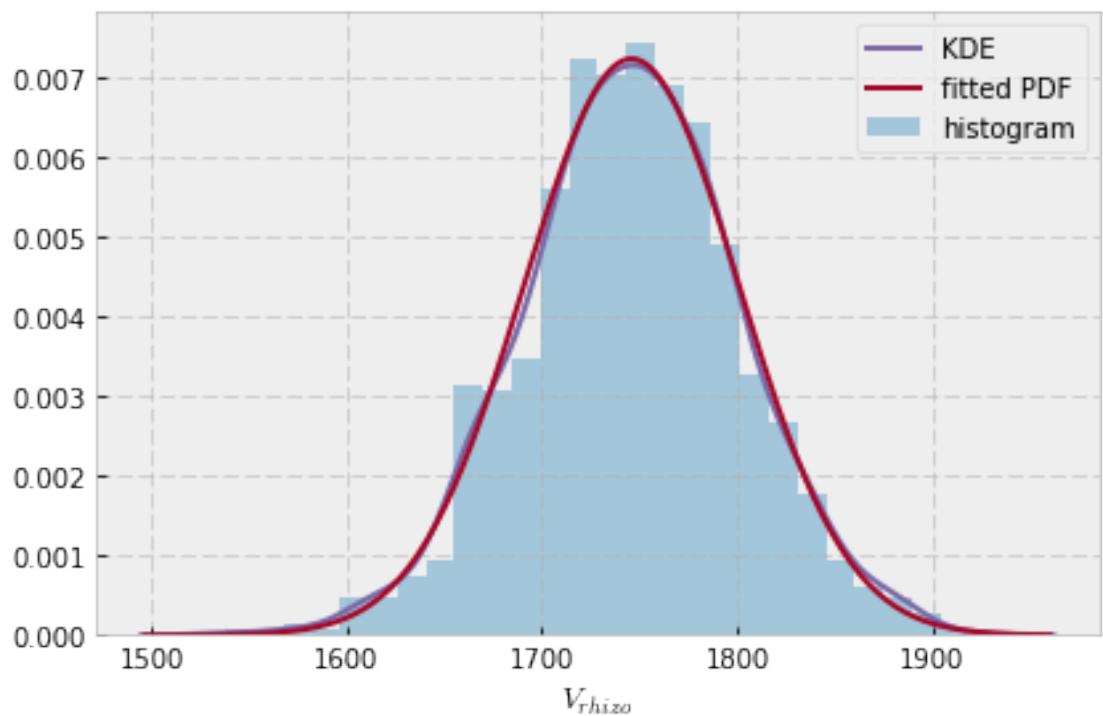


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(), loc=1744.2108820054136, scale=57.70396996020353)
DescribeResult(nobs=1000, minmax=(1558.691764756273, 1927.0850650814998)
               mean=1744.2108820054136, variance=3333.0812303984694,
               skewness=-0.031492115091371536, kurtosis=-0.21037848853803576)
```

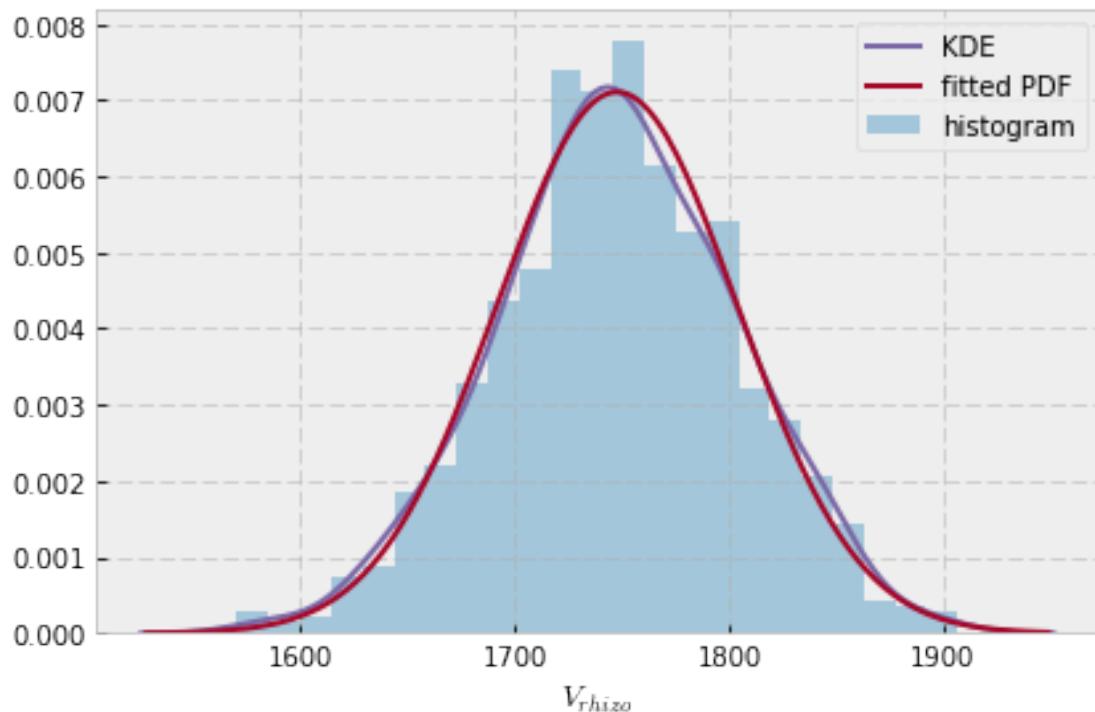


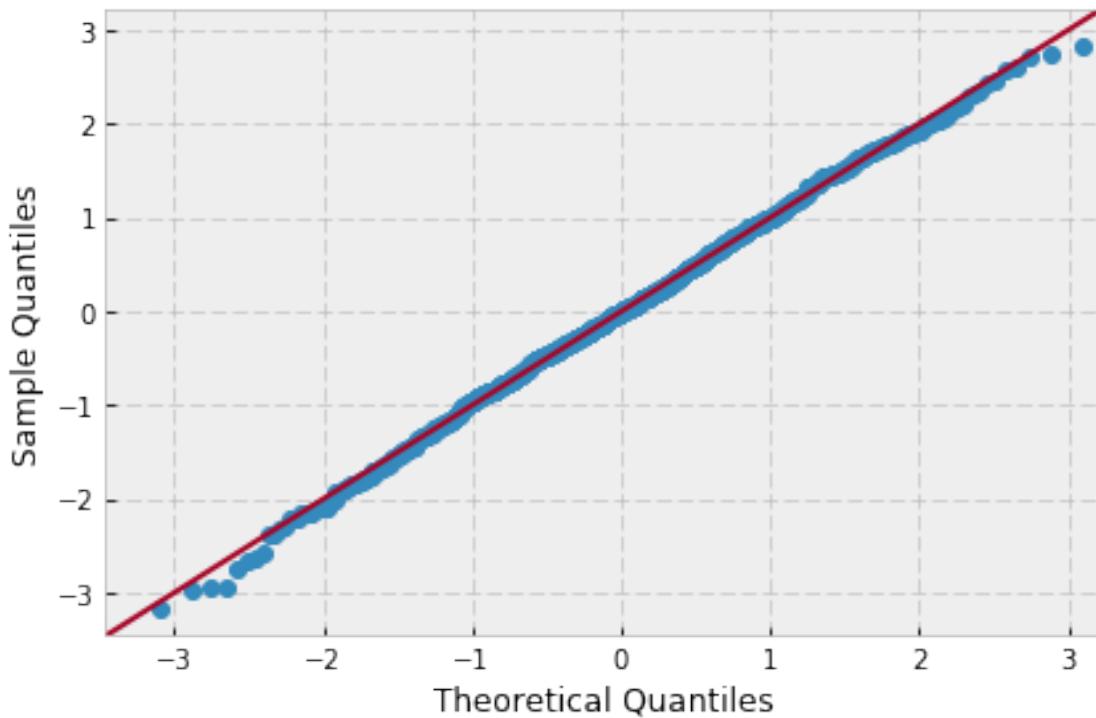


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(),loc=1746.0426533807897, scale=55.19429982399885)
DescribeResult(nobs=1000, minmax=(1538.1178128202328, 1918.9124494557502)
               mean=1746.0426533807897, variance=3049.4601932547339,
               skewness=-0.06533692330910552, kurtosis=0.1792366871931348)
```

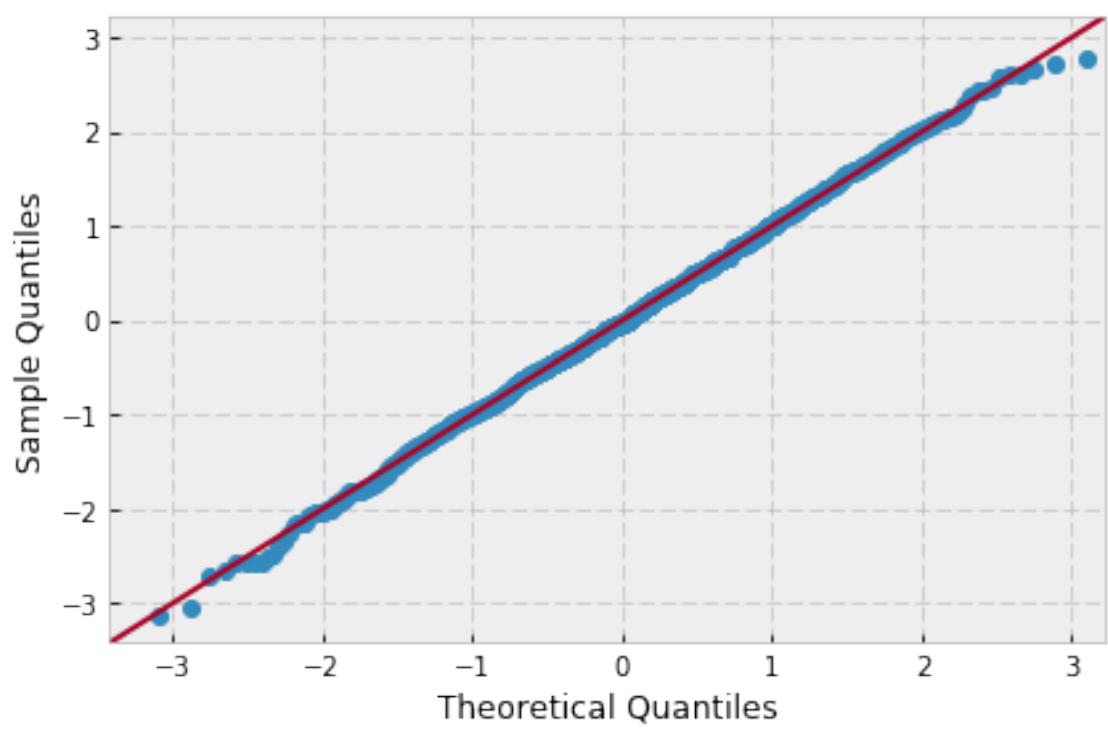
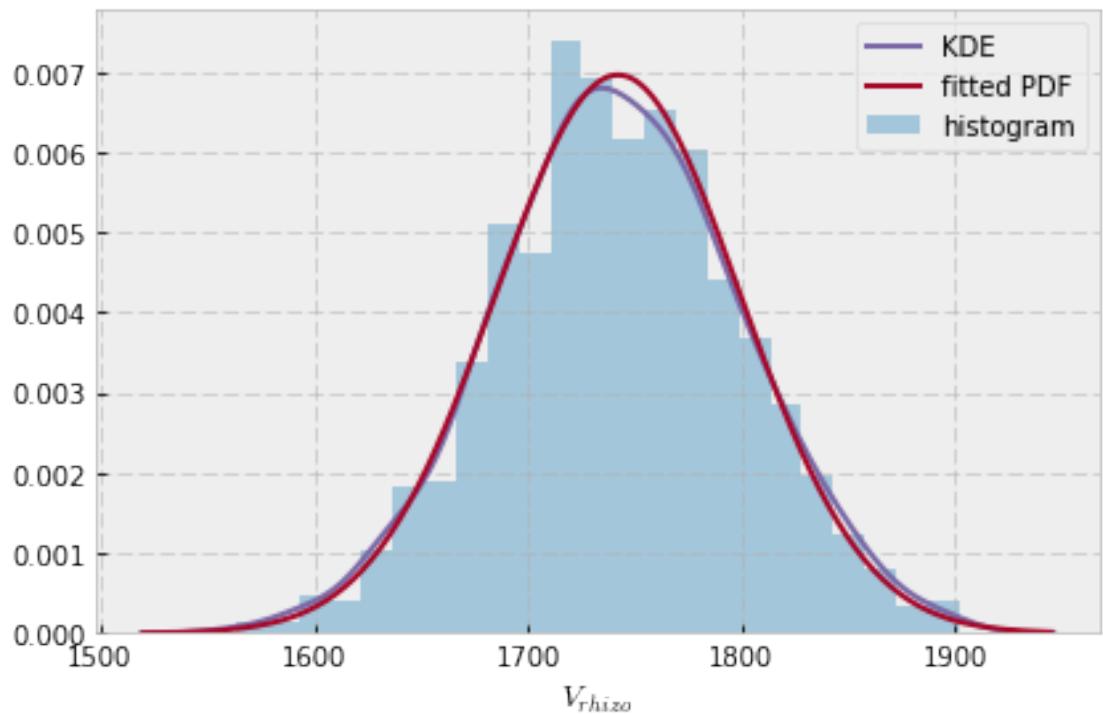


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=1747.6084551095917, scale=56.11784501026188)
DescribeResult(nobs=1000, minmax=(1570.2571051565981, 1906.5698809959454)
               mean=1747.6084551095917, variance=3152.3648934892635,
               skewness=-0.06903507436831137, kurtosis=-0.05960499140335784)
```

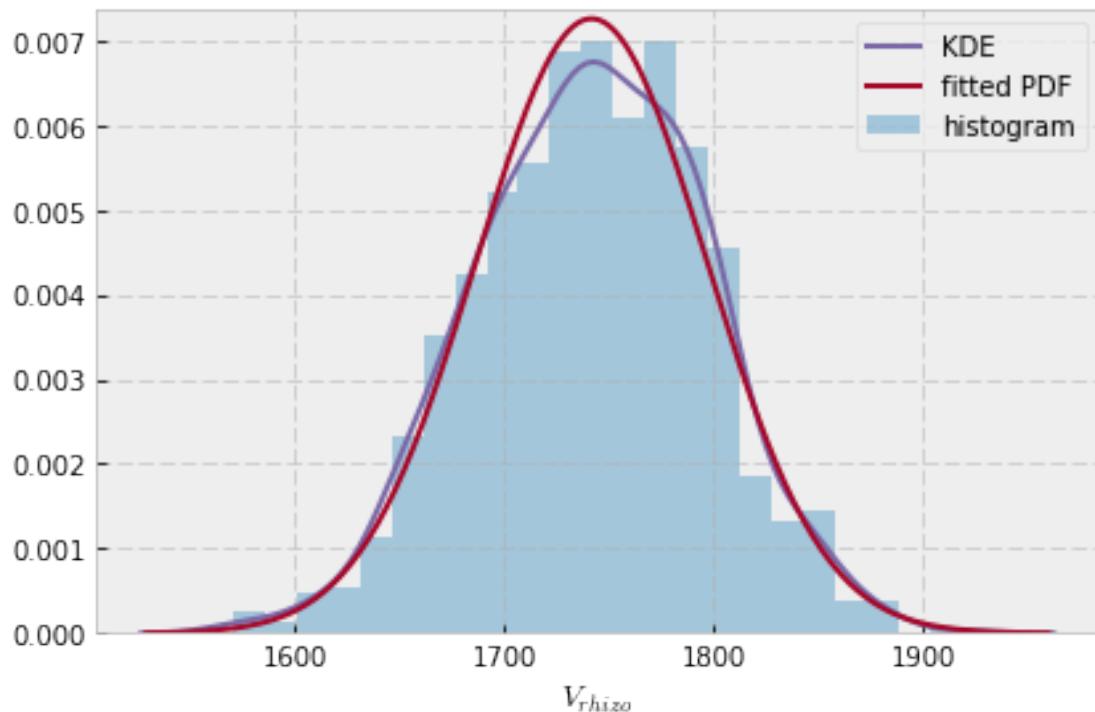


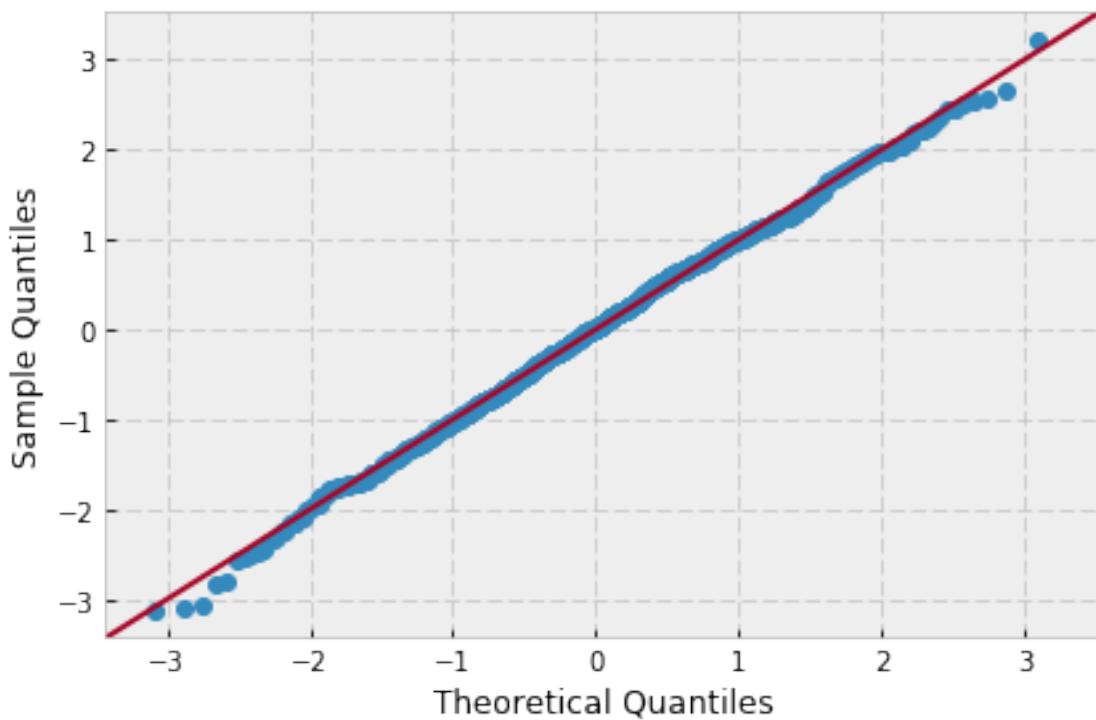


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=1742.087840461061, scale=57.28261386952508)
DescribeResult(nobs=1000, minmax=(1563.1300849866154, 1901.7357083019683)
               mean=1742.087840461061, variance=3284.5824341592665,
               skewness=-0.014021947323285, kurtosis=-0.077672556082645)
```

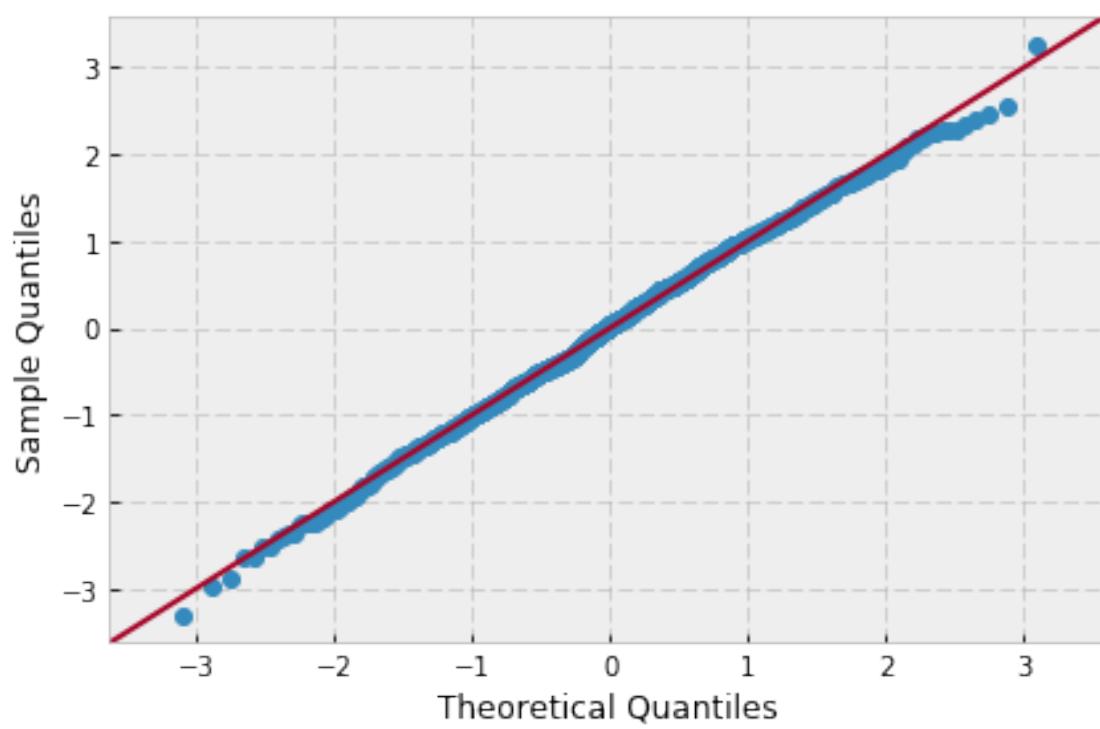
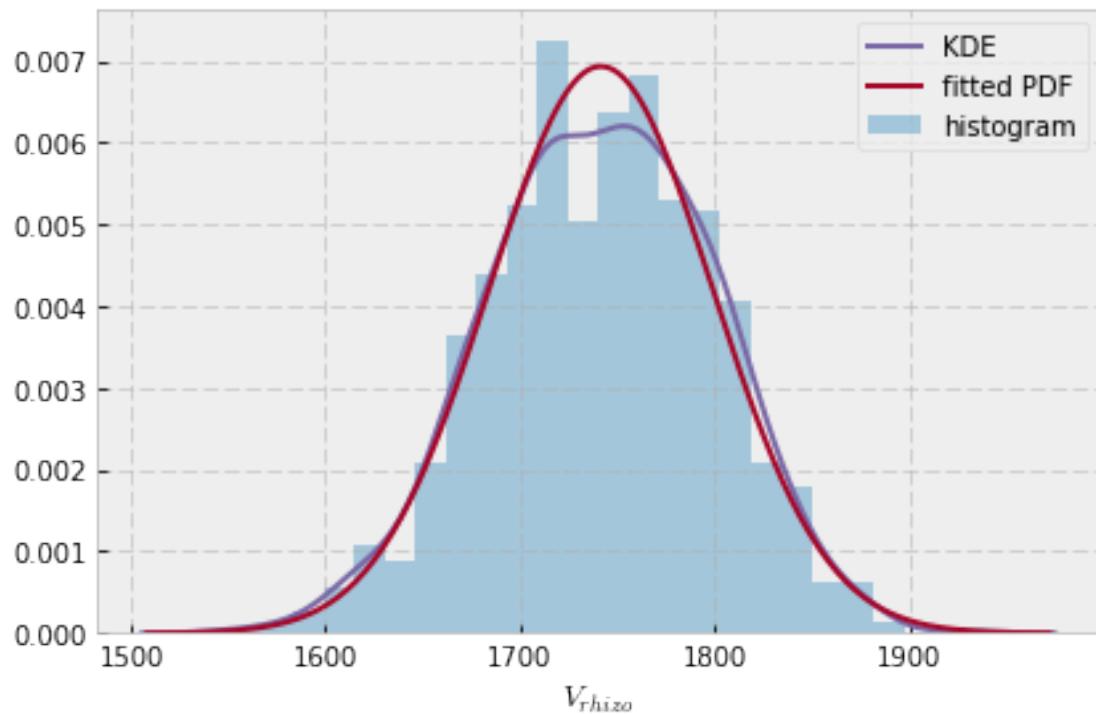


```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(),loc=1741.9328821604927, scale=54.96546380375735)
DescribeResult(nobs=1000, minmax=(1570.8133804575491, 1918.6135807601186)
               mean=1741.9328821604927, variance=3024.2264375997593,
               skewness=-0.09284348680364068, kurtosis=-0.12152102494177974)
```

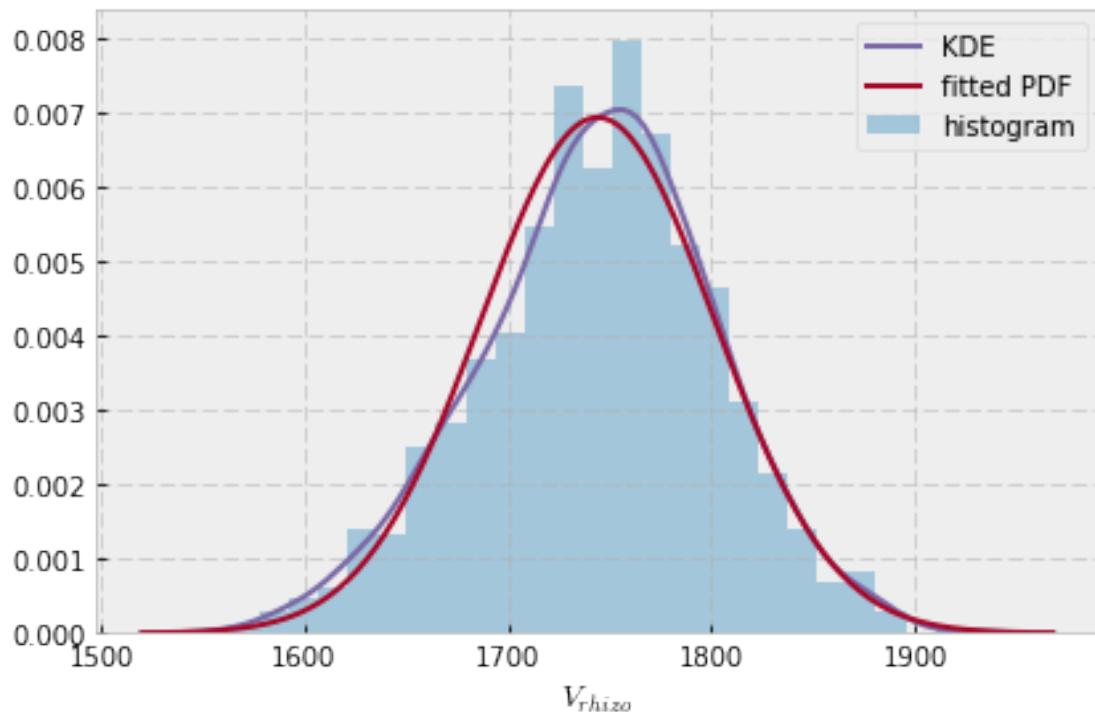


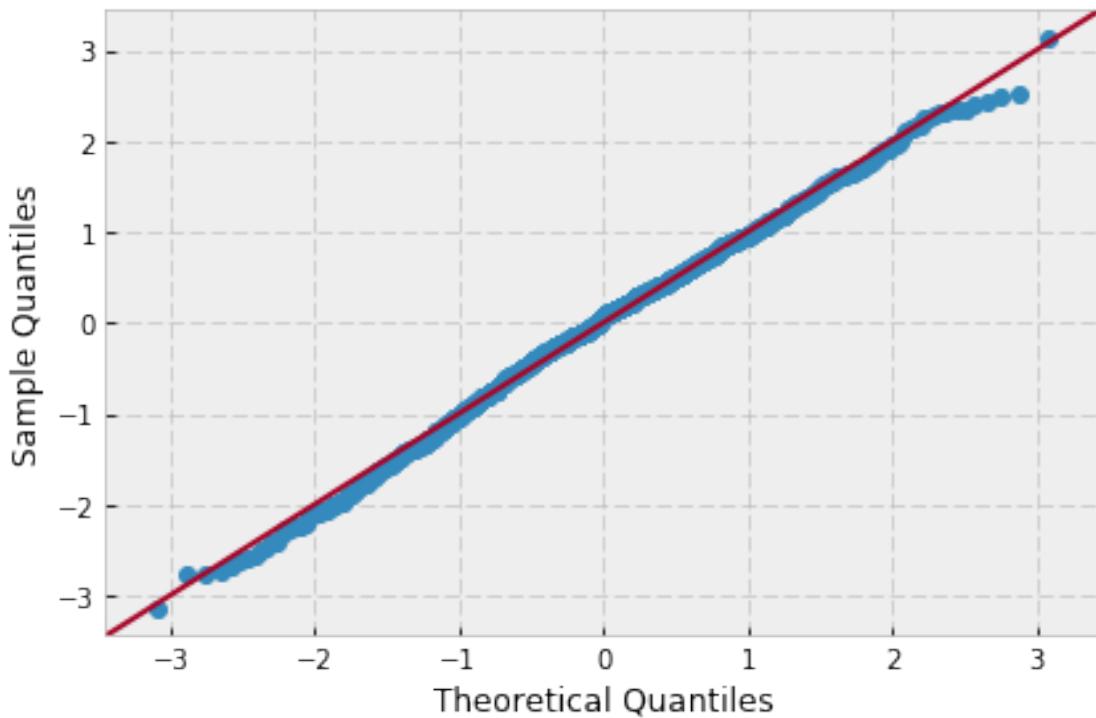


```
variable name: sigma
variable value: 0.0666666666666667
distribution: normal(shape=(),loc=1741.3861522819511, scale=57.521409595423314)
DescribeResult(nobs=1000, minmax=(1551.6954185739587, 1928.4792559350749)
               mean=1741.3861522819511, variance=3312.0245864308877,
               skewness=-0.1040910288376349, kurtosis=-0.20479351068945606)
```

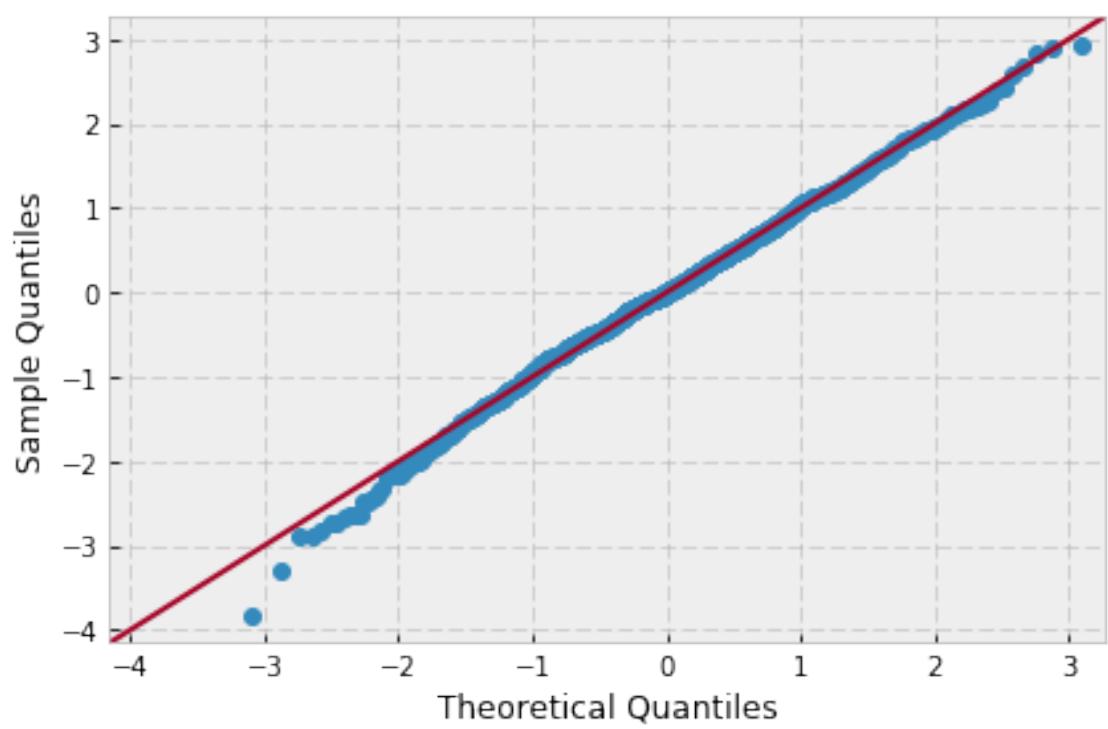
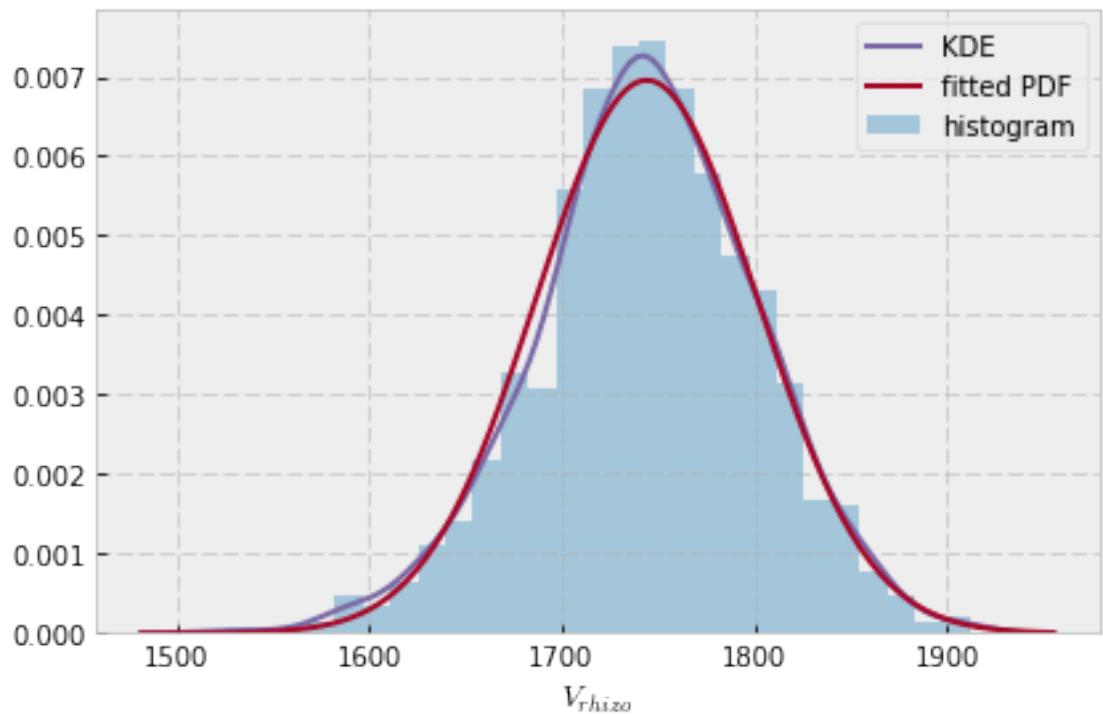


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=1743.9607649104921, scale=57.542377276285016)
DescribeResult(nobs=1000, minmax=(1563.6601466396251, 1924.0035528820385)
               mean=1743.9607649104921, variance=3314.4396222285513,
               skewness=-0.17368890980882, kurtosis=-0.0499684052135283)
```

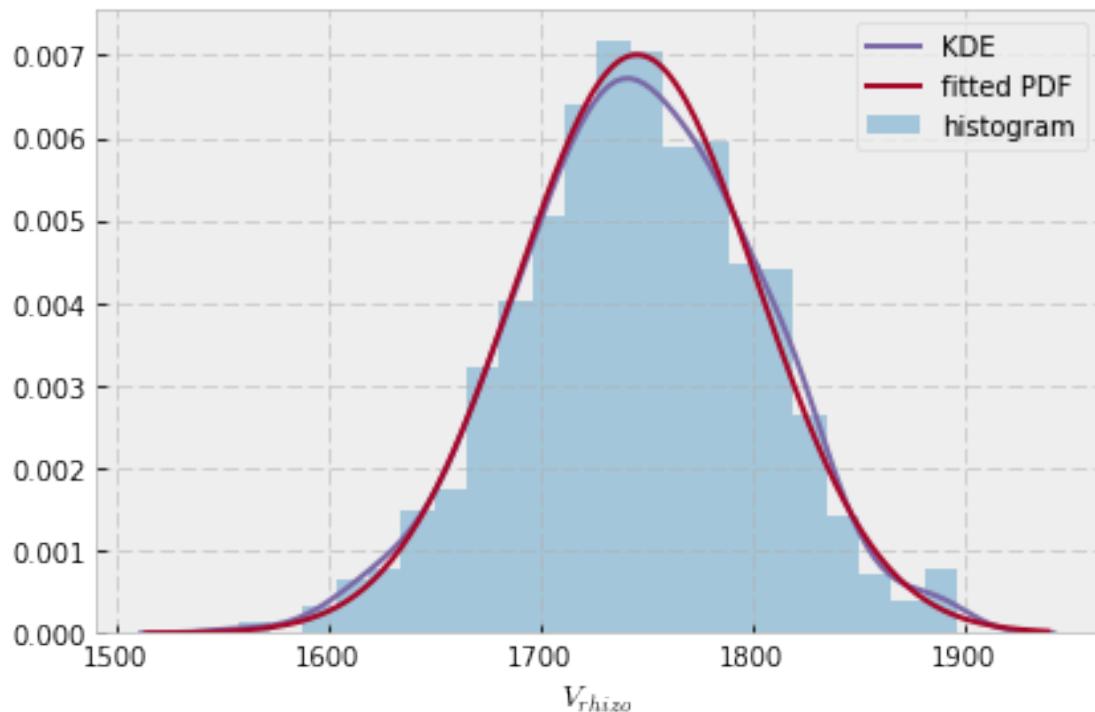


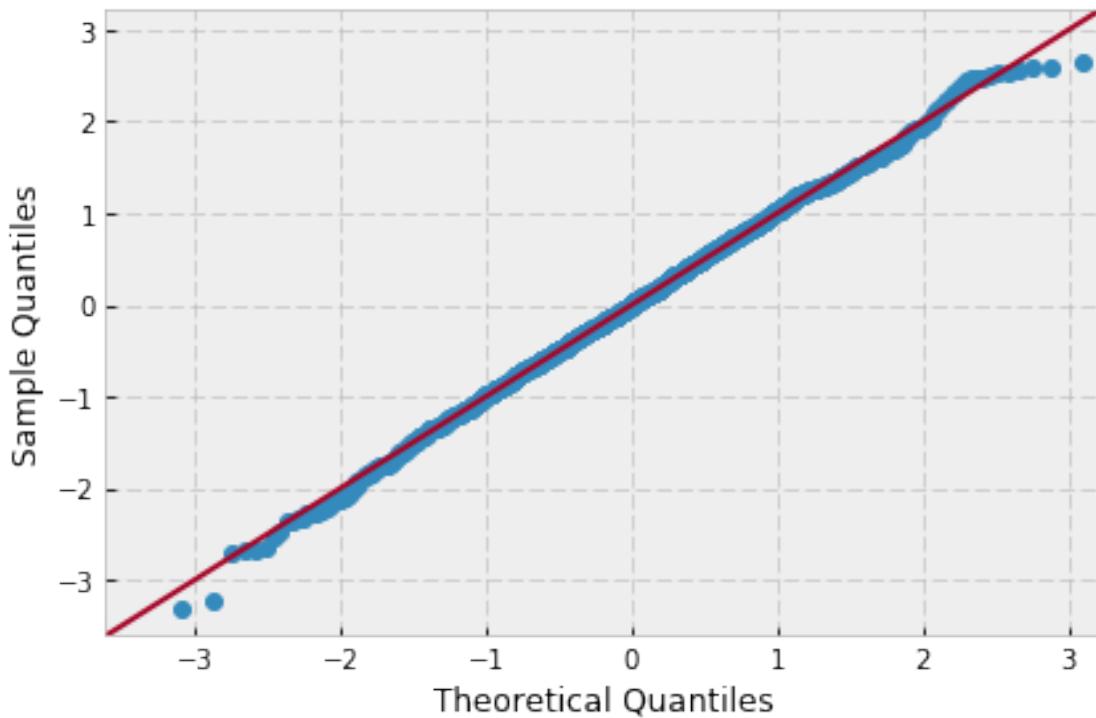


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(),loc=1743.982687591328, scale=57.359784048451914)
DescribeResult(nobs=1000, minmax=(1524.8855431830896, 1911.8627601077524)
               mean=1743.9826875913279, variance=3293.4382643493877,
               skewness=-0.1716017694600497, kurtosis=0.24917139525782206)
```

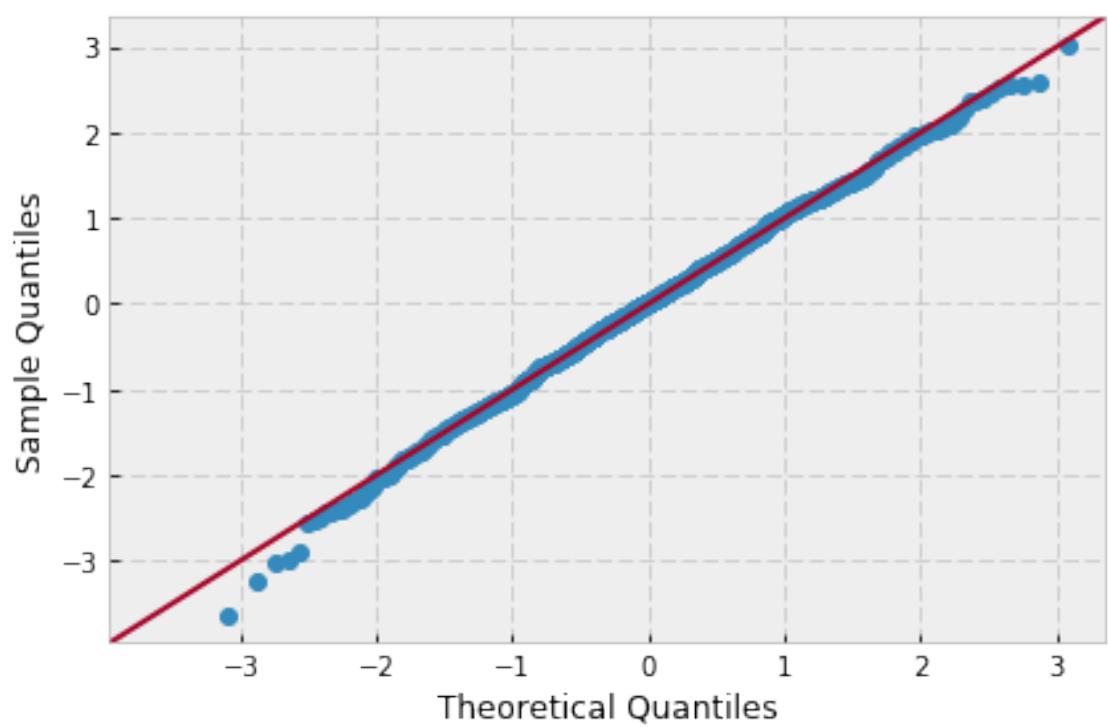
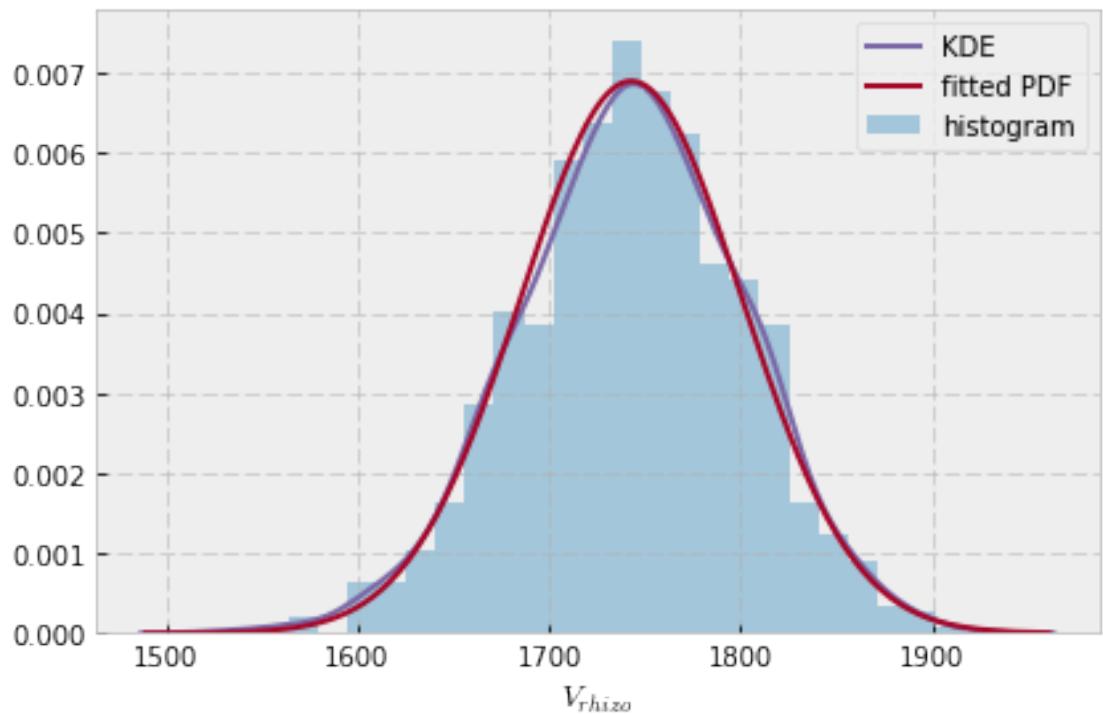


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(),loc=1745.7282285736208, scale=56.95740817133087)
DescribeResult(nobs=1000, minmax=(1557.1266830708048, 1896.9230789752203)
               mean=1745.7282285736208, variance=3247.3937393349229,
               skewness=-0.08066433910277883, kurtosis=-0.06552602129533325)
```

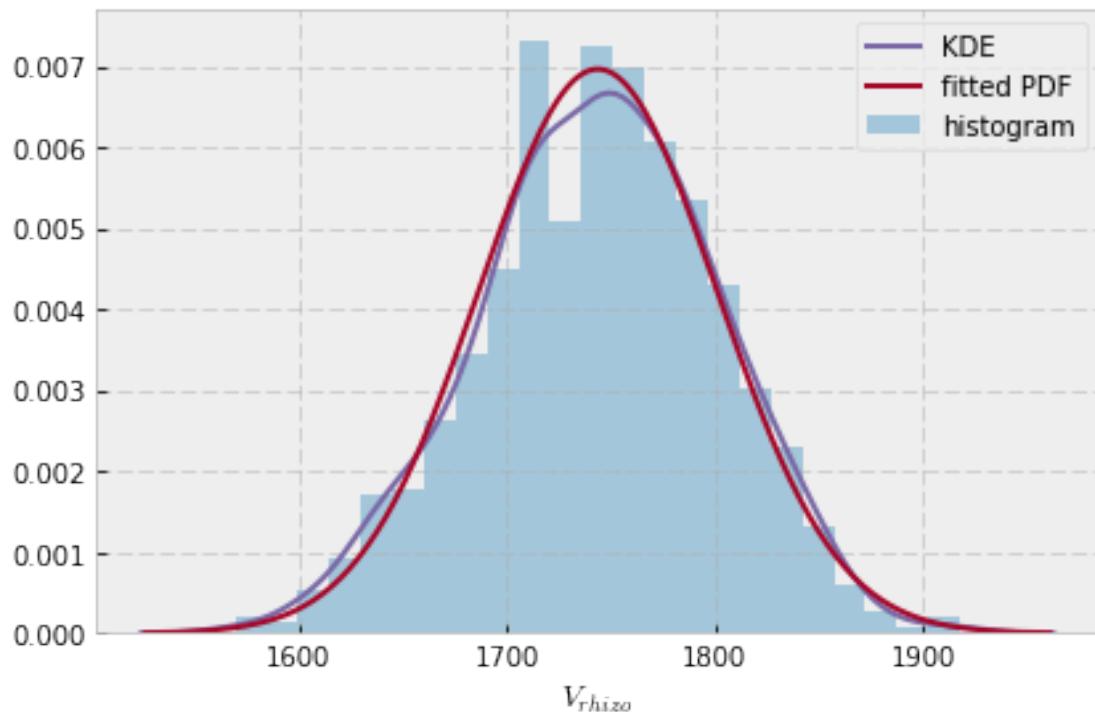


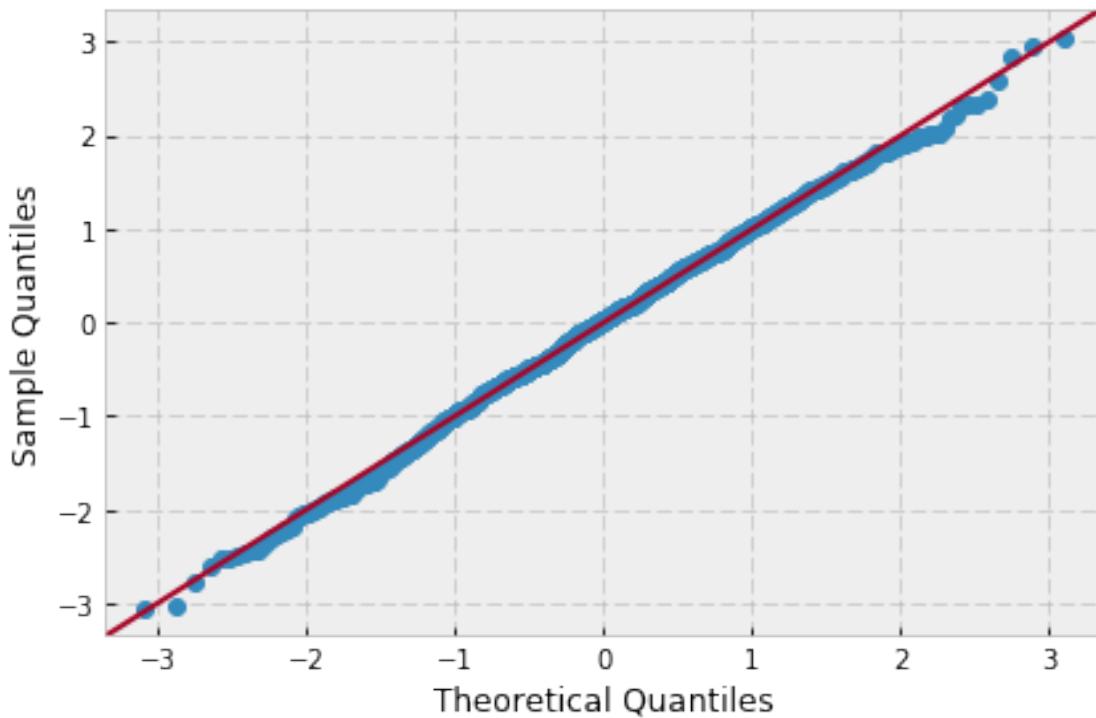


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(),loc=1742.965955572017, scale=57.87412610898681)
DescribeResult(nobs=1000, minmax=(1532.9526957172764, 1917.5427645518707)
               mean=1742.965955572017, variance=3352.767240119028,
               skewness=-0.13853078721777315, kurtosis=0.025819778355304912)
```

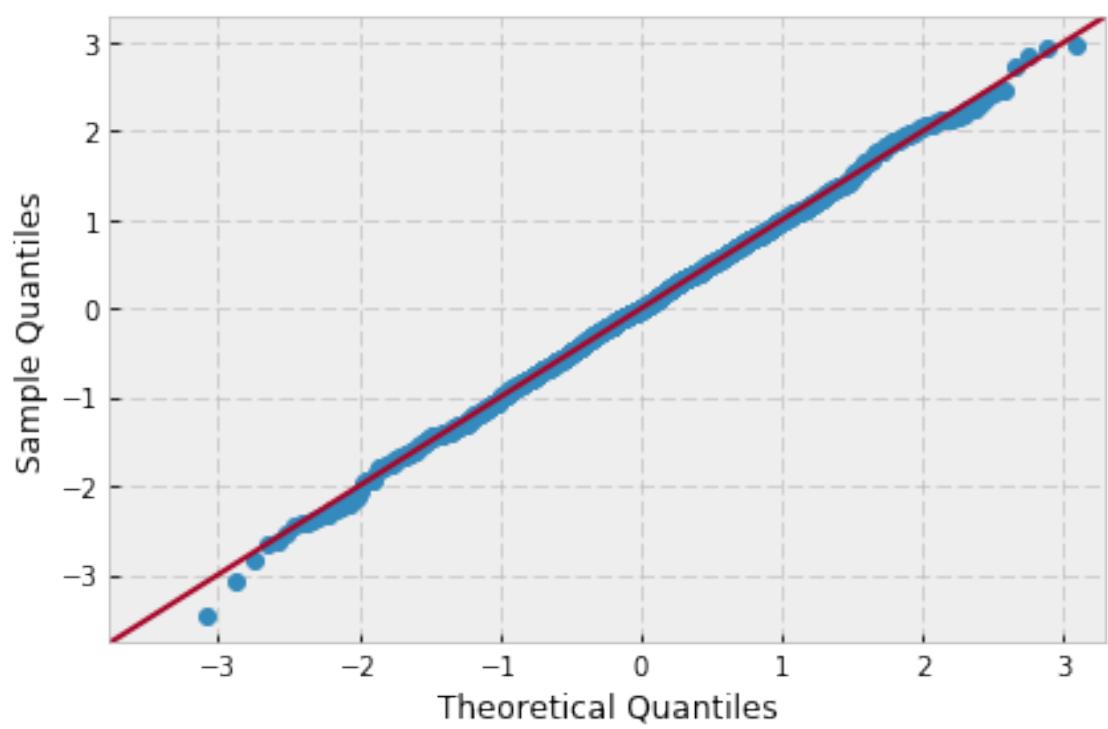
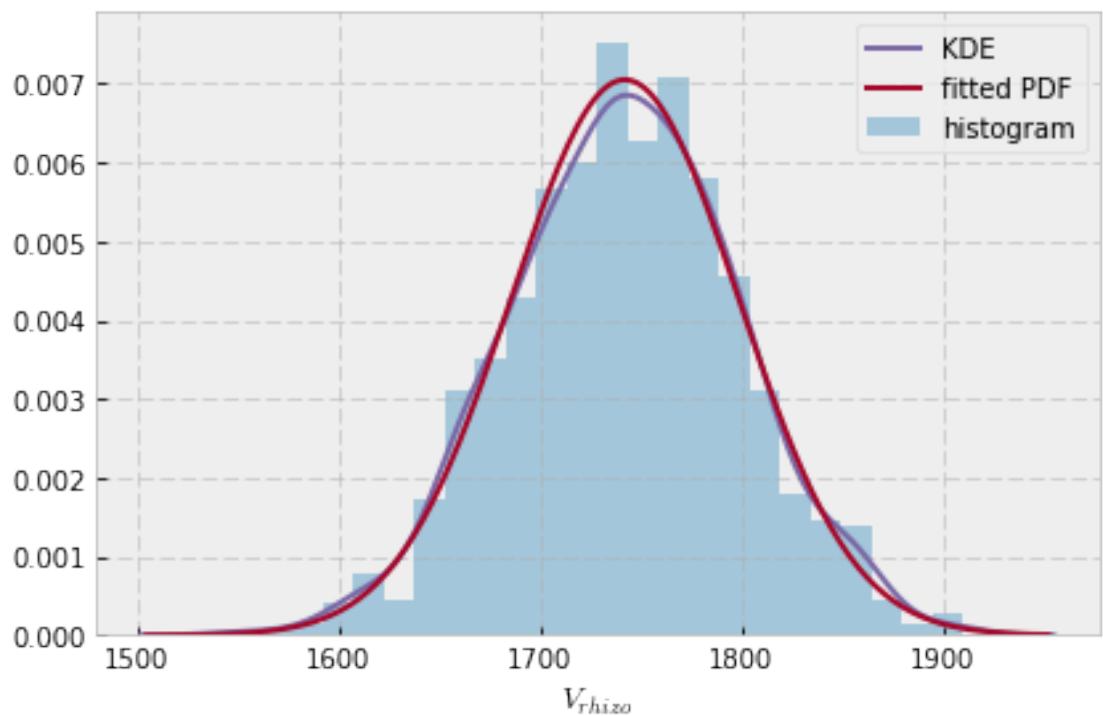


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=1743.8235194761226, scale=57.305009276598646)
DescribeResult(nobs=1000, minmax=(1569.3163550084253, 1917.8579980533721)
               mean=1743.8235194761226, variance=3287.1512394304877,
               skewness=-0.11596607520832979, kurtosis=-0.1432344518727766)
```

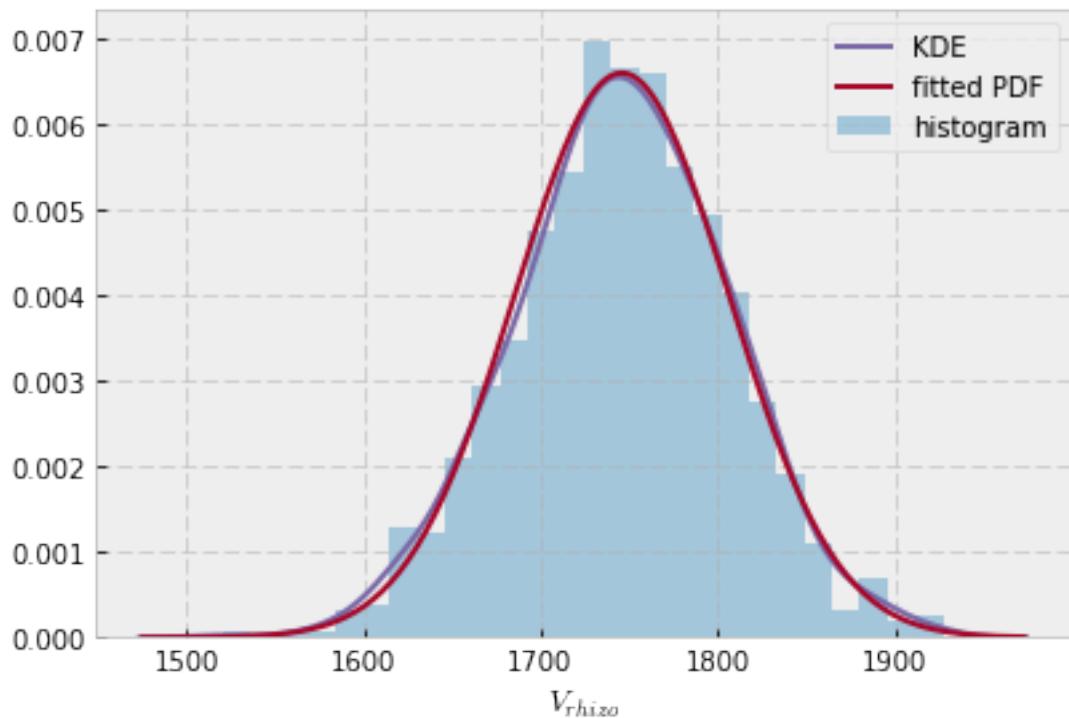


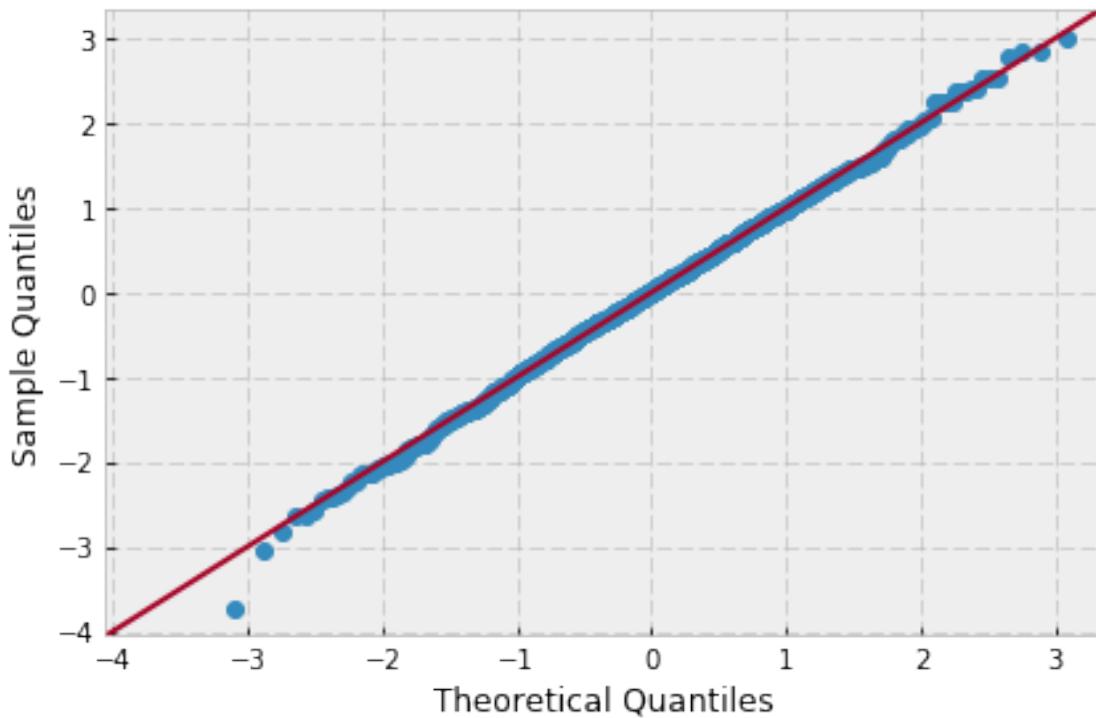


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(),loc=1742.0808233807402, scale=56.56624621314898)
DescribeResult(nobs=1000, minmax=(1546.8761167060591, 1909.9587787465348)
               mean=1742.0808233807402, variance=3202.9431538003914,
               skewness=-0.036844760009423584, kurtosis=-0.026591821922131054)
```

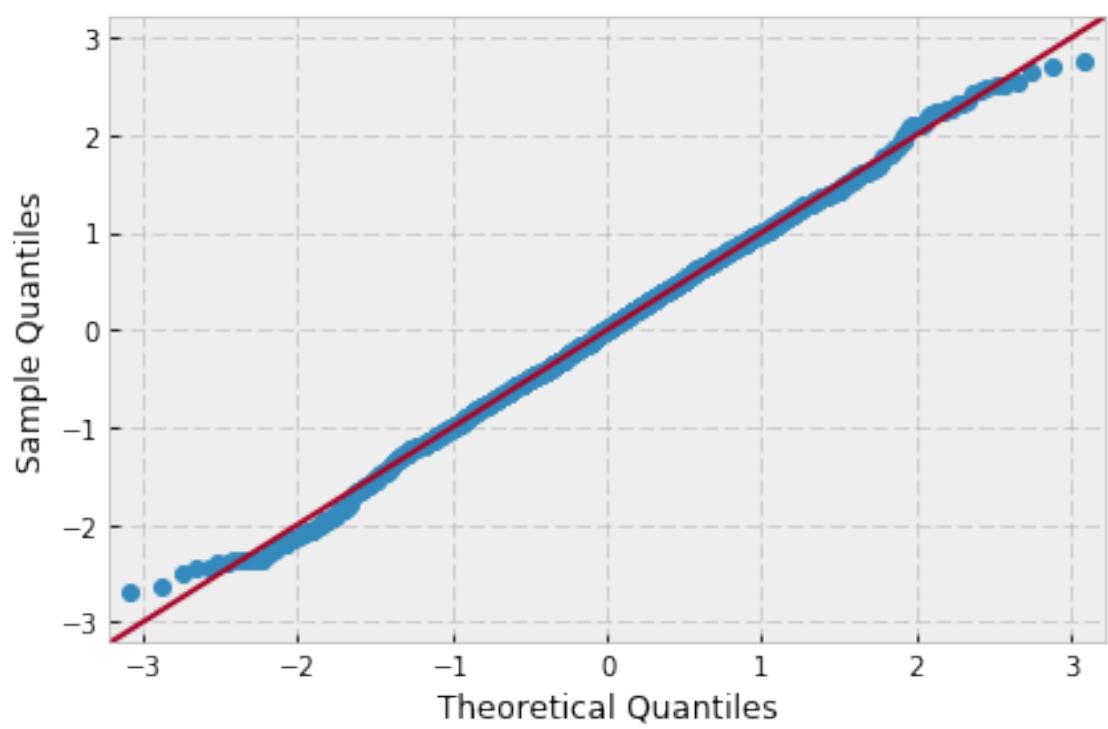
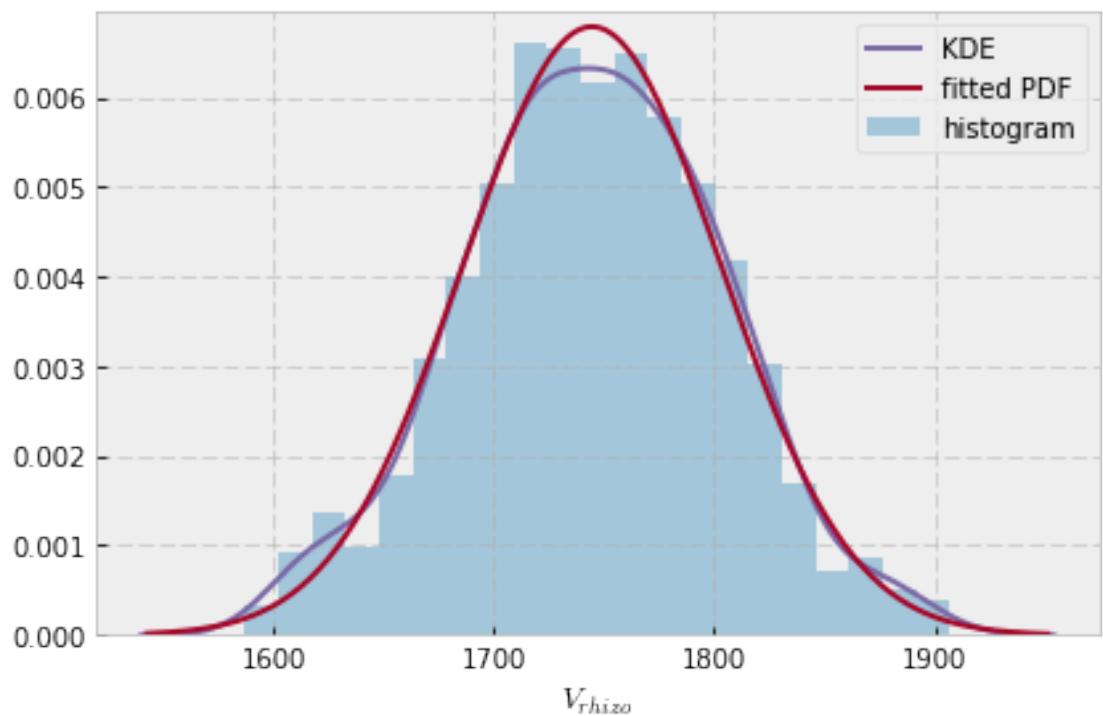


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(),loc=1745.7412747596597, scale=60.48555649923886)
DescribeResult(nobs=1000, minmax=(1520.8687949743012, 1926.8980440813959)
               mean=1745.7412747596597, variance=3662.1647097323485,
               skewness=-0.073137032710753, kurtosis=0.055694989872196565)
```





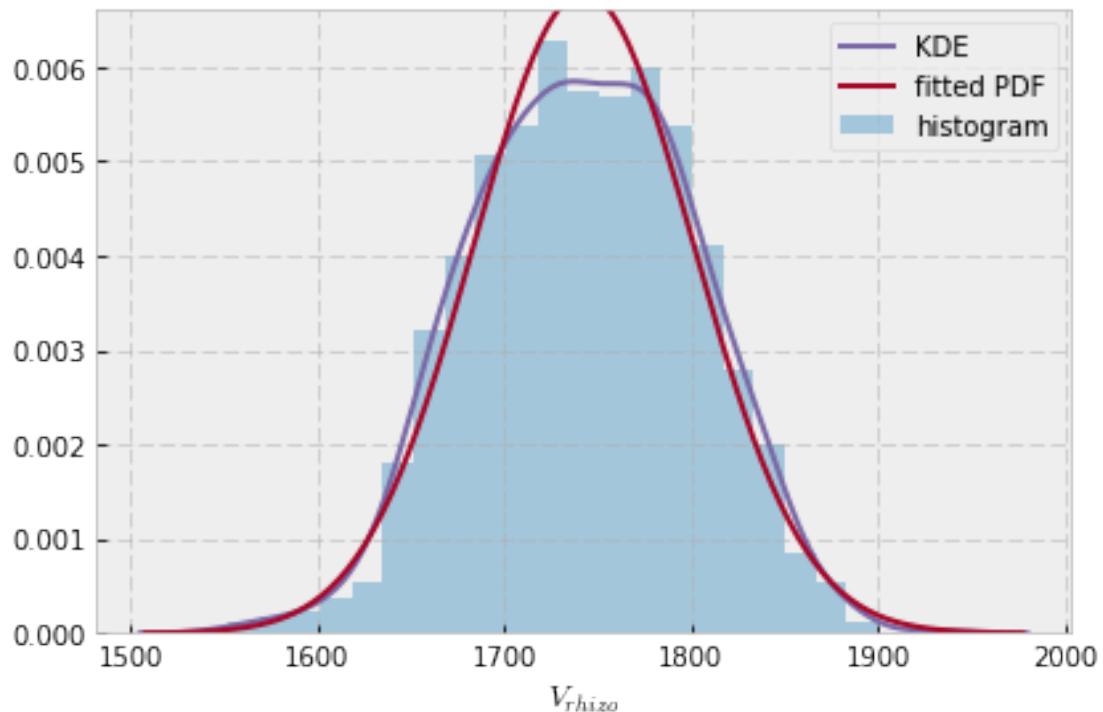
```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=1744.434538669596, scale=58.780993384022885)
DescribeResult(nobs=1000, minmax=(1586.6810983487533, 1907.0806987020533)
               mean=1744.4345386695959, variance=3458.6638470596022,
               skewness=-0.03360574753858457, kurtosis=-0.15614779091588638)
```

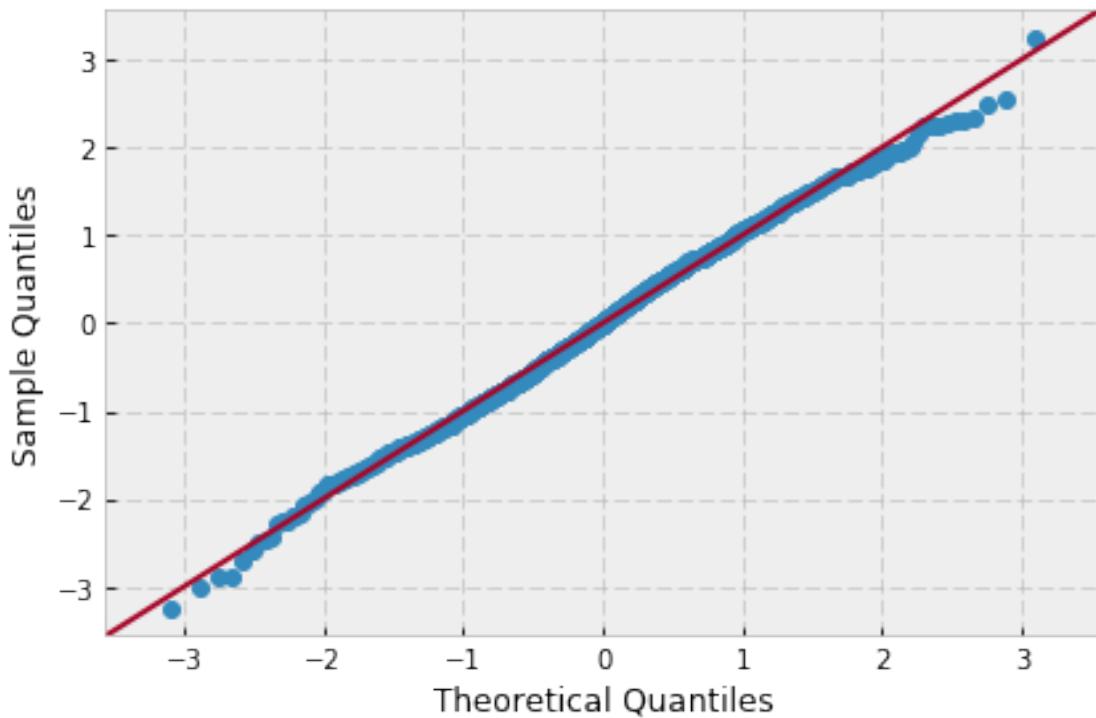


```

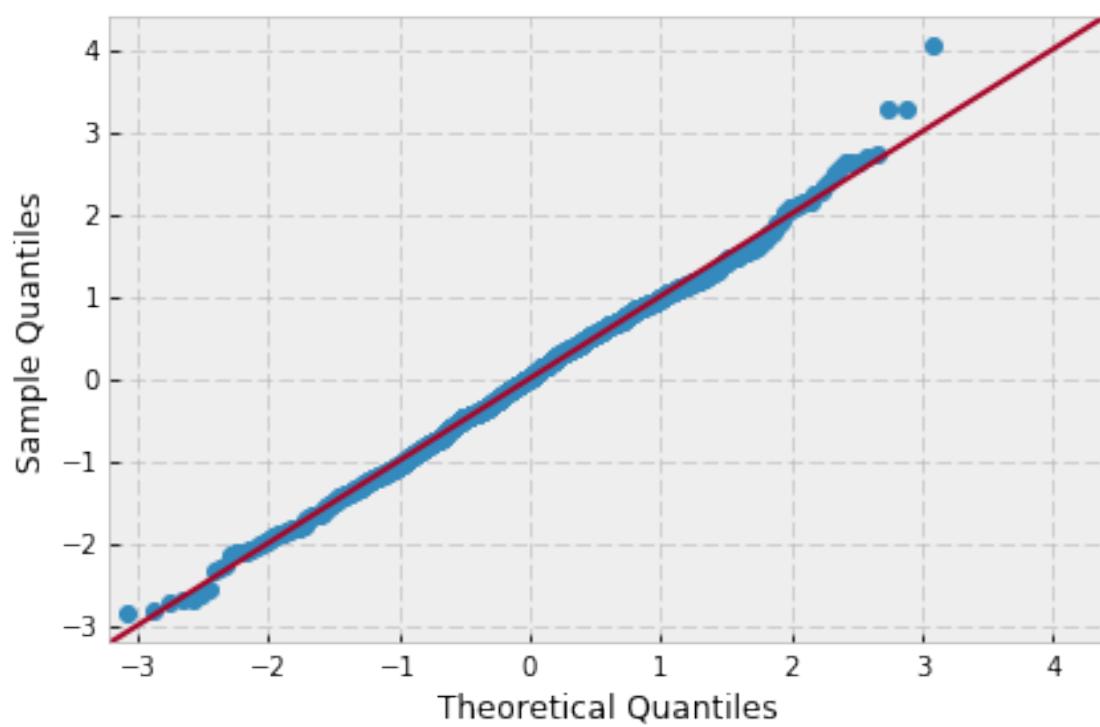
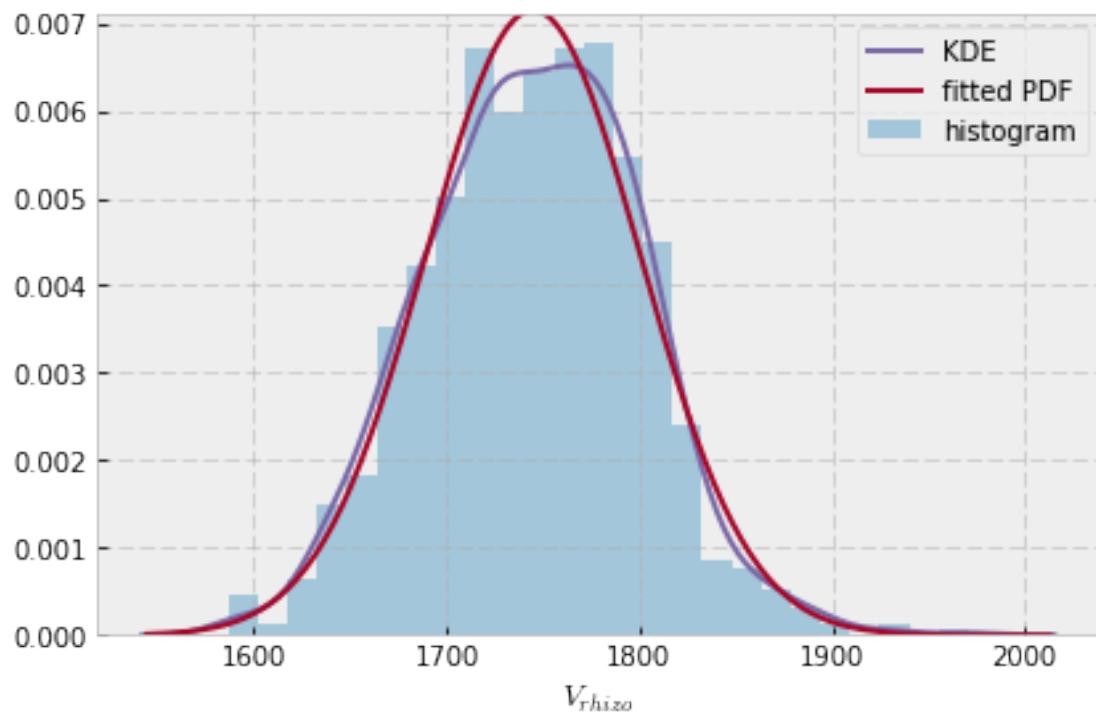
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(),loc=1742.5132220136336, scale=58.84043423105184)
DescribeResult(nobs=1000, minmax=(1552.2468716082542, 1932.7940041302741)
               mean=1742.5132220136336, variance=3465.6623628615985,
               skewness=-0.06337265159395698, kurtosis=-0.3205022157584376)

```

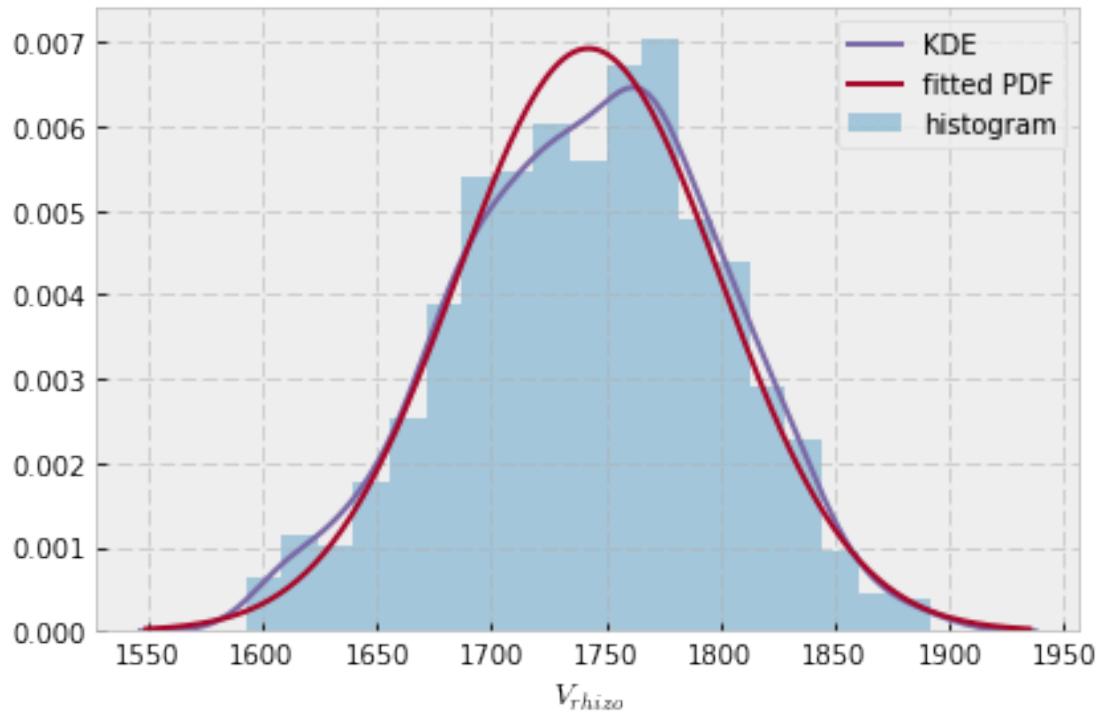


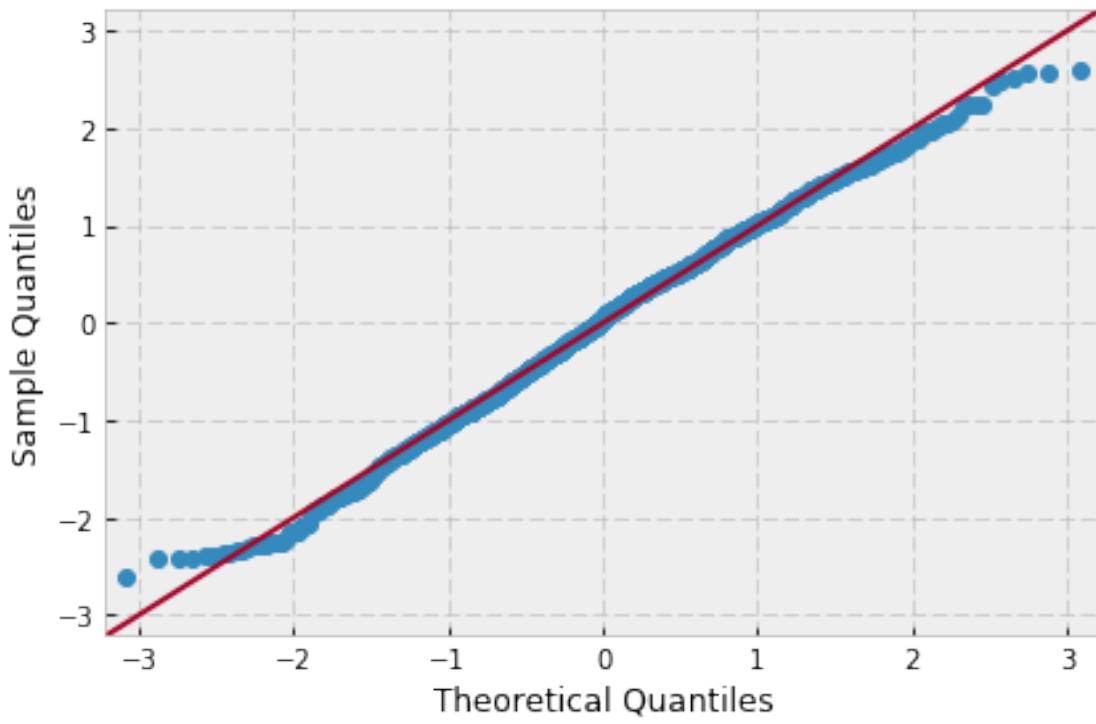


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(),loc=1744.8754330560132, scale=55.79615977685549)
DescribeResult(nobs=1000, minmax=(1586.6628339254091, 1970.7702733814087)
               mean=1744.8754330560132, variance=3116.3277736180048,
               skewness=0.042486310347704954, kurtosis=0.09269533445619693)
```

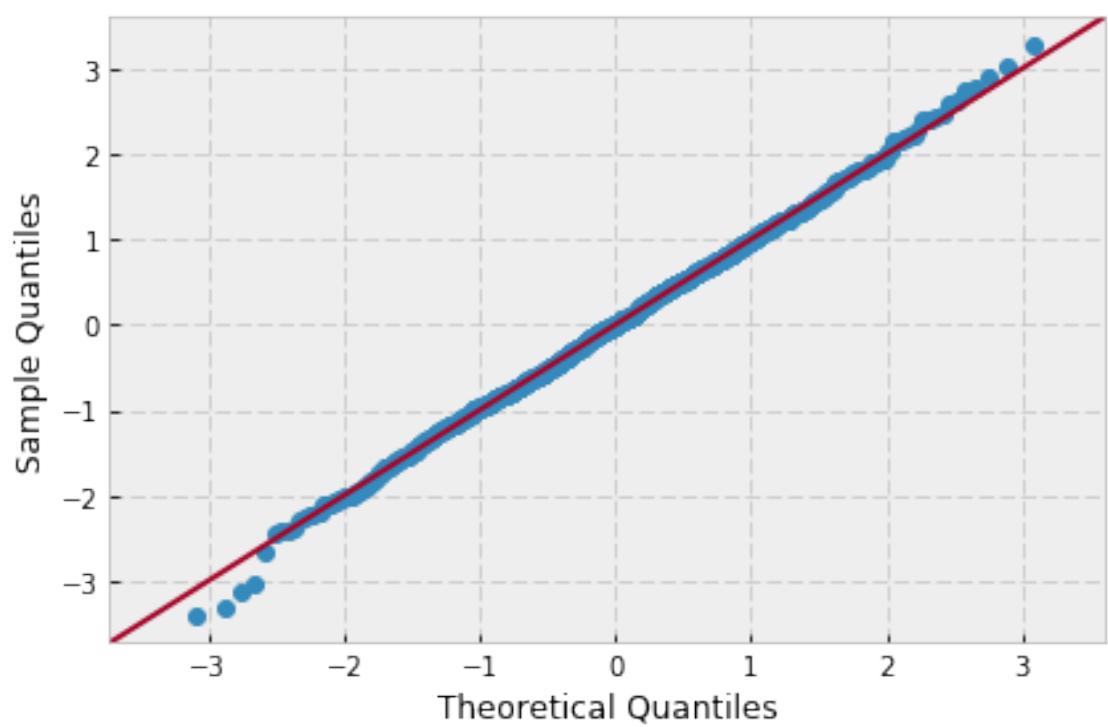
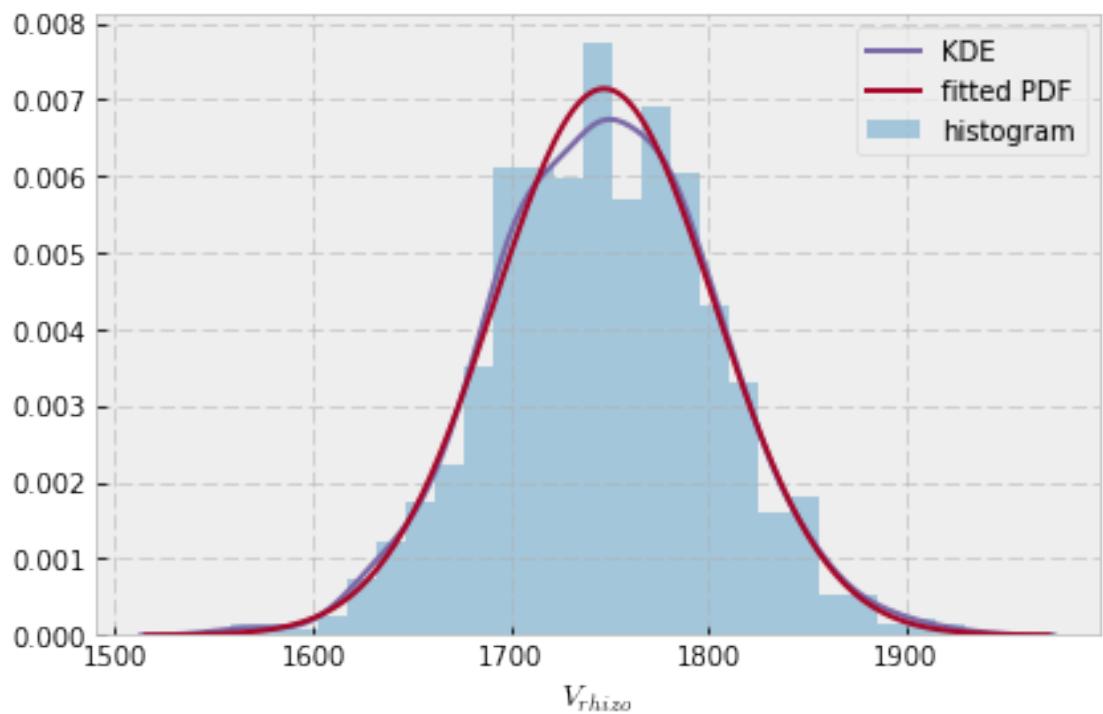


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=1742.443833939682, scale=57.570369388524426)
DescribeResult(nobs=1000, minmax=(1592.7268693962944, 1891.5589600895025)
               mean=1742.443833939682, variance=3317.665096627778,
               skewness=-0.1231138292683463, kurtosis=-0.38316748179687954)
```





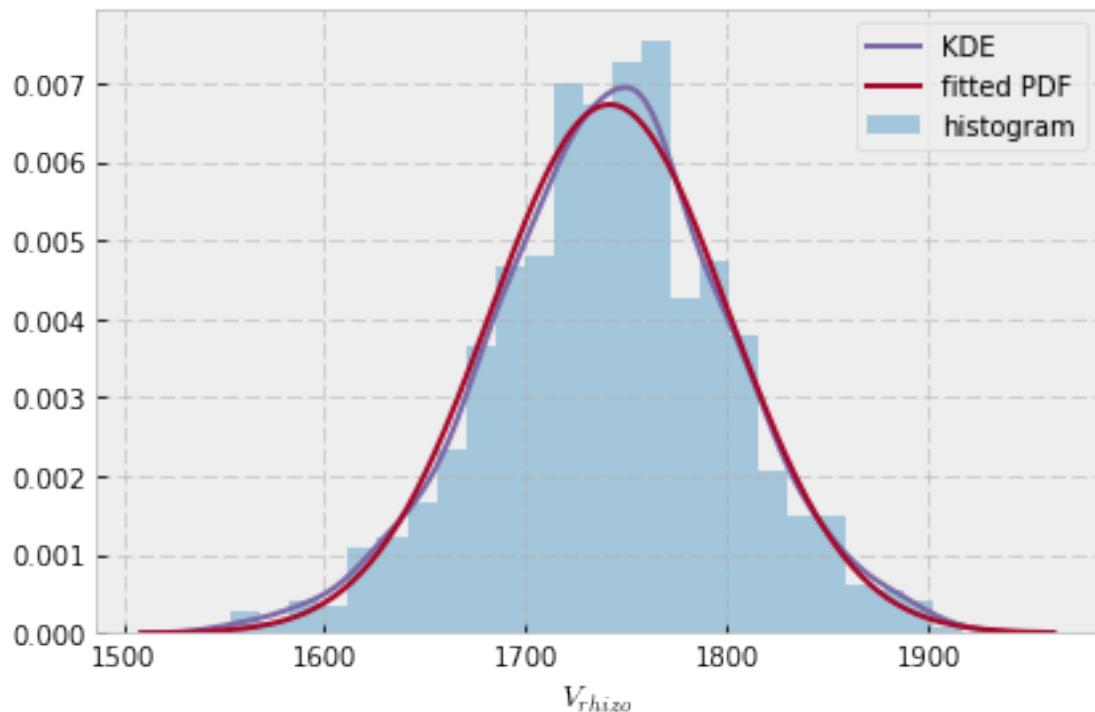
```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(),loc=1747.2015201219872, scale=55.84339812425714)
DescribeResult(nobs=1000, minmax=(1557.7791520841324, 1929.6929540674025)
               mean=1747.2015201219872, variance=3121.6067207850715,
               skewness=-0.007707661056550271, kurtosis=0.10830544613616988)
```

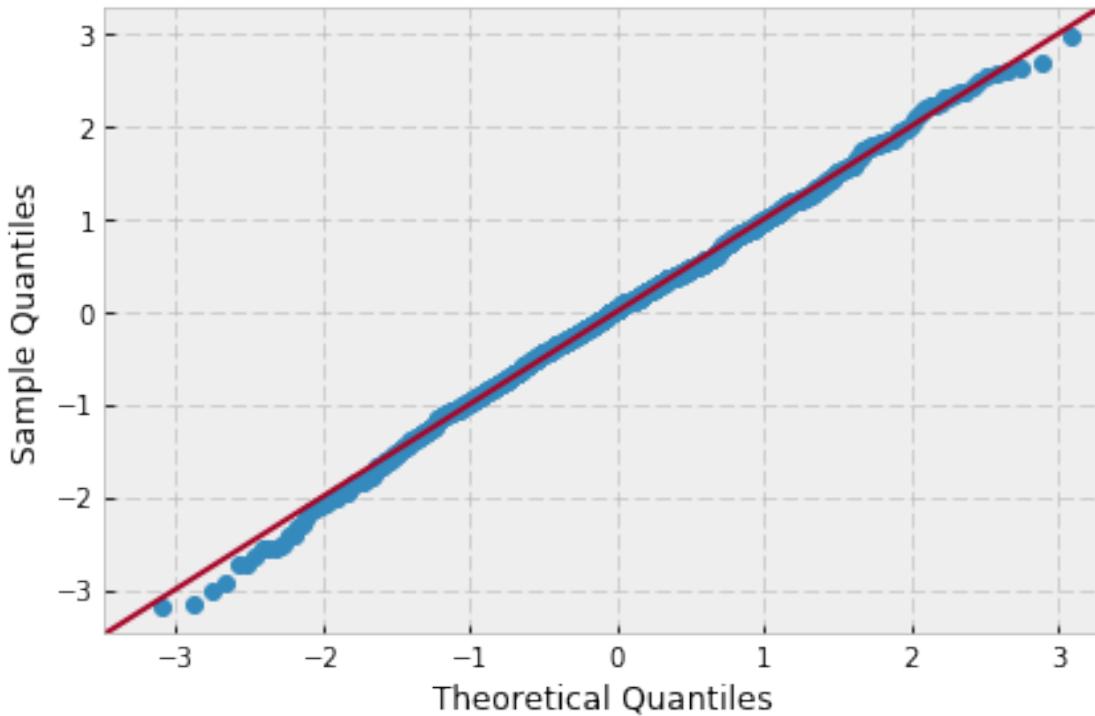


```

variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=1741.7189251327056, scale=59.318664938988114)
DescribeResult(nobs=1000, minmax=(1553.4937564911336, 1917.703726661108)
               mean=1741.7189251327056, variance=3522.2262363803179,
               skewness=-0.09338369956779327, kurtosis=0.13575480375463922)

```

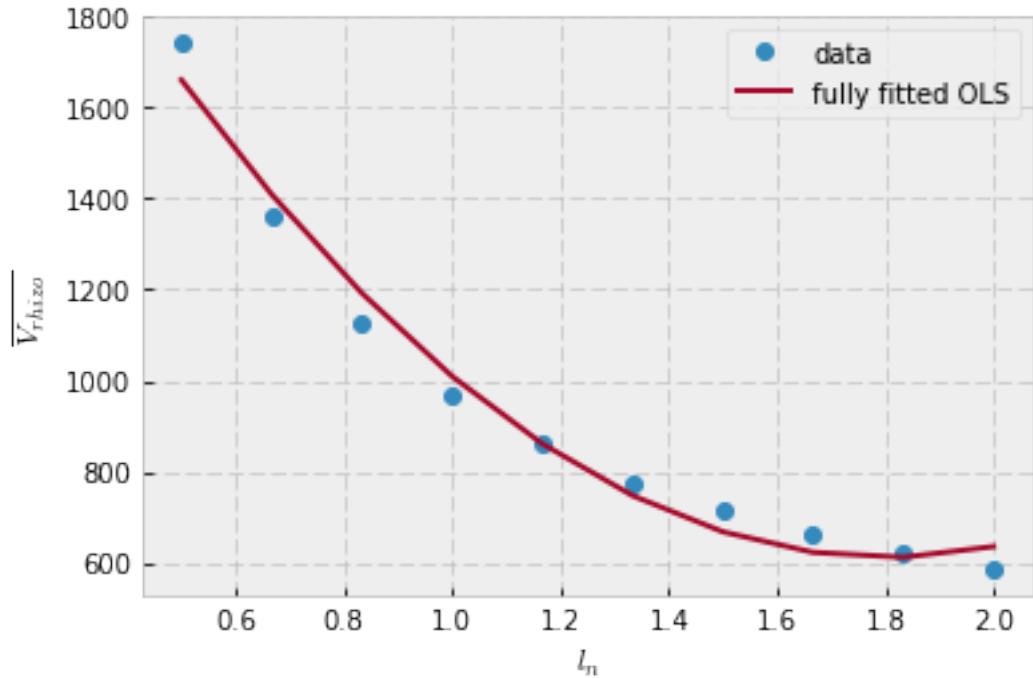




9.2 Parameter regression

```
In [24]: var_name = 'rhizosphere_volume_phosphate'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly1')
mean_reg('maxB', var_name, 'poly1')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly1')
mean_reg('N', var_name, 'poly1')
```

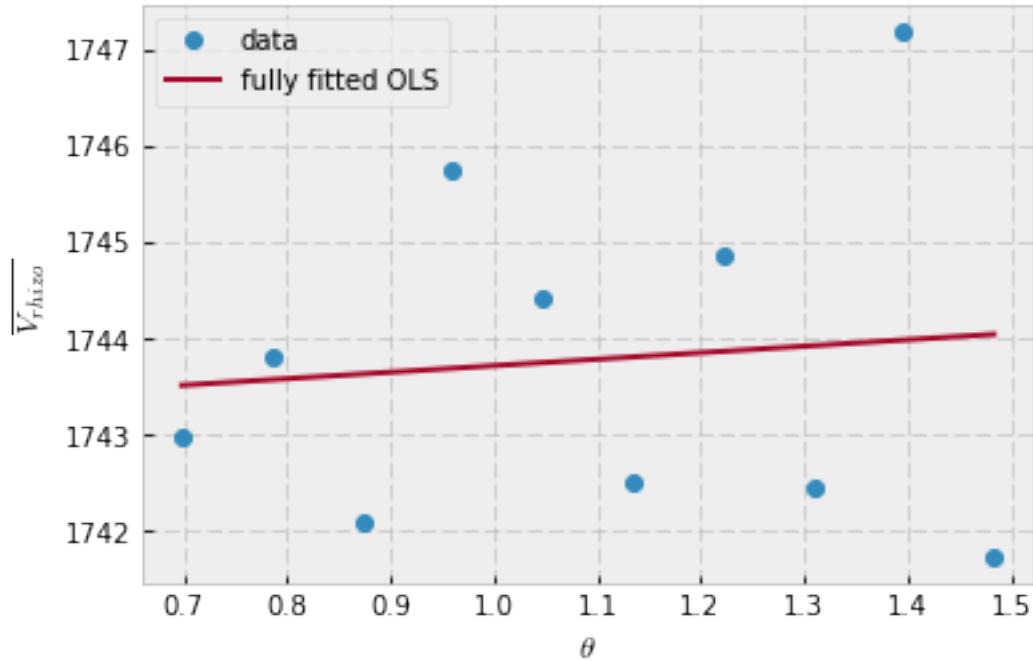
```
variable name: ln
model: poly2
r-squared: 0.9296452549541001
explained variance: 0.9297756460580479
```



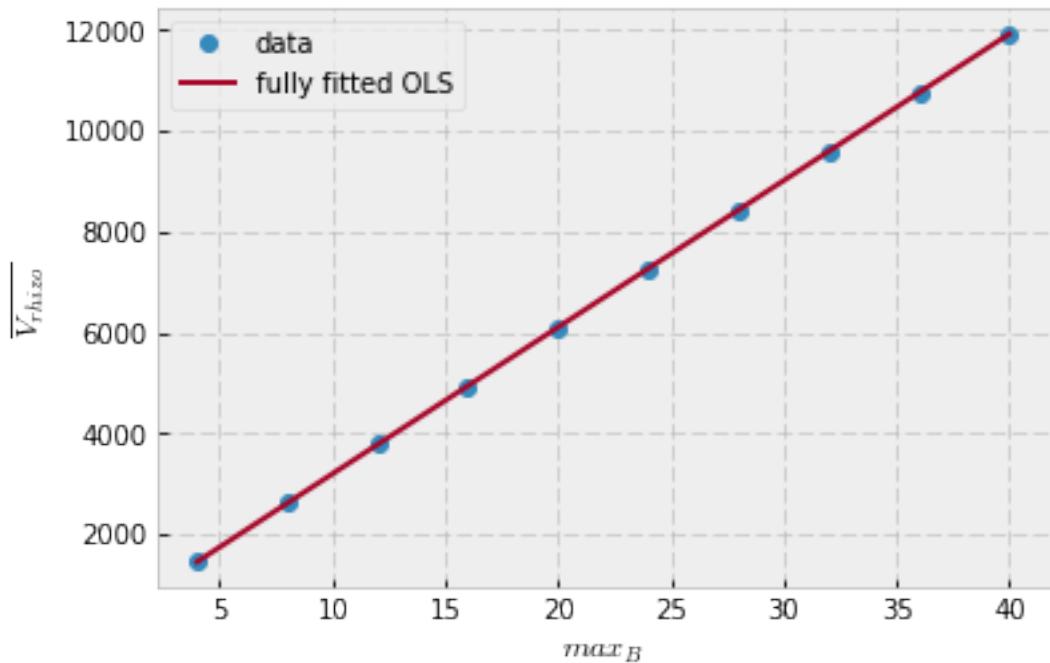
```

variable name: theta
model: poly1
r-squared: -0.6935905174218402
explained variance: -0.6921320396744748

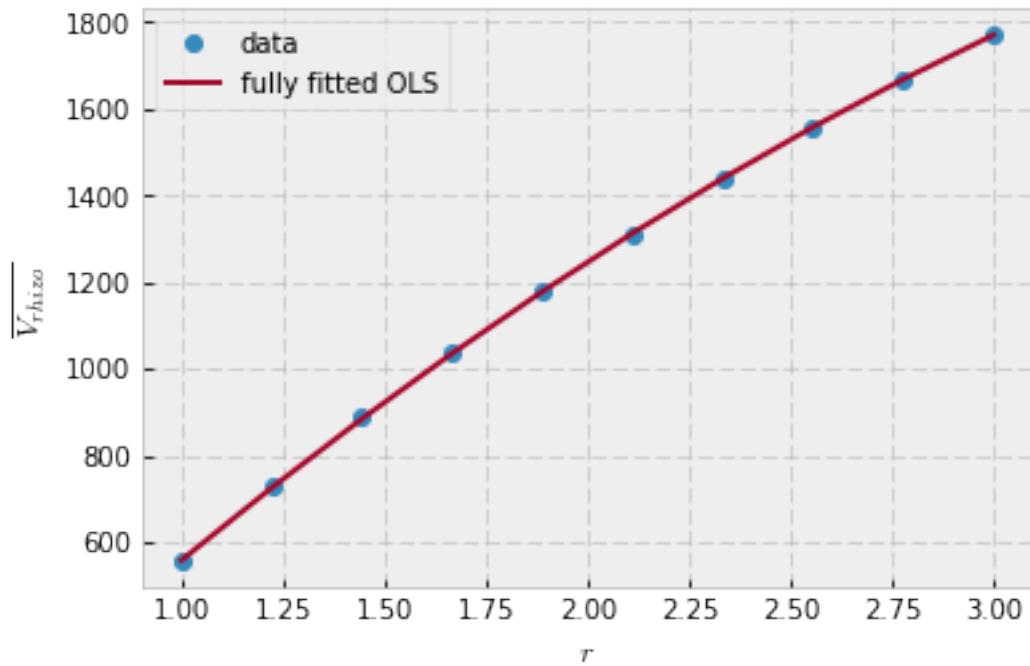
```



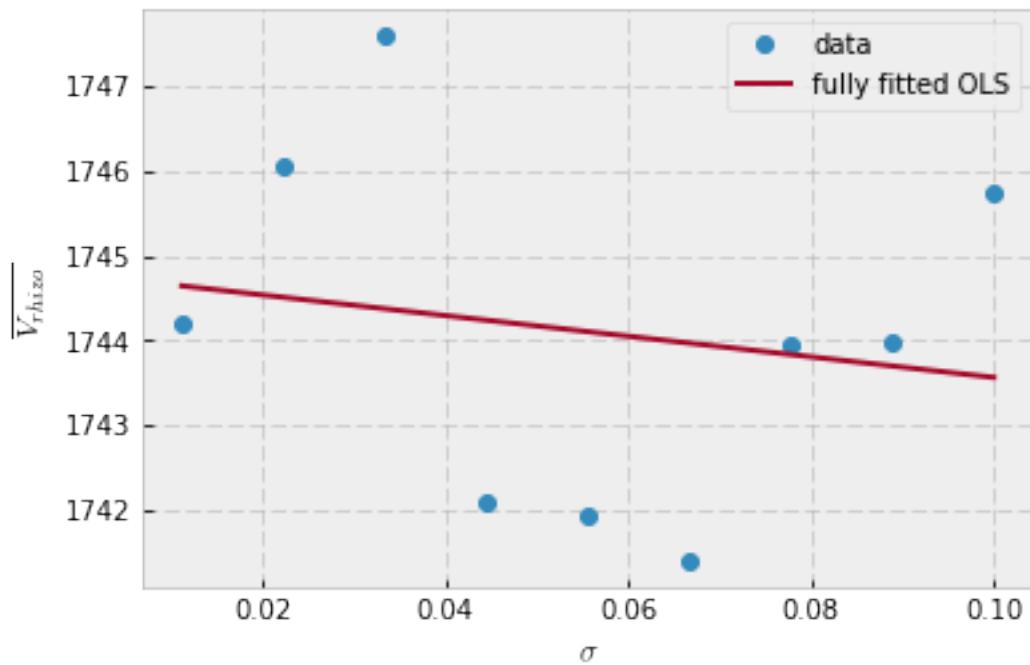
```
variable name: maxB  
model: poly1  
r-squared: 0.9999989115960336  
explained variance: 0.9999989135220785
```



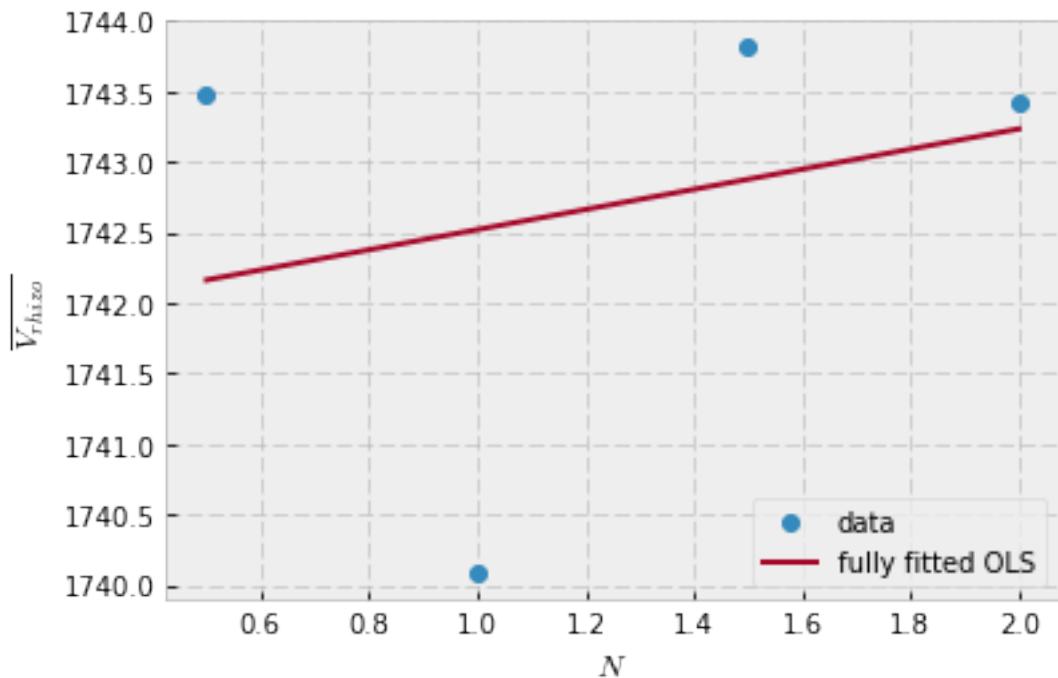
```
variable name: r  
model: poly2  
r-squared: 0.9998723695196802  
explained variance: 0.9998724993039633
```



```
variable name: sigma
model: poly1
r-squared: -0.5291843415548669
explained variance: -0.52245448652538
```



```
variable name: N
model: poly1
r-squared: -2.6205565151214705
explained variance: -2.4015435303753105
```

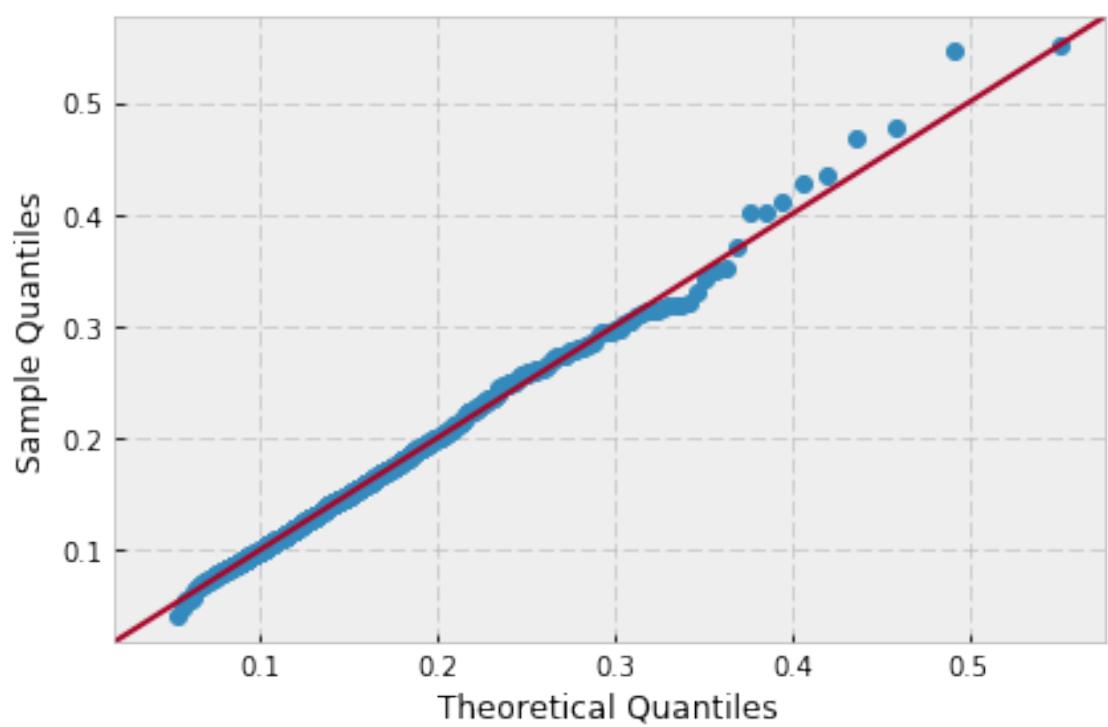
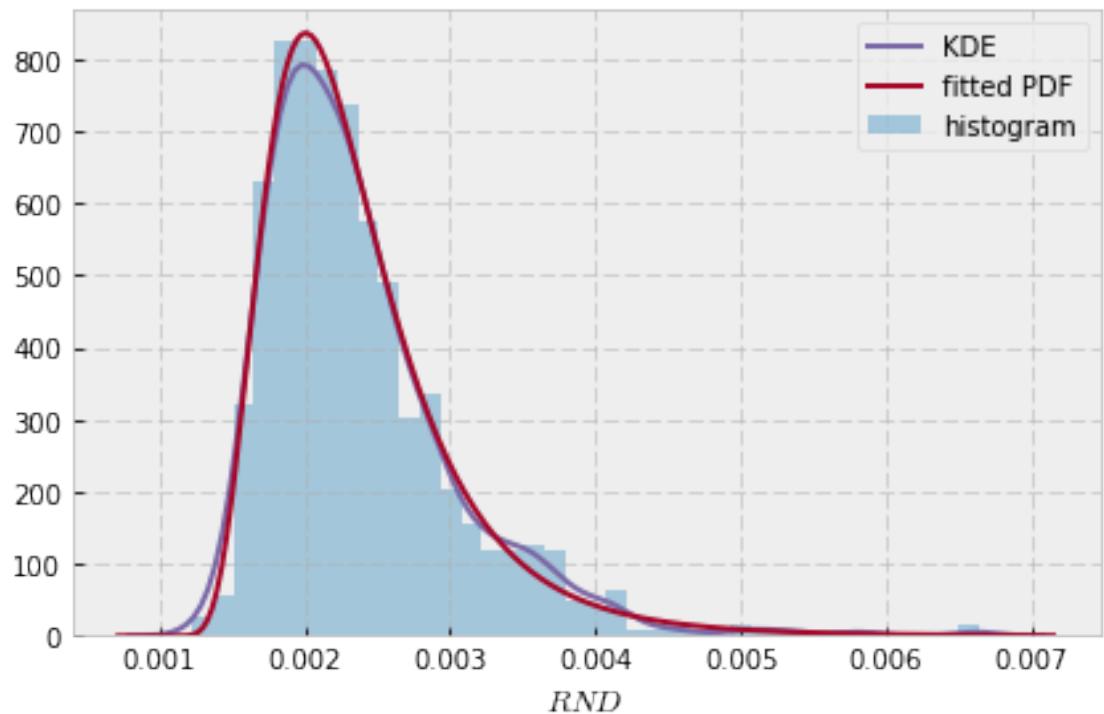


10 Root tip density

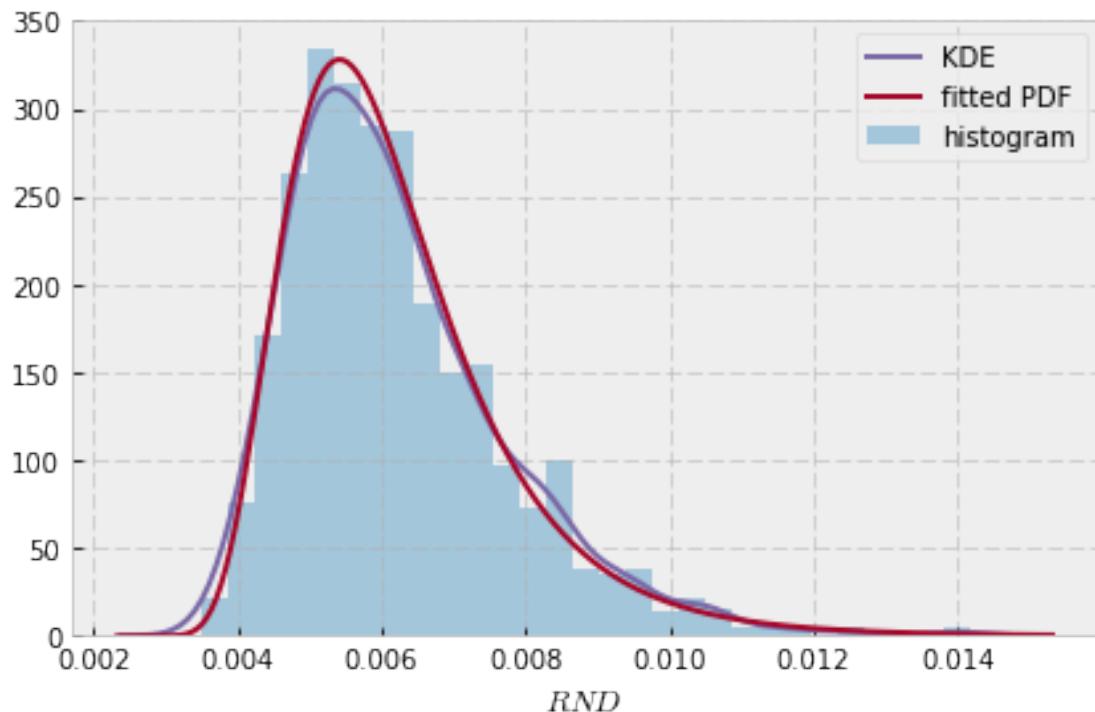
10.1 Probability distributions

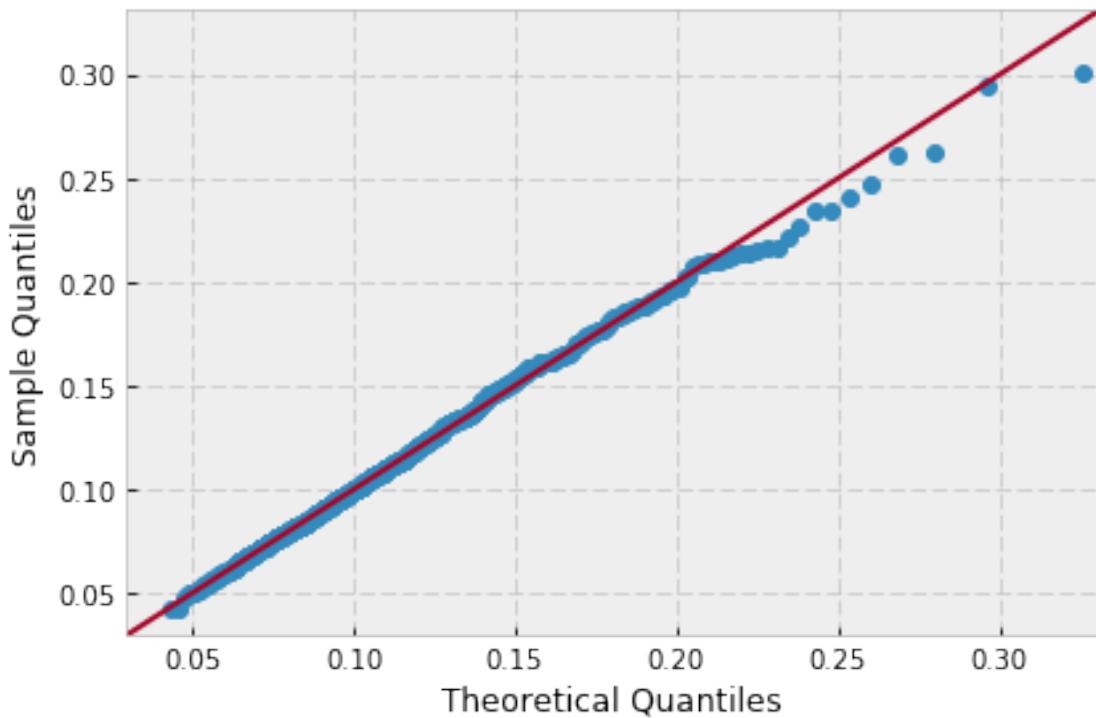
```
In [25]: plot_dist('root_tip_density', 'inverse-gamma')
```

```
variable name: N
variable value: 0.5
distribution: inverse-gamma(shape=(7.6597523723327594,), loc=0.0007738103072321231, scale=0.01065
DescribeResult(nobs=1000, minmax=(0.0012194689788932209, 0.0066462516400040165)
    mean=0.0023745120625213167, variance=4.4927062113585908e-07,
    skewness=1.7987372939645103, kurtosis=5.540434645730894)
```

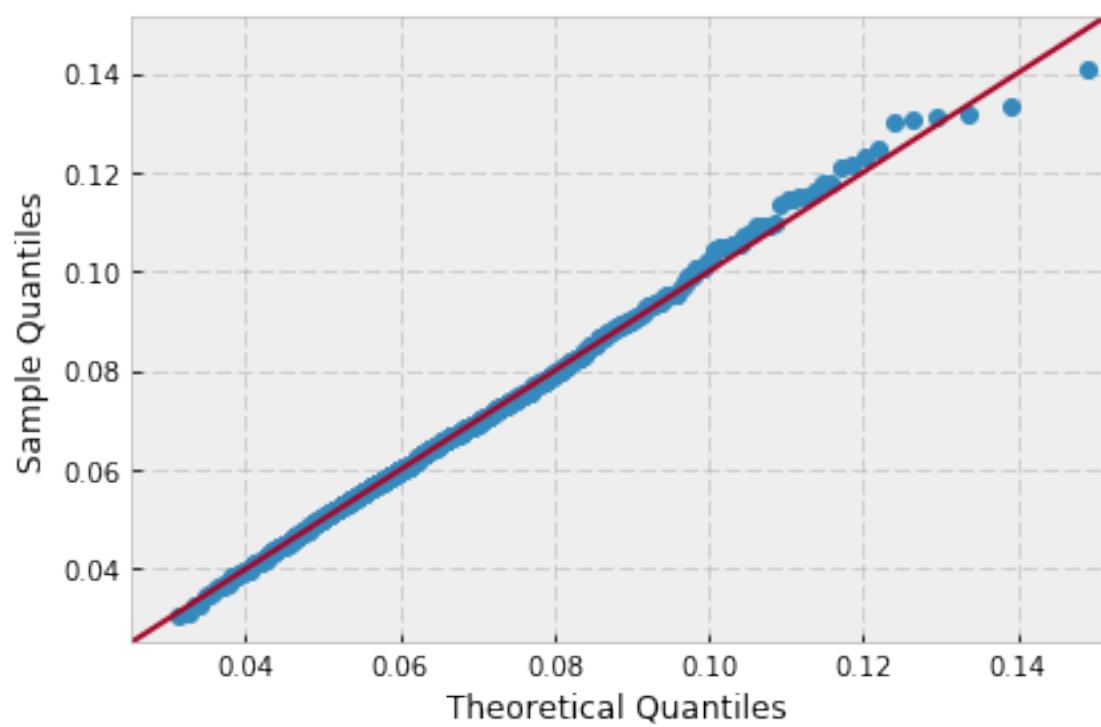
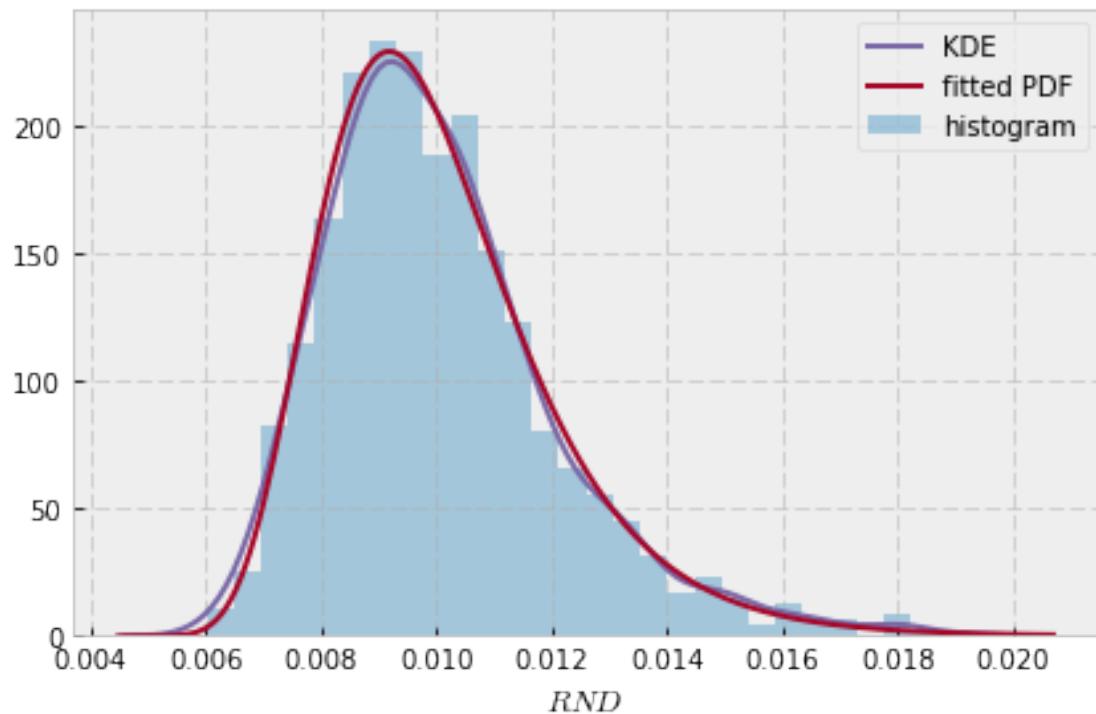


```
variable name: N
variable value: 1.0
distribution: inverse-gamma(shape=(10.217187108181434,),loc=0.001727217434534203, scale=0.041322
DescribeResult(nobs=1000, minmax=(0.003477241404873098, 0.014168429118523599)
               mean=0.006207526806693309, variance=2.3353382924956831e-06,
               skewness=1.2165848594478115, kurtosis=2.136160019340233)
```

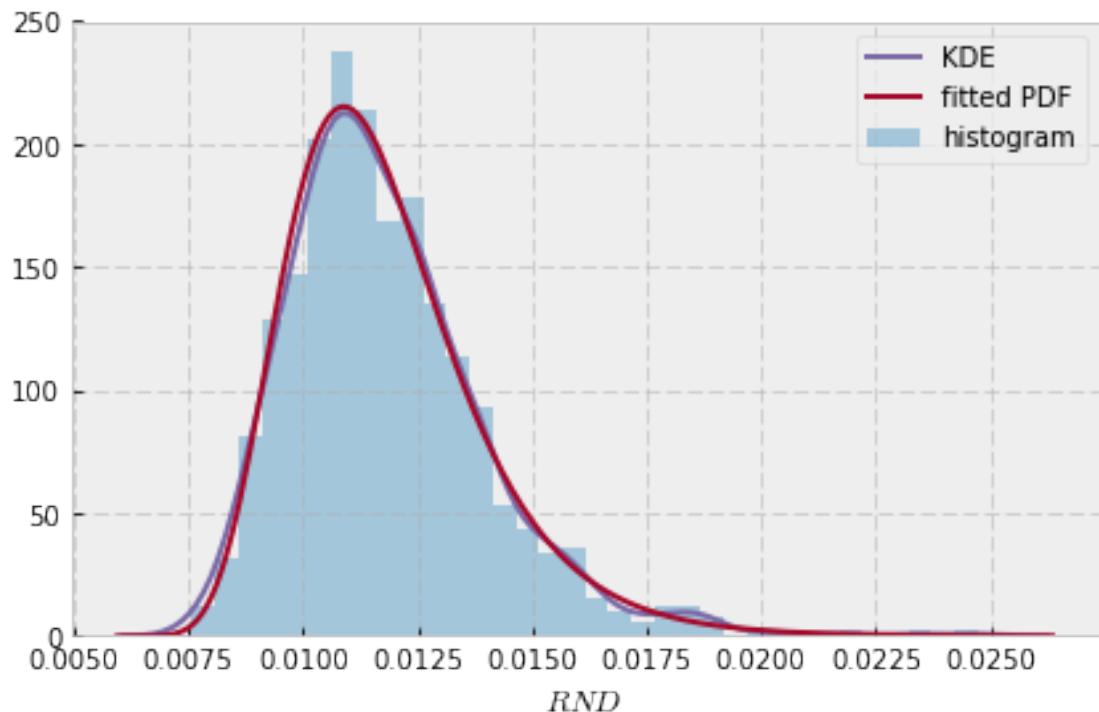


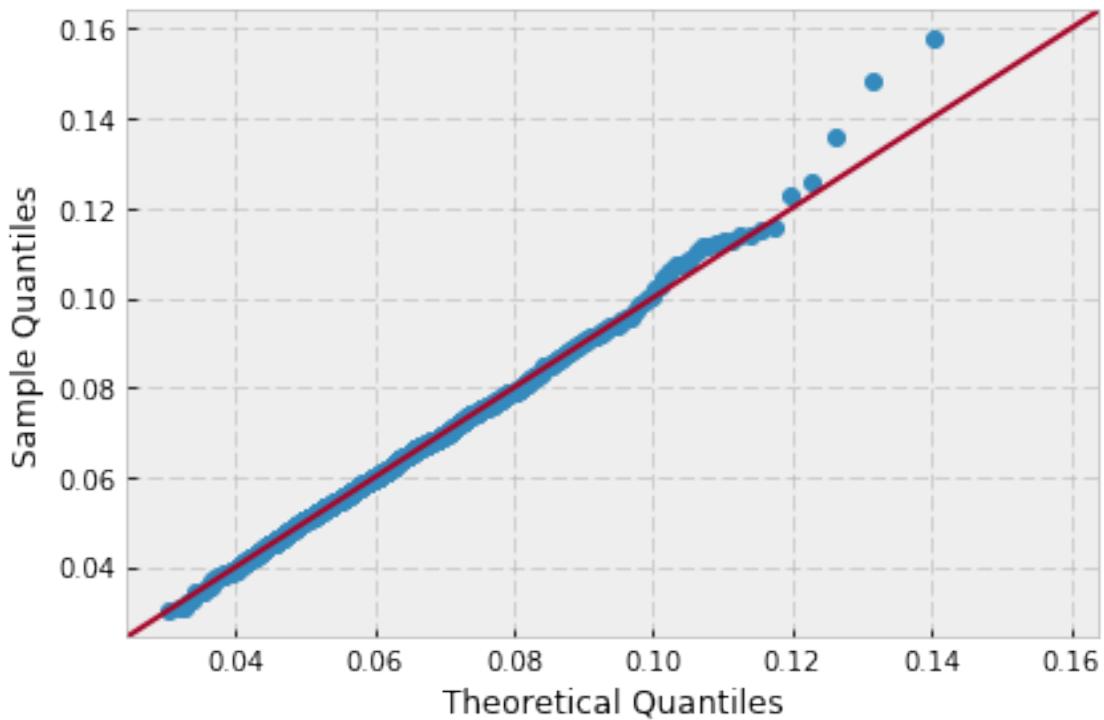


```
variable name: N
variable value: 1.5
distribution: inverse-gamma(shape=(16.497825997435555,),loc=0.0023547979153463808, scale=0.11935
DescribeResult(nobs=1000, minmax=(0.0060070844987077255, 0.019162329361833347)
               mean=0.010056338844834688, variance=4.0965361380441957e-06,
               skewness=1.077201066808754, kurtosis=1.739863332699911)
```

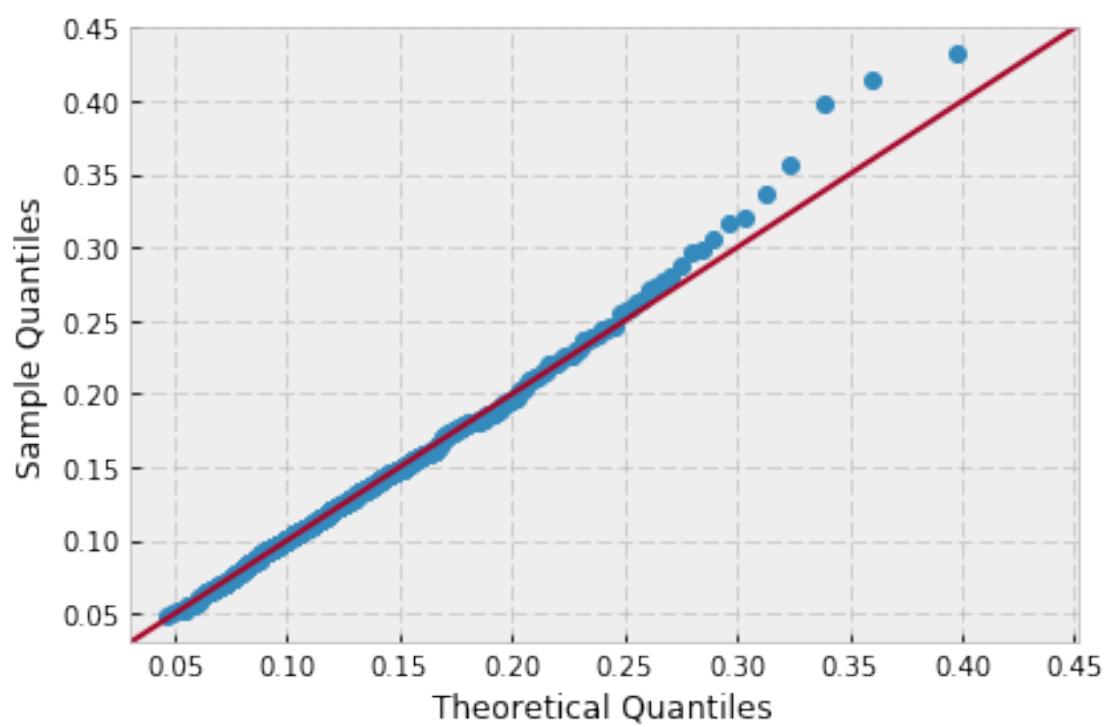
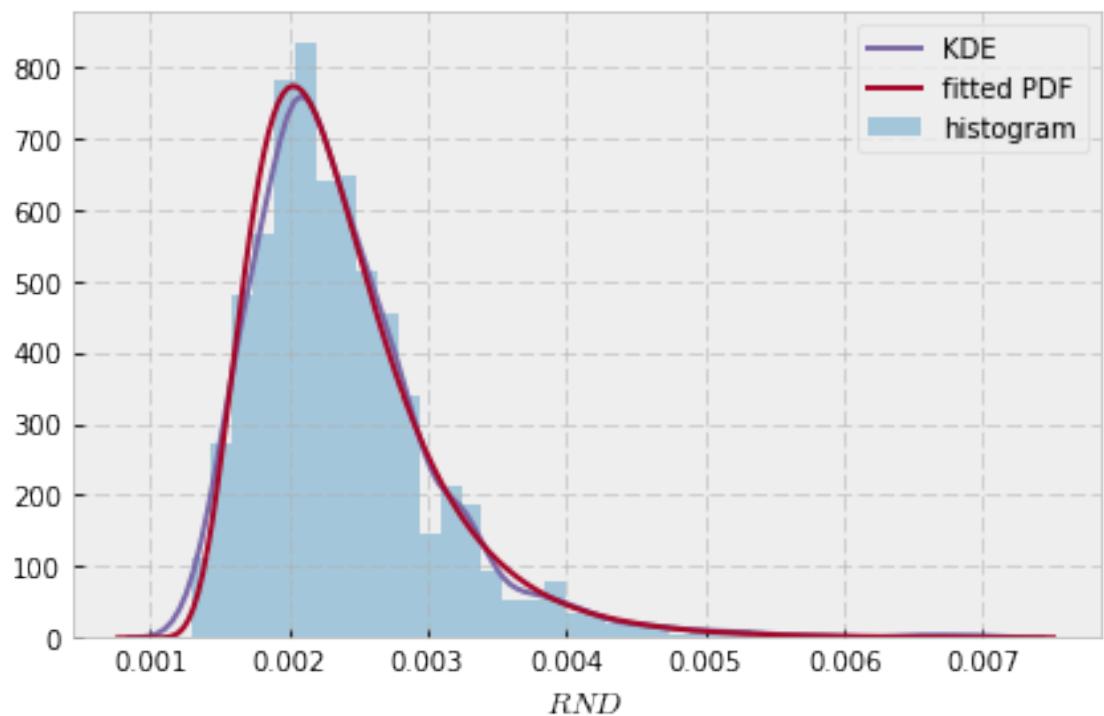


```
variable name: N
variable value: 2.0
distribution: inverse-gamma(shape=(17.142062306770406,),loc=0.0034535697946684206, scale=0.13482
DescribeResult(nobs=1000, minmax=(0.0075749480225817574, 0.024724846892995197)
    mean=0.011807156498800863, variance=4.6775603941161935e-06,
    skewness=1.206583363116419, kurtosis=2.9682170906933987)
```

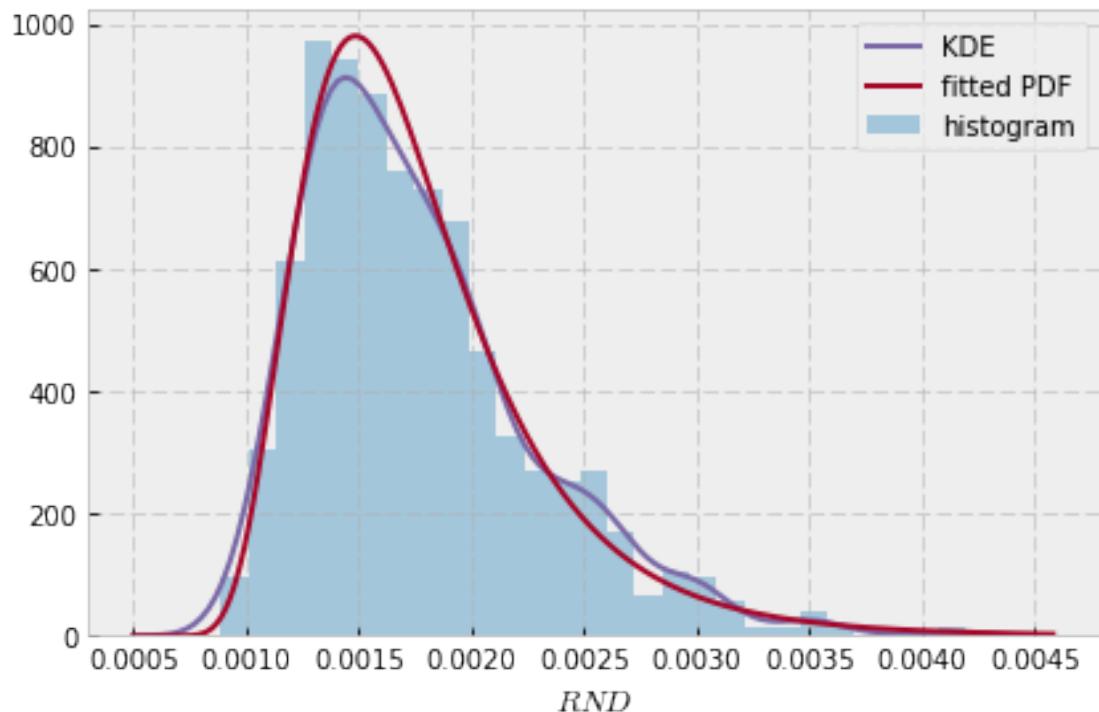


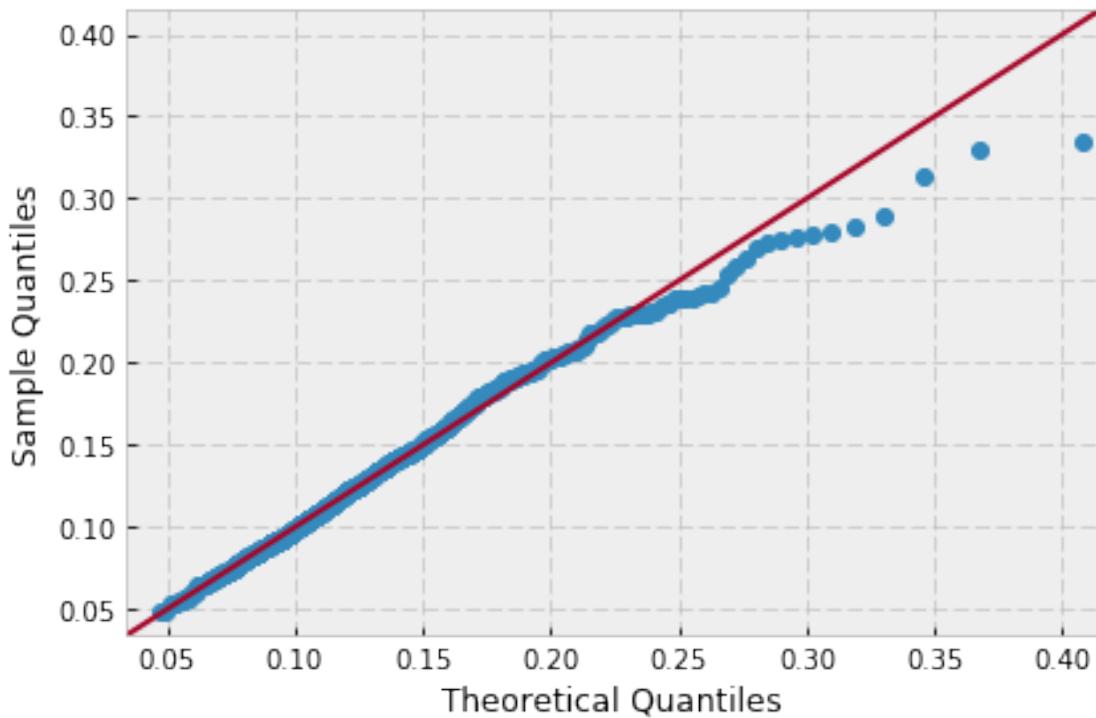


```
variable name: ln
variable value: 0.5
distribution: inverse-gamma(shape=(9.1210016782027523,),loc=0.0005645128220015788, scale=0.01485
DescribeResult(nobs=1000, minmax=(0.0012940575542343799, 0.0069827658368684679)
               mean=0.002394167679528496, variance=4.8250983542434575e-07,
               skewness=1.850199652612085, kurtosis=6.297771527742082)
```

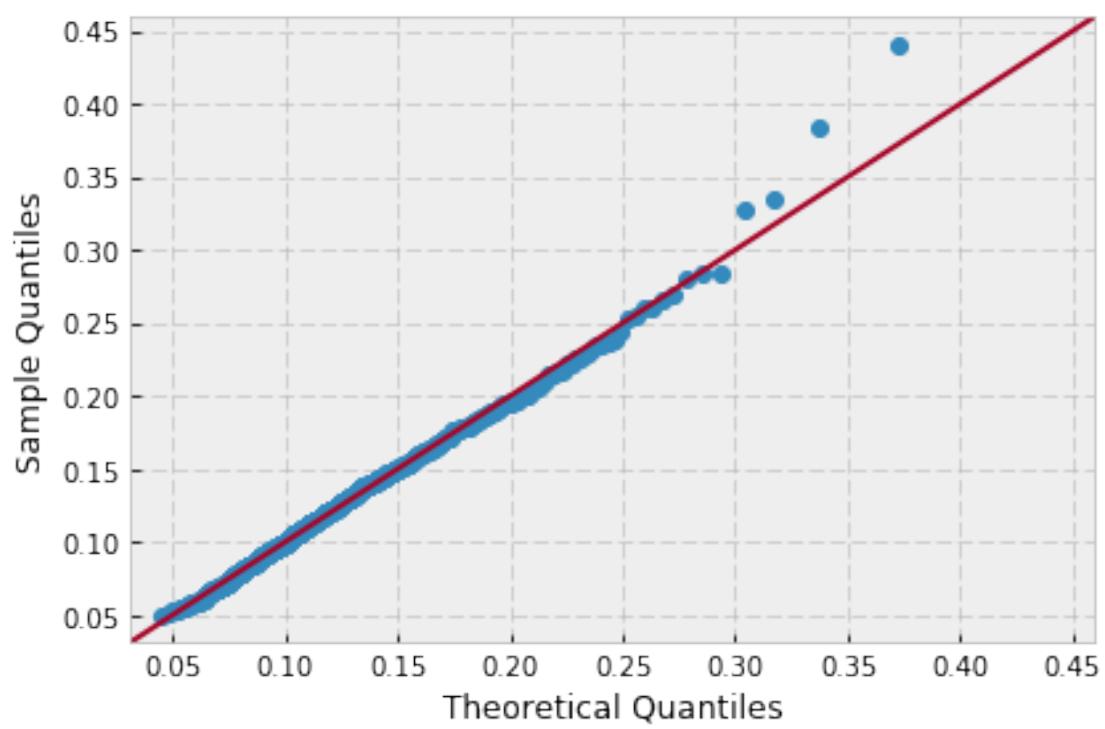
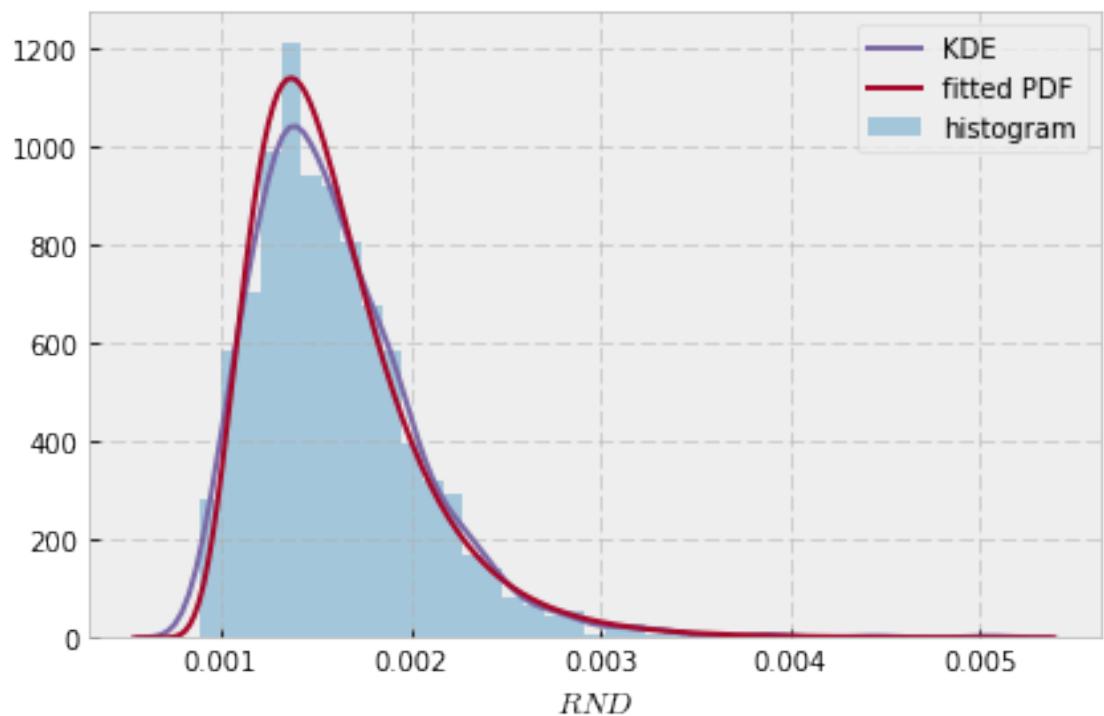


```
variable name: ln
variable value: 0.6666666666666666
distribution: inverse-gamma(shape=(9.0037156855751572,), loc=0.0003345351123713474, scale=0.01149
DescribeResult(nobs=1000, minmax=(0.00088785719367103985, 0.0041880970030089767)
               mean=0.0017690232568291877, variance=2.6977151491494418e-07,
               skewness=1.1355191590879237, kurtosis=1.5202045296035873)
```

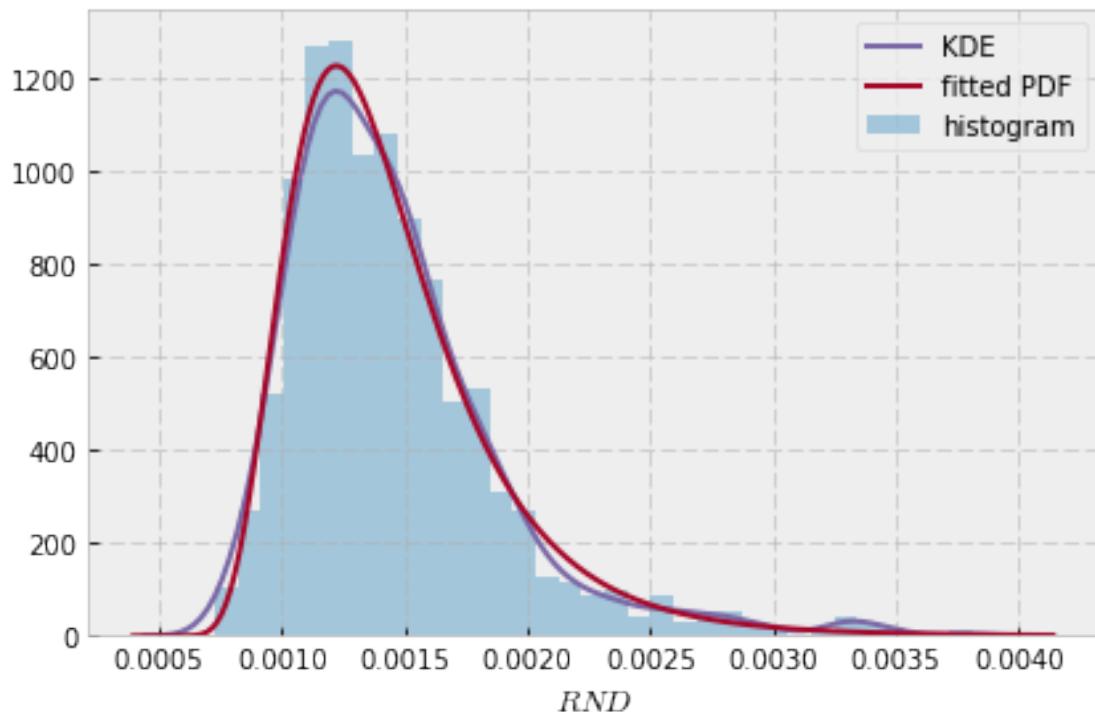


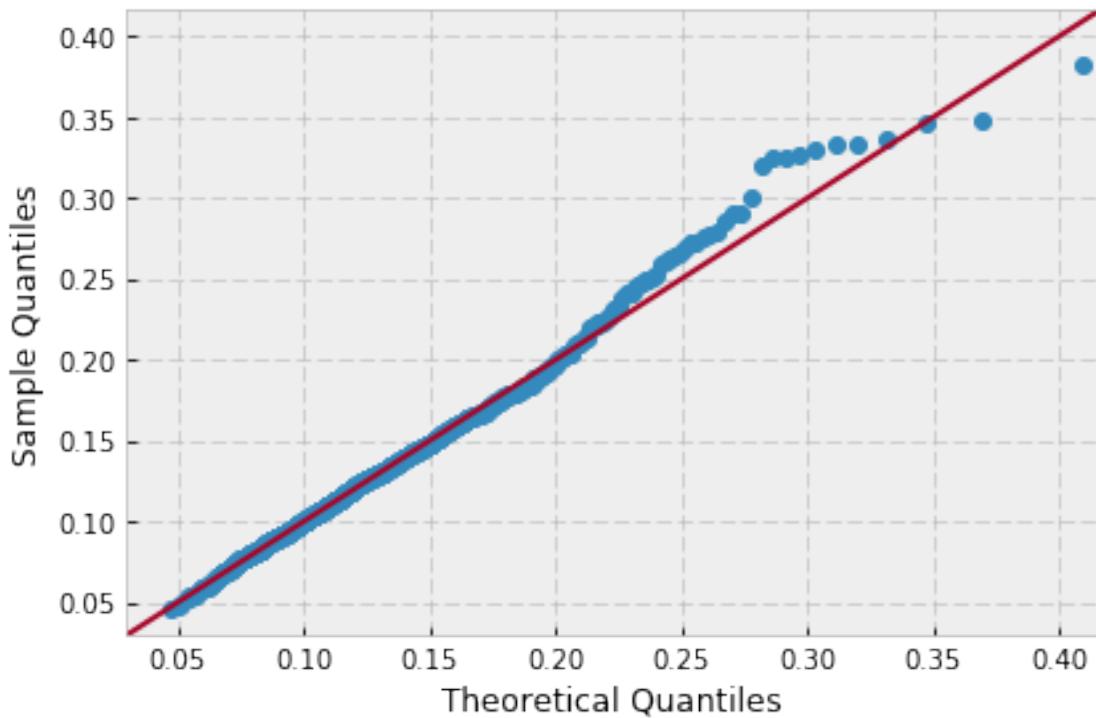


```
variable name: ln
variable value: 0.8333333333333333
distribution: inverse-gamma(shape=(9.4729965252635804,), loc=0.0003506674008919872, scale=0.01066
DescribeResult(nobs=1000, minmax=(0.00088882780346902853, 0.0050426455876741005)
               mean=0.001609482843270437, variance=2.1003323272646491e-07,
               skewness=1.6610518050490881, kurtosis=5.979772731131083)
```

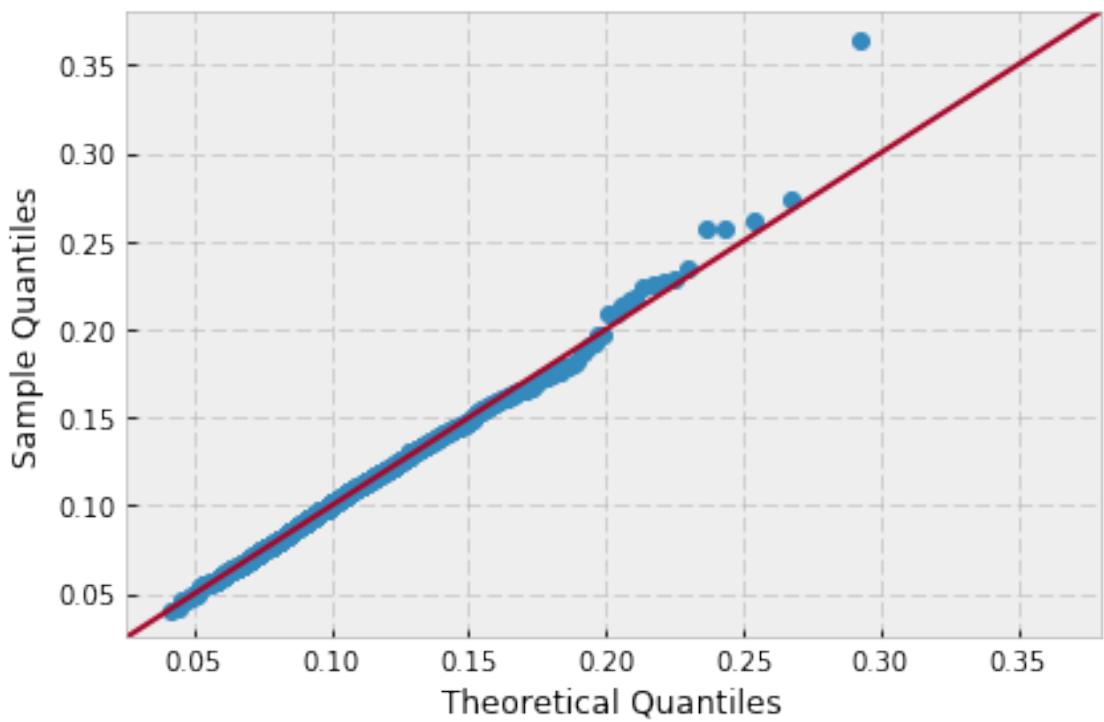
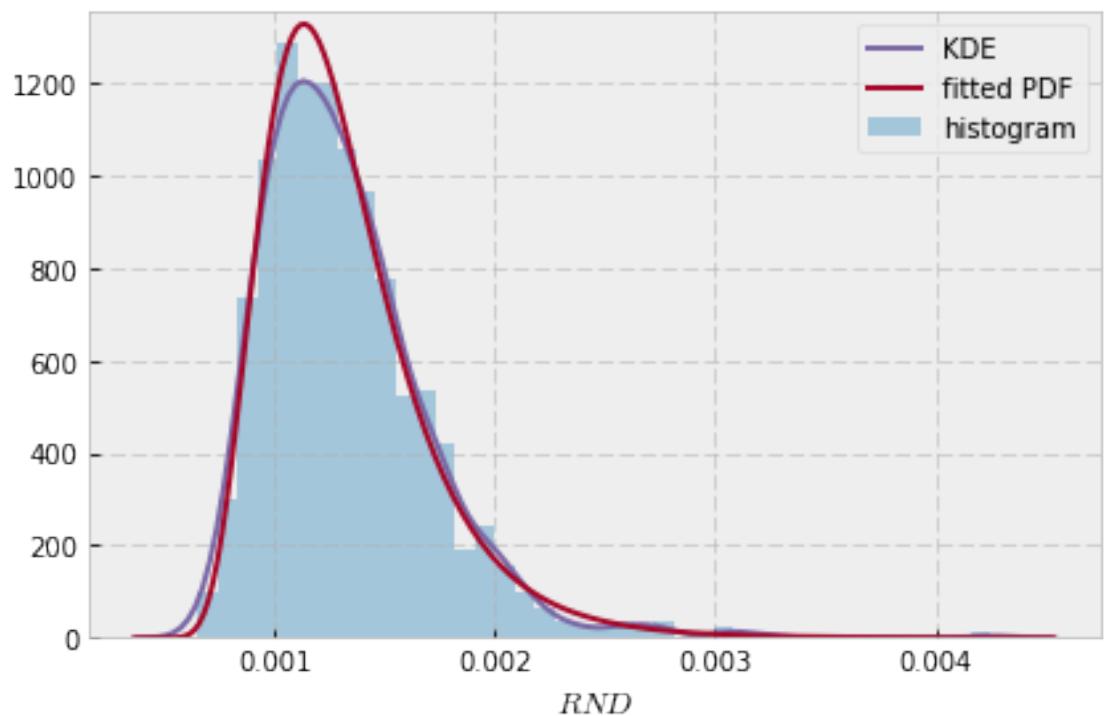


```
variable name: ln
variable value: 1.0
distribution: inverse-gamma(shape=(8.986494423448729,), loc=0.00030557152979978285, scale=0.00915
DescribeResult(nobs=1000, minmax=(0.00072521826947069665, 0.0038108885761589037)
    mean=0.0014521234386859203, variance=1.9152153667660772e-07,
    skewness=1.6776156320081734, kurtosis=4.239590077553565)
```

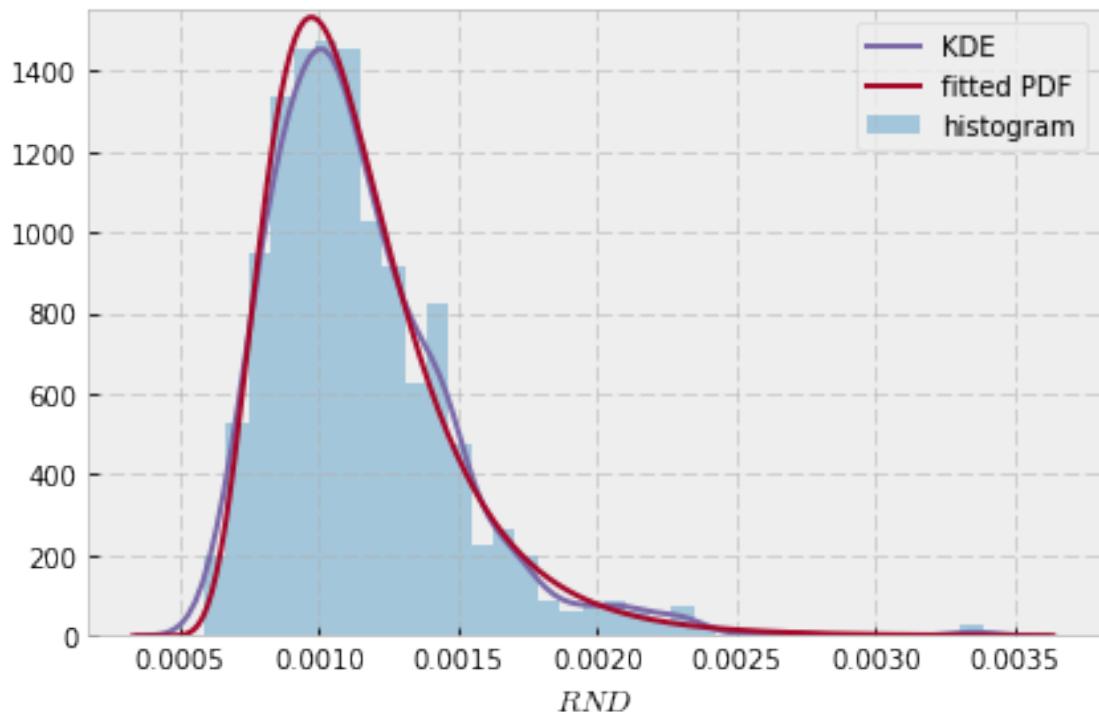


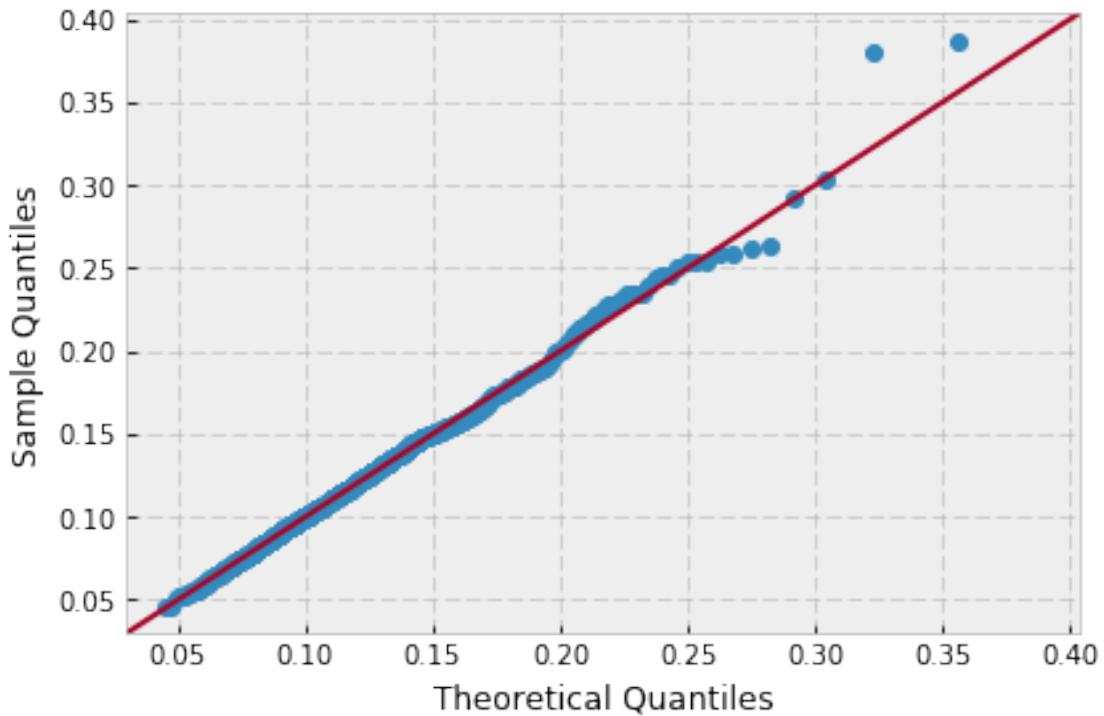


```
variable name: ln
variable value: 1.1666666666666665
distribution: inverse-gamma(shape=(10.857750316544212,), loc=0.0001926111208456953, scale=0.01115
DescribeResult(nobs=1000, minmax=(0.00065137940302353544, 0.0042507610747134624)
               mean=0.0013241123623787202, variance=1.4504561703332024e-07,
               skewness=1.5971218305009955, kurtosis=5.583879891518713)
```

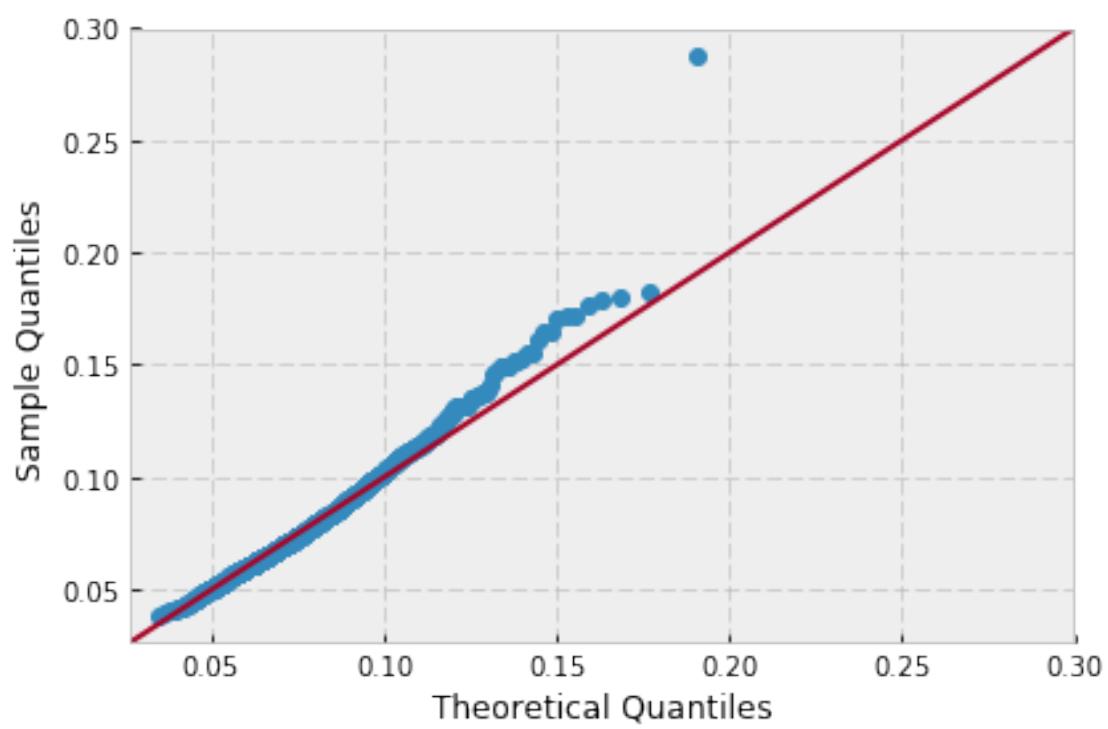
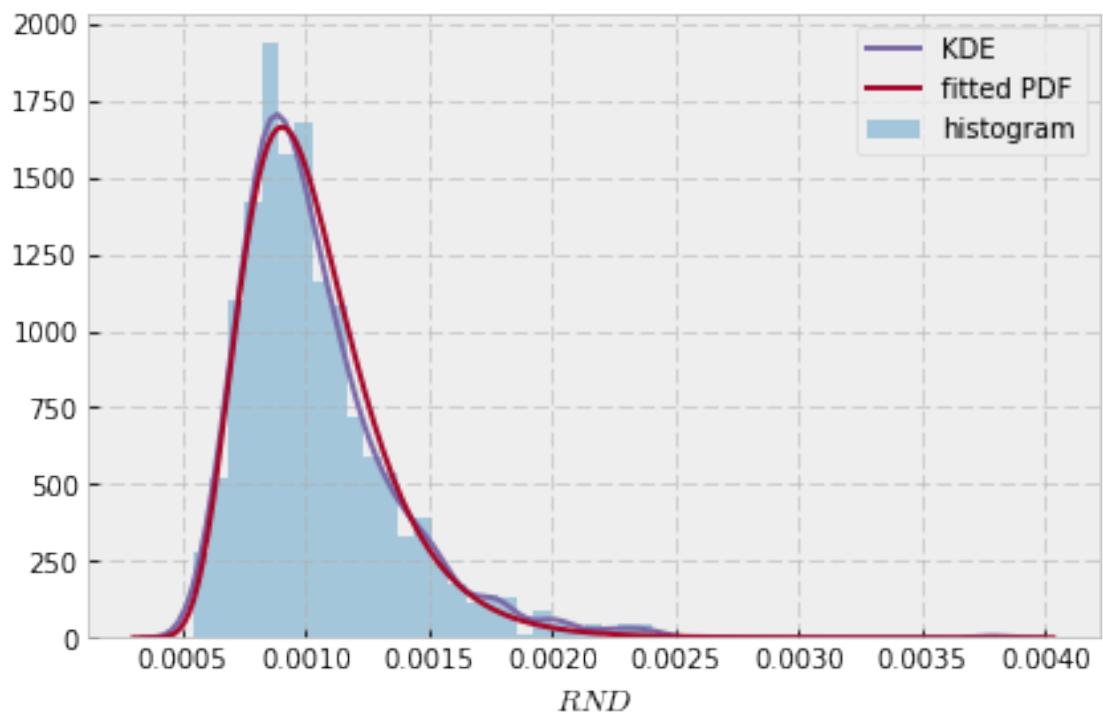


```
variable name: ln
variable value: 1.3333333333333333
distribution: inverse-gamma(shape=(9.7100570488957345,), loc=0.00020683093233207073, scale=0.0082
DescribeResult(nobs=1000, minmax=(0.00058759995682583683, 0.0033833777083214253)
               mean=0.0011484932467901645, variance=1.14330855487305e-07,
               skewness=1.5955571520748002, kurtosis=4.857350030483503)
```

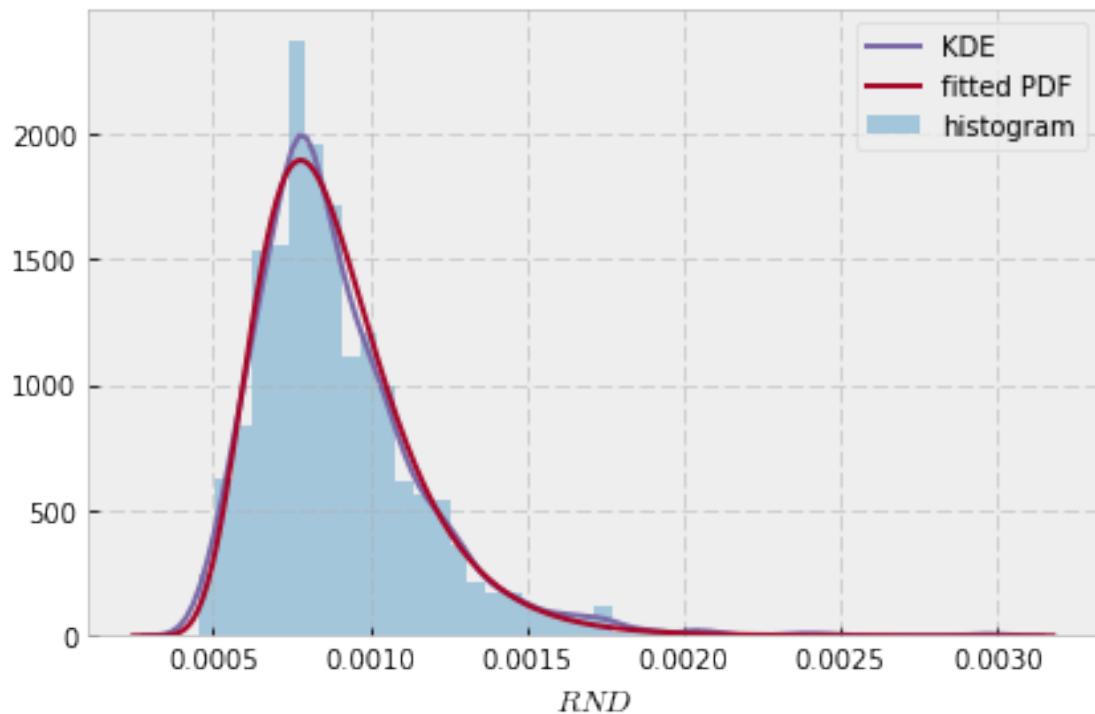


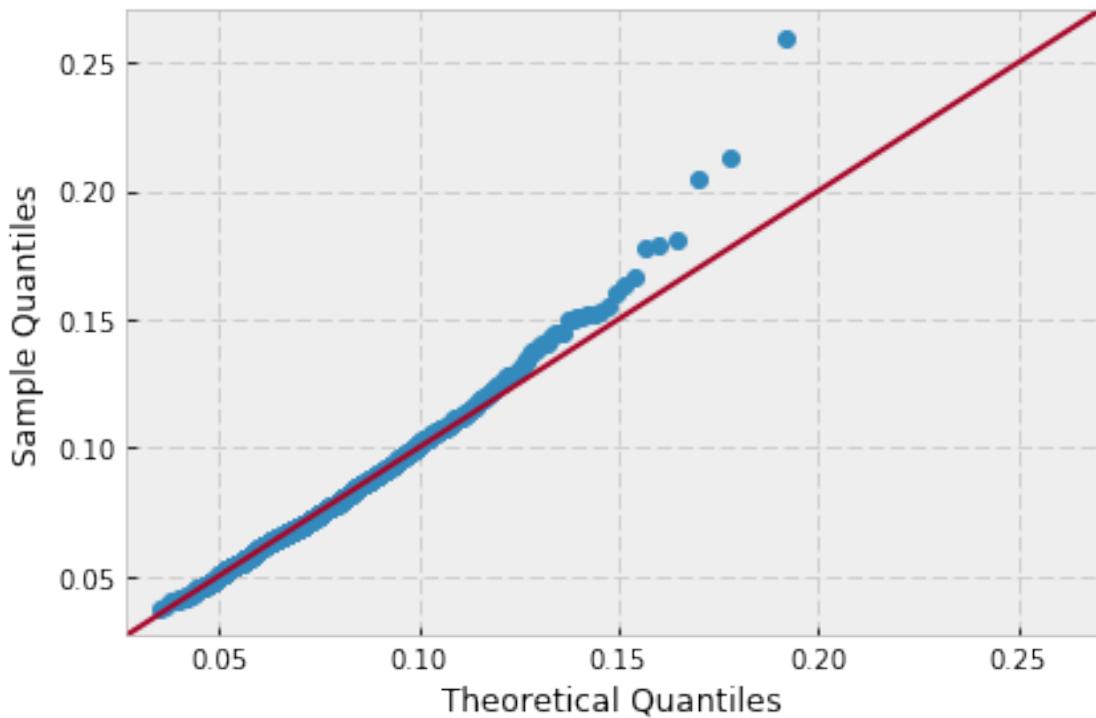


```
variable name: ln
variable value: 1.5
distribution: inverse-gamma(shape=(14.098366676974599,), loc=3.836548239606208e-05, scale=0.01306
DescribeResult(nobs=1000, minmax=(0.00054336439665528908, 0.0037910889184602971)
               mean=0.0010431340922836911, variance=1.0248459099133162e-07,
               skewness=1.8591718652867188, kurtosis=7.172608168152507)
```

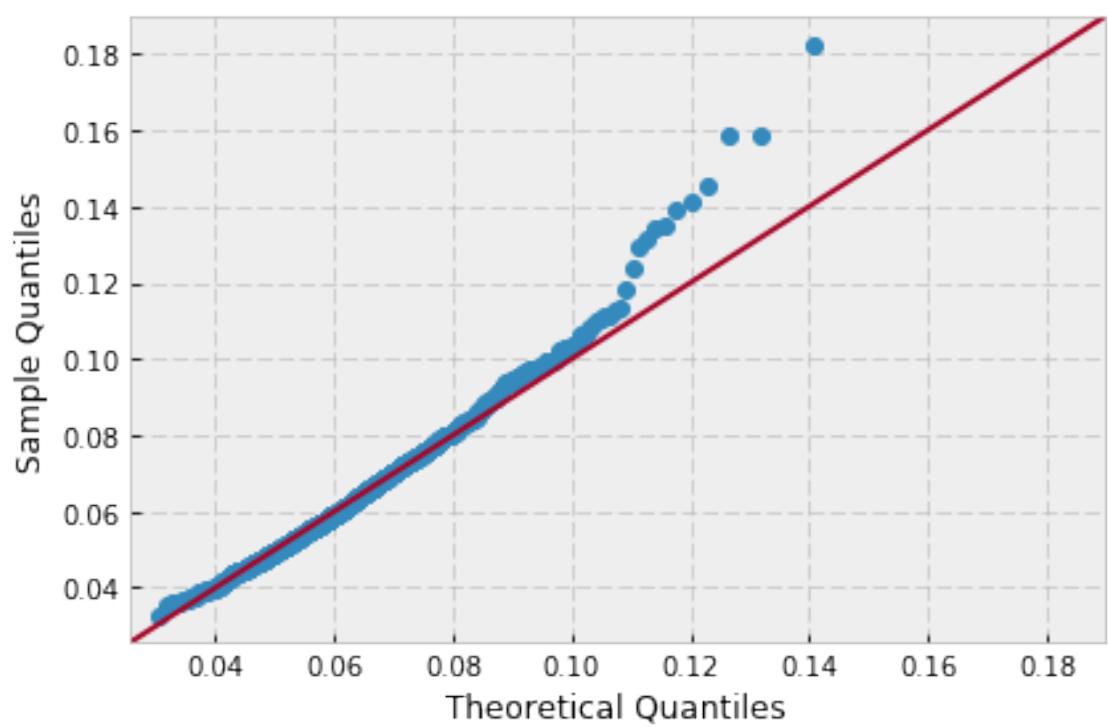
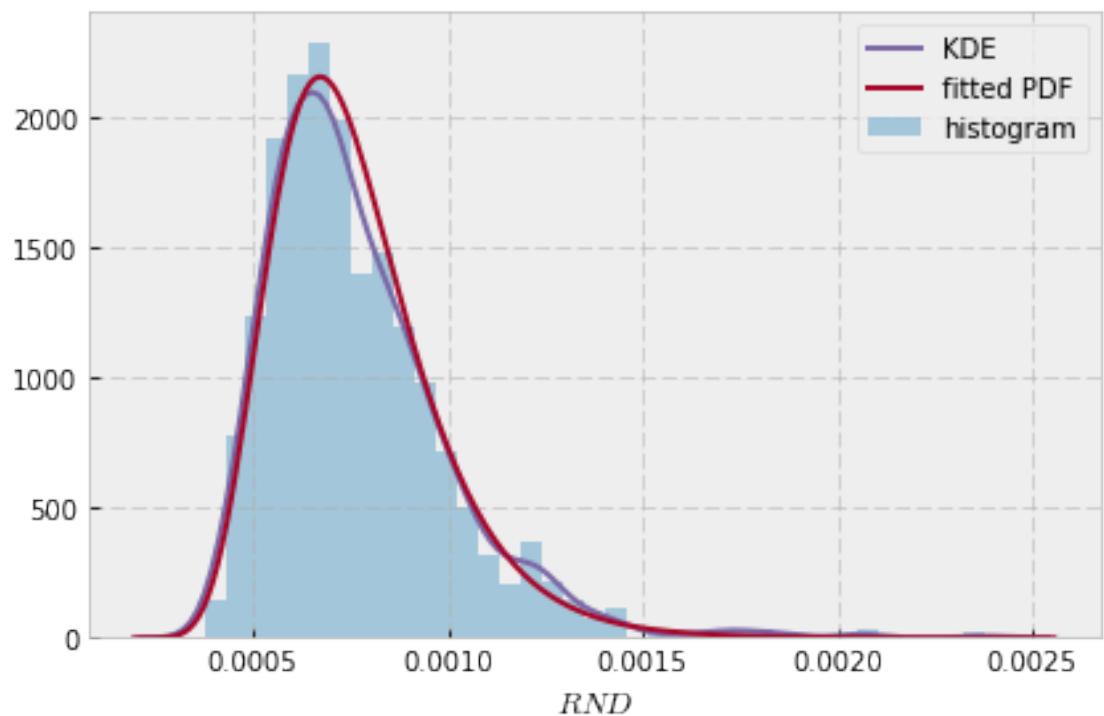


```
variable name: ln
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(14.021036219483733,),loc=2.1948851641173348e-05, scale=0.0113
DescribeResult(nobs=1000, minmax=(0.0004518266129493394, 0.0029715874314313745)
    mean=0.0008997161646819025, variance=7.4528321677554258e-08,
    skewness=1.7611585970234545, kurtosis=6.127348651580023)
```

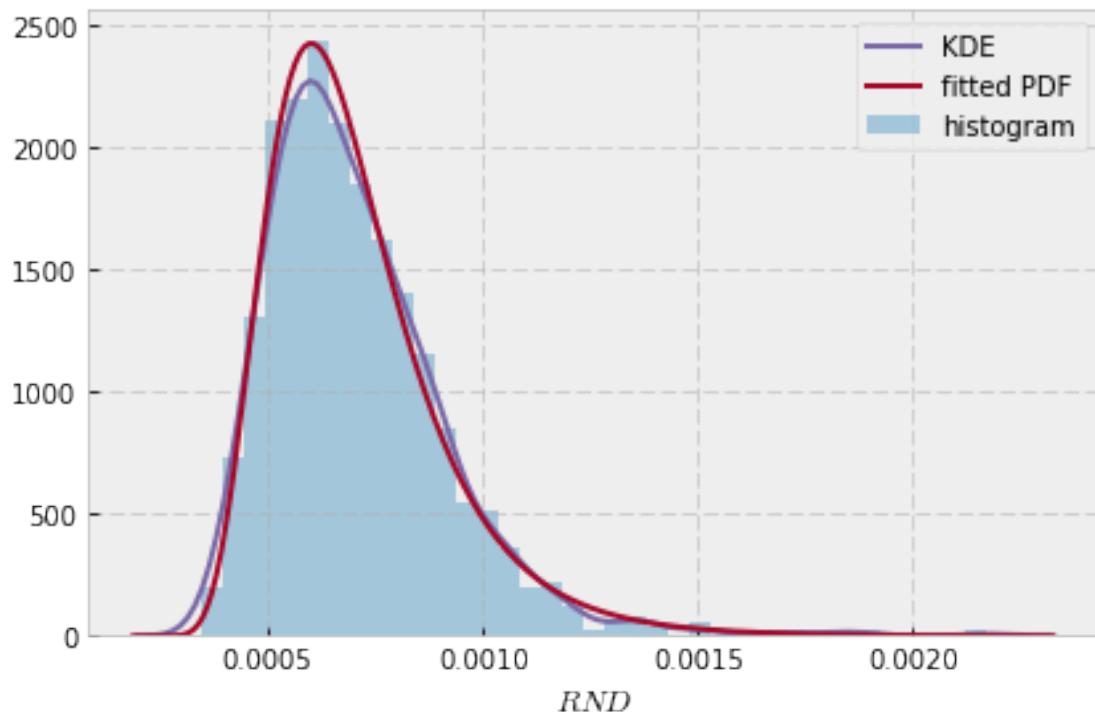


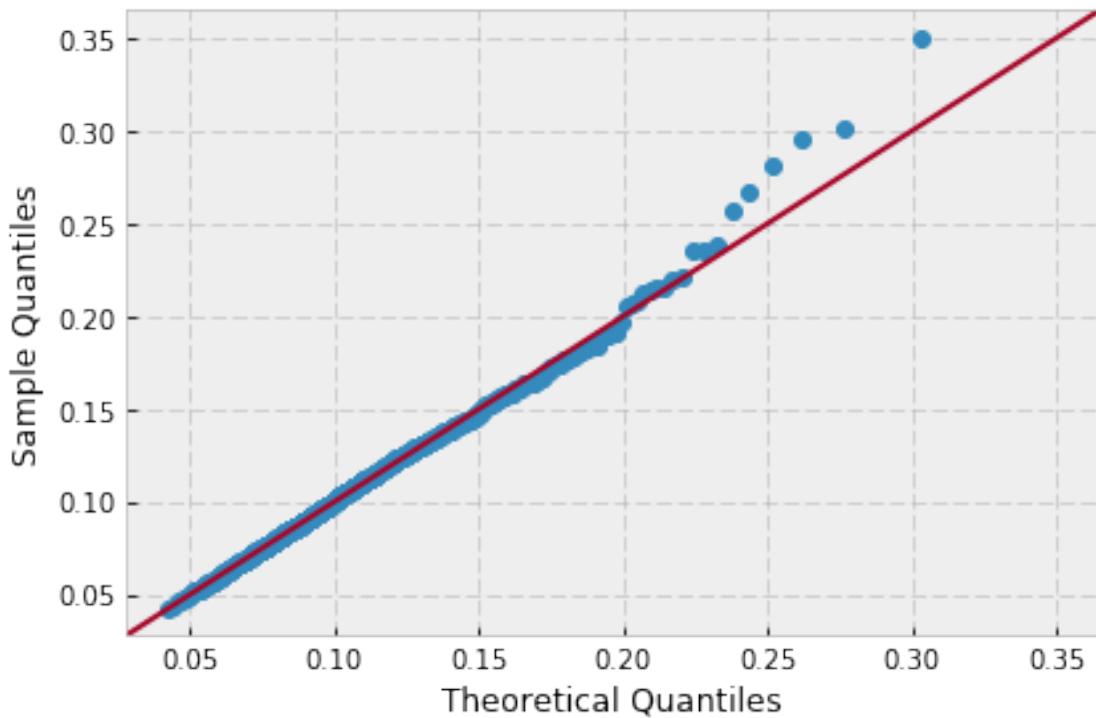


```
variable name: ln
variable value: 1.833333333333333
distribution: inverse-gamma(shape=(17.12109962975849,), loc=-6.819839462147471e-05, scale=0.01341
DescribeResult(nobs=1000, minmax=(0.00037406685655433261, 0.0023770105426163318)
               mean=0.00076884734564426778, variance=5.6203072210380379e-08,
               skewness=1.6707675280213647, kurtosis=5.212219530564875)
```

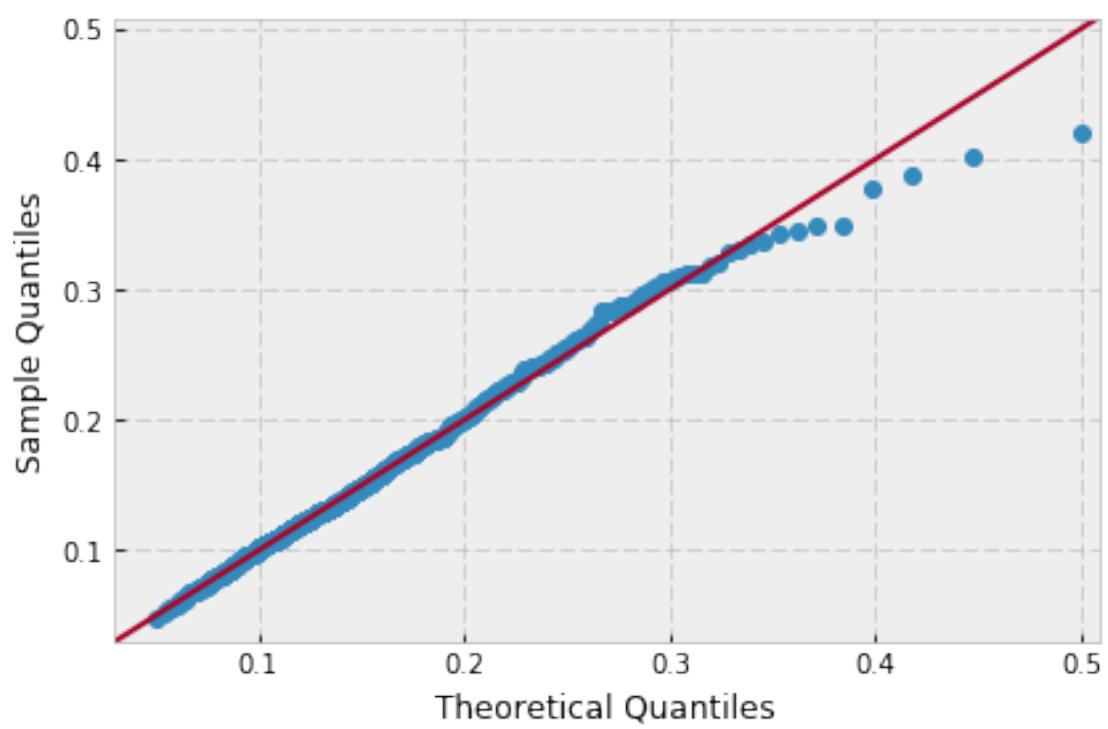
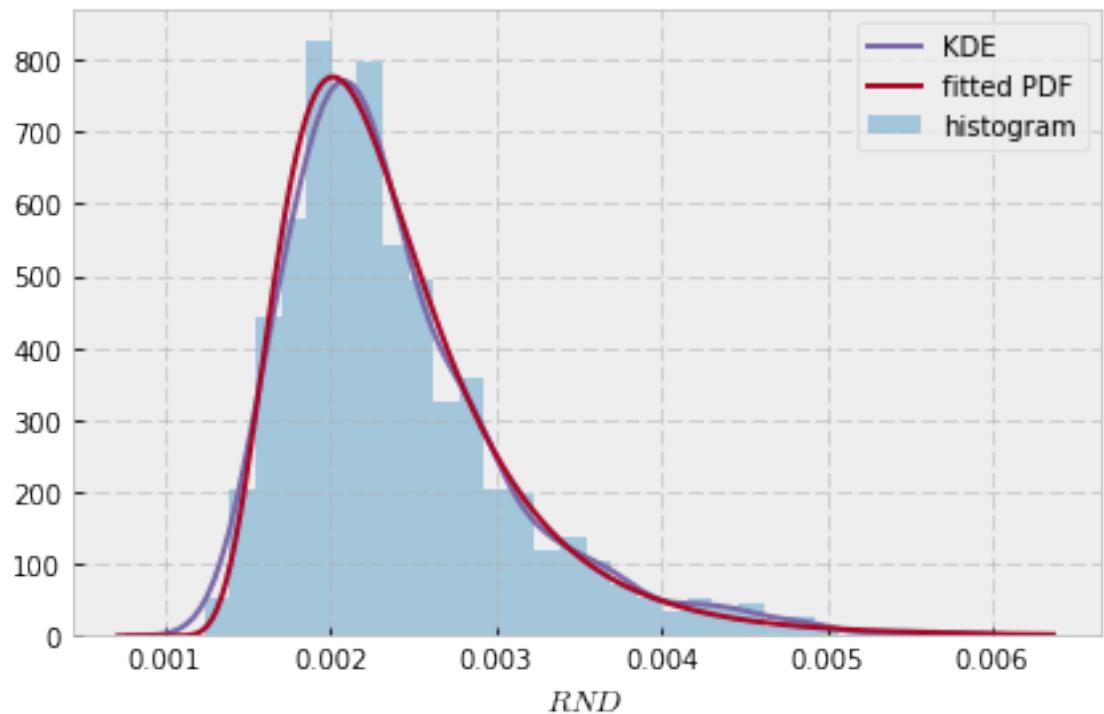


```
variable name: ln
variable value: 2.0
distribution: inverse-gamma(shape=(10.64234612532821,), loc=9.034227007348637e-05, scale=0.005936
DescribeResult(nobs=1000, minmax=(0.00034508684060869732, 0.0021669516222238266)
               mean=0.00070605342863764686, variance=4.4230207094265221e-08,
               skewness=1.6243558127645155, kurtosis=5.386548184033272)
```

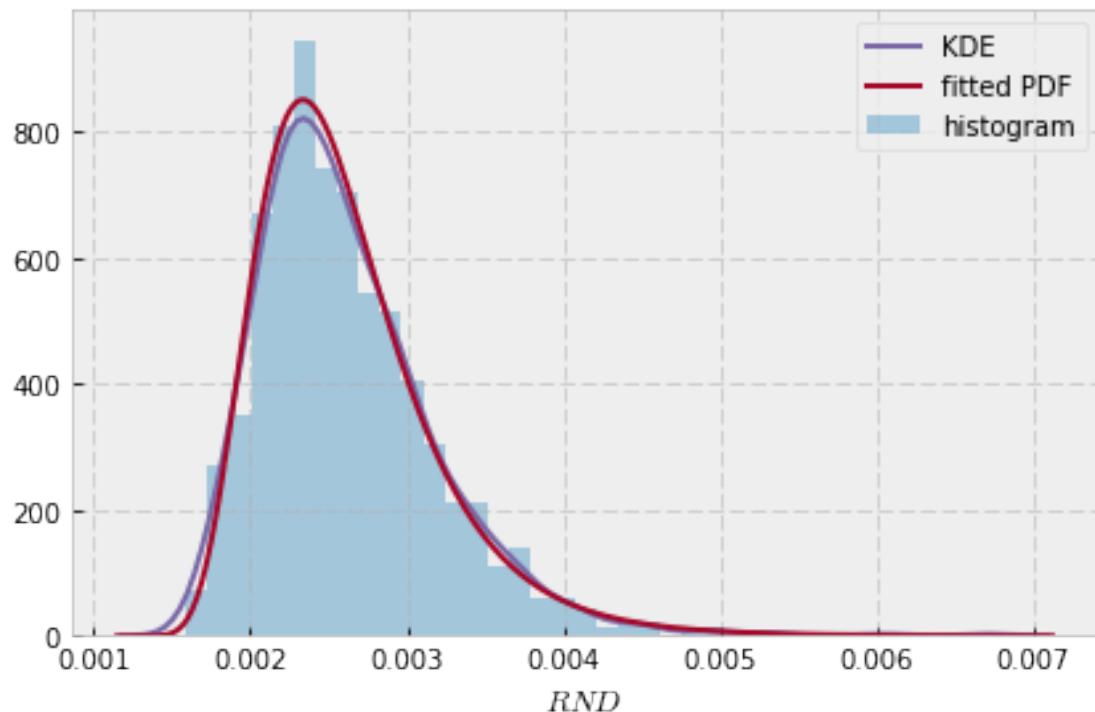


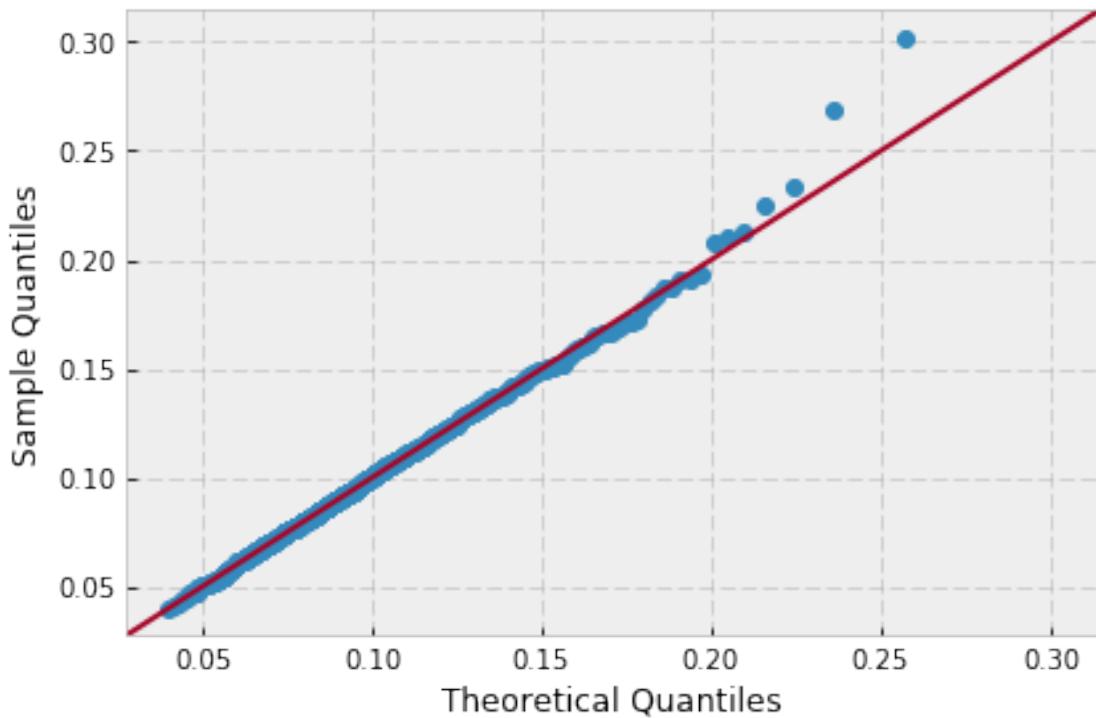


```
variable name: maxB
variable value: 4.0
distribution: inverse-gamma(shape=(8.0680377593288775,), loc=0.0006525093027268251, scale=0.01236
DescribeResult(nobs=1000, minmax=(0.0012422137011074792, 0.0058441863016140275)
               mean=0.0024015159751704629, variance=4.8187199064089205e-07,
               skewness=1.4356865924776931, kurtosis=2.6514091875881833)
```

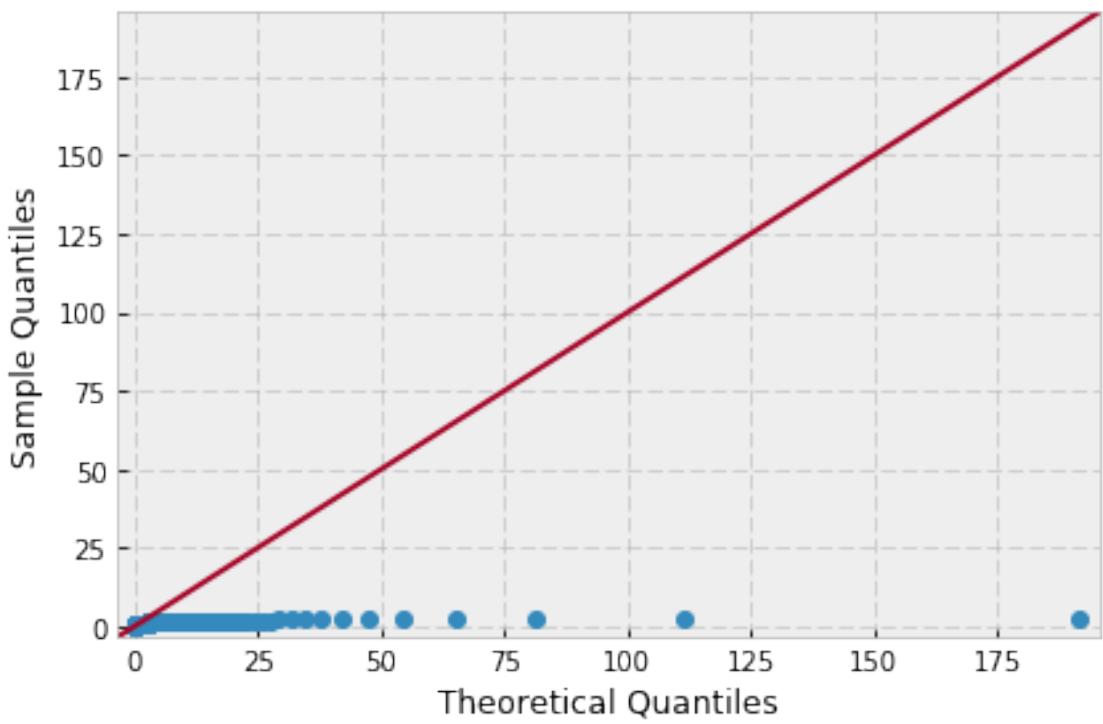
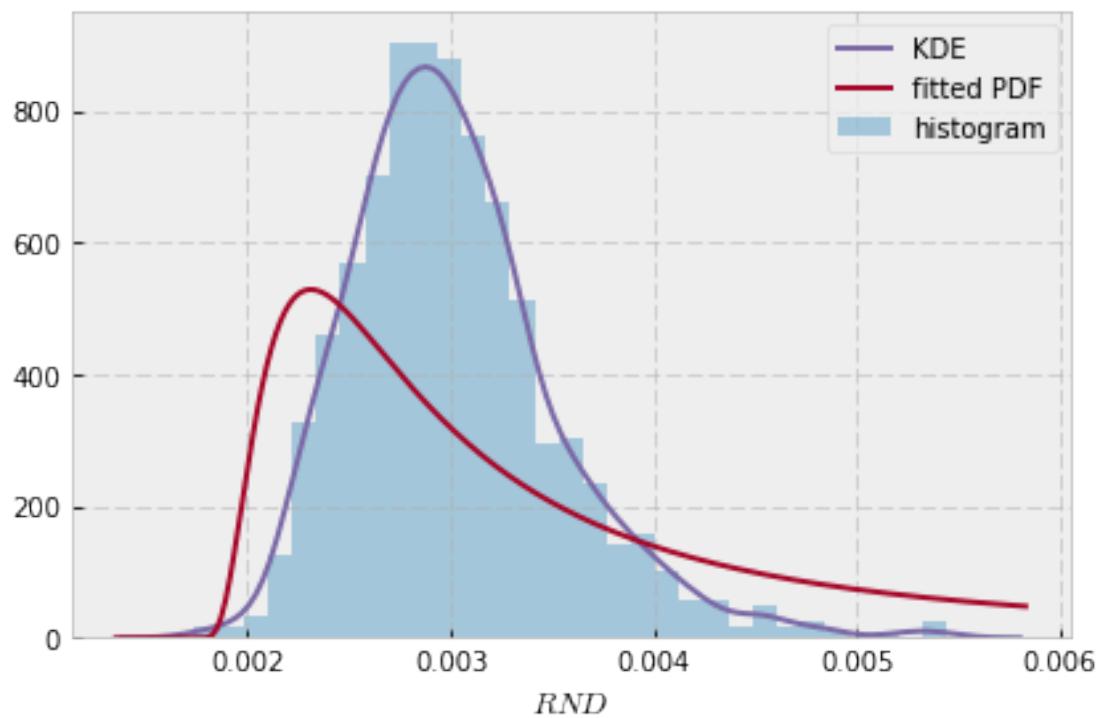


```
variable name: maxB
variable value: 8.0
distribution: inverse-gamma(shape=(11.719993724603537,), loc=0.000804064215589948, scale=0.019474
DescribeResult(nobs=1000, minmax=(0.0015903481584270416, 0.006677044478295896)
               mean=0.0026207204931535232, variance=3.4046715421678768e-07,
               skewness=1.4722877243809247, kurtosis=4.540812050335441)
```

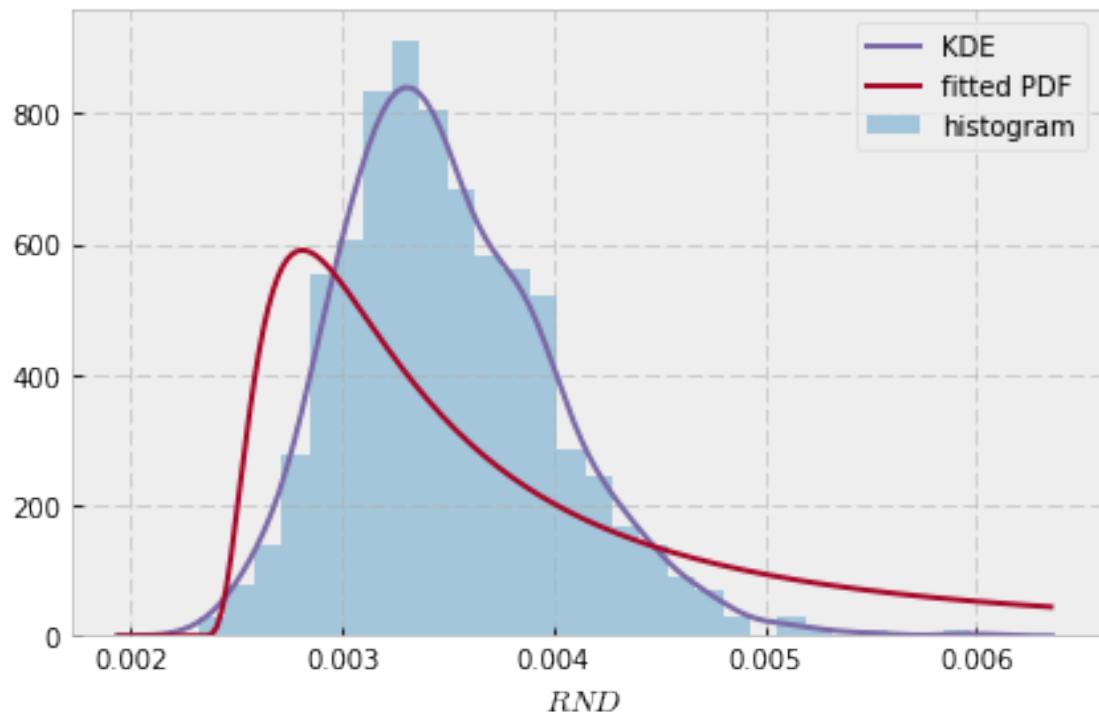


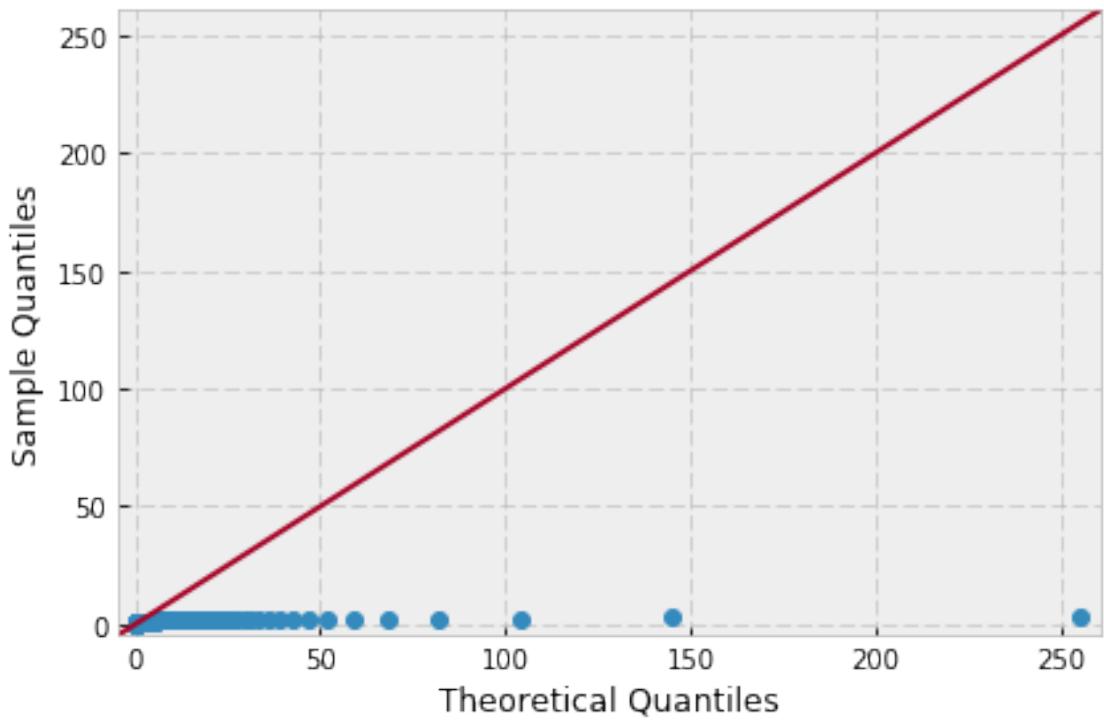


```
variable name: maxB
variable value: 12.0
distribution: inverse-gamma(shape=(1.2859230018078964,),loc=0.0016924177451570351, scale=0.00141
DescribeResult(nobs=1000, minmax=(0.0017398636850075092, 0.0054417863668430478)
               mean=0.0030160065724048825, variance=2.6487395463057792e-07,
               skewness=0.9715156047772535, kurtosis=1.826275161584931)
```

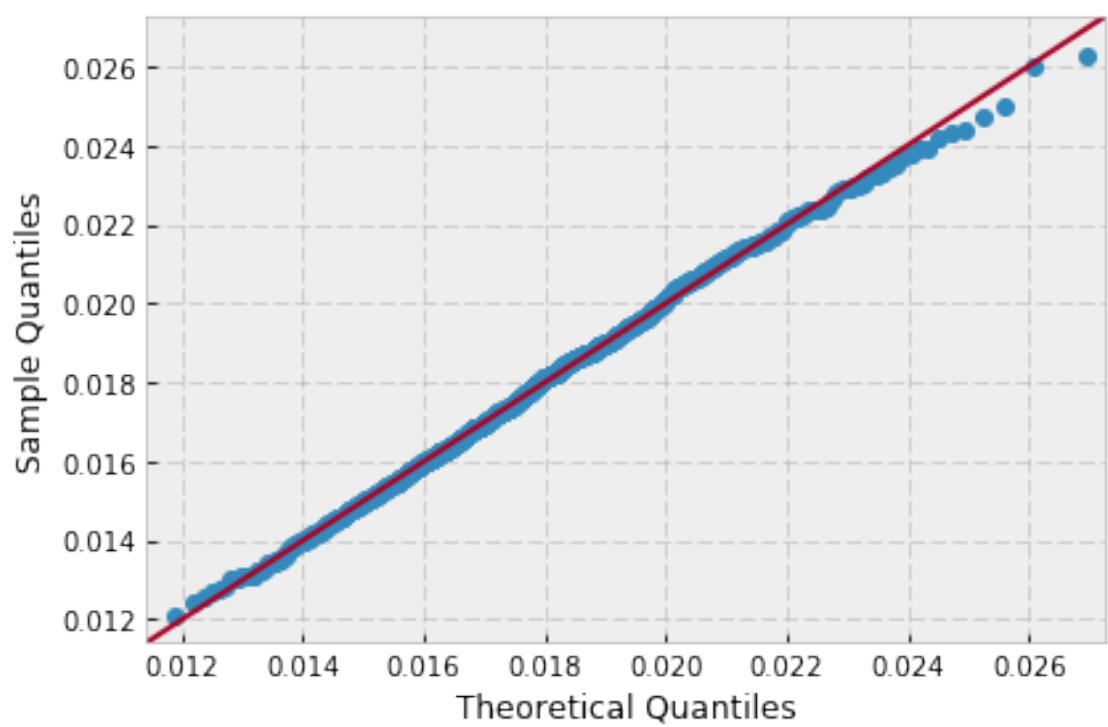
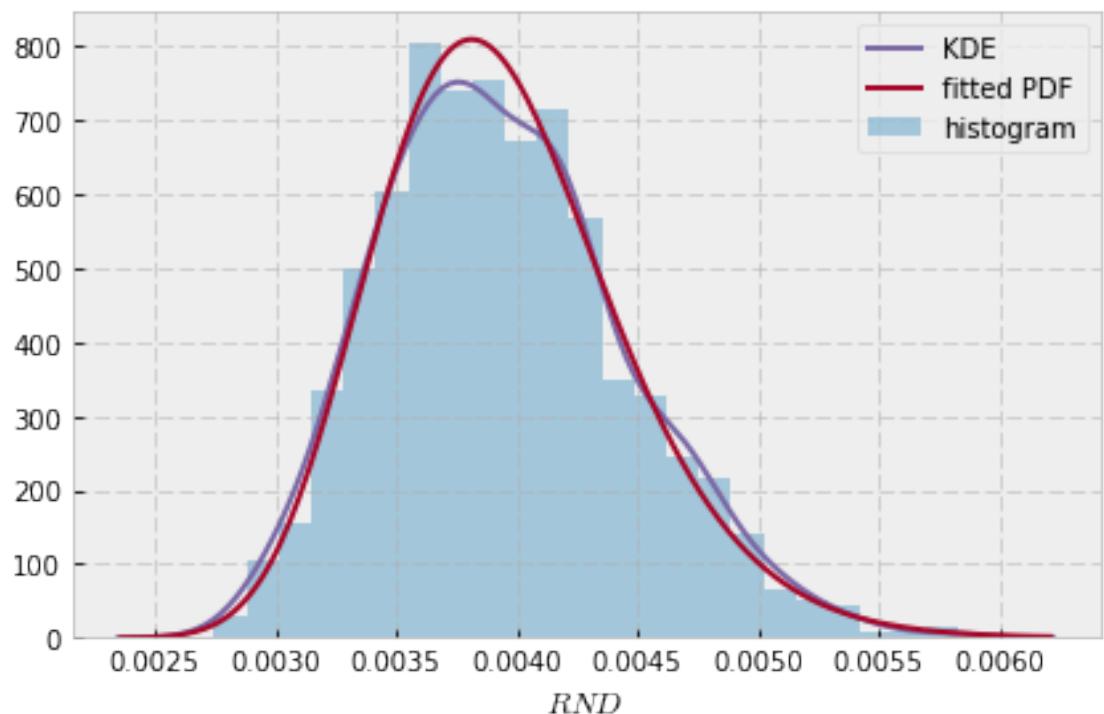


```
variable name: maxB
variable value: 16.0
distribution: inverse-gamma(shape=(1.2257949664607755,),loc=0.0022769165196521407, scale=0.00118
DescribeResult(nobs=1000, minmax=(0.0023220250300683334, 0.0059717612275542557)
    mean=0.0034974190553540463, variance=2.5337359894042704e-07,
    skewness=0.6843780390901987, kurtosis=0.7943908199035423)
```

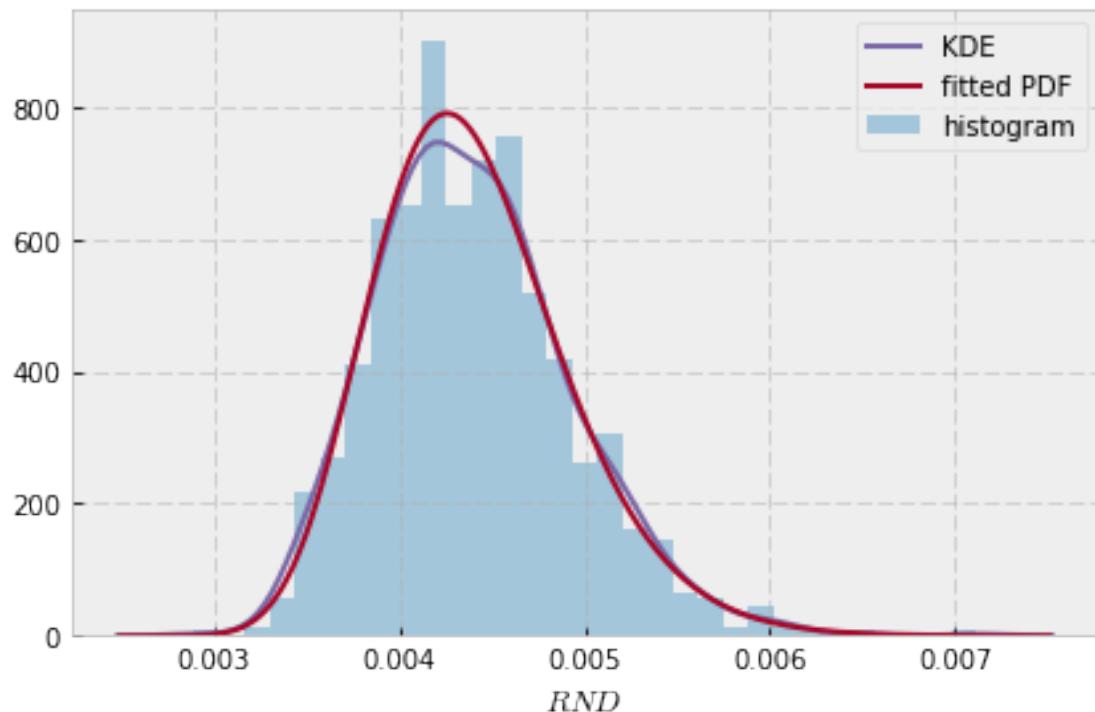


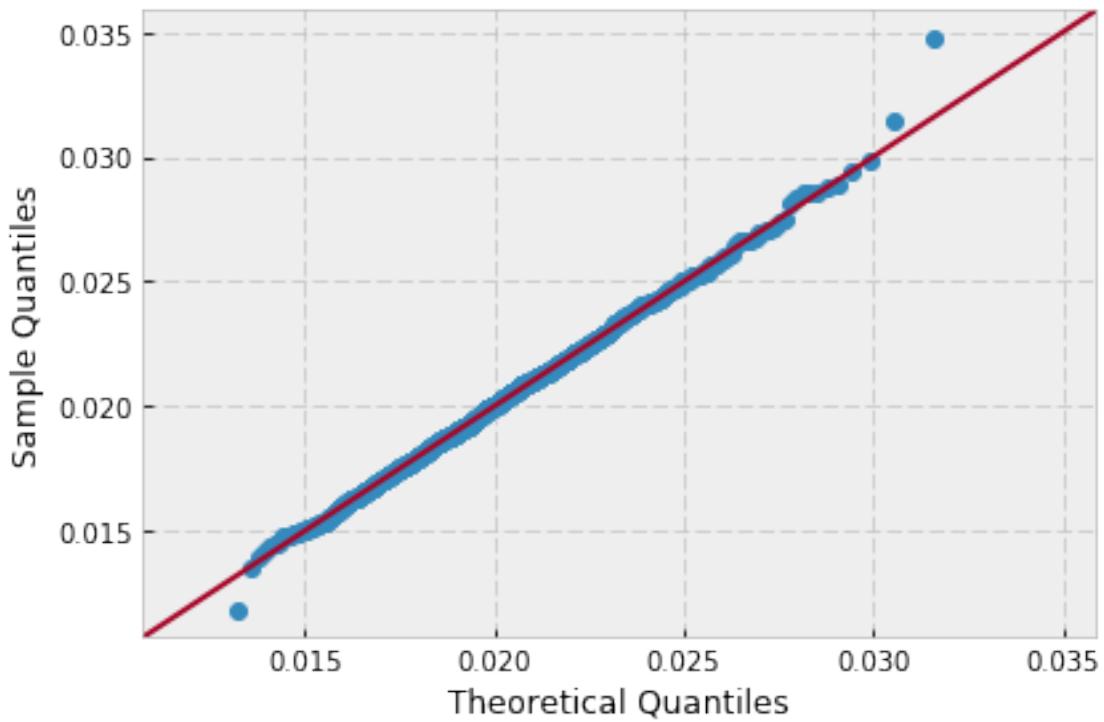


```
variable name: maxB
variable value: 20.0
distribution: inverse-gamma(shape=(57.796215371583102,), loc=0.00010503080289526307, scale=0.2179
DescribeResult(nobs=1000, minmax=(0.0027457529382975453, 0.005825568369789382)
               mean=0.0039431782604635252, variance=2.6223384416891871e-07,
               skewness=0.44992481882679264, kurtosis=0.03687918177954641)
```

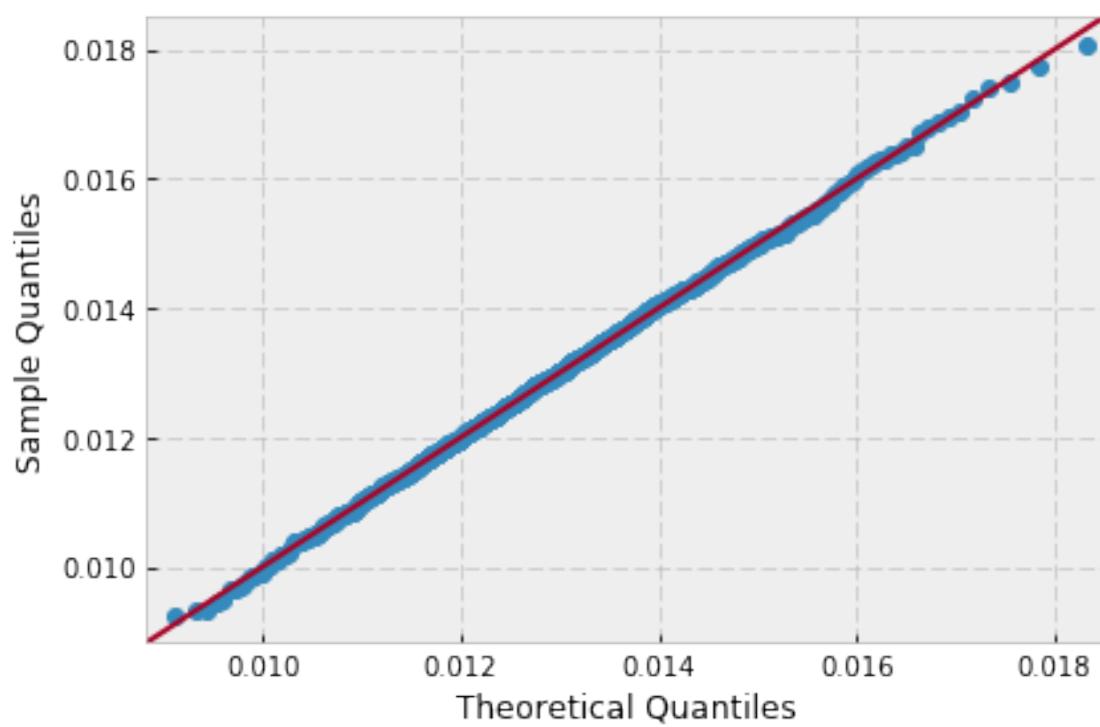
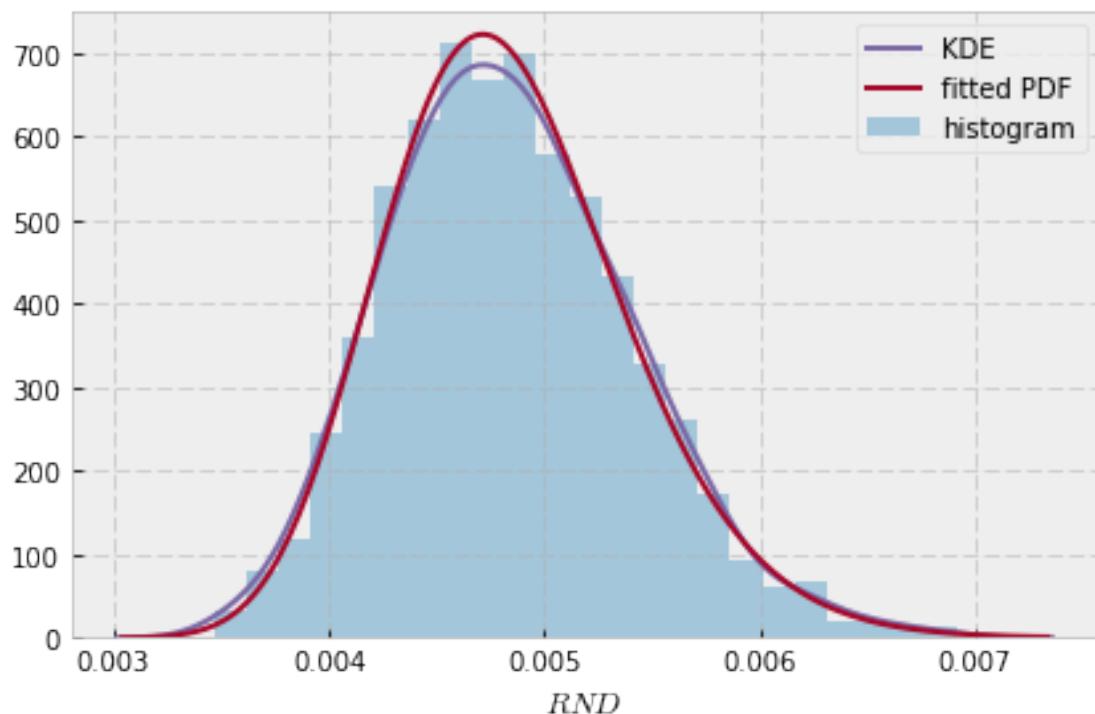


```
variable name: maxB
variable value: 24.0
distribution: inverse-gamma(shape=(50.890331848430115,),loc=0.0006988354743477779, scale=0.18461
DescribeResult(nobs=1000, minmax=(0.0028866815160225805, 0.0071131510838407059)
               mean=0.0043992852832388622, variance=2.8078836259060537e-07,
               skewness=0.6038646689792617, kurtosis=0.8073825115272273)
```

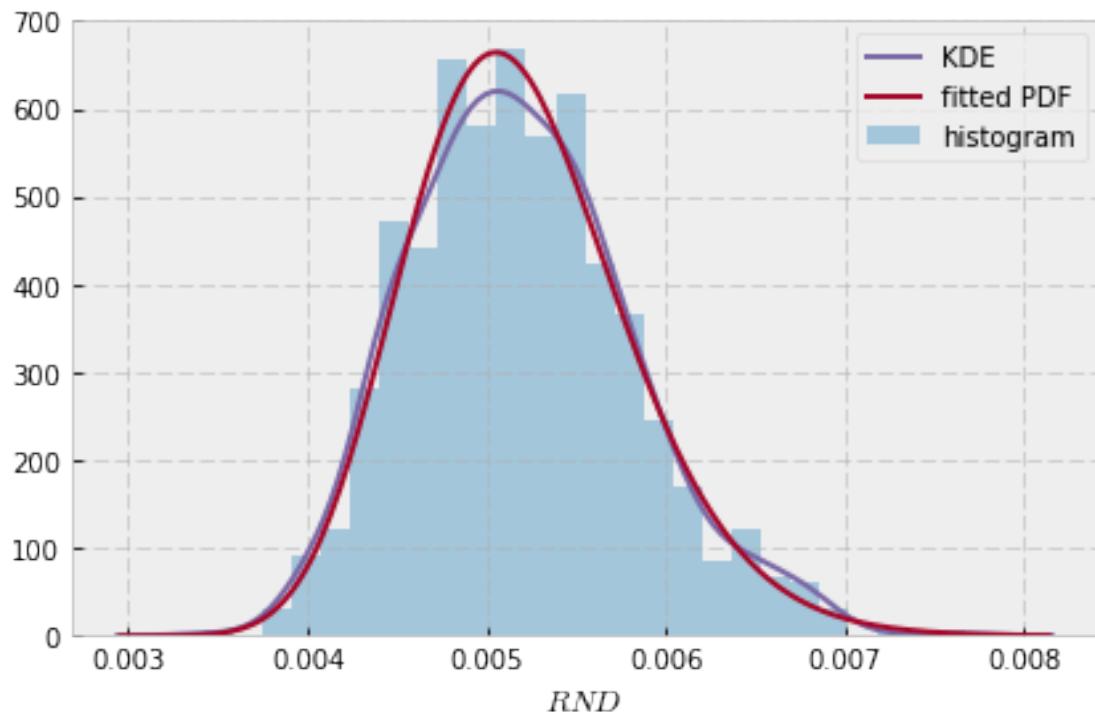


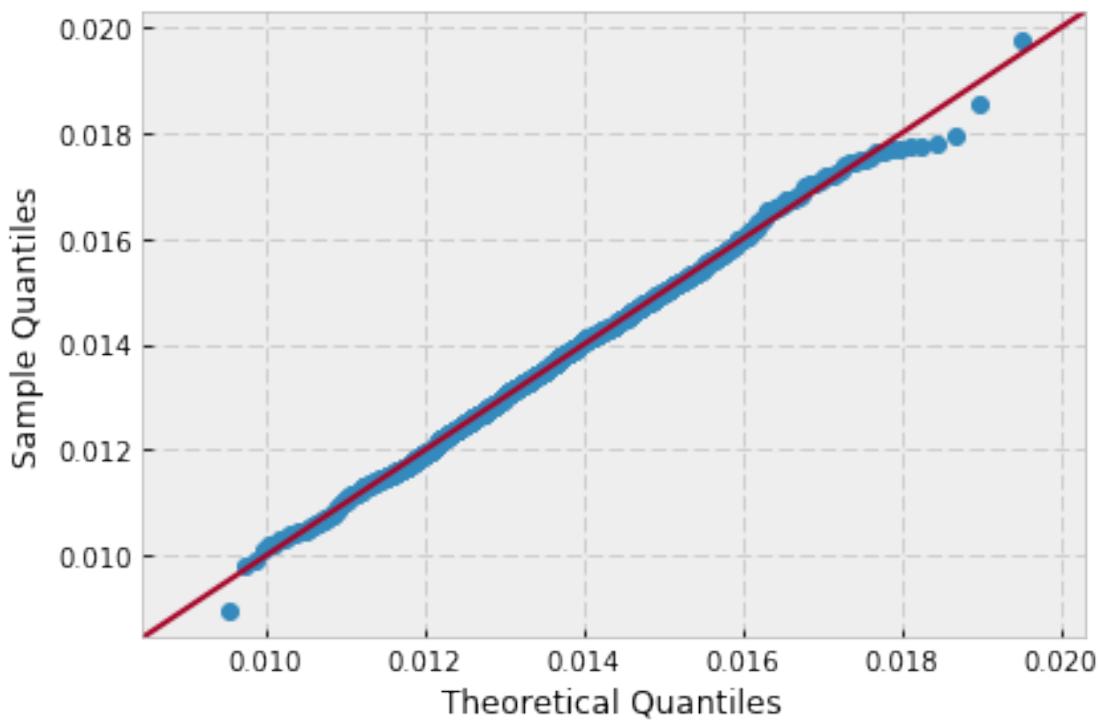


```
variable name:  maxB
variable value: 28.0
distribution: inverse-gamma(shape=(79.31449891635512,),loc=-0.00016586391732324008, scale=0.3919
DescribeResult(nobs=1000, minmax=(0.0034638257264915065, 0.0069116527290337118)
               mean=0.0048408908224029813, variance=3.2155837527474251e-07,
               skewness=0.3997857409663429, kurtosis=0.15855803835067306)
```

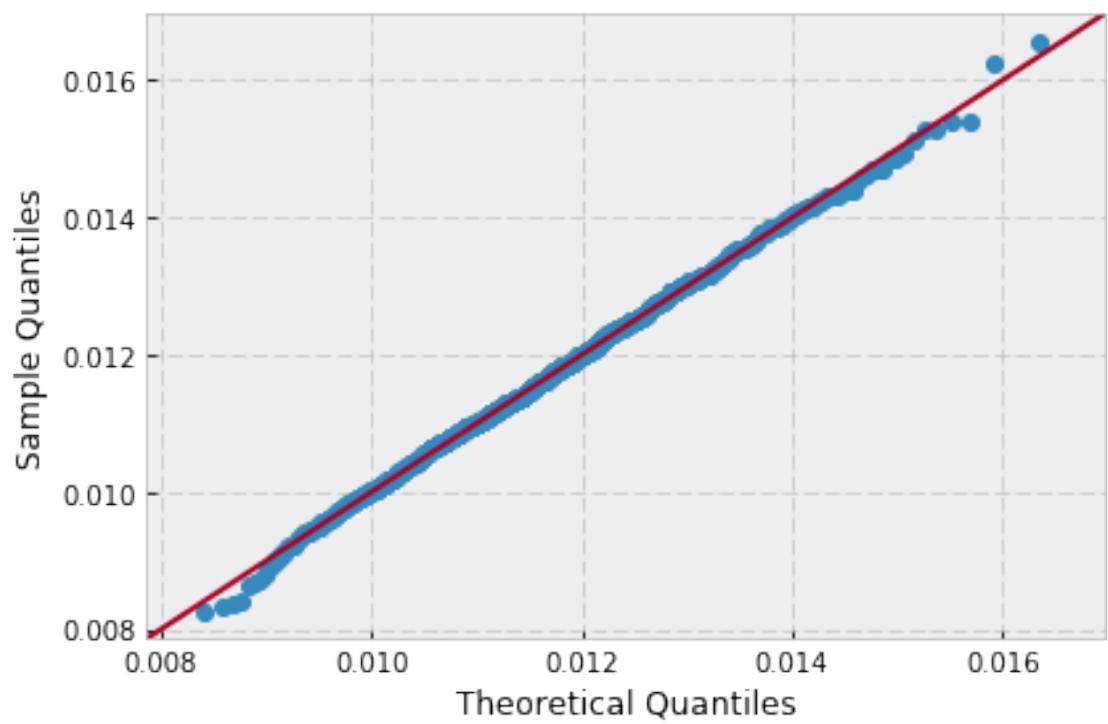
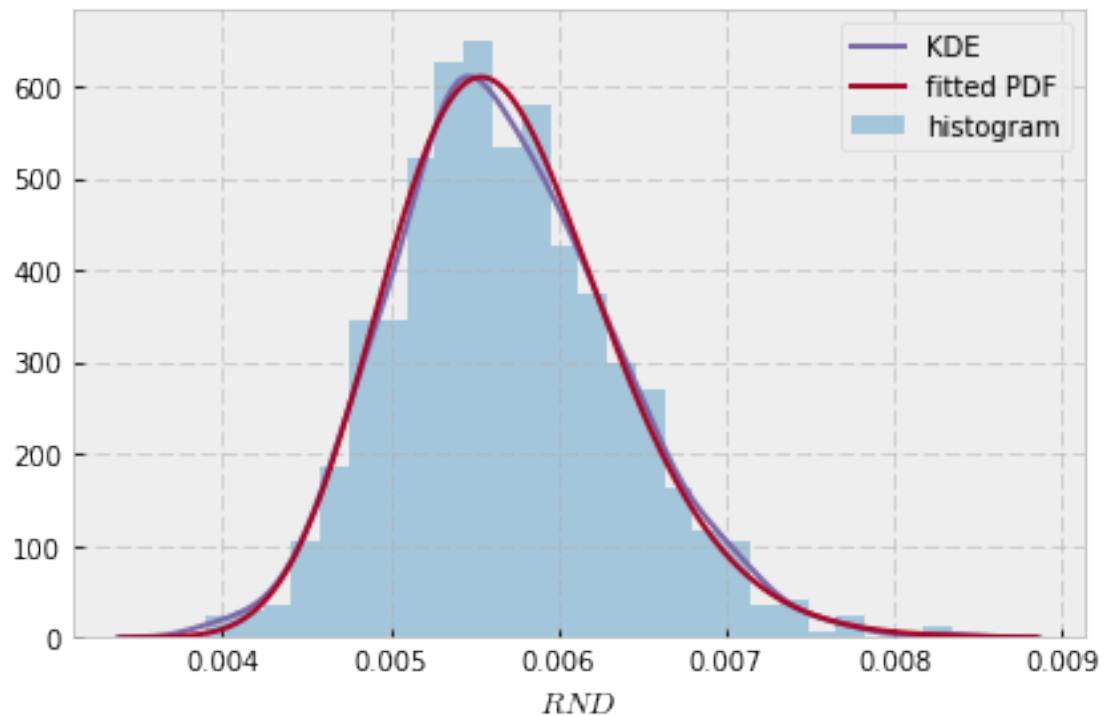


```
variable name: maxB
variable value: 32.0
distribution: inverse-gamma(shape=(75.27825192519677,), loc=-0.00012212589349136927, scale=0.3946
DescribeResult(nobs=1000, minmax=(0.0034216385066908328, 0.0076719208598780217)
    mean=0.0051907213583006241, variance=3.8367247518687185e-07,
    skewness=0.4215067182976278, kurtosis=0.12788993739953147)
```

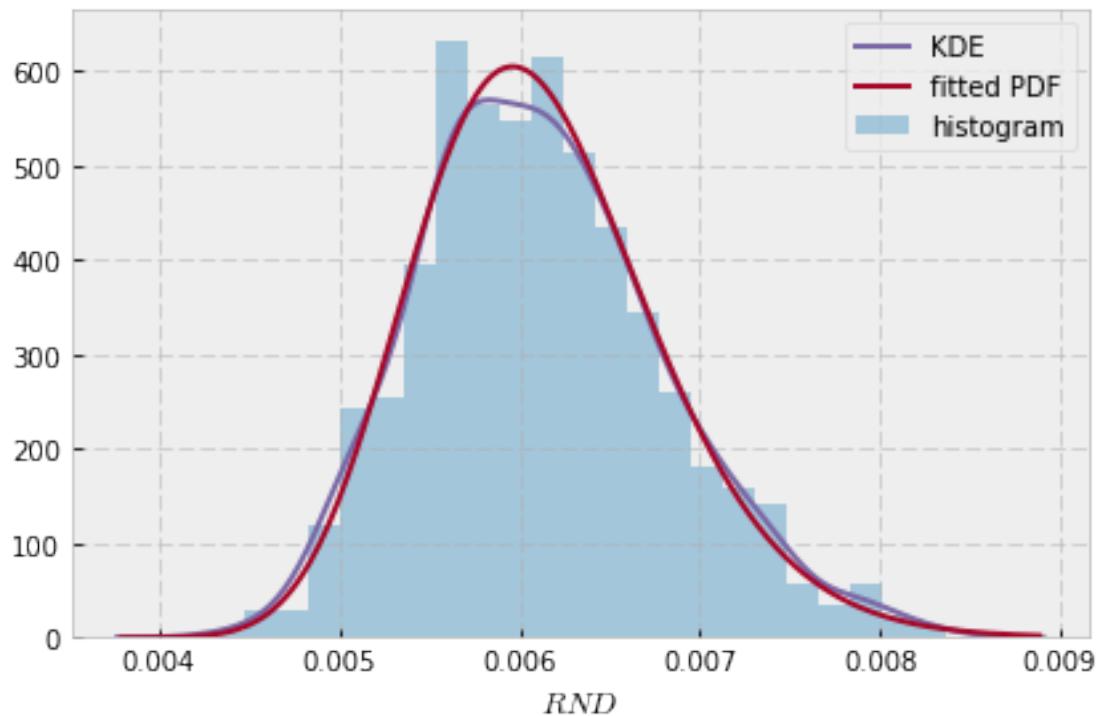


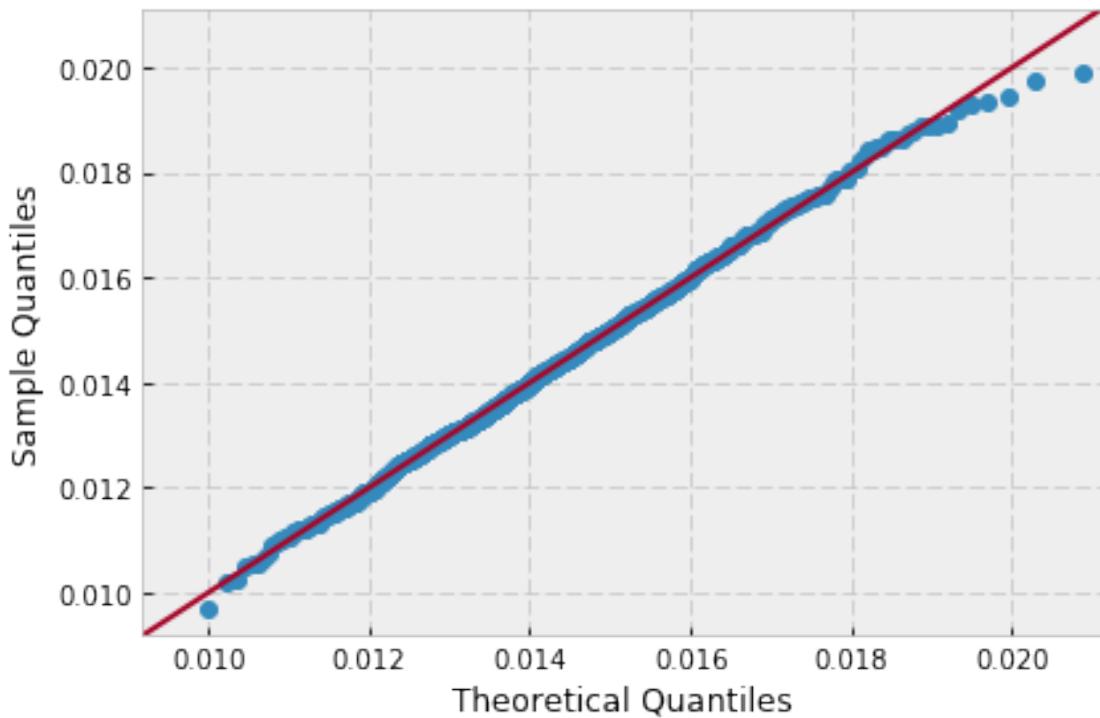


```
variable name: maxB
variable value: 36.0
distribution: inverse-gamma(shape=(87.265150547346281,), loc=-0.0005308410275796911, scale=0.5359
DescribeResult(nobs=1000, minmax=(0.0039000369540309466, 0.0083393340928508281)
               mean=0.0056856833219737111, variance=4.4666940428670374e-07,
               skewness=0.3790783749300781, kurtosis=0.236107664686096)
```

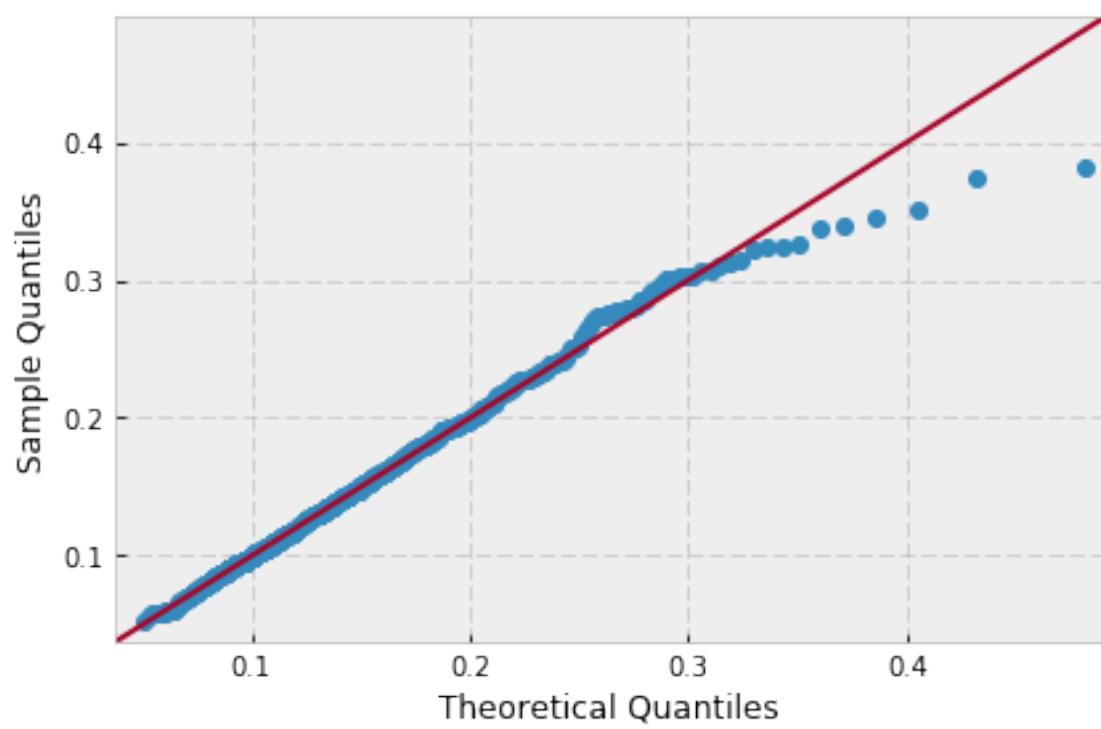
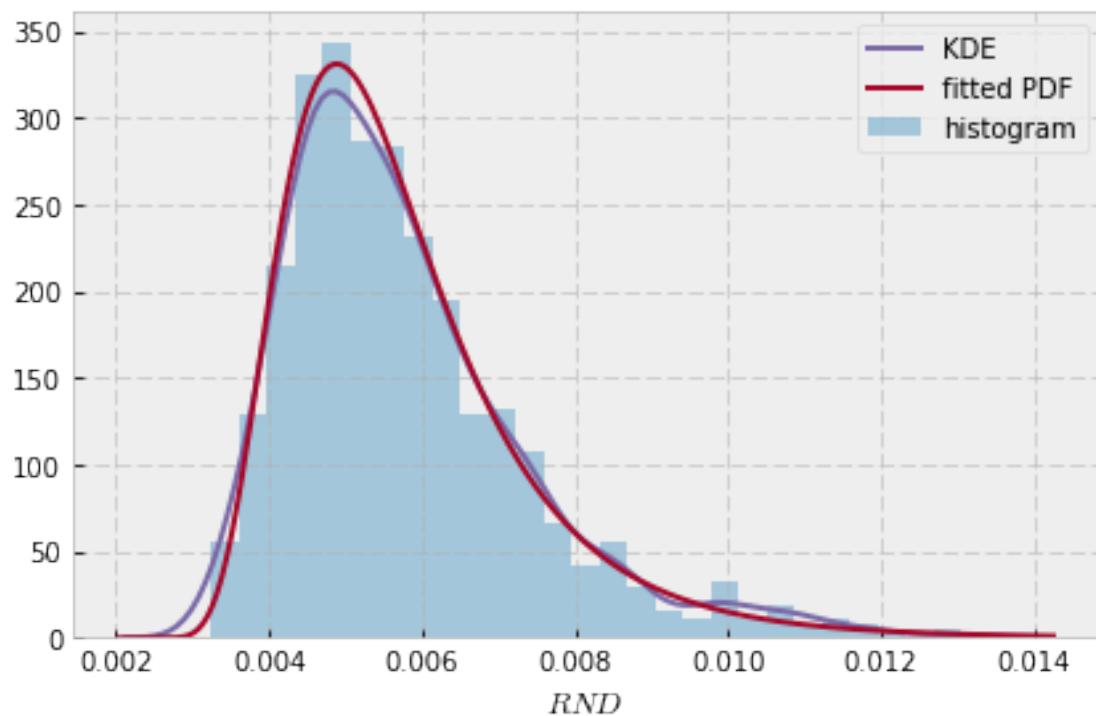


```
variable name: maxB
variable value: 40.0
distribution: inverse-gamma(shape=(71.156385983807411,),loc=0.0004433205310128627, scale=0.39817
DescribeResult(nobs=1000, minmax=(0.0043010952661228083, 0.0083700615215994197)
               mean=0.0061188149876124425, variance=4.6356930234884657e-07,
               skewness=0.425117281075665, kurtosis=0.05639620510888799)
```

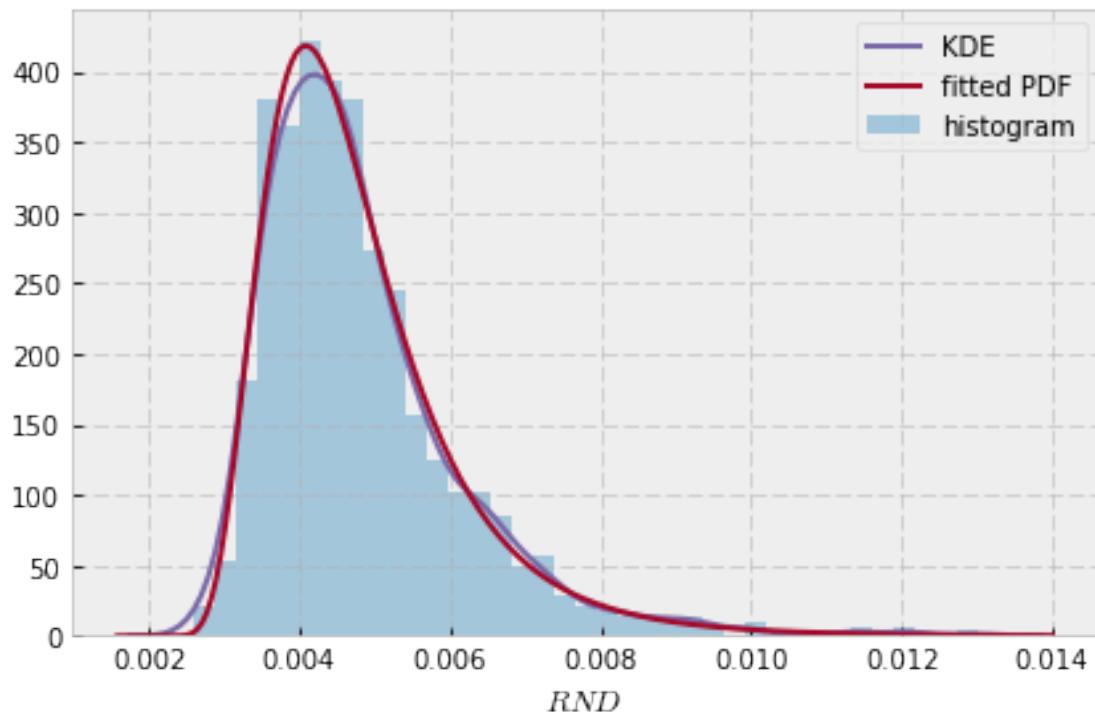


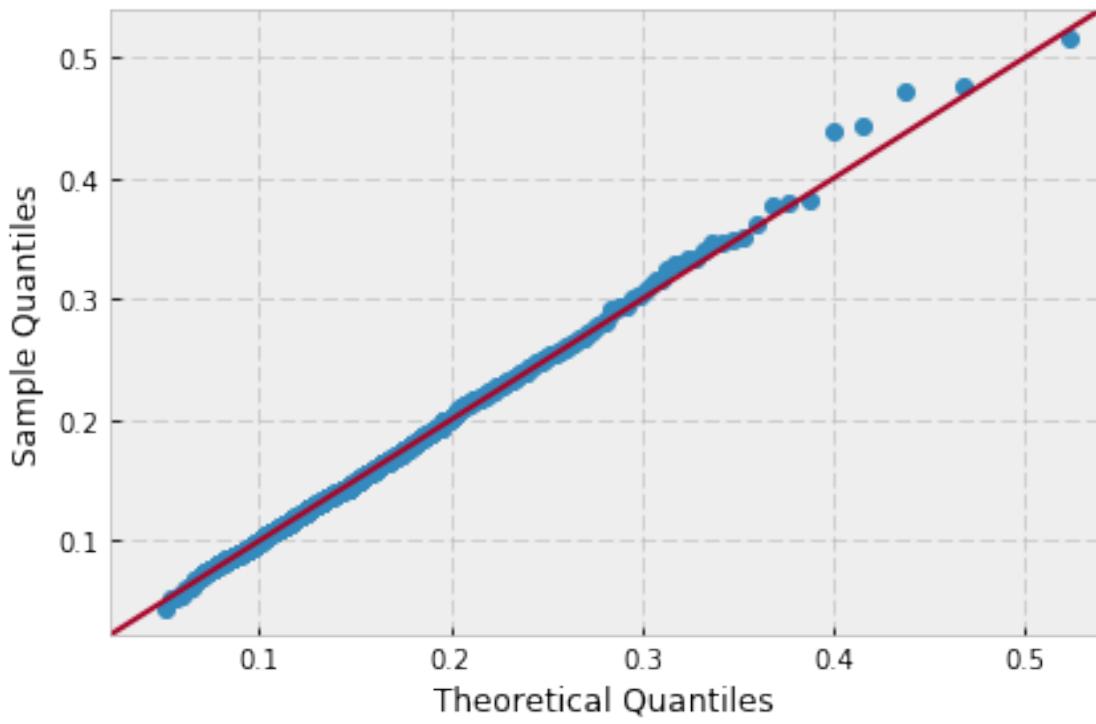


```
variable name: r
variable value: 1.0
distribution: inverse-gamma(shape=(8.2194931300272351,),loc=0.001650856878440292, scale=0.029801
DescribeResult(nobs=1000, minmax=(0.0032352343628683764, 0.013051754908406684)
               mean=0.0057741820232263662, variance=2.5592932869891839e-06,
               skewness=1.3271777986379898, kurtosis=2.1509965496938452)
```

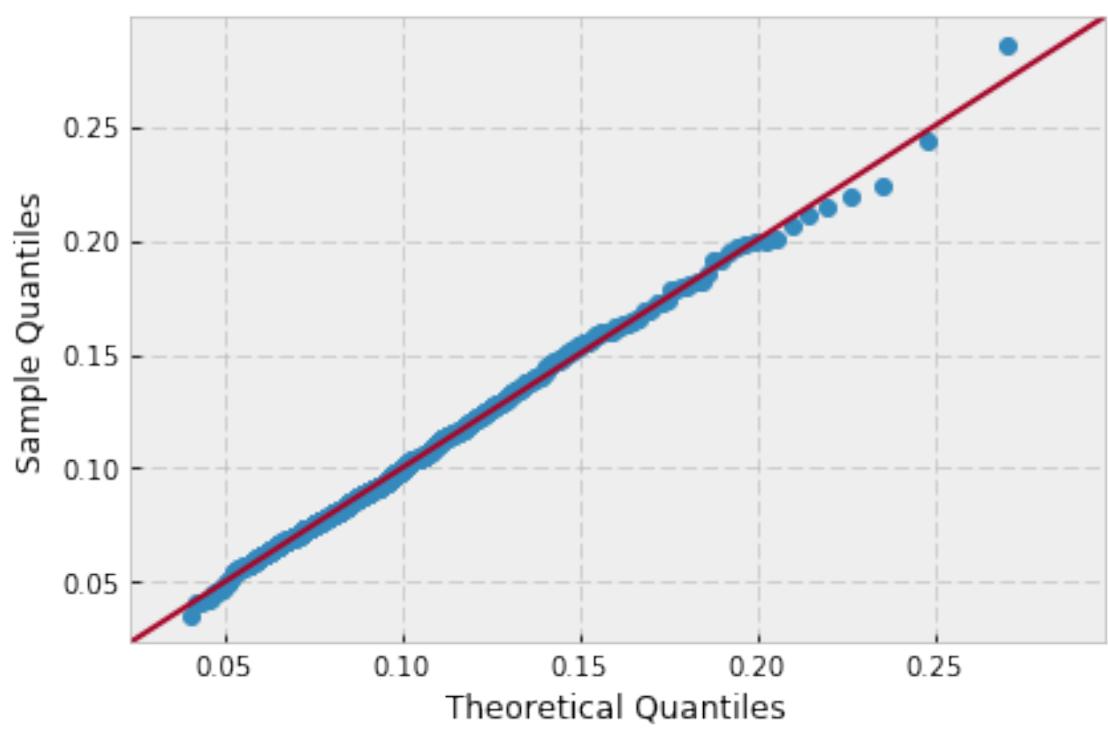
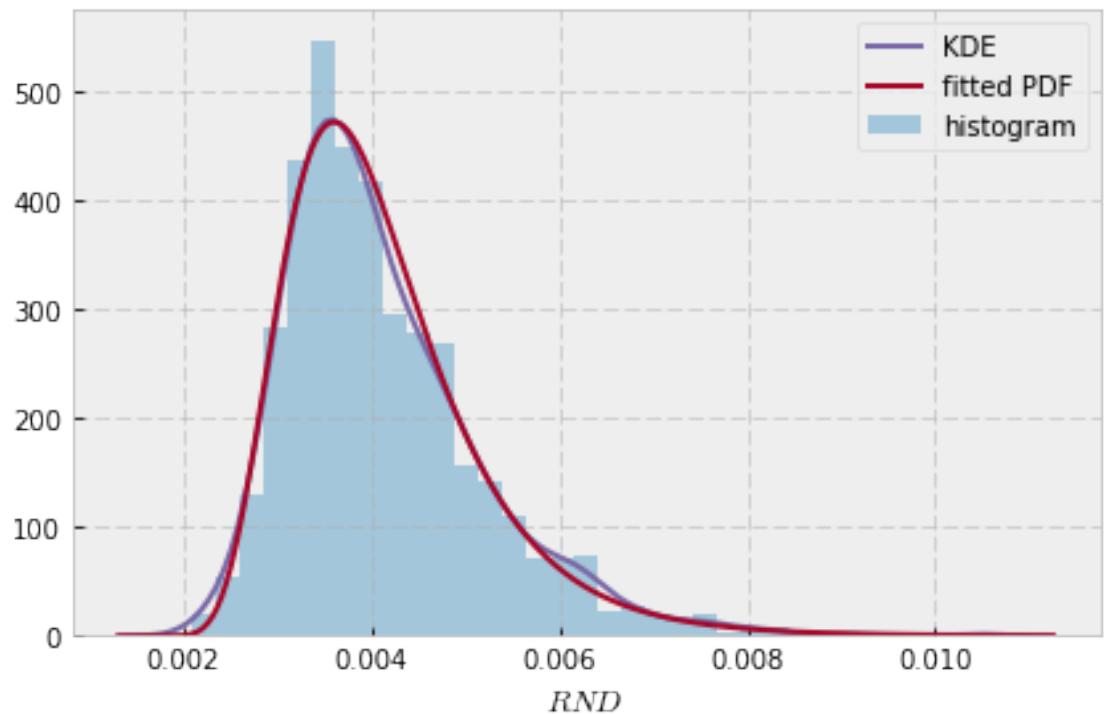


```
variable name: r
variable value: 1.222222222222223
distribution: inverse-gamma(shape=(7.8703938643099836,),loc=0.0015868223783364684, scale=0.02213
DescribeResult(nobs=1000, minmax=(0.002581876043244035, 0.013000971944696302)
               mean=0.0048095789917068973, variance=1.7620195825527052e-06,
               skewness=1.75597429123646, kurtosis=5.016949253524199)
```

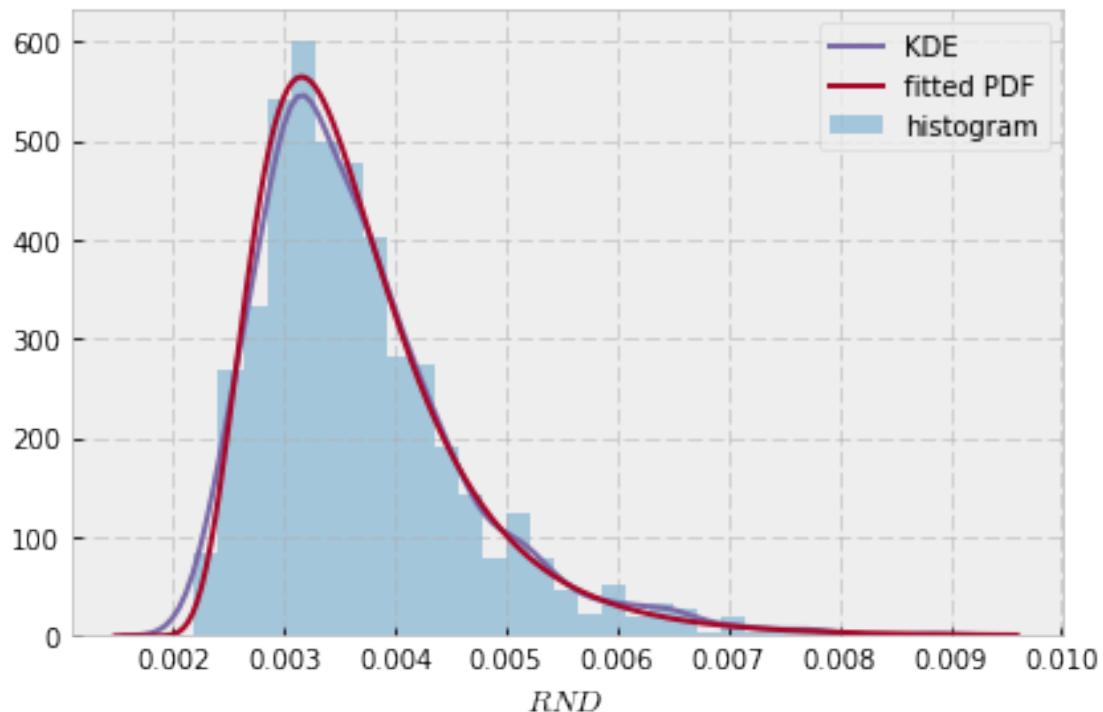


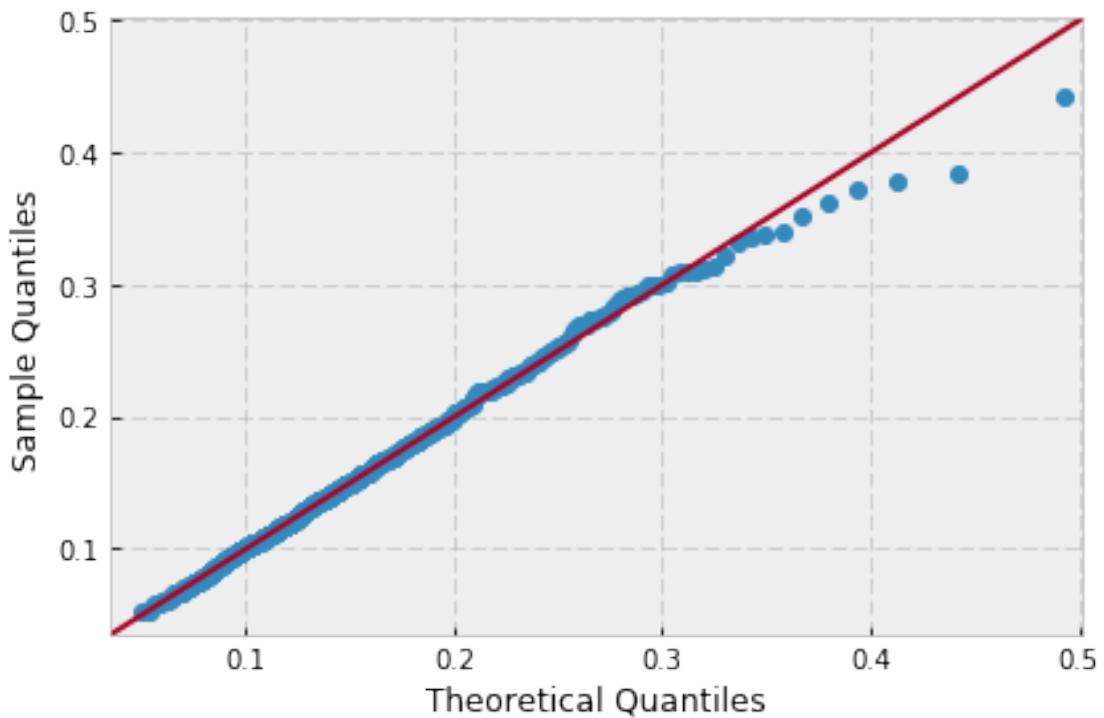


```
variable name: r
variable value: 1.444444444444444
distribution: inverse-gamma(shape=(11.374506397215765,), loc=0.0008911885312249372, scale=0.03354
DescribeResult(nobs=1000, minmax=(0.0020965926652662817, 0.010463548797291855)
               mean=0.0041245795081579877, variance=1.1064794444818589e-06,
               skewness=1.2962712673607448, kurtosis=2.6439559107667376)
```

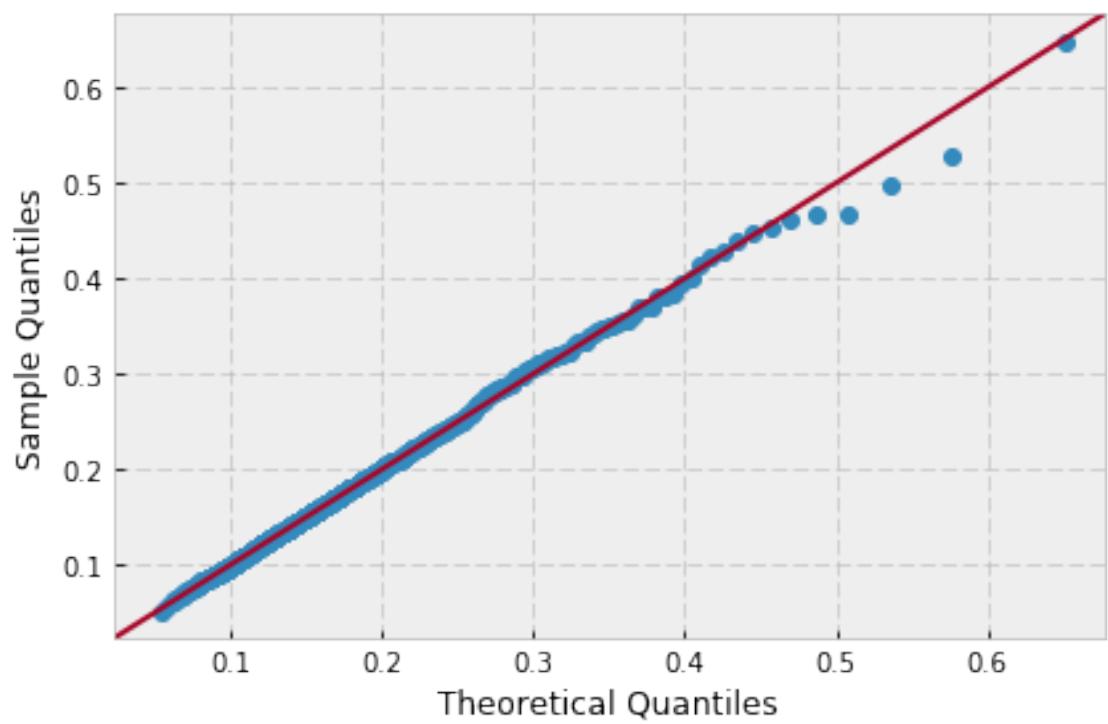
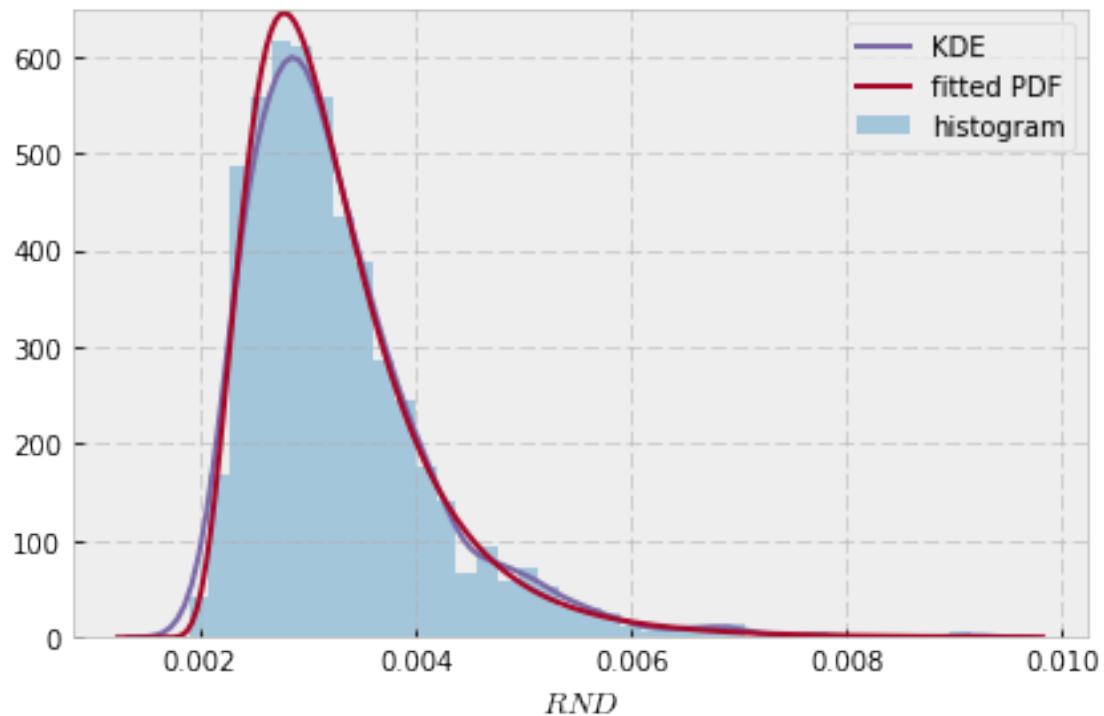


```
variable name: r
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(8.122302596582081,), loc=0.001271714501771777, scale=0.0171751
DescribeResult(nobs=1000, minmax=(0.0021914936792302238, 0.0088860377634421683)
    mean=0.0036808545353111555, variance=8.9598342361238596e-07,
    skewness=1.4227841502791032, kurtosis=2.8210368209107077)
```

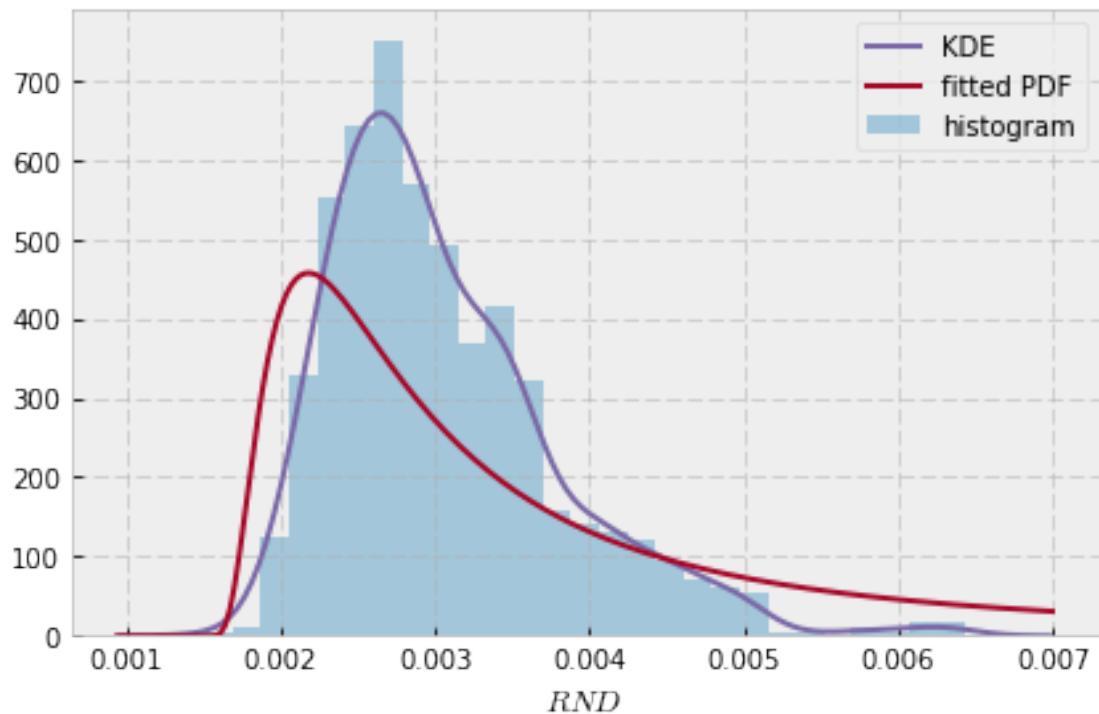


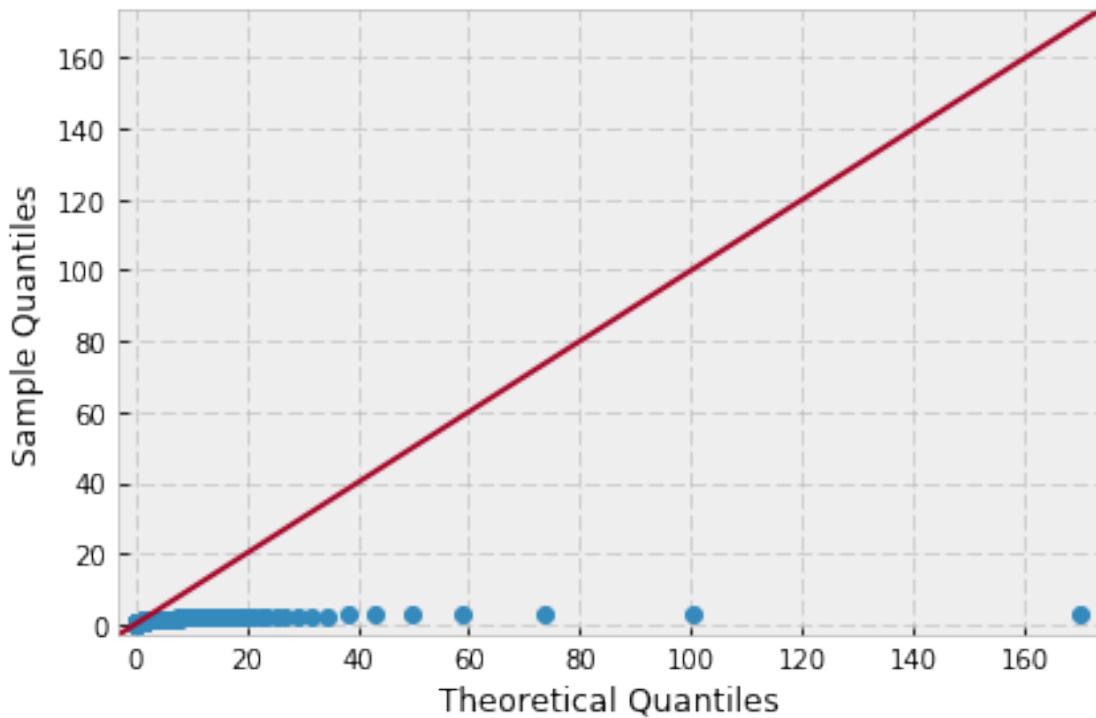


```
variable name: r
variable value: 1.888888888888888
distribution: inverse-gamma(shape=(7.0350544853897627,), loc=0.001260134768520139, scale=0.012209
DescribeResult(nobs=1000, minmax=(0.0018935231885065711, 0.0091621704877107024)
               mean=0.0032811473377947083, variance=7.6797103842598215e-07,
               skewness=1.6891047873353375, kurtosis=4.5729693904967395)
```

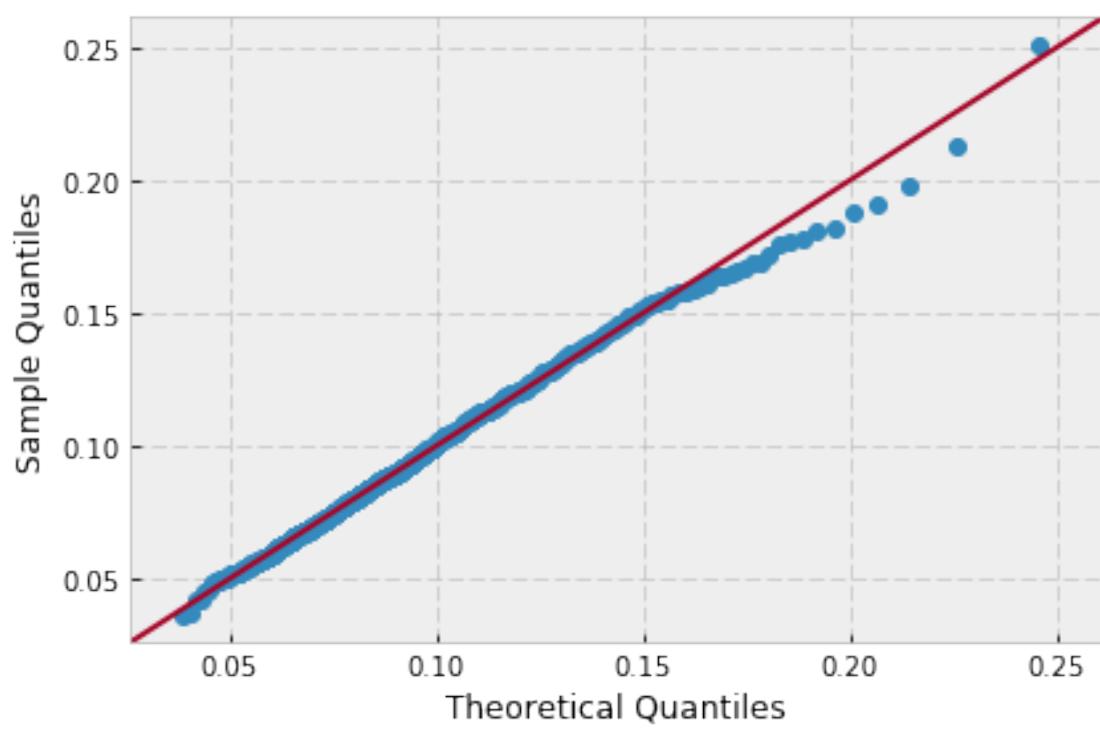
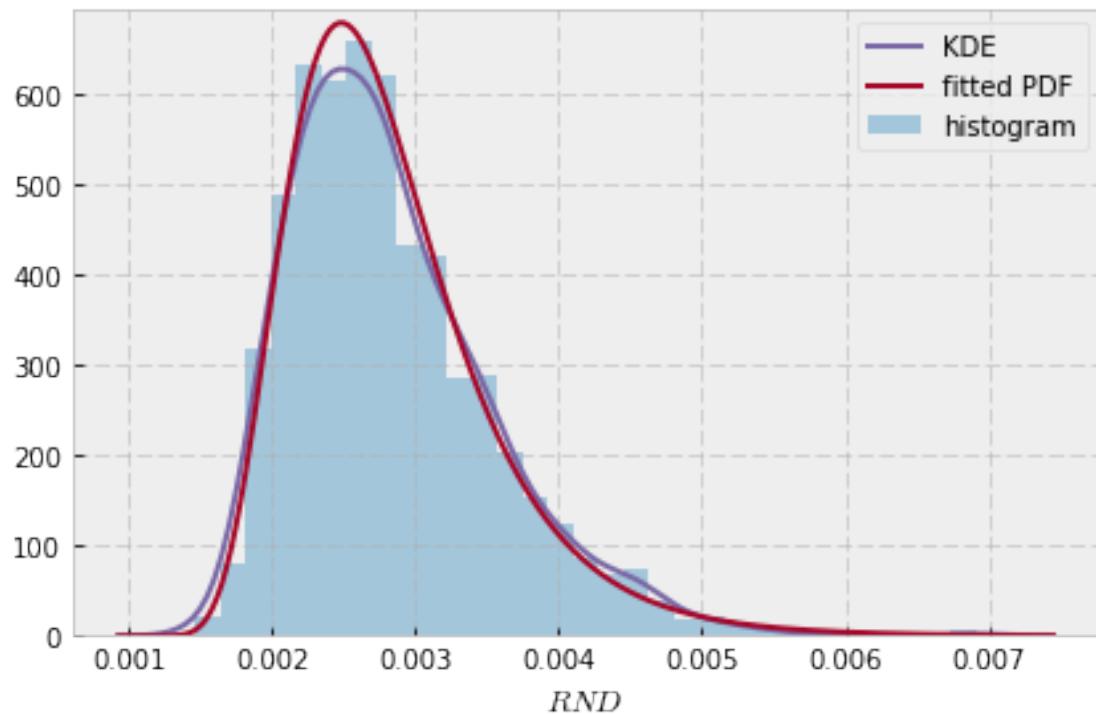


```
variable name: r
variable value: 2.11111111111111
distribution: inverse-gamma(shape=(1.3129419166321838,),loc=0.0014535968434145397, scale=0.00168
DescribeResult(nobs=1000, minmax=(0.0015075611083633278, 0.0064314030240019016)
    mean=0.0030582350931018069, variance=5.6520180306817787e-07,
    skewness=1.2146953119658233, kurtosis=1.9238148820582097)
```

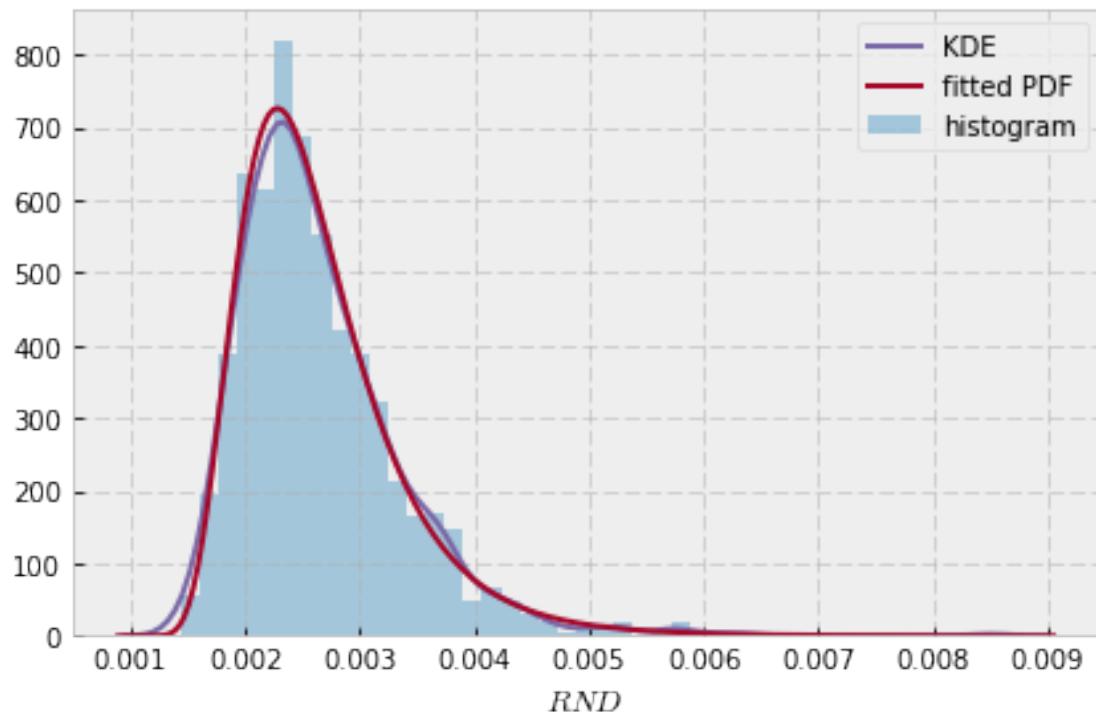


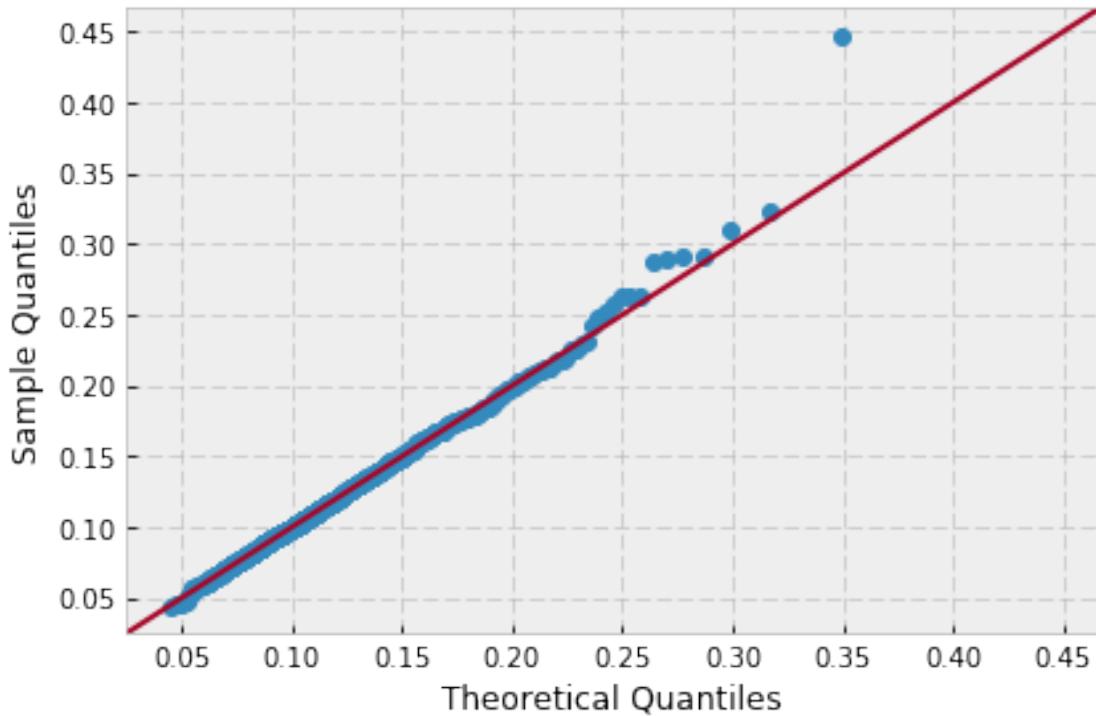


```
variable name: r
variable value: 2.333333333333333
distribution: inverse-gamma(shape=(12.058980290250858,), loc=0.0005313381013989833, scale=0.02546
DescribeResult(nobs=1000, minmax=(0.0014608917387780565, 0.006915191131676139)
               mean=0.0028332152674186302, variance=5.0593607458837639e-07,
               skewness=1.0876827445523898, kurtosis=1.773156194012329)
```

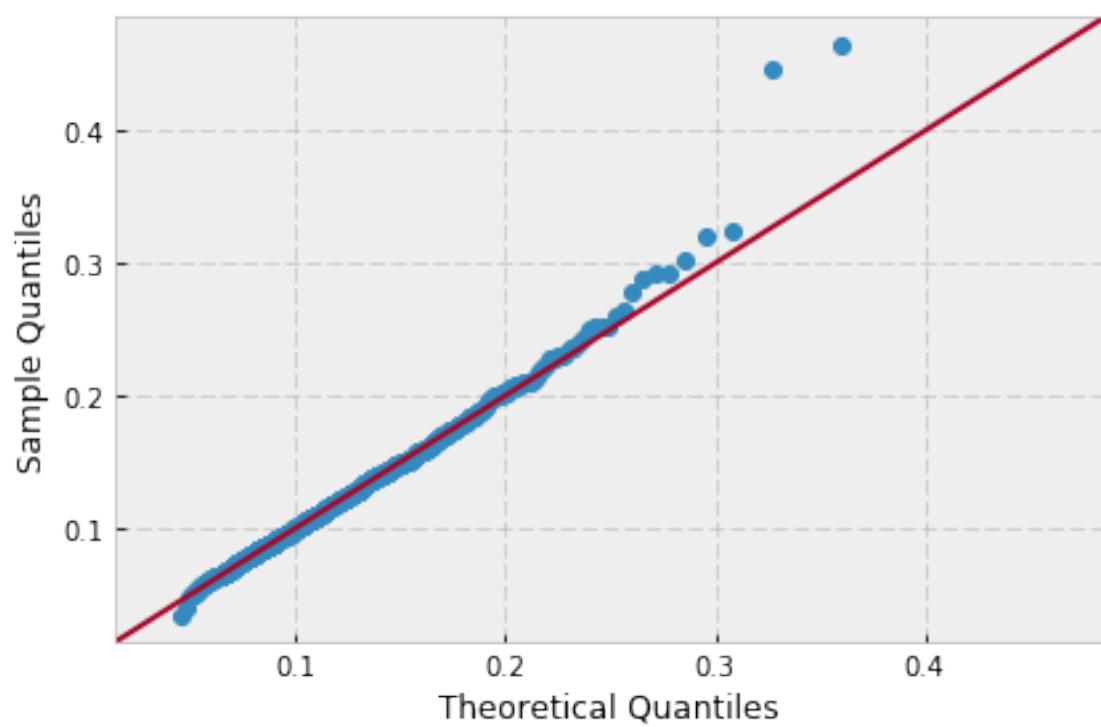
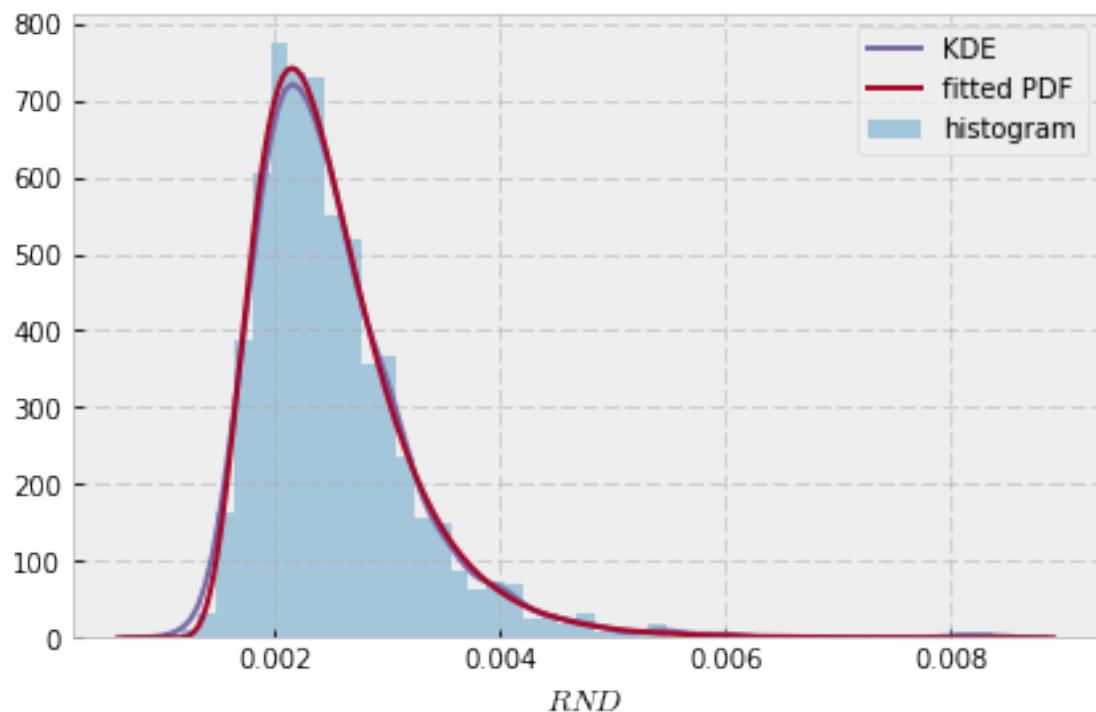


```
variable name: r
variable value: 2.5555555555555554
distribution: inverse-gamma(shape=(9.8109105602453575,),loc=0.0006533735958187208, scale=0.01757
DescribeResult(nobs=1000, minmax=(0.0014332872766533002, 0.0085060770813666781)
    mean=0.0026487938386748388, variance=5.1925167859330232e-07,
    skewness=1.772470651611544, kurtosis=6.682015102507471)
```

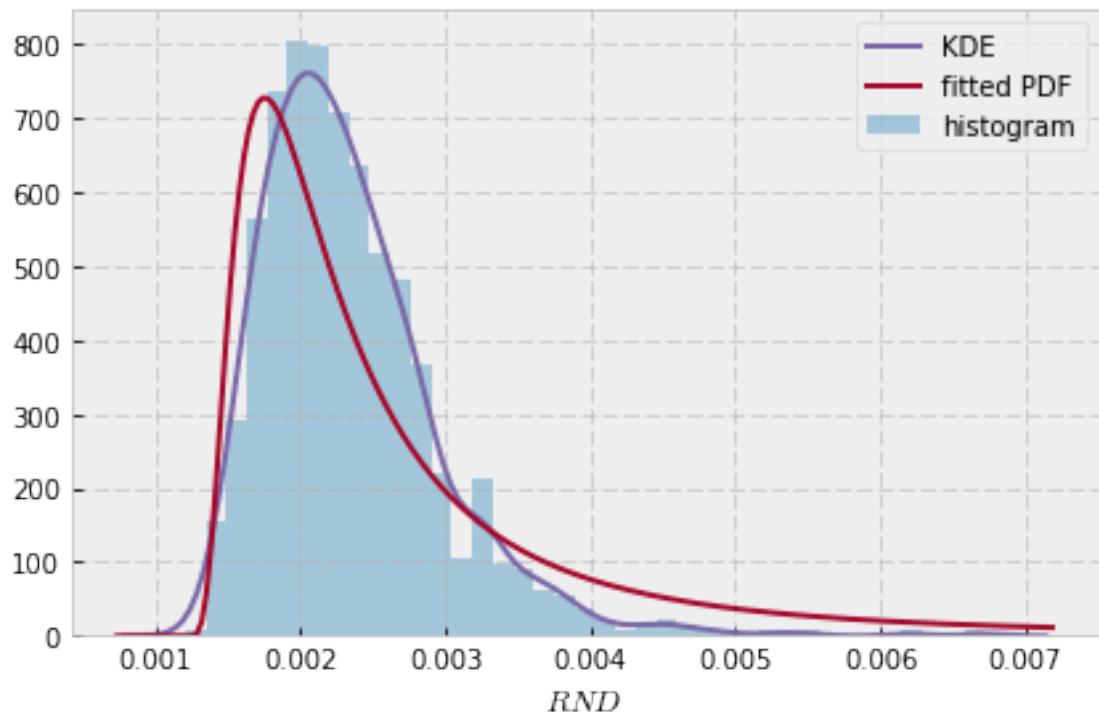


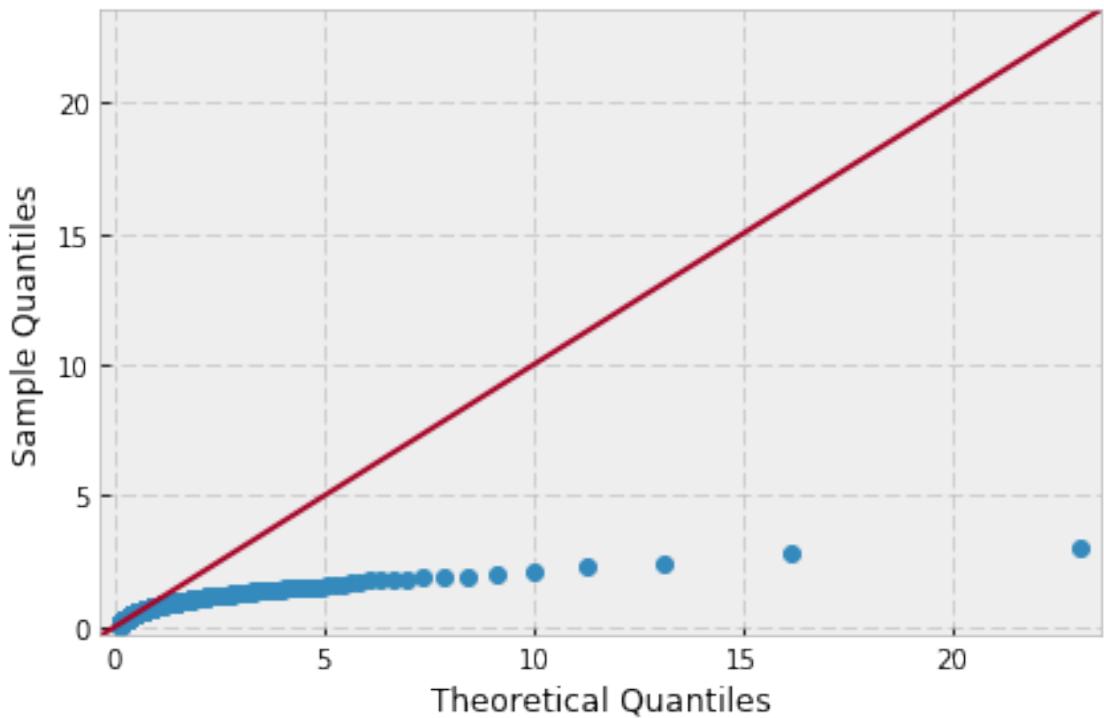


```
variable name: r
variable value: 2.777777777777777
distribution: inverse-gamma(shape=(9.6377047571794527,), loc=0.0005761964149588306, scale=0.01676
DescribeResult(nobs=1000, minmax=(0.0011579817530630607, 0.008362853345104726)
               mean=0.0025199244912648879, variance=5.3020197844960996e-07,
               skewness=2.0970849095140904, kurtosis=9.208075759175248)
```

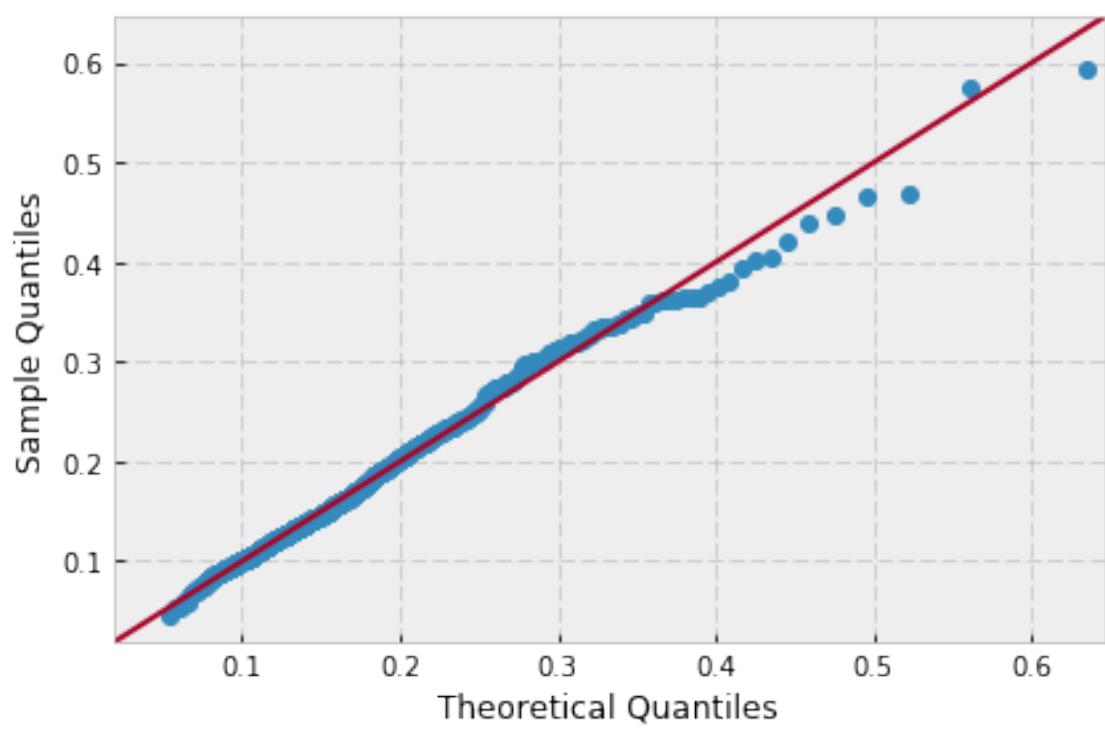
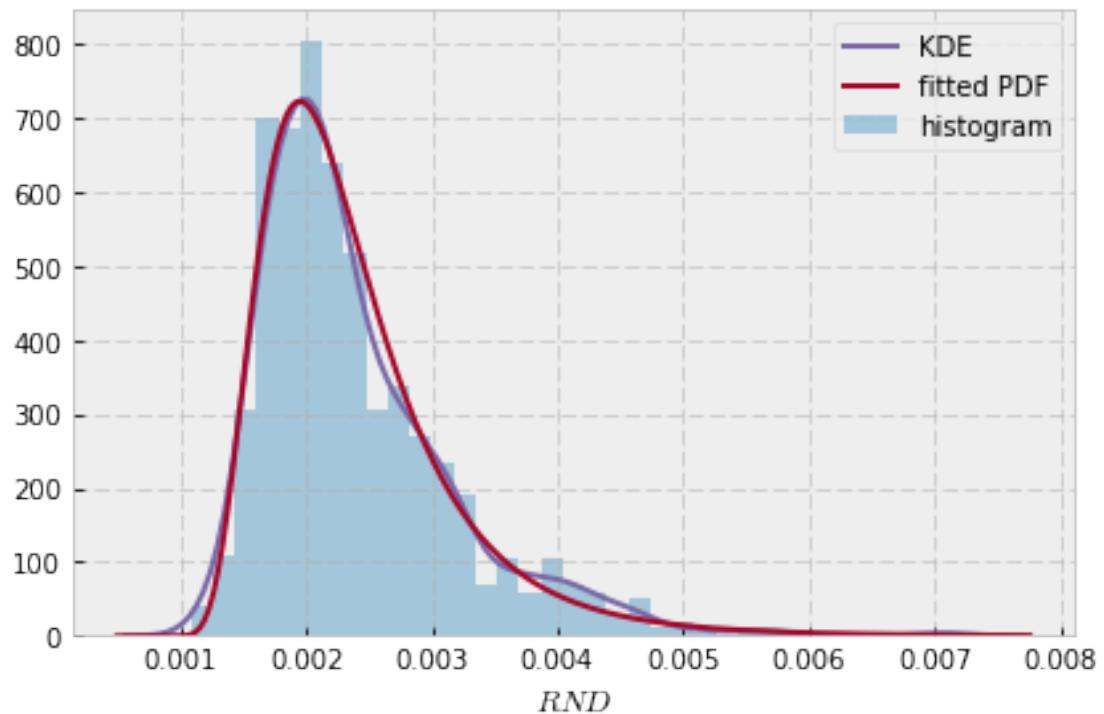


```
variable name: r
variable value: 3.0
distribution: inverse-gamma(shape=(1.97728366113635,), loc=0.0011418742166770372, scale=0.0018172
DescribeResult(nobs=1000, minmax=(0.0012034397731205697, 0.0067156737904481732)
               mean=0.0023406502949119318, variance=3.8638752640996922e-07,
               skewness=1.6515949318953824, kurtosis=5.501816572180807)
```

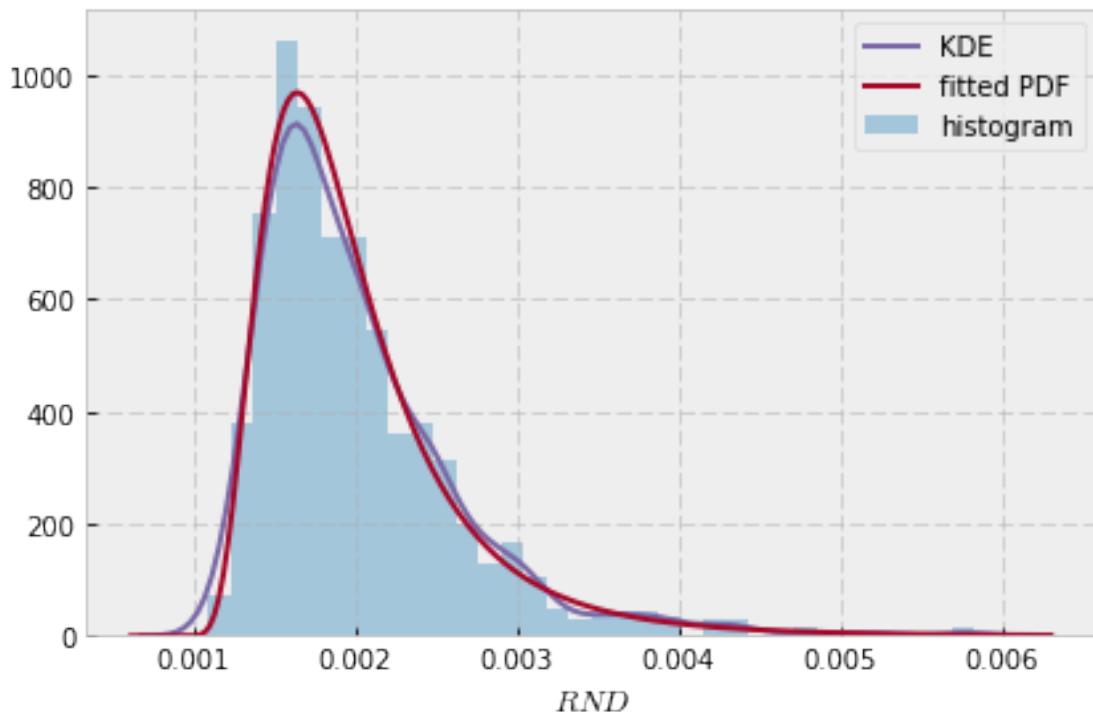


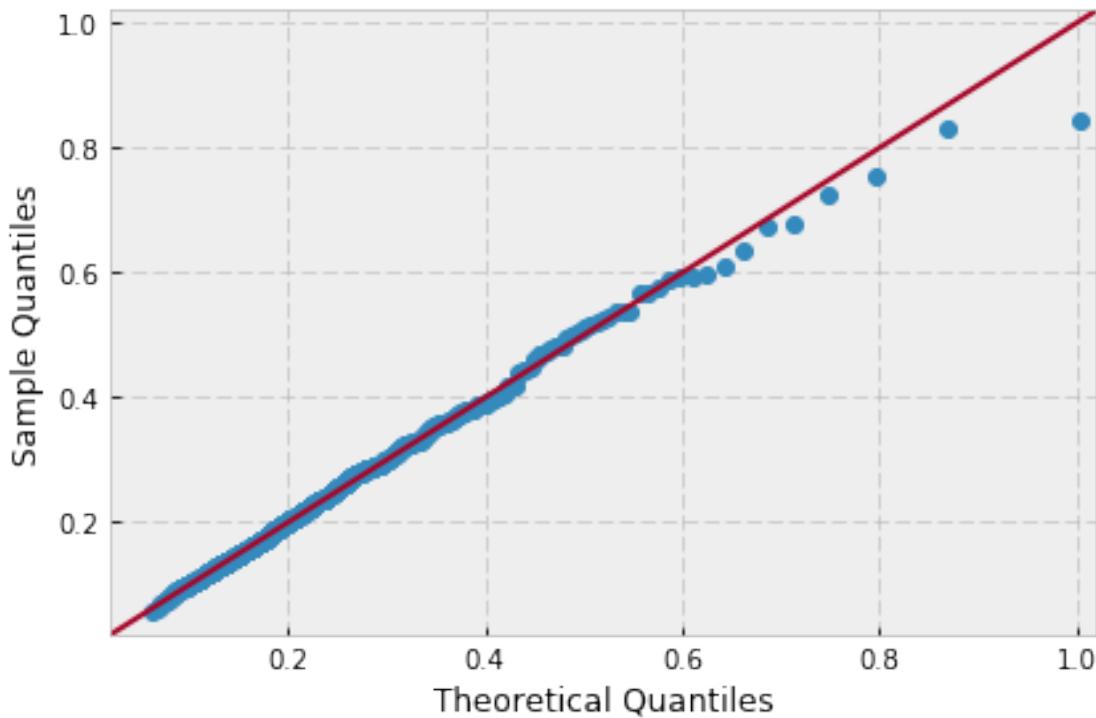


```
variable name: sigma
variable value: 0.0111111111111112
distribution: inverse-gamma(shape=(7.1294325718250864,), loc=0.0005831380926948795, scale=0.01110
DescribeResult(nobs=1000, minmax=(0.0010865724420020227, 0.0071652292481739349)
               mean=0.0023934841101512588, variance=6.1017347318825891e-07,
               skewness=1.606657353299011, kurtosis=3.8417525963303936)
```

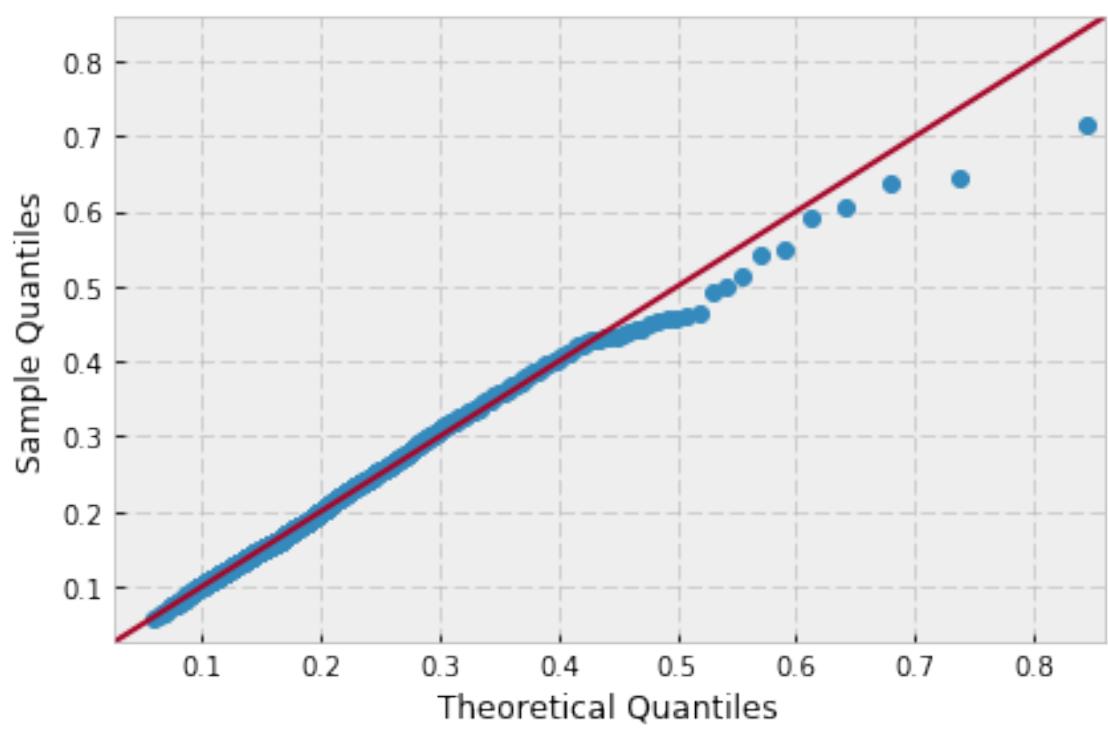
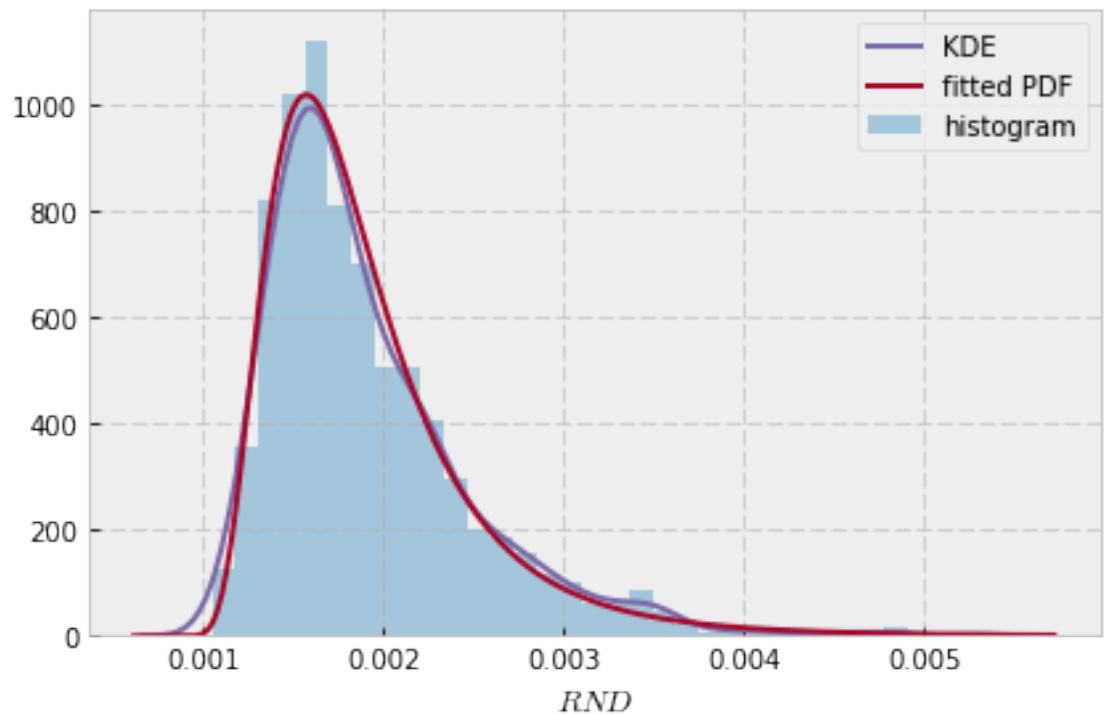


```
variable name: sigma
variable value: 0.0222222222222223
distribution: inverse-gamma(shape=(5.7184141132688131,),loc=0.0007465505796944842, scale=0.00602
DescribeResult(nobs=1000, minmax=(0.001092438282800187, 0.0058289831285405014)
               mean=0.0020219938655322935, variance=3.9902627249406751e-07,
               skewness=1.8687563039192818, kurtosis=5.299805870877238)
```

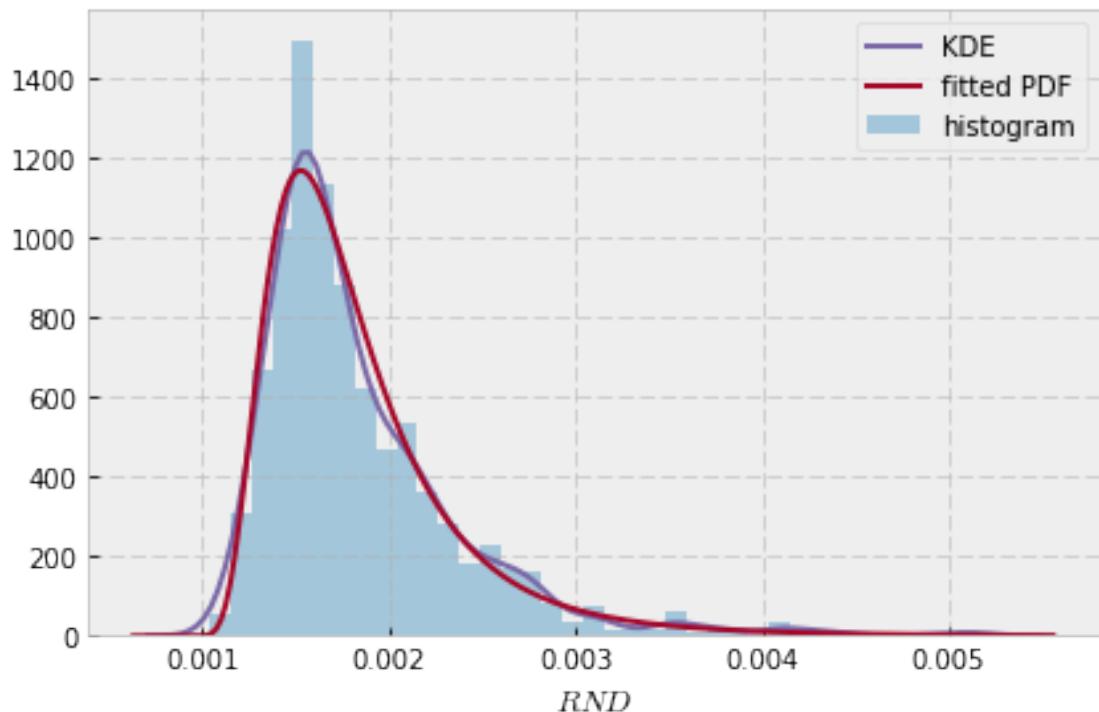


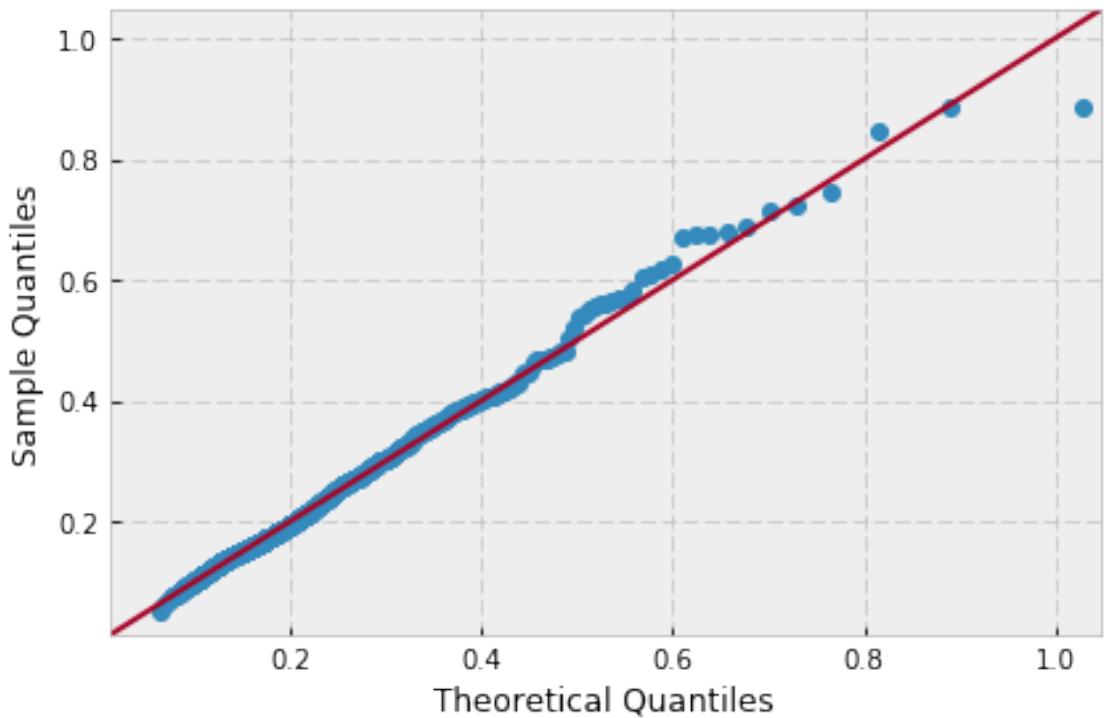


```
variable name: sigma
variable value: 0.0333333333333333
distribution: inverse-gamma(shape=(6.1987715411233886,), loc=0.0006773822543704434, scale=0.00644
DescribeResult(nobs=1000, minmax=(0.0010490725084385862, 0.0052885947319301621)
               mean=0.0019139670785234192, variance=3.2922851894993815e-07,
               skewness=1.6248083948632845, kurtosis=3.8069299543733903)
```

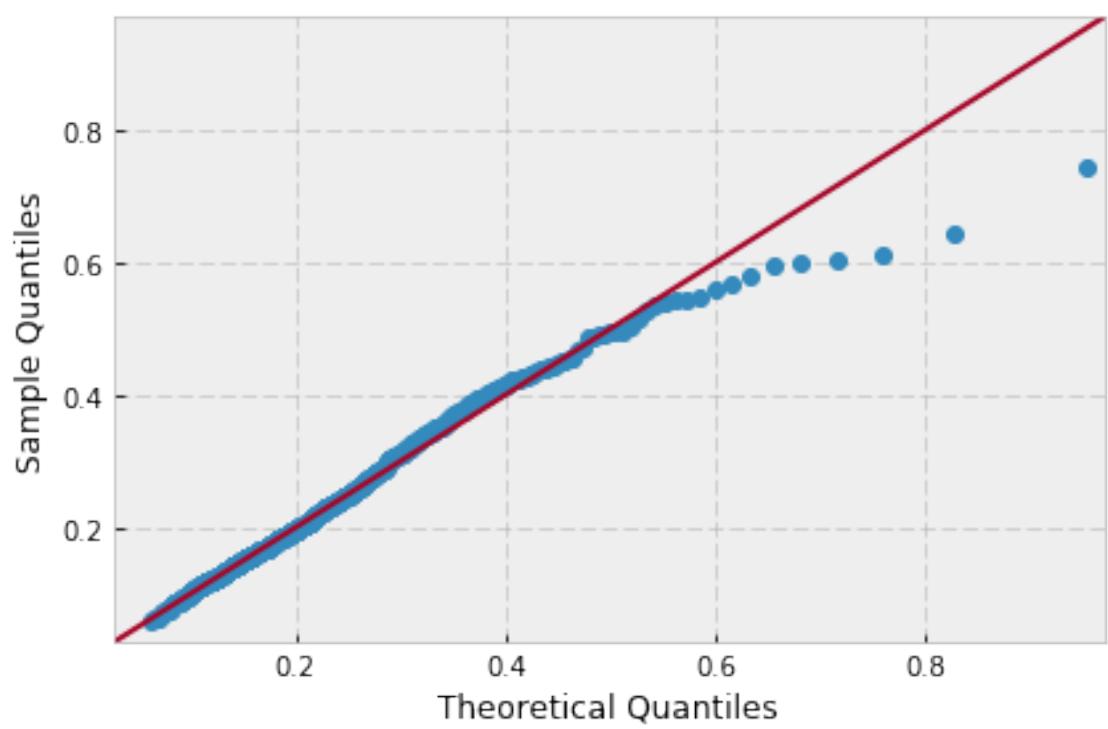
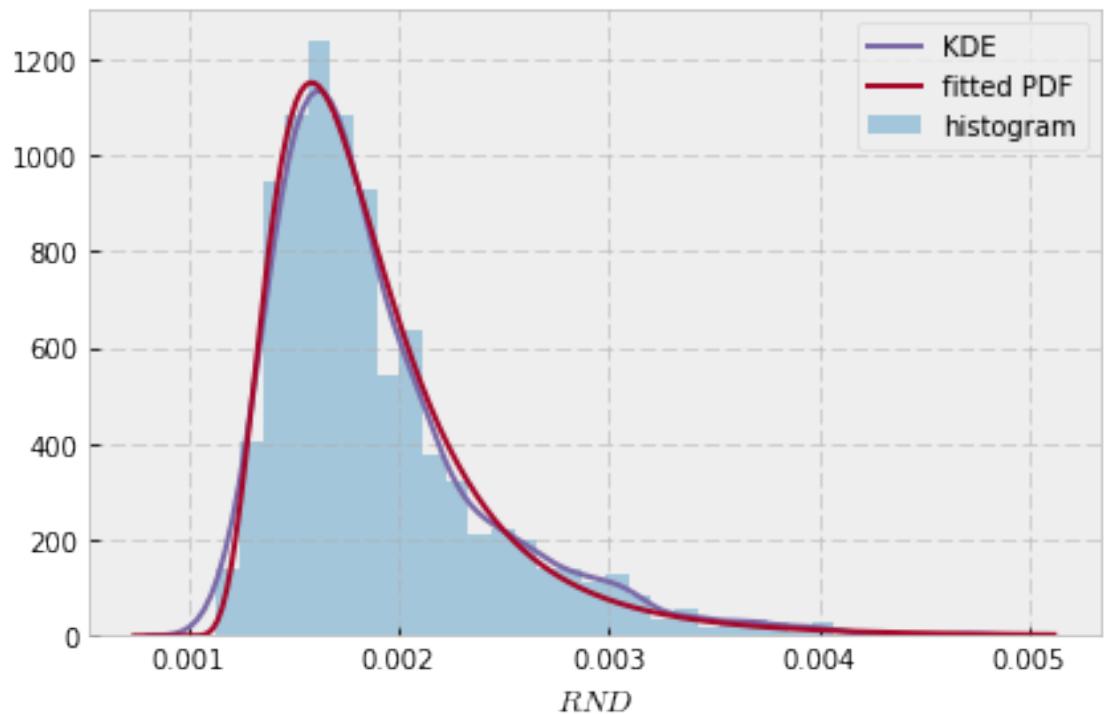


```
variable name: sigma
variable value: 0.04444444444444446
distribution: inverse-gamma(shape=(5.6513501930300549,),loc=0.0007878752380146173, scale=0.00491
DescribeResult(nobs=1000, minmax=(0.0010407518441419288, 0.0051476445036652838)
               mean=0.0018451963342749264, variance=2.9677450423618964e-07,
               skewness=2.082884751775205, kurtosis=6.425756523096034)
```

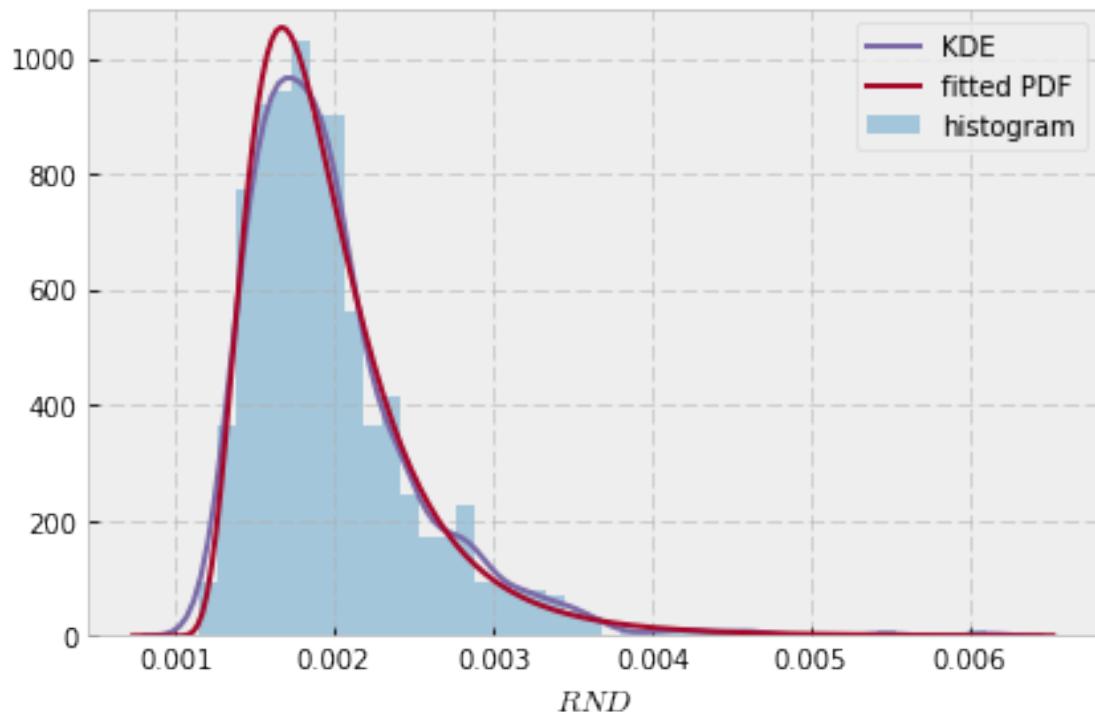


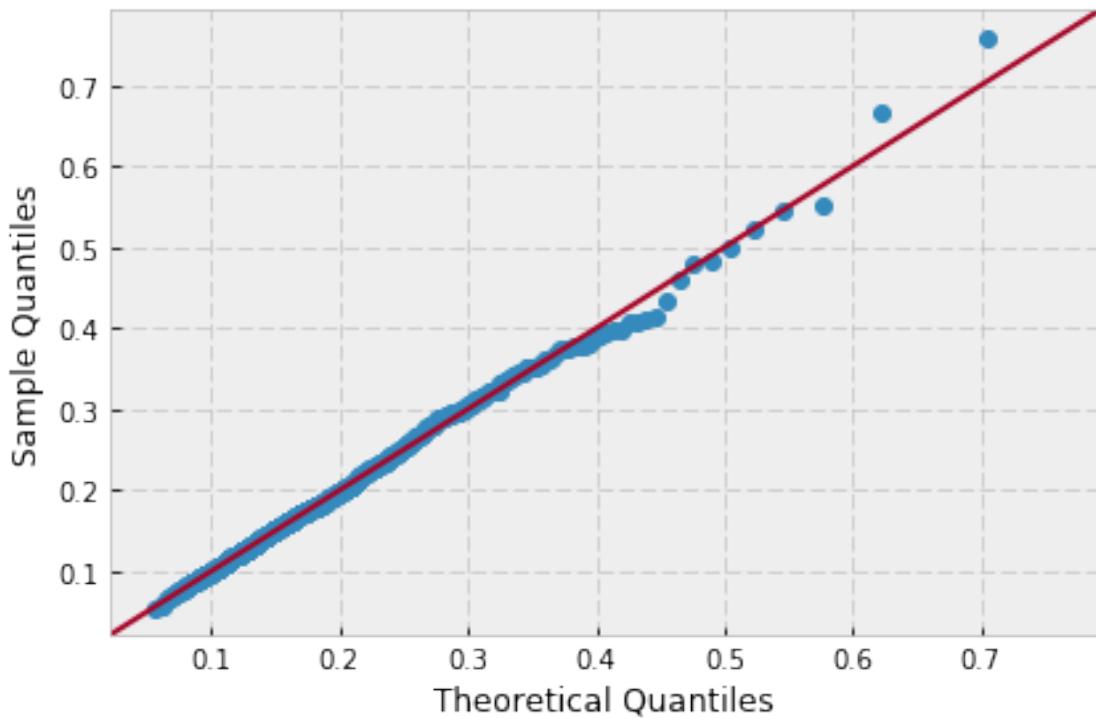


```
variable name: sigma
variable value: 0.05555555555555556
distribution: inverse-gamma(shape=(5.8553801168932162,), loc=0.0008217342823178373, scale=0.00524
DescribeResult(nobs=1000, minmax=(0.0011357964729575042, 0.0047253995324688883)
               mean=0.0018995172899466768, variance=2.6849807087637718e-07,
               skewness=1.567305608643505, kurtosis=2.954440443426229)
```

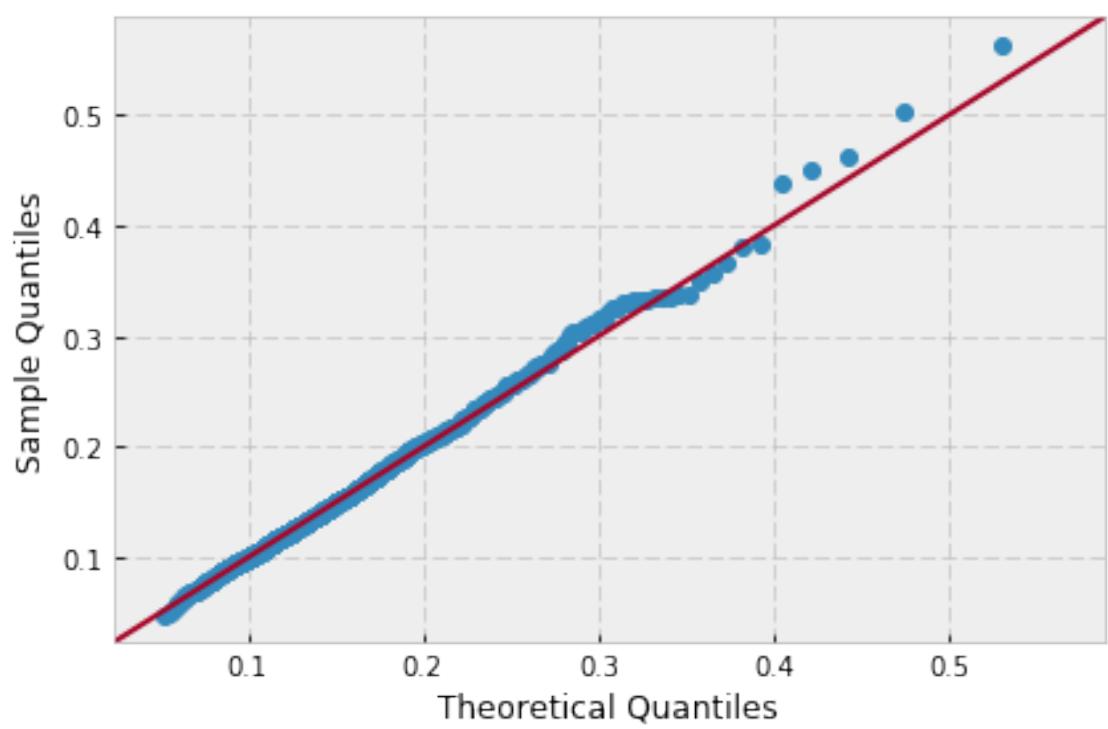
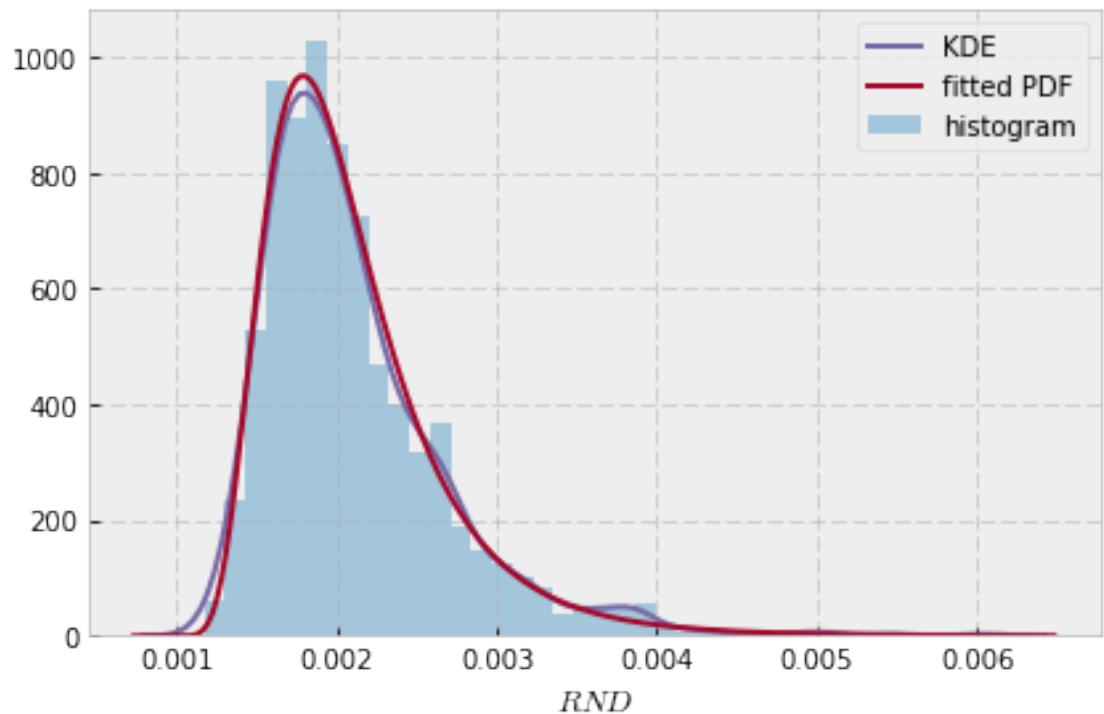


```
variable name: sigma
variable value: 0.06666666666666667
distribution: inverse-gamma(shape=(6.7613000035280049,),loc=0.0007621510254981579, scale=0.00705
DescribeResult(nobs=1000, minmax=(0.001149481007397167, 0.0061070873844520975)
               mean=0.0019857160783252402, variance=3.0259266304244608e-07,
               skewness=1.909236304197017, kurtosis=6.581794183496292)
```

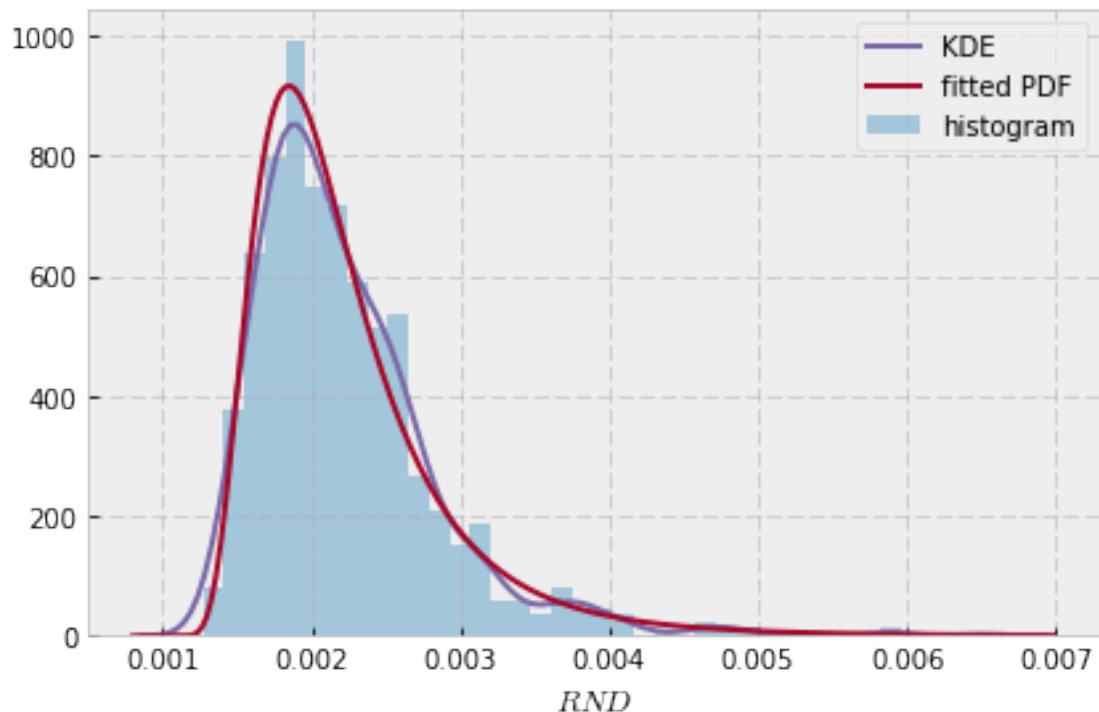


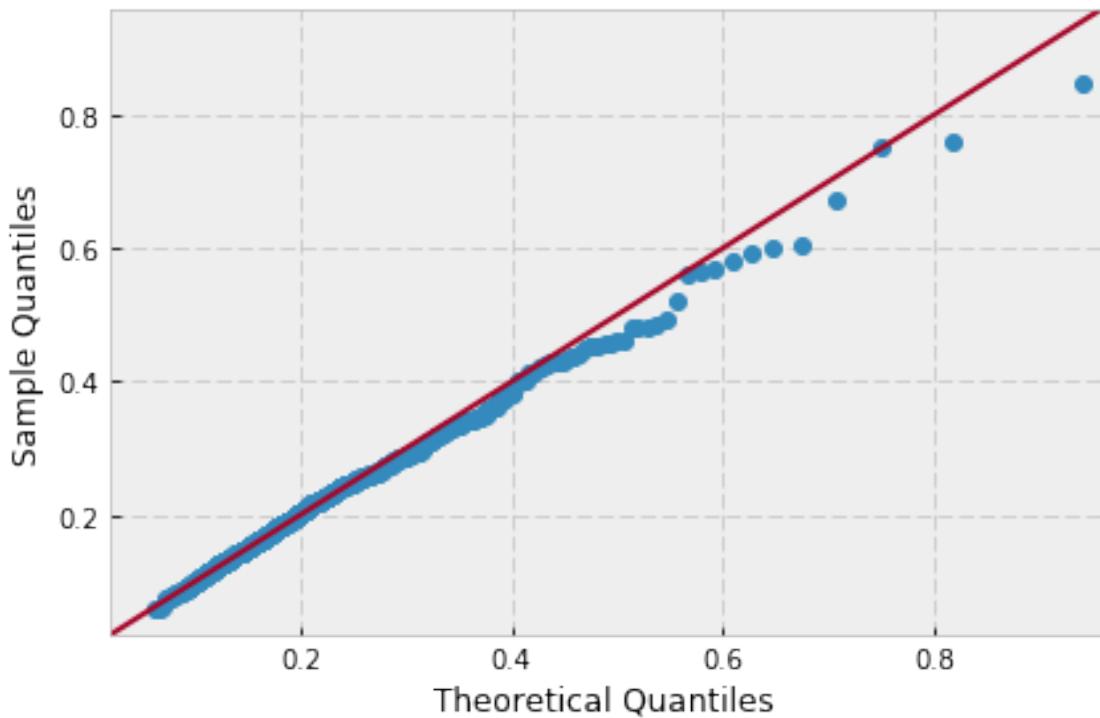


```
variable name: sigma
variable value: 0.07777777777777778
distribution: inverse-gamma(shape=(7.81275815202169,), loc=0.0007126825642379032, scale=0.0094680
DescribeResult(nobs=1000, minmax=(0.0011714650129341171, 0.006041586789568191)
               mean=0.0021057131517739263, variance=3.4123536989635804e-07,
               skewness=1.7931297343555197, kurtosis=5.326455249382628)
```

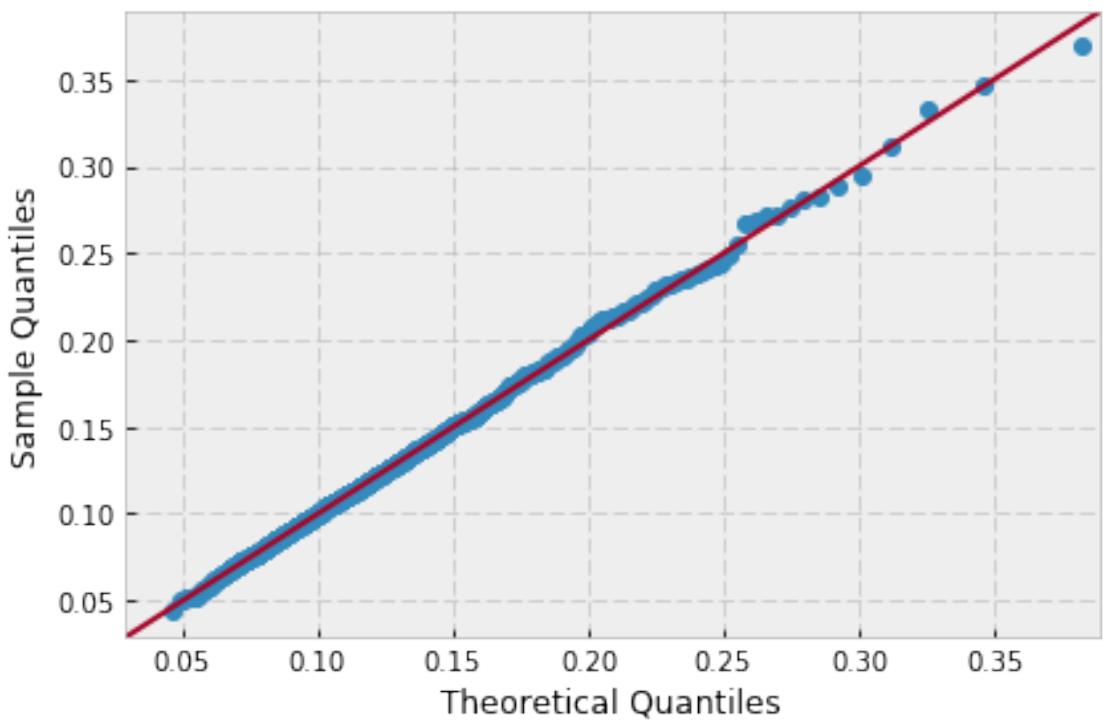
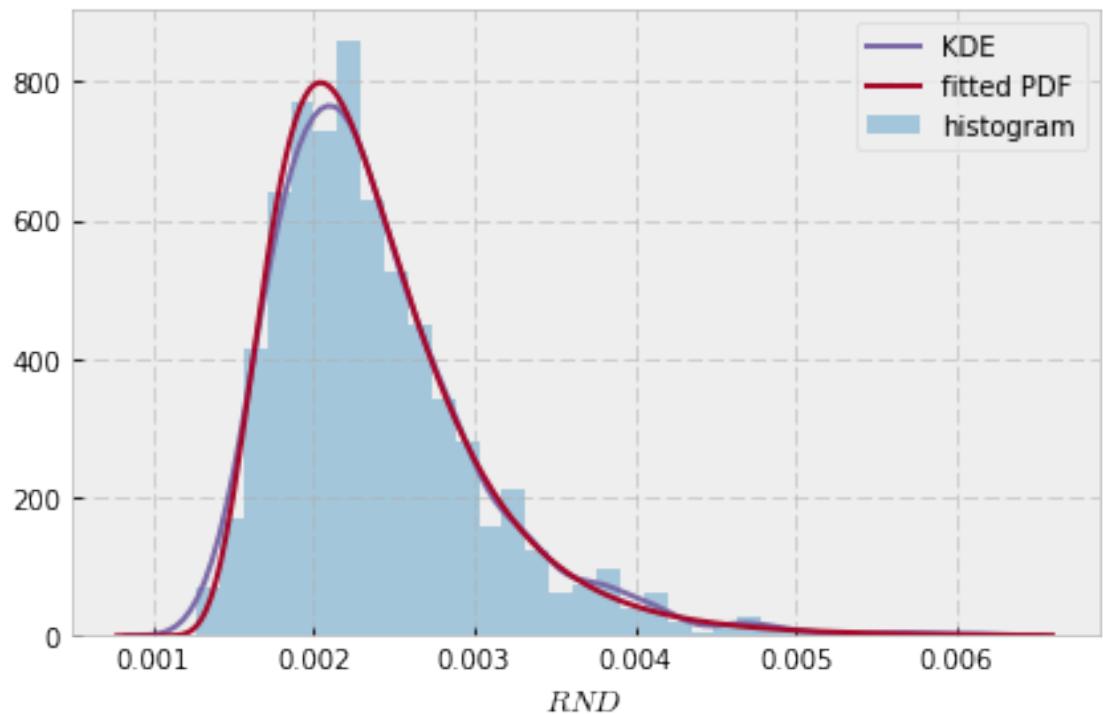


```
variable name: sigma
variable value: 0.0888888888888889
distribution: inverse-gamma(shape=(5.8931516085316105,),loc=0.0008810944300765879, scale=0.00665
DescribeResult(nobs=1000, minmax=(0.0012692813397541204, 0.0065102051164576628)
               mean=0.0022241555437955726, variance=3.8949087207058788e-07,
               skewness=1.9088010401973008, kurtosis=6.3994059679225685)
```

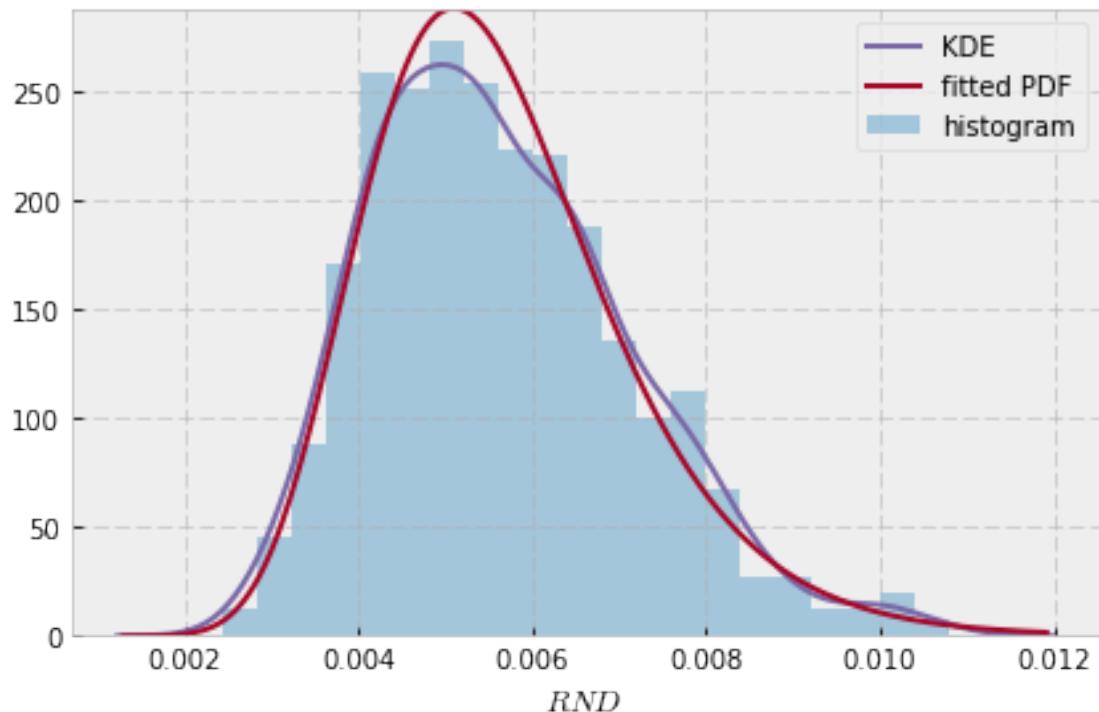


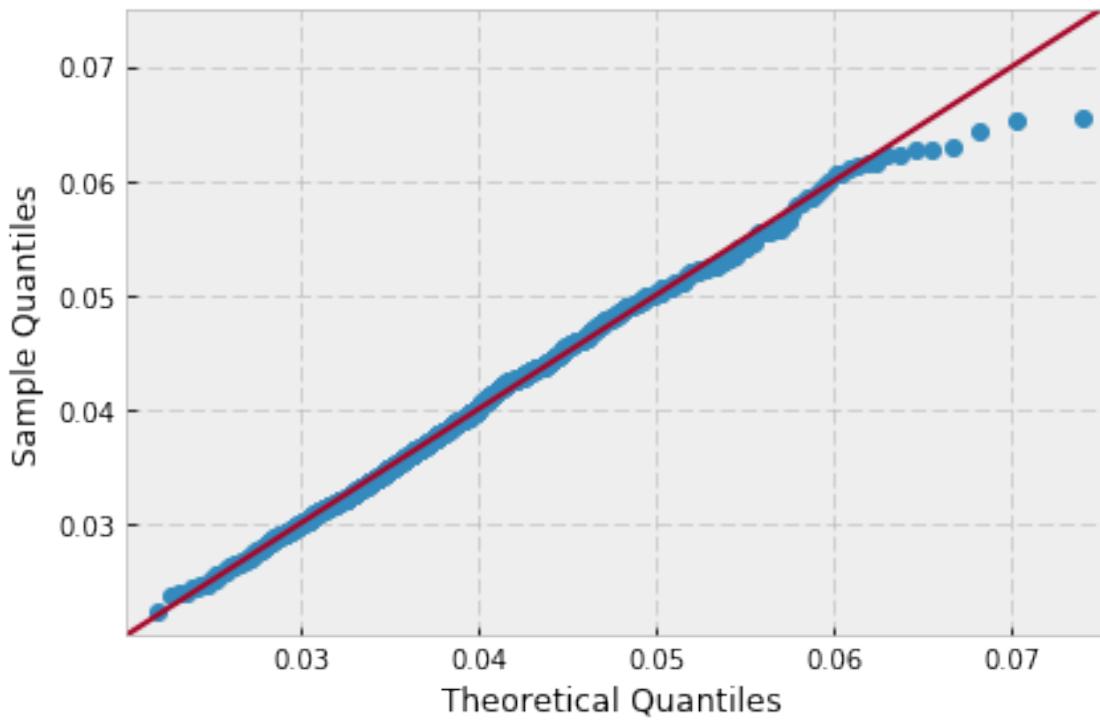


```
variable name: sigma
variable value: 0.1
distribution: inverse-gamma(shape=(9.3307500819871567,), loc=0.0006061780118854752, scale=0.01485
DescribeResult(nobs=1000, minmax=(0.0012709248223530109, 0.0061032717333038512)
               mean=0.0023891162320771289, variance=4.2518839798470721e-07,
               skewness=1.4643648567993106, kurtosis=3.366662762704647)
```

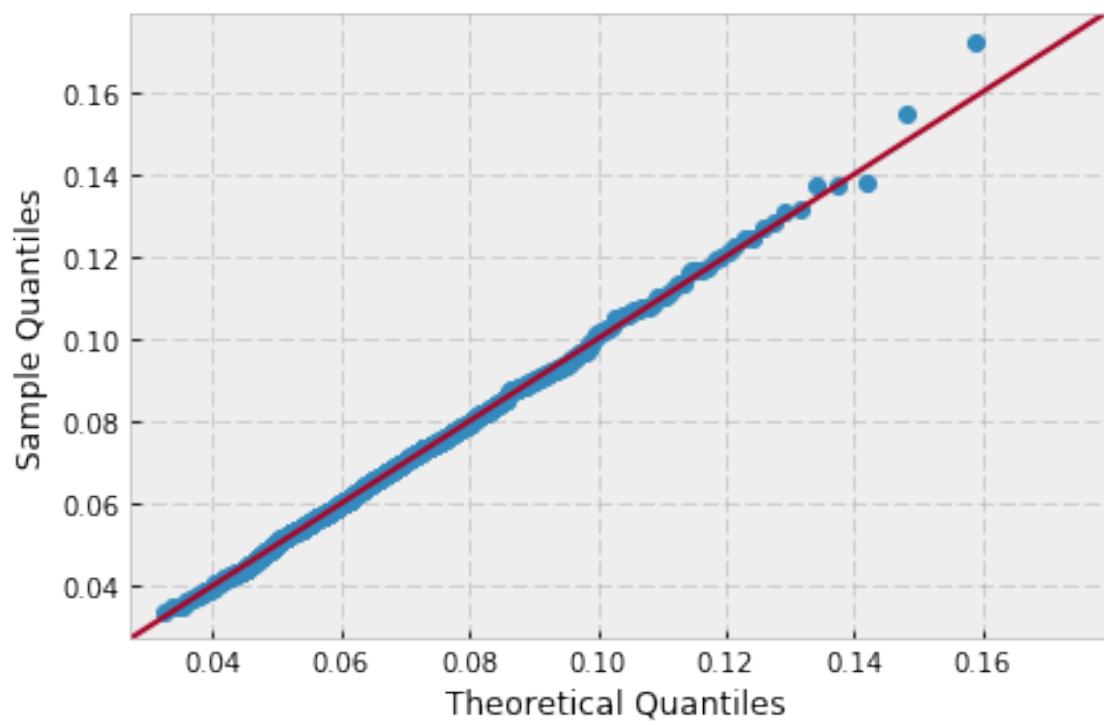
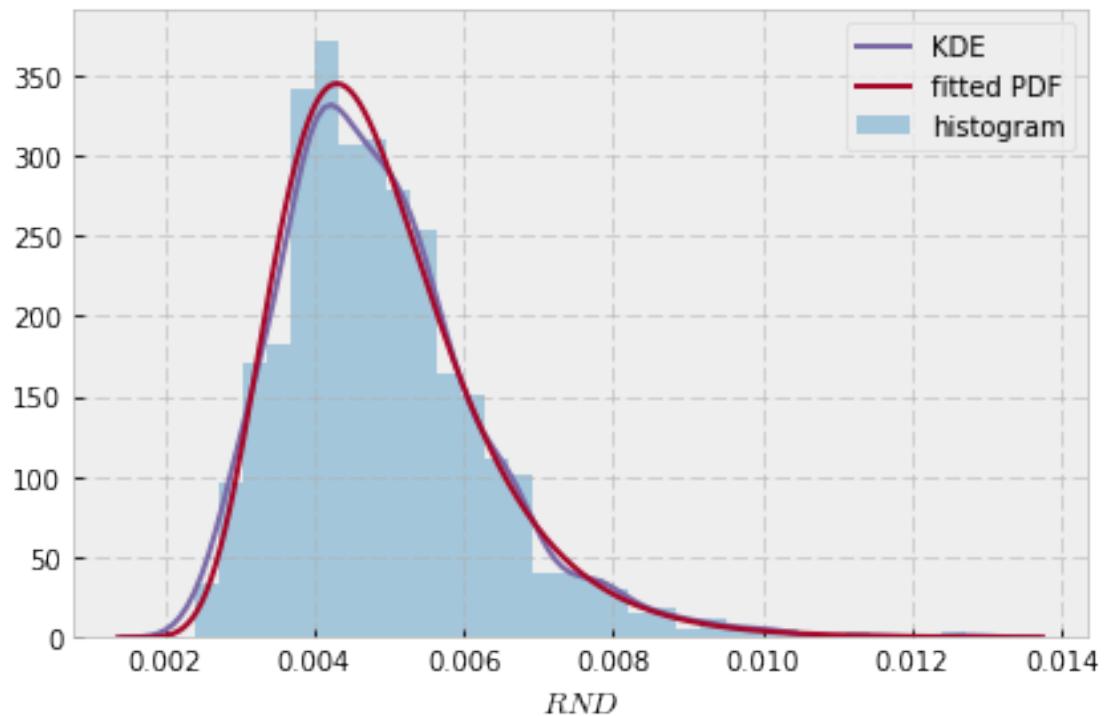


```
variable name: theta
variable value: 0.6981317007977318
distribution: inverse-gamma(shape=(26.713933111899252,),loc=-0.0019103165880605334, scale=0.1941
DescribeResult(nobs=1000, minmax=(0.0024181869166037548, 0.010784604980312279)
               mean=0.0056379442872697078, variance=2.2527427220307903e-06,
               skewness=0.6225823684465879, kurtosis=0.19145337638262783)
```

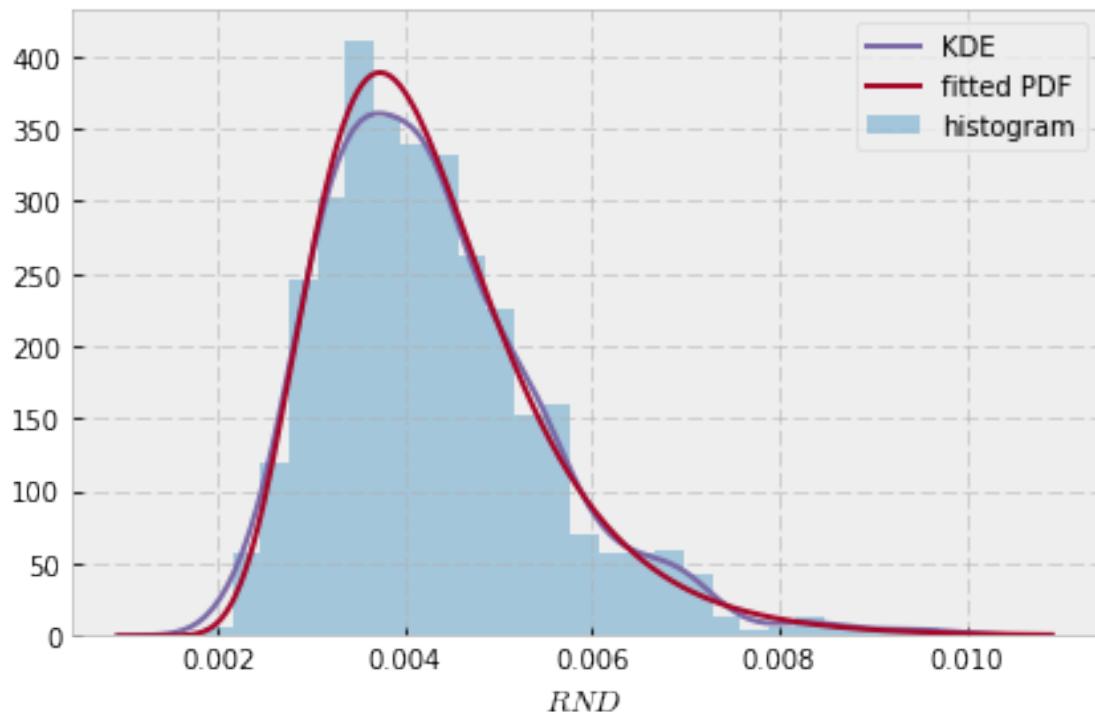


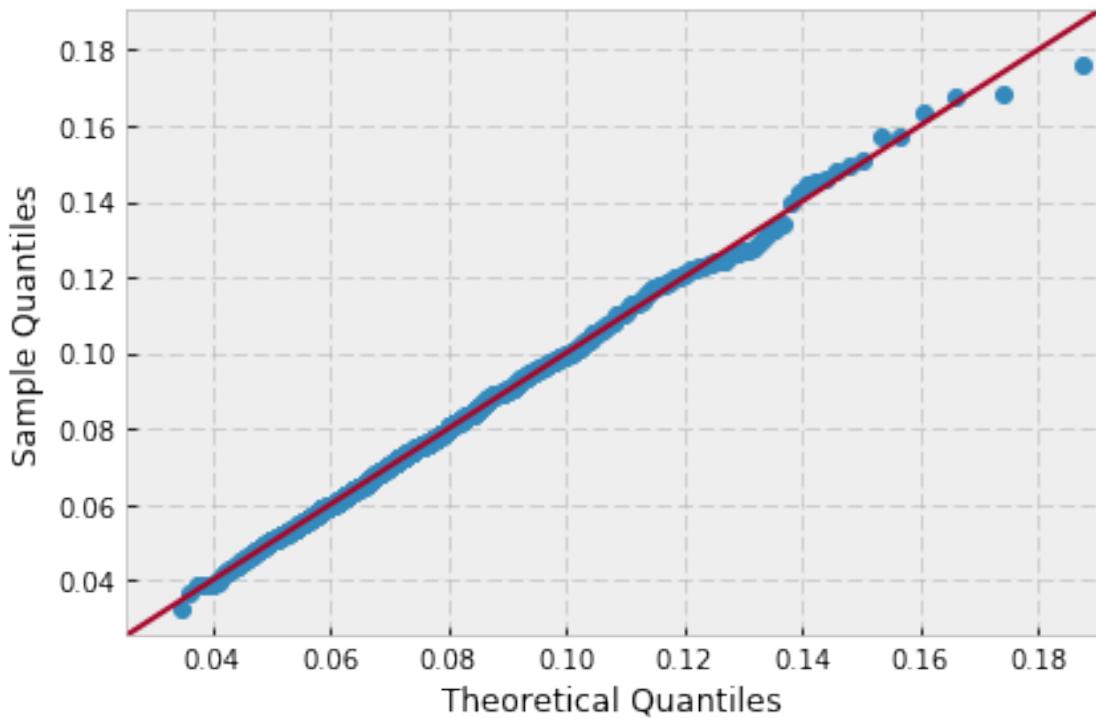


```
variable name: theta
variable value: 0.7853981633974483
distribution: inverse-gamma(shape=(15.828857201088734,), loc=-0.0001440462335362261, scale=0.0748
DescribeResult(nobs=1000, minmax=(0.0023951589065198536, 0.012725559087435757)
               mean=0.0049006001612865828, variance=1.8331766969831501e-06,
               skewness=1.1223557727944928, kurtosis=2.3183211448915824)
```

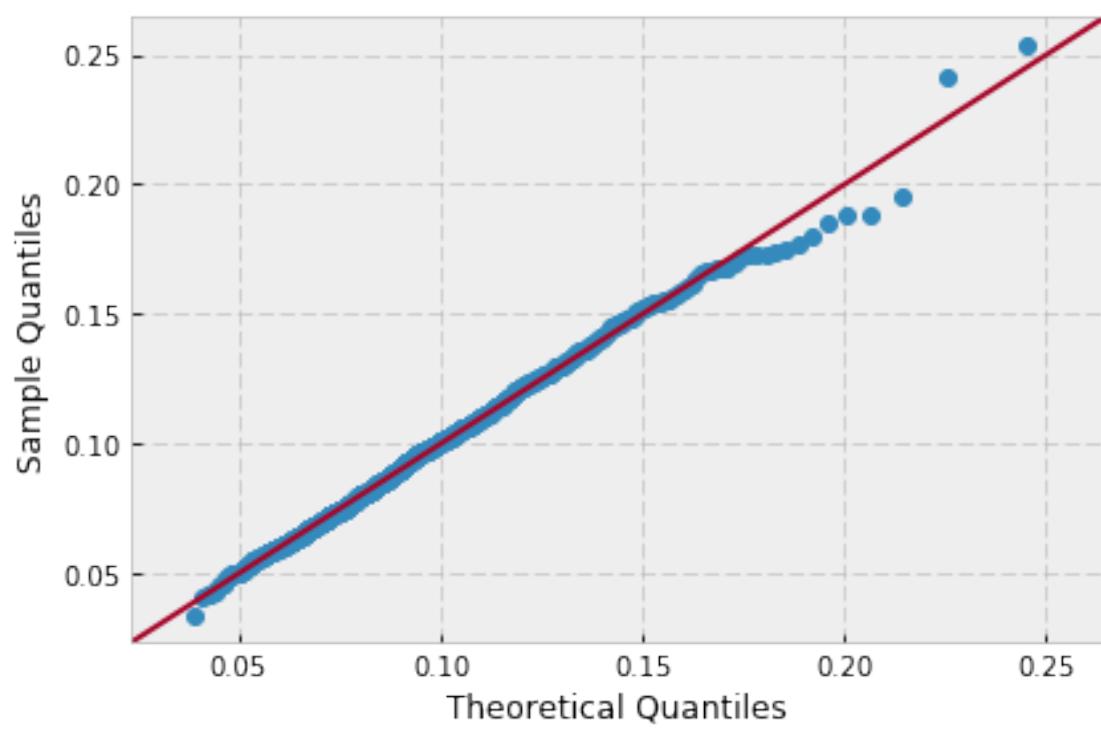
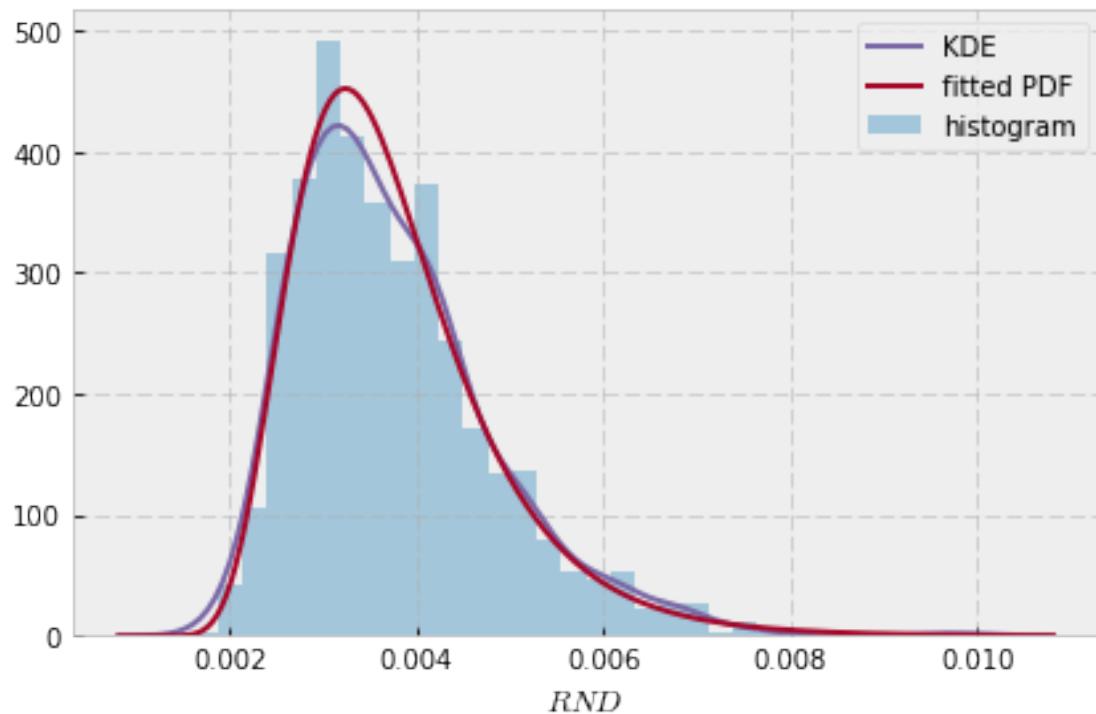


```
variable name: theta
variable value: 0.8726646259971648
distribution: inverse-gamma(shape=(14.23285417970331,), loc=1.0596465353663373e-05, scale=0.05670
DescribeResult(nobs=1000, minmax=(0.0018523613747456812, 0.0099929372413421931)
               mean=0.0042948176803821328, variance=1.4753655937822054e-06,
               skewness=1.0800849318329897, kurtosis=1.7452633307748702)
```

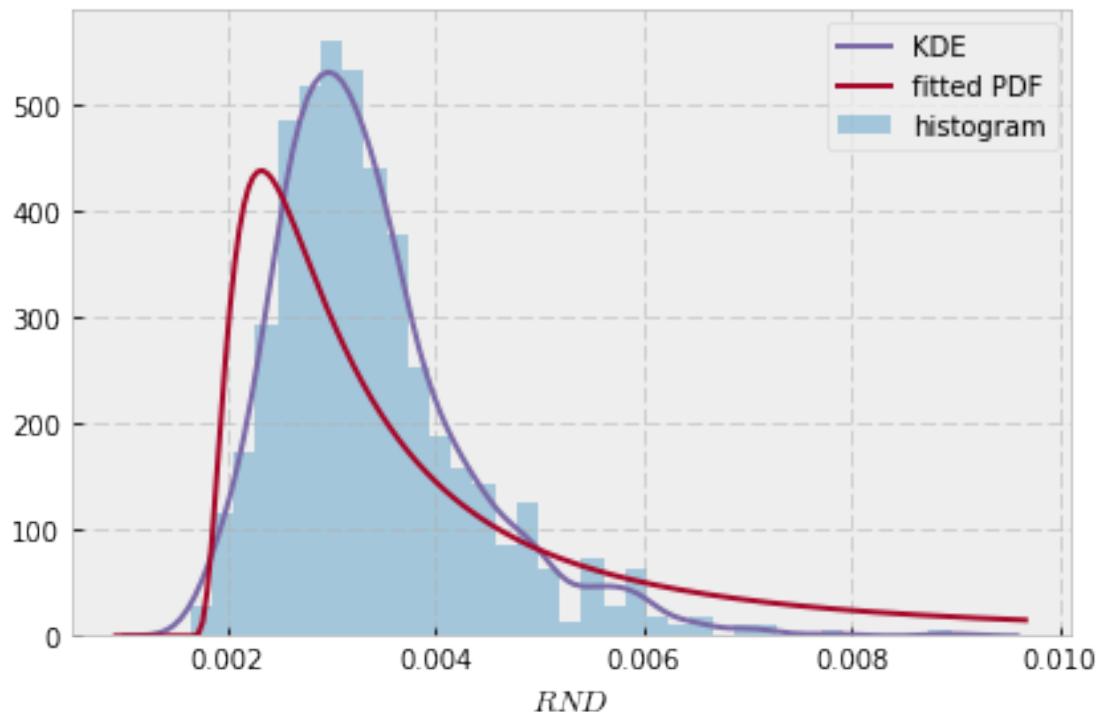


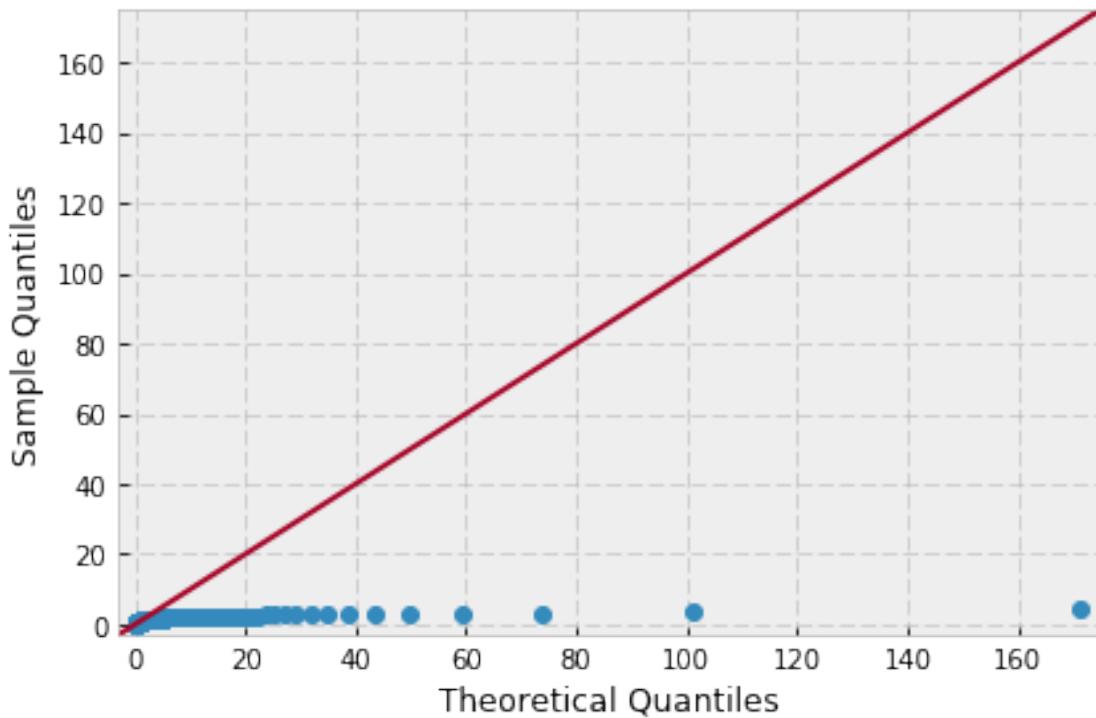


```
variable name: theta
variable value: 0.9599310885968813
distribution: inverse-gamma(shape=(12.059791939604422,), loc=0.00031513818644757566, scale=0.0382
DescribeResult(nobs=1000, minmax=(0.0016202226547197066, 0.010004912711584197)
               mean=0.0037690424208182182, variance=1.1540437021955503e-06,
               skewness=1.1747169935079307, kurtosis=2.3030263771807844)
```

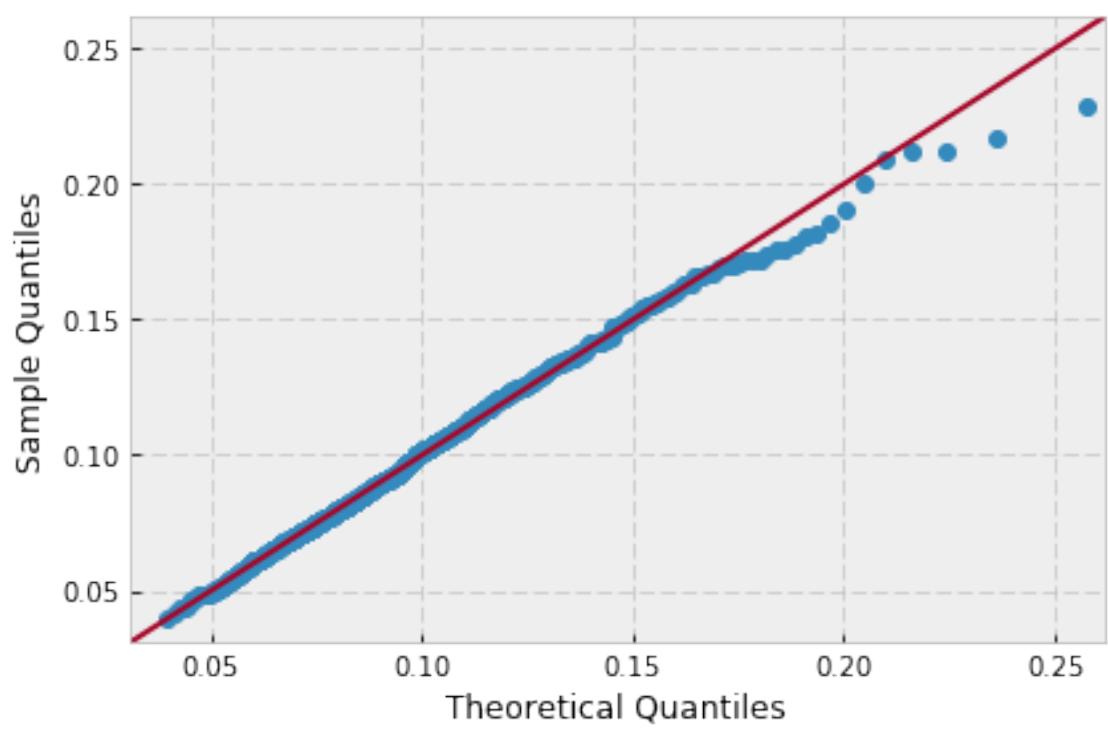
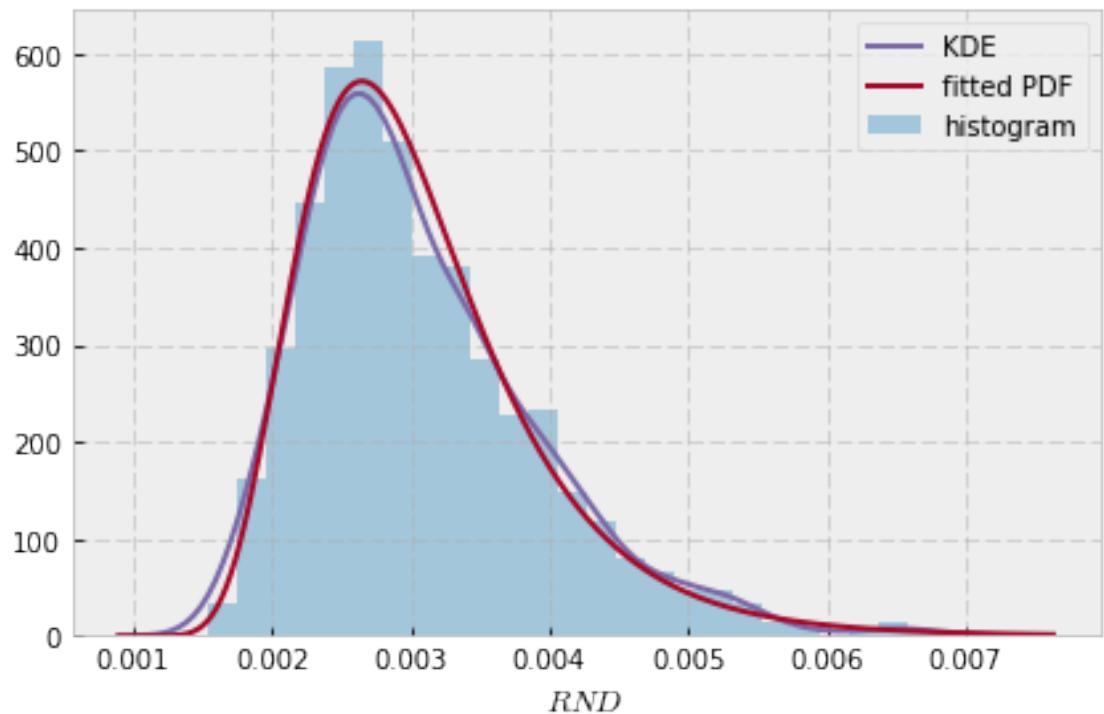


```
variable name: theta
variable value: 1.0471975511965979
distribution: inverse-gamma(shape=(1.3110443858986005,),loc=0.0015742771513559866, scale=0.00175
DescribeResult(nobs=1000, minmax=(0.0016530872347353986, 0.0089493947010920814)
    mean=0.0033901211261146385, variance=9.0358620254277982e-07,
    skewness=1.3107465510484149, kurtosis=2.5948595562643426)
```

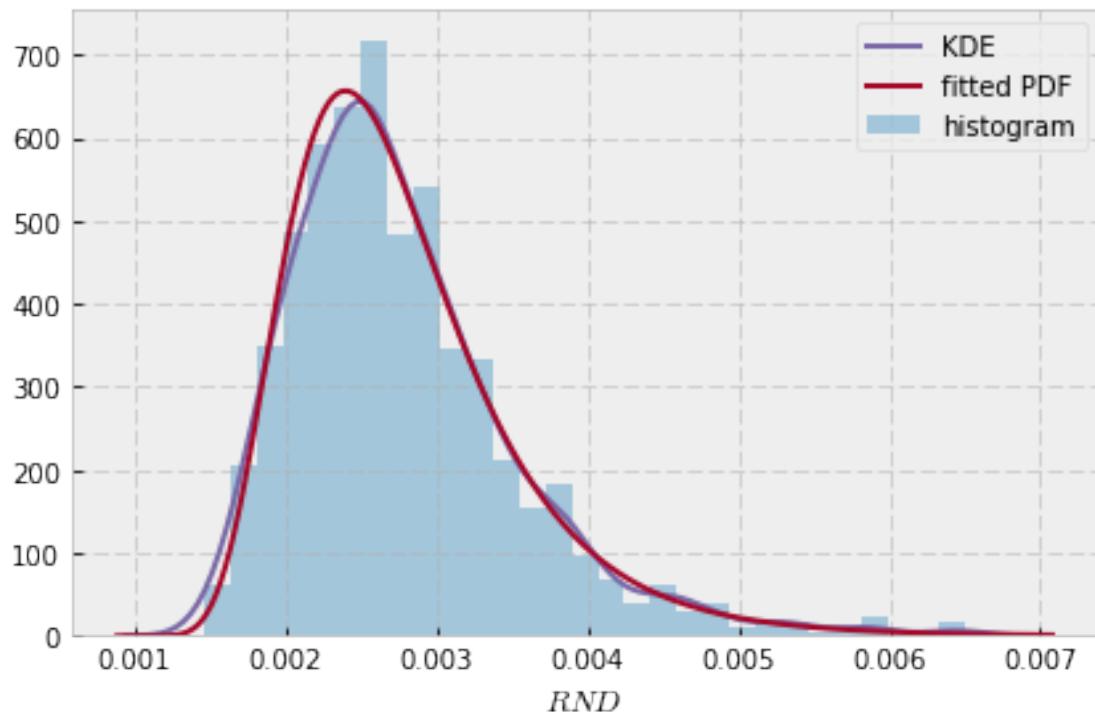


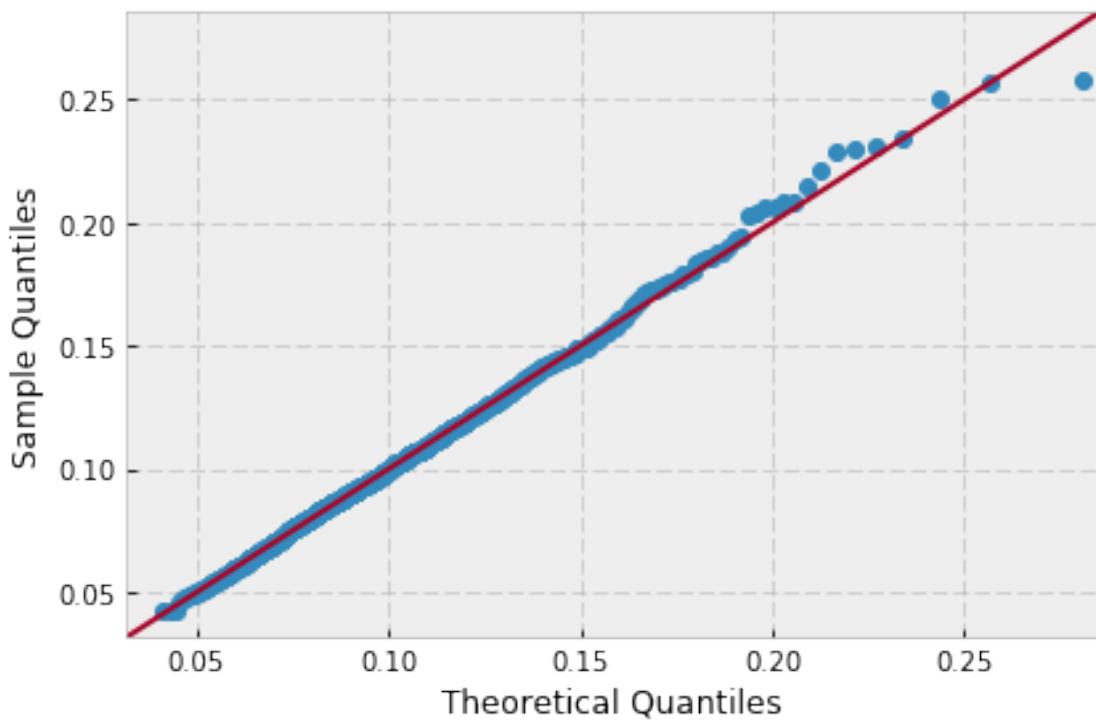


```
variable name: theta
variable value: 1.1344640137963142
distribution: inverse-gamma(shape=(11.721253650949407,), loc=0.0003692505372555832, scale=0.02899
DescribeResult(nobs=1000, minmax=(0.0015311130368986653, 0.0069934249523205113)
               mean=0.0030721290279496472, variance=7.1884169498416139e-07,
               skewness=1.0706043996156314, kurtosis=1.4598090239513741)
```

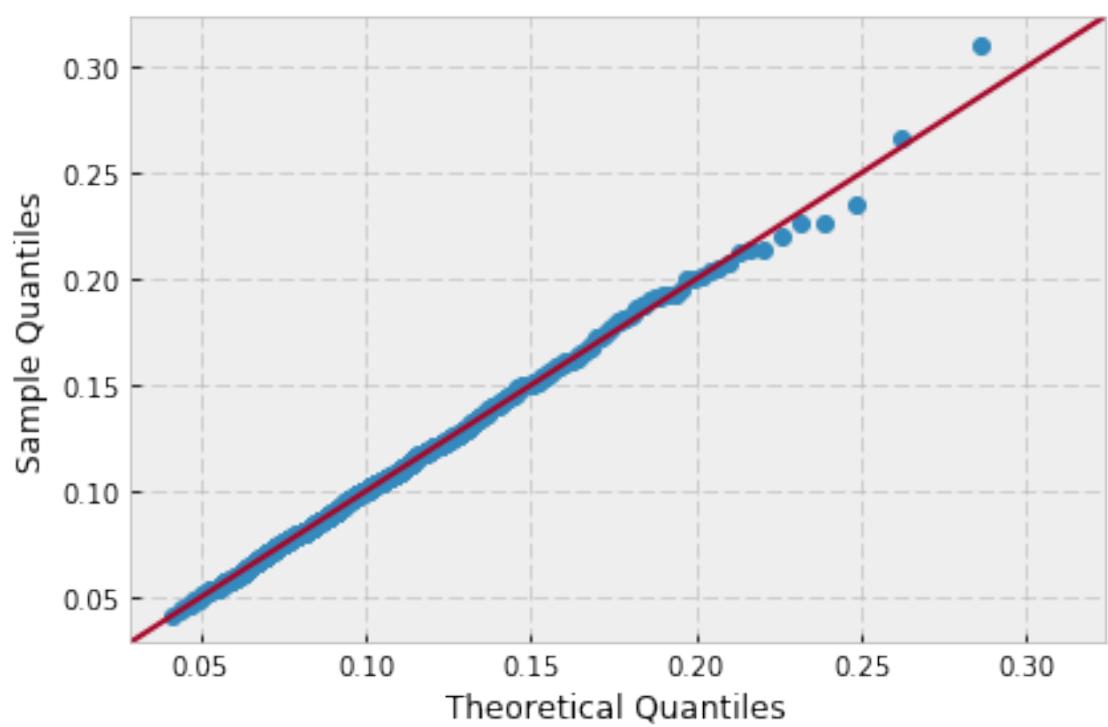
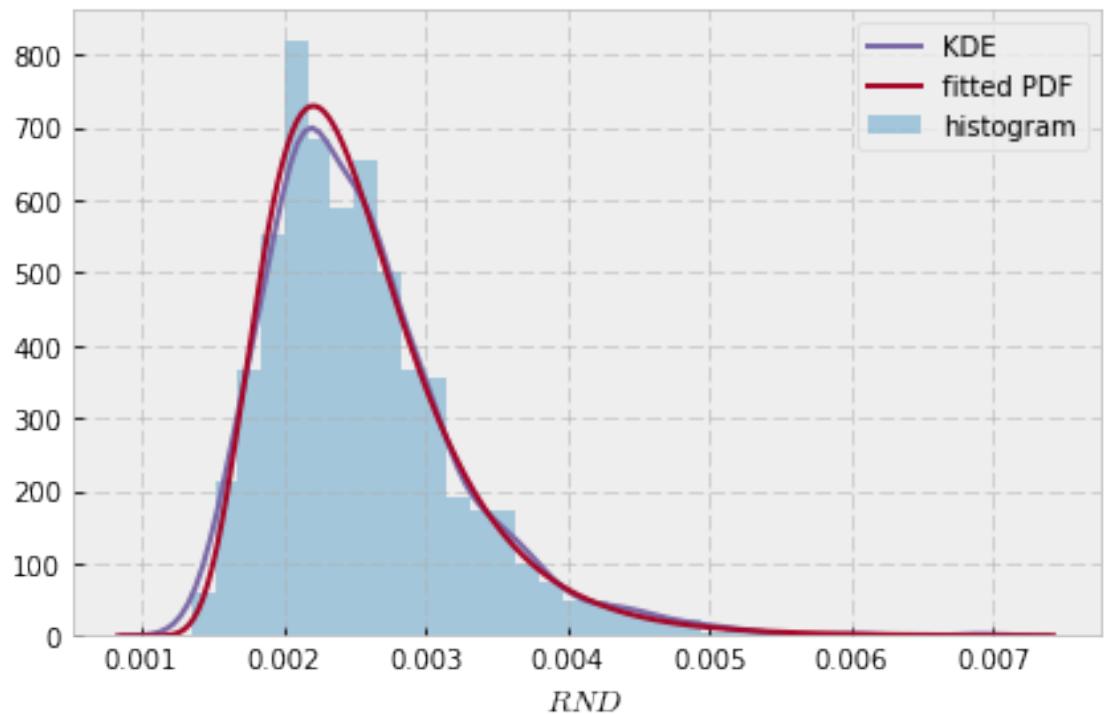


```
variable name: theta
variable value: 1.2217304763960306
distribution: inverse-gamma(shape=(11.13491250658209,), loc=0.000467767337694939, scale=0.0234080
DescribeResult(nobs=1000, minmax=(0.0014570444193502598, 0.0065063839915163509)
    mean=0.0027771135373555948, variance=5.7749213948061332e-07,
    skewness=1.3383413989379485, kurtosis=2.8085051300562993)
```

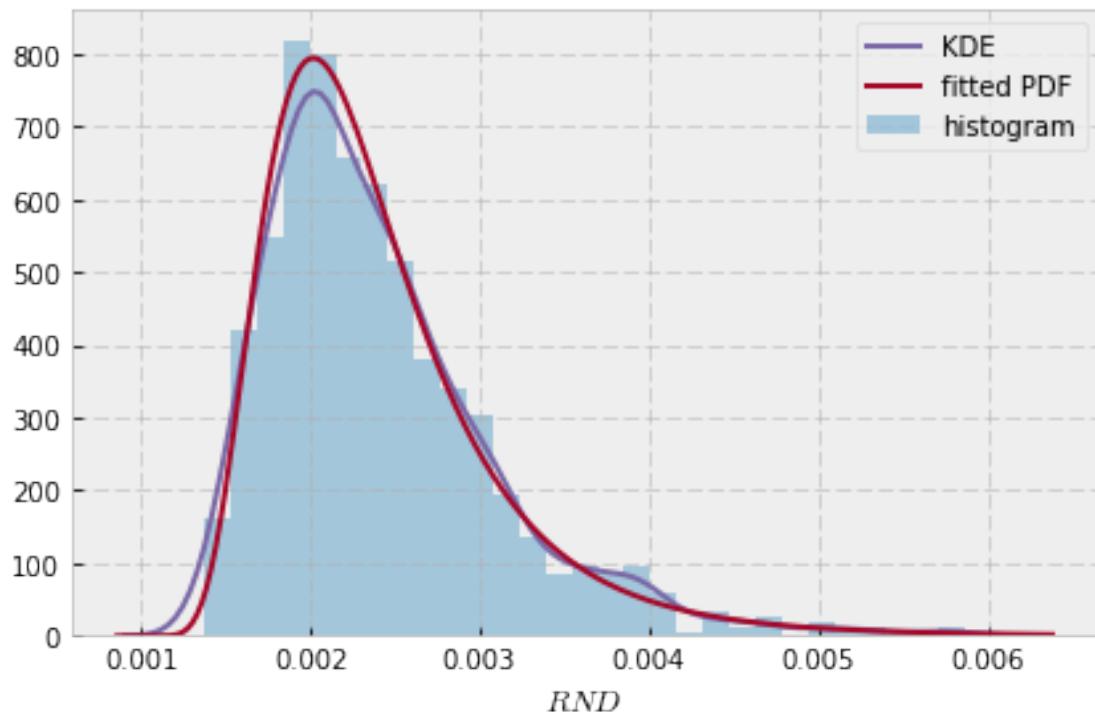


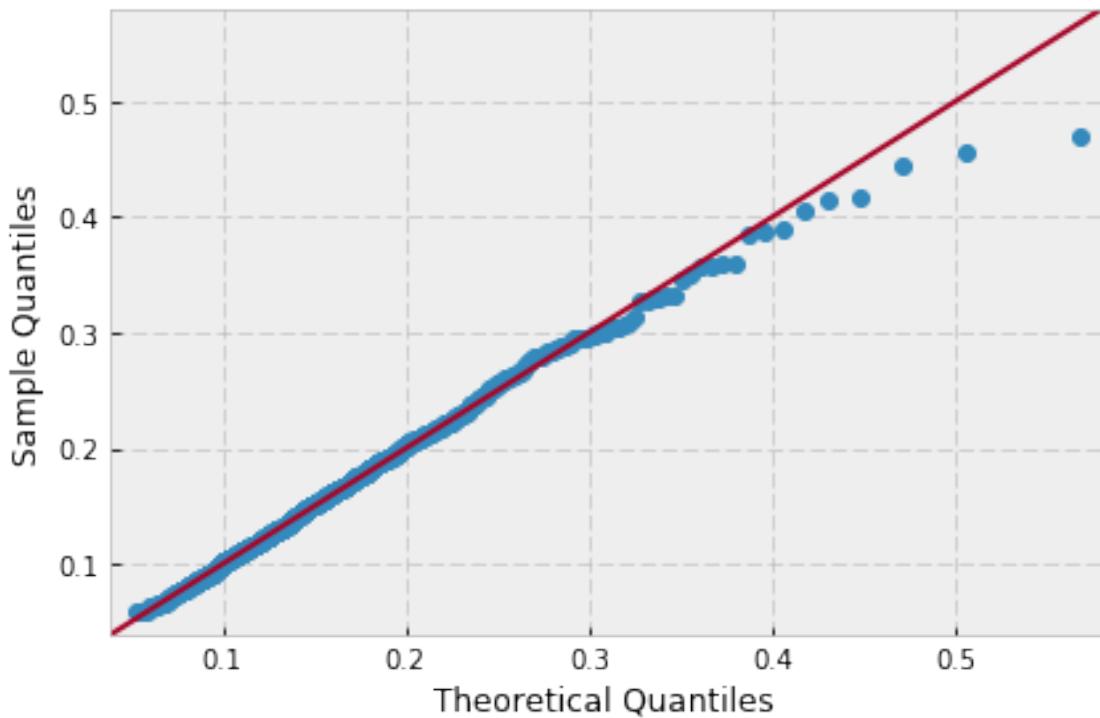


```
variable name: theta
variable value: 1.3089969389957472
distribution: inverse-gamma(shape=(11.003228169564551,), loc=0.00048624097993710334, scale=0.0207
DescribeResult(nobs=1000, minmax=(0.0013526884982854553, 0.0069067502792330071)
               mean=0.0025556880800398966, variance=4.6791106201495336e-07,
               skewness=1.3472785036617763, kurtosis=3.180295240120439)
```

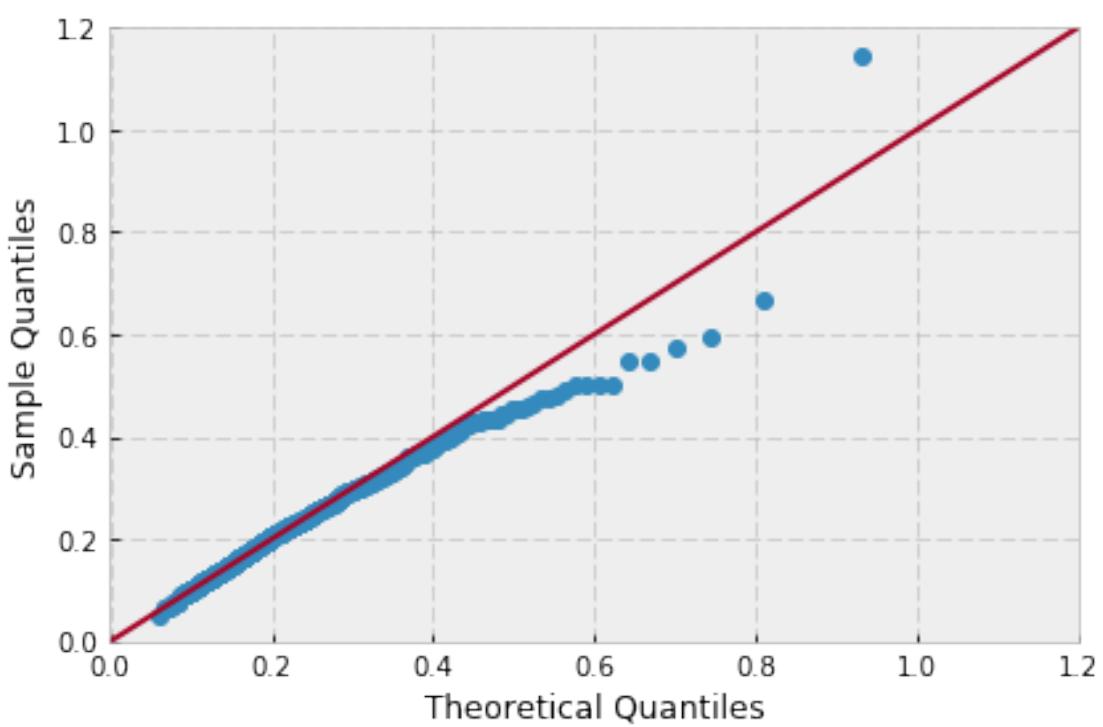
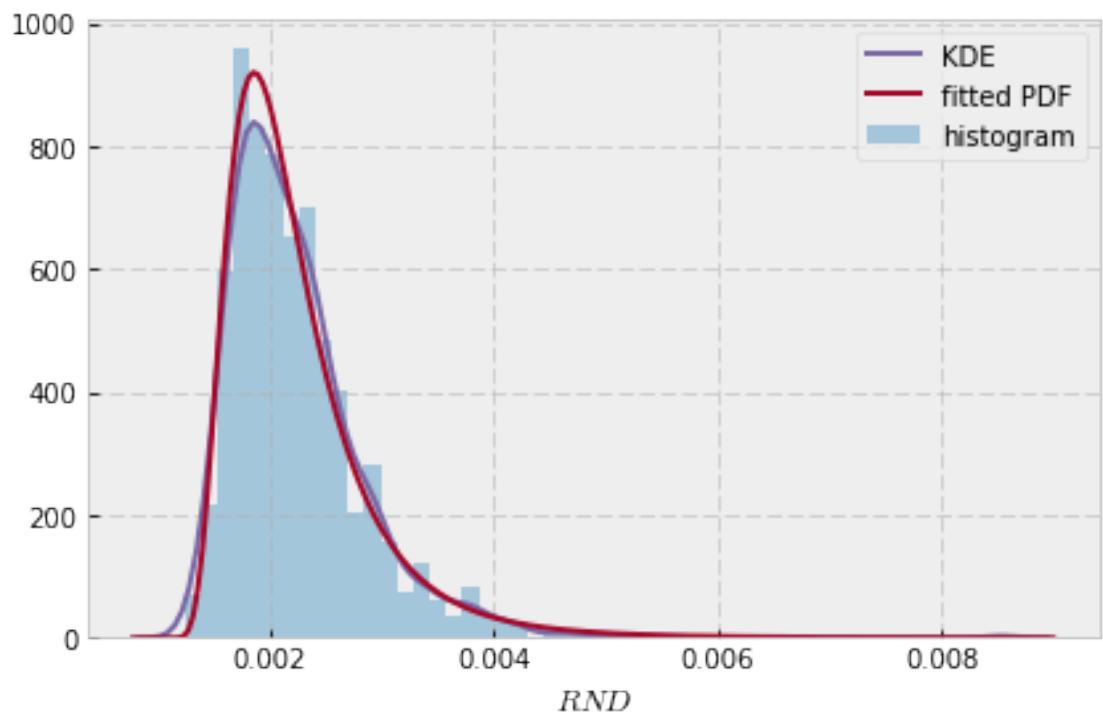


```
variable name: theta
variable value: 1.3962634015954638
distribution: inverse-gamma(shape=(7.5380958473641417,),loc=0.0007352666592390434, scale=0.01094
DescribeResult(nobs=1000, minmax=(0.0013736957558775464, 0.0058668524927937751)
    mean=0.0024071157285678033, variance=4.6546926831608823e-07,
    skewness=1.4313808780030552, kurtosis=2.8981562904327793)
```





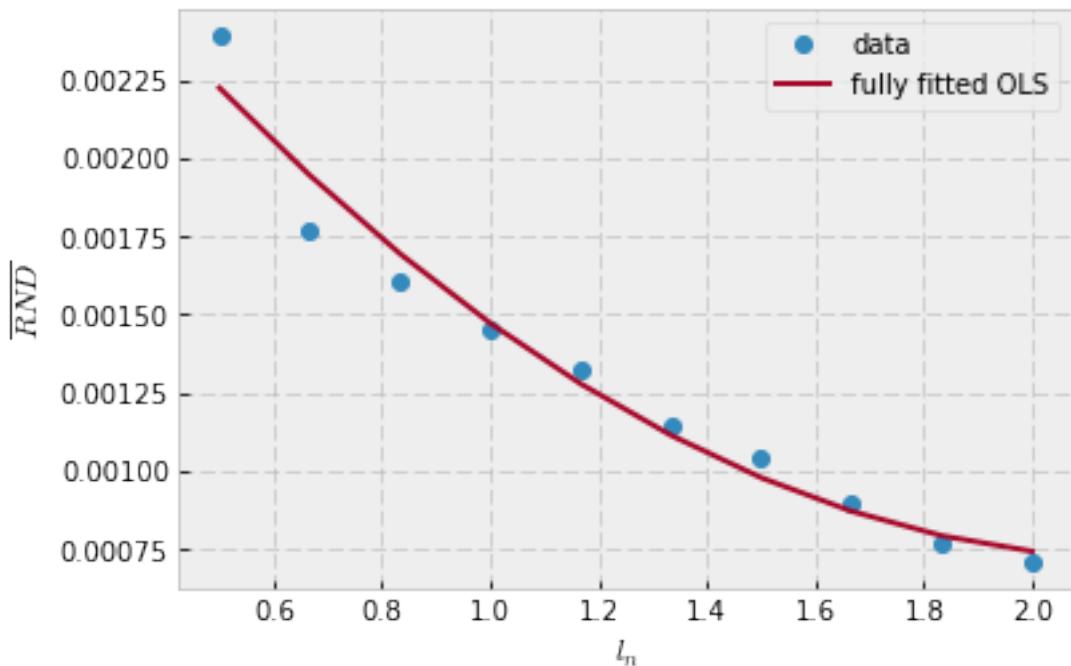
```
variable name: theta
variable value: 1.4835298641951802
distribution: inverse-gamma(shape=(5.9107895068292047,),loc=0.0008935974353300442, scale=0.00666
DescribeResult(nobs=1000, minmax=(0.0012357410278065327, 0.0085367874753306573)
               mean=0.0022298817559736881, variance=3.7436240712260079e-07,
               skewness=2.2031338172374855, kurtosis=12.653845078342659)
```



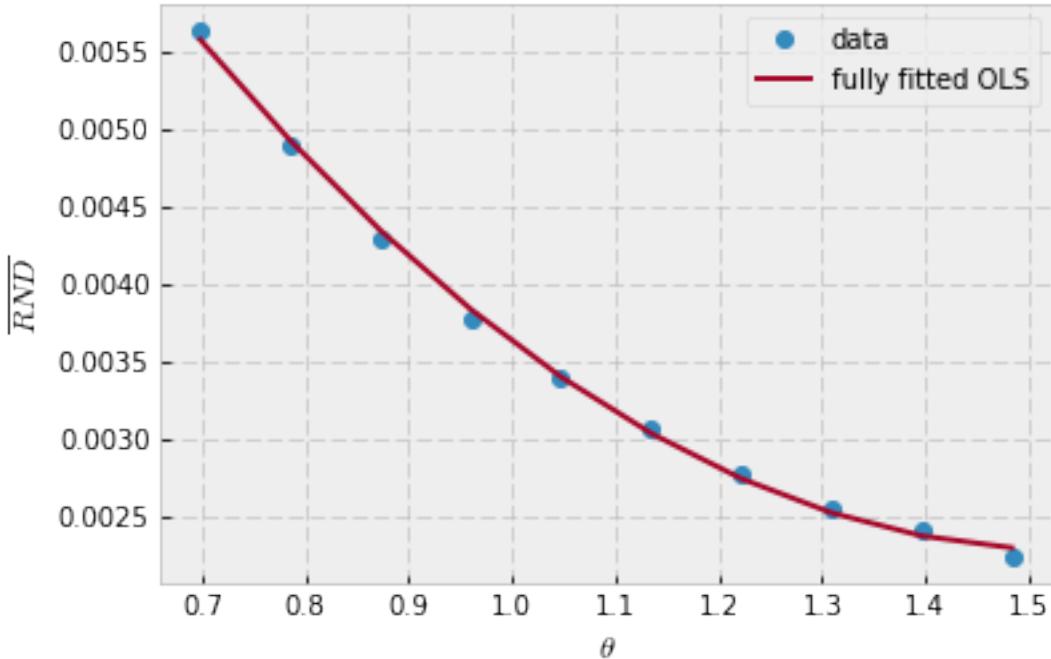
10.2 Parameter regression

```
In [26]: var_name = 'root_tip_density'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly2')  
        mean_reg('maxB', var_name, 'poly1')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly2')  
        mean_reg('N', var_name, 'poly1')
```

variable name: ln
model: poly2
r-squared: 0.8790453328713246
explained variance: 0.8800809921099316



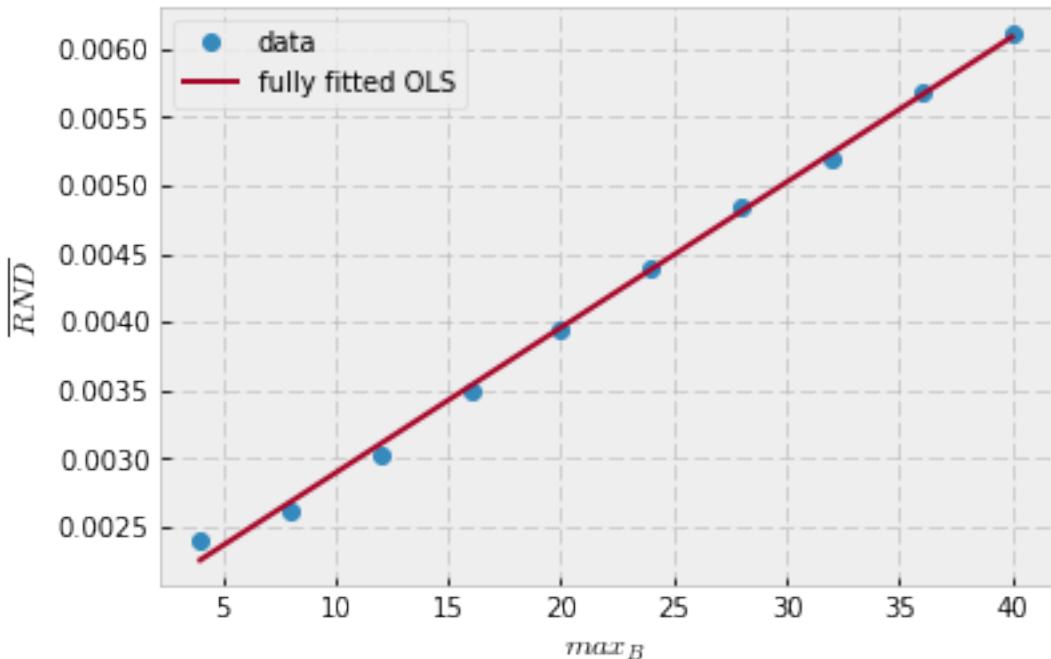
```
variable name: theta  
model: poly2  
r-squared: 0.9934182090244179  
explained variance: 0.9934182363856531
```



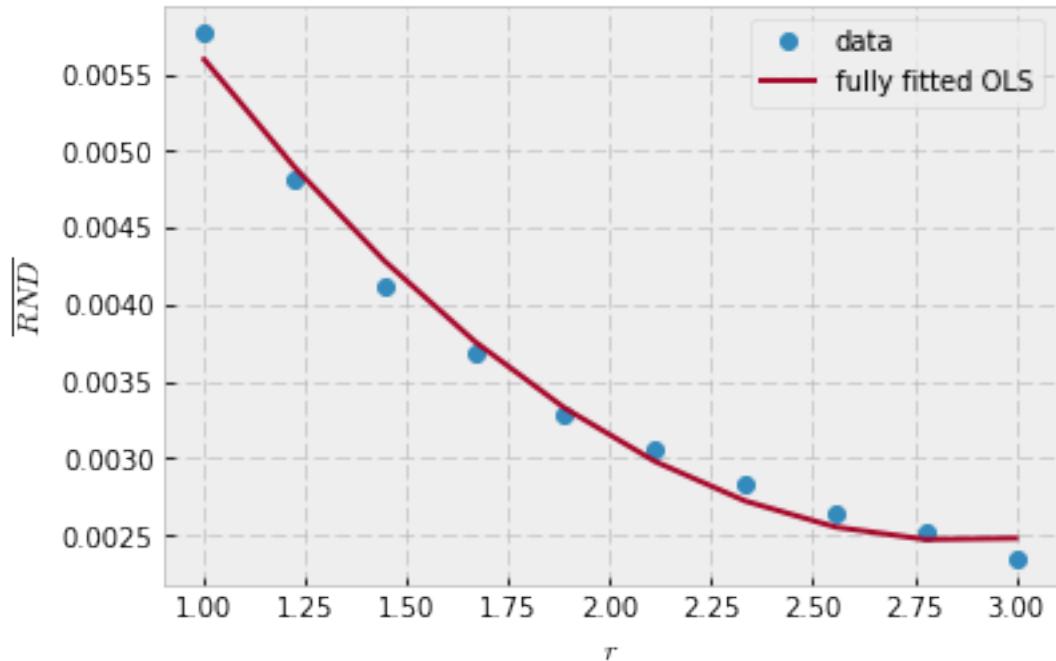
```

variable name: maxB
model: poly1
r-squared: 0.994778851400622
explained variance: 0.9947967989214536

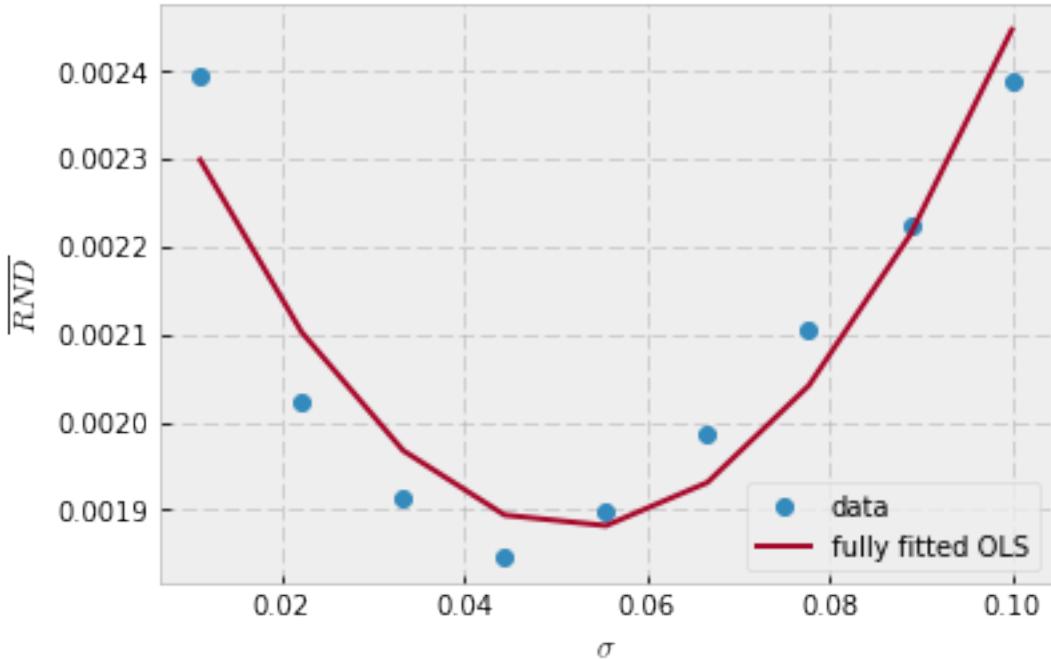
```



```
variable name: r
model: poly2
r-squared: 0.9574765352127267
explained variance: 0.9575002782739763
```



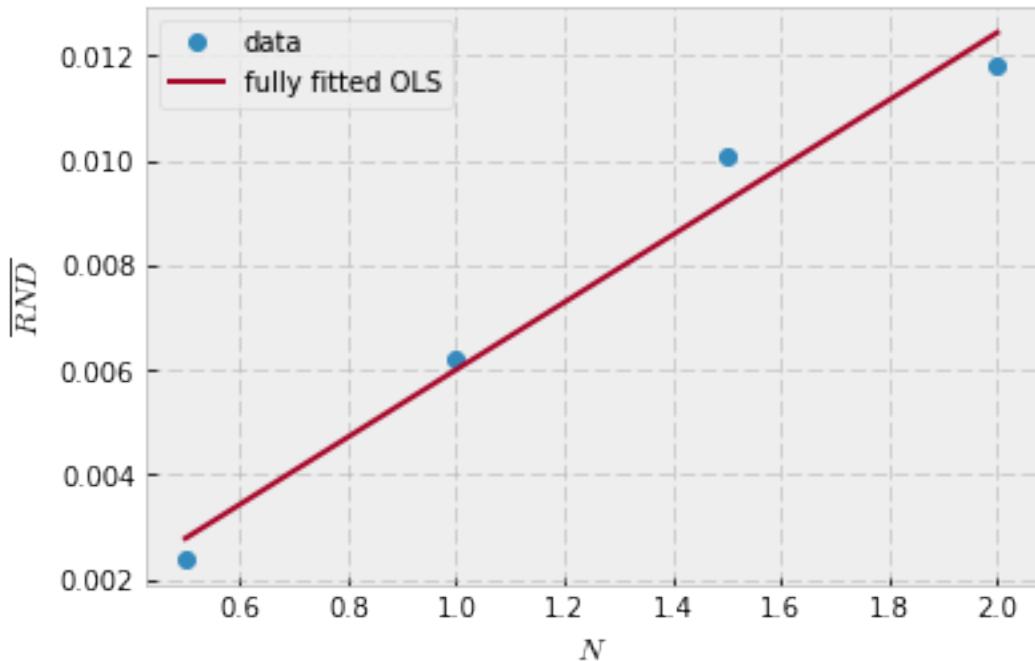
```
variable name: sigma
model: poly2
r-squared: 0.5901729182904394
explained variance: 0.5911316030940146
```



```

variable name: N
model: poly1
r-squared: 0.8530301495817243
explained variance: 0.8715872407350763

```

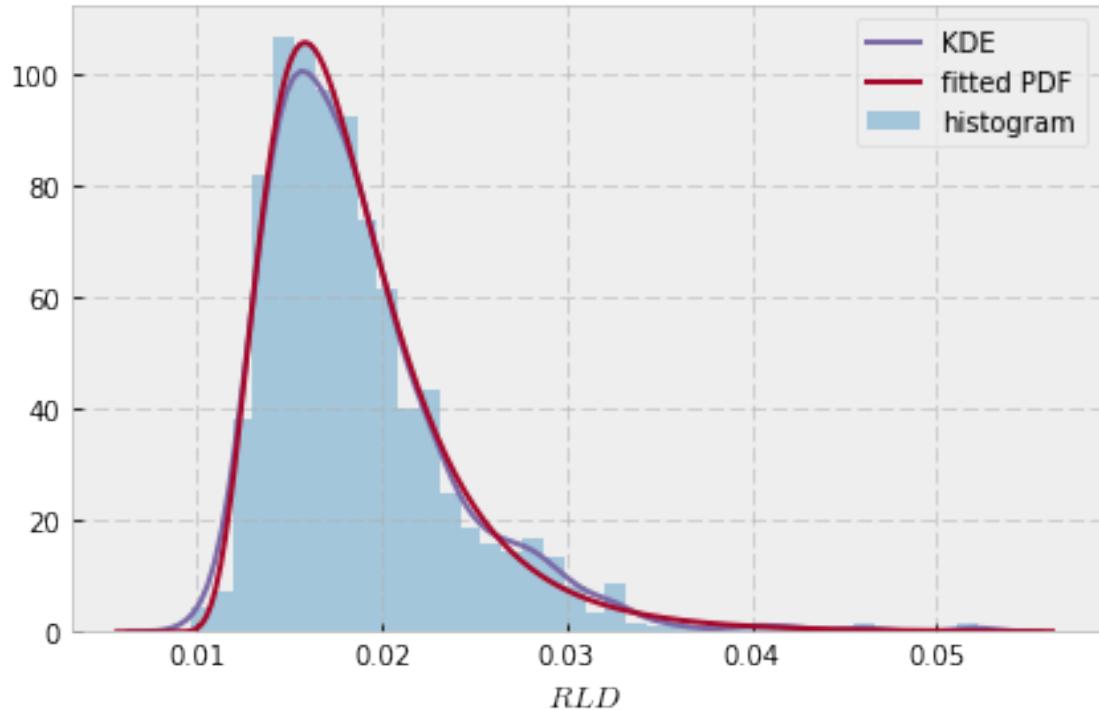


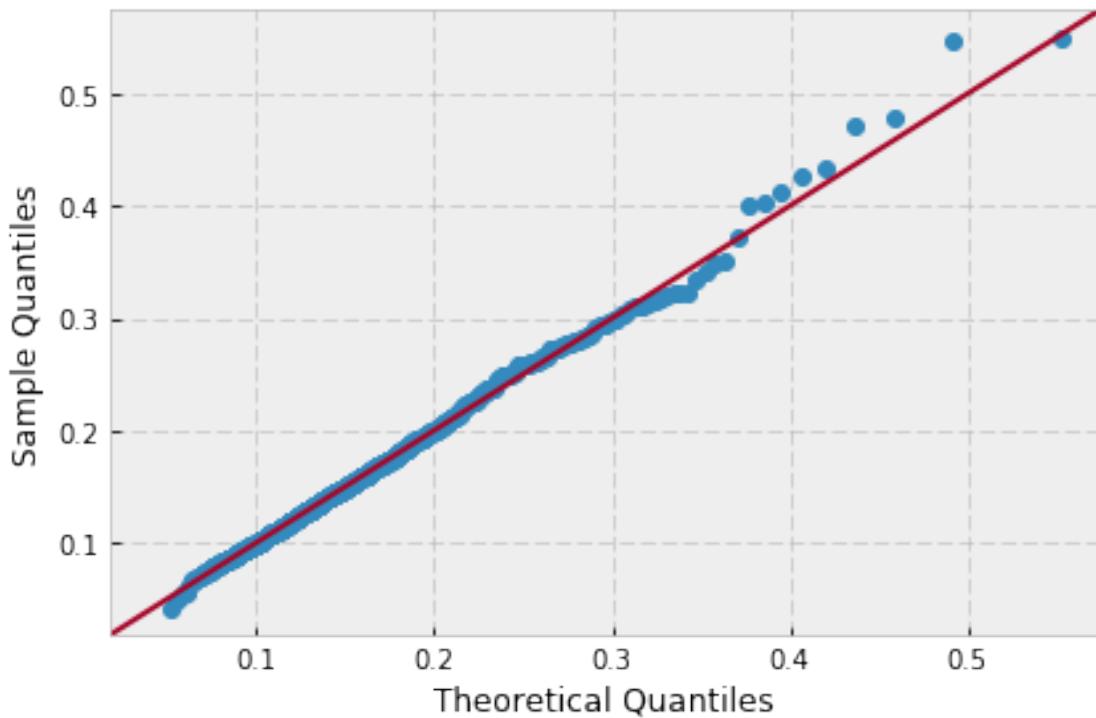
11 Root length relative to convex hull

11.1 Probability distributions

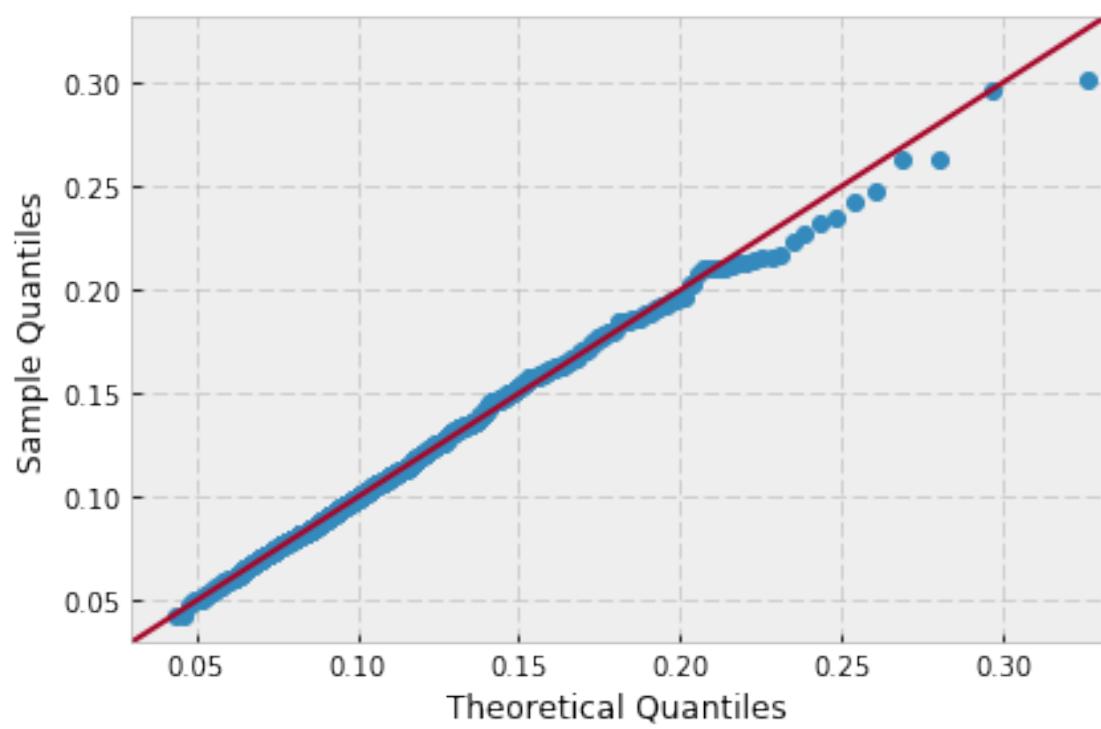
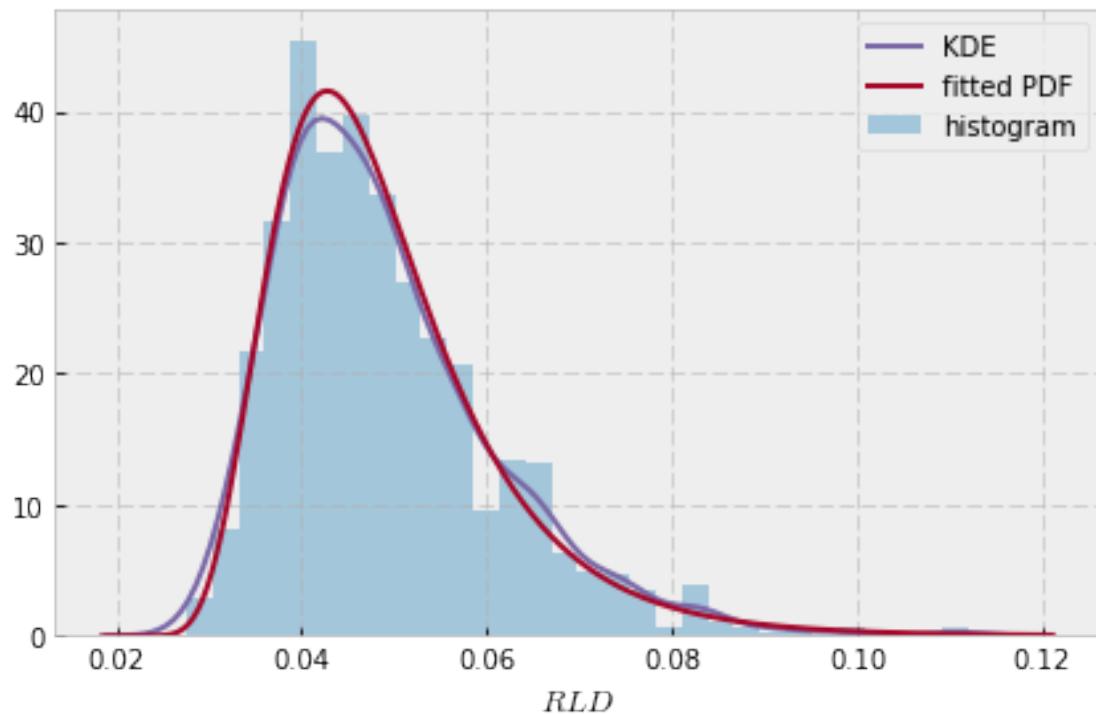
```
In [27]: plot_dist('total_root_length_per_convex_hull_volume', 'inverse-gamma')
```

```
variable name: N
variable value: 0.5
distribution: inverse-gamma(shape=(7.6518480402278515,), loc=0.006121216511868739, scale=0.084051
DescribeResult(nobs=1000, minmax=(0.009662484871099249, 0.052279764334324588)
    mean=0.018757069990123023, variance=2.8036194059545509e-05,
    skewness=1.7988100982753636, kurtosis=5.5147390189864876)
```

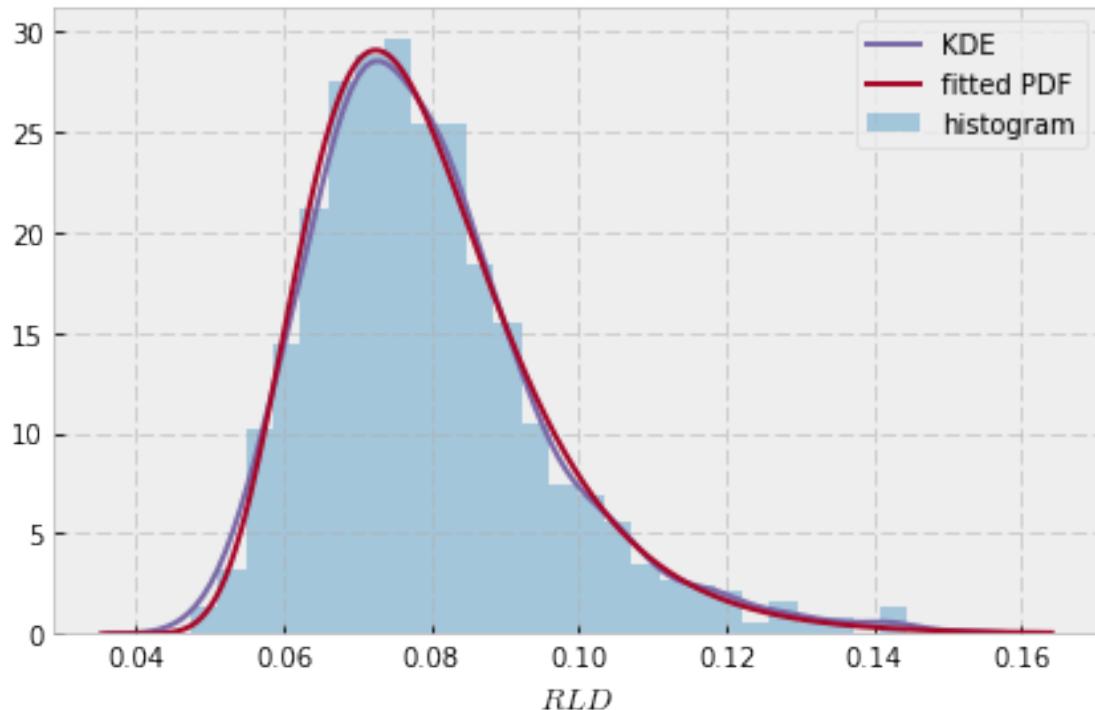


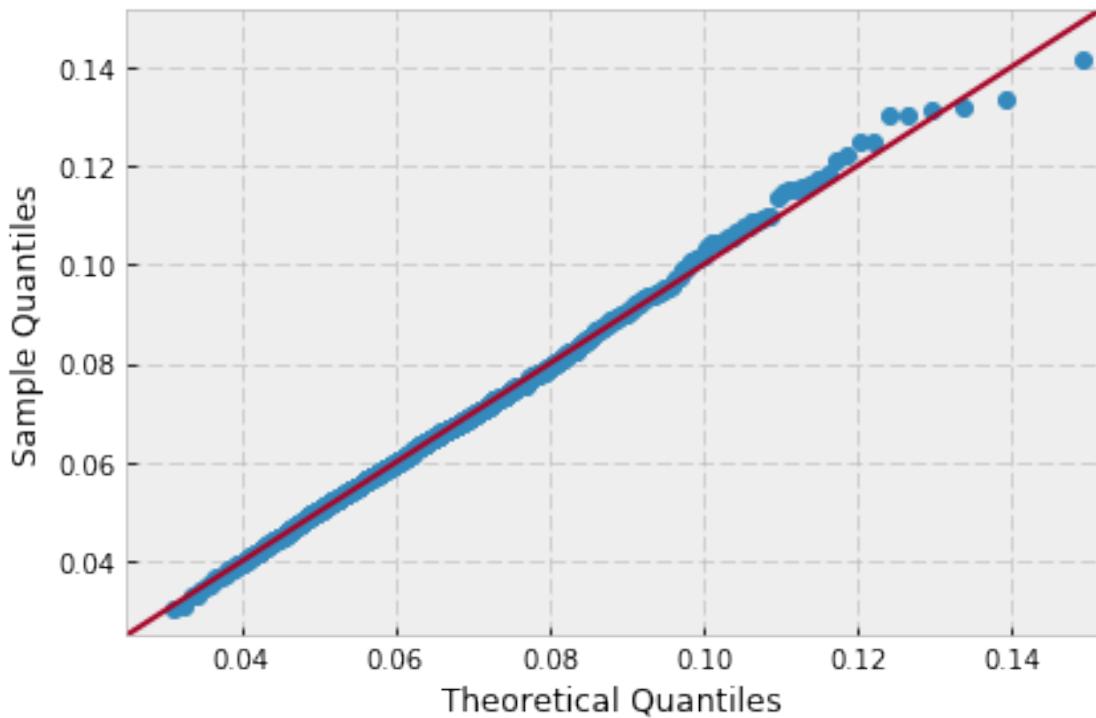


```
variable name: N
variable value: 1.0
distribution: inverse-gamma(shape=(10.203010749065053,), loc=0.013707658578072884, scale=0.325349
DescribeResult(nobs=1000, minmax=(0.027544827710664993, 0.11202021207302901)
               mean=0.049037917799901833, variance=0.00014562424930398182,
               skewness=1.2241468392845467, kurtosis=2.1793870550837644)
```

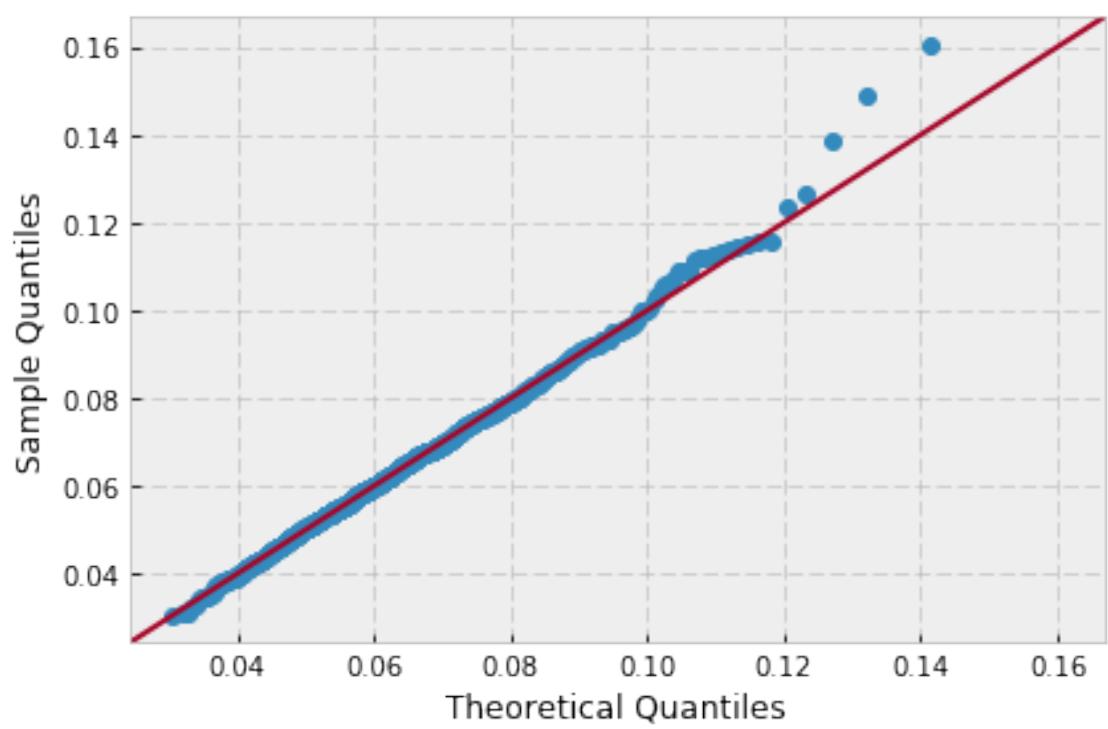
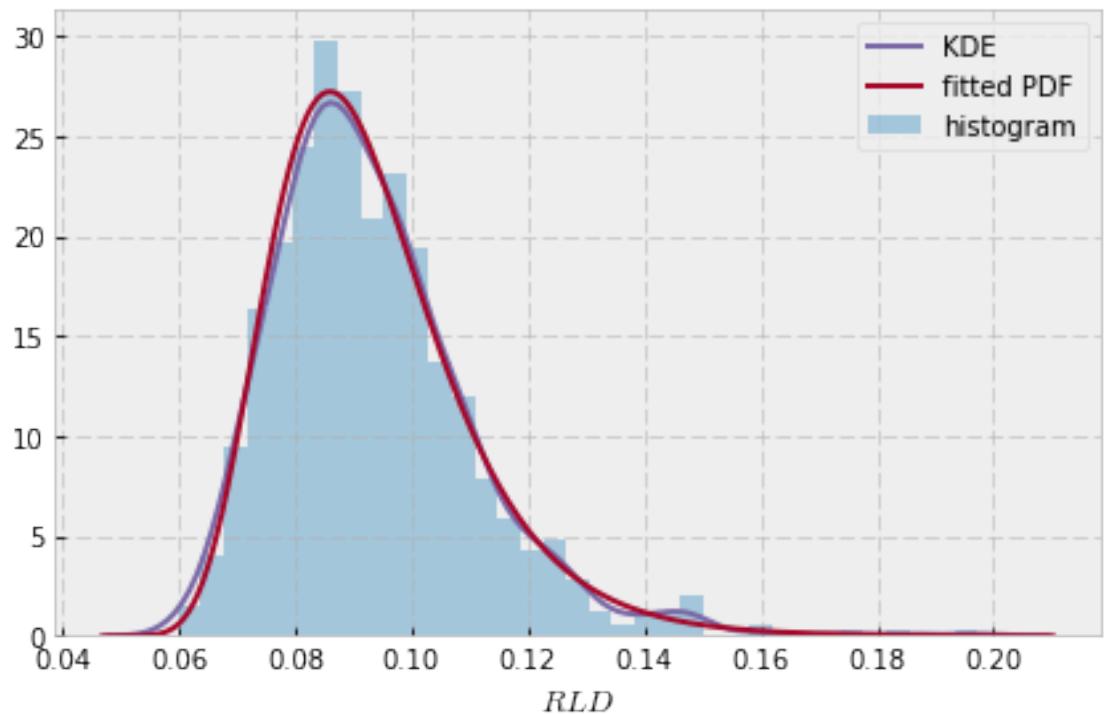


```
variable name: N
variable value: 1.5
distribution: inverse-gamma(shape=(16.487371145906451,),loc=0.018689014940395217, scale=0.940879
DescribeResult(nobs=1000, minmax=(0.047400724200290484, 0.15200886697524249)
    mean=0.079442722363798457, variance=0.00025523994985104373,
    skewness=1.0808304230113916, kurtosis=1.7555379141930878)
```

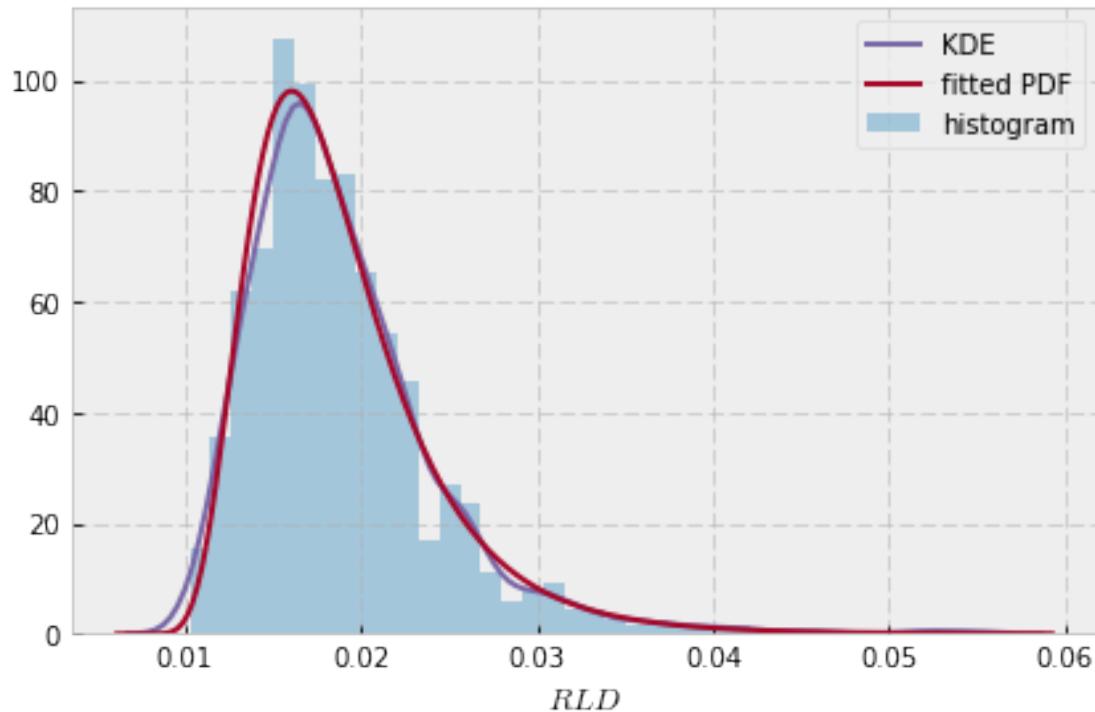


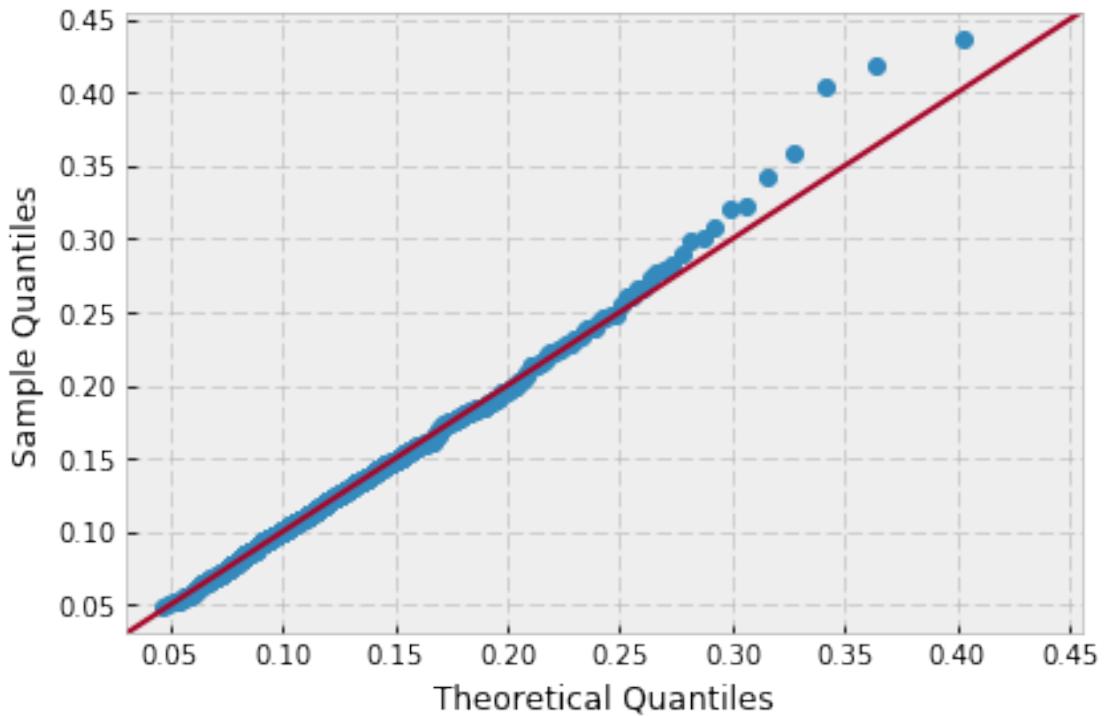


```
variable name: N
variable value: 2.0
distribution: inverse-gamma(shape=(17.075375170461868,),loc=0.027386483918896753, scale=1.059234
DescribeResult(nobs=1000, minmax=(0.059844527844415164, 0.19722867015622744)
               mean=0.093287143120182645, variance=0.00029271241190512057,
               skewness=1.2218224615447102, kurtosis=3.0919744339336486)
```

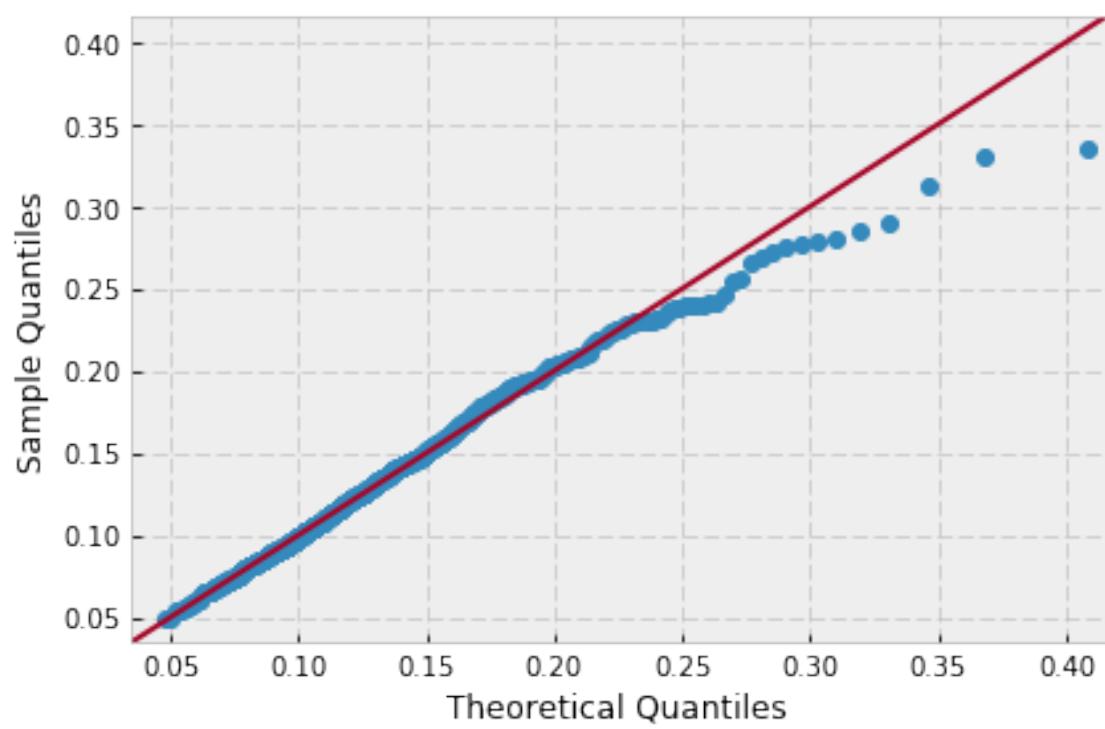
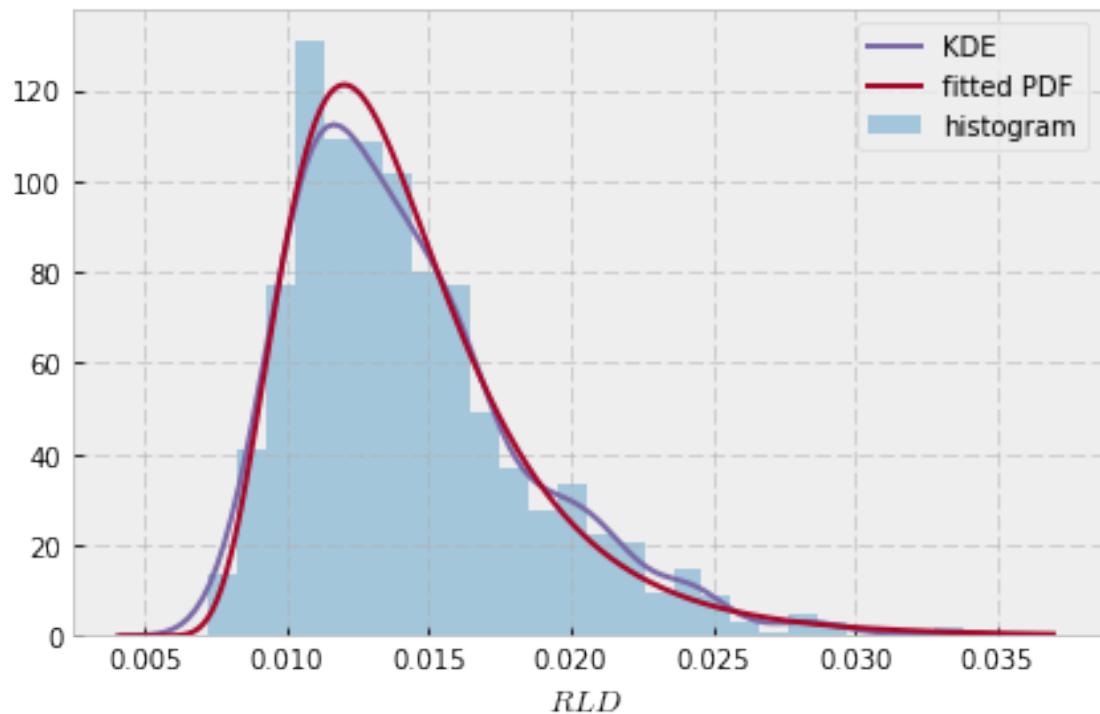


```
variable name: ln
variable value: 0.5
distribution: inverse-gamma(shape=(9.067090635144524,), loc=0.004523068667862116, scale=0.1160797
DescribeResult(nobs=1000, minmax=(0.01027839895125247, 0.055113833568200425)
    mean=0.018918595079620276, variance=3.0090361855107915e-05,
    skewness=1.8574896387724635, kurtosis=6.34888642962858)
```

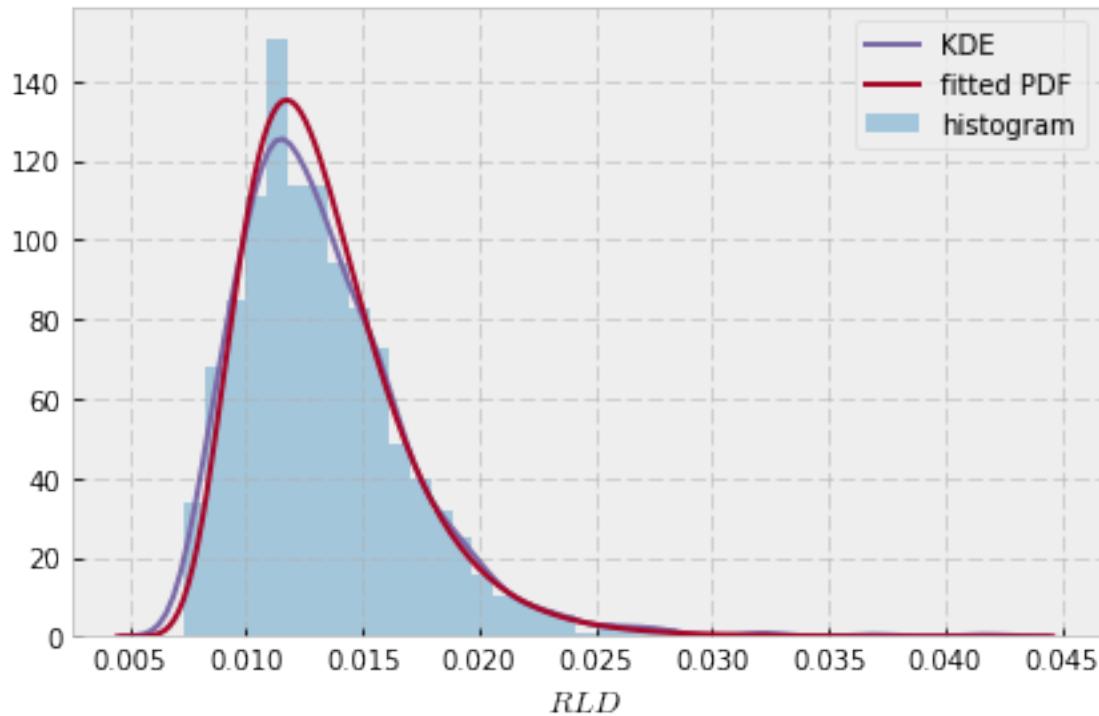


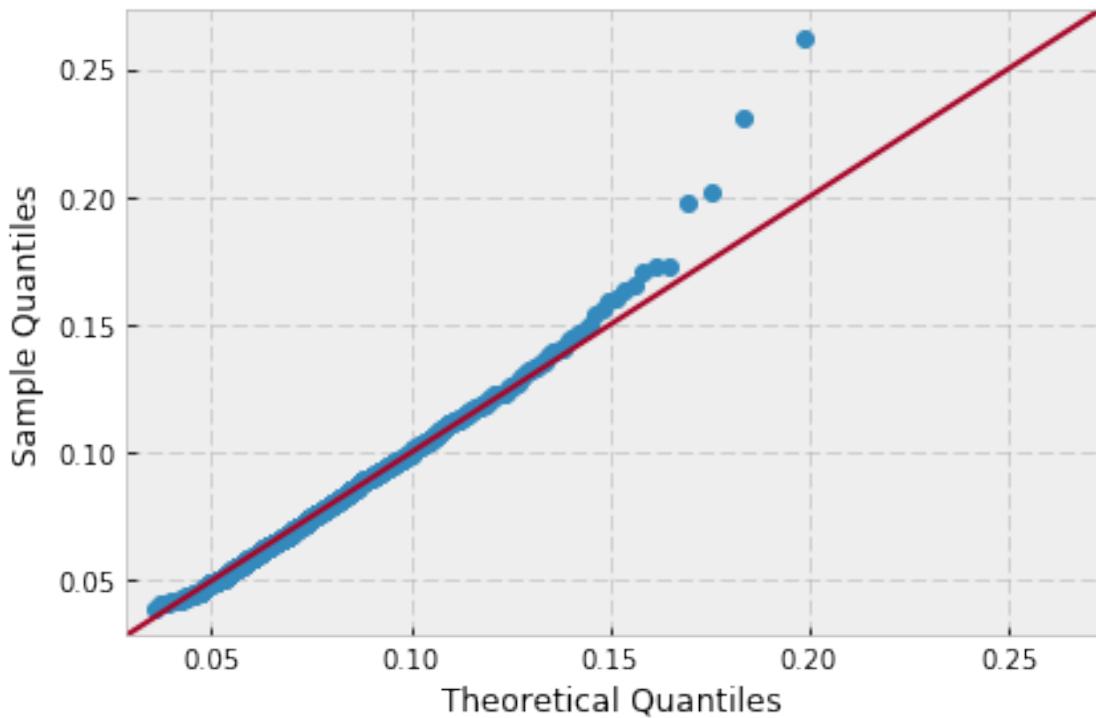


```
variable name: ln
variable value: 0.6666666666666666
distribution: inverse-gamma(shape=(8.9978185315812667,), loc=0.0027334134210720033, scale=0.09286
DescribeResult(nobs=1000, minmax=(0.0072383286619386346, 0.033817842464274939)
               mean=0.014328023656774831, variance=1.7618458918352365e-05,
               skewness=1.1309546653470595, kurtosis=1.4986232242821753)
```

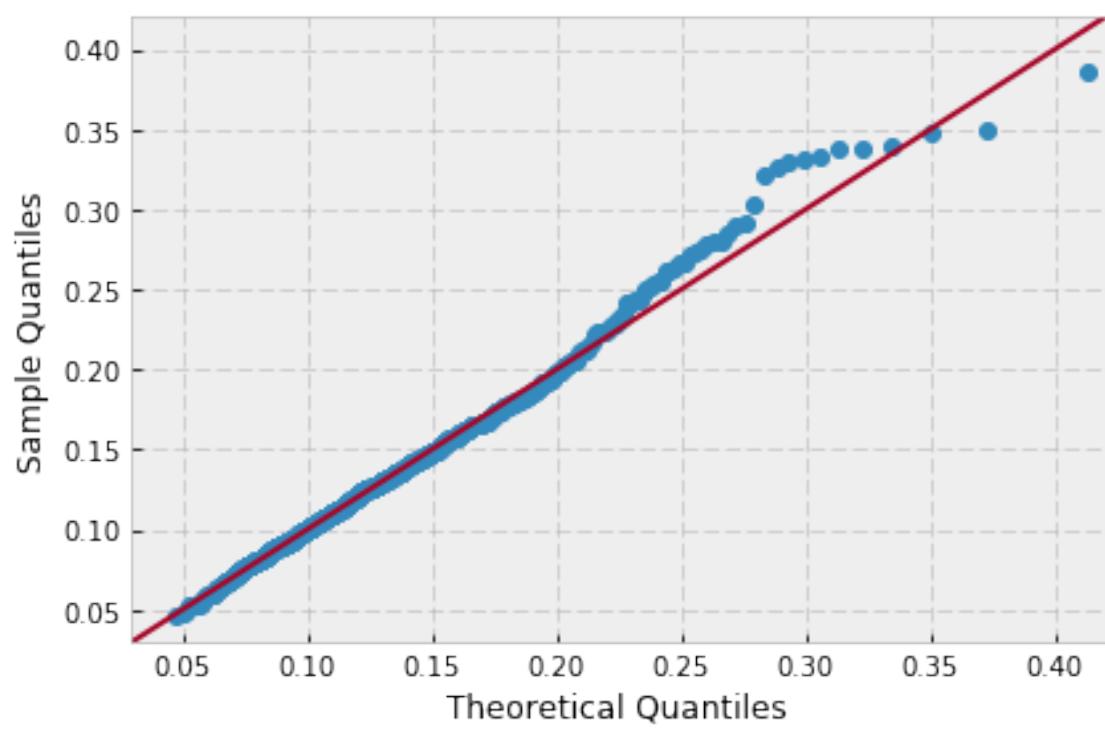
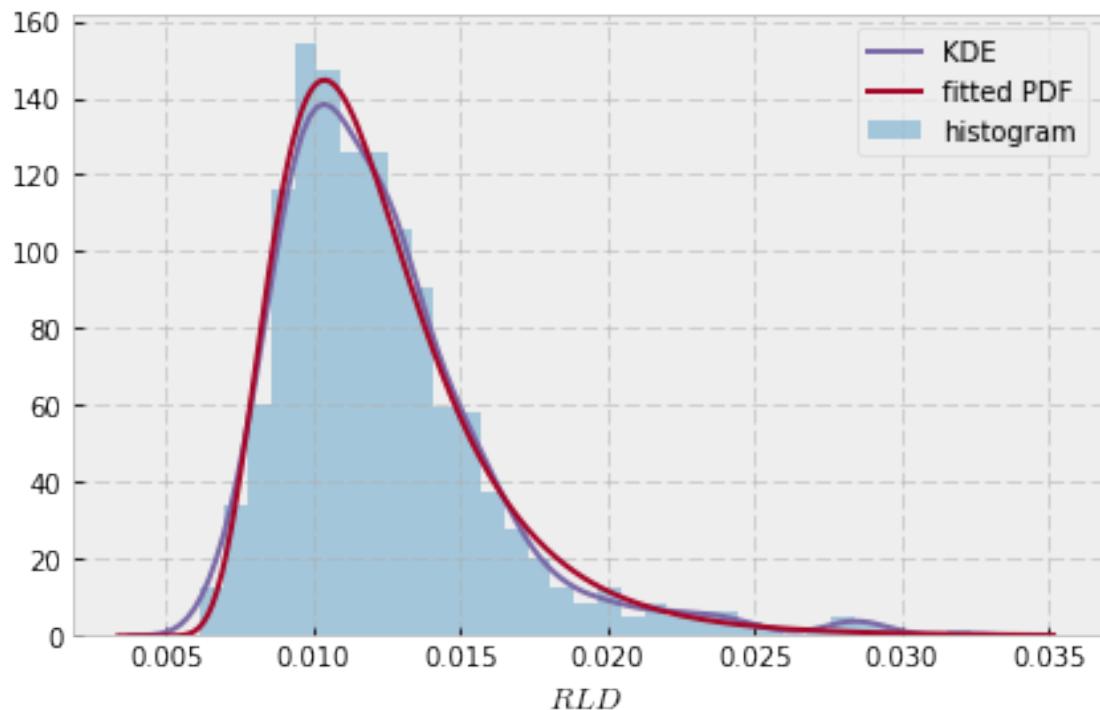


```
variable name: ln
variable value: 0.8333333333333333
distribution: inverse-gamma(shape=(13.735372852221587,),loc=0.001232715093535152, scale=0.154733
DescribeResult(nobs=1000, minmax=(0.0073530336810886856, 0.041750366291065606)
    mean=0.013359179305783399, variance=1.4443895773584995e-05,
    skewness=1.658890992818328, kurtosis=5.9537091384226475)
```

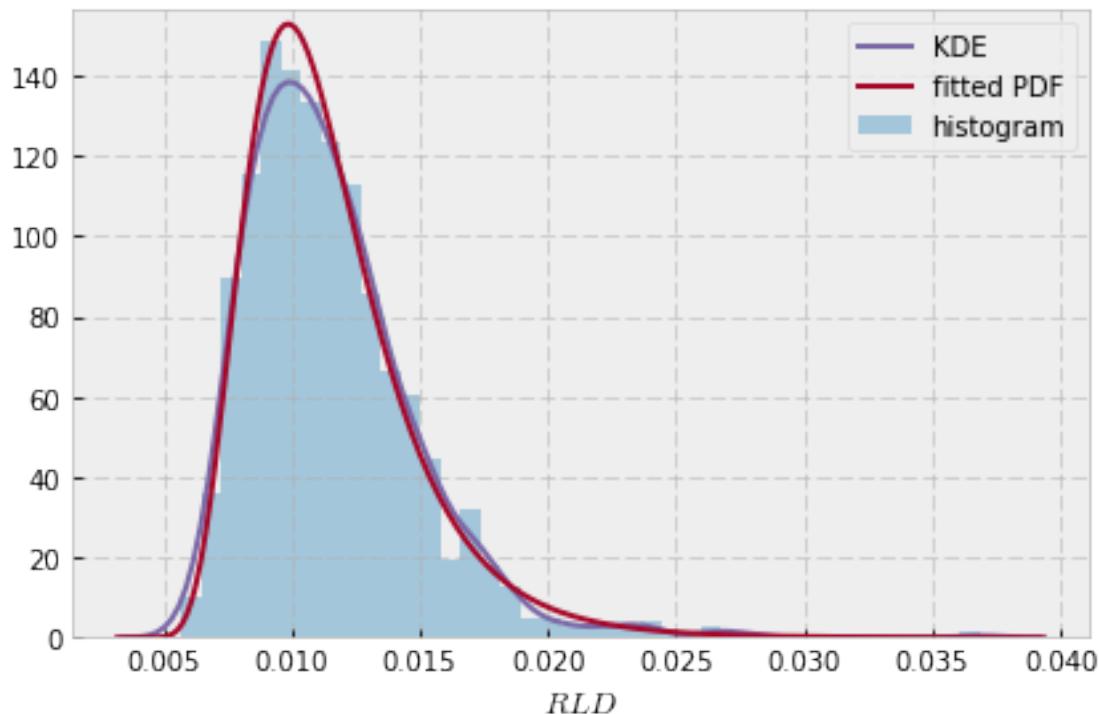


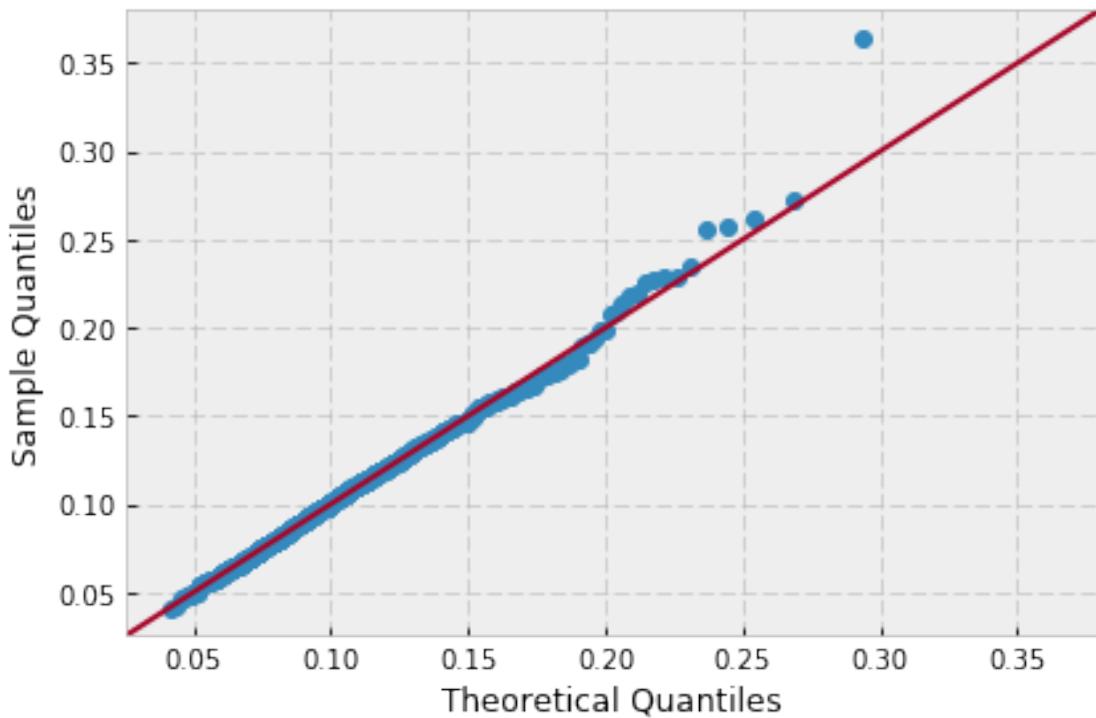


```
variable name: ln
variable value: 1.0
distribution: inverse-gamma(shape=(8.9400679694773864,), loc=0.0026274326783980607, scale=0.07708
DescribeResult(nobs=1000, minmax=(0.0061845049904421046, 0.03235391033141443)
               mean=0.012340746654129389, variance=1.3821562899332335e-05,
               skewness=1.6786244713449485, kurtosis=4.2453684974623505)
```

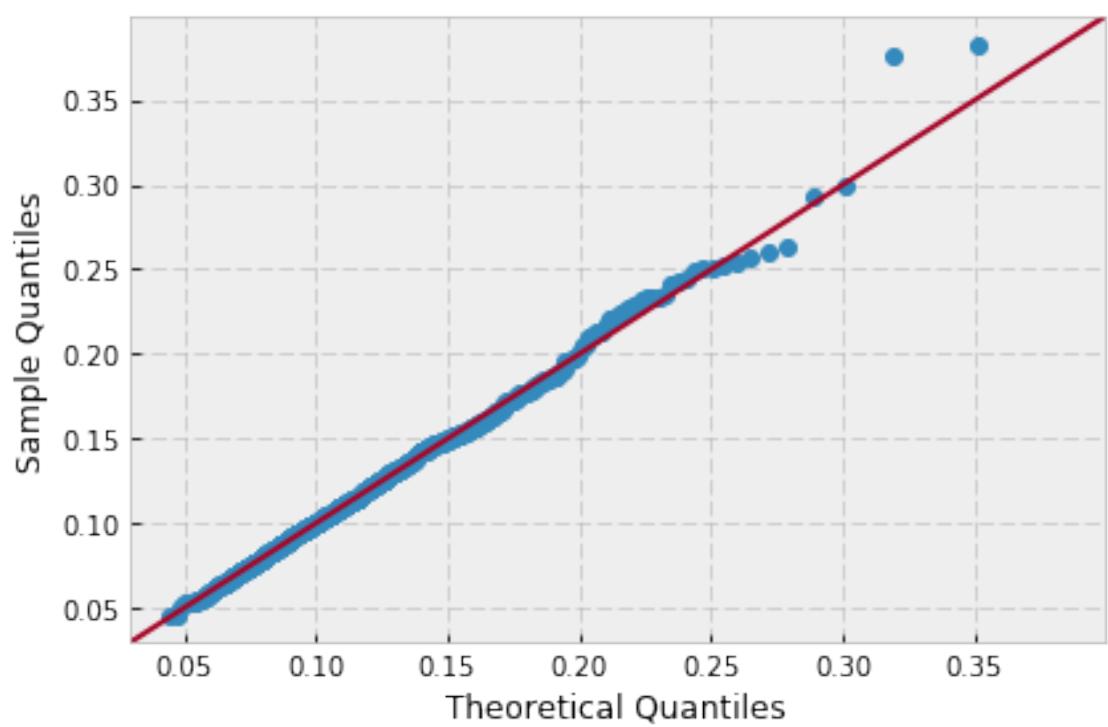
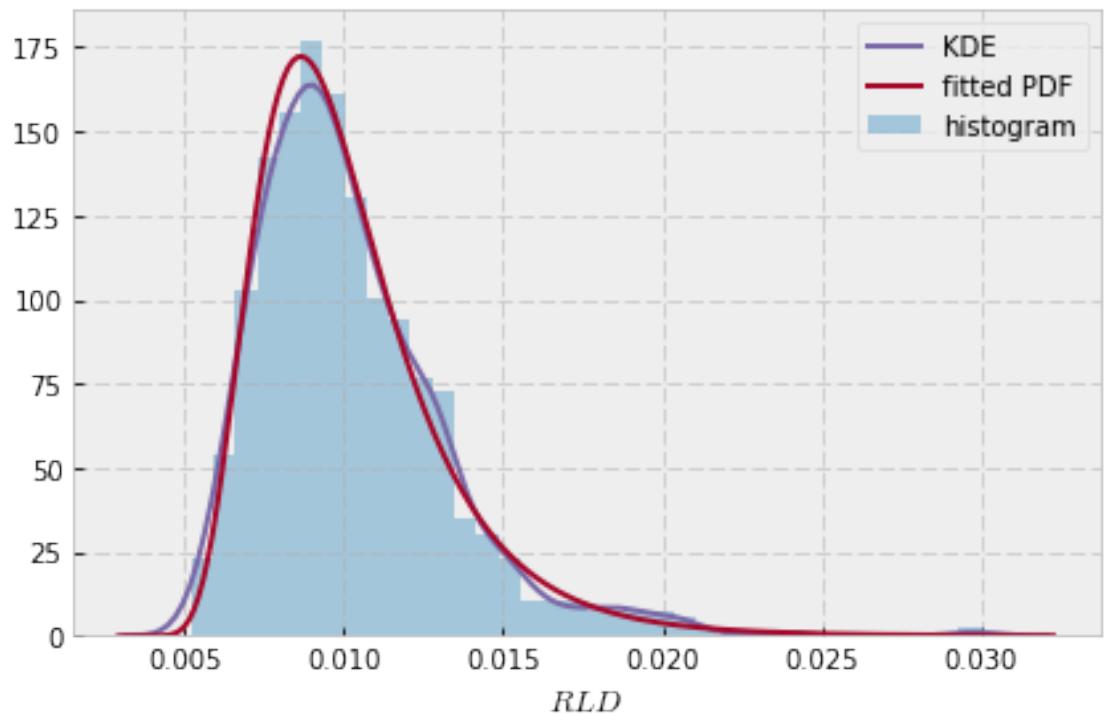


```
variable name: ln
variable value: 1.1666666666666665
distribution: inverse-gamma(shape=(10.832277490379902,), loc=0.0016978197922584513, scale=0.09656
DescribeResult(nobs=1000, minmax=(0.0056847187648307951, 0.036855574646734612)
    mean=0.01151840612734603, variance=1.0944674133806671e-05,
    skewness=1.5899038805728234, kurtosis=5.510056346616068)
```

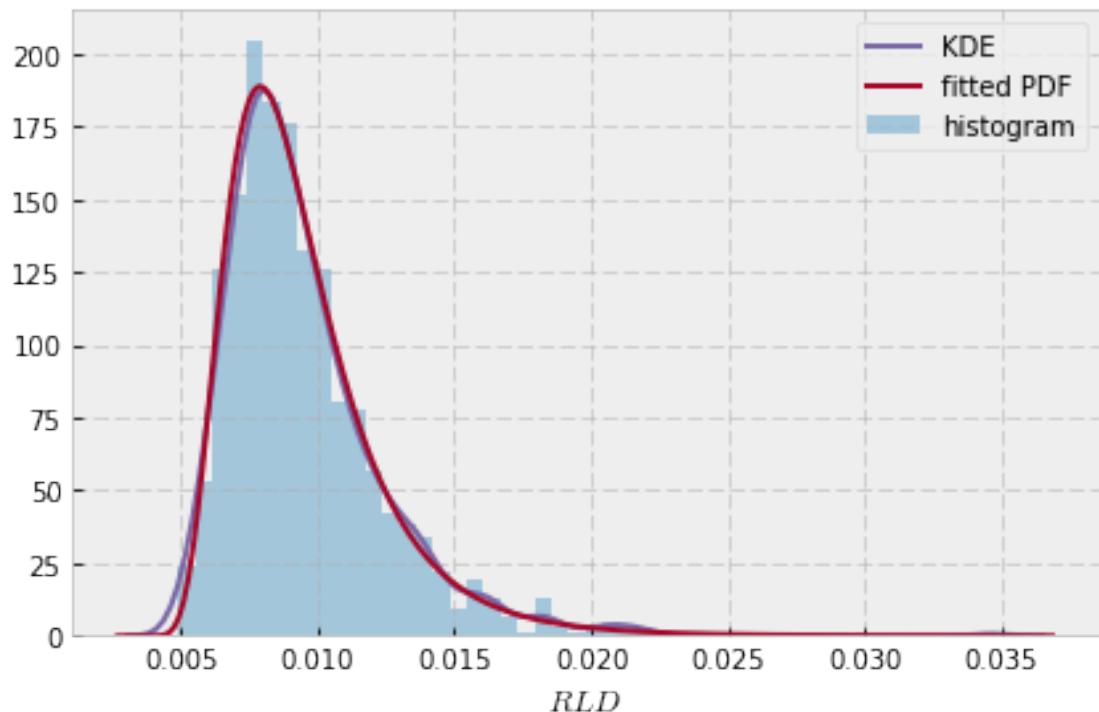


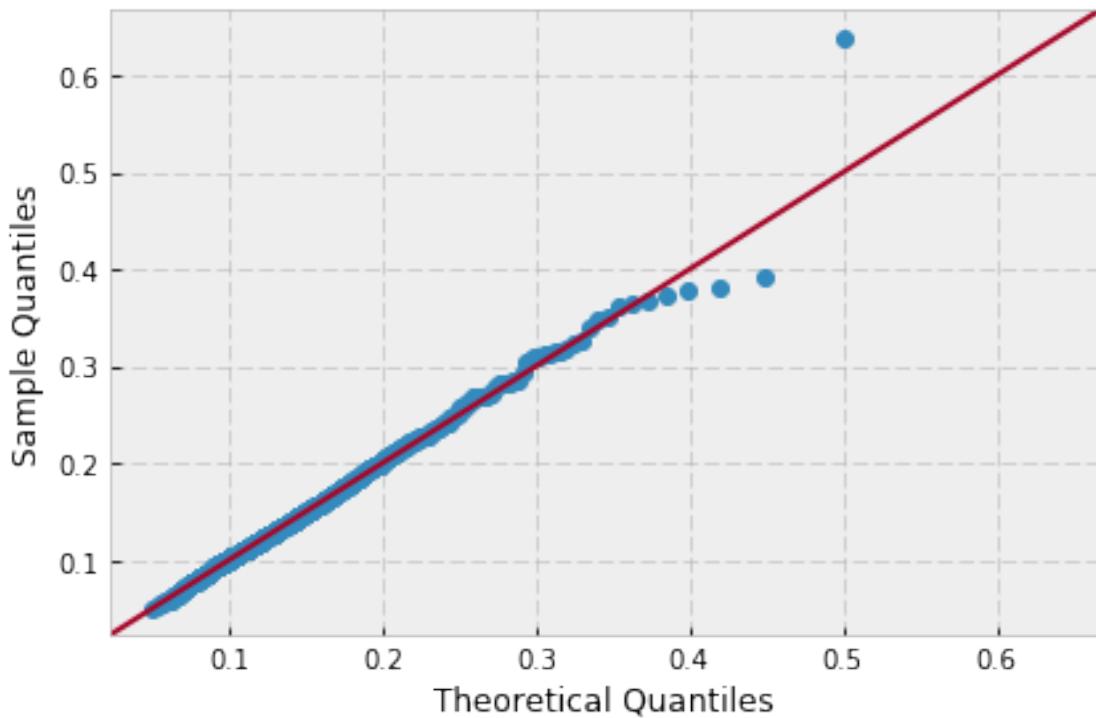


```
variable name: ln
variable value: 1.333333333333333
distribution: inverse-gamma(shape=(9.7754290369978865,), loc=0.0018158563018805525, scale=0.07371
DescribeResult(nobs=1000, minmax=(0.0052140969033877757, 0.029992160716120958)
               mean=0.010215604800915851, variance=9.0343429643139177e-06,
               skewness=1.5950229928242936, kurtosis=4.8421876168626135)
```

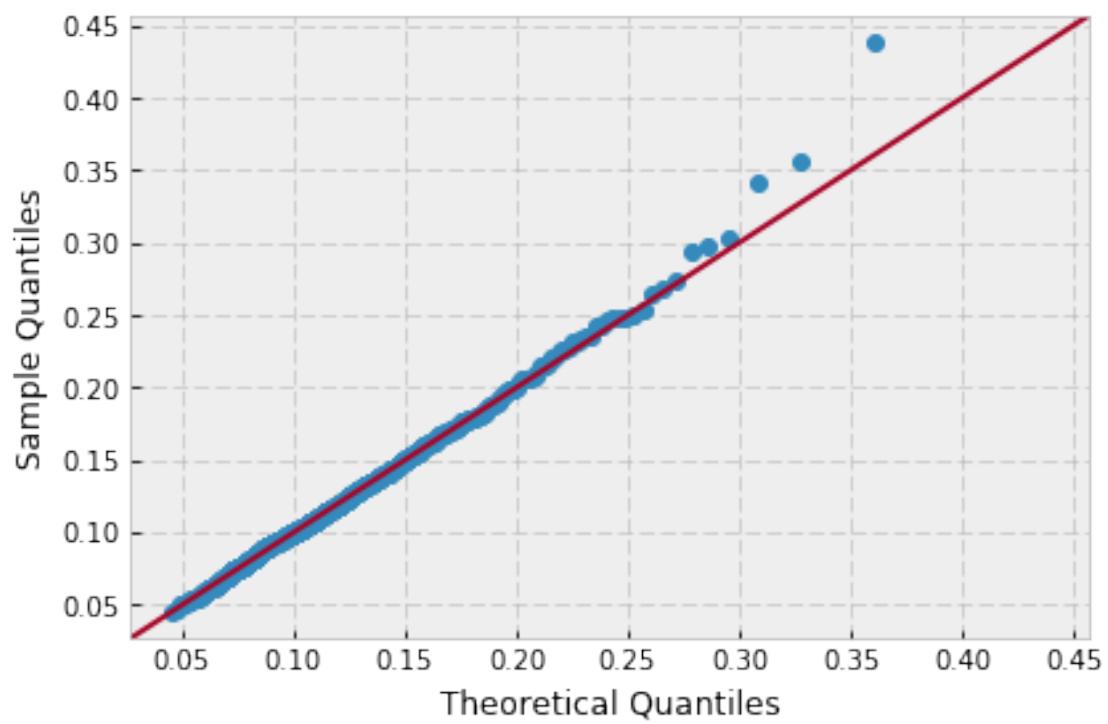
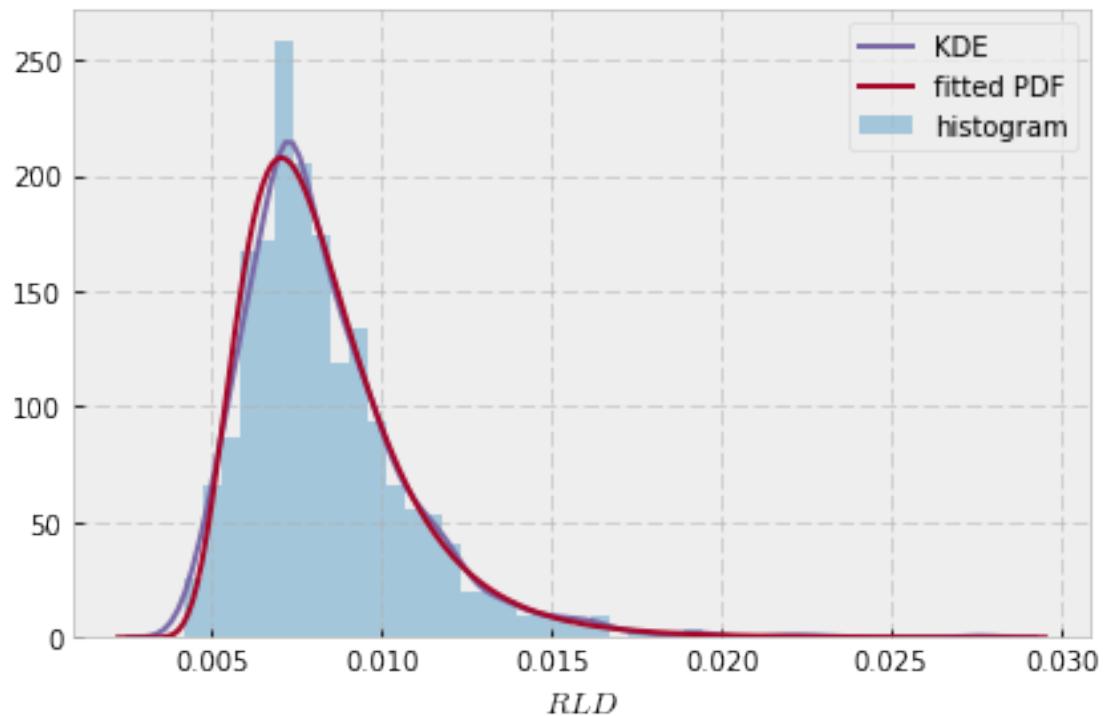


```
variable name: ln
variable value: 1.5
distribution: inverse-gamma(shape=(8.0548304453443631,), loc=0.0022996687881148992, scale=0.05071
DescribeResult(nobs=1000, minmax=(0.0048960332076242868, 0.034647388991293472)
               mean=0.0094876854721283408, variance=8.5004692748217146e-06,
               skewness=1.869434377423541, kurtosis=7.289415099938886)
```

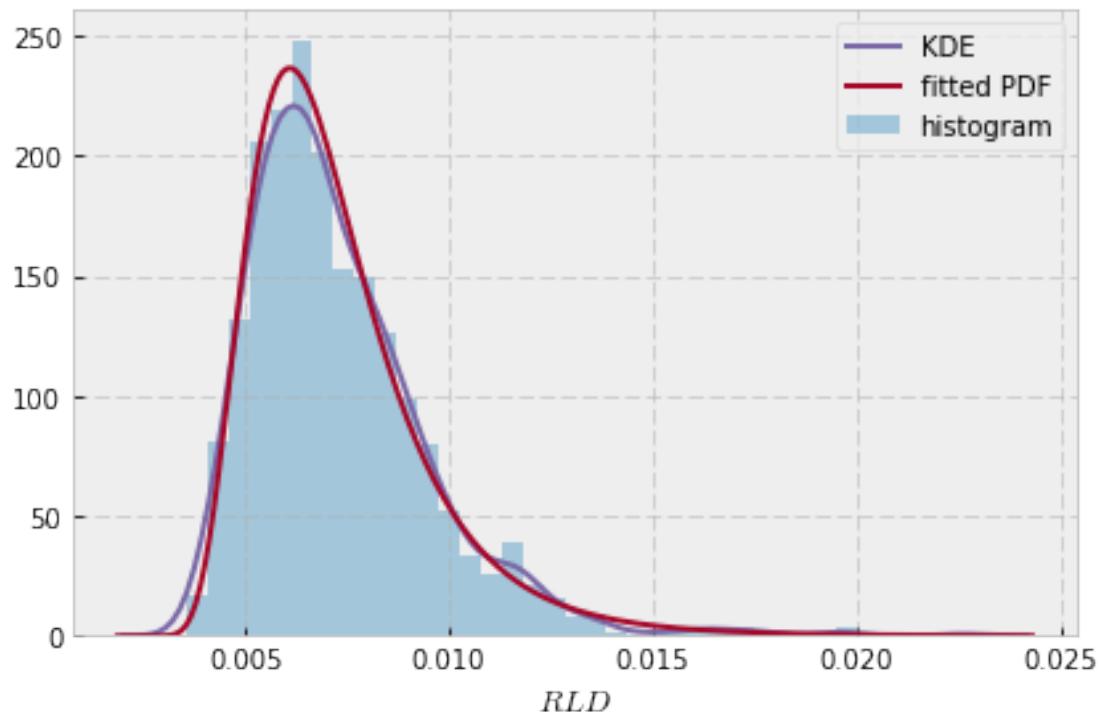


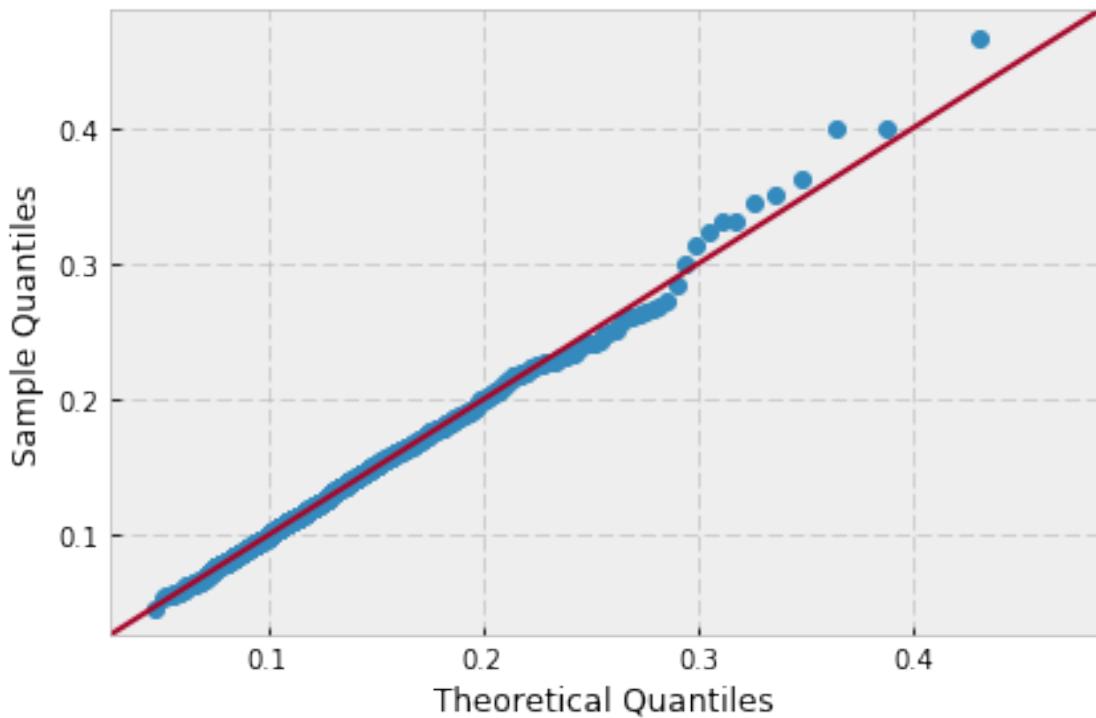


```
variable name: ln
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(9.6295263019626436,), loc=0.0014249408494441354, scale=0.05983
DescribeResult(nobs=1000, minmax=(0.0041673310377313231, 0.02760877517211692)
               mean=0.0083608916611420791, variance=6.432864246099307e-06,
               skewness=1.763407308553682, kurtosis=6.141610305895961)
```

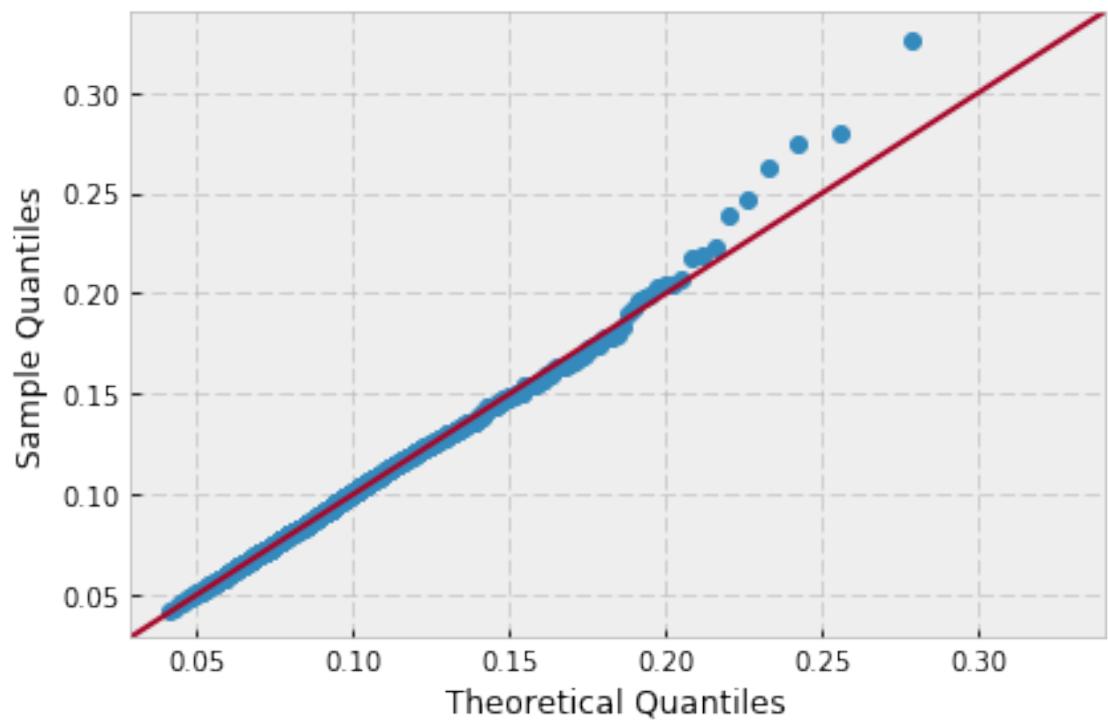
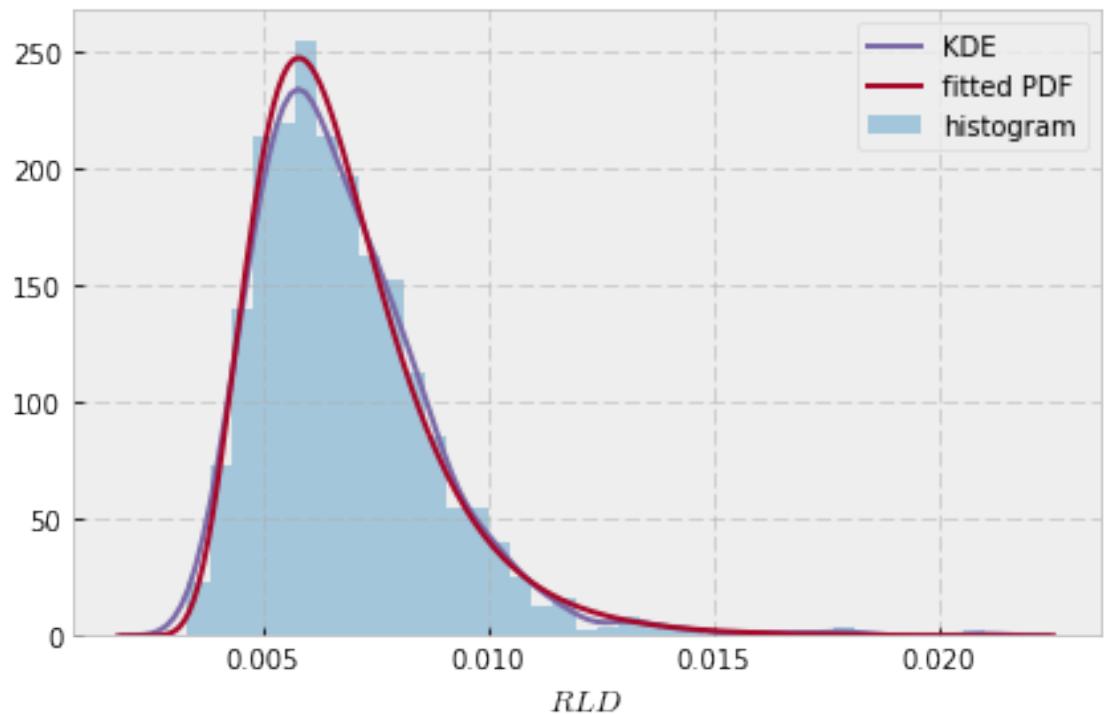


```
variable name: ln
variable value: 1.833333333333333
distribution: inverse-gamma(shape=(8.730551379470139,), loc=0.0014132463709173762, scale=0.045502
DescribeResult(nobs=1000, minmax=(0.0035555158388325256, 0.022616376617418794)
               mean=0.0072969695695494375, variance=5.0535217478446979e-06,
               skewness=1.6755805164044861, kurtosis=5.278246558590807)
```

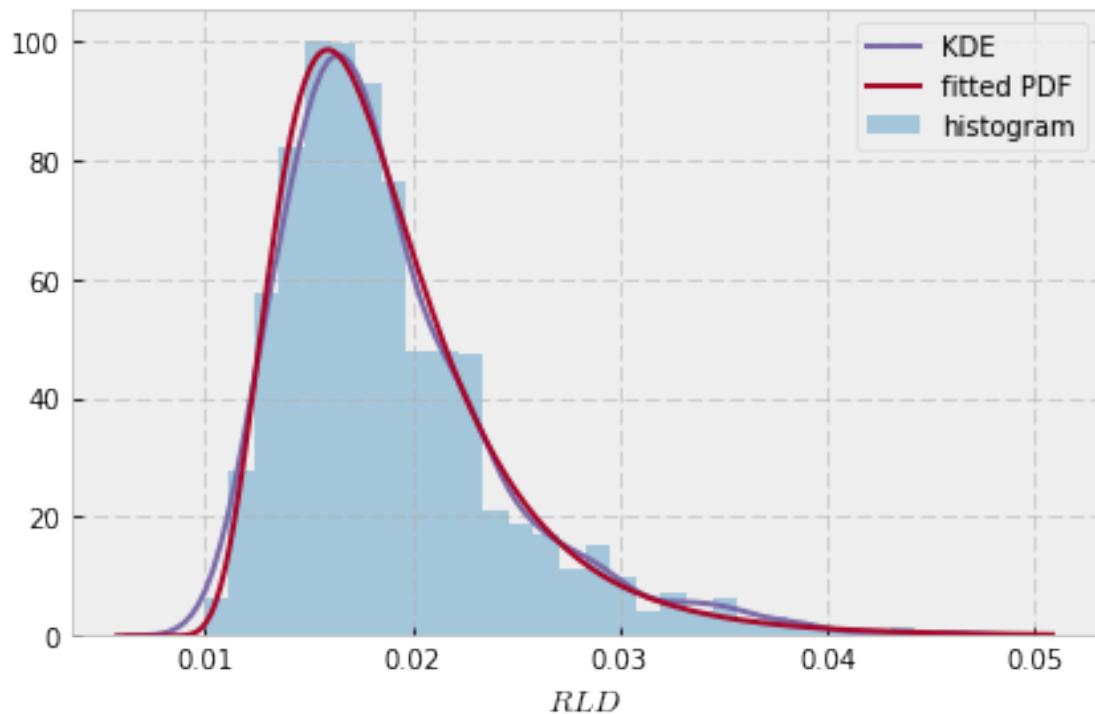


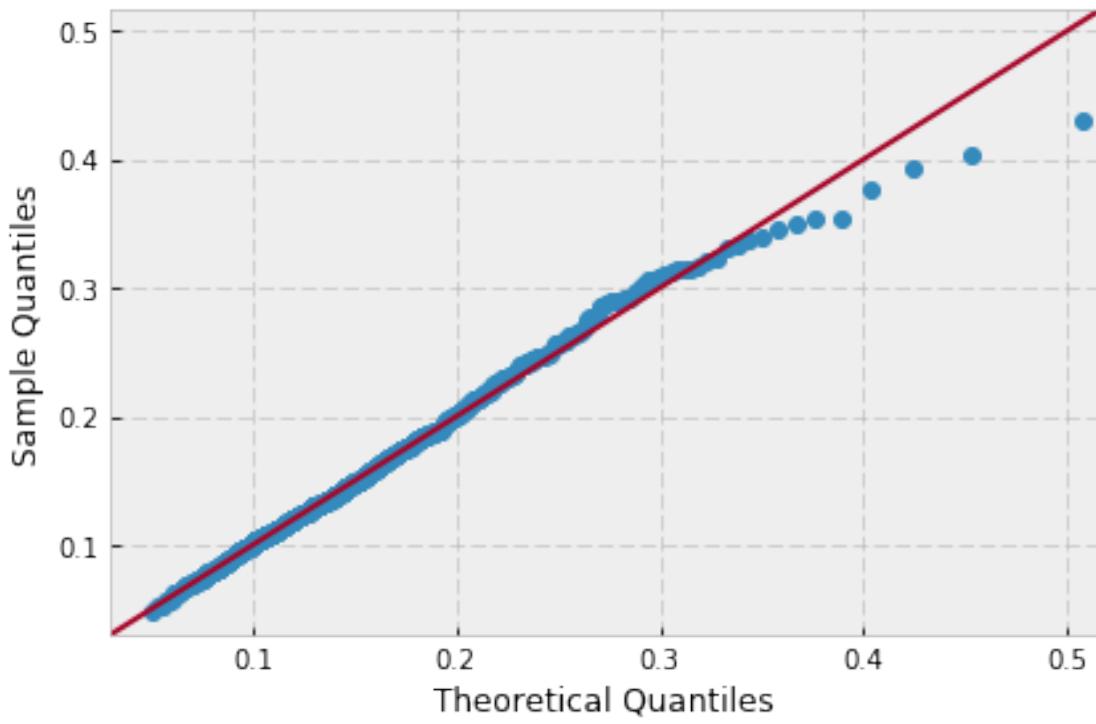


```
variable name: ln
variable value: 2.0
distribution: inverse-gamma(shape=(11.1739821932306,), loc=0.000691938928395688, scale=0.06235093
DescribeResult(nobs=1000, minmax=(0.003345573404739391, 0.021016079680042789)
               mean=0.0068383412951932886, variance=4.1364114506719687e-06,
               skewness=1.6252837932926059, kurtosis=5.410389946263745)
```

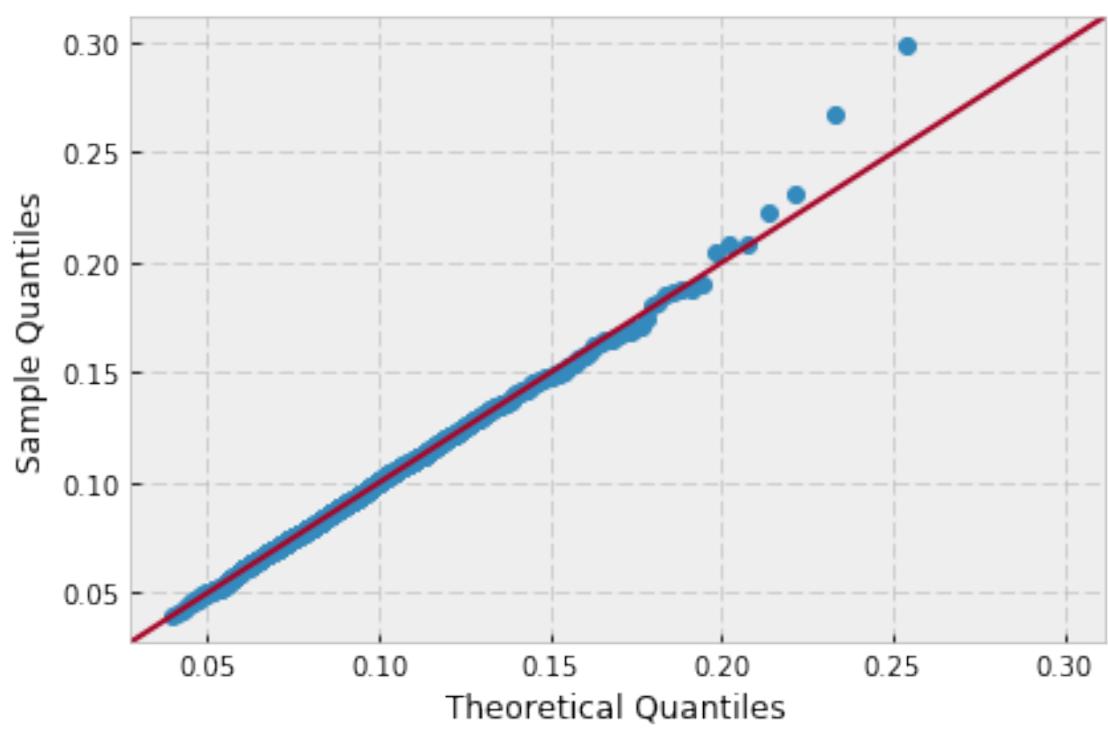
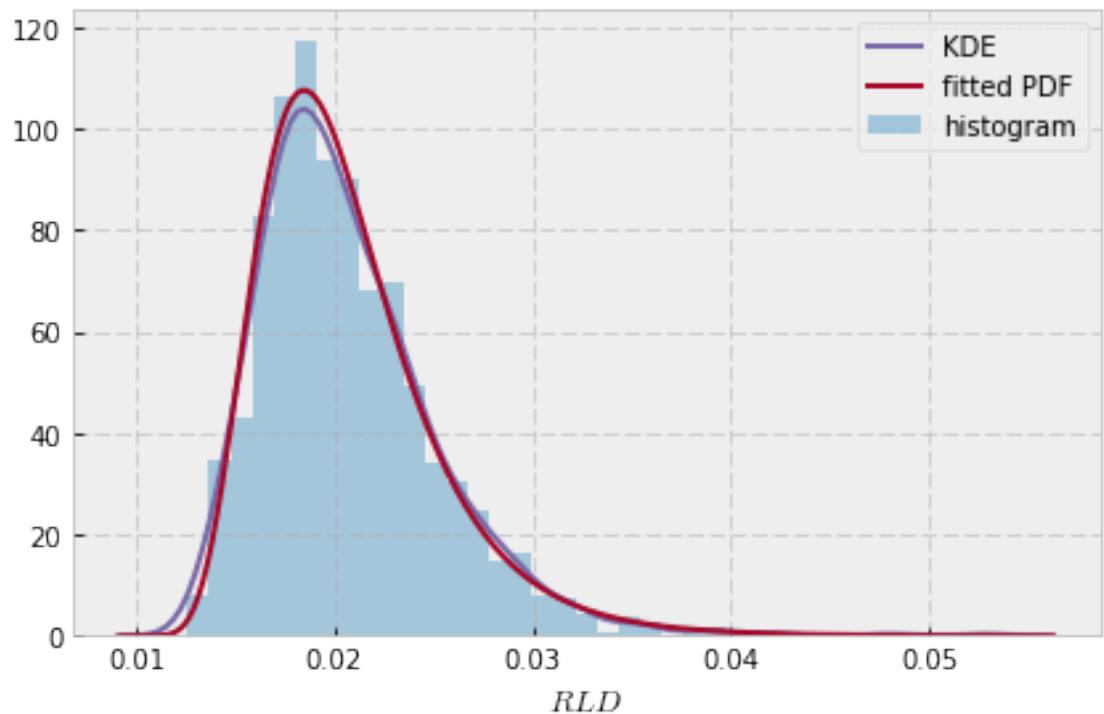


```
variable name: maxB
variable value: 4.0
distribution: inverse-gamma(shape=(8.0036279530913248,), loc=0.00523311797060554, scale=0.0962801
DescribeResult(nobs=1000, minmax=(0.0099026856795272501, 0.04671254058030435)
               mean=0.018971290732750206, variance=2.997792109154598e-05,
               skewness=1.4362054745126767, kurtosis=2.6574767120141036)
```

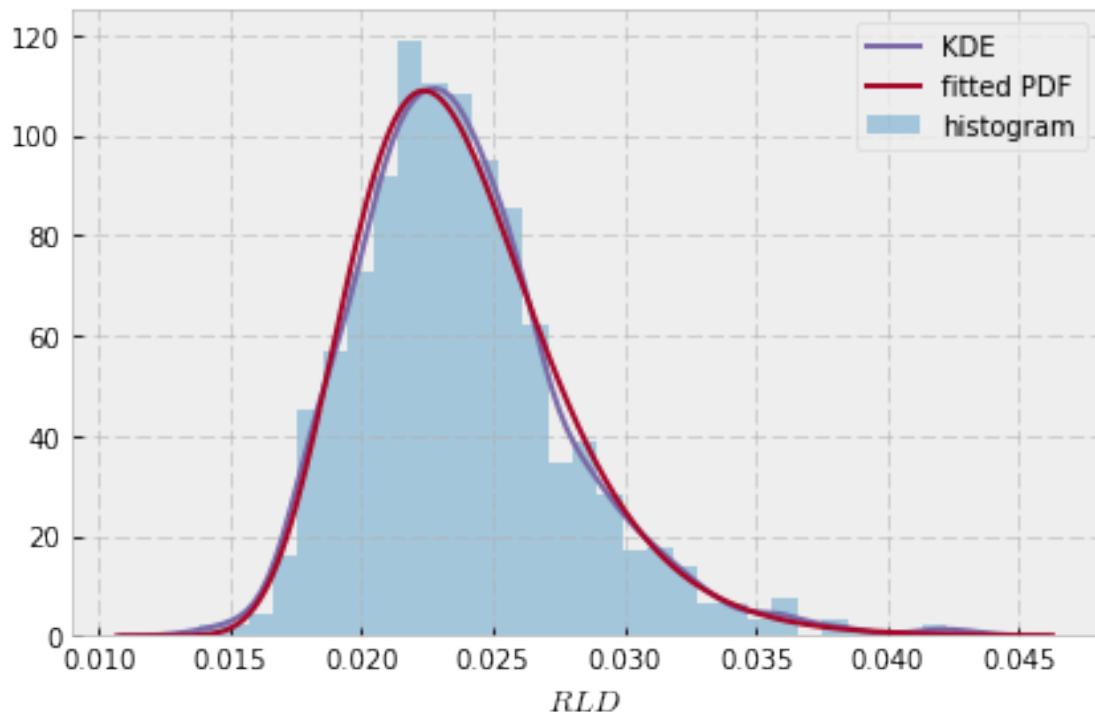


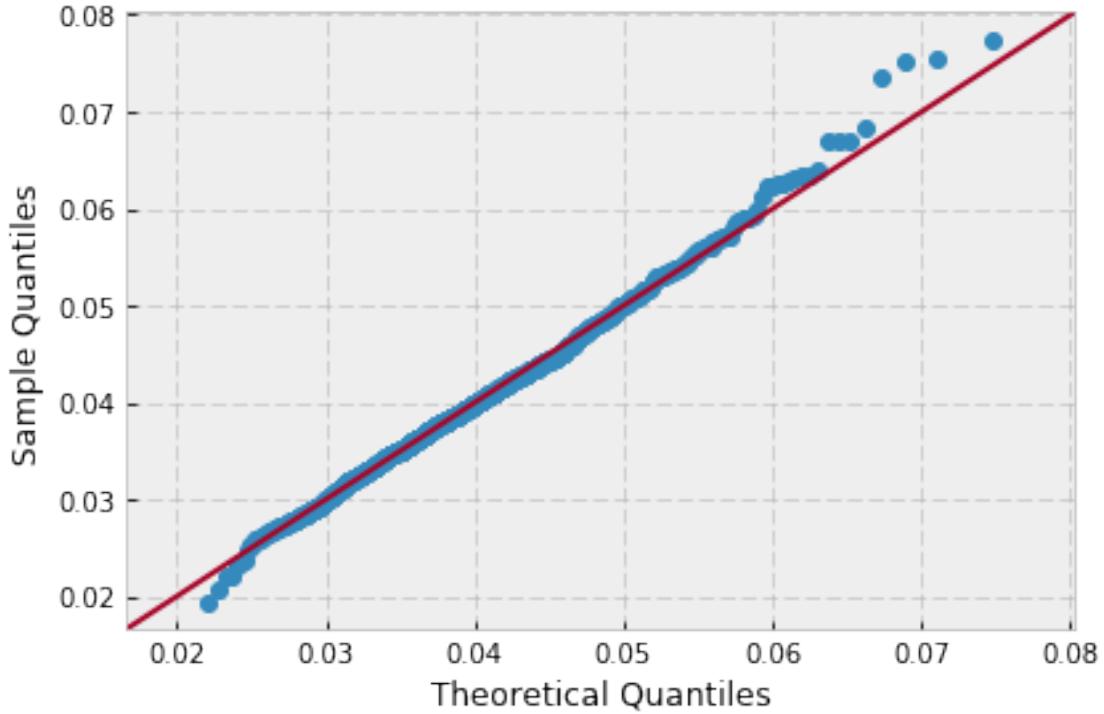


```
variable name: maxB
variable value: 8.0
distribution: inverse-gamma(shape=(11.81870955225591,), loc=0.006312035521577535, scale=0.1557433
DescribeResult(nobs=1000, minmax=(0.012561616283442488, 0.052824723730203624)
               mean=0.020707763239050884, variance=2.1192411458345449e-05,
               skewness=1.47546620328036, kurtosis=4.601212895123473)
```

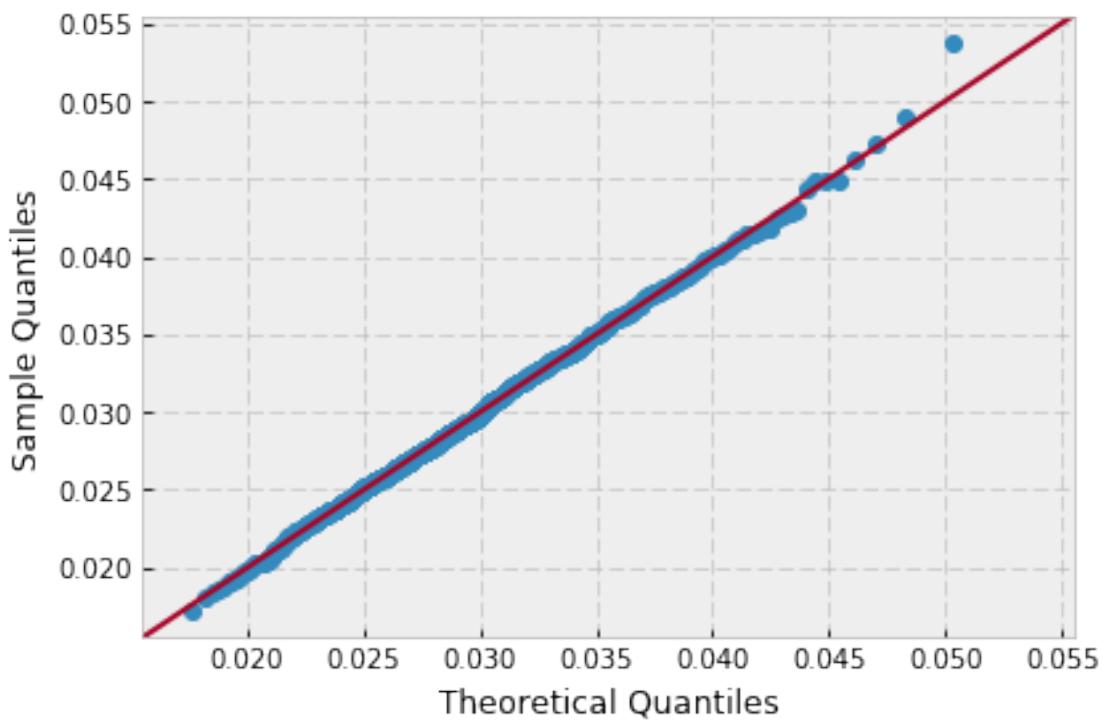
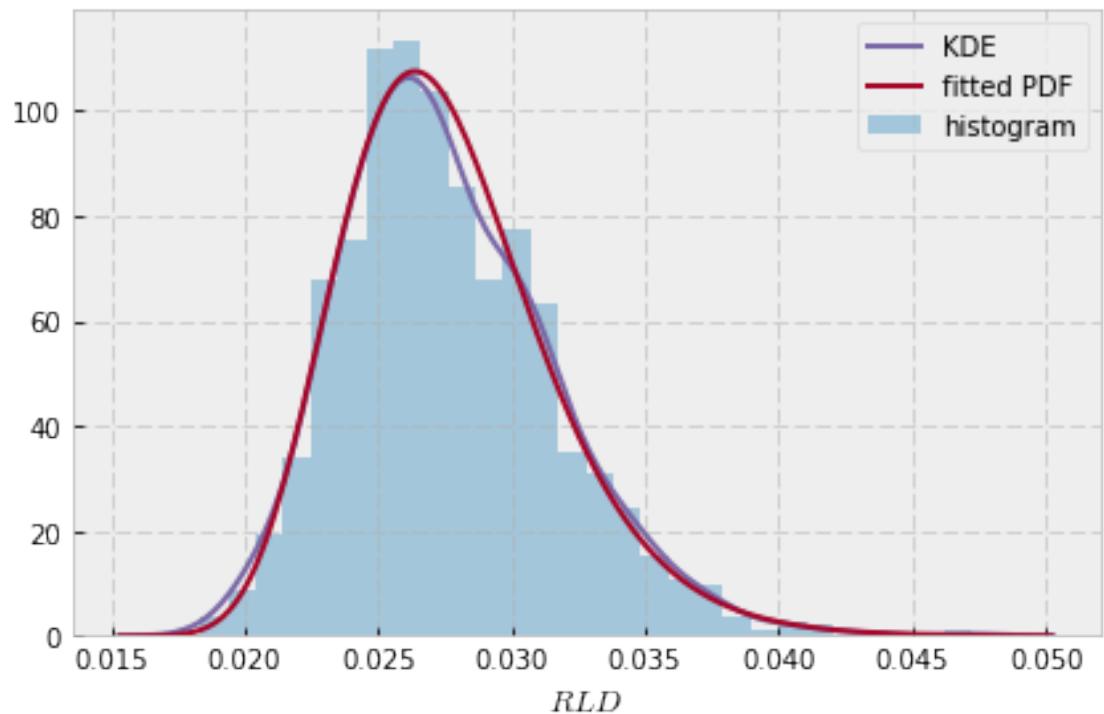


```
variable name: maxB
variable value: 12.0
distribution: inverse-gamma(shape=(26.505094786339903,), loc=0.003911555854224657, scale=0.507927
DescribeResult(nobs=1000, minmax=(0.013777507958225008, 0.043212377052028648)
    mean=0.023828806233171618, variance=1.6508885898339098e-05,
    skewness=0.9754351194817755, kurtosis=1.8561146164403173)
```

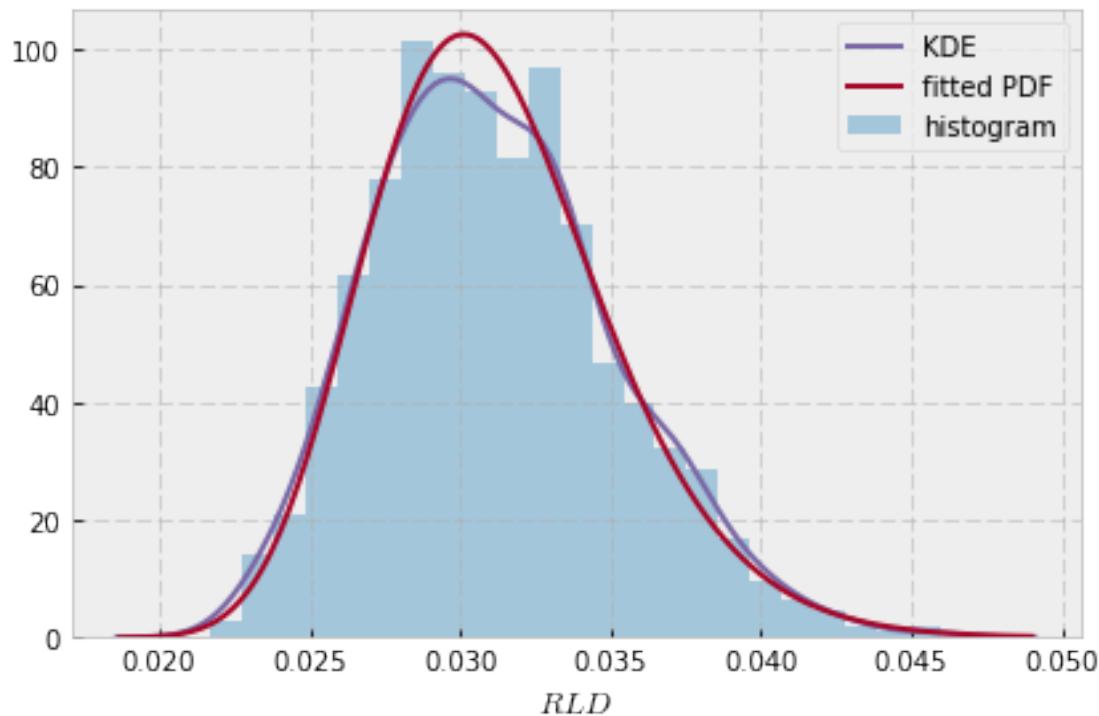


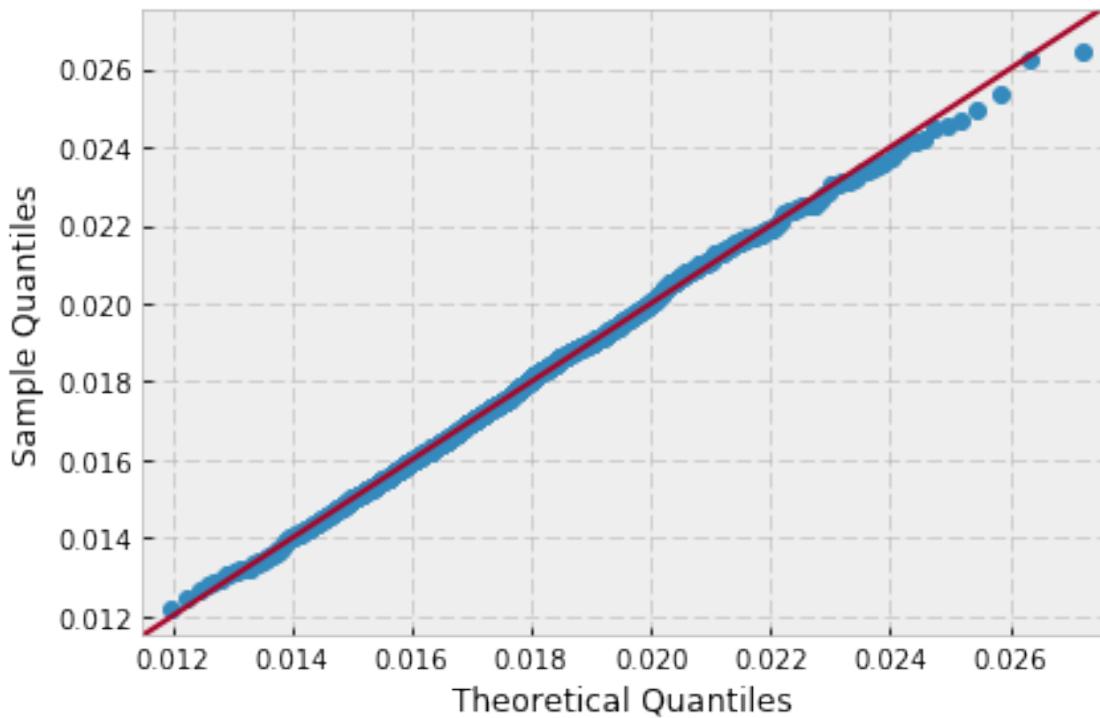


```
variable name: maxB
variable value: 16.0
distribution: inverse-gamma(shape=(35.459702967376366,), loc=0.0046295167258445106, scale=0.79269
DescribeResult(nobs=1000, minmax=(0.018338109086942587, 0.04720042166166627)
               mean=0.02763273757358313, variance=1.5795318500857504e-05,
               skewness=0.6874534395827142, kurtosis=0.8125279912480785)
```

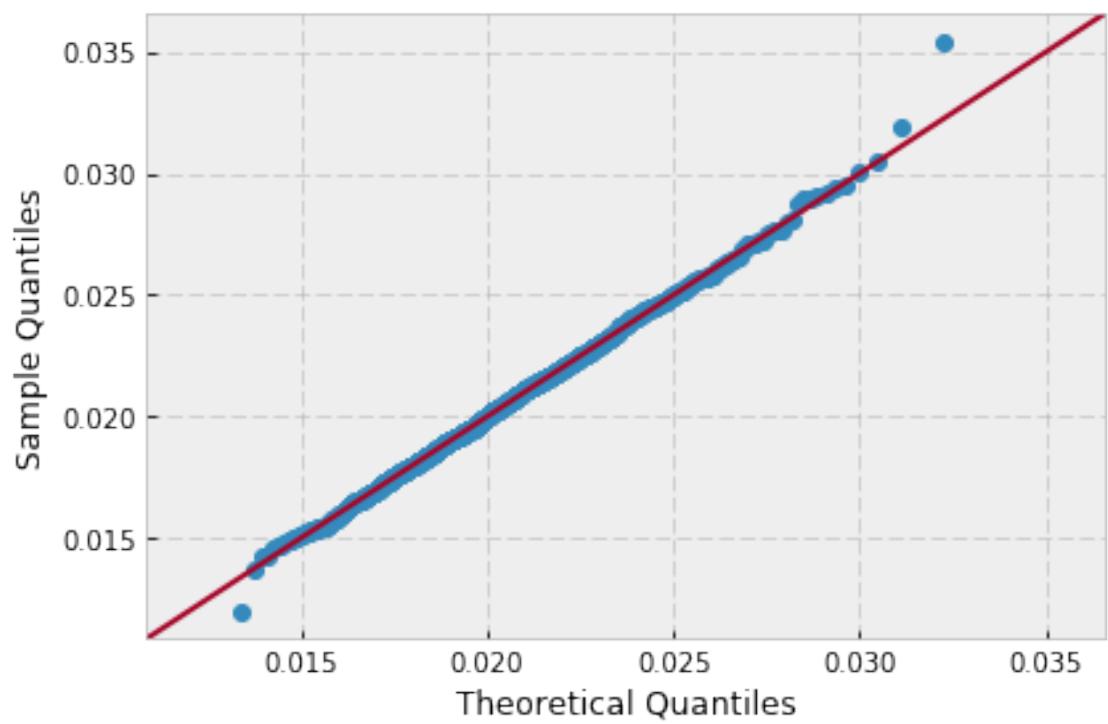
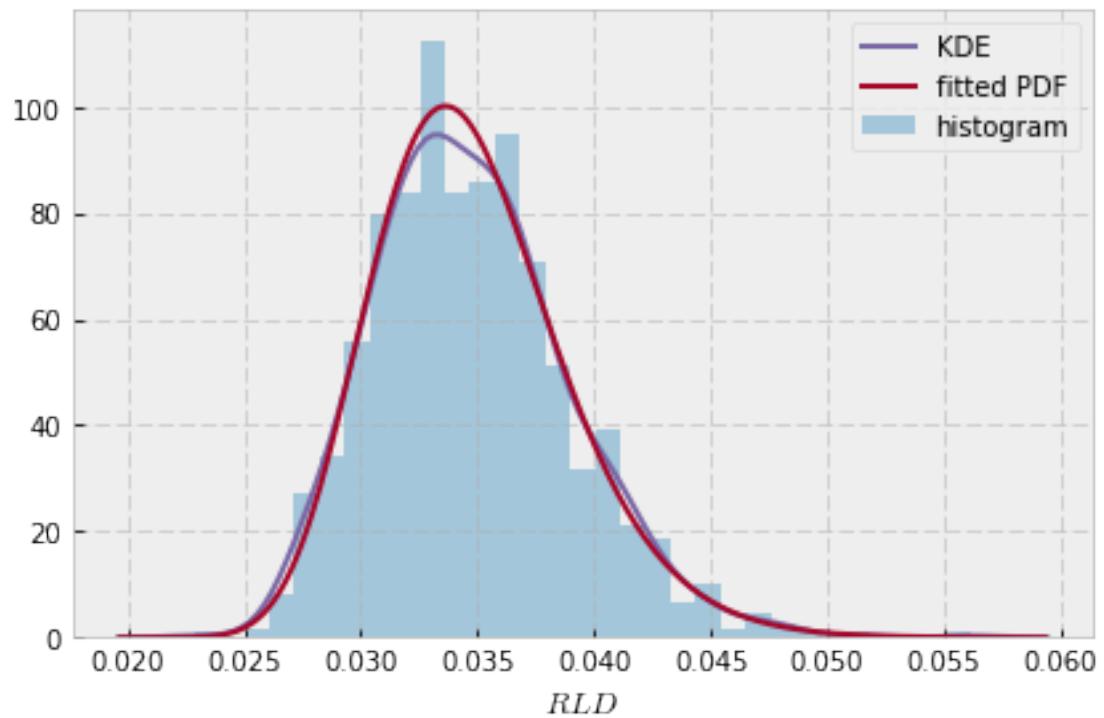


```
variable name: maxB
variable value: 20.0
distribution: inverse-gamma(shape=(57.346621616607791,), loc=0.0009520923362754206, scale=1.70207
DescribeResult(nobs=1000, minmax=(0.021727176992377729, 0.045957106746036501)
               mean=0.031158648230398756, variance=1.6374795607148748e-05,
               skewness=0.45296374024177766, kurtosis=0.04656612594356524)
```

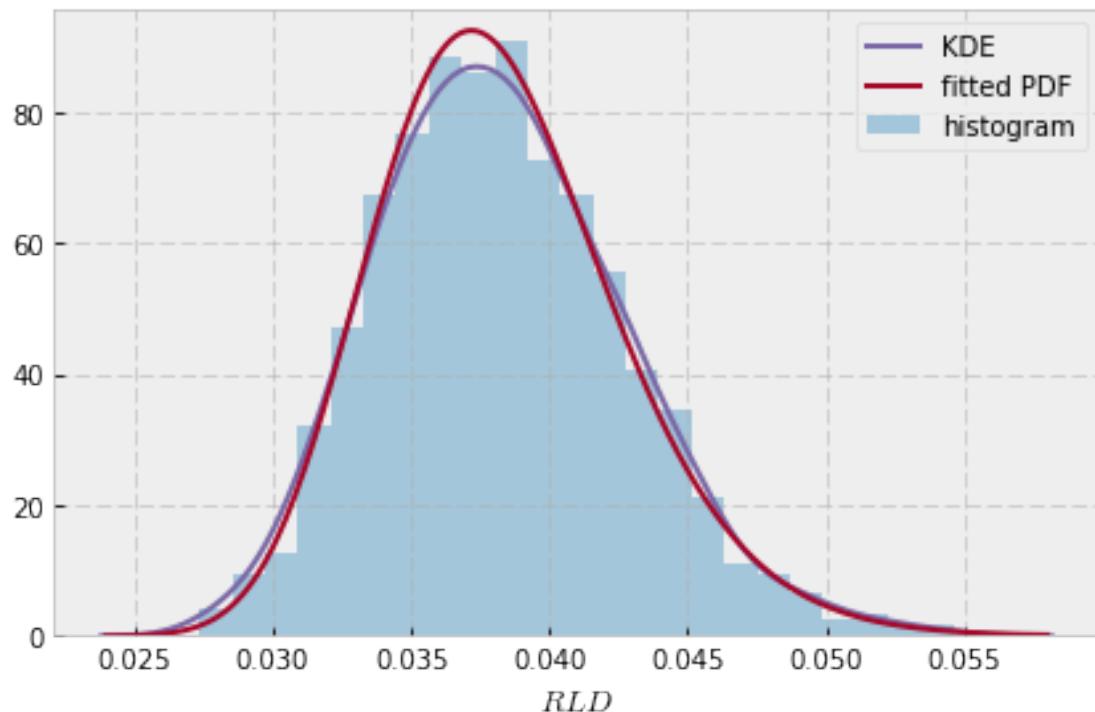


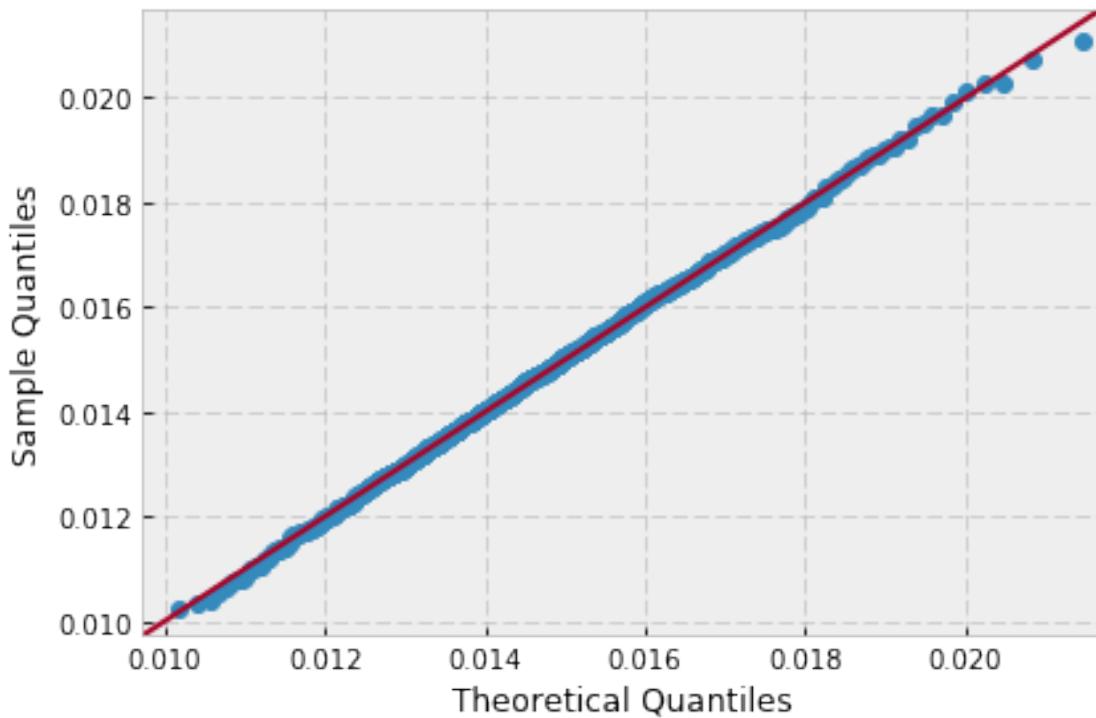


```
variable name: maxB
variable value: 24.0
distribution: inverse-gamma(shape=(50.100032354820705,), loc=0.005750808479756456, scale=1.424492
DescribeResult(nobs=1000, minmax=(0.02284458955053768, 0.056161994361394213)
               mean=0.034763003650078637, variance=1.75436056101198e-05,
               skewness=0.6087597611835883, kurtosis=0.813540901567579)
```

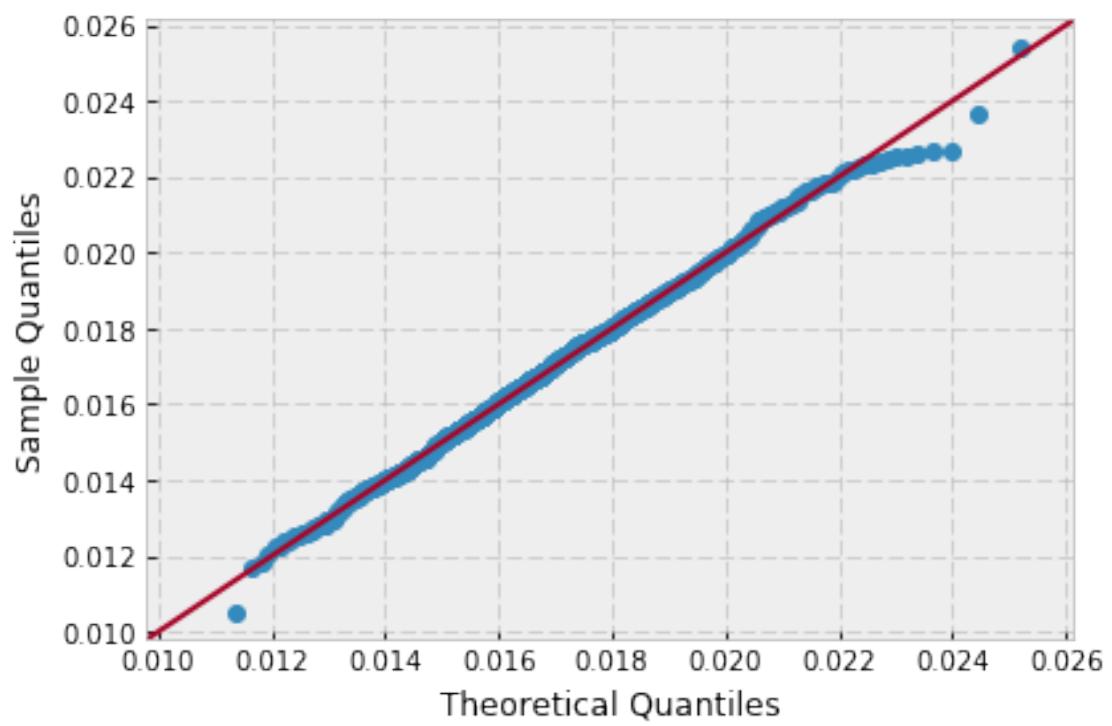
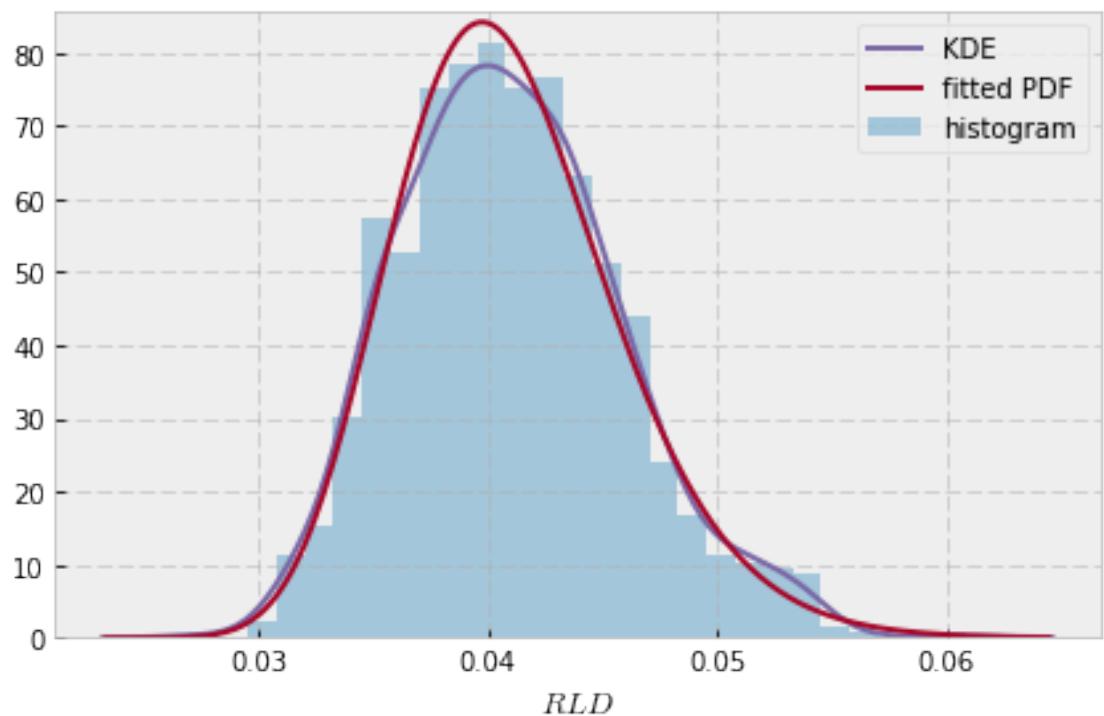


```
variable name: maxB
variable value: 28.0
distribution: inverse-gamma(shape=(69.579423413675386,), loc=0.0014741559736194607, scale=2.52013
DescribeResult(nobs=1000, minmax=(0.027331419449752117, 0.054627553850148093)
    mean=0.038246504508748995, variance=2.0018258052666299e-05,
    skewness=0.4037254615970546, kurtosis=0.17075349767530845)
```

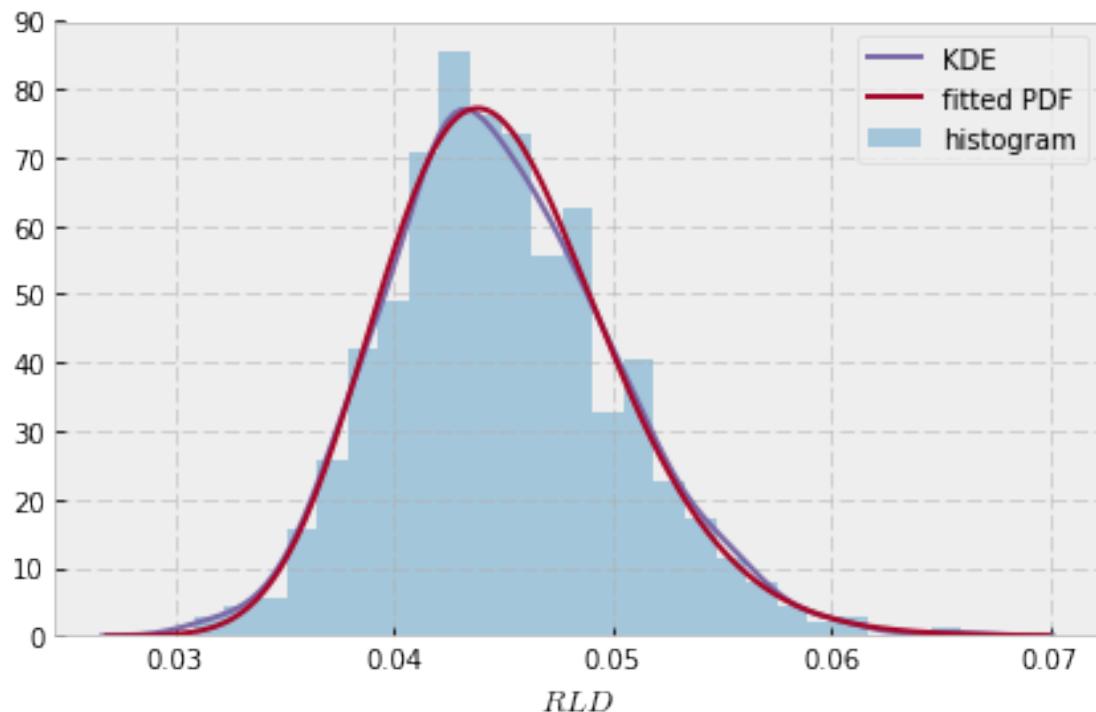


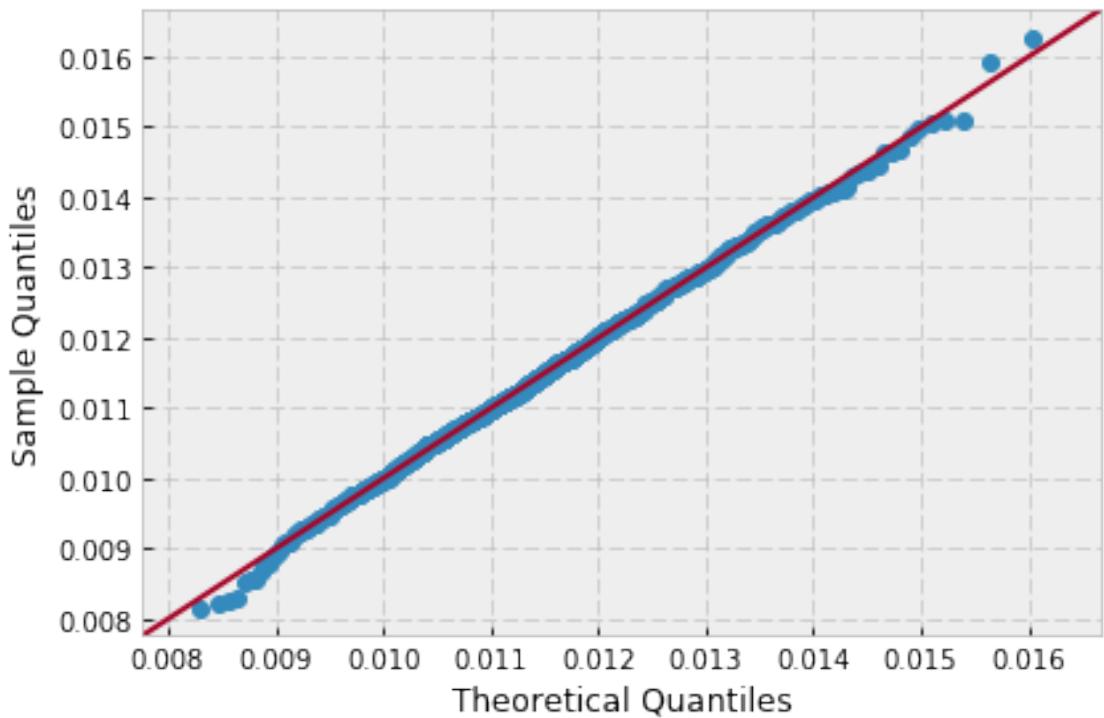


```
variable name: maxB
variable value: 32.0
distribution: inverse-gamma(shape=(60.981773878629056,), loc=0.003085992550640848, scale=2.272197
DescribeResult(nobs=1000, minmax=(0.027011426773548782, 0.060806095211313985)
               mean=0.041014545768920899, variance=2.3944126823533659e-05,
               skewness=0.4213200367565847, kurtosis=0.13275081054198745)
```

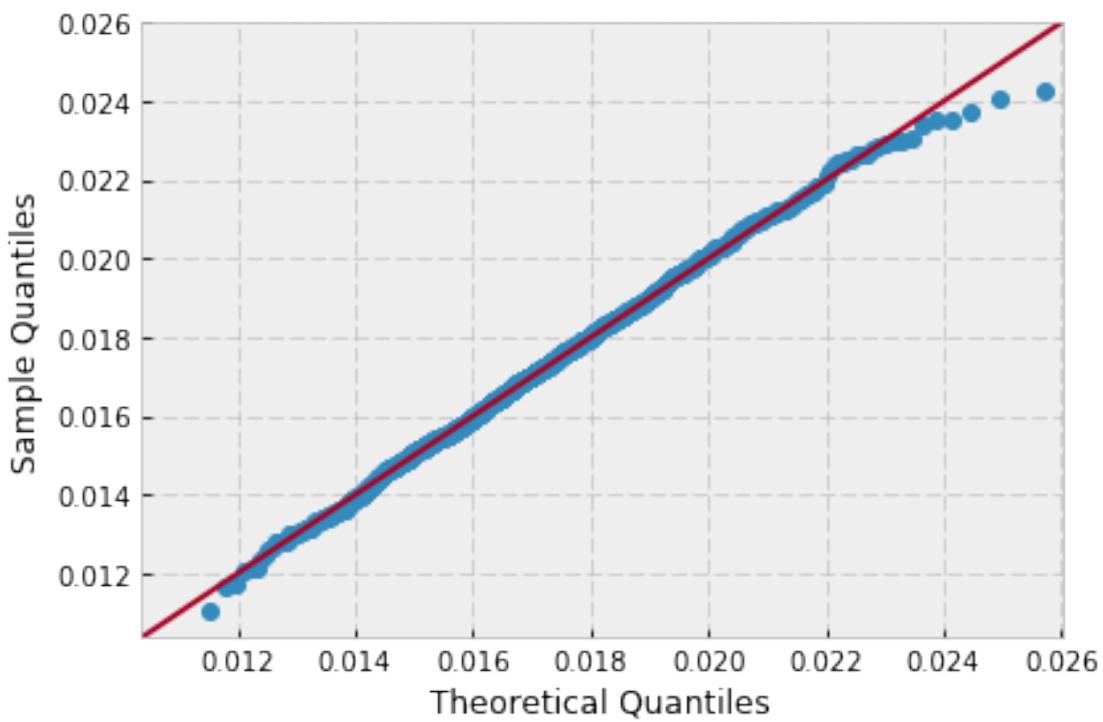
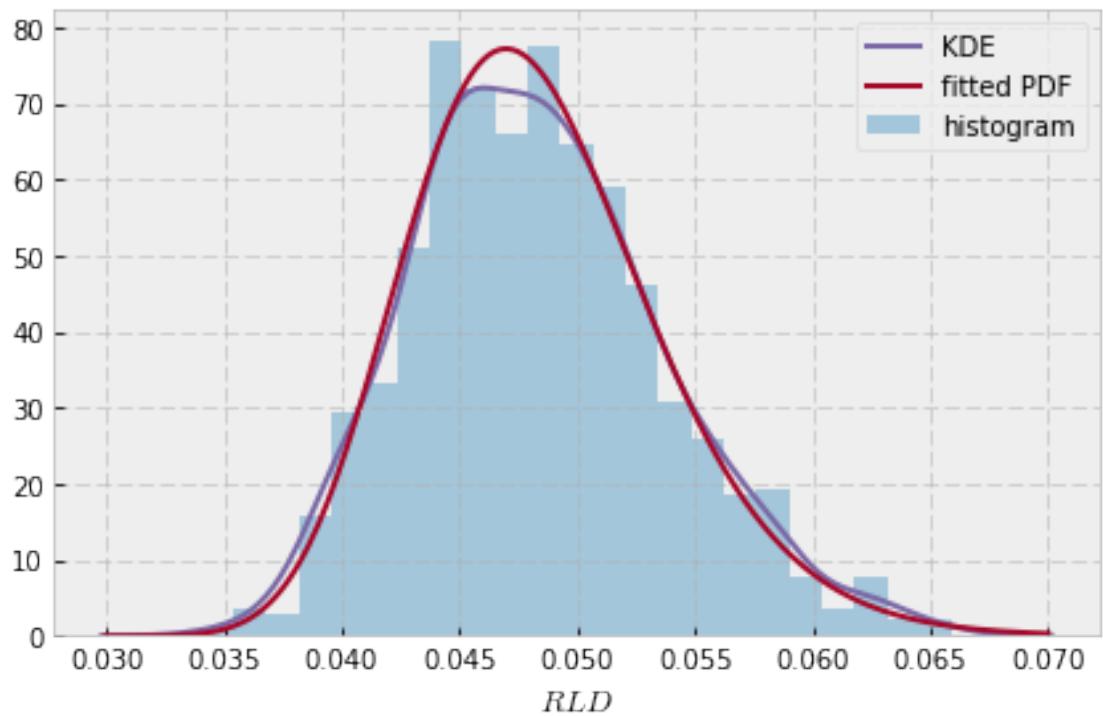


```
variable name: maxB
variable value: 36.0
distribution: inverse-gamma(shape=(88.61129249384777,), loc=-0.004516627932886272, scale=4.332316
DescribeResult(nobs=1000, minmax=(0.030855162076599333, 0.065924631249535959)
               mean=0.044922881333185428, variance=2.7877274682257962e-05,
               skewness=0.37933974182901237, kurtosis=0.2342638330887743)
```

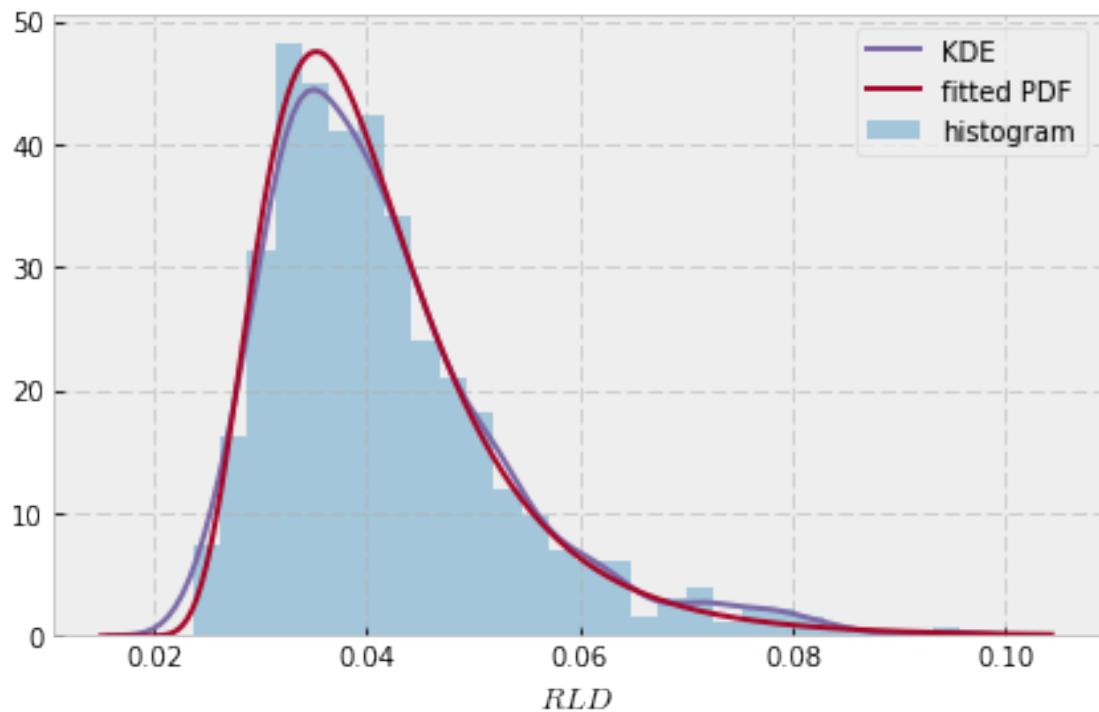


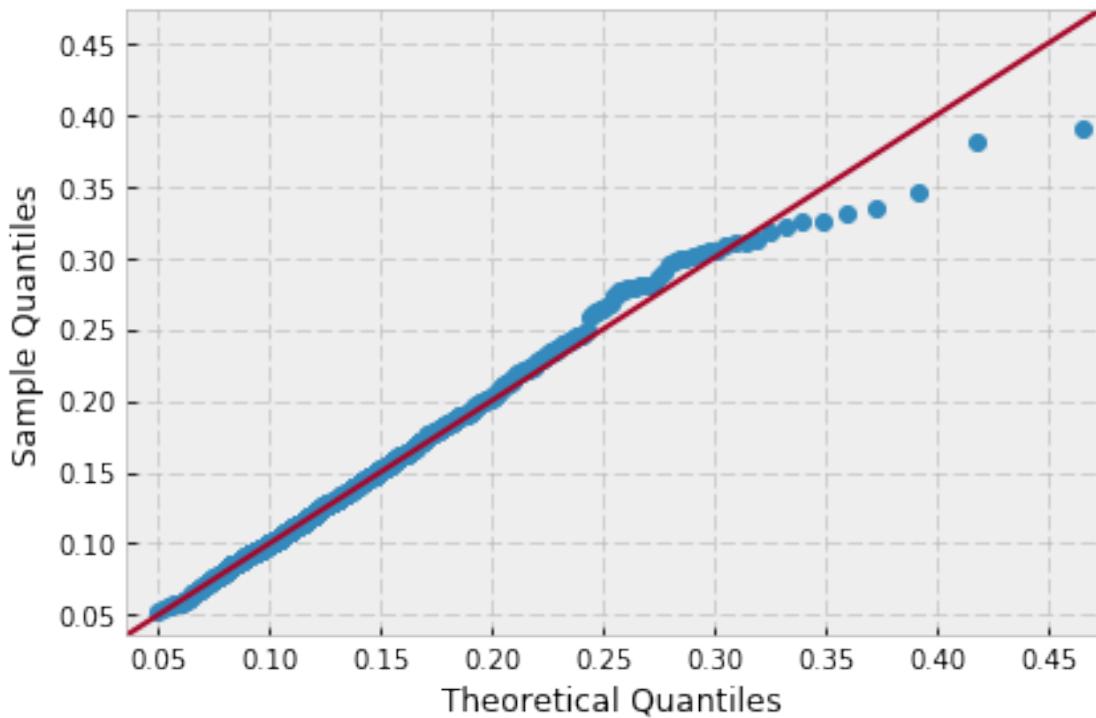


```
variable name: maxB
variable value: 40.0
distribution: inverse-gamma(shape=(59.997015442661024,), loc=0.007339250967365372, scale=2.417650
DescribeResult(nobs=1000, minmax=(0.033976188398847543, 0.065954766128591233)
               mean=0.048349071592438432, variance=2.8879386798858455e-05,
               skewness=0.42181961910128357, kurtosis=0.04788498469086999)
```

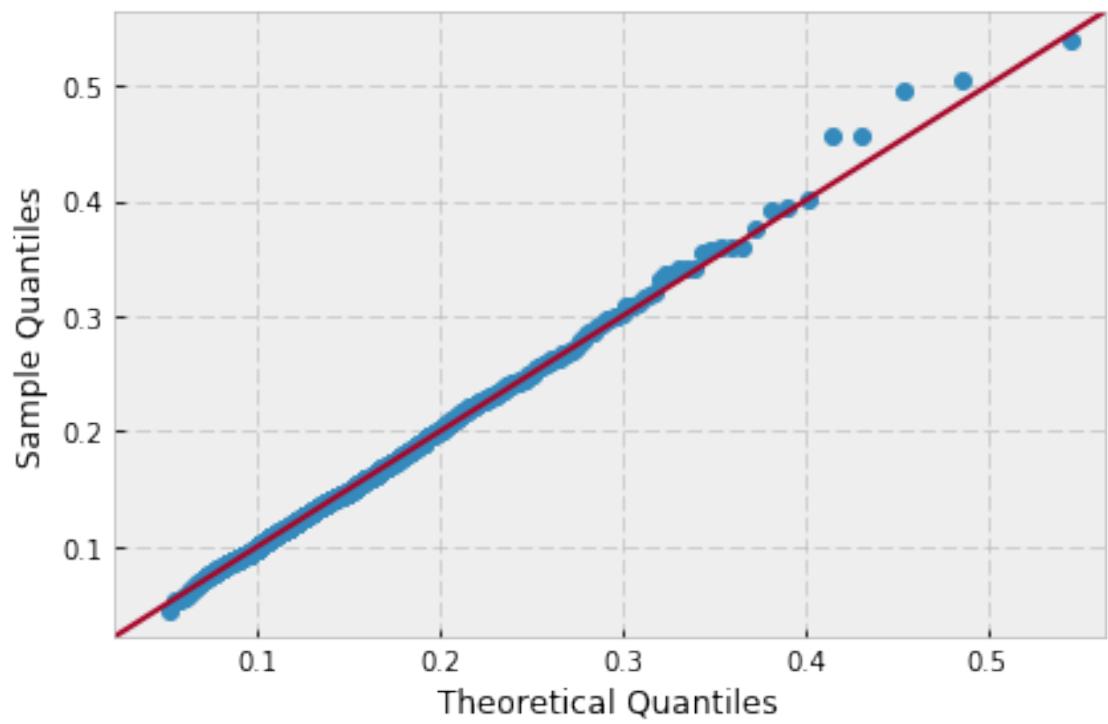
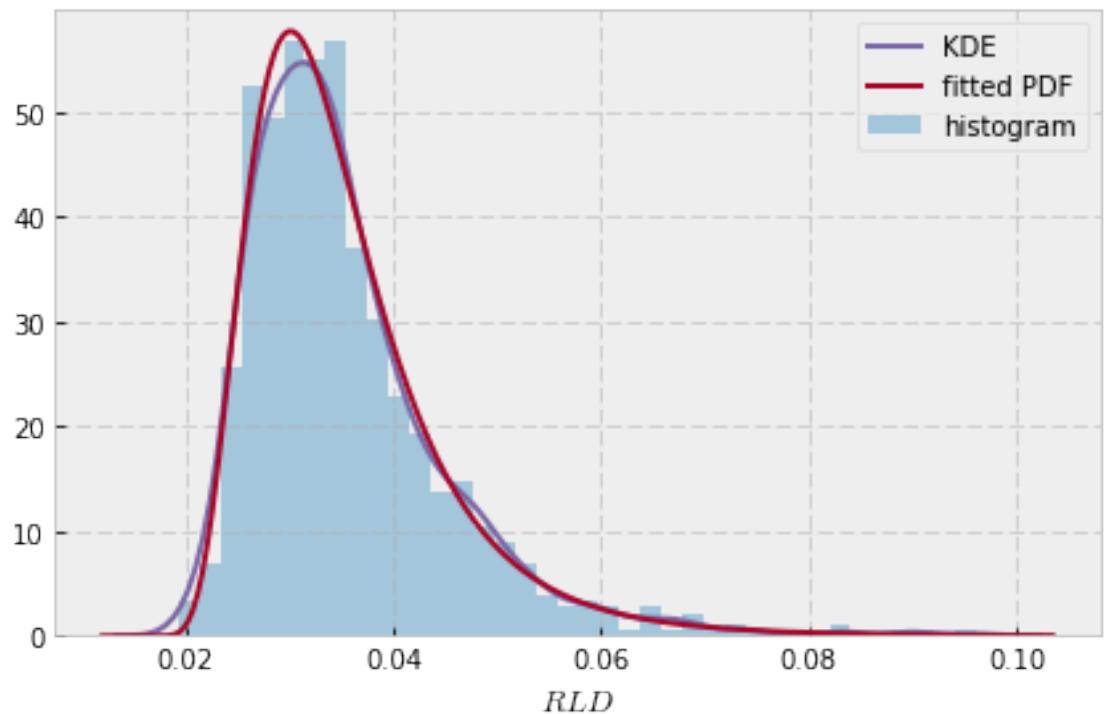


```
variable name: r
variable value: 1.0
distribution: inverse-gamma(shape=(8.3820531649231889,),loc=0.012500860145287954, scale=0.213617
DescribeResult(nobs=1000, minmax=(0.023652680443488706, 0.095748848144371776)
               mean=0.041719976411923175, variance=0.0001291460627467851,
               skewness=1.3513533344525228, kurtosis=2.2560842122734366)
```

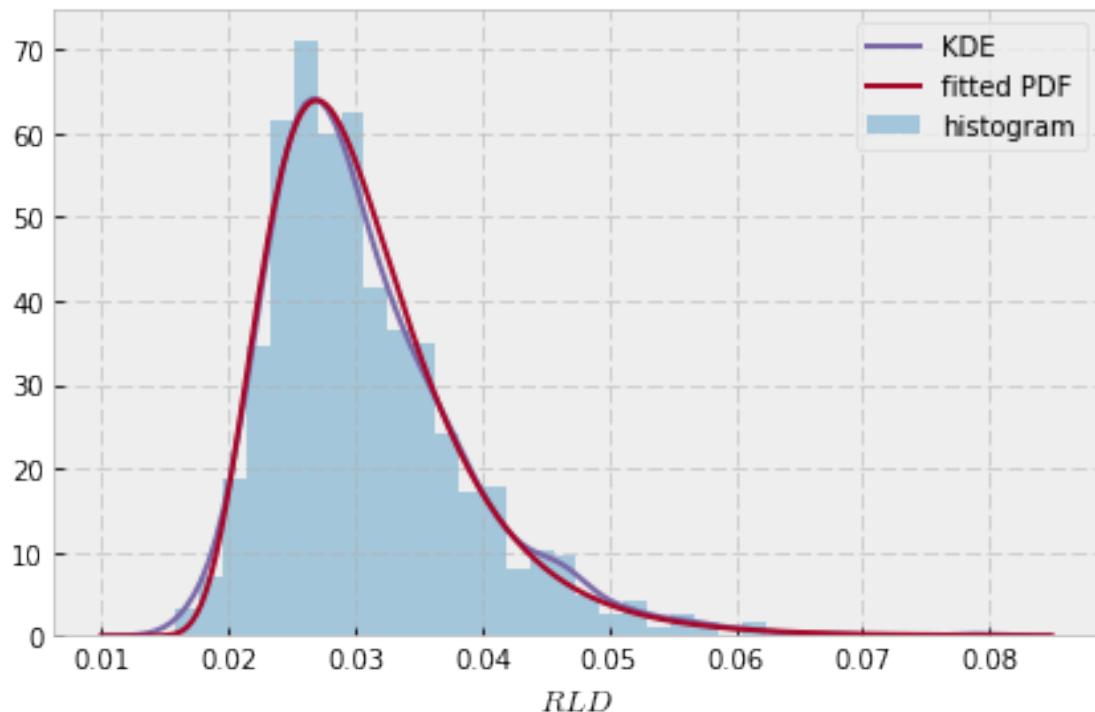


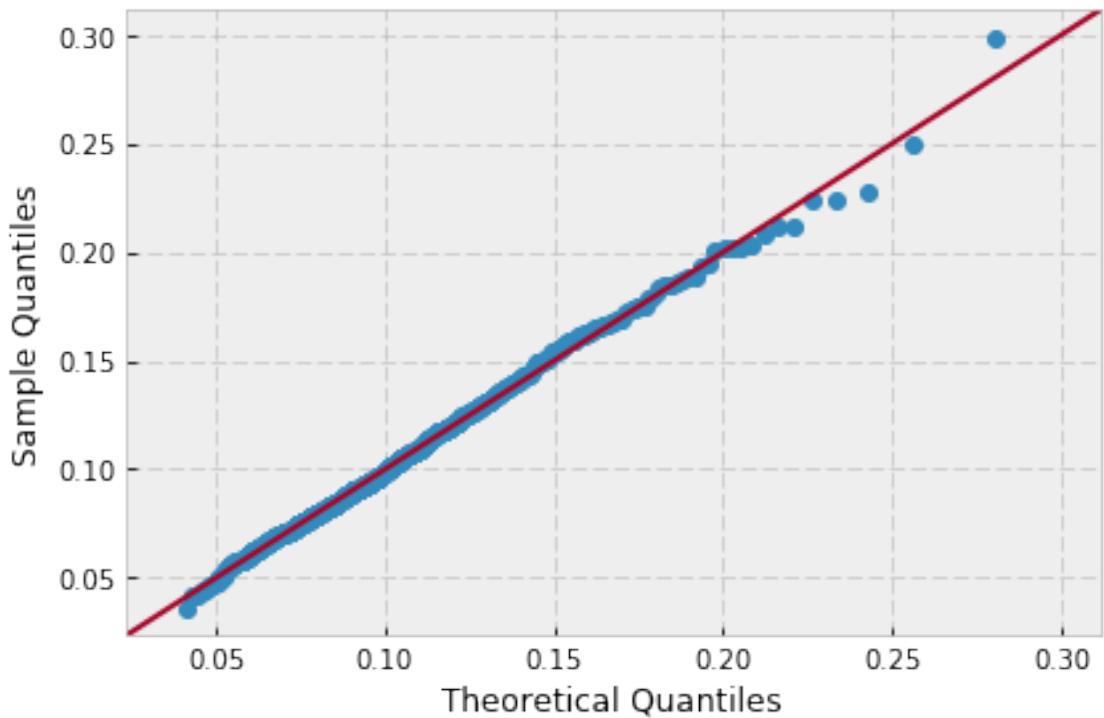


```
variable name: r
variable value: 1.2222222222222223
distribution: inverse-gamma(shape=(7.705338926635017,), loc=0.012241787751296713, scale=0.1556993
DescribeResult(nobs=1000, minmax=(0.019321490311306363, 0.096181203422360223)
               mean=0.035465943932726438, variance=9.4129475516480121e-05,
               skewness=1.7989149911465385, kurtosis=5.336814305415972)
```

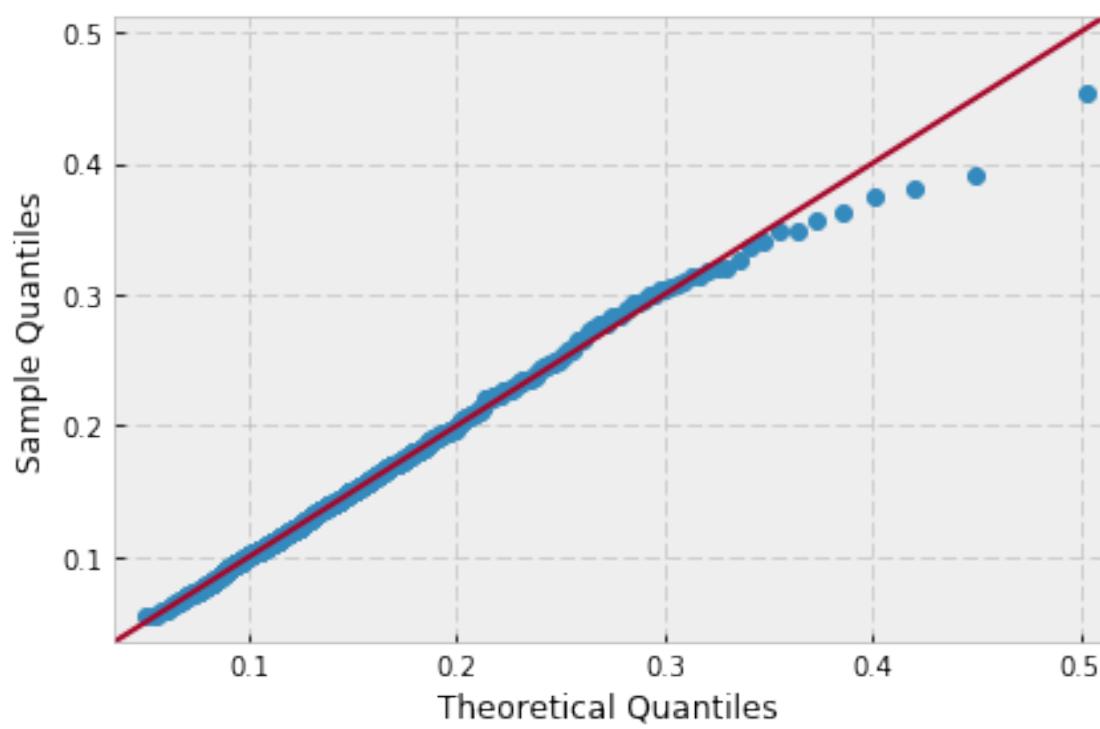
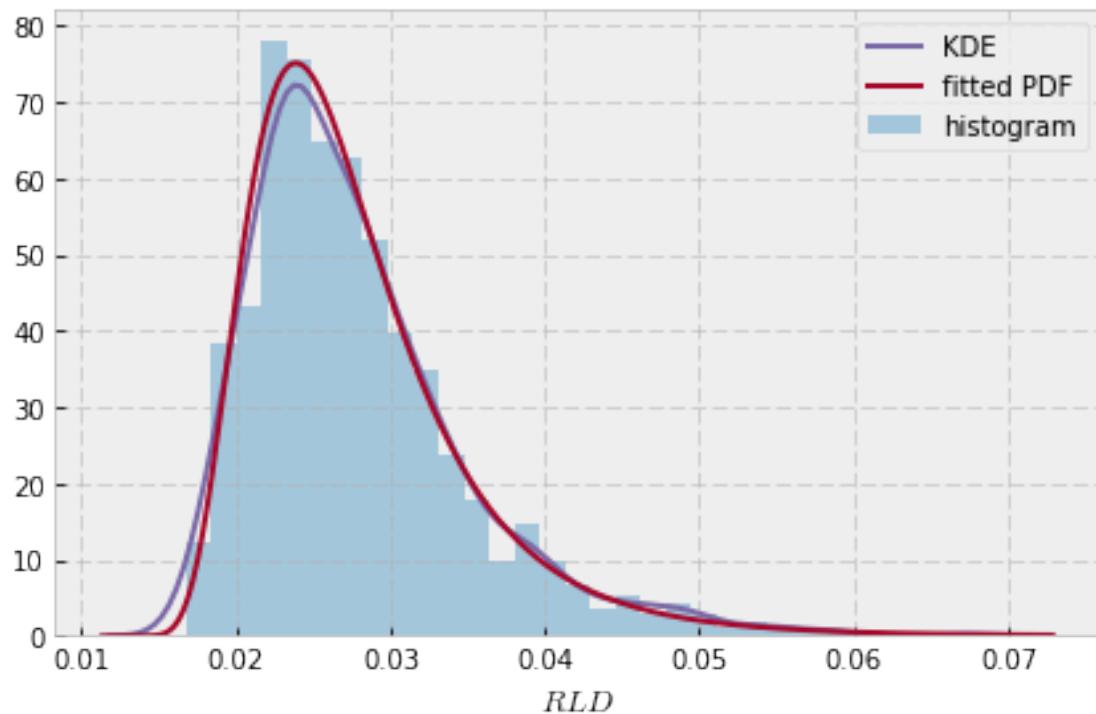


```
variable name: r
variable value: 1.444444444444444
distribution: inverse-gamma(shape=(11.147938040739501,),loc=0.007116623955728103, scale=0.240945
DescribeResult(nobs=1000, minmax=(0.015873761285682797, 0.078996682364022117)
               mean=0.030859247988892214, variance=6.1005347072948658e-05,
               skewness=1.3053427531508088, kurtosis=2.7087332559861927)
```

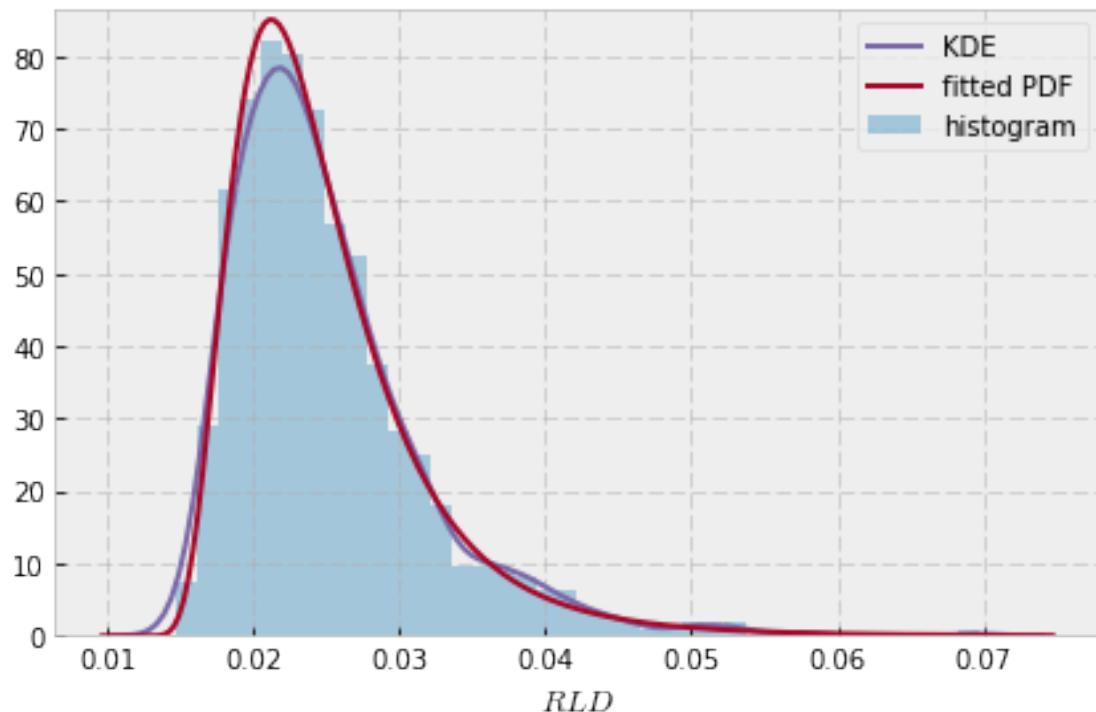


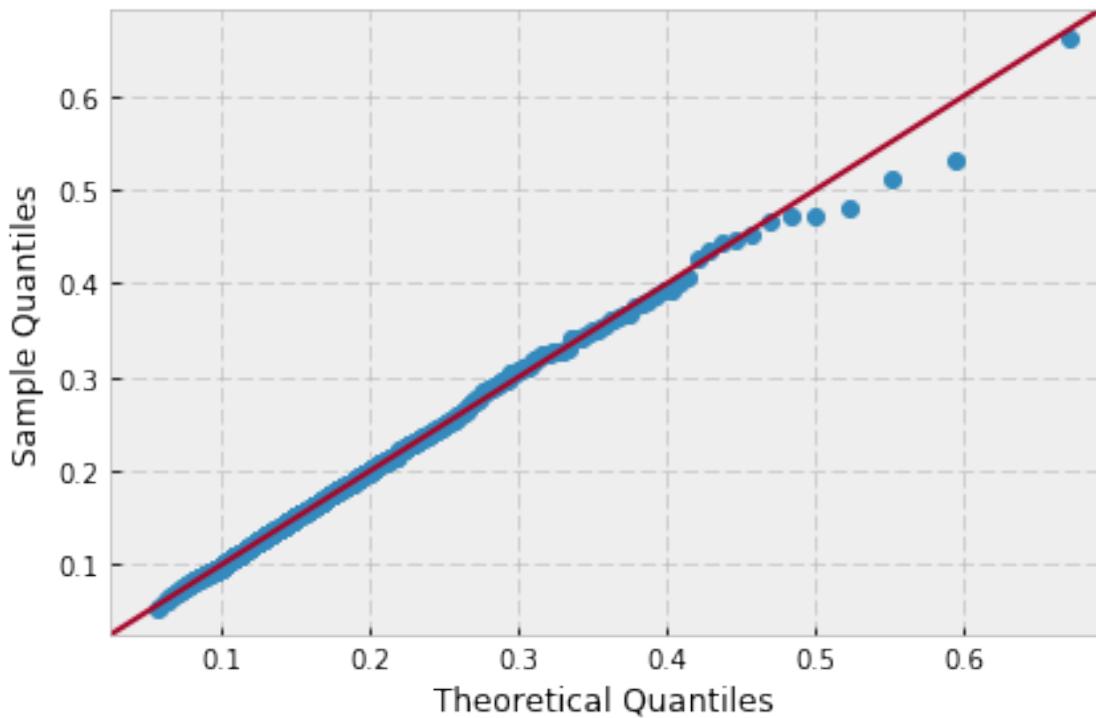


```
variable name: r
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(8.042247802453776,), loc=0.00980967709382725, scale=0.12722198
DescribeResult(nobs=1000, minmax=(0.016730339402314793, 0.067492196642389757)
               mean=0.027857191878979812, variance=5.0882400791852531e-05,
               skewness=1.4297804872081608, kurtosis=2.848512352889845)
```

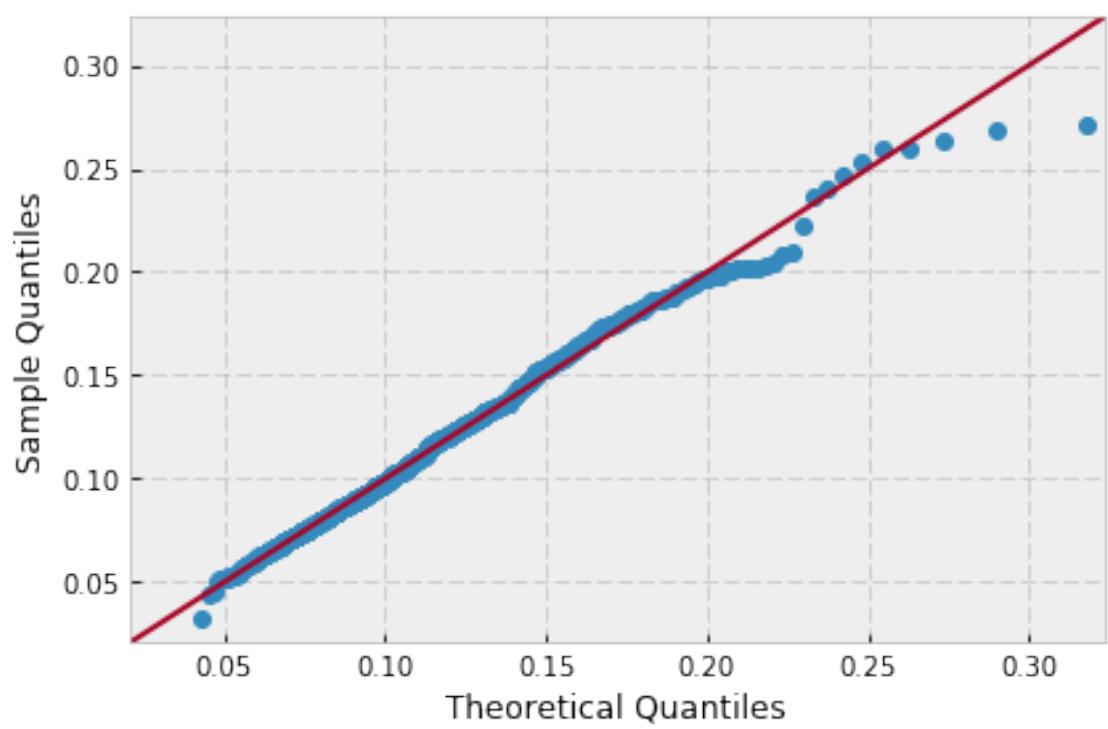
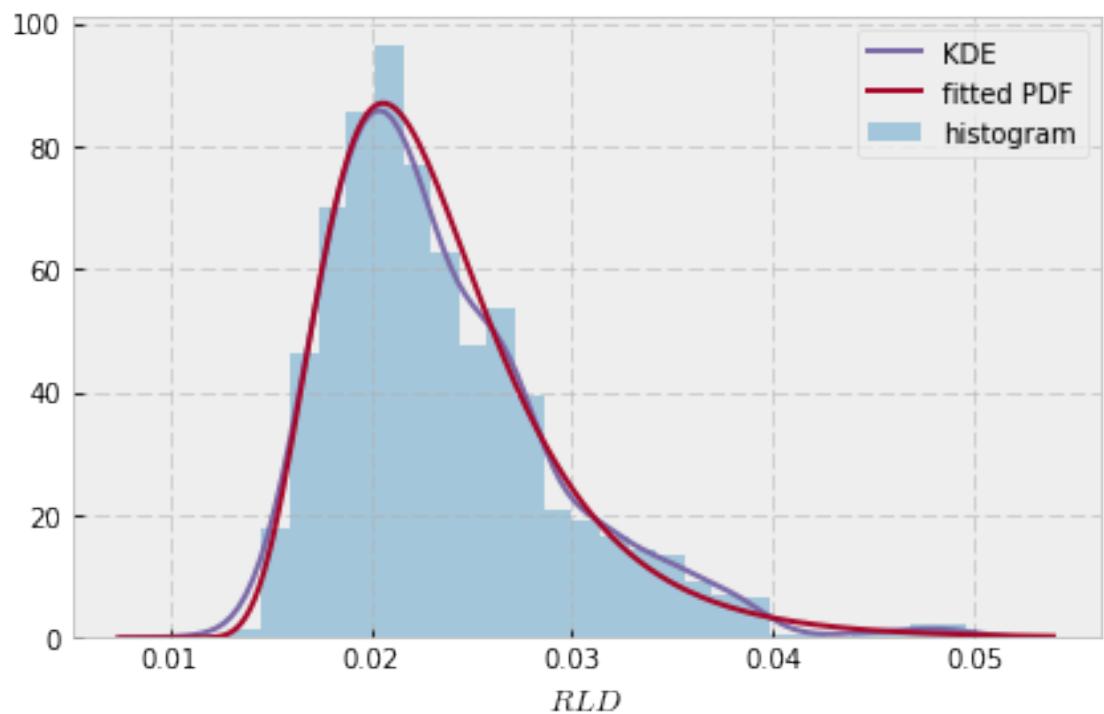


```
variable name: r
variable value: 1.888888888888888
distribution: inverse-gamma(shape=(6.9245933951232193,),loc=0.009819066790327183, scale=0.090466
DescribeResult(nobs=1000, minmax=(0.014706767538888421, 0.069683417543478621)
               mean=0.025070422338451238, variance=4.4432081615407945e-05,
               skewness=1.6845014538184304, kurtosis=4.519348813689556)
```

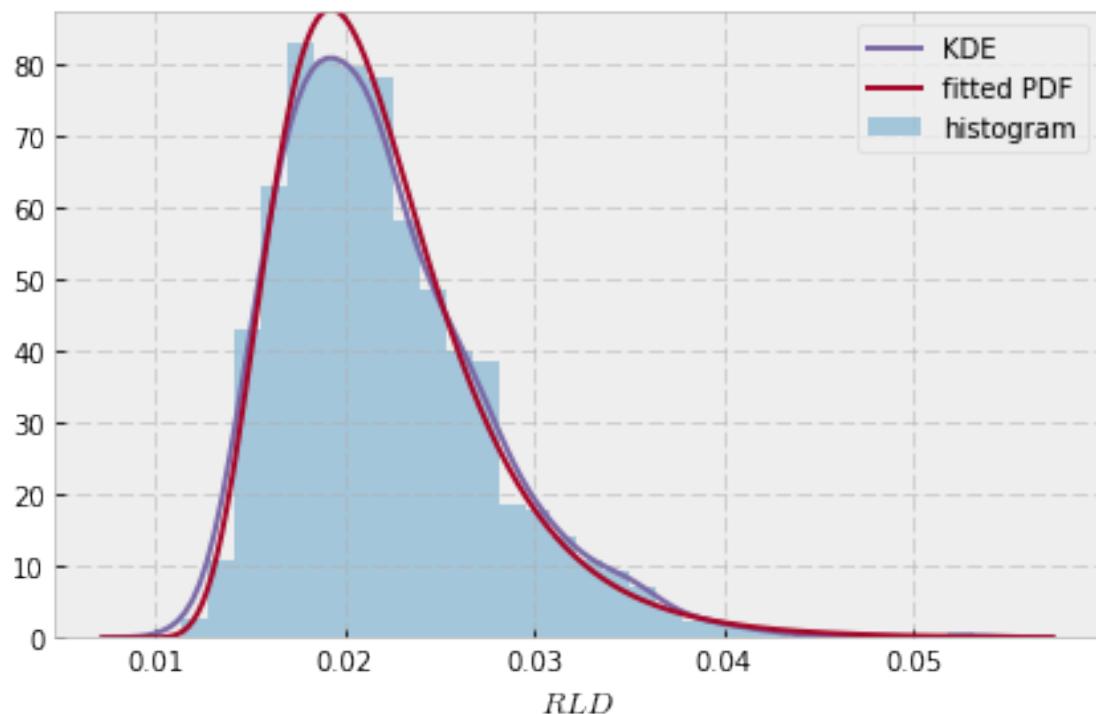


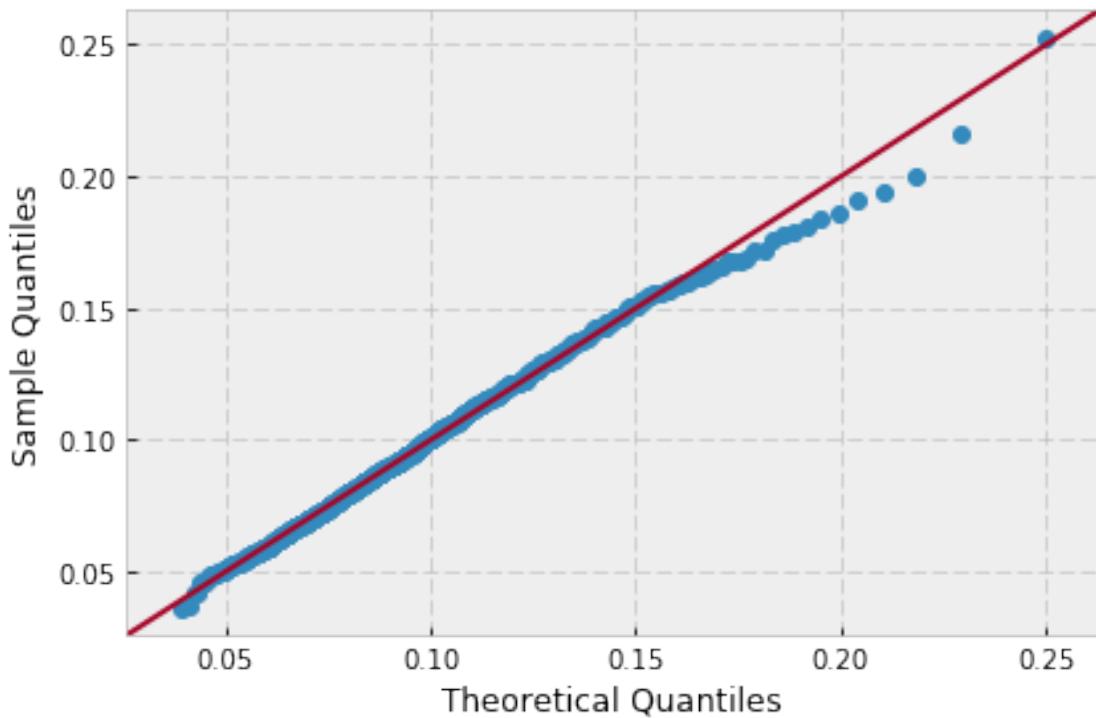


```
variable name: r
variable value: 2.111111111111111
distribution: inverse-gamma(shape=(10.35539452874475,), loc=0.006596717579518836, scale=0.1588665
DescribeResult(nobs=1000, minmax=(0.011756425827545072, 0.04958857338670572)
               mean=0.023572390557845269, variance=3.3380101975725419e-05,
               skewness=1.216474072364544, kurtosis=1.9385645860718368)
```

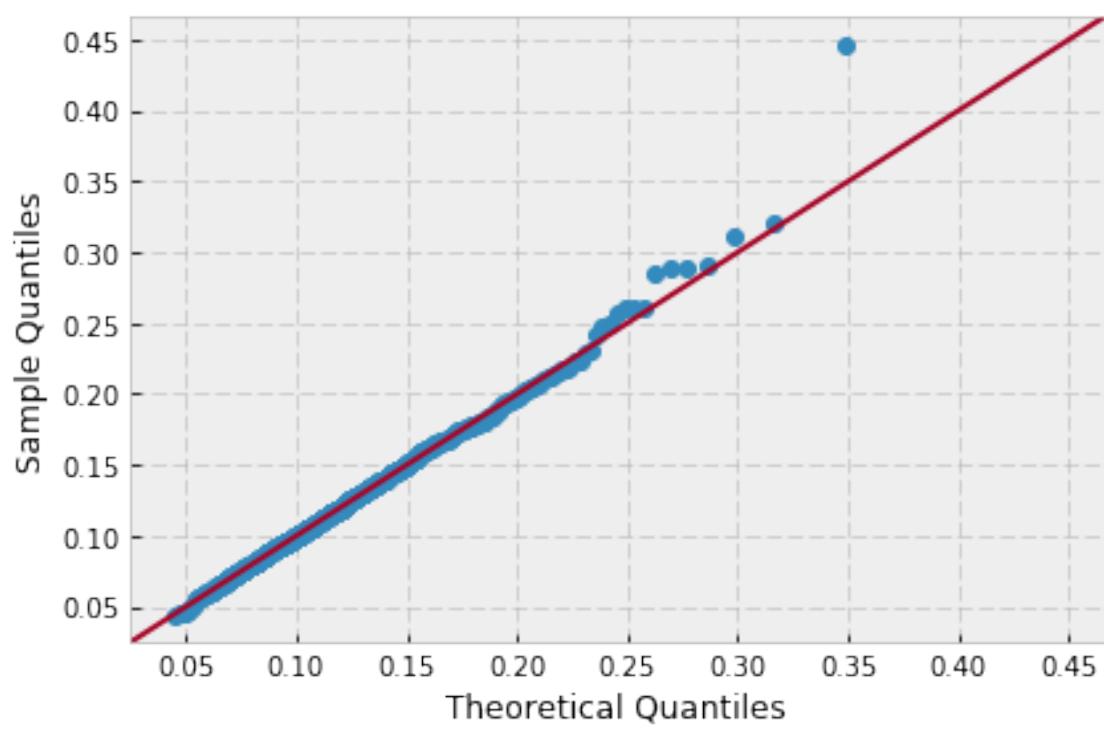
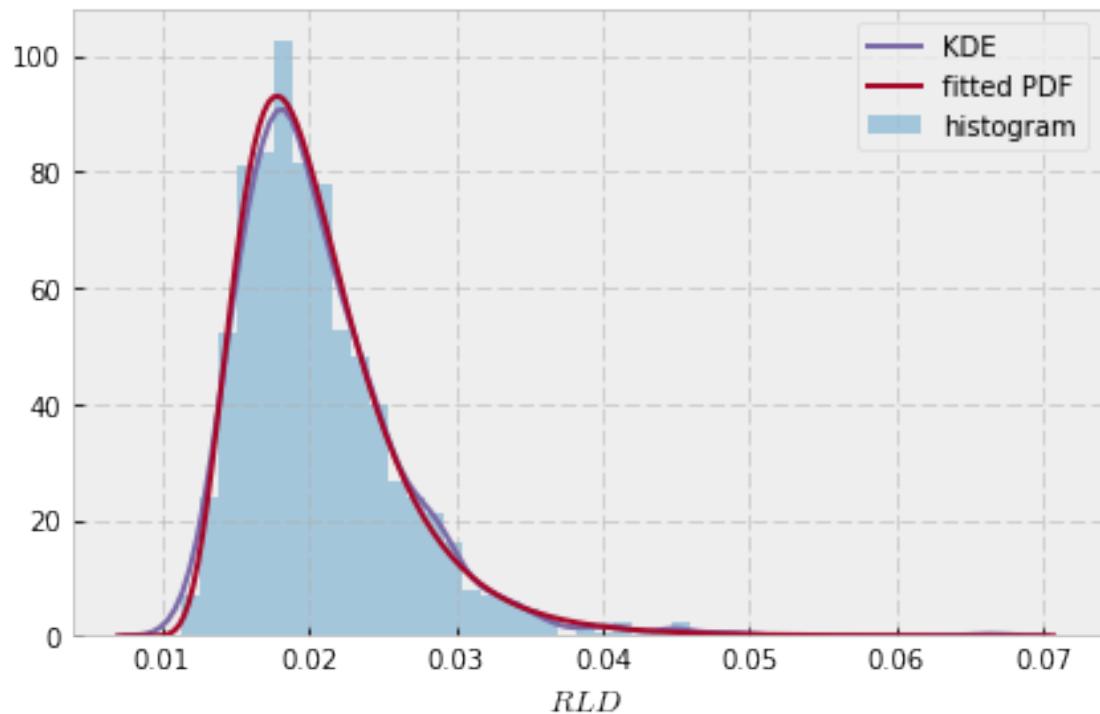


```
variable name: r
variable value: 2.333333333333333
distribution: inverse-gamma(shape=(11.933498426620837,),loc=0.0042667537981464125, scale=0.19397
DescribeResult(nobs=1000, minmax=(0.011349941844817578, 0.053206492799994905)
    mean=0.021997937874538662, variance=3.0331085900628036e-05,
    skewness=1.0828407917475362, kurtosis=1.7243881793004663)
```

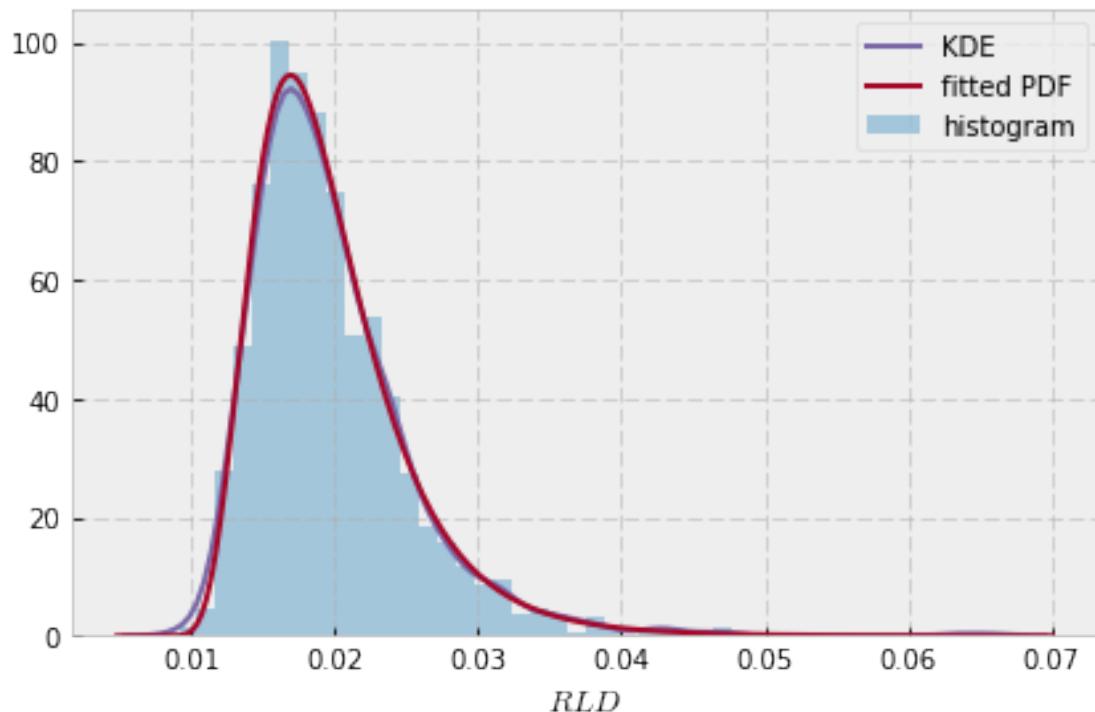


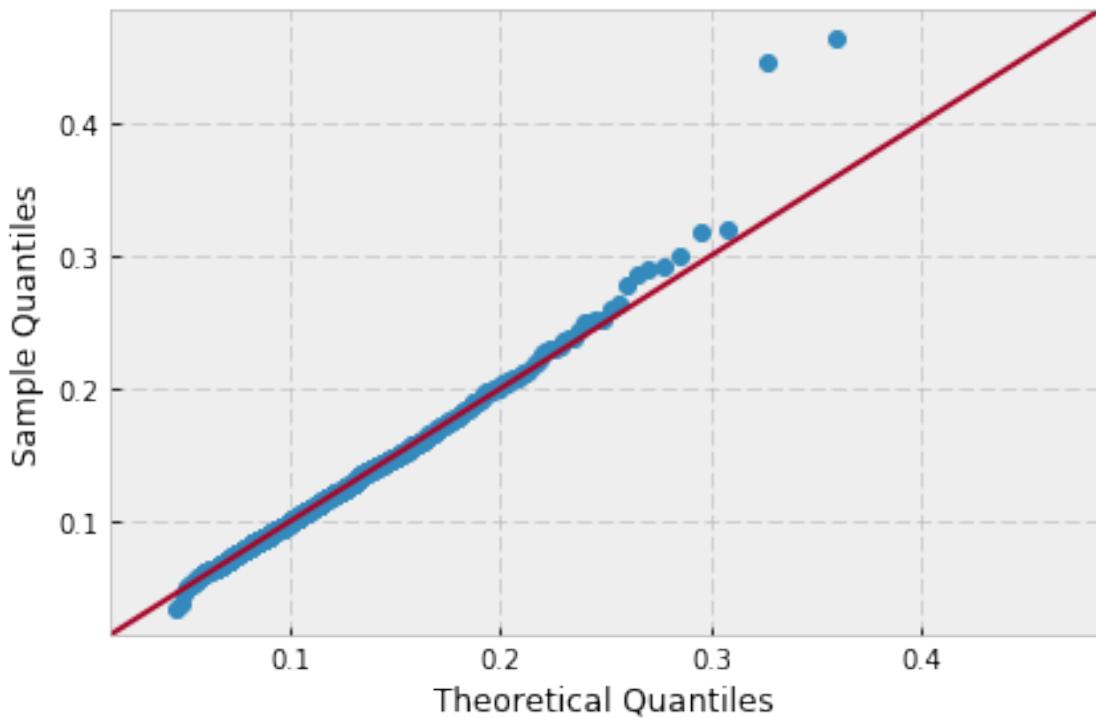


```
variable name: r
variable value: 2.5555555555555554
distribution: inverse-gamma(shape=(9.8290000708388732,), loc=0.005120271146880185, scale=0.137610
DescribeResult(nobs=1000, minmax=(0.011238532556233078, 0.066483765881497972)
               mean=0.020710387253990218, variance=3.1610074381752423e-05,
               skewness=1.770983106793816, kurtosis=6.694485036105739)
```

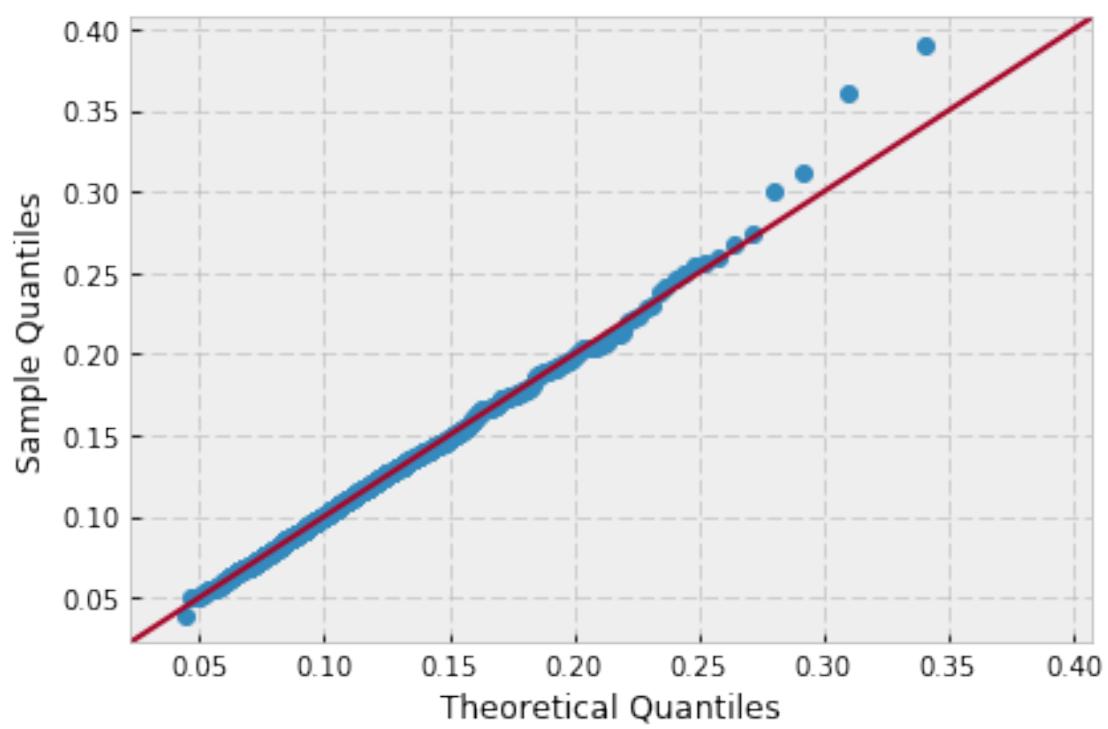
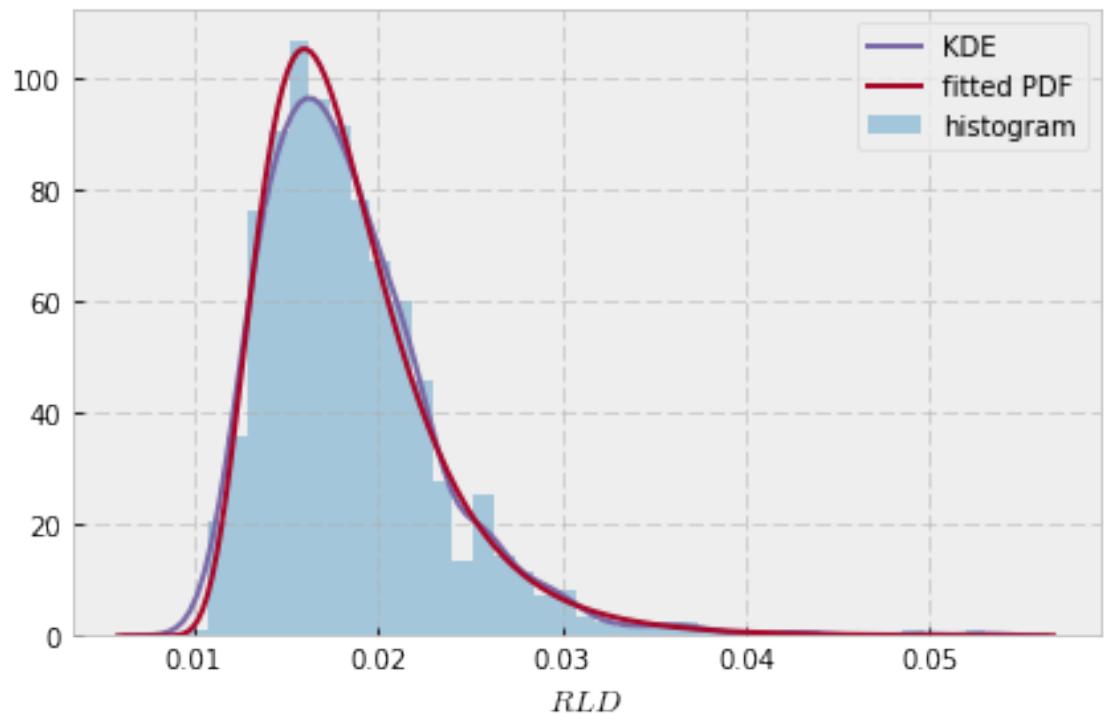


```
variable name: r
variable value: 2.777777777777777
distribution: inverse-gamma(shape=(9.6487670877842469,),loc=0.004554344839258315, scale=0.131897
DescribeResult(nobs=1000, minmax=(0.0091755718552603855, 0.06571422388555756)
               mean=0.01982289903813313, variance=3.2631691902018094e-05,
               skewness=2.088794971729858, kurtosis=9.148018004980427)
```

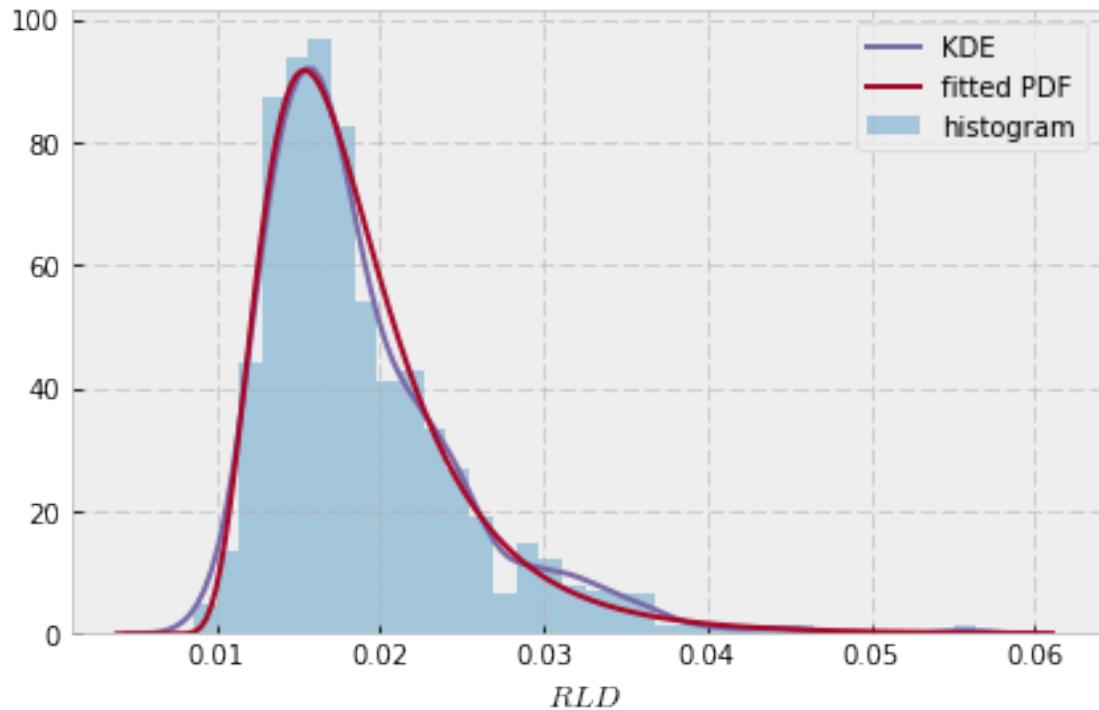


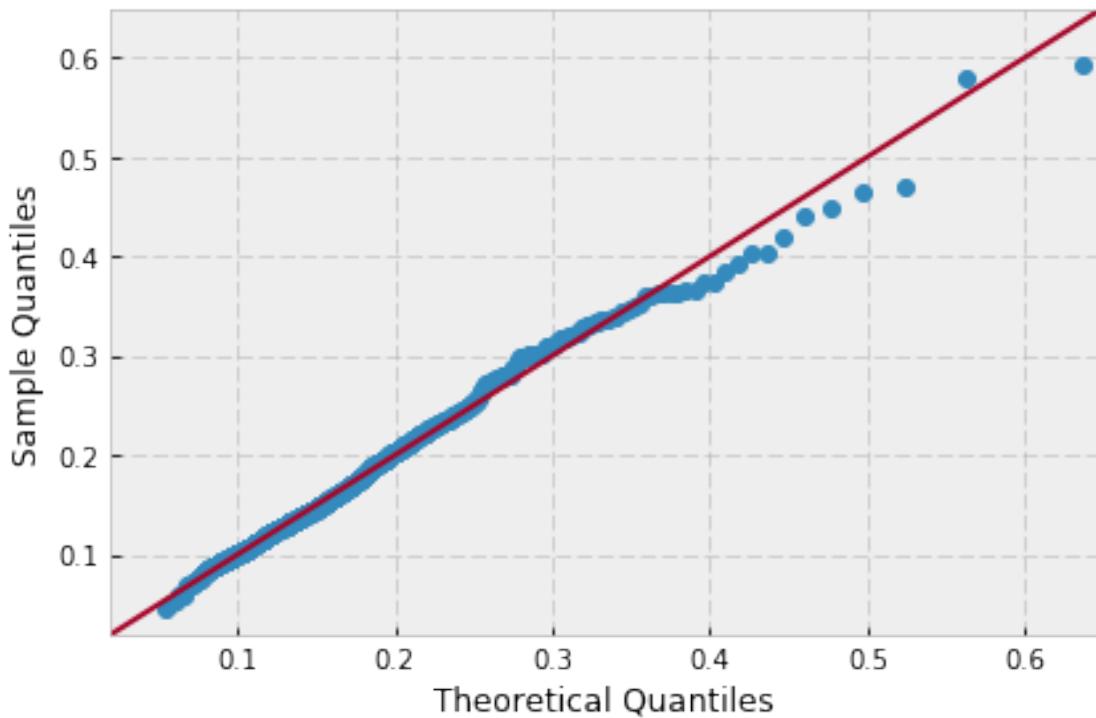


```
variable name: r
variable value: 3.0
distribution: inverse-gamma(shape=(9.9525182548618609,), loc=0.00468434564489378, scale=0.1238624
DescribeResult(nobs=1000, minmax=(0.0095848023187109499, 0.053002462534387229)
               mean=0.018519432536541931, variance=2.4141986425180587e-05,
               skewness=1.6571378598316135, kurtosis=5.542362808642213)
```

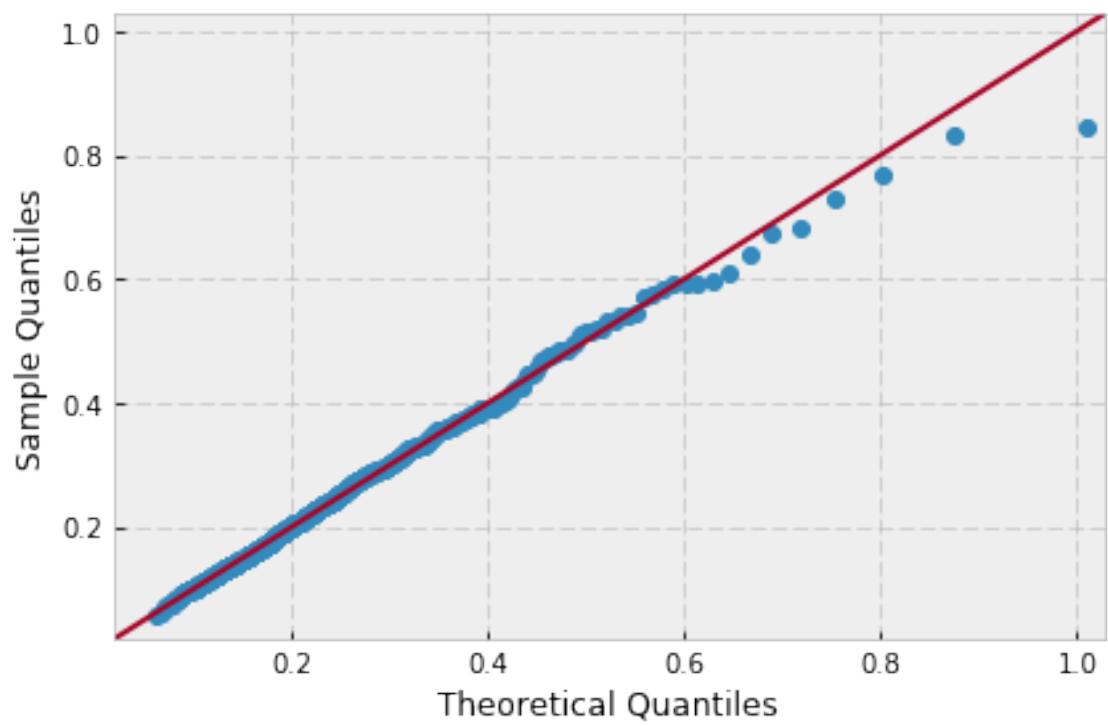
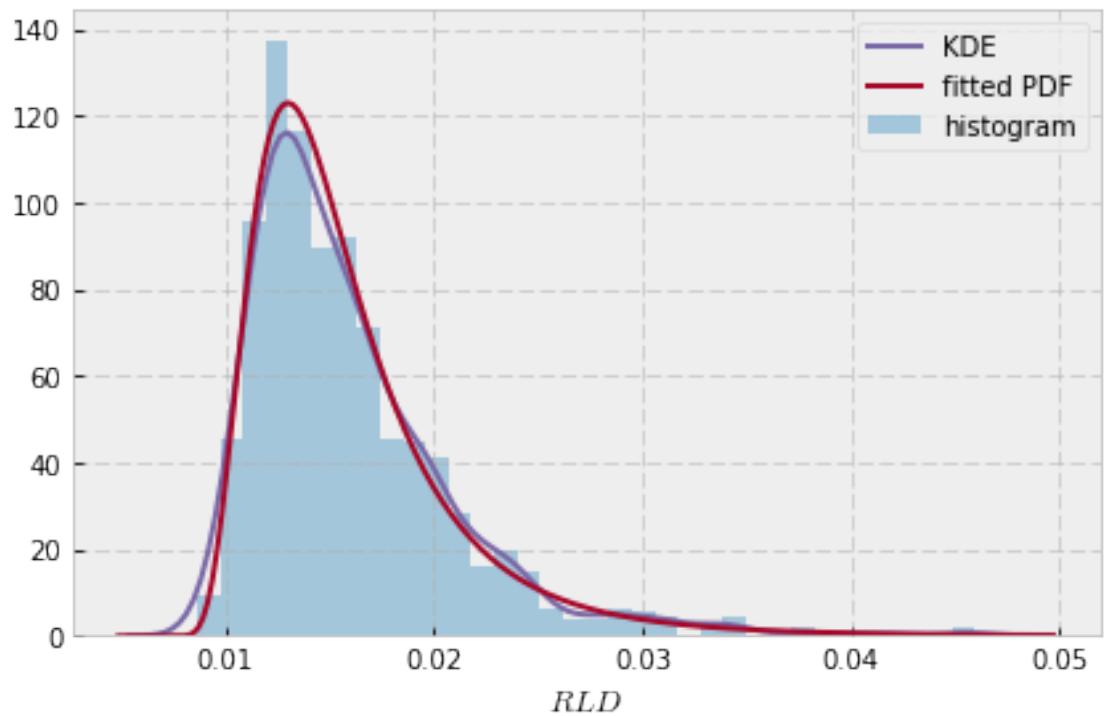


```
variable name: sigma
variable value: 0.01111111111111112
distribution: inverse-gamma(shape=(7.1199631756920958,),loc=0.004640431234000775, scale=0.087354
DescribeResult(nobs=1000, minmax=(0.0086014250182118116, 0.056426225140124374)
    mean=0.018907528974831053, variance=3.7937865890037595e-05,
    skewness=1.6056126679066198, kurtosis=3.8452192887515135)
```

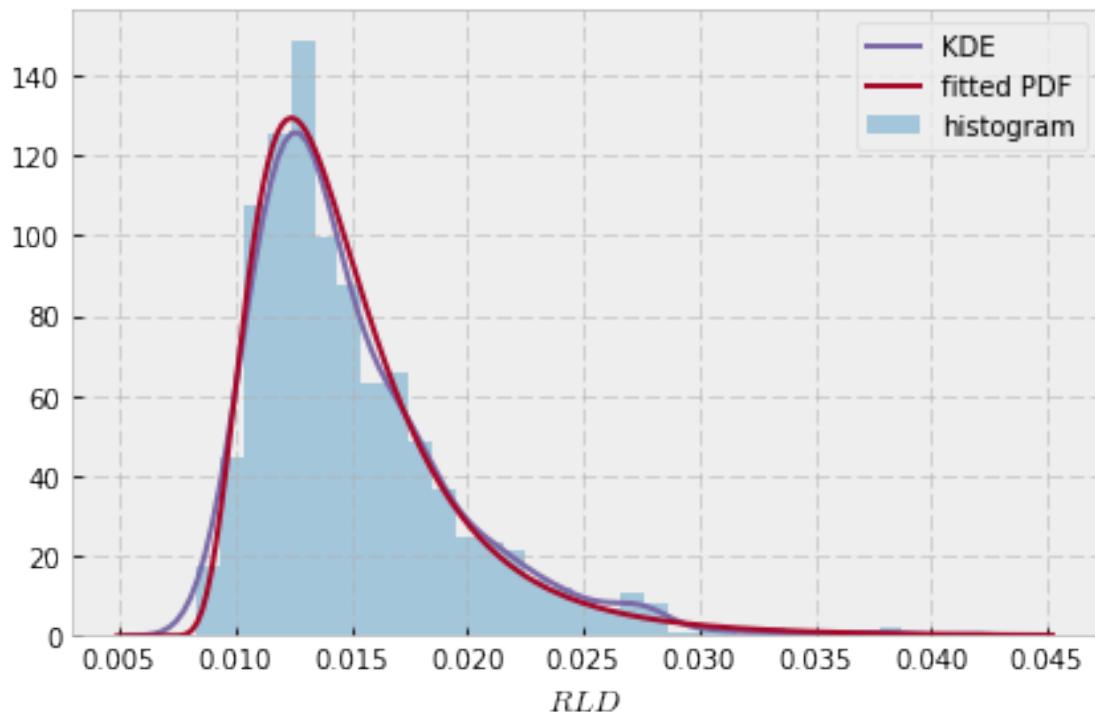


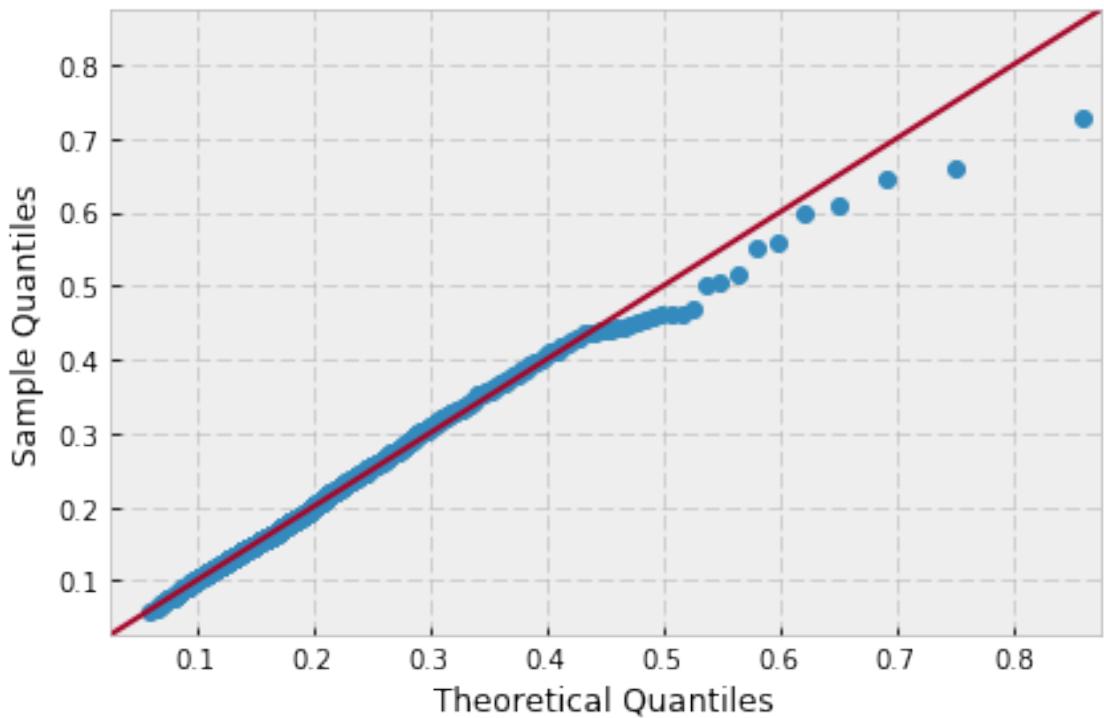


```
variable name: sigma
variable value: 0.02222222222222223
distribution: inverse-gamma(shape=(5.7001208498156721,), loc=0.0059358384822591394, scale=0.04726
DescribeResult(nobs=1000, minmax=(0.0086177404485816844, 0.045946738812753574)
               mean=0.015973393459432737, variance=2.4848825368540315e-05,
               skewness=1.8730956399782146, kurtosis=5.314236202310266)
```



```
variable name: sigma
variable value: 0.0333333333333333
distribution: inverse-gamma(shape=(6.1464510115453486,),loc=0.005409391865409506, scale=0.050094
DescribeResult(nobs=1000, minmax=(0.008336150464532158, 0.041811794786943288)
    mean=0.015122350075917217, variance=2.0517790375292094e-05,
    skewness=1.6293832082356412, kurtosis=3.8293680350769987)
```

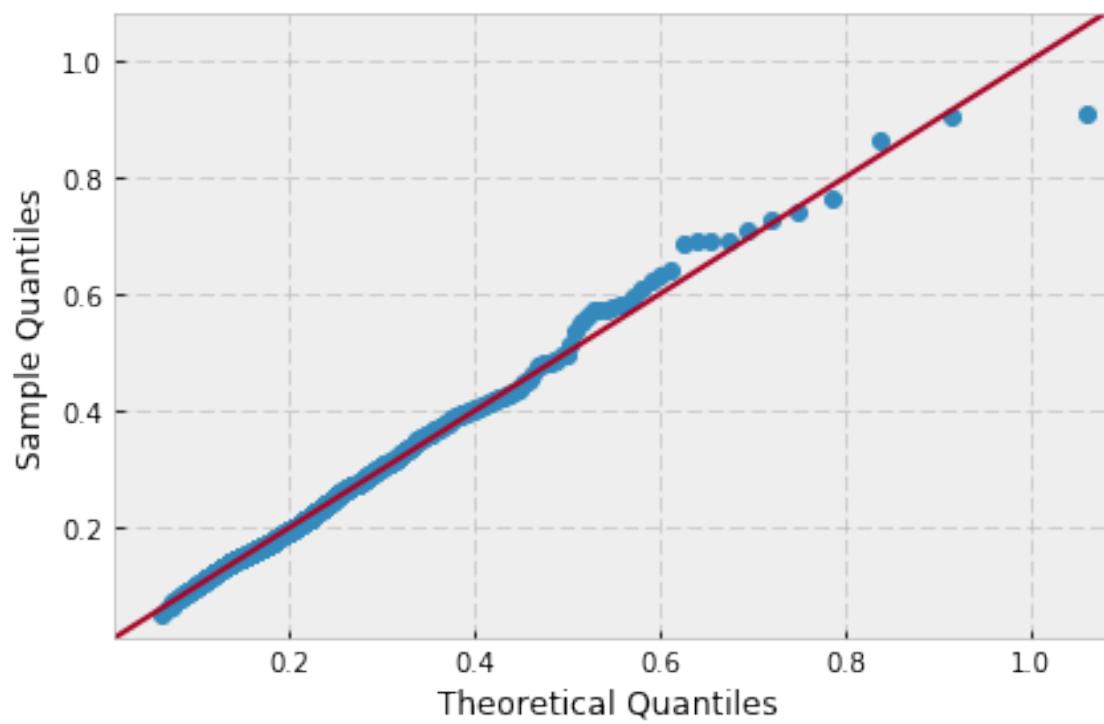
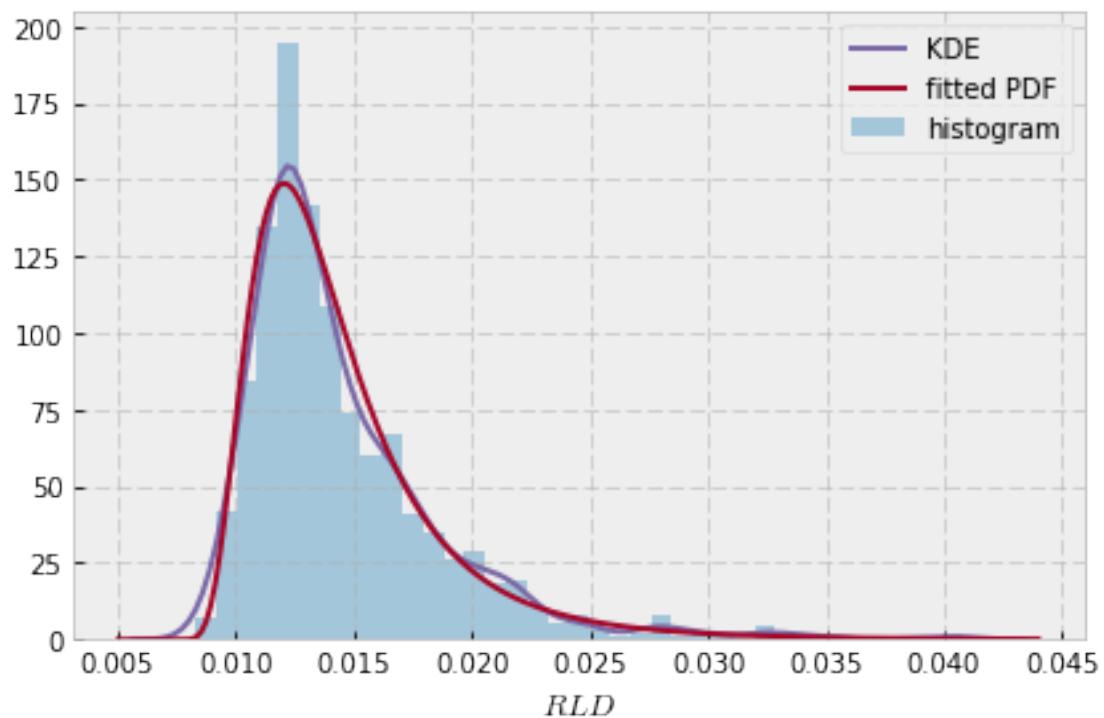




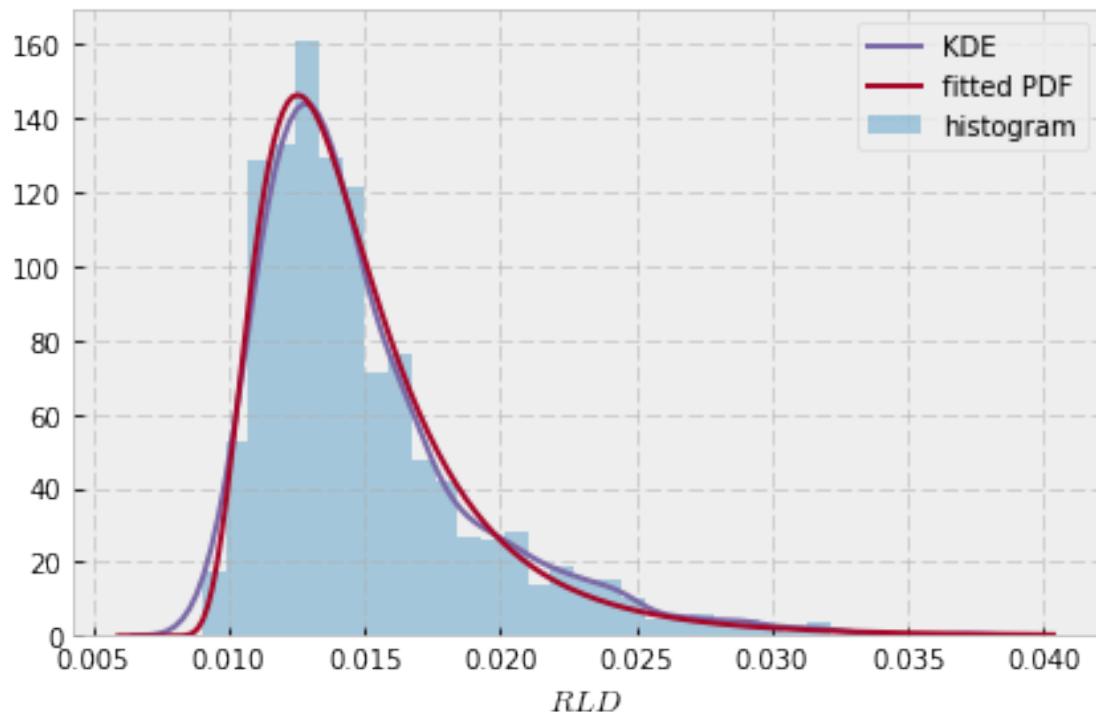
```

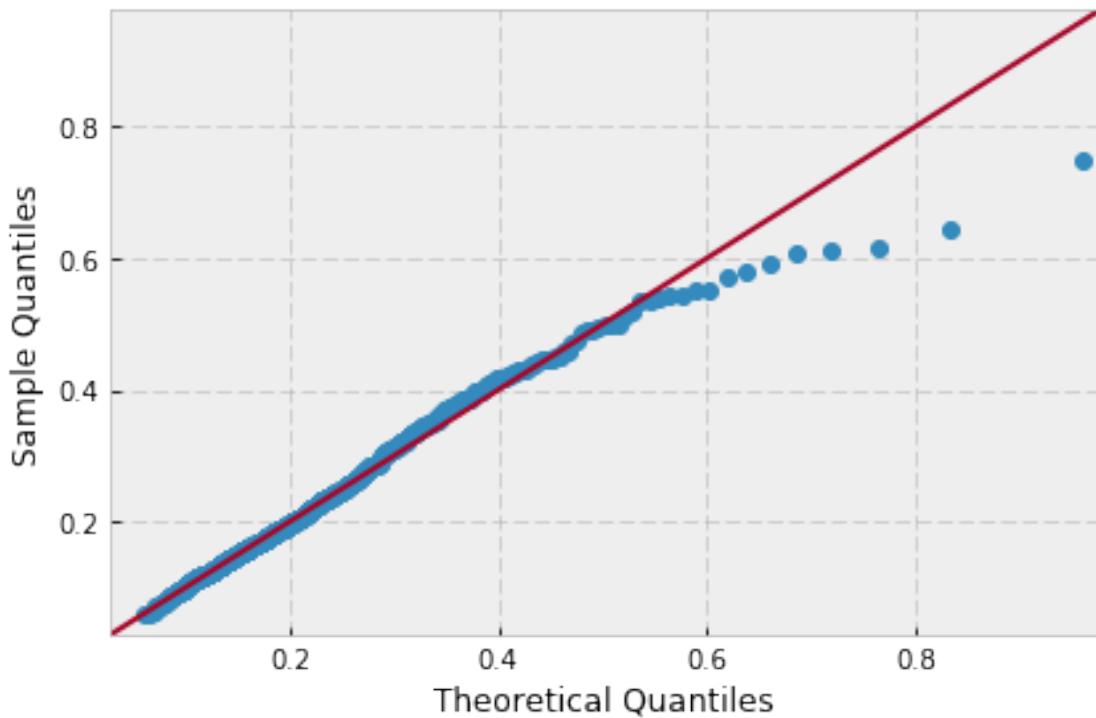
variable name: sigma
variable value: 0.04444444444444446
distribution: inverse-gamma(shape=(5.5721967664992835,), loc=0.006305161019191196, scale=0.037810
DescribeResult(nobs=1000, minmax=(0.0082815394857386759, 0.040740713128223083)
               mean=0.014576712213284425, variance=1.8466651551738495e-05,
               skewness=2.0874078980810395, kurtosis=6.437849658560738)

```

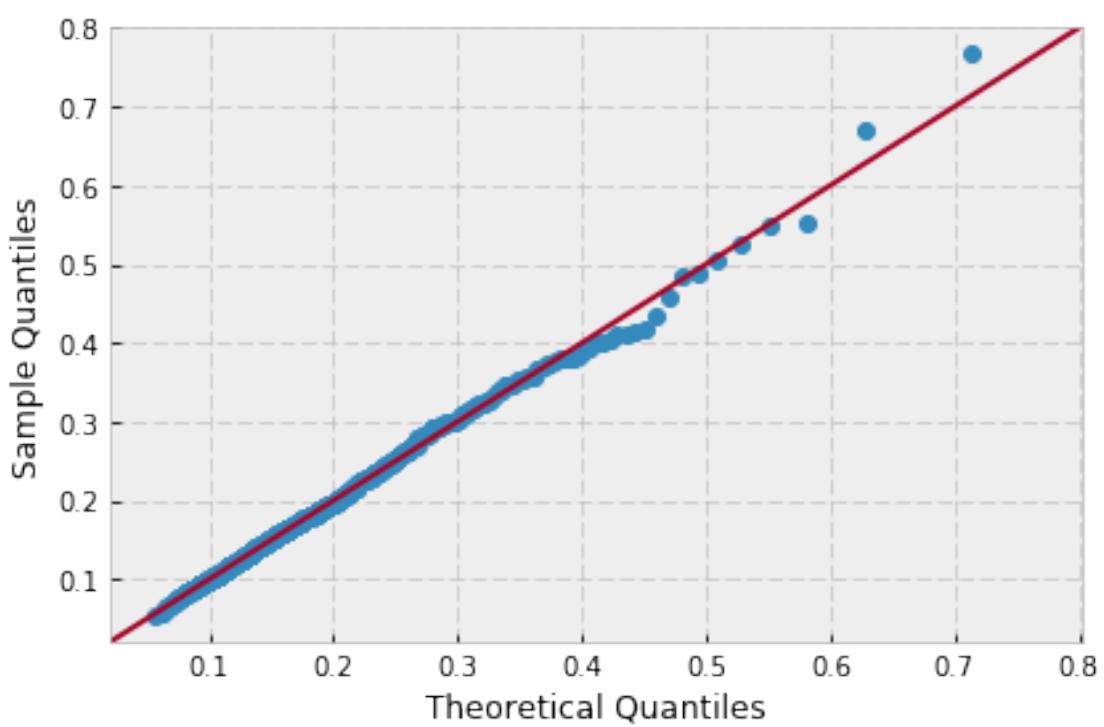
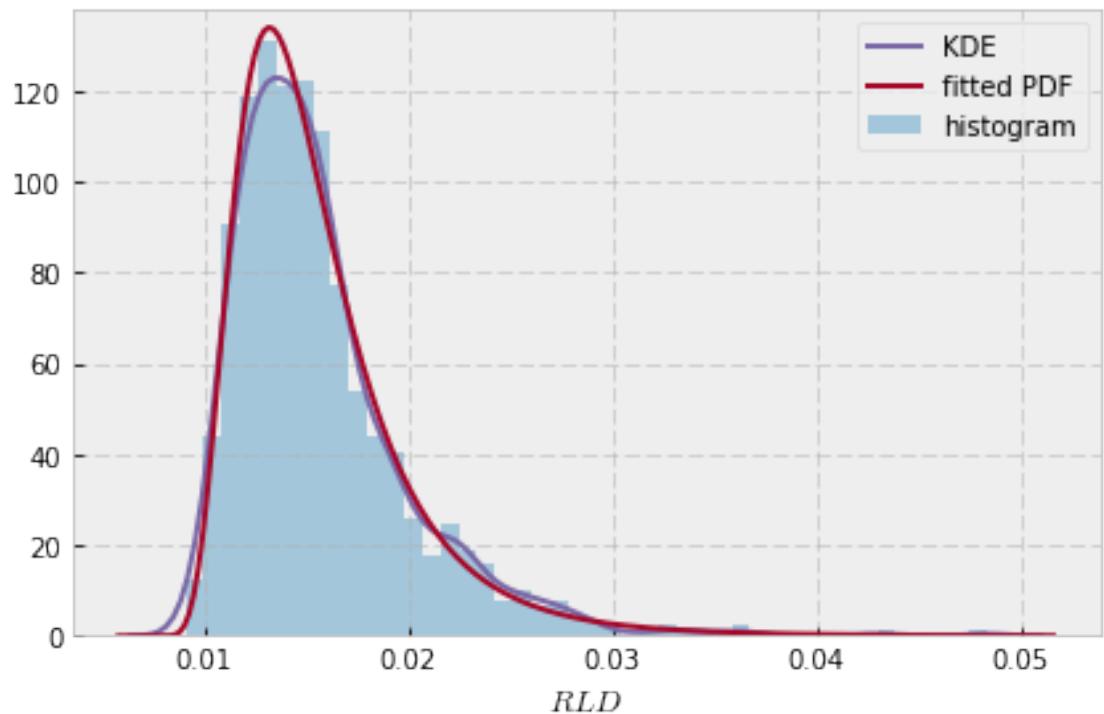


```
variable name: sigma
variable value: 0.0555555555555556
distribution: inverse-gamma(shape=(5.8341391366305055,),loc=0.006525400178599936, scale=0.041079
DescribeResult(nobs=1000, minmax=(0.0090226193139567104, 0.037264661335753145)
    mean=0.015004407328231042, variance=1.668606650383748e-05,
    skewness=1.5685854986187258, kurtosis=2.9569918023096395)
```

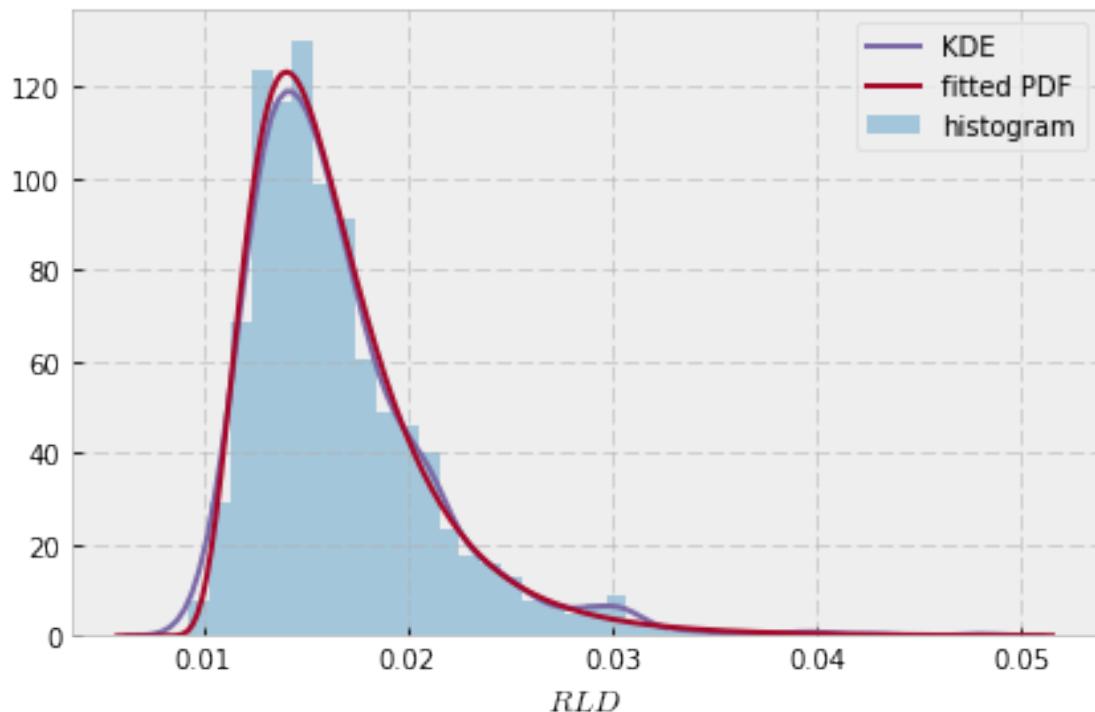


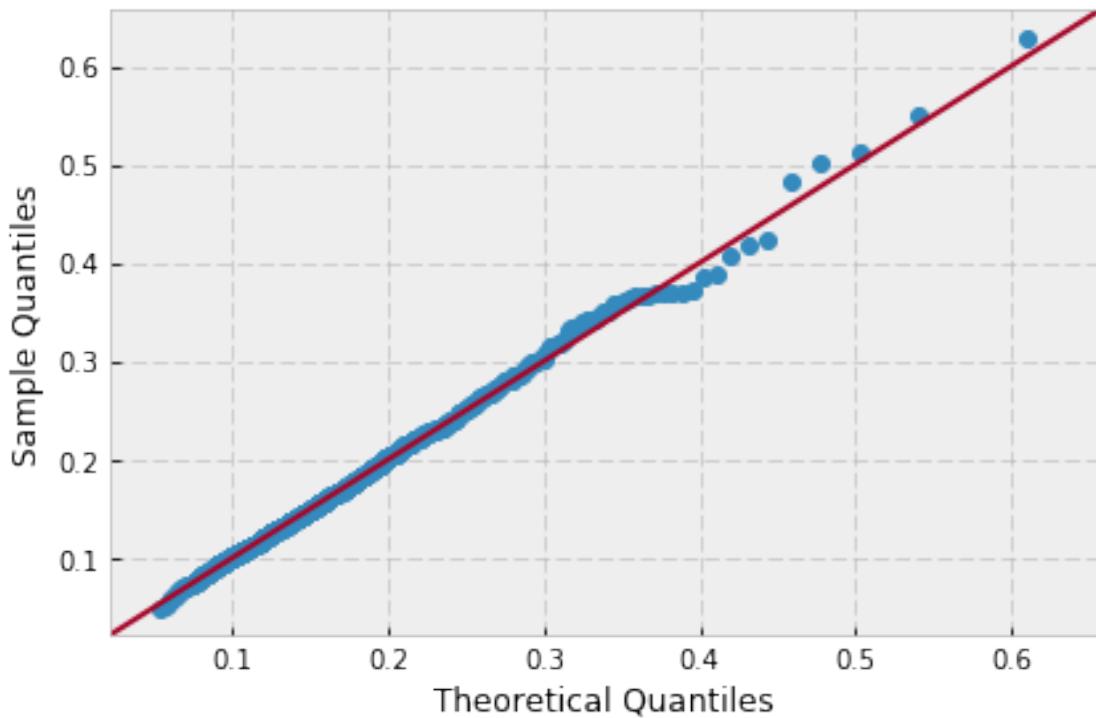


```
variable name: sigma
variable value: 0.06666666666666667
distribution: inverse-gamma(shape=(6.7276211835629169,), loc=0.006071364506935142, scale=0.055116
DescribeResult(nobs=1000, minmax=(0.0090801829343274305, 0.048321176240643006)
               mean=0.015687355922797446, variance=1.8817899667853744e-05,
               skewness=1.913245373642265, kurtosis=6.600784310473152)
```

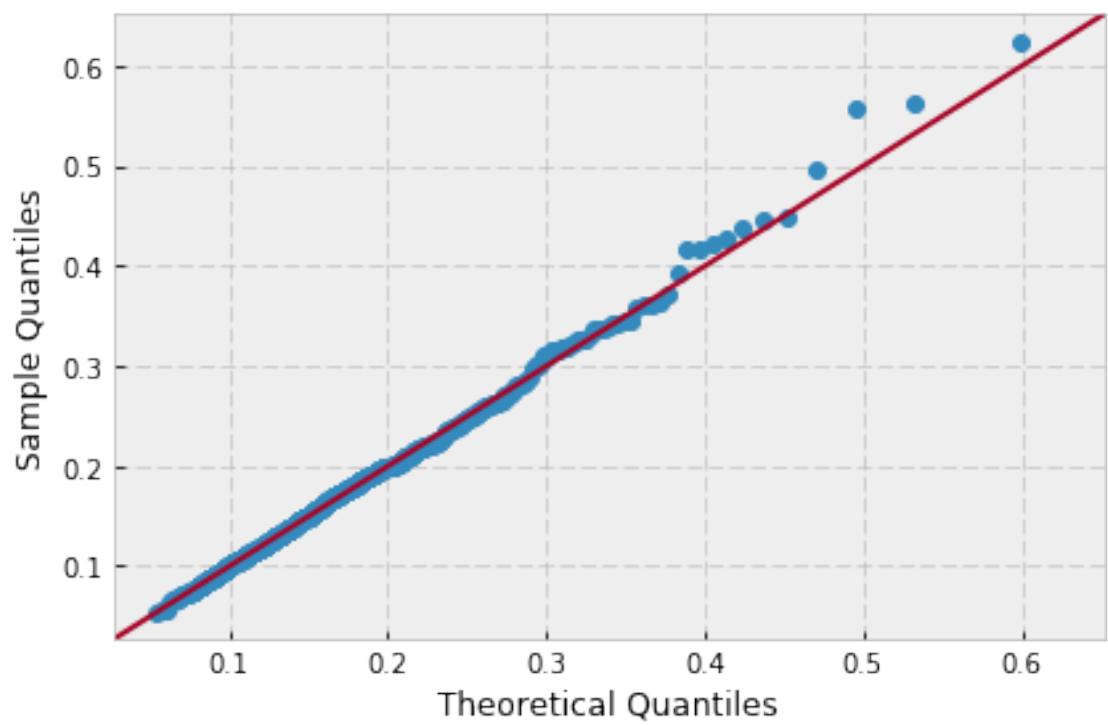
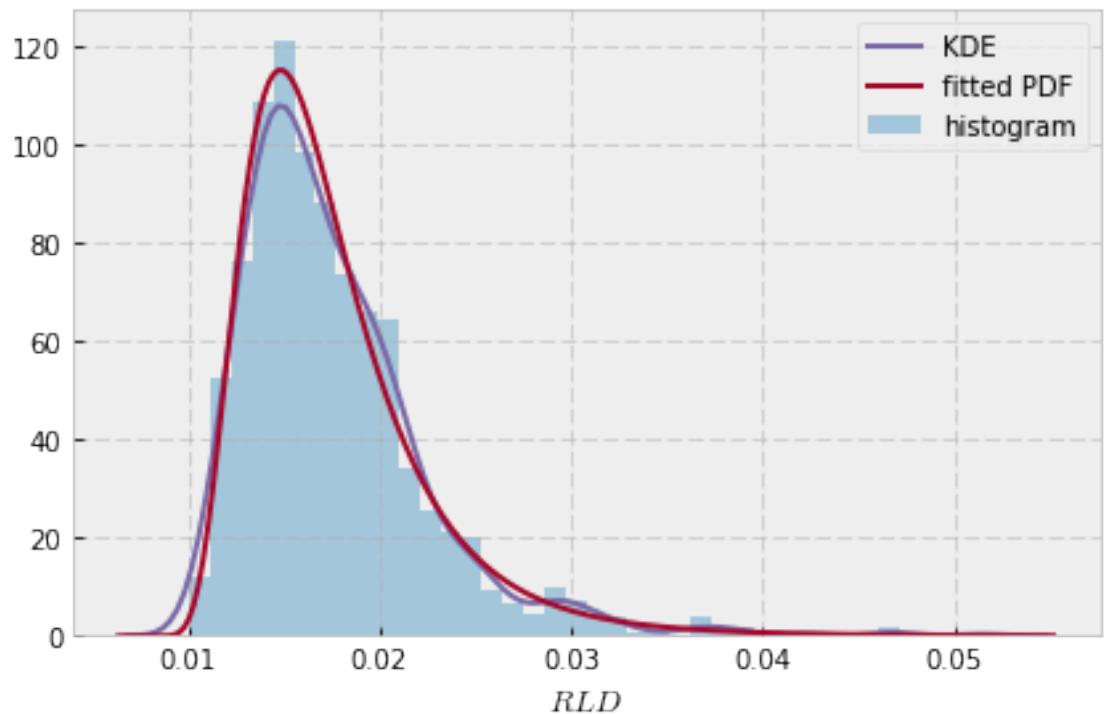


```
variable name: sigma
variable value: 0.0777777777777778
distribution: inverse-gamma(shape=(7.2738406784464669,),loc=0.005945260602309613, scale=0.067078
DescribeResult(nobs=1000, minmax=(0.009251217312919939, 0.048074074661452859)
    mean=0.016636177514124099, variance=2.1270929905547223e-05,
    skewness=1.8032411374778397, kurtosis=5.401658994636554)
```

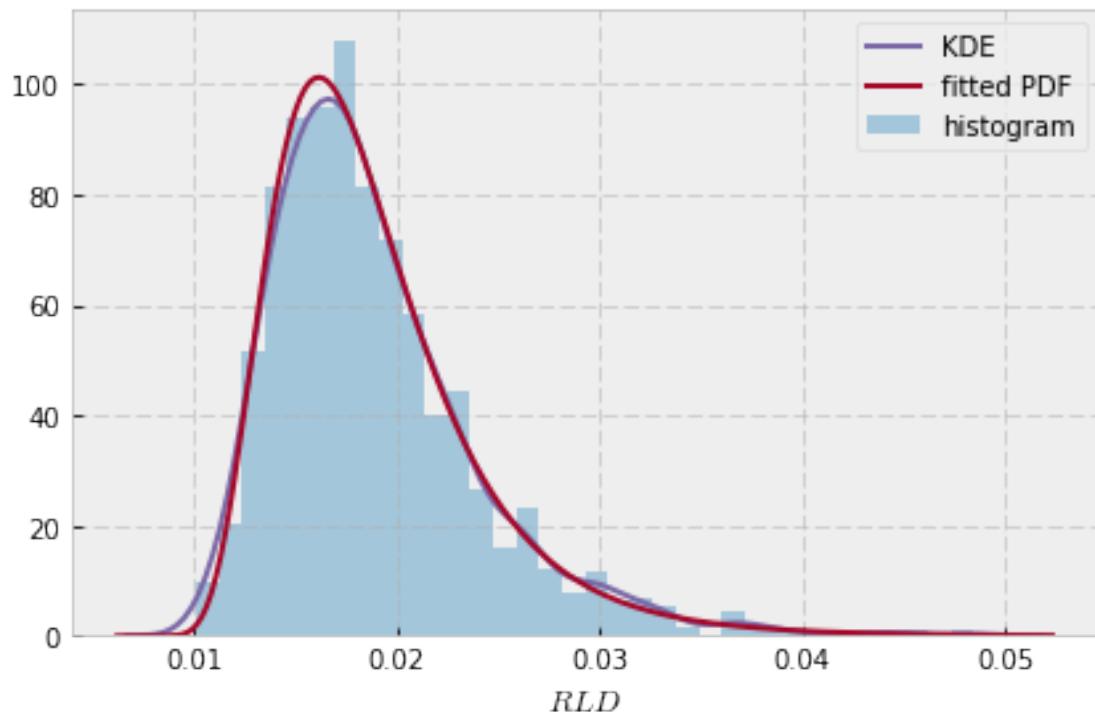


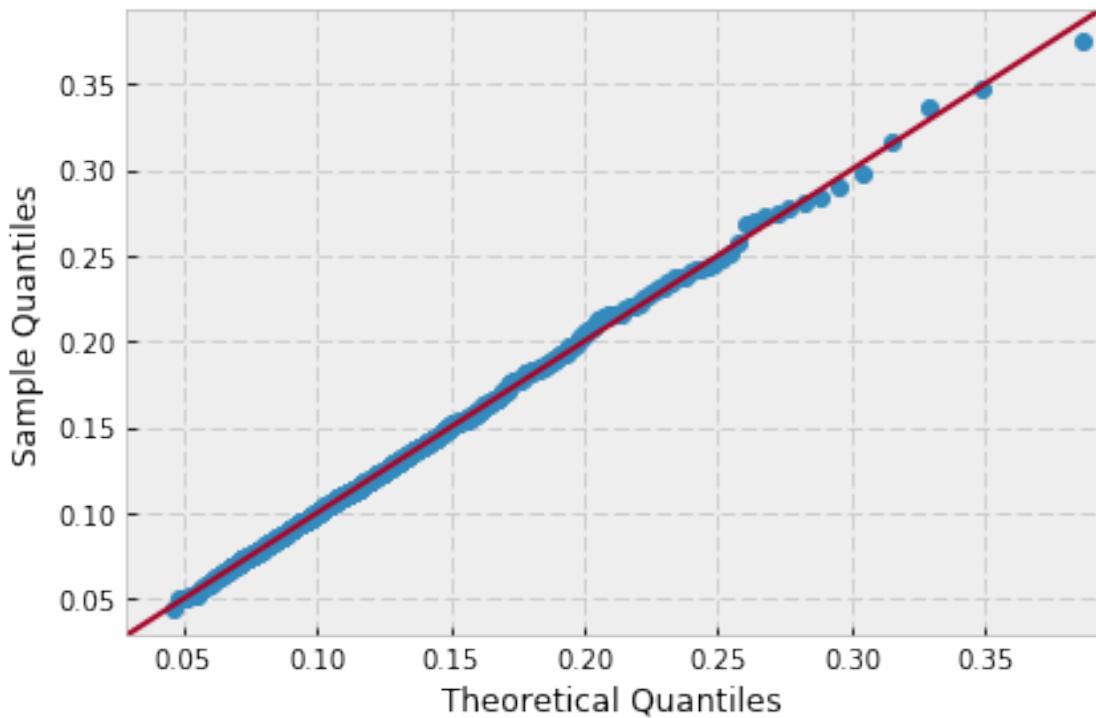


```
variable name: sigma
variable value: 0.0888888888888889
distribution: inverse-gamma(shape=(7.3415975778503899,), loc=0.00610120625500524, scale=0.0727626
DescribeResult(nobs=1000, minmax=(0.010081540555912752, 0.051465018191489298)
               mean=0.017572086627795074, variance=2.4307597949119932e-05,
               skewness=1.9207202338609026, kurtosis=6.499432950658321)
```

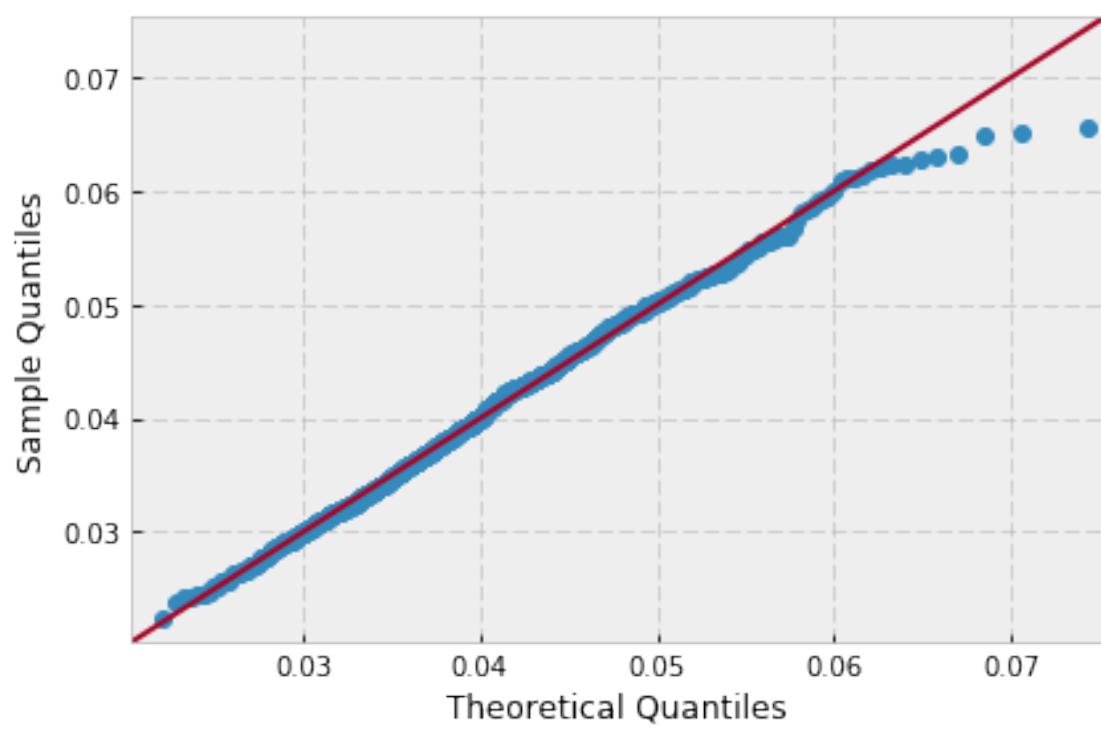
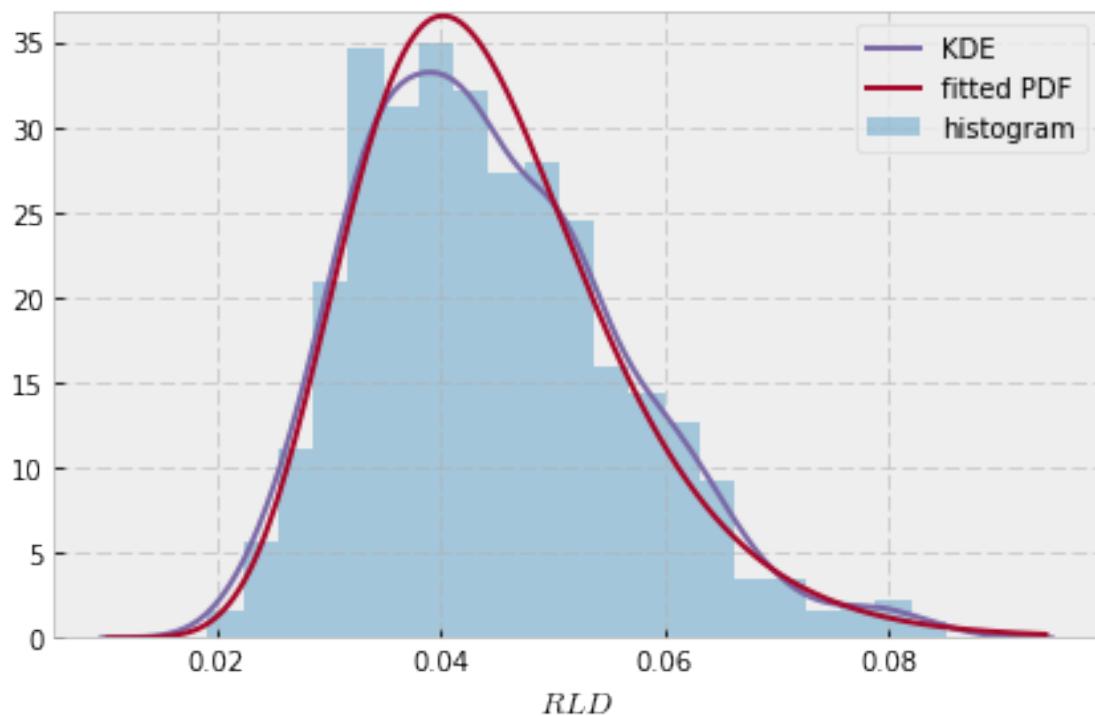


```
variable name: sigma
variable value: 0.1
distribution: inverse-gamma(shape=(9.2768513487201822,),loc=0.004846906446694442, scale=0.116198
DescribeResult(nobs=1000, minmax=(0.010085246382423779, 0.048467103472549171)
               mean=0.018882291622132556, variance=2.6515849178750486e-05,
               skewness=1.466064781182595, kurtosis=3.376317114699087)
```

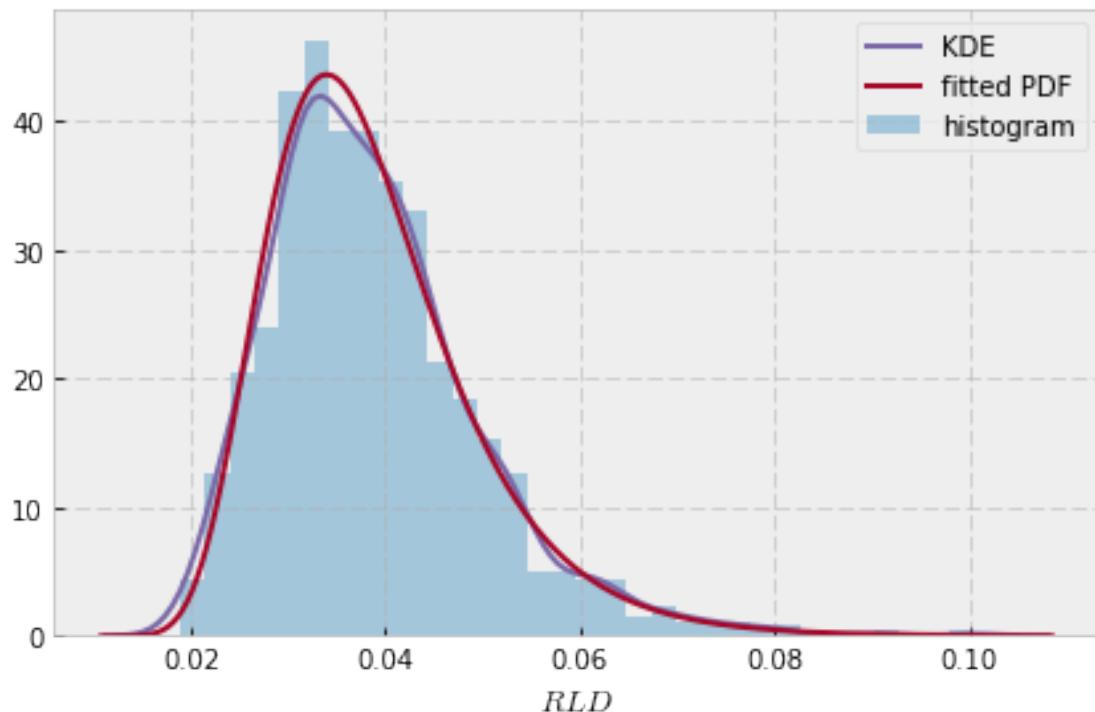


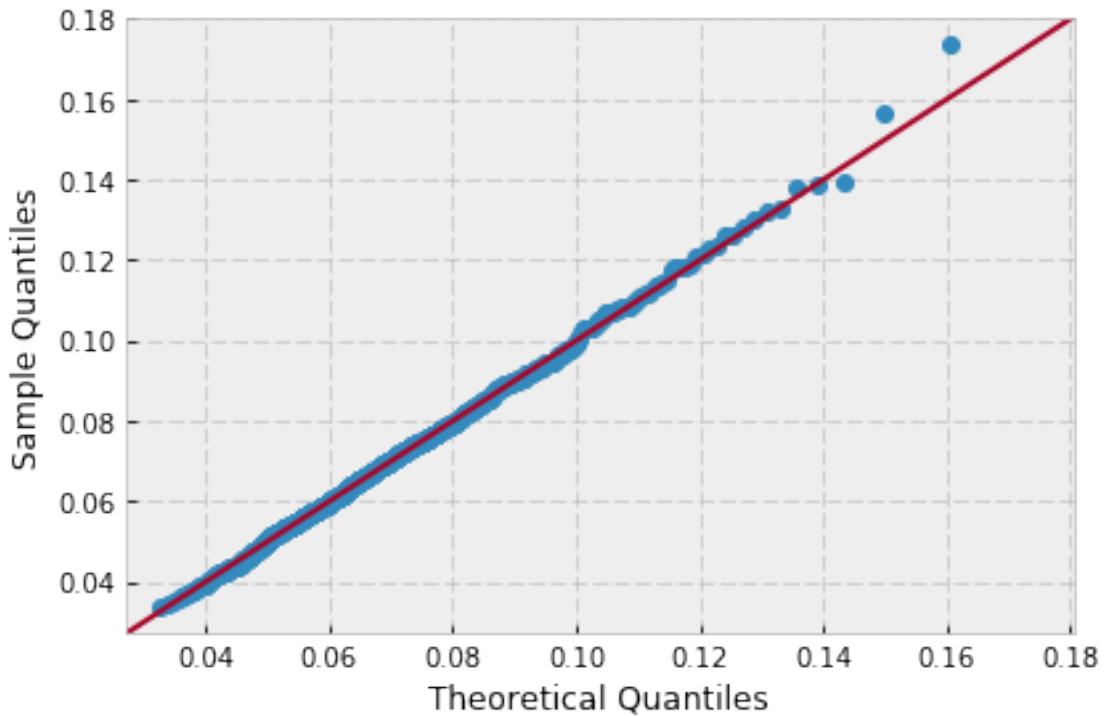


```
variable name: theta
variable value: 0.6981317007977318
distribution: inverse-gamma(shape=(26.616527180443711,), loc=-0.014913048900563487, scale=1.52314
DescribeResult(nobs=1000, minmax=(0.019177194841051681, 0.08518105115112011)
               mean=0.044537701375638505, variance=0.00014025640722225487,
               skewness=0.6221685561422626, kurtosis=0.1859801204788134)
```

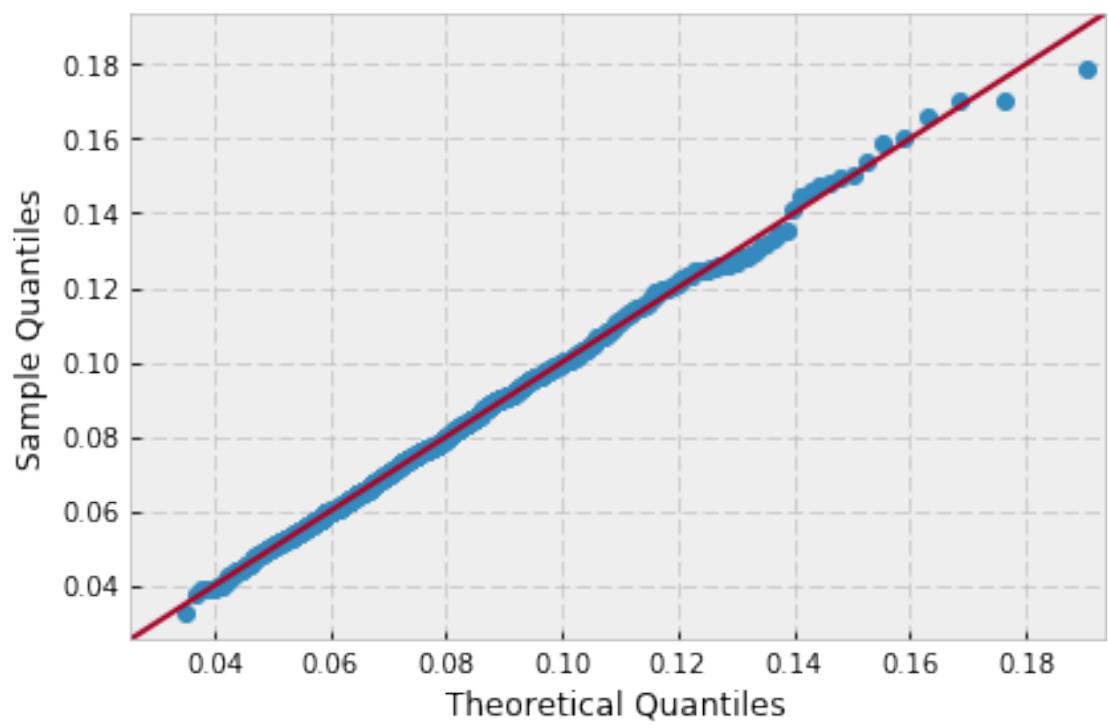
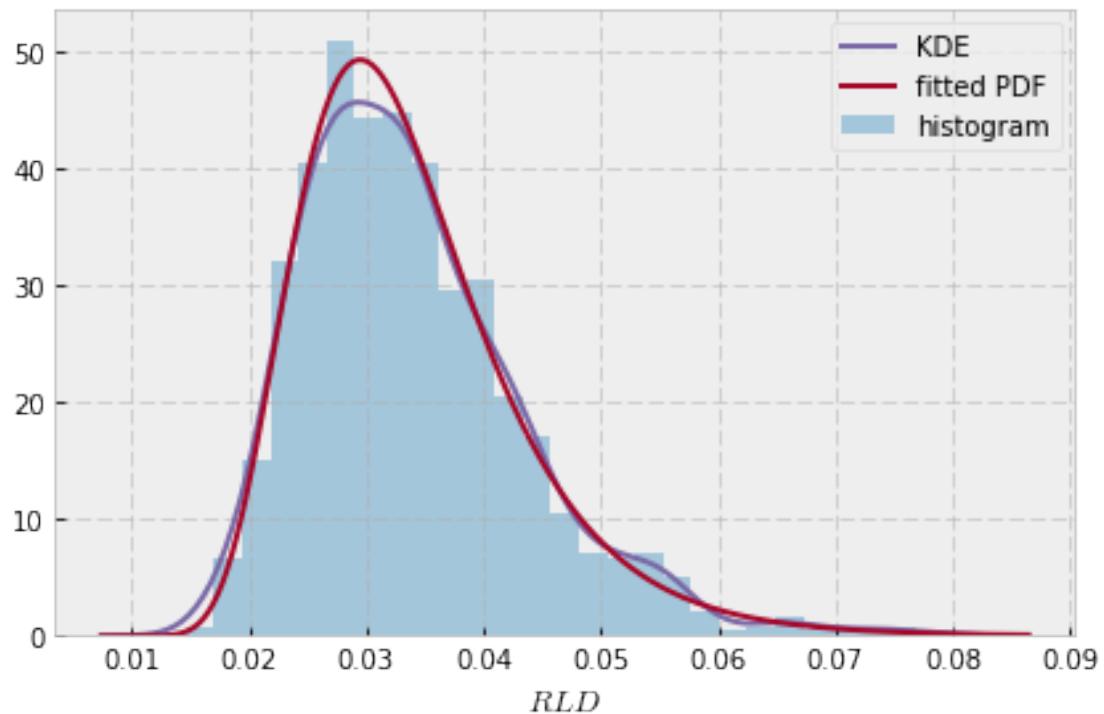


```
variable name: theta
variable value: 0.7853981633974483
distribution: inverse-gamma(shape=(15.705561784644878,),loc=-0.0009713023155058773, scale=0.5837
DescribeResult(nobs=1000, minmax=(0.018888408209669898, 0.10035799078646927)
    mean=0.038722692574654168, variance=0.00011446324636621822,
    skewness=1.1241662865988848, kurtosis=2.315186901963072)
```

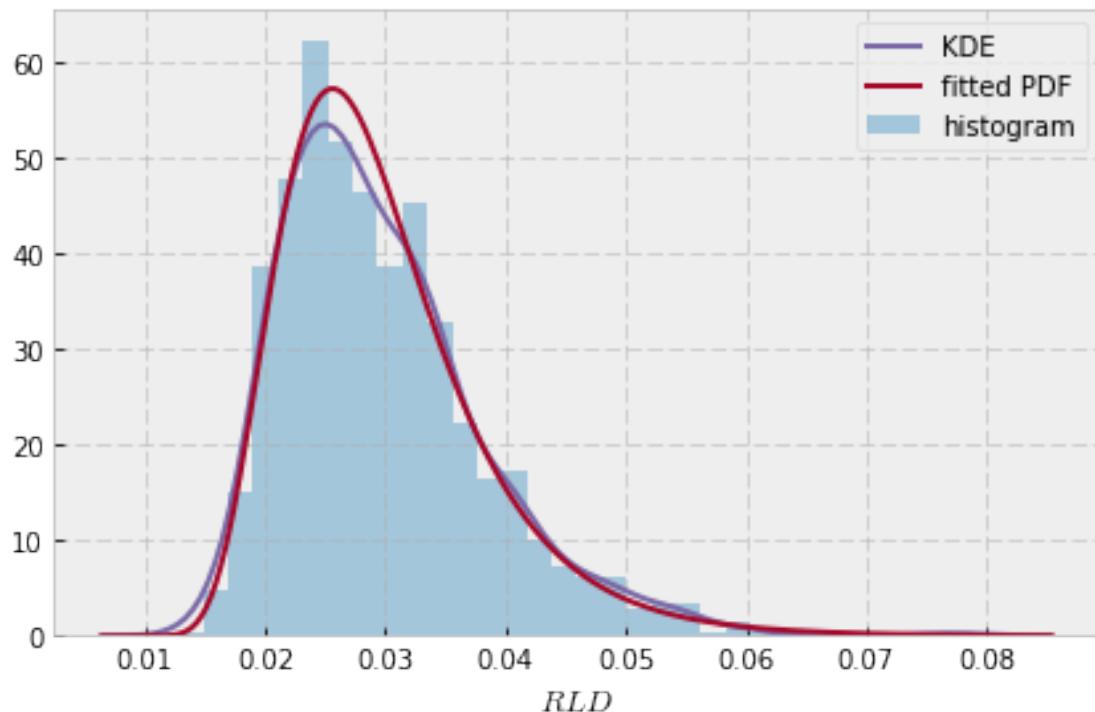


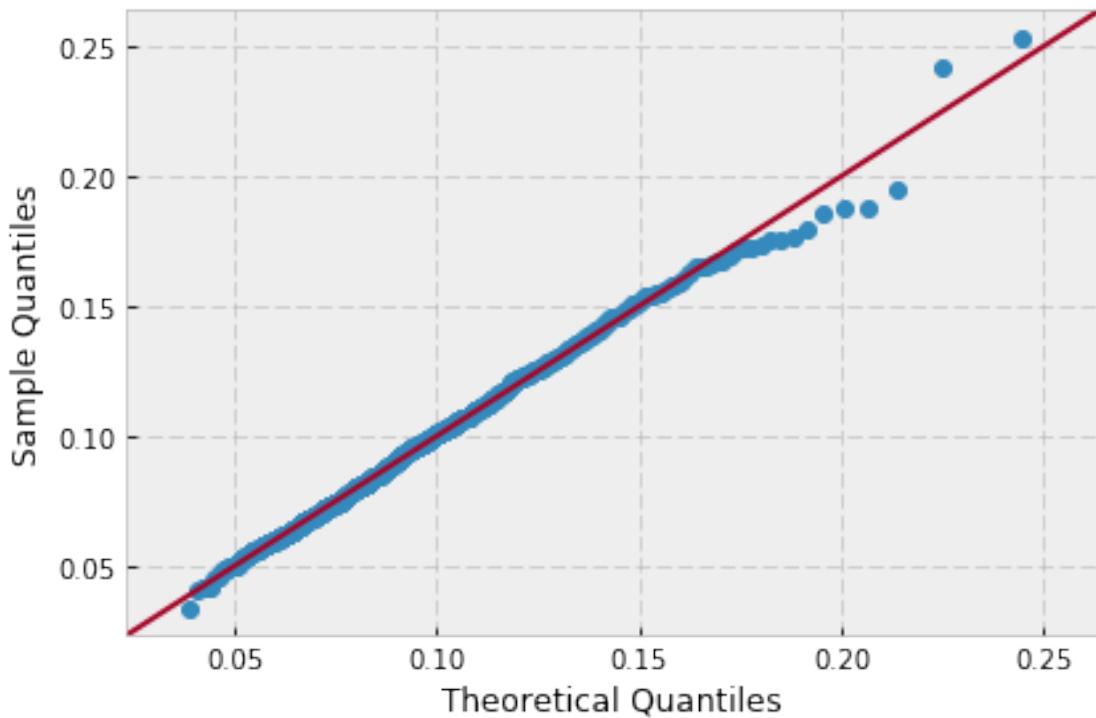


```
variable name: theta
variable value: 0.8726646259971648
distribution: inverse-gamma(shape=(14.094234353110028,),loc=0.00028978003010023655, scale=0.4404
DescribeResult(nobs=1000, minmax=(0.014657252324051925, 0.079208907324887959)
               mean=0.033922299081465108, variance=9.1933317316299541e-05,
               skewness=1.0853268597755297, kurtosis=1.7610050568804514)
```

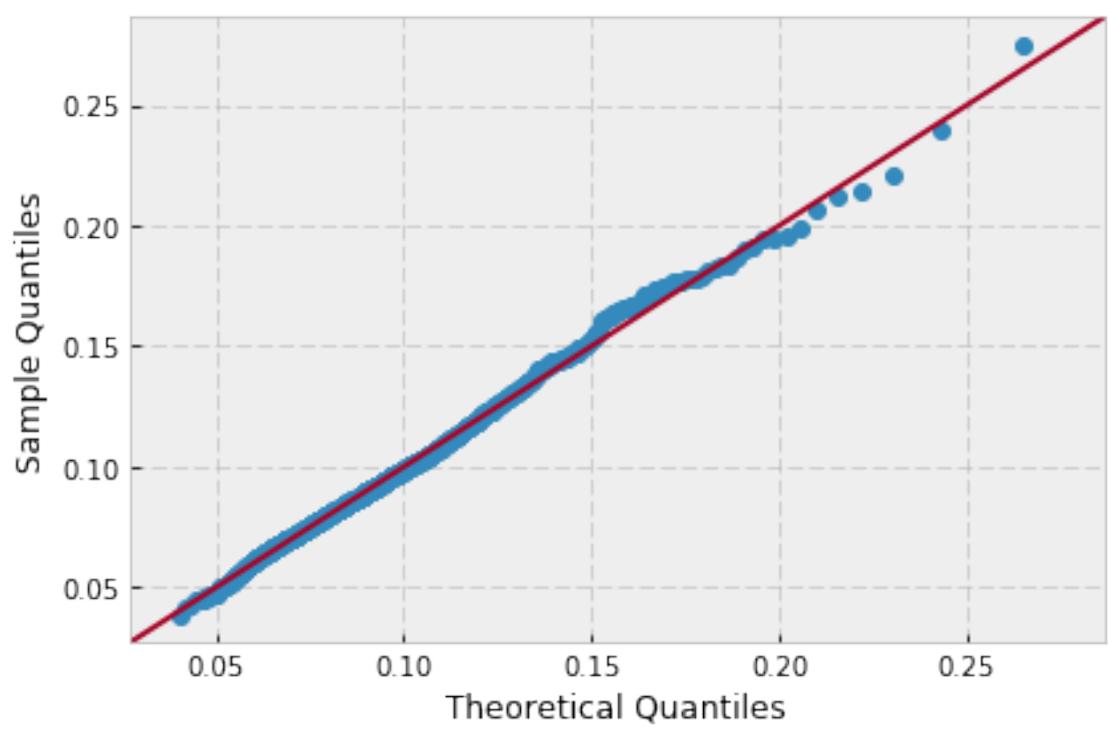
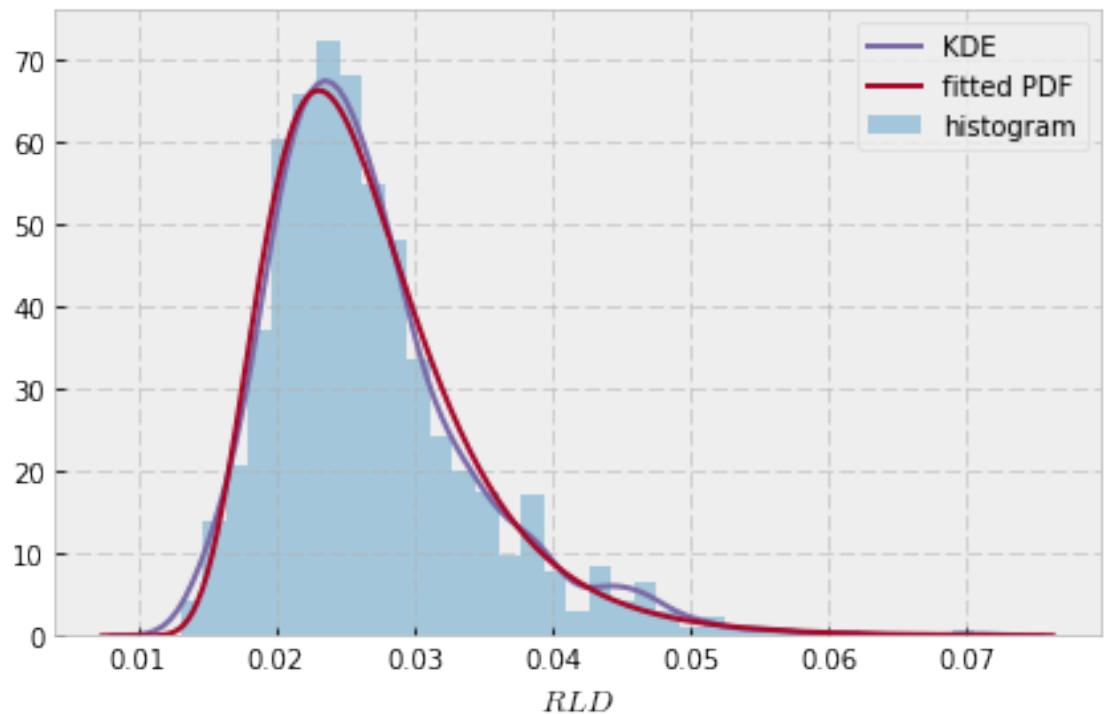


```
variable name: theta
variable value: 0.9599310885968813
distribution: inverse-gamma(shape=(12.076003758497453,),loc=0.002496062127979575, scale=0.302278
DescribeResult(nobs=1000, minmax=(0.012789822317704296, 0.078979441517859031)
               mean=0.029778397109200504, variance=7.1944391755180854e-05,
               skewness=1.1779086283003184, kurtosis=2.323093204666435)
```

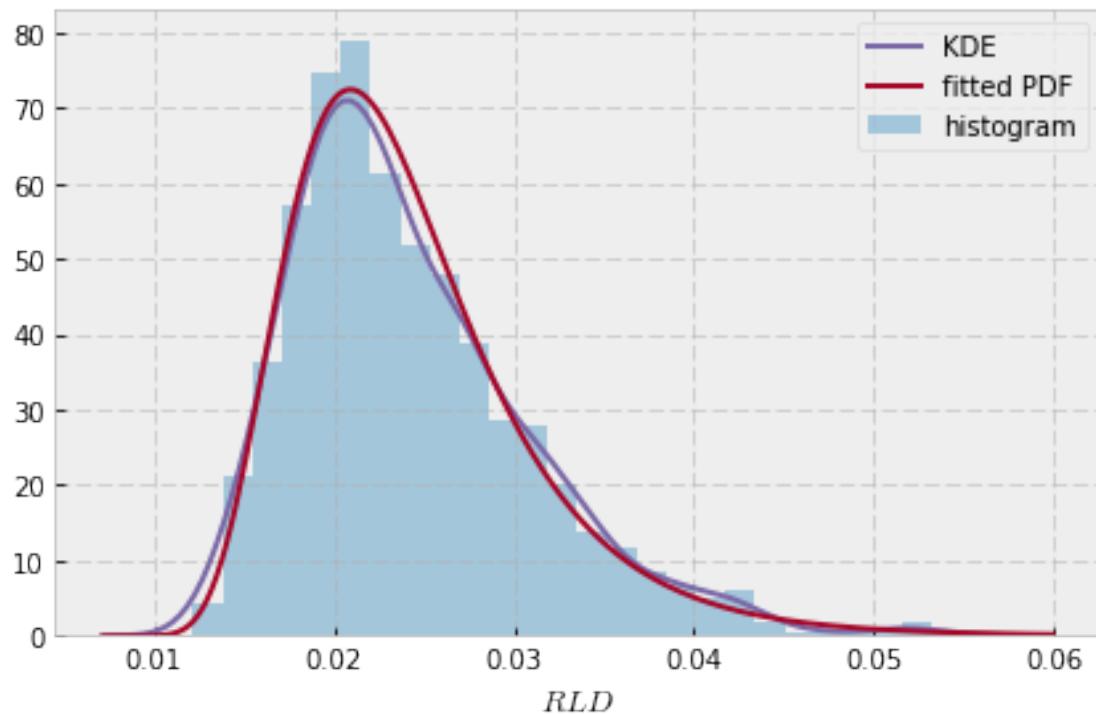


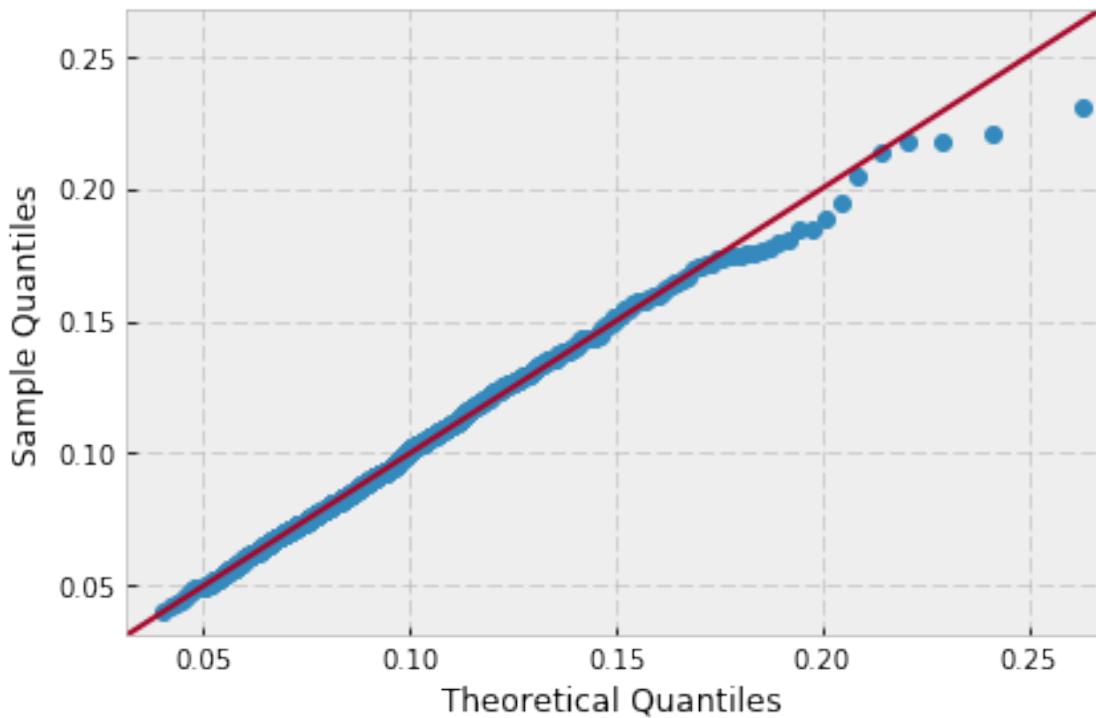


```
variable name: theta
variable value: 1.0471975511965979
distribution: inverse-gamma(shape=(11.520294395154988,),loc=0.0036133065675875344, scale=0.24370
DescribeResult(nobs=1000, minmax=(0.013062704557660215, 0.070564612974755997)
               mean=0.026780451295168983, variance=5.6275731561339643e-05,
               skewness=1.3132294893620362, kurtosis=2.599089897532158)
```

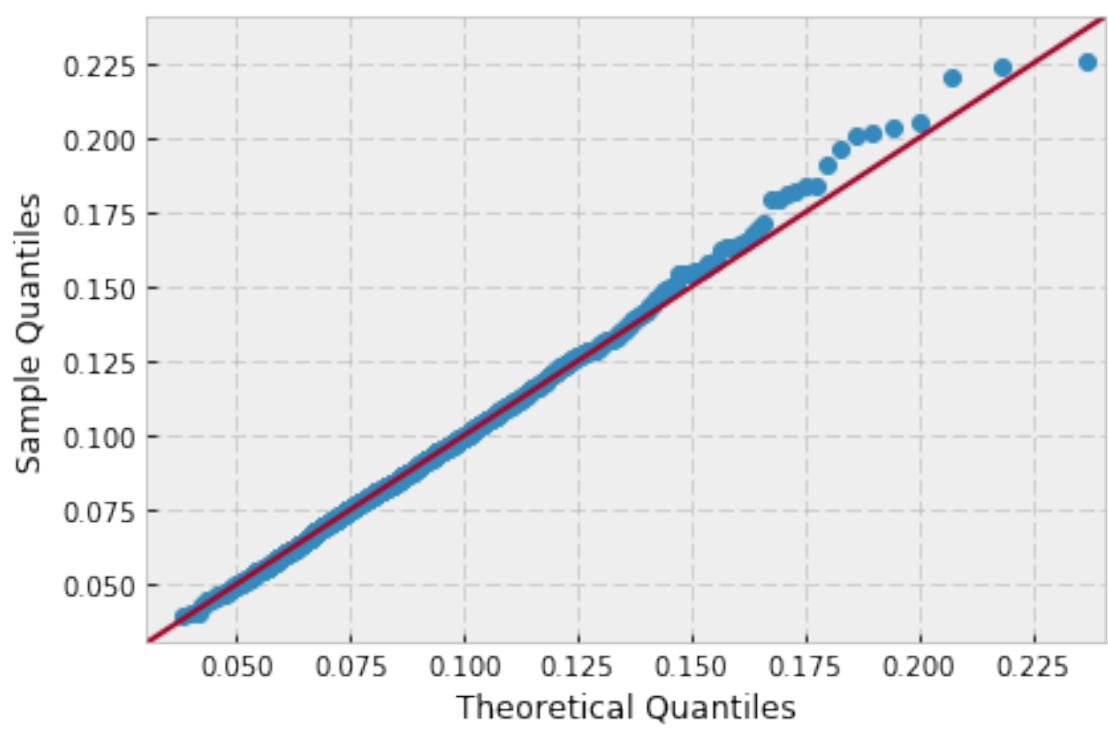
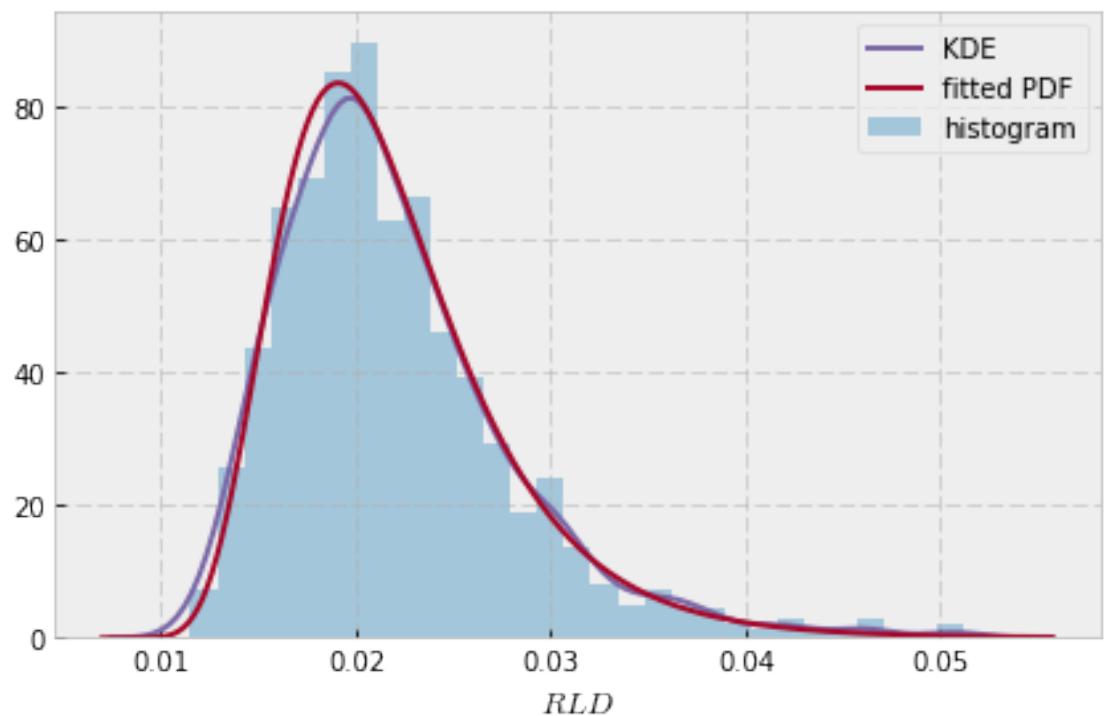


```
variable name: theta
variable value: 1.1344640137963142
distribution: inverse-gamma(shape=(11.571046003777646,),loc=0.0030679062939218565, scale=0.22422
DescribeResult(nobs=1000, minmax=(0.012159194789600265, 0.054889032965544975)
    mean=0.024266761295634487, variance=4.487049745365464e-05,
    skewness=1.0773354244410906, kurtosis=1.477081360664564)
```

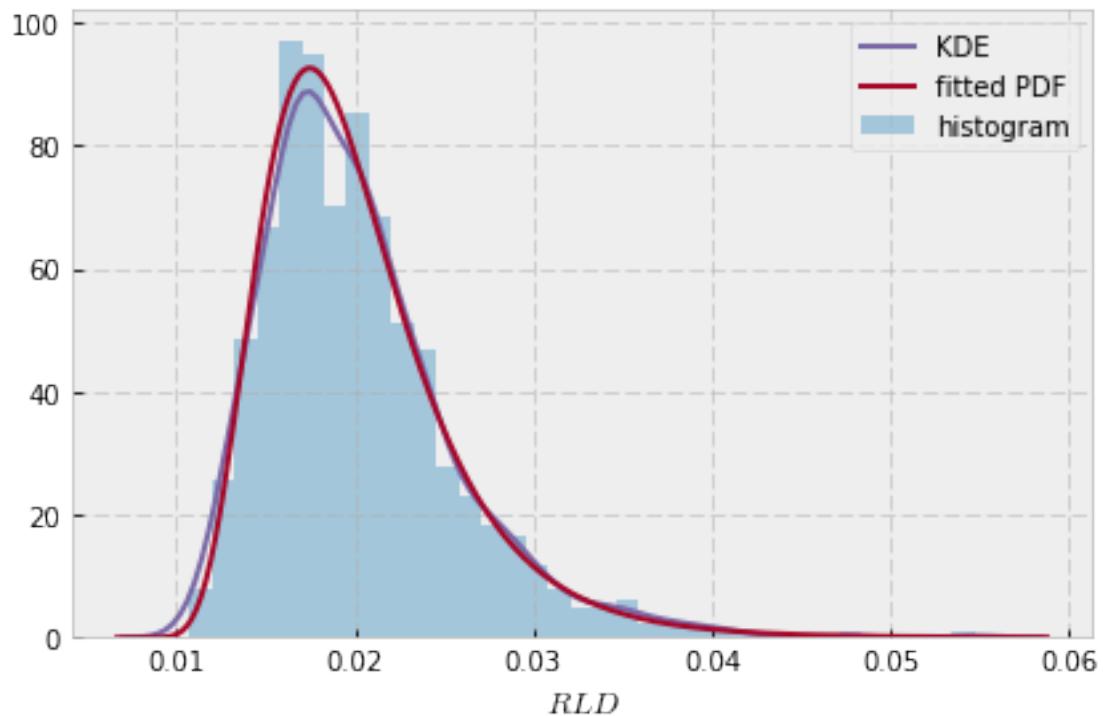


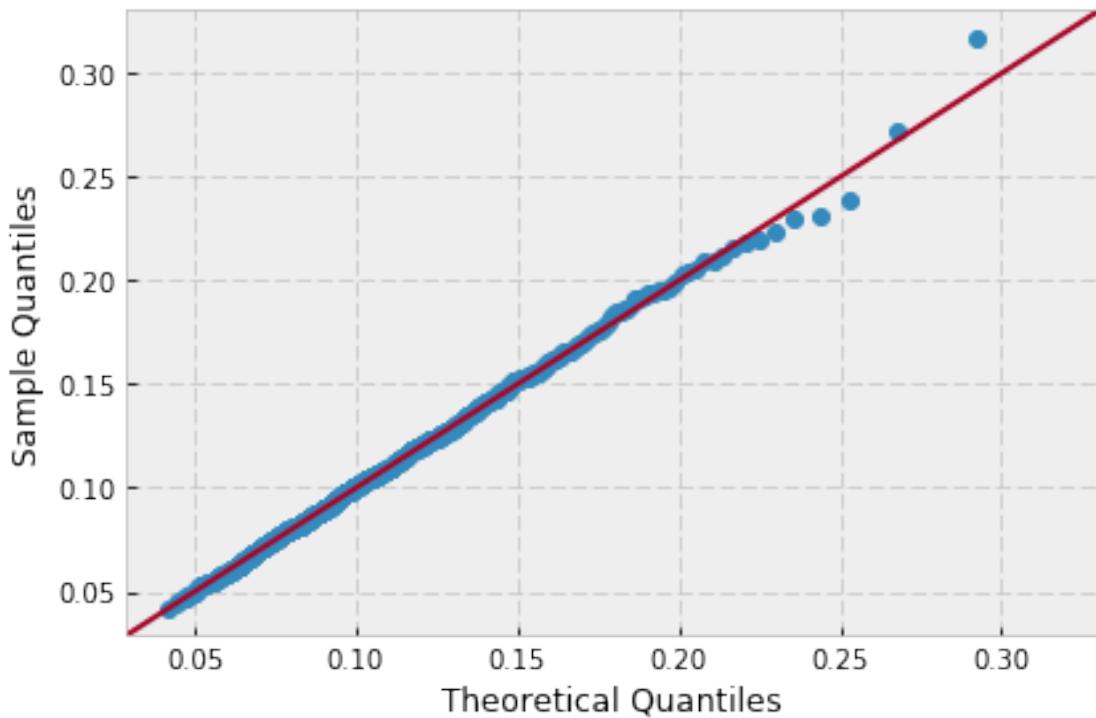


```
variable name: theta
variable value: 1.2217304763960306
distribution: inverse-gamma(shape=(12.327656010363837,), loc=0.003071780475877169, scale=0.213386
DescribeResult(nobs=1000, minmax=(0.011503466359629119, 0.051218782555127337)
               mean=0.0219406238664678, variance=3.6018958238112163e-05,
               skewness=1.3351699336488274, kurtosis=2.781752728371999)
```

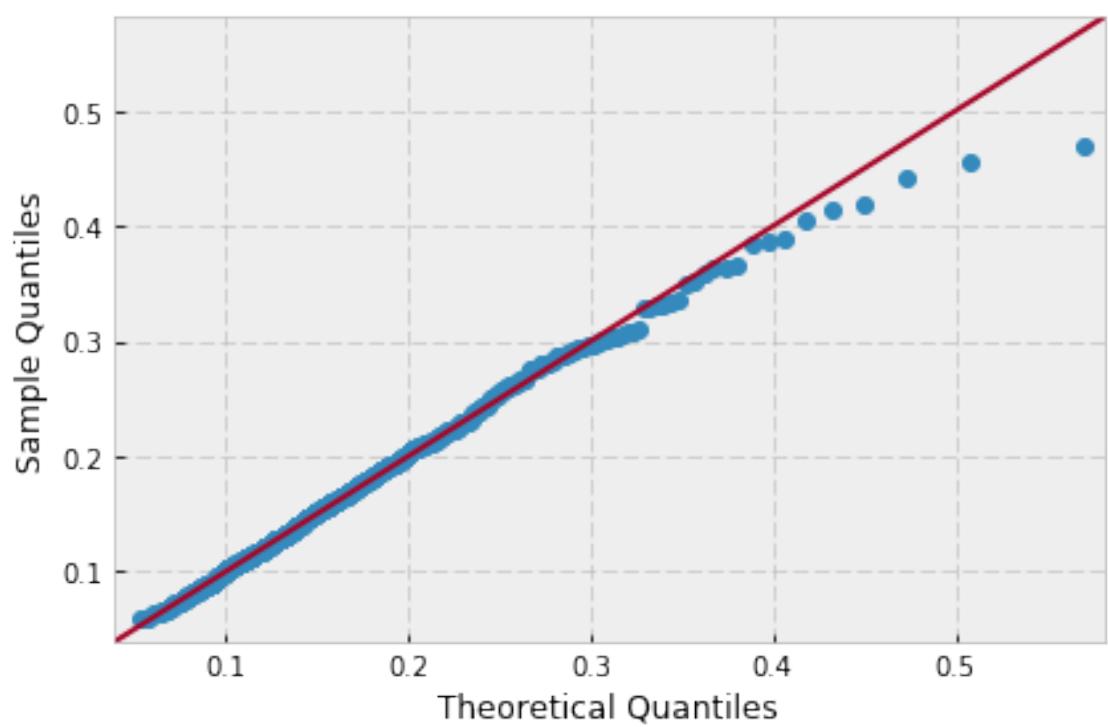
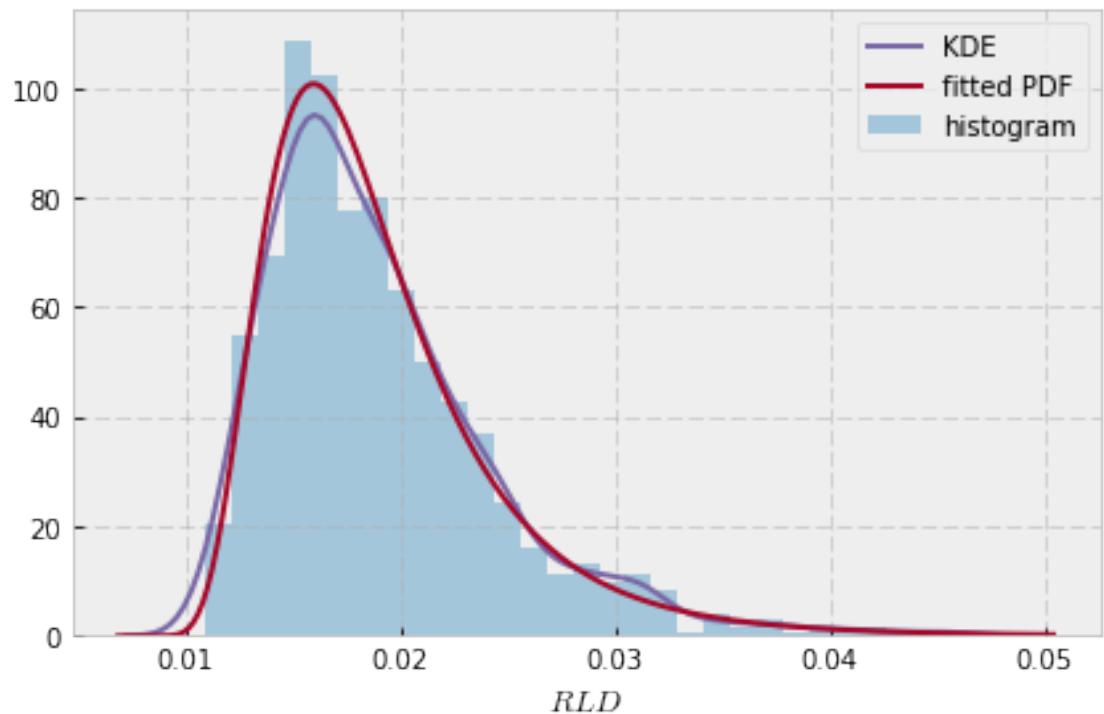


```
variable name: theta
variable value: 1.3089969389957472
distribution: inverse-gamma(shape=(10.86510304793806,), loc=0.003961070193092312, scale=0.160124
DescribeResult(nobs=1000, minmax=(0.010732754770822788, 0.054686738650277927)
    mean=0.020188264947368736, variance=2.918755073400164e-05,
    skewness=1.355159609435606, kurtosis=3.217103538141223)
```

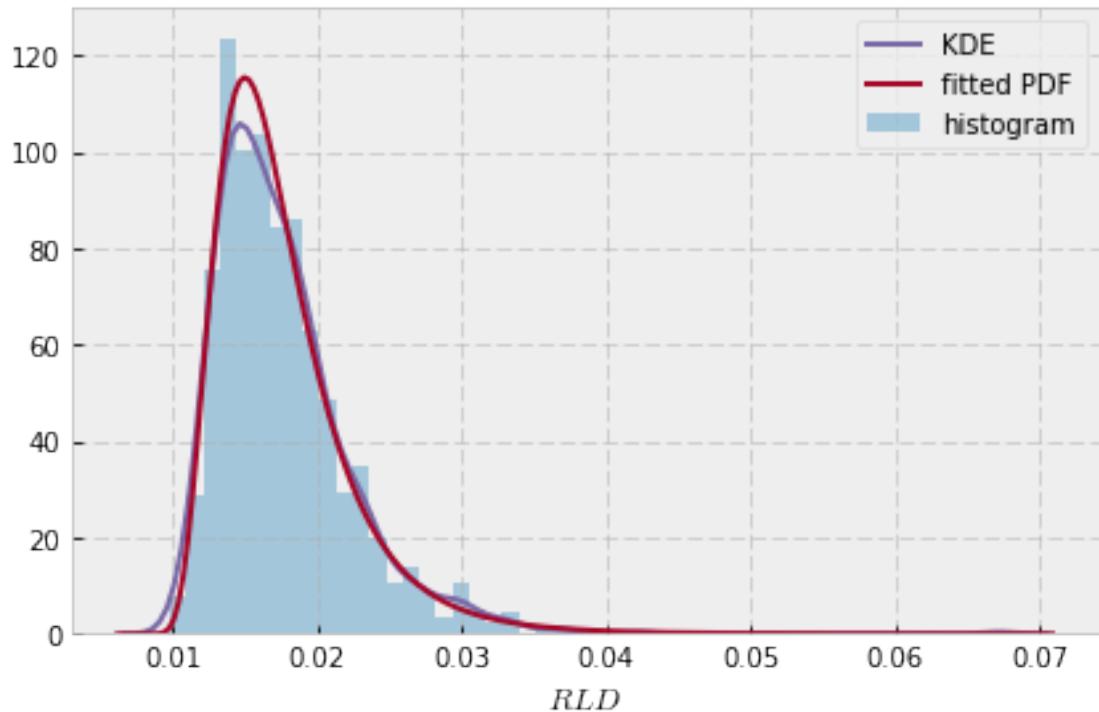


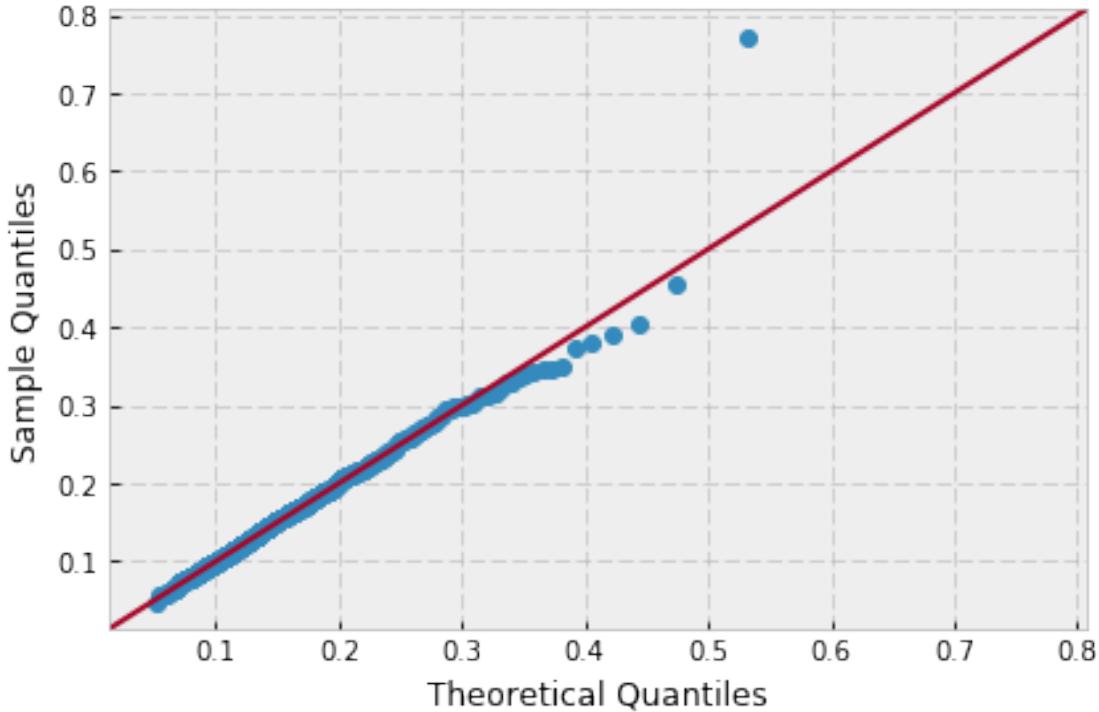


```
variable name: theta
variable value: 1.3962634015954638
distribution: inverse-gamma(shape=(7.5221983701573727,),loc=0.005837613408451714, scale=0.086116
DescribeResult(nobs=1000, minmax=(0.01089793134798423, 0.046359358295632043)
               mean=0.019019629259162847, variance=2.8985796919617022e-05,
               skewness=1.4276443404509866, kurtosis=2.8667054254102267)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: inverse-gamma(shape=(7.8039549017268932,),loc=0.0059671043156568175, scale=0.07927
DescribeResult(nobs=1000, minmax=(0.0097921197884409845, 0.067214019103406533)
    mean=0.017614011606907672, variance=2.3251602557532216e-05,
    skewness=2.1898875954541466, kurtosis=12.518481040243122)
```

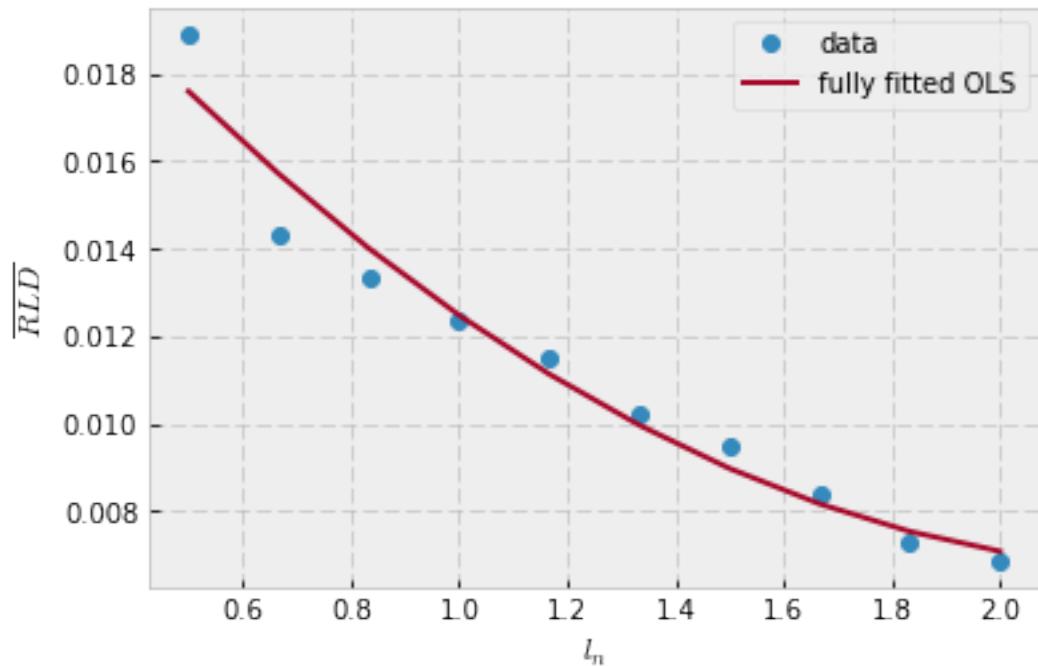




11.2 Parameter regression

```
In [28]: var_name = 'total_root_length_per_convex_hull_volume'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly2')
mean_reg('maxB', var_name, 'poly1')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly2')
mean_reg('N', var_name, 'poly1')
```

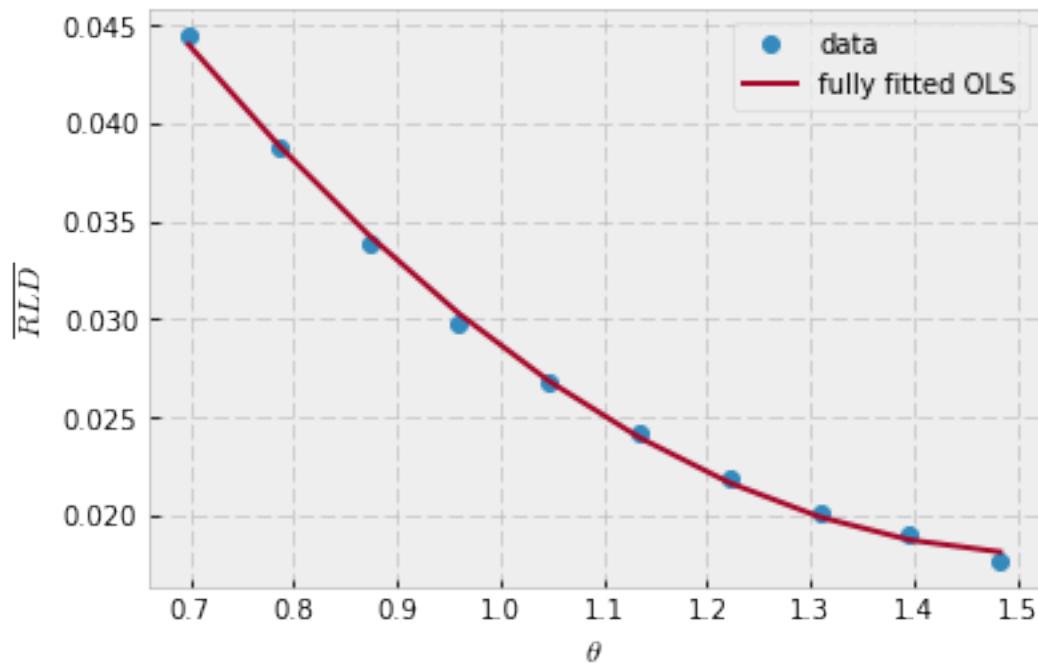
variable name: ln
model: poly2
r-squared: 0.8550551376625496
explained variance: 0.856407375530332



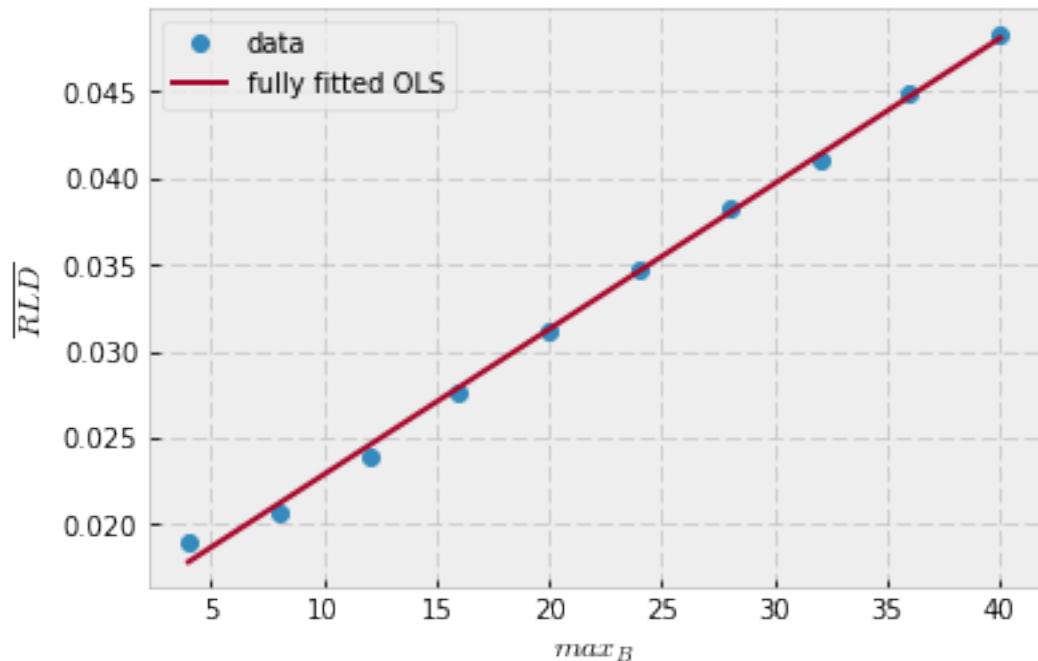
```

variable name: theta
model: poly2
r-squared: 0.9934248759598358
explained variance: 0.9934248870313761

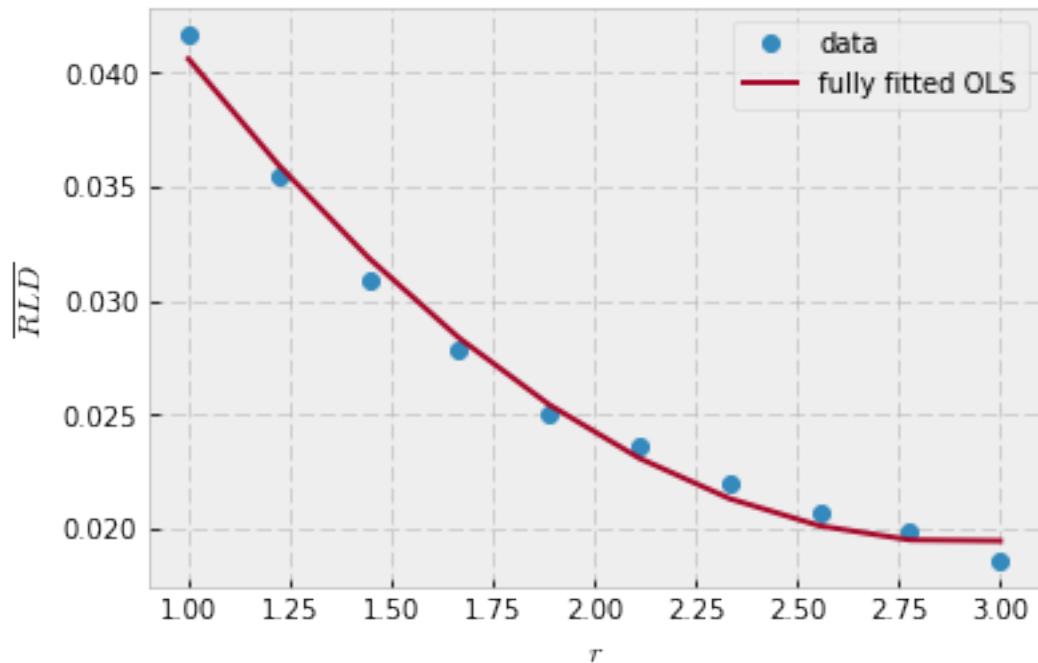
```



```
variable name: maxB  
model: poly1  
r-squared: 0.994796872680468  
explained variance: 0.9948147585803564
```



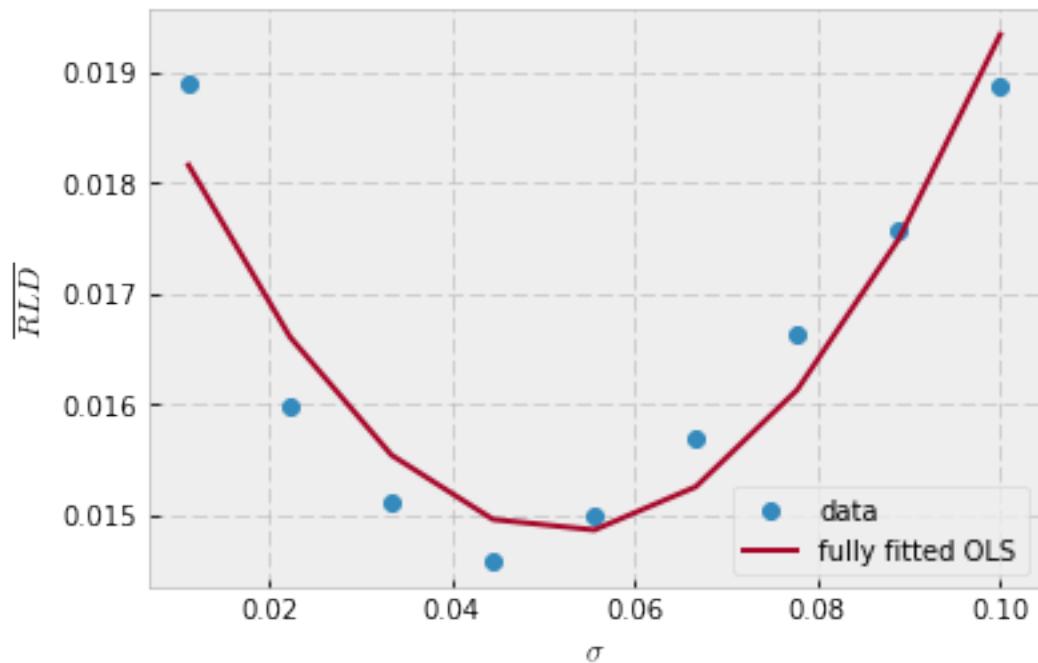
```
variable name: r  
model: poly2  
r-squared: 0.9629106057593358  
explained variance: 0.9629225439663144
```



```

variable name: sigma
model: poly2
r-squared: 0.5930821389234104
explained variance: 0.5940425481095111

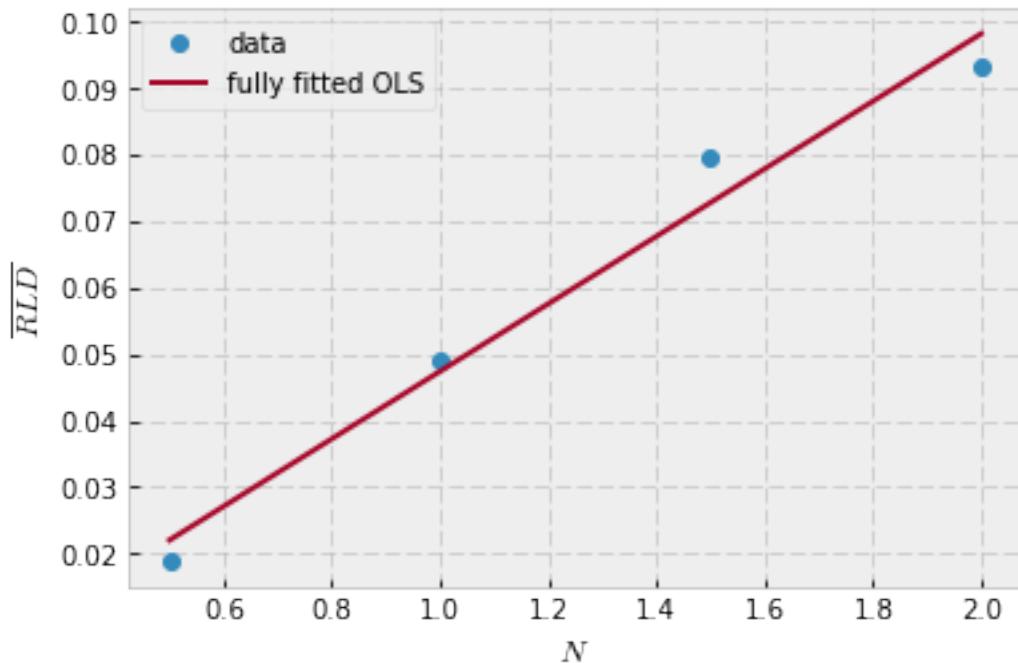
```



```

variable name: N
model: poly1
r-squared: 0.8532977932160297
explained variance: 0.8718213697893066

```



12 Root surface area relative to convex hull

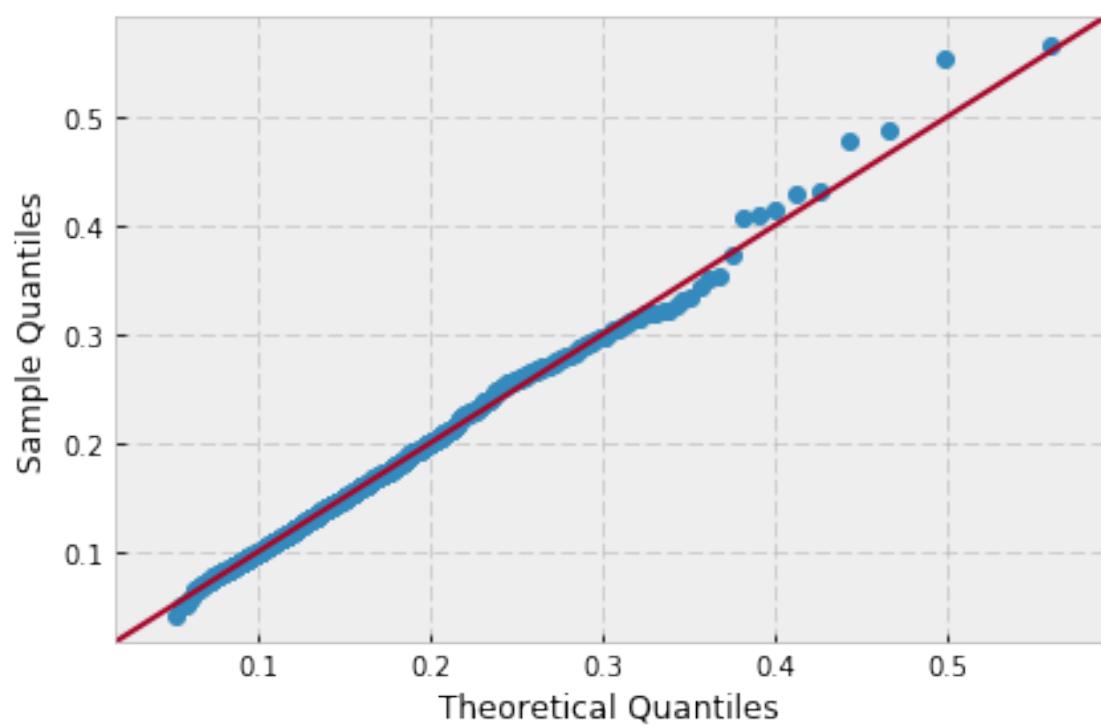
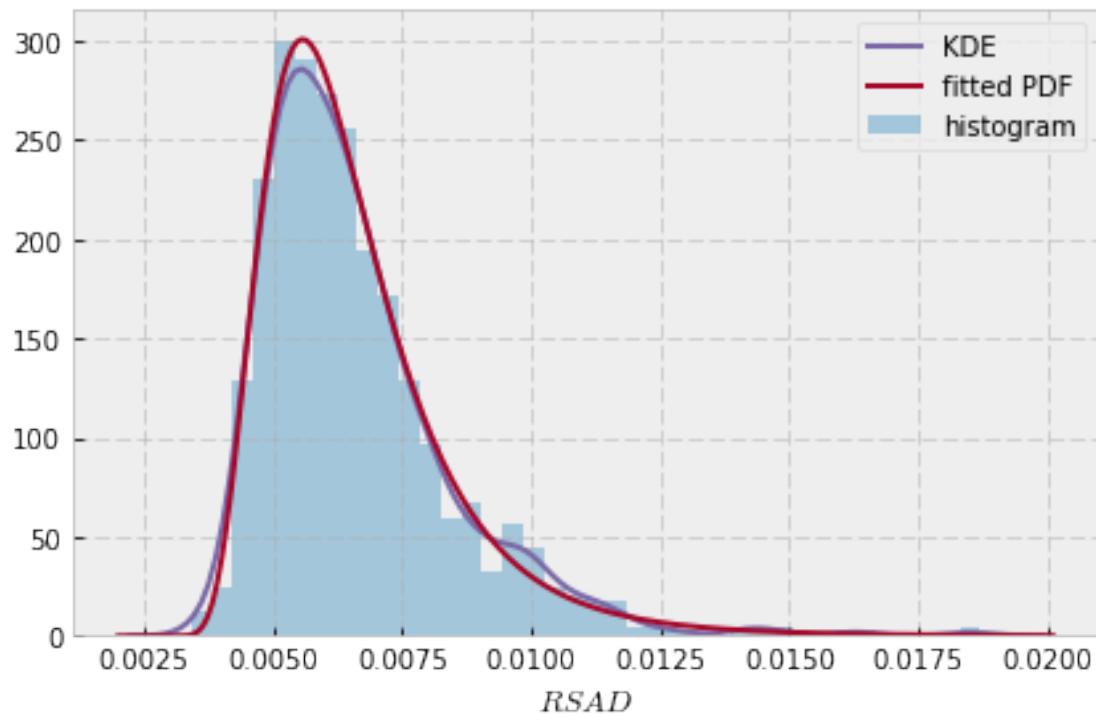
12.1 Probability distributions

```
In [29]: plot_dist('total_root_surface_area_per_convex_hull_volume', 'inverse-gamma')
```

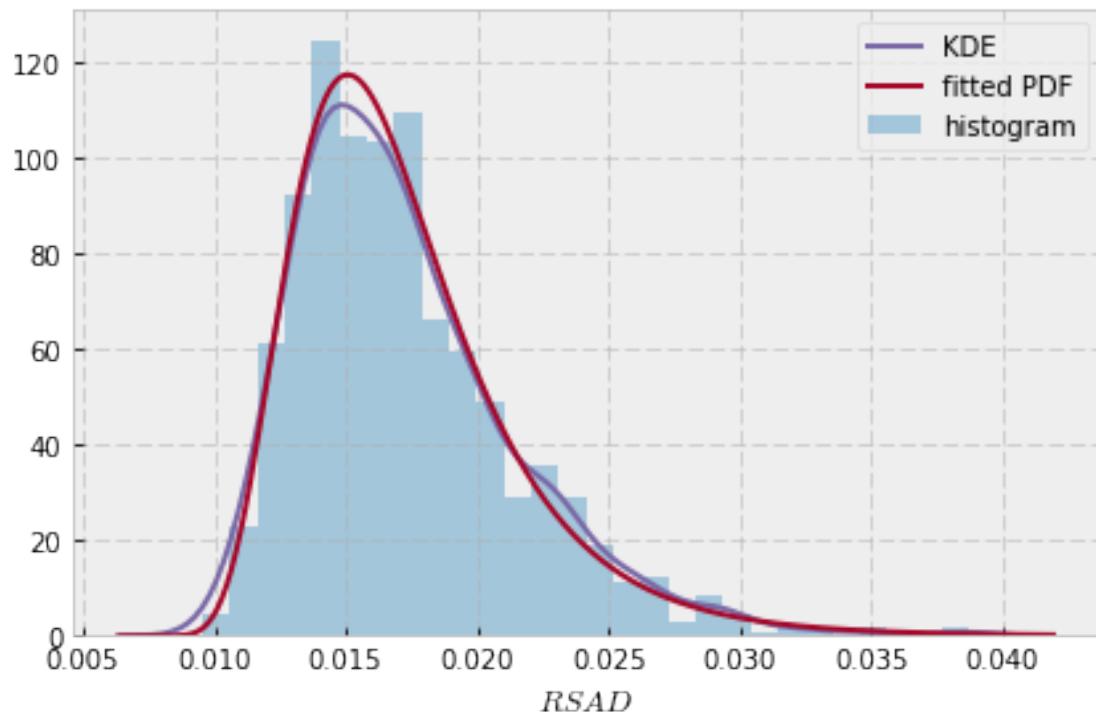
```

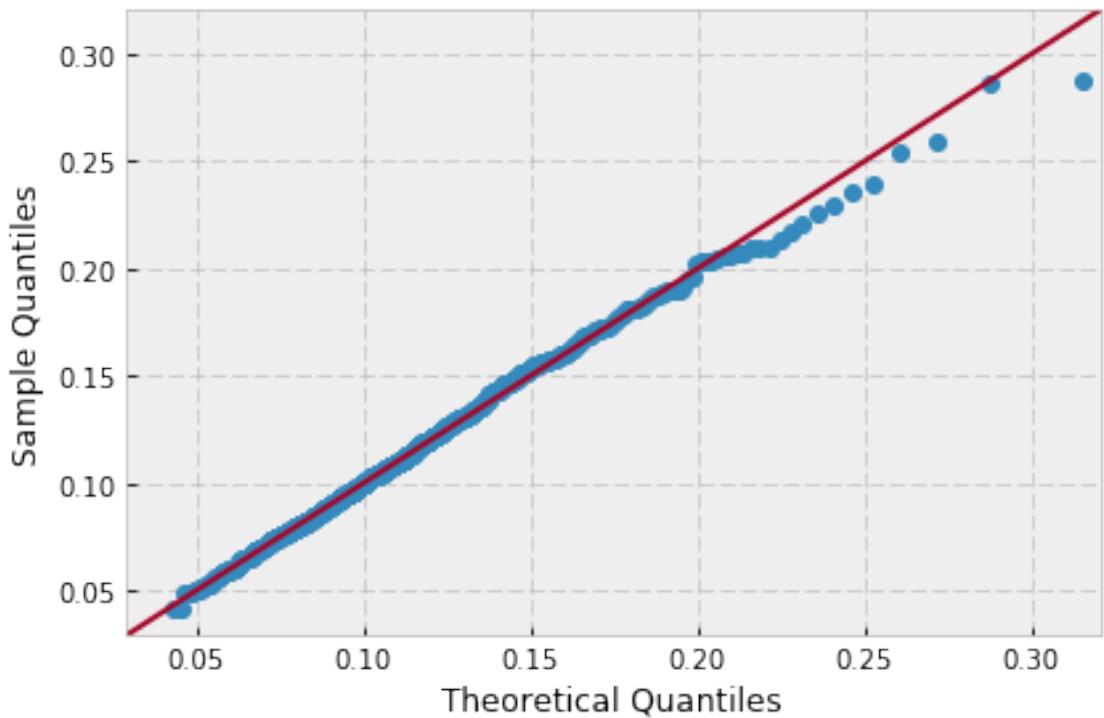
variable name: N
variable value: 0.5
distribution: inverse-gamma(shape=(7.5944667445317533,), loc=0.0021739215906456566, scale=0.02922
DescribeResult(nobs=1000, minmax=(0.003412130517475897, 0.018704416532773233)
    mean=0.0066064594153740436, variance=3.4819364471861154e-06,
    skewness=1.8093577644695733, kurtosis=5.631618268975794)

```

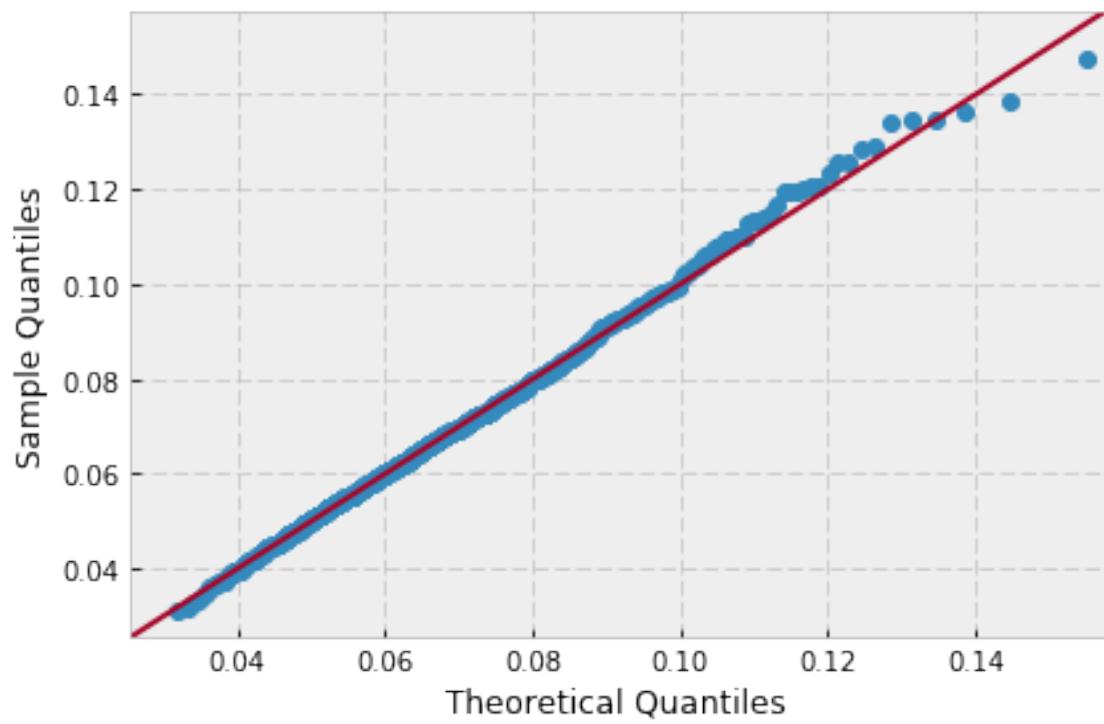
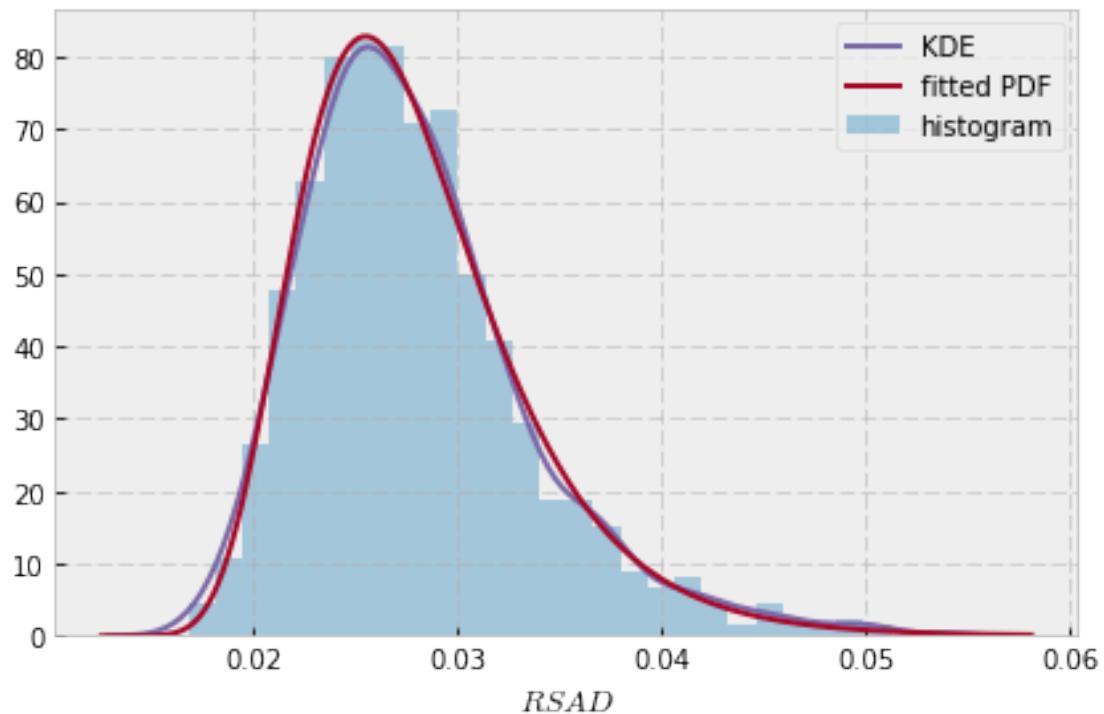


```
variable name: N
variable value: 1.0
distribution: inverse-gamma(shape=(10.407256918491104,),loc=0.004682771759653872, scale=0.118507
DescribeResult(nobs=1000, minmax=(0.0095502871317810264, 0.038726326947902852)
    mean=0.017272654783304, variance=1.8061621988564113e-05,
    skewness=1.204905063692676, kurtosis=2.0761032599397824)
```

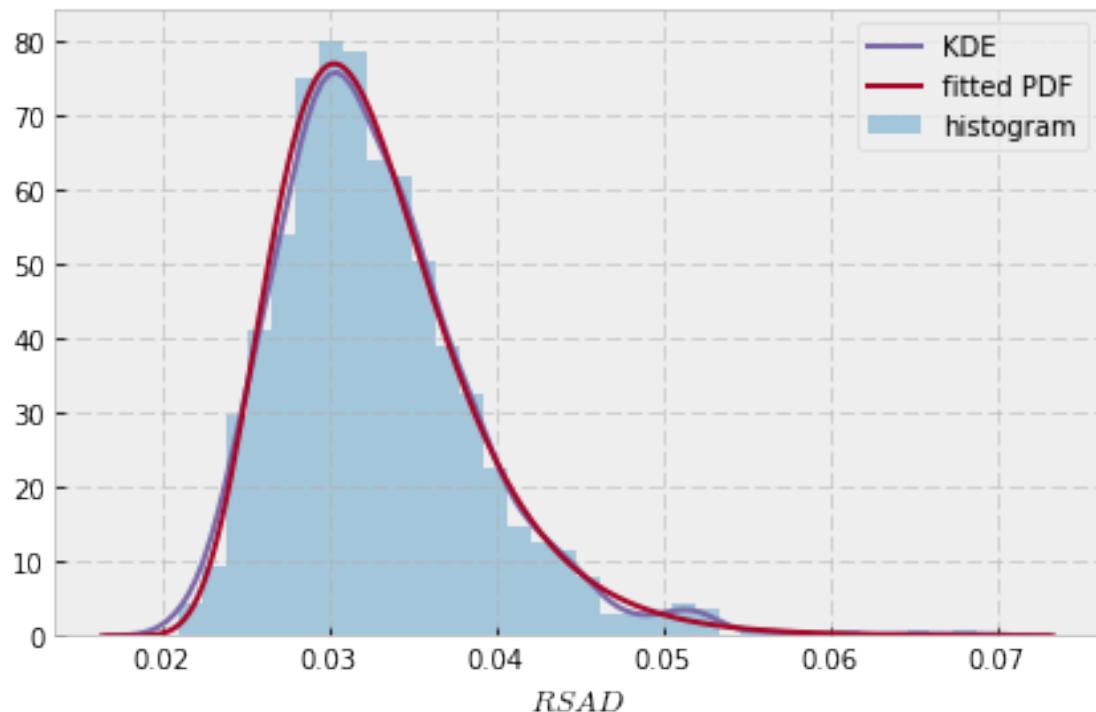


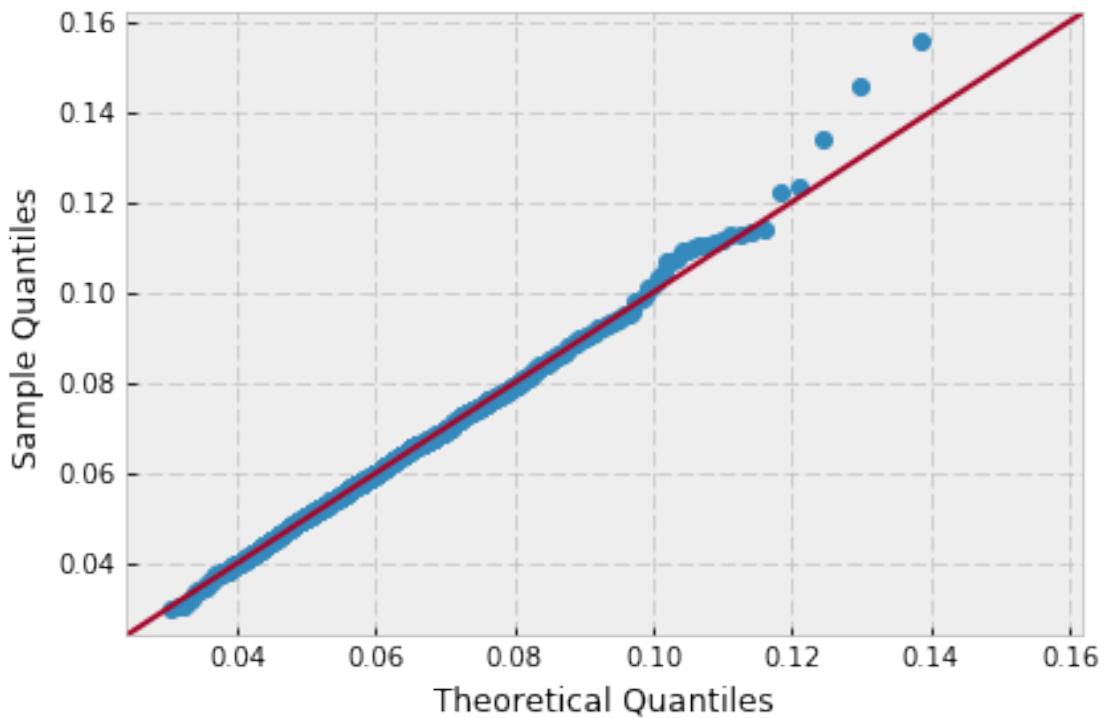


```
variable name: N
variable value: 1.5
distribution: inverse-gamma(shape=(16.082175986437353,), loc=0.006851710007435539, scale=0.318629
DescribeResult(nobs=1000, minmax=(0.016798626041433451, 0.053862611159962055)
               mean=0.0279801949287223, variance=3.177691659994143e-05,
               skewness=1.0923619375455855, kurtosis=1.7712562176911302)
```

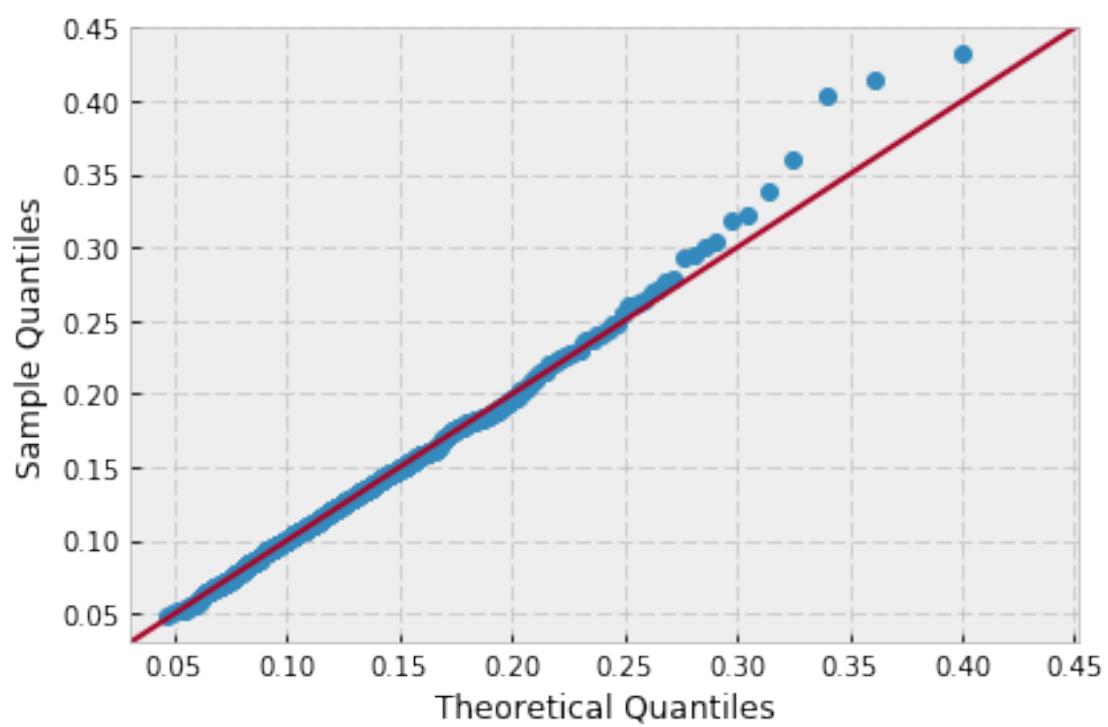
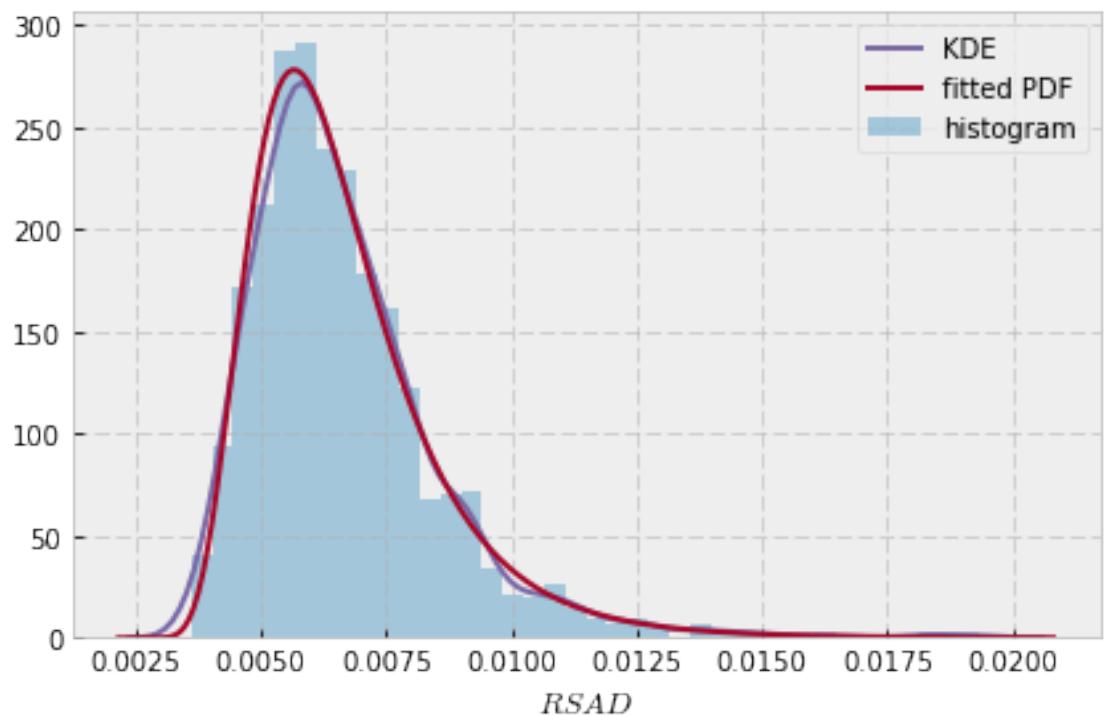


```
variable name: N
variable value: 2.0
distribution: inverse-gamma(shape=(17.30086561704757,), loc=0.009446148331349671, scale=0.3814597
DescribeResult(nobs=1000, minmax=(0.020942848447169697, 0.068808486629182208)
               mean=0.032850303352511601, variance=3.6334160741766985e-05,
               skewness=1.1993558160670863, kurtosis=2.9313469681182074)
```

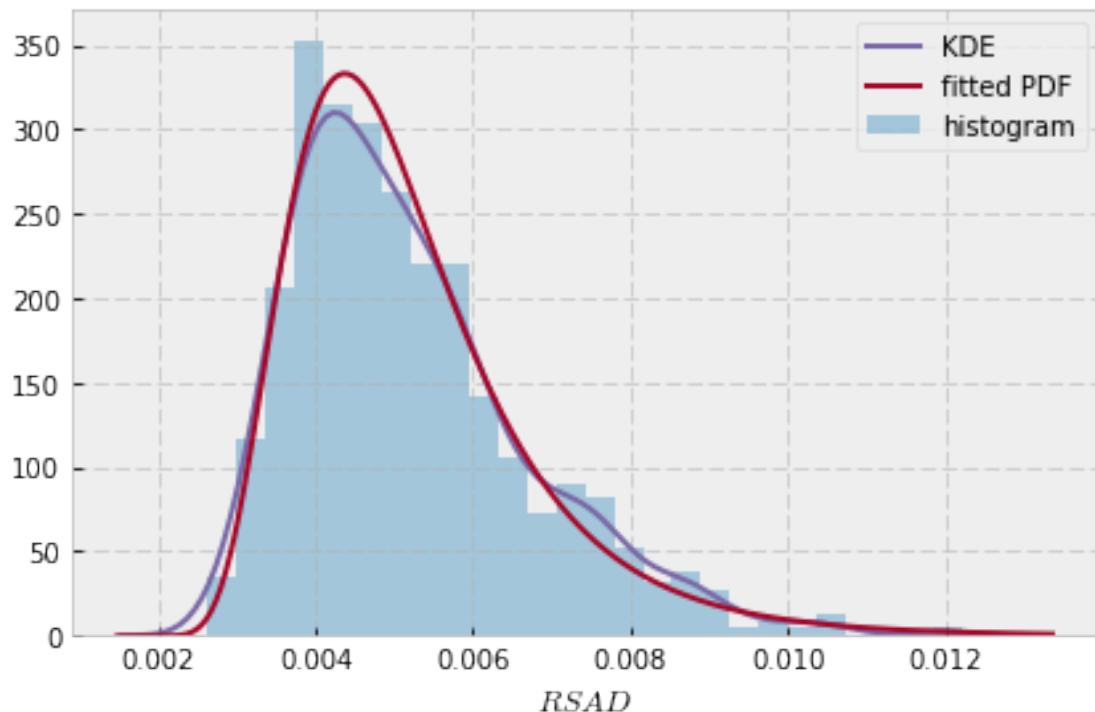


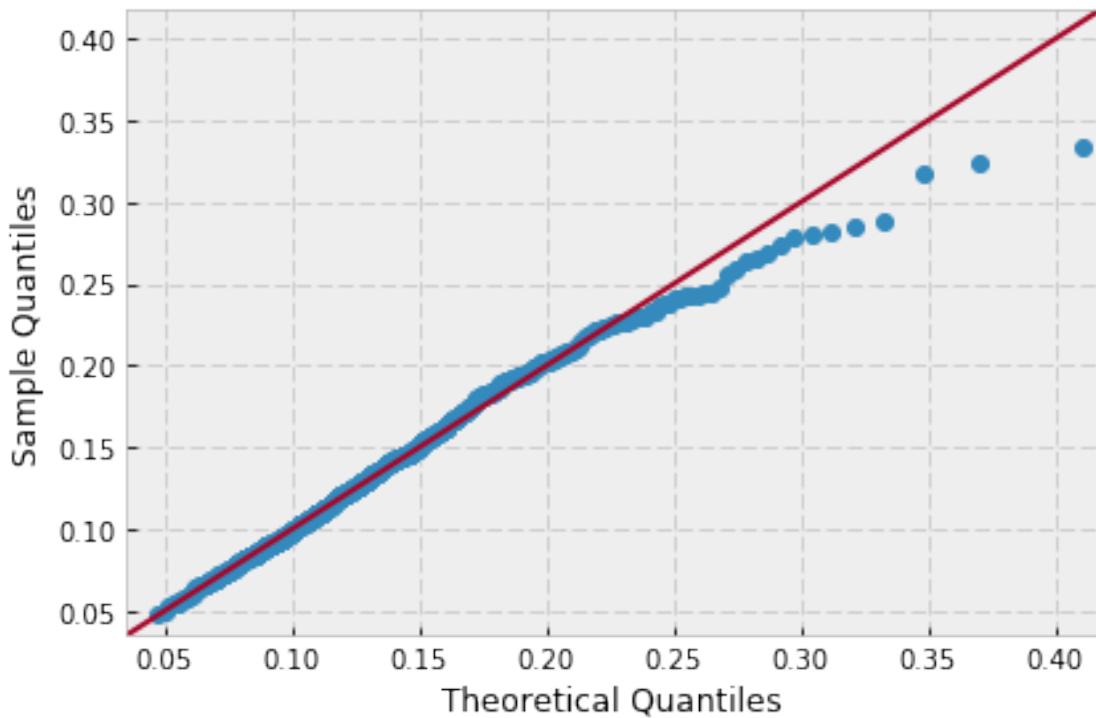


```
variable name: ln
variable value: 0.5
distribution: inverse-gamma(shape=(9.0949326049983519,), loc=0.0015837226295882952, scale=0.04108
DescribeResult(nobs=1000, minmax=(0.0036051847477412682, 0.01935417921448411)
               mean=0.0066611563524463652, variance=3.7235721508831977e-06,
               skewness=1.845186638725284, kurtosis=6.276479585117663)
```

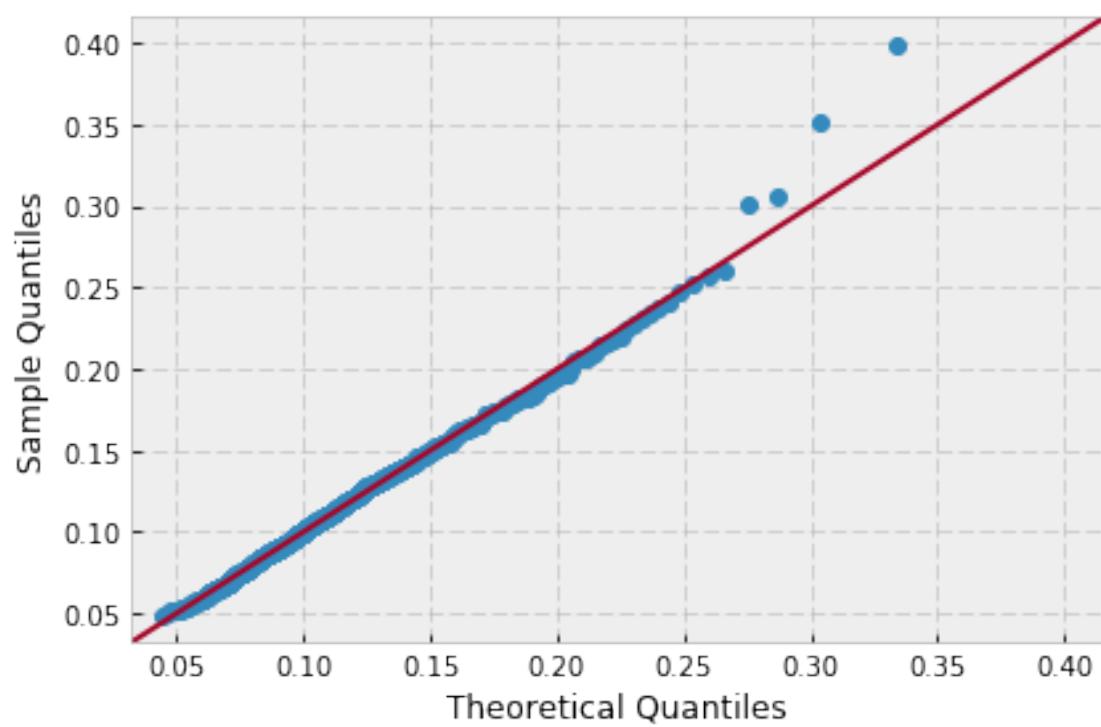
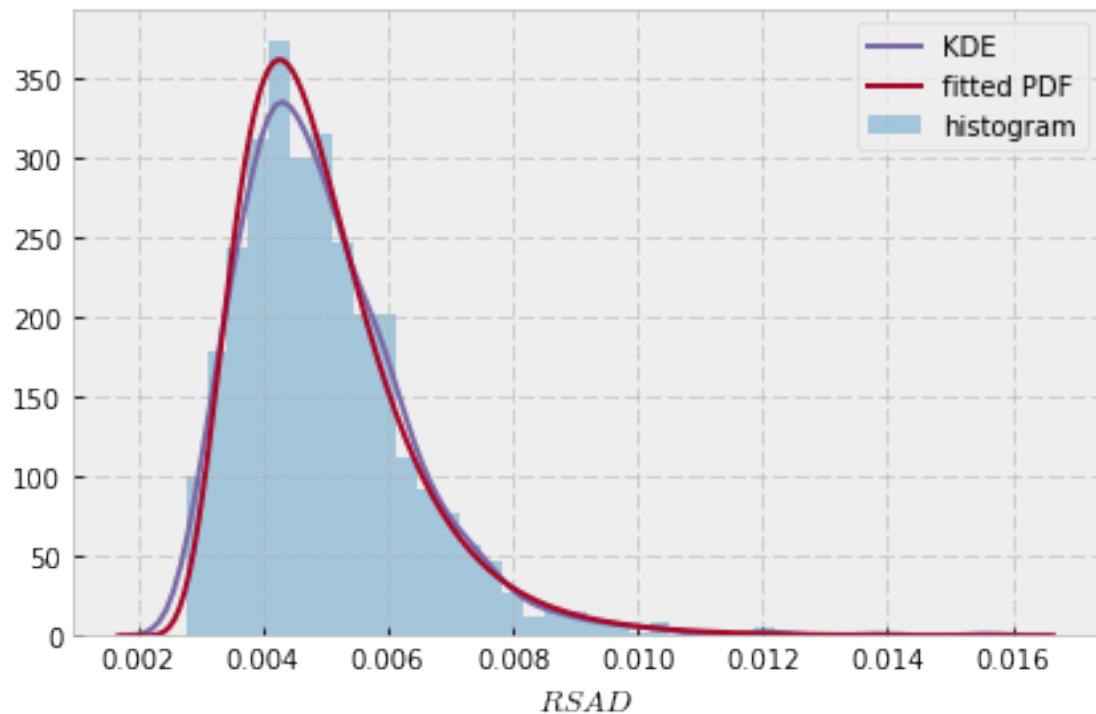


```
variable name: ln
variable value: 0.6666666666666666
distribution: inverse-gamma(shape=(8.9695865465901079,), loc=0.00099533575979842, scale=0.0336393
DescribeResult(nobs=1000, minmax=(0.002632968590387958, 0.012206543205992373)
    mean=0.0052098831984670347, variance=2.3306156593561044e-06,
    skewness=1.1180059582823503, kurtosis=1.4231085000294028)
```

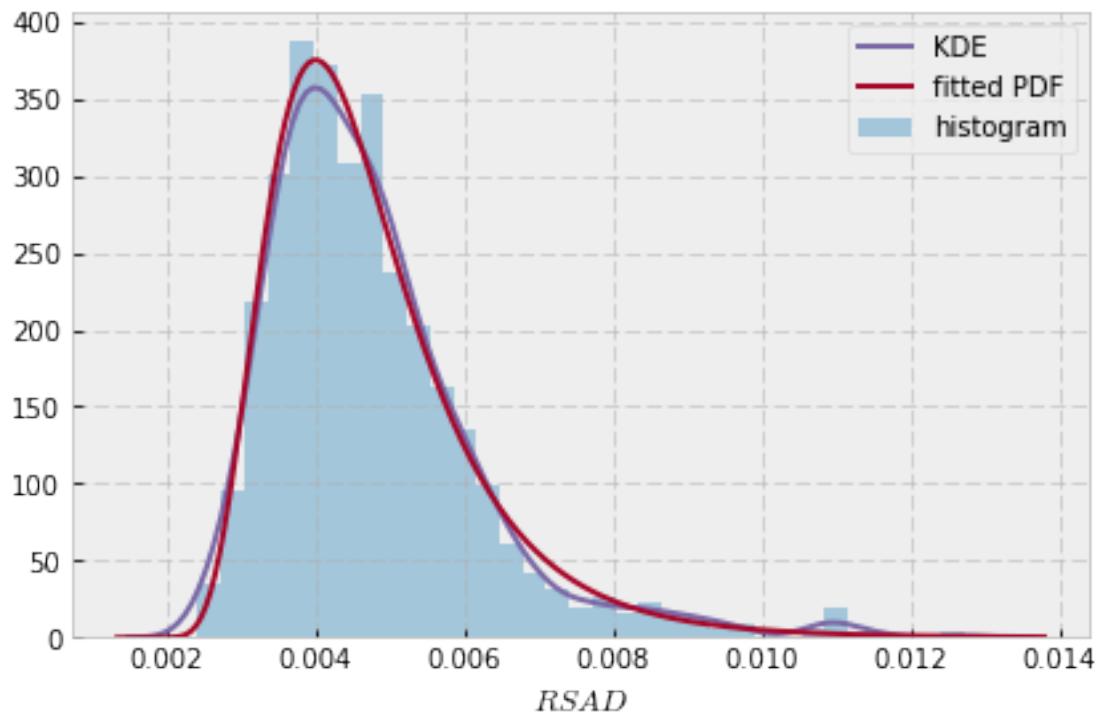


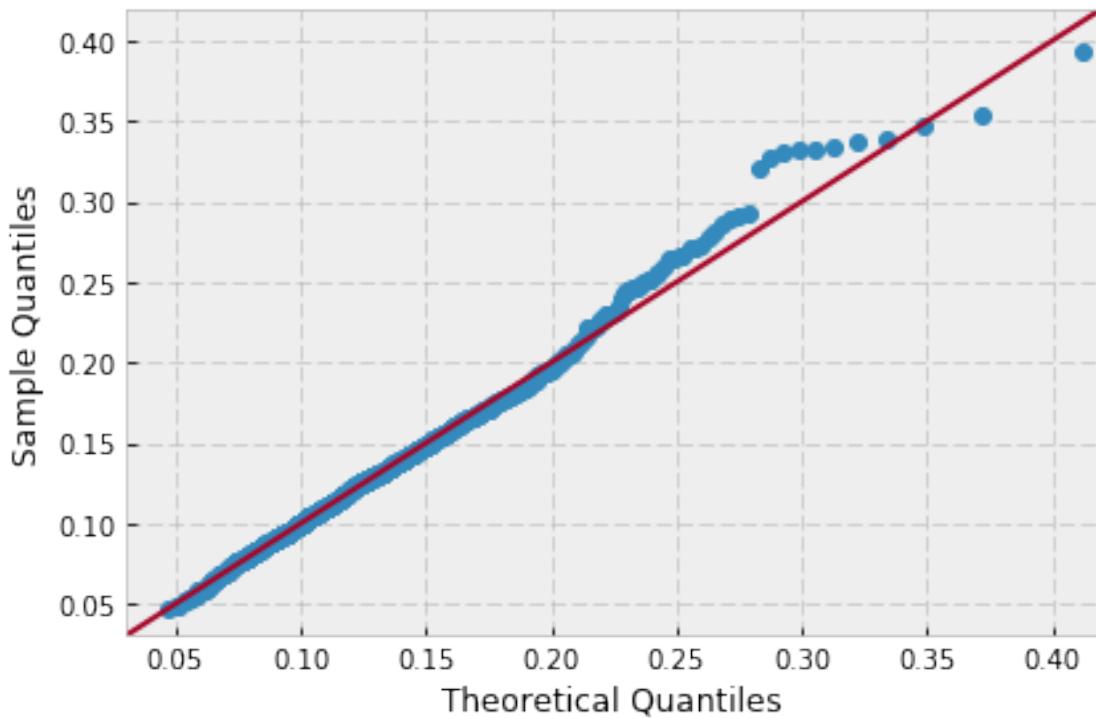


```
variable name: ln
variable value: 0.8333333333333333
distribution: inverse-gamma(shape=(10.062242981213792,), loc=0.0009544271668441174, scale=0.03669
DescribeResult(nobs=1000, minmax=(0.0027640078036797085, 0.015581220629984245)
               mean=0.0050025959714742217, variance=2.0169908920689899e-06,
               skewness=1.6456955073396902, kurtosis=5.935123366374722)
```

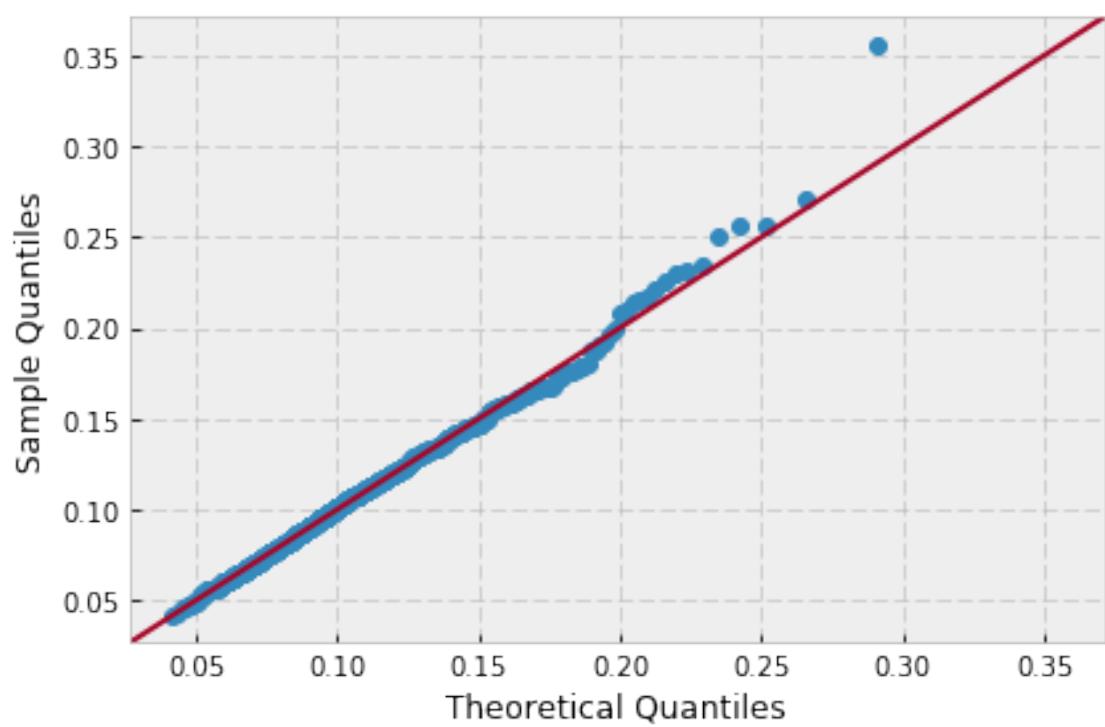
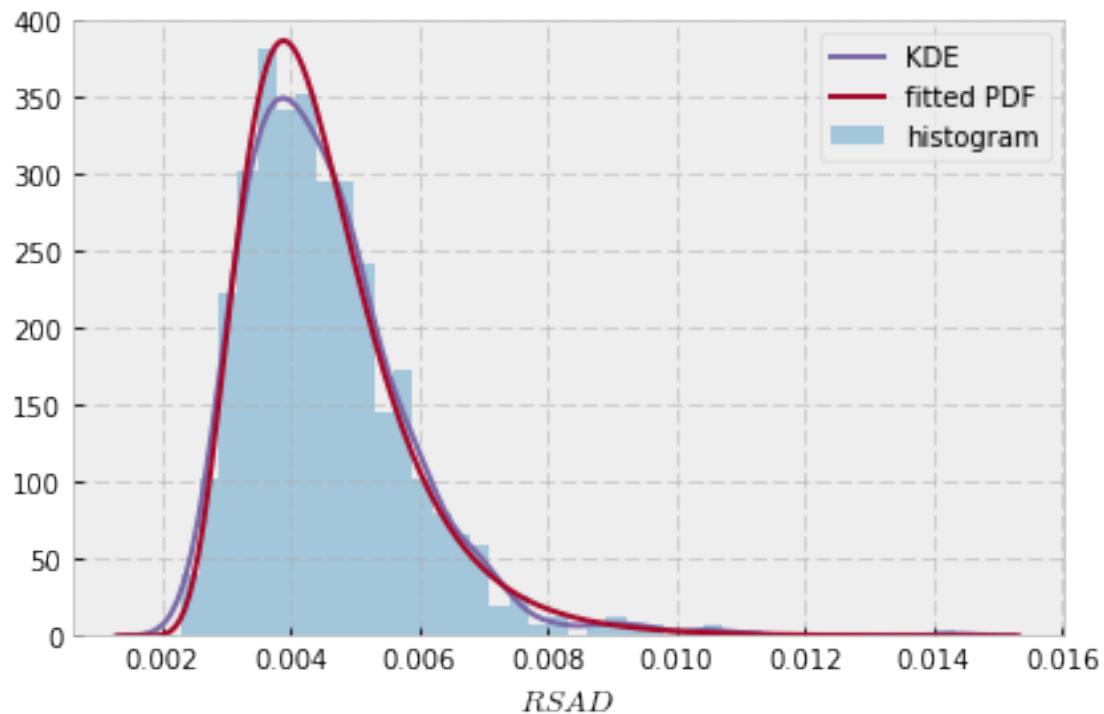


```
variable name: ln
variable value: 1.0
distribution: inverse-gamma(shape=(8.9514893344889099,),loc=0.0009998440305081261, scale=0.02978
DescribeResult(nobs=1000, minmax=(0.002407397715168901, 0.012707102545662482)
               mean=0.0047475673177228674, variance=2.0550385353085347e-06,
               skewness=1.6882037296542065, kurtosis=4.354142999987451)
```

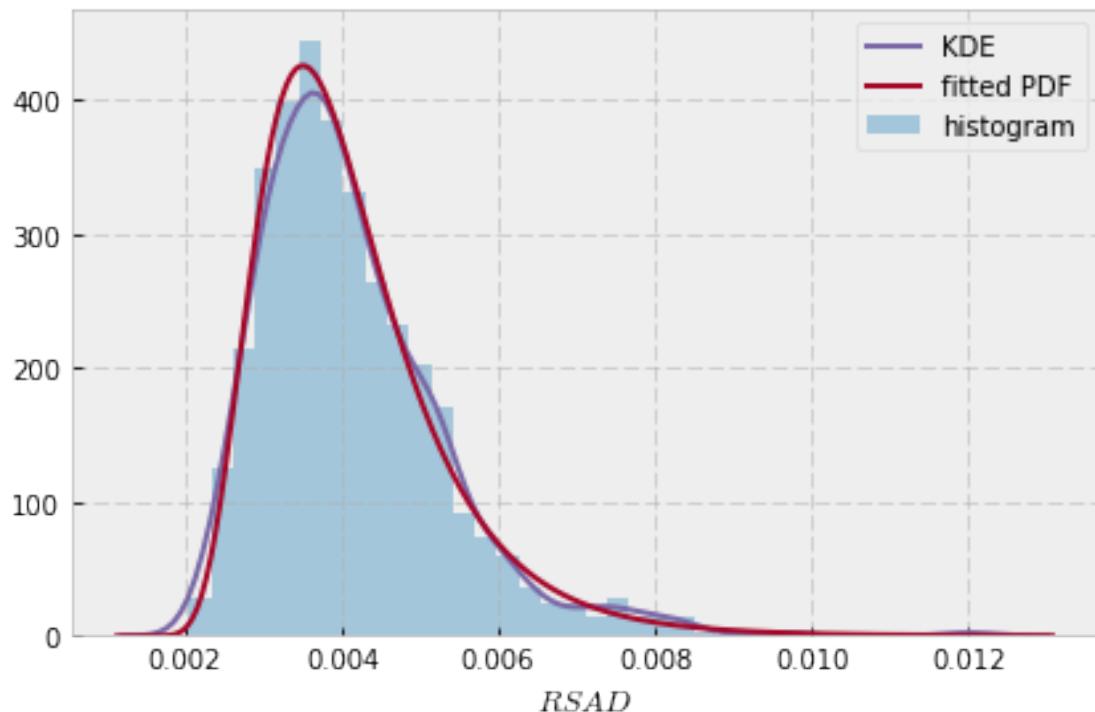


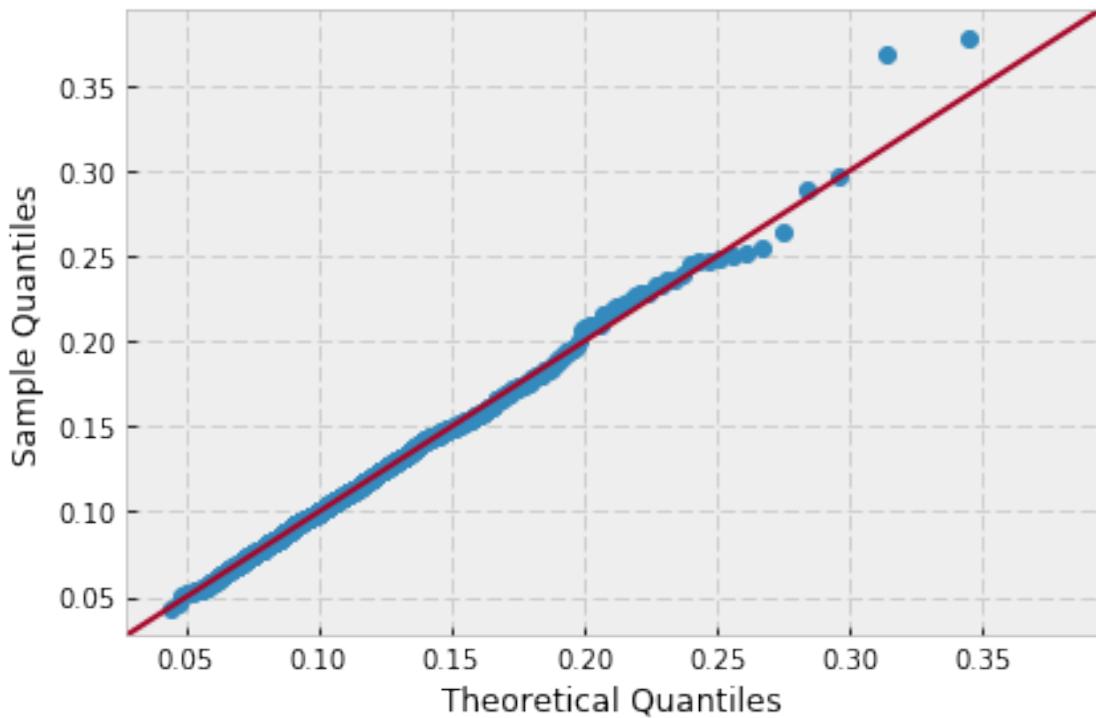


```
variable name: ln
variable value: 1.1666666666666665
distribution: inverse-gamma(shape=(10.899204276400397,), loc=0.0006549438632698617, scale=0.03847
DescribeResult(nobs=1000, minmax=(0.0022772122931898494, 0.01433430185111432)
               mean=0.0045414586483363394, variance=1.6977599465584493e-06,
               skewness=1.567588185659292, kurtosis=5.293544858372476)
```

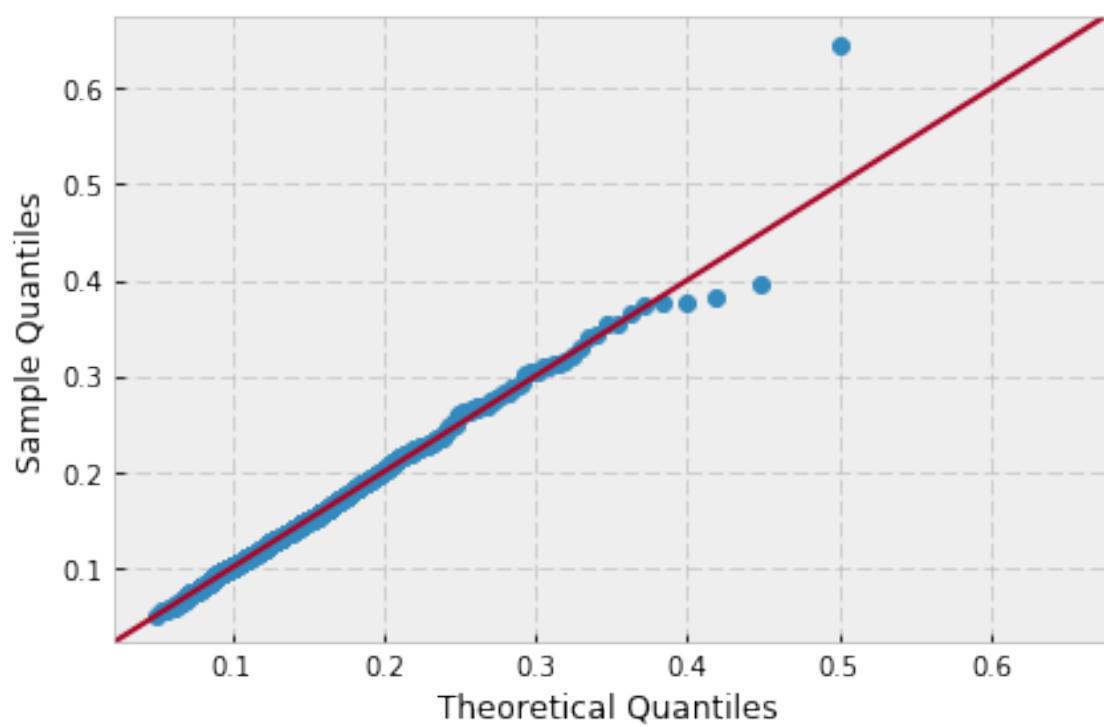
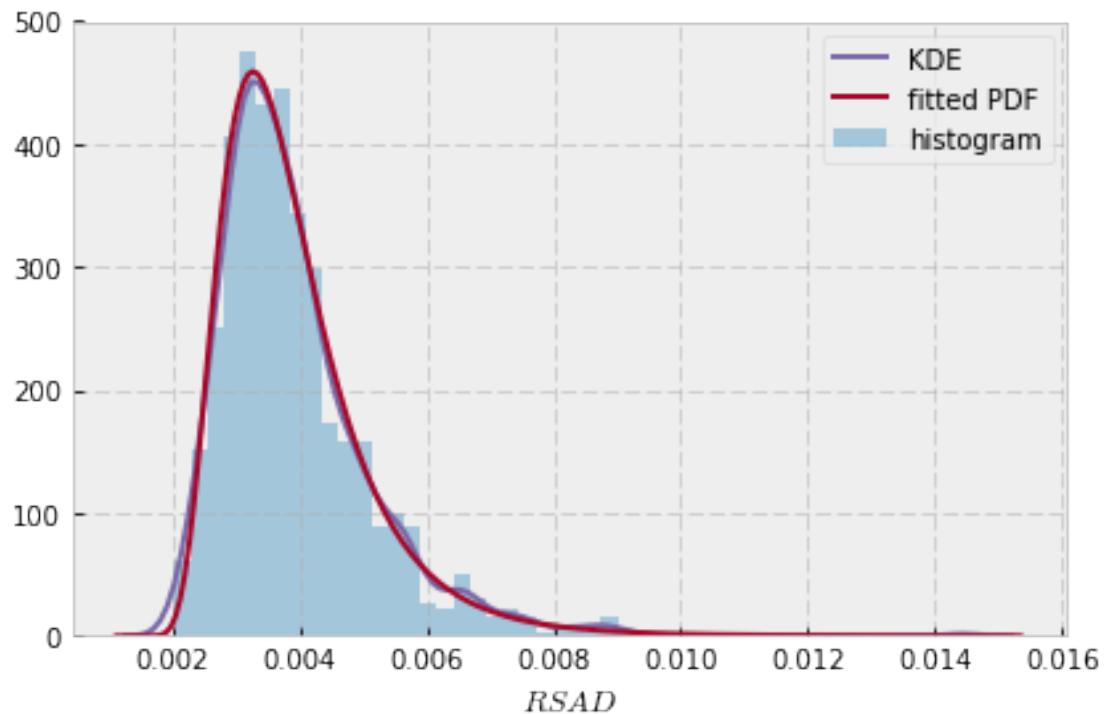


```
variable name: ln
variable value: 1.3333333333333333
distribution: inverse-gamma(shape=(9.8794816088259605,),loc=0.0007143517926419068, scale=0.03031
DescribeResult(nobs=1000, minmax=(0.0020429453207169673, 0.012161444936082341)
    mean=0.0041283936977009788, variance=1.4757275746303977e-06,
    skewness=1.591627517914968, kurtosis=4.822847871798885)
```

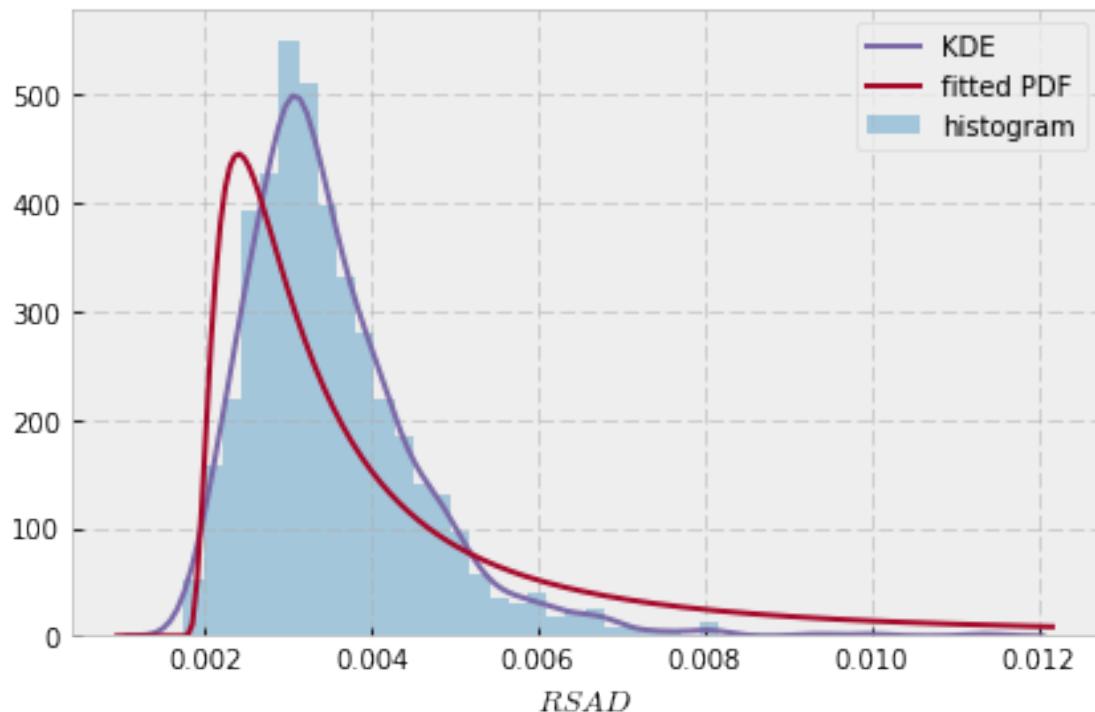


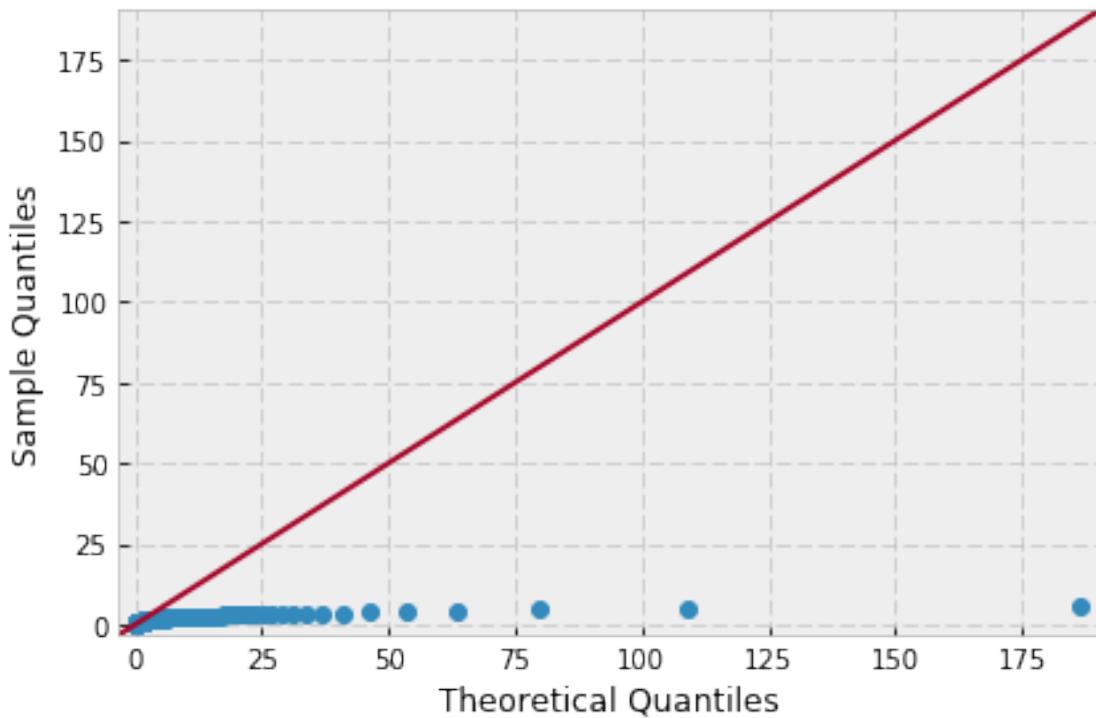


```
variable name: ln
variable value: 1.5
distribution: inverse-gamma(shape=(8.060484852469429,), loc=0.0009539098161319032, scale=0.020911
DescribeResult(nobs=1000, minmax=(0.0020192689438537528, 0.014439760026590767)
               mean=0.0039157750787993393, variance=1.4458799562287537e-06,
               skewness=1.896501870394956, kurtosis=7.612681358882897)
```

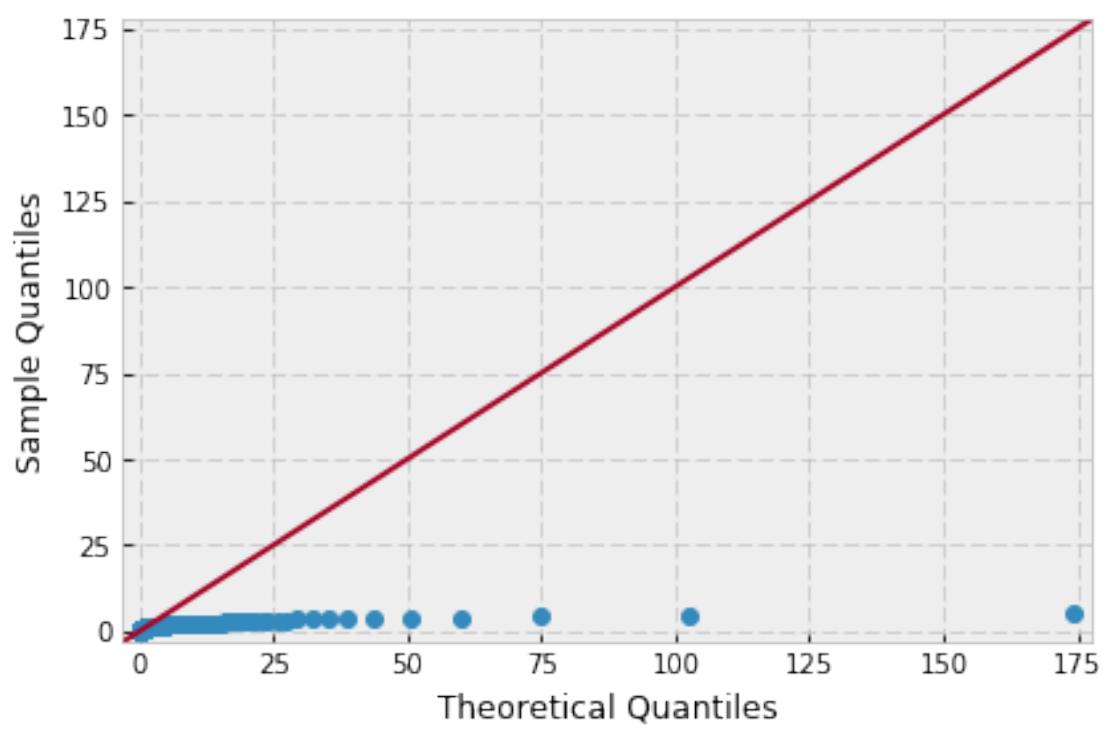
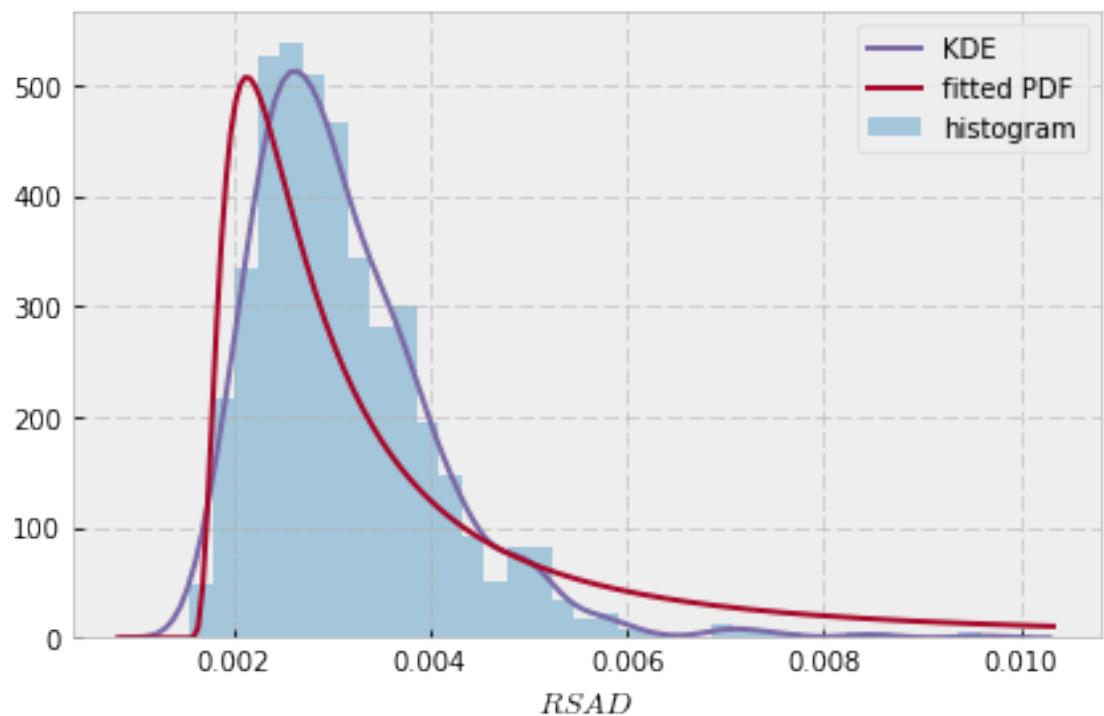


```
variable name: ln
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(1.2919307968307938,),loc=0.001670413656155391, scale=0.001689
DescribeResult(nobs=1000, minmax=(0.0017567794539327436, 0.011361430079422965)
    mean=0.0035274480928405212, variance=1.137514833575441e-06,
    skewness=1.7517802858270775, kurtosis=6.06130893397259)
```

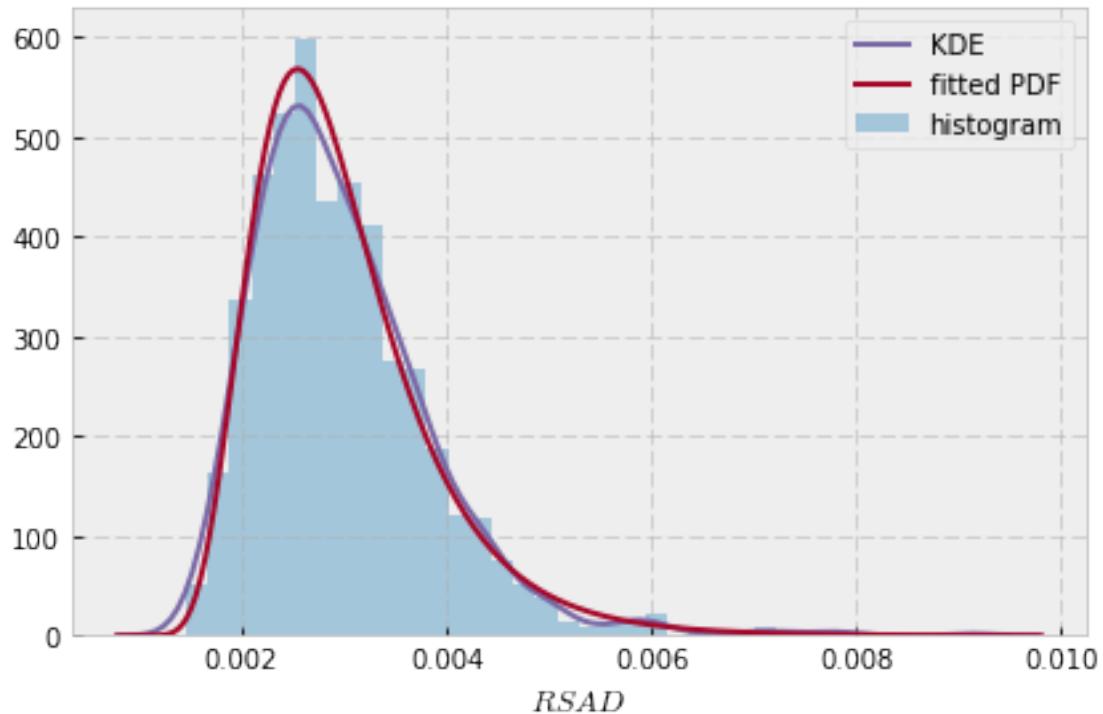


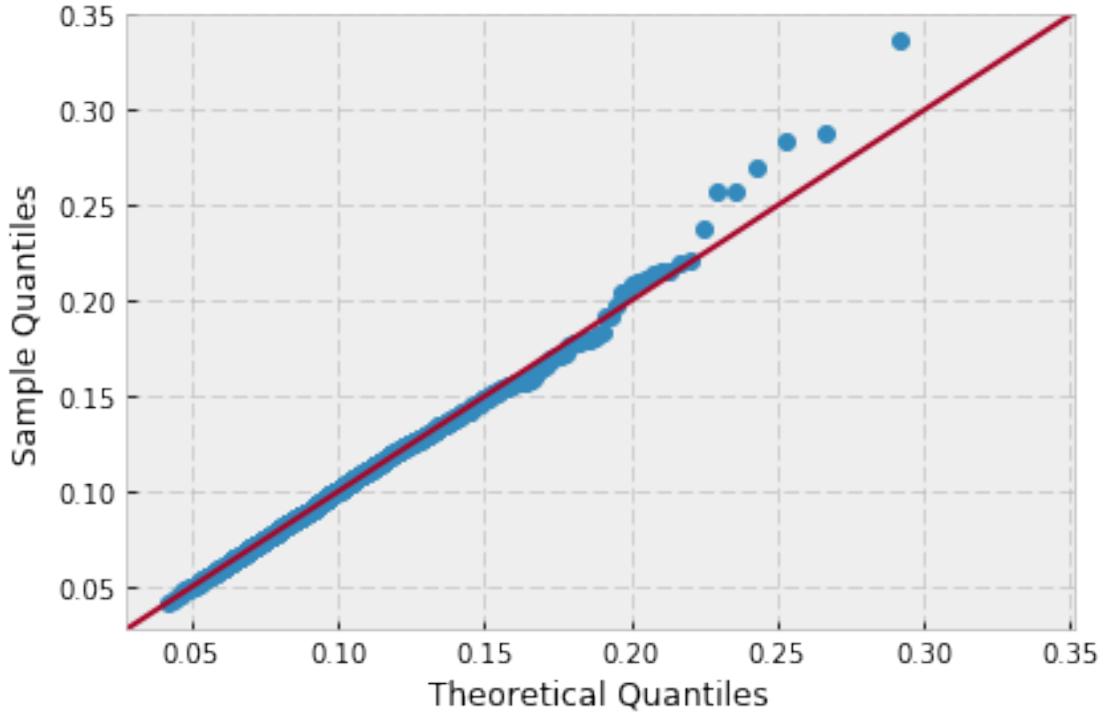


```
variable name: ln
variable value: 1.833333333333333
distribution: inverse-gamma(shape=(1.3074624568179734,), loc=0.0014798090407235273, scale=0.00150
DescribeResult(nobs=1000, minmax=(0.001552233228622794, 0.0096070095632407627)
               mean=0.003133966601054289, variance=9.3236107175269252e-07,
               skewness=1.6477585742616307, kurtosis=5.093013633034245)
```

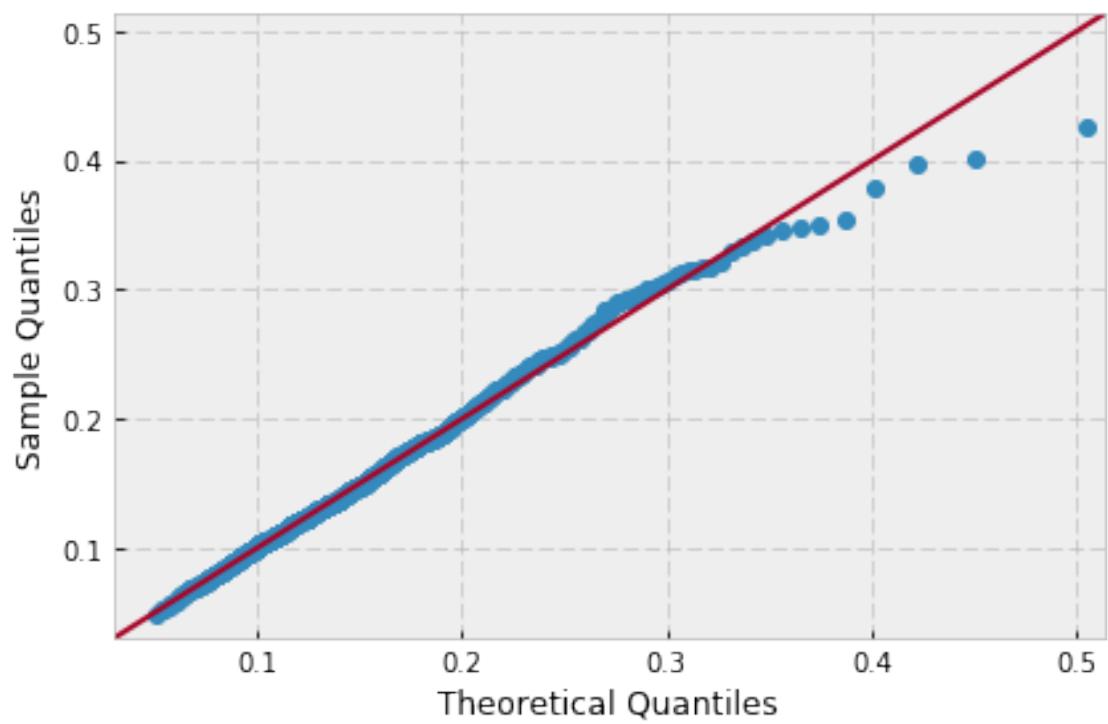
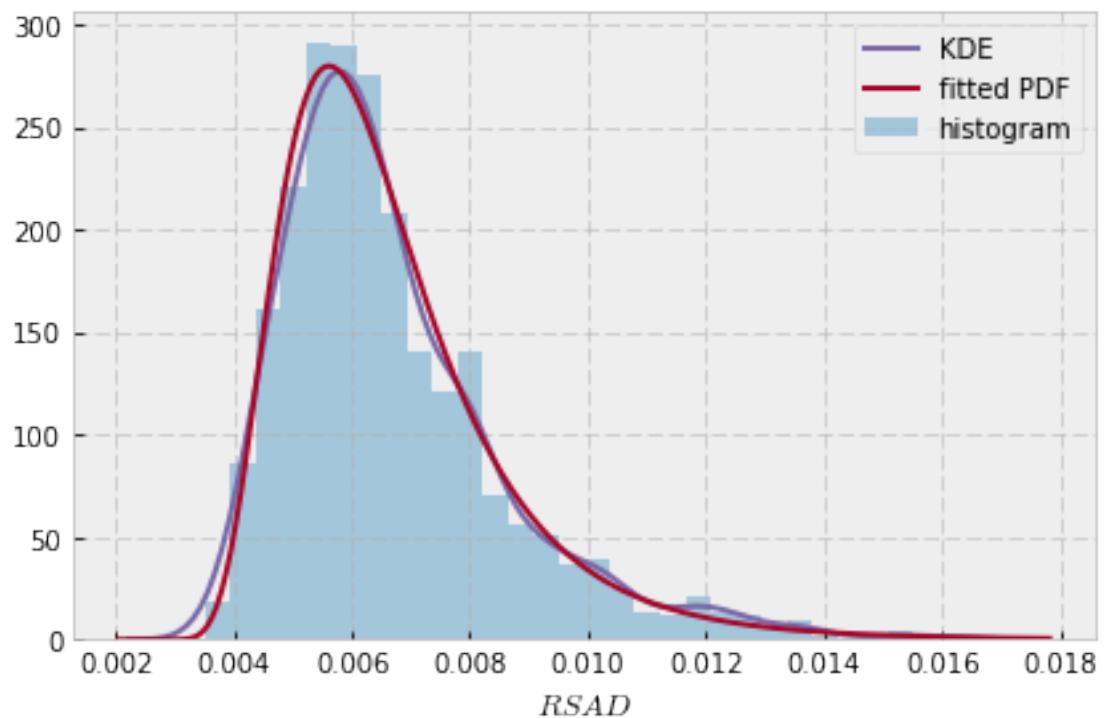


```
variable name: ln
variable value: 2.0
distribution: inverse-gamma(shape=(10.882776603546372,),loc=0.000344928159316251, scale=0.026138
DescribeResult(nobs=1000, minmax=(0.0014516691530308201, 0.0091366279737513396)
               mean=0.0029899760157405622, variance=7.963827302273691e-07,
               skewness=1.6149583905202263, kurtosis=5.3019248766101725)
```

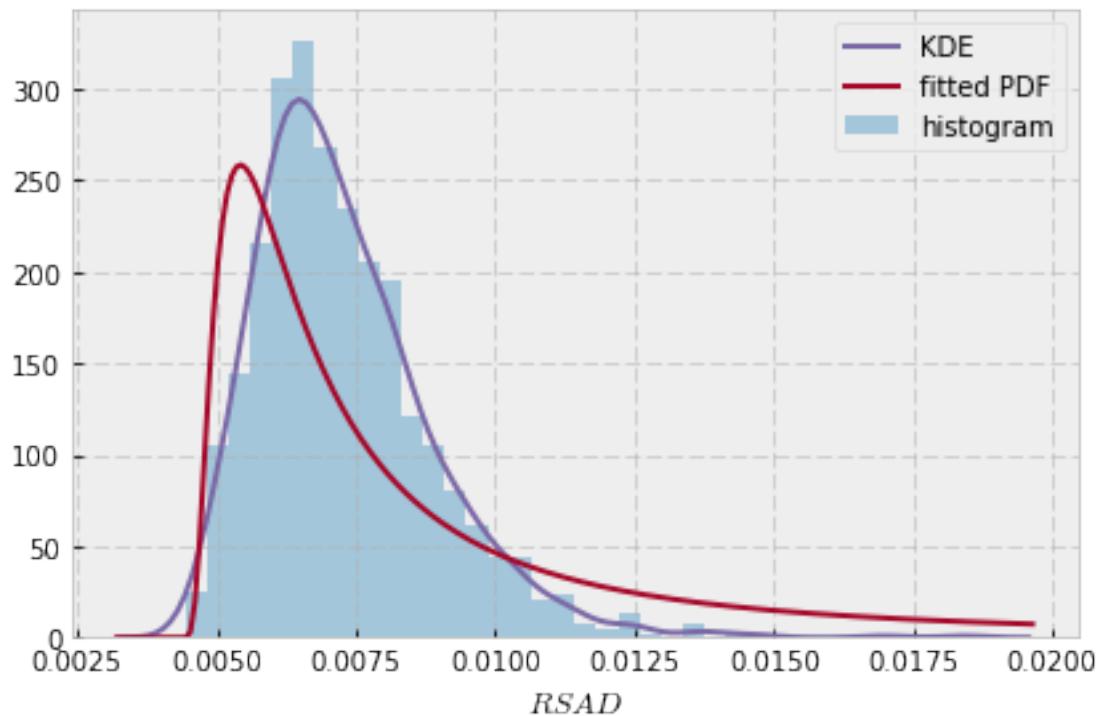


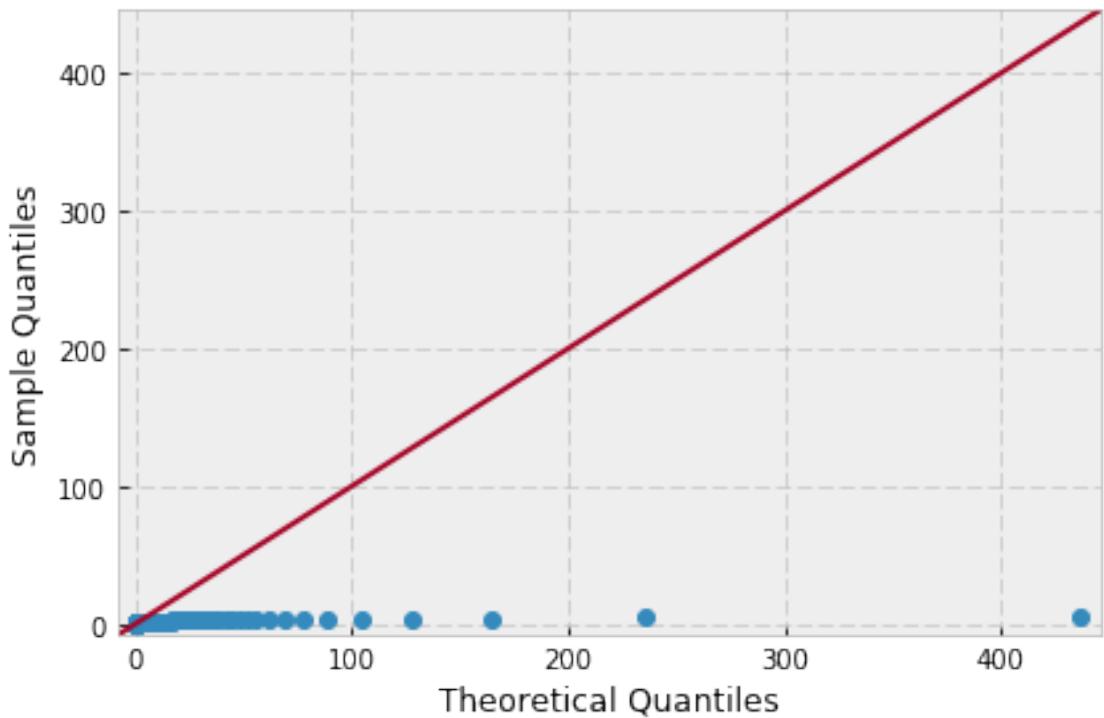


```
variable name: maxB
variable value: 4.0
distribution: inverse-gamma(shape=(8.02742507322988,), loc=0.0018368707077292198, scale=0.0340795
DescribeResult(nobs=1000, minmax=(0.0034990703815895789, 0.016365747718763601)
               mean=0.0066832776063180223, variance=3.7205020616408888e-06,
               skewness=1.4415189110679856, kurtosis=2.6959310639385077)
```

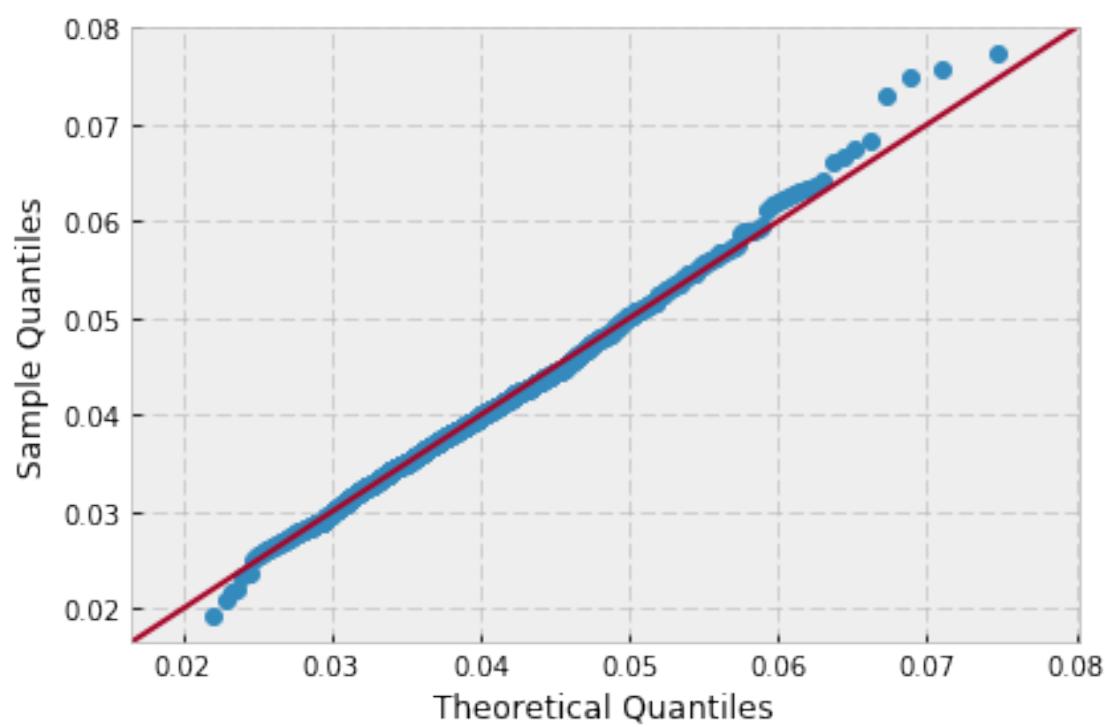
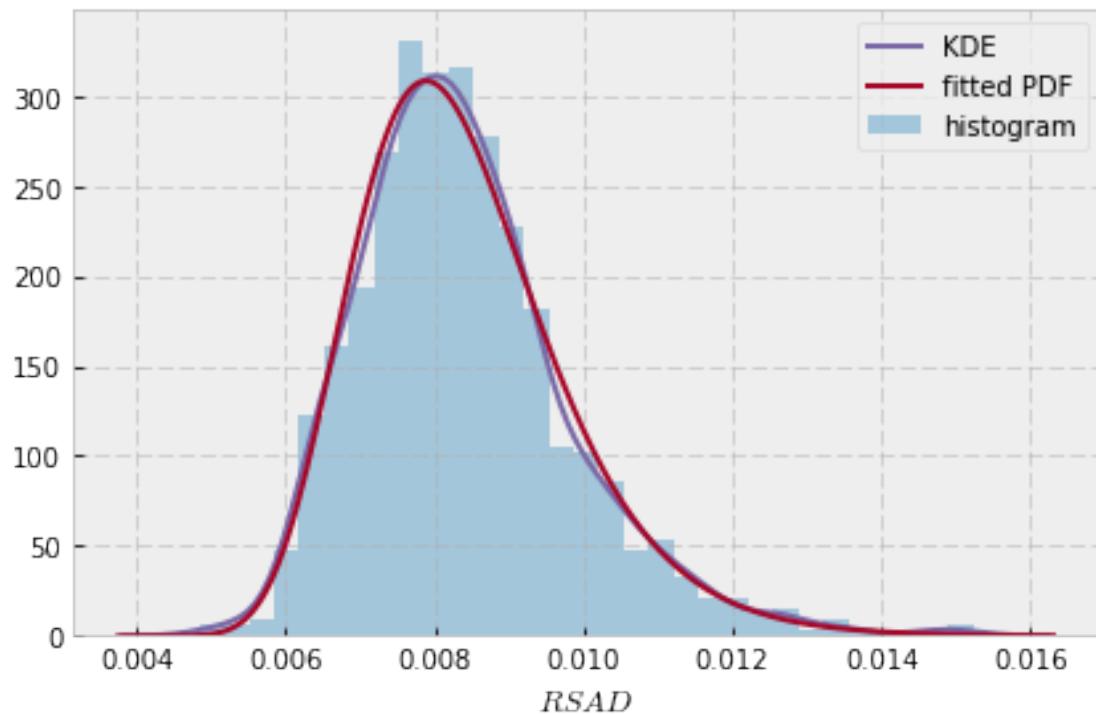


```
variable name: maxB
variable value: 8.0
distribution: inverse-gamma(shape=(1.1264462147973191,), loc=0.004278426322785342, scale=0.002437
DescribeResult(nobs=1000, minmax=(0.0044277644924108931, 0.018415062757330812)
               mean=0.0072939874805593285, variance=2.6259555571748564e-06,
               skewness=1.4568708147584148, kurtosis=4.4501167941910404)
```

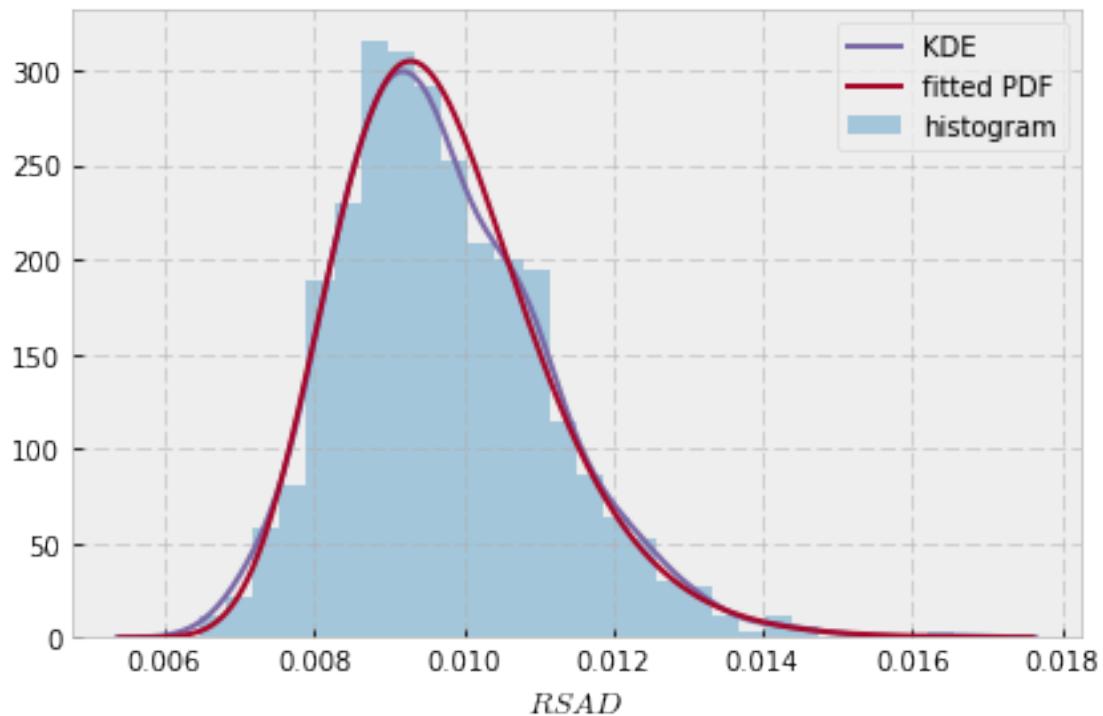


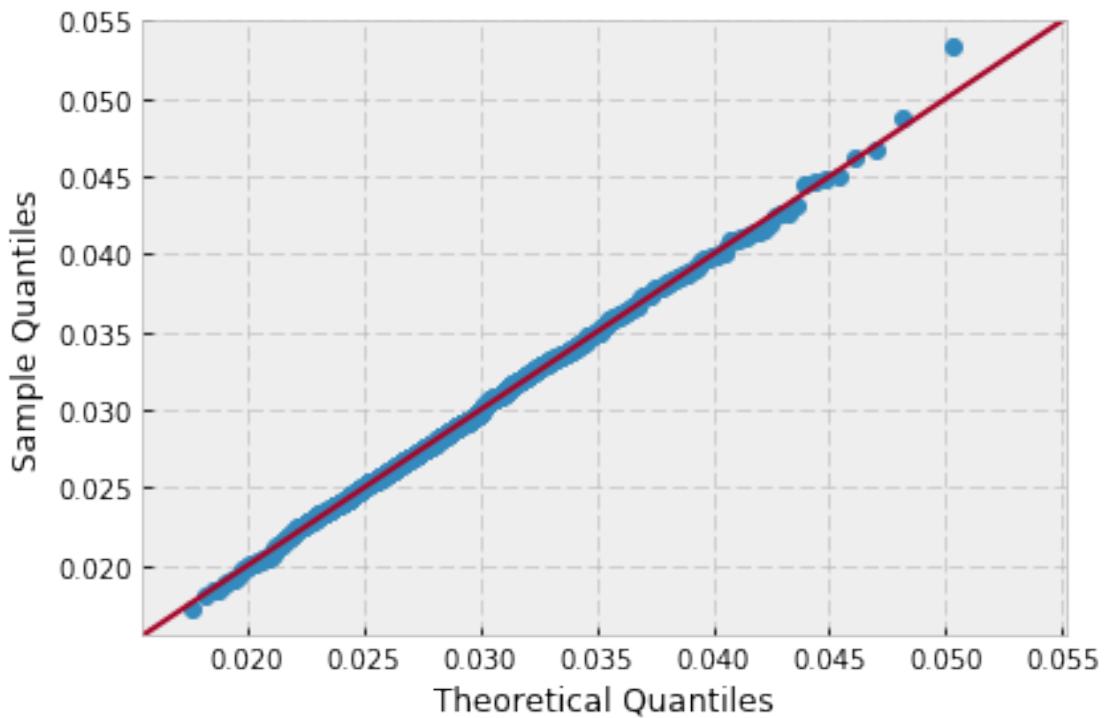


```
variable name: maxB
variable value: 12.0
distribution: inverse-gamma(shape=(26.513916118713126,), loc=0.0013612175018064801, scale=0.17938
DescribeResult(nobs=1000, minmax=(0.0048346555191728802, 0.015224129657372175)
               mean=0.0083928542174252323, variance=2.0569700156423505e-06,
               skewness=0.9748934141925566, kurtosis=1.8491181548794424)
```

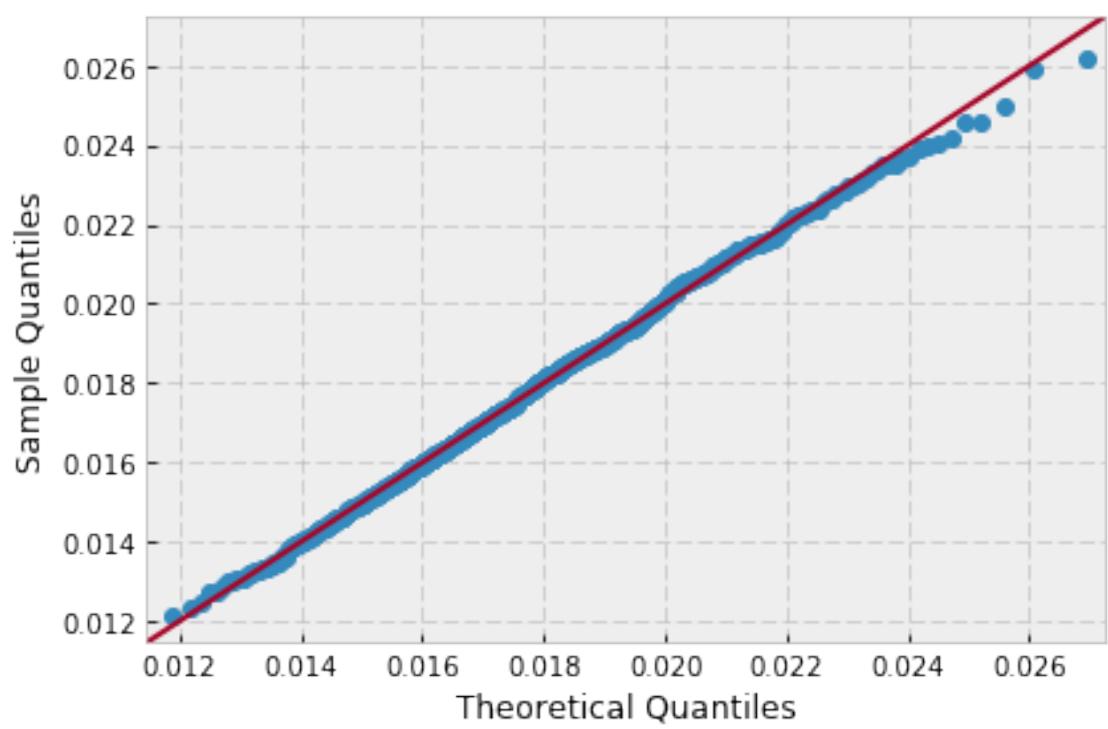
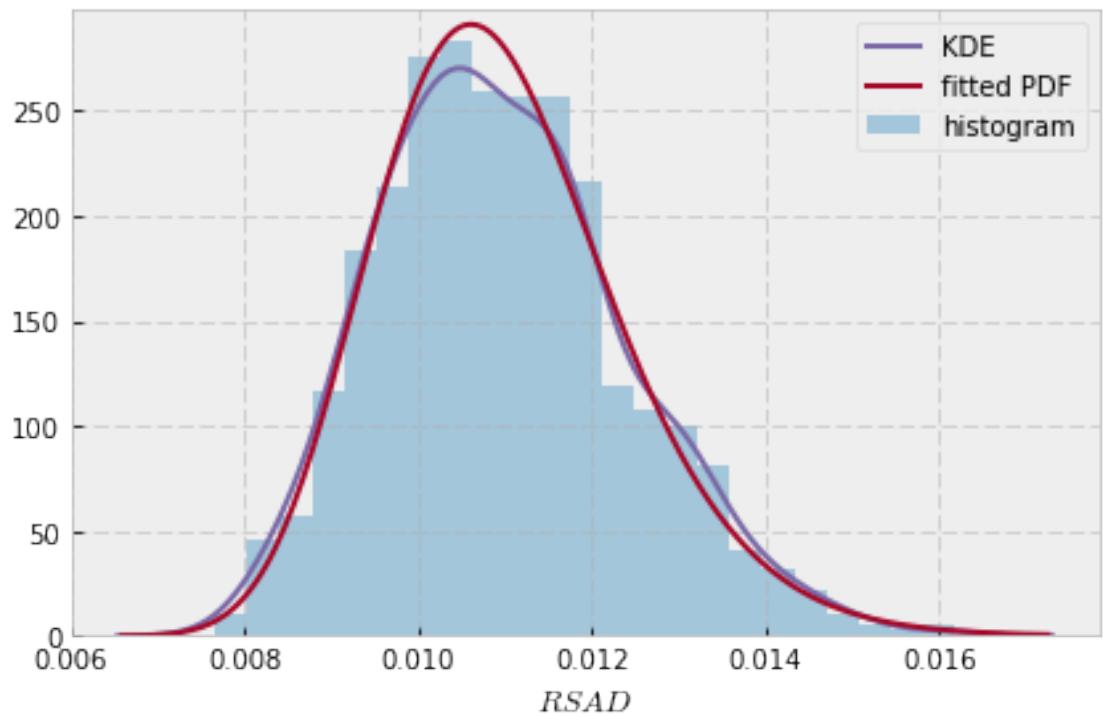


```
variable name: maxB
variable value: 16.0
distribution: inverse-gamma(shape=(35.50730214464663,), loc=0.0016190516067704948, scale=0.279988
DescribeResult(nobs=1000, minmax=(0.0064575259987842033, 0.016554085415759784)
               mean=0.0097328652005849257, variance=1.9619933716316191e-06,
               skewness=0.6839296808410613, kurtosis=0.7856091639347378)
```

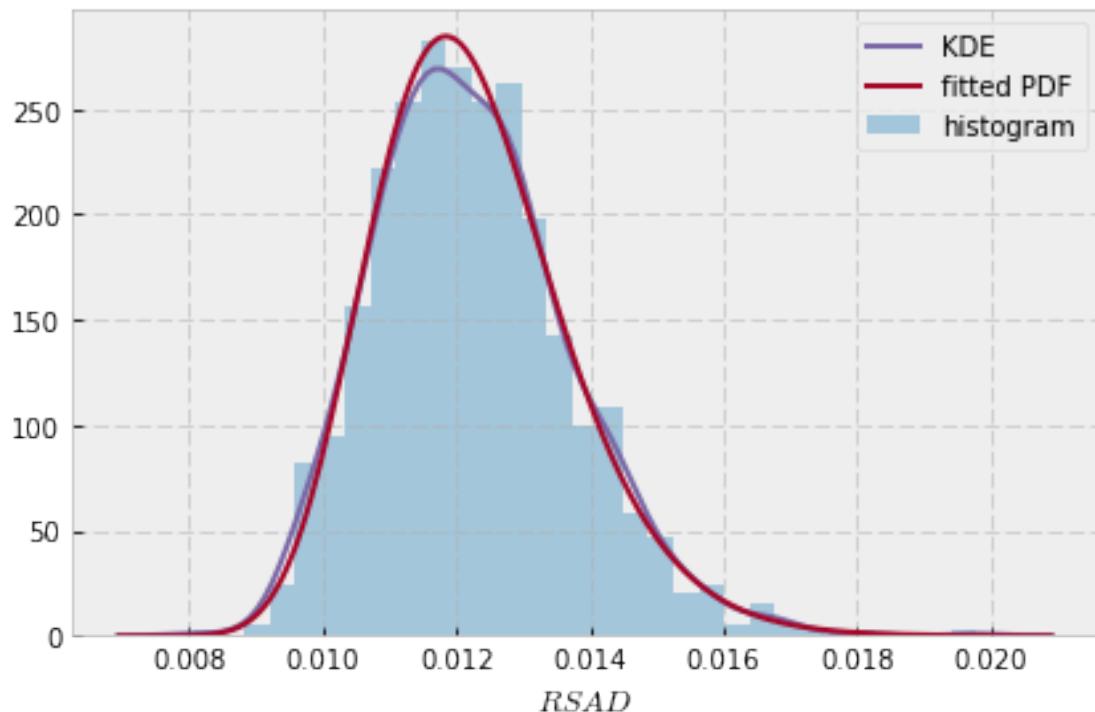


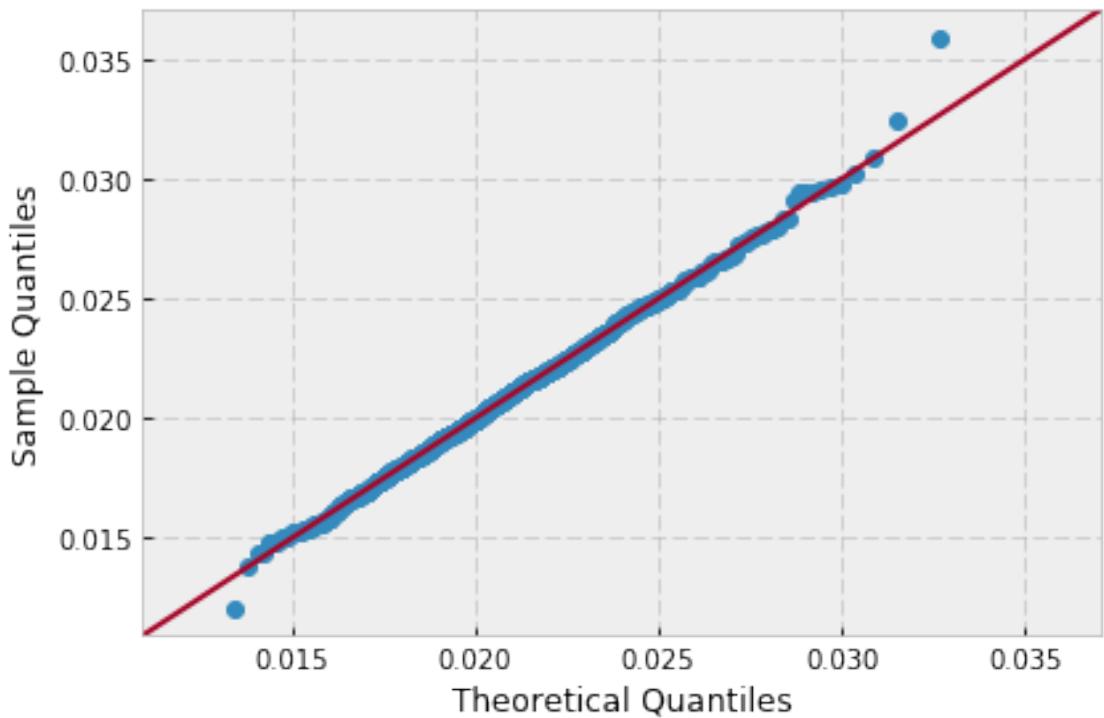


```
variable name: maxB
variable value: 20.0
distribution: inverse-gamma(shape=(57.825043765196327,),loc=0.00029110611753321717, scale=0.6070
DescribeResult(nobs=1000, minmax=(0.0076656679551990224, 0.016177976518571002)
               mean=0.010973699463210484, variance=2.0306015914343628e-06,
               skewness=0.4507710152318971, kurtosis=0.0371763165406227)
```

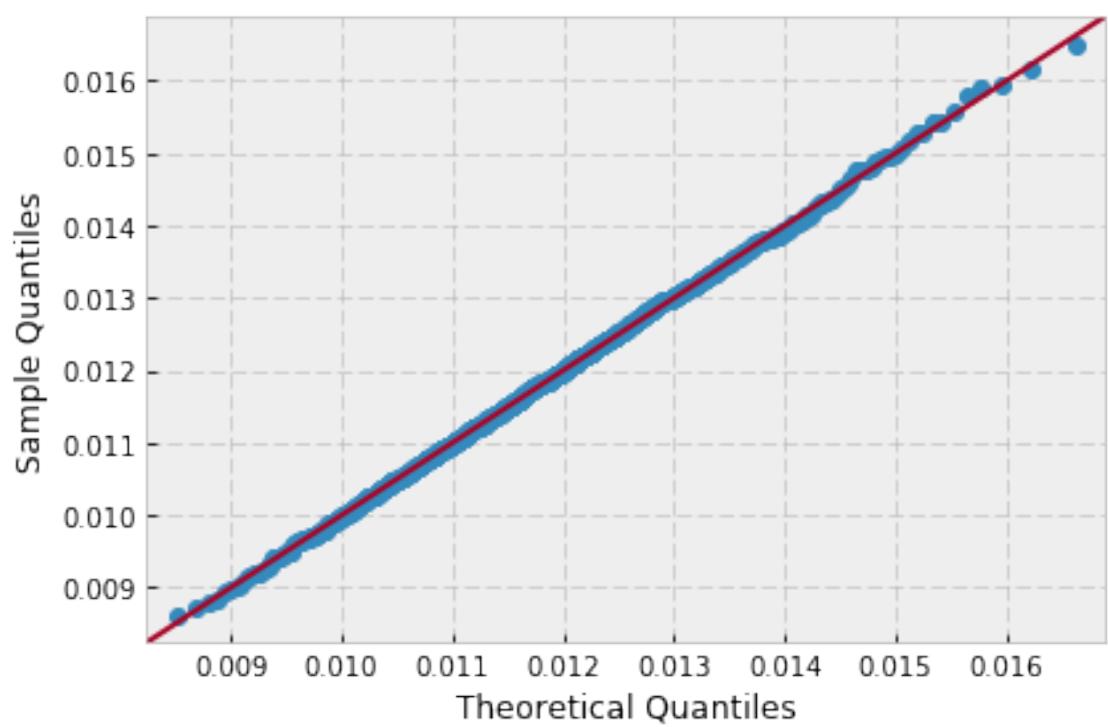
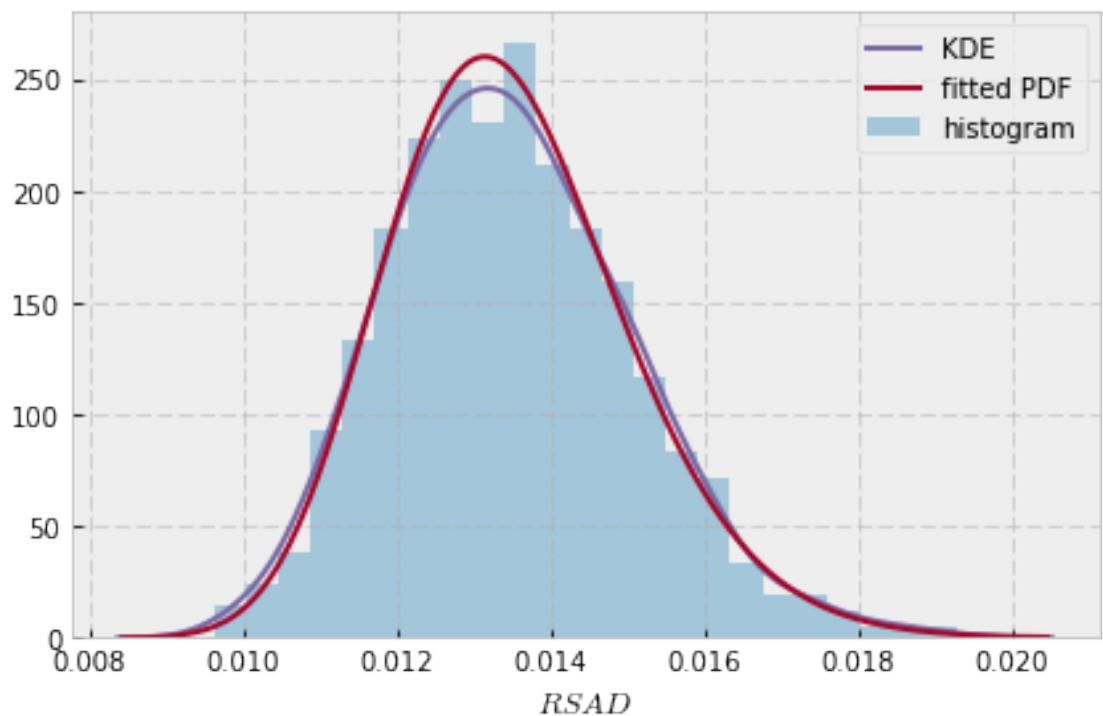


```
variable name: maxB
variable value: 24.0
distribution: inverse-gamma(shape=(49.59517474339097,), loc=0.0020807673650686457, scale=0.493850
DescribeResult(nobs=1000, minmax=(0.008059889540248855, 0.019786749556650753)
               mean=0.012243347947342557, variance=2.1751047074292555e-06,
               skewness=0.6104547786844097, kurtosis=0.8160312317035308)
```

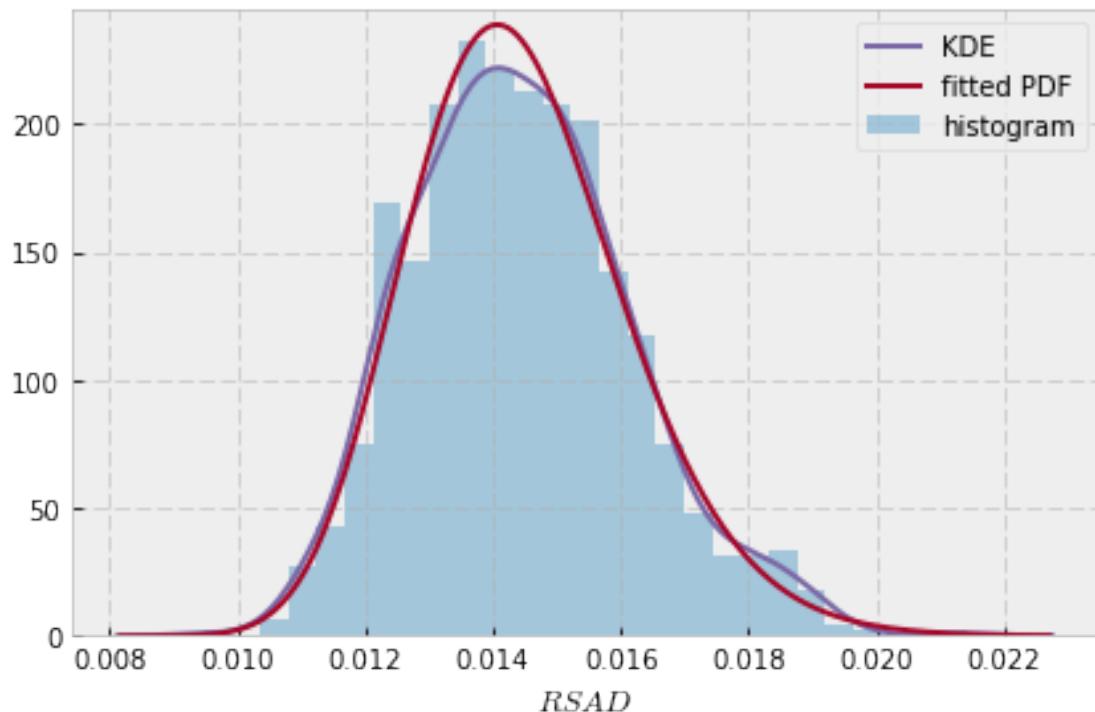


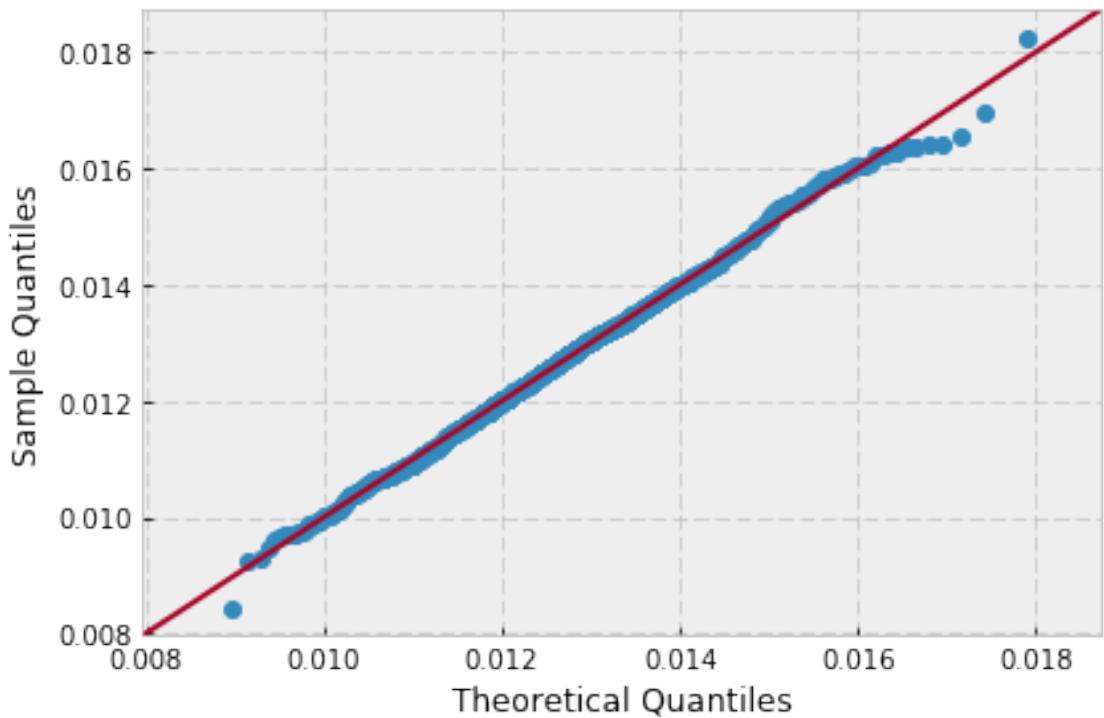


```
variable name: maxB
variable value: 28.0
distribution: inverse-gamma(shape=(85.96845676396299,), loc=-0.0009969350610204812, scale=1.22917
DescribeResult(nobs=1000, minmax=(0.0096073077605766301, 0.019268831746620838)
               mean=0.013469102496744764, variance=2.4897968041065446e-06,
               skewness=0.39907025674812785, kurtosis=0.16171211679277064)
```

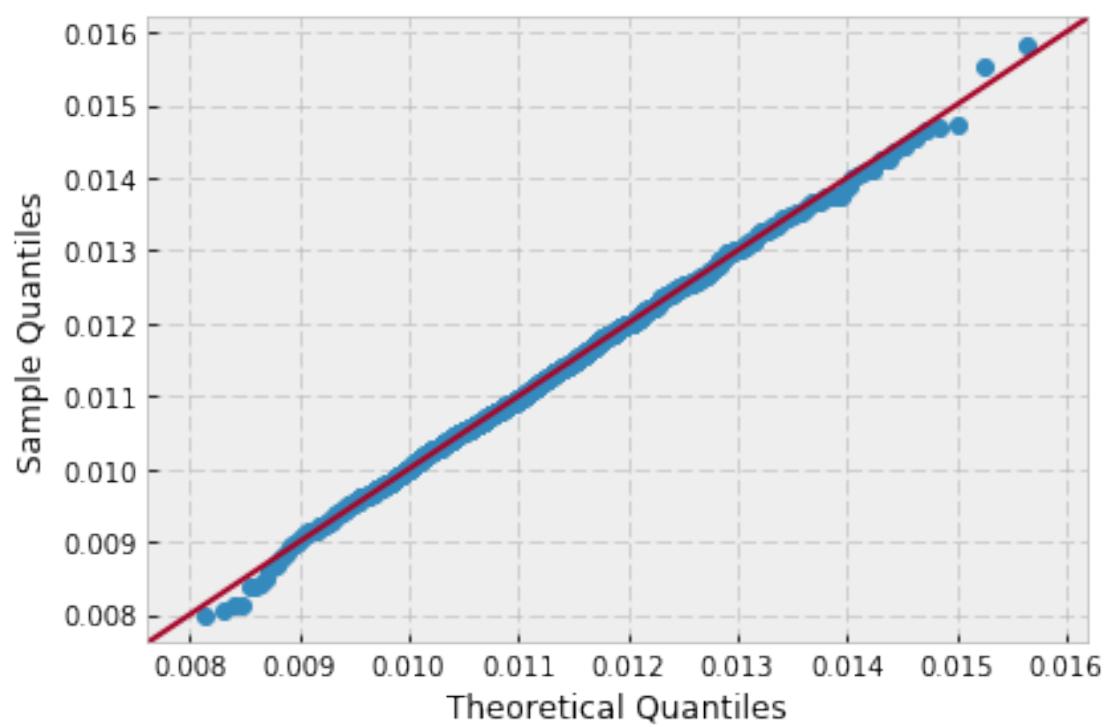
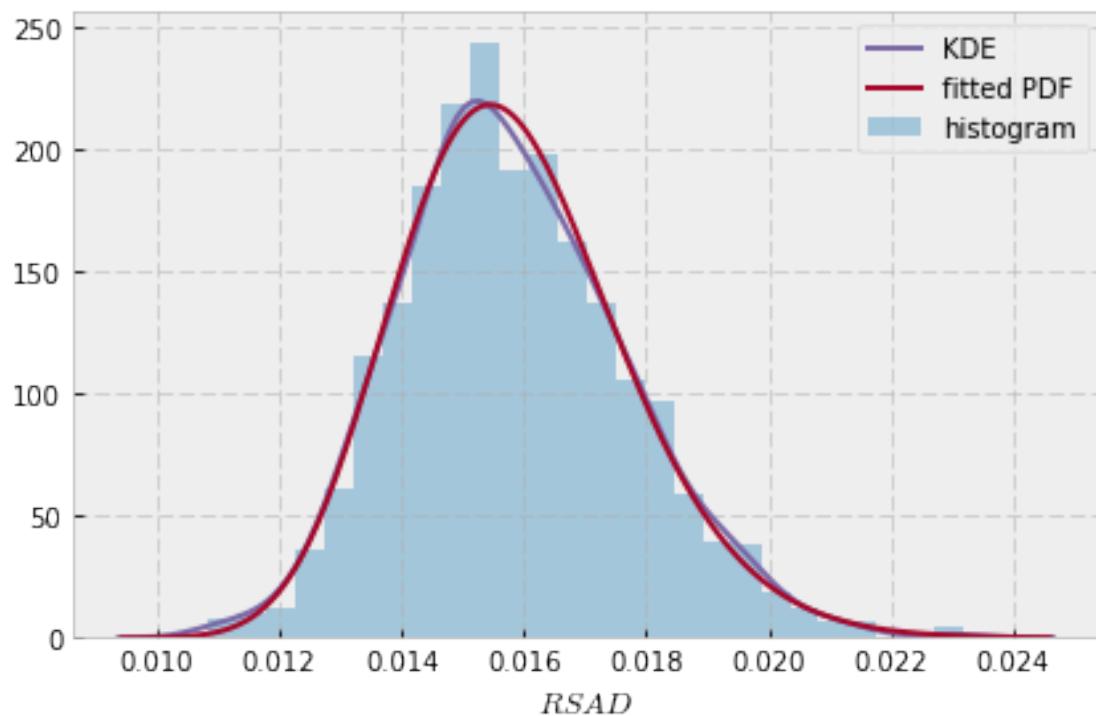


```
variable name: maxB
variable value: 32.0
distribution: inverse-gamma(shape=(80.852973226652409,), loc=-0.0008550708480592451, scale=1.2215
DescribeResult(nobs=1000, minmax=(0.0094658805510091874, 0.021413989077032725)
               mean=0.014441980730009947, variance=2.9627273215951875e-06,
               skewness=0.41647357363225324, kurtosis=0.13067074344071328)
```

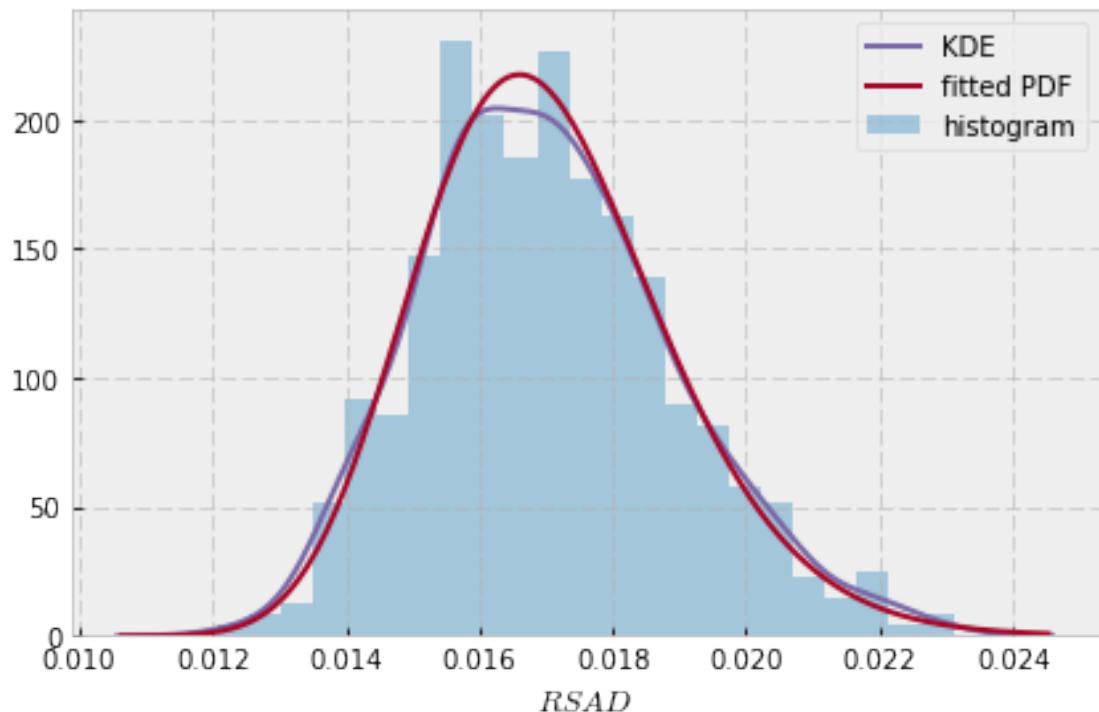


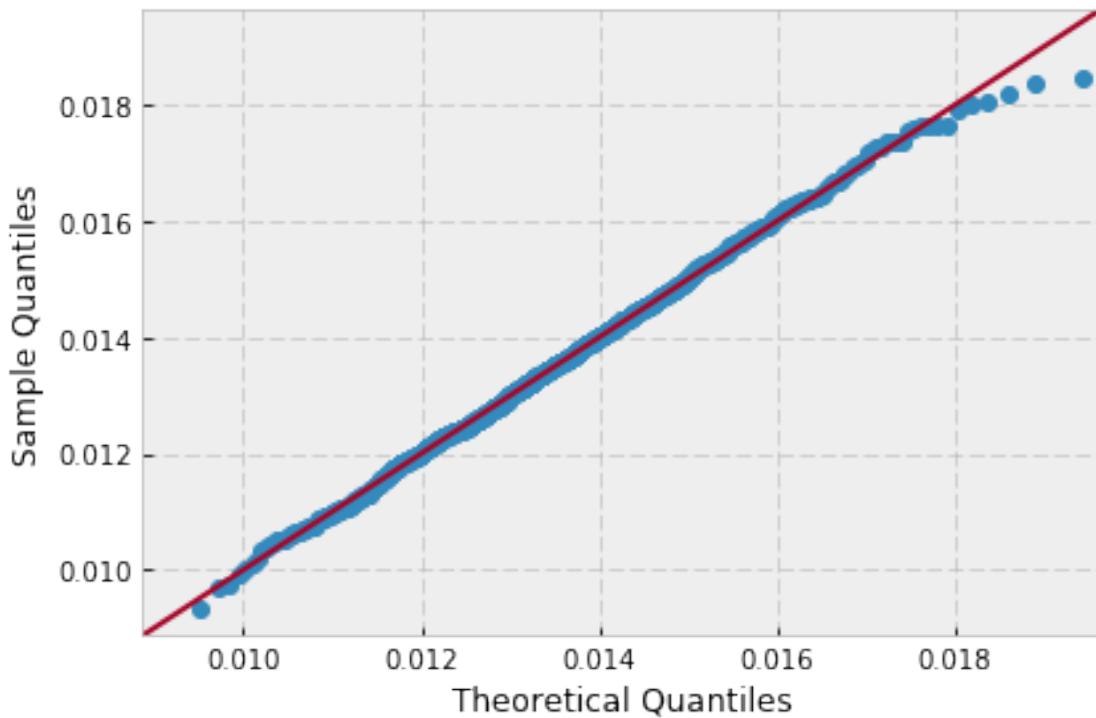


```
variable name: maxB
variable value: 36.0
distribution: inverse-gamma(shape=(90.557856496758404,), loc=-0.001864843462089565, scale=1.58503
DescribeResult(nobs=1000, minmax=(0.010822357636810775, 0.02320483589557306)
               mean=0.015823446142574131, variance=3.4646732488544428e-06,
               skewness=0.38419386775340314, kurtosis=0.2371120554631876)
```

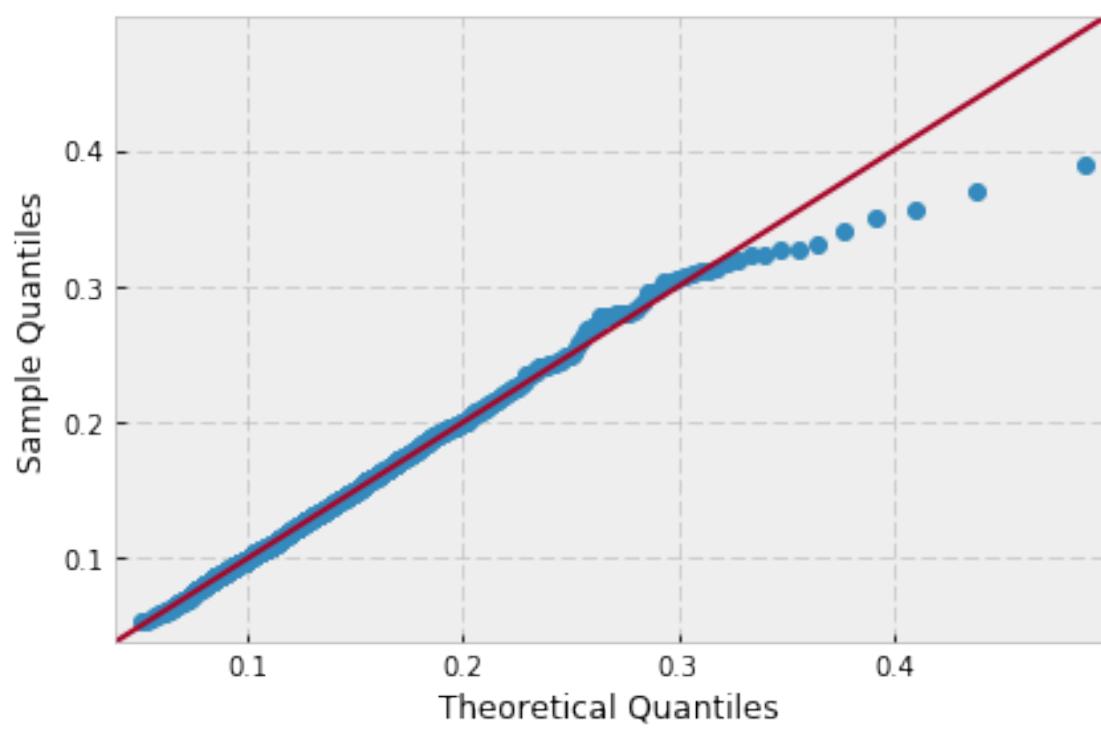
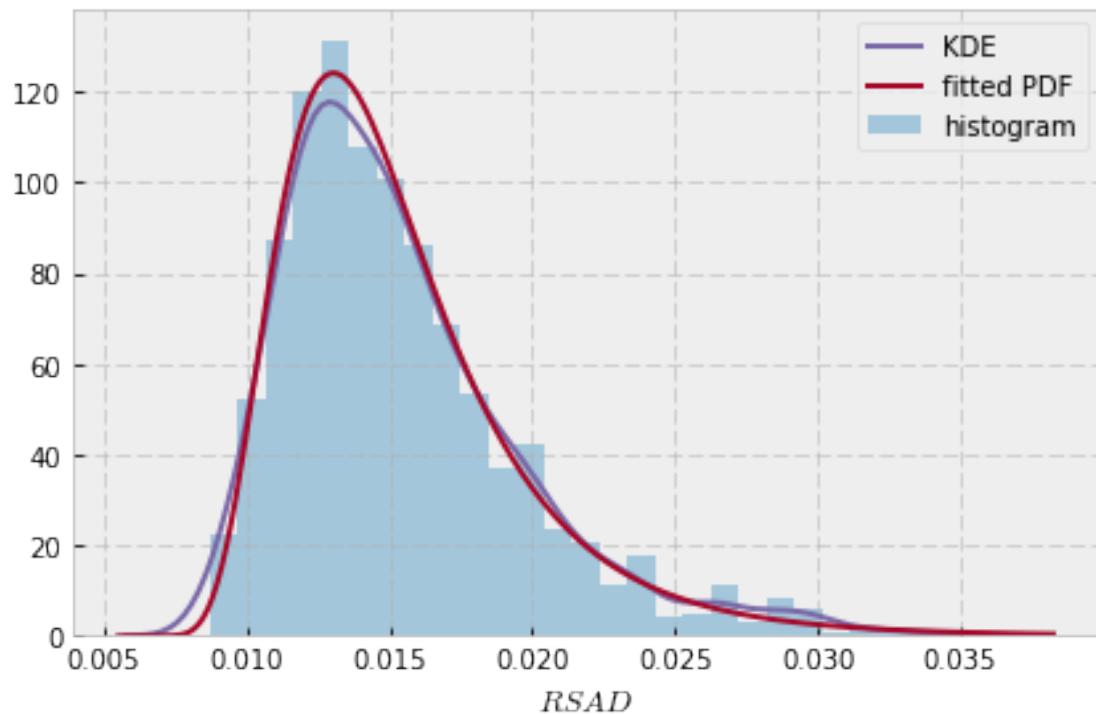


```
variable name: maxB
variable value: 40.0
distribution: inverse-gamma(shape=(75.478820490434572,), loc=0.0007955338250840059, scale=1.20887
DescribeResult(nobs=1000, minmax=(0.012057133405007683, 0.023105287618328533)
               mean=0.017026504052727268, variance=3.570724019025469e-06,
               skewness=0.41208659699791766, kurtosis=0.020266406316899577)
```

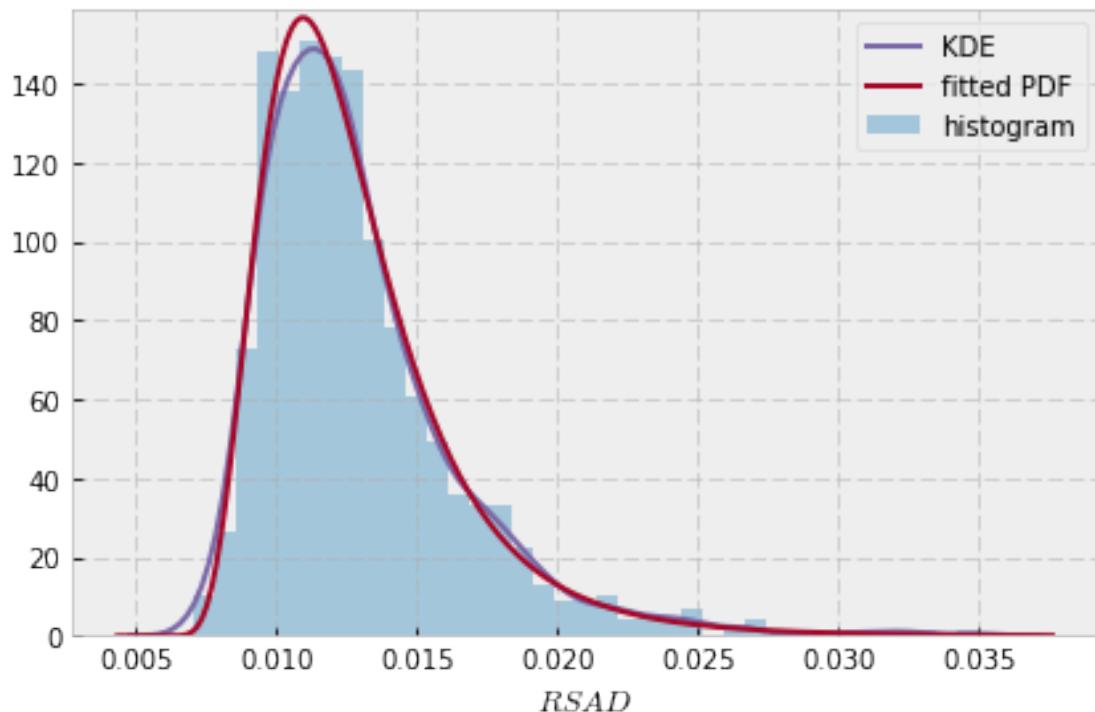


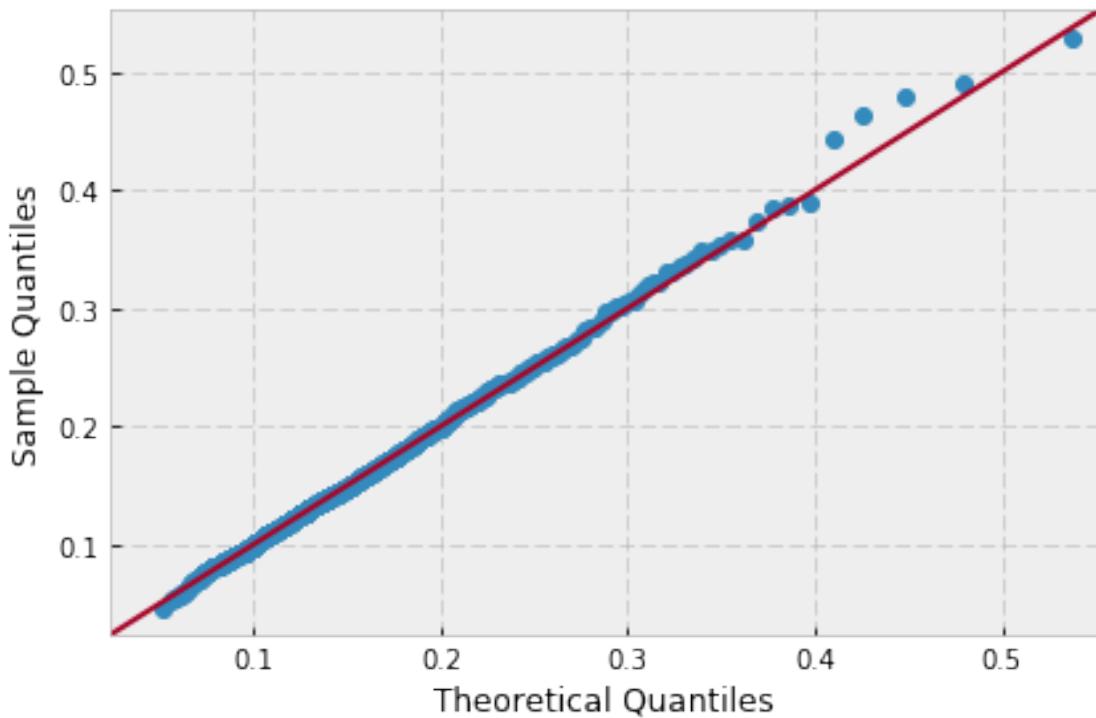


```
variable name: r
variable value: 1.0
distribution: inverse-gamma(shape=(8.1563366351904349,),loc=0.004457959798718492, scale=0.078603
DescribeResult(nobs=1000, minmax=(0.0087339803245598761, 0.03502693387496876)
               mean=0.015428158397178376, variance=1.8193898960348825e-05,
               skewness=1.3097990081876463, kurtosis=2.0686283379007264)
```

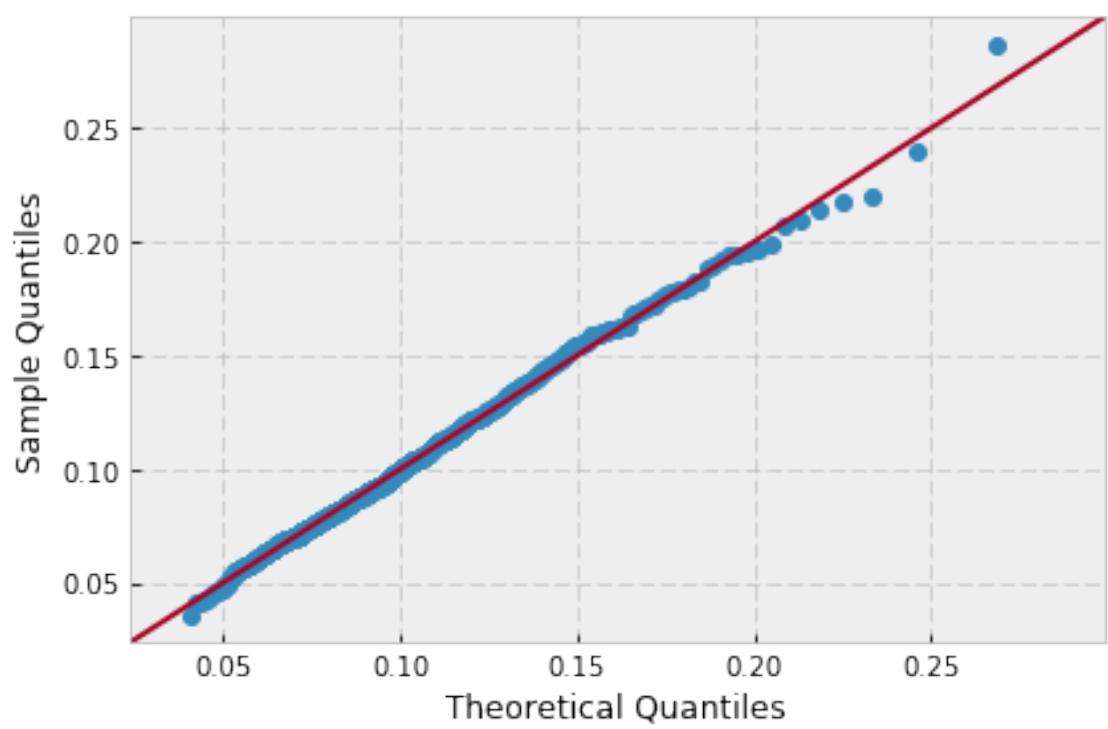
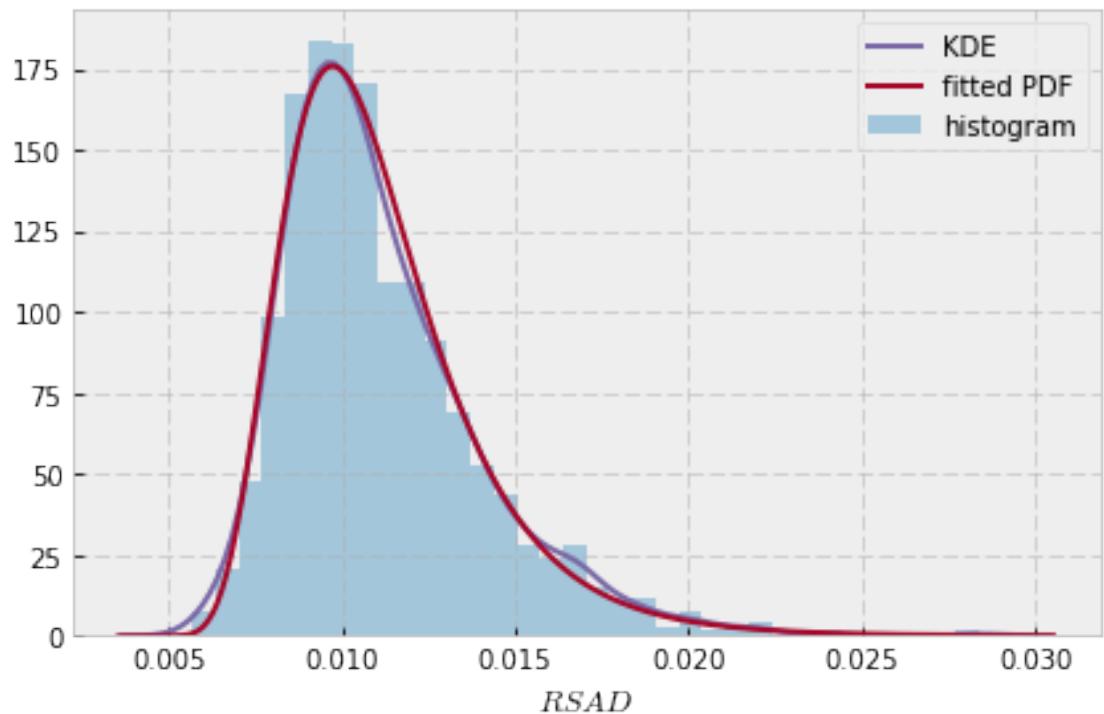


```
variable name: r
variable value: 1.22222222222223
distribution: inverse-gamma(shape=(7.7610241077300595,),loc=0.004333302170693927, scale=0.057931
DescribeResult(nobs=1000, minmax=(0.0070225889223939214, 0.034949522000928067)
               mean=0.012902845483428198, variance=1.2663127070609843e-05,
               skewness=1.7684297452852247, kurtosis=5.096435124957951)
```

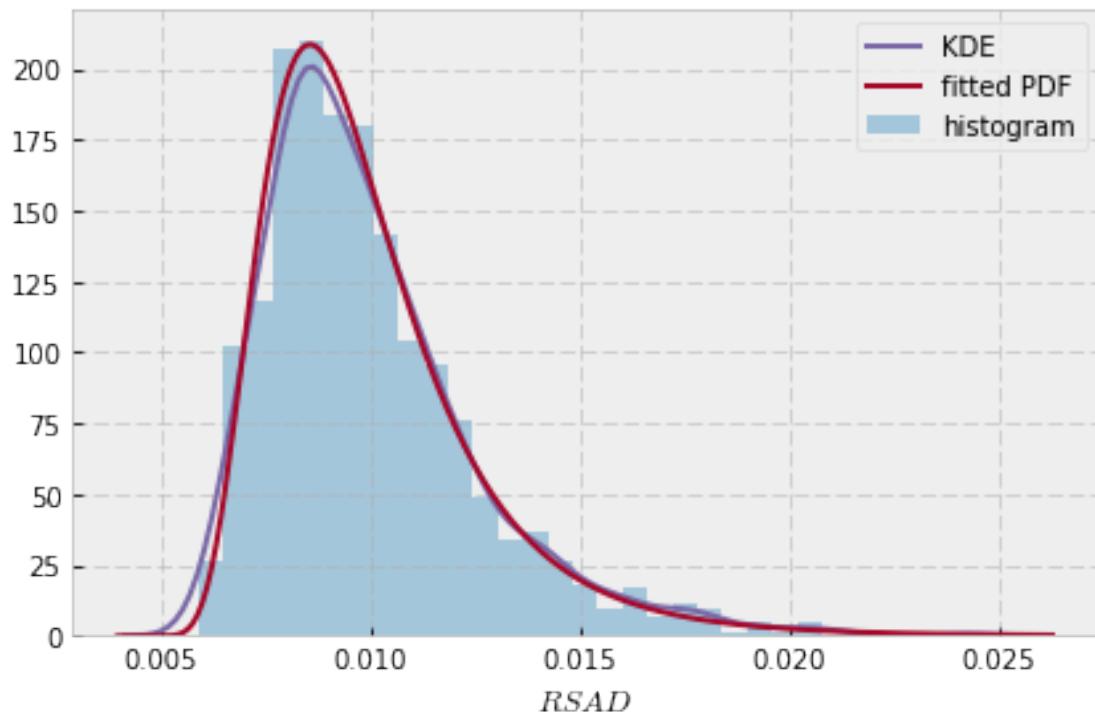


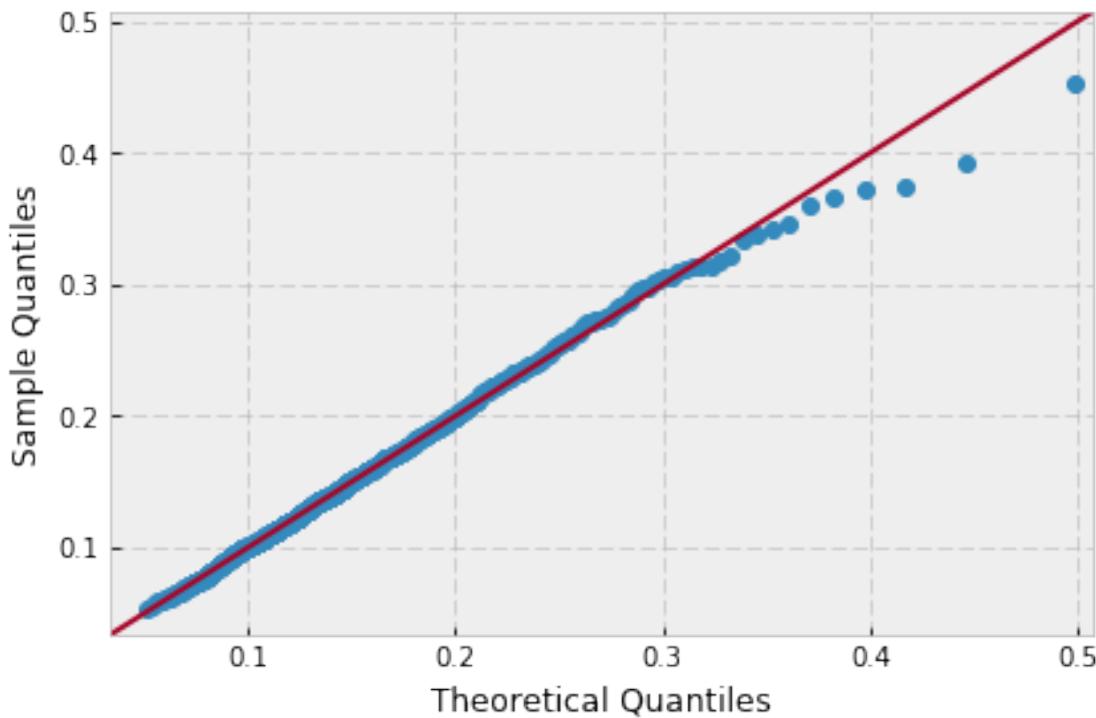


```
variable name: r
variable value: 1.444444444444444
distribution: inverse-gamma(shape=(11.424378901478057,),loc=0.0024168642108182616, scale=0.09069
DescribeResult(nobs=1000, minmax=(0.0056736300083330971, 0.028399665439922249)
               mean=0.01111768585239872, variance=7.9863459566852043e-06,
               skewness=1.3012244966091975, kurtosis=2.6704496573515595)
```

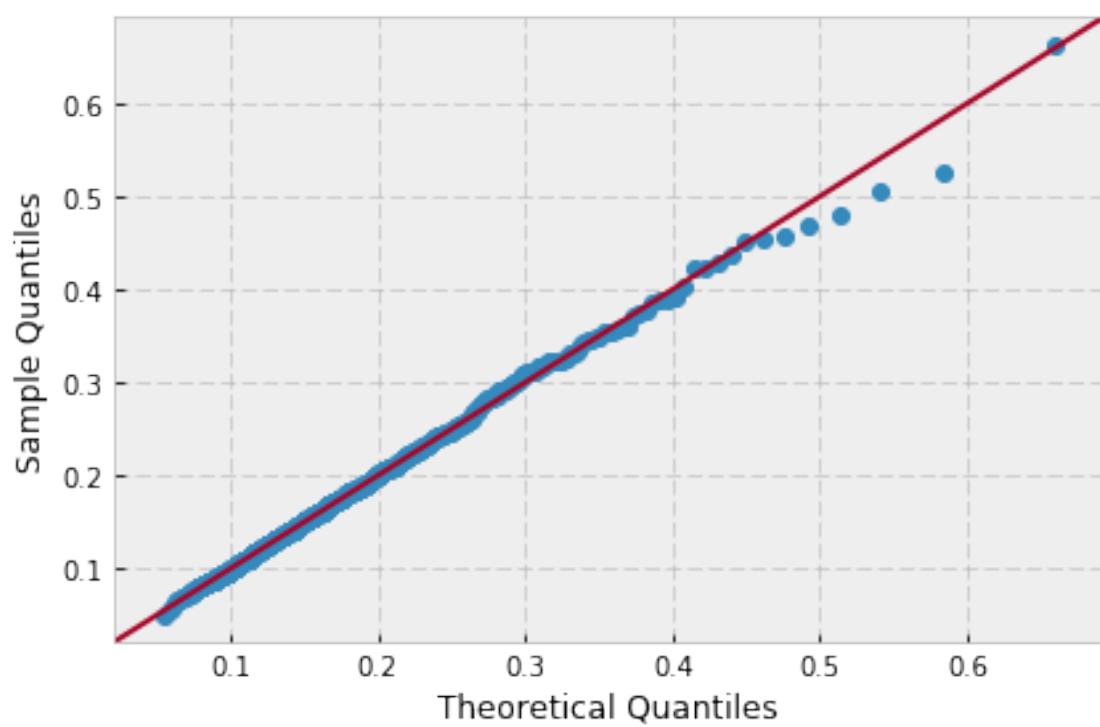
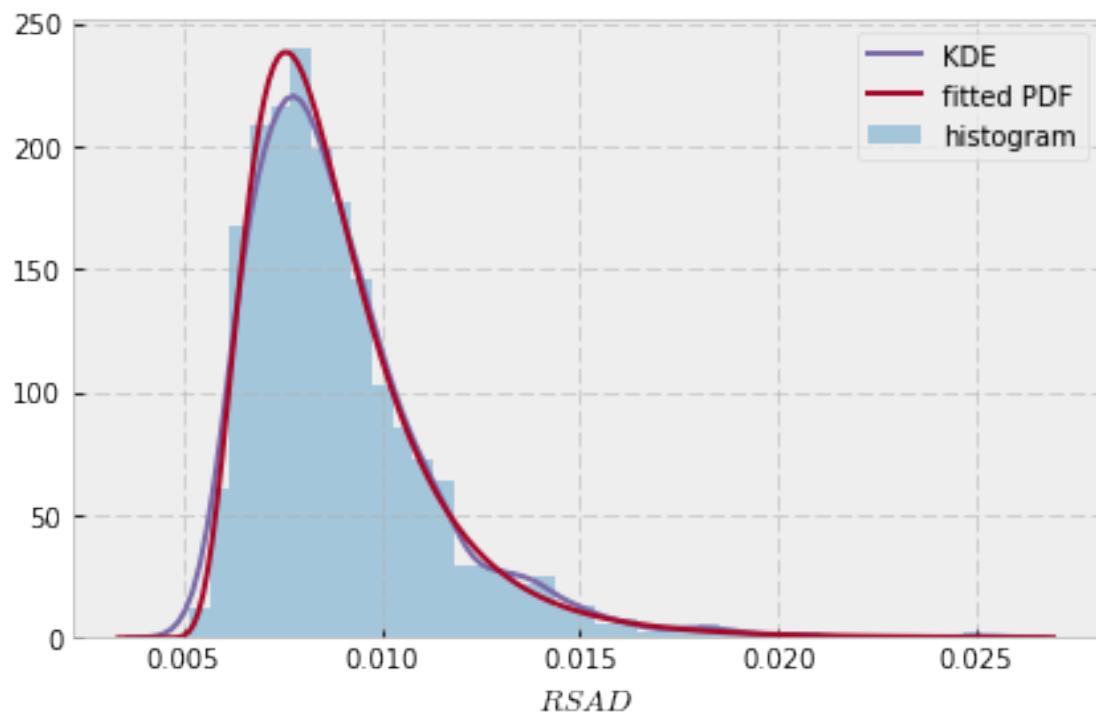


```
variable name: r
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(8.0780933085338091,), loc=0.0034731543827904374, scale=0.04609
DescribeResult(nobs=1000, minmax=(0.005901868174266584, 0.024323138404621372)
    mean=0.0099784072910387104, variance=6.5773760684445872e-06,
    skewness=1.4315676821710208, kurtosis=2.8868062731769877)
```

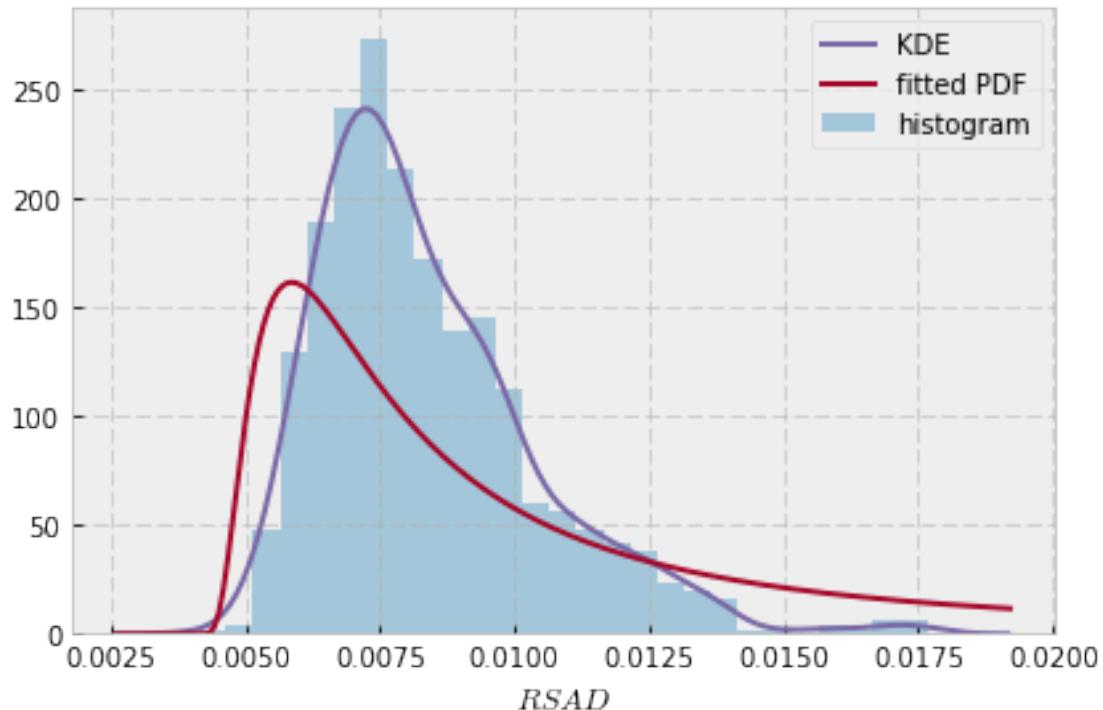


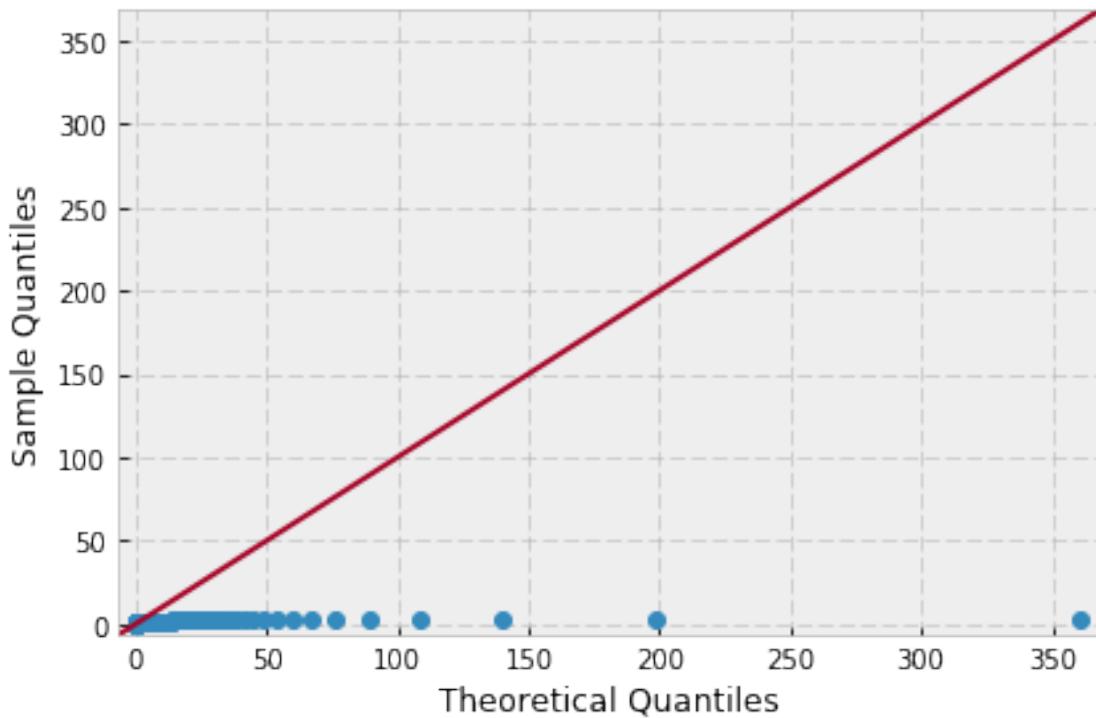


```
variable name: r
variable value: 1.888888888888888
distribution: inverse-gamma(shape=(6.9916147564269089,), loc=0.0034736644777170625, scale=0.03275
DescribeResult(nobs=1000, minmax=(0.0051405279502334429, 0.025154894149935891)
               mean=0.0089342012140586208, variance=5.6416951705792034e-06,
               skewness=1.6912978441614175, kurtosis=4.633155447558597)
```

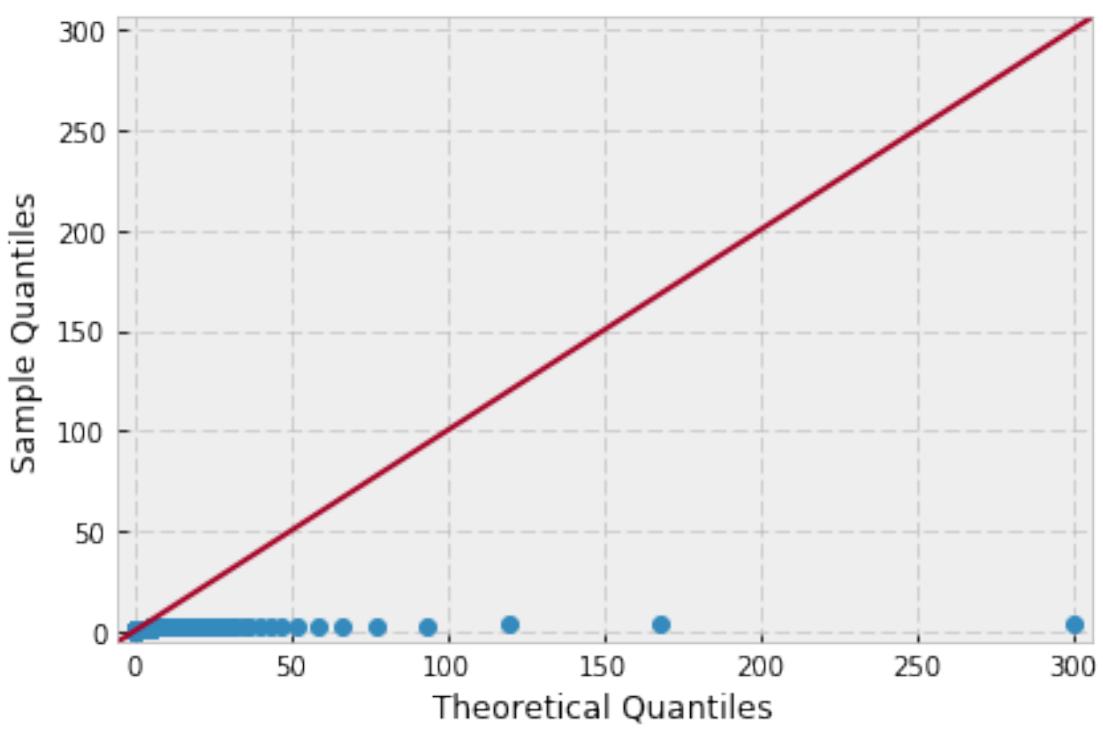
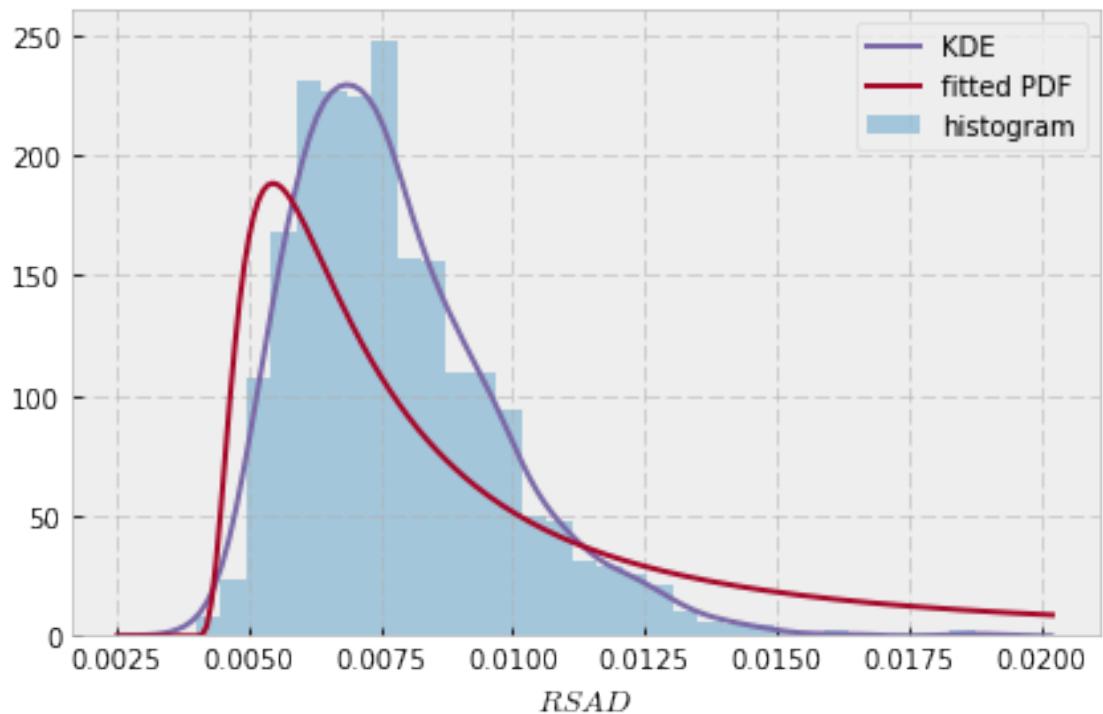


```
variable name: r
variable value: 2.11111111111111
distribution: inverse-gamma(shape=(1.1600502899275682,),loc=0.003983902481026335, scale=0.004056
DescribeResult(nobs=1000, minmax=(0.0041363696830300717, 0.017656586920700226)
    mean=0.0083709138090768392, variance=4.2123313634018187e-06,
    skewness=1.220263963066044, kurtosis=1.999298532495012)
```

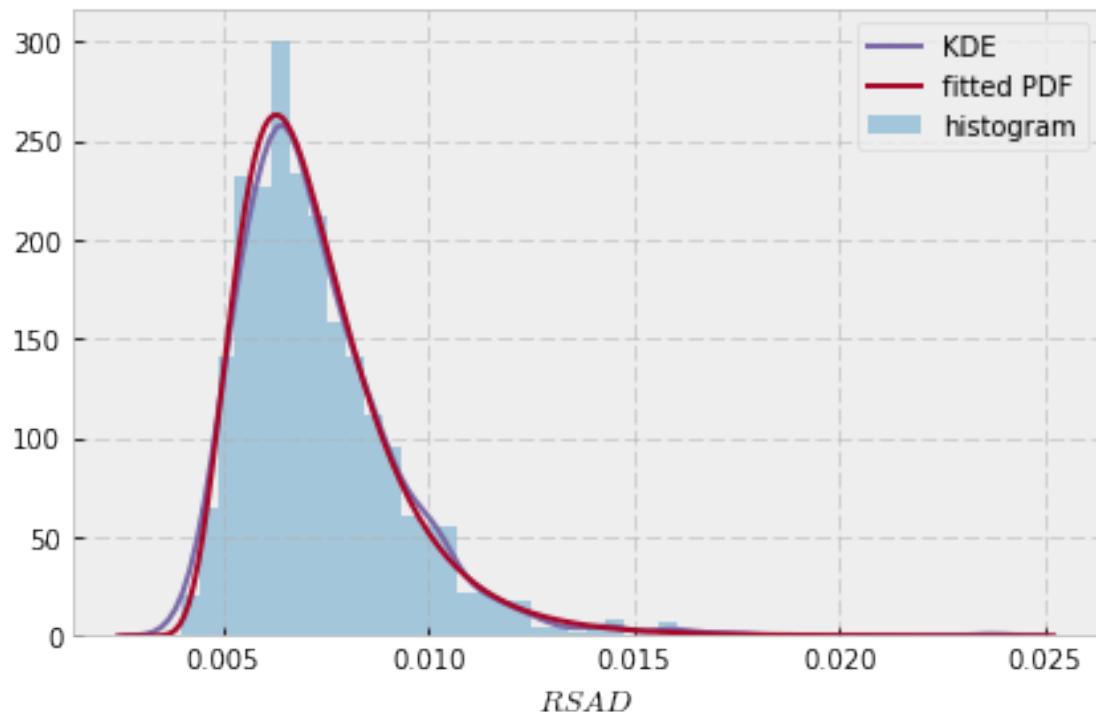


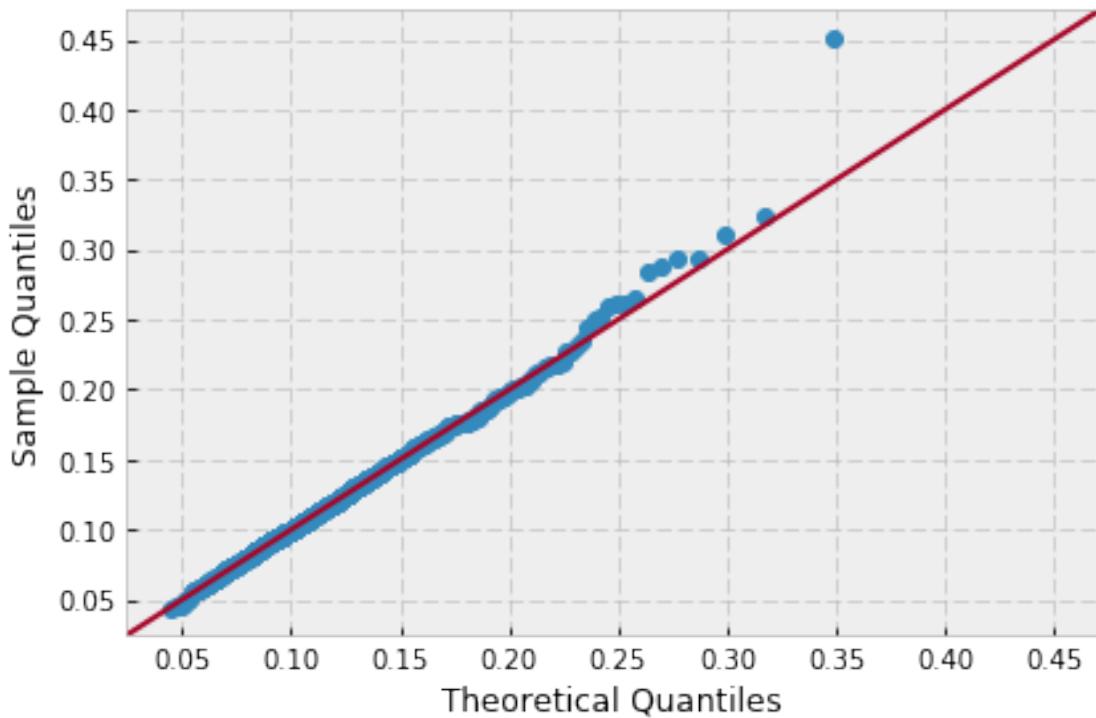


```
variable name: r
variable value: 2.333333333333333
distribution: inverse-gamma(shape=(1.1942624383031737,), loc=0.0038176042393212235, scale=0.00360
DescribeResult(nobs=1000, minmax=(0.0039926804228801833, 0.018738868225934954)
               mean=0.0077935931716035271, variance=3.8136235206448161e-06,
               skewness=1.0660683323359006, kurtosis=1.6290930638738725)
```

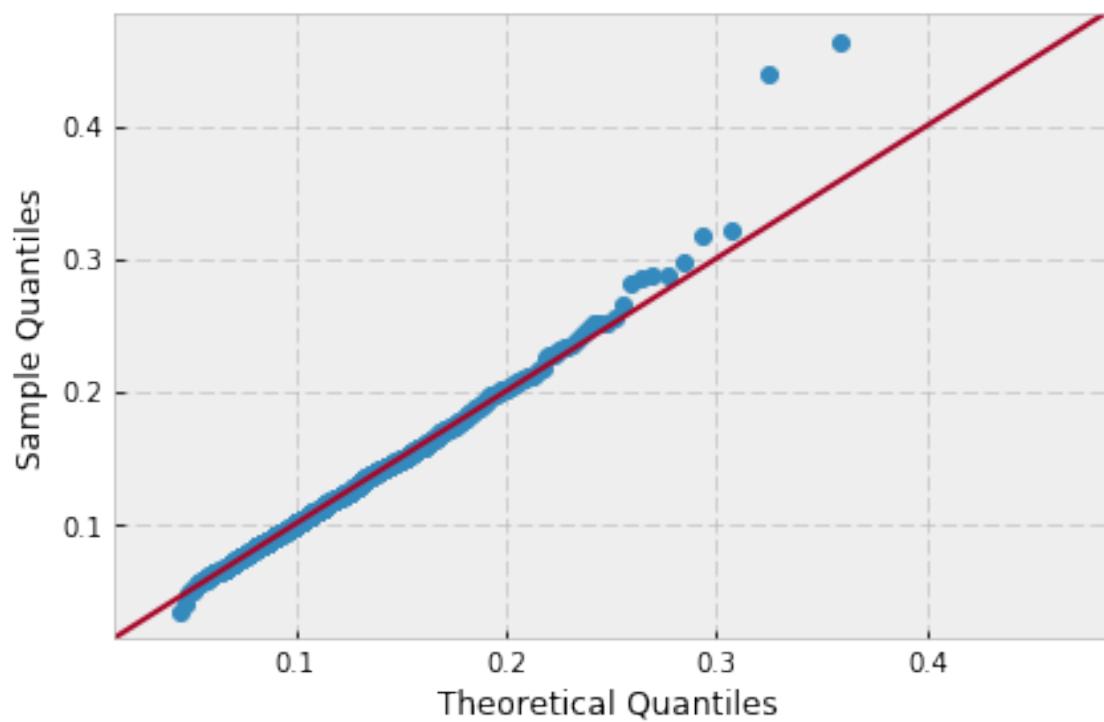
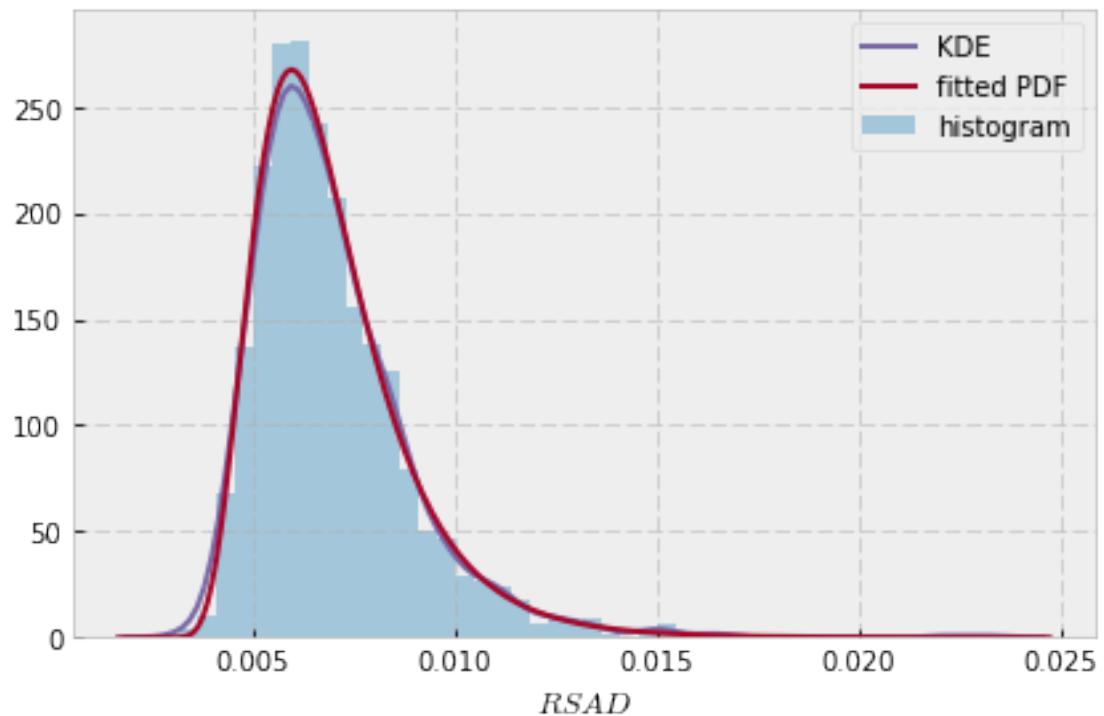


```
variable name: r
variable value: 2.5555555555555554
distribution: inverse-gamma(shape=(9.8168343844151273,),loc=0.0018066084553869008, scale=0.04857
DescribeResult(nobs=1000, minmax=(0.0039644356537438105, 0.023702439604706891)
    mean=0.0073177149086470095, variance=3.9705097300222463e-06,
    skewness=1.8008885766207356, kurtosis=6.94872495843007)
```

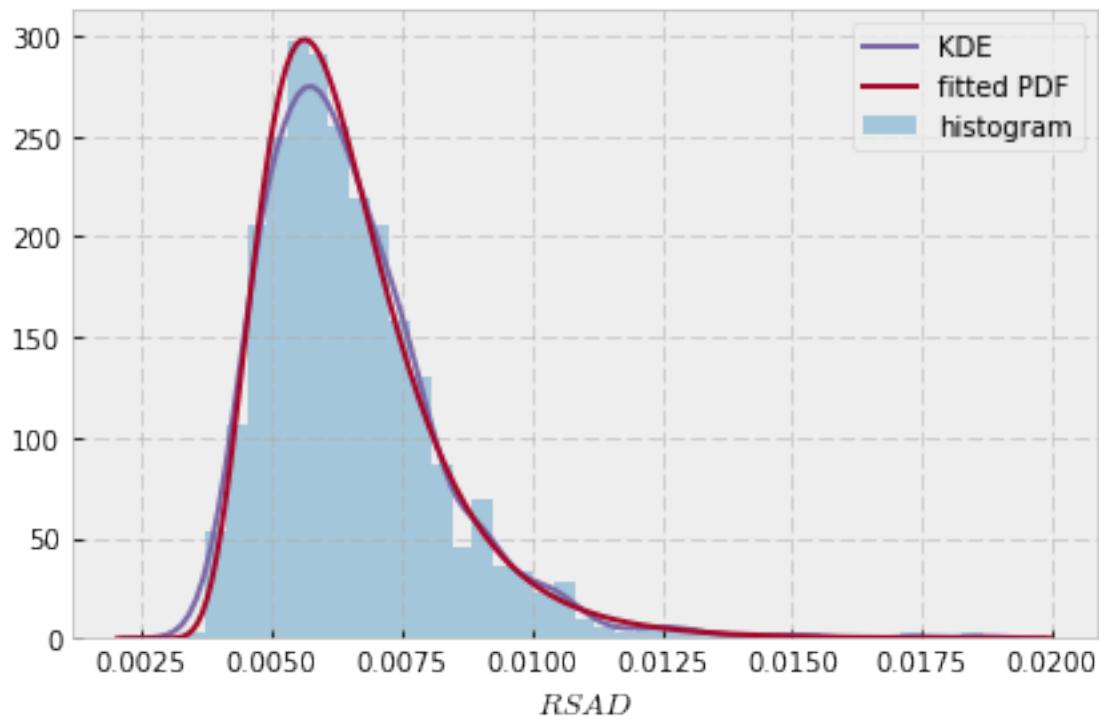


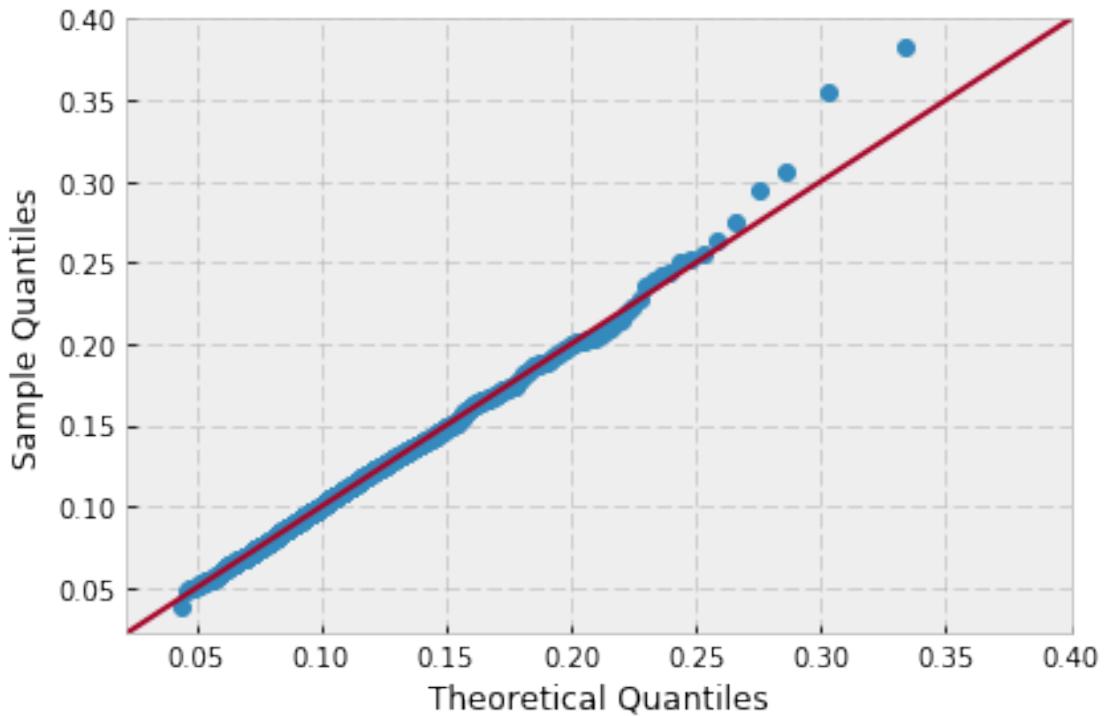


```
variable name: r
variable value: 2.777777777777777
distribution: inverse-gamma(shape=(9.6681604495039046,), loc=0.001594096813728357, scale=0.046730
DescribeResult(nobs=1000, minmax=(0.0032071272919654832, 0.023217522592267868)
               mean=0.0069914014039176983, variance=4.0591180036187449e-06,
               skewness=2.074933377114107, kurtosis=9.037283833058103)
```

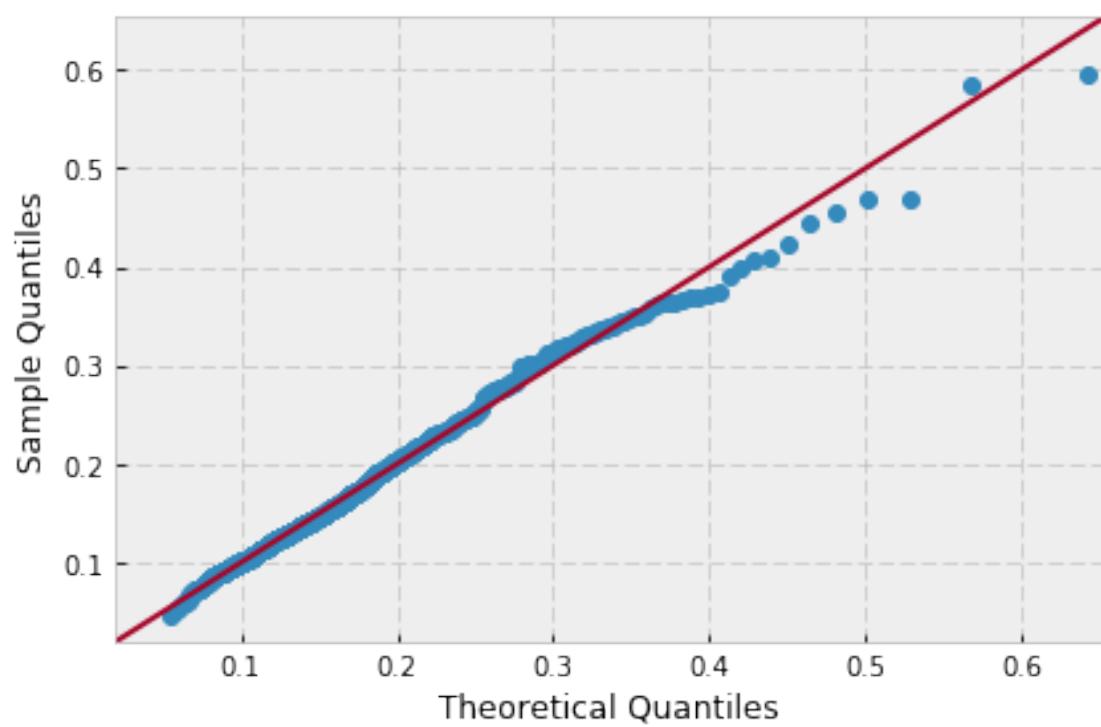
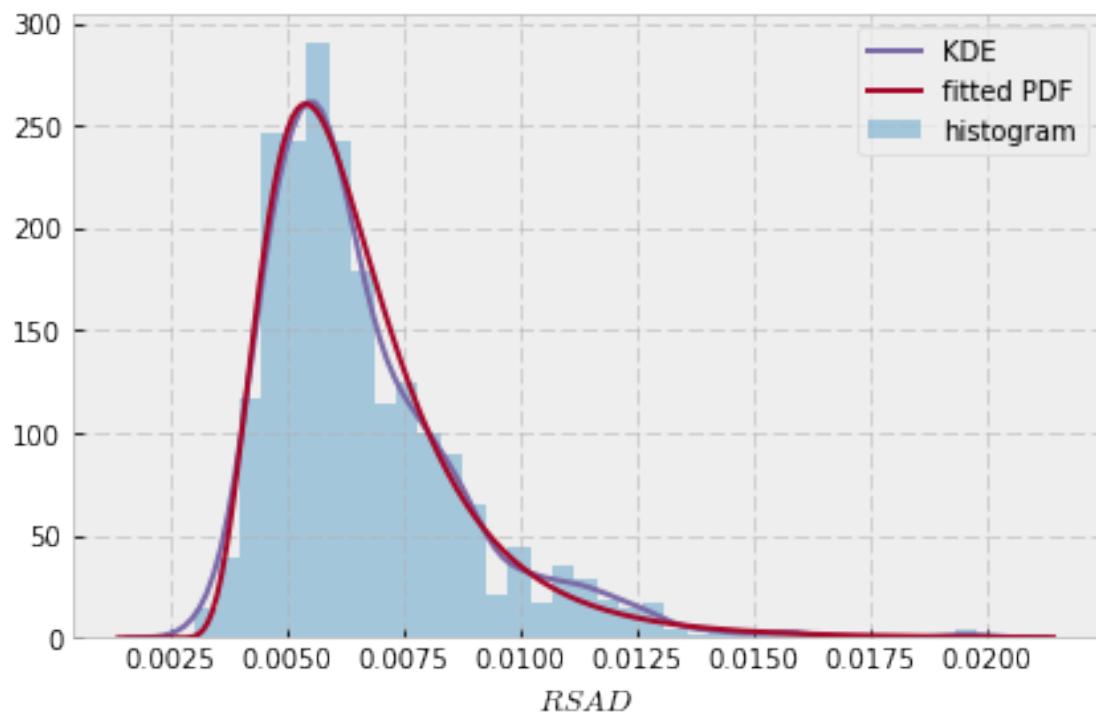


```
variable name: r
variable value: 3.0
distribution: inverse-gamma(shape=(10.072881967754455,),loc=0.0016132988282657559, scale=0.04450
DescribeResult(nobs=1000, minmax=(0.0033496172211548007, 0.018671024263326227)
    mean=0.0065191569392732858, variance=2.9995121204153563e-06,
    skewness=1.6593599862442925, kurtosis=5.565330412934774)
```

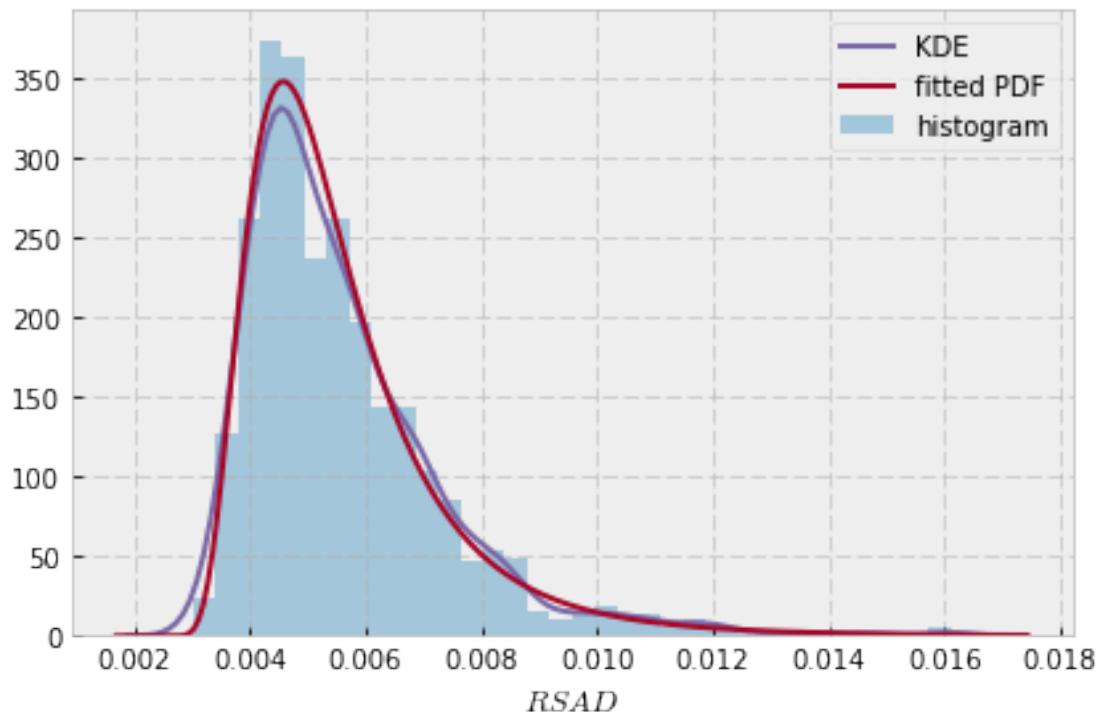


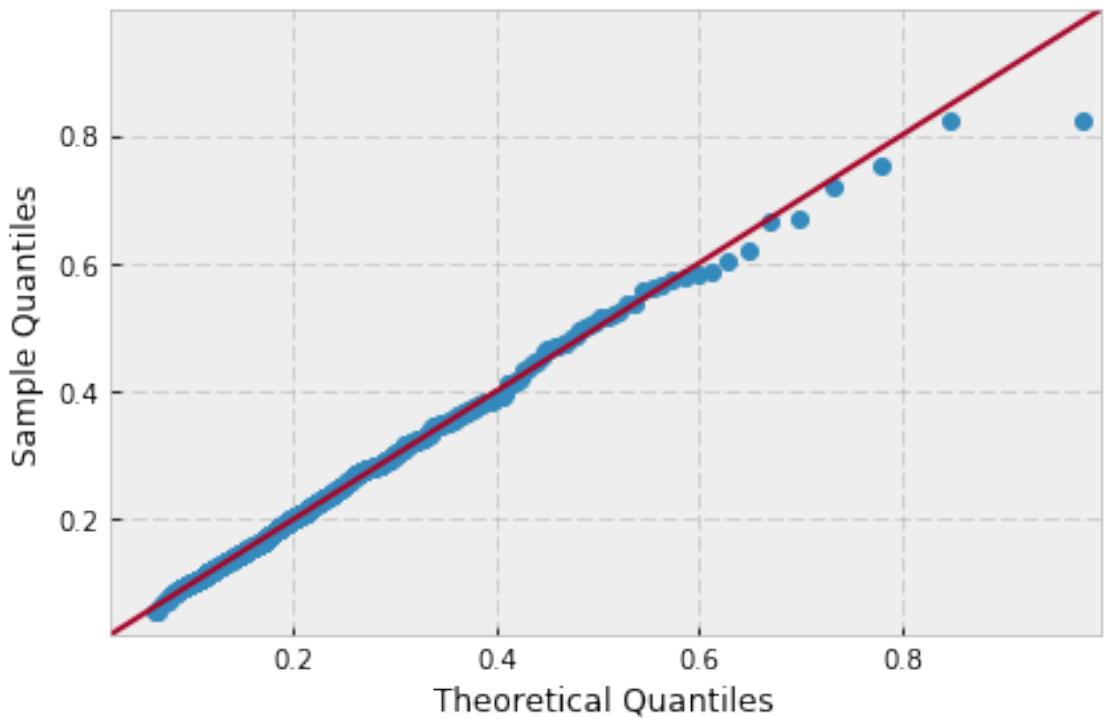


```
variable name: sigma
variable value: 0.0111111111111112
distribution: inverse-gamma(shape=(7.0887957726797417,), loc=0.001643935175597782, scale=0.030535
DescribeResult(nobs=1000, minmax=(0.003042136029024531, 0.01980850242669448)
               mean=0.0066564561036142099, variance=4.7050292000106014e-06,
               skewness=1.6082347945709212, kurtosis=3.852450766011435)
```

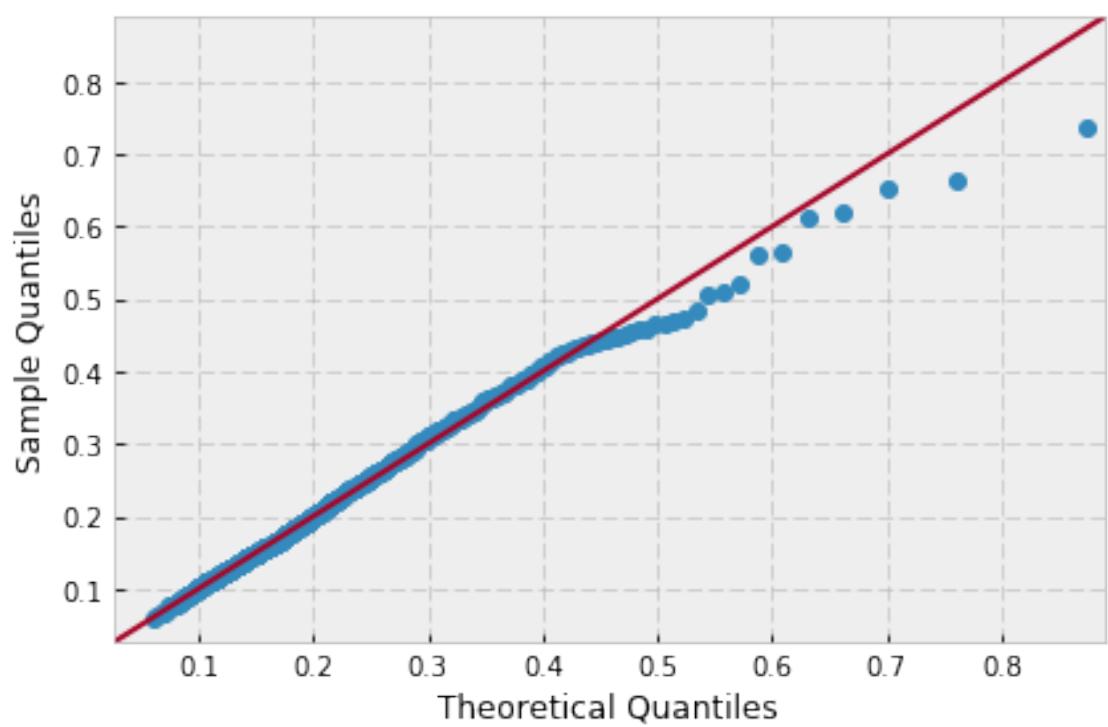
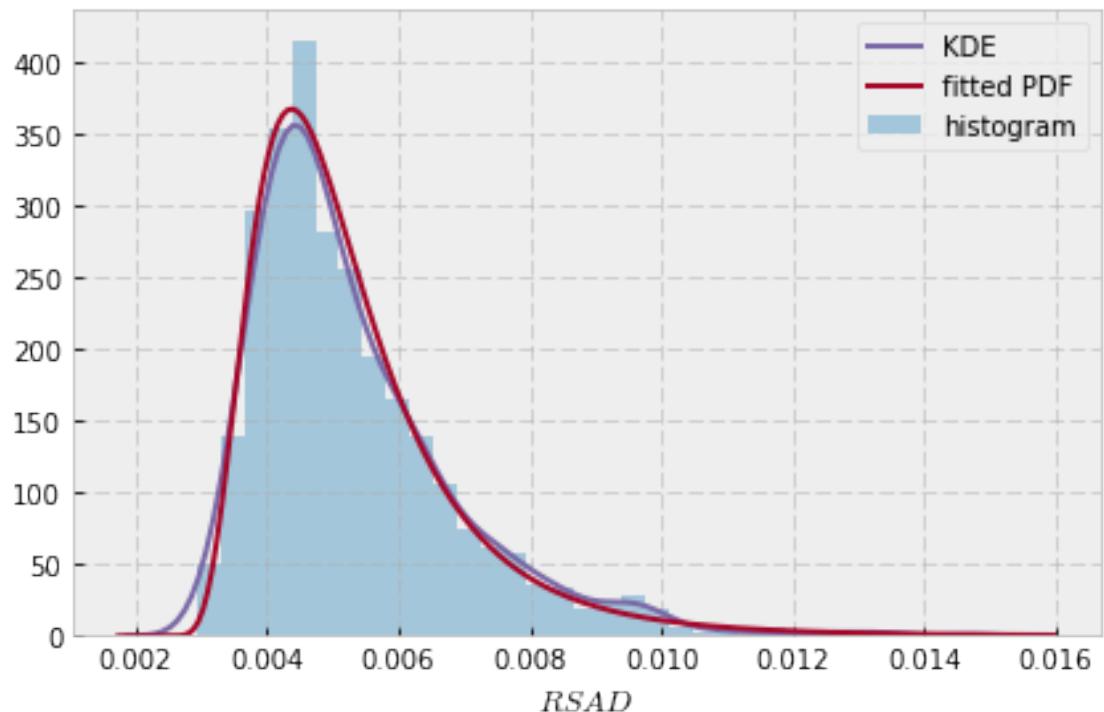


```
variable name: sigma
variable value: 0.0222222222222223
distribution: inverse-gamma(shape=(5.7843701323652397,), loc=0.002063030381956211, scale=0.017066
DescribeResult(nobs=1000, minmax=(0.003016238388258563, 0.01611088422424117)
    mean=0.005624147442104899, variance=3.0837872952404496e-06,
    skewness=1.8801218445717018, kurtosis=5.38507127858375)
```

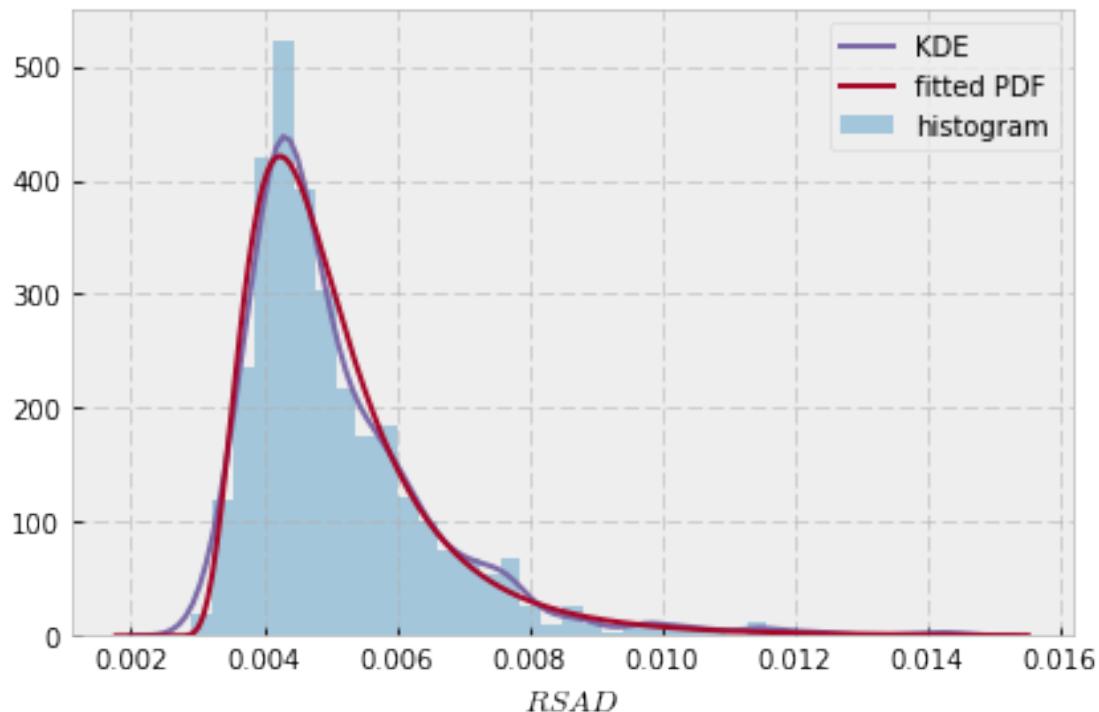


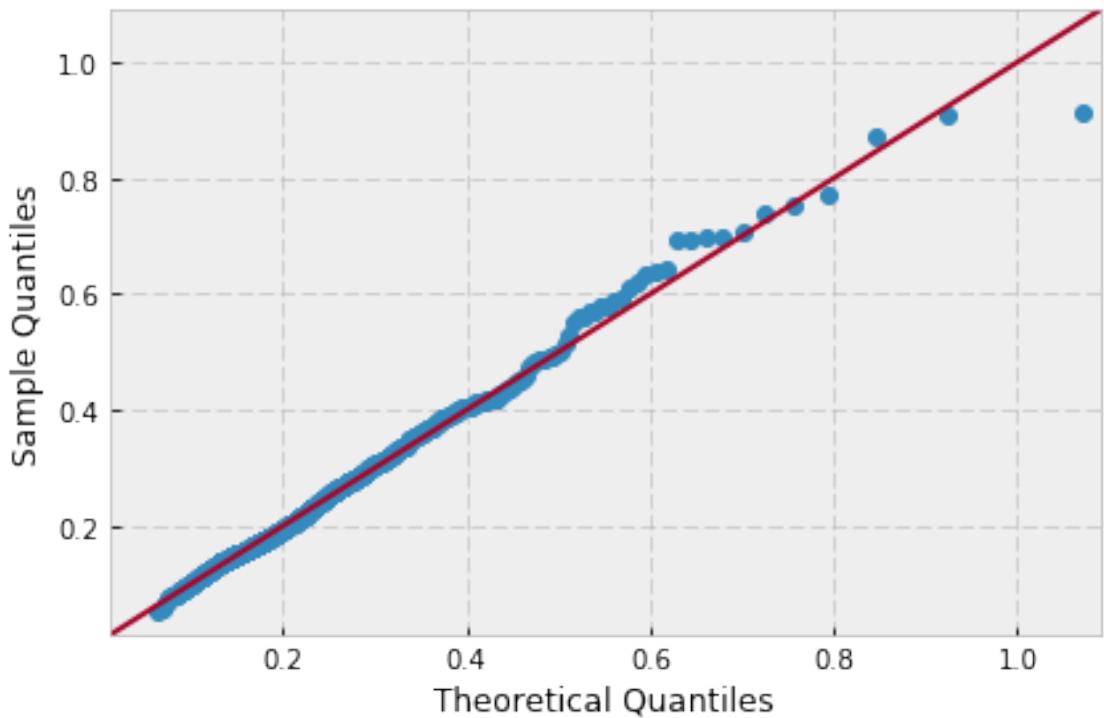


```
variable name: sigma
variable value: 0.0333333333333333
distribution: inverse-gamma(shape=(6.0974150057864245,), loc=0.0019188330819076968, scale=0.01740
DescribeResult(nobs=1000, minmax=(0.0029420345110439277, 0.014752830821415482)
               mean=0.005325601329502062, variance=2.5487102556837363e-06,
               skewness=1.6350227164441382, kurtosis=3.8492718386612292)
```

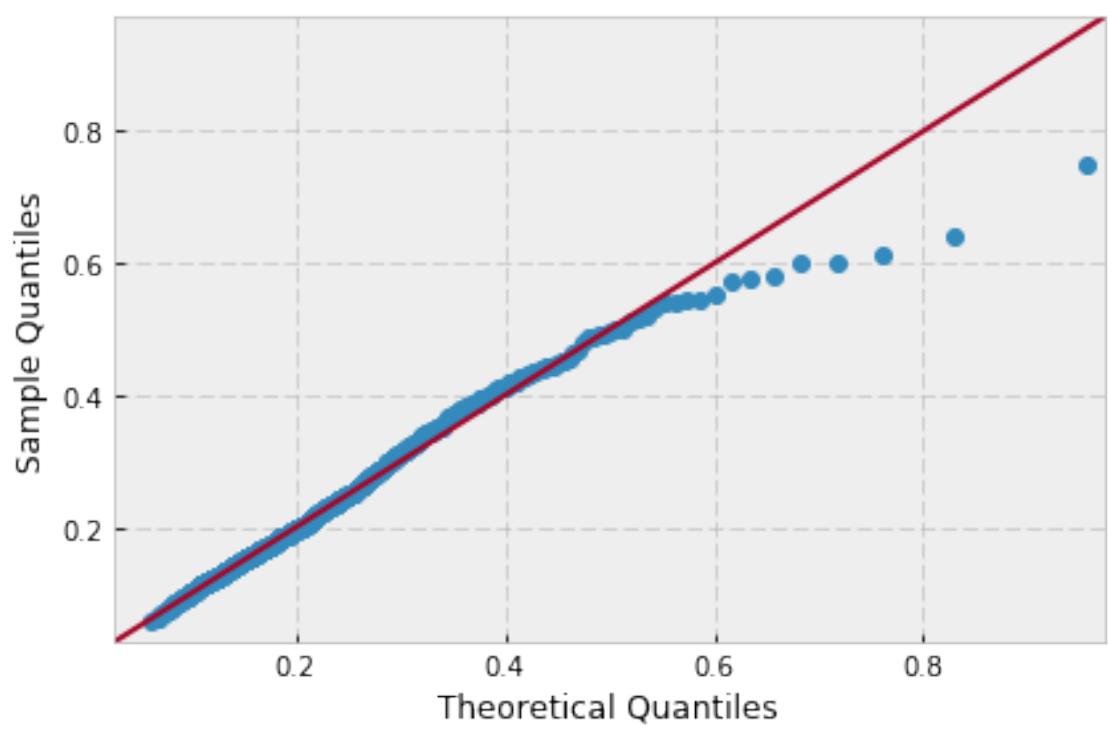
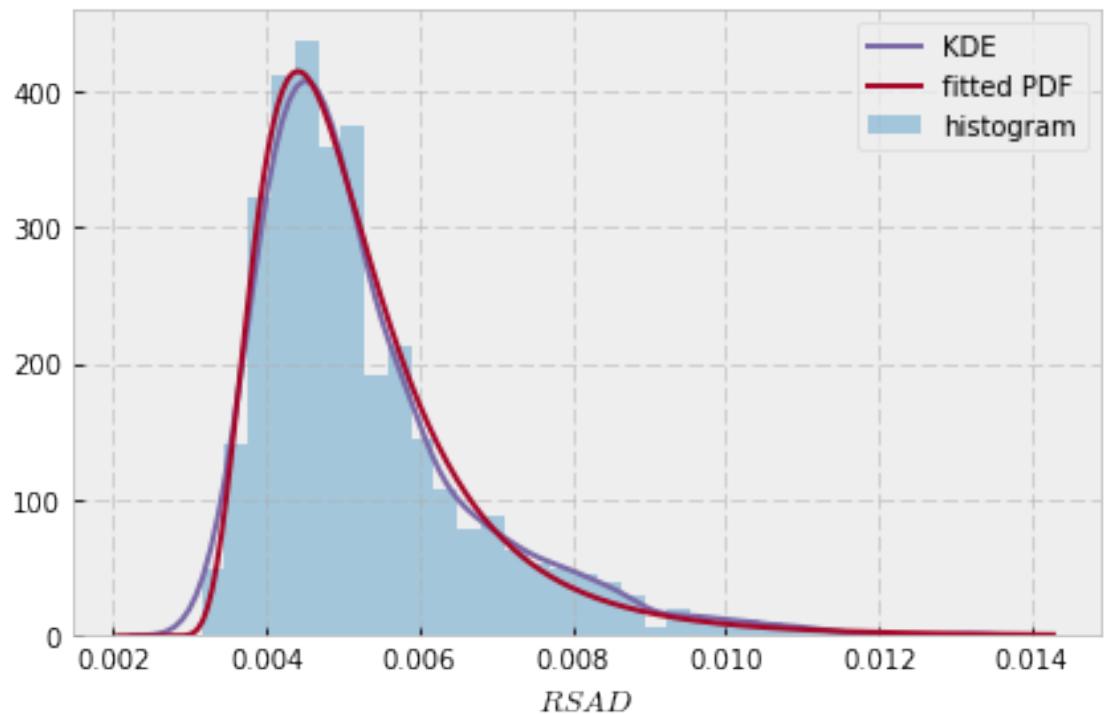


```
variable name: sigma
variable value: 0.04444444444444446
distribution: inverse-gamma(shape=(5.5477978614673304,), loc=0.0022176951858443386, scale=0.01327
DescribeResult(nobs=1000, minmax=(0.0029186591752678607, 0.014373906226720429)
    mean=0.0051371926863357674, variance=2.3144868655021382e-06,
    skewness=2.09474305826677, kurtosis=6.487239852944198)
```

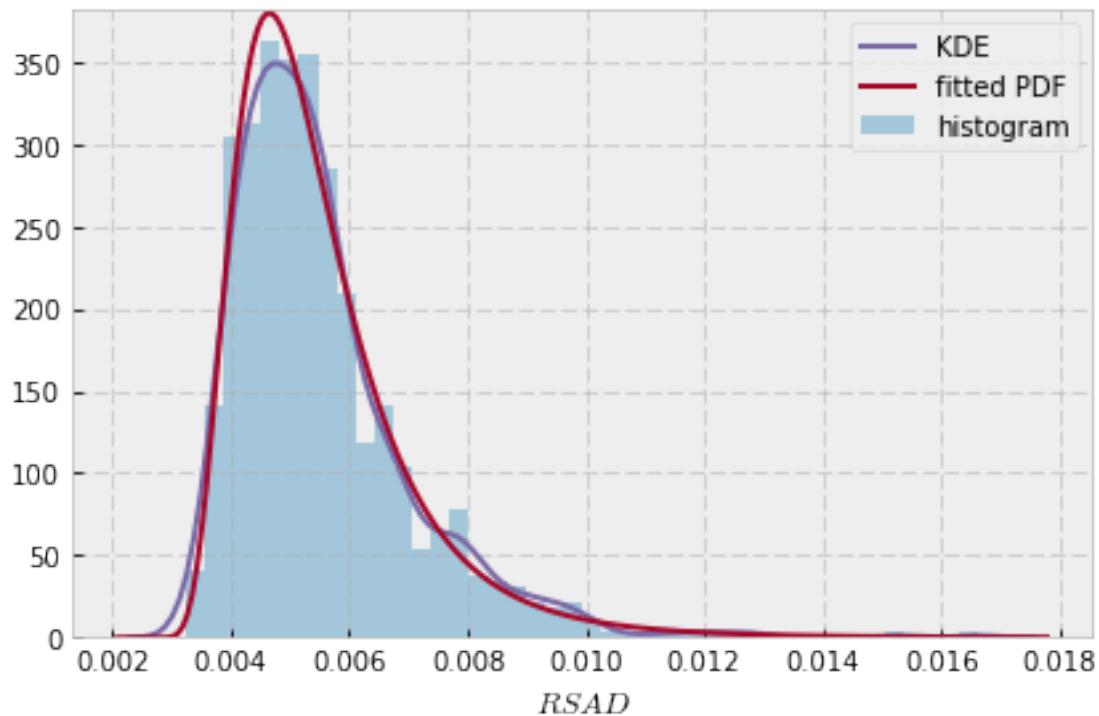


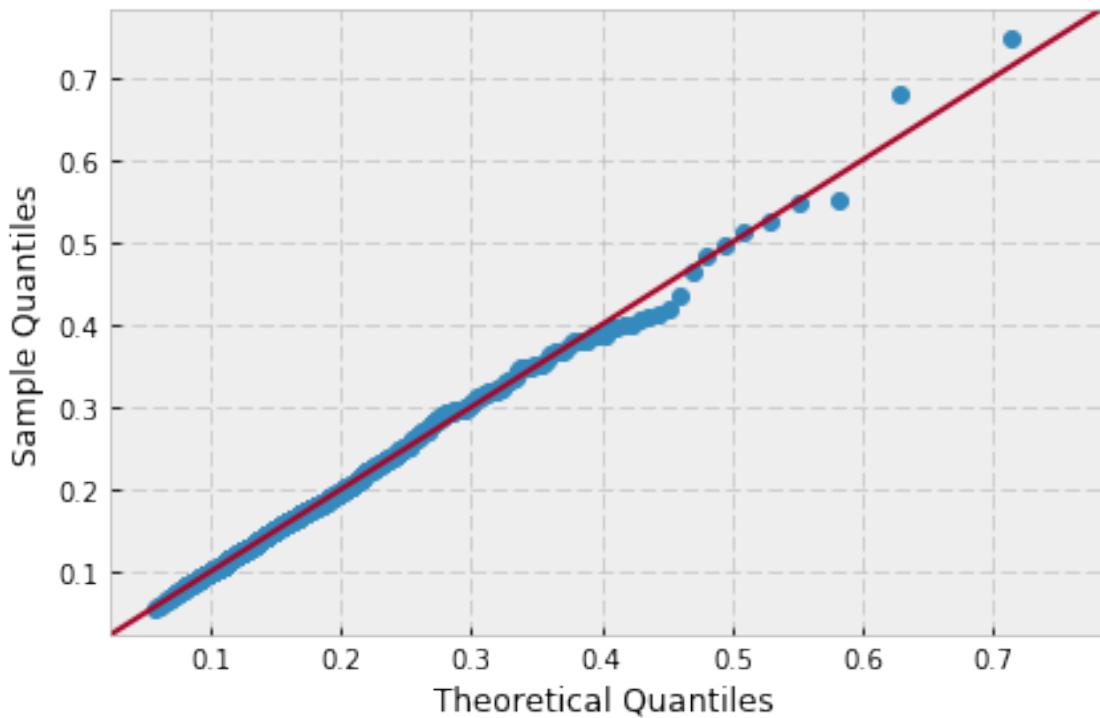


```
variable name: sigma
variable value: 0.05555555555555556
distribution: inverse-gamma(shape=(5.8468865587919554,), loc=0.0022878898711165448, scale=0.01455
DescribeResult(nobs=1000, minmax=(0.0031535290796158833, 0.013190813036371797)
               mean=0.0052847005521469096, variance=2.0770530429401927e-06,
               skewness=1.5610774849514775, kurtosis=2.9151397817826705)
```

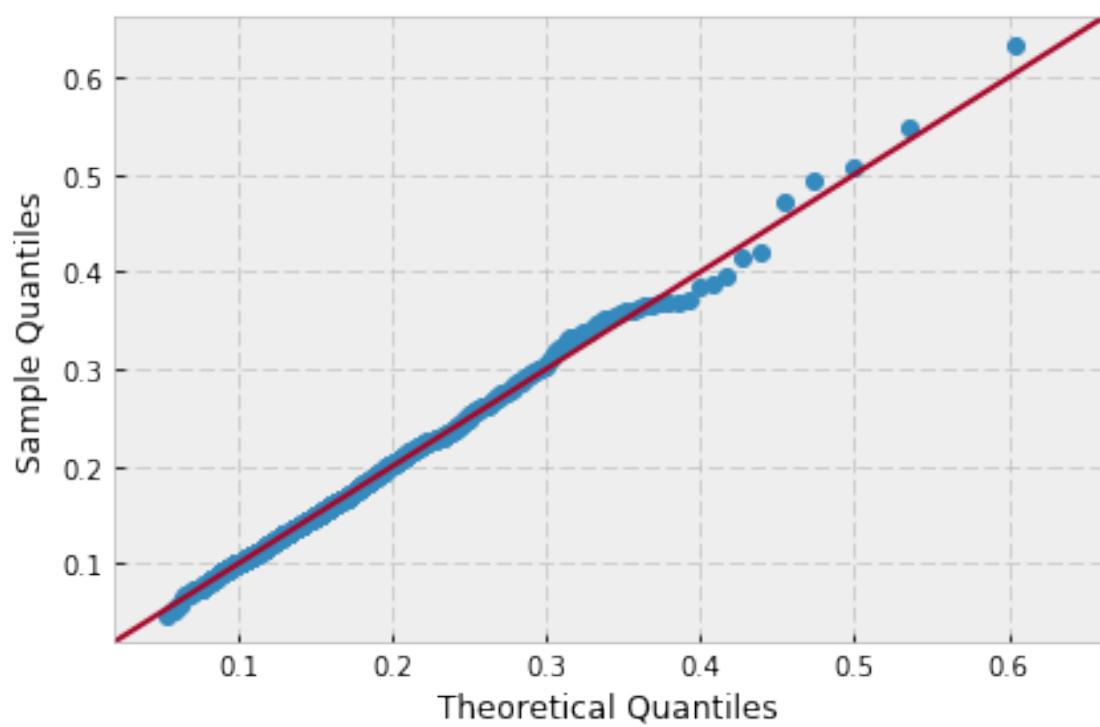
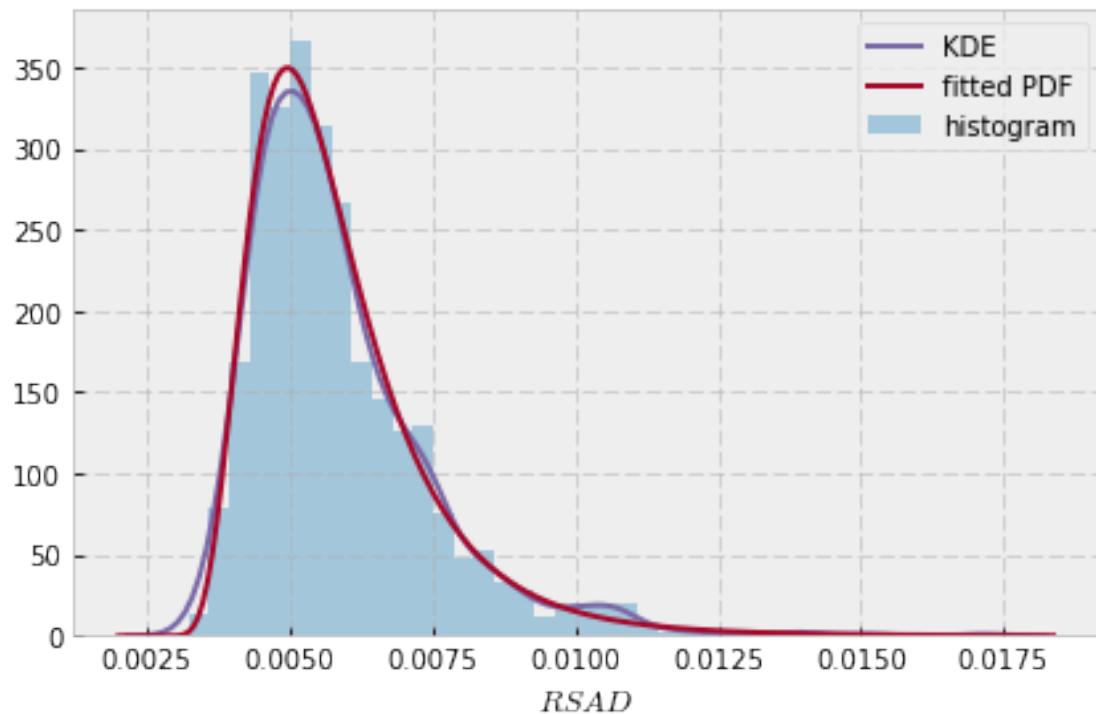


```
variable name: sigma
variable value: 0.06666666666666667
distribution: inverse-gamma(shape=(6.7238517051182196,),loc=0.002135502061605183, scale=0.019407
DescribeResult(nobs=1000, minmax=(0.0032323809387484275, 0.016635078991425786)
               mean=0.0055235866855012153, variance=2.3337211624046604e-06,
               skewness=1.8936915647498185, kurtosis=6.355124270678214)
```

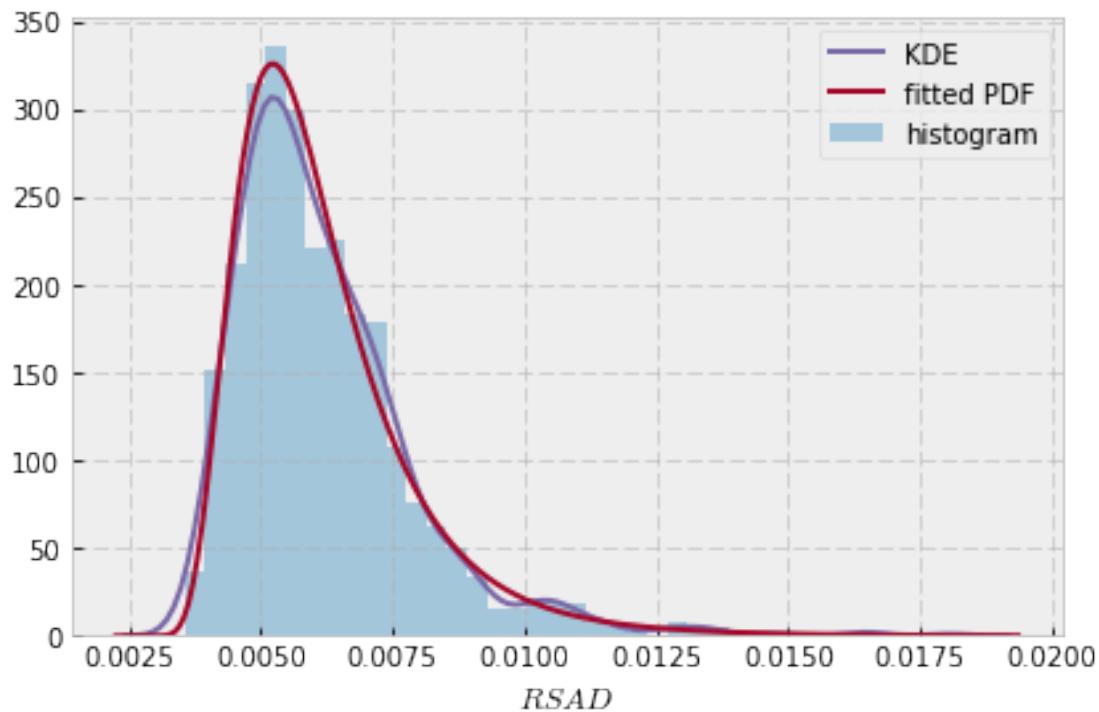


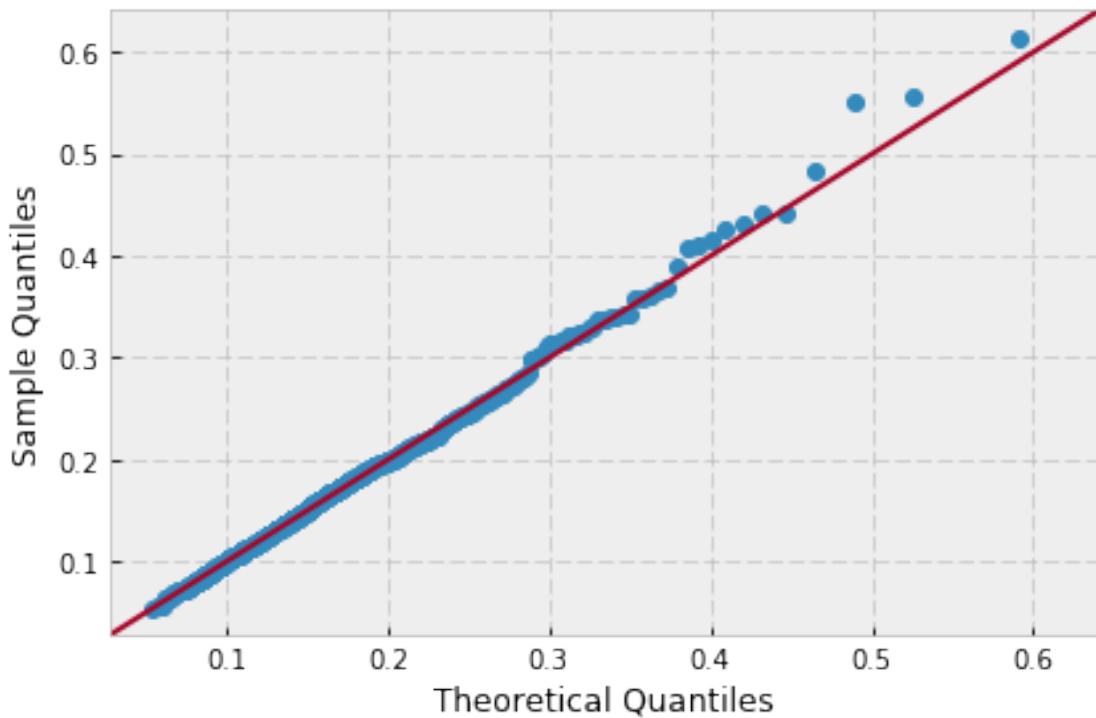


```
variable name: sigma
variable value: 0.07777777777777778
distribution: inverse-gamma(shape=(7.3098313826562986,), loc=0.0020873863701045504, scale=0.02380
DescribeResult(nobs=1000, minmax=(0.0032189430998368911, 0.017147385521457555)
               mean=0.0058594109474850329, variance=2.6363831838734408e-06,
               skewness=1.8121347656790445, kurtosis=5.5209521969883255)
```

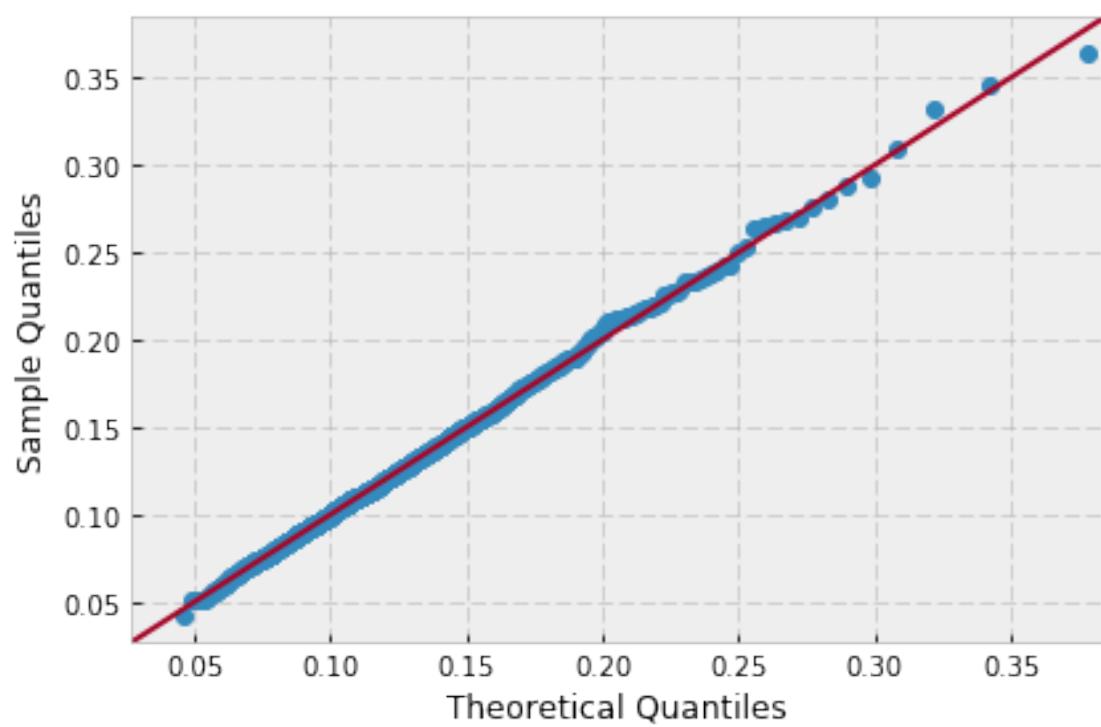
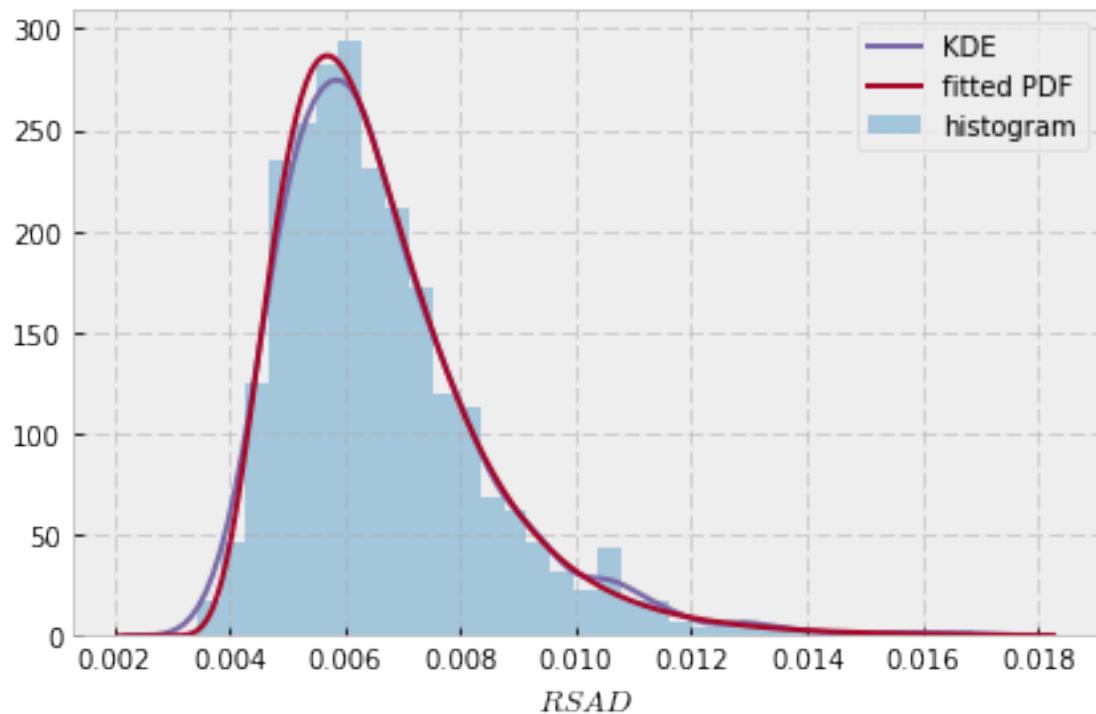


```
variable name: sigma
variable value: 0.0888888888888889
distribution: inverse-gamma(shape=(7.388058310132025,), loc=0.002129108429055588, scale=0.0259503
DescribeResult(nobs=1000, minmax=(0.0035579876674629472, 0.018047286920742624)
               mean=0.0061903490999341837, variance=3.01911112419829e-06,
               skewness=1.903264787799752, kurtosis=6.349107923672509)
```

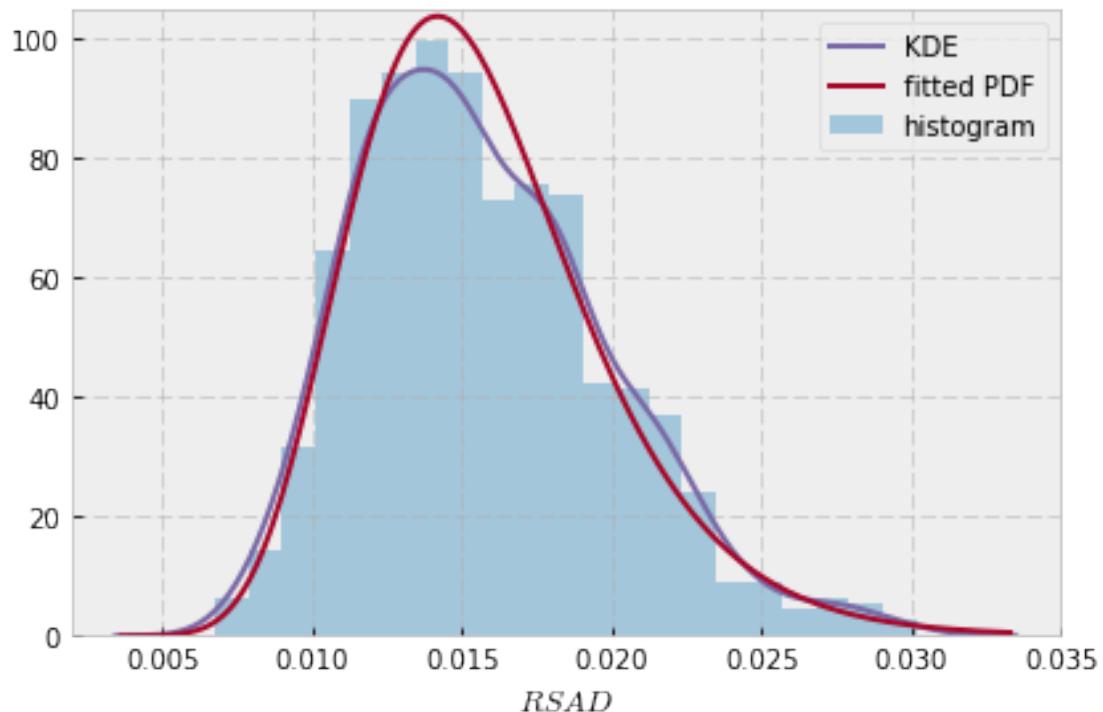


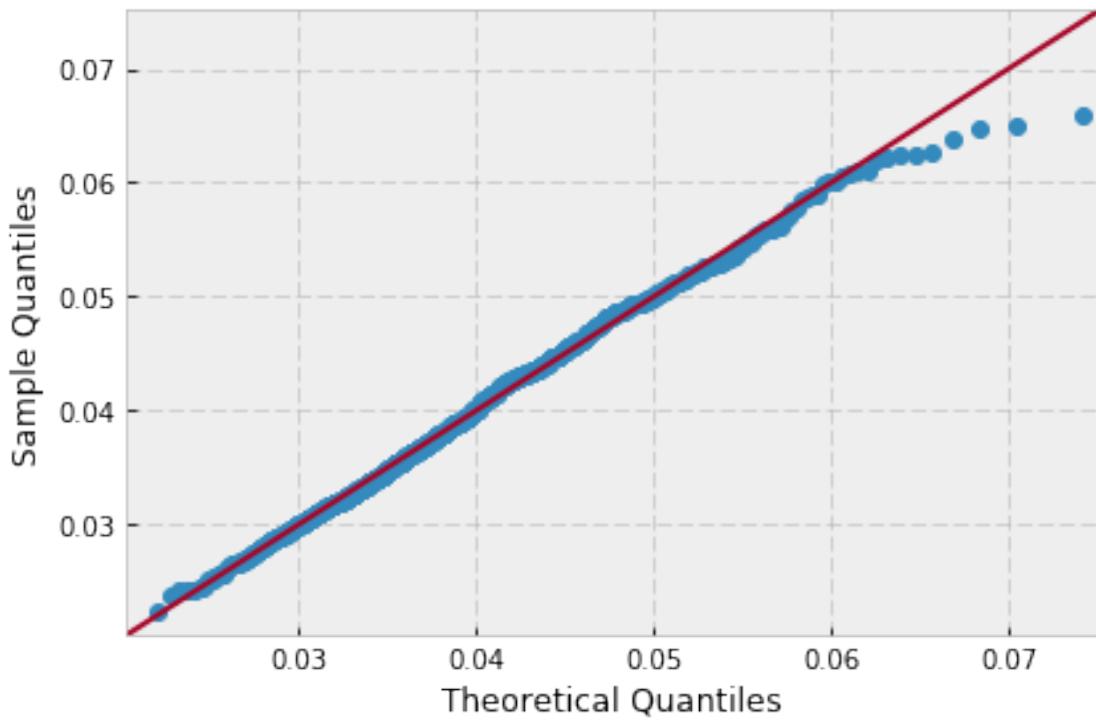


```
variable name: sigma
variable value: 0.1
distribution: inverse-gamma(shape=(9.3860105792156858,),loc=0.0016738831614609235, scale=0.04174
DescribeResult(nobs=1000, minmax=(0.0034492290362248485, 0.016894397481472503)
               mean=0.0066508934919163243, variance=3.2933475479889725e-06,
               skewness=1.4608156469988145, kurtosis=3.343935537007331)
```

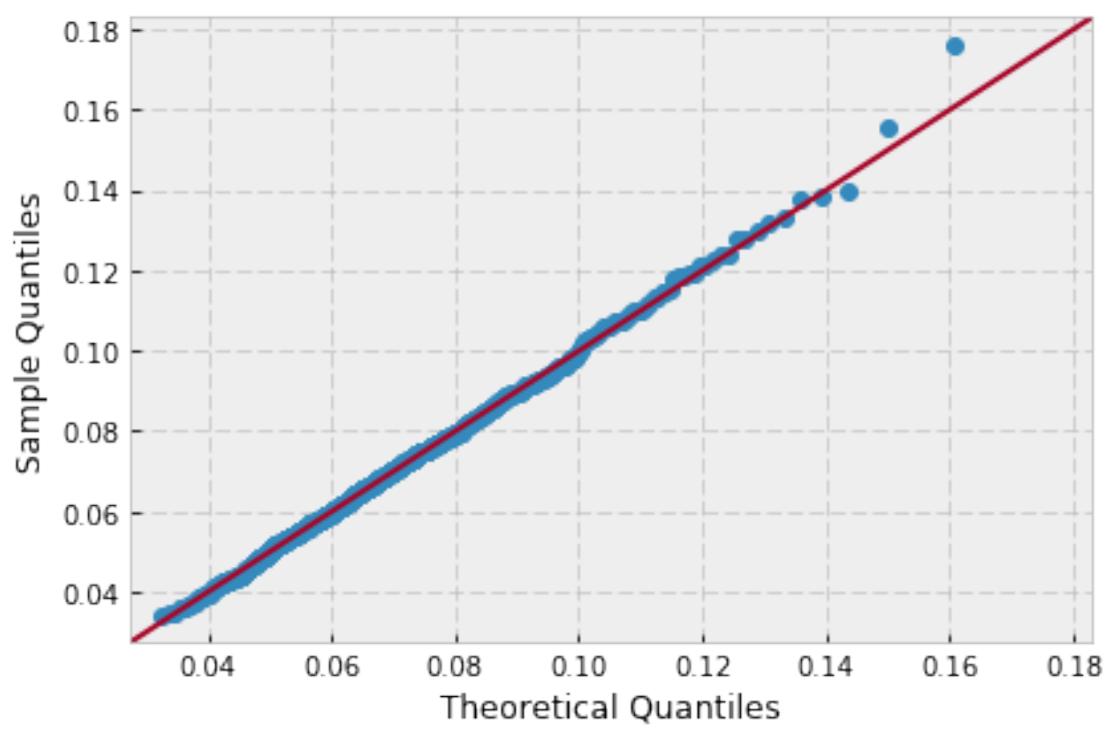
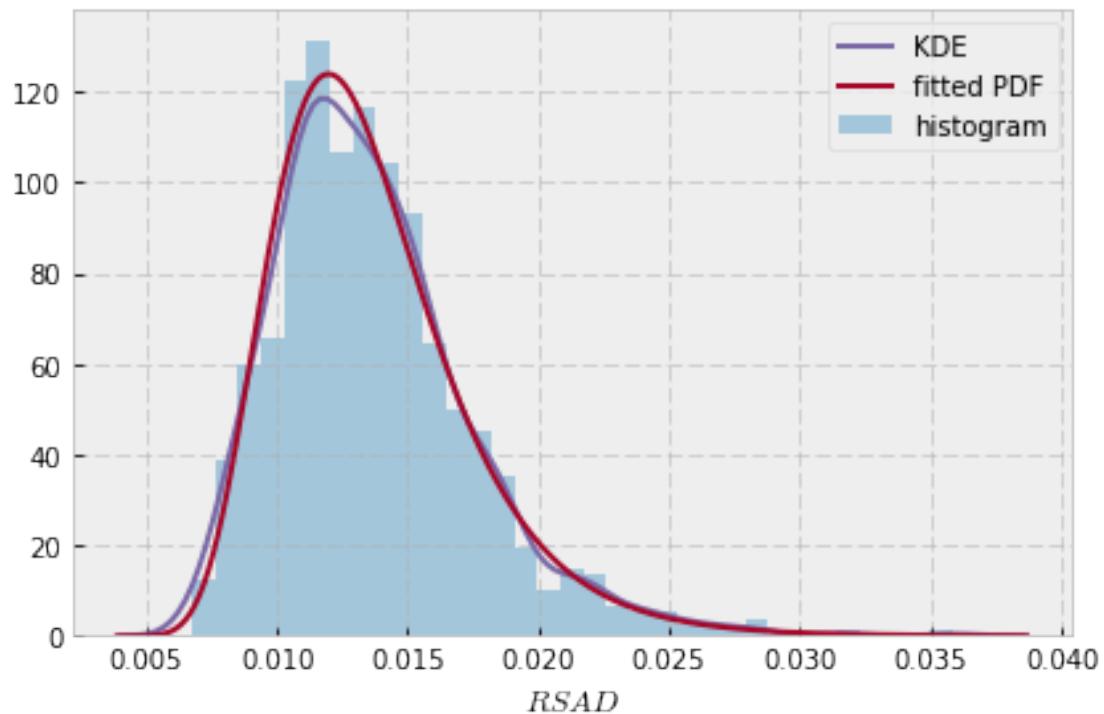


```
variable name: theta
variable value: 0.6981317007977318
distribution: inverse-gamma(shape=(26.659799566730445,),loc=-0.0052839815140676444, scale=0.5382
DescribeResult(nobs=1000, minmax=(0.0067544318665307803, 0.030140918278170194)
               mean=0.015689195390684592, variance=1.7424639600882804e-05,
               skewness=0.620708847138992, kurtosis=0.17957977282253035)
```

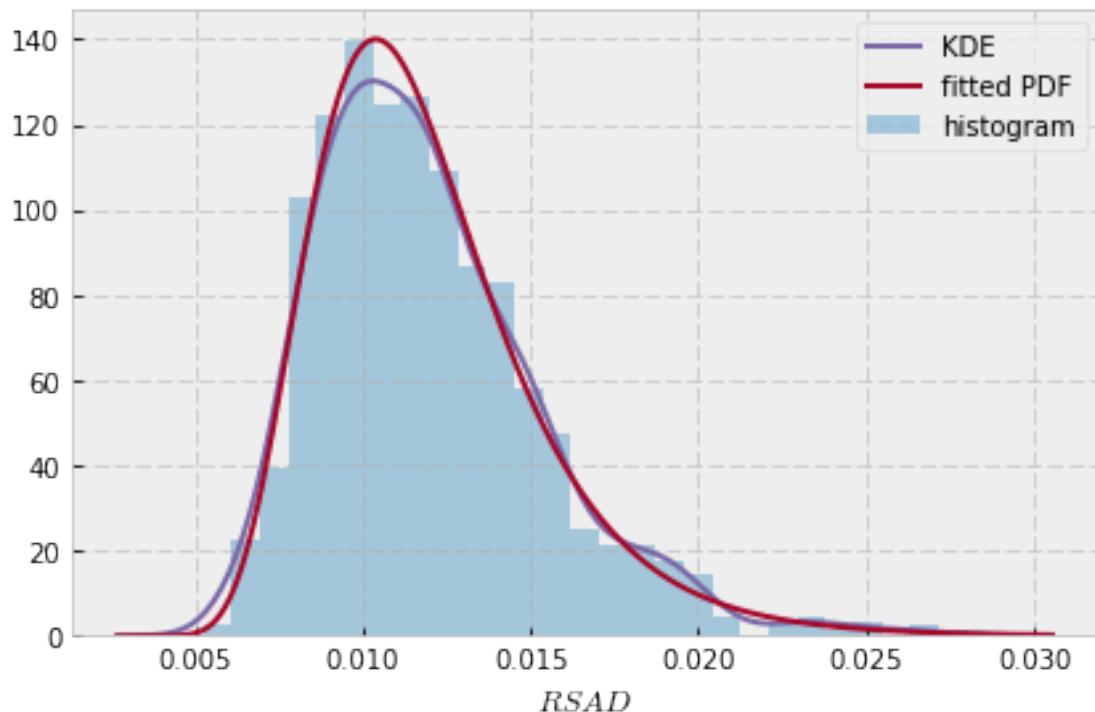


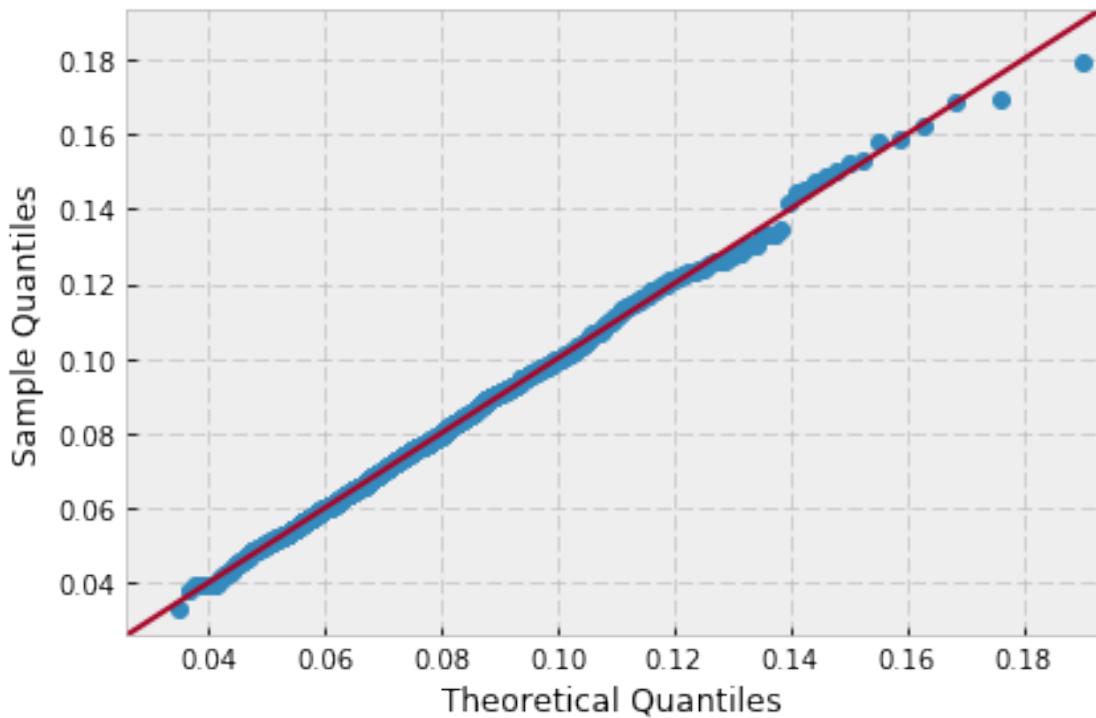


```
variable name: theta
variable value: 0.7853981633974483
distribution: inverse-gamma(shape=(15.702092302900443,), loc=-0.00033796722713517926, scale=0.205
DescribeResult(nobs=1000, minmax=(0.0067235449391562204, 0.035835966286857299)
               mean=0.013638549179209914, variance=1.4215805421813491e-05,
               skewness=1.1370335006273948, kurtosis=2.406991333666908)
```

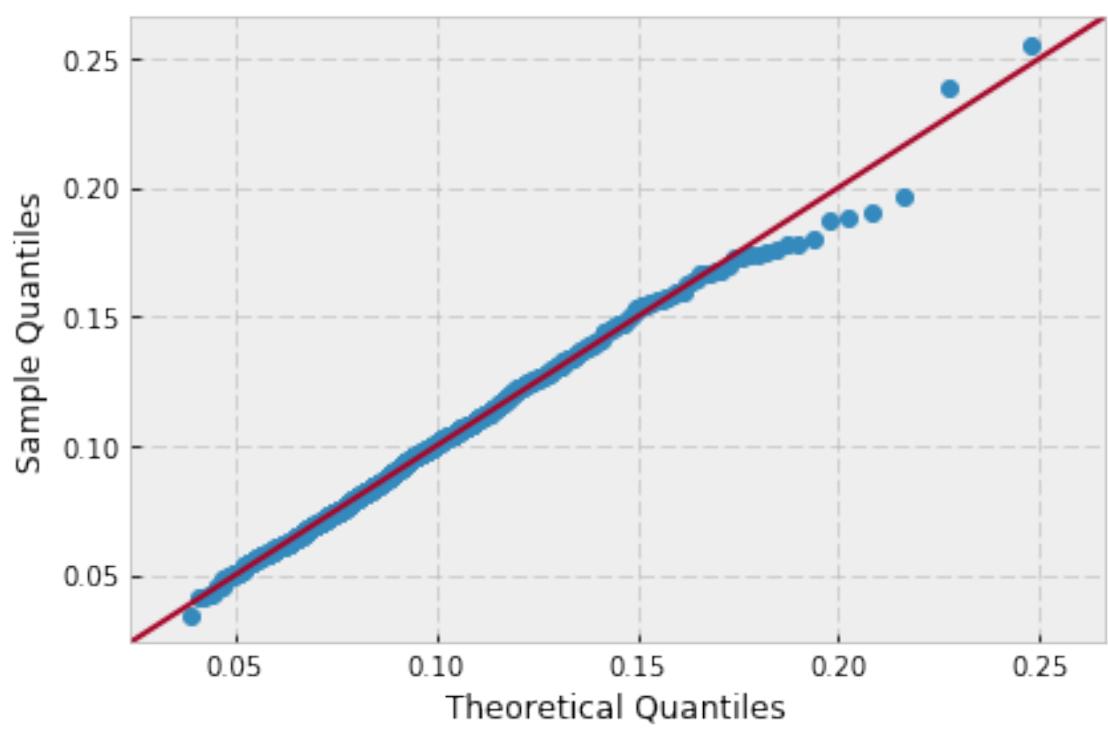
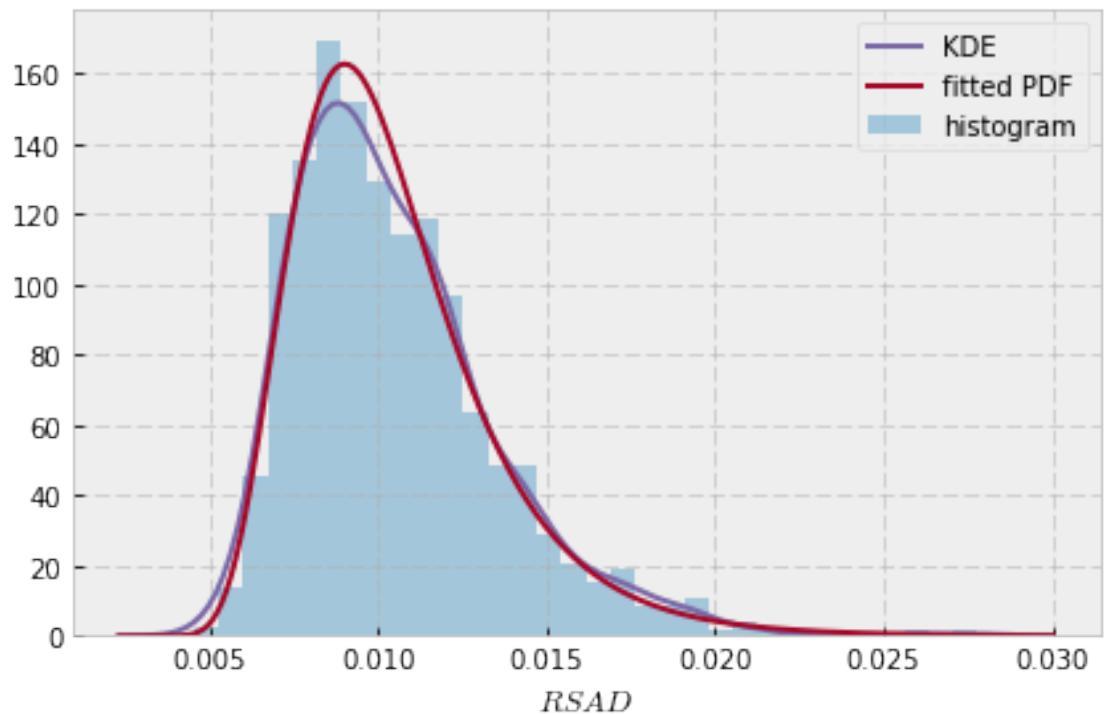


```
variable name: theta
variable value: 0.8726646259971648
distribution: inverse-gamma(shape=(14.108250414040032,),loc=9.541966273532998e-05, scale=0.15535
DescribeResult(nobs=1000, minmax=(0.0052291155896732567, 0.028005439319069663)
               mean=0.011945011333992768, variance=1.1381045604624589e-05,
               skewness=1.0757415007010172, kurtosis=1.7182212421659173)
```

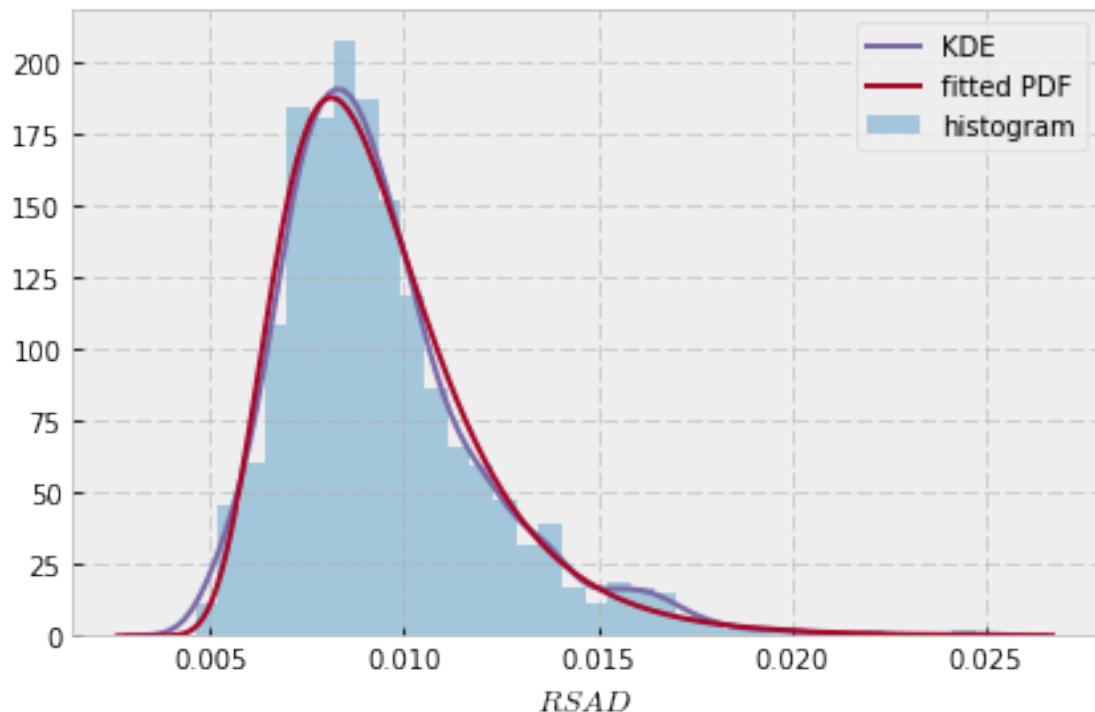


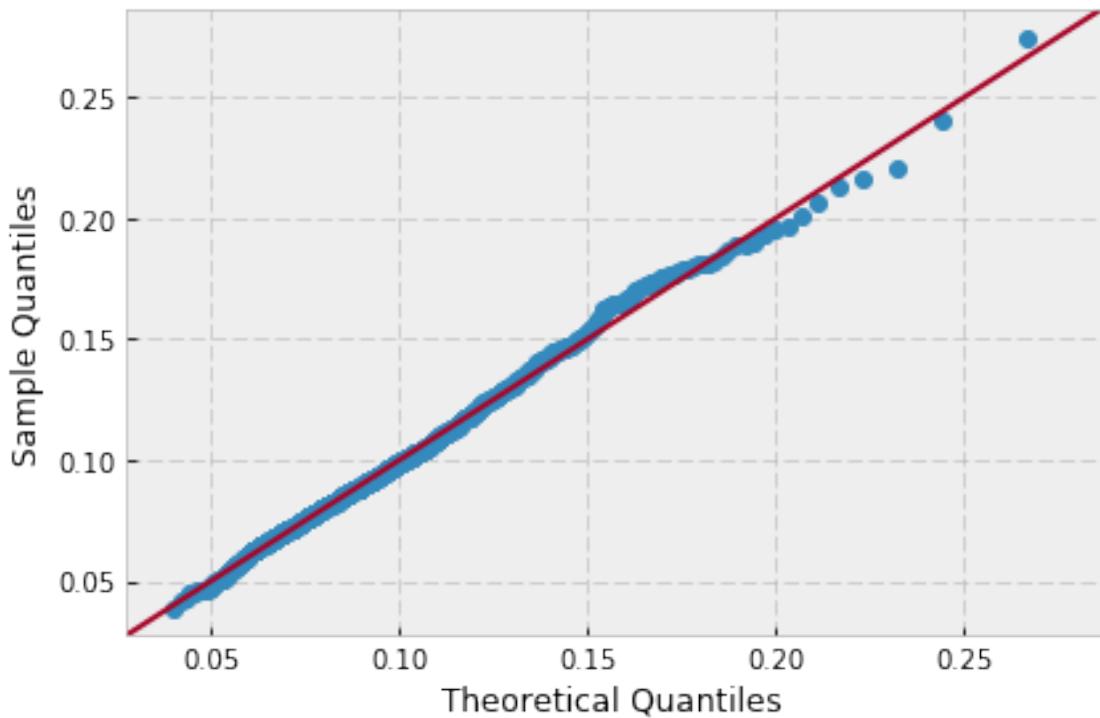


```
variable name: theta
variable value: 0.9599310885968813
distribution: inverse-gamma(shape=(11.993839008025148,), loc=0.000909372937979091, scale=0.105371
DescribeResult(nobs=1000, minmax=(0.0045630019286218458, 0.02778297963368791)
               mean=0.010490459642846157, variance=8.9225115708426385e-06,
               skewness=1.1661268590798095, kurtosis=2.2219373570467553)
```

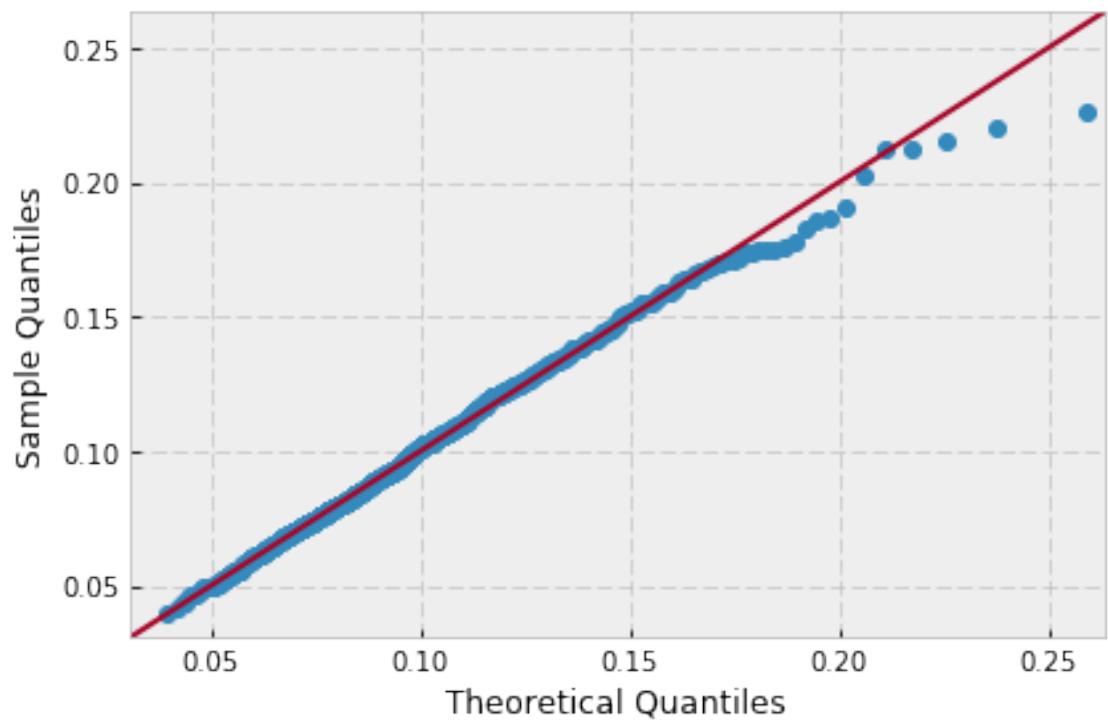
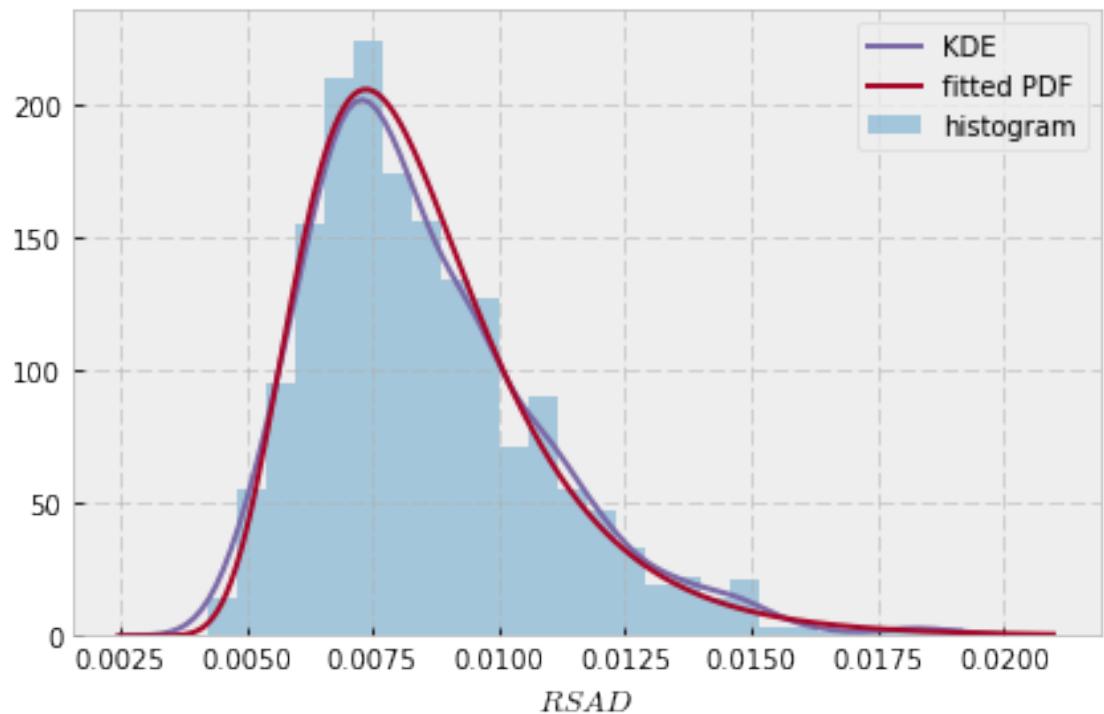


```
variable name: theta
variable value: 1.0471975511965979
distribution: inverse-gamma(shape=(11.476644756568321,), loc=0.0012767829397925003, scale=0.08544
DescribeResult(nobs=1000, minmax=(0.0046214173287148757, 0.024709193955350987)
               mean=0.0094325802310628142, variance=6.9940873717700856e-06,
               skewness=1.3055761135355906, kurtosis=2.544325511494052)
```

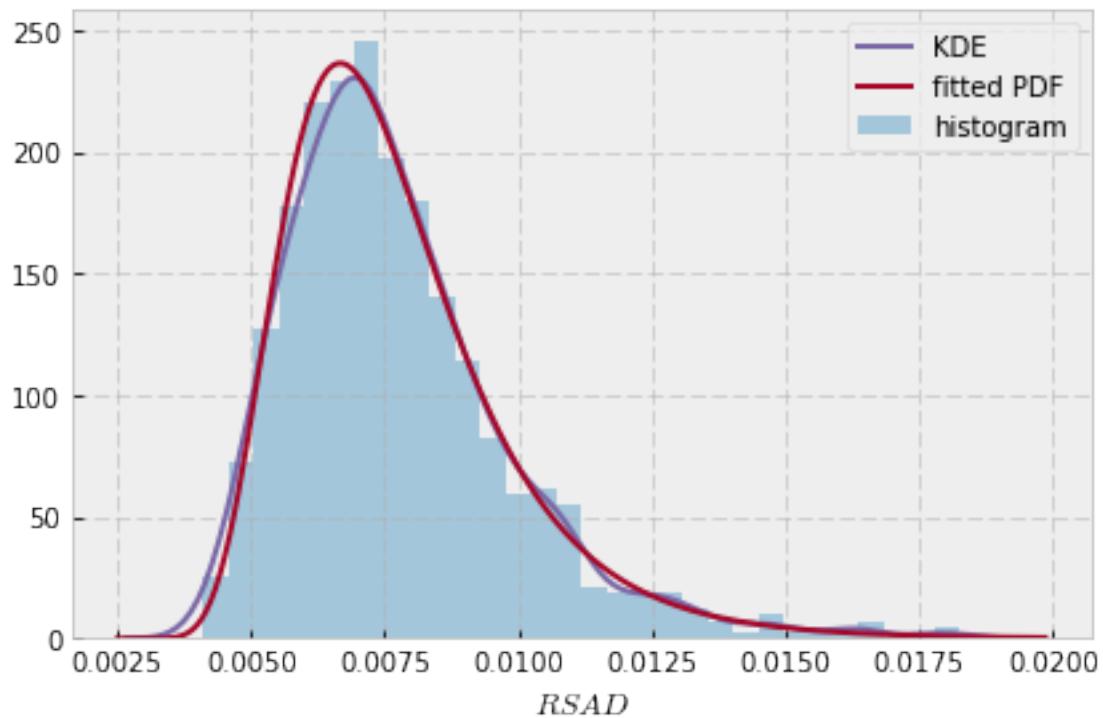


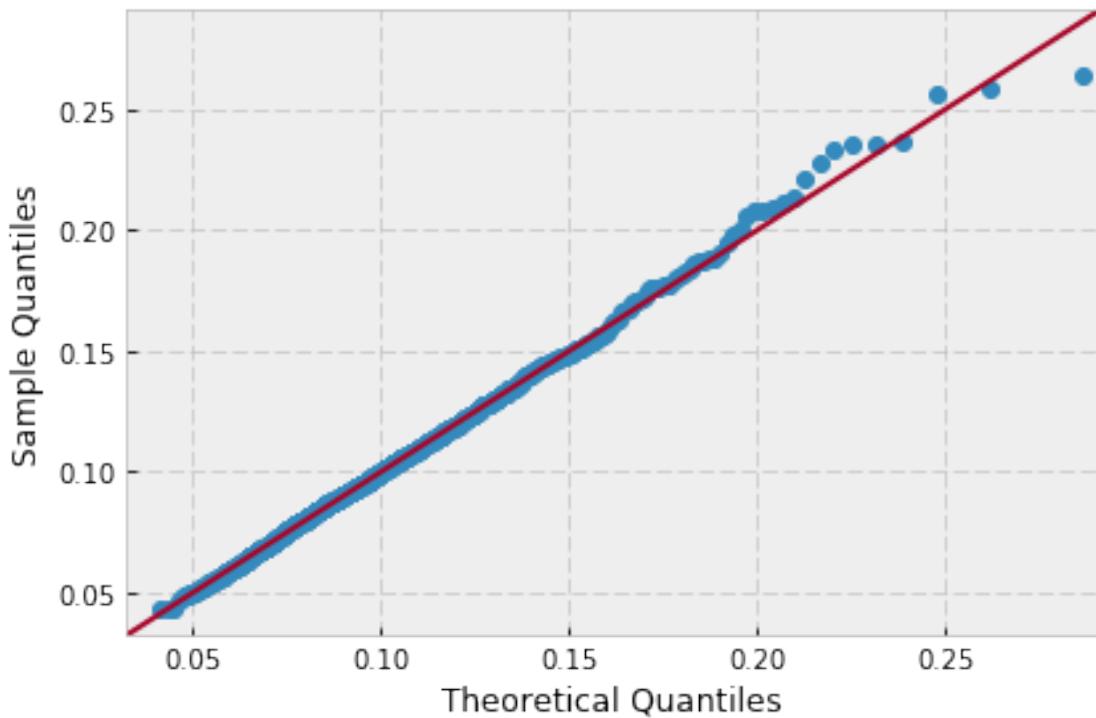


```
variable name: theta
variable value: 1.1344640137963142
distribution: inverse-gamma(shape=(11.679226954770002,),loc=0.00104894408118932, scale=0.0801252
DescribeResult(nobs=1000, minmax=(0.0042484448297063363, 0.019196272872643638)
               mean=0.0085475164028002669, variance=5.5572611408637265e-06,
               skewness=1.0748463272261208, kurtosis=1.4755378528936562)
```

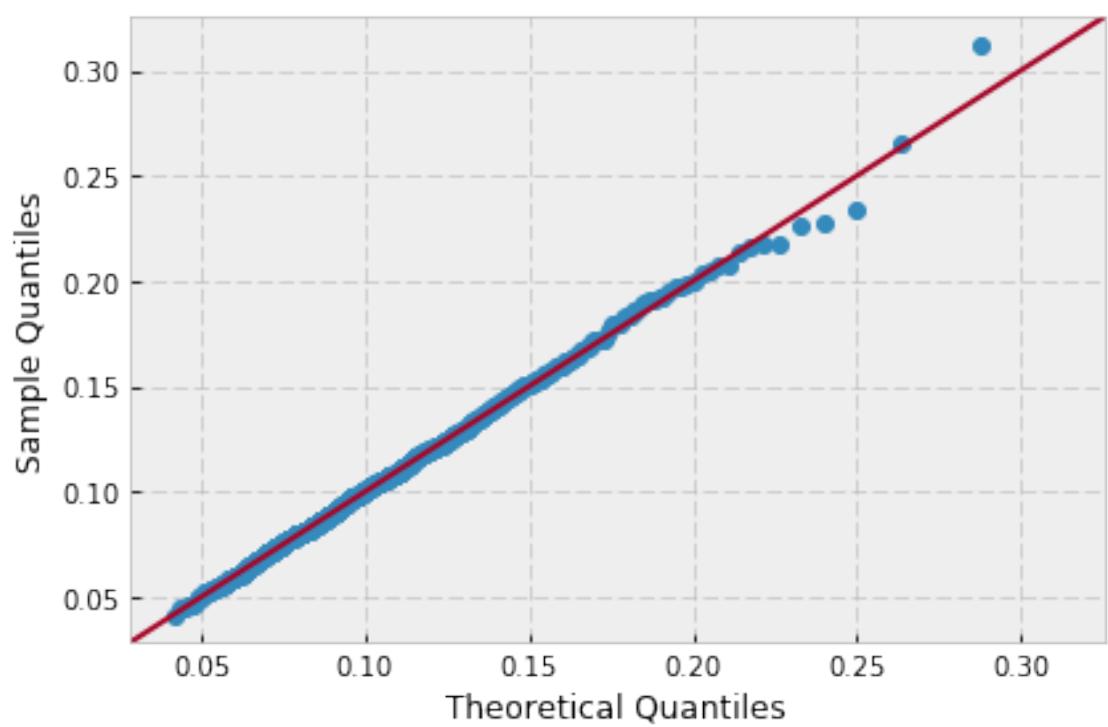
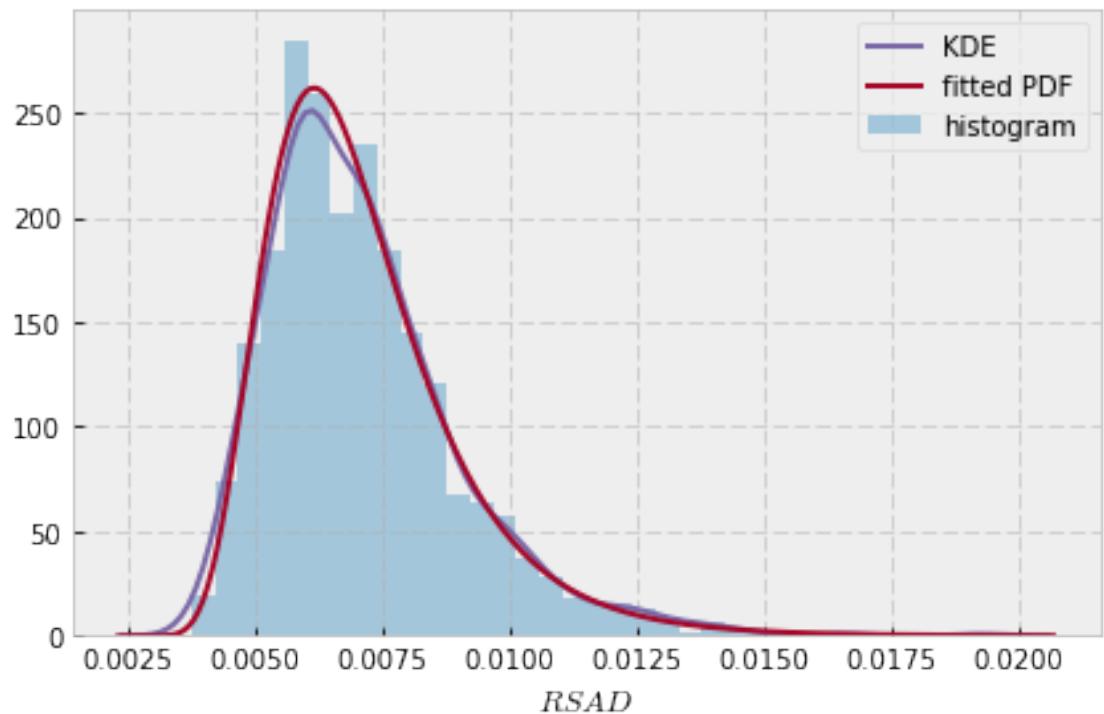


```
variable name: theta
variable value: 1.2217304763960306
distribution: inverse-gamma(shape=(11.001331782966279,),loc=0.0013436146047403883, scale=0.06388
DescribeResult(nobs=1000, minmax=(0.0040966503224696728, 0.018251851935865756)
               mean=0.0077302981031003261, variance=4.4773064689470104e-06,
               skewness=1.3467934758519475, kurtosis=2.853714791001252)
```

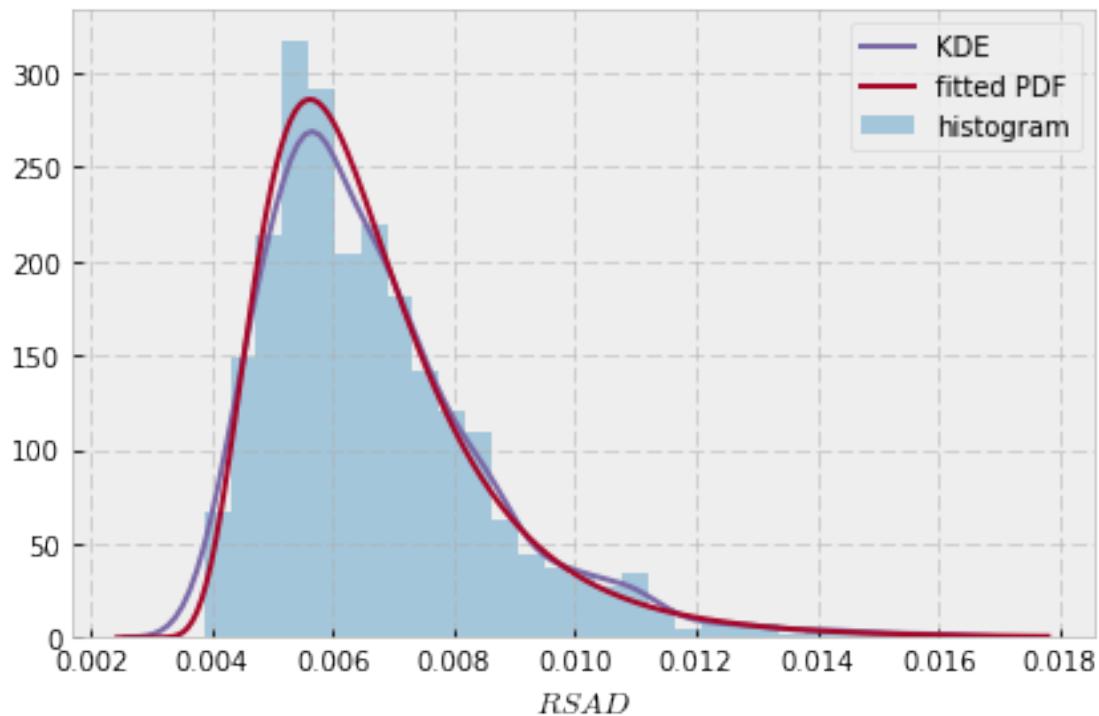


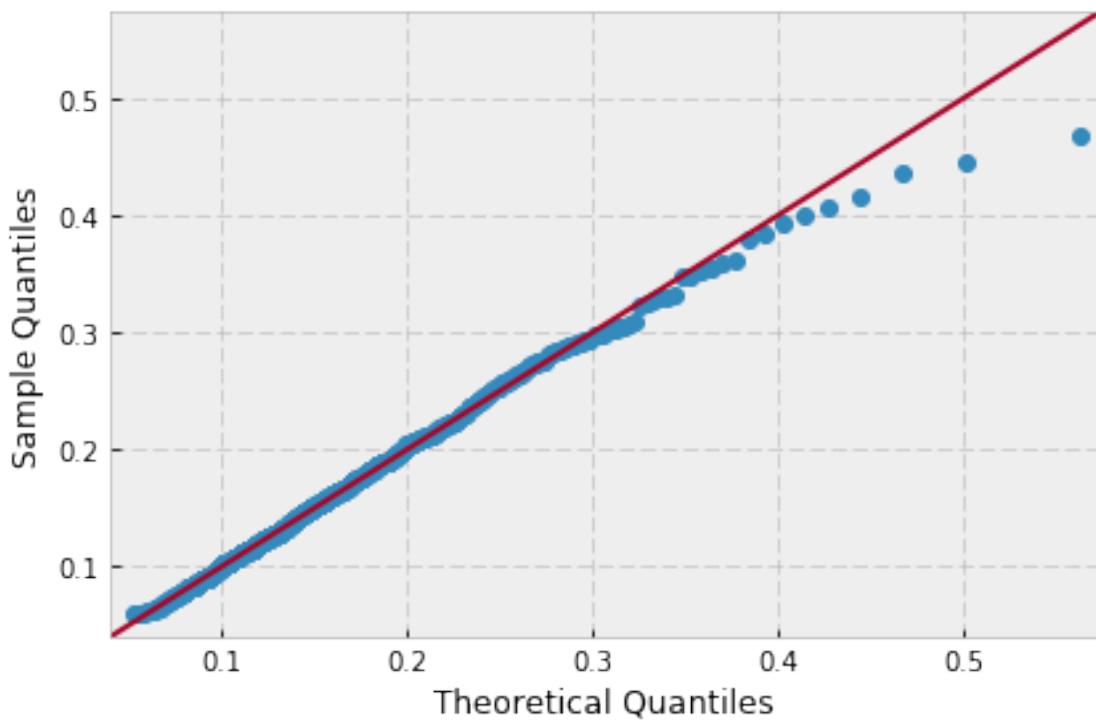


```
variable name: theta
variable value: 1.3089969389957472
distribution: inverse-gamma(shape=(10.968408980109459,), loc=0.0013537912488130962, scale=0.05741
DescribeResult(nobs=1000, minmax=(0.003739256508816749, 0.019259705142374316)
               mean=0.0071120620936239781, variance=3.6344209631949364e-06,
               skewness=1.343925705503349, kurtosis=3.146519474829028)
```

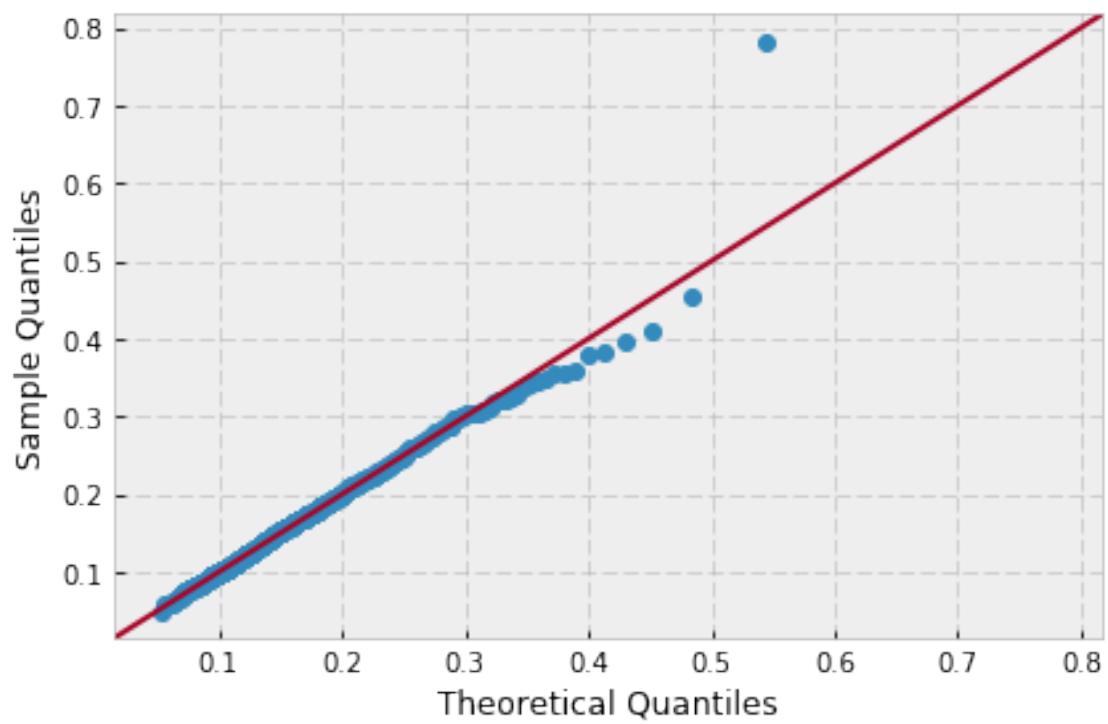
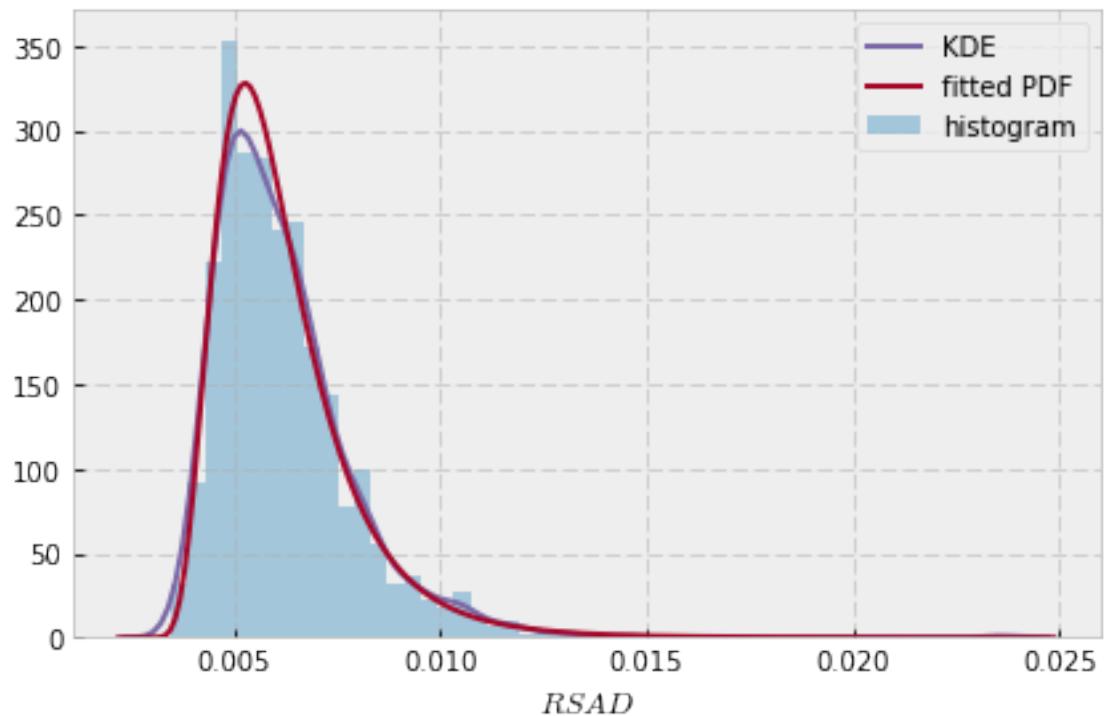


```
variable name: theta
variable value: 1.3962634015954638
distribution: inverse-gamma(shape=(7.5745299527271737,),loc=0.0020418687863107687, scale=0.03065
DescribeResult(nobs=1000, minmax=(0.0038629842414087123, 0.01638123120295383)
               mean=0.0066976825765582343, variance=3.5846544063849648e-06,
               skewness=1.4212442610945433, kurtosis=2.844975207721798)
```





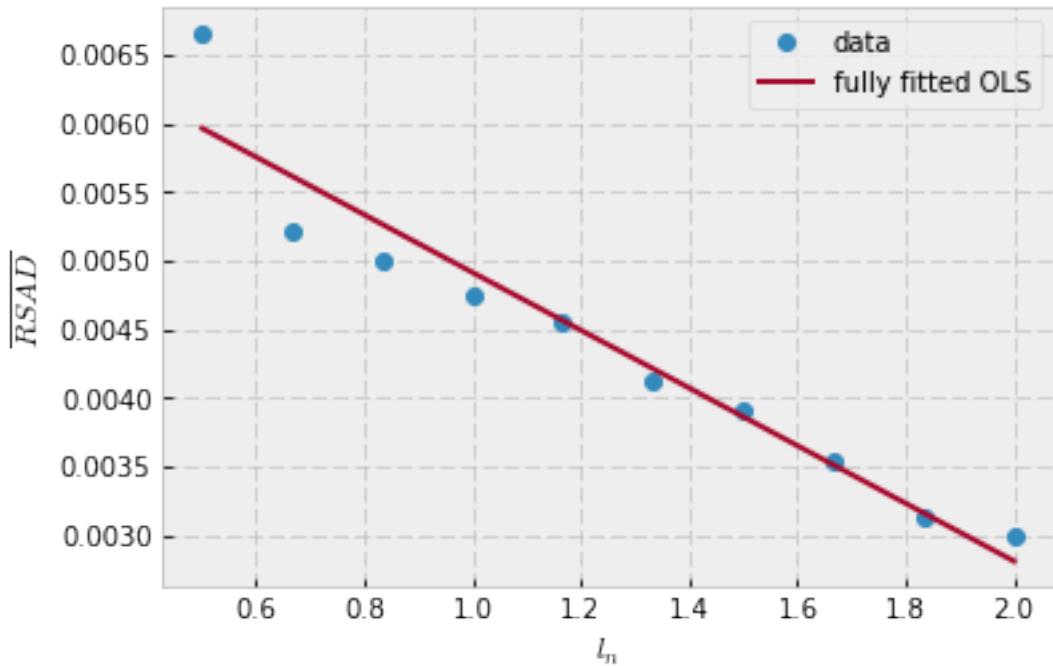
```
variable name: theta
variable value: 1.4835298641951802
distribution: inverse-gamma(shape=(7.721708819739785,), loc=0.00211685809148586, scale=0.02748802
DescribeResult(nobs=1000, minmax=(0.0034902810763933661, 0.023586002155740842)
               mean=0.0062041183967371282, variance=2.8872301075910453e-06,
               skewness=2.1679236322591873, kurtosis=12.238967181359723)
```



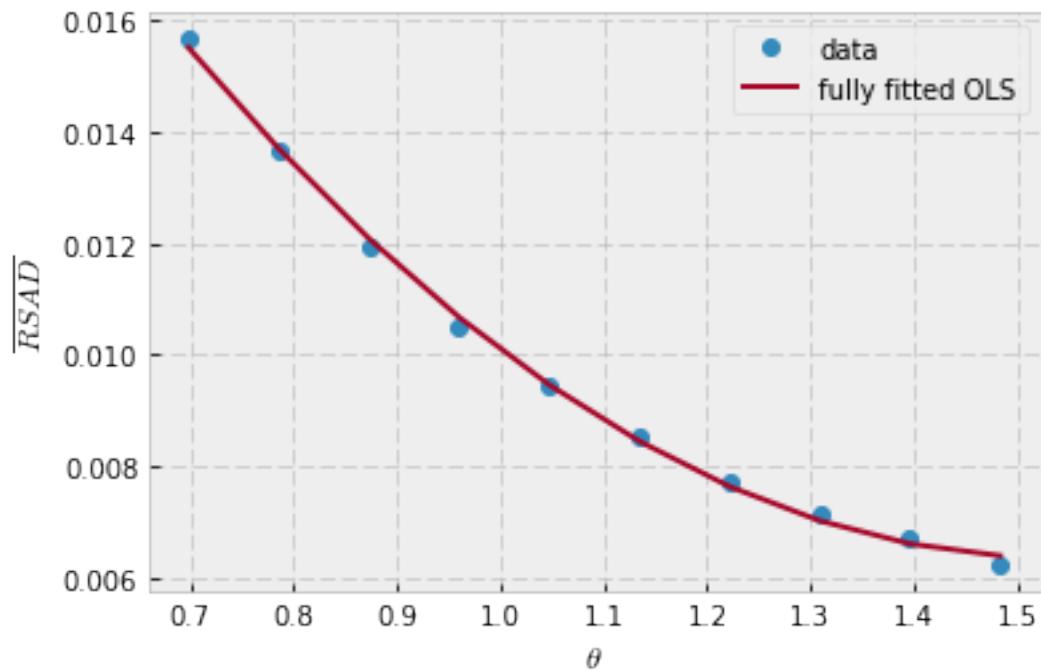
12.2 Parameter regression

```
In [30]: var_name = 'total_root_surface_area_per_convex_hull_volume'  
        mean_reg('ln', var_name, 'poly1')  
        mean_reg('theta', var_name, 'poly2')  
        mean_reg('maxB', var_name, 'poly1')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly2')  
        mean_reg('N', var_name, 'poly1')
```

variable name: ln
model: poly1
r-squared: 0.8490694776015657
explained variance: 0.8496128777323297



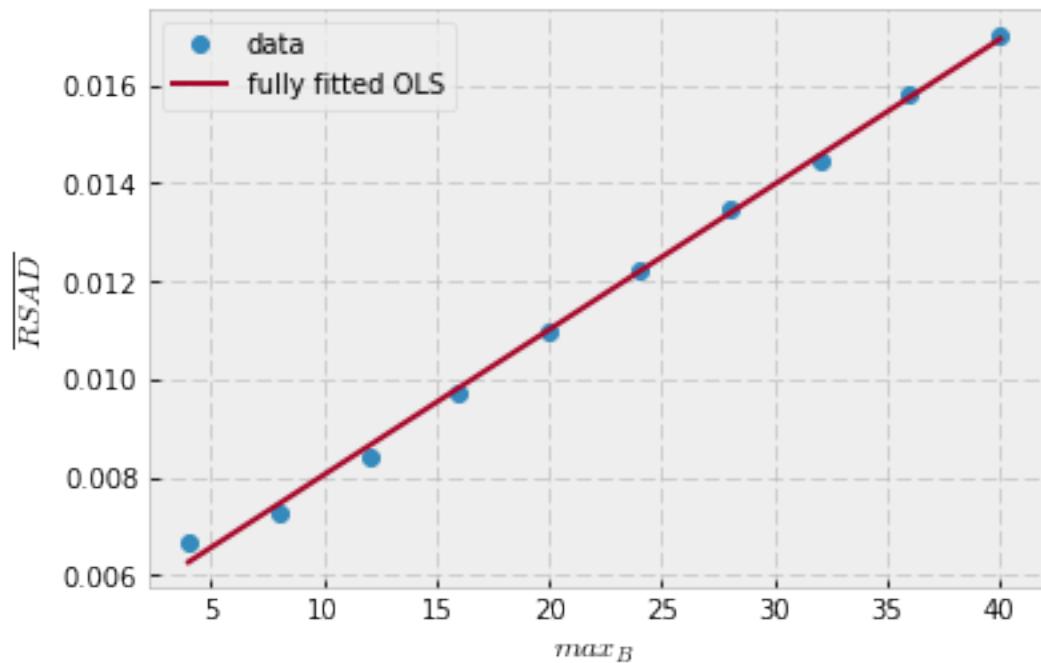
```
variable name: theta  
model: poly2  
r-squared: 0.993361583745013  
explained variance: 0.9933616149133653
```



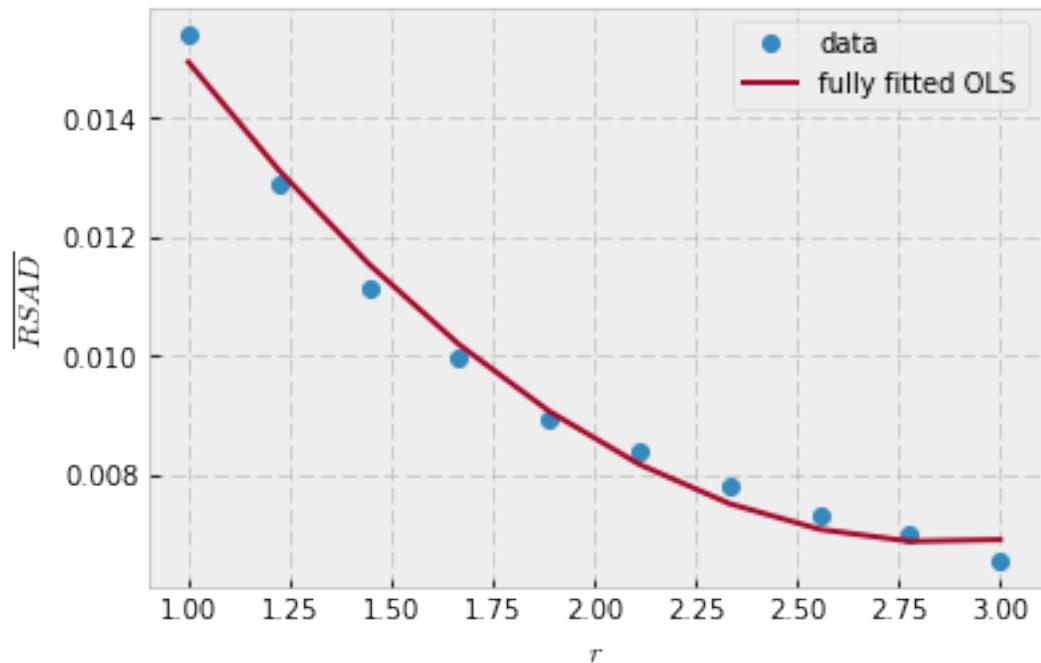
```

variable name: maxB
model: poly1
r-squared: 0.994774727842773
explained variance: 0.9947927122306905

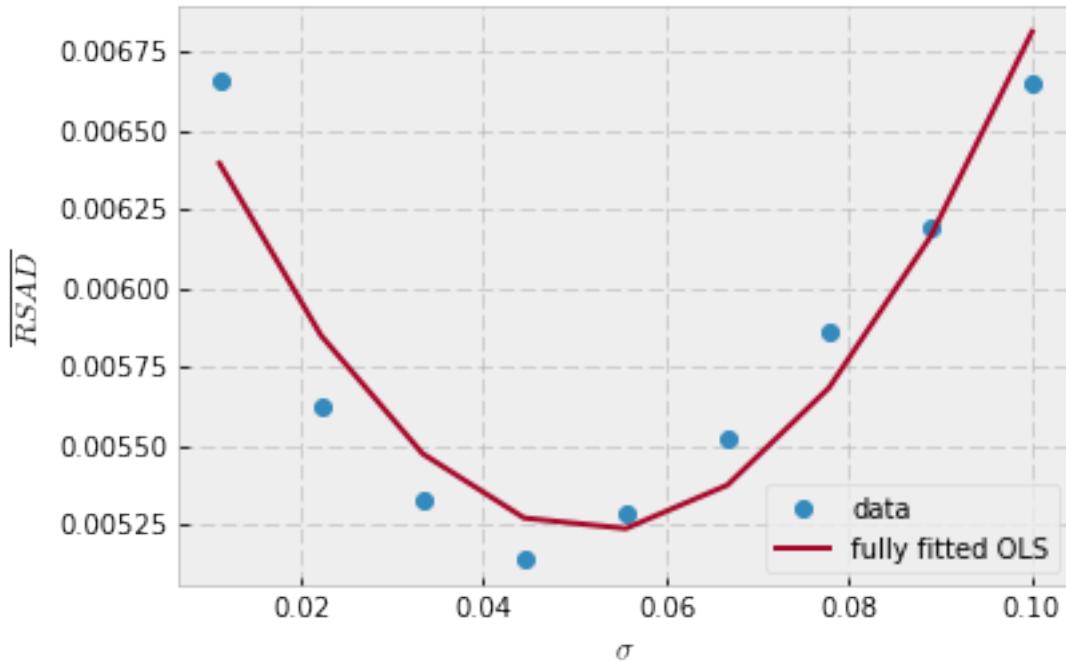
```



```
variable name: r
model: poly2
r-squared: 0.9550331540774906
explained variance: 0.9550578810043977
```



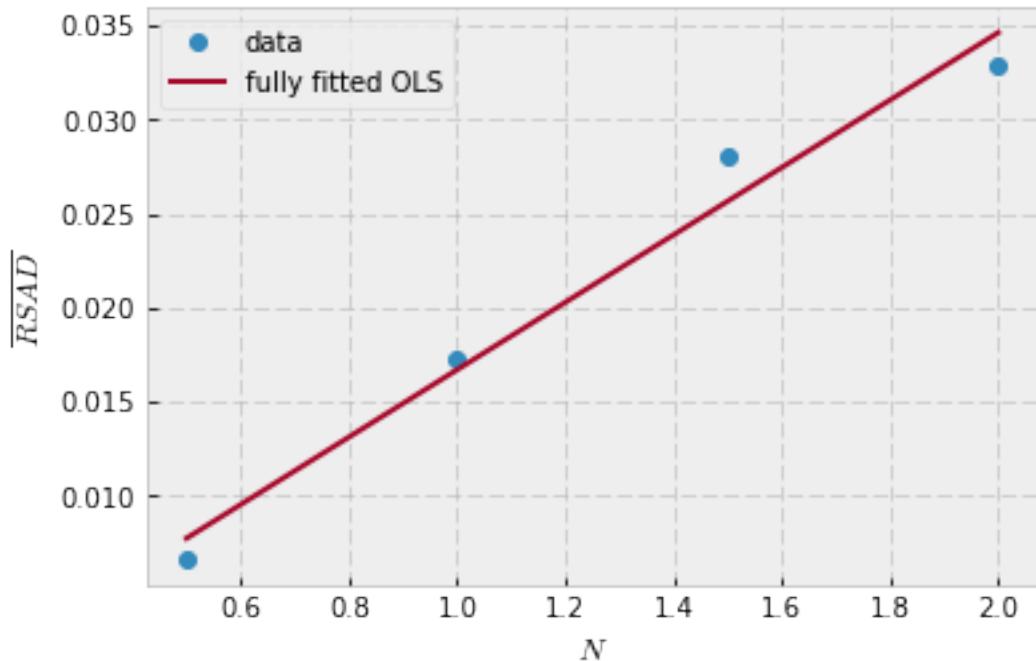
```
variable name: sigma
model: poly2
r-squared: 0.5943521464195411
explained variance: 0.5953124825961591
```



```

variable name: N
model: poly1
r-squared: 0.8529041364338754
explained variance: 0.8714799455587965

```

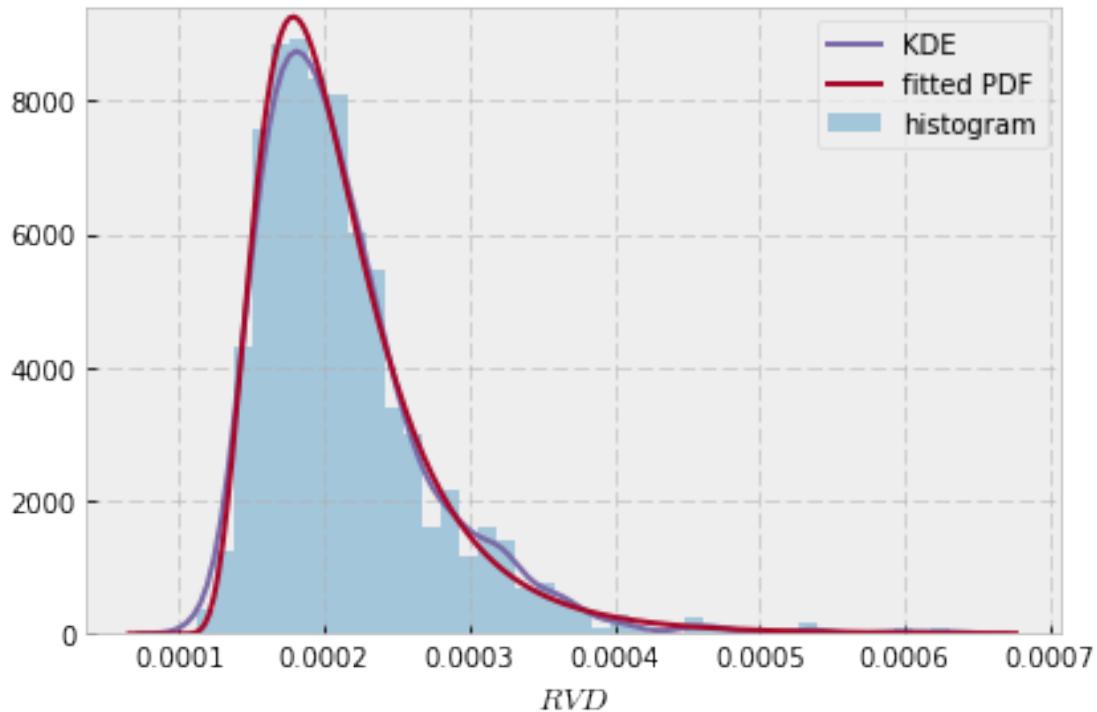


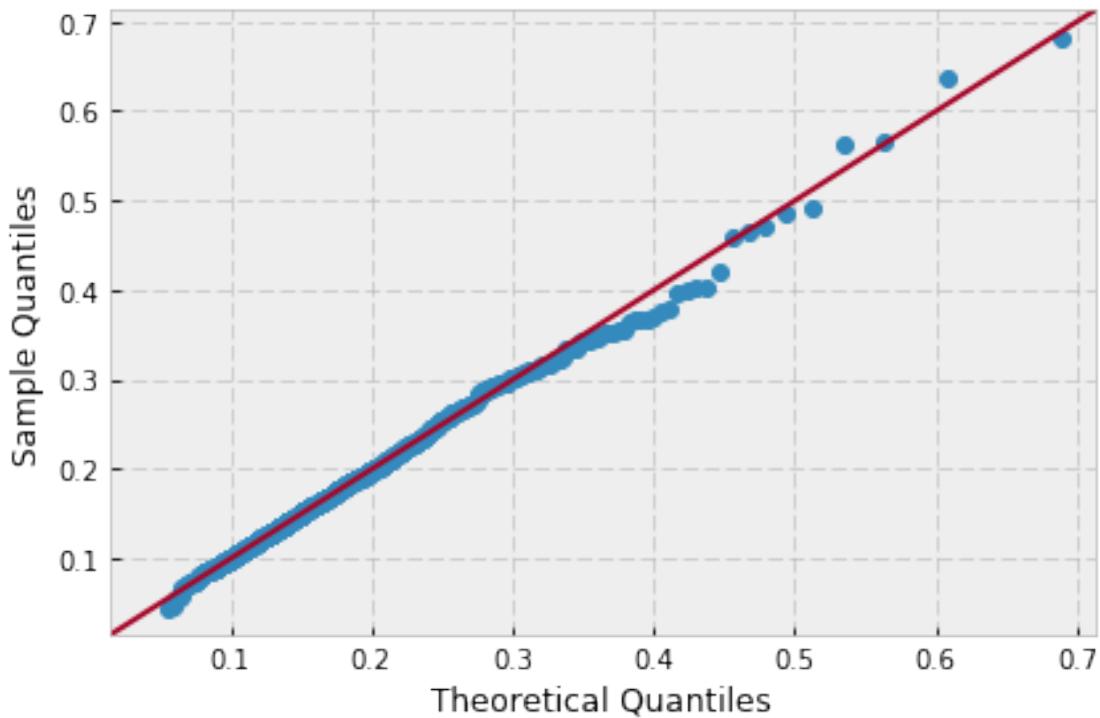
13 Root volume relative to convex hull

13.1 Probability distributions

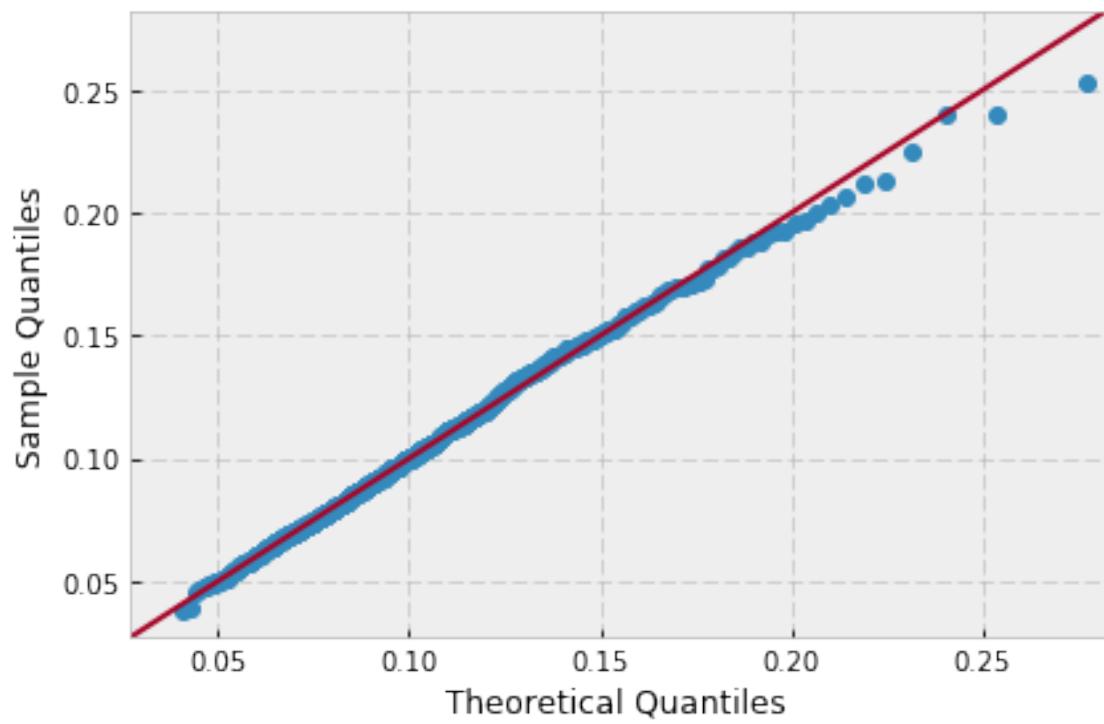
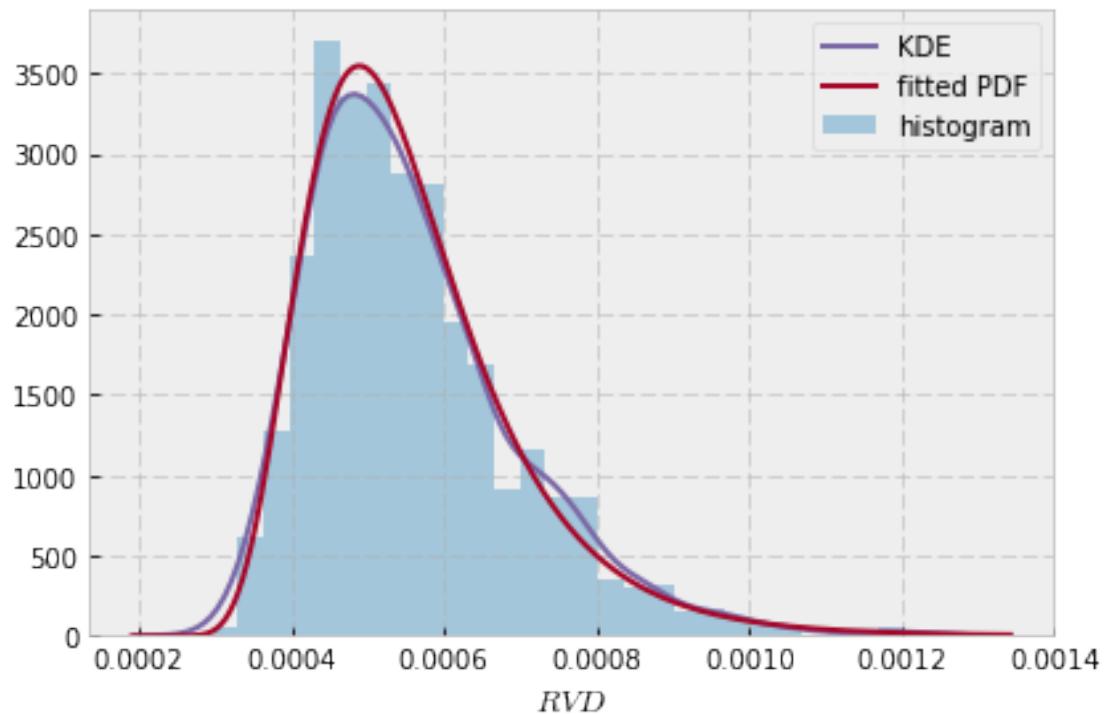
```
In [31]: plot_dist('total_root_volume_per_convex_hull_volume', 'inverse-gamma')
```

```
variable name: N
variable value: 0.5
distribution: inverse-gamma(shape=(6.8405266155592823,), loc=7.455192049209567e-05, scale=0.00081
DescribeResult(nobs=1000, minmax=(0.0001116955240372318, 0.00063046671666168452)
    mean=0.00021369454921087267, variance=3.7047017140764364e-09,
    skewness=1.8376426118988216, kurtosis=5.985918705967476)
```

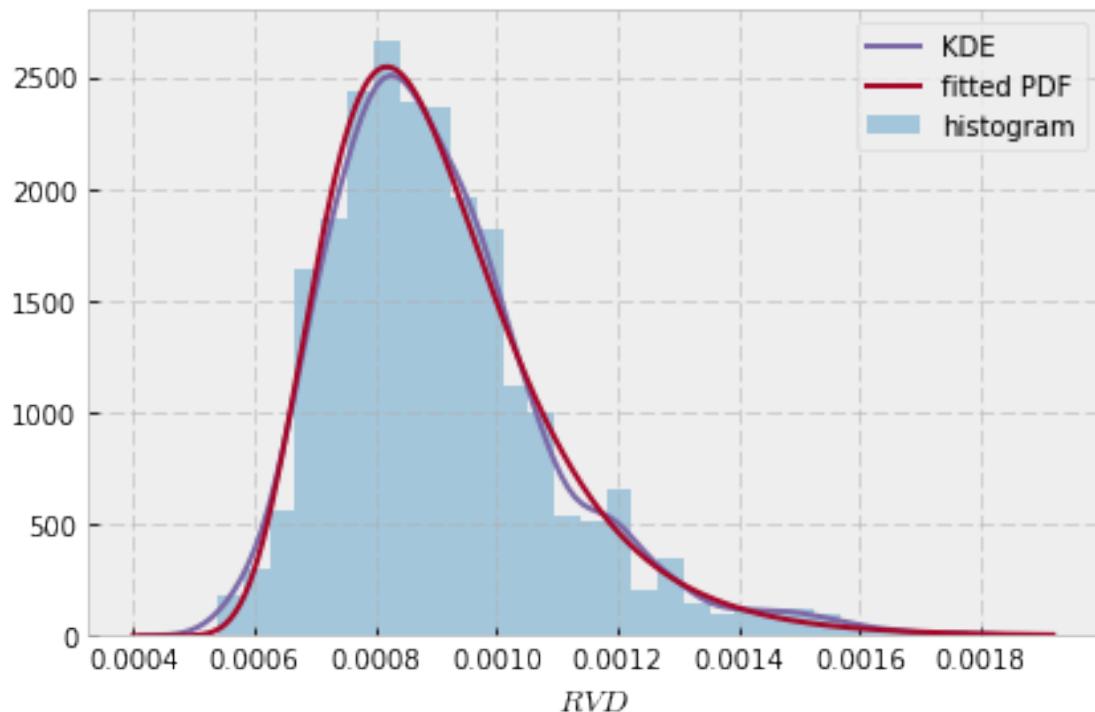


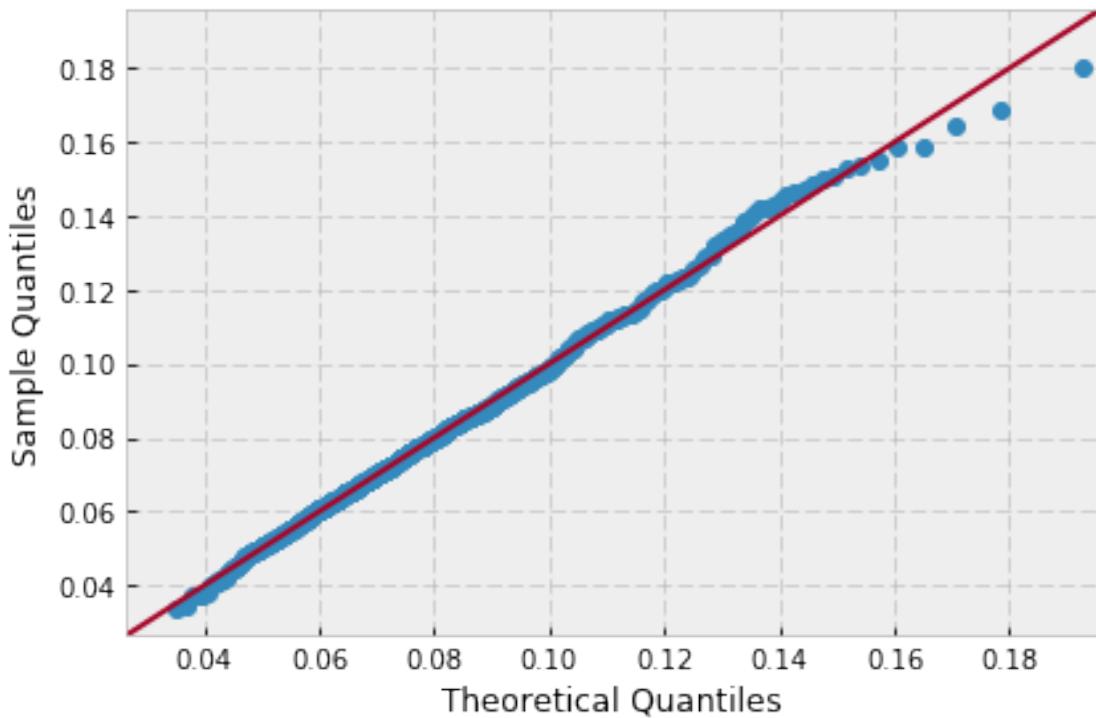


```
variable name: N
variable value: 1.0
distribution: inverse-gamma(shape=(11.219754949649939,), loc=0.00013046974558498608, scale=0.0043
DescribeResult(nobs=1000, minmax=(0.00029538674594706179, 0.0012384623509316248)
               mean=0.00055866914912388959, variance=1.9173992709138562e-08,
               skewness=1.1539672223580788, kurtosis=1.8291495284018708)
```

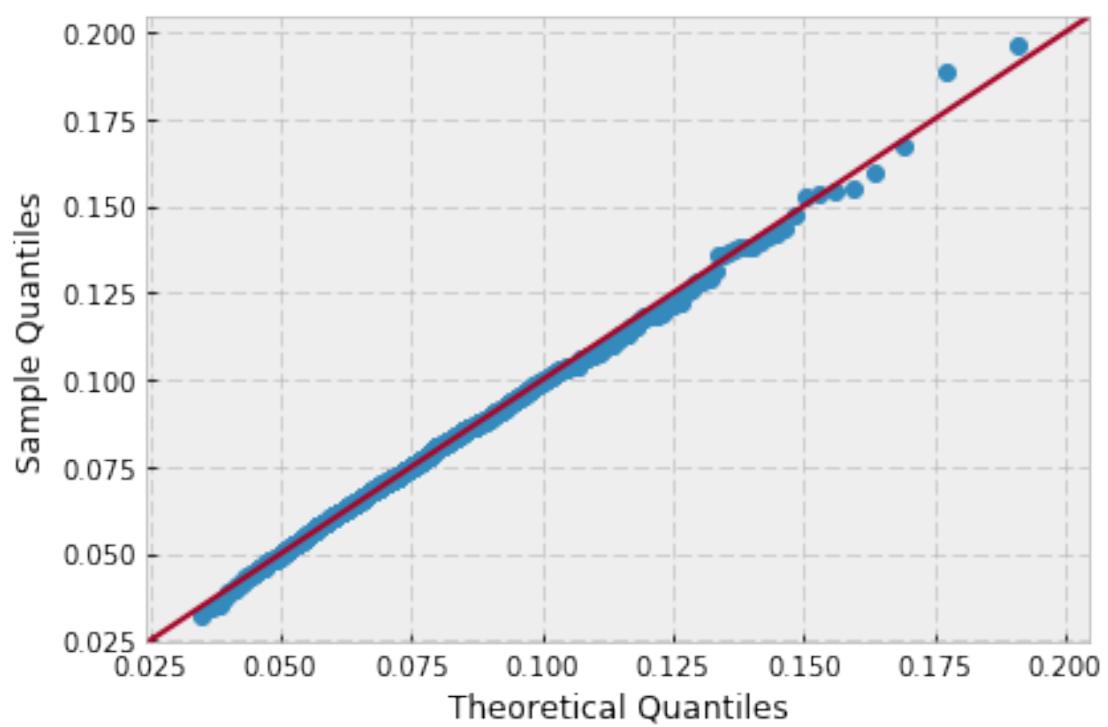
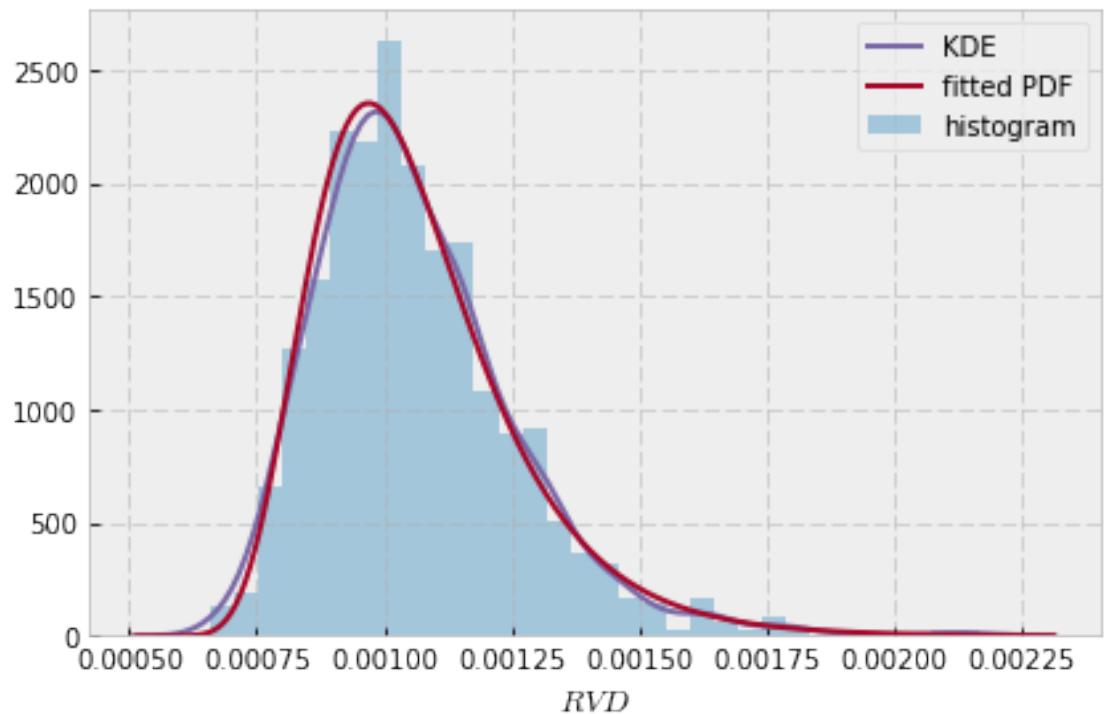


```
variable name: N
variable value: 1.5
distribution: inverse-gamma(shape=(13.995323004729833,),loc=0.00025559577060767257, scale=0.0084
DescribeResult(nobs=1000, minmax=(0.00053970073671131841, 0.0017798228332375342)
    mean=0.00090507727035041708, variance=3.430686090483061e-08,
    skewness=1.1221243156050151, kurtosis=1.8389564455754988)
```

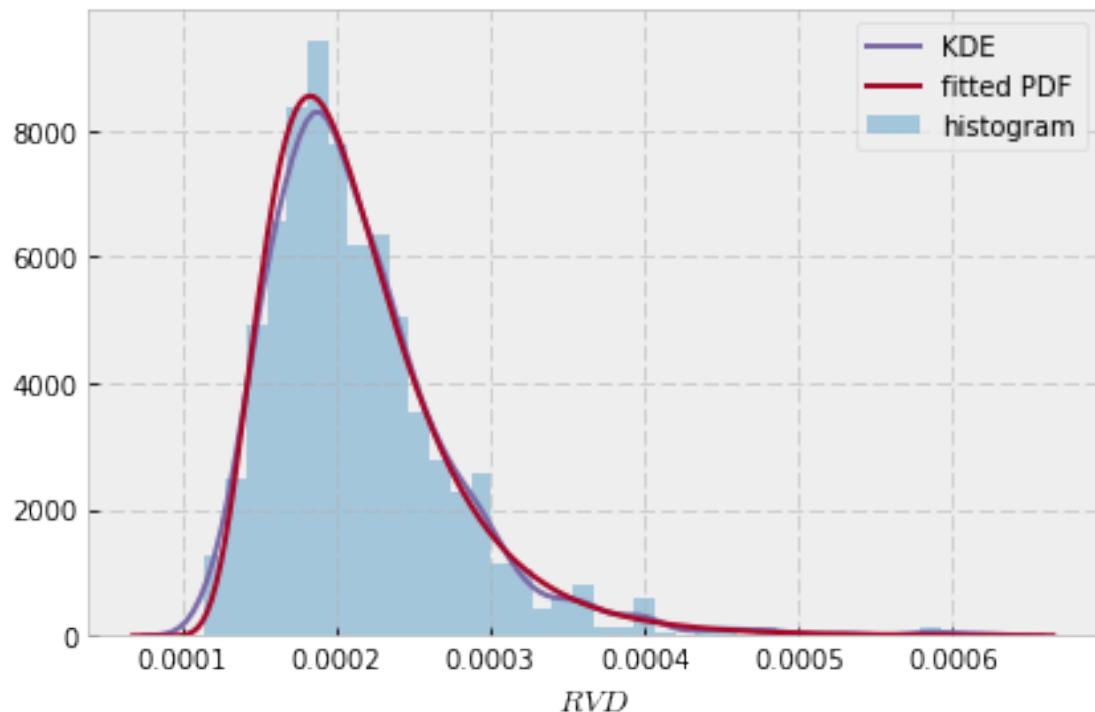


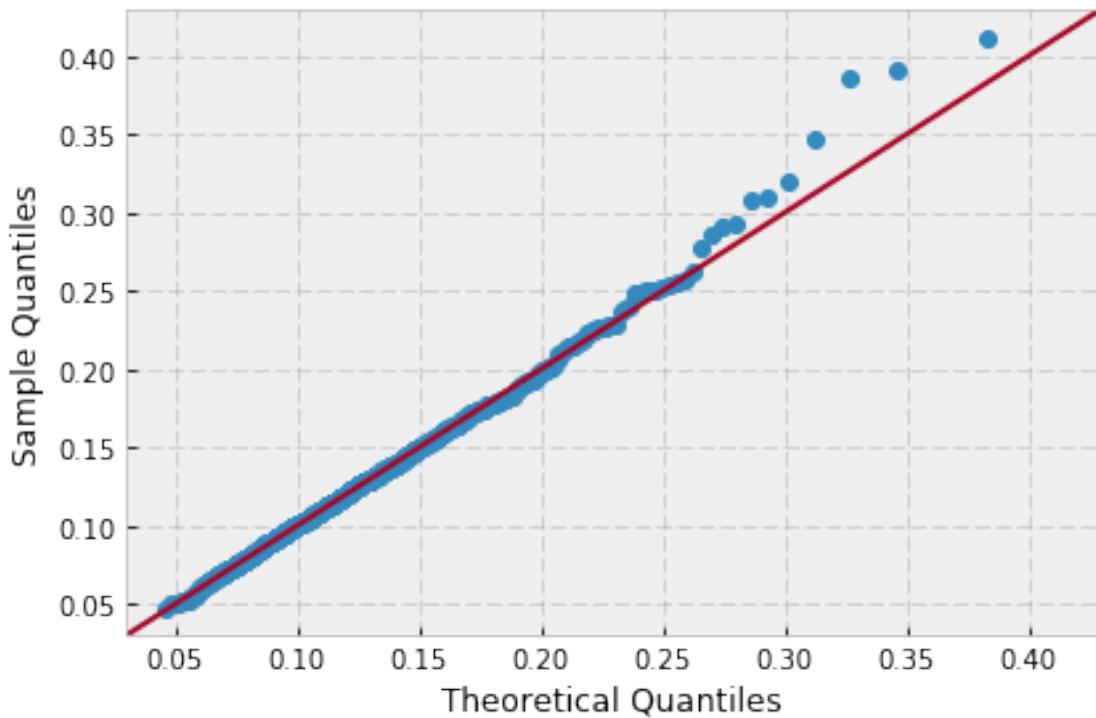


```
variable name: N
variable value: 2.0
distribution: inverse-gamma(shape=(14.071108899336183,),loc=0.00035846659985920684, scale=0.0092
DescribeResult(nobs=1000, minmax=(0.00065808838458192385, 0.0021654395670876346)
               mean=0.0010615877386503916, variance=3.8648589781930844e-08,
               skewness=1.1363791944028687, kurtosis=2.5363826321764176)
```

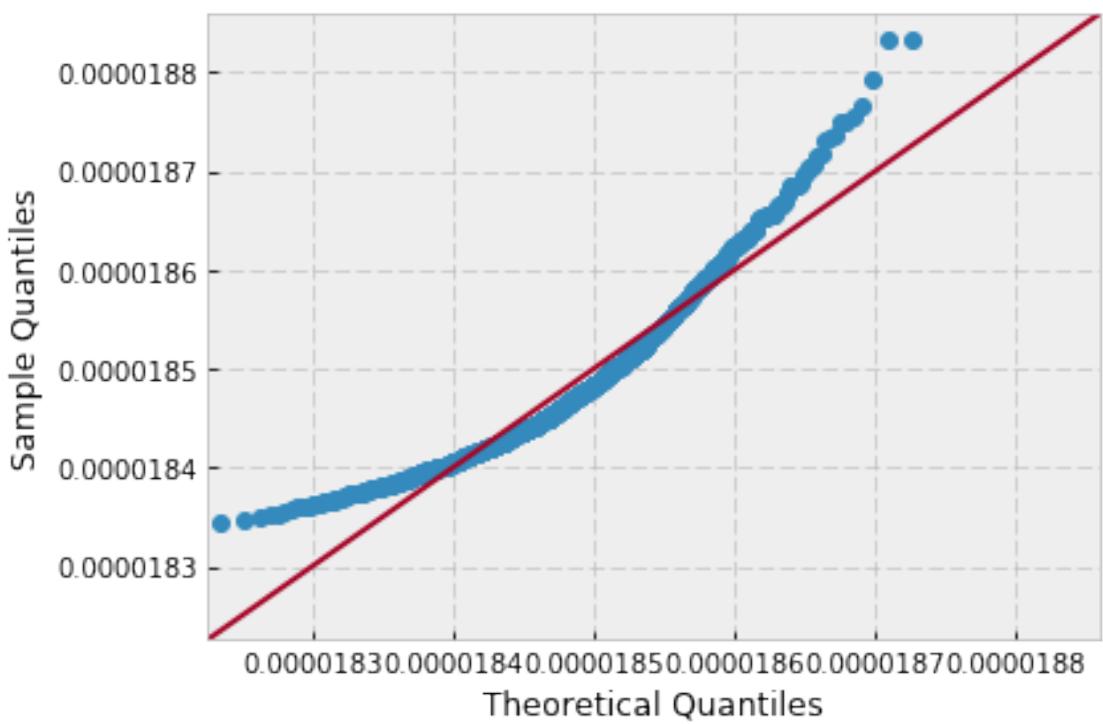
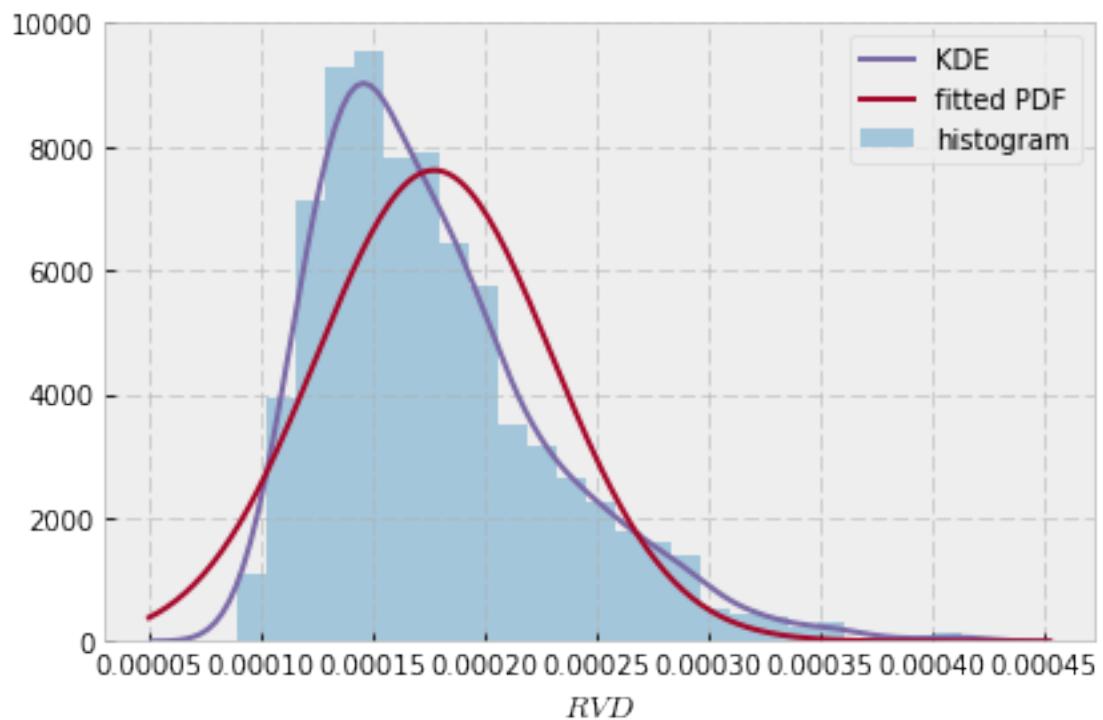


```
variable name: ln
variable value: 0.5
distribution: inverse-gamma(shape=(9.314884297759999,), loc=4.842049247220753e-05, scale=0.001385
DescribeResult(nobs=1000, minmax=(0.00011470526568819522, 0.00061836704723099481)
    mean=0.00021513592399024795, variance=3.8798464918276397e-09,
    skewness=1.7876493022142008, kurtosis=5.889301112758295)
```

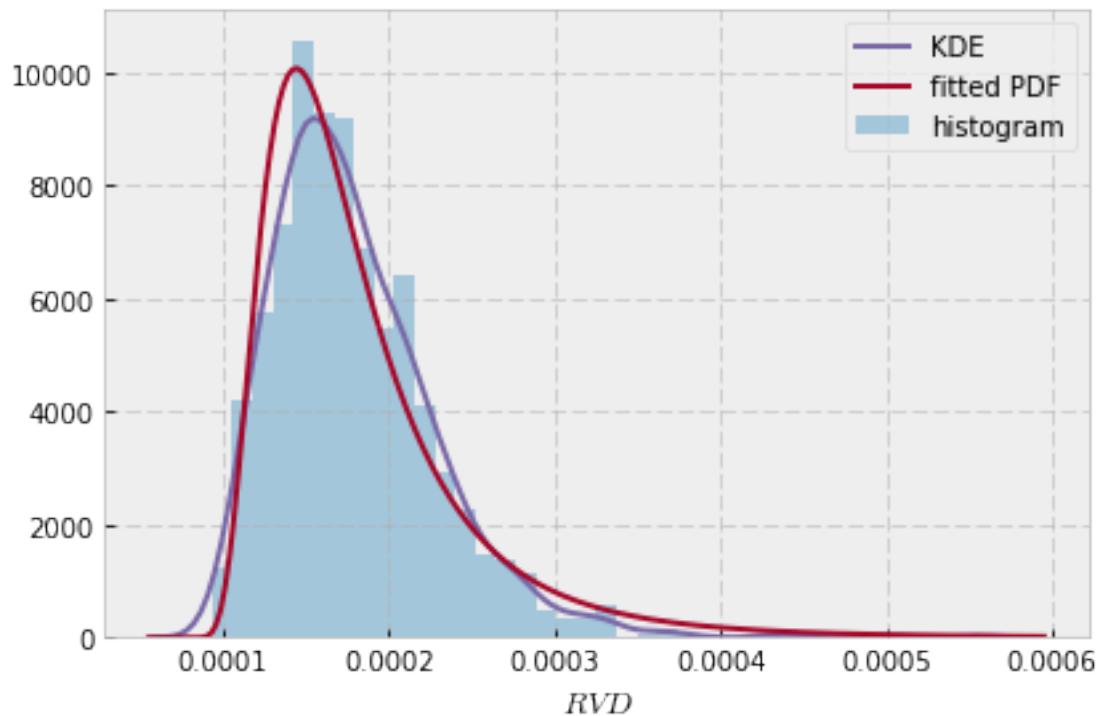


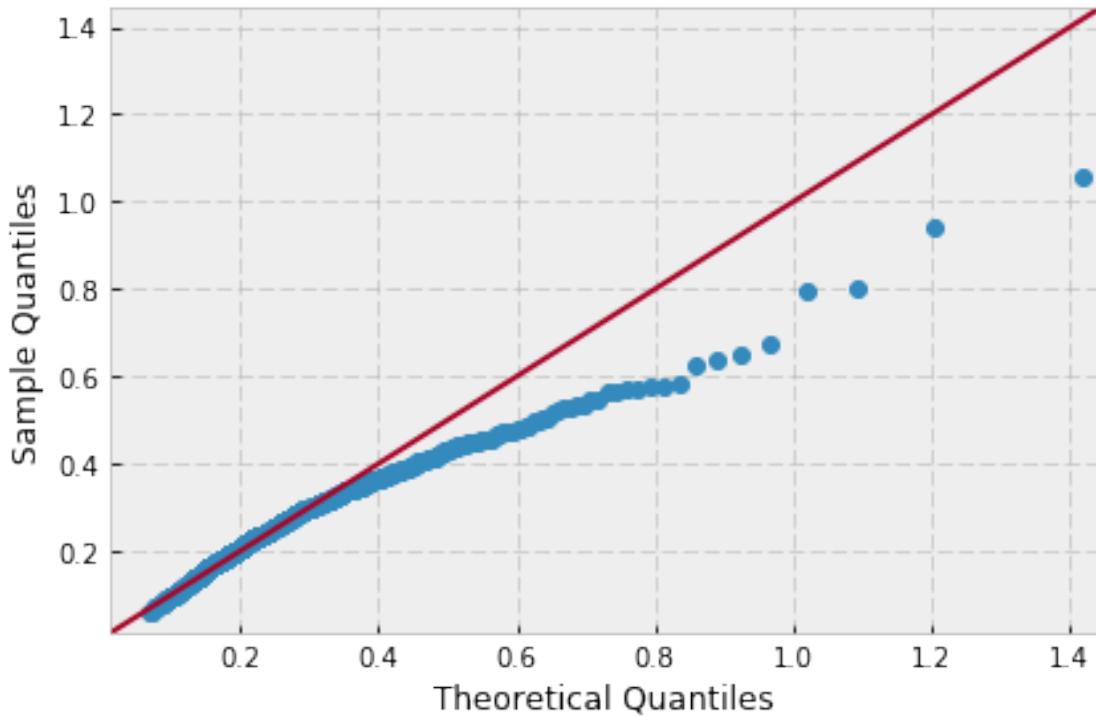


```
variable name: ln
variable value: 0.6666666666666666
distribution: inverse-gamma(shape=(54116.912553225819,), loc=-0.012007664375445894, scale=659.444
DescribeResult(nobs=1000, minmax=(9.0187736056421185e-05, 0.00041267087874957036)
               mean=0.00017810951770969496, variance=2.7637659029972354e-09,
               skewness=1.0889878445408354, kurtosis=1.2637265380535423)
```

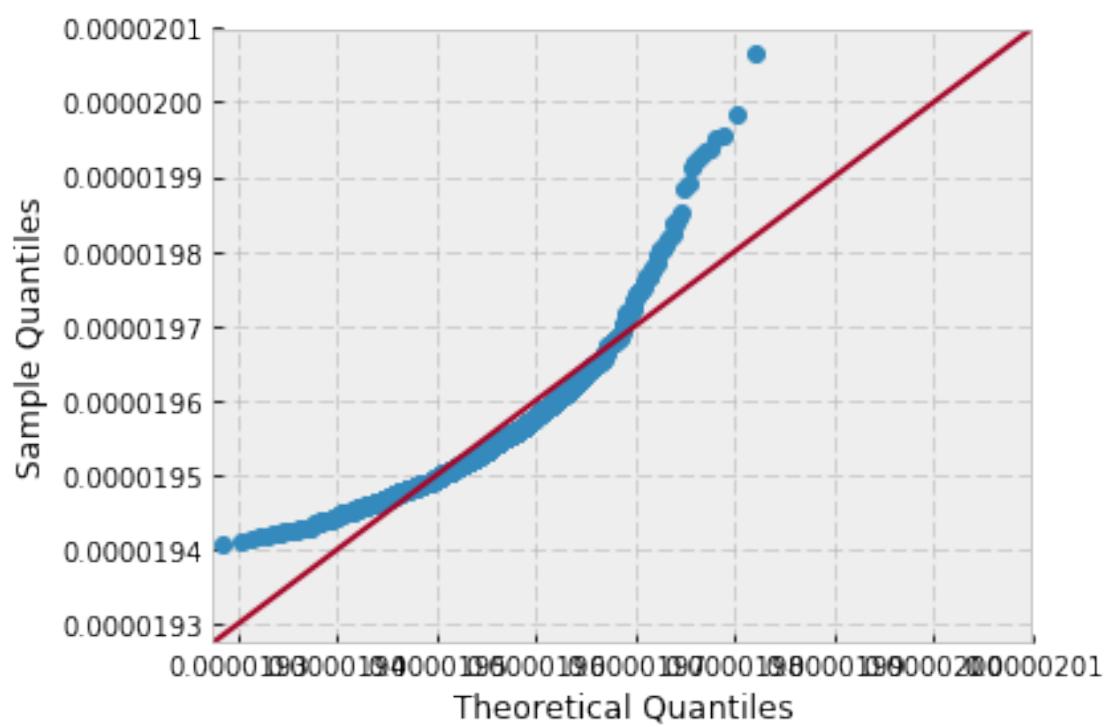
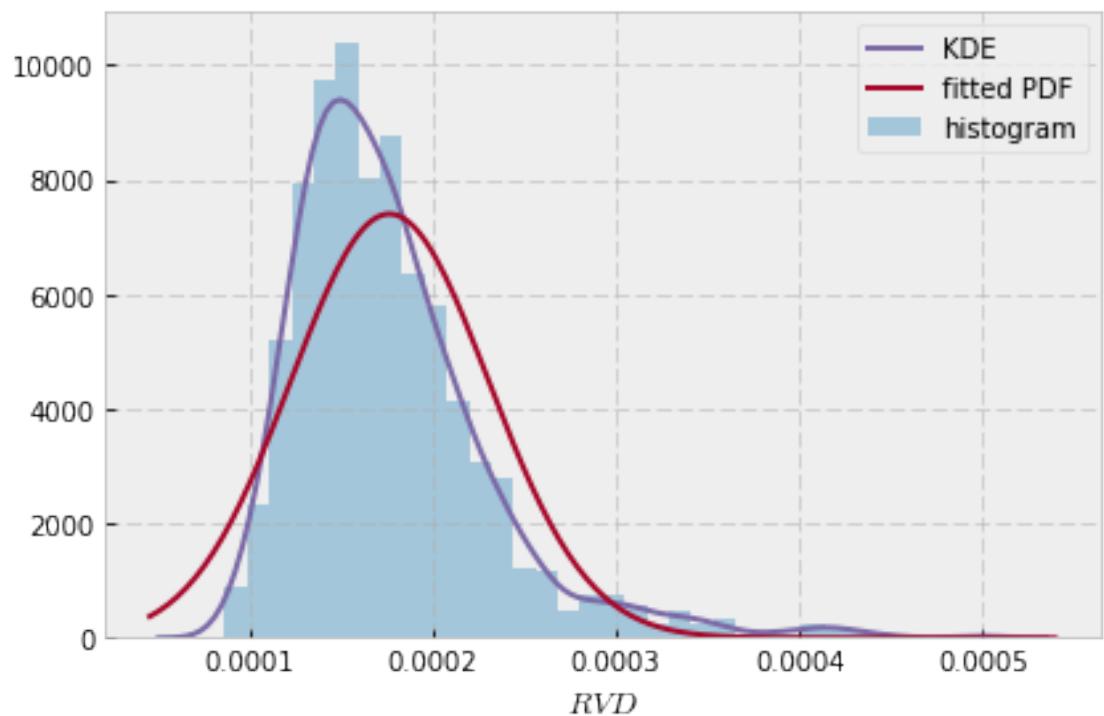


```
variable name: ln
variable value: 0.8333333333333333
distribution: inverse-gamma(shape=(4.9018923382726314,),loc=6.538771101004422e-05, scale=0.00046
DescribeResult(nobs=1000, minmax=(9.3483497733016951e-05, 0.00055663778026833542)
    mean=0.00017925940876054478, variance=2.5983533482089845e-09,
    skewness=1.608394942200798, kurtosis=5.855516855035242)
```

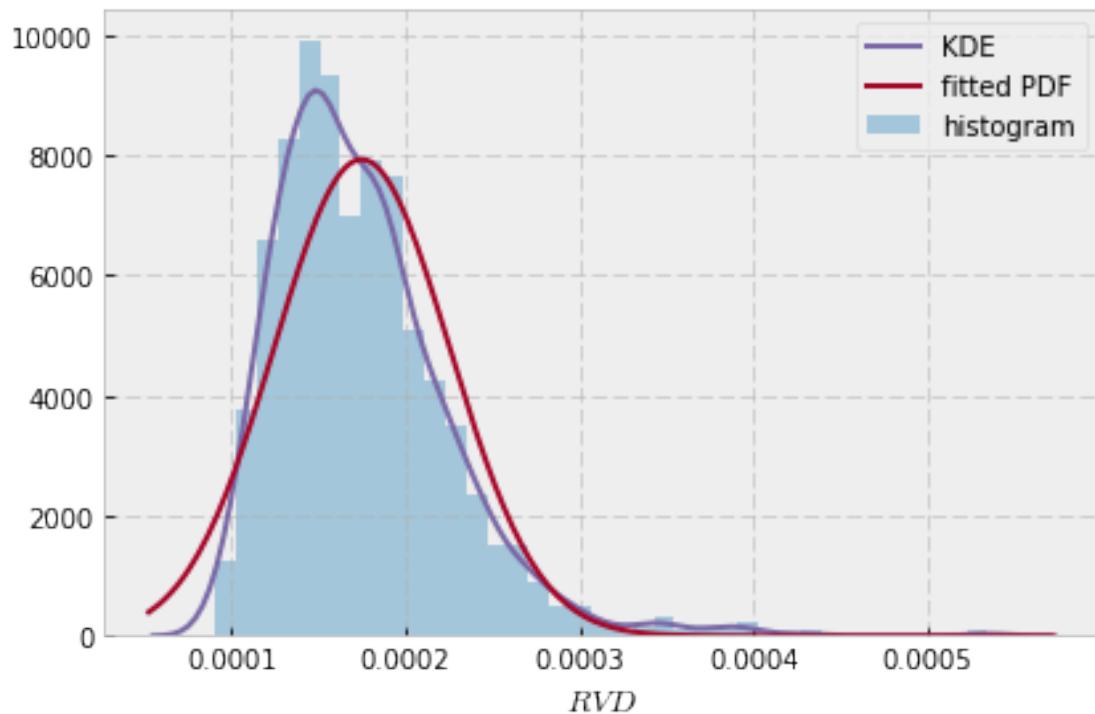


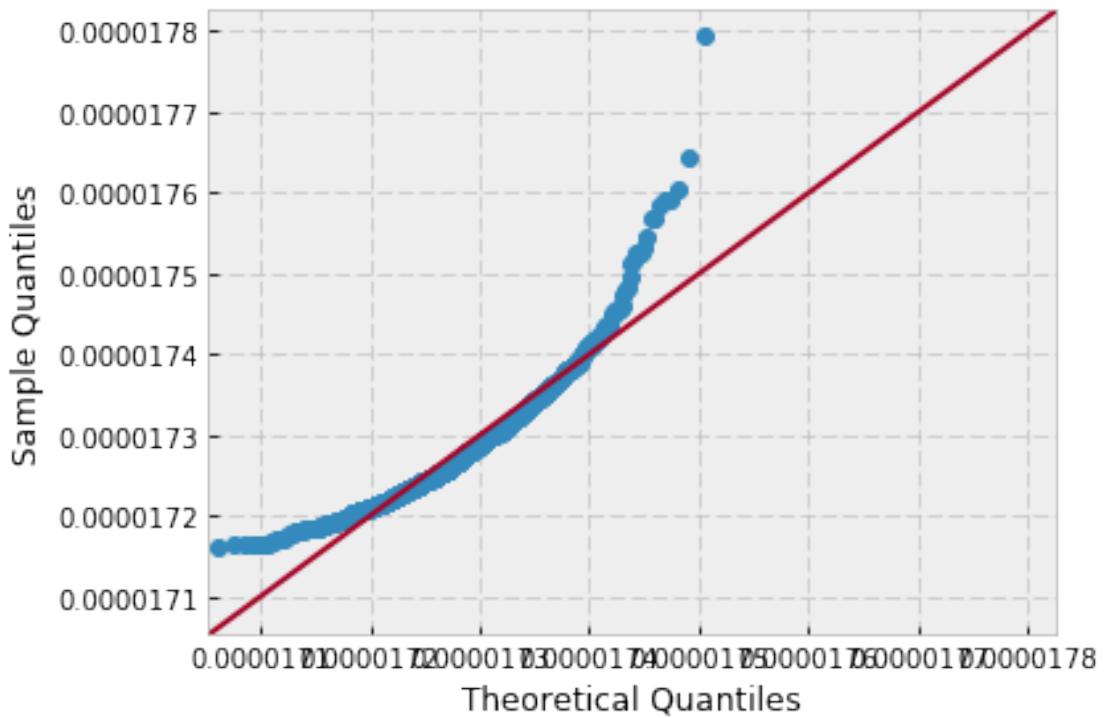


```
variable name: ln
variable value: 1.0
distribution: inverse-gamma(shape=(51150.388724660472,), loc=-0.012008736128258883, scale=623.293
DescribeResult(nobs=1000, minmax=(8.6799885276605529e-05, 0.00049831537816011806)
               mean=0.00017701461396052665, variance=2.9350425641817136e-09,
               skewness=1.7068026217879684, kurtosis=4.623486973090805)
```

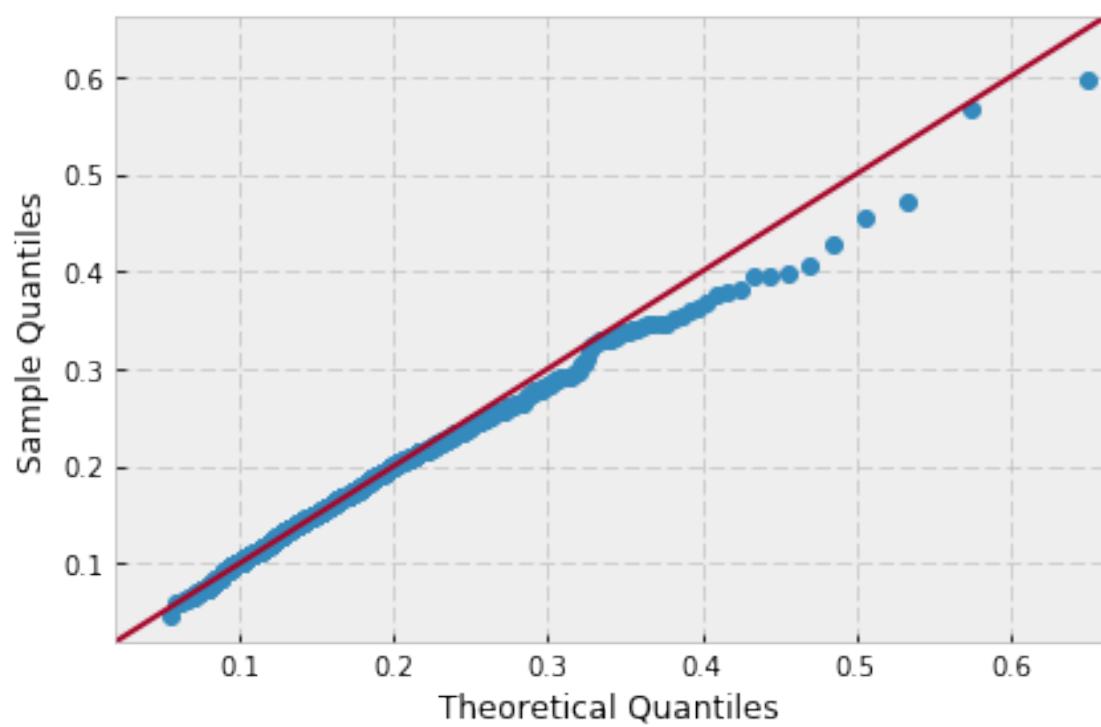
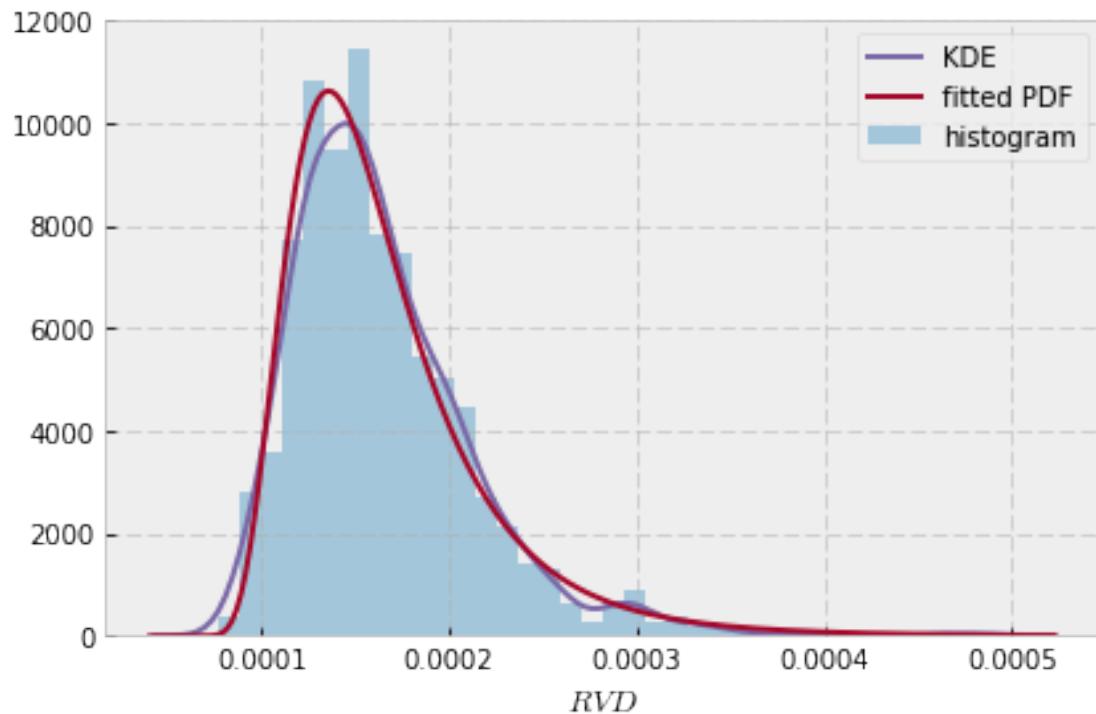


```
variable name: ln
variable value: 1.1666666666666665
distribution: inverse-gamma(shape=(57865.684914808604,),loc=-0.011924744150541775, scale=700.154
DescribeResult(nobs=1000, minmax=(9.0555825457972405e-05, 0.00053431140477748214)
    mean=0.00017520259994152439, variance=2.5609596359554585e-09,
    skewness=1.49296104086403, kurtosis=4.618383511130817)
```

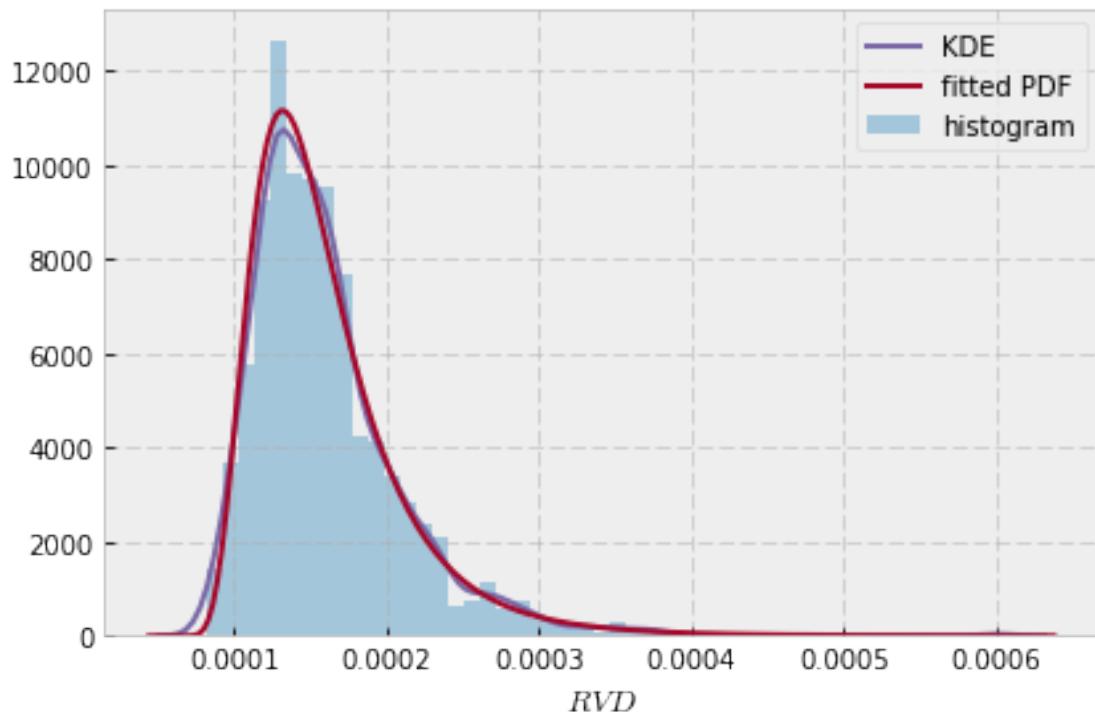


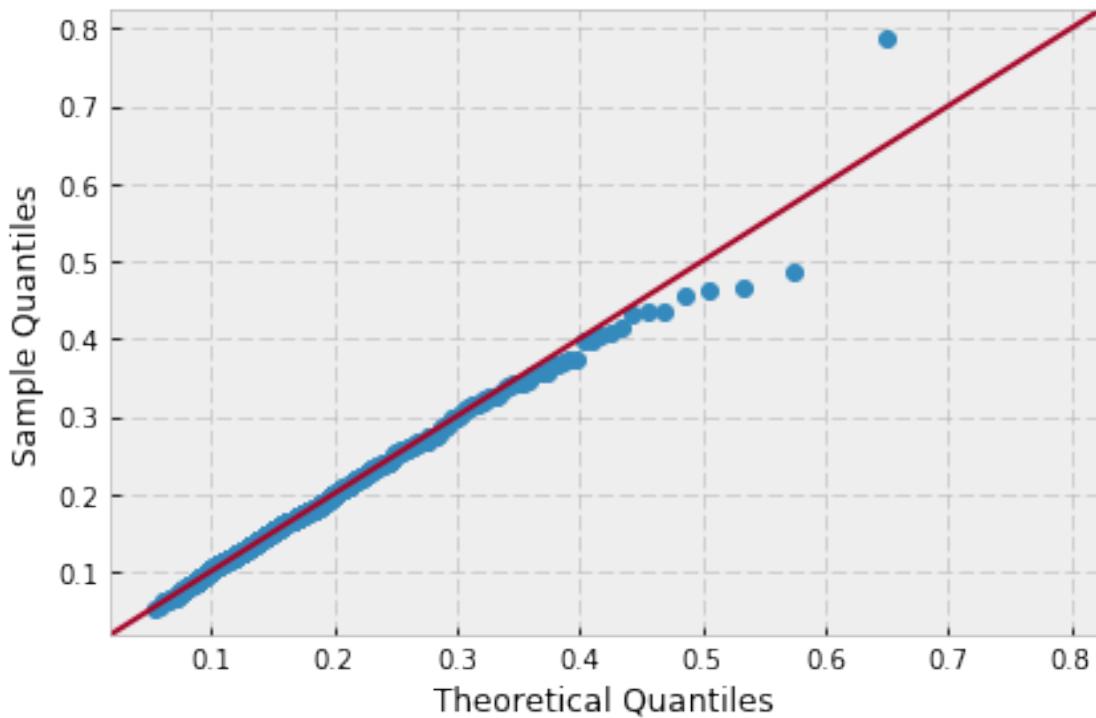


```
variable name: ln
variable value: 1.333333333333333
distribution: inverse-gamma(shape=(7.0455625096463201,), loc=4.3378623720665316e-05, scale=0.0007
DescribeResult(nobs=1000, minmax=(7.7311064916869656e-05, 0.00048613016625248398)
               mean=0.00016460806619096152, variance=2.3907077815150048e-09,
               skewness=1.5664092057402046, kurtosis=4.589895991054768)
```

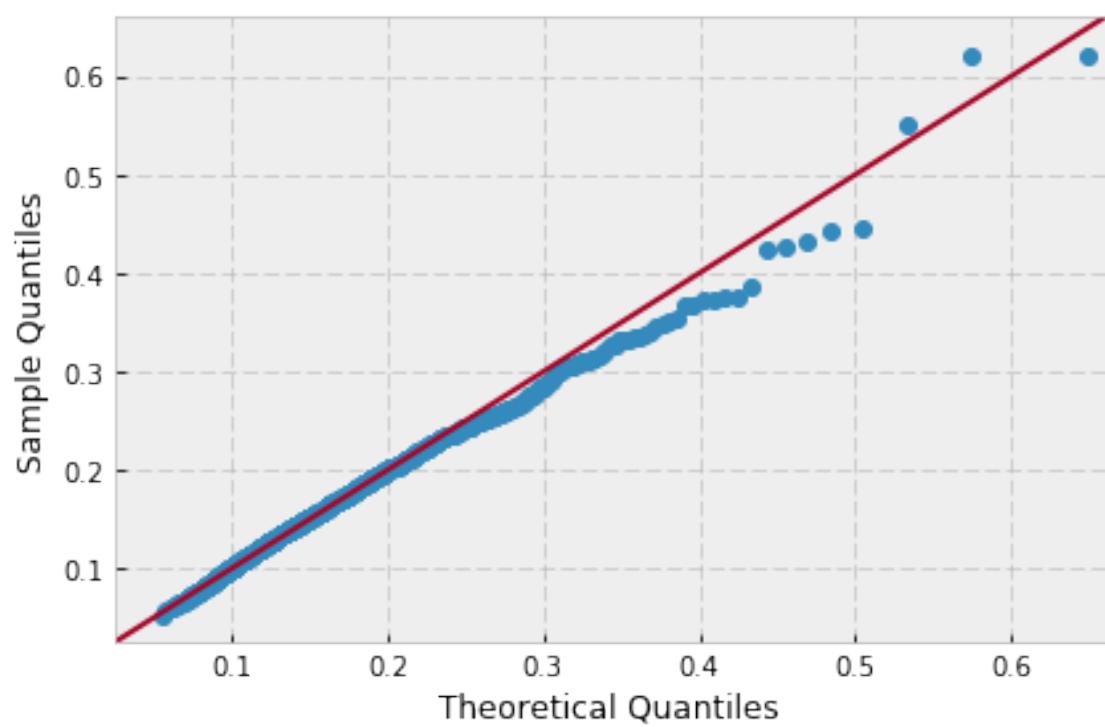
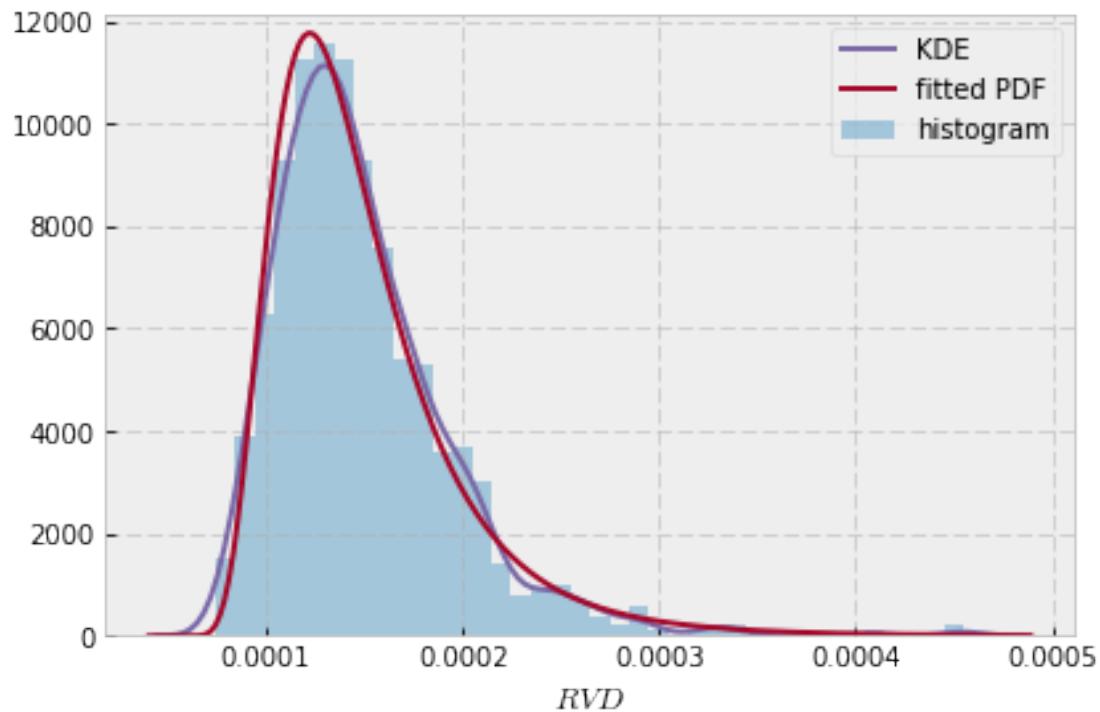


```
variable name: ln
variable value: 1.5
distribution: inverse-gamma(shape=(7.0451037650963997,), loc=4.389303288404551e-05, scale=0.00070
DescribeResult(nobs=1000, minmax=(8.1653982468577222e-05, 0.00060054936649852502)
               mean=0.00016021776776652558, variance=2.4612525529401303e-09,
               skewness=1.9274767779135005, kurtosis=8.019055803500375)
```

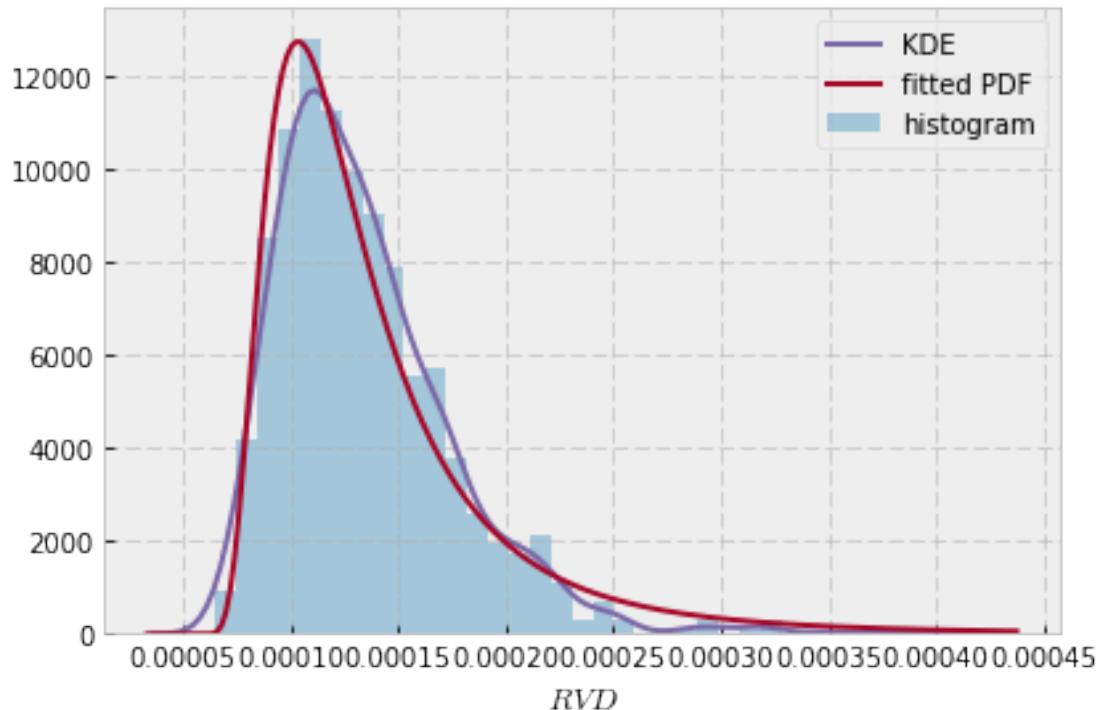


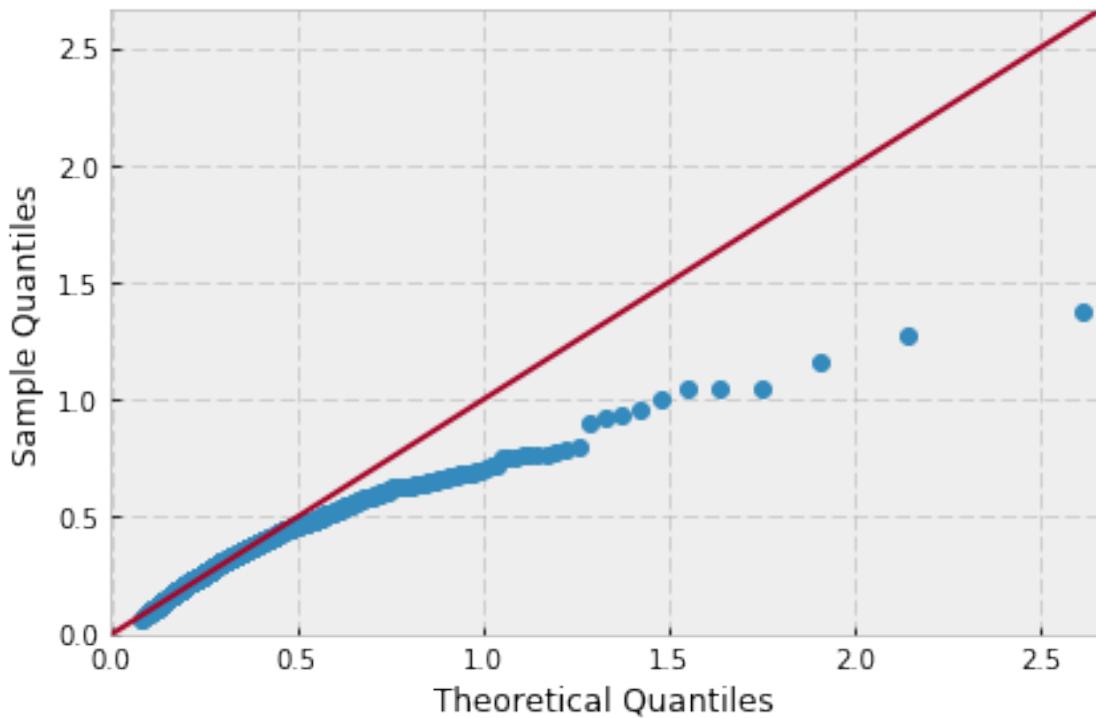


```
variable name: ln
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(7.052764185191517,), loc=3.861813013940955e-05, scale=0.000671
DescribeResult(nobs=1000, minmax=(7.4129646148812564e-05, 0.00045490766151130271)
               mean=0.00014839705639972987, variance=2.0327069727015455e-09,
               skewness=1.7420930931227658, kurtosis=6.117227620500042)
```

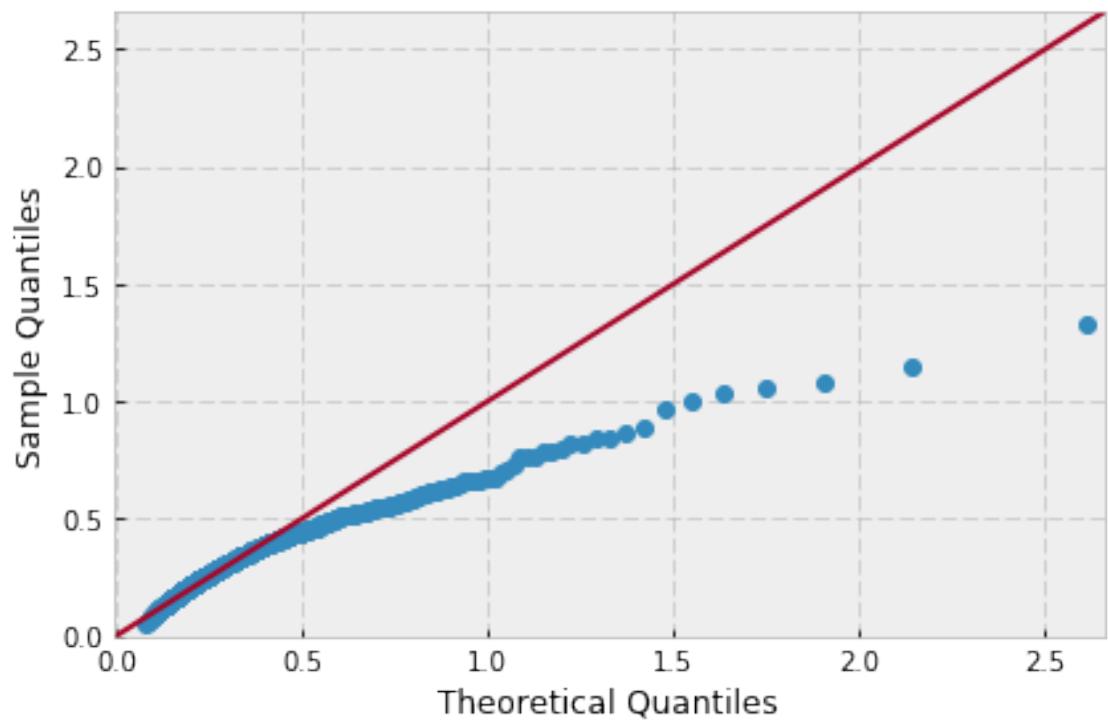
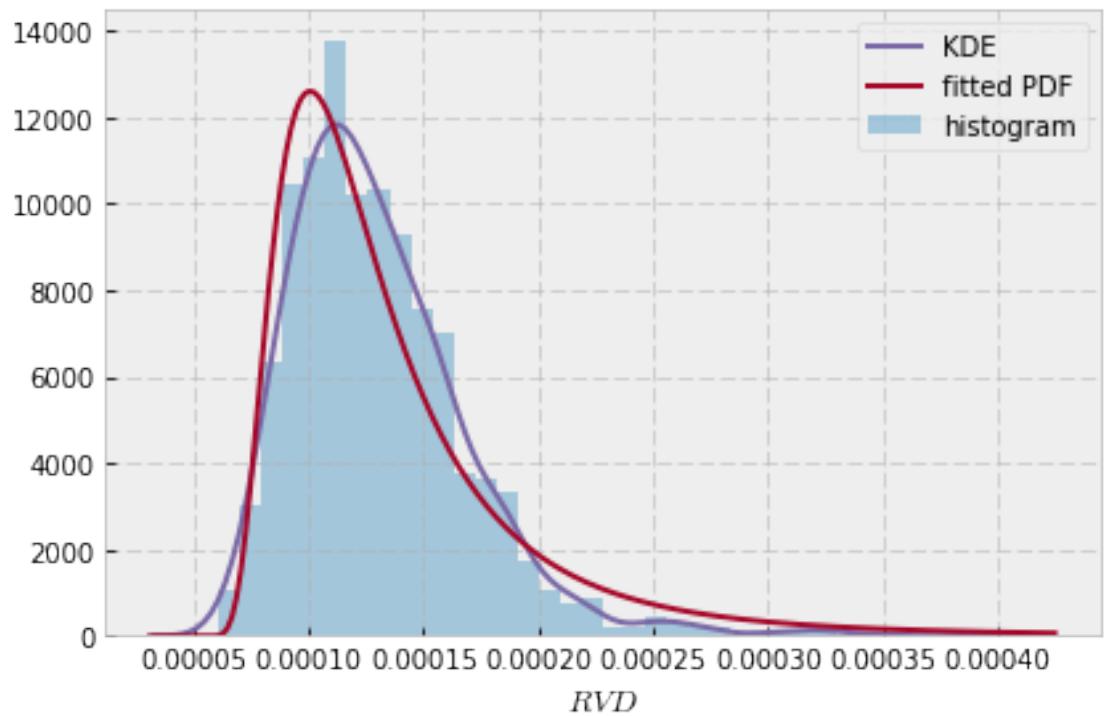


```
variable name: ln
variable value: 1.833333333333333
distribution: inverse-gamma(shape=(3.8336721983211057,),loc=4.985477764035136e-05, scale=0.00025
DescribeResult(nobs=1000, minmax=(6.5106057764683712e-05, 0.00040581120958171294)
    mean=0.0001344565318028169, variance=1.7637751870101043e-09,
    skewness=1.5914841242596767, kurtosis=4.7138898129318)
```

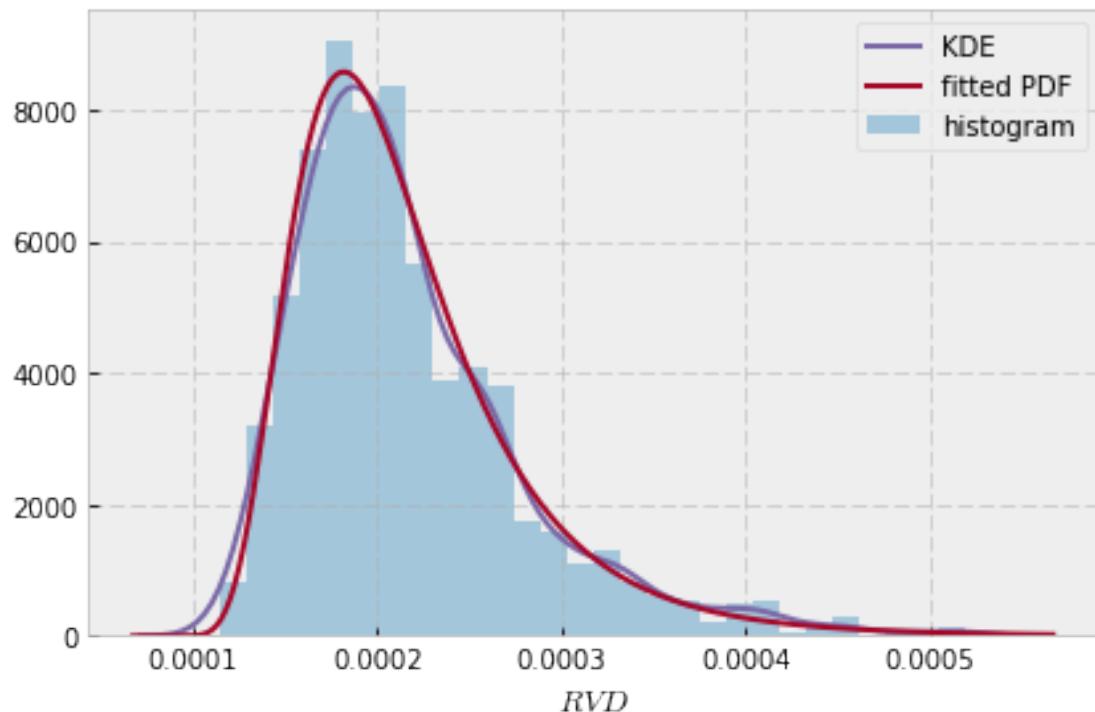


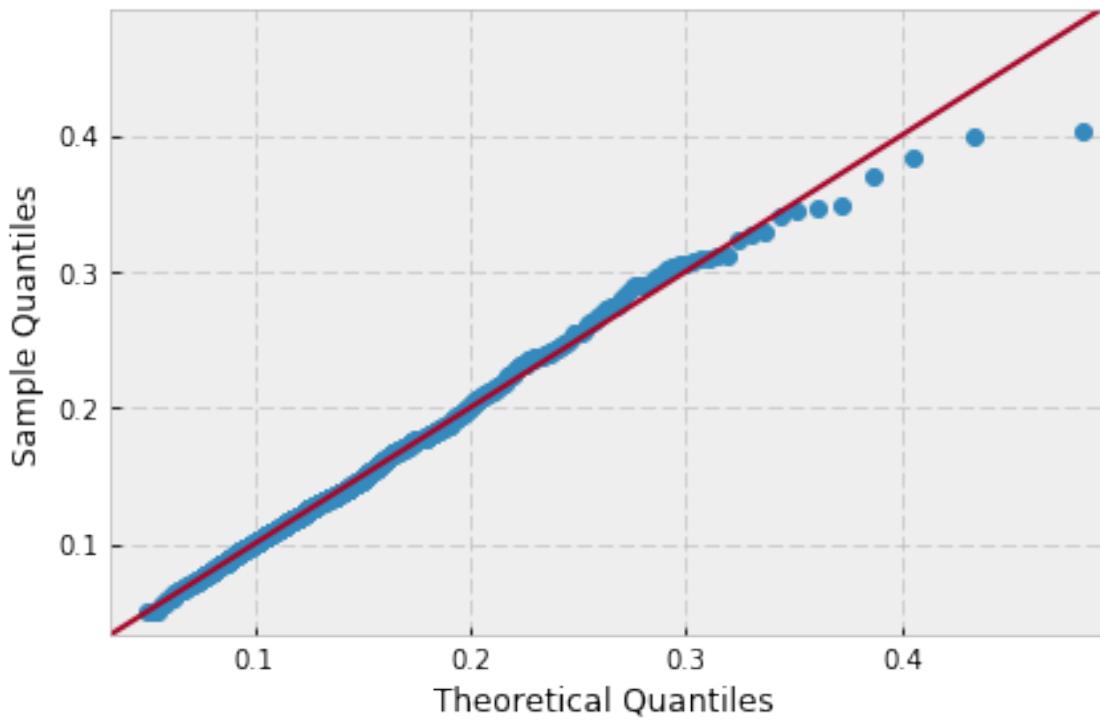


```
variable name: ln
variable value: 2.0
distribution: inverse-gamma(shape=(3.8346941887385517,), loc=4.637749340676849e-05, scale=0.00026
DescribeResult(nobs=1000, minmax=(6.0910669050188601e-05, 0.00039498777941612882)
               mean=0.0001307011891577029, variance=1.5791345077224251e-09,
               skewness=1.5832841282111212, kurtosis=5.006256278793174)
```

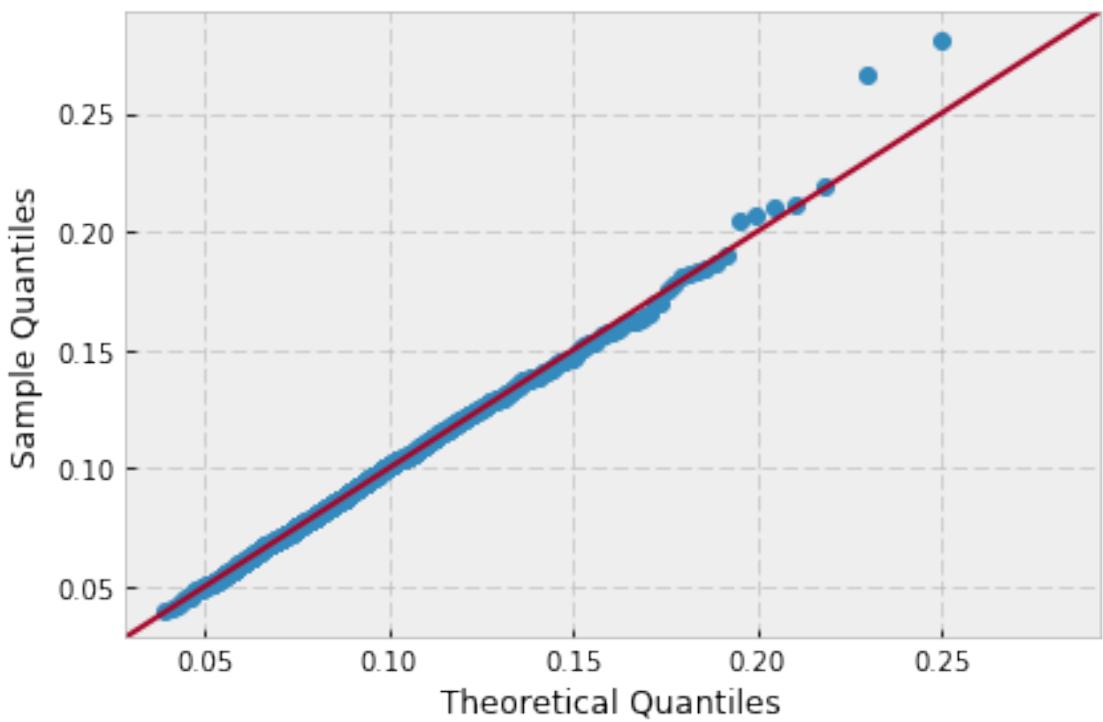
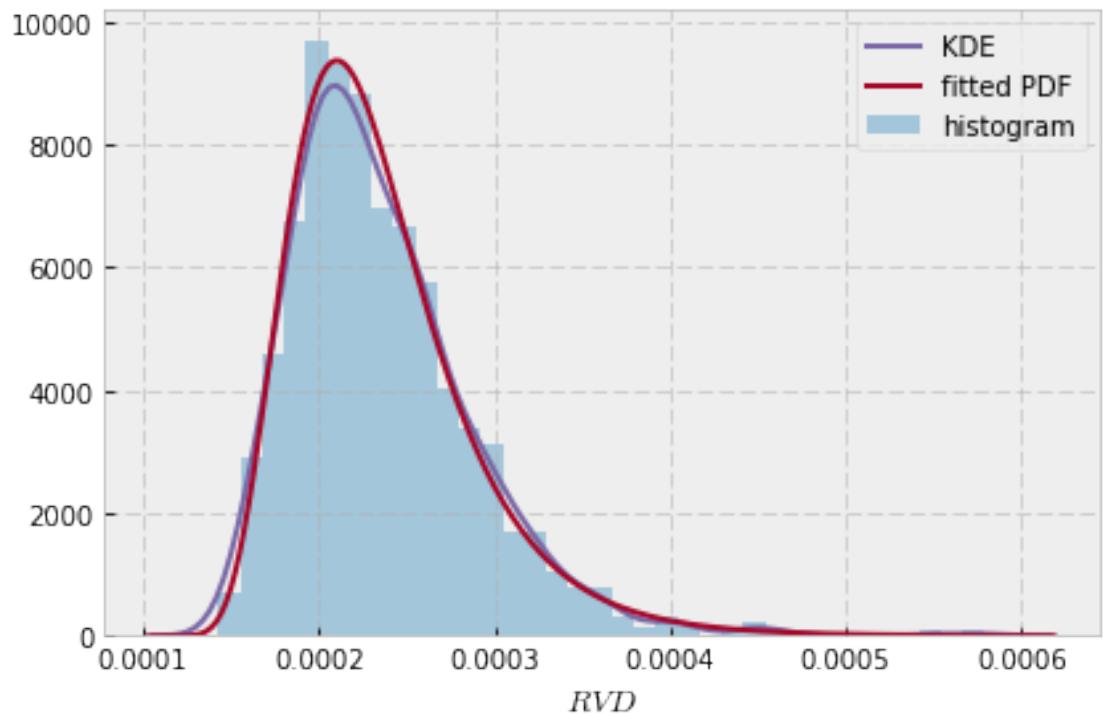


```
variable name: maxB
variable value: 4.0
distribution: inverse-gamma(shape=(8.2117623335983723,), loc=5.69695876378831e-05, scale=0.001149
DescribeResult(nobs=1000, minmax=(0.00011454531095721588, 0.00051943887930988846)
    mean=0.00021626509864017685, variance=3.9240854104555613e-09,
    skewness=1.4500755837444654, kurtosis=2.7858749613437617)
```

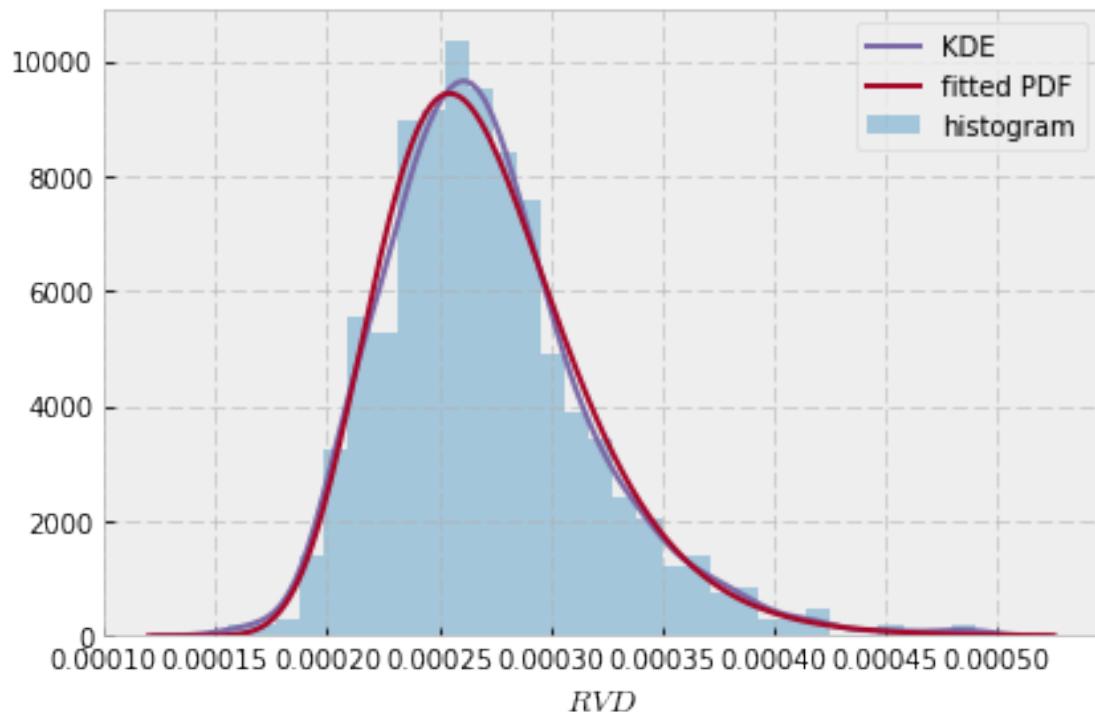


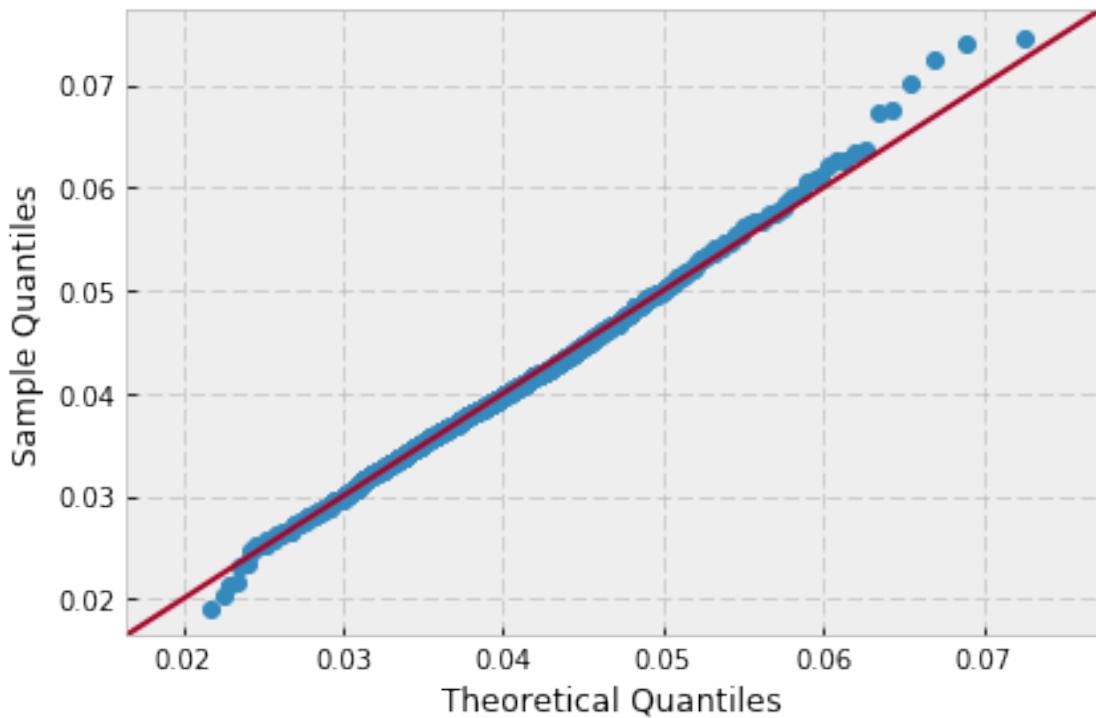


```
variable name: maxB
variable value: 8.0
distribution: inverse-gamma(shape=(11.922202559890735,), loc=6.99682587338426e-05, scale=0.001812
DescribeResult(nobs=1000, minmax=(0.000142749089979044, 0.00057893471340346718)
               mean=0.00023588117148707843, variance=2.7609651353838198e-09,
               skewness=1.4013264757136084, kurtosis=4.028429750354097)
```

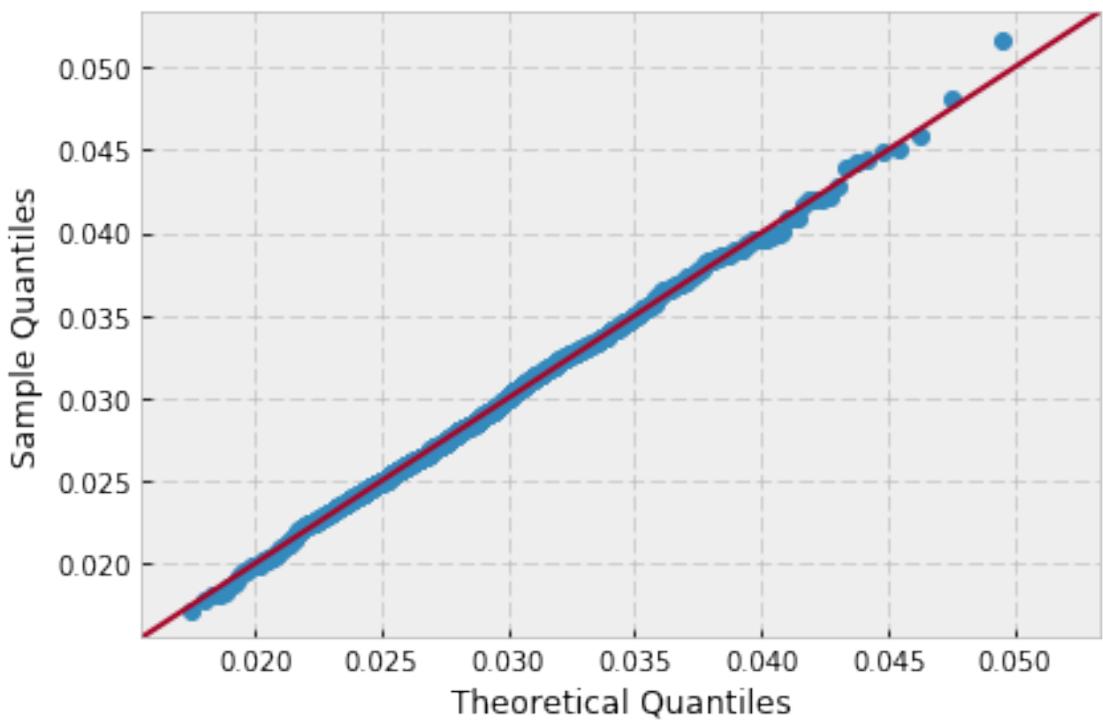
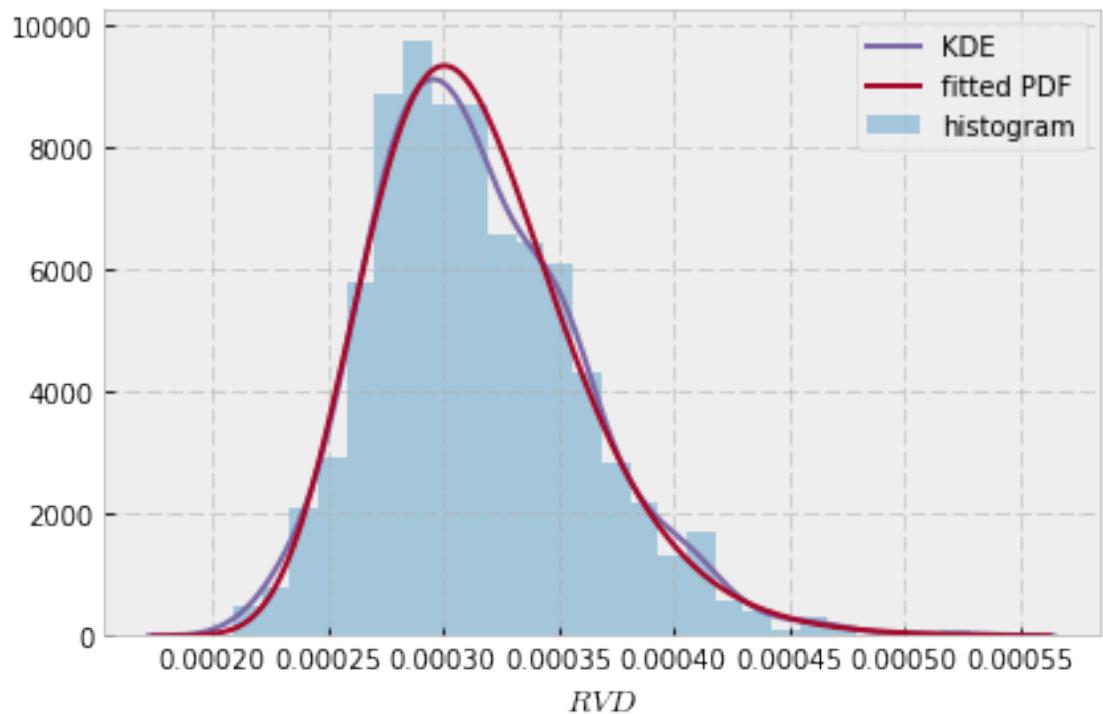


```
variable name: maxB
variable value: 12.0
distribution: inverse-gamma(shape=(27.087111717652469,),loc=3.934281150260411e-05, scale=0.00605
DescribeResult(nobs=1000, minmax=(0.00015533081970933838, 0.00049032653034933749)
    mean=0.00027145407956680954, variance=2.1890682155080053e-09,
    skewness=0.9612646890422718, kurtosis=1.7928899125891302)
```

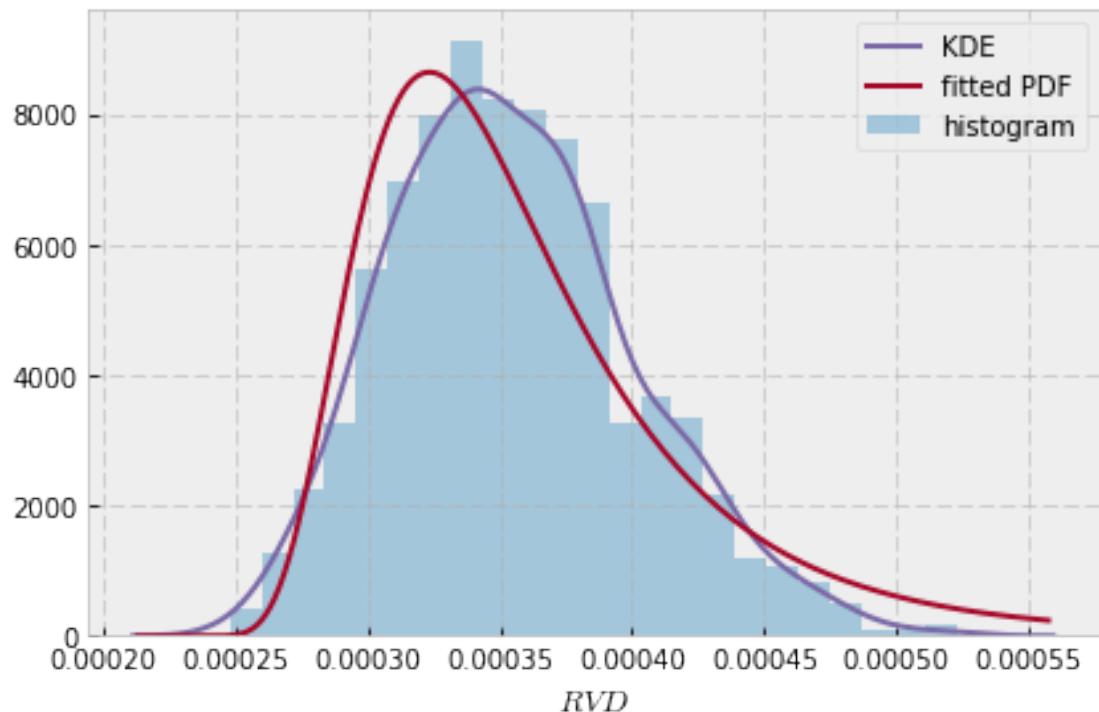


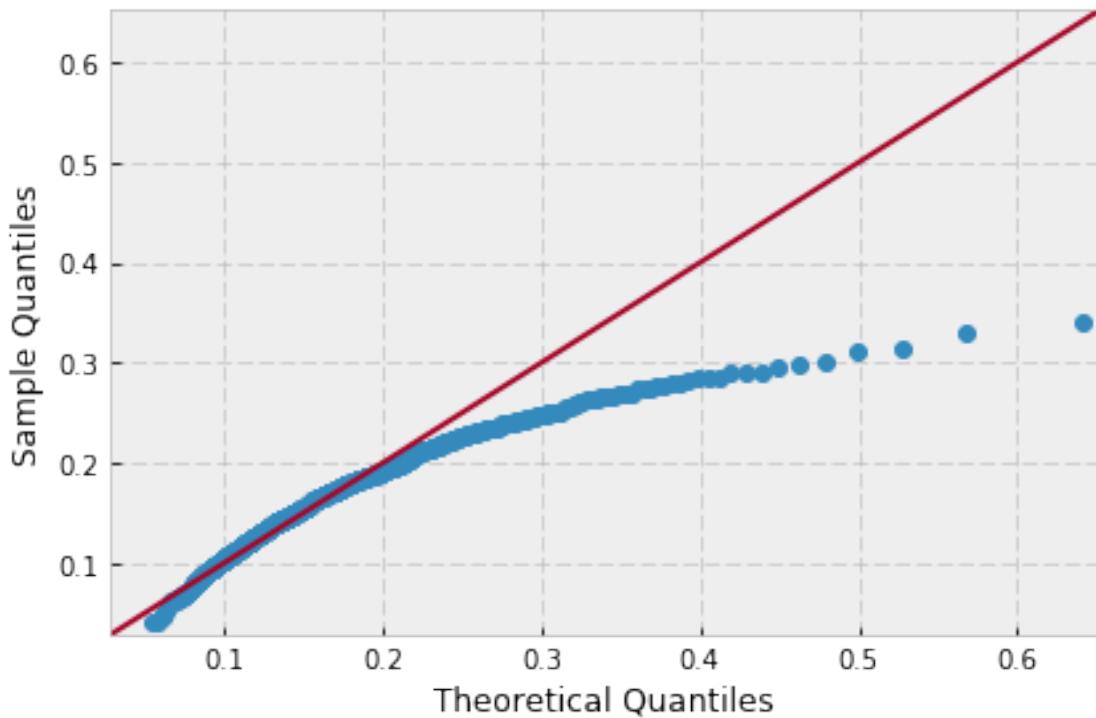


```
variable name: maxB
variable value: 16.0
distribution: inverse-gamma(shape=(35.909001058078346,), loc=4.883062686496505e-05, scale=0.00928
DescribeResult(nobs=1000, minmax=(0.00020888486233235212, 0.00052815077608160449)
               mean=0.00031492110561088572, variance=2.0843856029507976e-09,
               skewness=0.6730292018536278, kurtosis=0.7105149535104021)
```

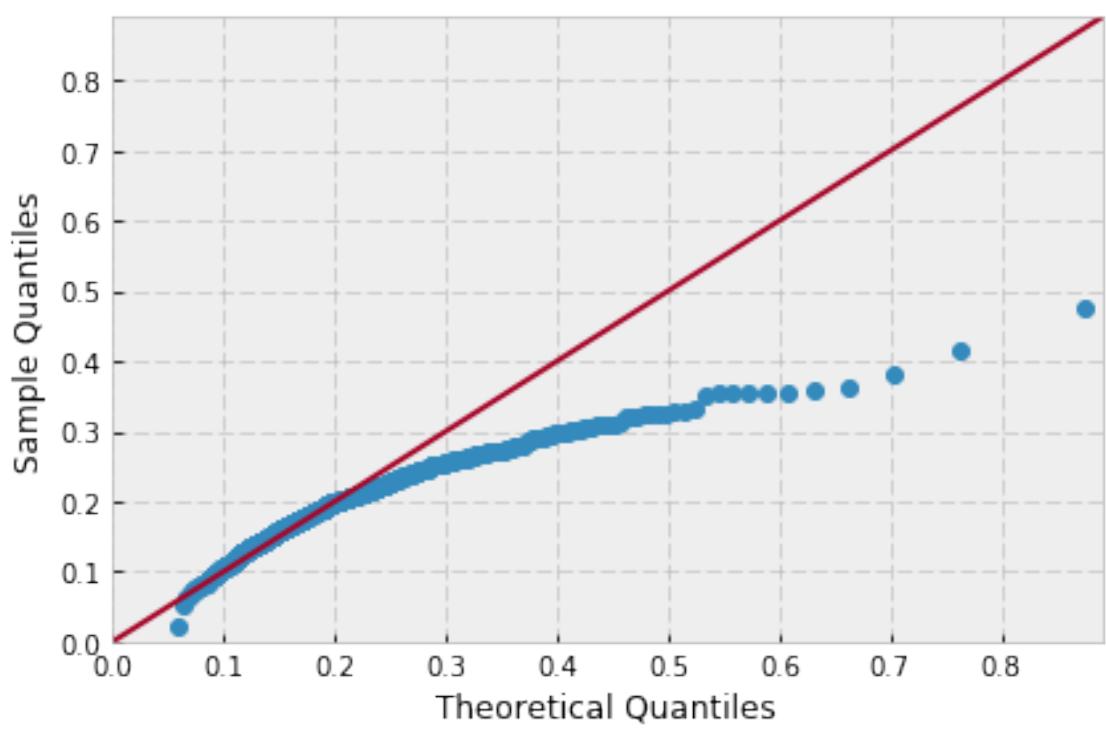
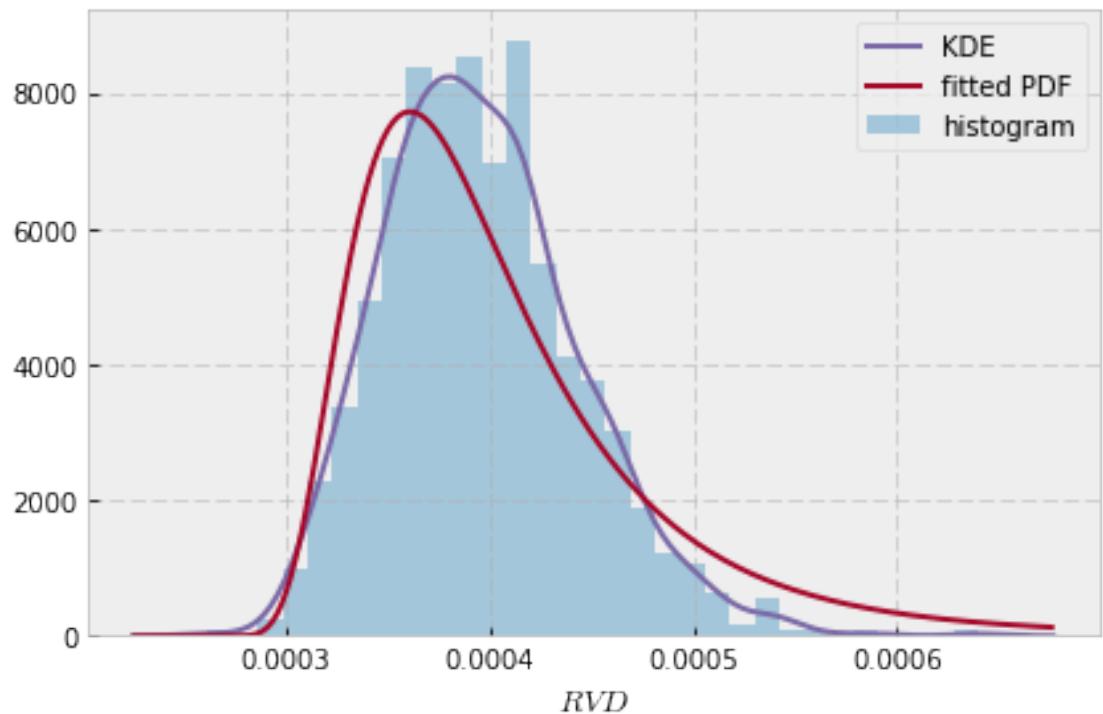


```
variable name: maxB
variable value: 20.0
distribution: inverse-gamma(shape=(7.0949875634206112,),loc=0.00020934321959873518, scale=0.0009
DescribeResult(nobs=1000, minmax=(0.00024795180091973917, 0.00052242896739253491)
    mean=0.0003548823577222328, variance=2.1442868407762261e-09,
    skewness=0.4473734729152729, kurtosis=0.031746320694193475)
```

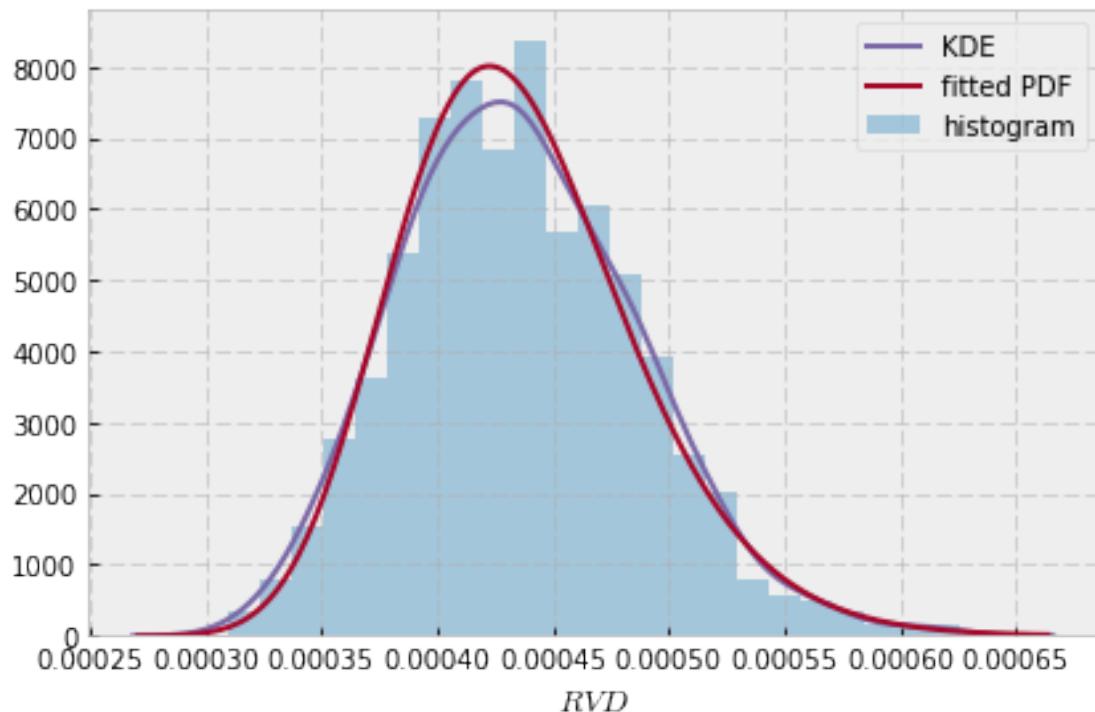


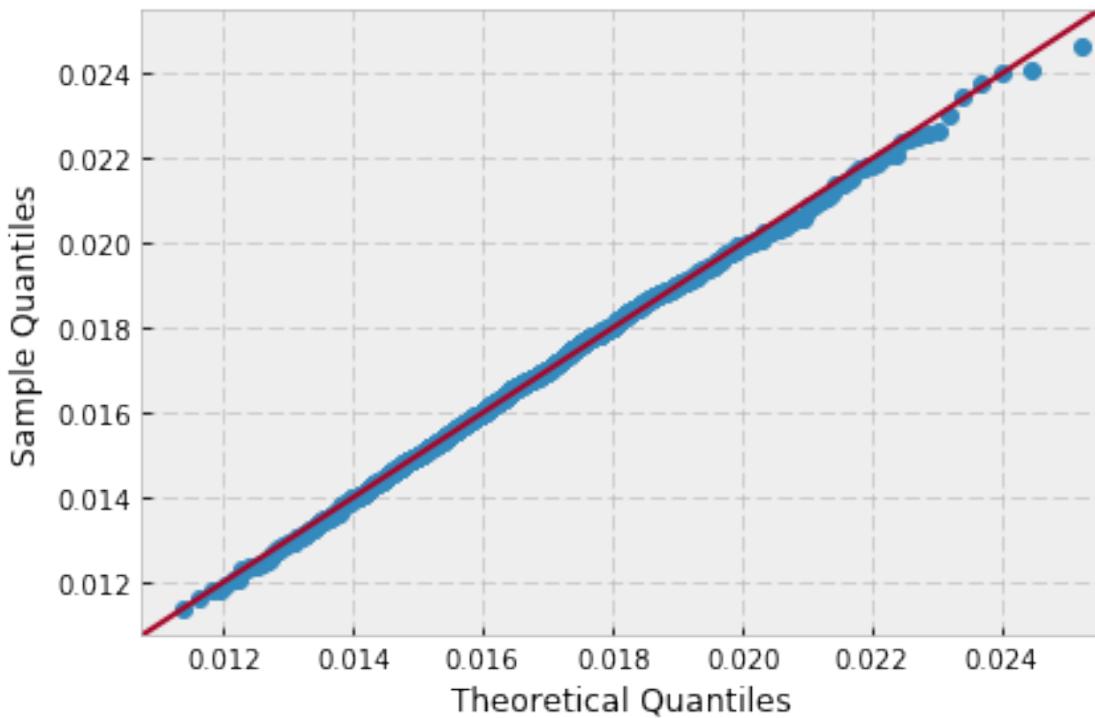


```
variable name: maxB
variable value: 24.0
distribution: inverse-gamma(shape=(6.0942691877404425,), loc=0.00024406192721983423, scale=0.0008
DescribeResult(nobs=1000, minmax=(0.00026176969017230729, 0.00063970313793276312)
               mean=0.00039589986390383401, variance=2.2948473918915552e-09,
               skewness=0.6146682336608799, kurtosis=0.7933941381927152)
```

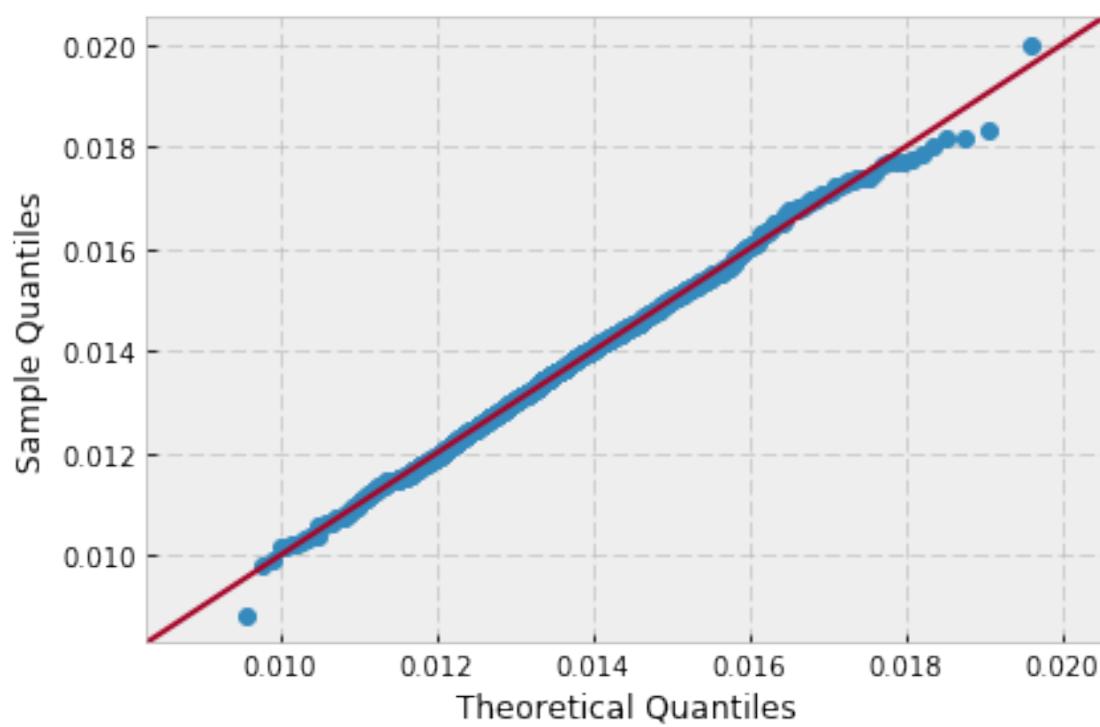
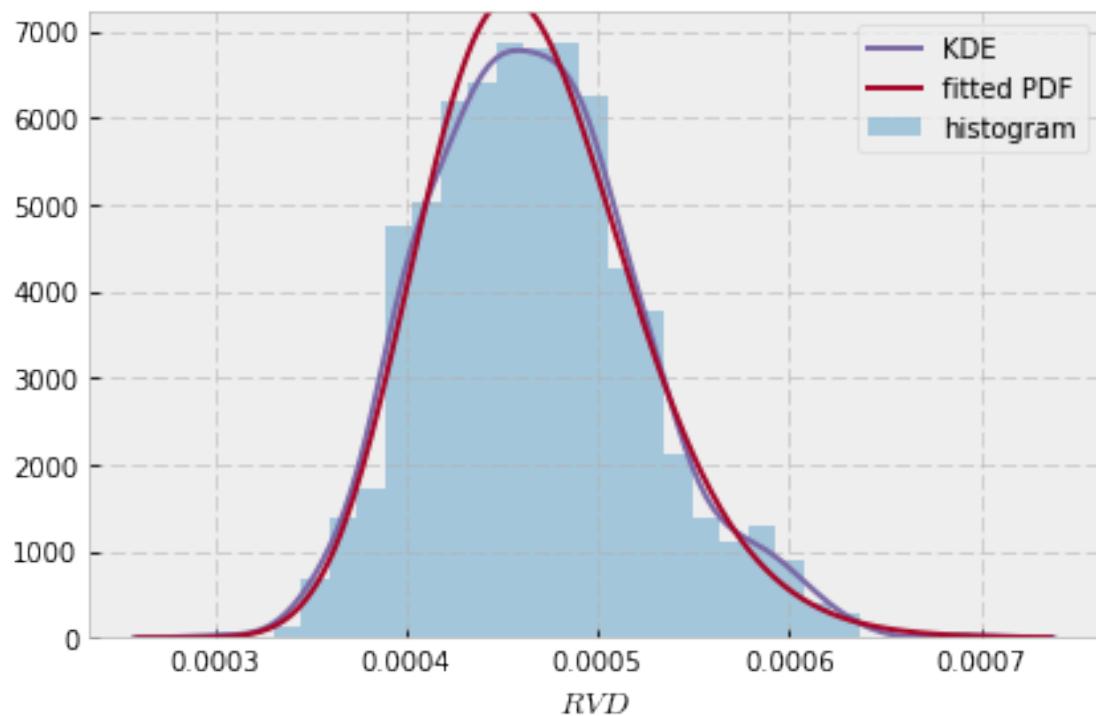


```
variable name: maxB
variable value: 28.0
distribution: inverse-gamma(shape=(60.975483454265301,), loc=3.732390434785872e-05, scale=0.02387
DescribeResult(nobs=1000, minmax=(0.00030947365811300381, 0.00062517219474582083)
               mean=0.000435496207206845, variance=2.6430746073880413e-09,
               skewness=0.3832700139151923, kurtosis=0.12540694407615405)
```

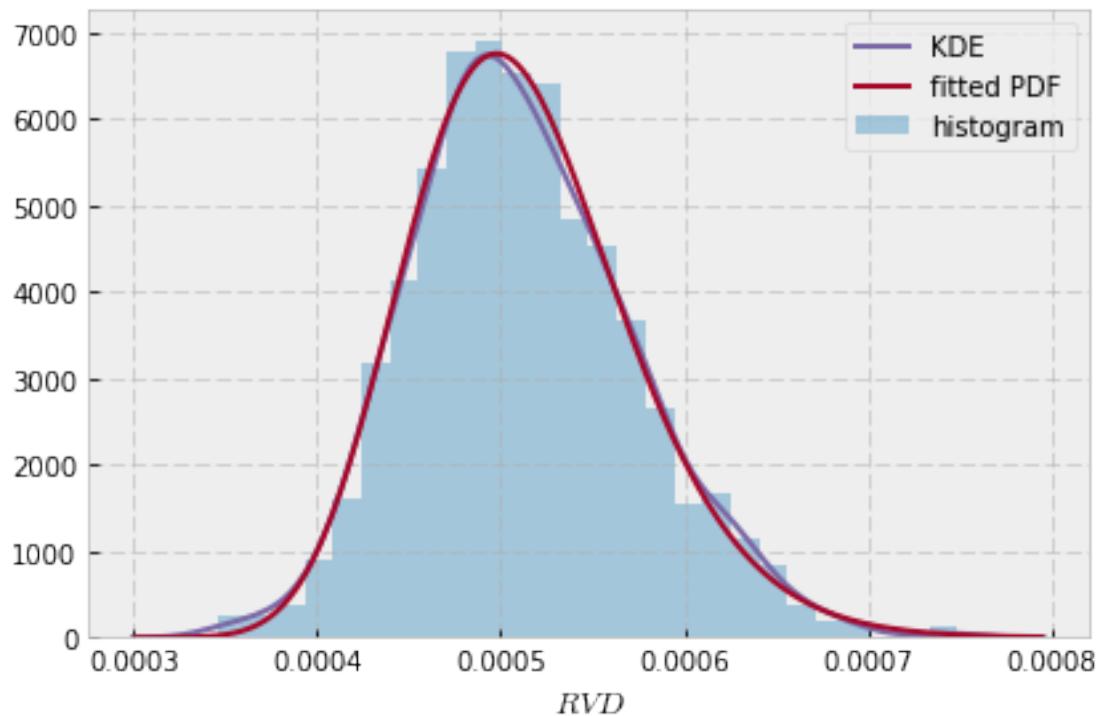


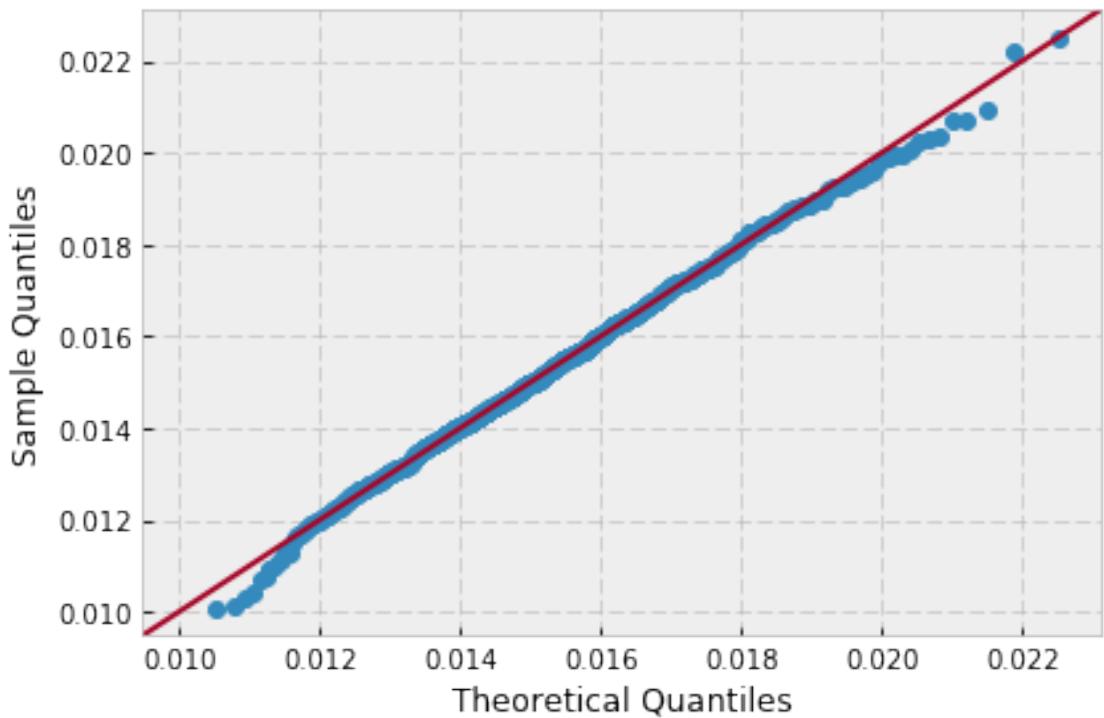


```
variable name: maxB
variable value: 32.0
distribution: inverse-gamma(shape=(74.902244140803589,), loc=-1.0457350319738084e-05, scale=0.035
DescribeResult(nobs=1000, minmax=(0.00030163477579445884, 0.00069434824770042865)
               mean=0.00046686712728952846, variance=3.1099726373685035e-09,
               skewness=0.4032394090844251, kurtosis=0.13241586237133118)
```

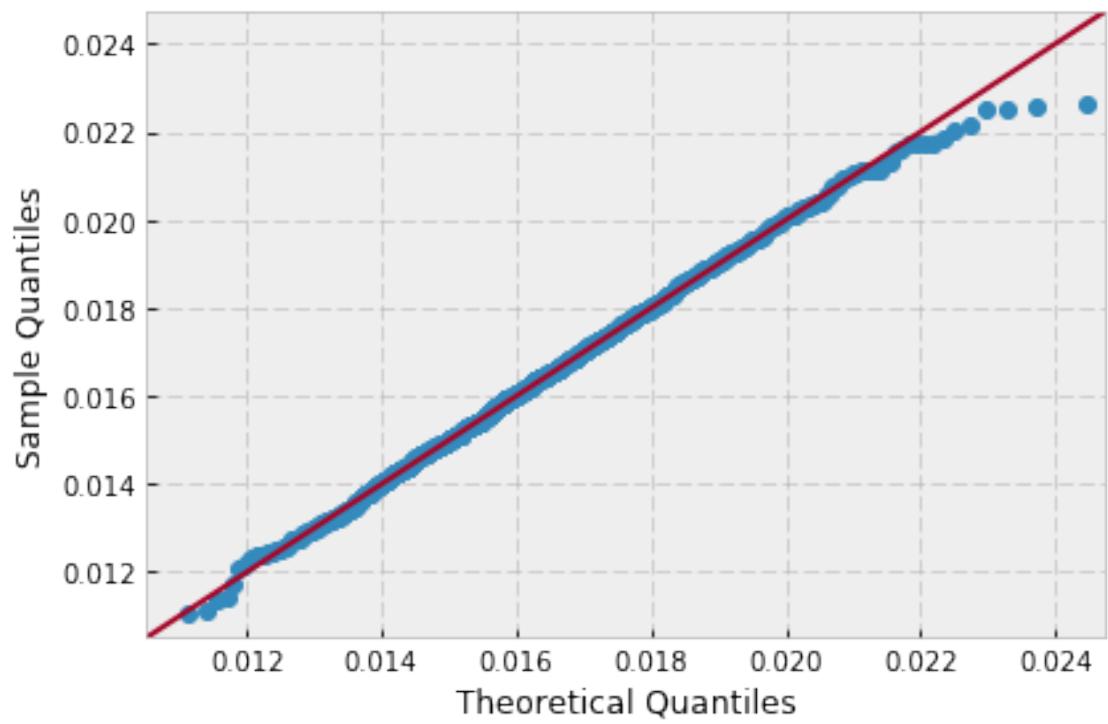
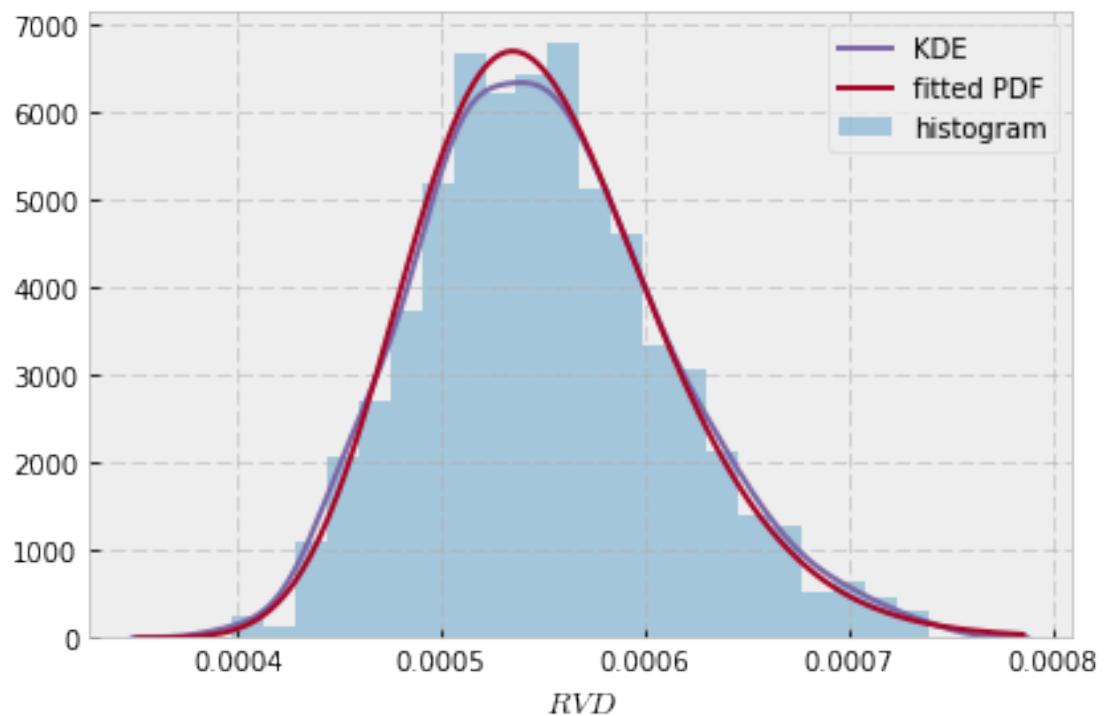


```
variable name: maxB
variable value: 36.0
distribution: inverse-gamma(shape=(66.825645613956567,), loc=1.9129264019982878e-05, scale=0.0324
DescribeResult(nobs=1000, minmax=(0.00034653265005259046, 0.0007488398838177365)
               mean=0.00051193335877610399, variance=3.6720691376199615e-09,
               skewness=0.3937706103351912, kurtosis=0.23398203026844078)
```

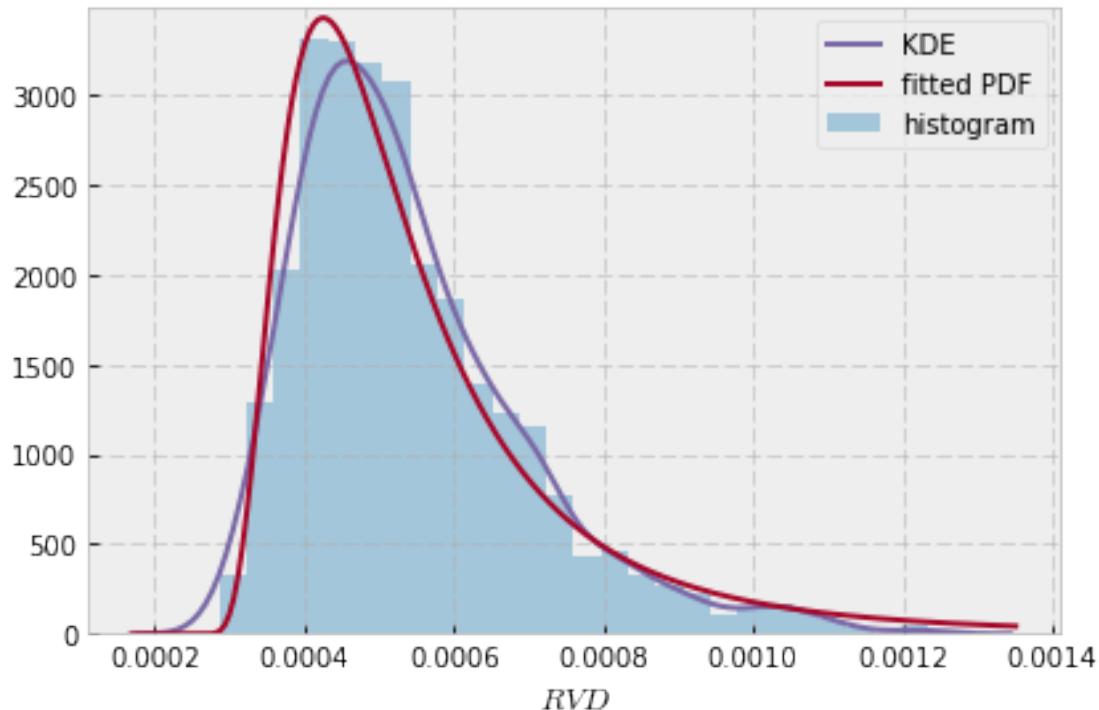


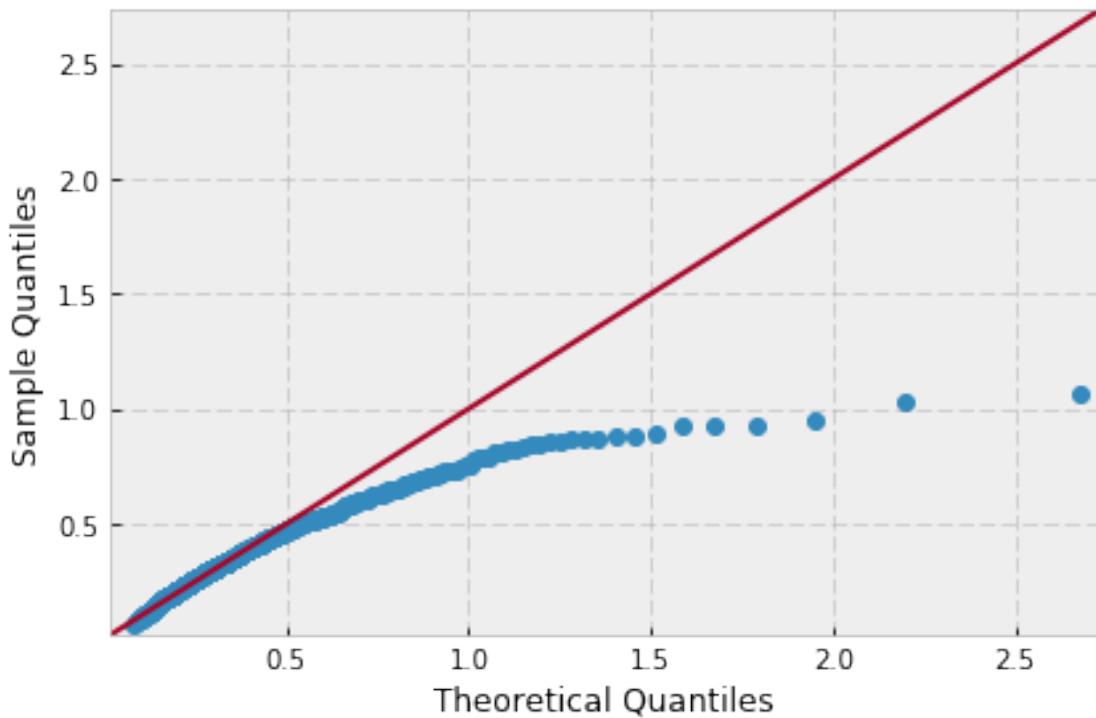


```
variable name: maxB
variable value: 40.0
distribution: inverse-gamma(shape=(62.50209511112169,), loc=6.824086081344438e-05, scale=0.029638
DescribeResult(nobs=1000, minmax=(0.00039668462726307358, 0.00073950597666672504)
               mean=0.00055048412944633132, variance=3.7279230666969871e-09,
               skewness=0.3897037308260911, kurtosis=-0.03577646771205423)
```

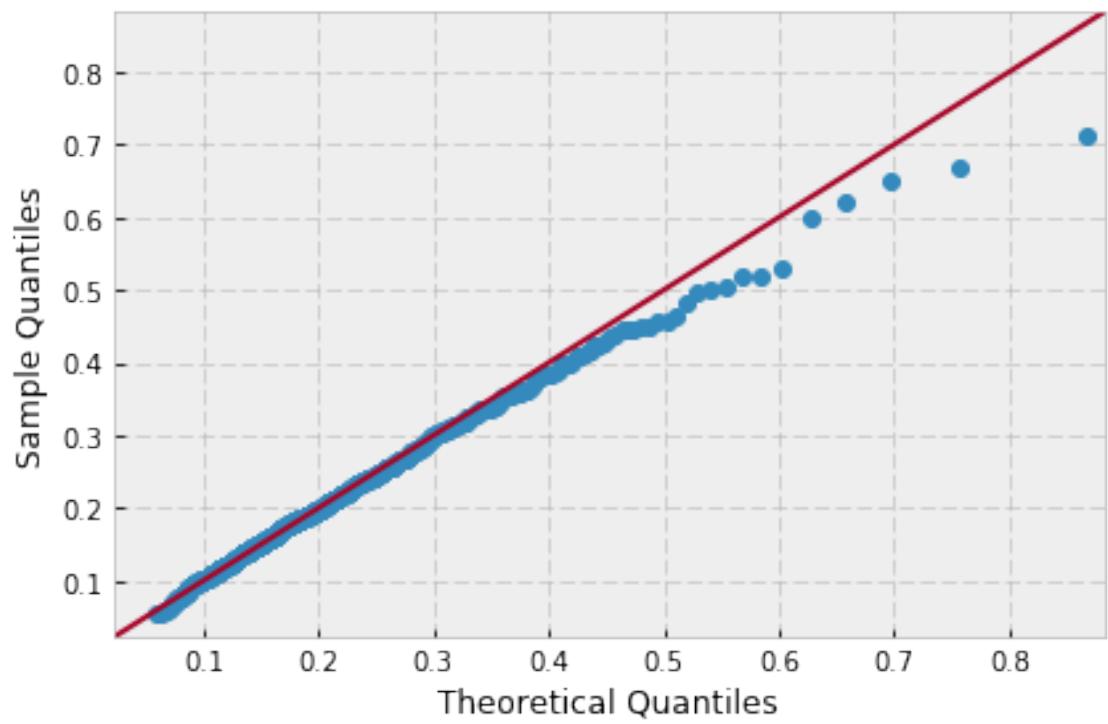
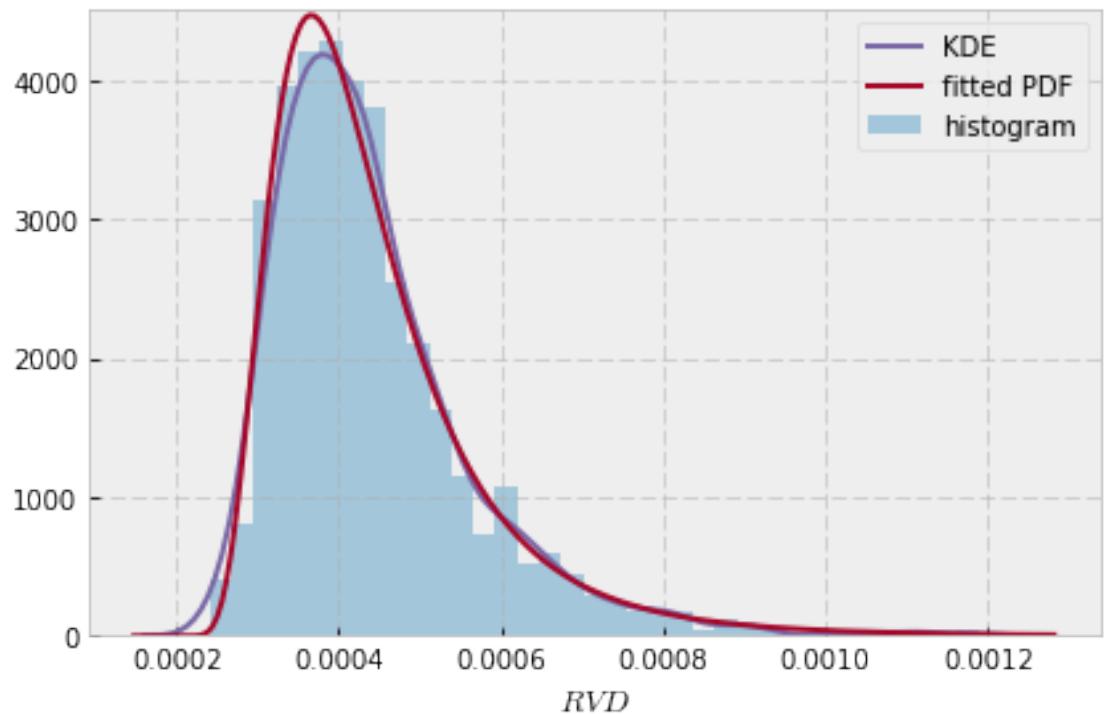


```
variable name: r
variable value: 1.0
distribution: inverse-gamma(shape=(3.7948289997073275,),loc=0.00022698401405493287, scale=0.0009
DescribeResult(nobs=1000, minmax=(0.0002857391183987921, 0.0012341119478371406)
    mean=0.00054160122797479969, variance=2.3983061554804747e-08,
    skewness=1.245527660446143, kurtosis=1.8316774846098296)
```

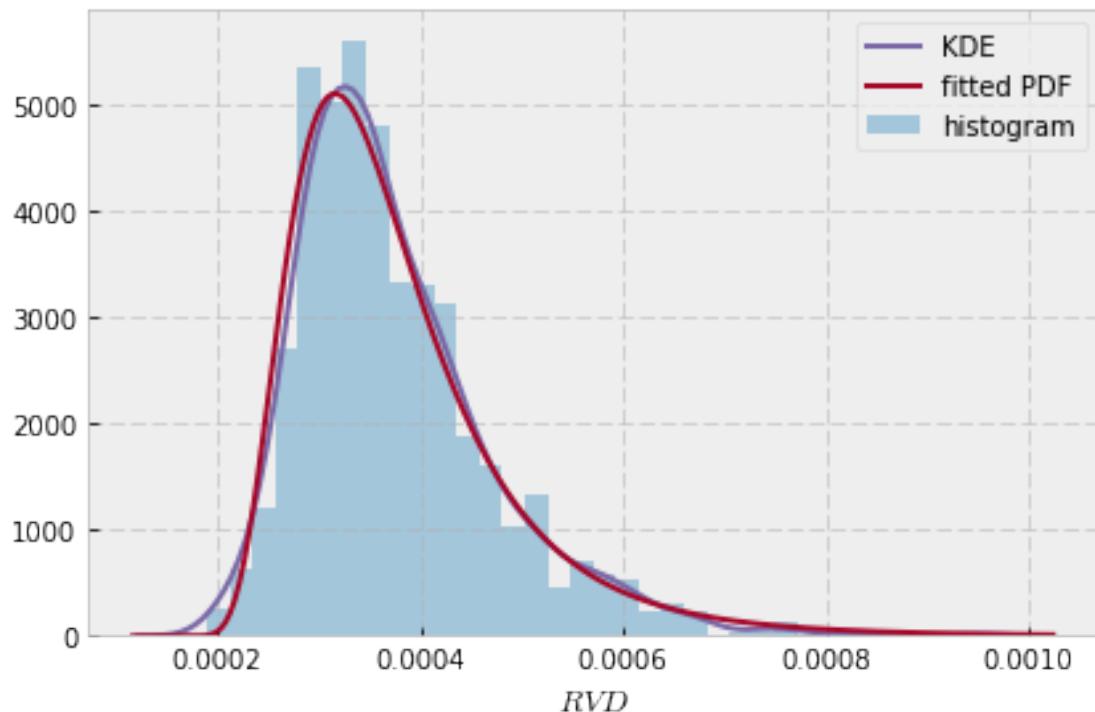


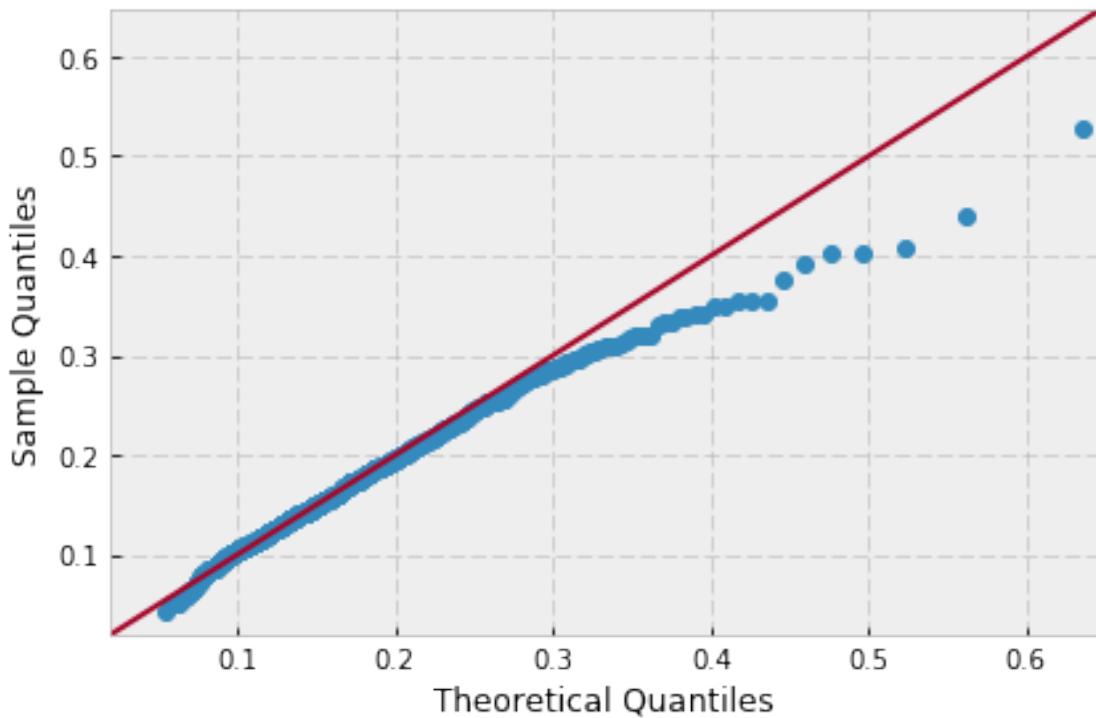


```
variable name: r
variable value: 1.2222222222222223
distribution: inverse-gamma(shape=(6.1207938710357919,),loc=0.00016358356791977904, scale=0.0014
DescribeResult(nobs=1000, minmax=(0.0002414275673415671, 0.0011878432506851665)
               mean=0.00044135553481712596, variance=1.5492475684927143e-08,
               skewness=1.7022526642619635, kurtosis=4.628391186970012)
```

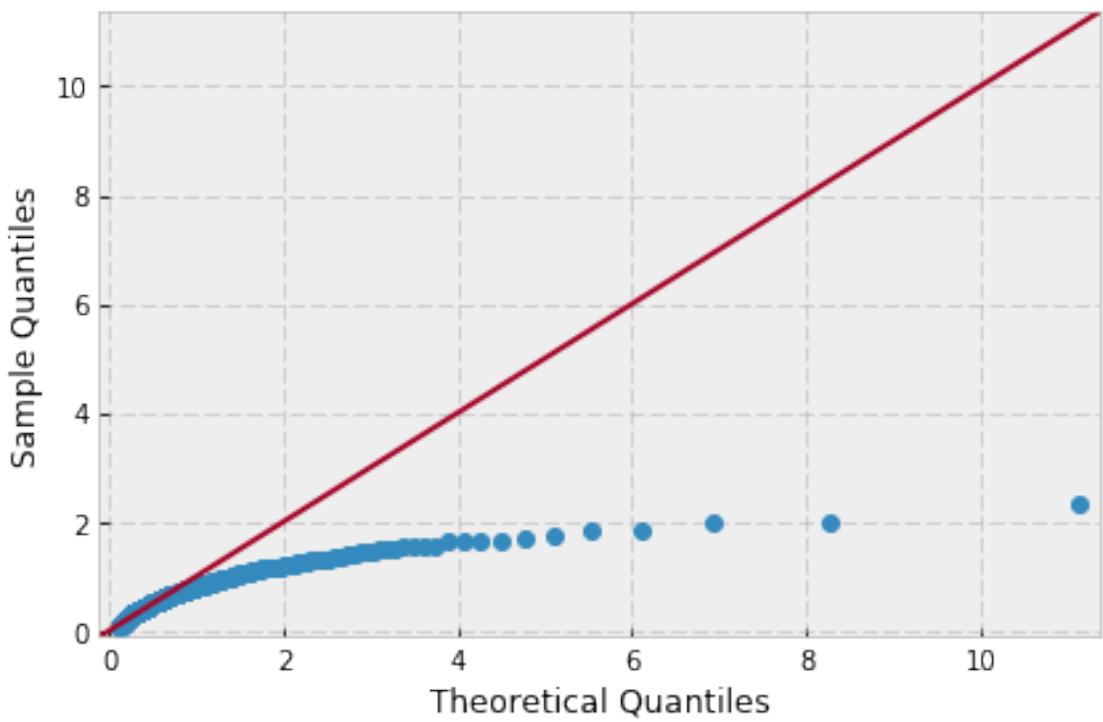
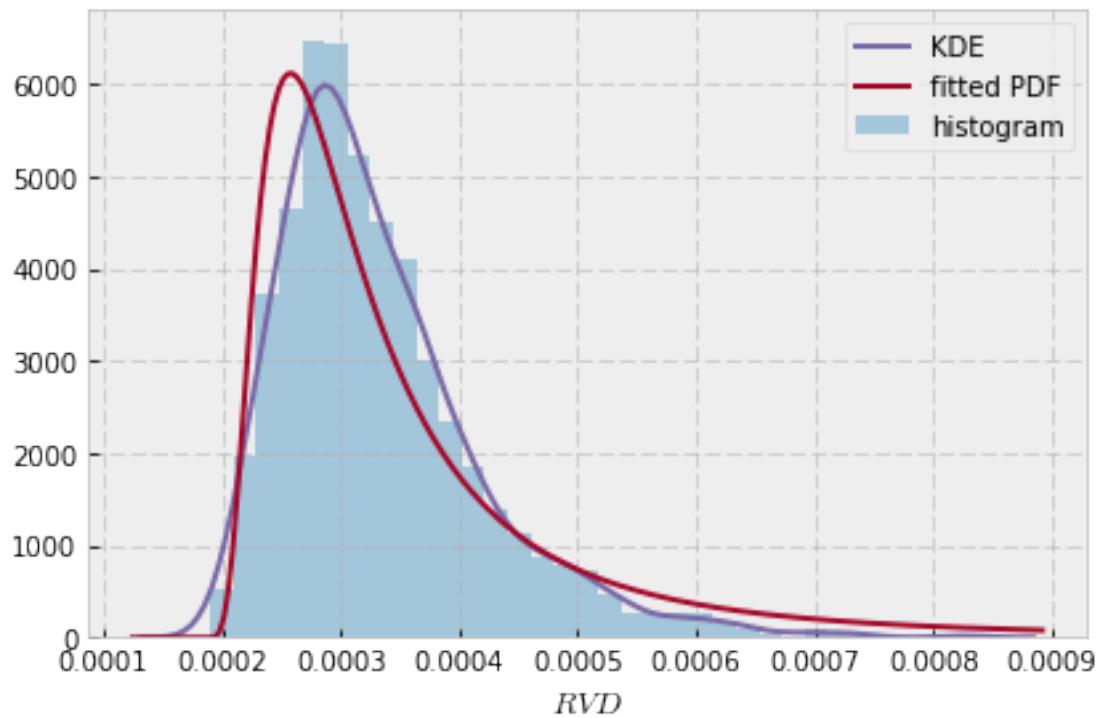


```
variable name: r
variable value: 1.444444444444444
distribution: inverse-gamma(shape=(7.1251094207746686,),loc=0.00012268734867916083, scale=0.0015
DescribeResult(nobs=1000, minmax=(0.00018915435980505557, 0.00095127503598741077)
    mean=0.00037414628150419084, variance=9.3541982886288566e-09,
    skewness=1.2895130629604032, kurtosis=2.565746084018066)
```

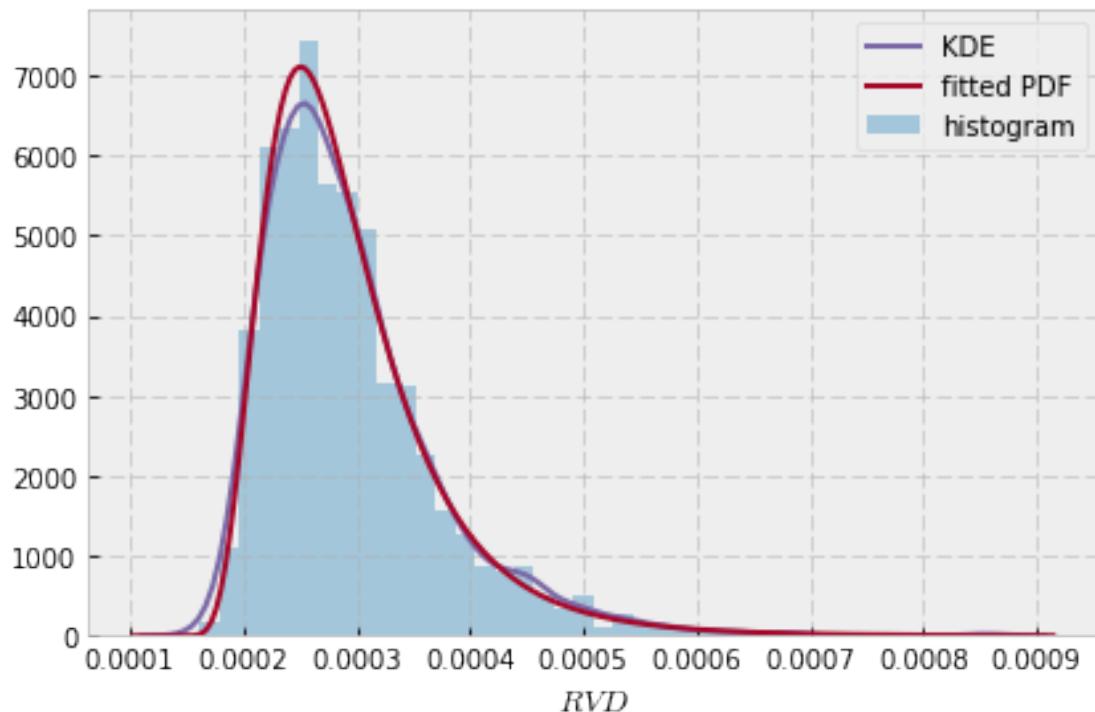


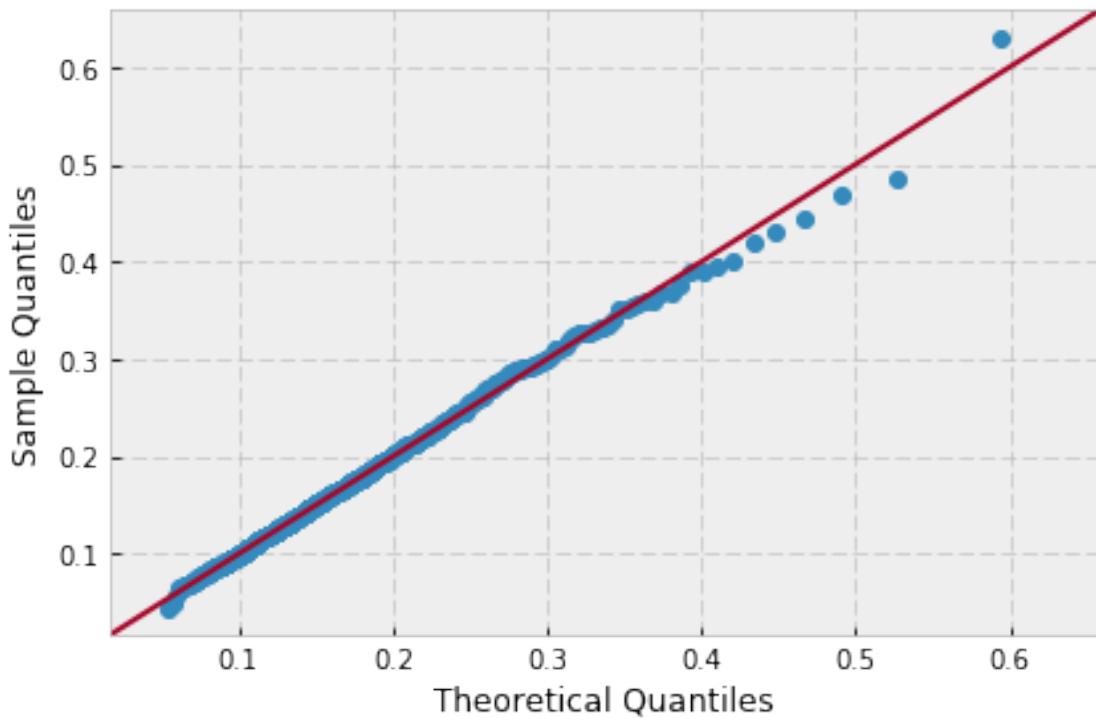


```
variable name: r
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(2.3904046859568107,), loc=0.00017497163304056021, scale=0.0002
DescribeResult(nobs=1000, minmax=(0.00018994775009548365, 0.00082668510520774341)
               mean=0.00033258765994860907, variance=7.5543636052480684e-09,
               skewness=1.4462310072267583, kurtosis=3.039455959023279)
```

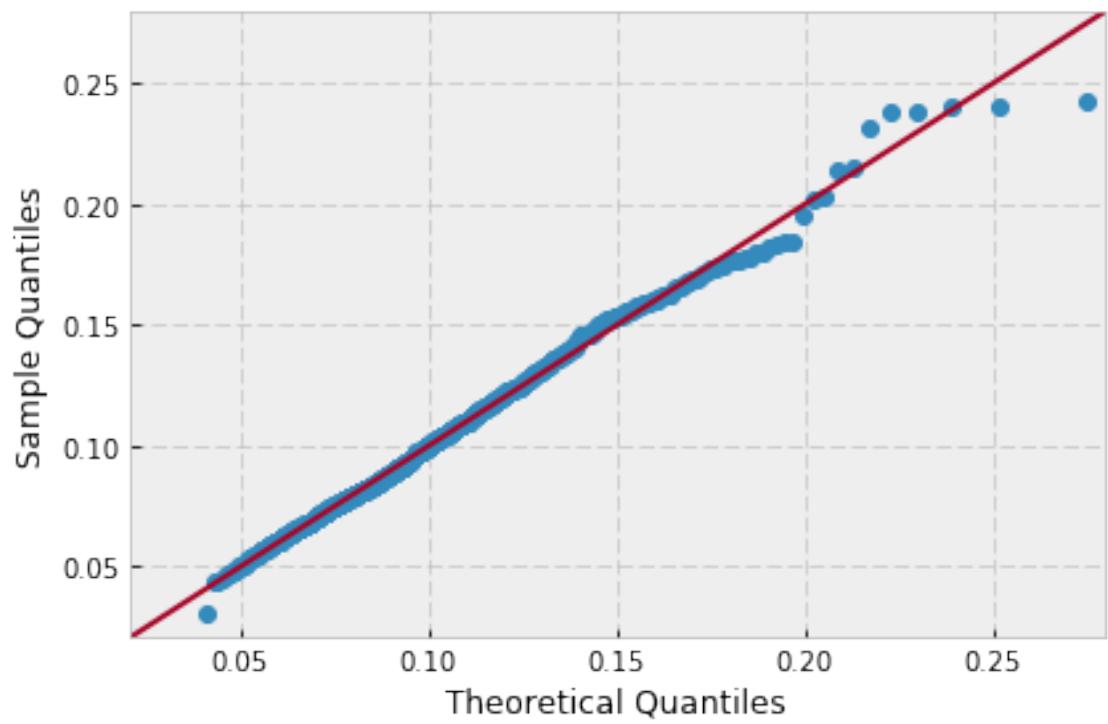
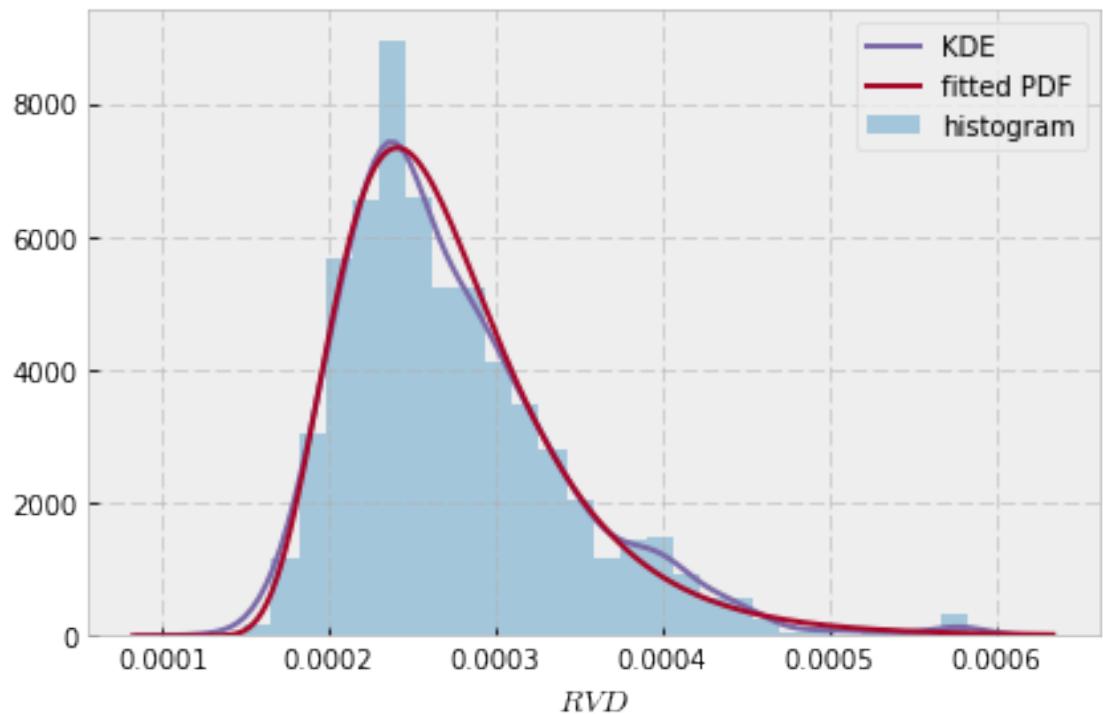


```
variable name: r
variable value: 1.888888888888888
distribution: inverse-gamma(shape=(7.373929083299545,), loc=0.00010894301129596825, scale=0.0011
DescribeResult(nobs=1000, minmax=(0.00016188500699774701, 0.00085509548224756409)
    mean=0.00029486991775246496, variance=6.1711423308878432e-09,
    skewness=1.6842913057119206, kurtosis=4.809081239926476)
```

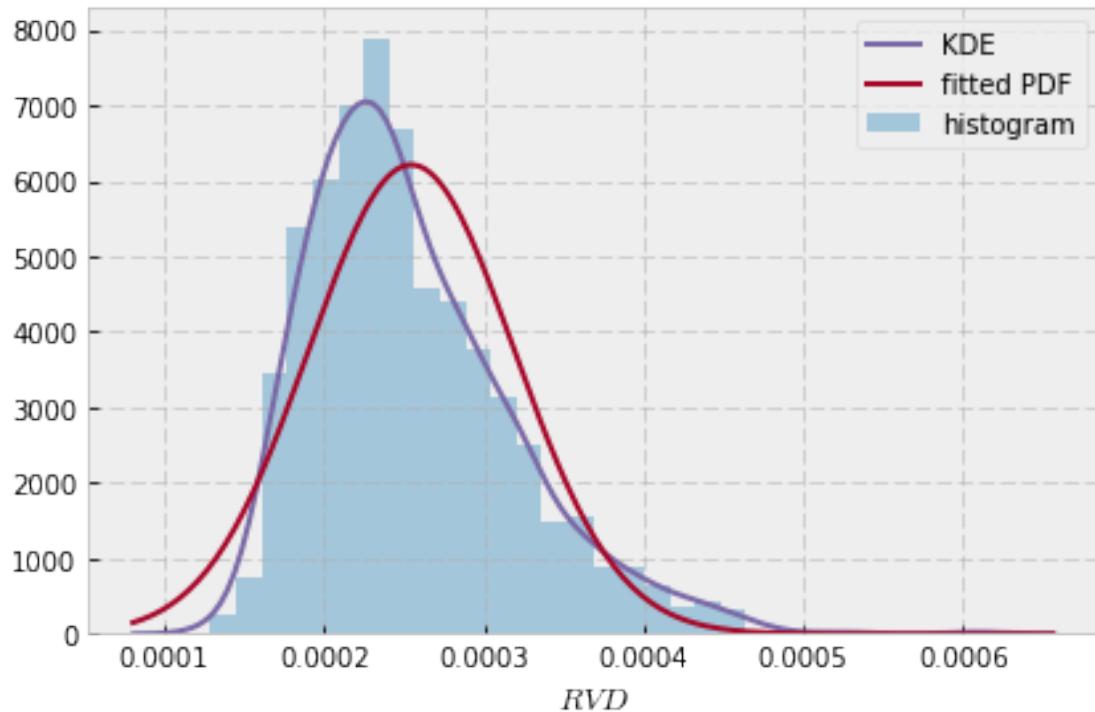


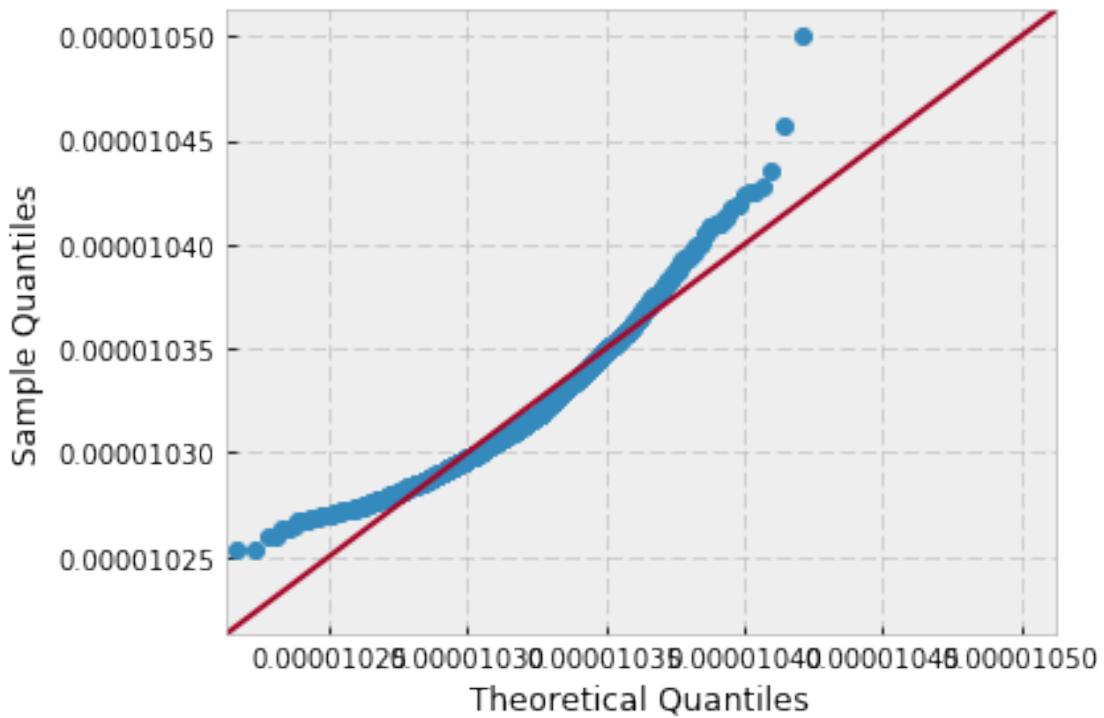


```
variable name: r
variable value: 2.111111111111111
distribution: inverse-gamma(shape=(11.275301138947592,), loc=6.702148922141604e-05, scale=0.00213
DescribeResult(nobs=1000, minmax=(0.00013325574007376275, 0.00058364350622420221)
               mean=0.00027475112039486638, variance=4.5708721737884842e-09,
               skewness=1.231905442222493, kurtosis=2.1794642515773512)
```

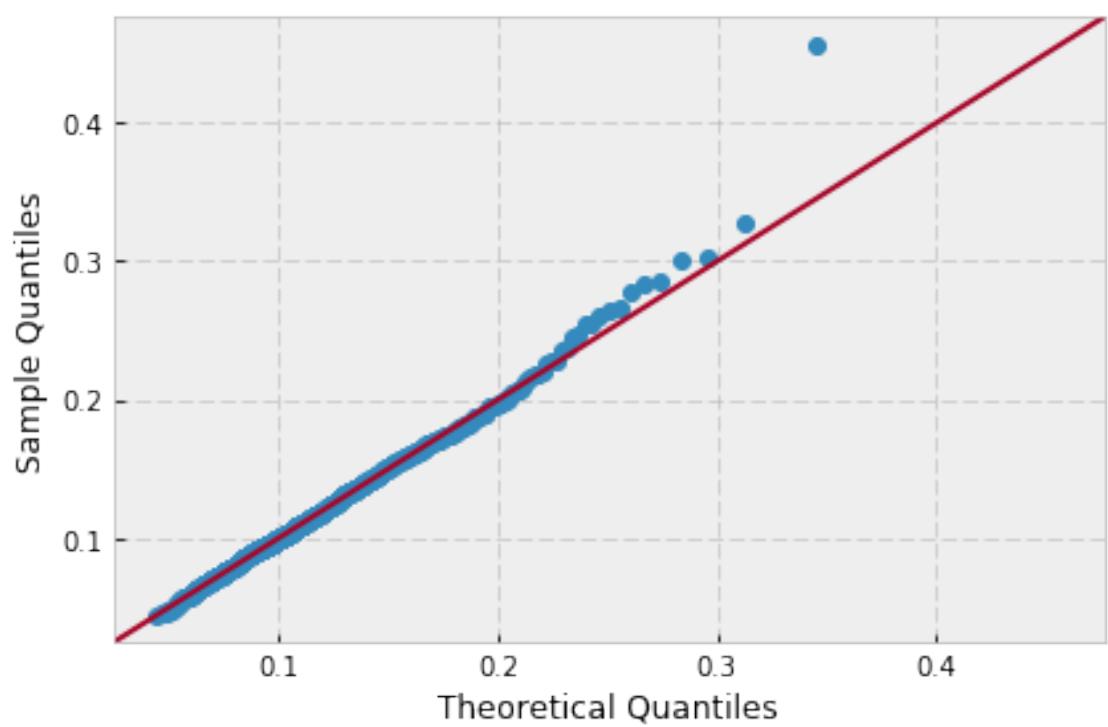
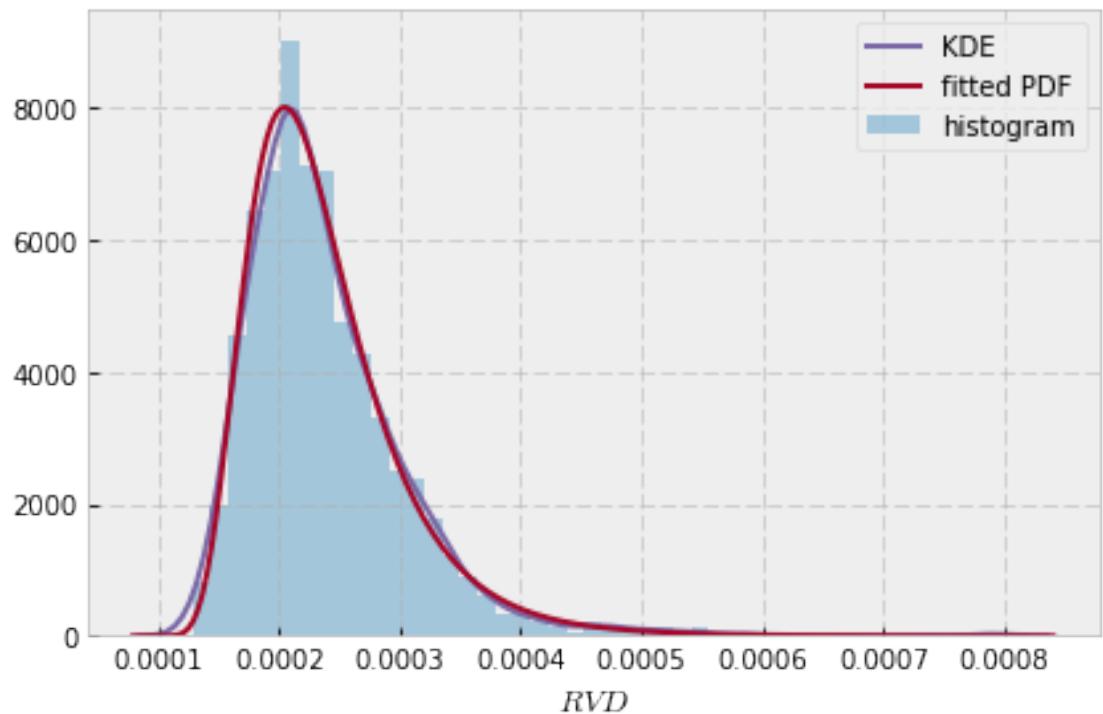


```
variable name: r
variable value: 2.333333333333333
distribution: inverse-gamma(shape=(96916.168152397673,),loc=-0.019731102971752908, scale=1936.94
DescribeResult(nobs=1000, minmax=(0.00012893712027707986, 0.00060765248954171671)
    mean=0.00025492588952378479, variance=4.1435840327232571e-09,
    skewness=1.0232117843476143, kurtosis=1.4161036718148763)
```

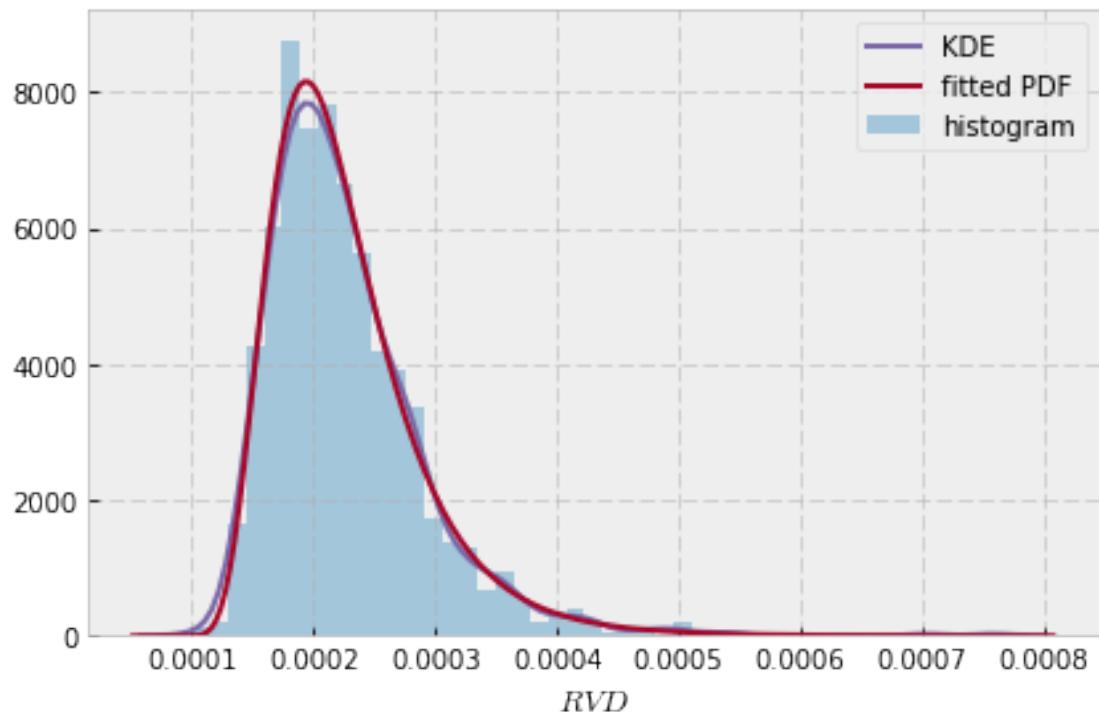


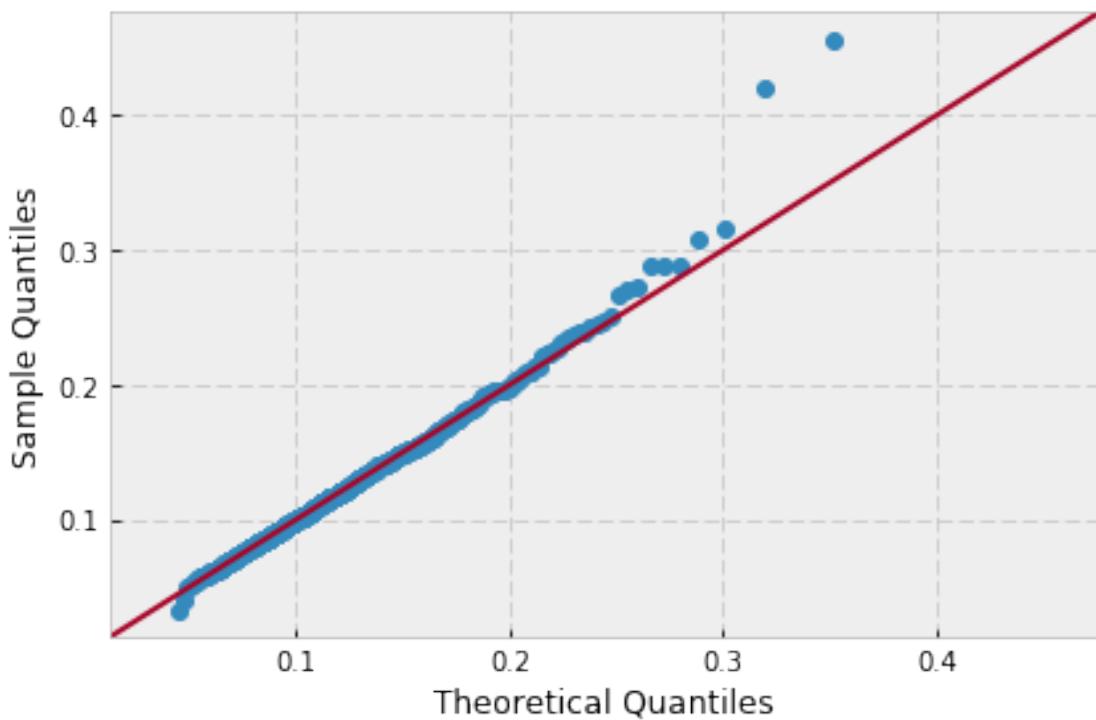


```
variable name: r
variable value: 2.5555555555555554
distribution: inverse-gamma(shape=(9.8861697508606738,), loc=5.670716280548041e-05, scale=0.00161
DescribeResult(nobs=1000, minmax=(0.00012874305258449688, 0.00079086325411914093)
               mean=0.00023798673901077934, variance=4.2933536921098776e-09,
               skewness=1.8622859509394, kurtosis=7.554463391452762)
```

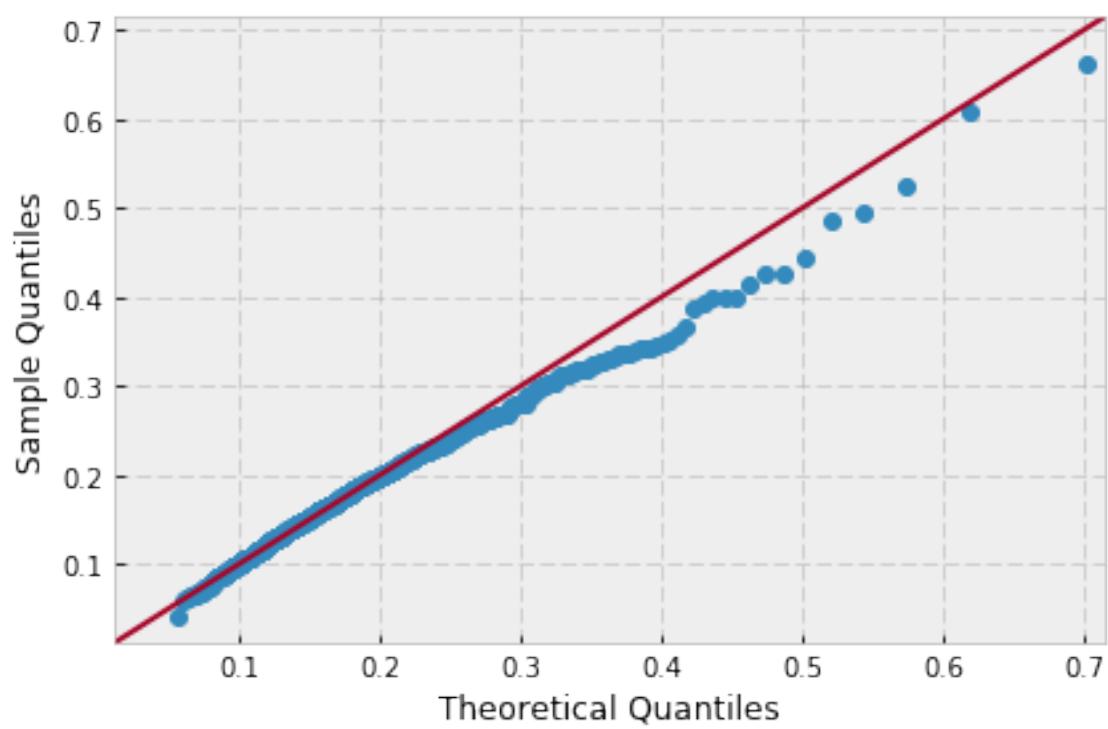
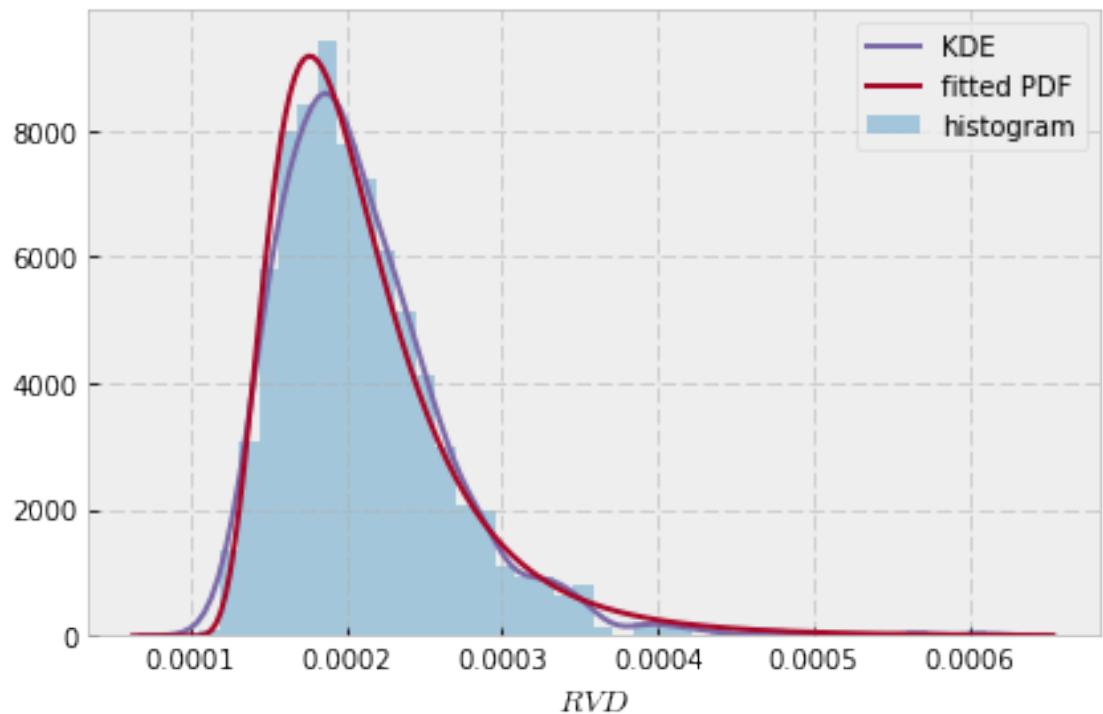


```
variable name: r
variable value: 2.777777777777777
distribution: inverse-gamma(shape=(9.7708047989539271,), loc=4.9198685379451556e-05, scale=0.0015
DescribeResult(nobs=1000, minmax=(0.00010090190426898683, 0.00075776601276126123)
    mean=0.00022677166742431556, variance=4.3061658495834299e-09,
    skewness=2.023282122855513, kurtosis=8.636358936344037)
```

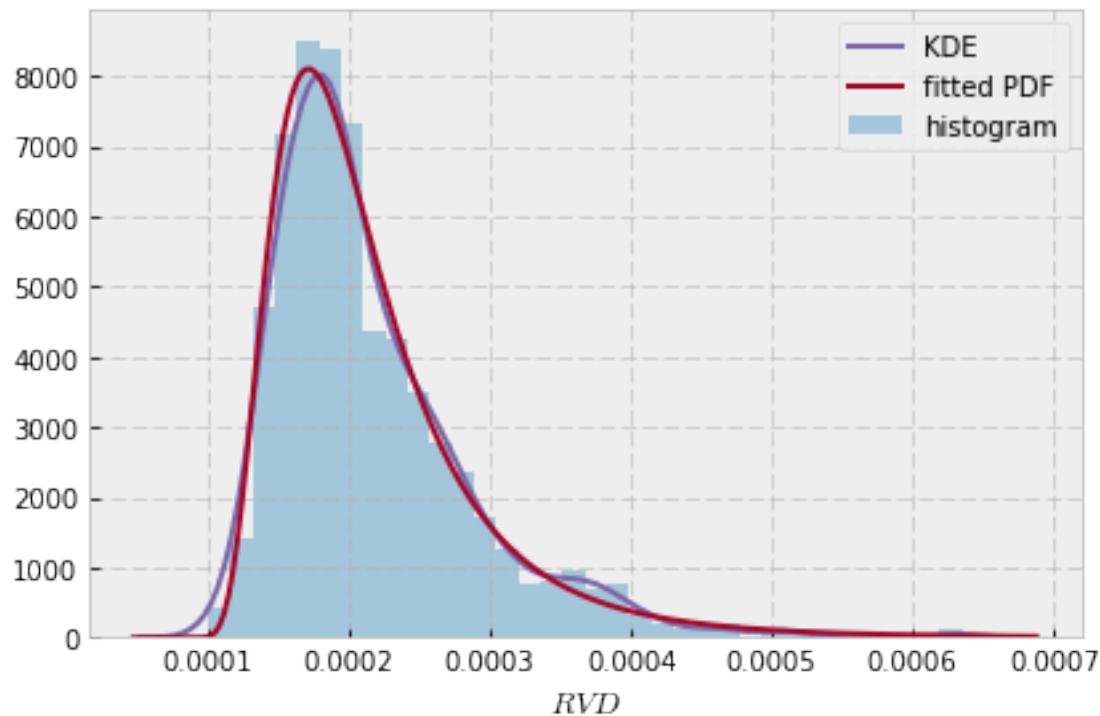


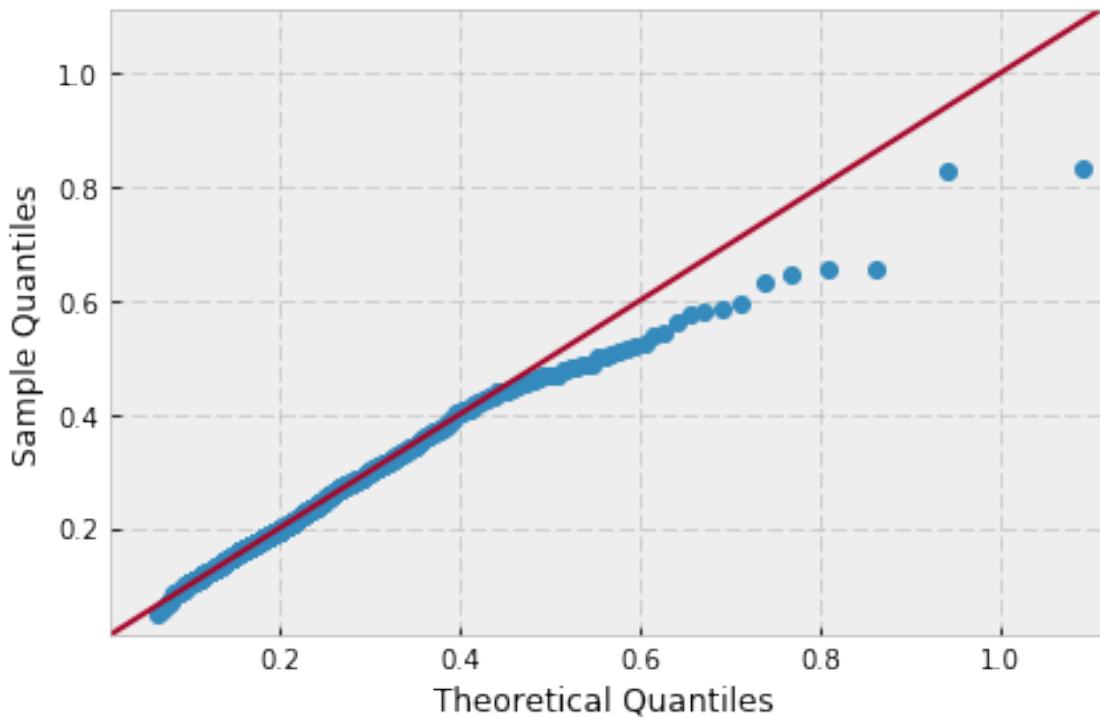


```
variable name: r
variable value: 3.0
distribution: inverse-gamma(shape=(6.7778543967318523,), loc=7.221423904499206e-05, scale=0.00081
DescribeResult(nobs=1000, minmax=(0.00010603627332818829, 0.00061052120943536244)
               mean=0.00021065129458814979, variance=3.1944921715473631e-09,
               skewness=1.6705371941099623, kurtosis=5.688860493199279)
```

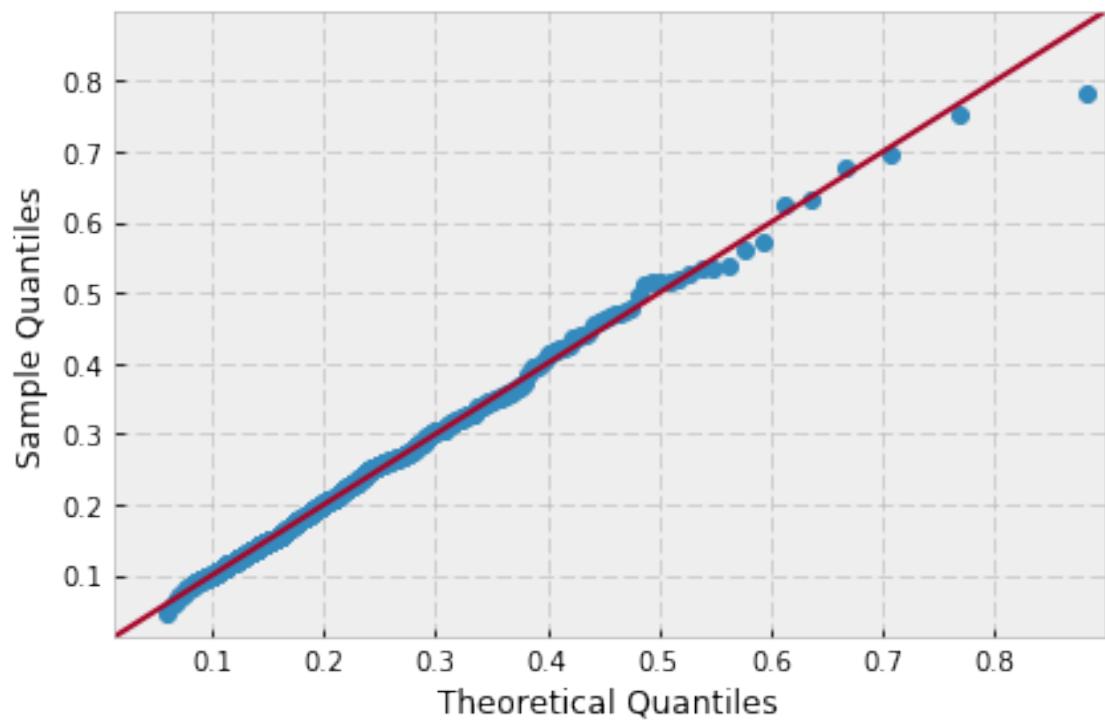
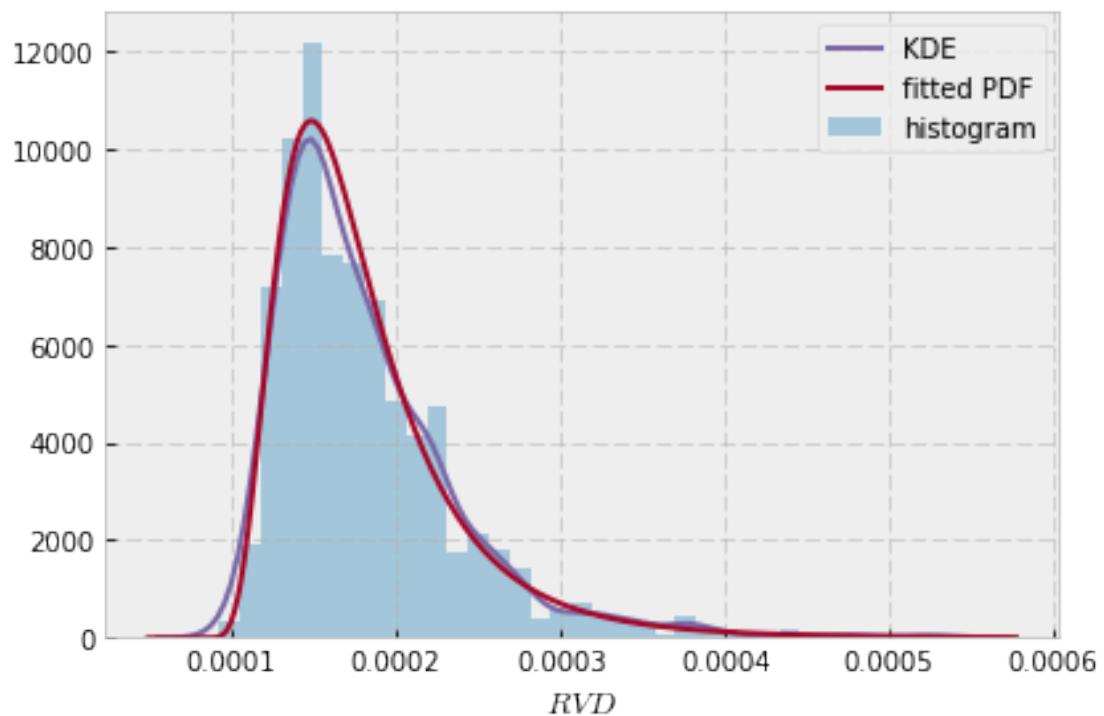


```
variable name: sigma
variable value: 0.0111111111111112
distribution: inverse-gamma(shape=(5.501245975752842,), loc=6.616676144875742e-05, scale=0.000681
DescribeResult(nobs=1000, minmax=(9.9690146005562515e-05, 0.00063462938062019275)
               mean=0.00021503920648848935, variance=4.9449265321587766e-09,
               skewness=1.6103109447063164, kurtosis=3.8564508040129306)
```

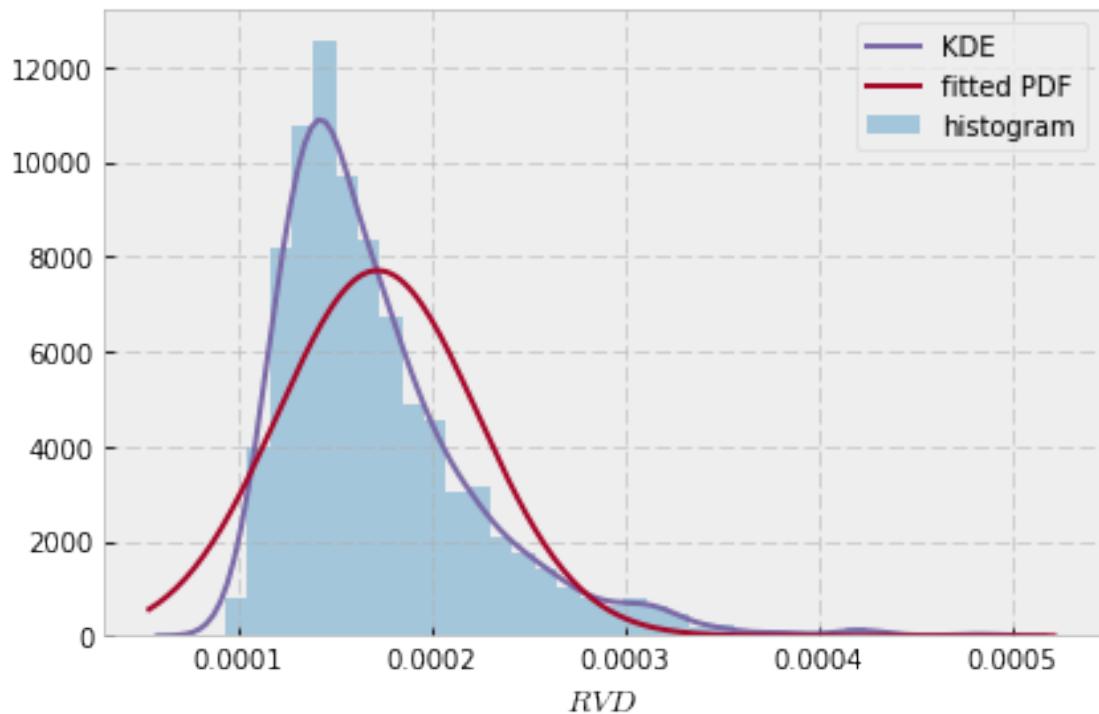


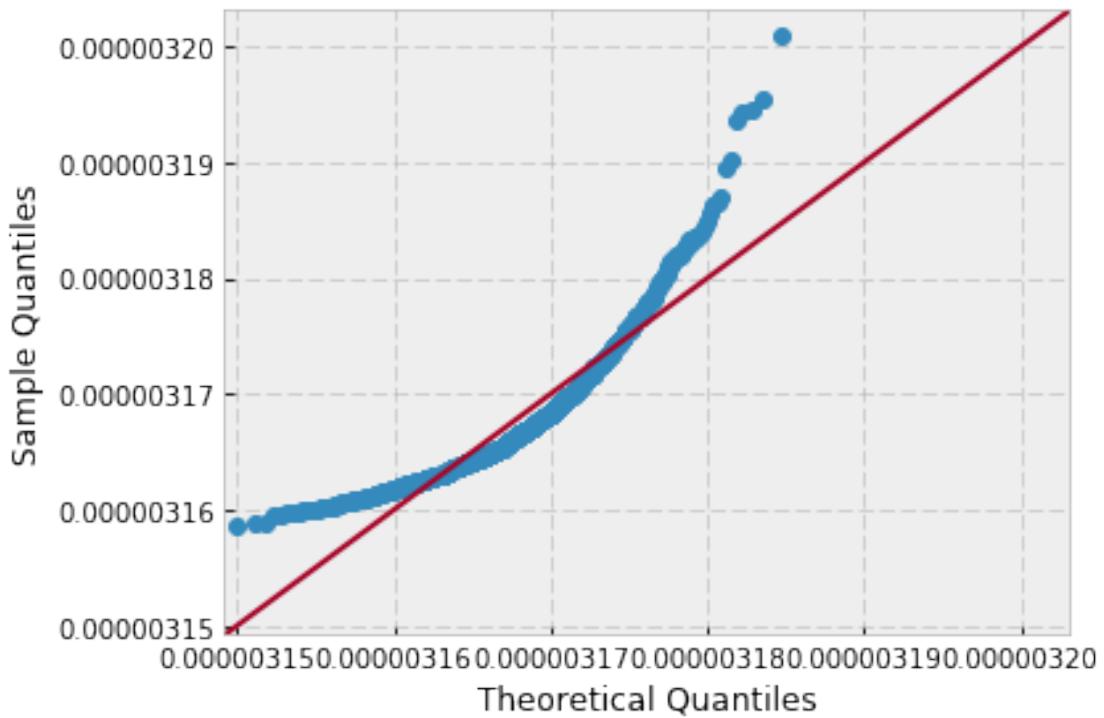


```
variable name: sigma
variable value: 0.02222222222222223
distribution: inverse-gamma(shape=(6.0703919321968307,), loc=6.315937094943387e-05, scale=0.00060
DescribeResult(nobs=1000, minmax=(9.2053423701236701e-05, 0.00053404334414177182)
               mean=0.00018166459368791865, variance=3.2543424387009611e-09,
               skewness=1.8906059310022776, kurtosis=5.525850576815367)
```

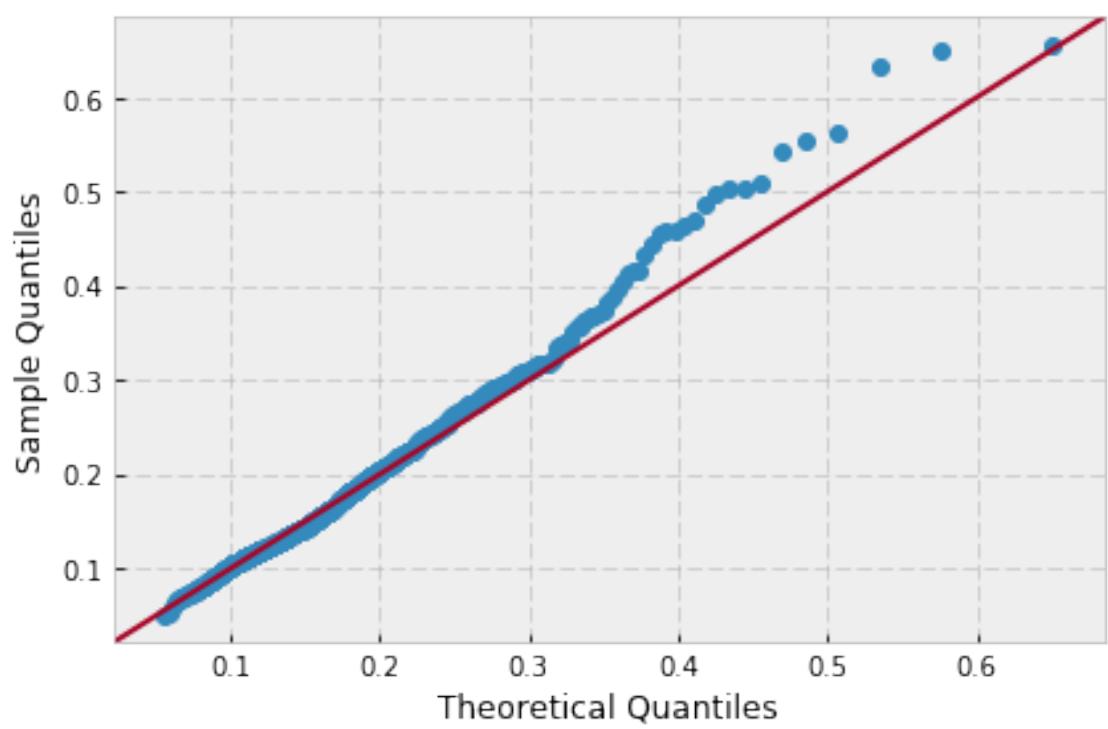
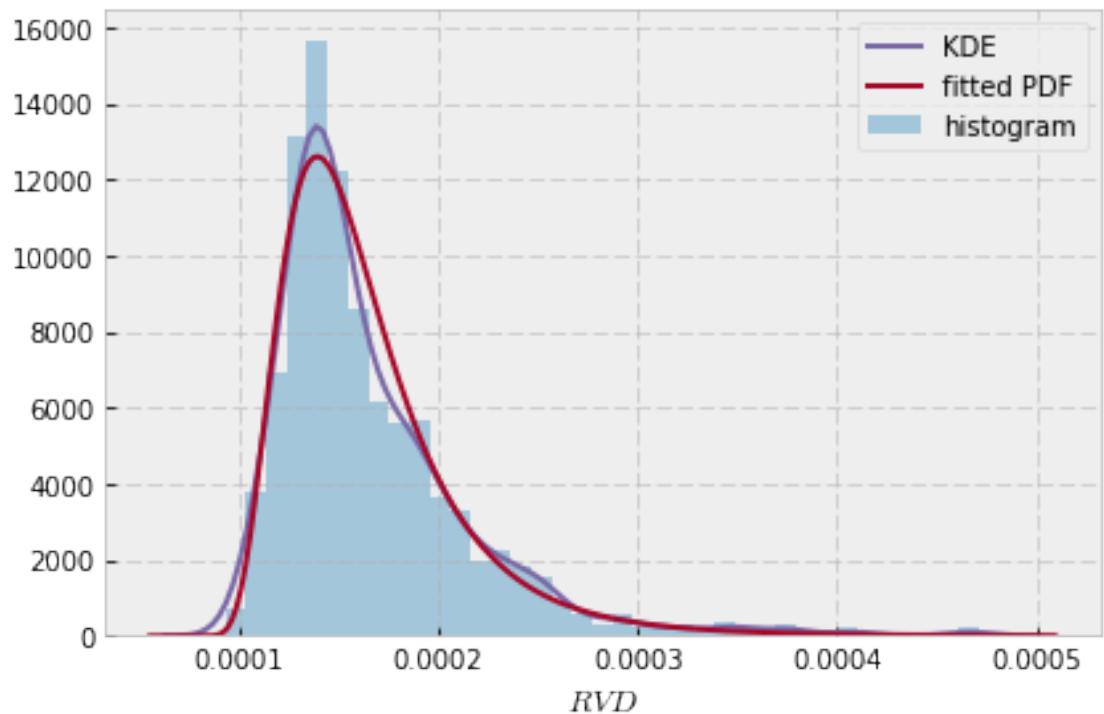


```
variable name: sigma
variable value: 0.0333333333333333
distribution: inverse-gamma(shape=(315733.2995607609,), loc=-0.028925699547413, scale=9187.164530
DescribeResult(nobs=1000, minmax=(9.3029343136950677e-05, 0.00048266886176622349)
    mean=0.00017225550321595207, variance=2.6948073415150883e-09,
    skewness=1.6483925864819957, kurtosis=3.922922266744181)
```

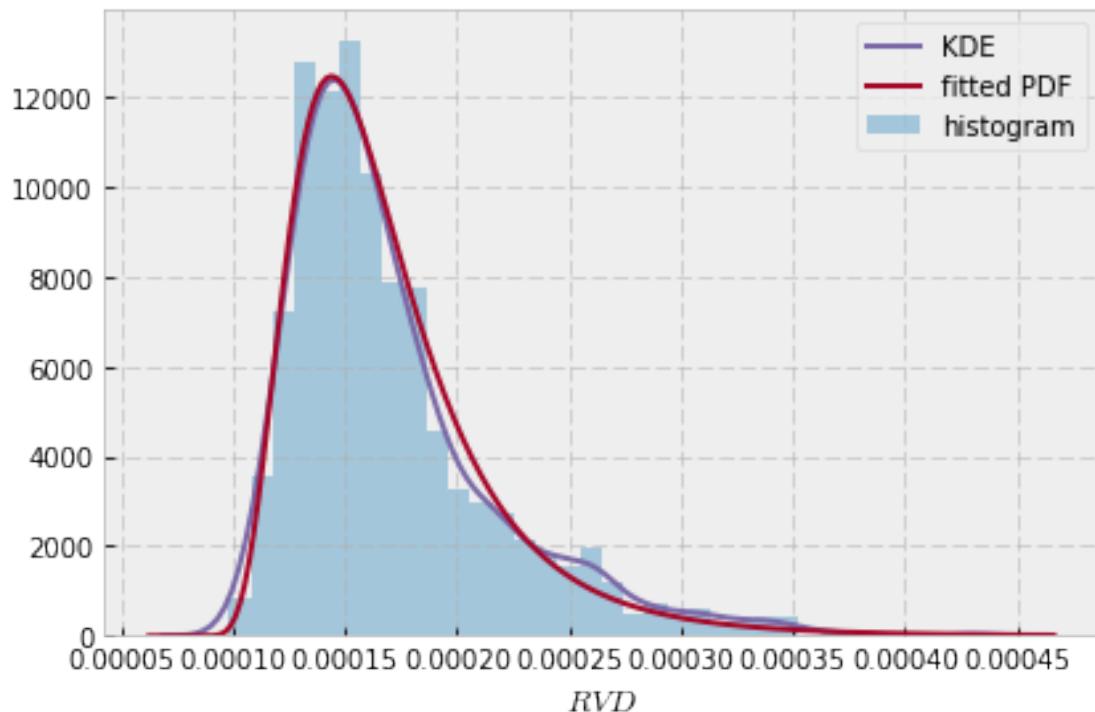


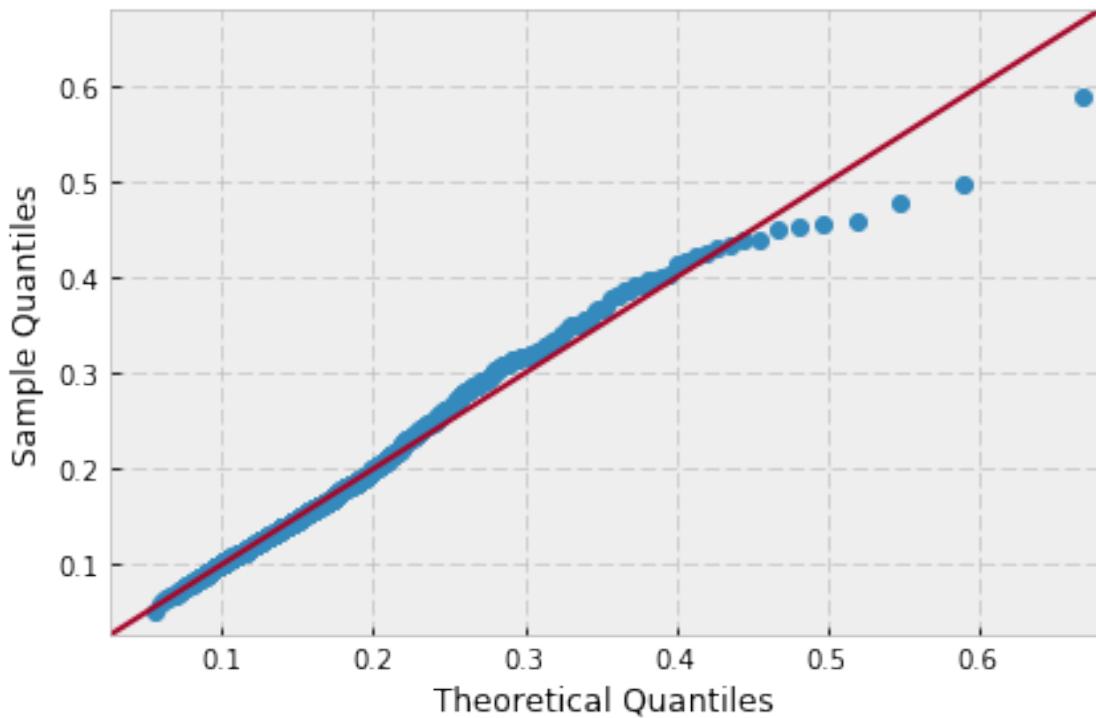


```
variable name: sigma
variable value: 0.04444444444444446
distribution: inverse-gamma(shape=(7.0391485828433851,), loc=6.179800231763817e-05, scale=0.00062
DescribeResult(nobs=1000, minmax=(9.3618585016859931e-05, 0.00047118200940351752)
               mean=0.00016651225800309196, variance=2.5149908783507719e-09,
               skewness=2.1188232736881116, kurtosis=6.6843787527406135)
```

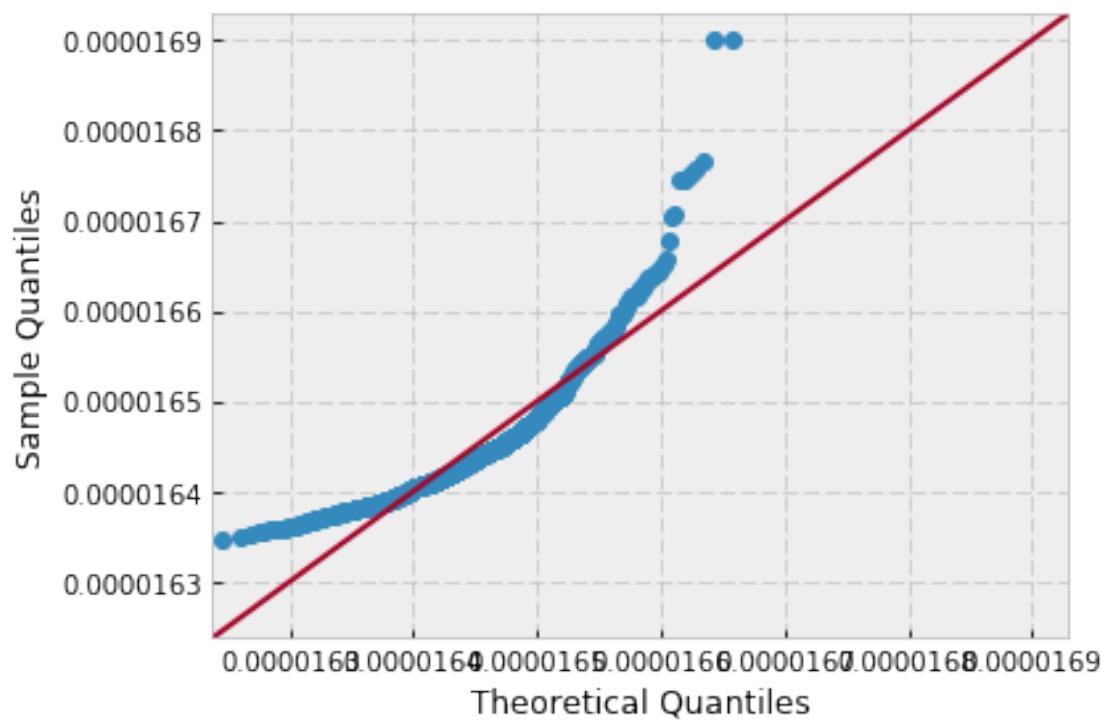
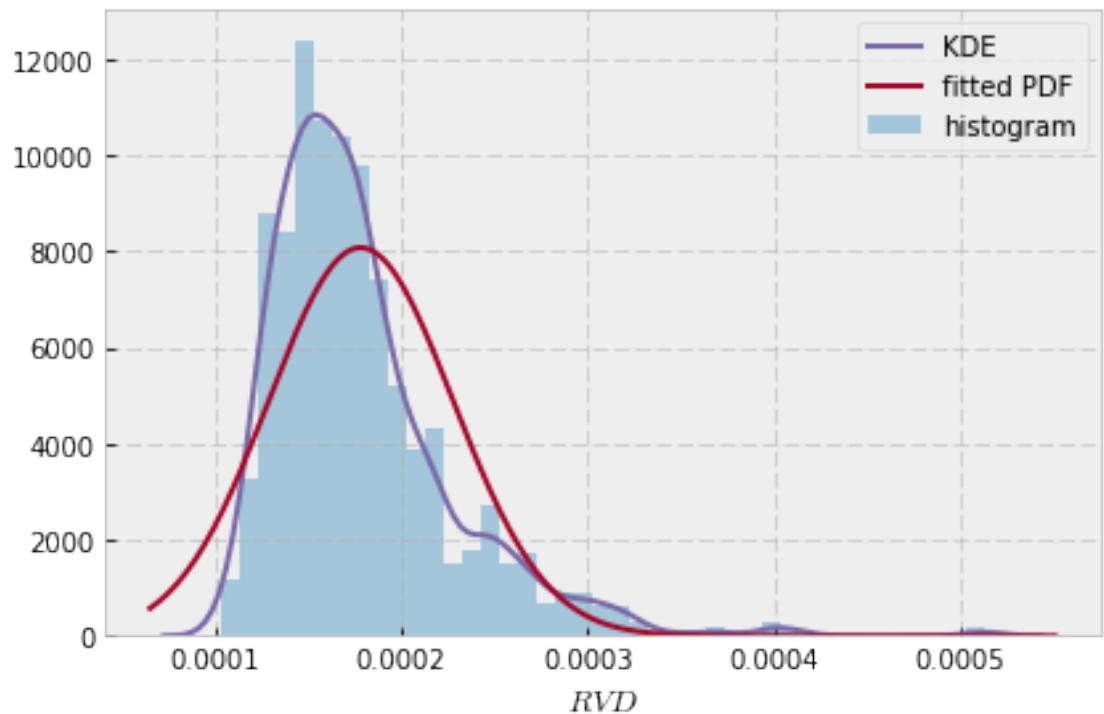


```
variable name: sigma
variable value: 0.05555555555555556
distribution: inverse-gamma(shape=(6.9460798872980583,),loc=6.60510610674006e-05, scale=0.000619
DescribeResult(nobs=1000, minmax=(9.8124301329047337e-05, 0.00043097646357016632)
               mean=0.00017086599610996515, variance=2.2153039219331675e-09,
               skewness=1.5450799318172783, kurtosis=2.841183358585588)
```

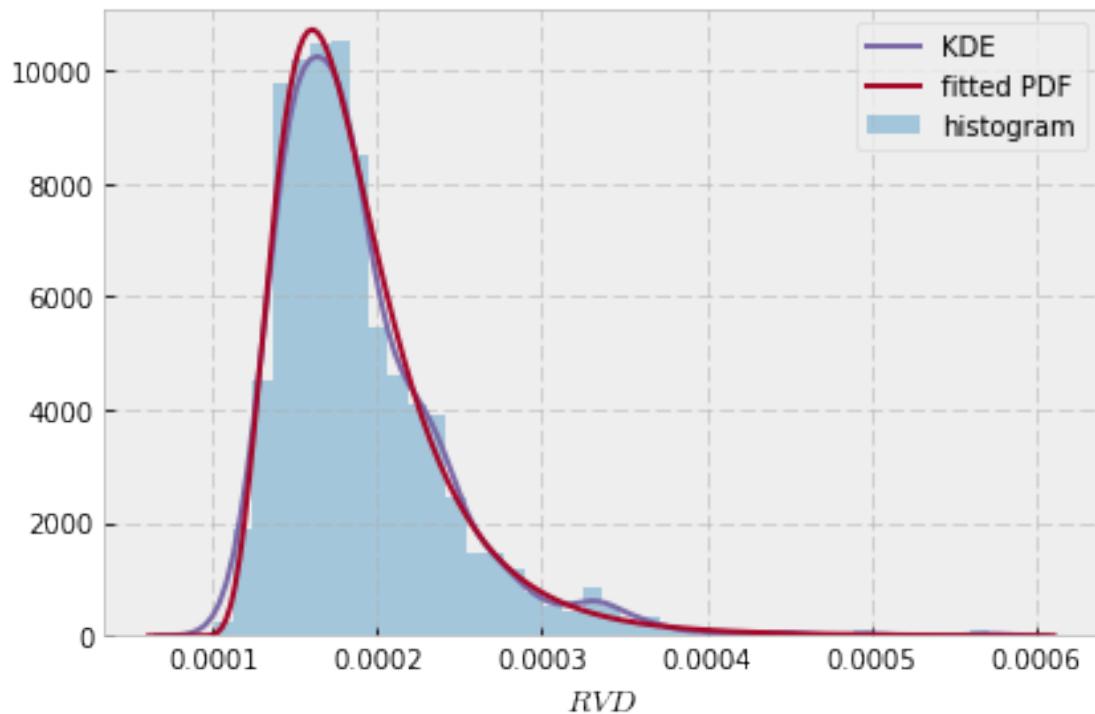


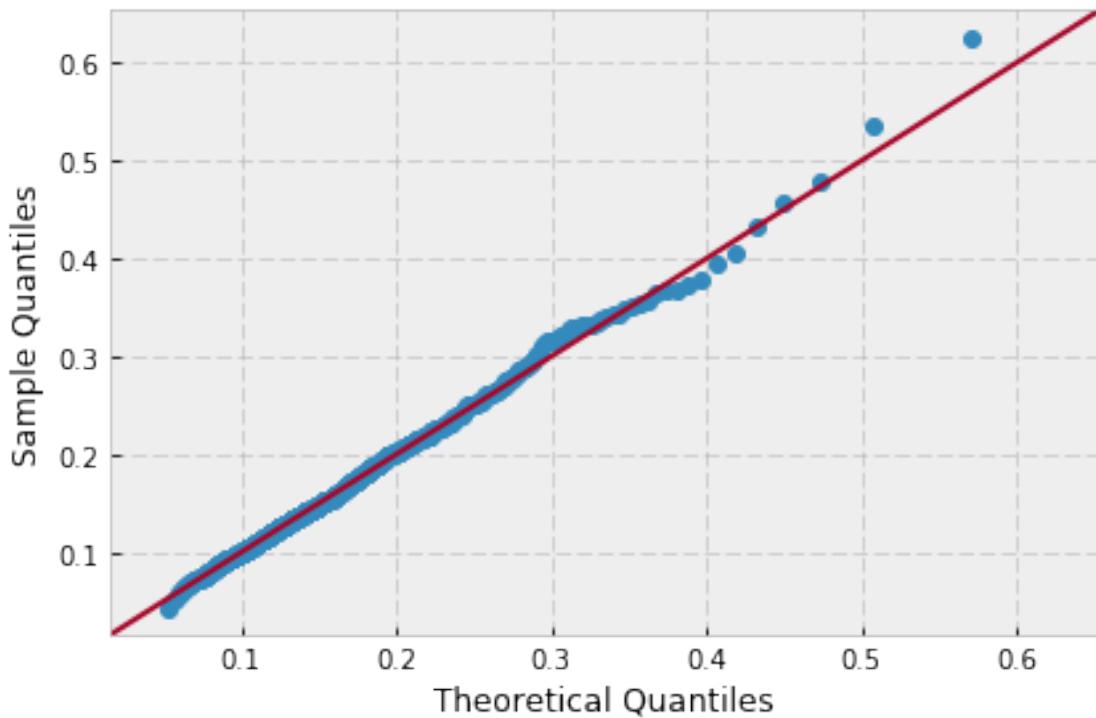


```
variable name: sigma
variable value: 0.0666666666666667
distribution: inverse-gamma(shape=(60791.248331857445,), loc=-0.012007356250877526, scale=740.782
DescribeResult(nobs=1000, minmax=(0.00010267002121520114, 0.00051292790436233702)
               mean=0.00017851972095312939, variance=2.469661946610575e-09,
               skewness=1.862363978747267, kurtosis=5.999162430302469)
```

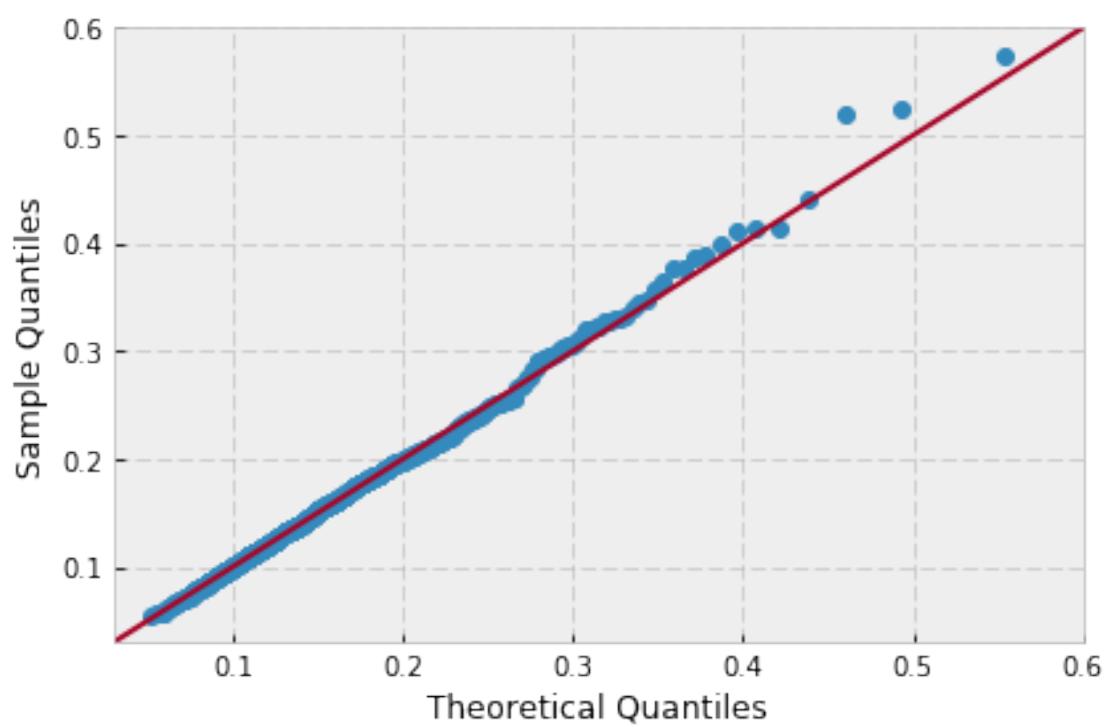
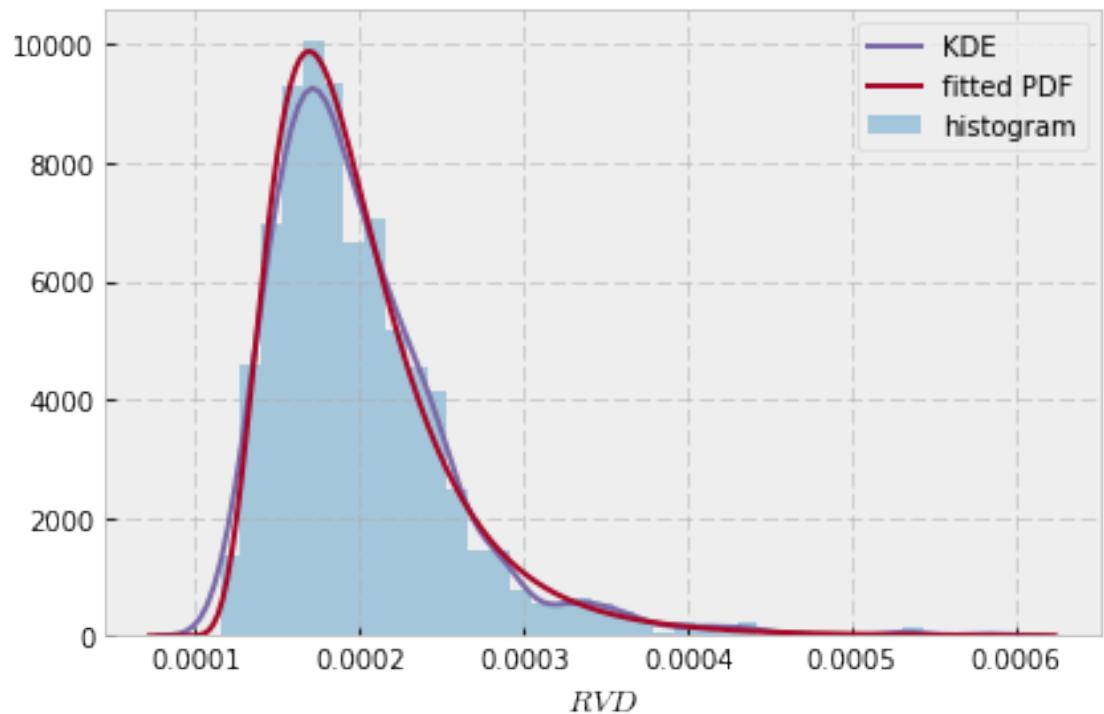


```
variable name: sigma
variable value: 0.0777777777777778
distribution: inverse-gamma(shape=(7.5274102637979592,),loc=6.534728190267806e-05, scale=0.00081
DescribeResult(nobs=1000, minmax=(0.0001007665664186771, 0.00057140717444815635)
               mean=0.00018946907434055498, variance=2.7675575733764984e-09,
               skewness=1.8308381514094156, kurtosis=5.845936088344624)
```

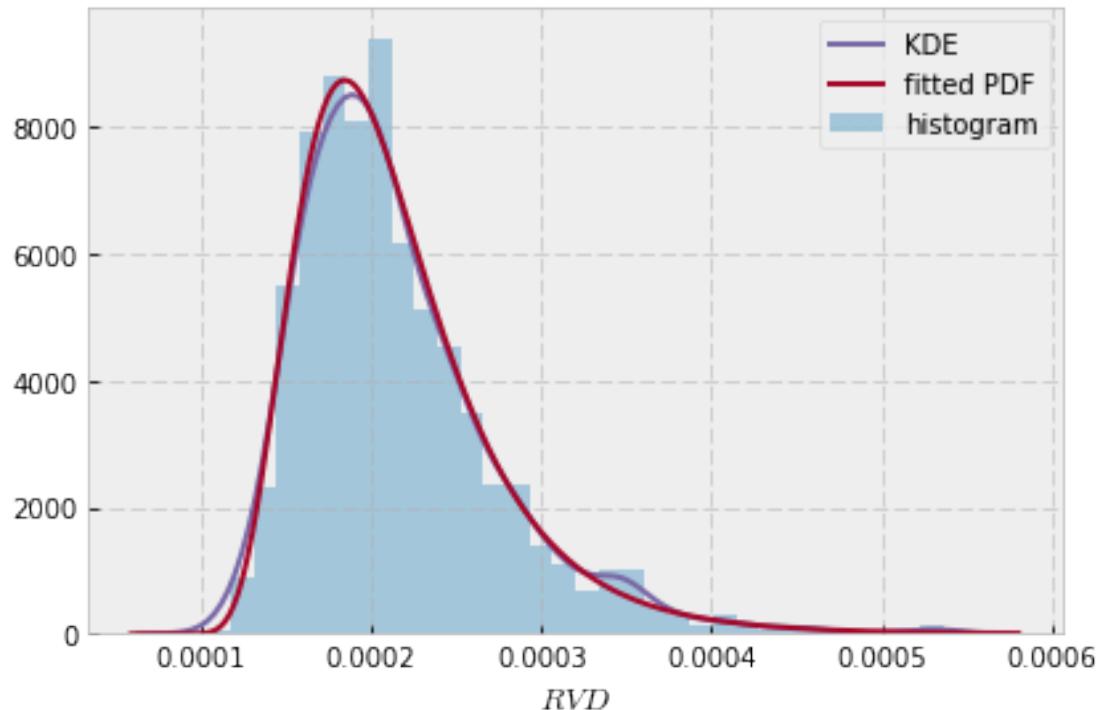


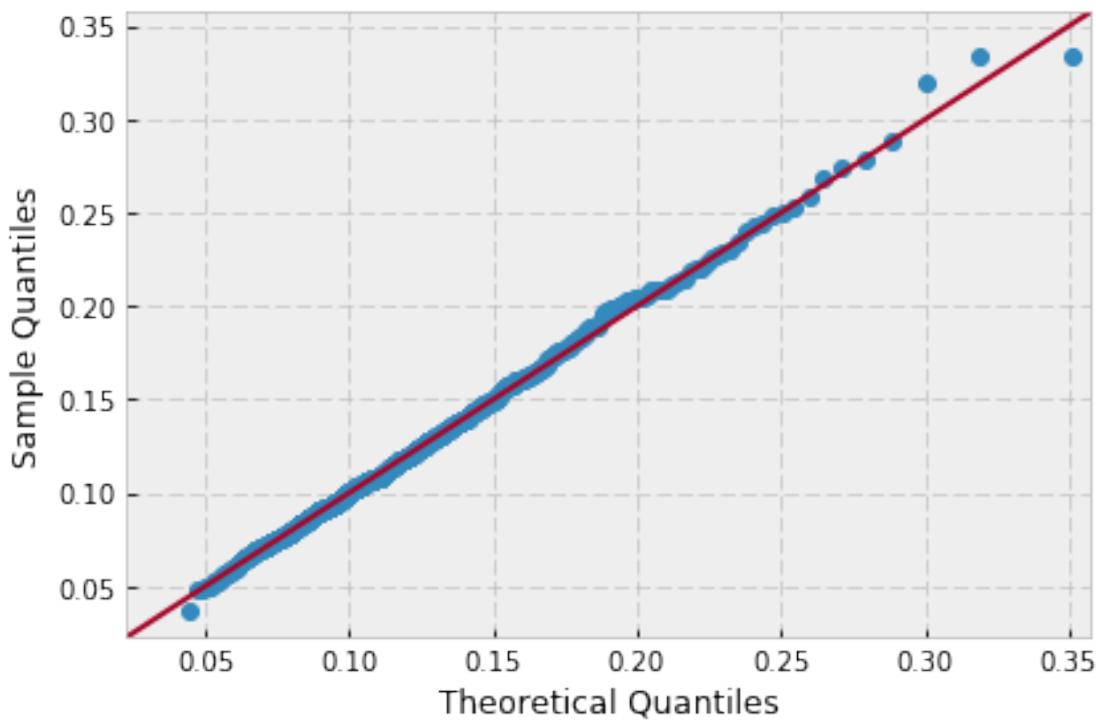


```
variable name: sigma
variable value: 0.0888888888888889
distribution: inverse-gamma(shape=(7.6412156267846605,), loc=6.519710388564133e-05, scale=0.00089
DescribeResult(nobs=1000, minmax=(0.00011472069141484329, 0.00058088202821115868)
               mean=0.00020033012054966604, variance=3.2051178669333627e-09,
               skewness=1.8637747755261418, kurtosis=6.050014286060023)
```

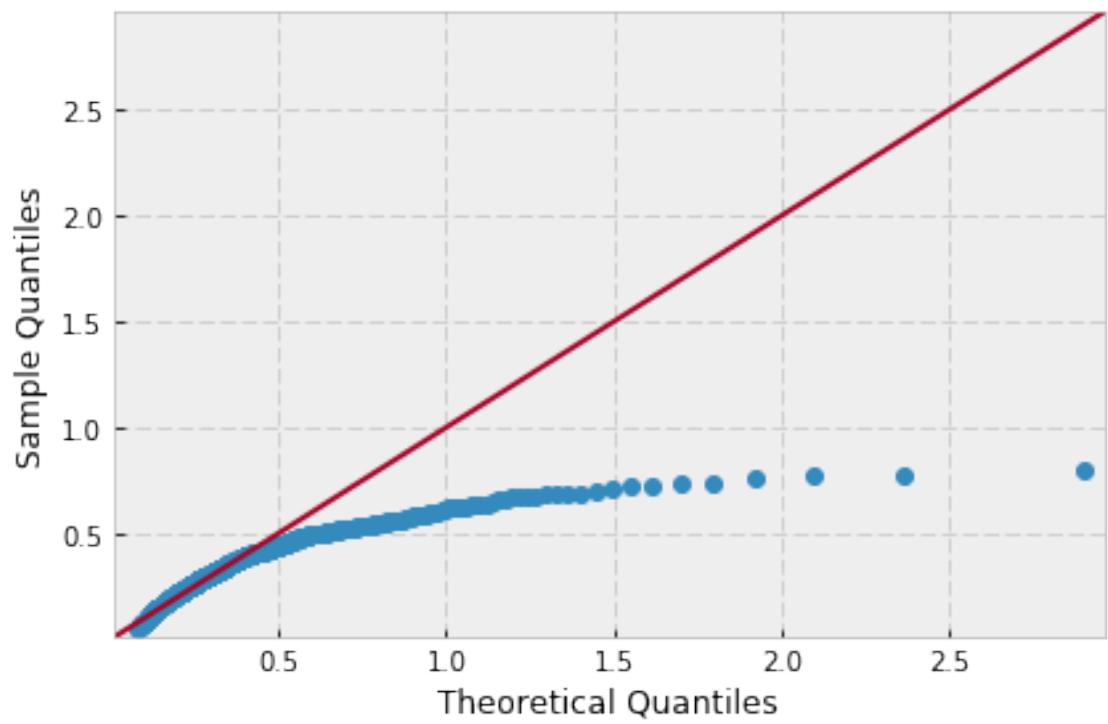
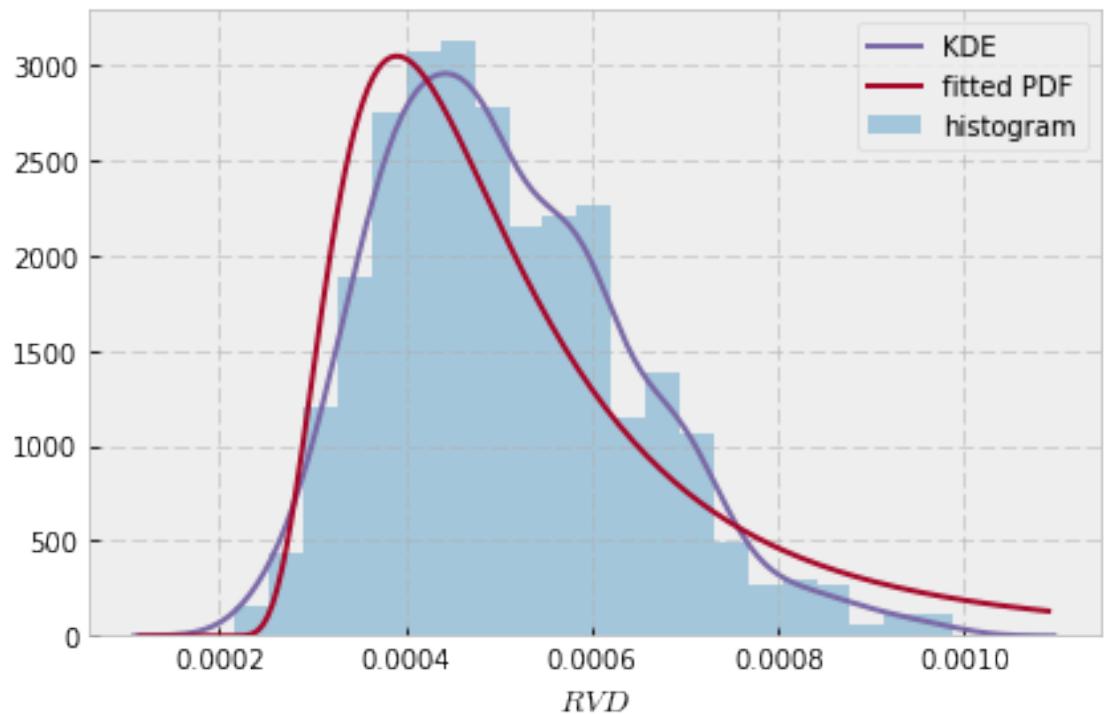


```
variable name: sigma
variable value: 0.1
distribution: inverse-gamma(shape=(9.7801399446157831,), loc=4.9590362771853116e-05, scale=0.0014
DescribeResult(nobs=1000, minmax=(0.00010370076465254685, 0.00053561175537209221)
               mean=0.00021519916253889945, variance=3.497189543929493e-09,
               skewness=1.4549378715669543, kurtosis=3.325504140894343)
```

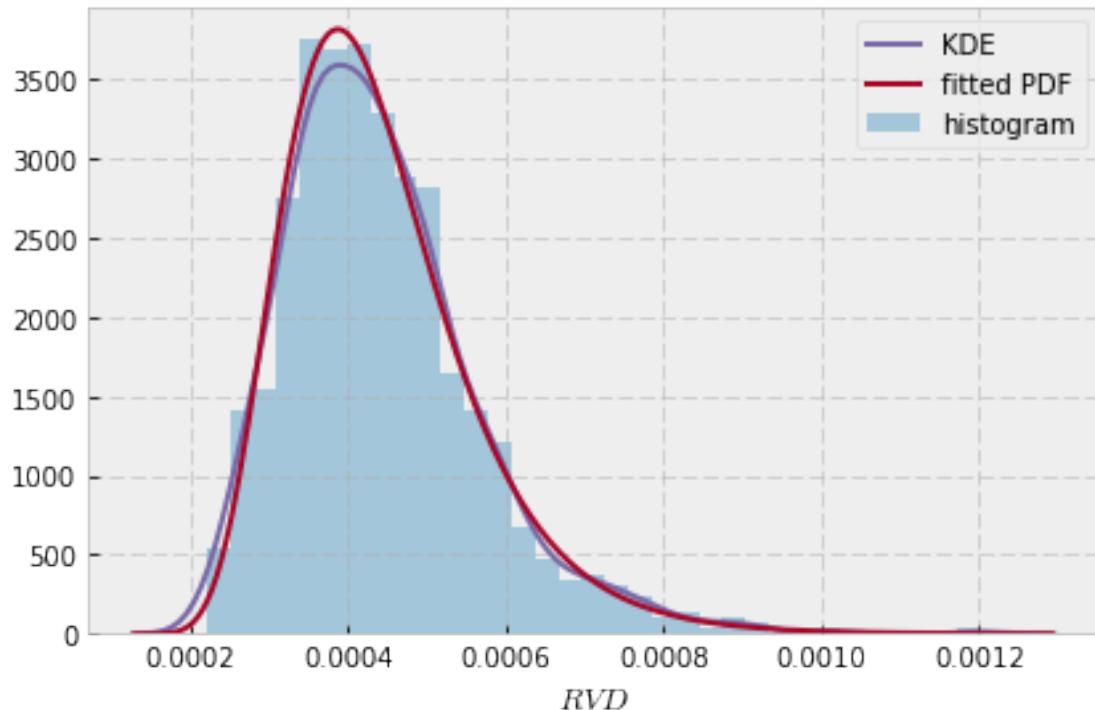


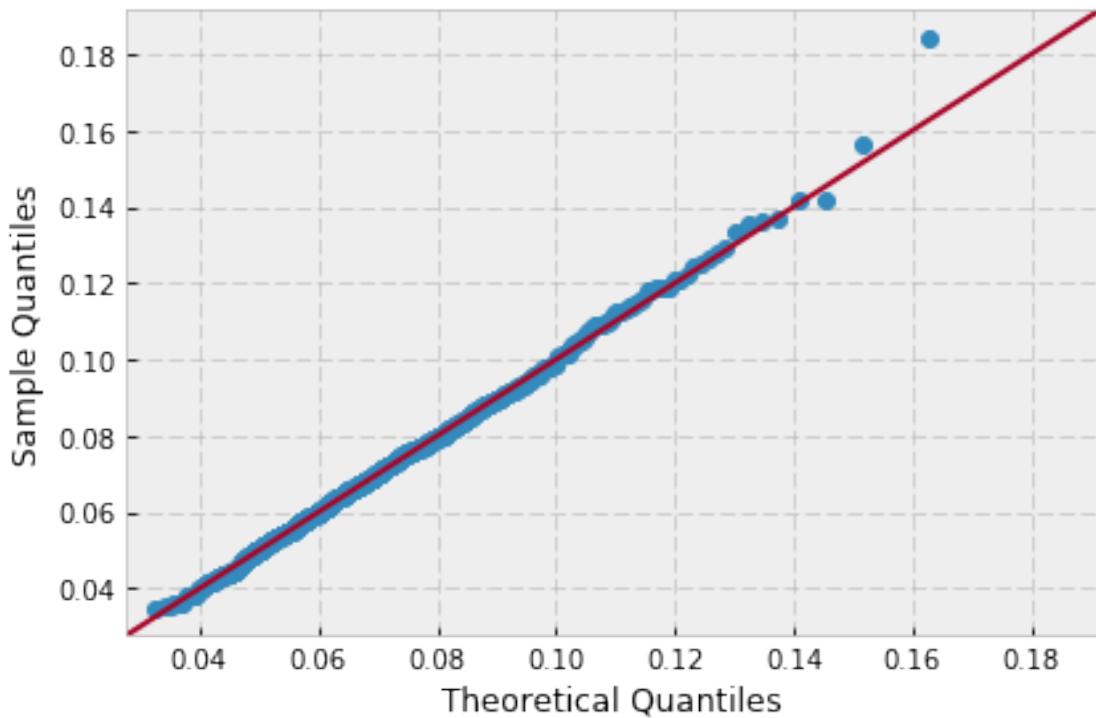


```
variable name: theta
variable value: 0.6981317007977318
distribution: inverse-gamma(shape=(3.6832481793337282,),loc=0.0001718873886398498, scale=0.00102
DescribeResult(nobs=1000, minmax=(0.00021730313111118535, 0.00098787394604101441)
               mean=0.00050753049857437673, variance=1.8447855215452052e-08,
               skewness=0.6289199318327575, kurtosis=0.19031840127250854)
```

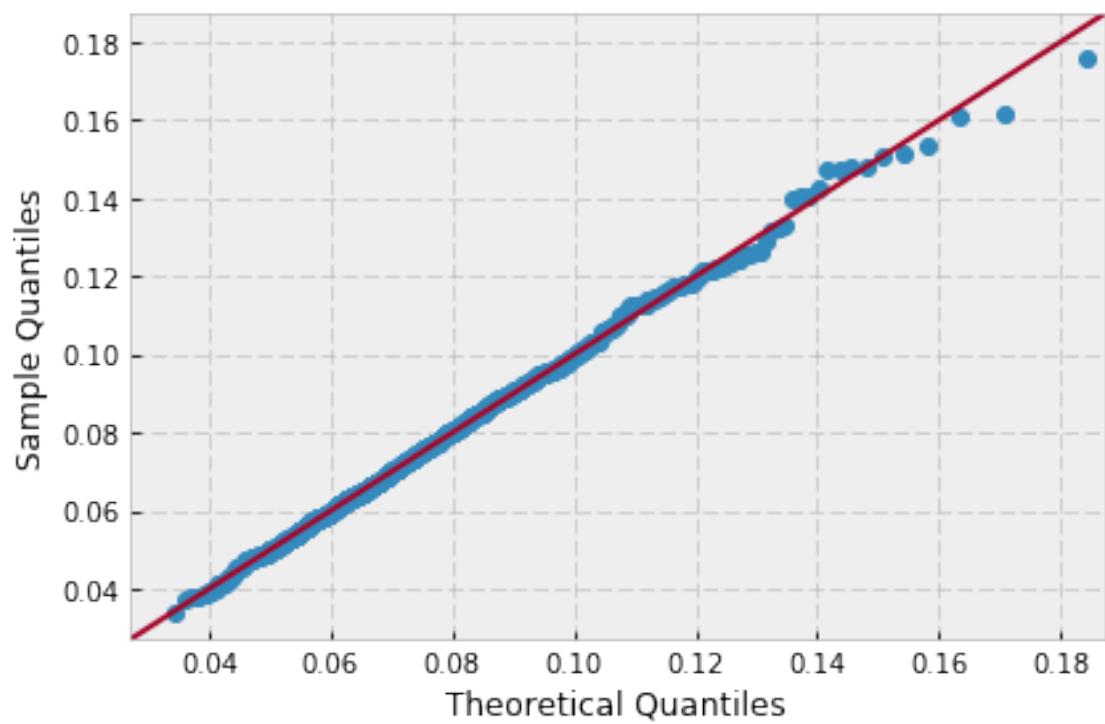
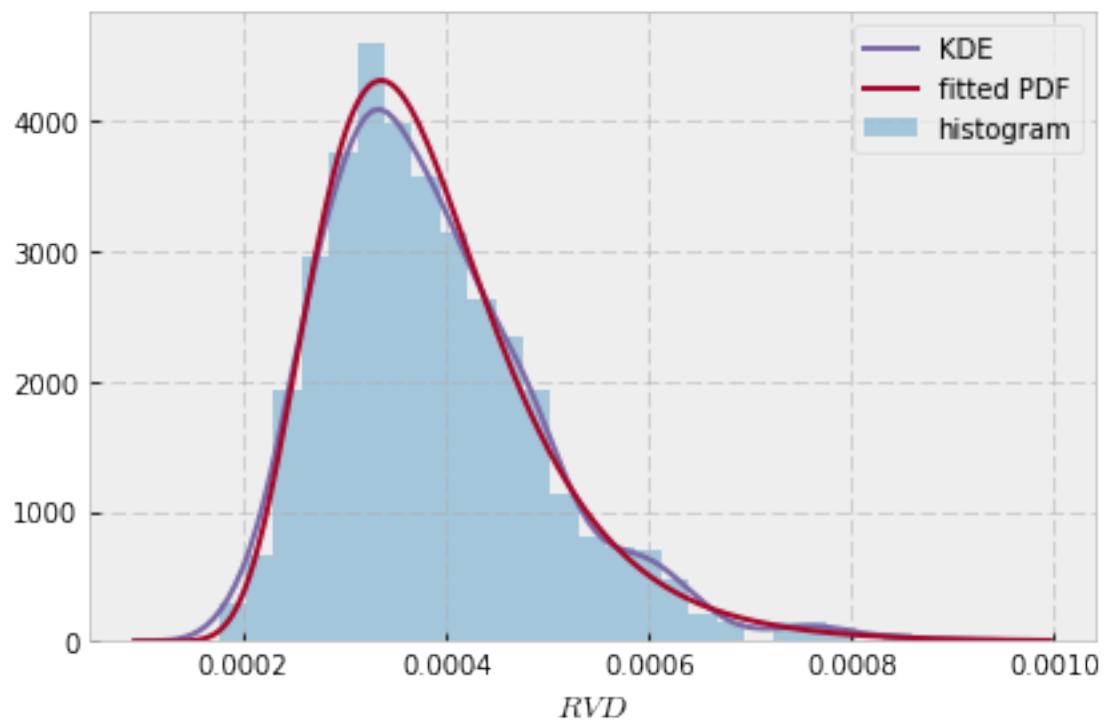


```
variable name: theta
variable value: 0.7853981633974483
distribution: inverse-gamma(shape=(15.56793939745911,),loc=-1.0814508628847151e-05, scale=0.0065
DescribeResult(nobs=1000, minmax=(0.00021971921983182855, 0.0012013461391123725)
               mean=0.00044119509606584475, variance=1.5060375260271788e-08,
               skewness=1.1715821243645255, kurtosis=2.6603715181469534)
```

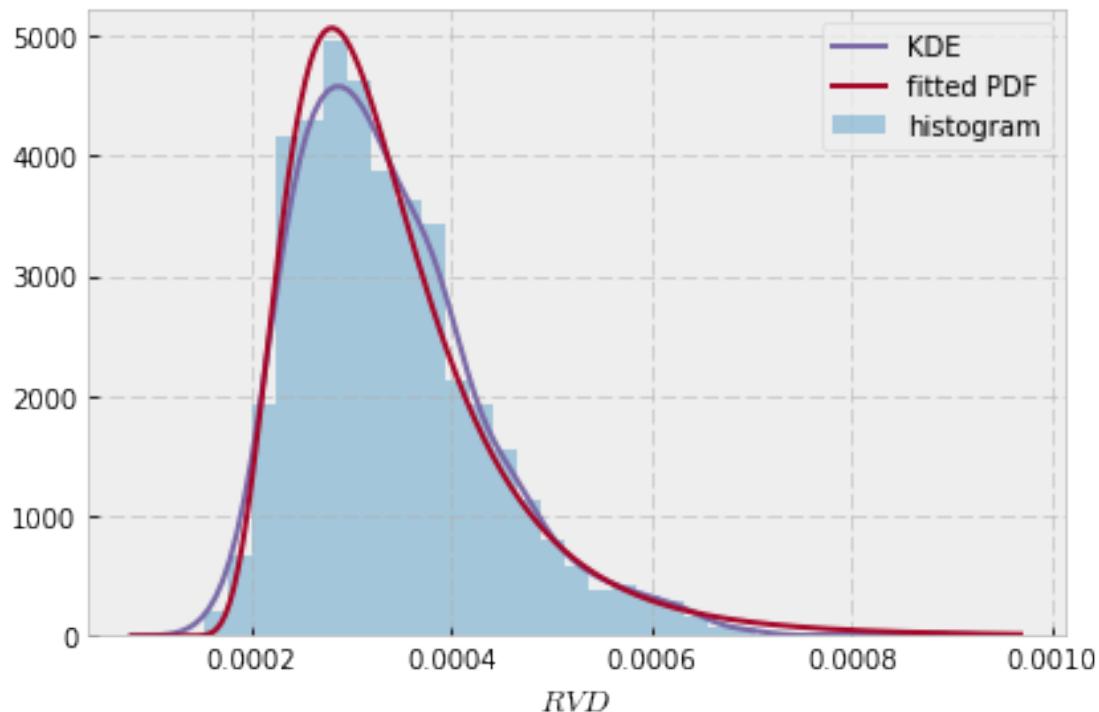


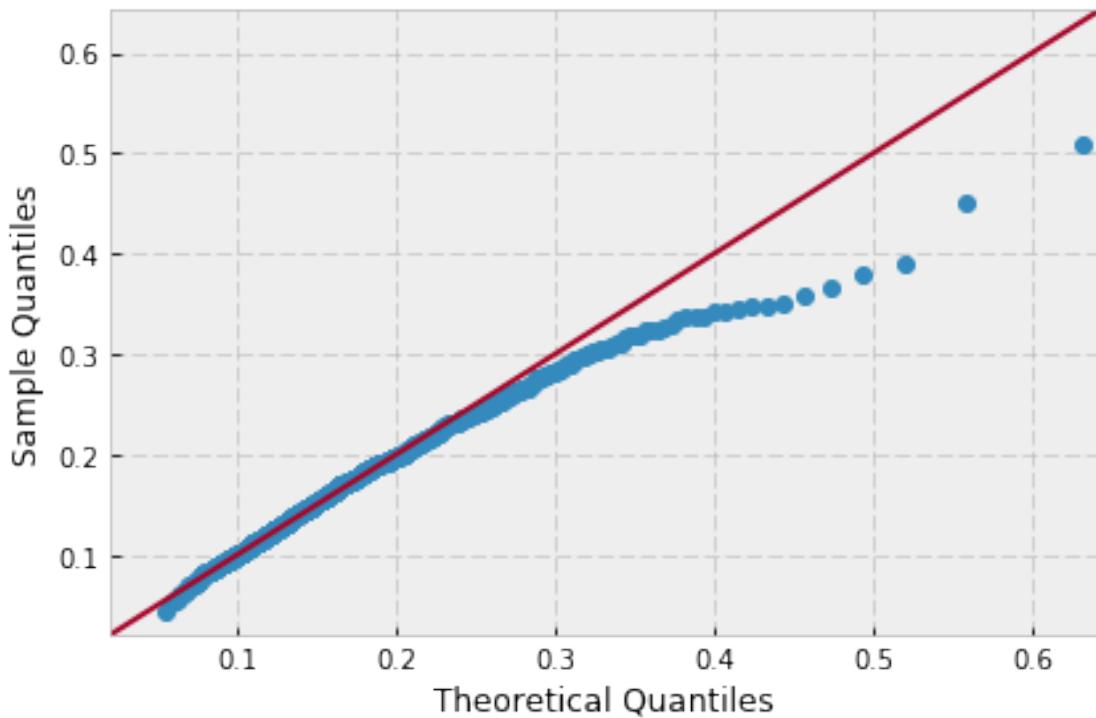


```
variable name: theta
variable value: 0.8726646259971648
distribution: inverse-gamma(shape=(14.397300950756049,), loc=-1.8758709138757787e-06, scale=0.005
DescribeResult(nobs=1000, minmax=(0.00017487548463644408, 0.00091317504432959784)
               mean=0.00038599541595977317, variance=1.1881707970537097e-08,
               skewness=1.046505585014973, kurtosis=1.601312132046366)
```

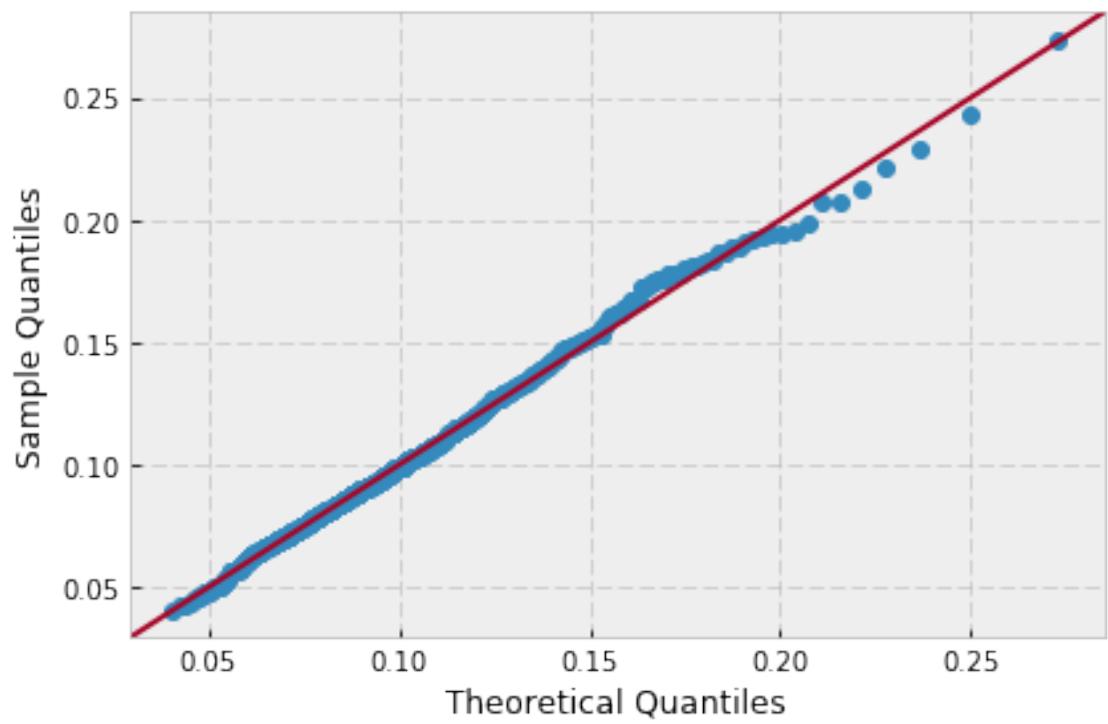
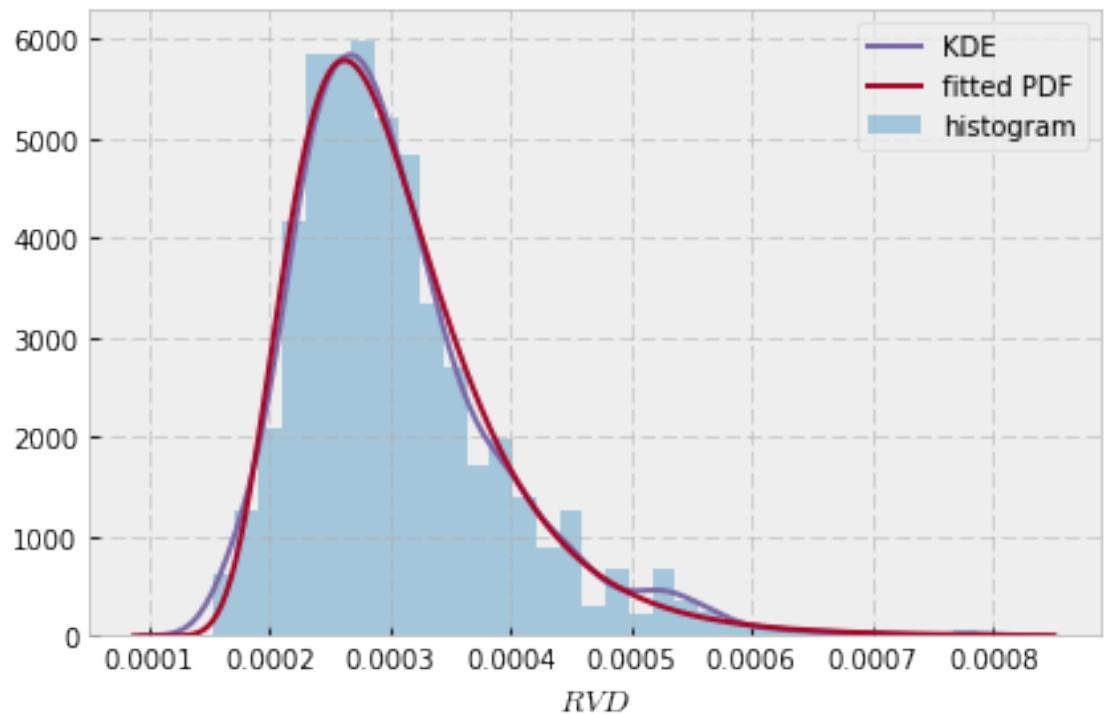


```
variable name: theta
variable value: 0.9599310885968813
distribution: inverse-gamma(shape=(7.1463041652359713,),loc=8.508153329143253e-05, scale=0.00158
DescribeResult(nobs=1000, minmax=(0.00015297450258788058, 0.00089538272499553291)
    mean=0.00033946791309349327, variance=9.4159704815027297e-09,
    skewness=1.1378799729800801, kurtosis=1.99422058835414)
```

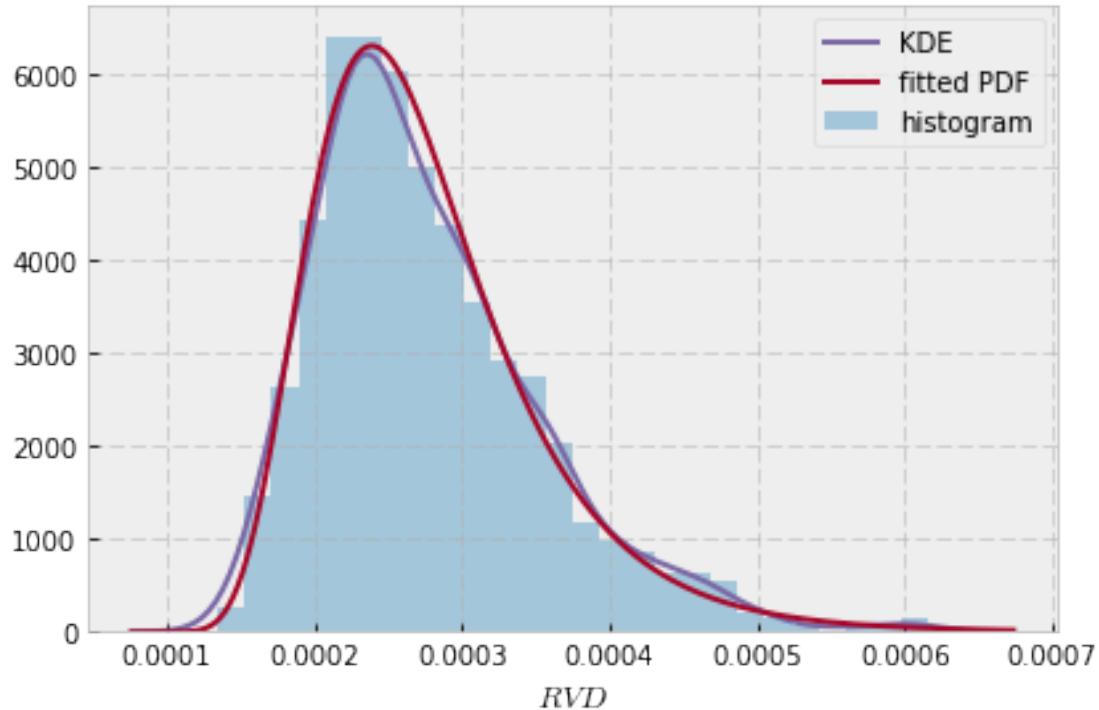


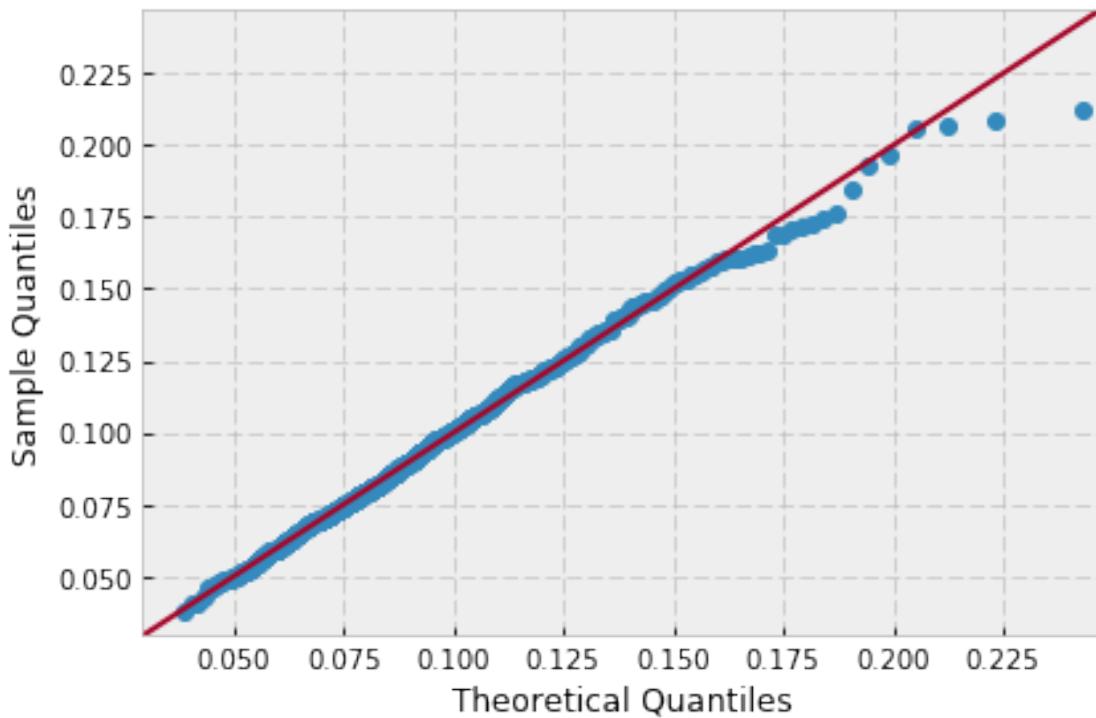


```
variable name: theta
variable value: 1.0471975511965979
distribution: inverse-gamma(shape=(11.331135722766131,),loc=4.114667855354841e-05, scale=0.002722
DescribeResult(nobs=1000, minmax=(0.00015249645291963329, 0.0007859921510821919)
               mean=0.00030493326788087418, variance=7.4000230714167897e-09,
               skewness=1.2908884292818252, kurtosis=2.4242650923175892)
```

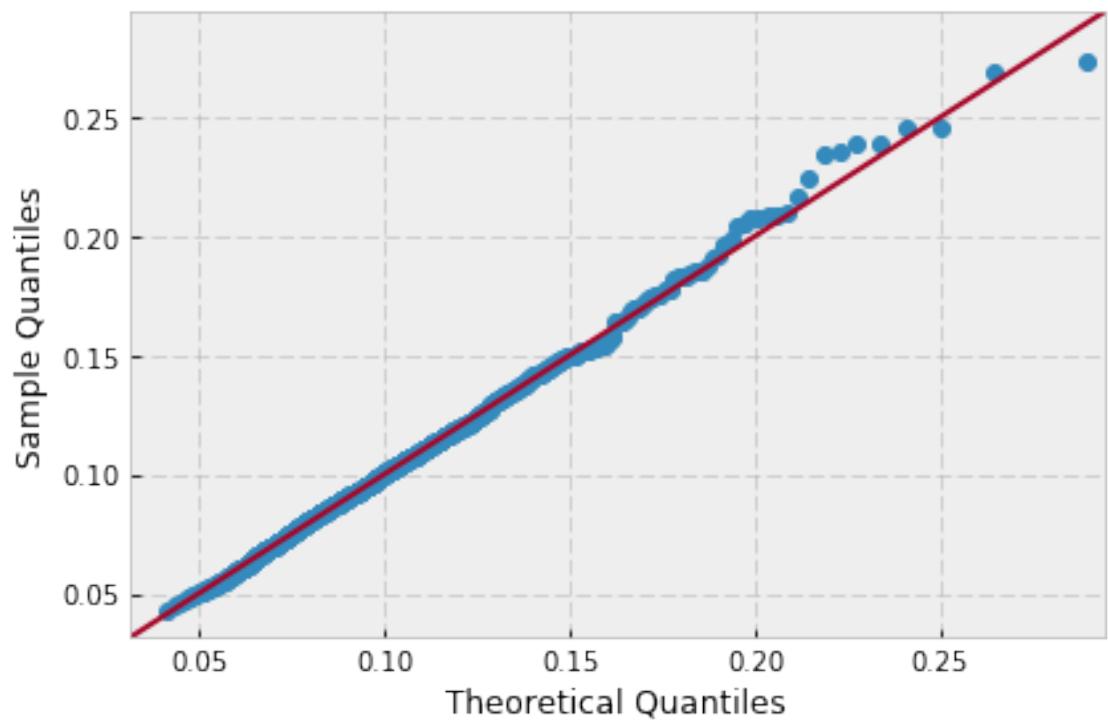
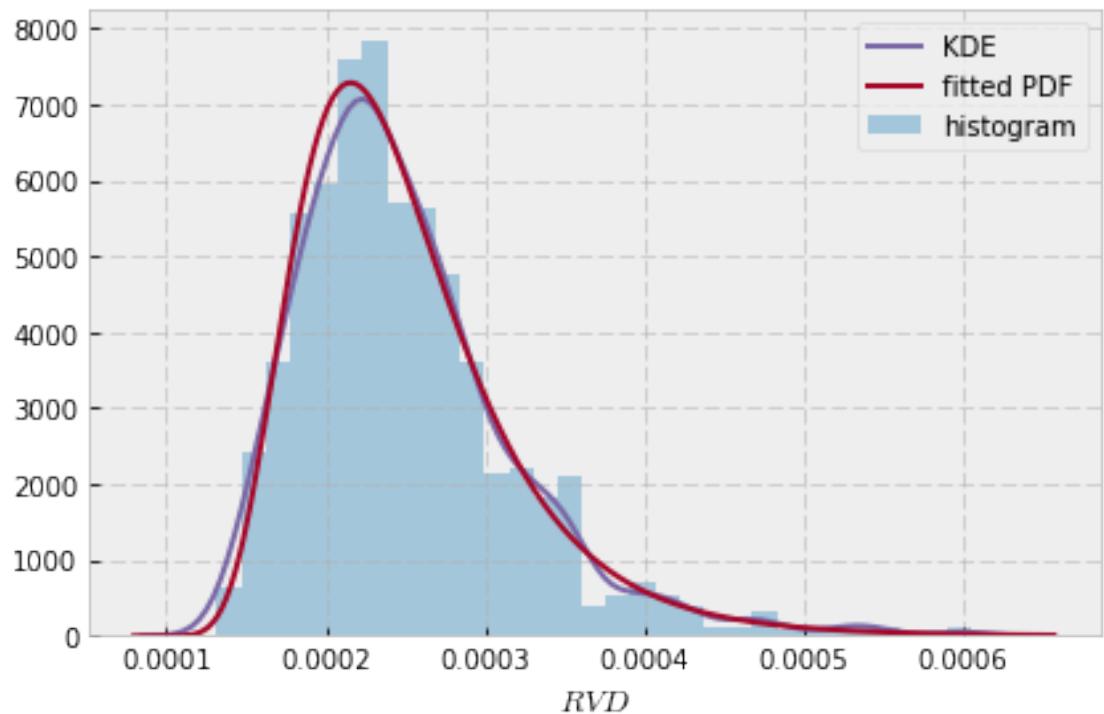


```
variable name: theta
variable value: 1.1344640137963142
distribution: inverse-gamma(shape=(12.134403758526876,),loc=2.841088555192341e-05, scale=0.00276
DescribeResult(nobs=1000, minmax=(0.00013348633936523989, 0.00061583248819473385)
               mean=0.00027652289996693091, variance=5.8457911934422563e-09,
               skewness=1.0737668772535522, kurtosis=1.5022803573136532)
```

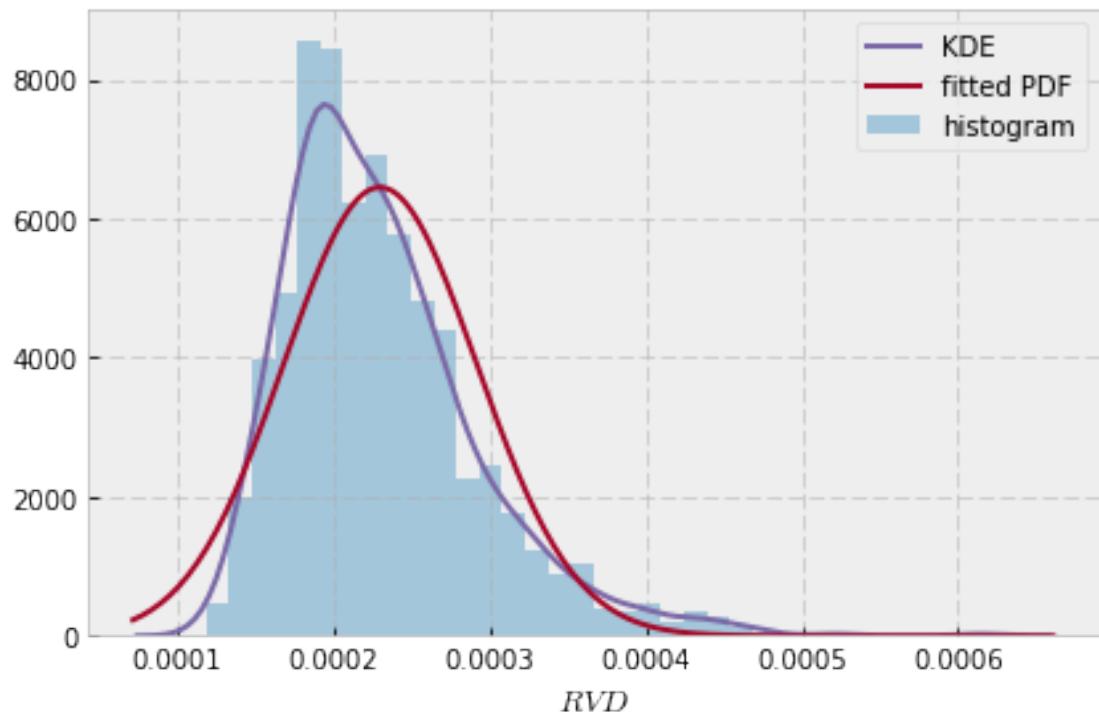


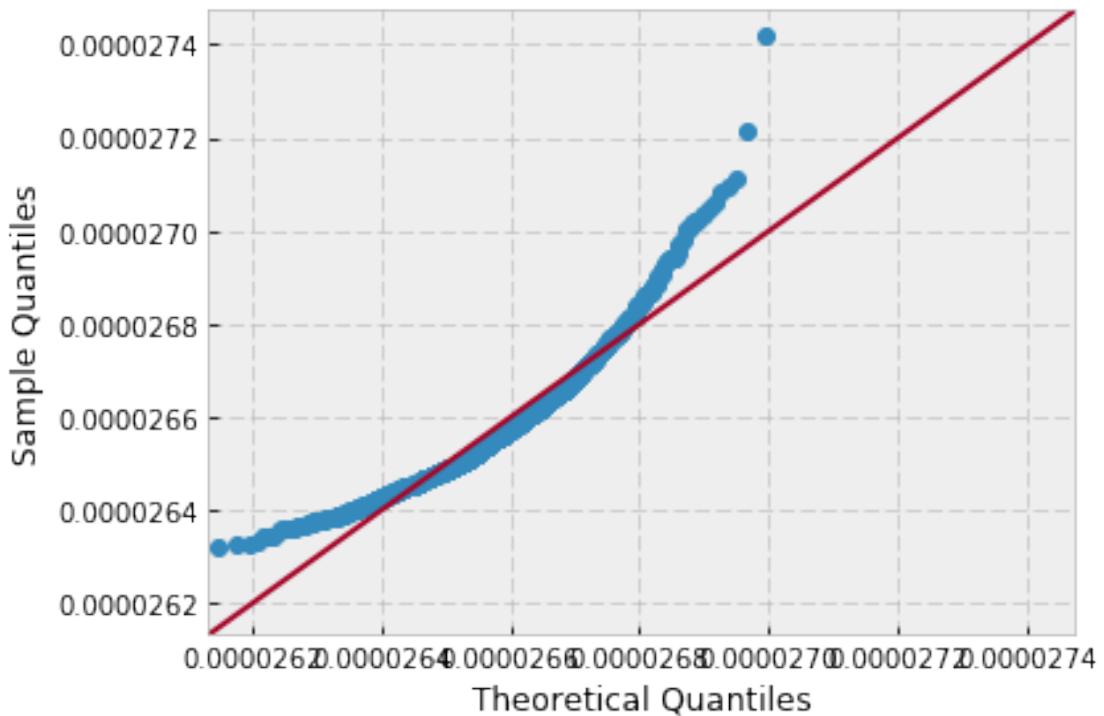


```
variable name: theta
variable value: 1.2217304763960306
distribution: inverse-gamma(shape=(10.940369122894209,), loc=4.3434979030799727e-05, scale=0.002000000000000001)
DescribeResult(nobs=1000, minmax=(0.00013147017164220871, 0.00060537369658281615)
               mean=0.00025020092483842444, variance=4.7302588692282077e-09,
               skewness=1.3690424848900427, kurtosis=3.0198320646743637)
```

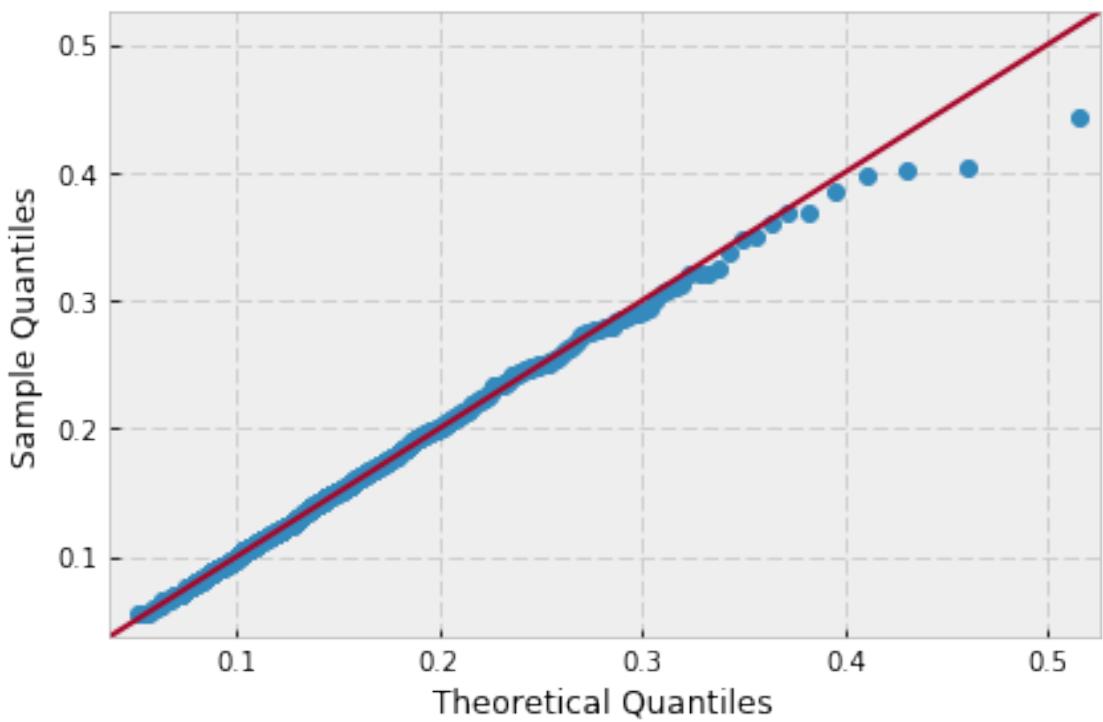
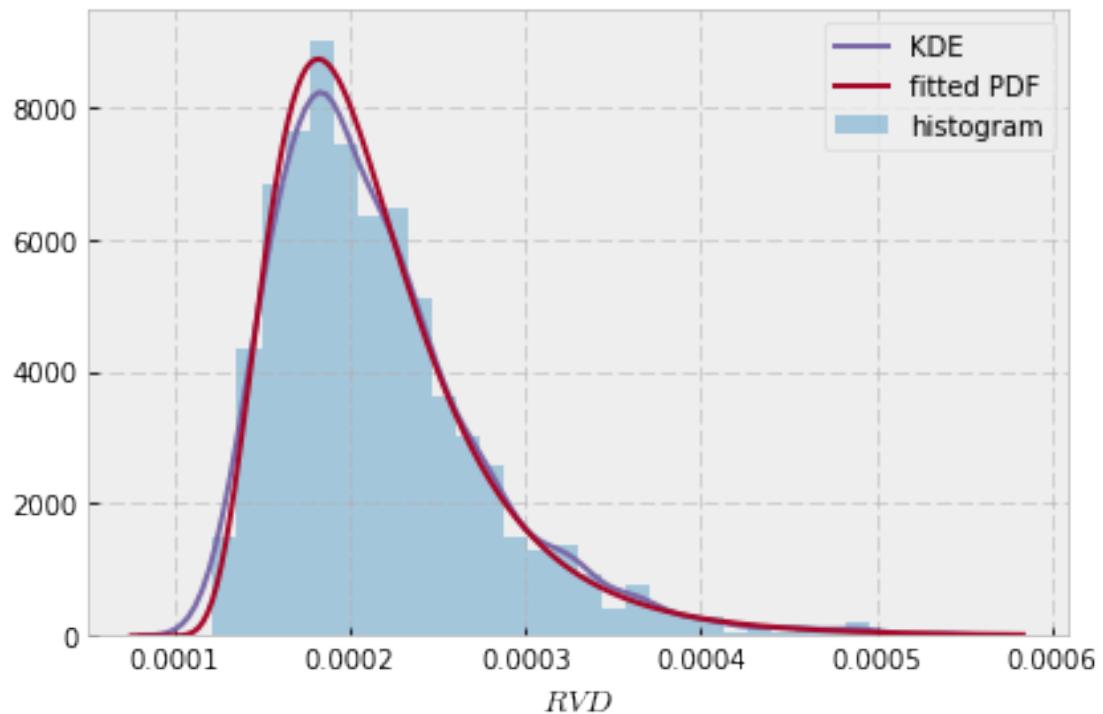


```
variable name: theta
variable value: 1.3089969389957472
distribution: inverse-gamma(shape=(37638.956017174845,),loc=-0.011767690780381423, scale=451.567
DescribeResult(nobs=1000, minmax=(0.00011801763535591254, 0.00061385057903962212)
               mean=0.0002301119814498238, variance=3.8697199069262696e-09,
               skewness=1.291023045984961, kurtosis=2.7988406564370285)
```

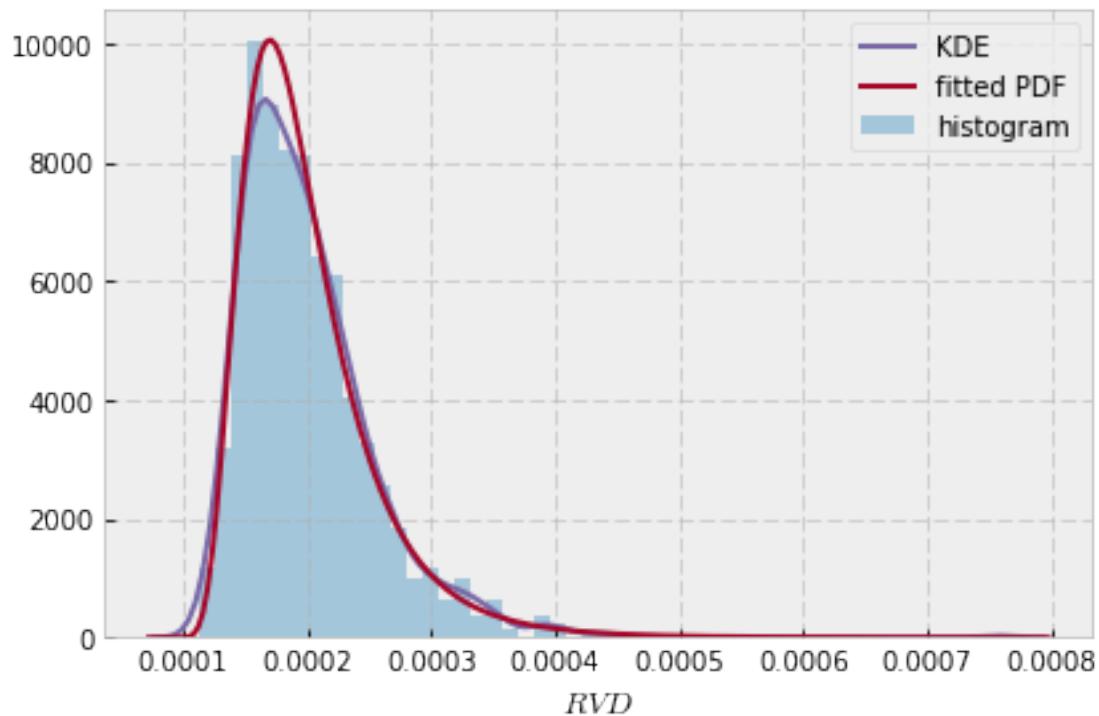


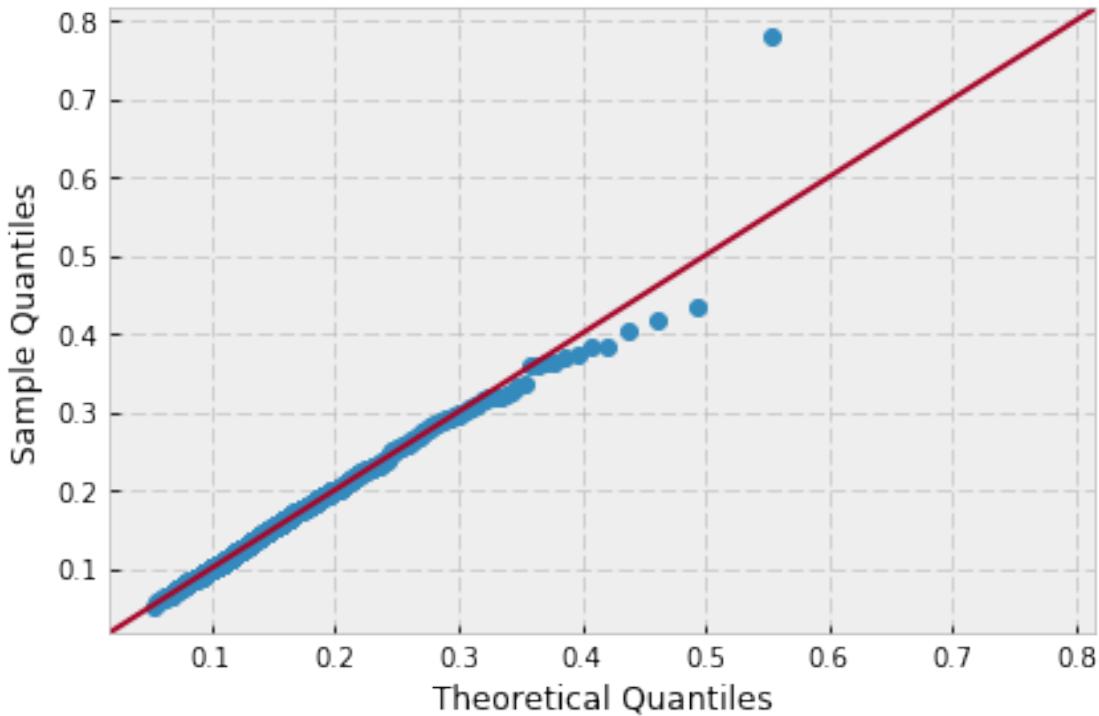


```
variable name: theta
variable value: 1.3962634015954638
distribution: inverse-gamma(shape=(7.9311956946770348,),loc=6.187805516106901e-05, scale=0.00107
DescribeResult(nobs=1000, minmax=(0.0001217609468431391, 0.00053747040618674024)
               mean=0.00021648839412676928, variance=3.7521416186004547e-09,
               skewness=1.4037312303920533, kurtosis=2.8149572628112836)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: inverse-gamma(shape=(7.6409986078566323,),loc=6.783082946632563e-05, scale=0.00088
DescribeResult(nobs=1000, minmax=(0.00011409481642380277, 0.0007556029972817205)
               mean=0.00020061244340085864, variance=3.0550886504342285e-09,
               skewness=2.099511480039028, kurtosis=11.37924072748378)
```

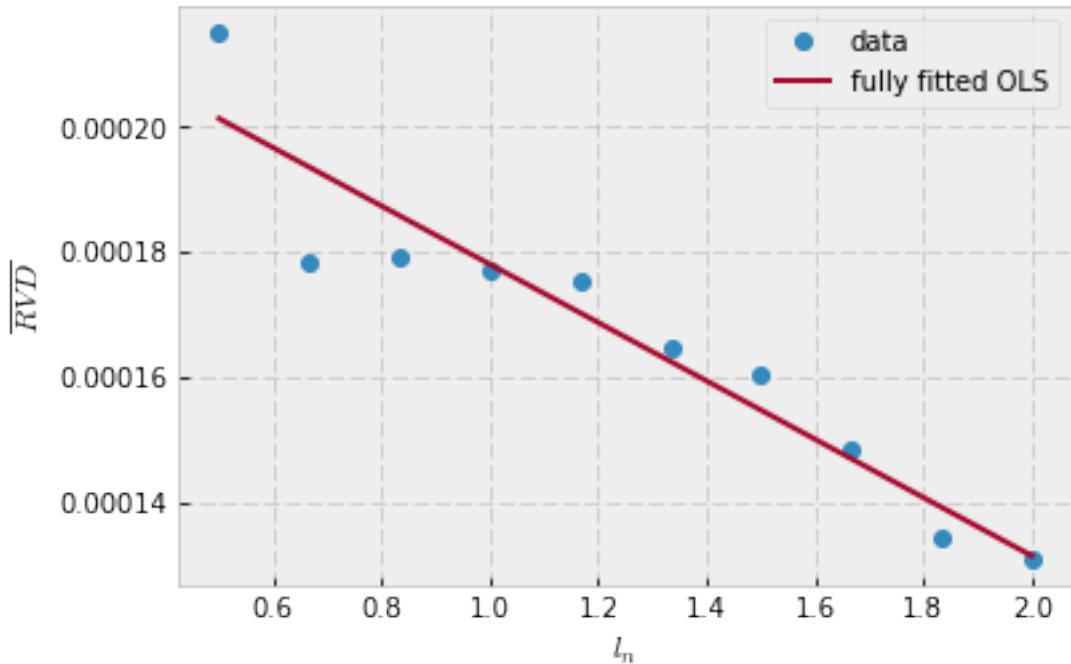




13.2 Parameter regression

```
In [32]: var_name = 'total_root_volume_per_convex_hull_volume'
mean_reg('ln', var_name, 'poly1')
mean_reg('theta', var_name, 'poly2')
mean_reg('maxB', var_name, 'poly1')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly2')
mean_reg('N', var_name, 'poly1')
```

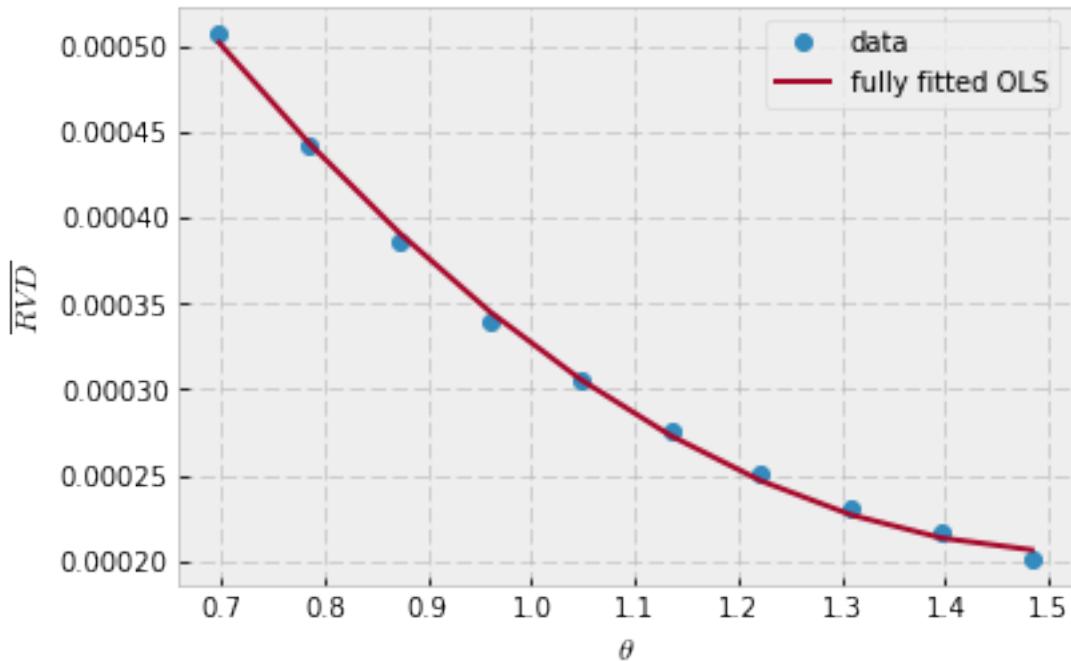
```
variable name: ln
model: poly1
r-squared: 0.8096614001910445
explained variance: 0.809672531407038
```



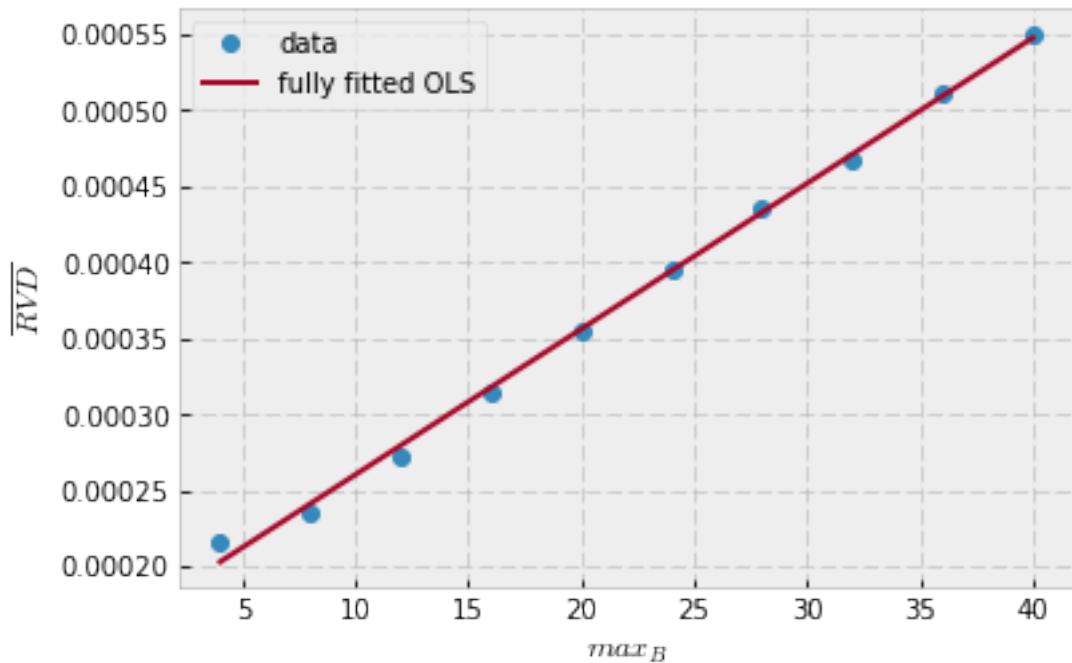
```

variable name: theta
model: poly2
r-squared: 0.9932210761499464
explained variance: 0.9932211511467577

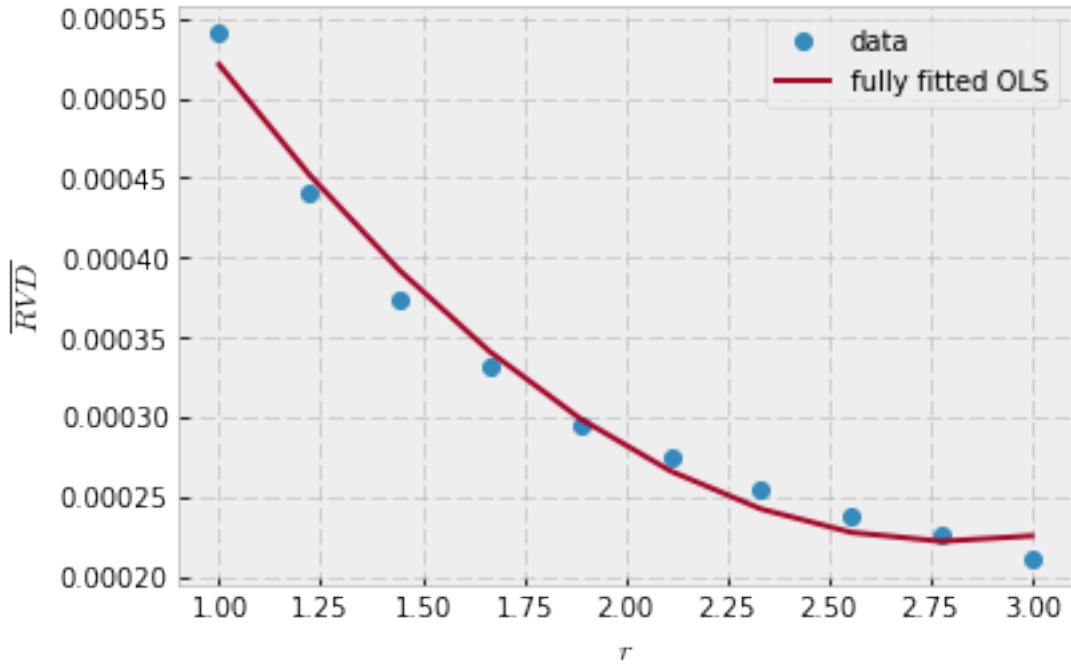
```



```
variable name: maxB  
model: poly1  
r-squared: 0.9947209785931284  
explained variance: 0.9947390963181044
```



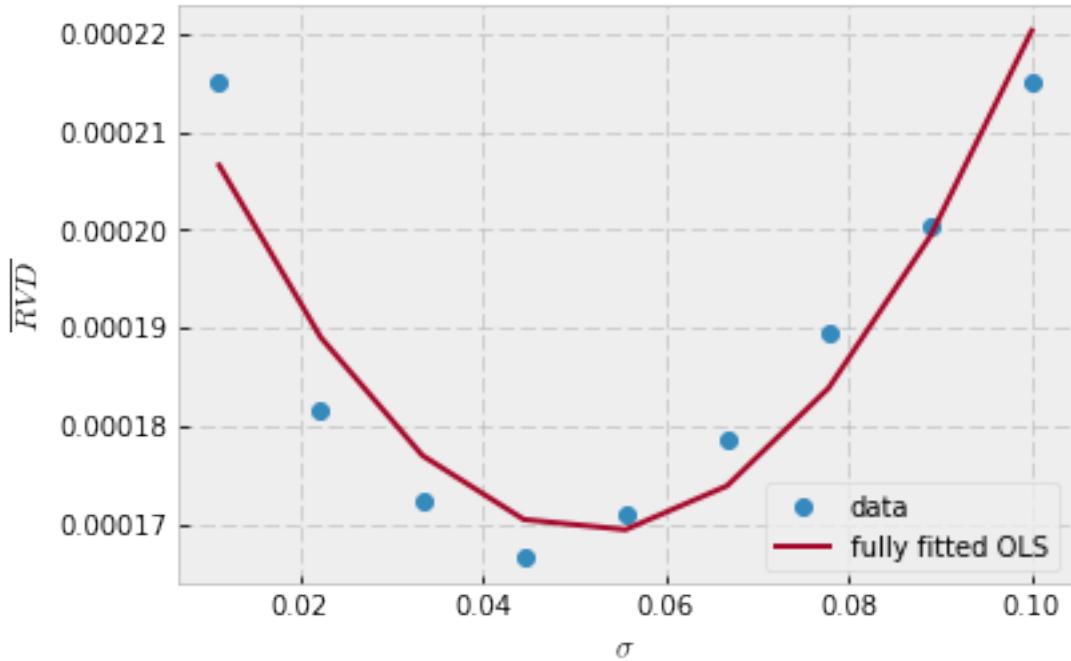
```
variable name: r  
model: poly2  
r-squared: 0.9423640905919581  
explained variance: 0.9424137457146899
```



```

variable name: sigma
model: poly2
r-squared: 0.5989946642199655
explained variance: 0.5999769888406405

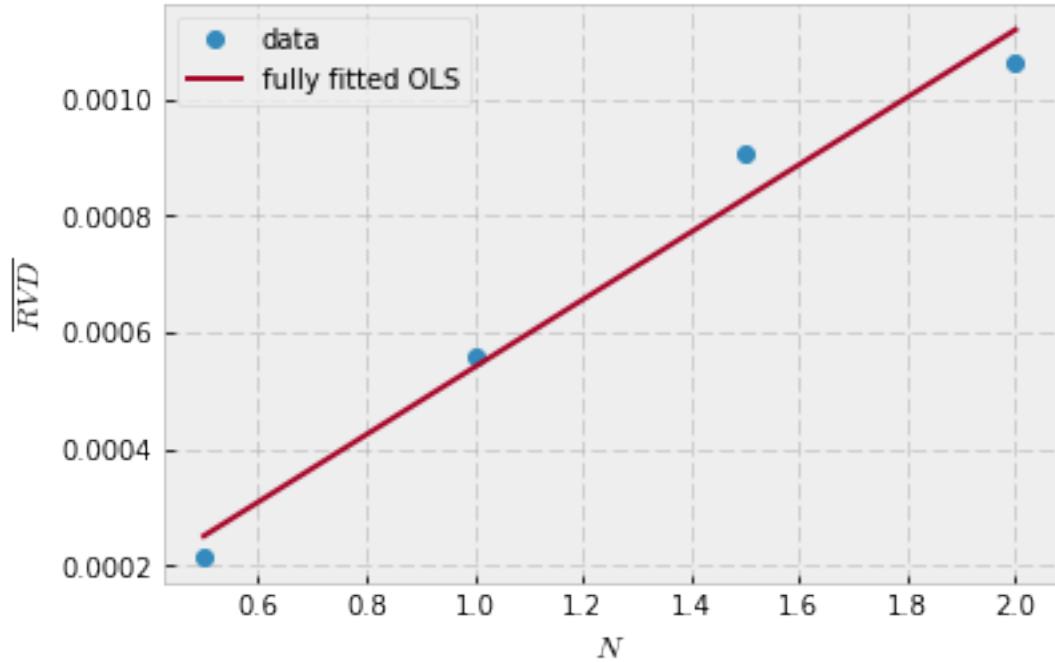
```



```

variable name: N
model: poly1
r-squared: 0.8510807106160571
explained variance: 0.869883824750038

```



14 Volume of the rhizosphere relative to convex hull: phosphate (0.2cm)

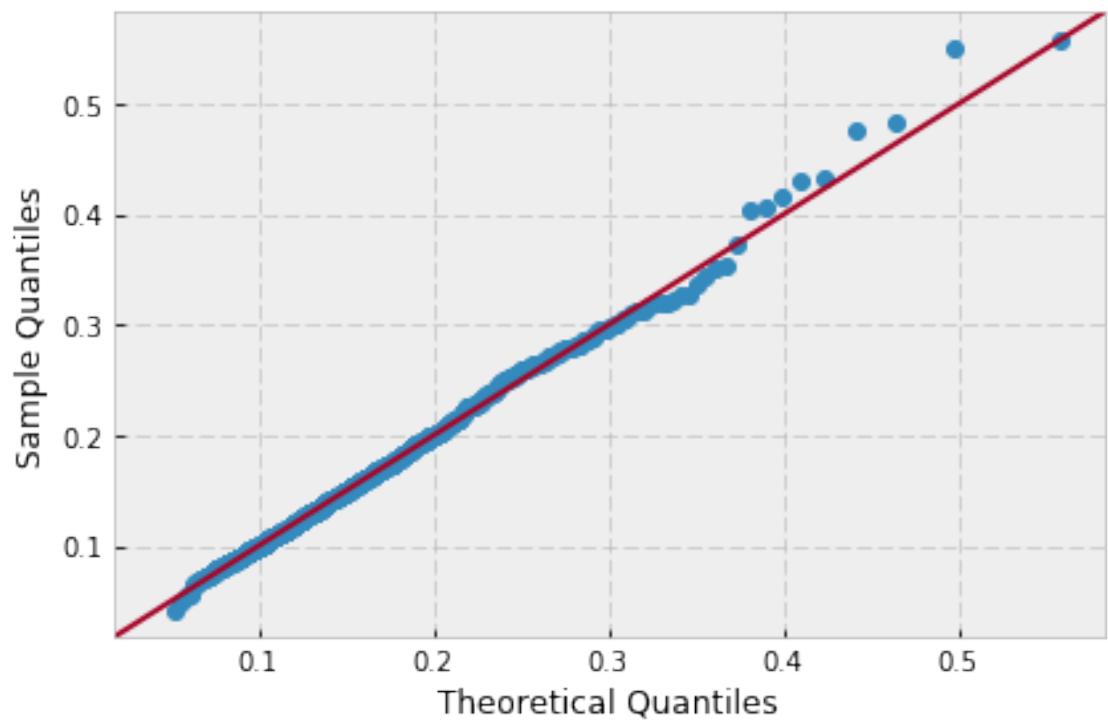
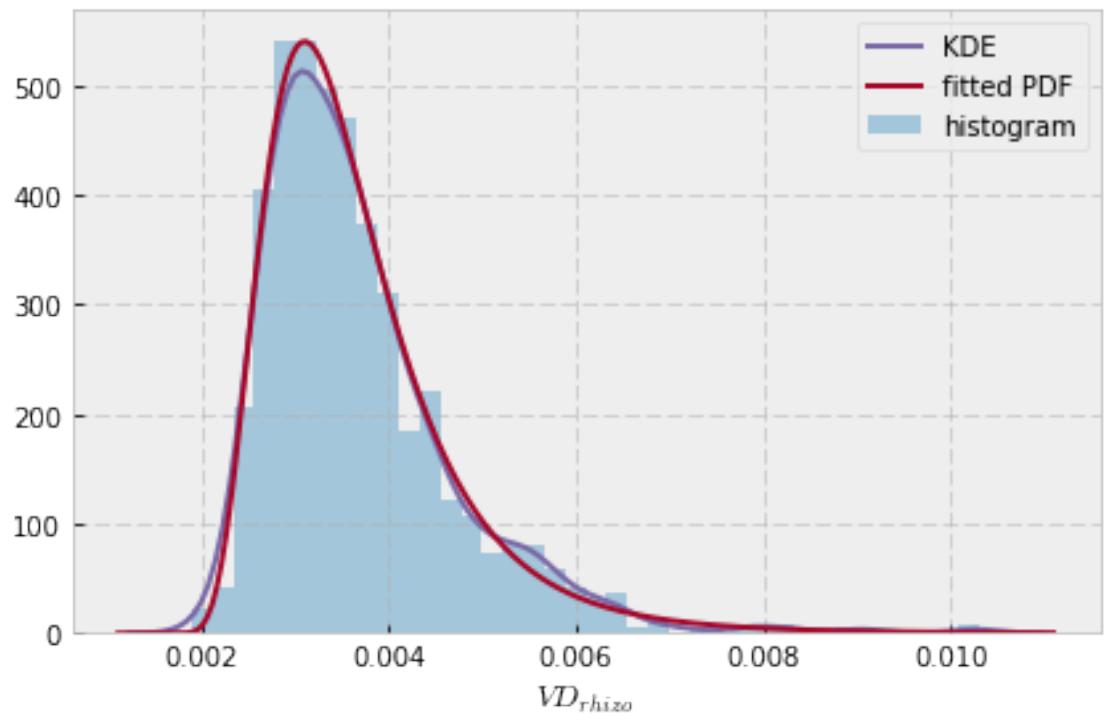
14.1 Probability distributions

```
In [33]: plot_dist('rhizosphere_volume_phosphate_per_convex_hull_volume', 'inverse-gamma')
```

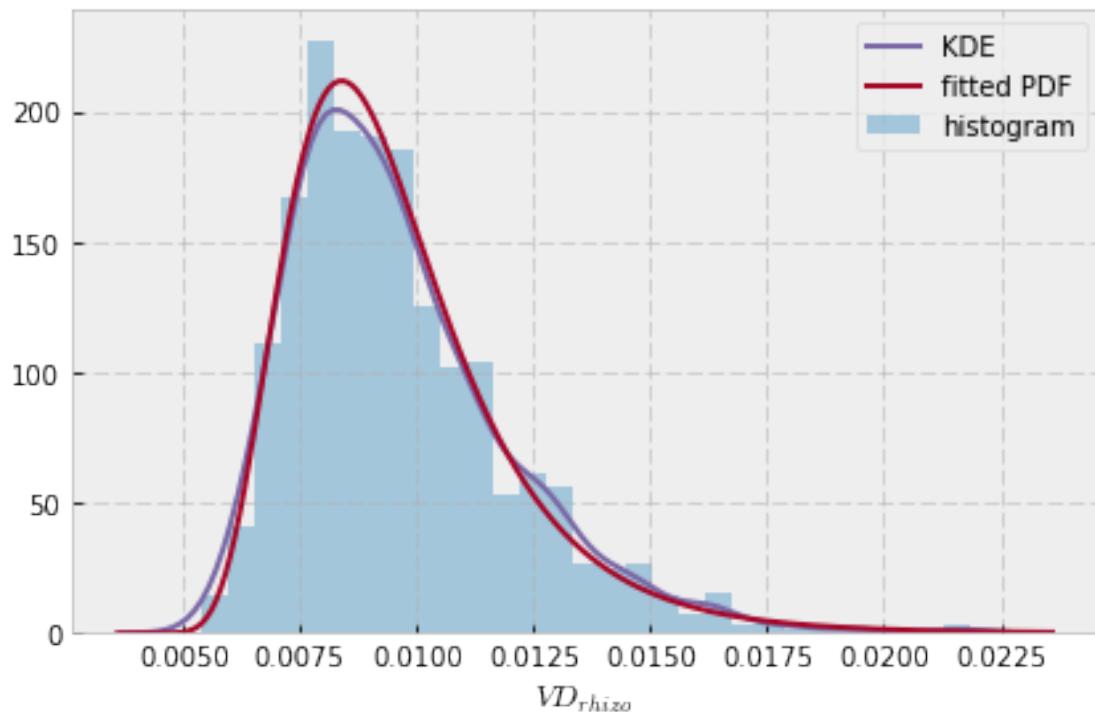
```

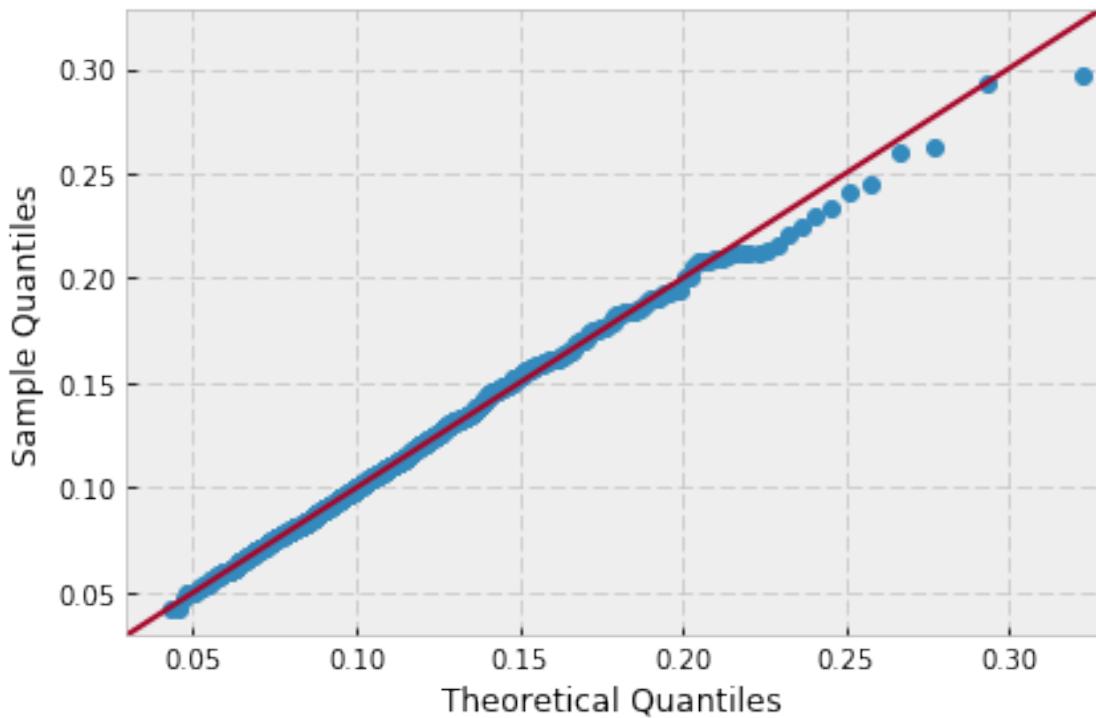
variable name: N
variable value: 0.5
distribution: inverse-gamma(shape=(7.6103011818812867,), loc=0.0012082430970182948, scale=0.01632
DescribeResult(nobs=1000, minmax=(0.0018966584652980495, 0.0103105436122346)
               mean=0.0036783740110576195, variance=1.0784352753835315e-06,
               skewness=1.8026677499285493, kurtosis=5.556231198388204)

```

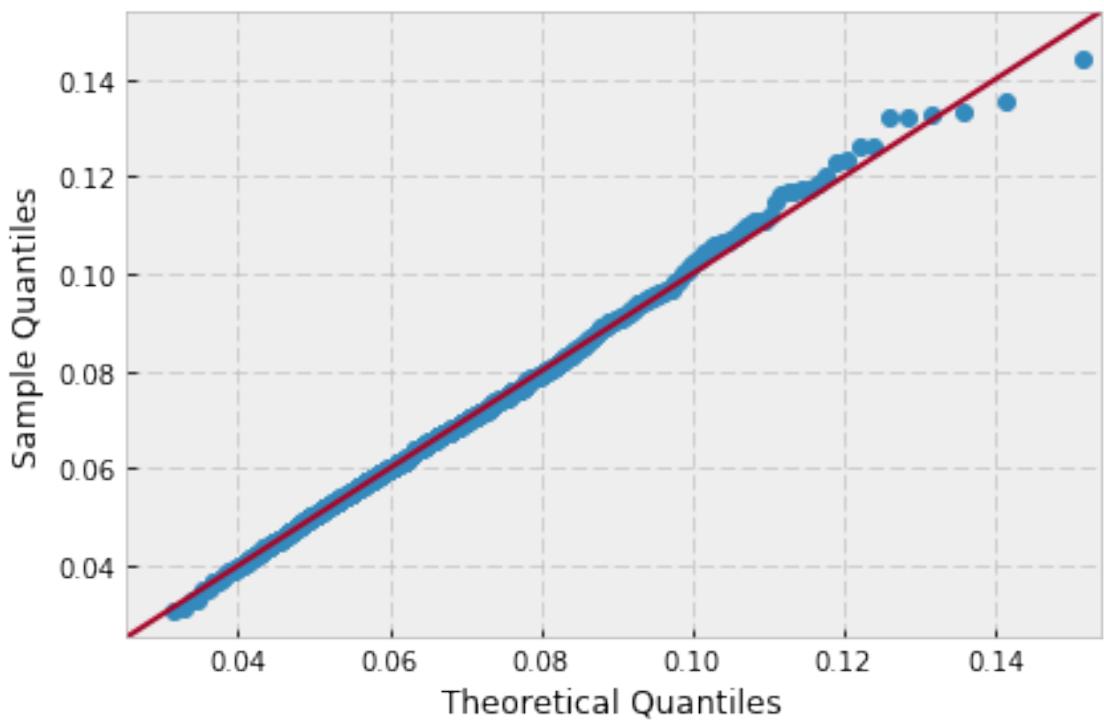
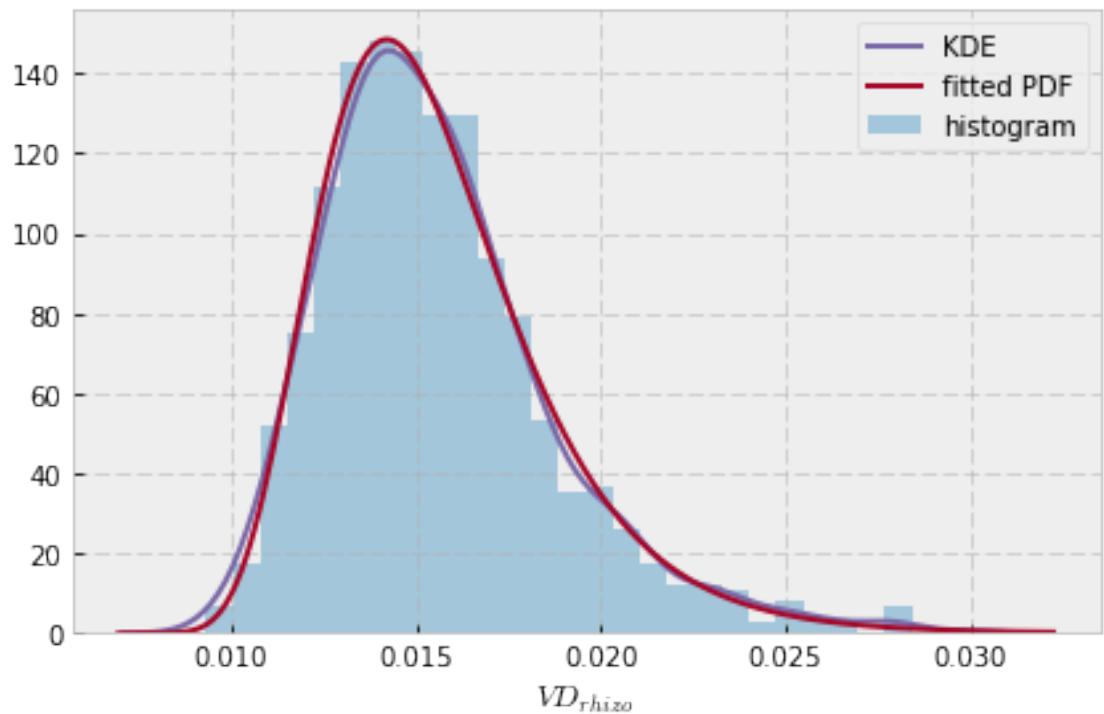


```
variable name: N
variable value: 1.0
distribution: inverse-gamma(shape=(10.270225995655117,),loc=0.0026621550771169754, scale=0.06451
DescribeResult(nobs=1000, minmax=(0.0053714243185457033, 0.021822017865906818)
    mean=0.0096168155816905507, variance=5.5985078175138158e-06,
    skewness=1.2172841858171801, kurtosis=2.1417758606490285)
```

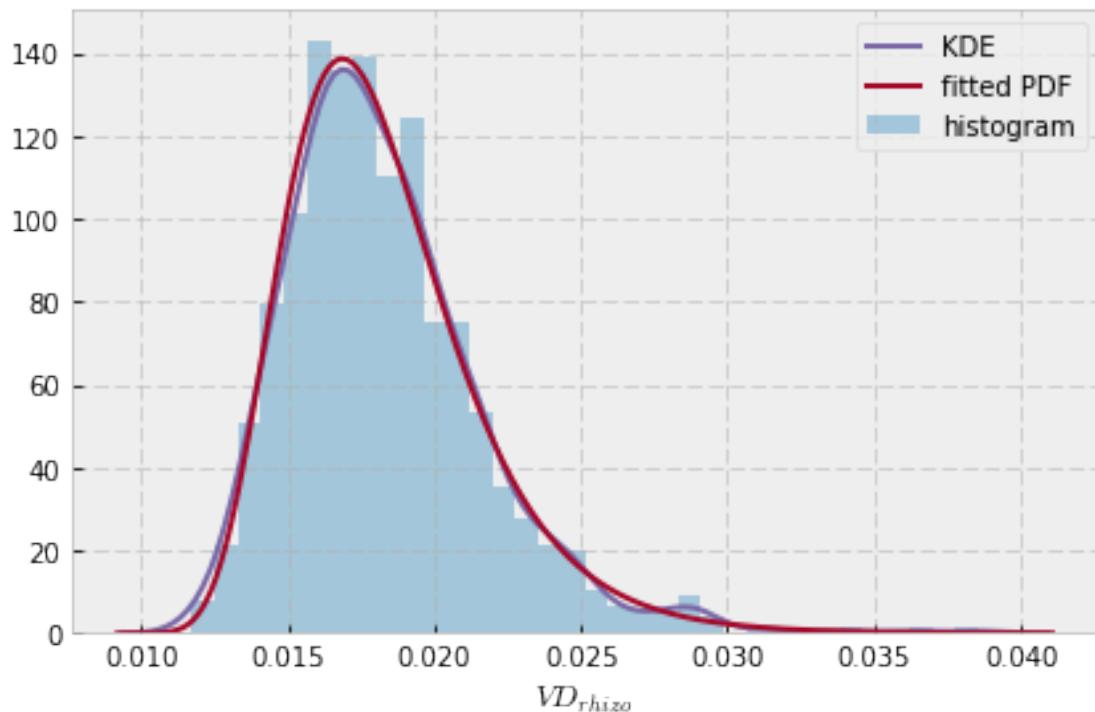


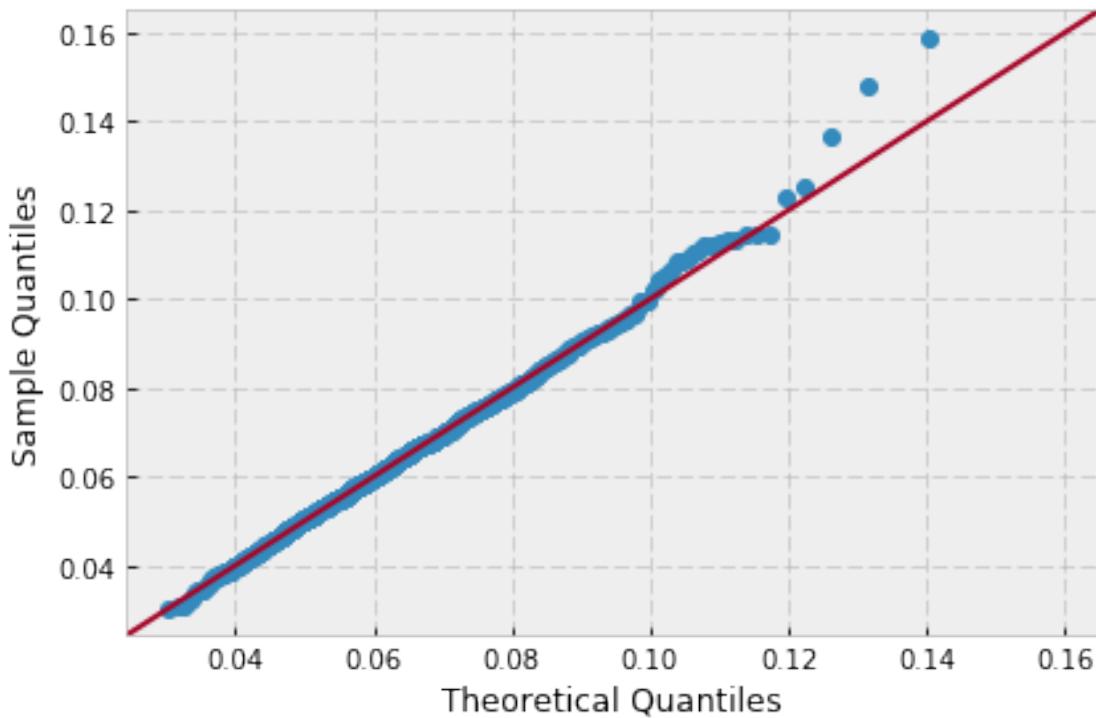


```
variable name: N
variable value: 1.5
distribution: inverse-gamma(shape=(16.318577455019689,),loc=0.0037277883495442153, scale=0.18153
DescribeResult(nobs=1000, minmax=(0.0093163927030555076, 0.029874584908317757)
               mean=0.015579108923111901, variance=9.8251000448028525e-06,
               skewness=1.085182041696817, kurtosis=1.7615652471551693)
```

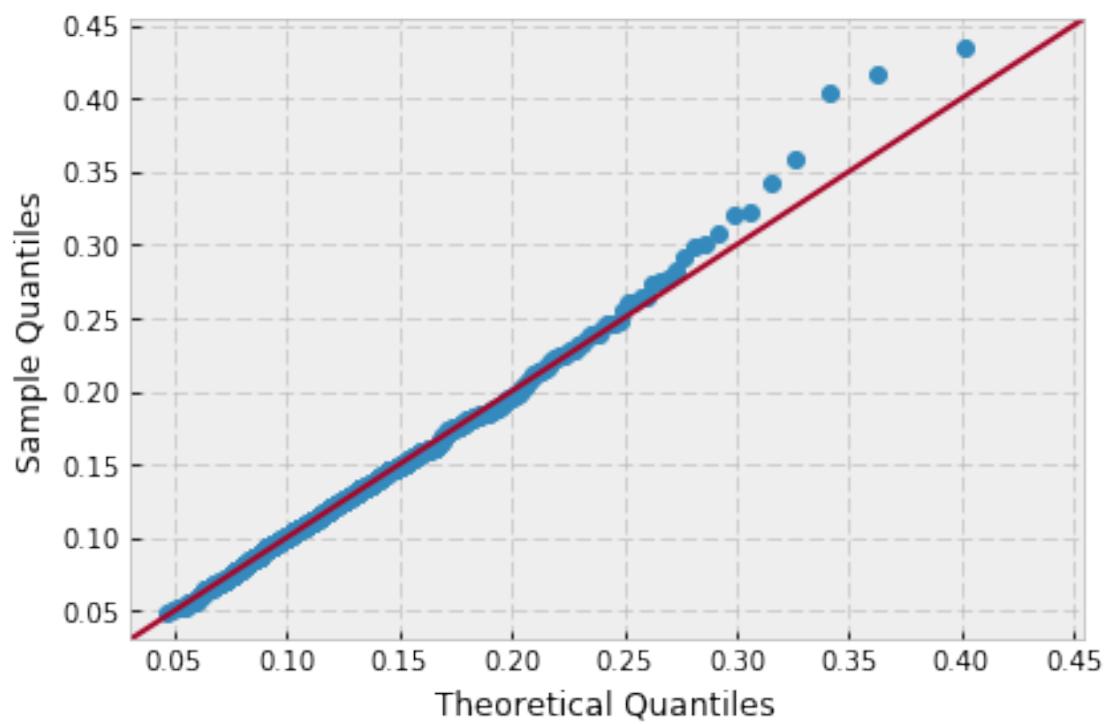
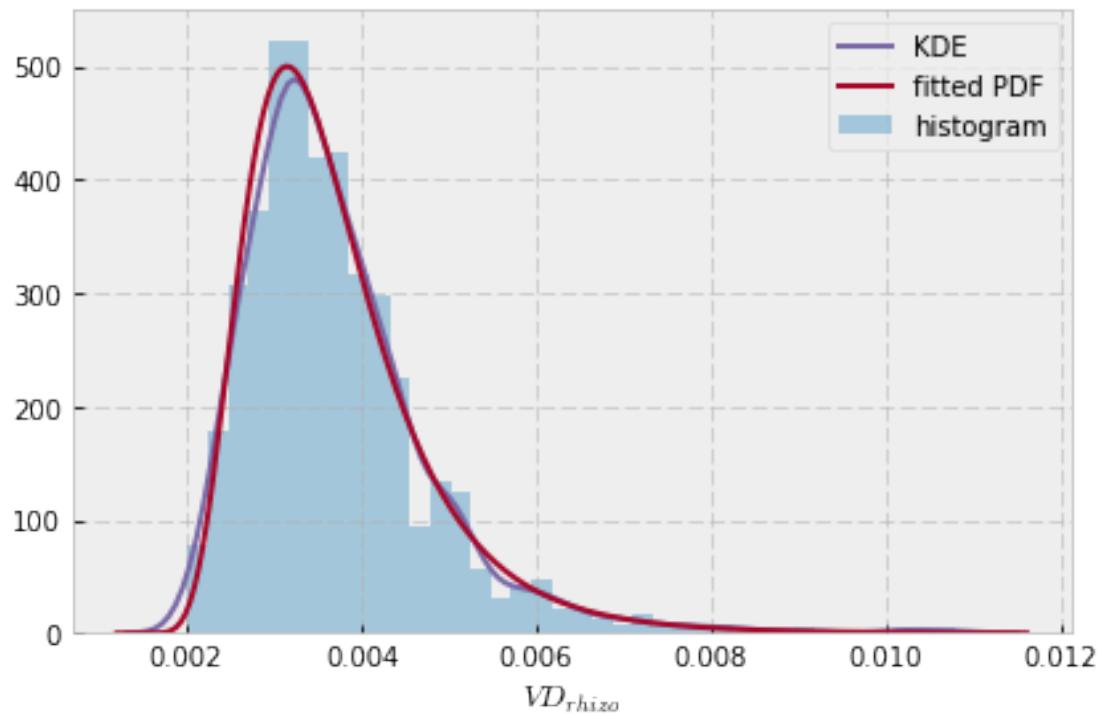


```
variable name: N
variable value: 2.0
distribution: inverse-gamma(shape=(17.15482858086034,), loc=0.005334030259684731, scale=0.2093201
DescribeResult(nobs=1000, minmax=(0.011722768791531862, 0.038545913074651848)
    mean=0.018292866452669032, variance=1.1254879057956187e-05,
    skewness=1.2138034968686122, kurtosis=3.0341408287344853)
```

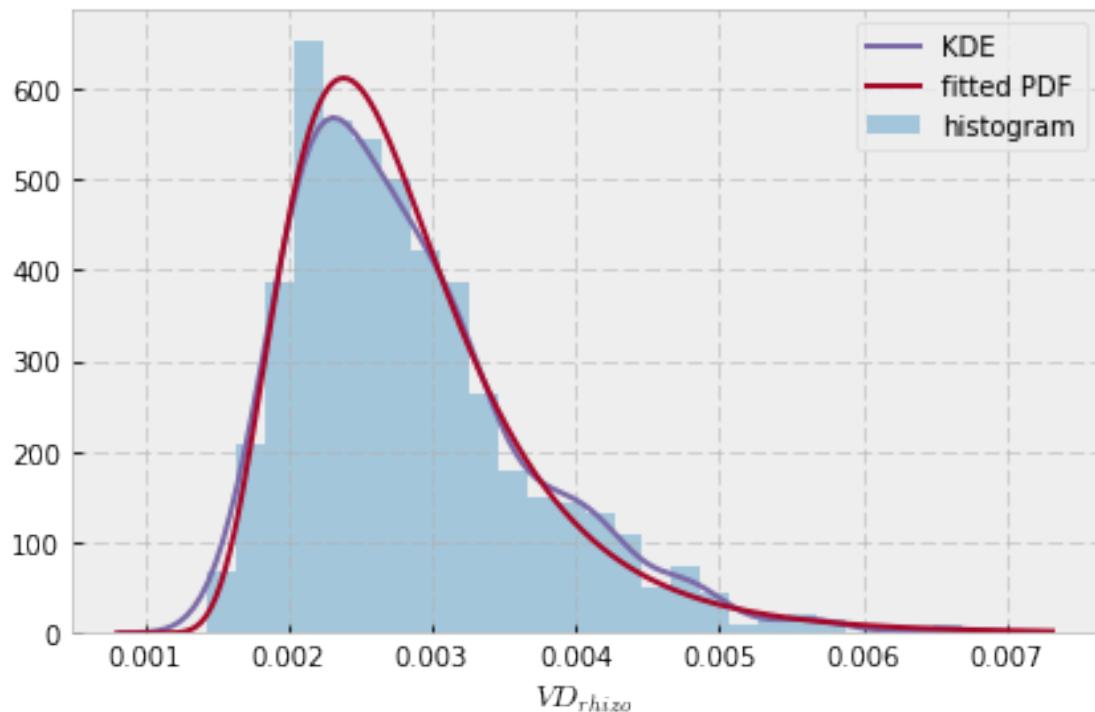


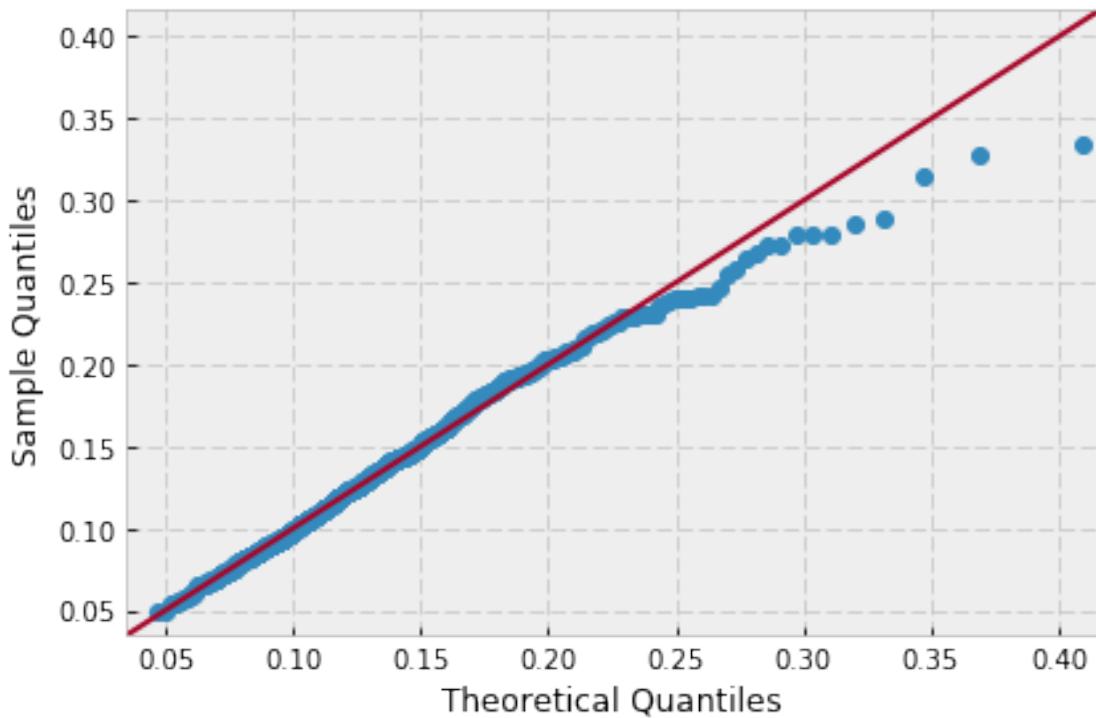


```
variable name: ln
variable value: 0.5
distribution: inverse-gamma(shape=(9.0751695159328918,), loc=0.0008857179942373535, scale=0.02279
DescribeResult(nobs=1000, minmax=(0.0020126917284518432, 0.01079668274823764)
               mean=0.0037096119855023523, variance=1.1559934797352866e-06,
               skewness=1.853290377527669, kurtosis=6.3242669375303056)
```

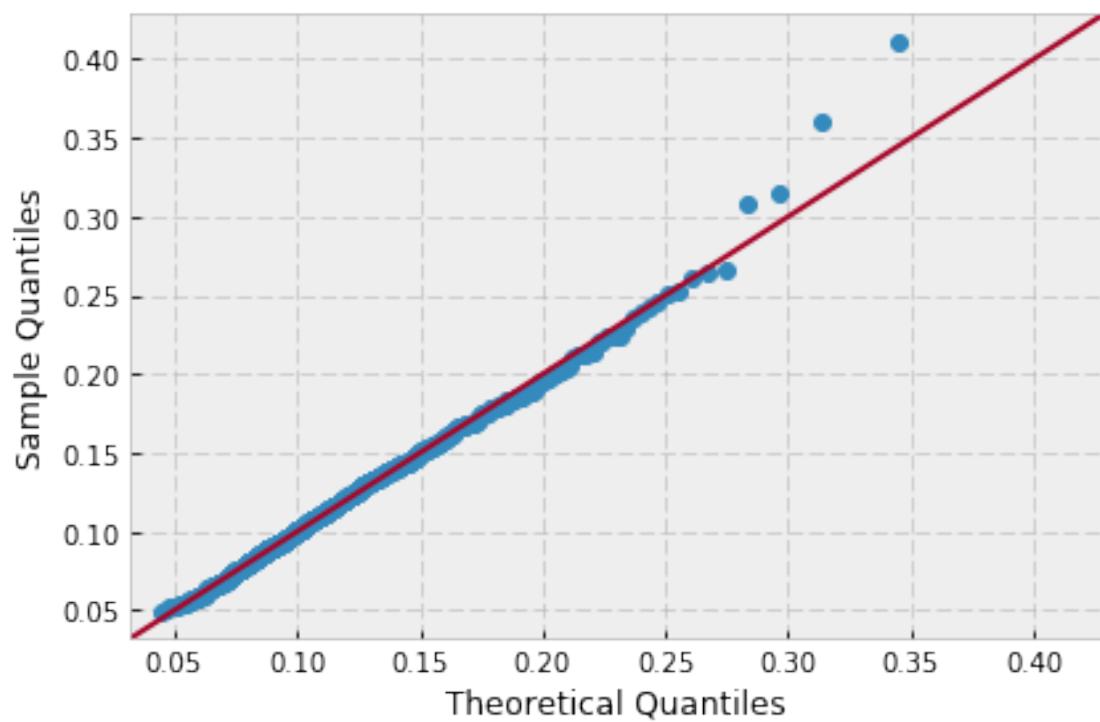
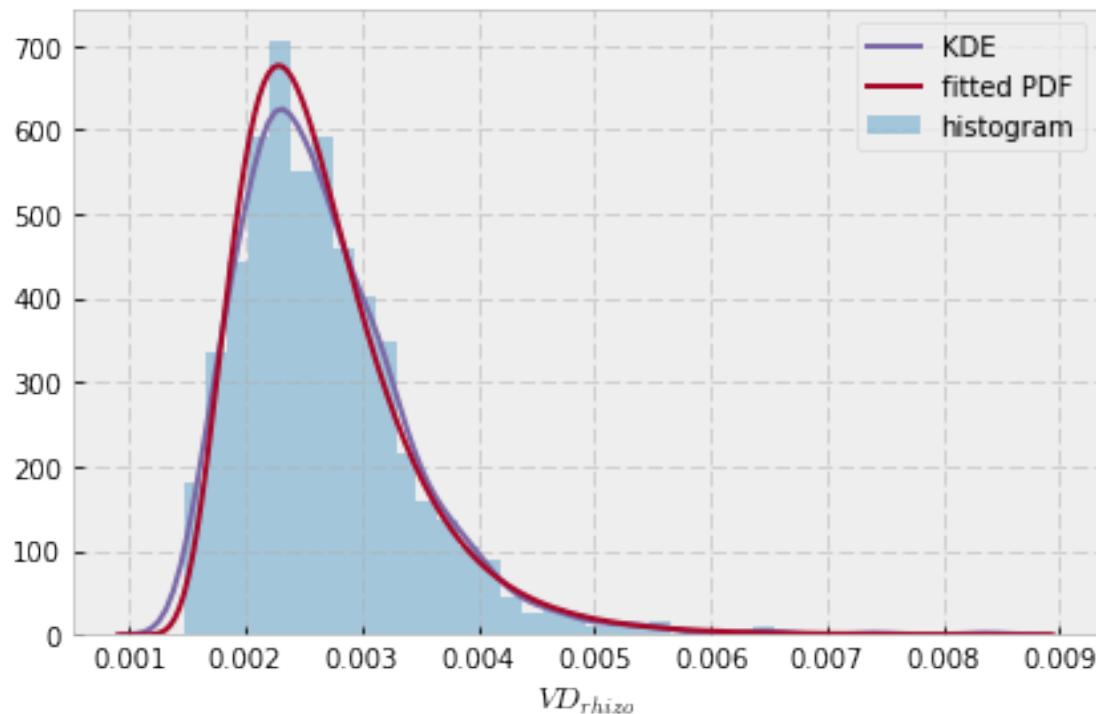


```
variable name: ln
variable value: 0.6666666666666666
distribution: inverse-gamma(shape=(8.9861145708073717,),loc=0.0005430487726158218, scale=0.01839
DescribeResult(nobs=1000, minmax=(0.0014362031464428516, 0.0066910493164055781)
               mean=0.0028424897778414663, variance=6.9334478038727693e-07,
               skewness=1.126070908175798, kurtosis=1.4697589656178636)
```

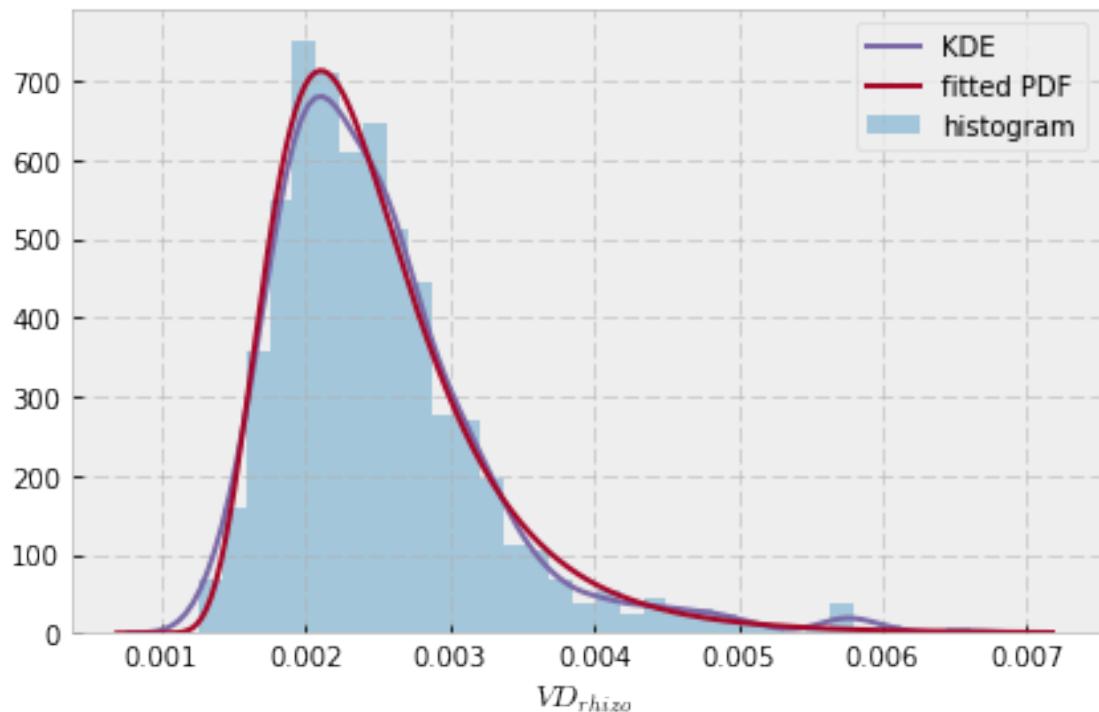


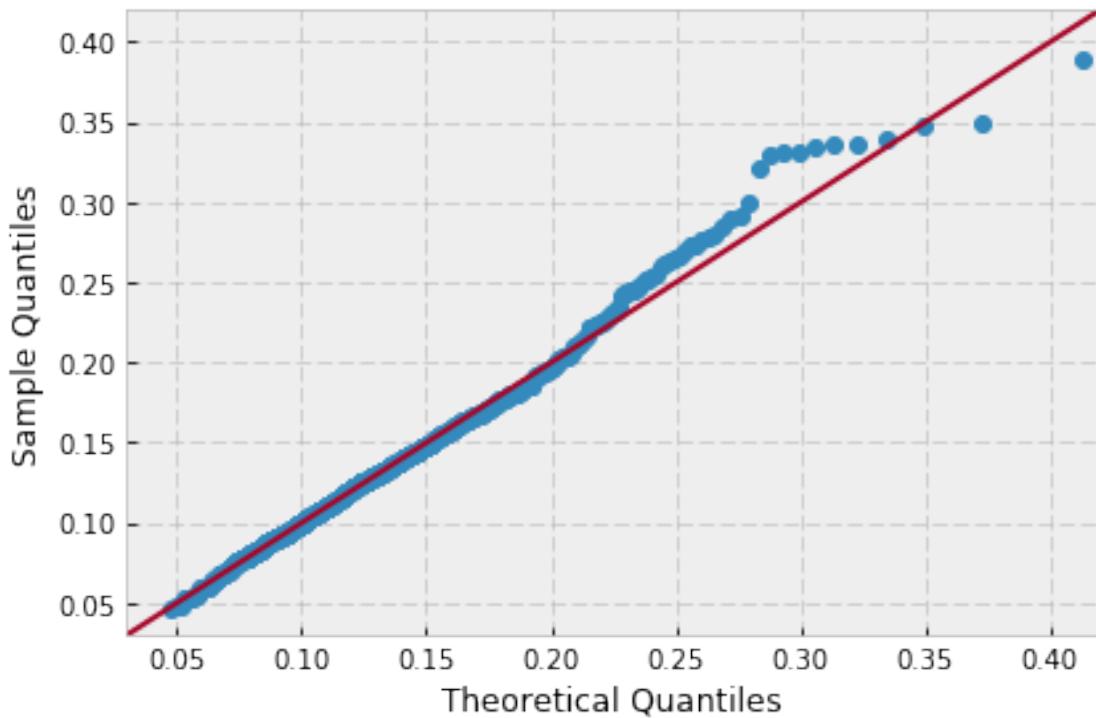


```
variable name: ln
variable value: 0.8333333333333333
distribution: inverse-gamma(shape=(9.8785674371762831,), loc=0.0005315980486835165, scale=0.01907
DescribeResult(nobs=1000, minmax=(0.0014805648371220544, 0.0083627264862101442)
               mean=0.0026792847956604113, variance=5.7985593310177553e-07,
               skewness=1.654312430145433, kurtosis=5.949861336987508)
```

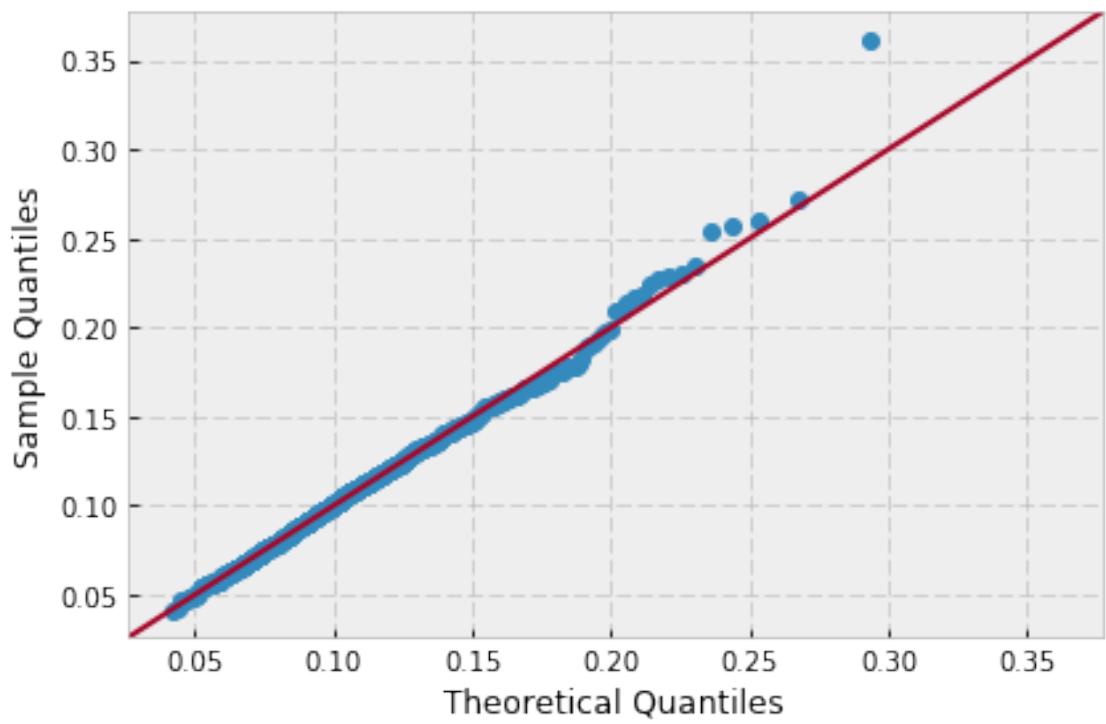
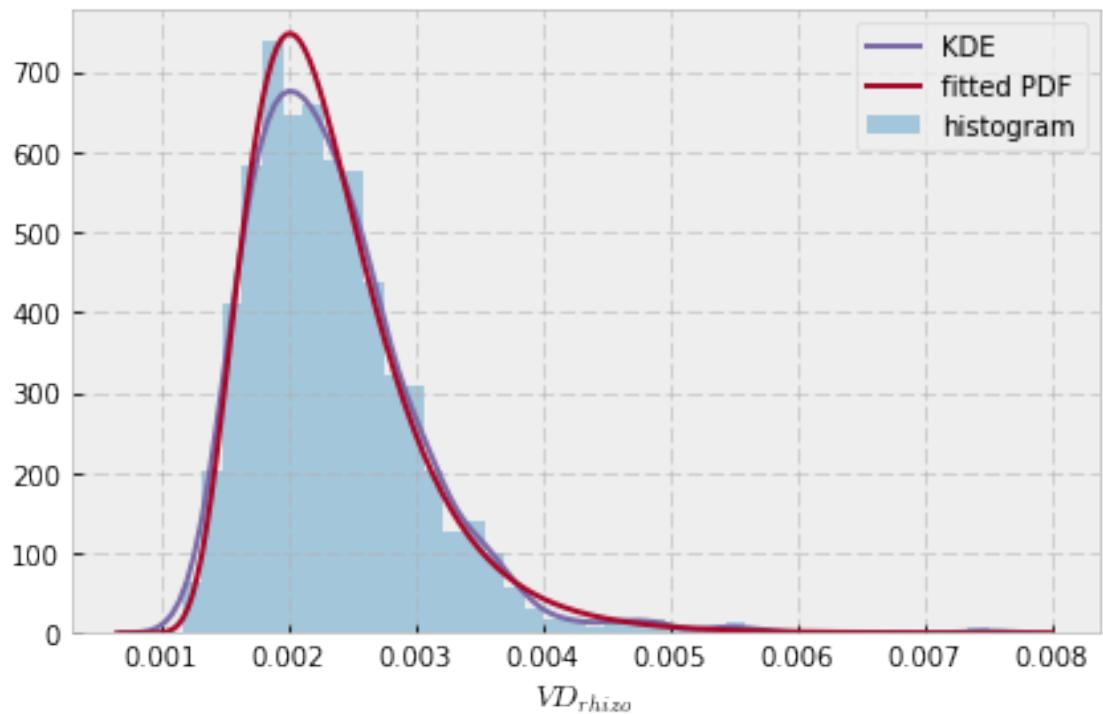


```
variable name: ln
variable value: 1.0
distribution: inverse-gamma(shape=(8.9426464479608132,),loc=0.0005308519679855053, scale=0.01563
DescribeResult(nobs=1000, minmax=(0.0012586663415426482, 0.0066069971037196109)
    mean=0.0025002986654465934, variance=5.6807578510000627e-07,
    skewness=1.6823158689081688, kurtosis=4.285803913782177)
```

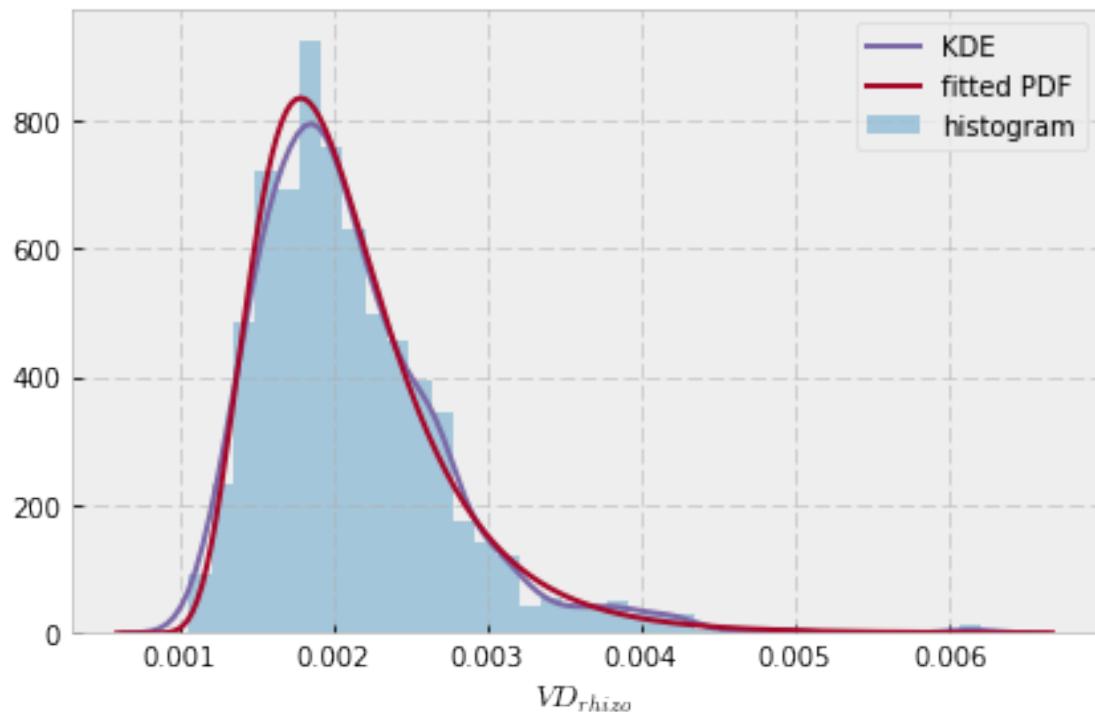


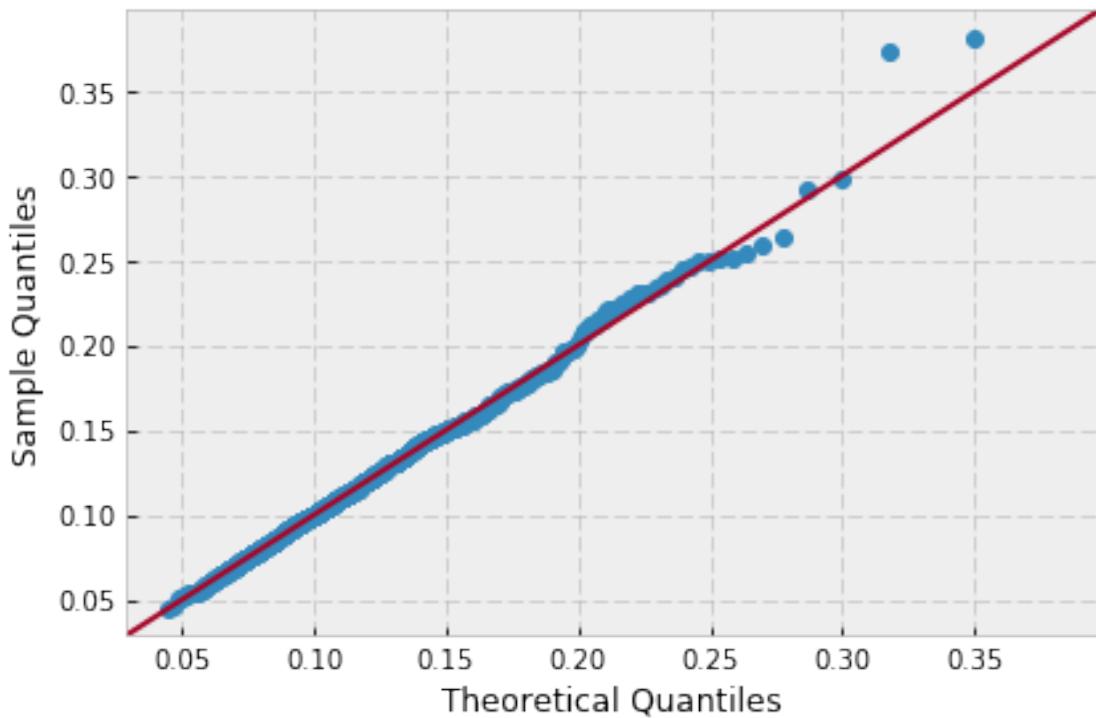


```
variable name: ln
variable value: 1.1666666666666665
distribution: inverse-gamma(shape=(10.852493036340688,), loc=0.00034555033175146226, scale=0.0198
DescribeResult(nobs=1000, minmax=(0.0011698390318036211, 0.0074981874522280188)
               mean=0.0023557356157343271, variance=4.5712370086504747e-07,
               skewness=1.581737872159245, kurtosis=5.430077058659025)
```

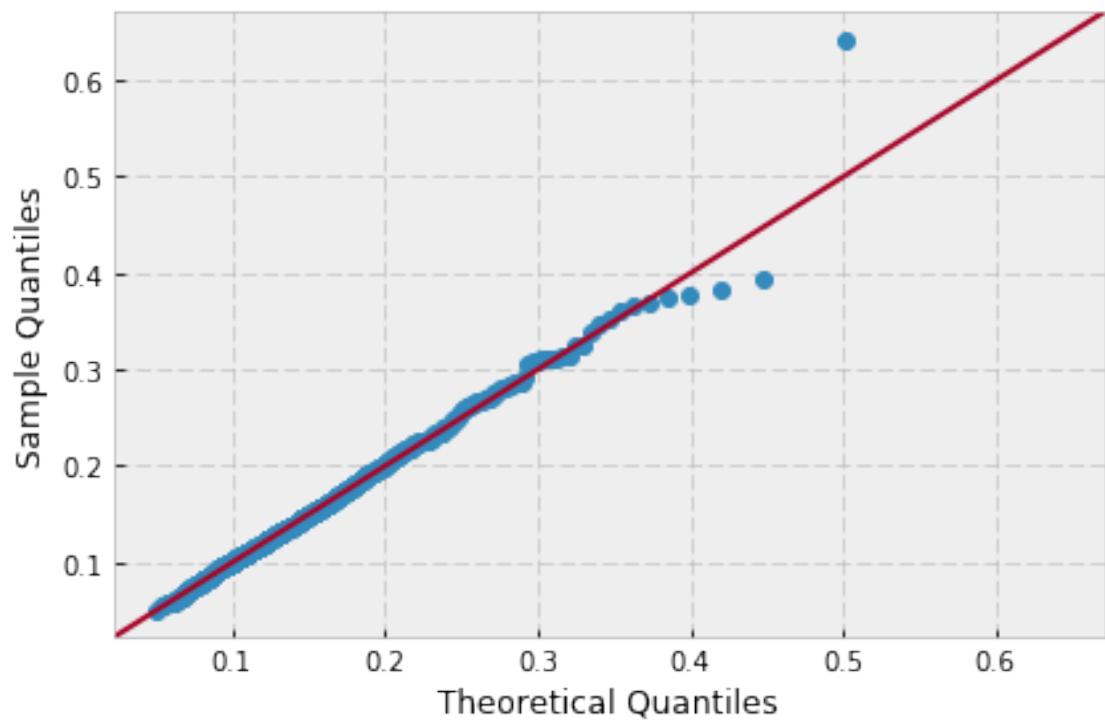
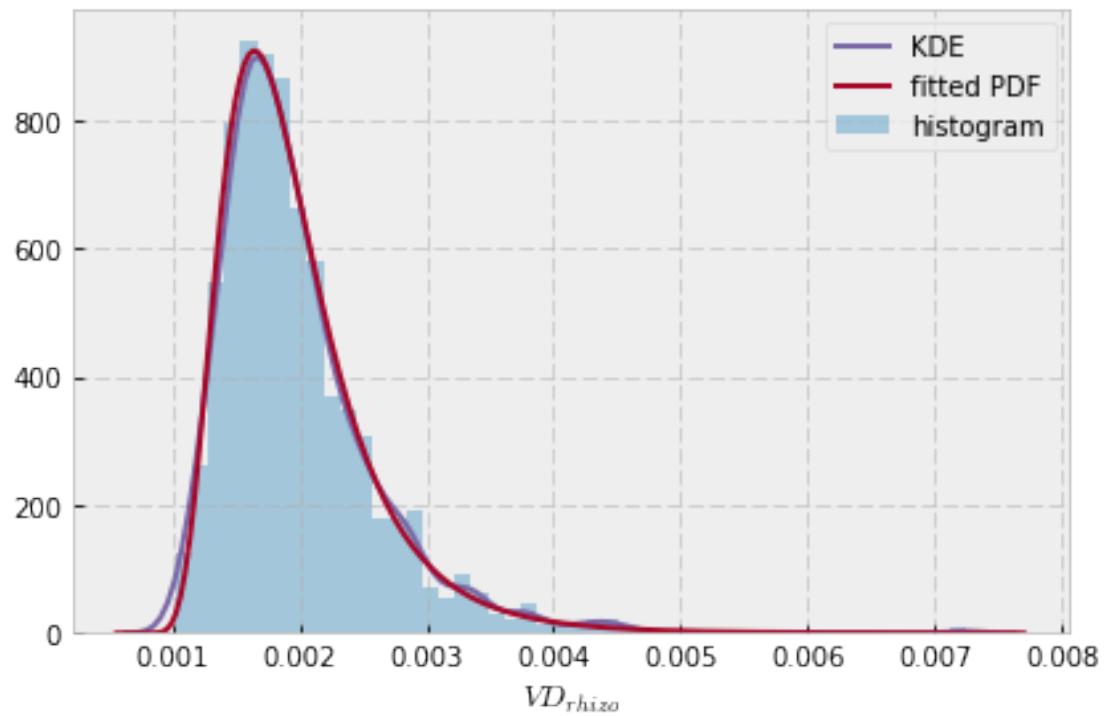


```
variable name: ln
variable value: 1.333333333333333
distribution: inverse-gamma(shape=(9.8099047144011315,),loc=0.000372255038013972, scale=0.015305
DescribeResult(nobs=1000, minmax=(0.0010647555451878778, 0.0062013628602897554)
    mean=0.0021094082191312302, variance=3.8497237120462655e-07,
    skewness=1.594291452957645, kurtosis=4.839290916315233)
```

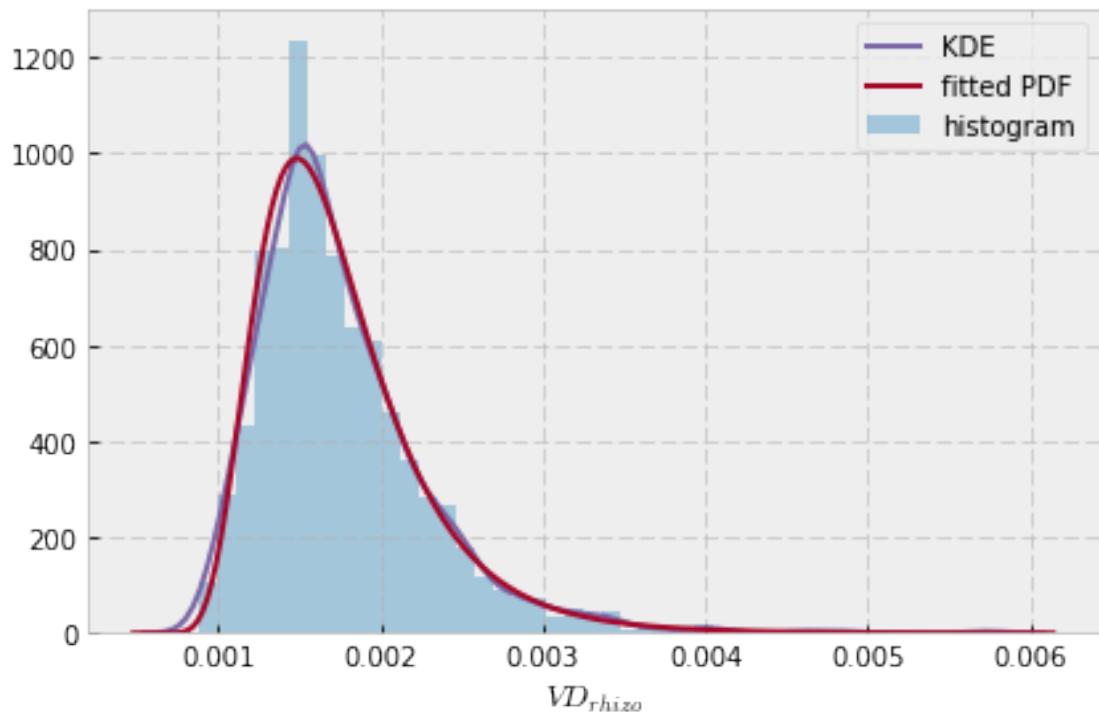


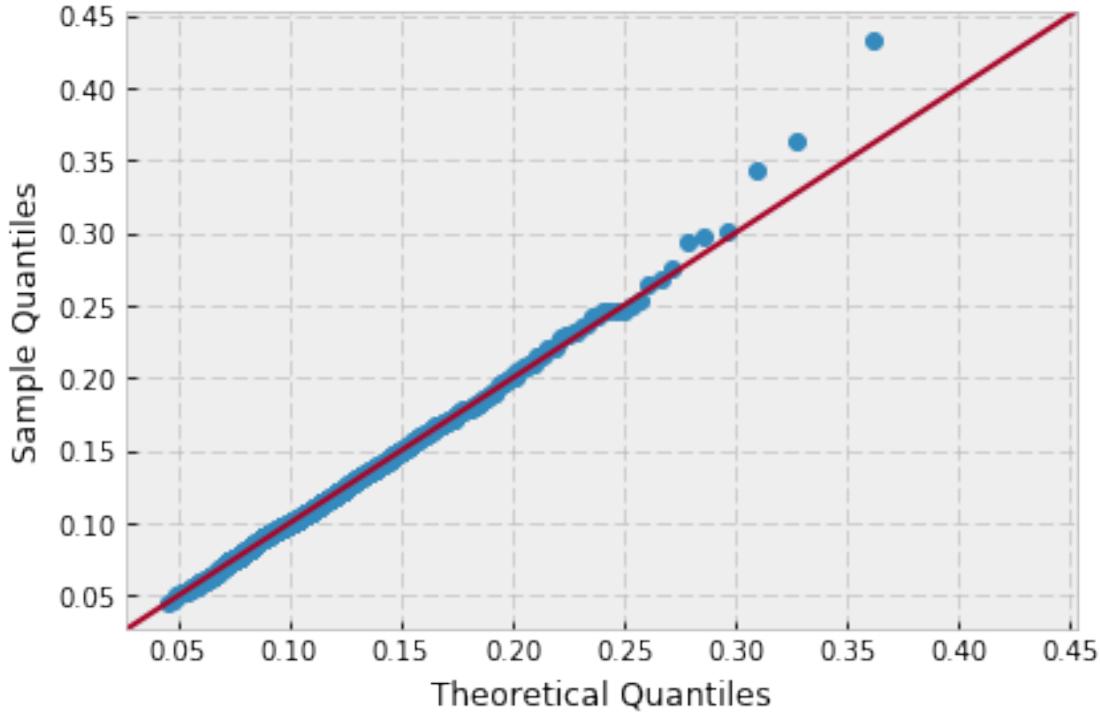


```
variable name: ln
variable value: 1.5
distribution: inverse-gamma(shape=(8.0534683426181388,), loc=0.00048073088196149143, scale=0.0105
DescribeResult(nobs=1000, minmax=(0.0010190874979945472, 0.0072416796741388567)
               mean=0.0019754130070529347, variance=3.6803834023646658e-07,
               skewness=1.8807870720668638, kurtosis=7.422311666638743)
```

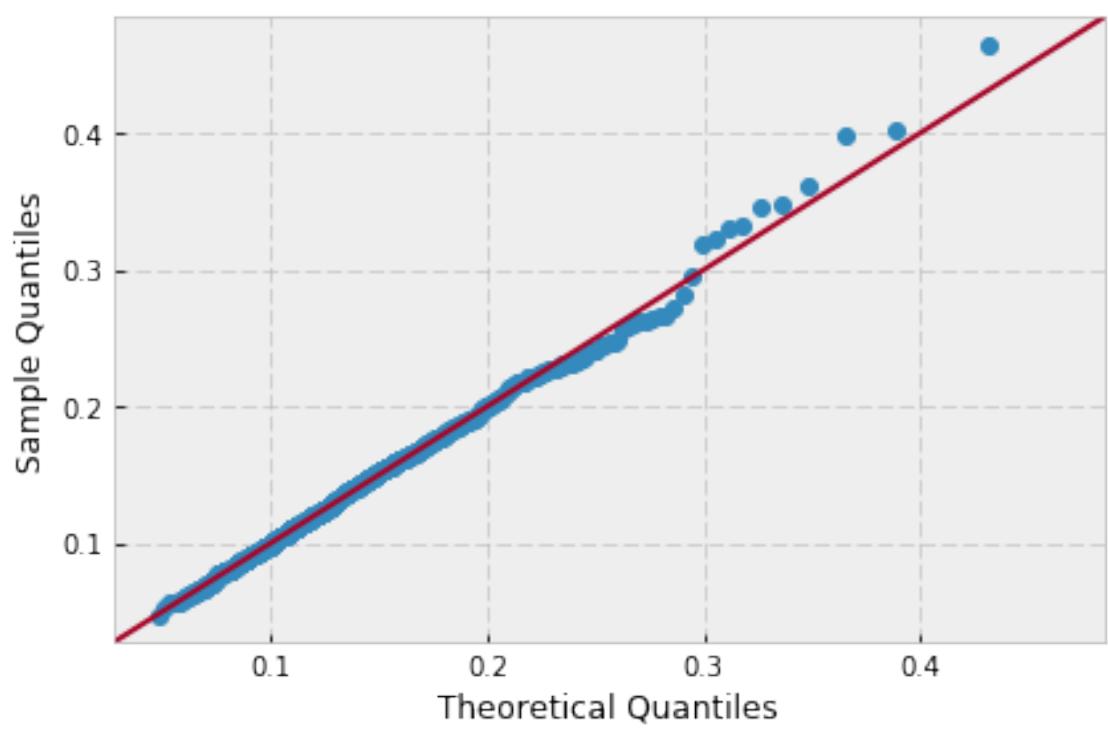
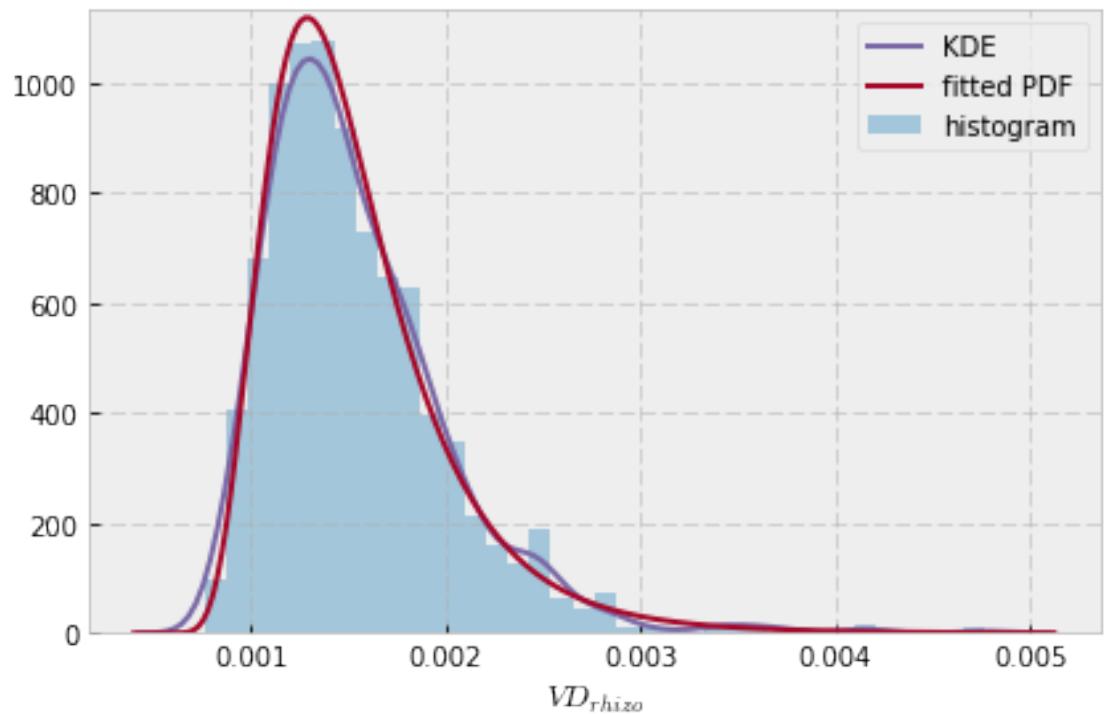


```
variable name: ln
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(9.6264963984703478,),loc=0.00030101017539544713, scale=0.0125
DescribeResult(nobs=1000, minmax=(0.00087504017859820675, 0.0057418750757117325)
               mean=0.0017561493741946107, variance=2.828197612983967e-07,
               skewness=1.7587021299110508, kurtosis=6.104545056051217)
```

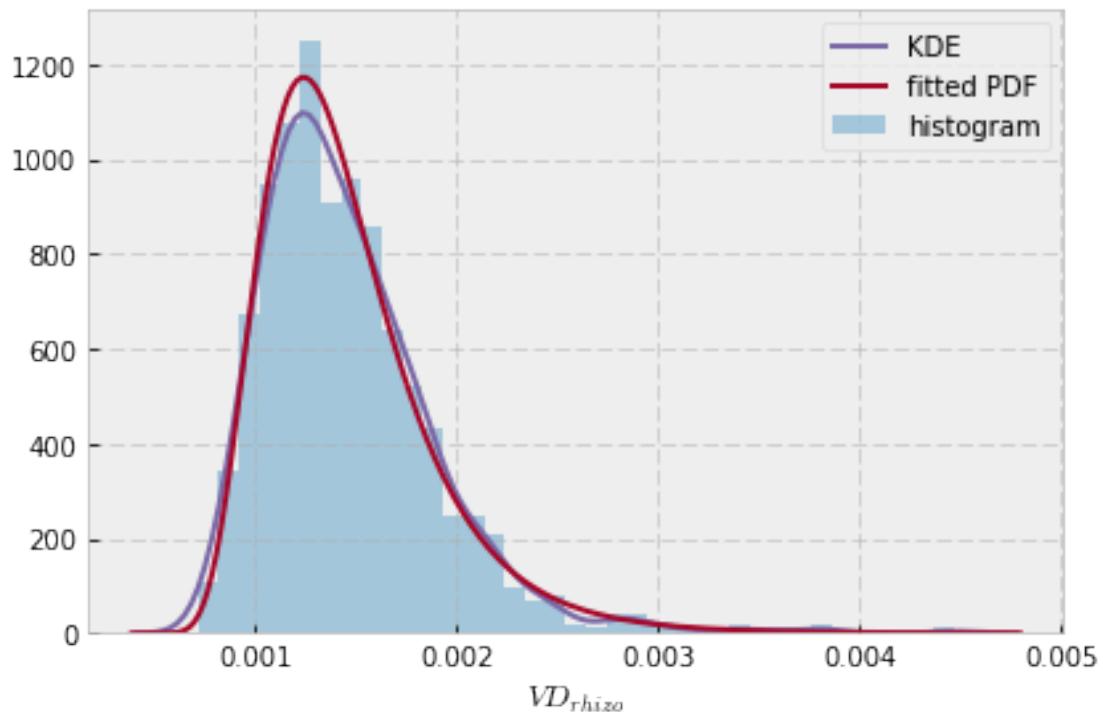


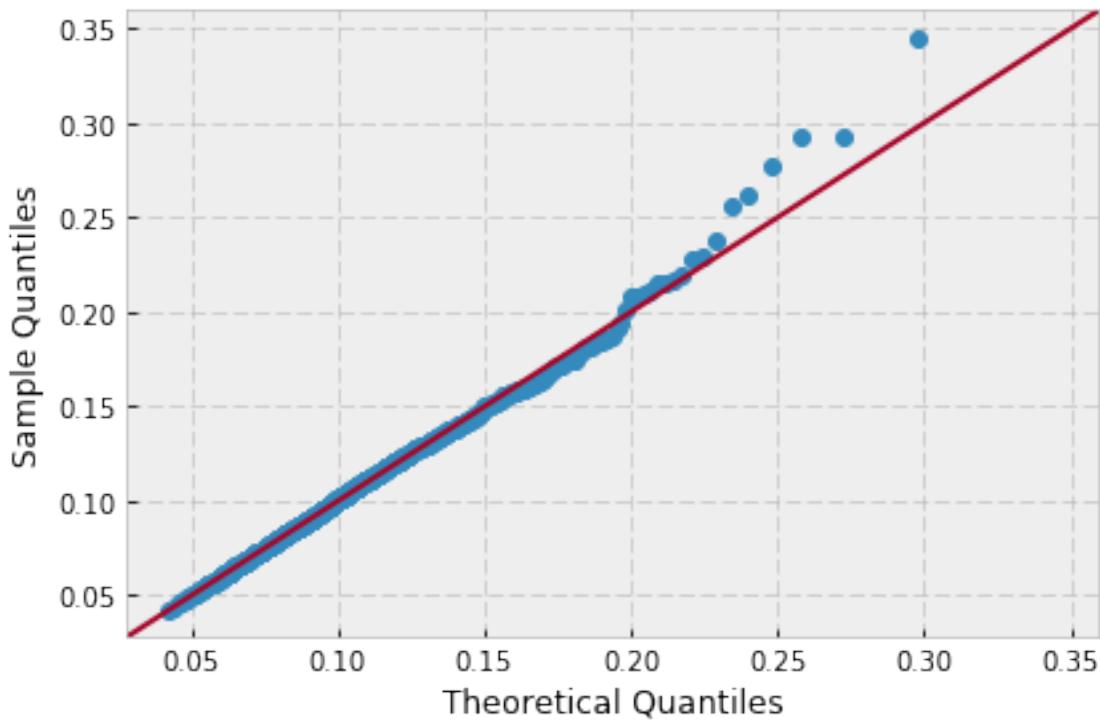


```
variable name: ln
variable value: 1.833333333333333
distribution: inverse-gamma(shape=(8.7239488404116976,), loc=0.0002989144894777813, scale=0.00961
DescribeResult(nobs=1000, minmax=(0.00075725769473970902, 0.0047634767881890652)
               mean=0.0015437568088381313, variance=2.2598112764729673e-07,
               skewness=1.6646591335134693, kurtosis=5.205766458029634)
```

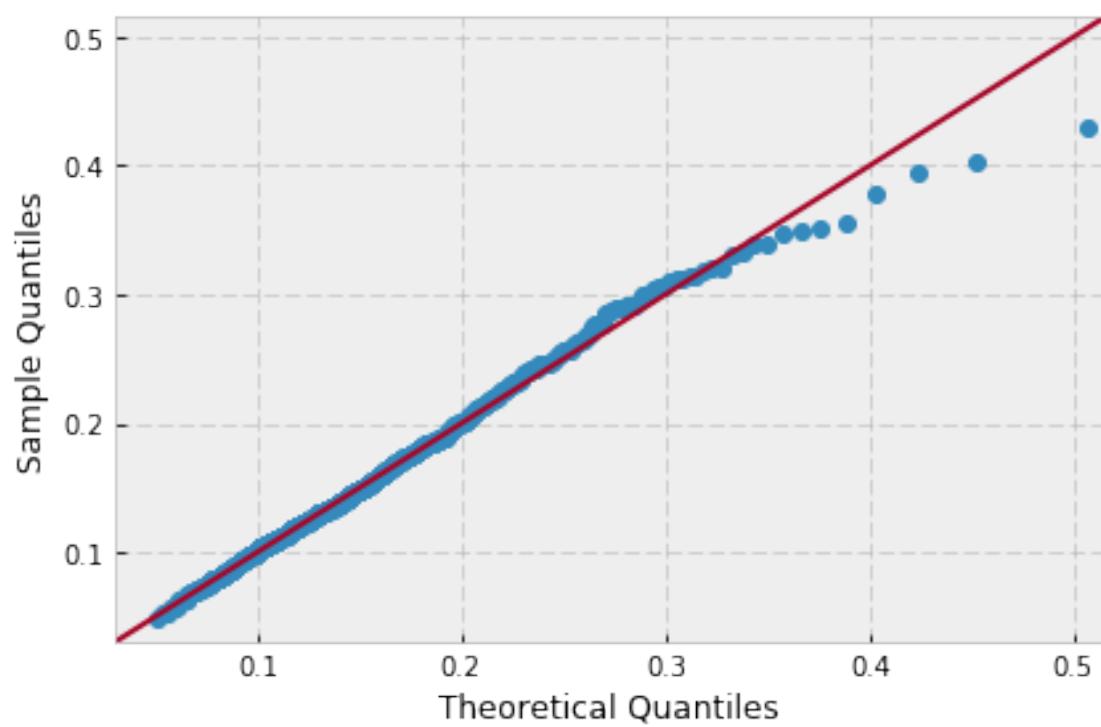
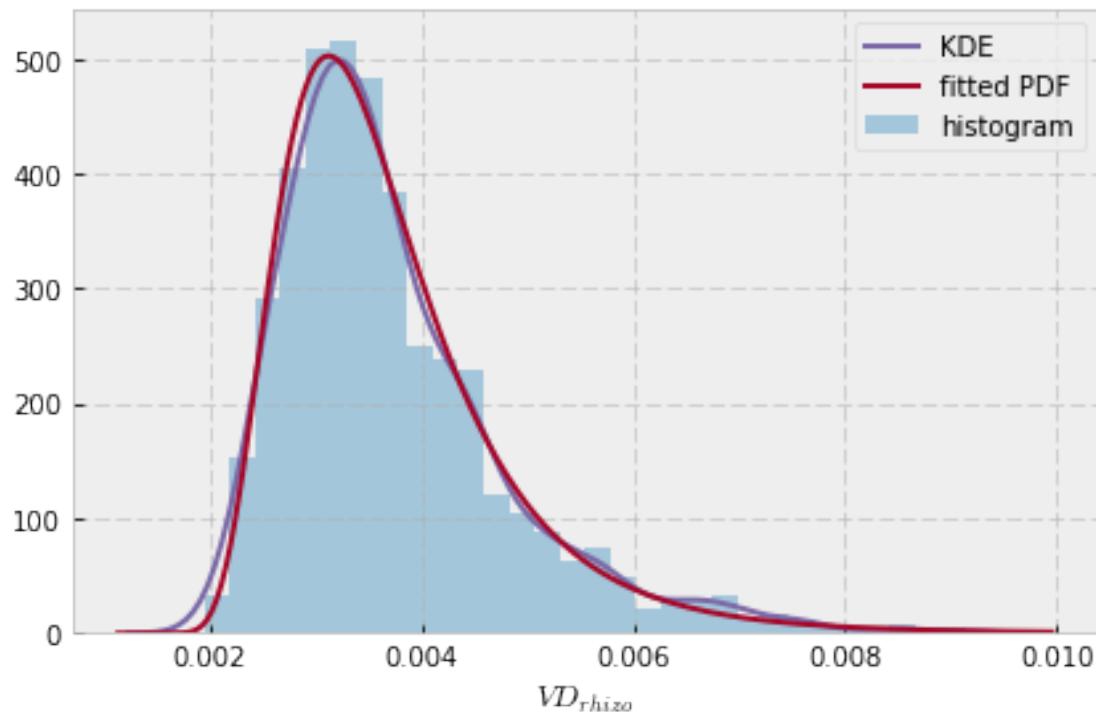


```
variable name: ln
variable value: 2.0
distribution: inverse-gamma(shape=(10.744565005180196,),loc=0.0001804332709162789, scale=0.01244
DescribeResult(nobs=1000, minmax=(0.00071342416007436147, 0.0044682760687837948)
    mean=0.0014573270875595133, variance=1.8820950318122591e-07,
    skewness=1.621462398114542, kurtosis=5.369067406334759)
```

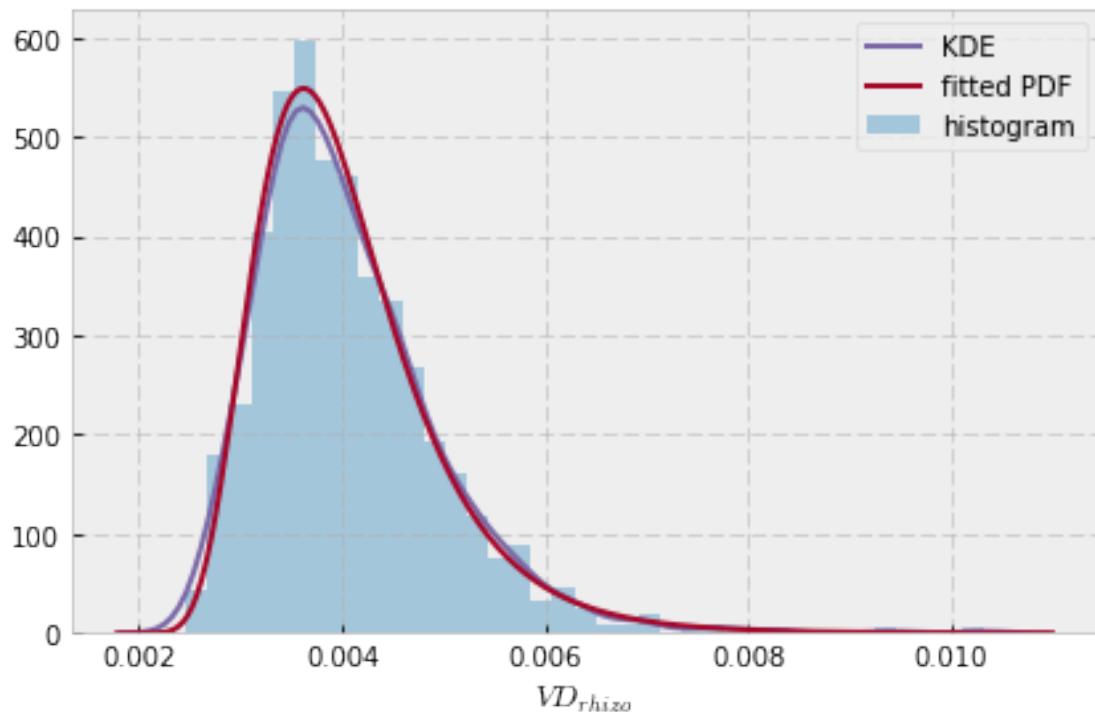


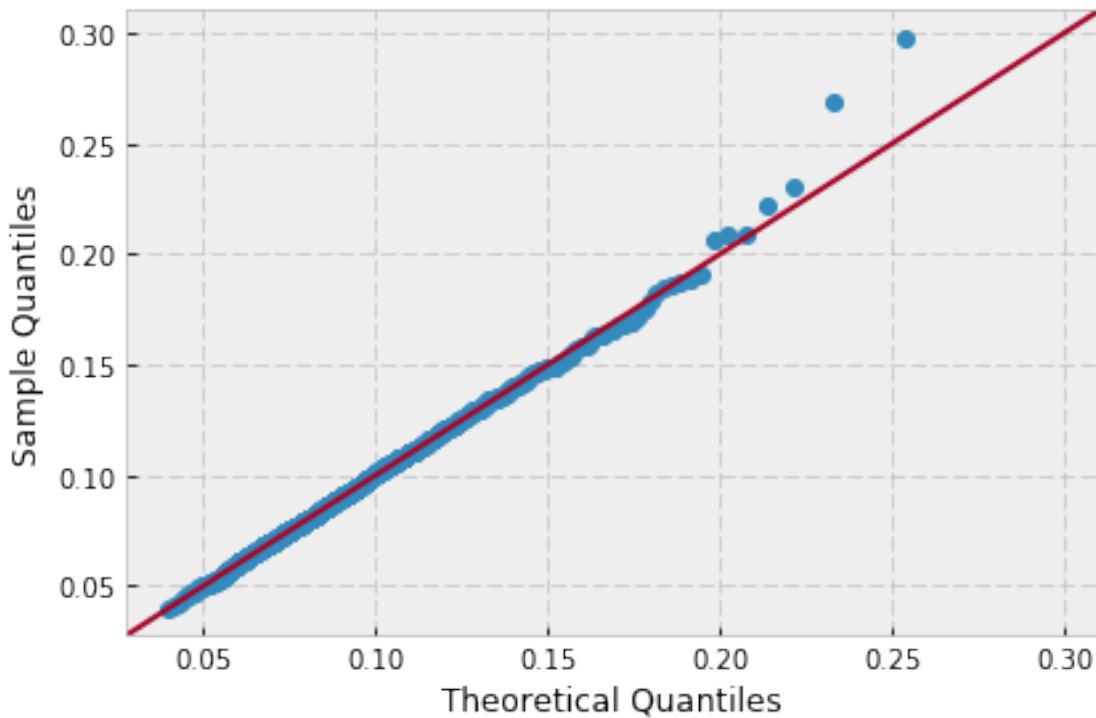


```
variable name: maxB
variable value: 4.0
distribution: inverse-gamma(shape=(8.0104726207798809,),loc=0.0010256204576204494, scale=0.01890
DescribeResult(nobs=1000, minmax=(0.0019442132076032911, 0.0091433130748723914)
               mean=0.0037206573322806614, variance=1.1528022519496199e-06,
               skewness=1.4381717653790576, kurtosis=2.671209164554921)
```

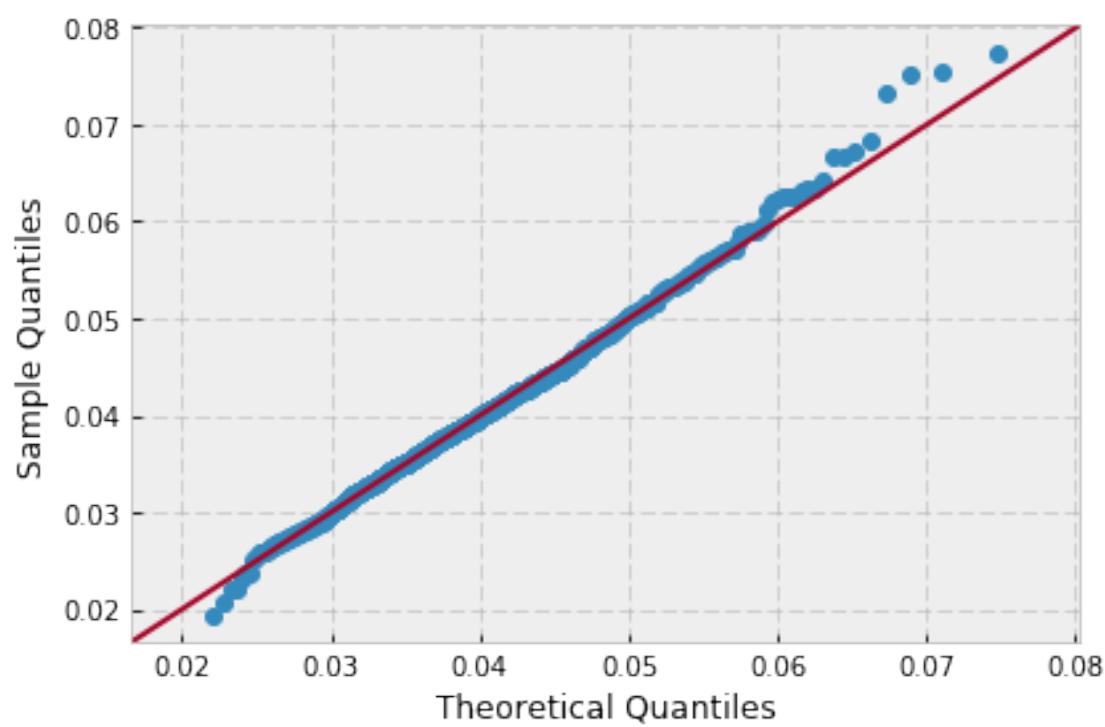
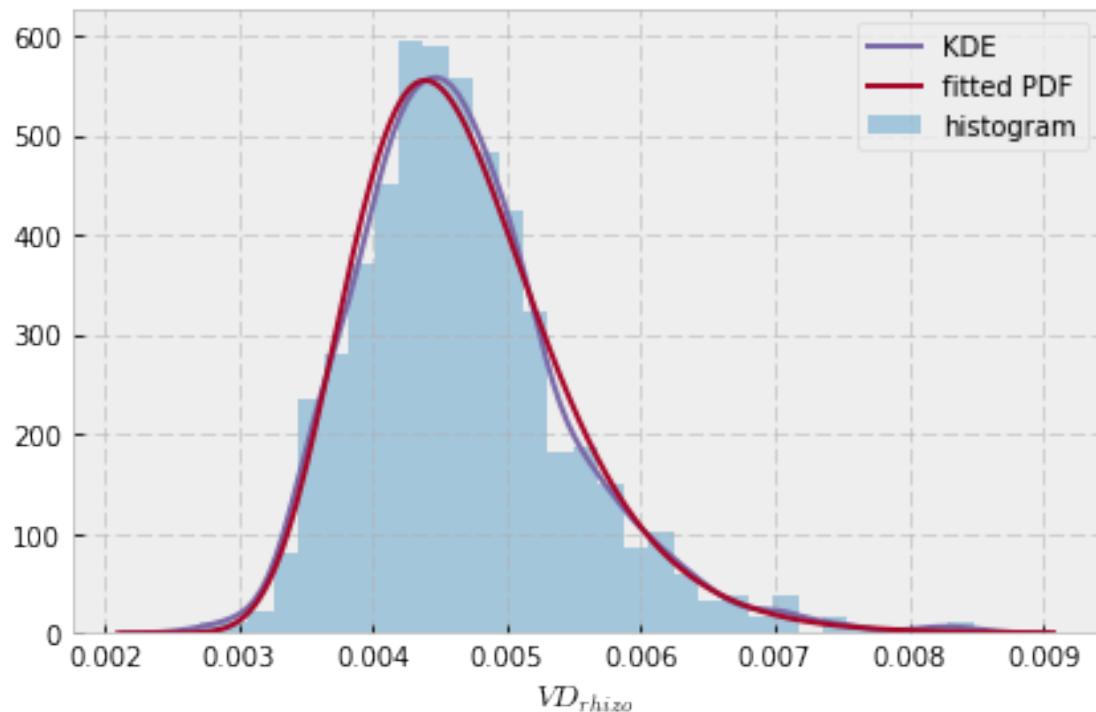


```
variable name: maxB
variable value: 8.0
distribution: inverse-gamma(shape=(11.813771181388681,),loc=0.0012381184789665768, scale=0.03052
DescribeResult(nobs=1000, minmax=(0.0024640916836820323, 0.010321124080785775)
               mean=0.0040610110789647044, variance=8.145070821511065e-07,
               skewness=1.468890581489218, kurtosis=4.547192793281773)
```

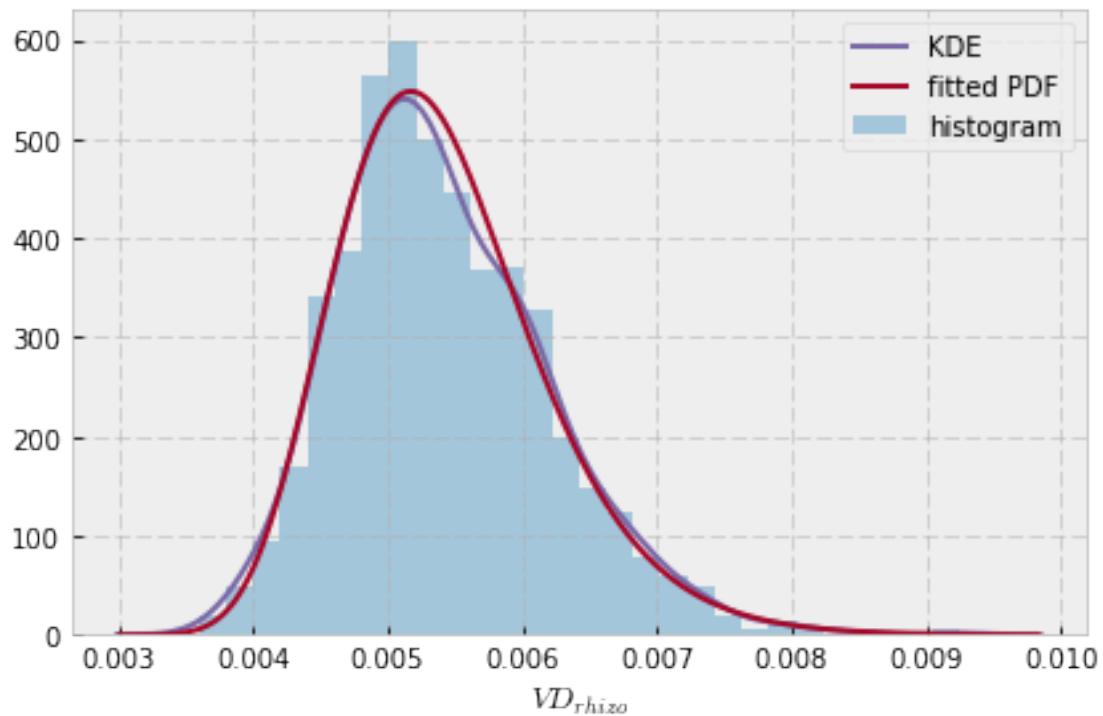


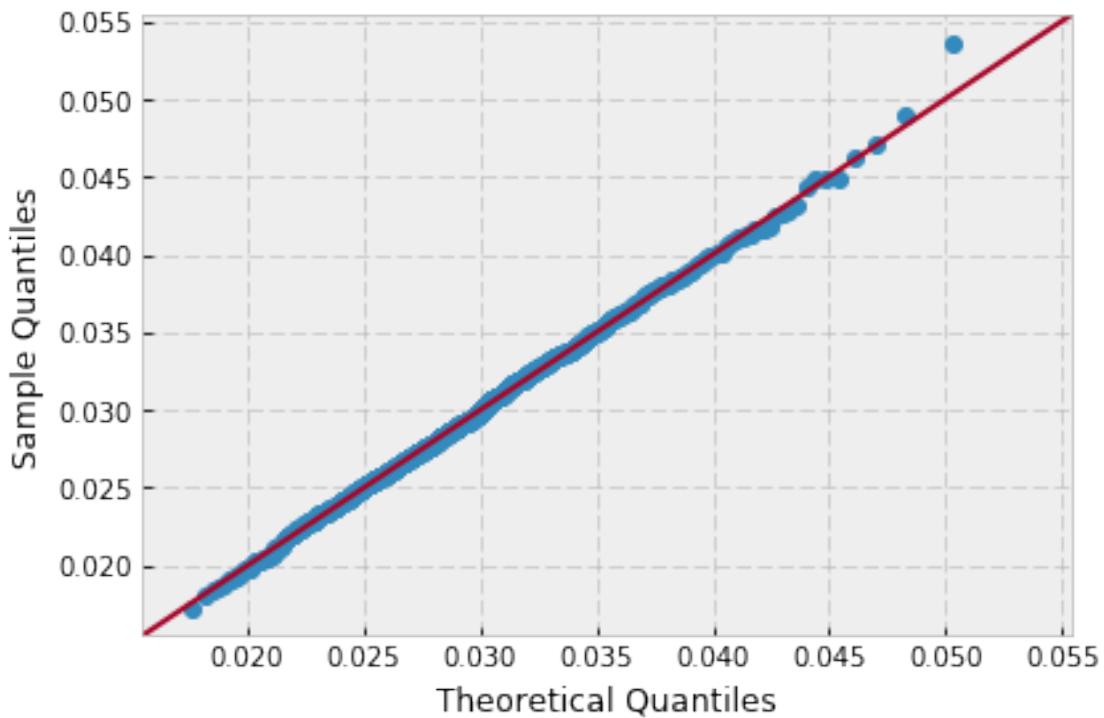


```
variable name: maxB
variable value: 12.0
distribution: inverse-gamma(shape=(26.500672873412647,), loc=0.0007648497354108912, scale=0.09964
DescribeResult(nobs=1000, minmax=(0.0026982531104859579, 0.0084750270495297913)
               mean=0.0046729864416953024, variance=6.3574605845967557e-07,
               skewness=0.9753962673878616, kurtosis=1.854037612080461)
```

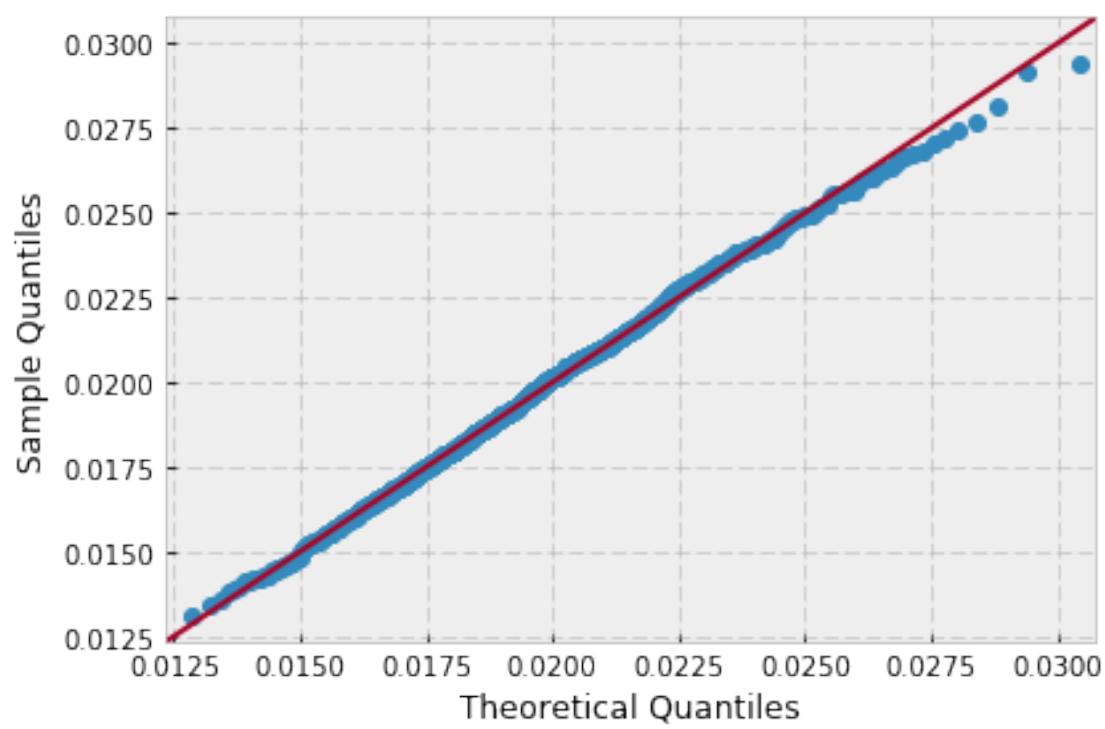
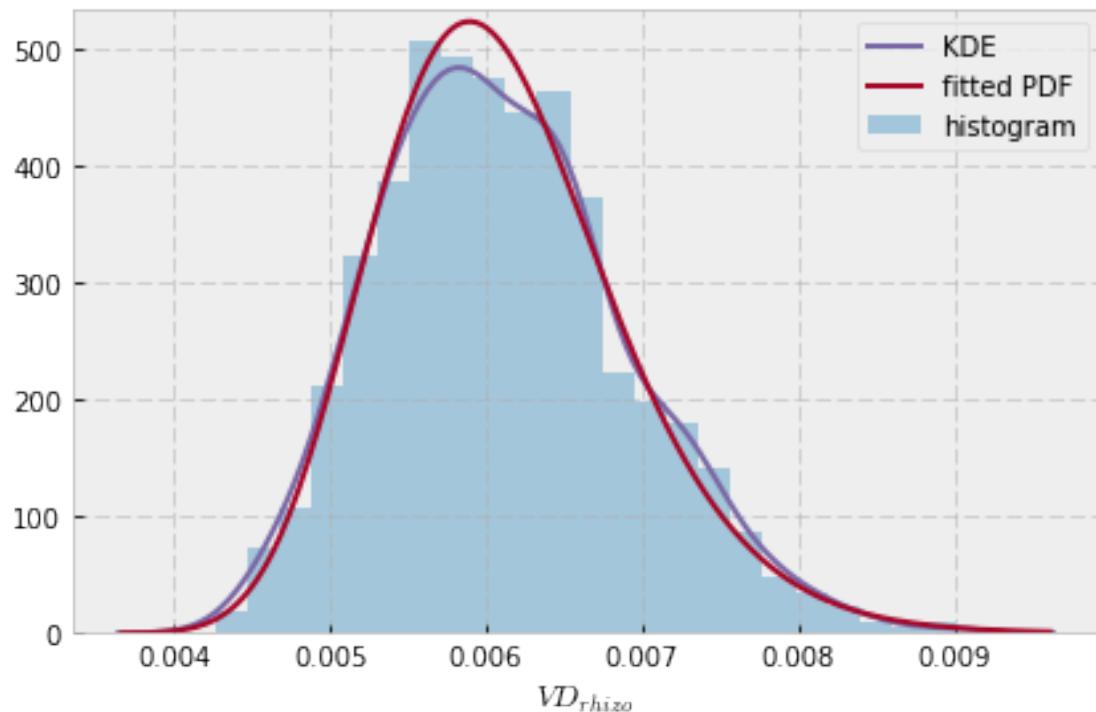


```
variable name: maxB
variable value: 16.0
distribution: inverse-gamma(shape=(35.473672611088219,), loc=0.0009063066499440092, scale=0.15557
DescribeResult(nobs=1000, minmax=(0.0035959427377552223, 0.0092421799339248798)
               mean=0.0054190059915920543, variance=6.0759107970053346e-07,
               skewness=0.6862141189443981, kurtosis=0.8028792244046334)
```

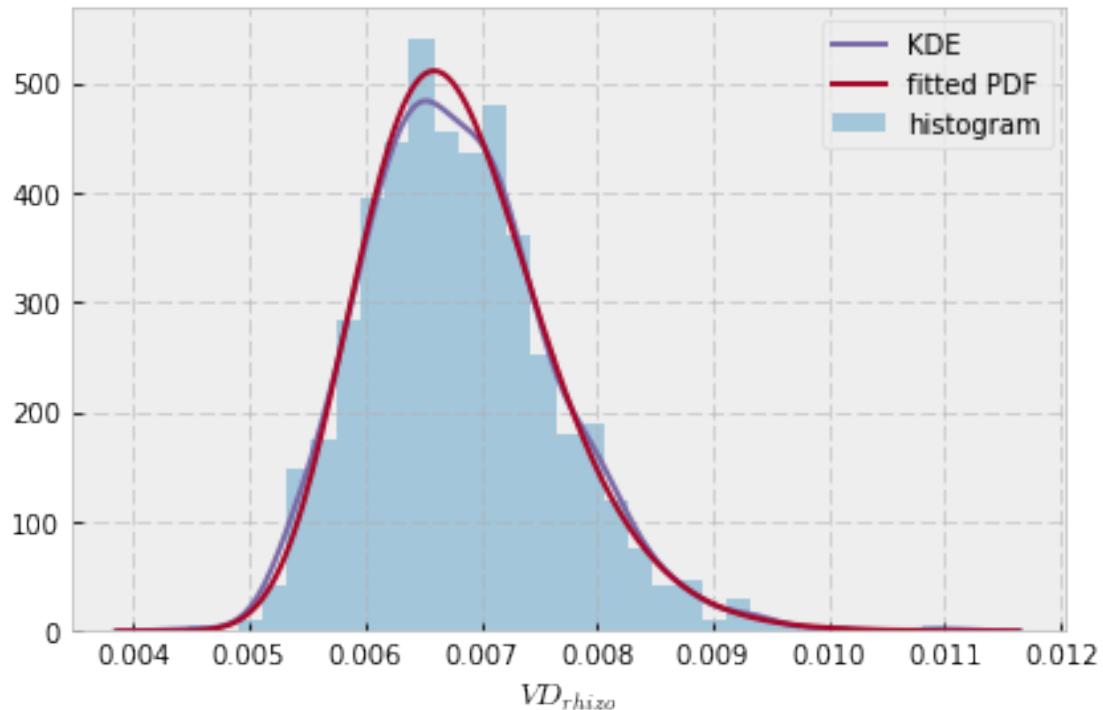


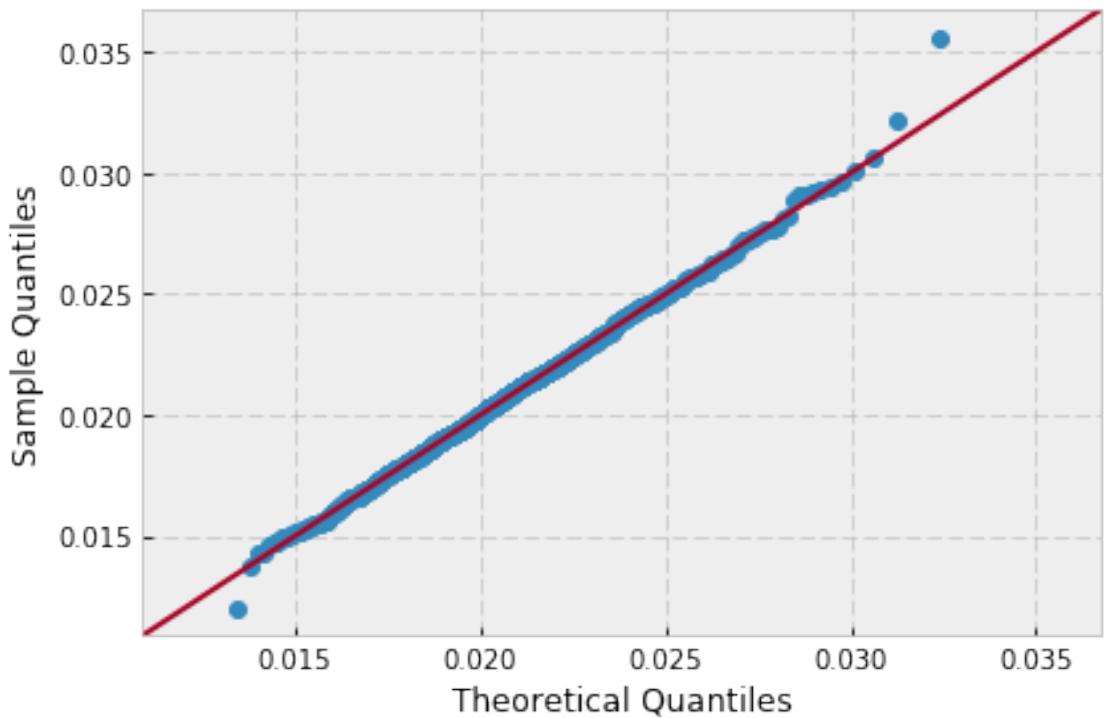


```
variable name: maxB
variable value: 20.0
distribution: inverse-gamma(shape=(52.431264340085477,), loc=0.00044394658549417525, scale=0.2911
DescribeResult(nobs=1000, minmax=(0.0042634676366660931, 0.0090107428867594944)
               mean=0.0061102519108633385, variance=6.2951101082064256e-07,
               skewness=0.45213154934831573, kurtosis=0.04288118373058358)
```

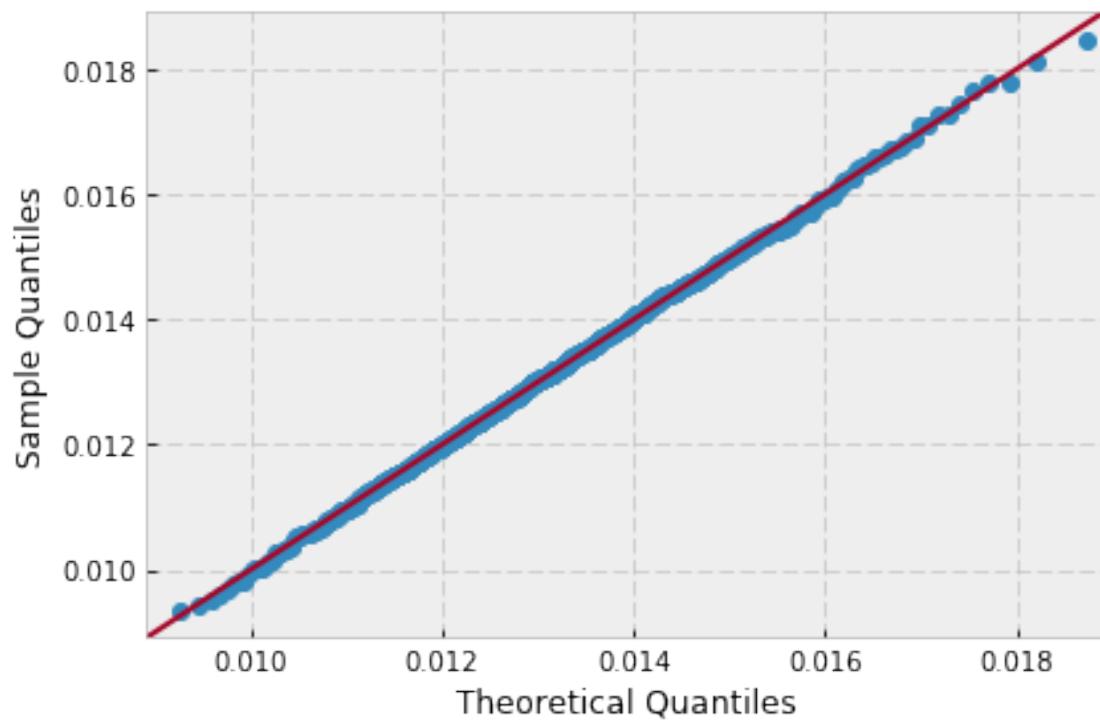
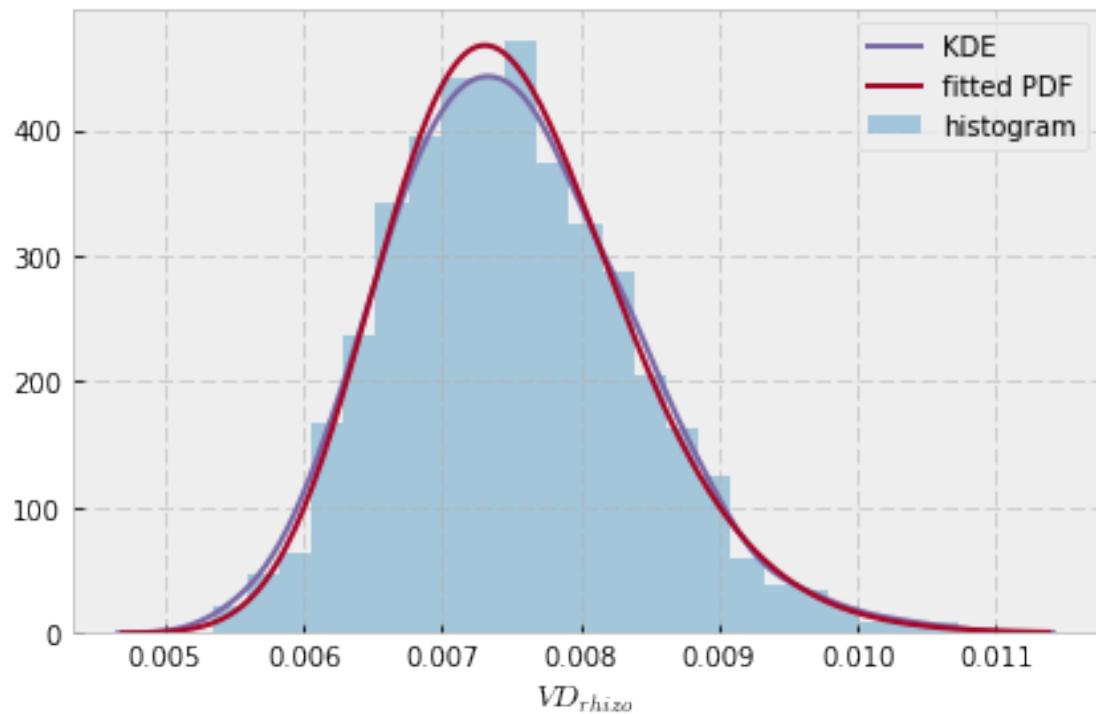


```
variable name: maxB
variable value: 24.0
distribution: inverse-gamma(shape=(49.92545523369224,), loc=0.001139094253517458, scale=0.2777984
DescribeResult(nobs=1000, minmax=(0.0044827267333018232, 0.011014849137745778)
               mean=0.006817116546687714, variance=6.7438979889019732e-07,
               skewness=0.6093968702604218, kurtosis=0.8148635511576097)
```

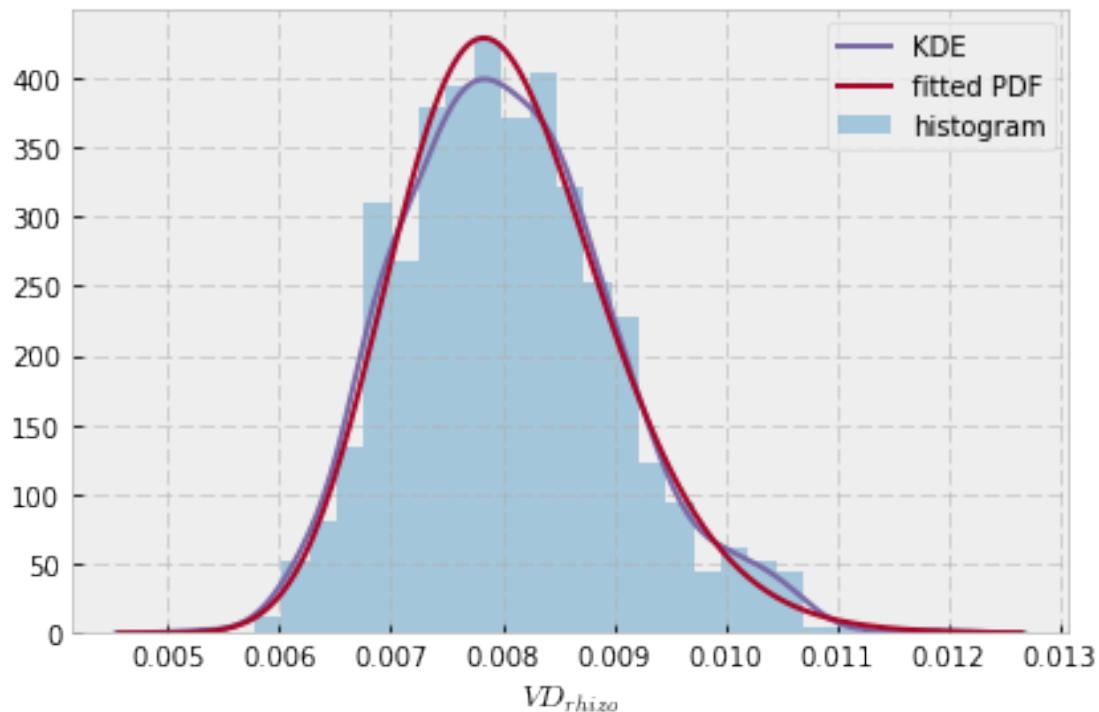


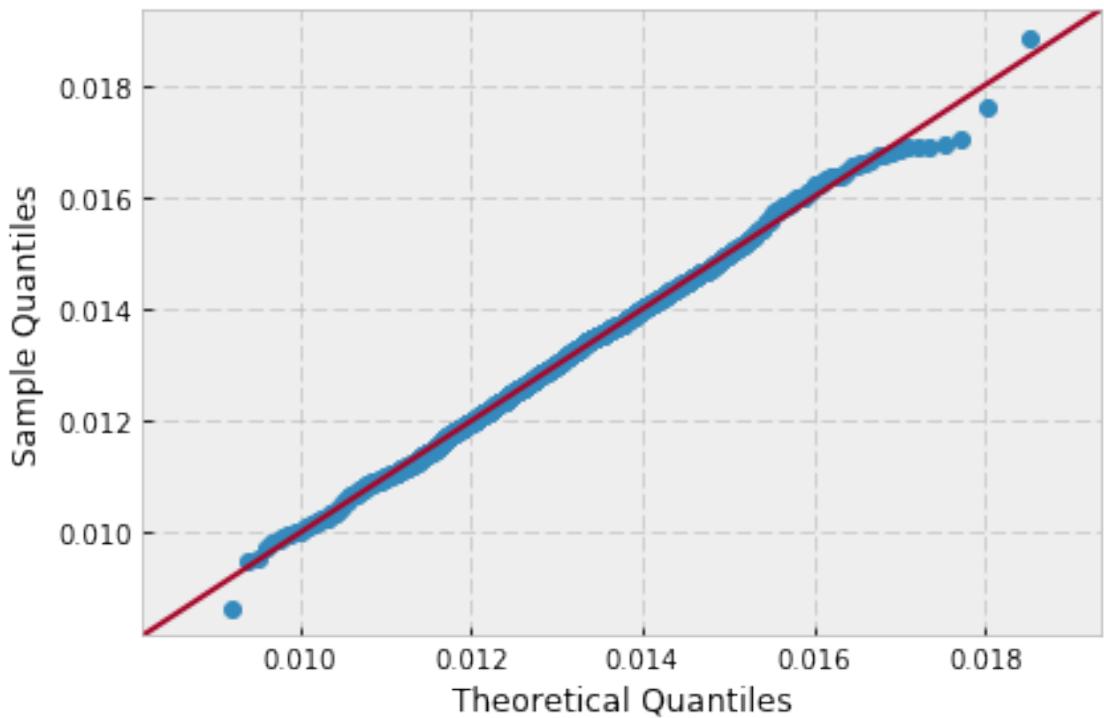


```
variable name: maxB
variable value: 28.0
distribution: inverse-gamma(shape=(77.896404189639441,), loc=-0.0001698337821993993, scale=0.5900
DescribeResult(nobs=1000, minmax=(0.0053560251767997159, 0.01071843165880697)
               mean=0.0075000166845963787, variance=7.7040677748922811e-07,
               skewness=0.40199372345433193, kurtosis=0.16752426936218567)
```

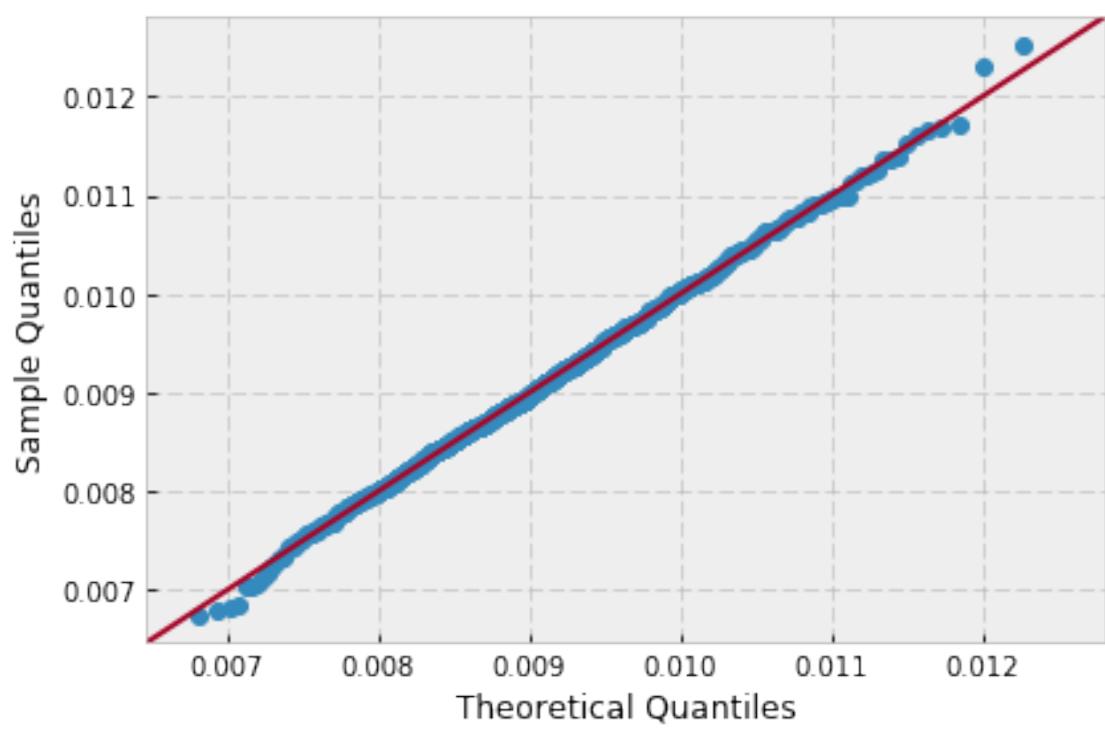
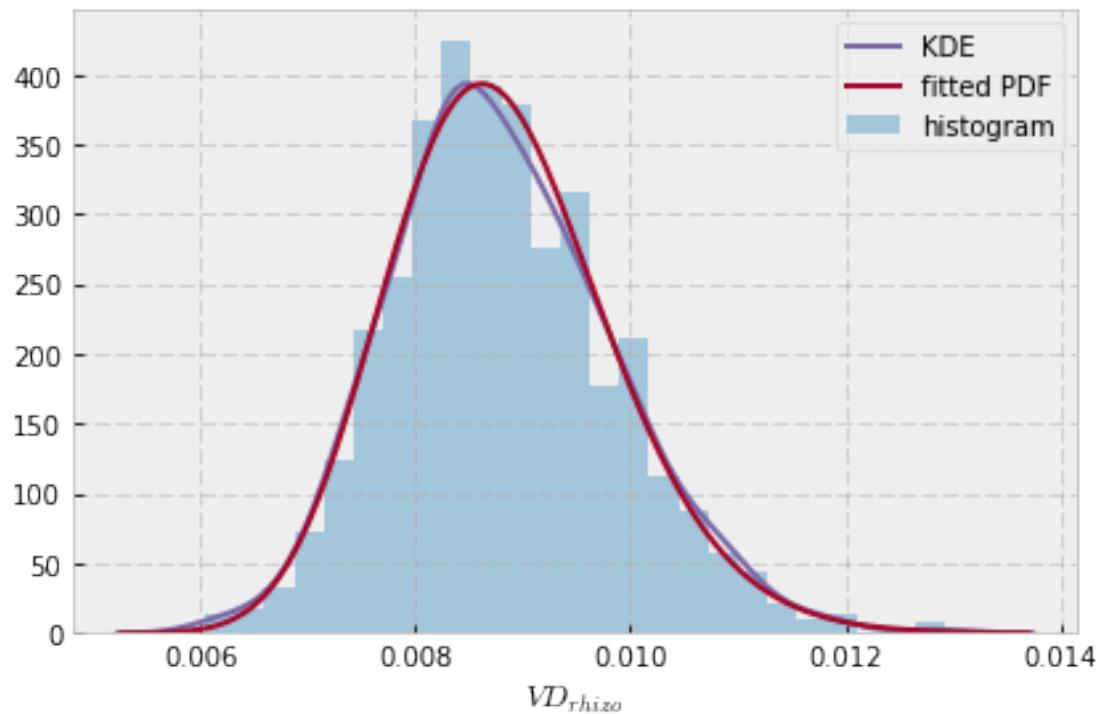


```
variable name: maxB
variable value: 32.0
distribution: inverse-gamma(shape=(78.603216565503601,), loc=-0.00036295549584243426, scale=0.652
DescribeResult(nobs=1000, minmax=(0.0052875250605549685, 0.011923982089949599)
               mean=0.0080424343532093864, variance=9.1981496058978906e-07,
               skewness=0.4195540882743349, kurtosis=0.13190397540692578)
```

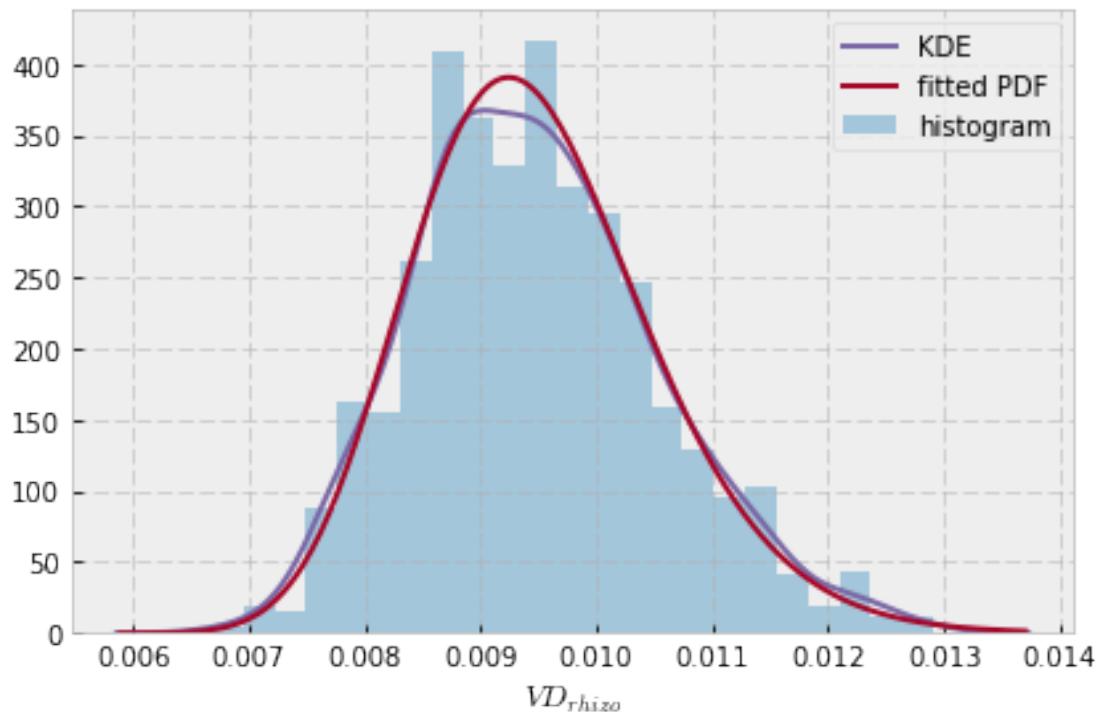


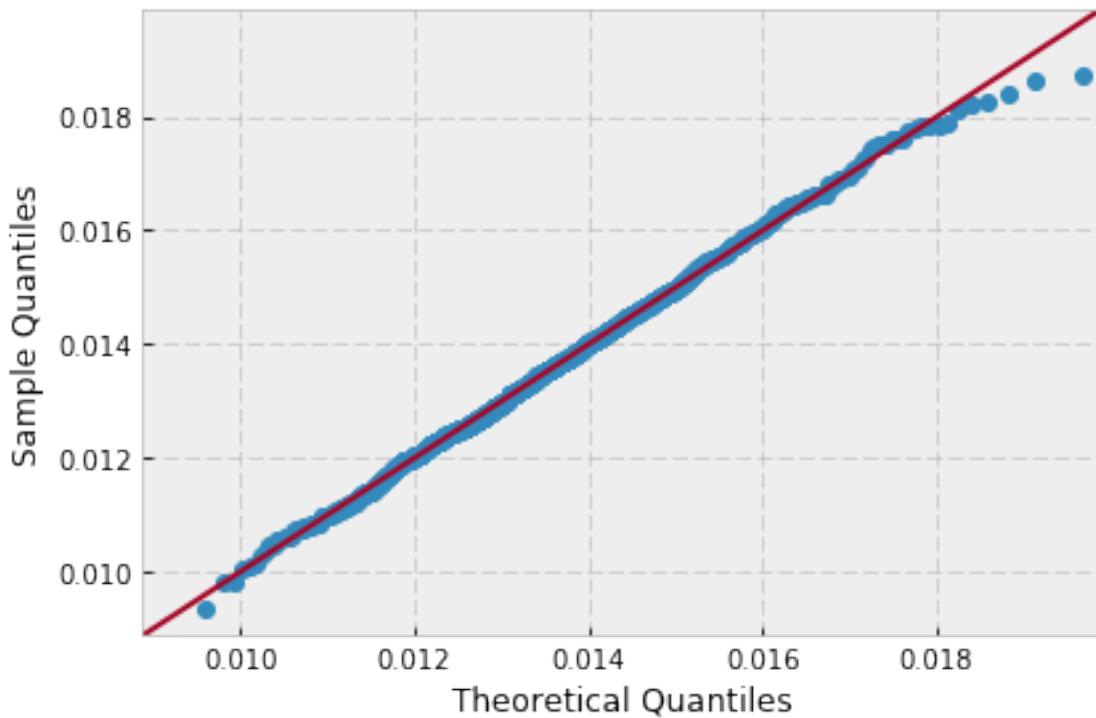


```
variable name: maxB
variable value: 36.0
distribution: inverse-gamma(shape=(111.21135967126452,), loc=-0.0020217593780128863, scale=1.1938
DescribeResult(nobs=1000, minmax=(0.0060418264499035321, 0.012925299871917815)
               mean=0.0088098648248777067, variance=1.0726152618374052e-06,
               skewness=0.38107176783736896, kurtosis=0.23540917368003544)
```

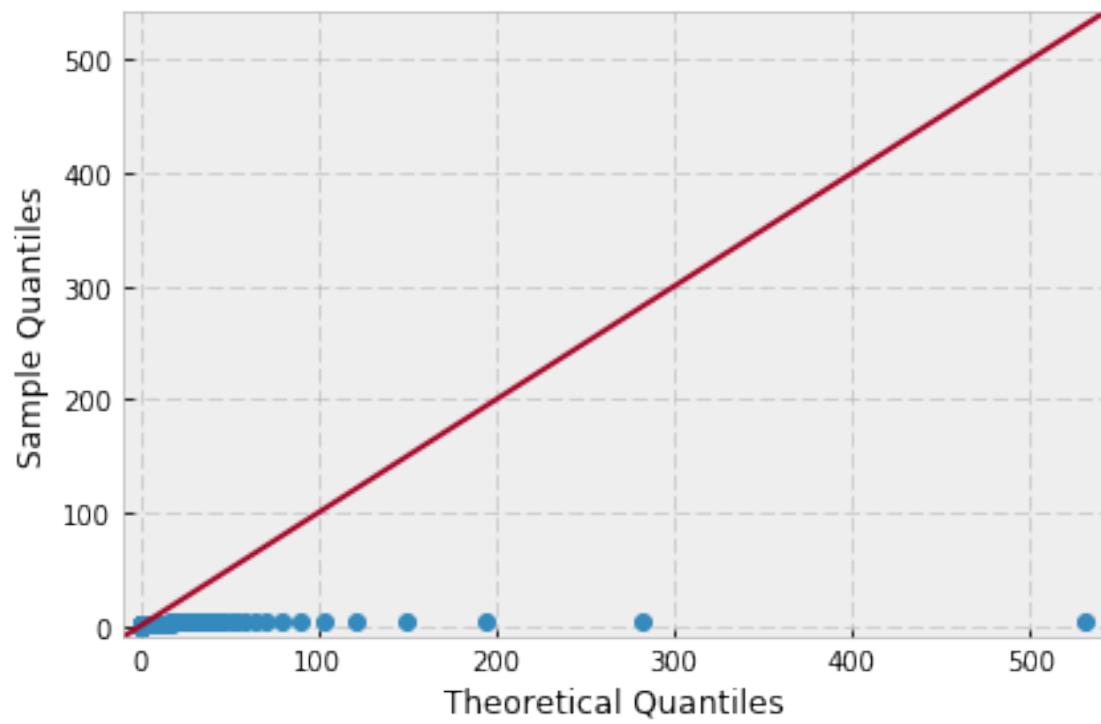
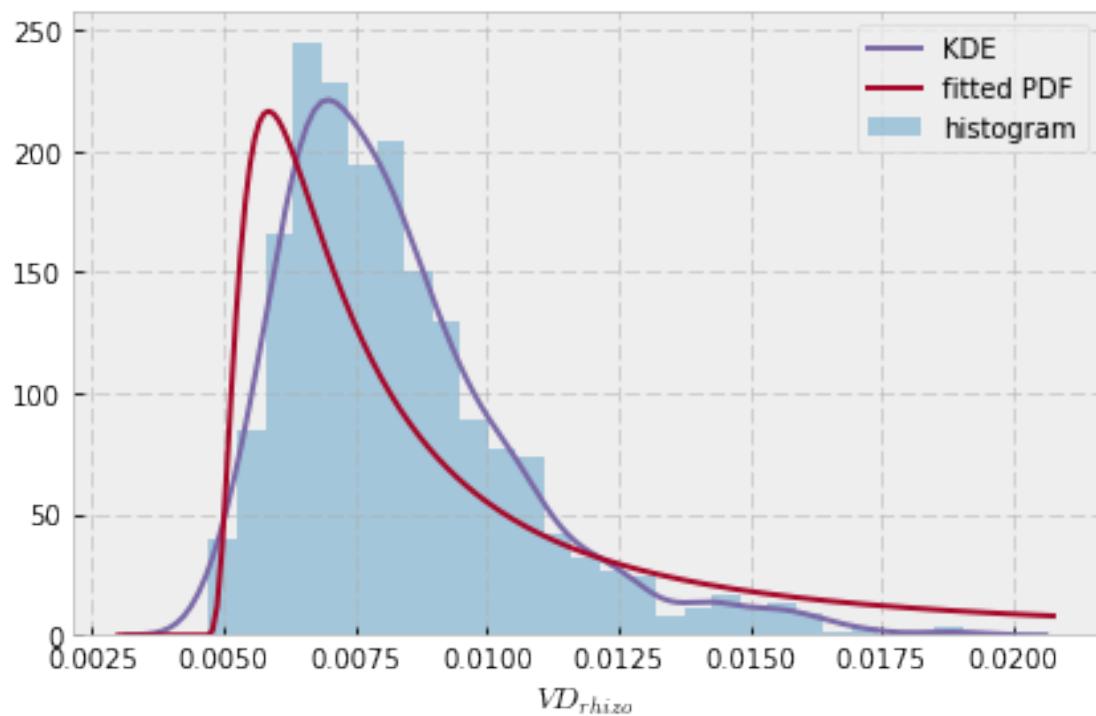


```
variable name: maxB
variable value: 40.0
distribution: inverse-gamma(shape=(74.730739003254513,), loc=0.00048679860210071486, scale=0.6631
DescribeResult(nobs=1000, minmax=(0.006680978128720514, 0.012909129528221531)
               mean=0.0094810256909844248, variance=1.1091195372722806e-06,
               skewness=0.41829159540583105, kurtosis=0.03770779990903472)
```

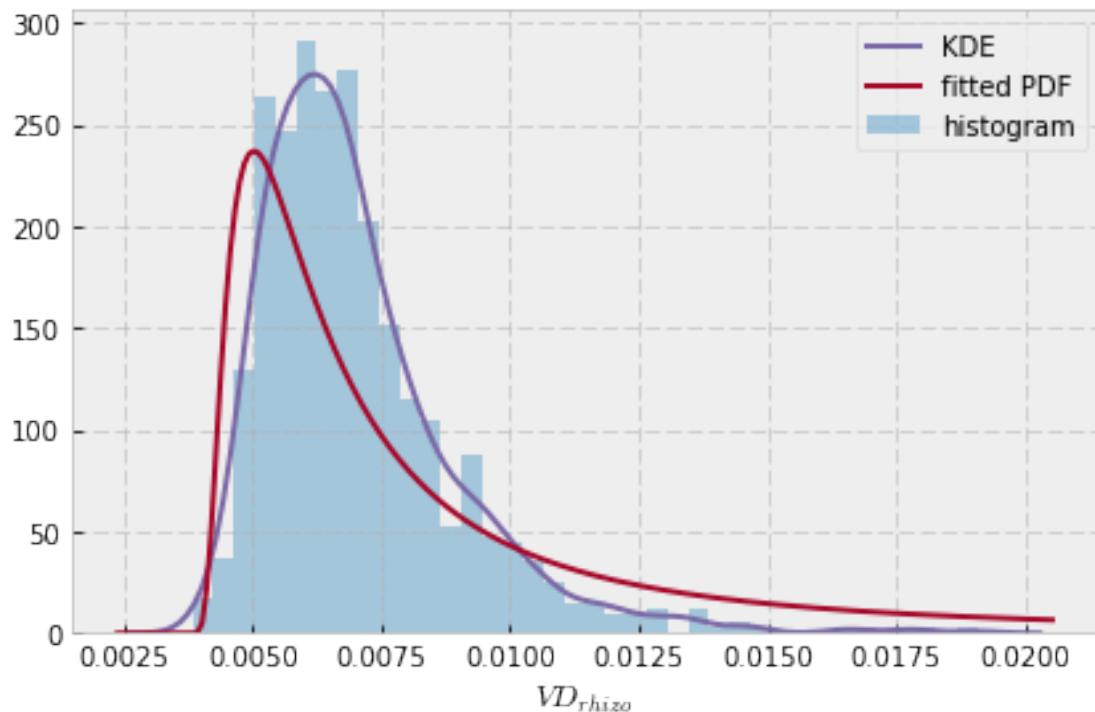


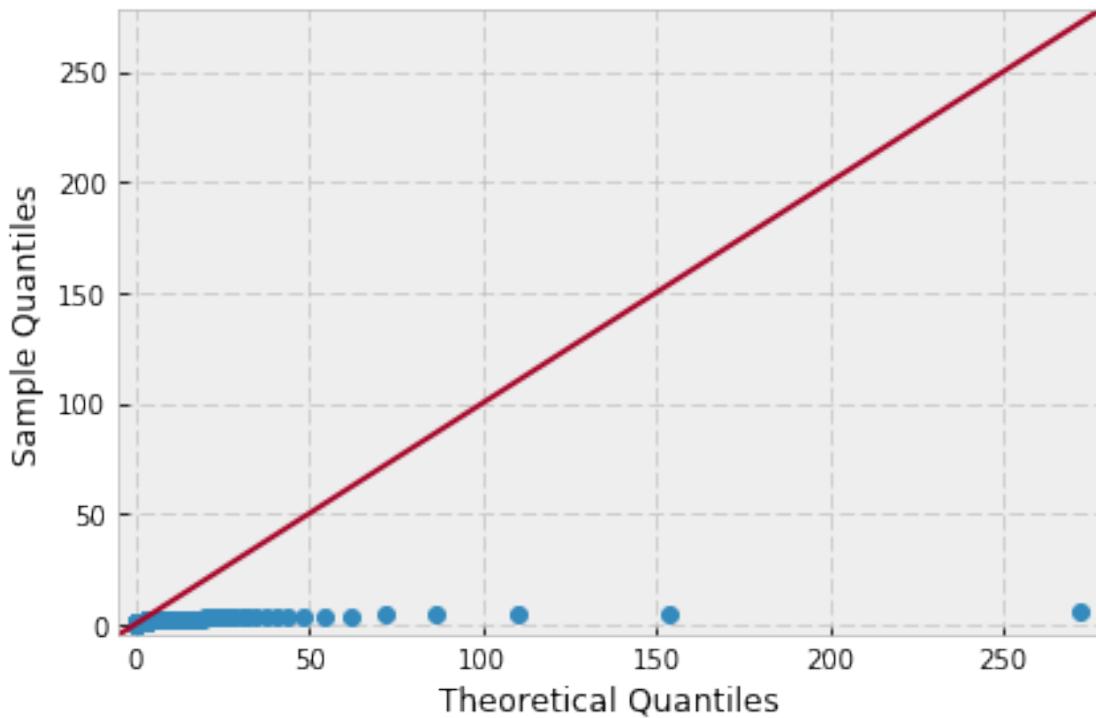


```
variable name: r
variable value: 1.0
distribution: inverse-gamma(shape=(1.0938643095302112,),loc=0.00451077861303528, scale=0.0028045
DescribeResult(nobs=1000, minmax=(0.0047190238817042705, 0.019037428911820985)
               mean=0.0083283160722629375, variance=5.1986464713326919e-06,
               skewness=1.3356445931500143, kurtosis=2.1838672114100435)
```

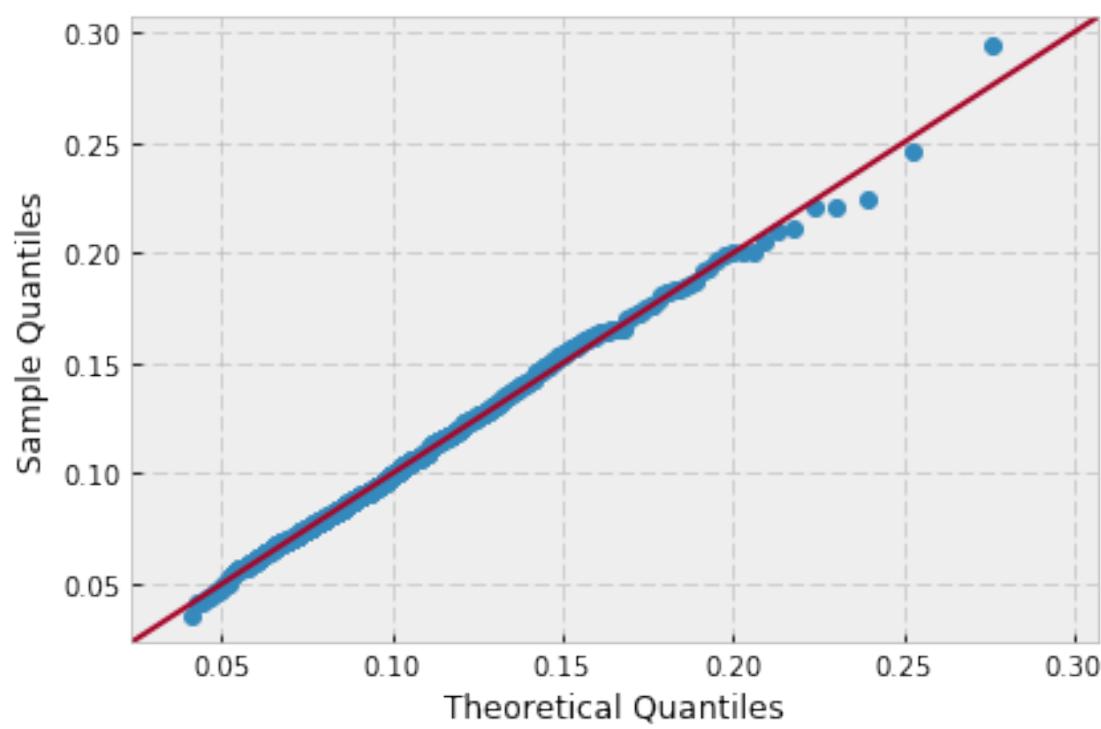
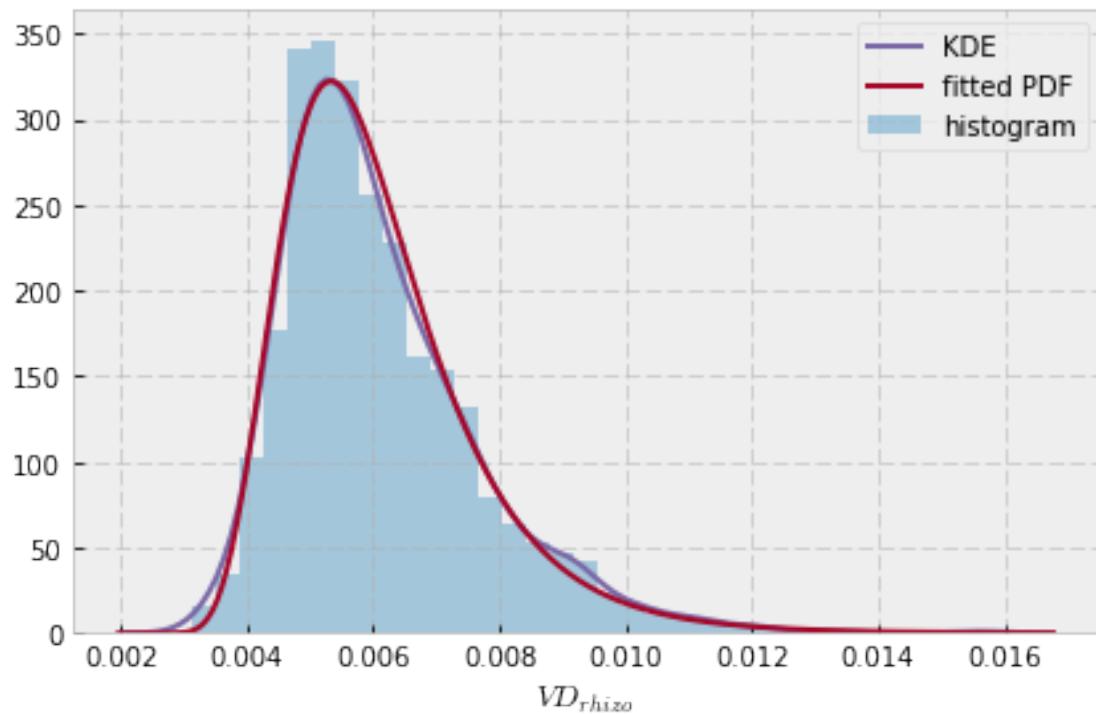


```
variable name: r
variable value: 1.222222222222223
distribution: inverse-gamma(shape=(1.2131204682921686,),loc=0.003704347731237405, scale=0.002926
DescribeResult(nobs=1000, minmax=(0.0038324891131462915, 0.019076250606701369)
    mean=0.0070373539385138262, variance=3.7265249406810239e-06,
    skewness=1.7879974048080902, kurtosis=5.248486205301537)
```

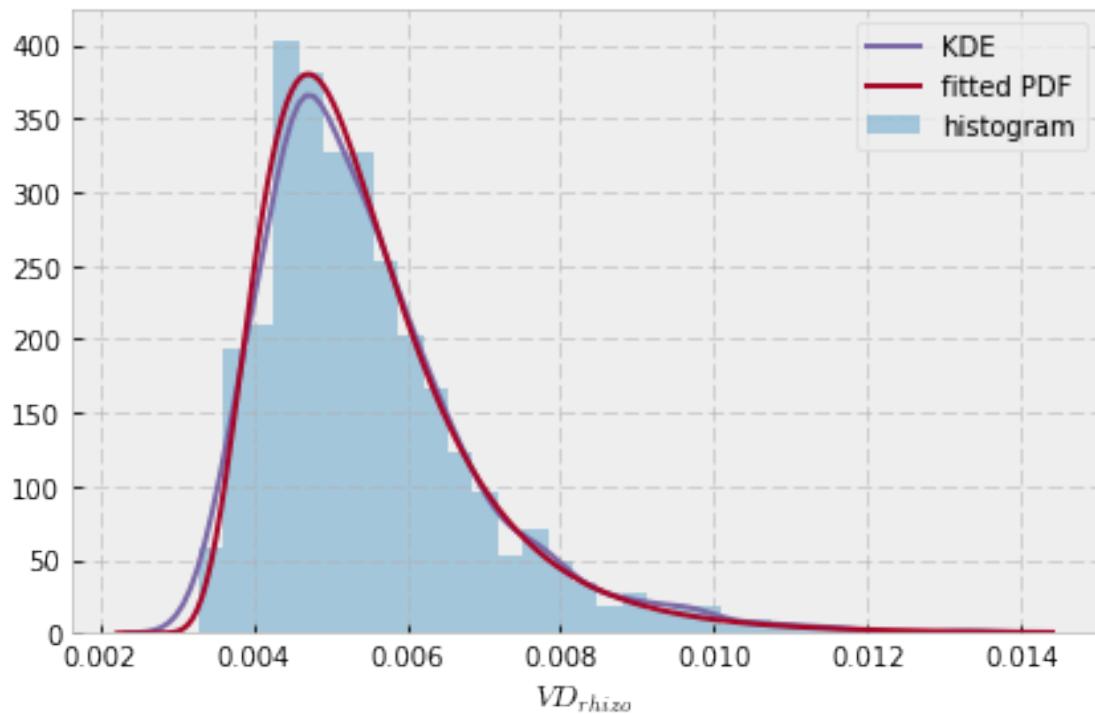


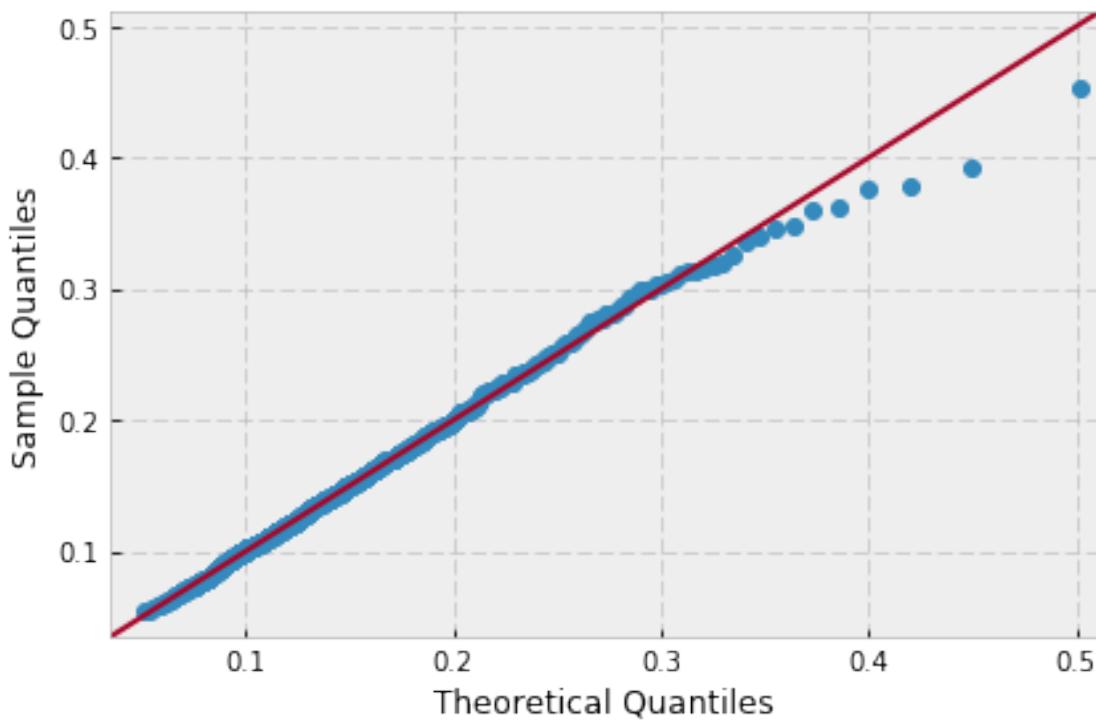


```
variable name: r
variable value: 1.444444444444444
distribution: inverse-gamma(shape=(11.247403361386162,), loc=0.0013790627502866105, scale=0.04839
DescribeResult(nobs=1000, minmax=(0.0031294580996540844, 0.015606680919136338)
               mean=0.0061014253250136277, variance=2.3913221456957216e-06,
               skewness=1.3040679674912916, kurtosis=2.695726343265669)
```

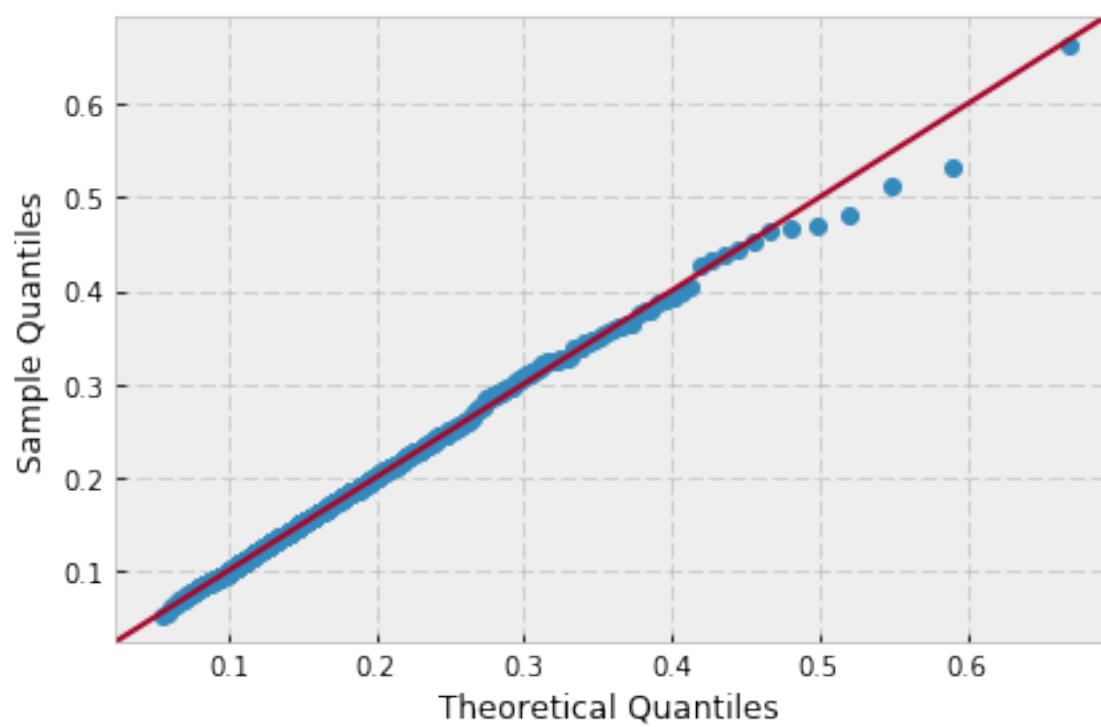
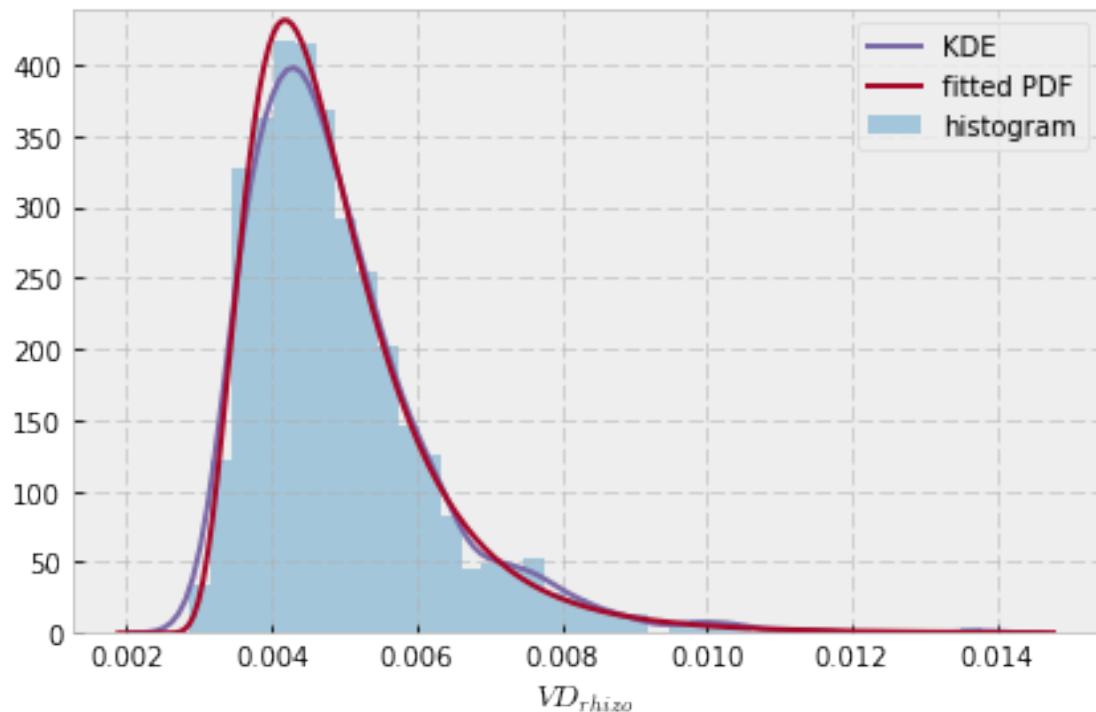


```
variable name: r
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(8.0481432009005793,),loc=0.0019296251886440547, scale=0.02516
DescribeResult(nobs=1000, minmax=(0.0032864345173158863, 0.013345832038683876)
               mean=0.0054963195634998756, variance=1.9854590264065984e-06,
               skewness=1.430428515243449, kurtosis=2.8614816560938747)
```

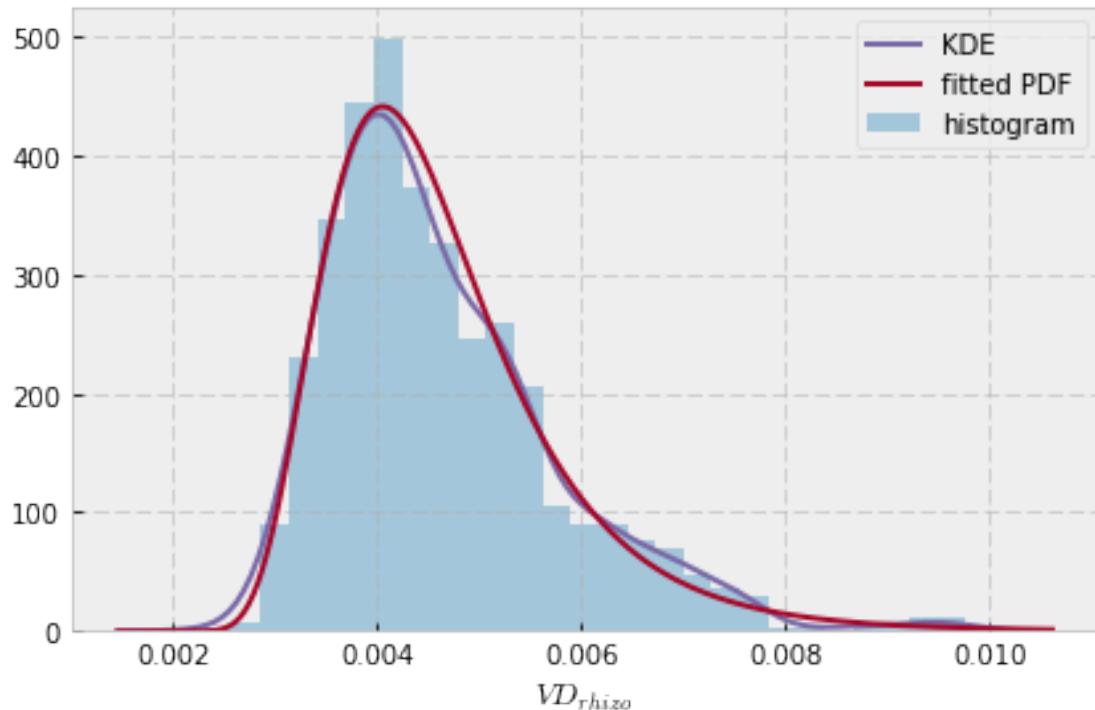


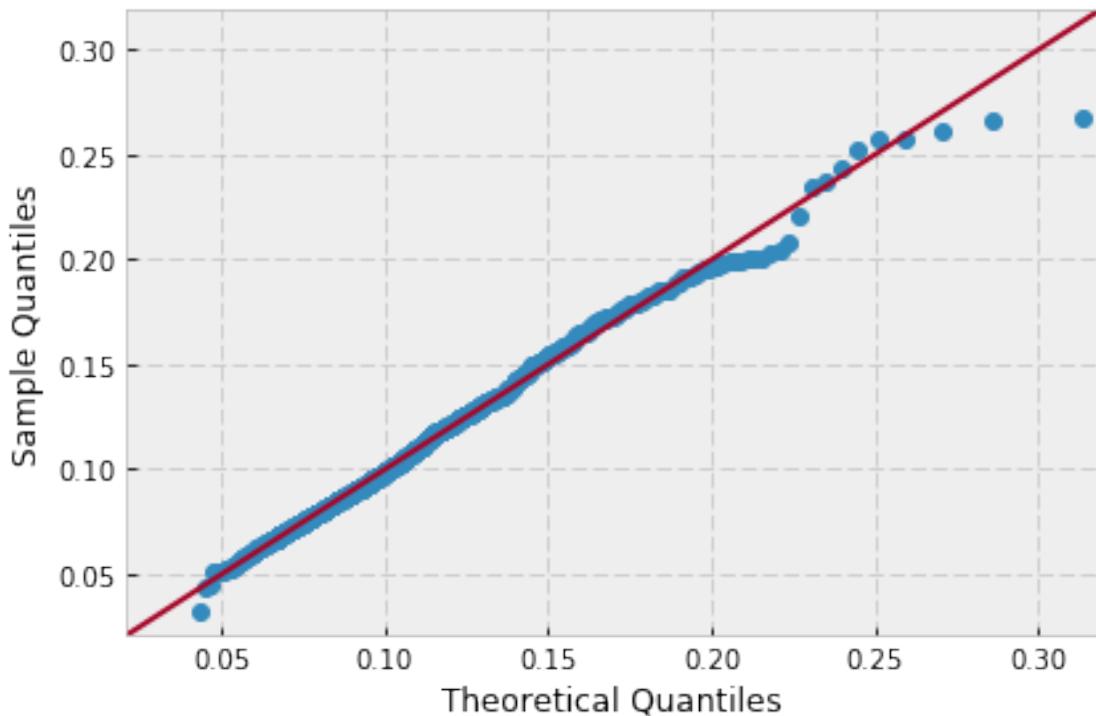


```
variable name: r
variable value: 1.888888888888888
distribution: inverse-gamma(shape=(6.9417803829681386,), loc=0.001930672665071207, scale=0.017885
DescribeResult(nobs=1000, minmax=(0.002876205929730894, 0.013787885739727954)
               mean=0.0049372833374327595, variance=1.7226624218826954e-06,
               skewness=1.6871520315716204, kurtosis=4.5607455666052035)
```

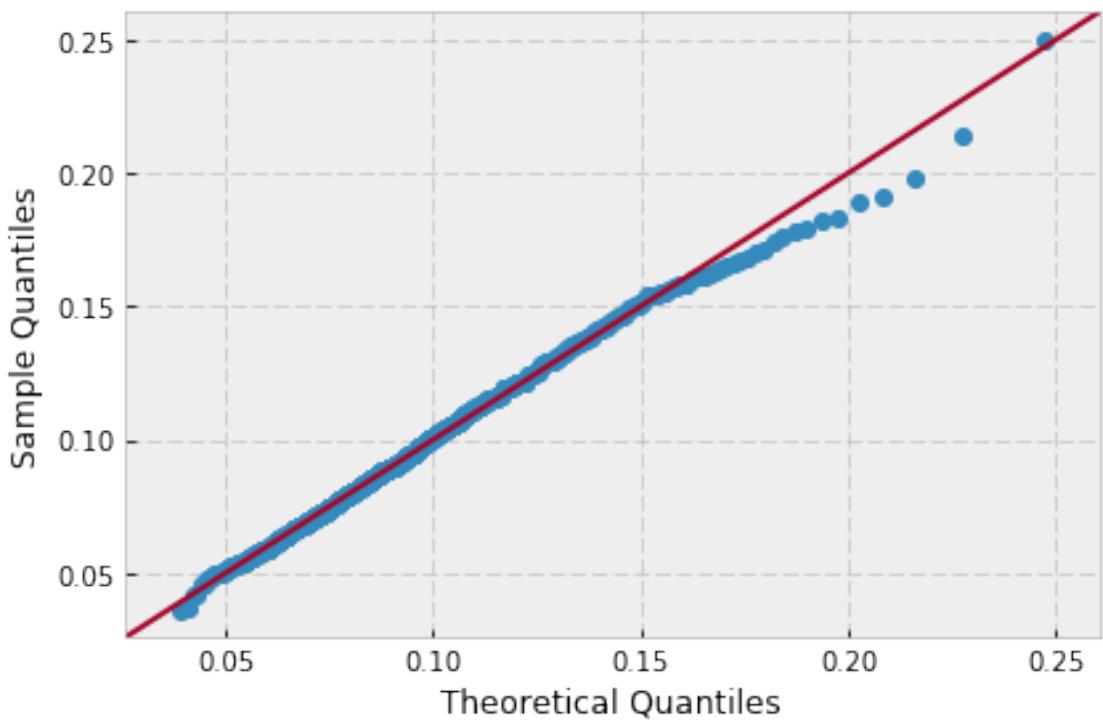
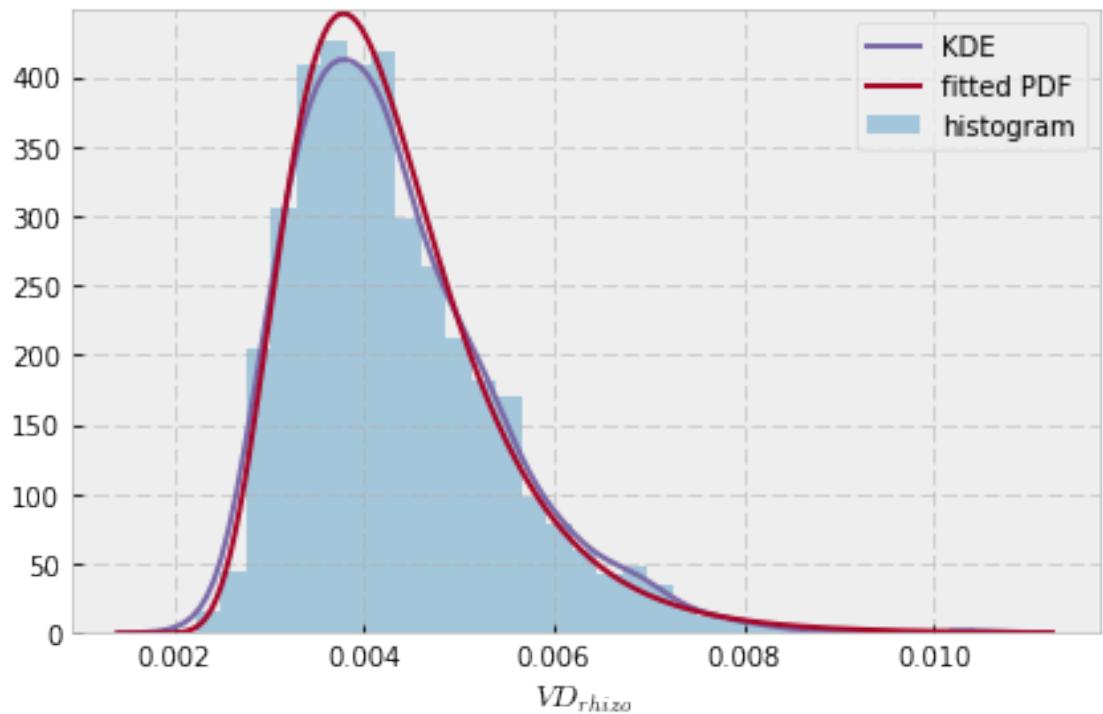


```
variable name: r
variable value: 2.11111111111111
distribution: inverse-gamma(shape=(10.436720462700194,),loc=0.0012839082615775013, scale=0.03164
DescribeResult(nobs=1000, minmax=(0.00230461702103323, 0.0097627688087468089)
    mean=0.0046363767680091747, variance=1.2912886721616033e-06,
    skewness=1.2179470613823444, kurtosis=1.9605638106848575)
```

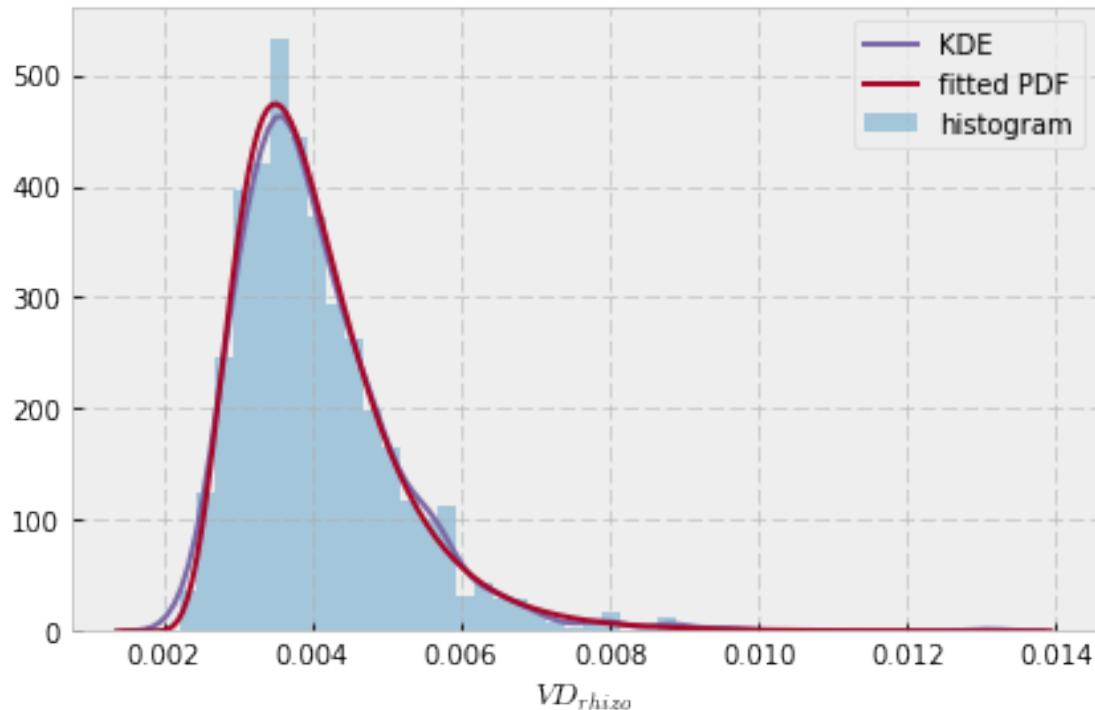


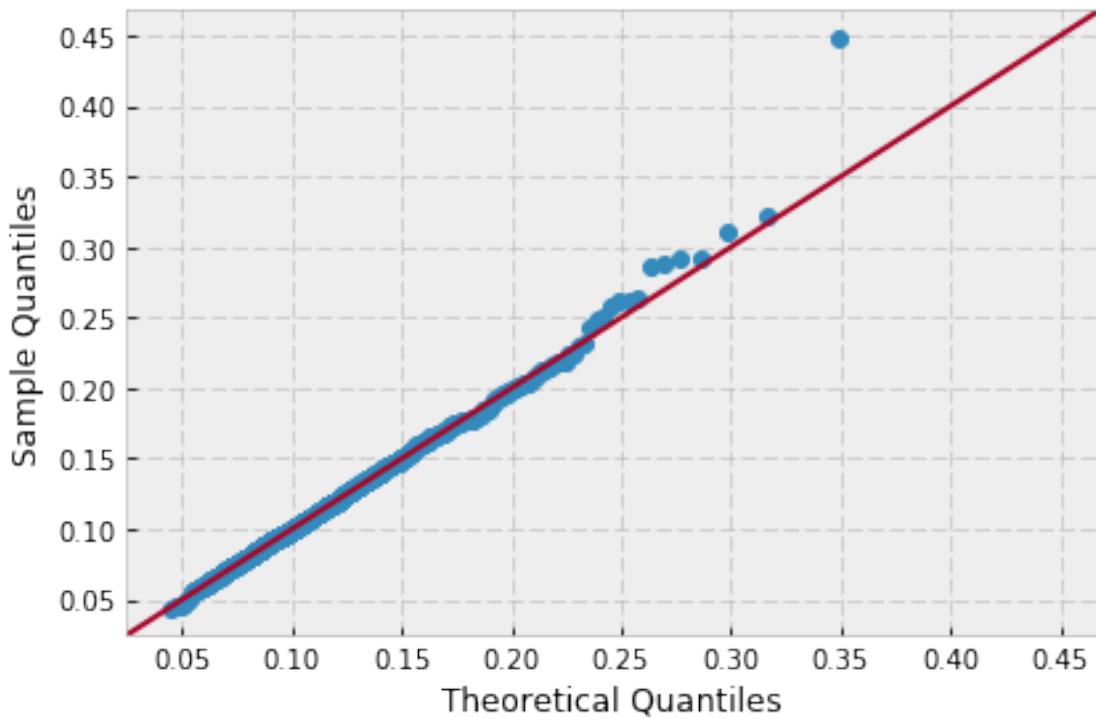


```
variable name: r
variable value: 2.333333333333333
distribution: inverse-gamma(shape=(12.000450121756892,), loc=0.0008264892873484918, scale=0.03848
DescribeResult(nobs=1000, minmax=(0.0022248245664479834, 0.01043387463253684)
               mean=0.0043230617324610705, variance=1.1718057654326208e-06,
               skewness=1.0768192797806933, kurtosis=1.6899487273565112)
```

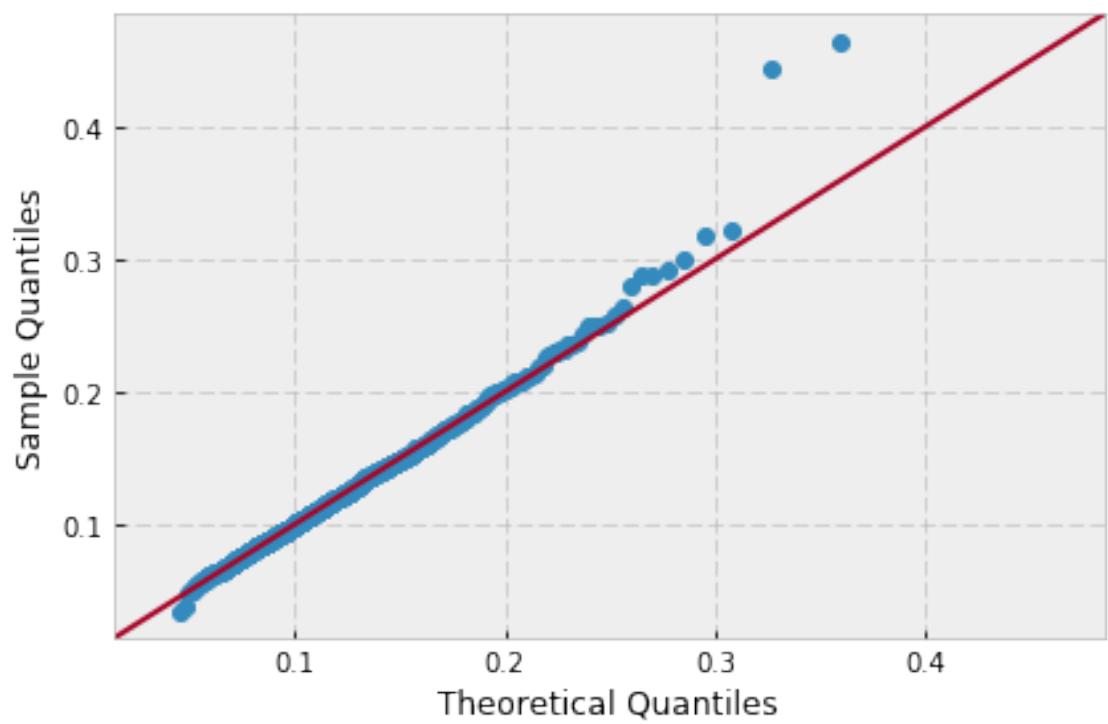
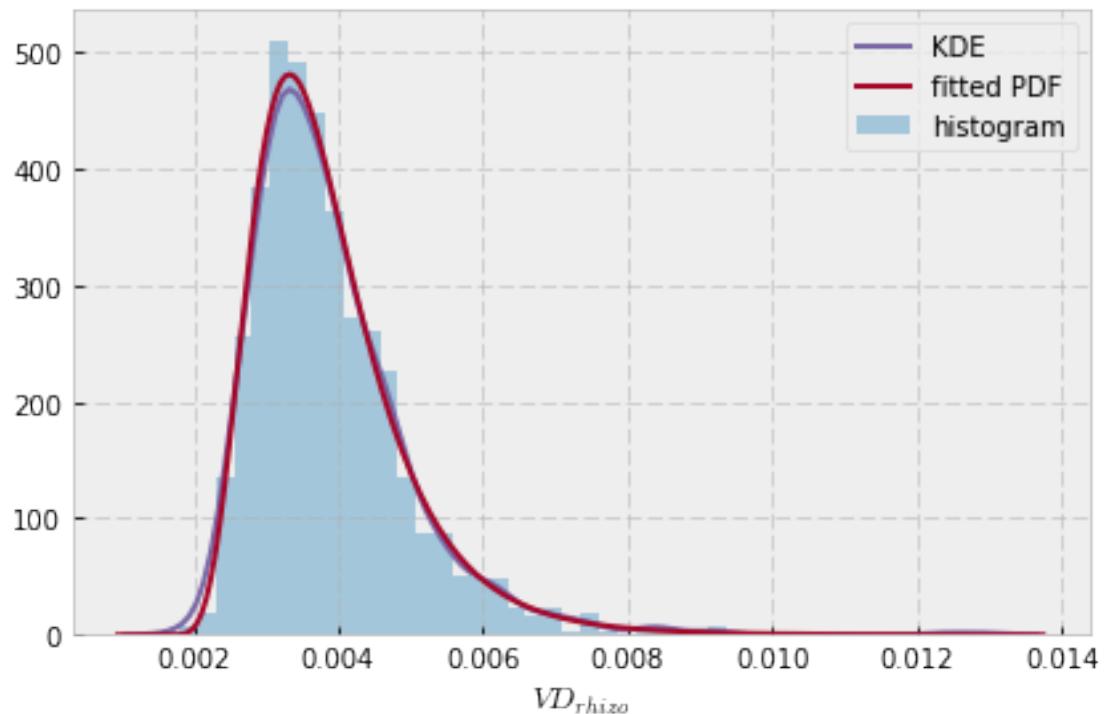


```
variable name: r
variable value: 2.5555555555555554
distribution: inverse-gamma(shape=(9.8229517690634616,),loc=0.0010053922507543111, scale=0.02699
DescribeResult(nobs=1000, minmax=(0.0022051818557029209, 0.013095108718633435)
               mean=0.0040660874426443625, variance=1.2208618804006043e-06,
               skewness=1.7819825191647027, kurtosis=6.787705979451719)
```

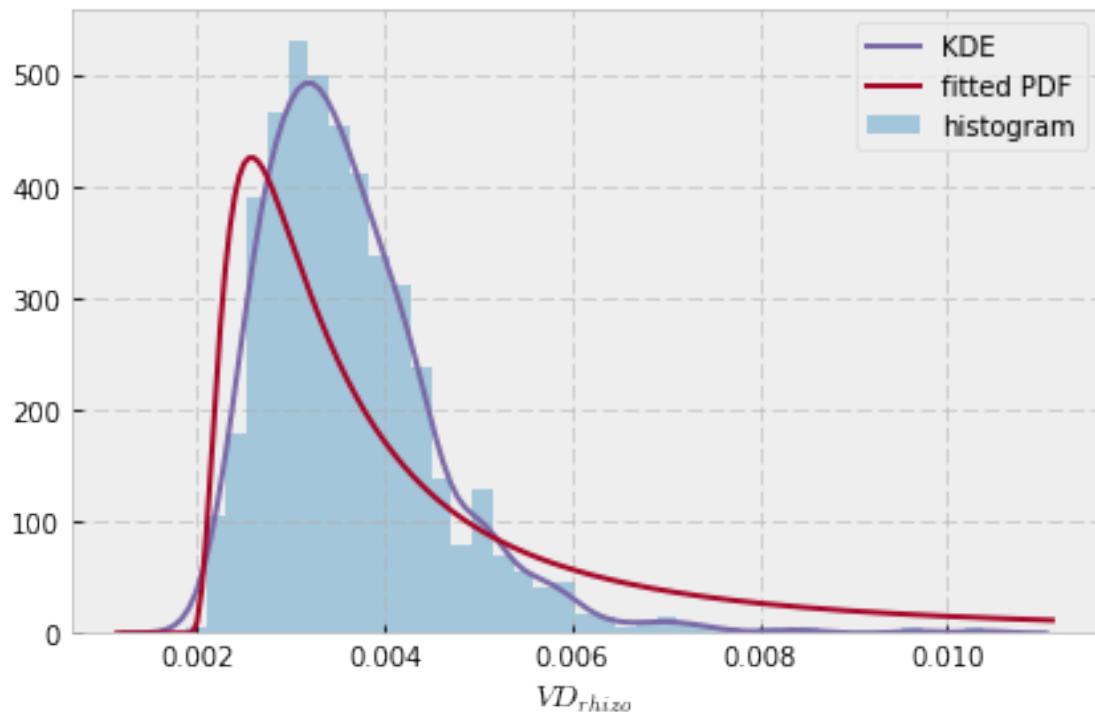


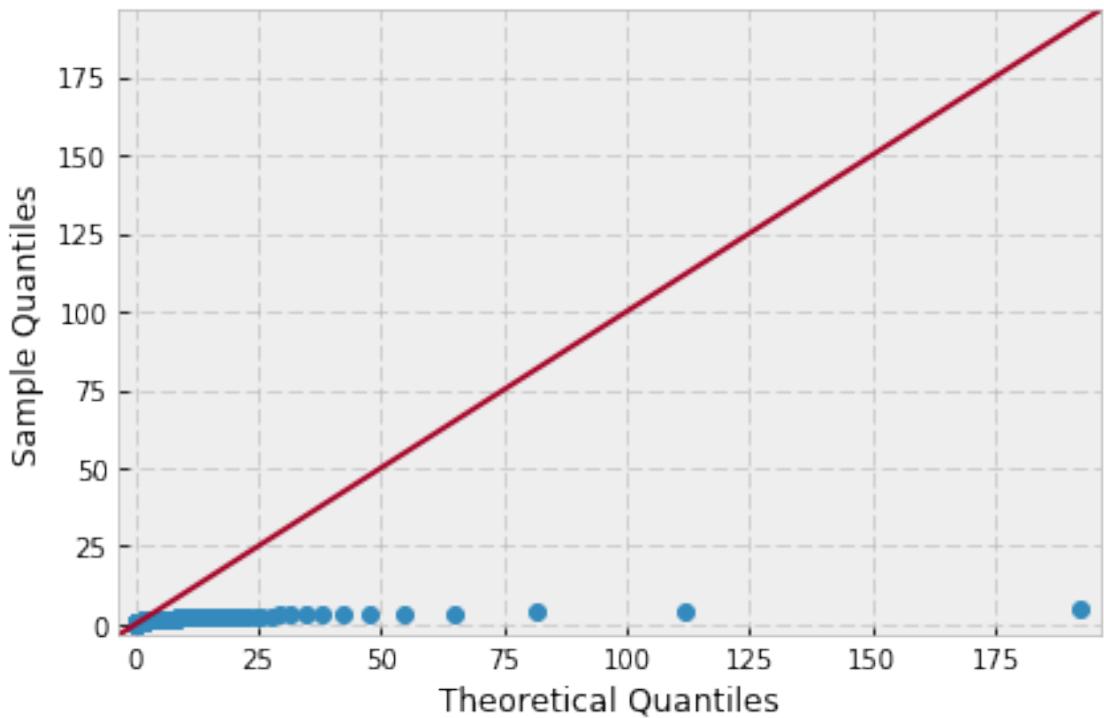


```
variable name: r
variable value: 2.777777777777777
distribution: inverse-gamma(shape=(9.6555901126528489,), loc=0.000891501225237665, scale=0.025917
DescribeResult(nobs=1000, minmax=(0.0017944464552660856, 0.012901341555732979)
               mean=0.0038892988512592231, variance=1.2559293689837088e-06,
               skewness=2.0840174927837247, kurtosis=9.109903010072713)
```

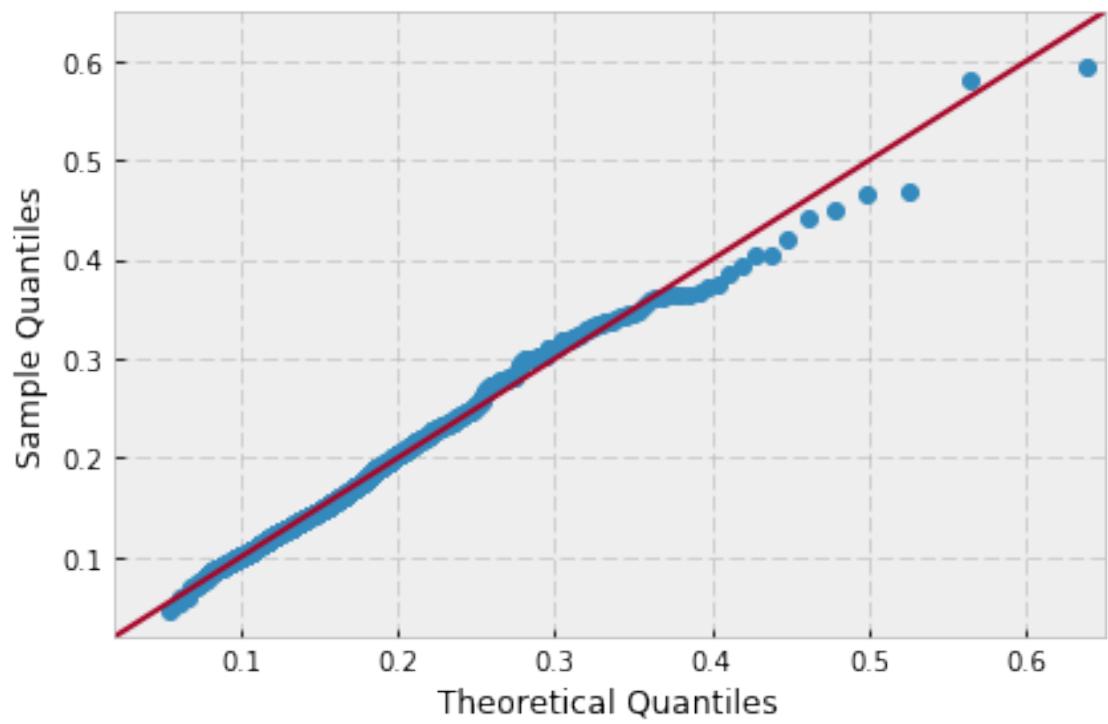
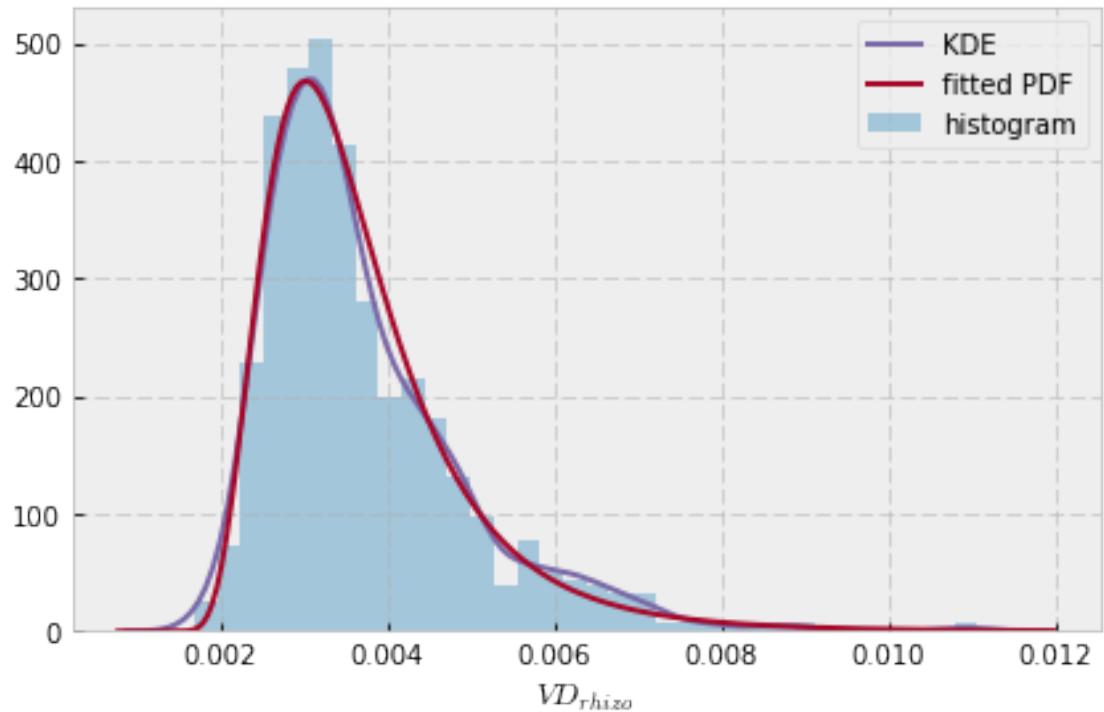


```
variable name: r
variable value: 3.0
distribution: inverse-gamma(shape=(1.2851854578637938,),loc=0.0018108733905792627, scale=0.00175
DescribeResult(nobs=1000, minmax=(0.0018743633965128113, 0.010394609499148427)
               mean=0.0036310522438424728, variance=9.287636368267442e-07,
               skewness=1.6581740344276792, kurtosis=5.552515488423943)
```

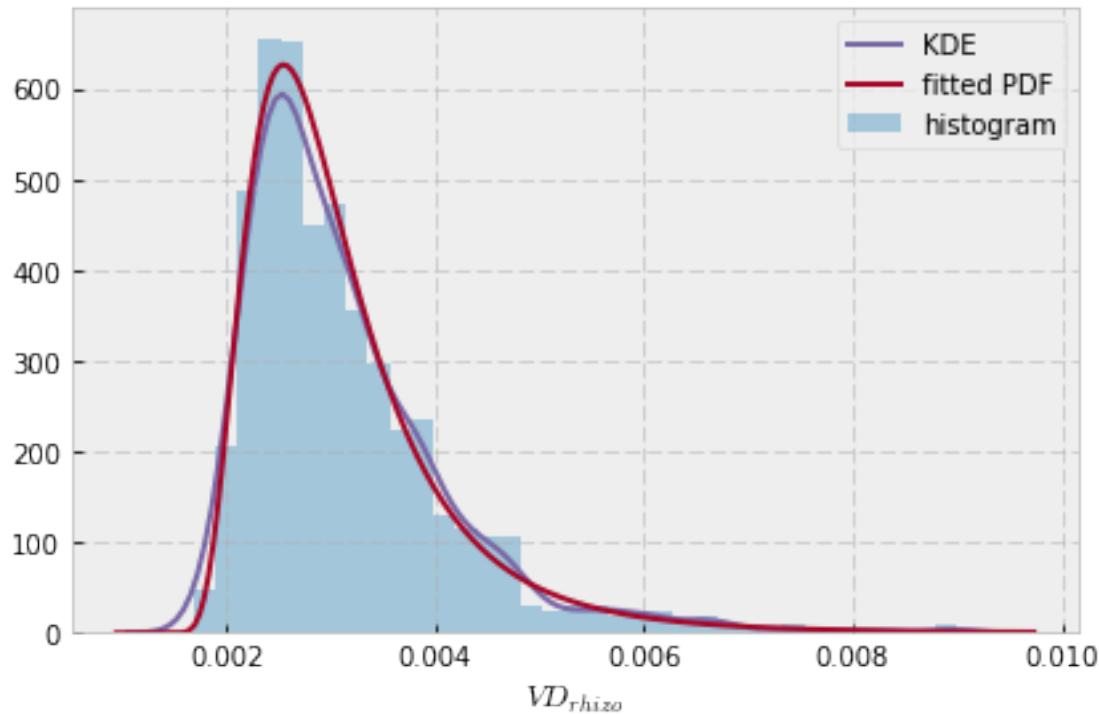


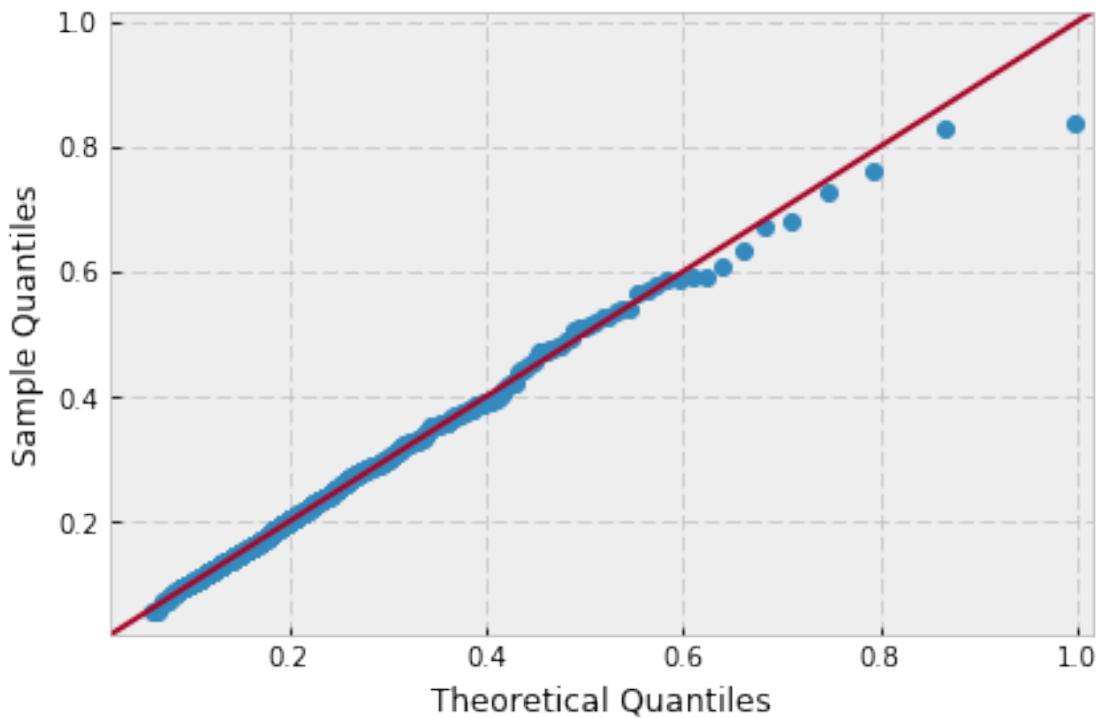


```
variable name: sigma
variable value: 0.0111111111111112
distribution: inverse-gamma(shape=(7.10810952543053,), loc=0.0009122908738682831, scale=0.0170803
DescribeResult(nobs=1000, minmax=(0.0016893106205795067, 0.011052612670757129)
               mean=0.0037072825022714203, variance=1.4586533655188623e-06,
               skewness=1.606631942440314, kurtosis=3.8482798541493004)
```

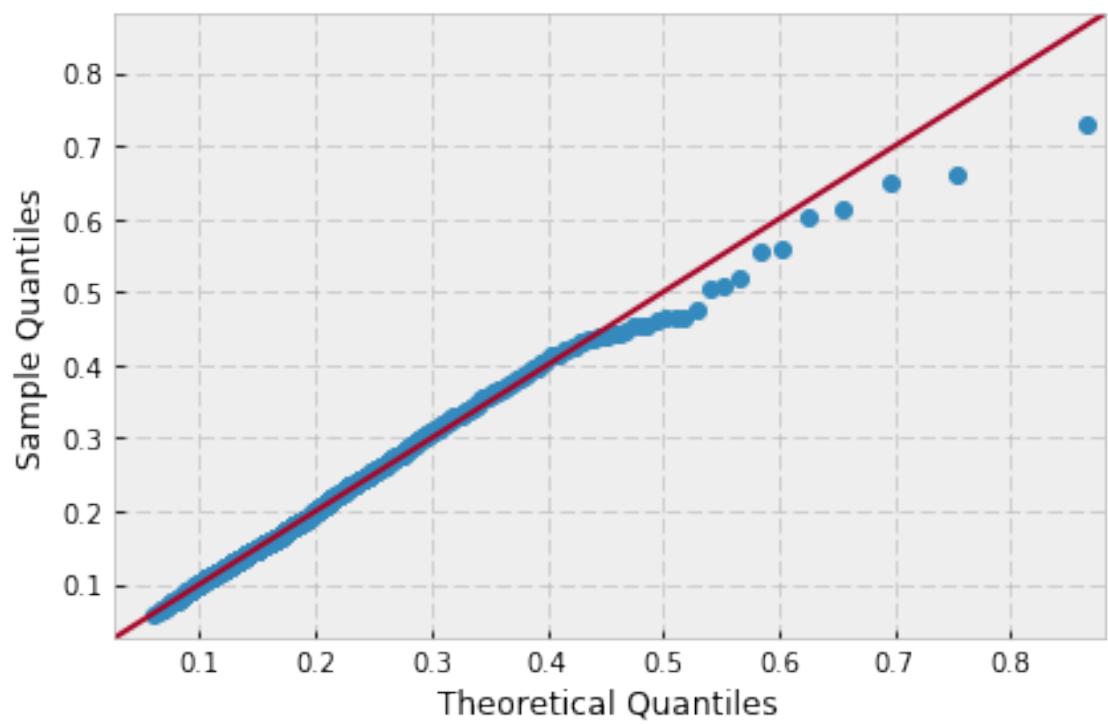
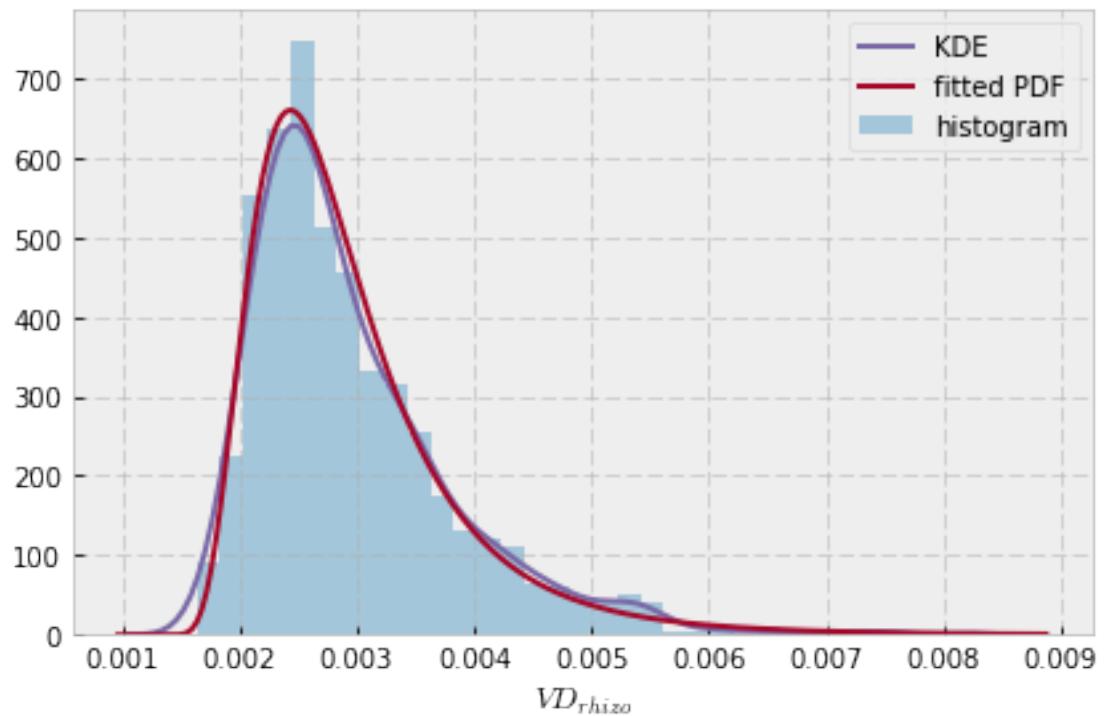


```
variable name: sigma
variable value: 0.0222222222222223
distribution: inverse-gamma(shape=(5.7275269825506481,),loc=0.0011592094309027614, scale=0.00934
DescribeResult(nobs=1000, minmax=(0.0016861859743800916, 0.0089961431860861932)
               mean=0.0031321058218281078, variance=9.5561758401250544e-07,
               skewness=1.8757528907969778, kurtosis=5.3402569799214135)
```

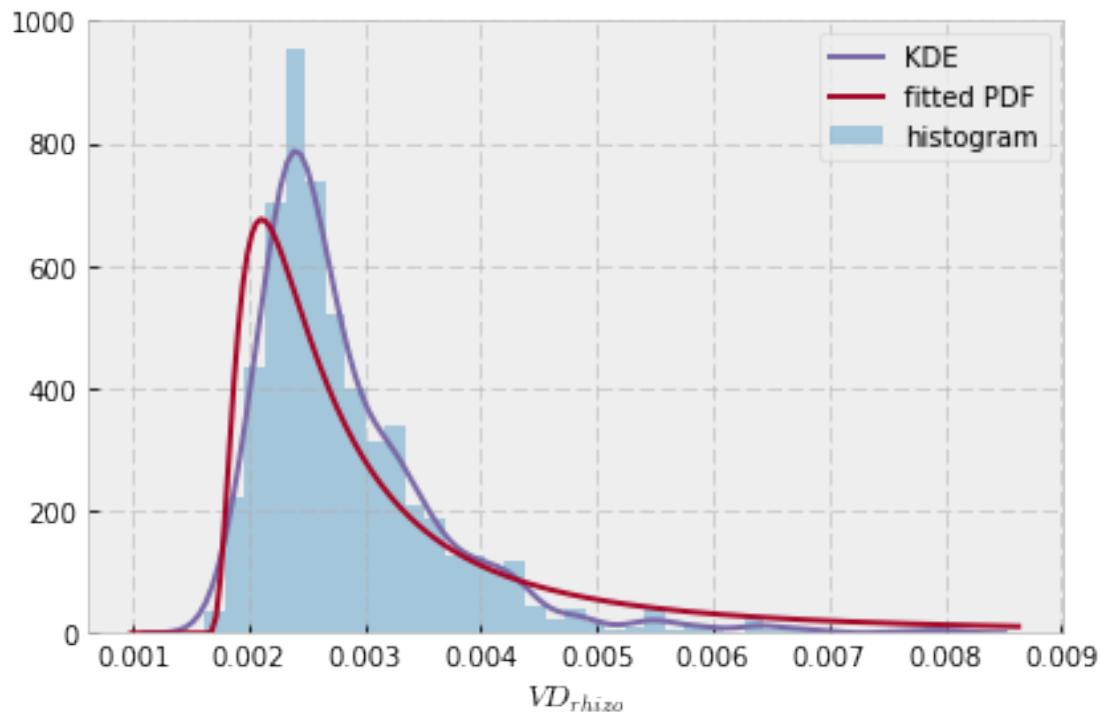


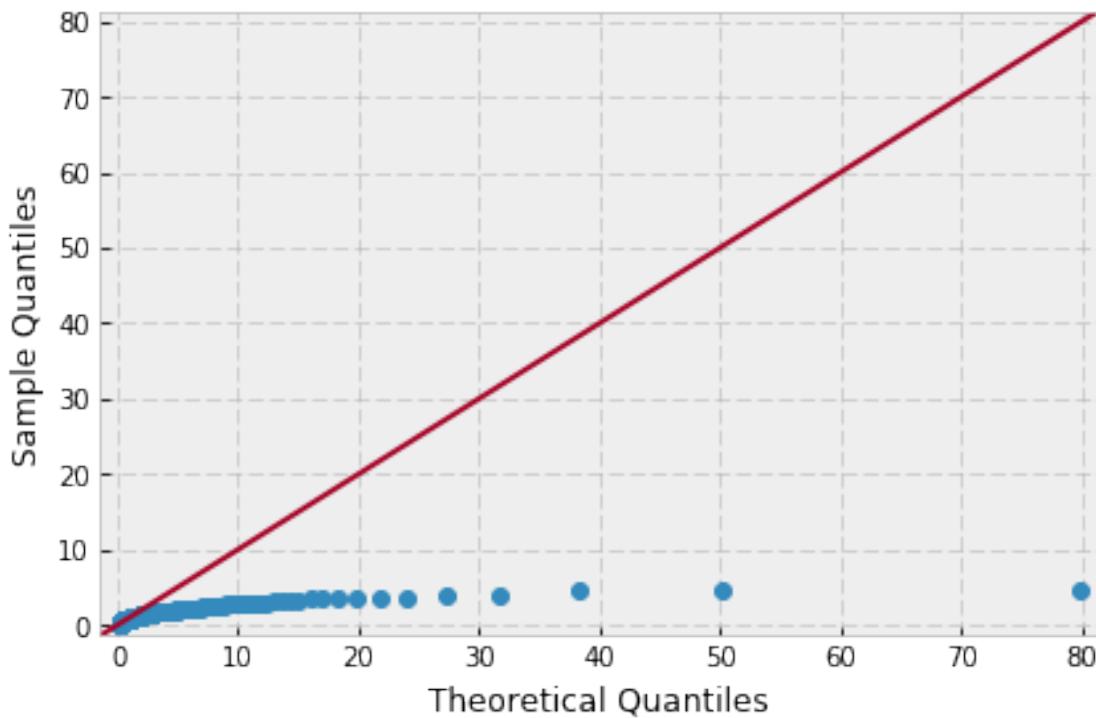


```
variable name: sigma
variable value: 0.0333333333333333
distribution: inverse-gamma(shape=(6.1266652585614523,), loc=0.0010640401613373962, scale=0.00976
DescribeResult(nobs=1000, minmax=(0.0016359404929755686, 0.0082047362163553943)
               mean=0.0029654508584627637, variance=7.8932494117826323e-07,
               skewness=1.6315430785405982, kurtosis=3.837135470469721)
```

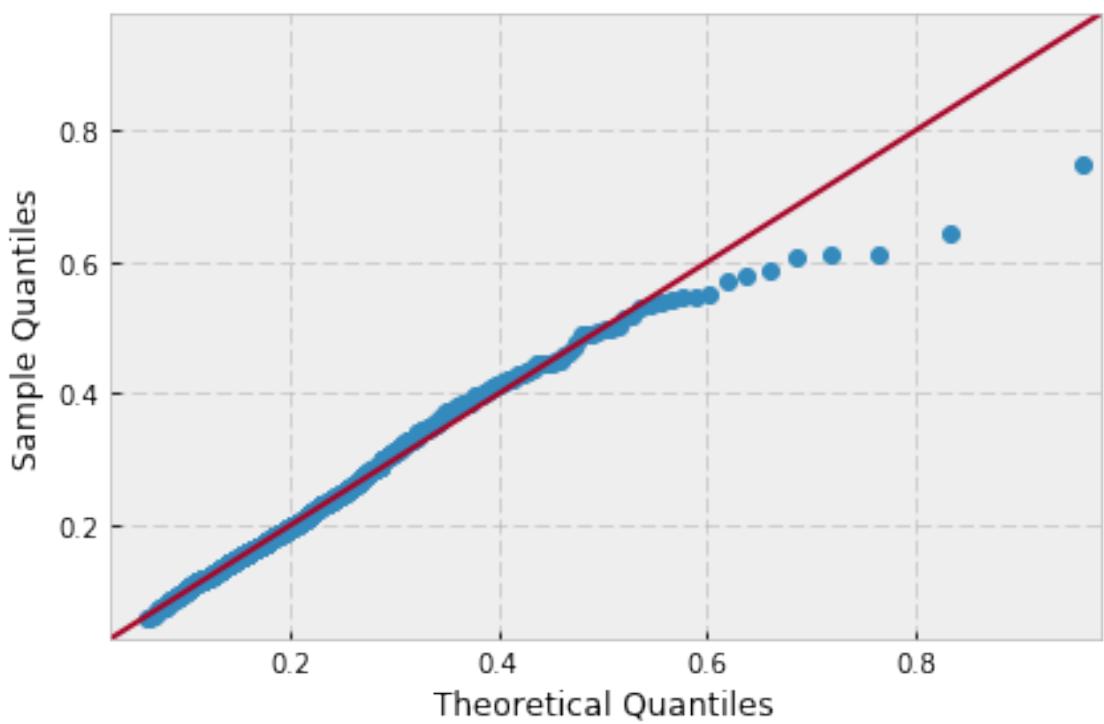
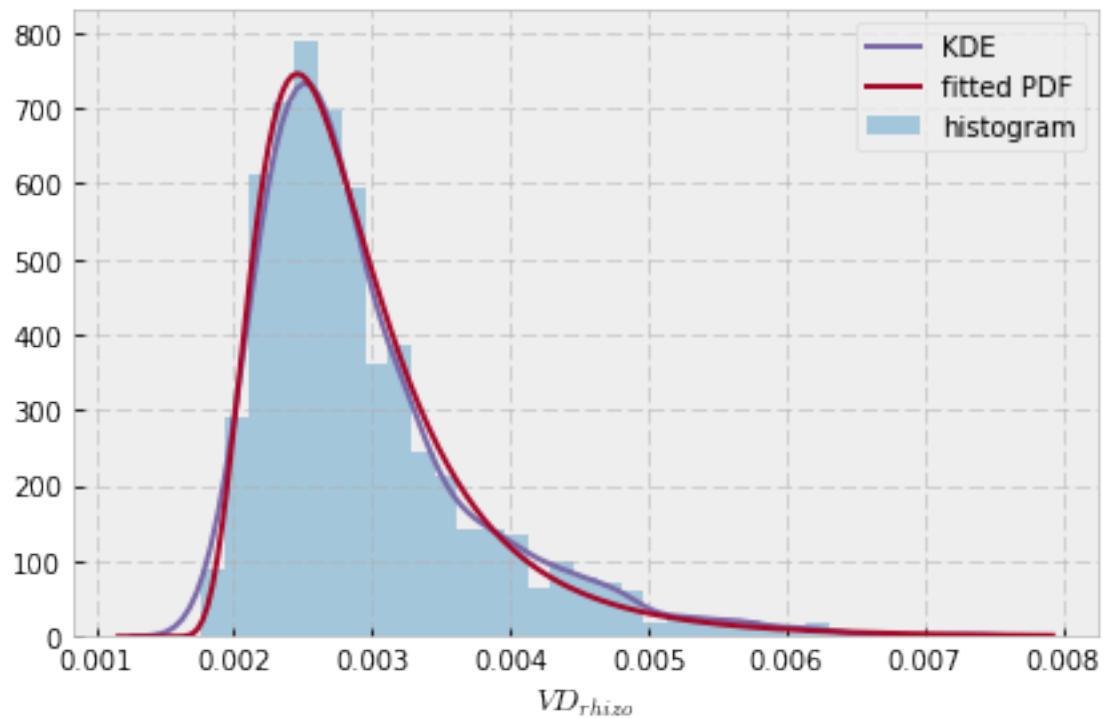


```
variable name: sigma
variable value: 0.04444444444444446
distribution: inverse-gamma(shape=(1.5093857845915739,),loc=0.0015649024624017362, scale=0.00136
DescribeResult(nobs=1000, minmax=(0.0016244228472926511, 0.0079944248705396386)
               mean=0.0028592013965255234, variance=7.1269090678444115e-07,
               skewness=2.0902761697372676, kurtosis=6.456887074828524)
```

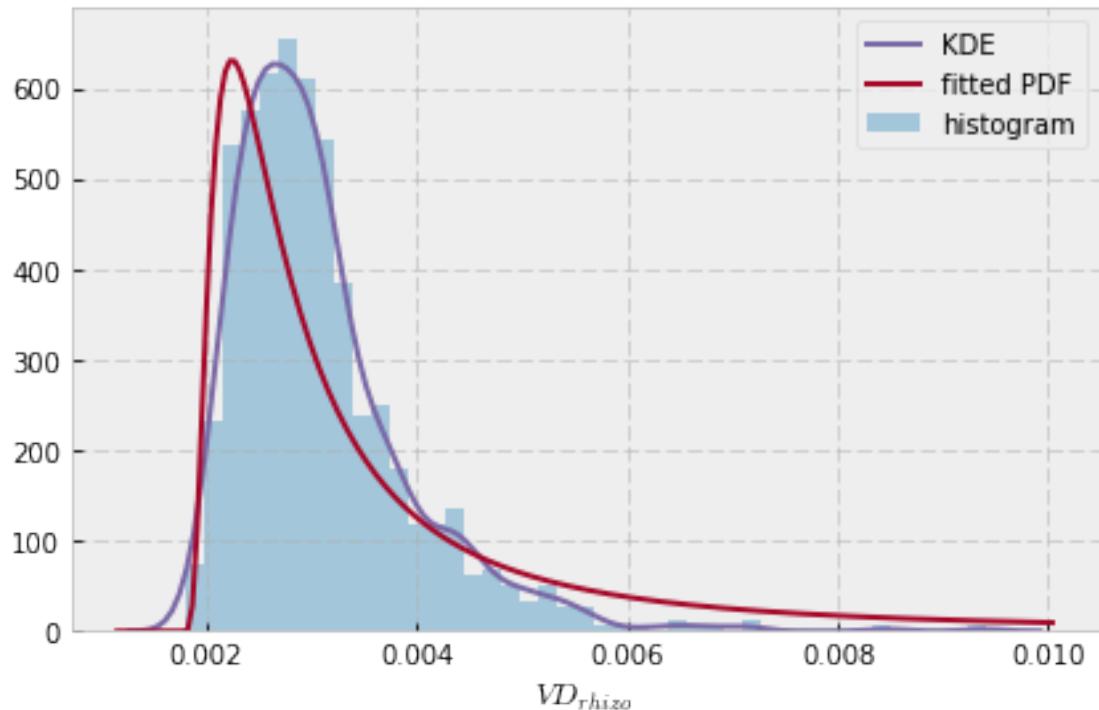


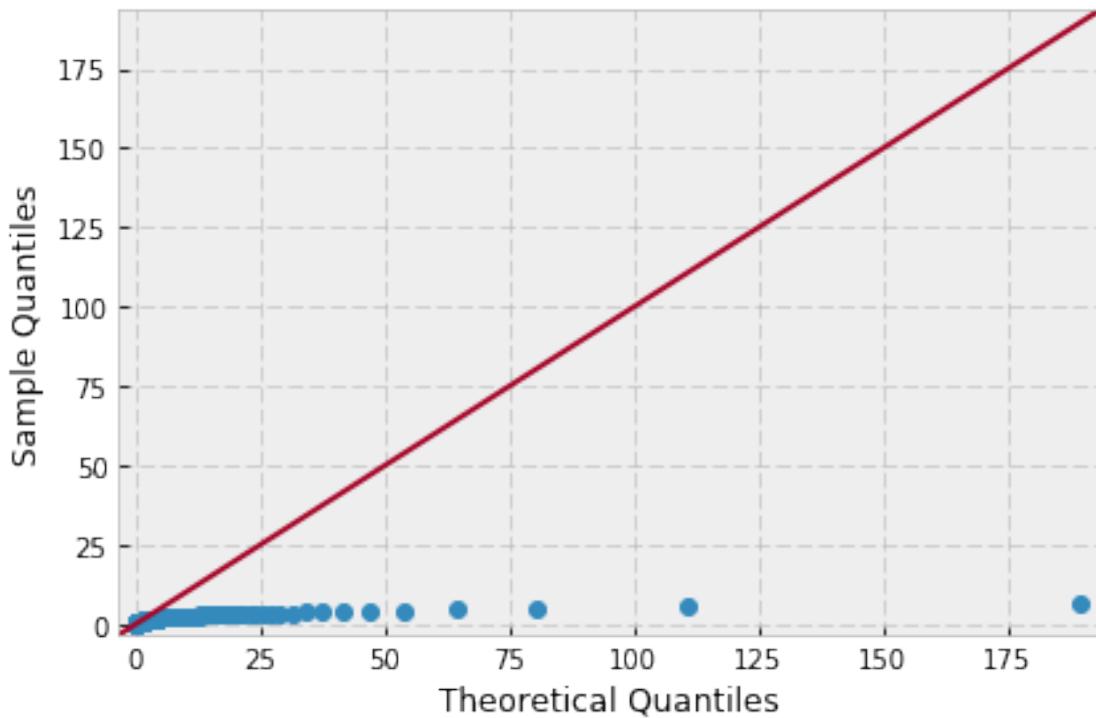


```
variable name: sigma
variable value: 0.05555555555555556
distribution: inverse-gamma(shape=(5.836822008338725,), loc=0.0012780889071907545, scale=0.008068
DescribeResult(nobs=1000, minmax=(0.0017674140994682153, 0.0073210570055445463)
               mean=0.0029424495022576077, variance=6.4236813858788419e-07,
               skewness=1.566006125389721, kurtosis=2.941864827520911)
```

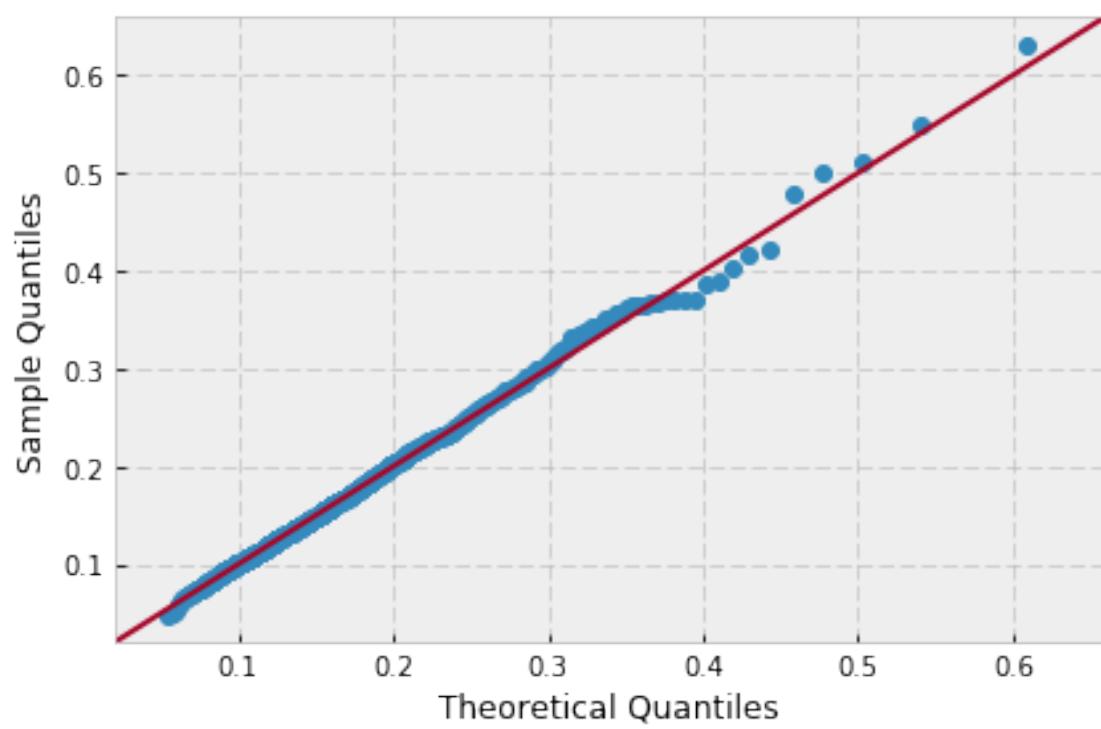
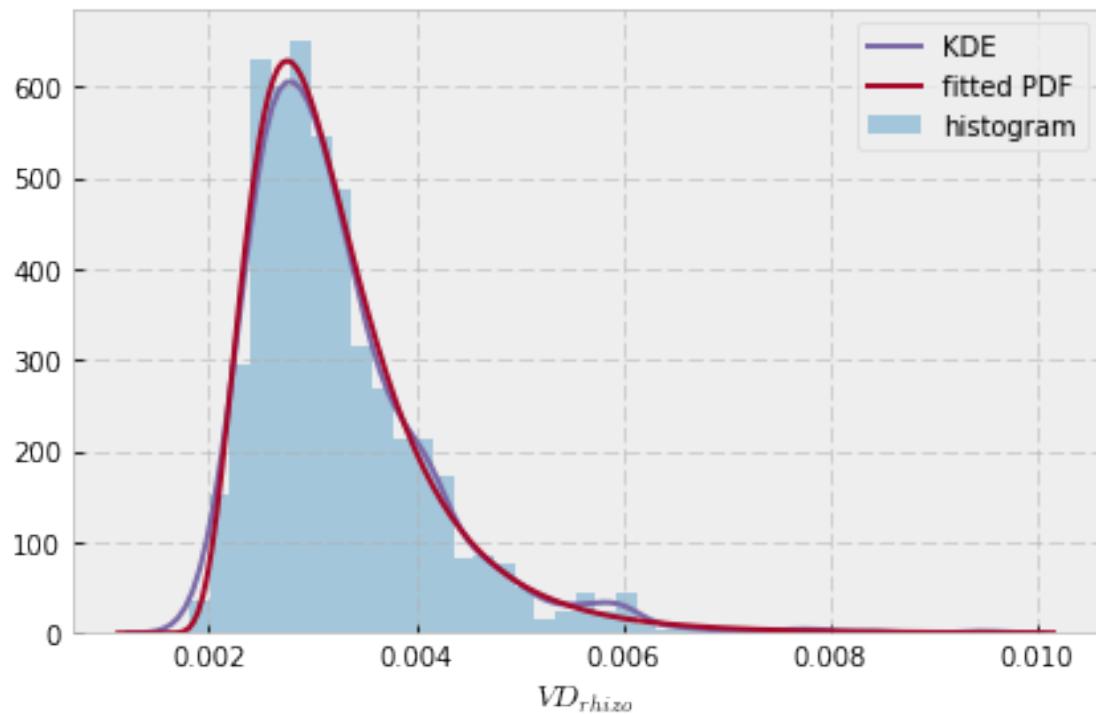


```
variable name: sigma
variable value: 0.06666666666666667
distribution: inverse-gamma(shape=(1.2884191339687763,),loc=0.0017229643640584826, scale=0.00118
DescribeResult(nobs=1000, minmax=(0.0017912037825022543, 0.0093993257187752269)
               mean=0.003076049192978081, variance=7.2347345035023666e-07,
               skewness=1.9061559901528455, kurtosis=6.50894388433411)
```

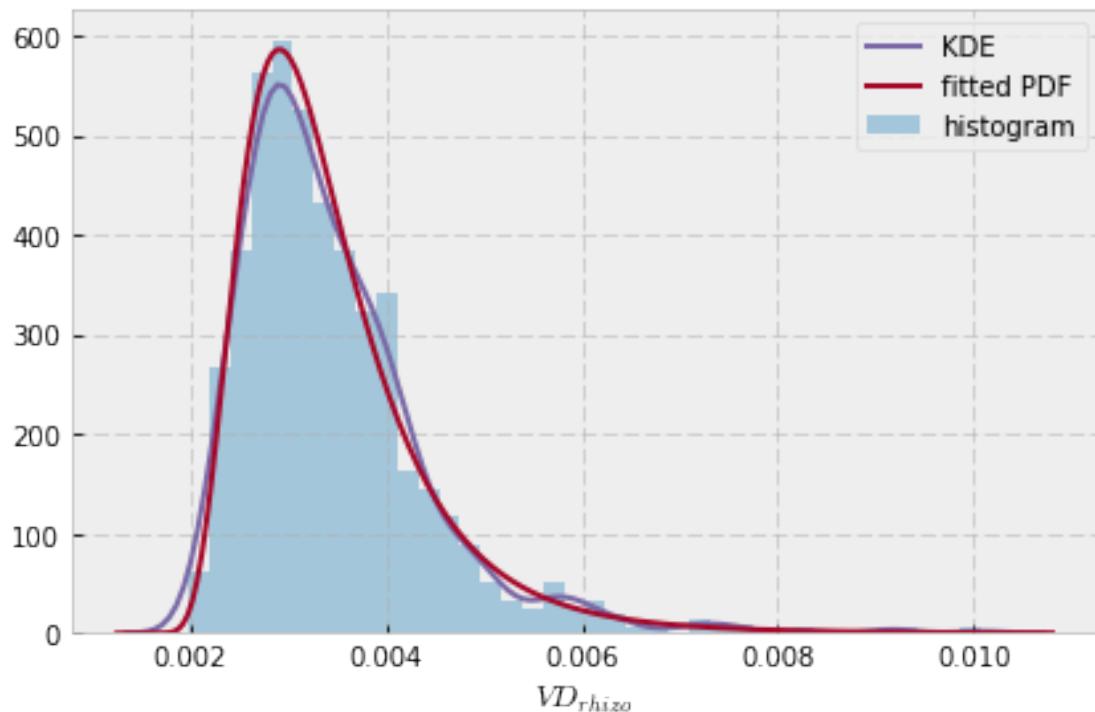


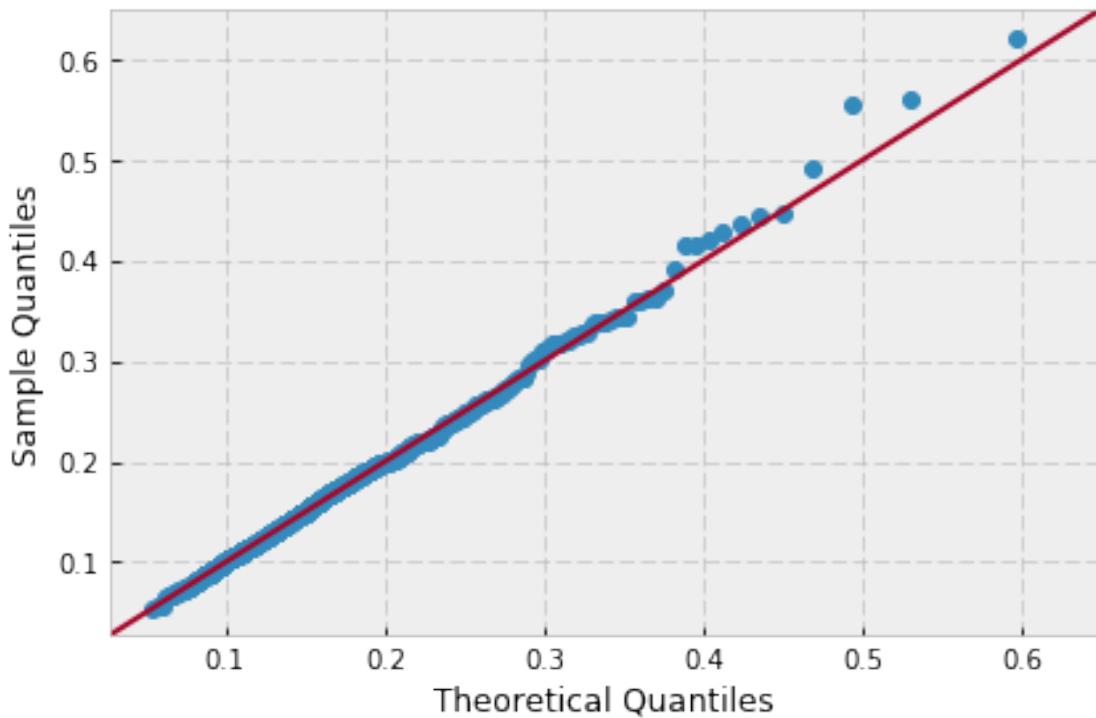


```
variable name: sigma
variable value: 0.07777777777777778
distribution: inverse-gamma(shape=(7.2832096844831096,), loc=0.001165358786954019, scale=0.013177
DescribeResult(nobs=1000, minmax=(0.0018063424991190927, 0.00947064883229164)
               mean=0.0032624470401502723, variance=8.1761652657359497e-07,
               skewness=1.806534916577904, kurtosis=5.444159123292465)
```

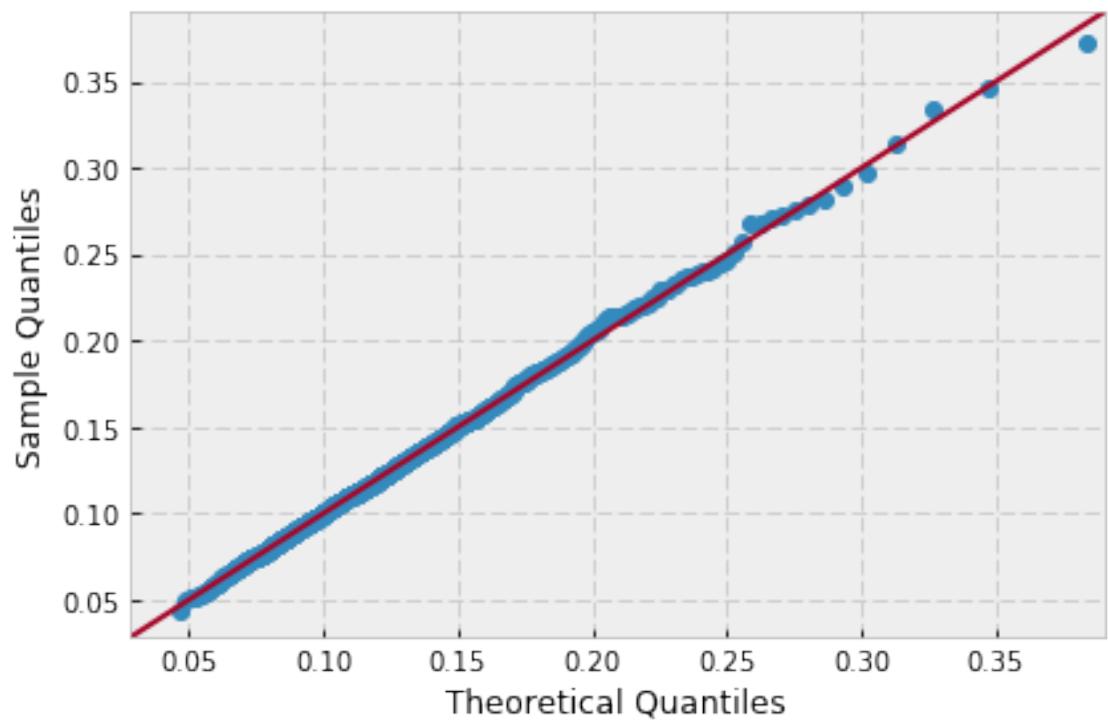
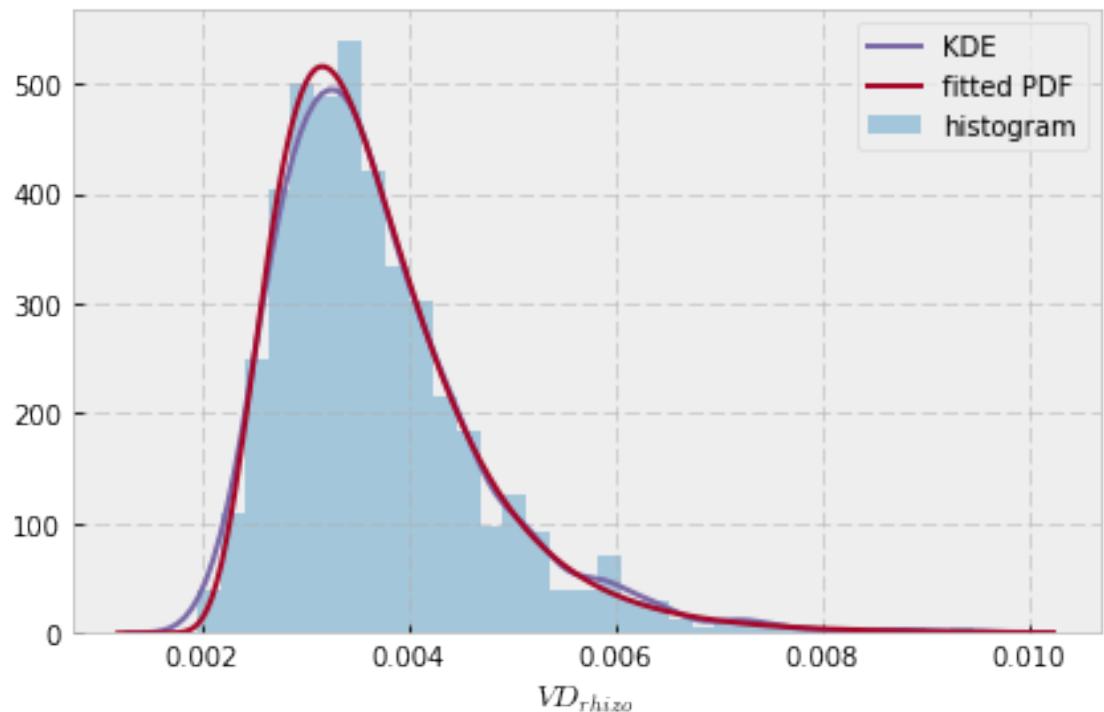


```
variable name: sigma
variable value: 0.0888888888888889
distribution: inverse-gamma(shape=(7.3561475784456487,),loc=0.0011931347516363493, scale=0.01432
DescribeResult(nobs=1000, minmax=(0.0019784857293162561, 0.010076738159161236)
               mean=0.0034462432327736908, variance=9.3504823260342616e-07,
               skewness=1.9146816349124816, kurtosis=6.446734373199323)
```





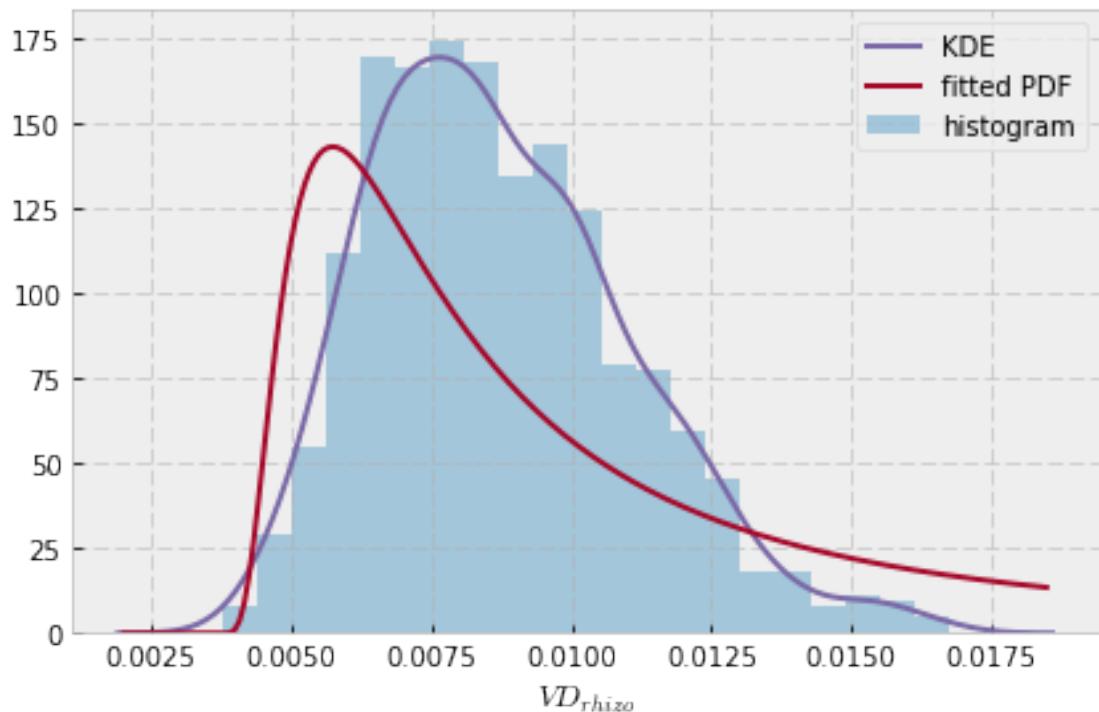
```
variable name: sigma
variable value: 0.1
distribution: inverse-gamma(shape=(9.3112893145902511,),loc=0.0009448625419084141, scale=0.02292
DescribeResult(nobs=1000, minmax=(0.0019572184805259152, 0.0094694656593463673)
               mean=0.0037029969437132495, variance=1.0199806576472797e-06,
               skewness=1.4642771440917934, kurtosis=3.3647734561245066)
```

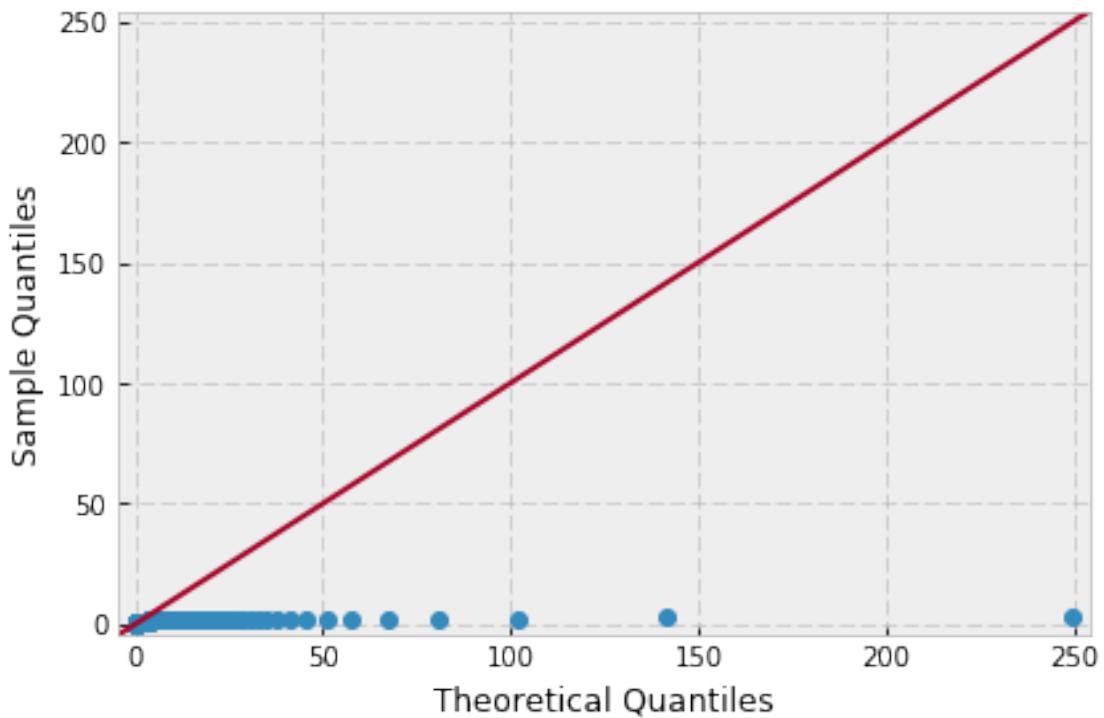


```

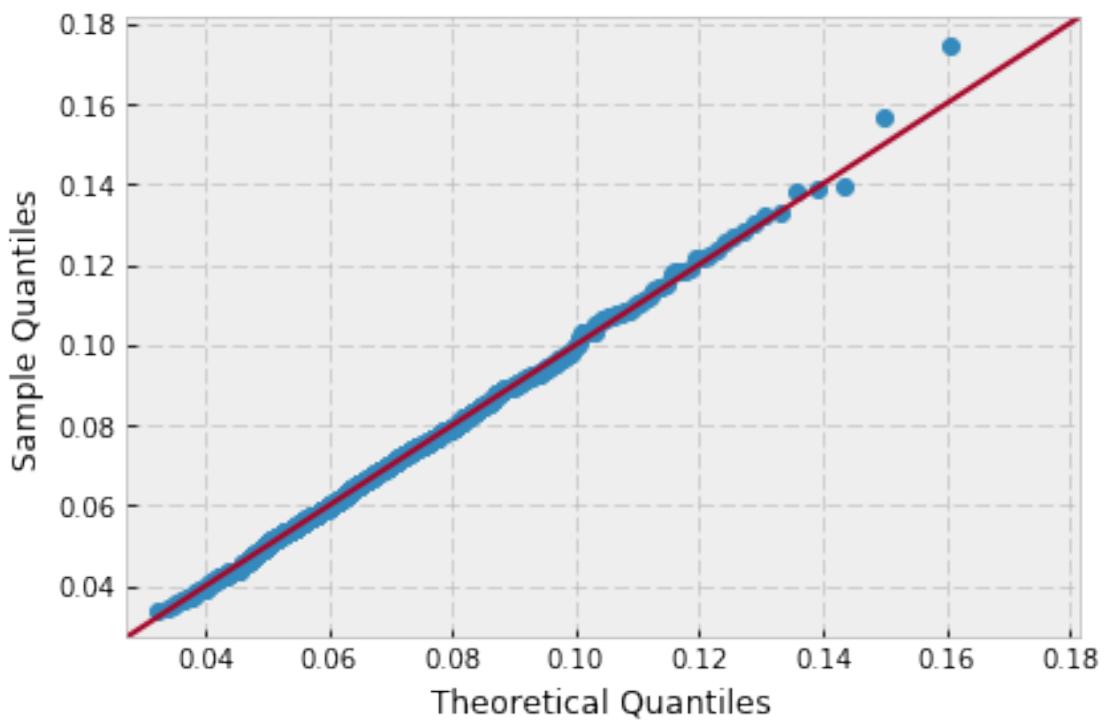
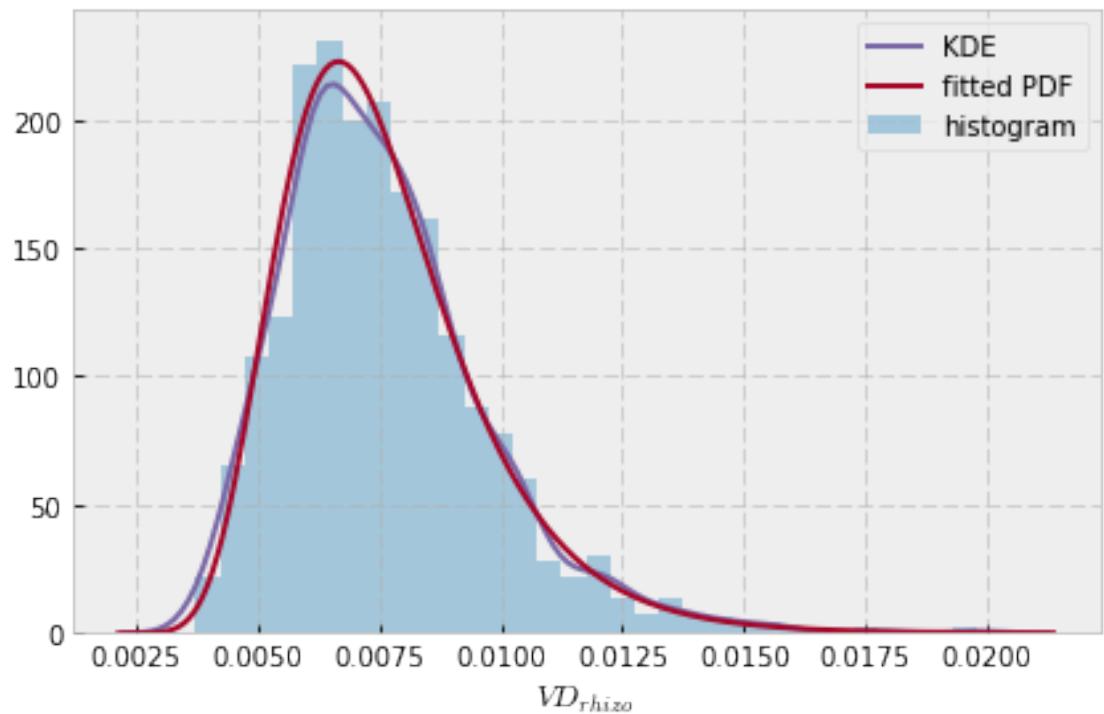
variable name: theta
variable value: 0.6981317007977318
distribution: inverse-gamma(shape=(1.2307663931687611,), loc=0.003532471225316033, scale=0.004931
DescribeResult(nobs=1000, minmax=(0.0037608089453414549, 0.016732081358806706)
               mean=0.0087346114890267772, variance=5.3956976625590969e-06,
               skewness=0.6214777255318995, kurtosis=0.18310970257810855)

```

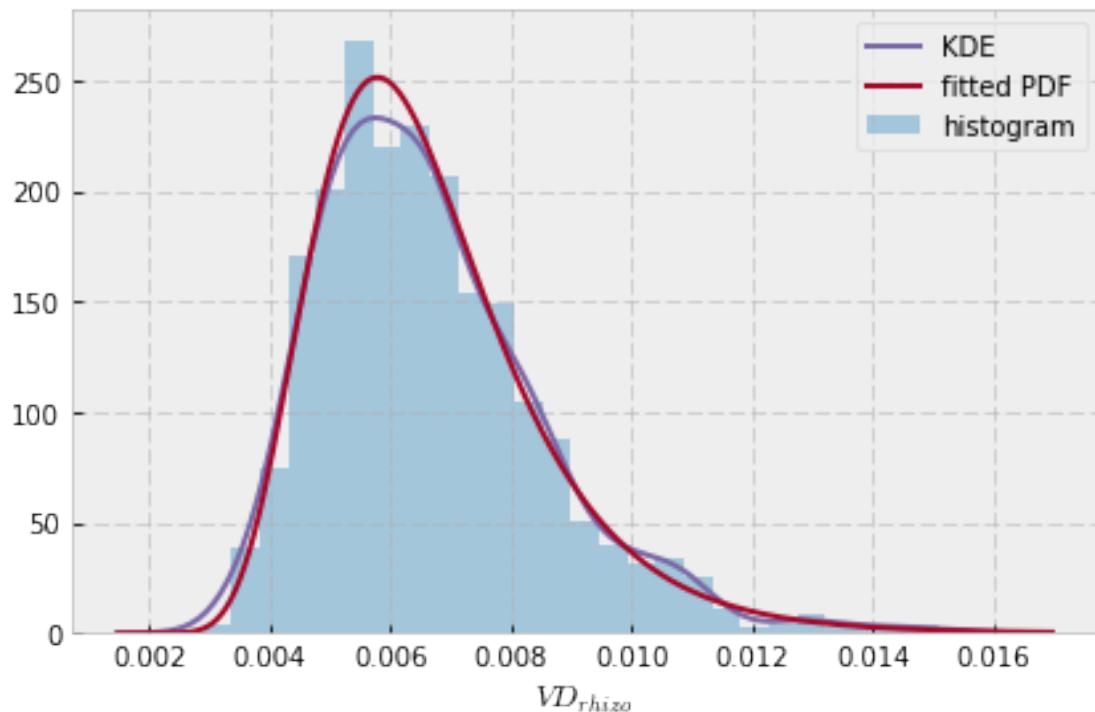


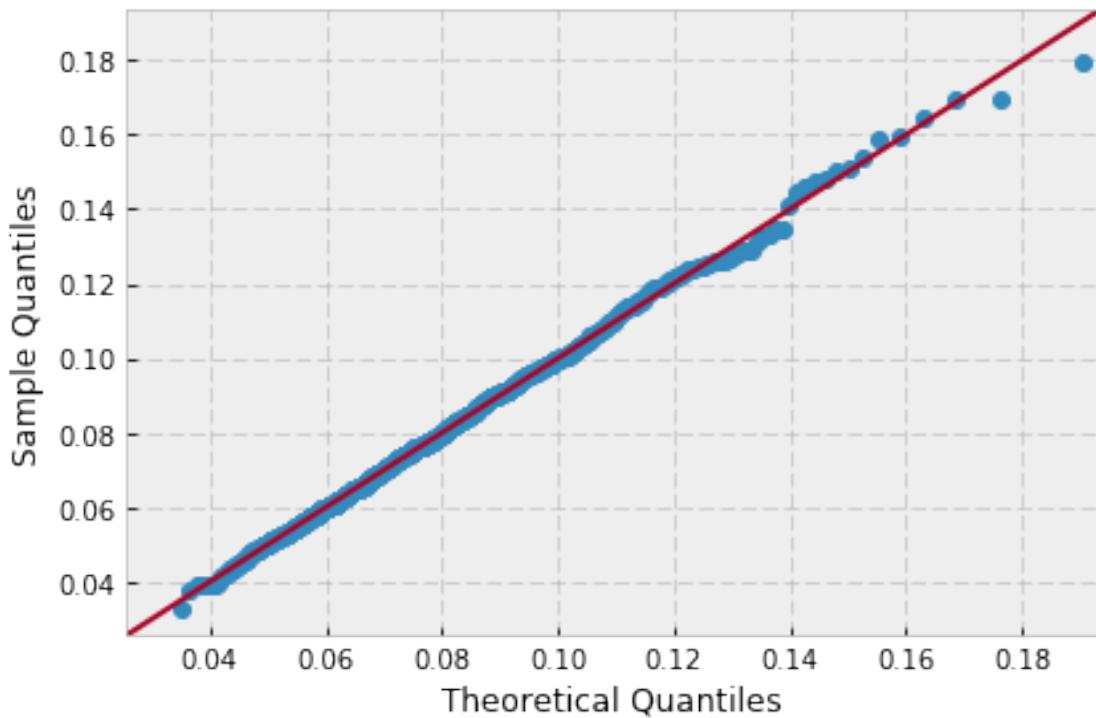


```
variable name: theta
variable value: 0.7853981633974483
distribution: inverse-gamma(shape=(15.70542855610255,), loc=-0.00018921553349609036, scale=0.1144
DescribeResult(nobs=1000, minmax=(0.003718300094256507, 0.0197783554744617)
               mean=0.0075937474293178413, variance=4.4029331088477259e-06,
               skewness=1.1287140456313176, kurtosis=2.347628227380257)
```

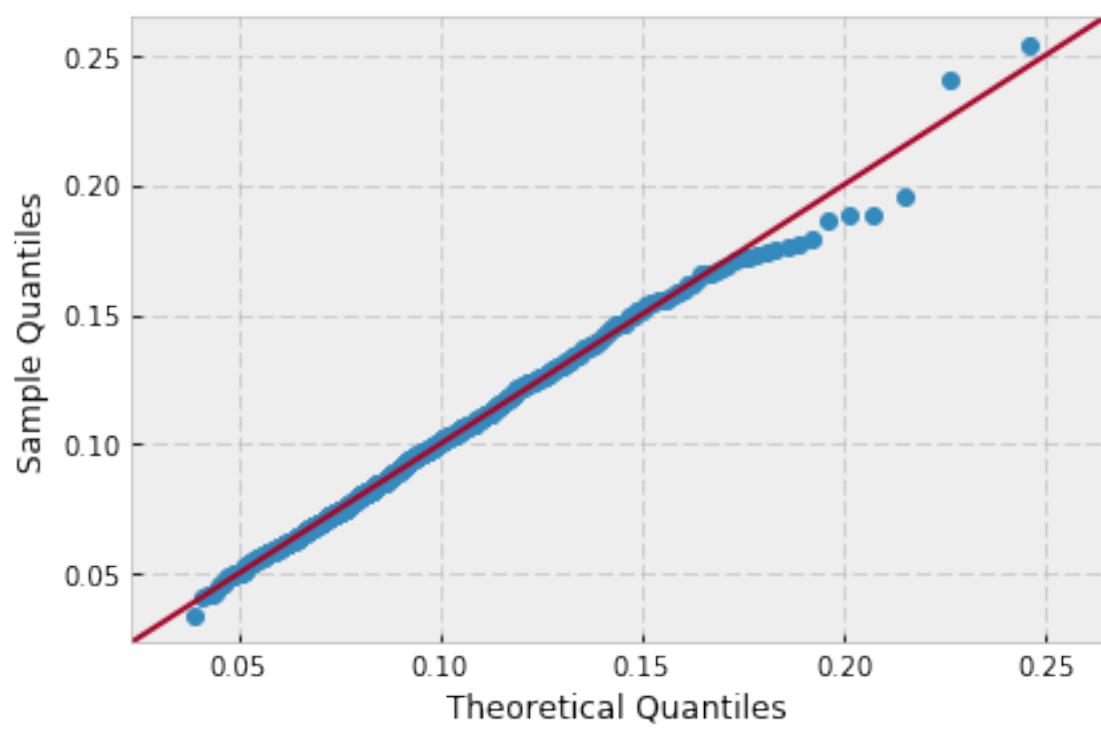
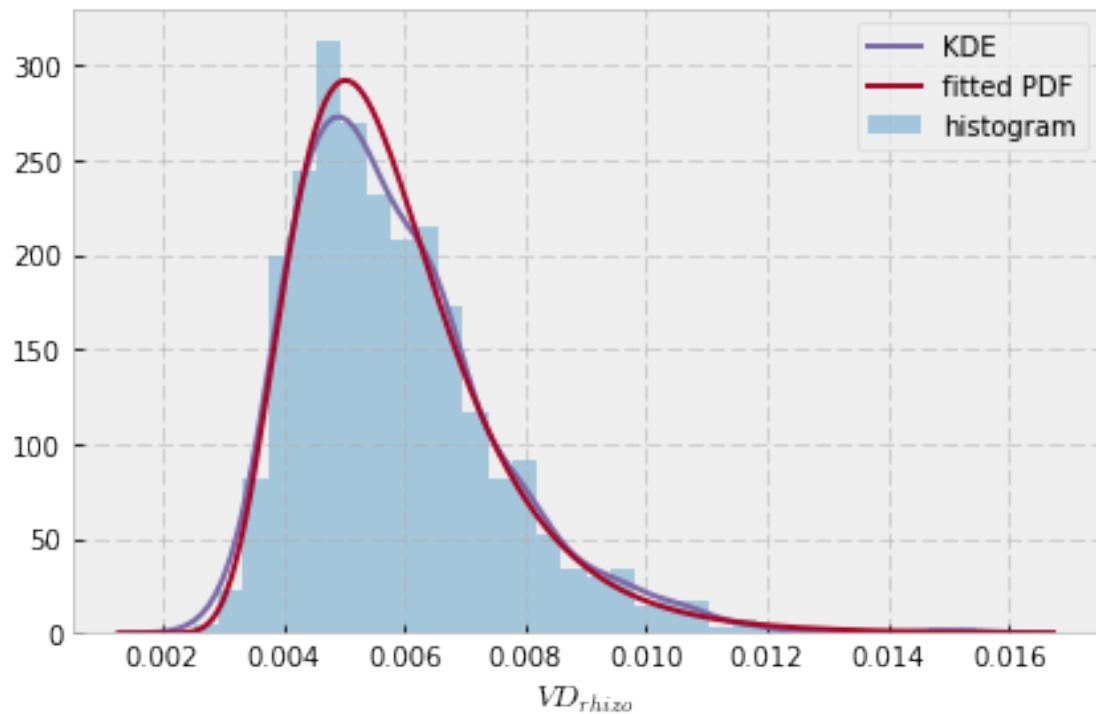


```
variable name: theta
variable value: 0.8726646259971648
distribution: inverse-gamma(shape=(14.09683369840938,), loc=5.662746913602786e-05, scale=0.086392)
DescribeResult(nobs=1000, minmax=(0.002887727069843938, 0.015554770810298948)
               mean=0.006651804663040598, variance=3.532289867555285e-06,
               skewness=1.0818124263748519, kurtosis=1.7450721607948516)
```

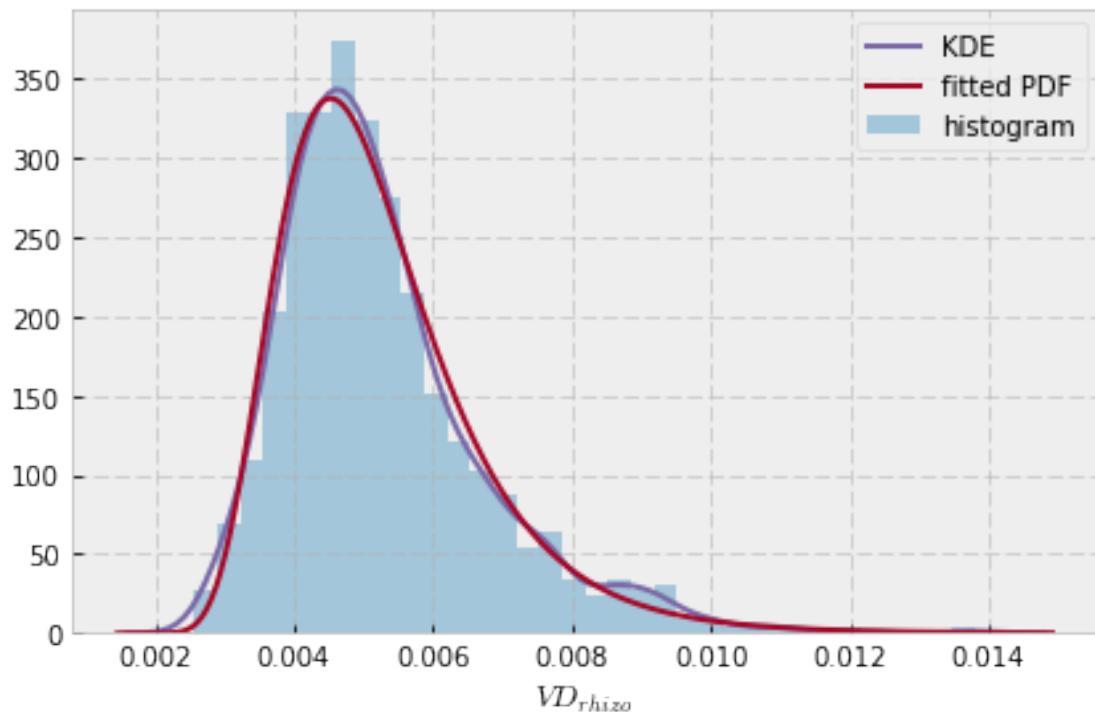


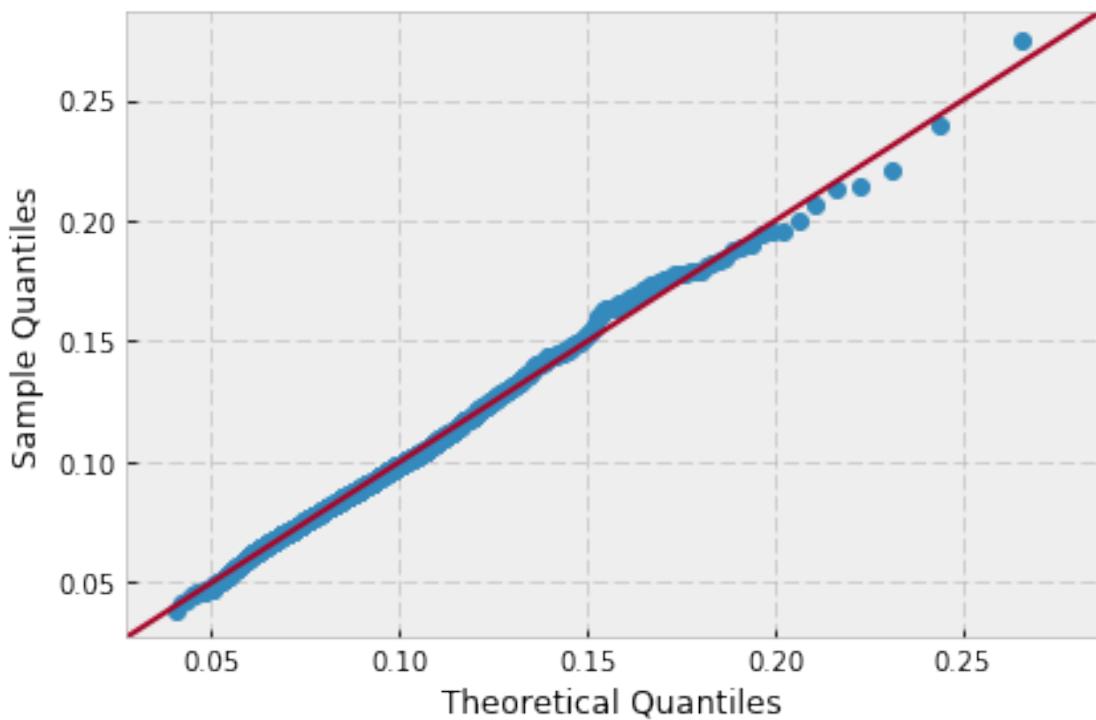


```
variable name: theta
variable value: 0.9599310885968813
distribution: inverse-gamma(shape=(12.046129961233337,),loc=0.0004961172240495228, scale=0.05905
DescribeResult(nobs=1000, minmax=(0.0025198359004930111, 0.015481515125034317)
               mean=0.0058401543644847724, variance=2.7659908653177378e-06,
               skewness=1.1735768659280152, kurtosis=2.285772043806145)
```

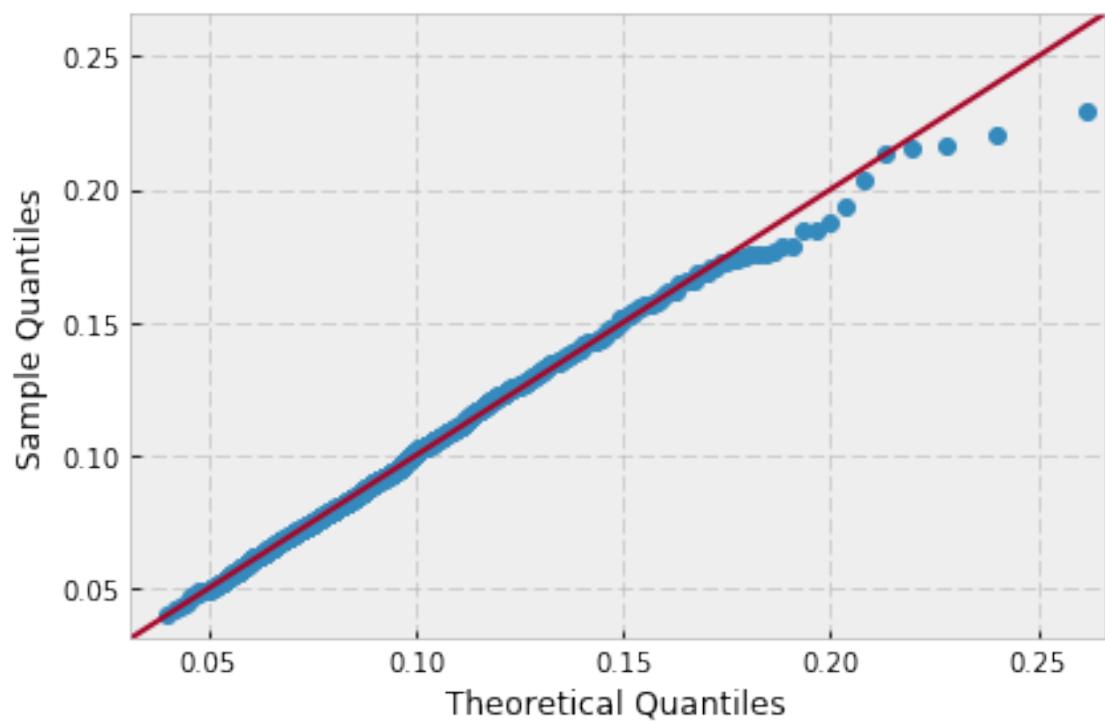
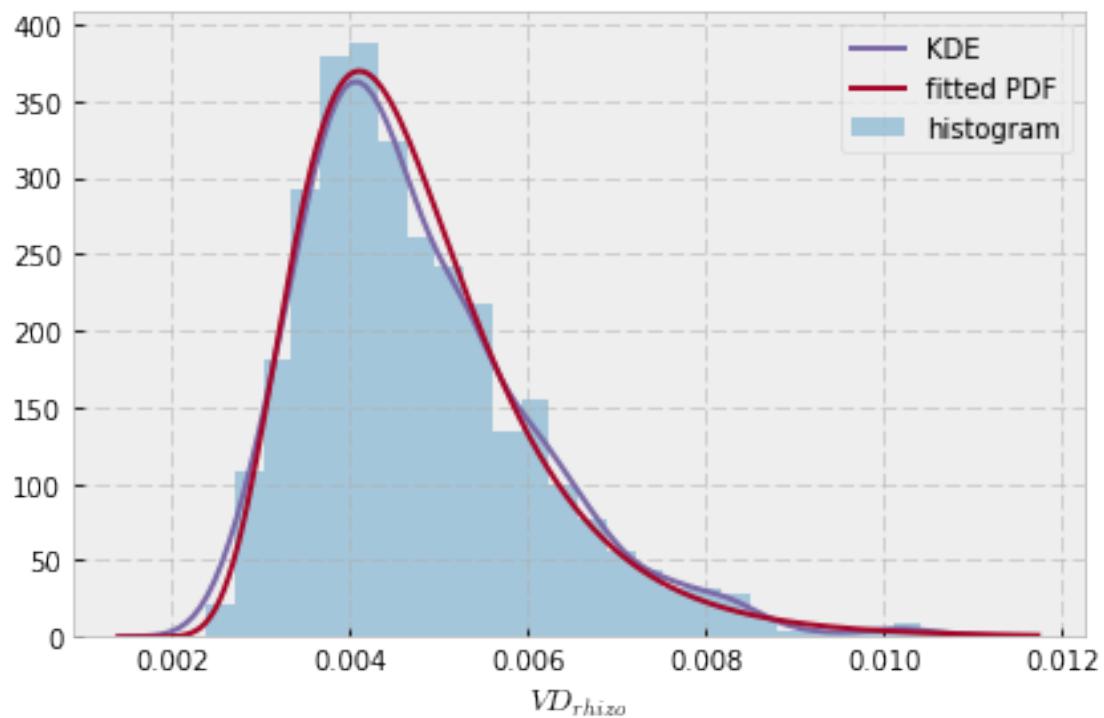


```
variable name: theta
variable value: 1.0471975511965979
distribution: inverse-gamma(shape=(11.503372102021705,),loc=0.0007101132231400918, scale=0.04770
DescribeResult(nobs=1000, minmax=(0.0025657769631552193, 0.013809295016725546)
               mean=0.0052518475843835654, variance=2.1652790298378616e-06,
               skewness=1.310523068057476, kurtosis=2.5793610804512994)
```

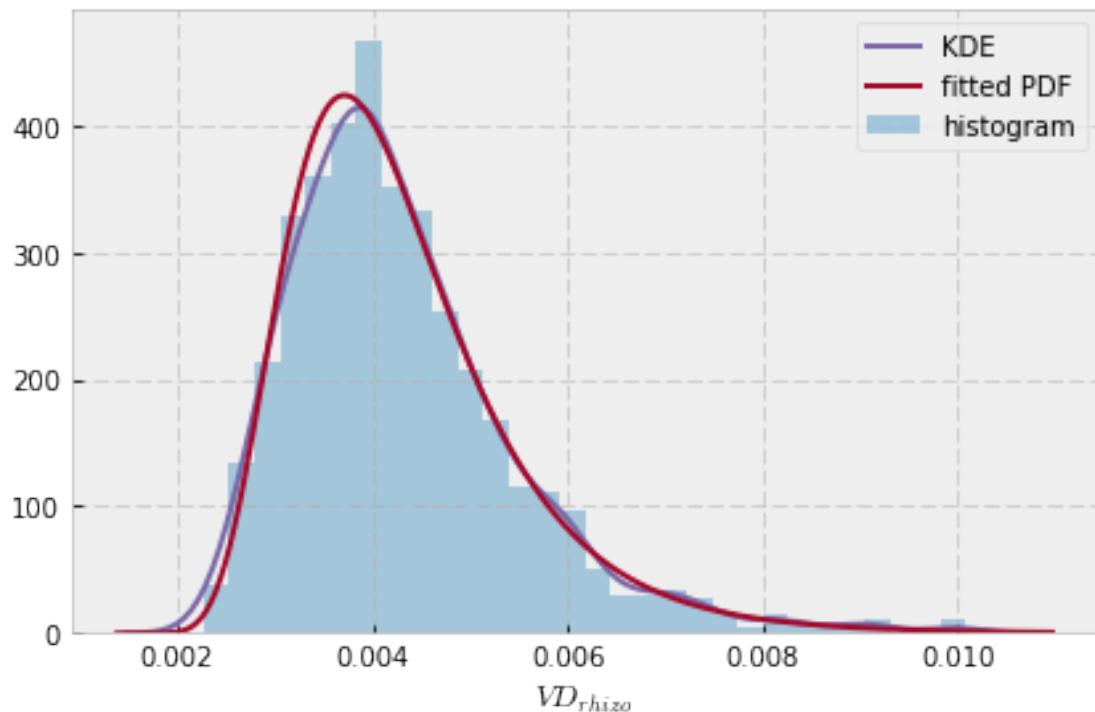


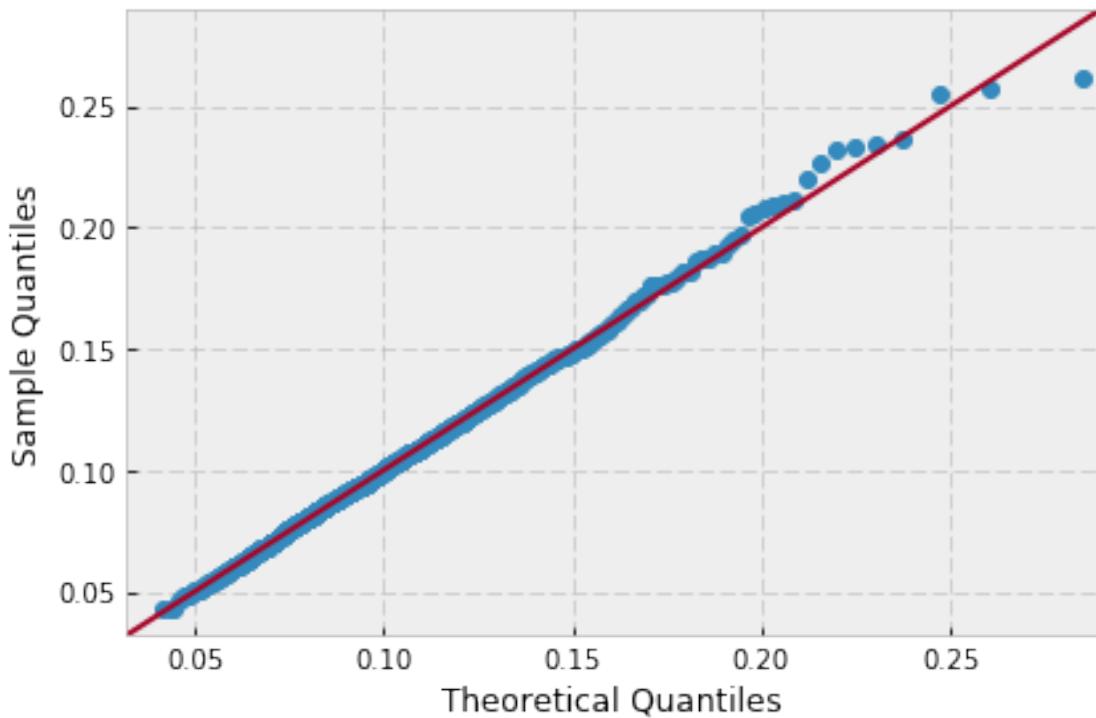


```
variable name: theta
variable value: 1.1344640137963142
distribution: inverse-gamma(shape=(11.607485850055793,), loc=0.0005961407585981307, scale=0.04418
DescribeResult(nobs=1000, minmax=(0.0023776751529159942, 0.010736886526612959)
               mean=0.0047589547625148214, variance=1.7242662523797189e-06,
               skewness=1.0764211272510884, kurtosis=1.4762476976076568)
```

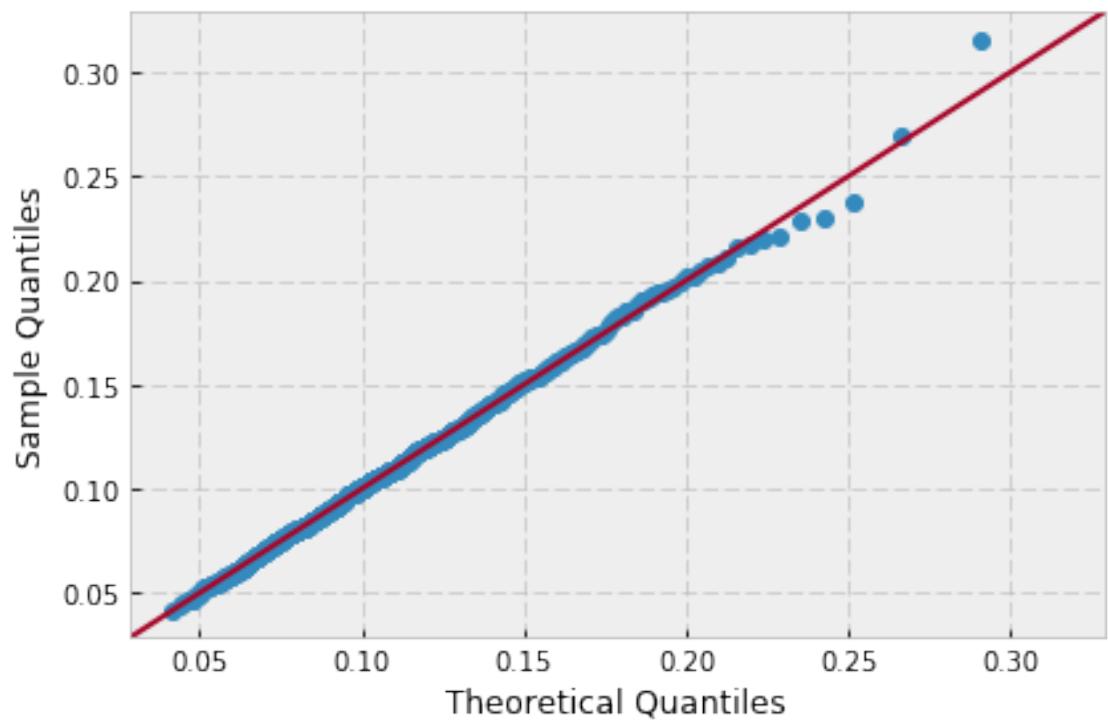
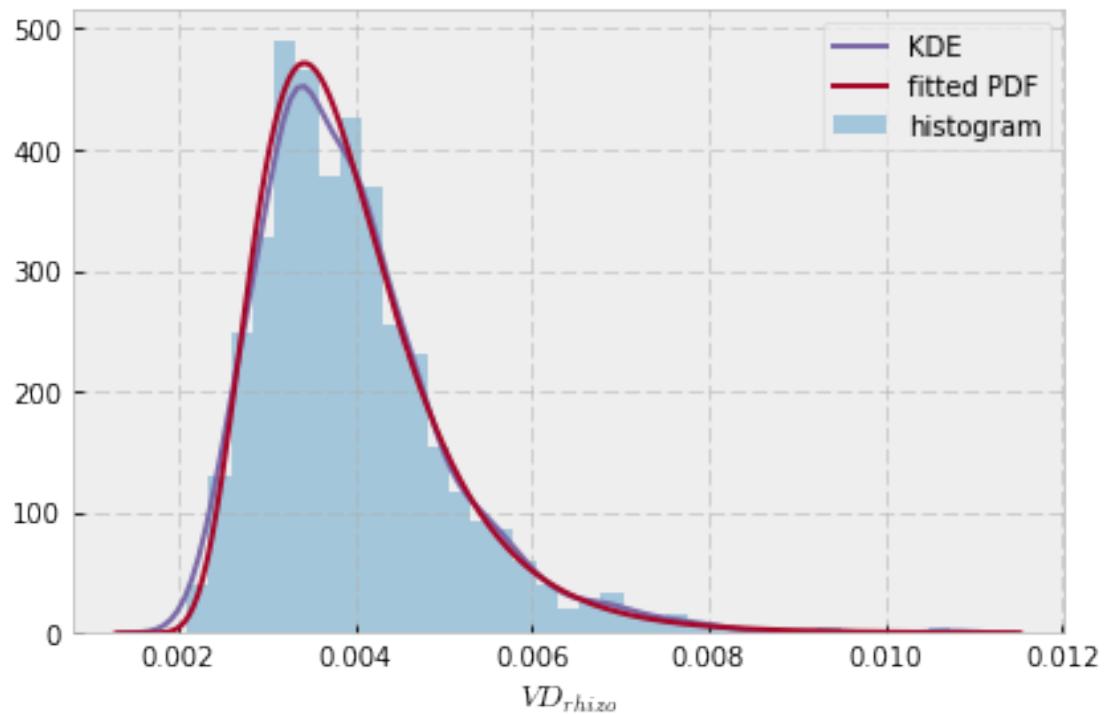


```
variable name: theta
variable value: 1.2217304763960306
distribution: inverse-gamma(shape=(11.038575635557656,),loc=0.0007419501596742319, scale=0.03575
DescribeResult(nobs=1000, minmax=(0.0022681631248651249, 0.010086695934138466)
               mean=0.0043031980184460644, variance=1.3859476890423934e-06,
               skewness=1.3393720942210663, kurtosis=2.8073121235081713)
```

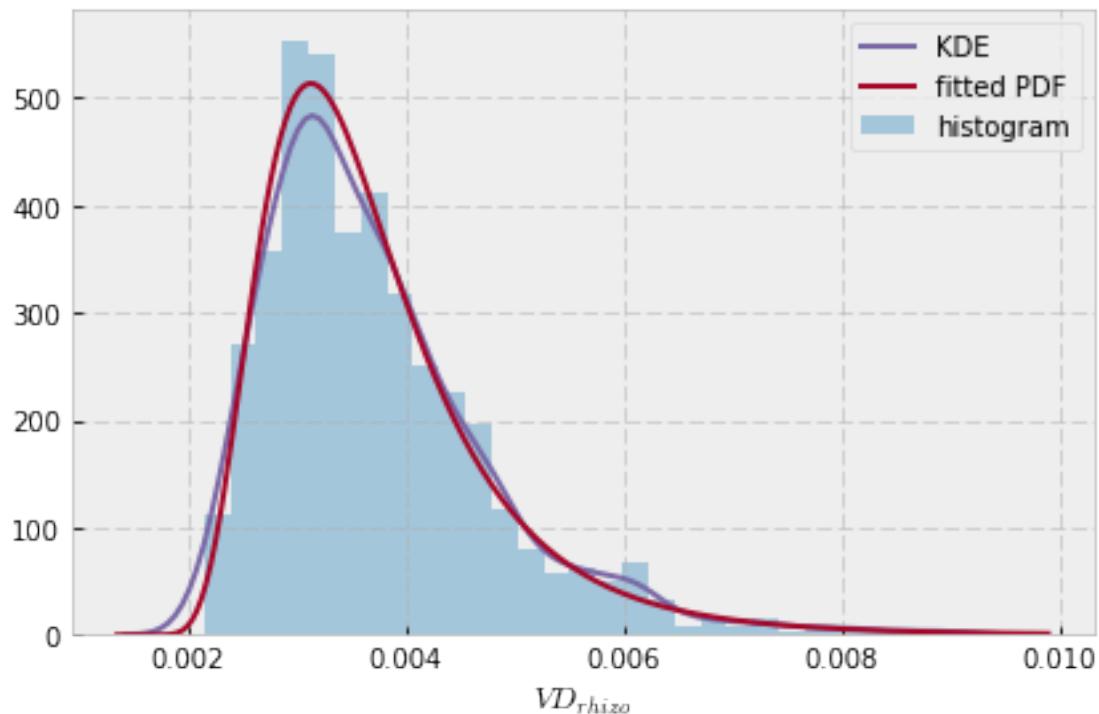


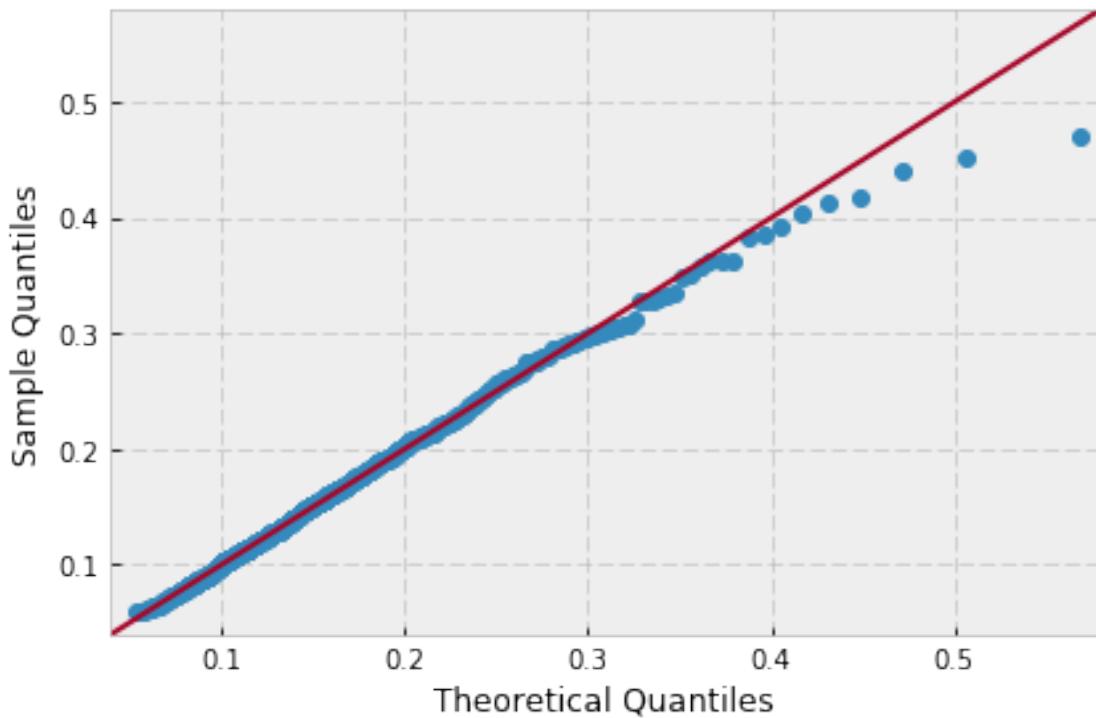


```
variable name: theta
variable value: 1.3089969389957472
distribution: inverse-gamma(shape=(10.898220679328542,), loc=0.0007694838713077968, scale=0.03158
DescribeResult(nobs=1000, minmax=(0.0020965931851537325, 0.010724158864890875)
               mean=0.0039593449978636329, variance=1.123772506035122e-06,
               skewness=1.3512424598595774, kurtosis=3.192166131335865)
```

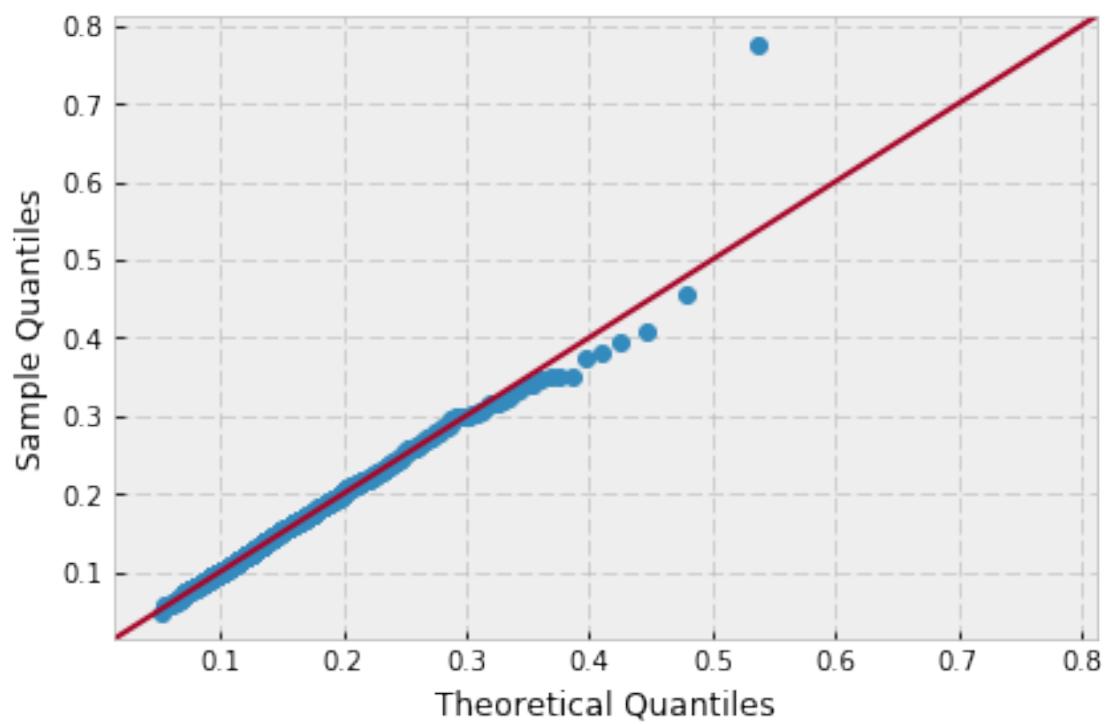
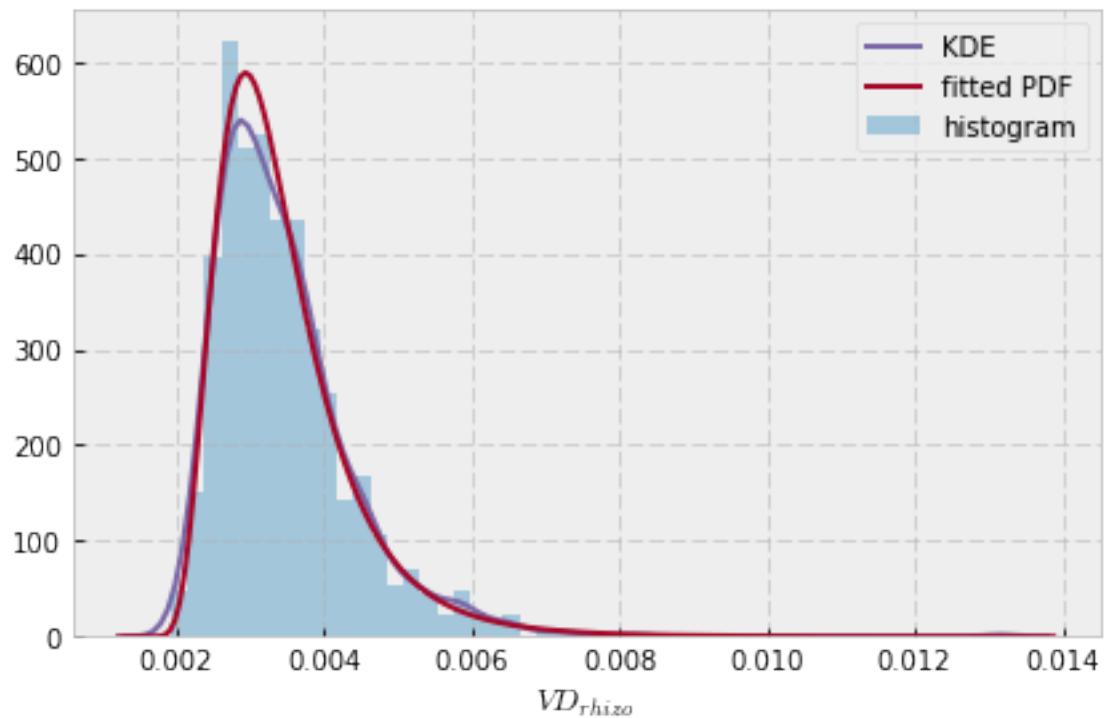


```
variable name: theta
variable value: 1.3962634015954638
distribution: inverse-gamma(shape=(7.5363503012667774,),loc=0.0011430464137846396, scale=0.01693
DescribeResult(nobs=1000, minmax=(0.002143360775687702, 0.0091020165919117602)
               mean=0.0037296145657502743, variance=1.1132588285082868e-06,
               skewness=1.4254576745224377, kurtosis=2.859211965663592)
```





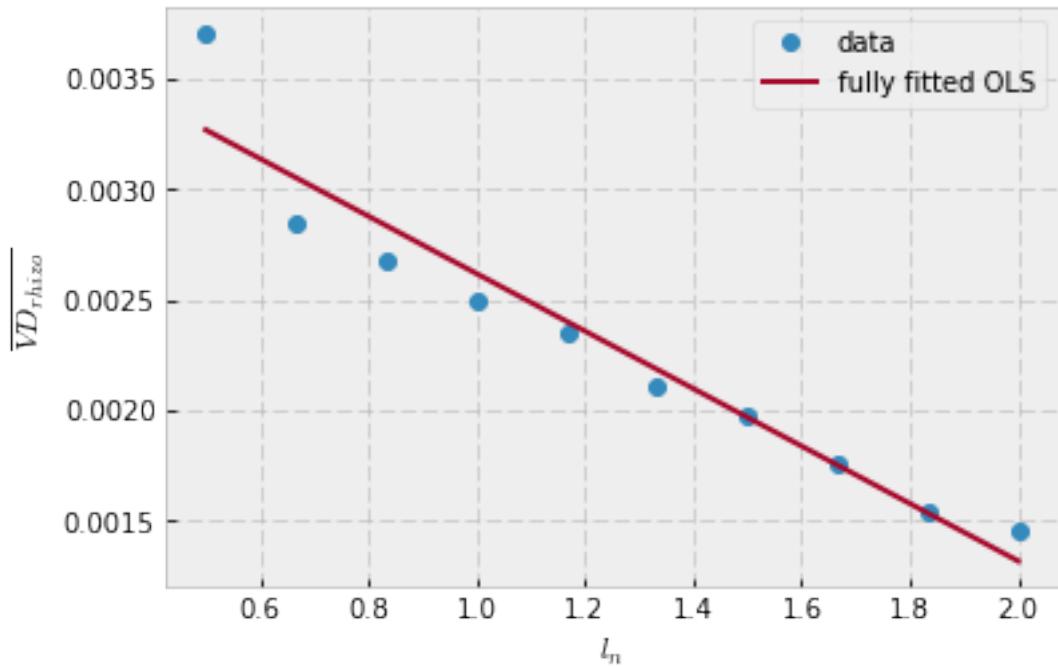
```
variable name: theta
variable value: 1.4835298641951802
distribution: inverse-gamma(shape=(7.7707112228133077,),loc=0.0011740394773986562, scale=0.01544
DescribeResult(nobs=1000, minmax=(0.0019285690458628849, 0.013163526825684602)
               mean=0.0034542655215307233, variance=8.9432717699323905e-07,
               skewness=2.1822673222614735, kurtosis=12.42133296982851)
```



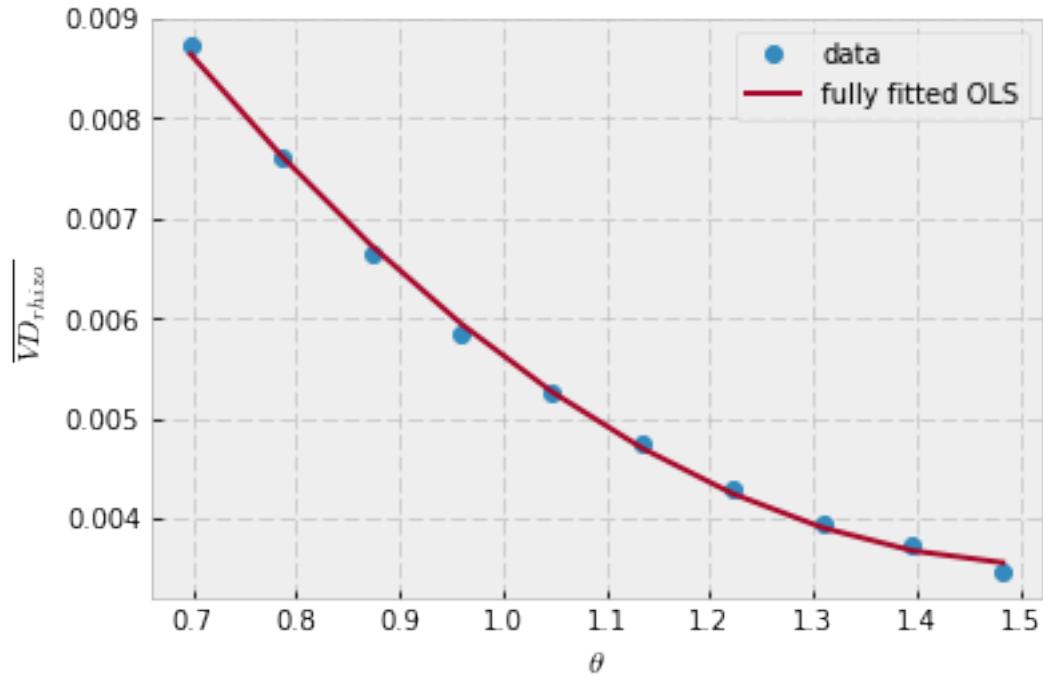
14.2 Parameter regression

```
In [34]: var_name = 'rhizosphere_volume_phosphate_per_convex_hull_volume'  
    mean_reg('ln', var_name, 'poly1')  
    mean_reg('theta', var_name, 'poly2')  
    mean_reg('maxB', var_name, 'poly1')  
    mean_reg('r', var_name, 'poly2')  
    mean_reg('sigma', var_name, 'poly2')  
    mean_reg('N', var_name, 'poly1')
```

variable name: ln
model: poly1
r-squared: 0.8469640536223522
explained variance: 0.8477260568878549



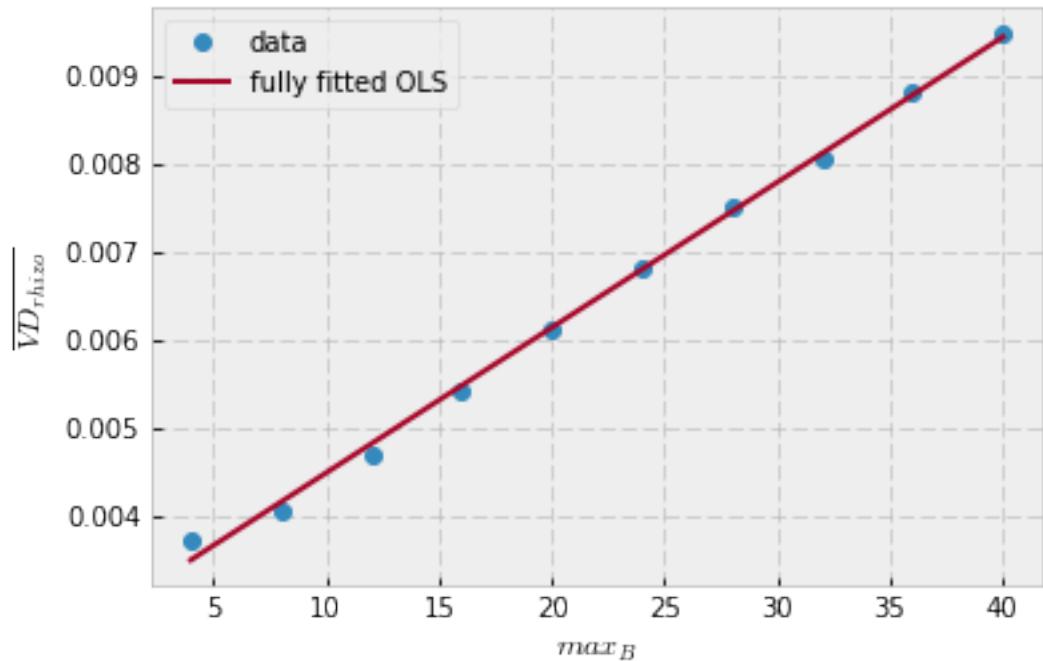
variable name: theta
model: poly2
r-squared: 0.9934022863281164
explained variance: 0.9934023034342006



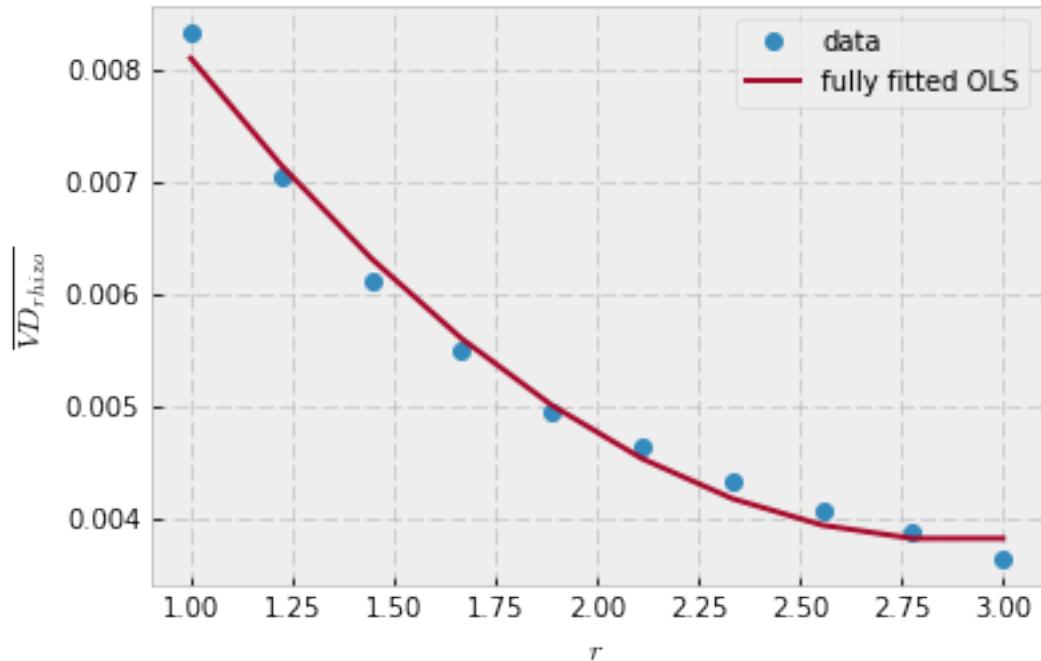
```

variable name: maxB
model: poly1
r-squared: 0.9947889721294337
explained variance: 0.9948068934853409

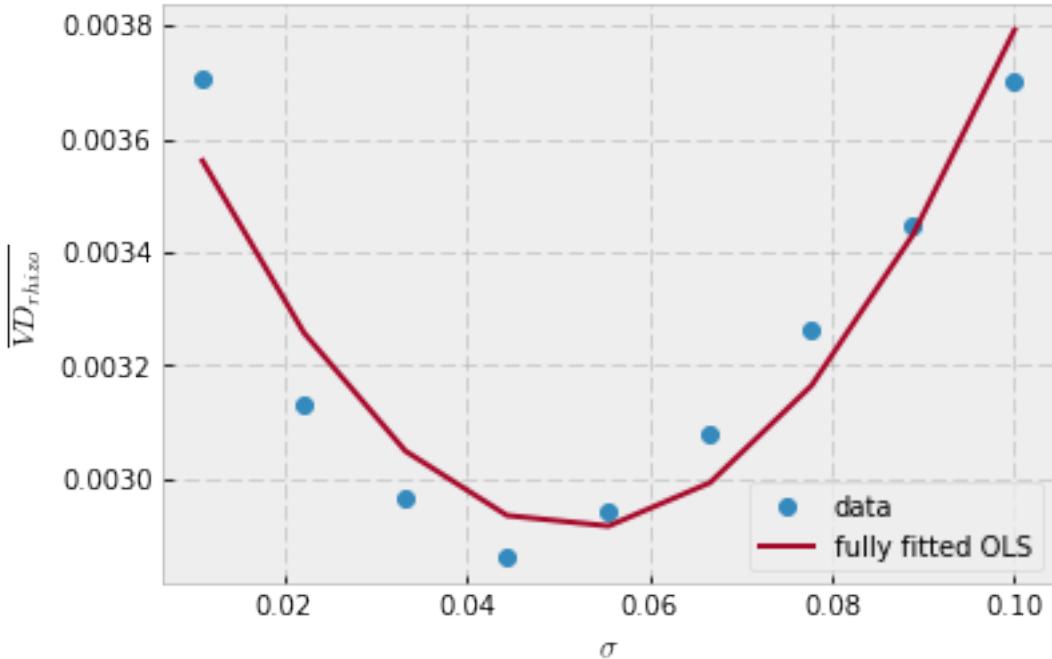
```



```
variable name: r
model: poly2
r-squared: 0.9600306650817725
explained variance: 0.9600469021315615
```



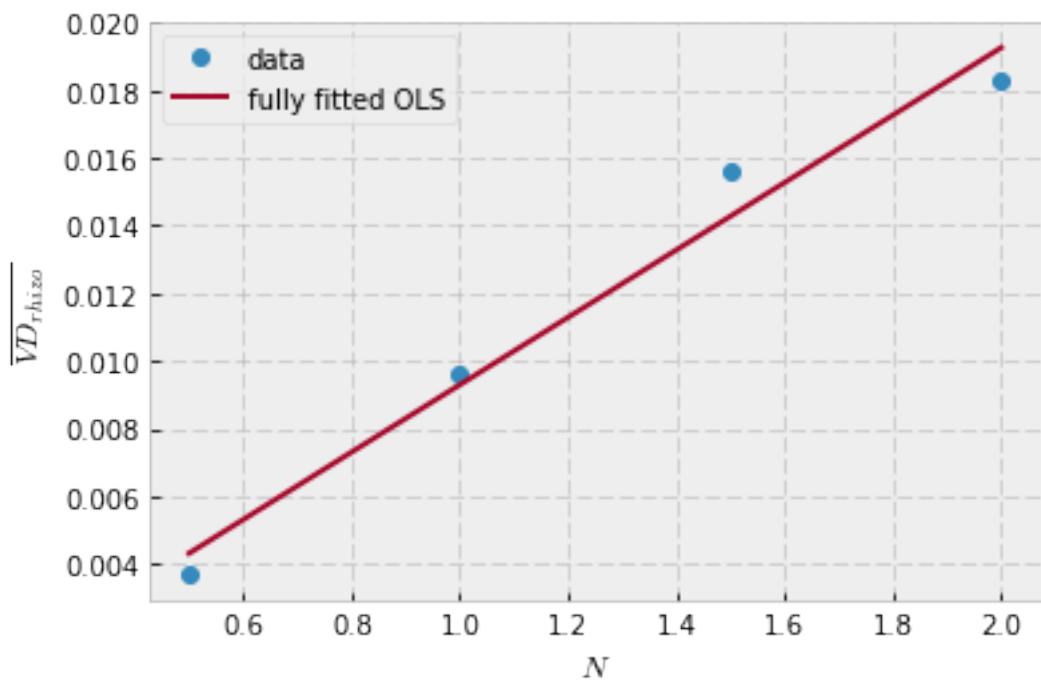
```
variable name: sigma
model: poly2
r-squared: 0.5935366076451365
explained variance: 0.5944969856533838
```



```

variable name: N
model: poly1
r-squared: 0.8531560016897919
explained variance: 0.8716983774172935

```

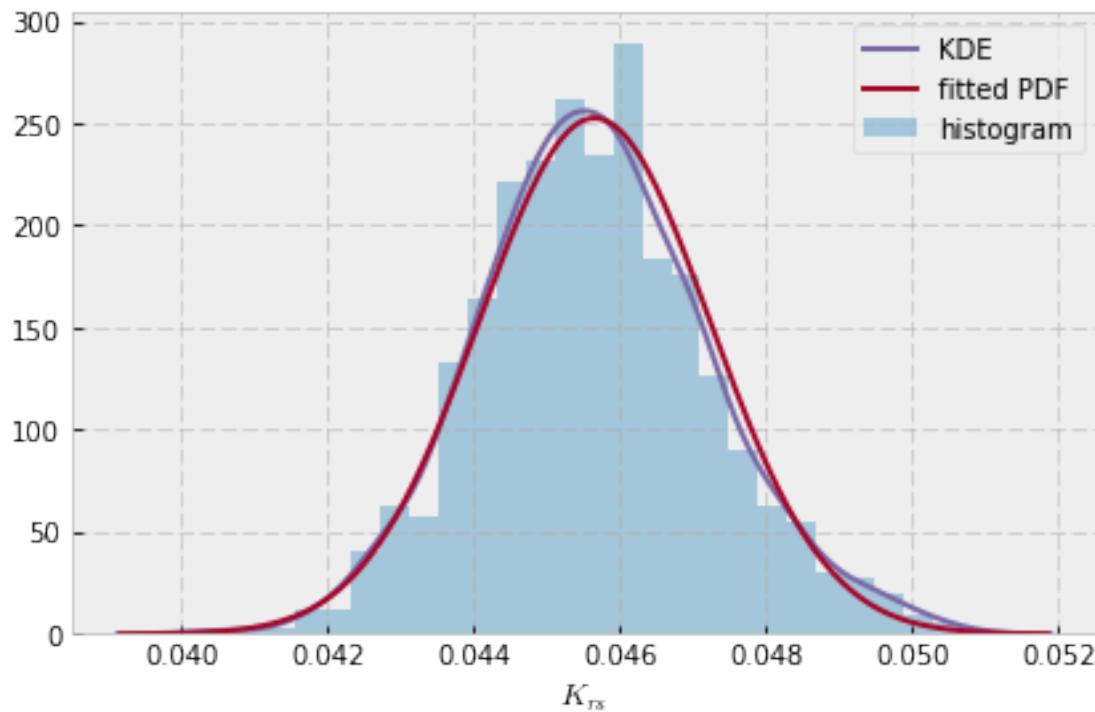


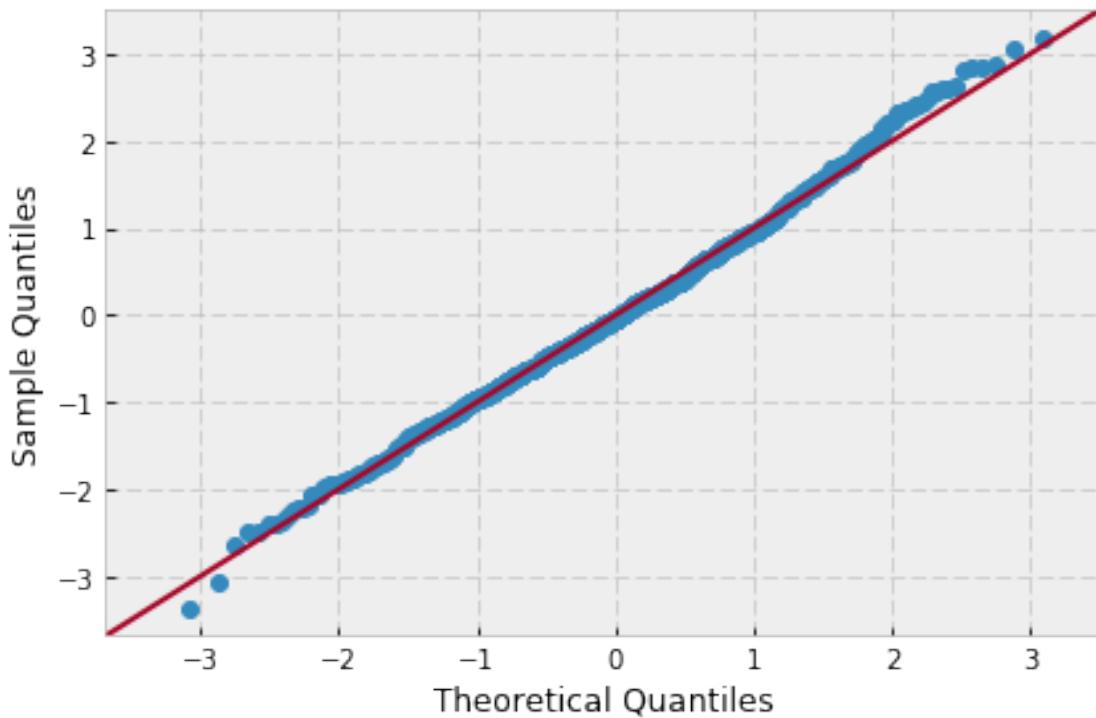
15 Root system total conductance

15.1 Probability distributions

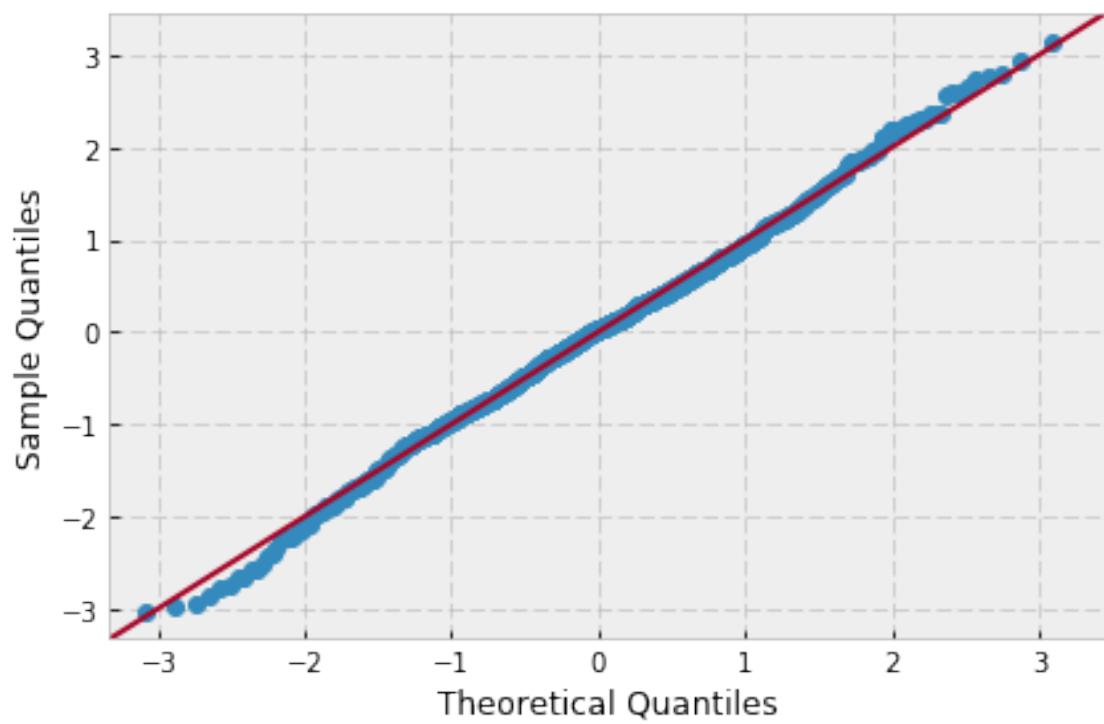
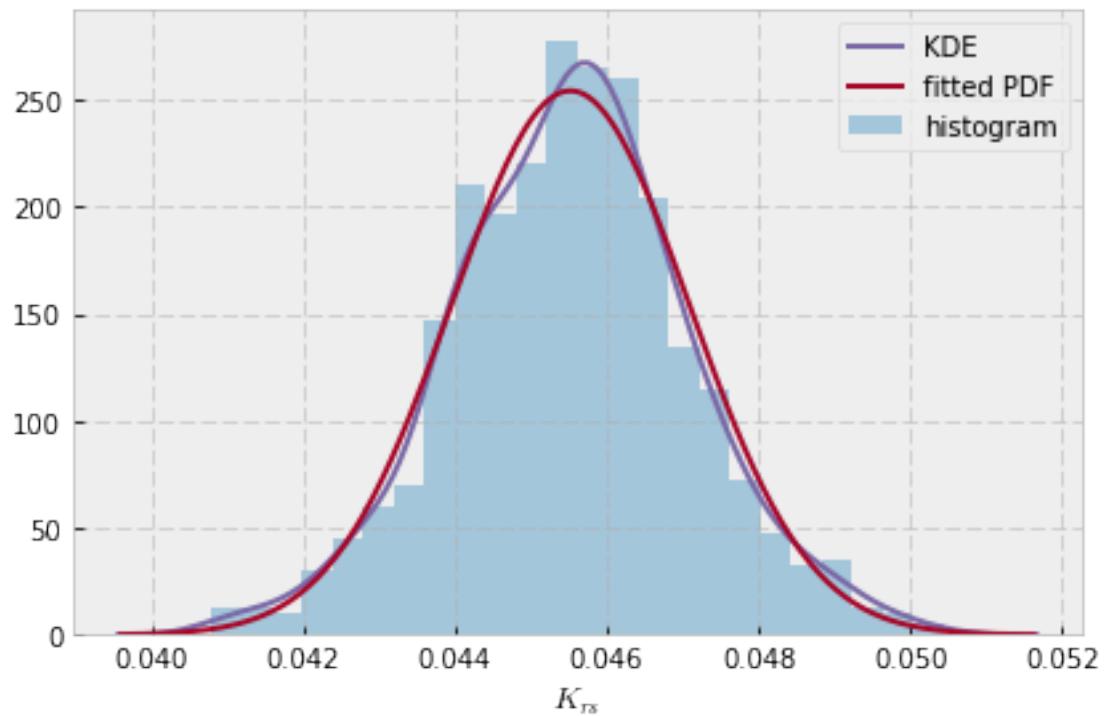
```
In [35]: plot_dist('root_system_total_conductance')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=0.04564901051487, scale=0.0015794061579788676)
DescribeResult(nobs=1000, minmax=(0.04034231597000001, 0.05066641919999998)
               mean=0.045649010514869998, variance=2.4970208326942622e-06,
               skewness=0.18277515883164833, kurtosis=0.17154175535701022)
```

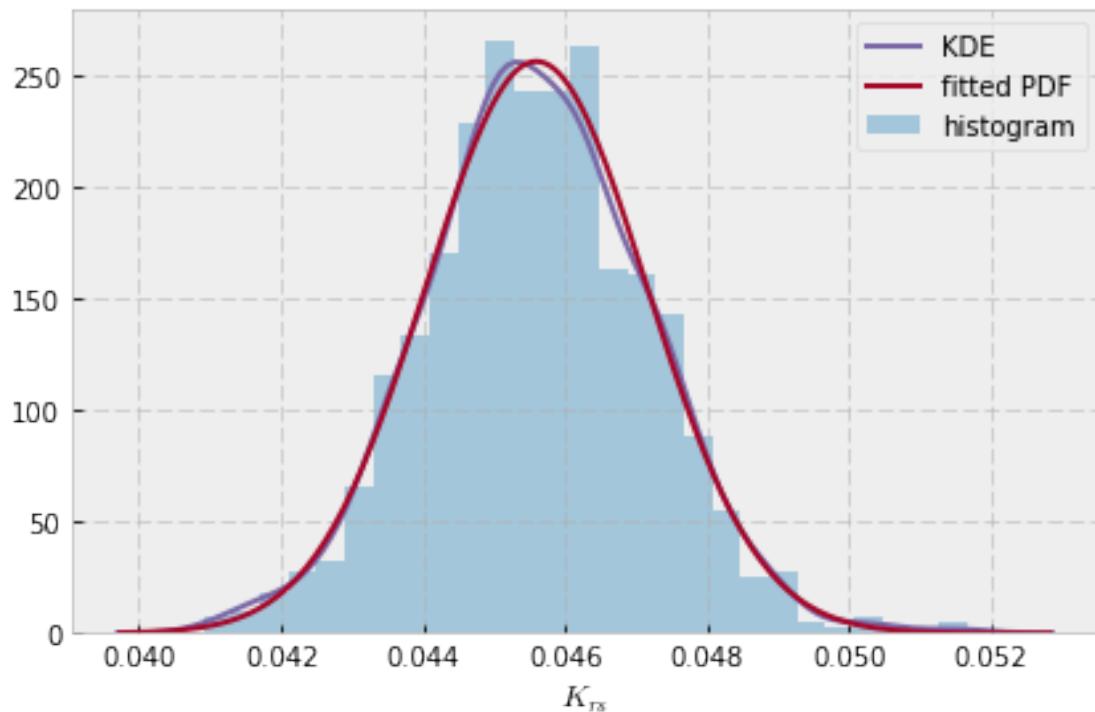


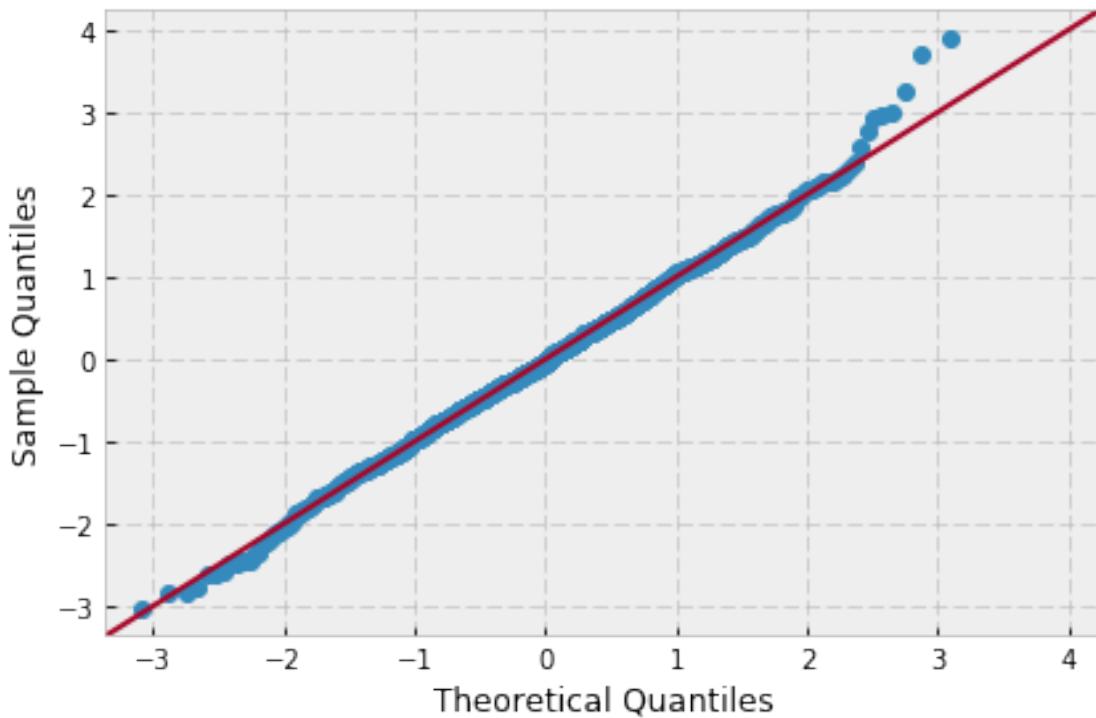


```
variable name: N
variable value: 1.0
distribution: normal(shape=(),loc=0.04551823857909, scale=0.0015691502512964934)
DescribeResult(nobs=1000, minmax=(0.04077490928, 0.050442095800000003)
               mean=0.045518238579090001, variance=2.4646972083522013e-06,
               skewness=-0.006880267557560633, kurtosis=0.23751006511327866)
```

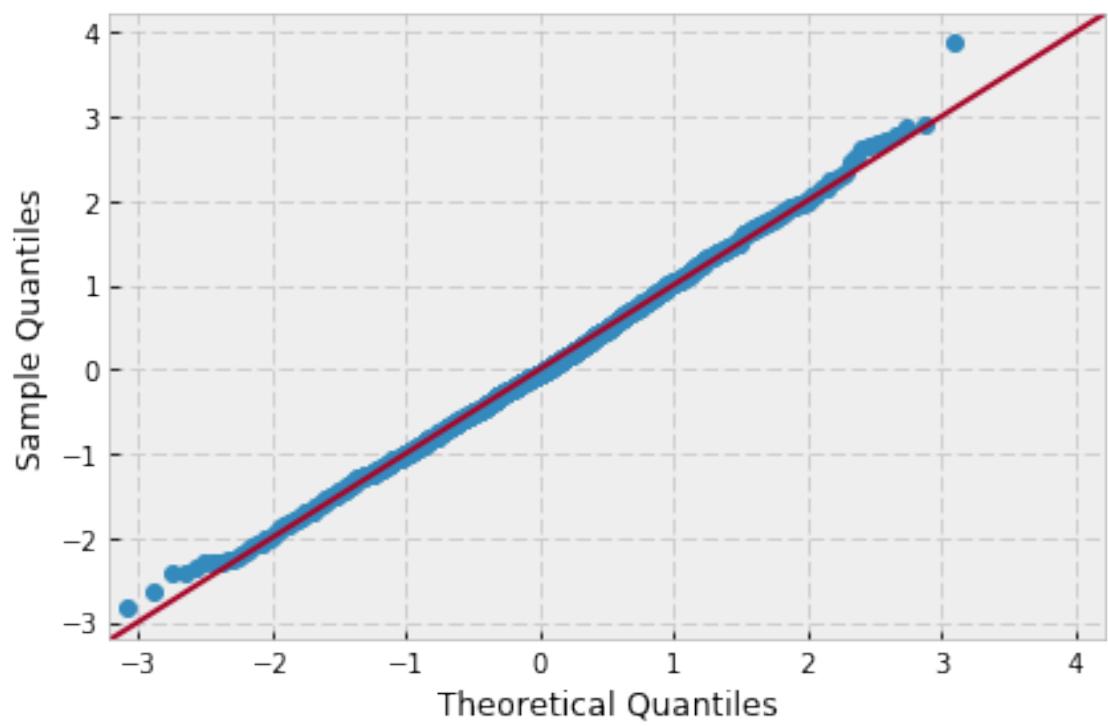
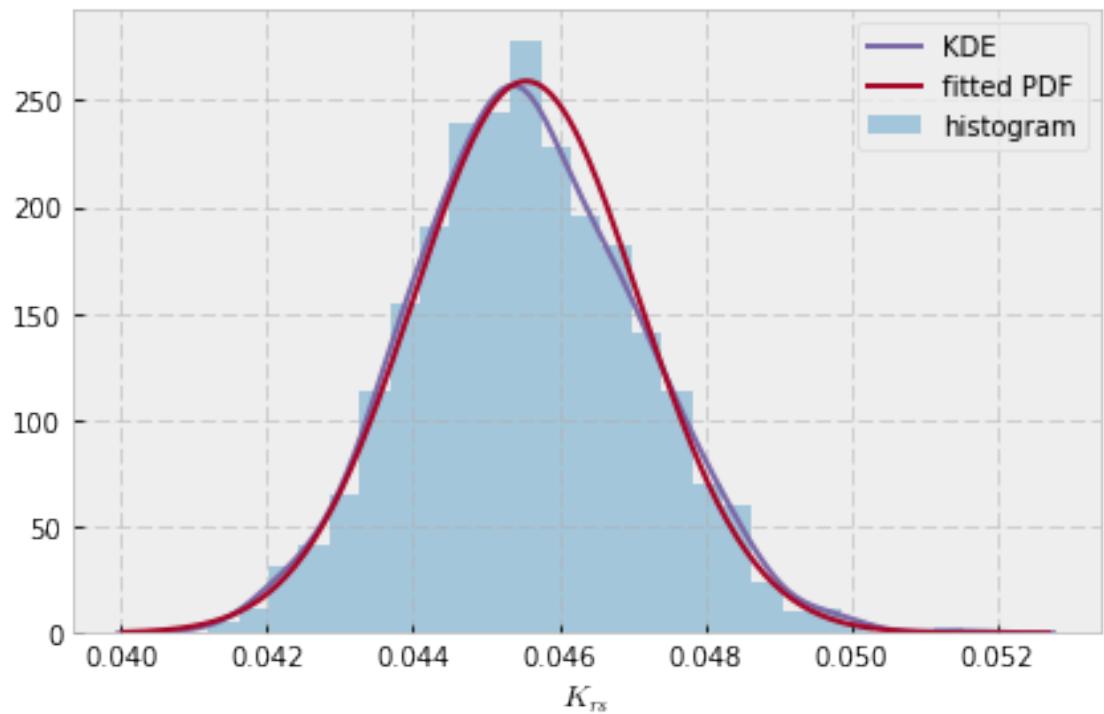


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=0.04559327550416001, scale=0.0015566813476997055)
DescribeResult(nobs=1000, minmax=(0.04089791986, 0.05165641925000002)
               mean=0.04559327550416008, variance=2.4256825007769481e-06,
               skewness=0.09091307187420154, kurtosis=0.3161975172025362)
```

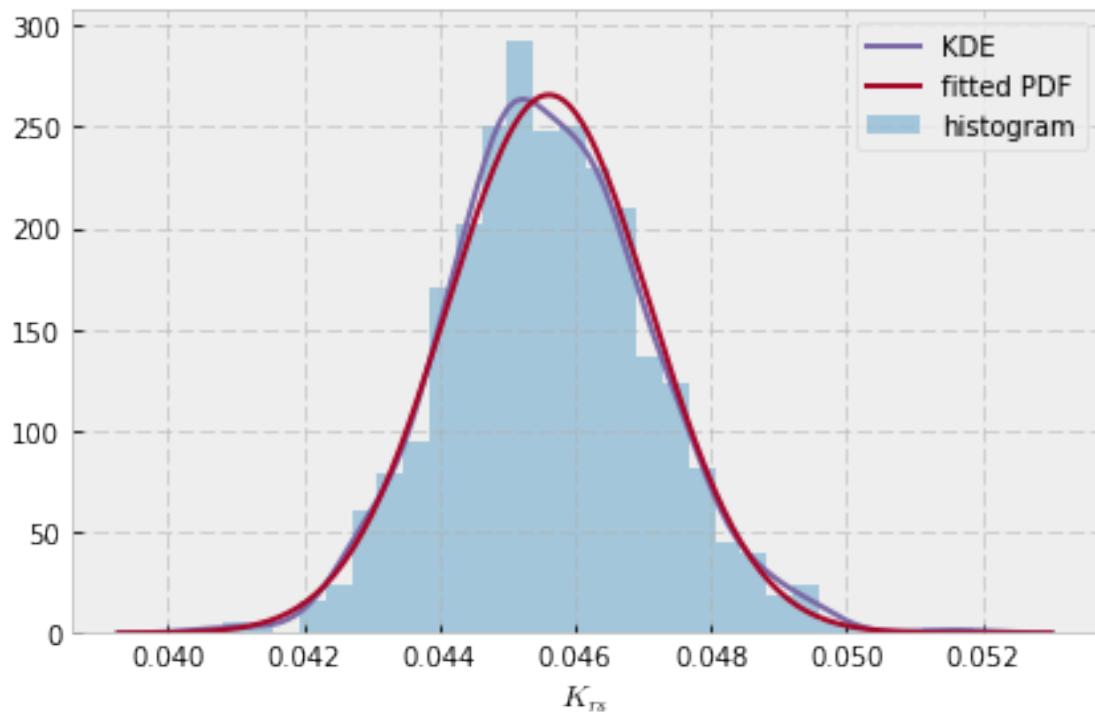


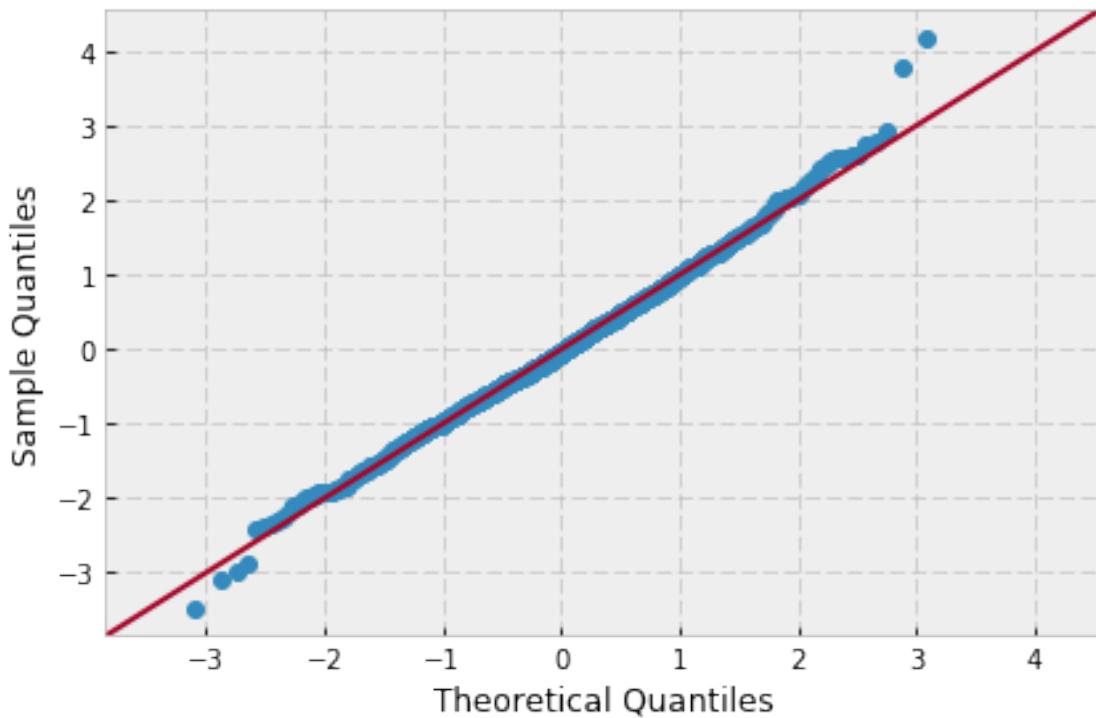


```
variable name: N
variable value: 2.0
distribution: normal(shape=(),loc=0.04555171280735, scale=0.0015405346637968448)
DescribeResult(nobs=1000, minmax=(0.041204862500000002, 0.05153108314999997)
               mean=0.045551712807350001, variance=2.3756226730326905e-06,
               skewness=0.16365100669709406, kurtosis=-0.042317883334735296)
```

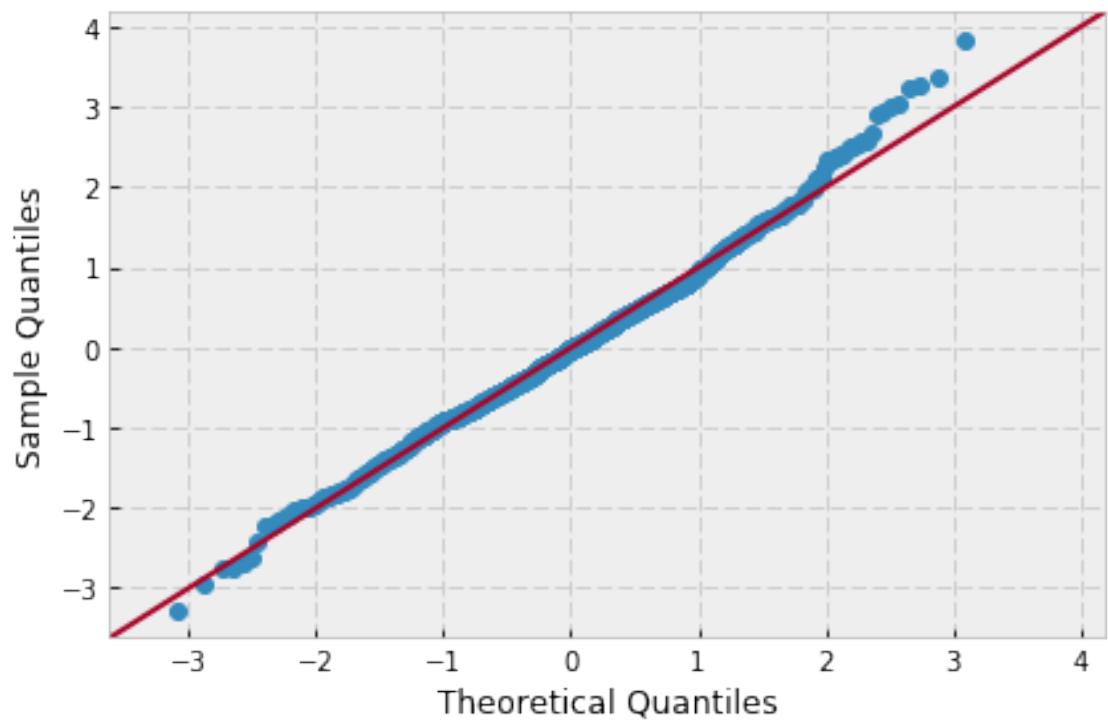
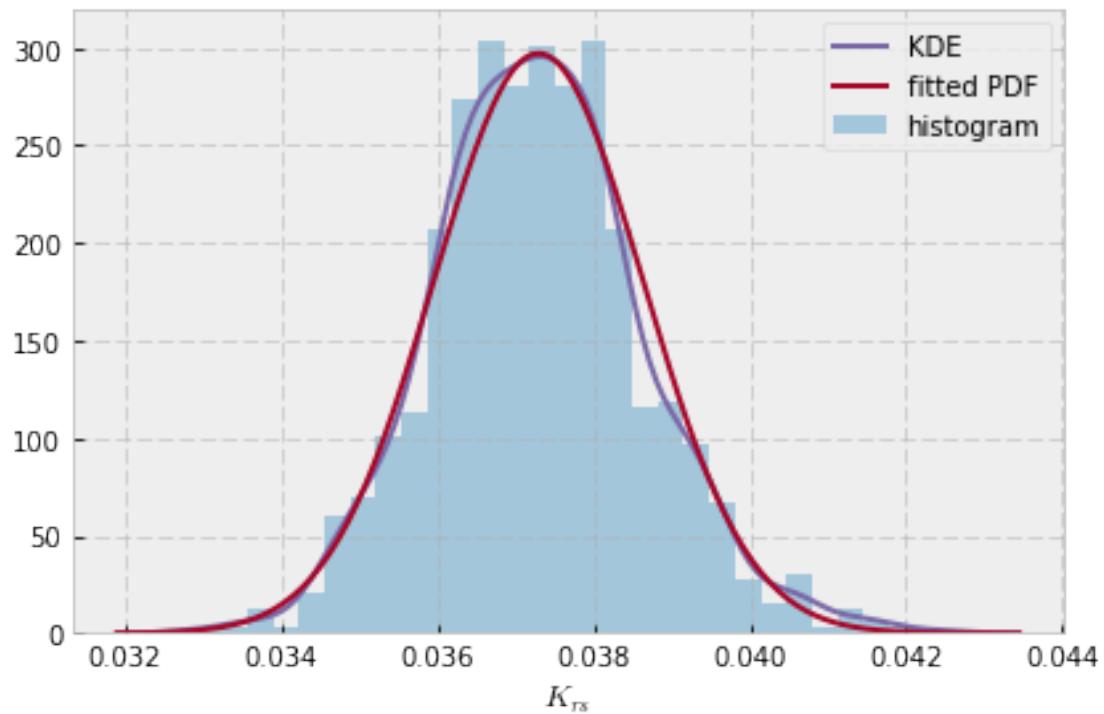


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=0.045609792761669994, scale=0.0015004481268919162)
DescribeResult(nobs=1000, minmax=(0.04040900220000002, 0.05187201781999998)
               mean=0.045609792761669994, variance=2.253598179673133e-06,
               skewness=0.18796517683555822, kurtosis=0.3944447067915968)
```

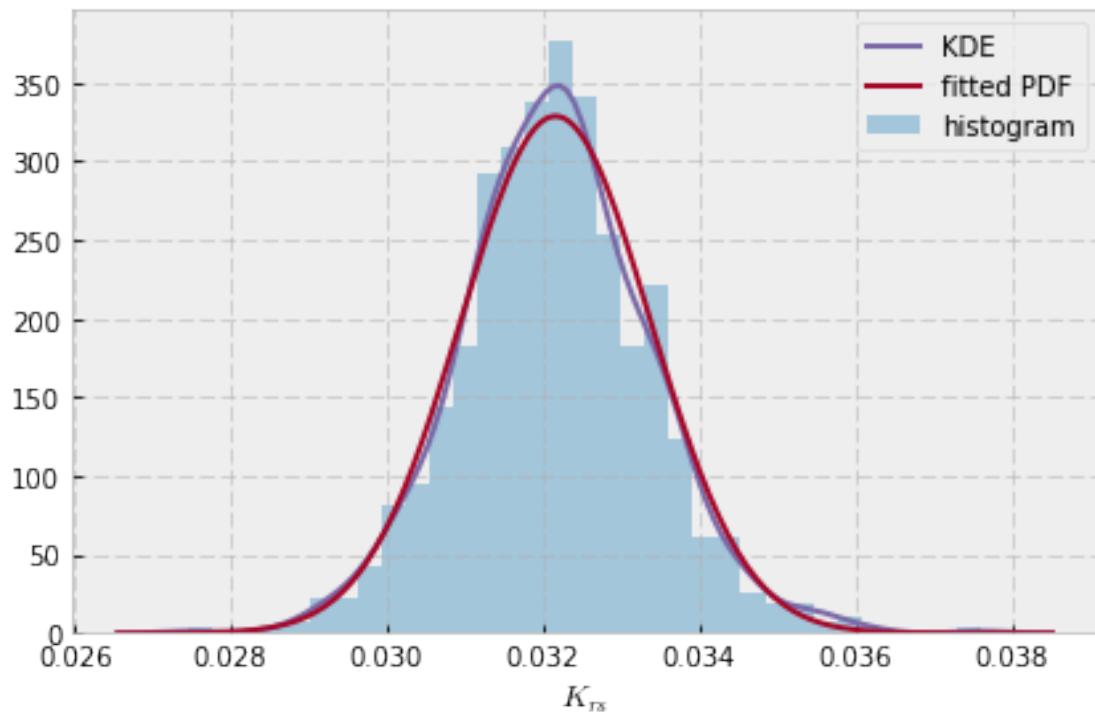


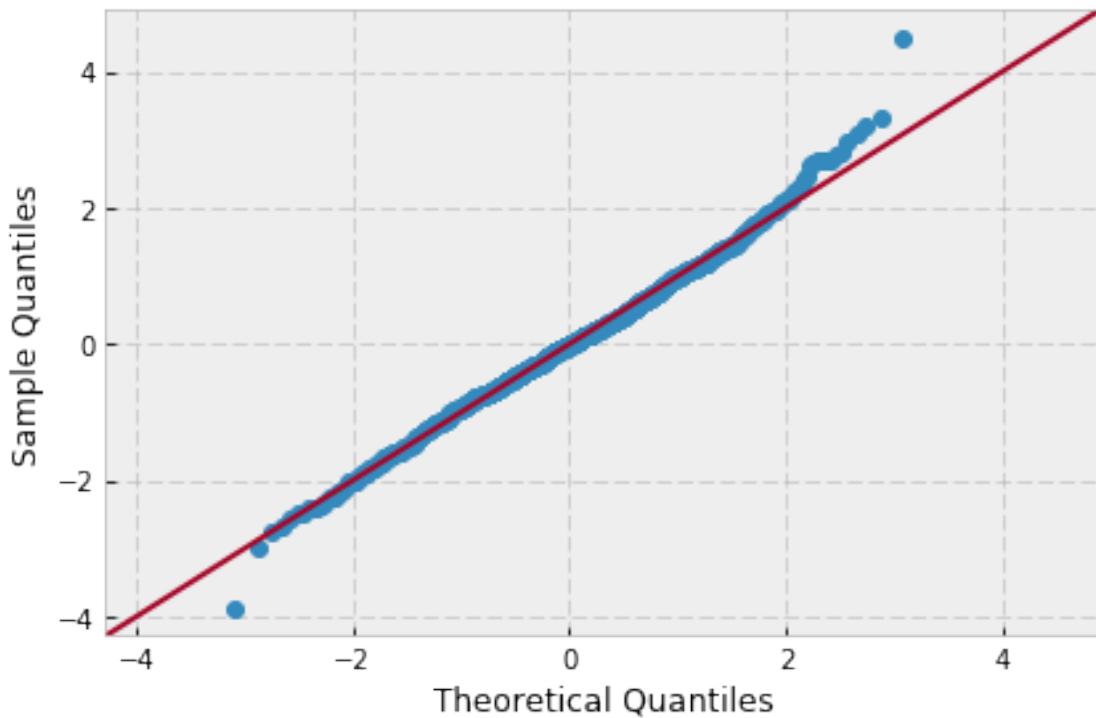


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(),loc=0.03729100269607, scale=0.0013420599428151856)
DescribeResult(nobs=1000, minmax=(0.03290402843000003, 0.04243511606)
               mean=0.037291002696069997, variance=1.8029278179270262e-06,
               skewness=0.24943143576515364, kurtosis=0.47372559036033124)
```

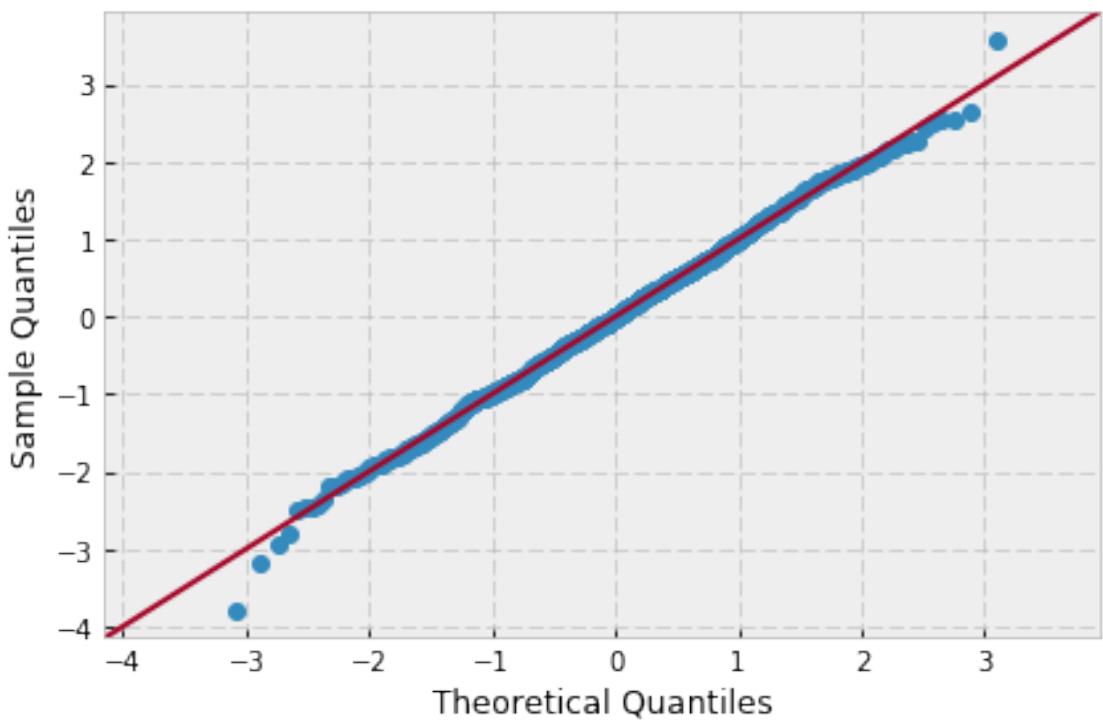
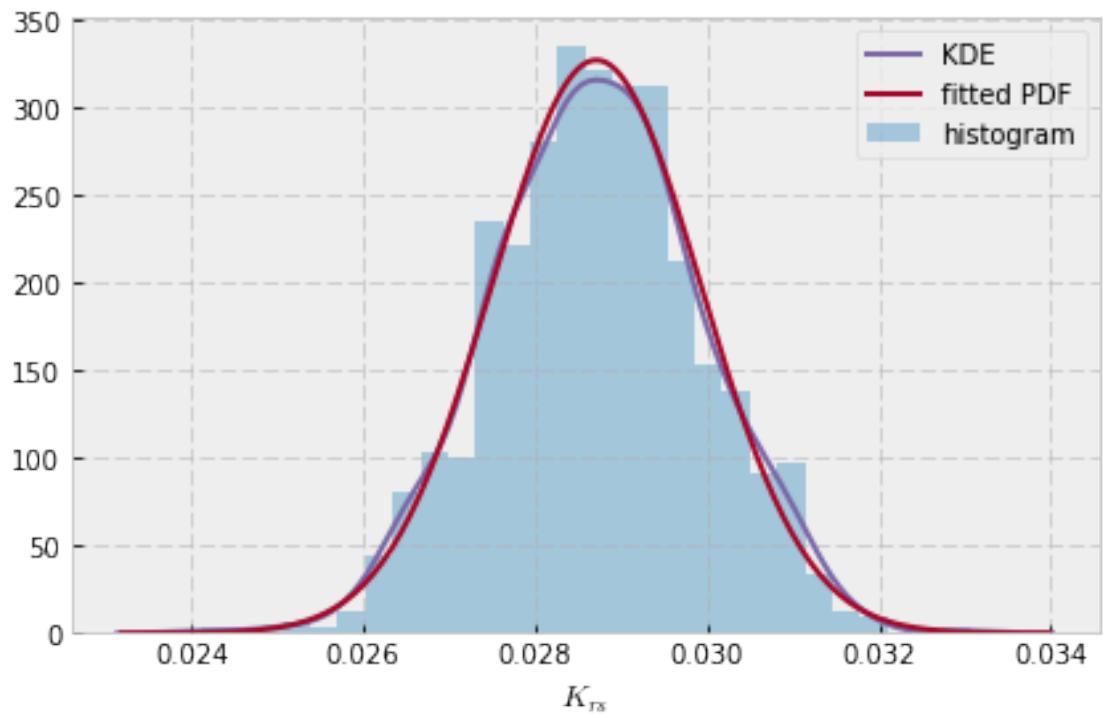


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=0.03216062897442, scale=0.0012135430961360482)
DescribeResult(nobs=1000, minmax=(0.02746550874999999, 0.03760885824000001)
               mean=0.03216062897442, variance=1.4741610071866525e-06,
               skewness=0.1608810197851539, kurtosis=0.7007935906816112)
```

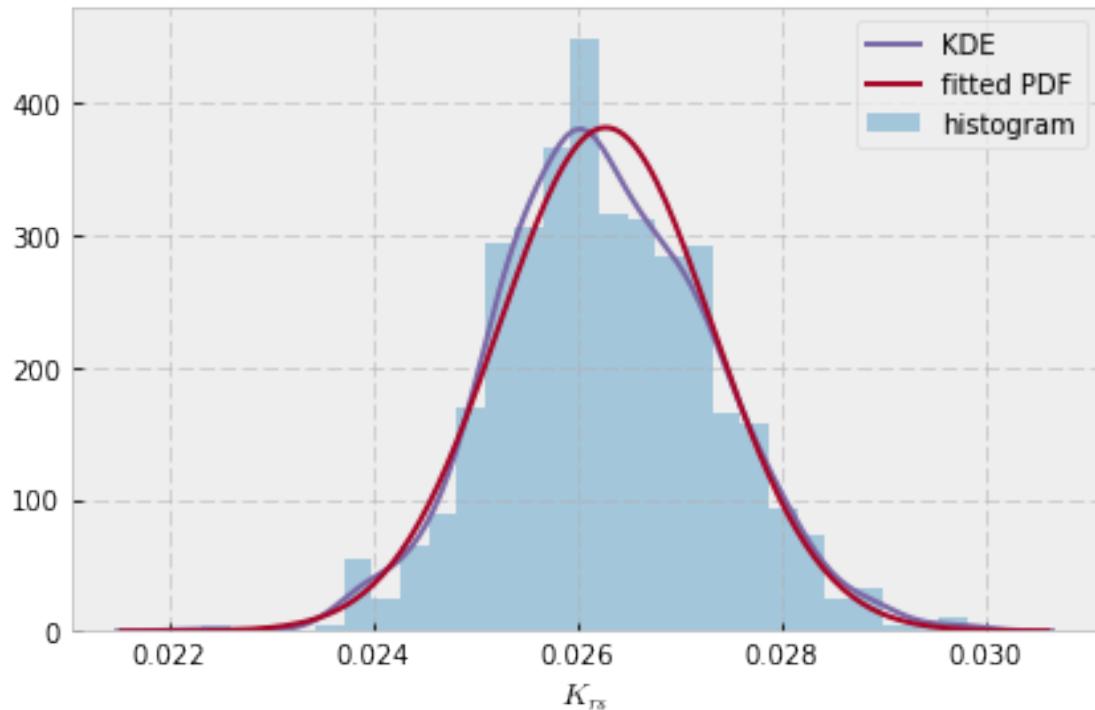


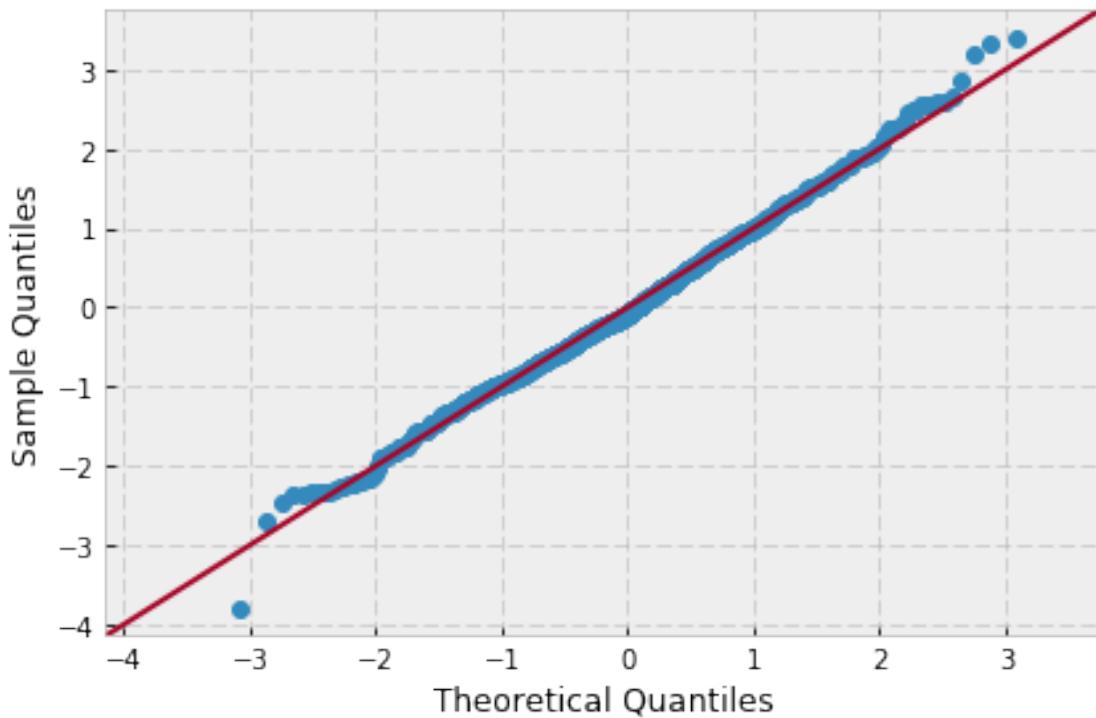


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=0.02871354146251, scale=0.0012179537027342513)
DescribeResult(nobs=1000, minmax=(0.024107460680000001, 0.033052989460000001)
               mean=0.028713541462510001, variance=1.4848961181221956e-06,
               skewness=-0.028638125643443296, kurtosis=0.0027326216798000402)
```

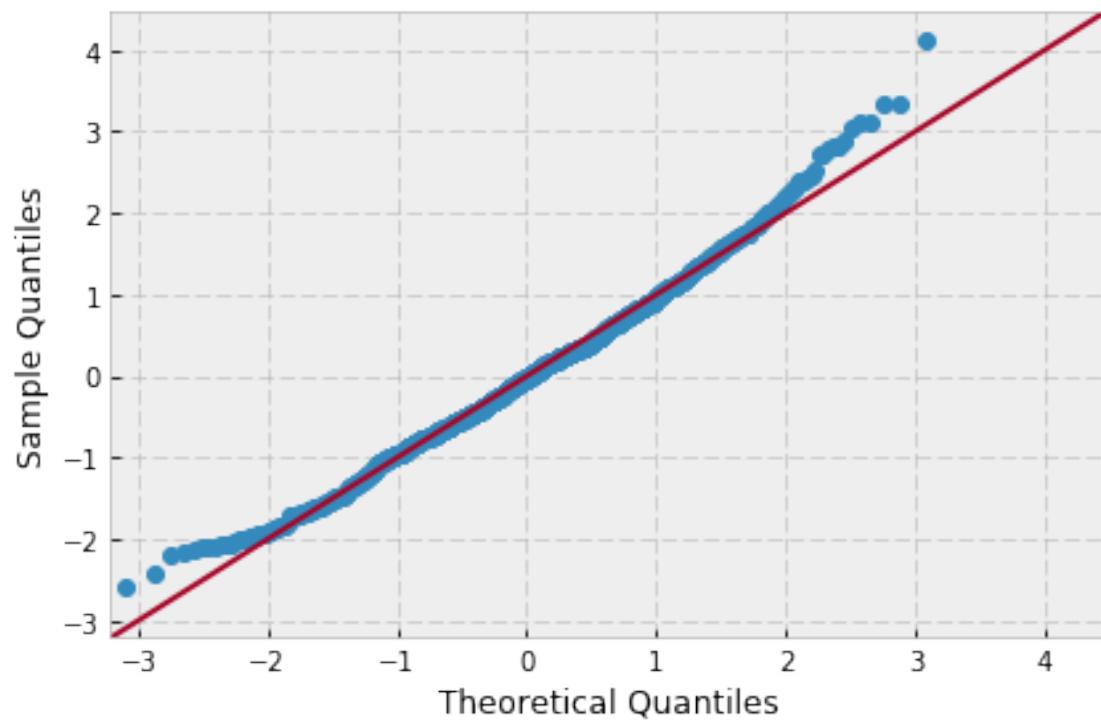
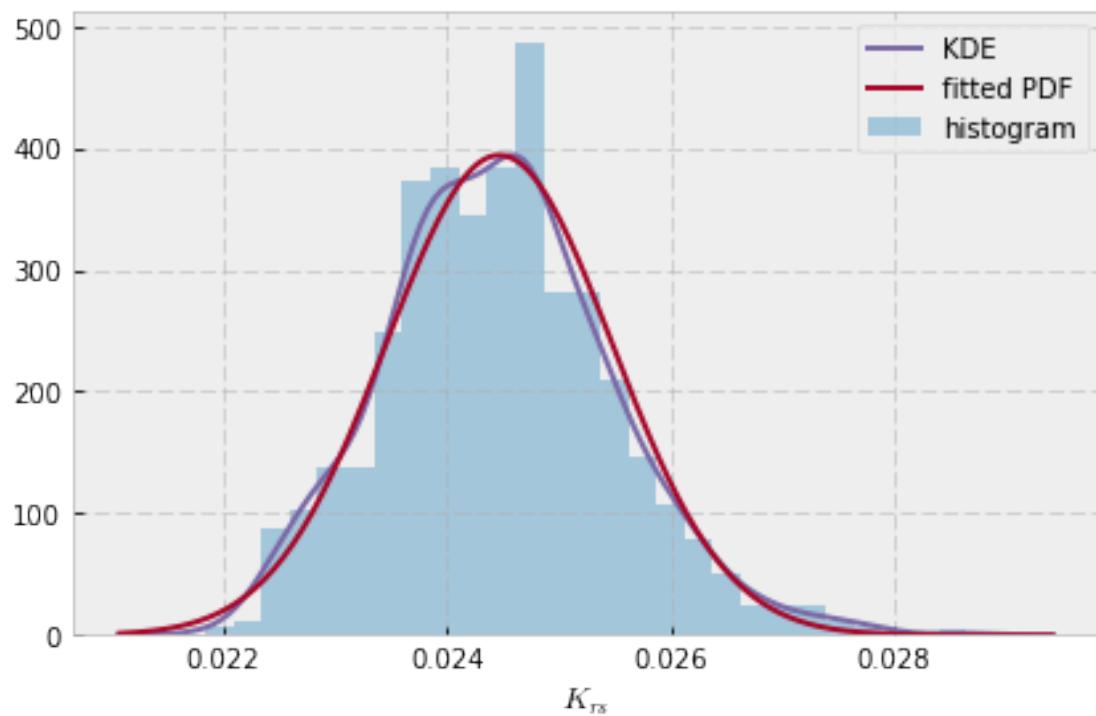


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=0.026275969500430003, scale=0.0010448558576742321)
DescribeResult(nobs=1000, minmax=(0.02232031647, 0.02982049904)
               mean=0.026275969500430003, variance=1.0928165798960512e-06,
               skewness=0.15380555297353268, kurtosis=0.14090933127990812)
```

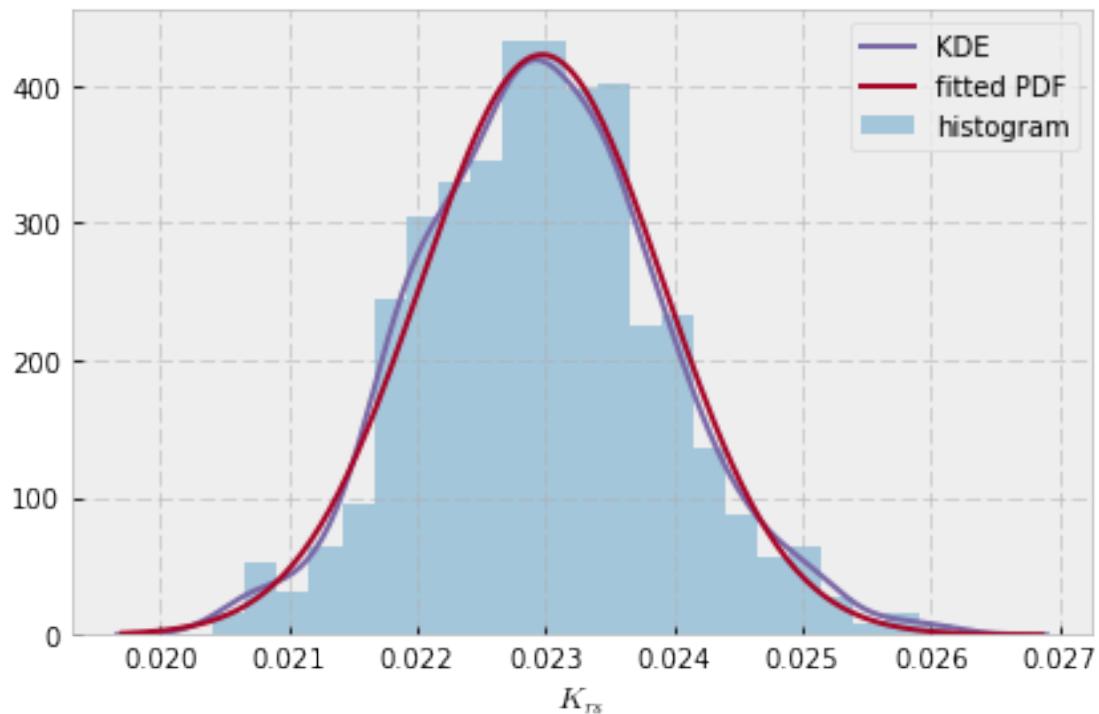


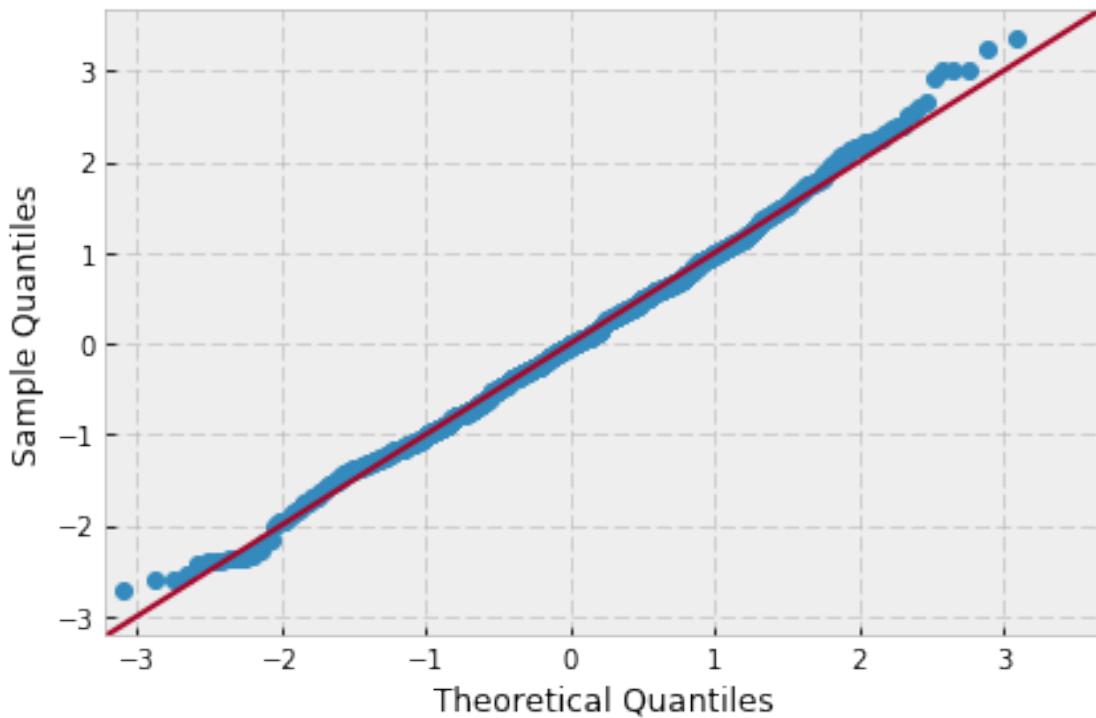


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=0.02446327367656, scale=0.0010124618973782924)
DescribeResult(nobs=1000, minmax=(0.02182957143, 0.02864572091999999)
               mean=0.02446327367656, variance=1.0261051988416937e-06,
               skewness=0.346522440042038, kurtosis=0.33275087934862846)
```

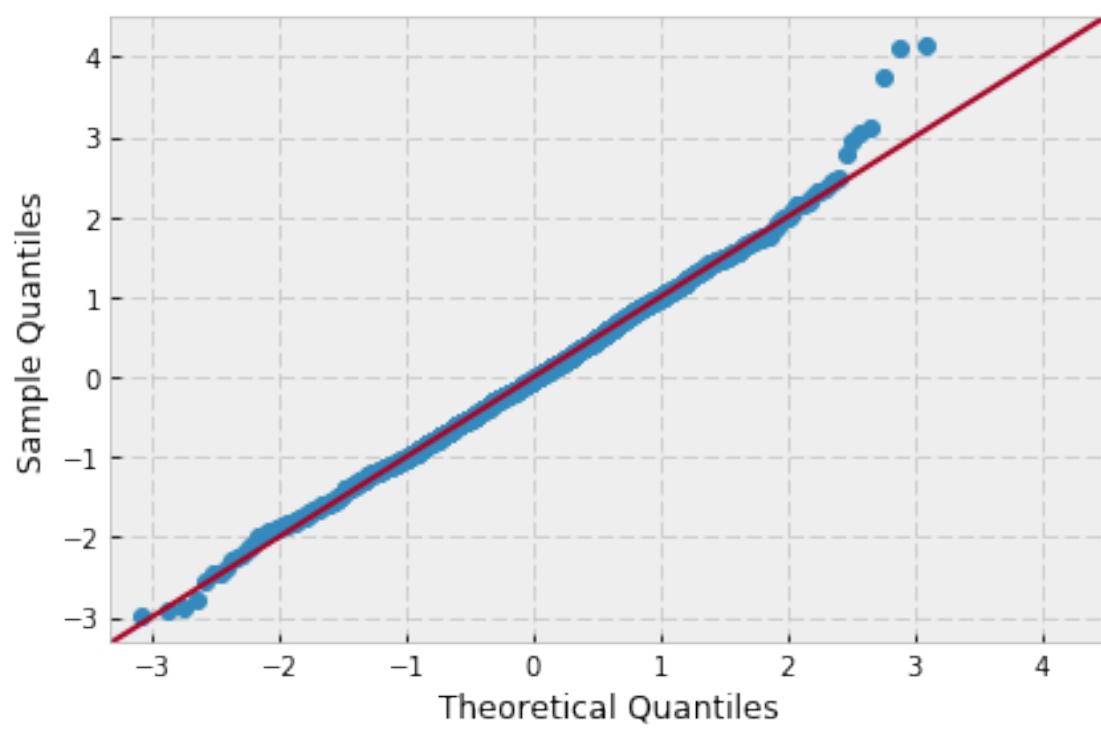
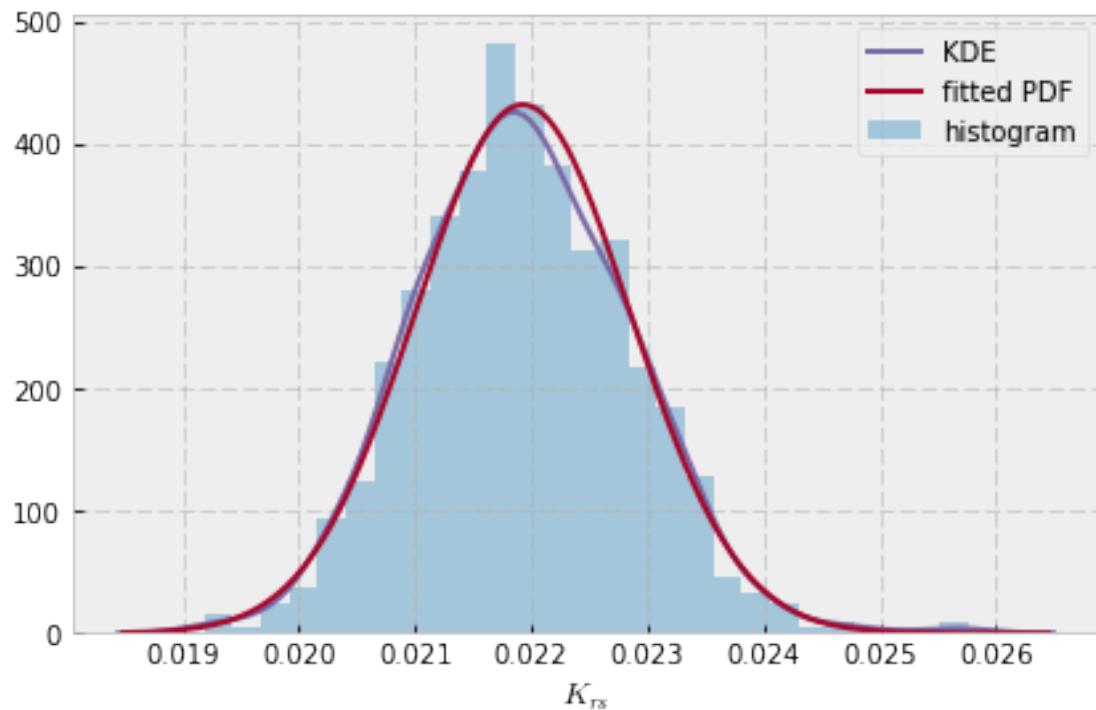


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=0.022974645377139998, scale=0.0009445958840763123)
DescribeResult(nobs=1000, minmax=(0.02041615102999999, 0.02615013294000001)
               mean=0.022974645377139998, variance=8.931545387526626e-07,
               skewness=0.19011527429066158, kurtosis=0.11463777699600453)
```

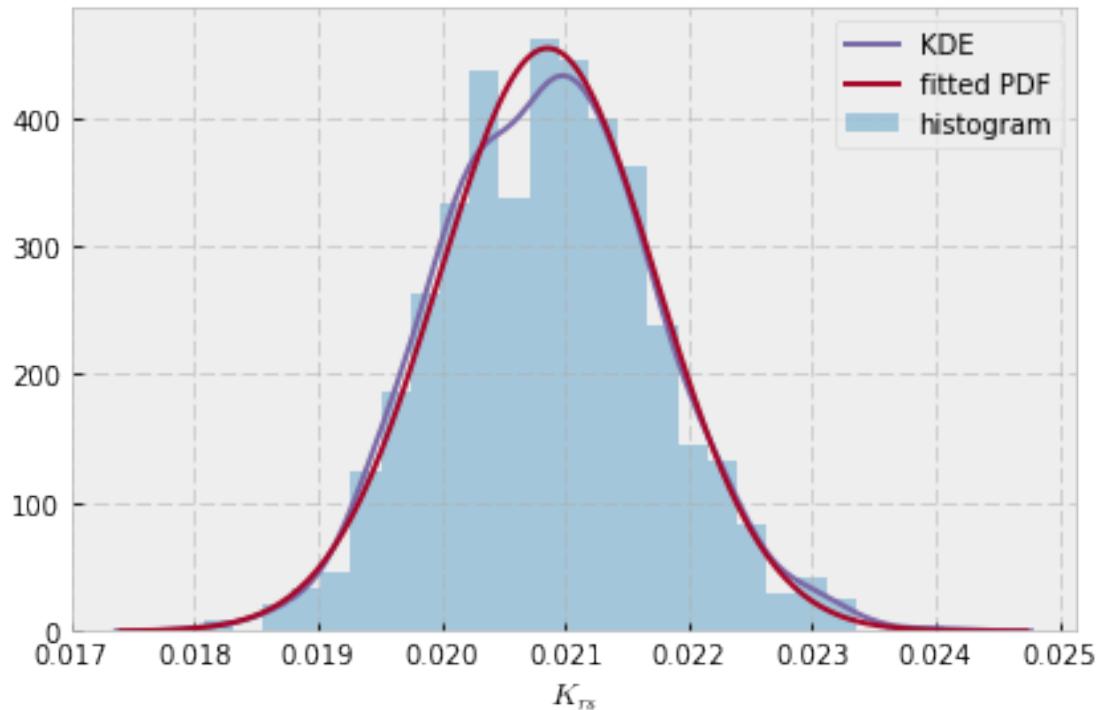


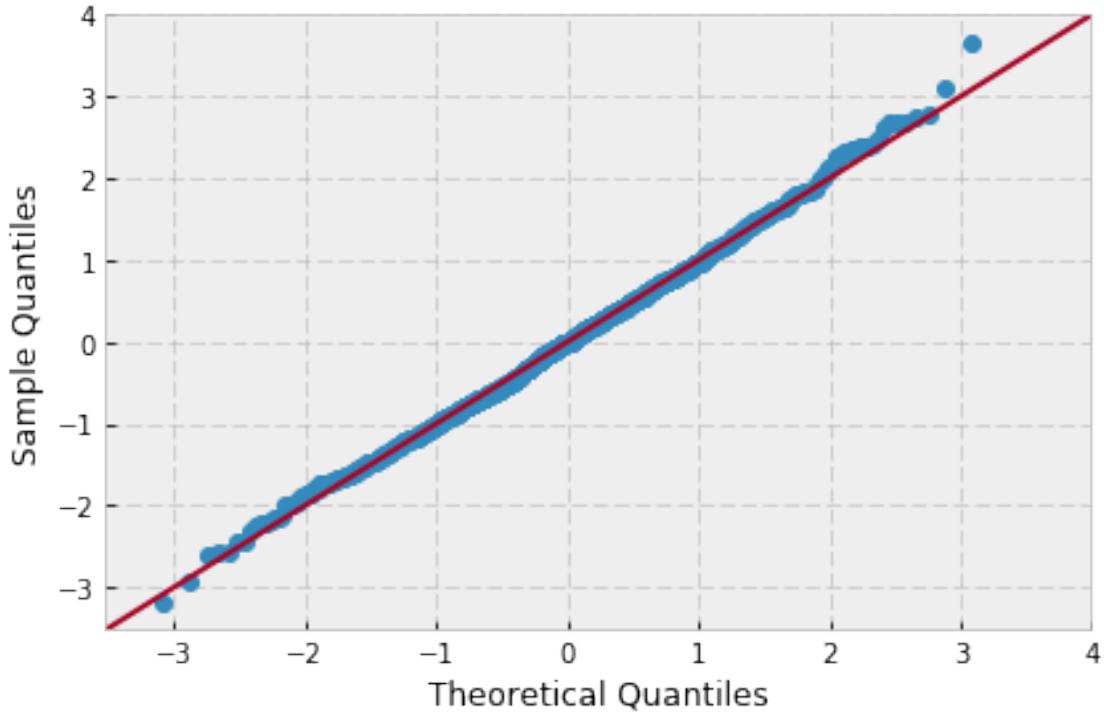


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=0.021925431591420002, scale=0.0009240593621583226)
DescribeResult(nobs=1000, minmax=(0.01918364314999999, 0.02575191365)
               mean=0.021925431591420002, variance=8.5474044523768371e-07,
               skewness=0.2159848011349591, kurtosis=0.42432628198503775)
```

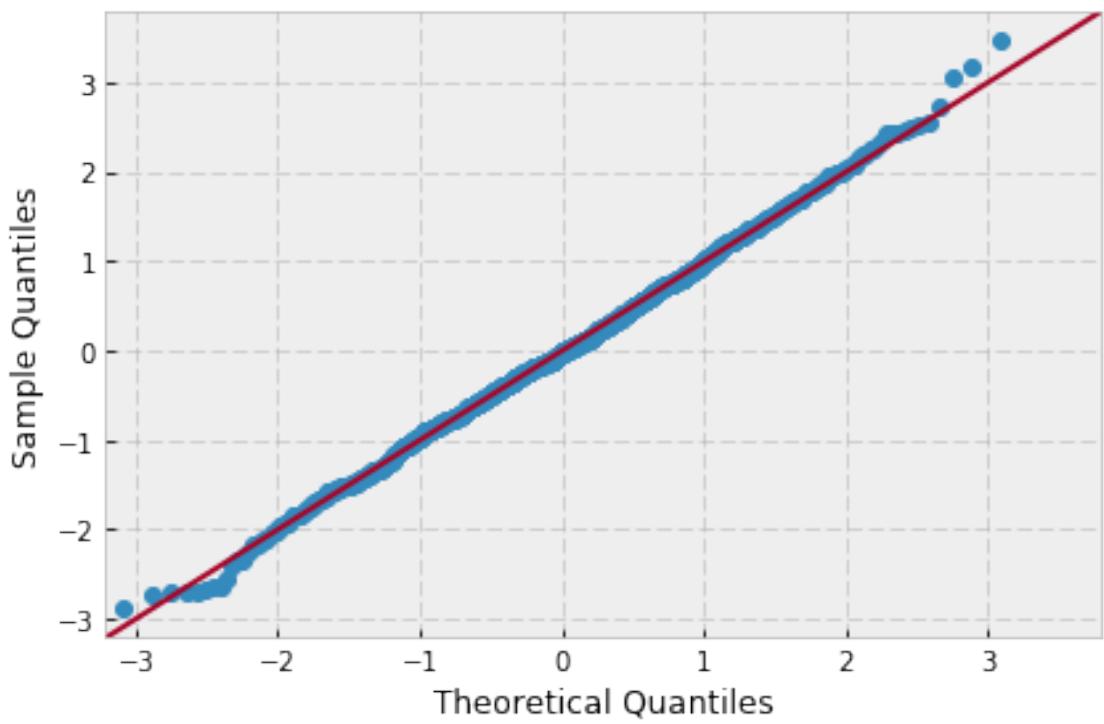
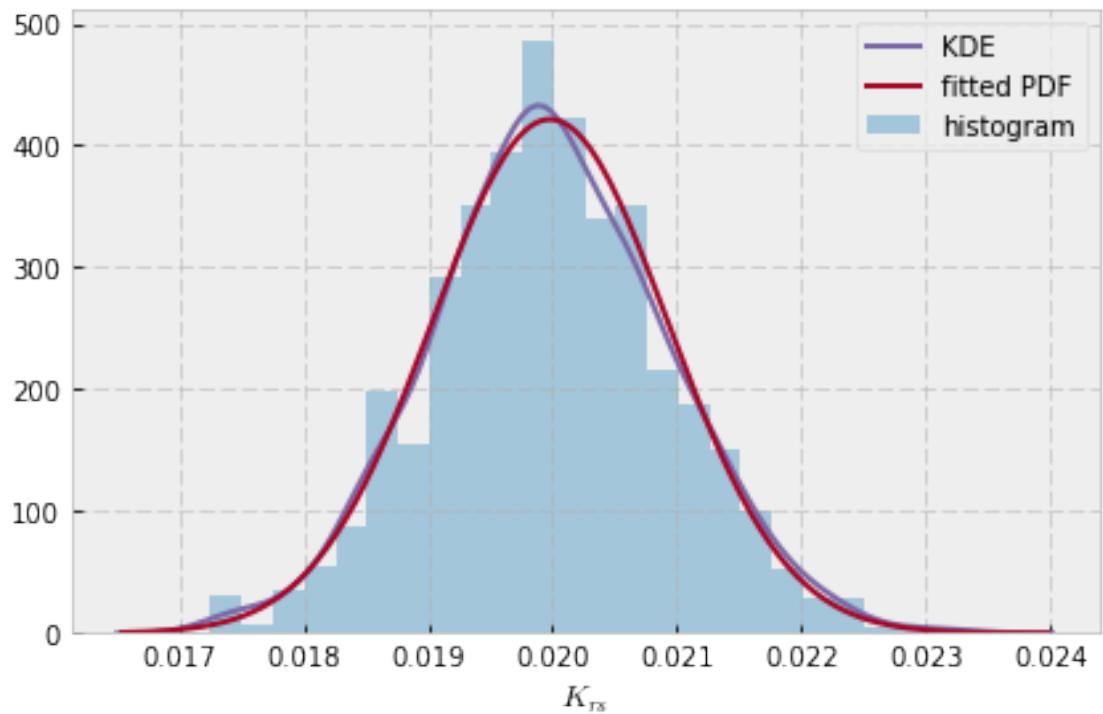


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=0.02085924896671, scale=0.0008786280470430236)
DescribeResult(nobs=1000, minmax=(0.01806812472999999, 0.02407139099)
               mean=0.020859248966710001, variance=7.7276000505569327e-07,
               skewness=0.1368835717550324, kurtosis=0.009893929444605654)
```

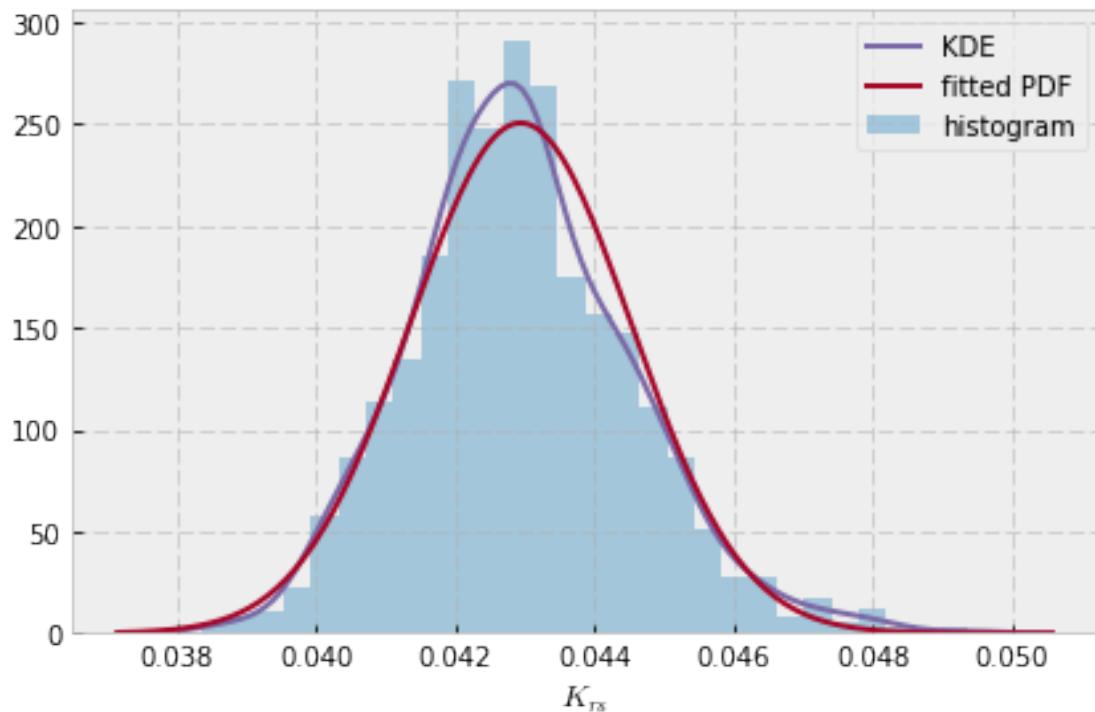


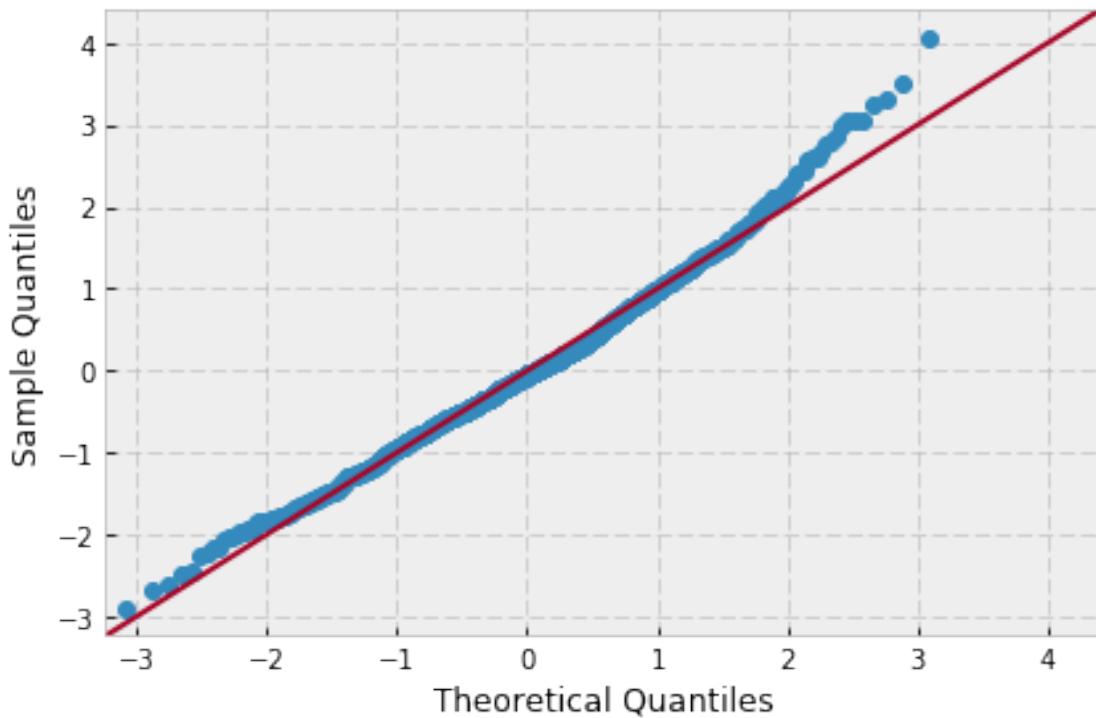


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=0.01997958996592, scale=0.000946427421668715)
DescribeResult(nobs=1000, minmax=(0.0172520821, 0.02326614743999999)
               mean=0.019979589965919999, variance=8.9662148597246426e-07,
               skewness=0.07049710334387325, kurtosis=0.06913438640563552)
```

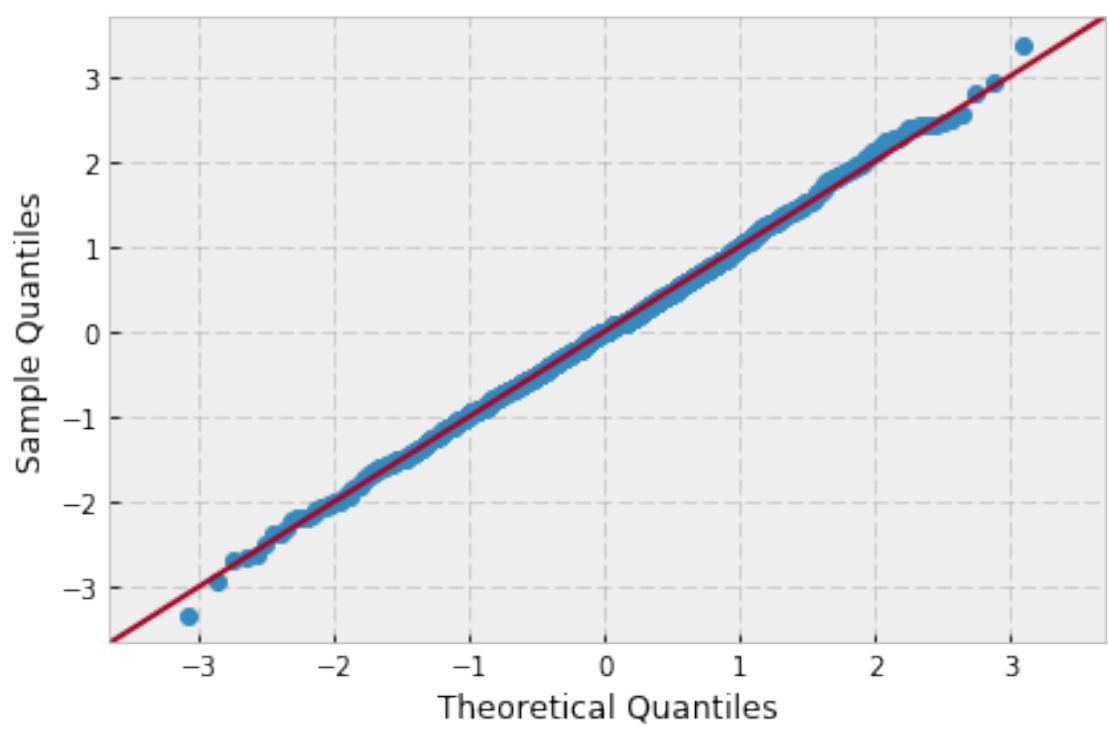
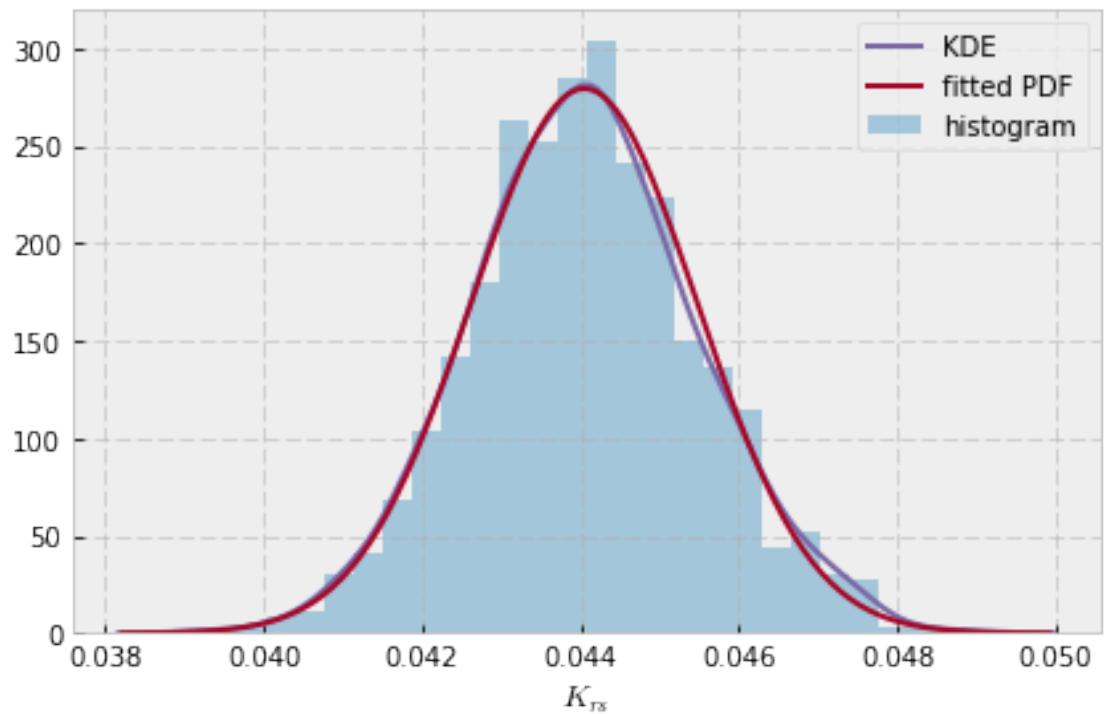


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(), loc=0.04293955460643, scale=0.0015912697029740479)
DescribeResult(nobs=1000, minmax=(0.038344034620000002, 0.049386237180000001)
               mean=0.042939554606430001, variance=2.534673941544659e-06,
               skewness=0.3892484102197144, kurtosis=0.4942360245607422)
```

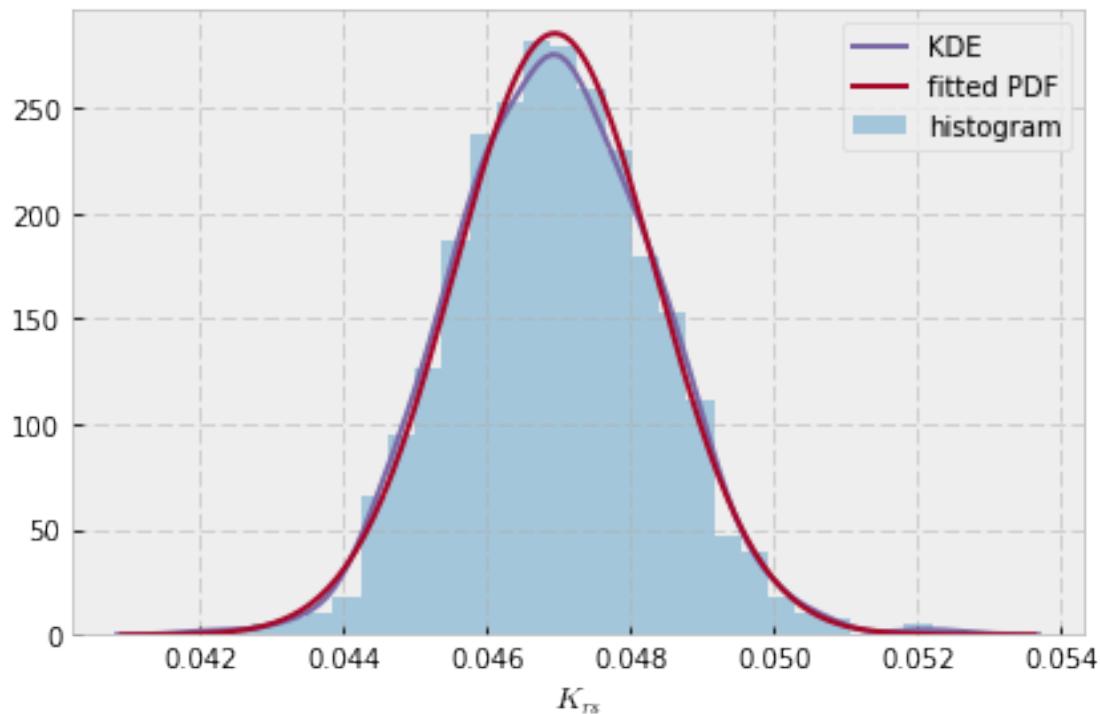


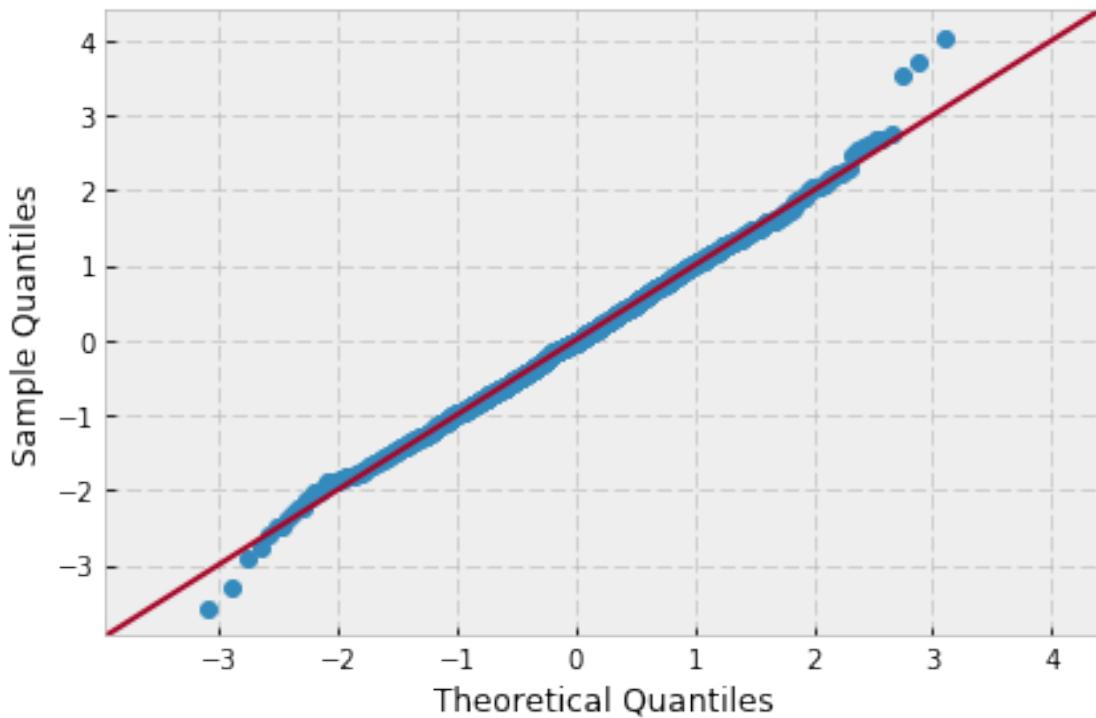


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(),loc=0.04404566881487, scale=0.0014273967538693069)
DescribeResult(nobs=1000, minmax=(0.039287242610000002, 0.048847015370000003)
               mean=0.044045668814870001, variance=2.0395009939505851e-06,
               skewness=0.08408540868614646, kurtosis=0.022295578389134985)
```

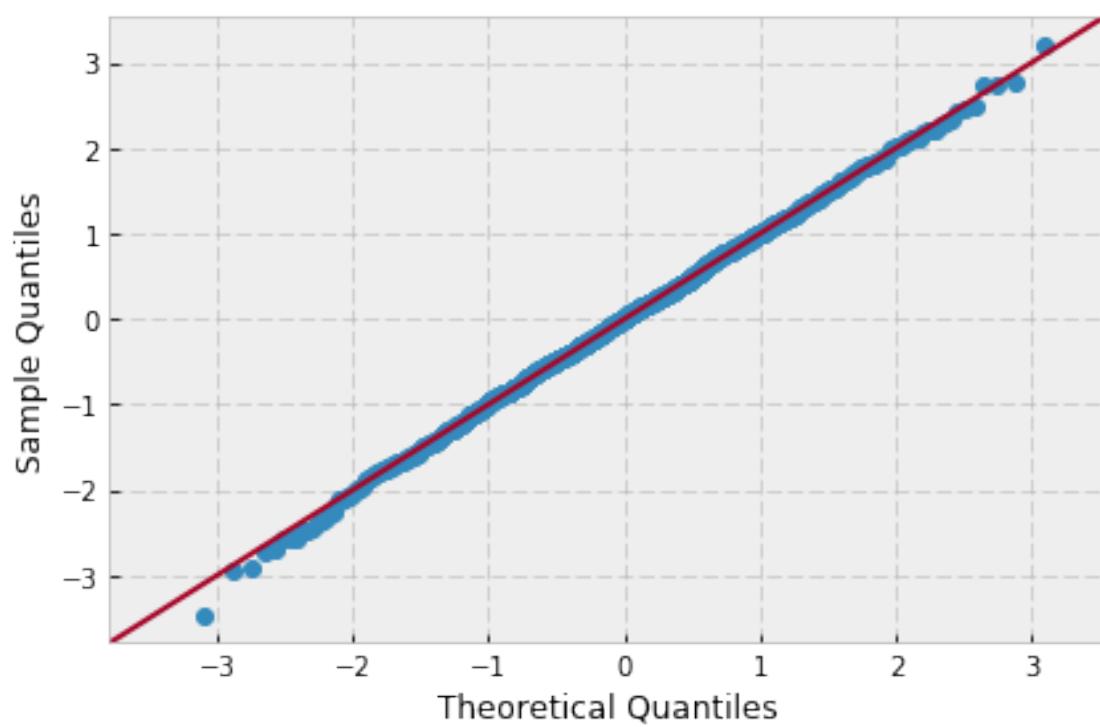
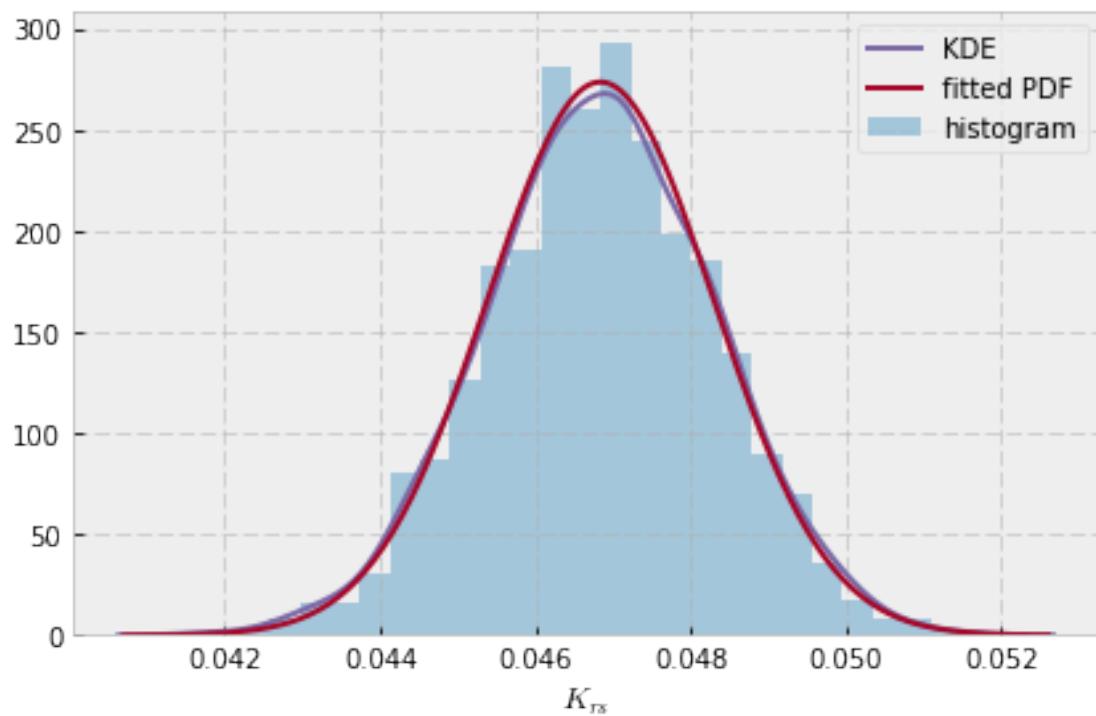


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(), loc=0.04694727939101, scale=0.0013976829622811167)
DescribeResult(nobs=1000, minmax=(0.04196397505999999, 0.05257290301999999)
               mean=0.046947279391009997, variance=1.9554731361871046e-06,
               skewness=0.1096794320445422, kurtosis=0.28790853088534973)
```

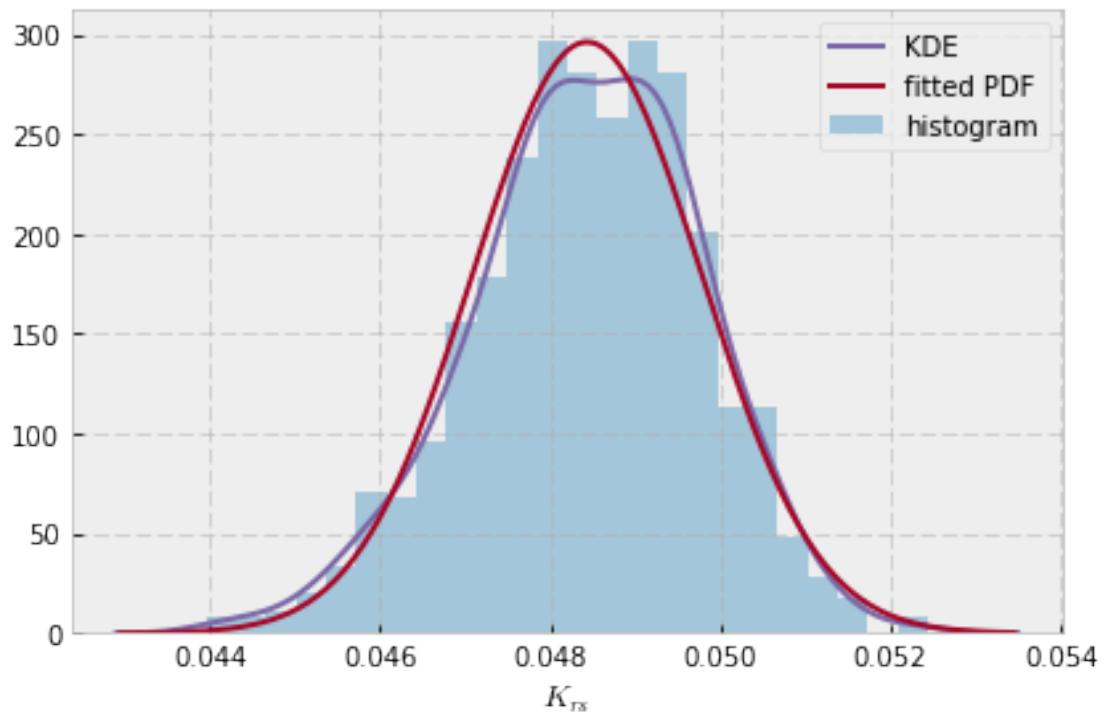


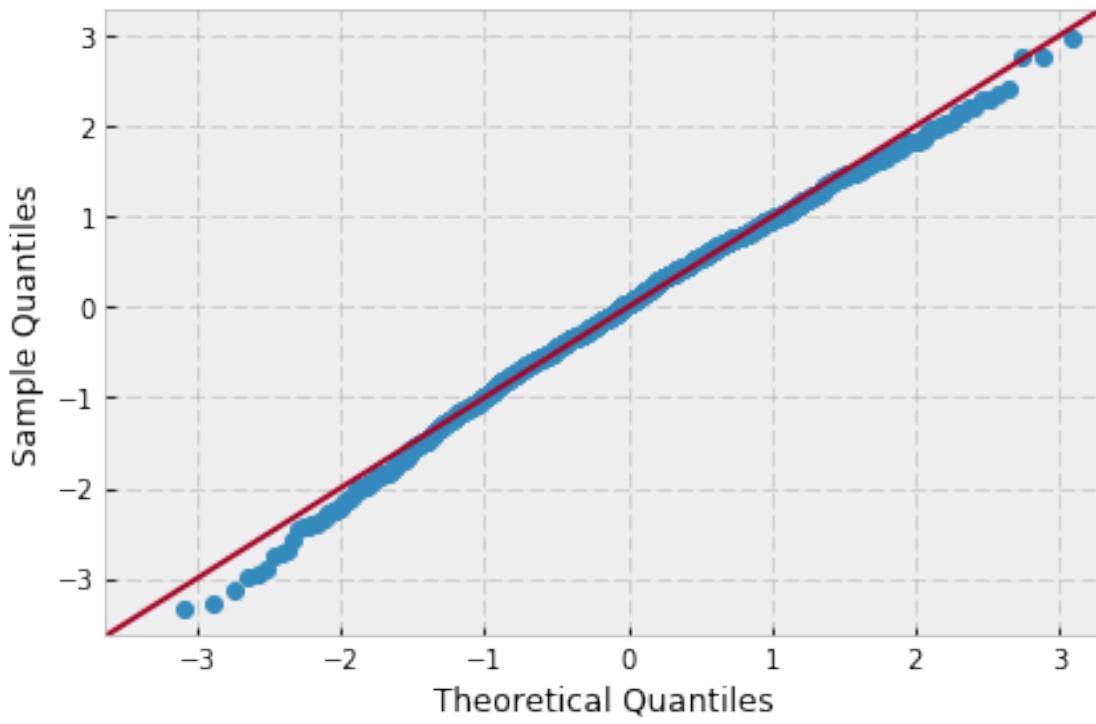


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(),loc=0.046837468422520005, scale=0.0014580700032537828)
DescribeResult(nobs=1000, minmax=(0.041795760600000002, 0.05150479758999998)
               mean=0.046837468422520005, variance=2.1280962306191054e-06,
               skewness=-0.05866846334283259, kurtosis=-0.0037584646743673567)
```

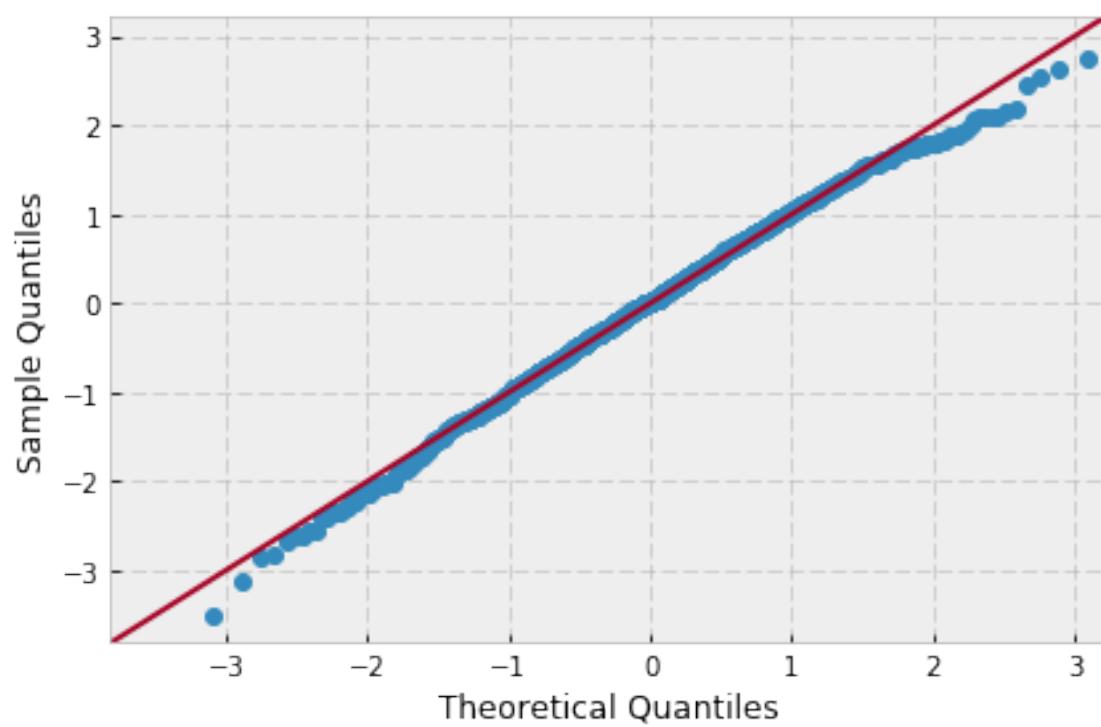
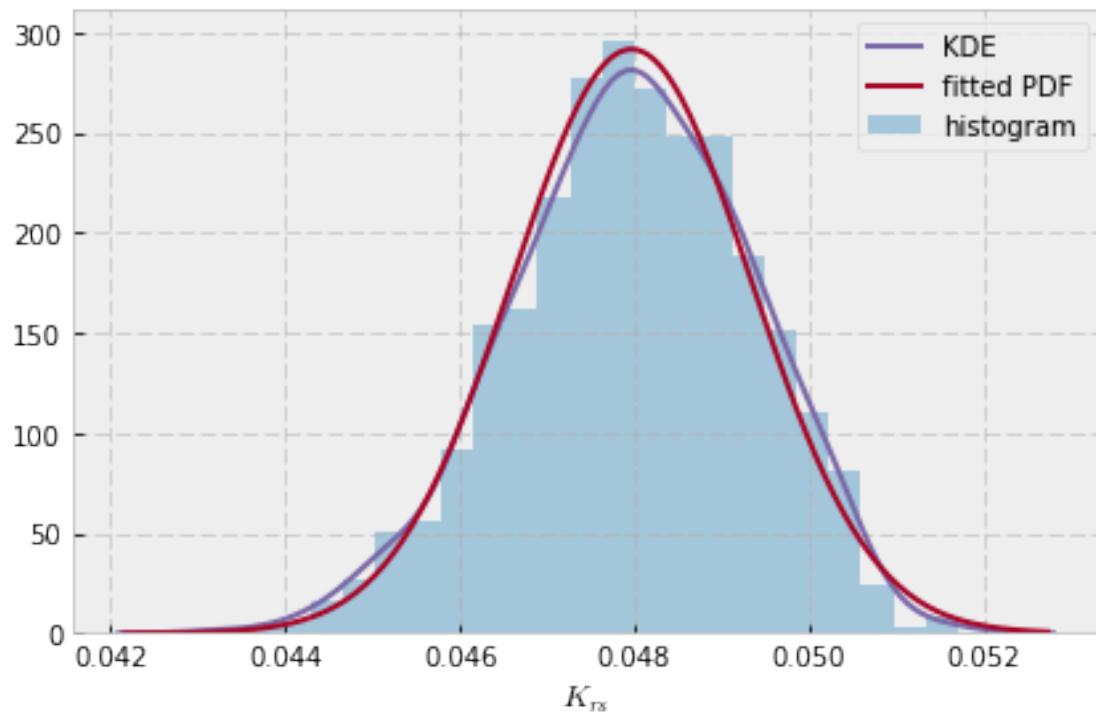


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(), loc=0.048431972777789996, scale=0.0013467521645224878)
DescribeResult(nobs=1000, minmax=(0.04395931924000003, 0.05243297587000002)
               mean=0.048431972777789996, variance=1.8155569495956014e-06,
               skewness=-0.28033595169392317, kurtosis=0.09892548243793442)
```

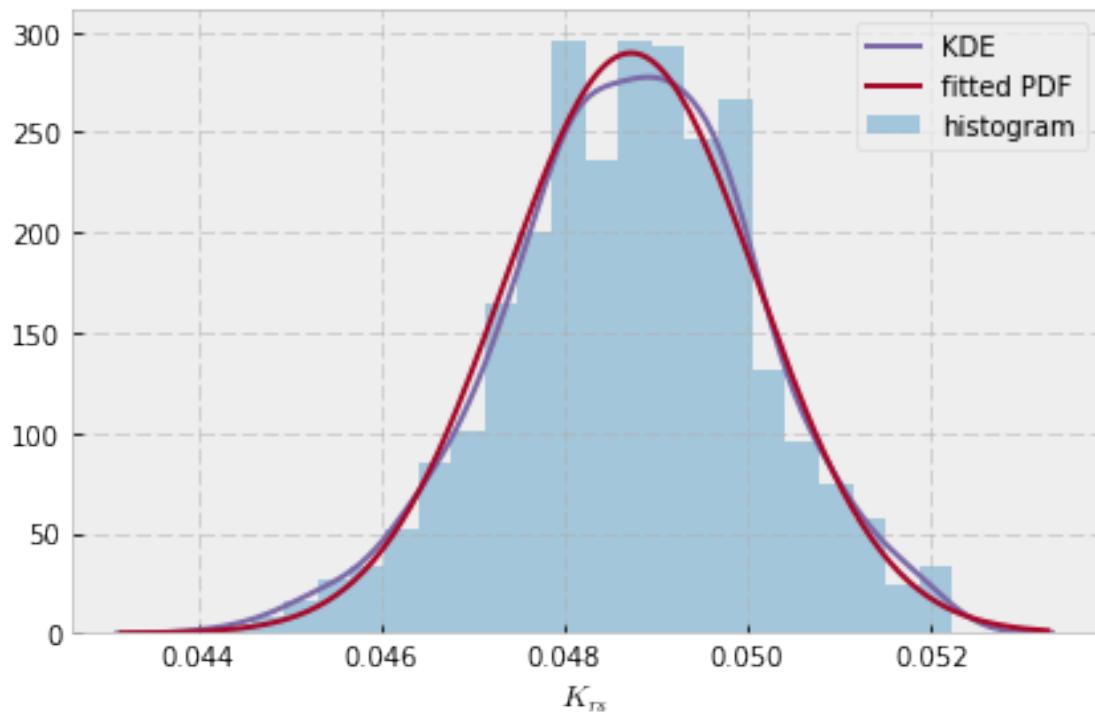


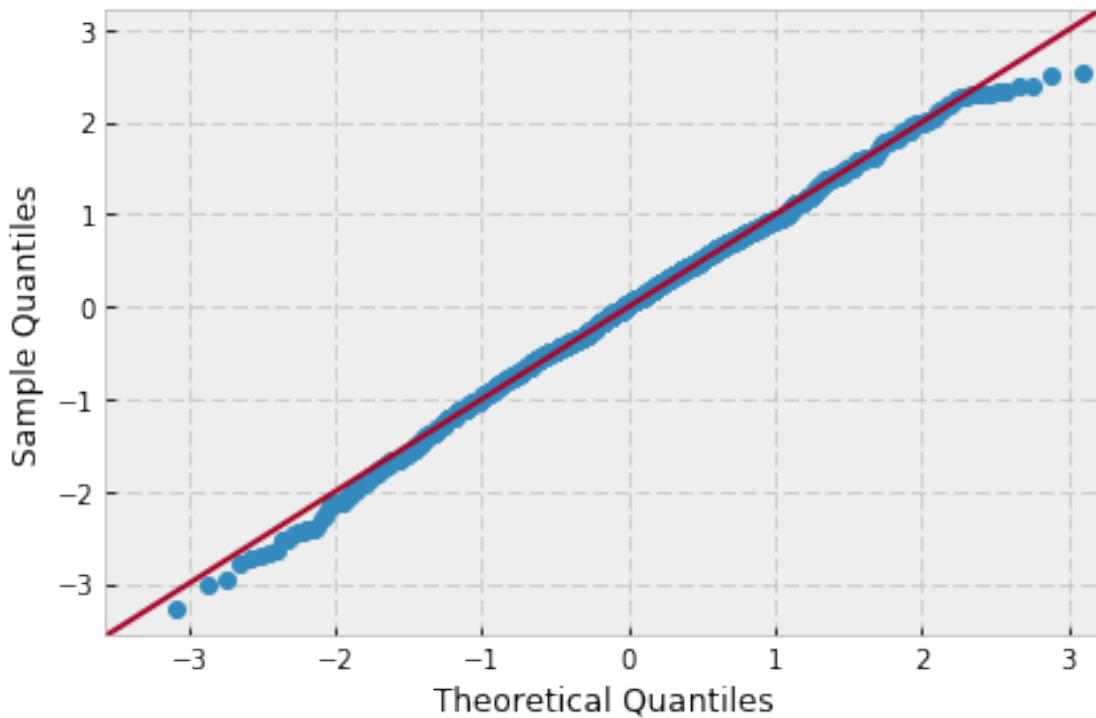


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(),loc=0.04796805953233, scale=0.0013674243932333579)
DescribeResult(nobs=1000, minmax=(0.04317930446999999, 0.05171141498000002)
               mean=0.04796805953233, variance=1.8717211924020188e-06,
               skewness=-0.2115815147416378, kurtosis=-0.14638910965036134)
```

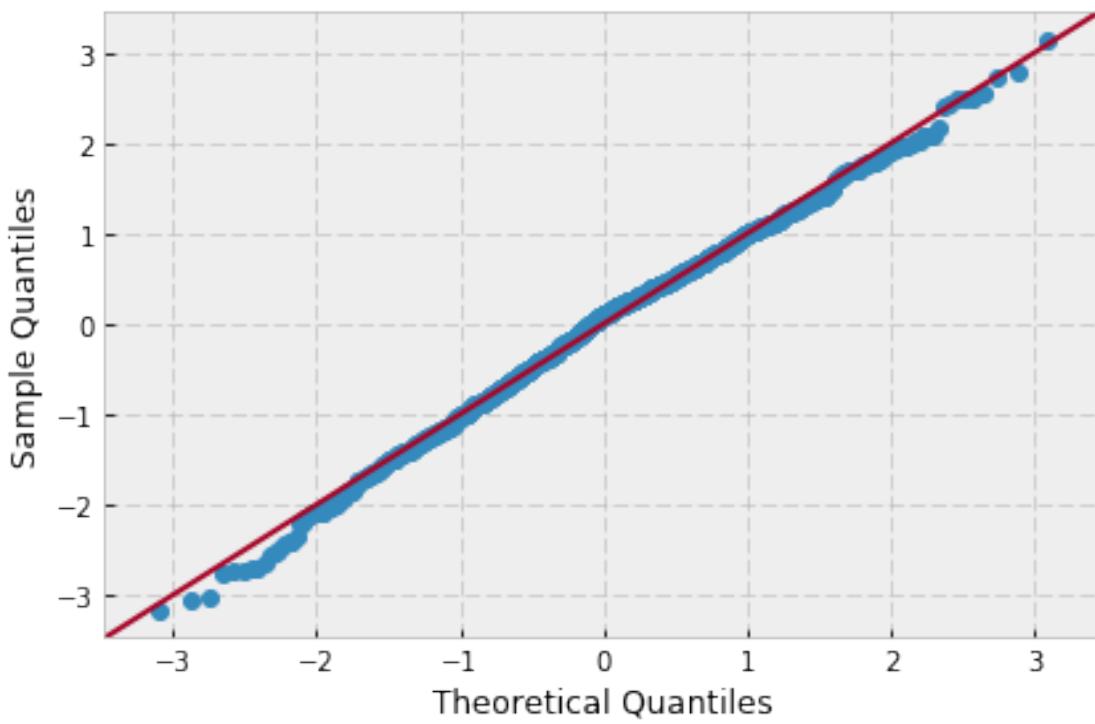
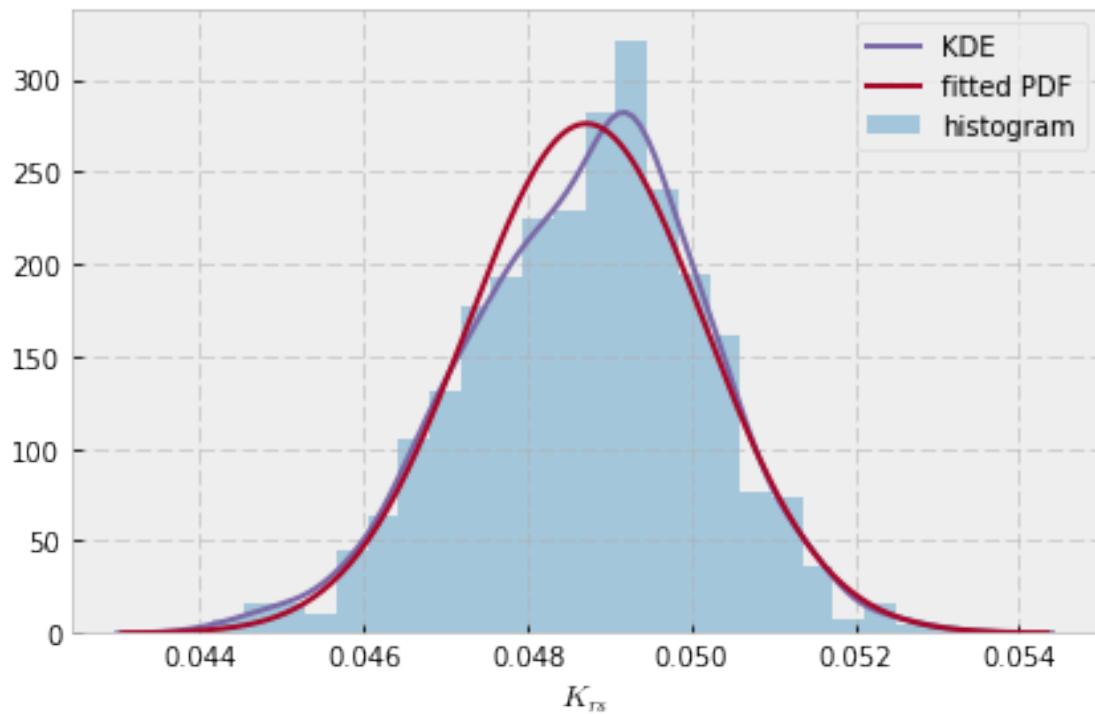


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(), loc=0.048727497763000005, scale=0.0013777963154207496)
DescribeResult(nobs=1000, minmax=(0.04421378547999997, 0.052235370660000002)
               mean=0.048727497763000005, variance=1.9002229096966906e-06,
               skewness=-0.16286347118775313, kurtosis=0.008073503783787128)
```

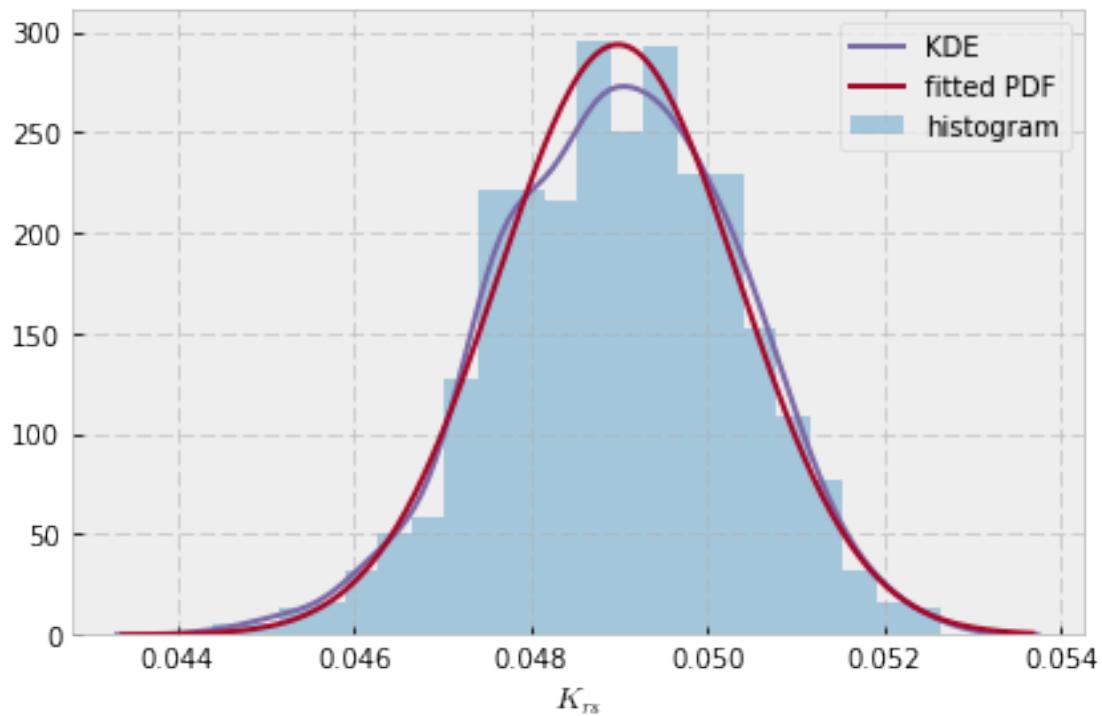


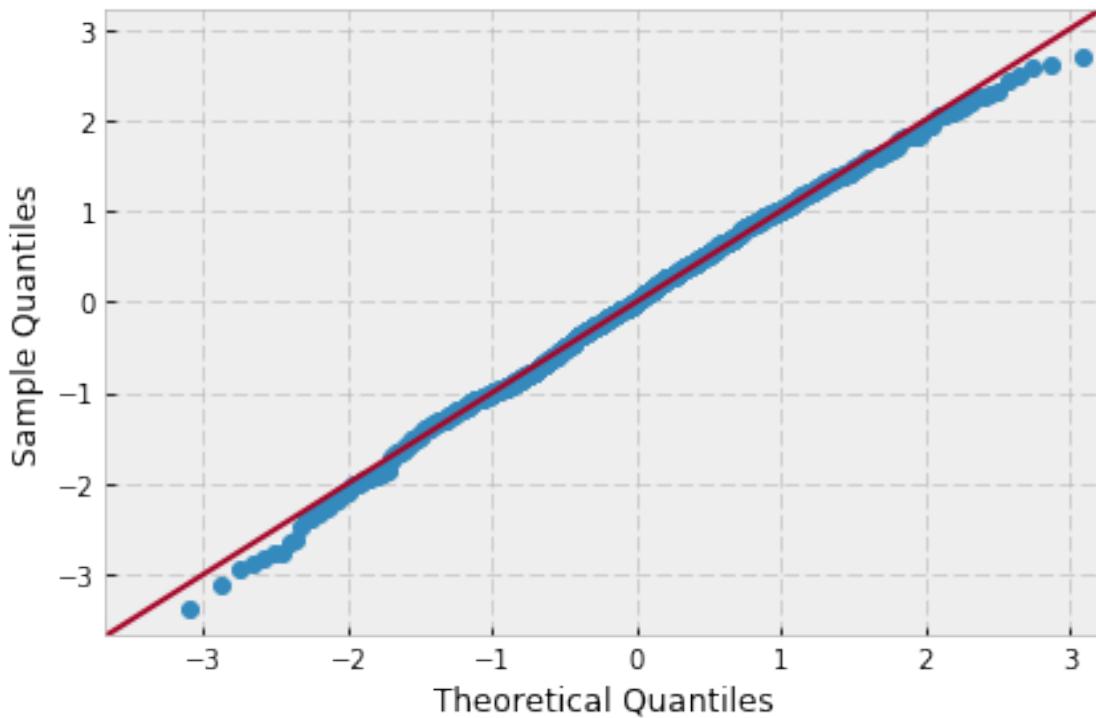


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(),loc=0.04871729049382, scale=0.0014450110013147412)
DescribeResult(nobs=1000, minmax=(0.04414728847999998, 0.05325141736999998)
               mean=0.04871729049382002, variance=2.0901469408614928e-06,
               skewness=-0.1922873897290026, kurtosis=0.0035692726234559657)
```

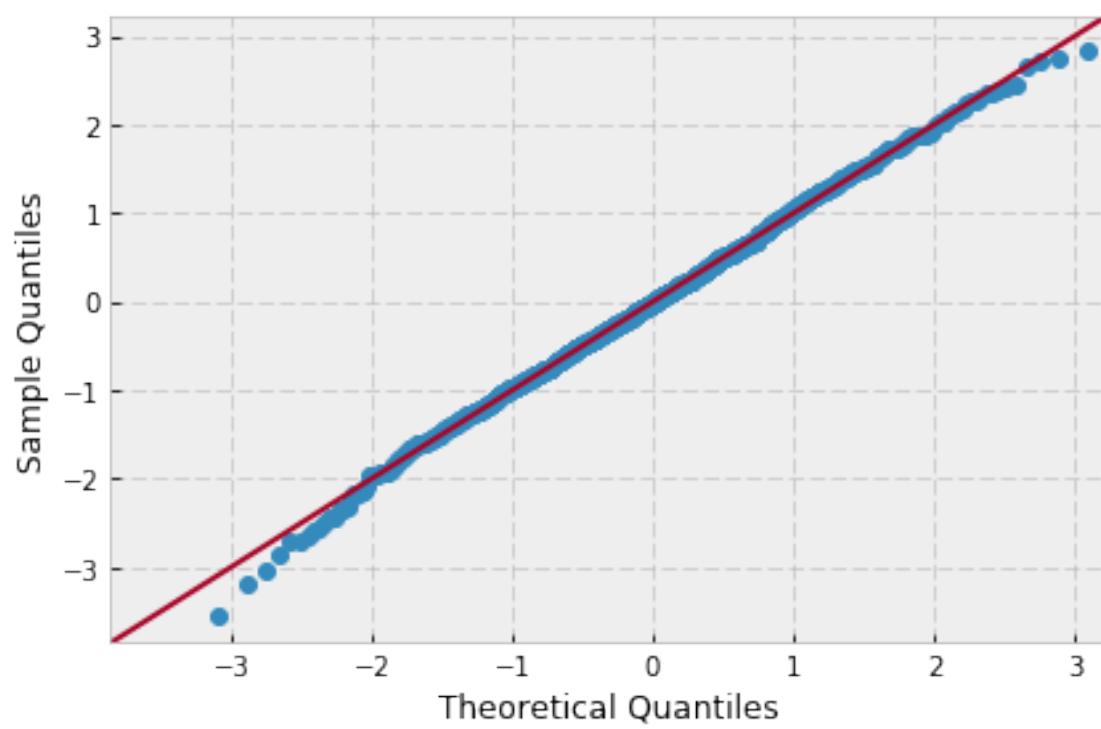
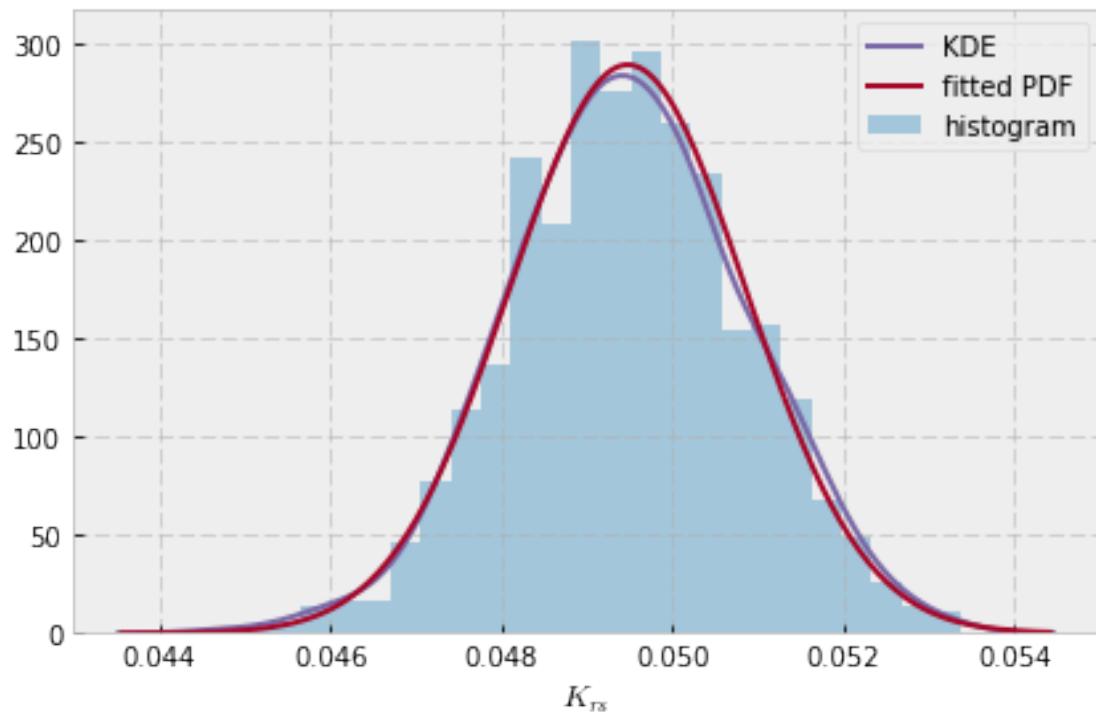


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(), loc=0.048983420635879994, scale=0.001358729542614874)
DescribeResult(nobs=1000, minmax=(0.04439797854000001, 0.05264912130999998)
               mean=0.048983420635879994, variance=1.8479939639383629e-06,
               skewness=-0.15256671139761854, kurtosis=-0.10838203683702563)
```

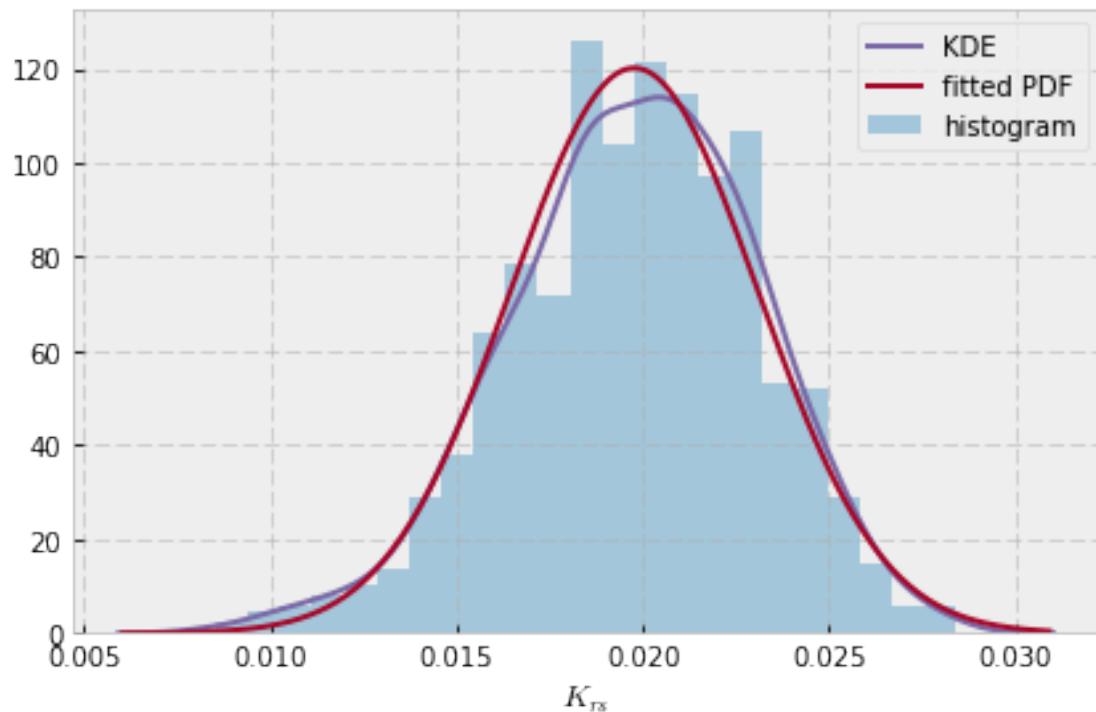


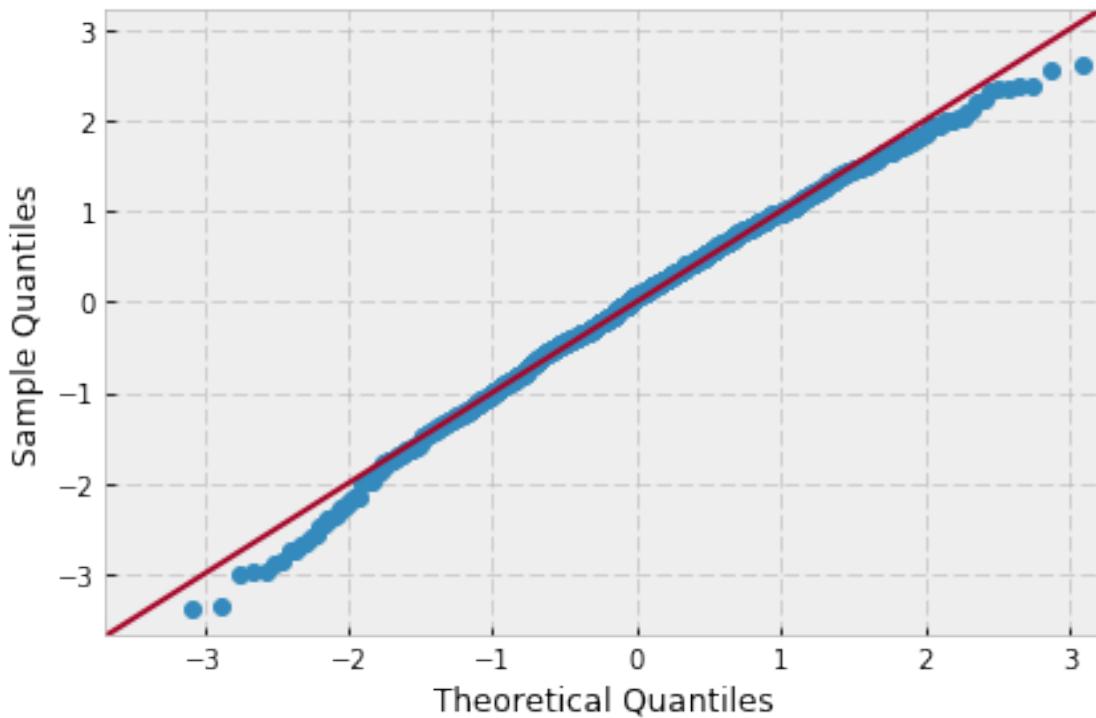


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(),loc=0.04948188922531, scale=0.0013801183984140457)
DescribeResult(nobs=1000, minmax=(0.04460332566999999, 0.05338308166999999)
               mean=0.049481889225310001, variance=1.9066334270680188e-06,
               skewness=-0.05172113845110719, kurtosis=0.016644601214522048)
```

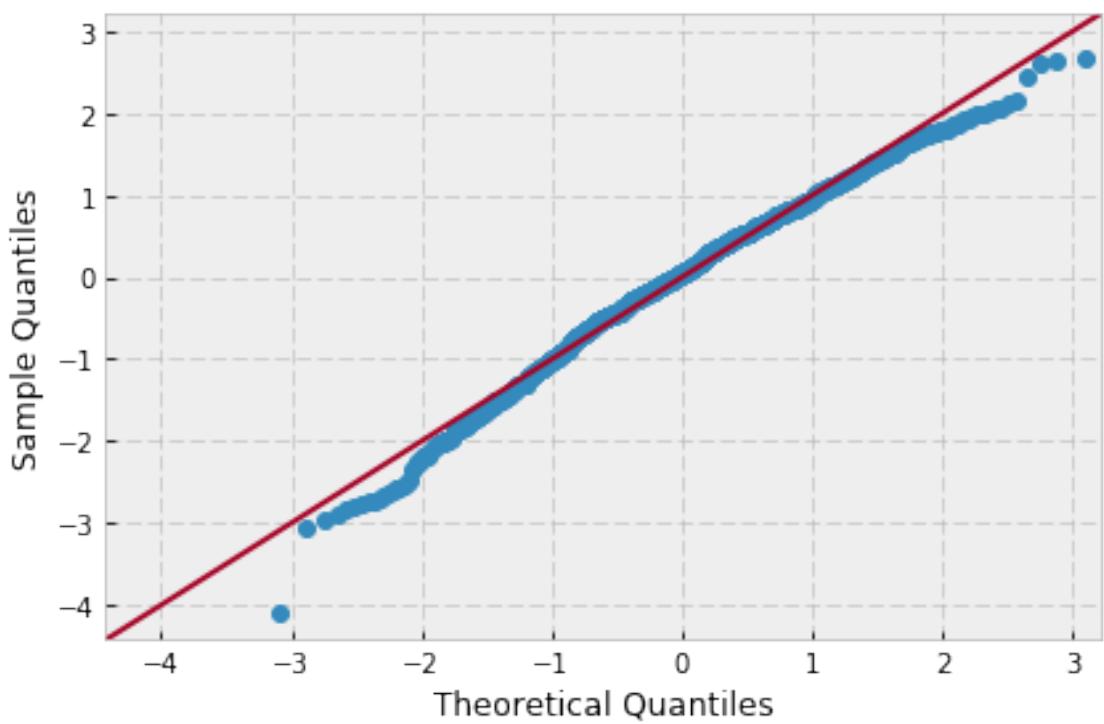
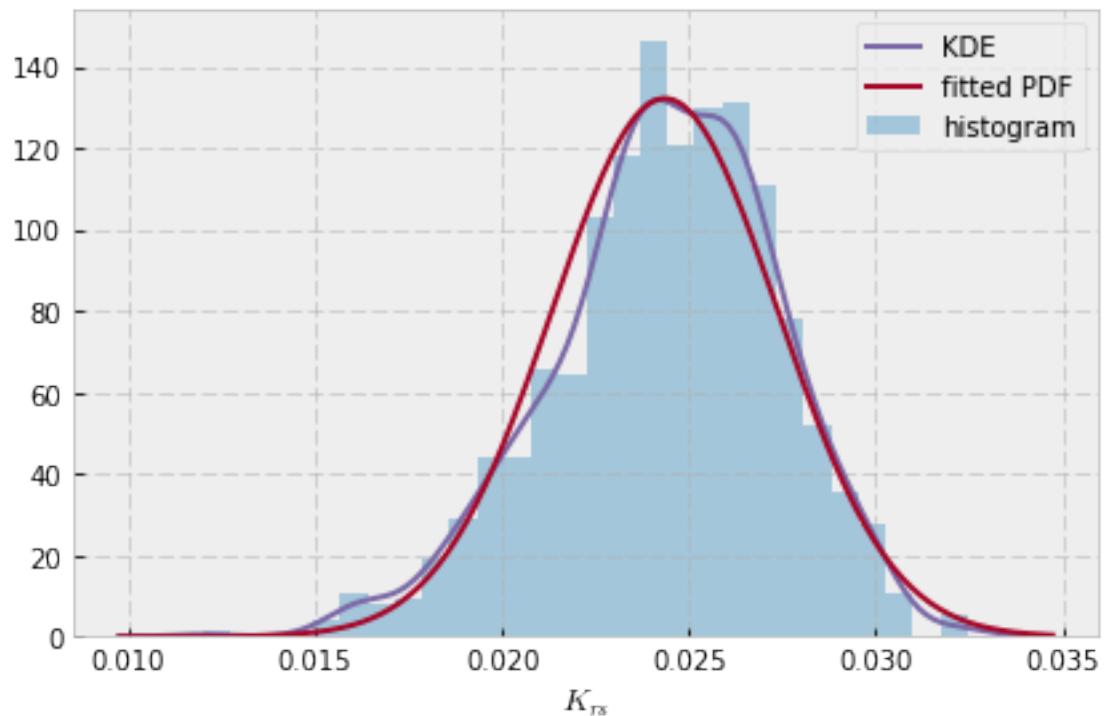


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=0.019766428767599, scale=0.003320106243935119)
DescribeResult(nobs=1000, minmax=(0.008537402500999997, 0.02840450645999999)
               mean=0.019766428767599001, variance=1.1034139610627592e-05,
               skewness=-0.28686696905452375, kurtosis=0.05399637769121712)
```

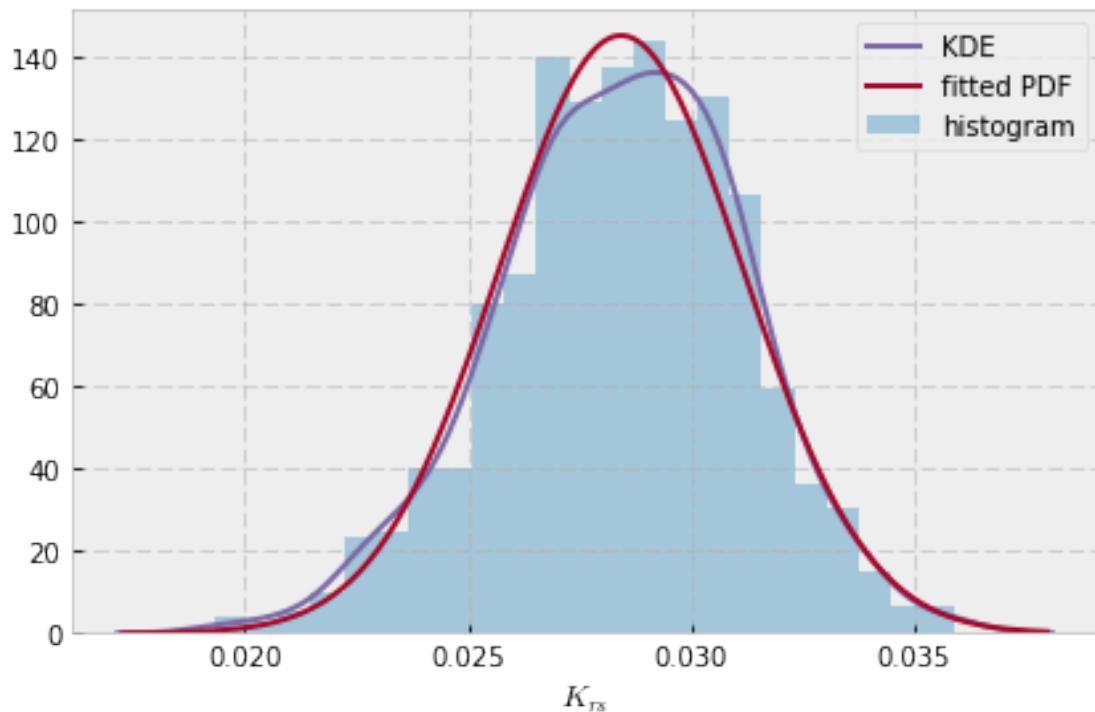


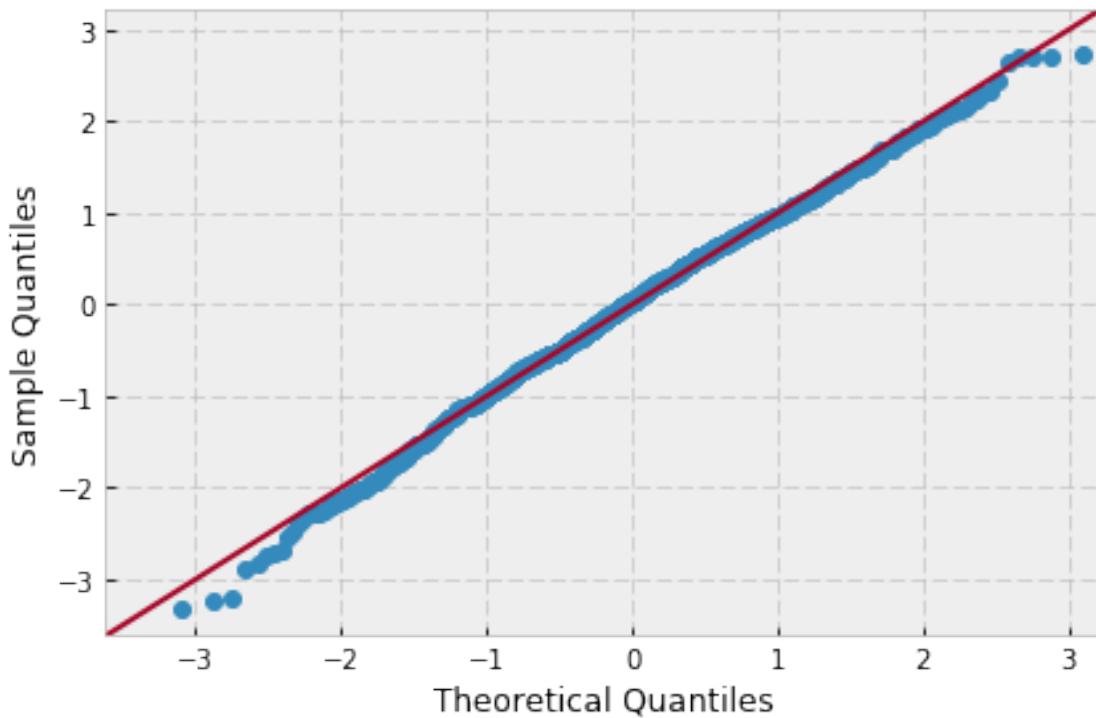


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(),loc=0.02435410454966, scale=0.0030175278078874907)
DescribeResult(nobs=1000, minmax=(0.01201551277999999, 0.03245159855000002)
               mean=0.024354104549659999, variance=9.1145886600343183e-06,
               skewness=-0.39819214516828794, kurtosis=0.21518999705948882)
```

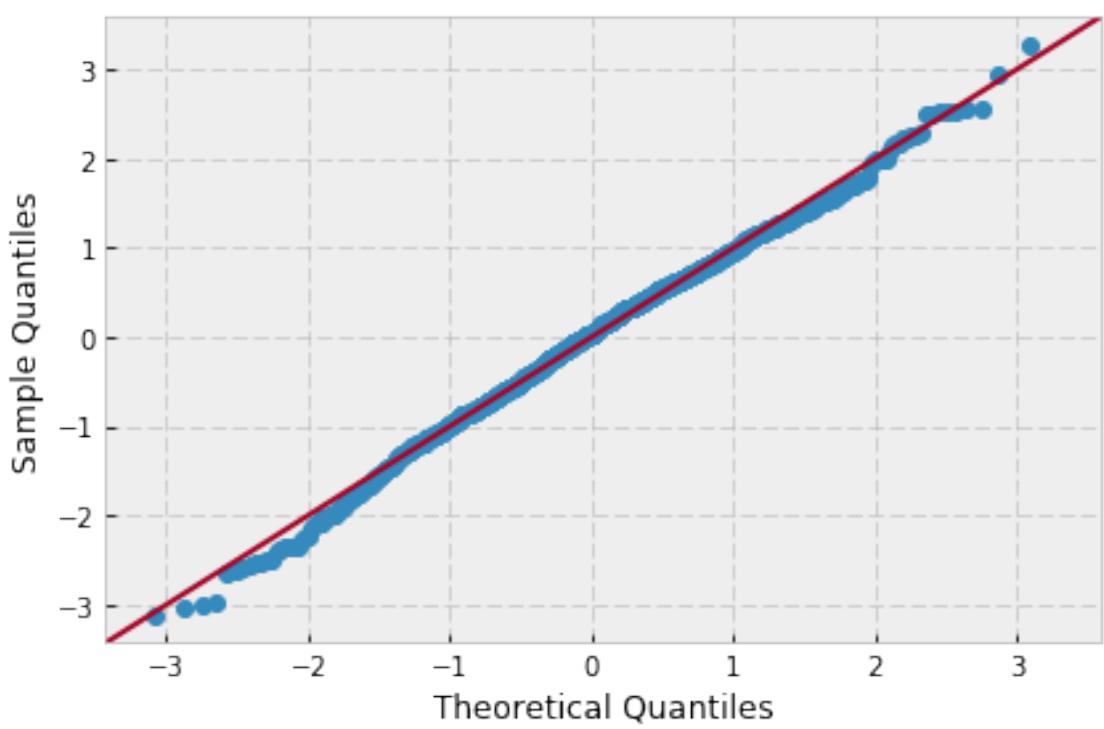
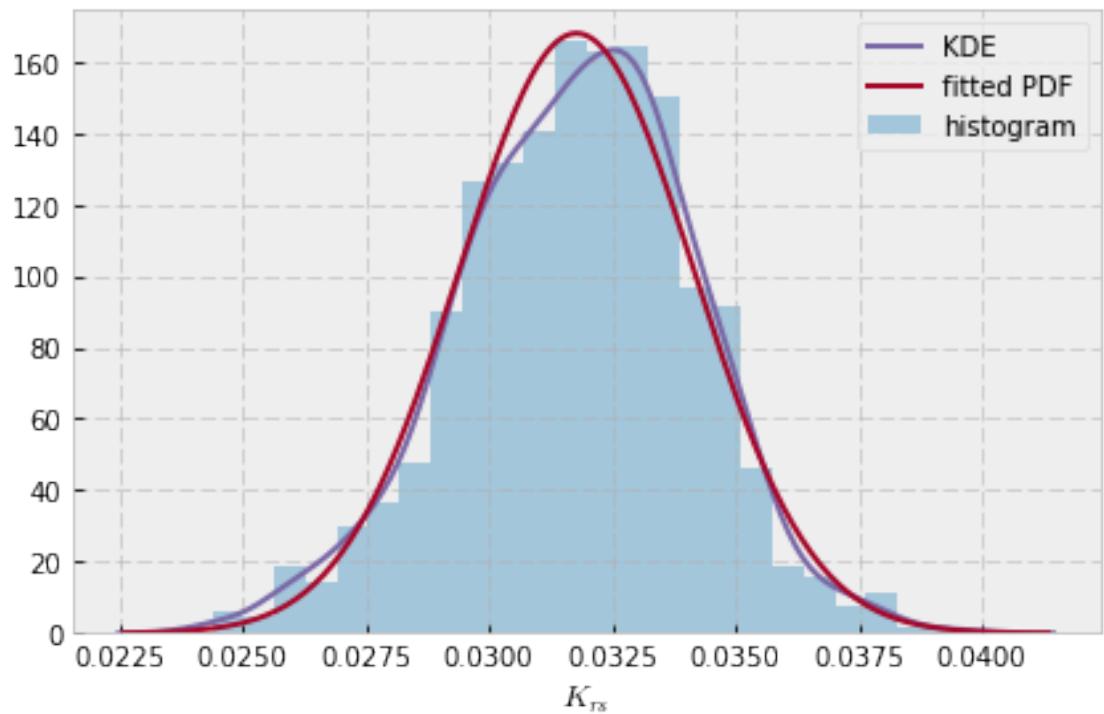


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(),loc=0.02843267487277, scale=0.0027462651129600037)
DescribeResult(nobs=1000, minmax=(0.019330983069999998, 0.03591221757000002)
               mean=0.02843267487277, variance=7.5495215922534762e-06,
               skewness=-0.21580061745843662, kurtosis=0.04335260148416653)
```

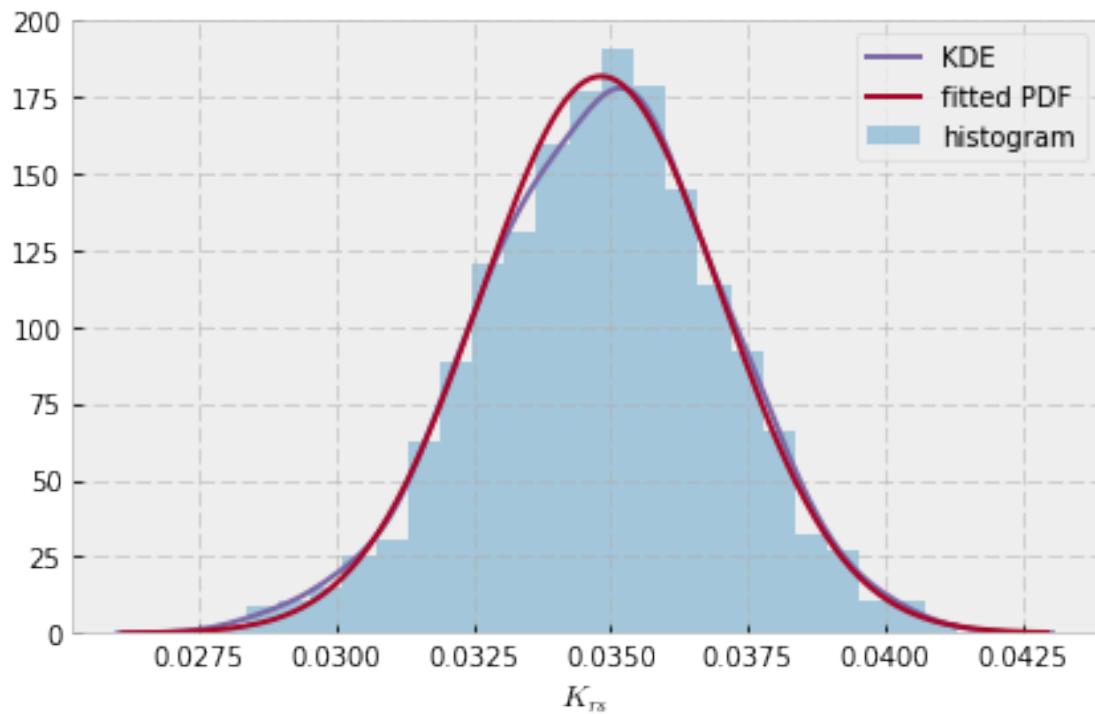


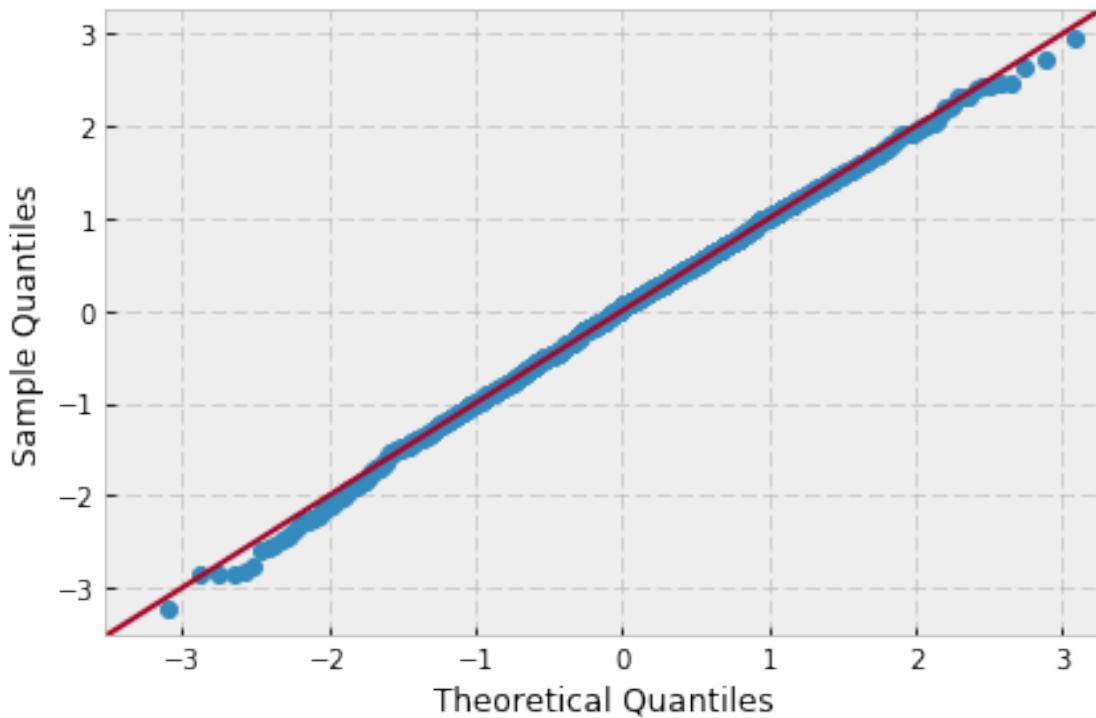


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=0.03177780327101, scale=0.00237564849638438)
DescribeResult(nobs=1000, minmax=(0.02438548057, 0.03955290308)
               mean=0.031777803271009999, variance=5.6493551335068716e-06,
               skewness=-0.17900957310421065, kurtosis=0.08945969276966492)
```

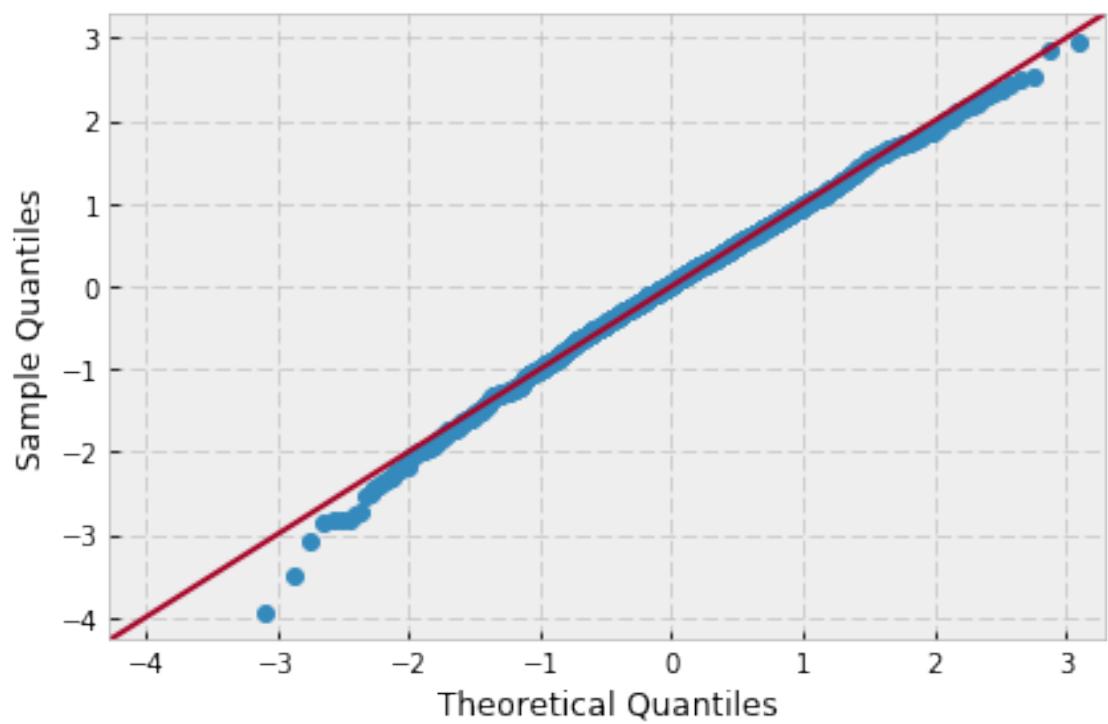
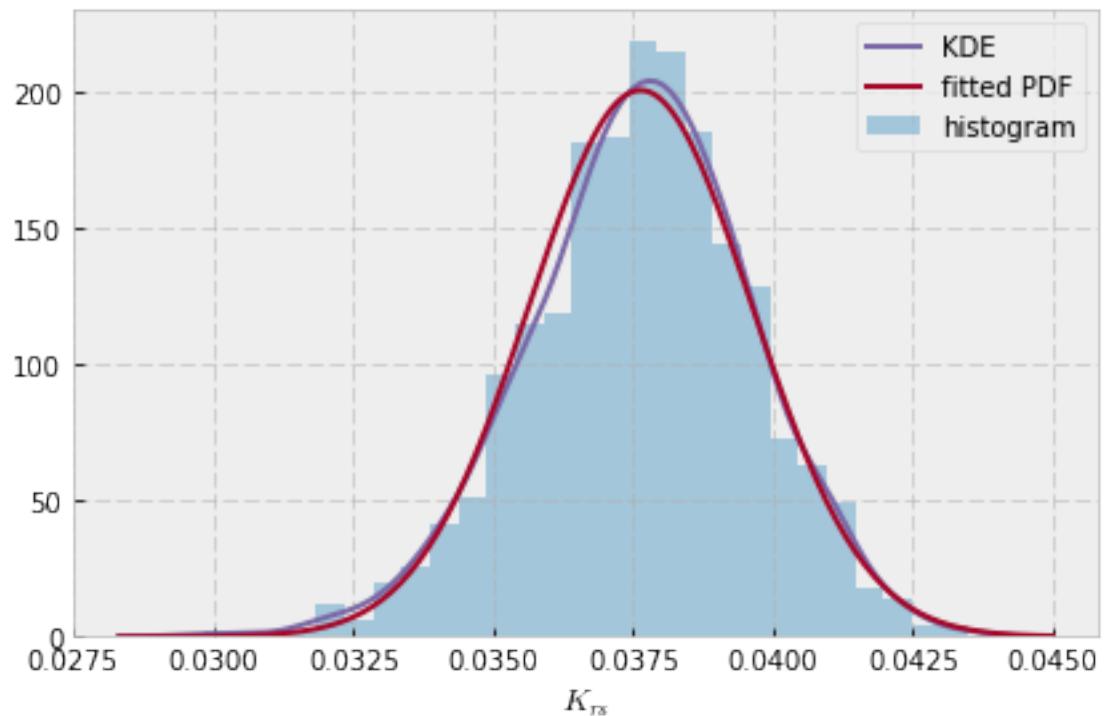


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(),loc=0.03482888684718, scale=0.002194110474894911)
DescribeResult(nobs=1000, minmax=(0.02777612715, 0.04129673234999997)
               mean=0.034828886847180003, variance=4.8189397157593302e-06,
               skewness=-0.11213366485183619, kurtosis=-0.04397066188687049)
```

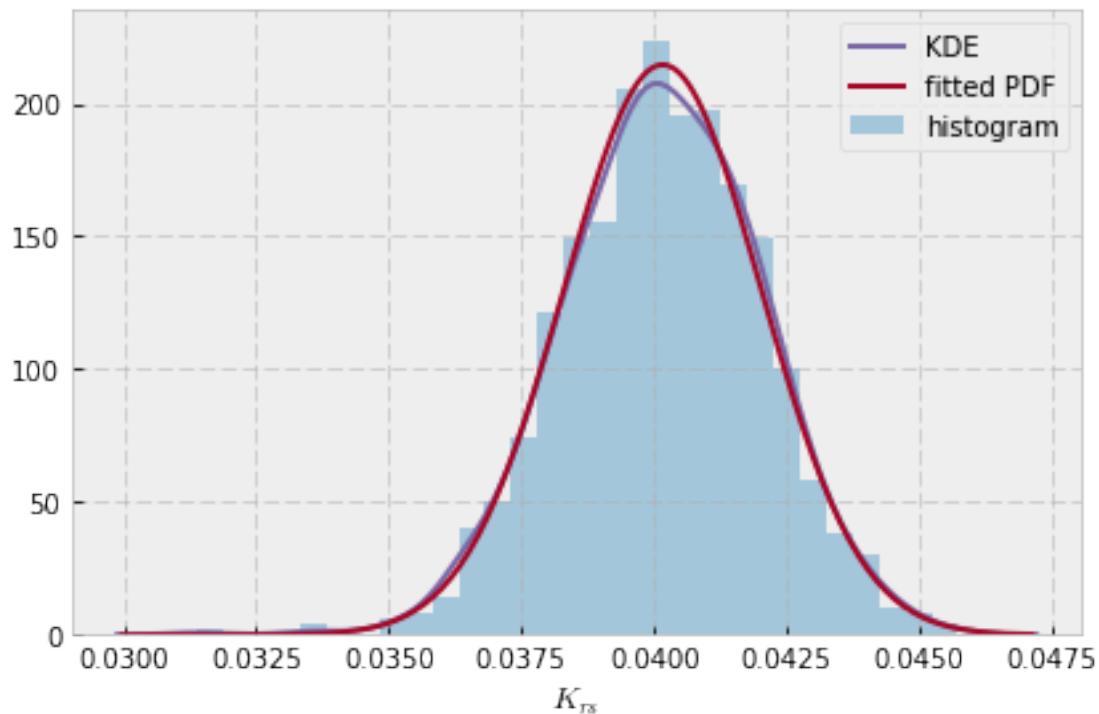


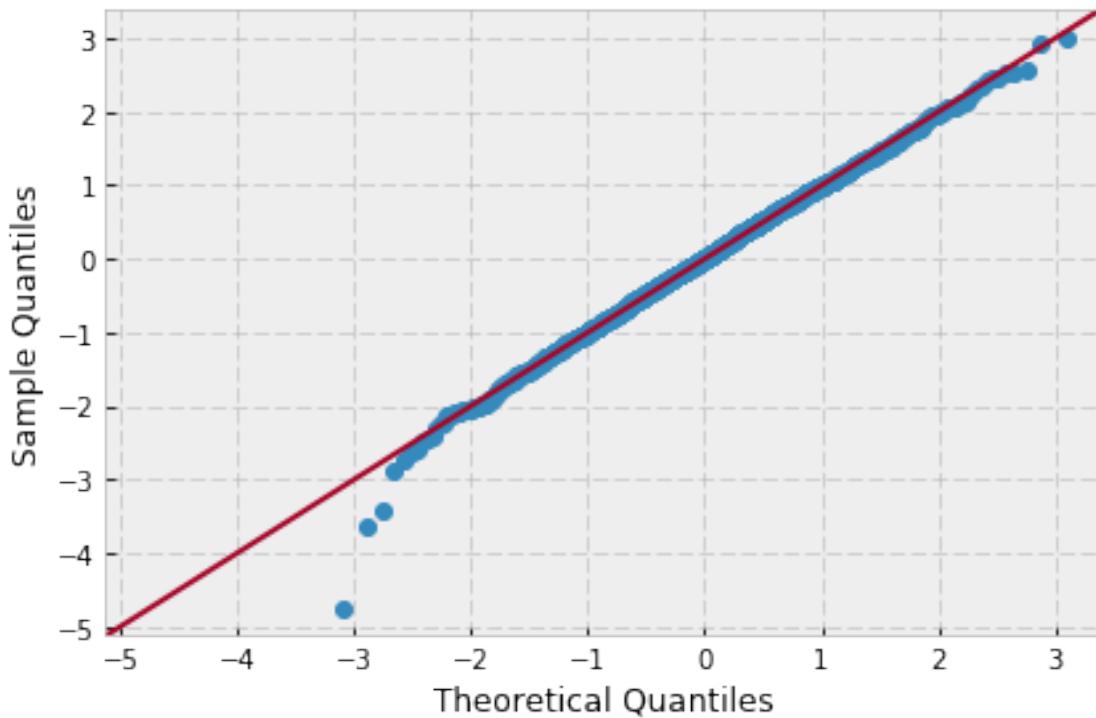


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(),loc=0.037628966105719995, scale=0.0019888909569438645)
DescribeResult(nobs=1000, minmax=(0.029810827580000001, 0.043492050550000001)
               mean=0.037628966105719995, variance=3.9596468854985787e-06,
               skewness=-0.22351976637524332, kurtosis=0.2323554950948279)
```

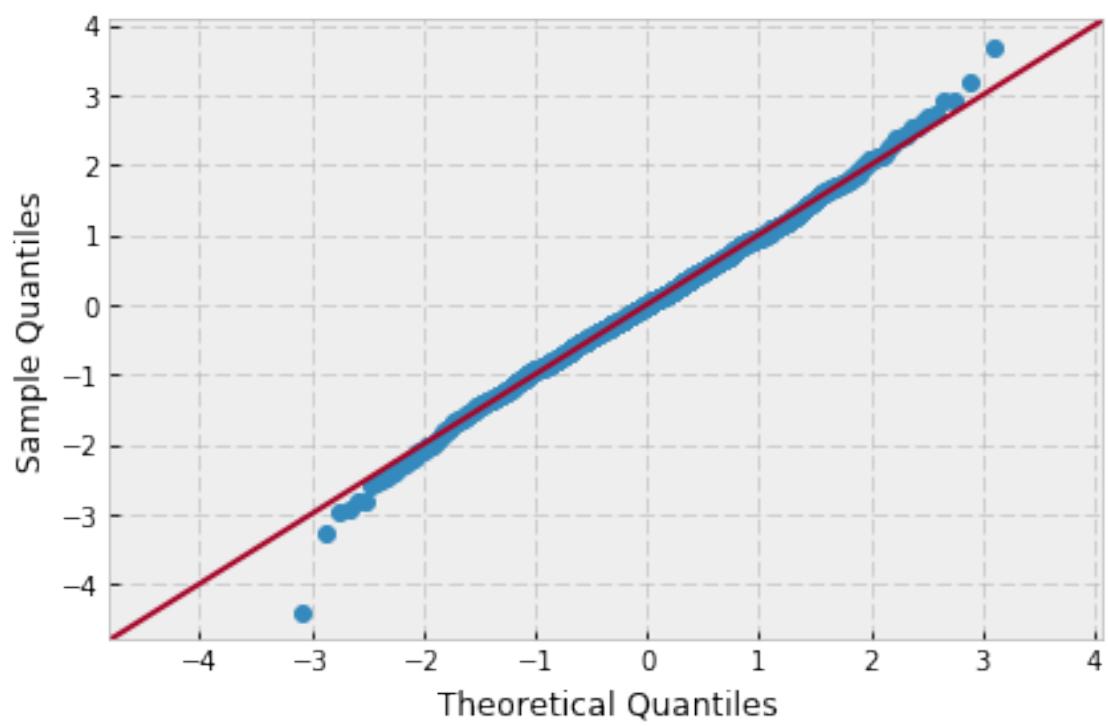
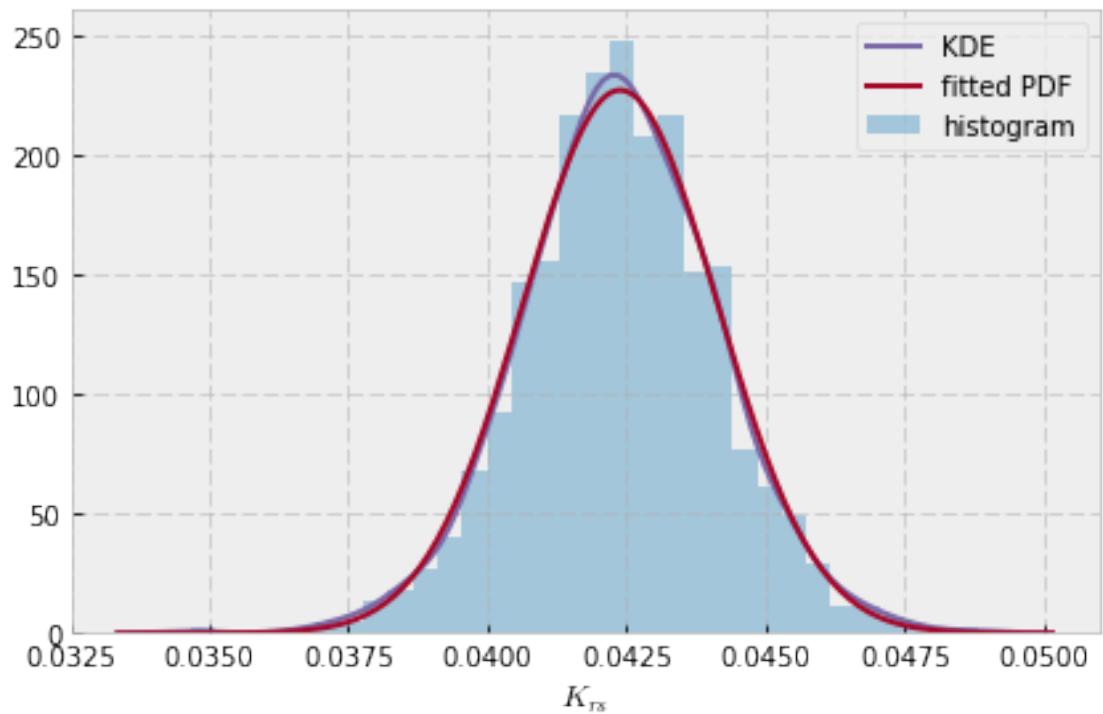


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=0.04016100659769, scale=0.0018590535309880806)
DescribeResult(nobs=1000, minmax=(0.031356727389999998, 0.045728761800000003)
               mean=0.040161006597690001, variance=3.4595395706499003e-06,
               skewness=-0.1830101098118199, kurtosis=0.39228245507128445)
```

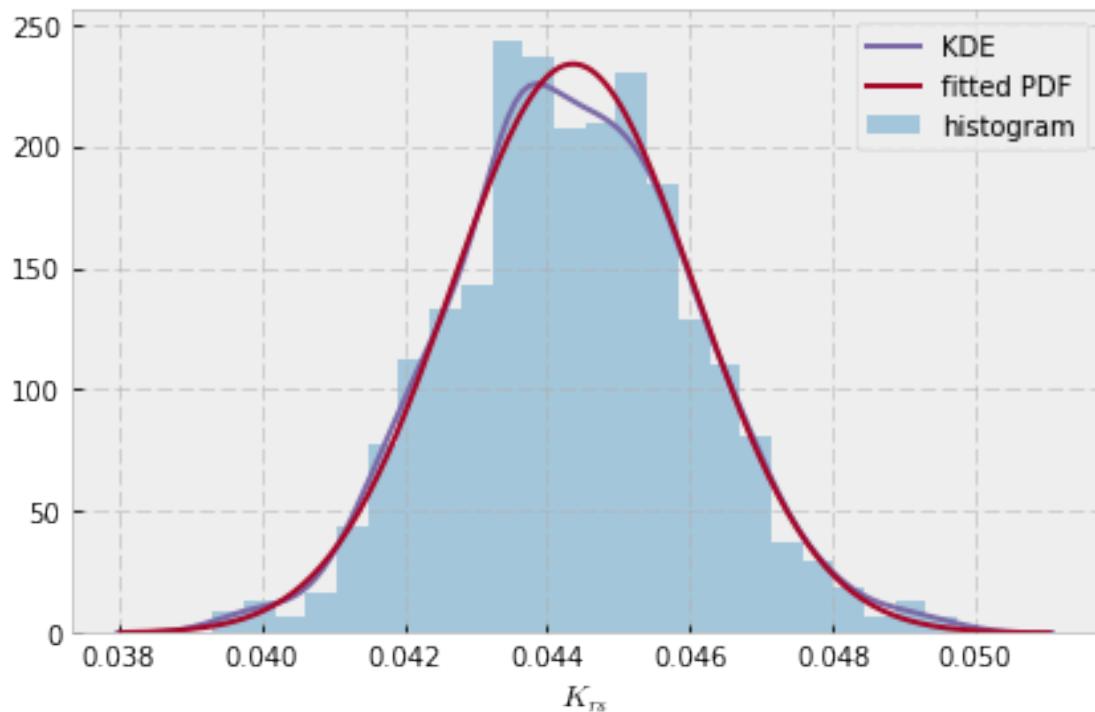


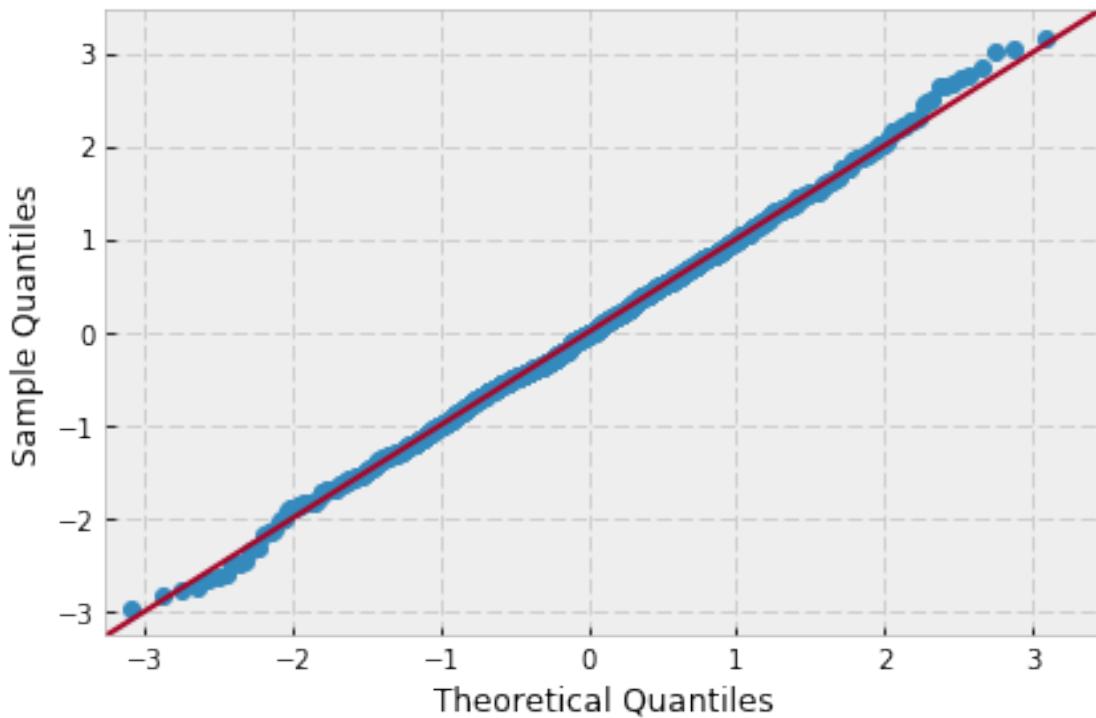


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=0.042383209055770005, scale=0.0017529651665427685)
DescribeResult(nobs=1000, minmax=(0.03467376434000003, 0.04882592613000002)
               mean=0.04238320905577005, variance=3.075962837950266e-06,
               skewness=-0.04164185342259237, kurtosis=0.5520857819926661)
```

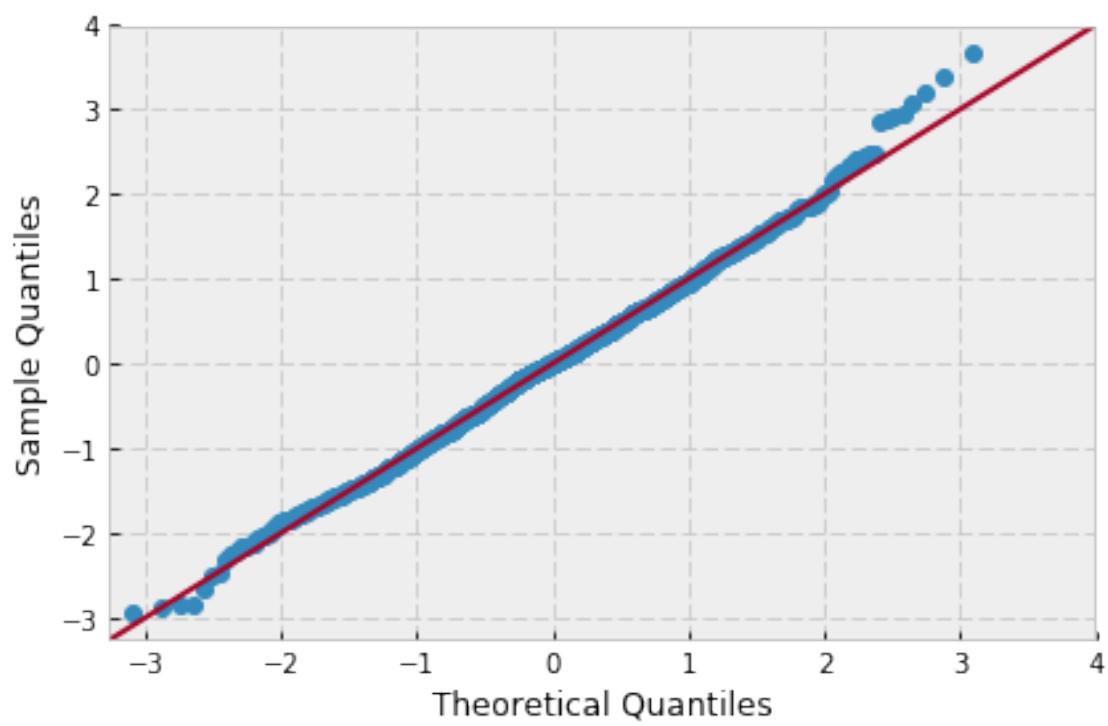
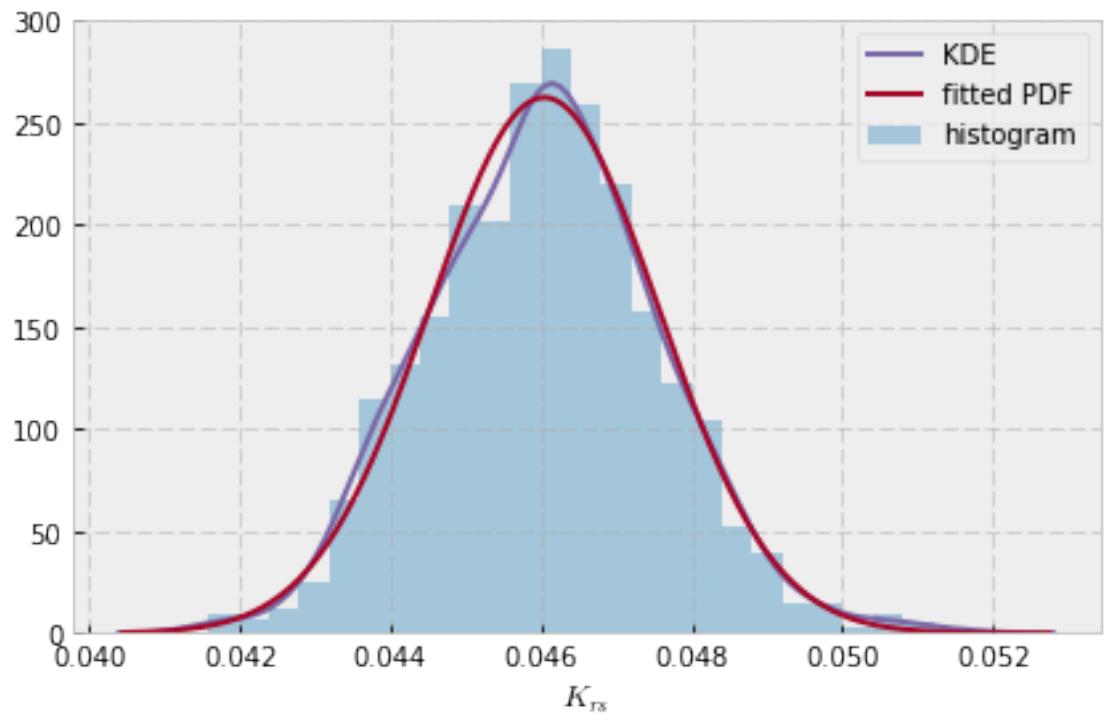


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=0.044360438648279994, scale=0.001707165603976573)
DescribeResult(nobs=1000, minmax=(0.03930548605999999, 0.04974643695)
               mean=0.044360438648279994, variance=2.9173317311318288e-06,
               skewness=0.07878272488293536, kurtosis=0.033687569211443336)
```

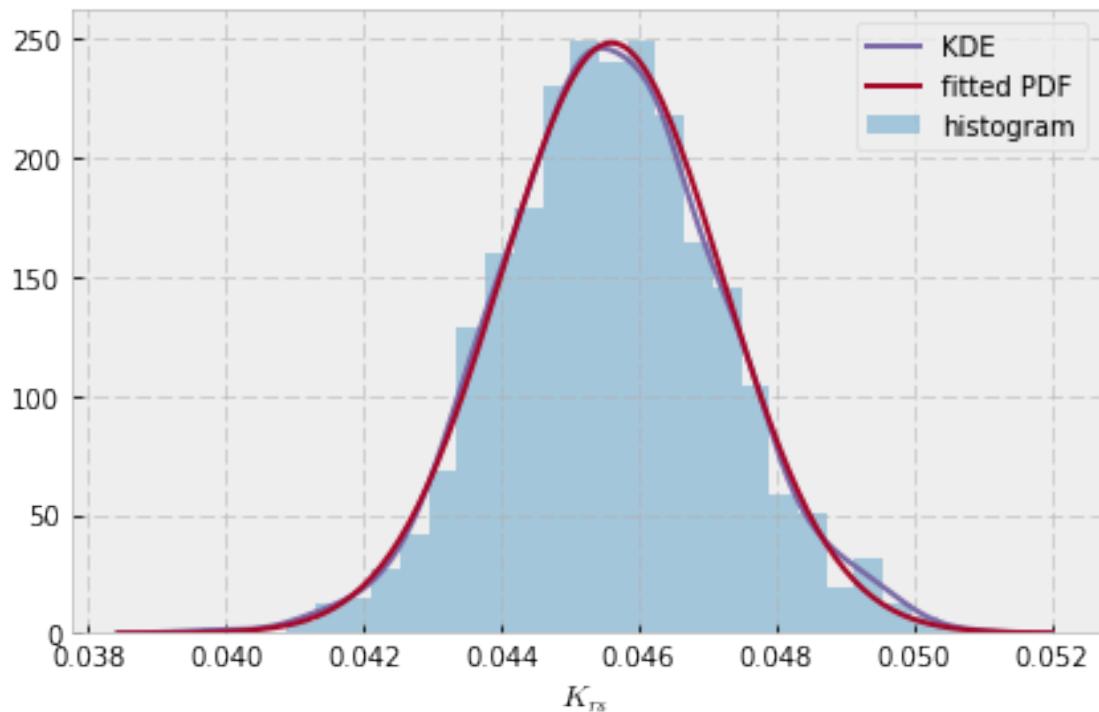


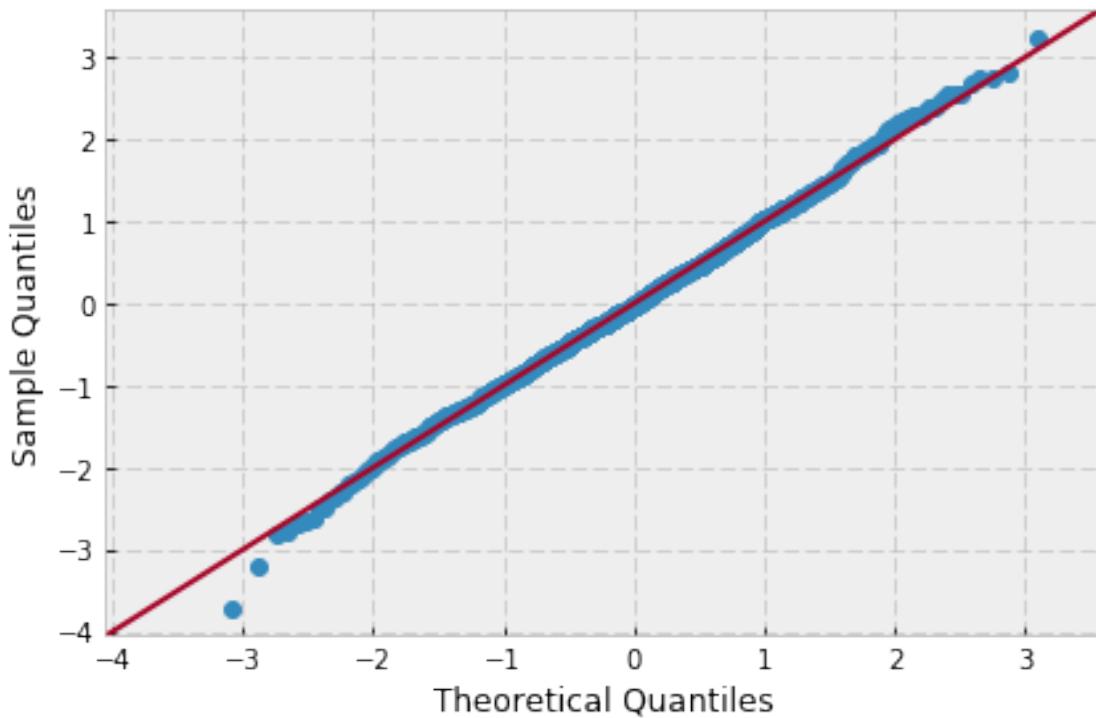


```
variable name: r
variable value: 3.0
distribution: normal(shape=(),loc=0.046041833474580005, scale=0.001520264247419762)
DescribeResult(nobs=1000, minmax=(0.041583065269999997, 0.05160900438999998)
               mean=0.046041833474580005, variance=2.3135168988816572e-06,
               skewness=0.12598704697025995, kurtosis=0.17540026358248495)
```

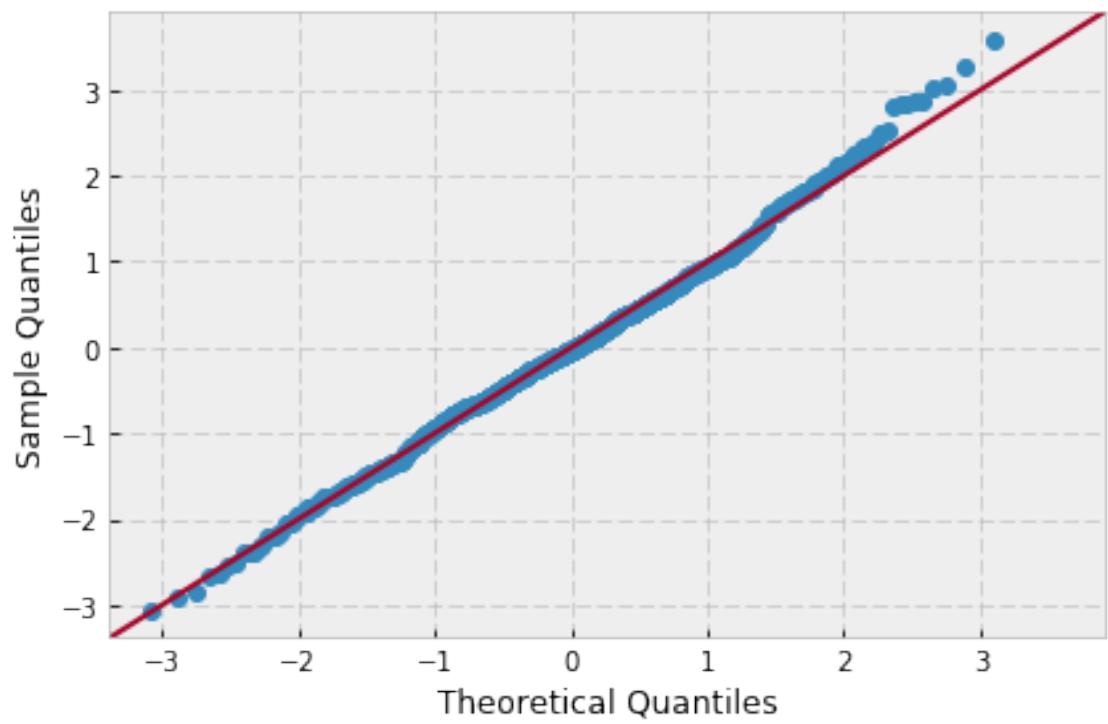
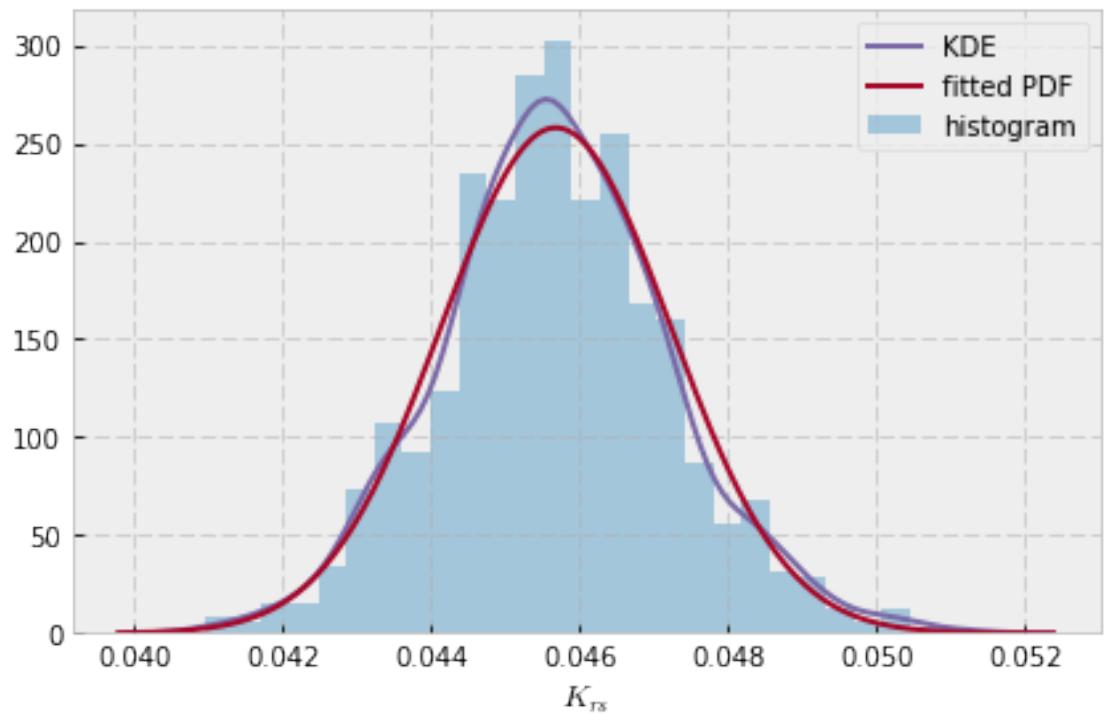


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(), loc=0.04560223593544, scale=0.0016048560220399465)
DescribeResult(nobs=1000, minmax=(0.03965187070000001, 0.05078030353000001)
               mean=0.045602235935440001, variance=2.5781409924703514e-06,
               skewness=0.040268507126563303, kurtosis=0.16494342387614713)
```

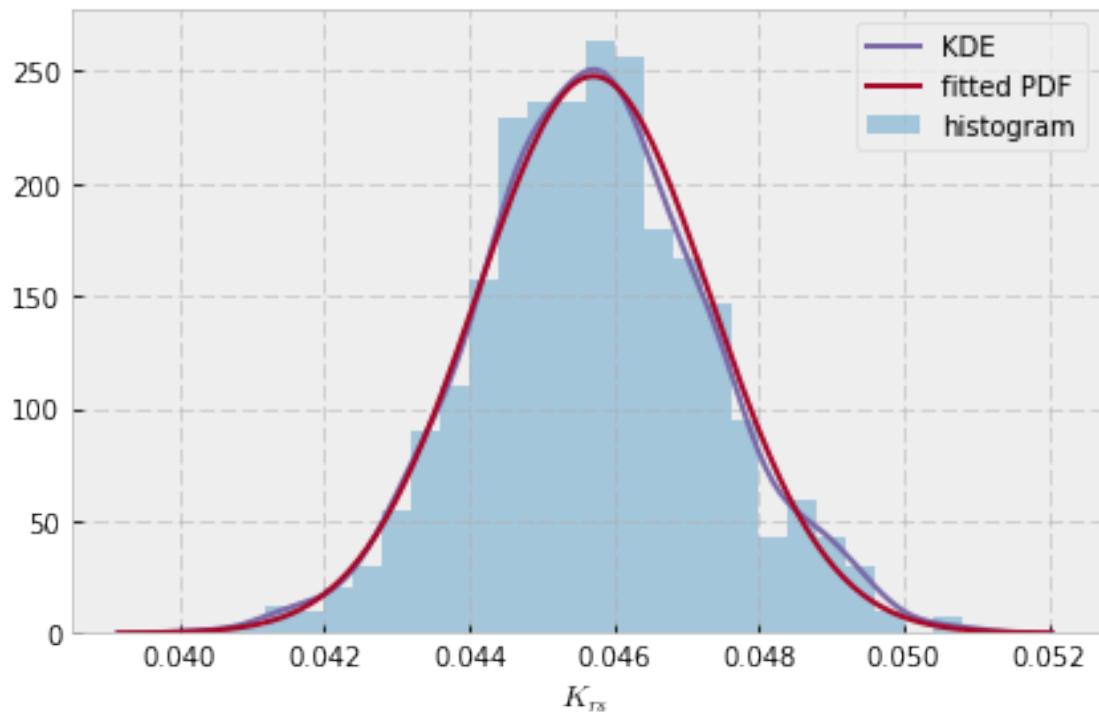


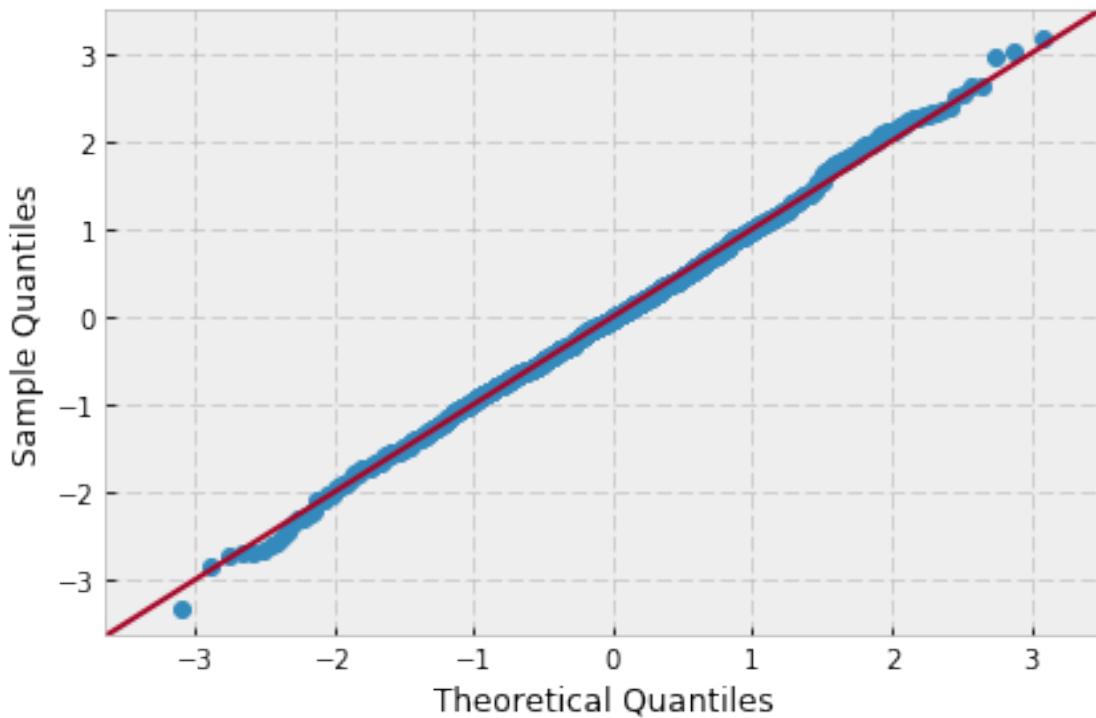


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=0.045676594685729995, scale=0.0015466841029225914)
DescribeResult(nobs=1000, minmax=(0.04095563311, 0.05120781203000001)
               mean=0.045676594685729995, variance=2.3946263405740352e-06,
               skewness=0.15489276123691764, kurtosis=0.3366218308887383)
```

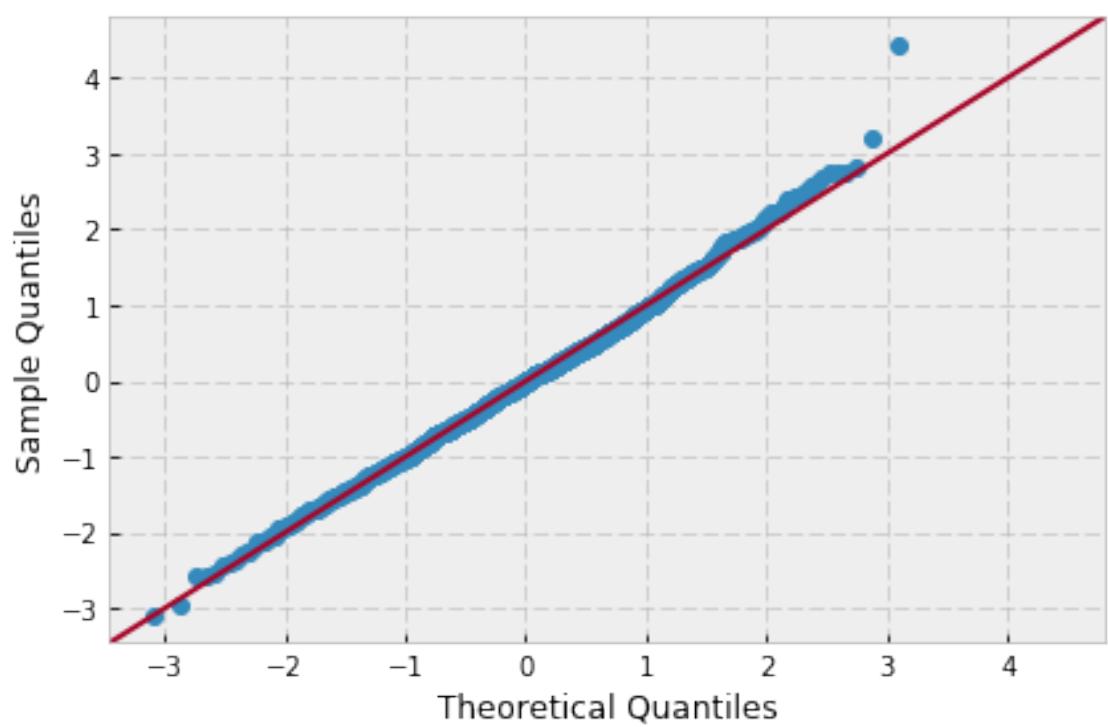
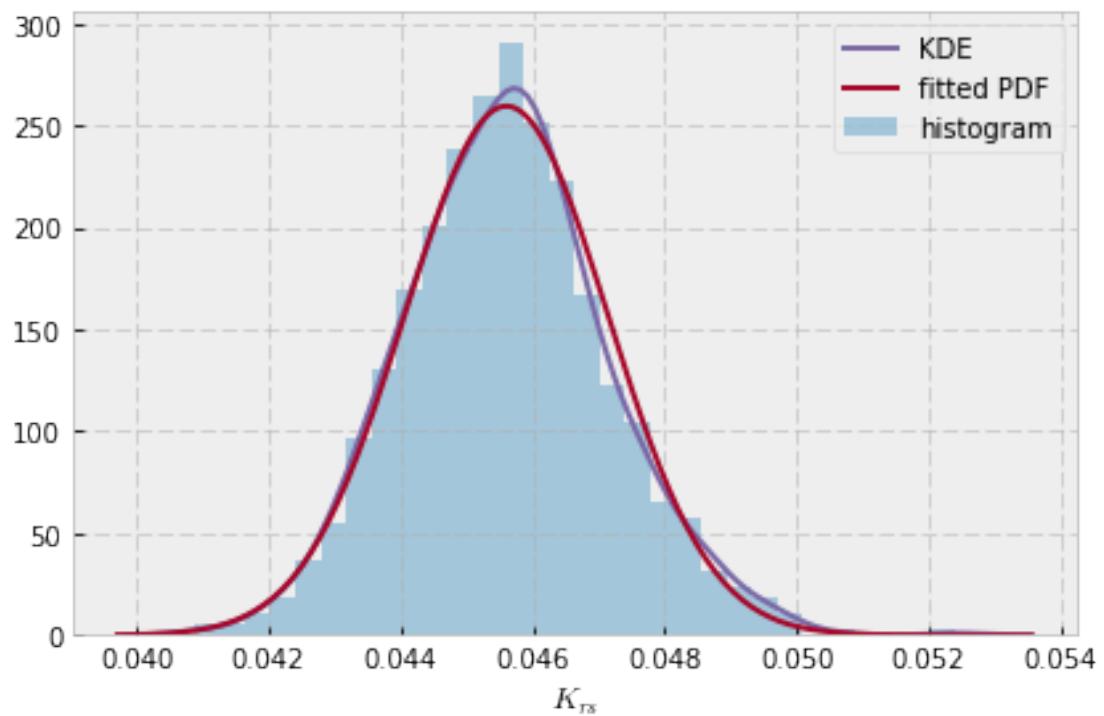


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=0.045713203396739996, scale=0.0016106211956234647)
DescribeResult(nobs=1000, minmax=(0.04037590421000001, 0.05081898883000002)
               mean=0.045713203396739996, variance=2.5966973331246841e-06,
               skewness=0.07156631288981853, kurtosis=0.09588373608255418)
```

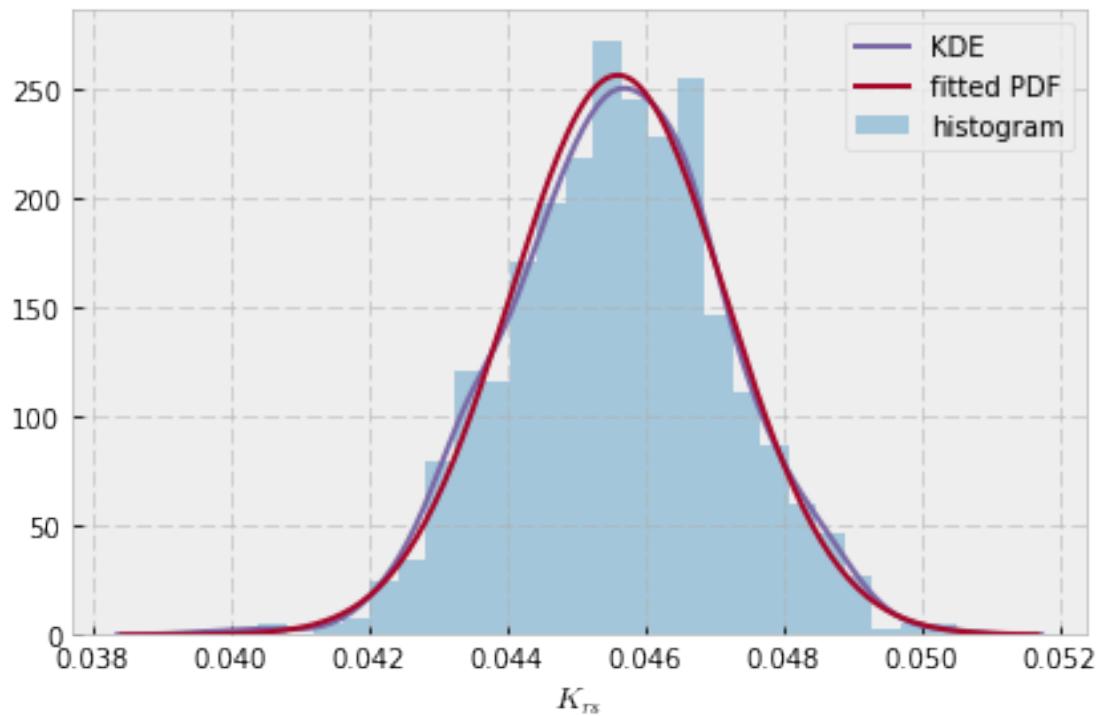


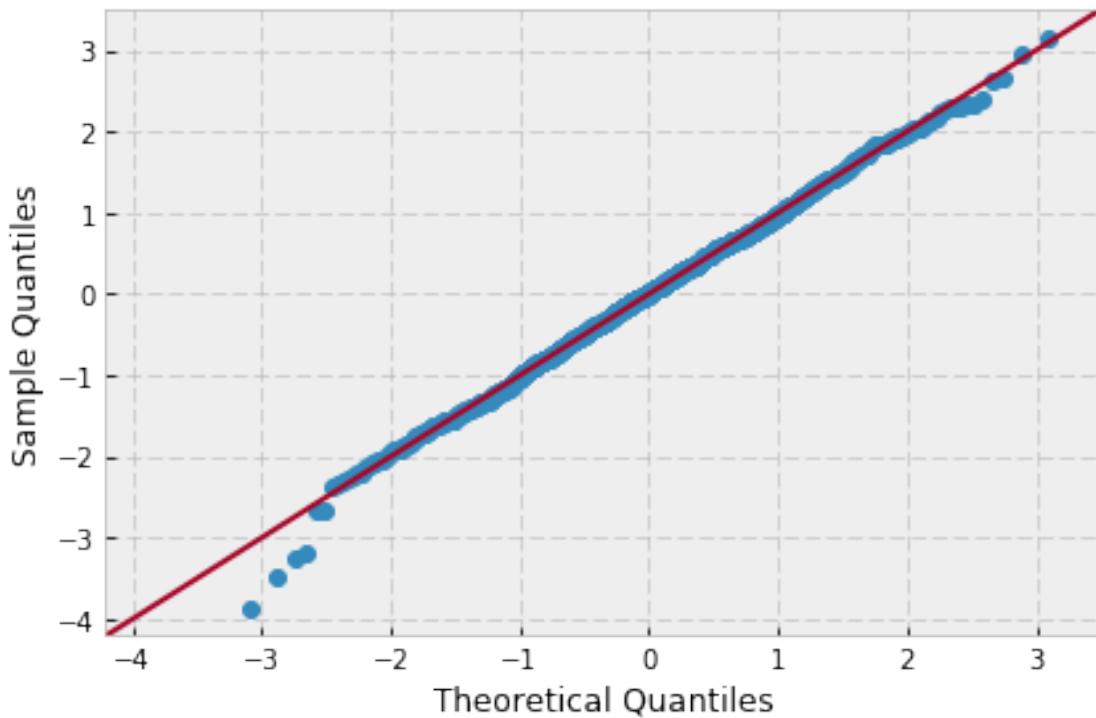


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(),loc=0.04560120043687, scale=0.0015365775684101216)
DescribeResult(nobs=1000, minmax=(0.040865455109999997, 0.052402352110000001)
               mean=0.04560120043687, variance=2.3634340577989612e-06,
               skewness=0.21059084917046403, kurtosis=0.3281665983781692)
```

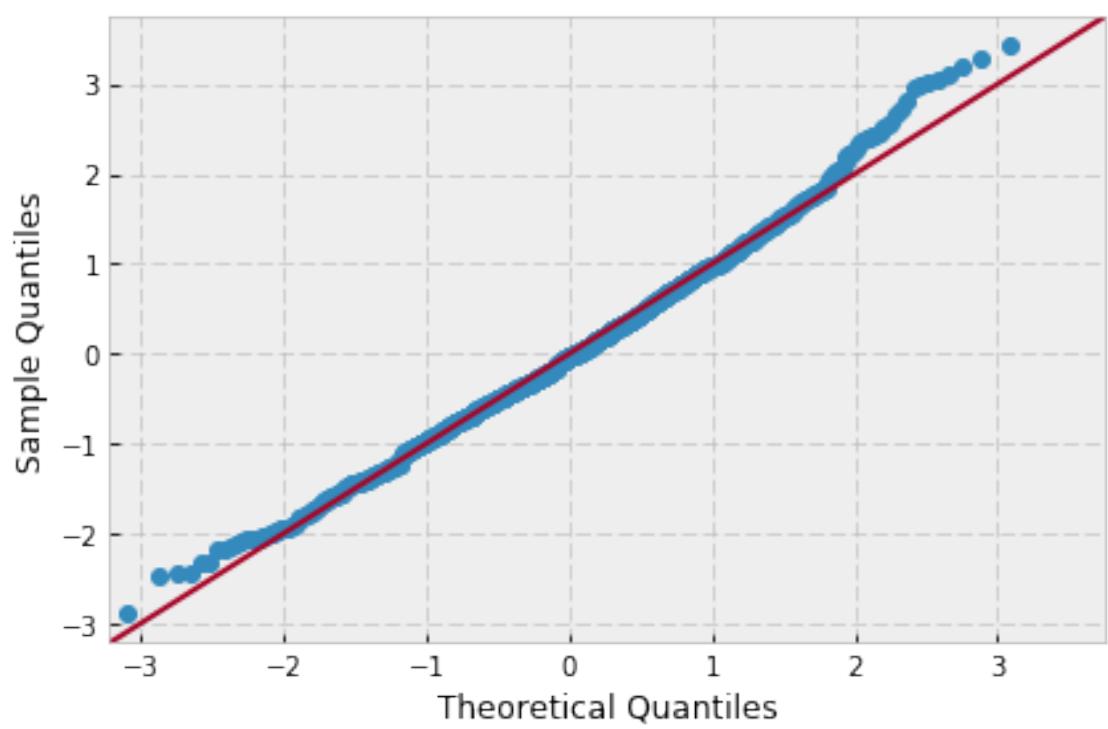
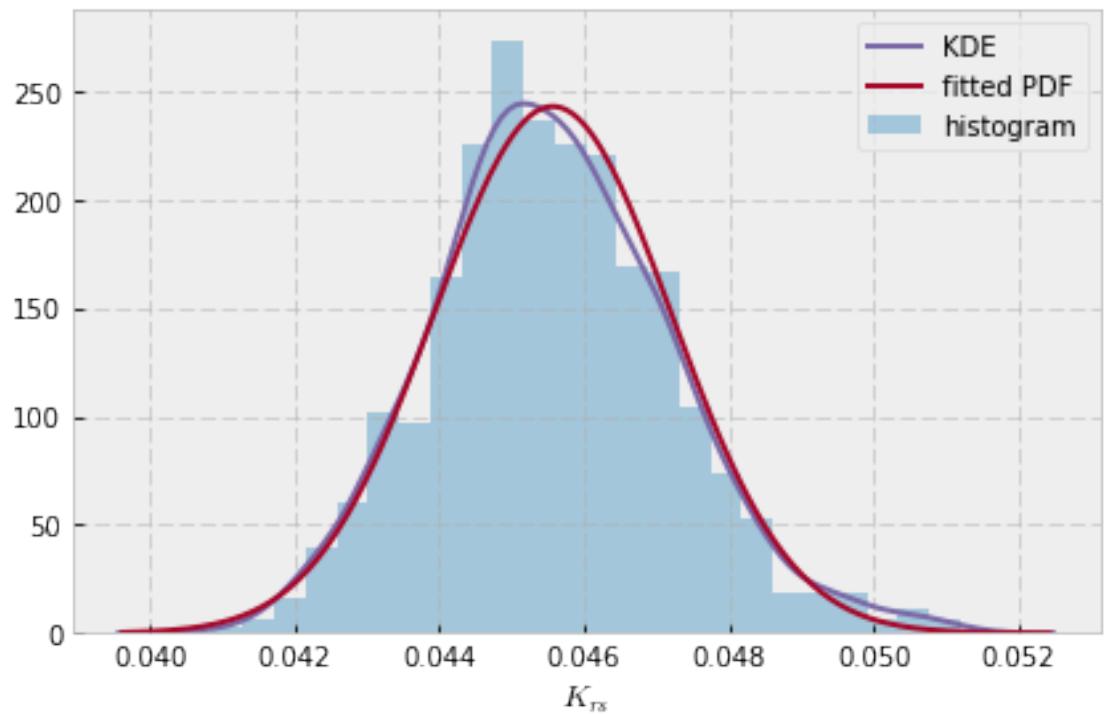


```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(), loc=0.04560199678325, scale=0.0015564764608756382)
DescribeResult(nobs=1000, minmax=(0.03959715116999997, 0.05049487335999998)
               mean=0.04560199678324998, variance=2.4250440172772296e-06,
               skewness=-0.08253530062371421, kurtosis=0.09736102580779615)
```

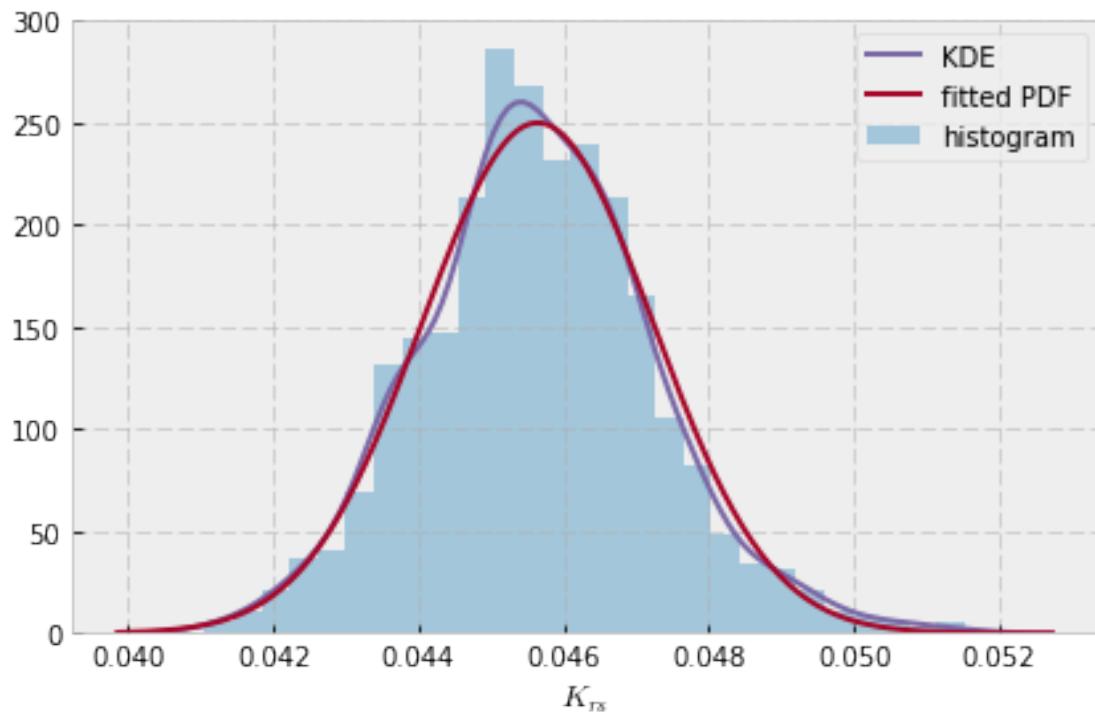


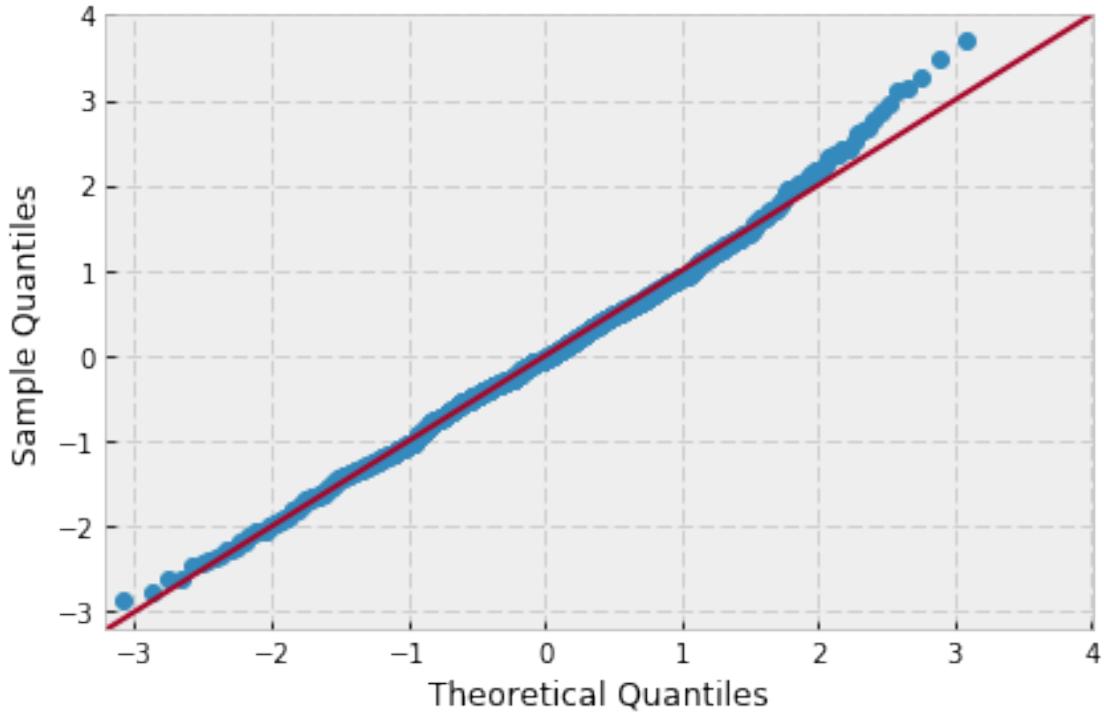


```
variable name: sigma
variable value: 0.0666666666666667
distribution: normal(shape=(),loc=0.04556630042634, scale=0.0016425982418571885)
DescribeResult(nobs=1000, minmax=(0.04085127303000003, 0.05120046560000002)
               mean=0.04556630042634002, variance=2.7008298139662927e-06,
               skewness=0.2992770928247034, kurtosis=0.2323830582272417)
```

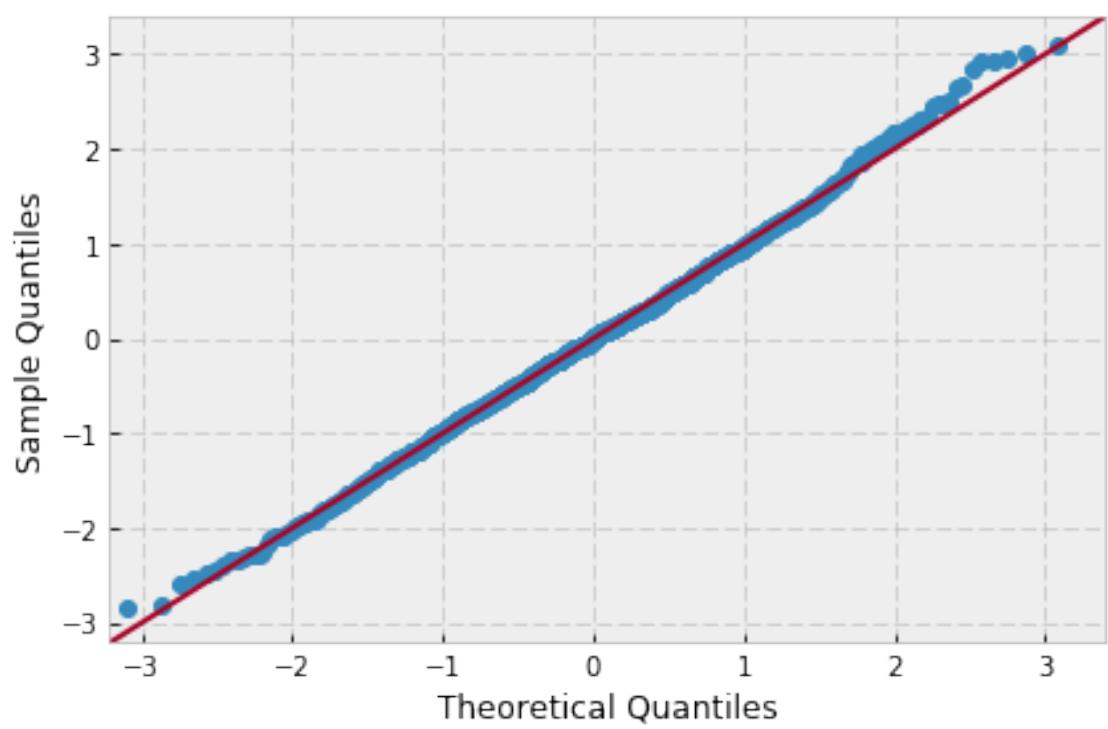
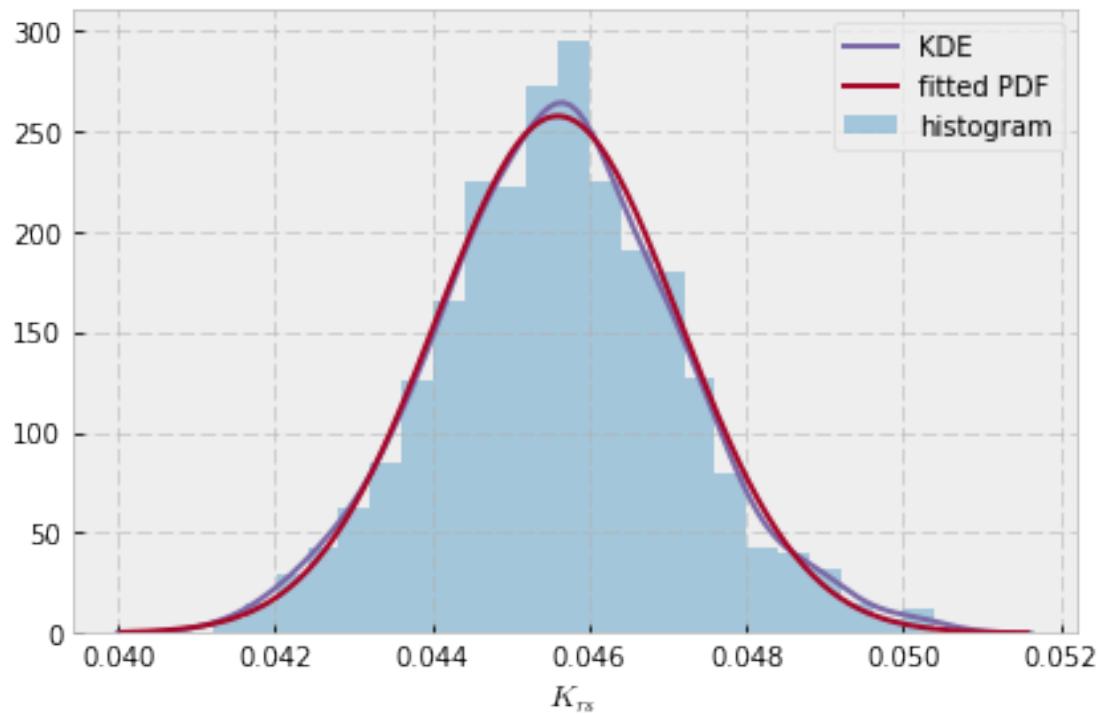


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=0.04563815191813, scale=0.0015958869582076794)
DescribeResult(nobs=1000, minmax=(0.04105566213999997, 0.05152496615999998)
               mean=0.045638151918129997, variance=2.5494045879653244e-06,
               skewness=0.20805200701594617, kurtosis=0.3634958805911461)
```

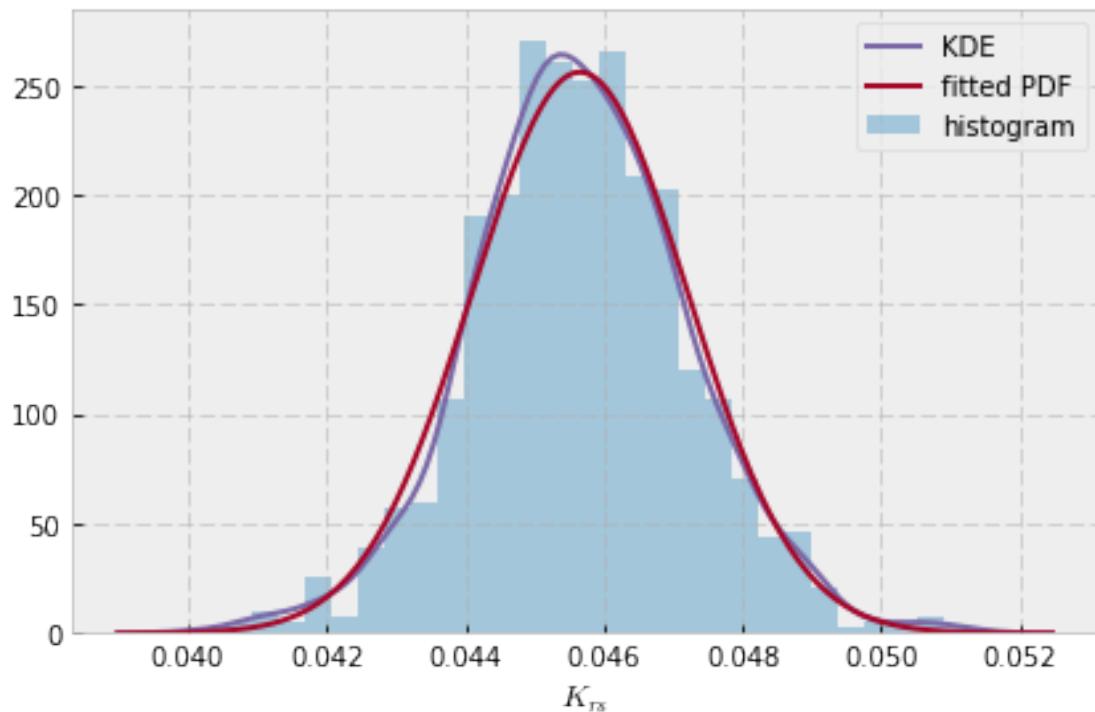


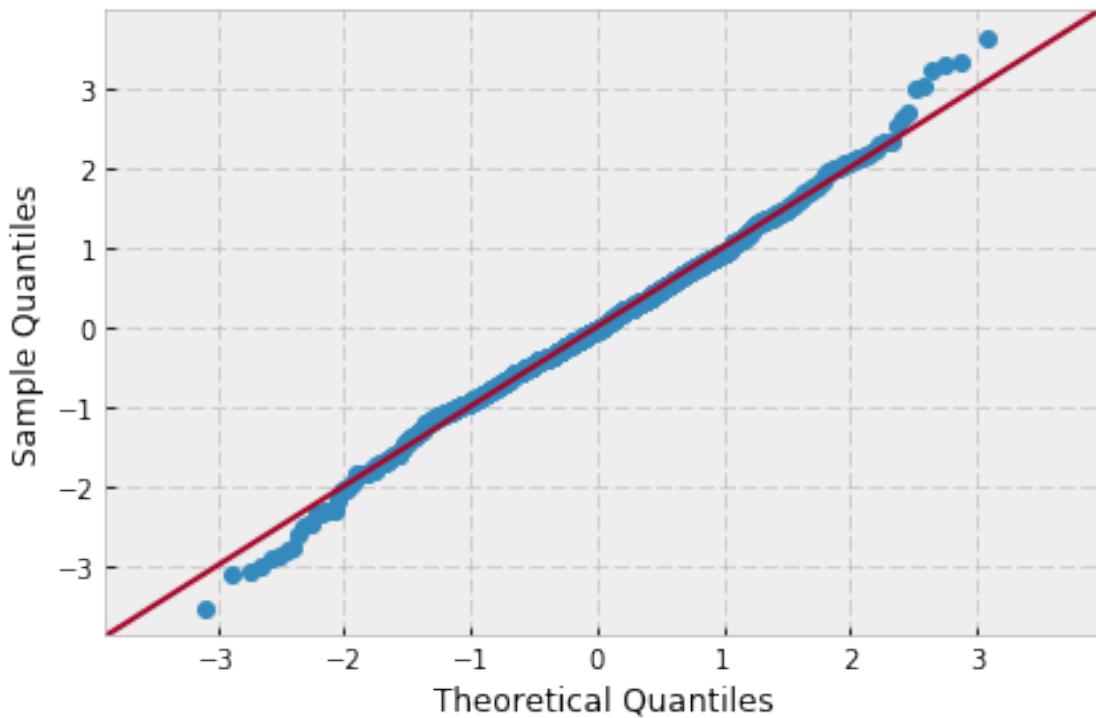


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(), loc=0.04560053773108, scale=0.00155083542601896)
DescribeResult(nobs=1000, minmax=(0.04121104682, 0.05040290642999998)
               mean=0.045600537731079999, variance=2.4074980166120214e-06,
               skewness=0.10513399030925294, kurtosis=0.08508438230469961)
```

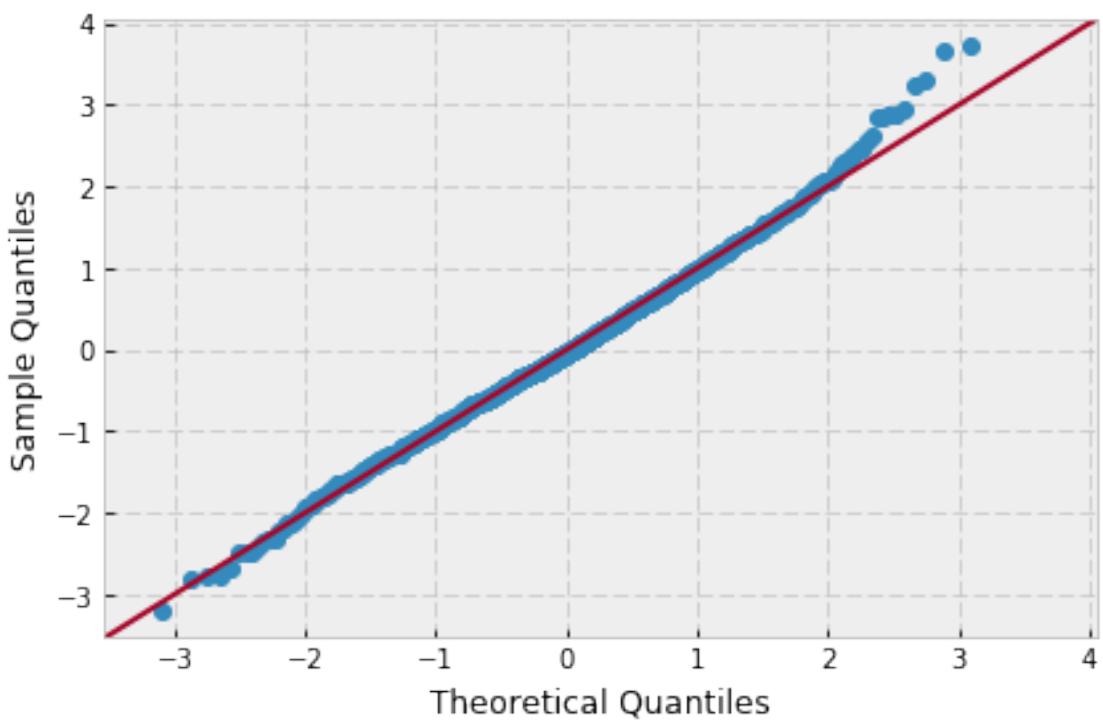
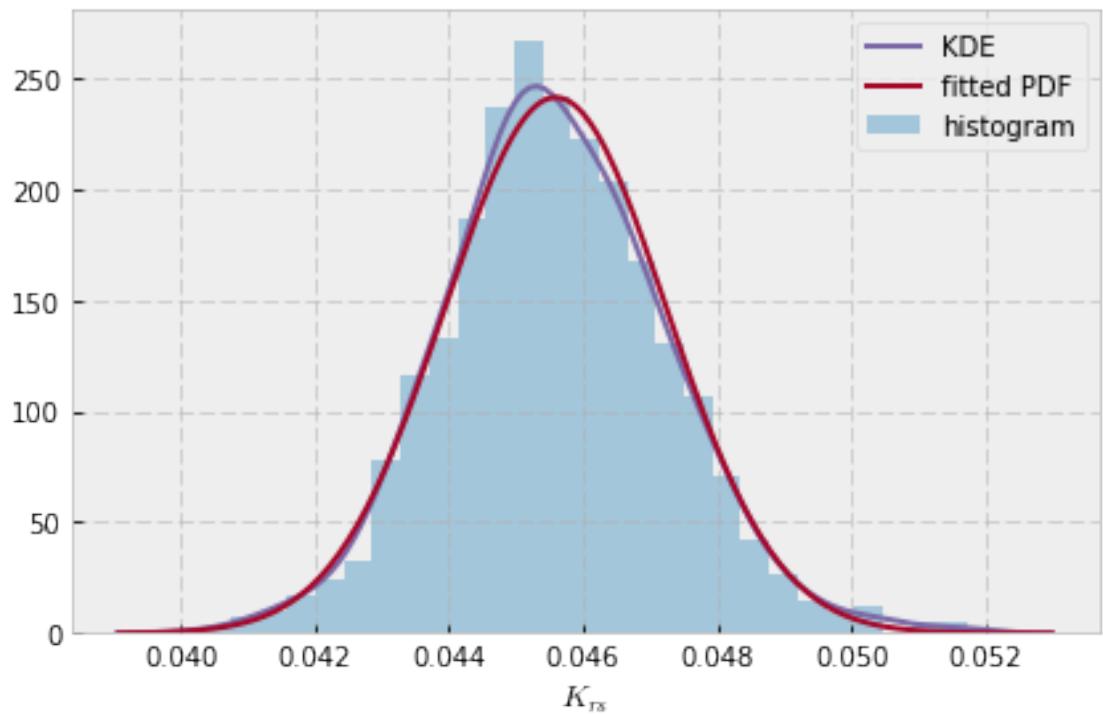


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=0.04565243272583, scale=0.001557751933818464)
DescribeResult(nobs=1000, minmax=(0.040158004329999997, 0.051290858459999998)
               mean=0.04565243272583, variance=2.429020107422587e-06,
               skewness=0.04623001313751583, kurtosis=0.5651416149234079)
```

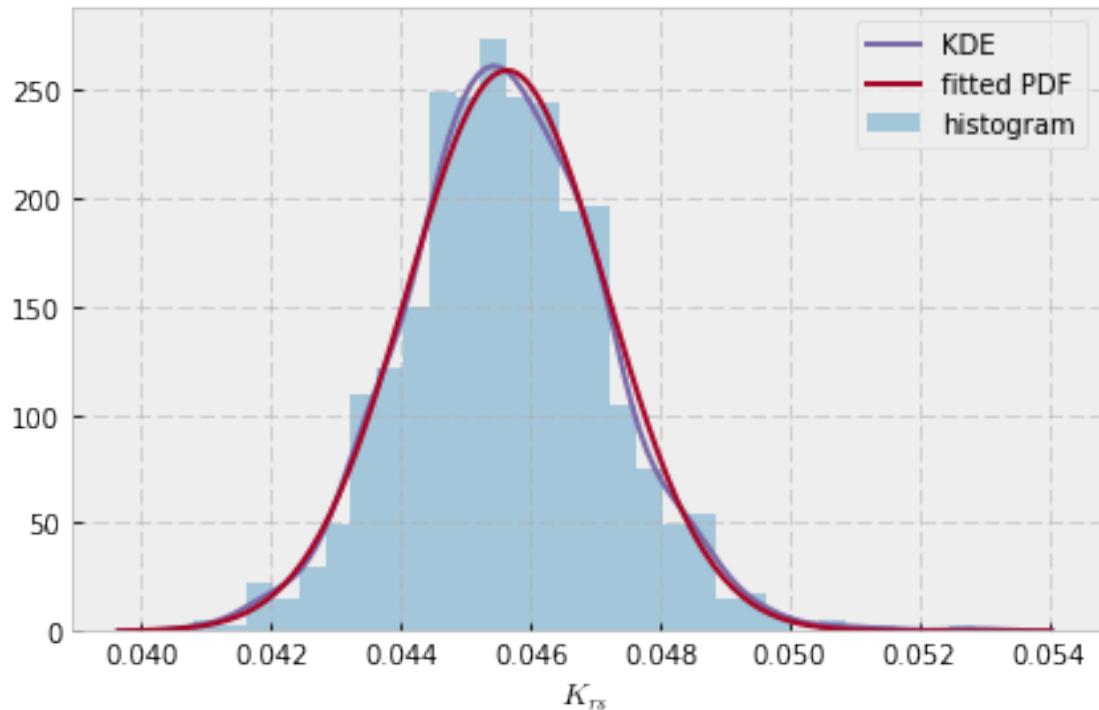


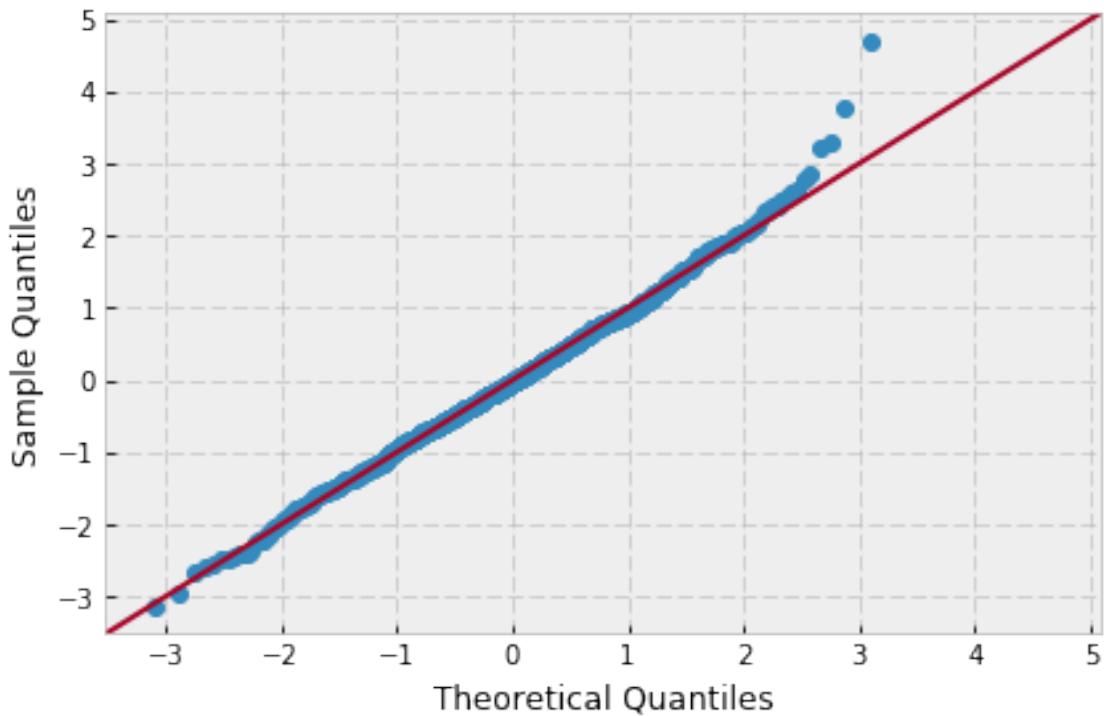


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(),loc=0.045588934641989994, scale=0.0016517248851208816)
DescribeResult(nobs=1000, minmax=(0.040323330960000001, 0.05172646311000002)
               mean=0.04558893464198994, variance=2.730926022149739e-06,
               skewness=0.1971492271645529, kurtosis=0.40548856421653534)
```

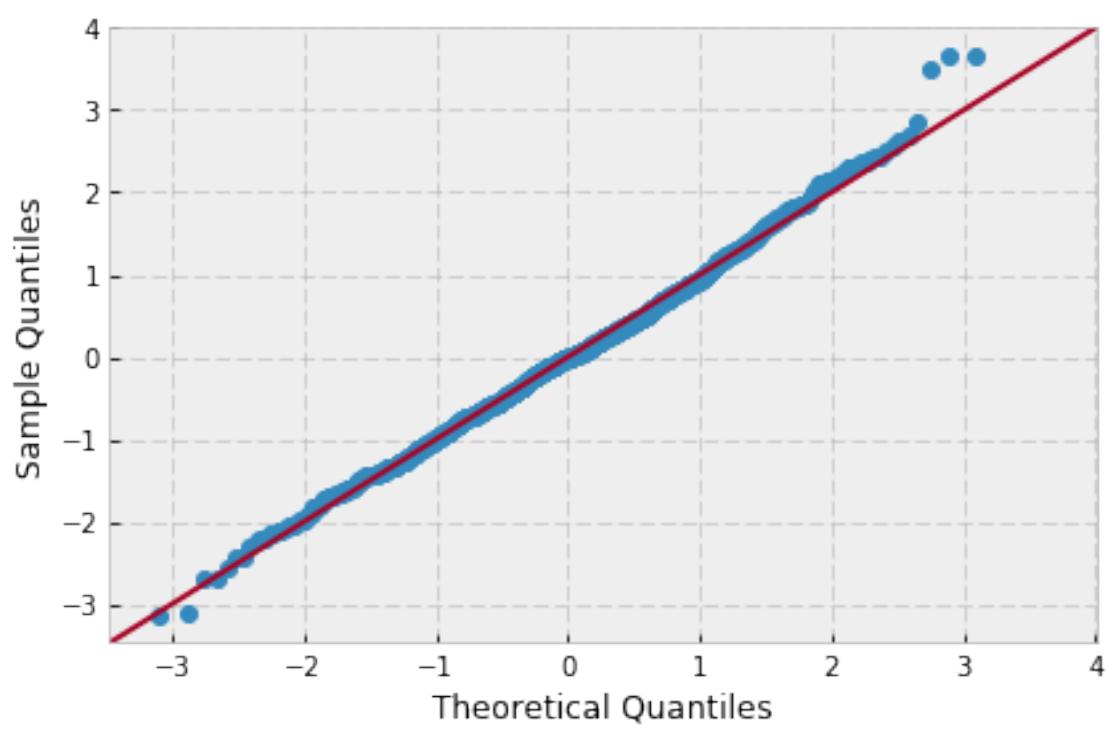
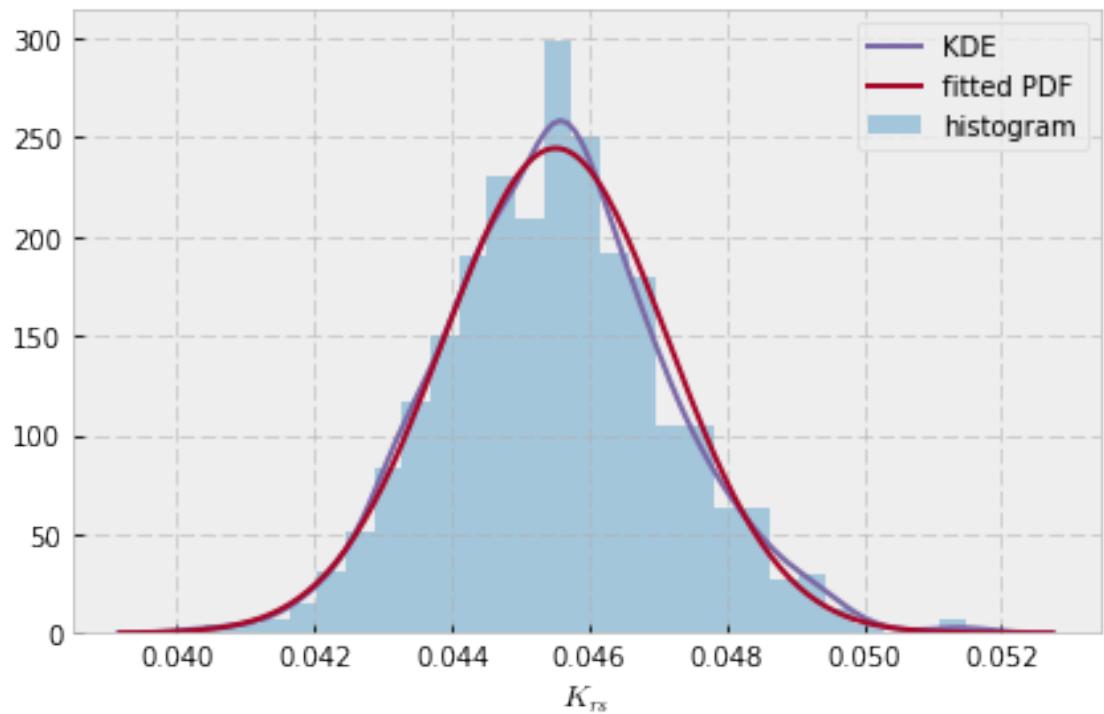


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=0.045637687437840004, scale=0.0015391625901485229)
DescribeResult(nobs=1000, minmax=(0.040826412970000003, 0.052854945200000003)
               mean=0.045637687437840004, variance=2.3713928717844943e-06,
               skewness=0.20407224825322587, kurtosis=0.5892740790141042)
```

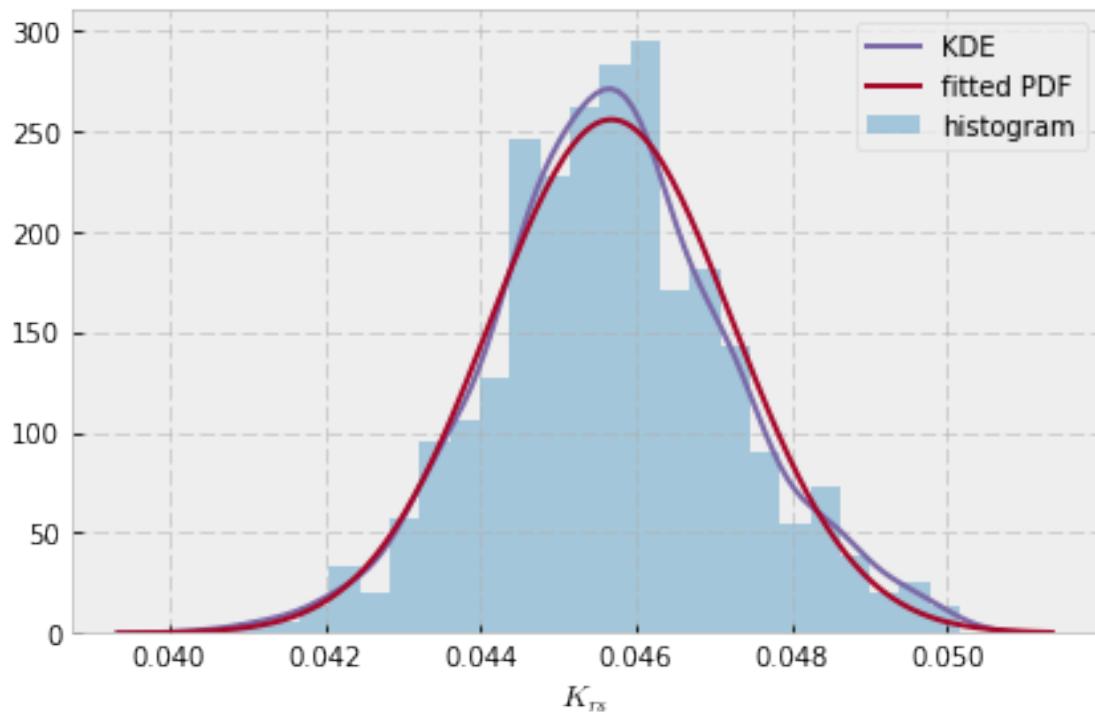


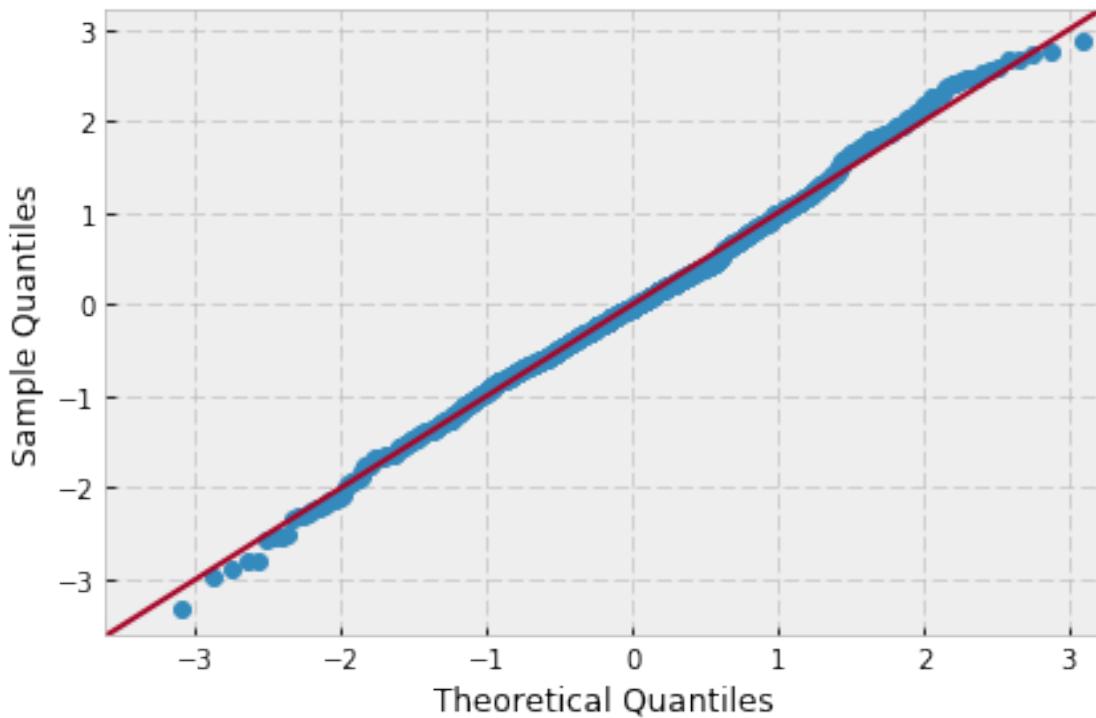


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=0.04551383420288001, scale=0.0016313422567704732)
DescribeResult(nobs=1000, minmax=(0.04040419693999998, 0.05150025919000001)
               mean=0.04551383420288007, variance=2.6639415002252055e-06,
               skewness=0.19734791096598311, kurtosis=0.2758481062309257)
```

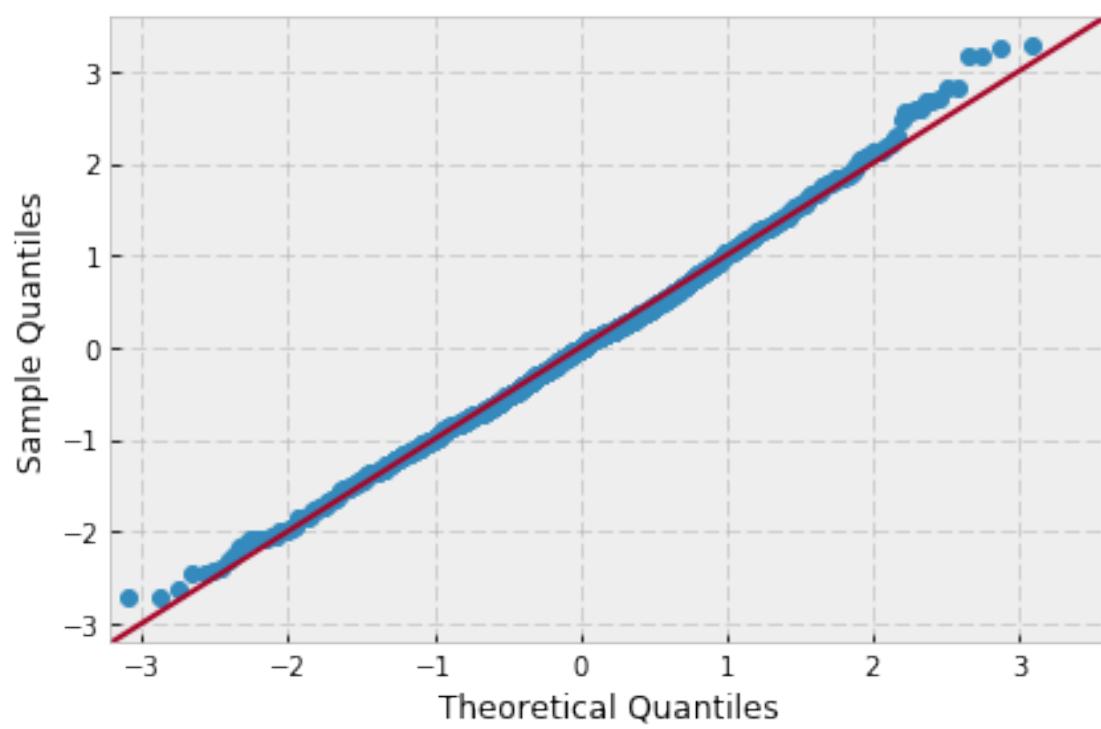
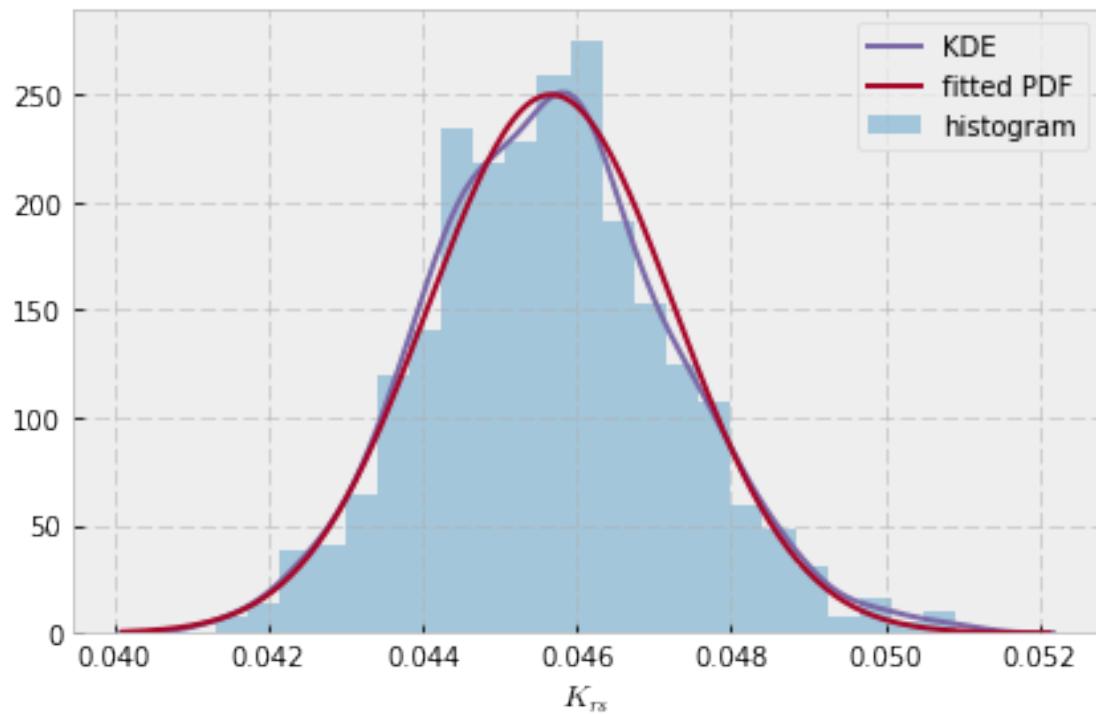


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=0.04568639010633, scale=0.0015607141374191602)
DescribeResult(nobs=1000, minmax=(0.04052064066000003, 0.05016930740000002)
               mean=0.04568639010633001, variance=2.4382668856256588e-06,
               skewness=0.08807508495003107, kurtosis=0.1668767045824553)
```

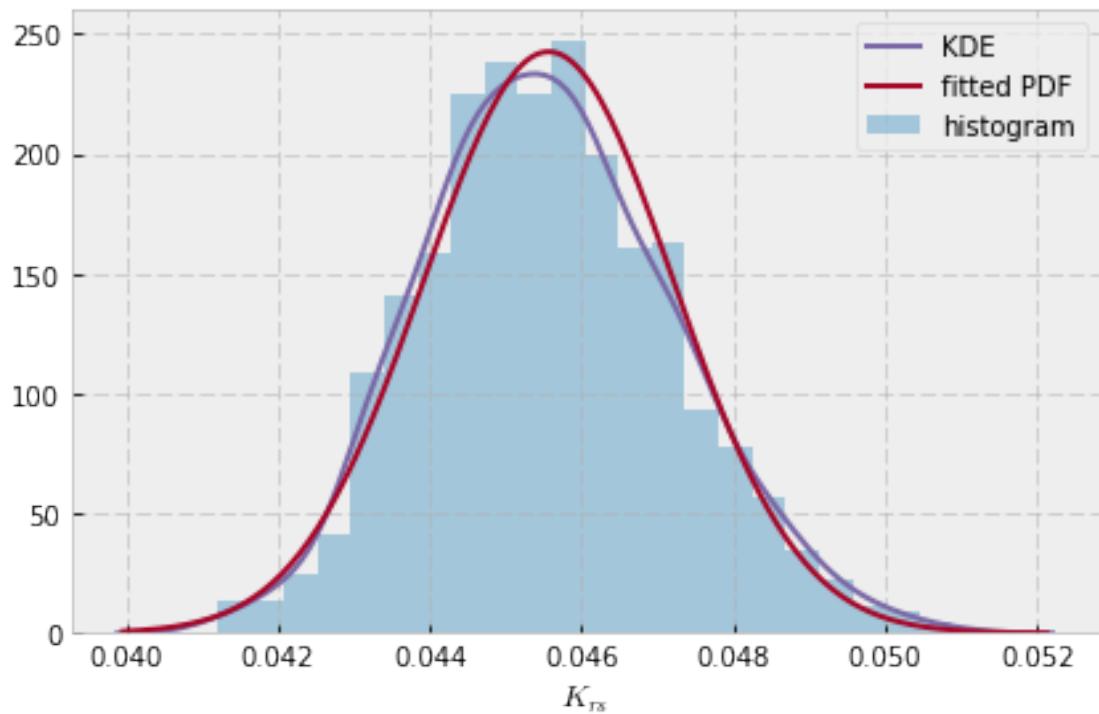


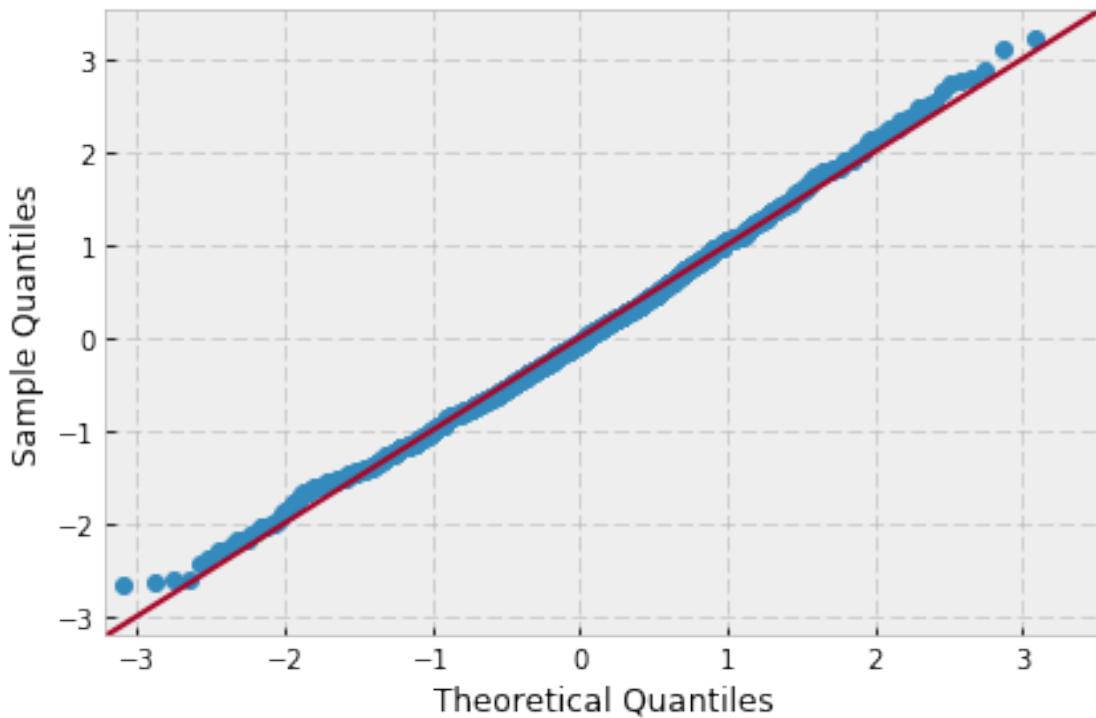


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(),loc=0.04566622343753, scale=0.001596060368841776)
DescribeResult(nobs=1000, minmax=(0.041316906469999998, 0.050918113760000001)
               mean=0.045666223437529997, variance=2.5499586596469928e-06,
               skewness=0.21280299992751395, kurtosis=0.09127460595522585)
```

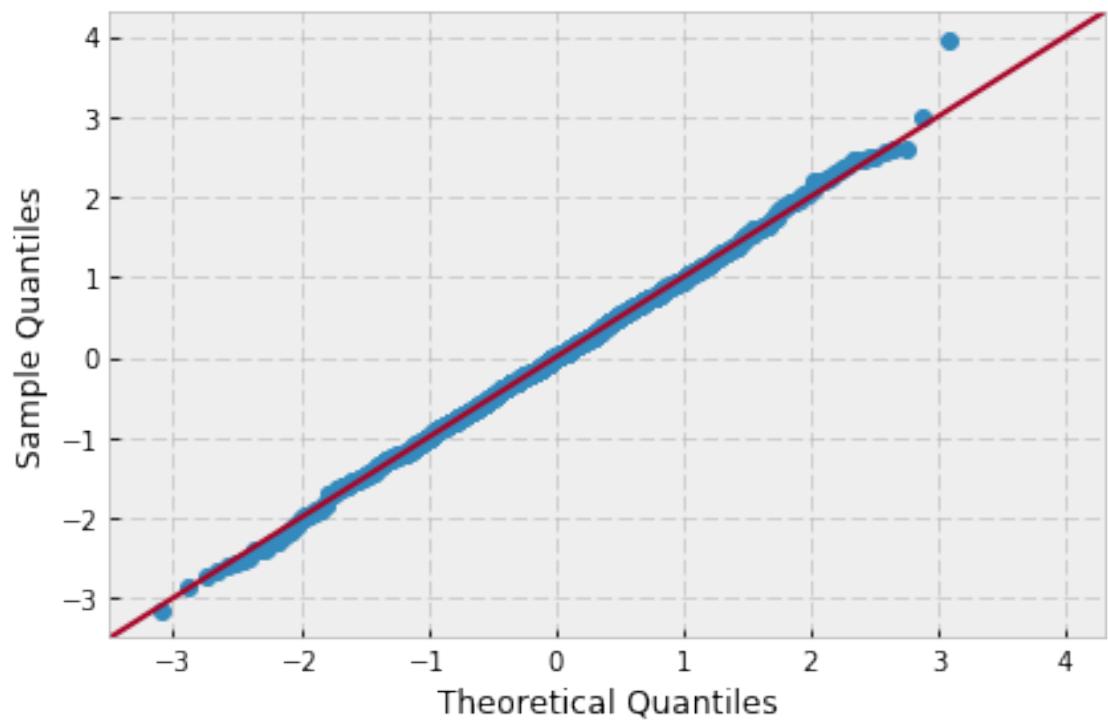
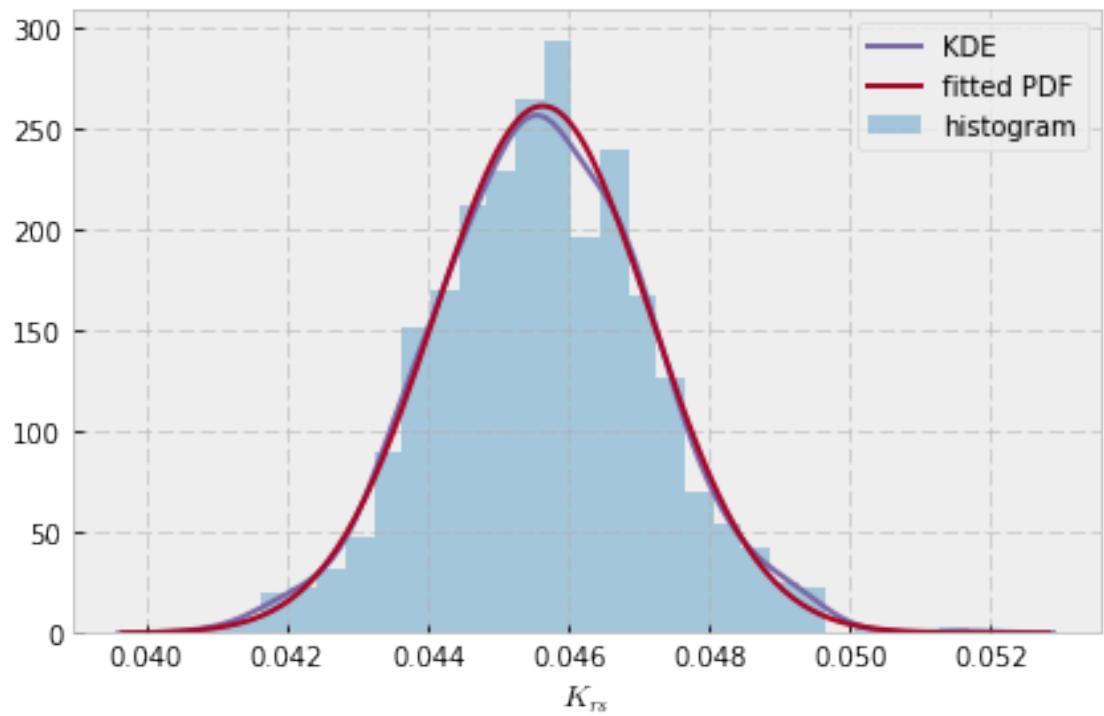


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(),loc=0.04556228111021, scale=0.0016447188955687573)
DescribeResult(nobs=1000, minmax=(0.041198063559999998, 0.050883373449999998)
               mean=0.045562281110209997, variance=2.7078080534944072e-06,
               skewness=0.22647696548054438, kurtosis=-0.09612690551014547)
```

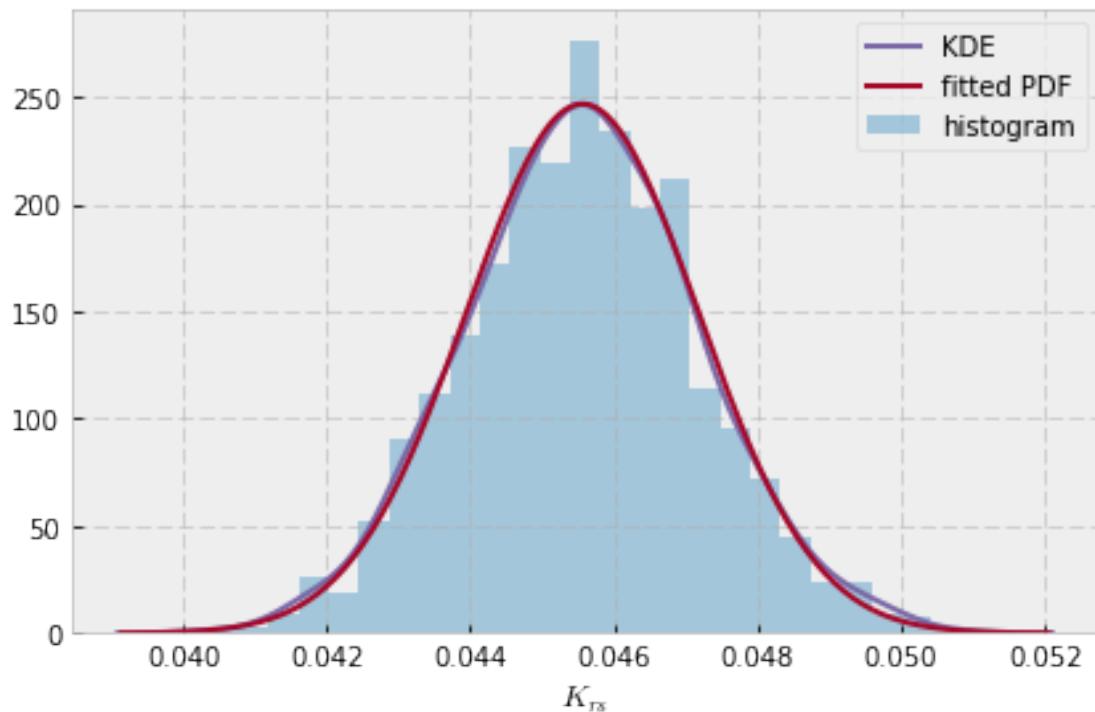


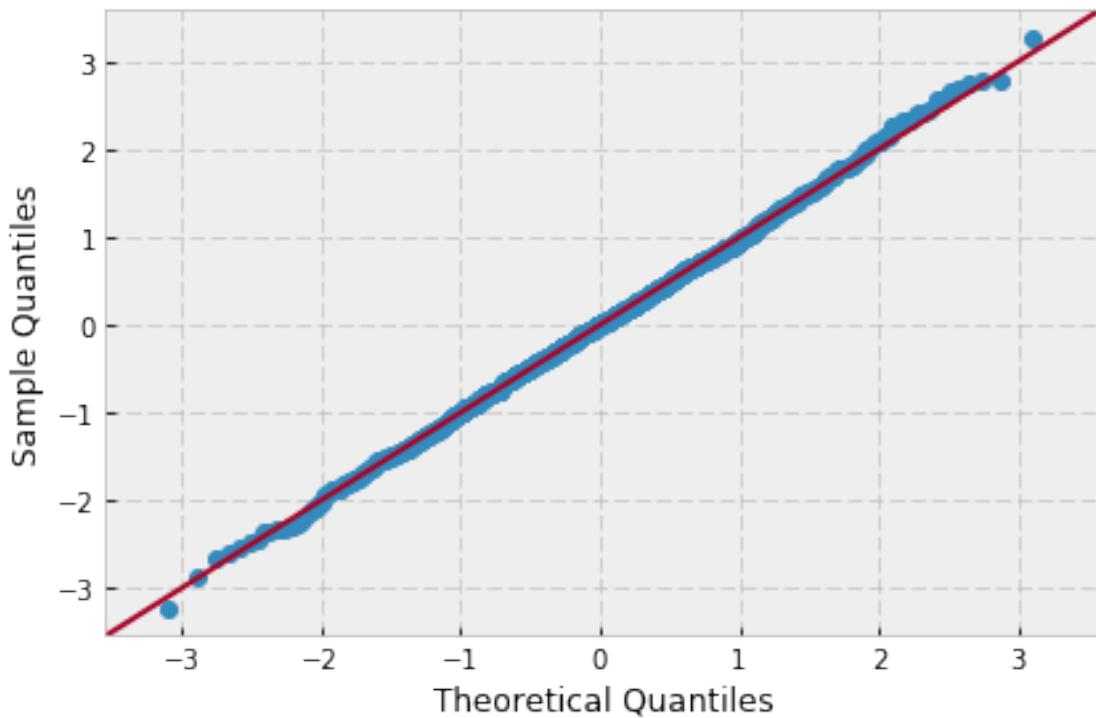


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(),loc=0.04562905396137001, scale=0.0015283257475941794)
DescribeResult(nobs=1000, minmax=(0.040823442510000002, 0.051665105660000002)
               mean=0.04562905396137007, variance=2.3381177084677752e-06,
               skewness=0.04981874255618288, kurtosis=0.12117955161884231)
```

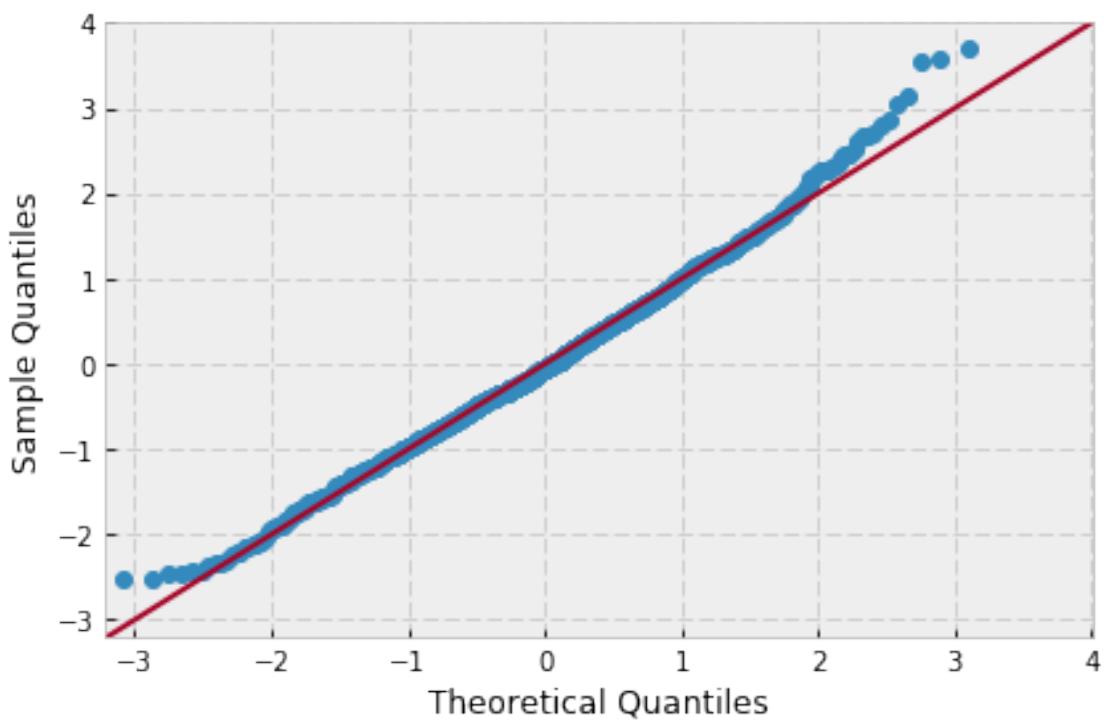
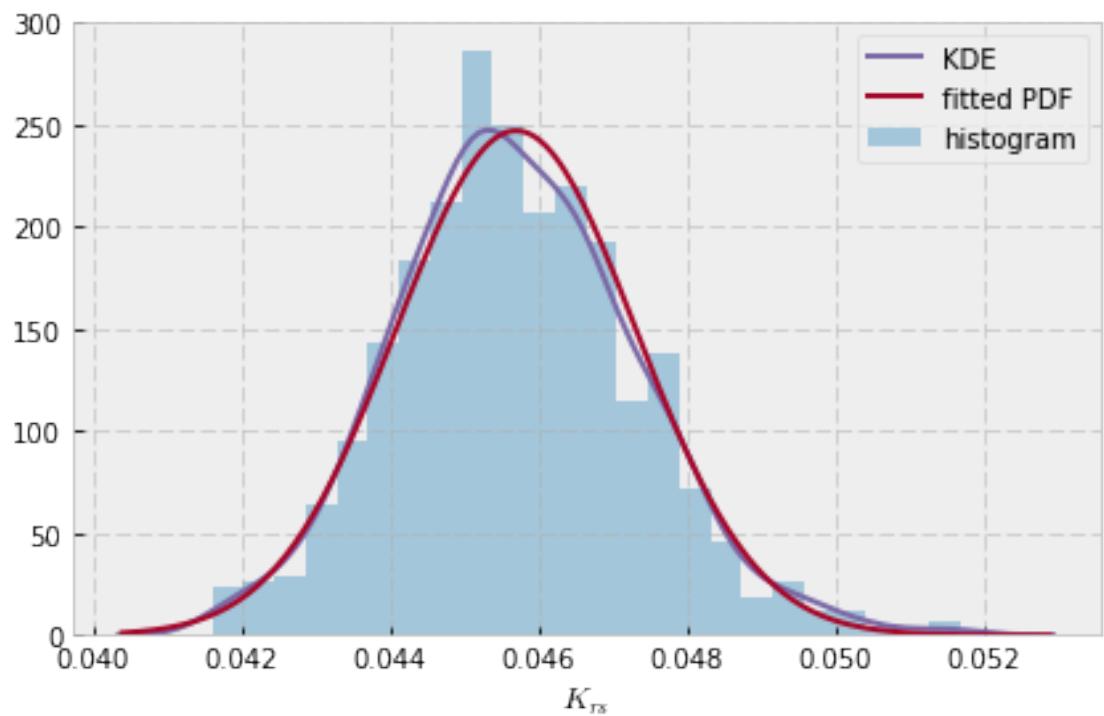


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=0.04556091566024, scale=0.0016174610016566737)
DescribeResult(nobs=1000, minmax=(0.040352040309999997, 0.050837455300000001)
               mean=0.045560915660240002, variance=2.6187988907709811e-06,
               skewness=0.041902746074035, kurtosis=0.018073445344831907)
```

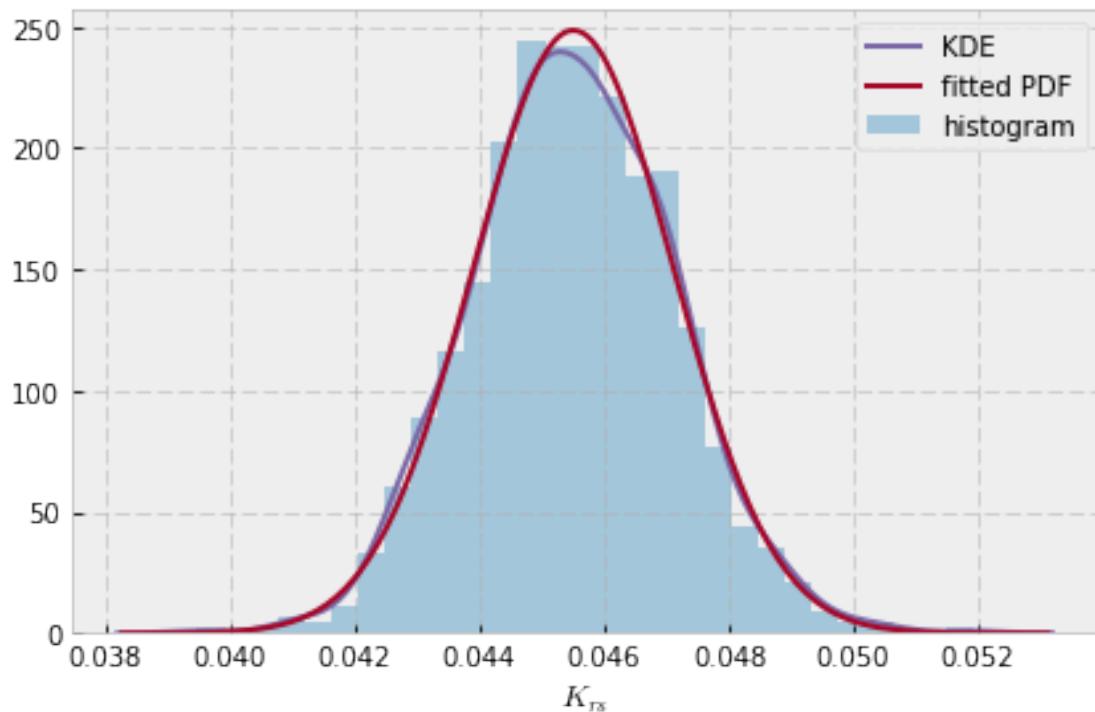


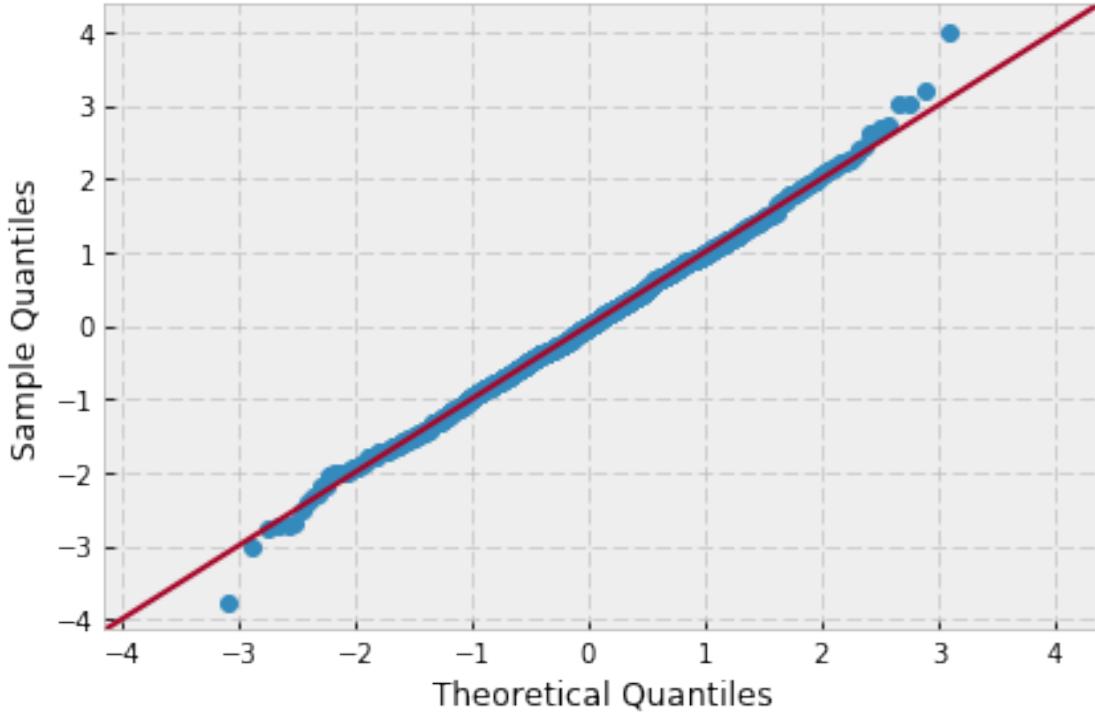


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(),loc=0.045694582975960005, scale=0.0016144883866672263)
DescribeResult(nobs=1000, minmax=(0.041604638950000002, 0.05166491264999999)
               mean=0.04569458297596005, variance=2.6091819326159594e-06,
               skewness=0.27725616530948516, kurtosis=0.2884507264288336)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=0.04550134957243, scale=0.0016044984313526131)
DescribeResult(nobs=1000, minmax=(0.03946665989999999, 0.05190718645999997)
               mean=0.045501349572430003, variance=2.5769922084214177e-06,
               skewness=0.06693818806415772, kurtosis=0.2373329946343845)
```

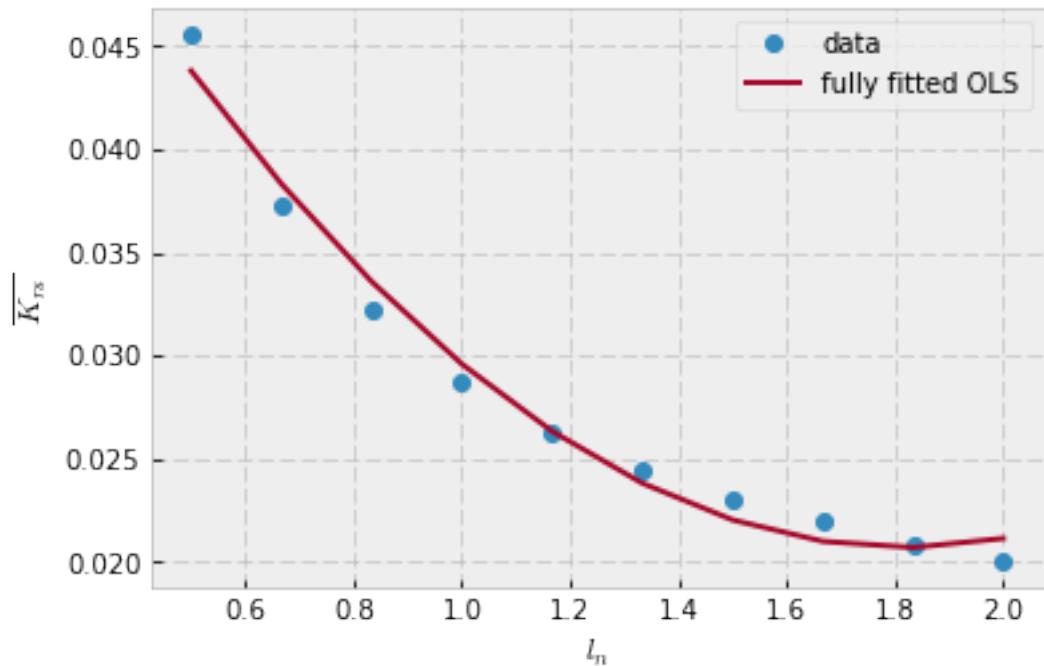




15.2 Parameter regression

```
In [36]: var_name = 'root_system_total_conductance'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly1')
mean_reg('maxB', var_name, 'poly2')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly1')
mean_reg('N', var_name, 'poly1')
```

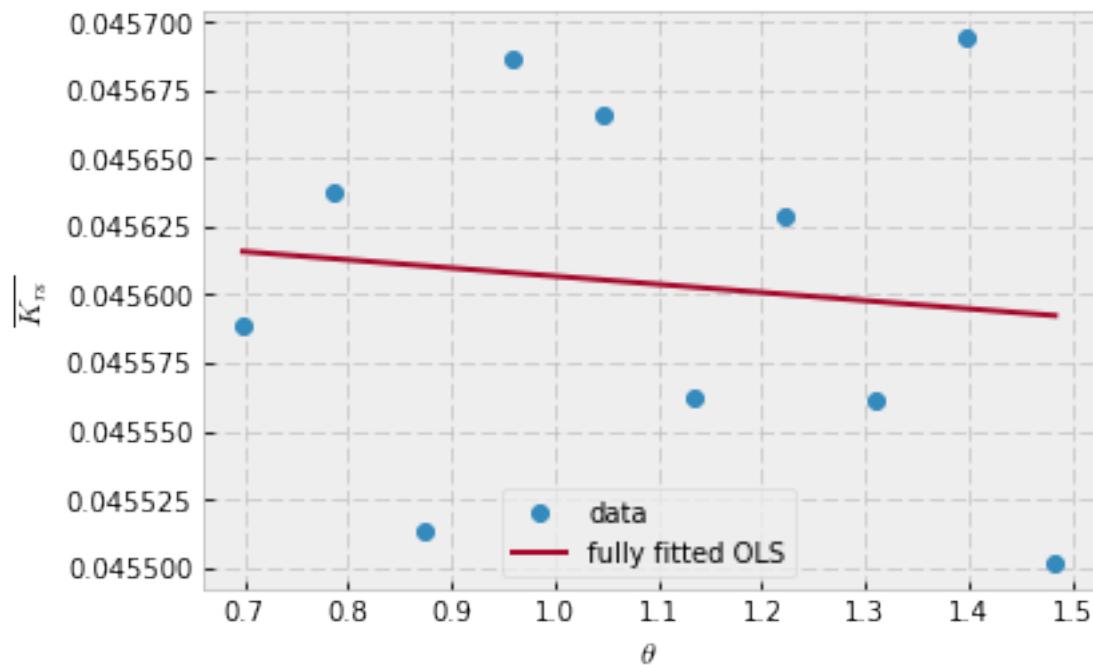
```
variable name: ln
model: poly2
r-squared: 0.9324525776830797
explained variance: 0.9325616281341976
```



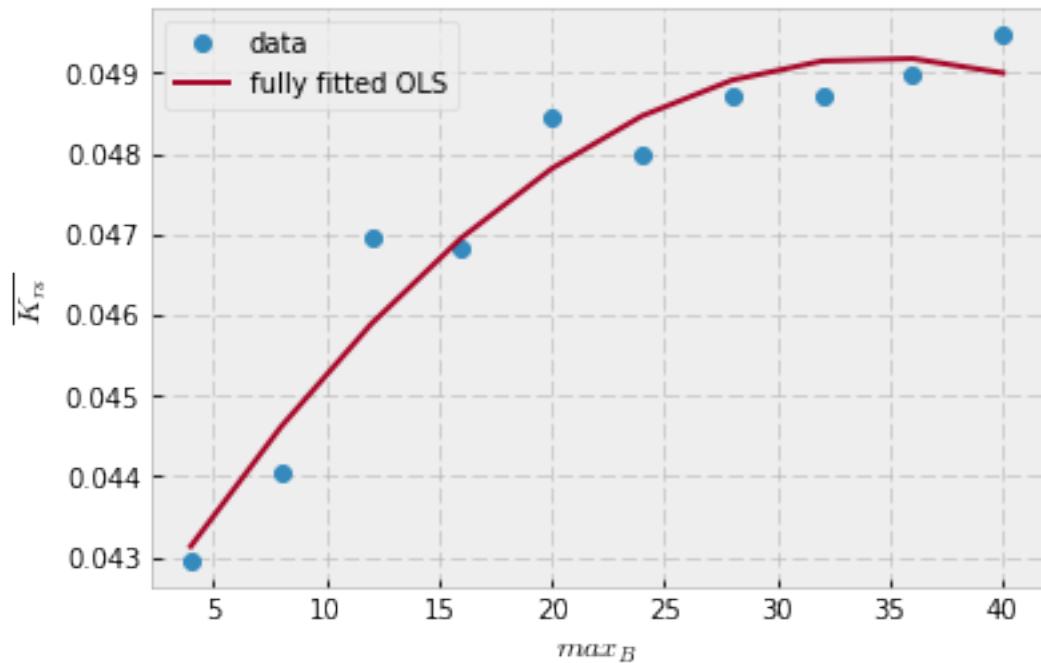
```

variable name: theta
model: poly1
r-squared: -0.6517451632042233
explained variance: -0.6494055309216382

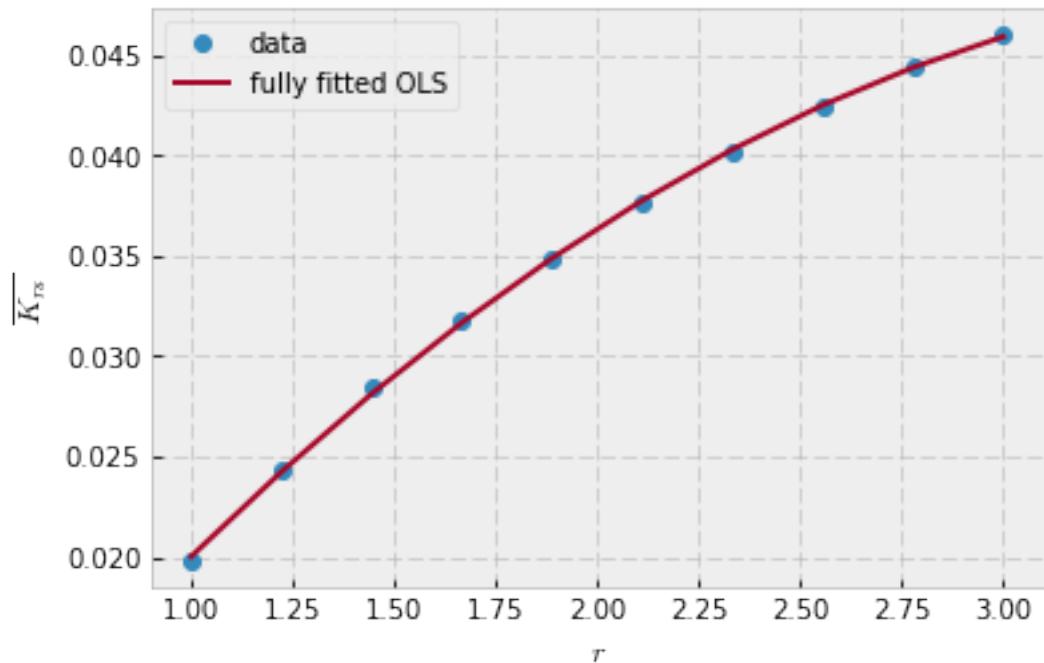
```



```
variable name: maxB  
model: poly2  
r-squared: 0.869747574724536  
explained variance: 0.8699413128268202
```



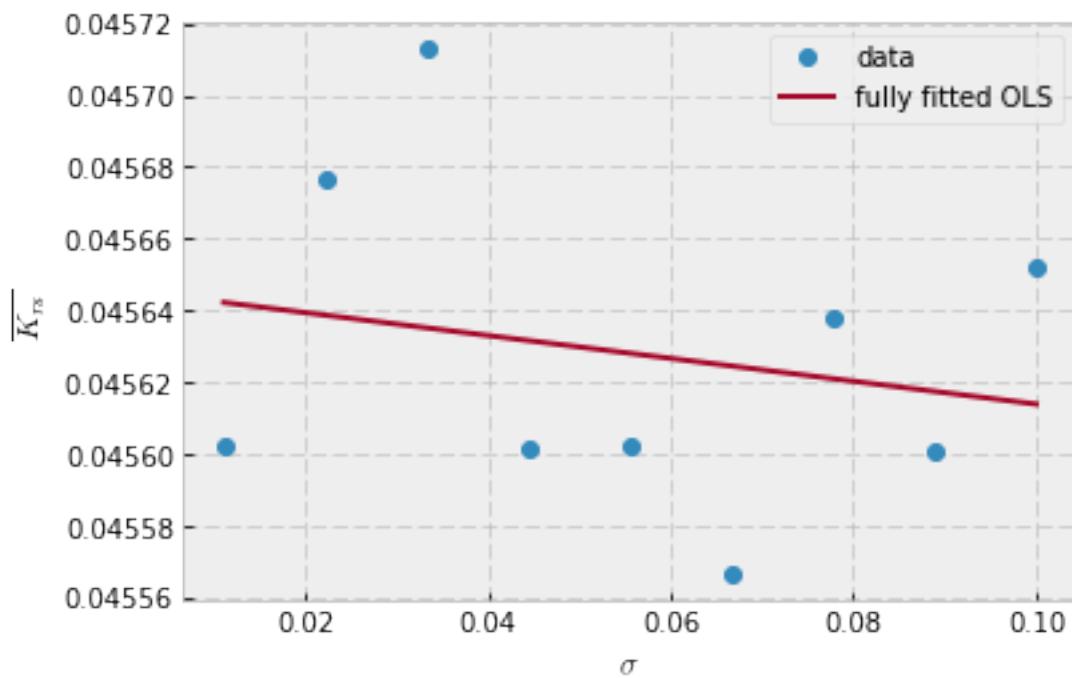
```
variable name: r  
model: poly2  
r-squared: 0.9987356371805631  
explained variance: 0.9987394969820559
```



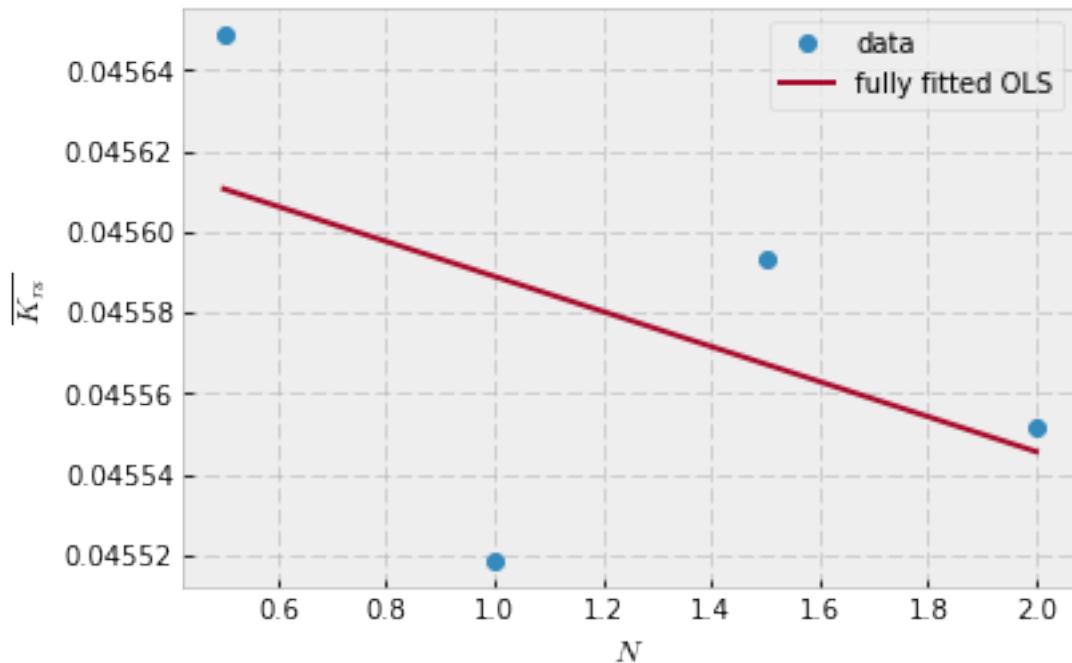
```

variable name: sigma
model: poly1
r-squared: -0.5997367417597548
explained variance: -0.5989631389350947

```



```
variable name: N
model: poly1
r-squared: -1.9779395716877812
explained variance: -1.7887436646216486
```

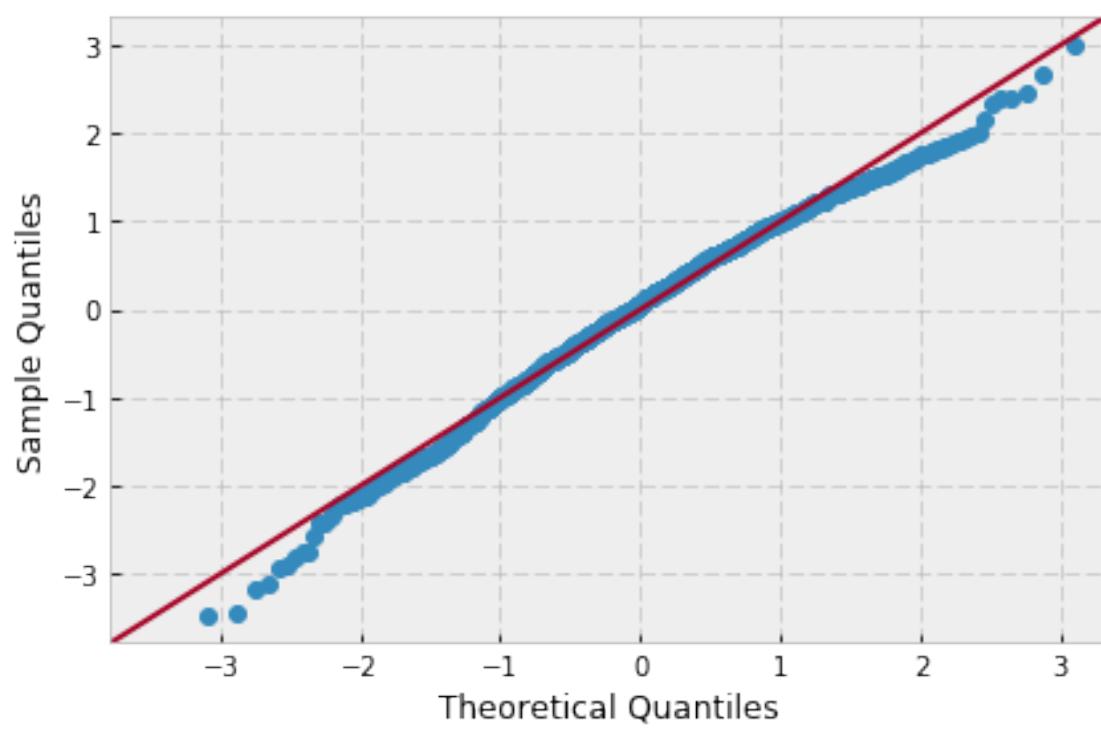
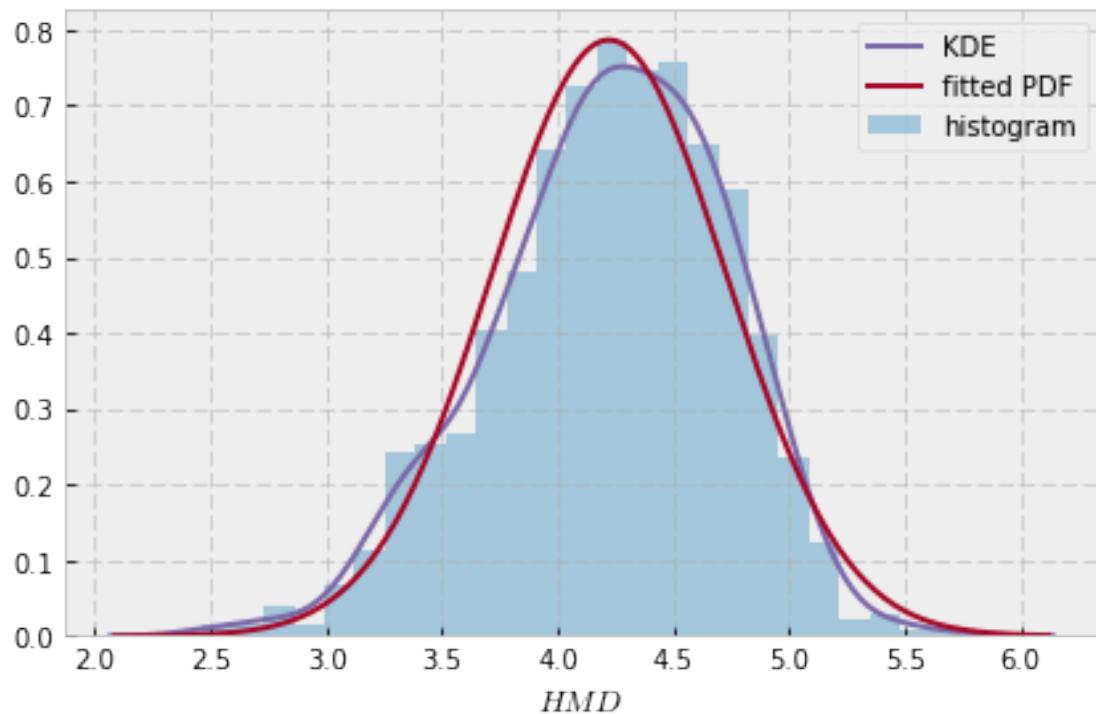


16 Half mean distance between roots

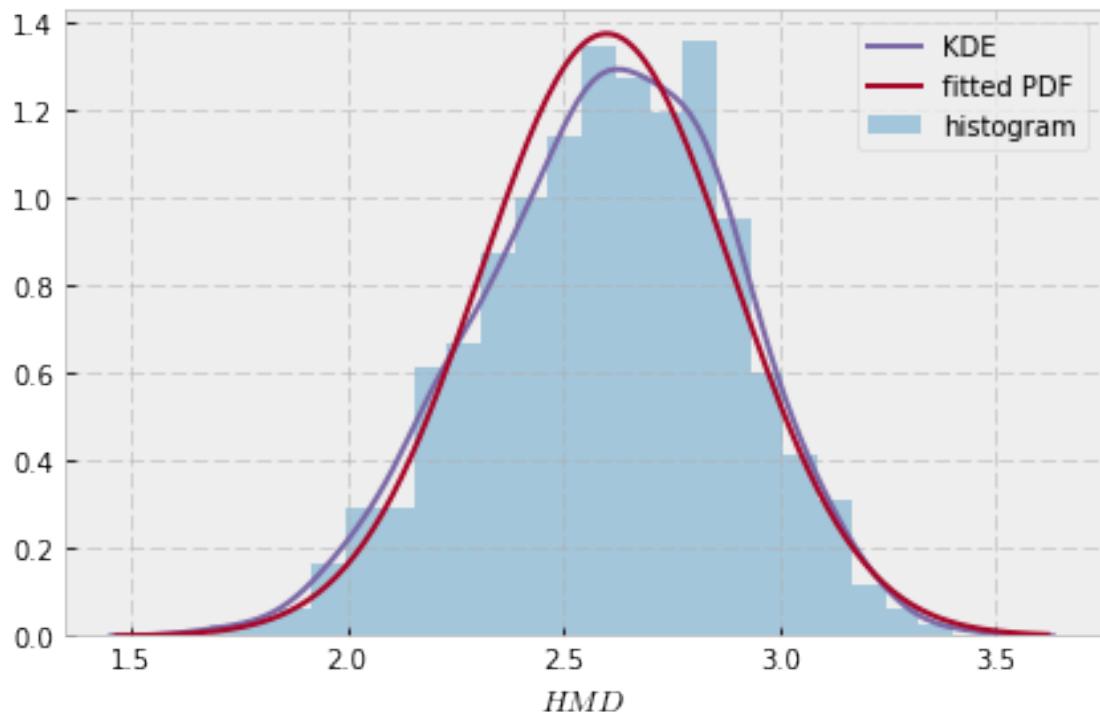
16.1 Probability distributions

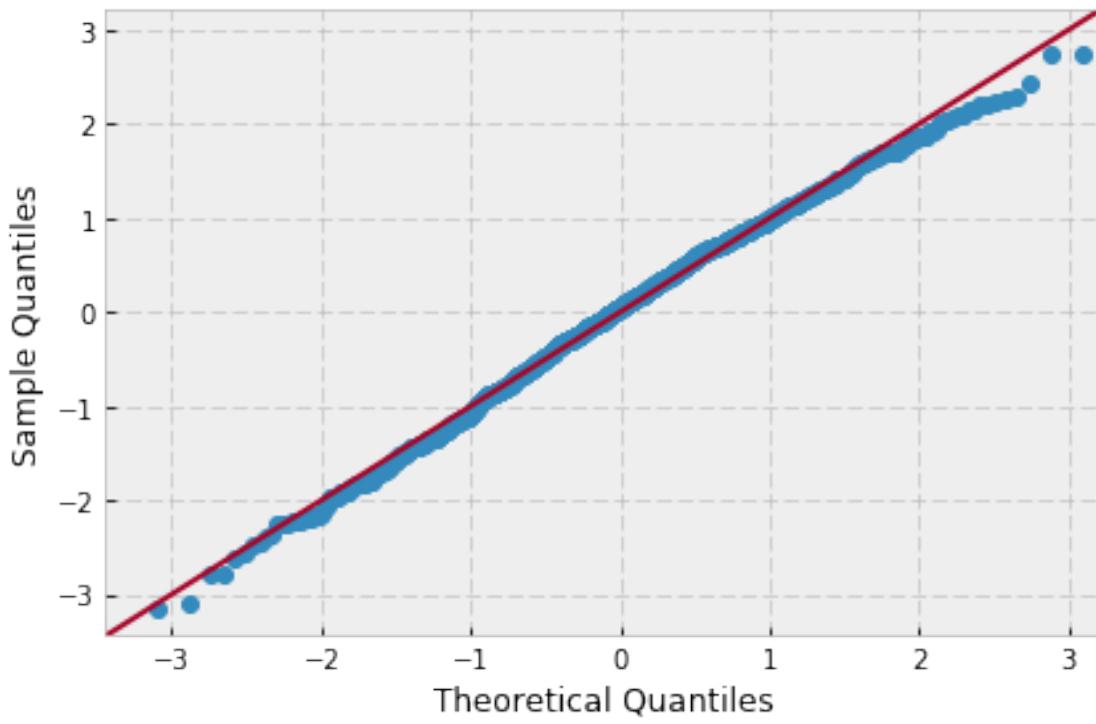
```
In [37]: plot_dist('half_mean_distance_between_roots')

variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=4.22114630996425, scale=0.507067135344299)
DescribeResult(nobs=1000, minmax=(2.4675062026804233, 5.7395870915673814)
               mean=4.2211463099642499, variance=0.25737445420047417,
               skewness=-0.37322431157225433, kurtosis=0.022369759610008977)
```

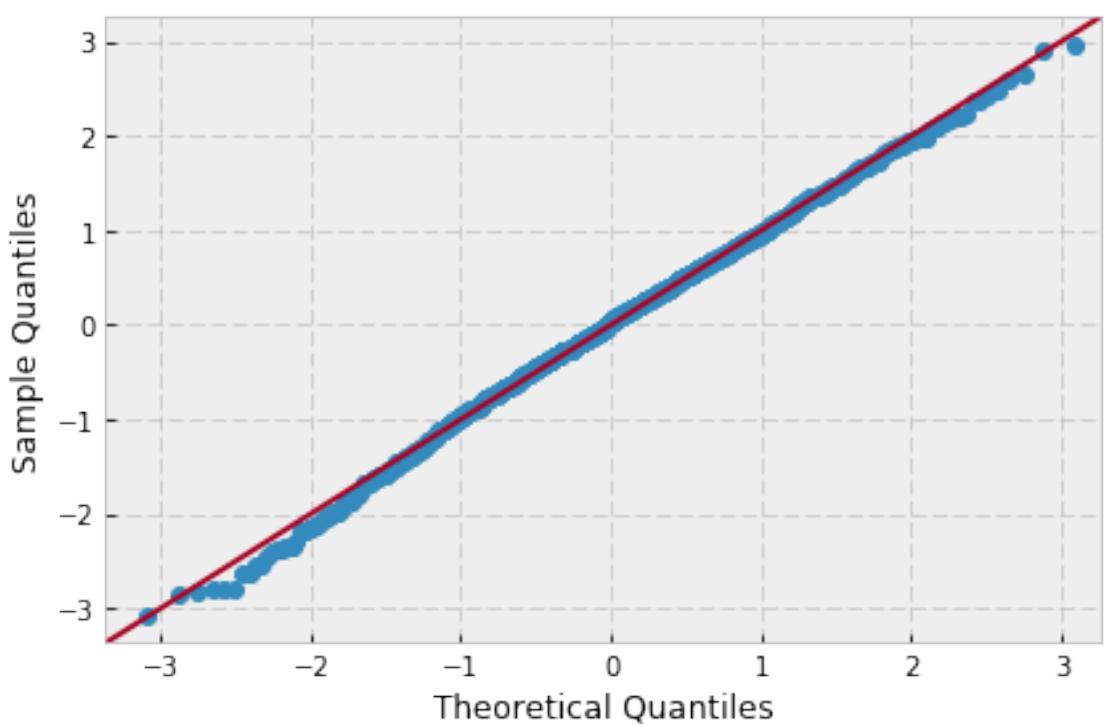
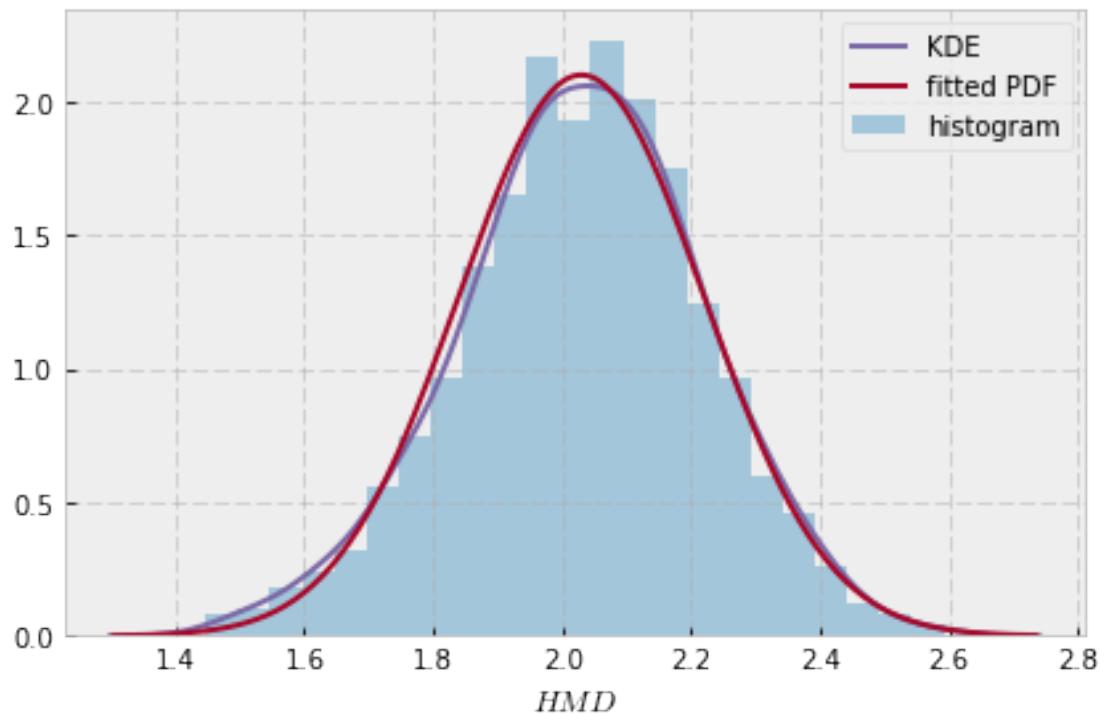


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=2.599485752649209, scale=0.2902010240582045)
DescribeResult(nobs=1000, minmax=(1.6856867315480197, 3.3994216306837743)
               mean=2.5994857526492088, variance=0.084300935299730331,
               skewness=-0.20460023491780407, kurtosis=-0.24728426040572682)
```

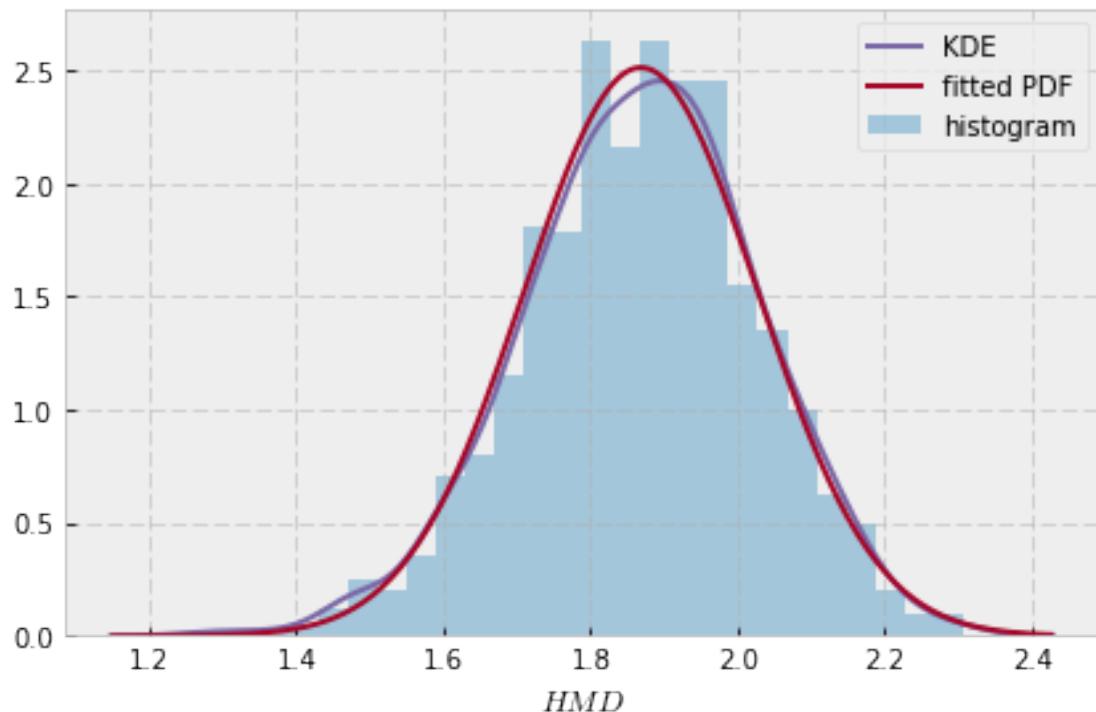


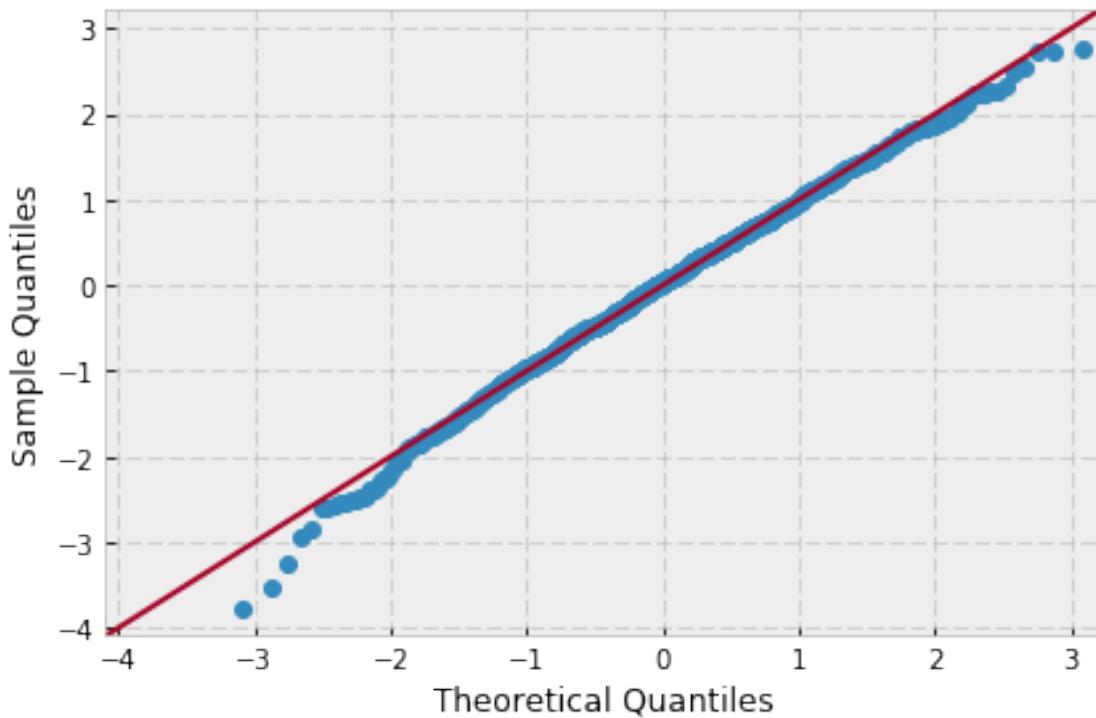


```
variable name: N
variable value: 1.5
distribution: normal(shape=(),loc=2.0295879587479693, scale=0.1900046788095221)
DescribeResult(nobs=1000, minmax=(1.4470735408149125, 2.5913887865649992)
               mean=2.0295879587479693, variance=0.036137915885395049,
               skewness=-0.14754341635305934, kurtosis=0.01844718076865437)
```

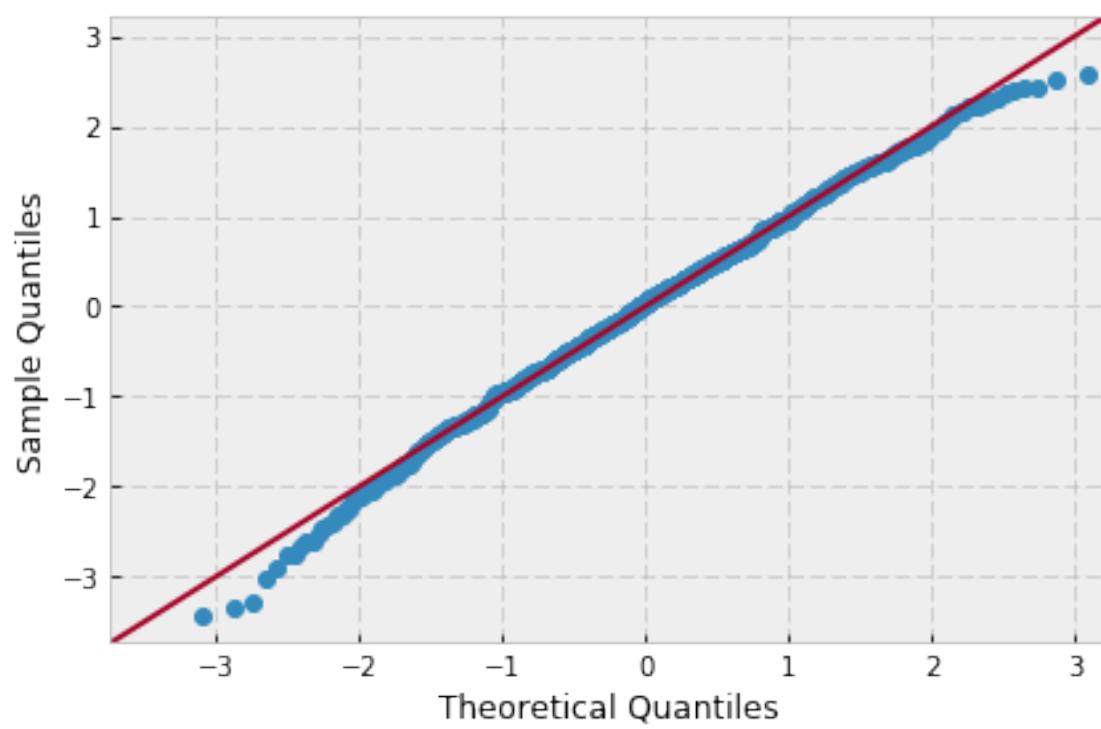
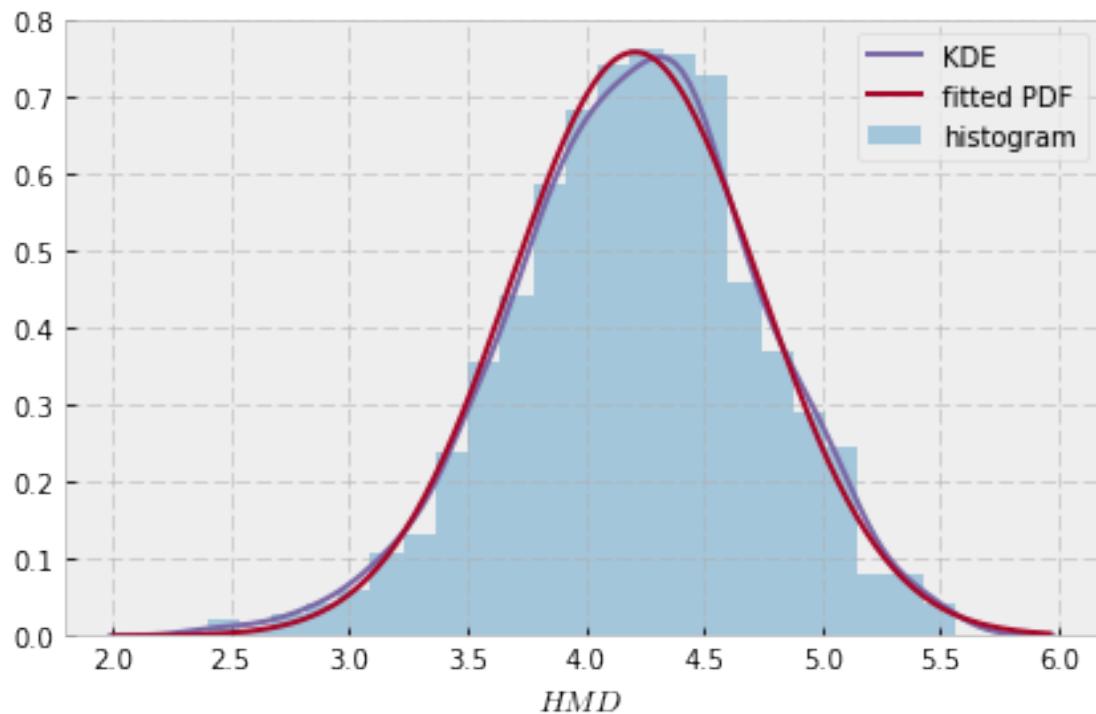


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=1.868453558808494, scale=0.15877276564561696)
DescribeResult(nobs=1000, minmax=(1.2703986993159275, 2.3062842928853287)
               mean=1.8684535588084941, variance=0.025234025135893894,
               skewness=-0.21744262354647687, kurtosis=0.1644553496021528)
```

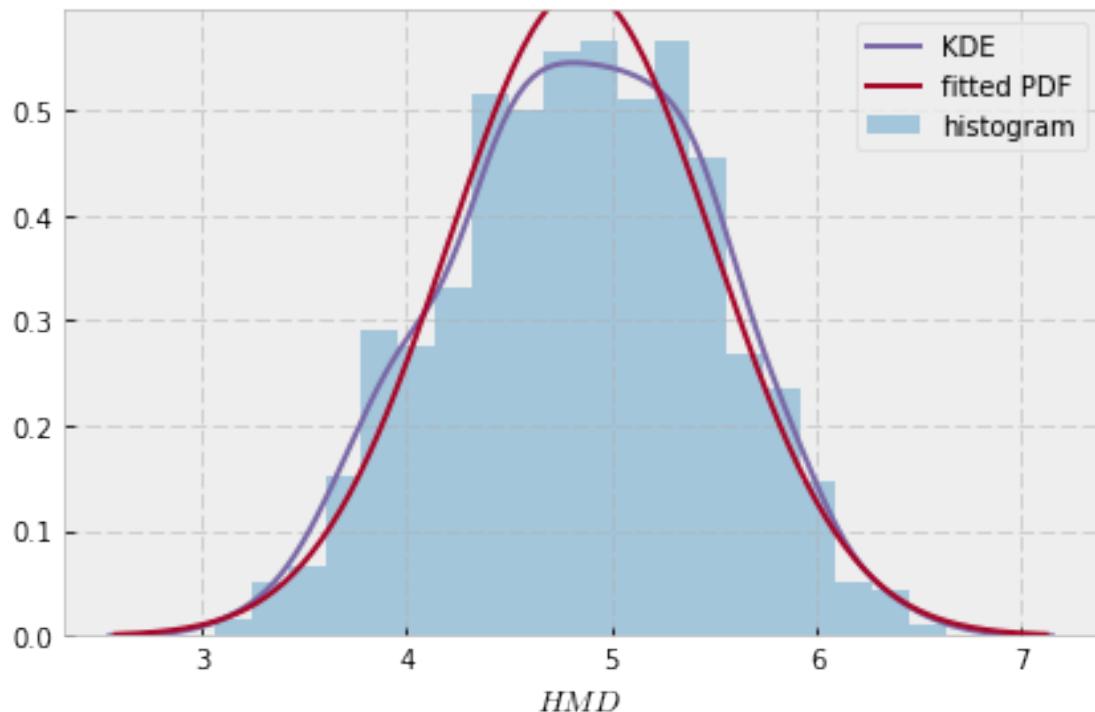


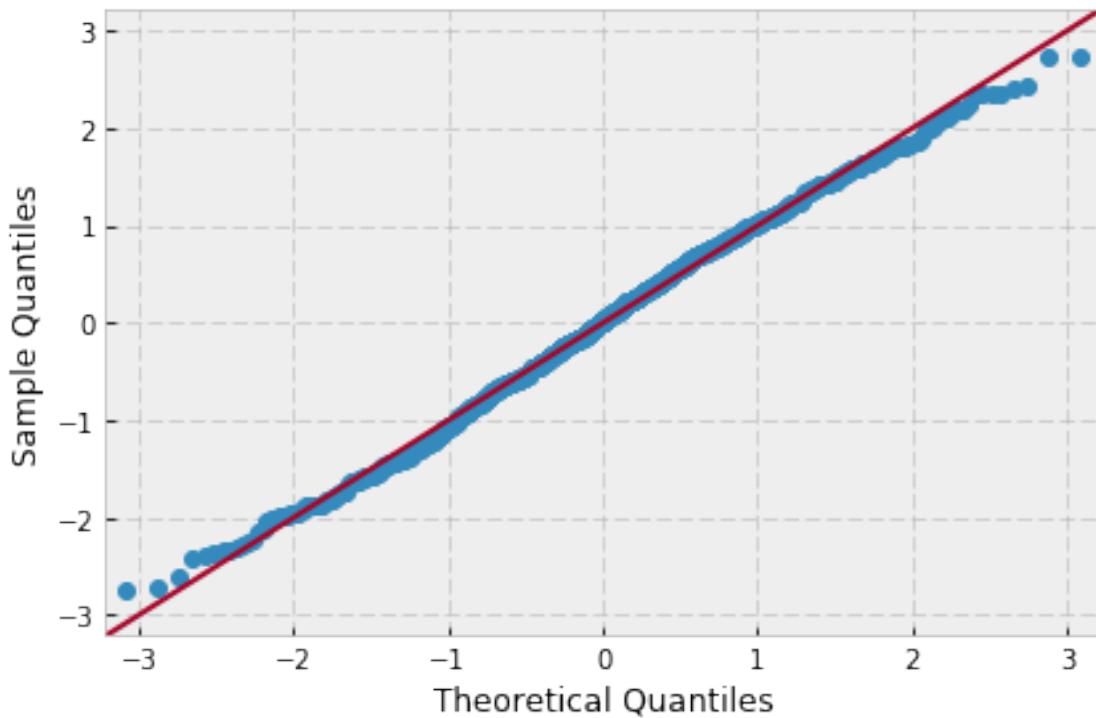


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(),loc=4.209383575851395, scale=0.525450622104719)
DescribeResult(nobs=1000, minmax=(2.4032267722366583, 5.5649636045227631)
               mean=4.2093835758513949, variance=0.27637473100123744,
               skewness=-0.2069145931485395, kurtosis=0.08889031999675101)
```

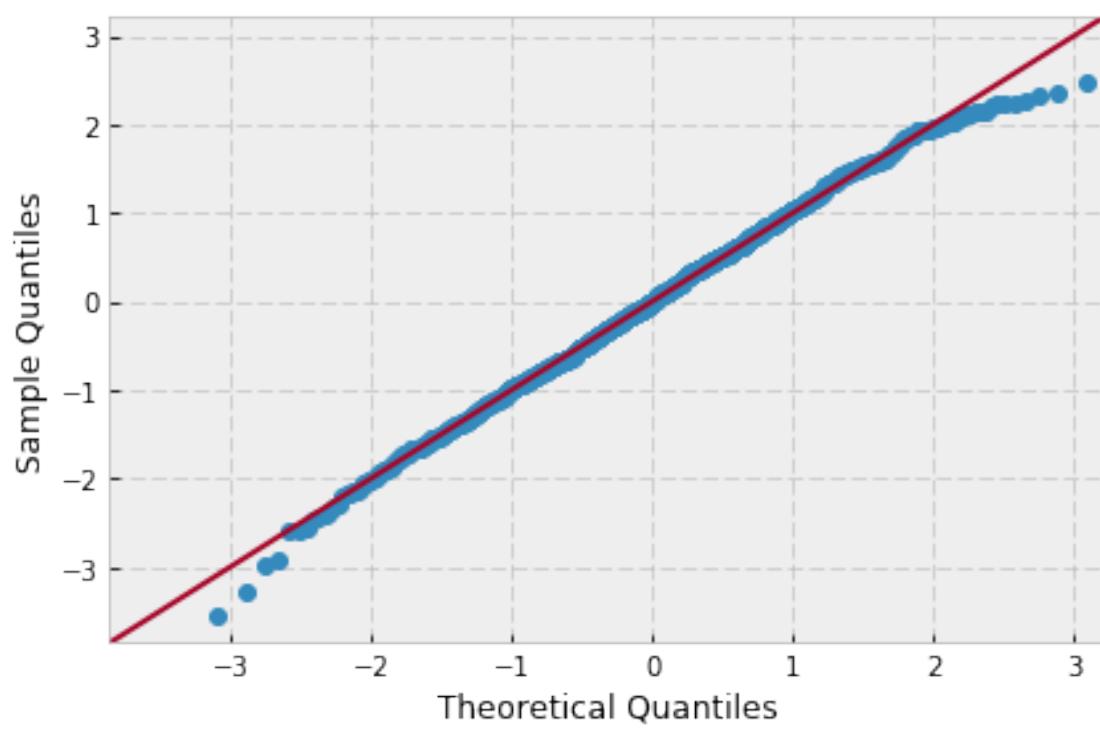
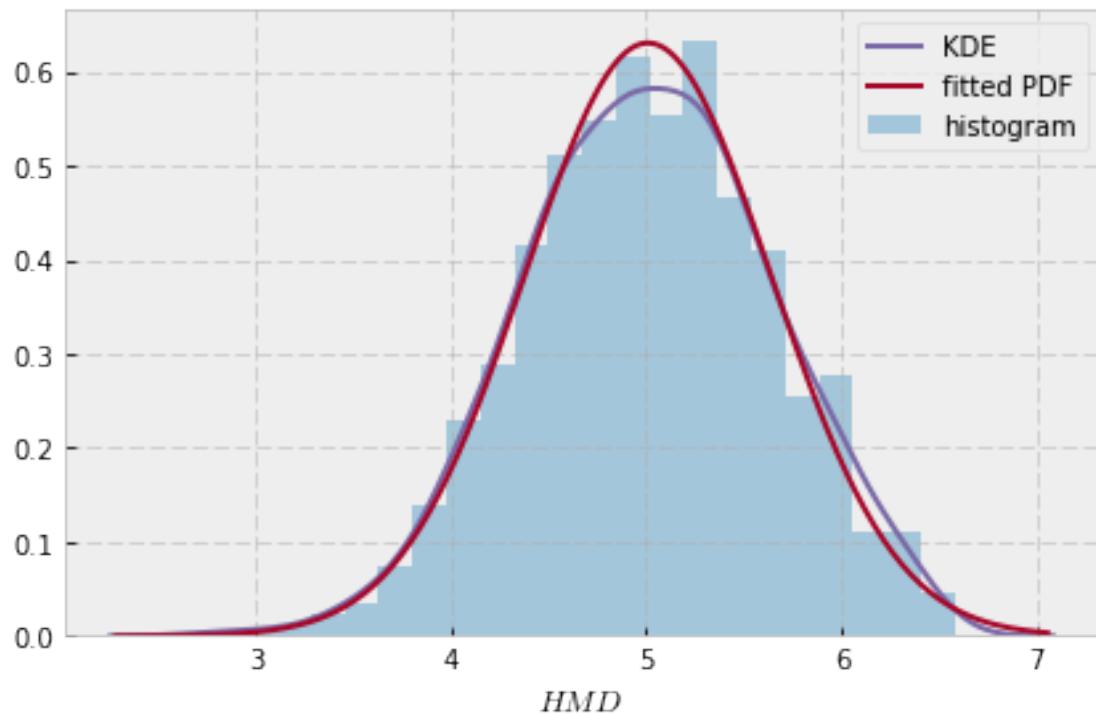


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=4.850365146449756, scale=0.6484199677526701)
DescribeResult(nobs=1000, minmax=(3.0679771118187471, 6.6314105260134824)
               mean=4.8503651464497564, variance=0.42086932390427806,
               skewness=-0.0772225933759459, kurtosis=-0.44273557916022055)
```

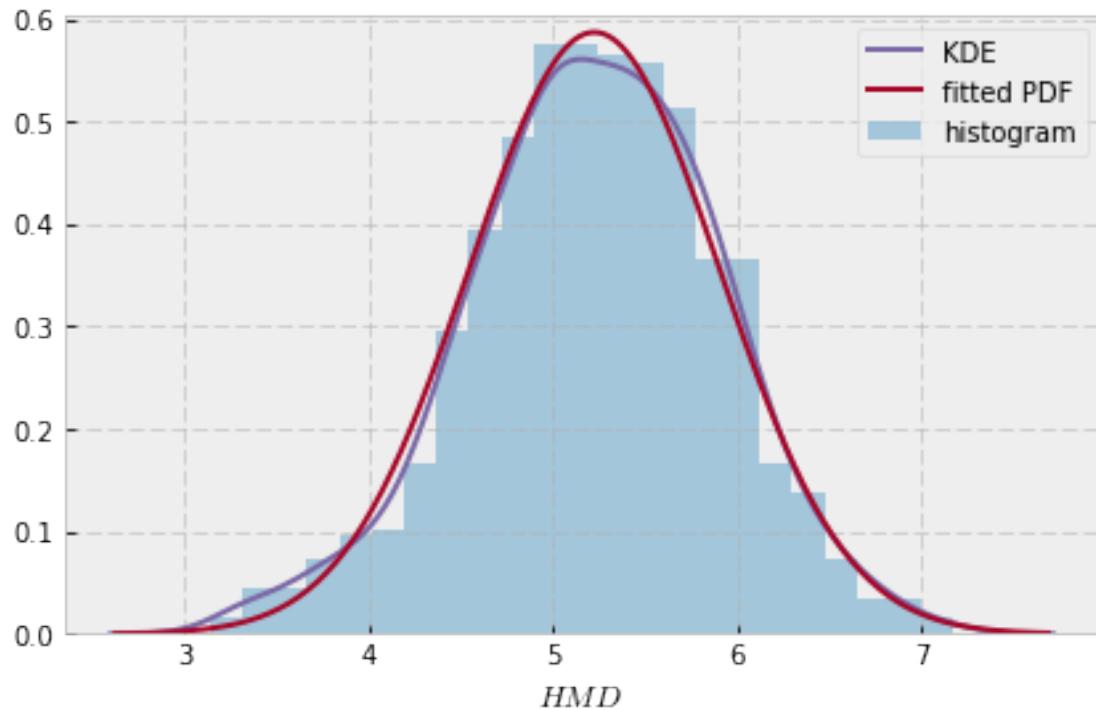


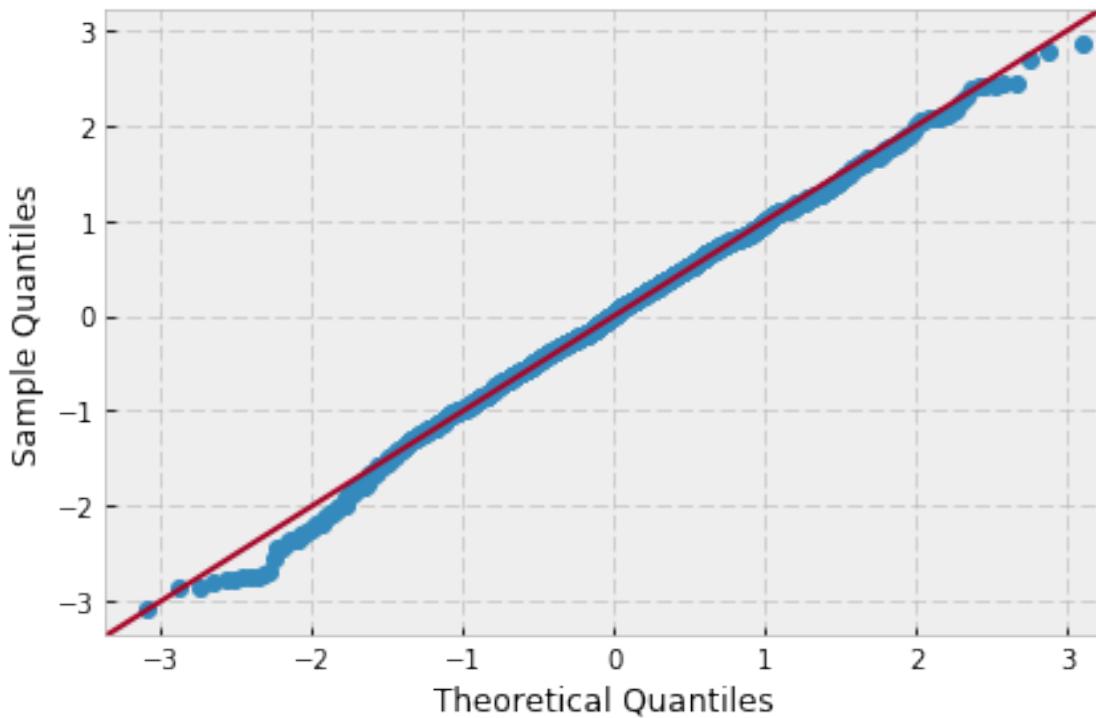


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=5.009237448109501, scale=0.6327855809005478)
DescribeResult(nobs=1000, minmax=(2.7611812691061632, 6.579483255593872)
               mean=5.0092374481095012, variance=0.40081840980544919,
               skewness=-0.10492953750511379, kurtosis=-0.18179643852701455)
```

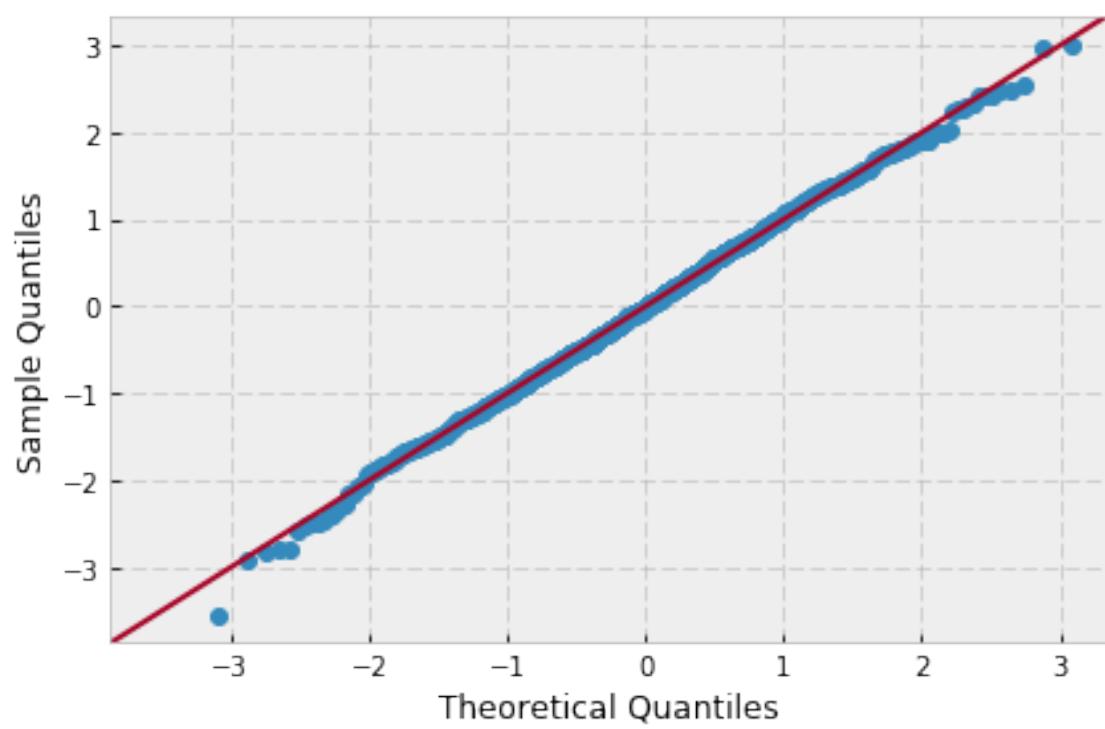
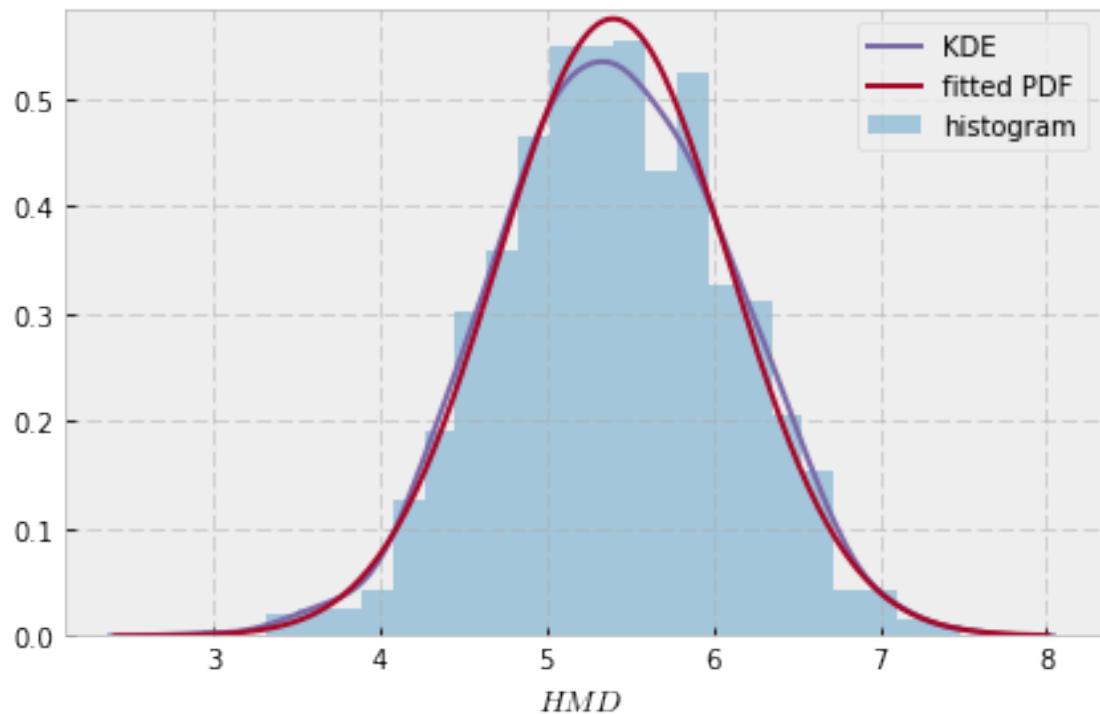


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=5.223531165169826, scale=0.6796283092330648)
DescribeResult(nobs=1000, minmax=(3.1366183213676821, 7.1741853288832393)
               mean=5.2235311651698257, variance=0.46235699570670113,
               skewness=-0.1837494033025925, kurtosis=0.08311508494747155)
```

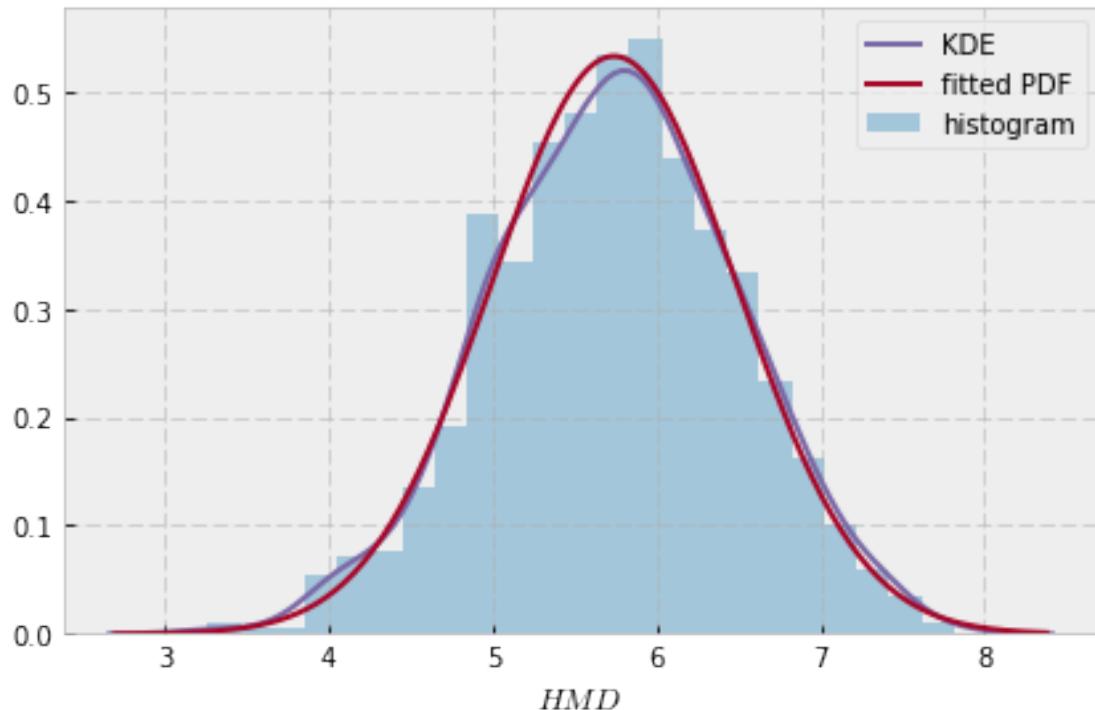


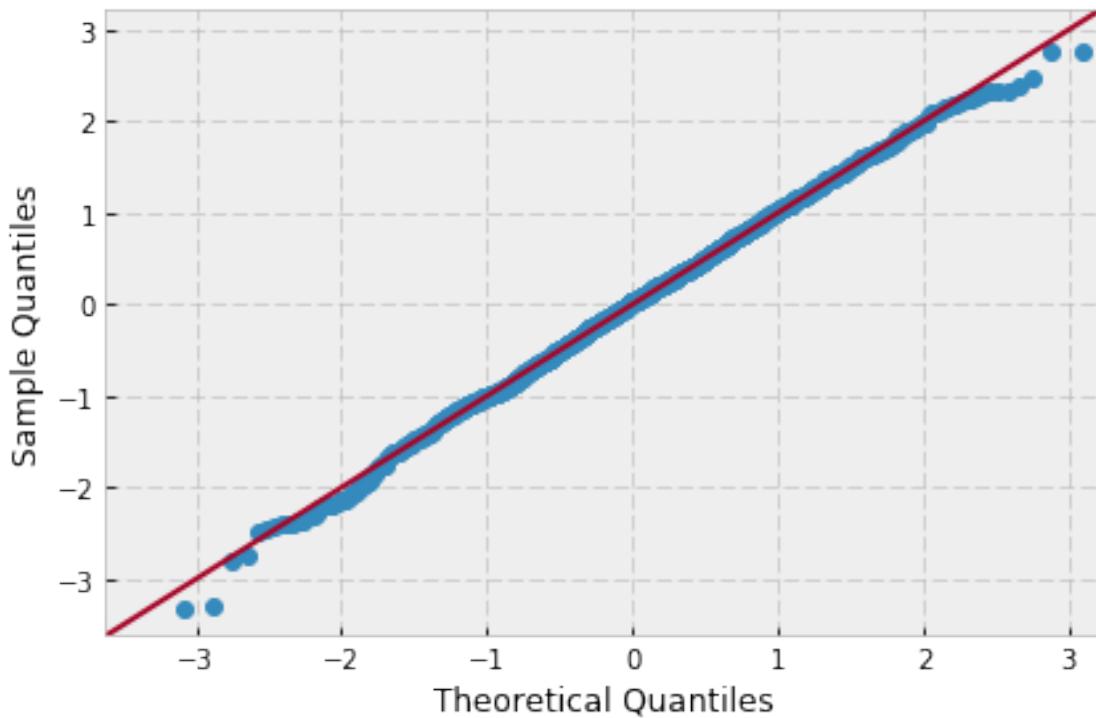


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=5.398794967355525, scale=0.6948533326089944)
DescribeResult(nobs=1000, minmax=(2.9388232709624176, 7.4829108579622936)
               mean=5.3987949673555251, variance=0.4833044582961219,
               skewness=-0.04321462925431739, kurtosis=-0.12957743803774902)
```

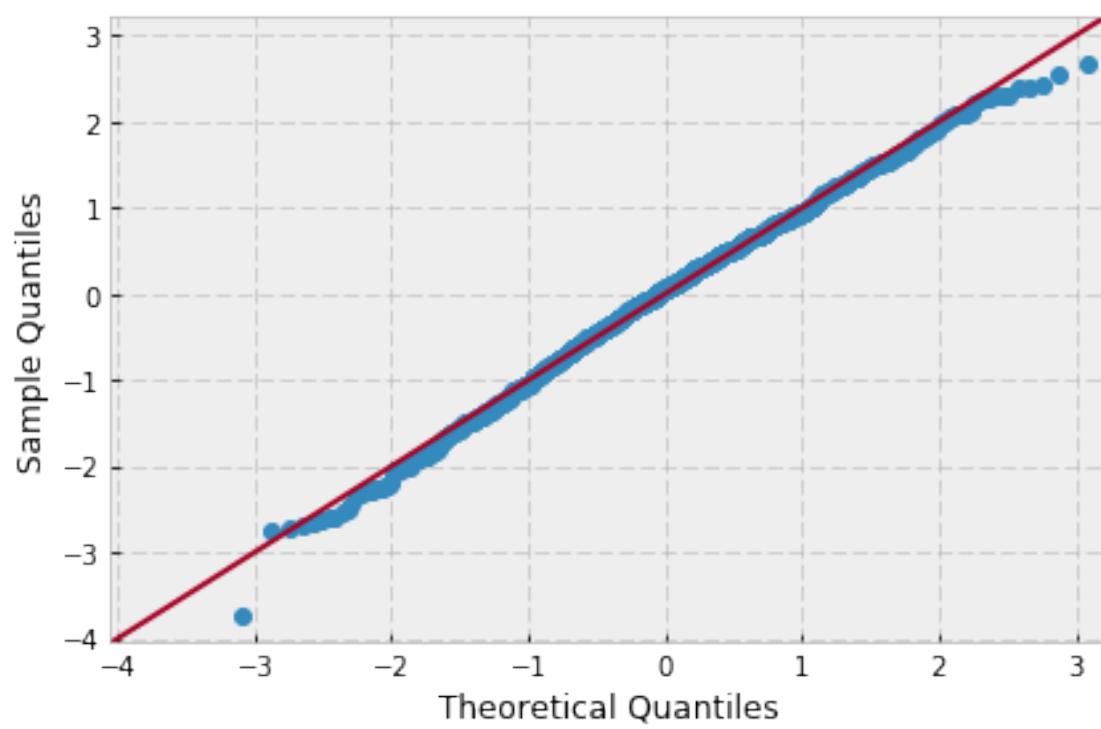
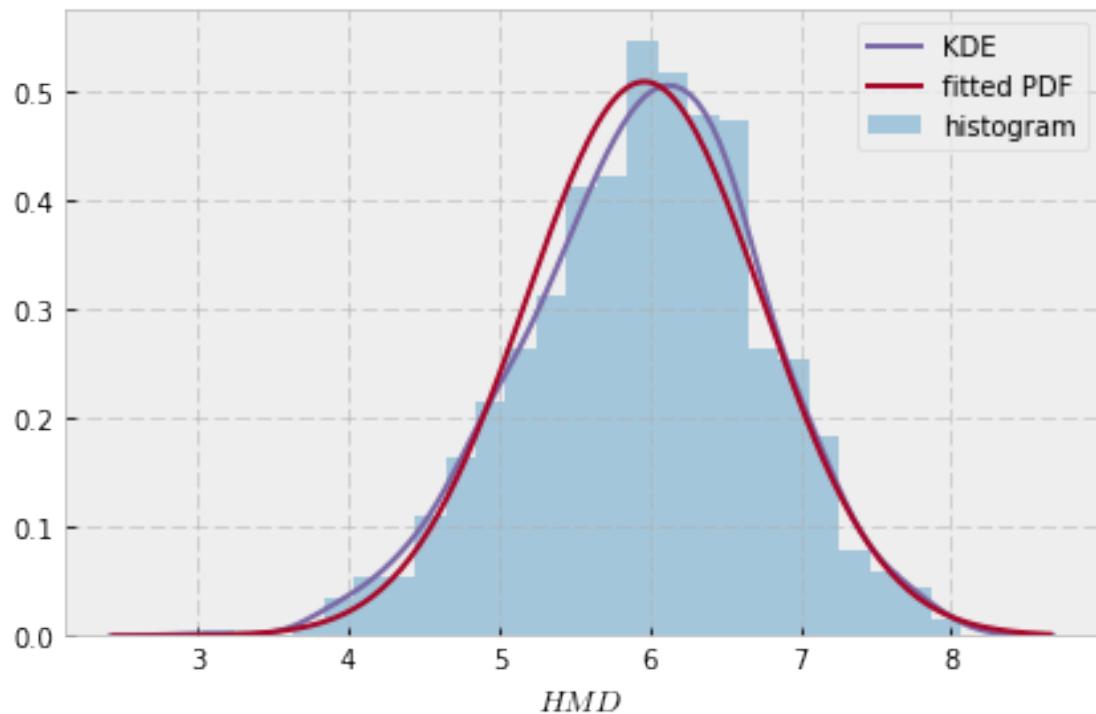


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=5.738321720134851, scale=0.7479695358645678)
DescribeResult(nobs=1000, minmax=(3.2577757509780589, 7.8133182555014793)
               mean=5.7383217201348513, variance=0.56001844502648357,
               skewness=-0.09092976858399848, kurtosis=-0.10851901459810964)
```

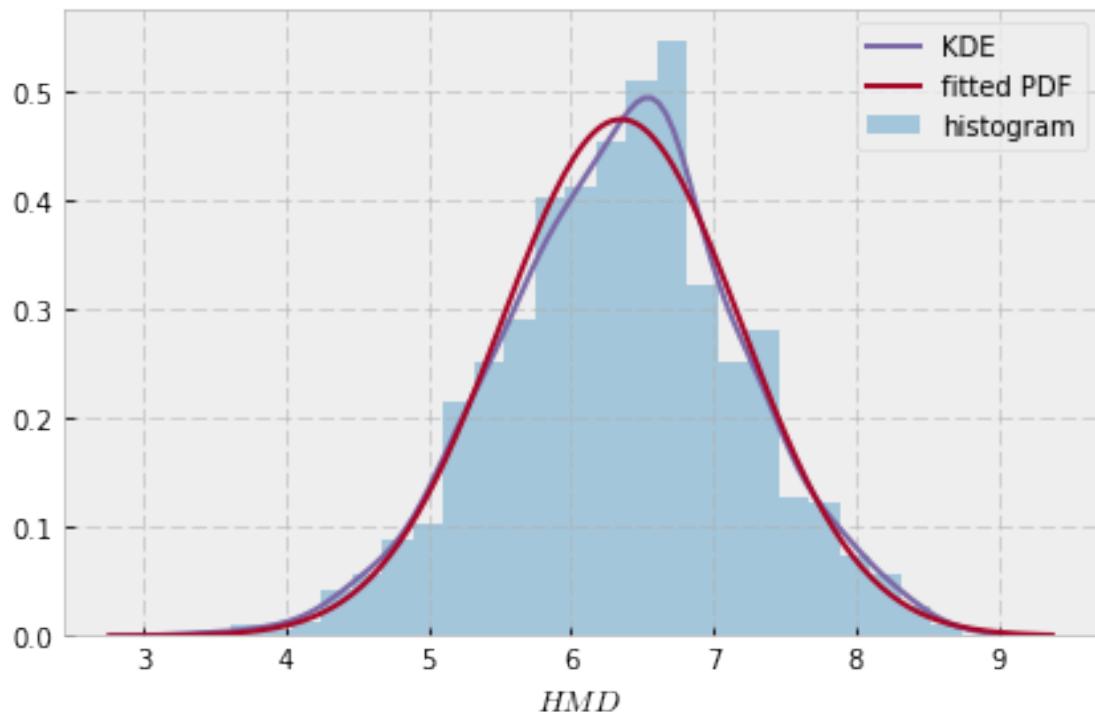


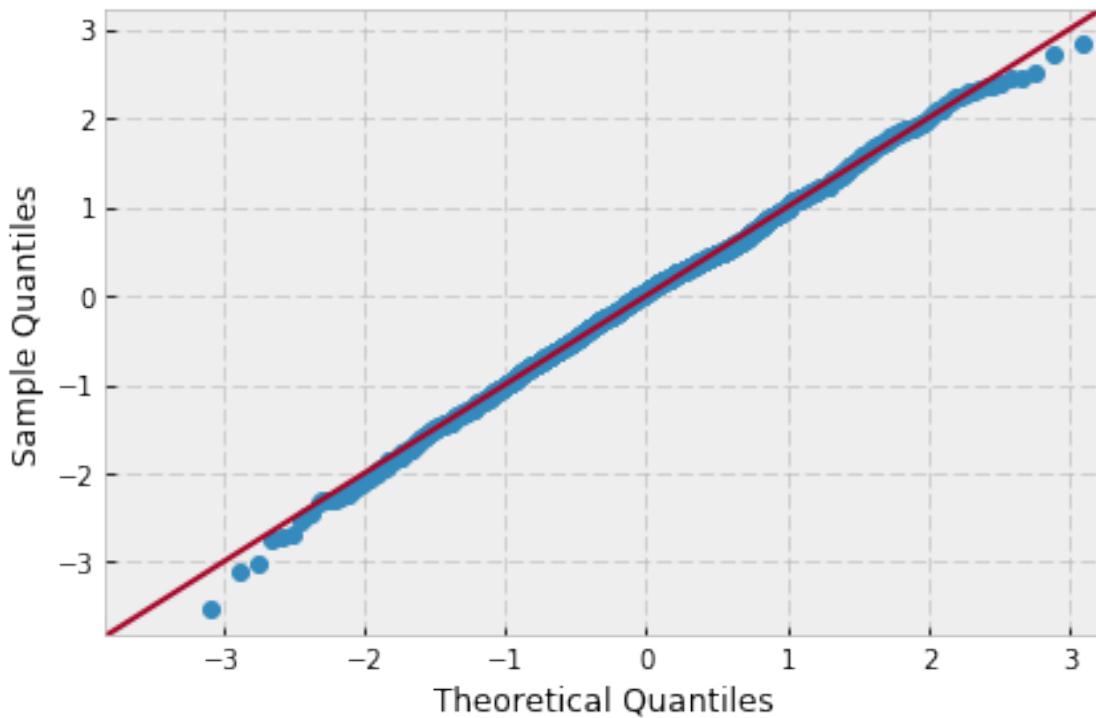


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(),loc=5.9623322649492625, scale=0.7852104945194603)
DescribeResult(nobs=1000, minmax=(3.0310270184110104, 8.063115599805462)
               mean=5.9623322649492625, variance=0.61717269339689229,
               skewness=-0.21550701897762536, kurtosis=-0.05250588052310157)
```

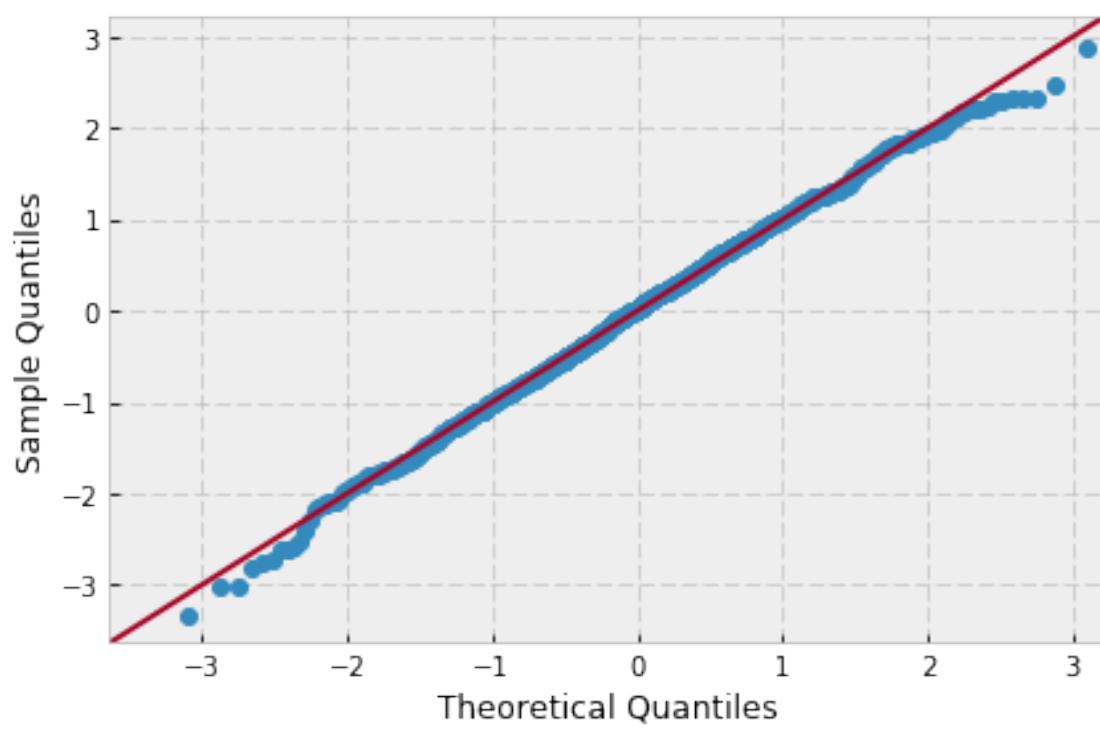
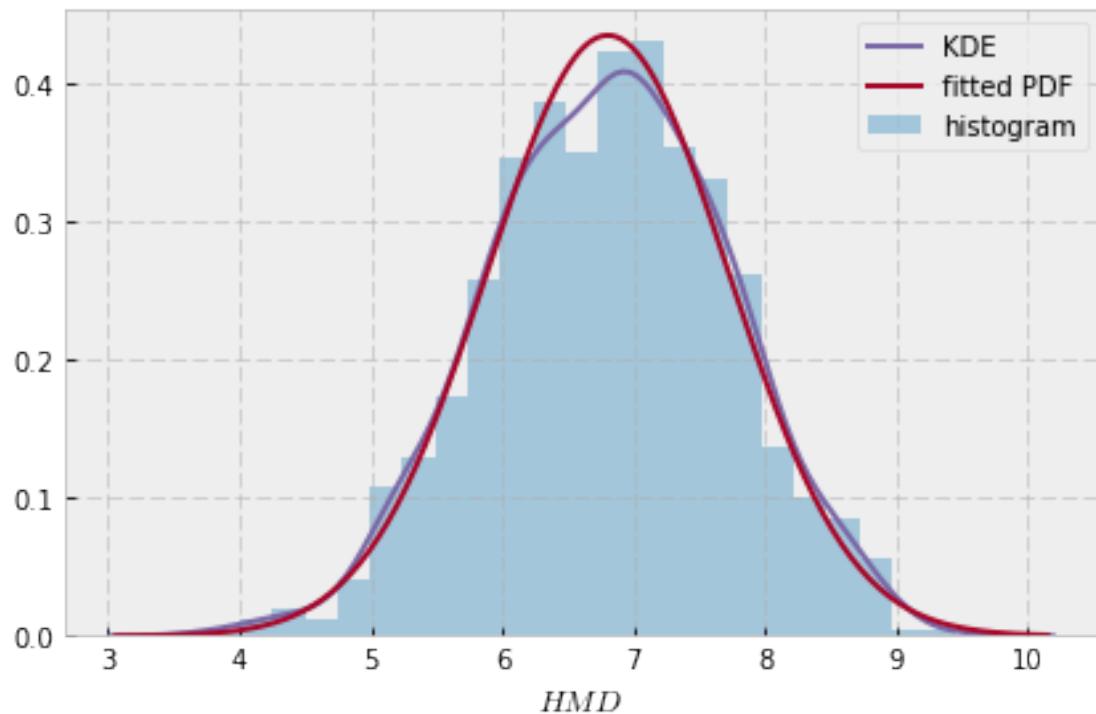


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=6.350196655763222, scale=0.8411761441096779)
DescribeResult(nobs=1000, minmax=(3.3954824771939944, 8.7396907047949597)
               mean=6.3501966557632219, variance=0.7082855910102358,
               skewness=-0.09894653404750207, kurtosis=0.016822147616560645)
```

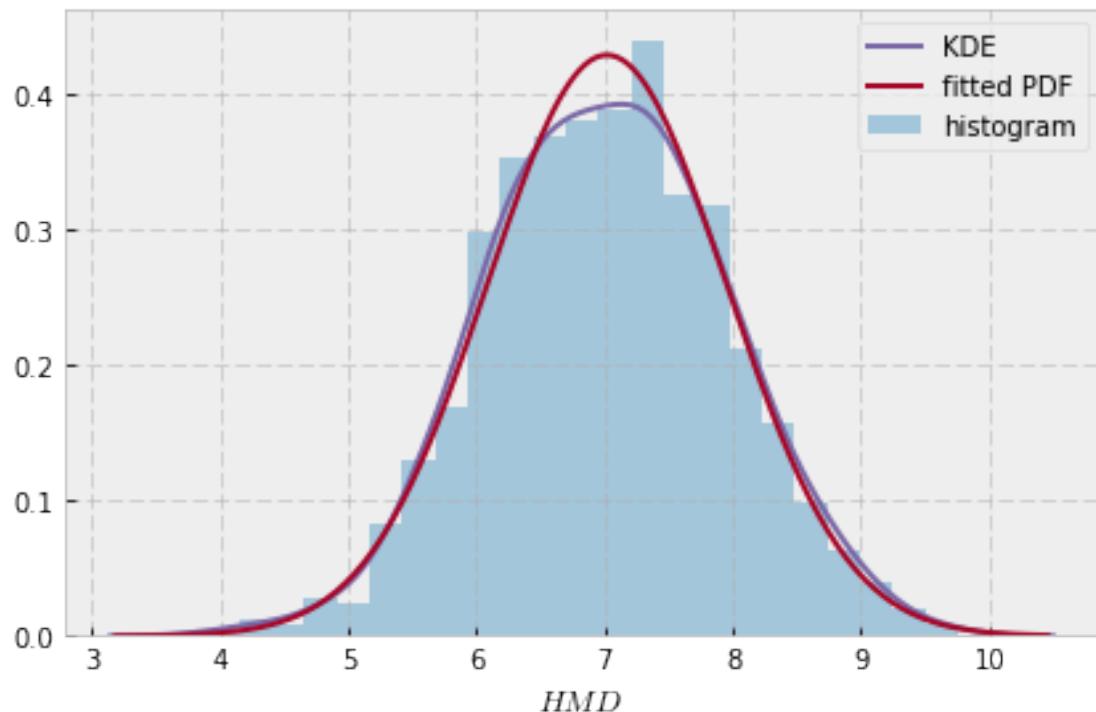


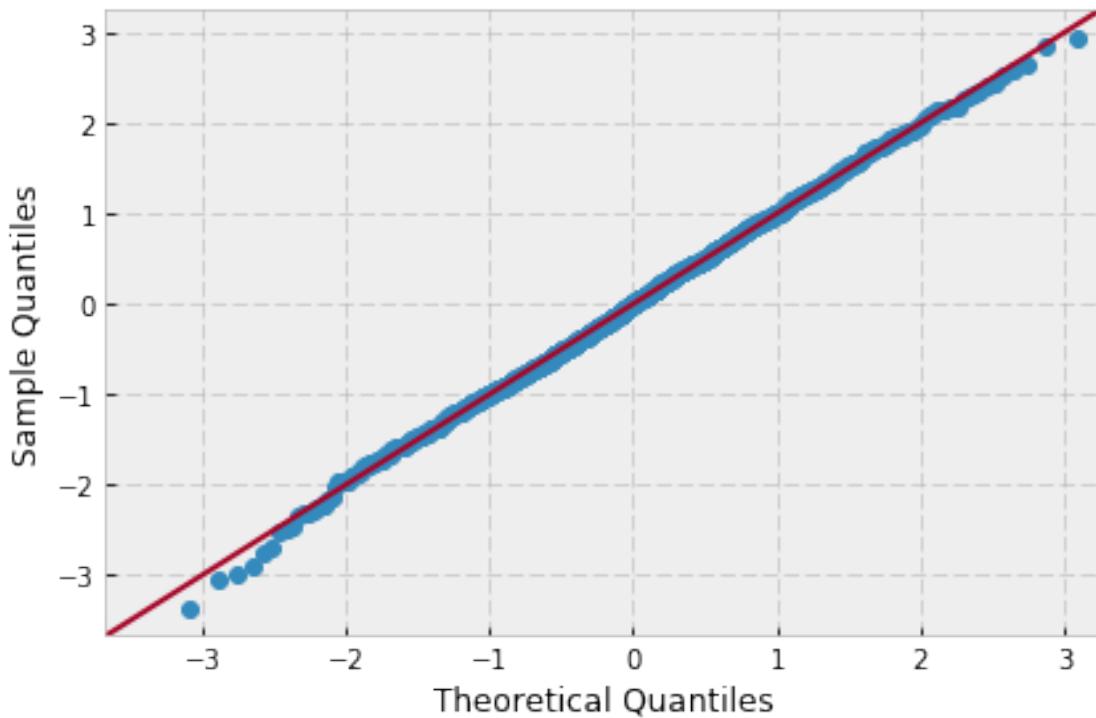


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=6.804886571952806, scale=0.9187529093393955)
DescribeResult(nobs=1000, minmax=(3.7515742807632106, 9.461799803428482)
               mean=6.8048865719528058, variance=0.84495186027988323,
               skewness=-0.11115397422314224, kurtosis=-0.17227647267426516)
```

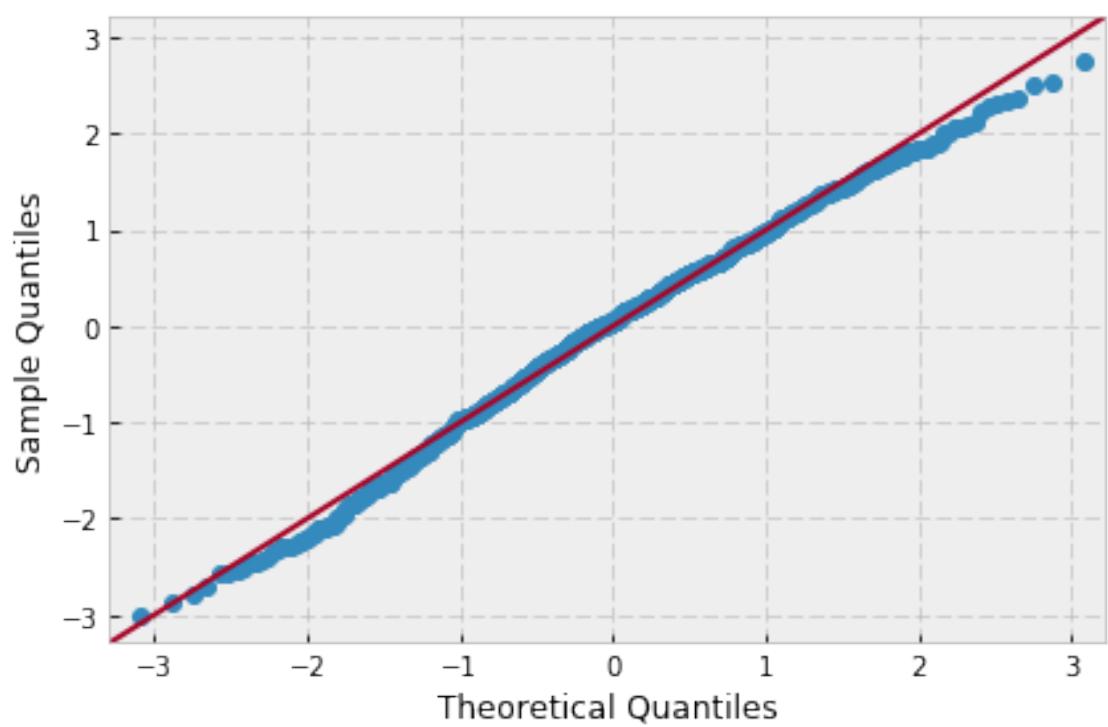
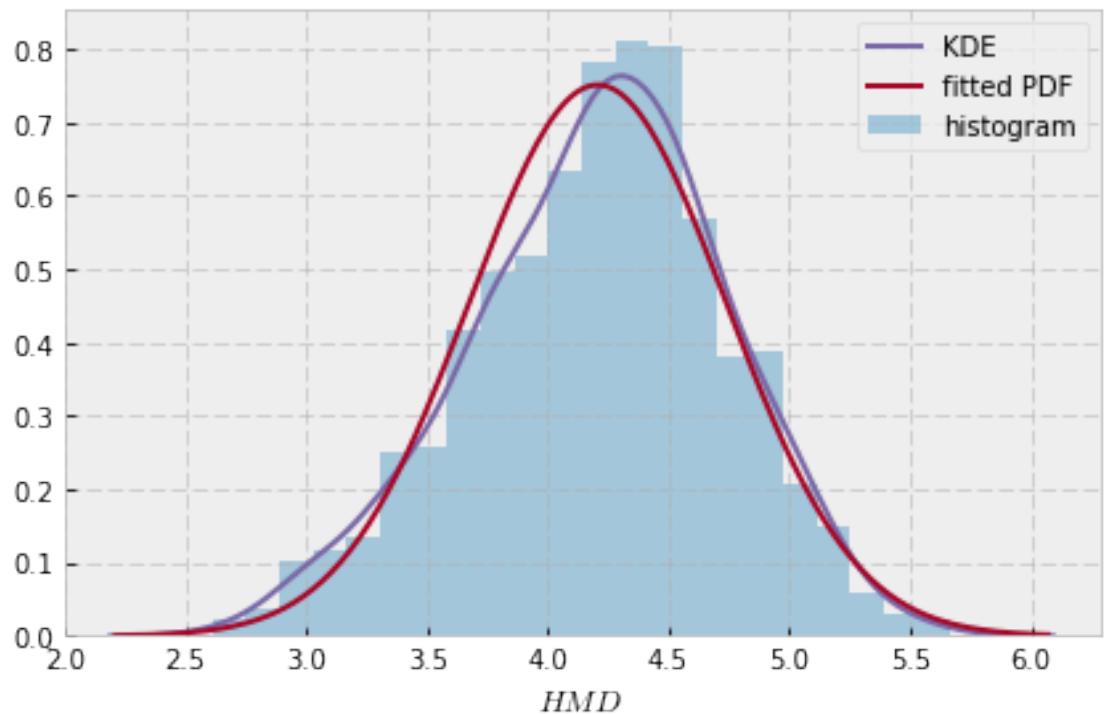


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=7.018910887139297, scale=0.9317163217516226)
DescribeResult(nobs=1000, minmax=(3.8917883212584754, 9.7541579098622542)
               mean=7.0189108871392971, variance=0.86896426848685993,
               skewness=-0.020570019890127324, kurtosis=-0.09656582655519053)
```

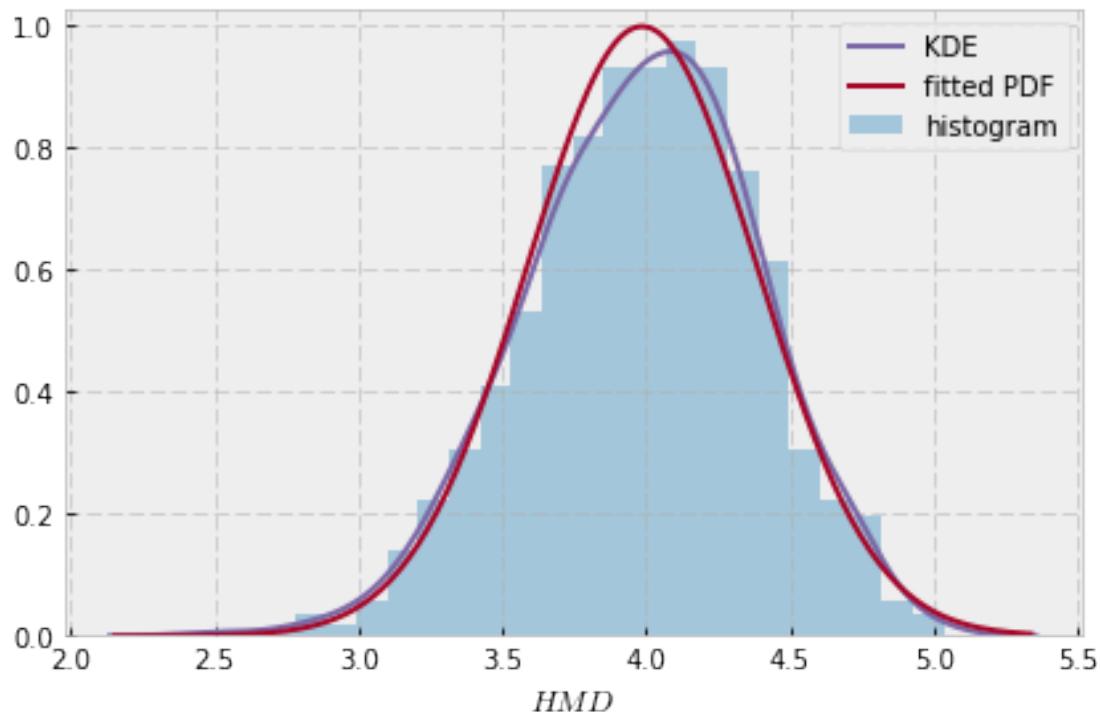


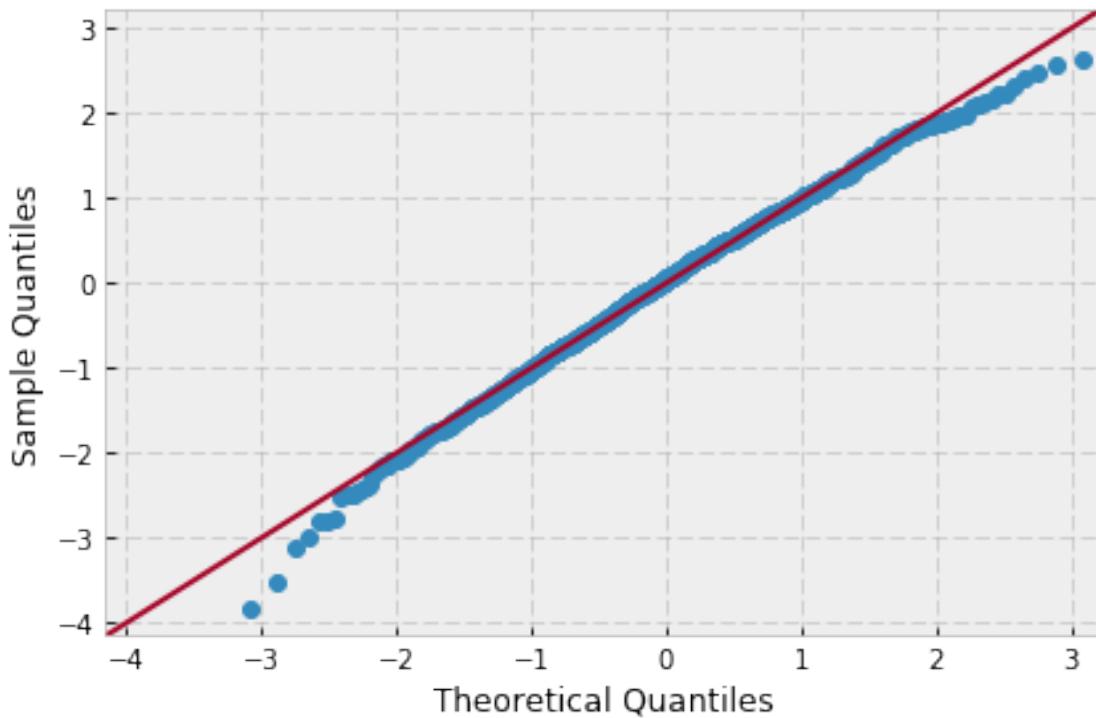


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(),loc=4.2058648285736, scale=0.5311314752207128)
DescribeResult(nobs=1000, minmax=(2.6104075662590538, 5.6695496966802006)
               mean=4.2058648285736, variance=0.28238302699712775,
               skewness=-0.2533974467767575, kurtosis=-0.15041114041119874)
```

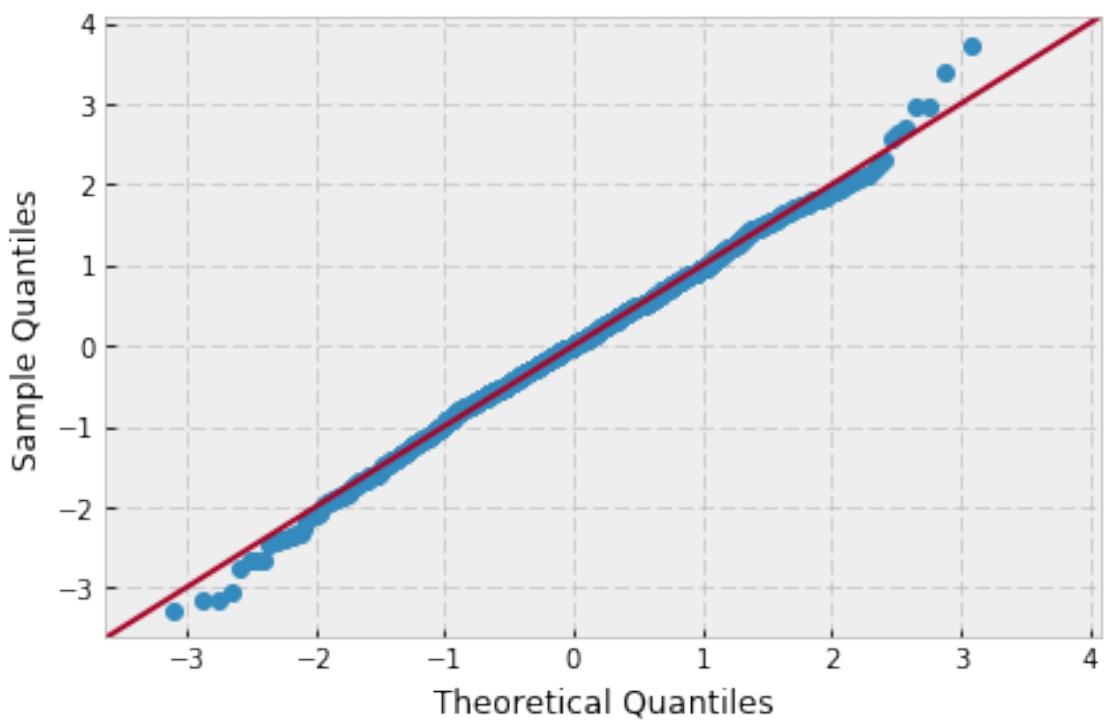
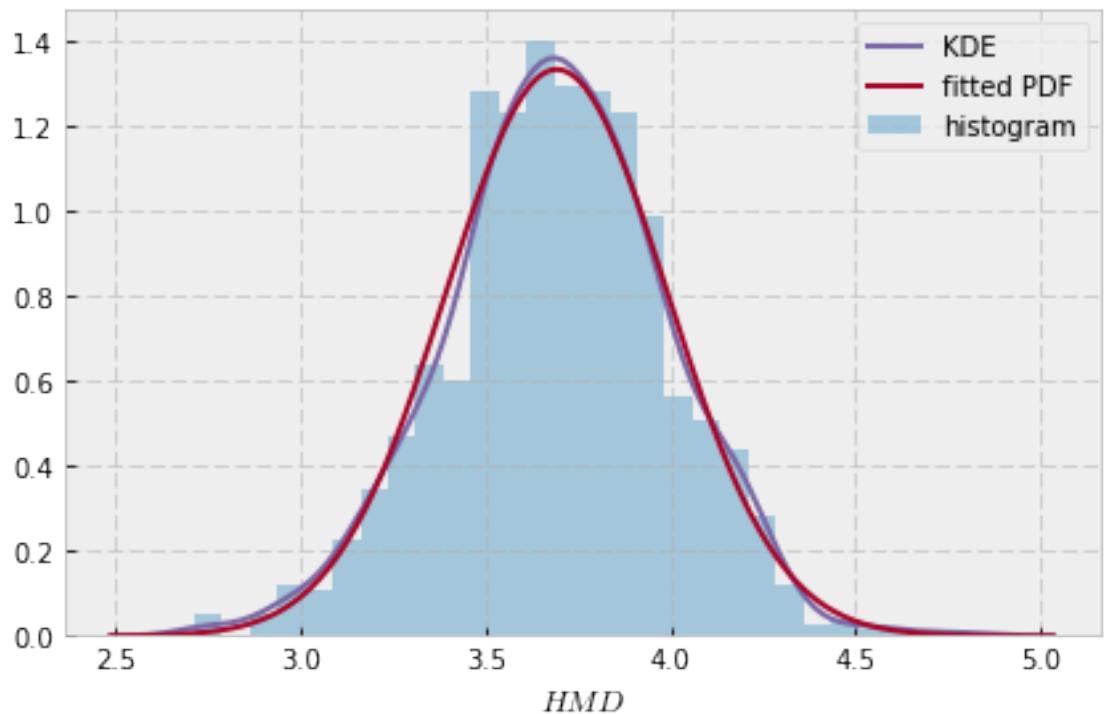


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(), loc=3.984758608760525, scale=0.39931507813638084)
DescribeResult(nobs=1000, minmax=(2.4547453524458547, 5.0338735524368641)
               mean=3.9847586087605249, variance=0.15961214377083477,
               skewness=-0.2478729687533784, kurtosis=0.029245646125278135)
```

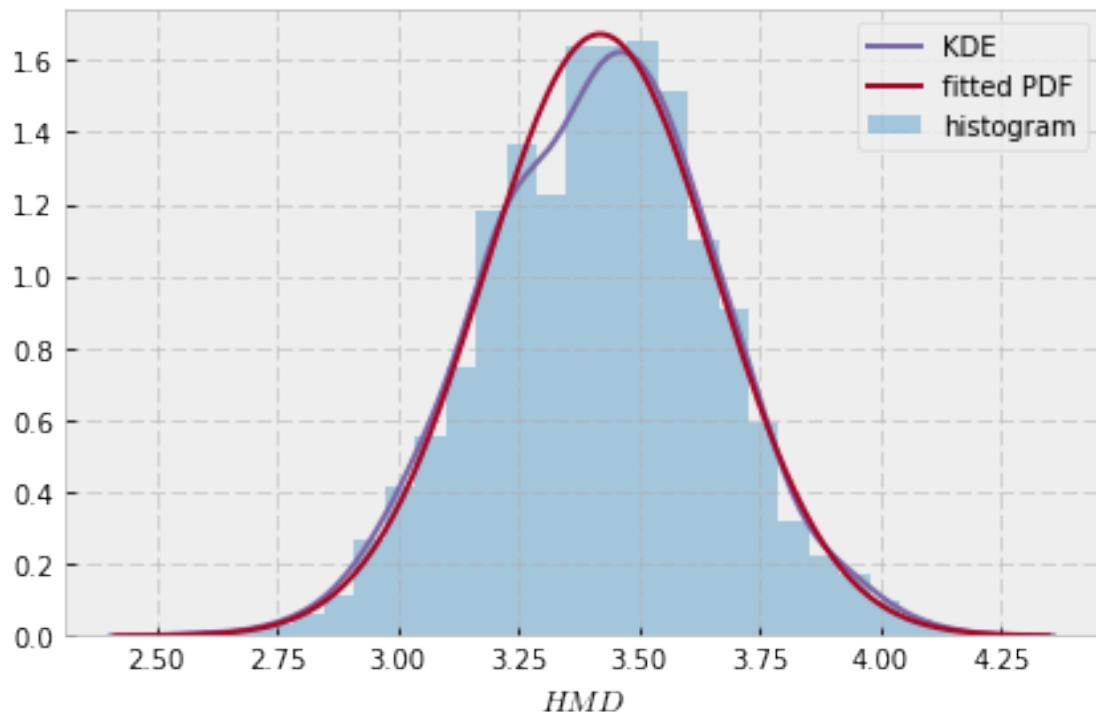


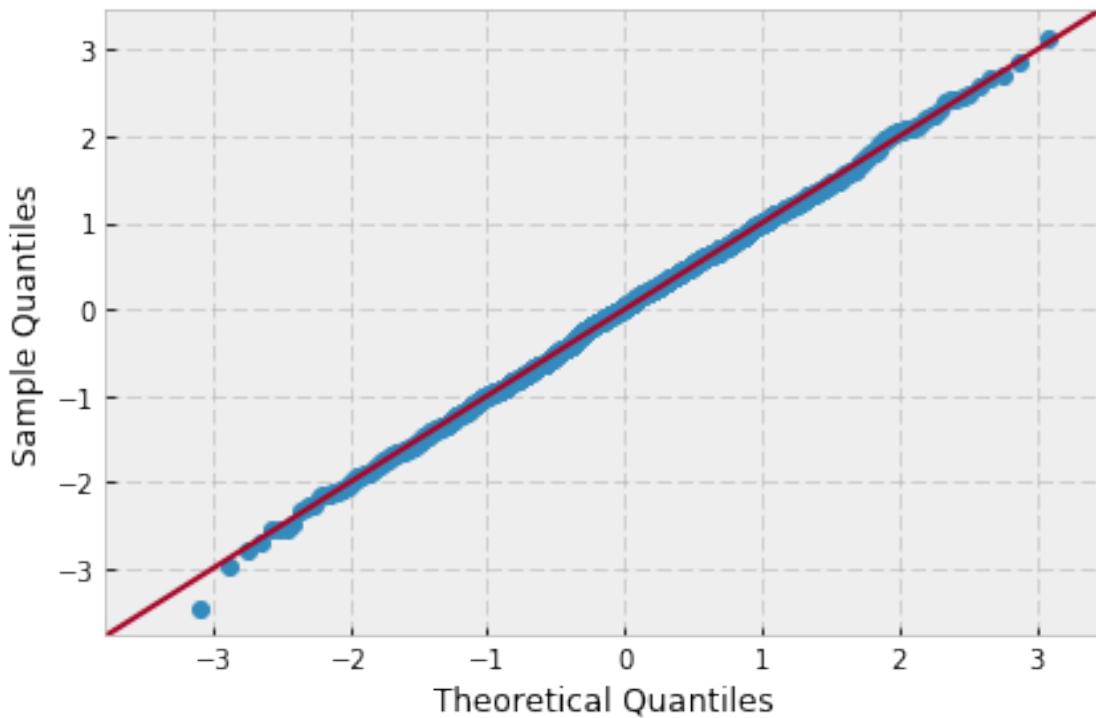


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(),loc=3.692268571074507, scale=0.2988142758872937)
DescribeResult(nobs=1000, minmax=(2.7140696289136512, 4.8066193268942028)
               mean=3.6922685710745071, variance=0.089379350824872542,
               skewness=-0.07422035929295794, kurtosis=0.29582458849808946)
```

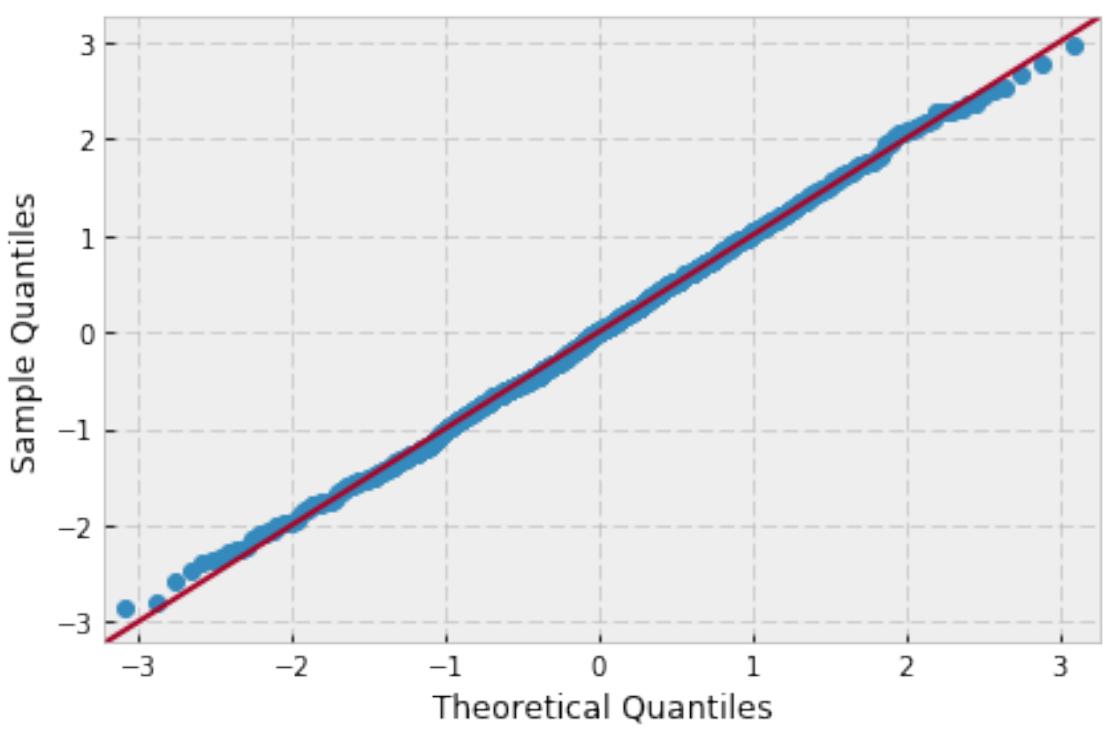
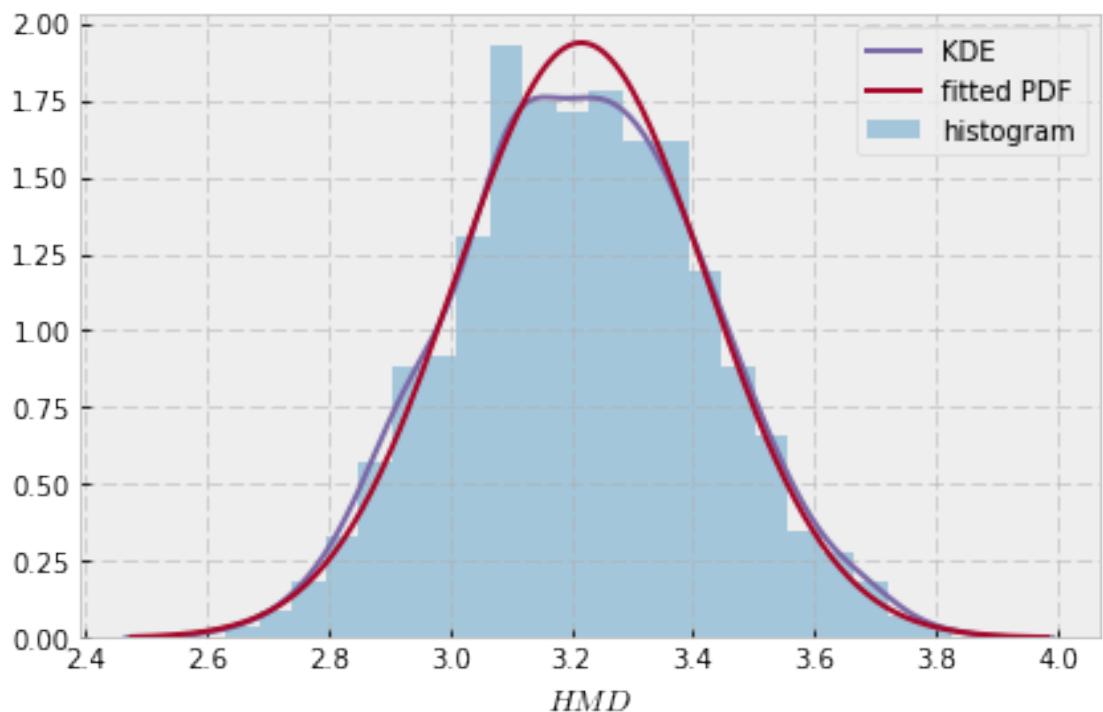


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(), loc=3.419399747651255, scale=0.2385369040632482)
DescribeResult(nobs=1000, minmax=(2.5968814518763041, 4.1662736092899824)
               mean=3.419399747651255, variance=0.056956811411490768,
               skewness=-0.04101250320746071, kurtosis=-0.06484791000410972)
```

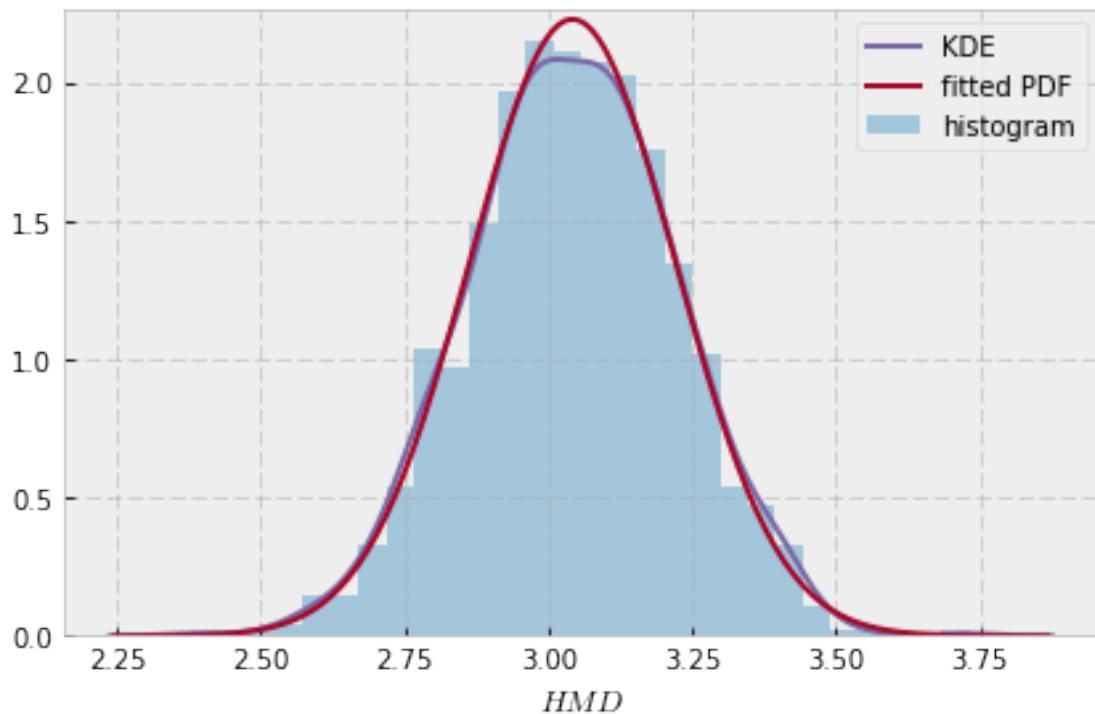


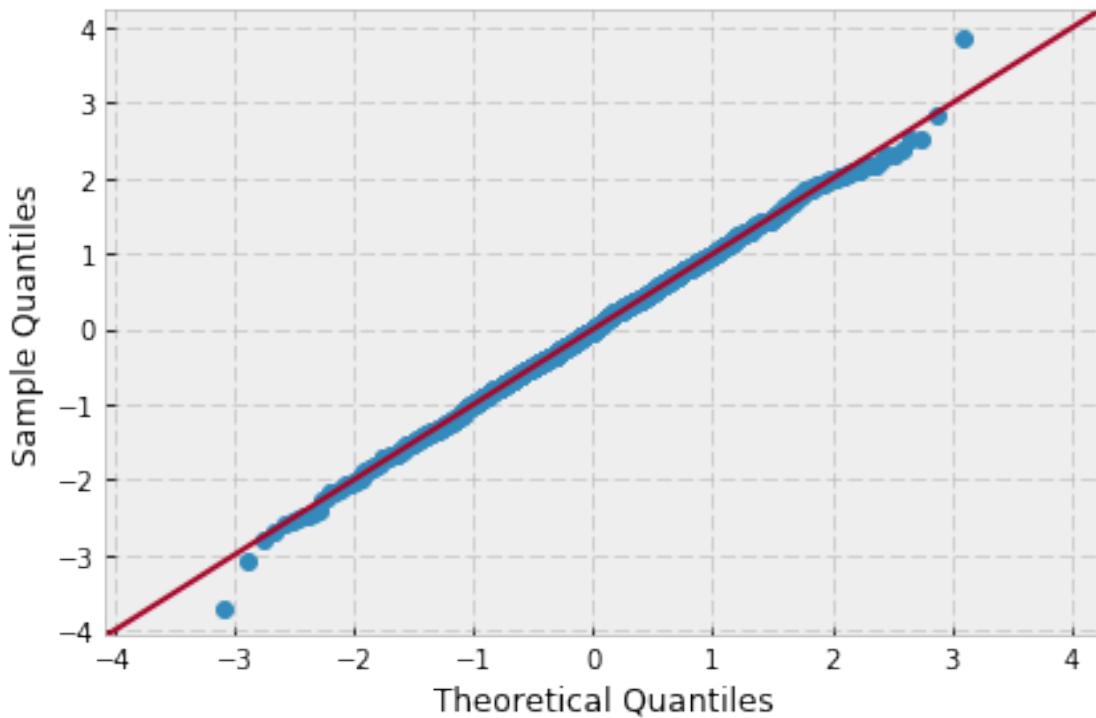


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(),loc=3.216105639738268, scale=0.20599864248600597)
DescribeResult(nobs=1000, minmax=(2.6317747983087507, 3.8275723846773984)
               mean=3.2161056397382679, variance=0.042477918624702002,
               skewness=0.05050222816730362, kurtosis=-0.28497655368431873)
```

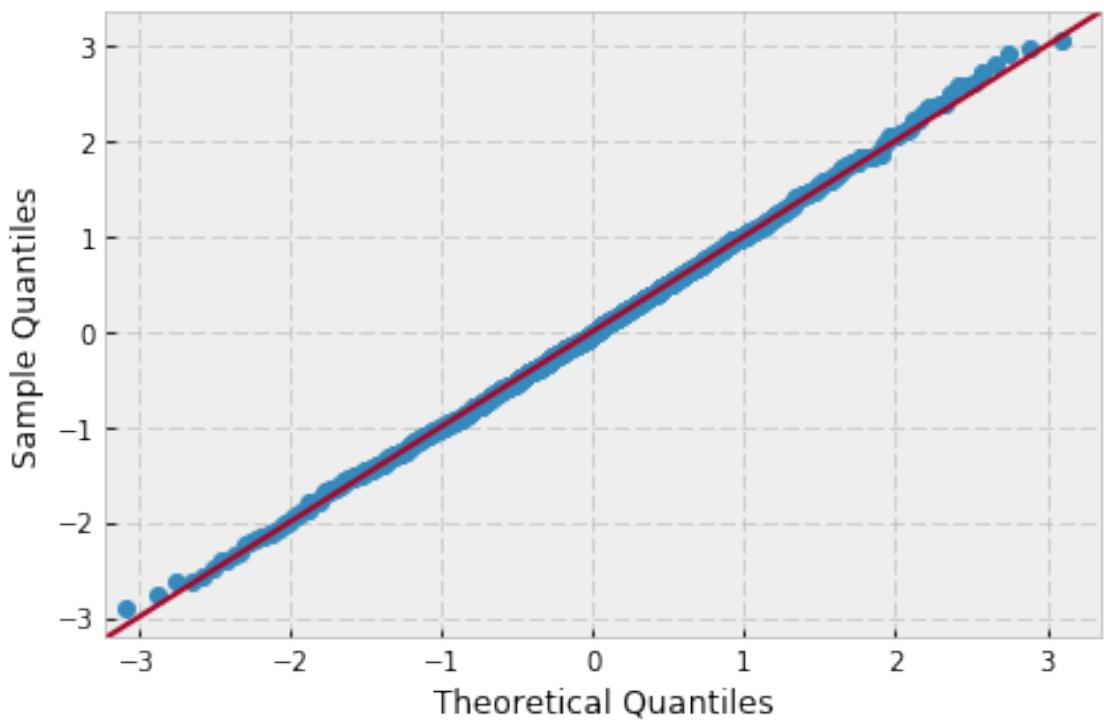
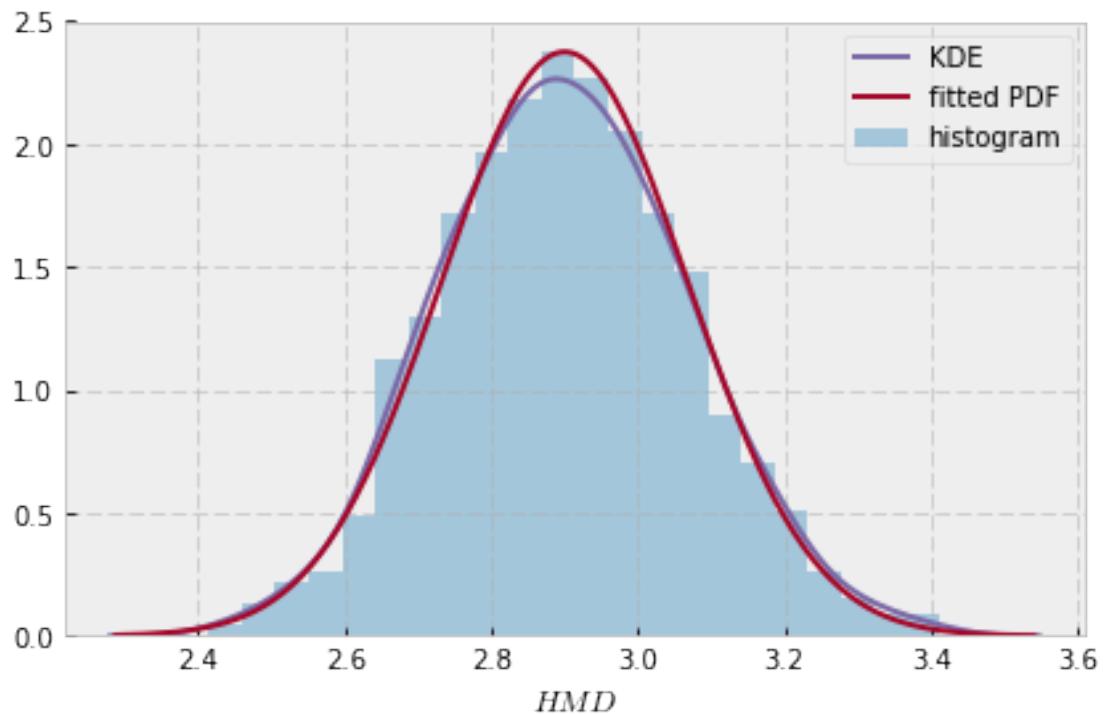


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(), loc=3.042002057705741, scale=0.1790845094042891)
DescribeResult(nobs=1000, minmax=(2.3806952359145548, 3.7327885037731843)
               mean=3.0420020577057412, variance=0.032103364873448358,
               skewness=-0.037573477586424306, kurtosis=0.02630169764423318)
```

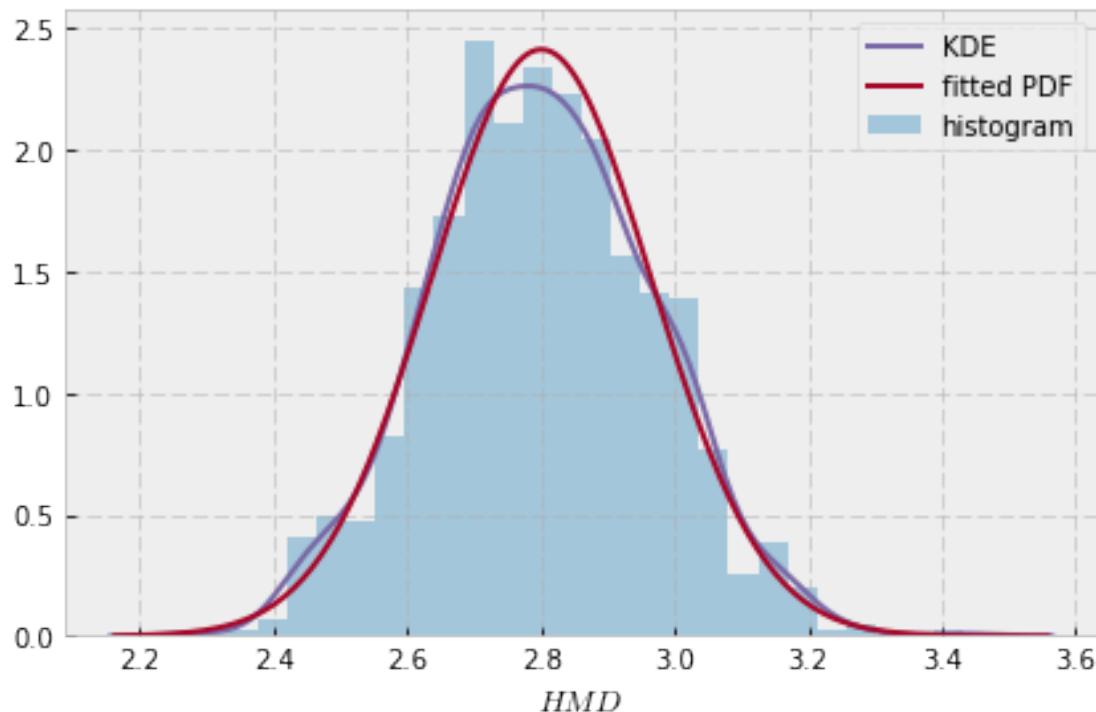


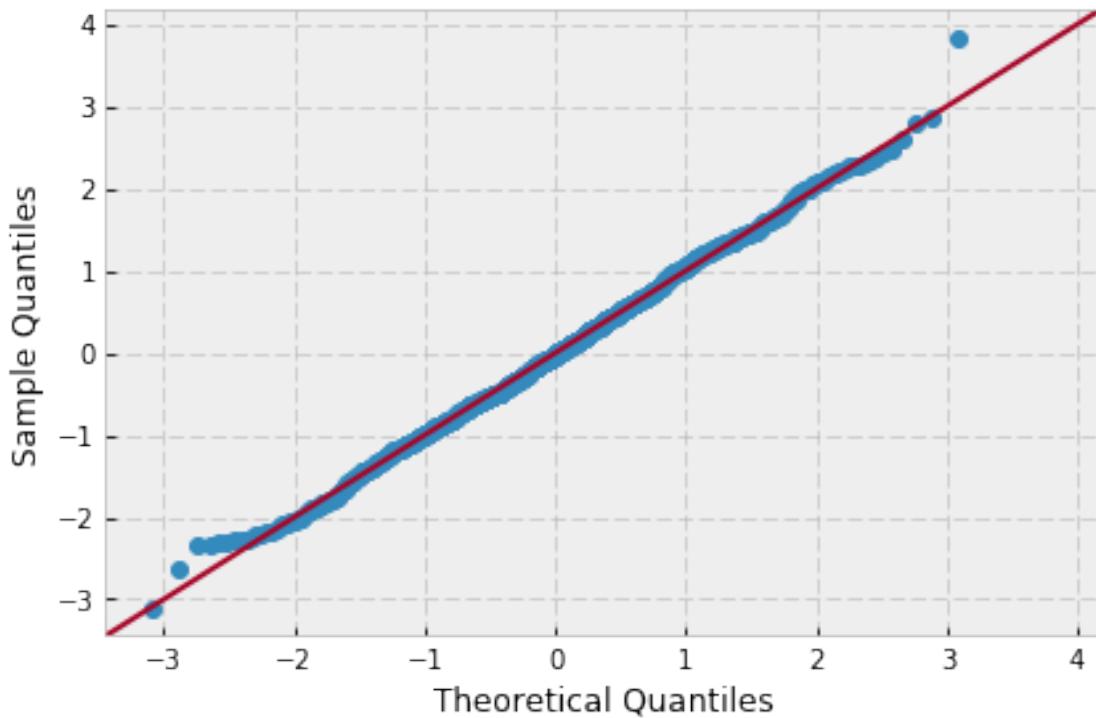


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(),loc=2.8995297970117035, scale=0.1681208369443936)
DescribeResult(nobs=1000, minmax=(2.4138995109574402, 3.4126674488236342)
               mean=2.8995297970117035, variance=0.028292908723606982,
               skewness=0.09529075656431303, kurtosis=-0.1051109850697185)
```

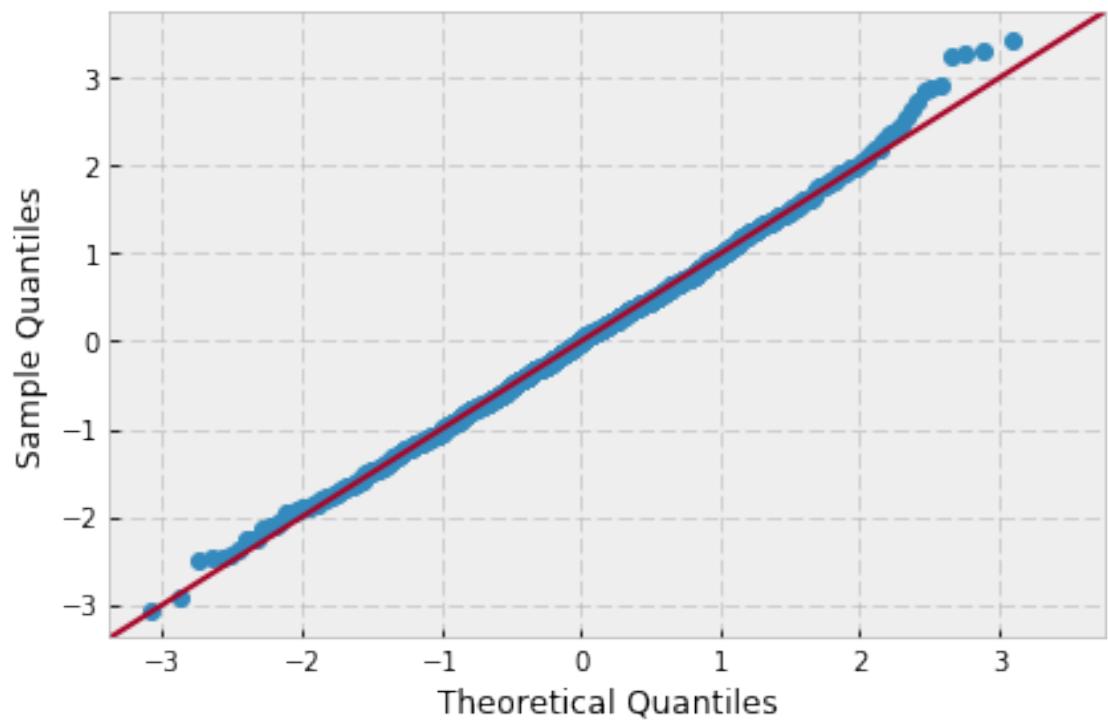
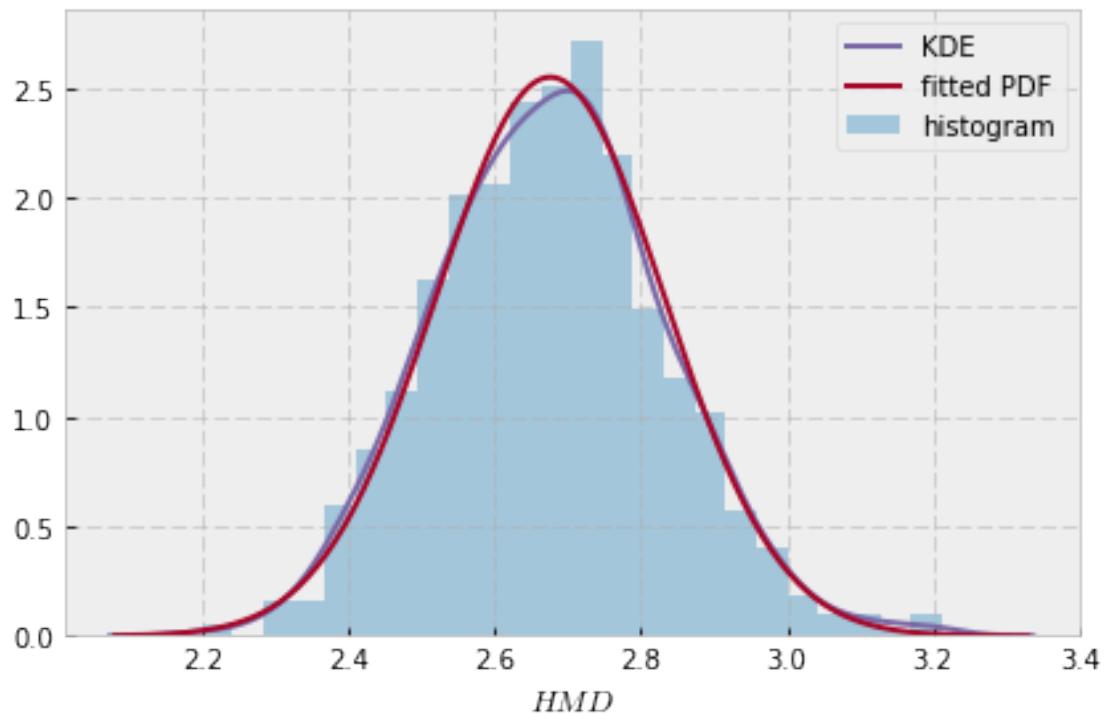


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(), loc=2.8005177412772735, scale=0.16532157563057565)
DescribeResult(nobs=1000, minmax=(2.2879762186105701, 3.4328221303590962)
               mean=2.8005177412772735, variance=0.027358581950927066,
               skewness=0.07809368351438514, kurtosis=-0.11593053758597893)
```

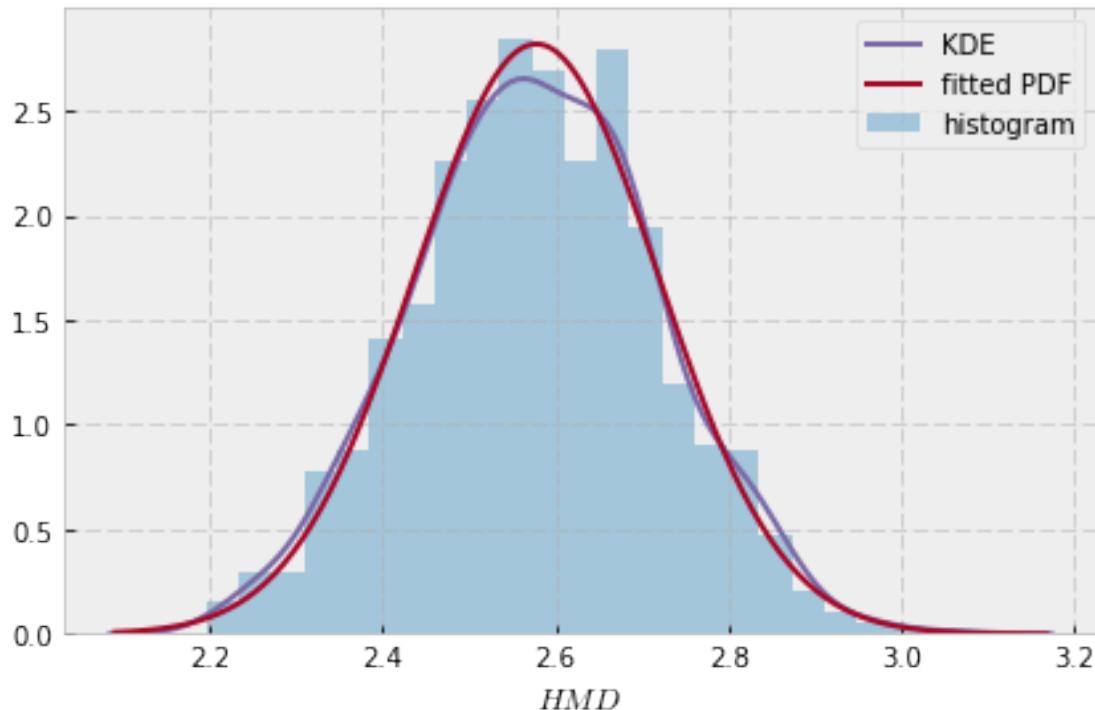


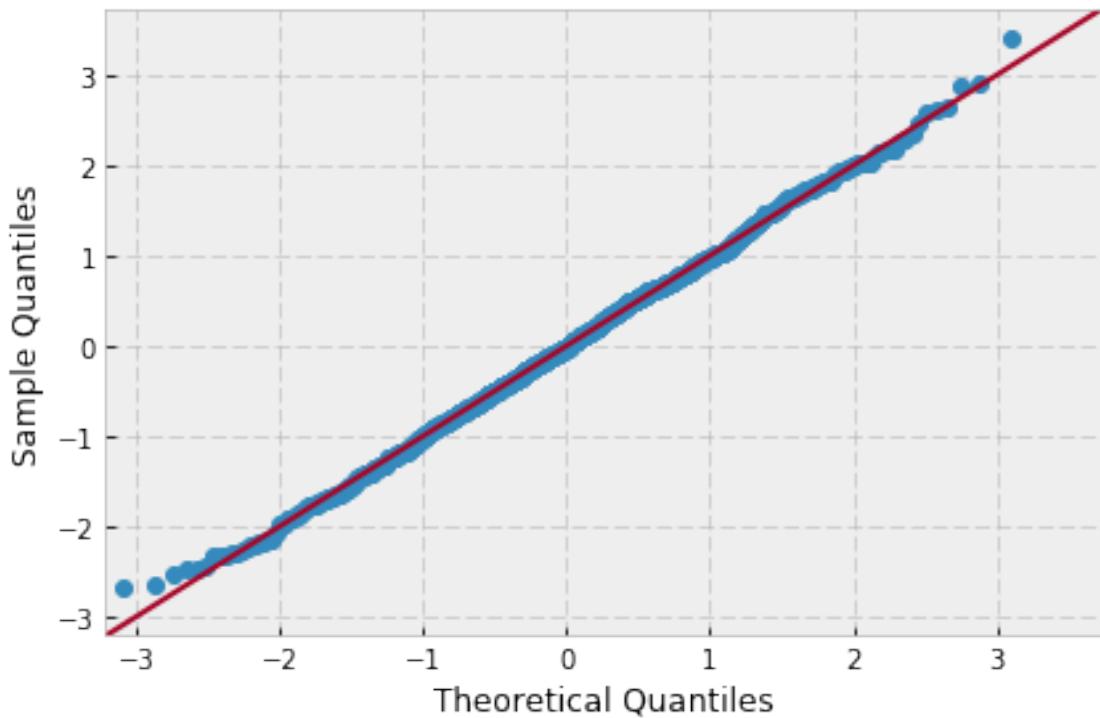


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(),loc=2.6755814487129825, scale=0.15643948717236886)
DescribeResult(nobs=1000, minmax=(2.1973599723330439, 3.2118935823527477)
               mean=2.6755814487129825, variance=0.024497810957711471,
               skewness=0.15848028952227952, kurtosis=0.11751847808799232)
```

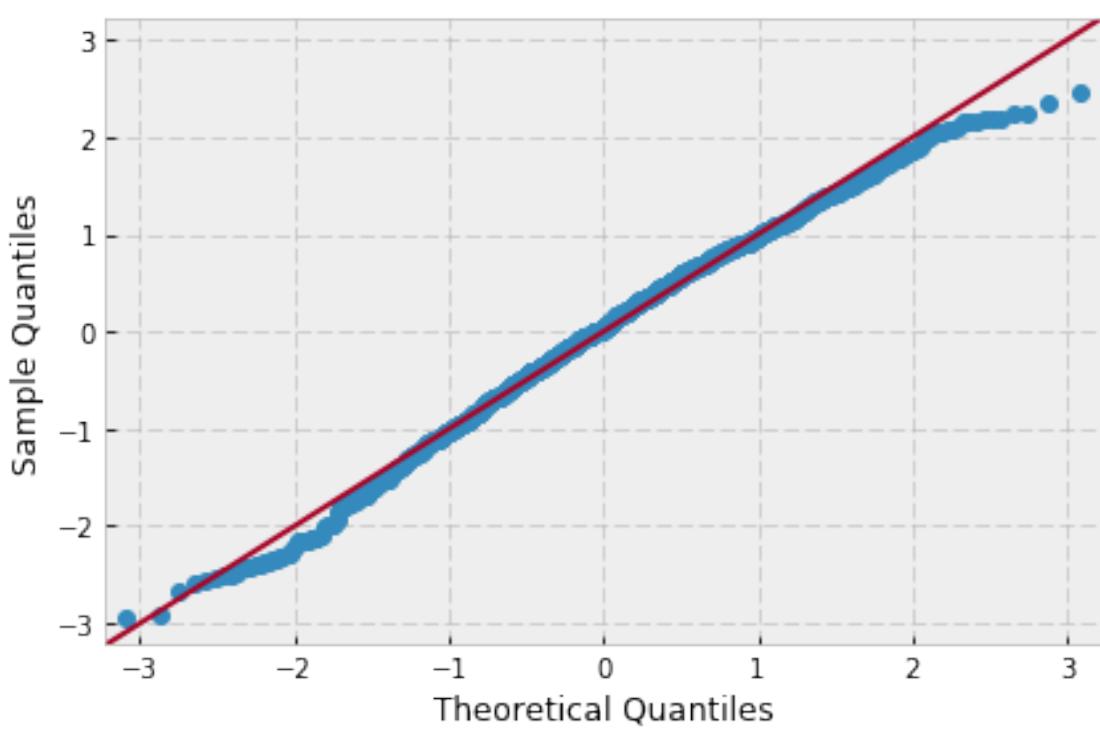
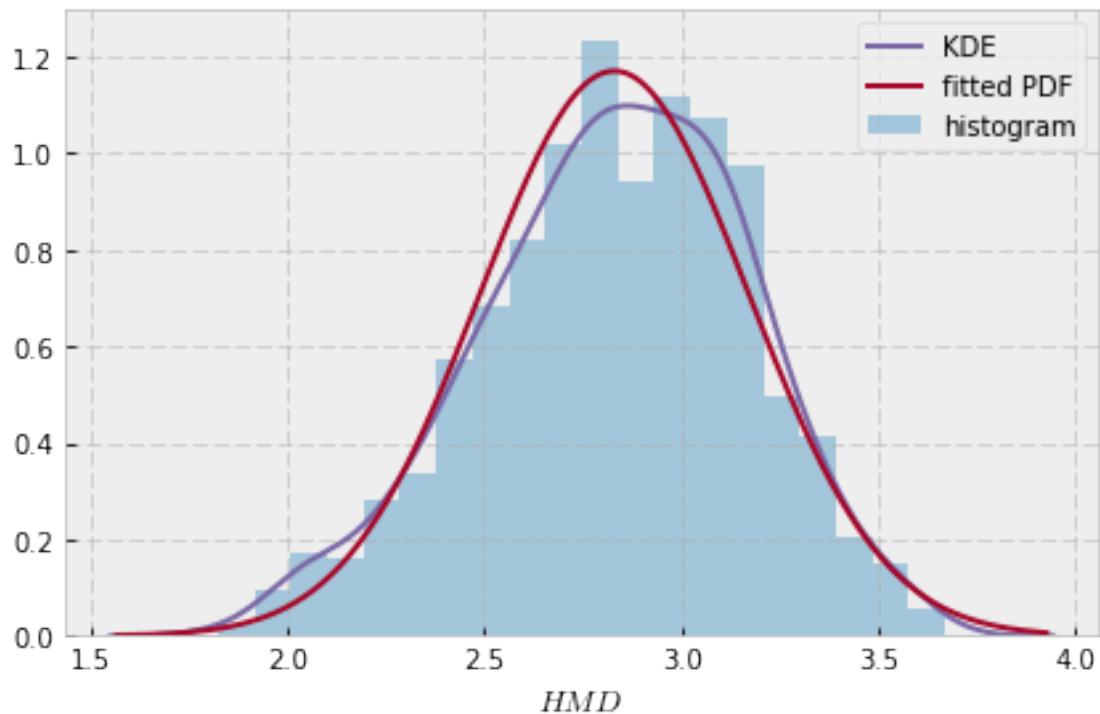


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(), loc=2.5775631736196294, scale=0.1415176489478837)
DescribeResult(nobs=1000, minmax=(2.1968579256553231, 3.0608196131683472)
               mean=2.5775631736196294, variance=0.020047292255992447,
               skewness=0.031716188503848844, kurtosis=-0.1579227121614335)
```

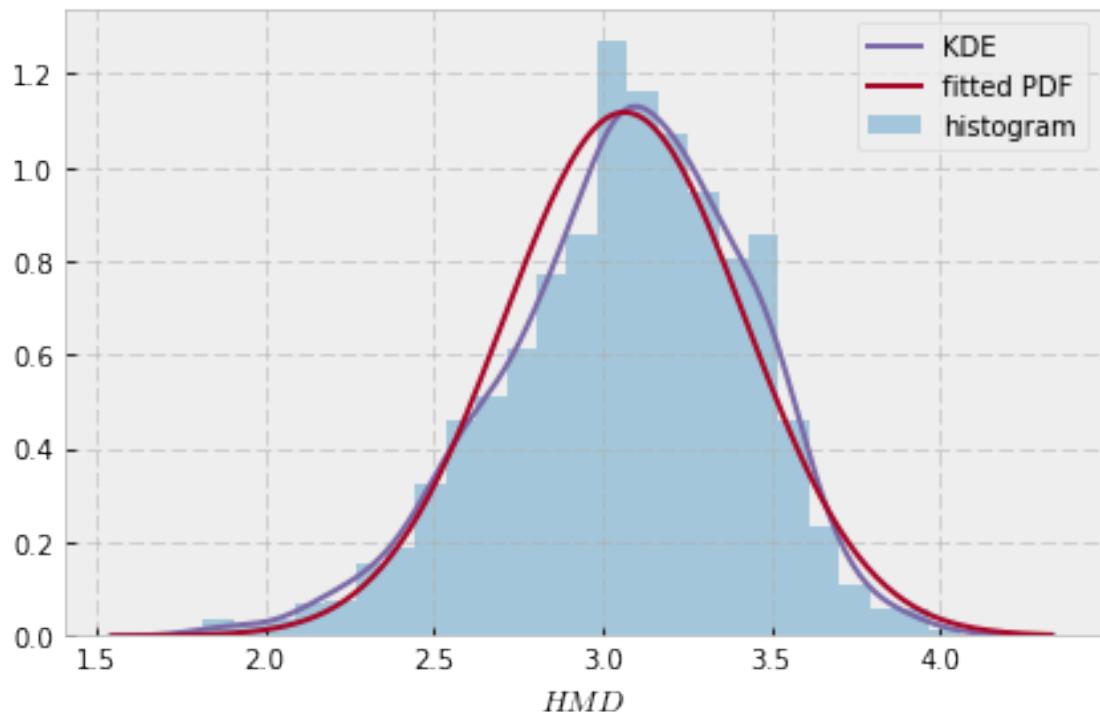


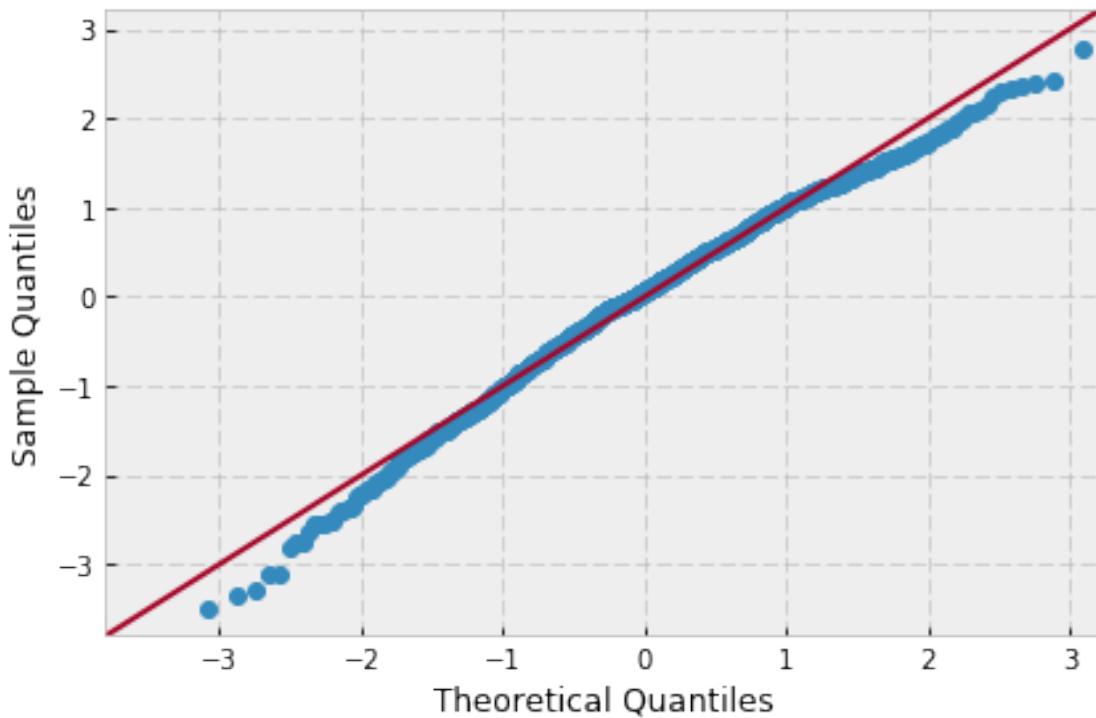


```
variable name: r
variable value: 1.0
distribution: normal(shape=(),loc=2.828959135378004, scale=0.34067513287443596)
DescribeResult(nobs=1000, minmax=(1.8233006407461041, 3.6684692366055591)
               mean=2.8289591353780041, variance=0.11617572188089548,
               skewness=-0.2718367045100893, kurtosis=-0.2220546871952438)
```

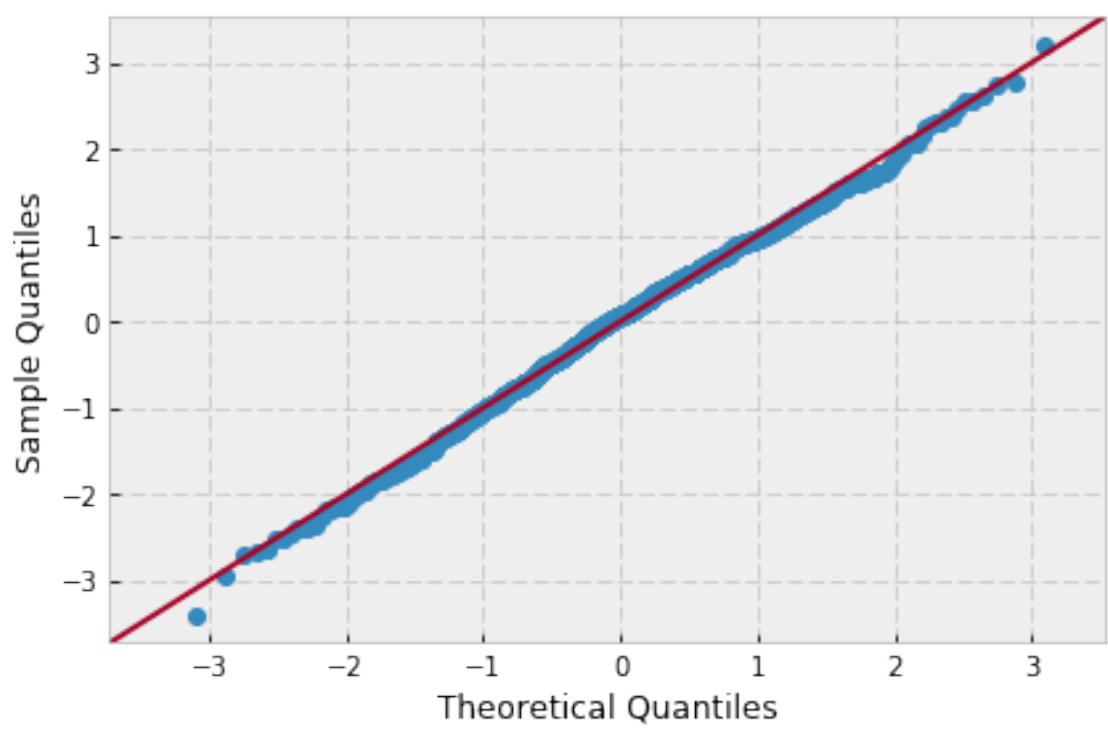
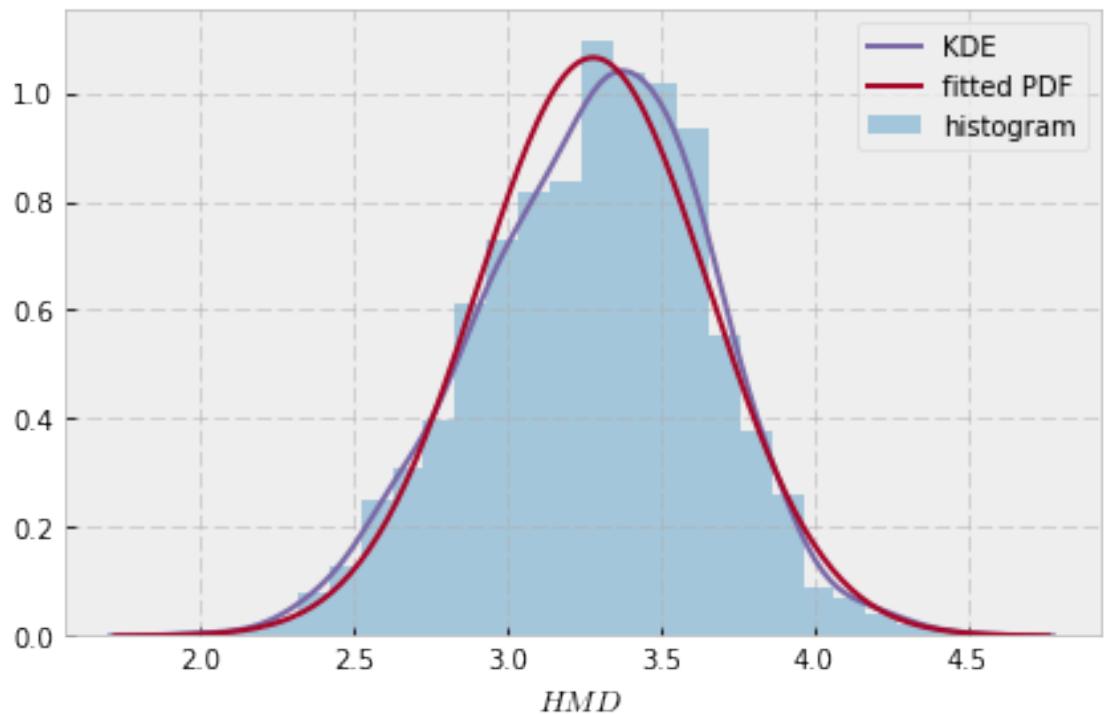


```
variable name: r
variable value: 1.22222222222223
distribution: normal(shape=(), loc=3.0652827462839722, scale=0.35692373507746744)
DescribeResult(nobs=1000, minmax=(1.8191979598993238, 4.0588663664104194)
               mean=3.0652827462839722, variance=0.12752207473638655,
               skewness=-0.39714360173479496, kurtosis=0.10131333657380637)
```

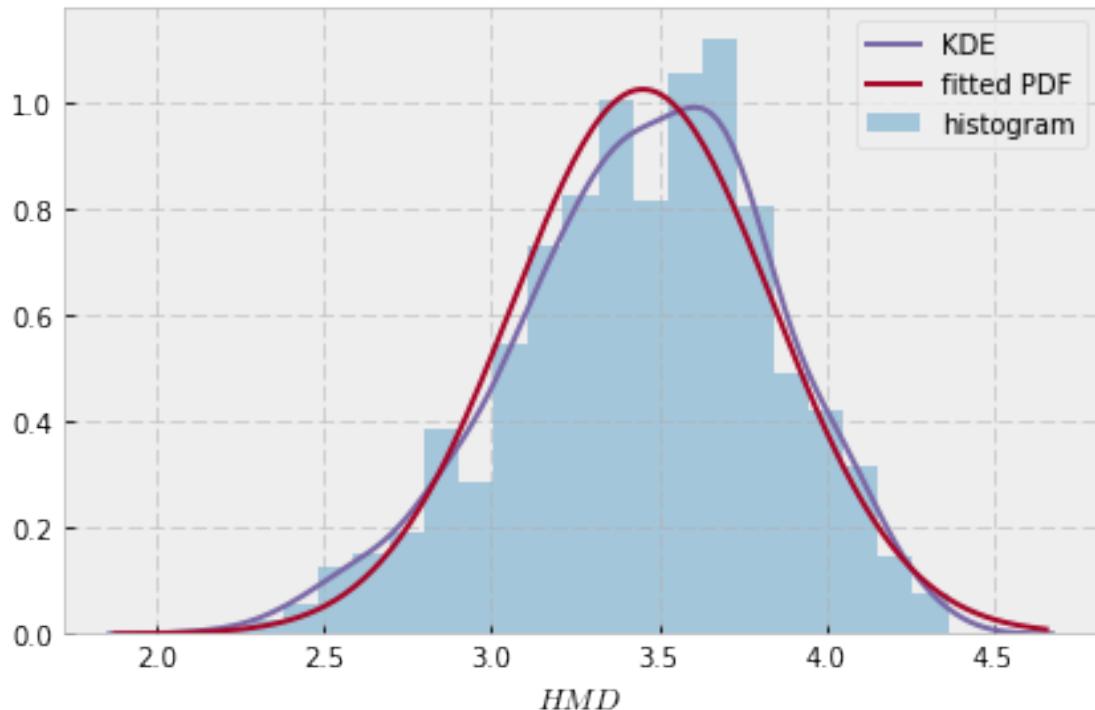


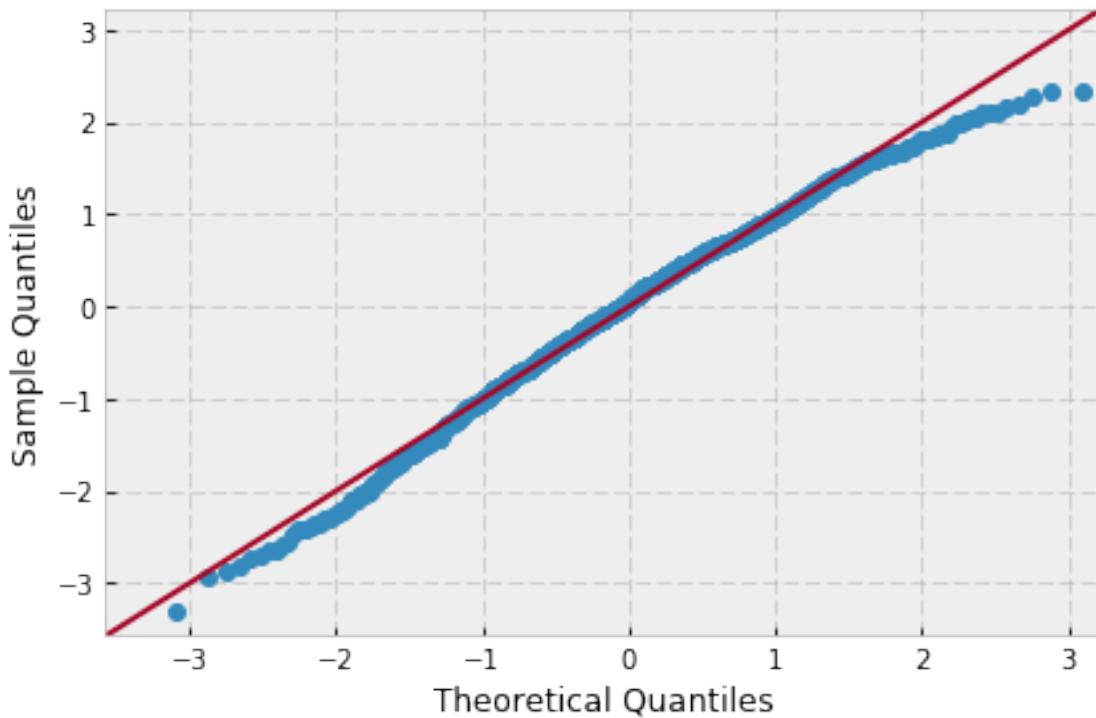


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(),loc=3.280088539993492, scale=0.37463268960295887)
DescribeResult(nobs=1000, minmax=(2.0073386067623087, 4.4780108463865584)
               mean=3.2800885399934918, variance=0.14049014226140832,
               skewness=-0.17680265957694985, kurtosis=-0.07346648297369507)
```

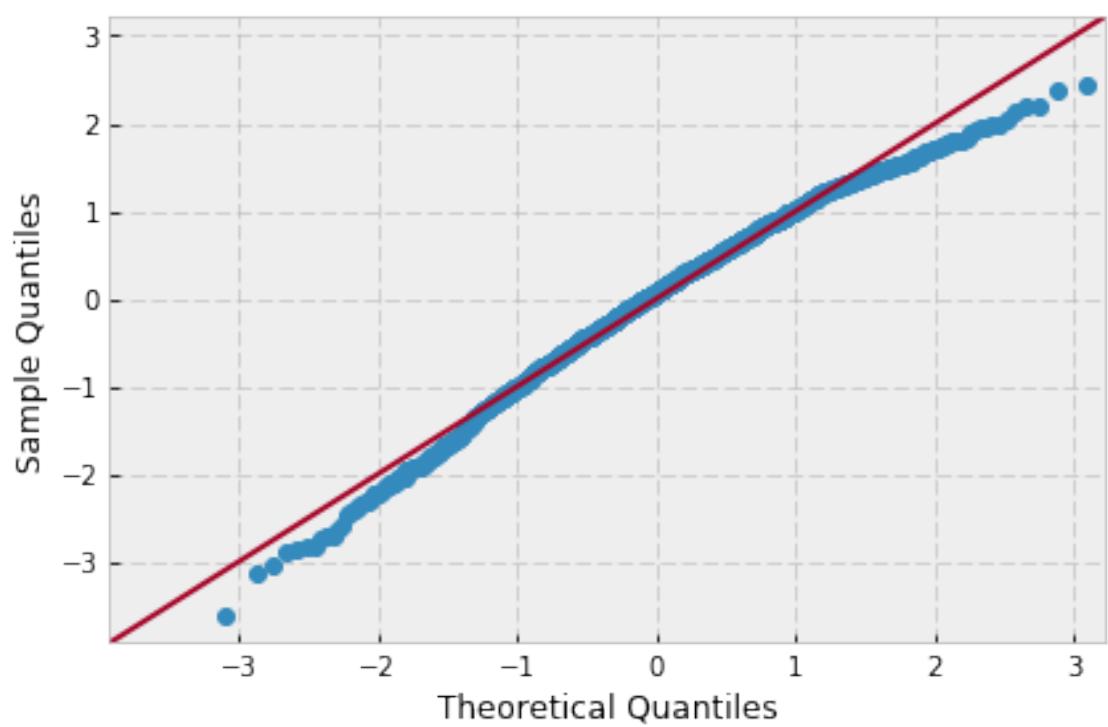
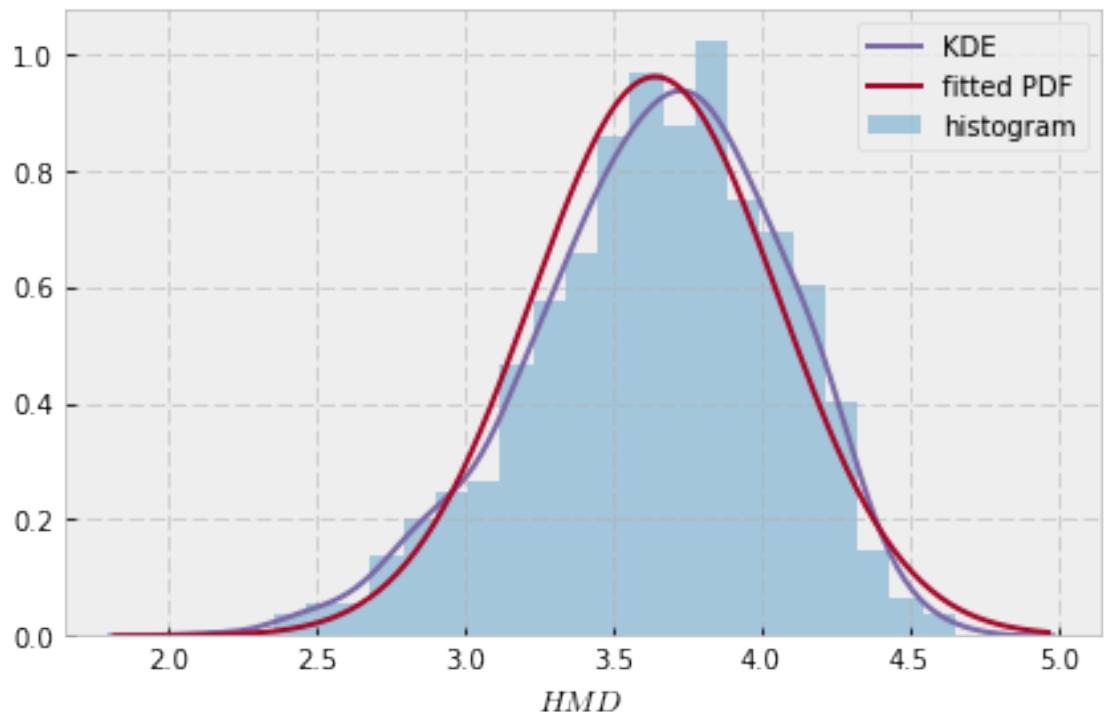


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=3.452087482155345, scale=0.38926647220755545)
DescribeResult(nobs=1000, minmax=(2.1716922527616593, 4.3618696996994295)
               mean=3.4520874821553451, variance=0.15168006645136692,
               skewness=-0.33519547550287565, kurtosis=-0.13162948640336714)
```

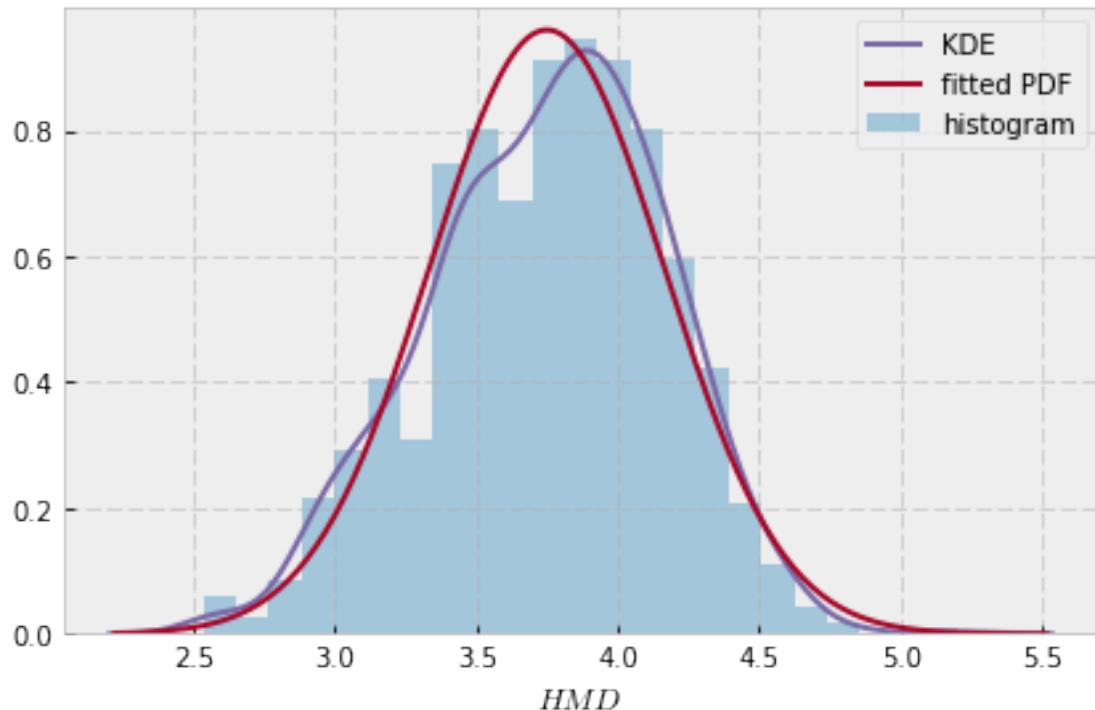


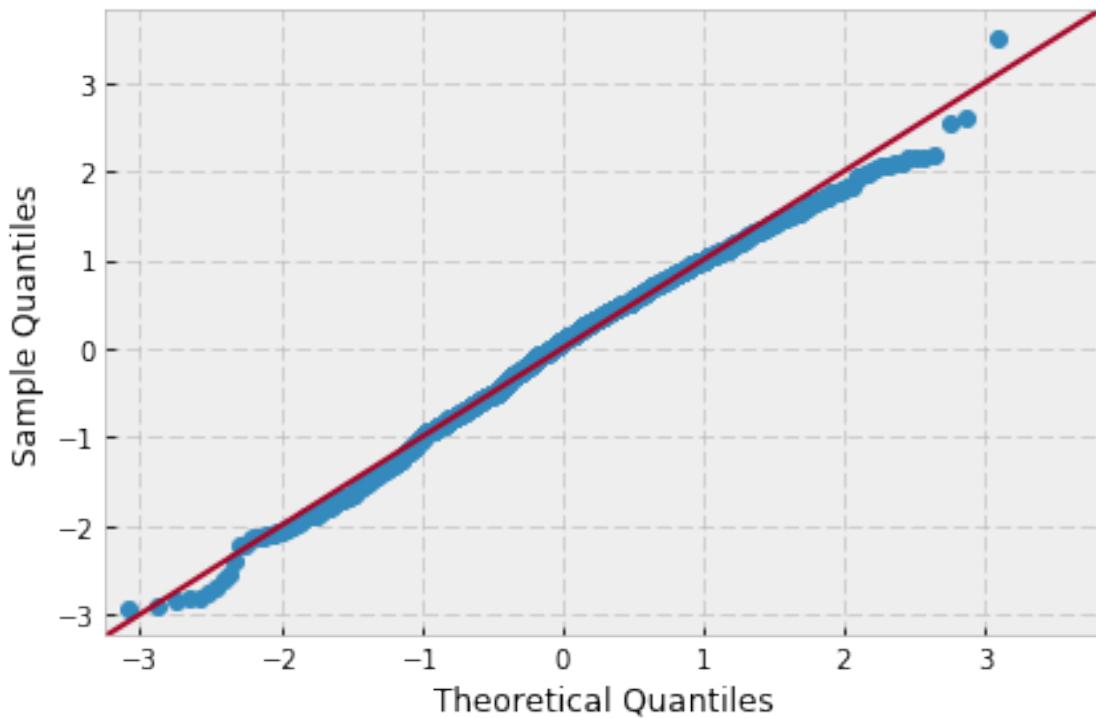


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(),loc=3.6421674323810365, scale=0.41523899627294747)
DescribeResult(nobs=1000, minmax=(2.137274689314387, 4.6522864770753989)
               mean=3.6421674323810365, variance=0.17259602004581071,
               skewness=-0.42796064237735915, kurtosis=-0.01953584481199311)
```

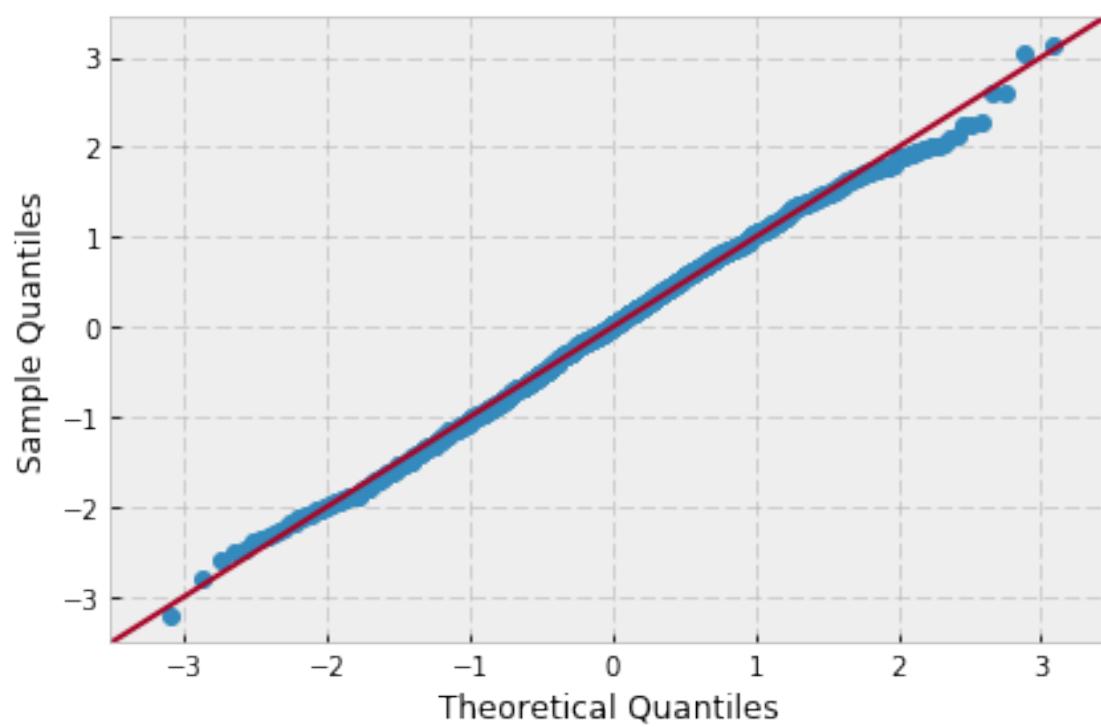
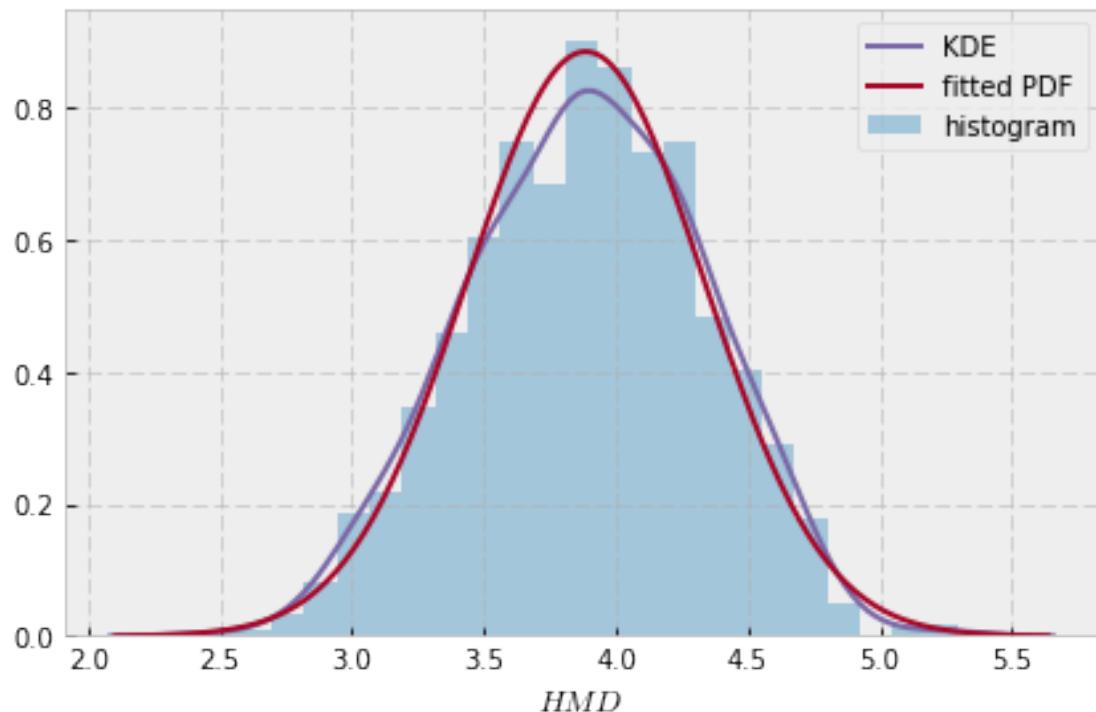


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=3.7485146288967672, scale=0.4154864141806172)
DescribeResult(nobs=1000, minmax=(2.5335778674864606, 5.2034023595666357)
               mean=3.7485146288967672, variance=0.17280176213079818,
               skewness=-0.2344497303059004, kurtosis=-0.20835746729530458)
```

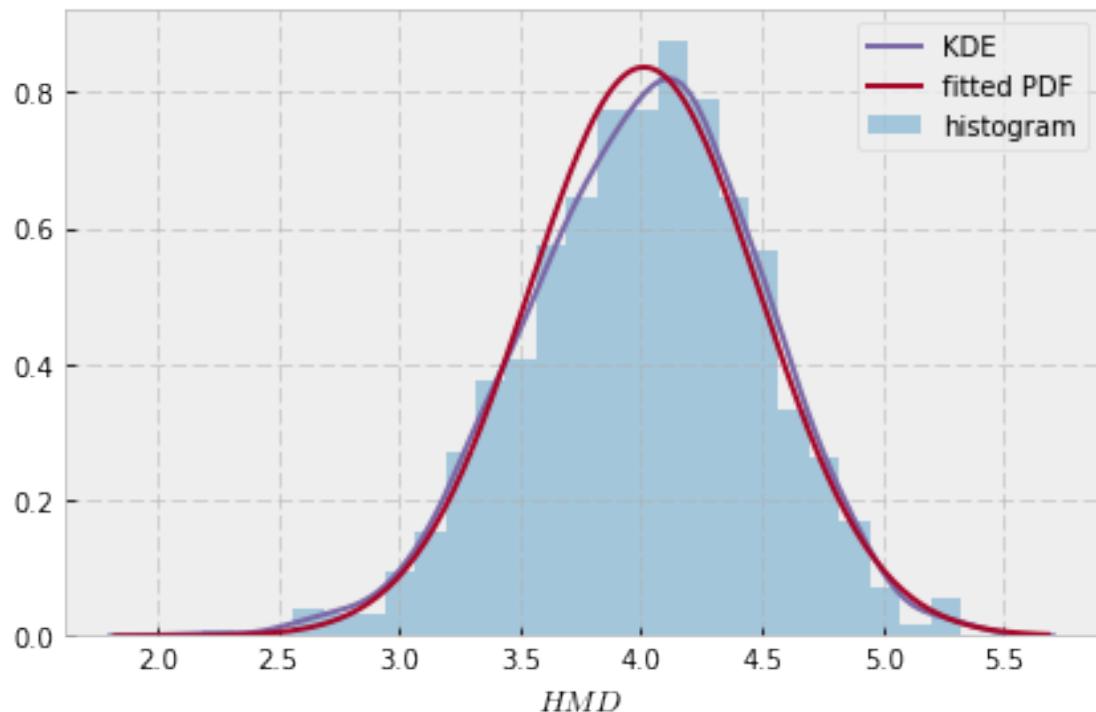


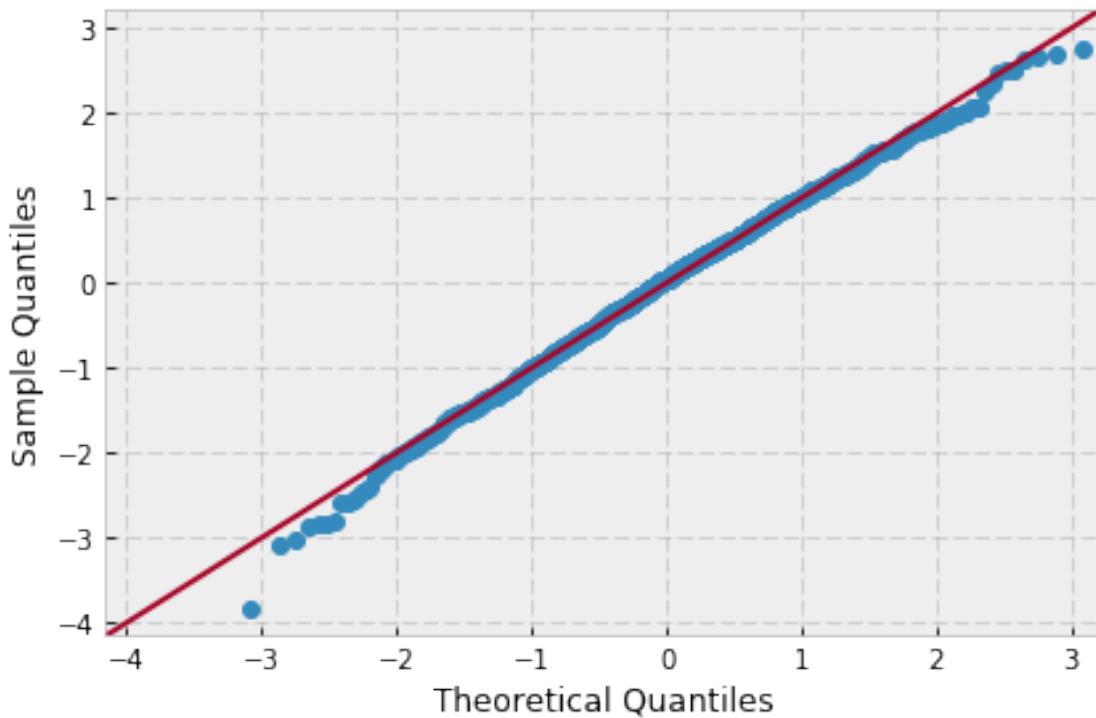


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(),loc=3.8857577084777595, scale=0.45024087697055054)
DescribeResult(nobs=1000, minmax=(2.4459228110707829, 5.2957593969412775)
               mean=3.8857577084777595, variance=0.20291976706227269,
               skewness=-0.08280239093025082, kurtosis=-0.32469015916931365)
```

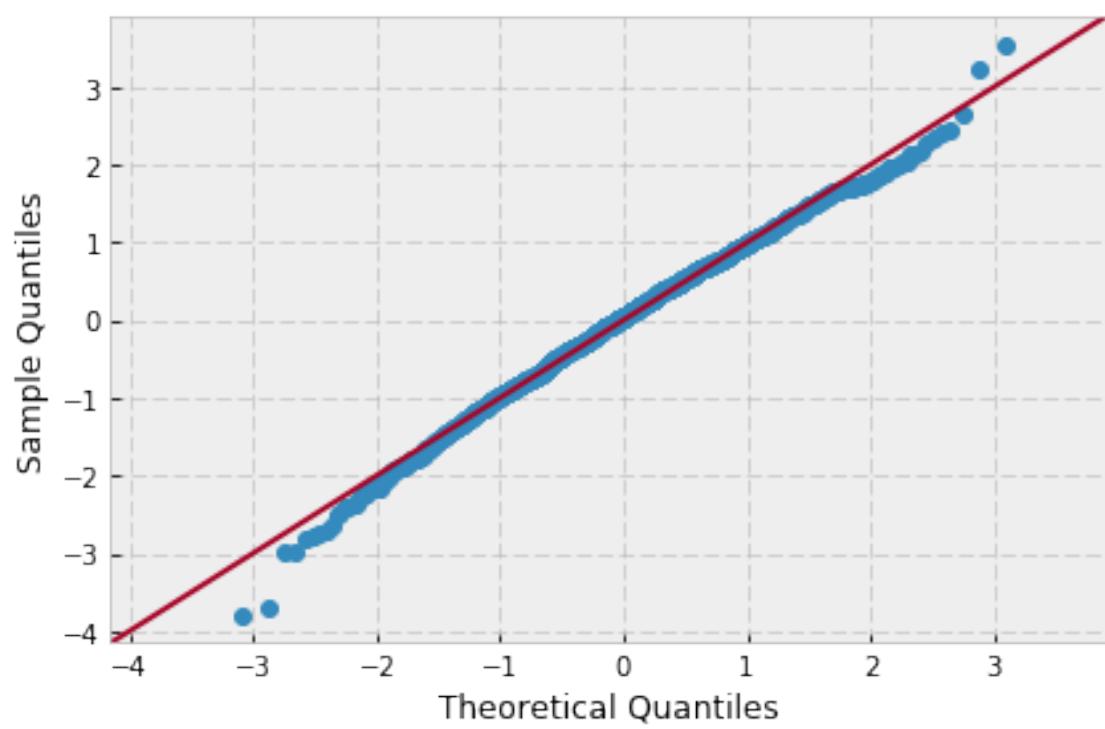
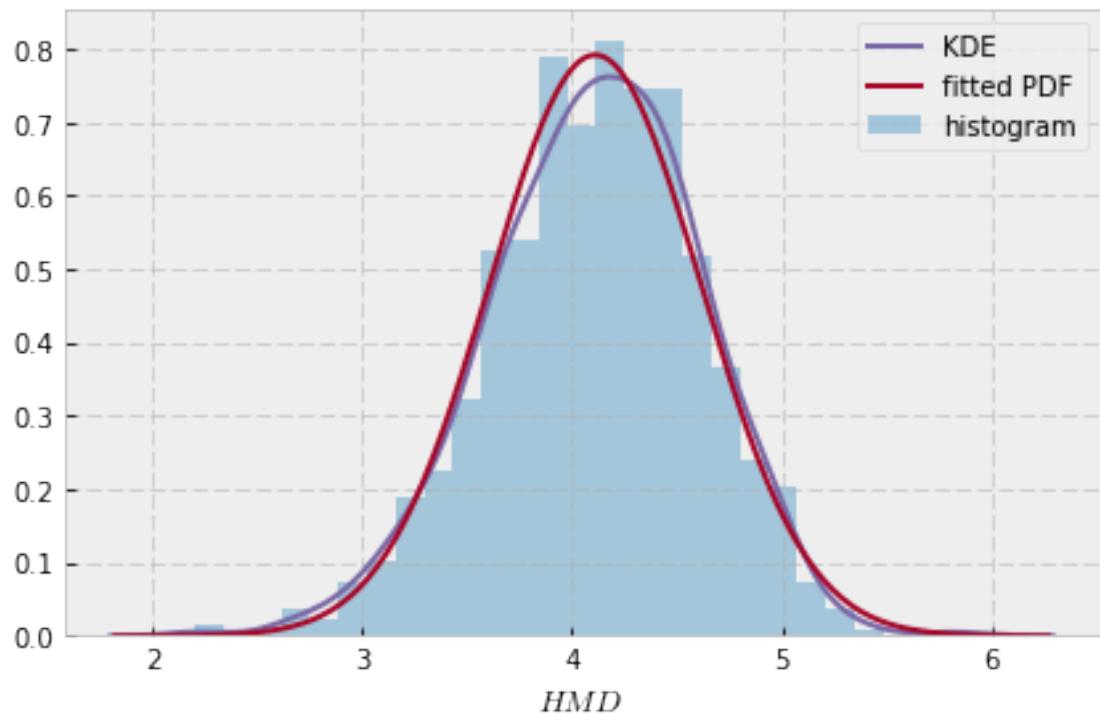


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=4.012346087580092, scale=0.47648608058034325)
DescribeResult(nobs=1000, minmax=(2.1881004623974087, 5.321943500815876)
               mean=4.0123460875800916, variance=0.22726625123805541,
               skewness=-0.2085430718042726, kurtosis=0.030902954893949897)
```

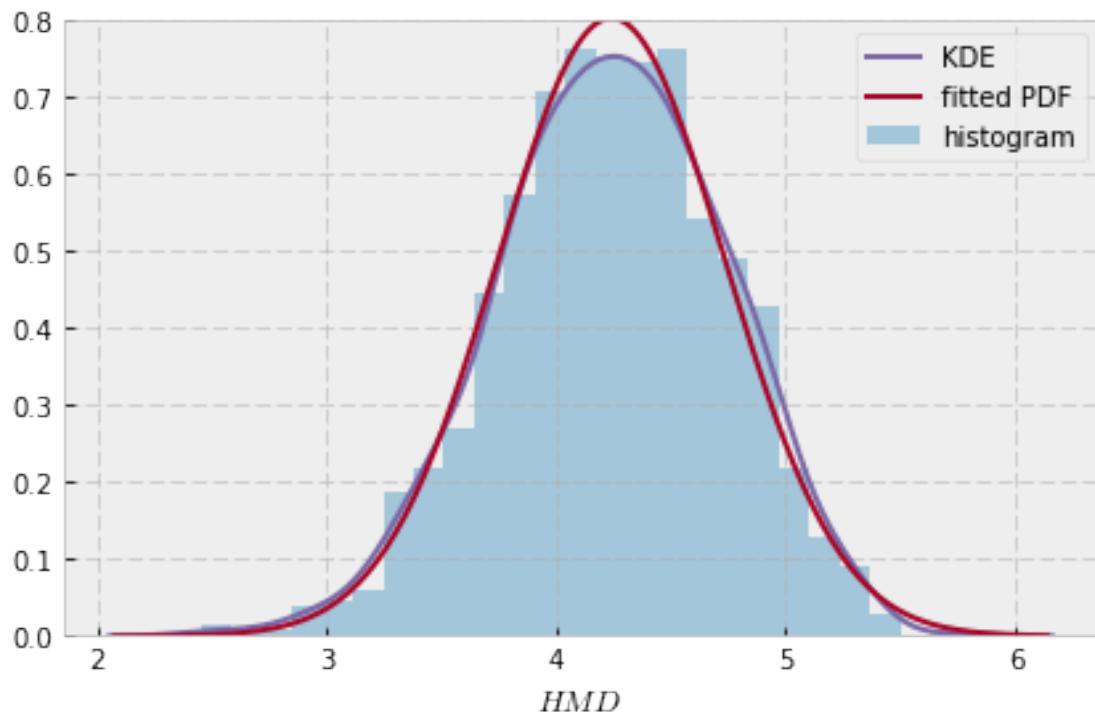


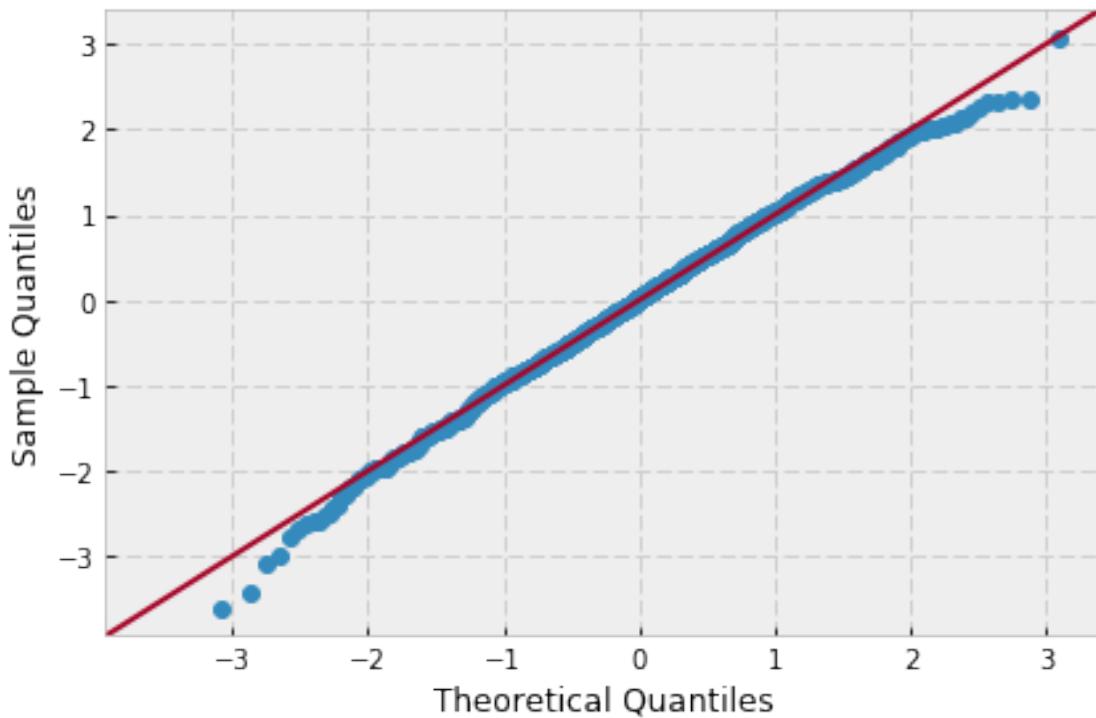


```
variable name: r
variable value: 2.7777777777777777
distribution: normal(shape=(), loc=4.109188646542356, scale=0.5038320851775705)
DescribeResult(nobs=1000, minmax=(2.2008749740828923, 5.8899077581938064)
               mean=4.1091886465423562, variance=0.25410087092530392,
               skewness=-0.23759759837713704, kurtosis=0.24094537843529196)
```

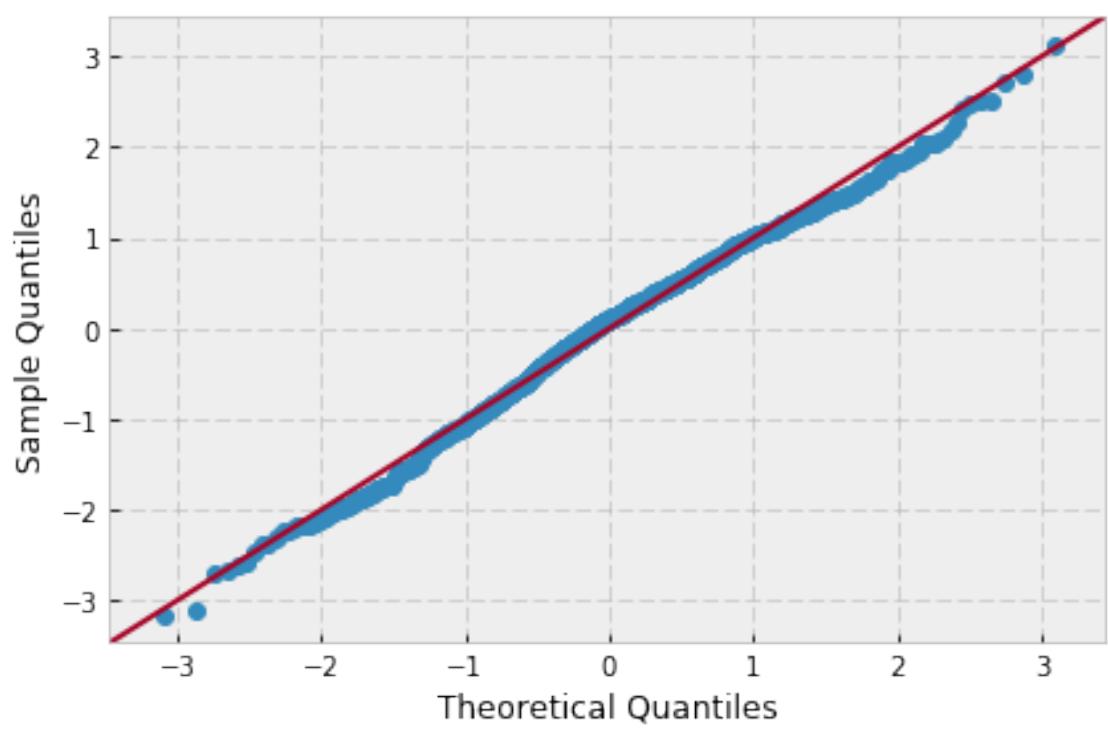
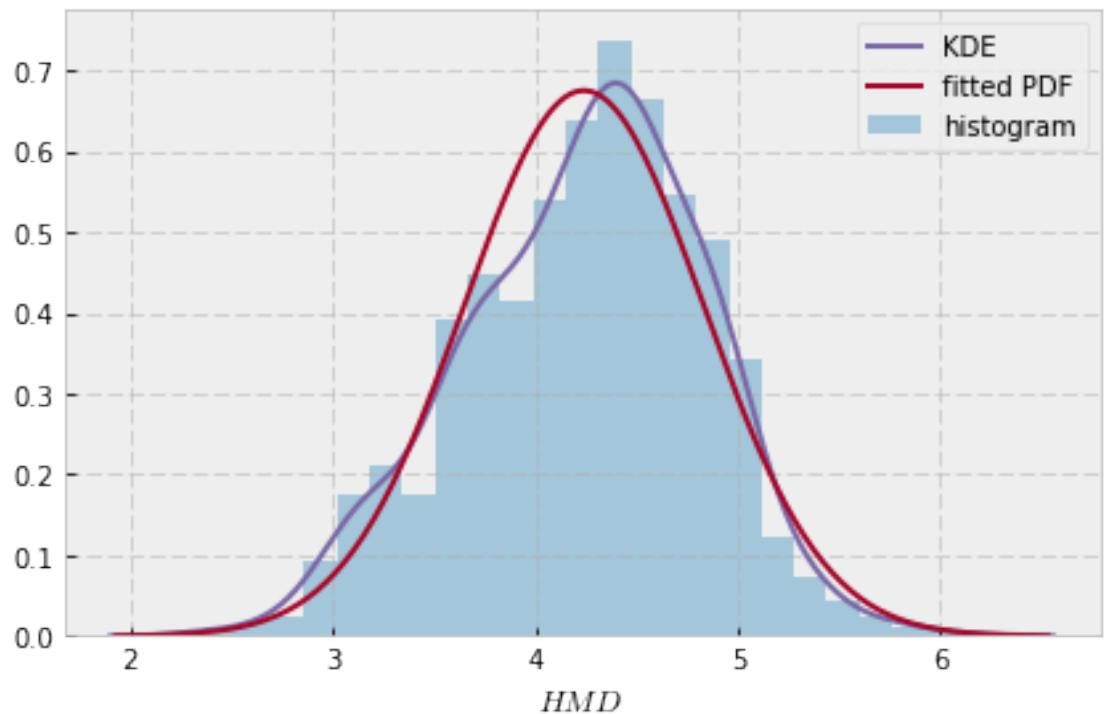


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=4.2398242290347845, scale=0.49662464514969884)
DescribeResult(nobs=1000, minmax=(2.4506260165416367, 5.762799151322386)
               mean=4.2398242290347845, variance=0.24688292109115545,
               skewness=-0.19553167120241333, kurtosis=-0.04127731426119485)
```

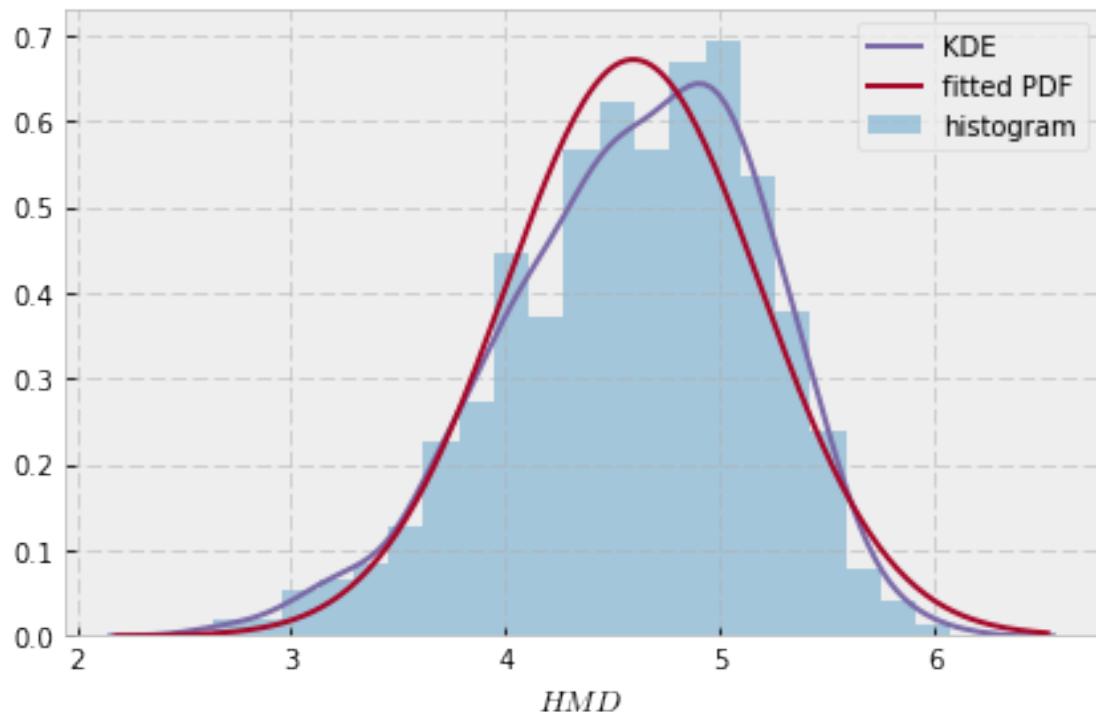


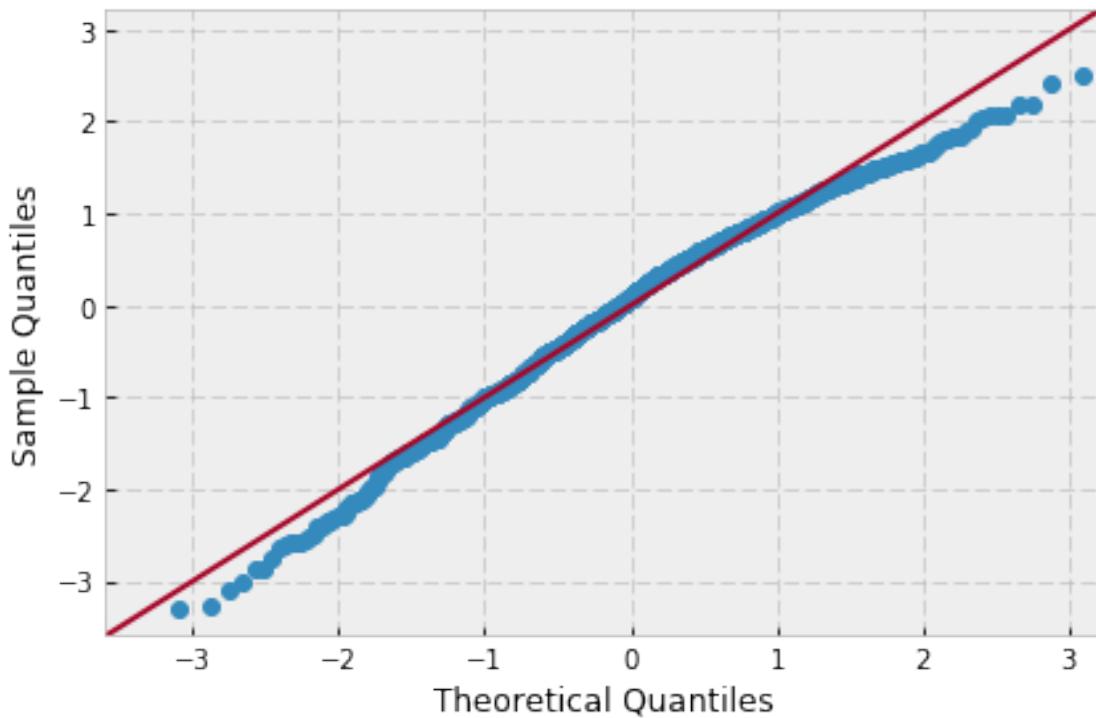


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(),loc=4.239426231776814, scale=0.590776327300123)
DescribeResult(nobs=1000, minmax=(2.3751145762946115, 6.0833087600229439)
               mean=4.2394262317768137, variance=0.34936603493315521,
               skewness=-0.2492253973435187, kurtosis=-0.16871022382358092)
```

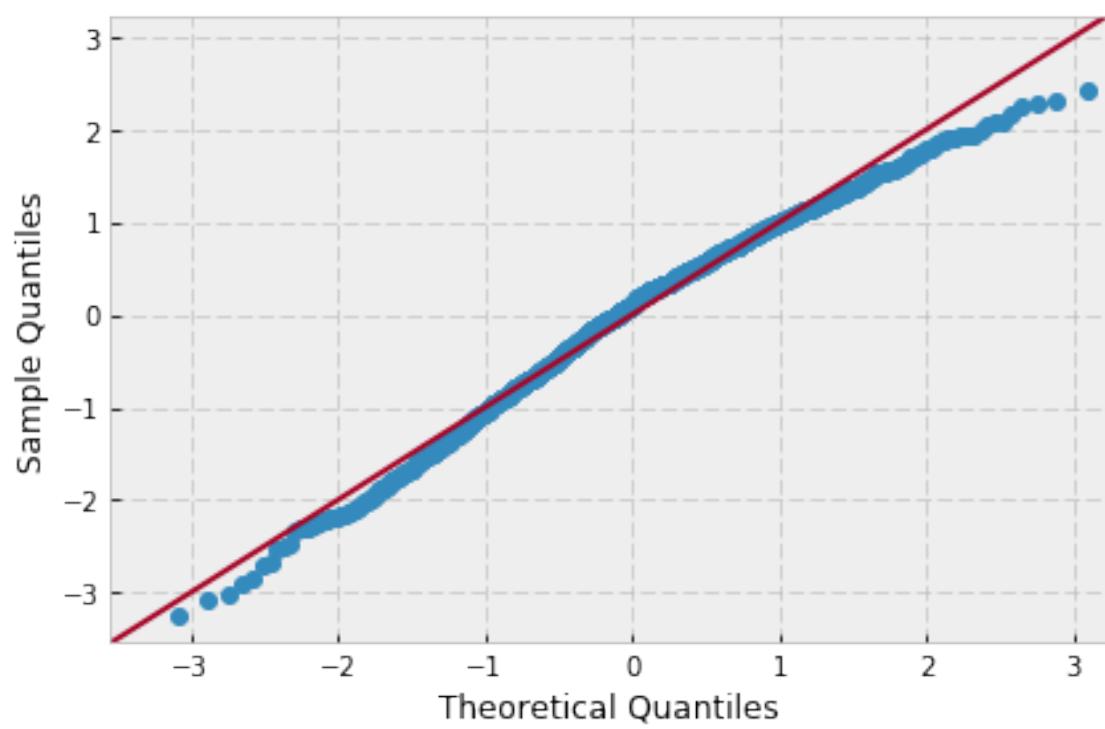
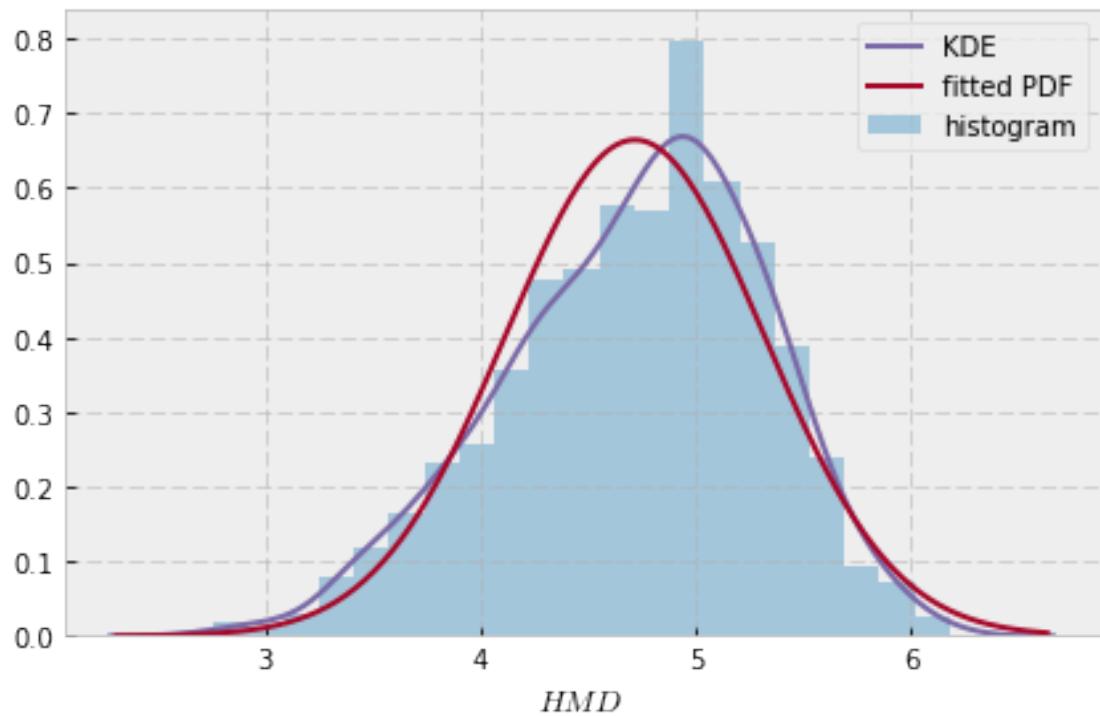


```
variable name: sigma
variable value: 0.0222222222222223
distribution: normal(shape=(), loc=4.5948598119520865, scale=0.5938231162763223)
DescribeResult(nobs=1000, minmax=(2.6320717130313231, 6.0775474573695742)
               mean=4.5948598119520865, variance=0.352978872296419,
               skewness=-0.44264372935309443, kurtosis=-0.07997964979085204)
```

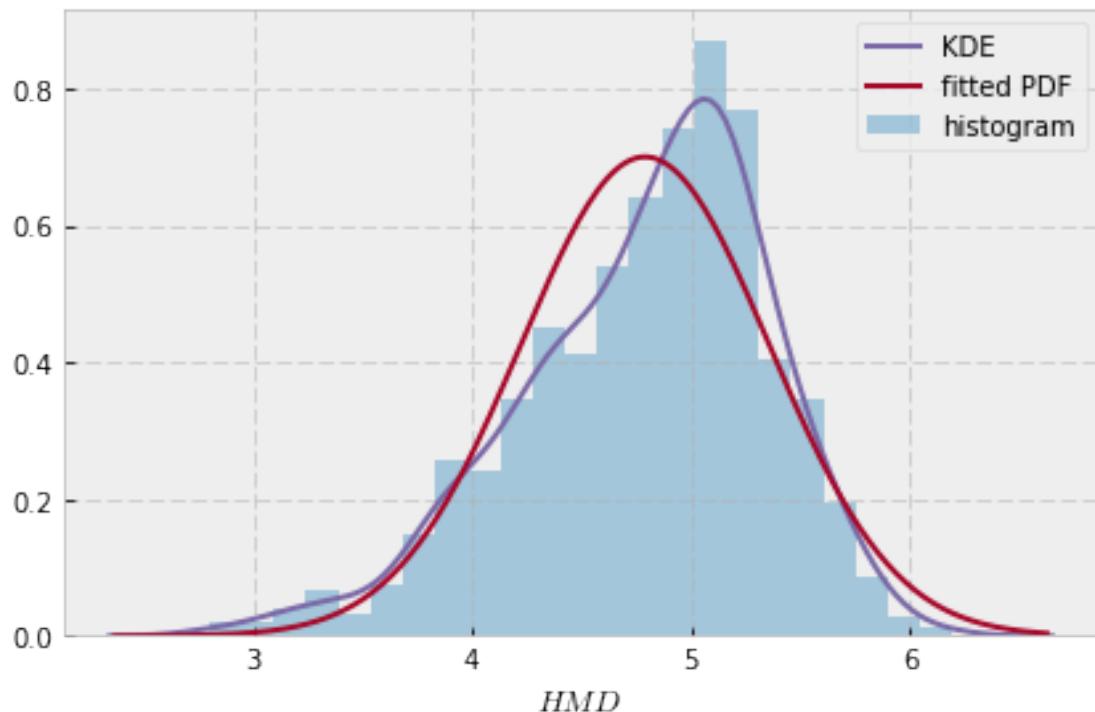


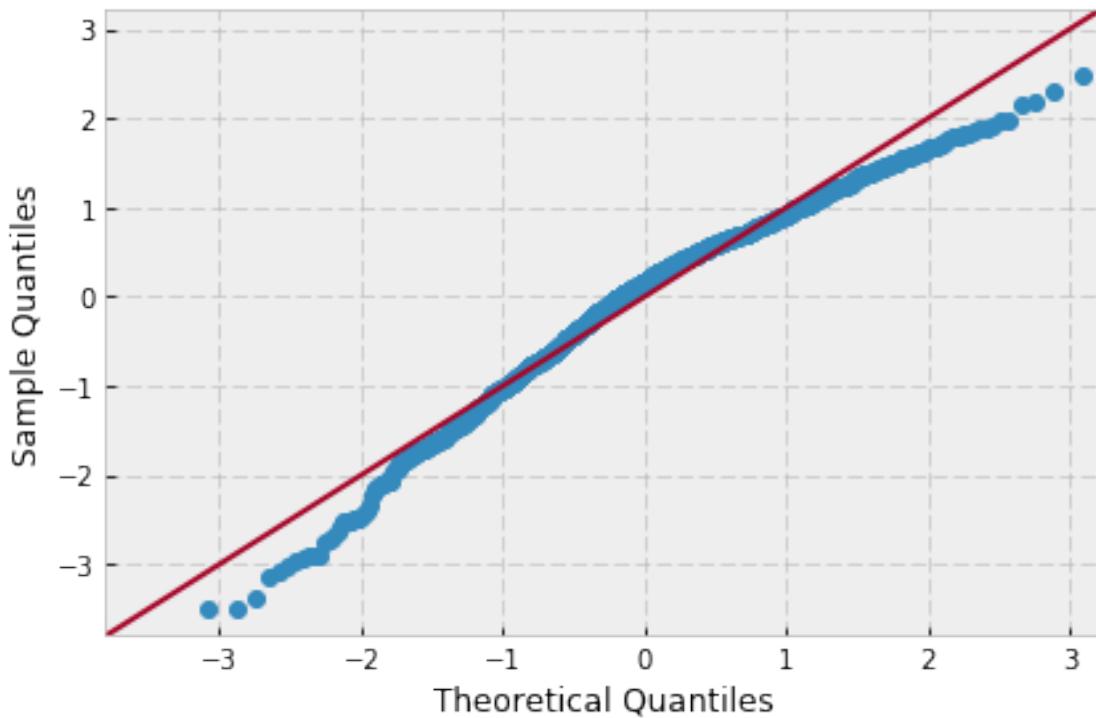


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=4.716092854372333, scale=0.6005996075563255)
DescribeResult(nobs=1000, minmax=(2.7591522058103704, 6.1793428394592542)
               mean=4.7160928543723326, variance=0.36108096956637858,
               skewness=-0.390338658846545, kurtosis=-0.19869738852230068)
```

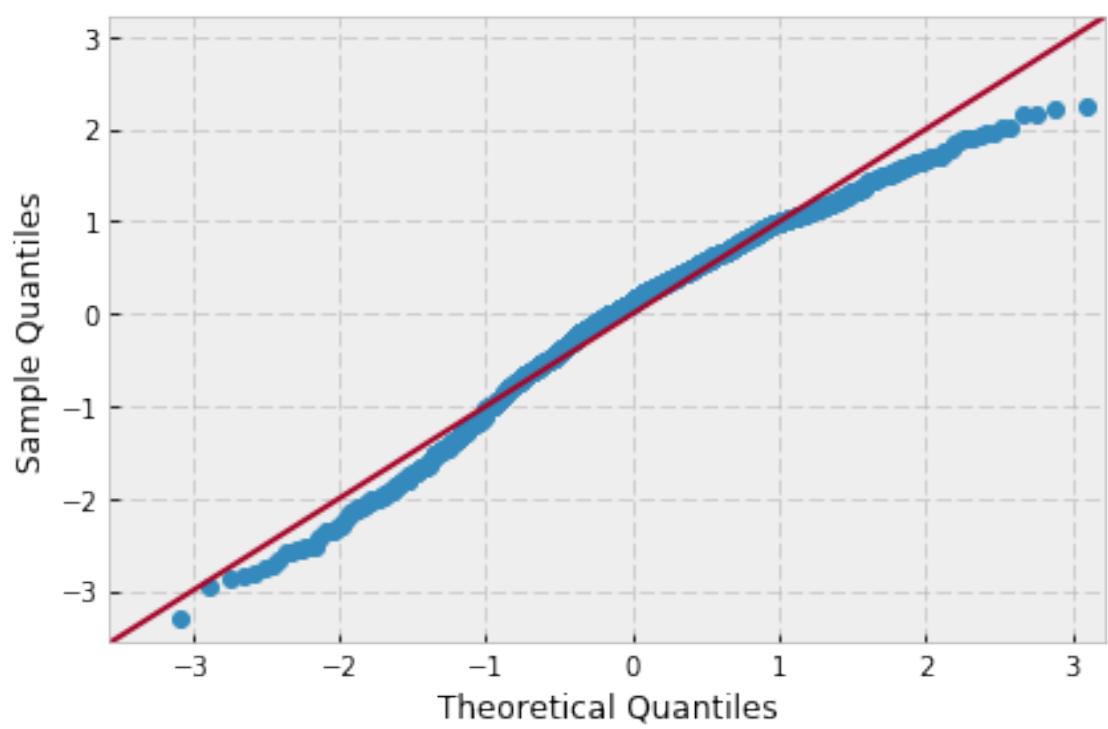
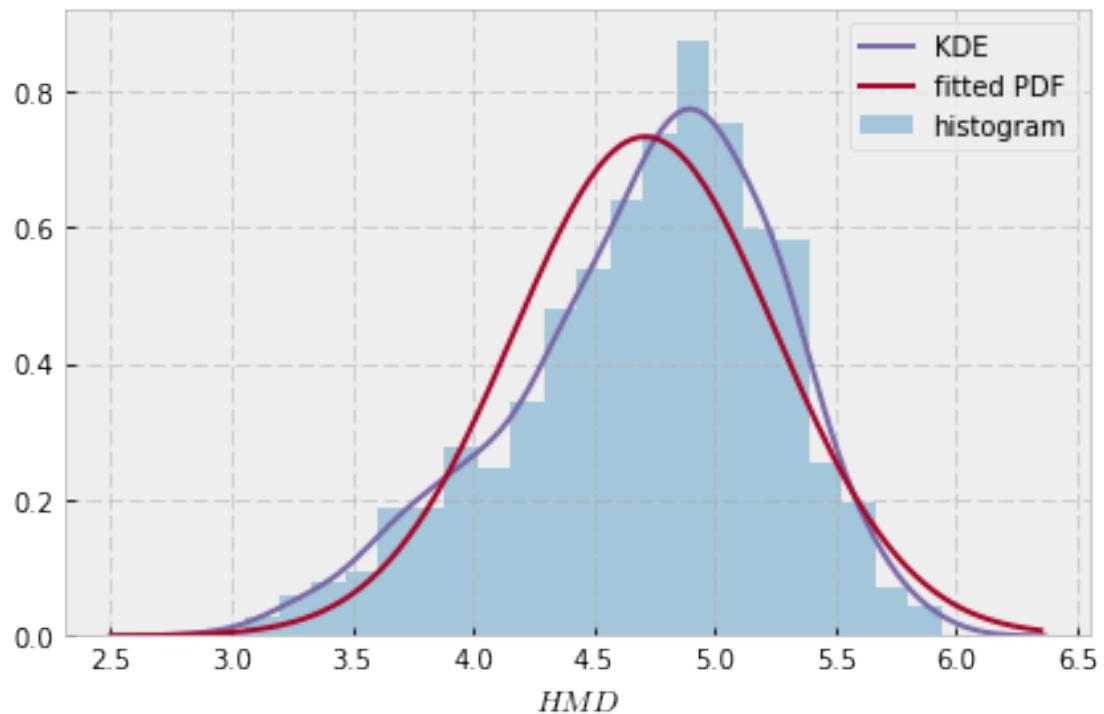


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=4.791409702892852, scale=0.5695929811866995)
DescribeResult(nobs=1000, minmax=(2.7951862439005368, 6.1996835901027882)
               mean=4.7914097028928522, variance=0.32476092514229415,
               skewness=-0.6361424826478432, kurtosis=0.29495504148541407)
```

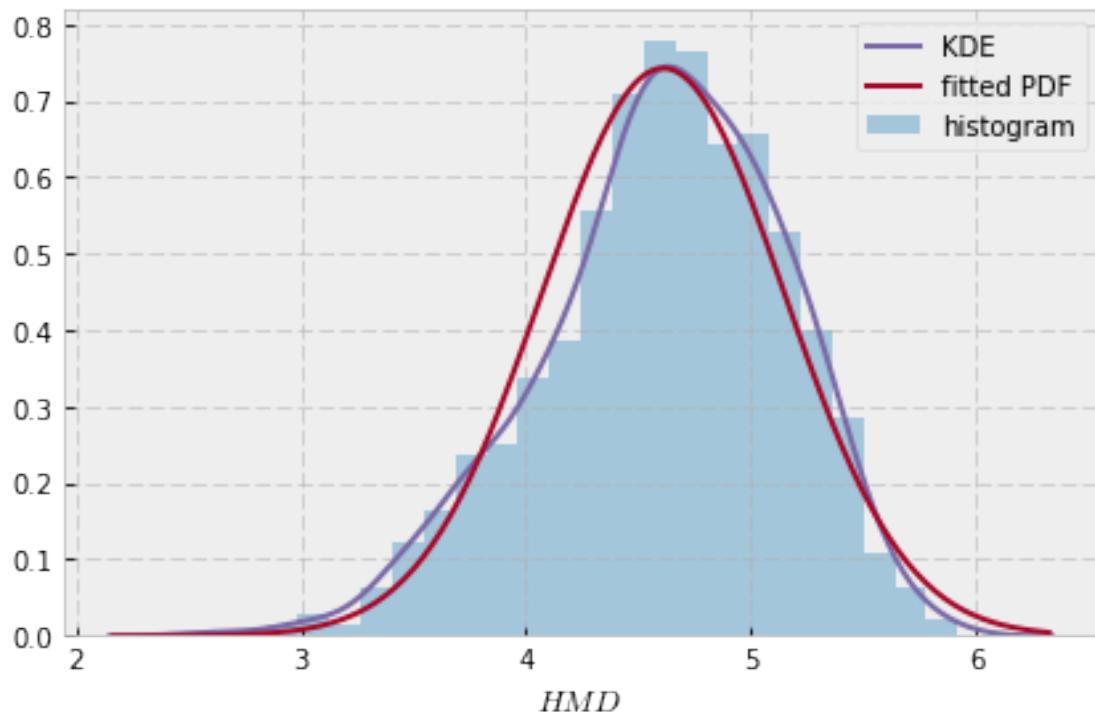


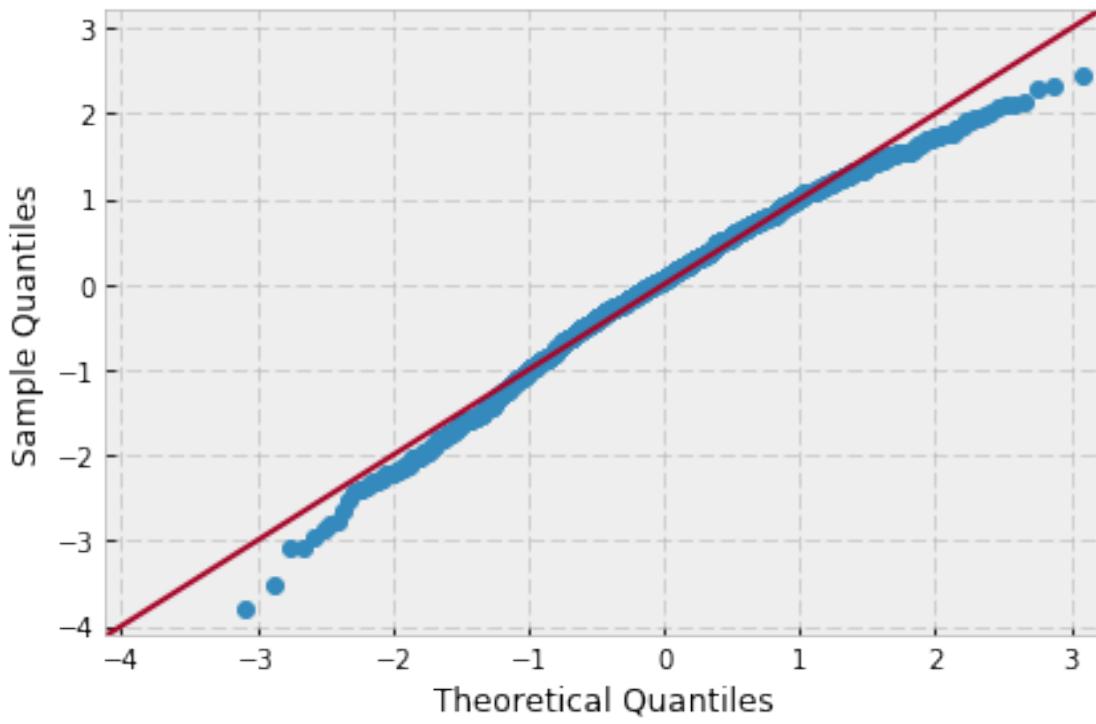


```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(),loc=4.712342392964504, scale=0.5445385205529809)
DescribeResult(nobs=1000, minmax=(2.9226477421153909, 5.9396211734039666)
               mean=4.7123423929645041, variance=0.29681901938541461,
               skewness=-0.5365663388743355, kurtosis=-0.07489319456747046)
```

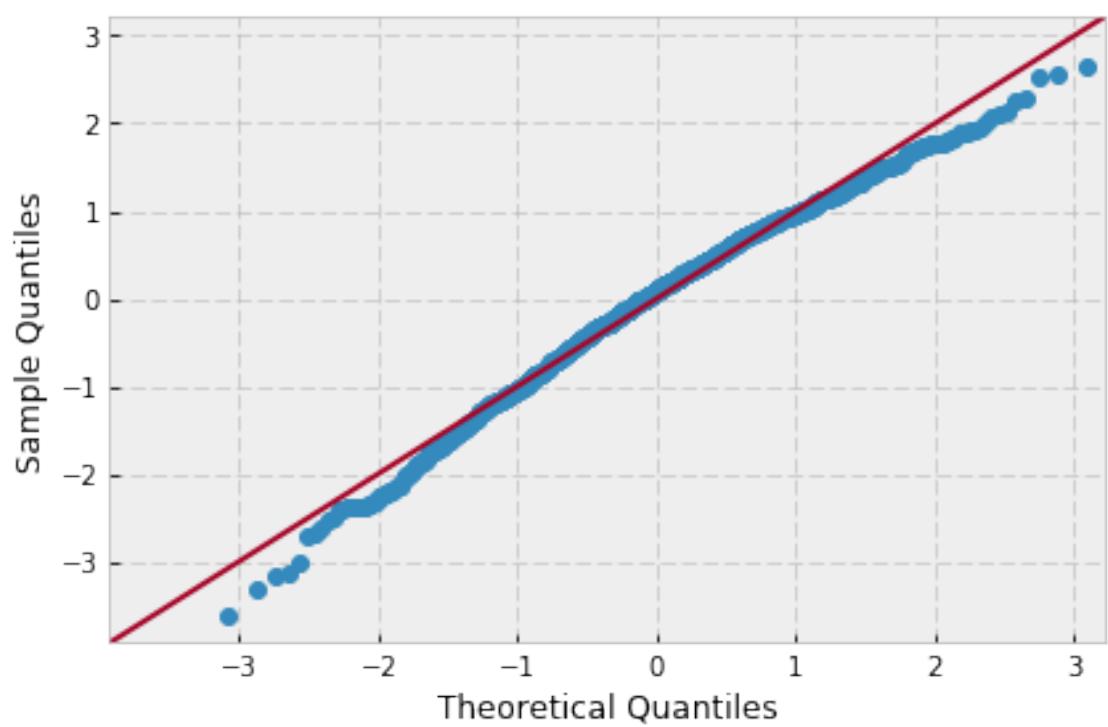
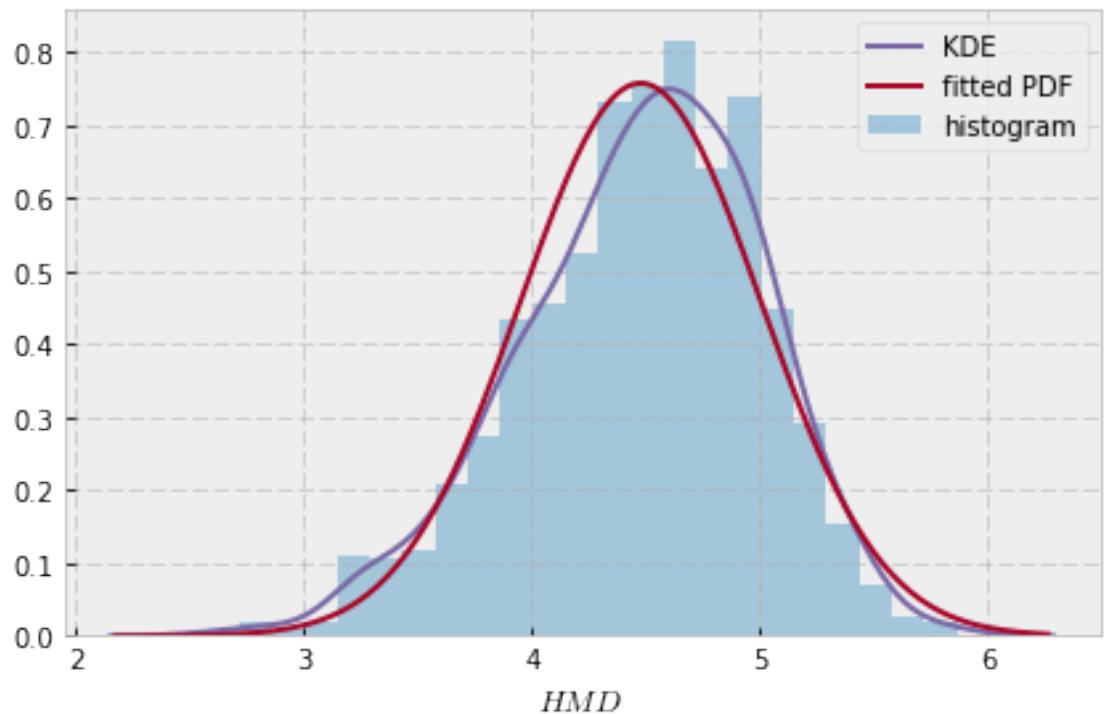


```
variable name: sigma
variable value: 0.06666666666666667
distribution: normal(shape=(), loc=4.609935449419434, scale=0.5370620462156217)
DescribeResult(nobs=1000, minmax=(2.5665889192641593, 5.9207641910148077)
               mean=4.6099354494194342, variance=0.28872436585116179,
               skewness=-0.4501241266825589, kurtosis=0.06663868316031163)
```

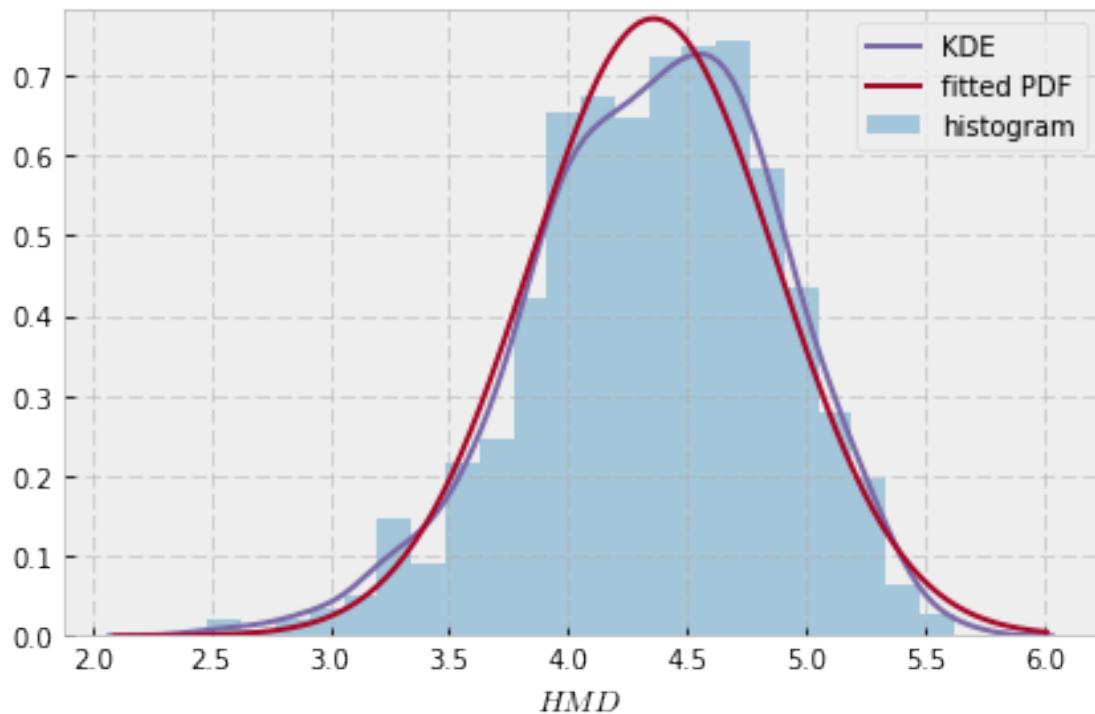


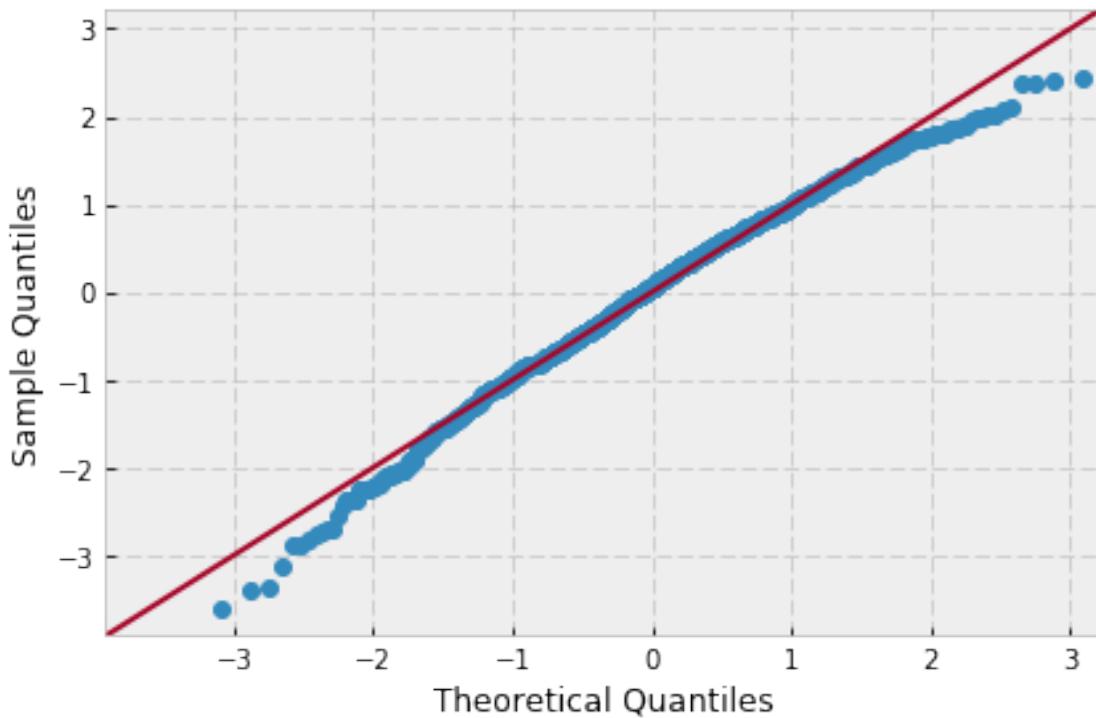


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=4.477988739789233, scale=0.5264685639570786)
DescribeResult(nobs=1000, minmax=(2.5731766207127356, 5.8657779988605743)
               mean=4.477988739789233, variance=0.27744659543045902,
               skewness=-0.41894511401851986, kurtosis=0.06290610308946398)
```

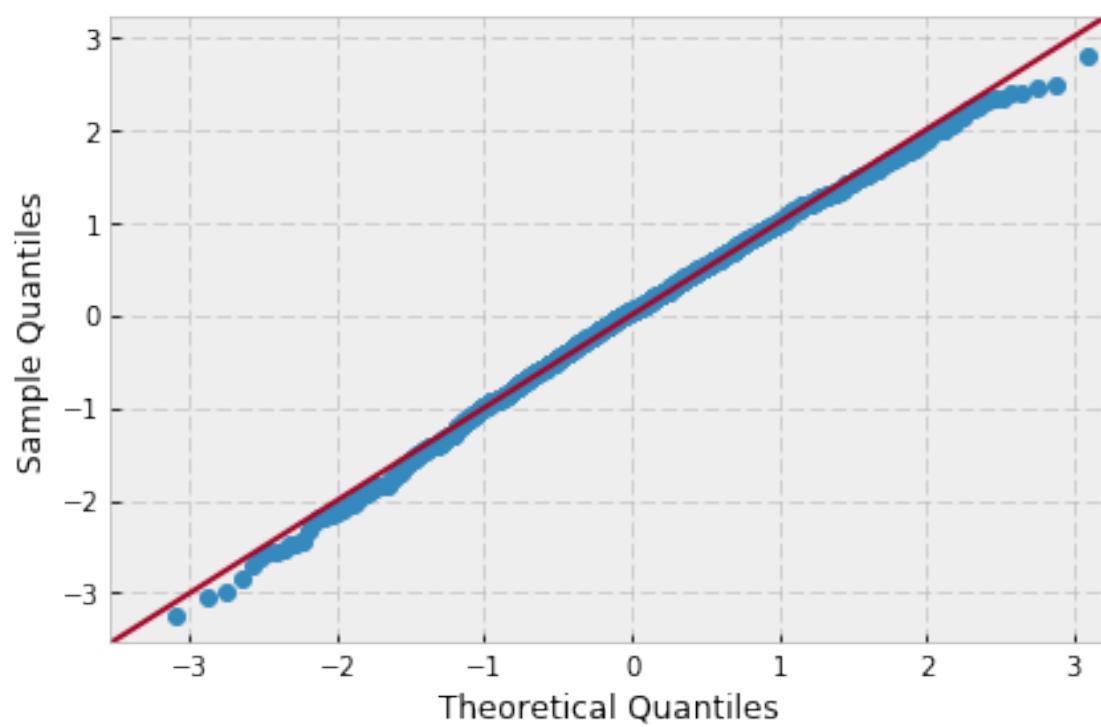
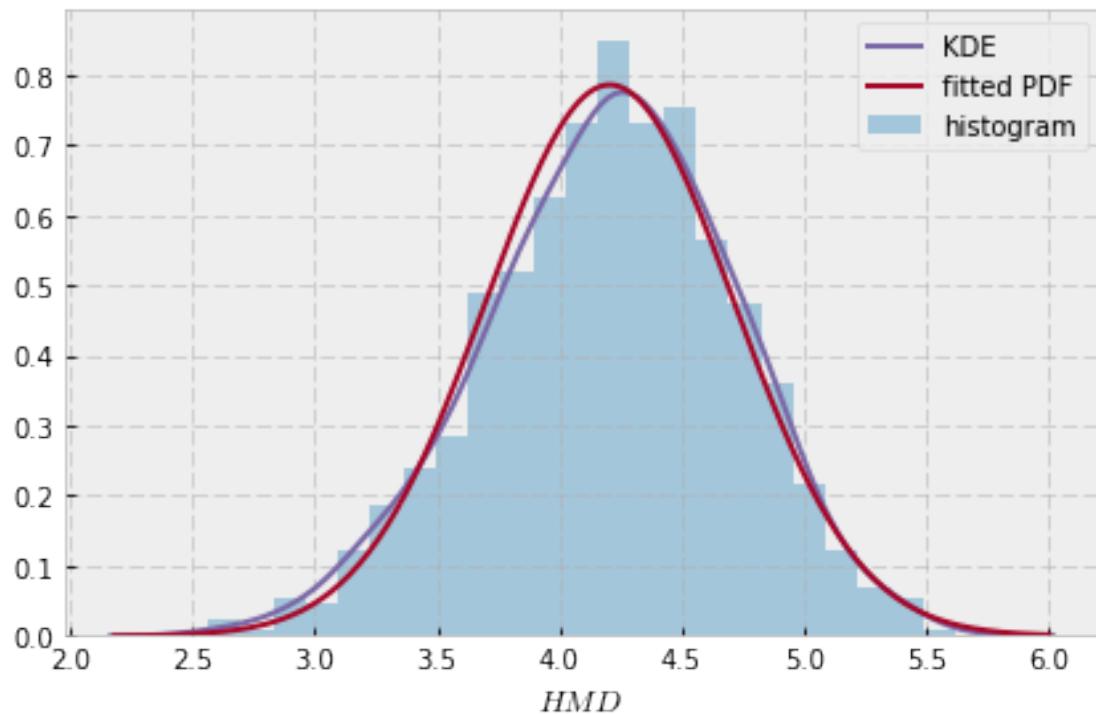


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(), loc=4.3588593523458465, scale=0.5174077372806418)
DescribeResult(nobs=1000, minmax=(2.4869611347593259, 5.6190333909485615)
               mean=4.3588593523458465, variance=0.26797874534321681,
               skewness=-0.37708863798008685, kurtosis=0.081263558369713)
```

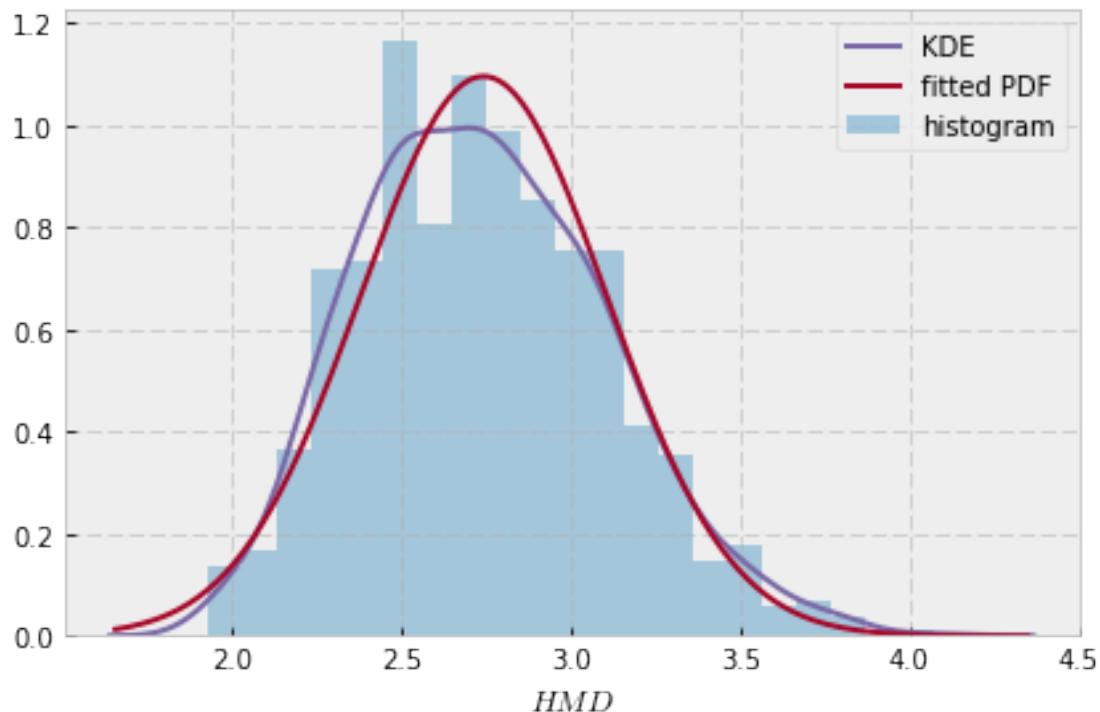


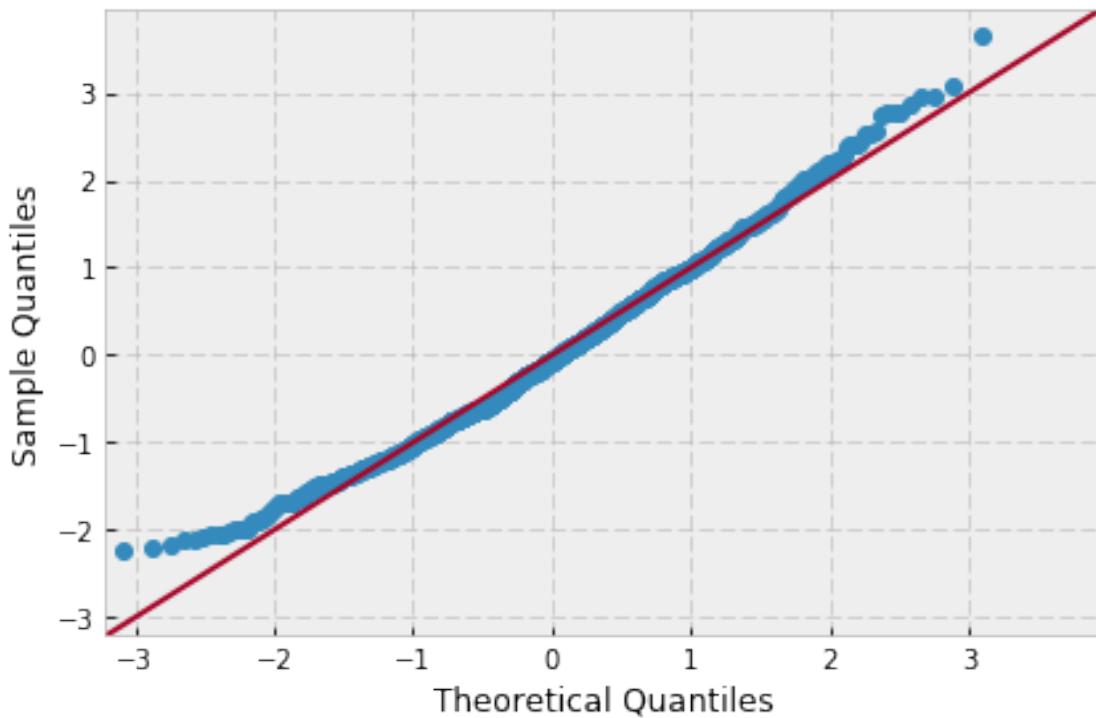


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(),loc=4.2048878319442995, scale=0.5072793435082414)
DescribeResult(nobs=1000, minmax=(2.5627221980014592, 5.6180009384264515)
               mean=4.2048878319442995, variance=0.25758992227242483,
               skewness=-0.2109751142974772, kurtosis=-0.09535086246889568)
```

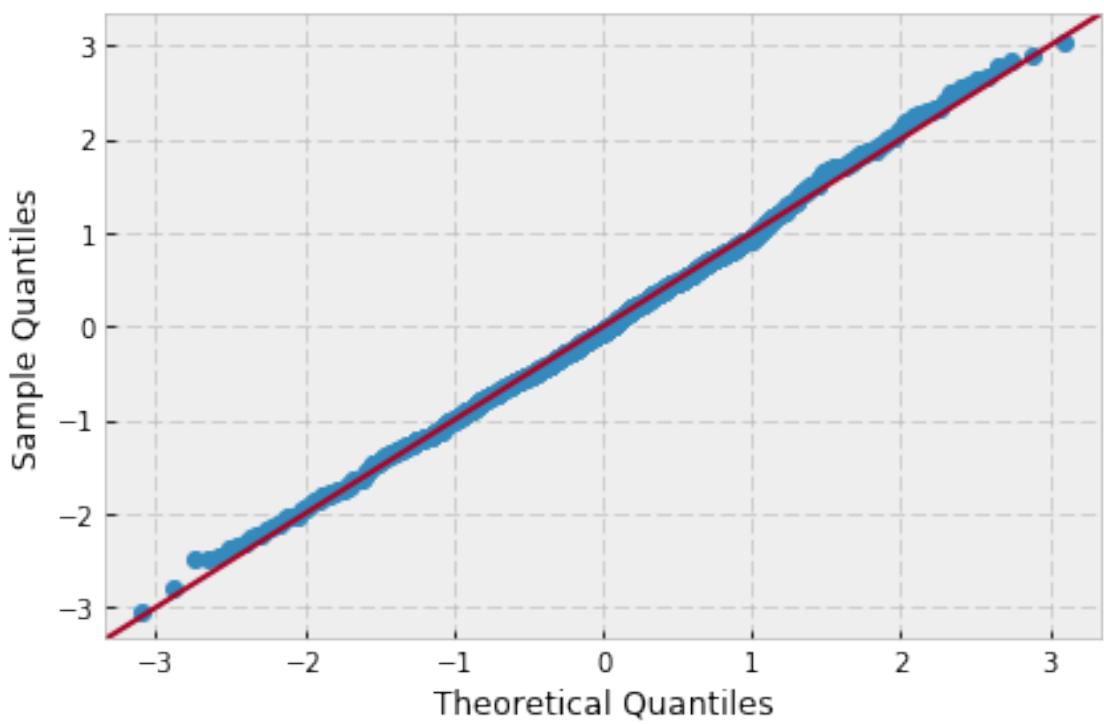
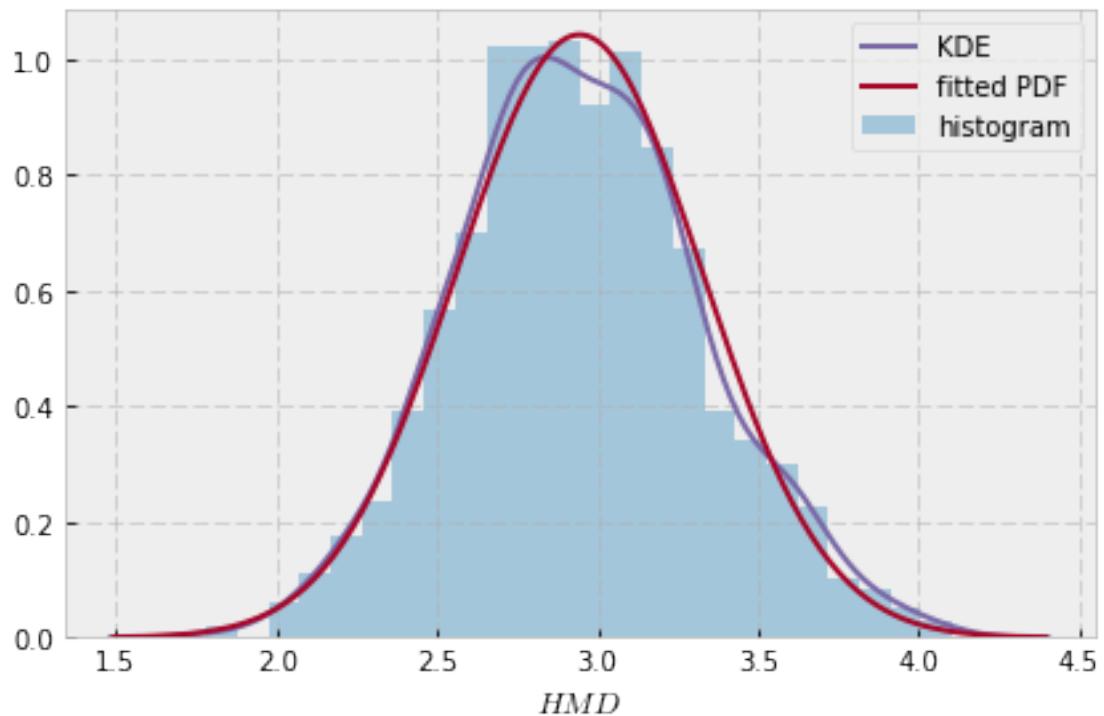


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=2.7440696649160334, scale=0.3641293630125752)
DescribeResult(nobs=1000, minmax=(1.9330966407308812, 4.0741078669767923)
               mean=2.7440696649160334, variance=0.13272291592386765,
               skewness=0.361316500044737, kurtosis=-0.11841480960681272)
```

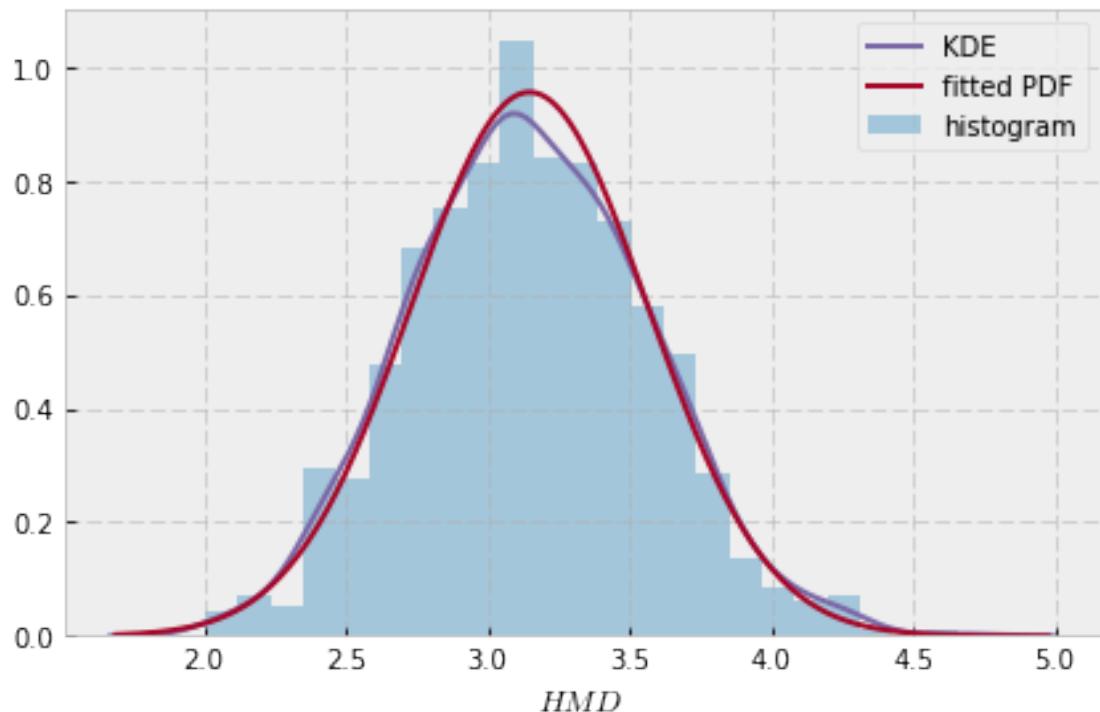


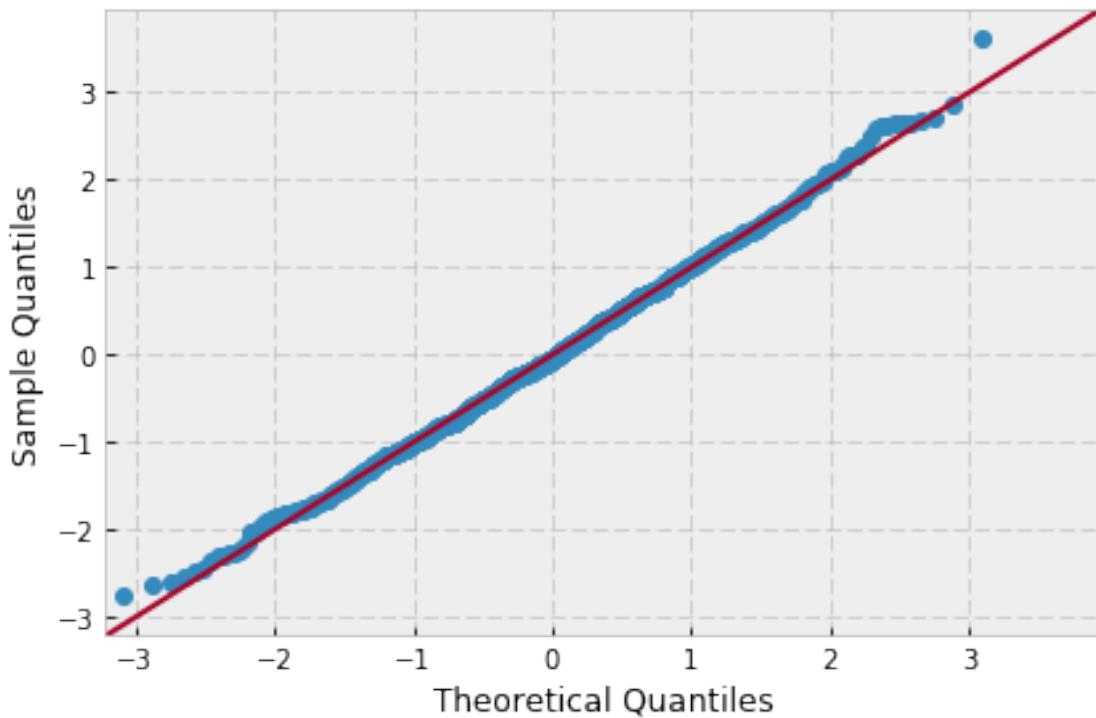


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(),loc=2.9432113849905672, scale=0.3829539819502022)
DescribeResult(nobs=1000, minmax=(1.7809391650024851, 4.1051344320372998)
               mean=2.9432113849905672, variance=0.14680055284436014,
               skewness=0.15810059679609995, kurtosis=-0.07336994544843378)
```

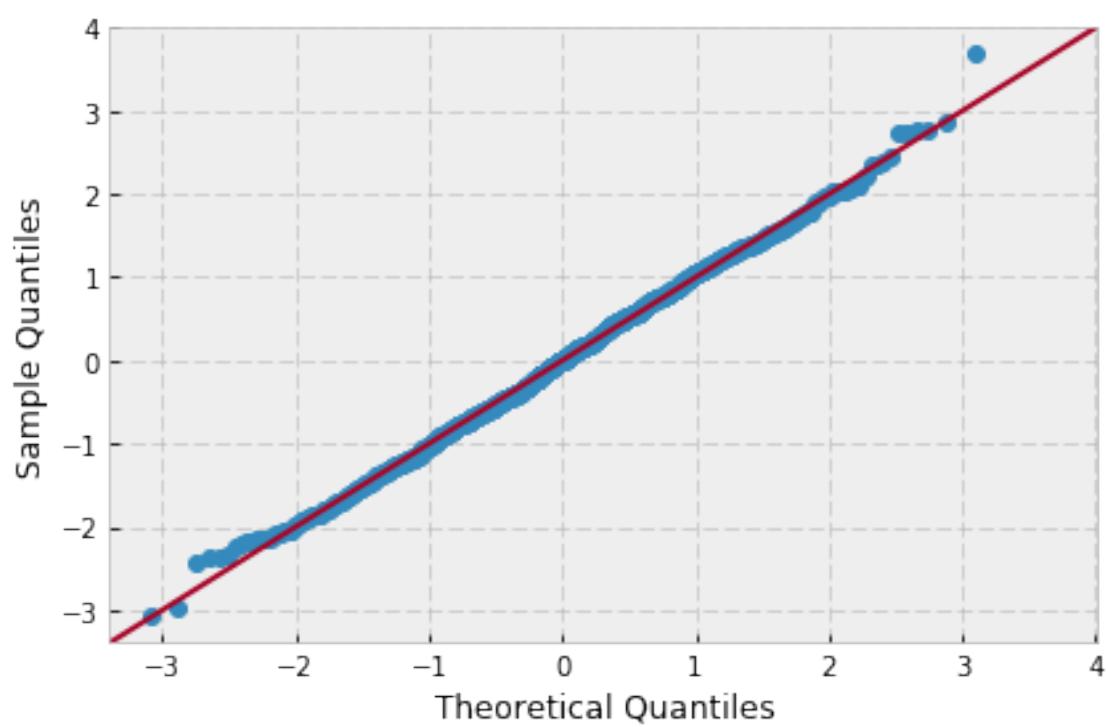
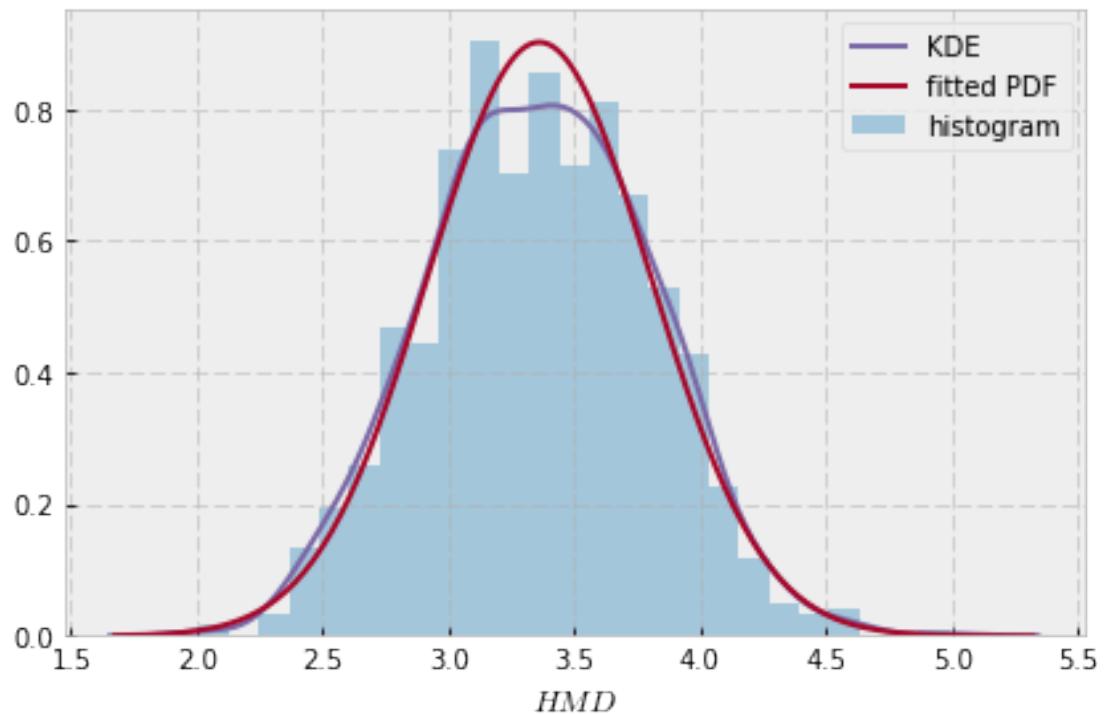


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=3.1481800254337435, scale=0.41731511178818054)
DescribeResult(nobs=1000, minmax=(2.0046476651392471, 4.6601380421992689)
               mean=3.1481800254337435, variance=0.17432622875553716,
               skewness=0.12501889845251435, kurtosis=-0.09970830901798289)
```

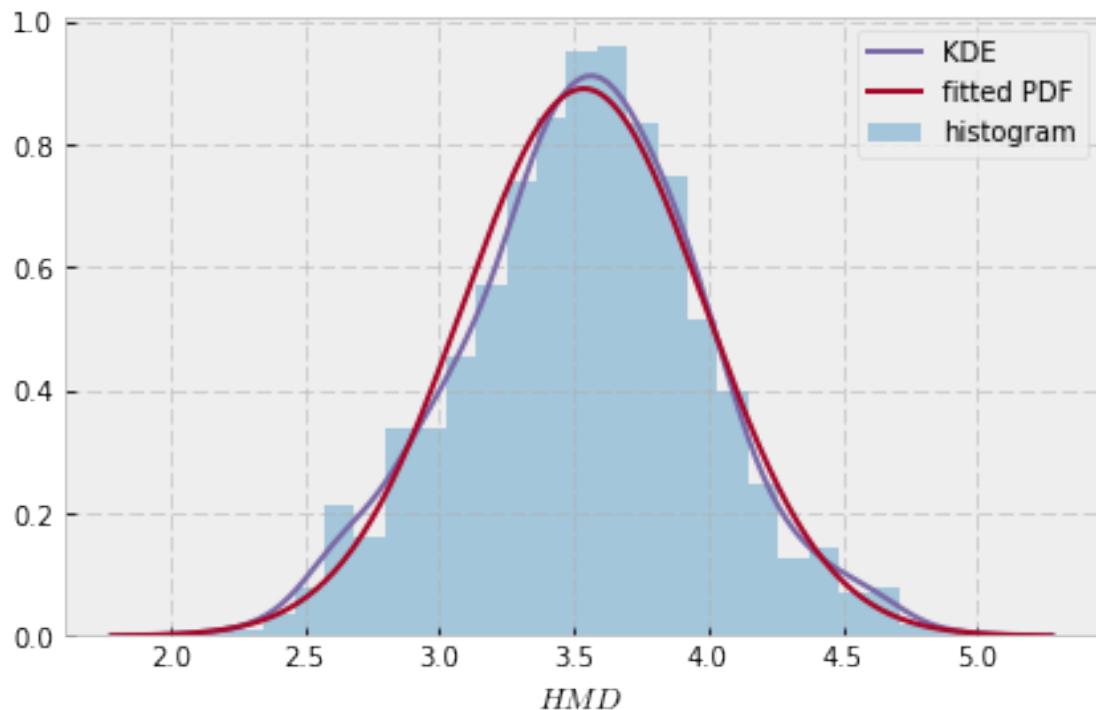


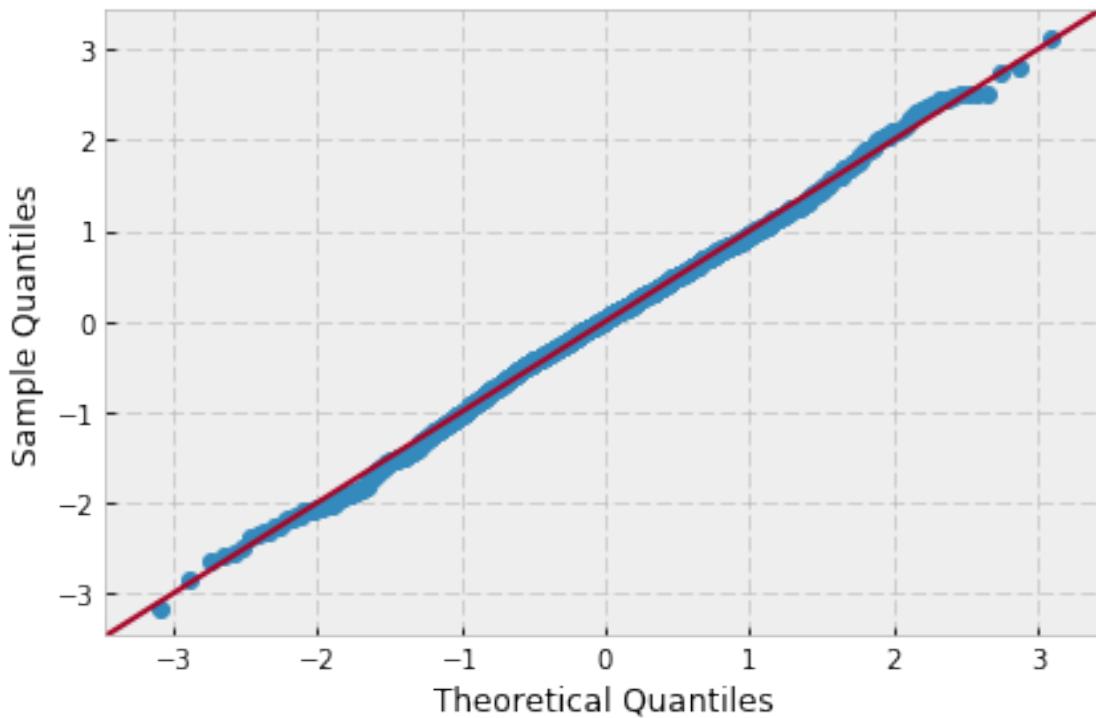


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(),loc=3.360037622630085, scale=0.4422142350906332)
DescribeResult(nobs=1000, minmax=(2.0075576911655038, 4.9887622602662143)
               mean=3.360037622630085, variance=0.19574917889568946,
               skewness=0.03730019767240711, kurtosis=-0.18778347218542235)
```

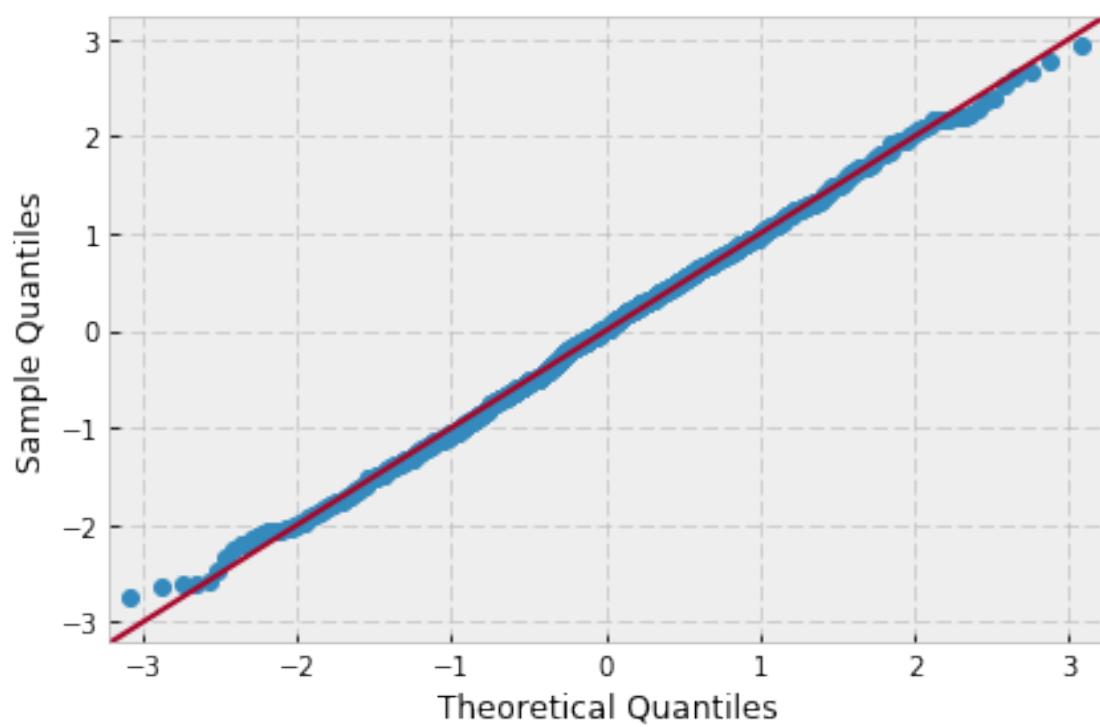
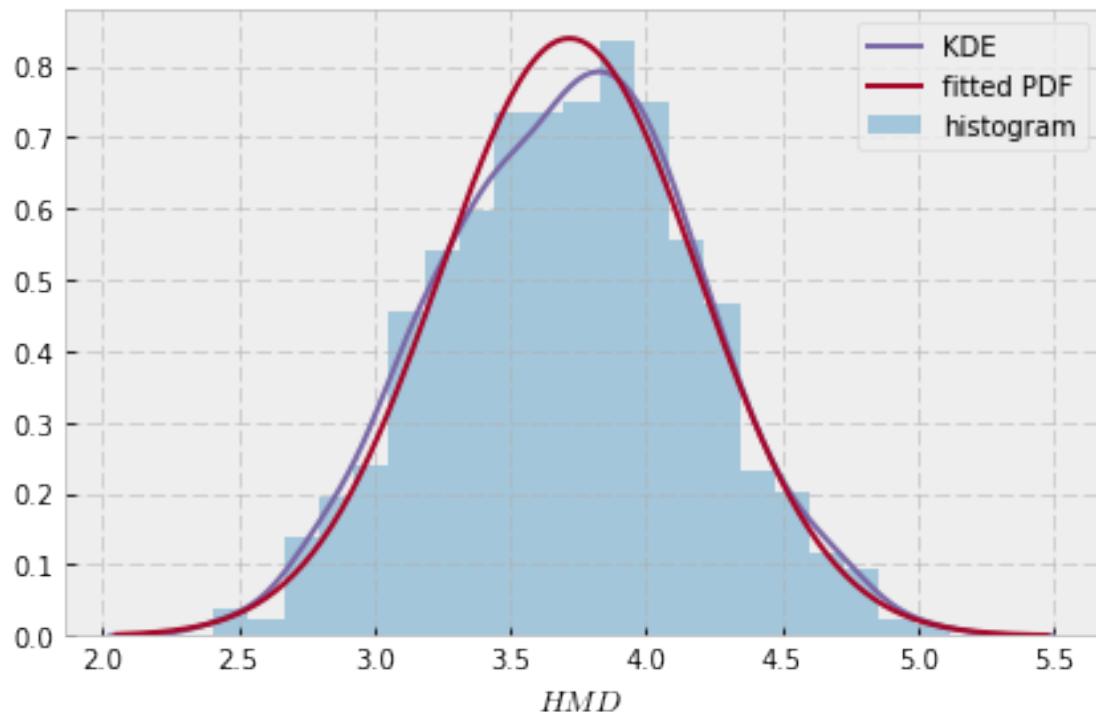


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=3.537594688805644, scale=0.44830893586381815)
DescribeResult(nobs=1000, minmax=(2.1238878559575336, 4.936379174408895)
               mean=3.537594688805644, variance=0.20118208405940843,
               skewness=-0.04651688699522977, kurtosis=0.0070344400591957346)
```

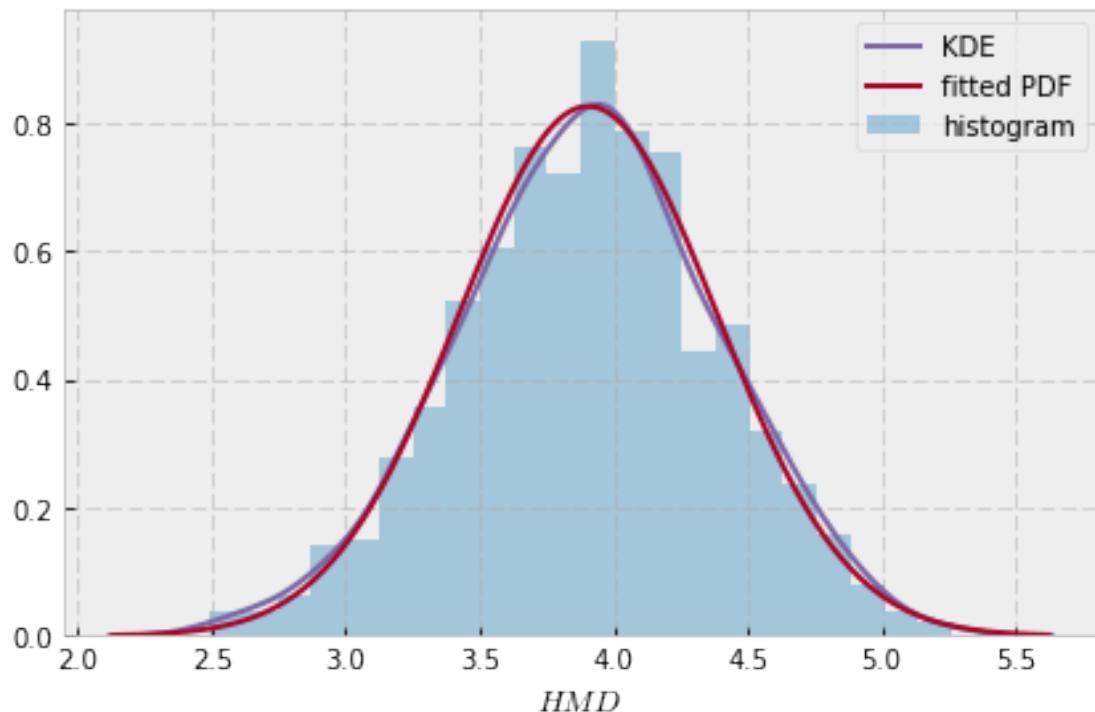


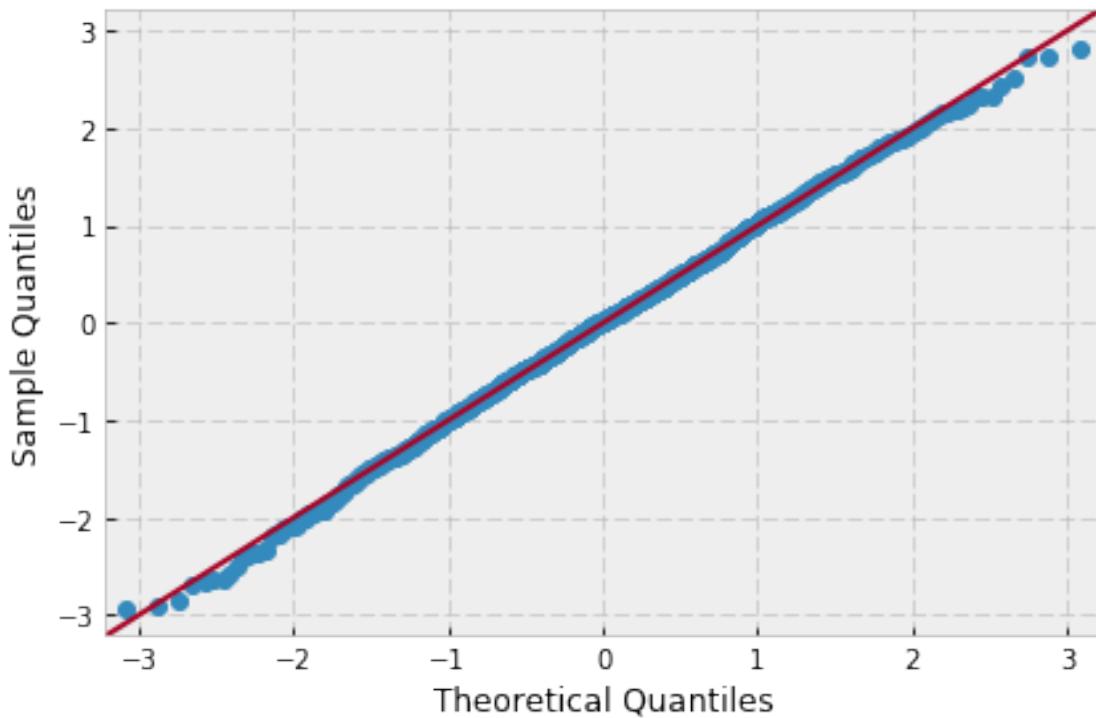


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(),loc=3.7165708251974183, scale=0.4752871630396627)
DescribeResult(nobs=1000, minmax=(2.4081430075559633, 5.1164961975679644)
               mean=3.7165708251974183, variance=0.22612401136165253,
               skewness=-0.005464772093466557, kurtosis=-0.3039500006348761)
```

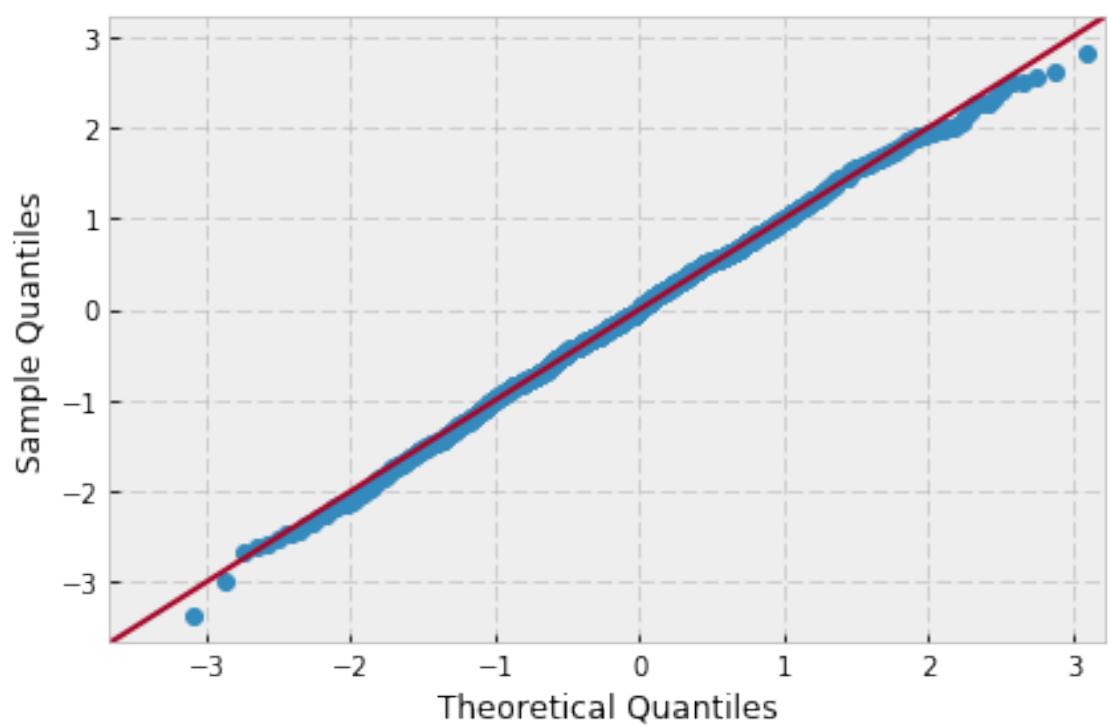
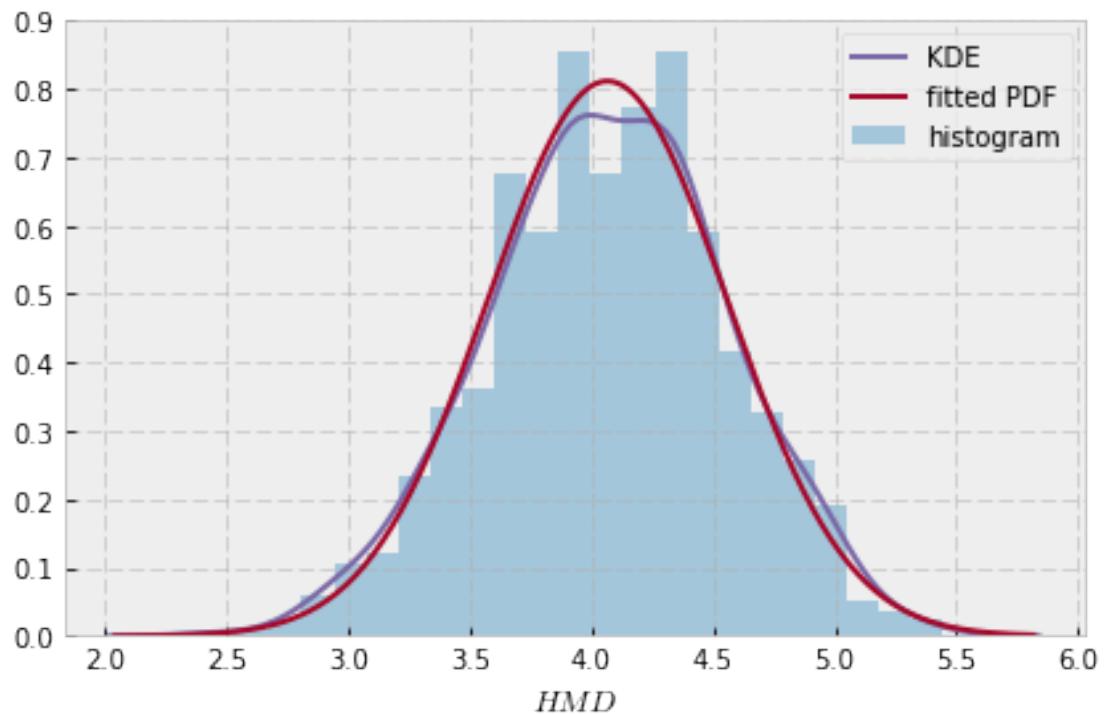


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(),loc=3.903636637772367, scale=0.48273581485803485)
DescribeResult(nobs=1000, minmax=(2.4929320323933308, 5.2603022669747022)
               mean=3.903636637772367, variance=0.23326713408073163,
               skewness=-0.07824369944784777, kurtosis=-0.11248232833761884)
```

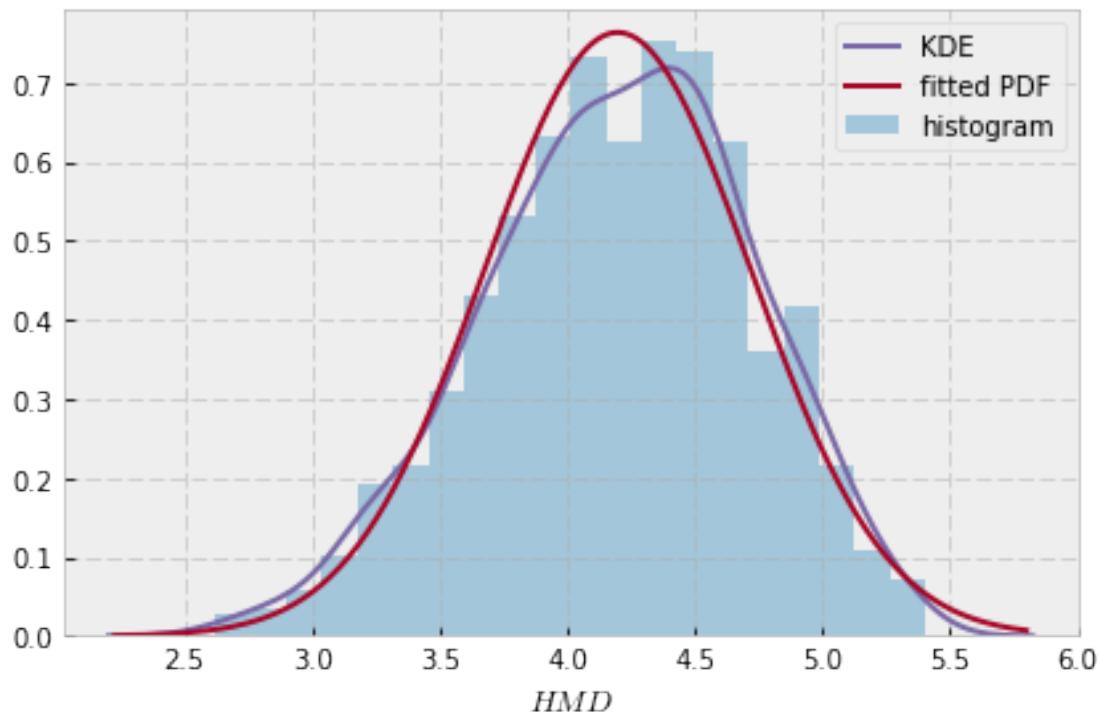


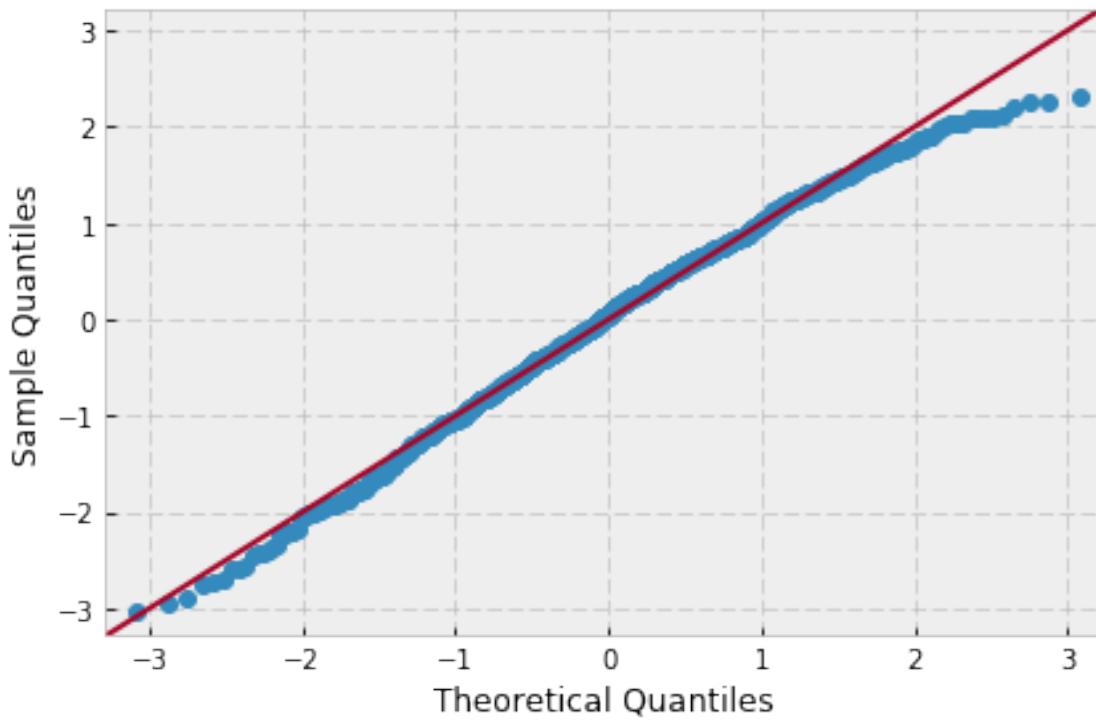


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(),loc=4.065272571271387, scale=0.4916028227731875)
DescribeResult(nobs=1000, minmax=(2.4125929342733716, 5.4458974678697416)
               mean=4.0652725712713869, variance=0.24191525060917521,
               skewness=-0.10600189464975605, kurtosis=-0.1501691641811509)
```

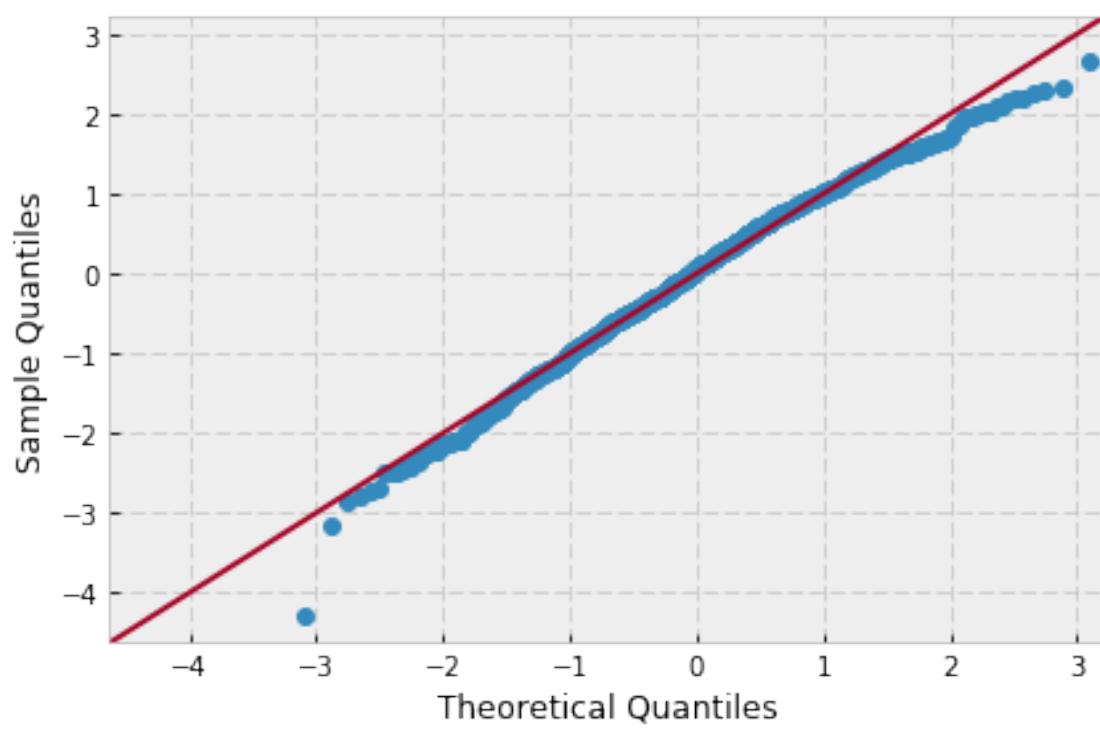
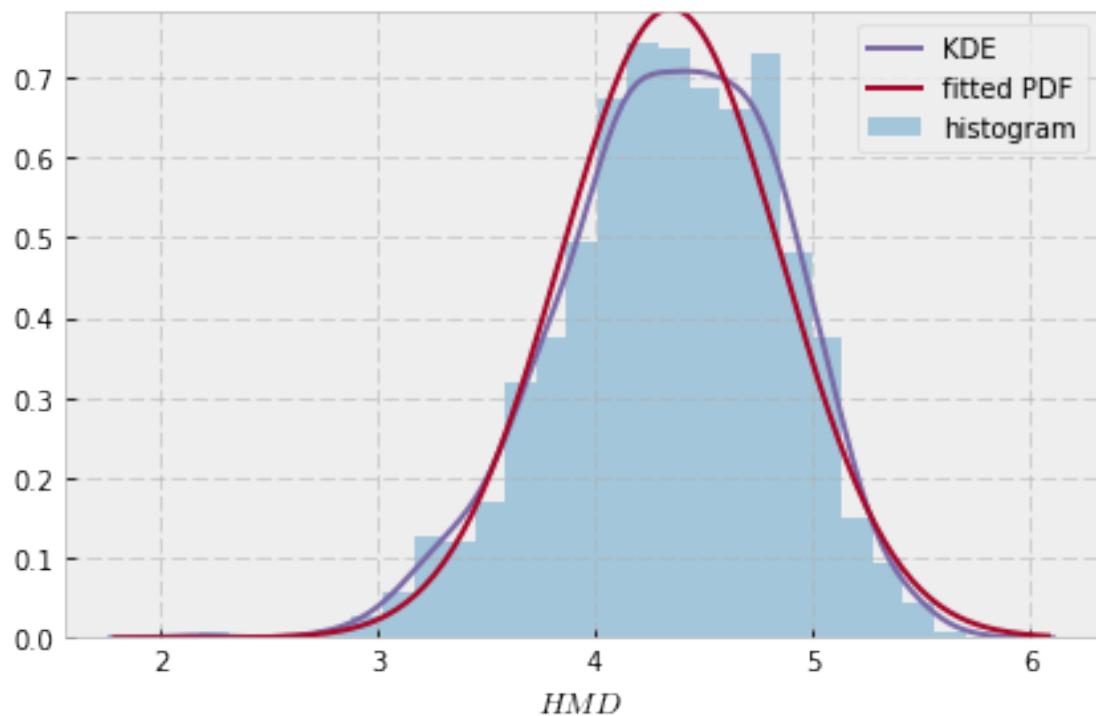


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=4.196971579402141, scale=0.5224667283770031)
DescribeResult(nobs=1000, minmax=(2.6203322121427779, 5.4044689964974246)
               mean=4.1969715794021409, variance=0.27324472698795721,
               skewness=-0.2571249075175762, kurtosis=-0.27170293549299274)
```





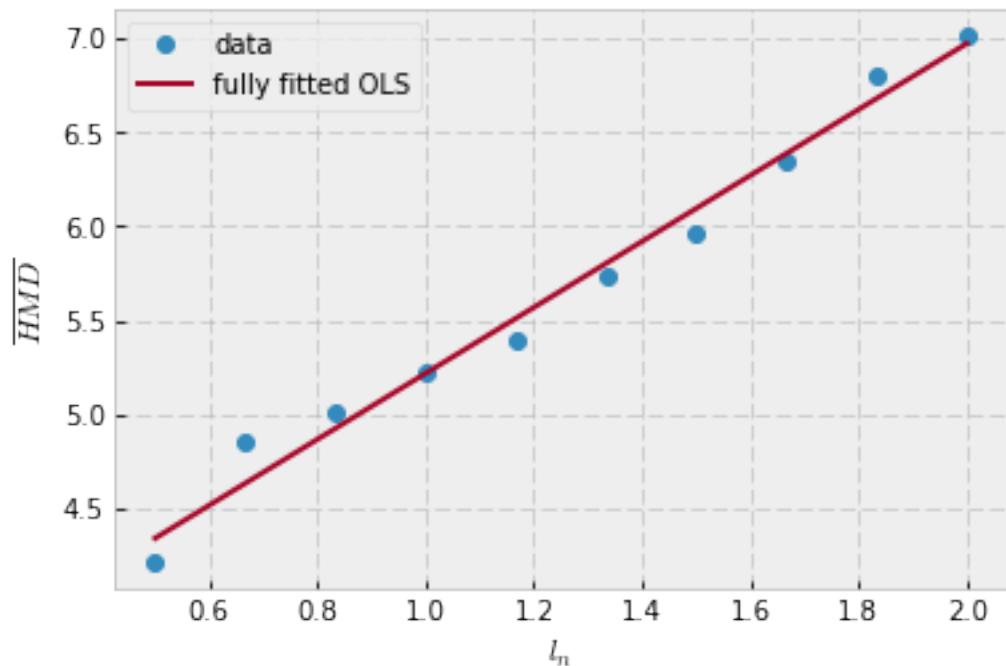
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(),loc=4.349139950263933, scale=0.5075636184127669)
DescribeResult(nobs=1000, minmax=(2.1761815858150424, 5.7014681797617861)
               mean=4.3491399502639334, variance=0.25787870544170255,
               skewness=-0.33461445442360394, kurtosis=-0.024614048610140493)
```



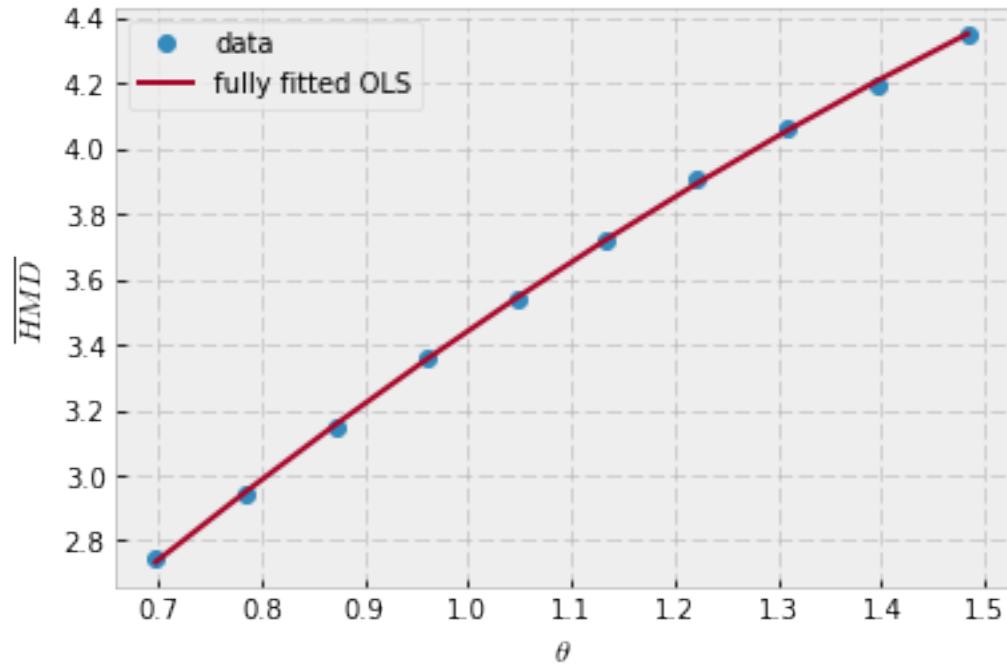
16.2 Parameter regression

```
In [38]: var_name = 'half_mean_distance_between_roots'  
        mean_reg('ln', var_name, 'poly1')  
        mean_reg('theta', var_name, 'poly2')  
        mean_reg('maxB', var_name, 'poly2')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly2')  
        mean_reg('N', var_name, 'poly1')
```

variable name: ln
model: poly1
r-squared: 0.9703289853093919
explained variance: 0.9703505825275665



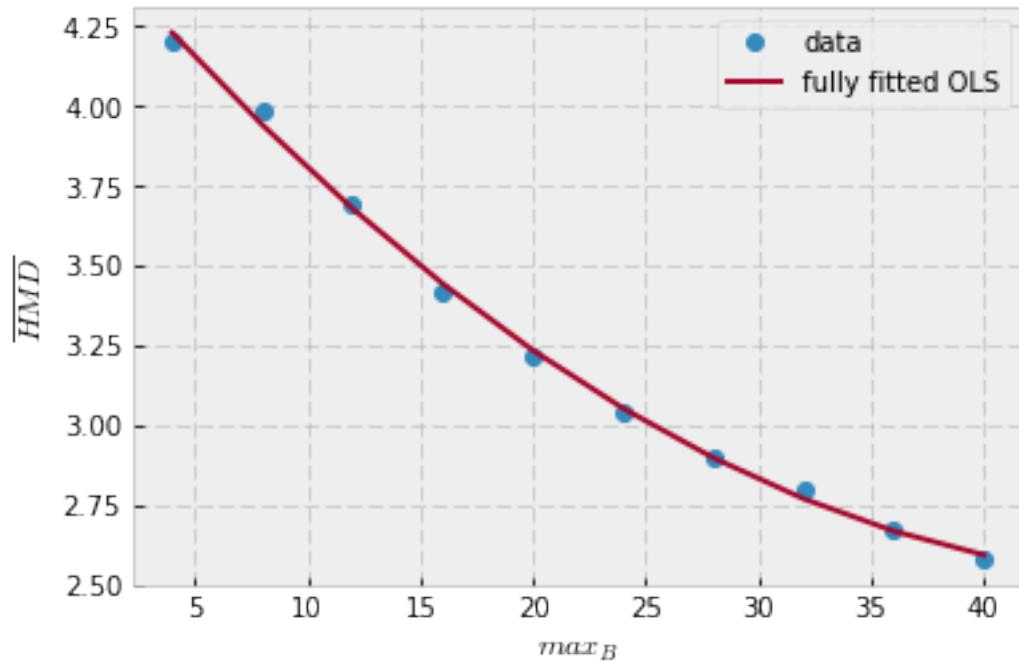
variable name: theta
model: poly2
r-squared: 0.9994437688058402
explained variance: 0.9994459390878516



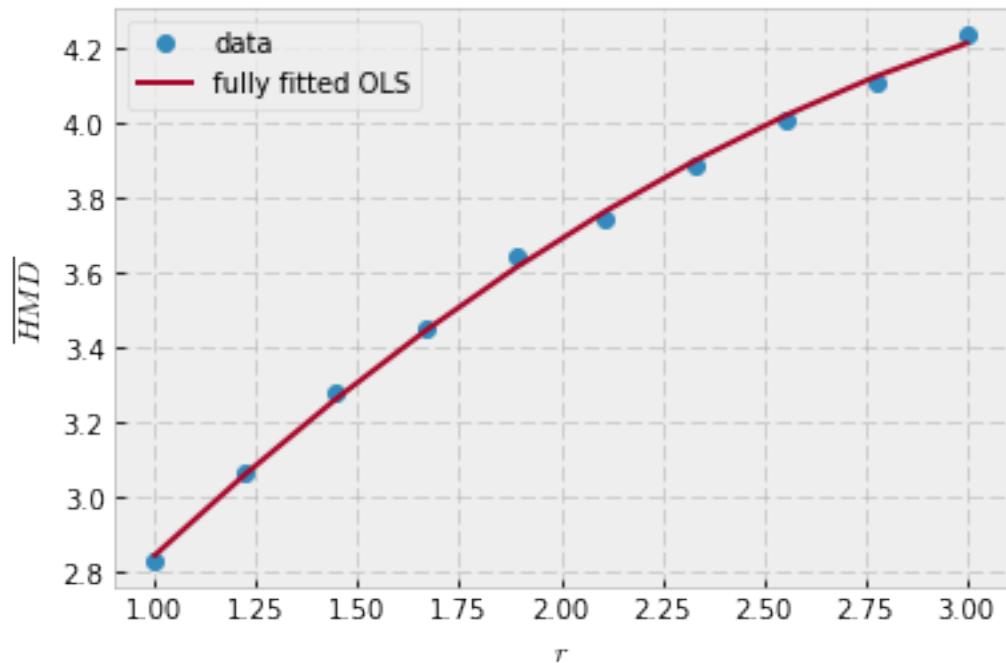
```

variable name: maxB
model: poly2
r-squared: 0.9953674902487221
explained variance: 0.9954610008257605

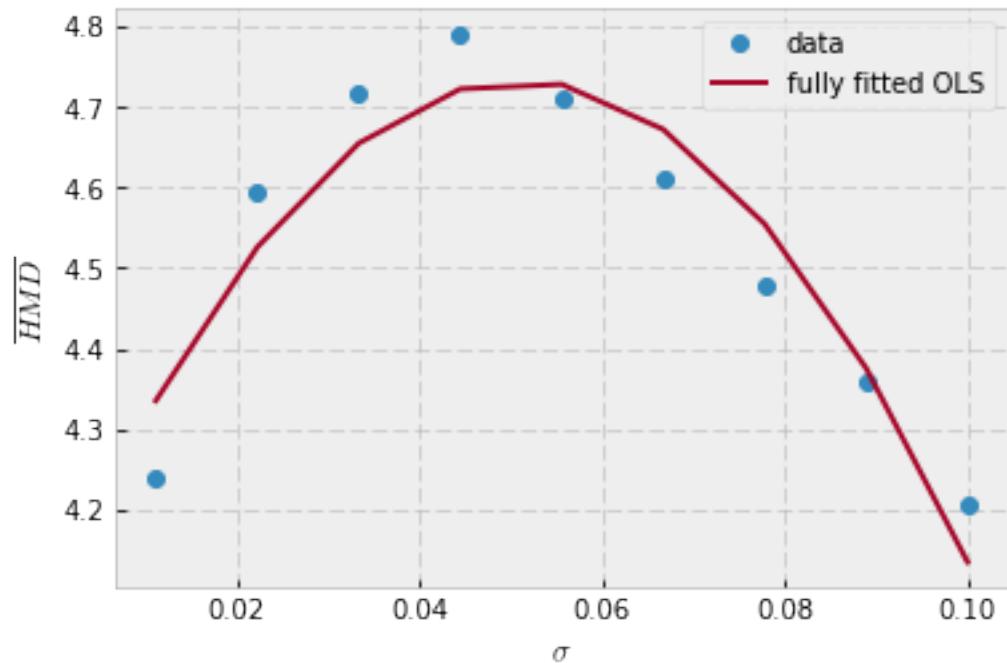
```



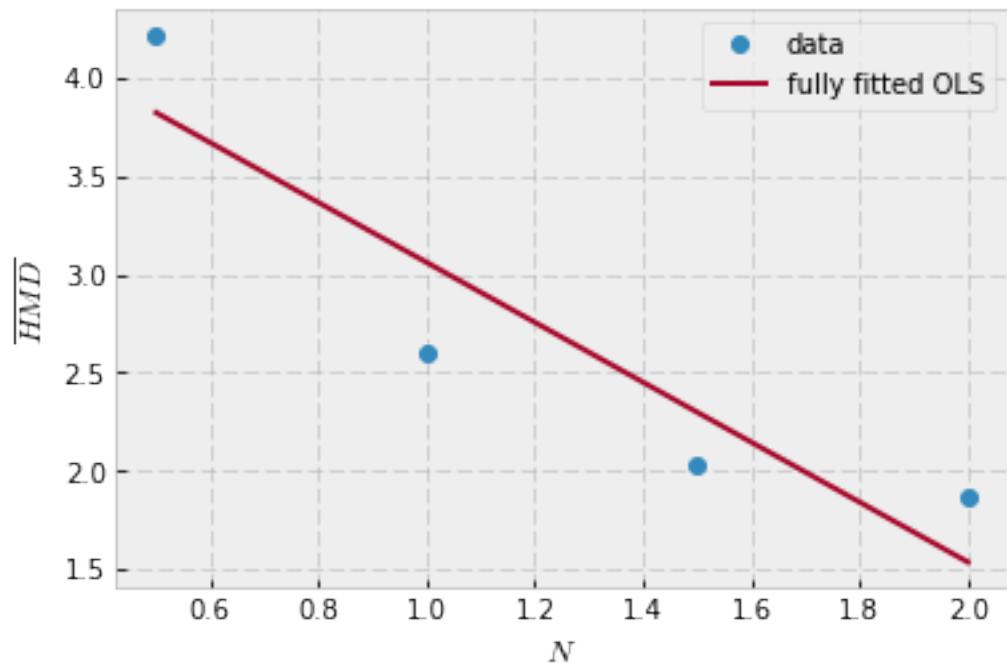
```
variable name: r
model: poly2
r-squared: 0.994812379749248
explained variance: 0.9948161429745567
```



```
variable name: sigma
model: poly2
r-squared: 0.5596462985625497
explained variance: 0.5600695811199354
```



```
variable name: N
model: poly1
r-squared: -0.030185132873665976
explained variance: 0.10948234980061311
```

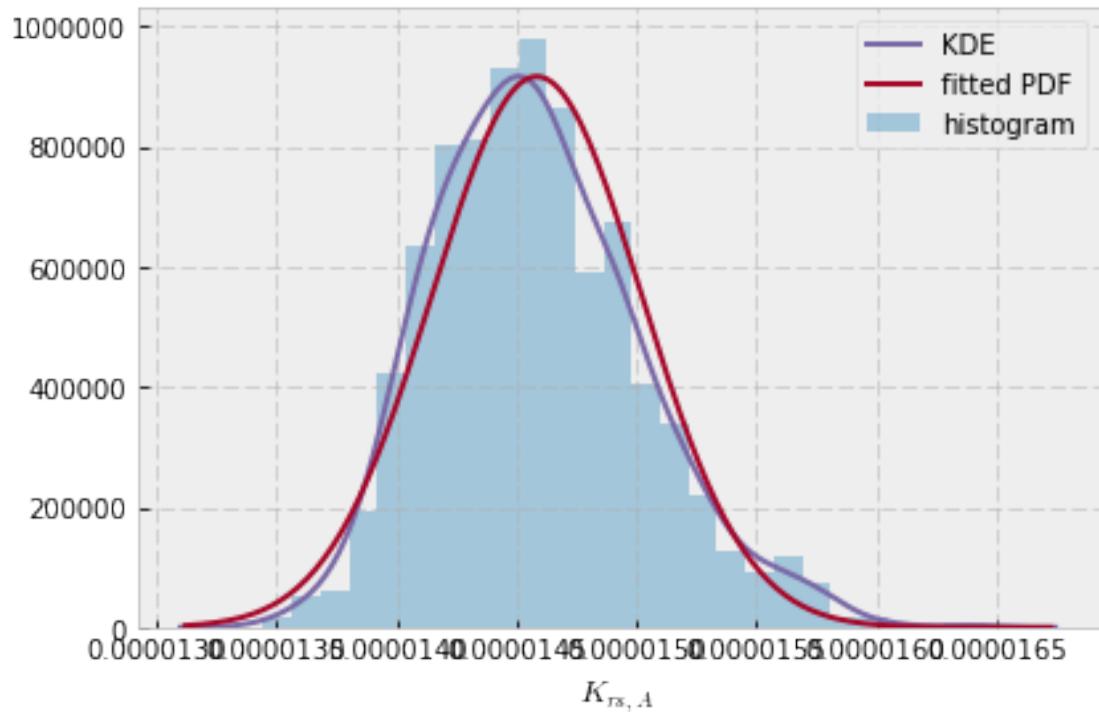


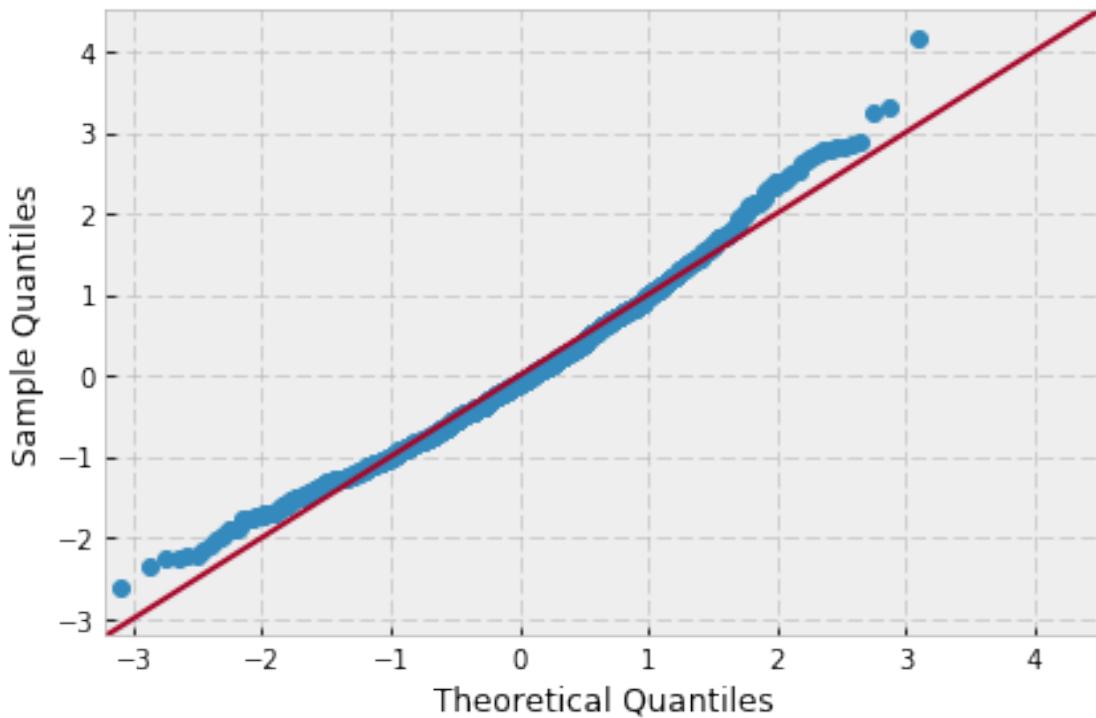
17 Root system conductance per unit of root area

17.1 Probability distributions

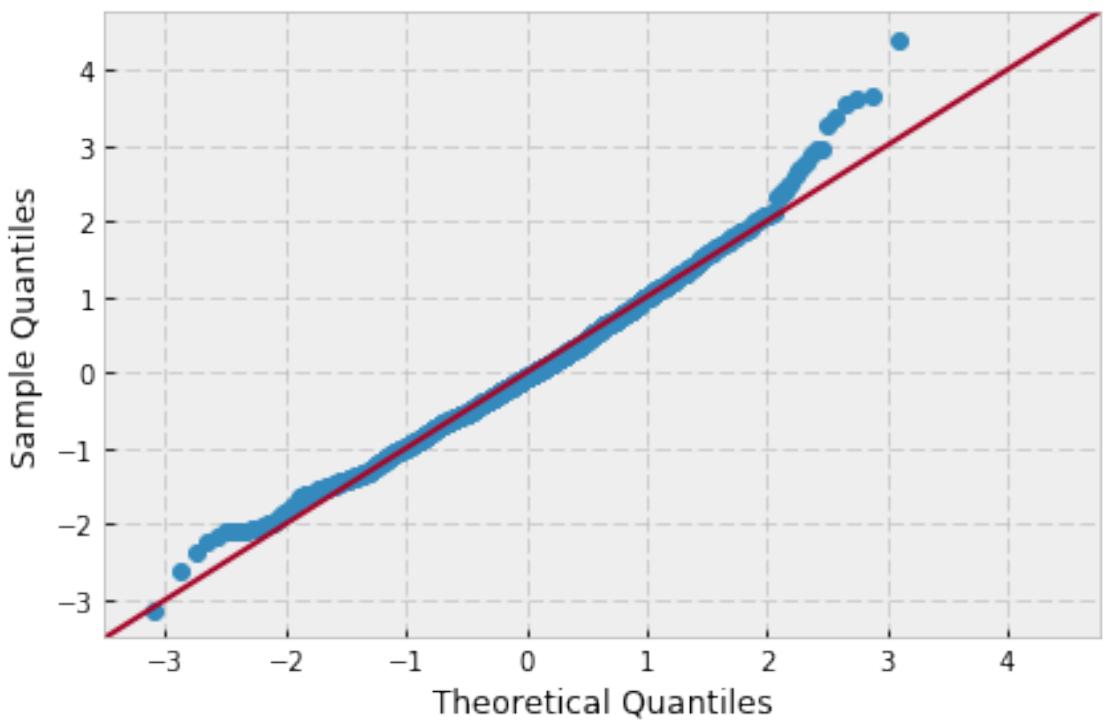
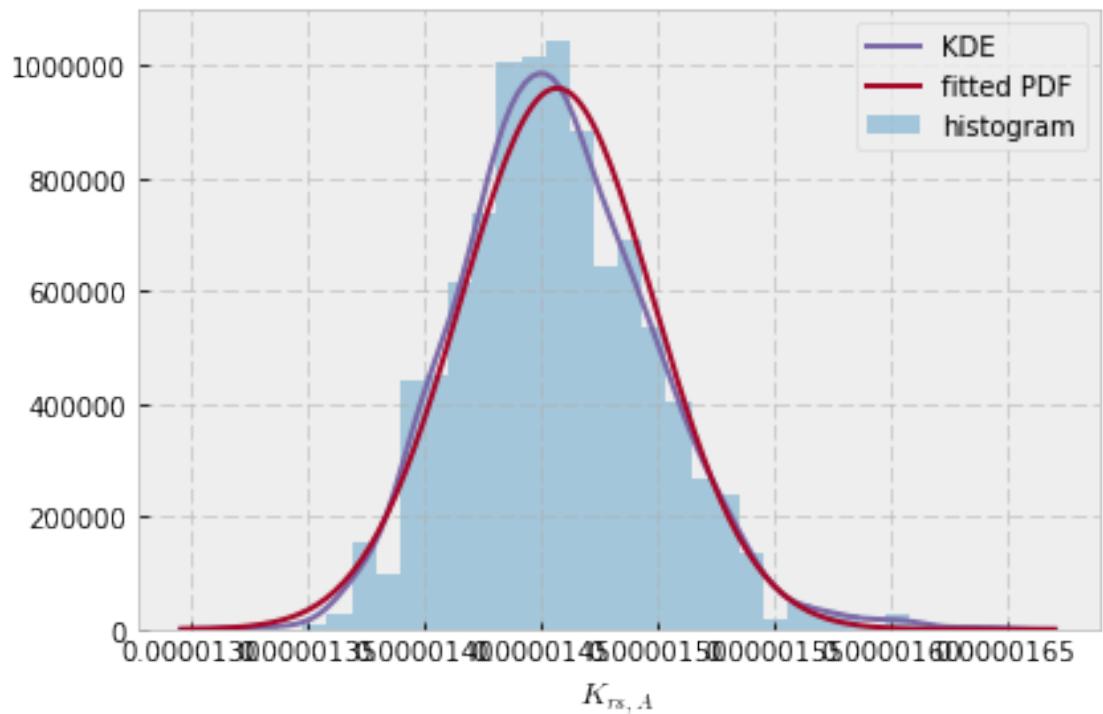
```
In [39]: plot_dist('root_system_conductance_per_unit_of_root_area')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=1.4583751258107807e-05, scale=4.352369257630005e-07)
DescribeResult(nobs=1000, minmax=(1.3445007597504146e-05, 1.6399645549551826e-05)
               mean=1.4583751258107807e-05, variance=1.8962080234997757e-13,
               skewness=0.5096976870757304, kurtosis=0.29640765043729456)
```





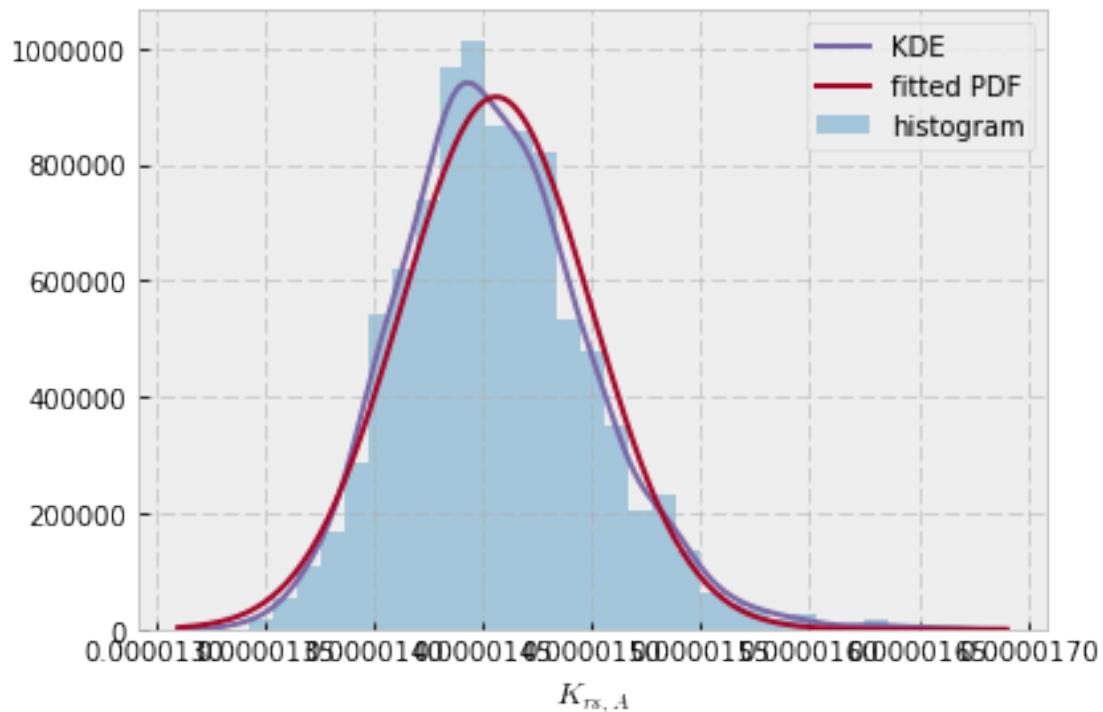
```
variable name: N
variable value: 1.0
distribution: normal(shape=(),loc=1.4570442384417457e-05, scale=4.154249833659862e-07)
DescribeResult(nobs=1000, minmax=(1.3269367767063472e-05, 1.6393413455803865e-05)
               mean=1.4570442384417457e-05, variance=1.7275066747210201e-13,
               skewness=0.4335722315106894, kurtosis=0.6077016636191481)
```

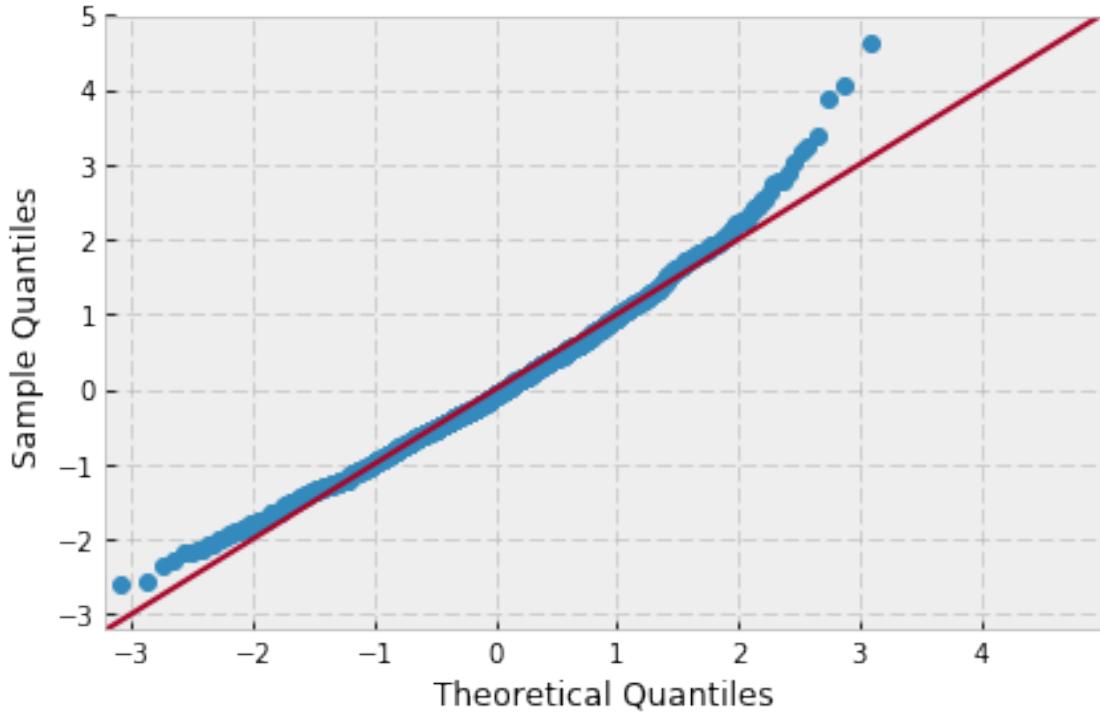


```

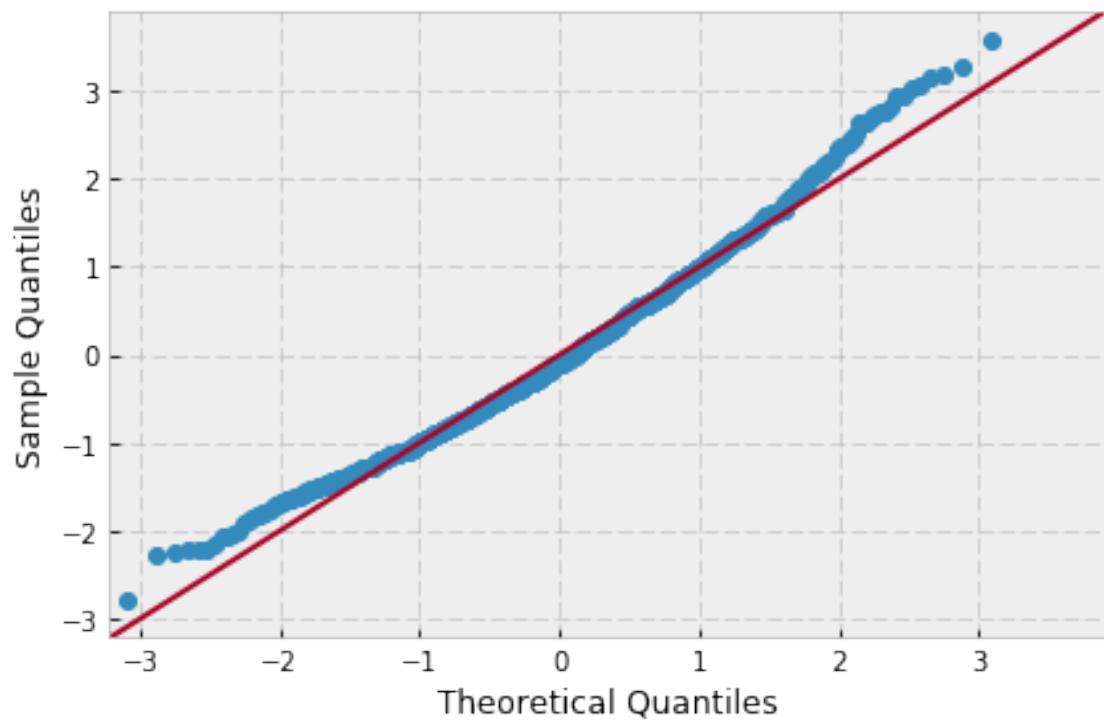
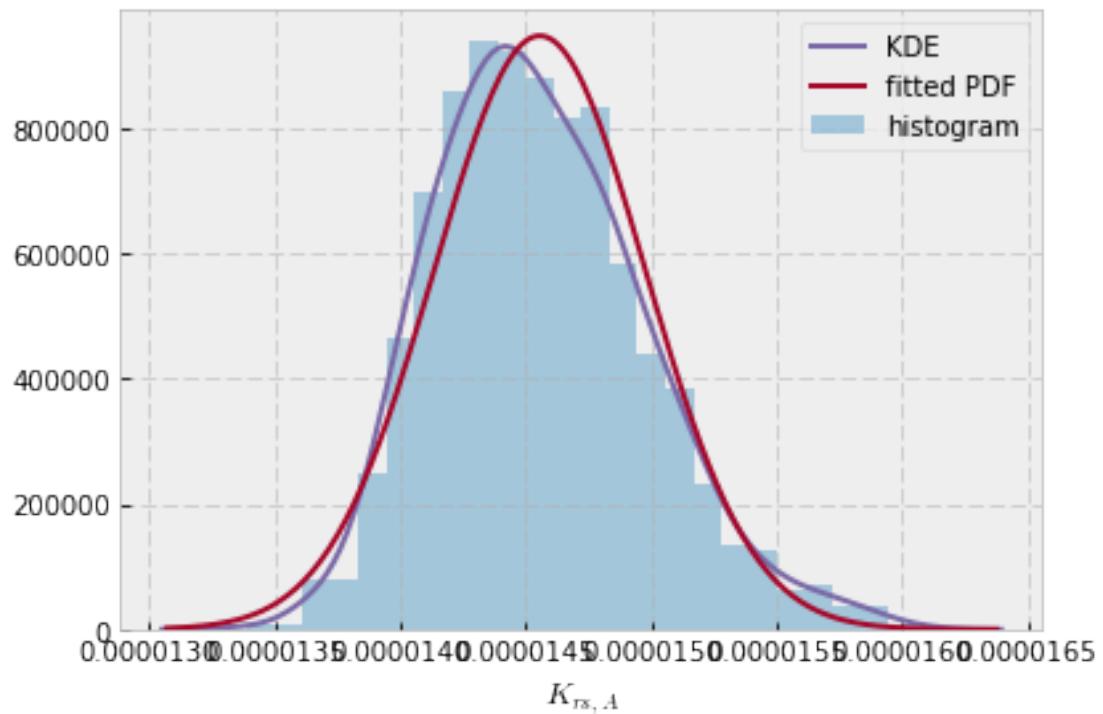
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=1.4563712783707934e-05, scale=4.3530118792947176e-07)
DescribeResult(nobs=1000, minmax=(1.343387252717082e-05, 1.6575858232423418e-05)
               mean=1.4563712783707934e-05, variance=1.8967680101382313e-13,
               skewness=0.5312592971835901, kurtosis=0.7662094570232583)

```





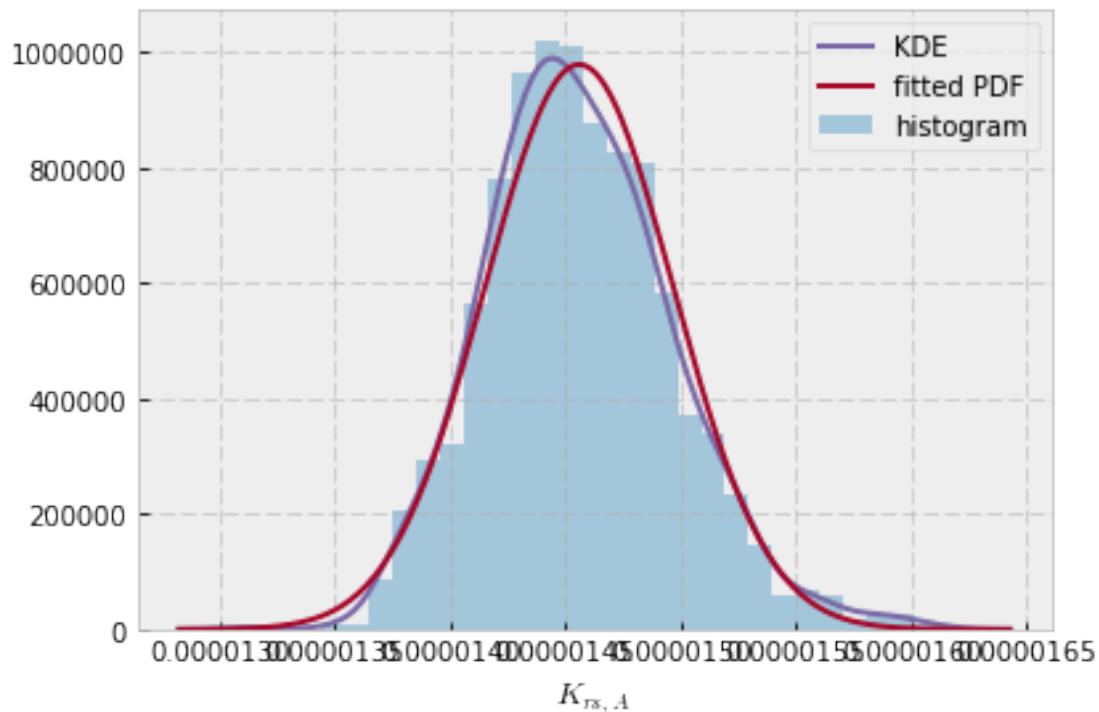
```
variable name: N
variable value: 2.0
distribution: normal(shape=(),loc=1.4555564753934058e-05, scale=4.2071298437845394e-07)
DescribeResult(nobs=1000, minmax=(1.3385323541432731e-05, 1.6059579368484459e-05)
               mean=1.4555564753934058e-05, variance=1.7717659181644168e-13,
               skewness=0.4862153330526284, kurtosis=0.20527511222125527)
```

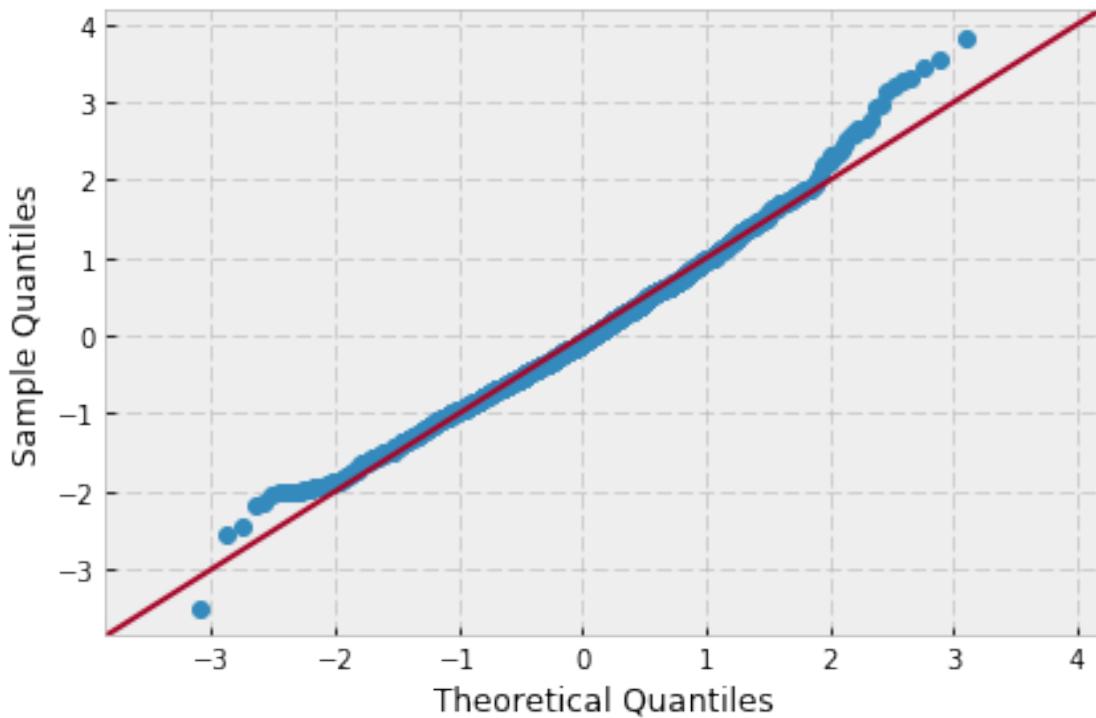


```

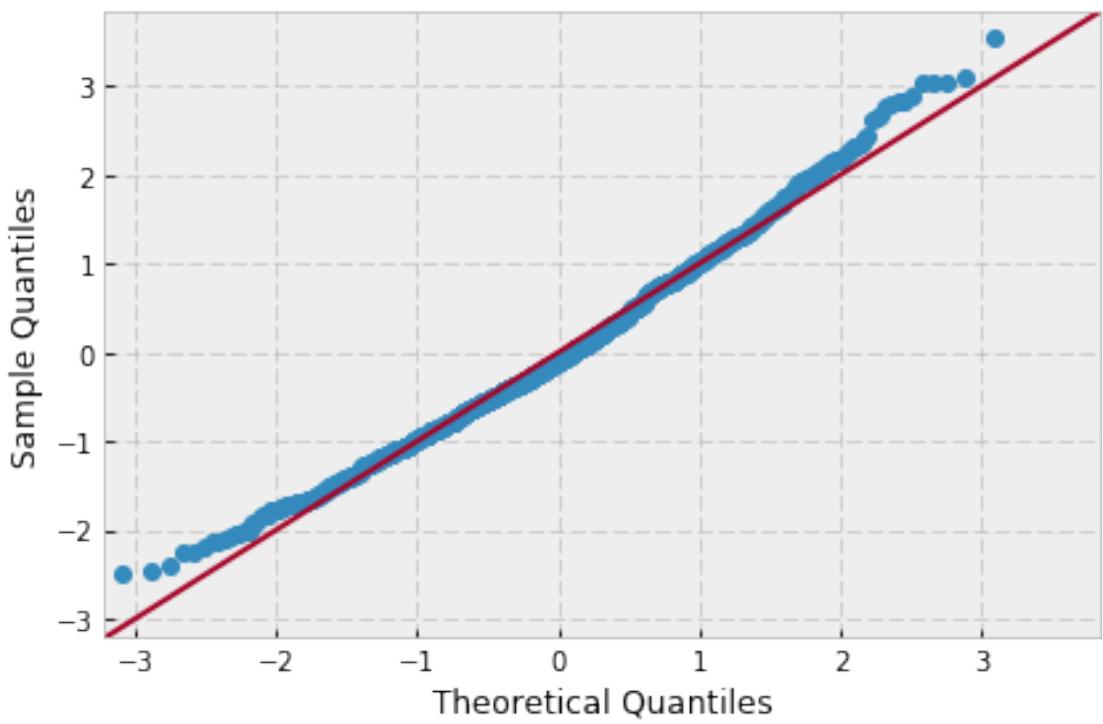
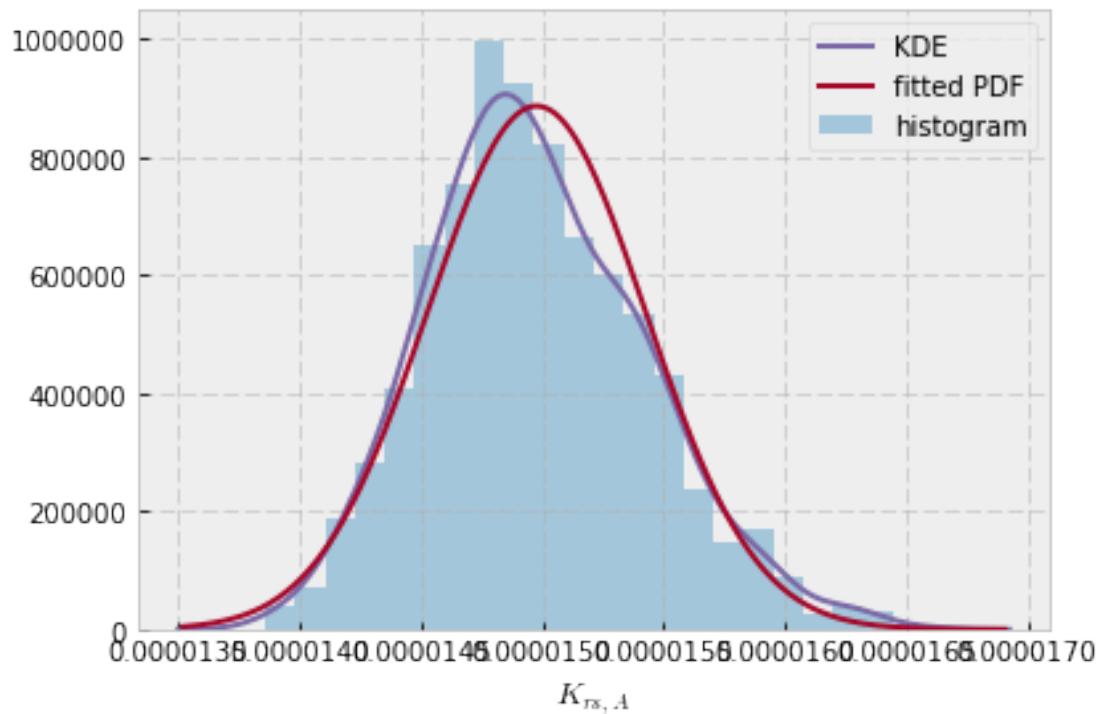
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=1.4558540518194788e-05, scale=4.0781037239270784e-07)
DescribeResult(nobs=1000, minmax=(1.3134705096058307e-05, 1.6115403441963763e-05)
               mean=1.4558540518194788e-05, variance=1.6647577560668571e-13,
               skewness=0.4119069934795062, kurtosis=0.4894682938941859)

```





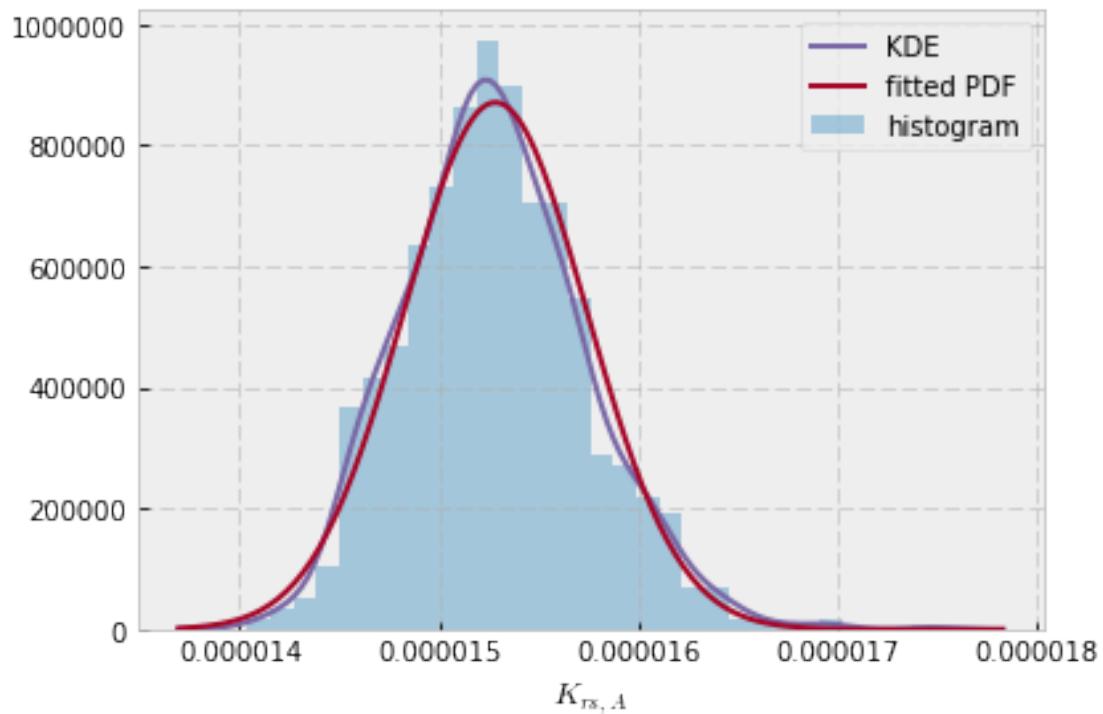
```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=1.4975886585840189e-05, scale=4.4996549727879583e-07)
DescribeResult(nobs=1000, minmax=(1.3857804740355526e-05, 1.656598160226226e-05)
               mean=1.4975886585840189e-05, variance=2.0267162036171574e-13,
               skewness=0.3891120116889821, kurtosis=0.06761074278597246)
```

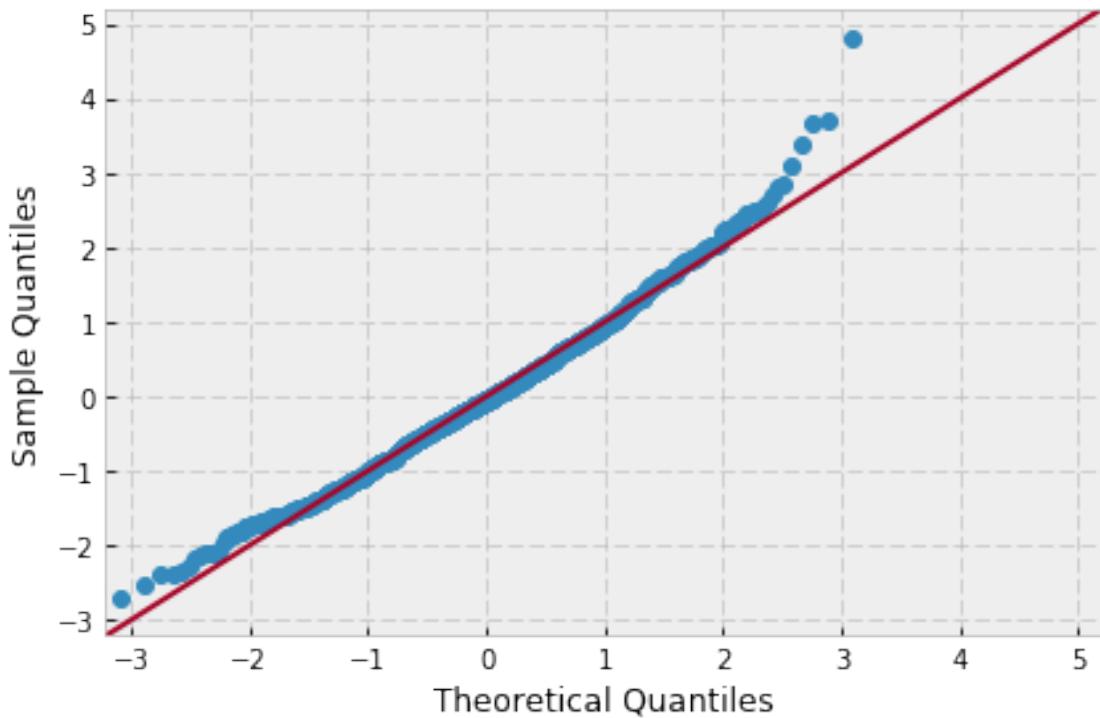


```

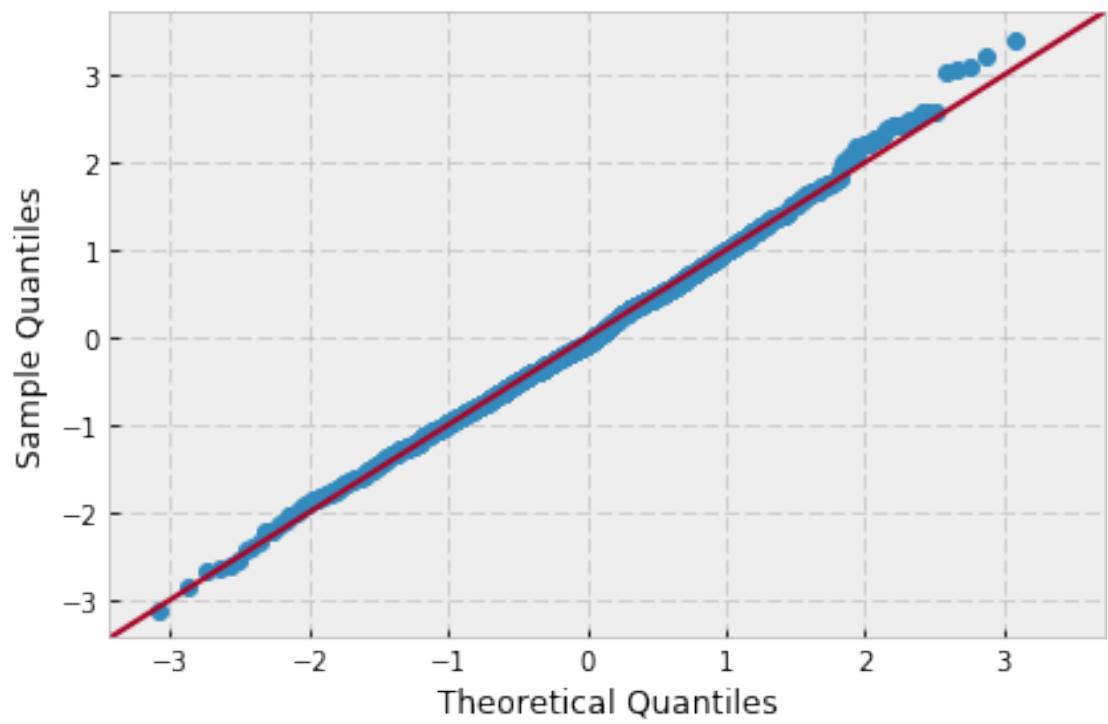
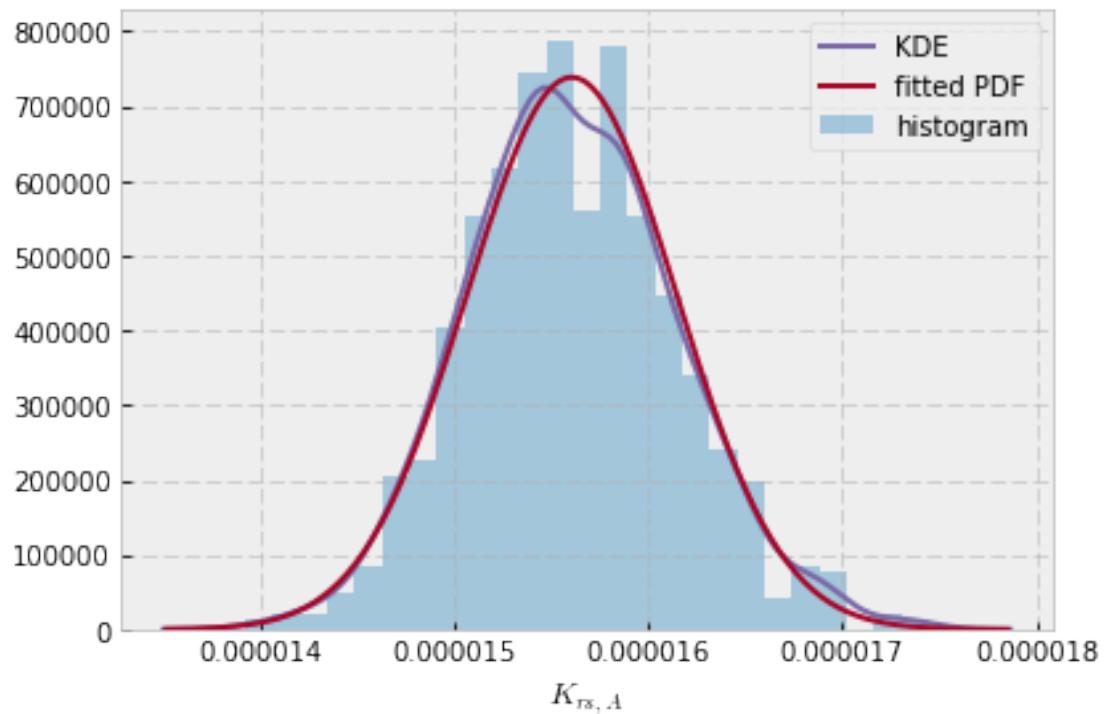
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=1.5282983588134047e-05, scale=4.5787157251854954e-07)
DescribeResult(nobs=1000, minmax=(1.4038597773558001e-05, 1.7483449355499501e-05)
               mean=1.5282983588134047e-05, variance=2.0985623315376312e-13,
               skewness=0.4190600689512046, kurtosis=0.5812945313666269)

```

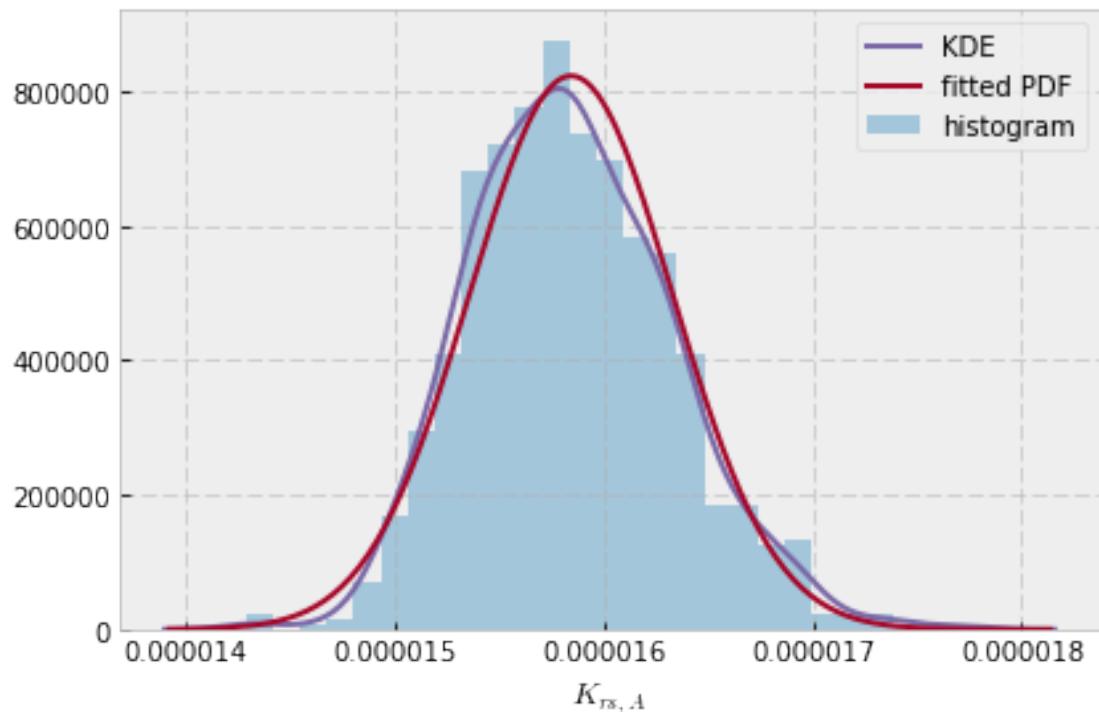


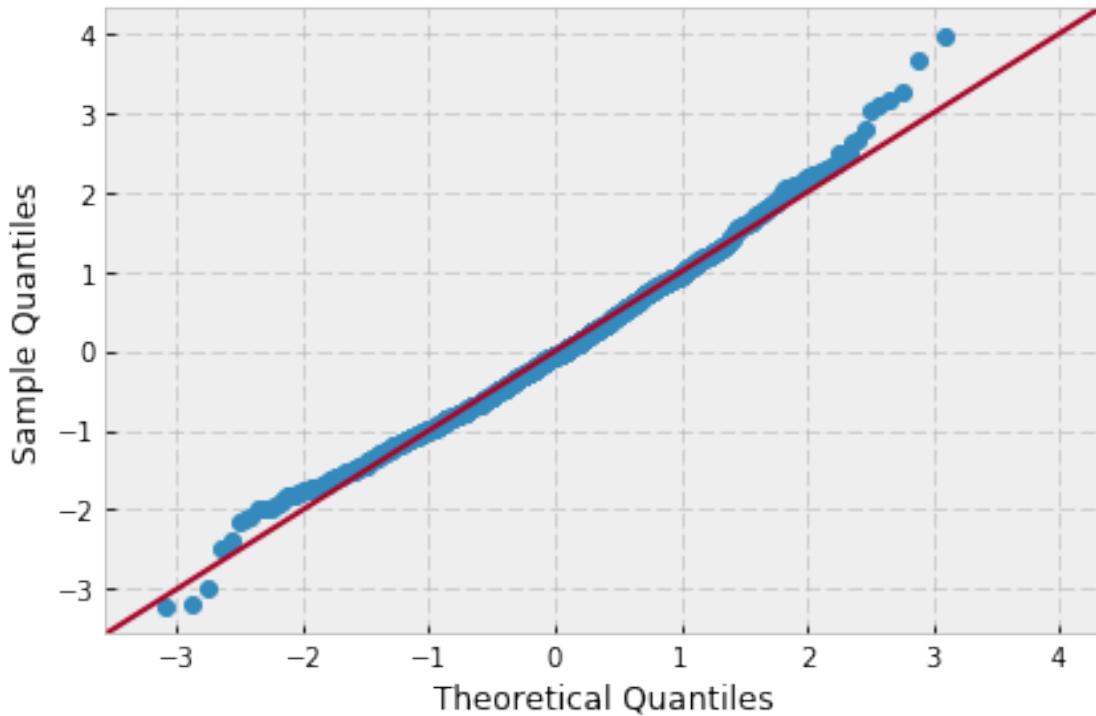


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=1.5606788548440496e-05, scale=5.401568475004497e-07)
DescribeResult(nobs=1000, minmax=(1.3925416971629453e-05, 1.7441808569704265e-05)
               mean=1.5606788548440496e-05, variance=2.9206148138300706e-13,
               skewness=0.1857032072170346, kurtosis=0.13342195211800734)
```

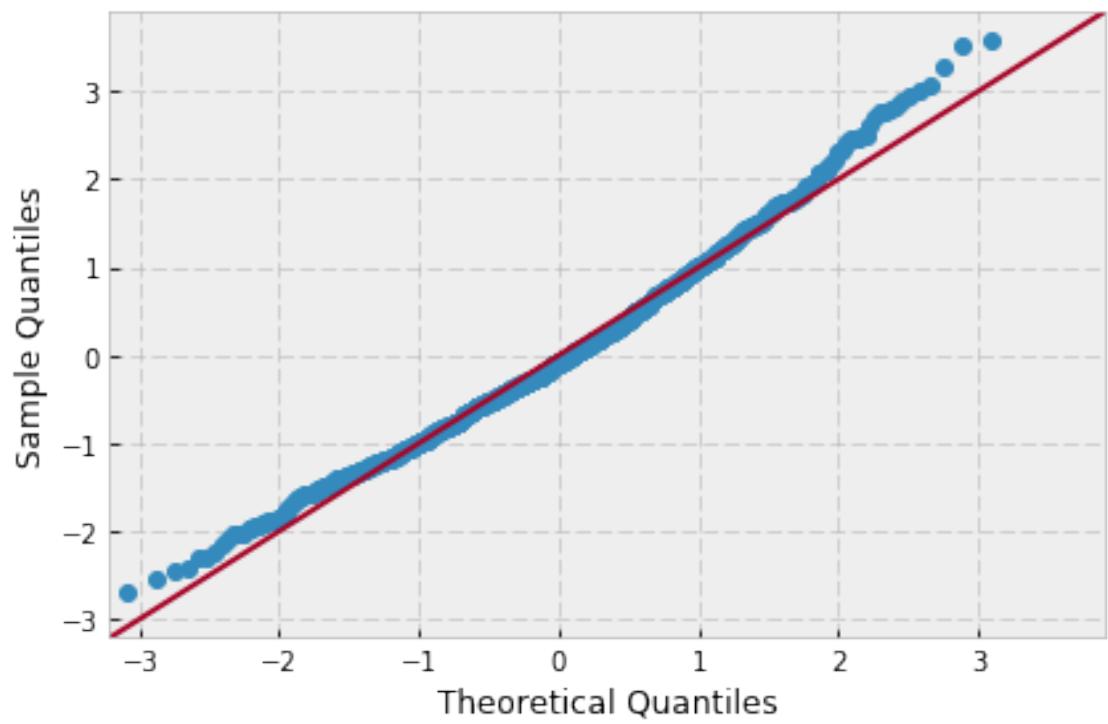
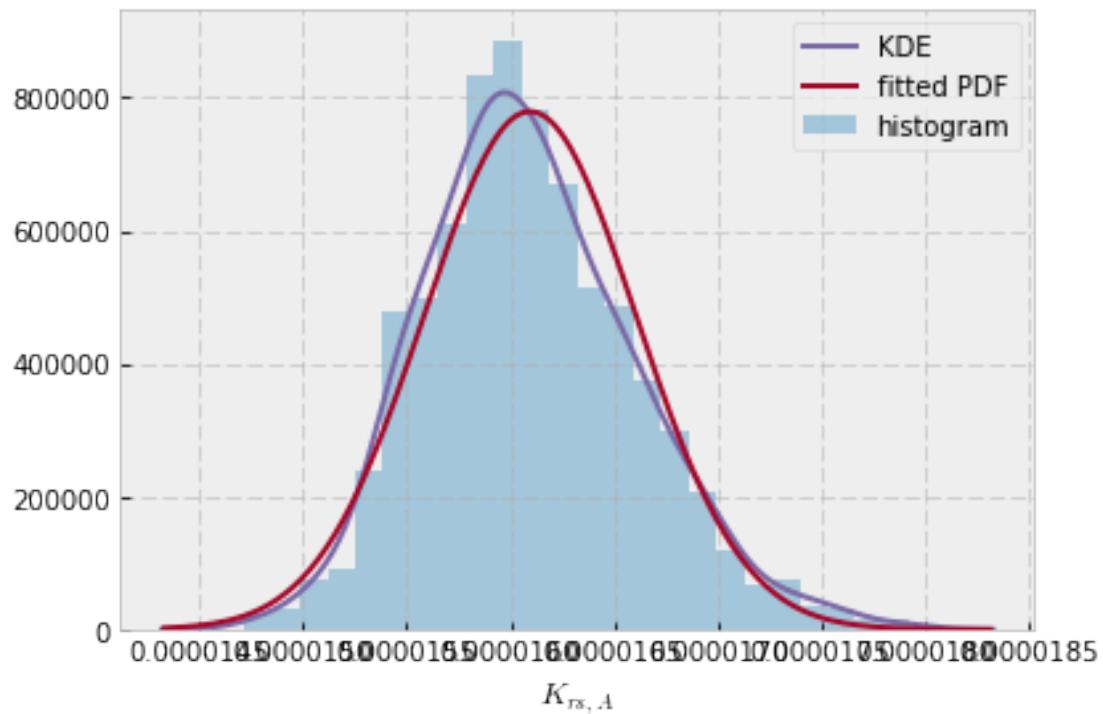


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=1.584607071981206e-05, scale=4.846739020858667e-07)
DescribeResult(nobs=1000, minmax=(1.4290846976689617e-05, 1.7769117373717976e-05)
               mean=1.584607071981206e-05, variance=2.3514393529843875e-13,
               skewness=0.34364275999542093, kurtosis=0.32675011395686004)
```

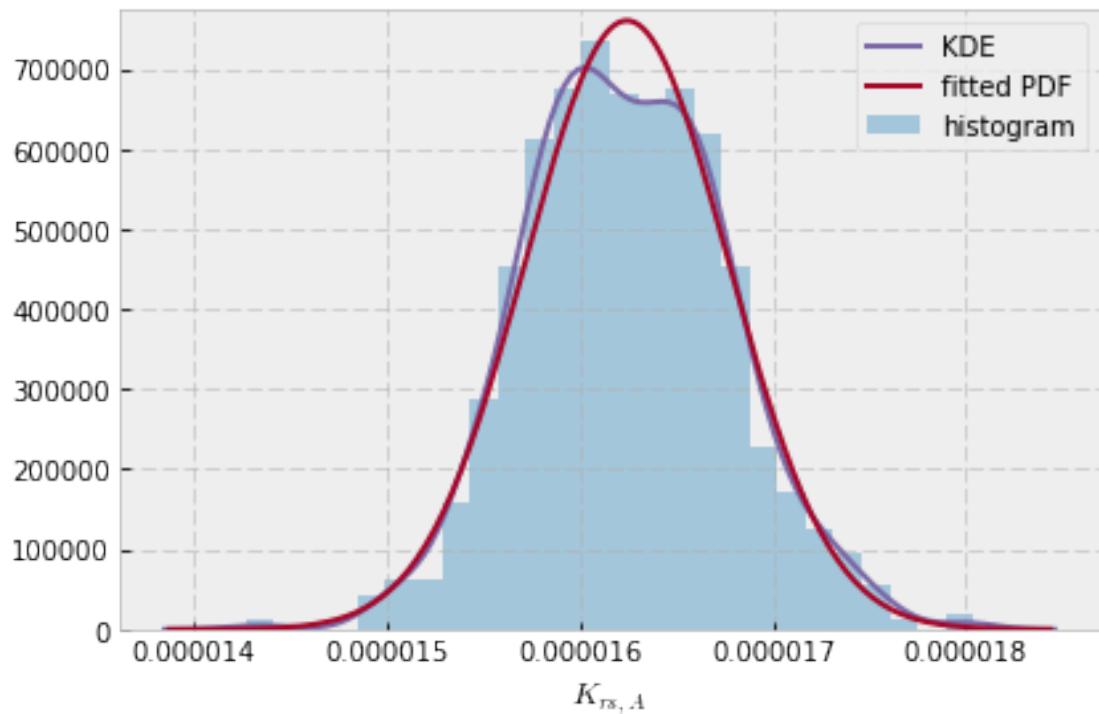


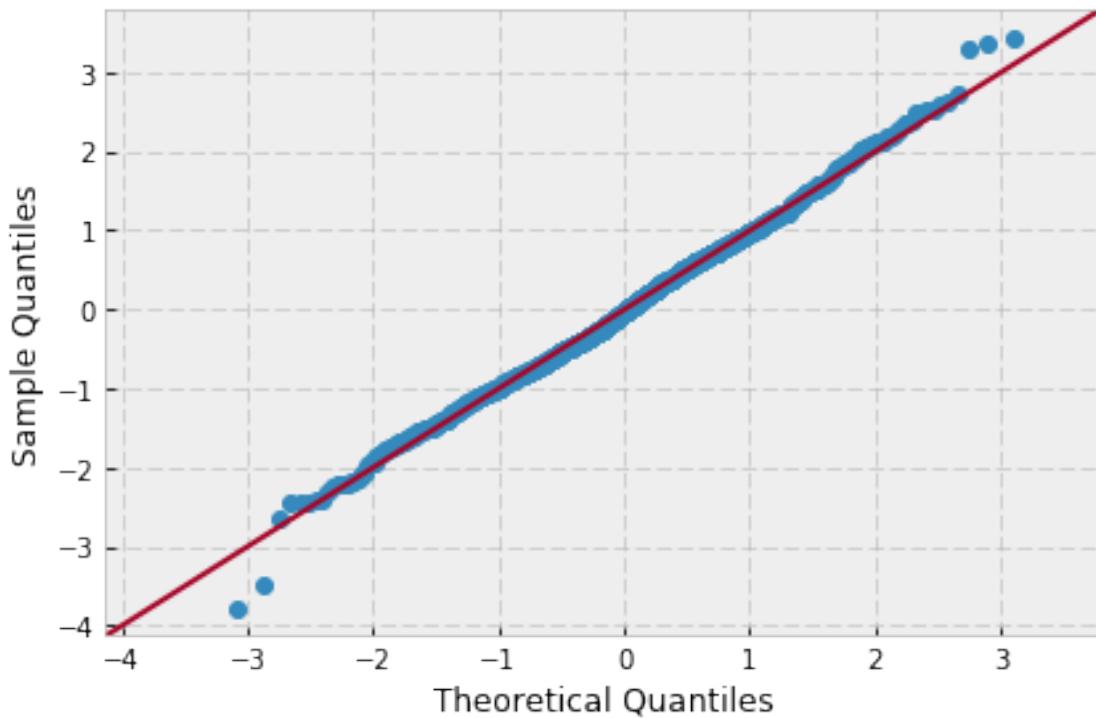


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=1.6094915483236845e-05, scale=5.11885554257729e-07)
DescribeResult(nobs=1000, minmax=(1.4714577253145952e-05, 1.7931963936086677e-05)
               mean=1.6094915483236845e-05, variance=2.6228911096451648e-13,
               skewness=0.43673698210164763, kurtosis=0.25532905480935586)
```

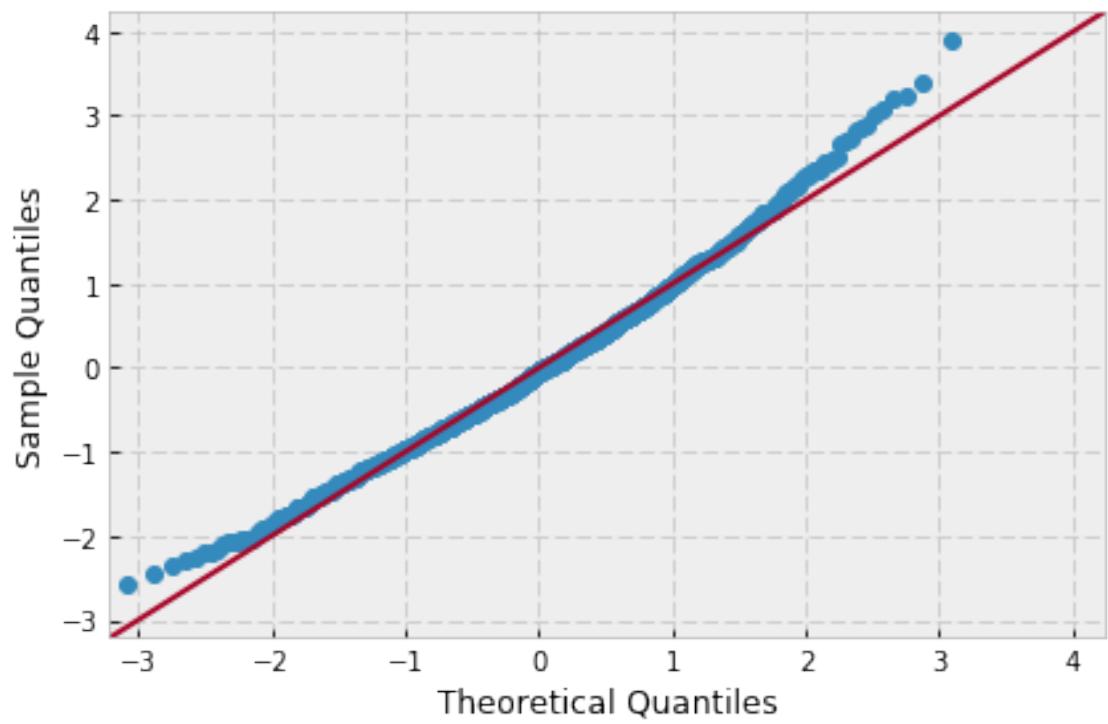
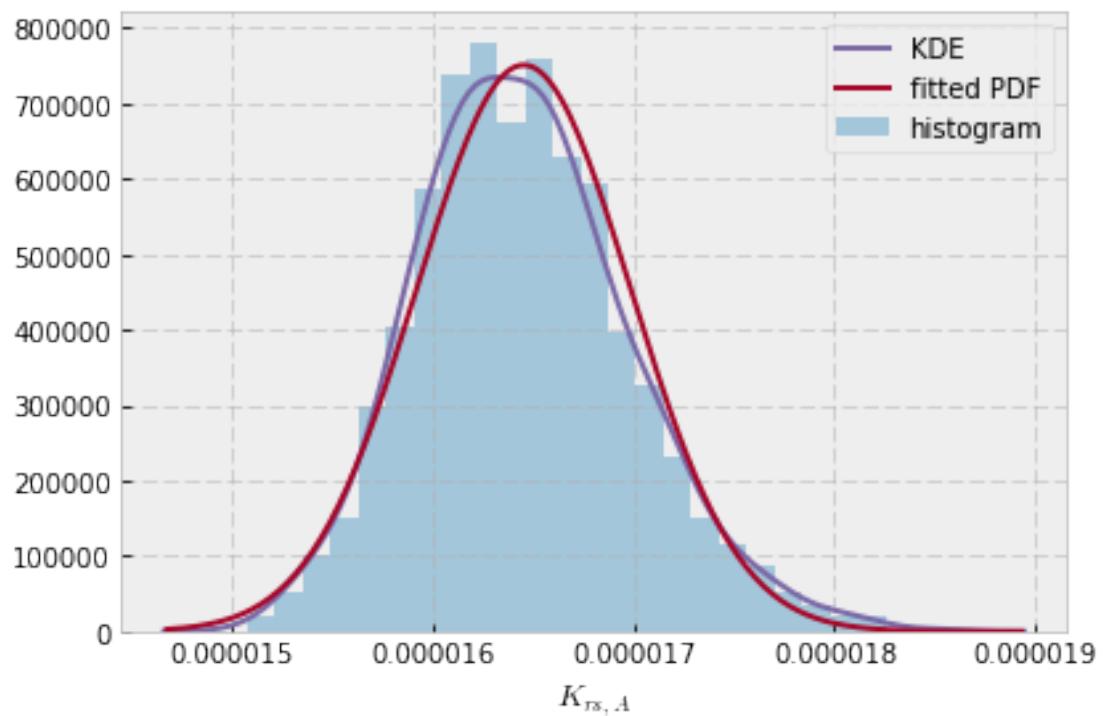


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=1.6243198080592188e-05, scale=5.24940643919321e-07)
DescribeResult(nobs=1000, minmax=(1.4264819896675884e-05, 1.8042666638237614e-05)
               mean=1.6243198080592188e-05, variance=2.7583851815658795e-13,
               skewness=0.11005629977648834, kurtosis=0.1654711585653006)
```

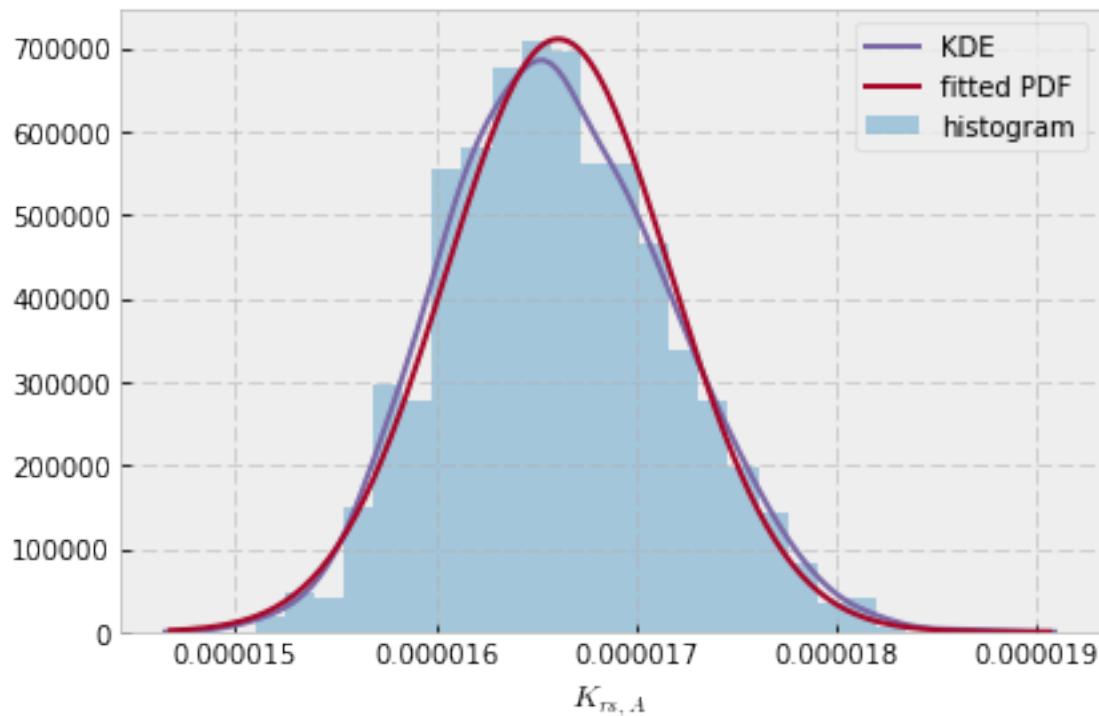


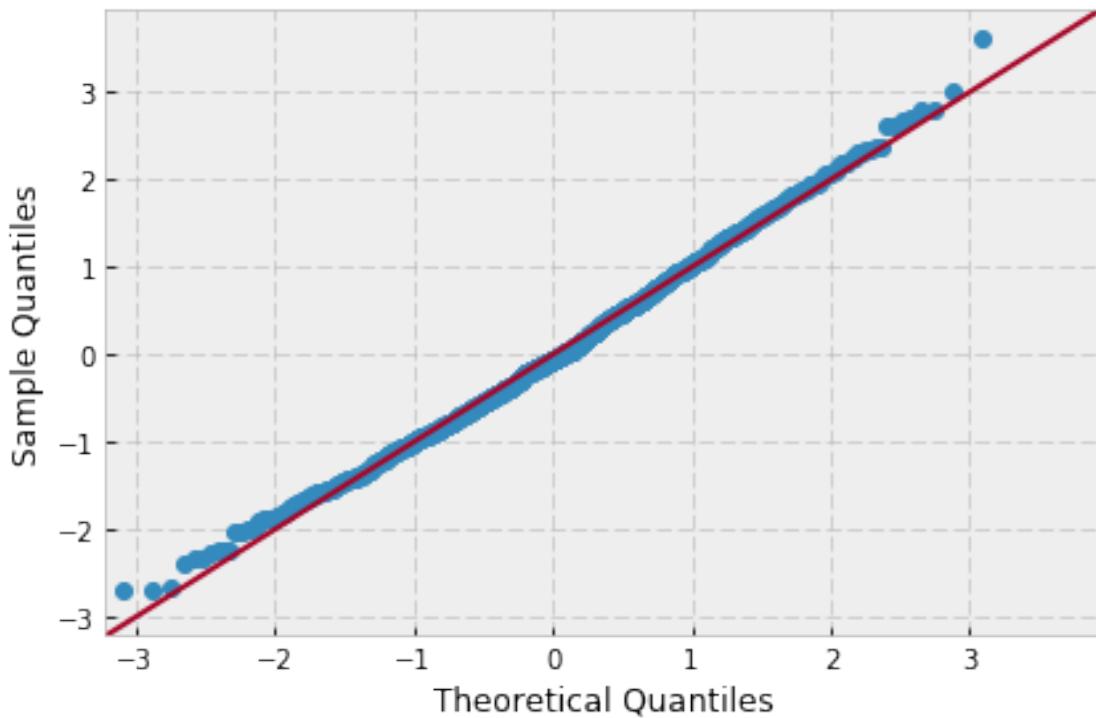


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=1.6452438176315064e-05, scale=5.313741798326965e-07)
DescribeResult(nobs=1000, minmax=(1.5076693747044741e-05, 1.852928147941188e-05)
               mean=1.6452438176315064e-05, variance=2.8264116015302396e-13,
               skewness=0.41227014478100676, kurtosis=0.2910924103237793)
```

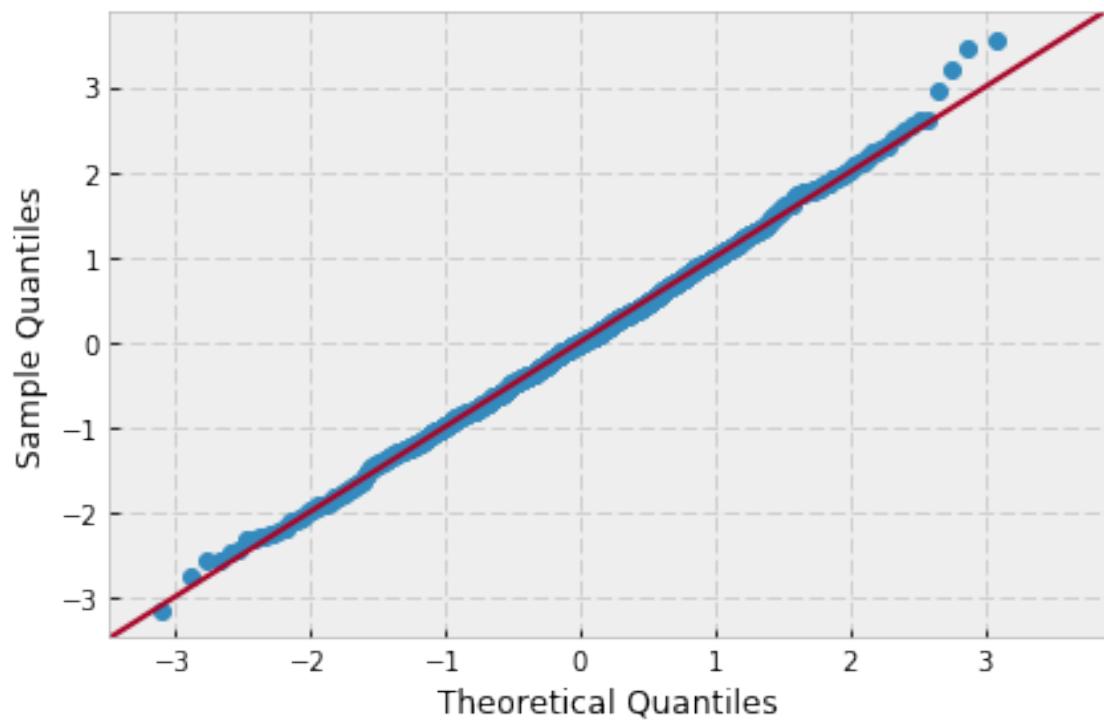
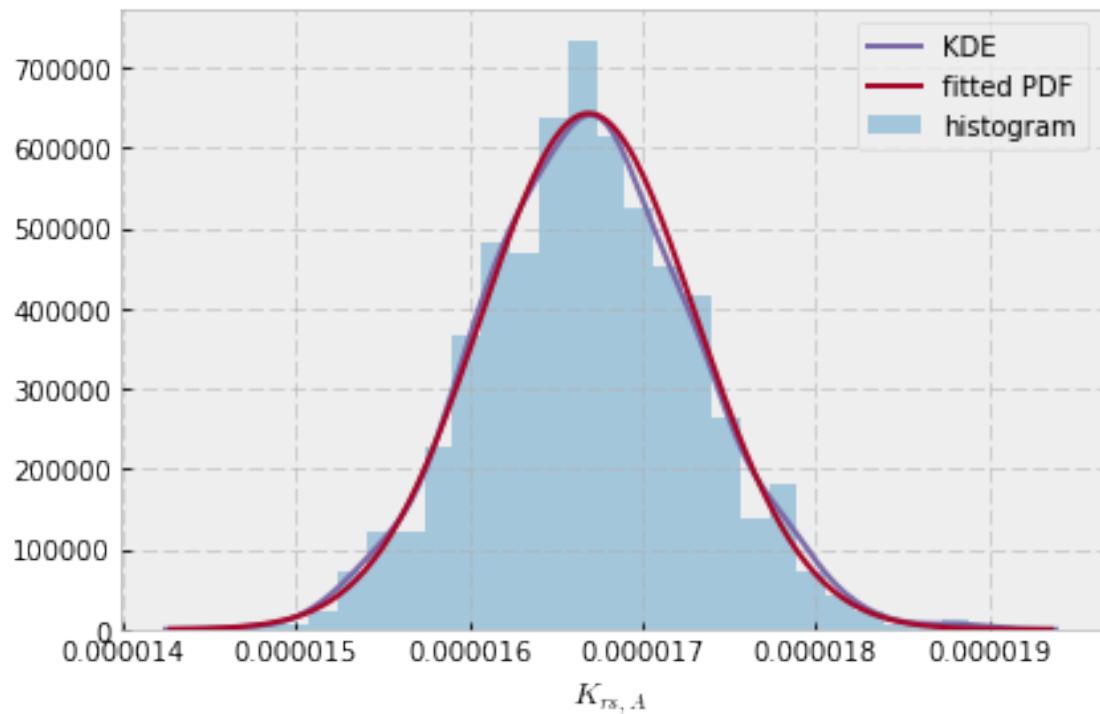


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=1.6613275866416636e-05, scale=5.614310319932939e-07)
DescribeResult(nobs=1000, minmax=(1.5096224464583243e-05, 1.8646218418802566e-05)
               mean=1.6613275866416636e-05, variance=3.1552032400906402e-13,
               skewness=0.22231423943703246, kurtosis=-0.14748420612160773)
```





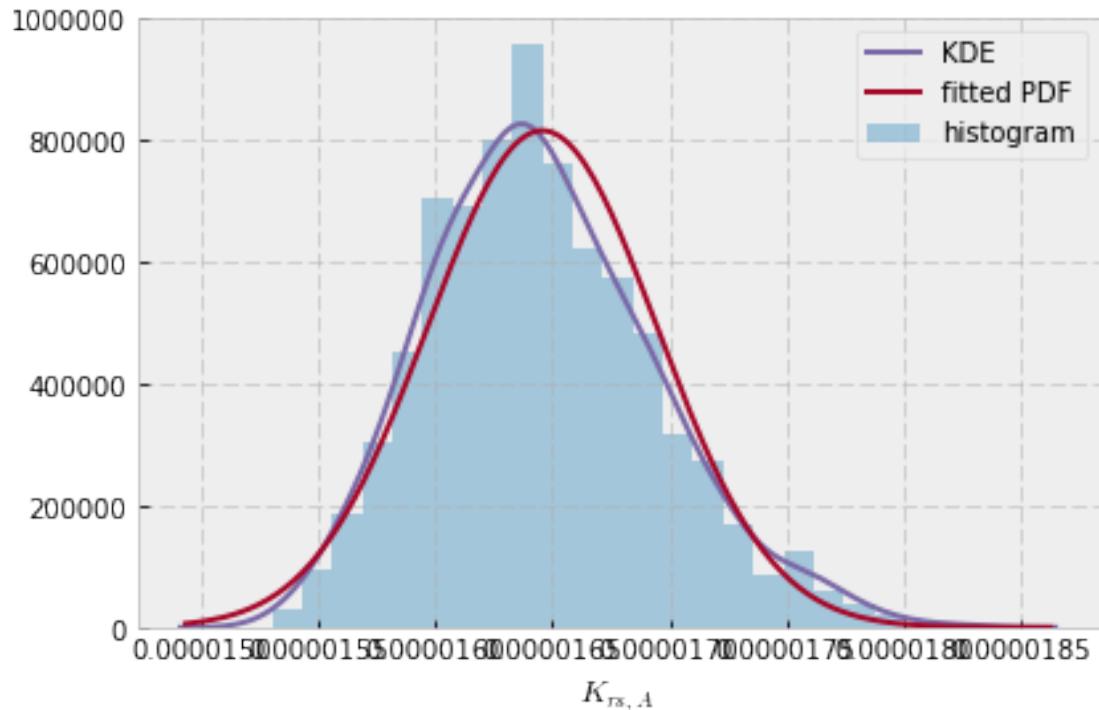
```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=1.6691476390858256e-05, scale=6.202358814984679e-07)
DescribeResult(nobs=1000, minmax=(1.4743979992778038e-05, 1.8892155842153047e-05)
               mean=1.6691476390858256e-05, variance=3.8507762632450605e-13,
               skewness=0.12025414509162909, kurtosis=0.07608887204391257)
```

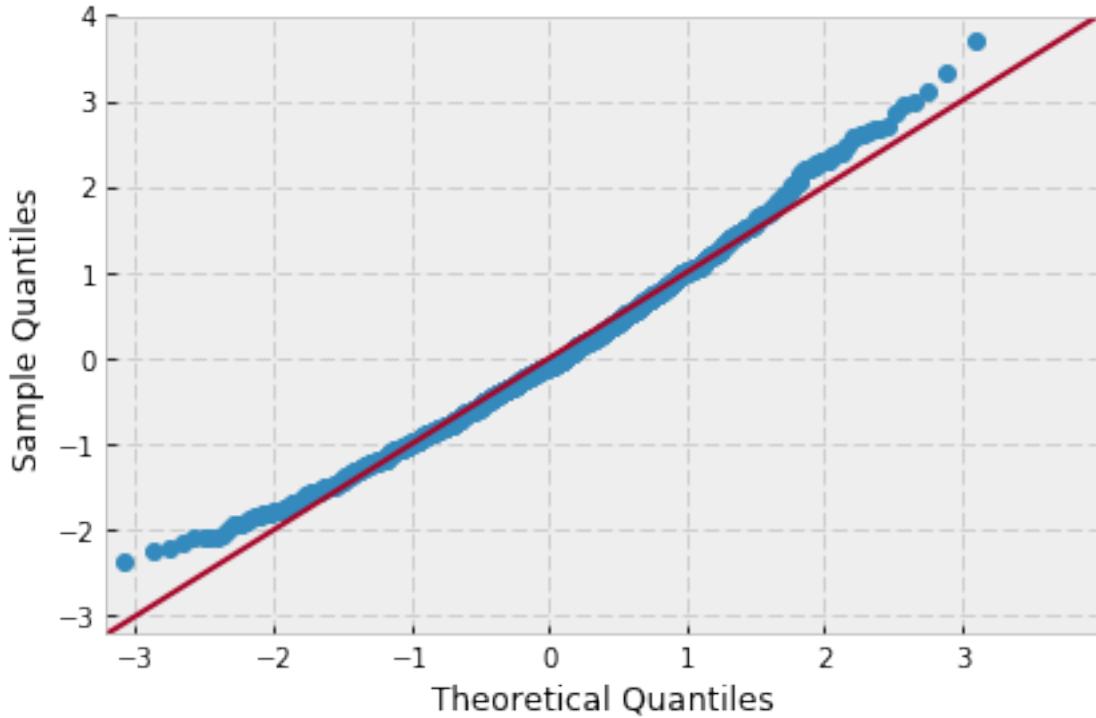


```

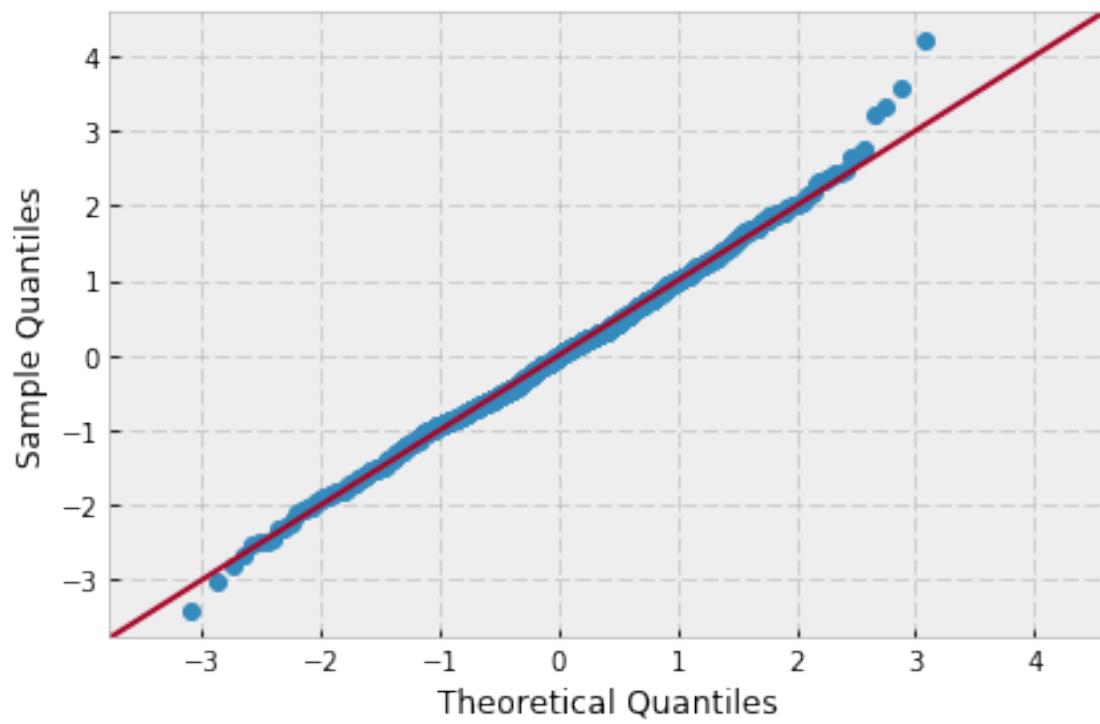
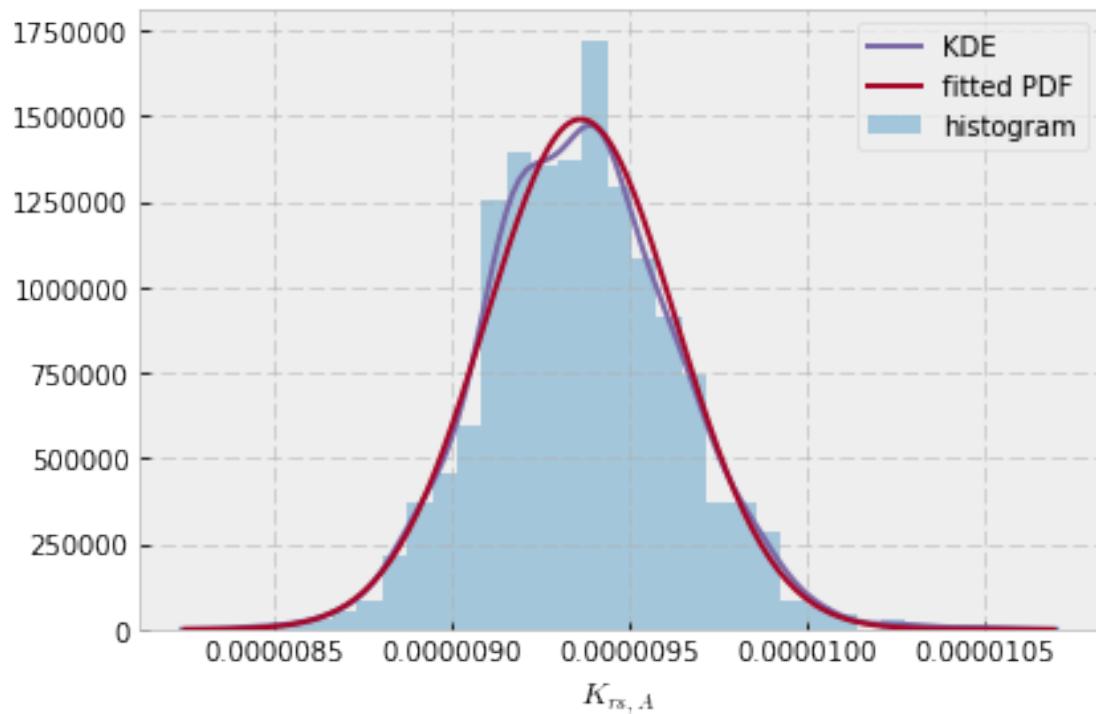
variable name: maxB
variable value: 4.0
distribution: normal(shape=(), loc=1.6456570281519693e-05, scale=4.88754329968327e-07)
DescribeResult(nobs=1000, minmax=(1.5300450423460153e-05, 1.8258780318695676e-05)
               mean=1.6456570281519693e-05, variance=2.391199149777661e-13,
               skewness=0.4441490961230089, kurtosis=0.11813317823265601)

```





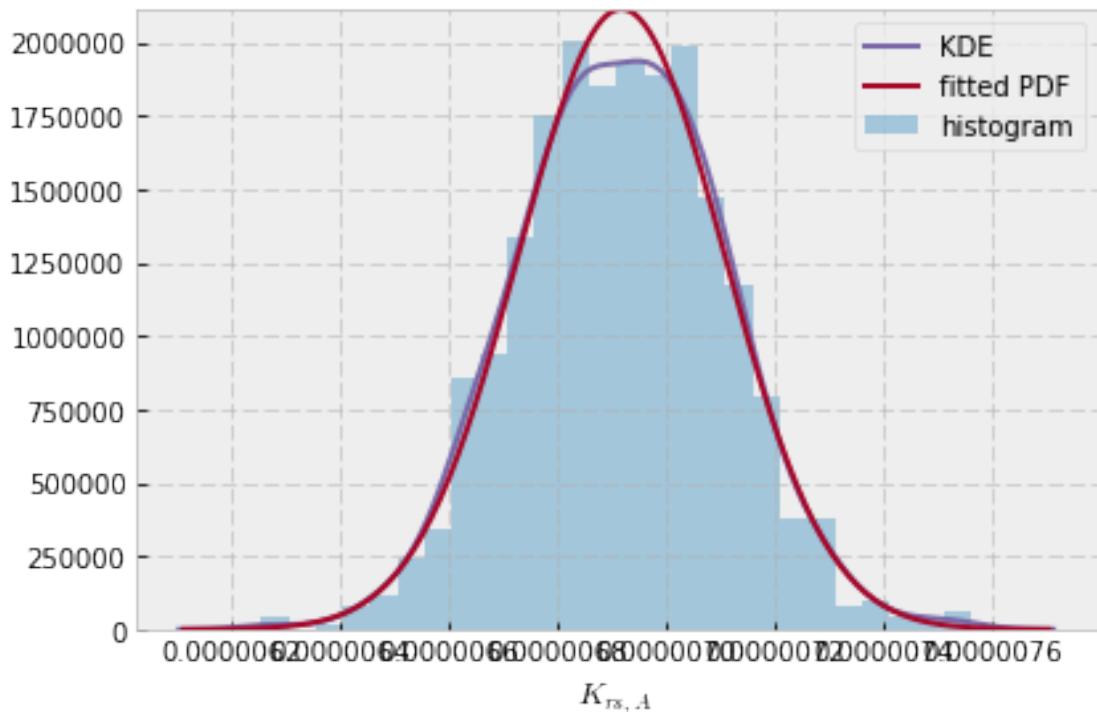
```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(), loc=9.364647330248162e-06, scale=2.6775829523028276e-07)
DescribeResult(nobs=1000, minmax=(8.4543102620387719e-06, 1.0491290607702414e-05)
               mean=9.3646473302481616e-06, variance=7.1766270935562814e-14,
               skewness=0.17327650600193725, kurtosis=0.37059456595138585)
```

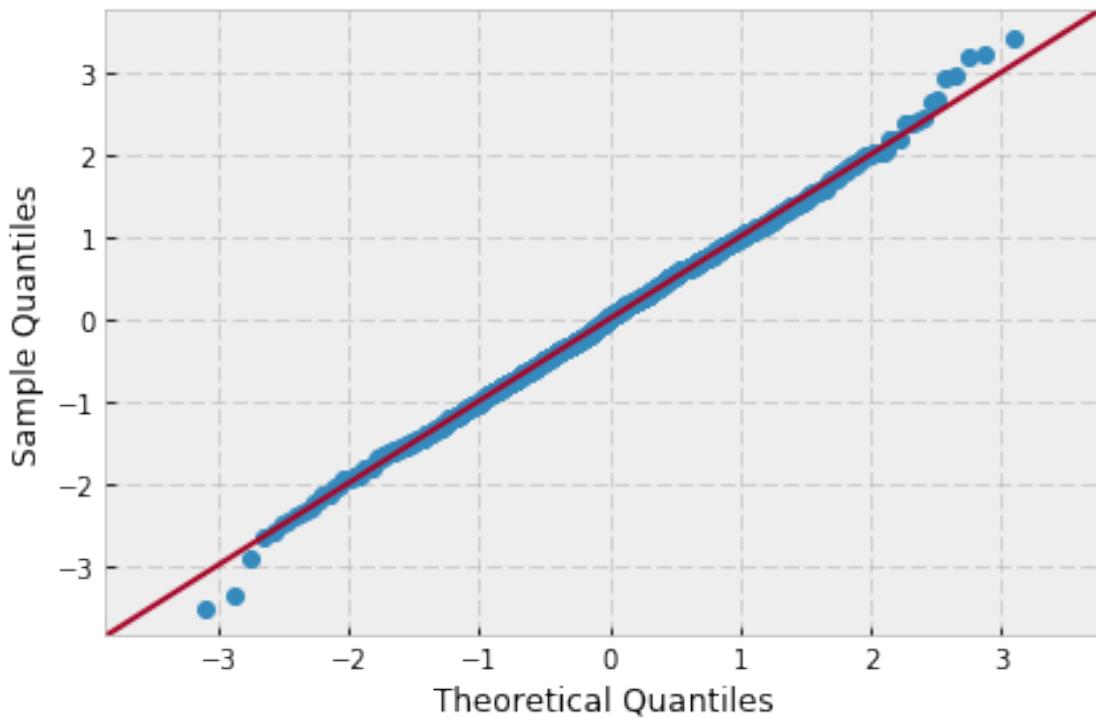


```

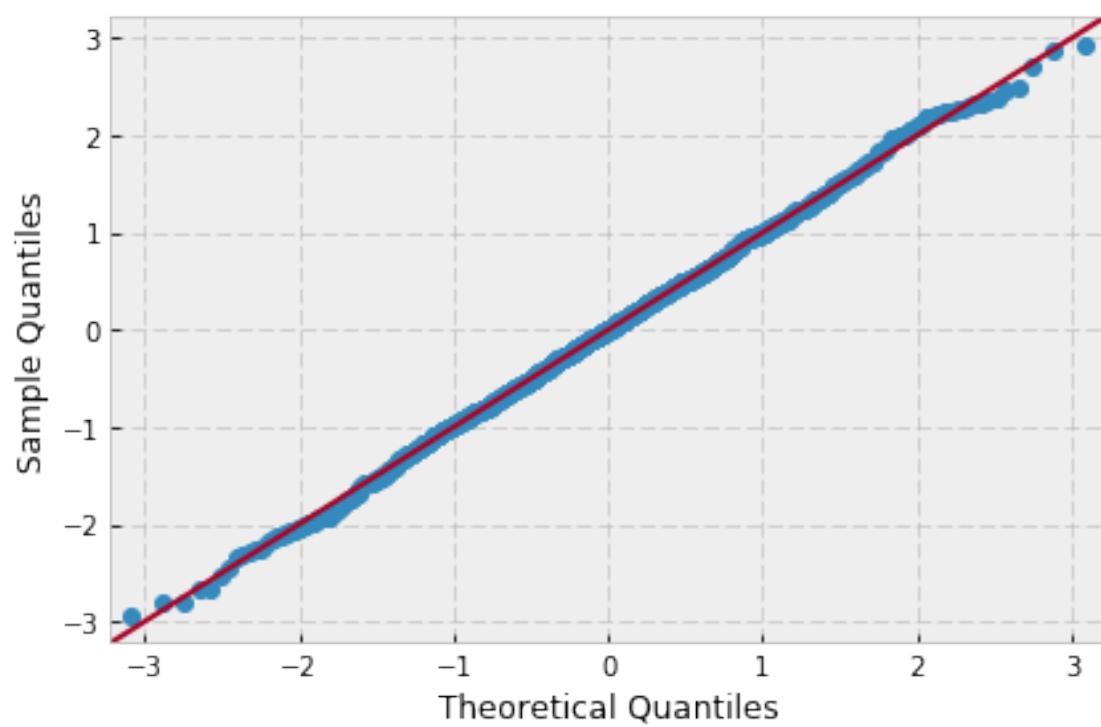
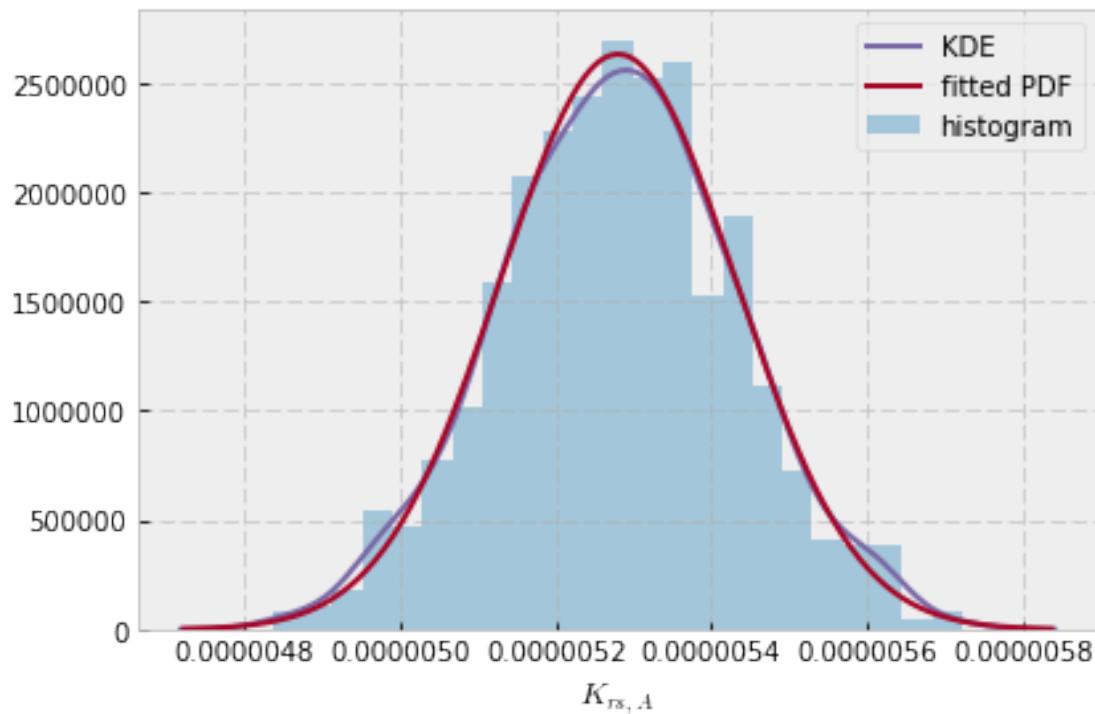
variable name: maxB
variable value: 12.0
distribution: normal(shape=(), loc=6.9184969641955675e-06, scale=1.8878278573184392e-07)
DescribeResult(nobs=1000, minmax=(6.2554313352605653e-06, 7.5622213337299514e-06)
               mean=6.9184969641955675e-06, variance=3.5674614803478779e-14,
               skewness=0.02425490787236573, kurtosis=0.10872789633379609)

```





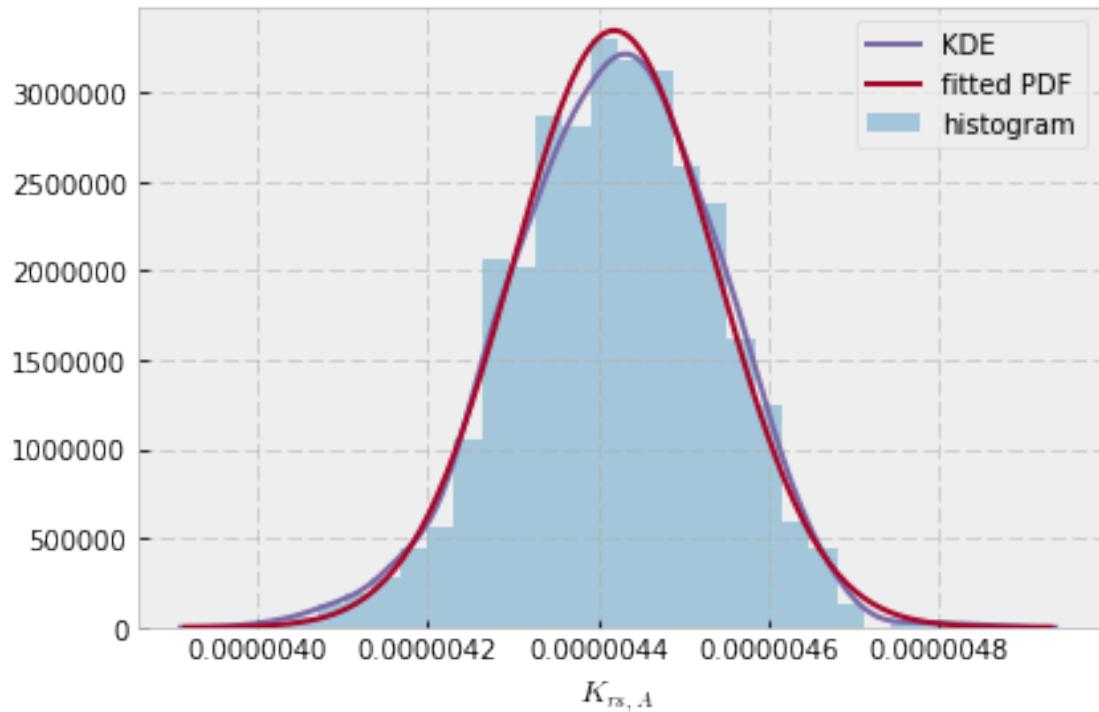
```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(),loc=5.279699532671825e-06, scale=1.5173723726224346e-07)
DescribeResult(nobs=1000, minmax=(4.8355088572763309e-06, 5.7226379977095728e-06)
               mean=5.2796995326718251e-06, variance=2.3047236408386749e-14,
               skewness=-0.006677422359127417, kurtosis=-0.10580272090157106)
```

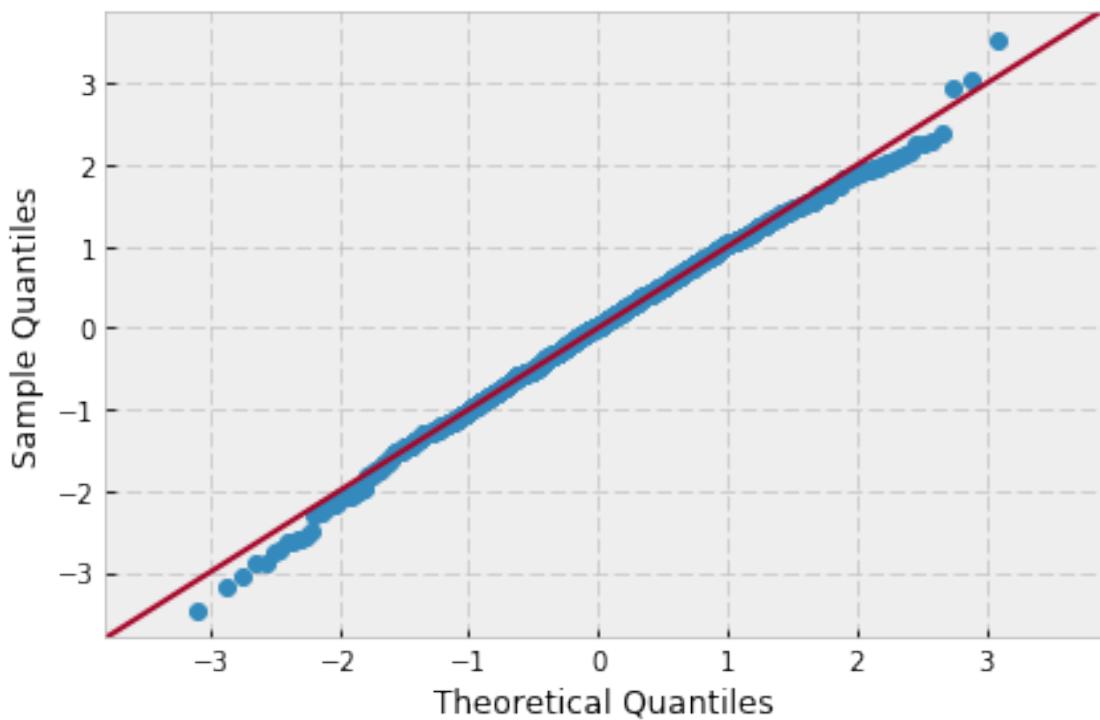


```

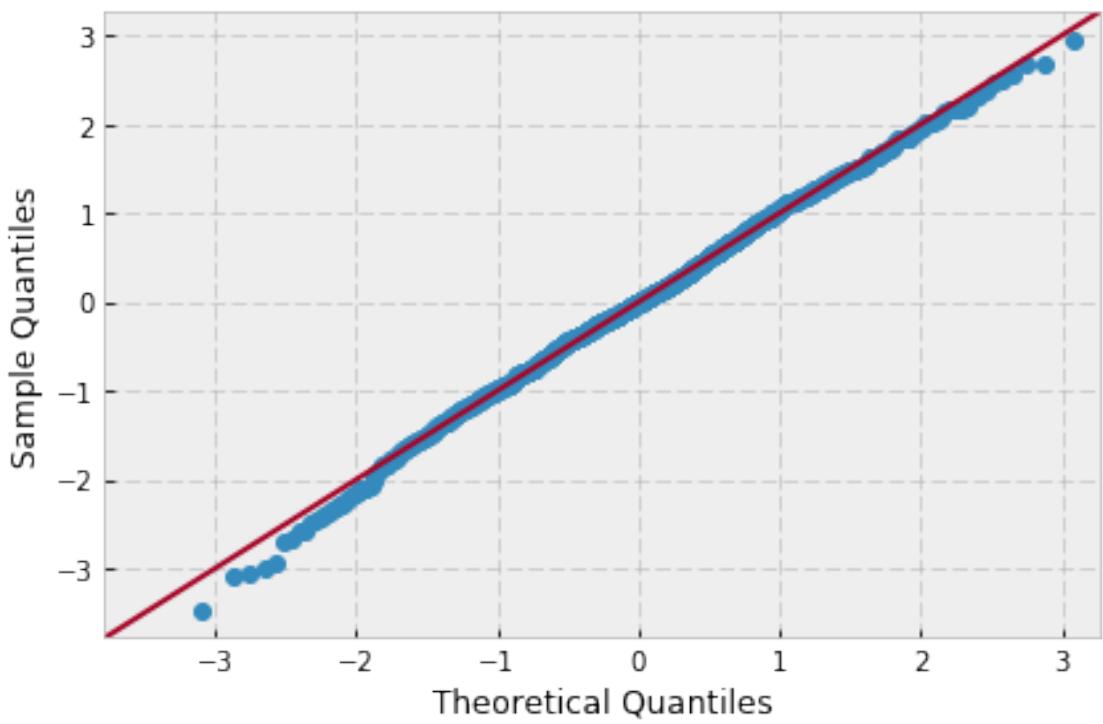
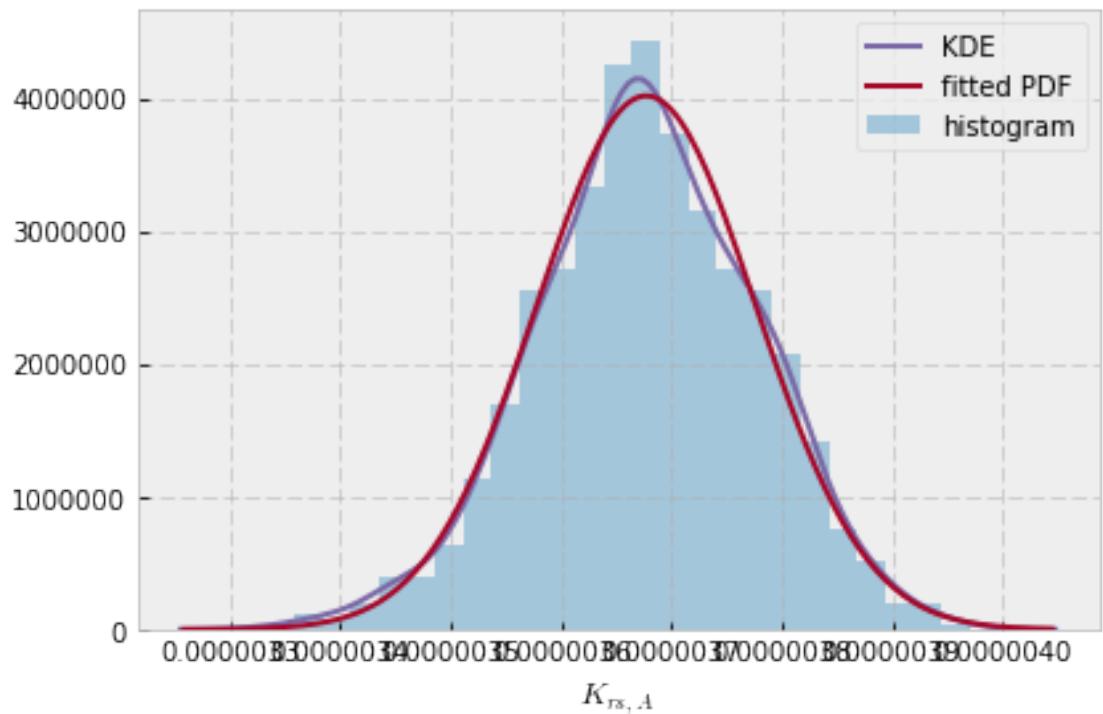
variable name: maxB
variable value: 20.0
distribution: normal(shape=(), loc=4.419154291395491e-06, scale=1.192157426402423e-07)
DescribeResult(nobs=1000, minmax=(4.006853740134043e-06, 4.8402156046159532e-06)
               mean=4.419154291395491e-06, variance=1.4226619913177662e-14,
               skewness=-0.17093636370294063, kurtosis=0.08916583394224586)

```

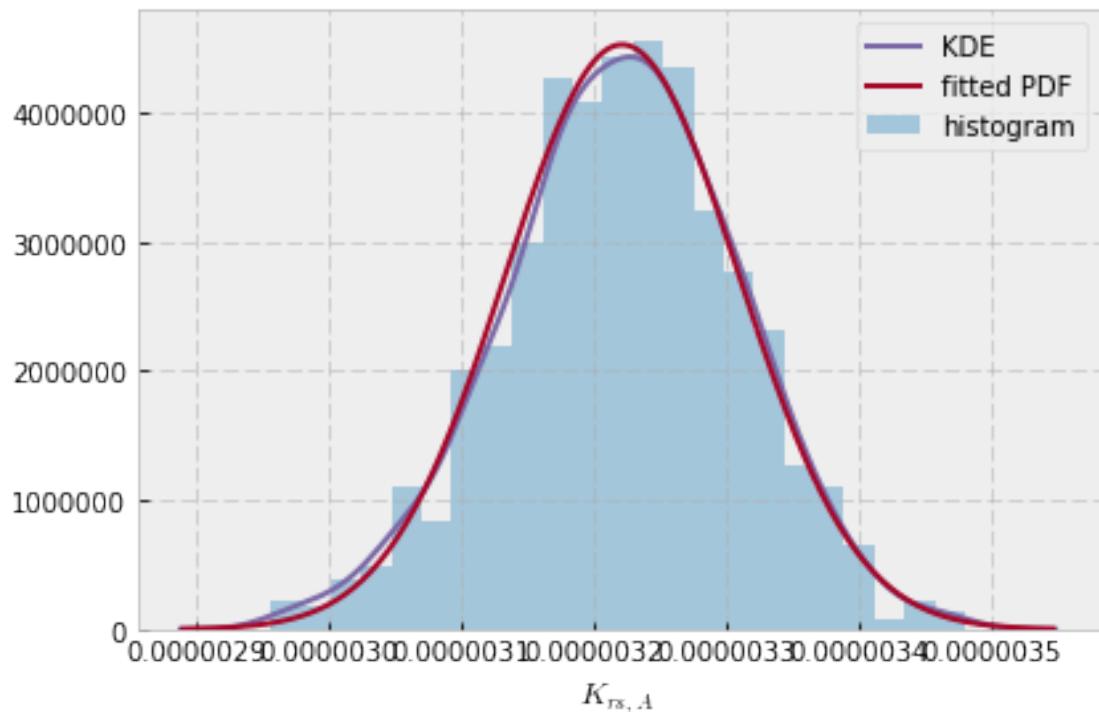


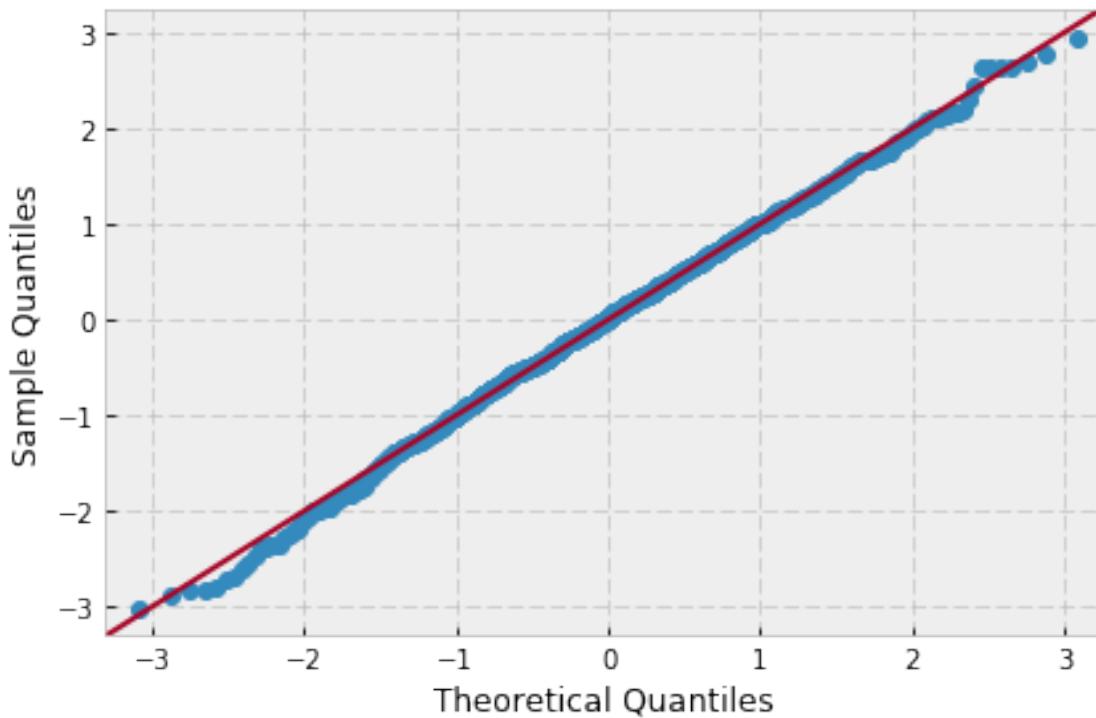


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(),loc=3.677186714913479e-06, scale=9.934968556738704e-08)
DescribeResult(nobs=1000, minmax=(3.3333043333243652e-06, 3.9697081858728368e-06)
               mean=3.6771867149134789e-06, variance=9.8802402626012741e-15,
               skewness=-0.13137643039781546, kurtosis=0.061695301947683134)
```

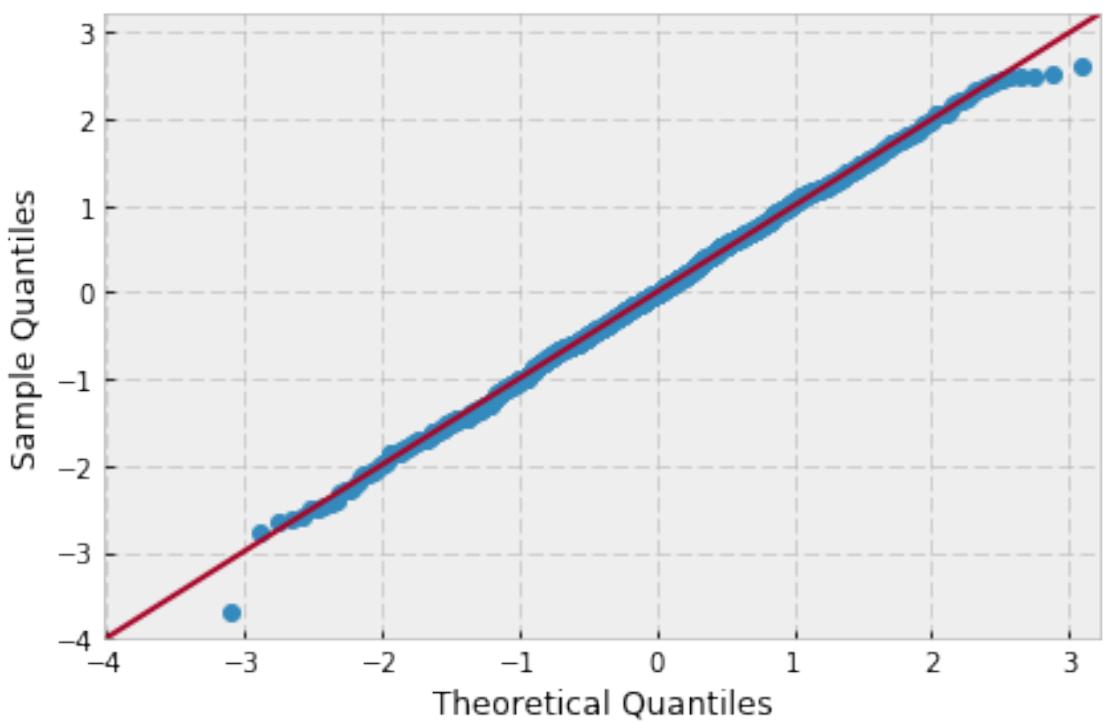
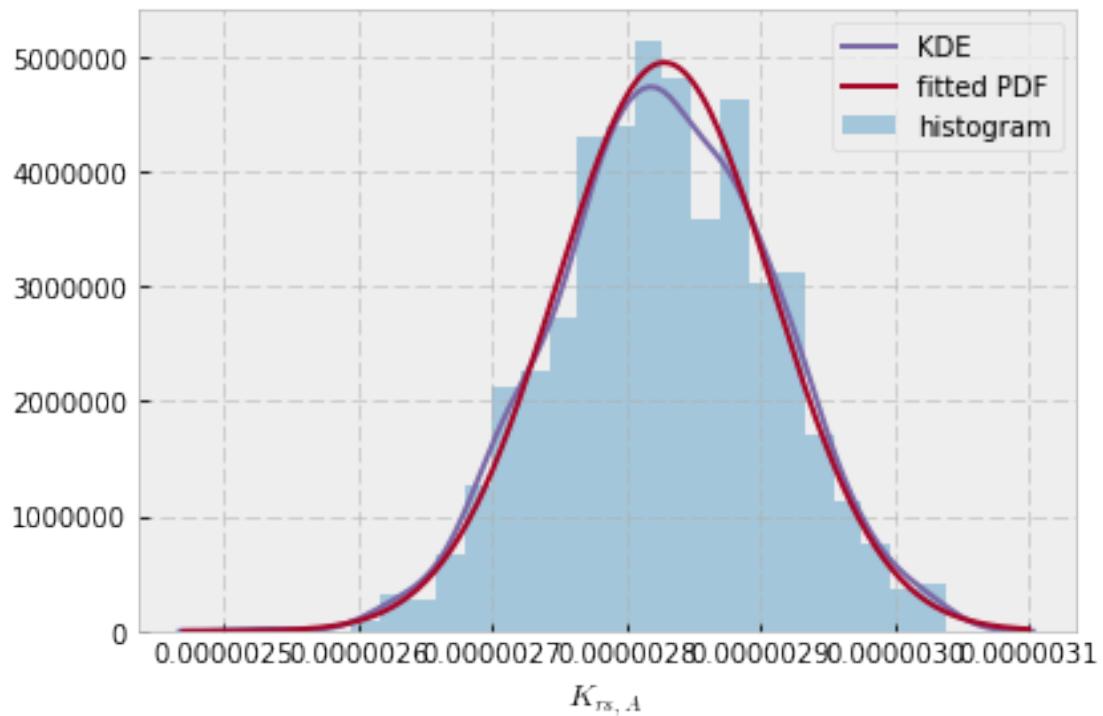


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(), loc=3.2215093636416624e-06, scale=8.812134925935197e-08)
DescribeResult(nobs=1000, minmax=(2.9564783587183725e-06, 3.4802628608344811e-06)
               mean=3.2215093636416624e-06, variance=7.7731453406293214e-15,
               skewness=-0.11710045795278202, kurtosis=-0.004846112556284865)
```

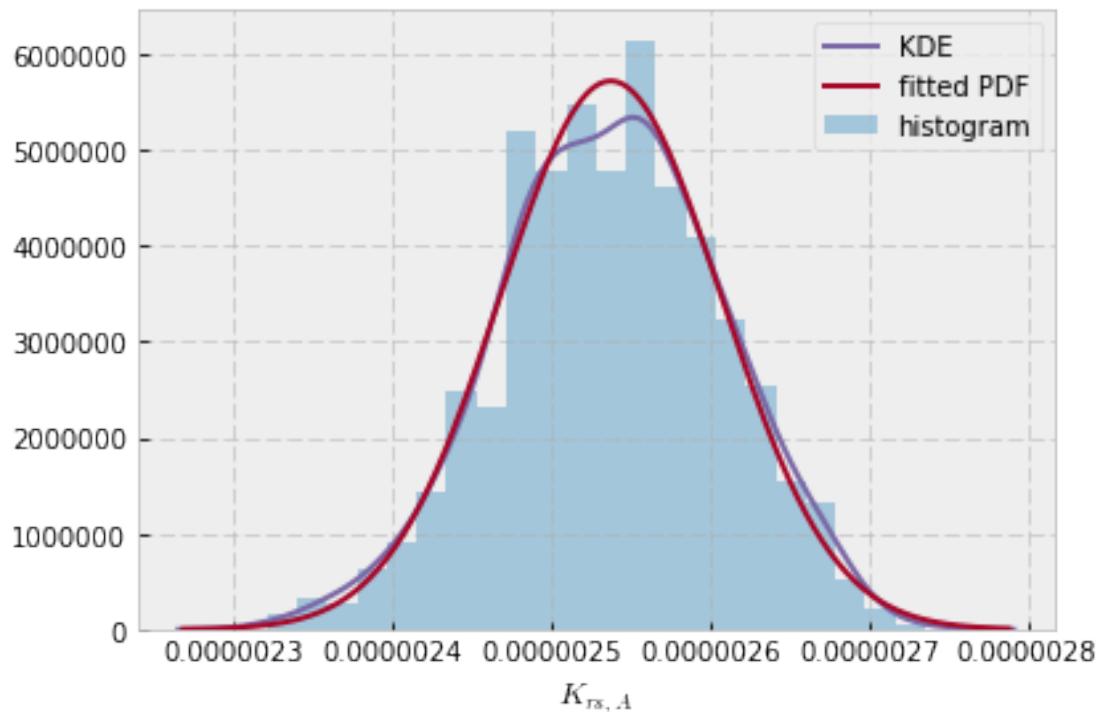


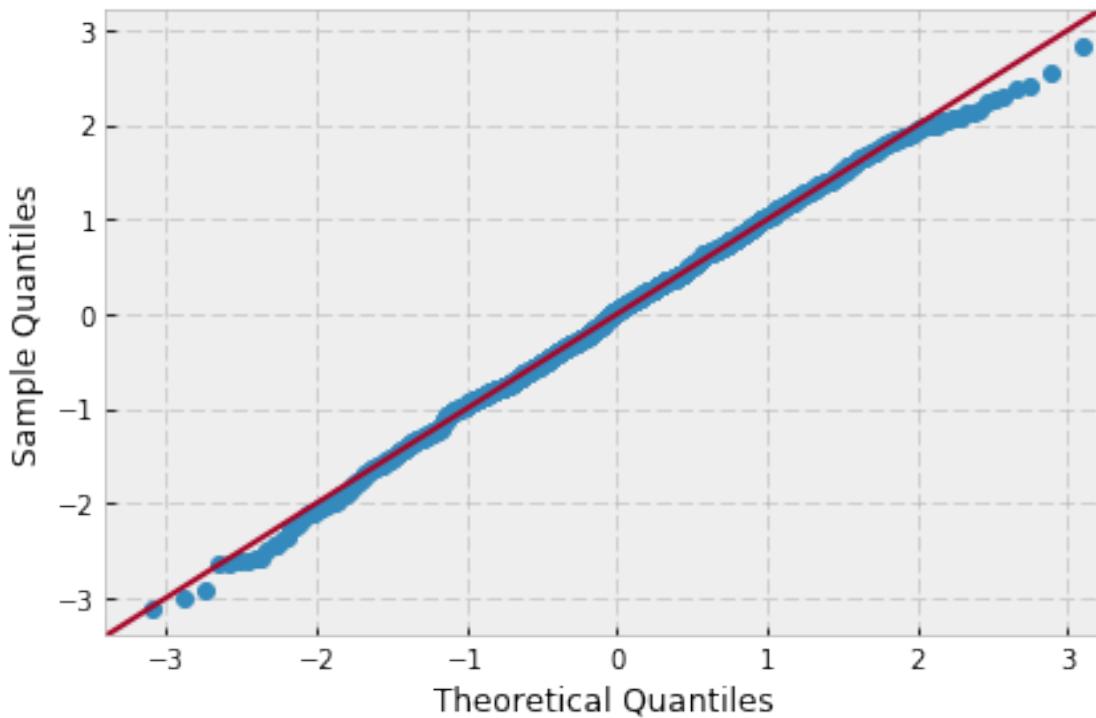


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(),loc=2.8283694497793994e-06, scale=8.066277923640715e-08)
DescribeResult(nobs=1000, minmax=(2.5308558927312633e-06, 3.0392802850276334e-06)
               mean=2.8283694497793994e-06, variance=6.5129969510924482e-15,
               skewness=-0.060410683399348054, kurtosis=-0.1702646637193137)
```

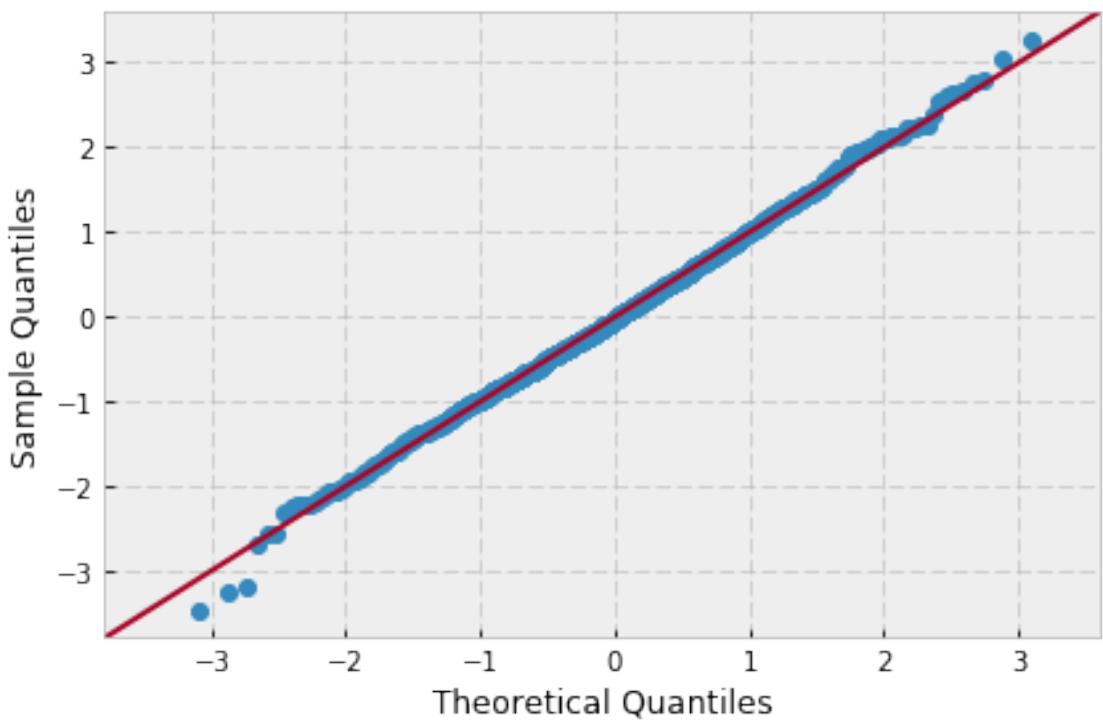
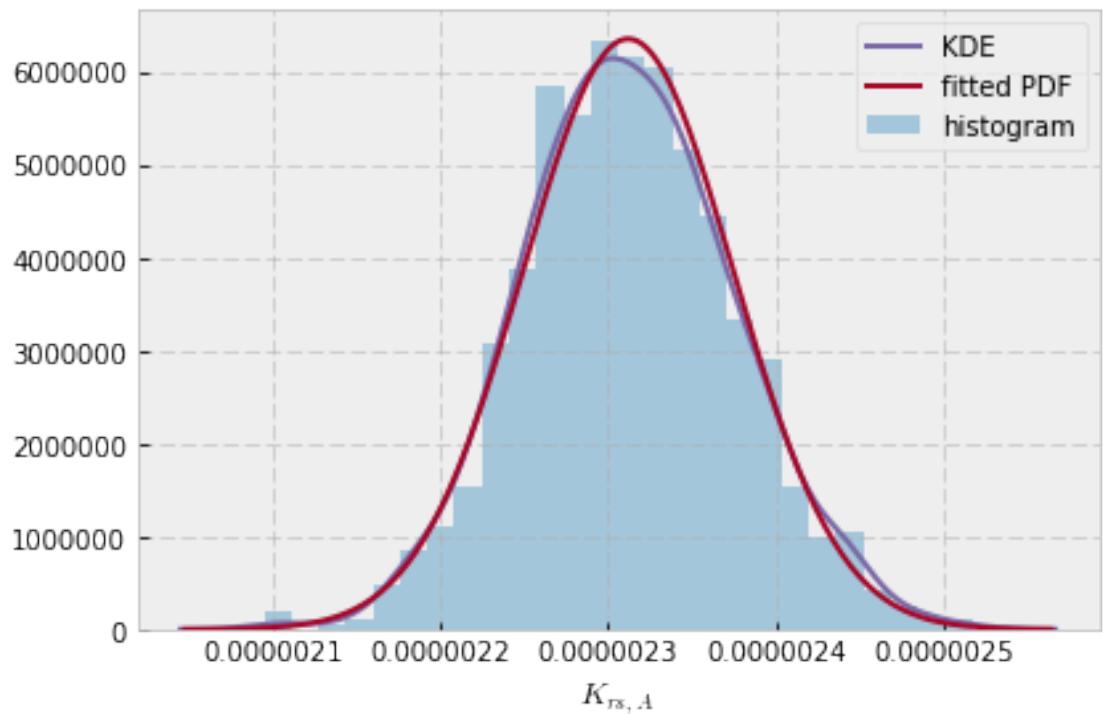


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(), loc=2.5375344694350135e-06, scale=6.974611660141559e-08)
DescribeResult(nobs=1000, minmax=(2.3209053207090186e-06, 2.7356742195749606e-06)
               mean=2.5375344694350135e-06, variance=4.8693901711494098e-15,
               skewness=-0.11507885102370406, kurtosis=-0.1754717946356723)
```





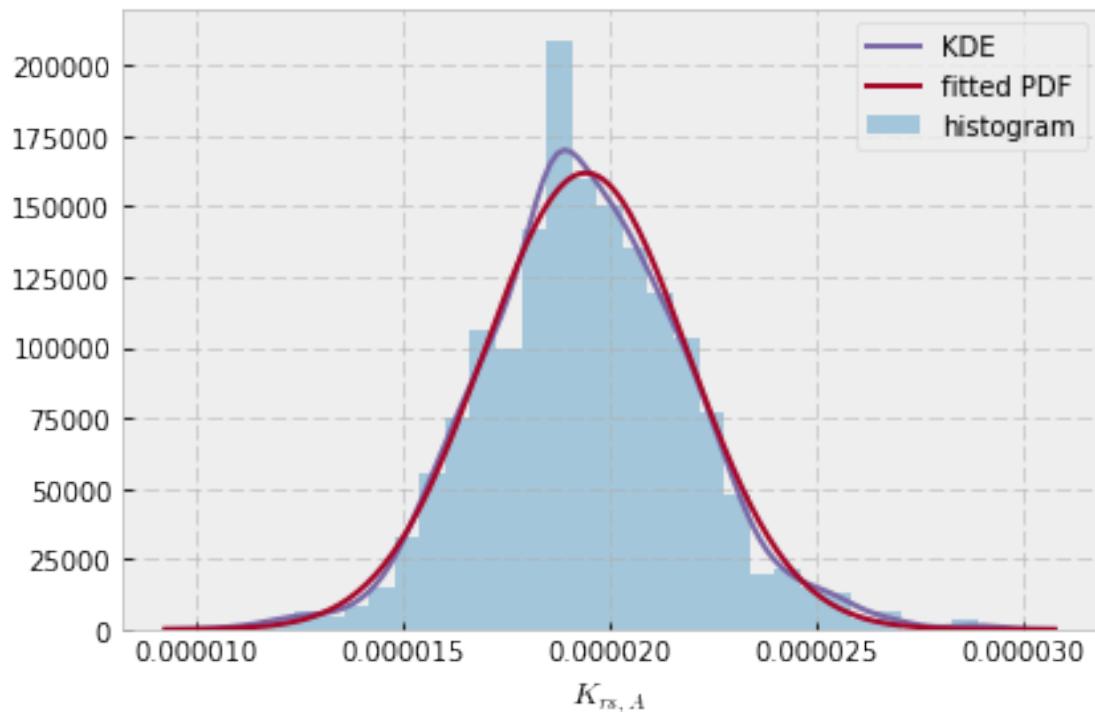
```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(),loc=2.311932079691834e-06, scale=6.280211536154675e-08)
DescribeResult(nobs=1000, minmax=(2.0947904289842009e-06, 2.5168243054353107e-06)
               mean=2.3119320796918339e-06, variance=3.94805374763266e-15,
               skewness=0.07699548973398283, kurtosis=0.0823435492460014)
```

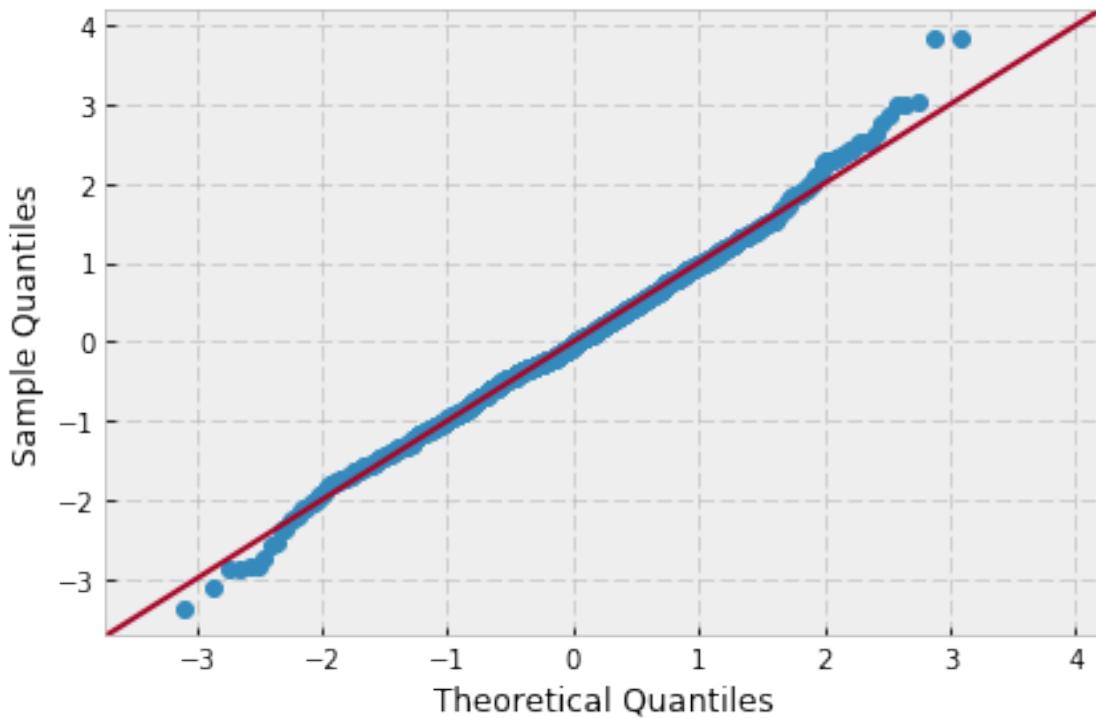


```

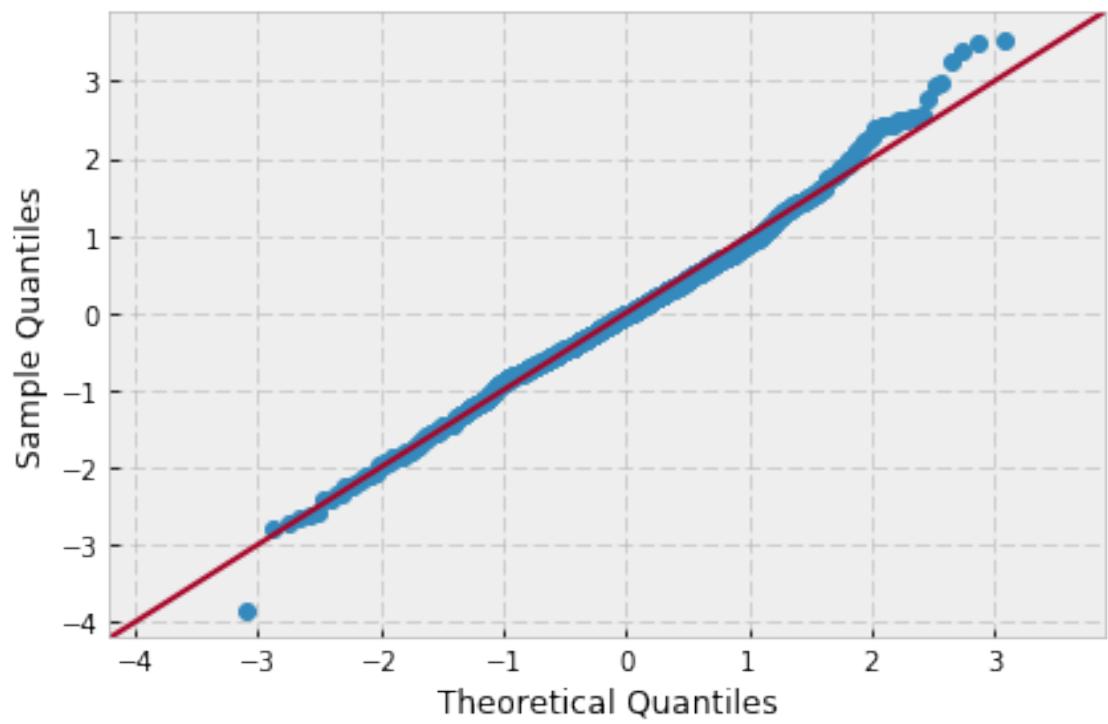
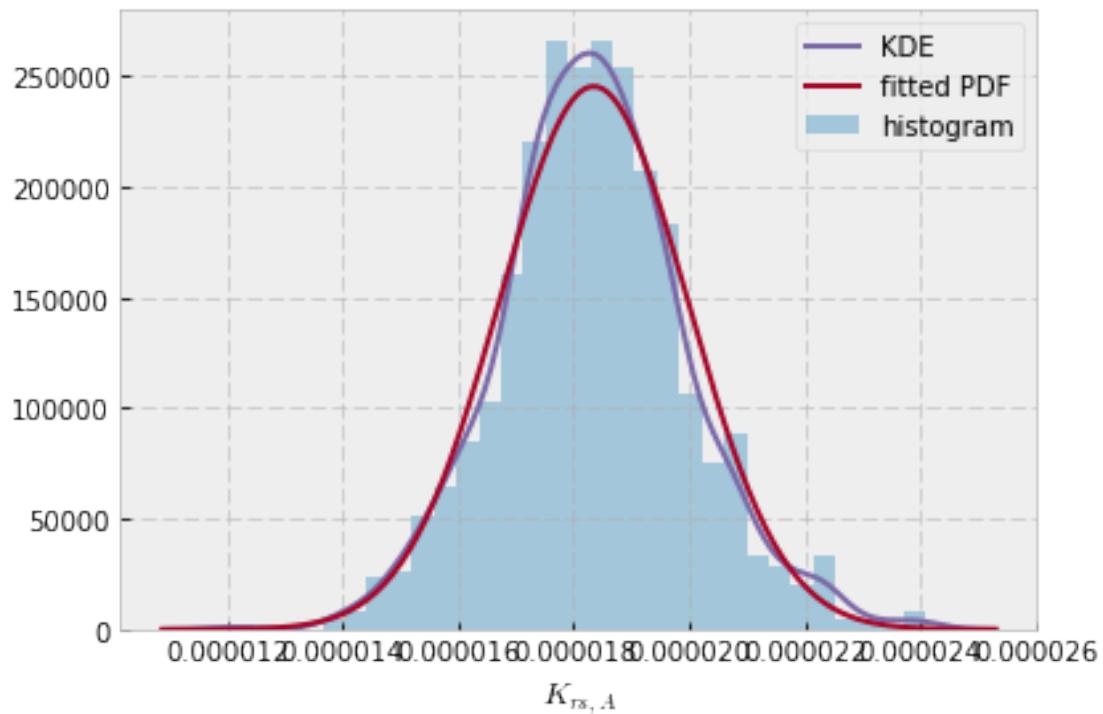
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=1.9435167601354466e-05, scale=2.464182412227445e-06)
DescribeResult(nobs=1000, minmax=(1.1144444322946498e-05, 2.8887147763766002e-05)
               mean=1.9435167601354466e-05, variance=6.0782732339650351e-12,
               skewness=0.15720043849149146, kurtosis=0.5089991224025914)

```

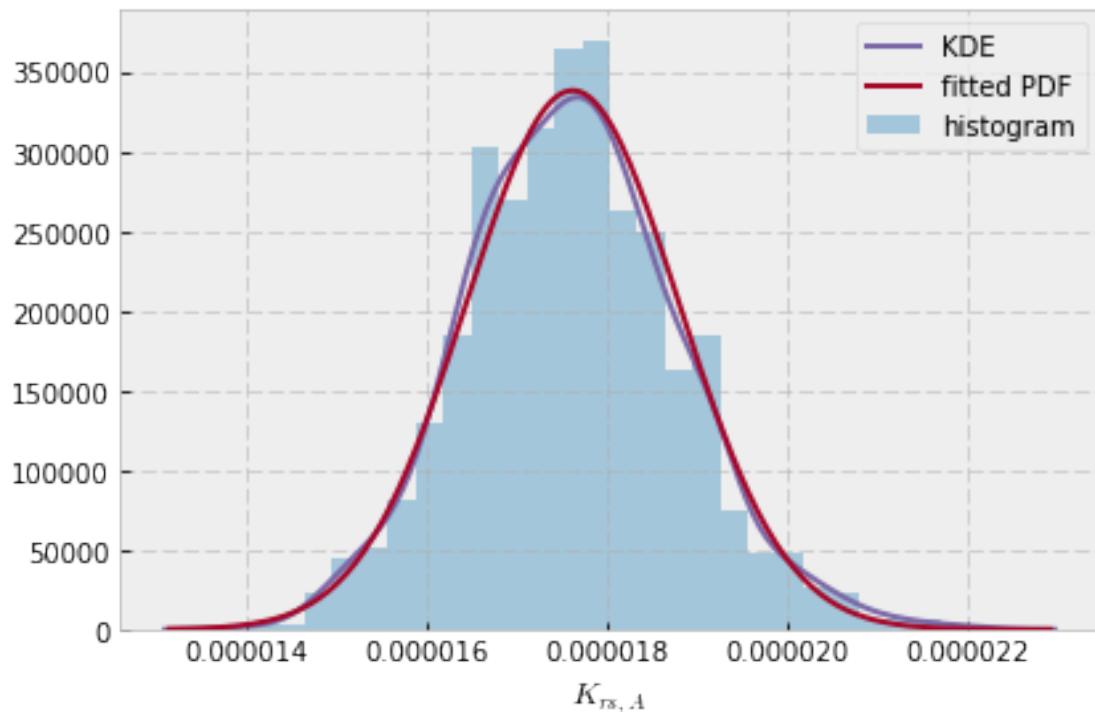


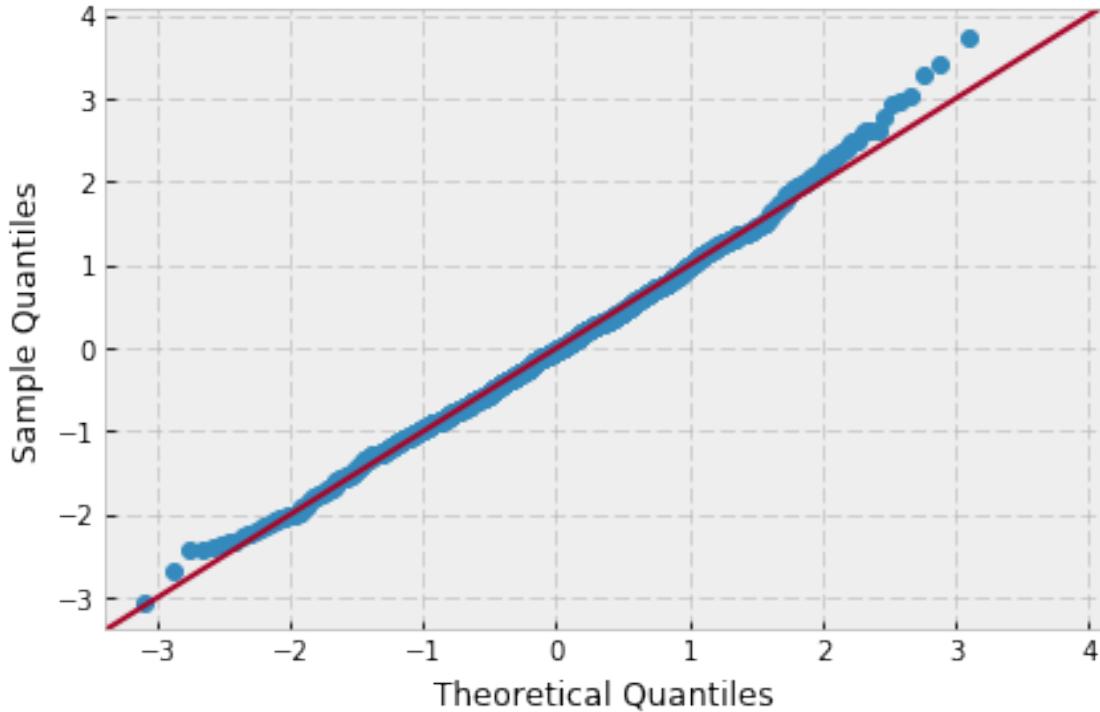


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(),loc=1.8342327010281072e-05, scale=1.6249440201089315e-06)
DescribeResult(nobs=1000, minmax=(1.210394952587236e-05, 2.4075198342368865e-05)
               mean=1.8342327010281072e-05, variance=2.6430861546424179e-12,
               skewness=0.18848479813709035, kurtosis=0.5888563824123545)
```

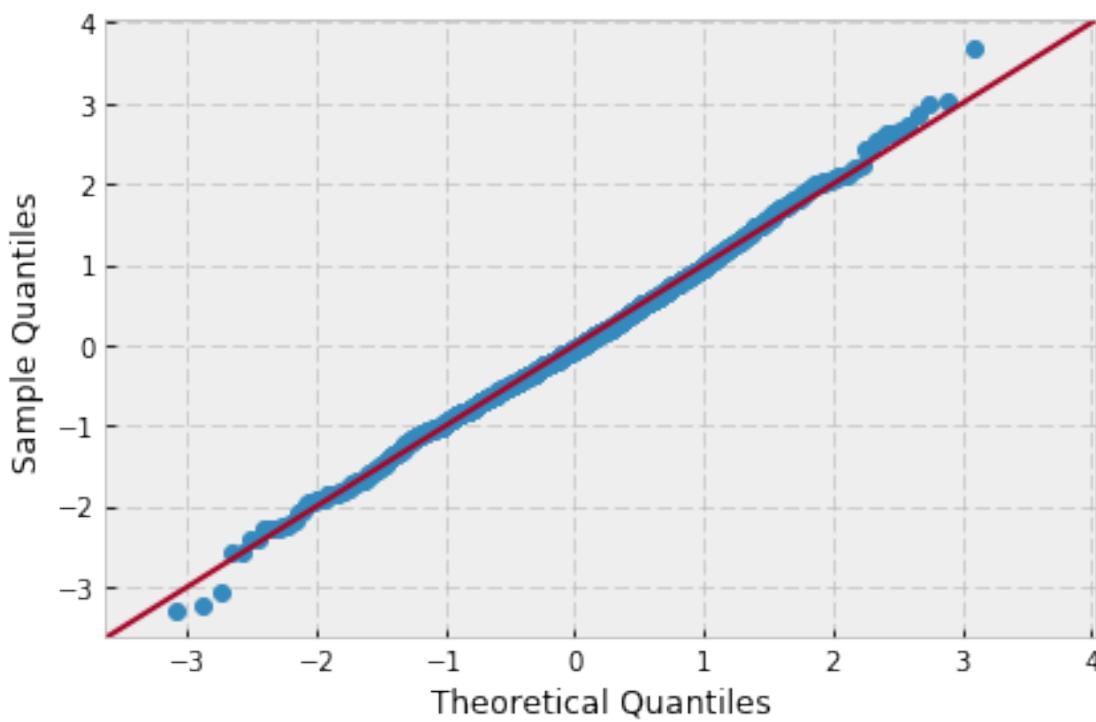
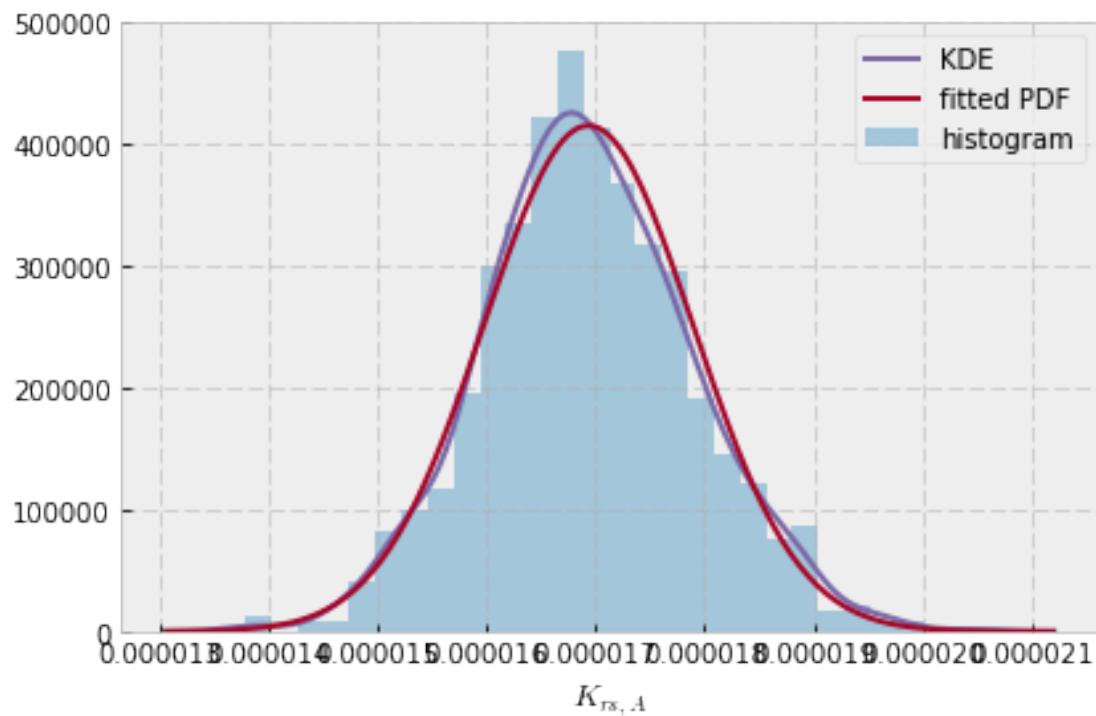


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=1.7618715678530148e-05, scale=1.1794174440142474e-06)
DescribeResult(nobs=1000, minmax=(1.4028344400999934e-05, 2.2033150420892641e-05)
               mean=1.7618715678530148e-05, variance=1.3924179251702704e-12,
               skewness=0.22796895370382697, kurtosis=0.2463166312057785)
```





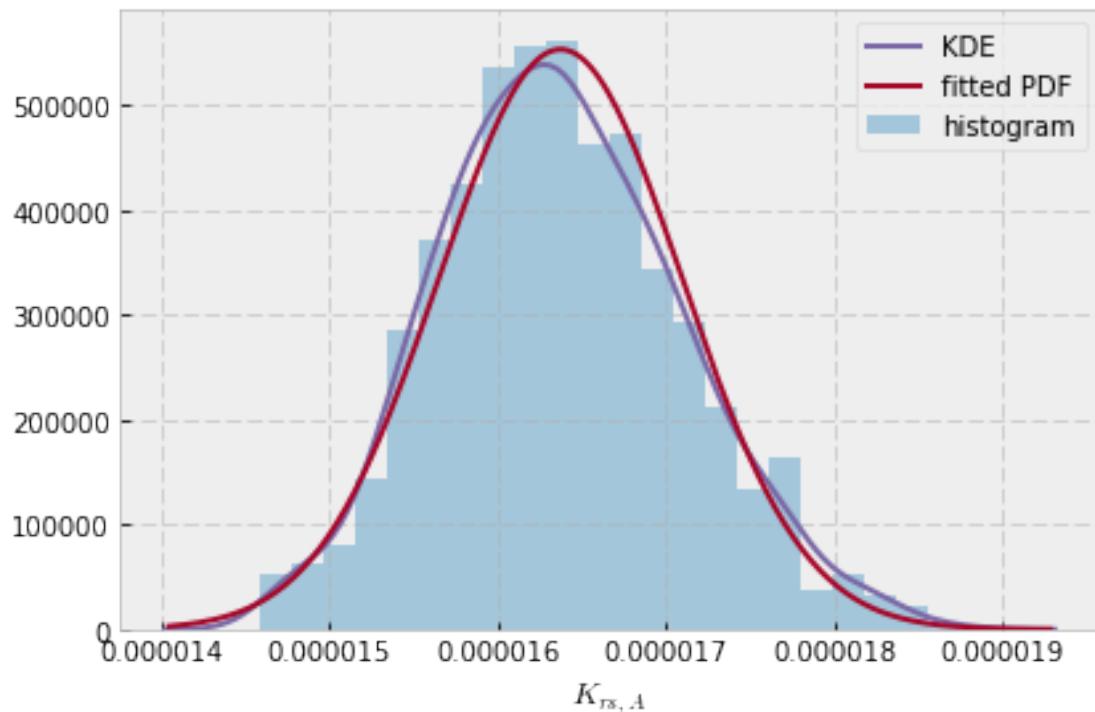
```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=1.693096331825533e-05, scale=9.60293151136992e-07)
DescribeResult(nobs=1000, minmax=(1.3776447392917167e-05, 2.04781475631855e-05)
               mean=1.693096331825533e-05, variance=9.2308602214275659e-13,
               skewness=0.135538788766632, kurtosis=0.2063336763628727)
```

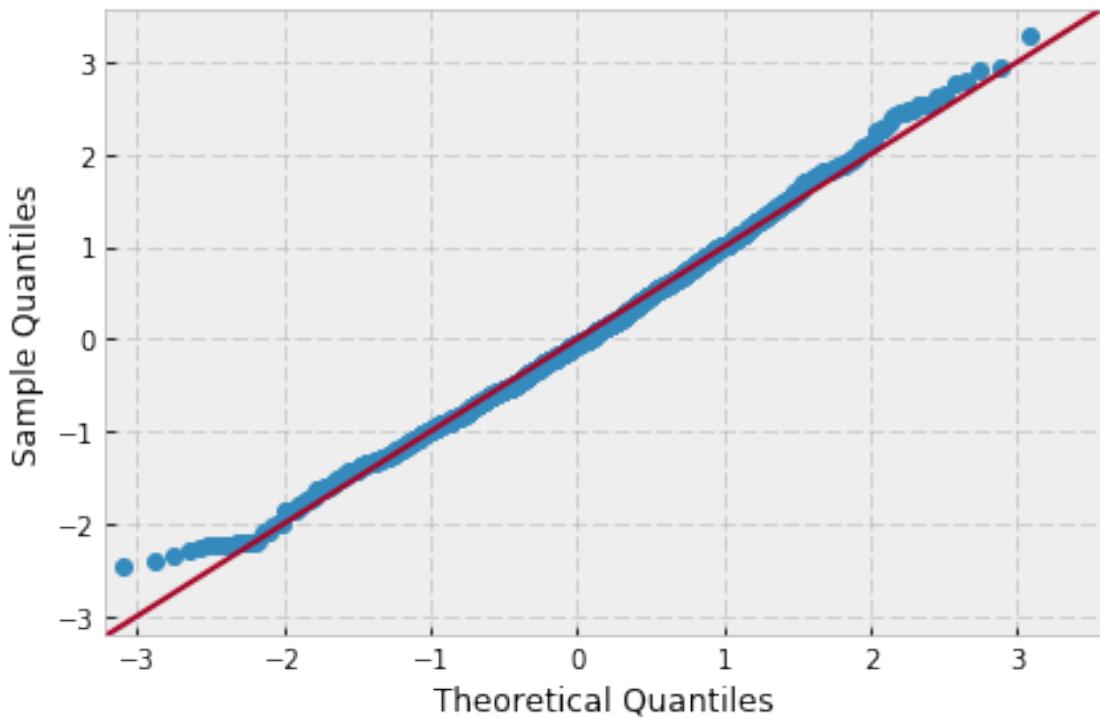


```

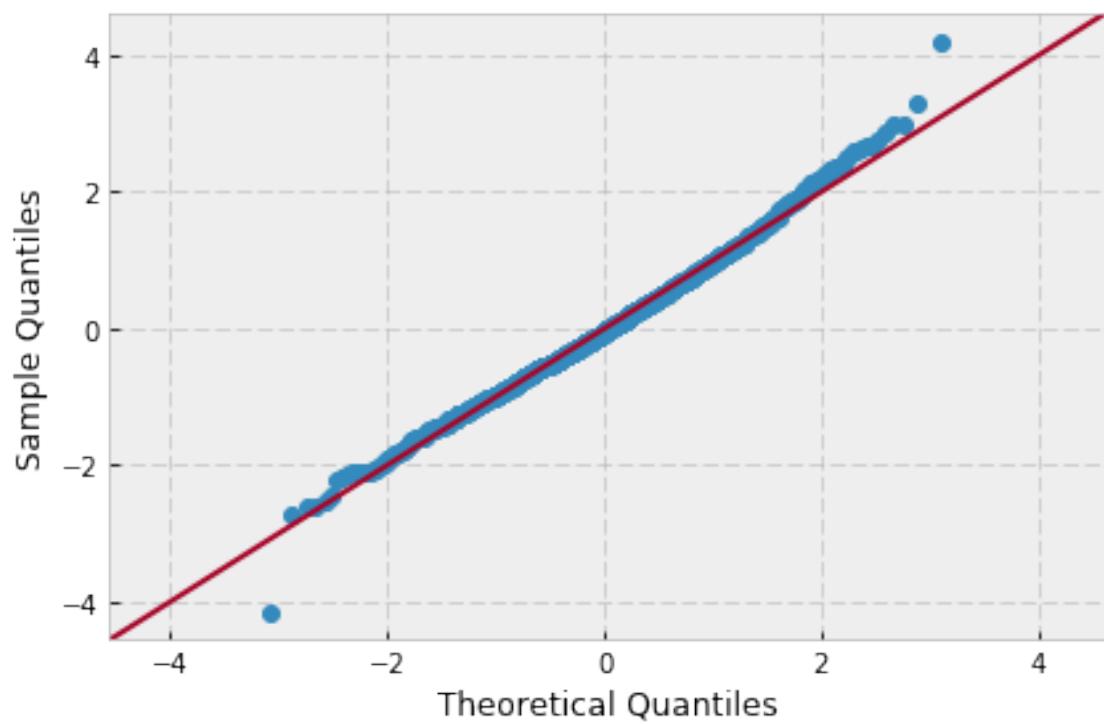
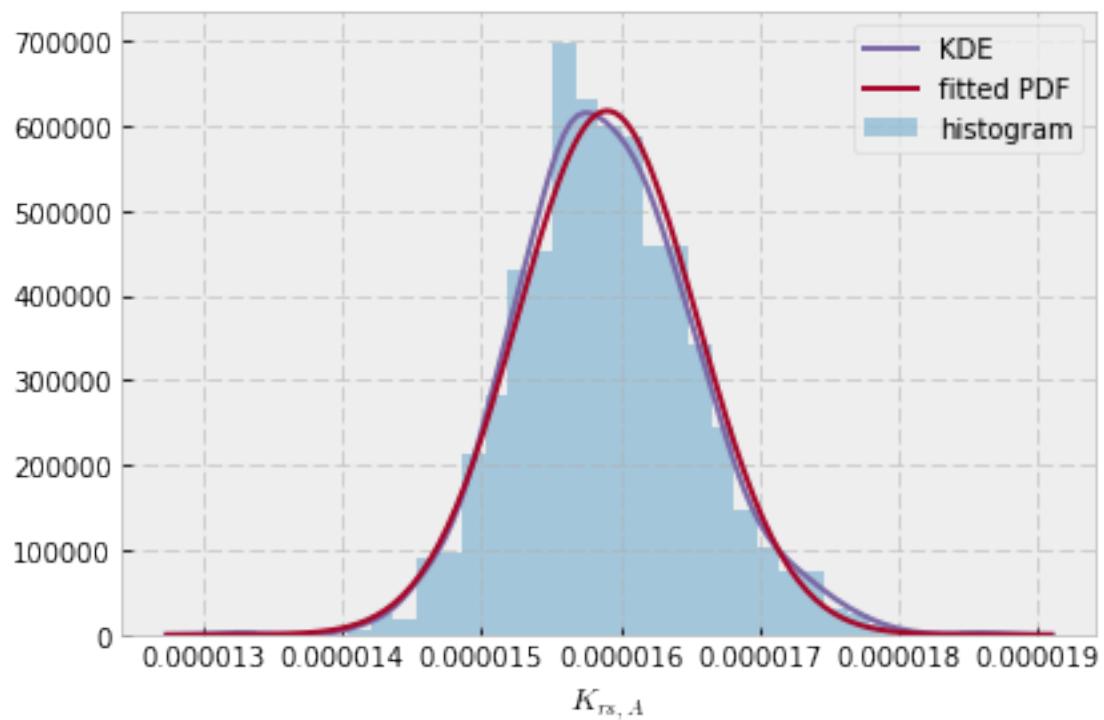
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=1.6370843725478095e-05, scale=7.214787588395908e-07)
DescribeResult(nobs=1000, minmax=(1.4589089379060314e-05, 1.8736028154259917e-05)
               mean=1.6370843725478095e-05, variance=5.2105265210882528e-13,
               skewness=0.2515851788000724, kurtosis=-0.1053407630778671)

```

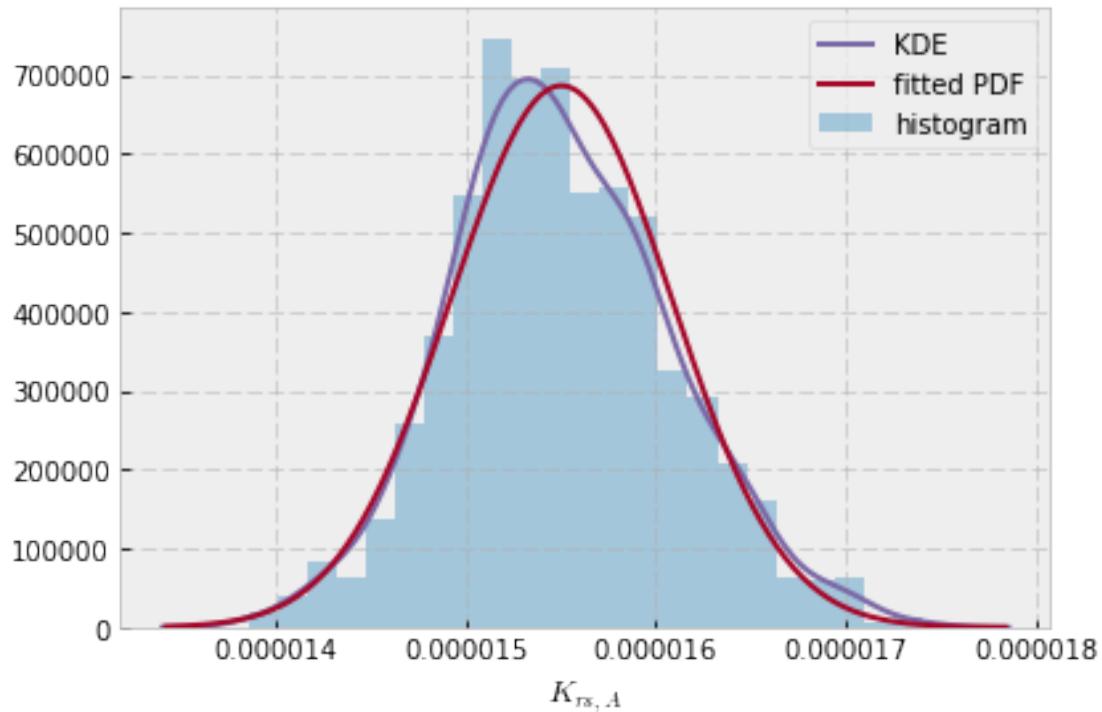


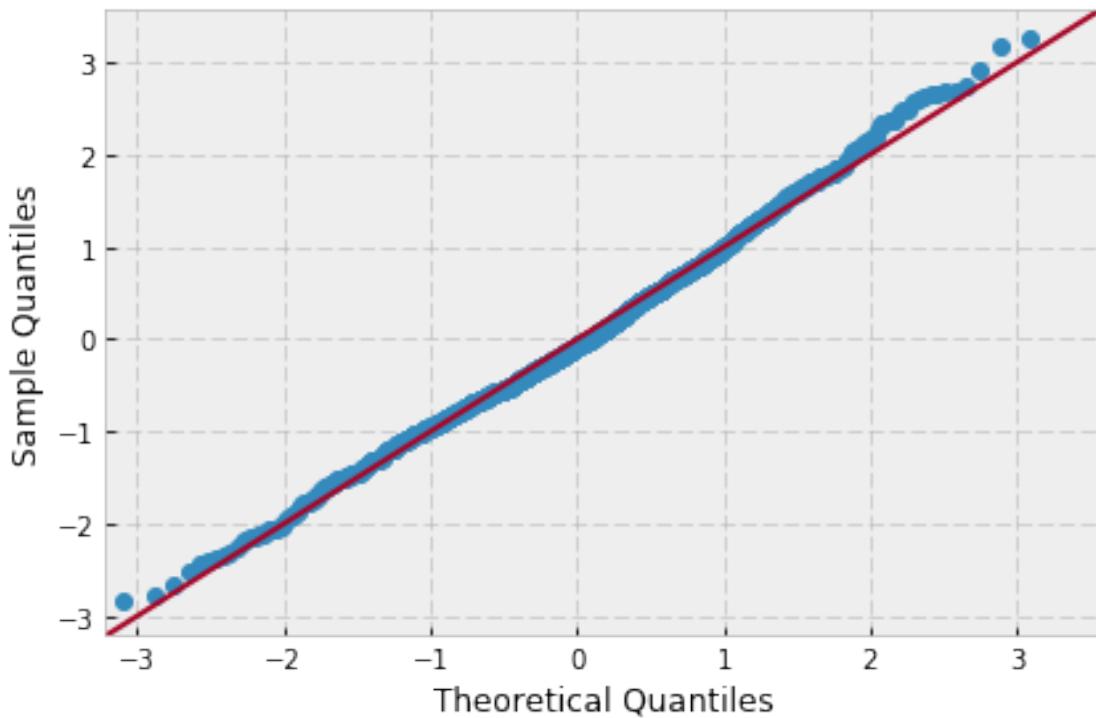


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(),loc=1.590422957073166e-05, scale=6.461804130590507e-07)
DescribeResult(nobs=1000, minmax=(1.3227510341413872e-05, 1.8611244710382243e-05)
               mean=1.5904229570731659e-05, variance=4.1796709331447994e-13,
               skewness=0.24152466857871607, kurtosis=0.47743130253762933)
```

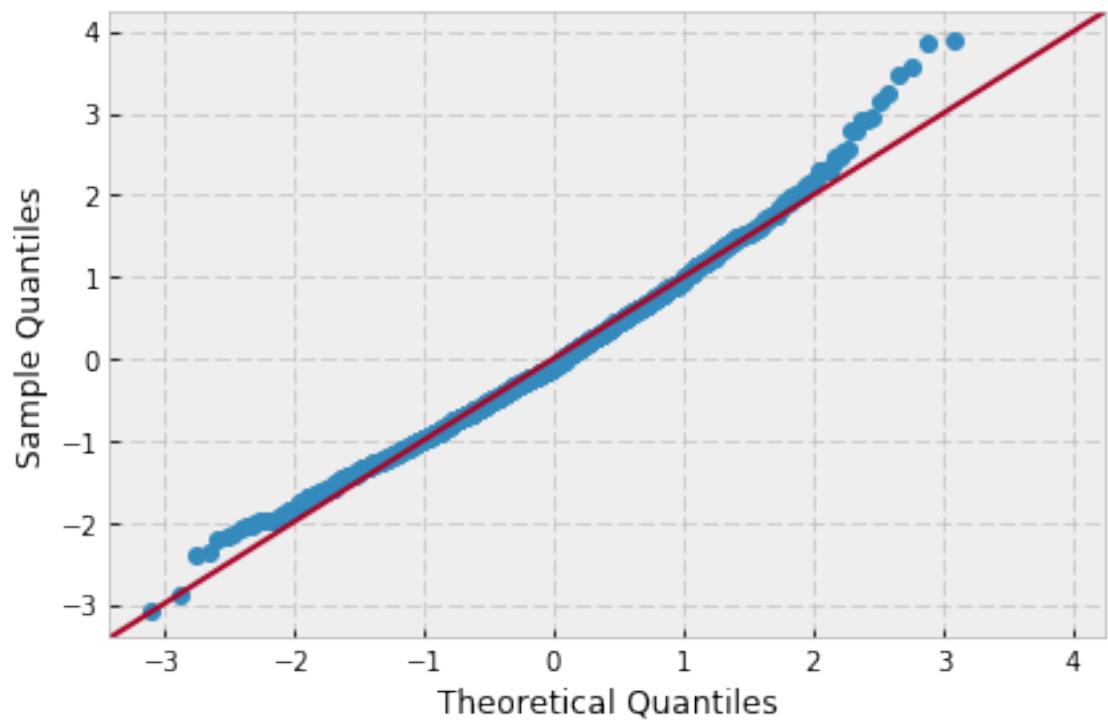
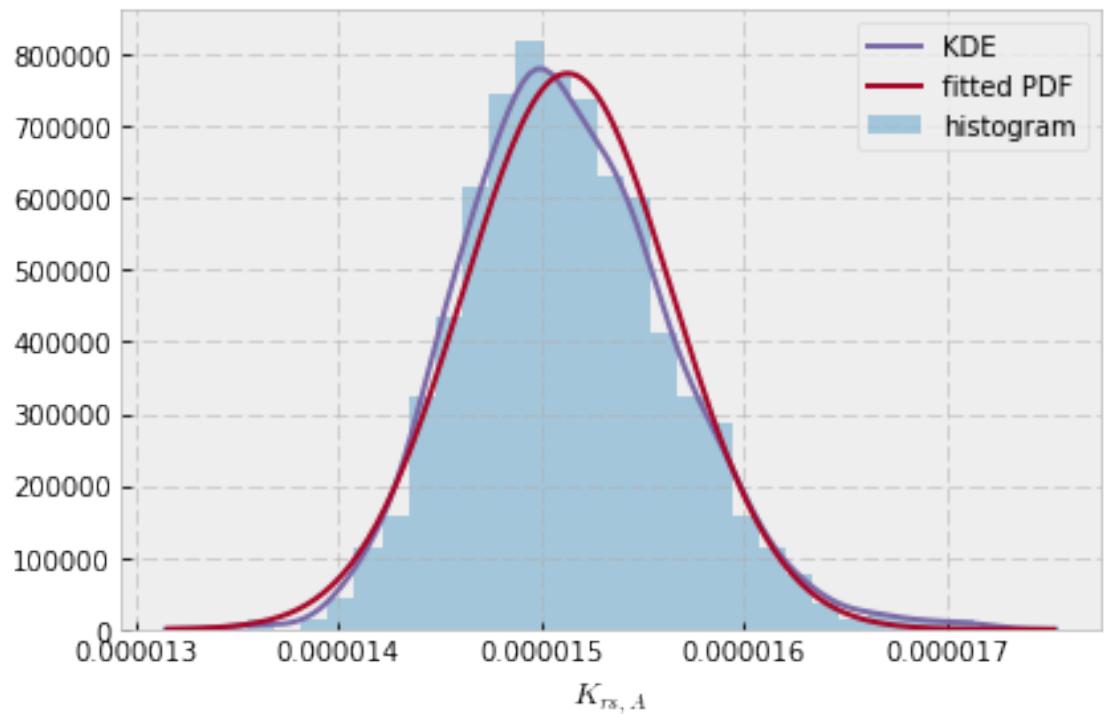


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=1.550295734876567e-05, scale=5.815611675842749e-07)
DescribeResult(nobs=1000, minmax=(1.38594607459055e-05, 1.7400385336298202e-05)
               mean=1.550295734876567e-05, variance=3.3855194358557066e-13,
               skewness=0.23859163585633666, kurtosis=0.053289247139681084)
```





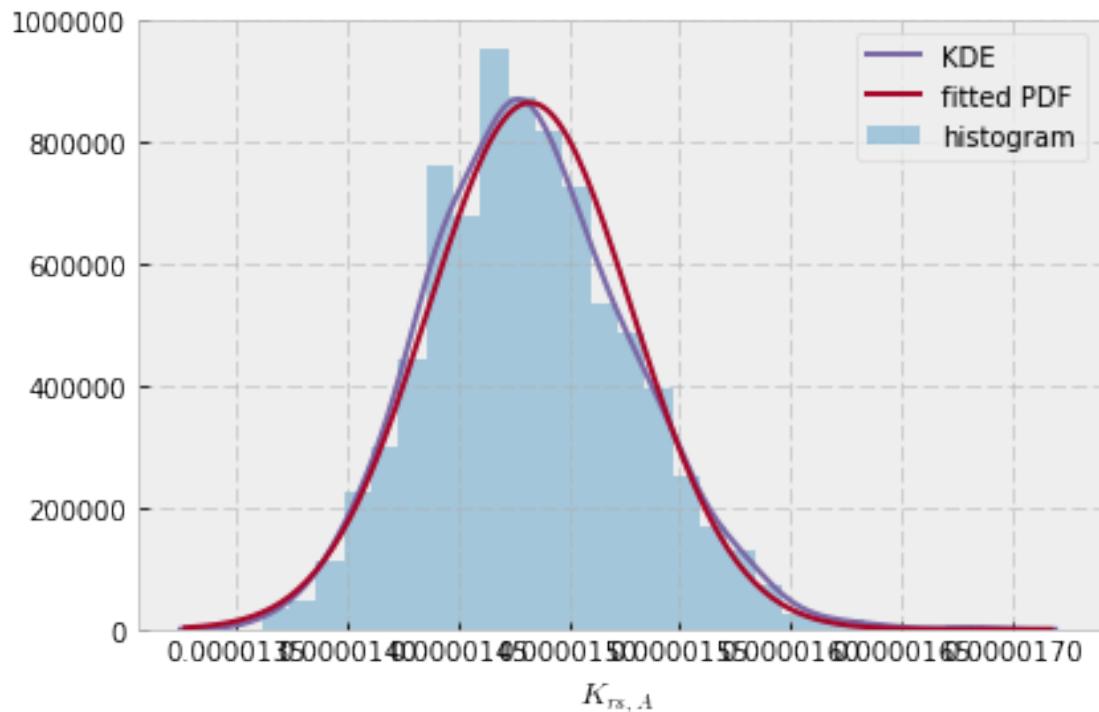
```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(),loc=1.5130935900019624e-05, scale=5.158632816795269e-07)
DescribeResult(nobs=1000, minmax=(1.3549954613942475e-05, 1.7136887946353615e-05)
               mean=1.5130935900019624e-05, variance=2.6638130669186275e-13,
               skewness=0.43088327892516776, kurtosis=0.48228597342910007)
```

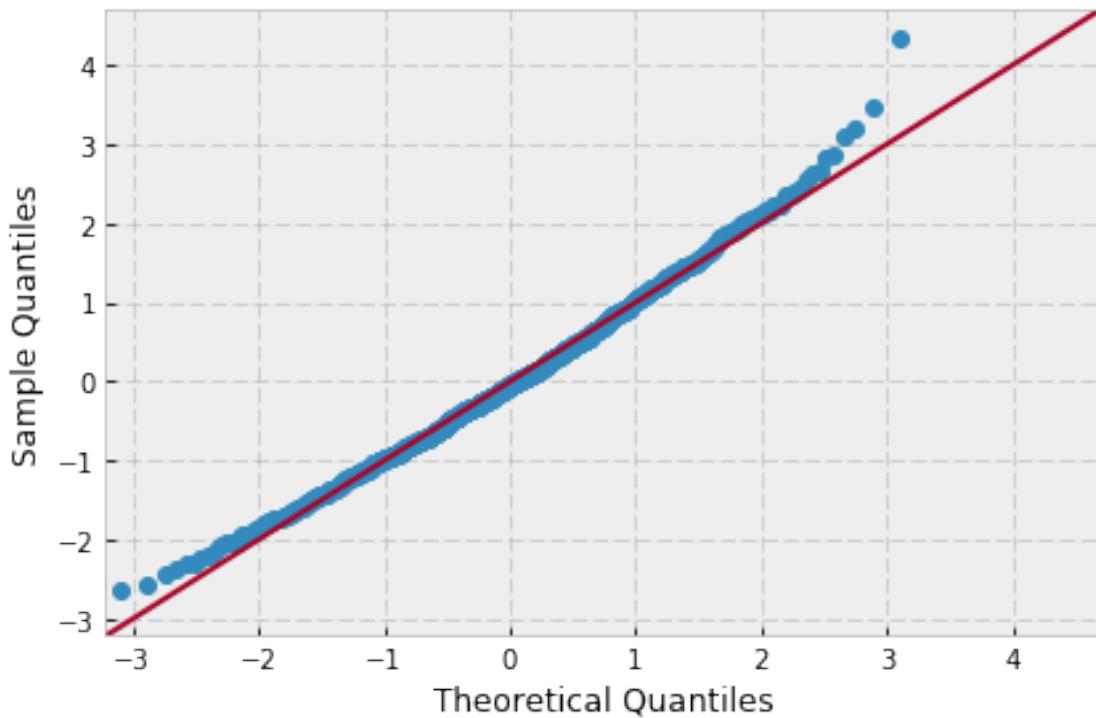


```

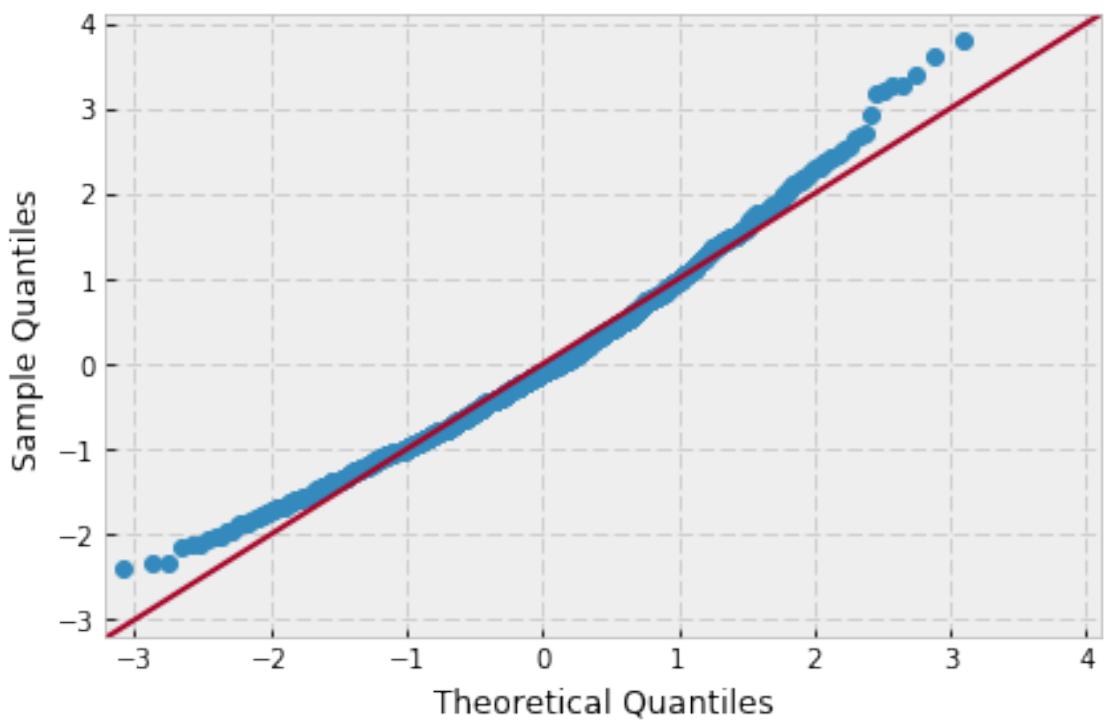
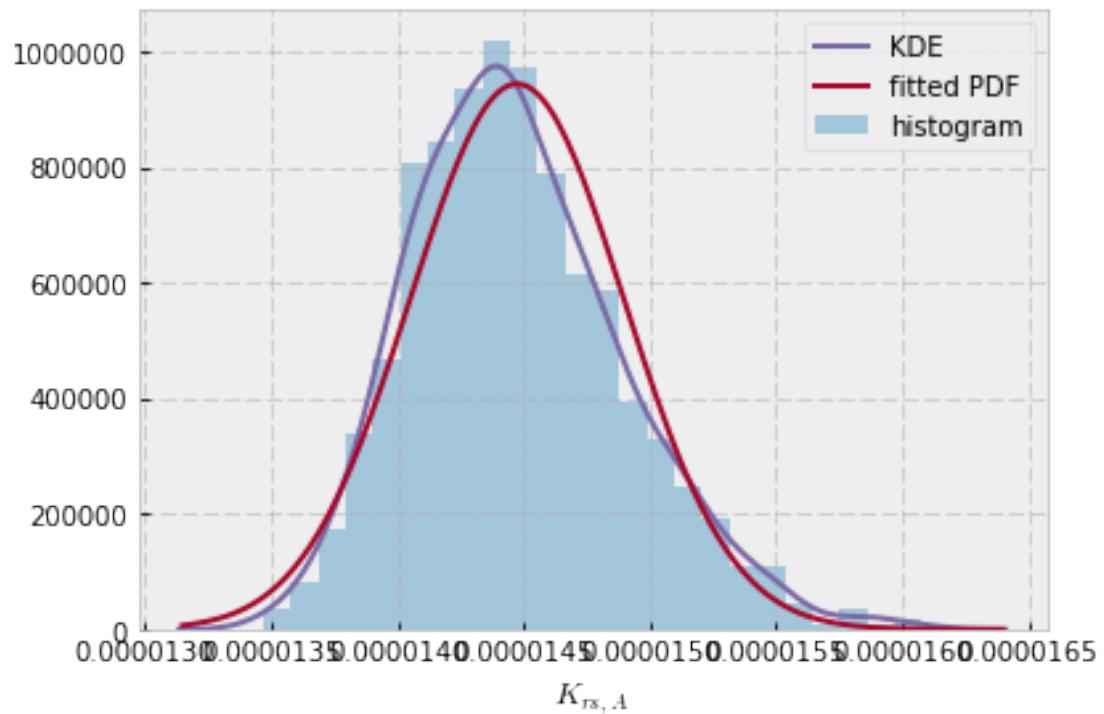
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=1.4825077183699578e-05, scale=4.610067536826433e-07)
DescribeResult(nobs=1000, minmax=(1.3615656915493237e-05, 1.6827846529832957e-05)
               mean=1.4825077183699578e-05, variance=2.1273996690791724e-13,
               skewness=0.3260945419989589, kurtosis=0.20946787347886087)

```





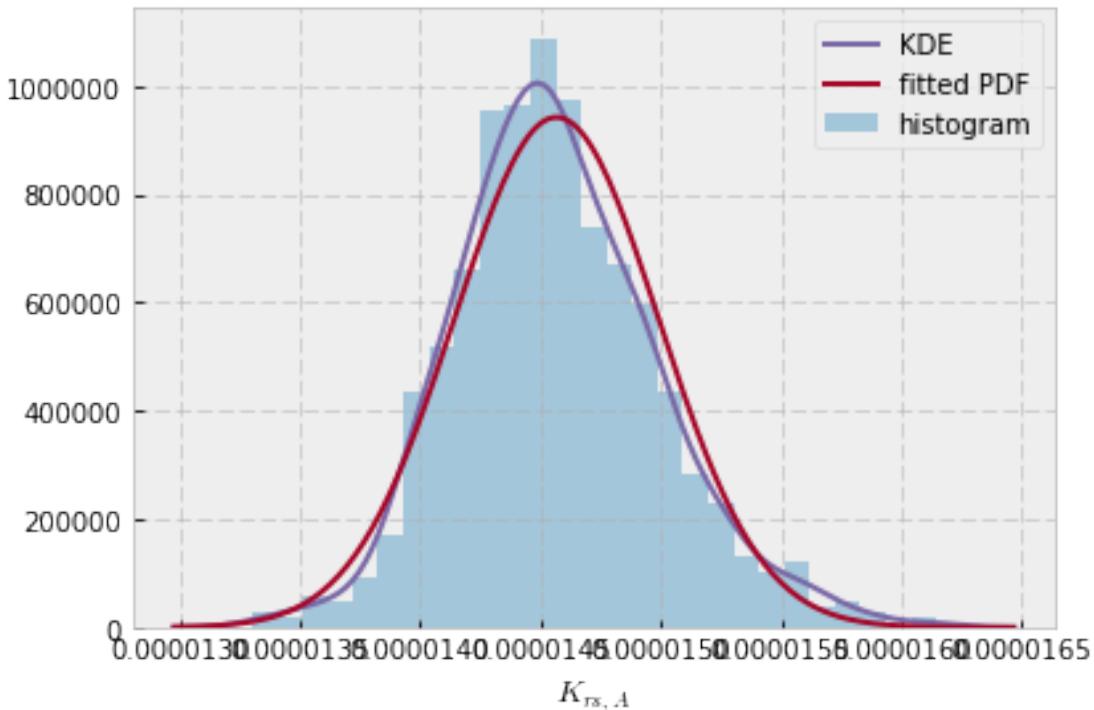
```
variable name: r
variable value: 3.0
distribution: normal(shape=(),loc=1.447285080399581e-05, scale=4.228673915249169e-07)
DescribeResult(nobs=1000, minmax=(1.3465600408982395e-05, 1.6076771747597163e-05)
               mean=1.447285080399581e-05, variance=1.7899582664172908e-13,
               skewness=0.5482546040502364, kurtosis=0.346543951246832)
```

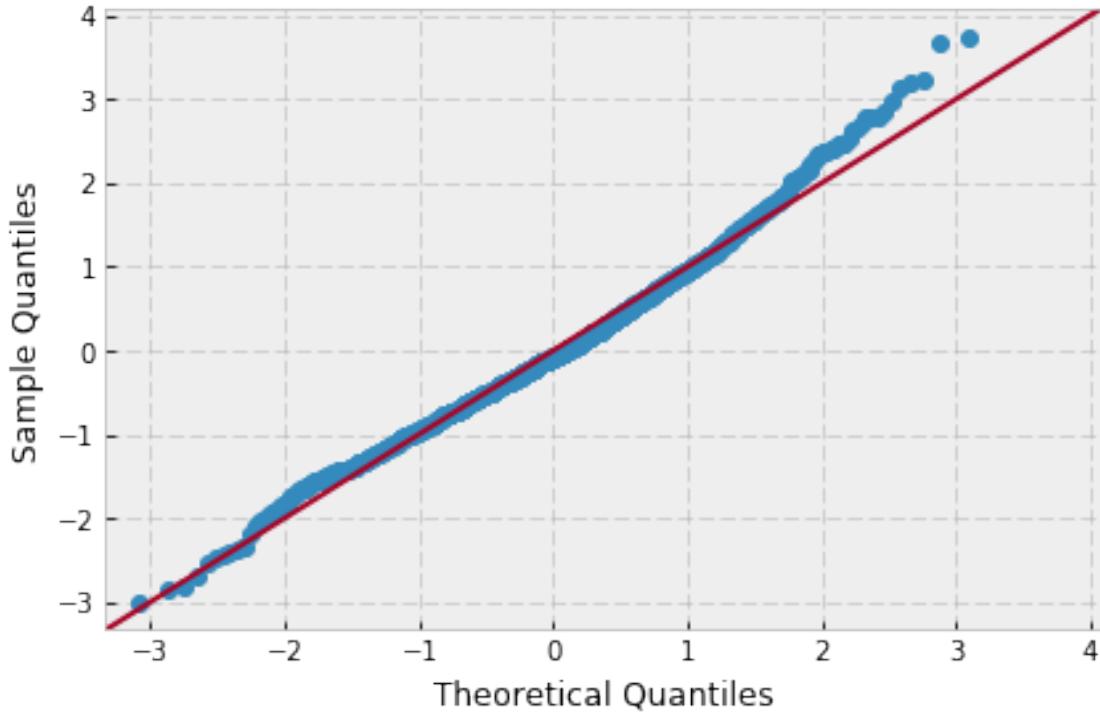


```

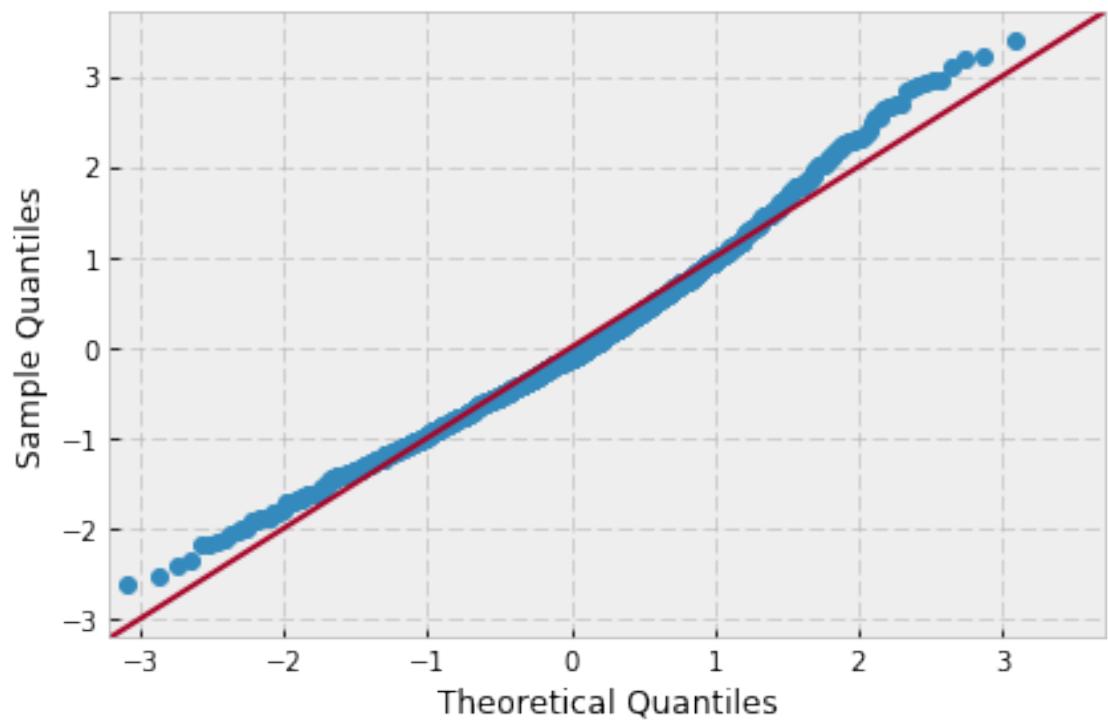
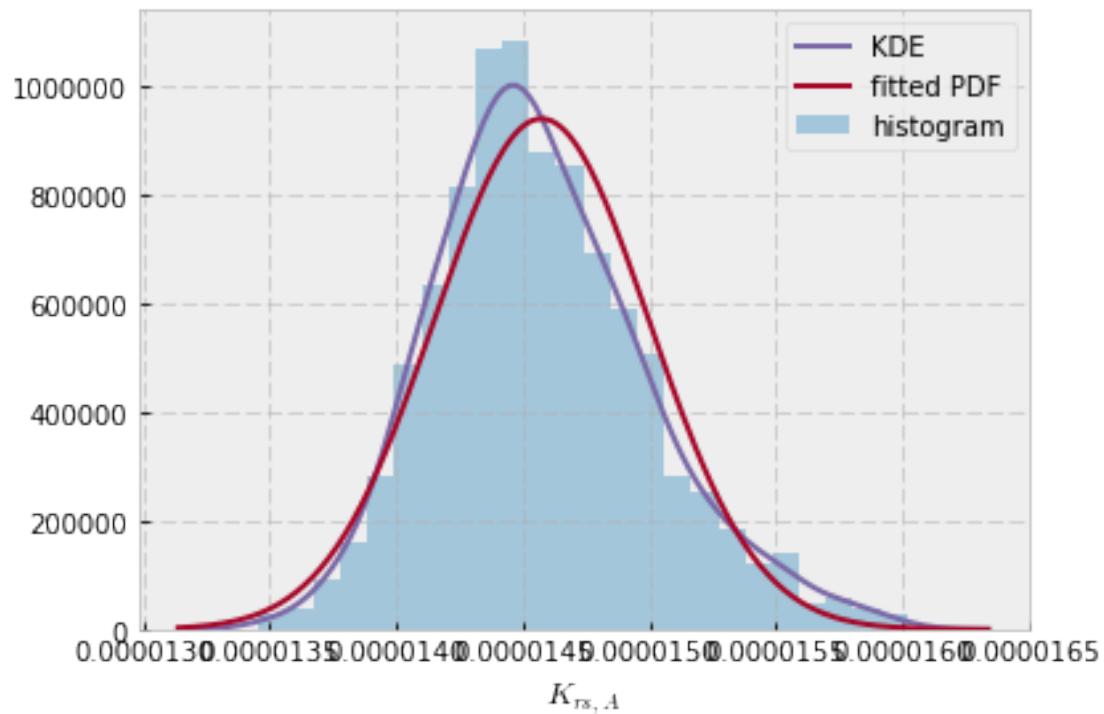
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(), loc=1.456657912188311e-05, scale=4.233851761345289e-07)
DescribeResult(nobs=1000, minmax=(1.3296920632768983e-05, 1.6146634056812856e-05)
               mean=1.456657912188311e-05, variance=1.7943444181227836e-13,
               skewness=0.3920177613991987, kurtosis=0.5343323560828952)

```





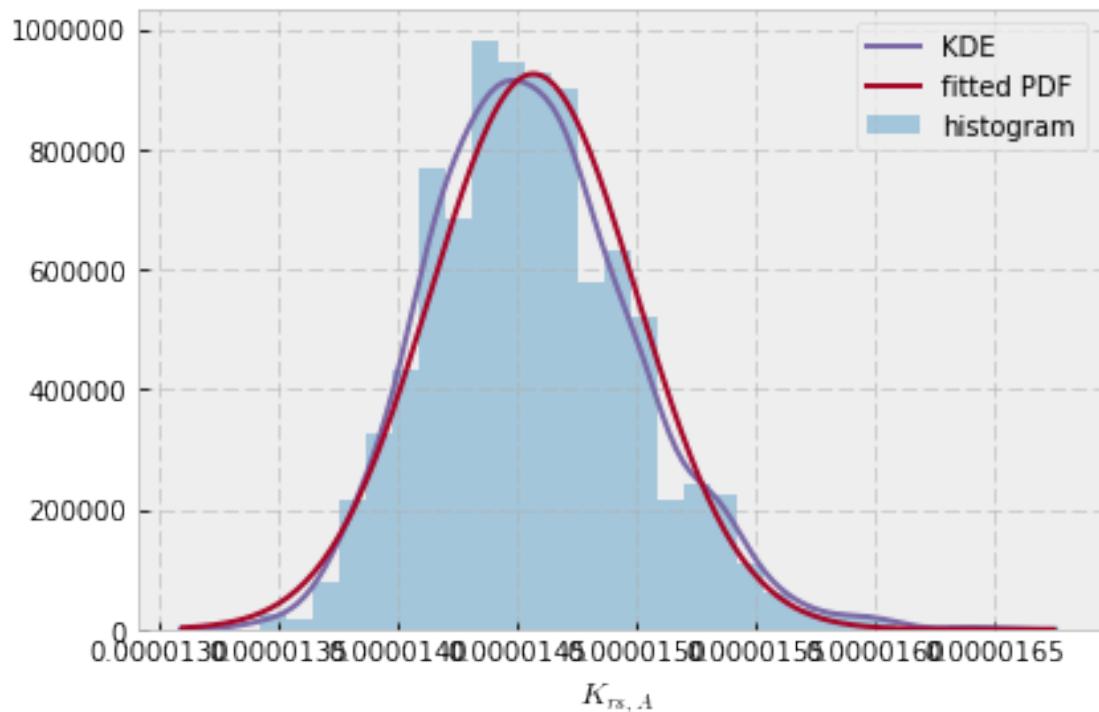
```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(),loc=1.457406053065869e-05, scale=4.23703094906332e-07)
DescribeResult(nobs=1000, minmax=(1.3458940991236256e-05, 1.6018665316539479e-05)
               mean=1.457406053065869e-05, variance=1.7970401664985406e-13,
               skewness=0.5078446139180889, kurtosis=0.3024178290351913)
```

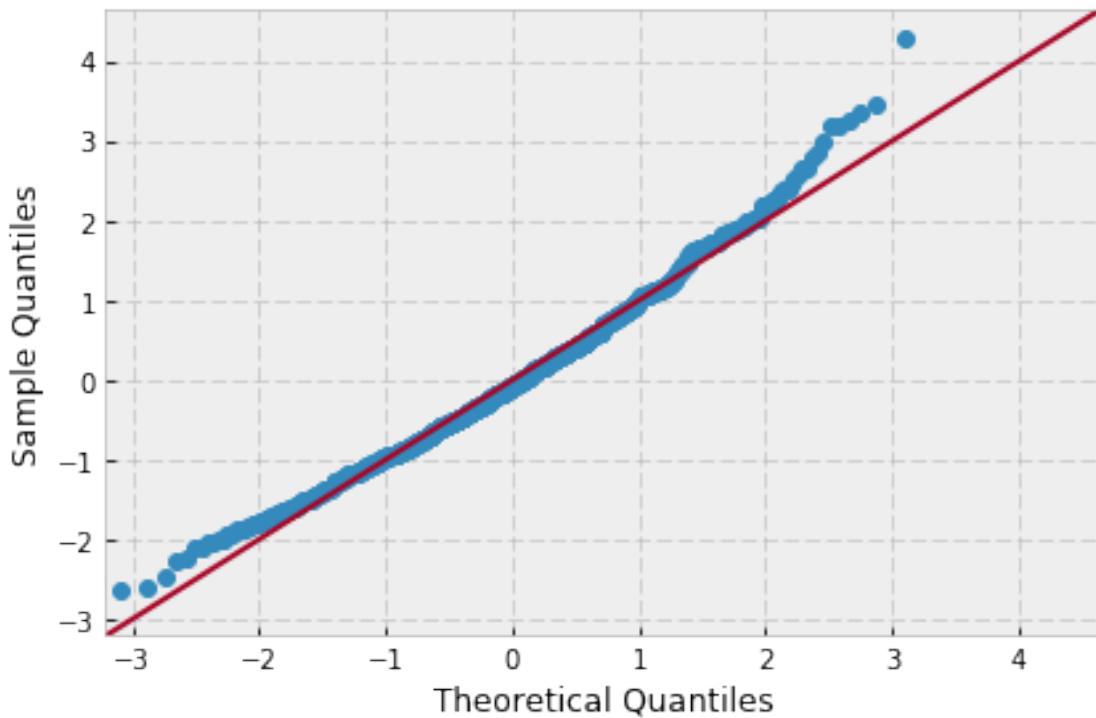


```

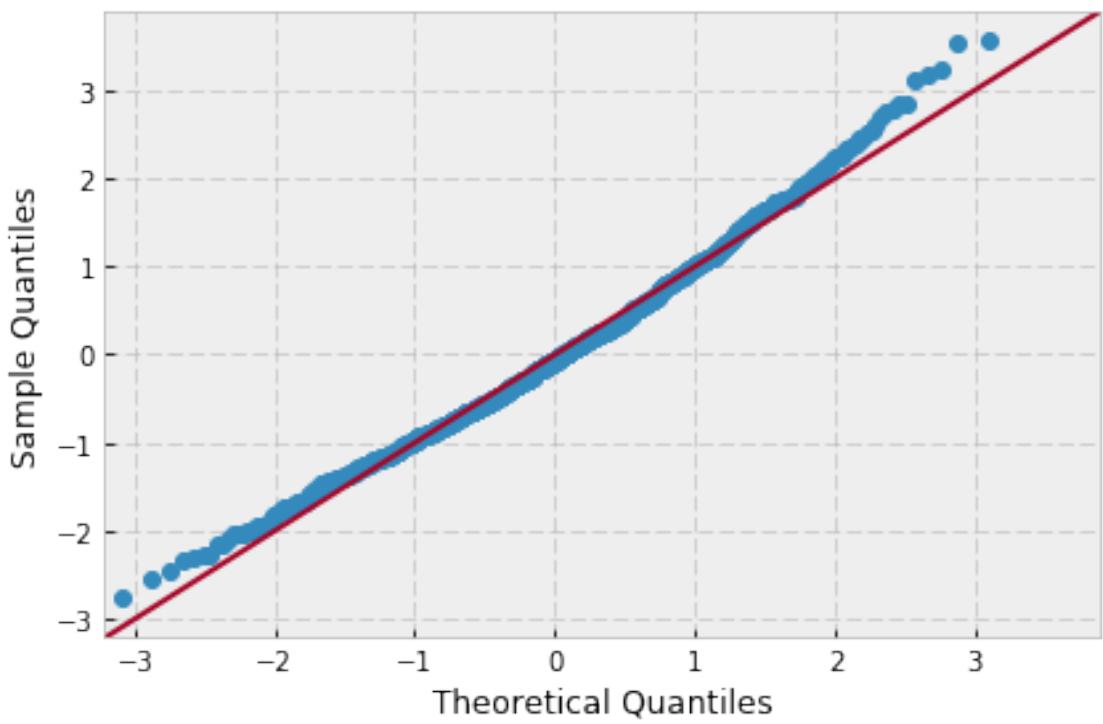
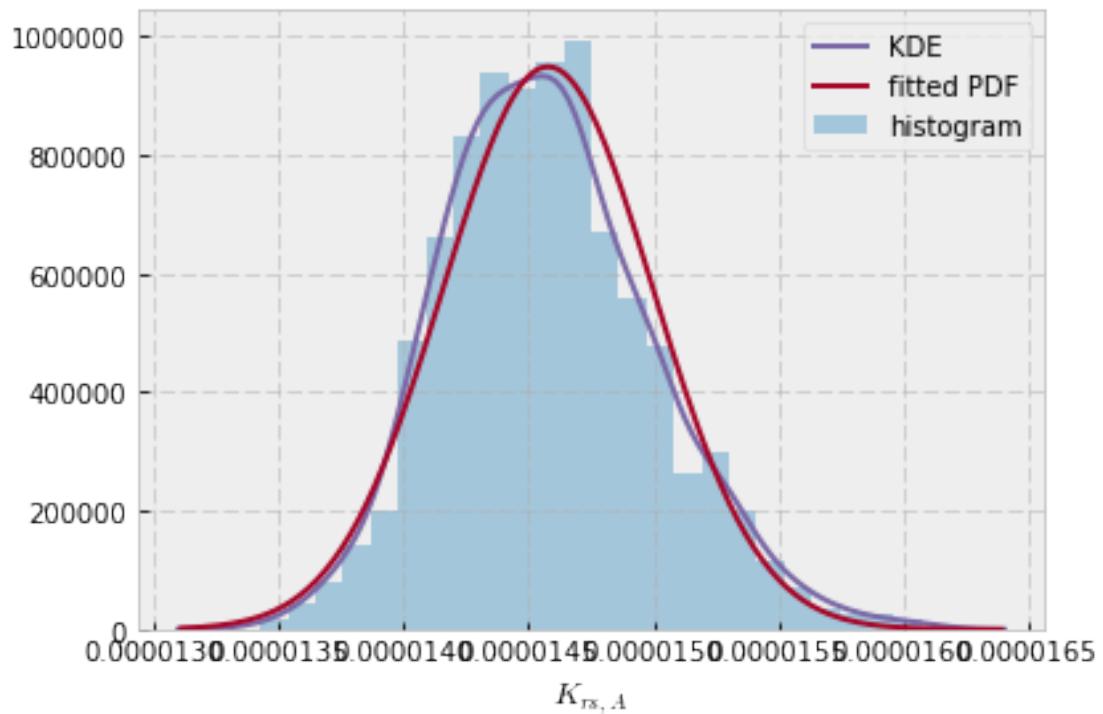
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(), loc=1.4570454718734615e-05, scale=4.309739190090135e-07)
DescribeResult(nobs=1000, minmax=(1.3427203180654178e-05, 1.6420897238629316e-05)
               mean=1.4570454718734615e-05, variance=1.8592444330929701e-13,
               skewness=0.45848361542055527, kurtosis=0.3863029987091404)

```





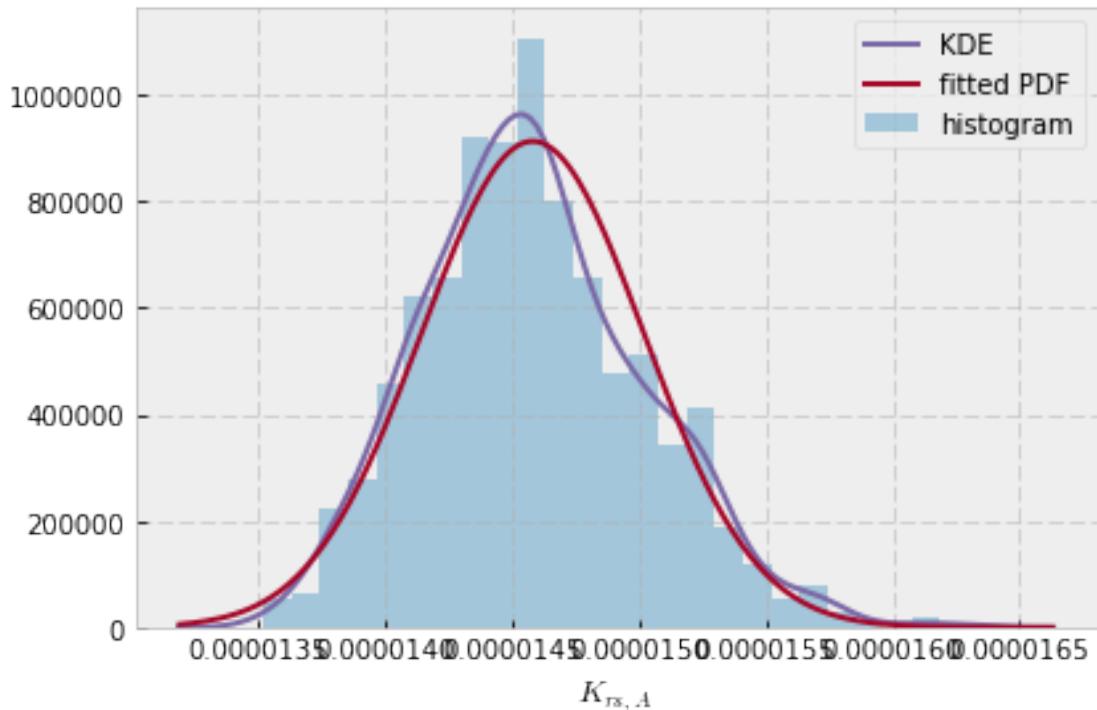
```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=1.4577166673013425e-05, scale=4.2043737428631013e-07)
DescribeResult(nobs=1000, minmax=(1.3423394296560741e-05, 1.6077384195074956e-05)
               mean=1.4577166673013425e-05, variance=1.7694453022699384e-13,
               skewness=0.4007318013177871, kurtosis=0.2000736366327187)
```

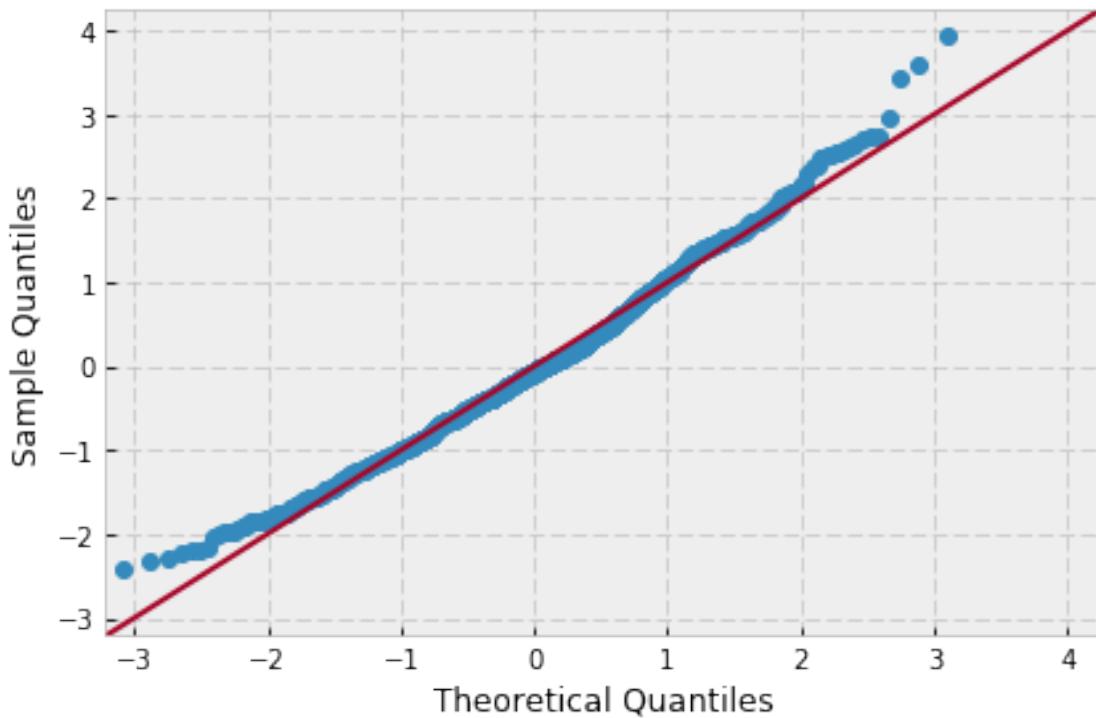


```

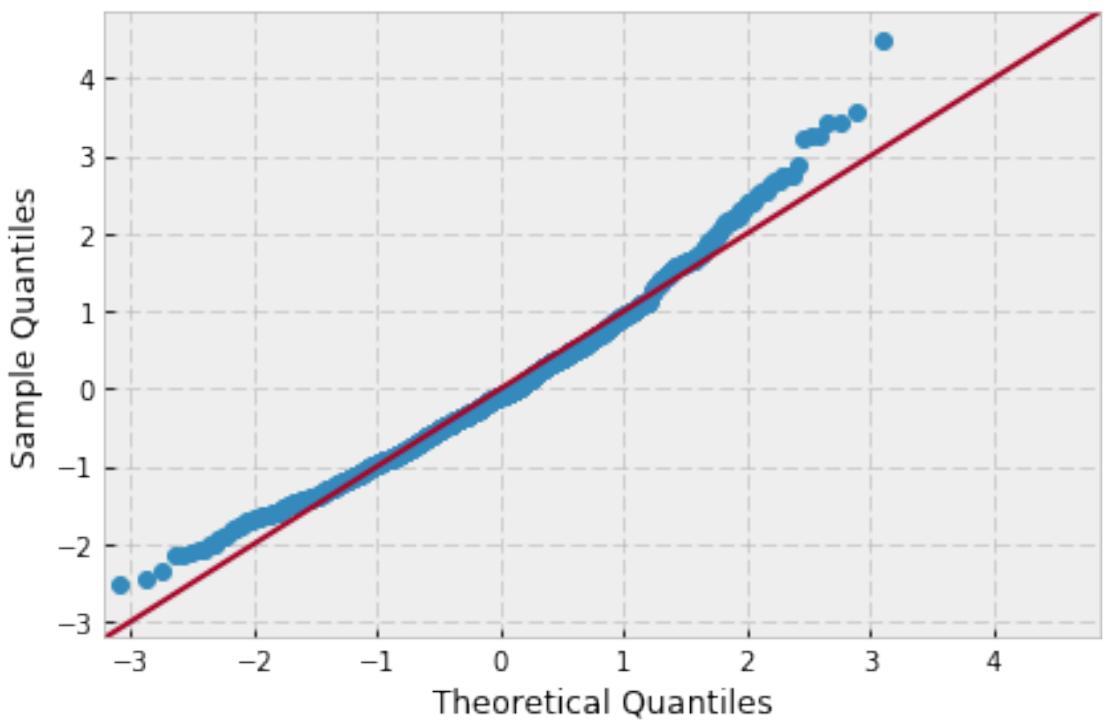
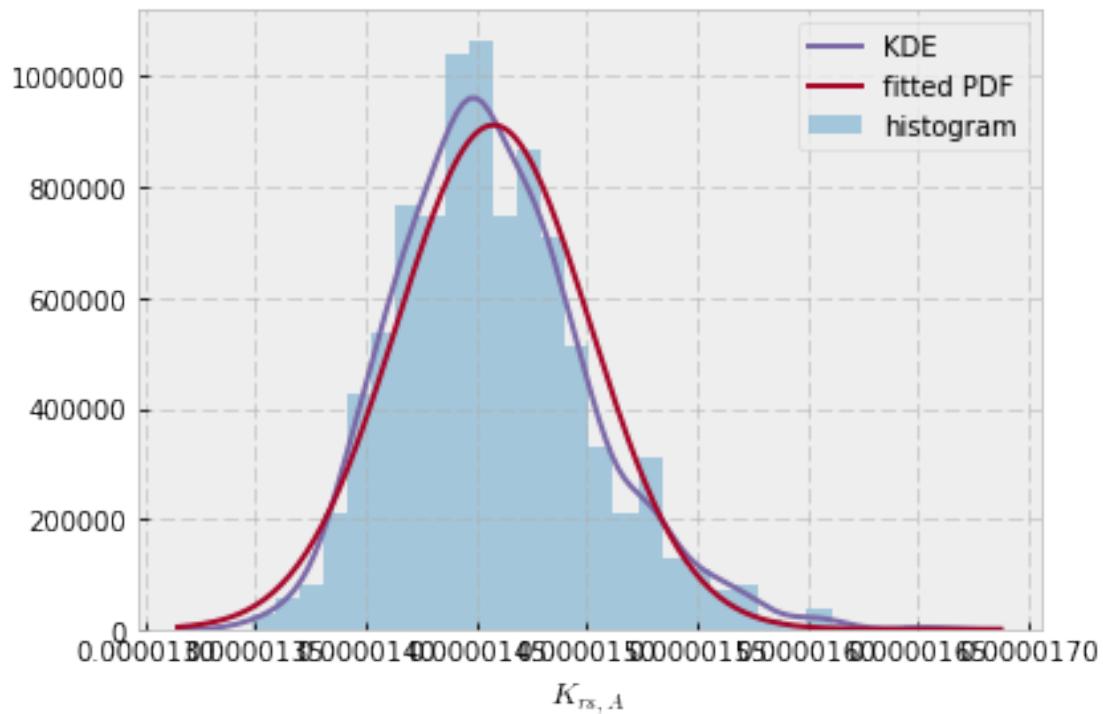
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(), loc=1.4582657420418822e-05, scale=4.3702837261810087e-07)
DescribeResult(nobs=1000, minmax=(1.3522086330150511e-05, 1.6298237143370559e-05)
               mean=1.4582657420418822e-05, variance=1.9118498345668227e-13,
               skewness=0.39599321061520326, kurtosis=0.11530269949588812)

```

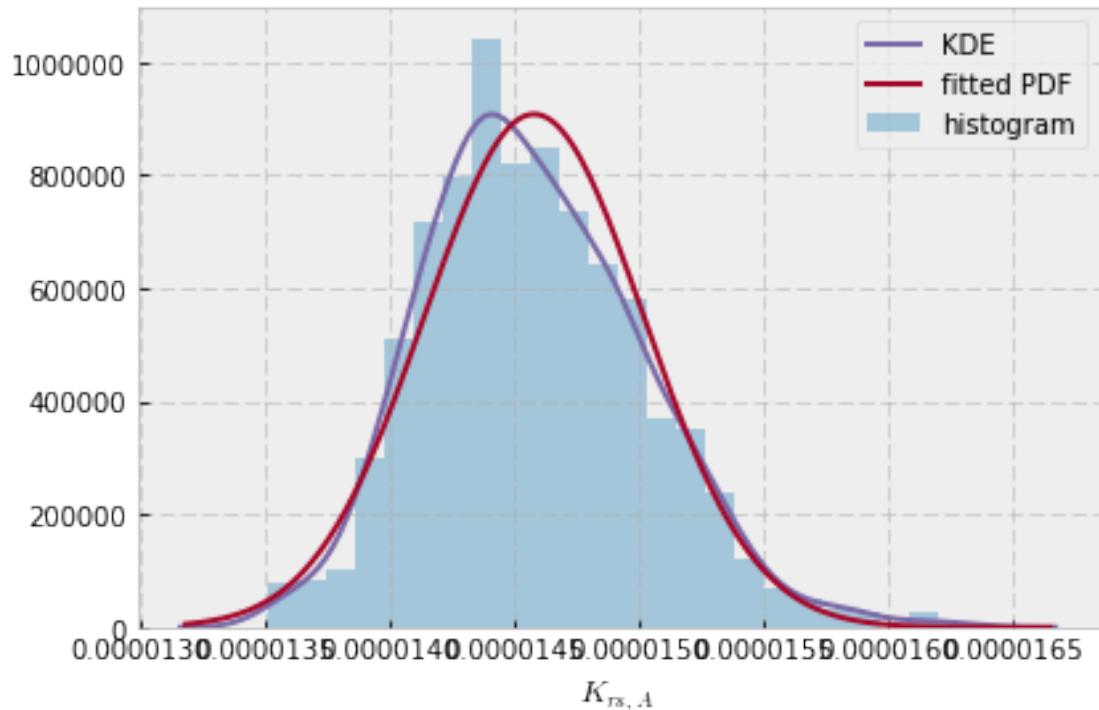


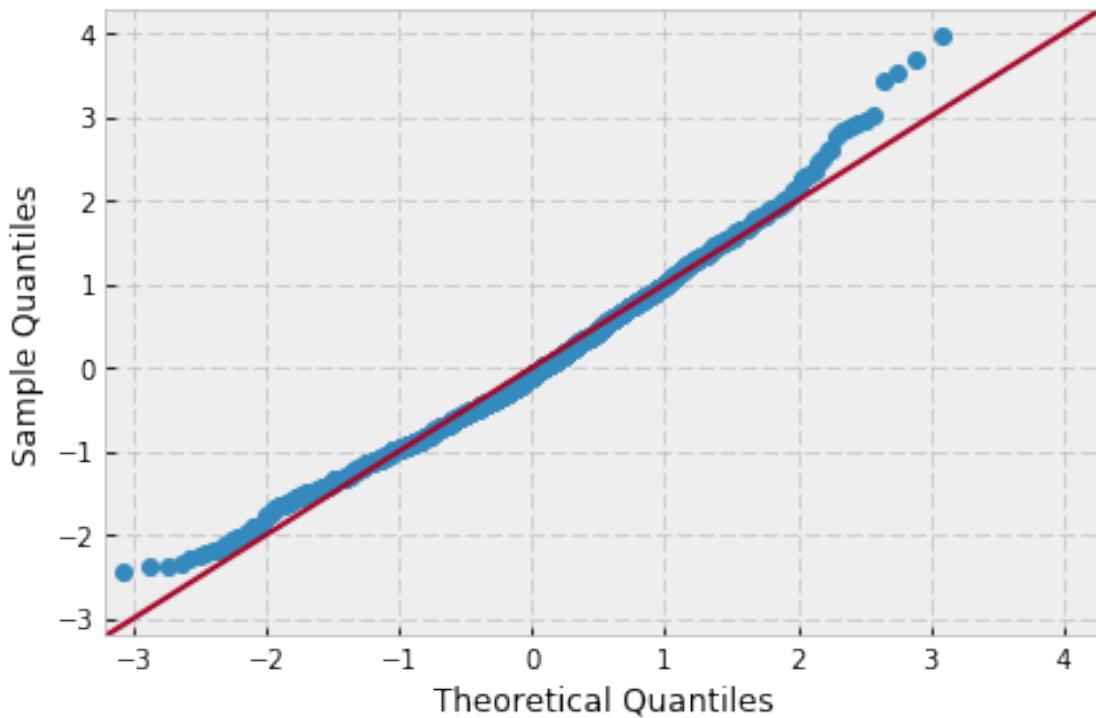


```
variable name: sigma
variable value: 0.06666666666666667
distribution: normal(shape=(), loc=1.4577618647473013e-05, scale=4.373039011538786e-07)
DescribeResult(nobs=1000, minmax=(1.347654235171577e-05, 1.6546895141222211e-05)
               mean=1.4577618647473013e-05, variance=1.9142612809249372e-13,
               skewness=0.6089496026560567, kurtosis=0.6914442298635604)
```

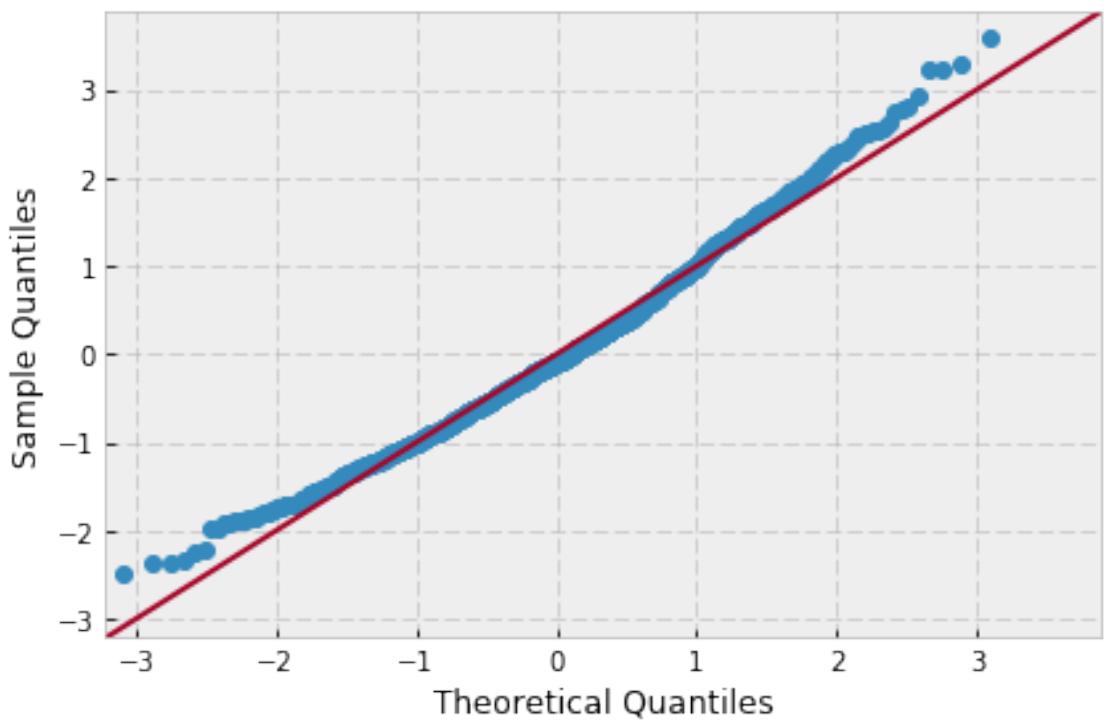
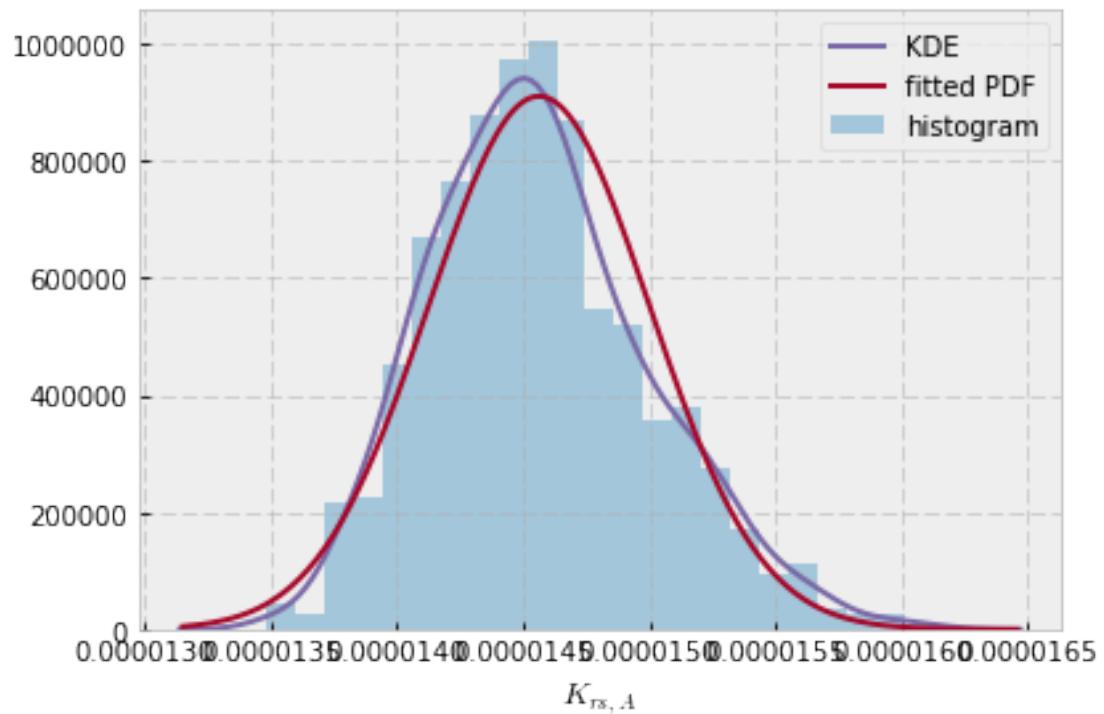


```
variable name: sigma
variable value: 0.0777777777777778
distribution: normal(shape=(), loc=1.457609950598749e-05, scale=4.39368888274027e-07)
DescribeResult(nobs=1000, minmax=(1.3509493918028471e-05, 1.6315034389615181e-05)
               mean=1.4576099505987491e-05, variance=1.9323825824139582e-13,
               skewness=0.4529668344967481, kurtosis=0.3203505262039079)
```

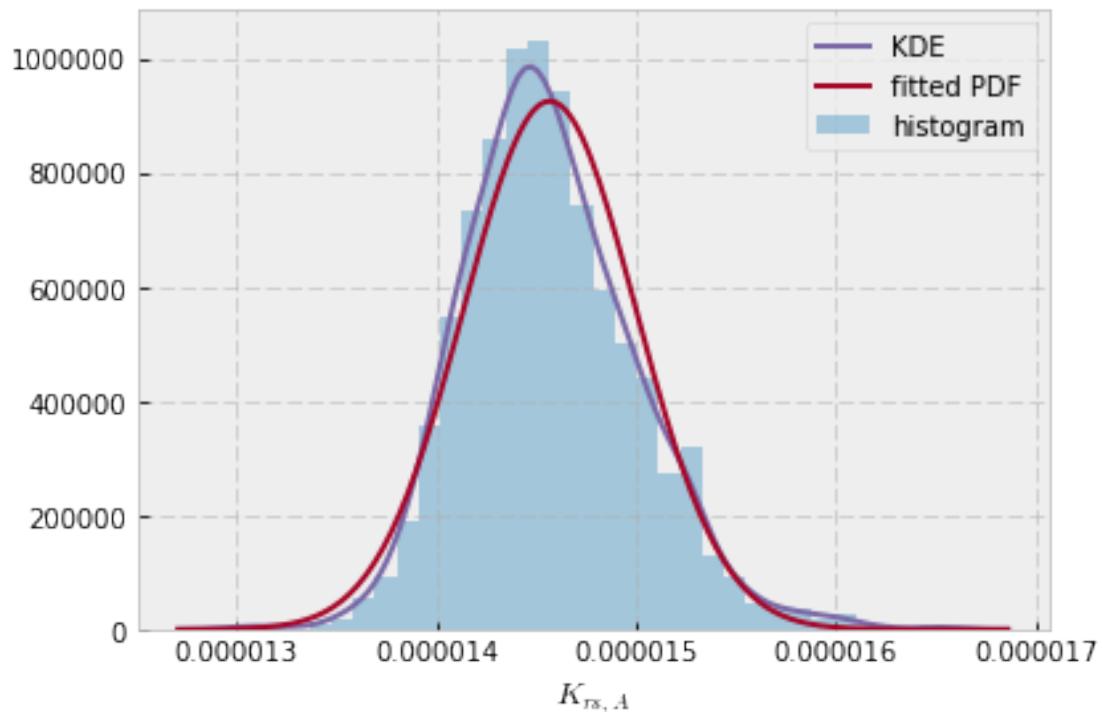


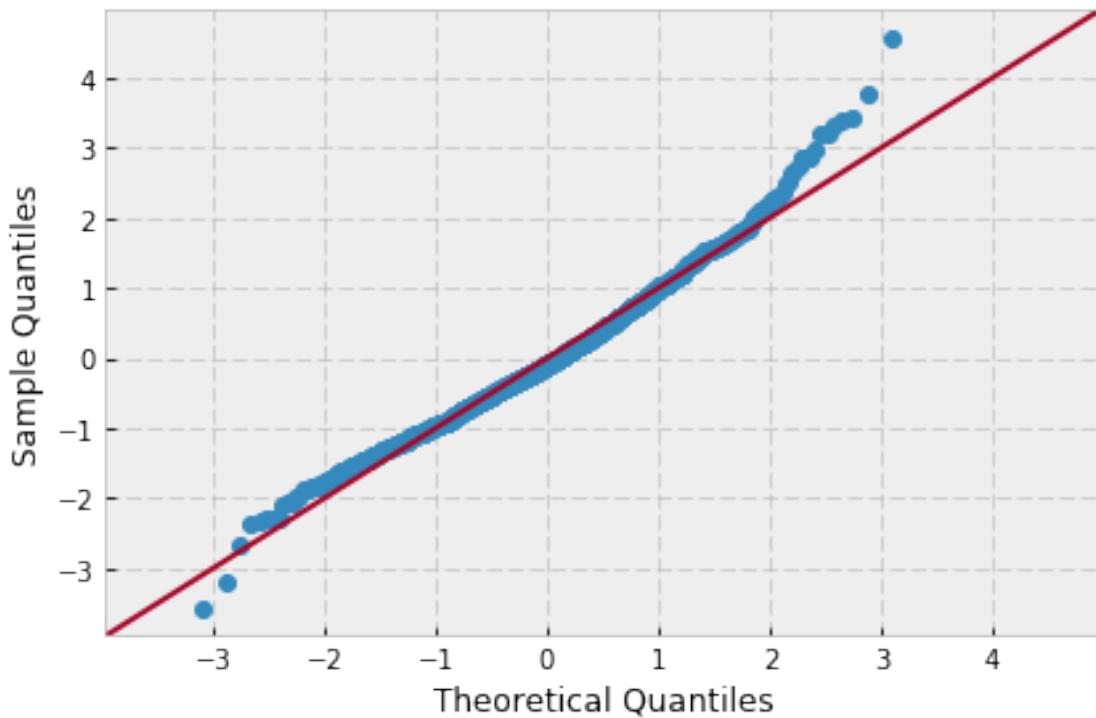


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(), loc=1.4563418078542561e-05, scale=4.381116526540486e-07)
DescribeResult(nobs=1000, minmax=(1.348089976197436e-05, 1.6129028198244298e-05)
               mean=1.4563418078542561e-05, variance=1.9213395414540718e-13,
               skewness=0.45548904171730004, kurtosis=0.09359445610310013)
```

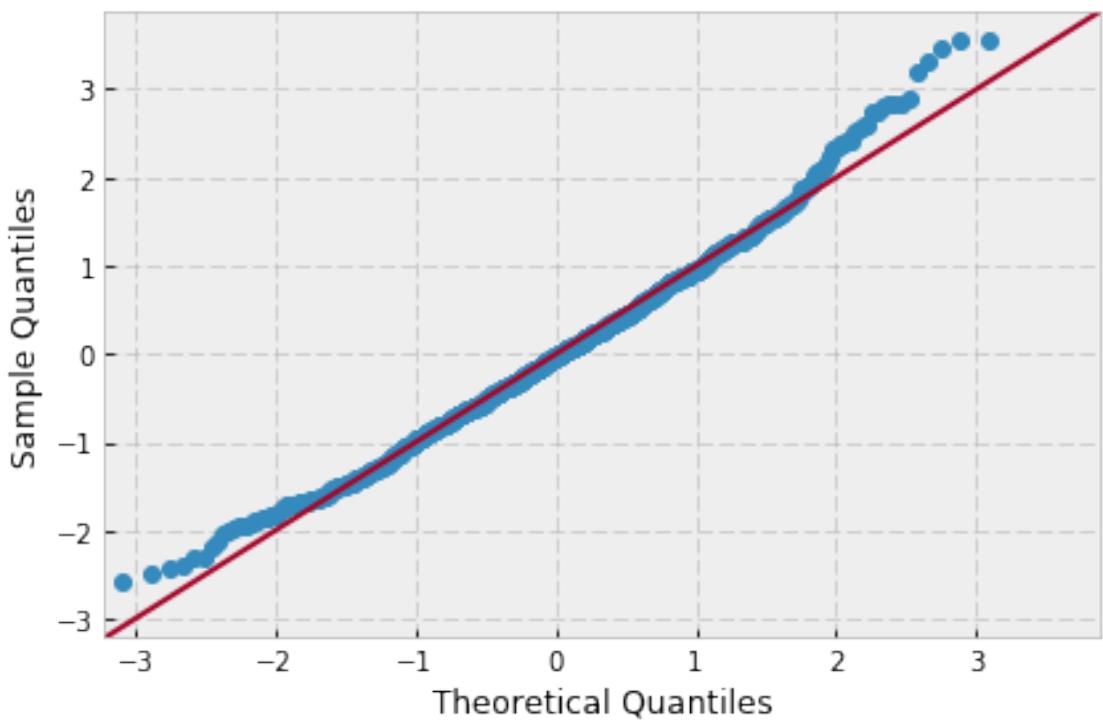
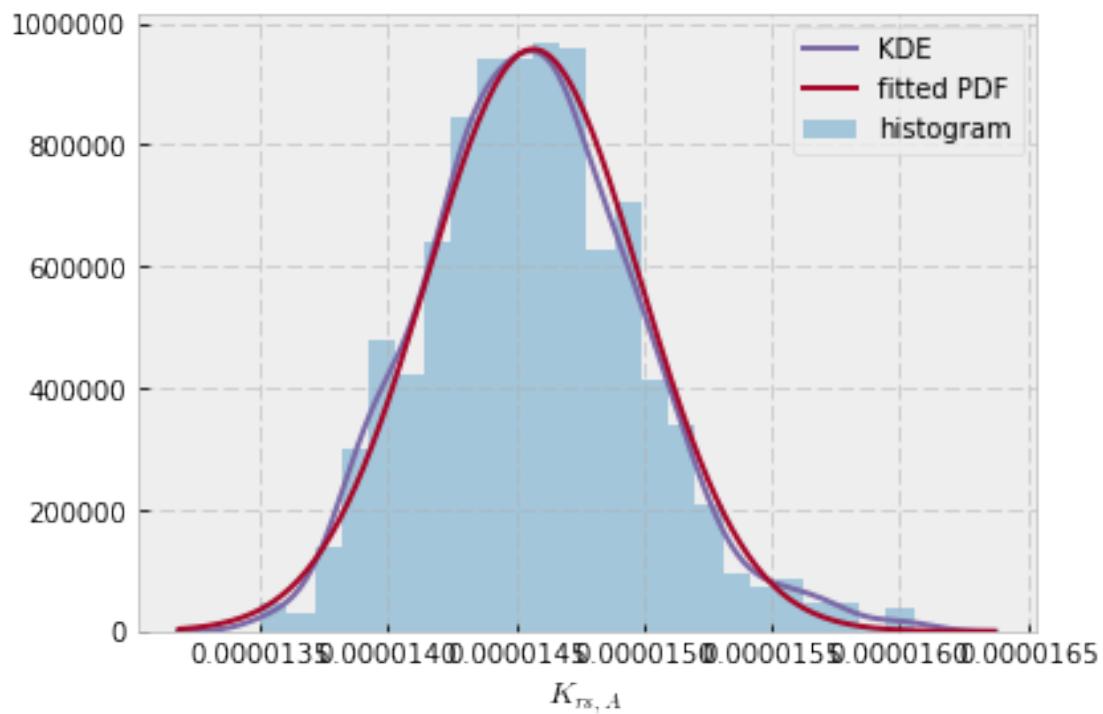


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=1.4566197839323826e-05, scale=4.3127657779171207e-07)
DescribeResult(nobs=1000, minmax=(1.3032730053392861e-05, 1.653287998625723e-05)
               mean=1.4566197839323826e-05, variance=1.8618567222395462e-13,
               skewness=0.4851217245949452, kurtosis=0.8332087369111938)
```





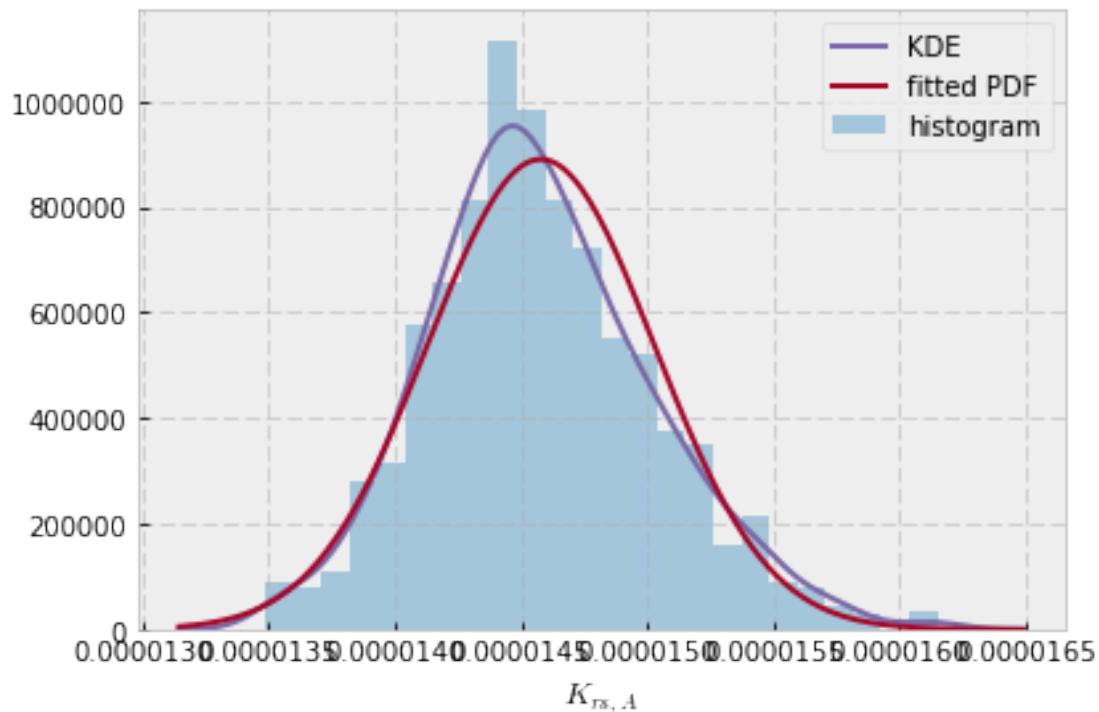
```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=1.4566731073761286e-05, scale=4.1673555552502573e-07)
DescribeResult(nobs=1000, minmax=(1.349832190327221e-05, 1.6052638453608122e-05)
               mean=1.4566731073761286e-05, variance=1.7384236560435618e-13,
               skewness=0.3633708829480881, kurtosis=0.3146686478210885)
```

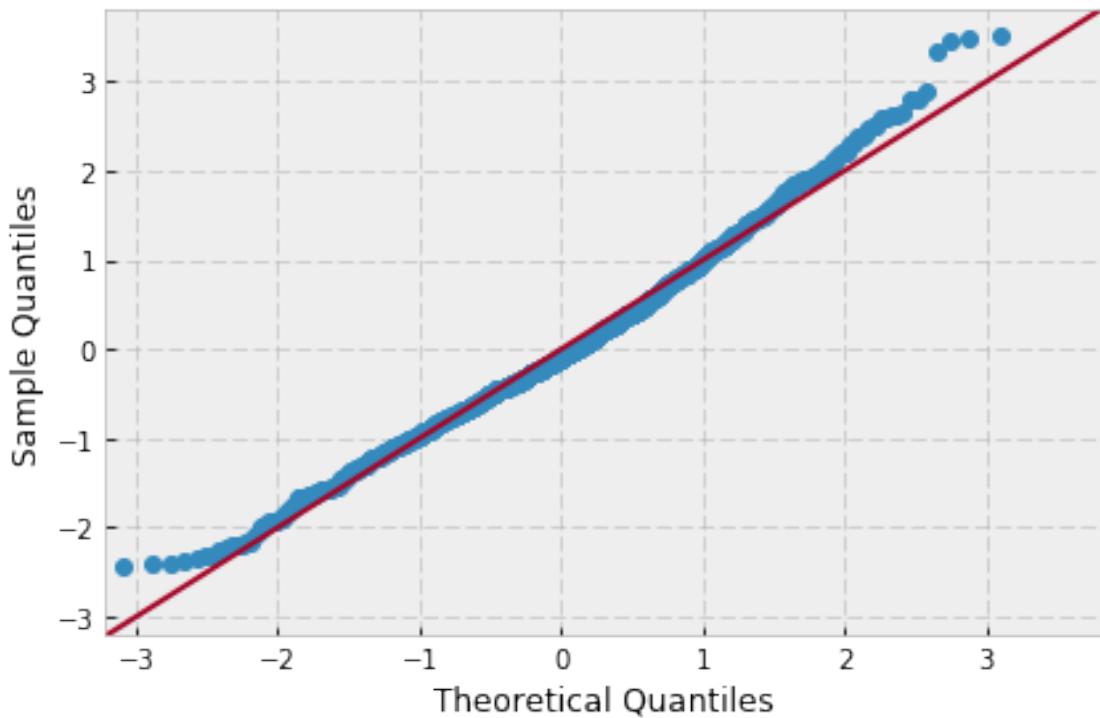


```

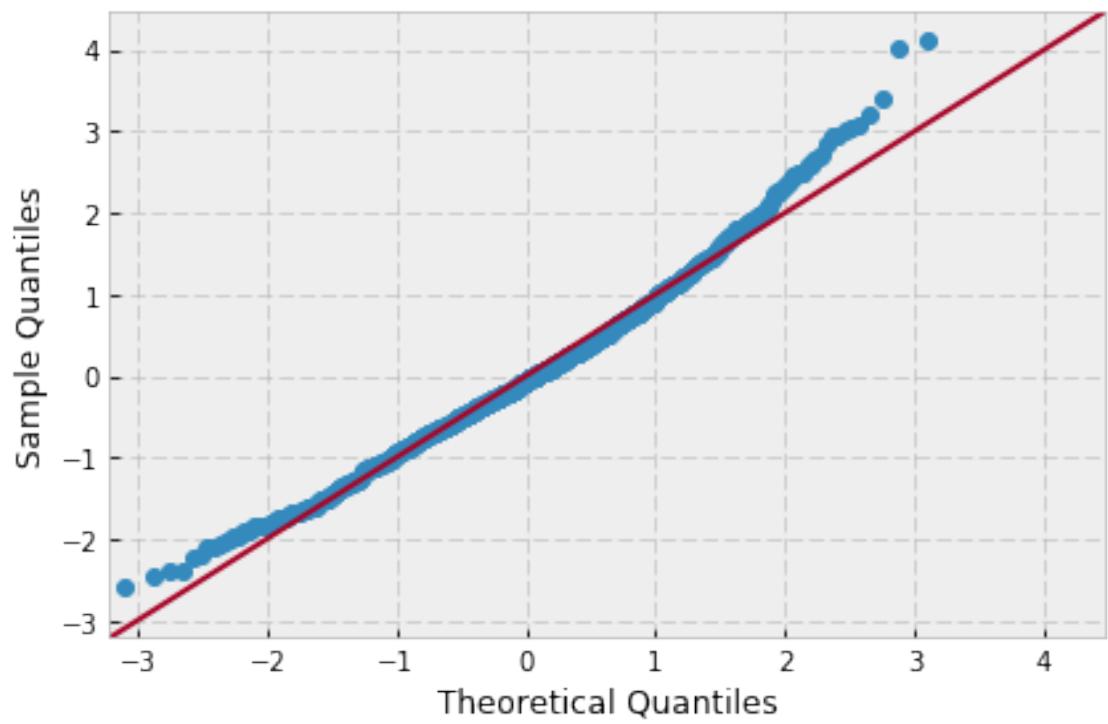
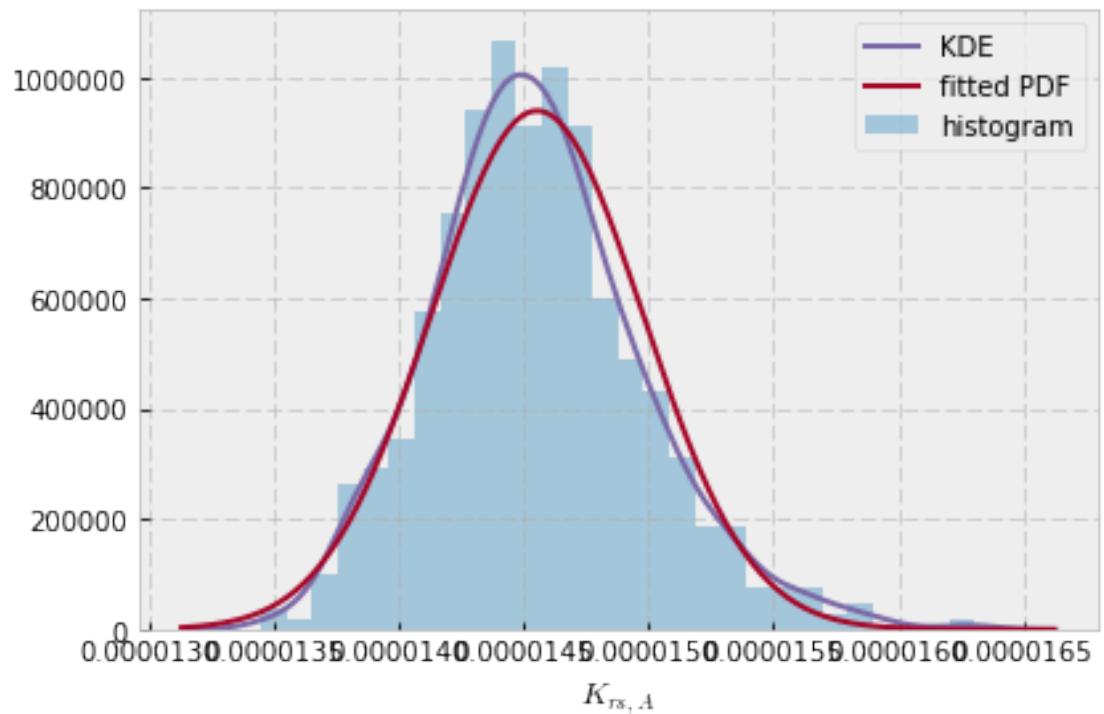
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=1.4578250079662663e-05, scale=4.4815809717361485e-07)
DescribeResult(nobs=1000, minmax=(1.3484142594226503e-05, 1.6148417804135691e-05)
               mean=1.4578250079662663e-05, variance=2.0104672678906428e-13,
               skewness=0.3919089296543544, kurtosis=0.25691796194093364)

```





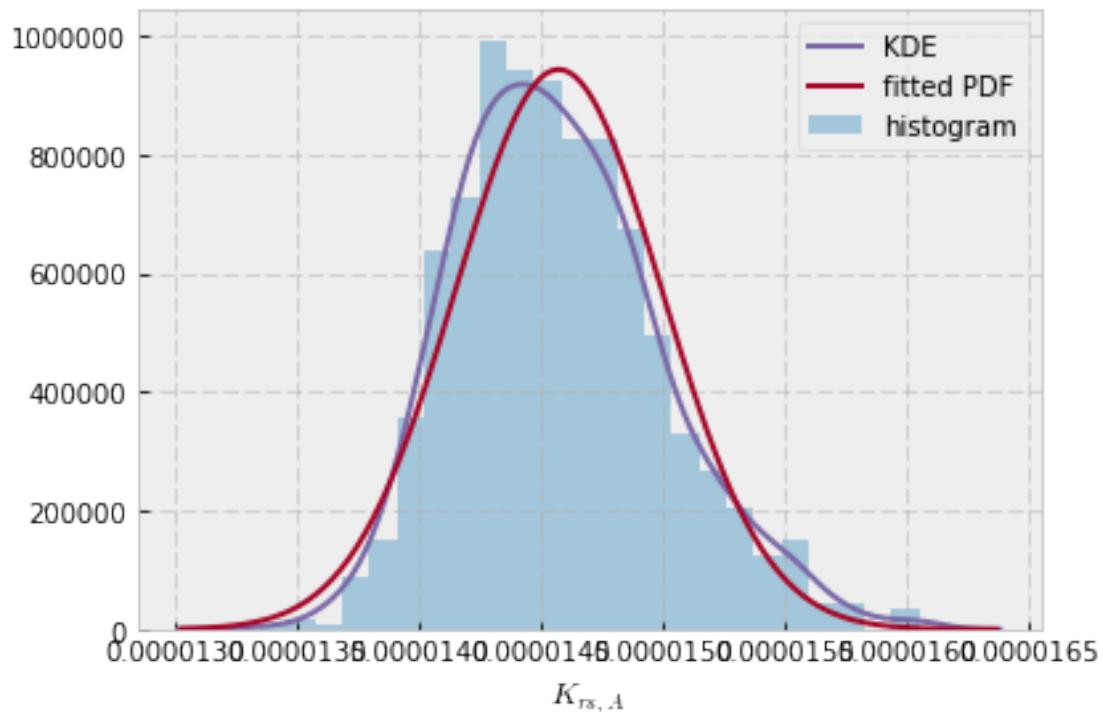
```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=1.455327870469303e-05, scale=4.247784238814979e-07)
DescribeResult(nobs=1000, minmax=(1.3449371530534409e-05, 1.6306425835761434e-05)
               mean=1.455327870469303e-05, variance=1.8061732672197146e-13,
               skewness=0.4845745906175721, kurtosis=0.6268595459546042)
```

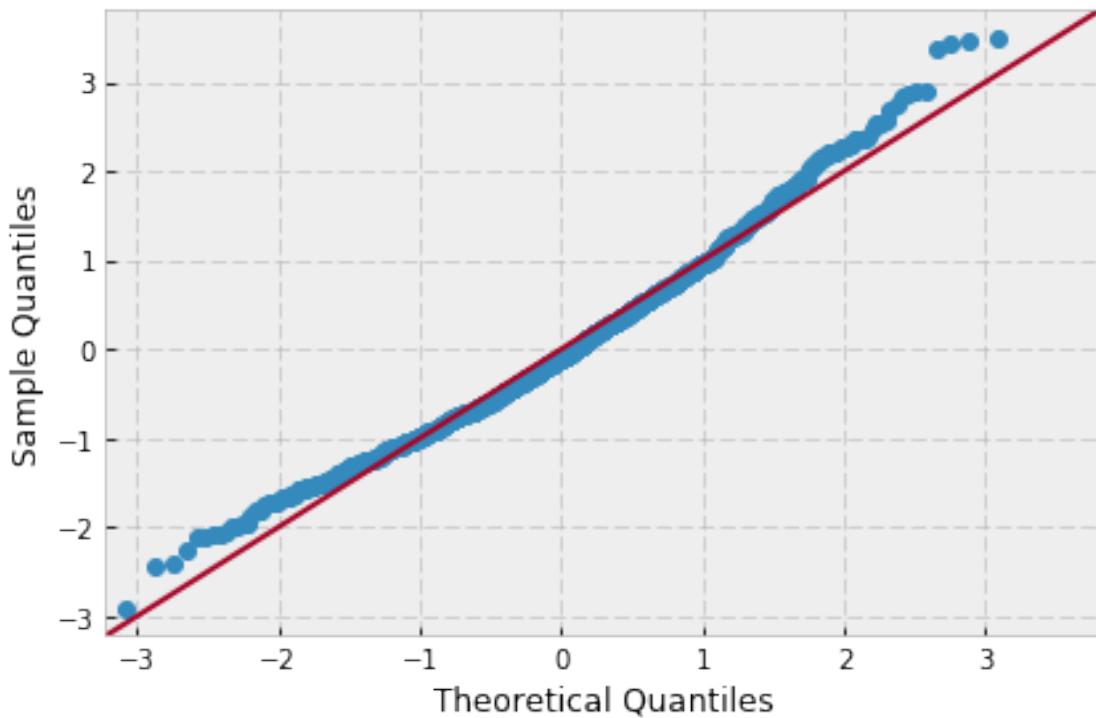


```

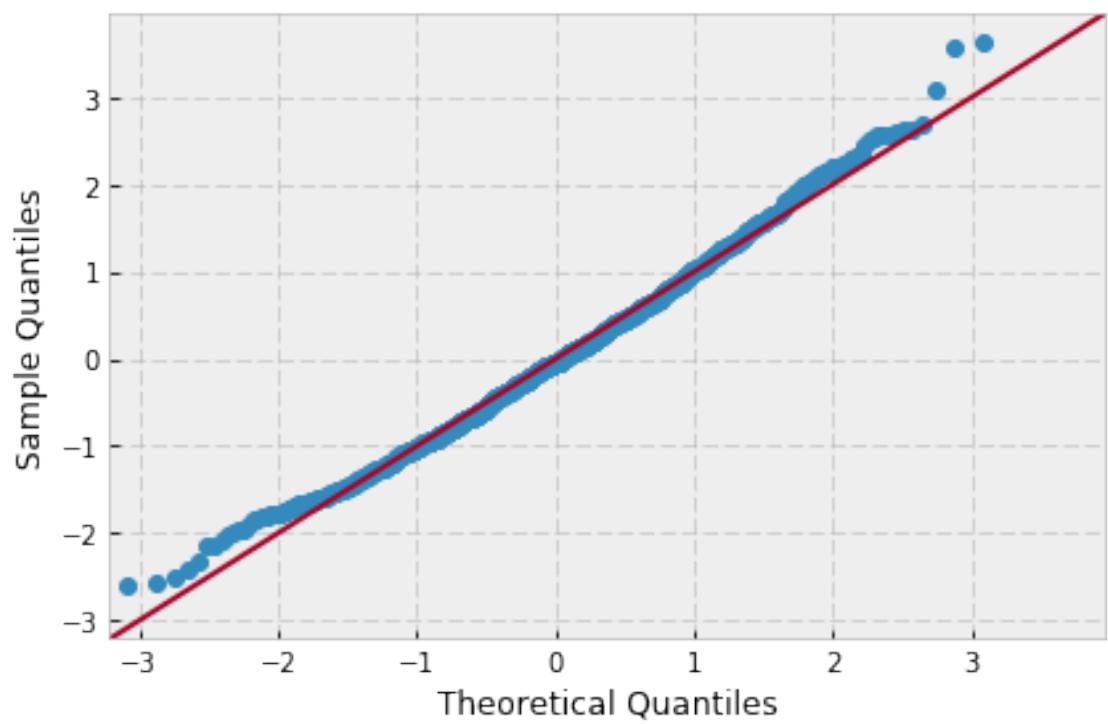
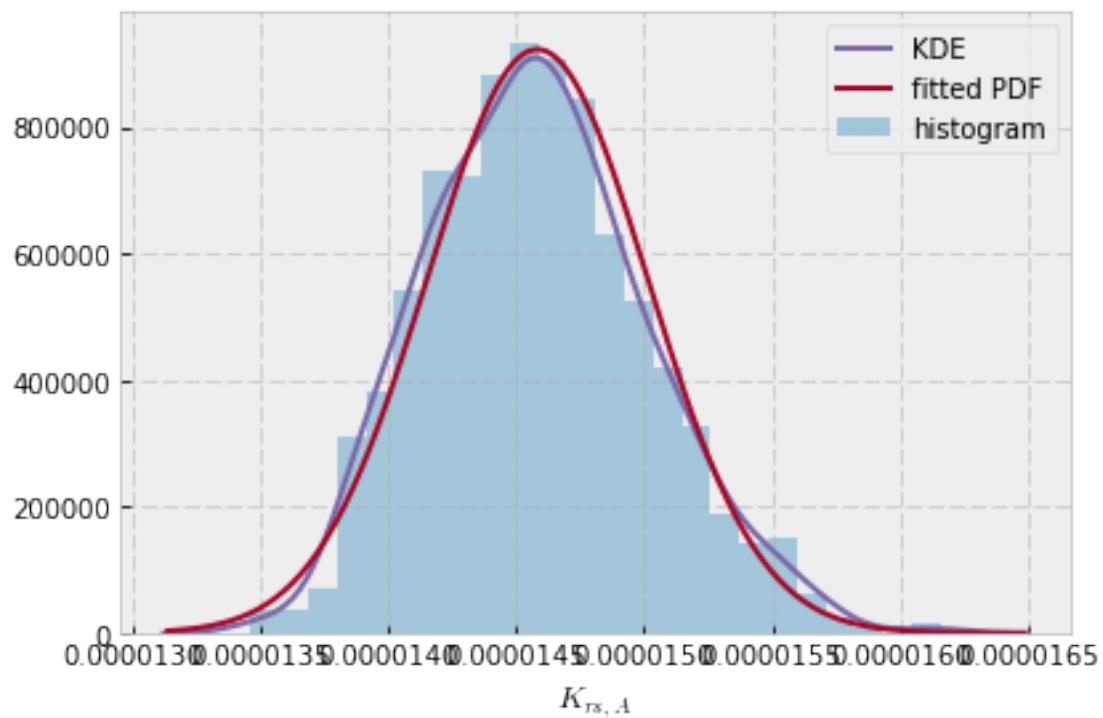
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=1.4575756893158682e-05, scale=4.2313674894724703e-07)
DescribeResult(nobs=1000, minmax=(1.3347250130314593e-05, 1.6053226201163893e-05)
               mean=1.4575756893158682e-05, variance=1.7922393224188744e-13,
               skewness=0.4939766545646995, kurtosis=0.22174189806571798)

```





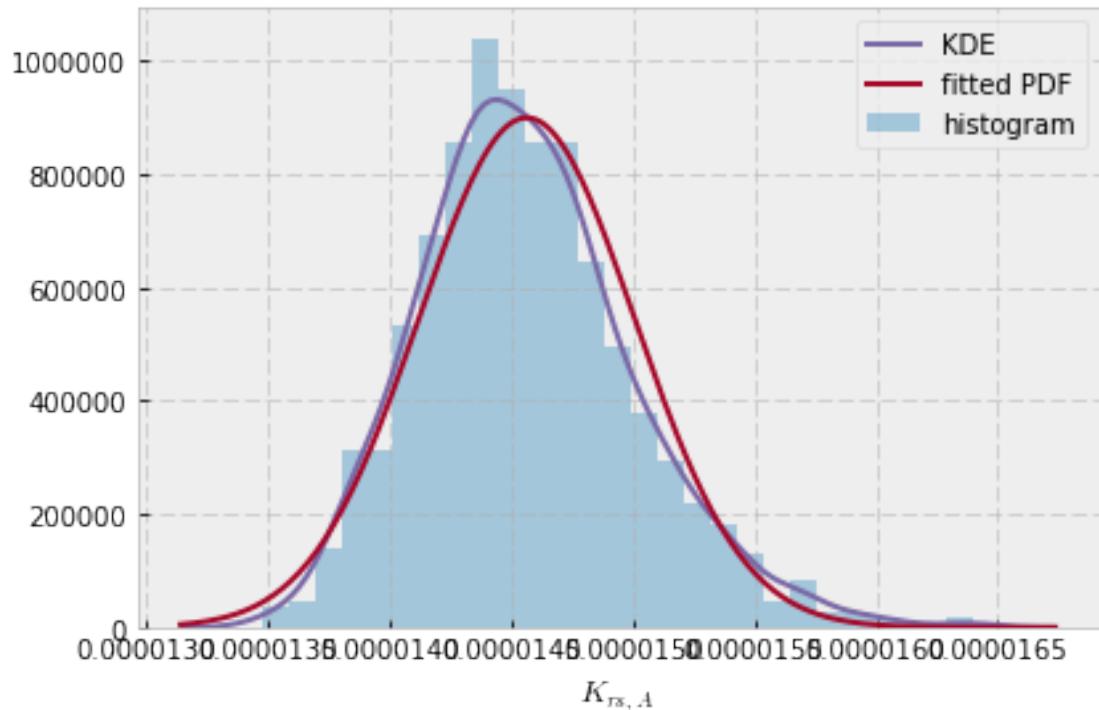
```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=1.4582148863396763e-05, scale=4.323638761082828e-07)
DescribeResult(nobs=1000, minmax=(1.3462225160052215e-05, 1.615744683137979e-05)
               mean=1.4582148863396763e-05, variance=1.871256470103889e-13,
               skewness=0.3131439227853409, kurtosis=-0.018648920667504232)
```

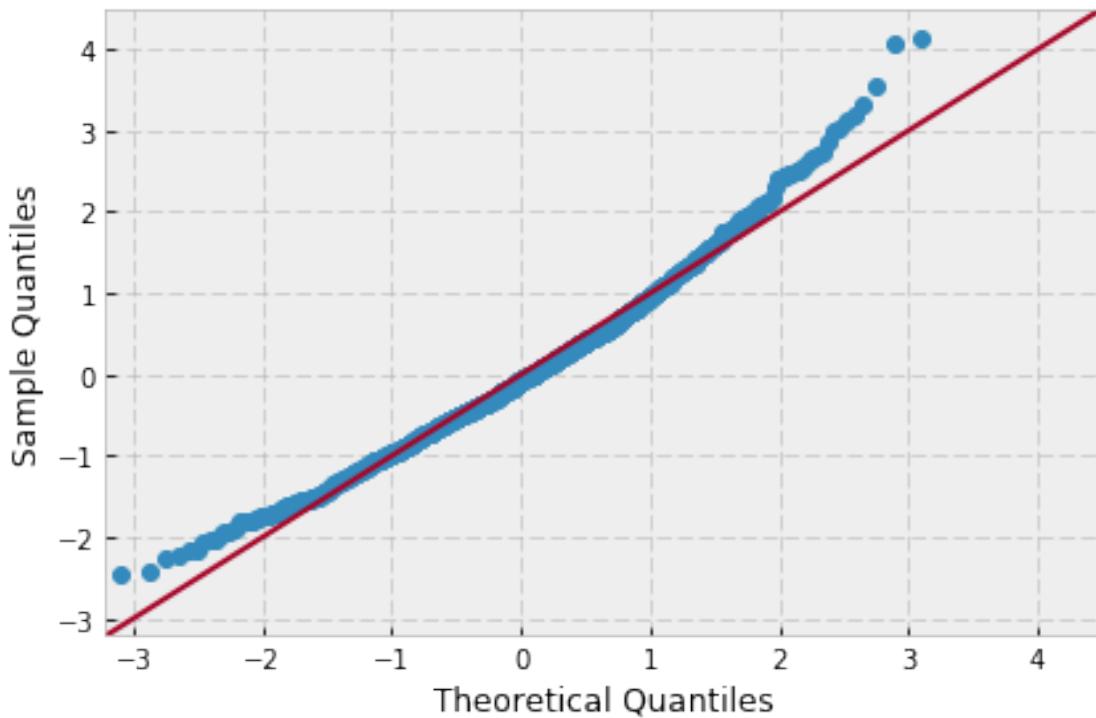


```

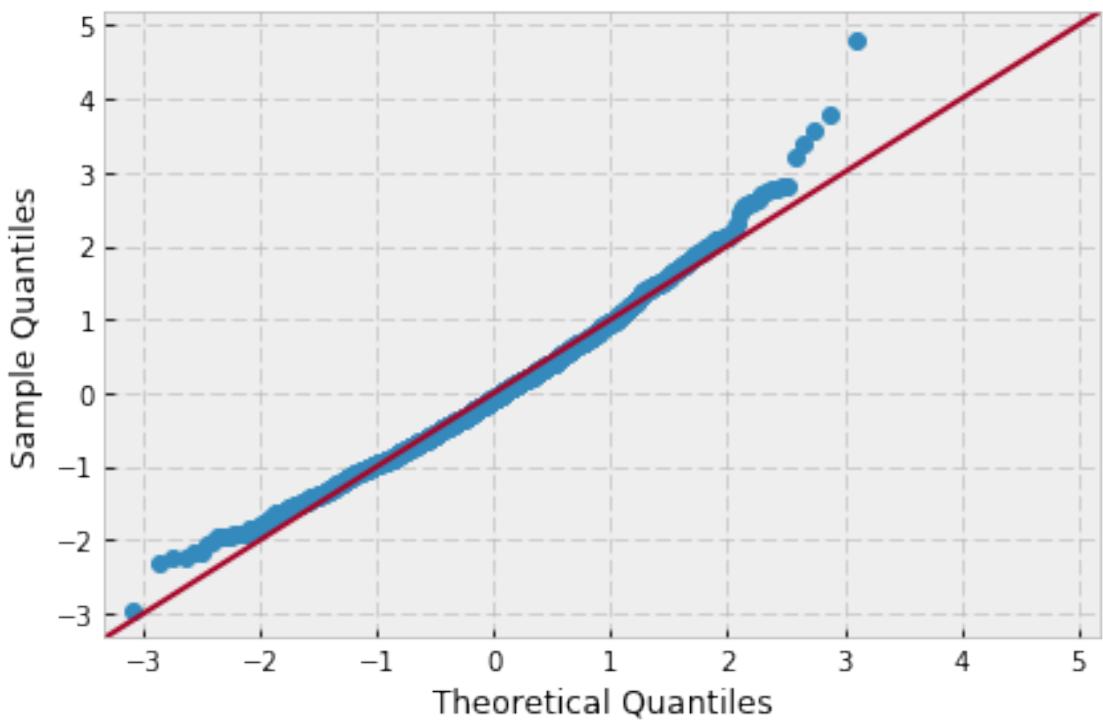
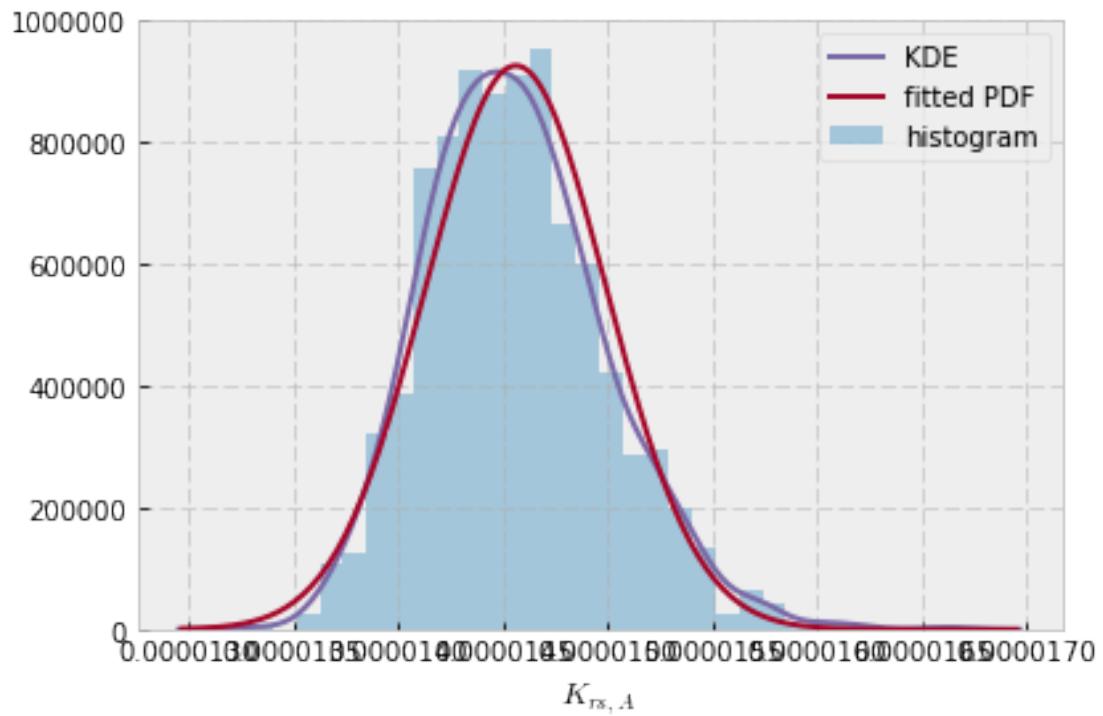
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=1.456323951582273e-05, scale=4.4315929429429894e-07)
DescribeResult(nobs=1000, minmax=(1.3473861853509188e-05, 1.6402298470970632e-05)
               mean=1.4563239515822731e-05, variance=1.9658674686628735e-13,
               skewness=0.5594911768147253, kurtosis=0.6246147438365437)

```





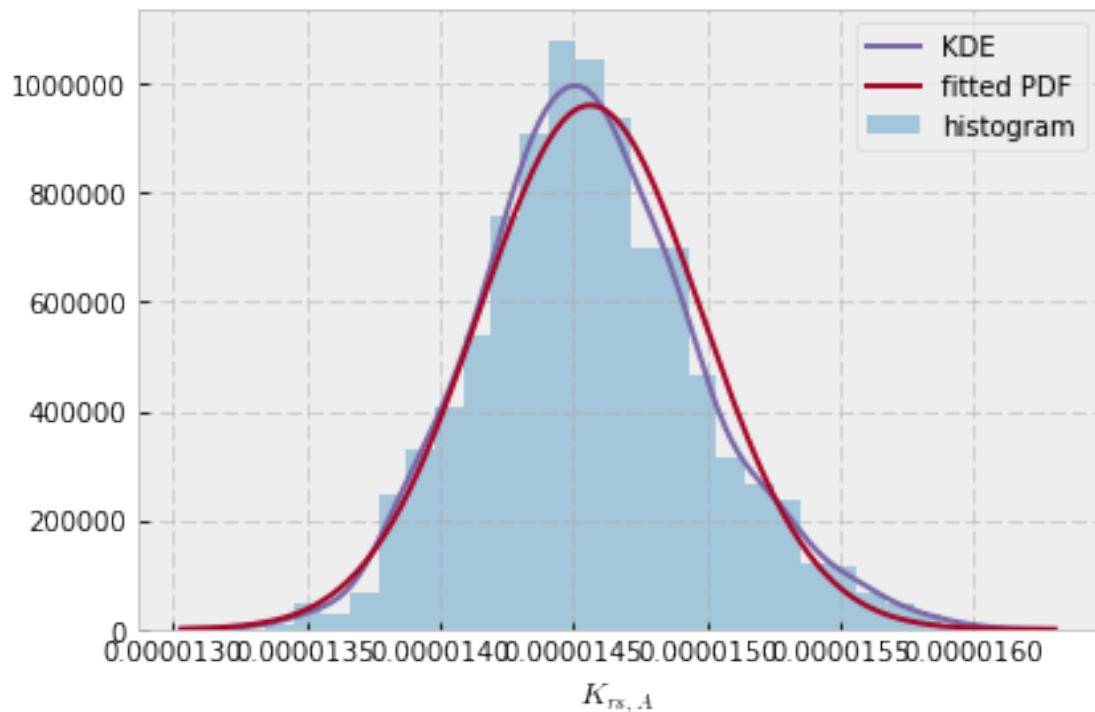
```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=1.4562757543972744e-05, scale=4.307511793972551e-07)
DescribeResult(nobs=1000, minmax=(1.3292931626272593e-05, 1.6627378441814238e-05)
               mean=1.4562757543972744e-05, variance=1.8573231086298928e-13,
               skewness=0.5258212647075221, kurtosis=0.6526155187162779)
```

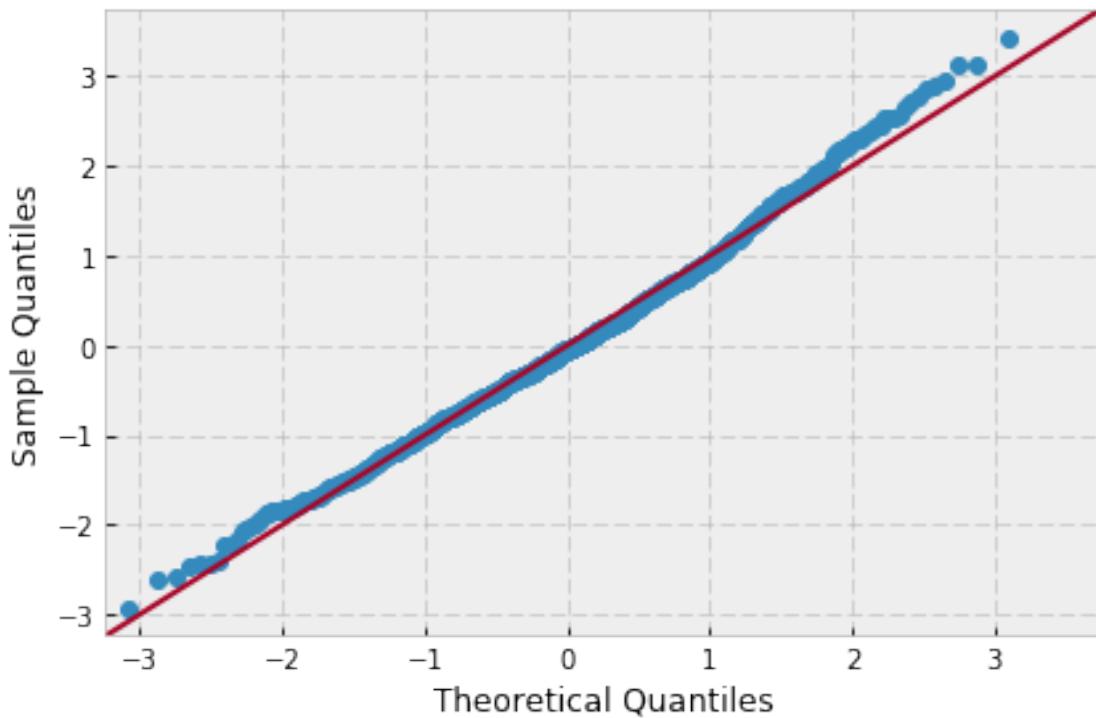


```

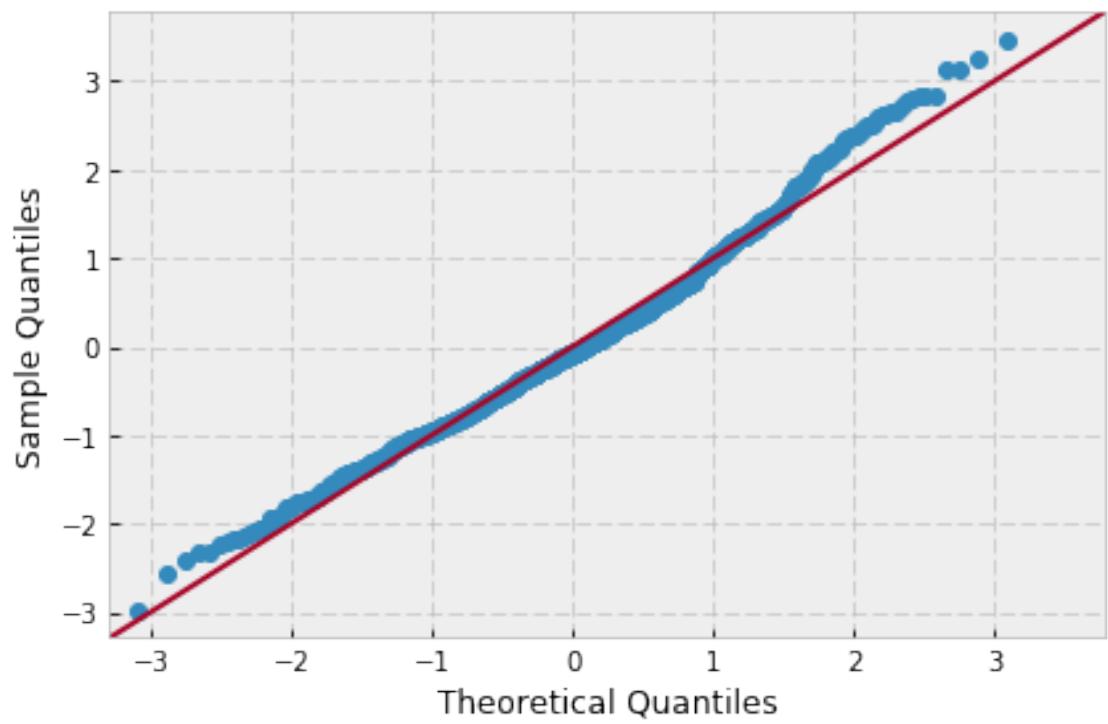
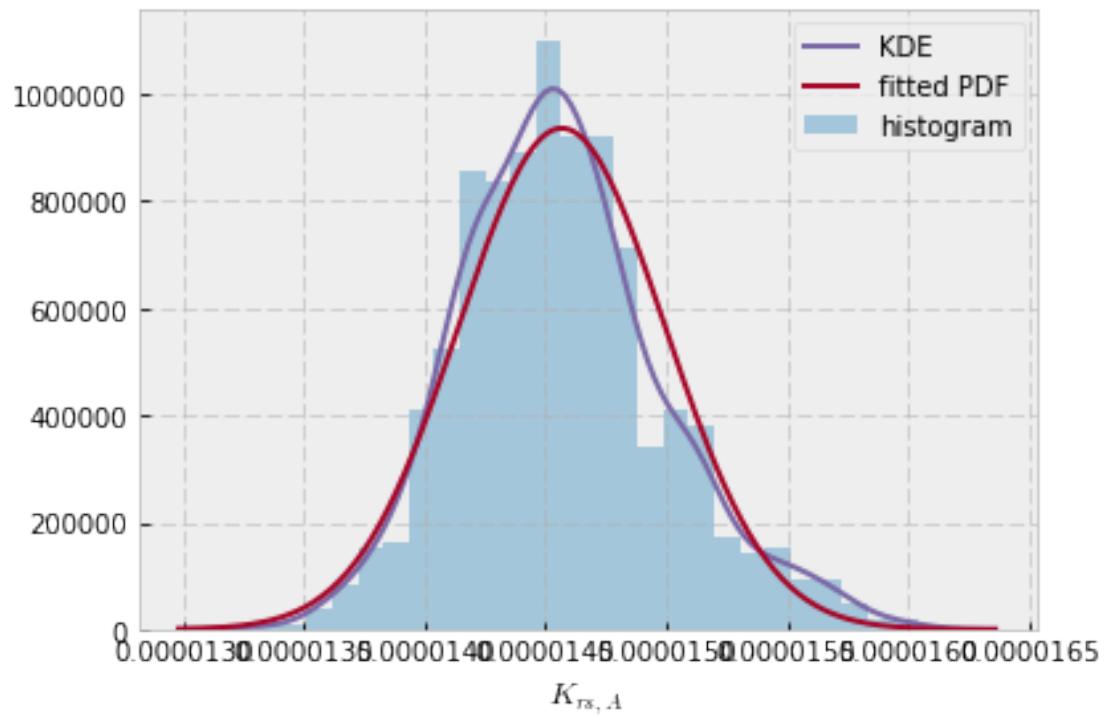
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=1.456278922501739e-05, scale=4.1570617032917763e-07)
DescribeResult(nobs=1000, minmax=(1.3347092089247182e-05, 1.598579622114115e-05)
               mean=1.456278922501739e-05, variance=1.7298460465440568e-13,
               skewness=0.3110945591987144, kurtosis=0.1764296029541237)

```





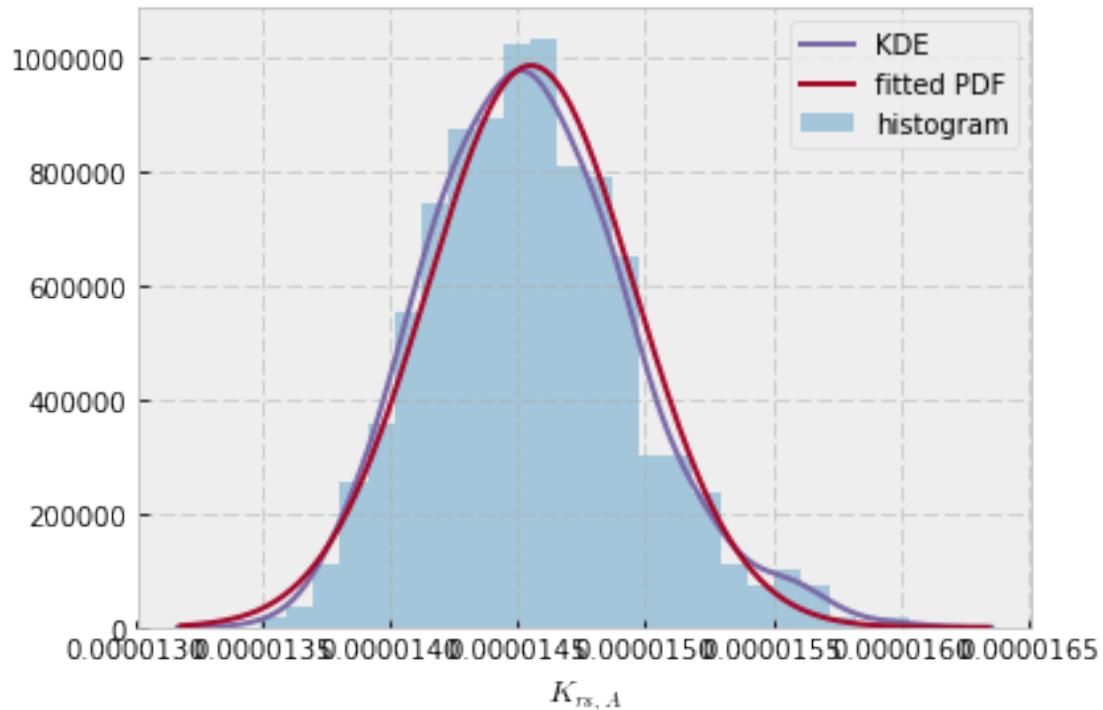
```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=1.4567477368375551e-05, scale=4.261057728499397e-07)
DescribeResult(nobs=1000, minmax=(1.3303221402228488e-05, 1.6040149638646671e-05)
               mean=1.4567477368375551e-05, variance=1.81747877533578e-13,
               skewness=0.45407425028374265, kurtosis=0.3478845234640273)
```

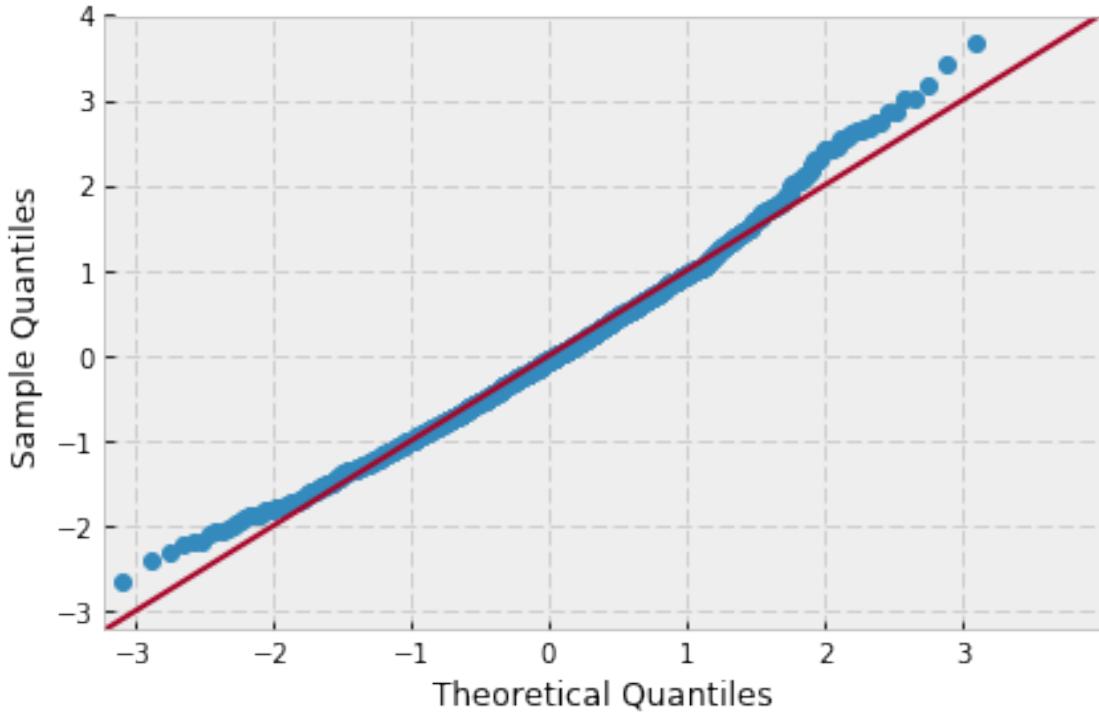


```

variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=1.4551050537857814e-05, scale=4.0497374392818366e-07)
DescribeResult(nobs=1000, minmax=(1.3484828726591847e-05, 1.6037841456568082e-05)
               mean=1.4551050537857814e-05, variance=1.6416790117238248e-13,
               skewness=0.42229903686757153, kurtosis=0.23541640689183163)

```

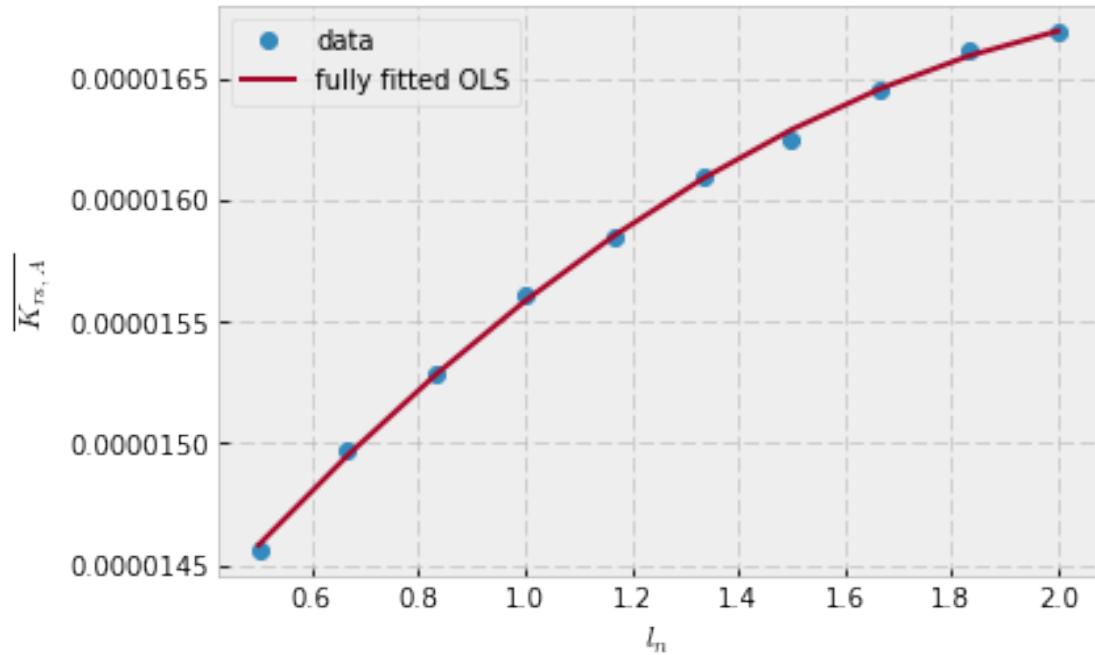




17.2 Parameter regression

```
In [40]: var_name = 'root_system_conductance_per_unit_of_root_area'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly1')
mean_reg('maxB', var_name, 'poly2')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly1')
mean_reg('N', var_name, 'poly1')
```

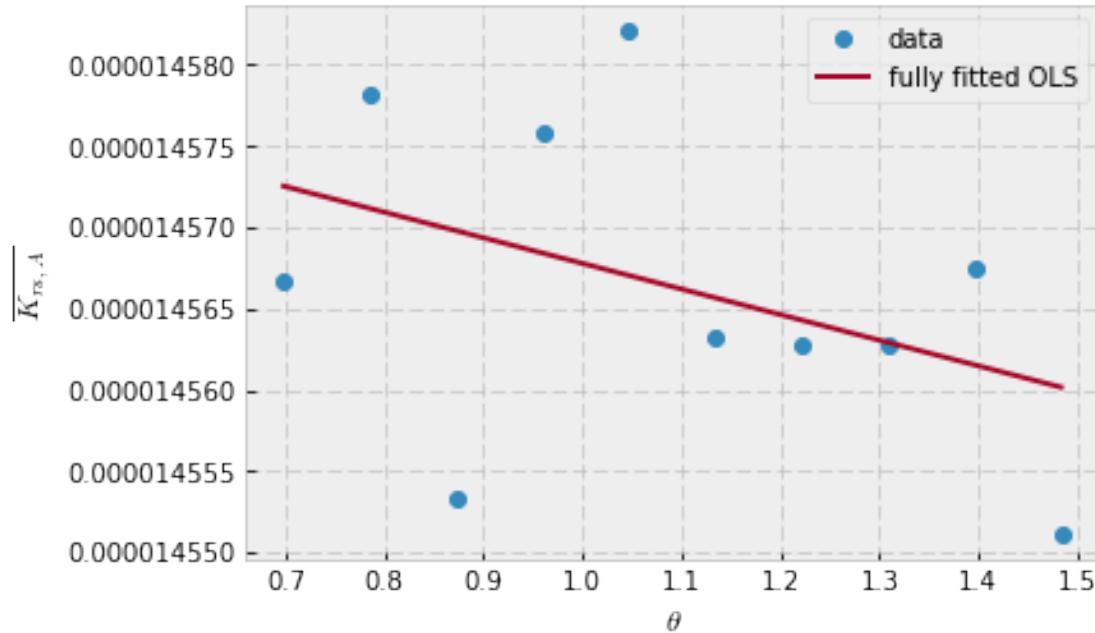
```
variable name: ln
model: poly2
r-squared: 0.9977542868308987
explained variance: 0.997771344460812
```



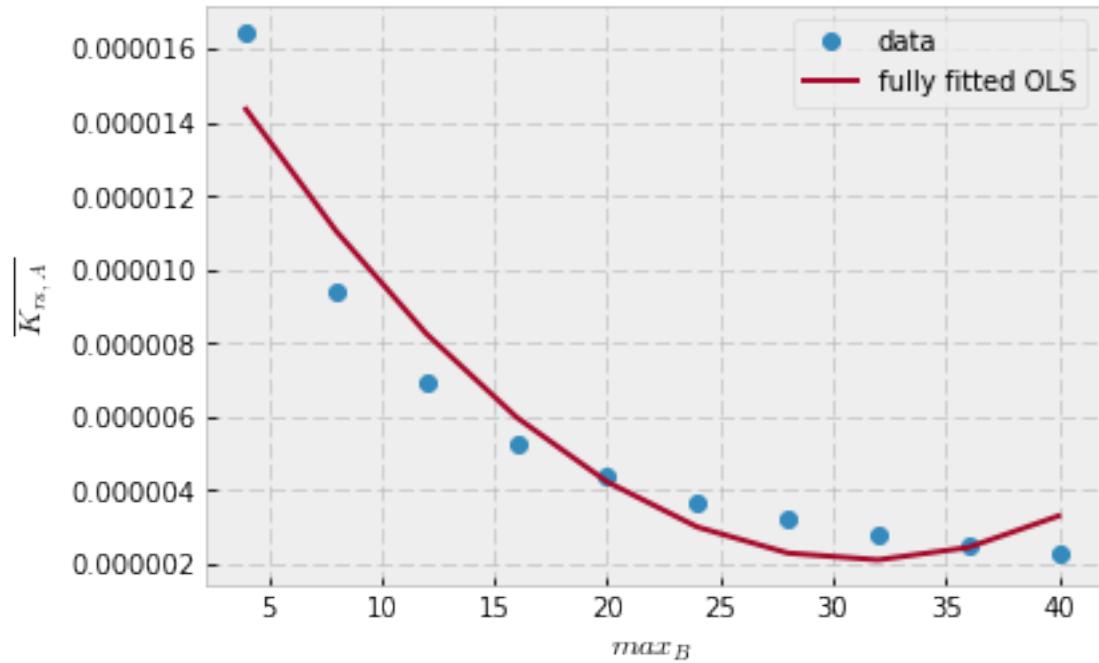
```

variable name: theta
model: poly1
r-squared: -0.29169053129454214
explained variance: -0.2891877681493418

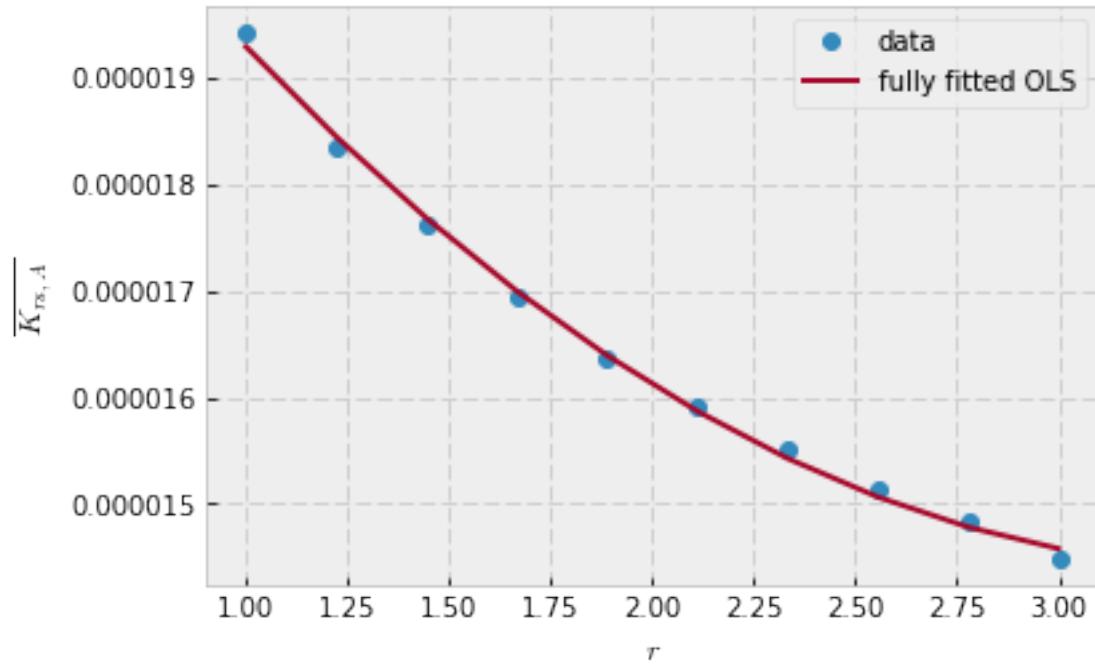
```



```
variable name: maxB
model: poly2
r-squared: 0.7161169664203737
explained variance: 0.7171816027791205
```



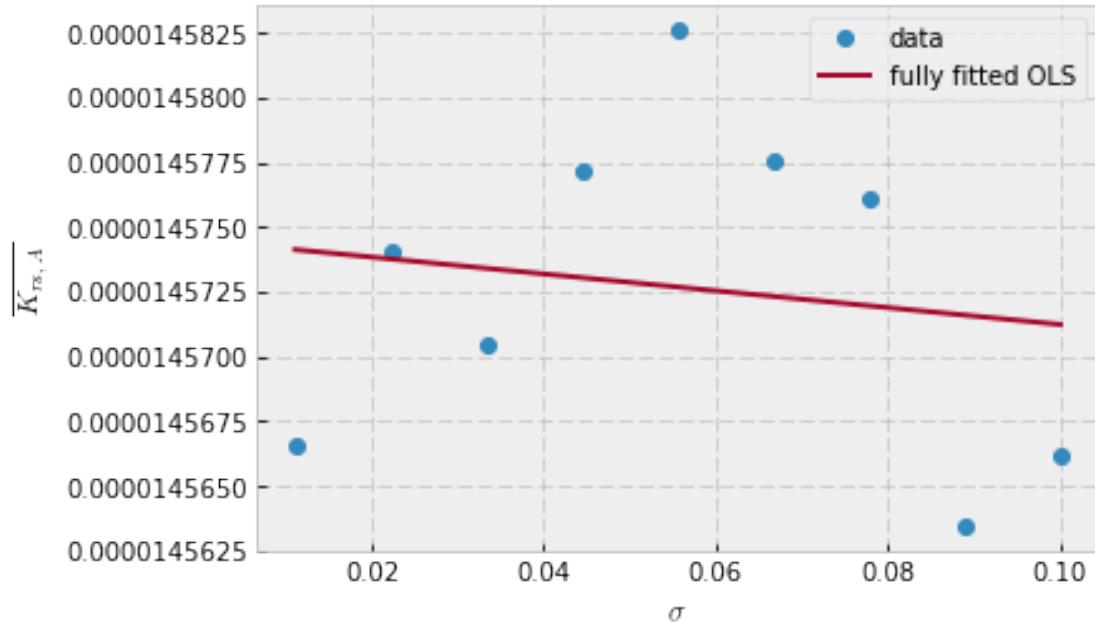
```
variable name: r
model: poly2
r-squared: 0.9885906835425083
explained variance: 0.9885972868340235
```



```

variable name: sigma
model: poly1
r-squared: -0.7073216318264504
explained variance: -0.6878392918876544

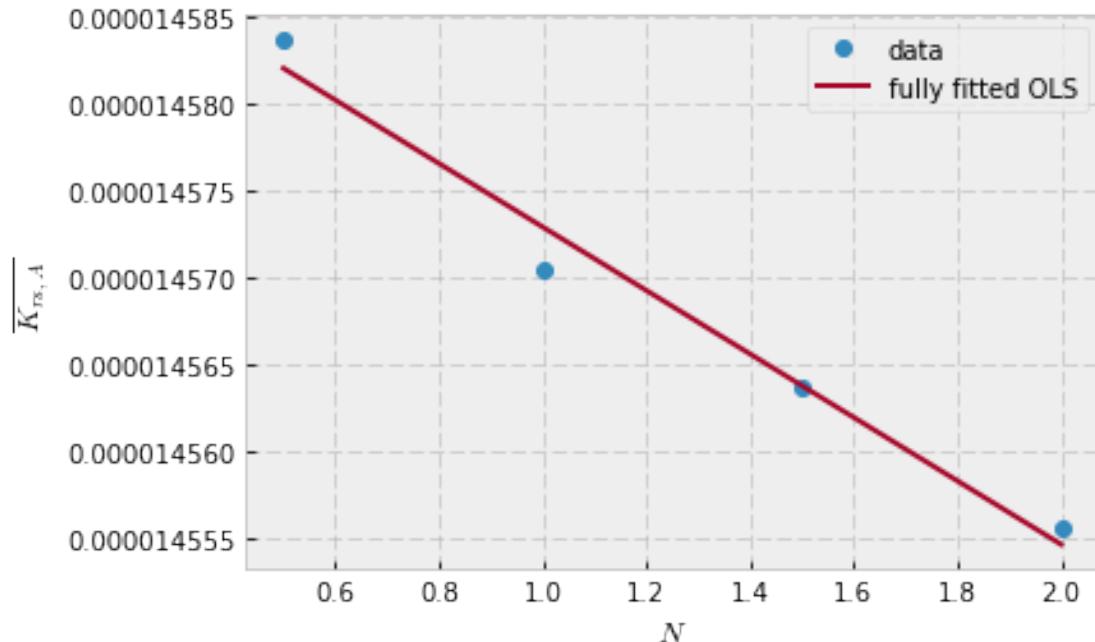
```



```

variable name: N
model: poly1
r-squared: 0.8752426000002083
explained variance: 0.8894018813789889

```



18 Root system conductance per unit of root length

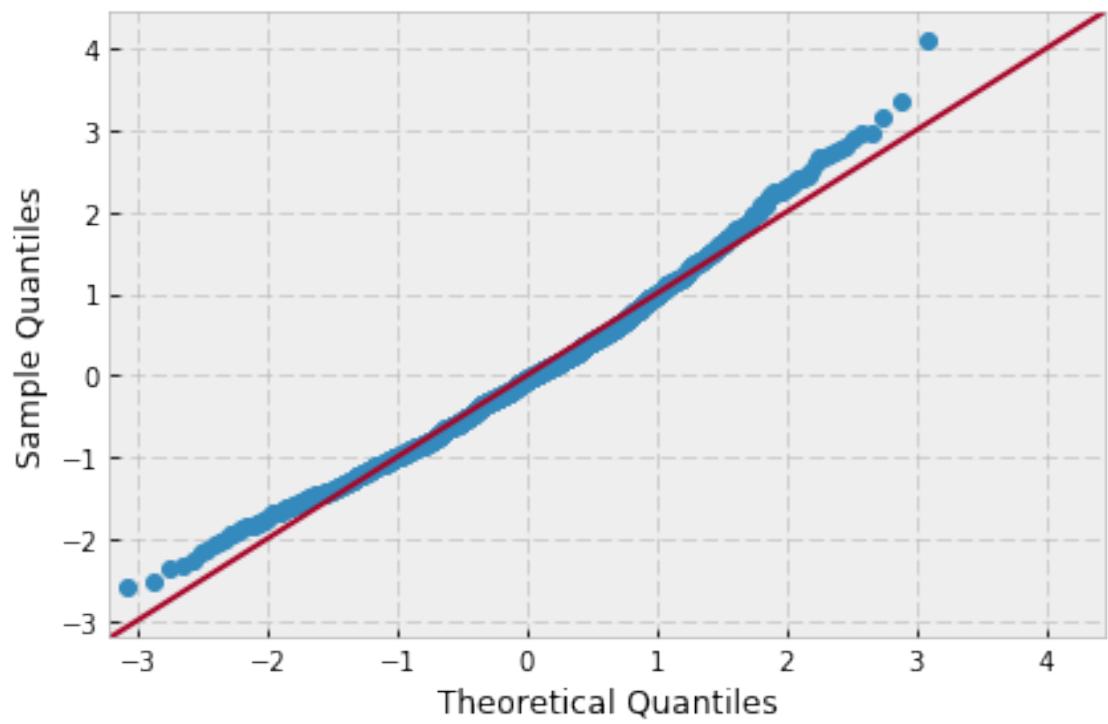
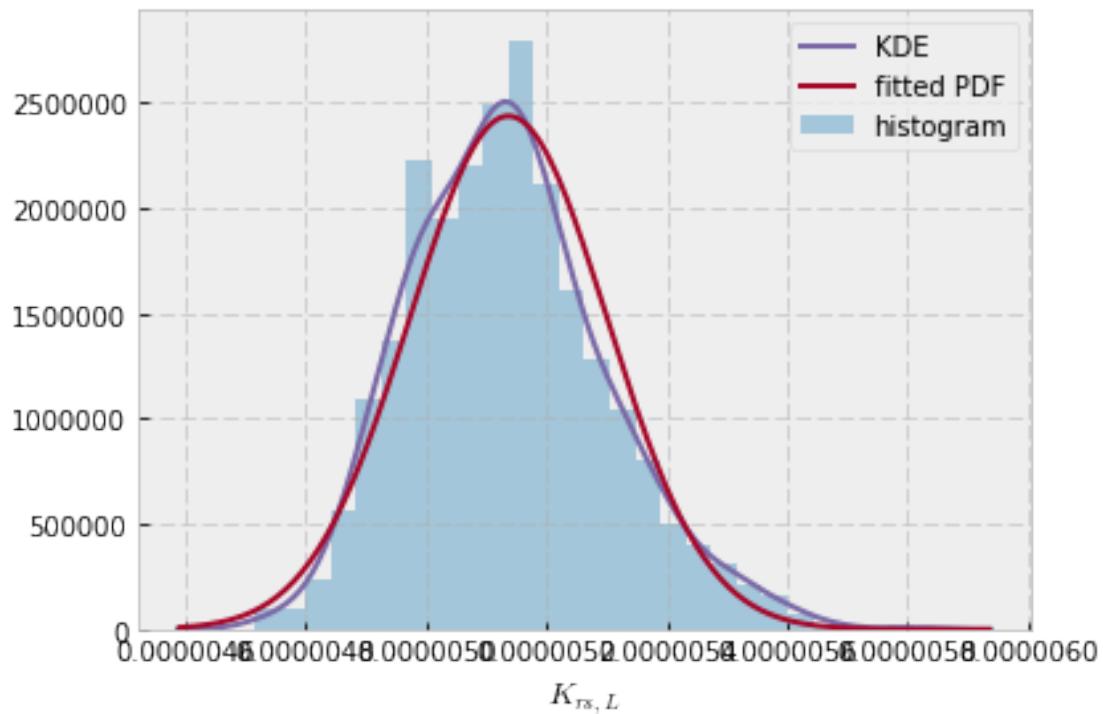
18.1 Probability distributions

```

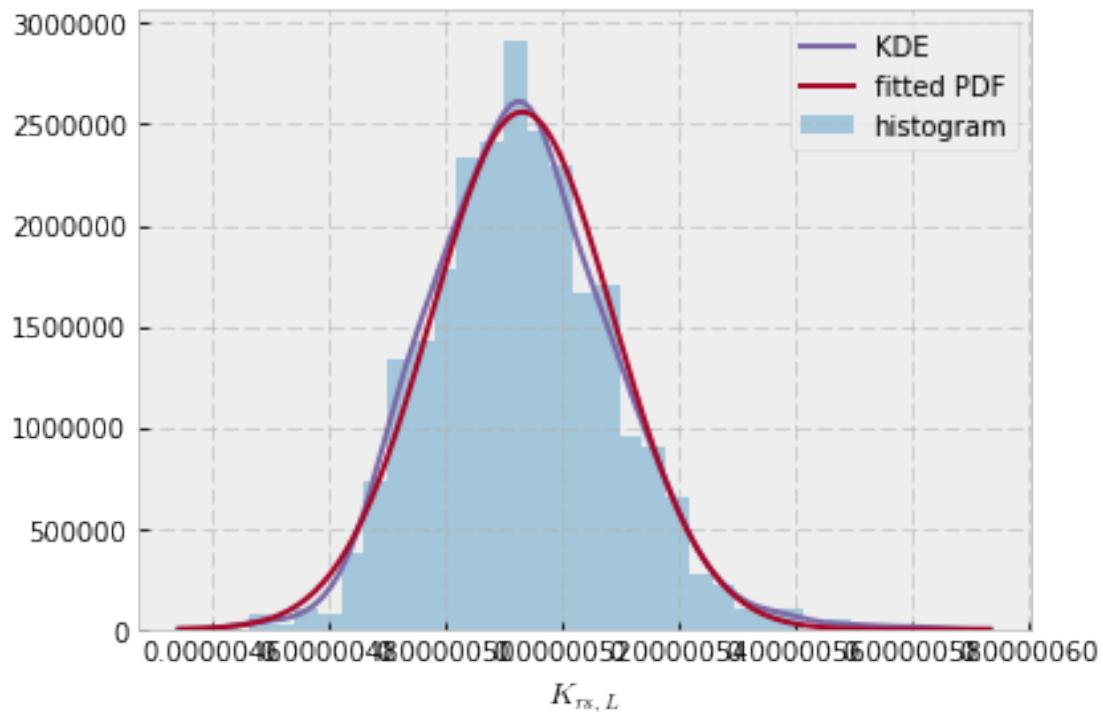
In [41]: plot_dist('root_system_conductance_per_unit_of_root_length')

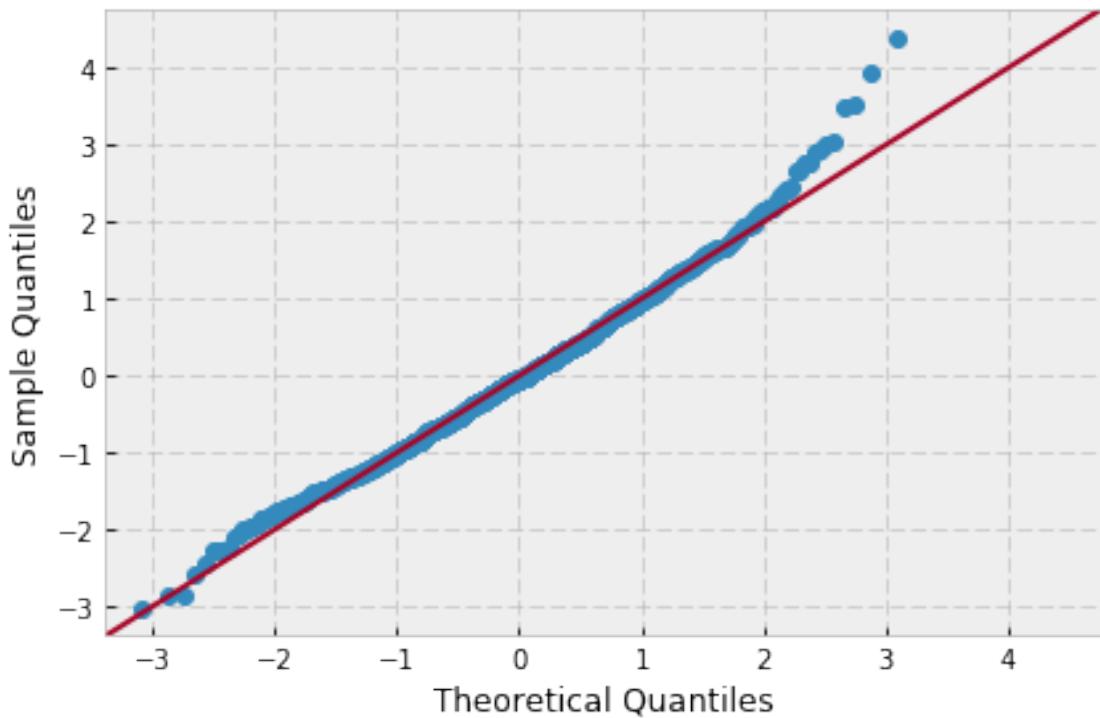
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=5.136819372335771e-06, scale=1.6377379022419562e-07)
DescribeResult(nobs=1000, minmax=(4.7147925254501379e-06, 5.8109222948429582e-06)
               mean=5.1368193723357707e-06, variance=2.6848703067466301e-14,
               skewness=0.4653443924060871, kurtosis=0.2860396292813454)

```

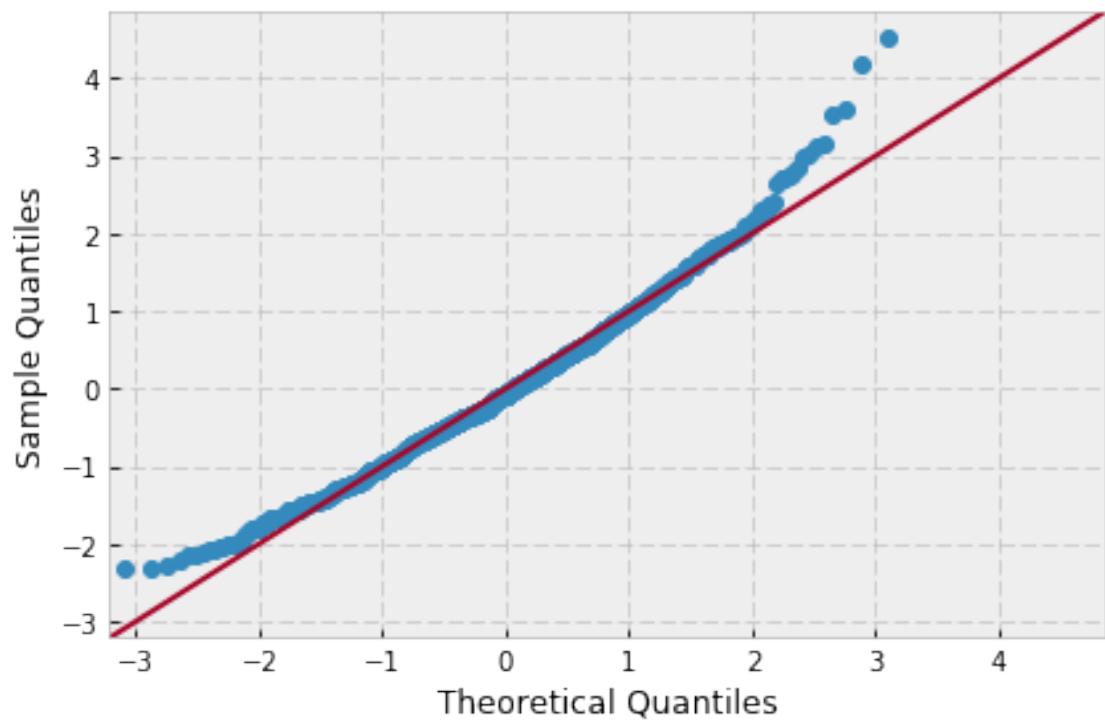
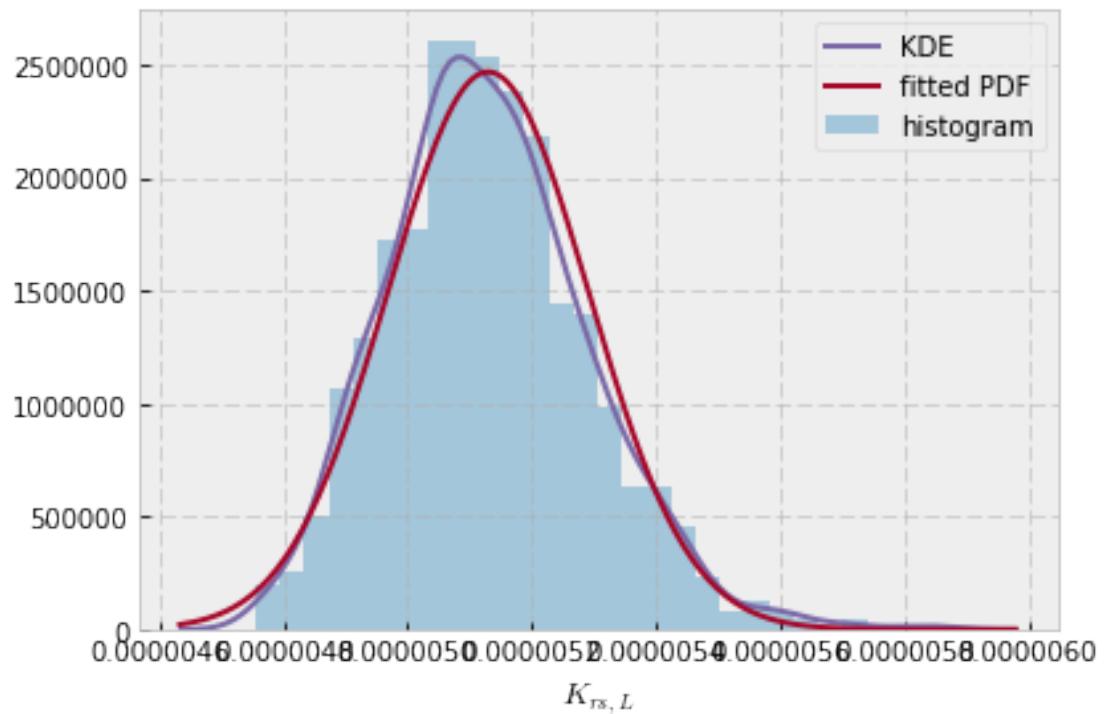


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=5.1321792966375575e-06, scale=1.5579756382277266e-07)
DescribeResult(nobs=1000, minmax=(4.6625274550788545e-06, 5.8148918725387936e-06)
               mean=5.1321792966375575e-06, variance=2.4297178071182105e-14,
               skewness=0.3637355198523635, kurtosis=0.5752954685499696)
```





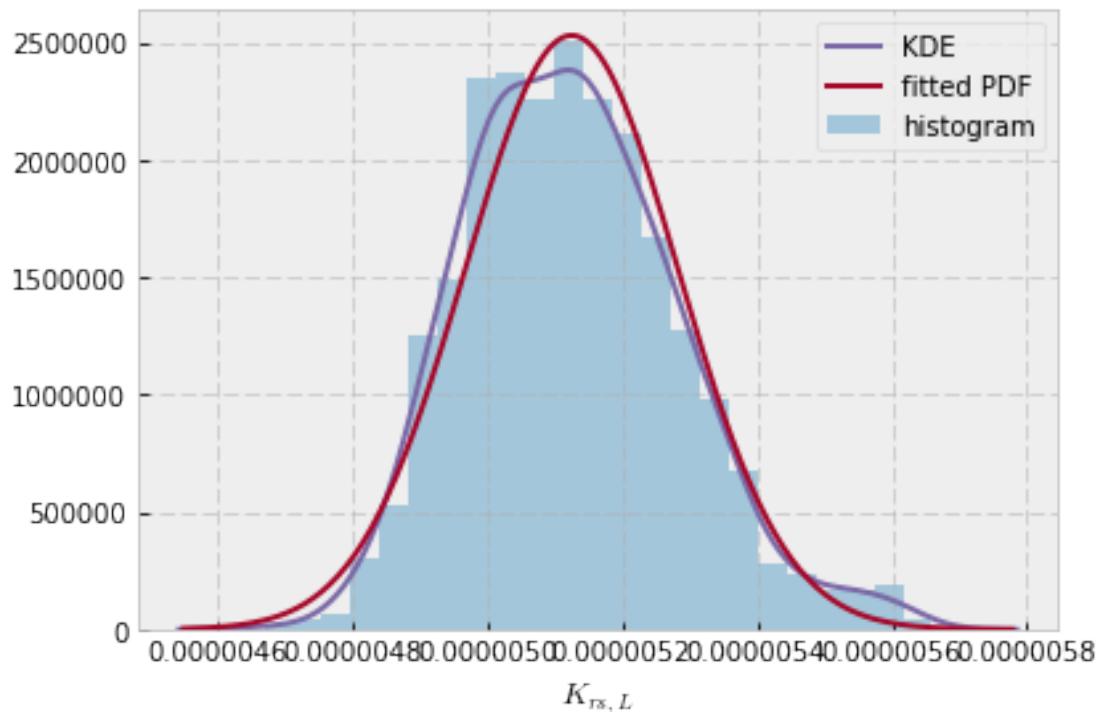
```
variable name: N
variable value: 1.5
distribution: normal(shape=(),loc=5.1294840153056306e-06, scale=1.615435667892132e-07)
DescribeResult(nobs=1000, minmax=(4.7543547503384262e-06, 5.857982585595107e-06)
               mean=5.1294840153056306e-06, variance=2.6122446417398392e-14,
               skewness=0.5109411045429784, kurtosis=0.6796802638568589)
```

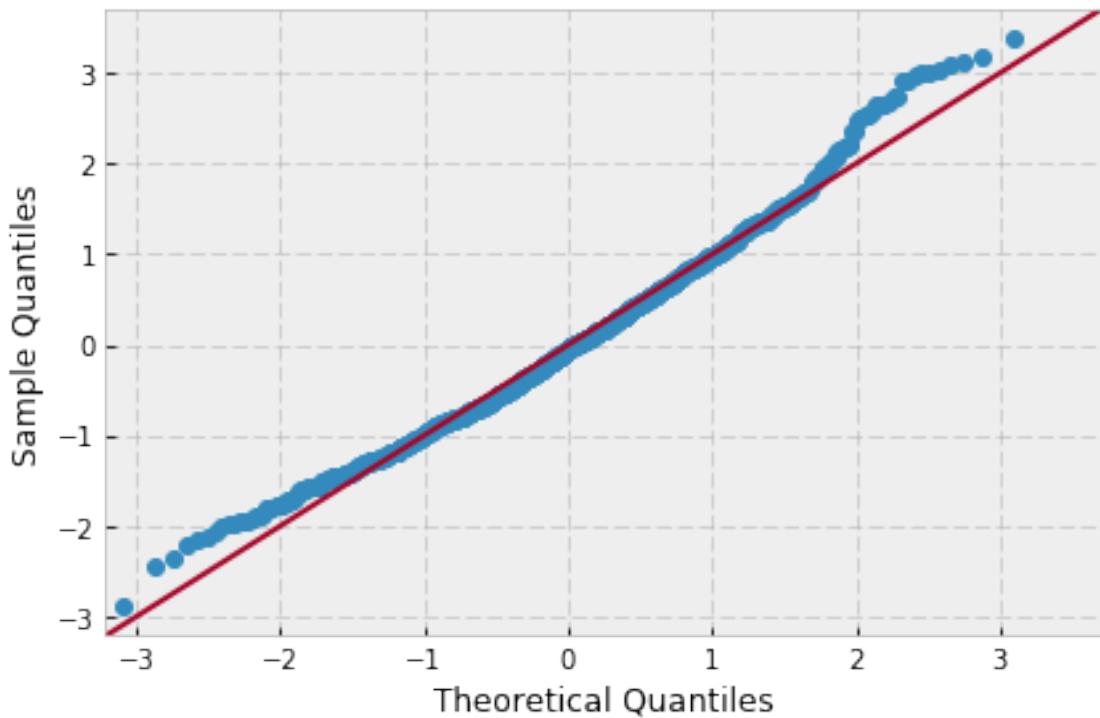


```

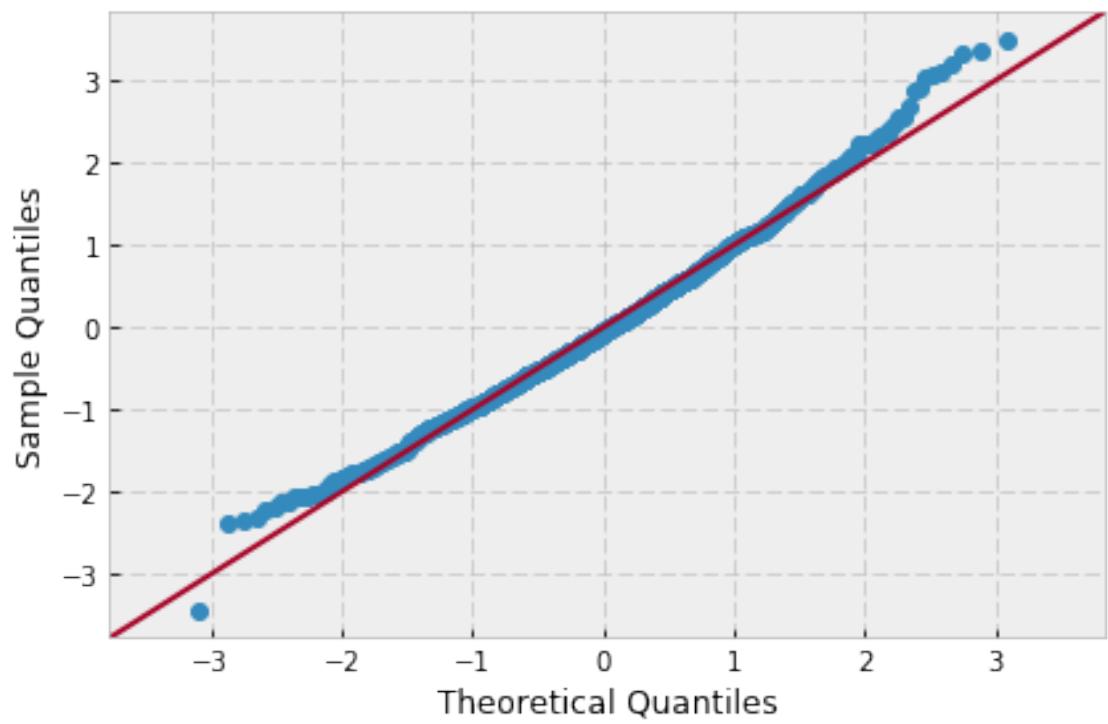
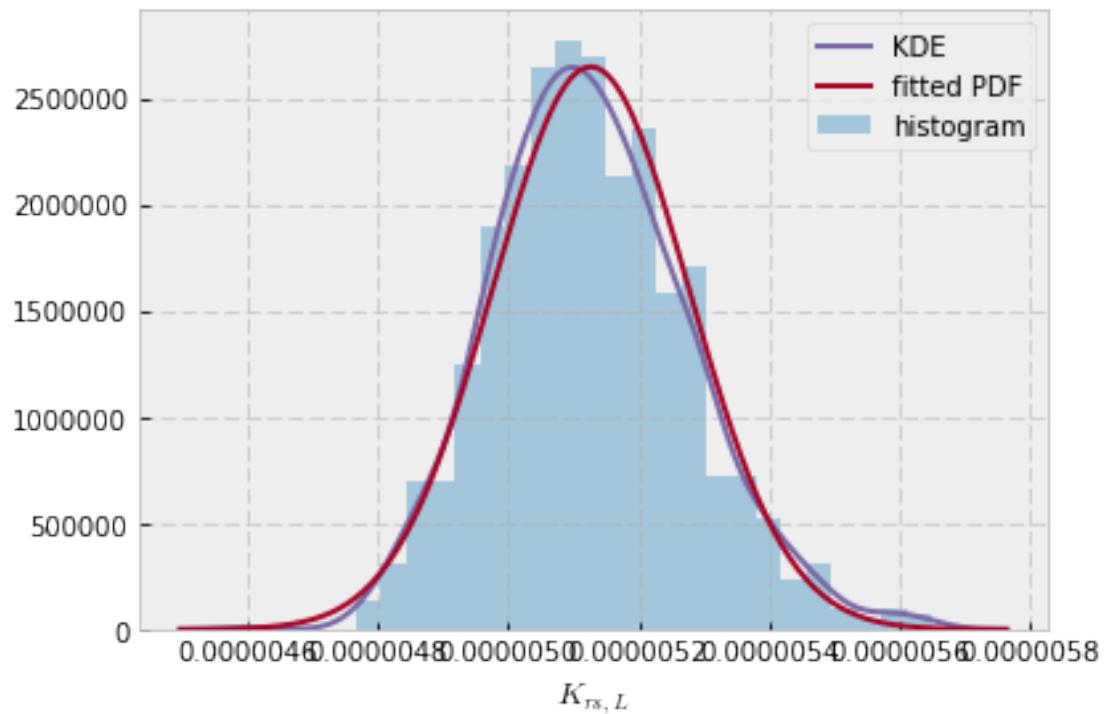
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=5.125696671632045e-06, scale=1.5765948054919972e-07)
DescribeResult(nobs=1000, minmax=(4.6702456767972925e-06, 5.6587561774700072e-06)
               mean=5.1256966716320452e-06, variance=2.4881393200243728e-14,
               skewness=0.4440172442509141, kurtosis=0.21127305592133583)

```





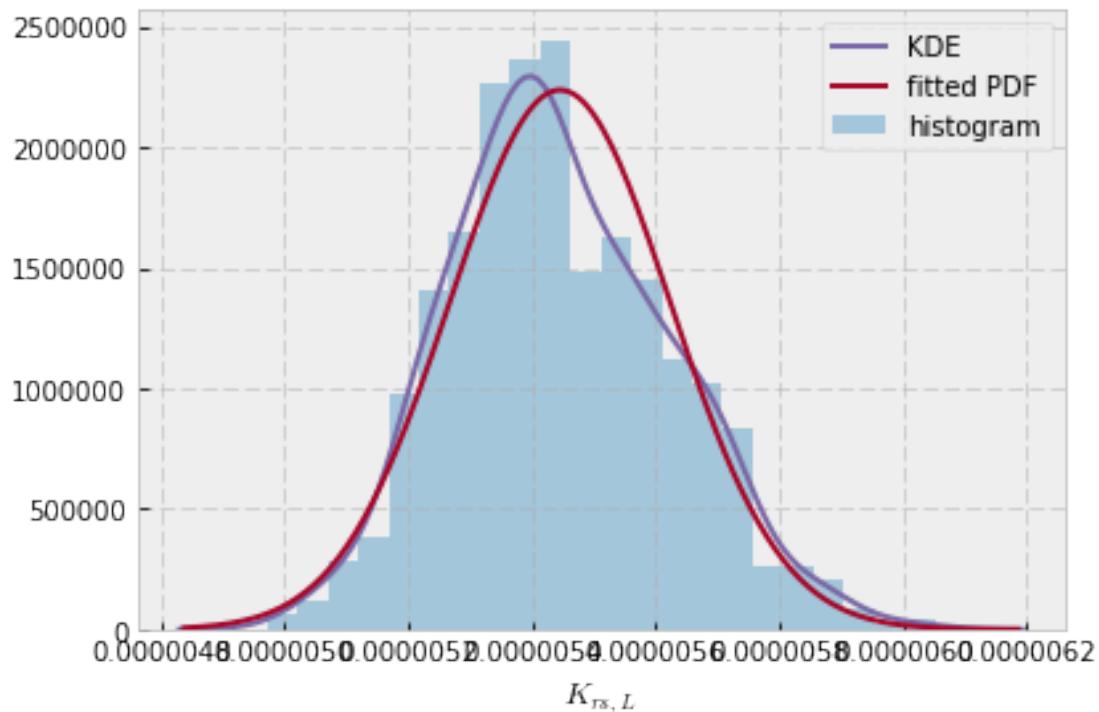
```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=5.126280215435601e-06, scale=1.5076991421938794e-07)
DescribeResult(nobs=1000, minmax=(4.609777306795153e-06, 5.6510449214591067e-06)
               mean=5.1262802154356014e-06, variance=2.2754321355076674e-14,
               skewness=0.3437870194693273, kurtosis=0.26538366004230696)
```

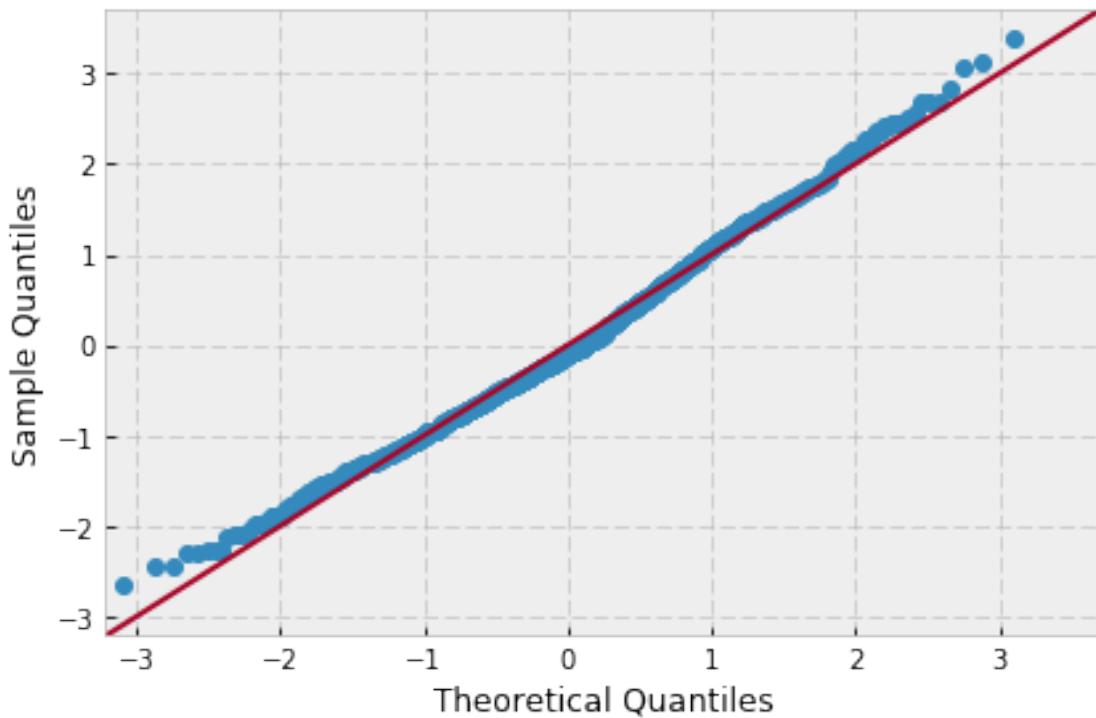


```

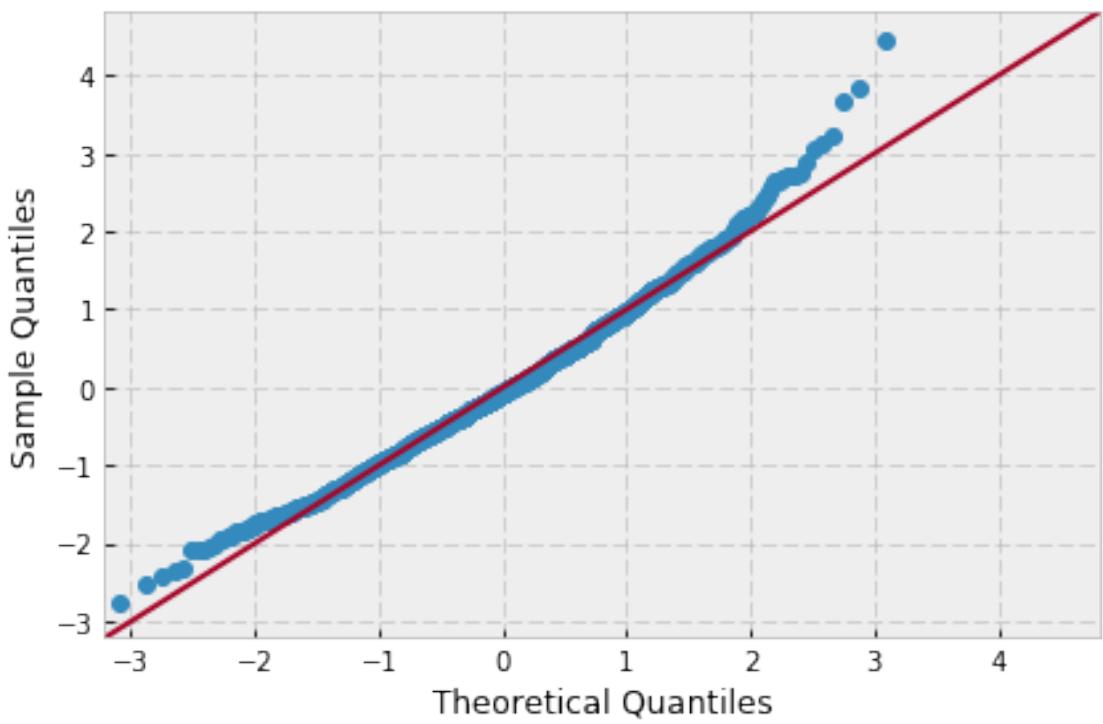
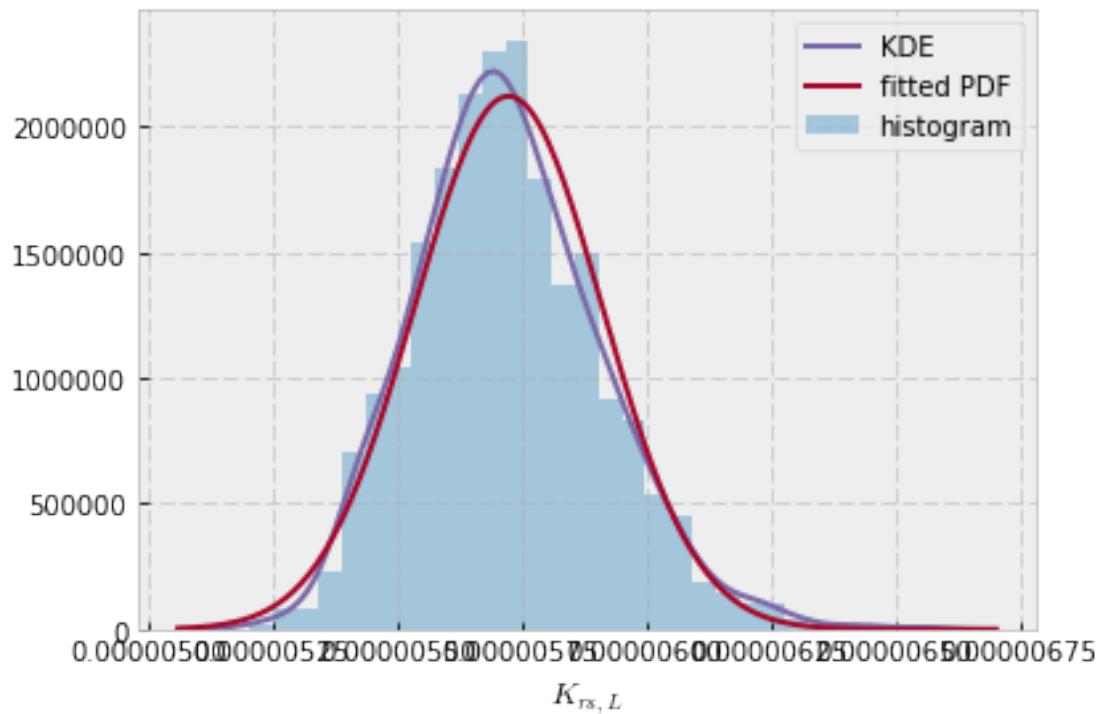
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=5.445544107463372e-06, scale=1.7836801277719195e-07)
DescribeResult(nobs=1000, minmax=(4.9720674710276138e-06, 6.0514157602307875e-06)
               mean=5.4455441074633717e-06, variance=3.1846994977061575e-14,
               skewness=0.3047367942813329, kurtosis=-0.13427034740172195)

```





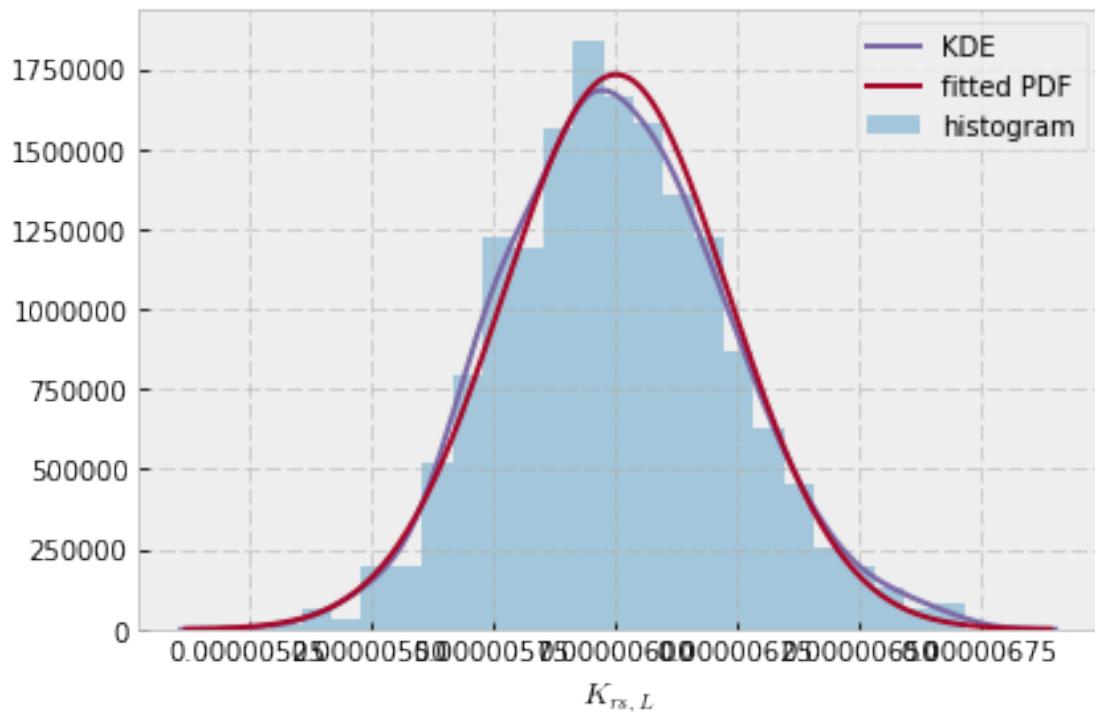
```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=5.723841281099328e-06, scale=1.8808181909779862e-07)
DescribeResult(nobs=1000, minmax=(5.20192822350784e-06, 6.5613745885937651e-06)
               mean=5.7238412810993276e-06, variance=3.5410180855993041e-14,
               skewness=0.46842431477947866, kurtosis=0.5703525447754028)
```

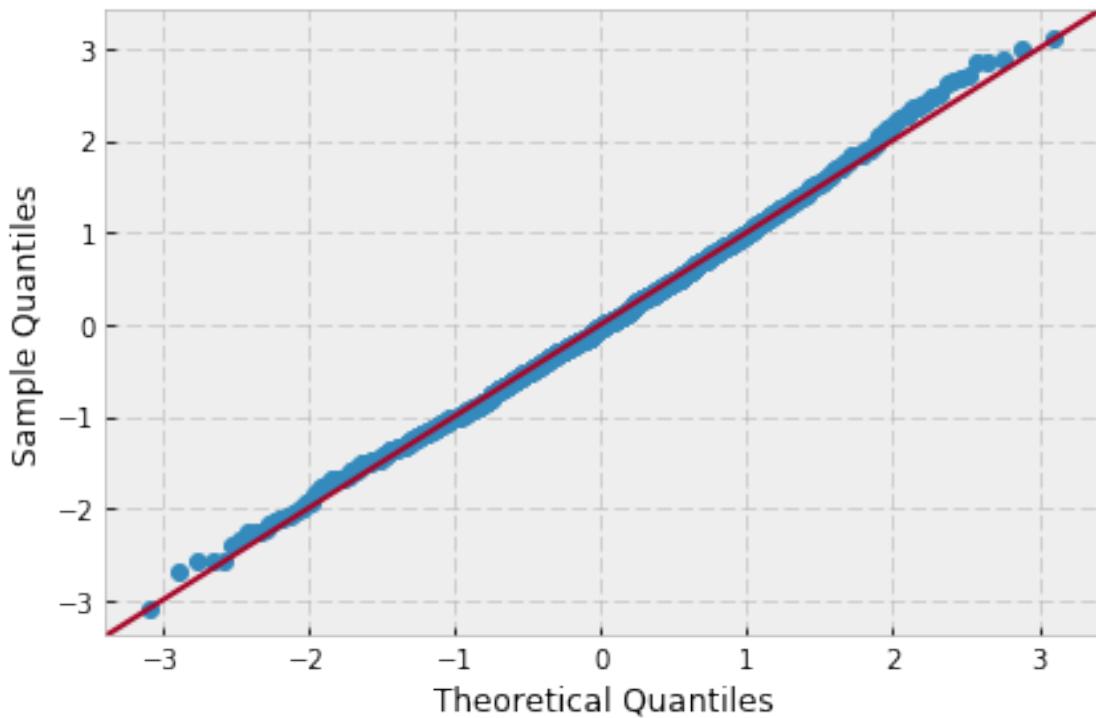


```

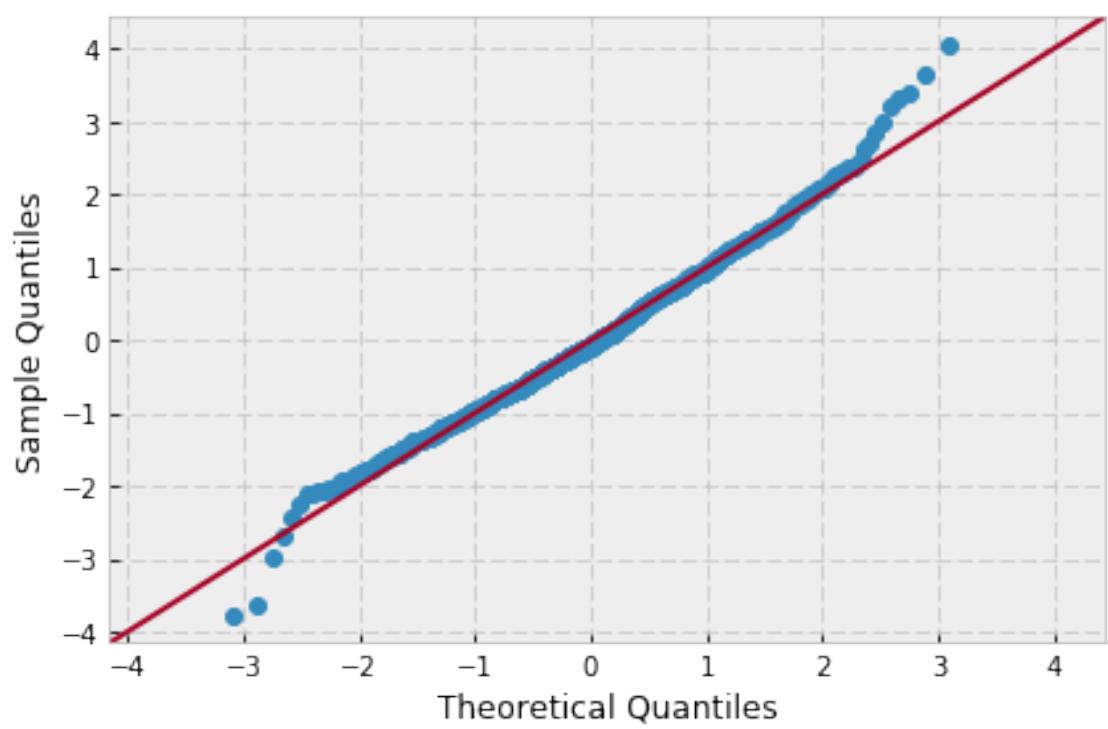
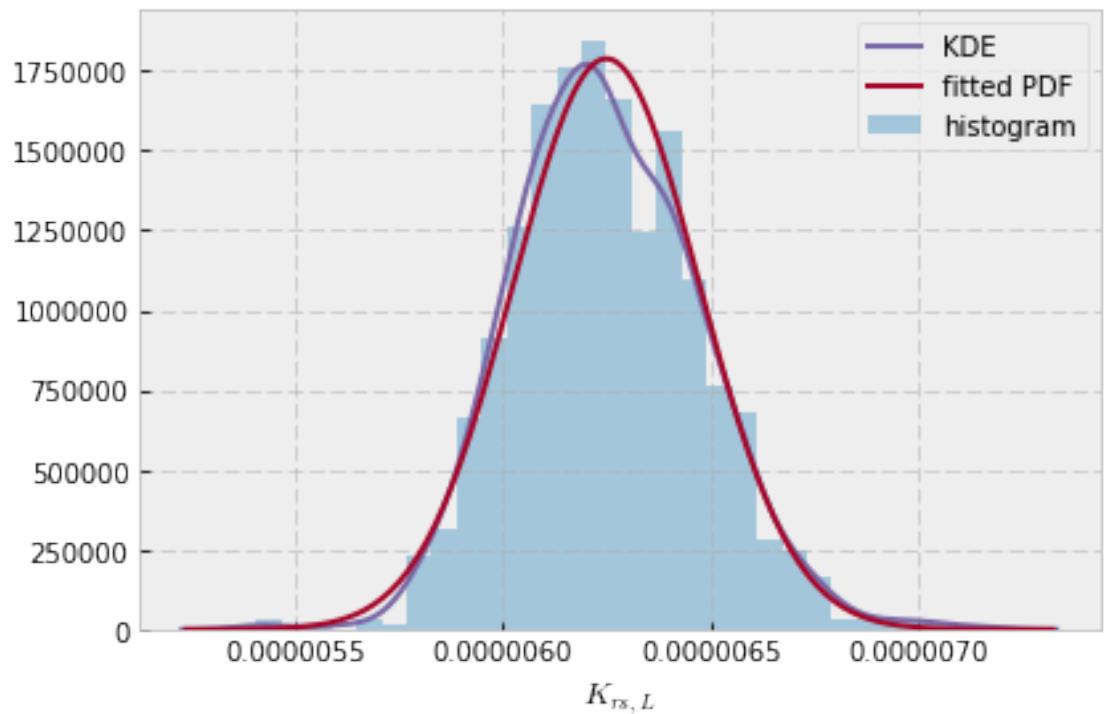
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=6.003883785543642e-06, scale=2.3030117052491595e-07)
DescribeResult(nobs=1000, minmax=(5.296119897026012e-06, 6.7199390866084046e-06)
               mean=6.003883785543642e-06, variance=5.3091720866012433e-14,
               skewness=0.18857461089811367, kurtosis=-0.0196163731468042)

```





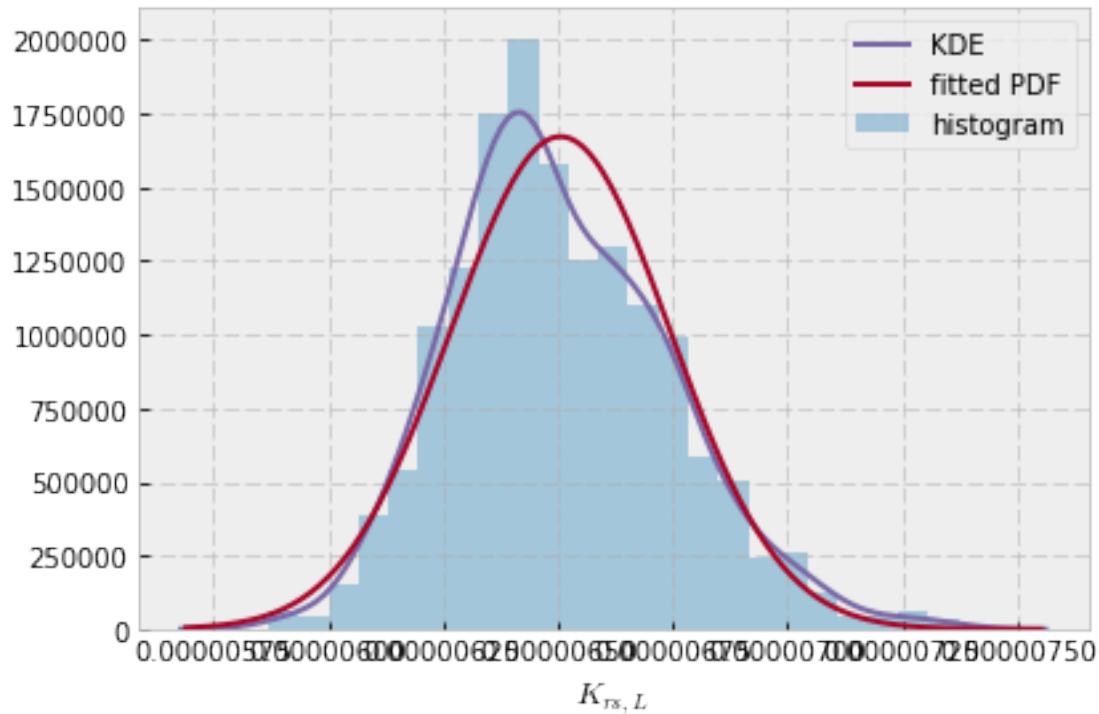
```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(),loc=6.248825343644636e-06, scale=2.2305403624391032e-07)
DescribeResult(nobs=1000, minmax=(5.4089705029322311e-06, 7.1504662403441401e-06)
               mean=6.2488253436446356e-06, variance=4.980290599069035e-14,
               skewness=0.256360794857108, kurtosis=0.4971887841849303)
```

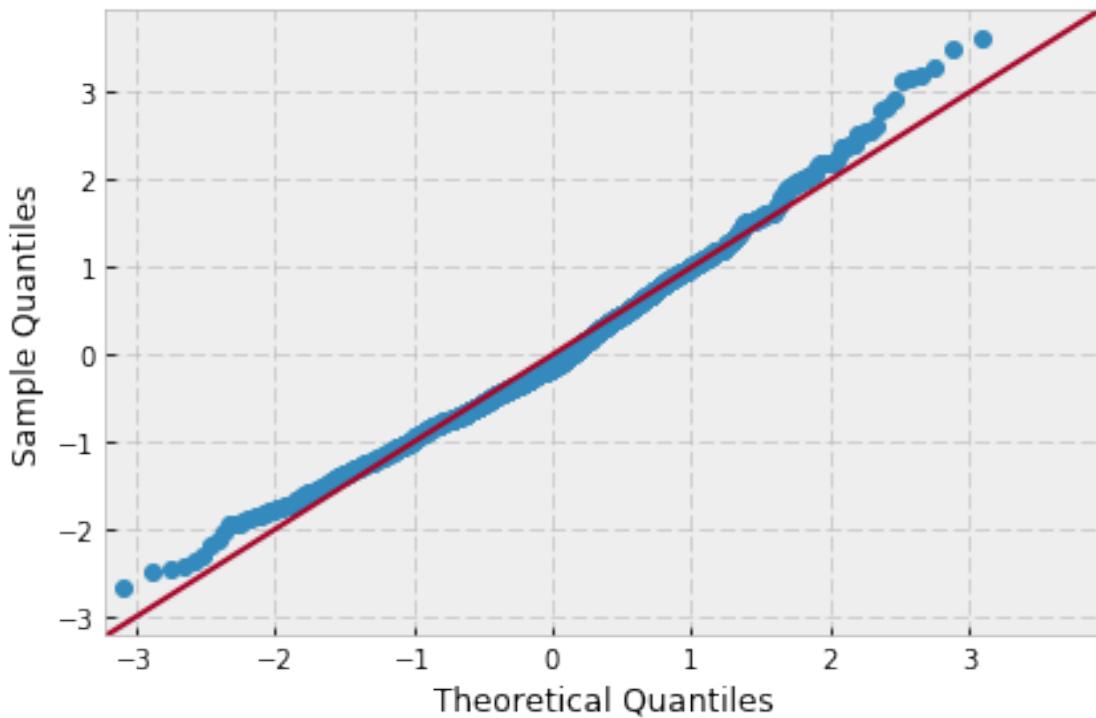


```

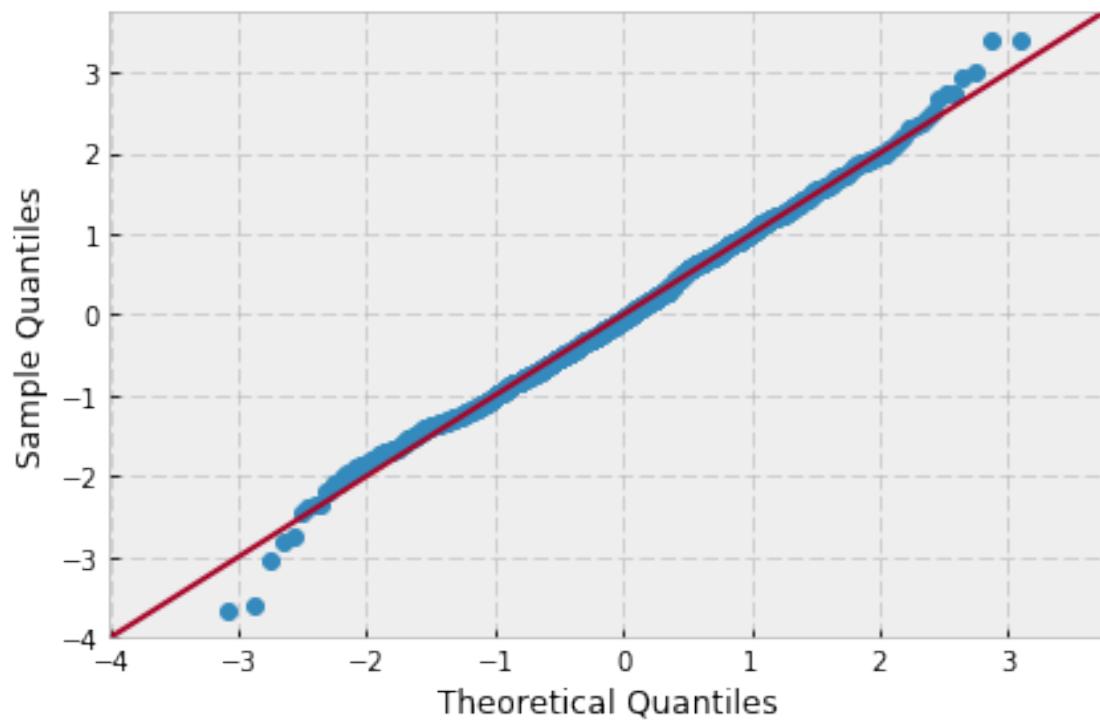
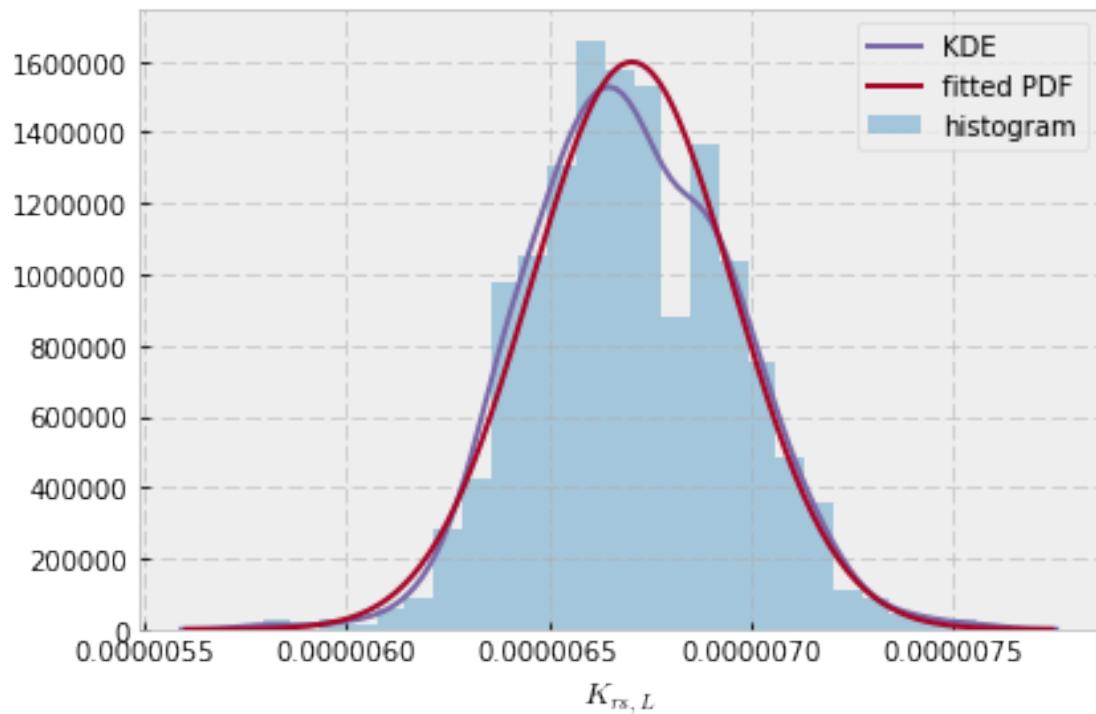
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=6.505282590142195e-06, scale=2.385089253014024e-07)
DescribeResult(nobs=1000, minmax=(5.8701471123396337e-06, 7.3701618054007927e-06)
               mean=6.5052825901421948e-06, variance=5.6943450899329281e-14,
               skewness=0.44618600278633586, kurtosis=0.21435977276076068)

```





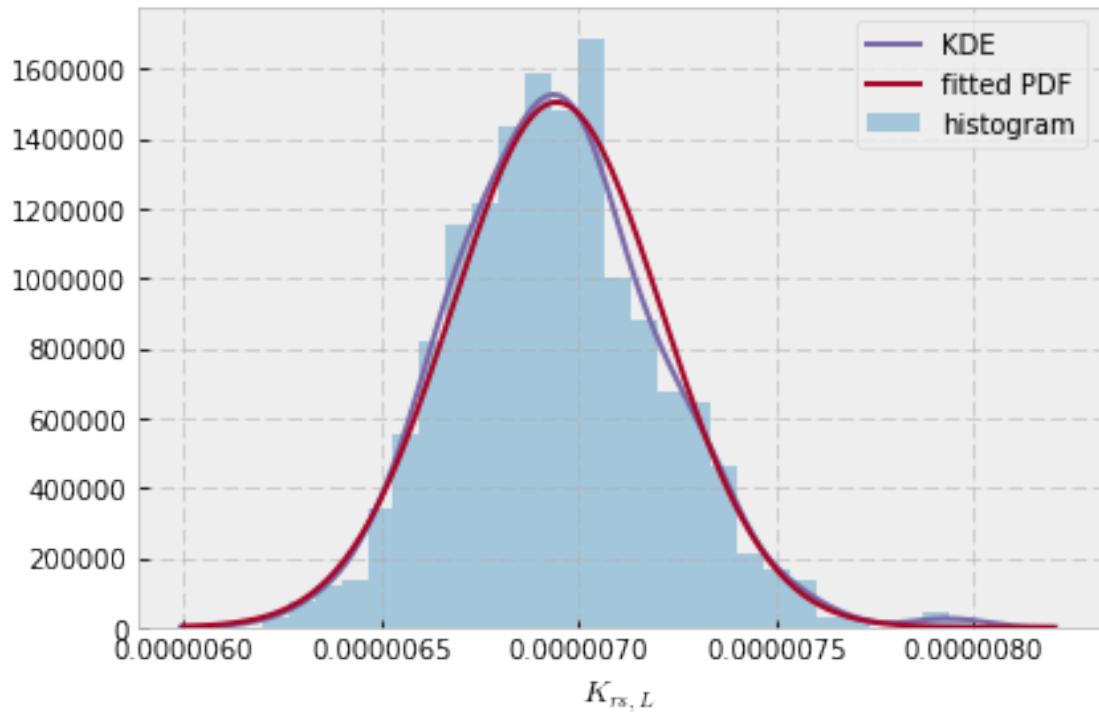
```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=6.705925555807249e-06, scale=2.496443507141575e-07)
DescribeResult(nobs=1000, minmax=(5.7938383516648872e-06, 7.554522113520397e-06)
               mean=6.7059255558072487e-06, variance=6.2384686530023308e-14,
               skewness=0.10891599777542205, kurtosis=0.11292301127446347)
```

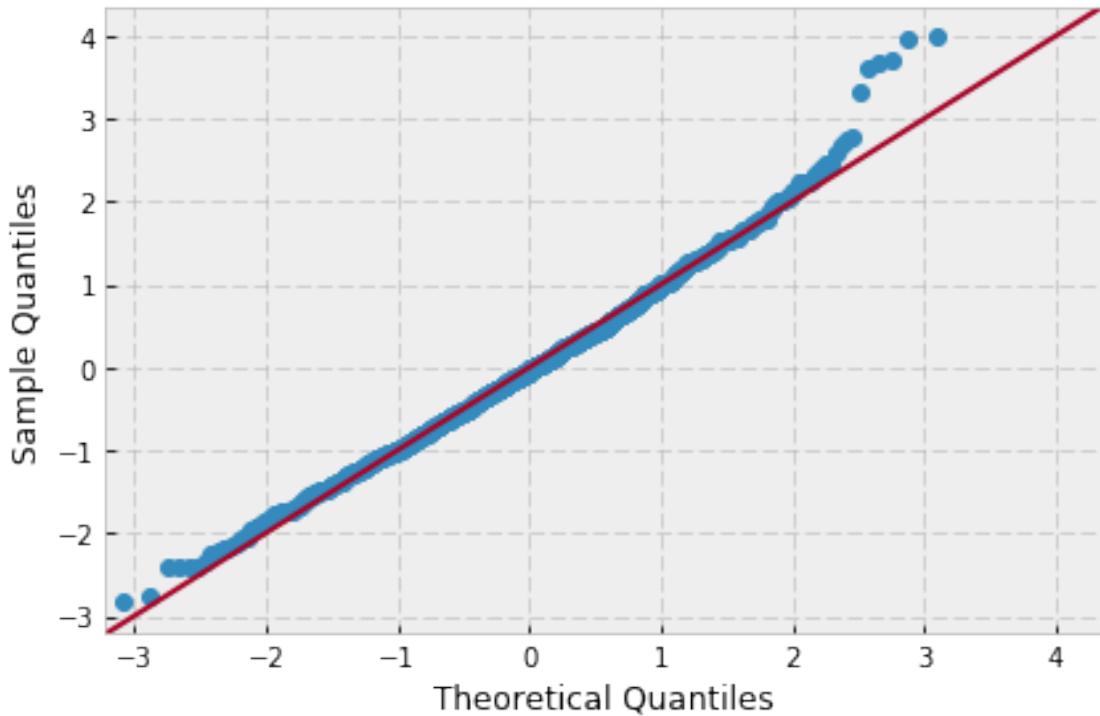


```

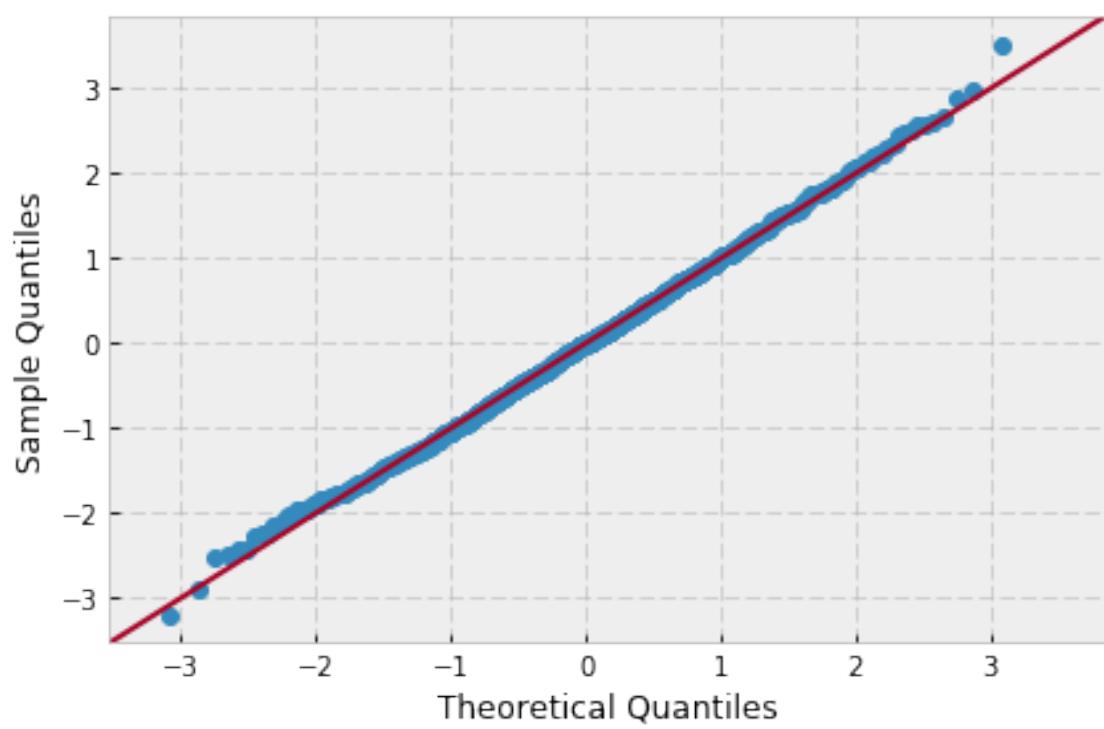
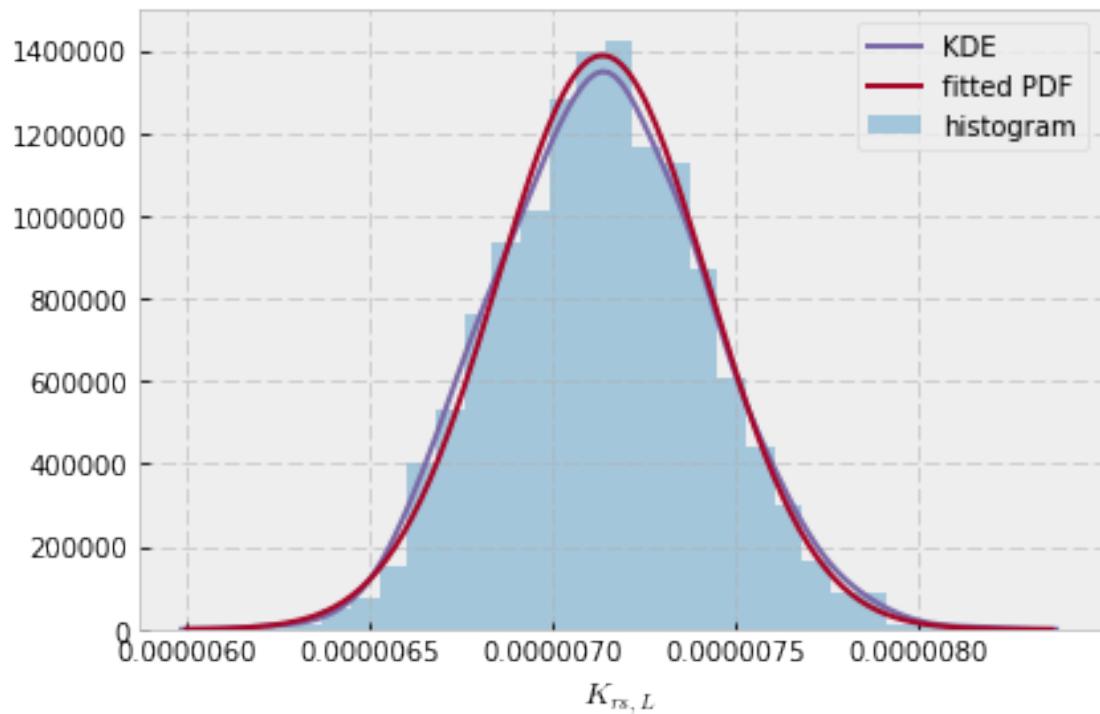
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=6.943948580535977e-06, scale=2.651983051918339e-07)
DescribeResult(nobs=1000, minmax=(6.1969047464951517e-06, 8.0023377462590175e-06)
               mean=6.9439485805359772e-06, variance=7.0400541618239321e-14,
               skewness=0.36017991076565364, kurtosis=0.567285045494788)

```

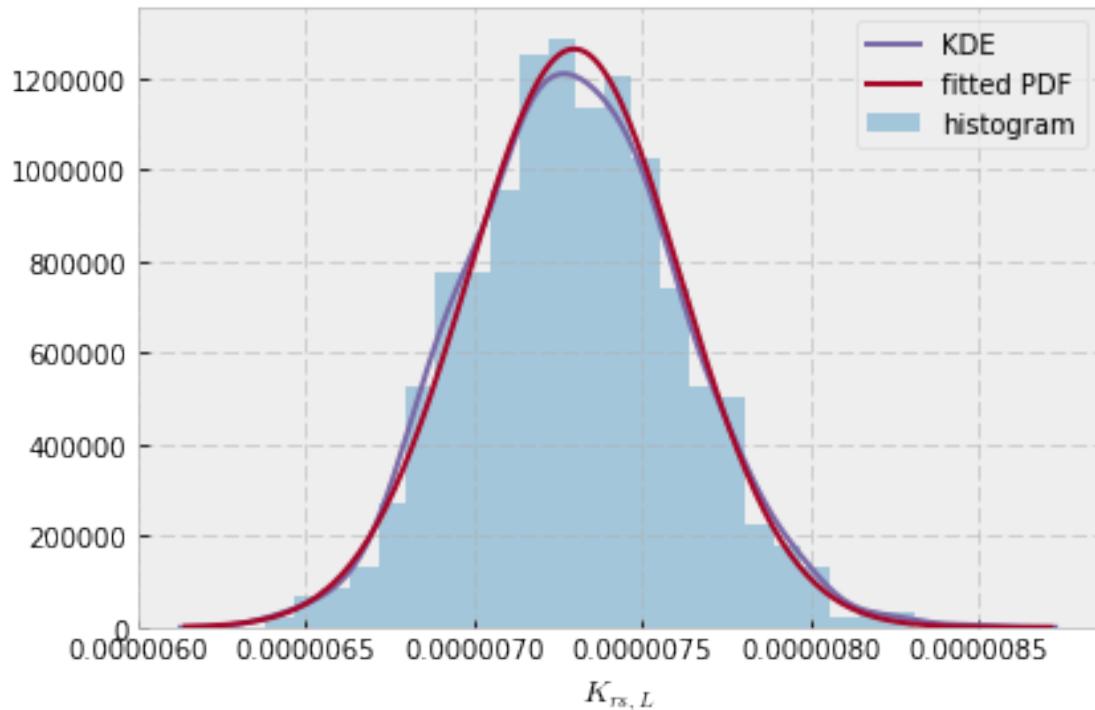


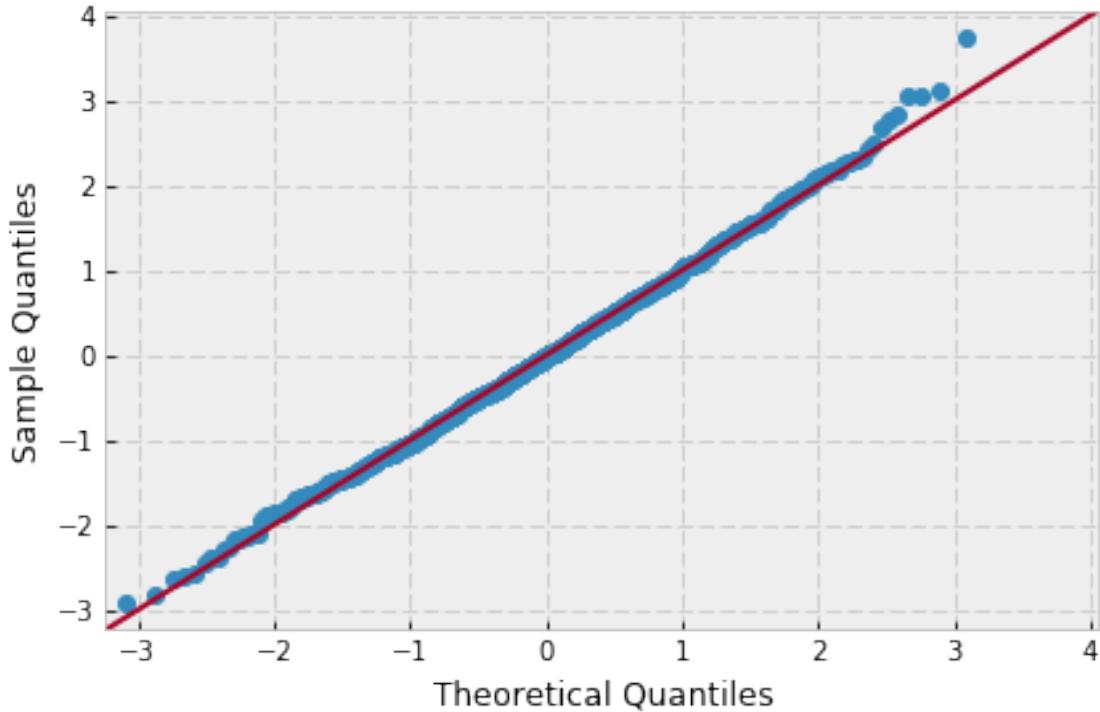


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=7.13603168990032e-06, scale=2.876012366458457e-07)
DescribeResult(nobs=1000, minmax=(6.2167663211898527e-06, 8.1455594425598901e-06)
               mean=7.1360316899003198e-06, variance=8.2797268588808567e-14,
               skewness=0.10635358185202445, kurtosis=-0.10612582241200874)
```

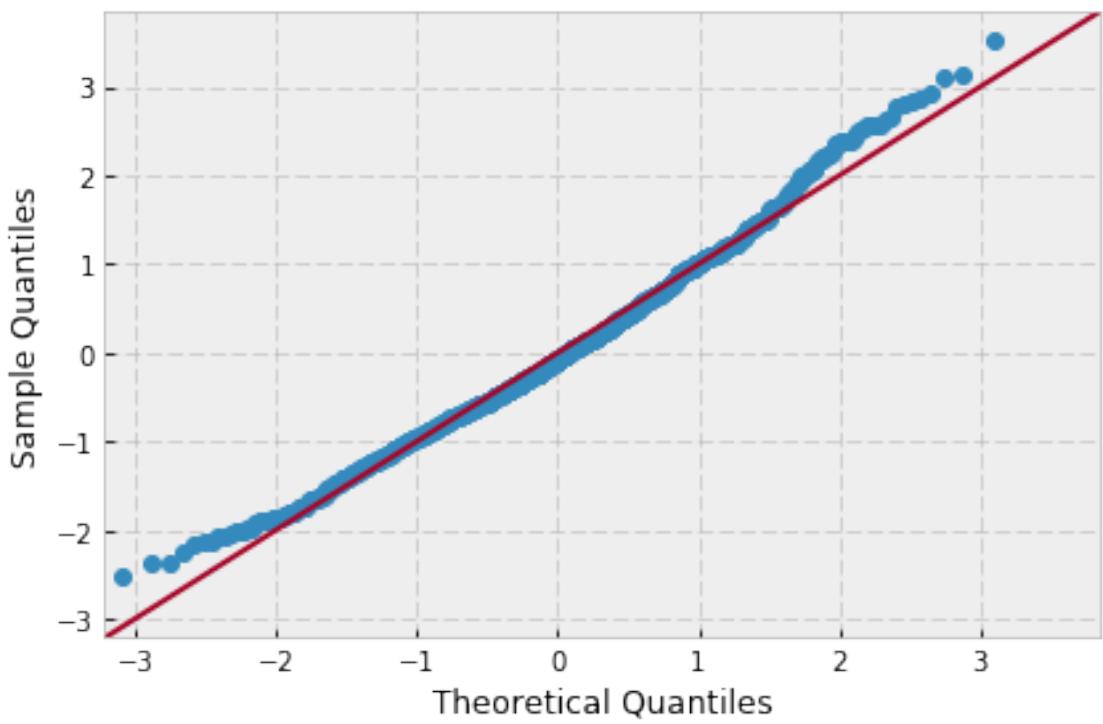
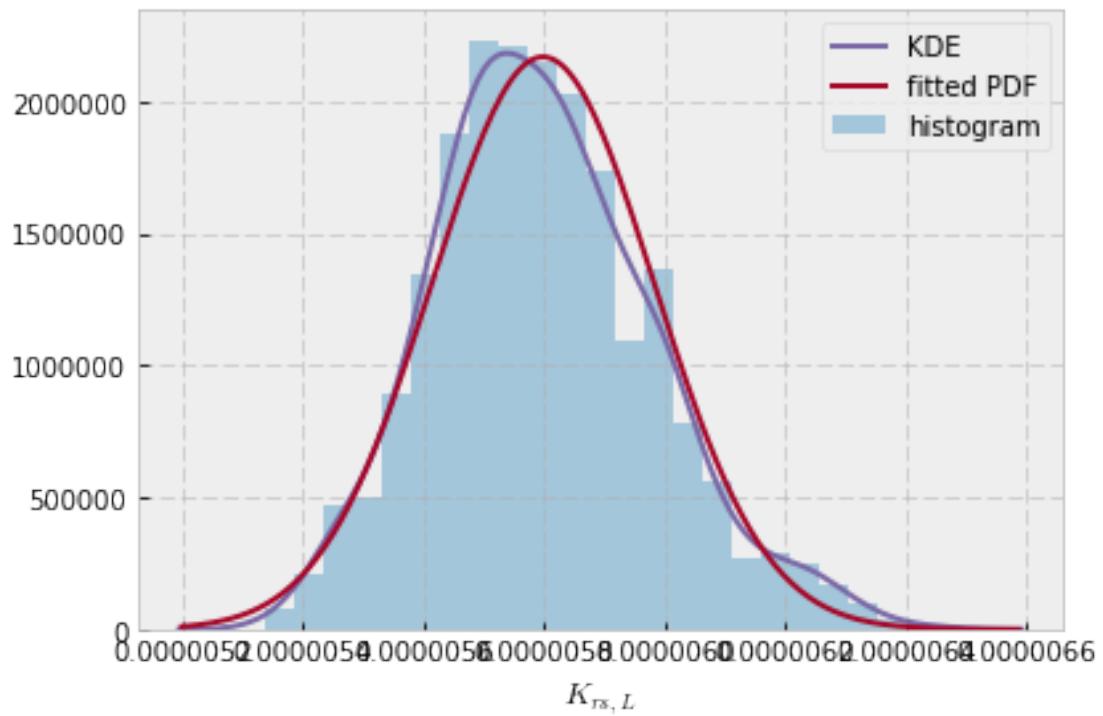


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=7.297555969273288e-06, scale=3.154954243044617e-07)
DescribeResult(nobs=1000, minmax=(6.3809525134716275e-06, 8.474812548415905e-06)
               mean=7.2975559692732876e-06, variance=9.9636999756809142e-14,
               skewness=0.1559354985398343, kurtosis=0.01765592290232698)
```

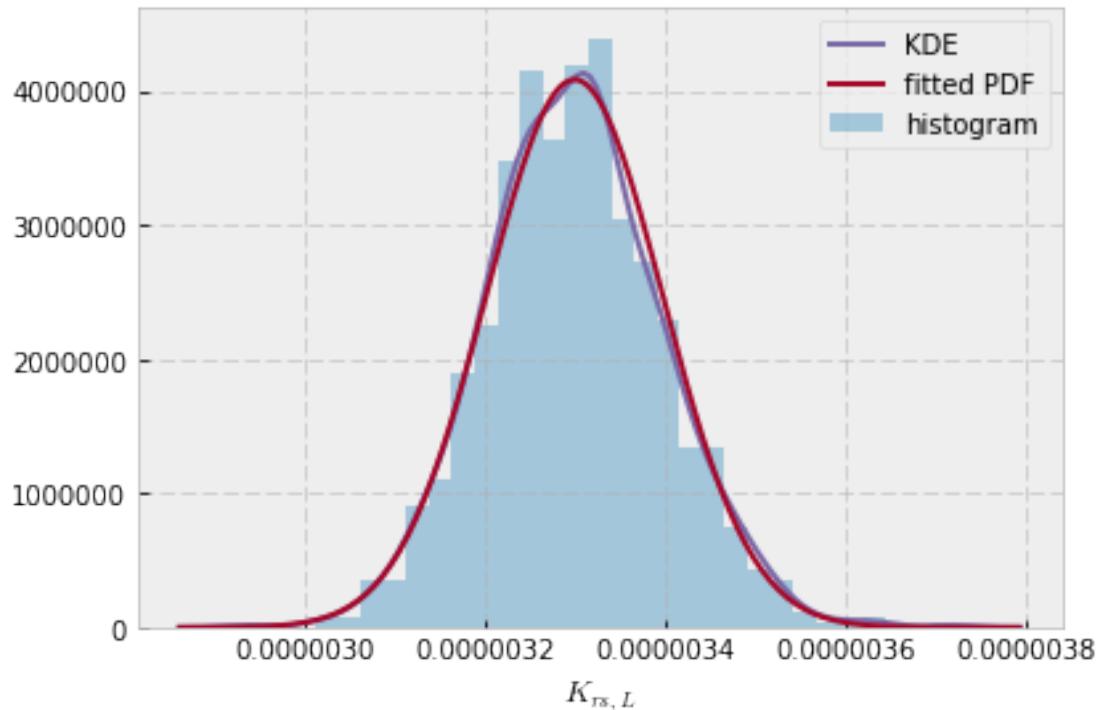


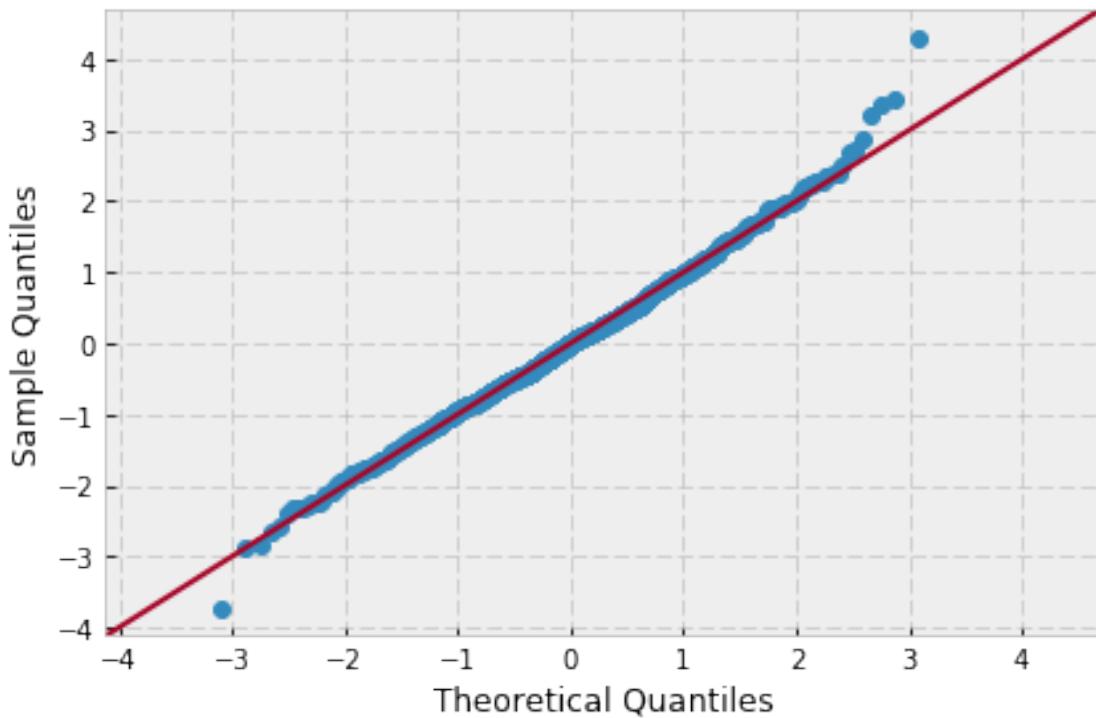


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(),loc=5.797803598118172e-06, scale=1.839259546815837e-07)
DescribeResult(nobs=1000, minmax=(5.3374042106269566e-06, 6.4487989518900234e-06)
               mean=5.7978035981181716e-06, variance=3.3862619424956936e-14,
               skewness=0.4014453891168891, kurtosis=0.14576999422186132)
```

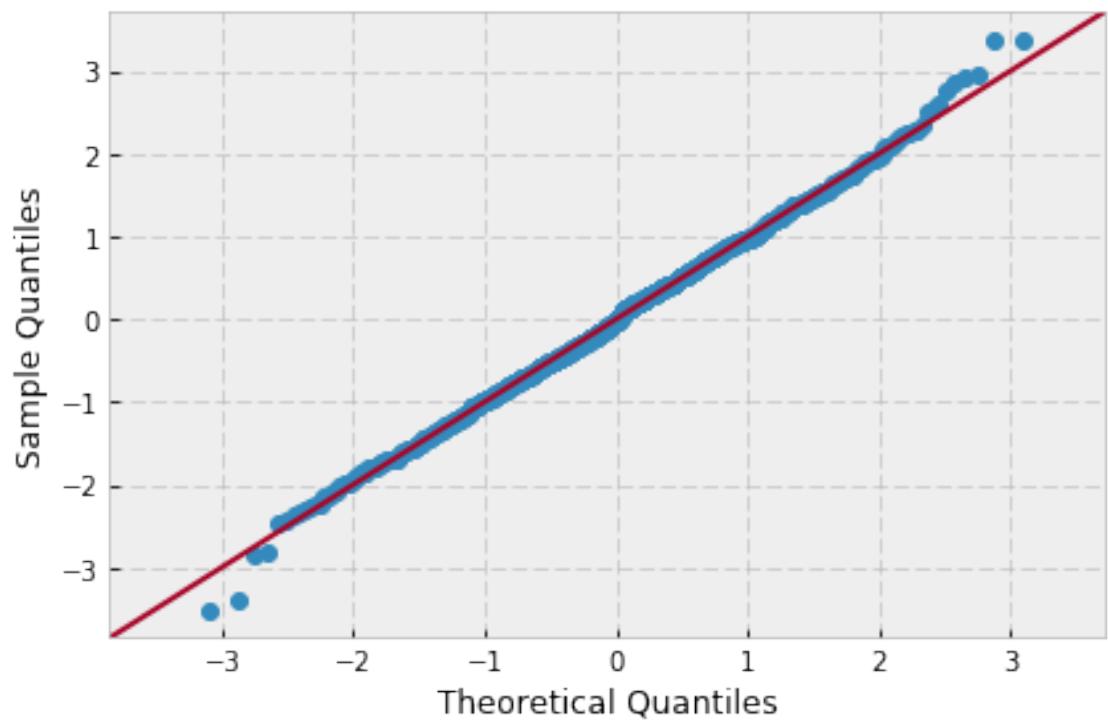
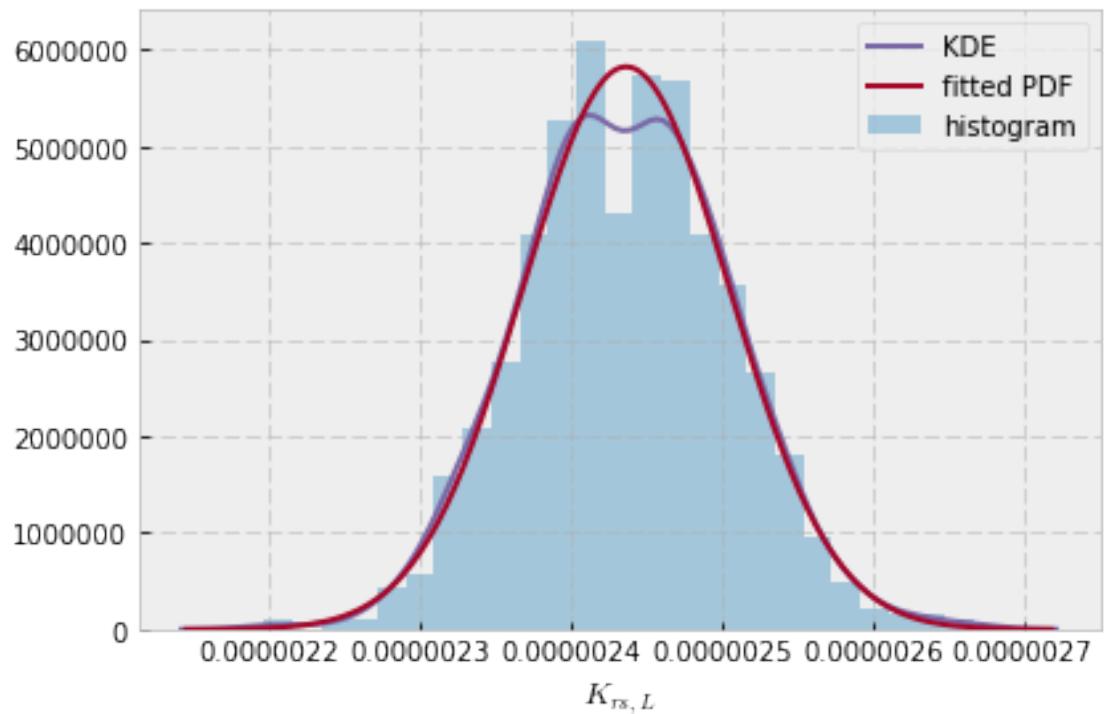


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(), loc=3.2986347815434287e-06, scale=9.76776879929178e-08)
DescribeResult(nobs=1000, minmax=(2.9352856384528542e-06, 3.7177123358303756e-06)
               mean=3.2986347815434287e-06, variance=9.5504812128546536e-15,
               skewness=0.17527216328231673, kurtosis=0.442542102323626)
```

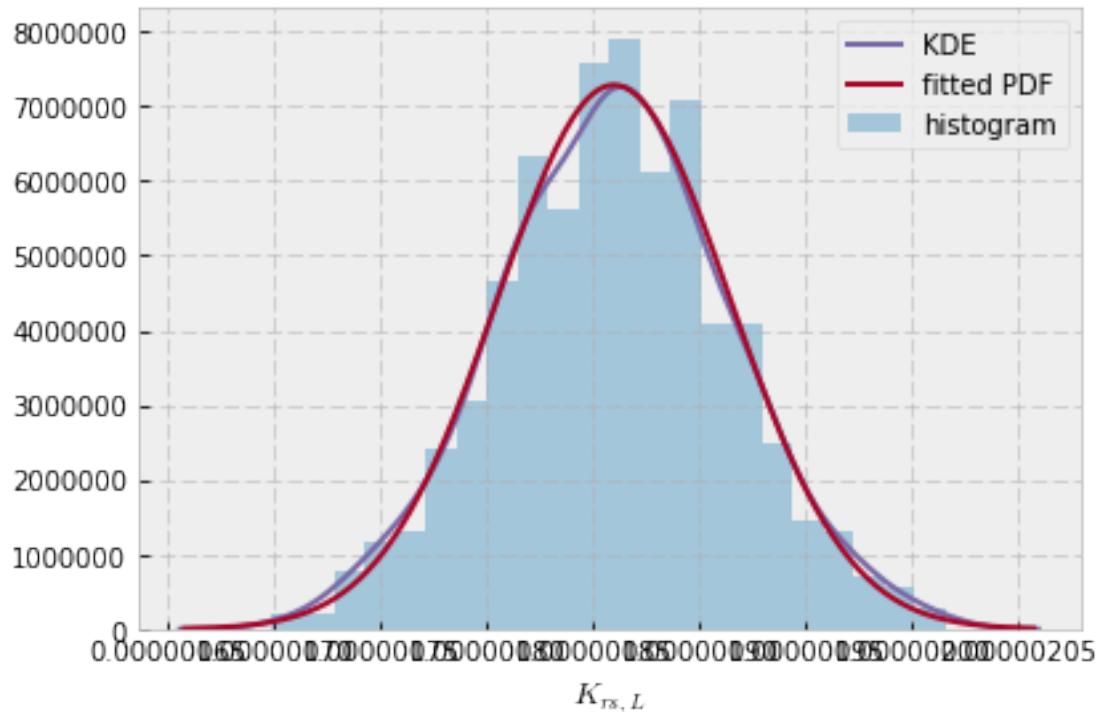


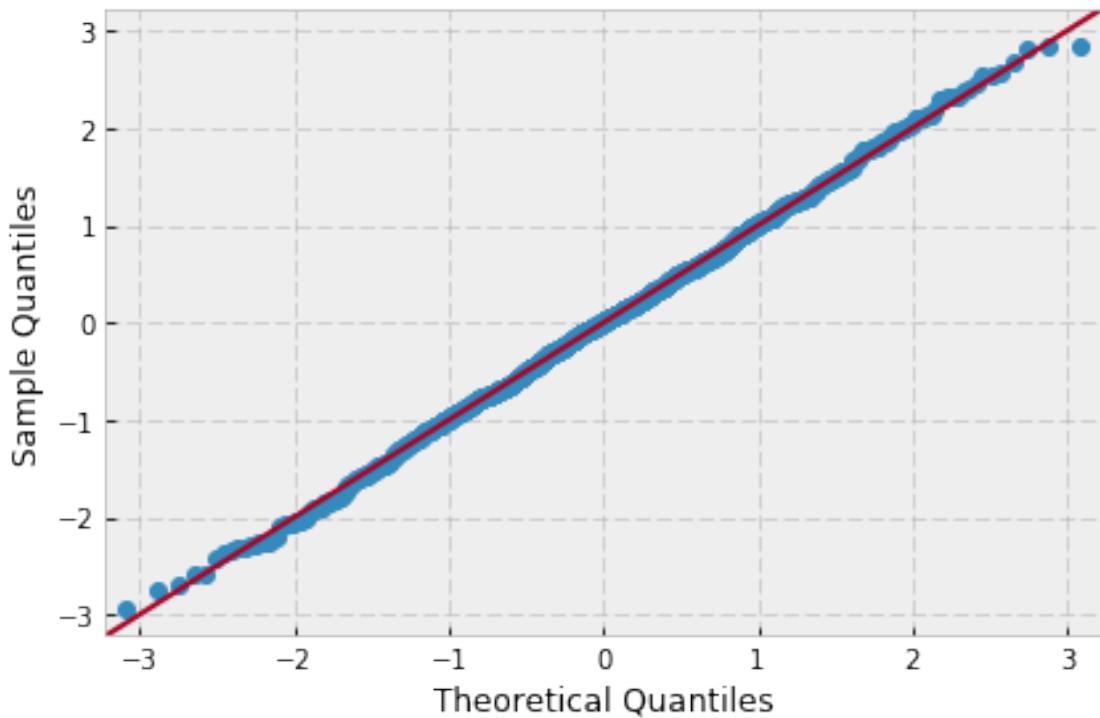


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(),loc=2.4367054350359072e-06, scale=6.848815768129505e-08)
DescribeResult(nobs=1000, minmax=(2.1964391938014478e-06, 2.6672168934409144e-06)
               mean=2.4367054350359072e-06, variance=4.6953230656435781e-15,
               skewness=0.03920502276709106, kurtosis=0.0793688229850007)
```

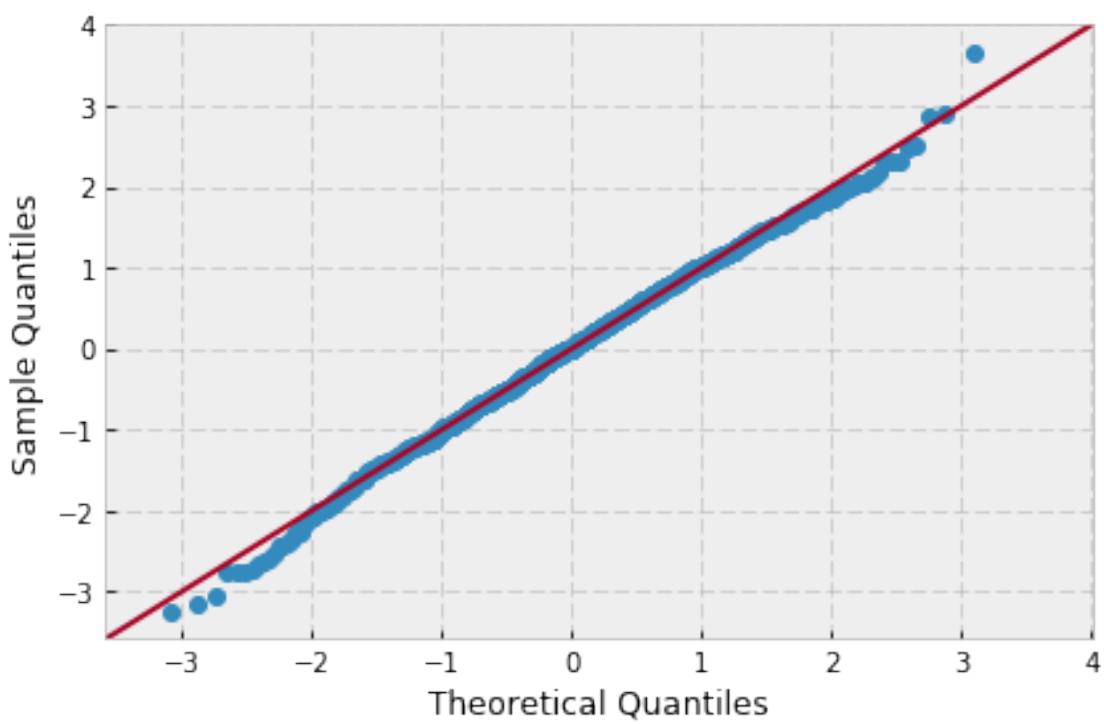
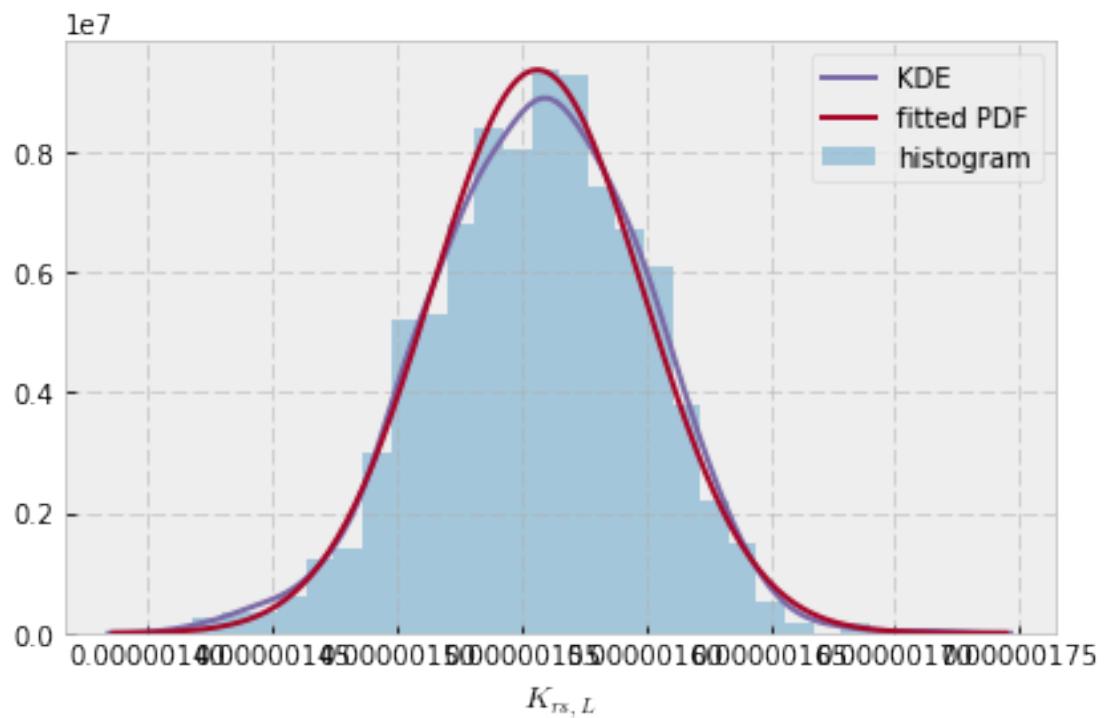


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(), loc=1.8596401018114856e-06, scale=5.478675905657108e-08)
DescribeResult(nobs=1000, minmax=(1.6989885636302368e-06, 2.0157938762177184e-06)
               mean=1.8596401018114856e-06, variance=3.0045935614842575e-15,
               skewness=0.013814558262906456, kurtosis=-0.08279598803025623)
```





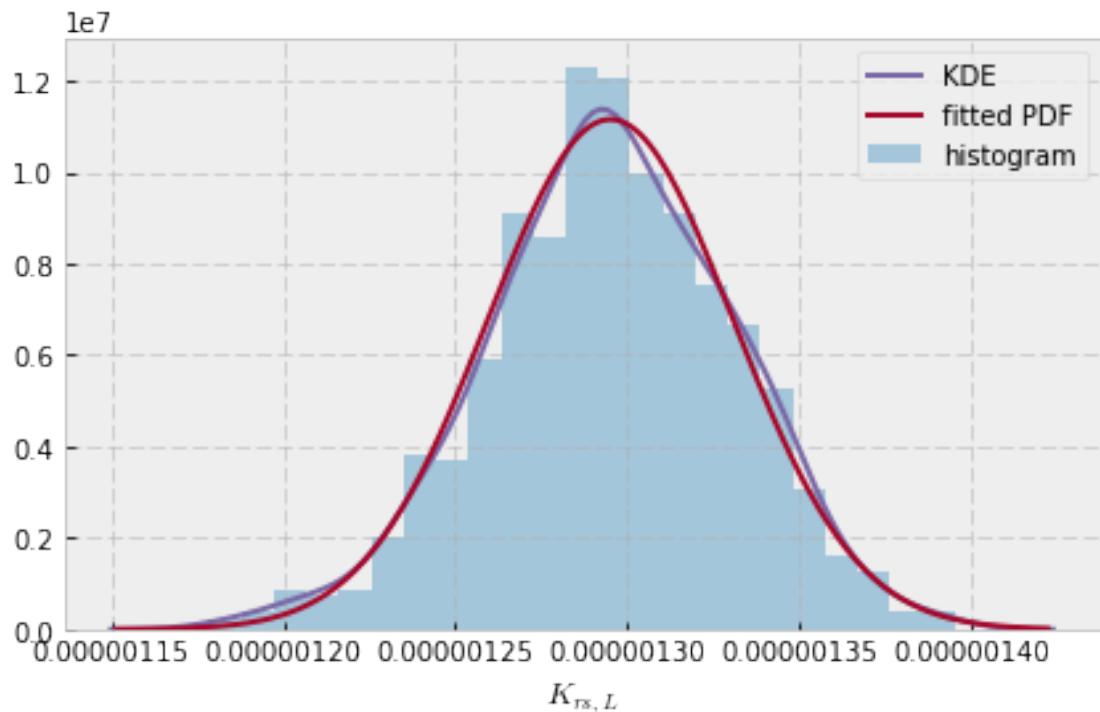
```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(),loc=1.5563903505975328e-06, scale=4.2608016828887144e-08)
DescribeResult(nobs=1000, minmax=(1.4182249742959054e-06, 1.7123822964778948e-06)
               mean=1.5563903505975328e-06, variance=1.8172603584491793e-15,
               skewness=-0.13810025488012825, kurtosis=0.04237988872575027)
```

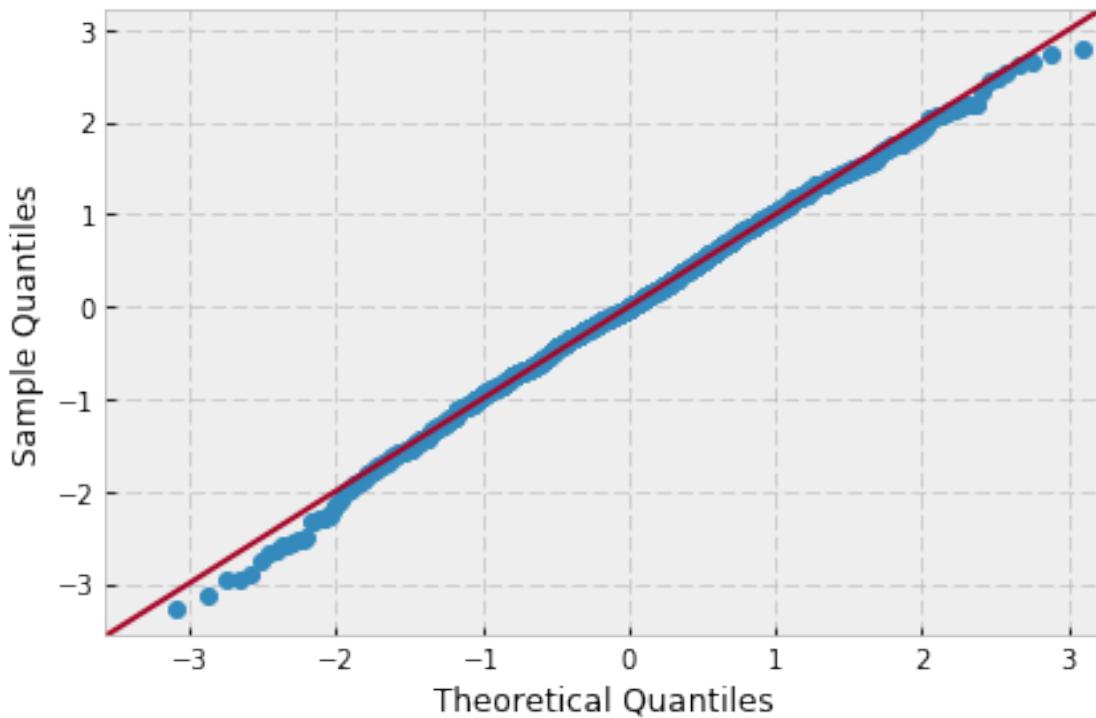


```

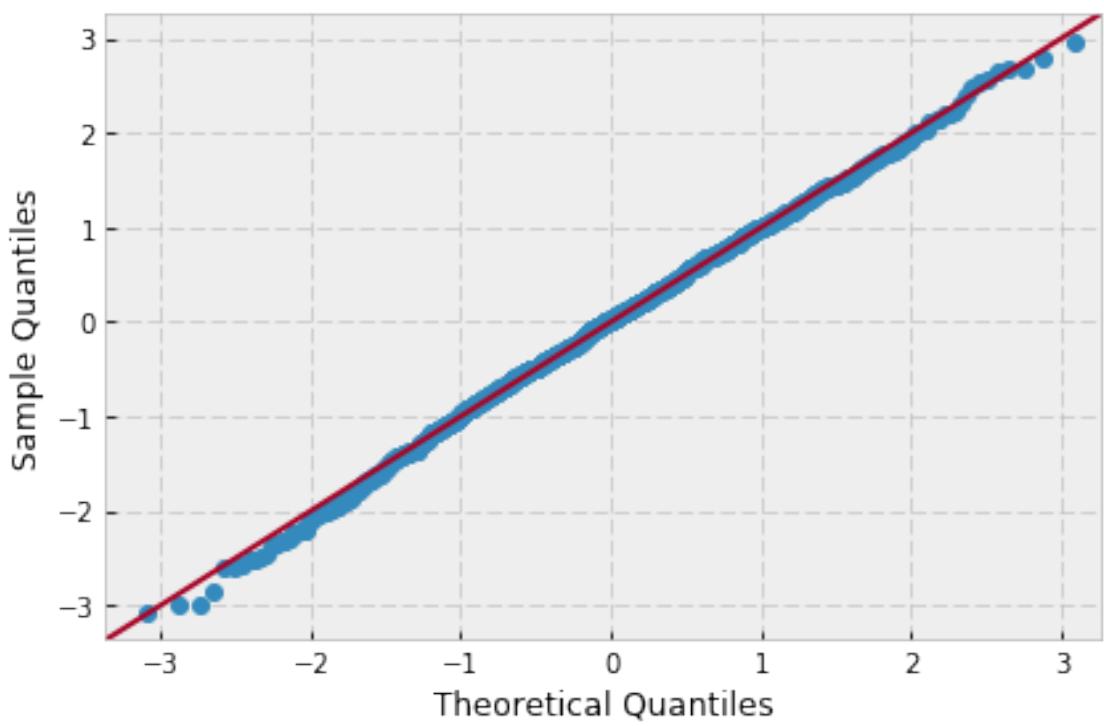
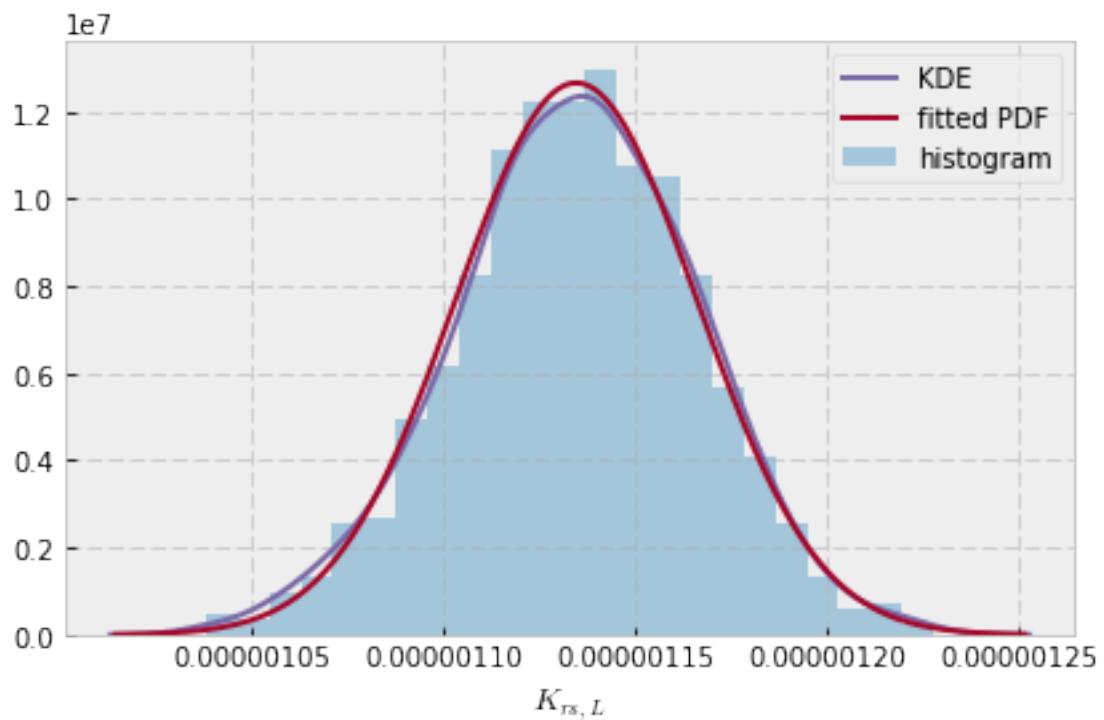
variable name: maxB
variable value: 24.0
distribution: normal(shape=(), loc=1.295112534992594e-06, scale=3.574825227874915e-08)
DescribeResult(nobs=1000, minmax=(1.1783620607663381e-06, 1.3952958734603198e-06)
               mean=1.2951125349925939e-06, variance=1.2792167577428363e-15,
               skewness=-0.15375625748141777, kurtosis=0.04617733553836478)

```





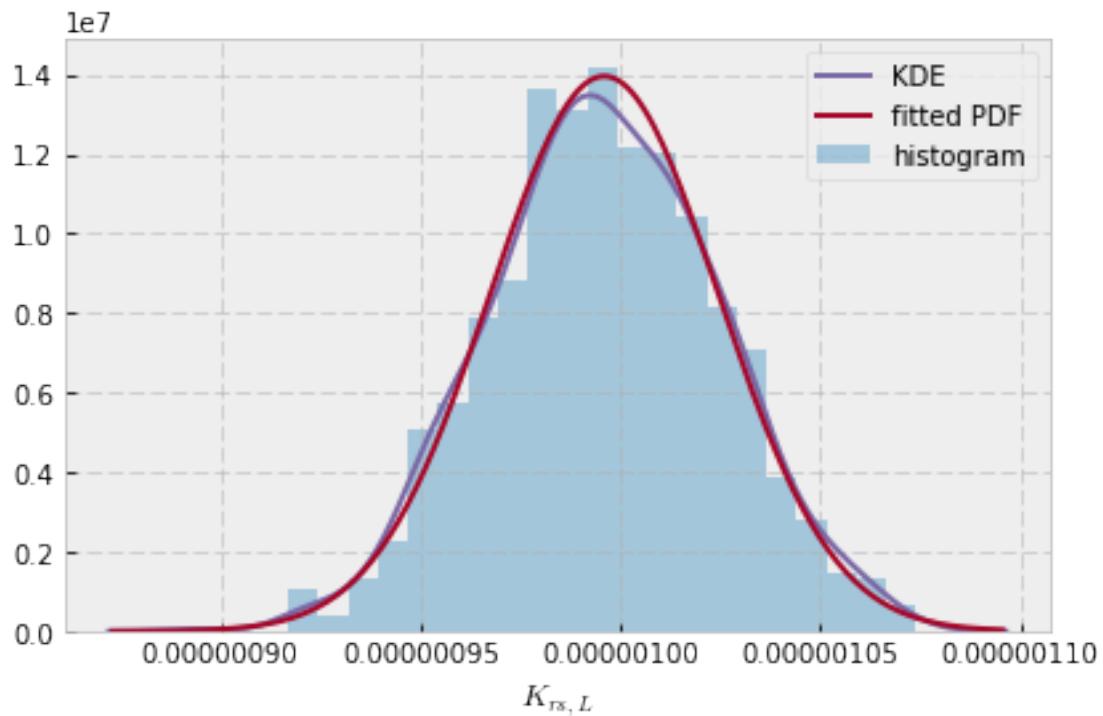
```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(),loc=1.1344936148075805e-06, scale=3.1514754444688455e-08)
DescribeResult(nobs=1000, minmax=(1.0378584640527932e-06, 1.2277283332242391e-06)
               mean=1.1344936148075805e-06, variance=9.9417392163064158e-16,
               skewness=-0.1170576825240064, kurtosis=0.004399425956741698)
```

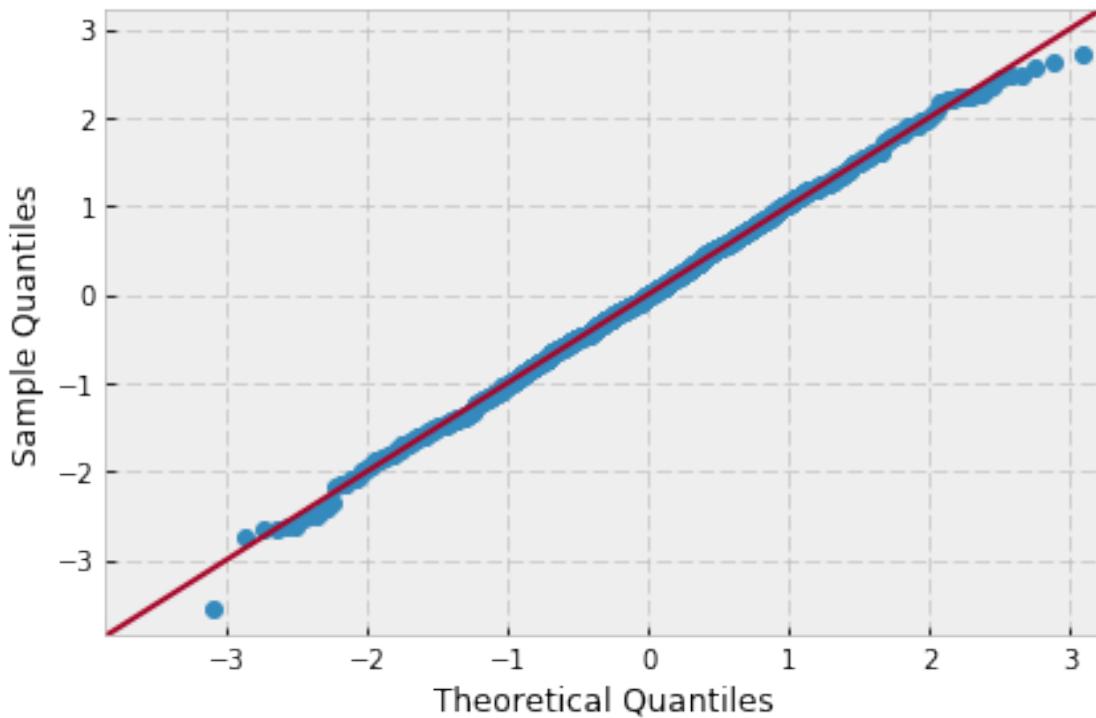


```

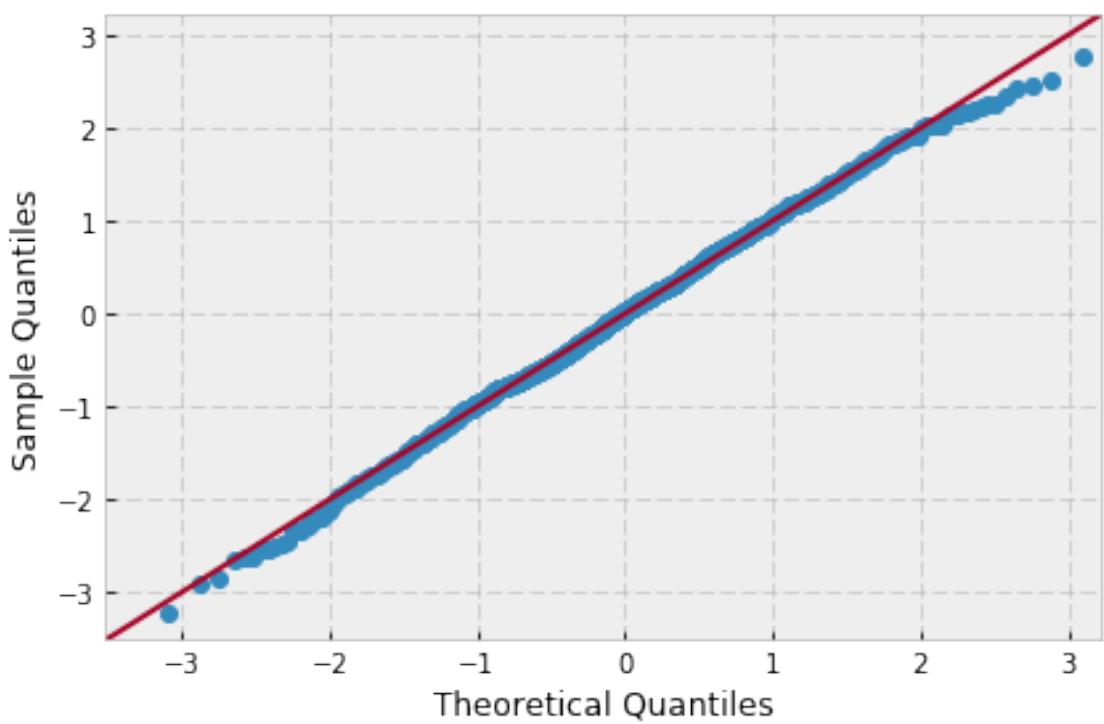
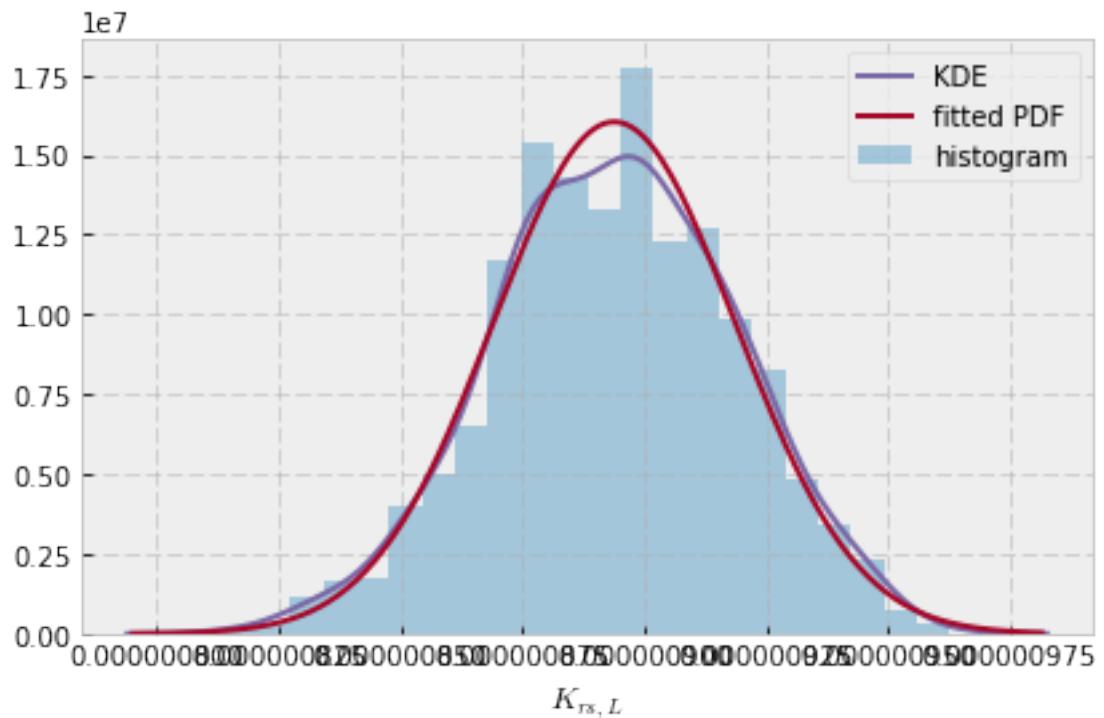
variable name: maxB
variable value: 32.0
distribution: normal(shape=(), loc=9.959376125958946e-07, scale=2.8581520017253244e-08)
DescribeResult(nobs=1000, minmax=(8.945211816988792e-07, 1.073961280525488e-06)
               mean=9.9593761259589459e-07, variance=8.1772100750415209e-16,
               skewness=-0.03769071371273488, kurtosis=-0.15376079538180987)

```





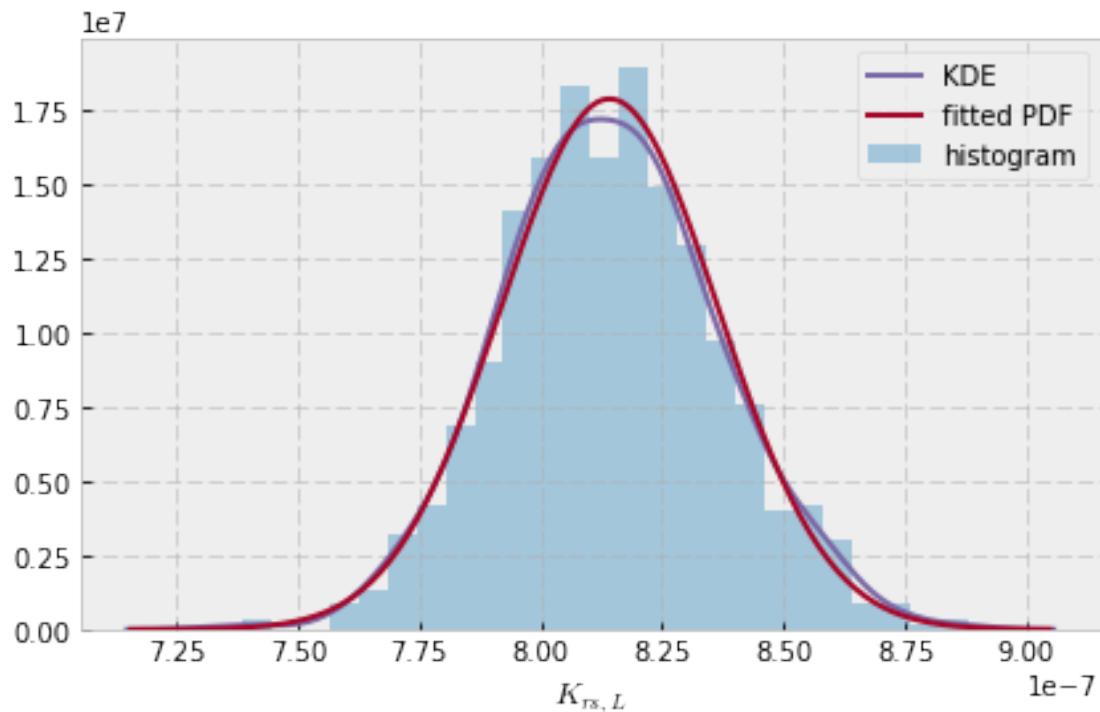
```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(),loc=8.938106074992762e-07, scale=2.4876954827163646e-08)
DescribeResult(nobs=1000, minmax=(8.138194713762846e-07, 9.627135494300952e-07)
               mean=8.9381060749927622e-07, variance=6.1948236383657702e-16,
               skewness=-0.10070300114518142, kurtosis=-0.17976625295737136)
```

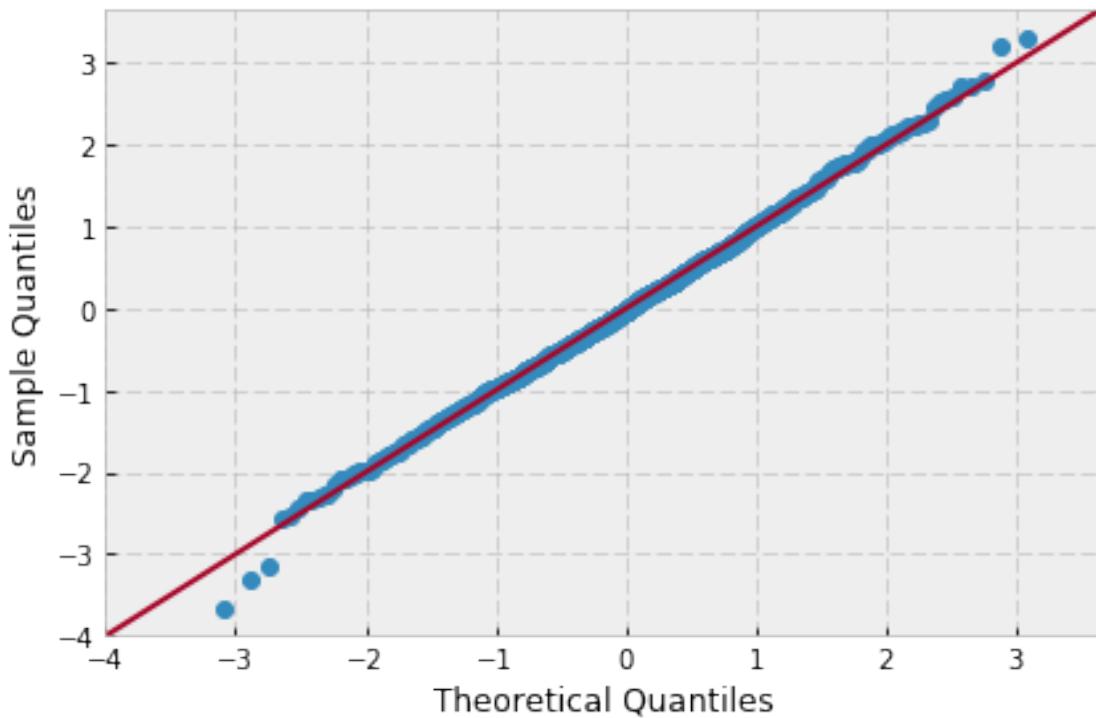


```

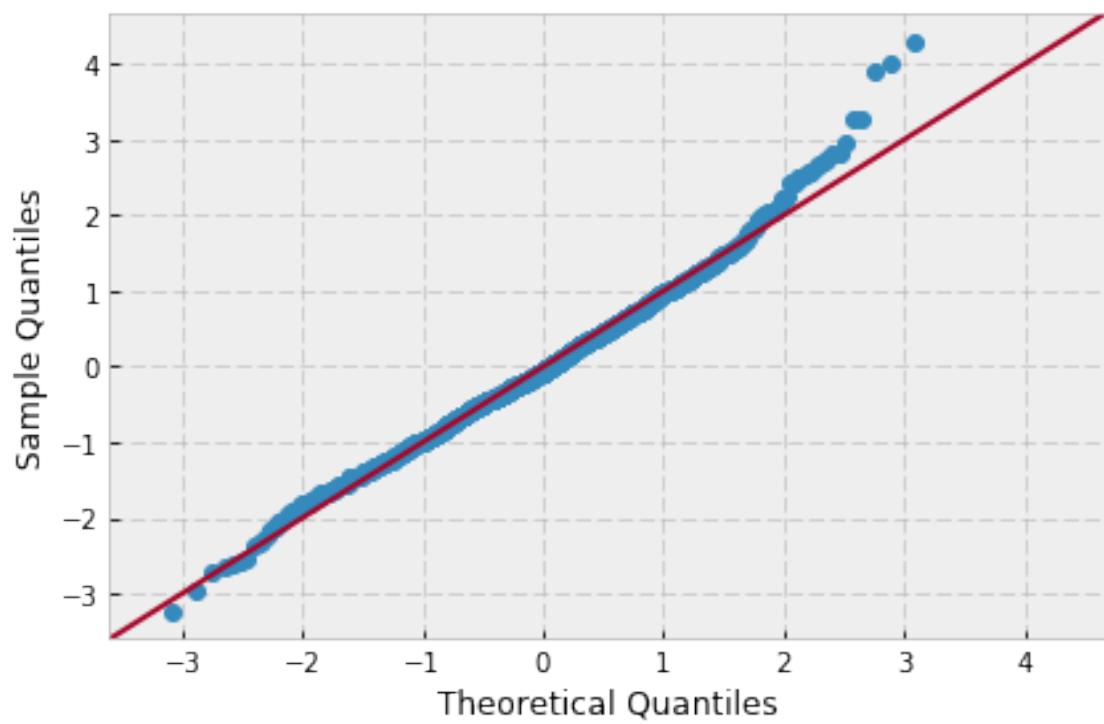
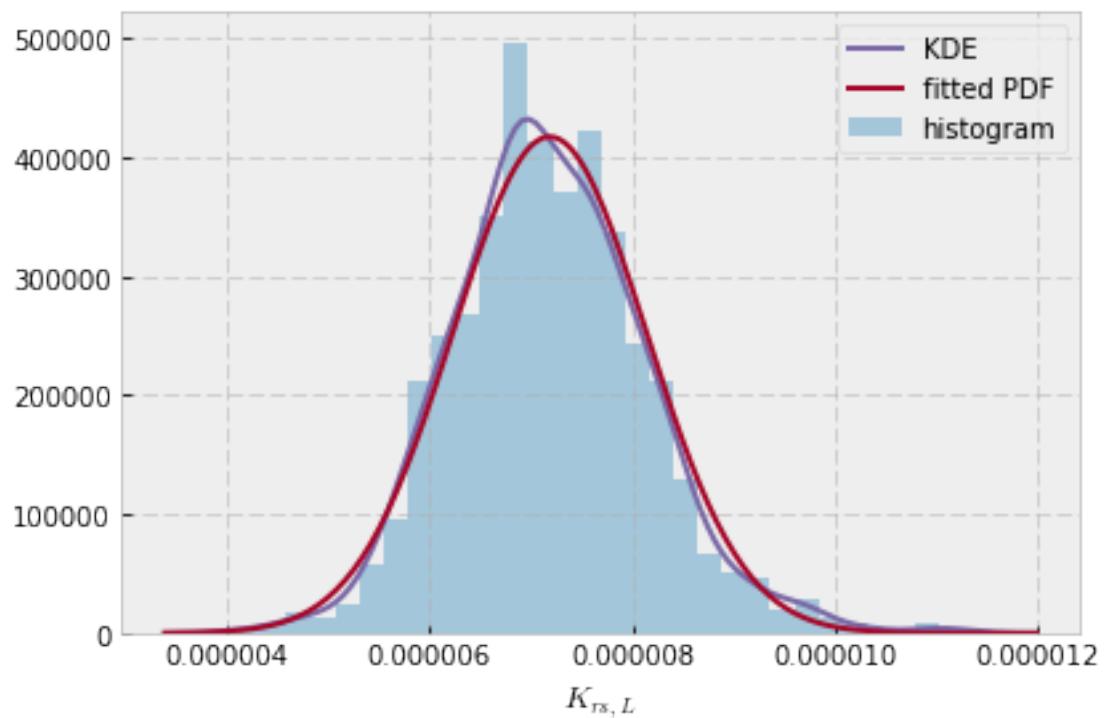
variable name: maxB
variable value: 40.0
distribution: normal(shape=(), loc=8.141833435839741e-07, scale=2.2351457295247898e-08)
DescribeResult(nobs=1000, minmax=(7.3255524385089917e-07, 8.879202601735415e-07)
               mean=8.1418334358397406e-07, variance=5.000877309522428e-16,
               skewness=0.07293929423543259, kurtosis=0.11592271754157935)

```





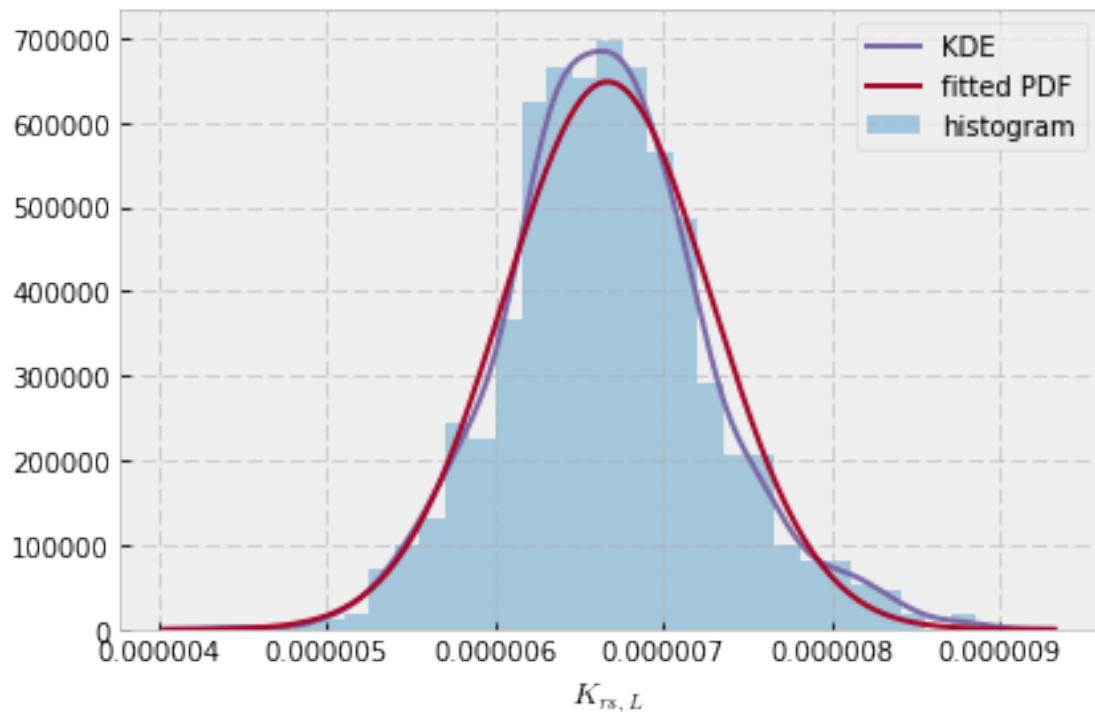
```
variable name: r
variable value: 1.0
distribution: normal(shape=(),loc=7.183911252559954e-06, scale=9.556959315261641e-07)
DescribeResult(nobs=1000, minmax=(4.1013372139816013e-06, 1.1278017242877861e-05)
               mean=7.1839112525599537e-06, variance=9.1426898251818069e-13,
               skewness=0.35534460581342975, kurtosis=0.797758552251183)
```

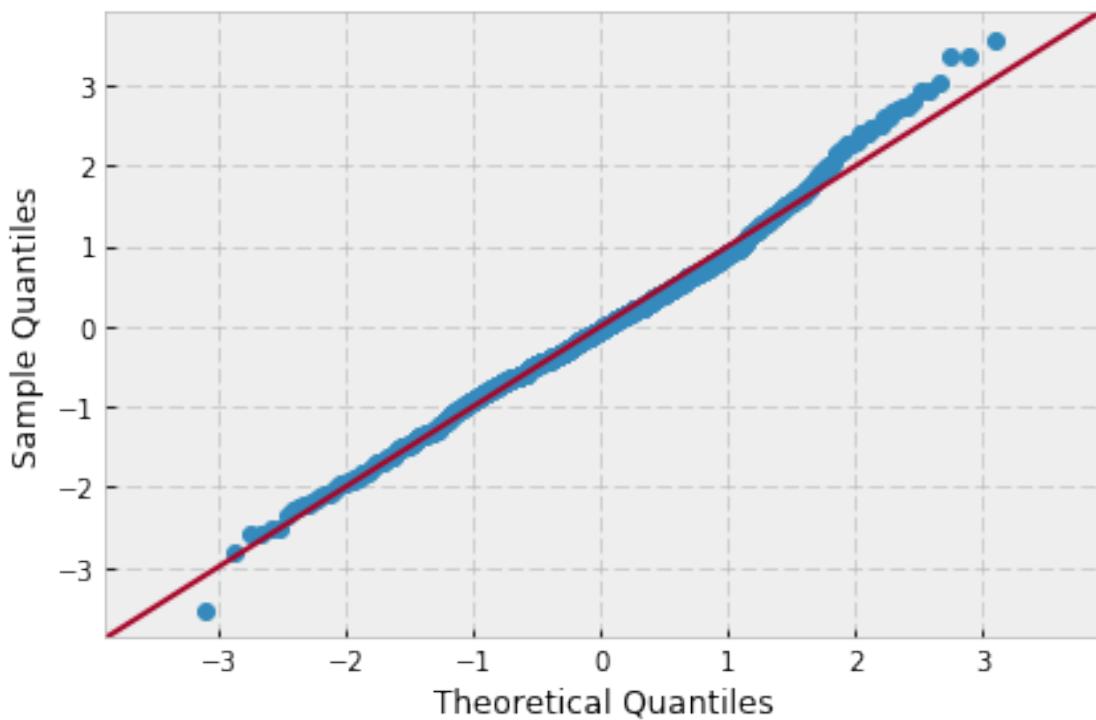


```

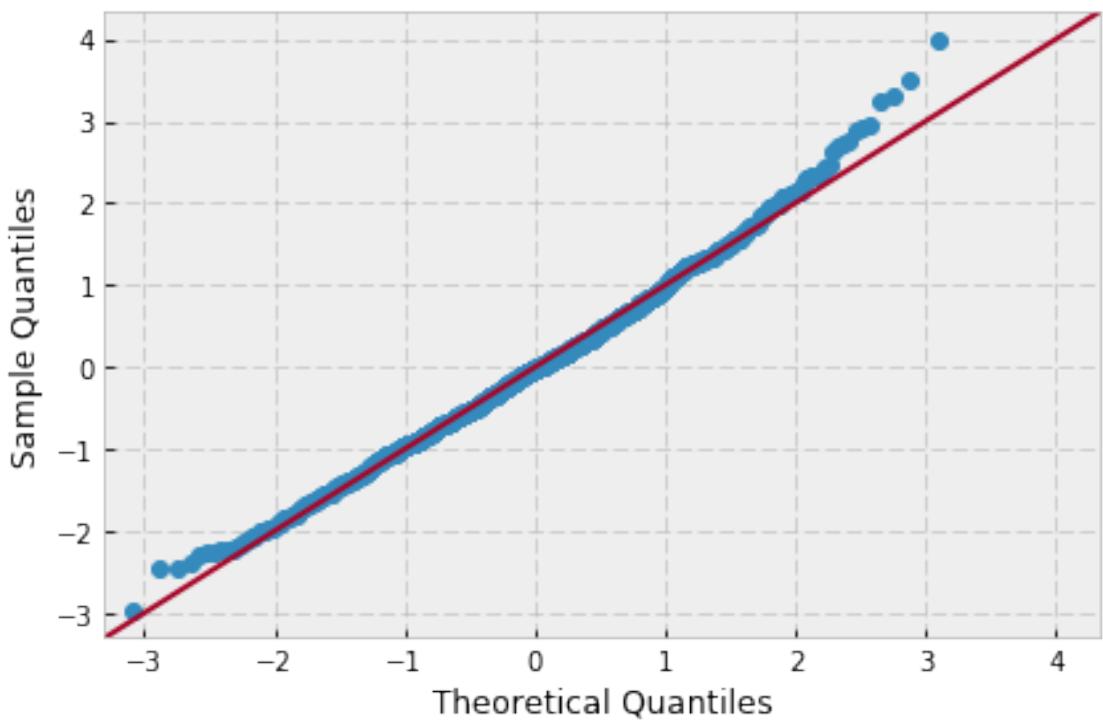
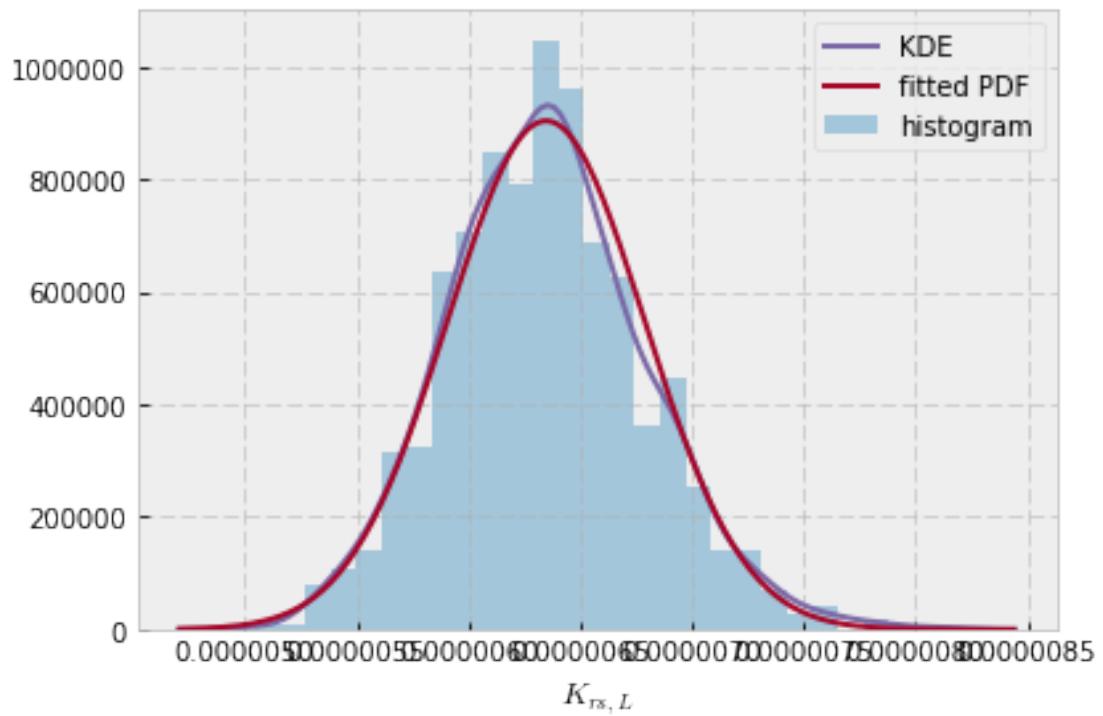
variable name: r
variable value: 1.22222222222223
distribution: normal(shape=(), loc=6.6712664879091044e-06, scale=6.156161570230887e-07)
DescribeResult(nobs=1000, minmax=(4.5018257959279097e-06, 8.864187373807161e-06)
               mean=6.6712664879091044e-06, variance=3.793626154032795e-13,
               skewness=0.2790130516653367, kurtosis=0.5393220269803876)

```





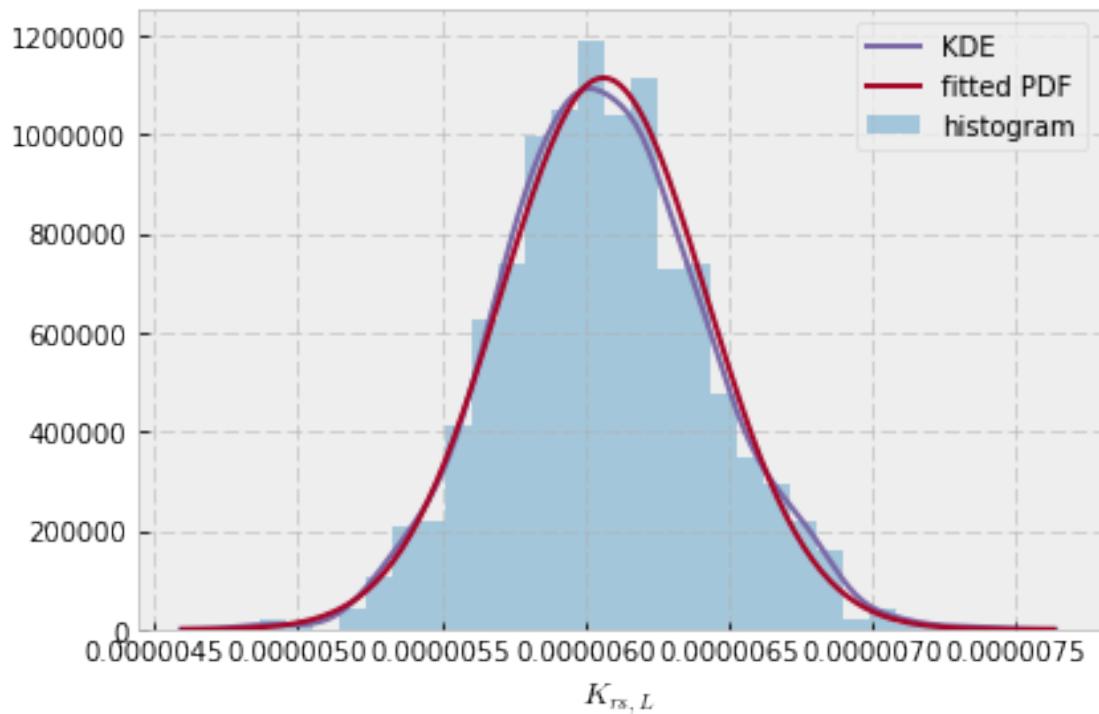
```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(),loc=6.346944153354838e-06, scale=4.414099273362624e-07)
DescribeResult(nobs=1000, minmax=(5.0409540377210848e-06, 8.1058584962091629e-06)
               mean=6.3469441533548379e-06, variance=1.9503776171271716e-13,
               skewness=0.29144602776183465, kurtosis=0.3068864016444235)
```

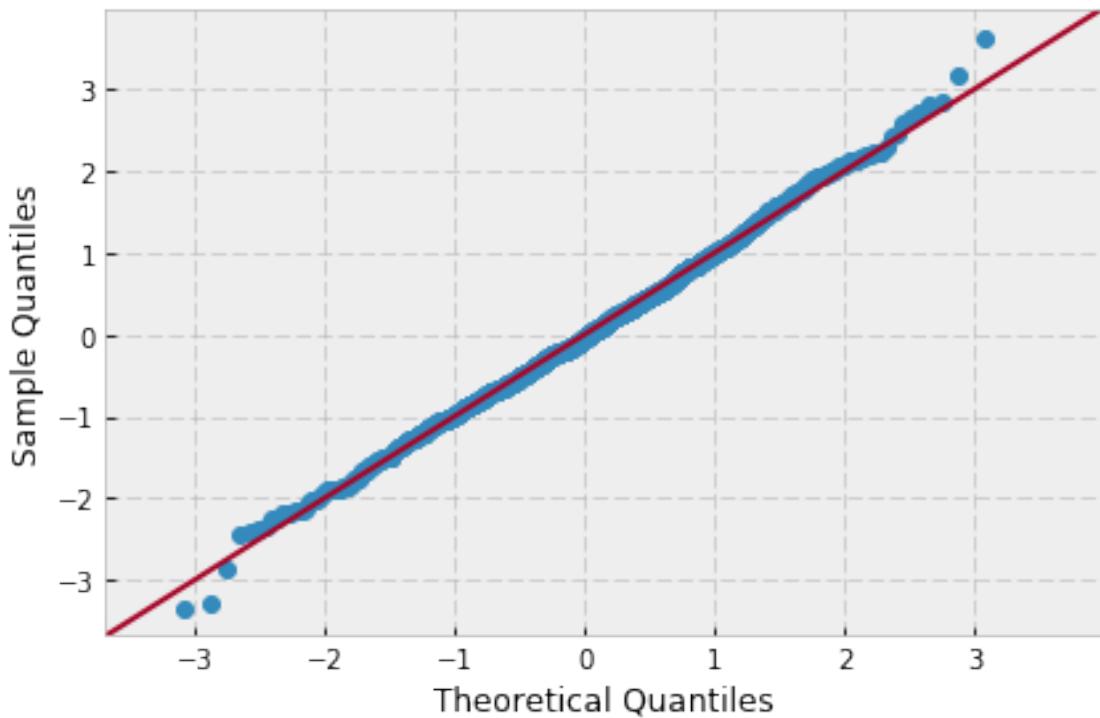


```

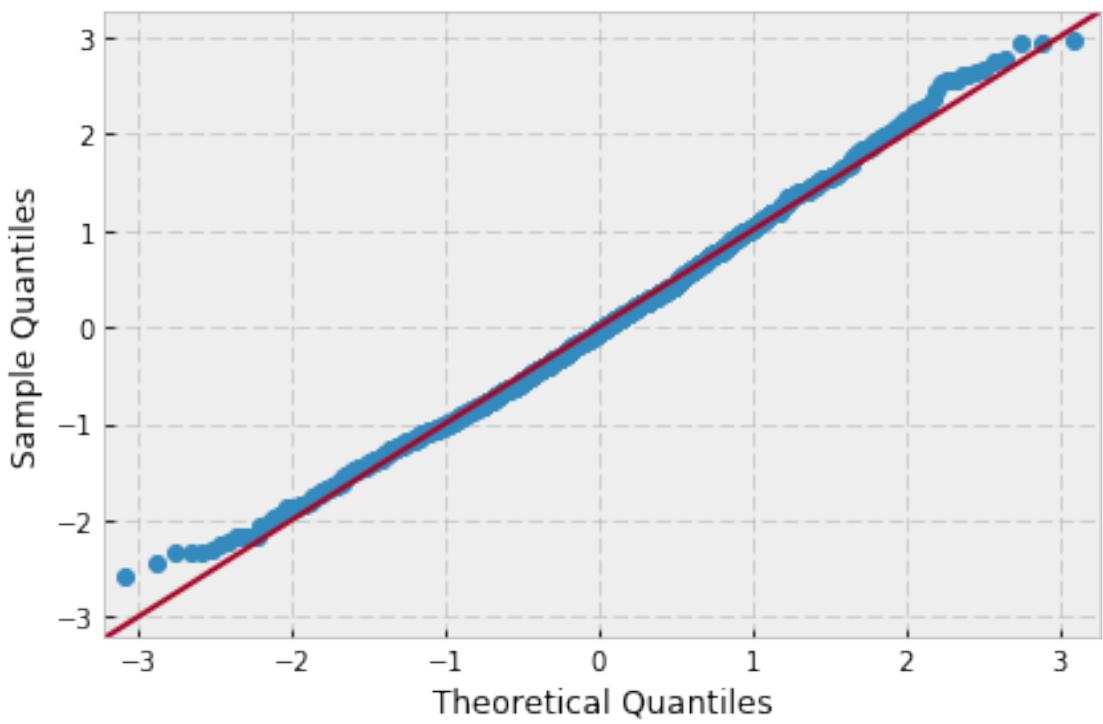
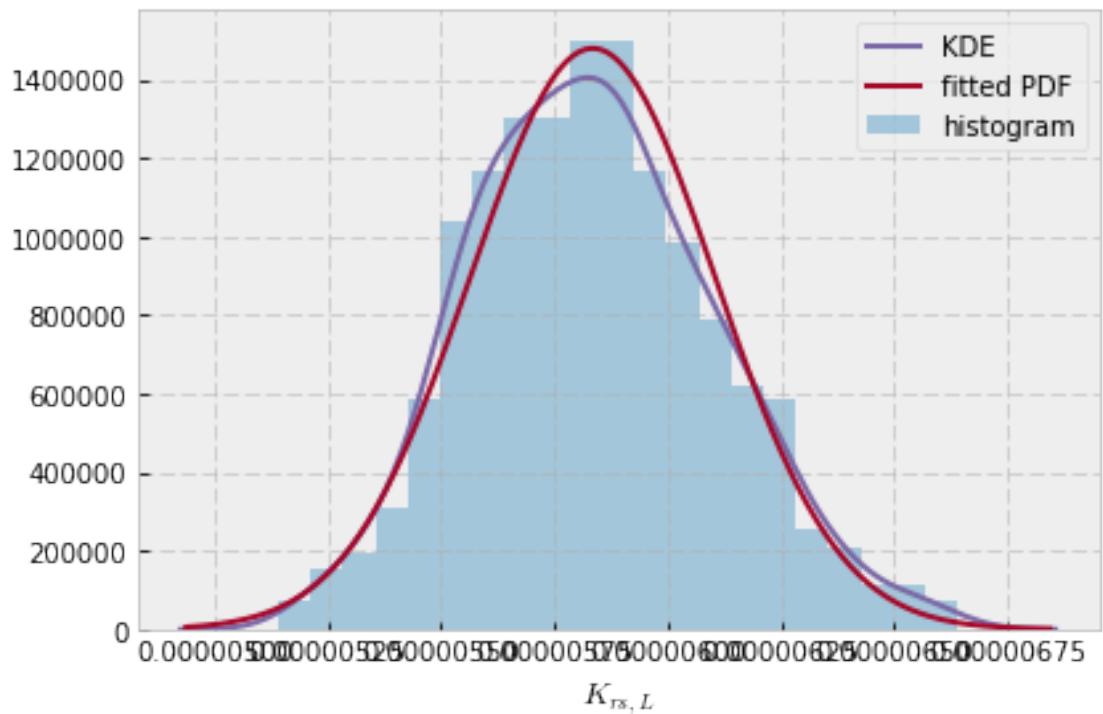
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=6.06416229151387e-06, scale=3.5802137314465457e-07)
DescribeResult(nobs=1000, minmax=(4.8689678961350805e-06, 7.3610261336843045e-06)
               mean=6.0641622915138703e-06, variance=1.283076112396236e-13,
               skewness=0.12653132357236077, kurtosis=0.0949001859755807)

```





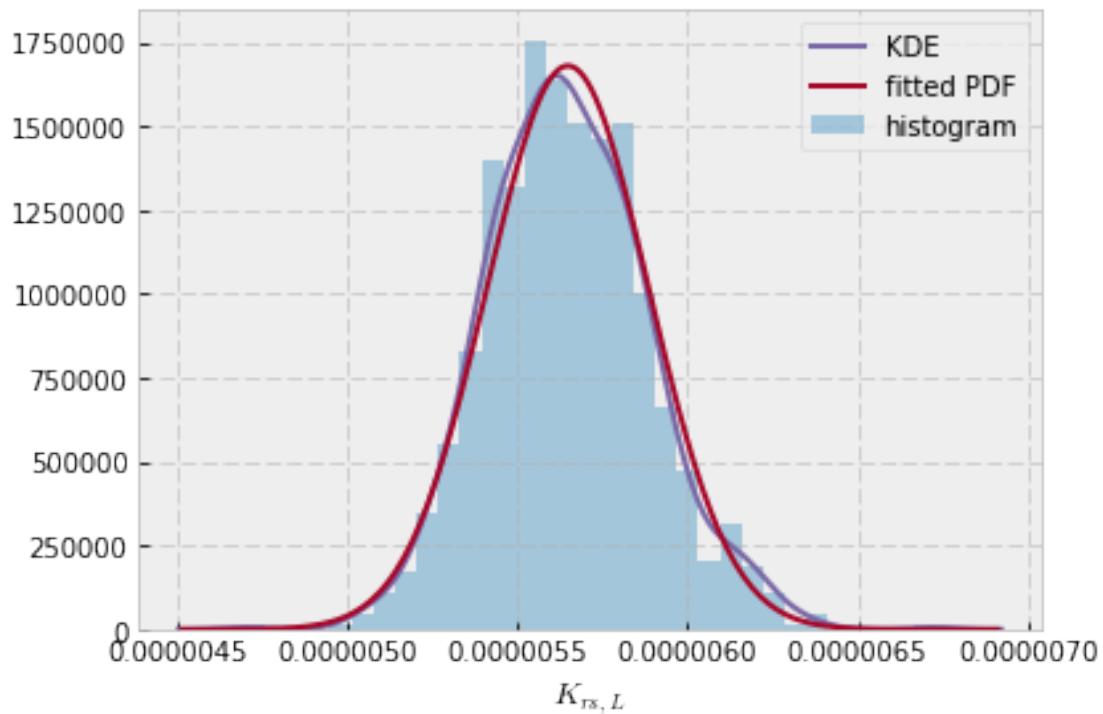
```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(),loc=5.8345928418283565e-06, scale=2.6975244723590955e-07)
DescribeResult(nobs=1000, minmax=(5.142147094715605e-06, 6.6387083011691872e-06)
               mean=5.8345928418283565e-06, variance=7.2839222011773941e-14,
               skewness=0.24906768127306958, kurtosis=-0.16127280560557855)
```

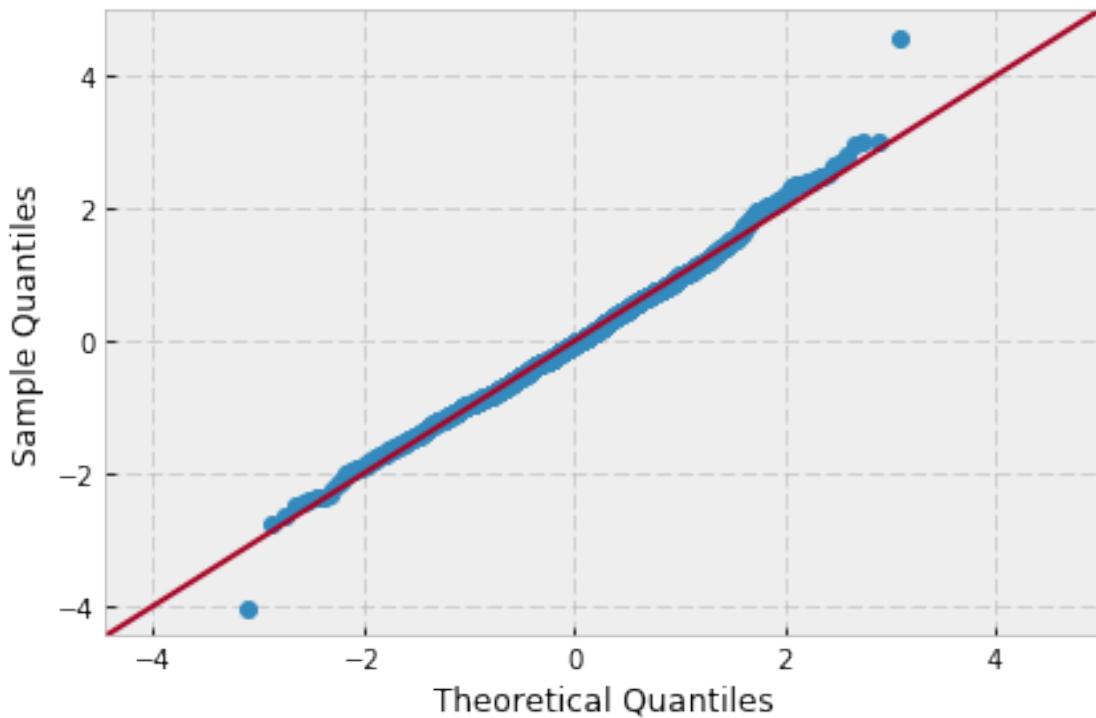


```

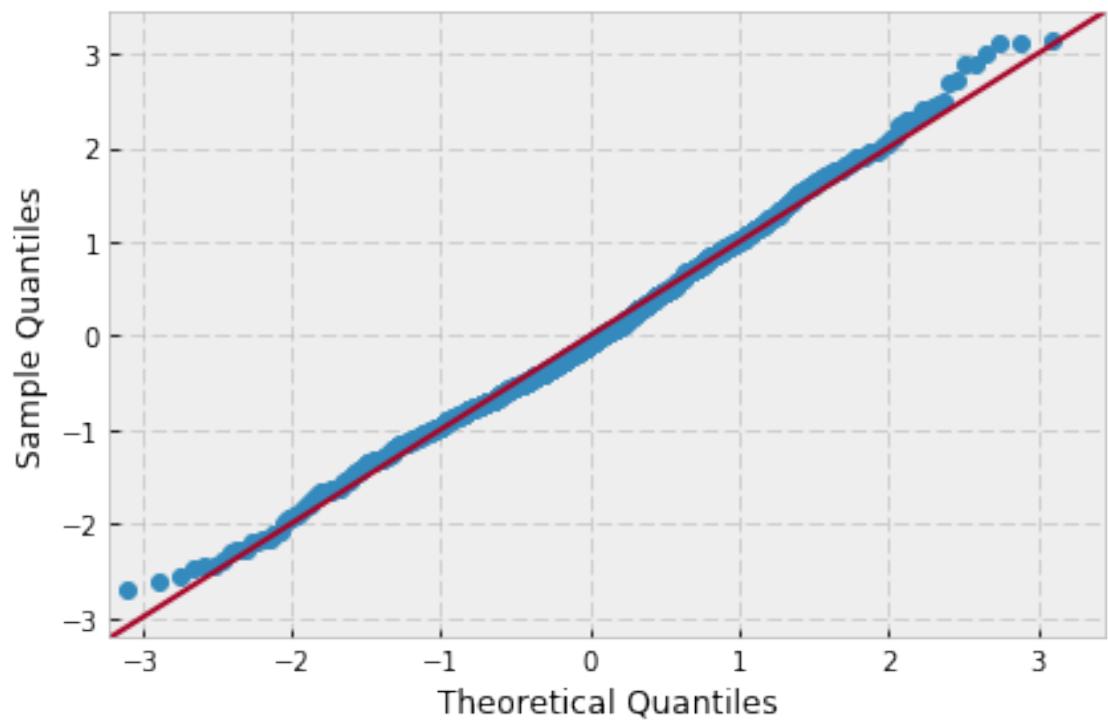
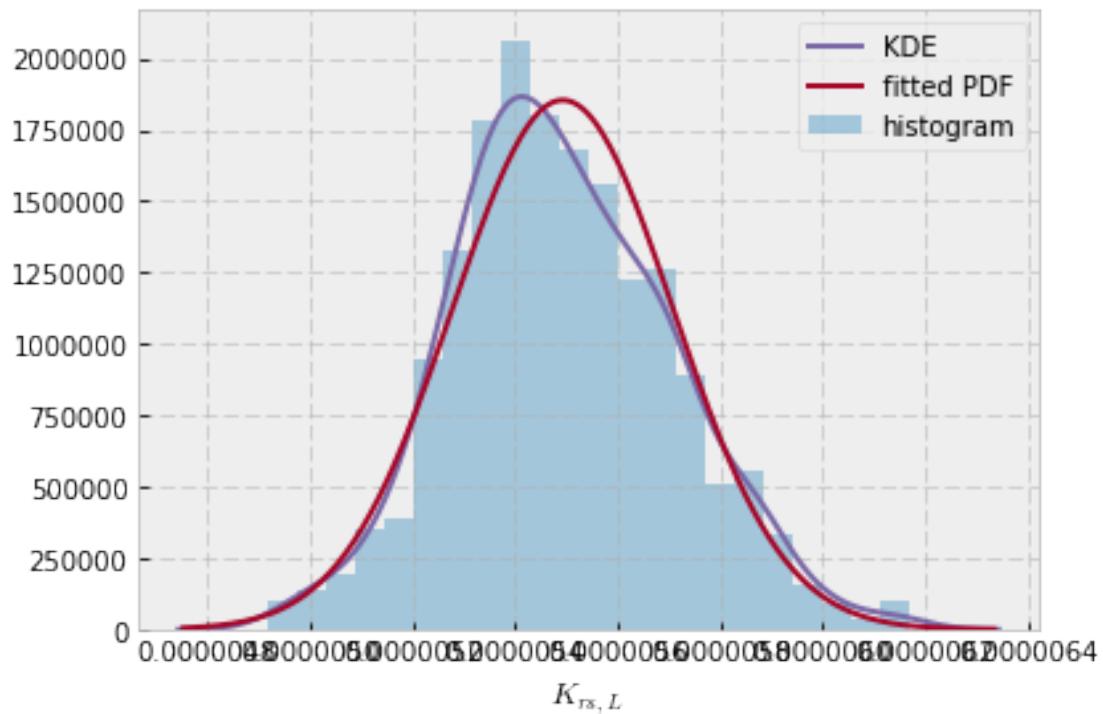
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=5.648015126207983e-06, scale=2.3743991487597214e-07)
DescribeResult(nobs=1000, minmax=(4.693624448747521e-06, 6.7318772534546503e-06)
               mean=5.648015126207983e-06, variance=5.6434147323632537e-14,
               skewness=0.2444549767391054, kurtosis=0.4888391276493089)

```





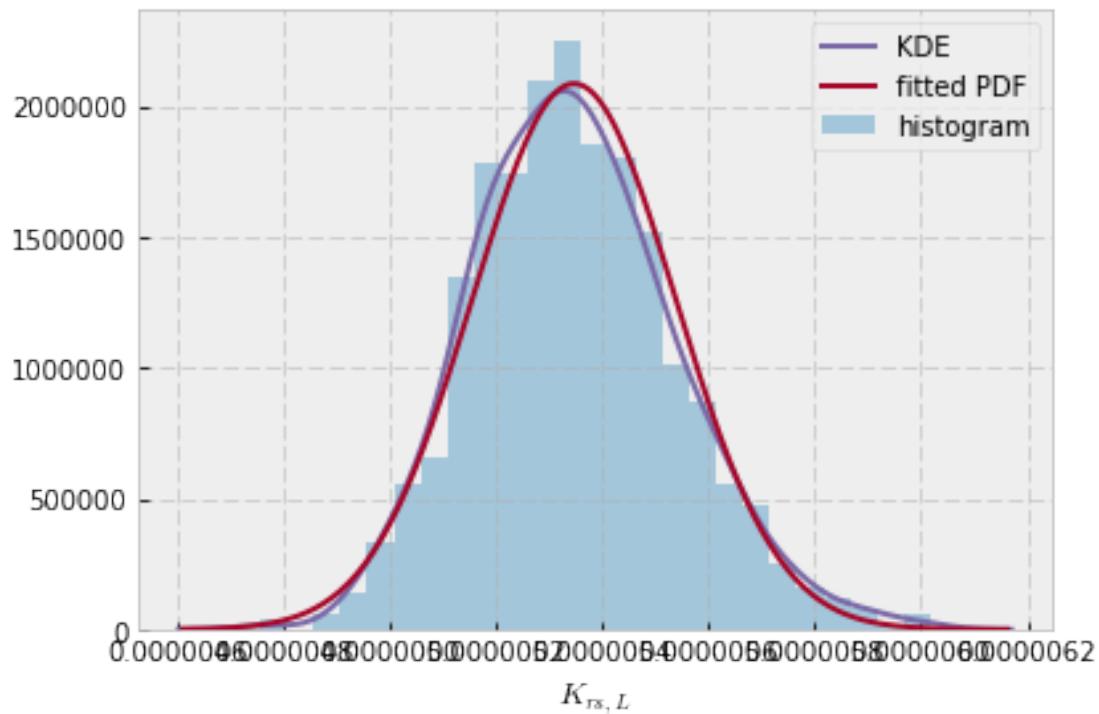
```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(),loc=5.492303735016215e-06, scale=2.1515615945718632e-07)
DescribeResult(nobs=1000, minmax=(4.9144426911104521e-06, 6.1706497984979328e-06)
               mean=5.4923037350162152e-06, variance=4.6338511463830015e-14,
               skewness=0.2544581801892668, kurtosis=-0.0059132591647972355)
```

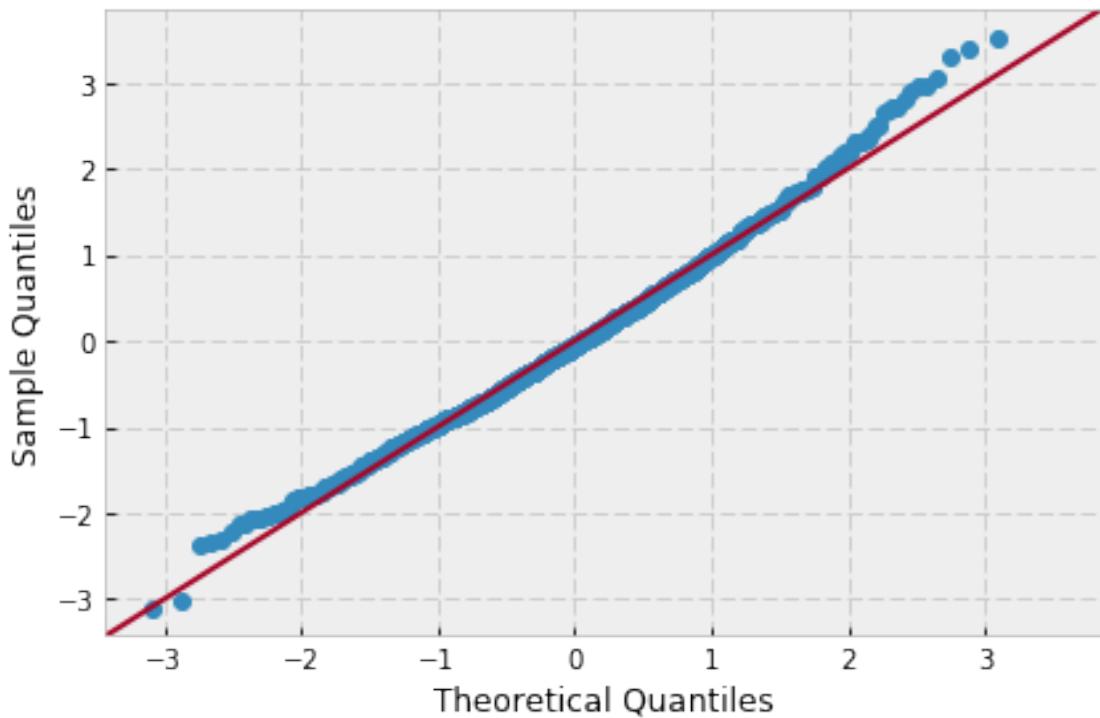


```

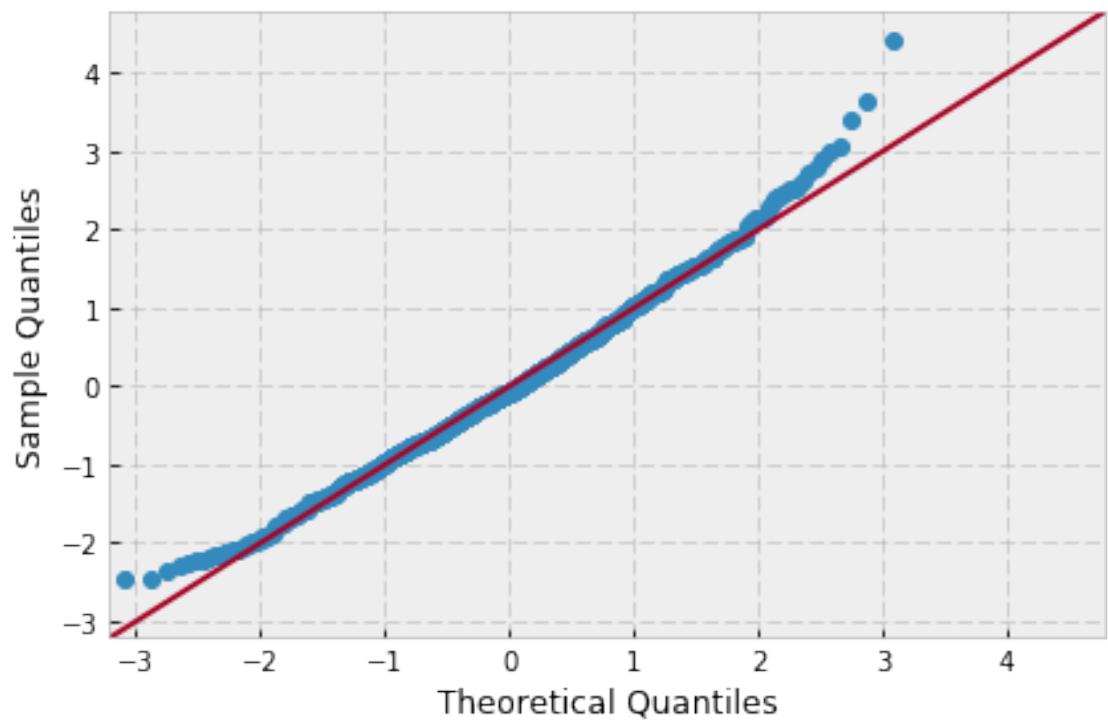
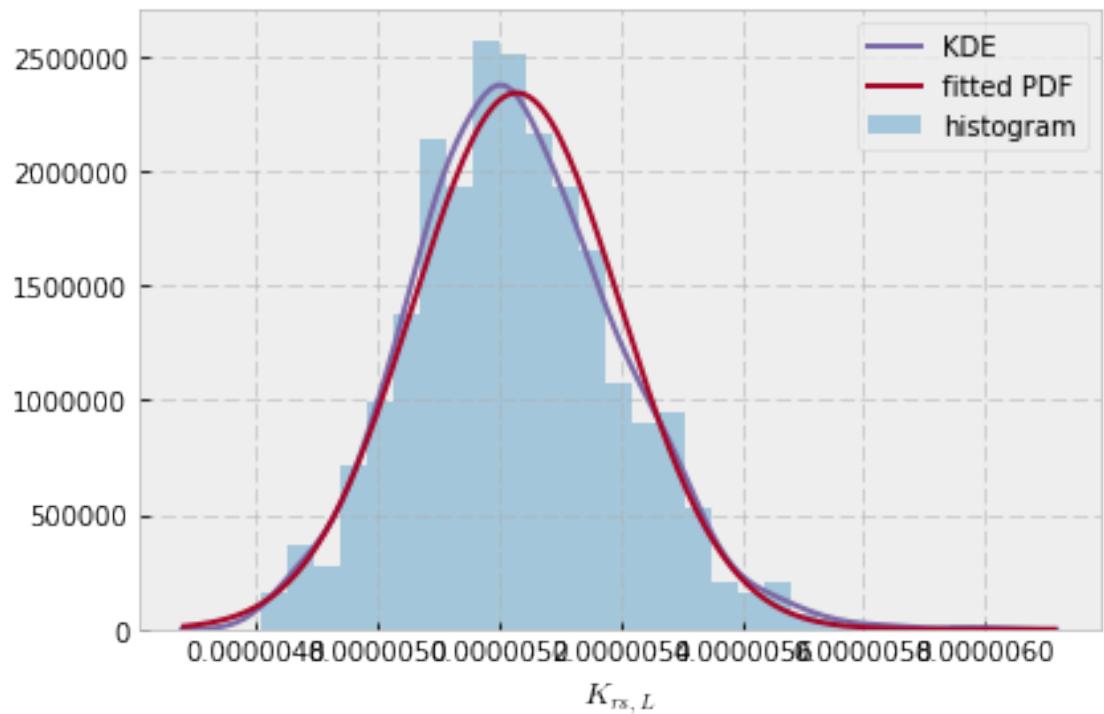
variable name: r
variable value: 2.555555555555554
distribution: normal(shape=(), loc=5.346032226561337e-06, scale=1.9131547766796356e-07)
DescribeResult(nobs=1000, minmax=(4.7535053489590139e-06, 6.0178754703041498e-06)
               mean=5.3460322265613371e-06, variance=3.6638250245566633e-14,
               skewness=0.33200146748889003, kurtosis=0.22169921410682258)

```





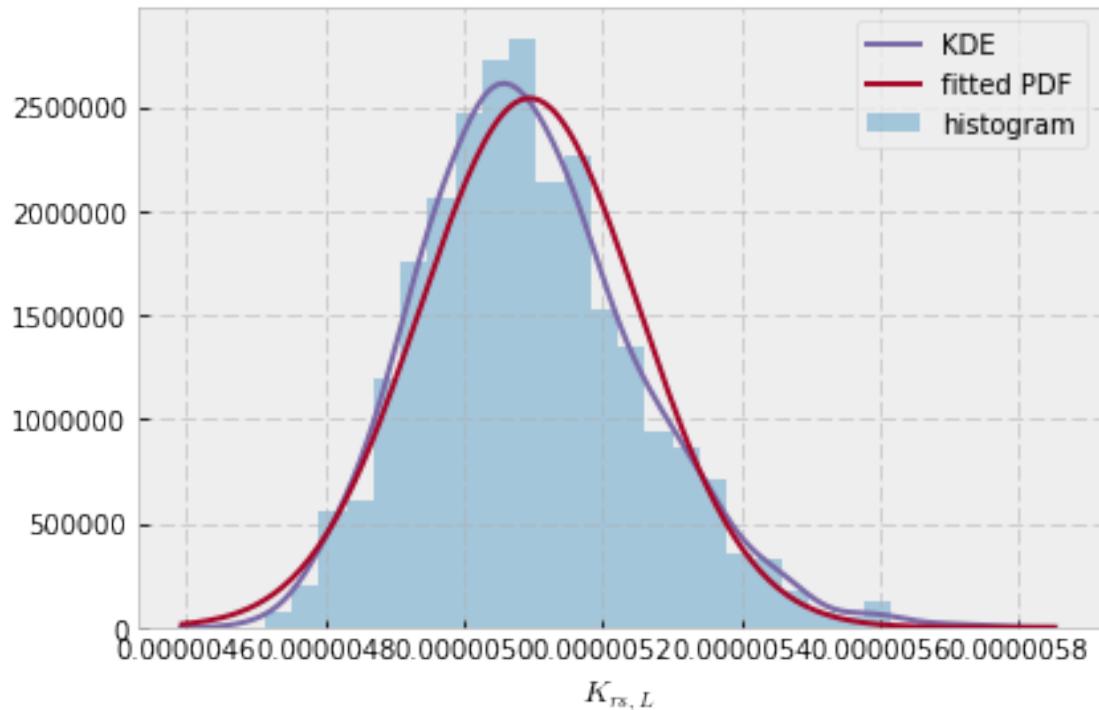
```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(),loc=5.228691264912813e-06, scale=1.704276260643278e-07)
DescribeResult(nobs=1000, minmax=(4.8097446808118126e-06, 5.9847349683509963e-06)
               mean=5.2286912649128128e-06, variance=2.9074650376298641e-14,
               skewness=0.35292350207305323, kurtosis=0.34878202004110204)
```

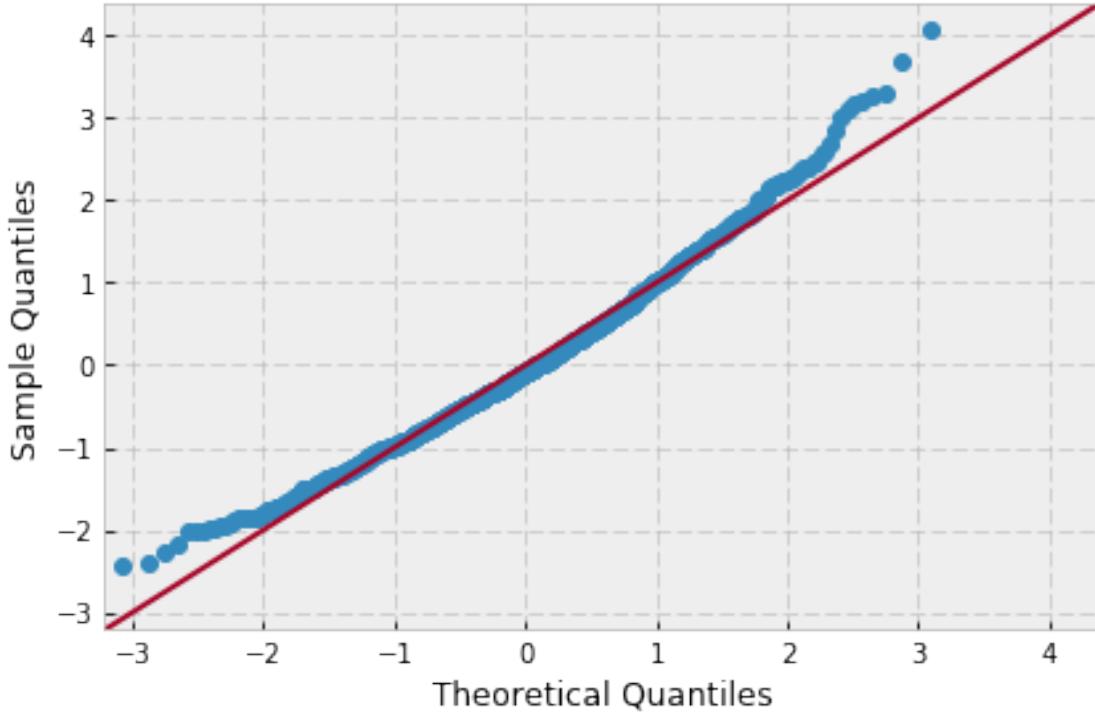


```

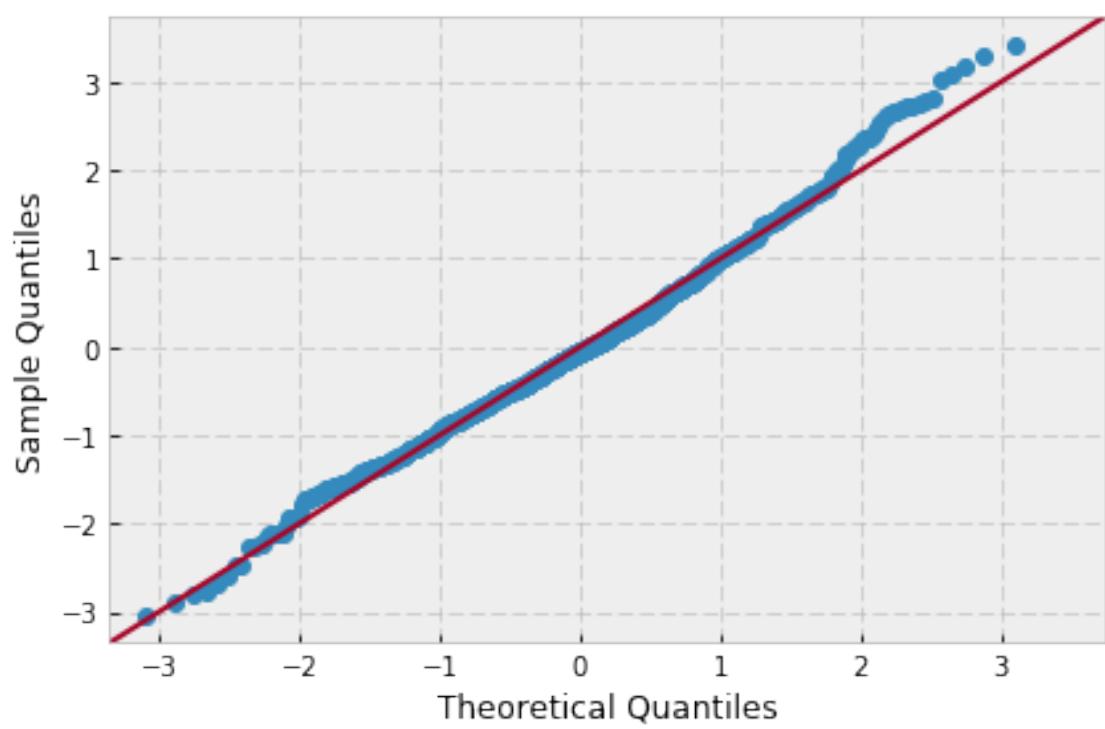
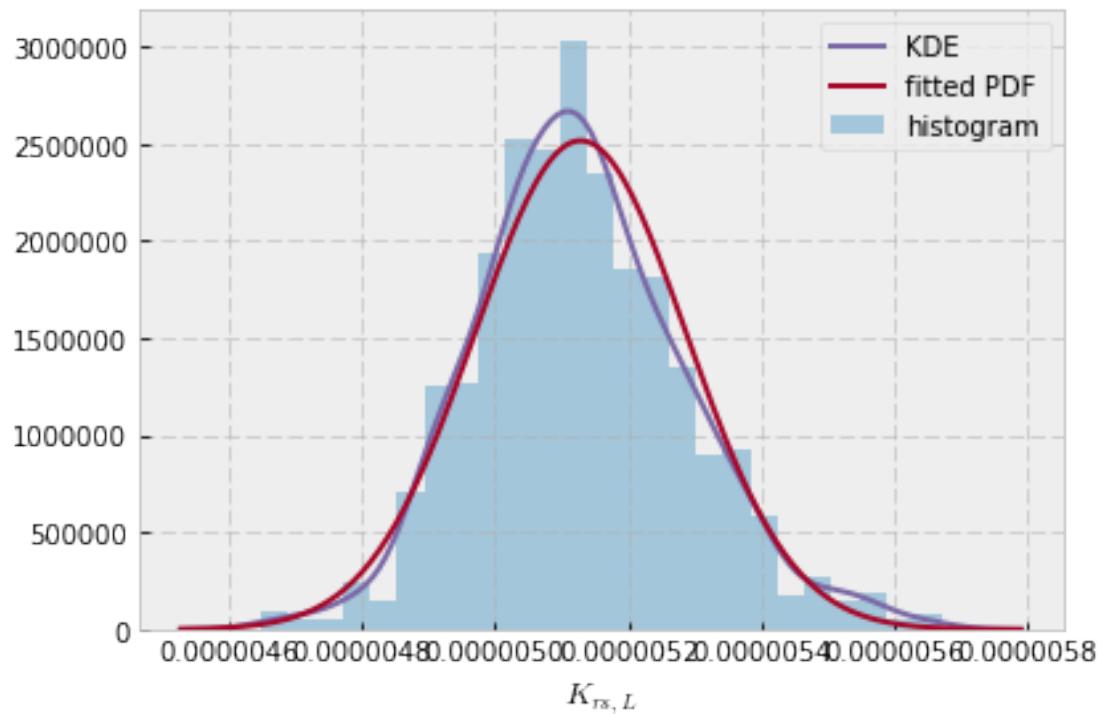
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=5.094536242143038e-06, scale=1.571056628539857e-07)
DescribeResult(nobs=1000, minmax=(4.7122621241924857e-06, 5.7317488119935458e-06)
               mean=5.0945362421430377e-06, variance=2.4706896196987208e-14,
               skewness=0.5163016233047991, kurtosis=0.3798960221614869)

```





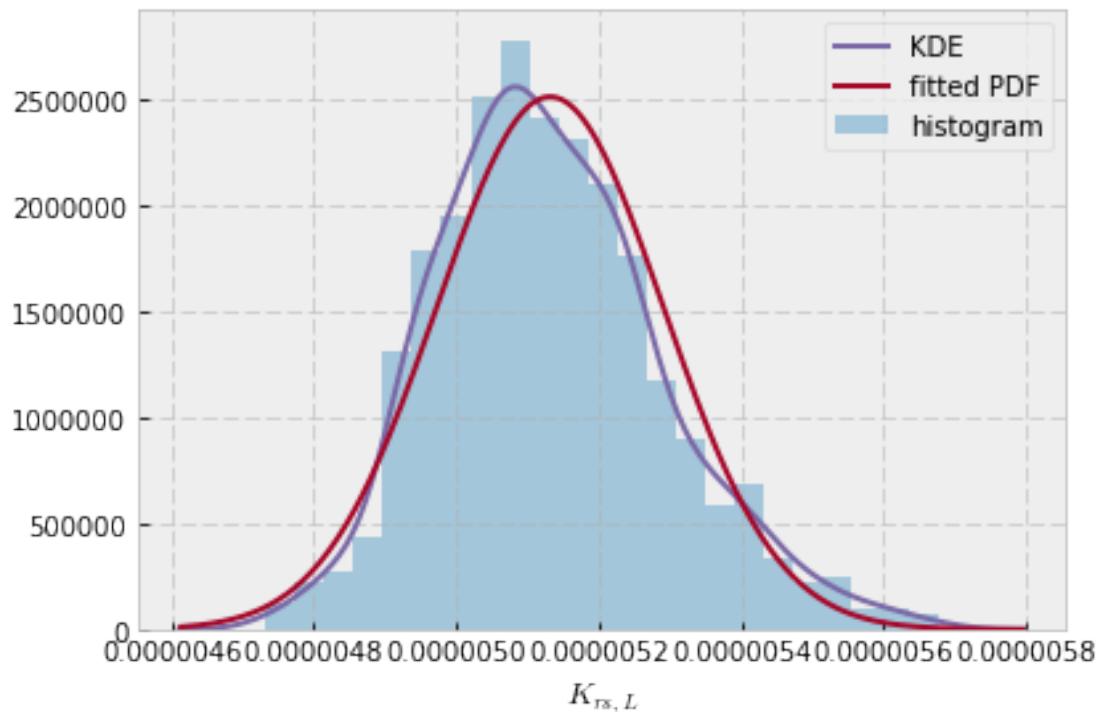
```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(),loc=5.128432787652644e-06, scale=1.58592220778978e-07)
DescribeResult(nobs=1000, minmax=(4.6482635560268412e-06, 5.6693207561380973e-06)
               mean=5.1284327876526443e-06, variance=2.5176669160768874e-14,
               skewness=0.3110905863933188, kurtosis=0.37849974002845954)
```

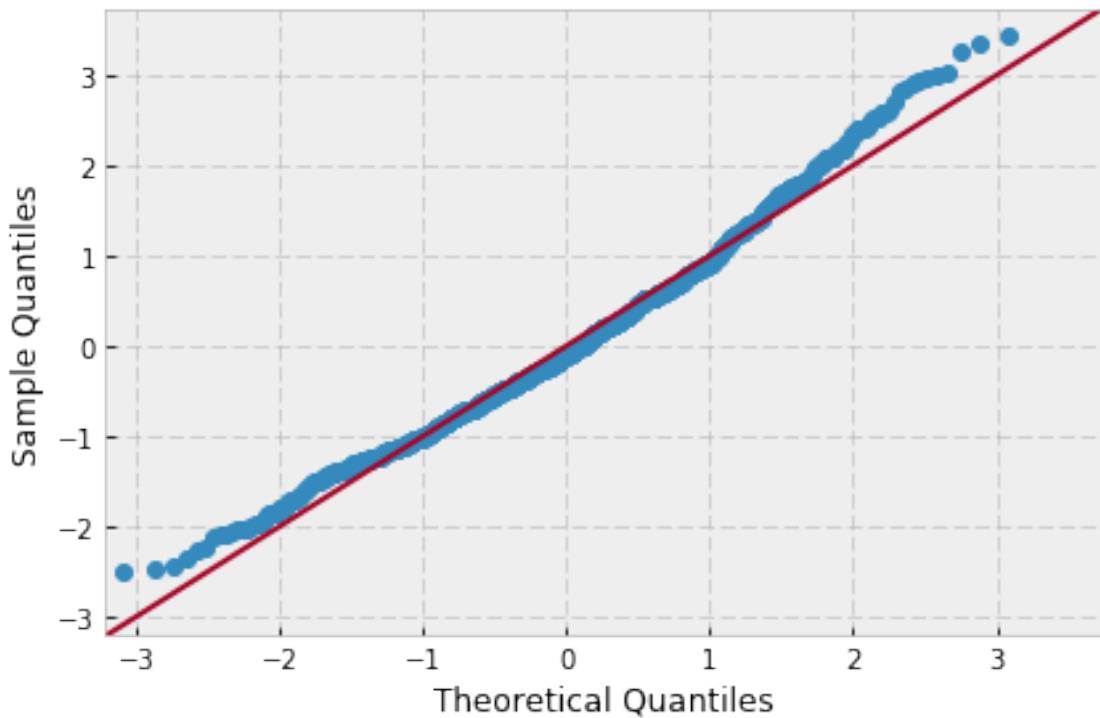


```

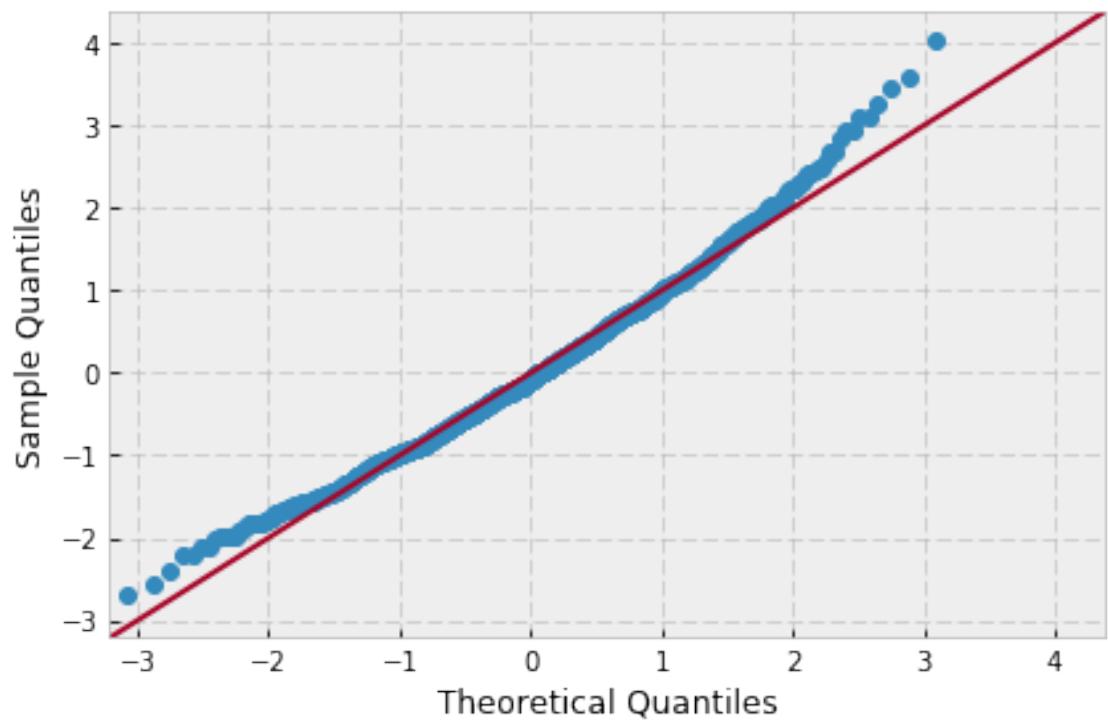
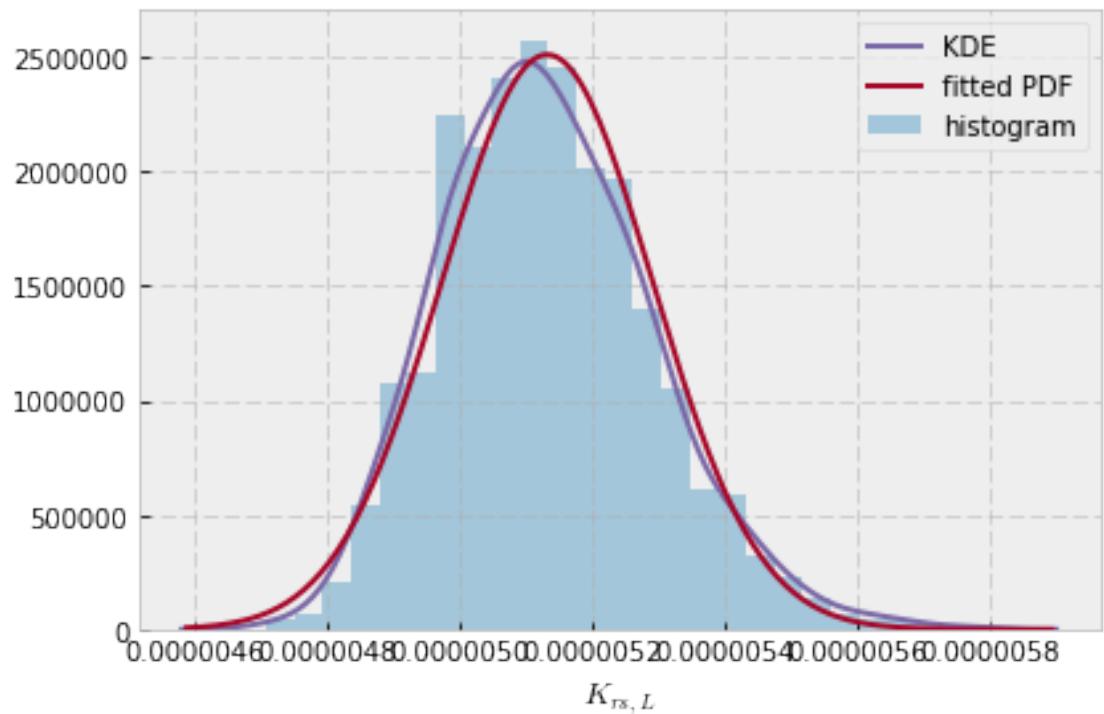
variable name: sigma
variable value: 0.0222222222222223
distribution: normal(shape=(), loc=5.131585701754121e-06, scale=1.58711625834444e-07)
DescribeResult(nobs=1000, minmax=(4.73332207937374e-06, 5.6751728114165217e-06)
               mean=5.1315857017541209e-06, variance=2.5214594769782333e-14,
               skewness=0.4808872122605223, kurtosis=0.25941279898516534)

```





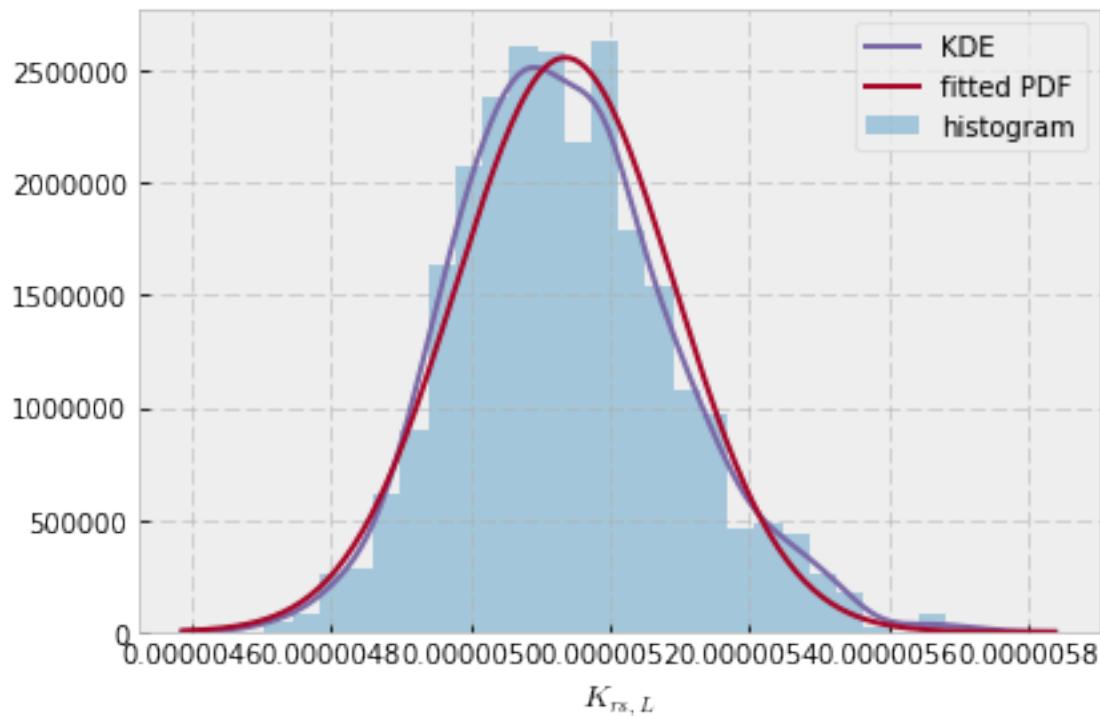
```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=5.131319781452394e-06, scale=1.588368048989729e-07)
DescribeResult(nobs=1000, minmax=(4.7064600954228993e-06, 5.7732524484888106e-06)
               mean=5.131319781452394e-06, variance=2.525438497548987e-14,
               skewness=0.4510188085674612, kurtosis=0.3399975400588744)
```

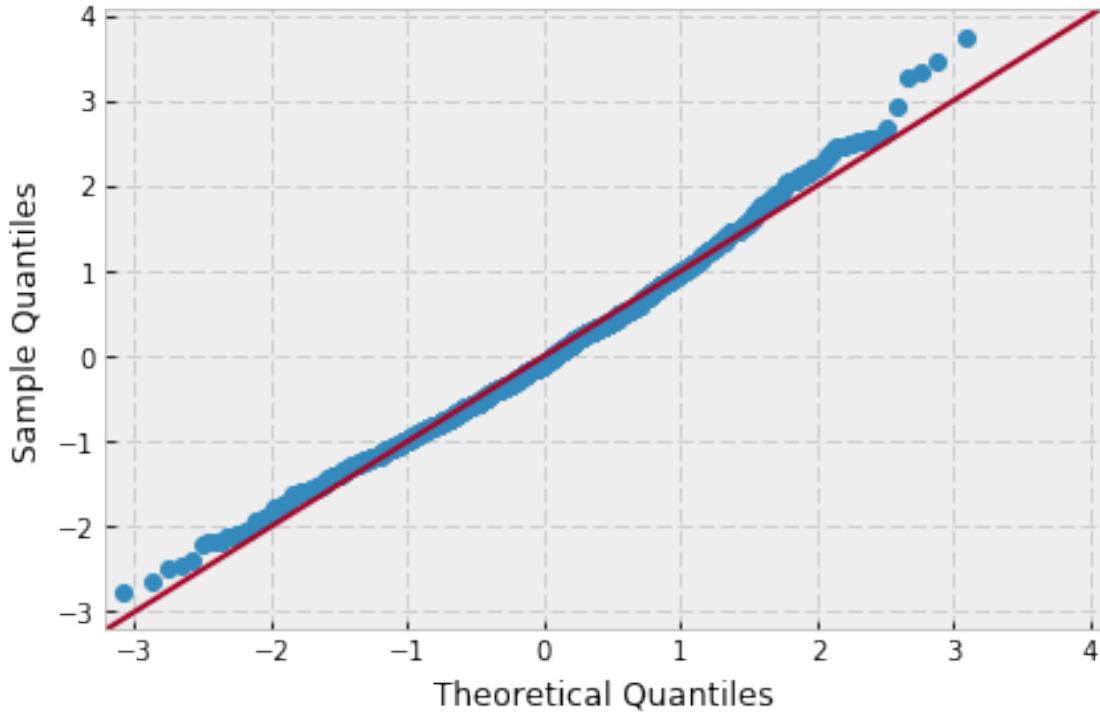


```

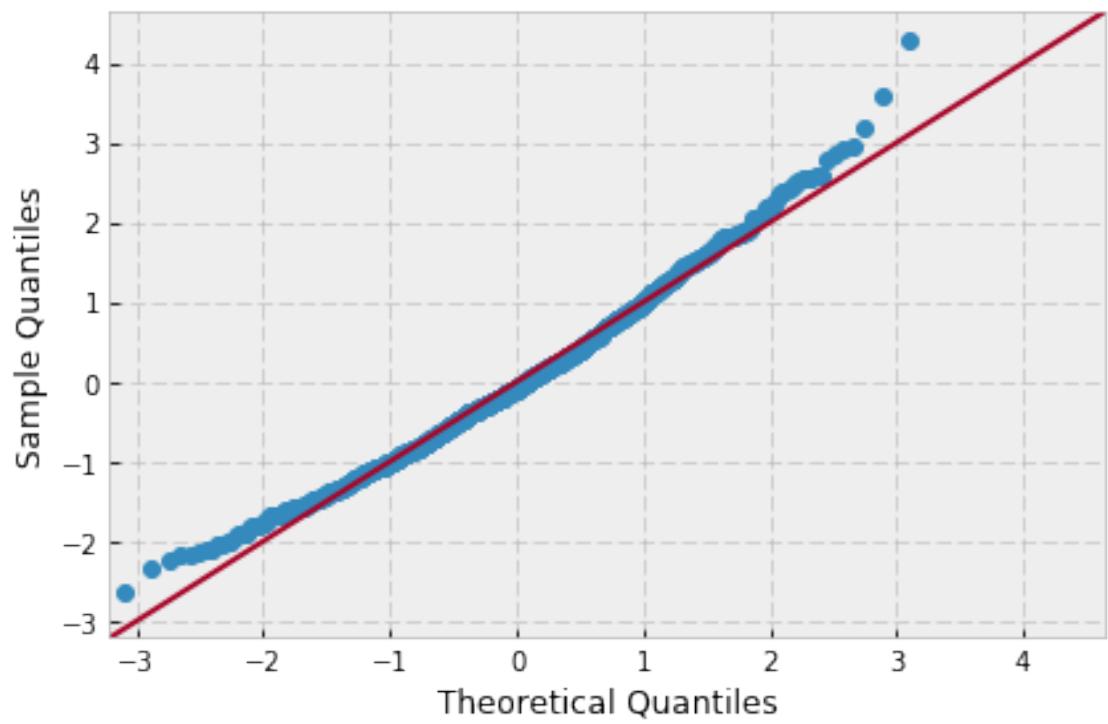
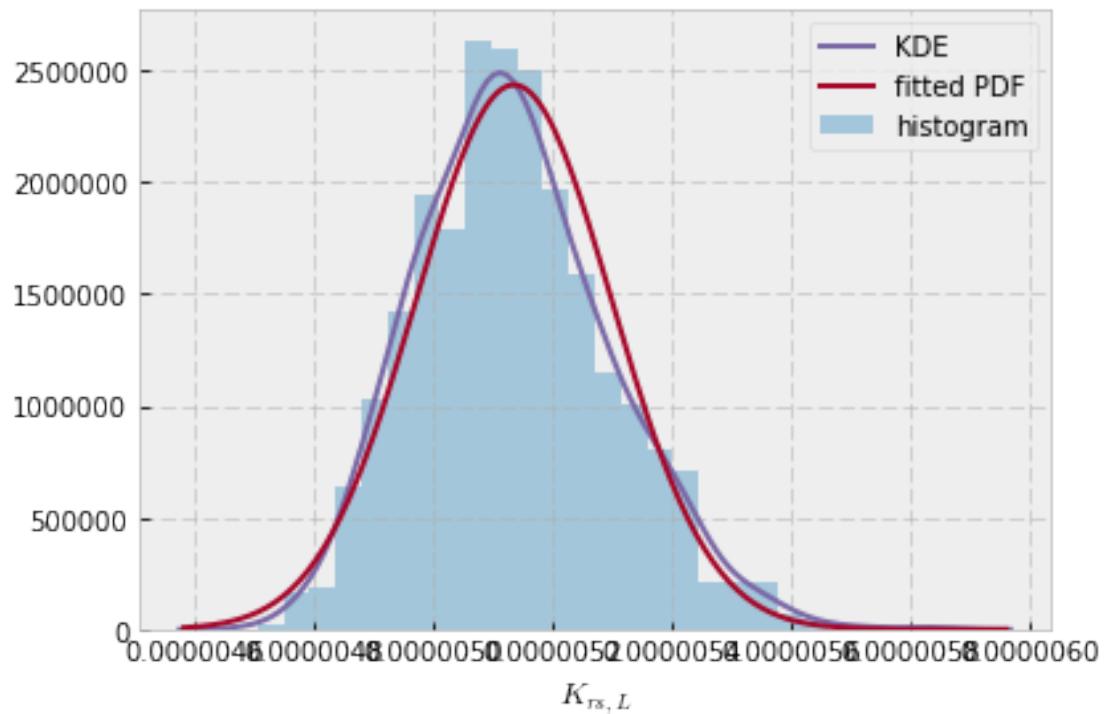
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=5.136175701925101e-06, scale=1.5591729504687543e-07)
DescribeResult(nobs=1000, minmax=(4.7051128440996742e-06, 5.7200411741298577e-06)
               mean=5.1361757019251013e-06, variance=2.4334537432166576e-14,
               skewness=0.3830361890402258, kurtosis=0.22317623351701243)

```





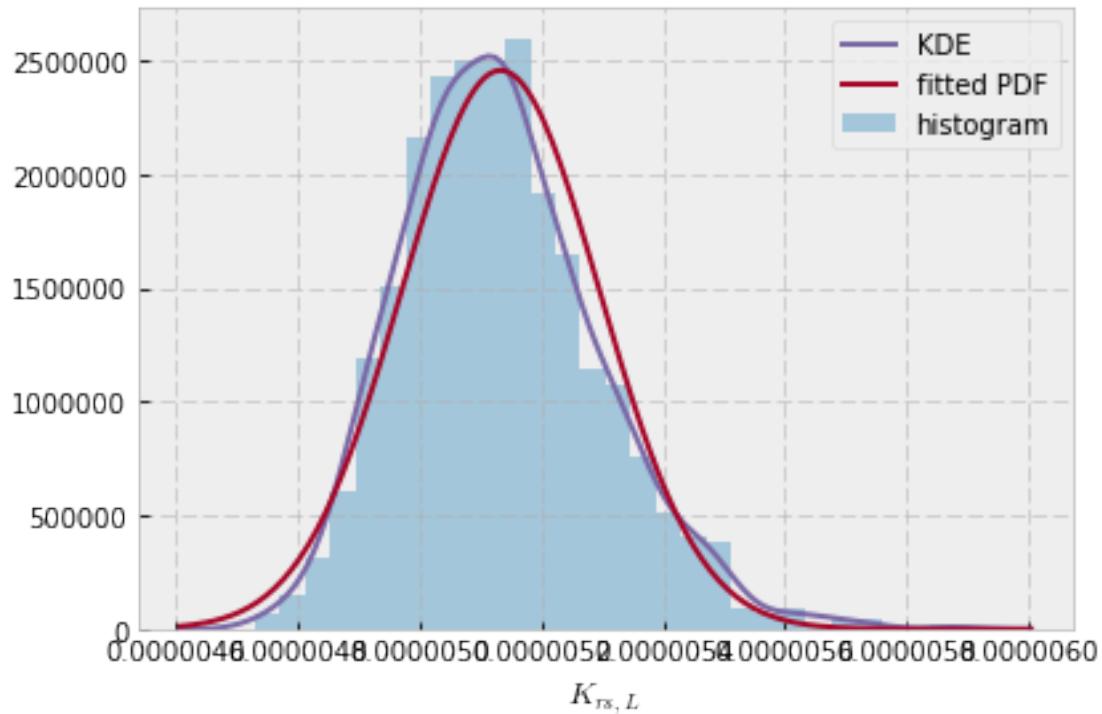
```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(), loc=5.135795420895345e-06, scale=1.637691791292173e-07)
DescribeResult(nobs=1000, minmax=(4.7054085335299515e-06, 5.8384608617299631e-06)
               mean=5.1357954208953451e-06, variance=2.6847191223881538e-14,
               skewness=0.4320208662975798, kurtosis=0.218805011587468)
```

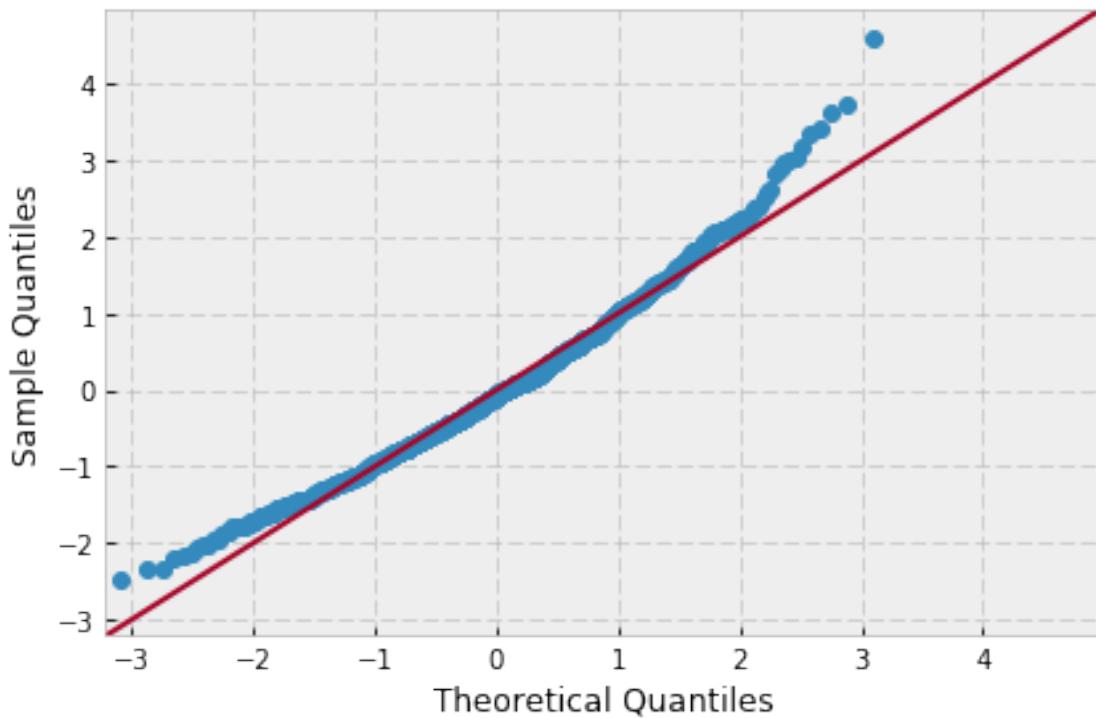


```

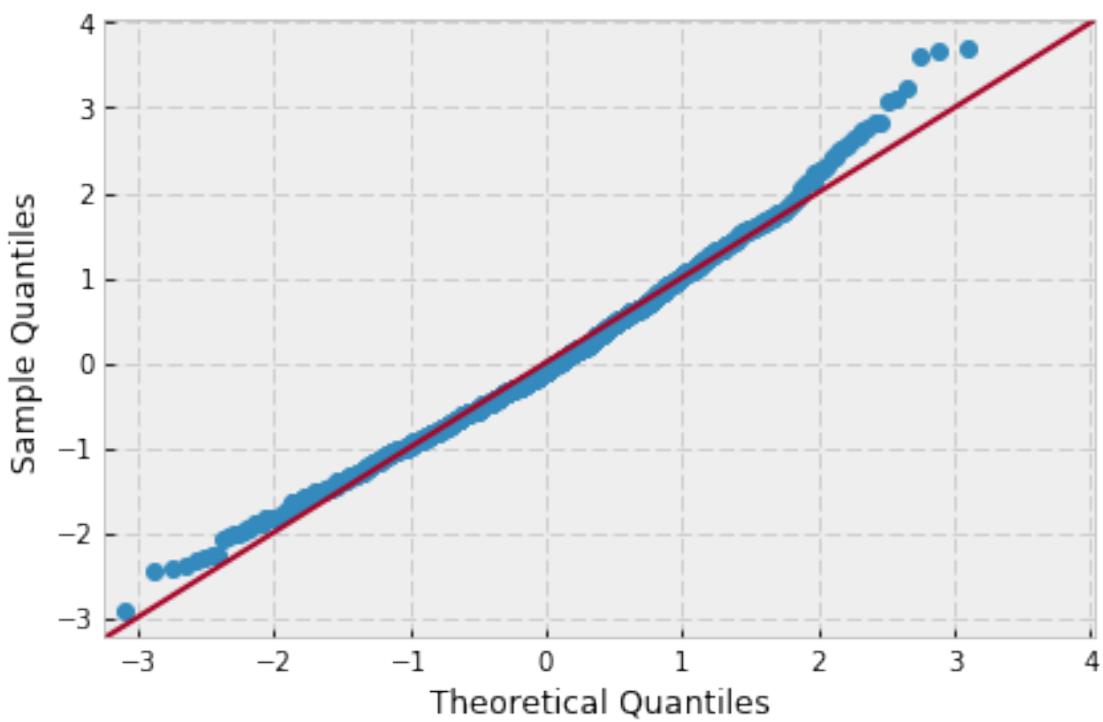
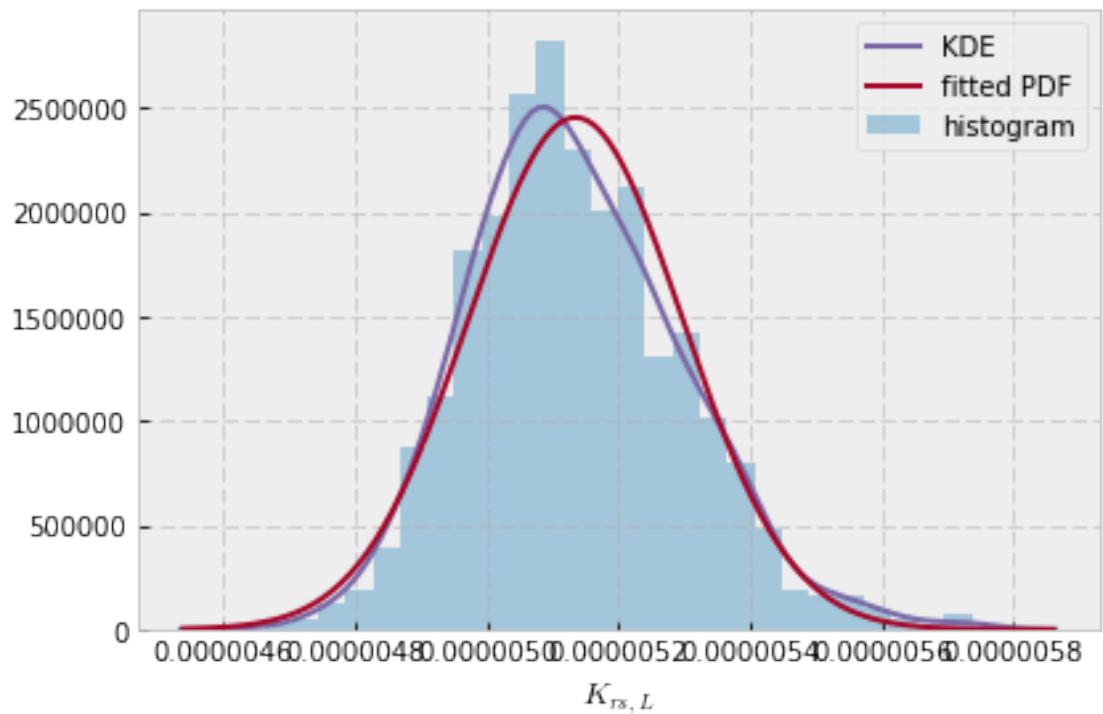
variable name: sigma
variable value: 0.06666666666666667
distribution: normal(shape=(), loc=5.132868328025852e-06, scale=1.6234605864501015e-07)
DescribeResult(nobs=1000, minmax=(4.7296243045587678e-06, 5.881207095868826e-06)
               mean=5.1328683280258522e-06, variance=2.6382625382952026e-14,
               skewness=0.5916728785518045, kurtosis=0.6704901032190631)

```





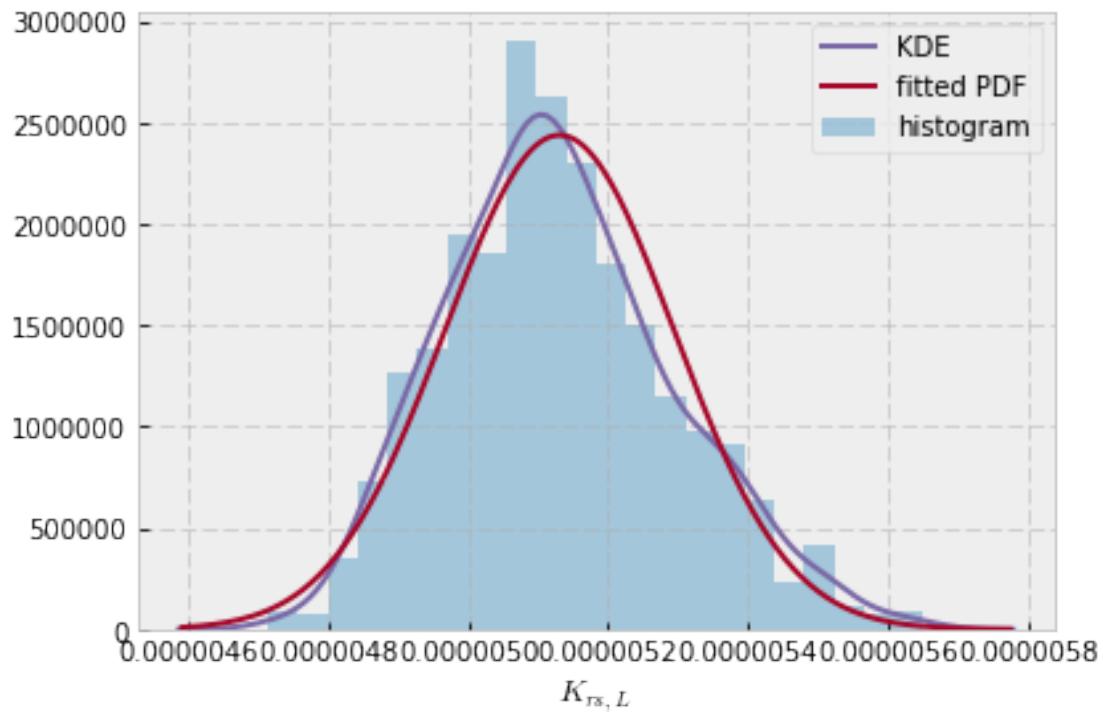
```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=5.134276401055037e-06, scale=1.6250519726878384e-07)
DescribeResult(nobs=1000, minmax=(4.6612293331712302e-06, 5.7365653574656909e-06)
               mean=5.1342764010550372e-06, variance=2.6434373512879229e-14,
               skewness=0.4217681466263346, kurtosis=0.3248027893150618)
```

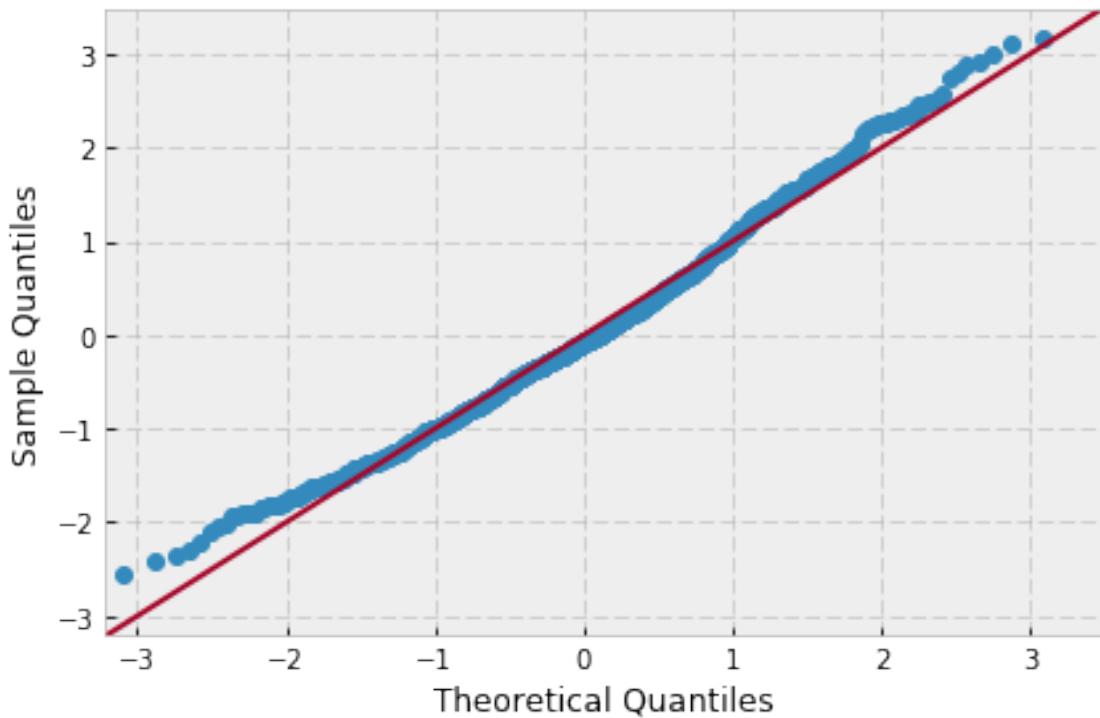


```

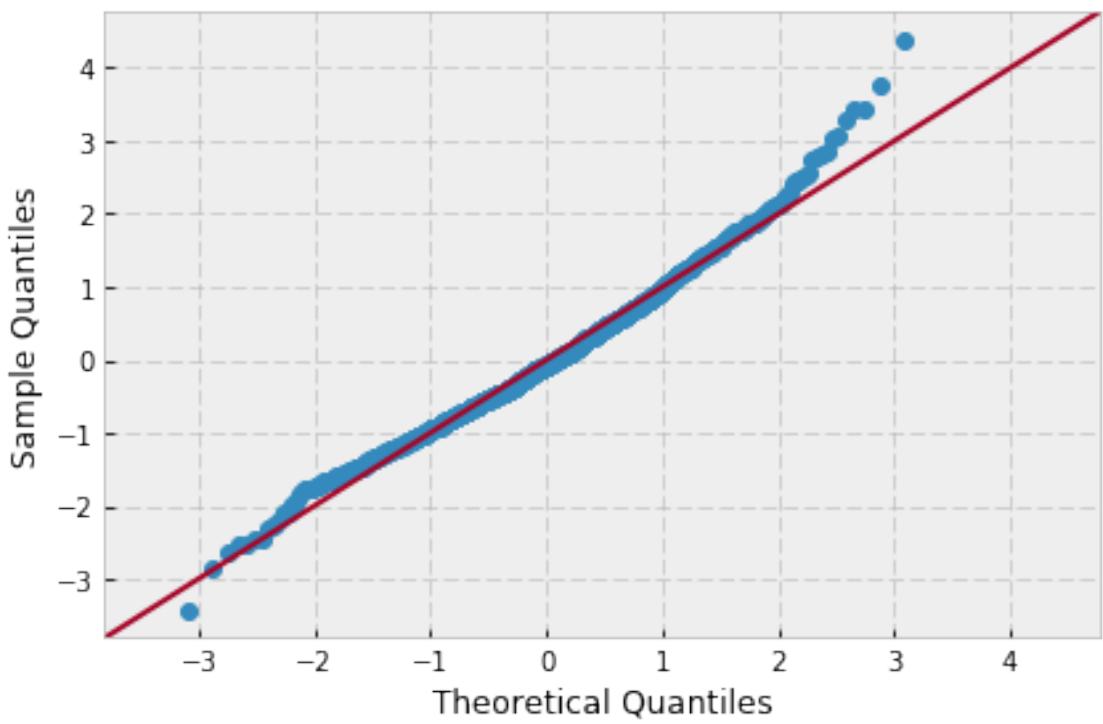
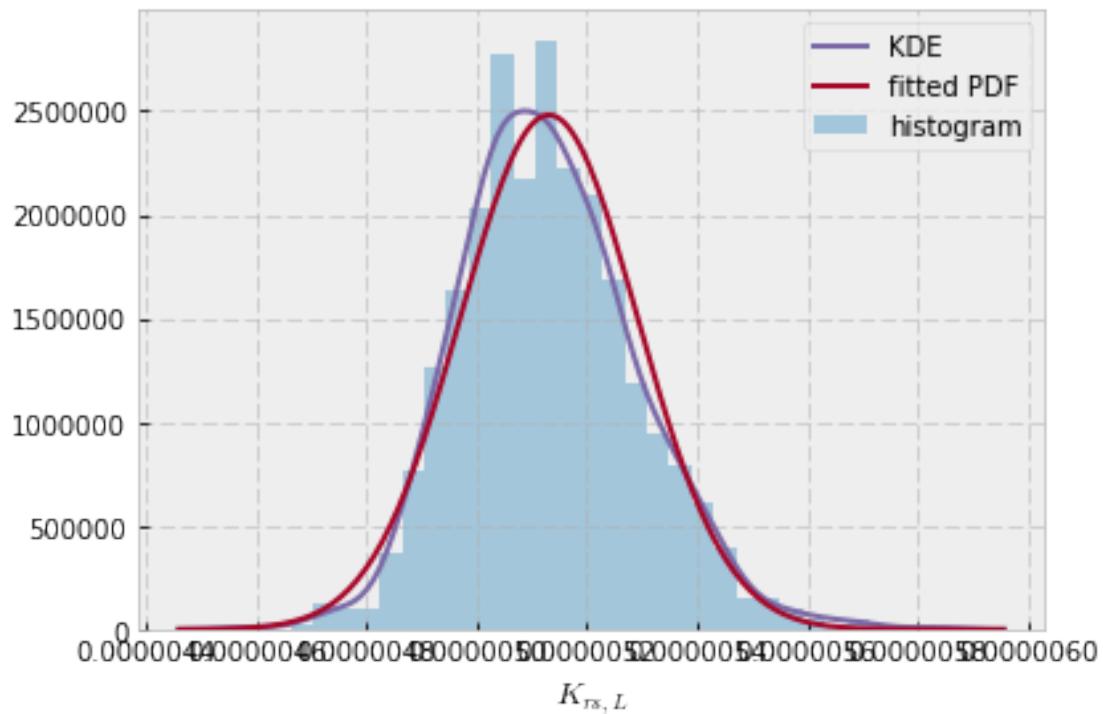
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(), loc=5.130336305123606e-06, scale=1.6358642551565547e-07)
DescribeResult(nobs=1000, minmax=(4.7136745411286467e-06, 5.6515584109825988e-06)
               mean=5.1303363051236064e-06, variance=2.6787305918908003e-14,
               skewness=0.3791305167577935, kurtosis=-0.08204513433808502)

```





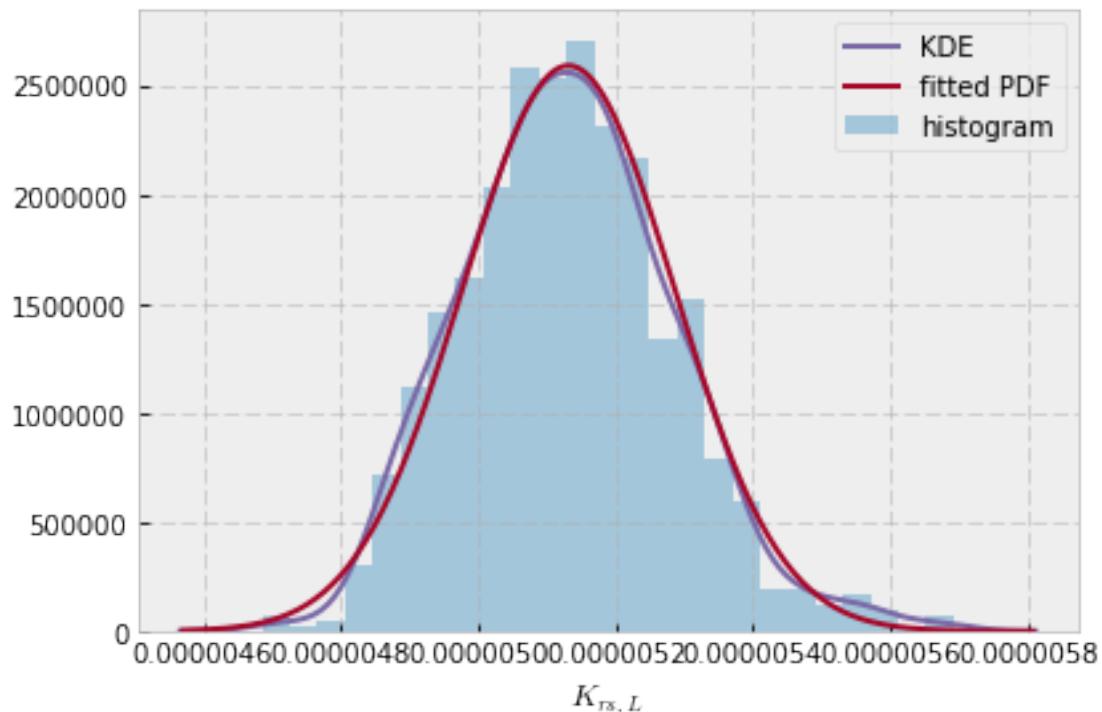
```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(),loc=5.130635602612961e-06, scale=1.6080247303670128e-07)
DescribeResult(nobs=1000, minmax=(4.5809594817261983e-06, 5.8340084985763946e-06)
               mean=5.1306356026129611e-06, variance=2.5883318653372412e-14,
               skewness=0.42231210169642447, kurtosis=0.6543424235583188)
```

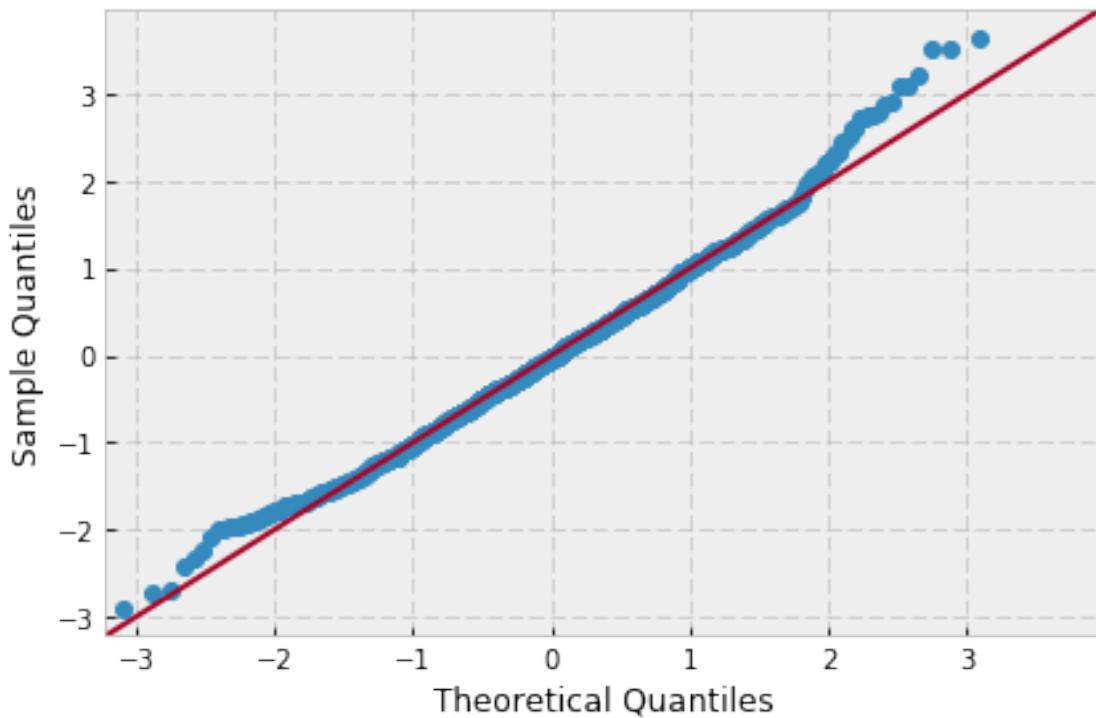


```

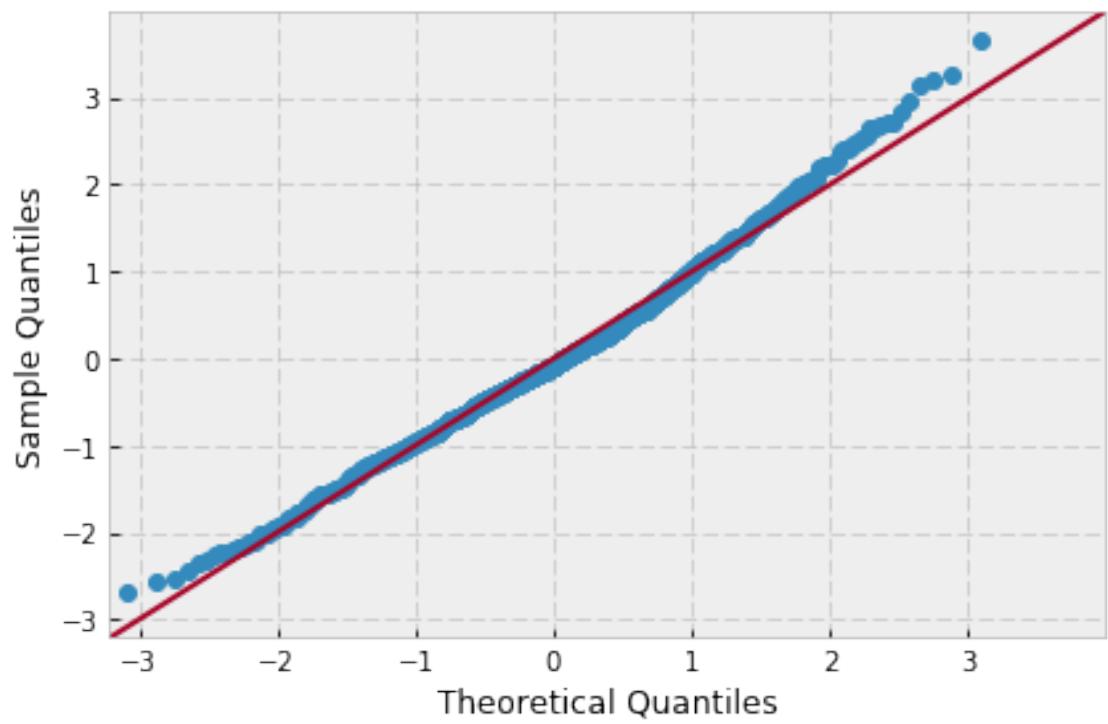
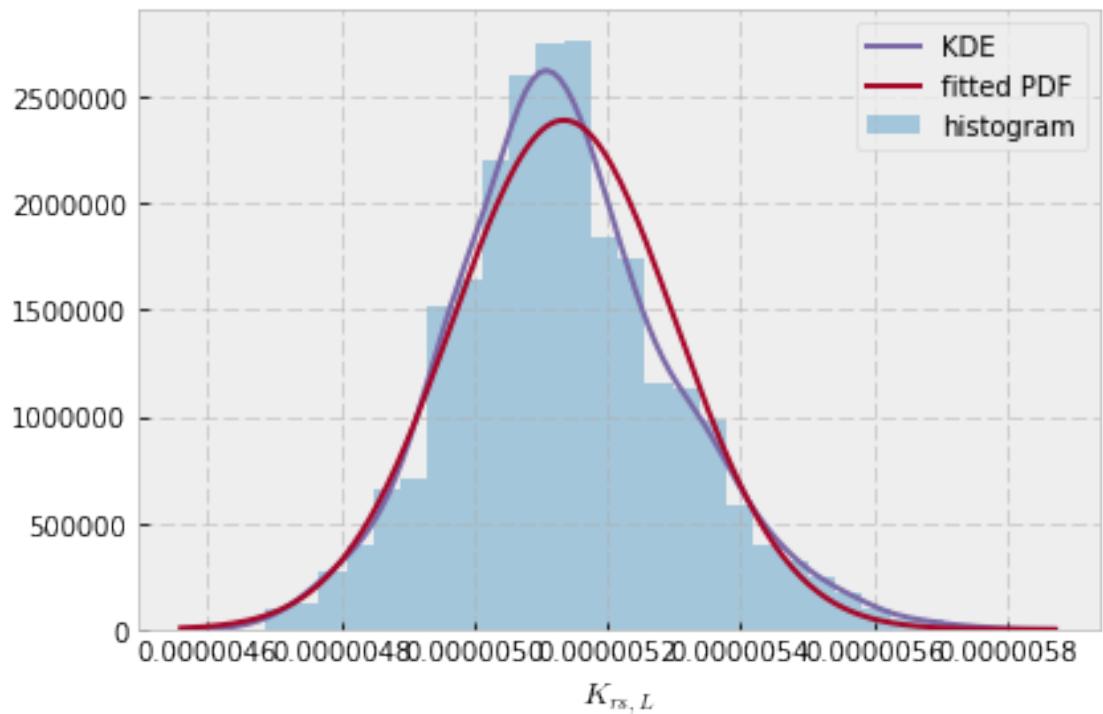
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=5.131362247824867e-06, scale=1.5401951678963124e-07)
DescribeResult(nobs=1000, minmax=(4.6855858245775502e-06, 5.6914172353644209e-06)
               mean=5.1313622478248666e-06, variance=2.3745757309420921e-14,
               skewness=0.3279201549247228, kurtosis=0.3179532178498583)

```

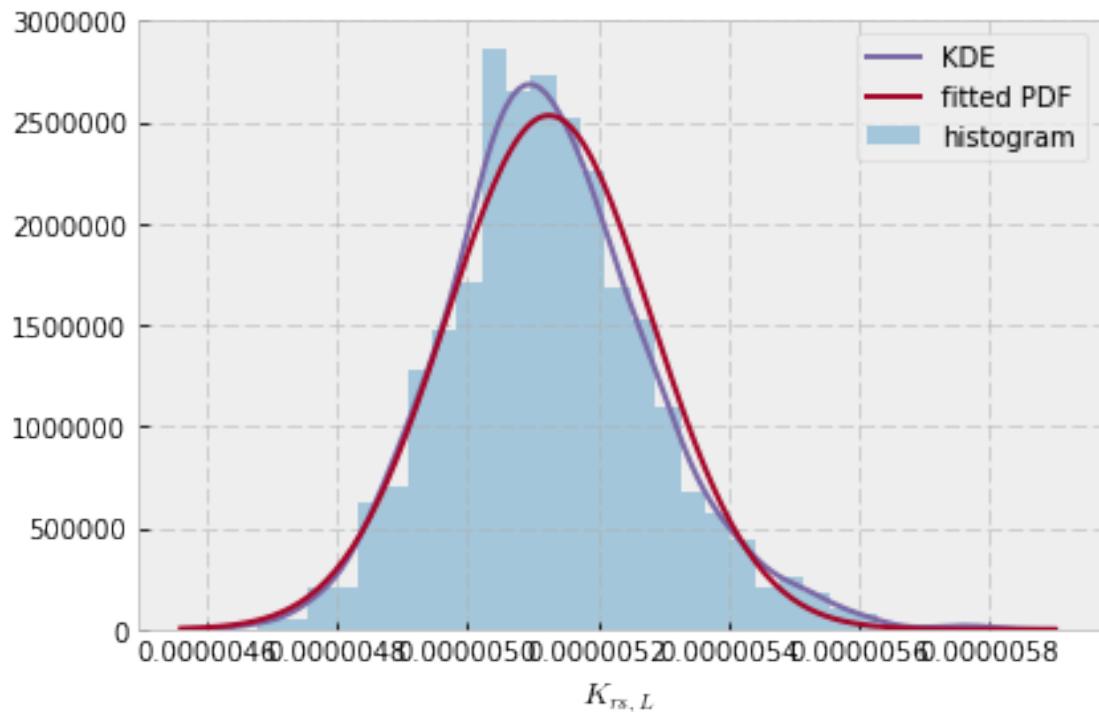


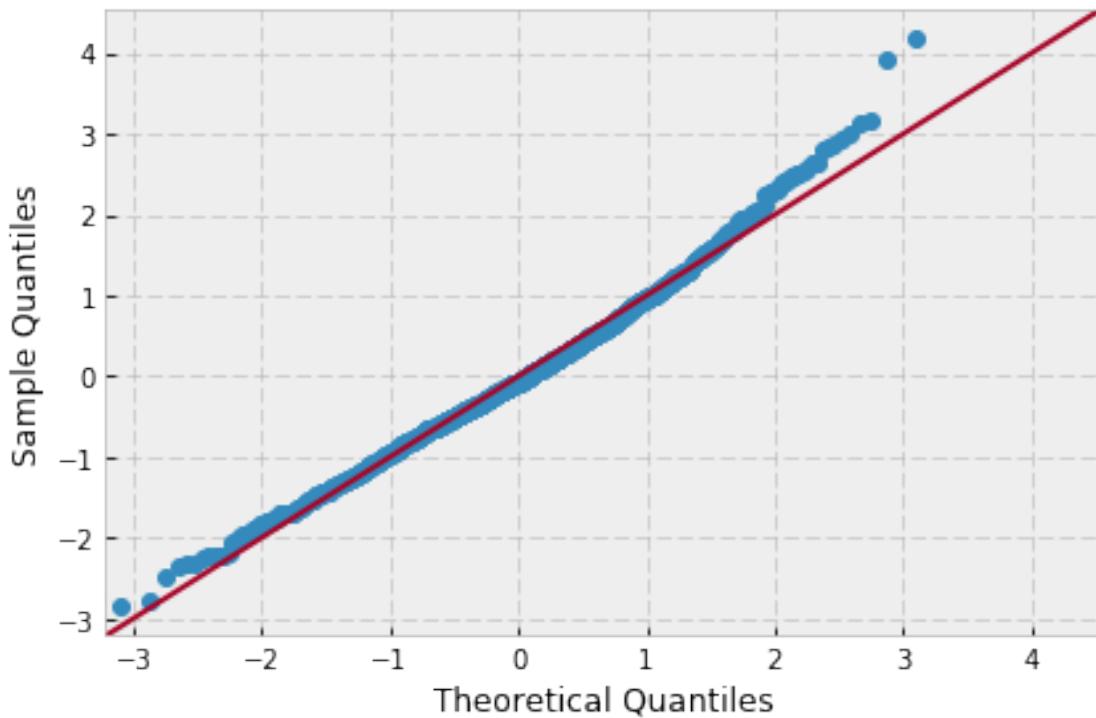


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=5.134893538269826e-06, scale=1.6687664257547272e-07)
DescribeResult(nobs=1000, minmax=(4.6857877096882925e-06, 5.746580458578384e-06)
               mean=5.1348935382698264e-06, variance=2.7875689526788861e-14,
               skewness=0.35173359359854284, kurtosis=0.27922385555497575)
```

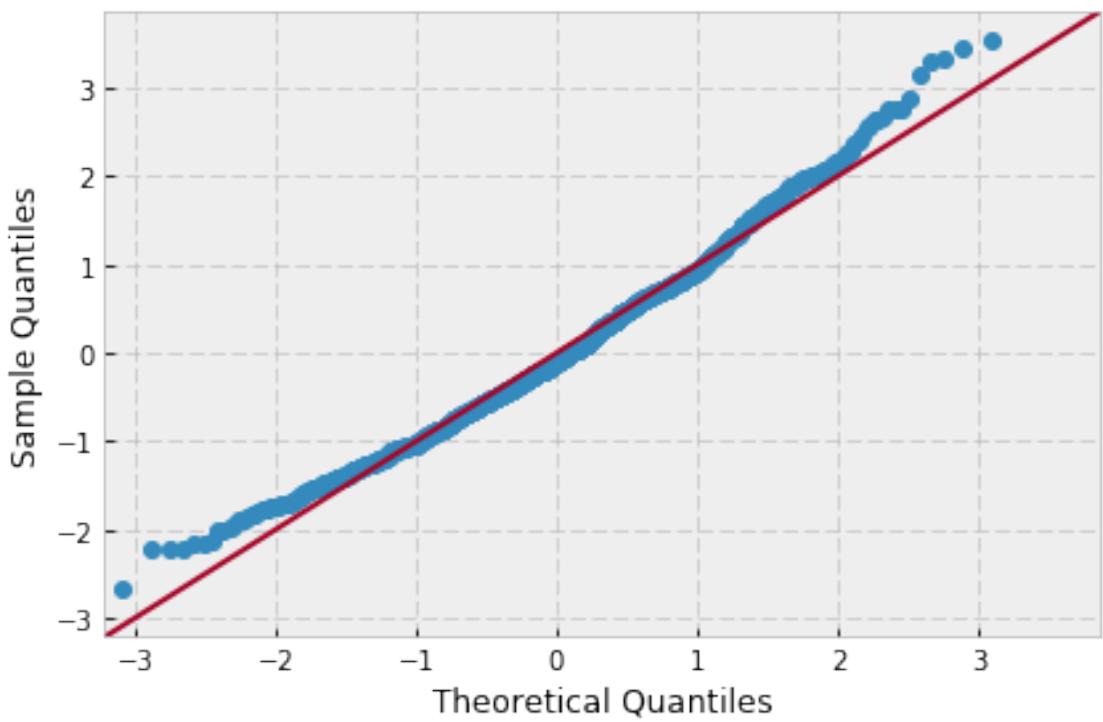
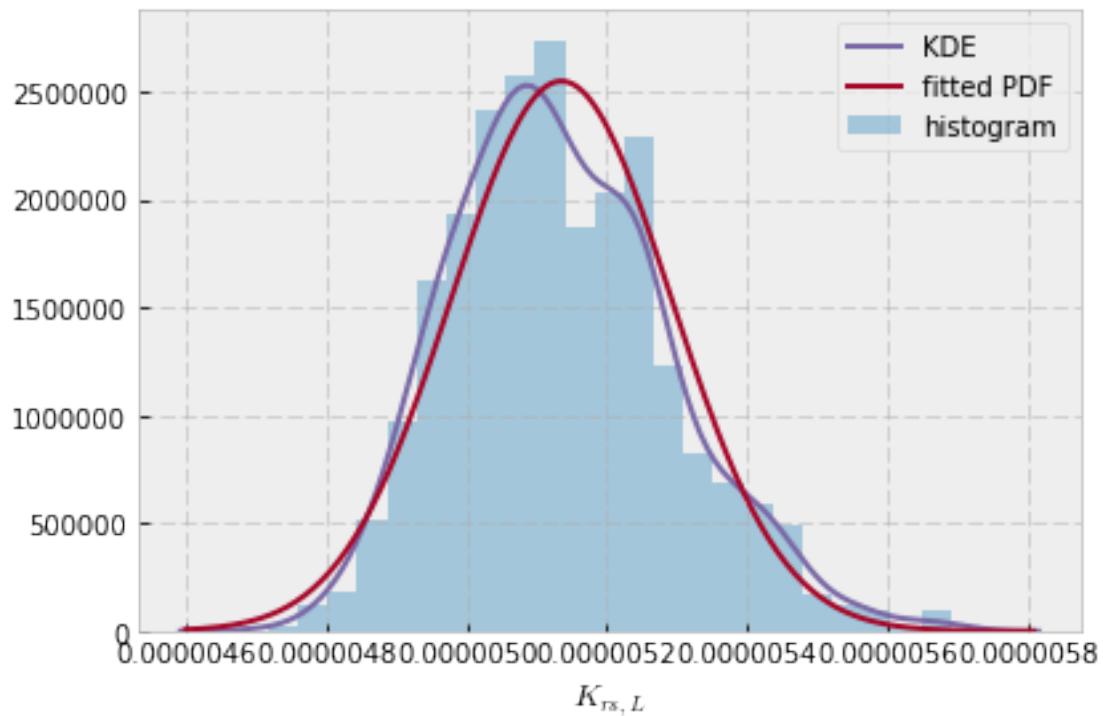


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=5.124951988083768e-06, scale=1.5751293714637465e-07)
DescribeResult(nobs=1000, minmax=(4.6783419707542135e-06, 5.783786749416861e-06)
               mean=5.1249519880837681e-06, variance=2.483516052900678e-14,
               skewness=0.4070655177356903, kurtosis=0.5454292105585967)
```





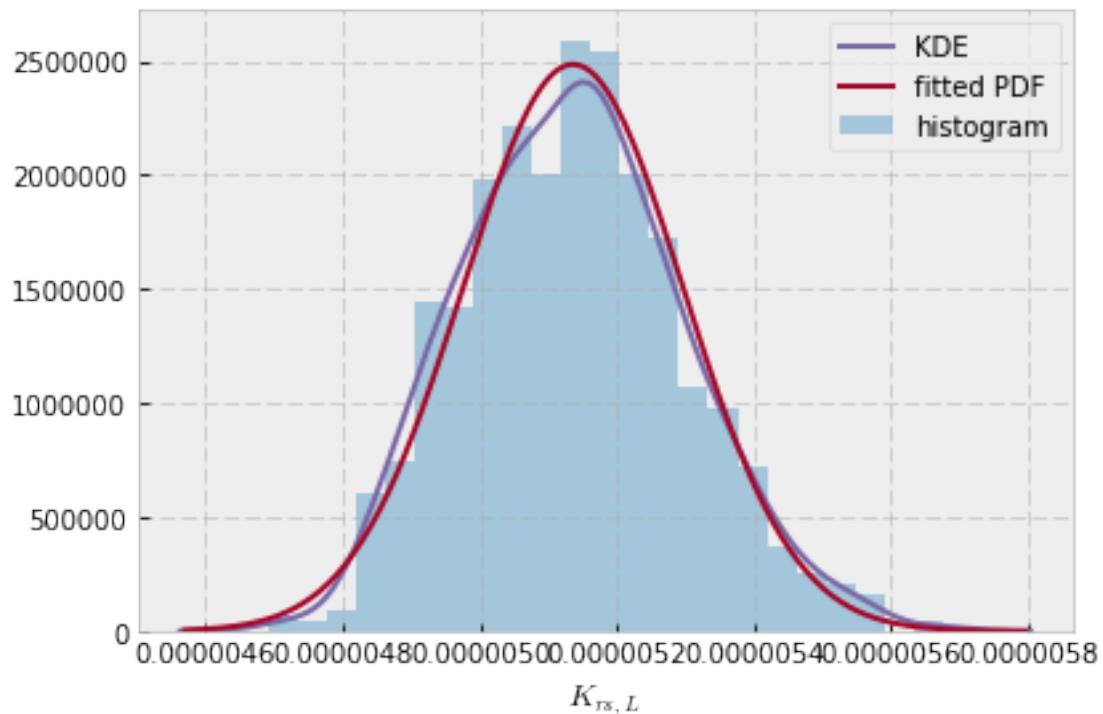
```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=5.135004301885885e-06, scale=1.5660492683254174e-07)
DescribeResult(nobs=1000, minmax=(4.7165972298297779e-06, 5.6904081788490417e-06)
               mean=5.1350043018858849e-06, variance=2.4549652760986735e-14,
               skewness=0.4590352577789928, kurtosis=0.13198629998768618)
```

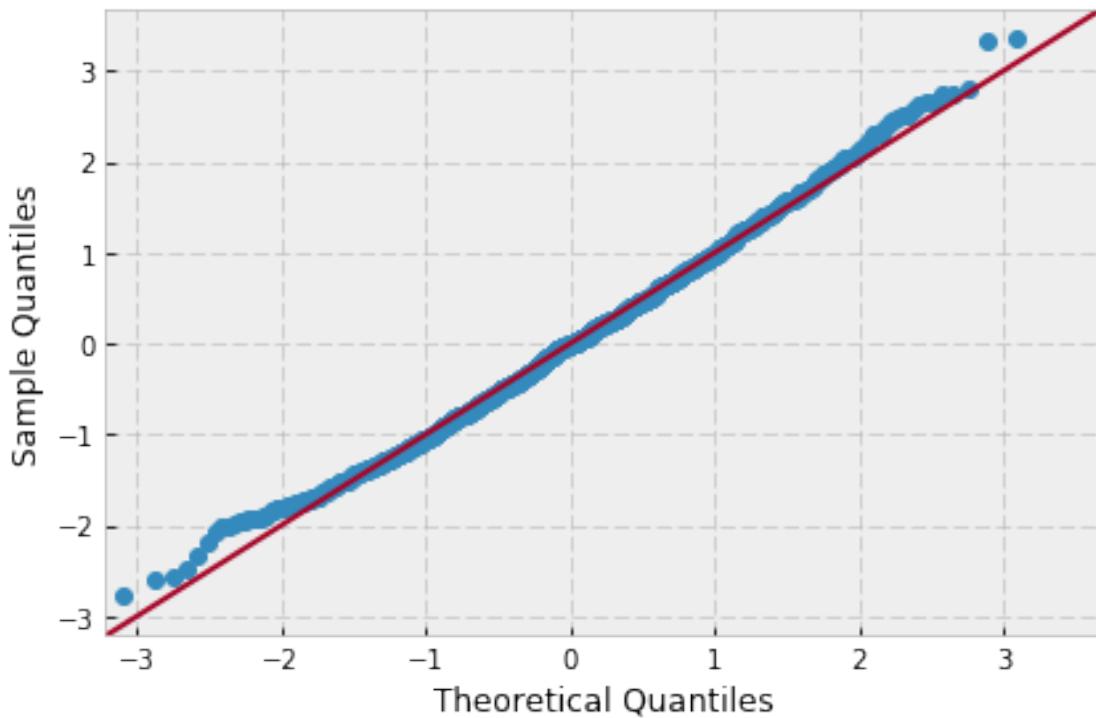


```

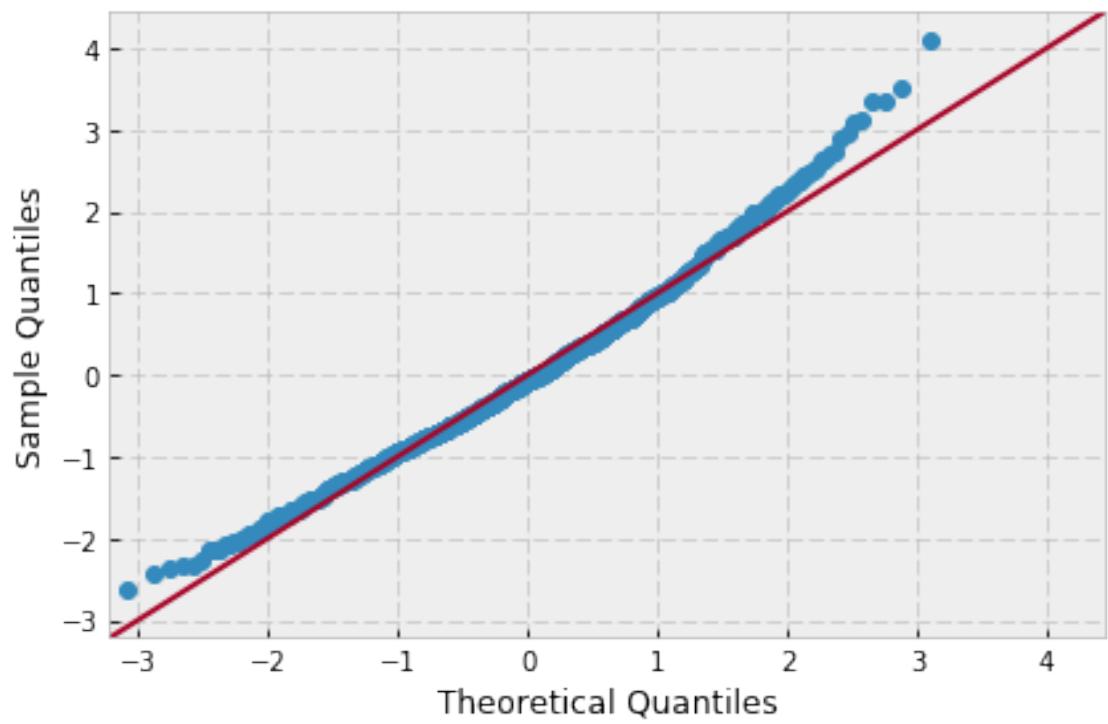
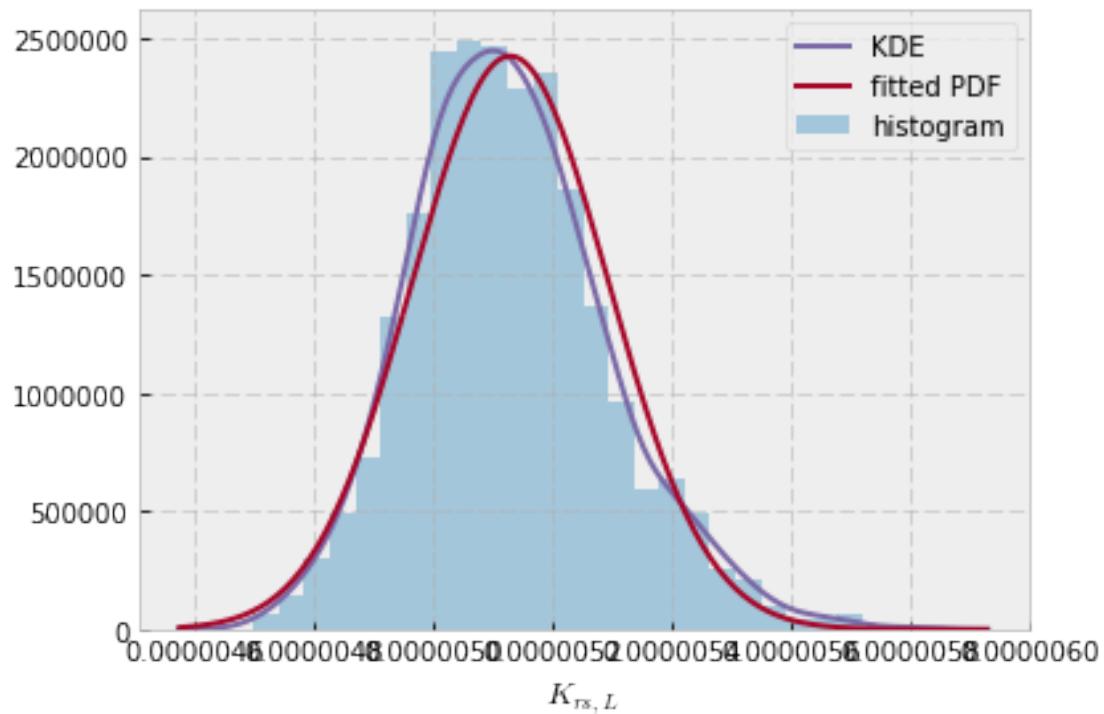
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=5.135918073507662e-06, scale=1.6048125092462085e-07)
DescribeResult(nobs=1000, minmax=(4.6909511248563738e-06, 5.6749972007890238e-06)
               mean=5.1359180735076616e-06, variance=2.5780011910241358e-14,
               skewness=0.23500860698132145, kurtosis=-0.11016418081554136)

```





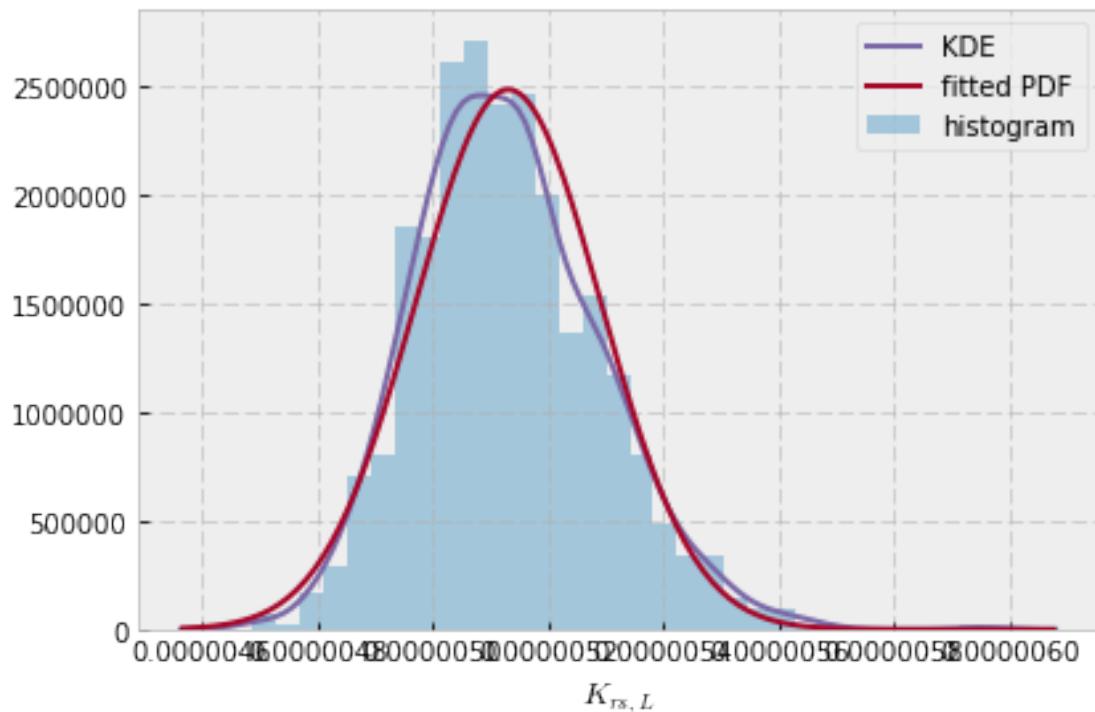
```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=5.130105632493544e-06, scale=1.6446070918387043e-07)
DescribeResult(nobs=1000, minmax=(4.7003439627405276e-06, 5.8046157688669483e-06)
               mean=5.1301056324935444e-06, variance=2.7074399264526129e-14,
               skewness=0.4695291689437227, kurtosis=0.43735781289341835)
```

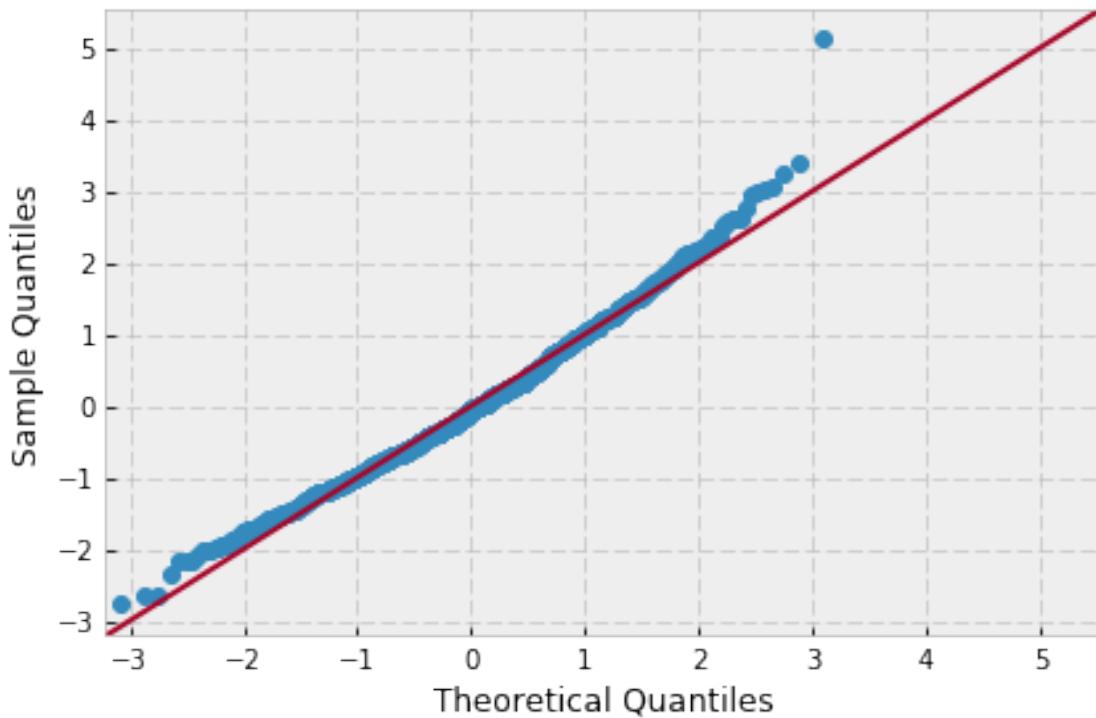


```

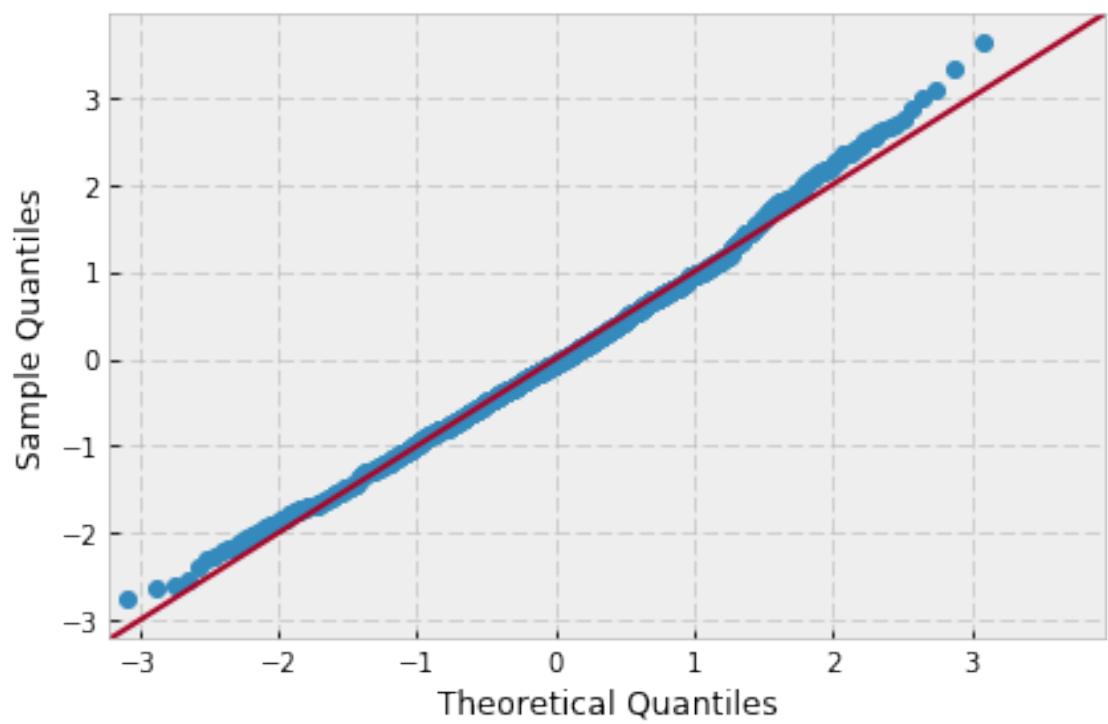
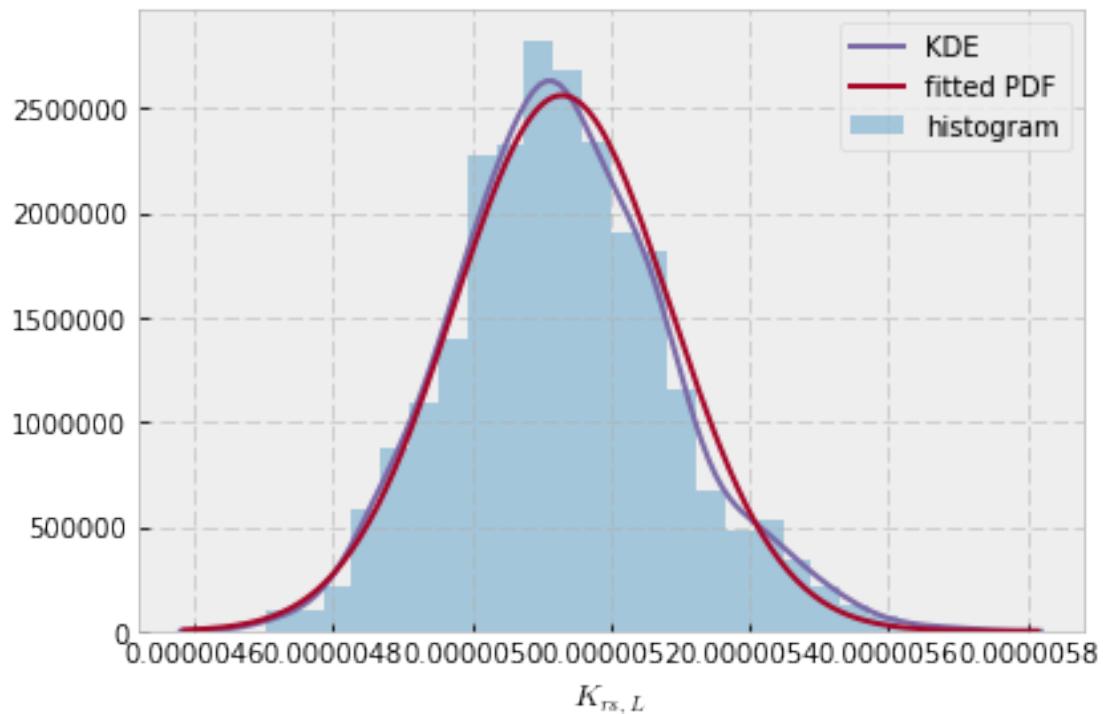
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=5.131092446700991e-06, scale=1.6035314306222574e-07)
DescribeResult(nobs=1000, minmax=(4.6883859959723625e-06, 5.9542355882285265e-06)
               mean=5.1310924467009907e-06, variance=2.5738869359293923e-14,
               skewness=0.47919643689153824, kurtosis=0.6274927993688957)

```





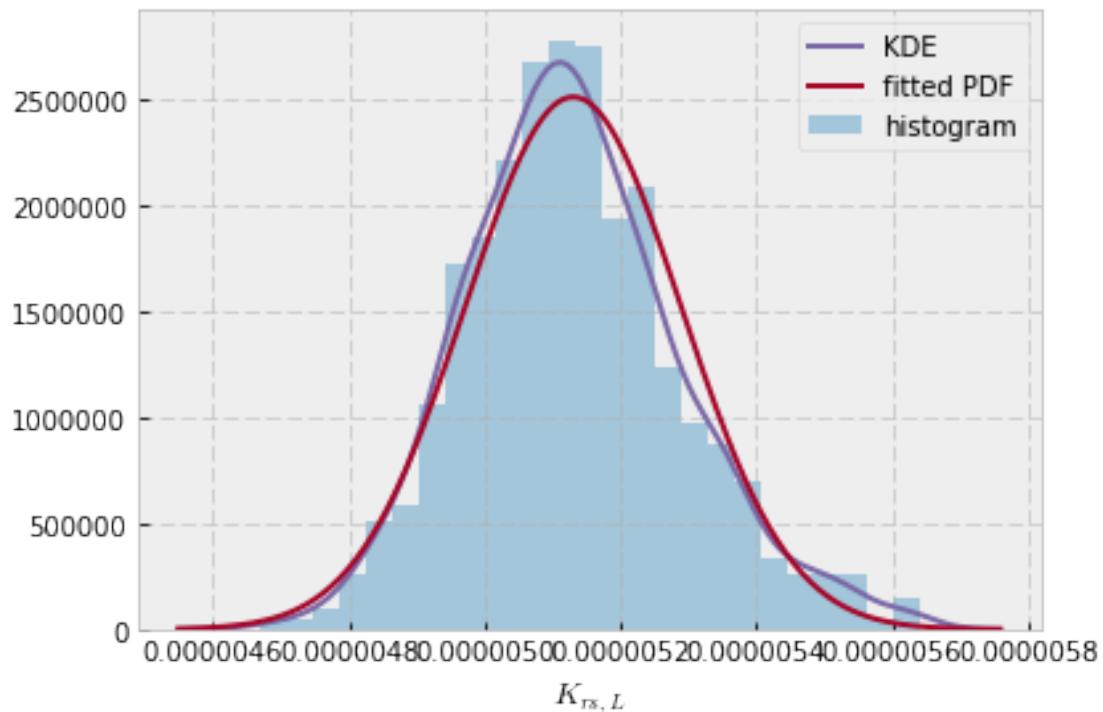
```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(),loc=5.129821726769263e-06, scale=1.5579976151788255e-07)
DescribeResult(nobs=1000, minmax=(4.7030445653875035e-06, 5.6962924983116942e-06)
               mean=5.1298217267692633e-06, variance=2.4297863552581657e-14,
               skewness=0.29555617214177055, kurtosis=0.18439505895697428)
```

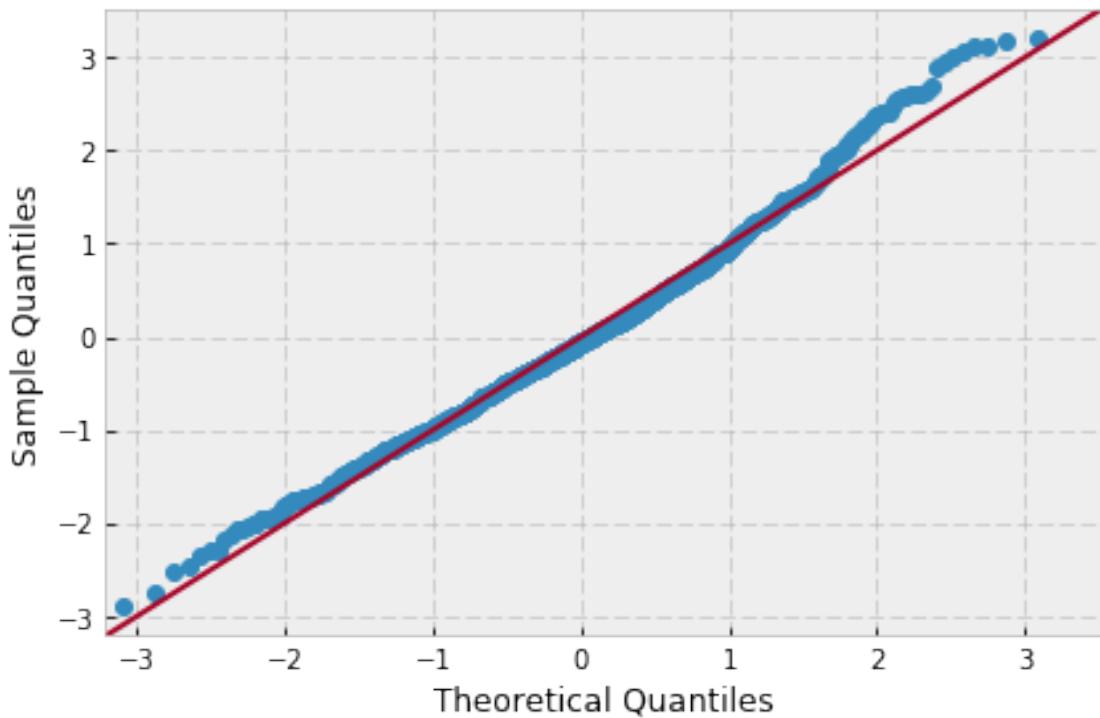


```

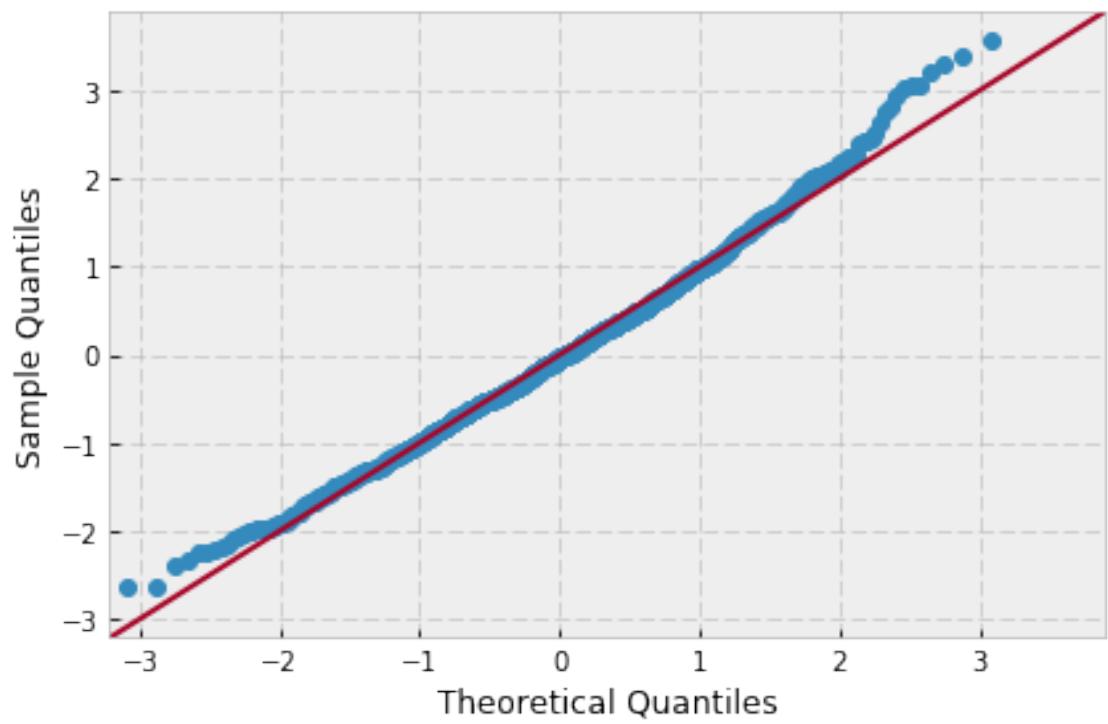
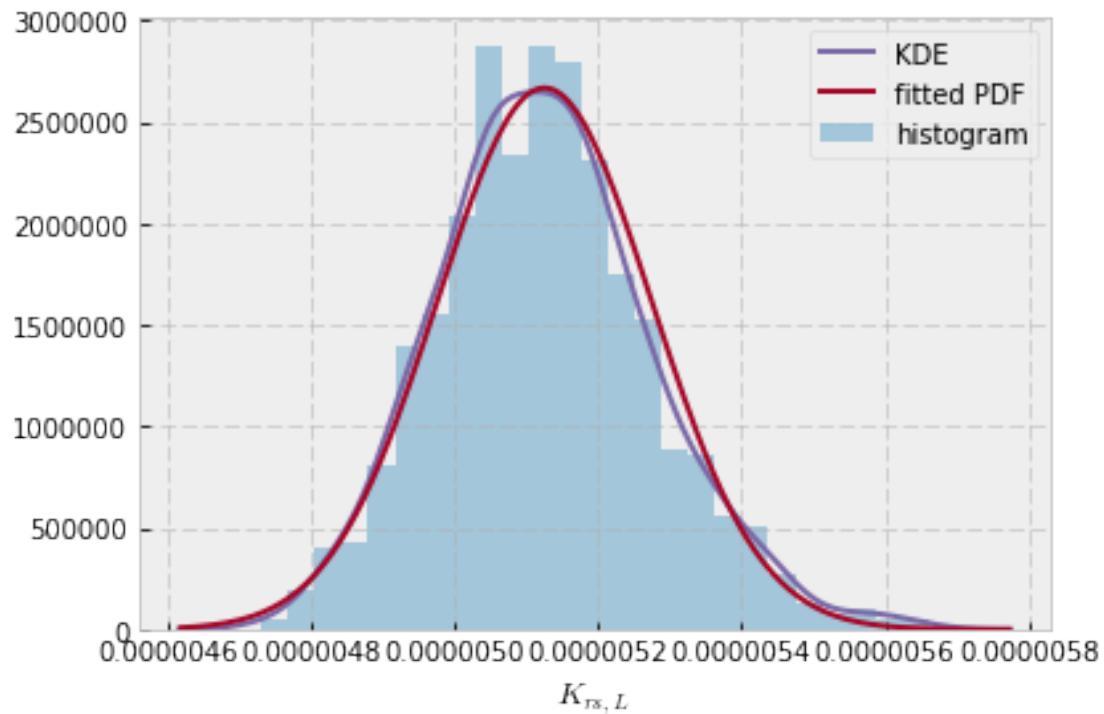
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=5.1304542872400925e-06, scale=1.589226034544062e-07)
DescribeResult(nobs=1000, minmax=(4.6688053481075507e-06, 5.6395460898000944e-06)
               mean=5.1304542872400925e-06, variance=2.5281675564290733e-14,
               skewness=0.38865831190492256, kurtosis=0.29942718238736266)

```





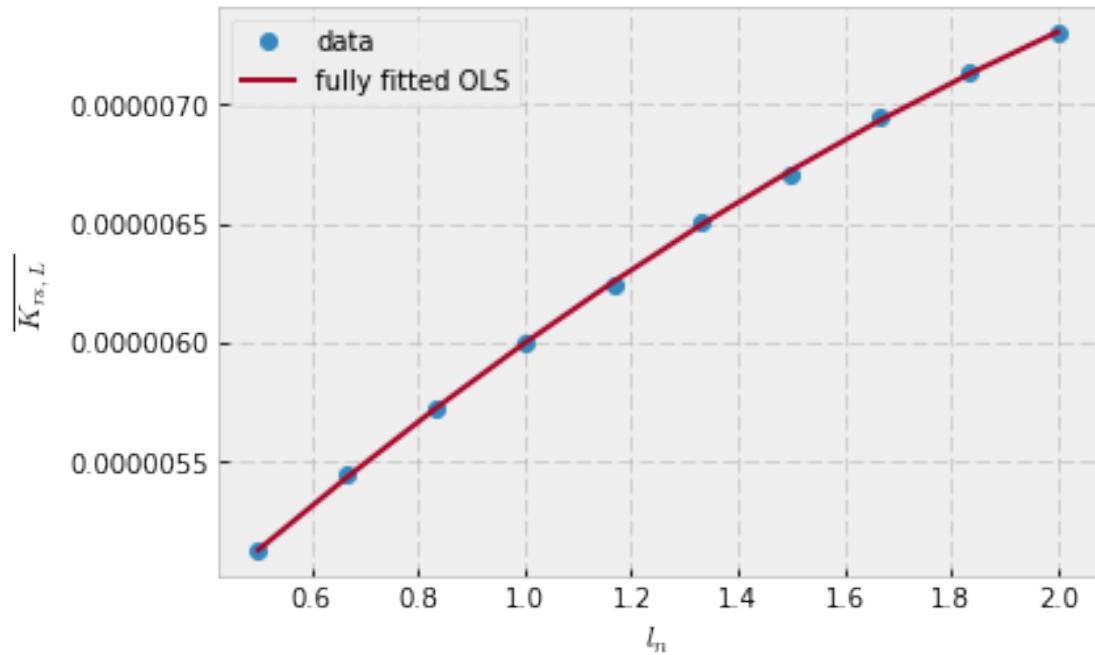
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(),loc=5.12509736112718e-06, scale=1.4944440798822444e-07)
DescribeResult(nobs=1000, minmax=(4.7305362499141158e-06, 5.6602373905725468e-06)
               mean=5.1250973611271798e-06, variance=2.2355987066016896e-14,
               skewness=0.349562472566311, kurtosis=0.25710592458299386)
```



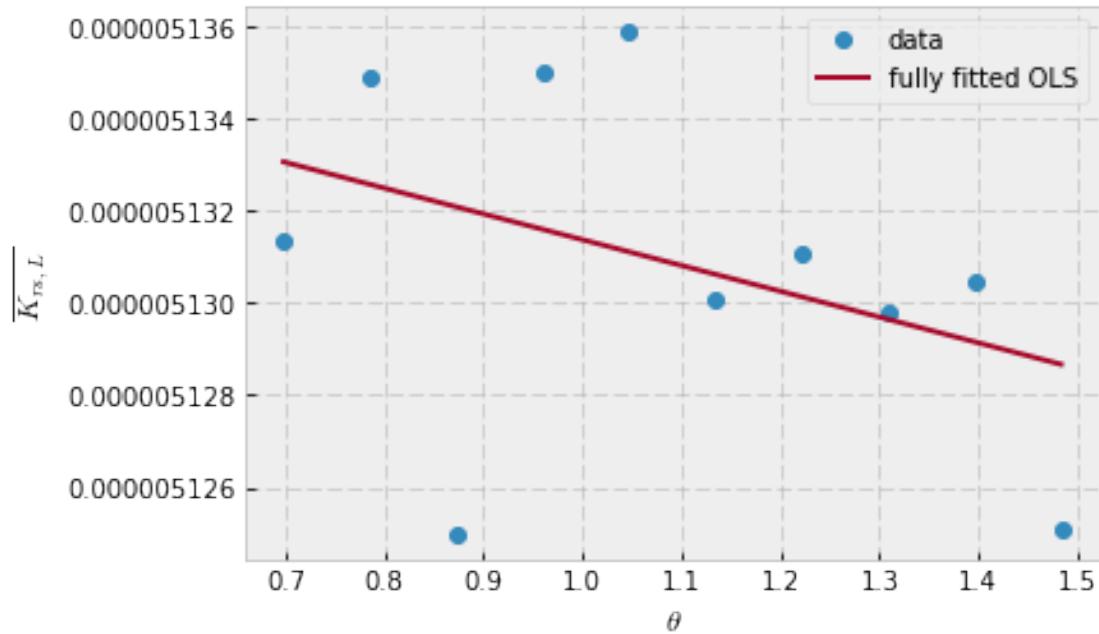
18.2 Parameter regression

```
In [42]: var_name = 'root_system_conductance_per_unit_of_root_length'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly1')  
        mean_reg('maxB', var_name, 'poly2')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly1')  
        mean_reg('N', var_name, 'poly1')
```

variable name: ln
model: poly2
r-squared: 0.9996480856782238
explained variance: 0.9996522062319211



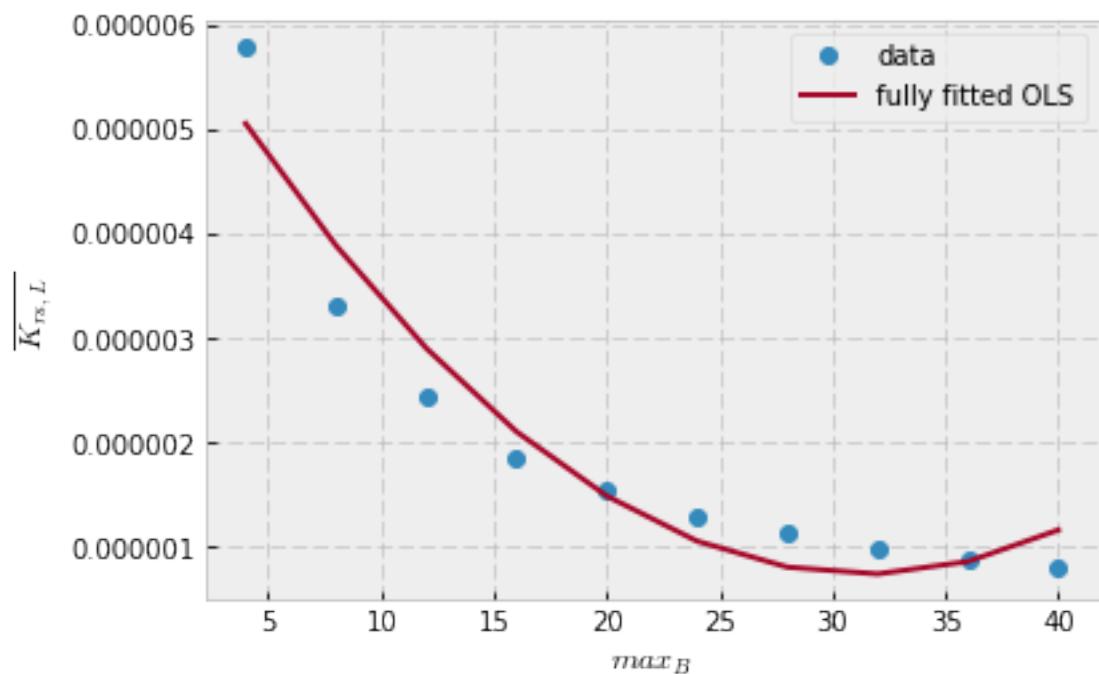
```
variable name: theta  
model: poly1  
r-squared: -0.3115671908309312  
explained variance: -0.30861803539480914
```



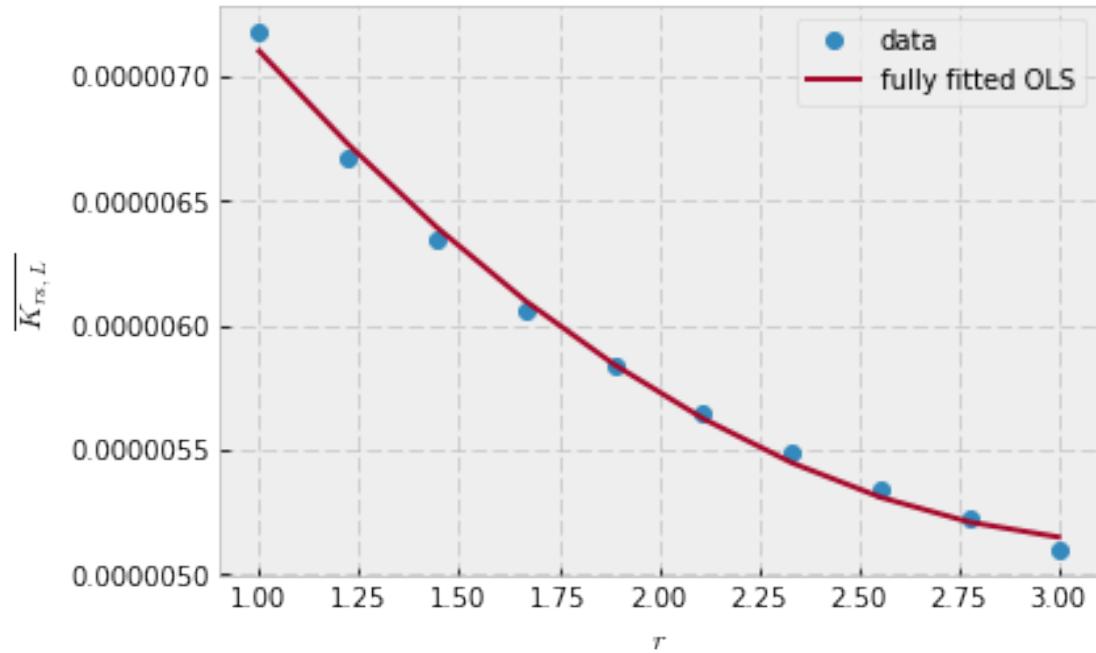
```

variable name: maxB
model: poly2
r-squared: 0.7159420665248539
explained variance: 0.7170075054160309

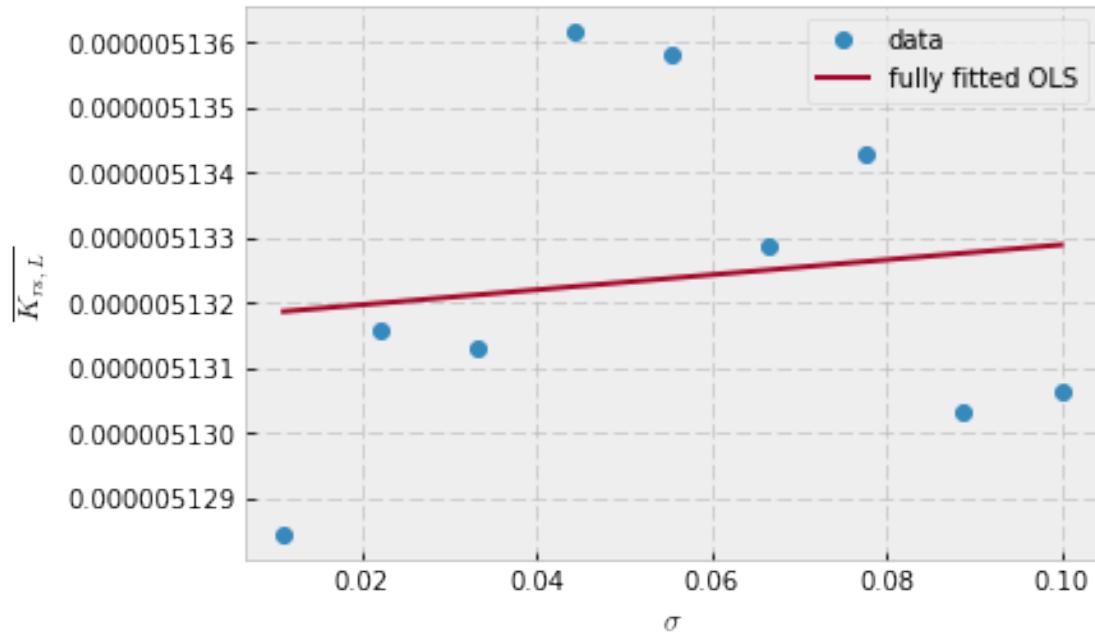
```



```
variable name: r
model: poly2
r-squared: 0.9795107219579947
explained variance: 0.979533425889127
```



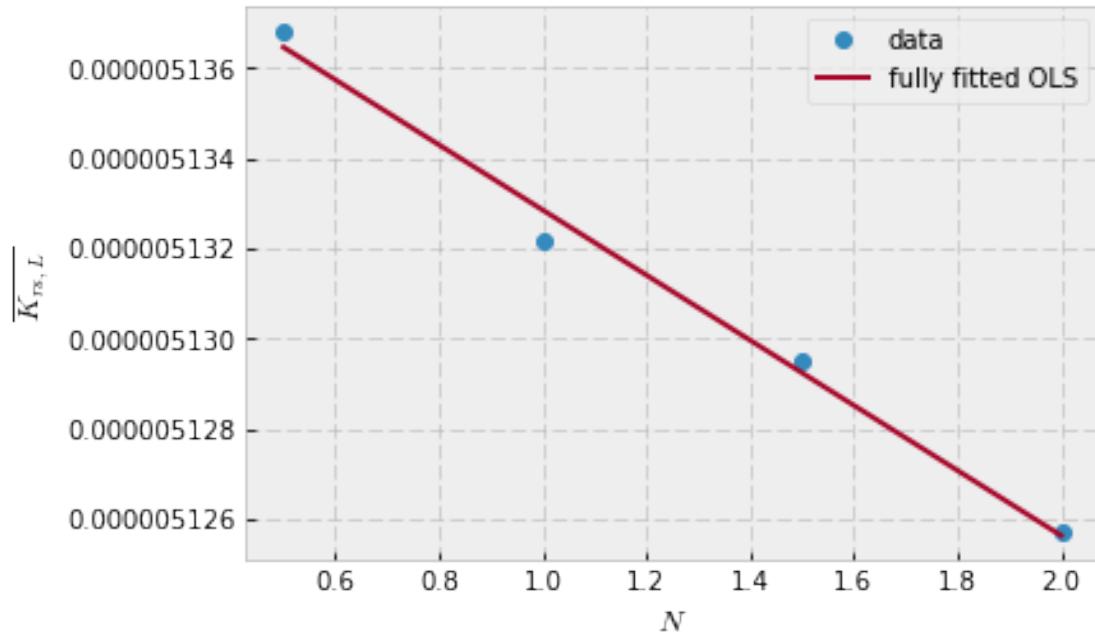
```
variable name: sigma
model: poly1
r-squared: -0.7416633648999937
explained variance: -0.7204438227514063
```



```

variable name: N
model: poly1
r-squared: 0.9610978225775526
explained variance: 0.9636086160001215

```

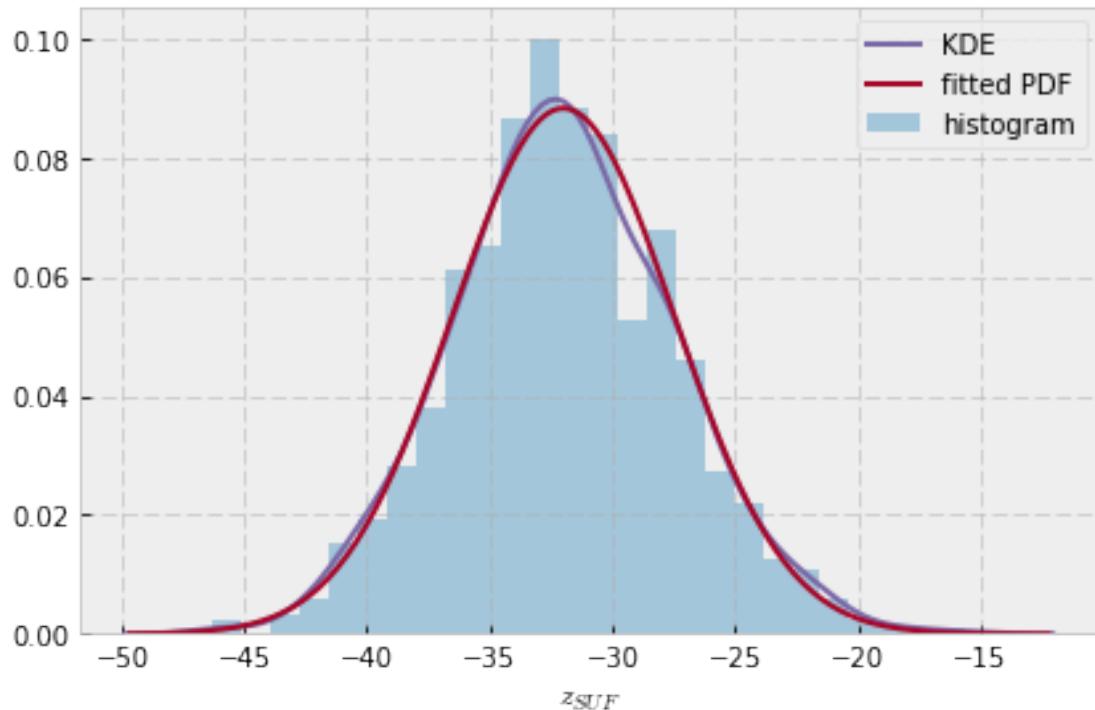


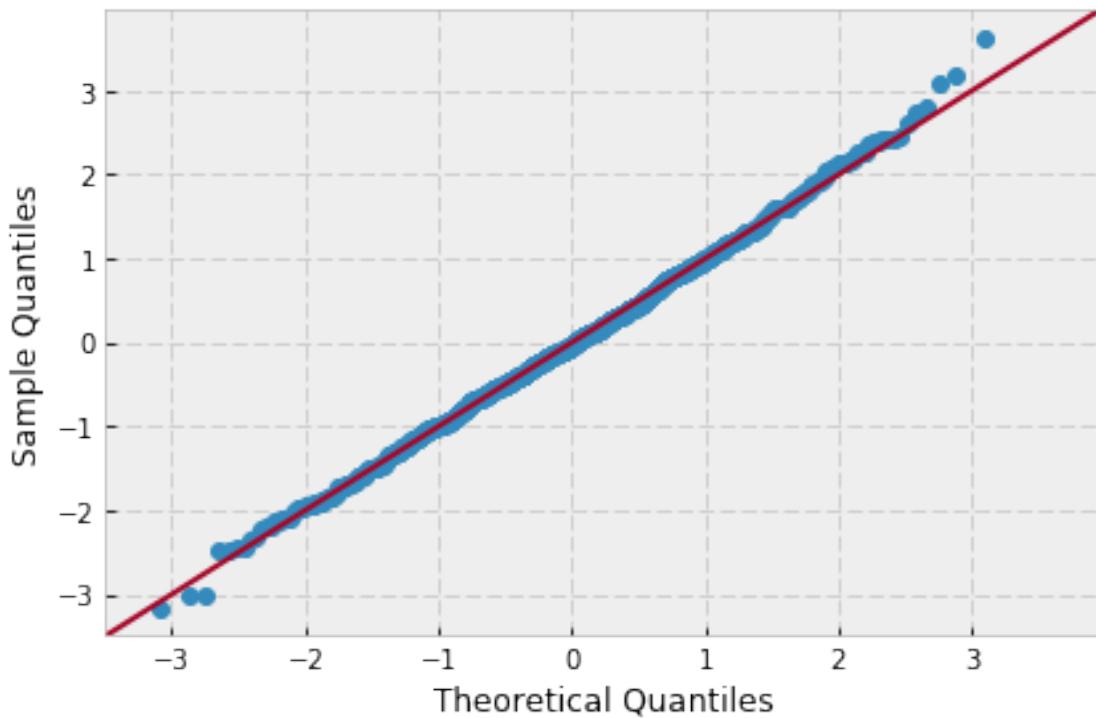
19 Mean depth of standard root water uptake

19.1 Probability distributions

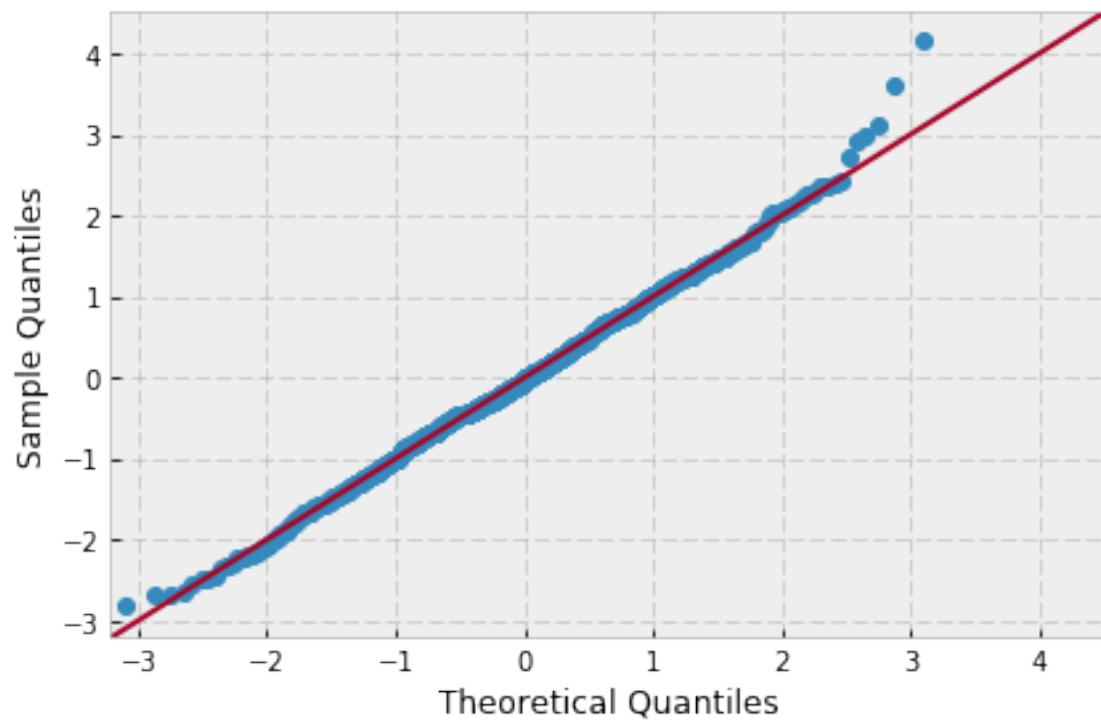
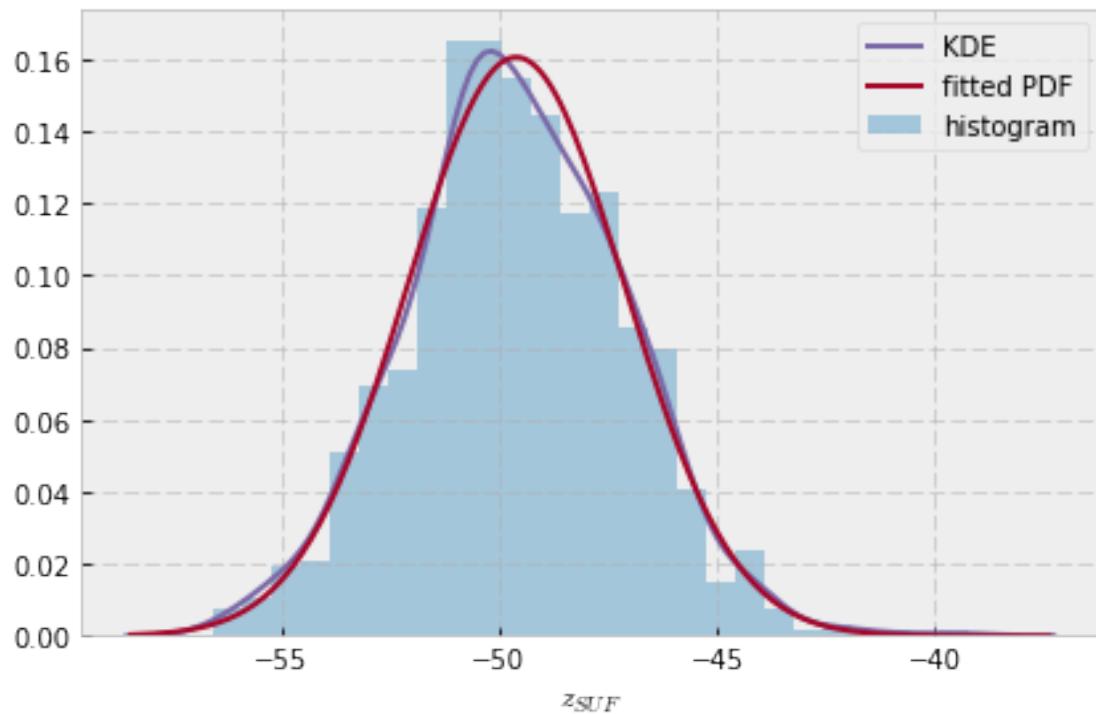
```
In [43]: plot_dist('mean_depth_of_standard_root_water_uptake')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=-32.00072324974, scale=4.518623399915355)
DescribeResult(nobs=1000, minmax=(-46.273955960000002, -15.63054546)
               mean=-32.000723249739998, variance=20.438395826088691,
               skewness=0.10158224881523667, kurtosis=0.11186527355733977)
```

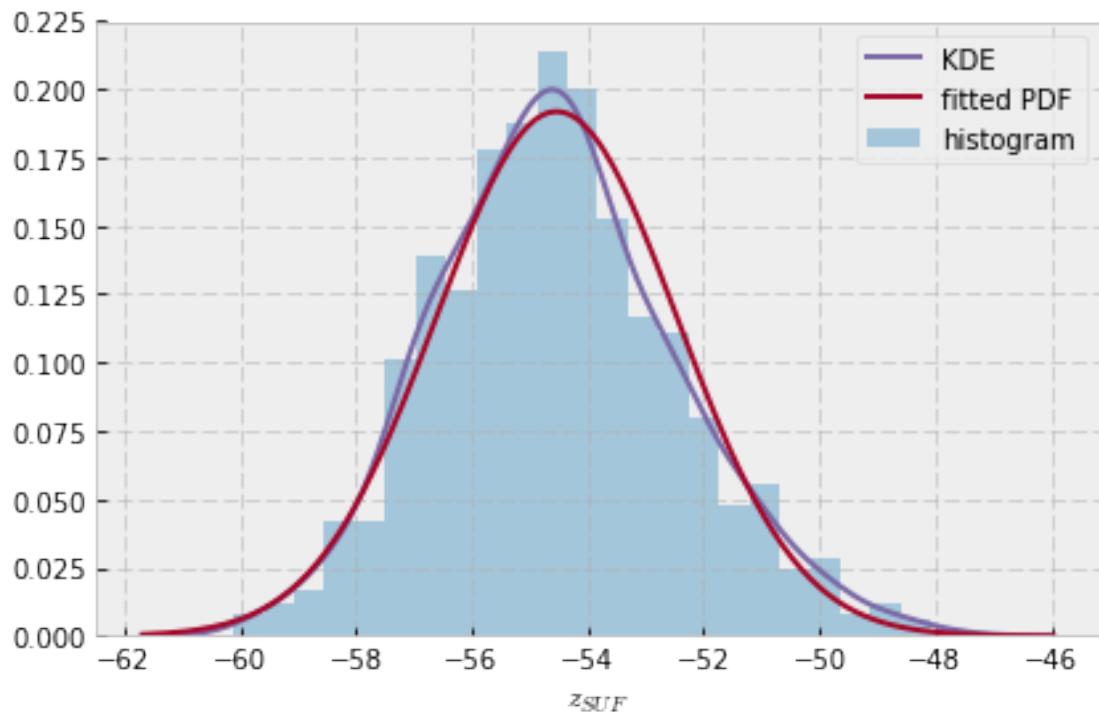


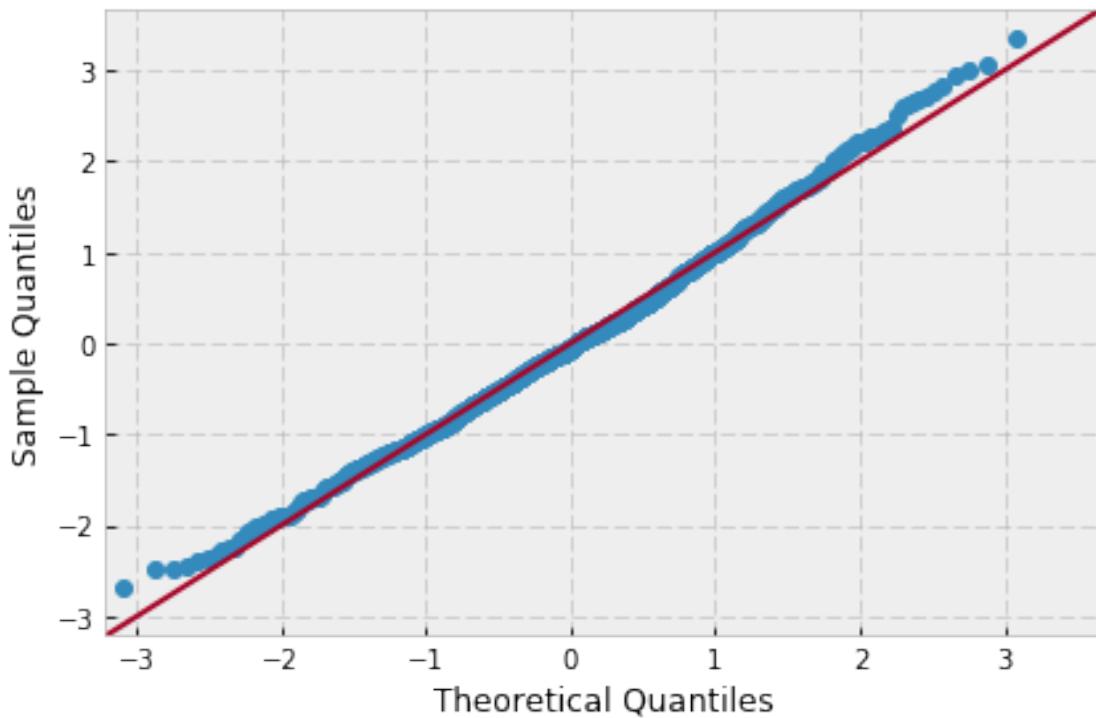


```
variable name: N
variable value: 1.0
distribution: normal(shape=(),loc=-49.60578726716, scale=2.4864198201780274)
DescribeResult(nobs=1000, minmax=(-56.55981414, -39.254587979999997)
               mean=-49.605787267159997, variance=6.1884719941683013,
               skewness=0.10446849274374957, kurtosis=0.23461207425816433)
```

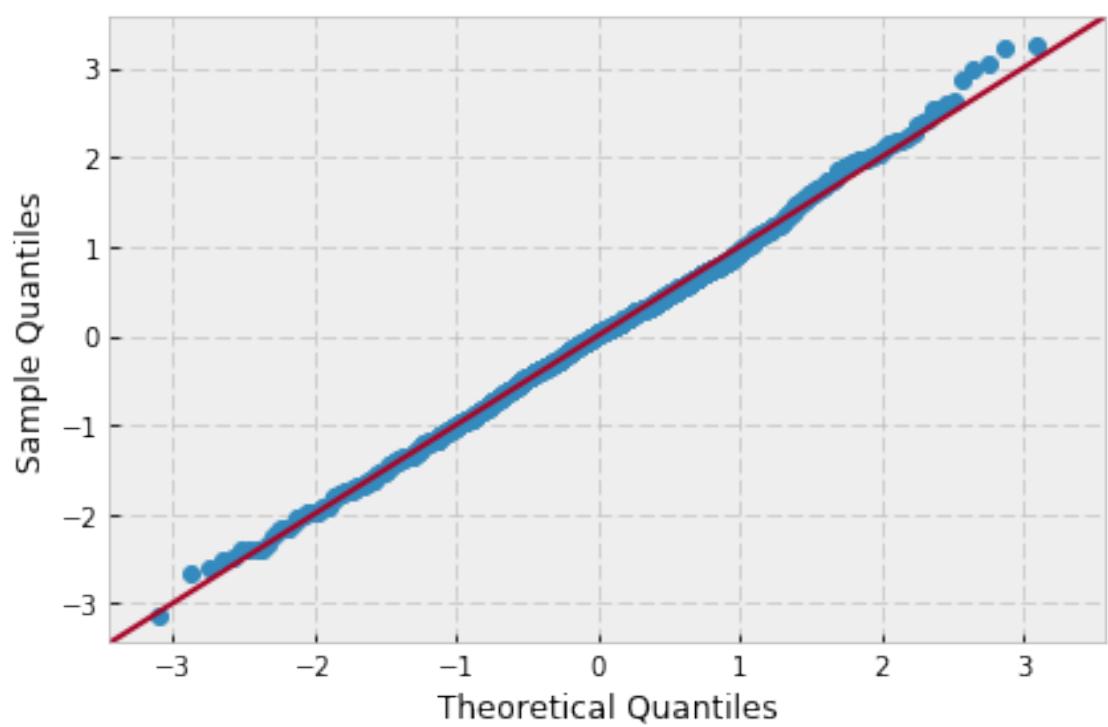
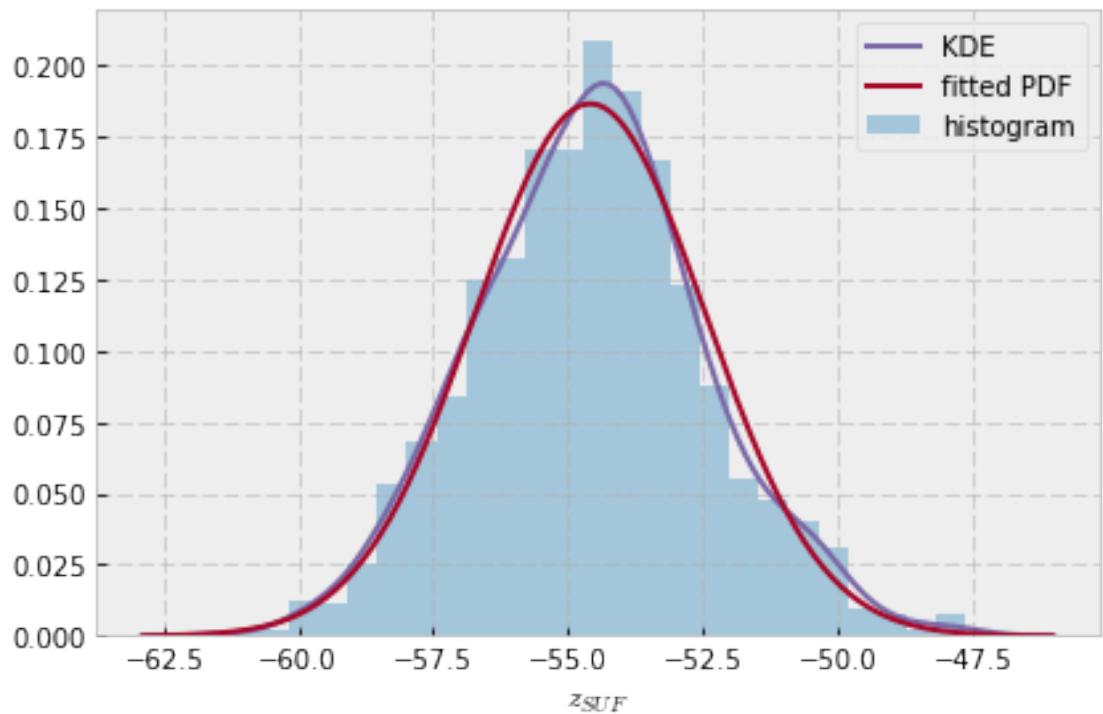


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=-54.533689614550006, scale=2.0808596736902807)
DescribeResult(nobs=1000, minmax=(-60.10773294999999, -47.54344291999997)
               mean=-54.533689614550006, variance=4.3343112928833047,
               skewness=0.2782190641752982, kurtosis=0.04147931191821819)
```

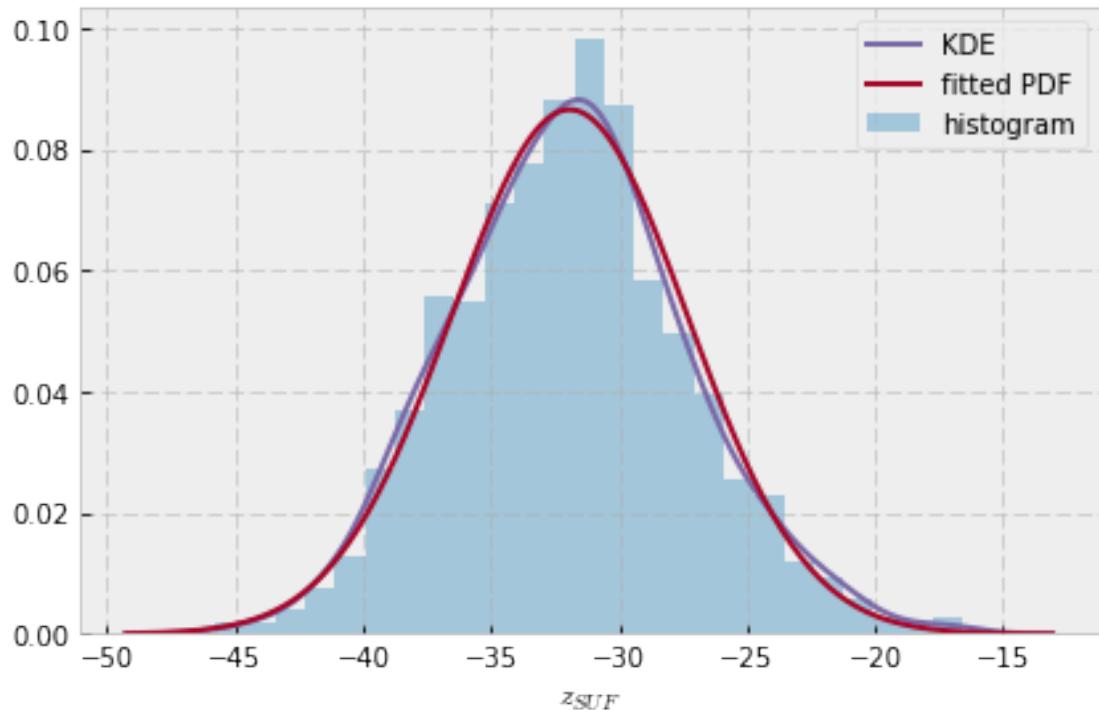


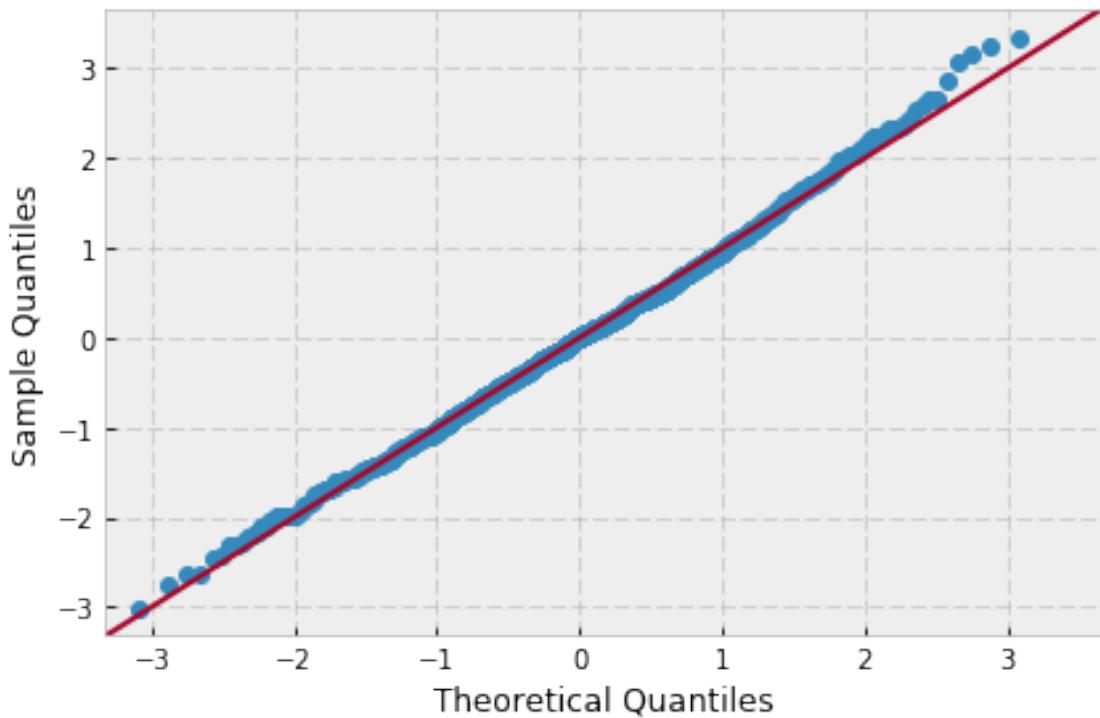


```
variable name: N
variable value: 2.0
distribution: normal(shape=(),loc=-54.587555135230005, scale=2.136542779228207)
DescribeResult(nobs=1000, minmax=(-61.261709430000003, -47.635762380000003)
               mean=-54.587555135230005, variance=4.569384431904095,
               skewness=0.1176381426086149, kurtosis=0.07998417126262547)
```

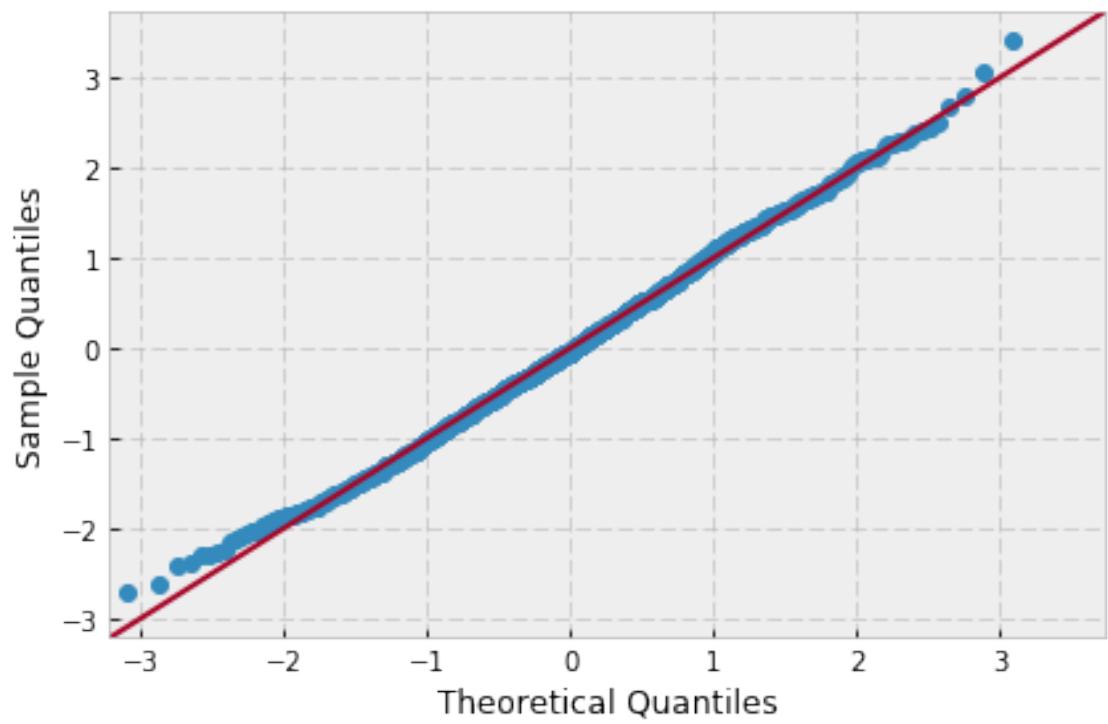
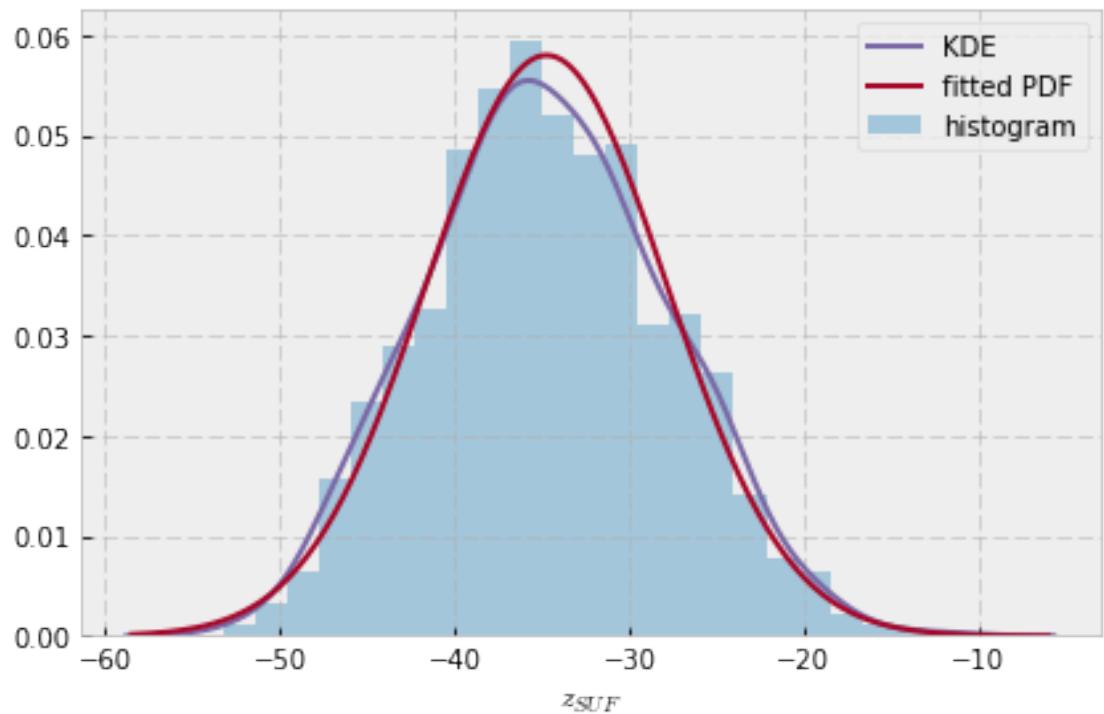


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=-31.93716019632, scale=4.610519394507342)
DescribeResult(nobs=1000, minmax=(-45.788869560000002, -16.563757160000002)
               mean=-31.937160196320001, variance=21.278167254382733,
               skewness=0.17063086601226113, kurtosis=0.08568222223616662)
```

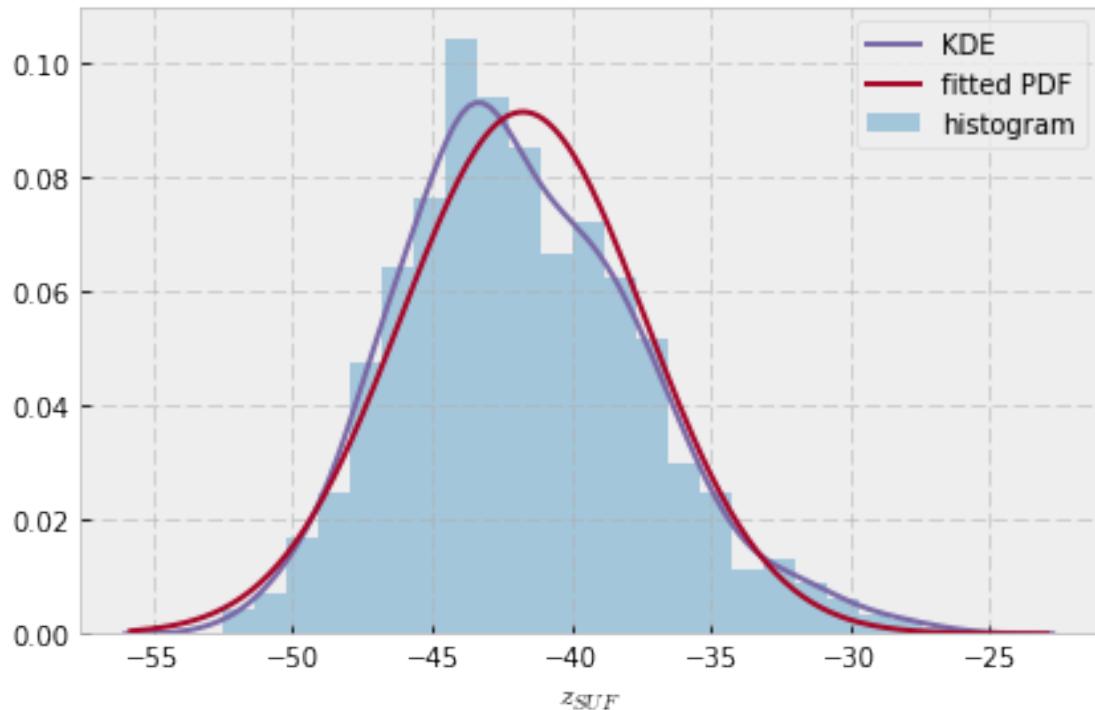


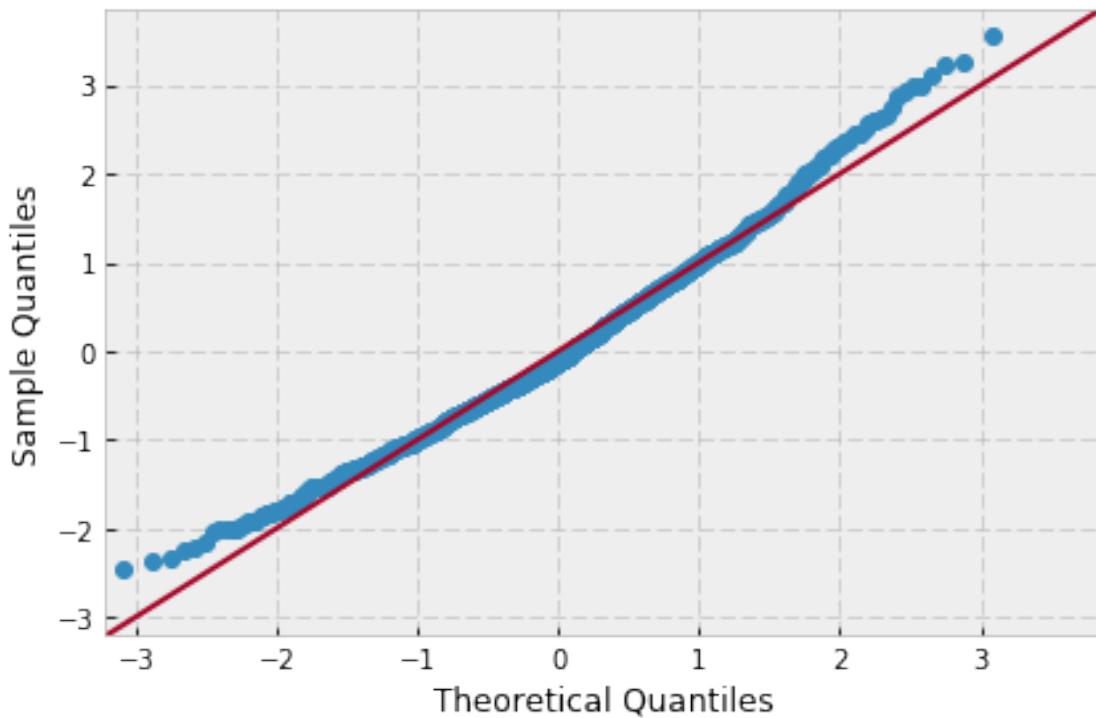


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(),loc=-34.72887064347999, scale=6.882555158308726)
DescribeResult(nobs=1000, minmax=(-53.29629284, -11.16789039999999)
               mean=-34.72887064347993, variance=47.416982489651701,
               skewness=0.11495401733947026, kurtosis=-0.27991302437274435)
```

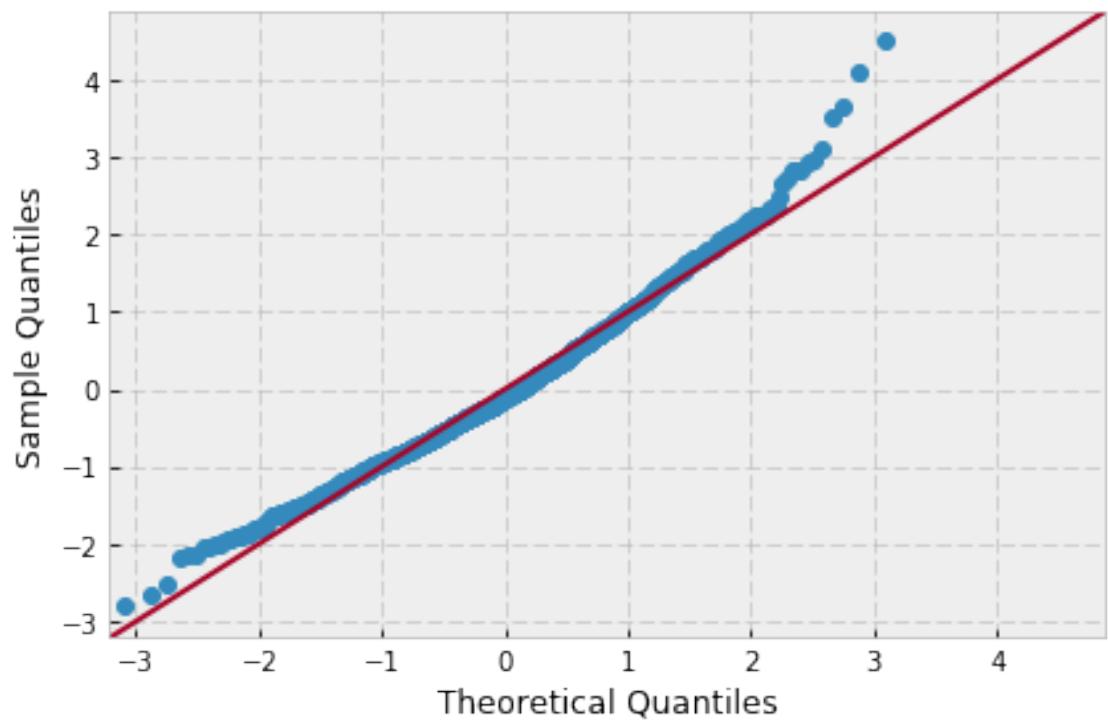
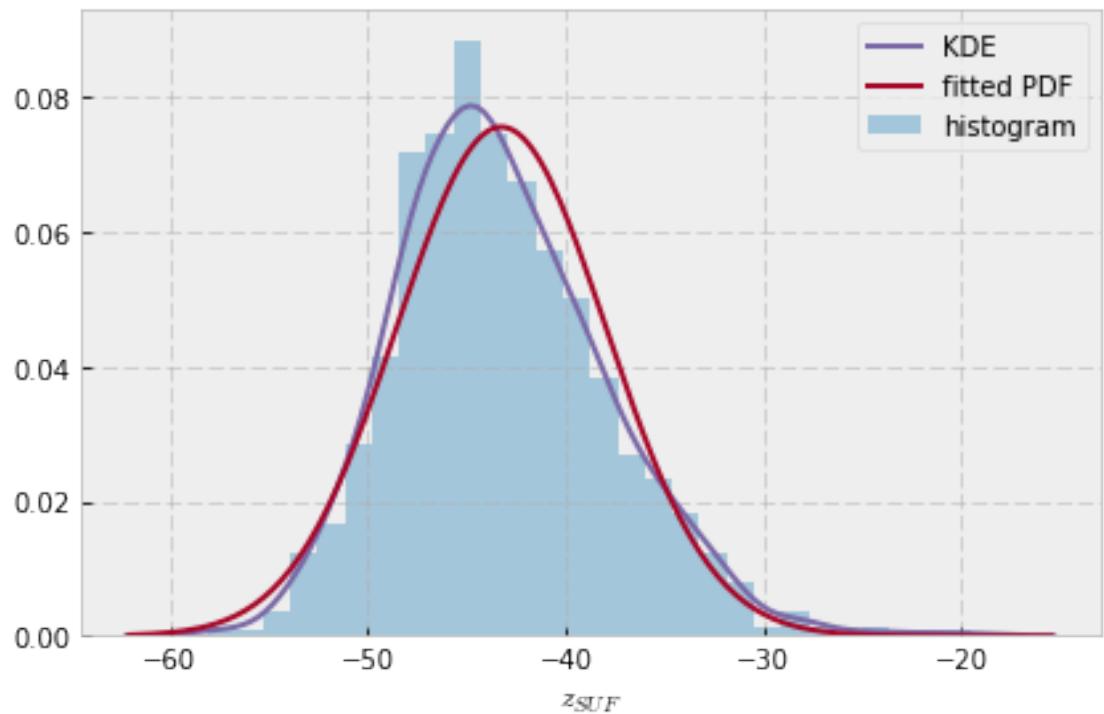


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=-41.73650043030005, scale=4.371467743558859)
DescribeResult(nobs=1000, minmax=(-52.49815478, -26.234480510000001)
               mean=-41.73650043030005, variance=19.128859092067646,
               skewness=0.44860559797561284, kurtosis=0.15056641447328278)
```

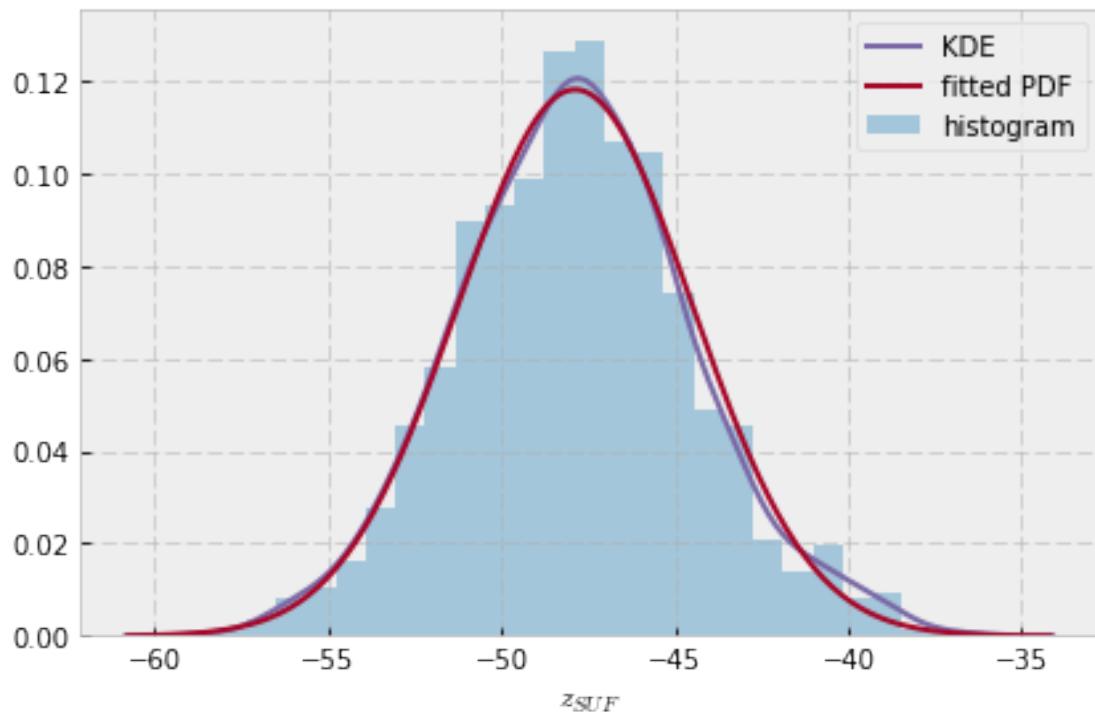


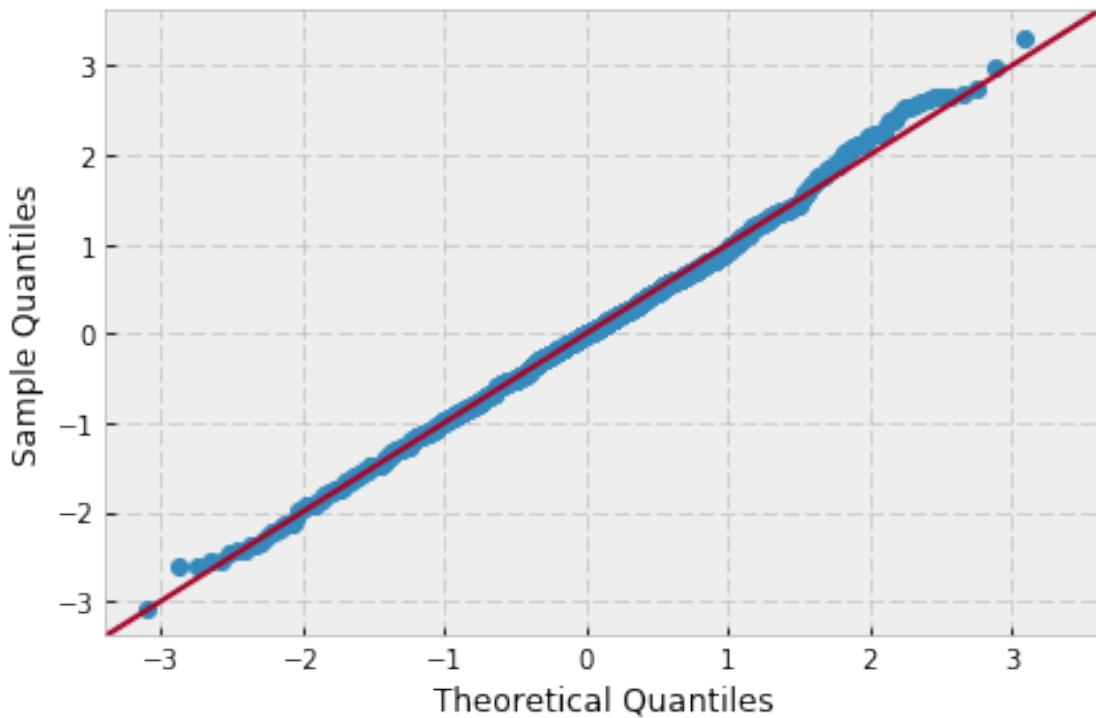


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(),loc=-43.245710245059996, scale=5.277577925815016)
DescribeResult(nobs=1000, minmax=(-58.056874520000001, -19.471868440000001)
               mean=-43.245710245059996, variance=27.880709472522451,
               skewness=0.539240595787454, kurtosis=0.6387154819401646)
```

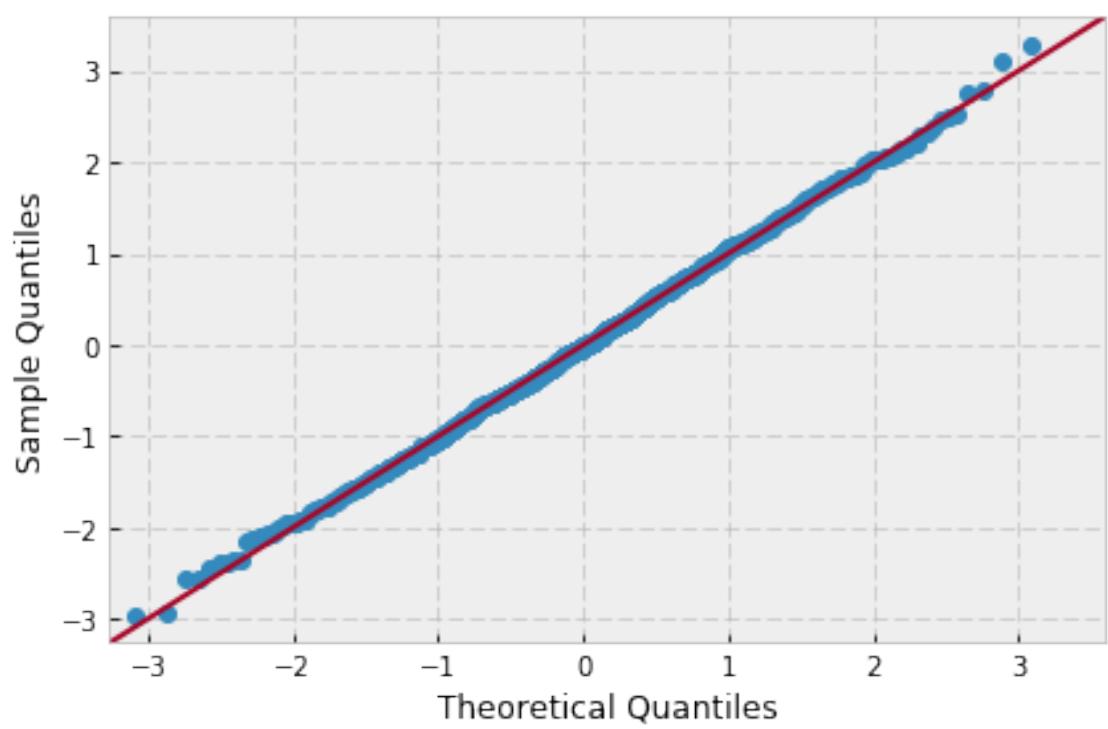
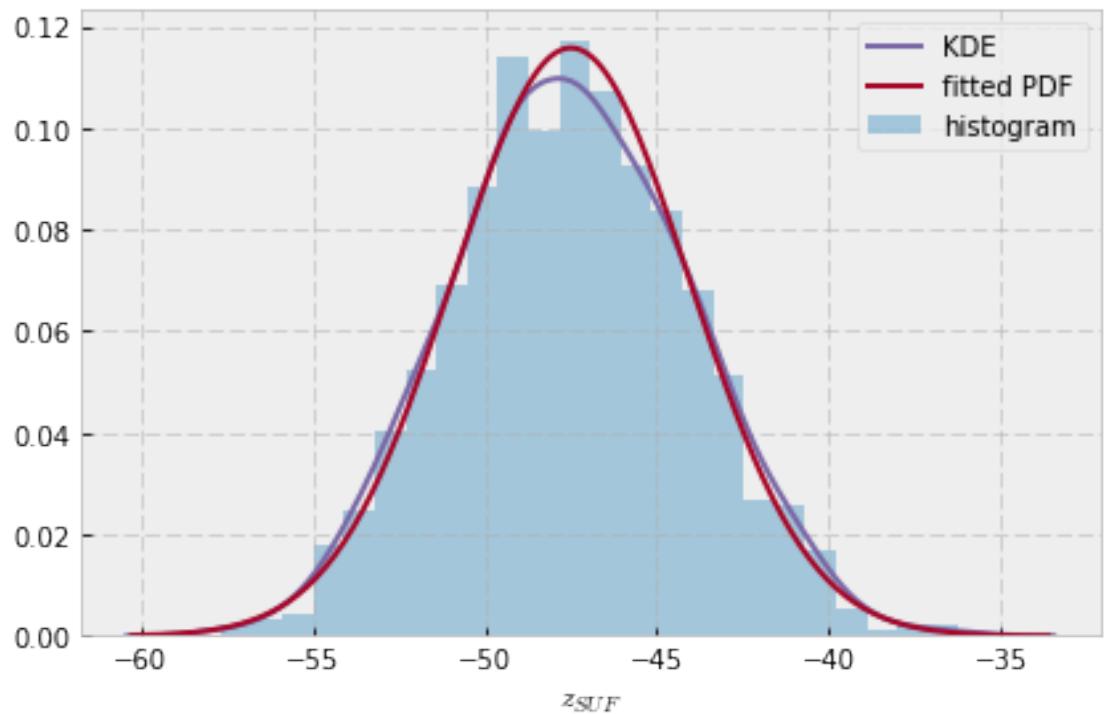


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=-47.87308997536999, scale=3.3738595648496537)
DescribeResult(nobs=1000, minmax=(-58.224323920000003, -36.715280100000001)
               mean=-47.873089975369993, variance=11.394322686013506,
               skewness=0.13279252634302258, kurtosis=0.11371011998257208)
```

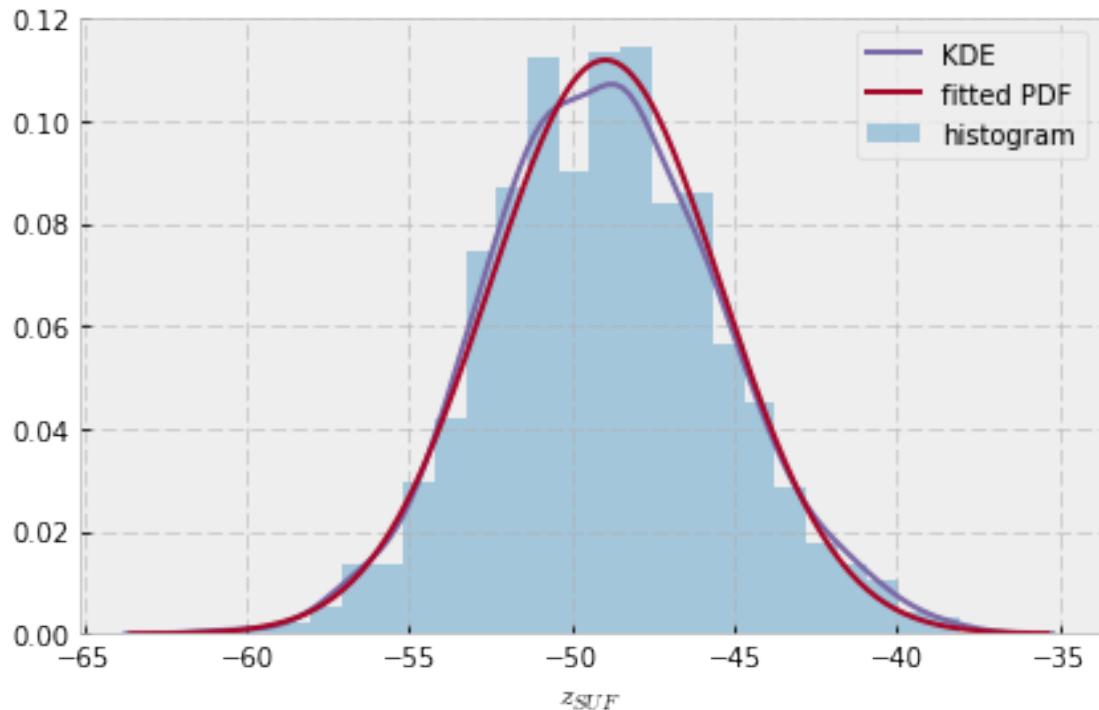


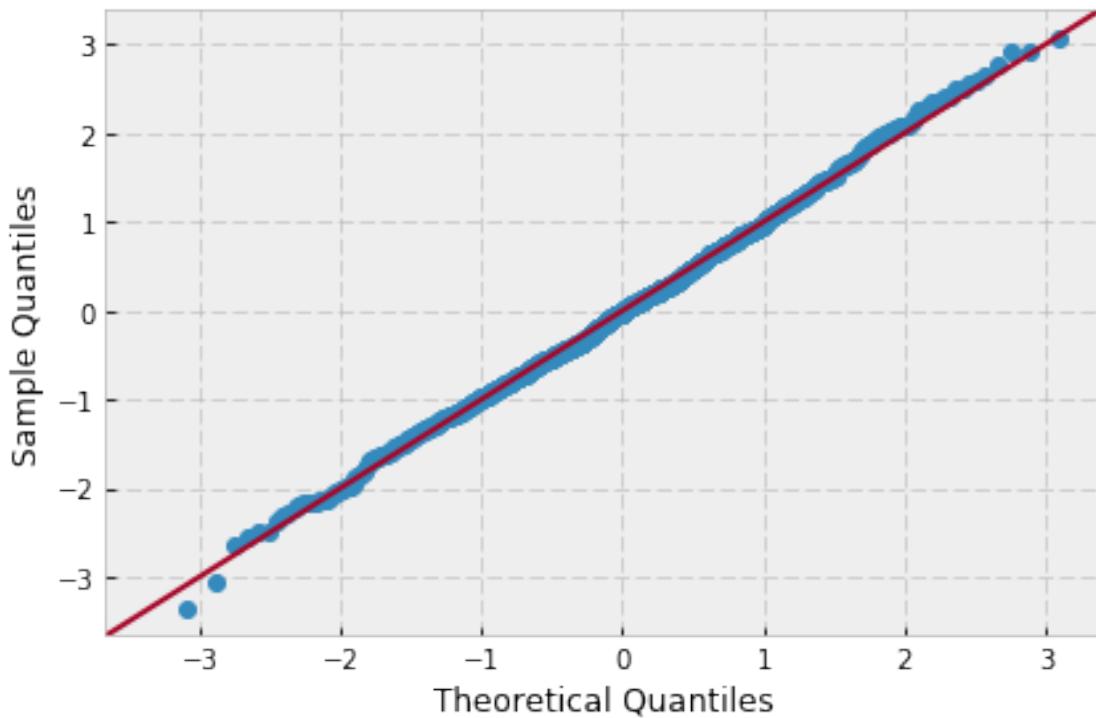


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(),loc=-47.49725687814, scale=3.445179705631458)
DescribeResult(nobs=1000, minmax=(-57.66962080999998, -36.203440180000001)
               mean=-47.49725687814, variance=11.881144348443305,
               skewness=0.05923717611460461, kurtosis=-0.19124822558917032)
```

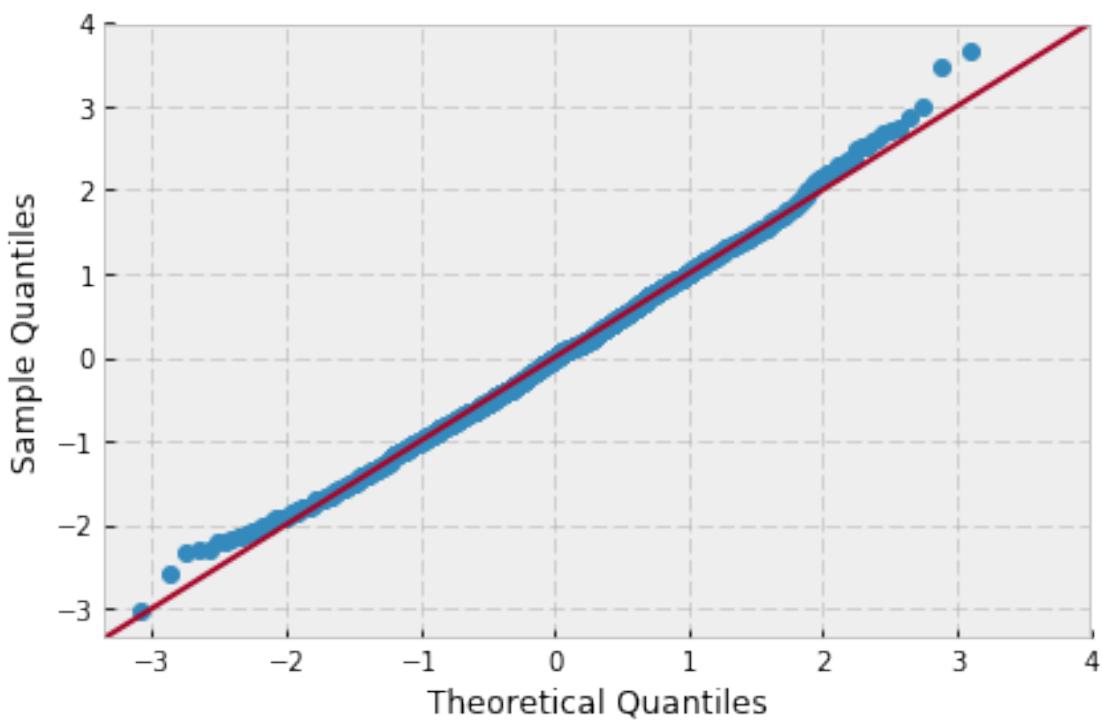
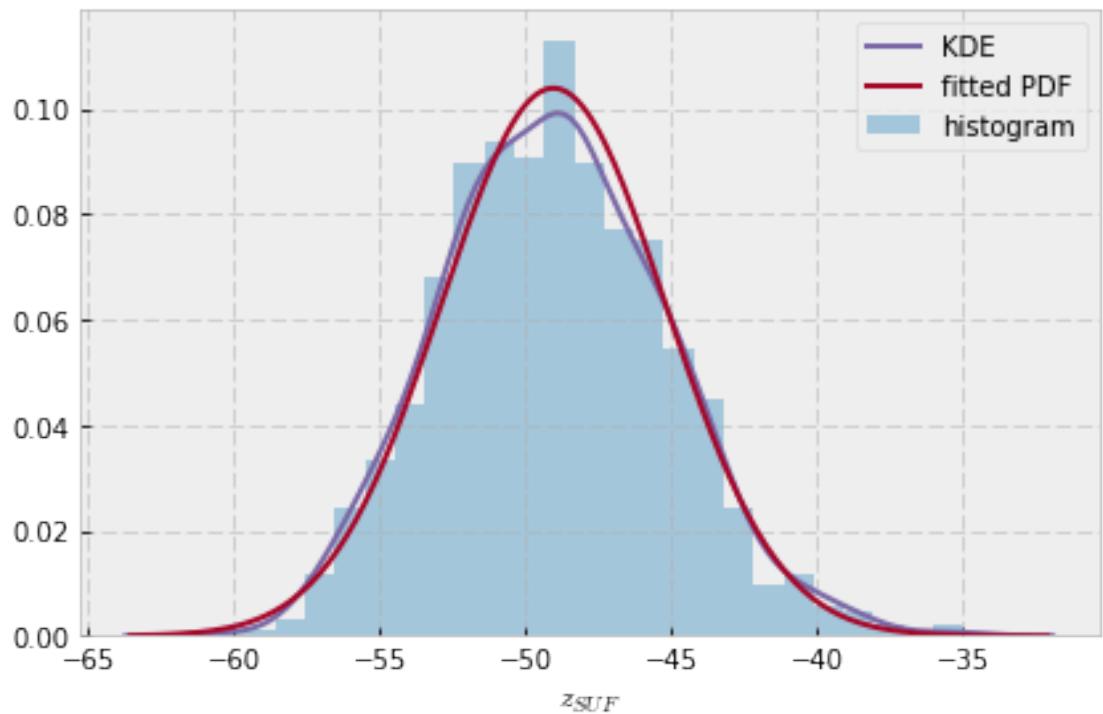


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=-48.969339005820004, scale=3.5639012788796207)
DescribeResult(nobs=1000, minmax=(-60.883241429999998, -38.042538489999998)
               mean=-48.969339005820004, variance=12.714106432031828,
               skewness=0.11699487956722274, kurtosis=0.0013337491706013616)
```

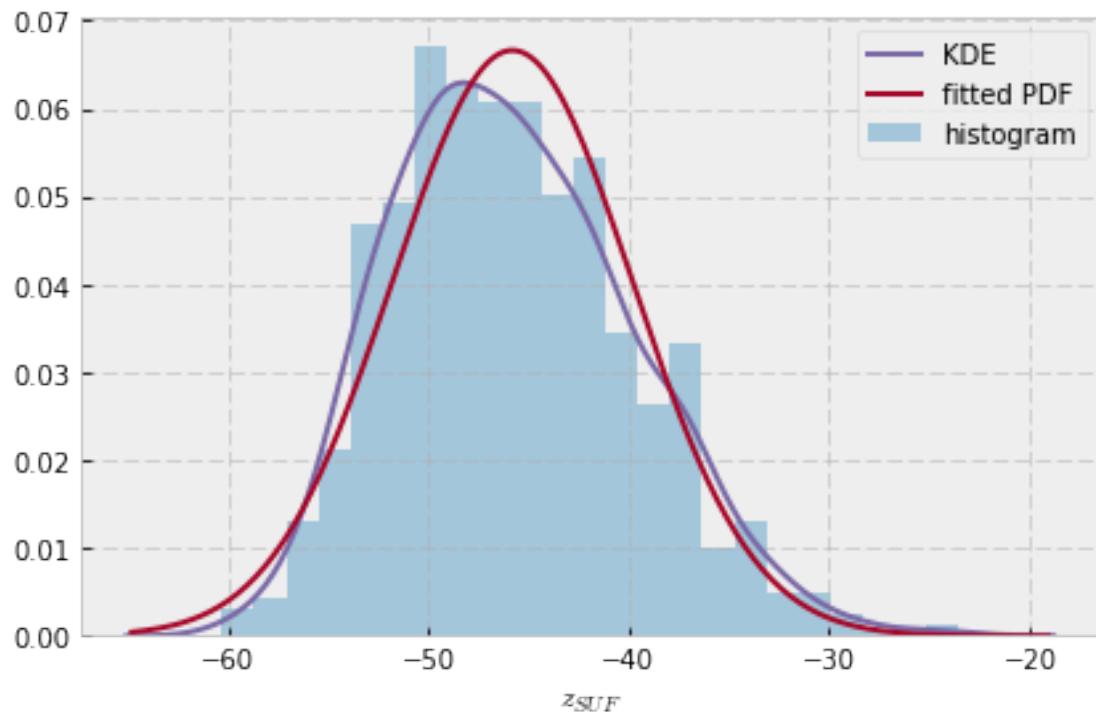


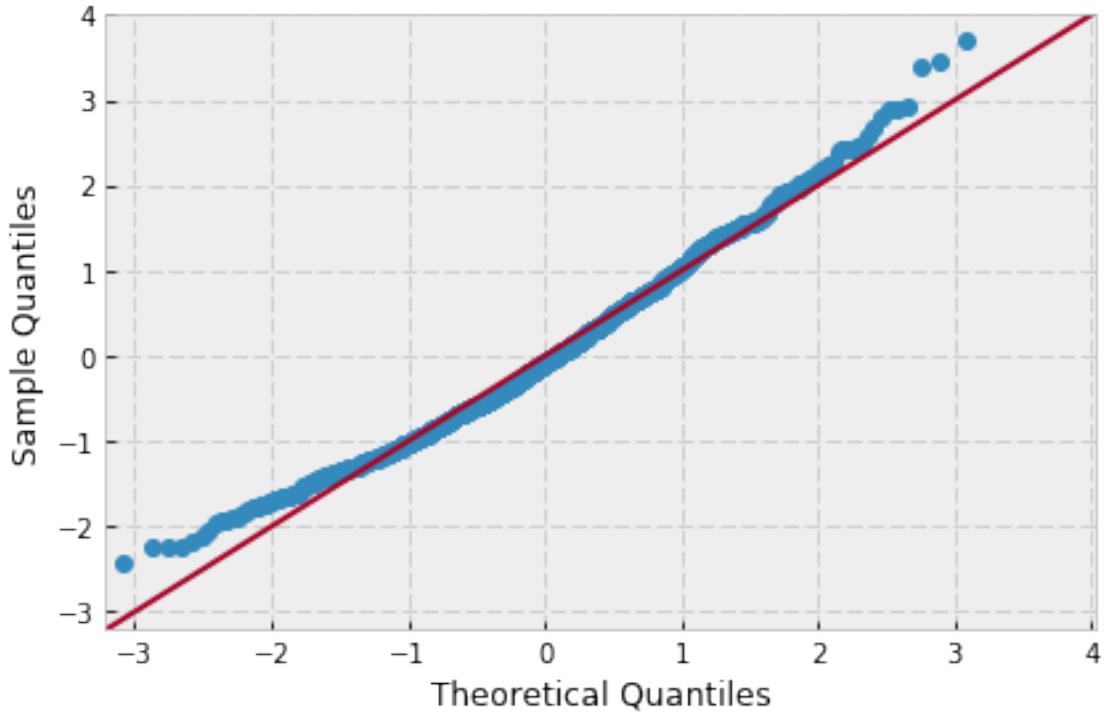


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=-49.03673807033, scale=3.8382004515674044)
DescribeResult(nobs=1000, minmax=(-60.63422706000001, -34.99596534999999)
               mean=-49.03673807033003, variance=14.746529235647875,
               skewness=0.20227227936376874, kurtosis=-0.0038831961380543234)
```

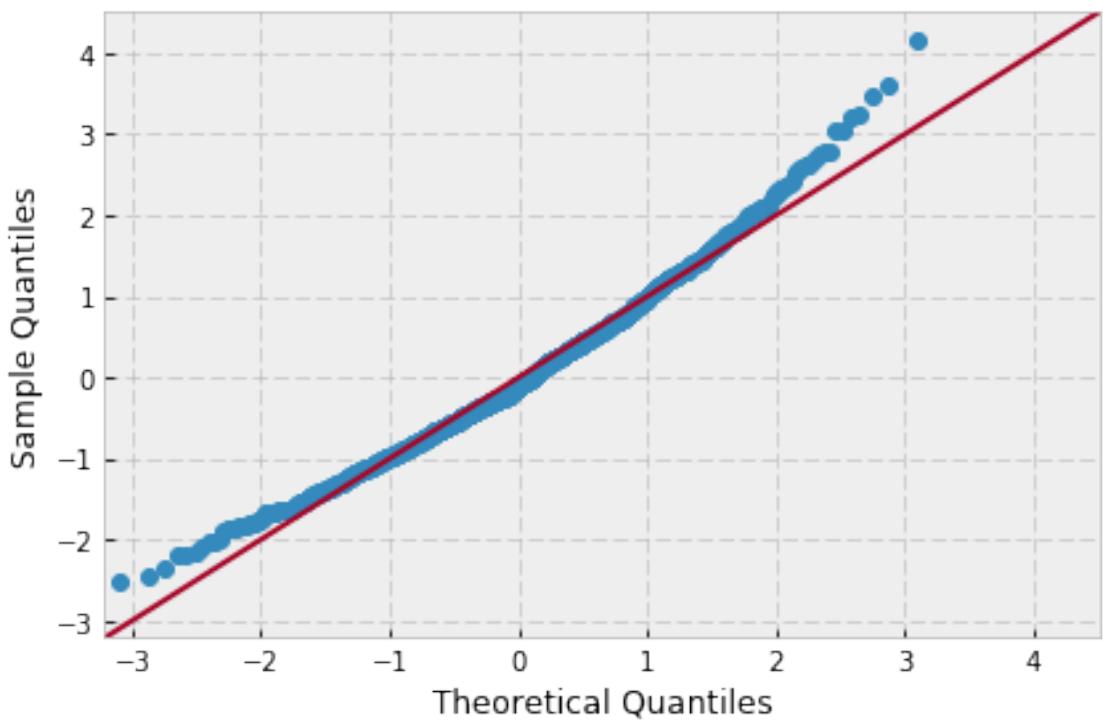
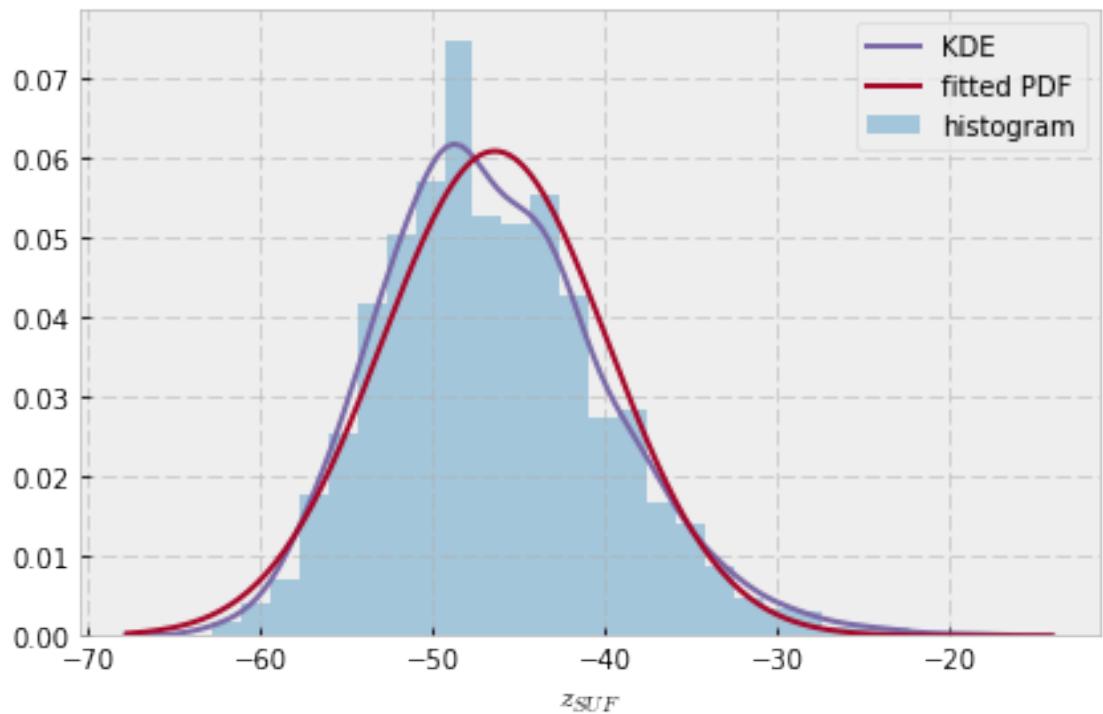


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=-45.81068216829001, scale=5.986318499988607)
DescribeResult(nobs=1000, minmax=(-60.304076940000002, -23.562845249999999)
               mean=-45.810682168290008, variance=35.871881064370214,
               skewness=0.4244259645129395, kurtosis=-0.05613301806157622)
```

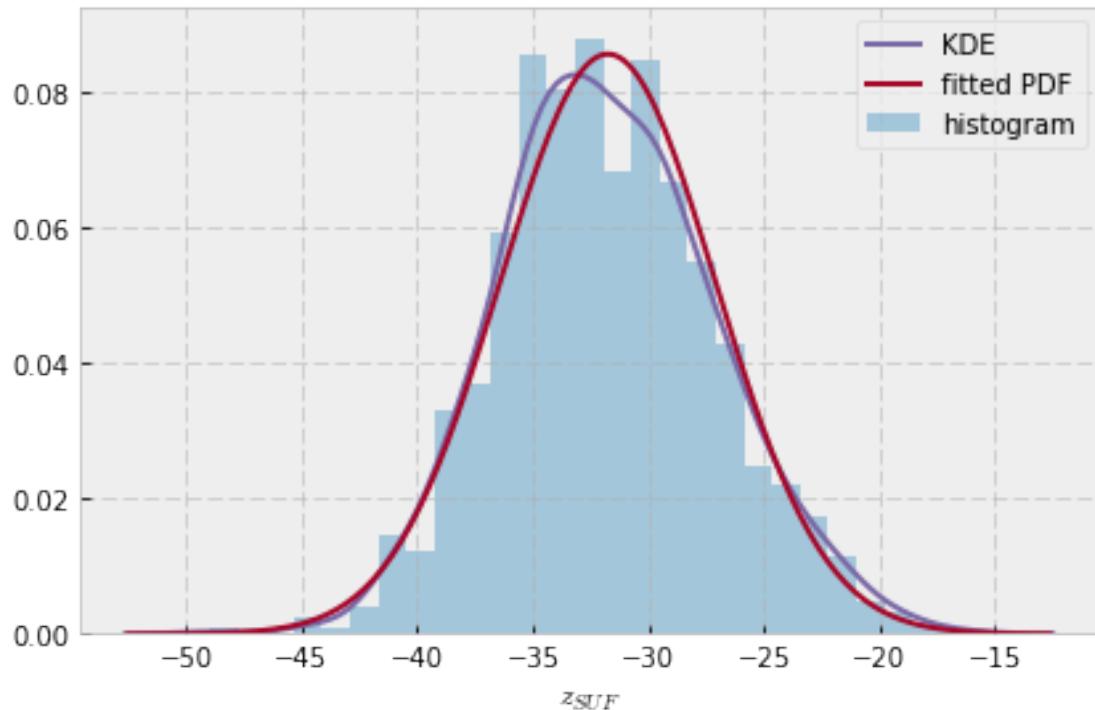


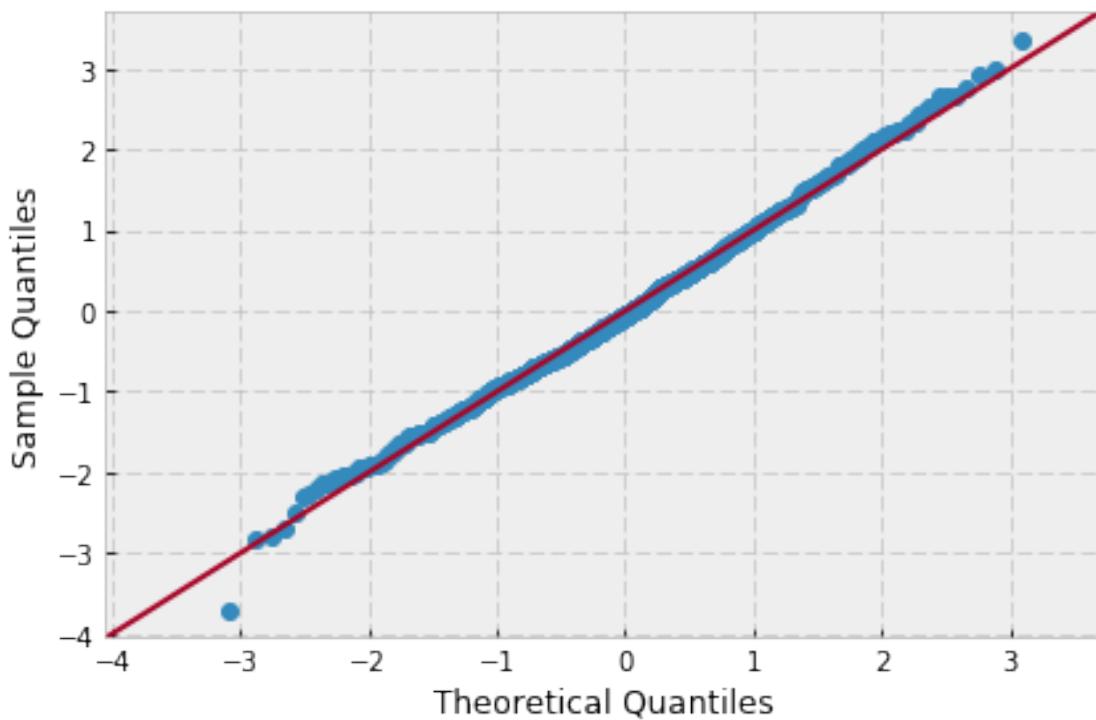


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(),loc=-46.38260209806, scale=6.545446934621923)
DescribeResult(nobs=1000, minmax=(-62.78255621999999, -19.06683759999999)
               mean=-46.382602098059998, variance=42.885761335286816,
               skewness=0.5102803381687228, kurtosis=0.4005693390762337)
```

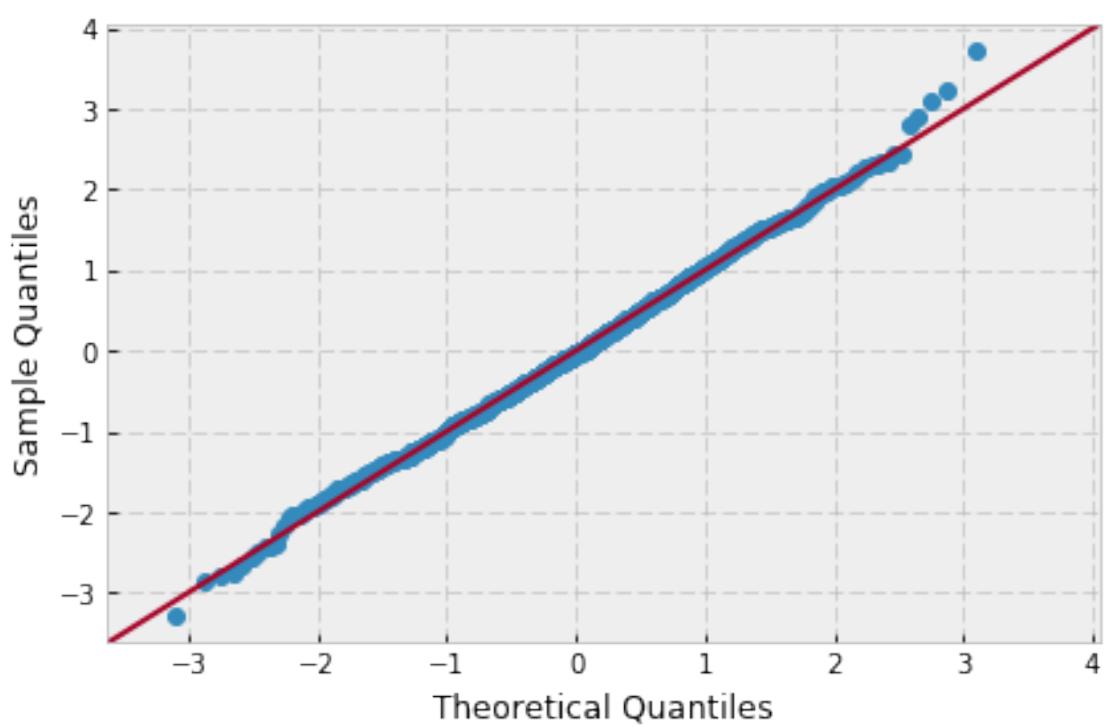
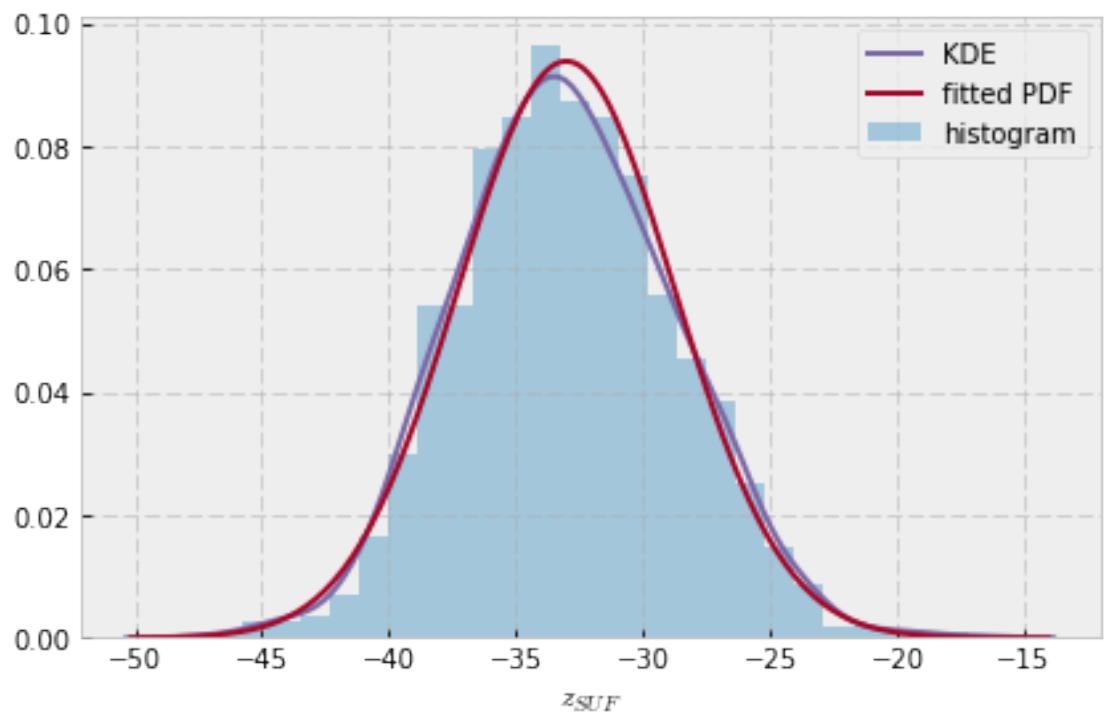


```
variable name: maxB
variable value: 4.0
distribution: normal(shape=(), loc=-31.75389260831, scale=4.666240187129613)
DescribeResult(nobs=1000, minmax=(-49.001878720000001, -16.13233889)
               mean=-31.75389260831, variance=21.795593077060474,
               skewness=0.17233270968561915, kurtosis=0.04339168414167549)
```

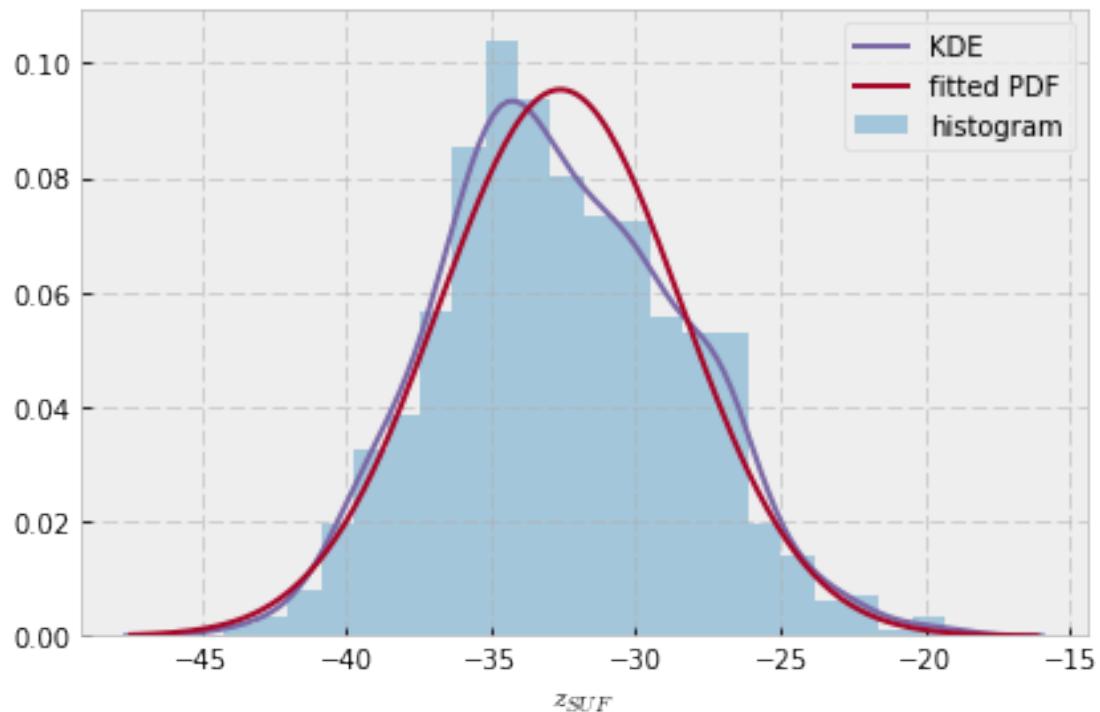


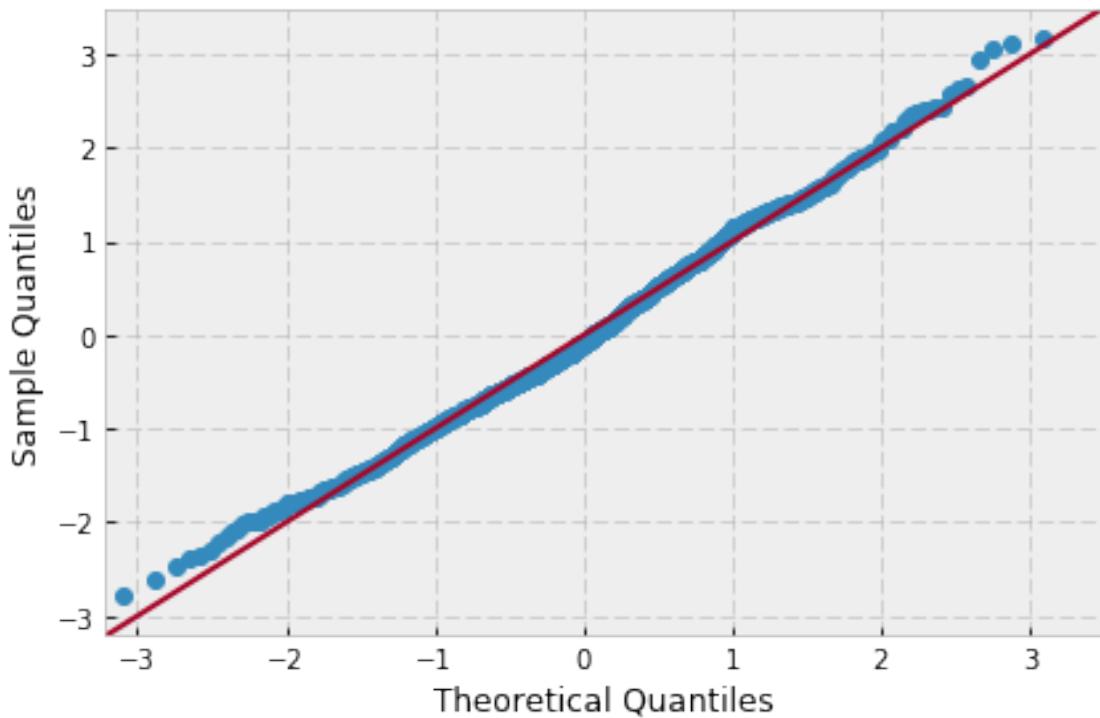


```
variable name: maxB
variable value: 8.0
distribution: normal(shape=(),loc=-33.01969339527, scale=4.25332684615452)
DescribeResult(nobs=1000, minmax=(-46.94122398999997, -17.22502179)
               mean=-33.019693395269996, variance=18.108898158377134,
               skewness=0.11742051570916107, kurtosis=0.01962087871371354)
```

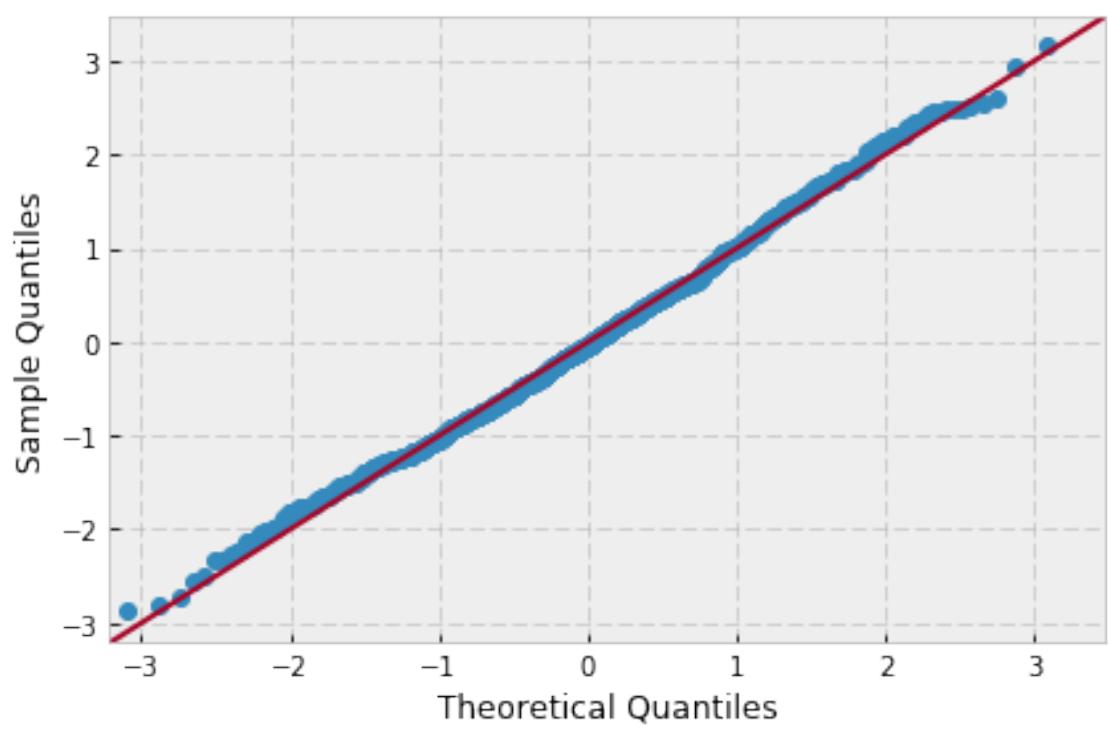
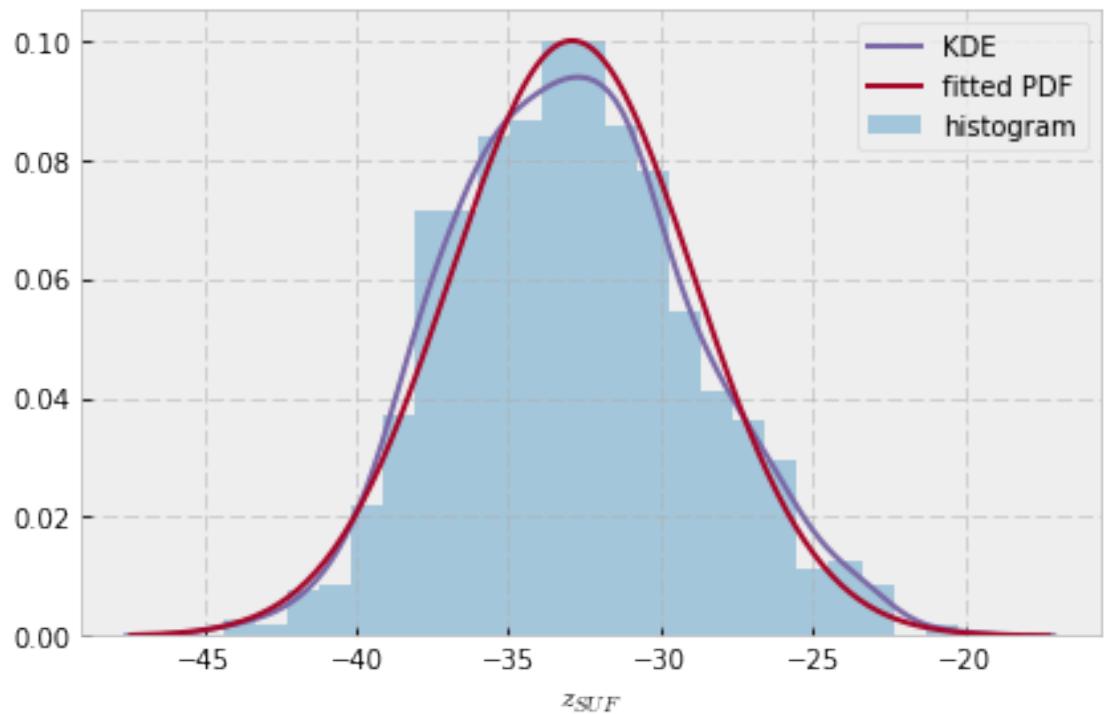


```
variable name: maxB
variable value: 12.0
distribution: normal(shape=(), loc=-32.5969170562, scale=4.183909270840132)
DescribeResult(nobs=1000, minmax=(-44.264772710000003, -19.310768209999999)
               mean=-32.596917056199999, variance=17.522619406028042,
               skewness=0.20634218589156164, kurtosis=-0.2514061198564672)
```

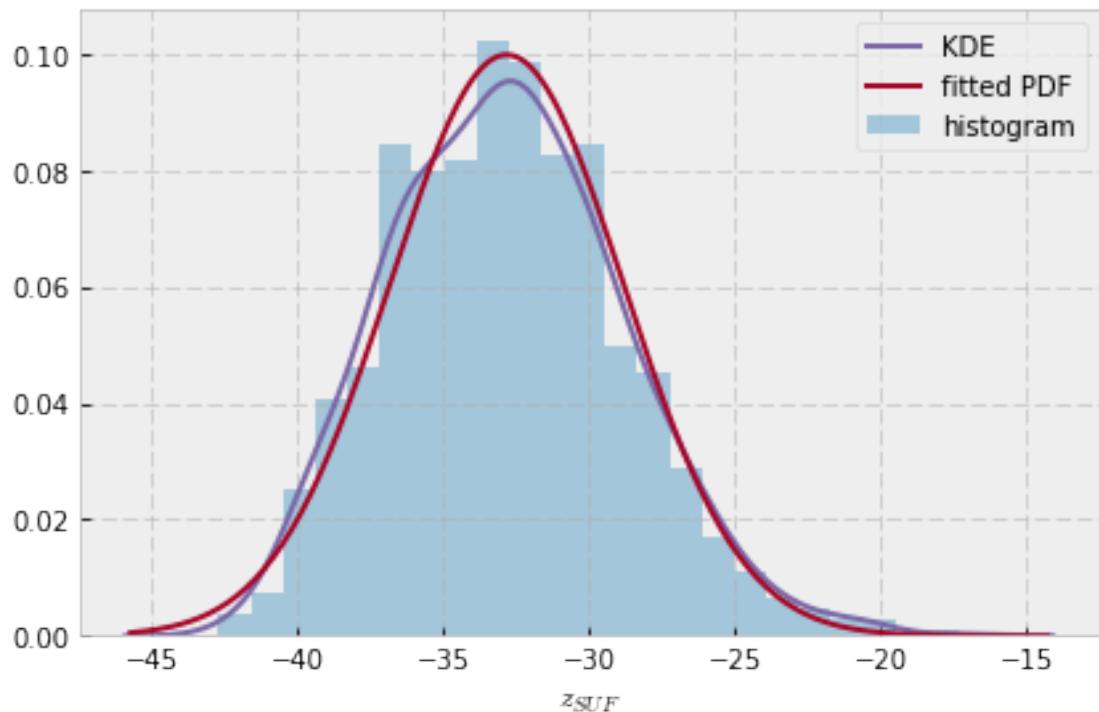


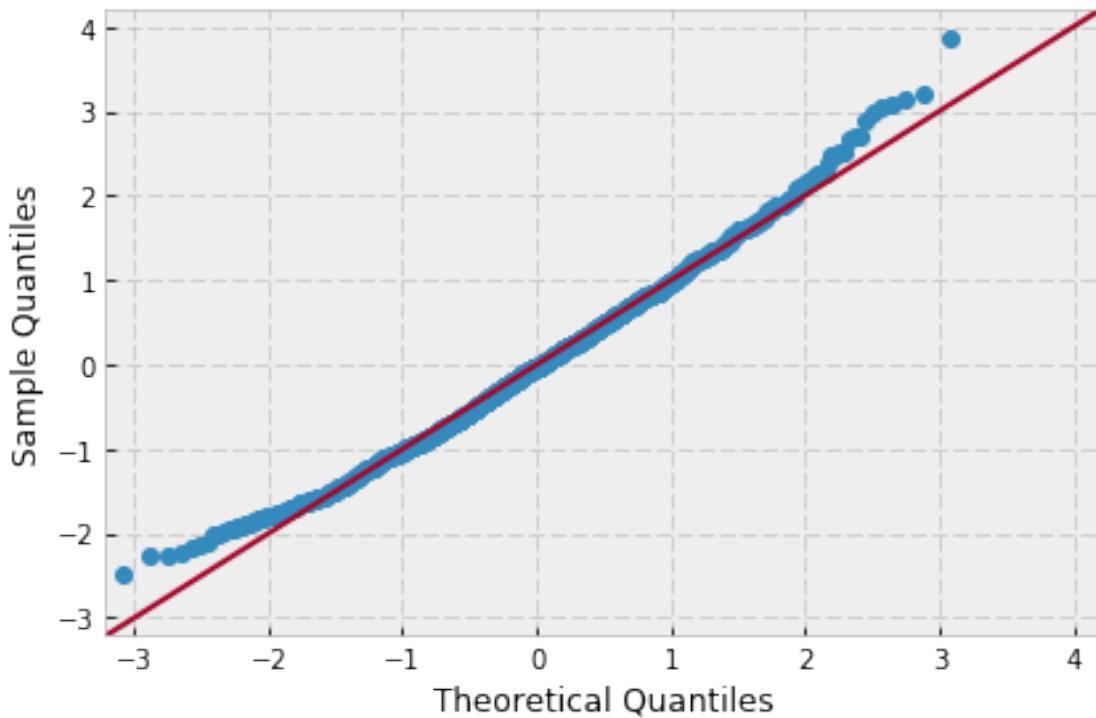


```
variable name: maxB
variable value: 16.0
distribution: normal(shape=(),loc=-32.900874763500006, scale=3.989561247154807)
DescribeResult(nobs=1000, minmax=(-44.37048008, -20.25416442999999)
               mean=-32.90087476350006, variance=15.932531476275694,
               skewness=0.18510806908320773, kurtosis=-0.1751401430394548)
```

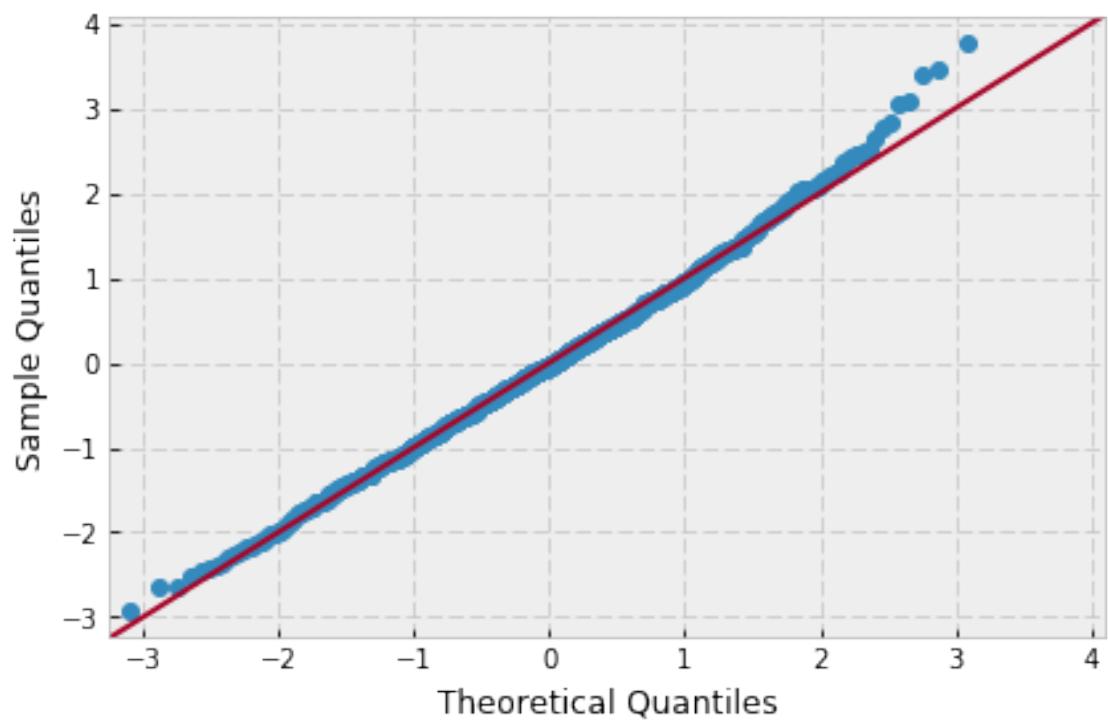
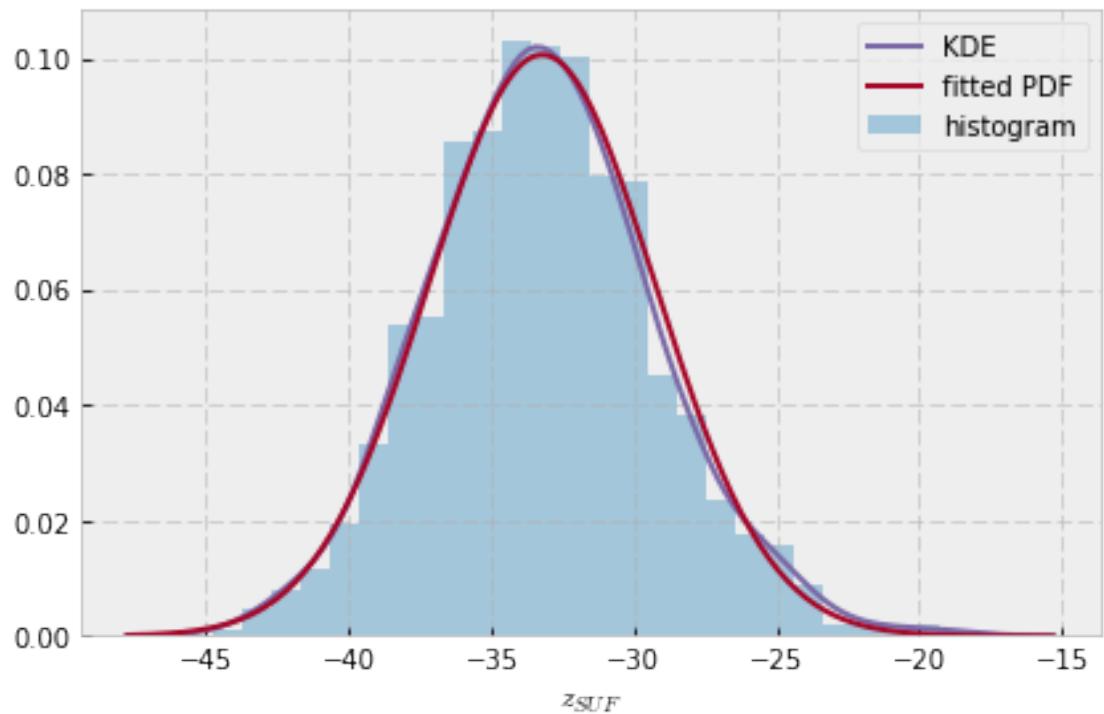


```
variable name: maxB
variable value: 20.0
distribution: normal(shape=(), loc=-32.81545656654, scale=3.997116777205032)
DescribeResult(nobs=1000, minmax=(-42.685499020000002, -17.30025771)
               mean=-32.81545656654, variance=15.992935466080022,
               skewness=0.31976609275246265, kurtosis=0.026032858816614546)
```

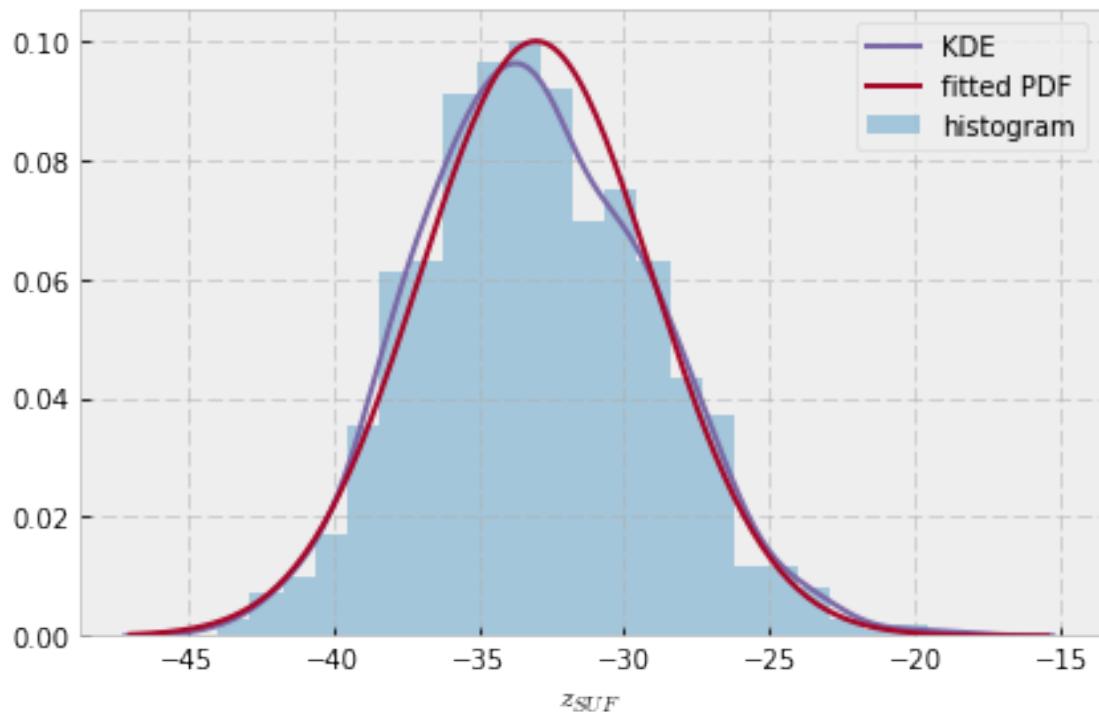


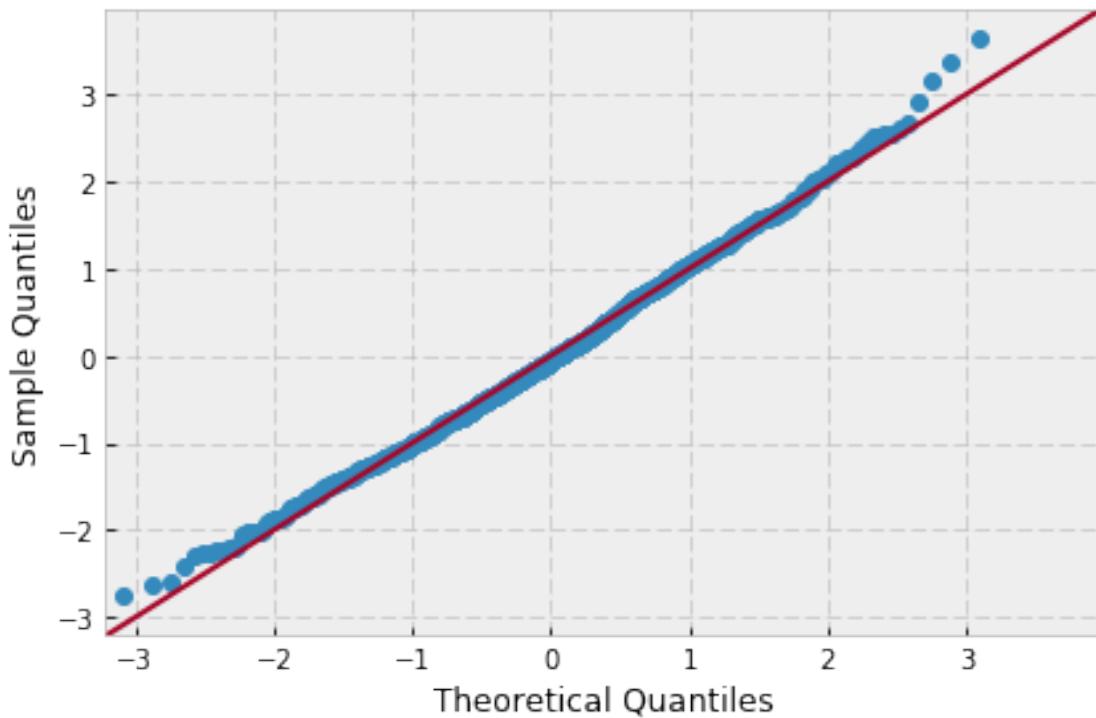


```
variable name: maxB
variable value: 24.0
distribution: normal(shape=(),loc=-33.22031188893, scale=3.965836134251512)
DescribeResult(nobs=1000, minmax=(-44.76503457999999, -18.307320860000001)
               mean=-33.220311888929999, variance=15.743599843578554,
               skewness=0.22191107319314668, kurtosis=0.2621536835148768)
```

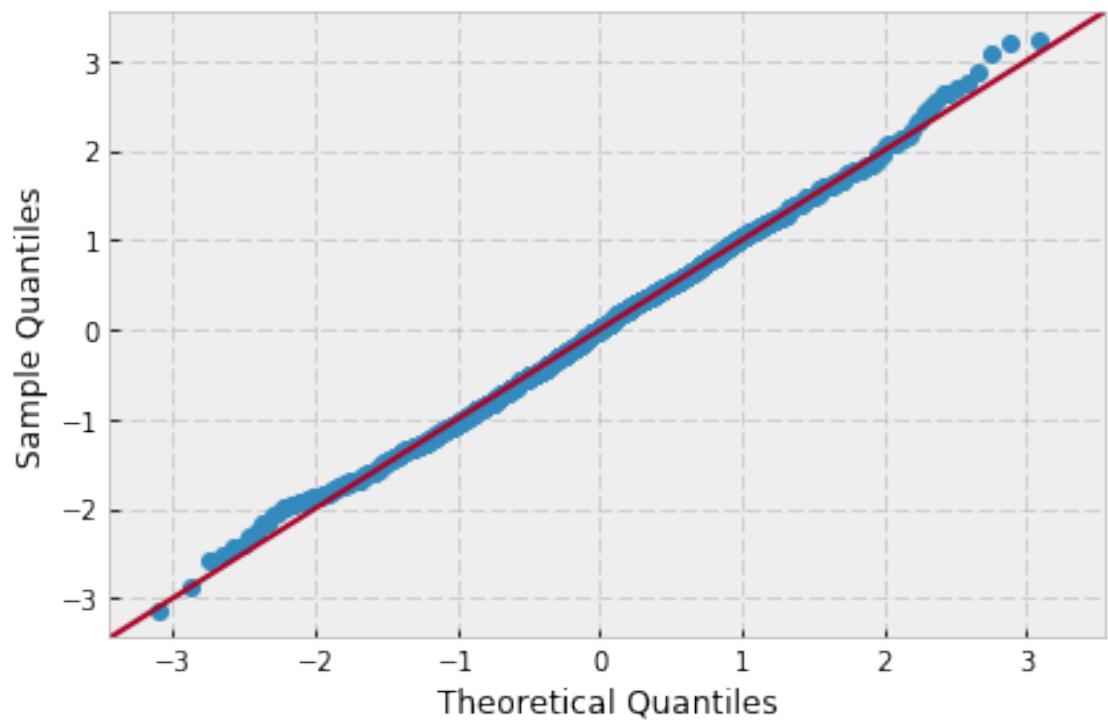
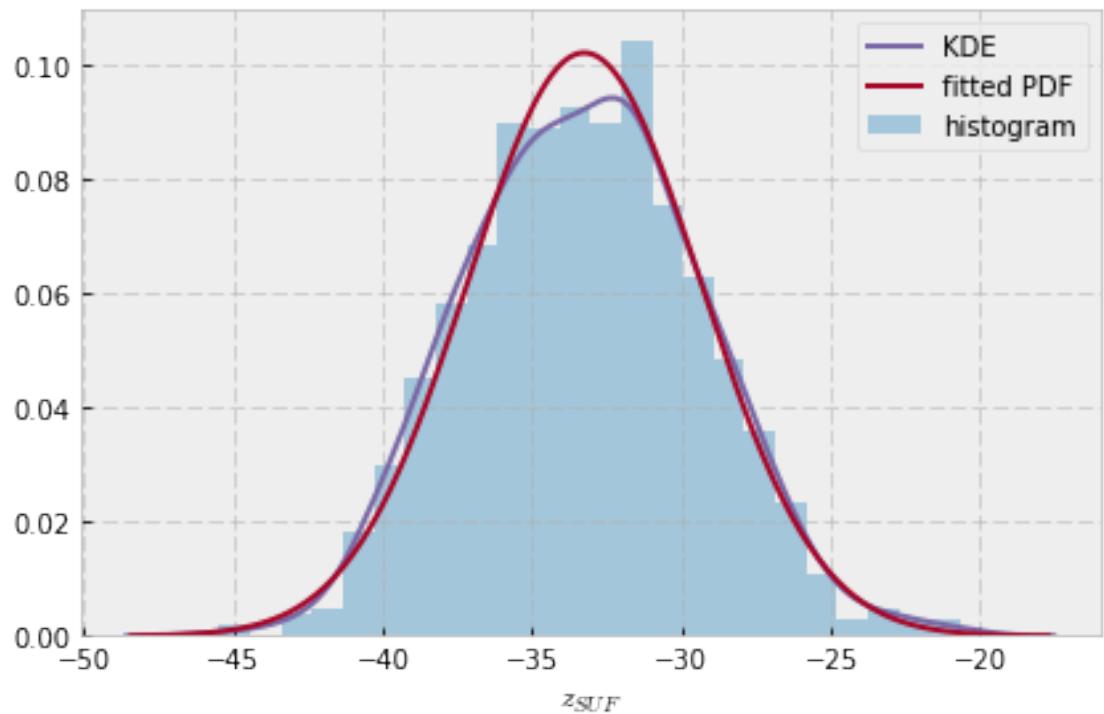


```
variable name: maxB
variable value: 28.0
distribution: normal(shape=(), loc=-33.028290165630004, scale=3.9883624114214604)
DescribeResult(nobs=1000, minmax=(-43.940552689999997, -18.475734939999999)
               mean=-33.028290165630004, variance=15.922957682522128,
               skewness=0.22835683301191126, kurtosis=-0.07092944772174059)
```

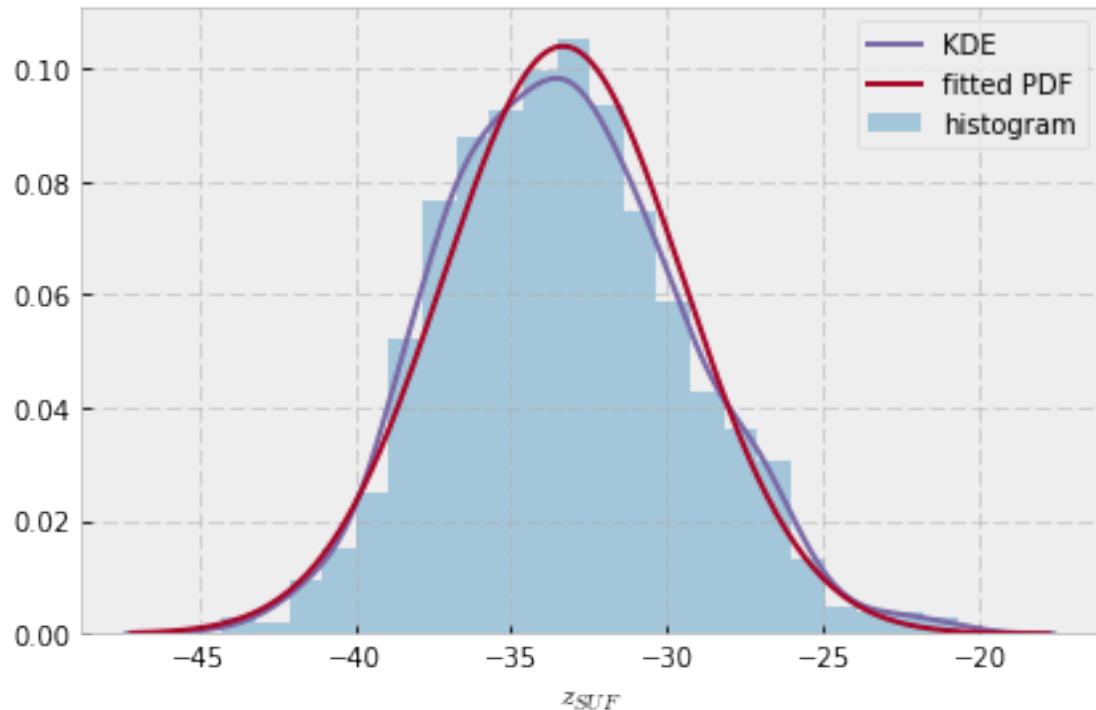


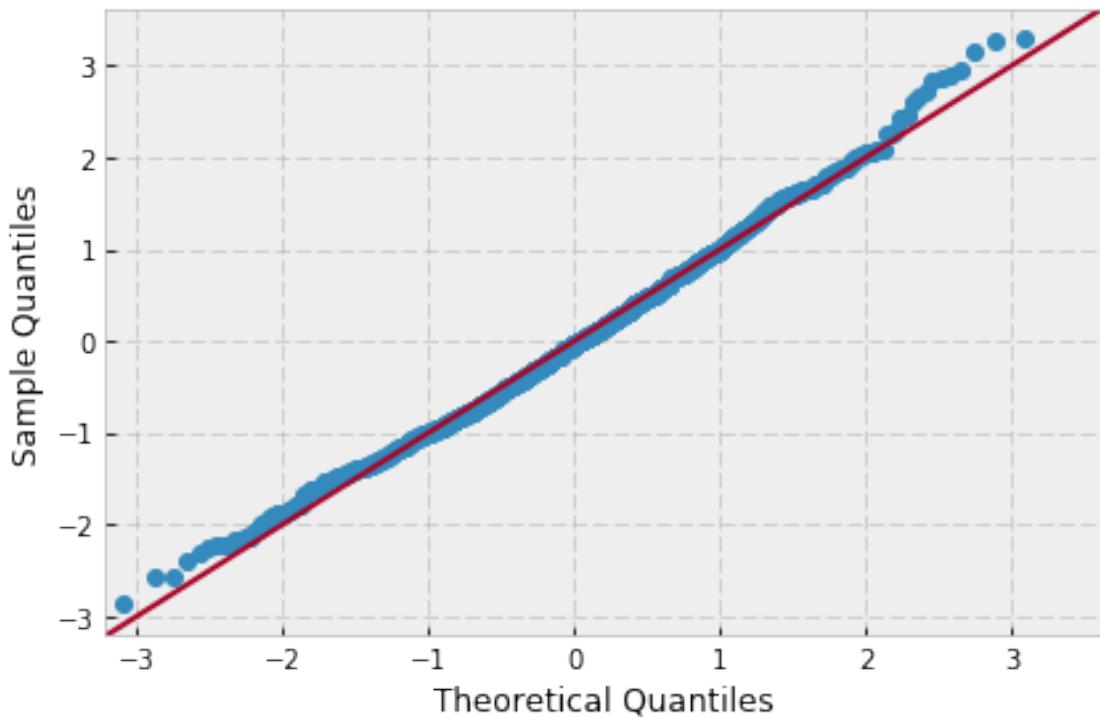


```
variable name: maxB
variable value: 32.0
distribution: normal(shape=(),loc=-33.261248829820005, scale=3.902756434007384)
DescribeResult(nobs=1000, minmax=(-45.467207629999997, -20.660573530000001)
               mean=-33.261248829820005, variance=15.246754537723756,
               skewness=0.10351902958500257, kurtosis=-0.14201571824063963)
```

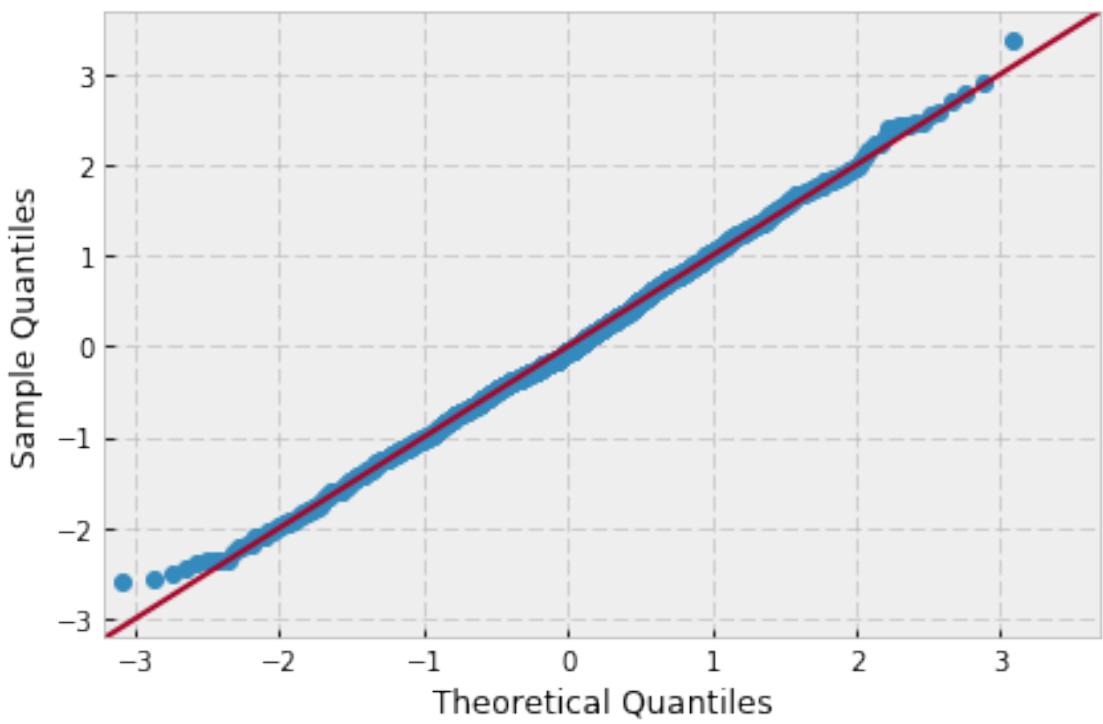
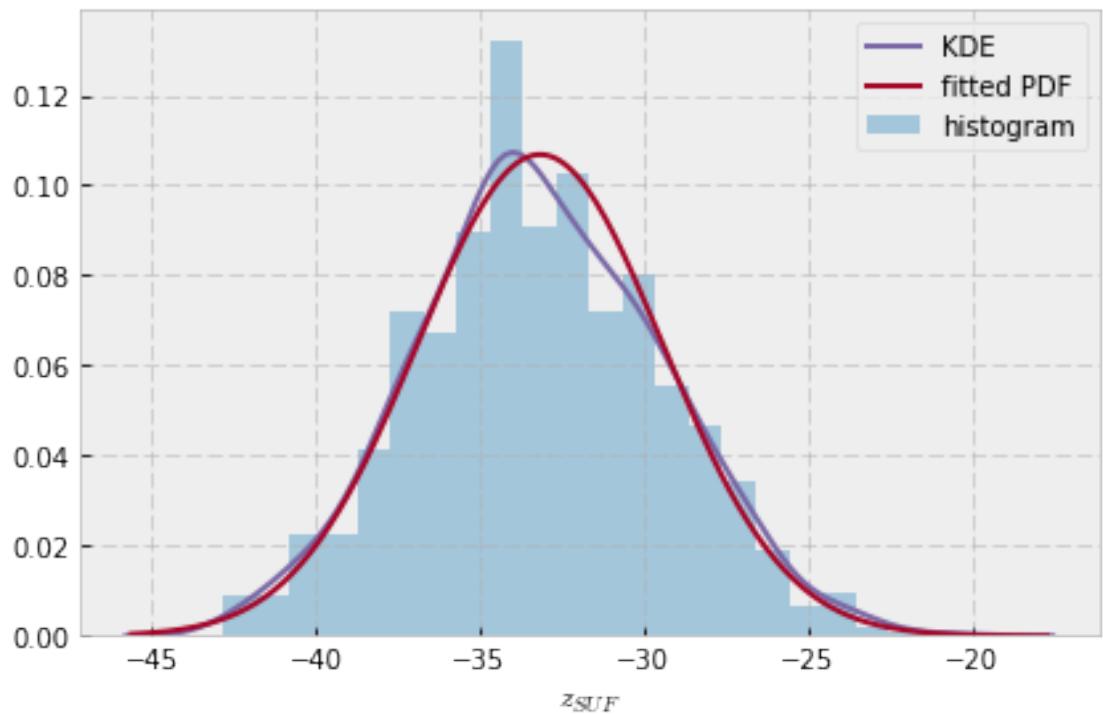


```
variable name: maxB
variable value: 36.0
distribution: normal(shape=(), loc=-33.33919935315001, scale=3.8383353085570446)
DescribeResult(nobs=1000, minmax=(-44.262255170000003, -20.682595249999999)
               mean=-33.339199353150008, variance=14.747565506422125,
               skewness=0.2568803719082728, kurtosis=-0.06375946038969049)
```

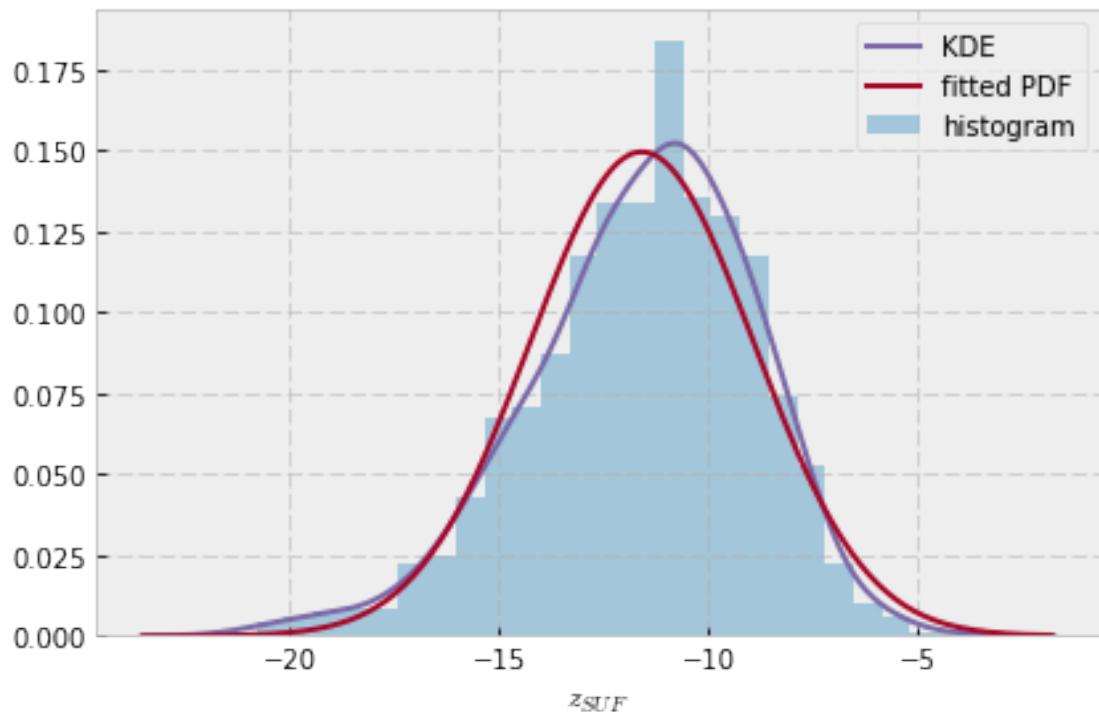


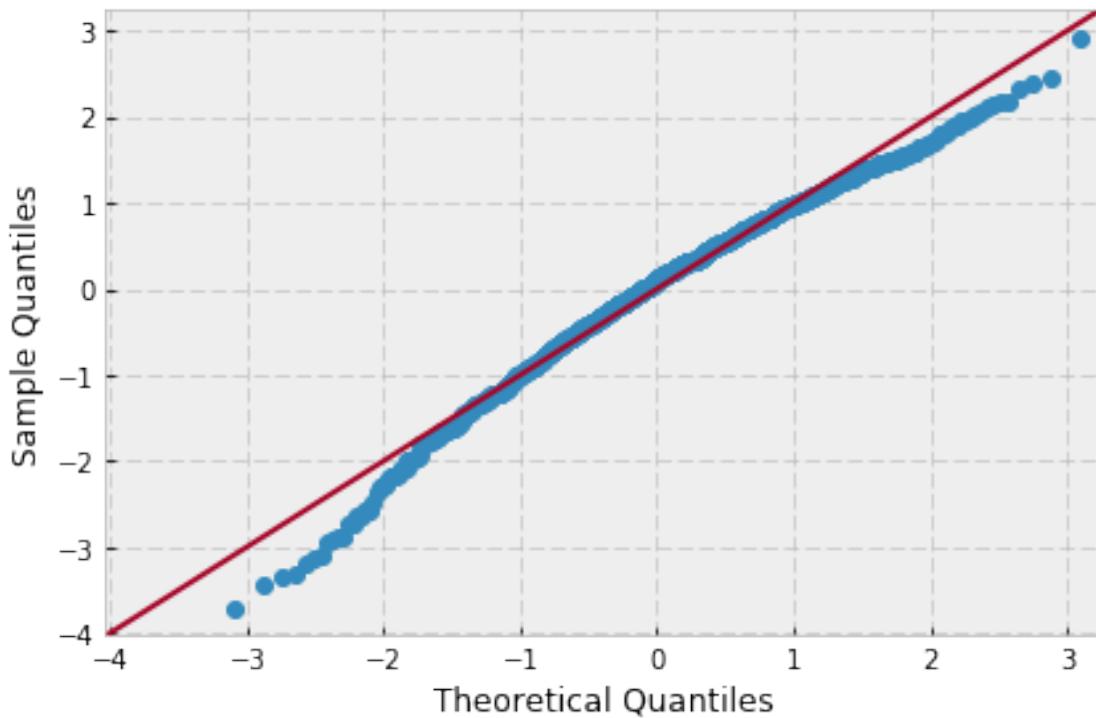


```
variable name: maxB
variable value: 40.0
distribution: normal(shape=(),loc=-33.153479231700004, scale=3.7340835813772957)
DescribeResult(nobs=1000, minmax=(-42.797441720000002, -20.514132239999999)
               mean=-33.153479231700004, variance=13.957337530241732,
               skewness=0.11488411063686266, kurtosis=-0.1600684825652472)
```

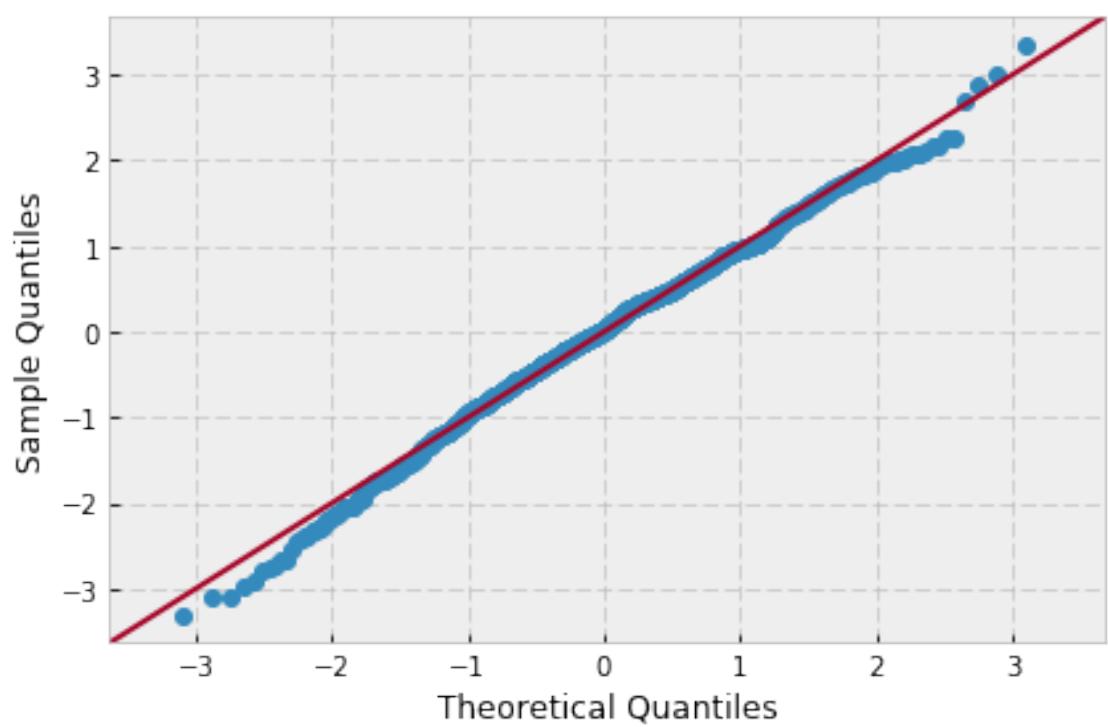
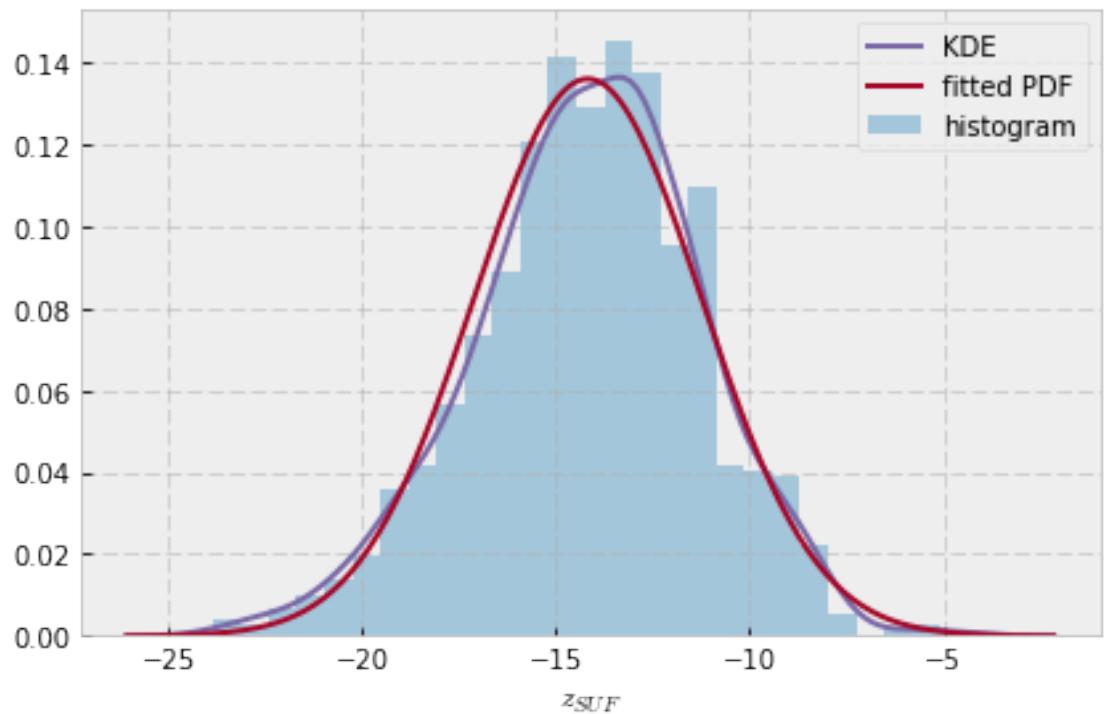


```
variable name: r
variable value: 1.0
distribution: normal(shape=(),loc=-11.591235789775, scale=2.667951157626695)
DescribeResult(nobs=1000, minmax=(-21.471458120000001, -3.821486063)
               mean=-11.591235789775, variance=7.1250884679495723,
               skewness=-0.5166406888683749, kurtosis=0.39168136602036174)
```

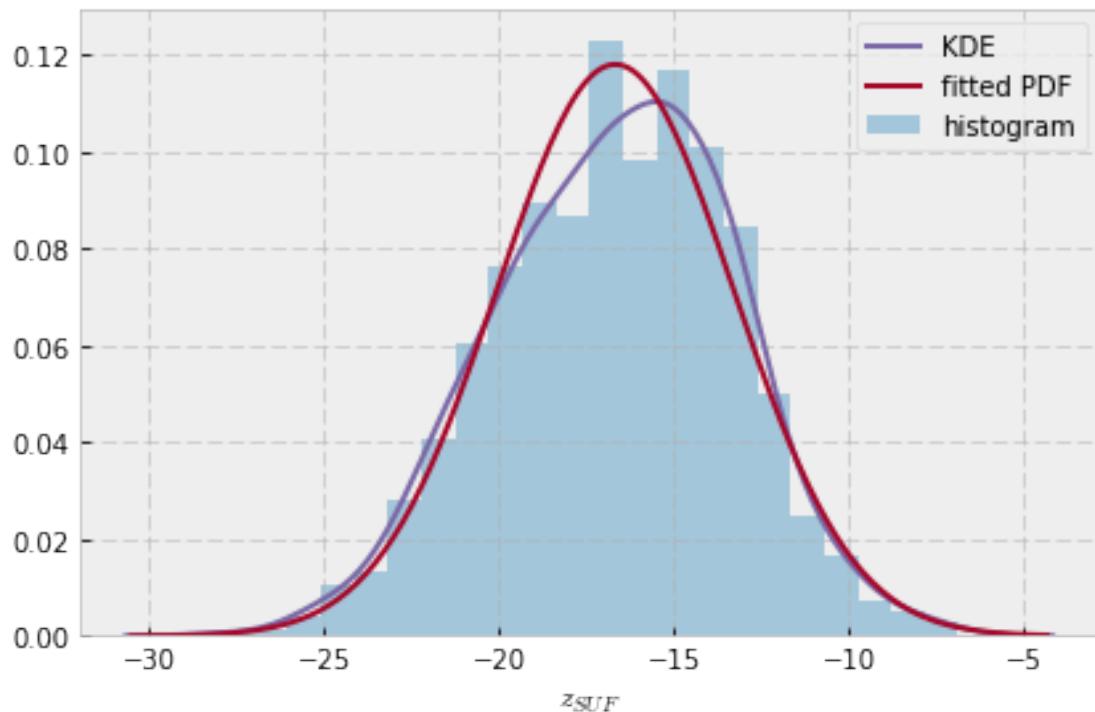


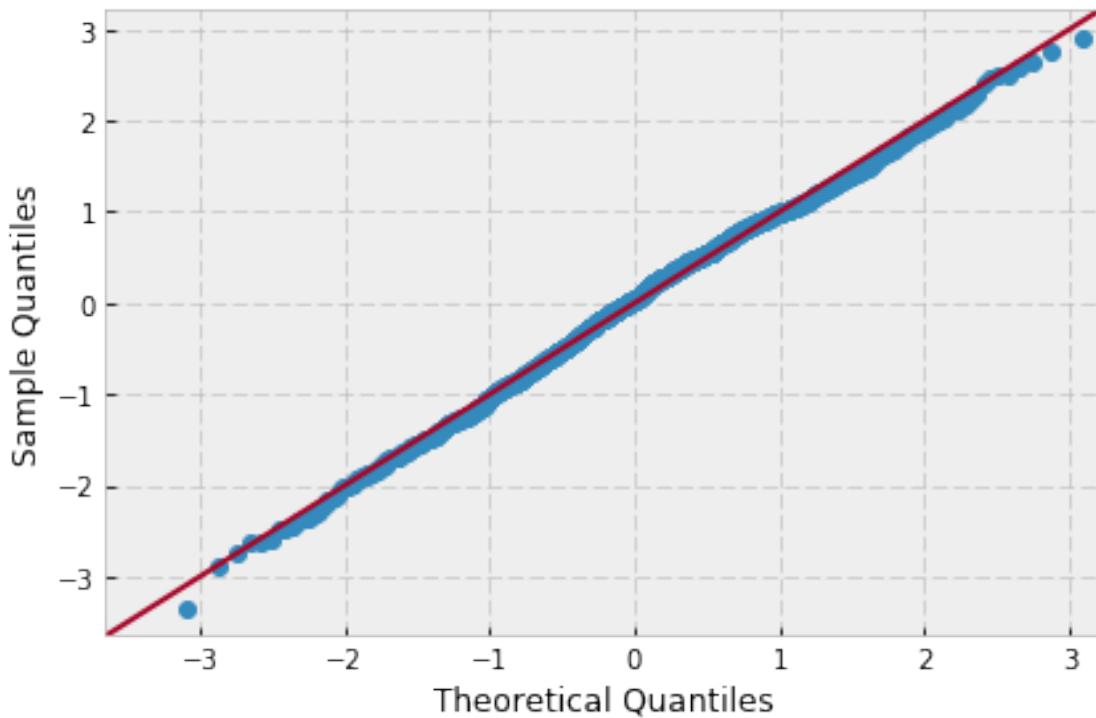


```
variable name: r
variable value: 1.222222222222223
distribution: normal(shape=(),loc=-14.177006214292, scale=2.9321227434692263)
DescribeResult(nobs=1000, minmax=(-23.847446170000001, -4.3762168480000003)
               mean=-14.177006214292, variance=8.6059497325020029,
               skewness=-0.22833733056179129, kurtosis=0.1831186710730437)
```

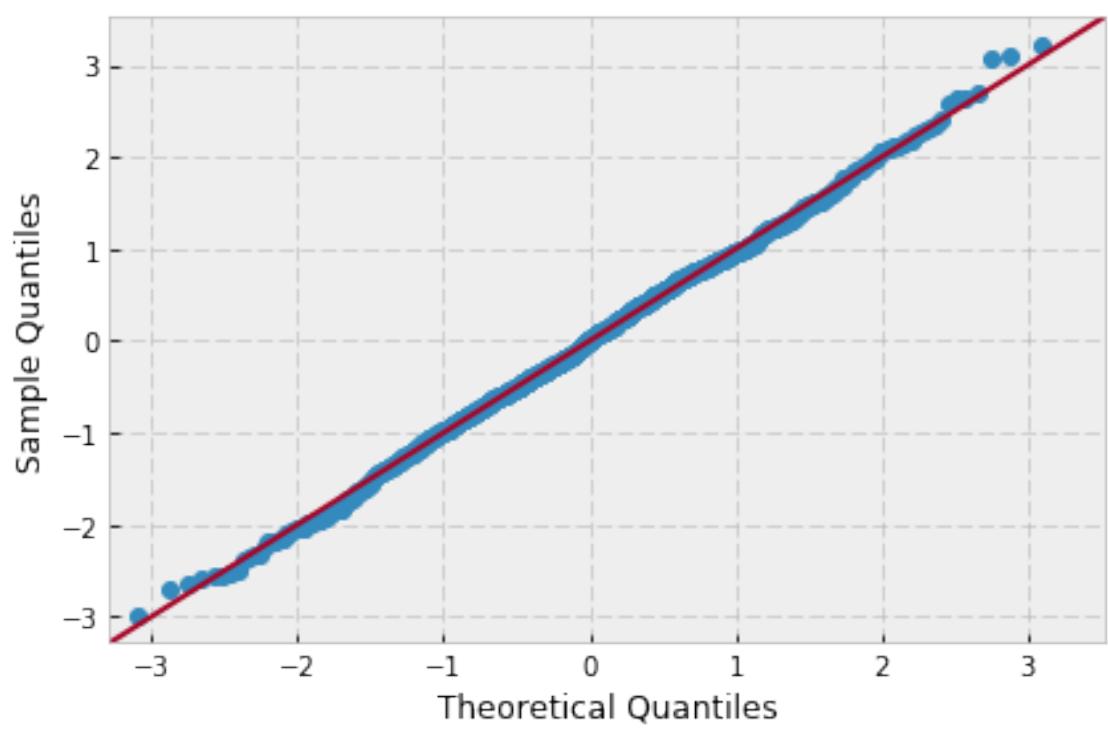
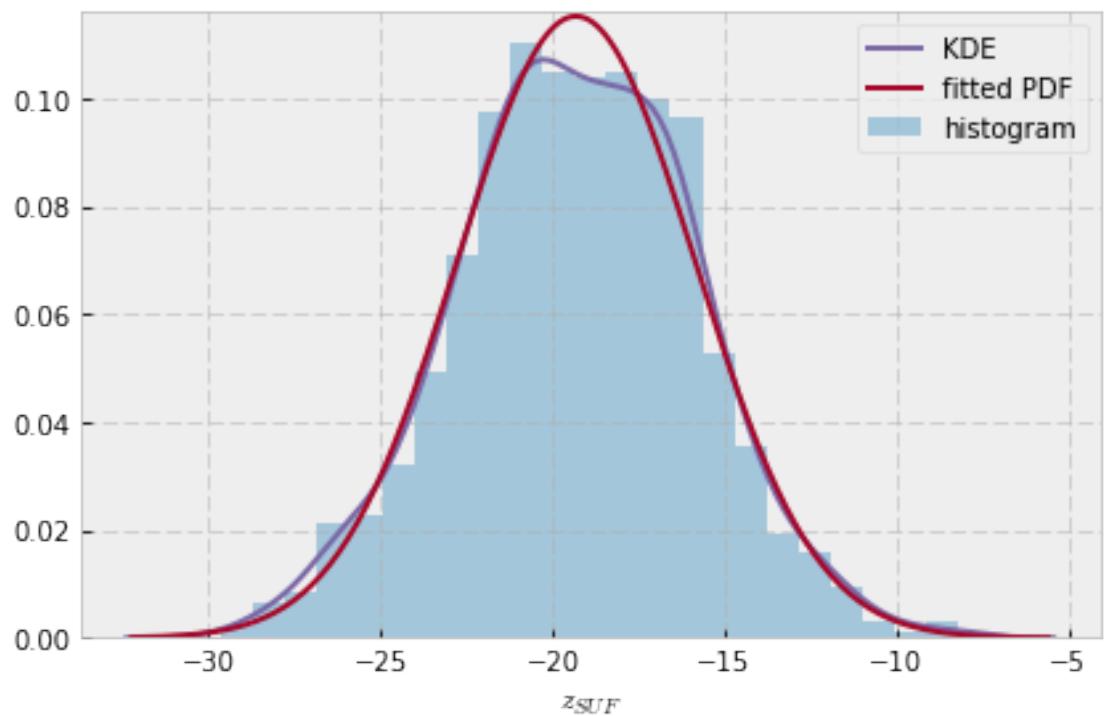


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=-16.676086386456, scale=3.378796130675062)
DescribeResult(nobs=1000, minmax=(-27.95913043999999, -6.880135683999999)
               mean=-16.676086386455999, variance=11.427690983648418,
               skewness=-0.1532788967344575, kurtosis=-0.19383431631754267)
```

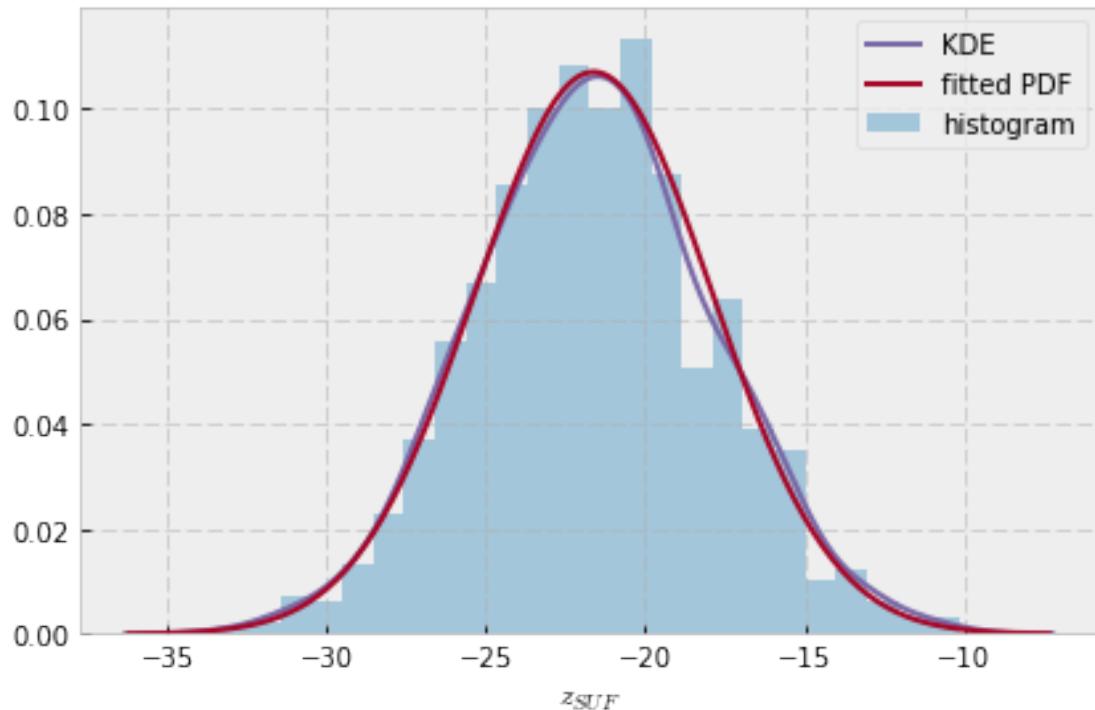


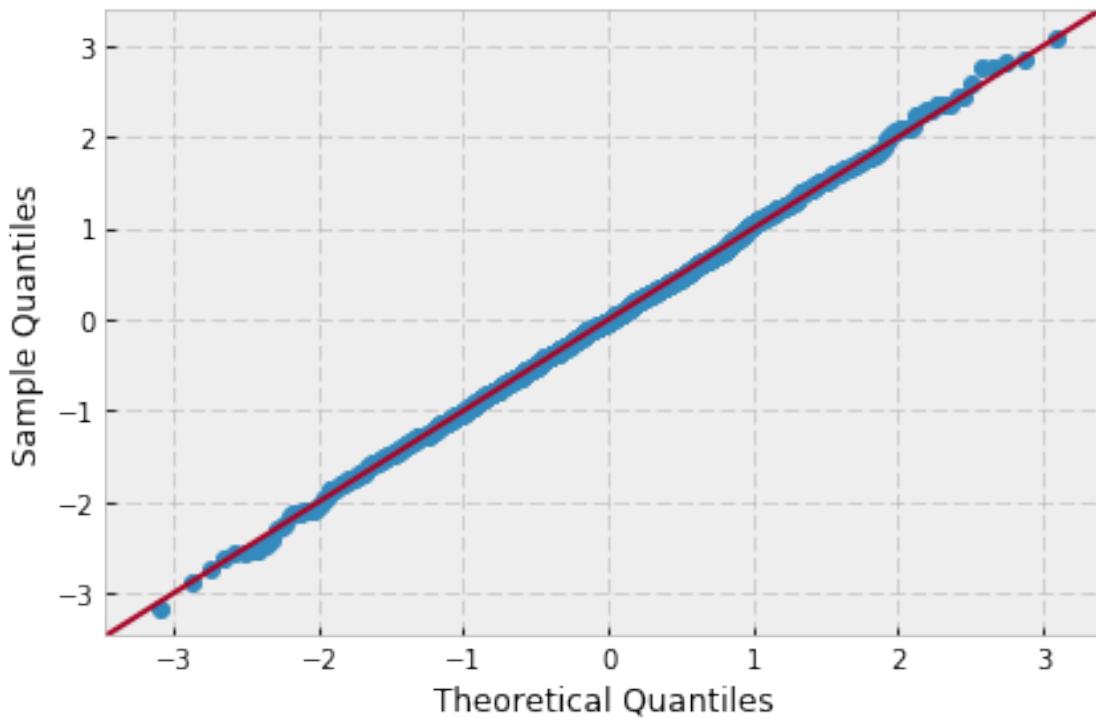


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(),loc=-19.314520846761, scale=3.4625955628644376)
DescribeResult(nobs=1000, minmax=(-29.626628090000001, -8.181916693999999)
               mean=-19.314520846760999, variance=12.001569601570063,
               skewness=-0.014962072229165884, kurtosis=-0.02913077731933944)
```

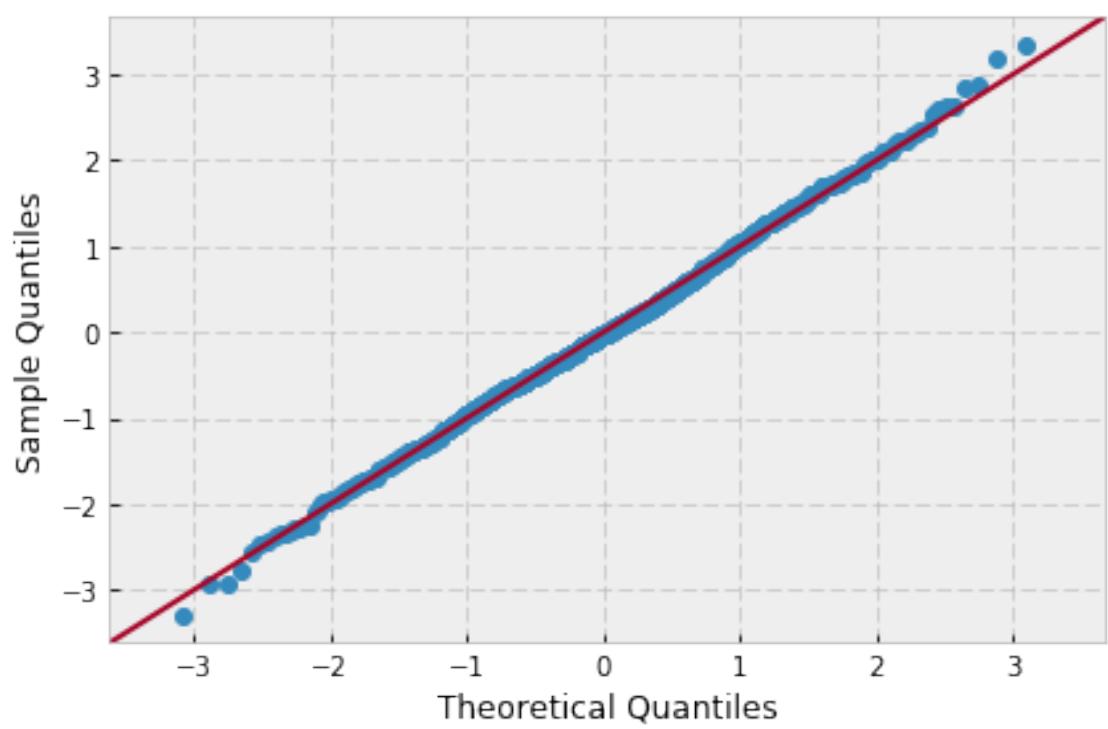
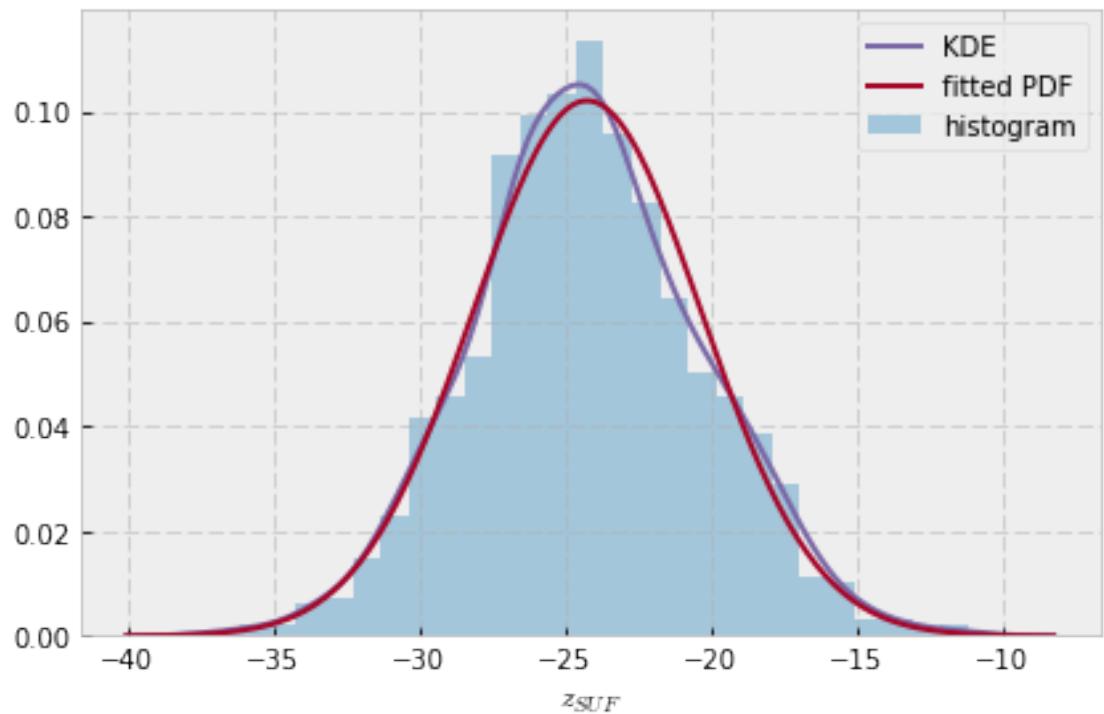


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=-21.61686044807, scale=3.7276579376789463)
DescribeResult(nobs=1000, minmax=(-33.38736033, -10.131370609999999)
               mean=-21.61686044807, variance=13.909343043384238,
               skewness=0.04425163083091408, kurtosis=-0.04795805496220673)
```

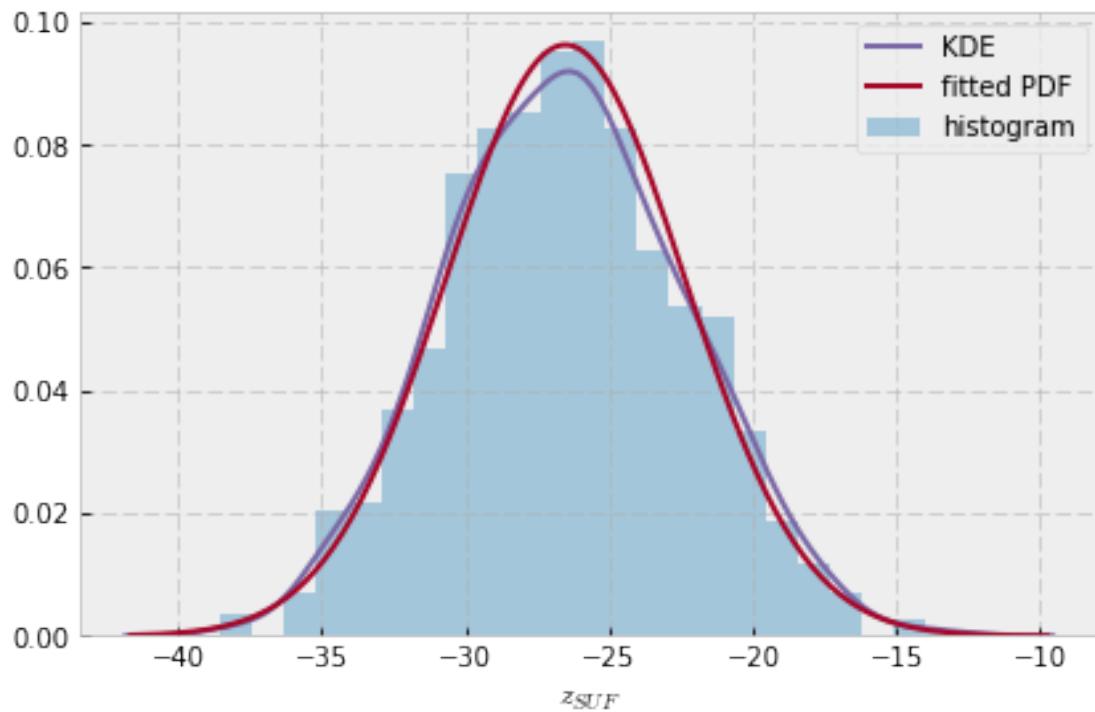


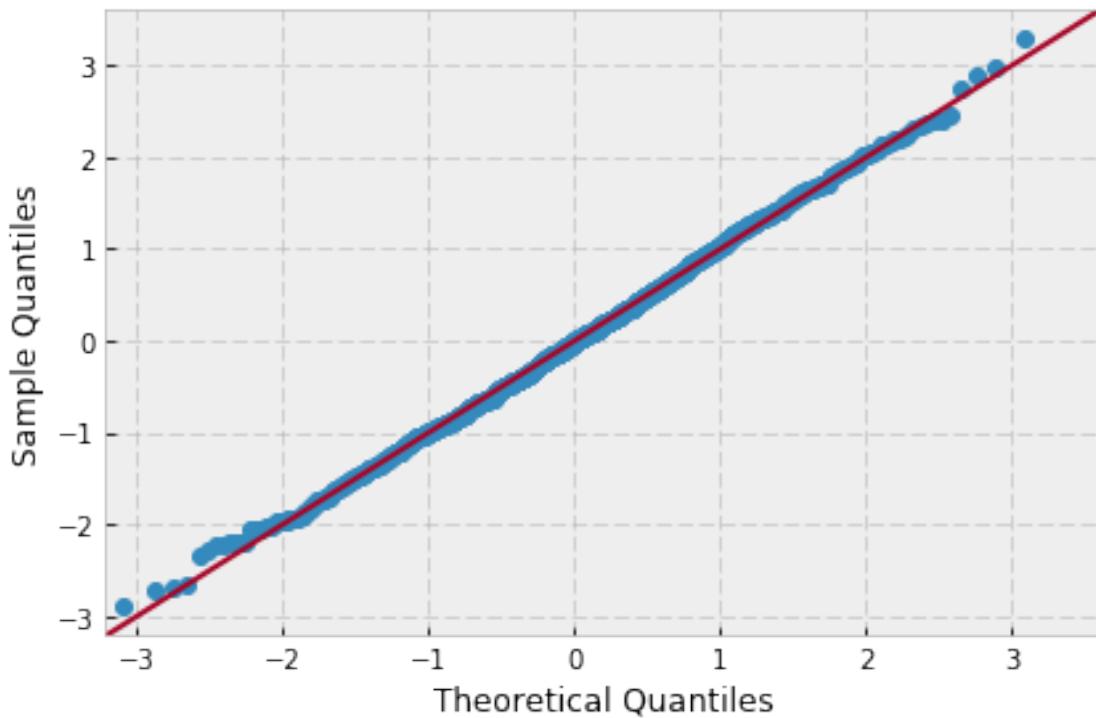


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(),loc=-24.230380311139996, scale=3.910793249868697)
DescribeResult(nobs=1000, minmax=(-37.080680899999997, -11.17739675)
               mean=-24.230380311139996, variance=15.309613456675239,
               skewness=0.08167576381314906, kurtosis=0.06662120135632721)
```

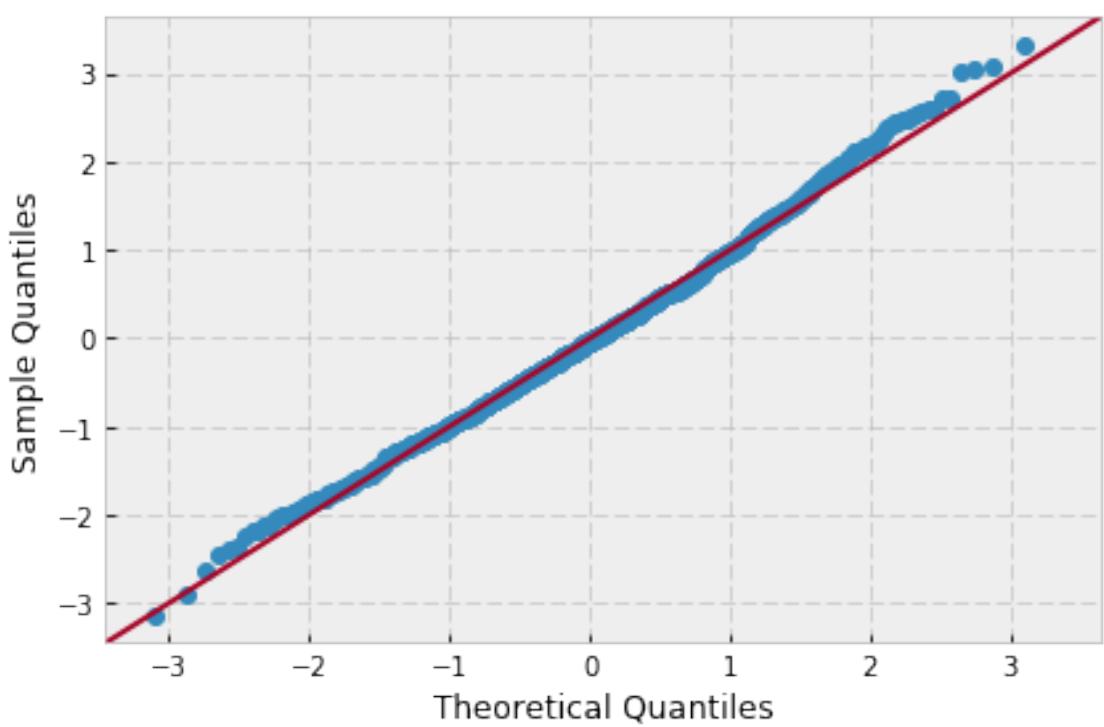
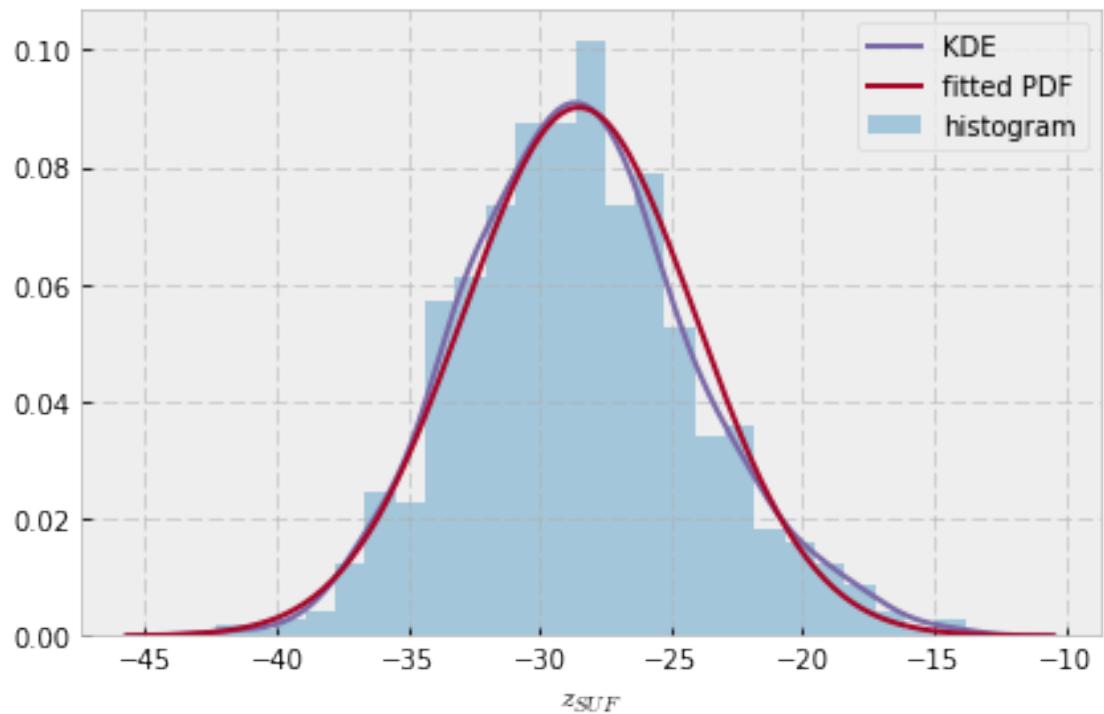


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=-26.520748238699998, scale=4.1477306468106505)
DescribeResult(nobs=1000, minmax=(-38.528175840000003, -12.865756559999999)
               mean=-26.520748238699998, variance=17.220890408901194,
               skewness=0.08372638701155213, kurtosis=-0.2147989552350409)
```

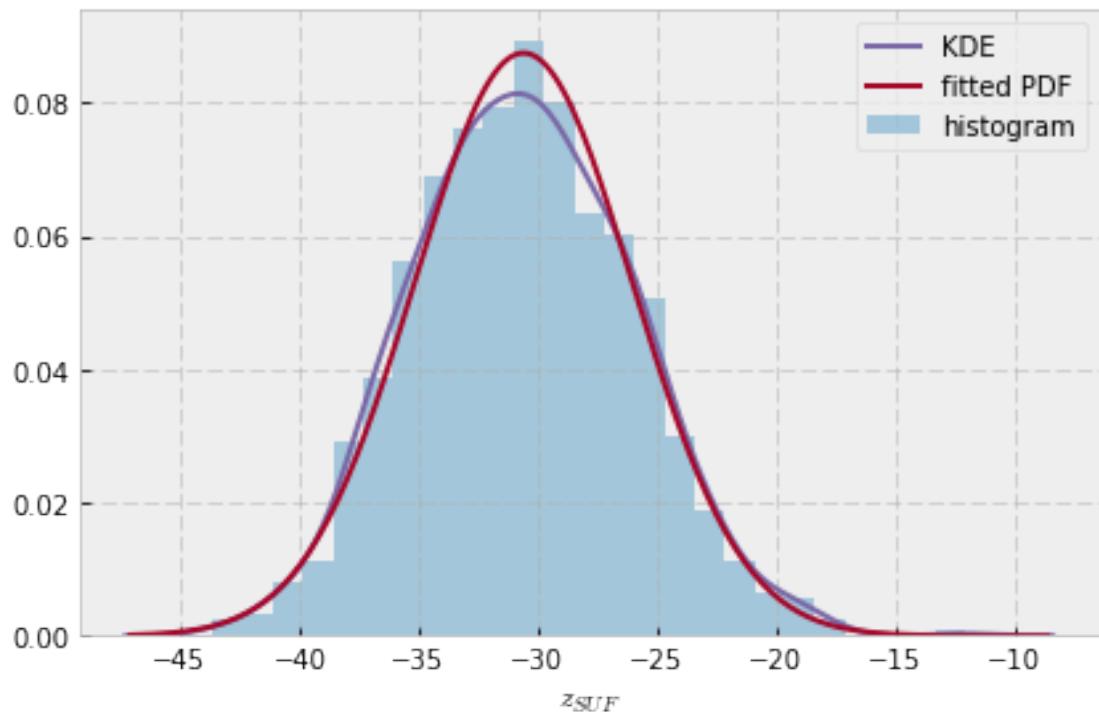


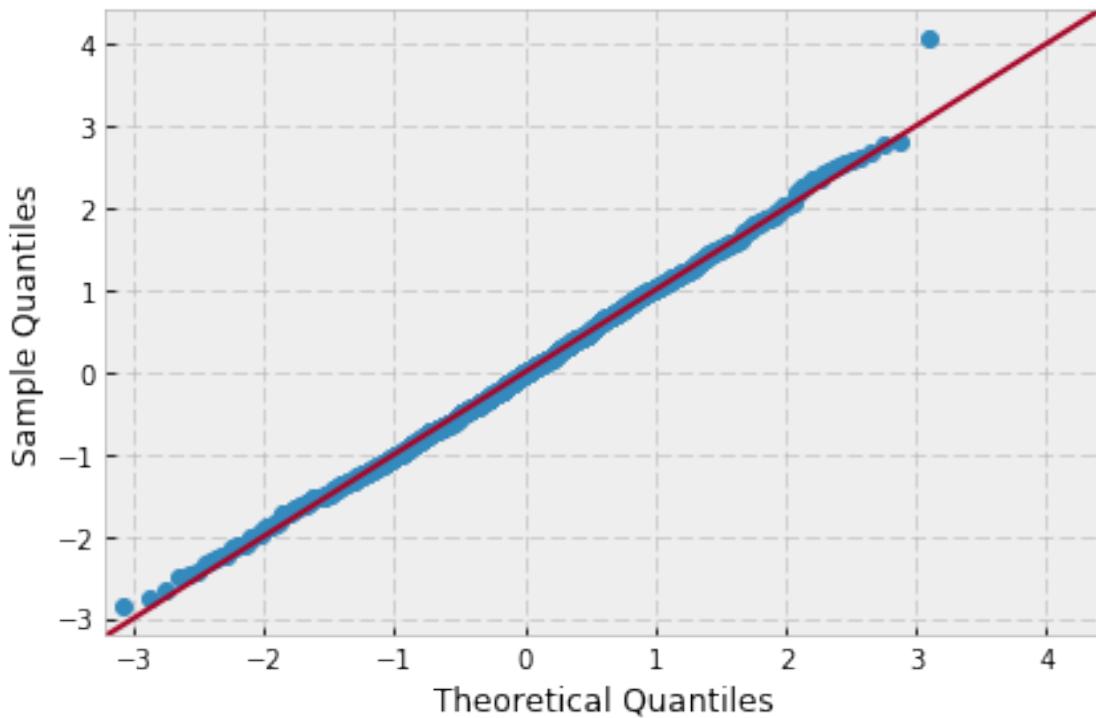


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(),loc=-28.4976012204, scale=4.423093427954631)
DescribeResult(nobs=1000, minmax=(-42.330466719999997, -13.82919148)
               mean=-28.4976012204, variance=19.583338811226675,
               skewness=0.2529939248973862, kurtosis=0.09971378476488146)
```

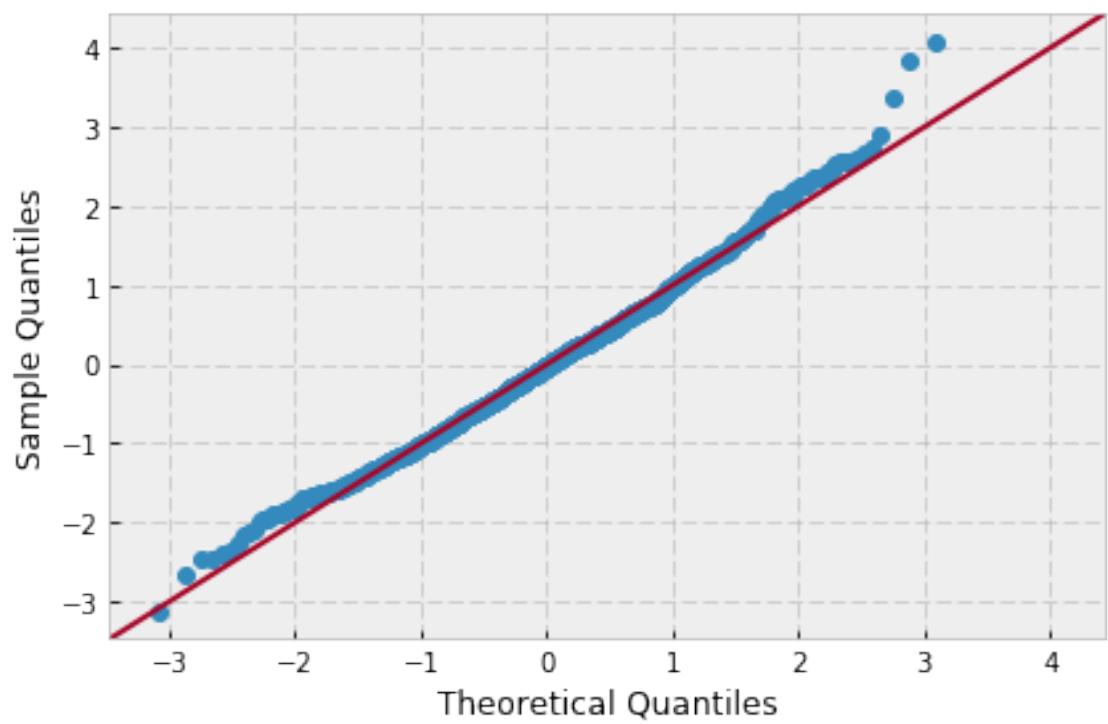
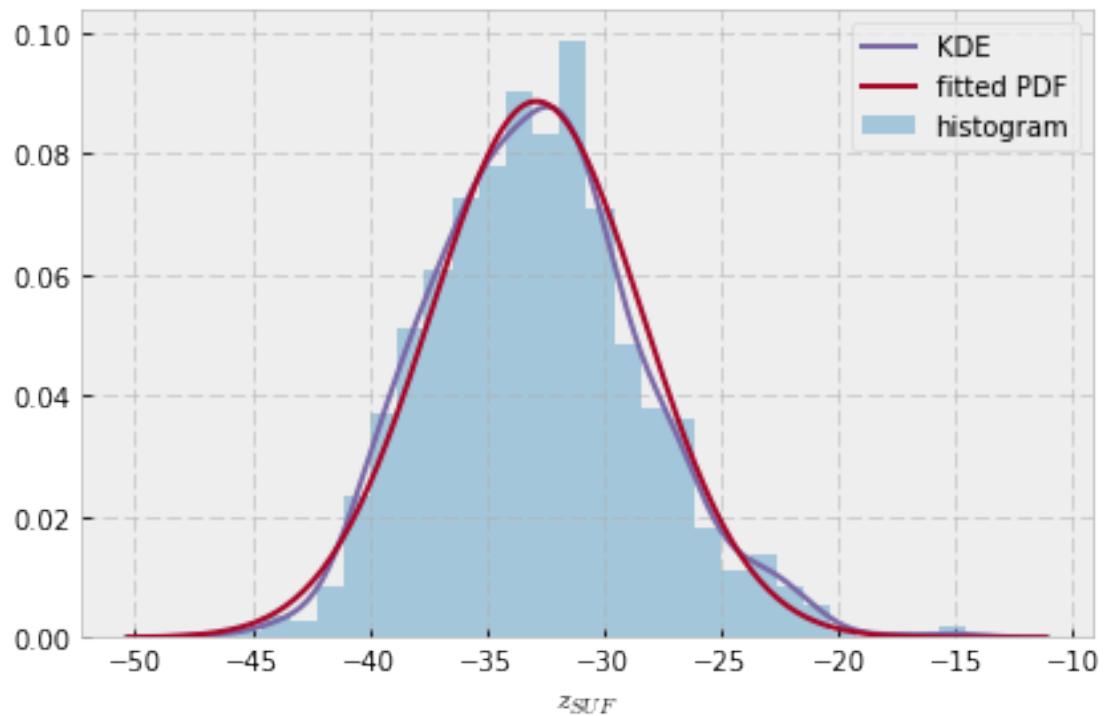


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(),loc=-30.59433547853, scale=4.561200765666767)
DescribeResult(nobs=1000, minmax=(-43.63777383, -12.05399177)
               mean=-30.594335478529999, variance=20.825377802521622,
               skewness=0.13611541549862446, kurtosis=-0.06045781932218075)
```

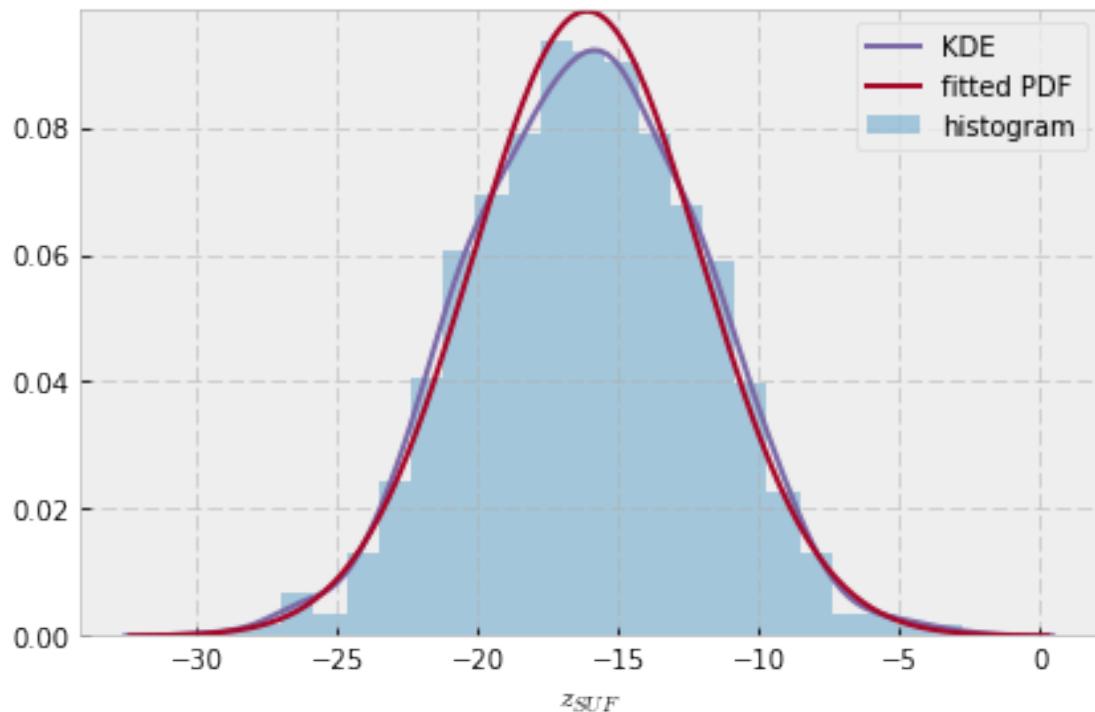


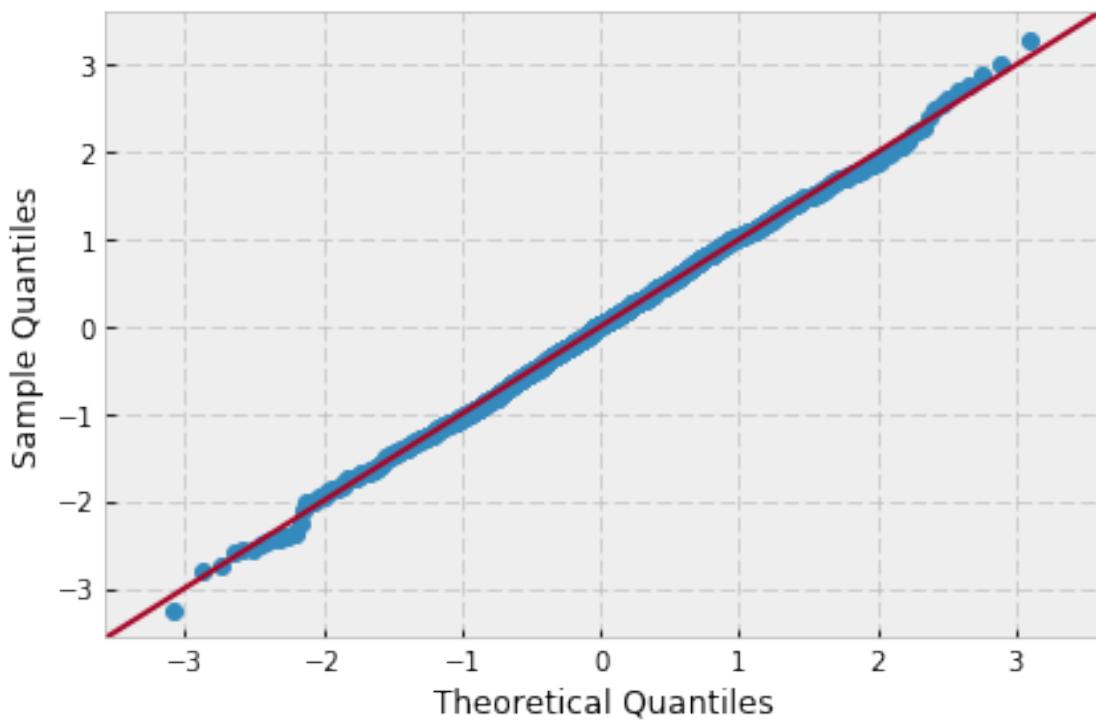


```
variable name: r
variable value: 3.0
distribution: normal(shape=(),loc=-32.86308004628, scale=4.502671186097614)
DescribeResult(nobs=1000, minmax=(-46.87840906999999, -14.549591250000001)
               mean=-32.863080046279997, variance=20.294342152265958,
               skewness=0.3264915170807752, kurtosis=0.27396687401038733)
```

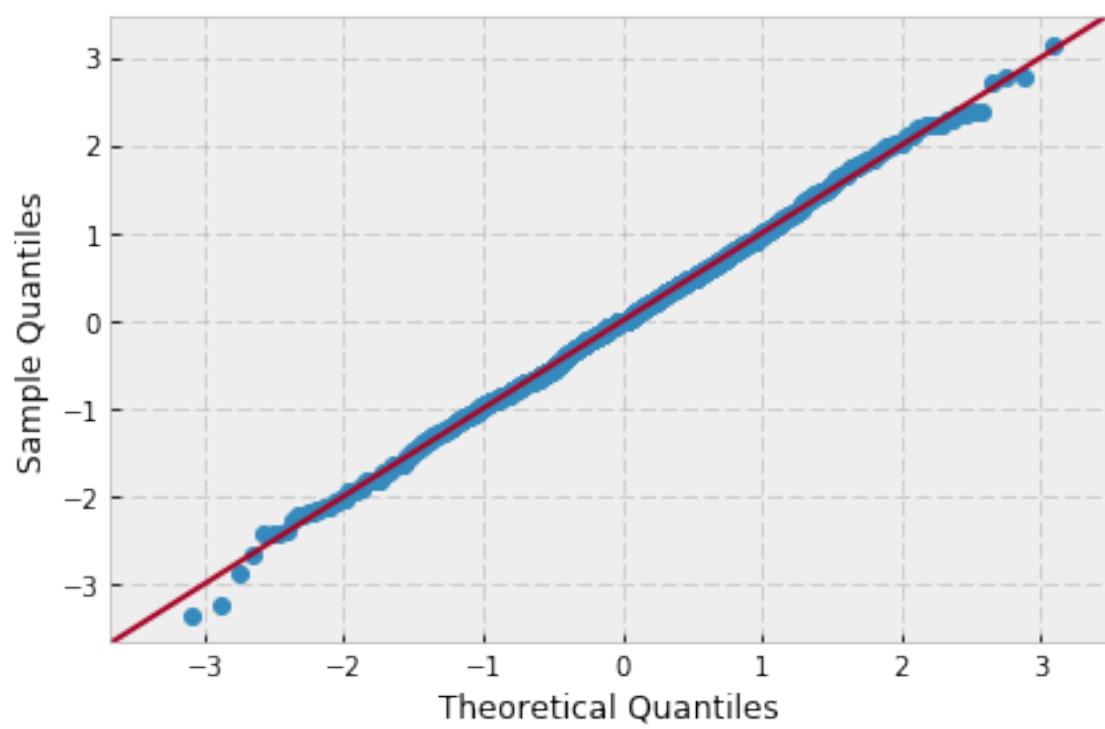
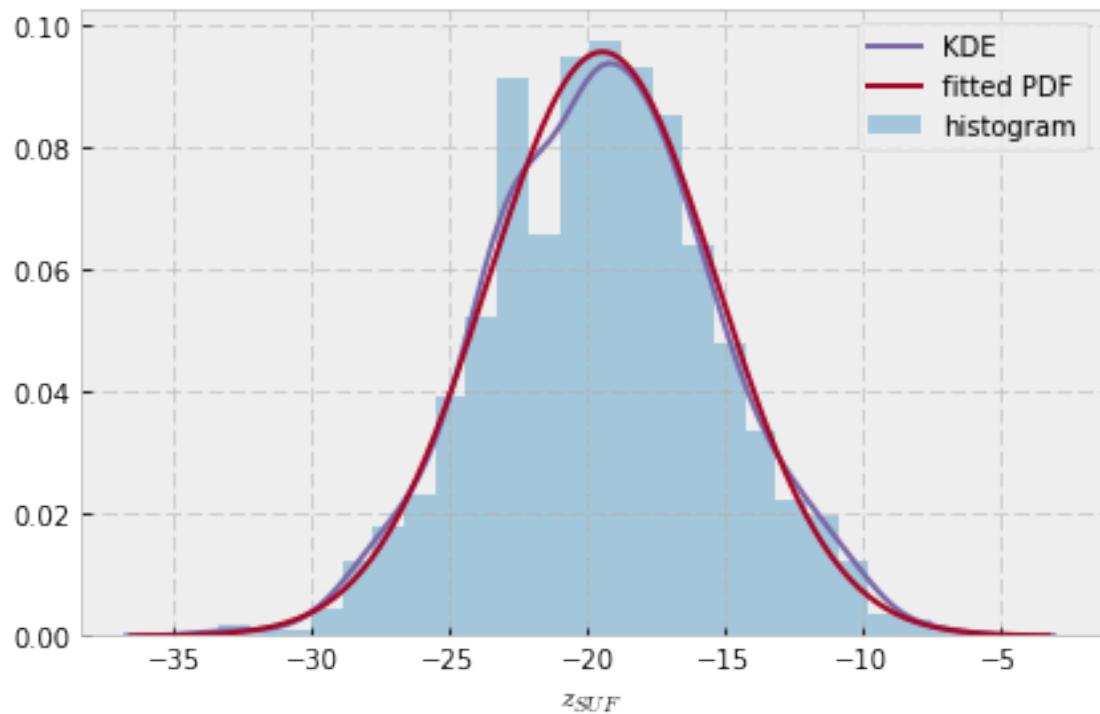


```
variable name: sigma
variable value: 0.0111111111111112
distribution: normal(shape=(), loc=-16.099682727831002, scale=4.059561929485104)
DescribeResult(nobs=1000, minmax=(-29.275856510000001, -2.7818094059999998)
               mean=-16.099682727831002, variance=16.496539598923743,
               skewness=-0.00413562099115514, kurtosis=-0.13193209914433623)
```

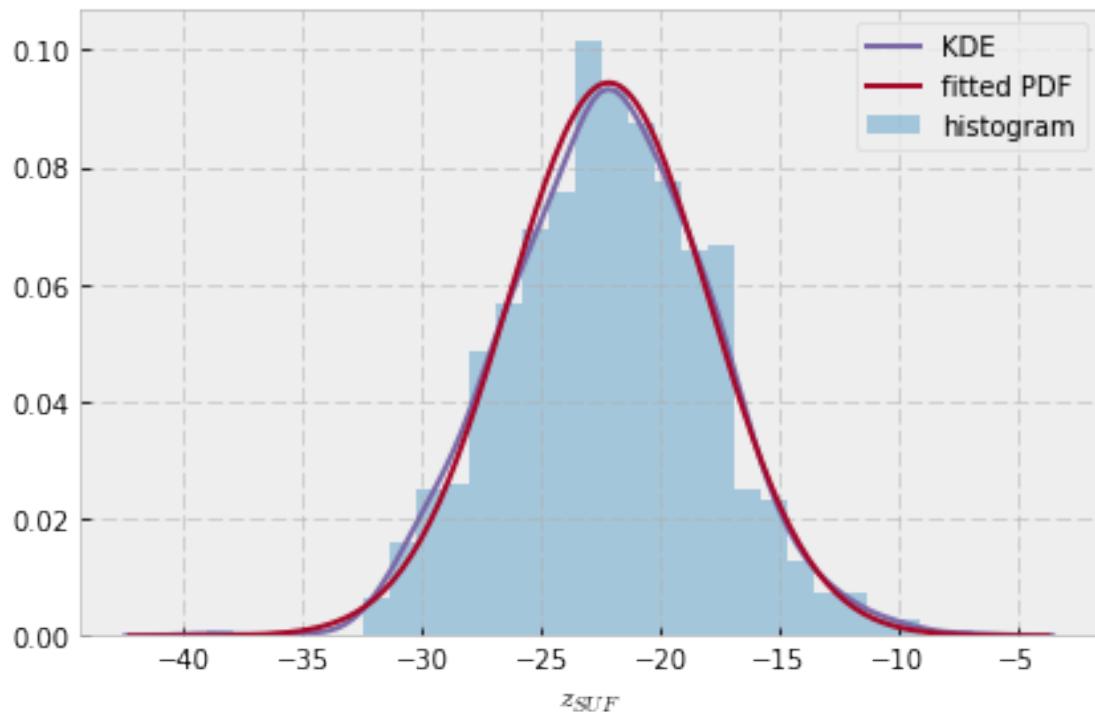


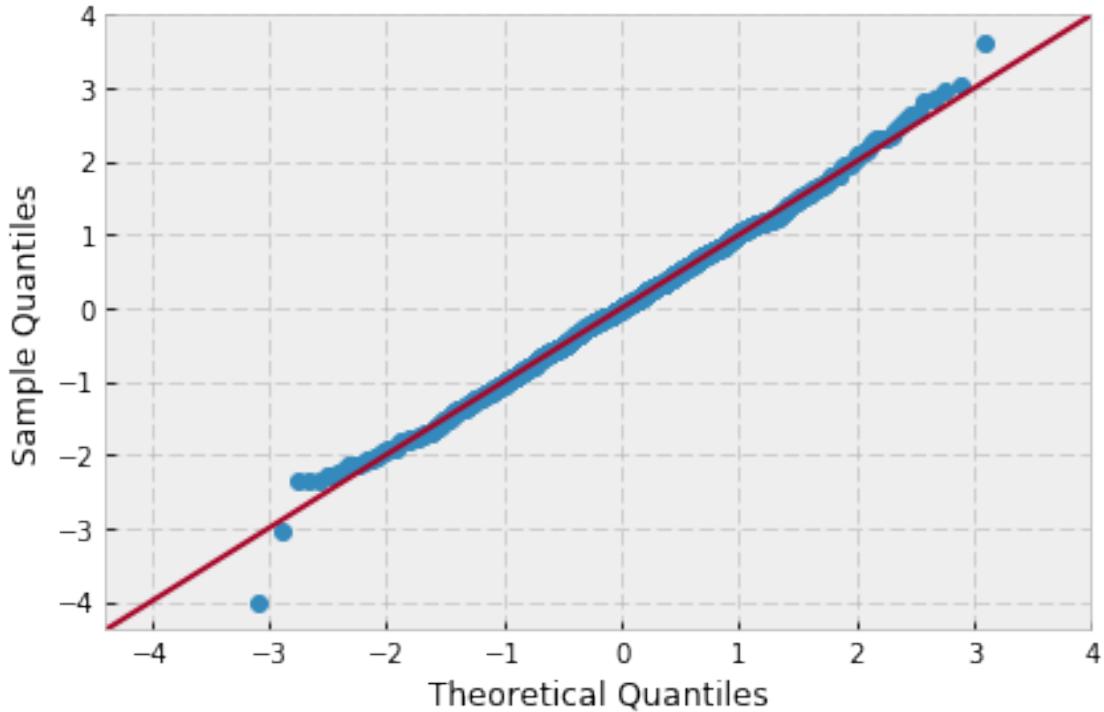


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(),loc=-19.426177801812997, scale=4.167048261569078)
DescribeResult(nobs=1000, minmax=(-33.381373619999998, -6.3654723219999996)
               mean=-19.426177801812997, variance=17.381672887133007,
               skewness=0.021337311178422937, kurtosis=-0.04894437990112044)
```

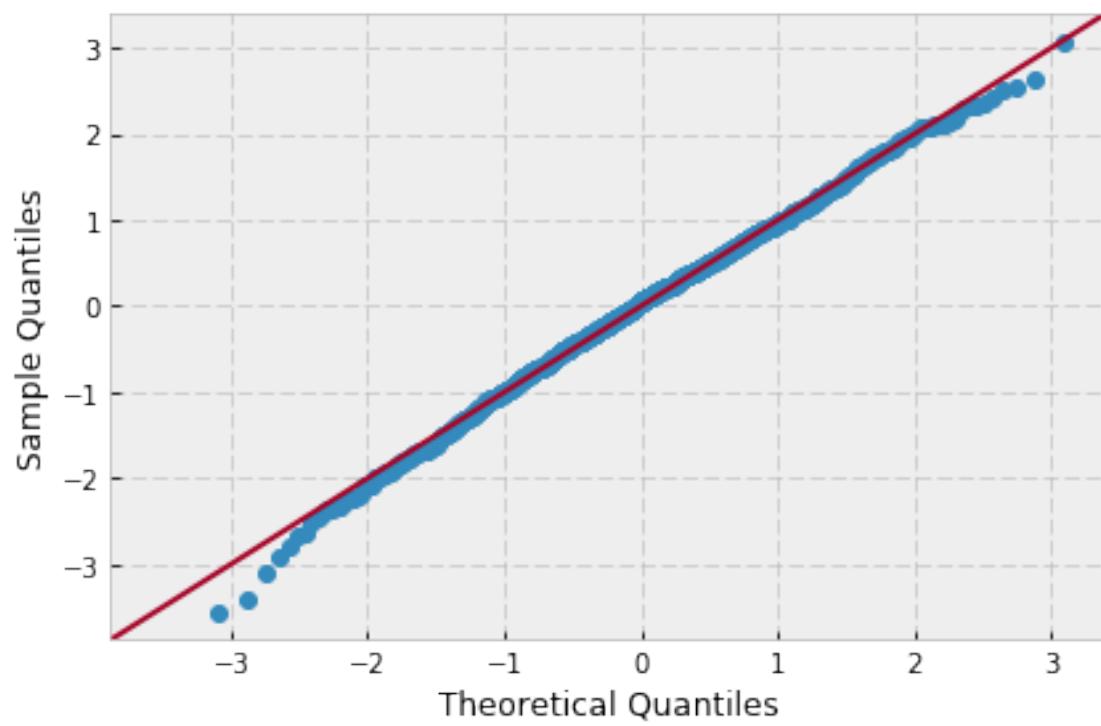
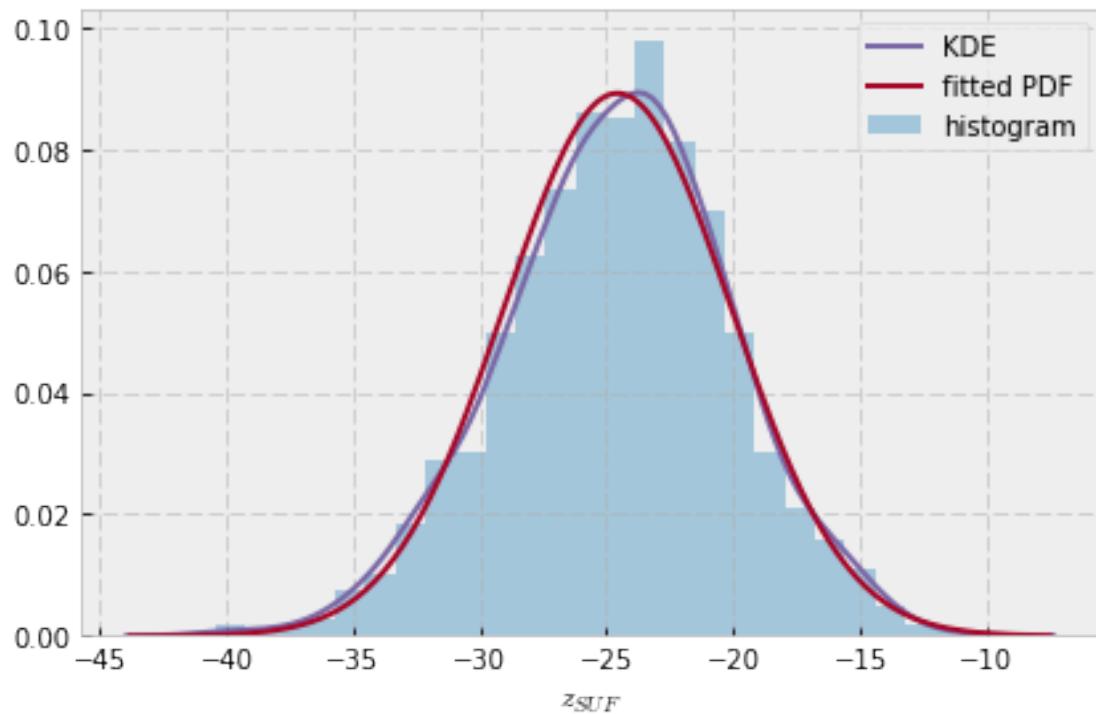


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(),loc=-22.155468374409, scale=4.226201785247434)
DescribeResult(nobs=1000, minmax=(-39.09220673000001, -6.876115021999996)
               mean=-22.155468374409001, variance=17.878660189818419,
               skewness=0.04527251562476141, kurtosis=0.11359207983904396)
```

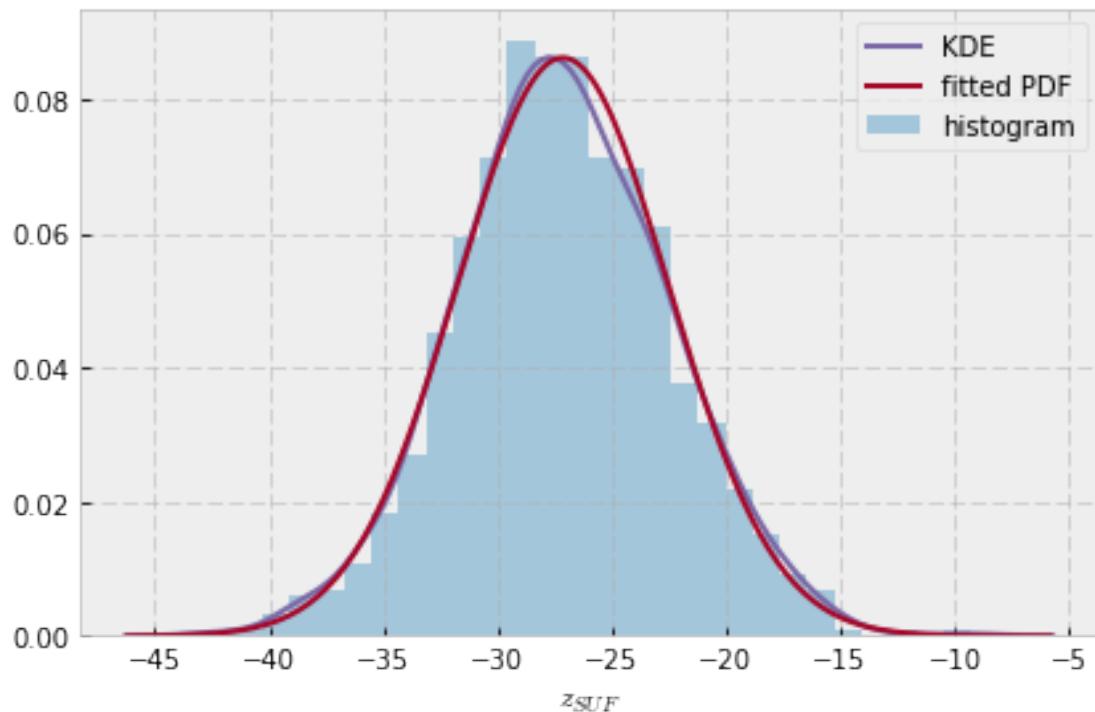


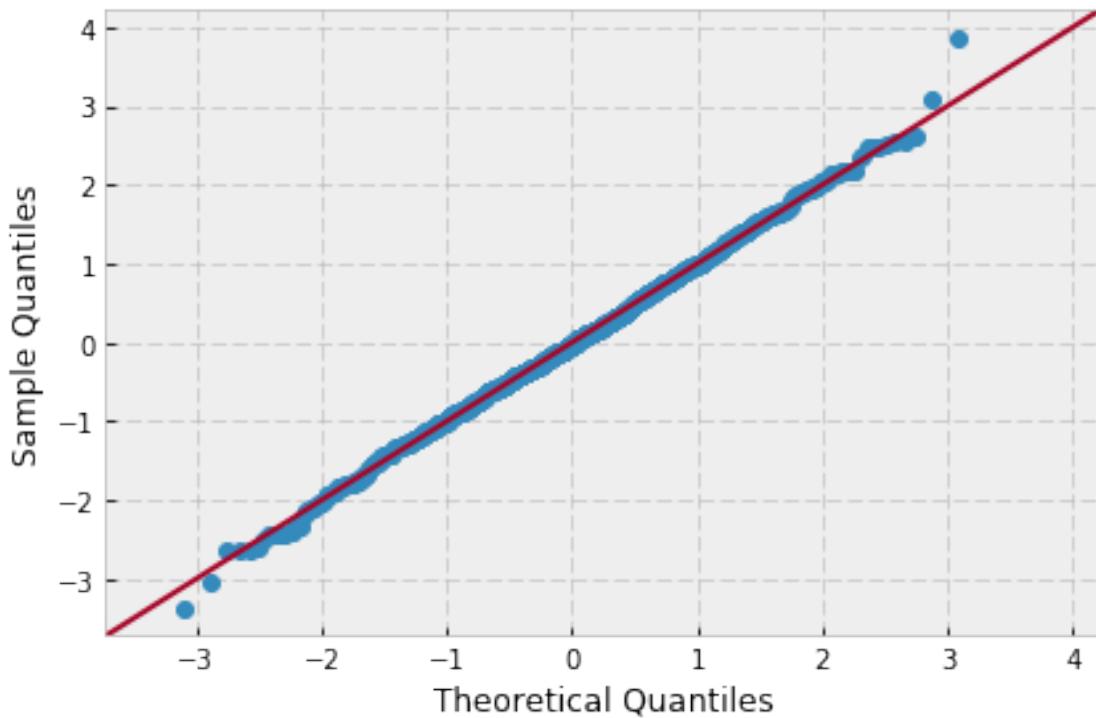


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(),loc=-24.56386907415, scale=4.470164468633611)
DescribeResult(nobs=1000, minmax=(-40.436687790000001, -10.85727041)
               mean=-24.563869074149999, variance=20.002372749383799,
               skewness=-0.16577890992185623, kurtosis=0.09735386521394629)
```

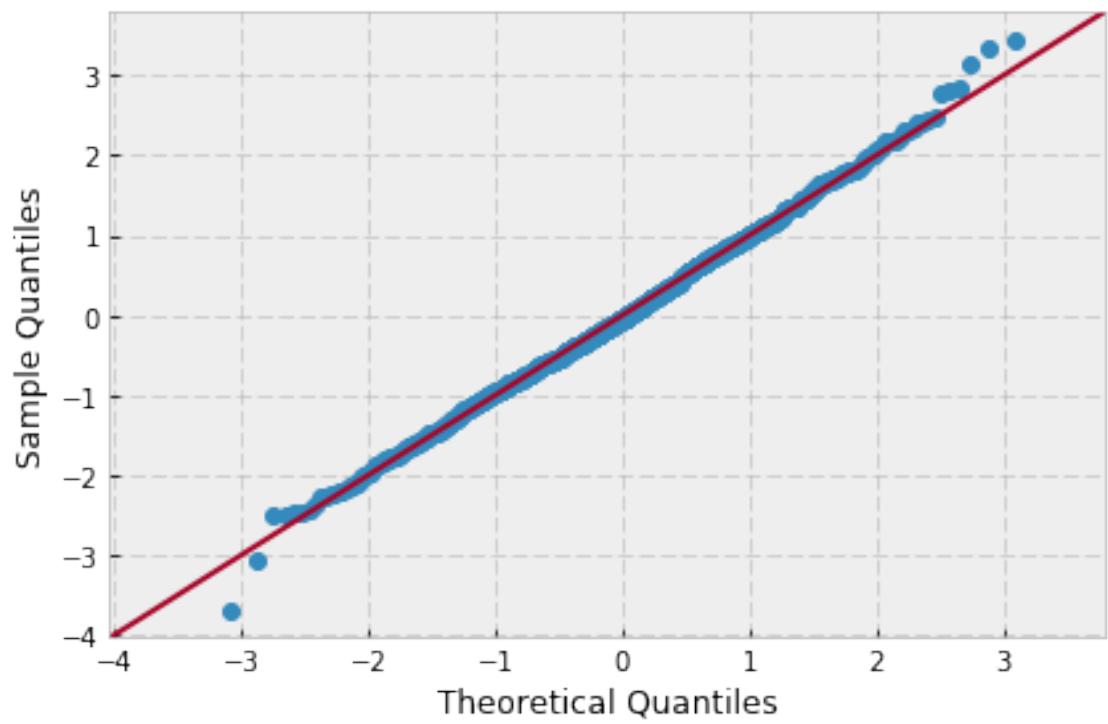
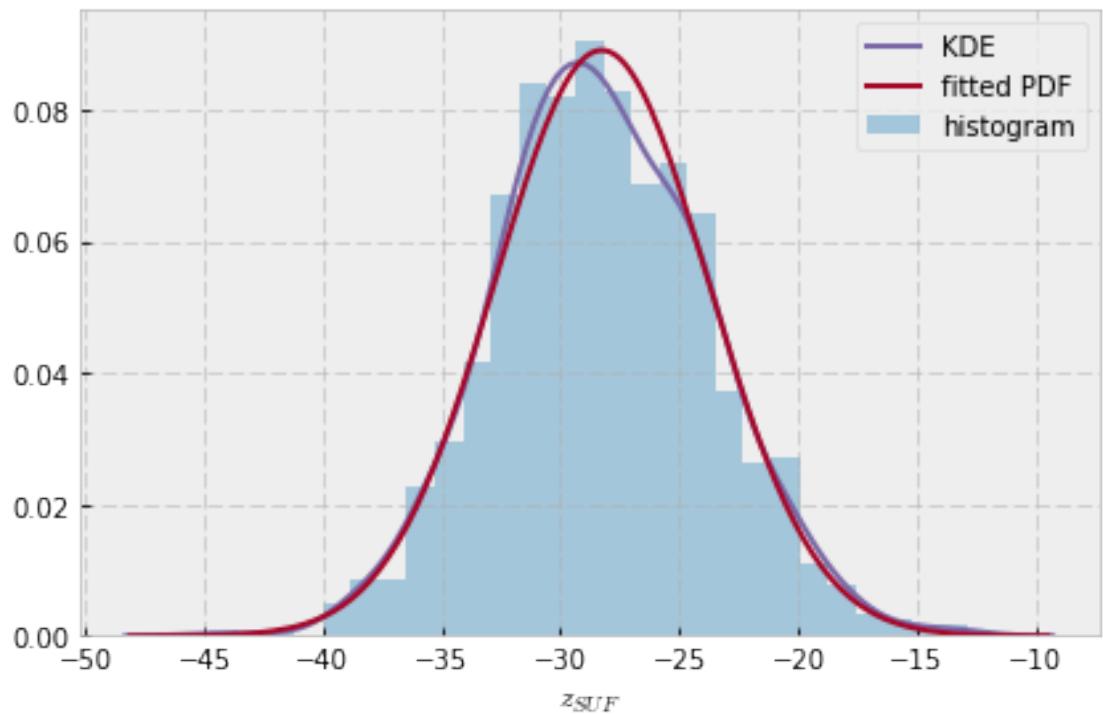


```
variable name: sigma
variable value: 0.05555555555555556
distribution: normal(shape=(),loc=-27.133159017997, scale=4.631496149063794)
DescribeResult(nobs=1000, minmax=(-42.697276090000003, -9.275280636999998)
               mean=-27.133159017996999, variance=21.472228807600352,
               skewness=0.04331079290780273, kurtosis=0.11812920446603048)
```

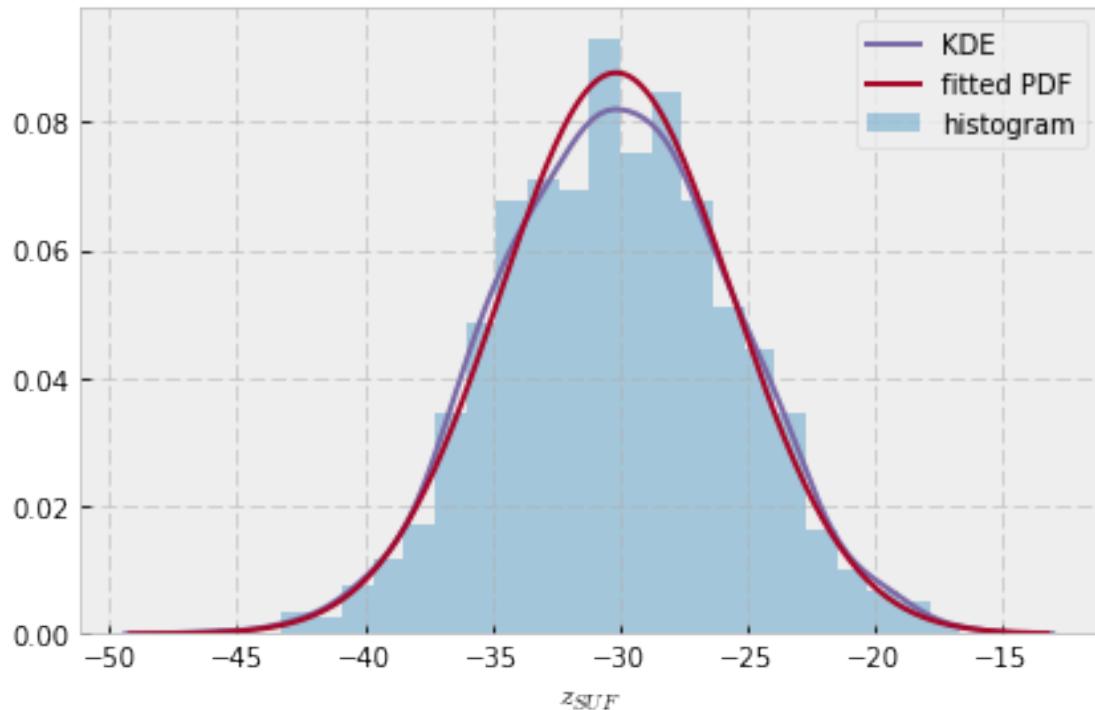


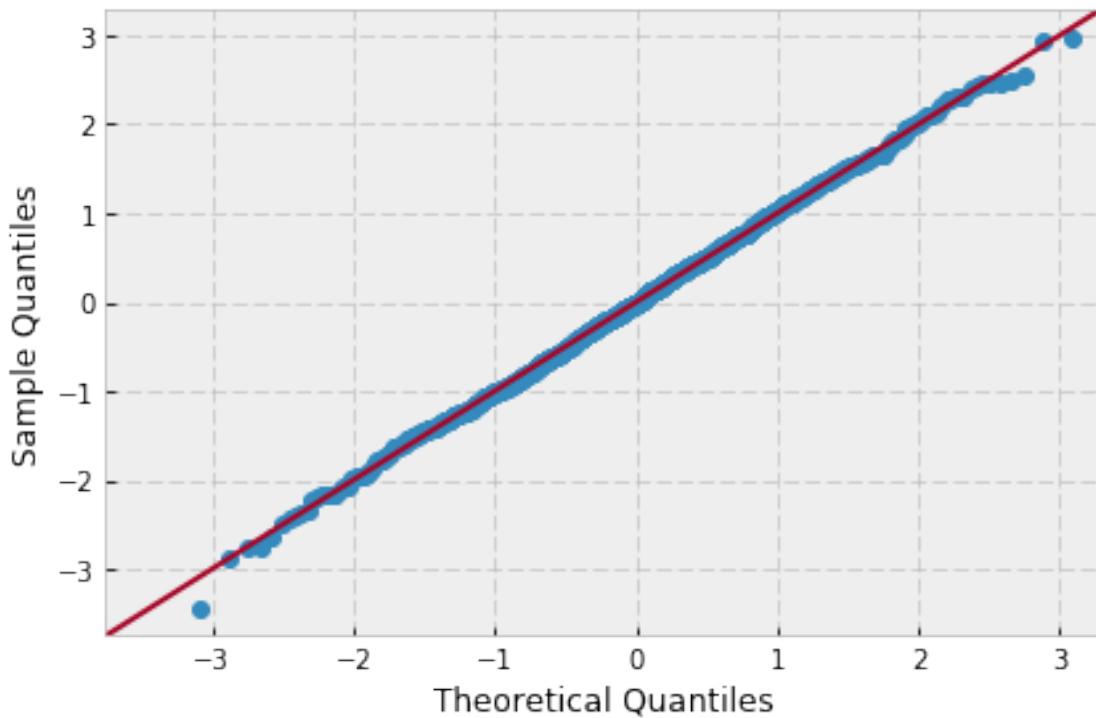


```
variable name: sigma
variable value: 0.0666666666666667
distribution: normal(shape=(),loc=-28.26045647754, scale=4.4794191441547655)
DescribeResult(nobs=1000, minmax=(-44.72728280999999, -12.85536894)
               mean=-28.26045647754, variance=20.085281150170378,
               skewness=0.10480242465546906, kurtosis=0.1175017258164206)
```

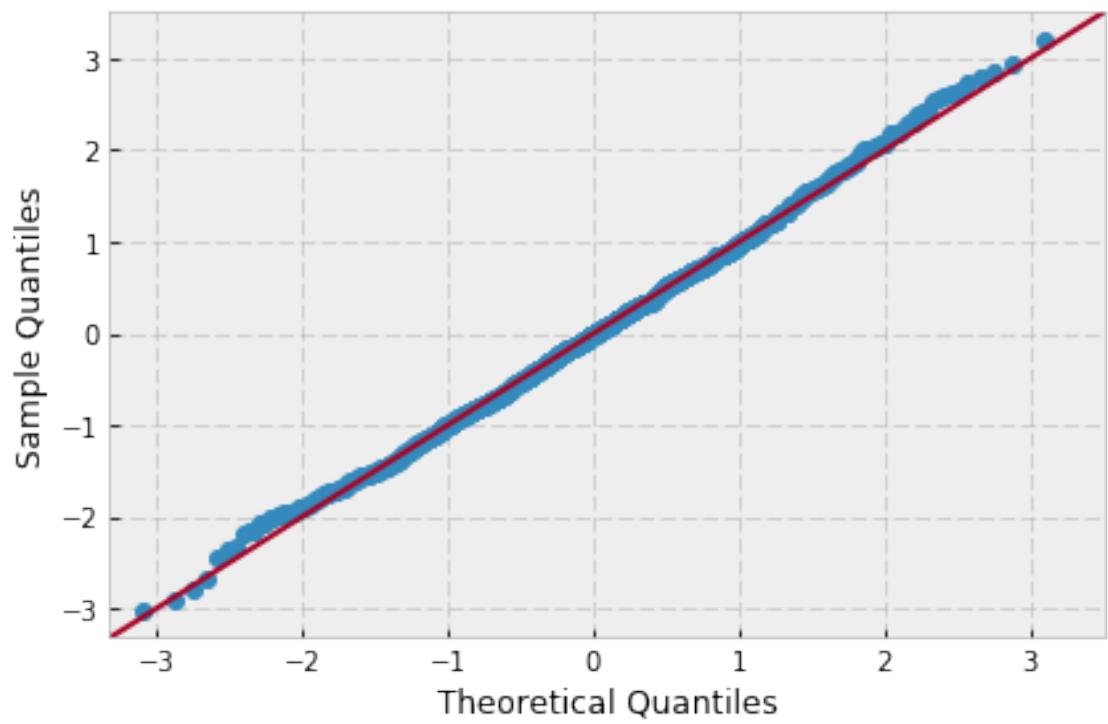
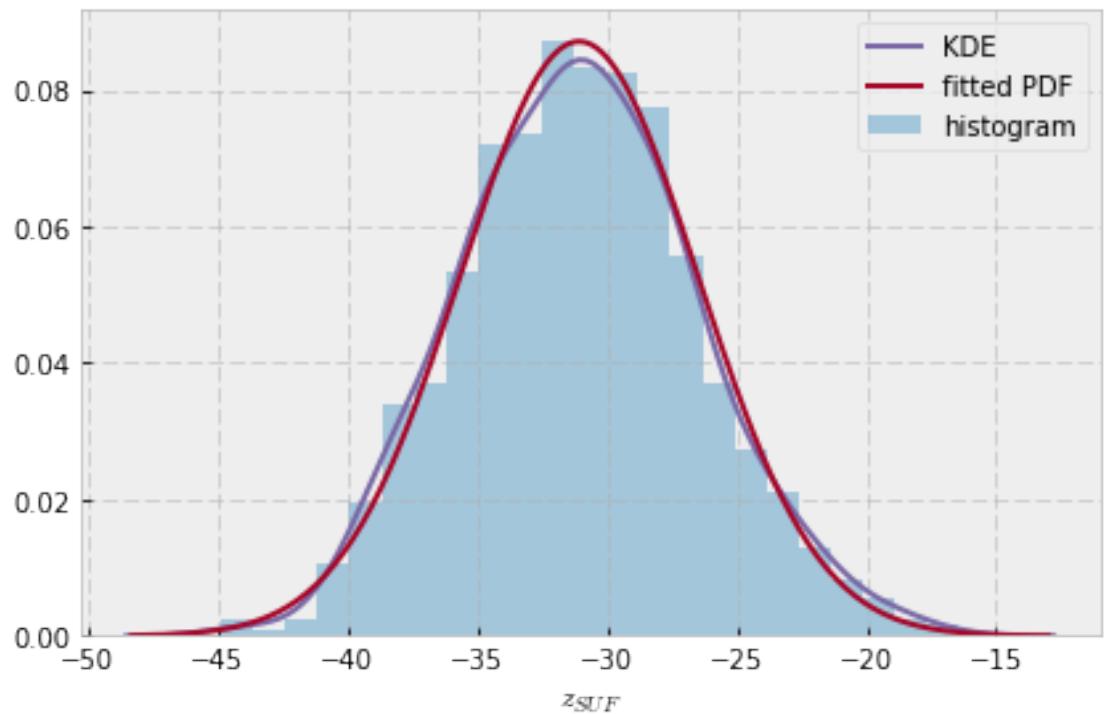


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=-30.126021530040003, scale=4.549401715337685)
DescribeResult(nobs=1000, minmax=(-45.748461059999997, -16.63371643)
               mean=-30.126021530040003, variance=20.717773741258728,
               skewness=0.00012016768079073046, kurtosis=-0.1501033981761215)
```

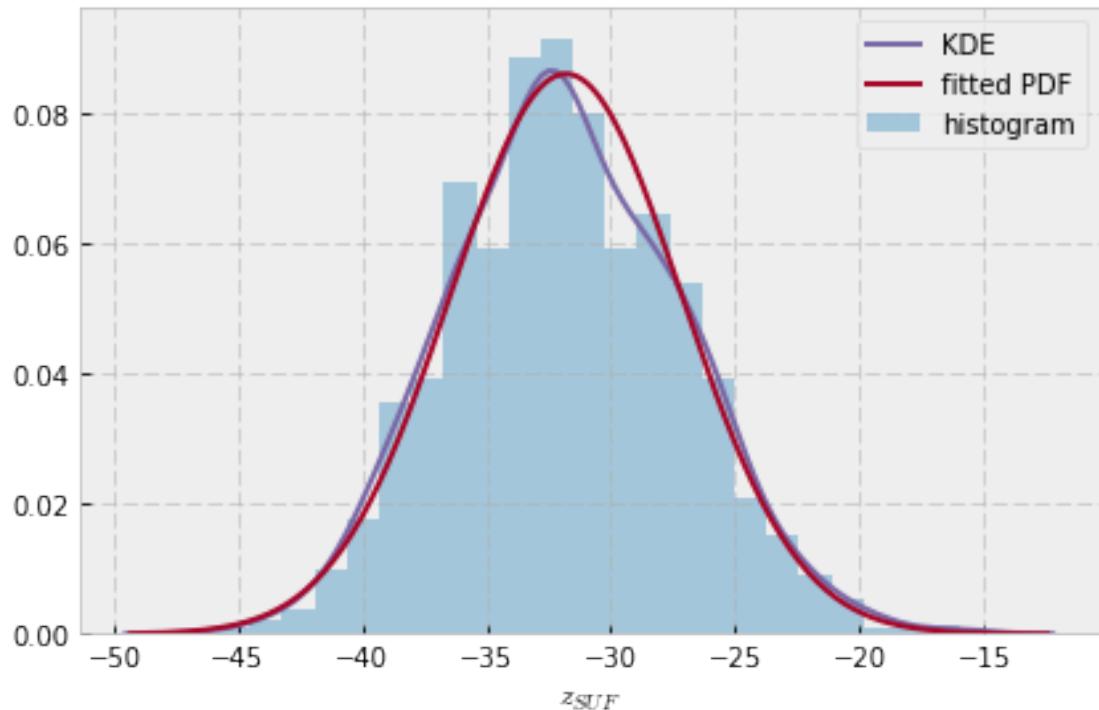


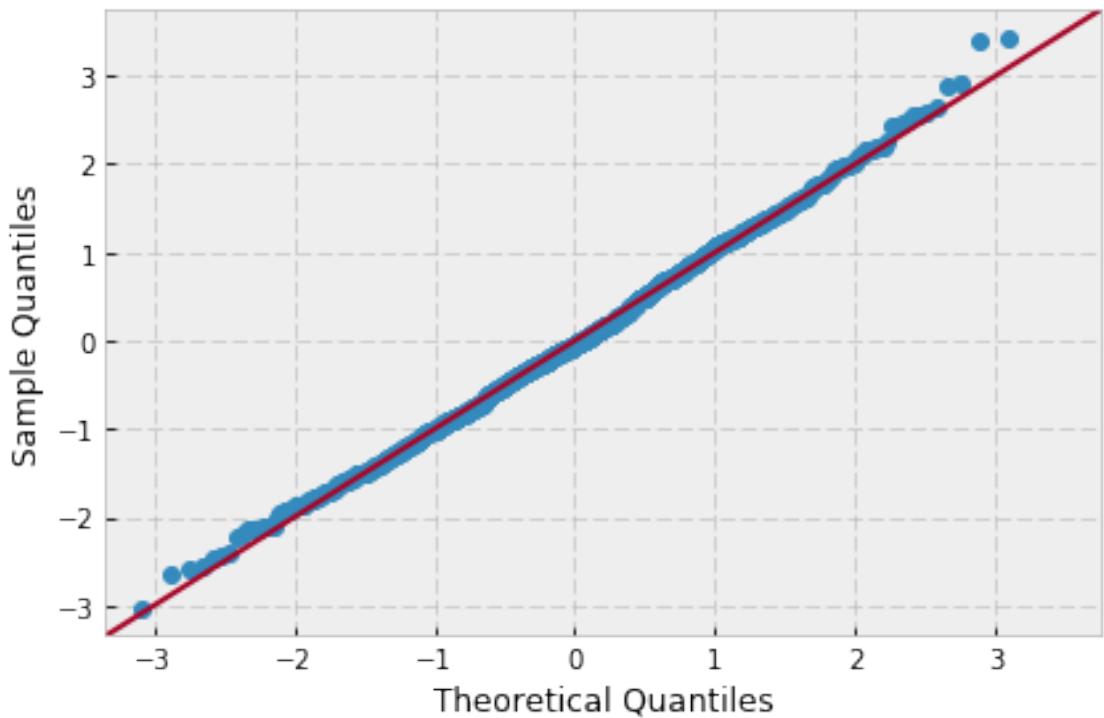


```
variable name: sigma
variable value: 0.0888888888888889
distribution: normal(shape=(),loc=-31.07777140939, scale=4.5767638450916985)
DescribeResult(nobs=1000, minmax=(-44.86401812999999, -16.44575699)
               mean=-31.077771409389999, variance=20.967735028767315,
               skewness=0.1382781012766355, kurtosis=-0.05915320530828527)
```

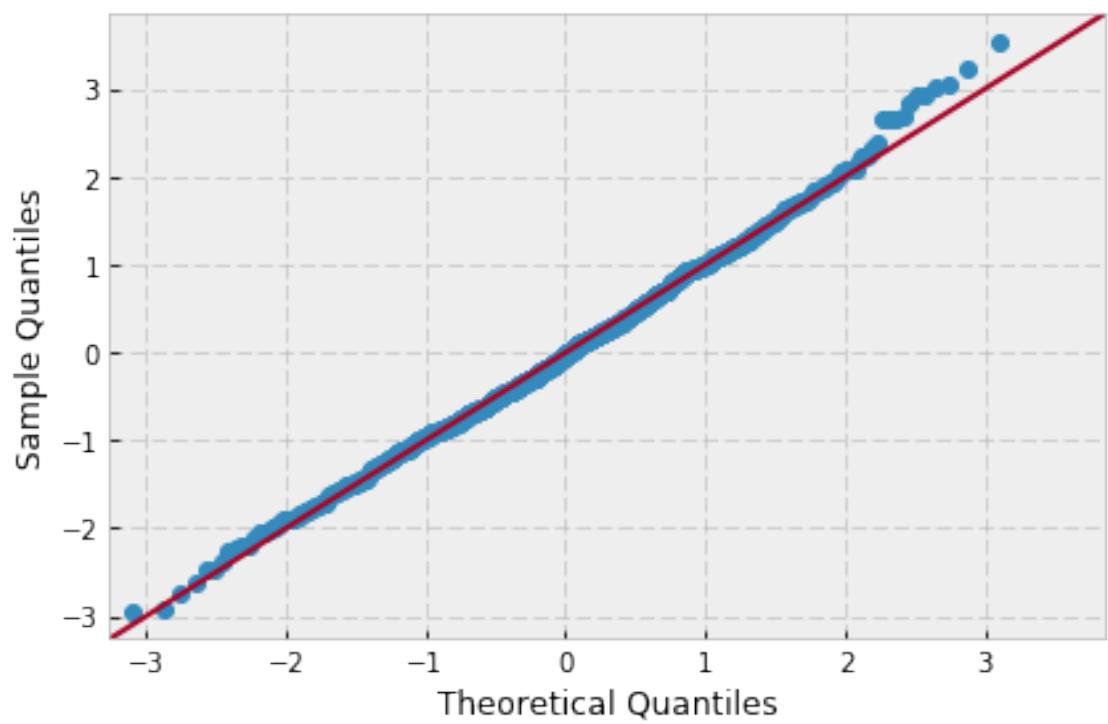
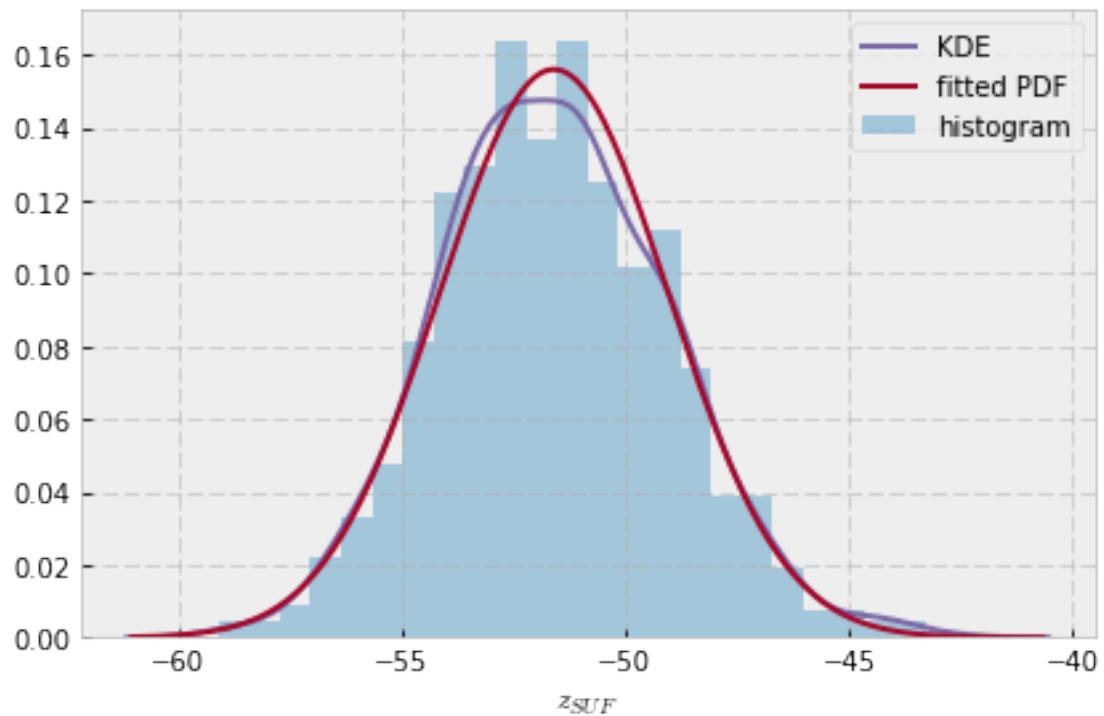


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=-31.80918807846, scale=4.64087571087459)
DescribeResult(nobs=1000, minmax=(-45.834588500000002, -15.922513220000001)
               mean=-31.80918807846, variance=21.559286650436164,
               skewness=0.14094883921761642, kurtosis=-0.0817944722226699)
```

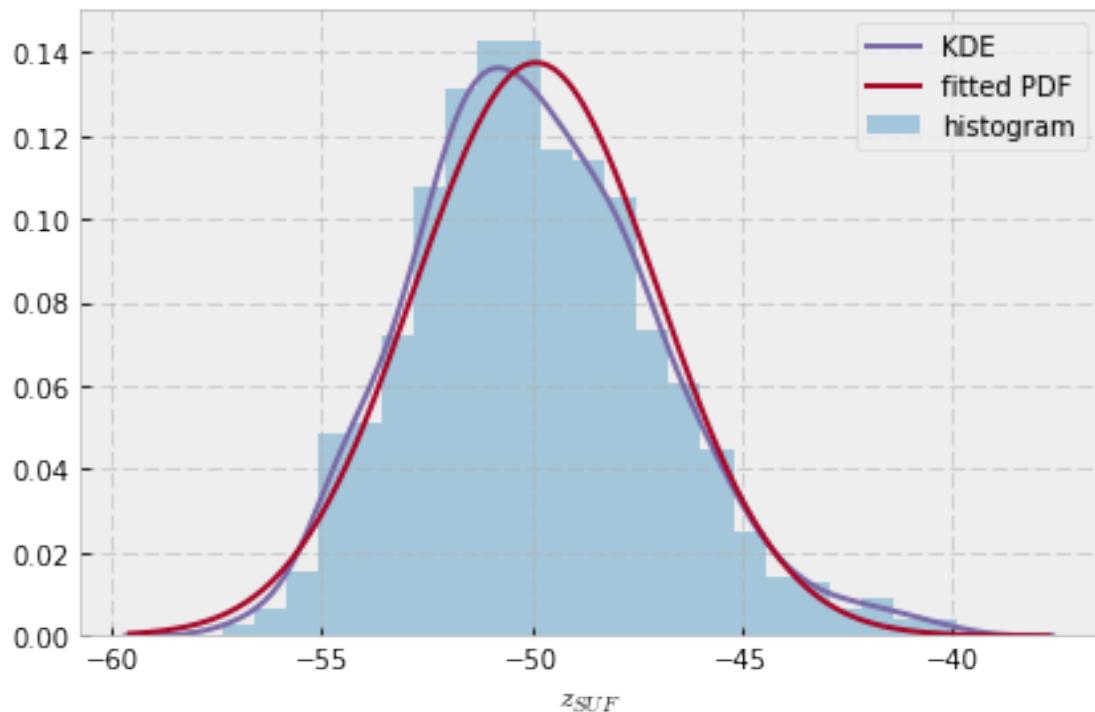


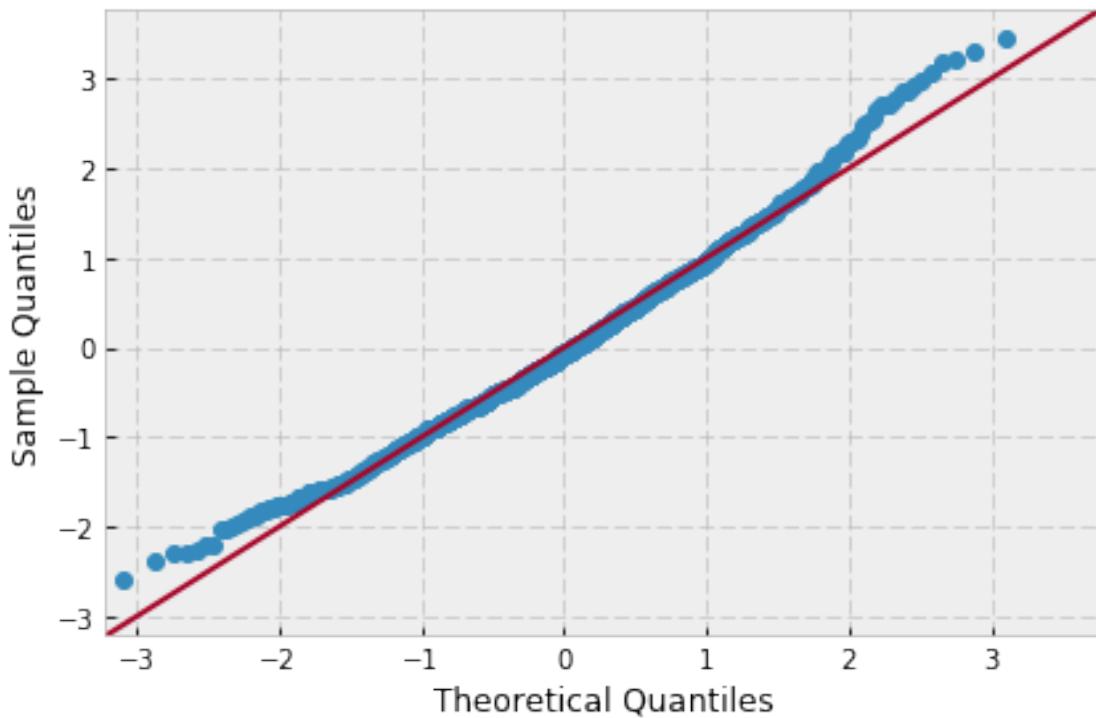


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(),loc=-51.59046151224, scale=2.55730862842361)
DescribeResult(nobs=1000, minmax=(-59.101023470000001, -42.573181419999997)
               mean=-51.590461512239997, variance=6.5463737948046496,
               skewness=0.1852340148387277, kurtosis=0.09752971912653274)
```

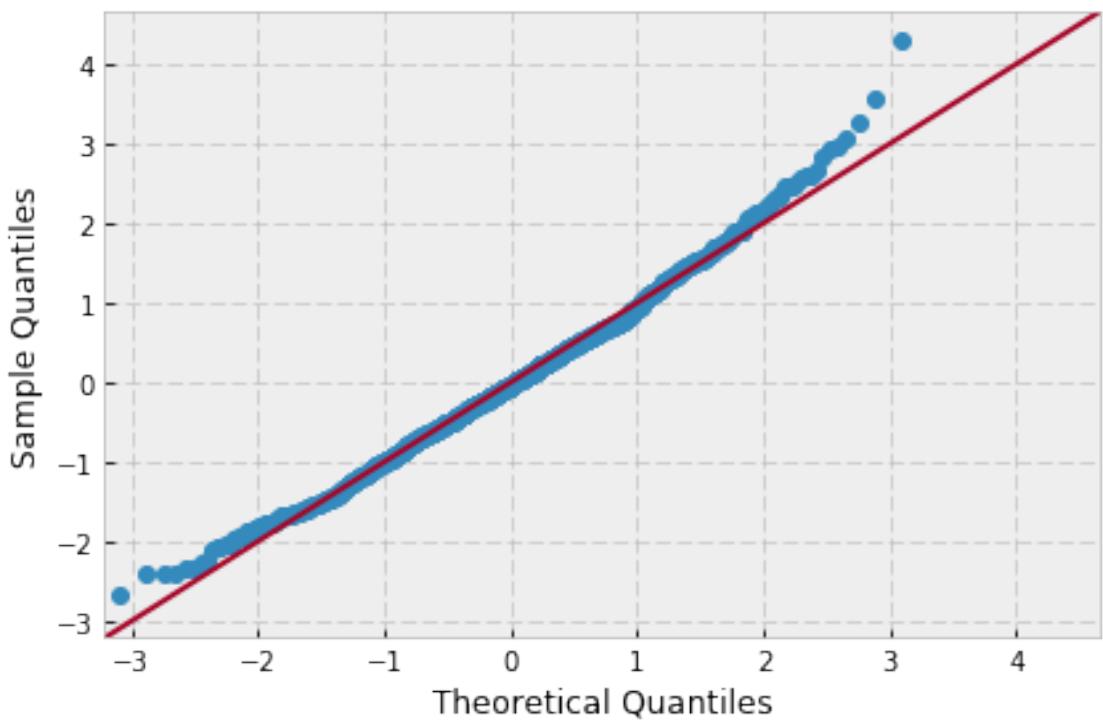
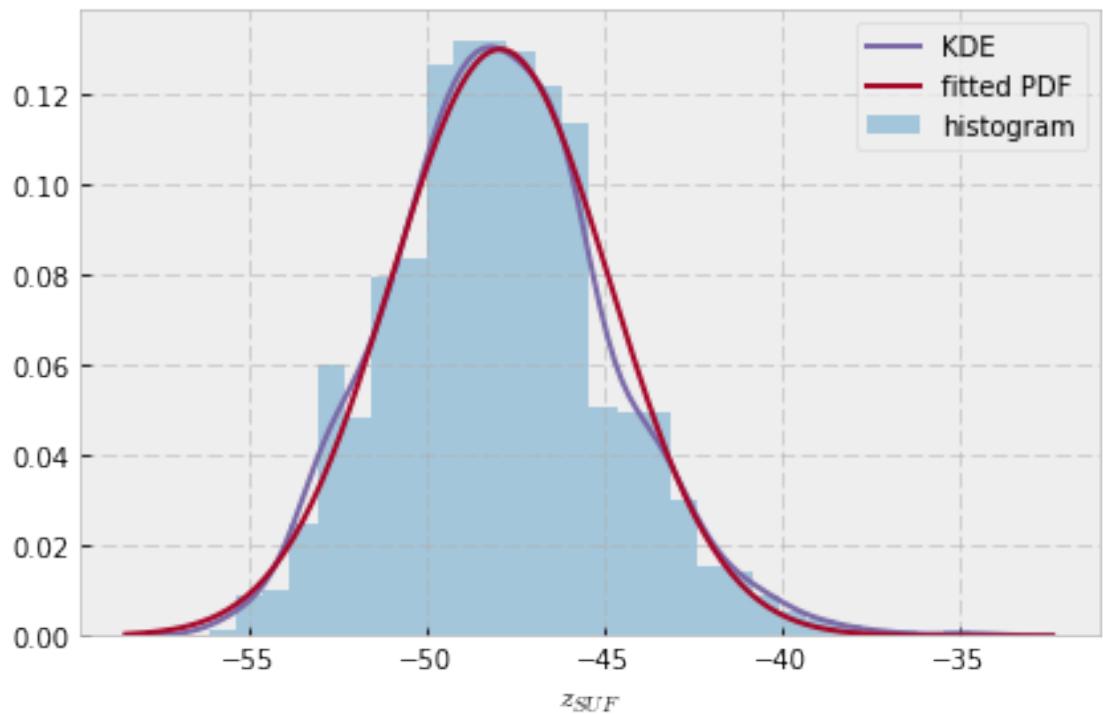


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=-49.900596653250005, scale=2.9007123222156803)
DescribeResult(nobs=1000, minmax=(-57.39342477999997, -39.86552417)
               mean=-49.900596653250005, variance=8.4225545307846712,
               skewness=0.4074481285312719, kurtosis=0.18202623602578472)
```

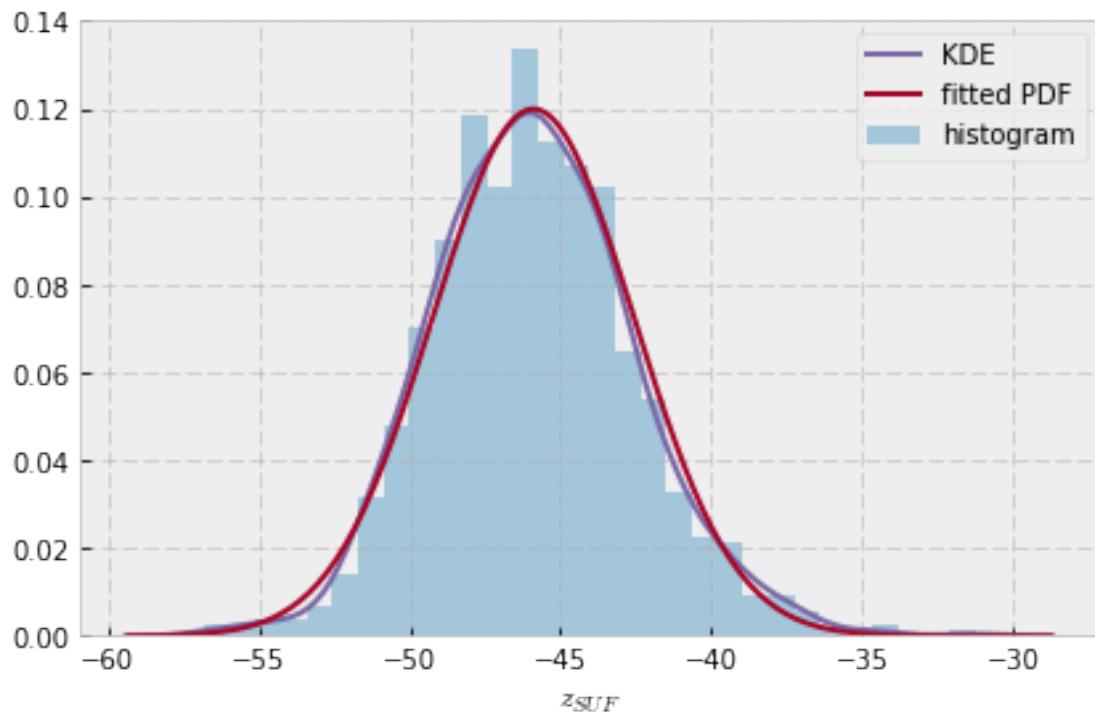


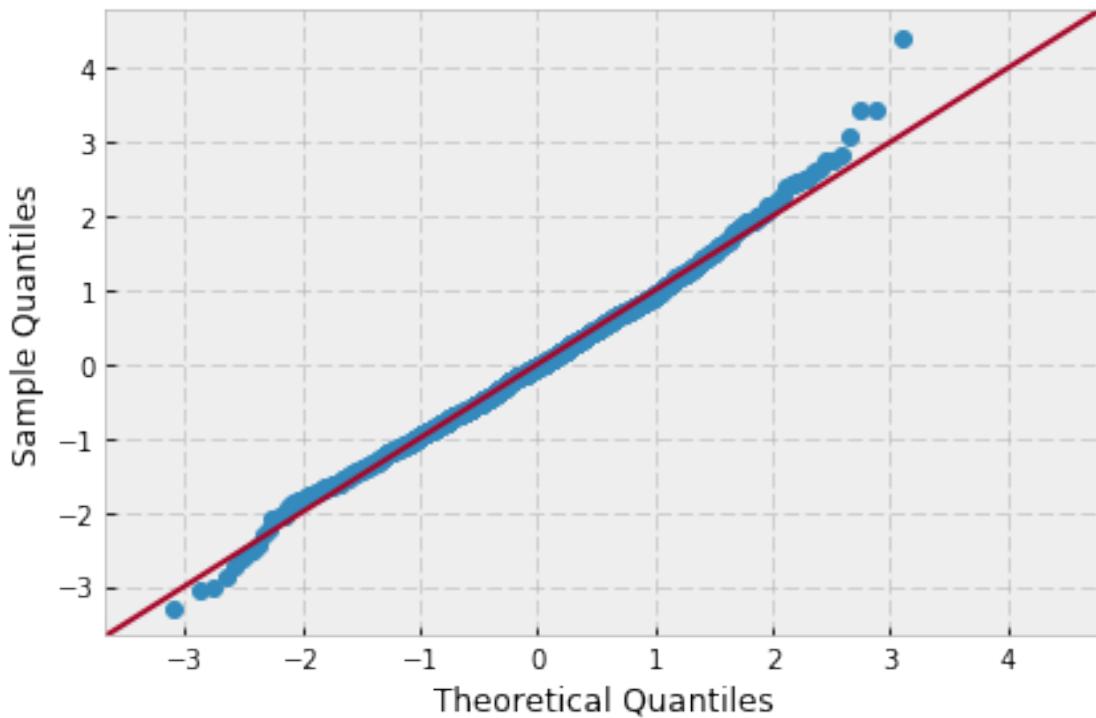


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(),loc=-47.90042641672, scale=3.0743290207144667)
DescribeResult(nobs=1000, minmax=(-56.118717320000002, -34.668823109999998)
               mean=-47.900426416720002, variance=9.460959887494667,
               skewness=0.32607262564890727, kurtosis=0.3325835767069938)
```

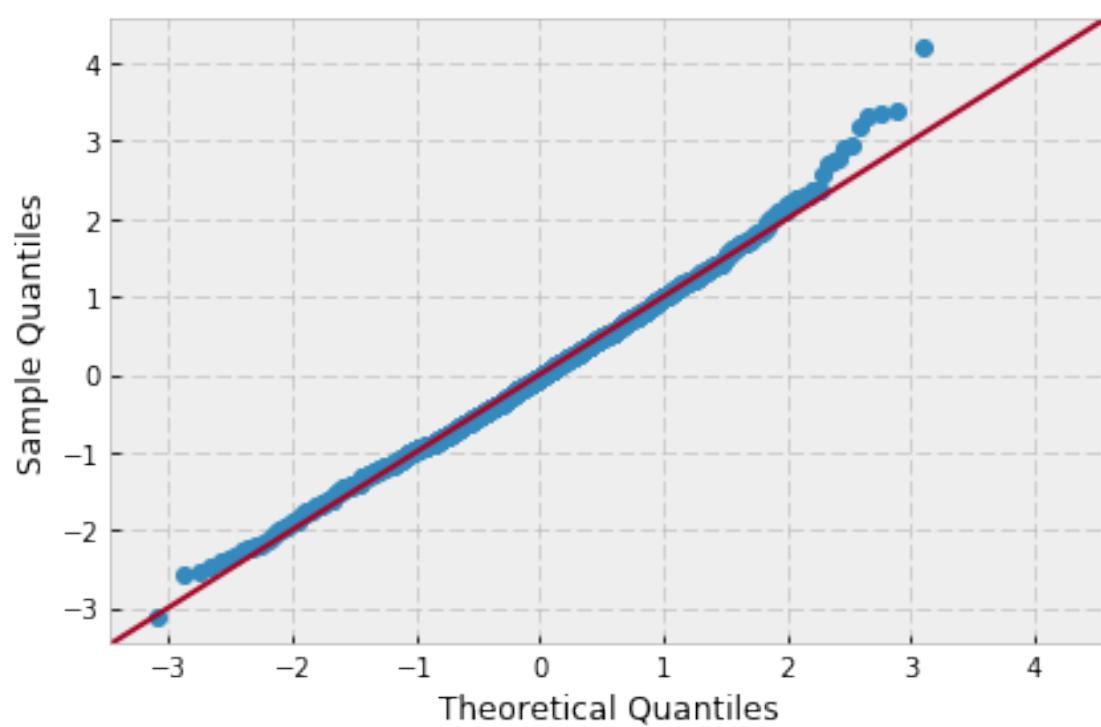
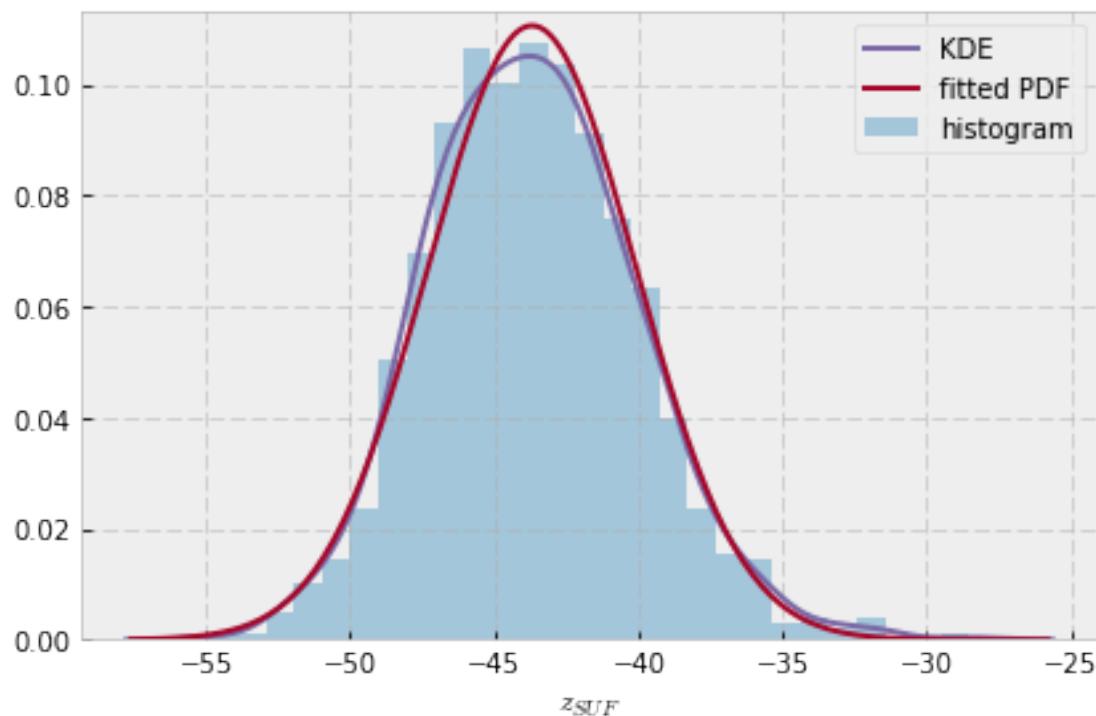


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(),loc=-45.900869858830006, scale=3.3269301368465607)
DescribeResult(nobs=1000, minmax=(-56.82767252, -31.2698684)
               mean=-45.900869858830006, variance=11.079543679137011,
               skewness=0.2505095838456253, kurtosis=0.5378100557441057)
```

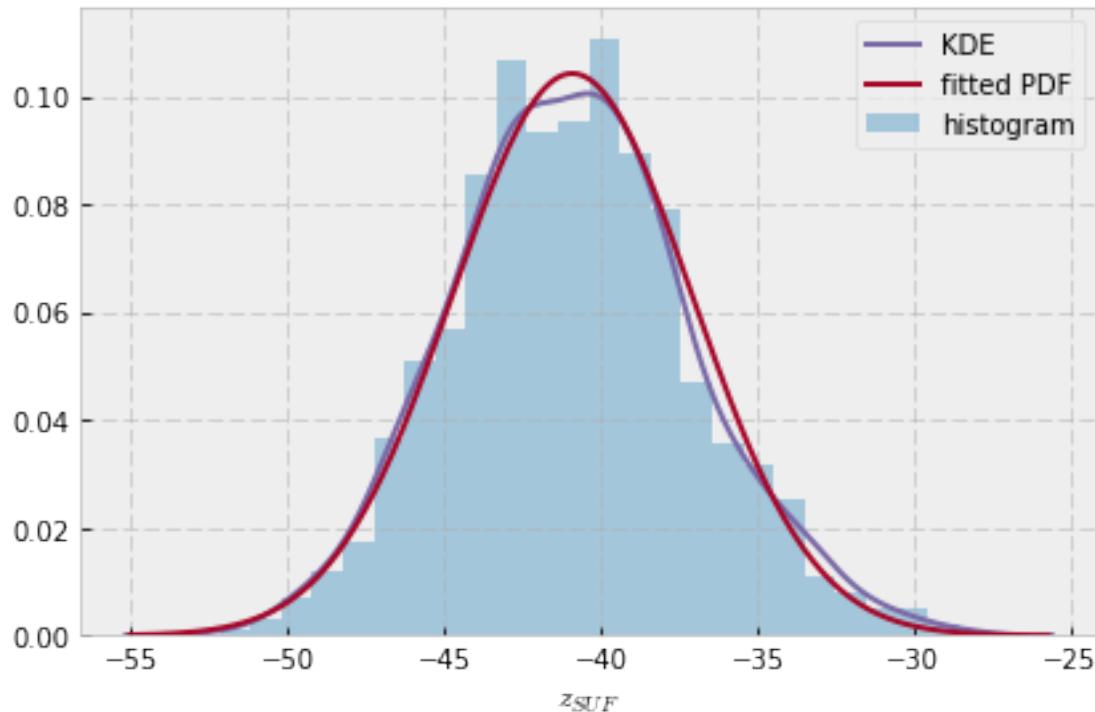


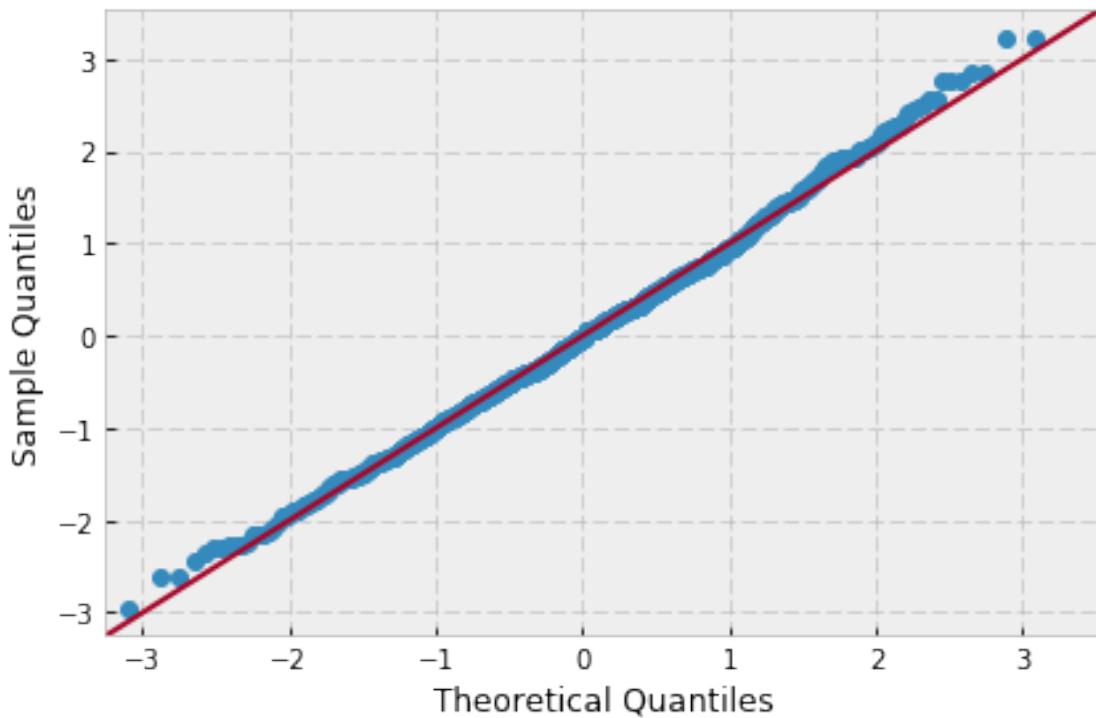


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(),loc=-43.70157718175, scale=3.608992819346678)
DescribeResult(nobs=1000, minmax=(-54.88280535999999, -28.53601518)
               mean=-43.701577181749997, variance=13.037867037133017,
               skewness=0.2993064539091108, kurtosis=0.3577749363582612)
```

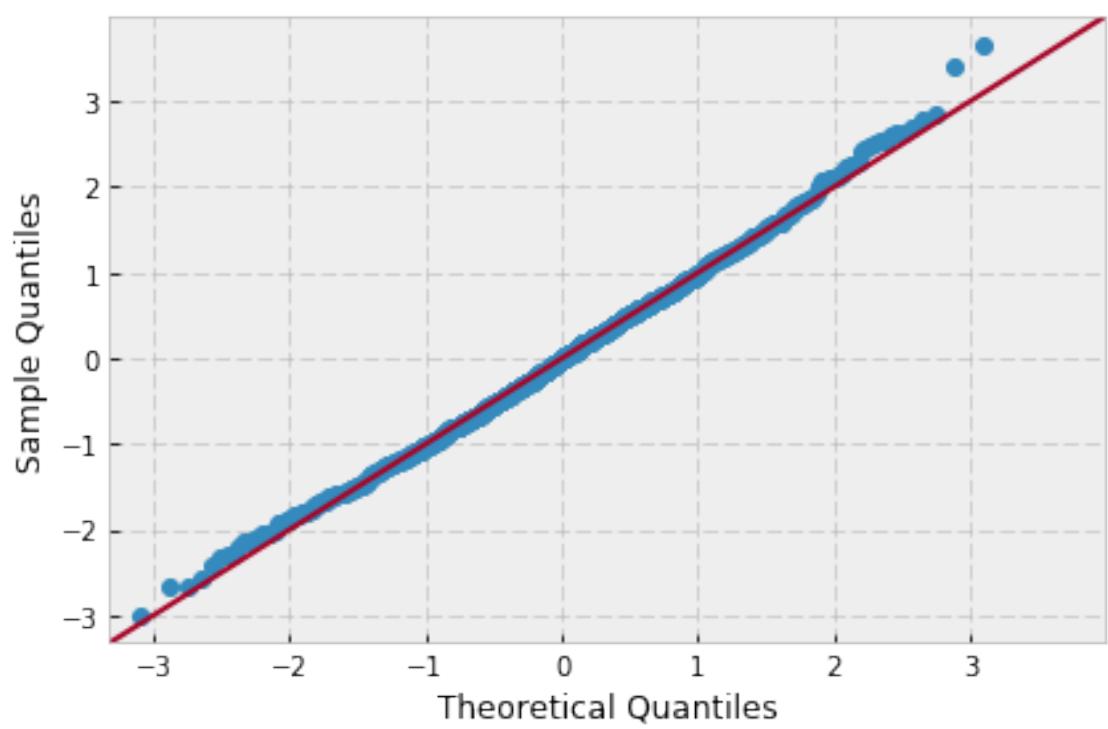
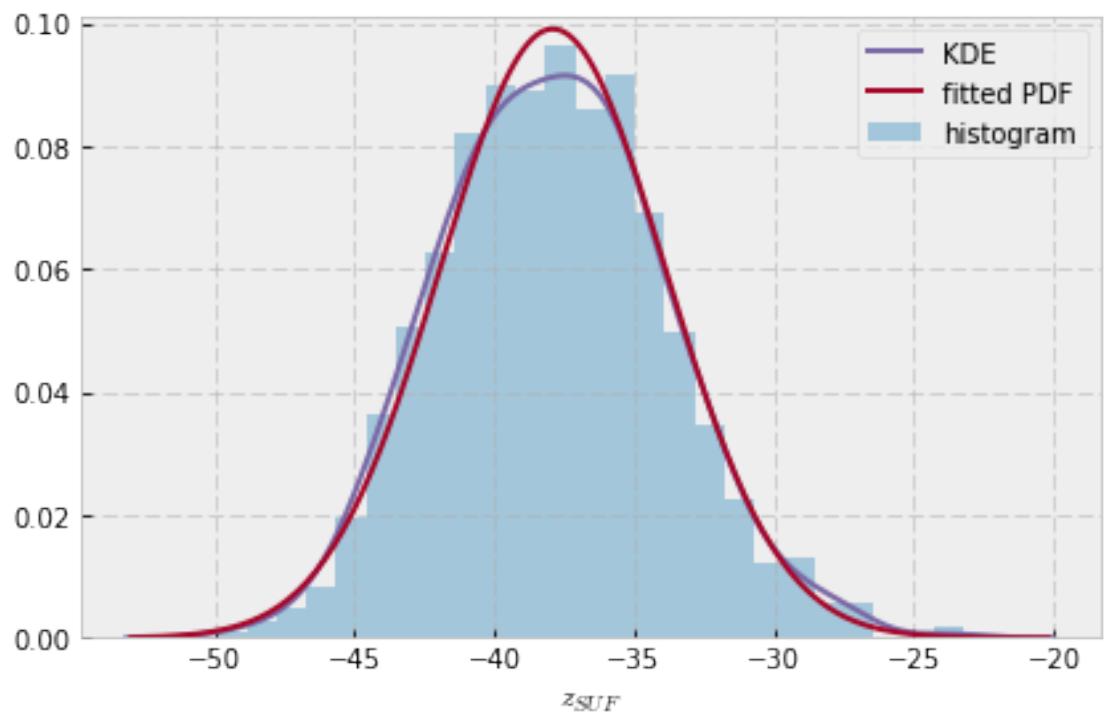


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=-40.904056081019995, scale=3.827872924546665)
DescribeResult(nobs=1000, minmax=(-52.176996879999997, -28.56956164)
               mean=-40.904056081019995, variance=14.667278404882323,
               skewness=0.1874662979884482, kurtosis=0.02296766636265346)
```

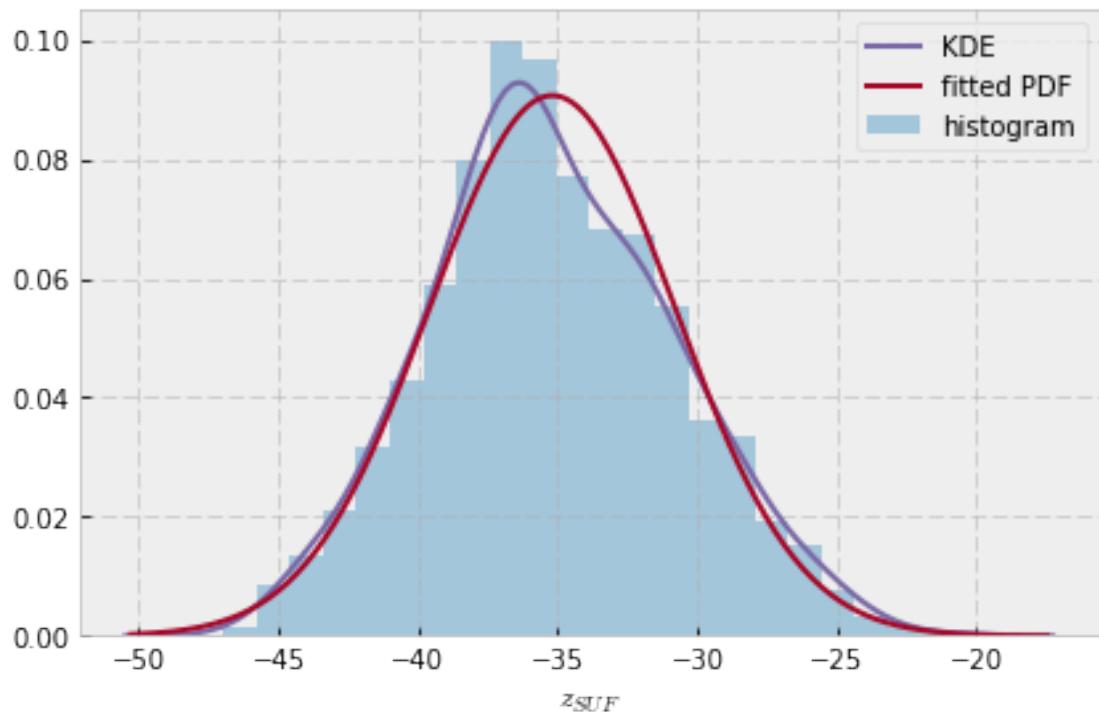


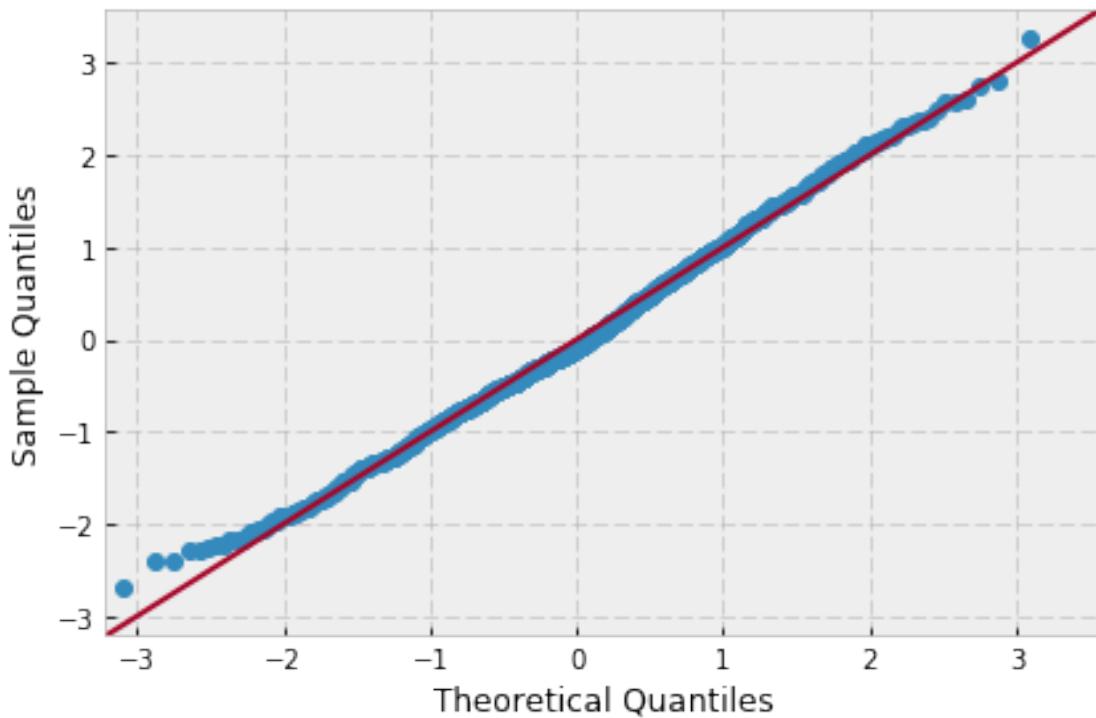


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(),loc=-37.905234185610006, scale=4.027593554397524)
DescribeResult(nobs=1000, minmax=(-49.940468410000001, -23.20245439)
               mean=-37.905234185610006, variance=16.237747587011494,
               skewness=0.17273156250240412, kurtosis=-0.012135504993052493)
```

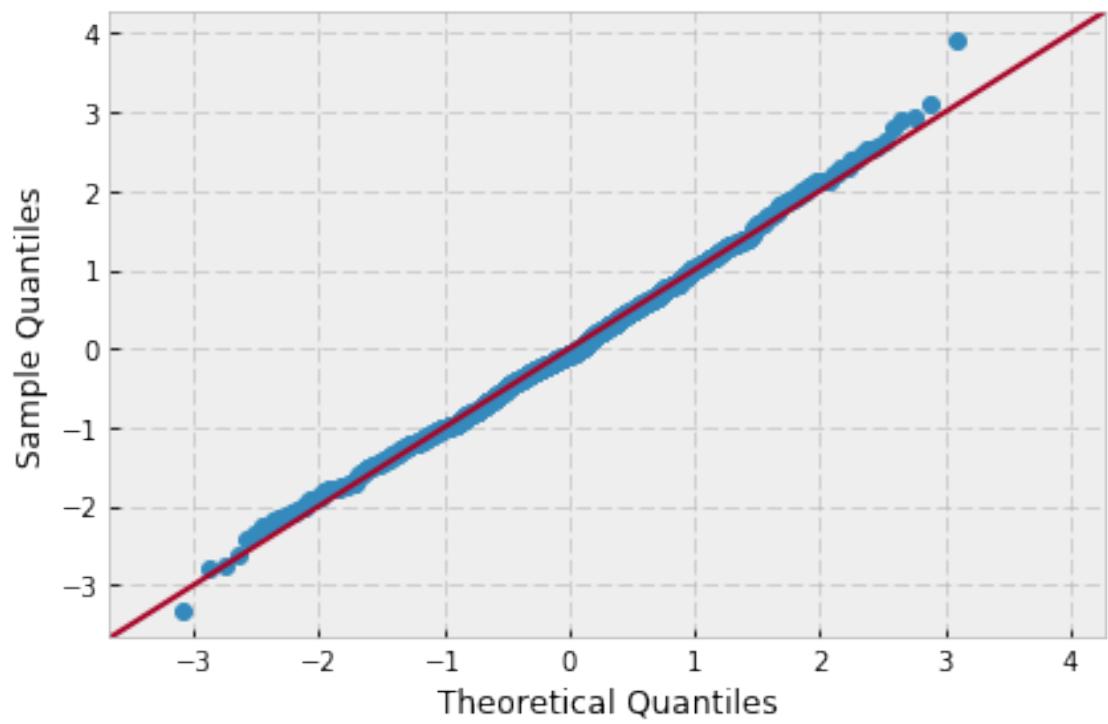
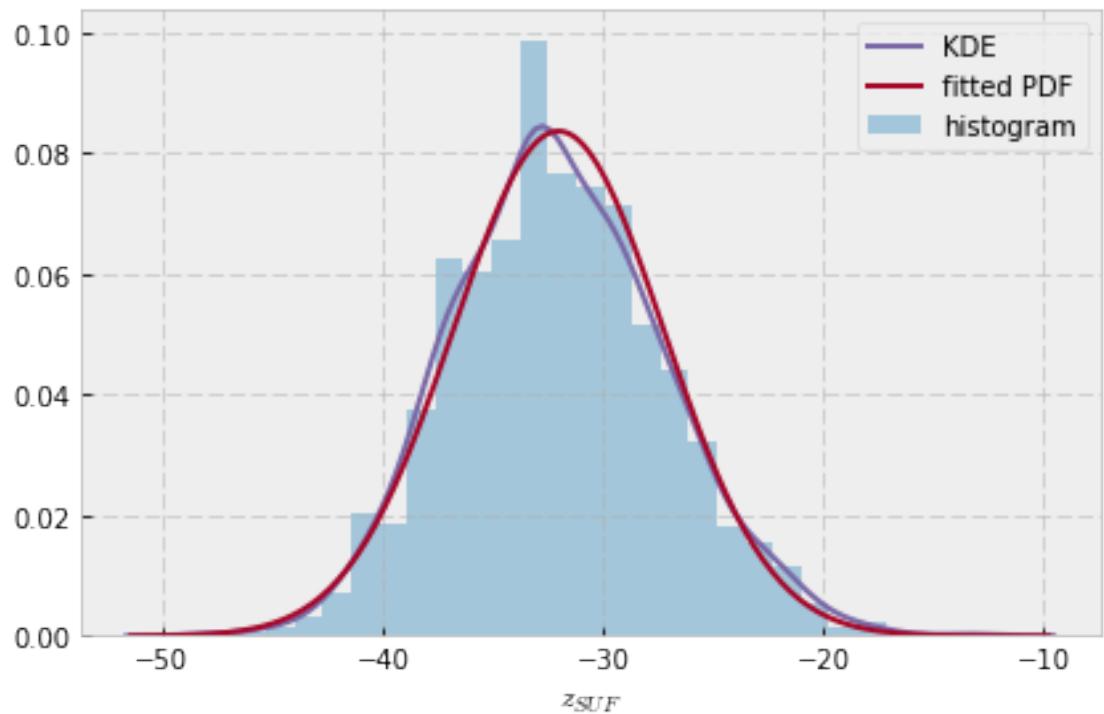


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(),loc=-35.19173212767, scale=4.399794001856719)
DescribeResult(nobs=1000, minmax=(-46.963836780000001, -20.787455980000001)
               mean=-35.191732127670001, variance=19.377564823597961,
               skewness=0.18646897107165492, kurtosis=-0.2254974383707018)
```

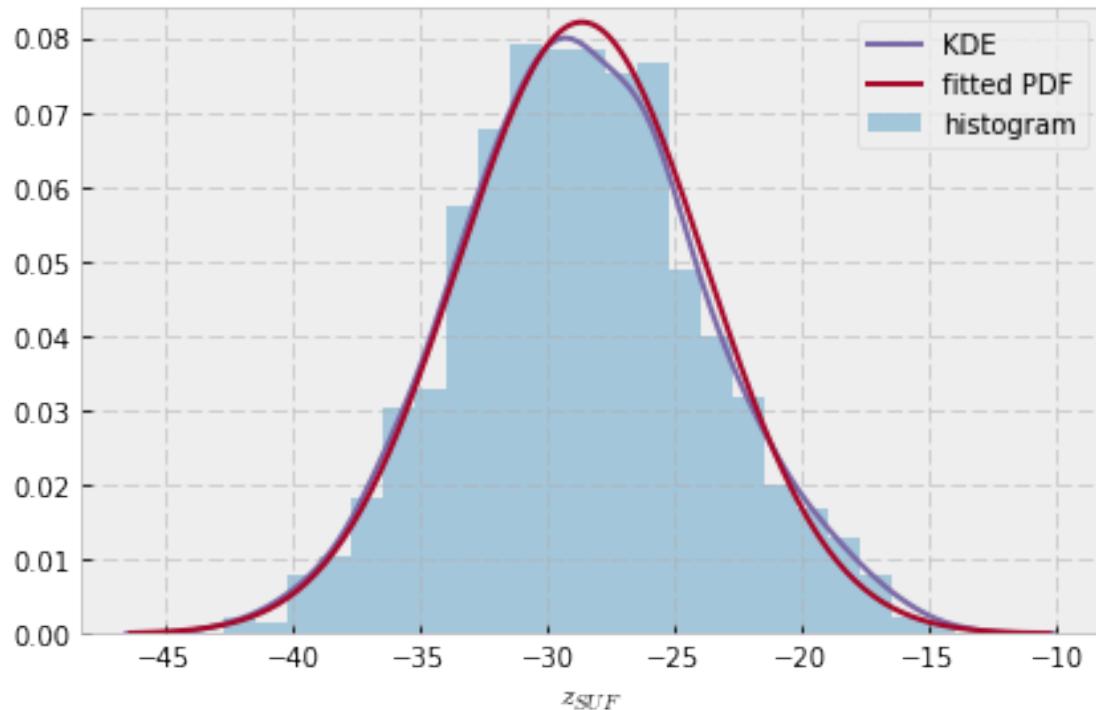


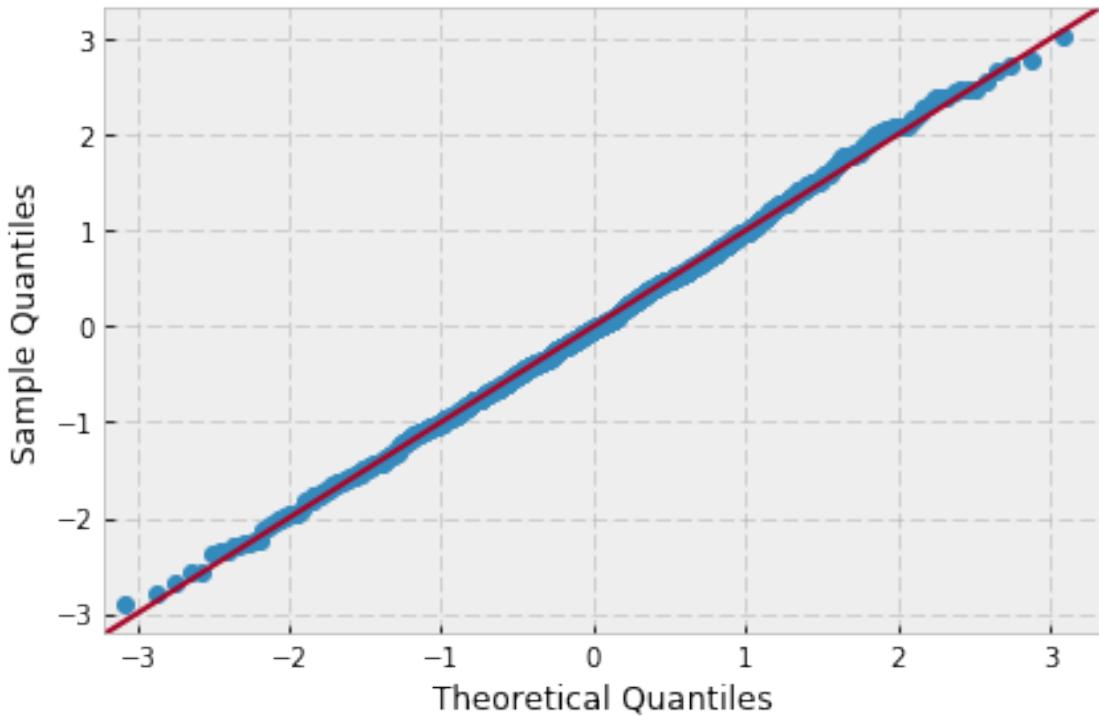


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(),loc=-32.00626315716, scale=4.772154878520703)
DescribeResult(nobs=1000, minmax=(-47.80611294999999, -13.34582466)
               mean=-32.006263157159999, variance=22.796258443031981,
               skewness=0.20349136899082923, kurtosis=0.08980464662606069)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=-28.616662349589998, scale=4.858165211851241)
DescribeResult(nobs=1000, minmax=(-42.69451234999997, -13.900715959999999)
               mean=-28.616662349589998, variance=23.625394620261876,
               skewness=0.10202333341882726, kurtosis=-0.10961524860517002)
```



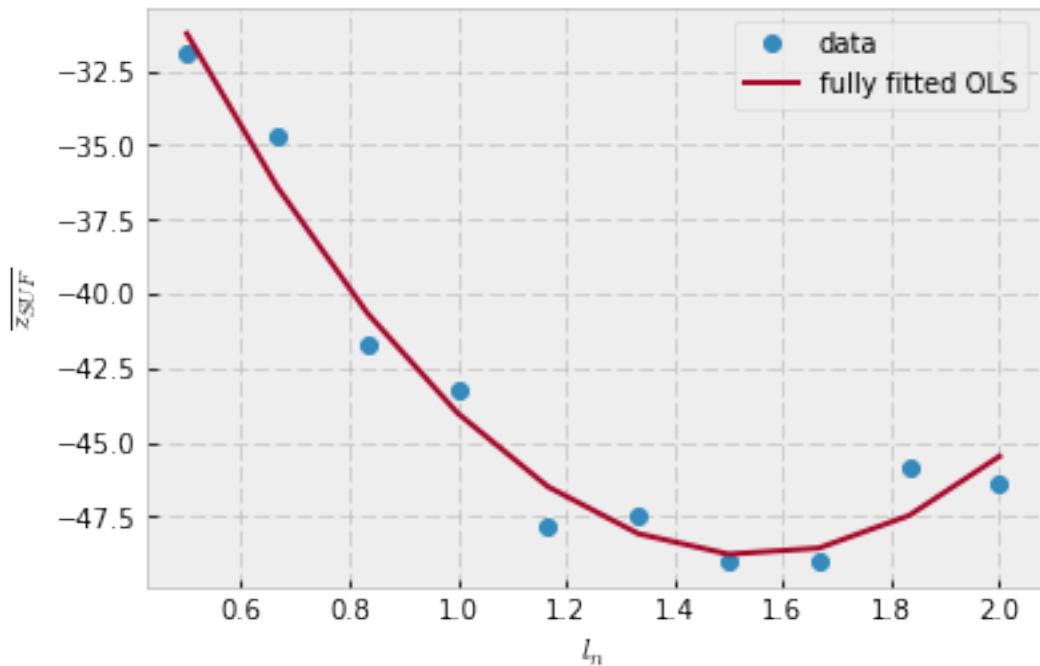


19.2 Parameter regression

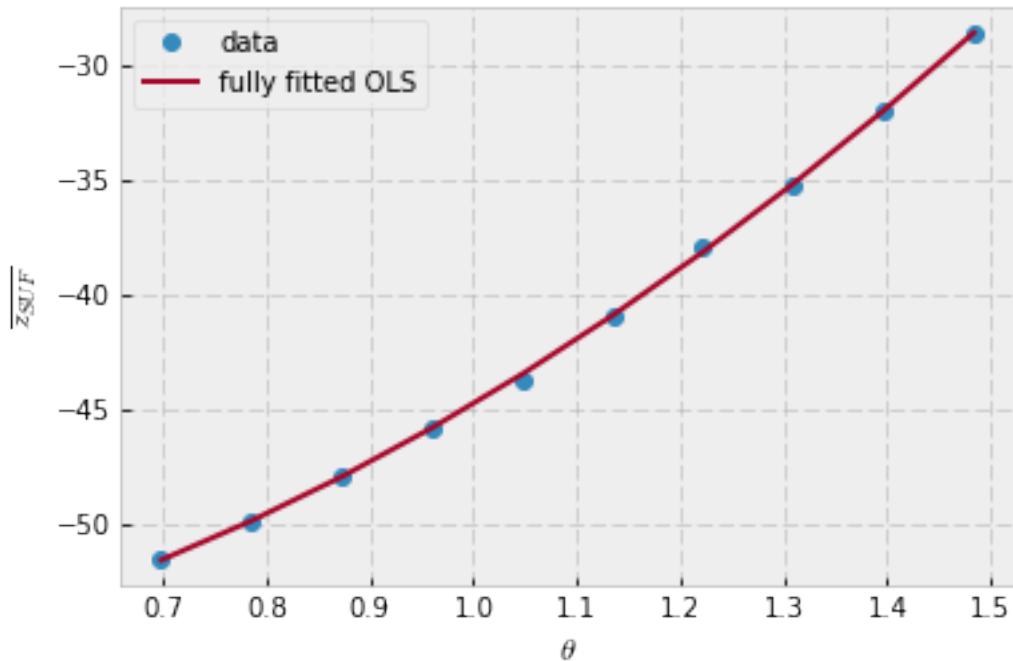
```
In [44]: var_name = 'mean_depth_of_standard_root_water_uptake'

mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly2')
mean_reg('maxB', var_name, 'poly1')
mean_reg('r', var_name, 'poly1')
mean_reg('sigma', var_name, 'poly2')
mean_reg('N', var_name, 'poly2')

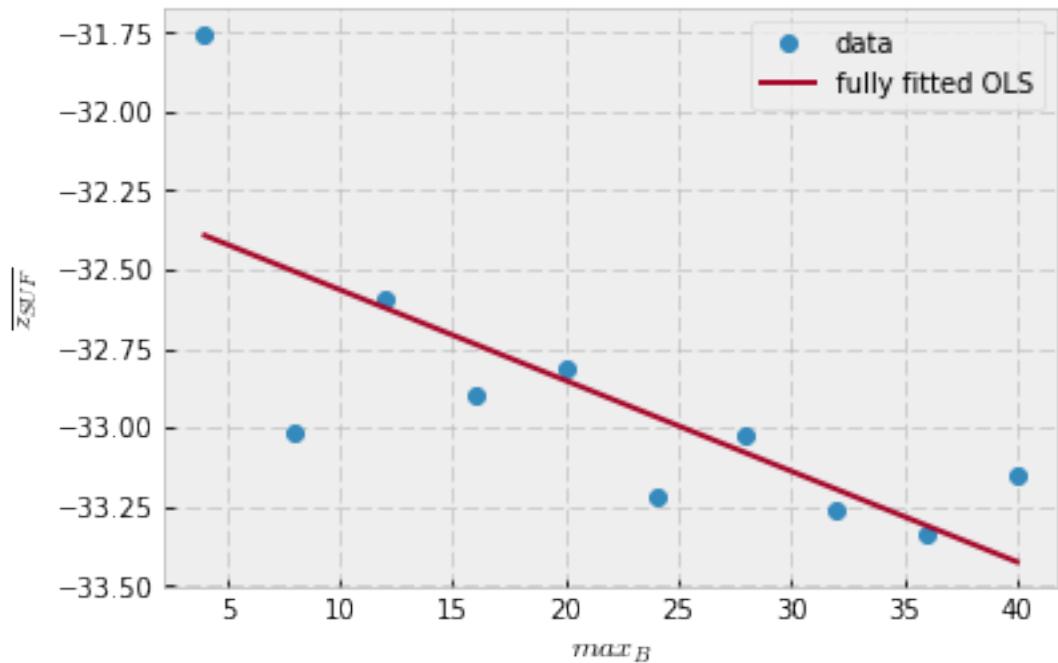
variable name: ln
model: poly2
r-squared: 0.9184665827479067
explained variance: 0.9193871700696293
```



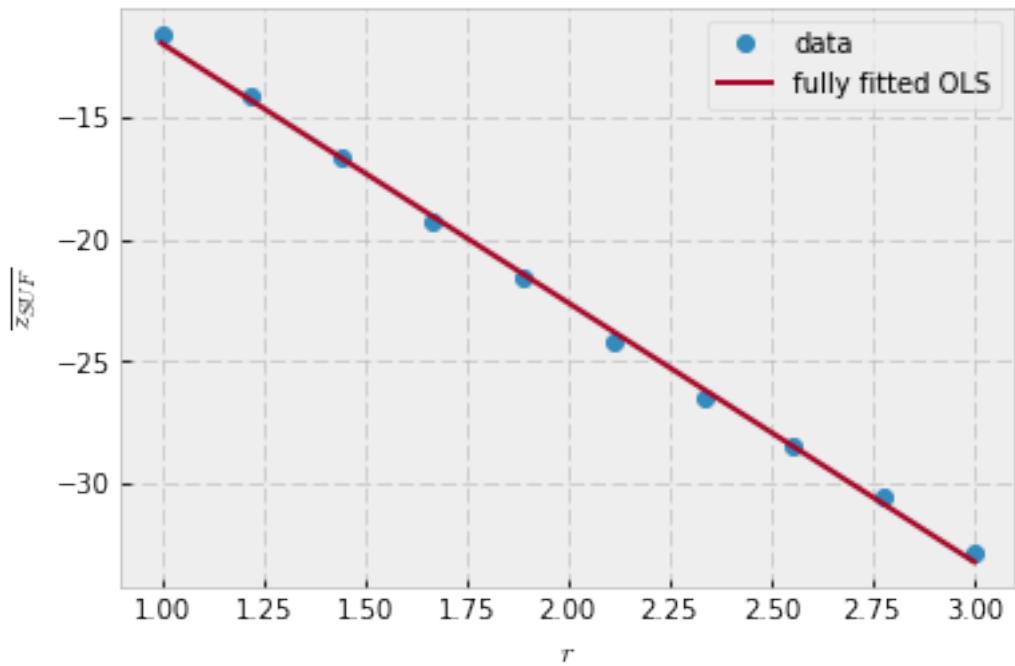
```
variable name: theta
model: poly2
r-squared: 0.9996231518681564
explained variance: 0.9996234211736845
```



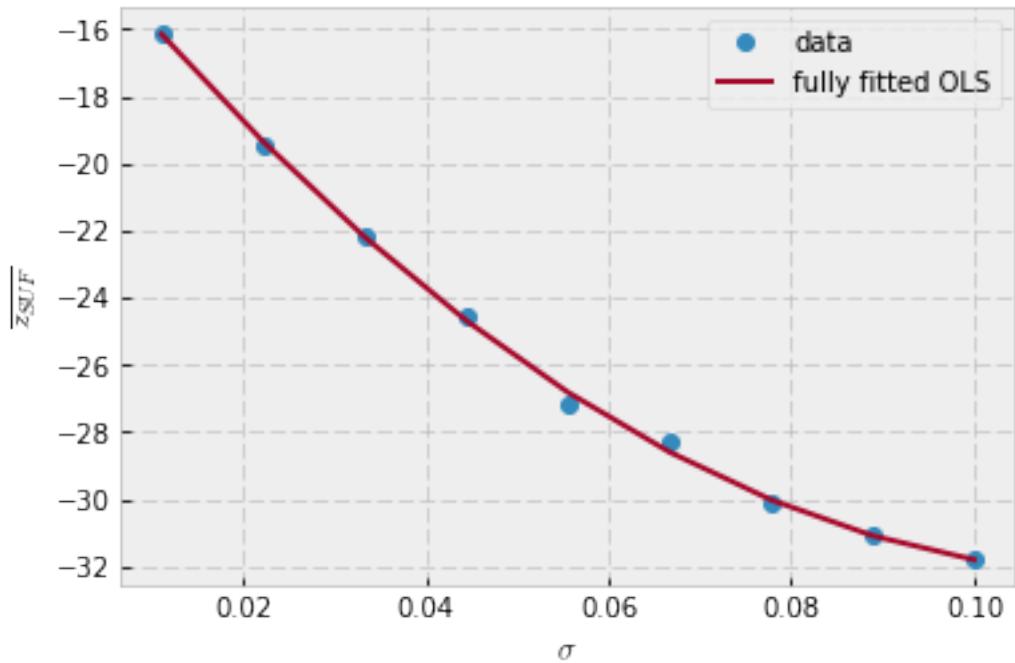
```
variable name: maxB  
model: poly1  
r-squared: 0.11072169026650547  
explained variance: 0.11400222069589105
```



```
variable name: r  
model: poly1  
r-squared: 0.9969323010758074  
explained variance: 0.9969662667438383
```



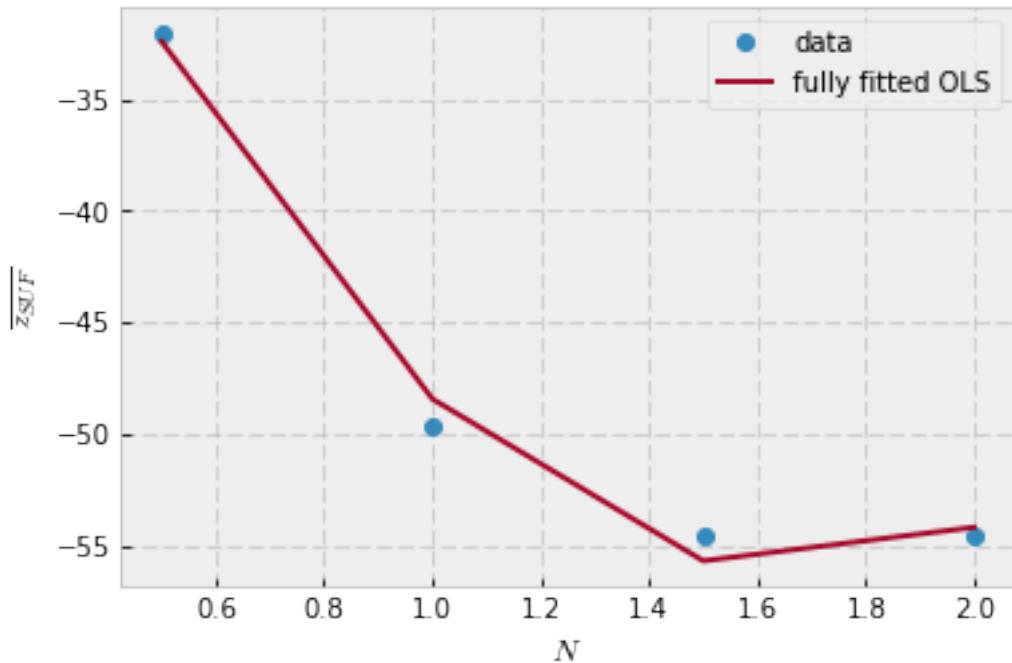
```
variable name: sigma
model: poly2
r-squared: 0.9981443859702446
explained variance: 0.998145196823463
```



```

variable name: N
model: poly2
r-squared: 0.6069311411366989
explained variance: 0.6069311411366989

```



20 Correlation

20.1 Selected root system measures

```

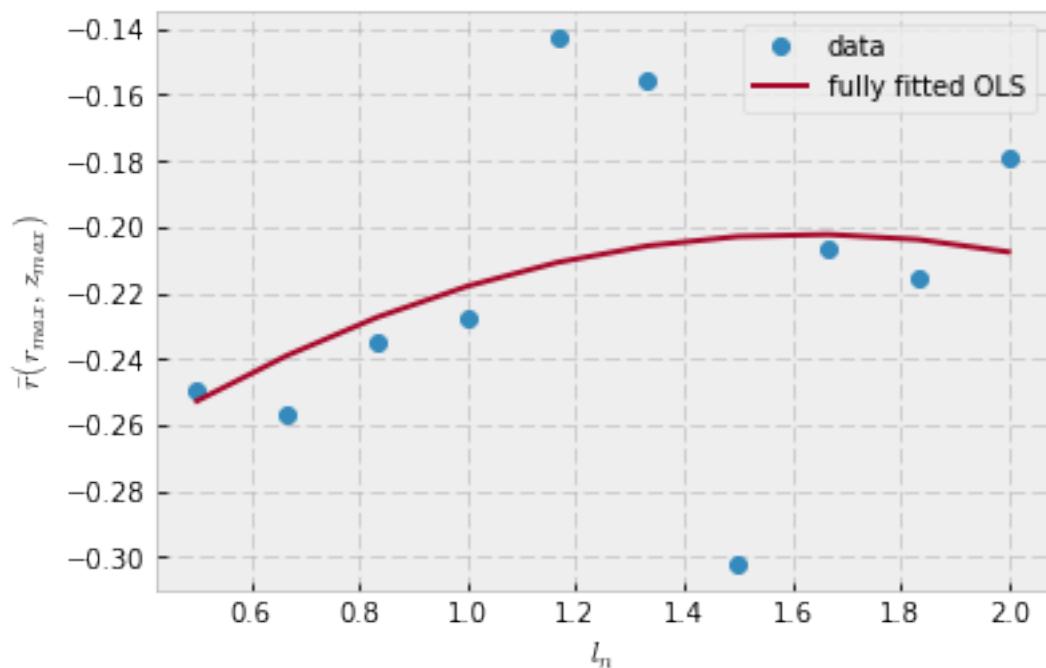
In [45]: y1_name, y2_name = ('maximal_horizontal_spread', 'maximal_rooting_depth')

corr_reg('ln', y1_name, y2_name, 'poly2')
corr_reg('theta', y1_name, y2_name, 'poly2')
corr_reg('maxB', y1_name, y2_name, 'poly2')
corr_reg('r', y1_name, y2_name, 'poly1')
corr_reg('sigma', y1_name, y2_name, 'poly2')
corr_reg('N', y1_name, y2_name, 'poly1')

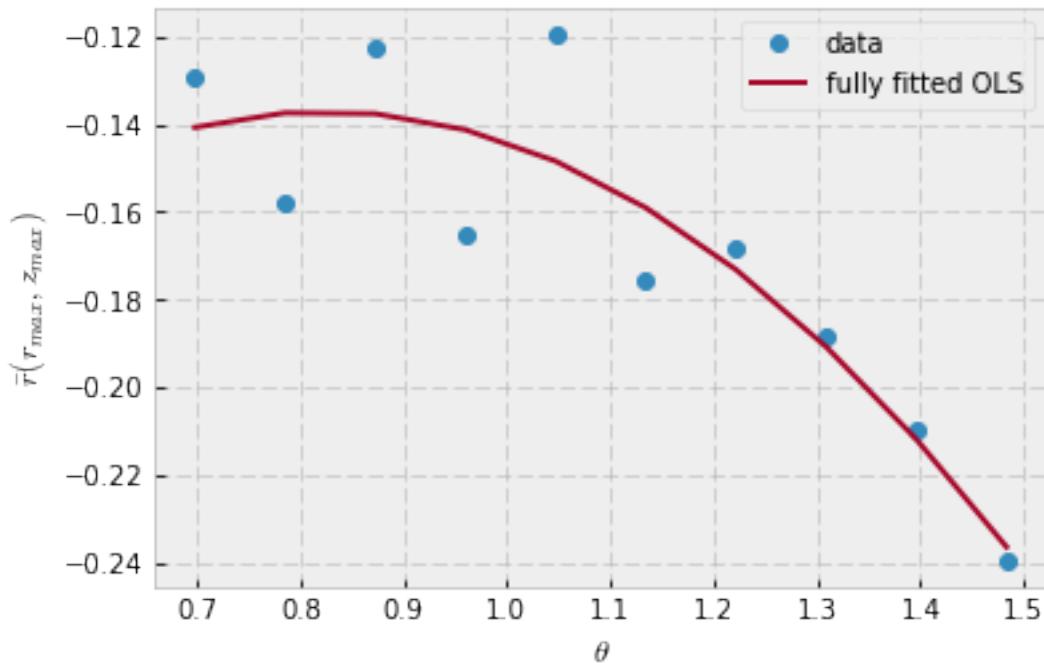
variable name: ln
model: poly2
r-squared: -0.6041147988013835

```

explained variance: -0.5943094122236416



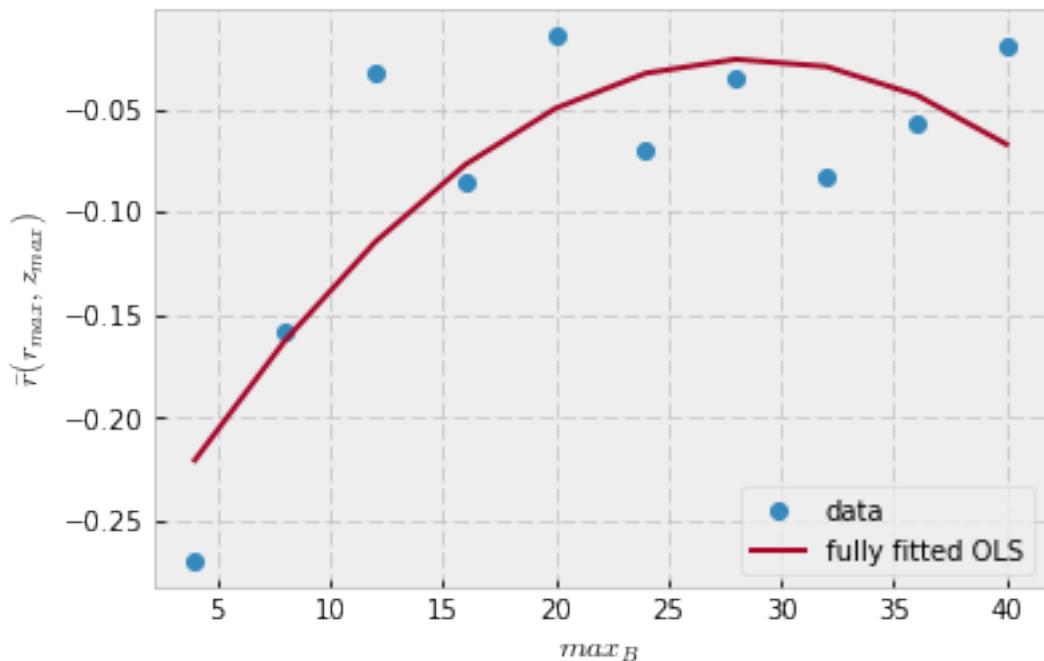
variable name: theta
model: poly2
r-squared: 0.6359426037415894
explained variance: 0.6364614300983983



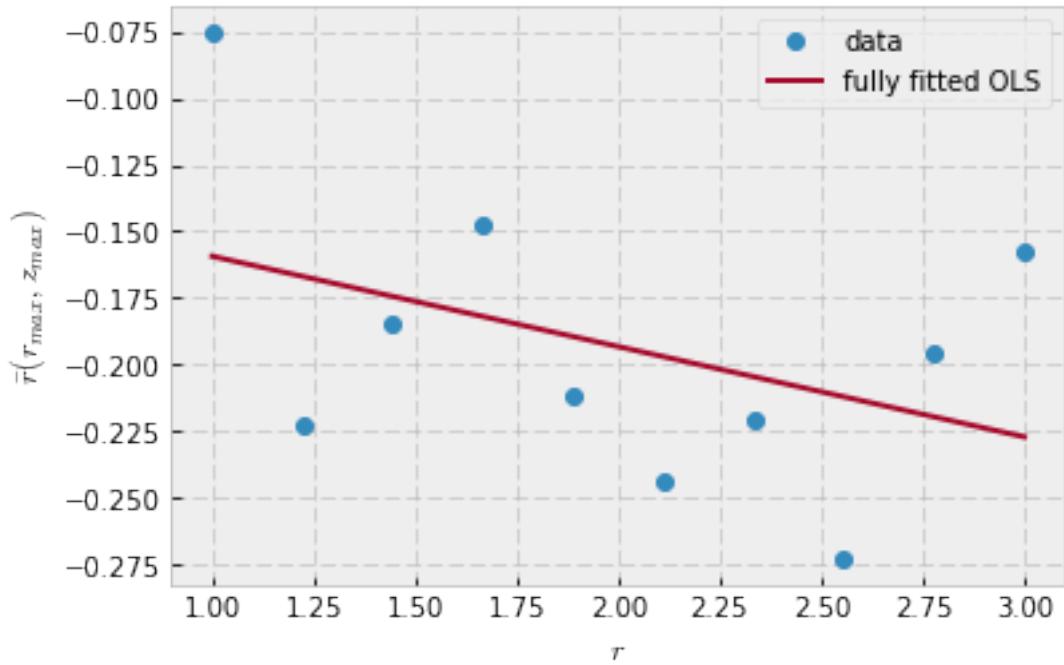
```

variable name: maxB
model: poly2
r-squared: 0.05964956974923885
explained variance: 0.059679379206102445

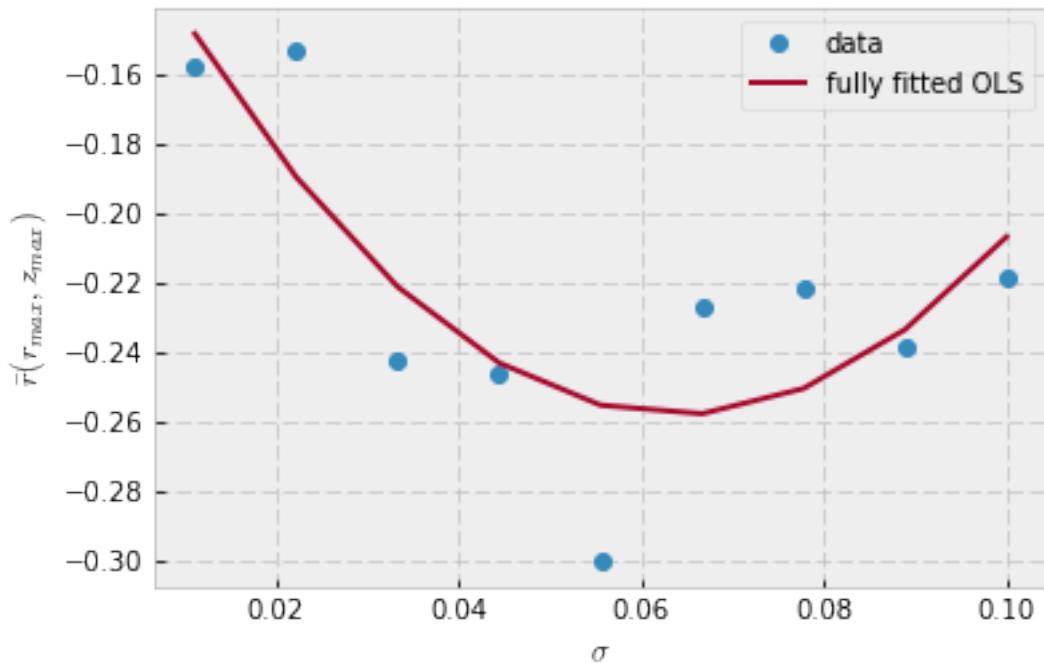
```



```
variable name: r
model: poly1
r-squared: -0.6034488770230966
explained variance: -0.5944946040967922
```



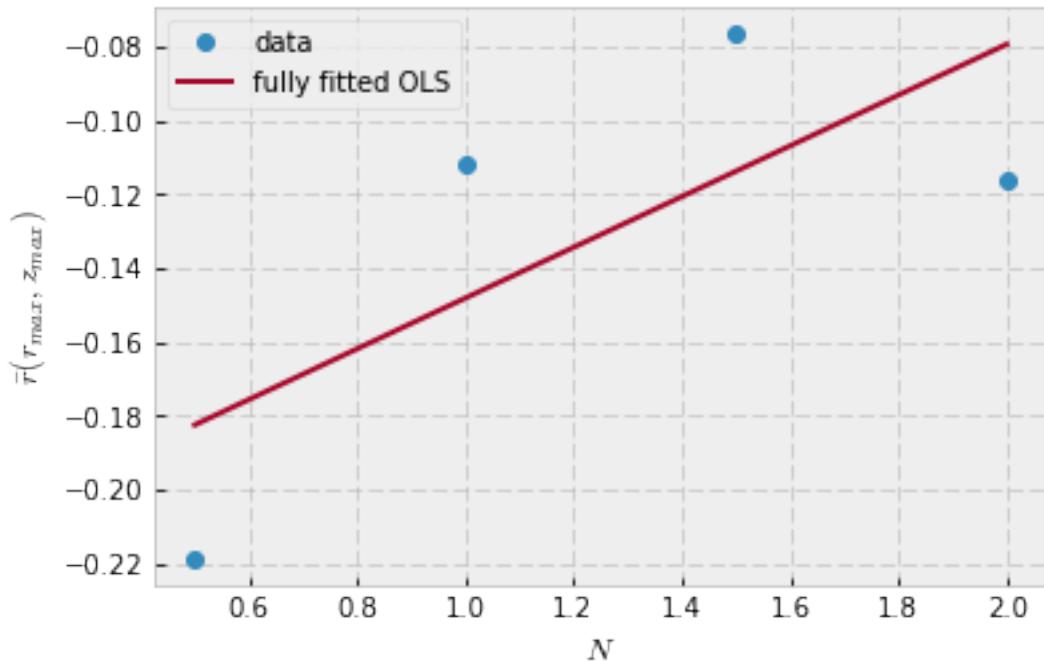
```
variable name: sigma
model: poly2
r-squared: 0.2716030705171678
explained variance: 0.27968571300295497
```



```

variable name: N
model: poly1
r-squared: -2.1512581257376713
explained variance: -1.7166395775561143

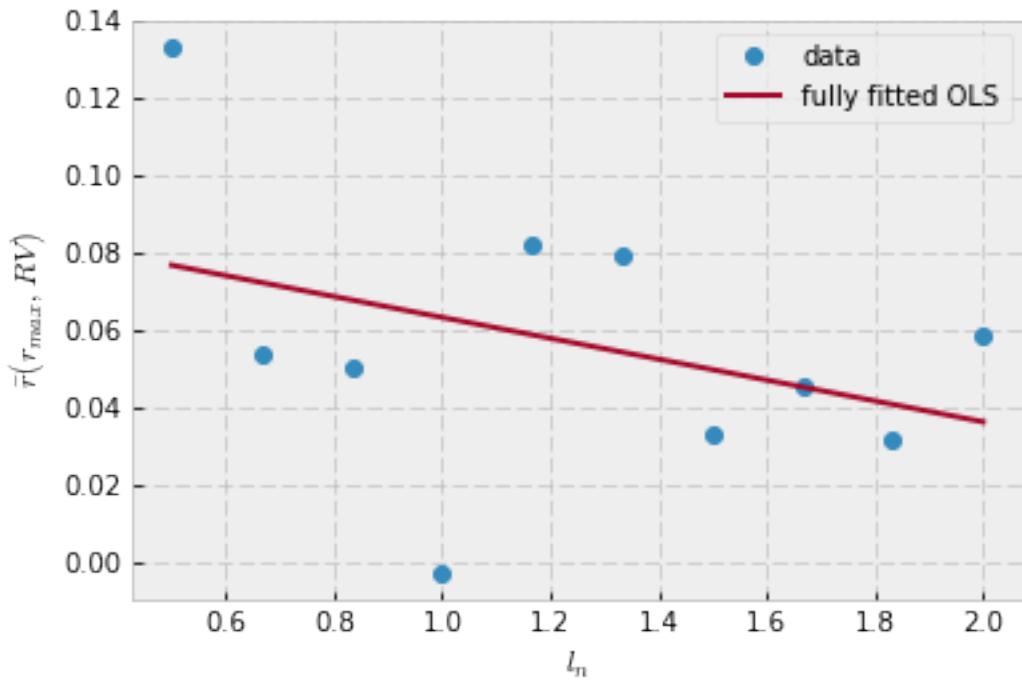
```



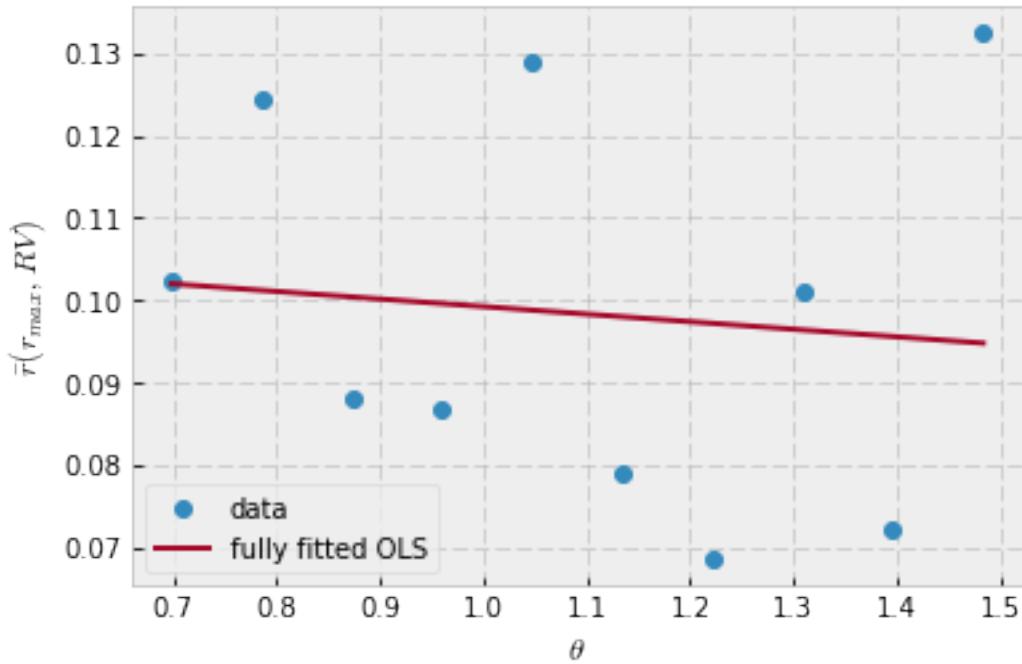
```
In [46]: y1_name, y2_name = ('maximal_horizontal_spread', 'total_root_volume')

corr_reg('ln', y1_name, y2_name, 'poly1')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('maxB', y1_name, y2_name, 'poly2')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly2')
corr_reg('N', y1_name, y2_name, 'poly1')

variable name: ln
model: poly1
r-squared: -0.45887292223685083
explained variance: -0.45463782736712965
```



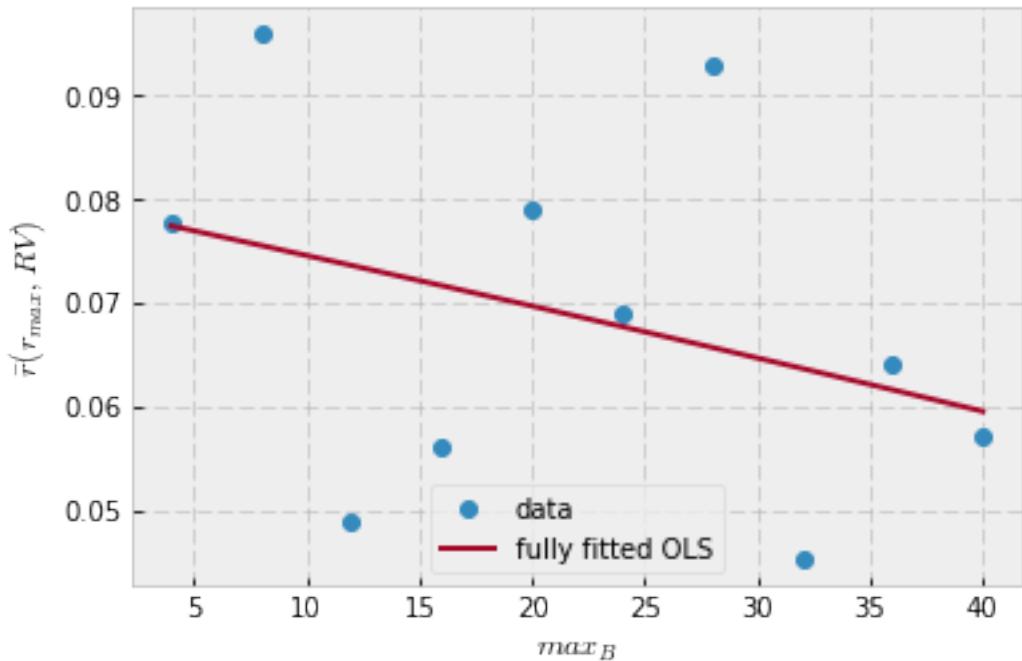
```
variable name: theta
model: poly1
r-squared: -0.6779806636160561
explained variance: -0.6742605384688309
```



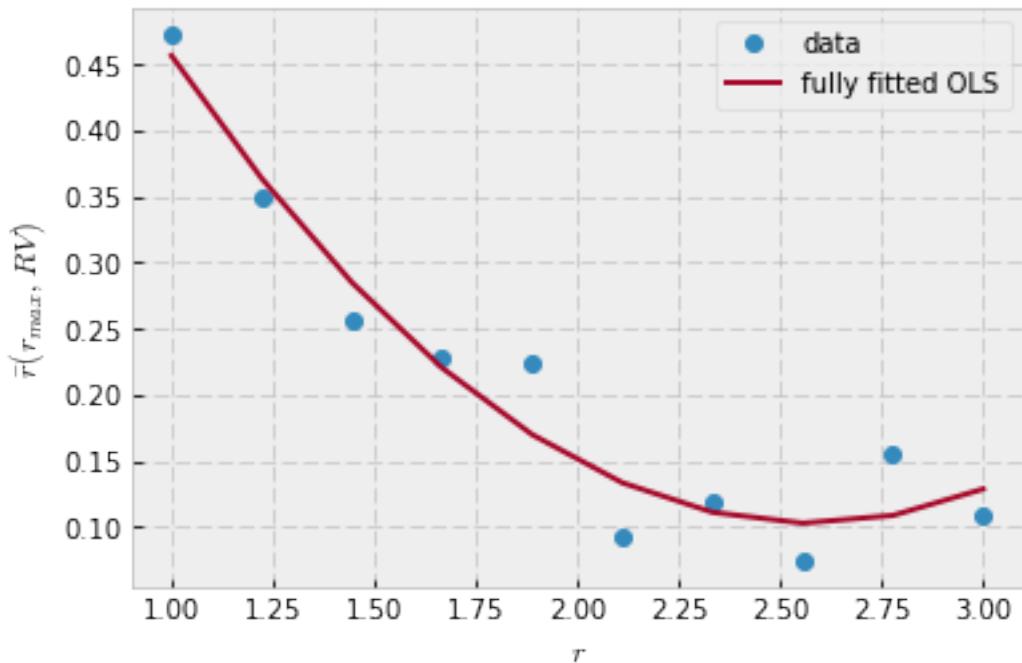
```

variable name: maxB
model: poly2
r-squared: -0.4208426610977436
explained variance: -0.4207283989312116

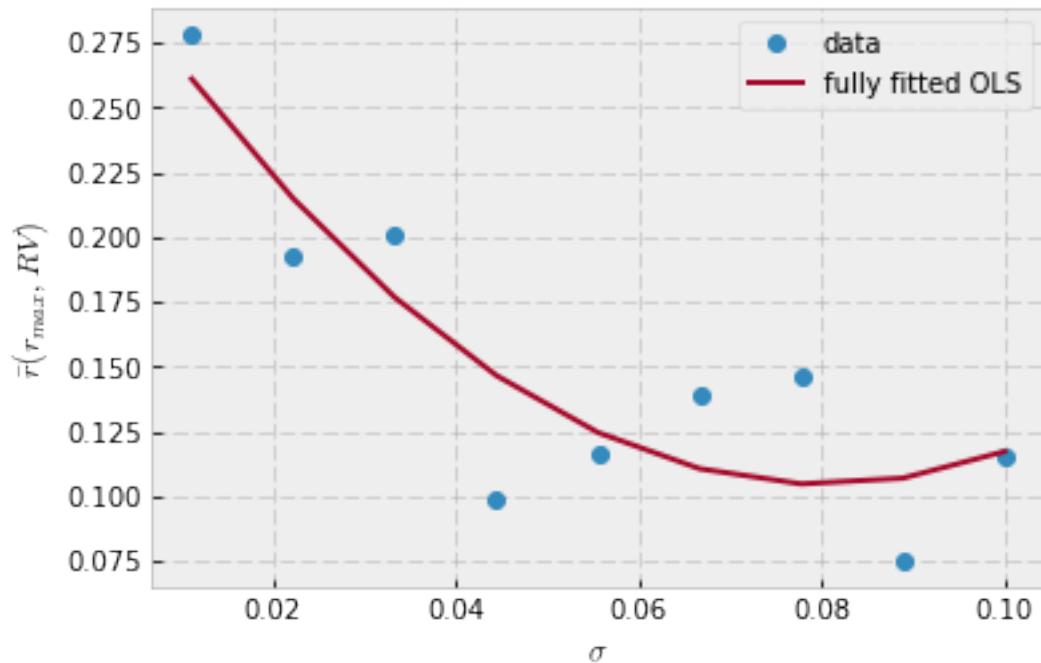
```



```
variable name: r
model: poly2
r-squared: 0.8654747755706115
explained variance: 0.8654775657884958
```



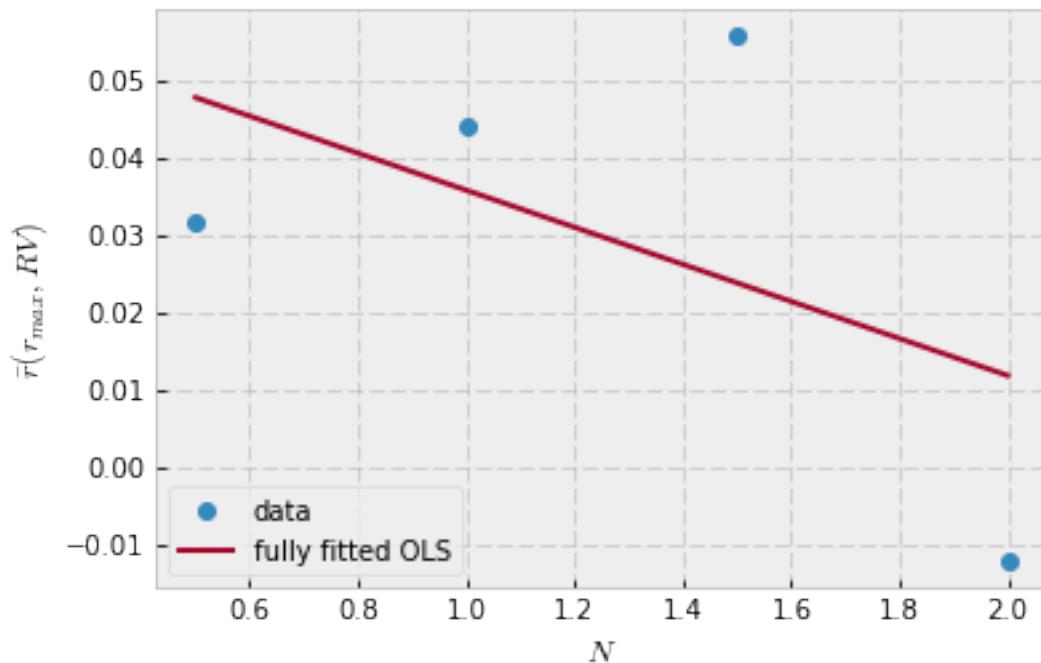
```
variable name: sigma
model: poly2
r-squared: 0.5413233228630012
explained variance: 0.5423141301184167
```



```

variable name: N
model: poly1
r-squared: -3.362009766639968
explained variance: -2.809211772751154

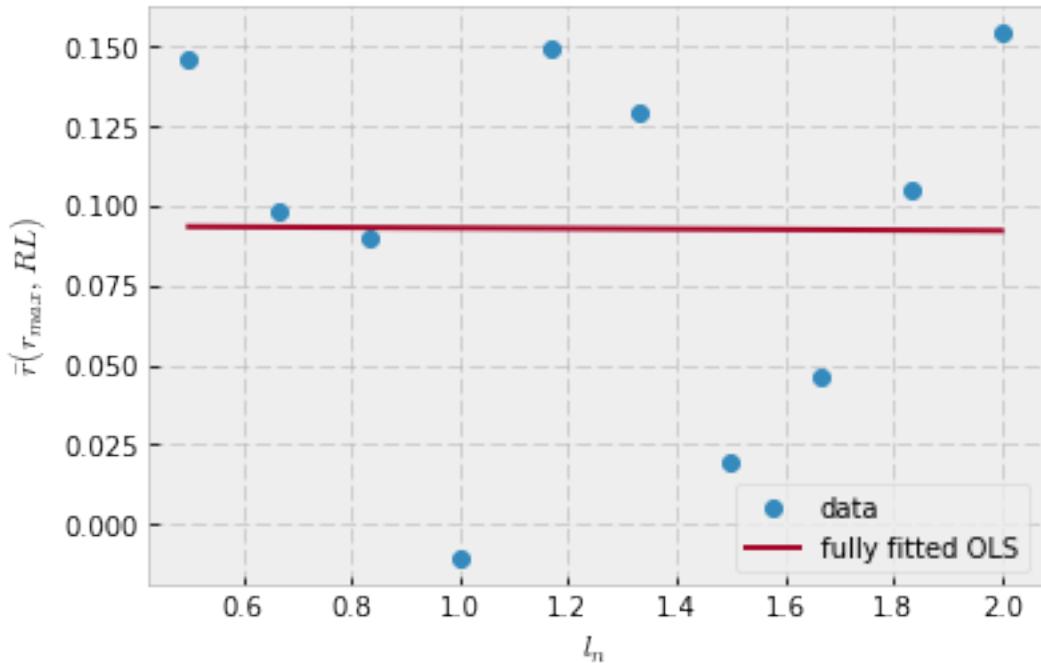
```



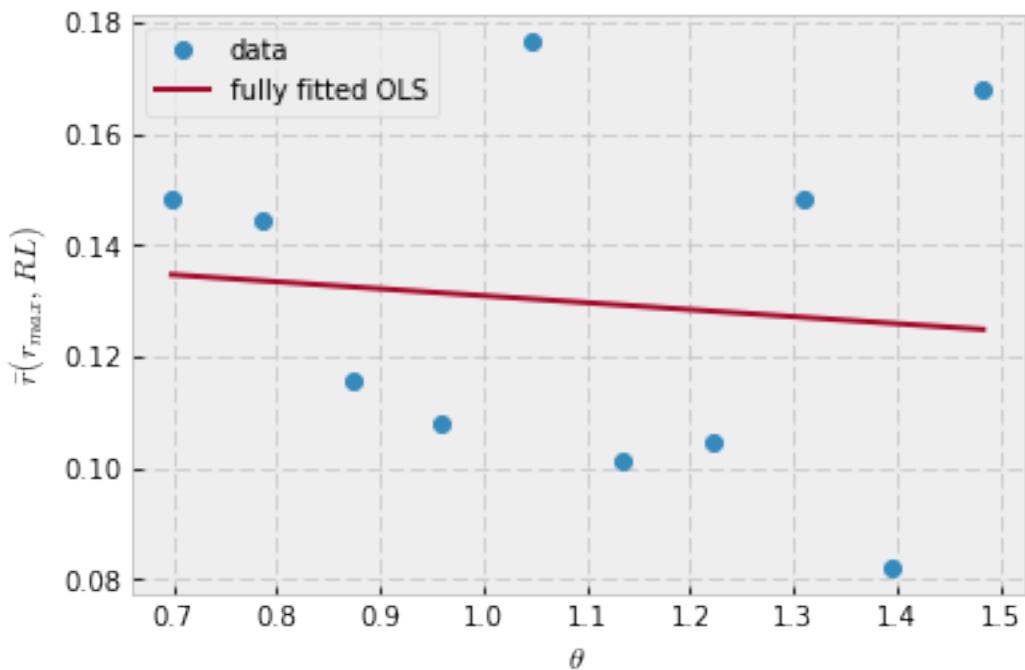
```
In [47]: y1_name, y2_name = ('maximal_horizontal_spread', 'total_root_length')

corr_reg('ln', y1_name, y2_name, 'poly1')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('maxB', y1_name, y2_name, 'poly1')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly1')
corr_reg('N', y1_name, y2_name, 'poly1')

variable name: ln
model: poly1
r-squared: -0.5449297755826334
explained variance: -0.5393028127215396
```



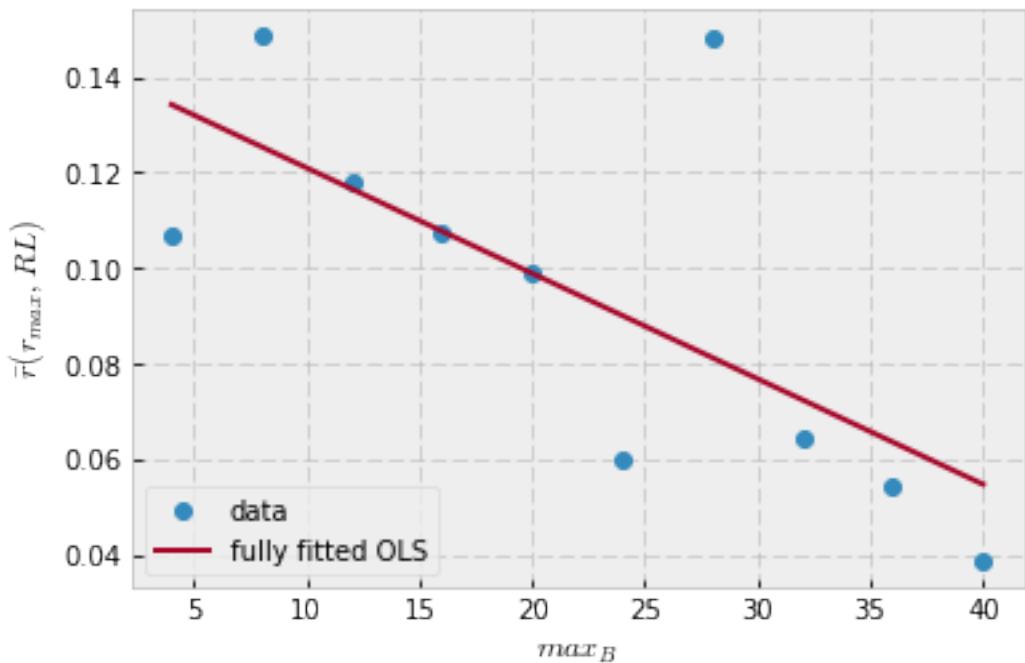
```
variable name: theta
model: poly1
r-squared: -0.6274287061938941
explained variance: -0.6248153887522665
```



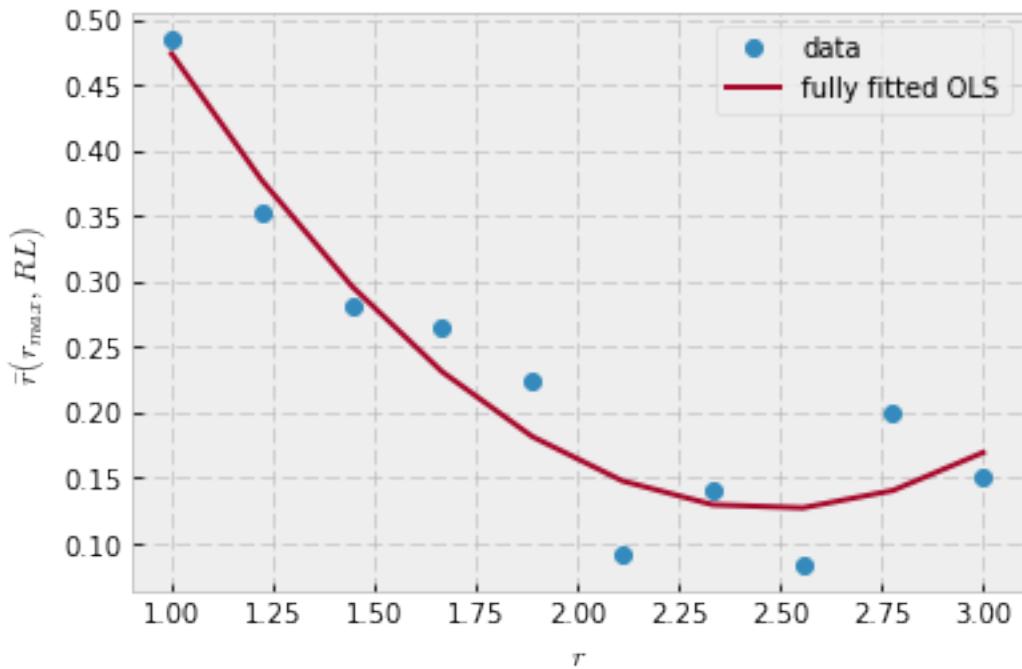
```

variable name: maxB
model: poly1
r-squared: 0.21539379826381444
explained variance: 0.21675549134847683

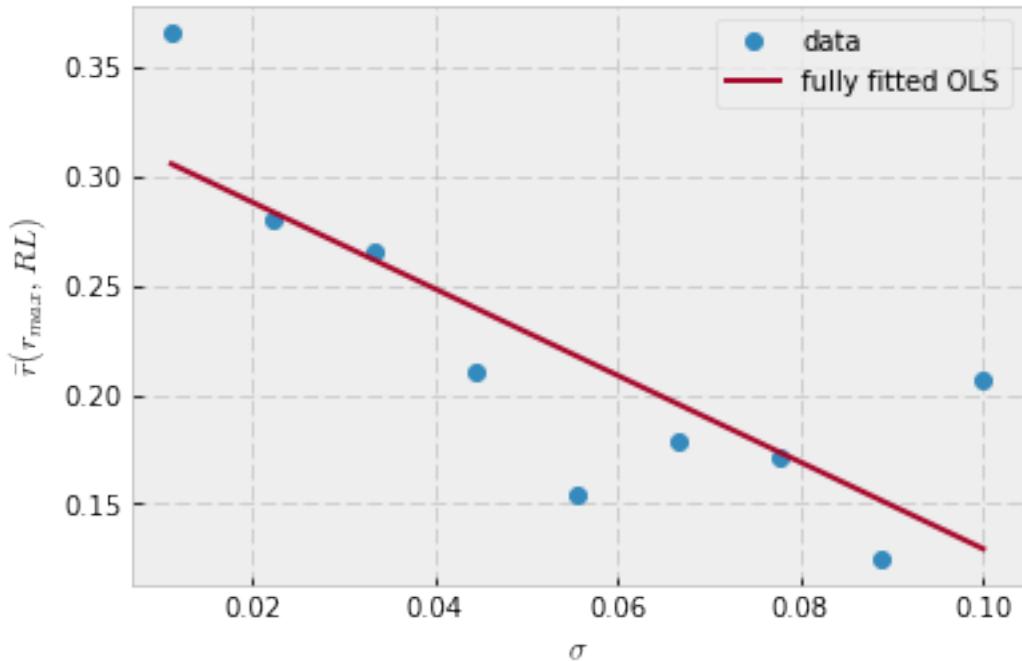
```



```
variable name: r
model: poly2
r-squared: 0.8219053012426474
explained variance: 0.8219223330216674
```



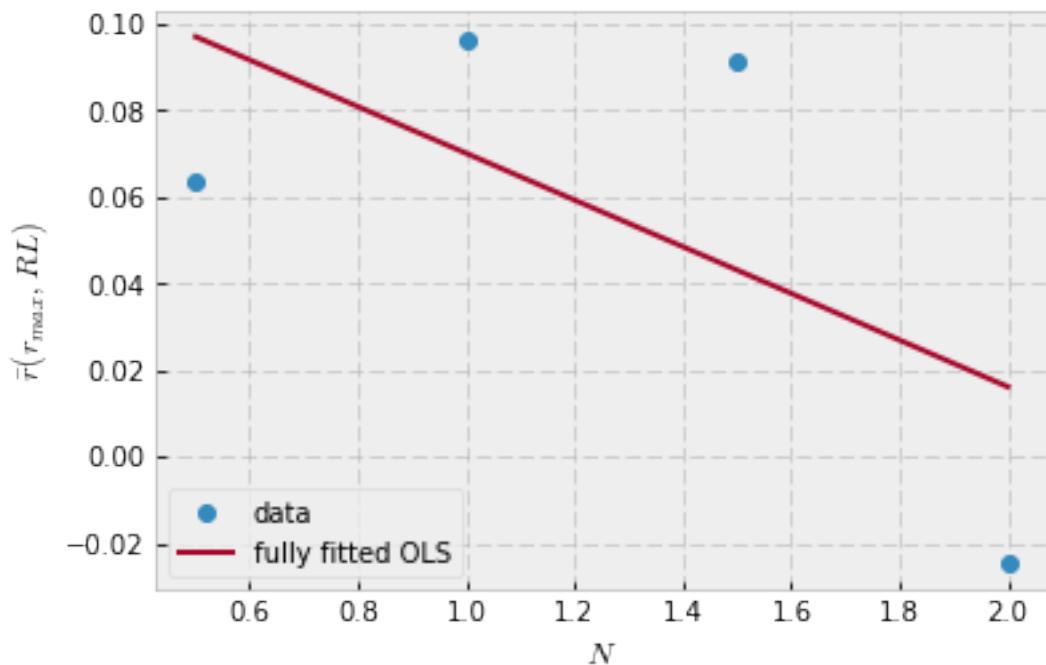
```
variable name: sigma
model: poly1
r-squared: 0.26101812776299116
explained variance: 0.26961209260580266
```



```

variable name: N
model: poly1
r-squared: -2.9524707597056246
explained variance: -2.418553557436615

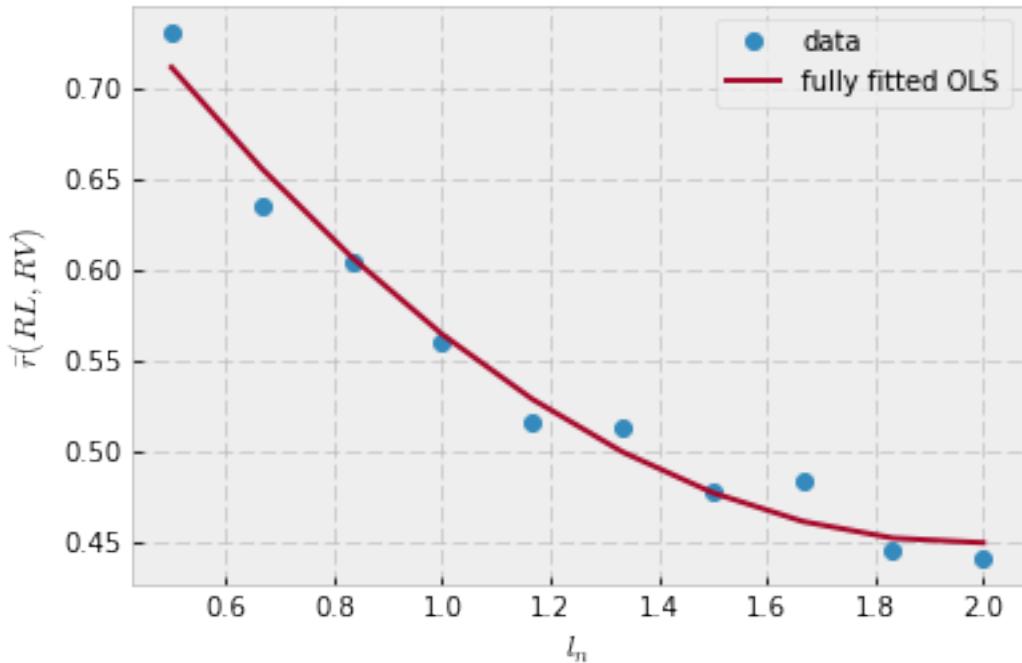
```



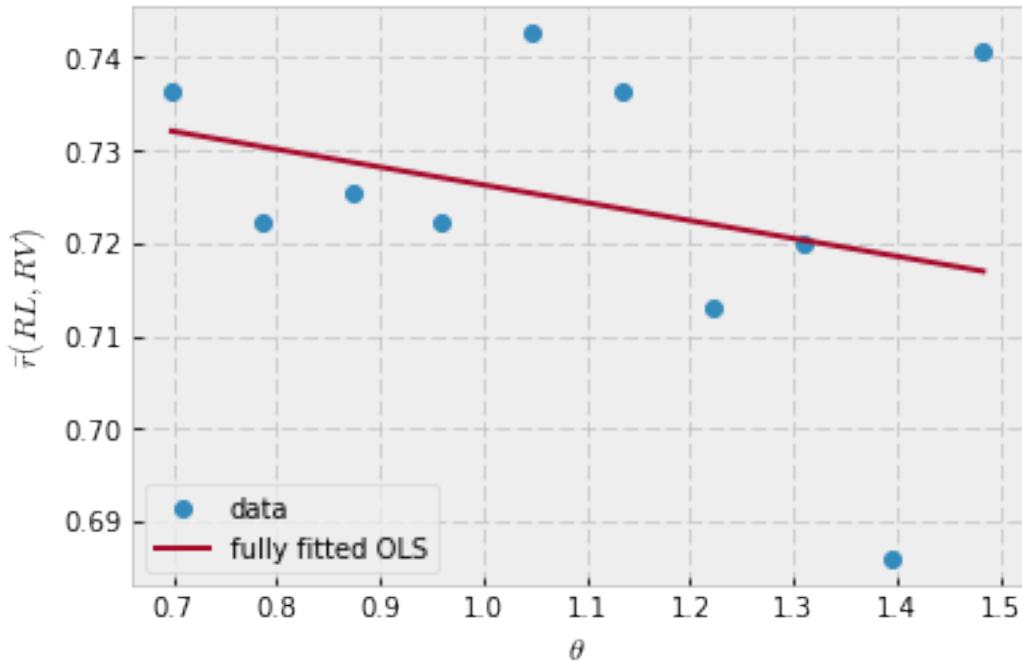
```
In [48]: y1_name, y2_name = ('total_root_length', 'total_root_volume')
```

```
corr_reg('ln', y1_name, y2_name, 'poly2')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('maxB', y1_name, y2_name, 'poly1')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly1')
corr_reg('N', y1_name, y2_name, 'poly2')
```

```
variable name: ln
model: poly2
r-squared: 0.9301505998033829
explained variance: 0.9302904182687706
```



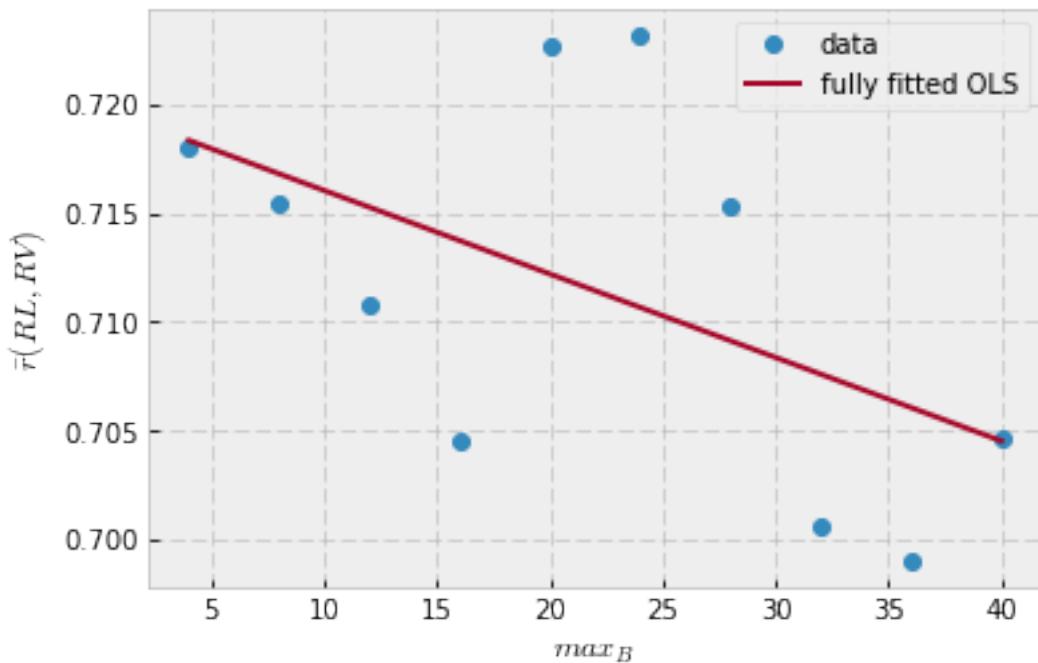
```
variable name: theta
model: poly1
r-squared: -0.6219891602409313
explained variance: -0.6218293266434534
```



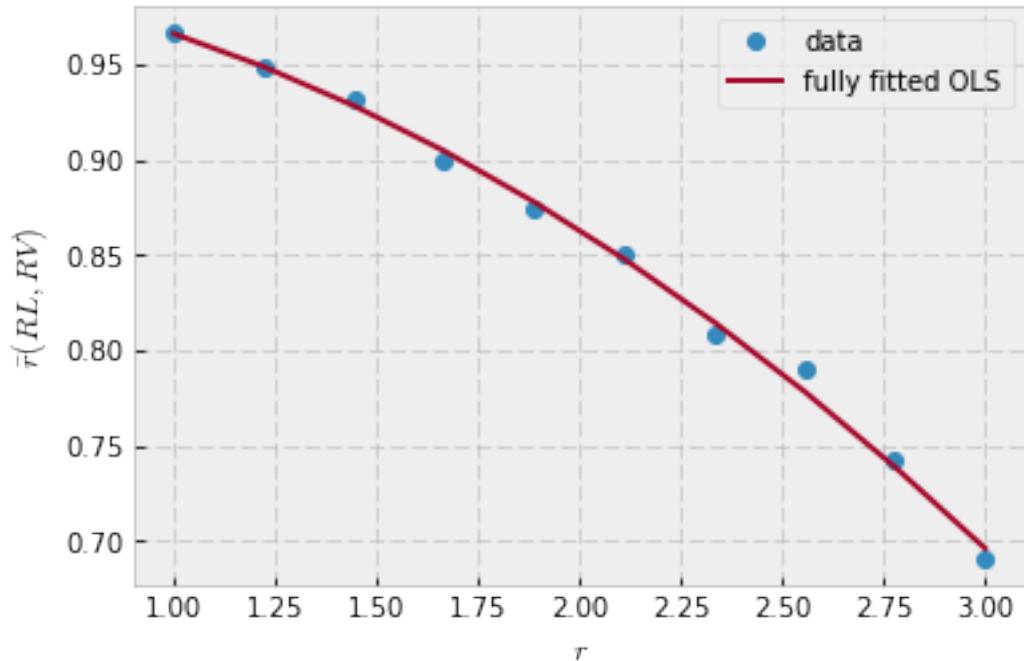
```

variable name: maxB
model: poly1
r-squared: 0.02580111086606718
explained variance: 0.02718593028960259

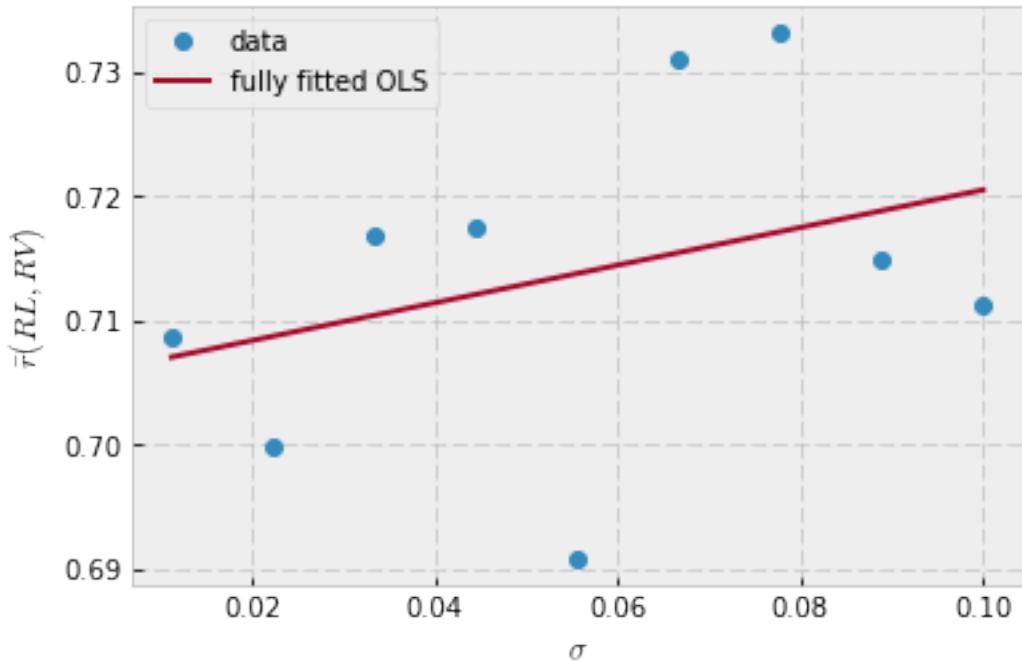
```



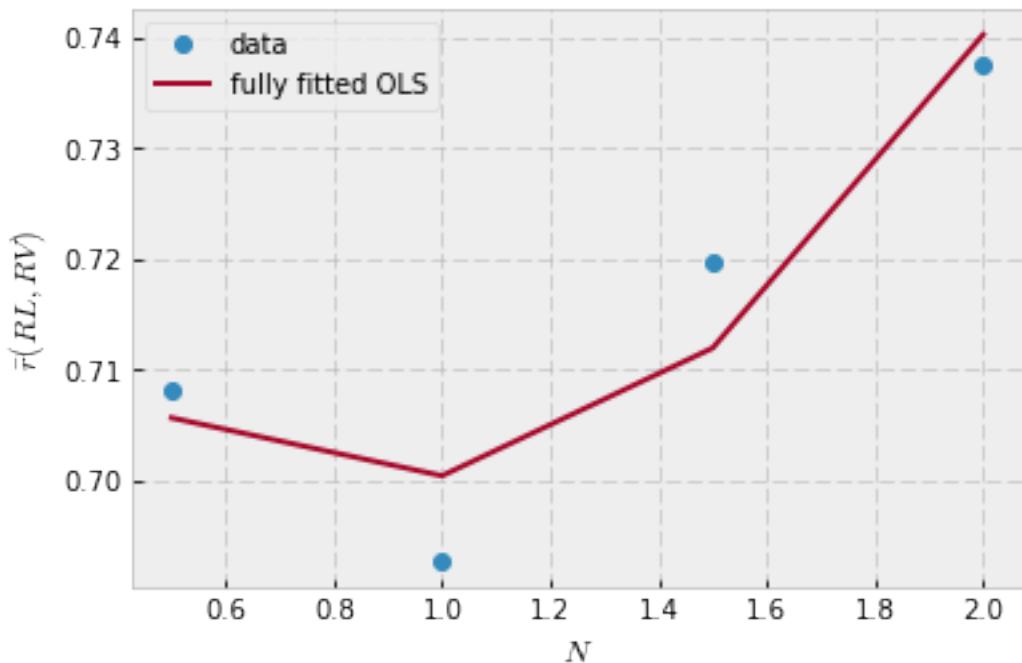
```
variable name: r
model: poly2
r-squared: 0.9914416863479607
explained variance: 0.9915049948580617
```



```
variable name: sigma
model: poly1
r-squared: -0.28829000447673403
explained variance: -0.2870178616175325
```



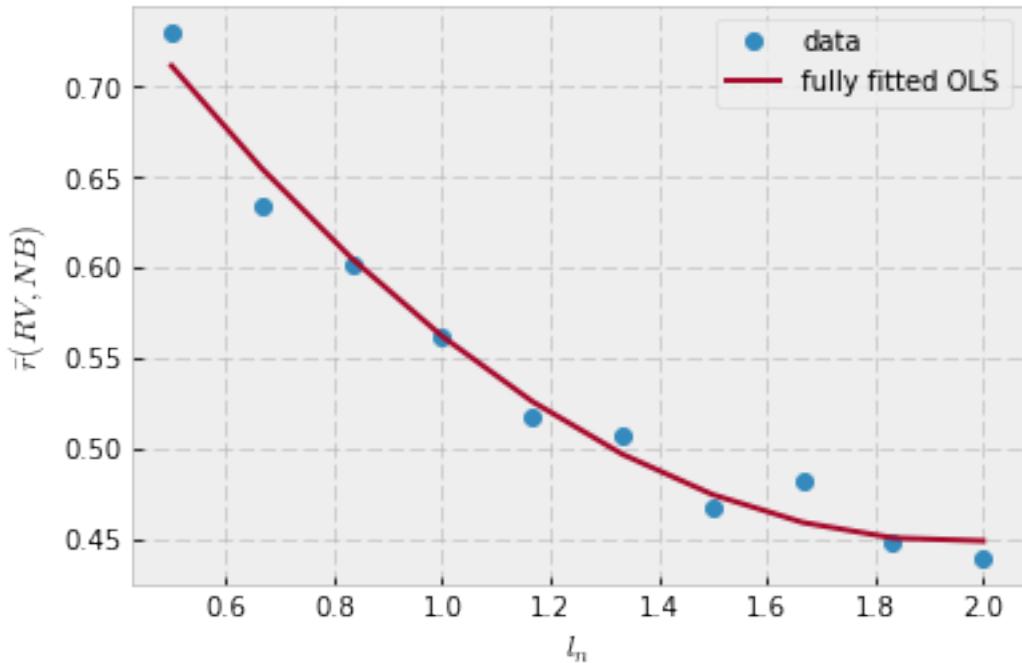
```
variable name: N
model: poly2
r-squared: -4.545107952290249
explained variance: -4.545107952290249
```



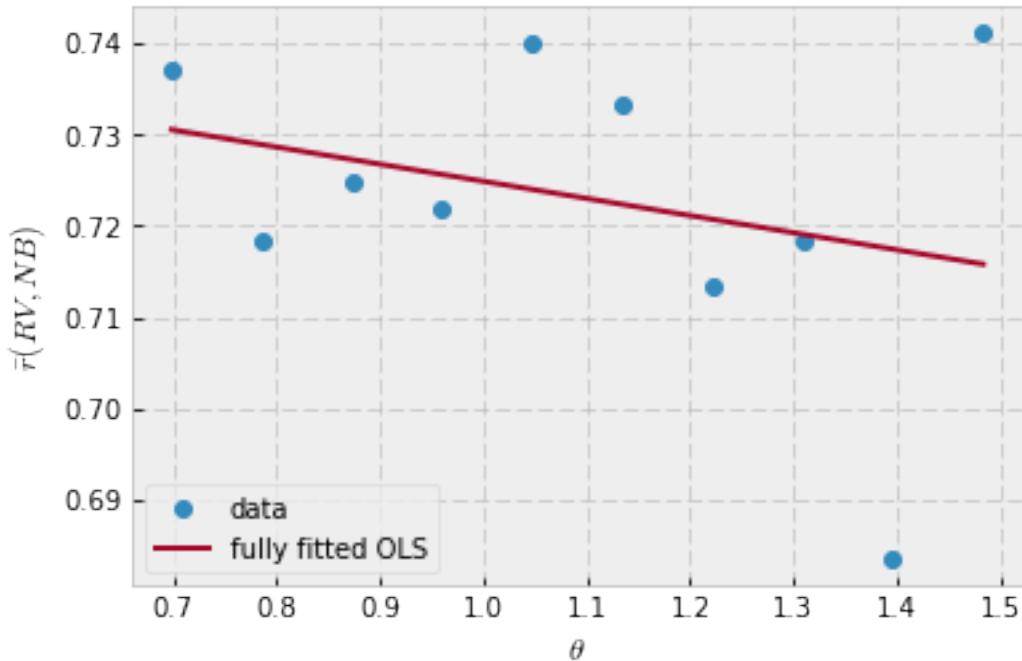
```
In [49]: y1_name, y2_name = ('total_root_volume', 'number_of_root_branches')

corr_reg('ln', y1_name, y2_name, 'poly2')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('maxB', y1_name, y2_name, 'poly1')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly2')
corr_reg('N', y1_name, y2_name, 'poly2')

variable name: ln
model: poly2
r-squared: 0.9332566538355056
explained variance: 0.9333471469035082
```



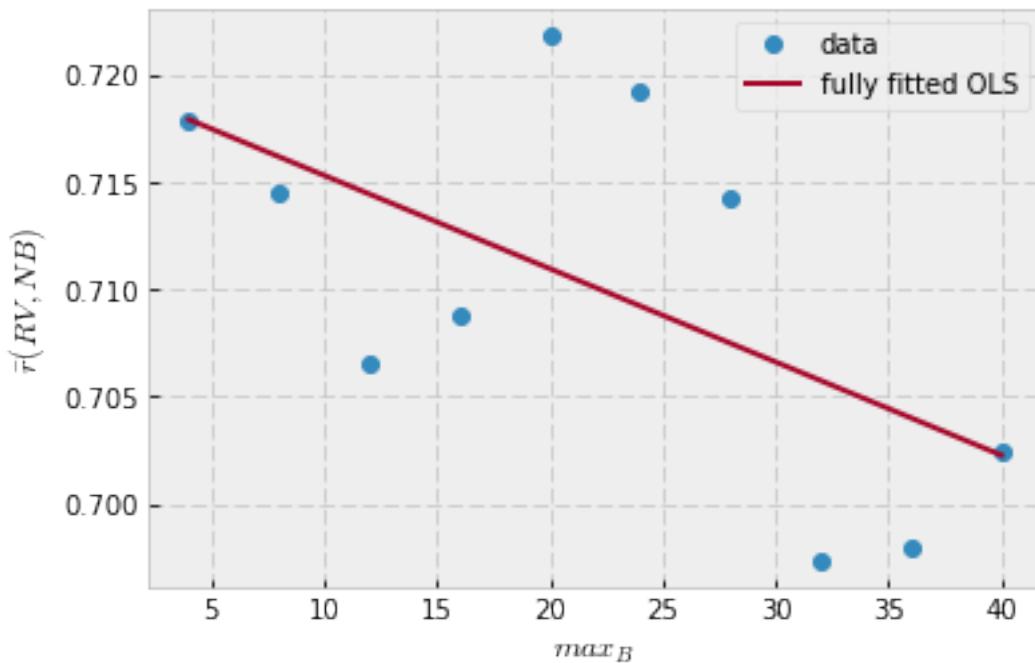
```
variable name: theta
model: poly1
r-squared: -0.6829057874103717
explained variance: -0.6825734441114688
```



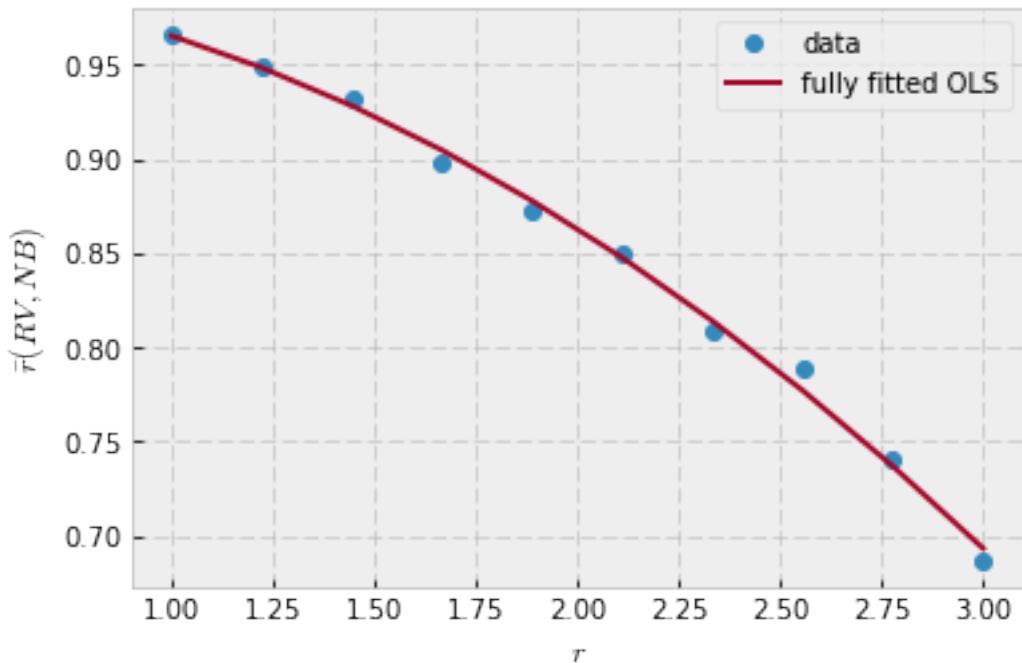
```

variable name: maxB
model: poly1
r-squared: 0.11970752546138919
explained variance: 0.12110404190084334

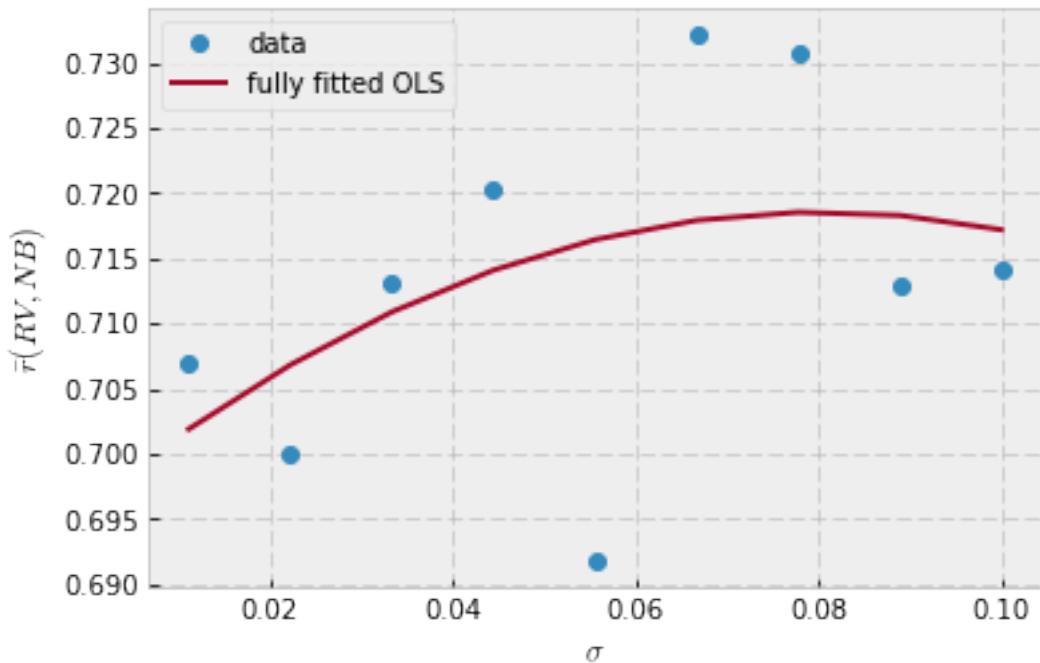
```



```
variable name: r
model: poly2
r-squared: 0.9900058892340838
explained variance: 0.9900933047649824
```



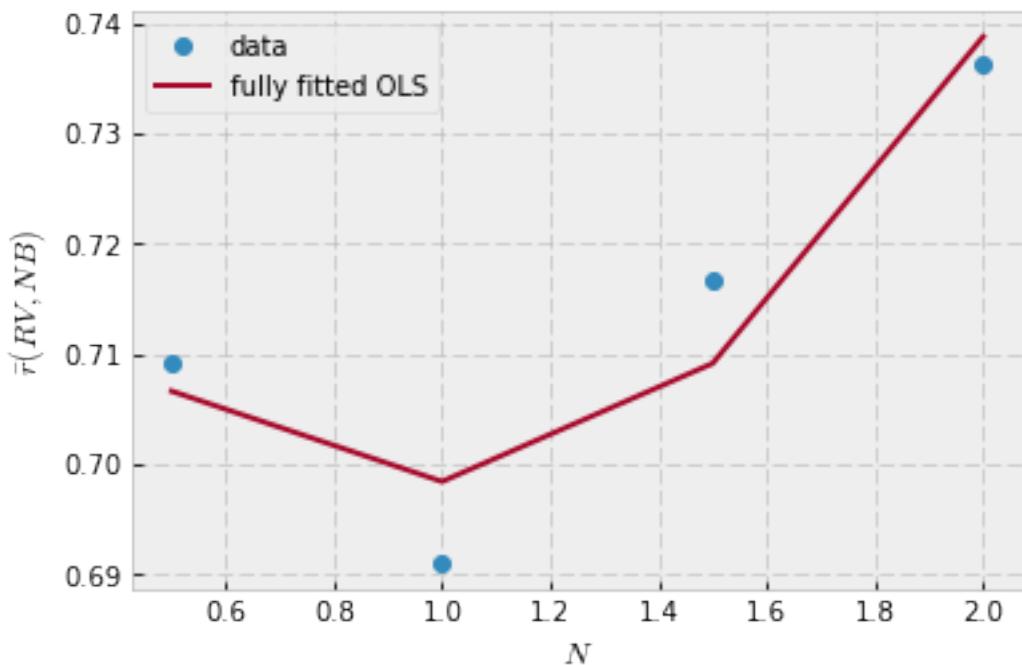
```
variable name: sigma
model: poly2
r-squared: -0.5842527935368456
explained variance: -0.584236033475459
```



```

variable name: N
model: poly2
r-squared: -4.2319392747508875
explained variance: -4.2319392747508875

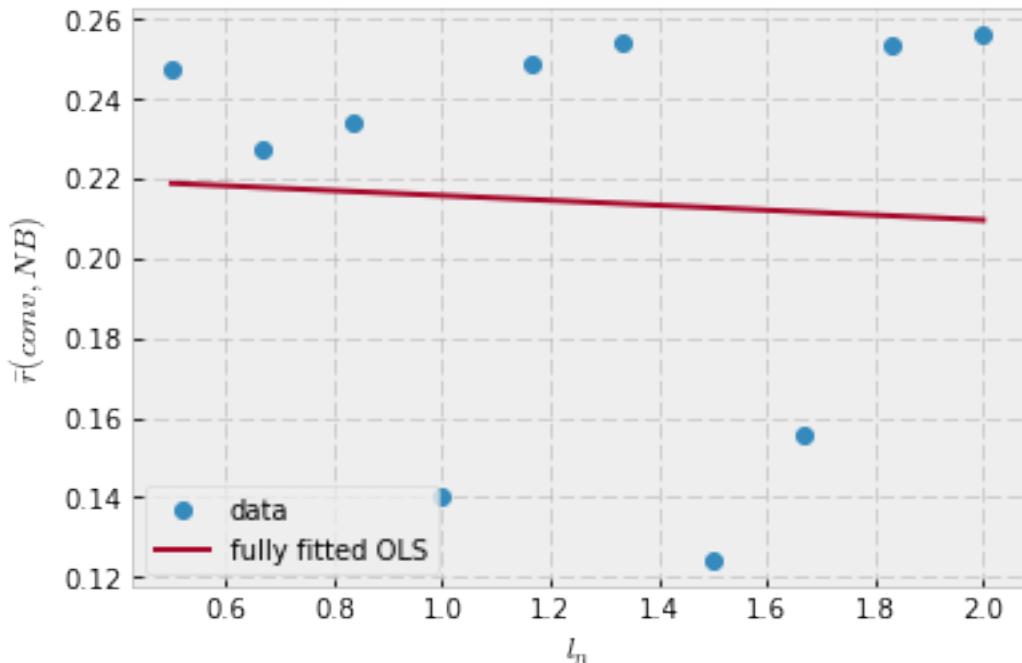
```



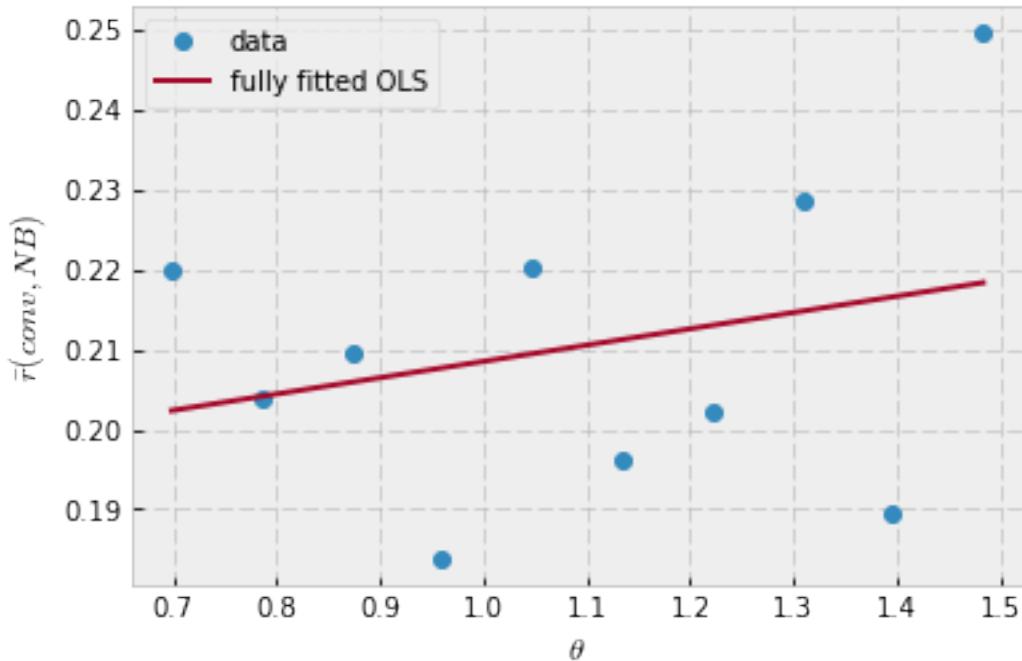
```
In [50]: y1_name, y2_name = ('convex_hull_volume', 'number_of_root_branches')

corr_reg('ln', y1_name, y2_name, 'poly1')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('maxB', y1_name, y2_name, 'poly1')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly1')
corr_reg('N', y1_name, y2_name, 'poly1')

variable name: ln
model: poly1
r-squared: -0.48166605640022975
explained variance: -0.47713377220862996
```



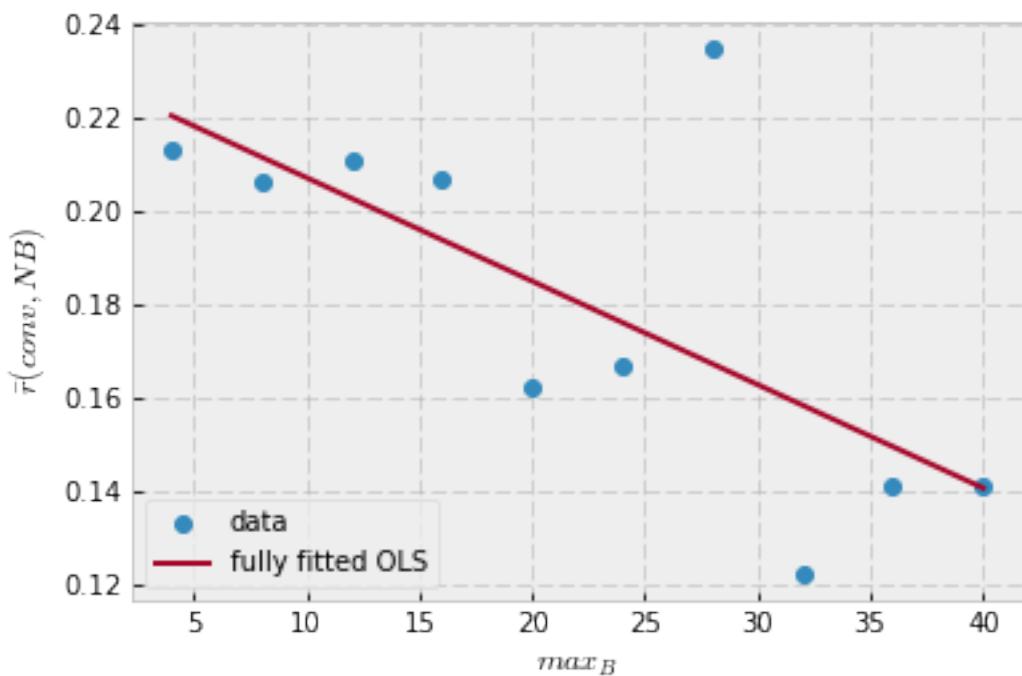
```
variable name: theta
model: poly1
r-squared: -0.6887039729777646
explained variance: -0.6823805537552114
```



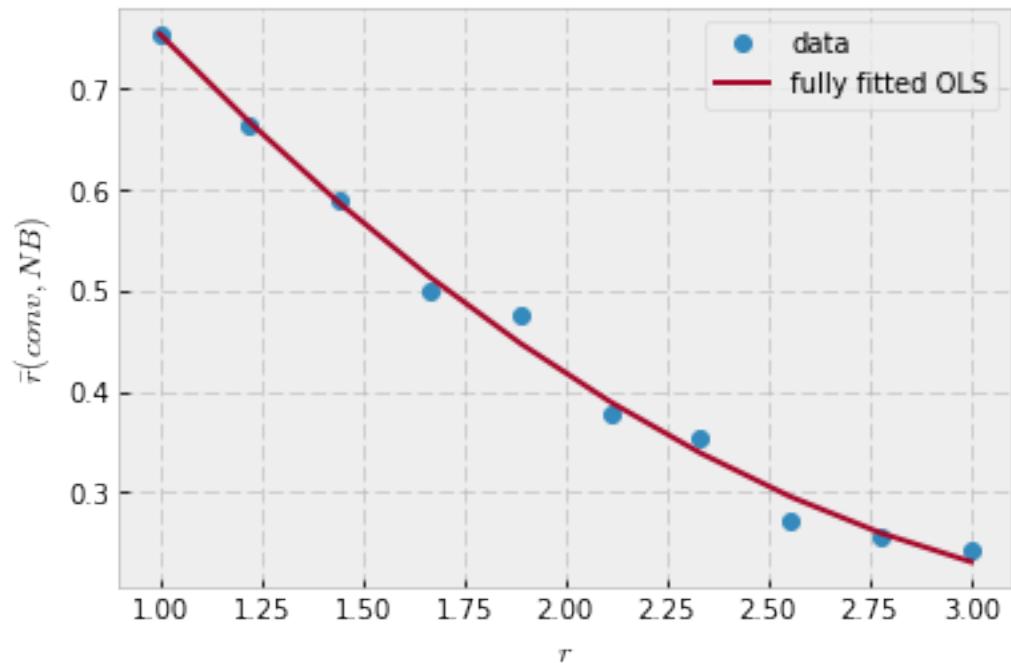
```

variable name: maxB
model: poly1
r-squared: 0.3021277623715234
explained variance: 0.30240226496291045

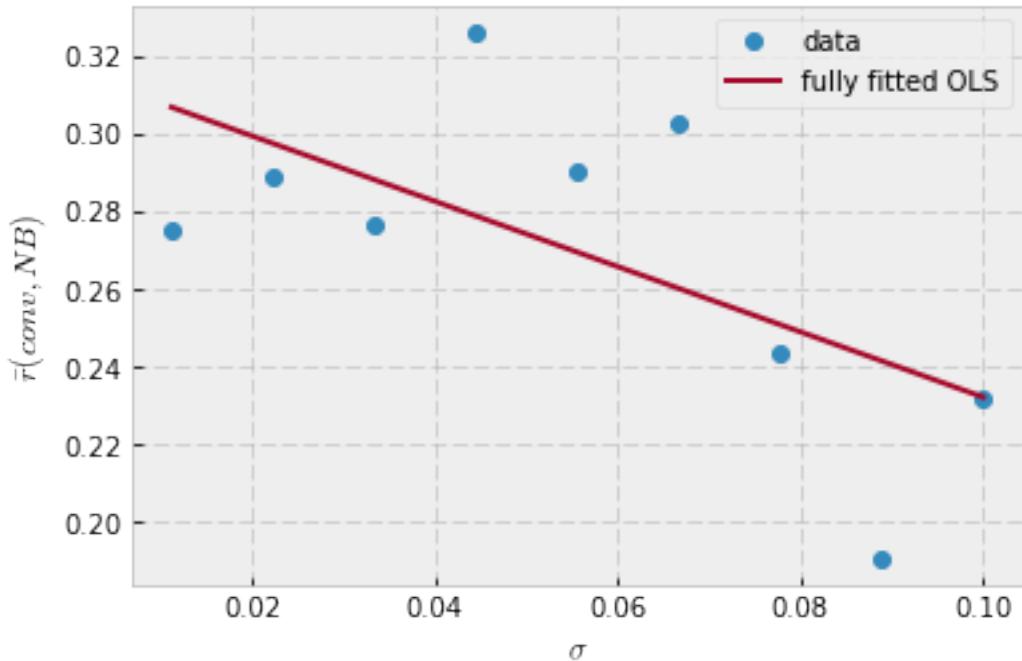
```



```
variable name: r
model: poly2
r-squared: 0.9867117617601608
explained variance: 0.9867881738016635
```



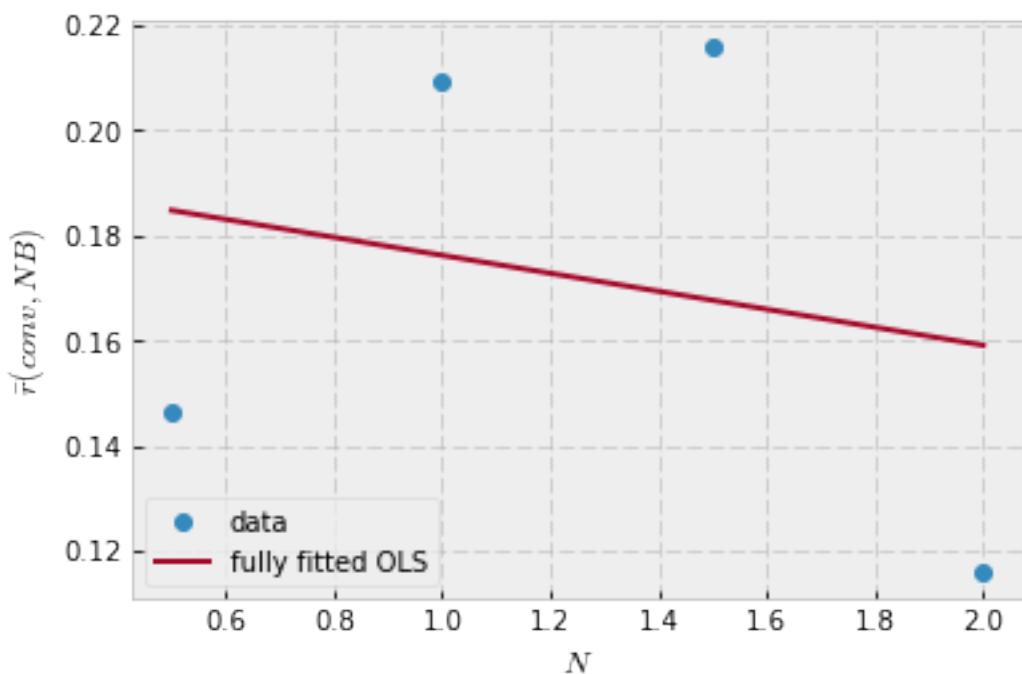
```
variable name: sigma
model: poly1
r-squared: -0.007184676190029604
explained variance: -0.00031197407053928927
```



```

variable name: N
model: poly1
r-squared: -5.1759510756002305
explained variance: -4.3312583988014755

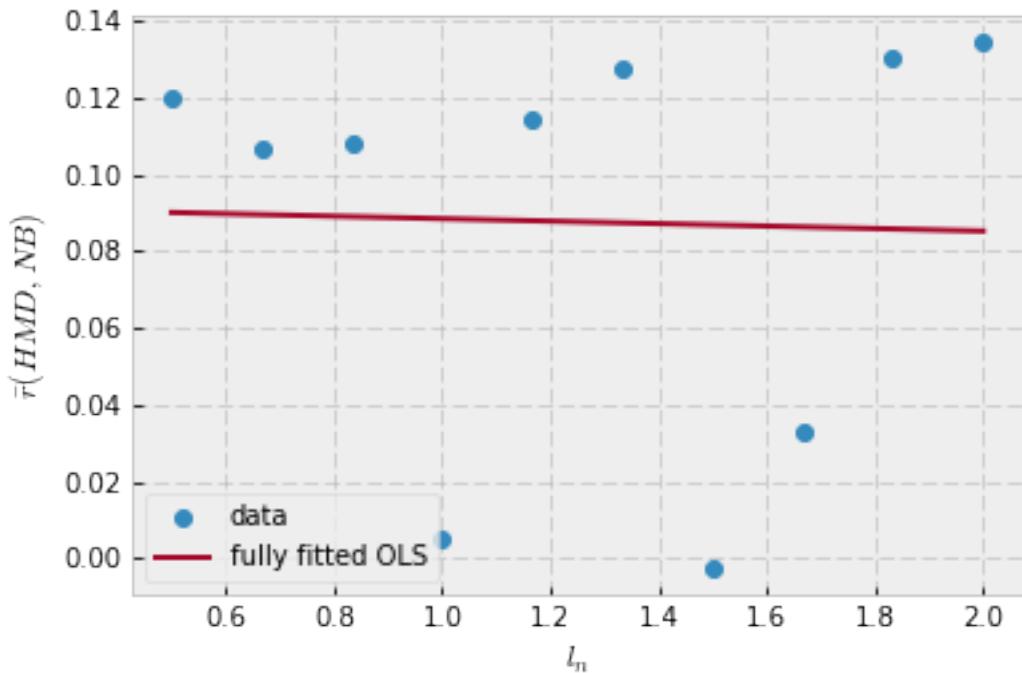
```



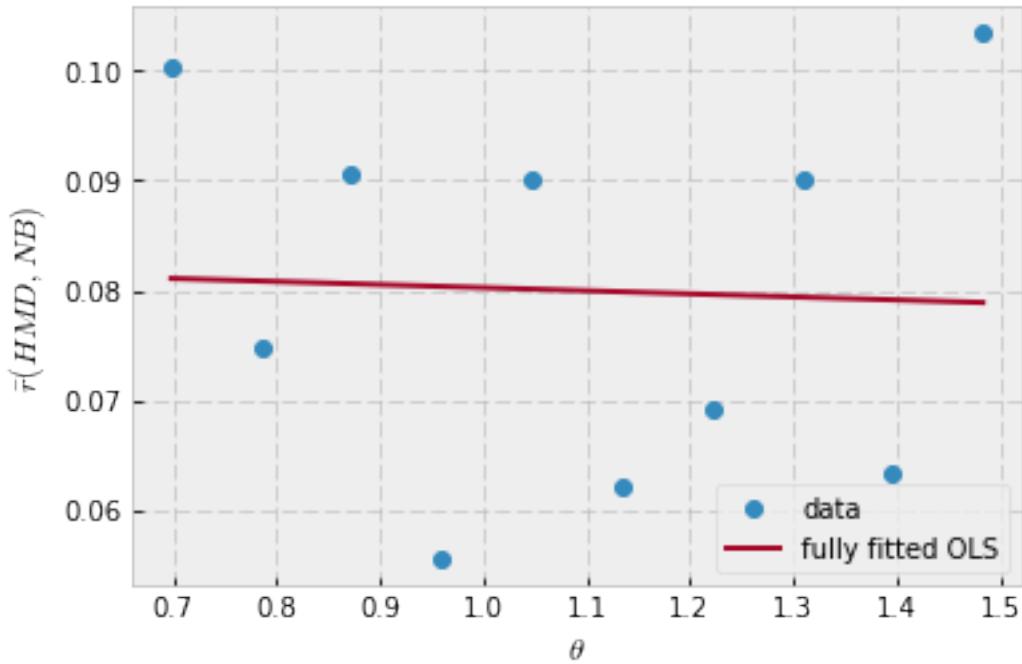
```
In [51]: y1_name, y2_name = ('half_mean_distance_between_roots', 'number_of_root_branches')

corr_reg('ln', y1_name, y2_name, 'poly1')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('maxB', y1_name, y2_name, 'poly1')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly1')
corr_reg('N', y1_name, y2_name, 'poly1')

variable name: ln
model: poly1
r-squared: -0.49577111839232857
explained variance: -0.49044967718467425
```



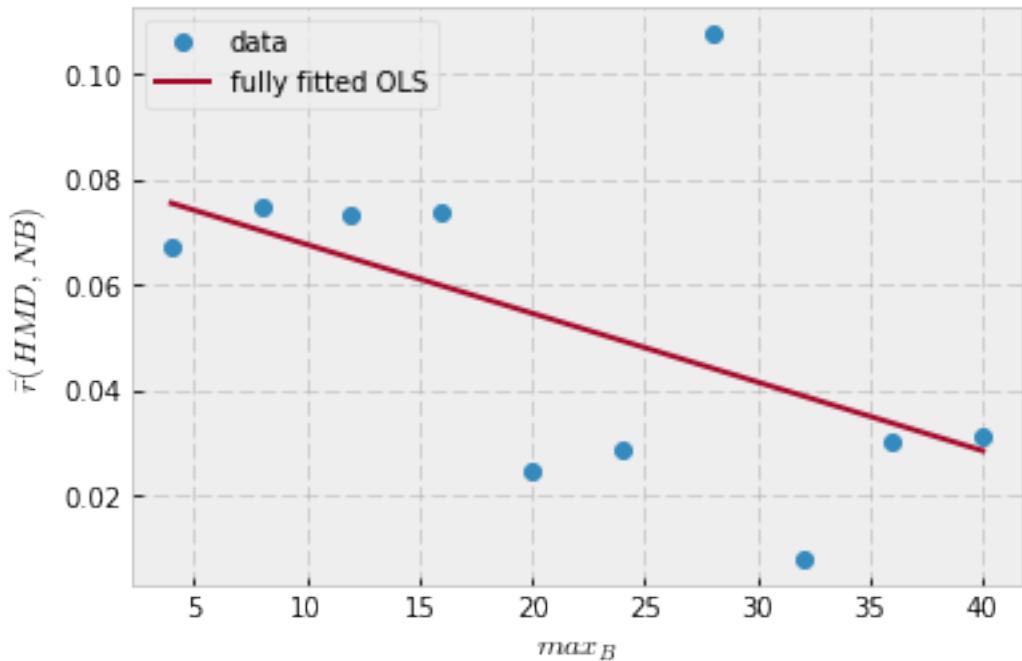
```
variable name: theta
model: poly1
r-squared: -0.742318535665935
explained variance: -0.734464735242847
```



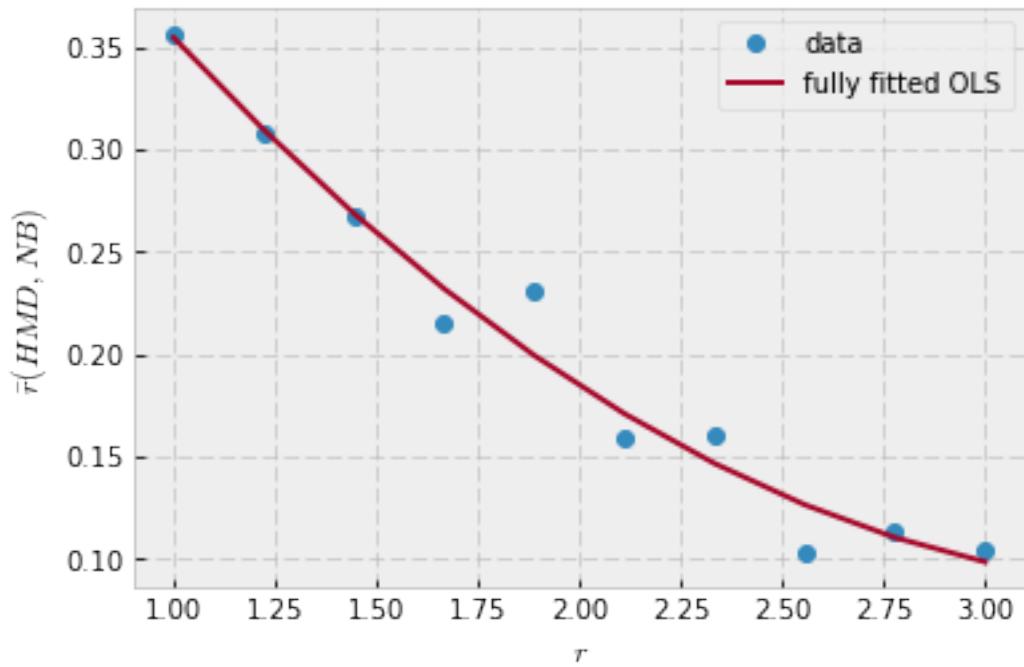
```

variable name: maxB
model: poly1
r-squared: -0.0022674093433165954
explained variance: -0.002231391051582321

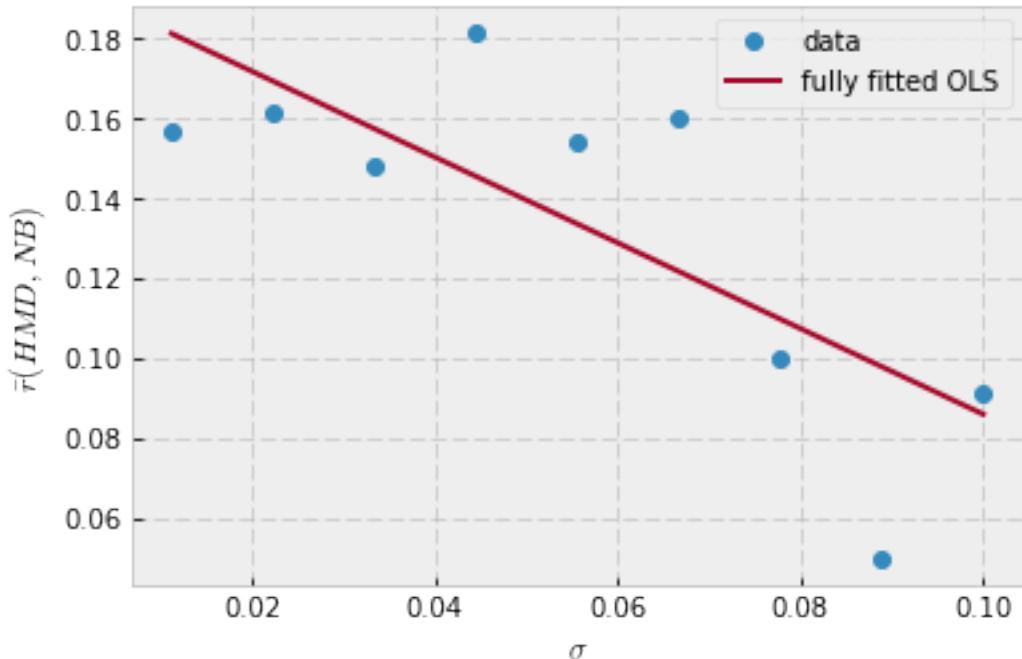
```



```
variable name: r
model: poly2
r-squared: 0.9484018825139207
explained variance: 0.9485807424928059
```



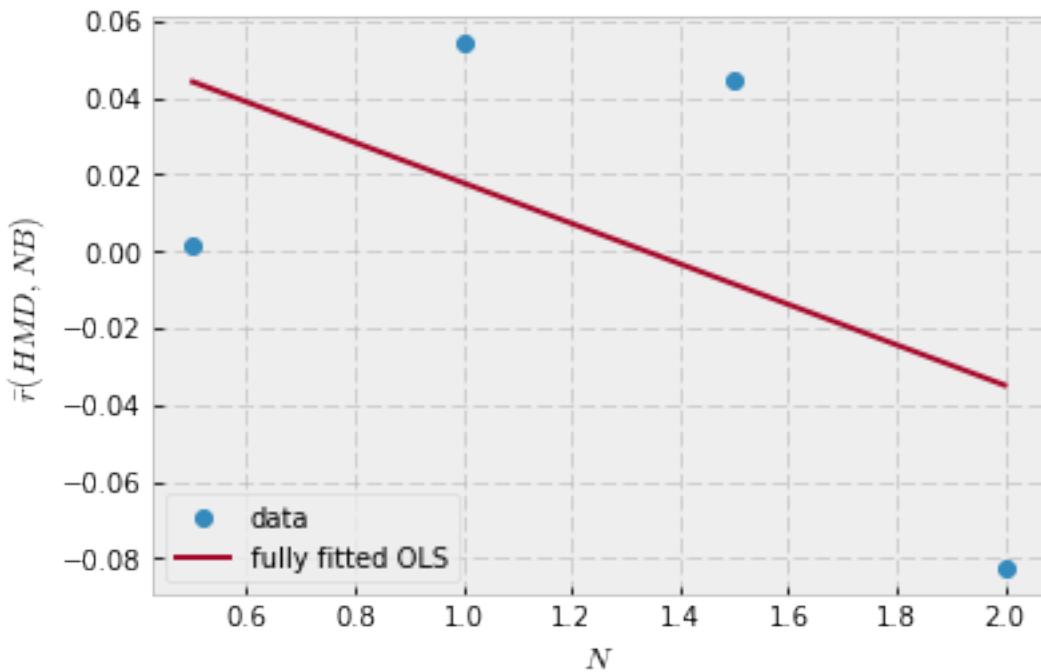
```
variable name: sigma
model: poly1
r-squared: 0.29609087300004244
explained variance: 0.29978998571676185
```



```

variable name: N
model: poly1
r-squared: -3.586546260535158
explained variance: -2.9593070928546674

```



```
In [52]: y1_name, y2_name = ('mean_depth_of_standard_root_water_uptake',
                           'maximal_horizontal_spread')
```

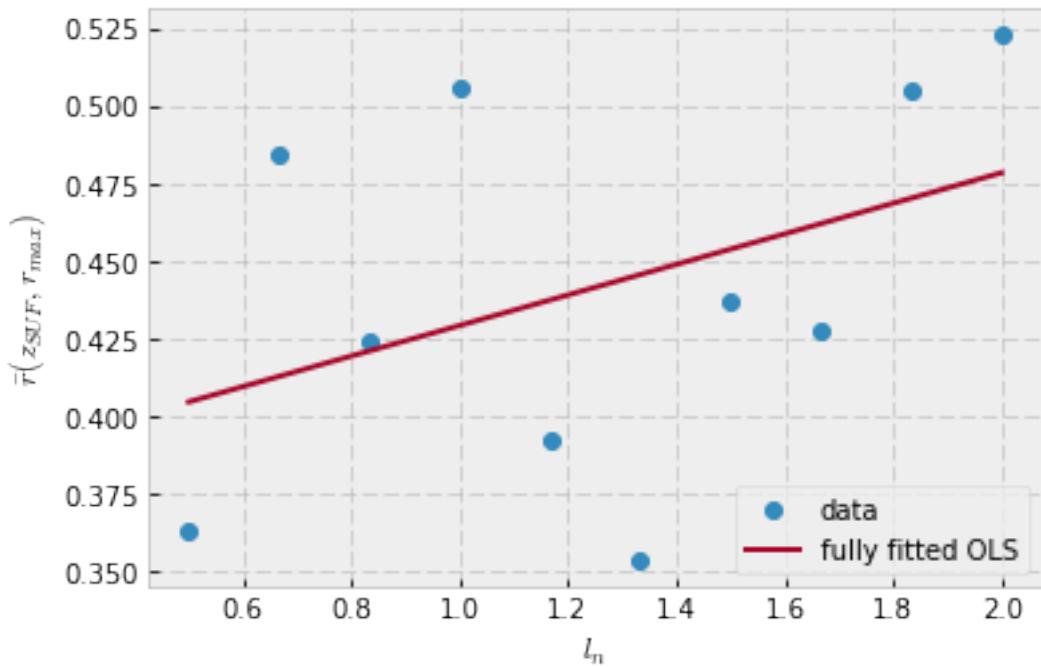
```
corr_reg('ln', y1_name, y2_name, 'poly1')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('maxB', y1_name, y2_name, 'poly2')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly2')
corr_reg('N', y1_name, y2_name, 'poly1')
```

variable name: ln

model: poly1

r-squared: -0.27874347557872126

explained variance: -0.2771542385817316

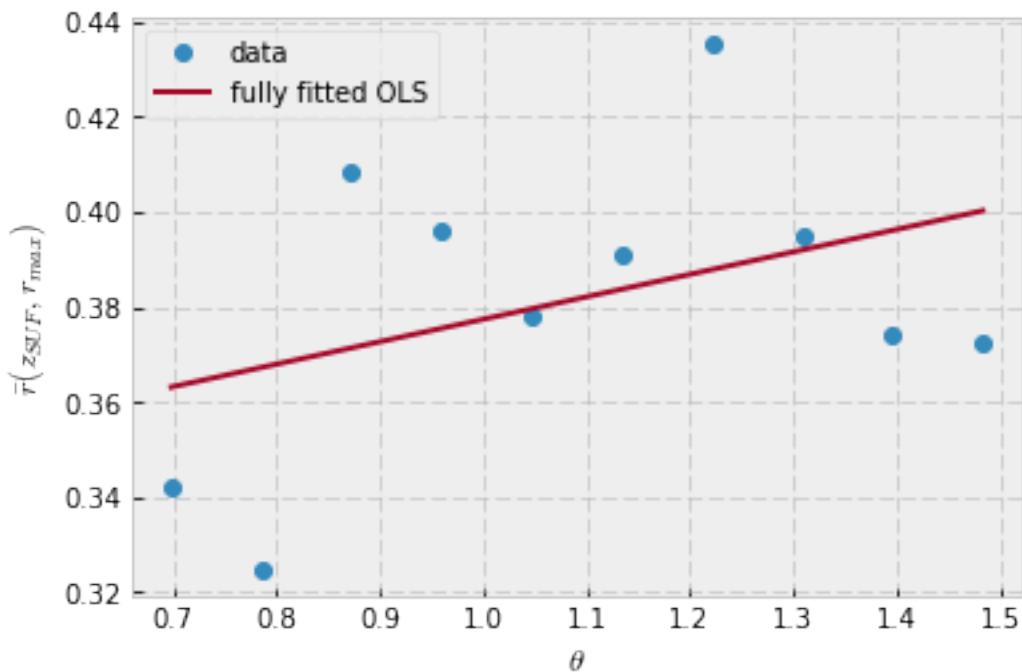


variable name: theta

model: poly1

r-squared: -0.38837947341724255

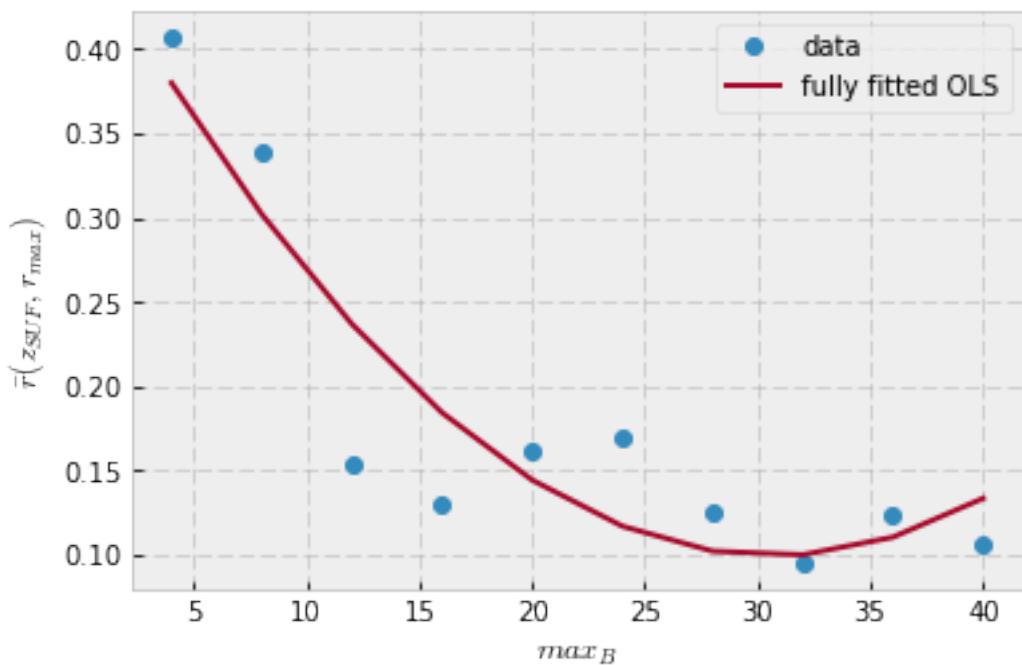
explained variance: -0.379615529088124



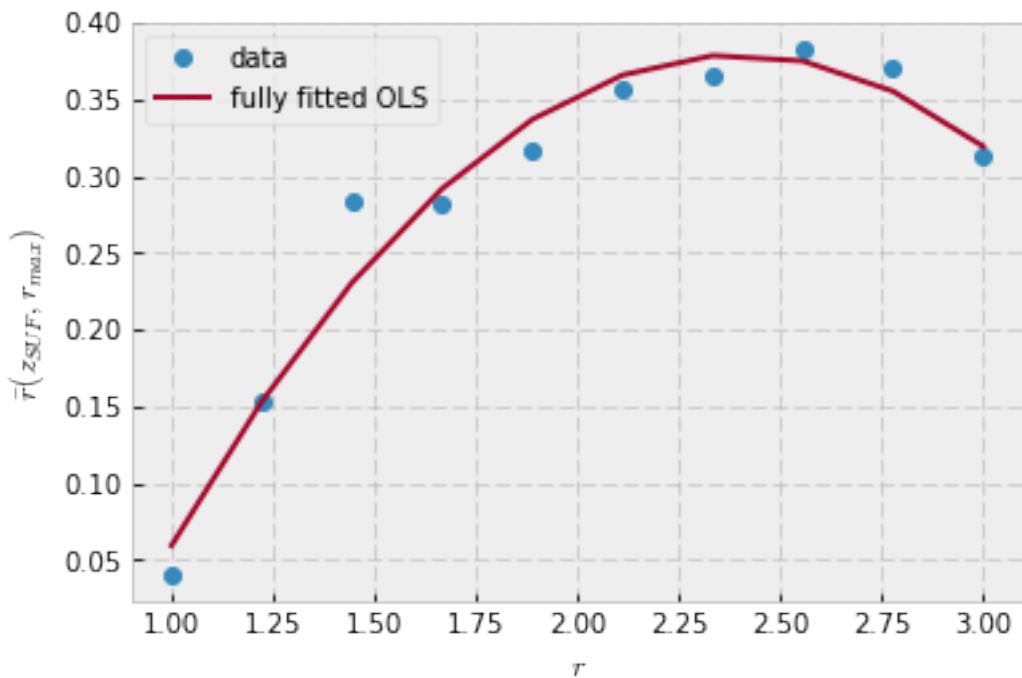
```

variable name: maxB
model: poly2
r-squared: 0.6555169906205093
explained variance: 0.6556523079204208

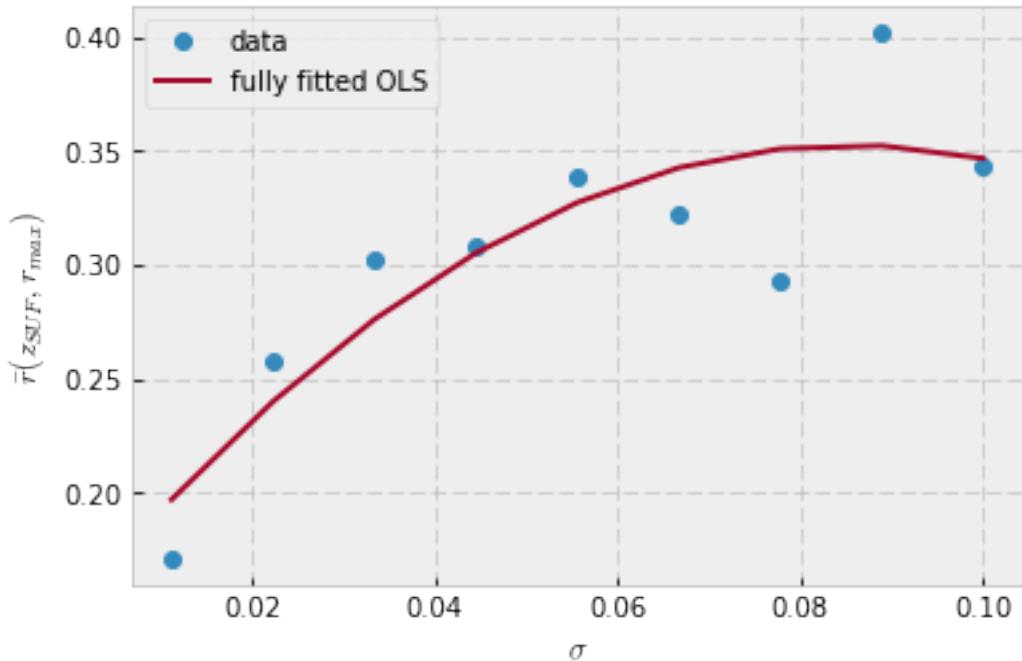
```



```
variable name: r
model: poly2
r-squared: 0.9157477922000973
explained variance: 0.9169572442983476
```



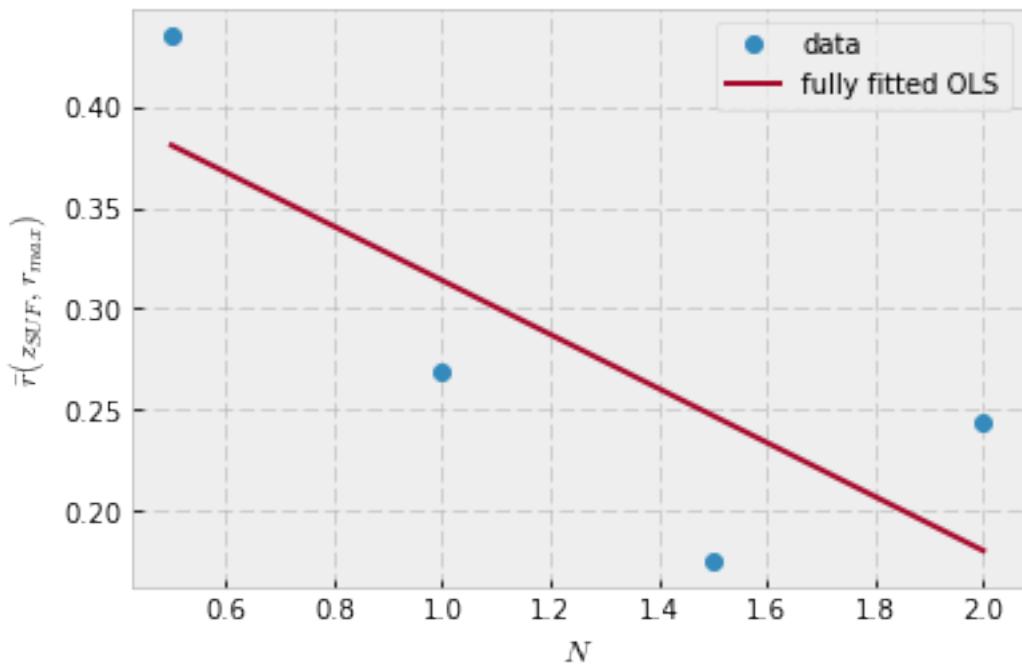
```
variable name: sigma
model: poly2
r-squared: 0.42717222381070263
explained variance: 0.43268569569560944
```



```

variable name: N
model: poly1
r-squared: -1.5361553779928028
explained variance: -1.1907483884539363

```

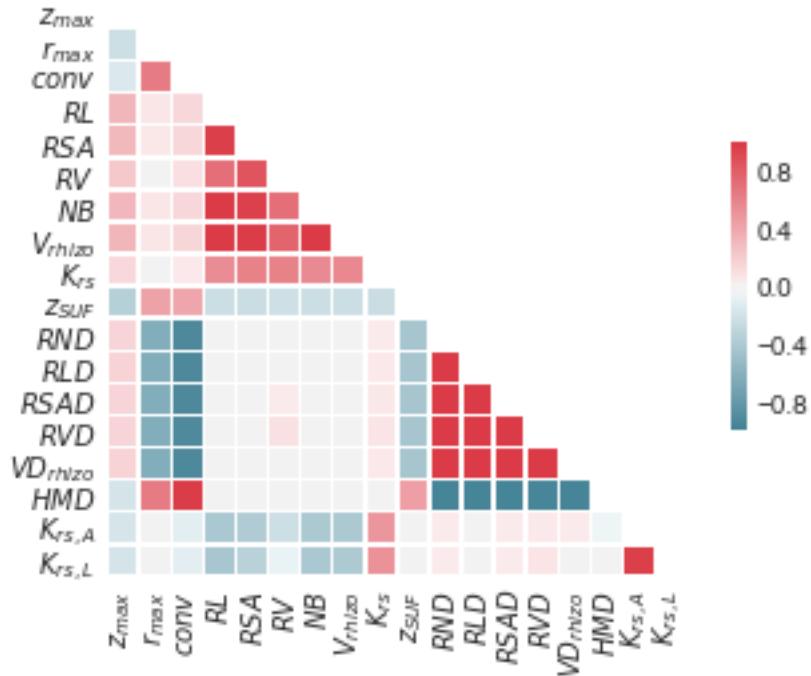


20.2 Correlation matrices

In [53]: `plot_corr_matrix()`

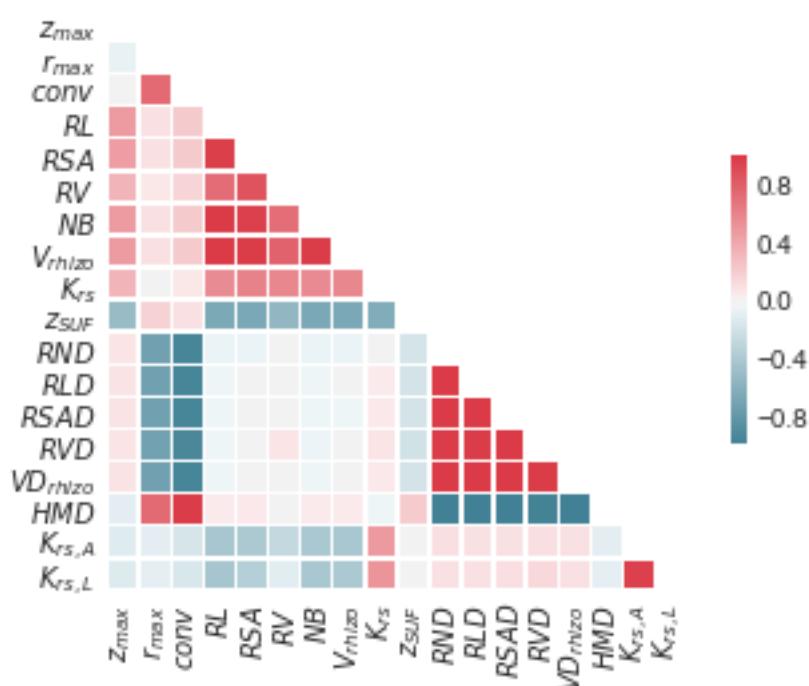
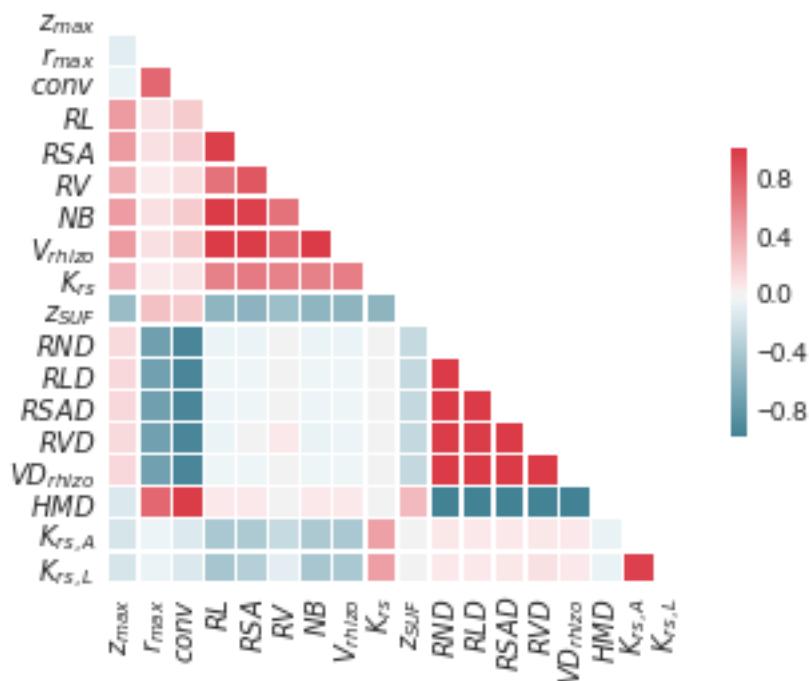
variable name: N

variable value: 0.5



variable name: N

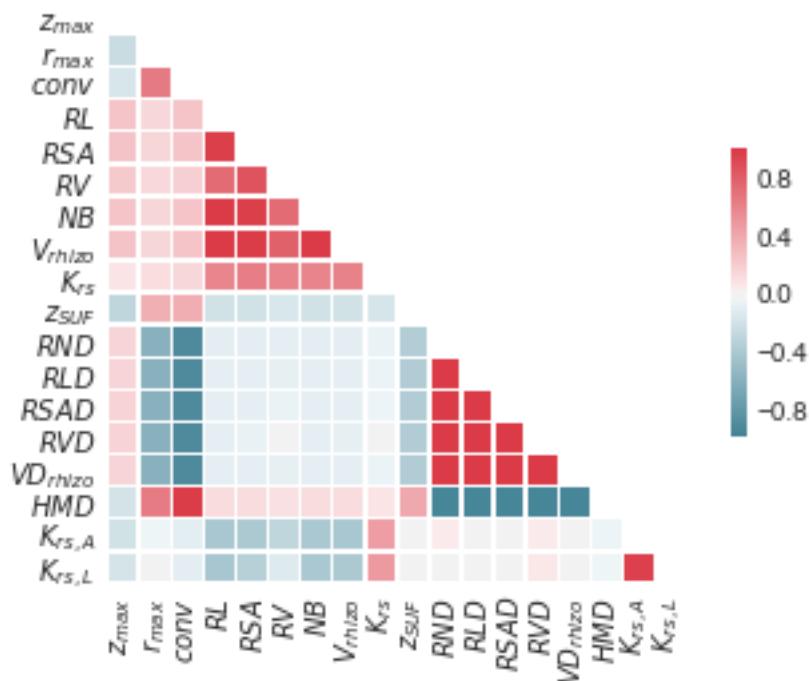
variable value: 1.0



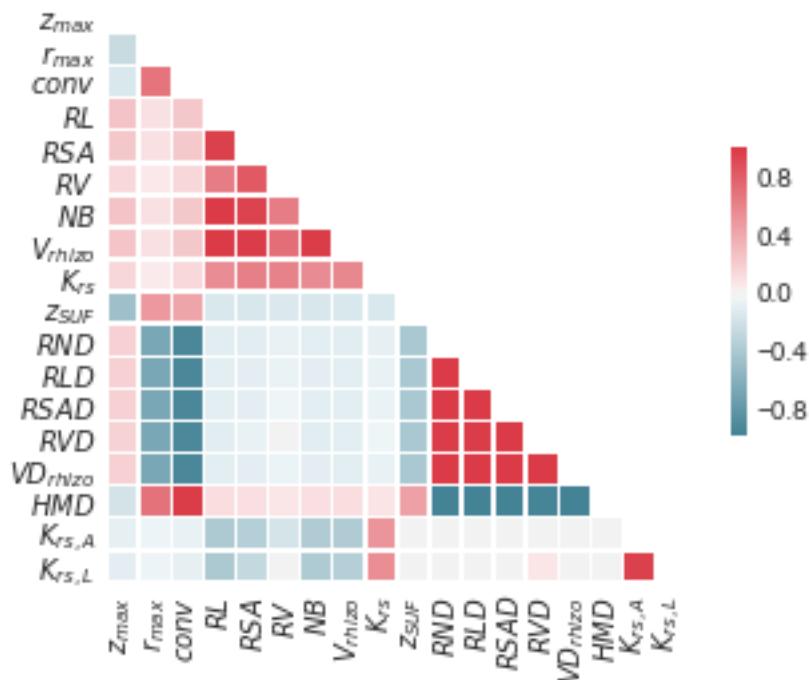
variable name: N
variable value: 2.0



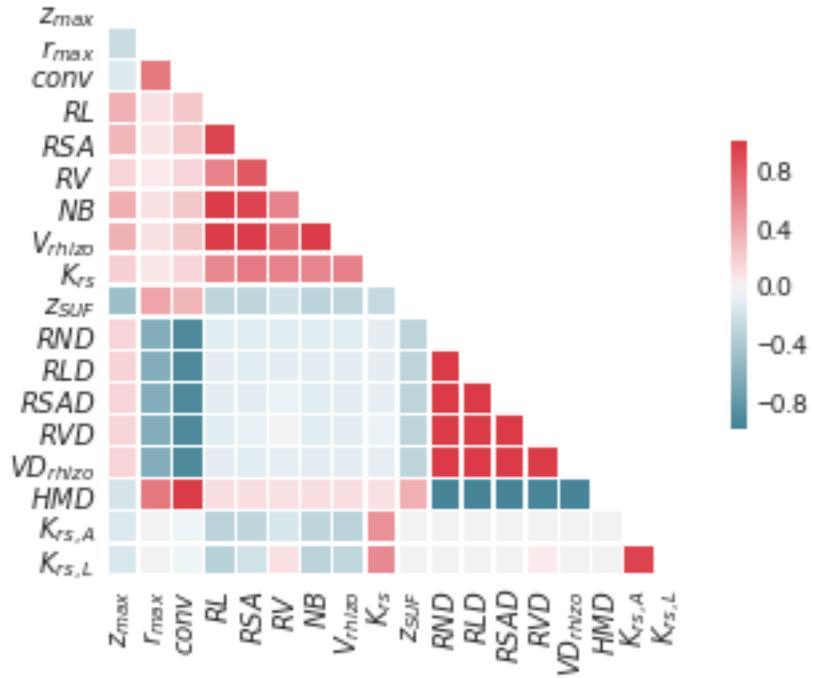
variable name: ln
variable value: 0.5



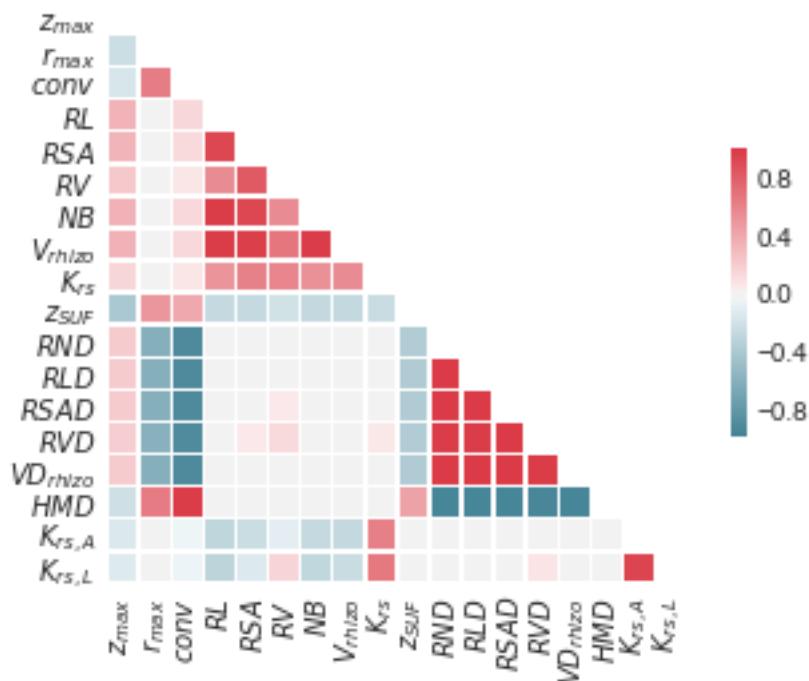
```
variable name: ln
variable value: 0.6666666666666666
```



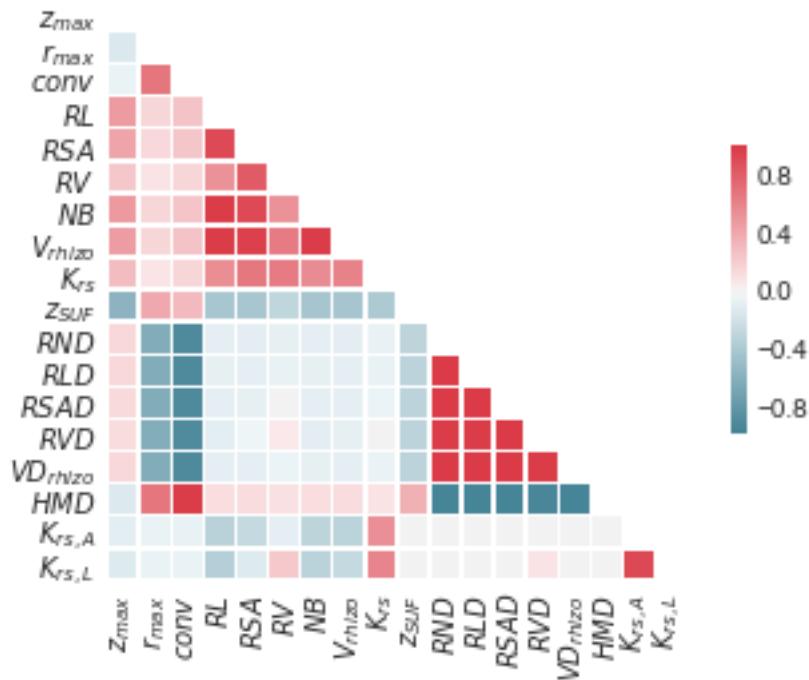
```
variable name: ln  
variable value: 0.8333333333333333
```



```
variable name: ln  
variable value: 1.0
```



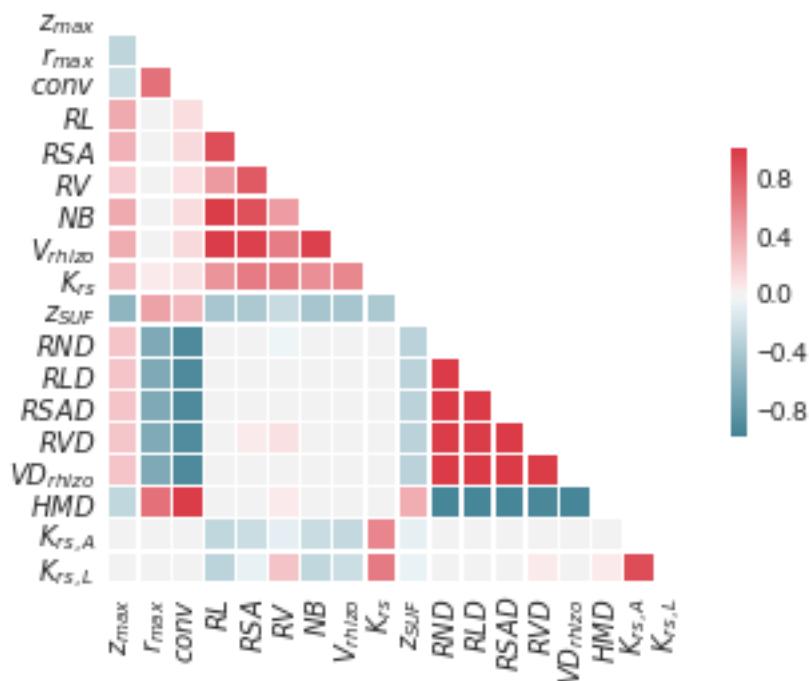
```
variable name: ln
variable value: 1.1666666666666665
```



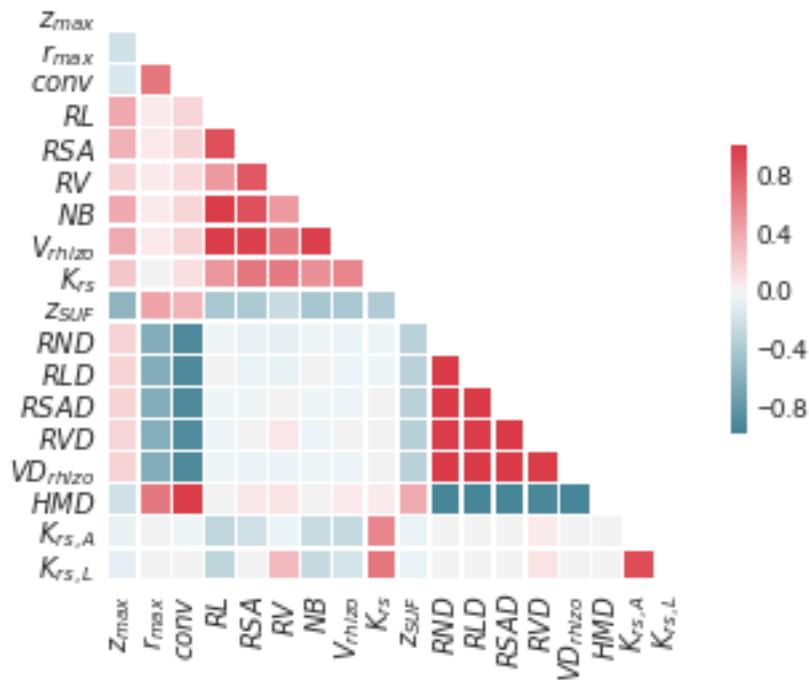
```
variable name: ln  
variable value: 1.333333333333333
```



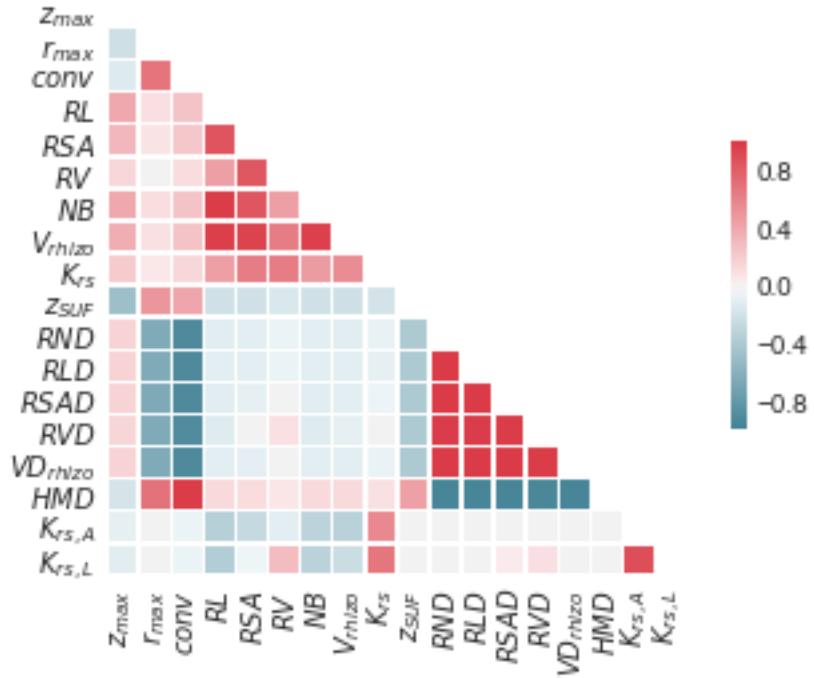
```
variable name: ln  
variable value: 1.5
```



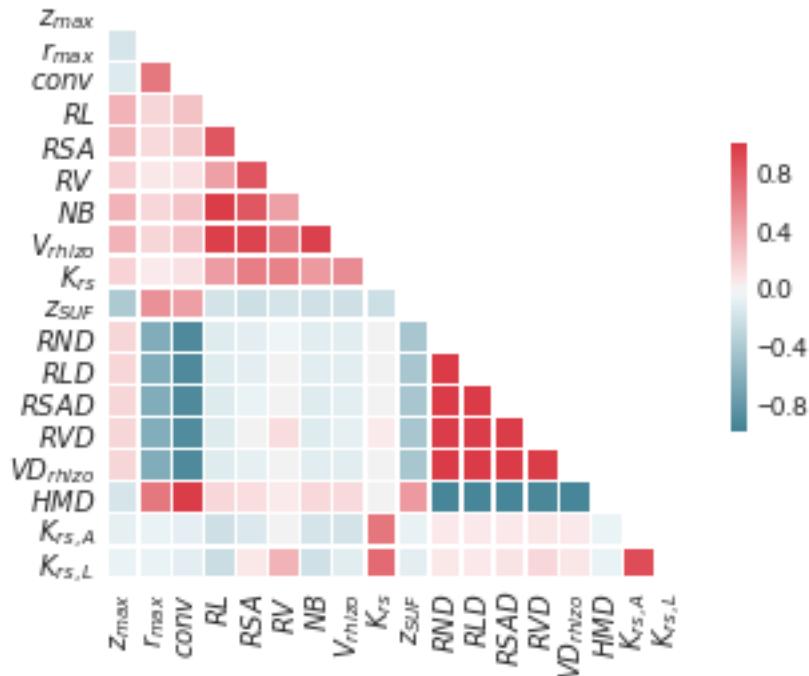
variable name: ln
 variable value: 1.6666666666666665



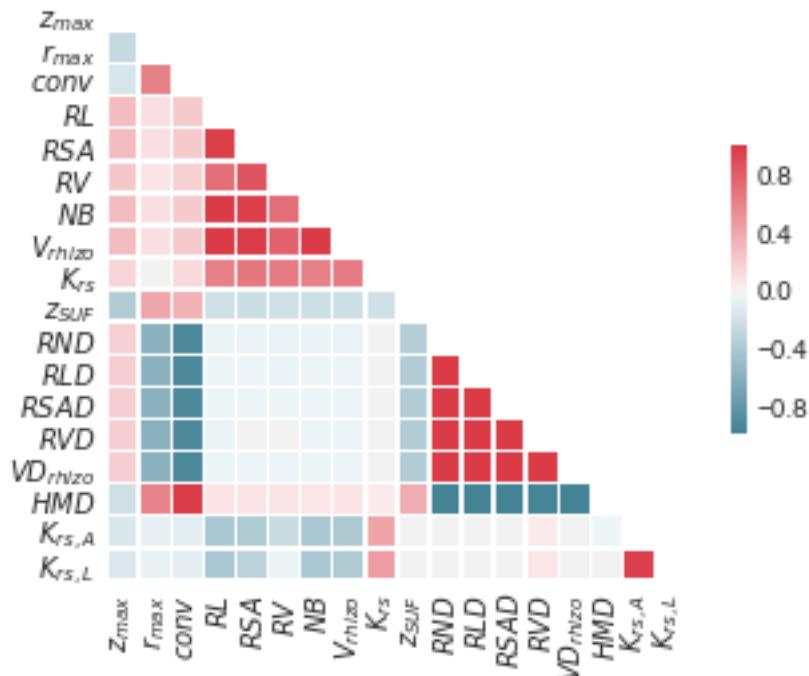
```
variable name: ln  
variable value: 1.833333333333333
```



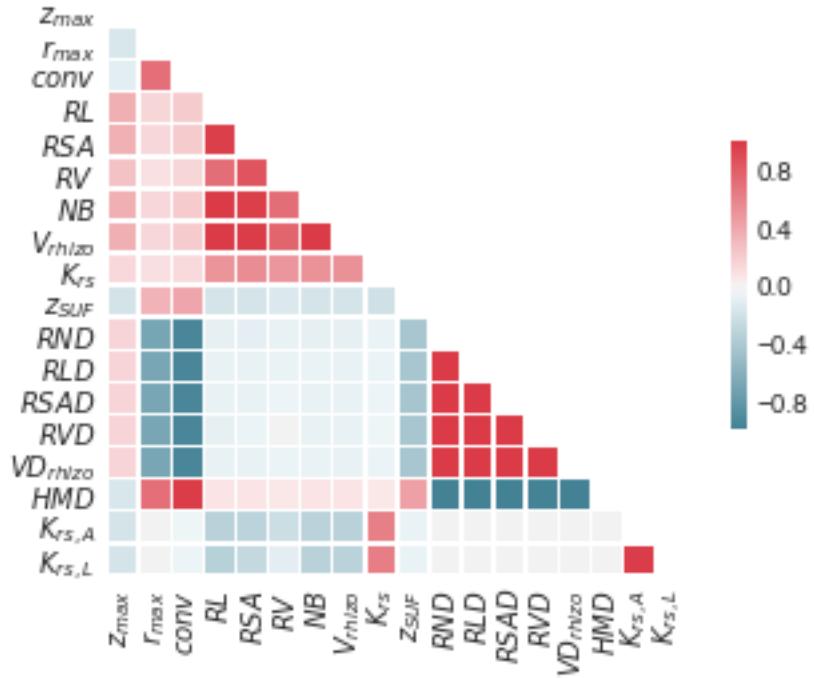
```
variable name: ln  
variable value: 2.0
```



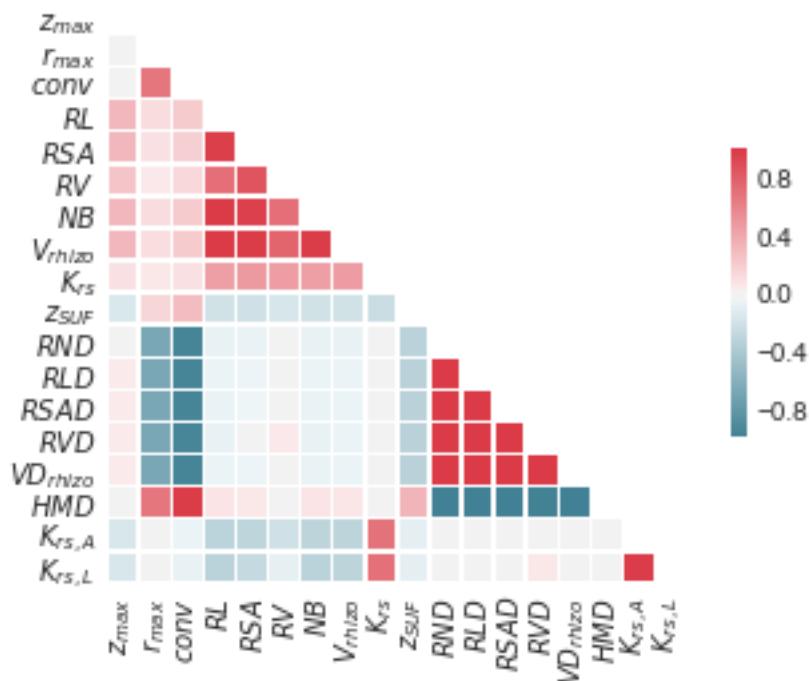
variable name: maxB
 variable value: 4.0



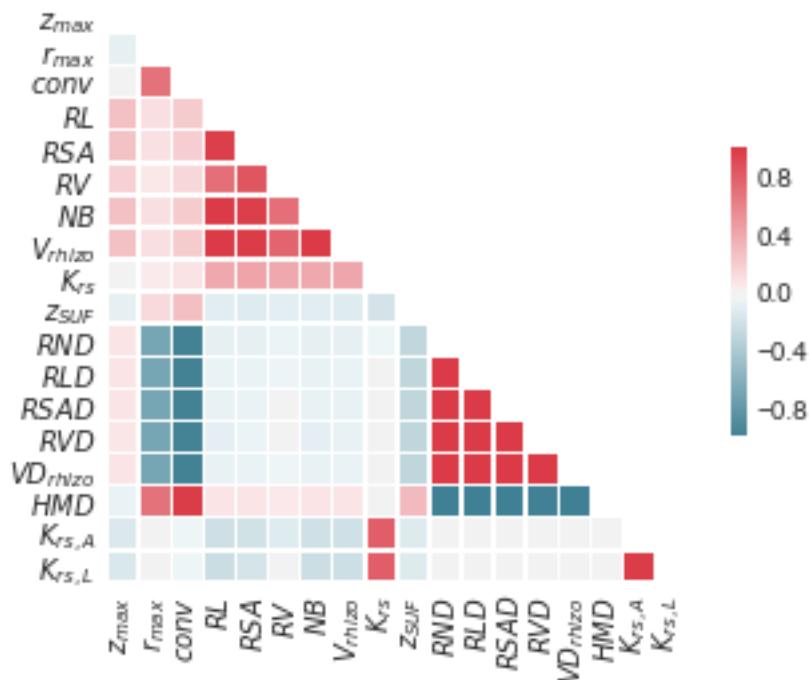
variable name: maxB
variable value: 8.0



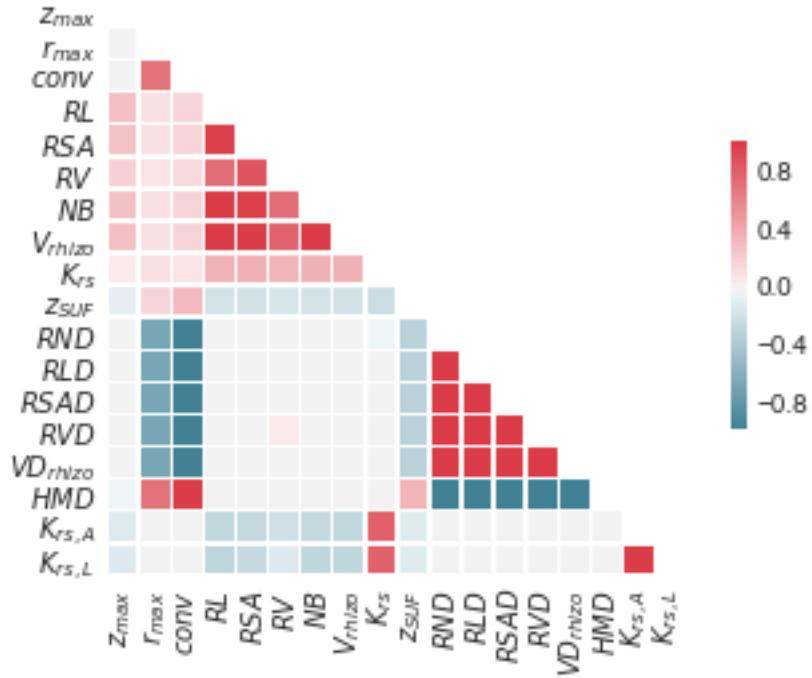
variable name: maxB
variable value: 12.0



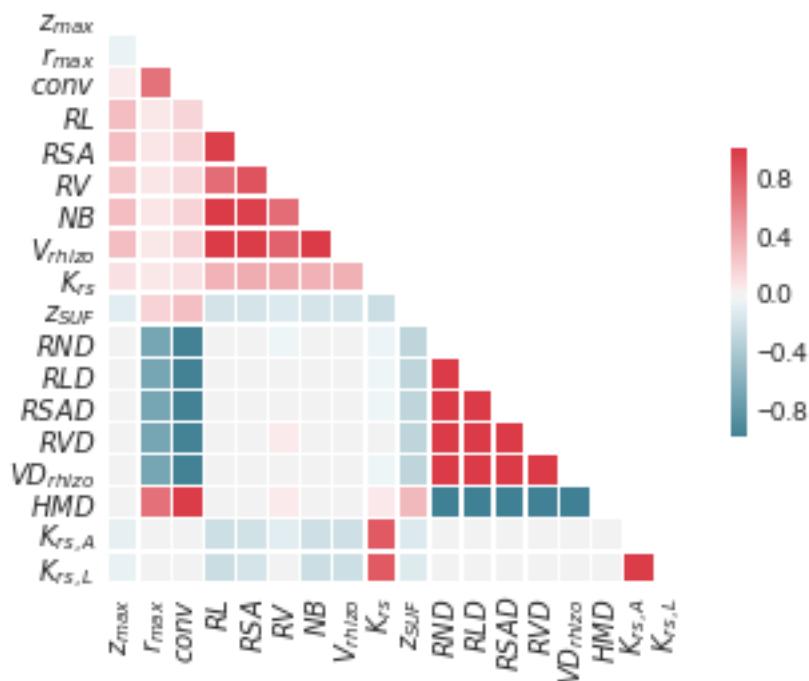
variable name: maxB
 variable value: 16.0



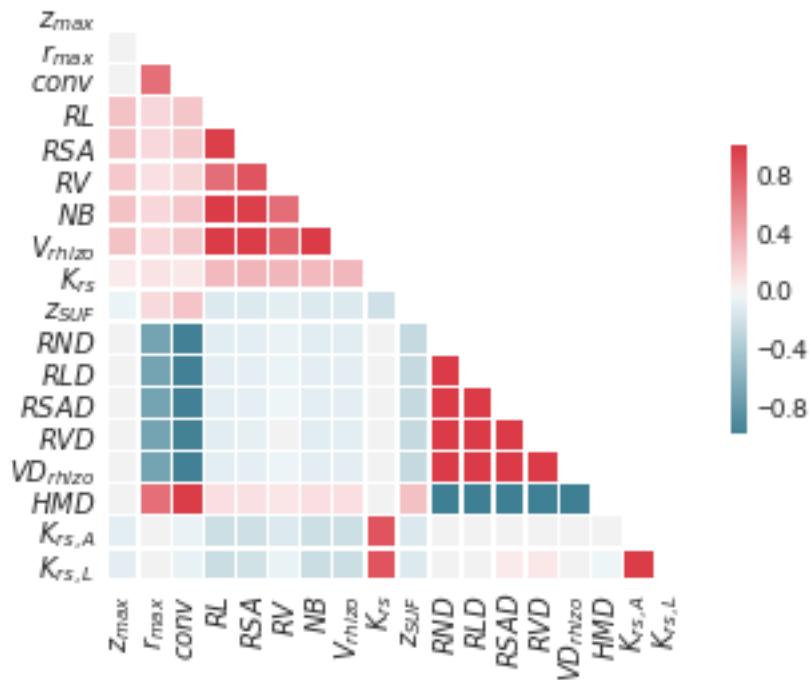
```
variable name: maxB  
variable value: 20.0
```



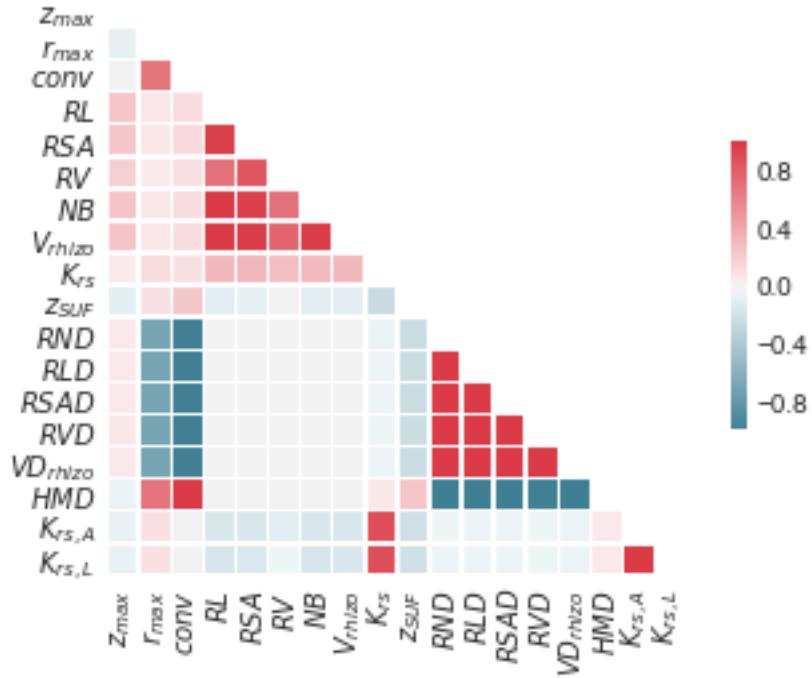
```
variable name: maxB  
variable value: 24.0
```



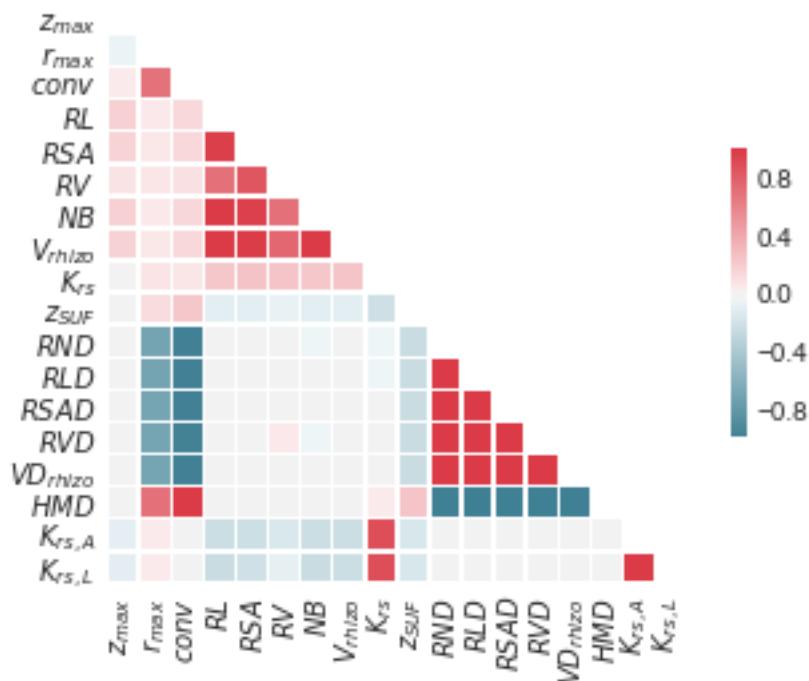
variable name: maxB
 variable value: 28.0



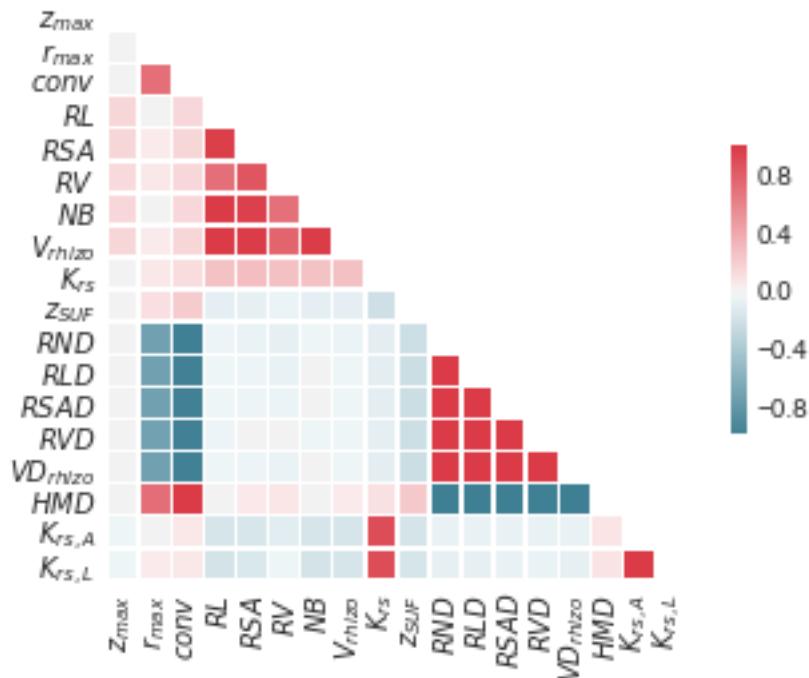
```
variable name: maxB  
variable value: 32.0
```



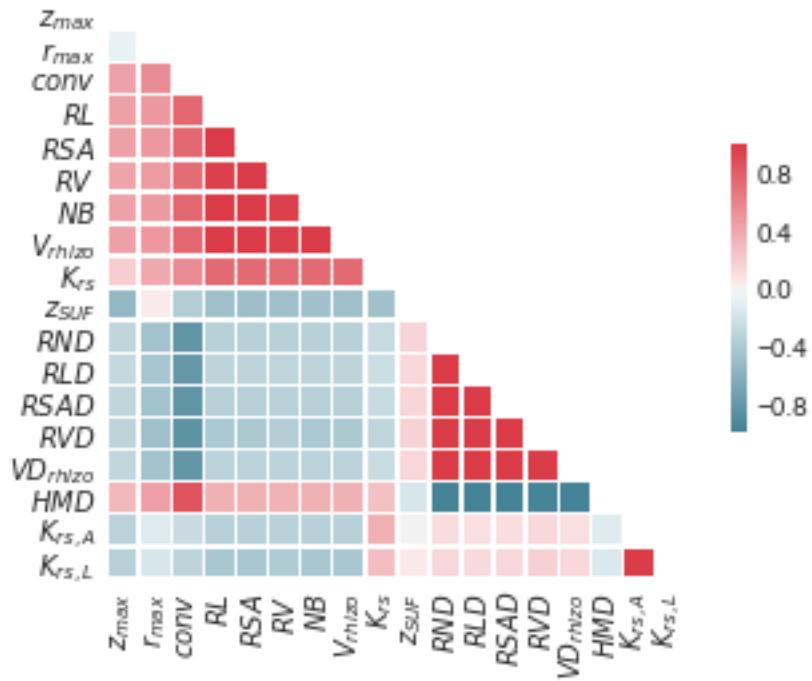
```
variable name: maxB  
variable value: 36.0
```



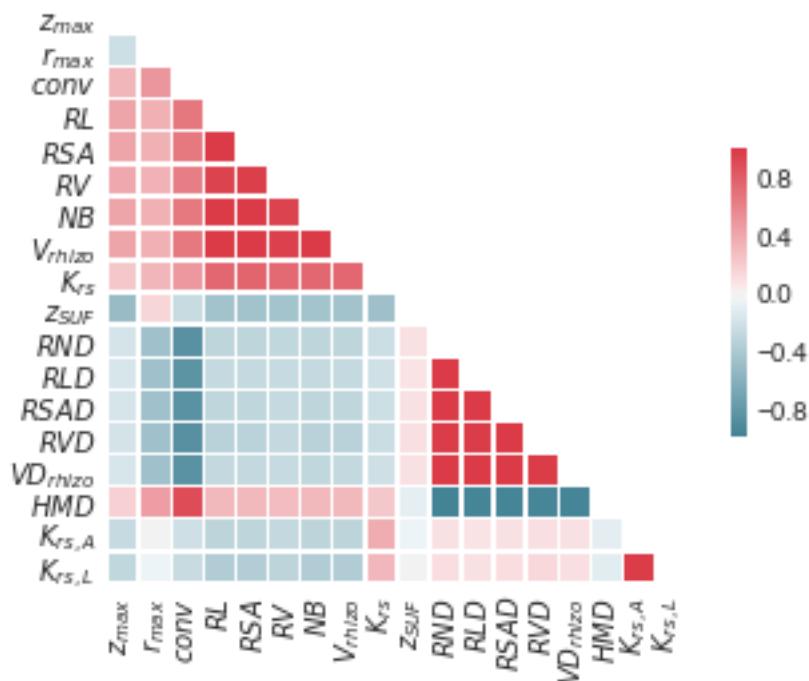
variable name: maxB
 variable value: 40.0



```
variable name: r  
variable value: 1.0
```

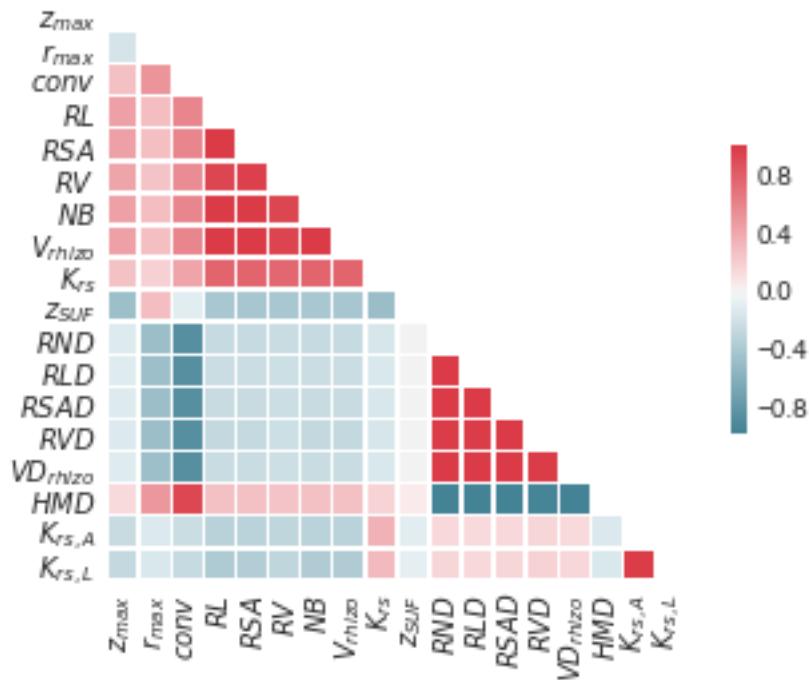


```
variable name: r  
variable value: 1.222222222222223
```



variable name: r

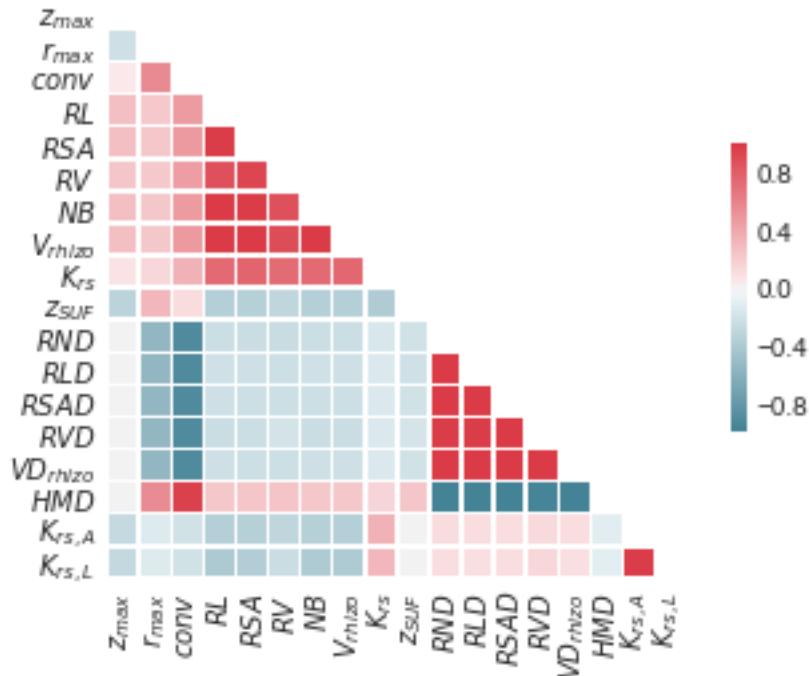
variable value: 1.4444444444444444



```
variable name: r  
variable value: 1.6666666666666665
```

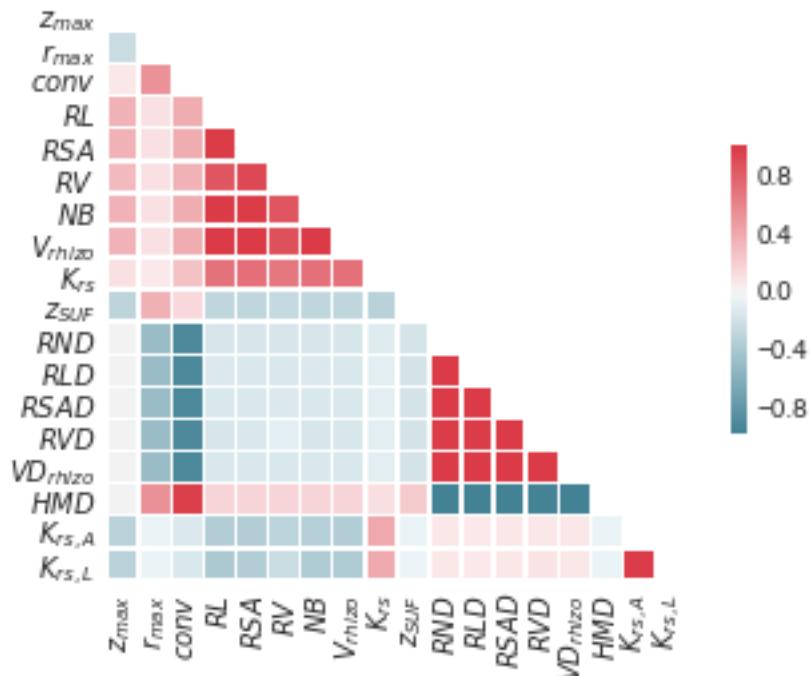


```
variable name: r  
variable value: 1.8888888888888888
```

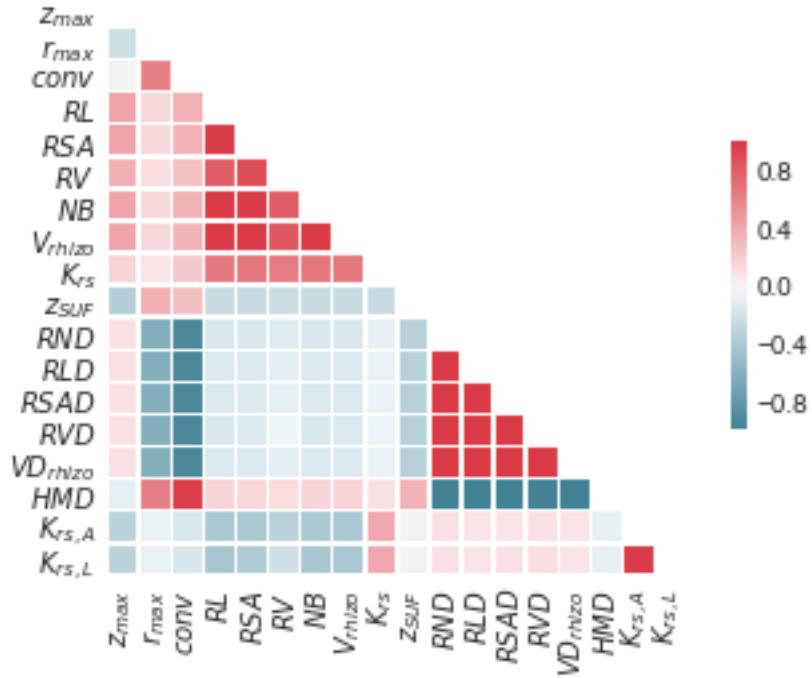


variable name: r

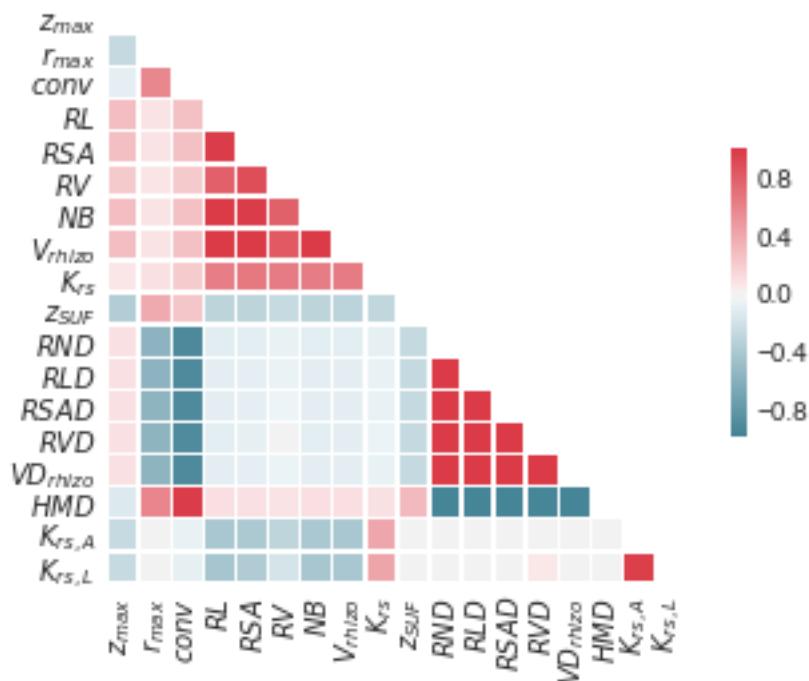
variable value: 2.111111111111111



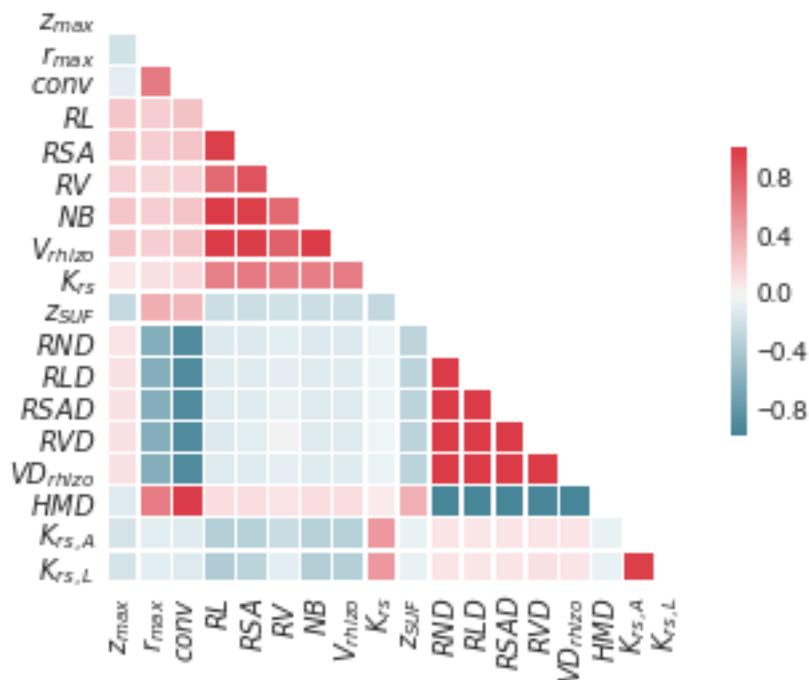
```
variable name: r  
variable value: 2.333333333333333
```



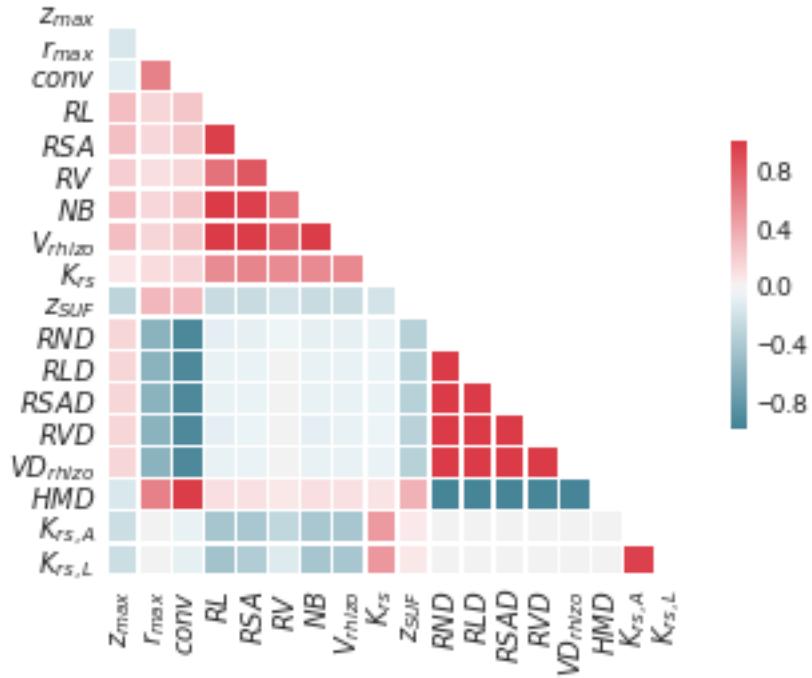
```
variable name: r  
variable value: 2.5555555555555554
```



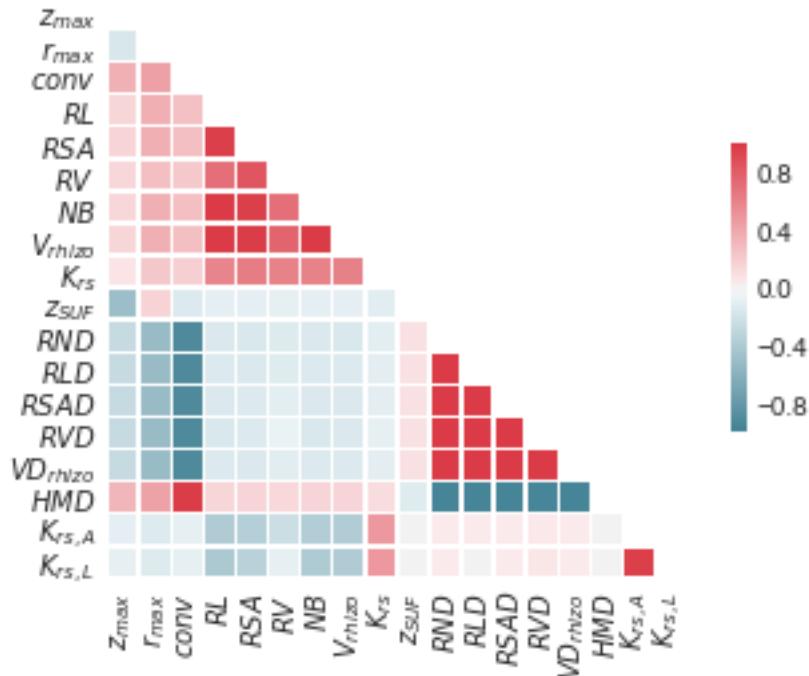
```
variable name: r
variable value: 2.7777777777777777
```



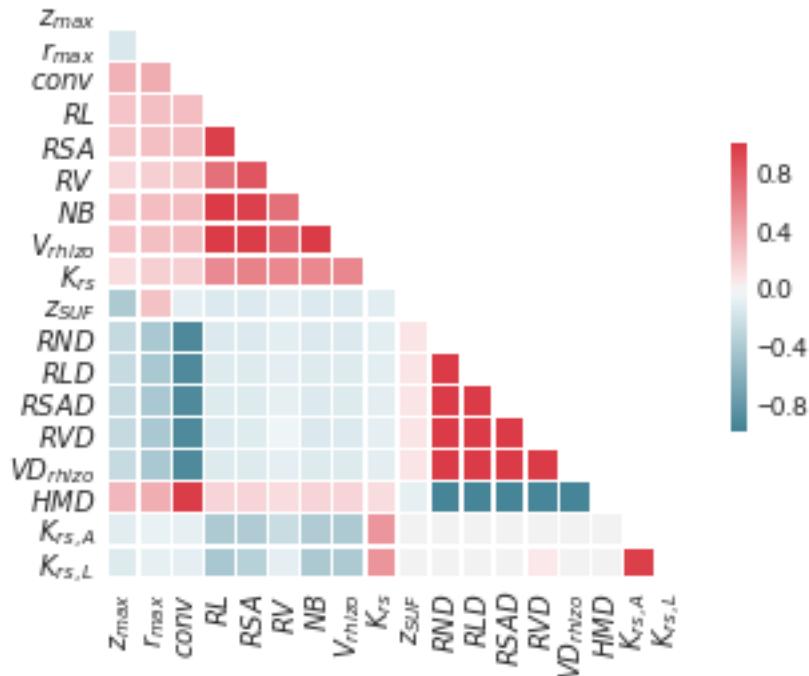
```
variable name: r  
variable value: 3.0
```



```
variable name: sigma  
variable value: 0.01111111111111112
```



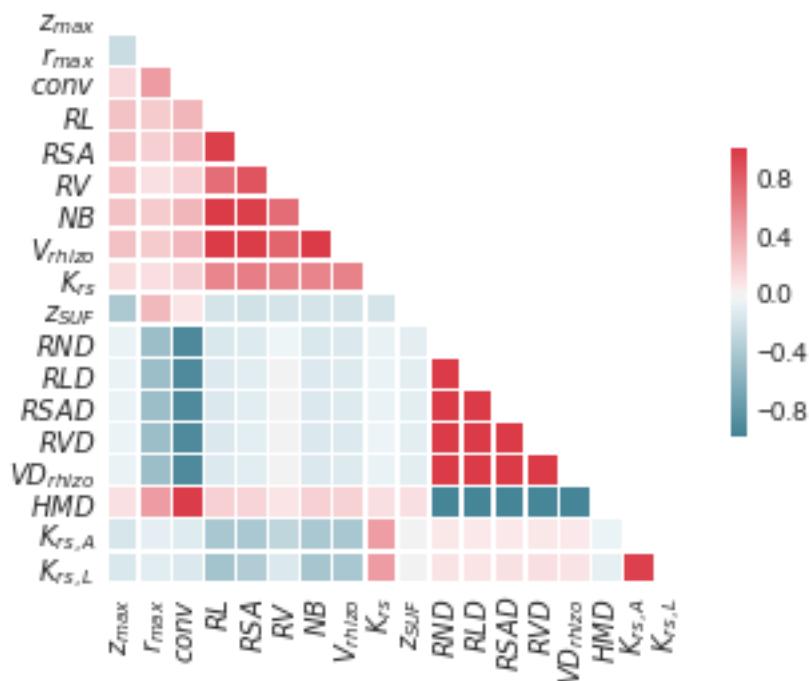
```
variable name: sigma
variable value: 0.0222222222222223
```



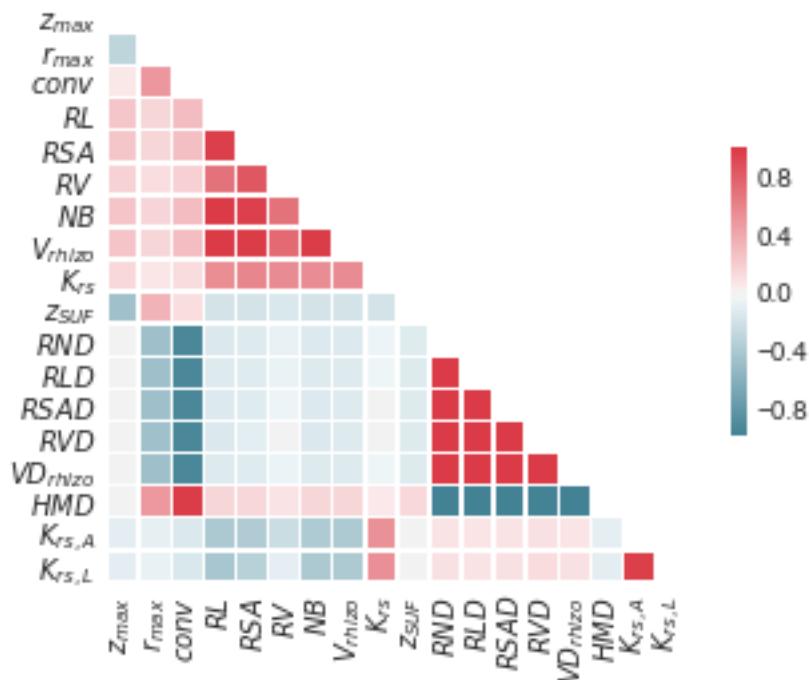
```
variable name: sigma
variable value: 0.03333333333333333
```



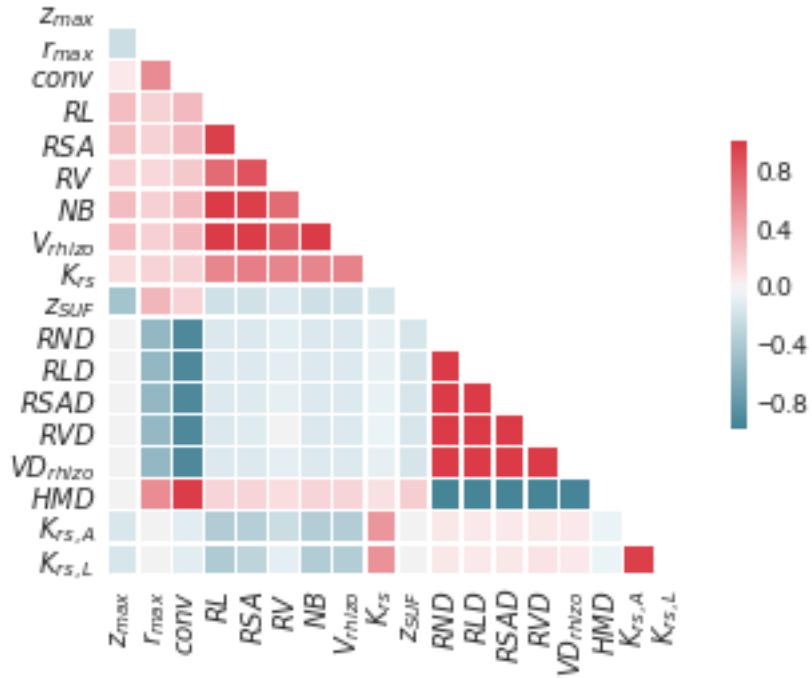
```
variable name: sigma
variable value: 0.04444444444444444
```



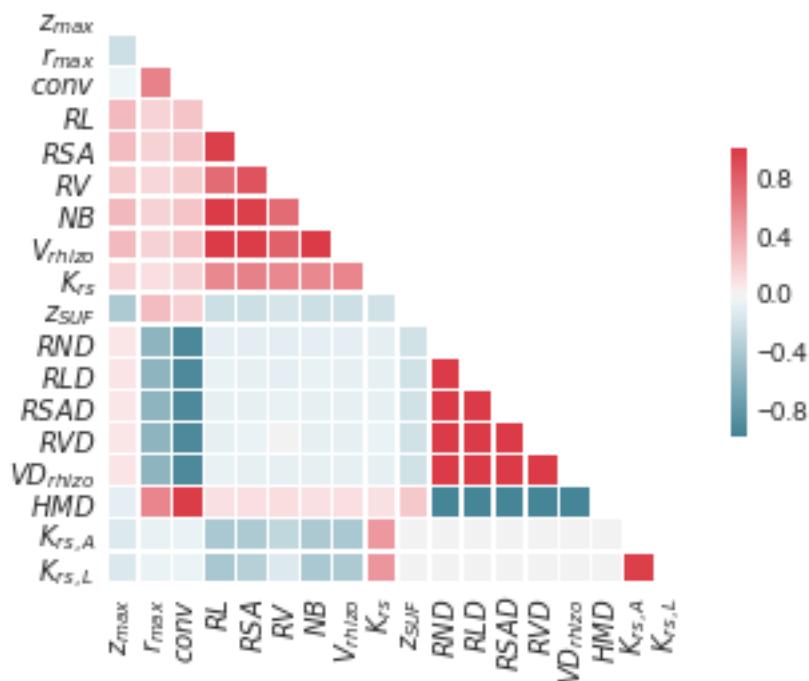
```
variable name: sigma
variable value: 0.05555555555555556
```



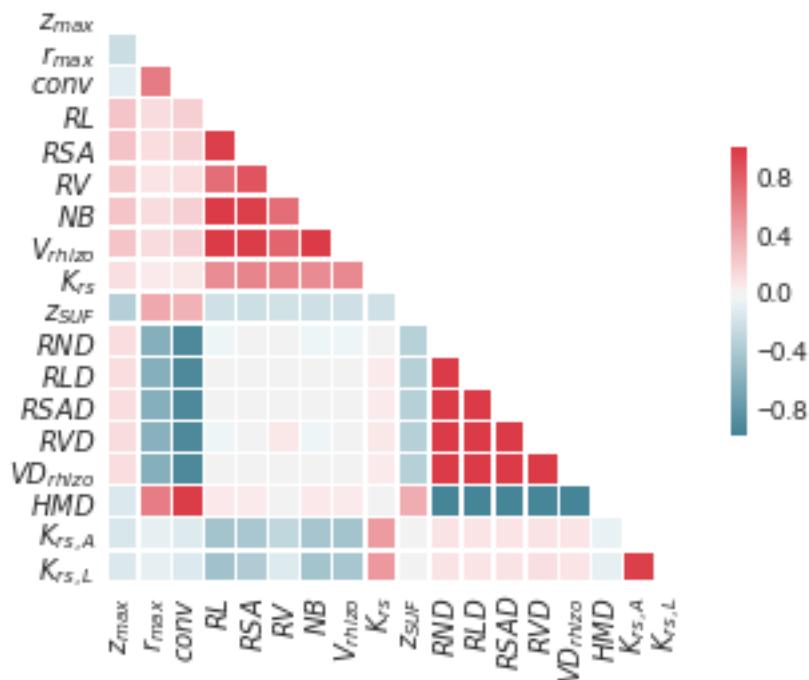
```
variable name: sigma
variable value: 0.06666666666666667
```



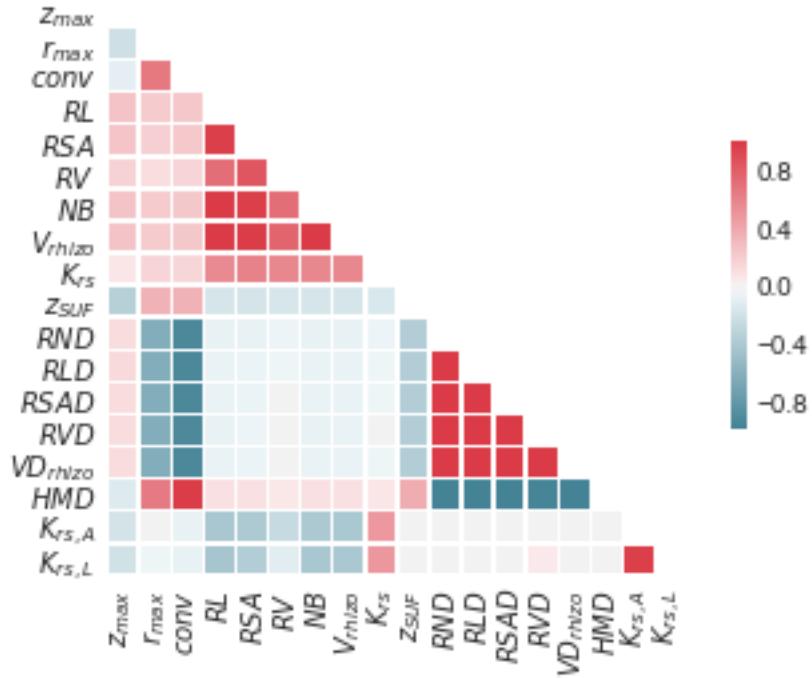
```
variable name: sigma
variable value: 0.07777777777777778
```



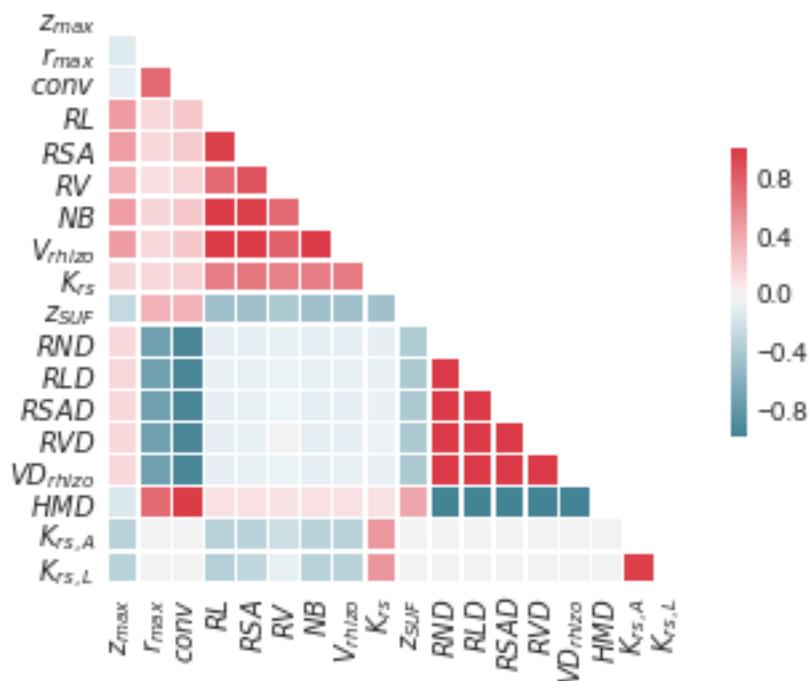
```
variable name: sigma  
variable value: 0.0888888888888889
```



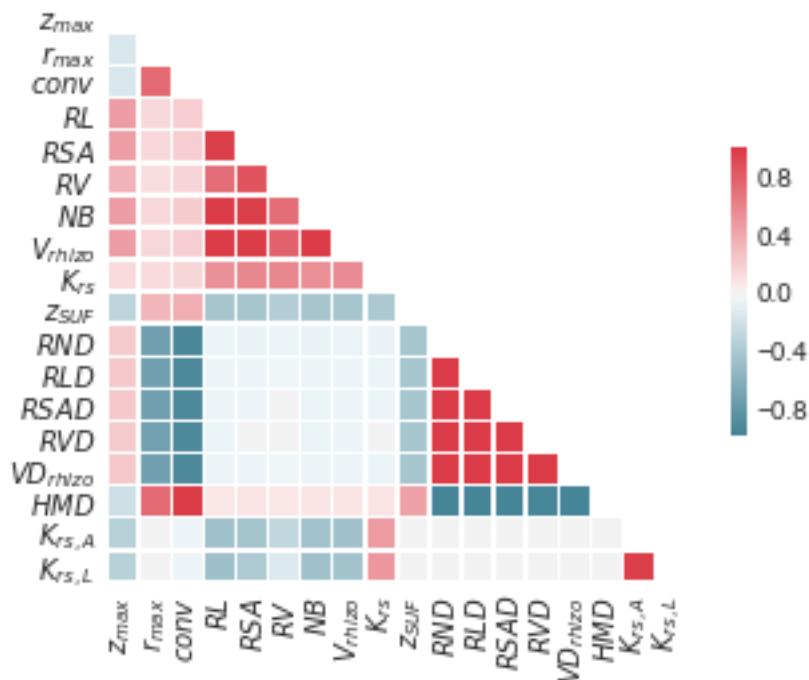
```
variable name: sigma  
variable value: 0.1
```



```
variable name: theta  
variable value: 0.6981317007977318
```



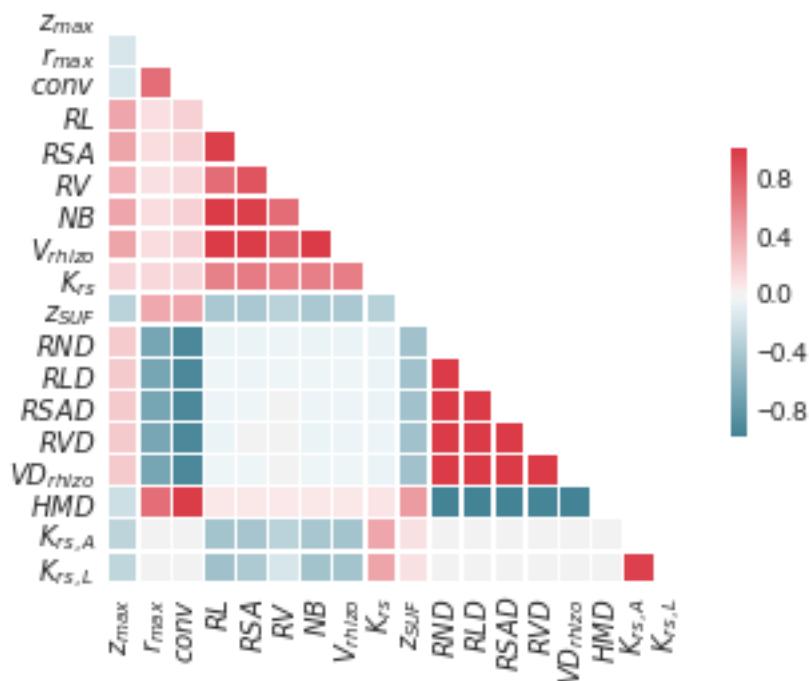
```
variable name: theta
variable value: 0.7853981633974483
```



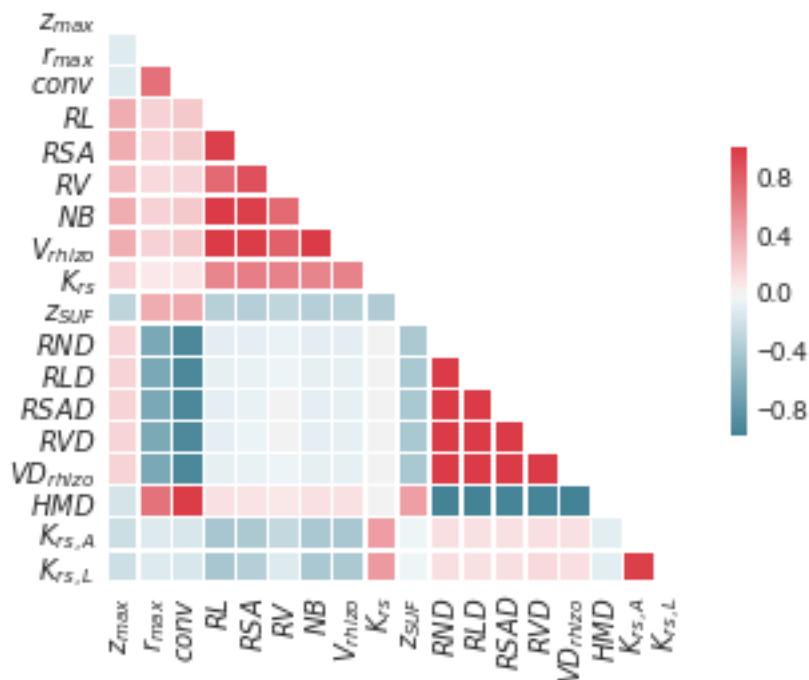
```
variable name: theta  
variable value: 0.8726646259971648
```



```
variable name: theta  
variable value: 0.9599310885968813
```



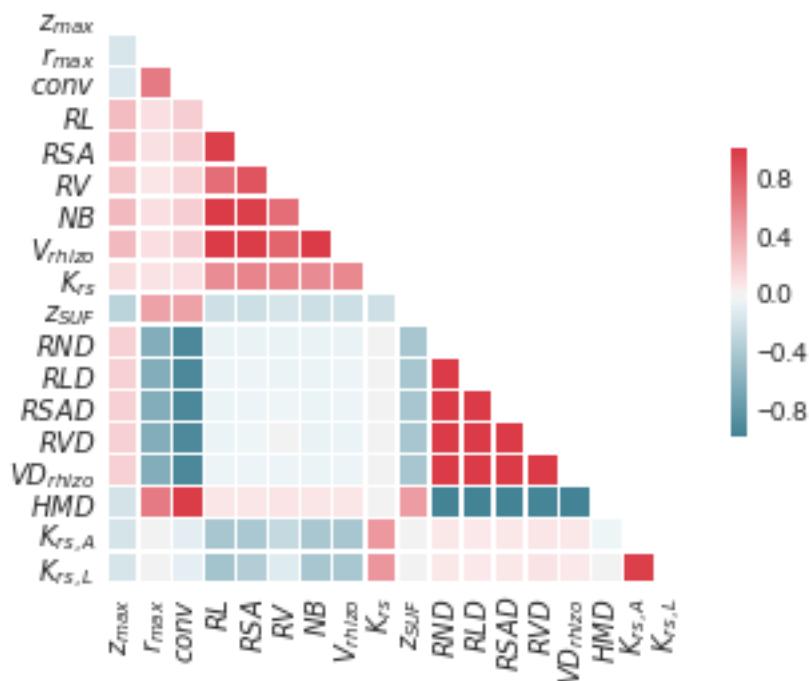
```
variable name: theta
variable value: 1.0471975511965979
```



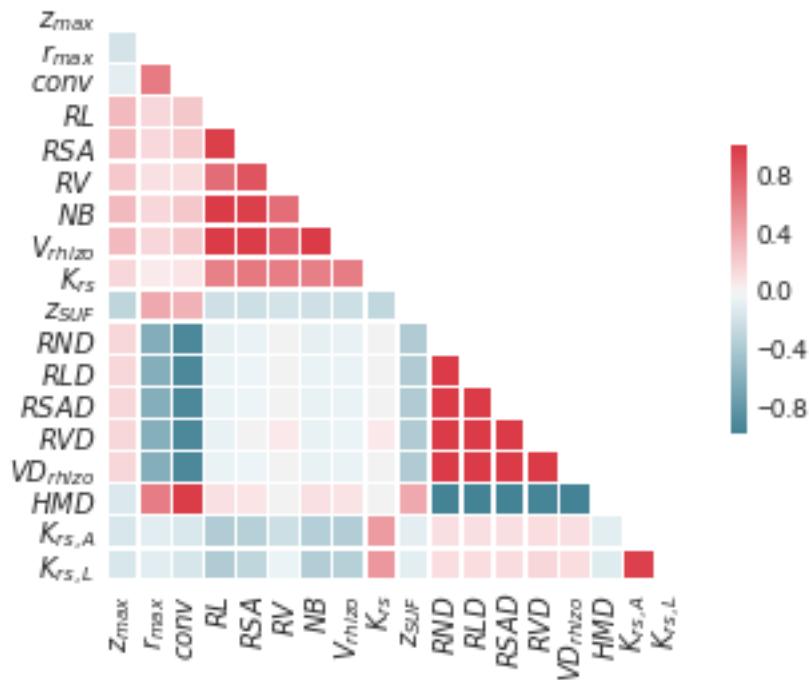
```
variable name: theta  
variable value: 1.1344640137963142
```



```
variable name: theta  
variable value: 1.2217304763960306
```



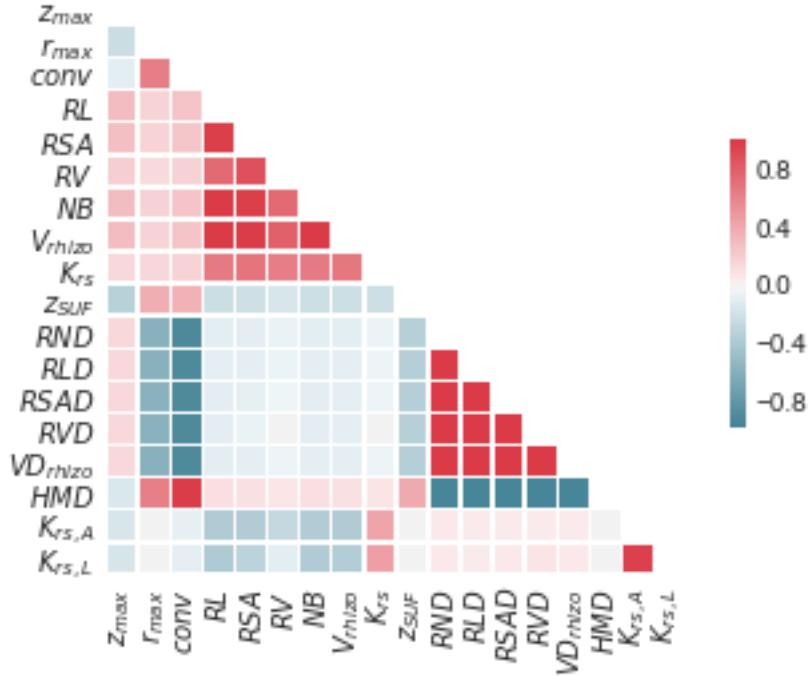
```
variable name: theta
variable value: 1.3089969389957472
```



```
variable name: theta
variable value: 1.3962634015954638
```



```
variable name: theta
variable value: 1.4835298641951802
```



21 Appendix

```
In [54]: curr_plot_style = plt.style.library[plot_style]
curr_plot_style.update({'font.size': 15})
with plt.style.context(curr_plot_style):
    fig, ((ax11, ax12), (ax21, ax22)) = plt.subplots(2,2, figsize=(13, 13))

    x_name, var_name = 'root_tip_density', 'maxB'
    var_value, dist_name = 4, 'inverse-gamma'
    dist = param_dists[dist_name]
    var_names = np.unique(root_data.variable_name)
    plot_data = root_data.loc[(root_data.variable_name == var_name) &
                               (root_data.variable_value == var_value), x_name]

    dist_params = dist.fit(plot_data)
    sb.distplot(plot_data, kde=True, fit=dist, ax=ax11,
                hist_kws={'color': plot_color_dict['data'], 'label':'histogram'},
                kde_kws={'color': plot_color_dict['kde'], 'label':'KDE'},
                fit_kws={'color': plot_color_dict['fitted_pred'], 'label':'fitted PDF'})
    ax11.set_xlabel(measure_label_dict[x_name])
    ax11.legend()
    ax11.set_title('(a)')
```

```

probplot = sm.ProbPlot(plot_data, dist, fit=True)
probplot.qqplot(ax=ax12, color=plot_color_dict['data'])
sm.qqline(line='45', ax=ax12, fmt=plot_color_dict['fitted_pred'])
ax12.set_title('(a)')

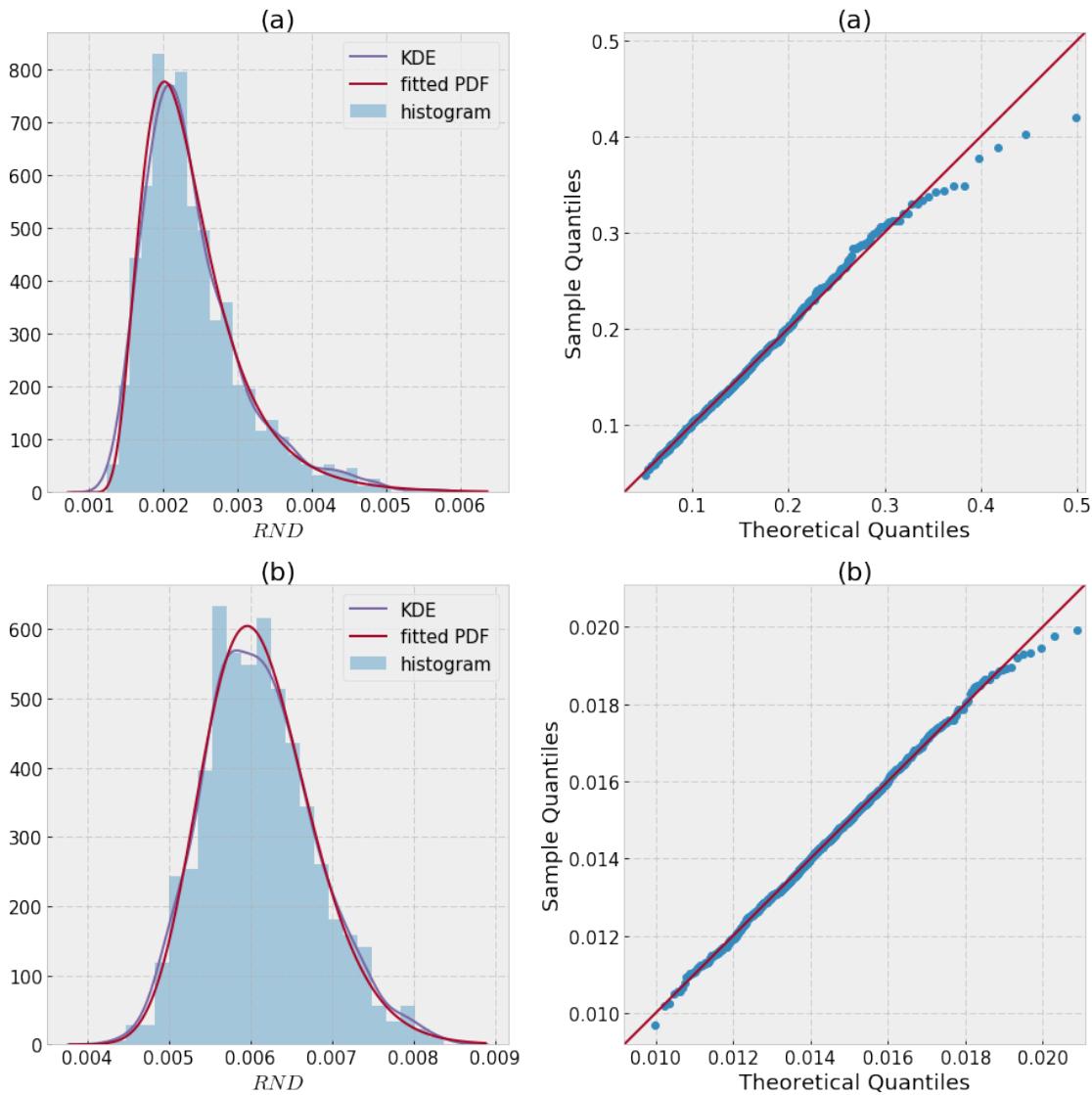
x_name, var_name = 'root_tip_density', 'maxB'
var_value, dist_name = 40, 'inverse-gamma'
dist = param_dists[dist_name]
var_names = np.unique(root_data.variable_name)
plot_data = root_data.loc[(root_data.variable_name == var_name) &
                           (root_data.variable_value == var_value), x_name]

dist_params = dist.fit(plot_data)
sb.distplot(plot_data, kde=True, fit=dist, ax=ax21,
            hist_kws={'color': plot_color_dict['data'], 'label':'histogram'},
            kde_kws={'color': plot_color_dict['kde'], 'label':'KDE'},
            fit_kws={'color': plot_color_dict['fitted_pred'], 'label':'fitted PDF'})
ax21.set_xlabel(measure_label_dict[x_name])
ax21.legend()
ax21.set_title('(b)')

probplot = sm.ProbPlot(plot_data, dist, fit=True)
probplot.qqplot(ax=ax22, color=plot_color_dict['data'])
sm.qqline(line='45', ax=ax22, fmt=plot_color_dict['fitted_pred'])
ax22.set_title('(b)')

fig.tight_layout()
plt.show(fig)
plt.close(fig)

```



```
In [55]: from sklearn.model_selection import LeaveOneOut, cross_val_predict
        from sklearn.metrics import r2_score, explained_variance_score
```

```
data = root_covariances

curr_plot_style = plt.style.library[plot_style]
curr_plot_style.update({'font.size': 15})
with plt.style.context(curr_plot_style):
    var_name, y1_name = 'sigma', 'maximal_horizontal_spread'
    y2_name, model_name = 'maximal_rooting_depth', 'poly2'

    y_name, reg_data = compute_correlation(y1_name, y2_name)
```

```

y_label = r'$\bar{{r}}({:s}, {:s})$'.format(measure_label_dict[y1_name].strip('$'),
                                         measure_label_dict[y2_name].strip('$'))
reg_data = reg_data.loc[data.variable_name == var_name]
X = reg_data.loc[:, ['variable_value']]
y = reg_data.loc[:, [y_name]]
model = reg_models[model_name]
y_loo_pred = cross_val_predict(reg_models[model_name], X, y, cv=LeaveOneOut())
y_fitted_pred = model.fit(X,y).predict(X)
X_plot = X.iloc[:,0]

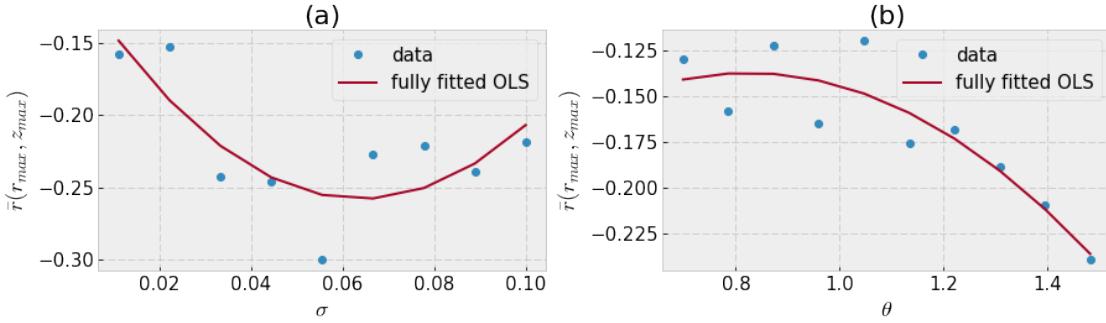
fig, (ax1, ax2) = plt.subplots(1,2, figsize=(13, 4))
ax1.plot(X_plot, y, color=plot_color_dict['data'], linestyle=' ', 
          marker='o', label="data")
ax1.plot(X_plot, y_fitted_pred, color=plot_color_dict['fitted_pred'],
          linestyle='--', marker='', label="fully fitted OLS")
ax1.legend(loc='best')
ax1.set_xlabel(input_param_label_dict[var_name])
ax1.set_ylabel(y_label)
ax1.set_title('(a)')

var_name, y1_name = 'theta', 'maximal_horizontal_spread'
y2_name, model_name = 'maximal_rooting_depth', 'poly2'
y_name, reg_data = compute_correlation(y1_name, y2_name)
y_label = r'$\bar{{r}}({:s}, {:s})$'.format(measure_label_dict[y1_name].strip('$'),
                                         measure_label_dict[y2_name].strip('$'))
reg_data = reg_data.loc[data.variable_name == var_name]
X = reg_data.loc[:, ['variable_value']]
y = reg_data.loc[:, [y_name]]
model = reg_models[model_name]
y_loo_pred = cross_val_predict(reg_models[model_name], X, y, cv=LeaveOneOut())
y_fitted_pred = model.fit(X,y).predict(X)
X_plot = X.iloc[:,0]

ax2.plot(X_plot, y, color=plot_color_dict['data'], linestyle=' ', 
          marker='o', label="data")
ax2.plot(X_plot, y_fitted_pred, color=plot_color_dict['fitted_pred'],
          linestyle='--', marker='', label="fully fitted OLS")
ax2.legend(loc='best')
ax2.set_xlabel(input_param_label_dict[var_name])
ax2.set_ylabel(y_label)
ax2.set_title('(b)')

fig.tight_layout()
plt.show(fig)
plt.close(fig)

```



```
In [59]: diff_i = len(input_param_label_dict) +1
diff_j = 10 +1
corr_mat = np.full((len(measure_label_dict)*
                     diff_i, len(measure_label_dict)*diff_j), np.nan)
max_i, max_j = corr_mat.shape

var_names = np.unique(root_data.variable_name)
measures = None
for var_i, var_name in enumerate(var_names):
    var_values = np.unique(
        root_data.variable_value.loc[root_data.variable_name == var_name])
    for var_j, var_value in enumerate(var_values):

        plot_data = root_data.loc[(root_data.variable_name == var_name) &
                                   (root_data.variable_value == var_value)]
        plot_data = plot_data.drop(['variable_name', 'variable_value'], axis=1)
        plot_data.rename(columns=measure_label_dict,
                         index=measure_label_dict, inplace=True)
        corr_data = plot_data.corr()
        if measures is None:
            measures = corr_data.index
        else:
            assert((measures == corr_data.index).all())
        corr_mat[var_i:max_i:diff_i, var_j:max_j:diff_j] = corr_data.values

index = pd.MultiIndex.from_tuples(
    [(m, i)
     for m in measures
     for i in (list(input_param_label_dict.values()) + [''])])
corr_df = pd.DataFrame(corr_mat, index=index)

cmap = sb.diverging_palette(220, 10, as_cmap=True)
```

```

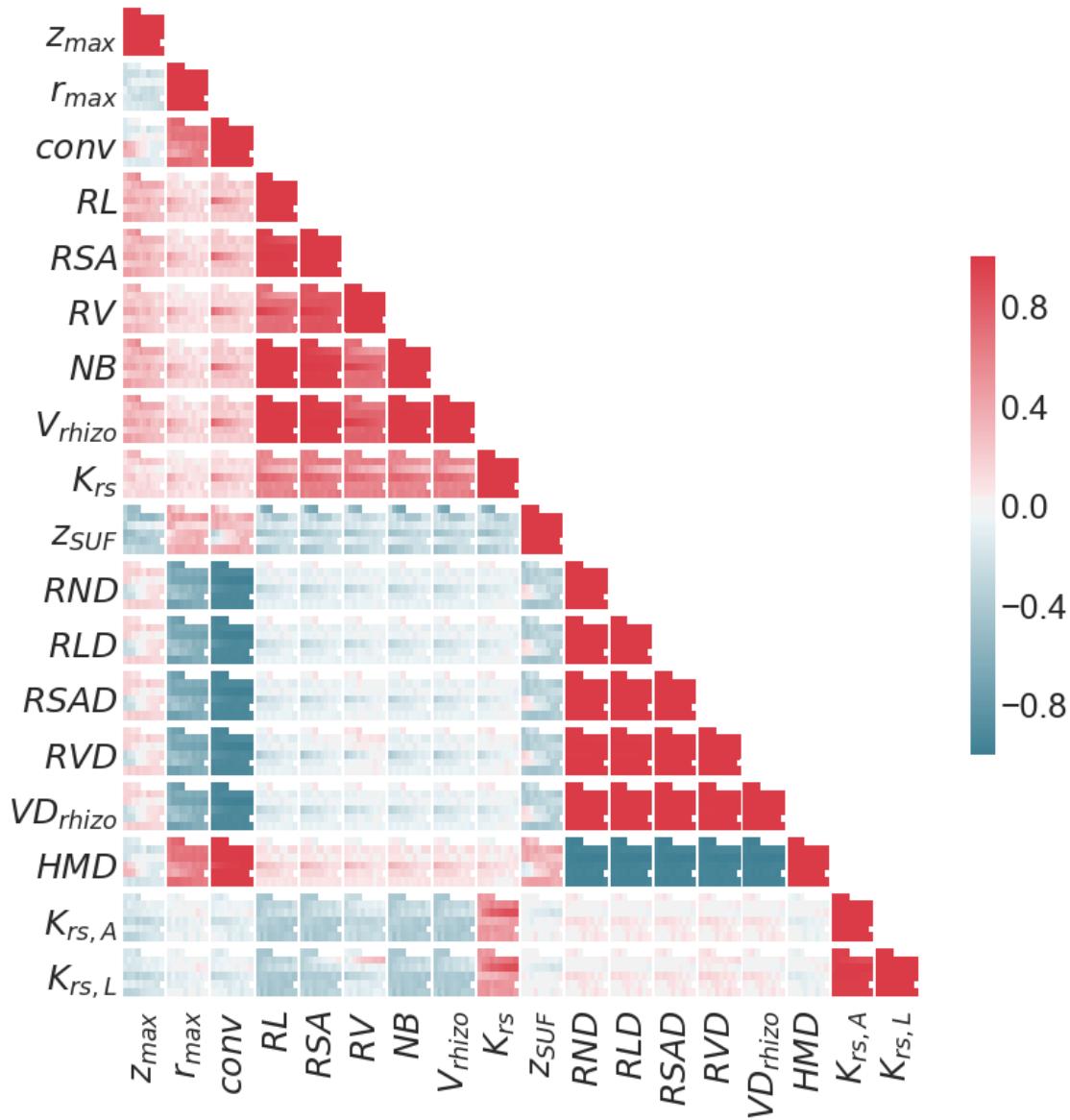
mask = np.zeros_like(corr_mat, dtype=np.bool)
for ind in range(len(measure_label_dict)):
    i = diff_i * ind
    j = diff_j * ind
    mask[0:i, j:(j+diff_j)] = True

curr_plot_style = sb.axes_style("white")
curr_plot_style.update({'font.size': 25})

with curr_plot_style:
    fig, ax = plt.subplots(figsize=(12,12))
    sb.heatmap(corr_mat, mask=mask, cmap=cmap, vmin=-1, ax=ax, vmax=1,
               cbar_kws={"shrink": .5})
    ax.set_yticks((np.arange(0, 1, step=diff_i/max_i) + (diff_i/max_i/2))*max_i)
    ax.set_yticklabels(measures)
    ax.set_xticks((np.arange(0, 1, step=diff_j/max_j) + (diff_j/max_j/2))*max_j)
    ax.set_xticklabels(measures)

fig.tight_layout()
plt.show(fig)
plt.close(fig)

```



In []: