

Analysis

November 15, 2017

1 Preamble

```
In [1]: import pandas as pd
        import numpy as np
        import scipy.stats as stats
        import scipy.optimize as optim
        import seaborn as sb
        from matplotlib import pyplot as plt

        import statsmodels.api as sm

pd.set_option('display.max_columns', None)

plot_style = 'bmh'

plot_color_dict = {
    'data': '#348ABD',
    'fitted_pred': '#A60628',
    'loo_pred': '#E24A33',
    'kde': '#7A68A6',
}

measure_label_dict = {
    'total_root_length':'$RL$',
    'total_root_surface_area':'$RSA$',
    'total_root_volume':'$RV$',
    'maximal_rooting_depth':'$z_{max}$',
    'maximal_horizontal_spread':'$r_{max}$',
    'convex_hull_volume':'$conv$',
    'number_of_root_branches':'$NB$',
    'rhizosphere_volume_phosphate':'$V_{rhizo}$',
    'root_tip_density':'$RND$',
    'total_root_length_per_convex_hull_volume':'$RLD$',
    'total_root_surface_area_per_convex_hull_volume':'$RSAD$',
    'total_root_volume_per_convex_hull_volume':'$RVD$',
    'rhizosphere_volume_phosphate_per_convex_hull_volume':'$VD_{rhizo}$',
    'root_system_total_conductance':'$K_{rs}$',
    'half_mean_distance_between_roots':'$K_{rs,A}$',
    'root_system_conductance_per_unit_of_root_area':'$K_{rs,L}$',
    'root_system_conductance_per_unit_of_root_length':'$z_{SUF}$',
    'mean_depth_of_standard_root_water_uptake':'$R$',
}
```

```

input_param_label_dict = {
    'N': '$N$',
    'ln': '$\ln$',
    'nB': '$nB$',
    'r': '$r$',
    'sigma': r'$\sigma$',
    'theta': r'$\theta$',
}
}

/home/lpetrich/.local/share/anaconda3/envs/Master/lib/python3.6/site-packages/statsmodels/compat/pandas
from pandas.core import datetools

```

1.1 Import data

```
In [2]: root_data = pd.read_hdf('/data/simulations/root_data.h5', '/data')
```

```
In [3]: root_data.describe()
```

```

Out[3]:      variable_value  maximal_rooting_depth  maximal_horizontal_spread  \
count      53000.000000          53000.000000          53000.000000
mean       5.073742           96.360813          140.675772
std        9.593408          18.432926          40.466511
min        0.011111          17.556674          31.591177
25%        0.785398          89.052064          113.039483
50%        1.333333          101.731758          141.611663
75%        2.333333          109.150982          170.431956
max        40.000000          133.102523          246.381642

      convex_hull_volume  total_root_length  total_root_surface_area  \
count      5.300000e+04          53000.000000          53000.000000
mean       5.068756e+05          12327.403791          4376.284805
std        3.113330e+05          13000.233771          4556.386593
min        2.037828e+04          928.977099          386.926864
25%        2.902084e+05          6480.670294          2310.025457
50%        4.430118e+05          8751.795049          3082.694361
75%        6.386111e+05          9183.583233          3232.672234
max        1.787740e+06          63098.115898          22217.951110

      total_root_volume  number_of_root_branches  \
count      53000.000000          53000.000000
mean       143.558379          1555.515453
std        146.169334          1648.684200
min        16.361829          139.000000
25%        77.564691          828.000000
50%        99.248805          1110.000000
75%        105.408625          1159.000000
max        724.107142          7956.000000

      rhizosphere_volume_phosphate  root_system_total_conductance  \
count          53000.000000          53000.000000
mean         2424.364206          0.040605
std          2544.861981          0.009128
min          194.122440          0.008537

```

25%	1276.985390	0.036379
50%	1716.621970	0.044755
75%	1800.116067	0.046622
max	12372.691625	0.053383
 mean_depth_of_standard_root_water_uptake root_tip_density \		
count	53000.000000	53000.000000
mean	-34.478992	0.003286
std	11.486981	0.002252
min	-62.782556	0.000345
25%	-43.842738	0.001821
50%	-33.646199	0.002680
75%	-26.704811	0.004176
max	-2.781809	0.024725
 total_root_length_per_convex_hull_volume \		
count	53000.000000	
mean	0.025915	
std	0.017492	
min	0.003346	
25%	0.014535	
50%	0.021064	
75%	0.032514	
max	0.197229	
 total_root_surface_area_per_convex_hull_volume \		
count	53000.000000	
mean	0.009240	
std	0.006119	
min	0.001452	
25%	0.005205	
50%	0.007459	
75%	0.011538	
max	0.068808	
 total_root_volume_per_convex_hull_volume \		
count	53000.000000	
mean	0.000305	
std	0.000196	
min	0.000061	
25%	0.000175	
50%	0.000245	
75%	0.000377	
max	0.002165	
 rhizosphere_volume_phosphate_per_convex_hull_volume \		
count	53000.000000	
mean	0.005105	
std	0.003421	
min	0.000713	
25%	0.002865	
50%	0.004137	
75%	0.006399	
max	0.038546	

```

    half_mean_distance_between_roots \
count          53000.000000
mean           4.013829
std            1.208540
min            1.270399
25%           3.128878
50%           3.887375
75%           4.679627
max            9.754158

root_system_conductance_per_unit_of_root_area \
count          53000.000000
mean           0.000013
std            0.000004
min            0.000002
25%           0.000014
50%           0.000015
75%           0.000016
max            0.000029

root_system_conductance_per_unit_of_root_length
count          5.300000e+04
mean           4.908577e-06
std            1.671638e-06
min            7.325552e-07
25%           4.976969e-06
50%           5.187365e-06
75%           5.695776e-06
max            1.127802e-05

```

In [4]: `root_means = root_data.groupby(['variable_name', 'variable_value']).mean().dropna().reset_index()`
`root_covariances = root_data.groupby(['variable_name', 'variable_value']).cov().unstack().reset_index()`

1.2 Code for probability distributions

```

In [5]: param_dists = {
    'normal': stats.norm,
    'inverse-gamma': stats.invgamma,
}

def plot_dist(x_name, dist_name='normal'):
    '''Plot KDE, histogram and fitted distribution'''
    dist = param_dists[dist_name]
    var_names = np.unique(root_data.variable_name)

    for var_name in var_names:
        var_values = np.unique(root_data.variable_value.loc[root_data.variable_name == var_name])
        for var_value in var_values:
            plot_data = root_data.loc[(root_data.variable_name == var_name) & (root_data.variable_value == var_value)]

            dist_params = dist.fit(plot_data)

            print(f"variable name: {str(var_name)}")

```

```

print(f"variable value: {var_value}")
print(f"distribution: {dist_name}(shape={dist_params[:-2]}, loc={dist_params[-2]}, s")
print(stats.describe(plot_data))

with plt.style.context(plot_style):
    fig, ax = plt.subplots()
    sb.distplot(plot_data, kde=True, fit=dist, ax=ax,
                hist_kws={'color': plot_color_dict['data'], 'label':'histogram'},
                kde_kws={'color': plot_color_dict['kde'], 'label':'KDE'},
                fit_kws={'color': plot_color_dict['fitted_pred'], 'label':'fitted P
    ax.set_xlabel(measure_label_dict[x_name])
    ax.legend()
    fig.tight_layout()
    plt.show(fig)
    plt.close(fig)

    fig, ax = plt.subplots()
    probplot = sm.ProbPlot(plot_data, dist, fit=True)
    probplot.qqplot(ax=ax, color=plot_color_dict['data'])
    sm.qqline(line='45', ax=ax, fmt=plot_color_dict['fitted_pred'])
    fig.tight_layout()
    plt.show(fig)
    plt.close(fig)

```

1.3 Code for parameter regression

```

In [6]: from sklearn.linear_model import LinearRegression
        from sklearn.pipeline import Pipeline
        from sklearn.preprocessing import PolynomialFeatures

        regression_poly1 = Pipeline([
            ('ols', LinearRegression())
        ])
        regression_poly2 = Pipeline([
            ('poly', PolynomialFeatures(degree=2, include_bias=False)),
            ('ols', LinearRegression())
        ])

        reg_models = {
            'poly1': regression_poly1,
            'poly2': regression_poly2,
        }

def eval_reg(data, var_name, y_name, model_name, y_label=''):
    '''Plot parameter regression models'''
    from sklearn.model_selection import LeaveOneOut, cross_val_predict
    from sklearn.metrics import r2_score, explained_variance_score

    reg_data = data.loc[data.variable_name == var_name]

    X = reg_data.loc[:, ['variable_value']]
    y = reg_data.loc[:, [y_name]]

```

```

model = reg_models[model_name]

y_loo_pred = cross_val_predict(reg_models[model_name], X, y, cv=LeaveOneOut())
y_fitted_pred = model.fit(X,y).predict(X)

print(f"variable name: {var_name}")
print(f"model: {model_name}")
print(f"r-squared: {r2_score(y, y_loo_pred)}")
print(f"explained variance: {explained_variance_score(y, y_loo_pred)}")

with plt.style.context(plot_style):
    X_plot = X.iloc[:,0]
    fig, ax = plt.subplots()
    ax.plot(X, y, color=plot_color_dict['data'], linestyle=' ', marker='o', label="data")
    ax.plot(X, y_fitted_pred, color=plot_color_dict['fitted_pred'], linestyle='-', marker='')
    # ax.plot(X, y_loo_pred, color=plot_color_dict['loo_pred'], linestyle=' ', marker='*')
    ax.legend(loc='best')
    ax.set_xlabel(input_param_label_dict[var_name])
    ax.set_ylabel(y_label)
    plt.show(fig)
    plt.close(fig)

def mean_reg(x_name, y_name, model_name):
    y_label = r'$\overline{{{:s}}}$'.format(measure_label_dict[y_name].strip('$'))
    eval_reg(root_means, x_name, y_name, model_name, y_label)

```

1.4 Code for correlations

```

In [7]: def compute_correlation(x_name: str, y_name: str):
    var_x = np.sqrt(root_covariances.loc[:, (x_name, x_name)])
    var_y = np.sqrt(root_covariances.loc[:, (y_name, y_name)])
    corr = root_covariances.loc[:, (x_name, y_name)] / (var_x * var_y)

    col_name = f"{{x_name}}X{{y_name}}"
    data = pd.DataFrame({
        'variable_name': root_covariances.variable_name,
        'variable_value': root_covariances.variable_value,
        col_name: corr,
    })

    return col_name, data

def corr_reg(x_name, y1_name, y2_name, model_name):
    var_name, corr_data = compute_correlation(y1_name, y2_name)
    y_label = r'$\bar{{r}}$'.format(measure_label_dict[y1_name].strip('$'),
                                  measure_label_dict[y2_name].strip('$'))
    eval_reg(corr_data, x_name, var_name, model_name, y_label)

In [8]: def plot_corr_matrix():
    var_names = np.unique(root_data.variable_name)
    for var_name in var_names:
        var_values = np.unique(root_data.variable_value.loc[root_data.variable_name == var_name])
        for var_value in var_values:

```

```

print(f"variable name: {str(var_name)}")
print(f"variable value: {var_value}")

plot_data = root_data.loc[(root_data.variable_name == var_name) & (root_data.variable_value == var_value)]
plot_data = plot_data.drop(['variable_name', 'variable_value'], axis=1)
plot_data.rename(columns=measure_label_dict, index=measure_label_dict, inplace=True)
corr_data = plot_data.corr()

mask = np.zeros_like(corr_data, dtype=np.bool)
mask[np.triu_indices_from(mask)] = True

cmap = sb.diverging_palette(220, 10, as_cmap=True)
with sb.axes_style("white"):
    fig, ax = plt.subplots()
    sb.heatmap(corr_data, cmap=cmap, mask=mask, center=0, square=True,
                linewidths=.5, cbar_kws={"shrink": .5}, vmin=-1, vmax=1, ax=ax)
    plt.show(fig)
    plt.close(fig)

```

2 Maximal rooting depth

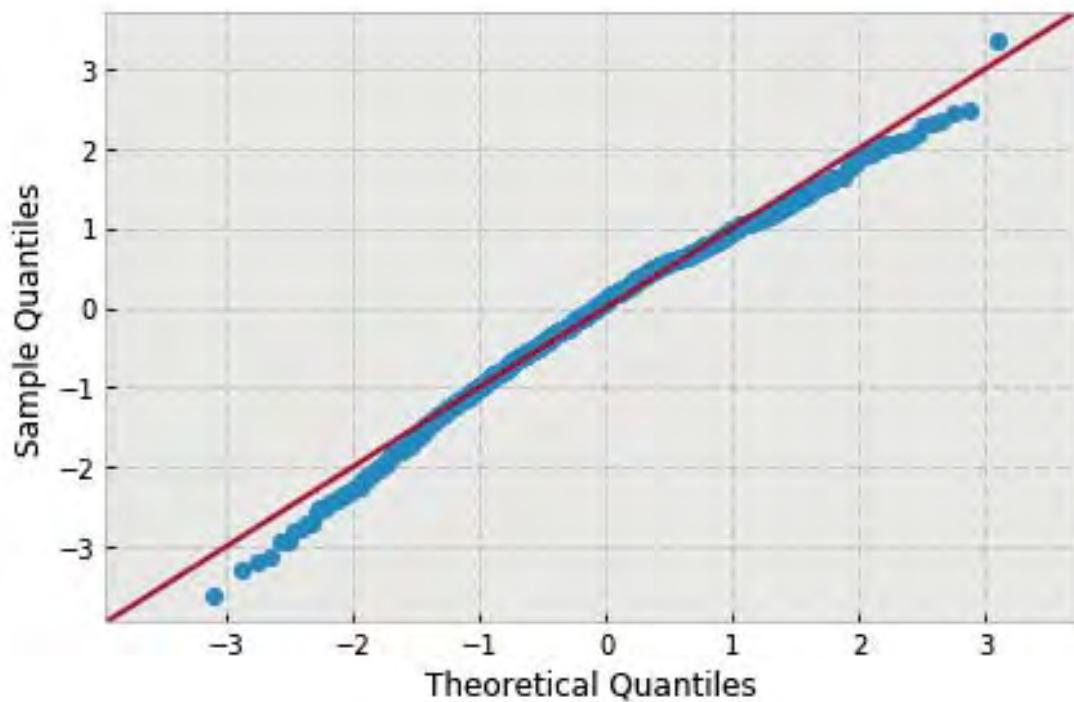
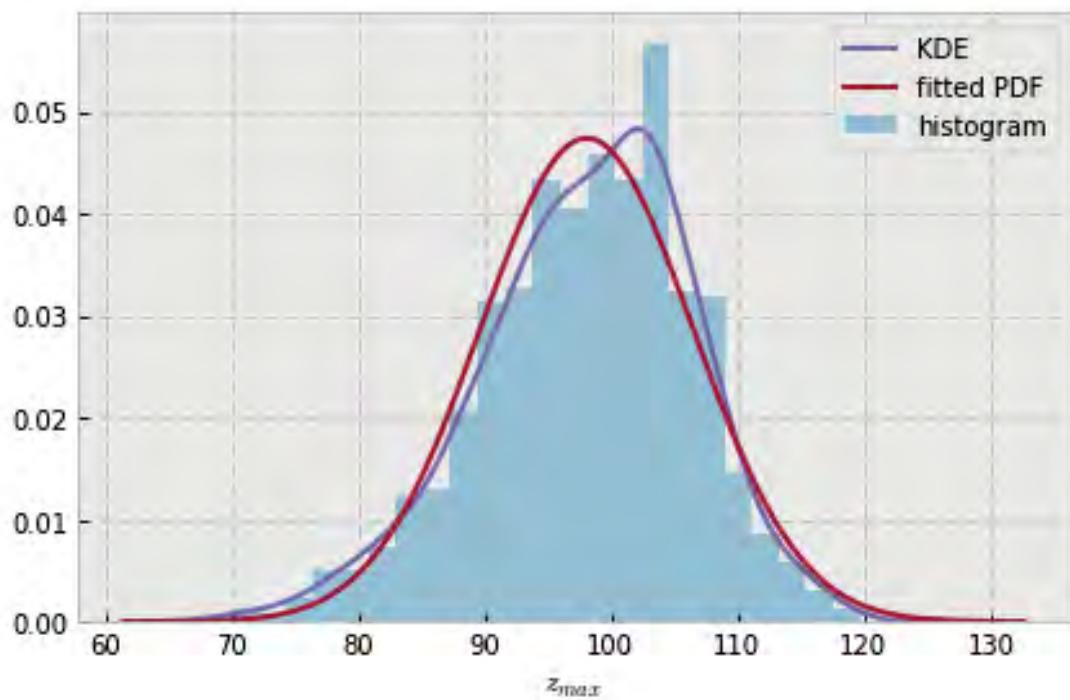
2.1 Probability distributions

In [9]: `plot_dist('maximal_rooting_depth')`

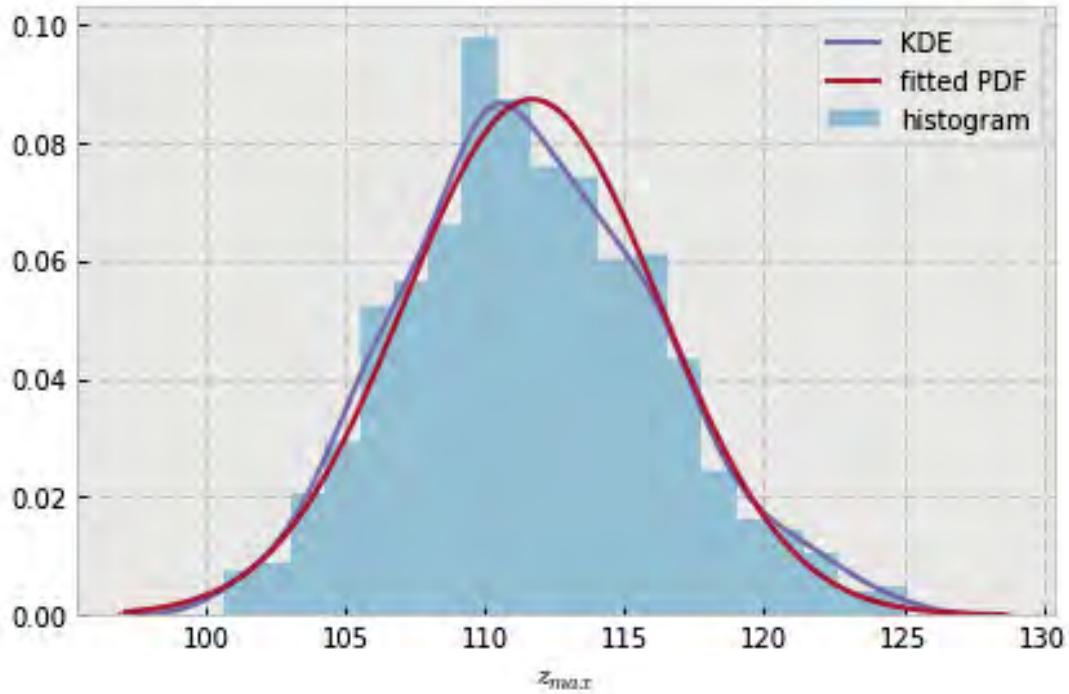
```

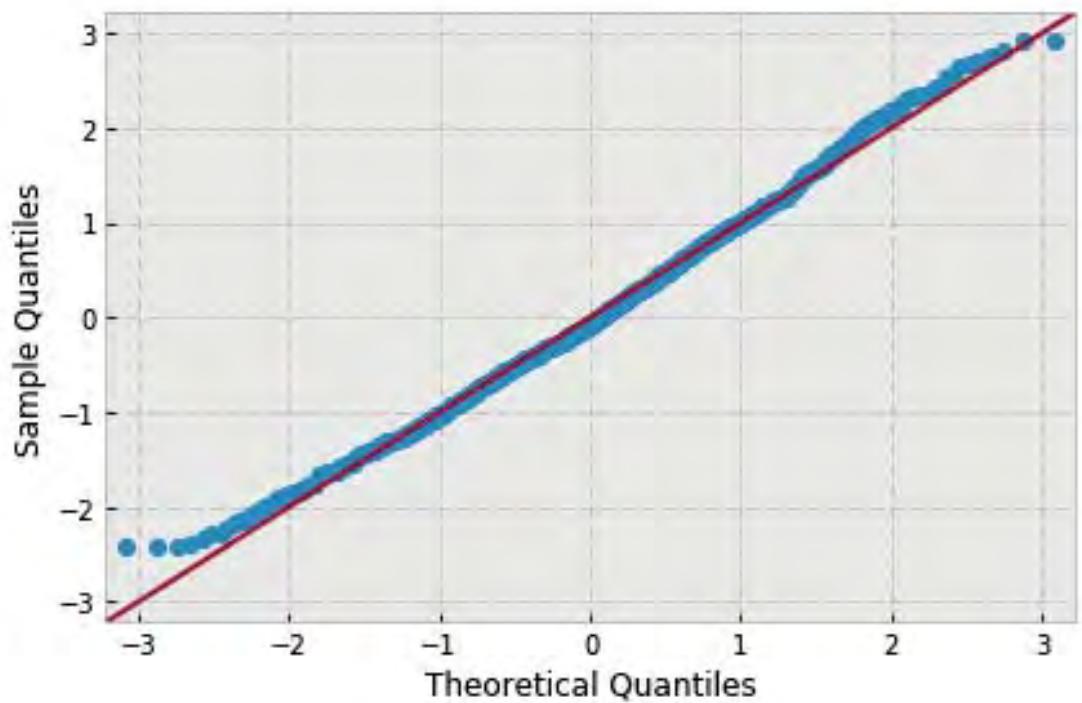
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=98.06661357456376, scale=8.41378708881183)
DescribeResult(nobs=1000, minmax=(67.811421795526883, 126.24922860625531)
               mean=98.06661357456376, variance=70.862675851708374,
               skewness=-0.4037788075684776, kurtosis=0.24472839692745607)

```

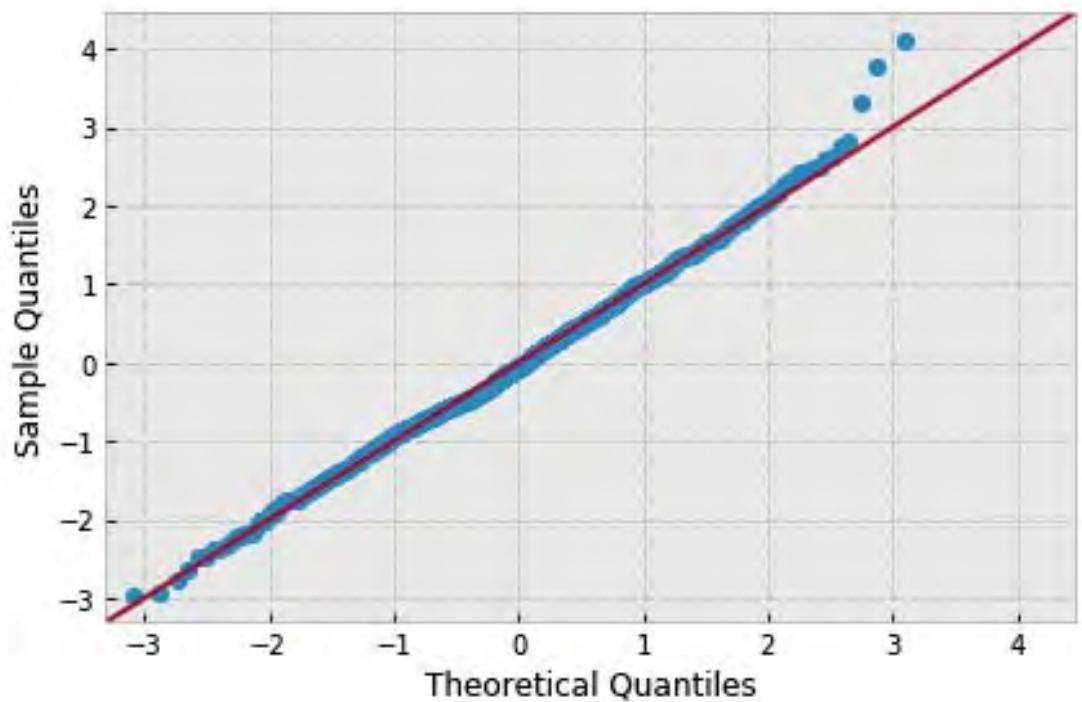
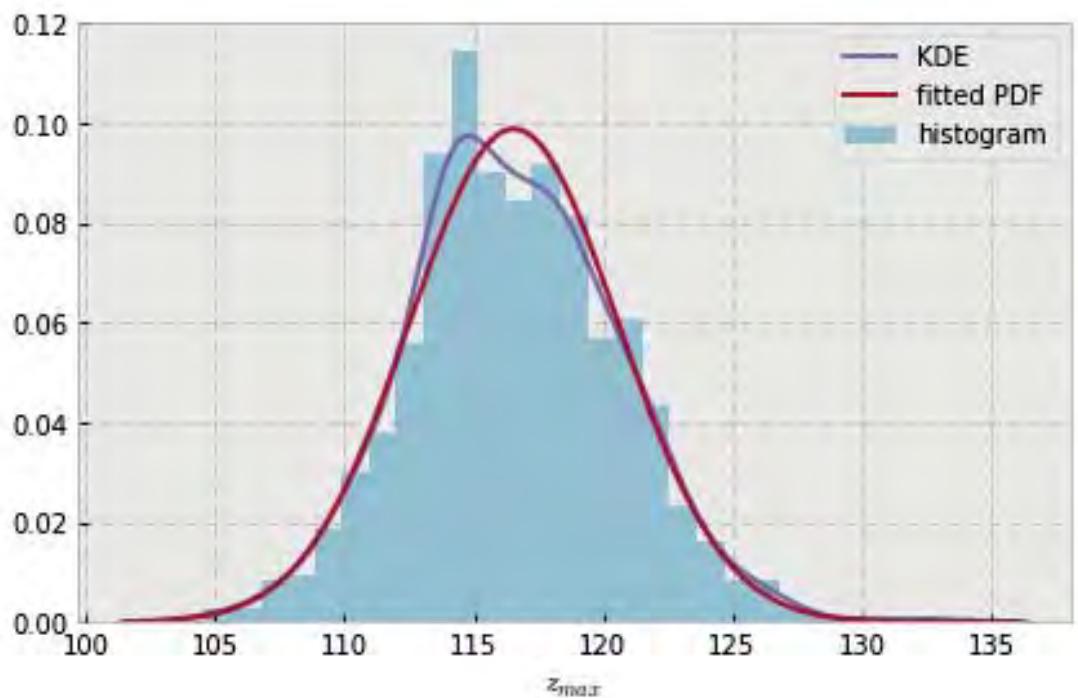


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=111.6824343109389, scale=4.5735361508299635)
DescribeResult(nobs=1000, minmax=(100.61035374317909, 125.11799583794614)
               mean=111.6824343109389, variance=20.938171094042605,
               skewness=0.23179243643468922, kurtosis=-0.1766191864831259)
```

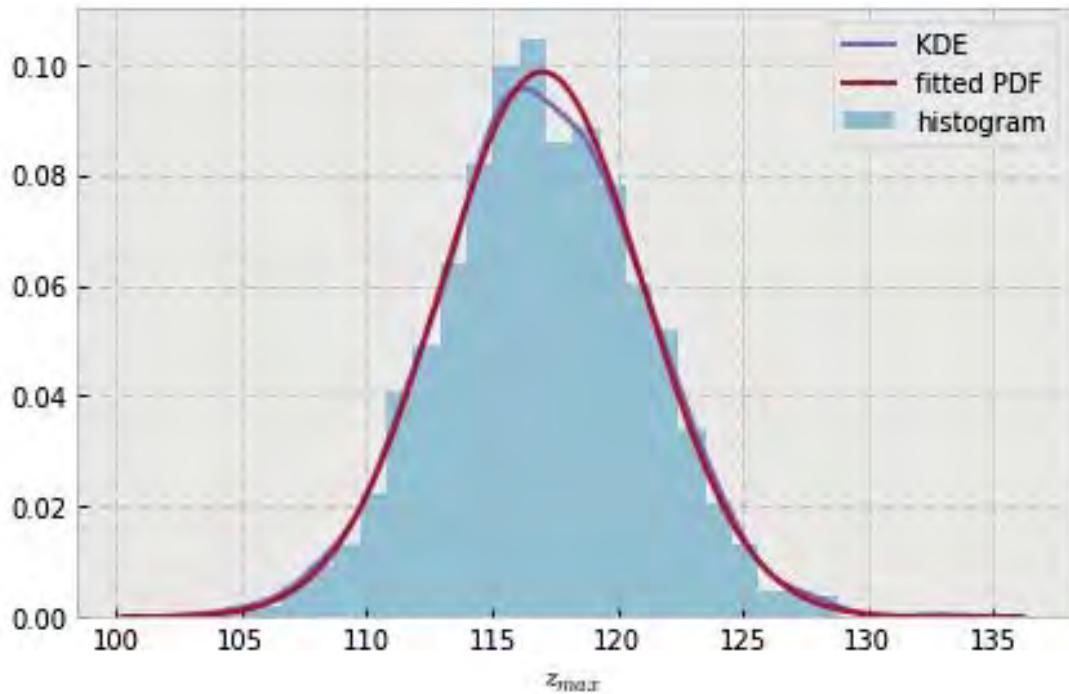


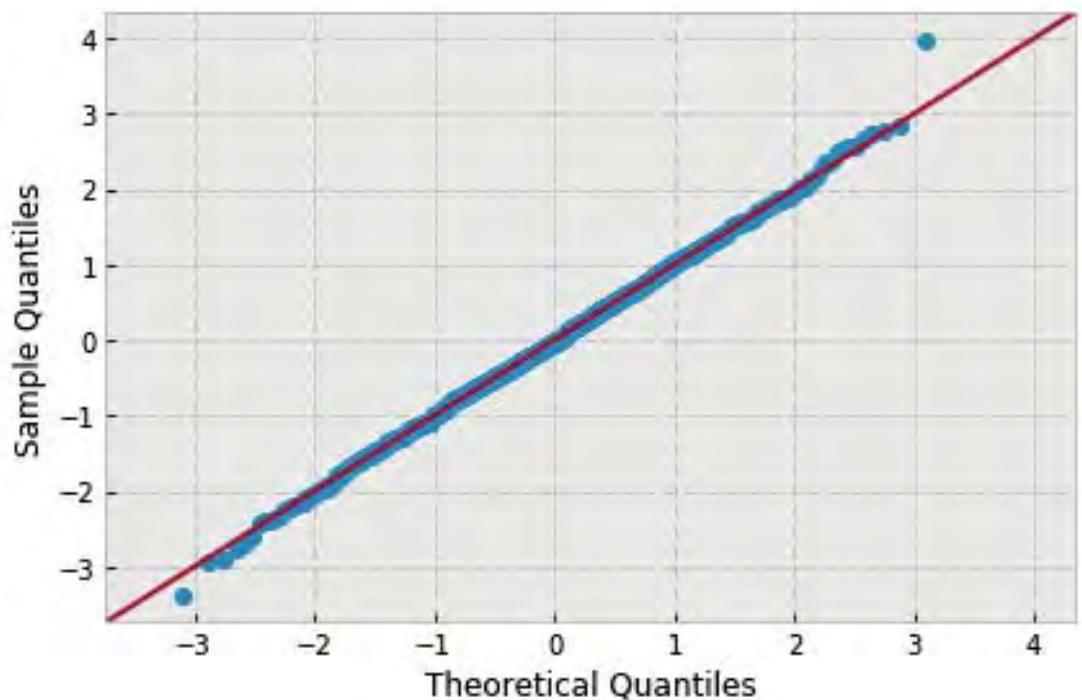


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=116.54189435753531, scale=4.034838584000744)
DescribeResult(nobs=1000, minmax=(104.61675284833174, 133.1025230840801)
               mean=116.54189435753531, variance=16.296218617558694,
               skewness=0.1977161007700751, kurtosis=0.26571479848197077)
```

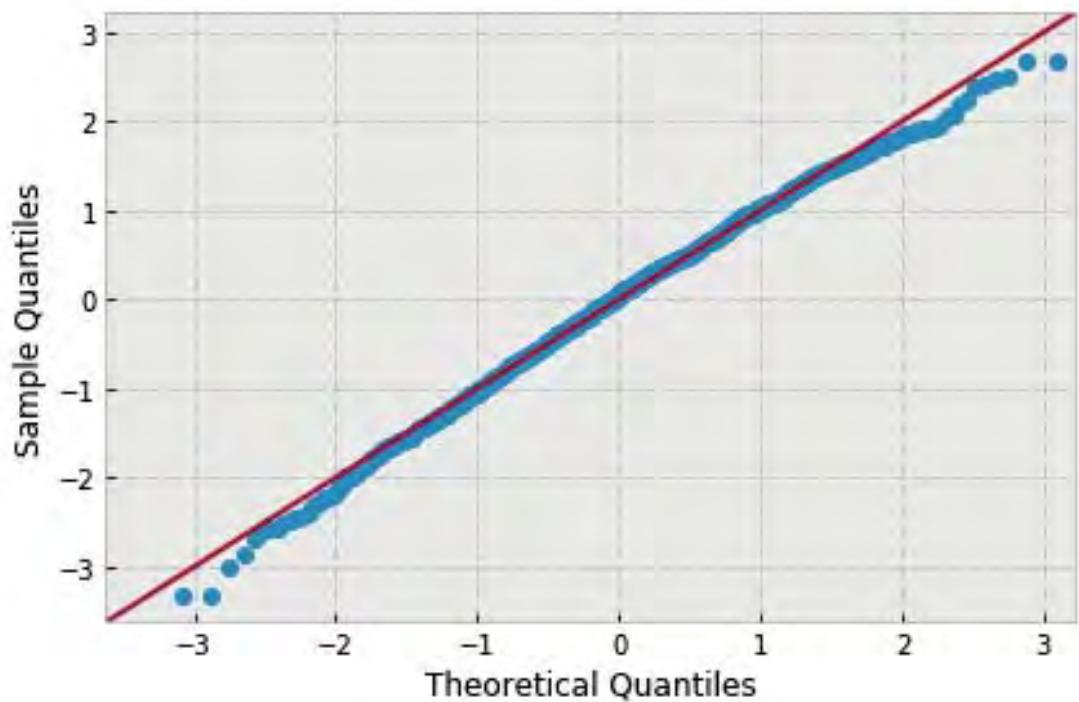
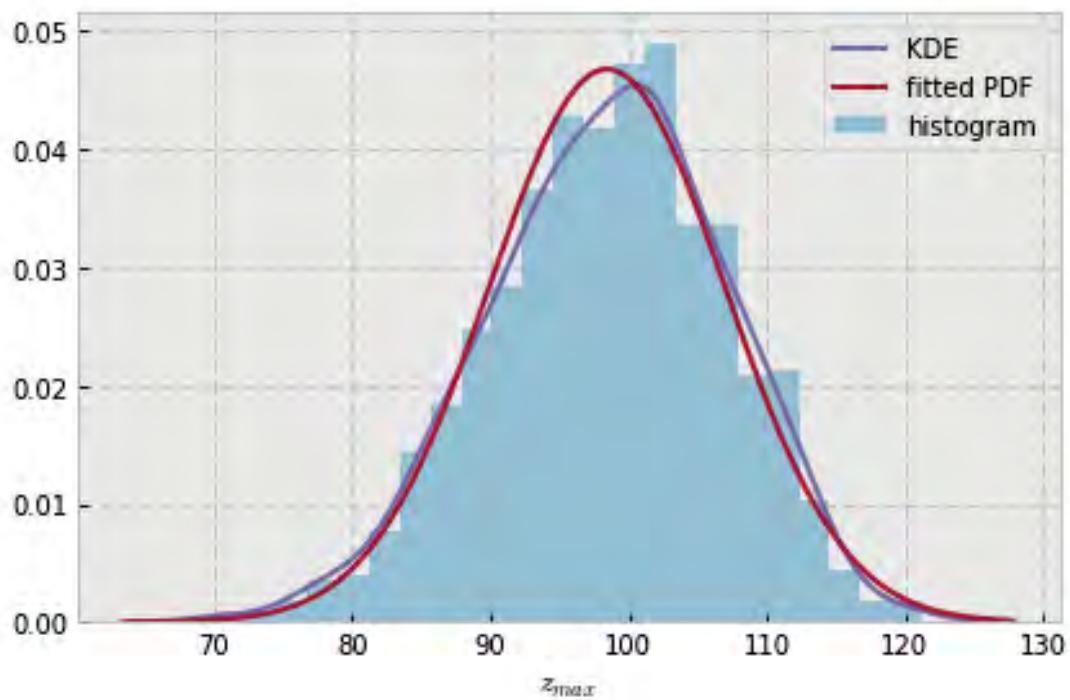


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=117.00625048326003, scale=4.045286398861881)
DescribeResult(nobs=1000, minmax=(103.37558044223572, 133.05445251997489)
               mean=117.00625048326003, variance=16.380722771588516,
               skewness=0.025668814080644642, kurtosis=0.1152173615807679)
```

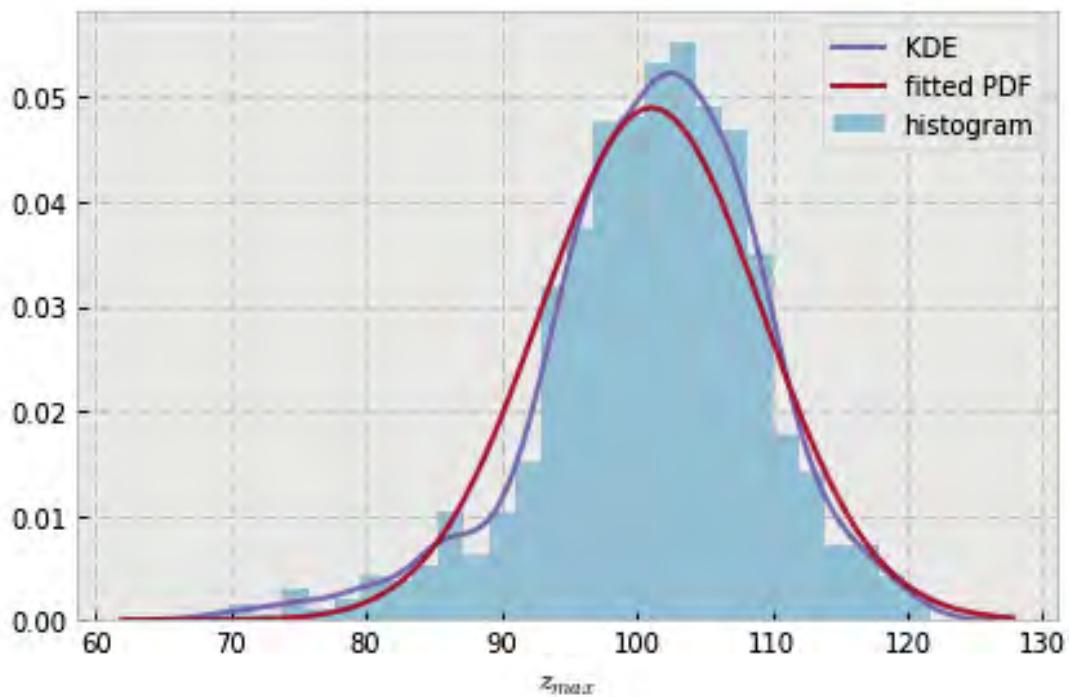


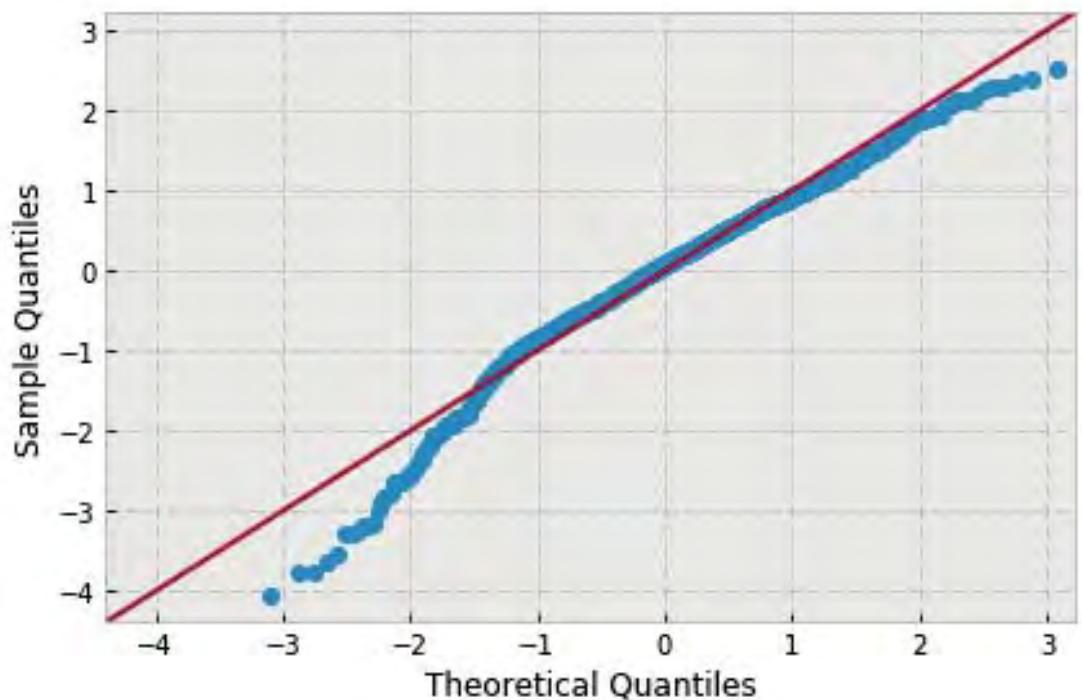


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=98.4236915370448, scale=8.528023124097365)
DescribeResult(nobs=1000, minmax=(70.09408222777455, 121.18070579475602)
               mean=98.423691537044803, variance=72.799978383522884,
               skewness=-0.2298389624751914, kurtosis=-0.12429895900797483)
```

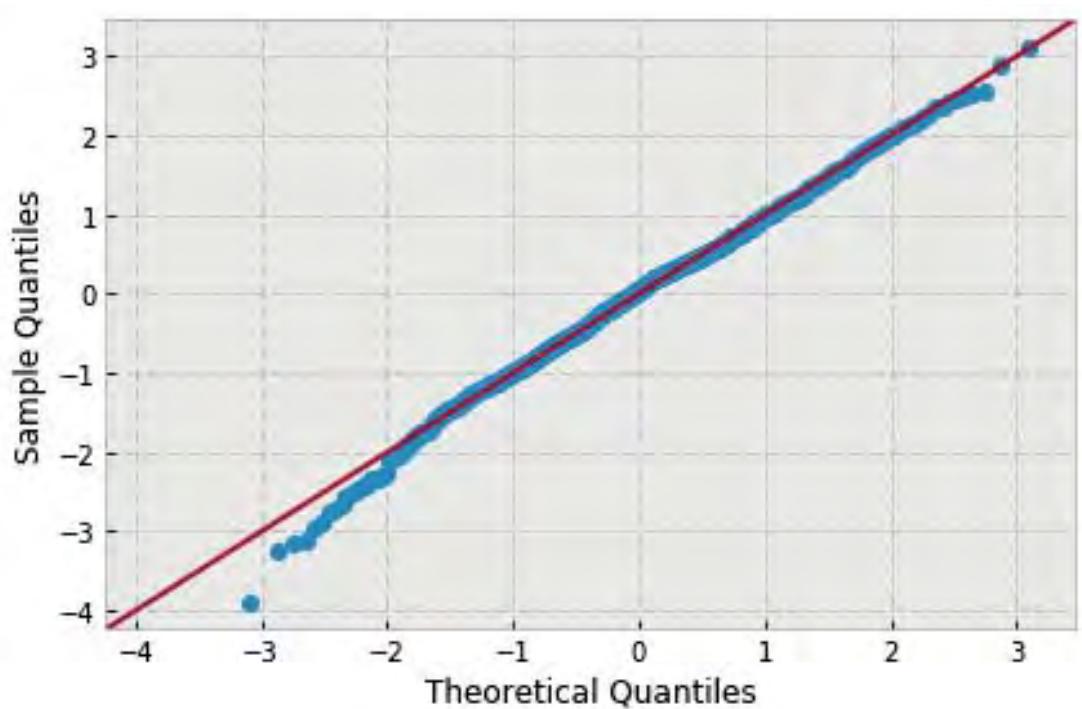
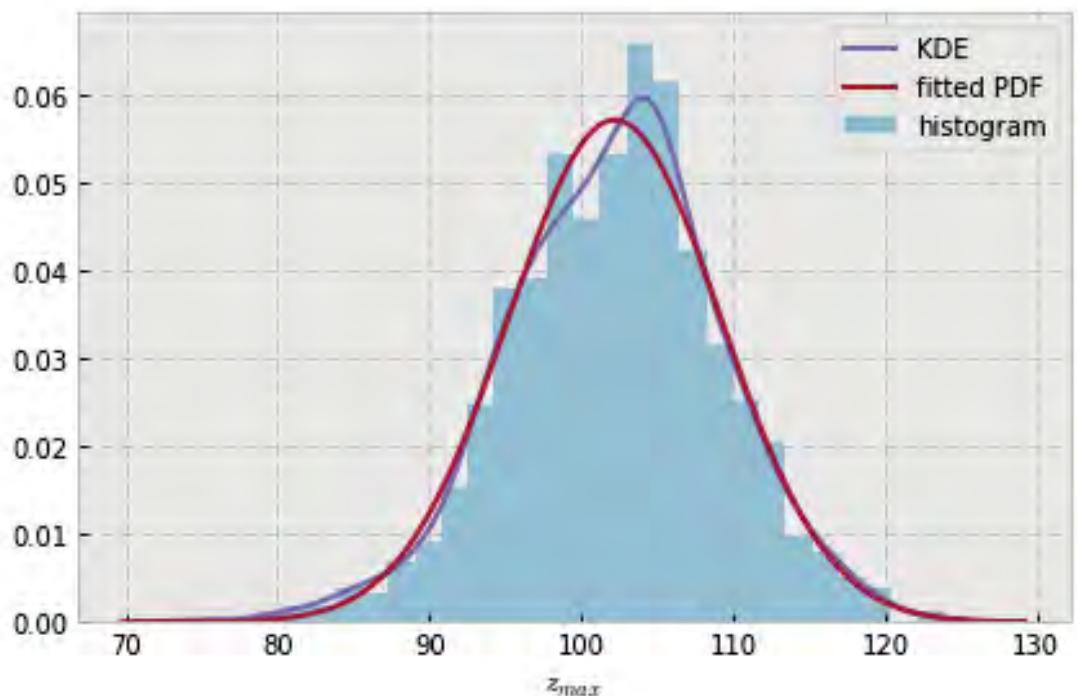


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=101.08709539269374, scale=8.145232724298712)
DescribeResult(nobs=1000, minmax=(68.039121574816008, 121.64257942547901)
               mean=101.08709539269374, variance=66.411227360346956,
               skewness=-0.6620871704156552, kurtosis=1.161046370735698)
```

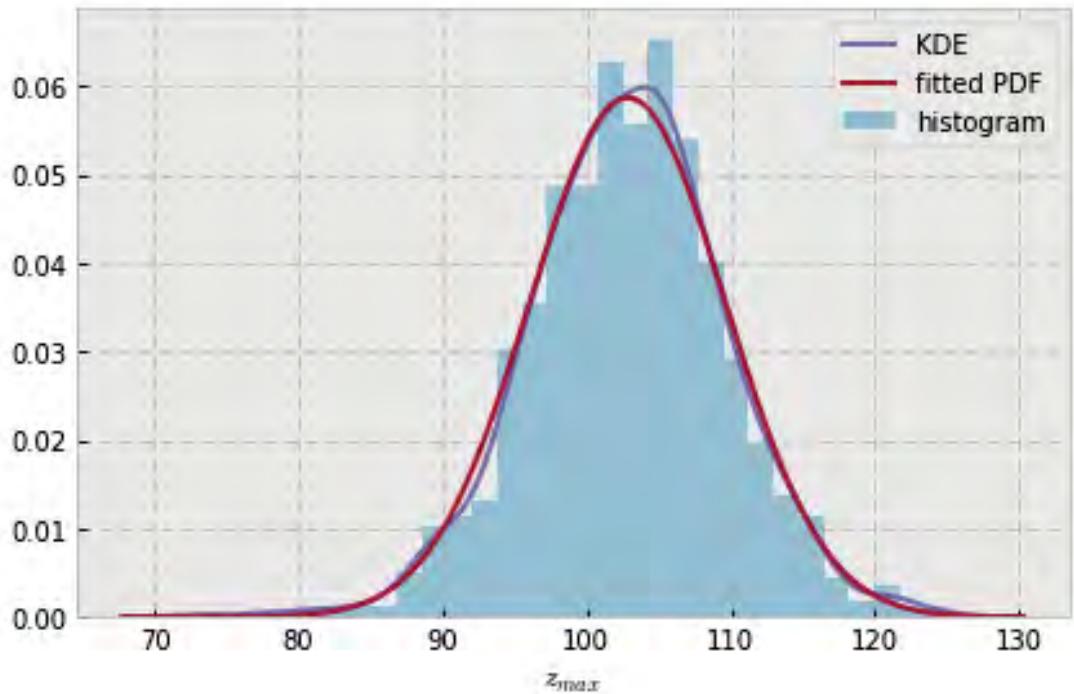


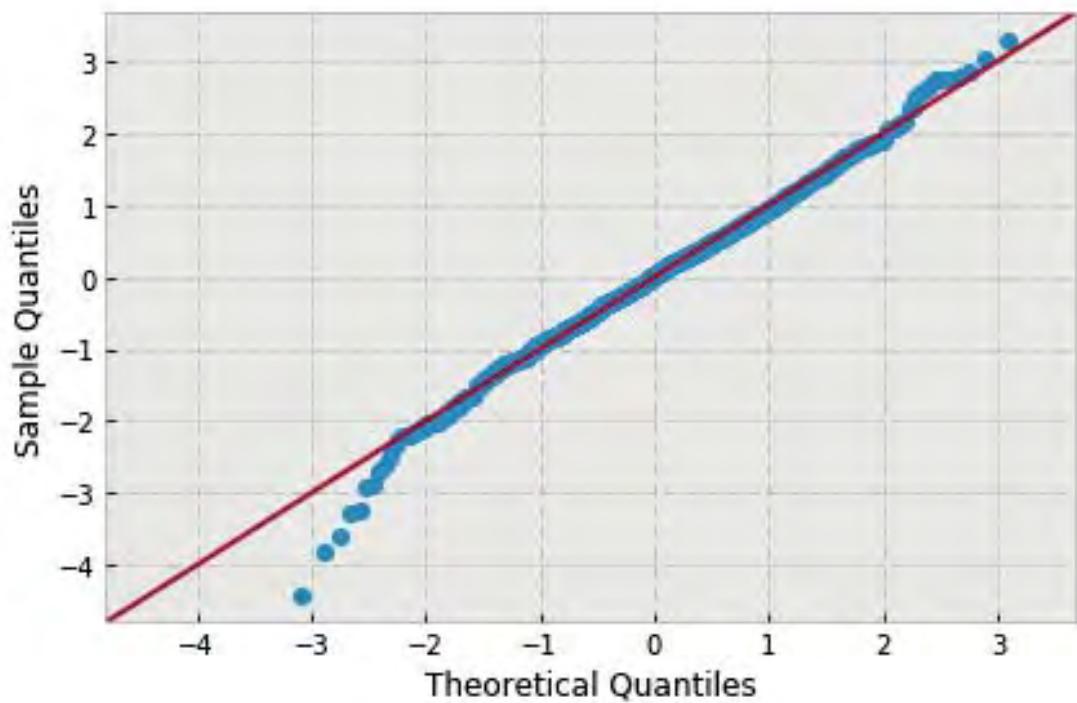


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=102.2083509387657, scale=6.982024993765703)
DescribeResult(nobs=1000, minmax=(75.100300909797767, 123.89422842115083)
               mean=102.2083509387657, variance=48.797470484053008,
               skewness=-0.18042755603725819, kurtosis=0.3049628948890639)
```

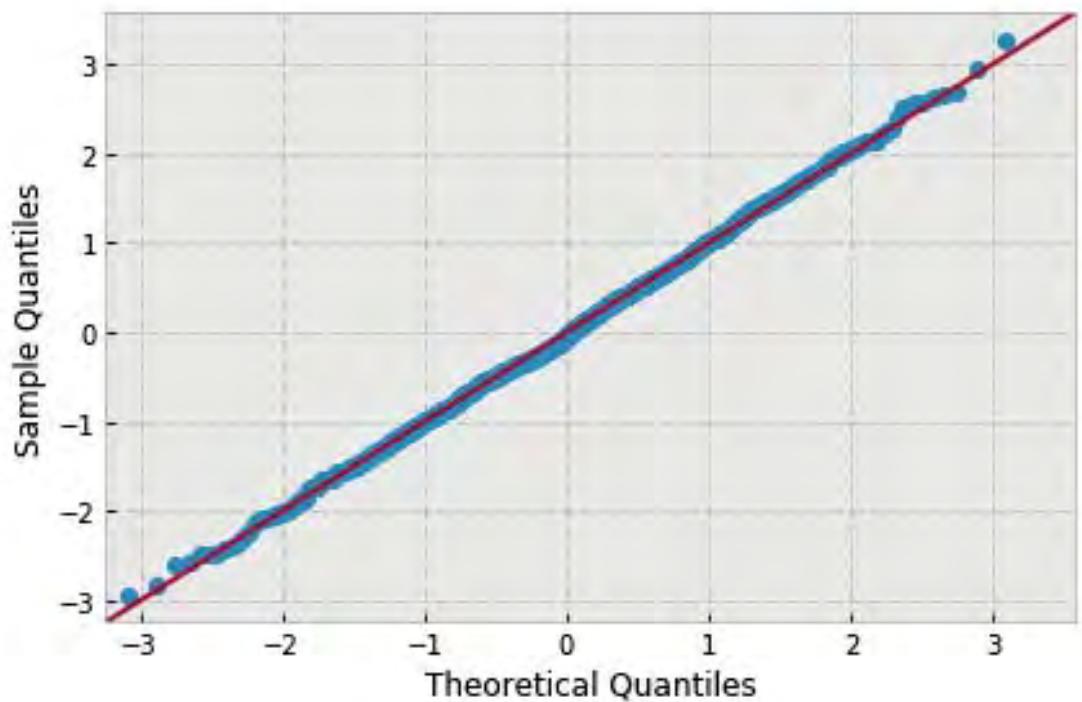
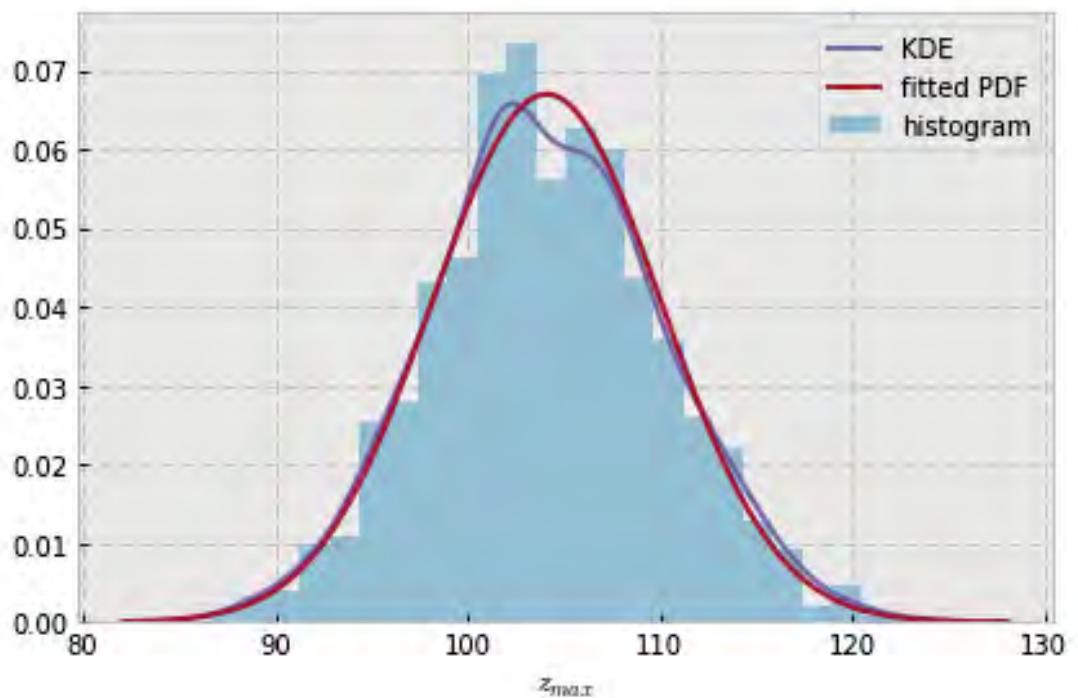


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=102.8204842914613, scale=6.798791620660571)
DescribeResult(nobs=1000, minmax=(72.858706874798344, 125.15416026039316)
               mean=102.8204842914613, variance=46.269837338502903,
               skewness=-0.15998265665302436, kurtosis=0.7517261444040466)
```

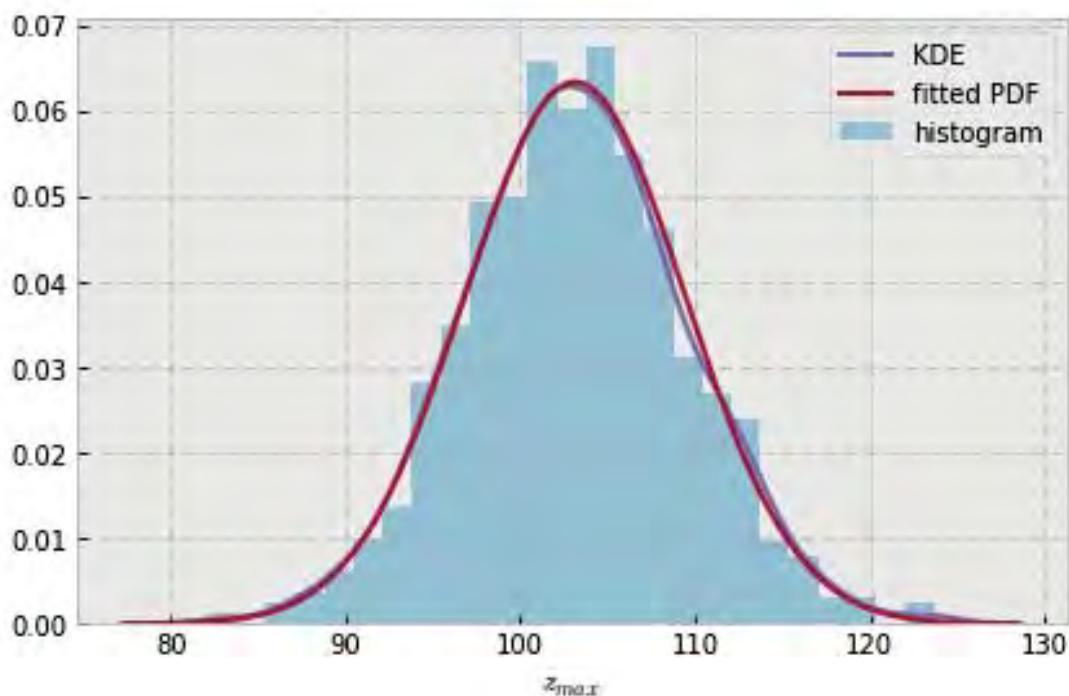


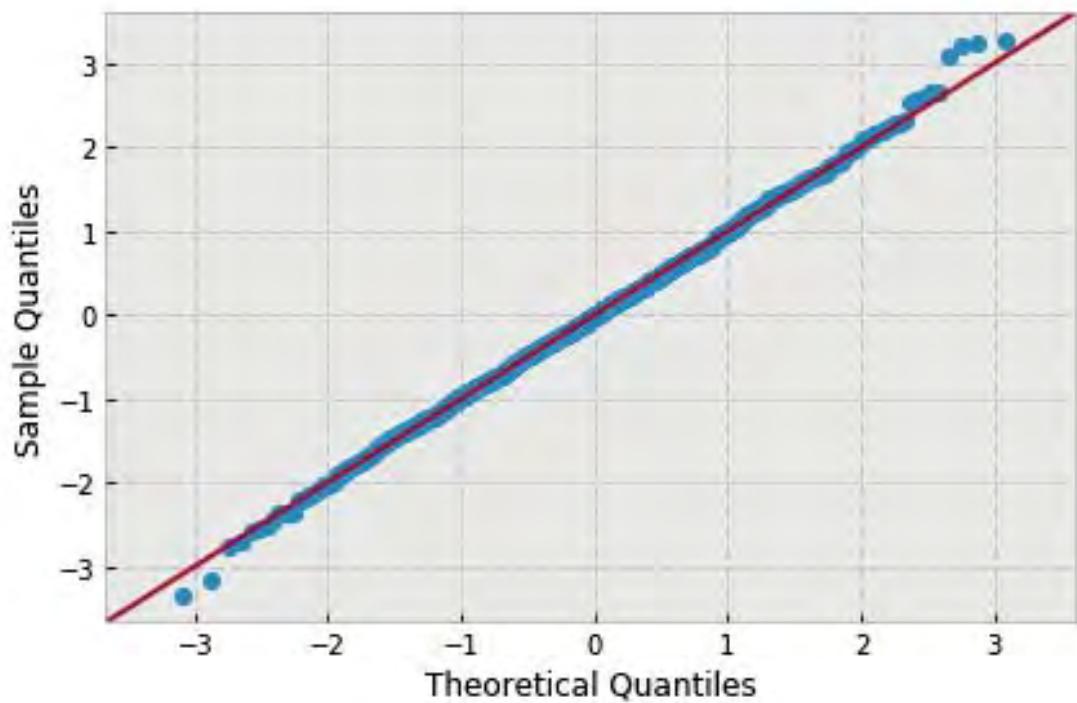


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=104.10638908666206, scale=5.9562156551704994)
DescribeResult(nobs=1000, minmax=(86.660471061730931, 123.51387453580392)
               mean=104.10638908666206, variance=35.512016947845986,
               skewness=0.07735478337023846, kurtosis=-0.10855563399660806)
```

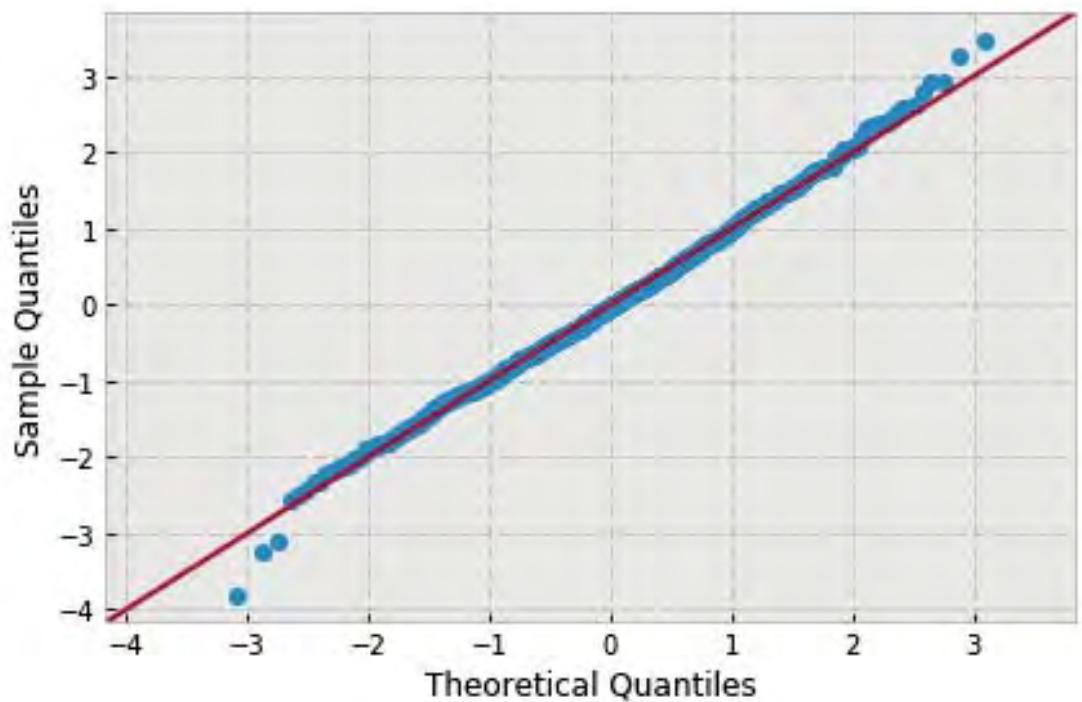
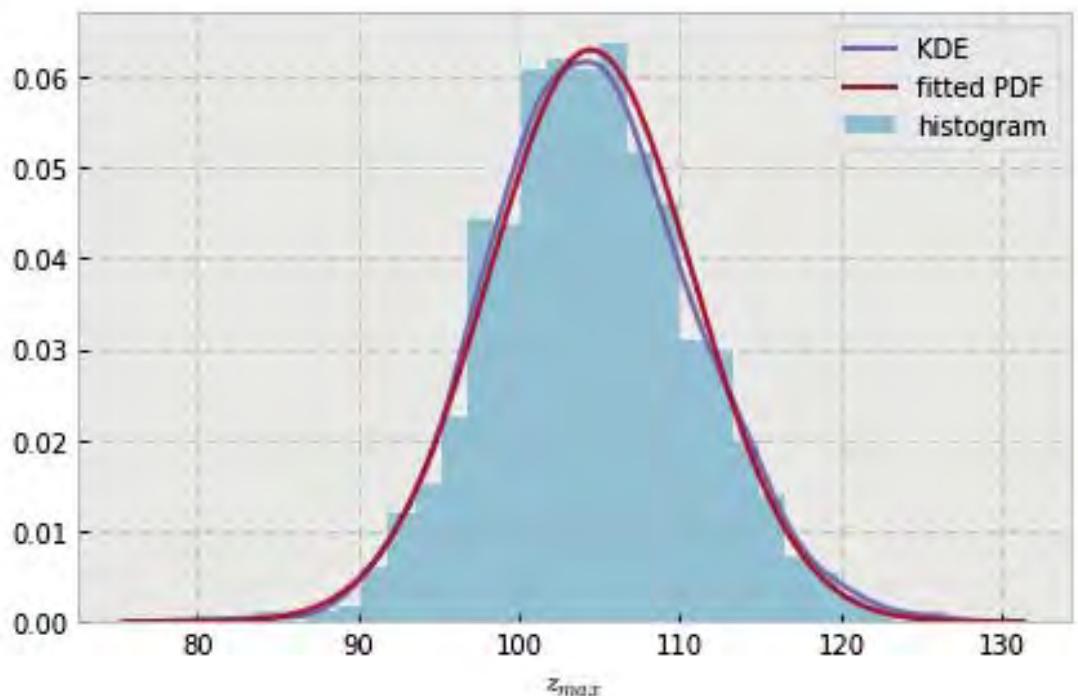


```
variable name: ln
variable value: 1.3333333333333333
distribution: normal(shape=(), loc=103.13484990889526, scale=6.30015739743069)
DescribeResult(nobs=1000, minmax=(82.132745973266353, 123.72029145742846)
               mean=103.13484990889526, variance=39.731714947347989,
               skewness=0.06167579792066425, kurtosis=0.14005412212456836)
```

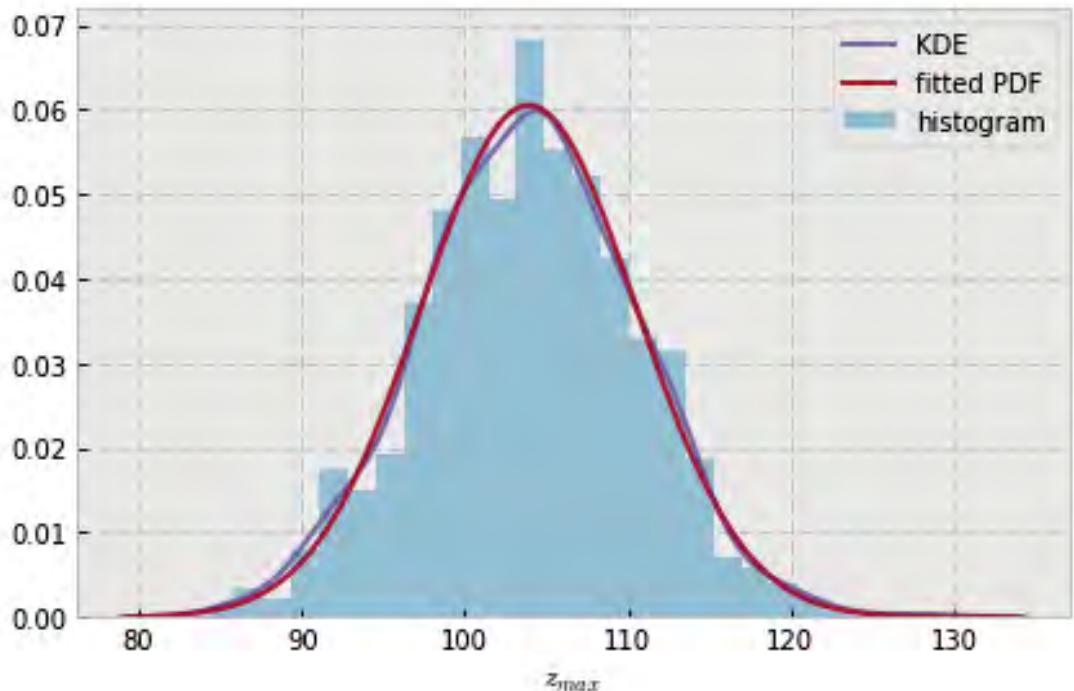


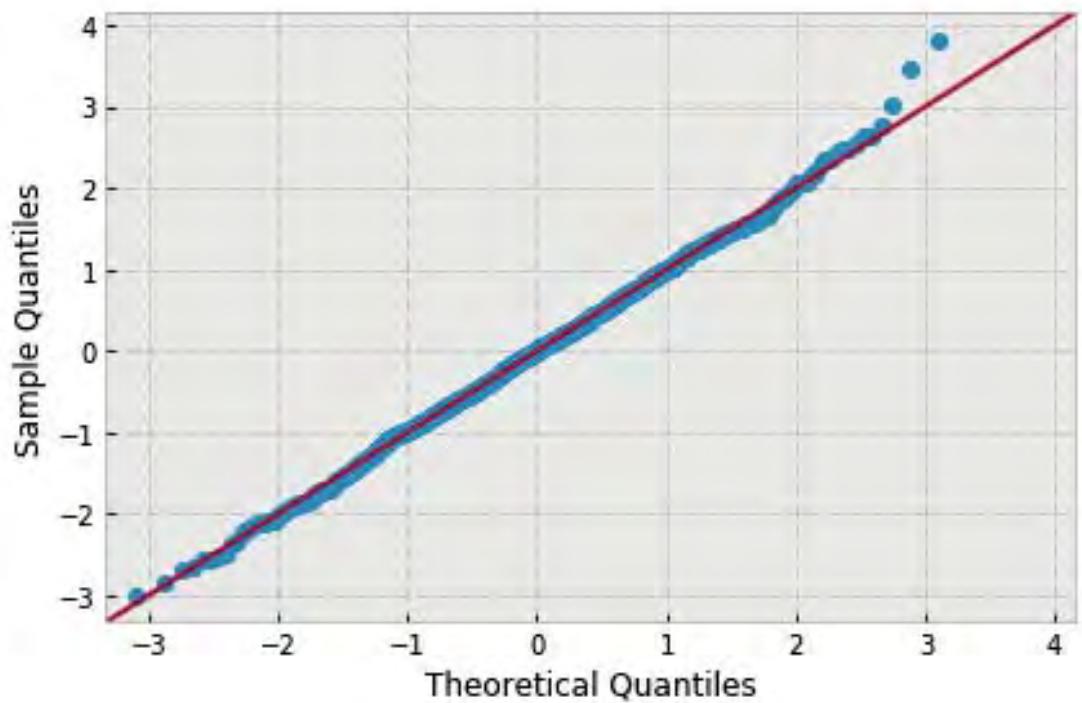


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=104.45294921425058, scale=6.3409783187082756)
DescribeResult(nobs=1000, minmax=(80.330215700741959, 126.44056072868223)
               mean=104.45294921425058, variance=40.248254292621048,
               skewness=0.1272666678465081, kurtosis=0.1987289332606399)
```

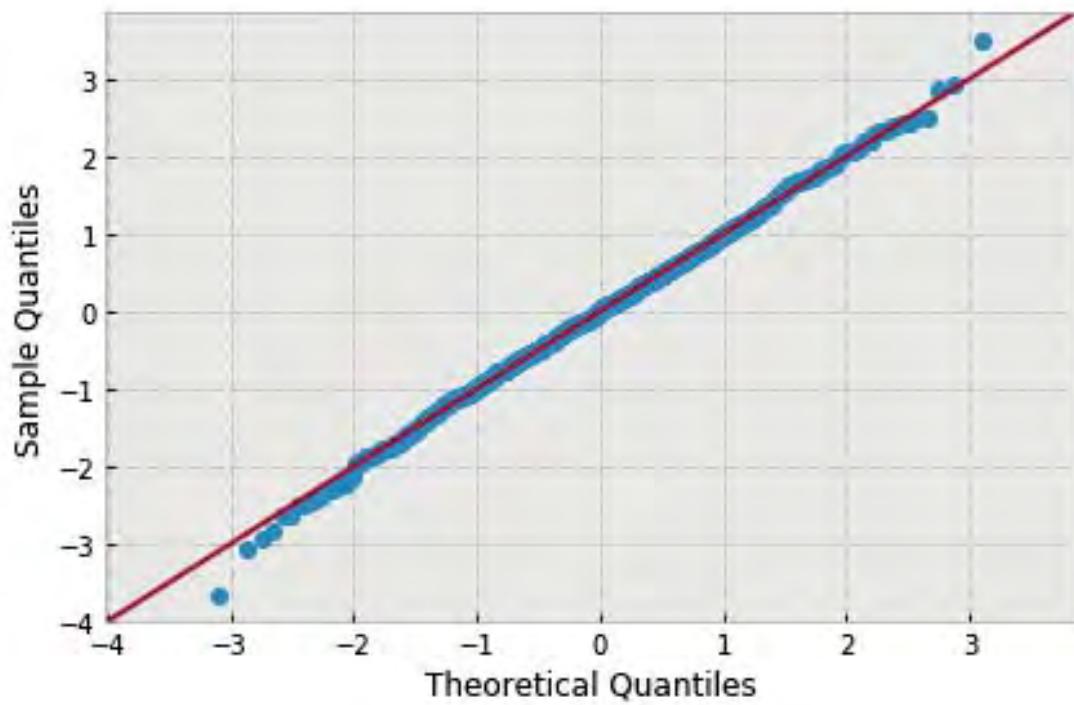
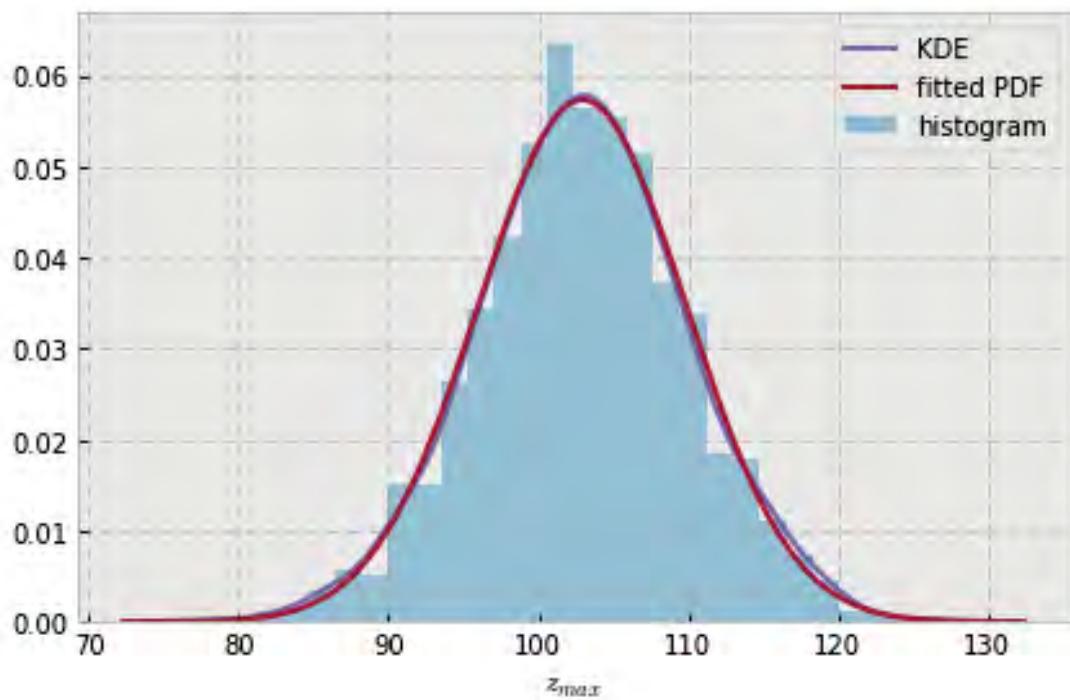


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=103.9117614445074, scale=6.594152948501469)
DescribeResult(nobs=1000, minmax=(84.200666262724098, 129.02345453489252)
               mean=103.91176144450741, variance=43.52637948771833,
               skewness=0.041902939947205, kurtosis=0.11852497021606645)
```

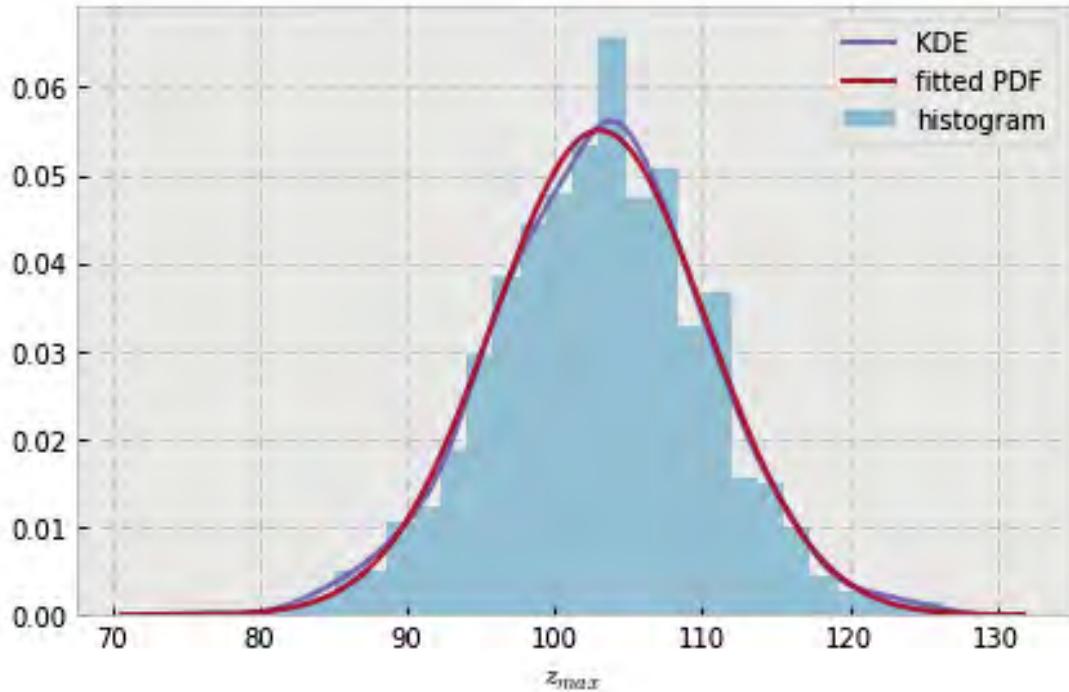


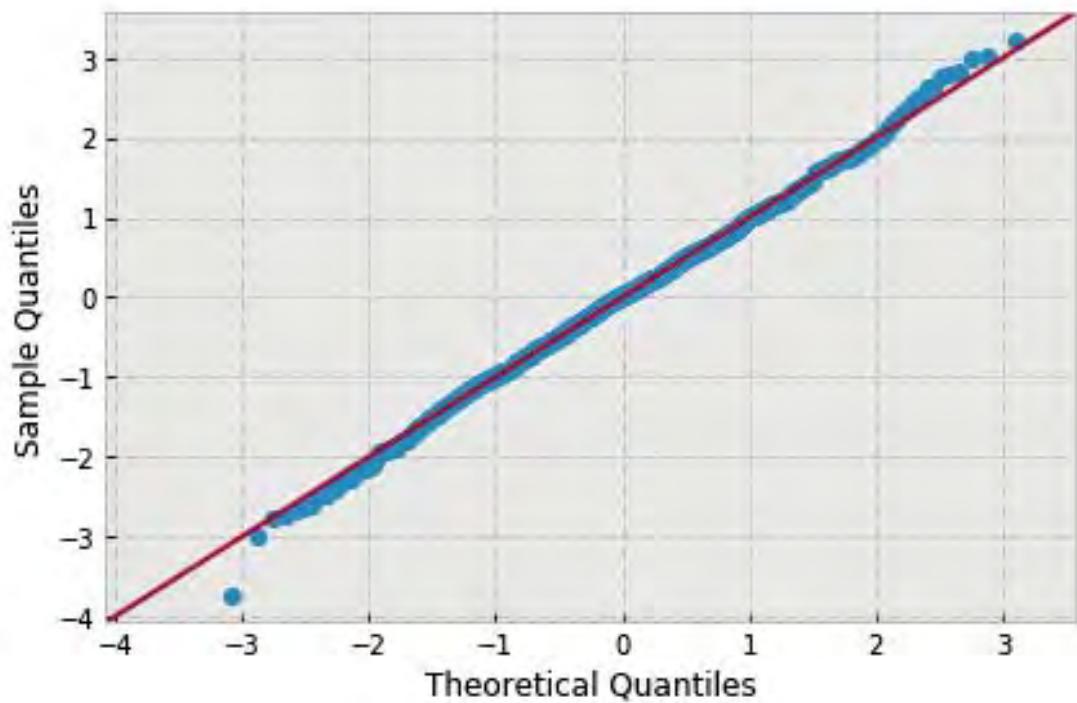


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=102.93562838632057, scale=6.9588822453592645)
DescribeResult(nobs=1000, minmax=(77.512672894604208, 127.21263818356731)
               mean=102.93562838632057, variance=48.474516621397797,
               skewness=-0.03489237457361211, kurtosis=0.1682974668437267)
```

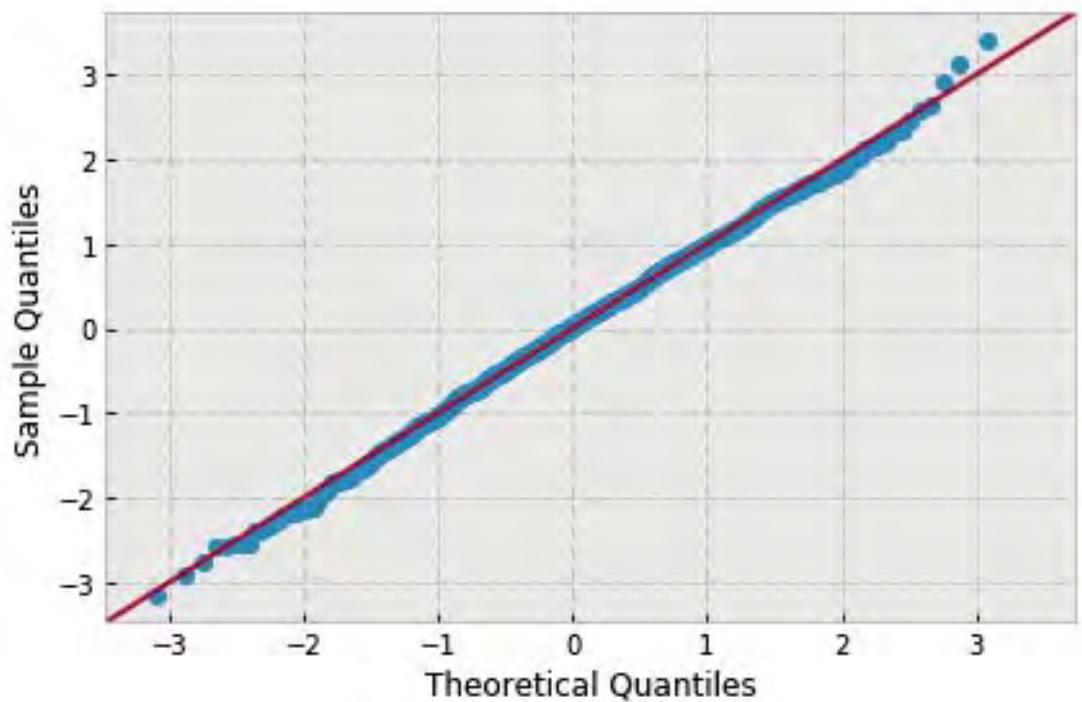
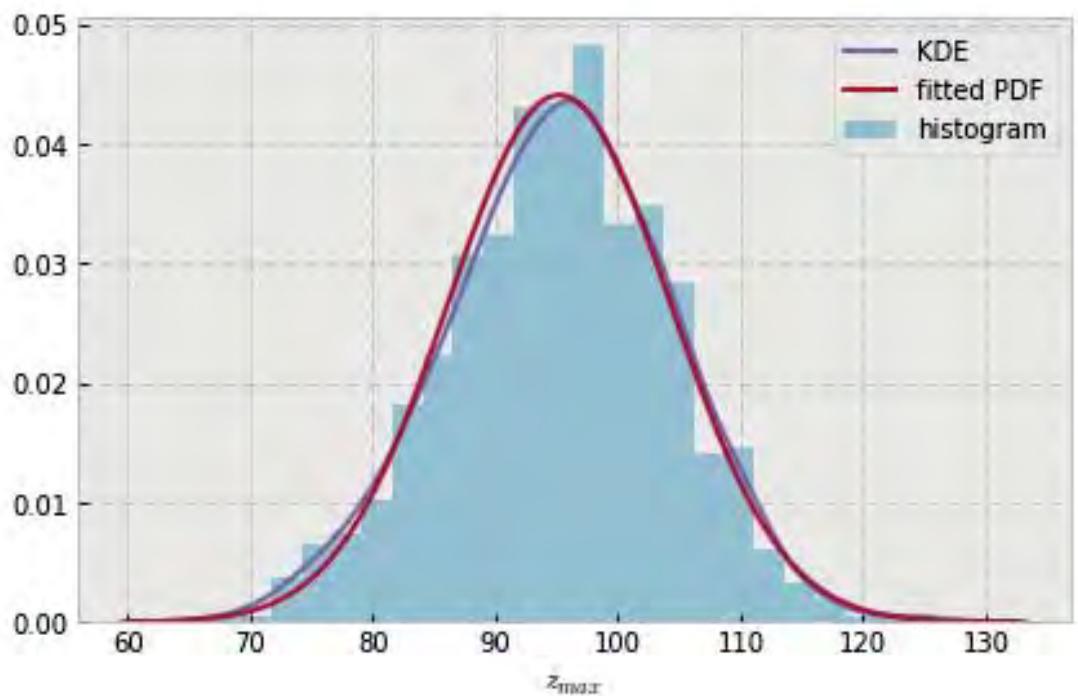


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=103.0551998594697, scale=7.244605932846012)
DescribeResult(nobs=1000, minmax=(76.097407121320217, 126.32986159809586)
               mean=103.0551998594697, variance=52.536851974201838,
               skewness=-0.03854570962836135, kurtosis=0.23603443273151337)
```

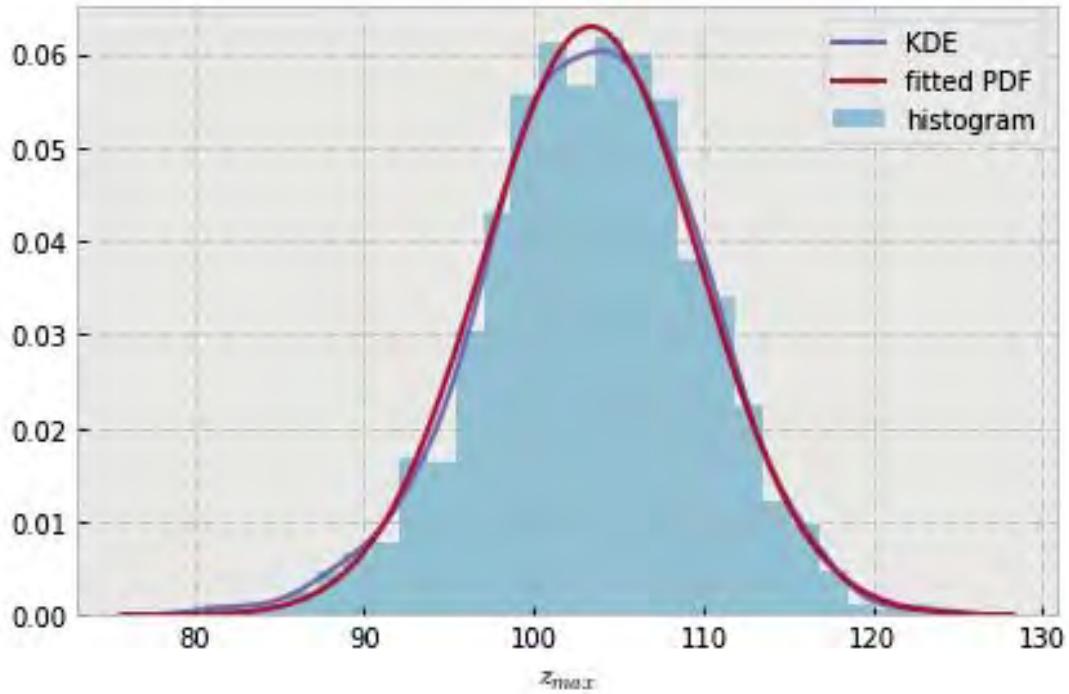


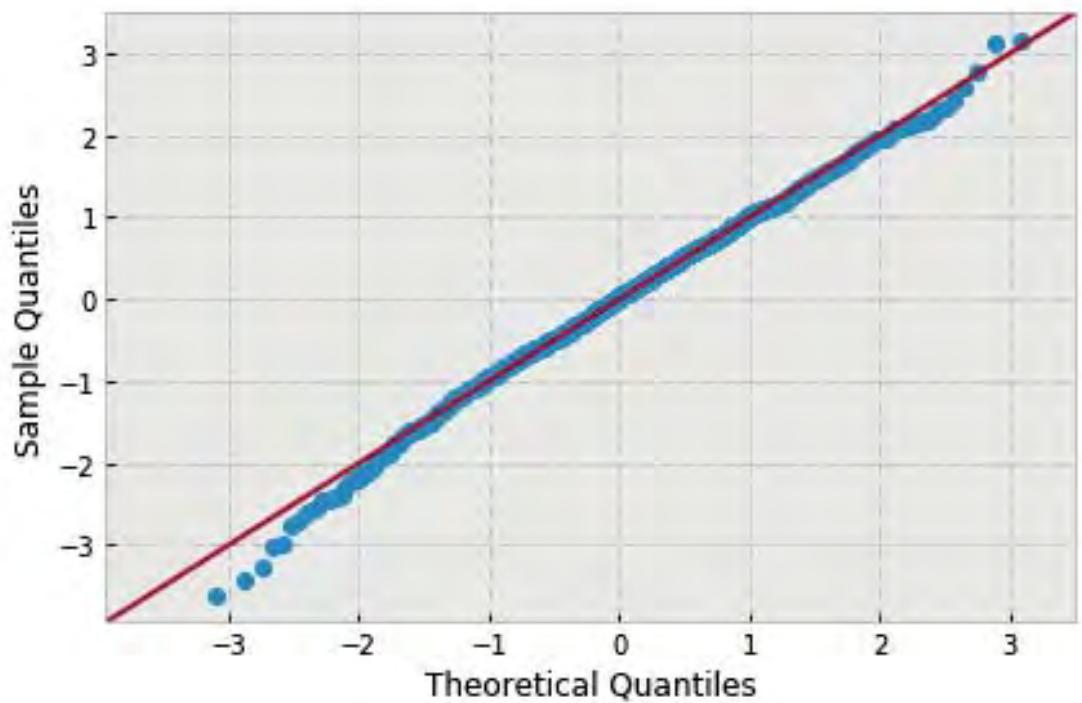


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=95.206438040821, scale=9.054750143151974)
DescribeResult(nobs=1000, minmax=(66.776645922882508, 125.98658663894506)
               mean=95.206438040820998, variance=82.070570725636316,
               skewness=-0.09390555385536832, kurtosis=-0.03157057437860811)
```

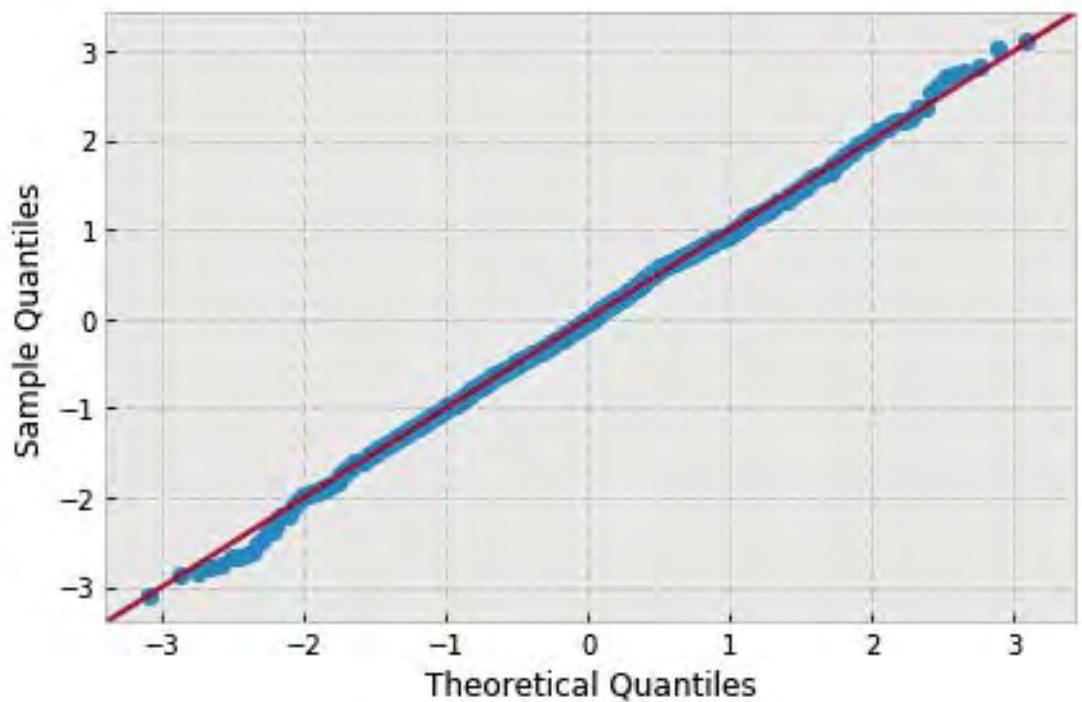
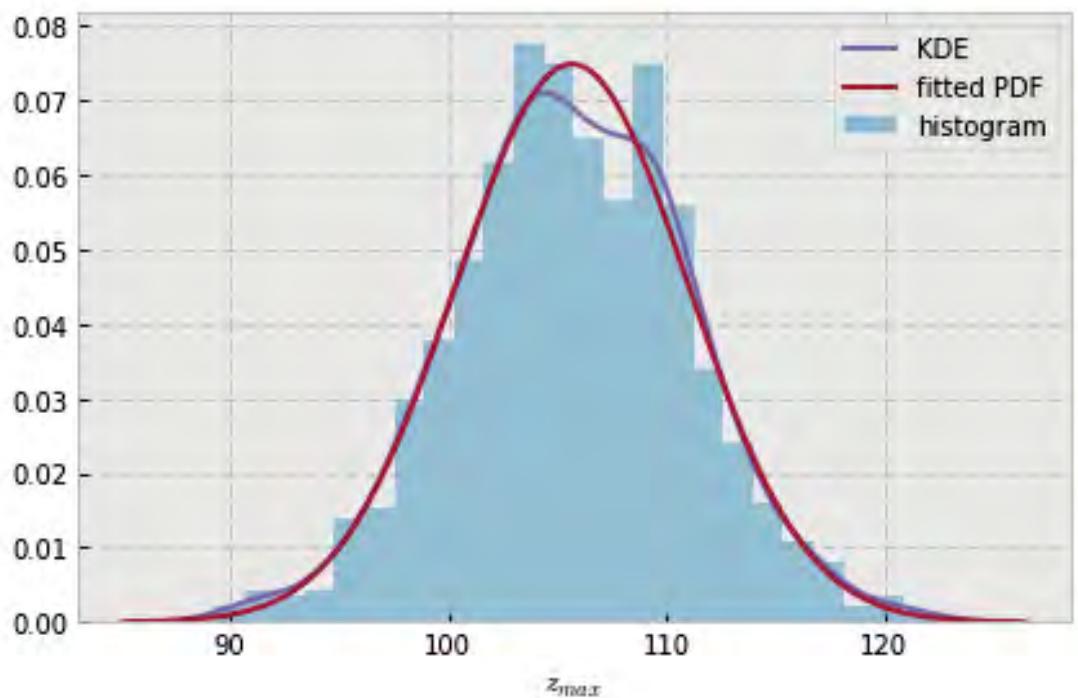


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=103.39298842529463, scale=6.34302424350122)
DescribeResult(nobs=1000, minmax=(80.551740679899922, 123.39654391986384)
               mean=103.39298842529463, variance=40.274230784428653,
               skewness=-0.1840813343761588, kurtosis=0.2312380070886837)
```

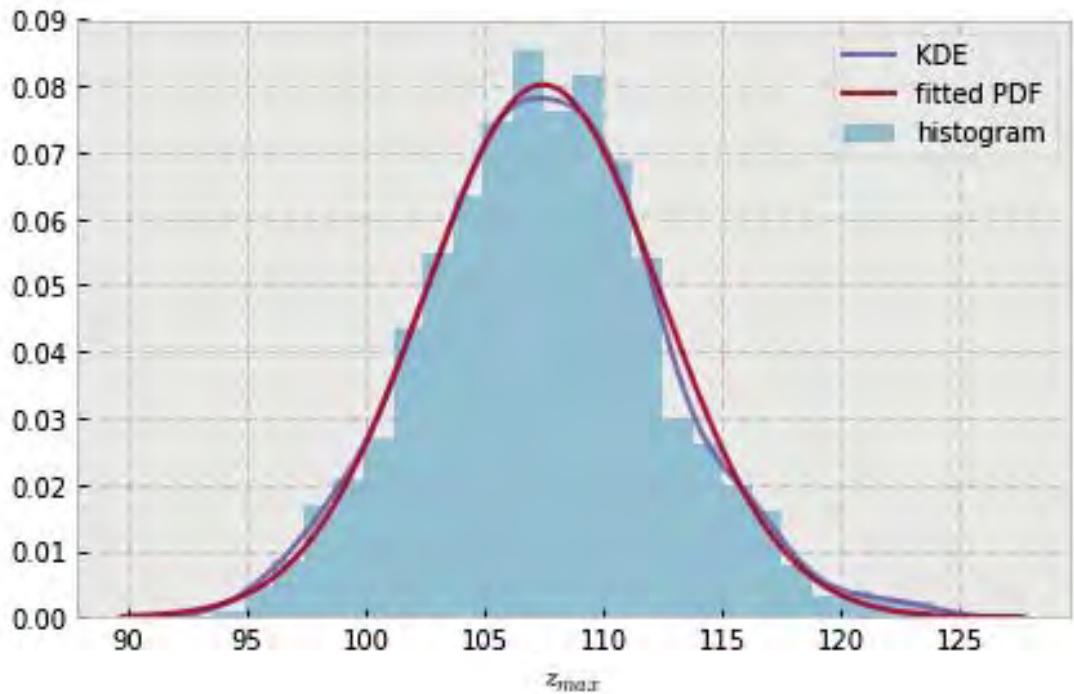


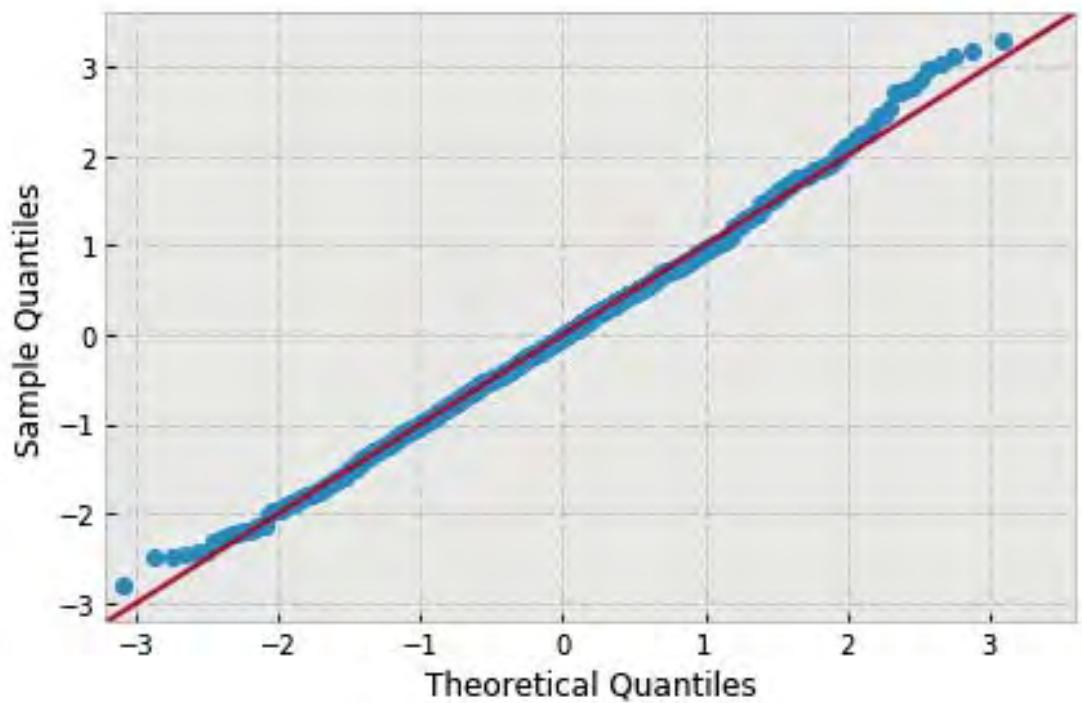


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=105.68183585527572, scale=5.324592553022331)
DescribeResult(nobs=1000, minmax=(89.262521951028191, 122.24514834321495)
               mean=105.68183585527572, variance=28.379665521222083,
               skewness=-0.020347359245835434, kurtosis=0.03334617716783406)
```

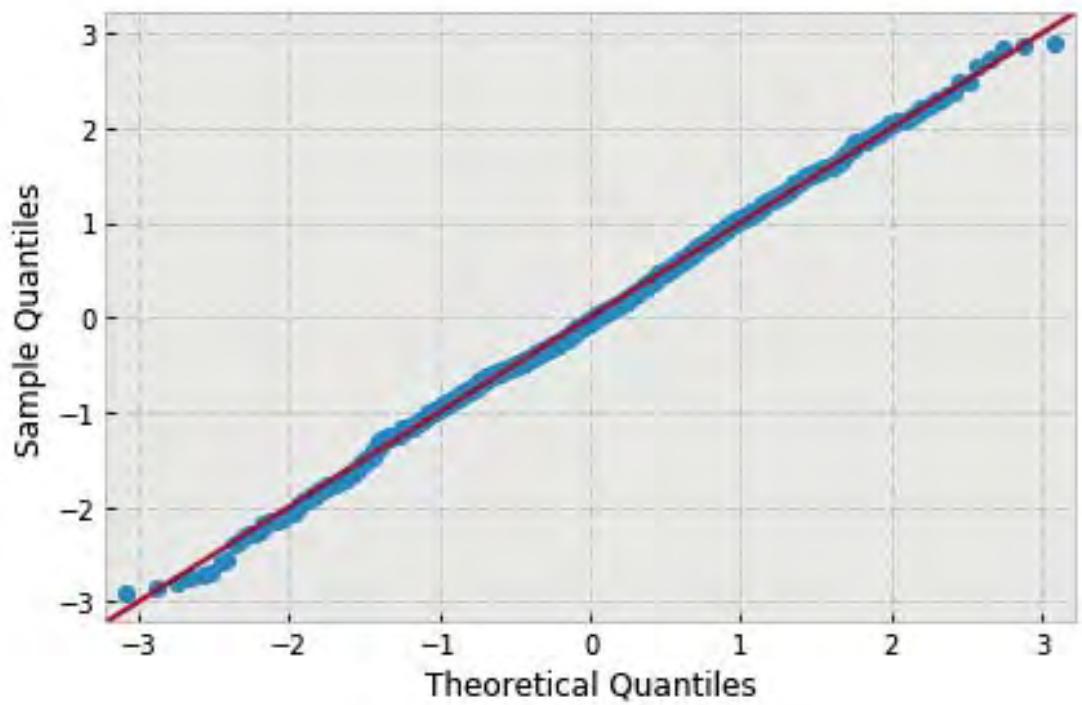
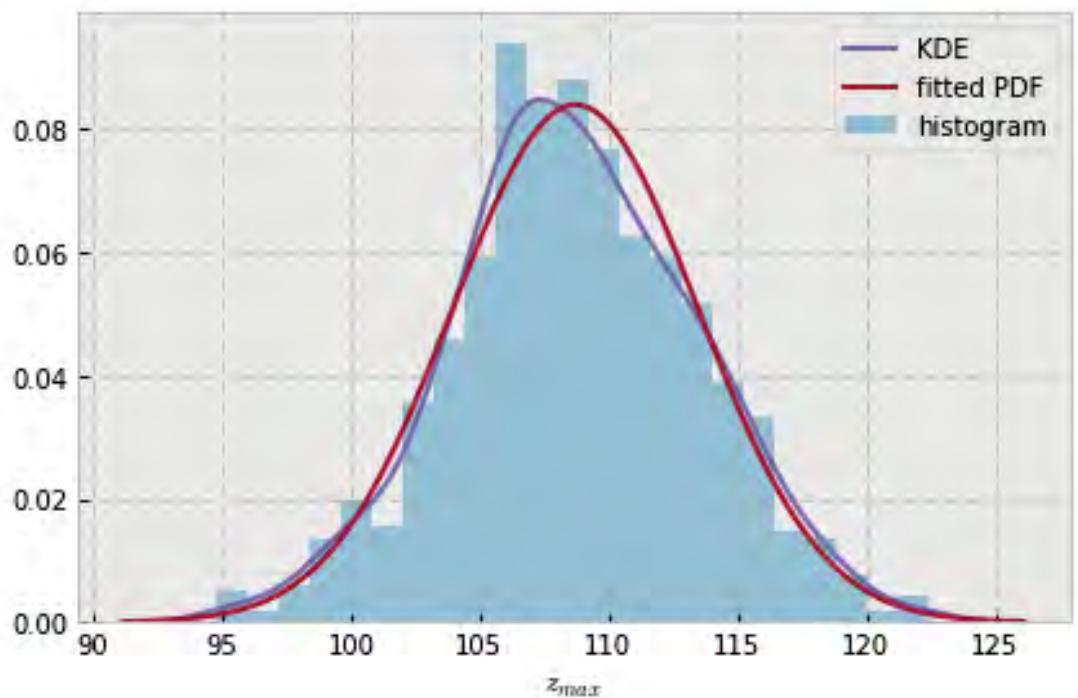


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=107.45602735059164, scale=4.976427983060977)
DescribeResult(nobs=1000, minmax=(93.570670530785875, 123.84931162099116)
               mean=107.45602735059164, variance=24.789625095688031,
               skewness=0.1652875779076341, kurtosis=0.10839432172309582)
```

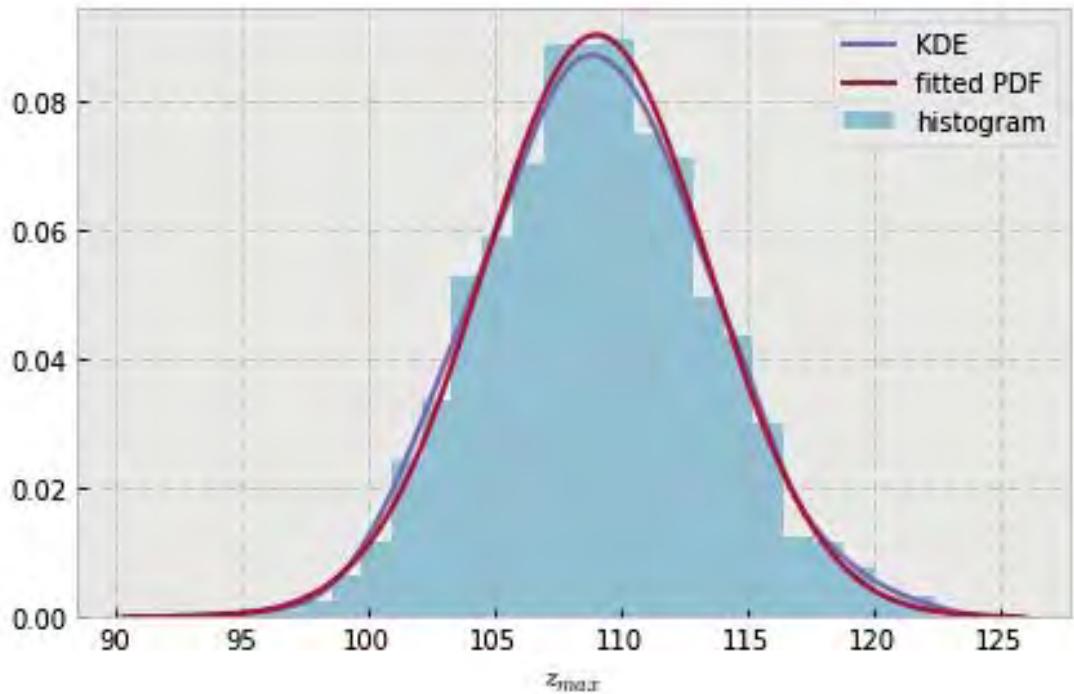


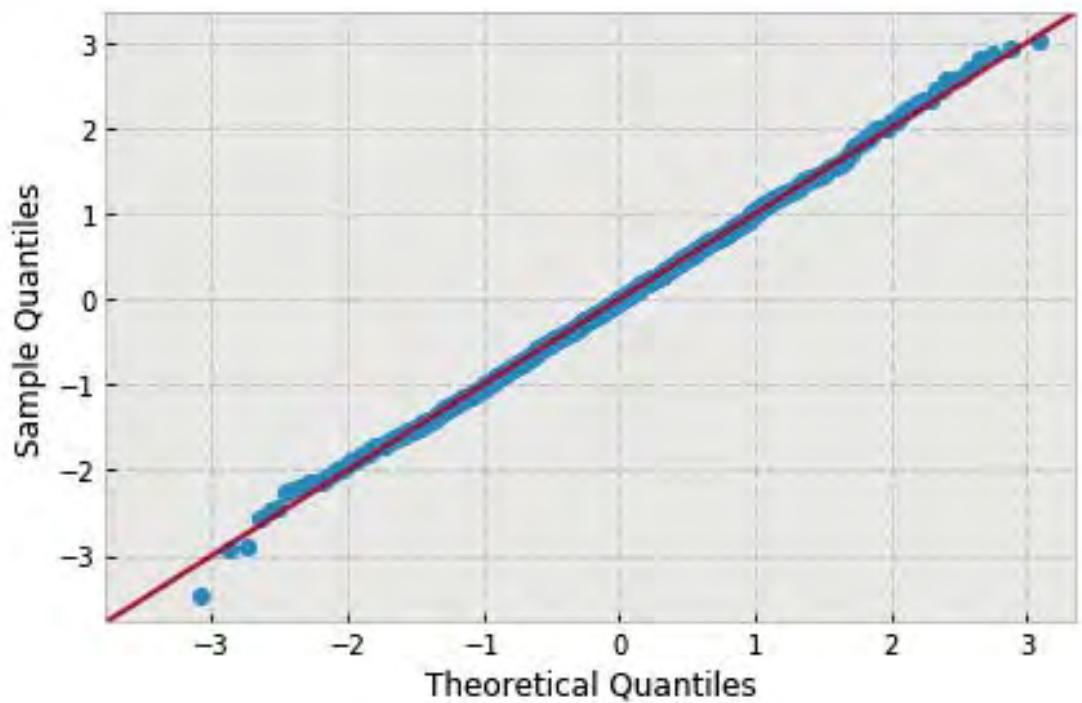


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=108.70573735279149, scale=4.757006801628118)
DescribeResult(nobs=1000, minmax=(94.81809009611726, 122.46056498903984)
               mean=108.70573735279149, variance=22.651765476212393,
               skewness=0.02547152522431889, kurtosis=-0.04595438126928153)
```

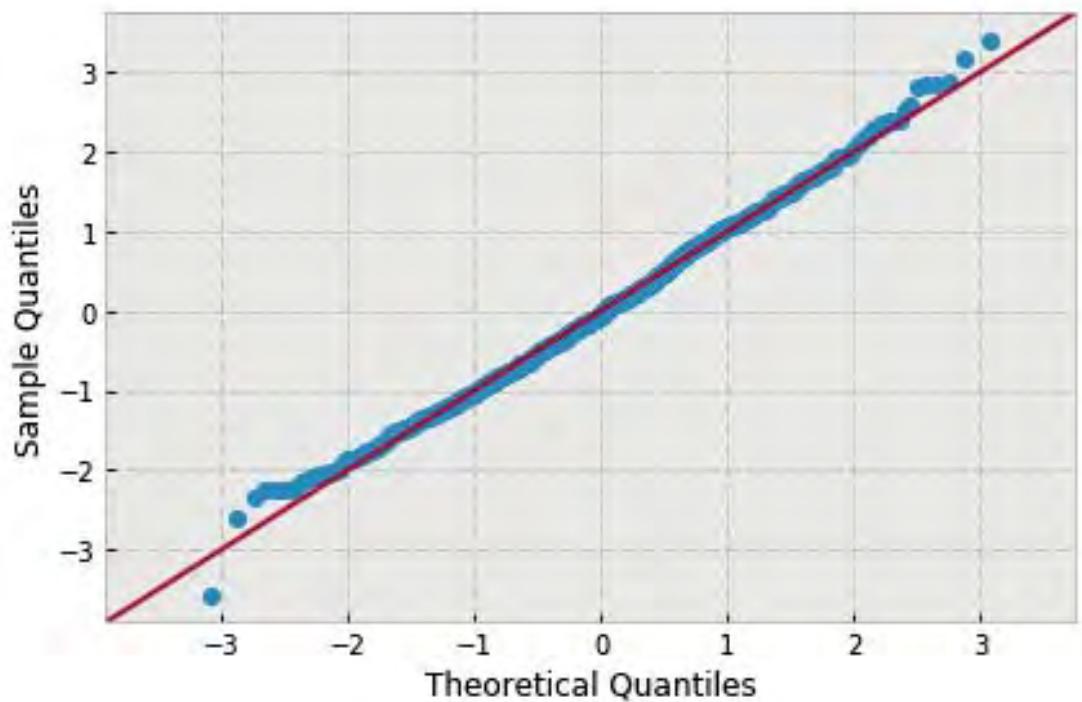
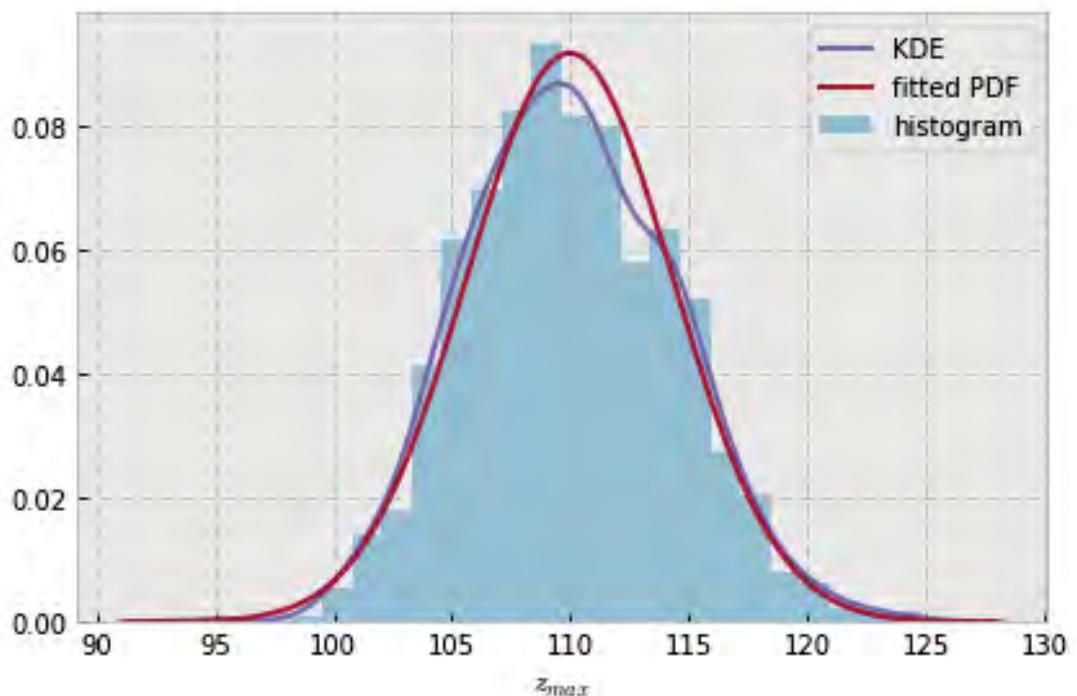


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=109.05680922380455, scale=4.424041252219403)
DescribeResult(nobs=1000, minmax=(93.785495860383392, 122.42535185226852)
               mean=109.05680922380455, variance=19.591732734073091,
               skewness=0.06627686934457923, kurtosis=-0.06266588954495145)
```

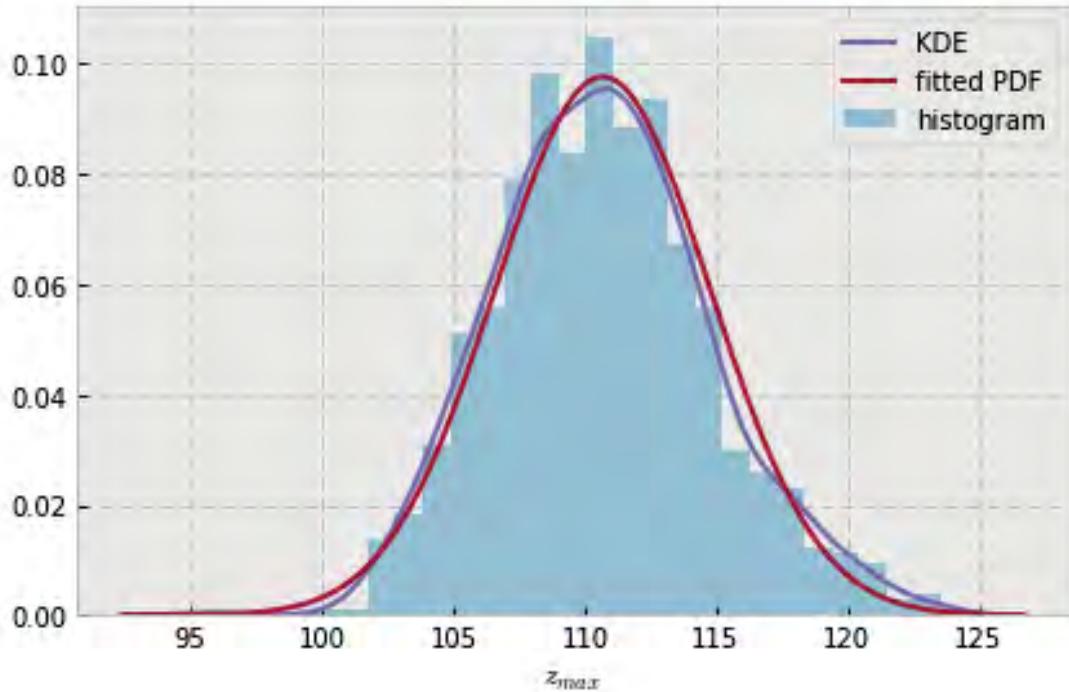


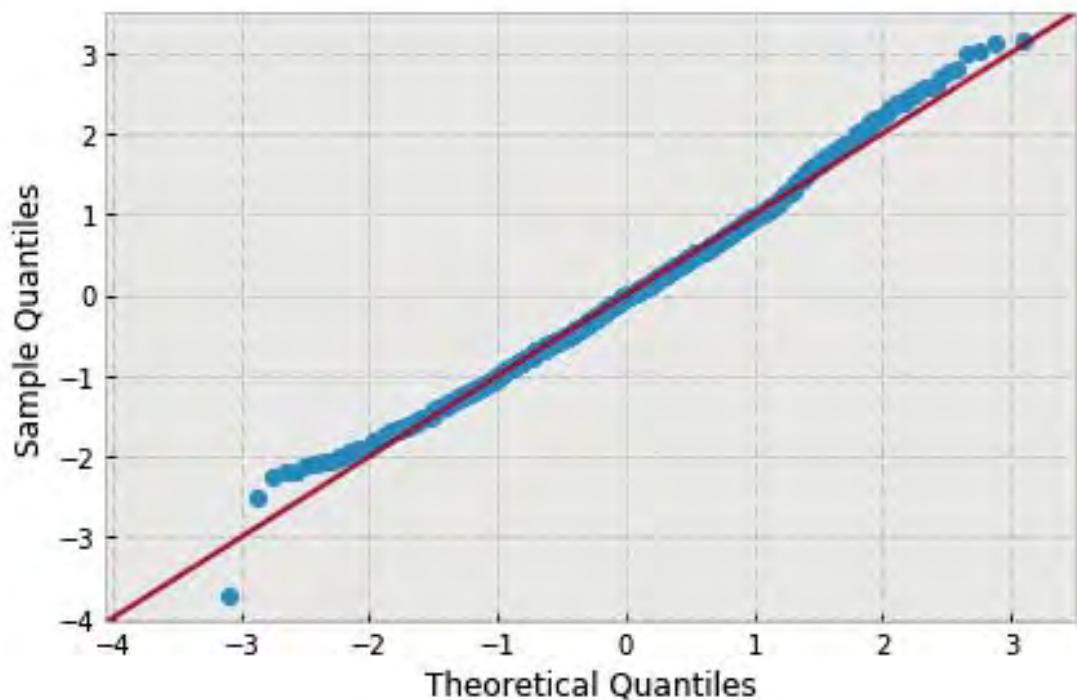


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=109.968309494804, scale=4.343796241353436)
DescribeResult(nobs=1000, minmax=(94.477441495342291, 124.72923731883311)
               mean=109.968309494804, variance=18.887453239635878,
               skewness=0.160659917899985, kurtosis=-0.11123910267374137)
```

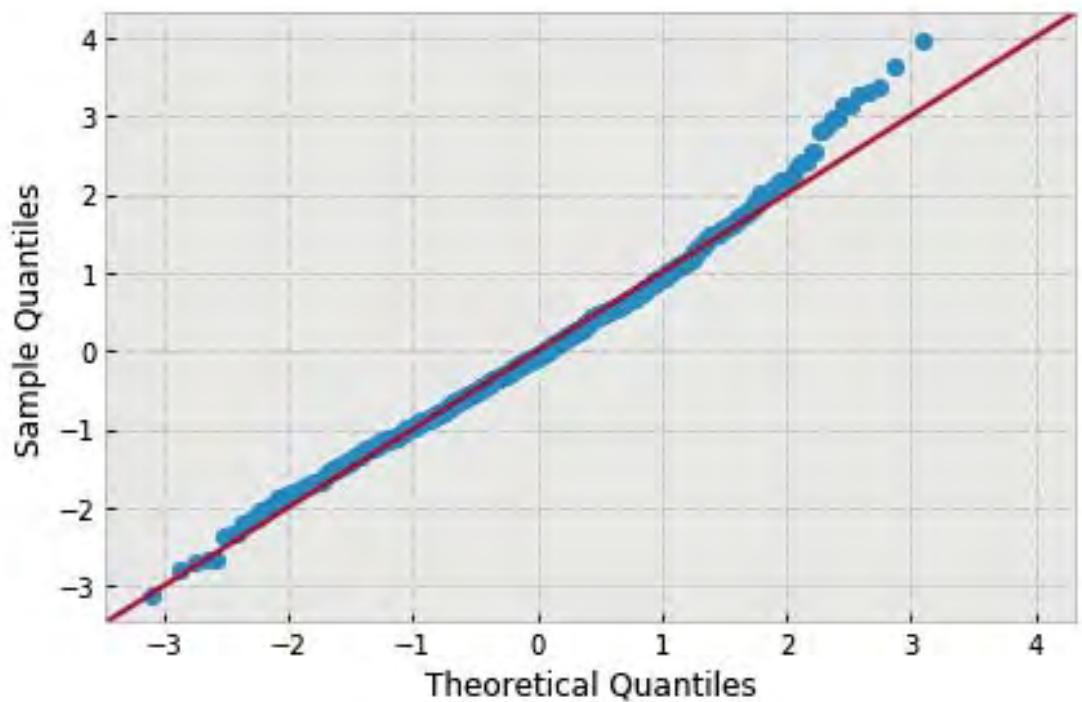
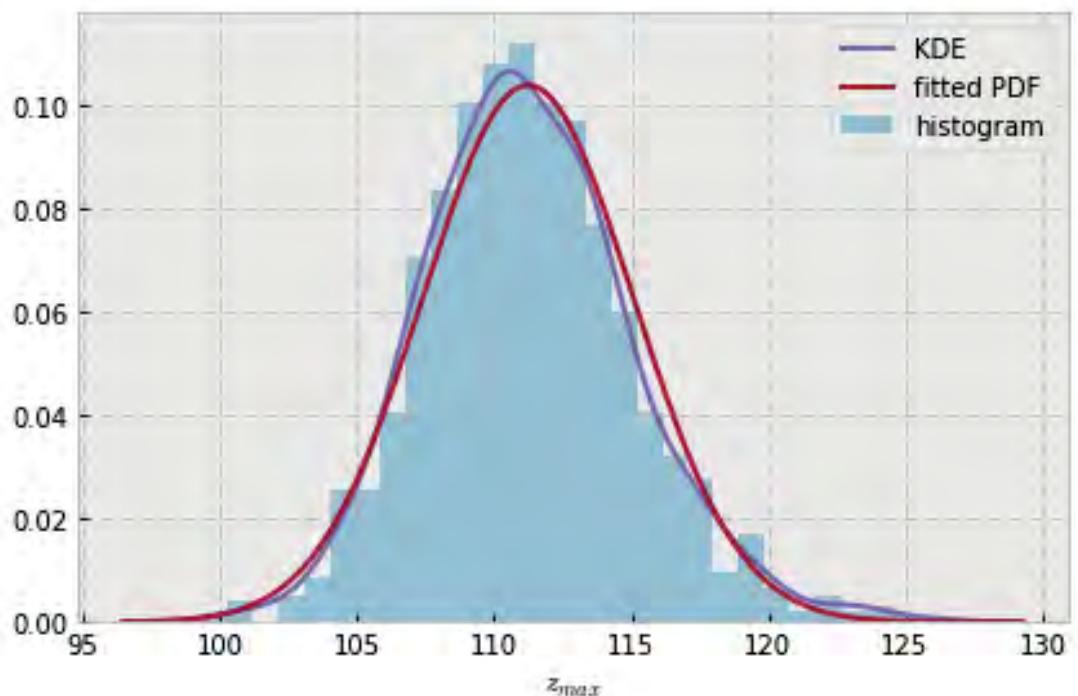


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=110.68294486149487, scale=4.0880241262735355)
DescribeResult(nobs=1000, minmax=(95.523275268892974, 123.5539518929744)
               mean=110.68294486149487, variance=16.728669926921427,
               skewness=0.27018391968920885, kurtosis=0.0862430519709636)
```

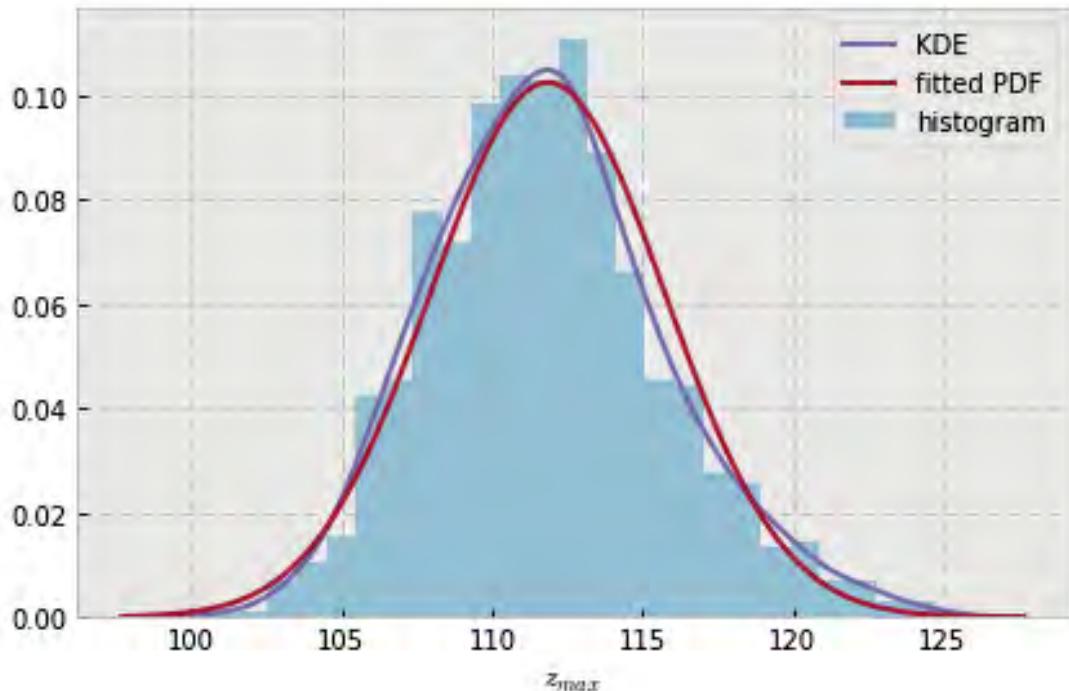


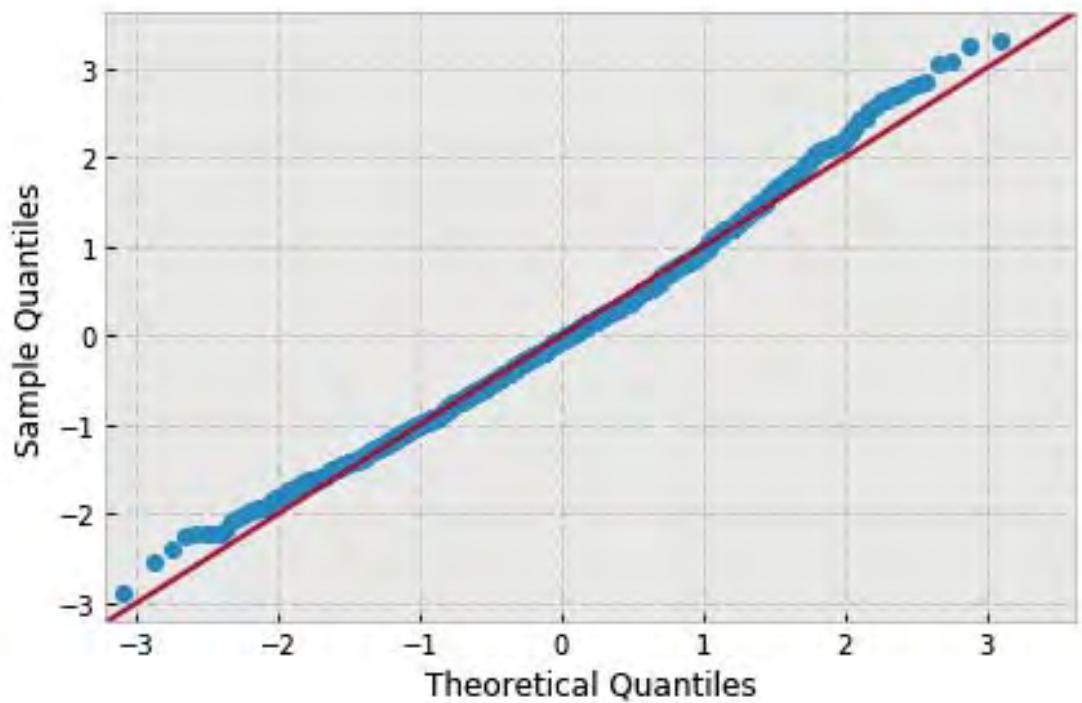


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=111.23261398397614, scale=3.8330112278812467)
DescribeResult(nobs=1000, minmax=(99.318421937405148, 126.40027758925274)
               mean=111.23261398397614, variance=14.70668175481852,
               skewness=0.3834501711923076, kurtosis=0.6059753359286639)
```

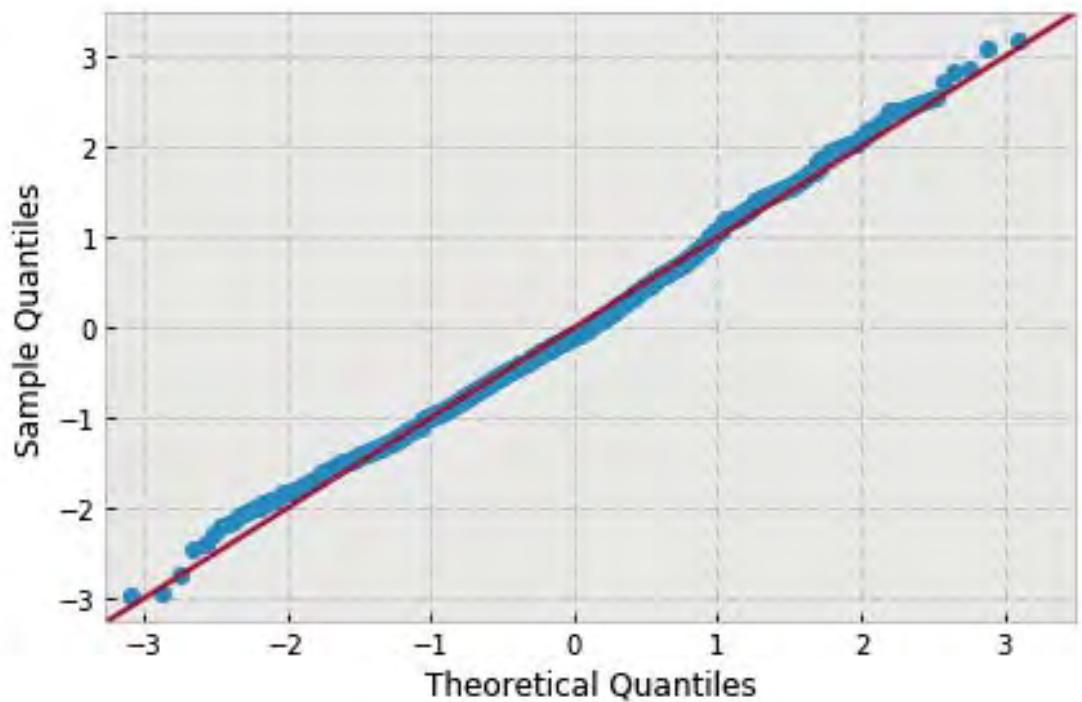
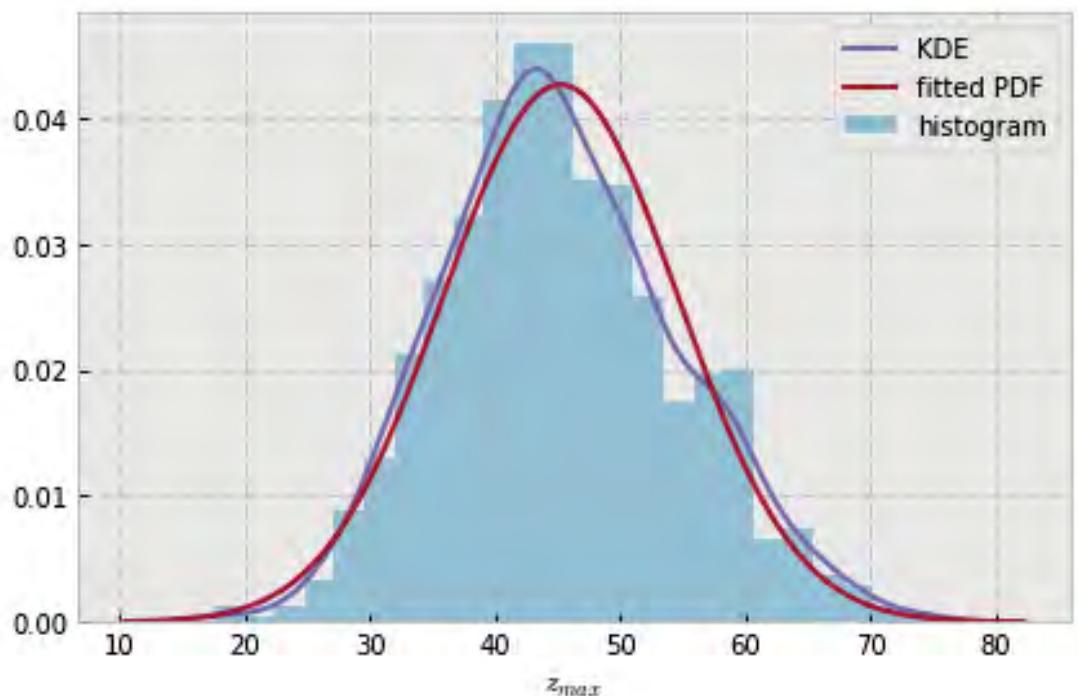


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=111.8509104248681, scale=3.8921714236231866)
DescribeResult(nobs=1000, minmax=(100.6307411543829, 124.7295389013818)
               mean=111.8509104248681, variance=15.164162553422365,
               skewness=0.36104973635143695, kurtosis=0.13531156326625915)
```

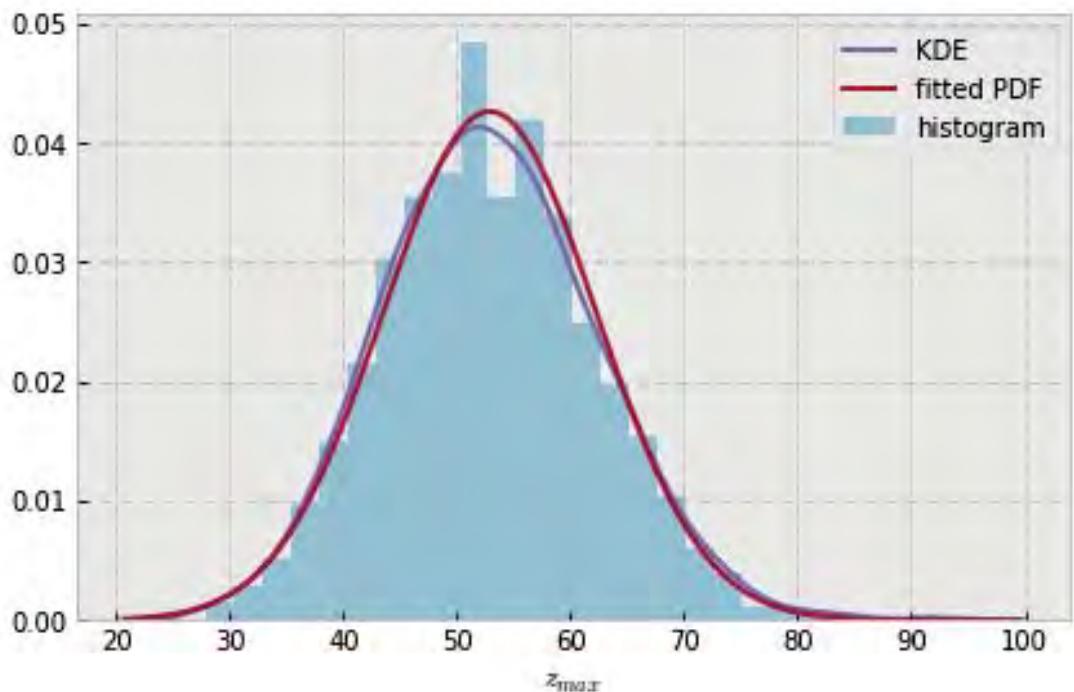


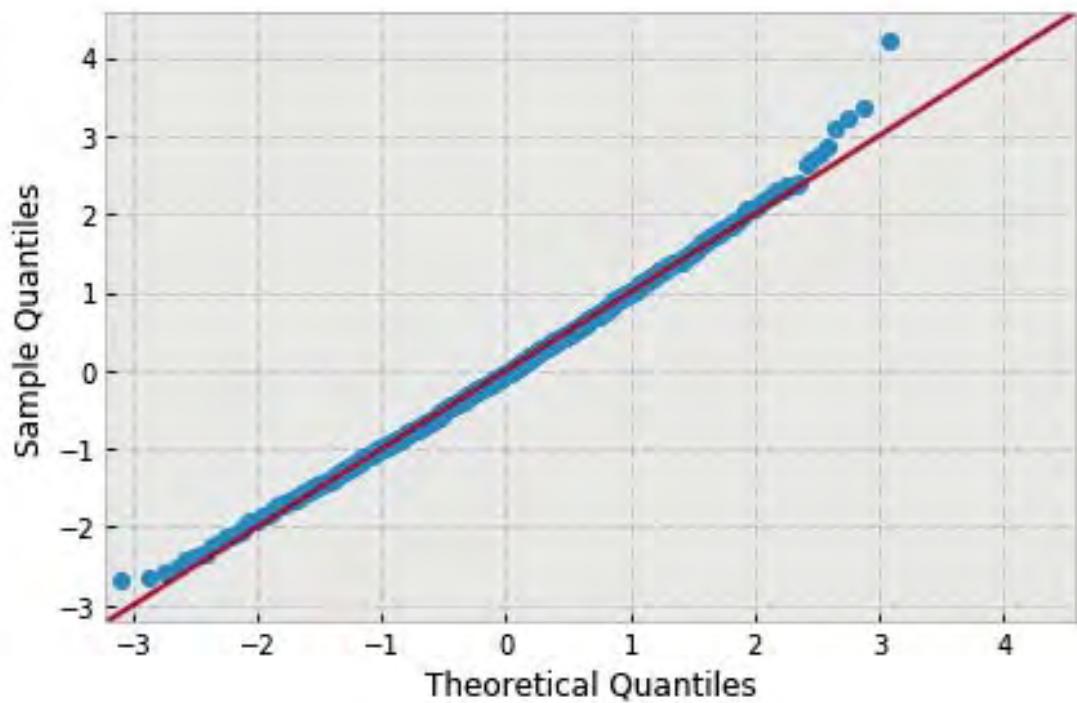


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=45.205285166459504, scale=9.360406526335002)
DescribeResult(nobs=1000, minmax=(17.556673588177649, 74.947419994626387)
               mean=45.205285166459504, variance=87.704915253508403,
               skewness=0.24973157484222006, kurtosis=-0.10575855713740667)
```

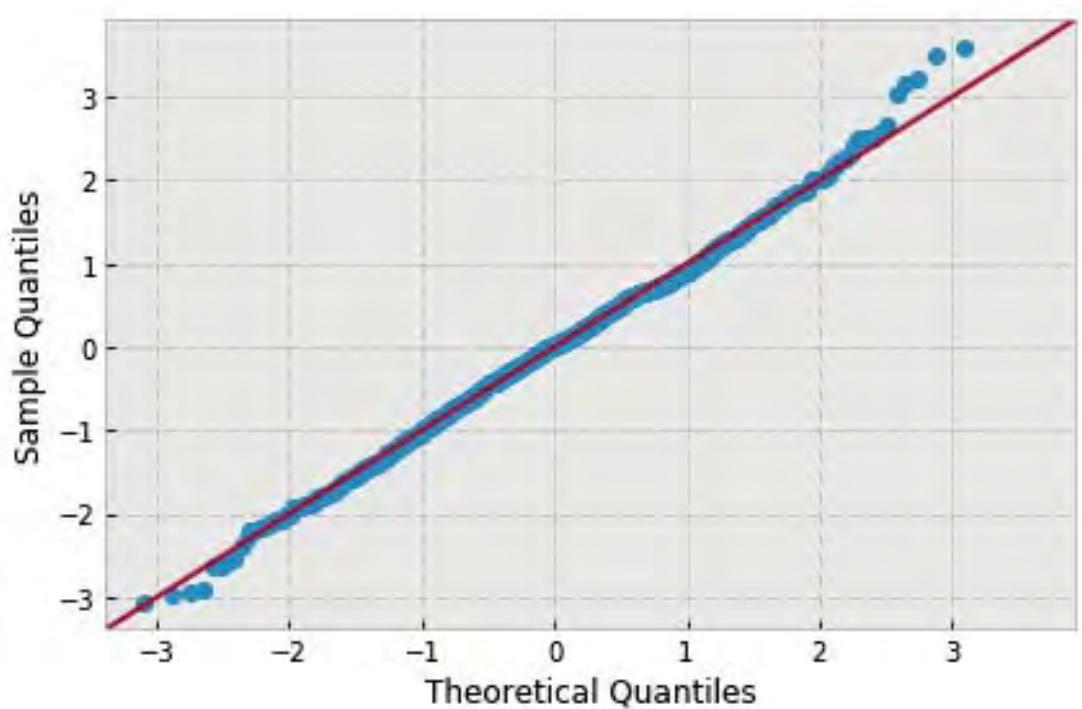
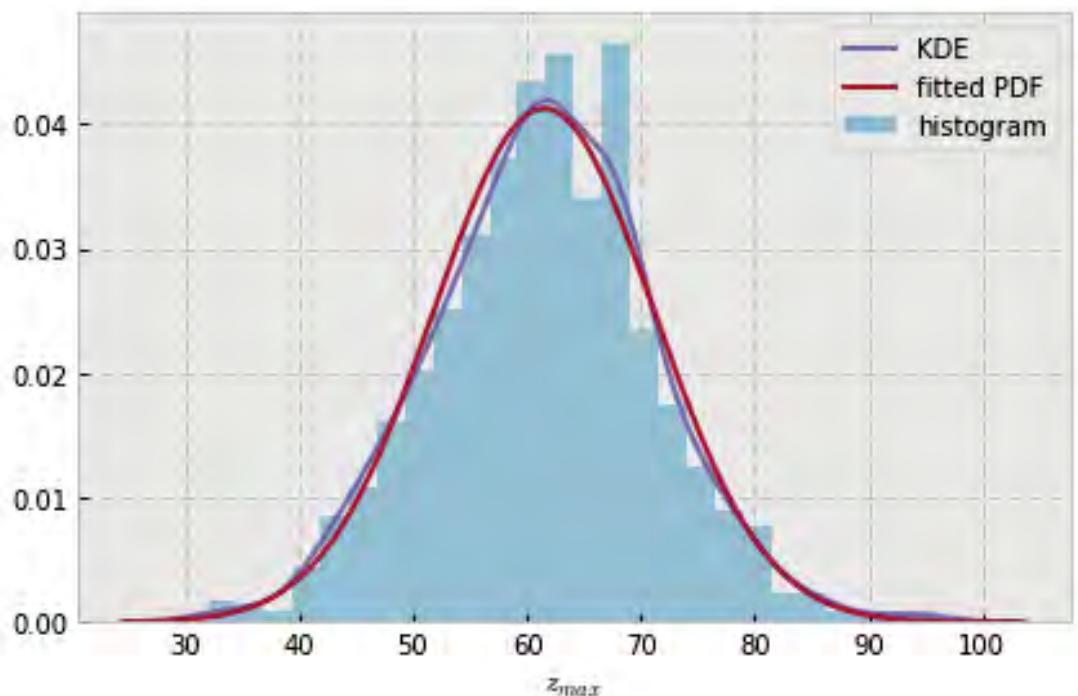


```
variable name: r
variable value: 1.222222222222223
distribution: normal(shape=(), loc=52.88795893855268, scale=9.363198545472523)
DescribeResult(nobs=1000, minmax=(27.928625415728927, 92.431641181066141)
               mean=52.887958938552678, variance=87.757244246184953,
               skewness=0.23129303304236967, kurtosis=0.15886171075258204)
```

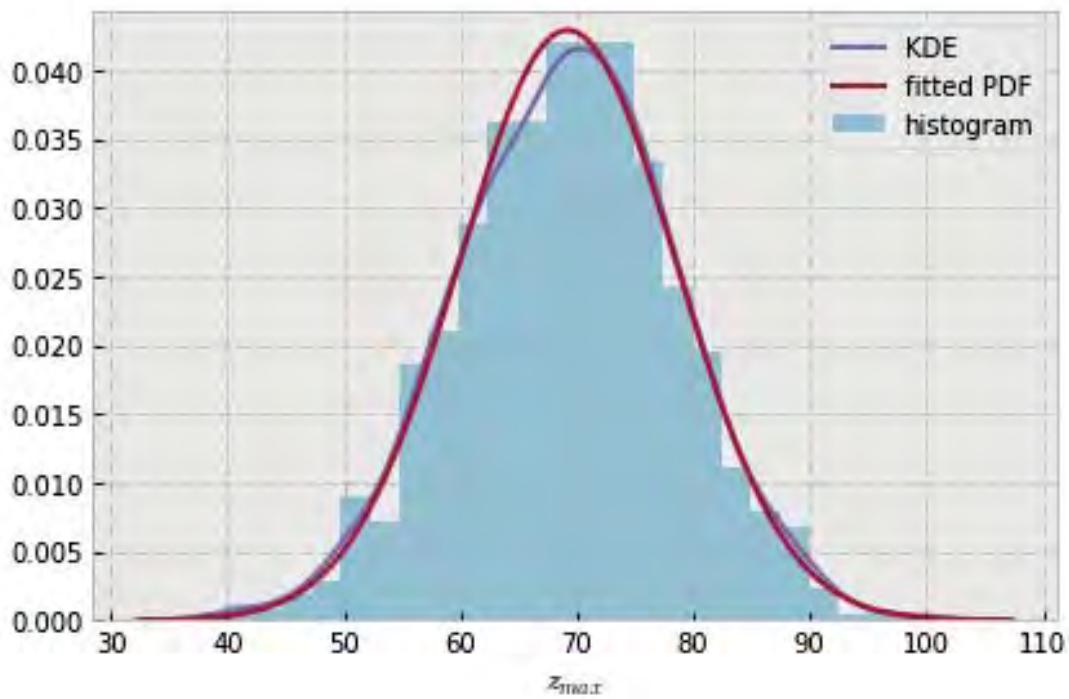


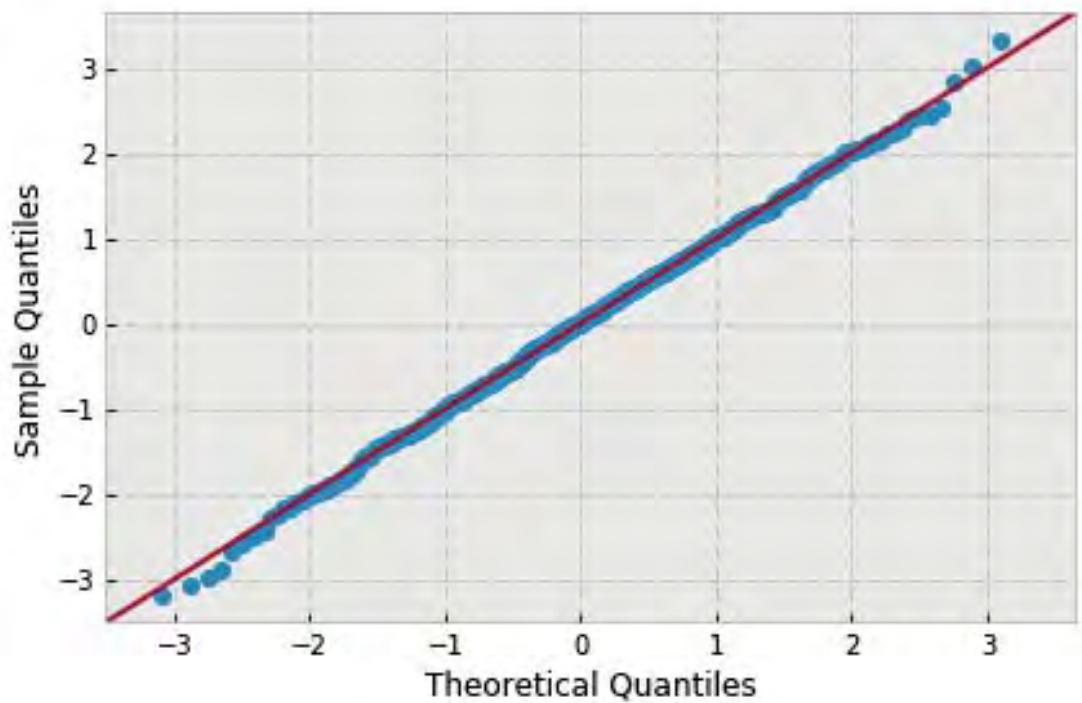


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=61.501683008721045, scale=9.66663395214342)
DescribeResult(nobs=1000, minmax=(32.0246927982827, 96.19057480013791)
               mean=61.501683008721045, variance=93.53734931404594,
               skewness=0.04231019971385085, kurtosis=0.2775895511256352)
```

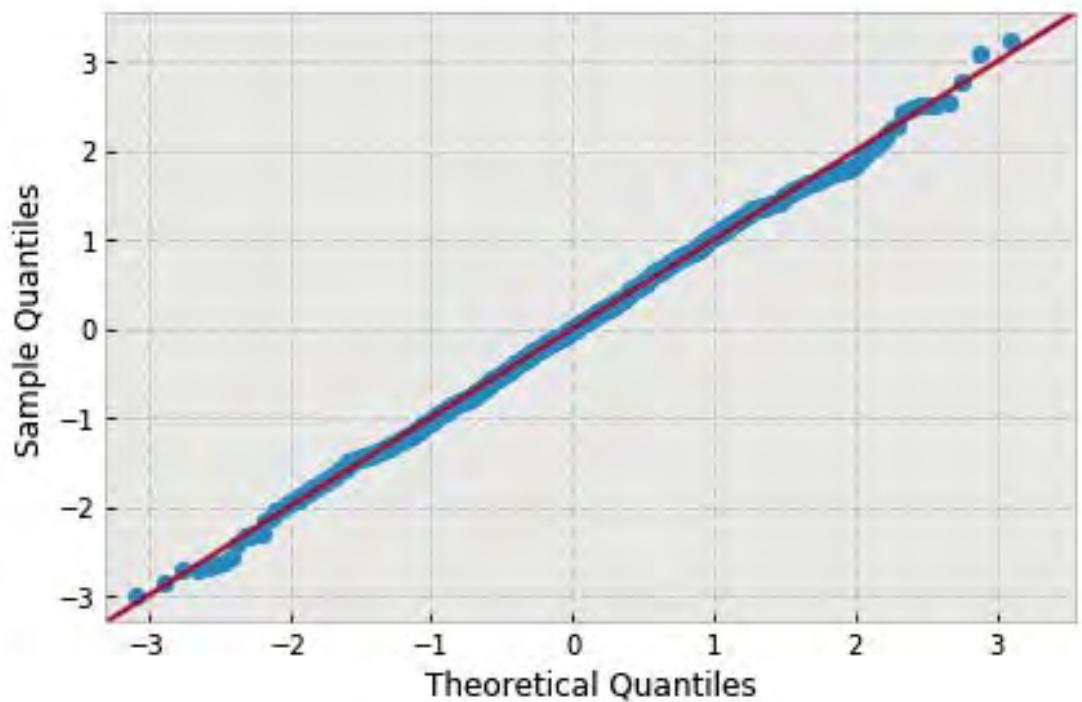
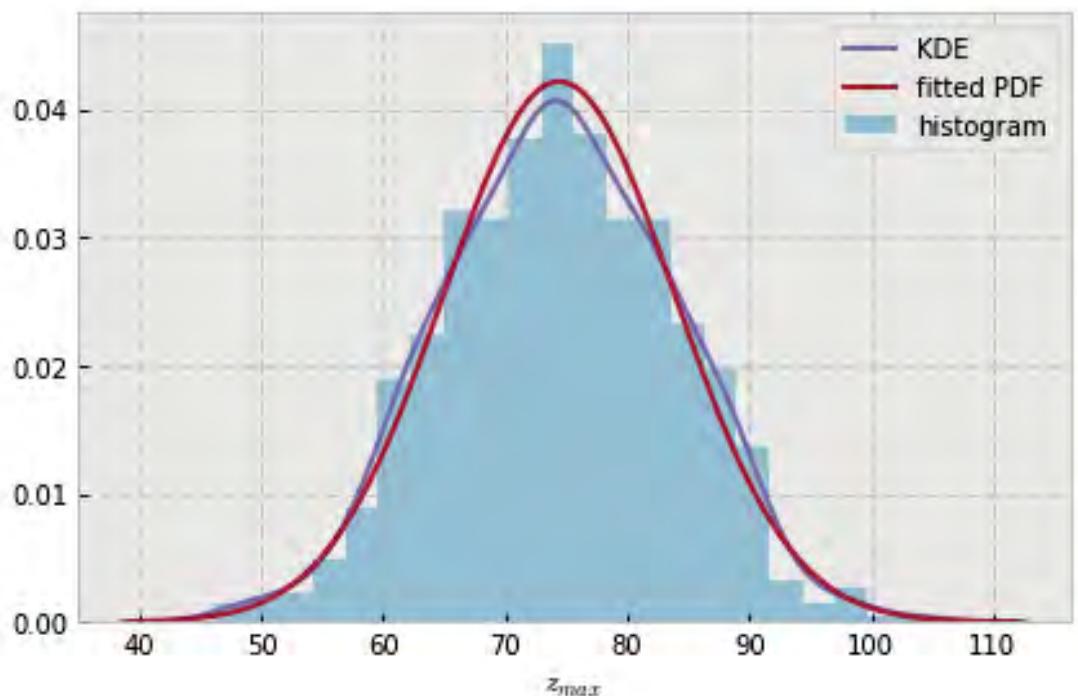


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=69.21249252966805, scale=9.299893201120295)
DescribeResult(nobs=1000, minmax=(39.63679603766851, 100.05602341445064)
               mean=69.212492529668054, variance=86.574588140383852,
               skewness=-0.048738357756973806, kurtosis=-0.023720104287471067)
```

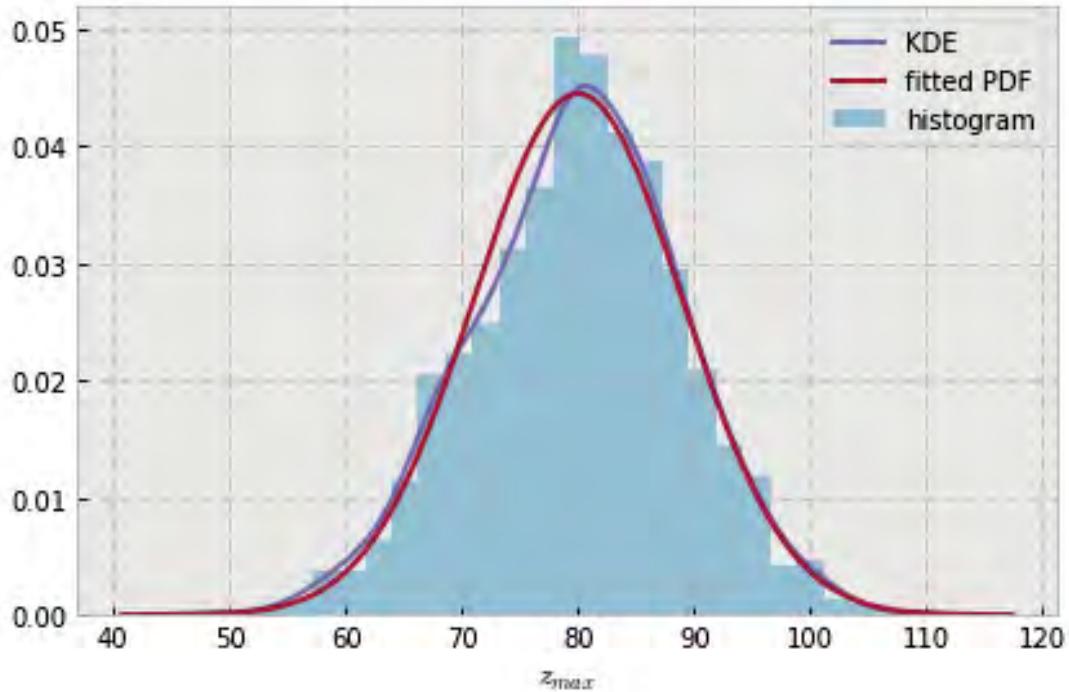


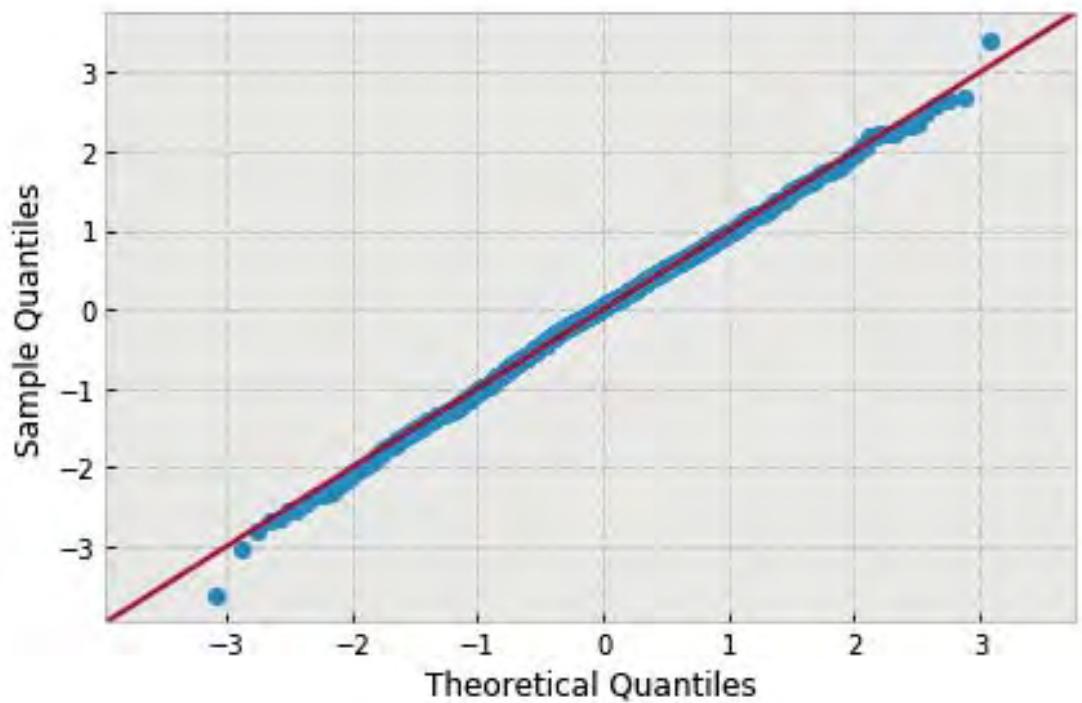


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=74.42791690784797, scale=9.468511709709516)
DescribeResult(nobs=1000, minmax=(46.168046723299909, 105.05020204305494)
               mean=74.427916907847973, variance=89.742456453359566,
               skewness=-0.008624385637929118, kurtosis=-0.1656982866322383)
```

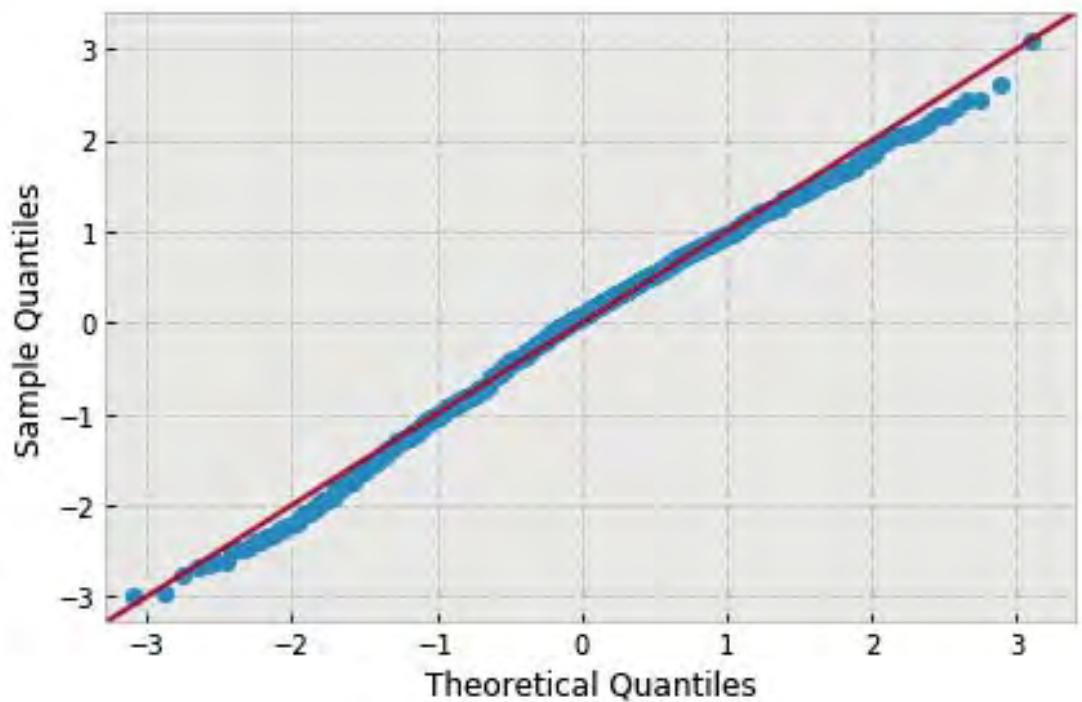
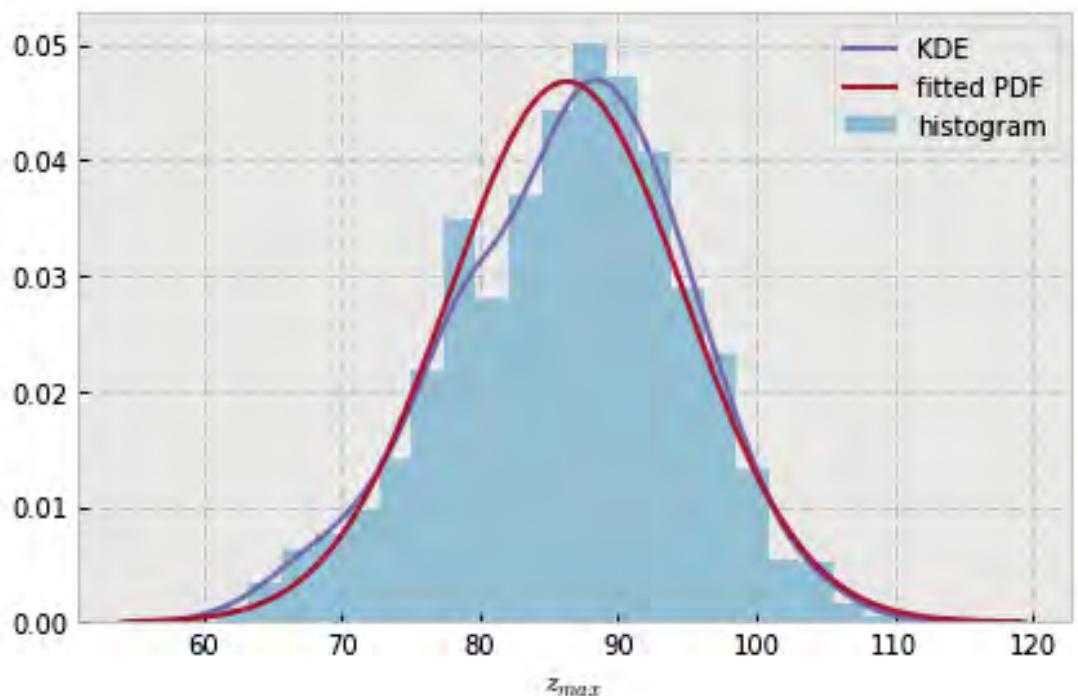


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=80.02078601959352, scale=8.962672524951222)
DescribeResult(nobs=1000, minmax=(47.669684283026825, 110.49258859696749)
               mean=80.020786019593515, variance=80.409908698213712,
               skewness=-0.12161739385737447, kurtosis=0.017119662632304244)
```

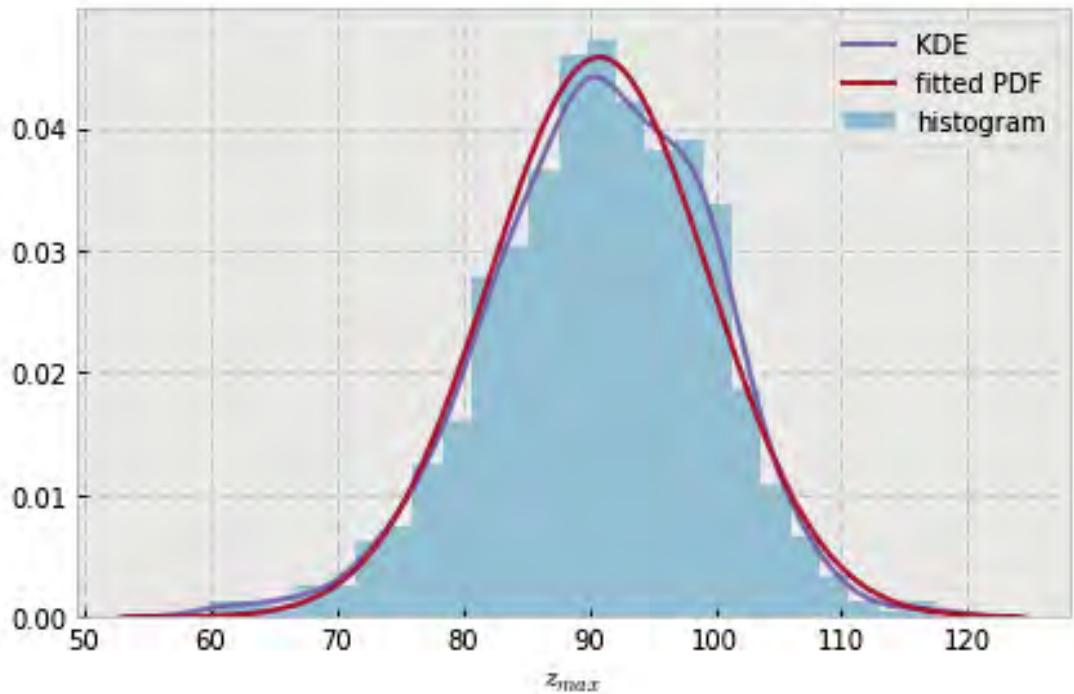


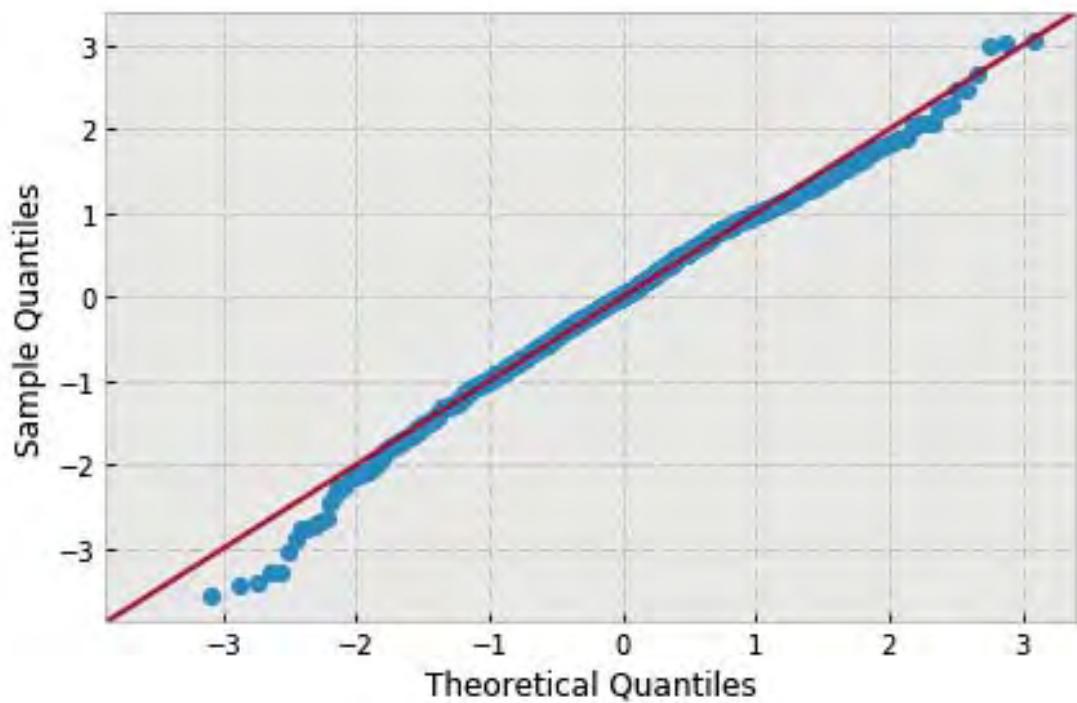


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=86.29576113872088, scale=8.522301980386318)
DescribeResult(nobs=1000, minmax=(60.947885935078268, 112.63299760682918)
               mean=86.295761138720877, variance=72.702333378274844,
               skewness=-0.25876063359601054, kurtosis=-0.12267861318030171)
```

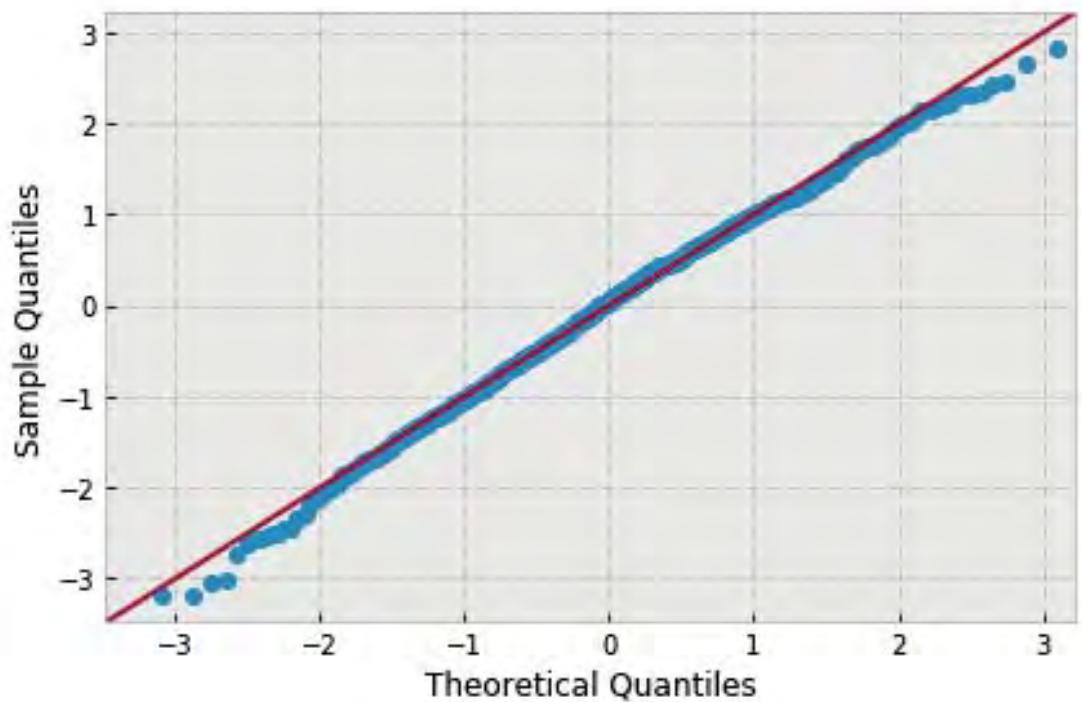
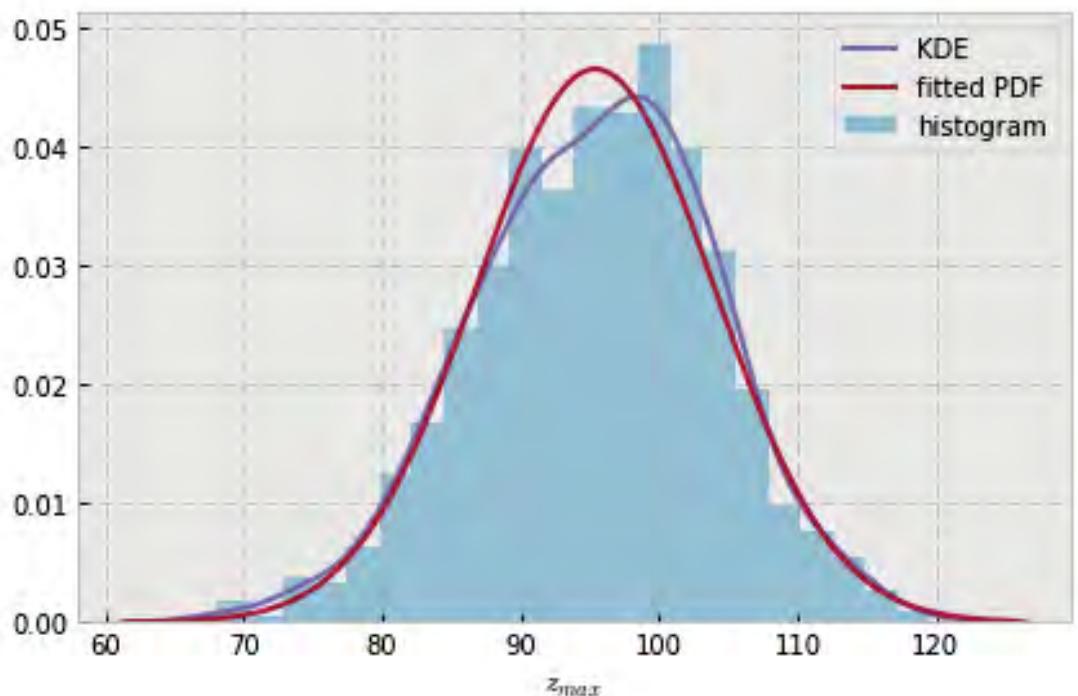


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=90.83499151541906, scale=8.710627833386317)
DescribeResult(nobs=1000, minmax=(59.93836635455235, 117.47312310192567)
               mean=90.834991515419063, variance=75.950988240004421,
               skewness=-0.2930929132723749, kurtosis=0.31830796321898935)
```

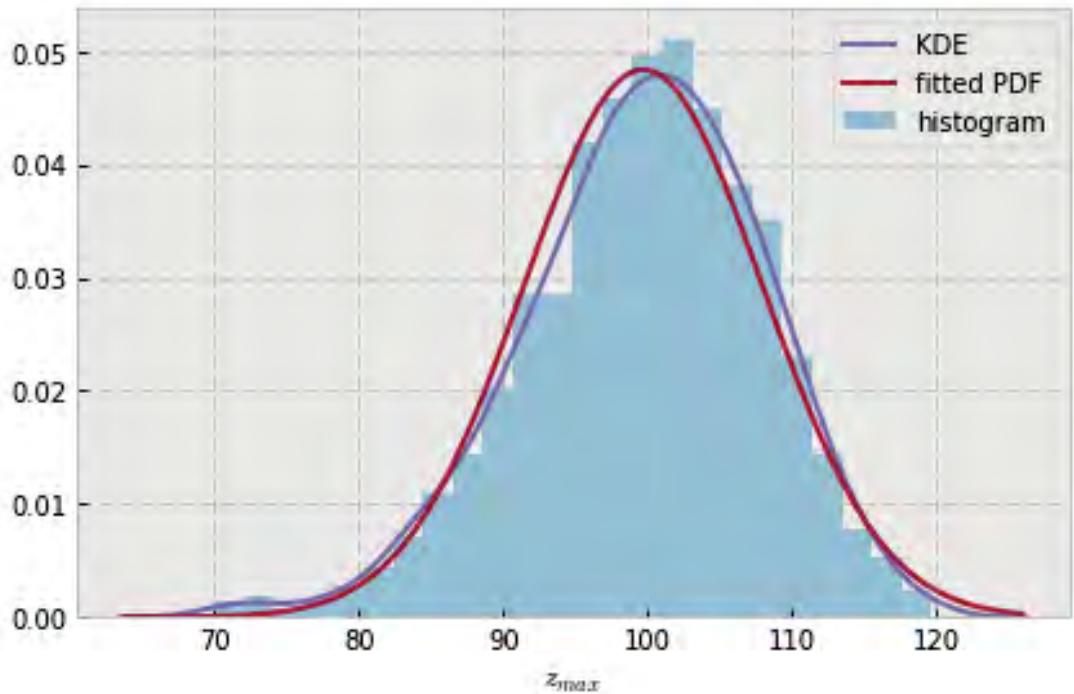


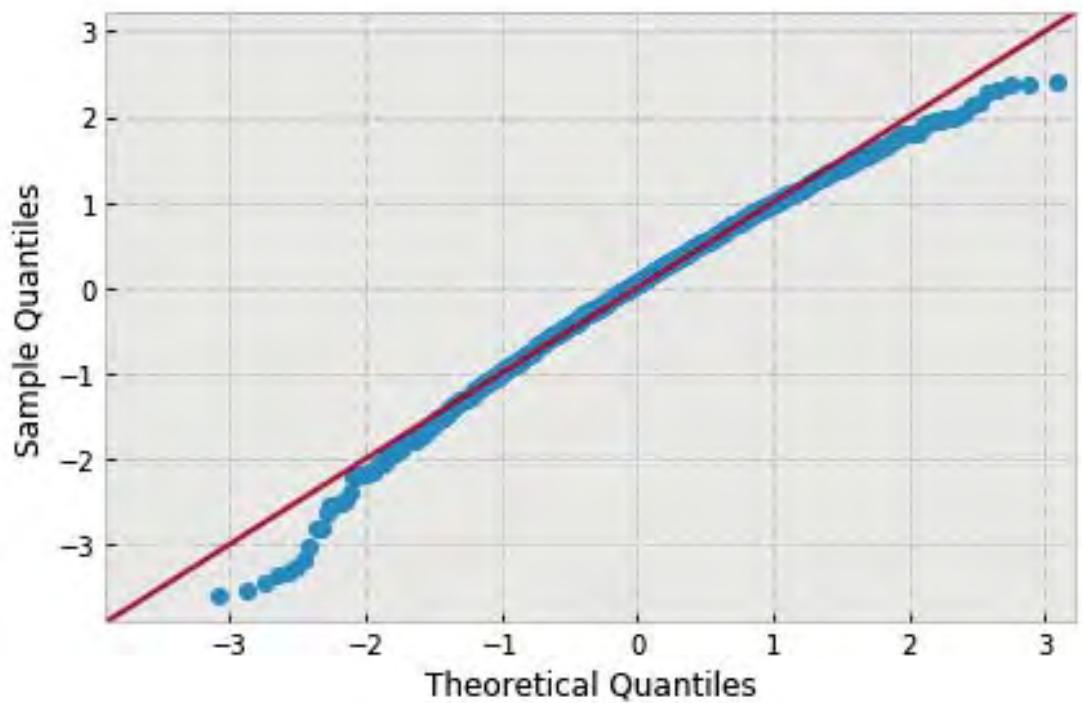


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=95.38613846331187, scale=8.588411253169193)
DescribeResult(nobs=1000, minmax=(68.080635877401491, 119.56377372342365)
               mean=95.386138463311866, variance=73.834642496059303,
               skewness=-0.19066909042504146, kurtosis=-0.07786581080024346)
```

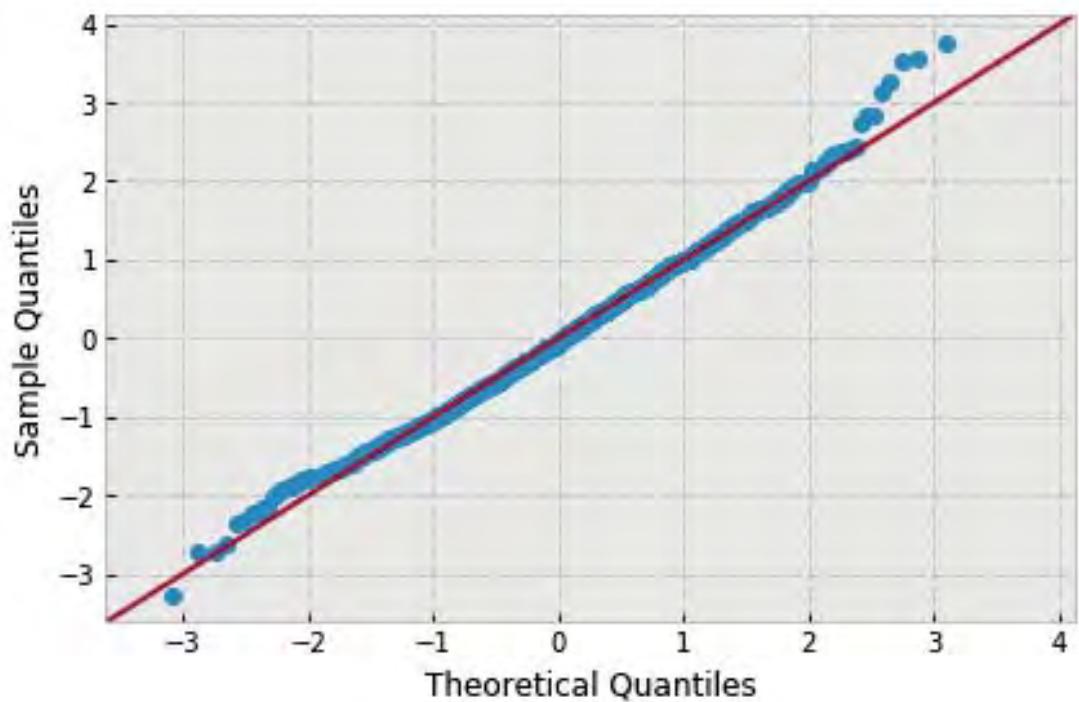
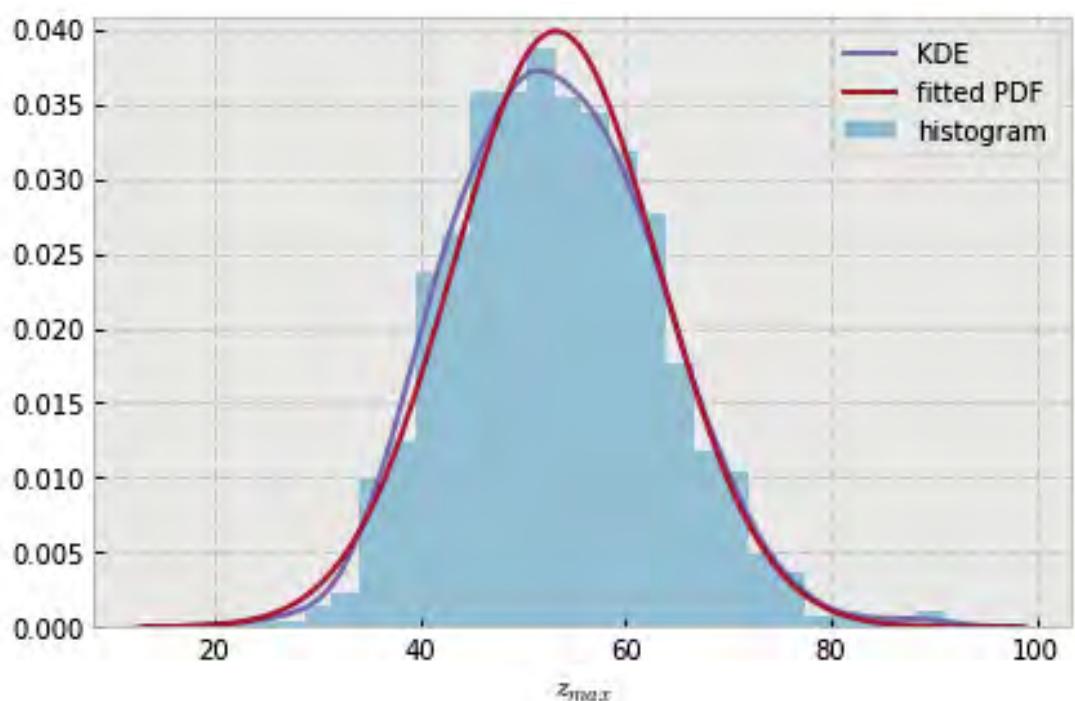


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=99.6946059357553, scale=8.247815757622336)
DescribeResult(nobs=1000, minmax=(69.940457934179065, 119.67036036898186)
               mean=99.694605935755305, variance=68.094559331014324,
               skewness=-0.43501632855243383, kurtosis=0.3101412779718955)
```

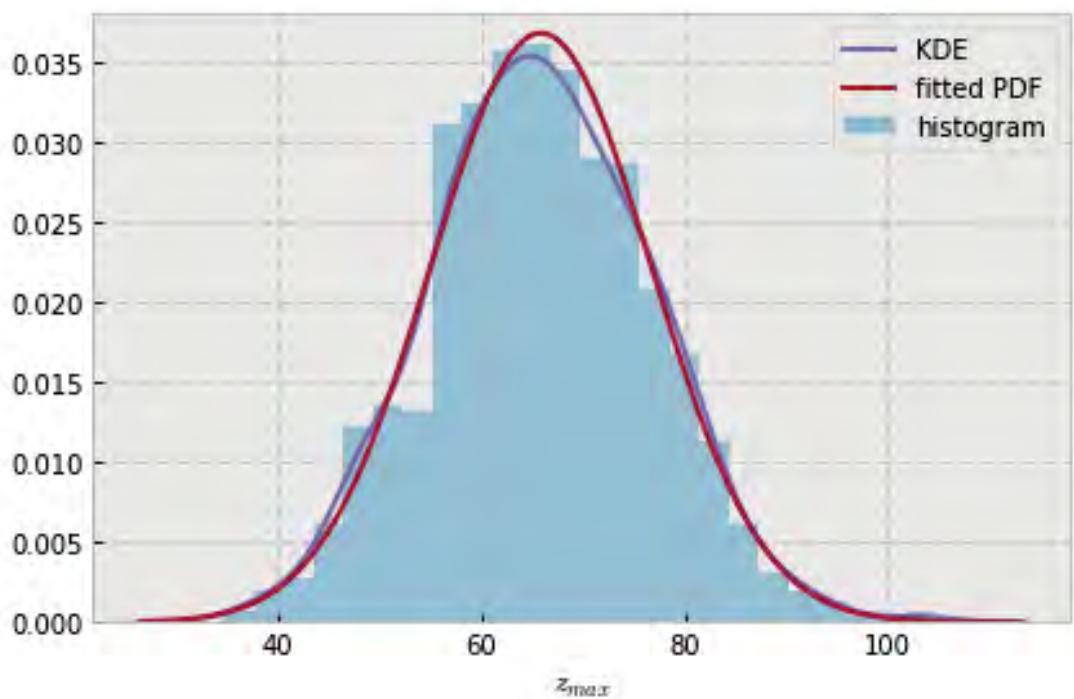


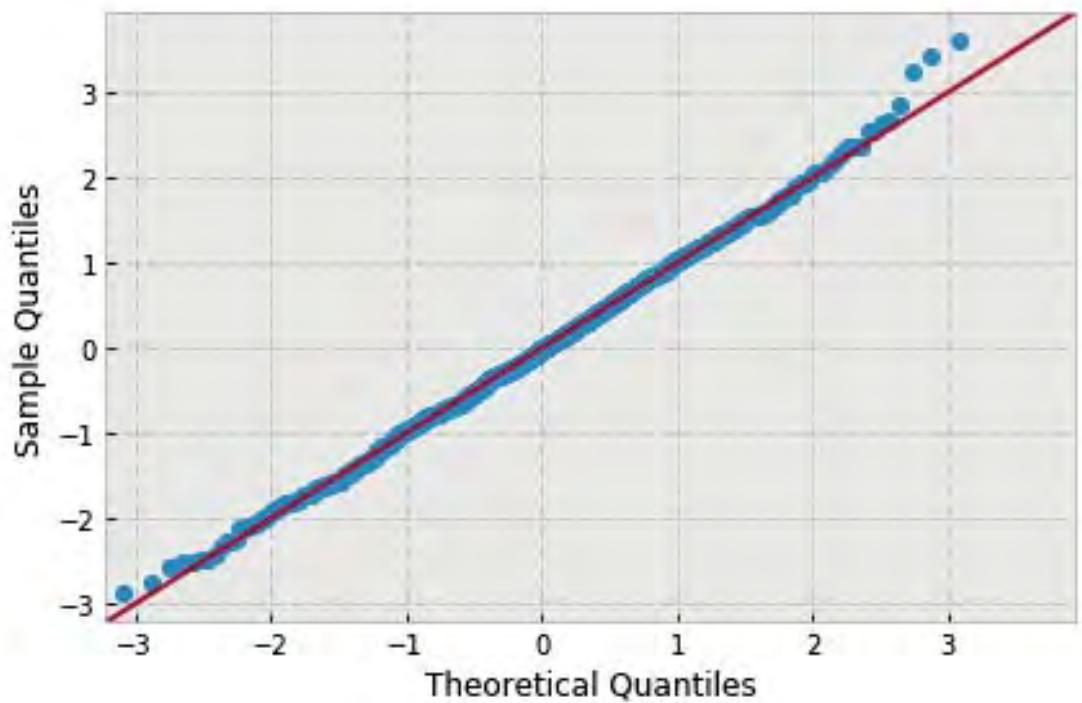


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=53.25820727057305, scale=10.003563733286029)
DescribeResult(nobs=1000, minmax=(20.58574701345233, 90.947315232995948)
               mean=53.258207270573052, variance=100.17145882474026,
               skewness=0.2523089023038684, kurtosis=0.1643538088733143)
```

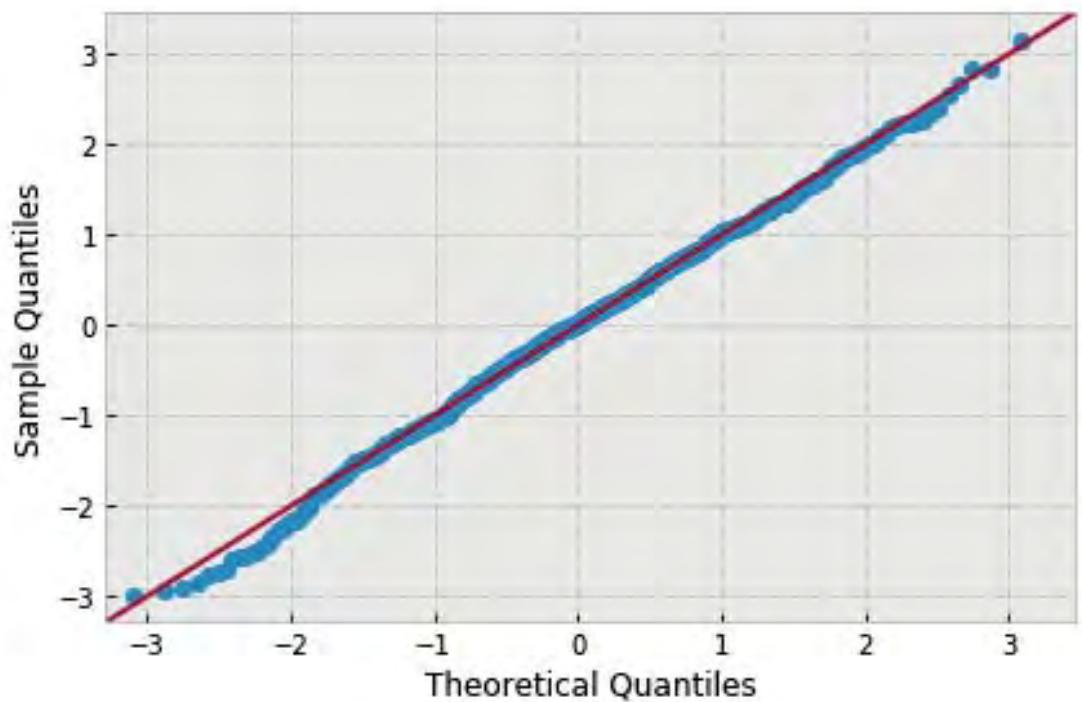
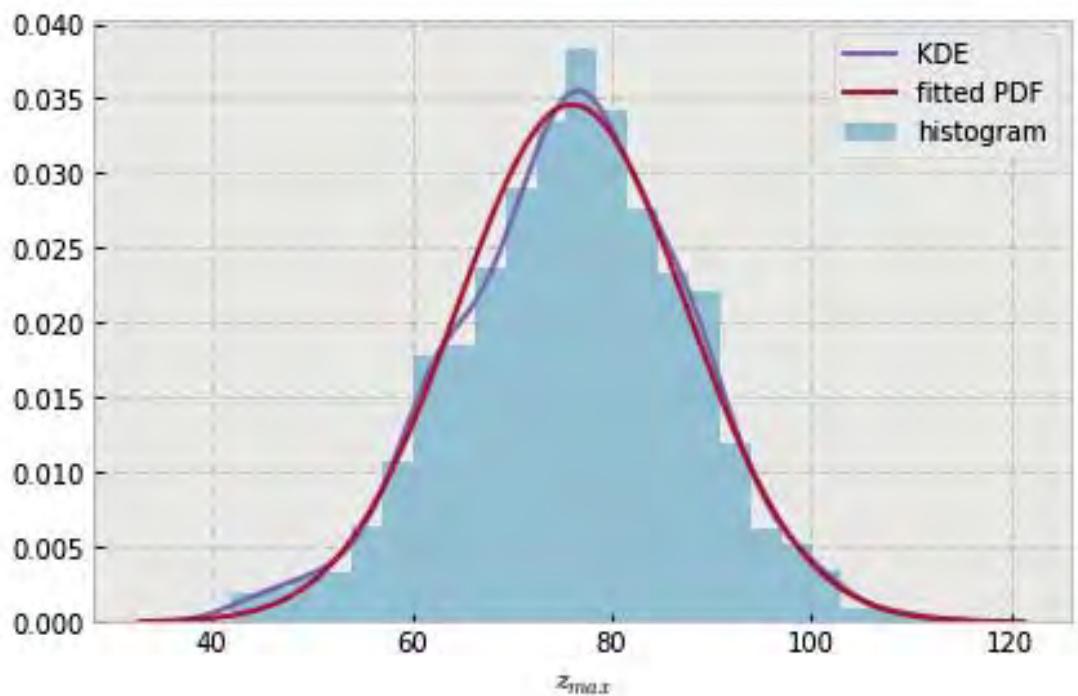


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=65.82811425988608, scale=10.813440799213247)
DescribeResult(nobs=1000, minmax=(34.685781715317383, 104.8351075017038)
               mean=65.828114259886078, variance=117.04754946755716,
               skewness=0.09670946461300595, kurtosis=0.00501826228983937)
```

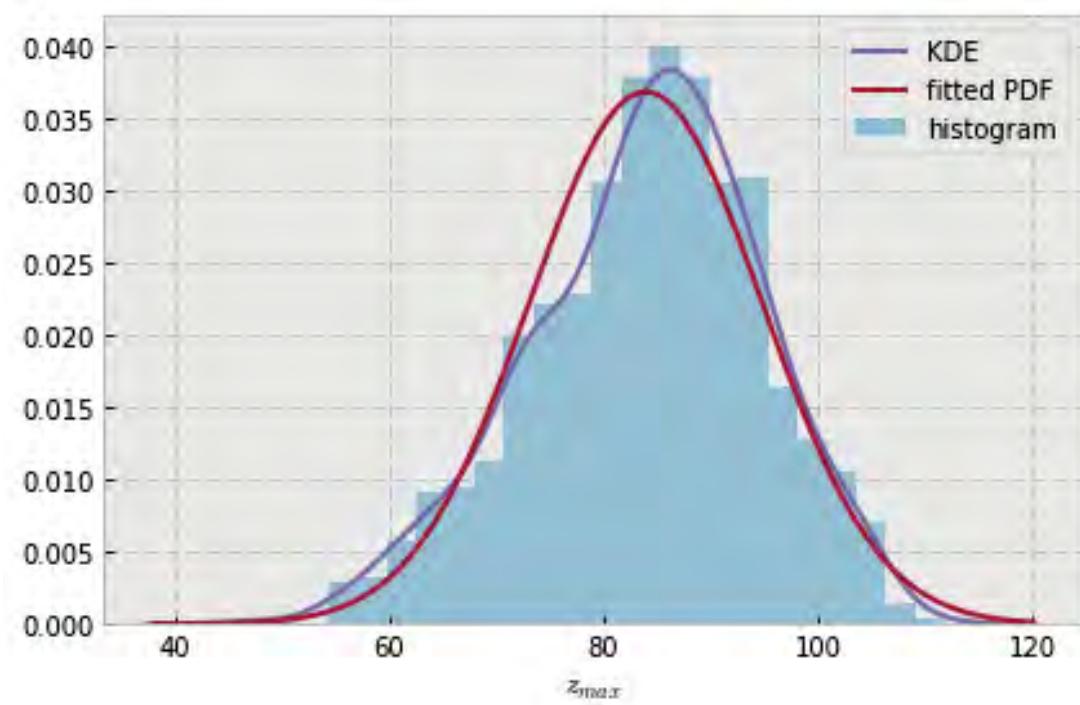


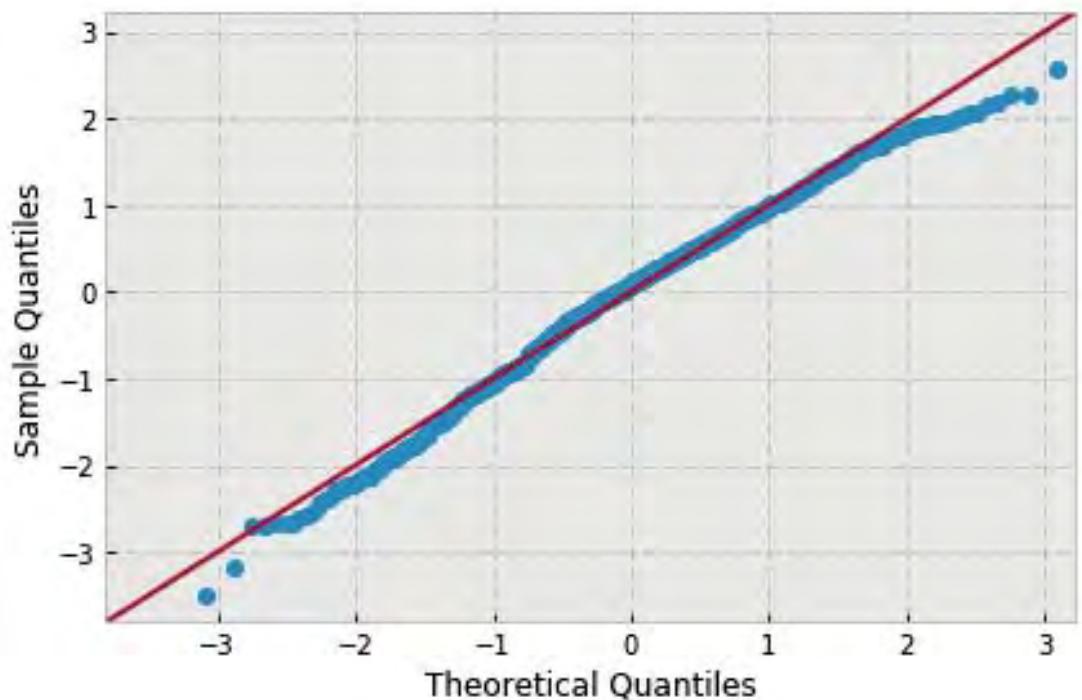


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(), loc=76.05183429297685, scale=11.530845153545284)
DescribeResult(nobs=1000, minmax=(41.691474355196426, 112.37481818395885)
               mean=76.051834292976849, variance=133.09348343847725,
               skewness=-0.14147139367396772, kurtosis=0.05053770451815787)
```

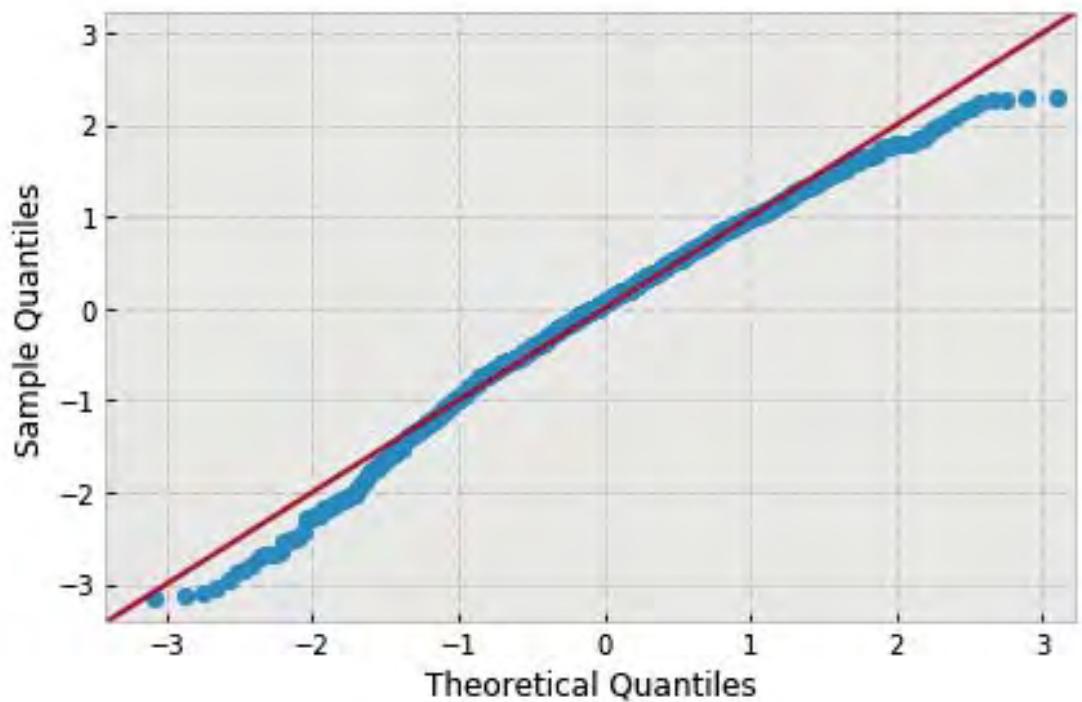
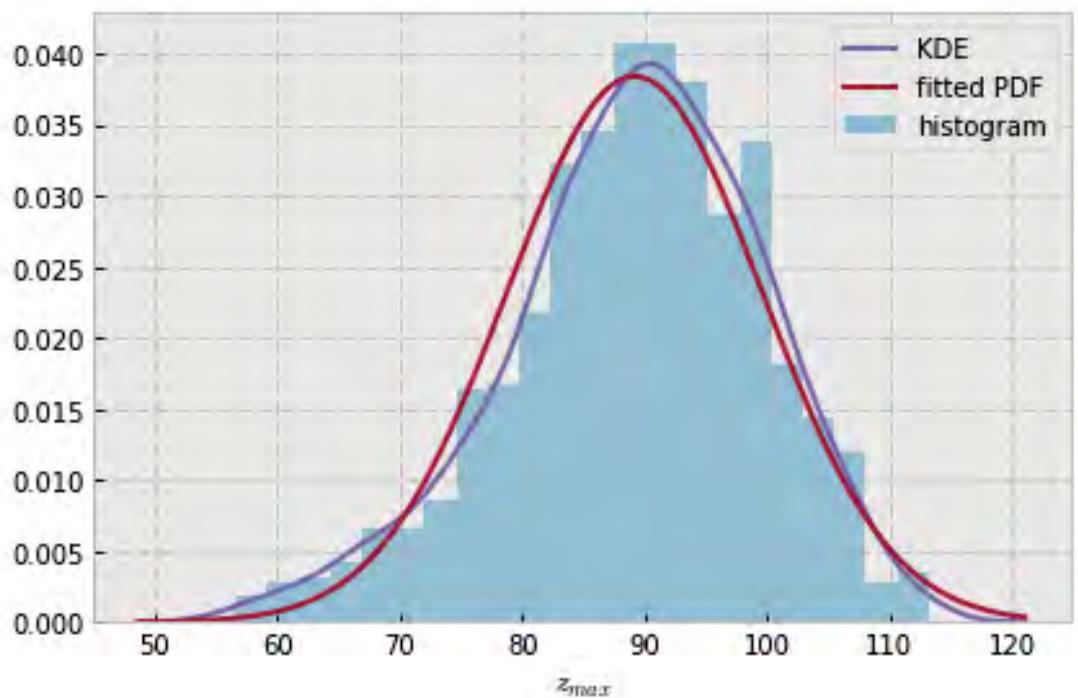


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=83.94823062944394, scale=10.83682393913526)
DescribeResult(nobs=1000, minmax=(45.962462236411952, 111.87485720735332)
               mean=83.948230629443941, variance=117.55430739521023,
               skewness=-0.35003014377849867, kurtosis=-0.09268331770222638)
```

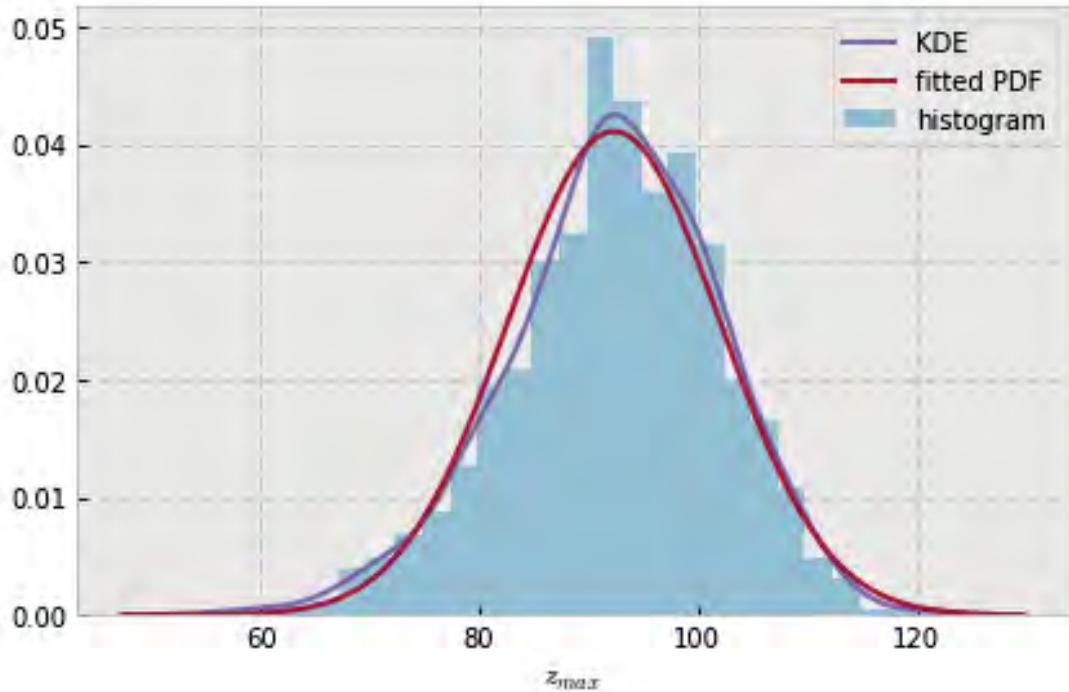


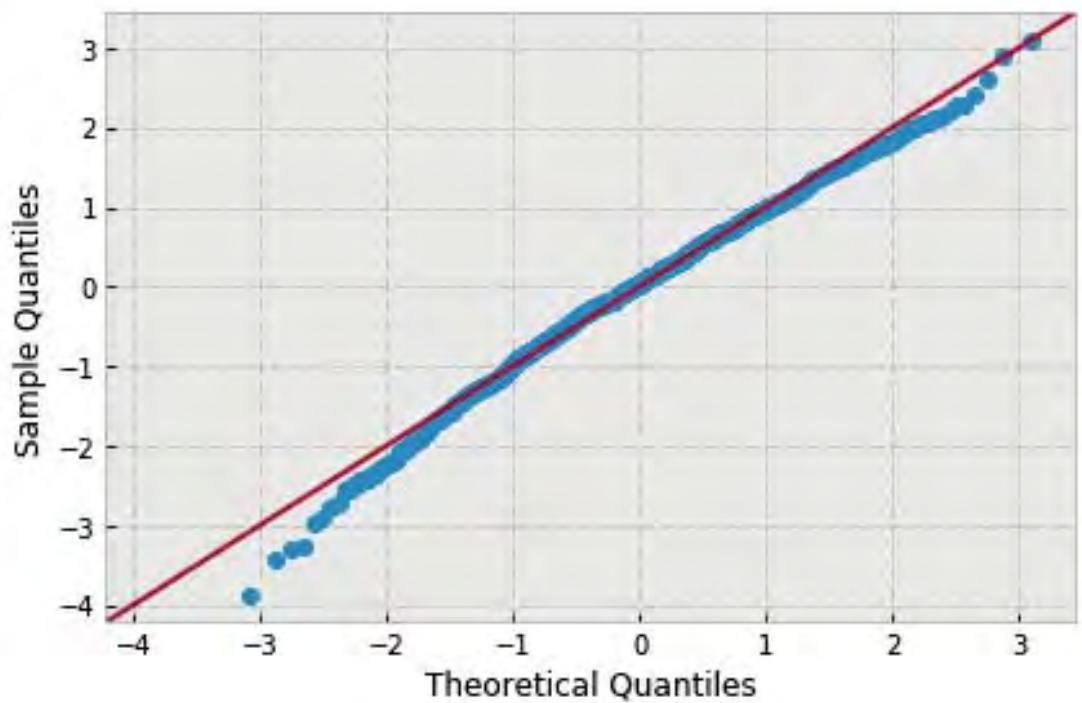


```
variable name: sigma
variable value: 0.0555555555555555
distribution: normal(shape=(), loc=89.18925484603722, scale=10.386577110070284)
DescribeResult(nobs=1000, minmax=(56.592457361096471, 113.1781357597005)
               mean=89.189254846037215, variance=107.98897303647244,
               skewness=-0.4338974288274305, kurtosis=0.12149538359127465)
```

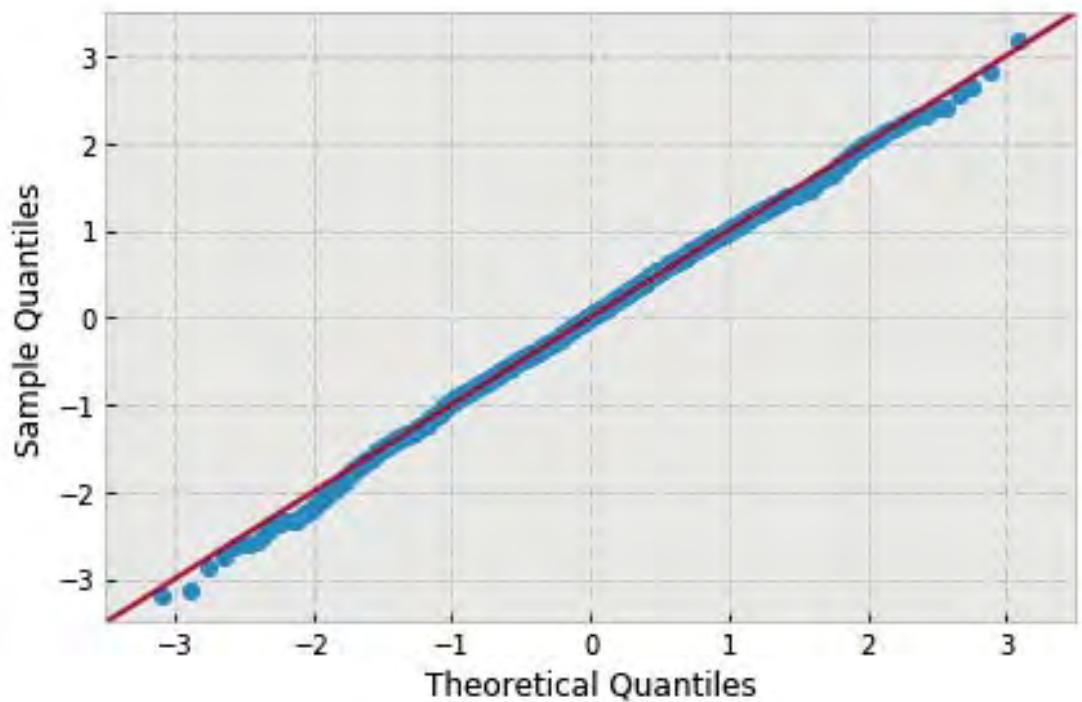
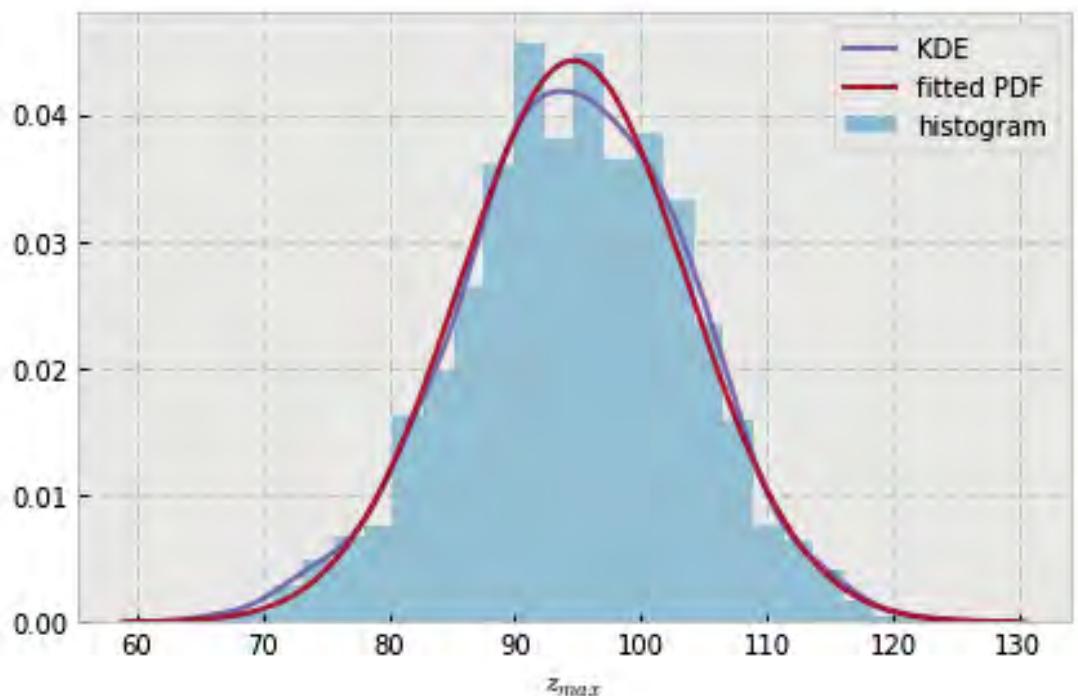


```
variable name: sigma
variable value: 0.066666666666666667
distribution: normal(shape=(), loc=92.3363376953257, scale=9.70902925389129)
DescribeResult(nobs=1000, minmax=(54.791630793994074, 122.31737433228407)
    mean=92.336337695325696, variance=94.359608661578434,
    skewness=-0.3412048573989796, kurtosis=0.32437900150778054)
```

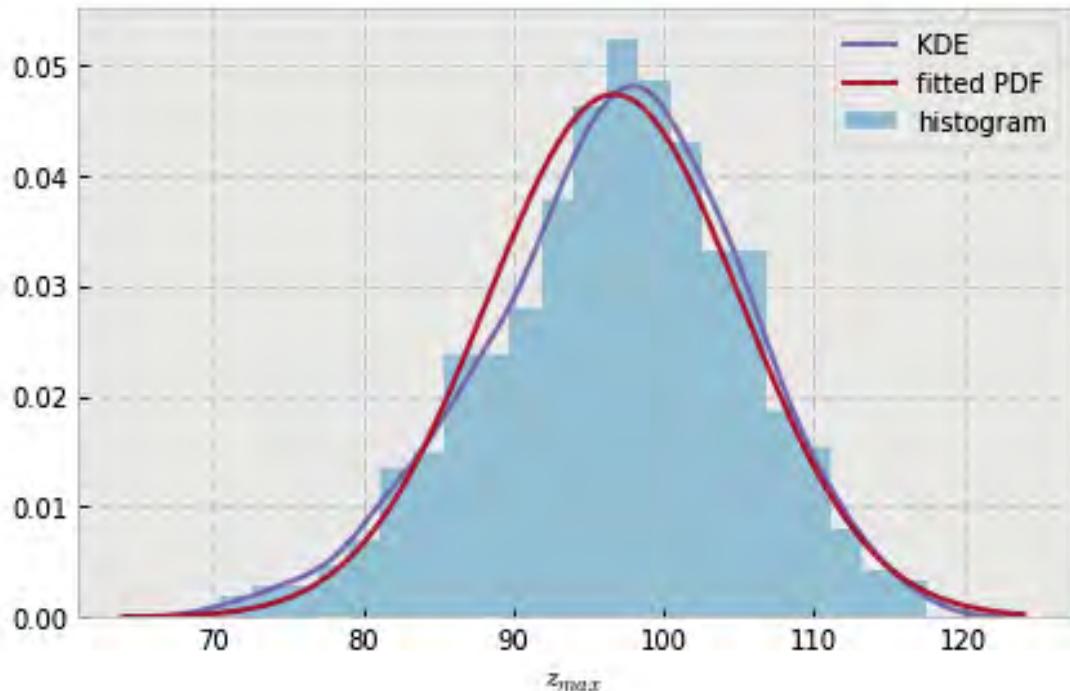


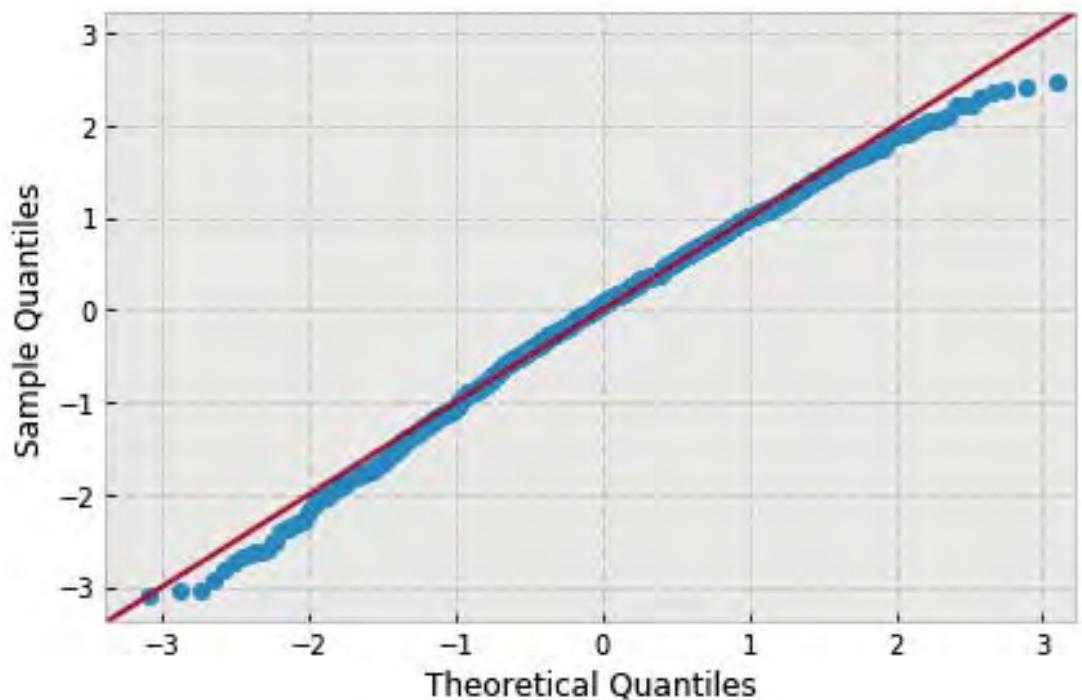


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=94.68581574791783, scale=9.018698823092159)
DescribeResult(nobs=1000, minmax=(66.045526114269066, 123.30089761104692)
               mean=94.685815747917829, variance=81.418346808452355,
               skewness=-0.1199383142900634, kurtosis=-0.014342960396485527)
```

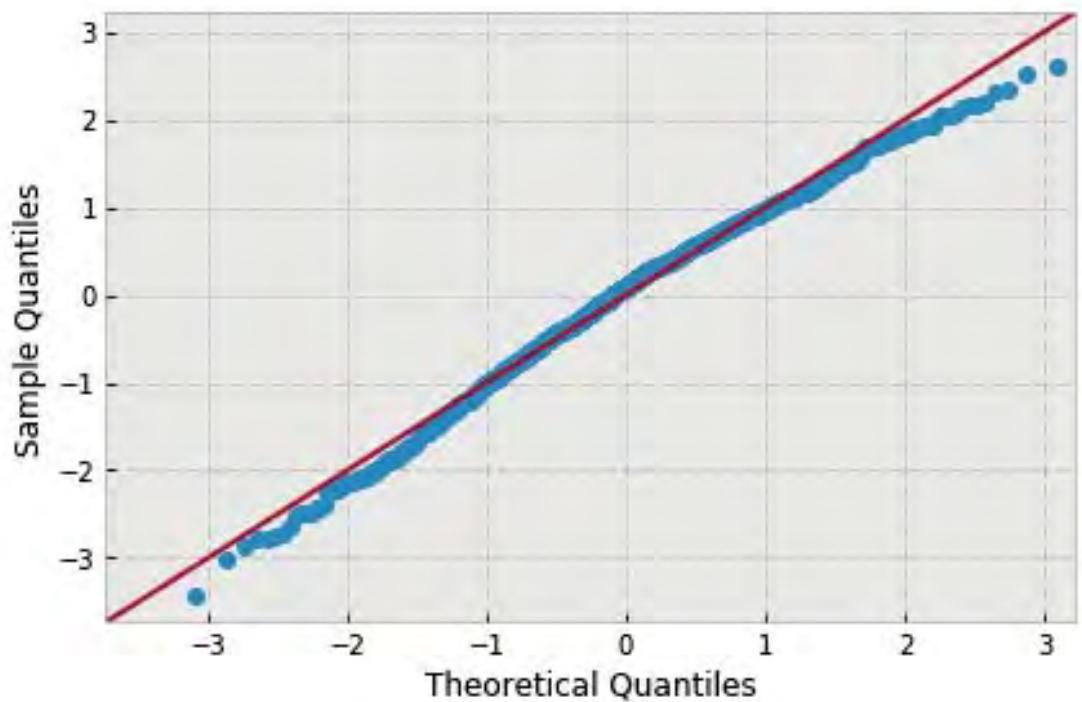
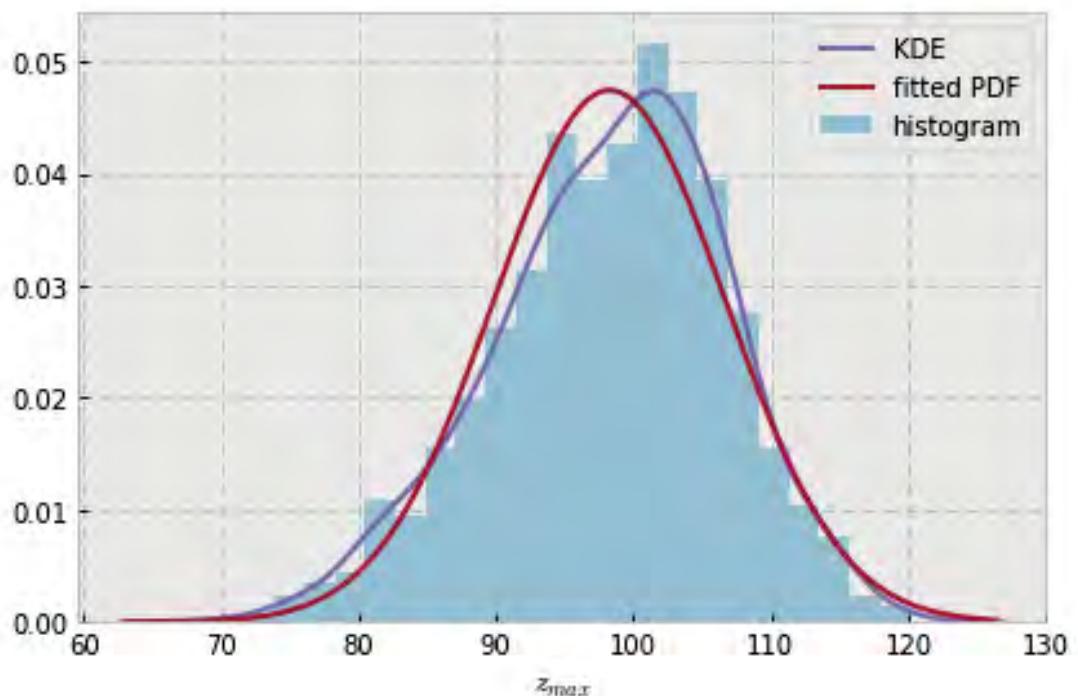


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=96.68031540470055, scale=8.438940073106293)
DescribeResult(nobs=1000, minmax=(70.462503882156398, 117.51155212176558)
               mean=96.680315404700551, variance=71.28699655403328,
               skewness=-0.30462660357898436, kurtosis=-0.05508815040866111)
```

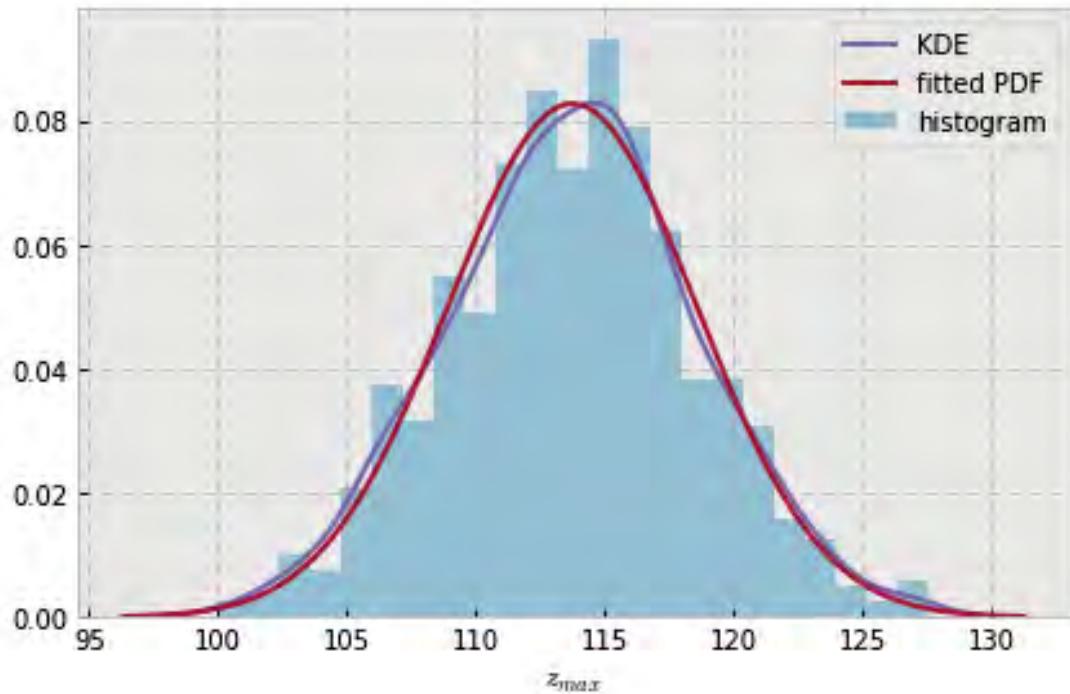


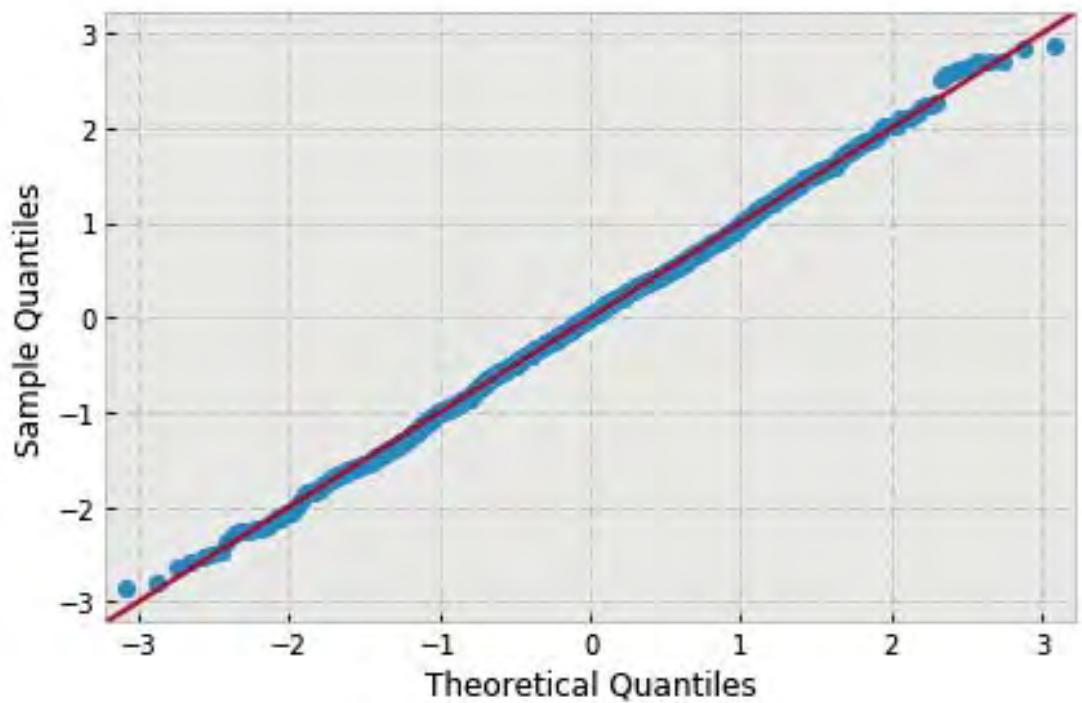


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=98.27628994146184, scale=8.408310282713316)
DescribeResult(nobs=1000, minmax=(69.434516026966577, 120.11294504698445)
               mean=98.27628994146184, variance=70.770452262645136,
               skewness=-0.34643730176474963, kurtosis=-0.0813633532822573)
```

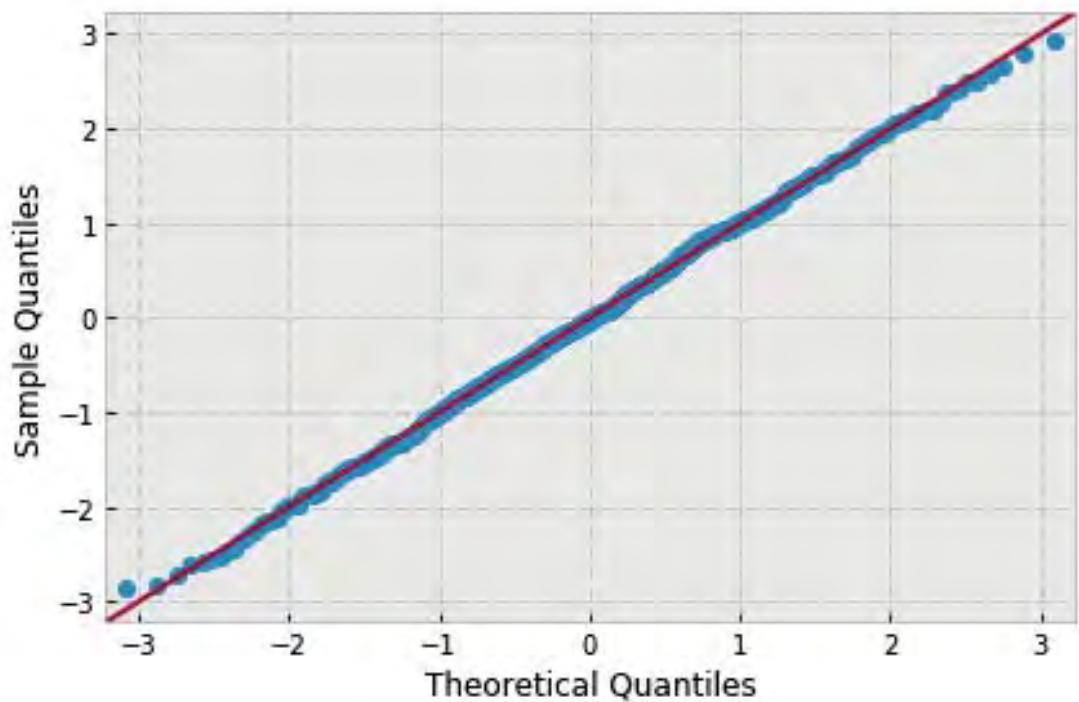
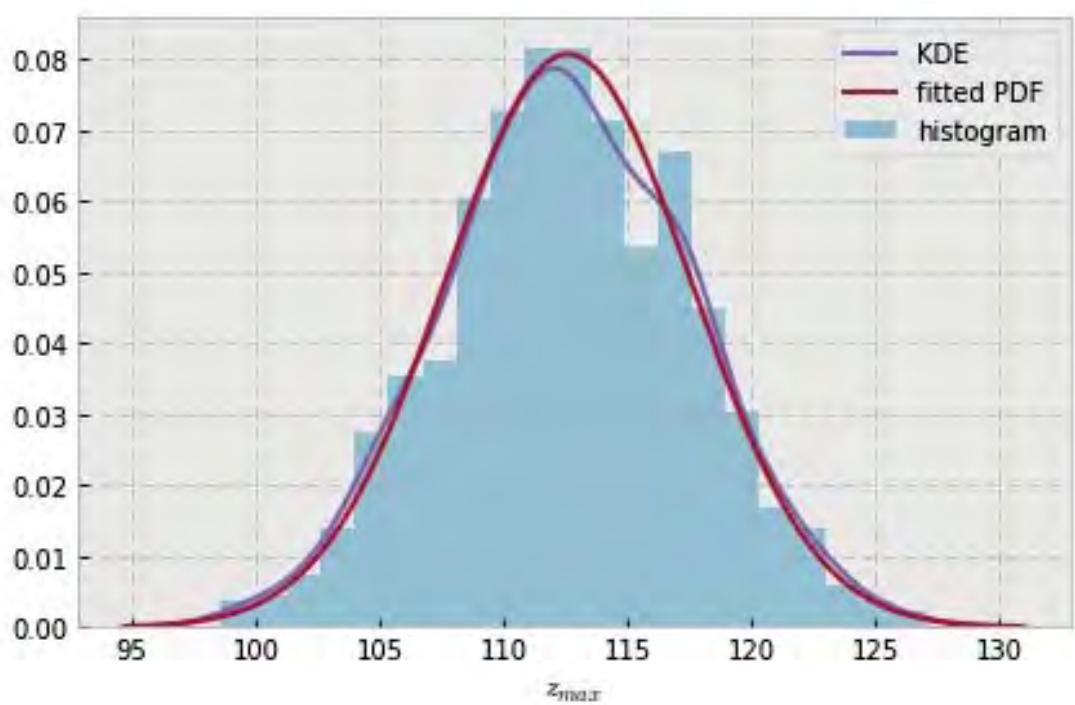


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=113.75613846789926, scale=4.824611361253967)
DescribeResult(nobs=1000, minmax=(99.997773847745933, 127.61737017492753)
               mean=113.75613846789926, variance=23.300174962102957,
               skewness=0.012851131180975597, kurtosis=-0.10413580982941983)
```

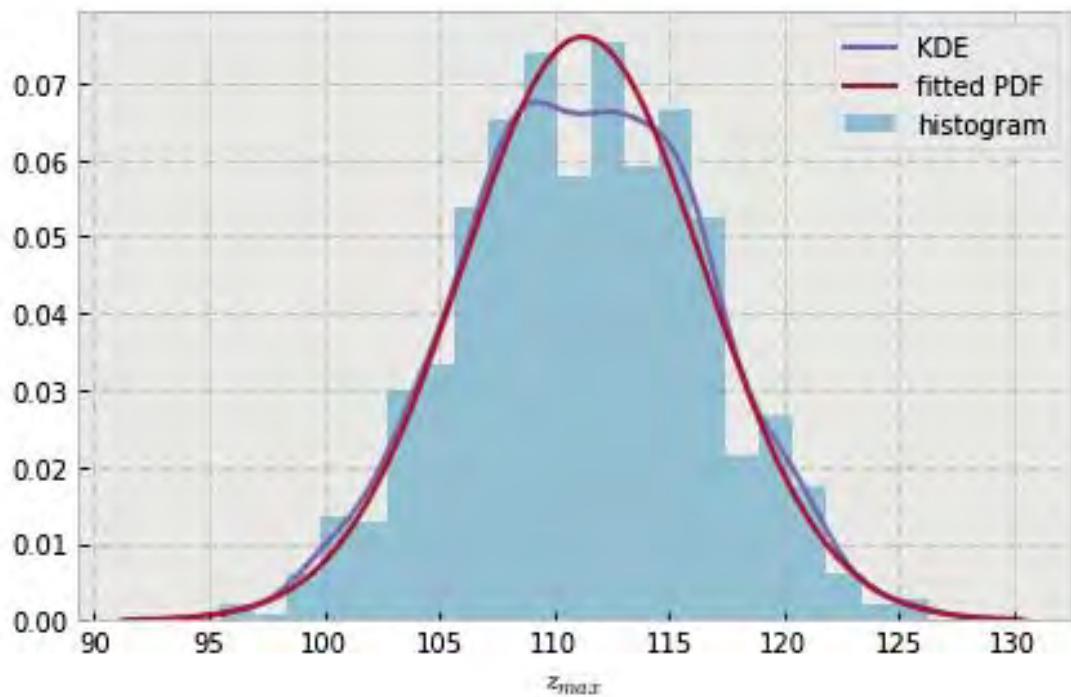


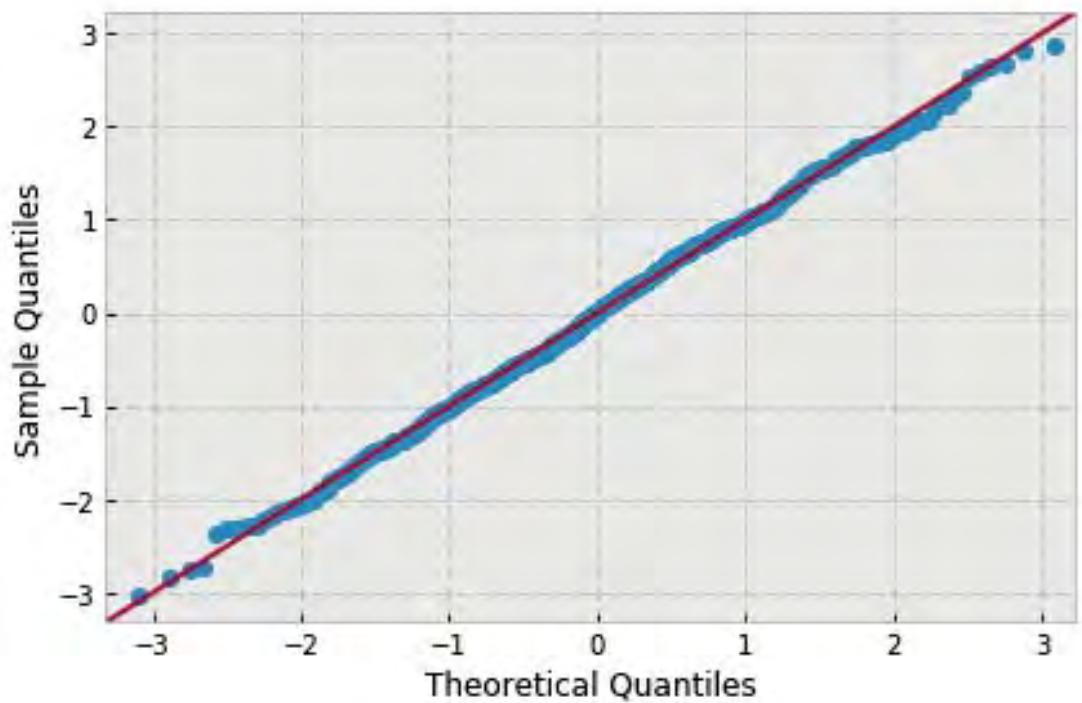


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=112.61517499968441, scale=4.943762098225543)
DescribeResult(nobs=1000, minmax=(98.571356626562363, 127.10558824425472)
               mean=112.61517499968441, variance=24.465248932784203,
               skewness=-0.016192117636339053, kurtosis=-0.1978695286948824)
```

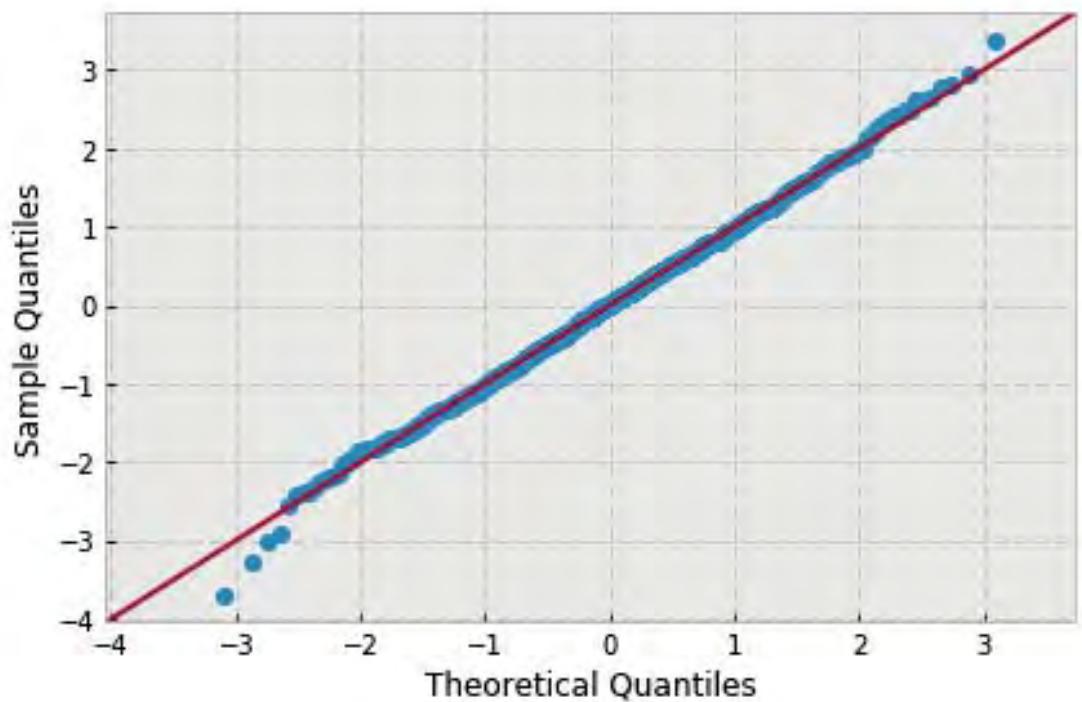
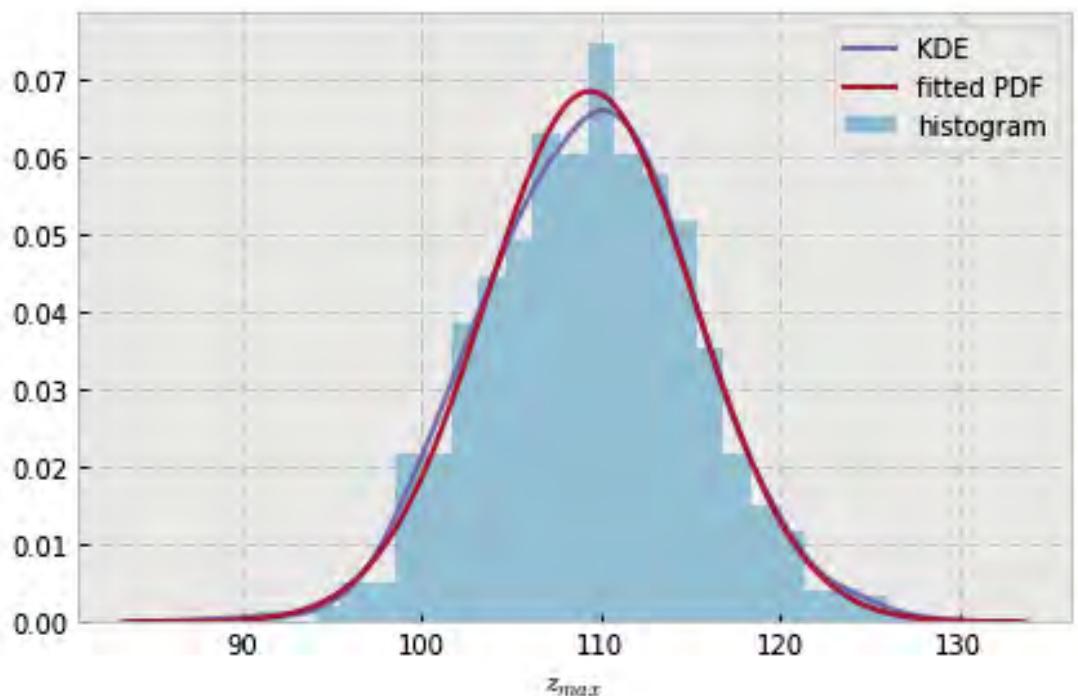


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=111.26135547213534, scale=5.248360006096953)
DescribeResult(nobs=1000, minmax=(95.410623431763284, 126.2935477884815)
               mean=111.26135547213534, variance=27.57285560920722,
               skewness=-0.031022284909504344, kurtosis=-0.2754189295893874)
```

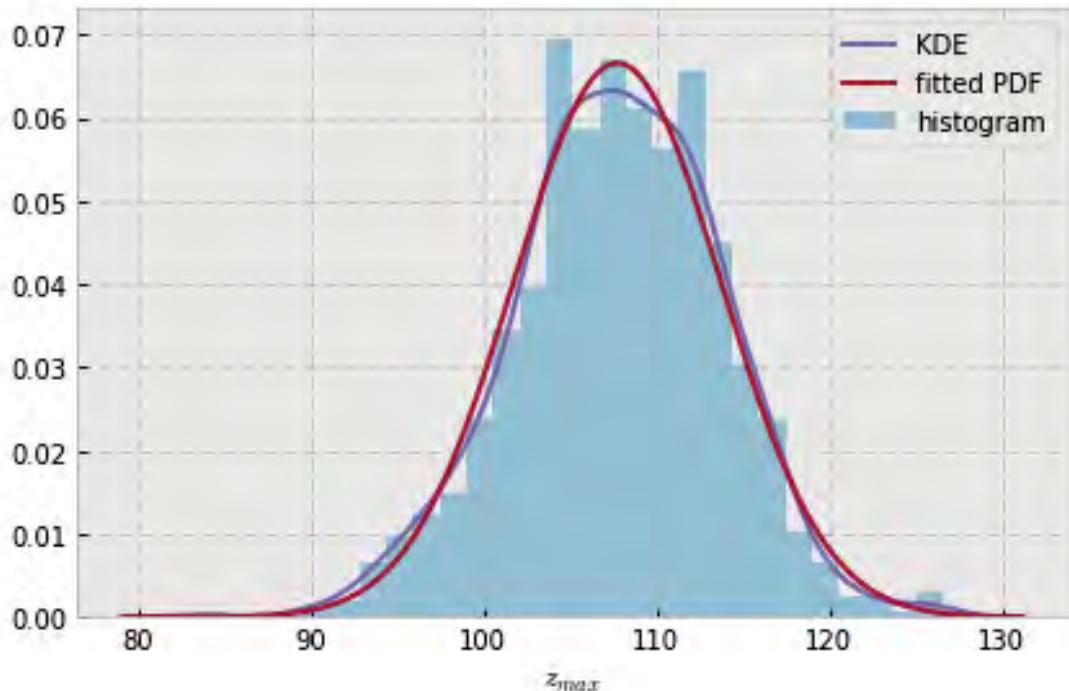


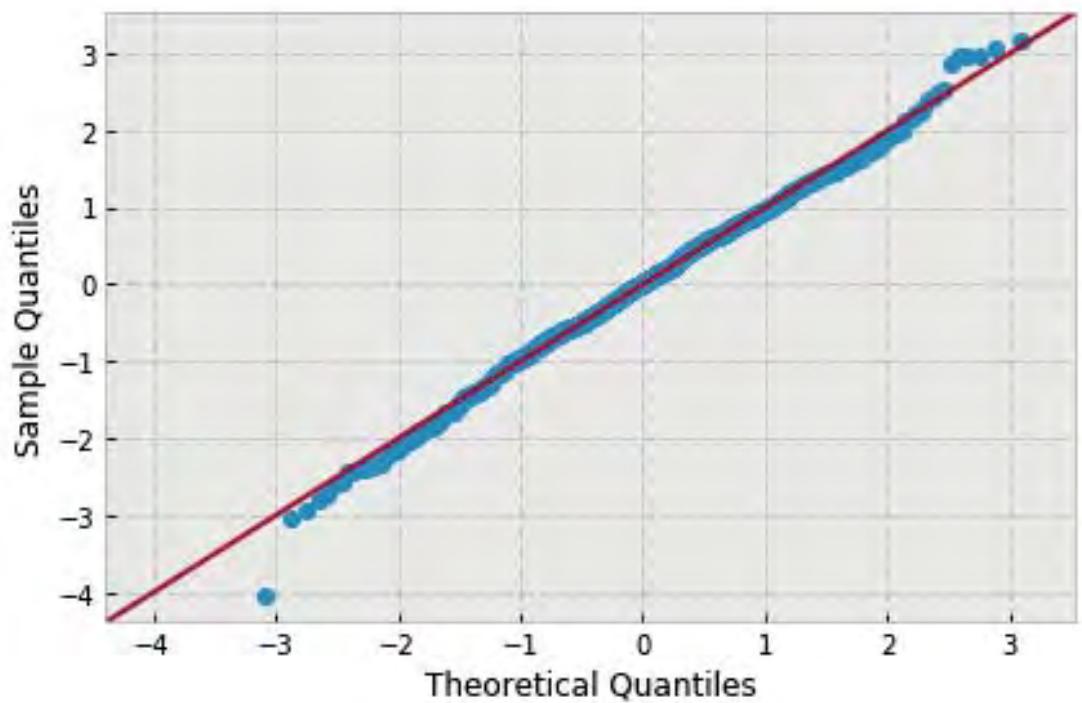


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=109.41224444222787, scale=5.82222664925973)
DescribeResult(nobs=1000, minmax=(87.936901192111392, 129.05011937586397)
               mean=109.41224444222787, variance=33.932208968946647,
               skewness=0.014930706934920558, kurtosis=0.08112124748901639)
```

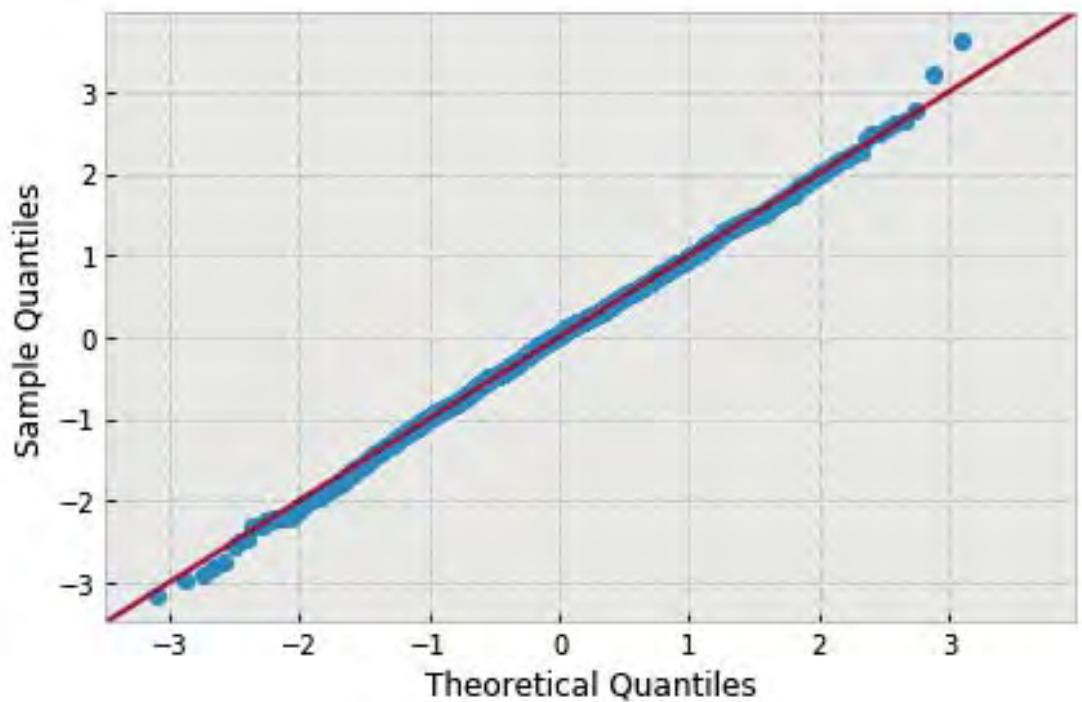
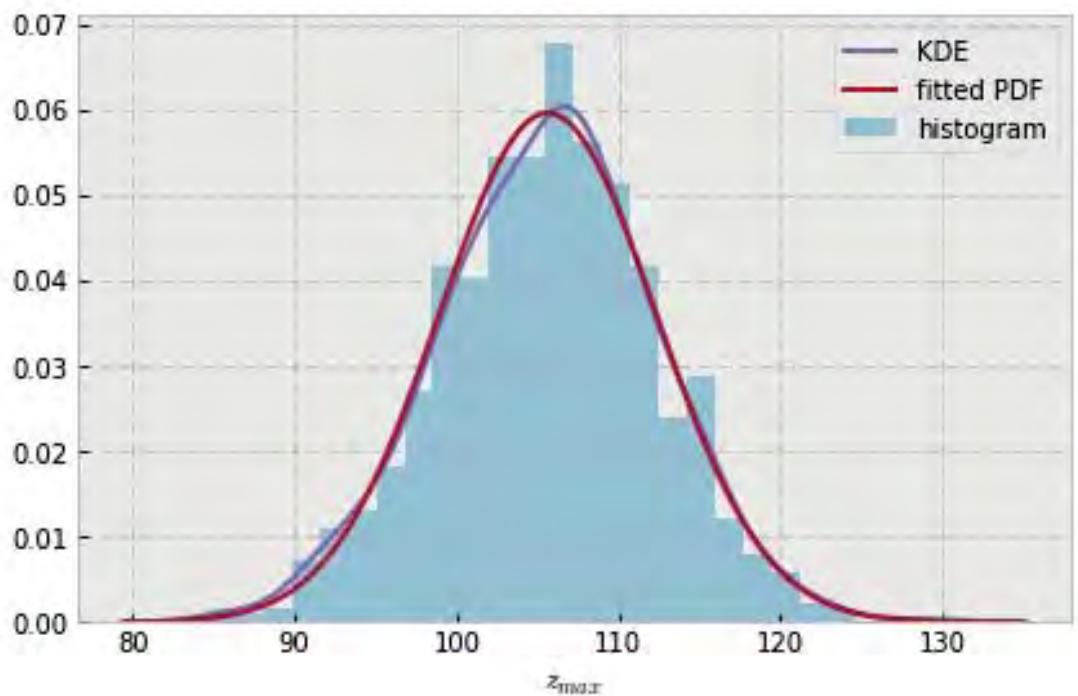


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=107.73206779251451, scale=5.9923332276339645)
DescribeResult(nobs=1000, minmax=(83.619660779368004, 126.68198941577324)
               mean=107.73206779251451, variance=35.944001512518604,
               skewness=-0.13077213553878544, kurtosis=0.2561801900542191)
```

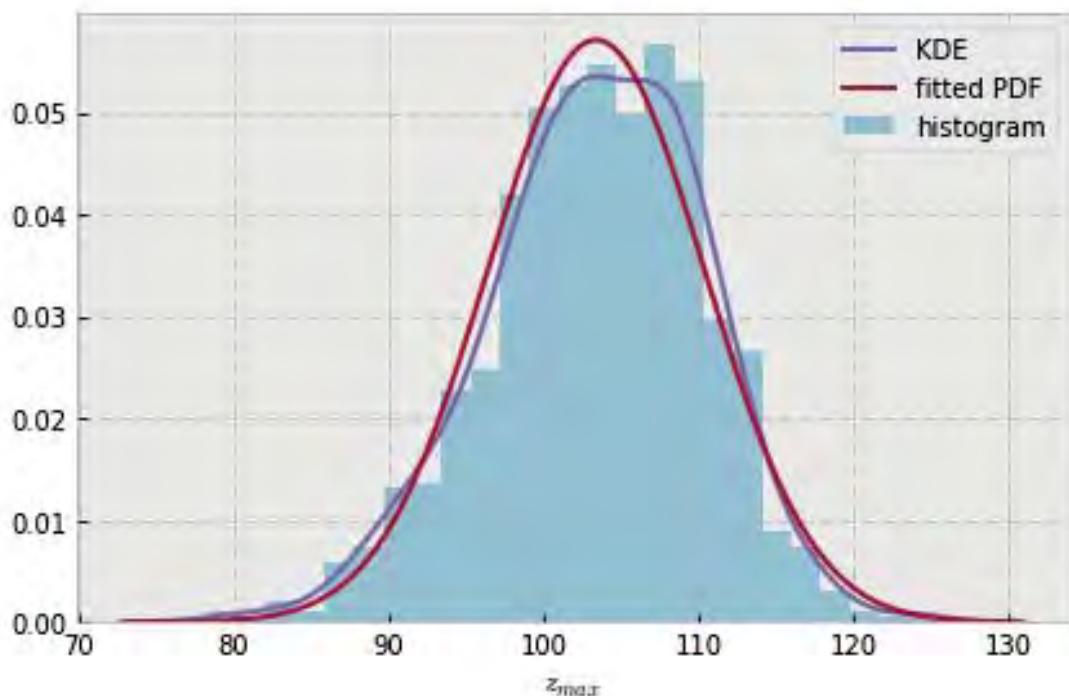


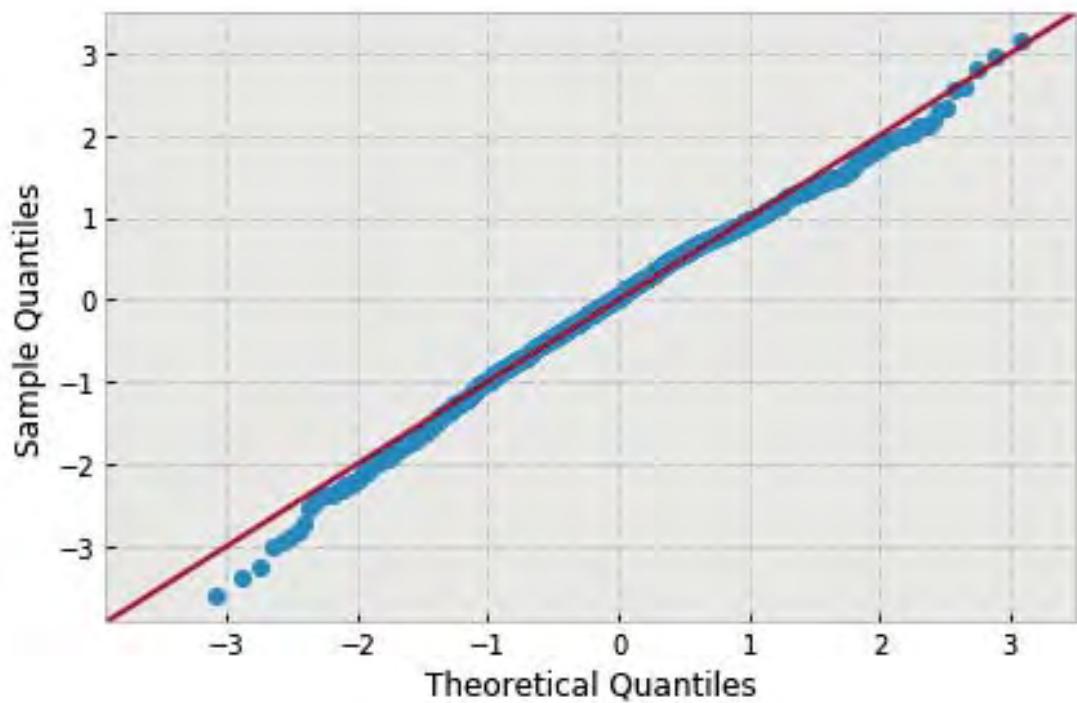


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=105.62295535944867, scale=6.694946121232342)
DescribeResult(nobs=1000, minmax=(84.543148955944005, 129.89500674636938)
               mean=105.62295535944867, variance=44.867170736940928,
               skewness=-0.05152944294715397, kurtosis=0.09977455145523484)
```

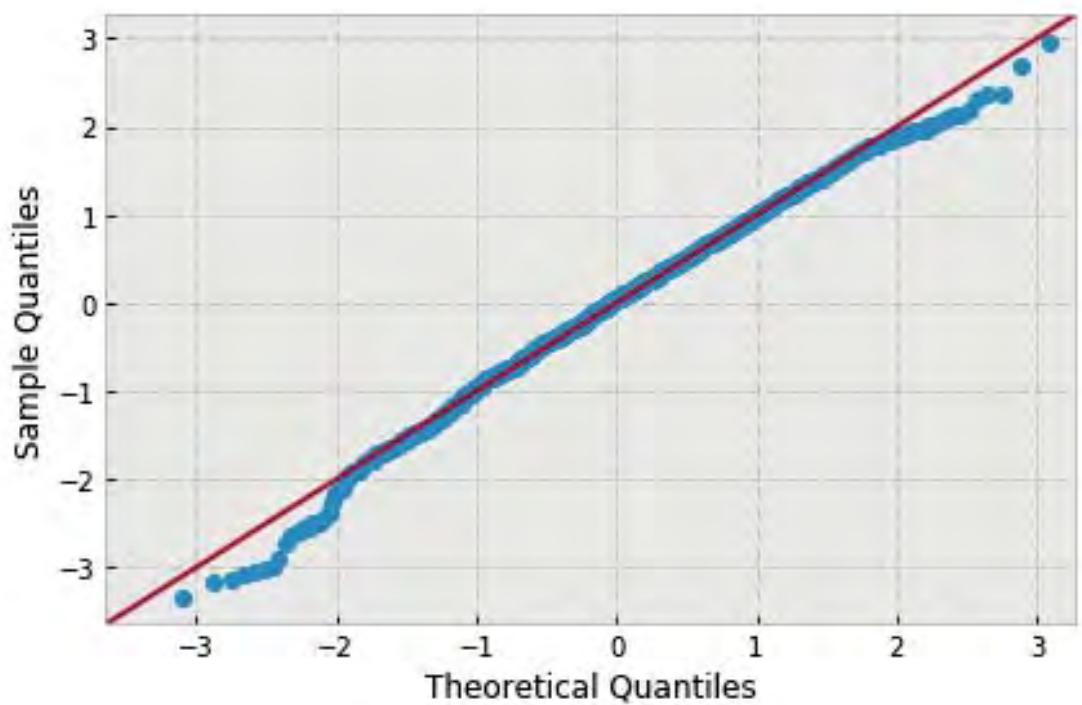
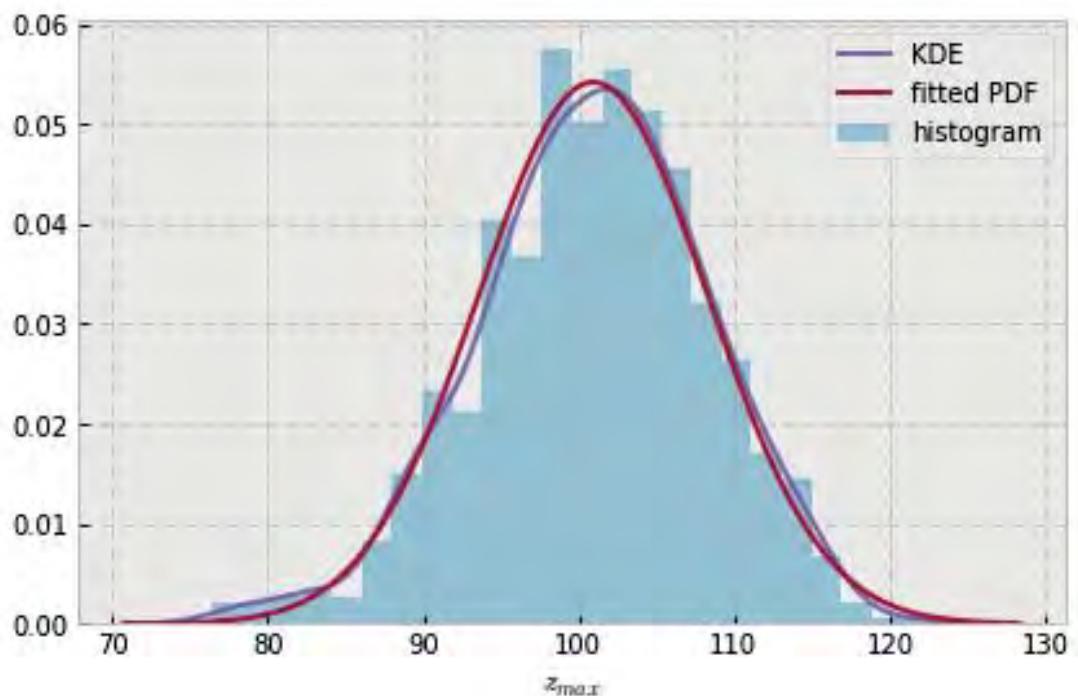


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=103.38930087823523, scale=6.985086050939321)
DescribeResult(nobs=1000, minmax=(78.32854403557964, 125.41789456451492)
               mean=103.38930087823523, variance=48.840267406433519,
               skewness=-0.2952671524217359, kurtosis=0.17177853296810808)
```

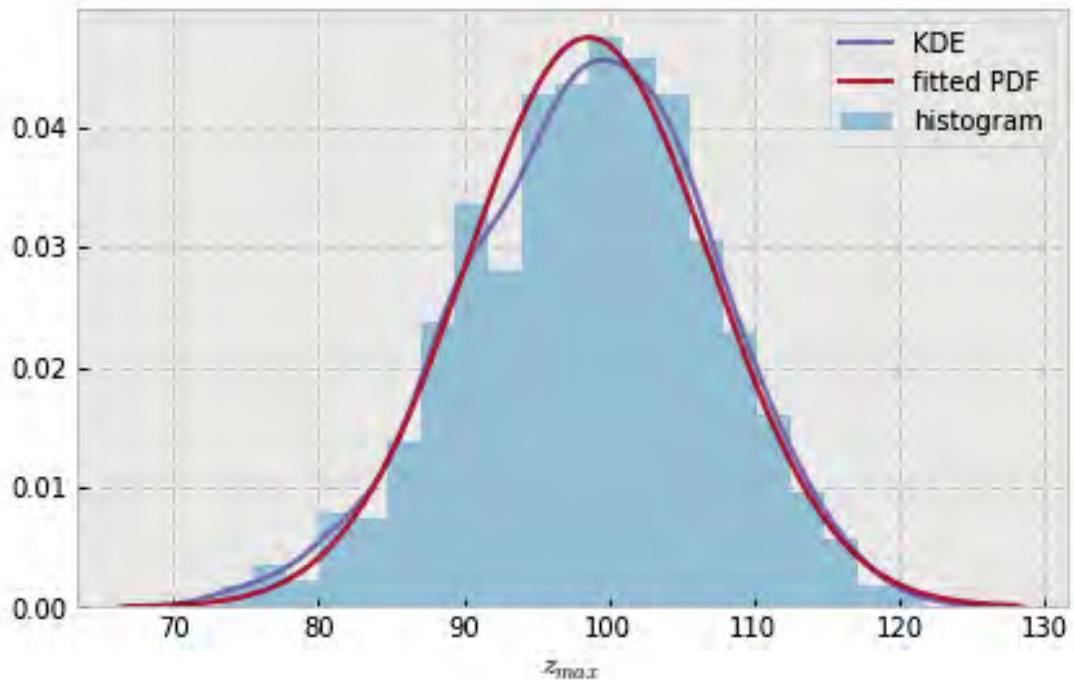


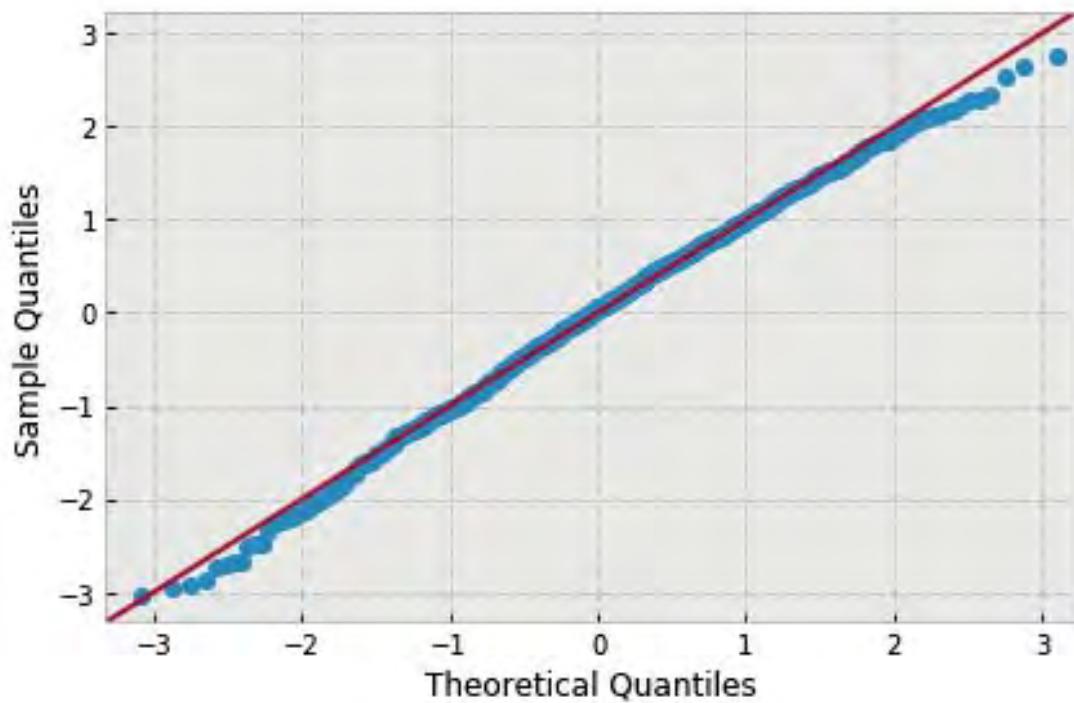


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=100.9254592090742, scale=7.364918299262674)
DescribeResult(nobs=1000, minmax=(76.419007032369265, 122.66190166680514)
               mean=100.92545920907421, variance=54.29631787268687,
               skewness=-0.26884274903822325, kurtosis=0.13654815060043912)
```

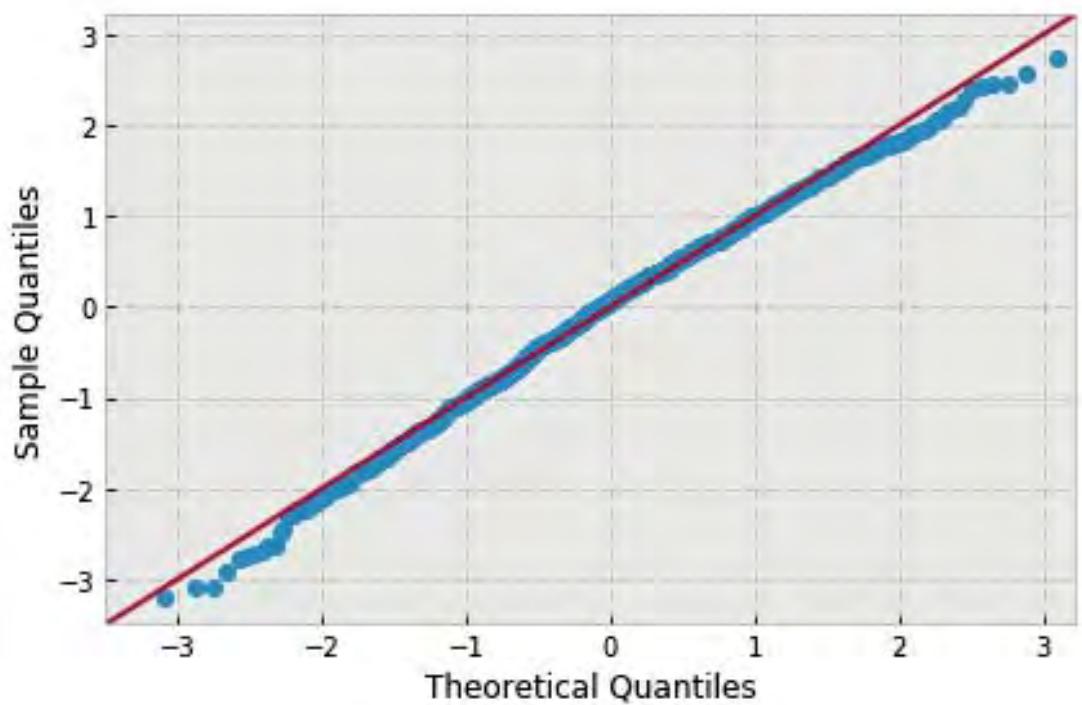
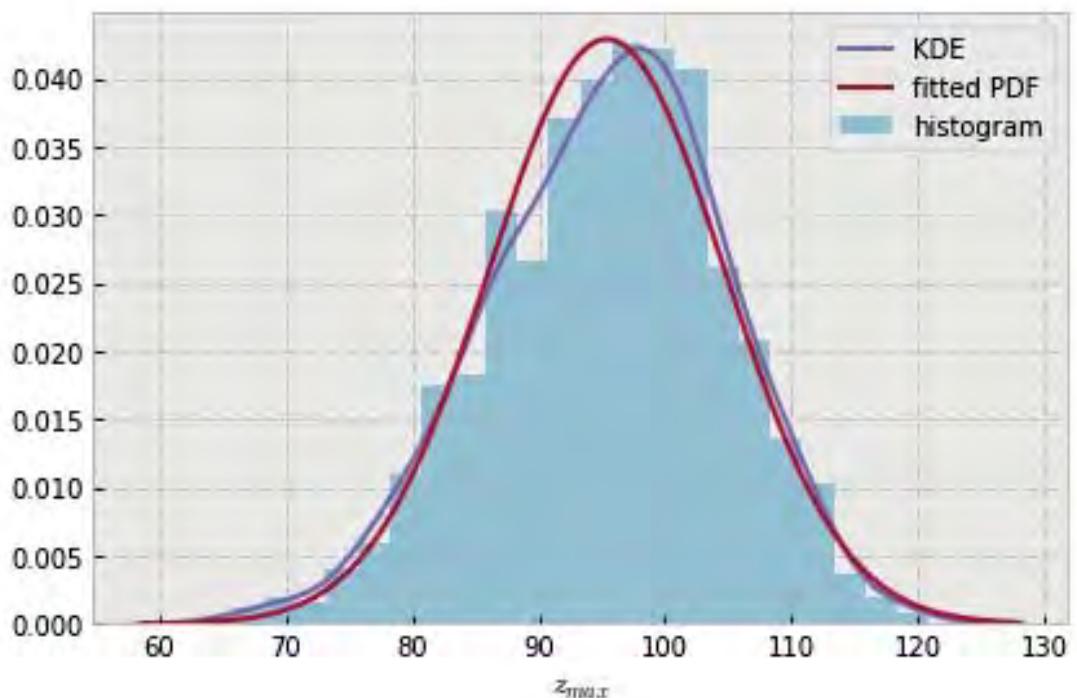


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=98.62311877353746, scale=8.40534983906537)
DescribeResult(nobs=1000, minmax=(73.145891239168904, 121.76698799378813)
               mean=98.623118773537456, variance=70.720626543619844,
               skewness=-0.19169823044996068, kurtosis=-0.14325041321901466)
```





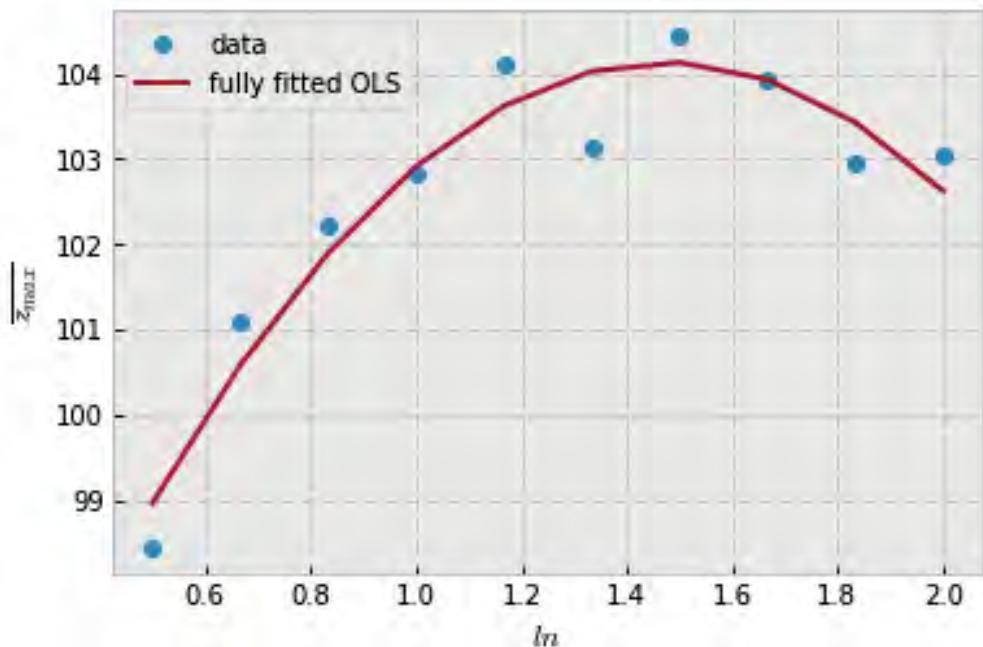
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=95.39502366609055, scale=9.299356995380673)
DescribeResult(nobs=1000, minmax=(65.693974642697455, 120.85966567410048)
               mean=95.395023666090552, variance=86.564605132668135,
               skewness=-0.2473434943618572, kurtosis=-0.10269905935028945)
```



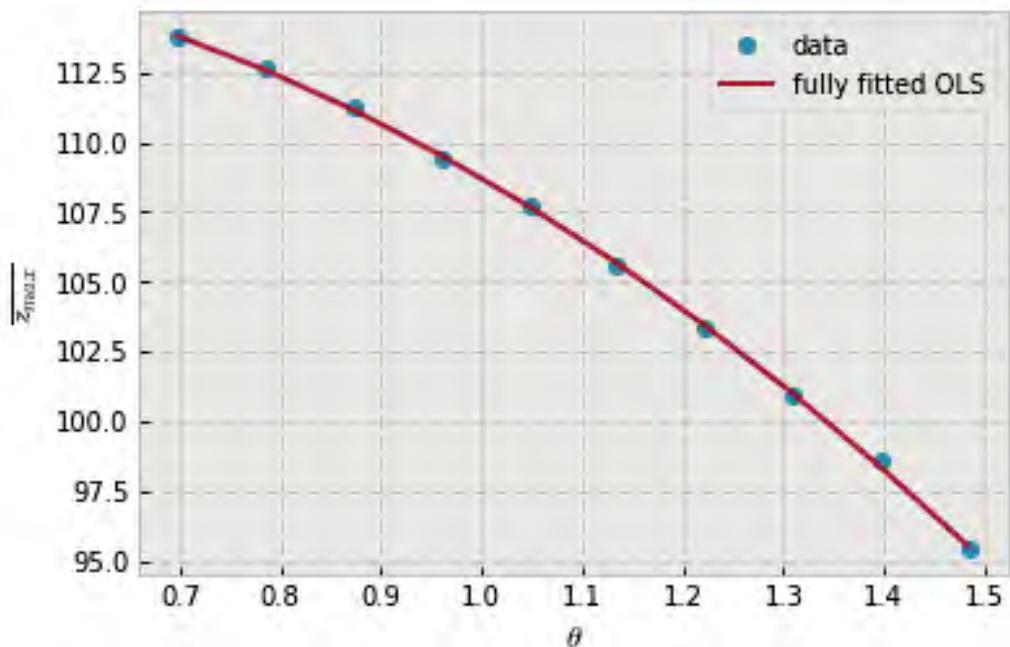
2.2 Parameter regression

```
In [10]: var_name = 'maximal_rooting_depth'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly2')  
        mean_reg('nB', var_name, 'poly2')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly2')  
        mean_reg('N', var_name, 'poly2')
```

```
variable name: ln  
model: poly2  
r-squared: 0.7753816325517864  
explained variance: 0.7754962177441531
```



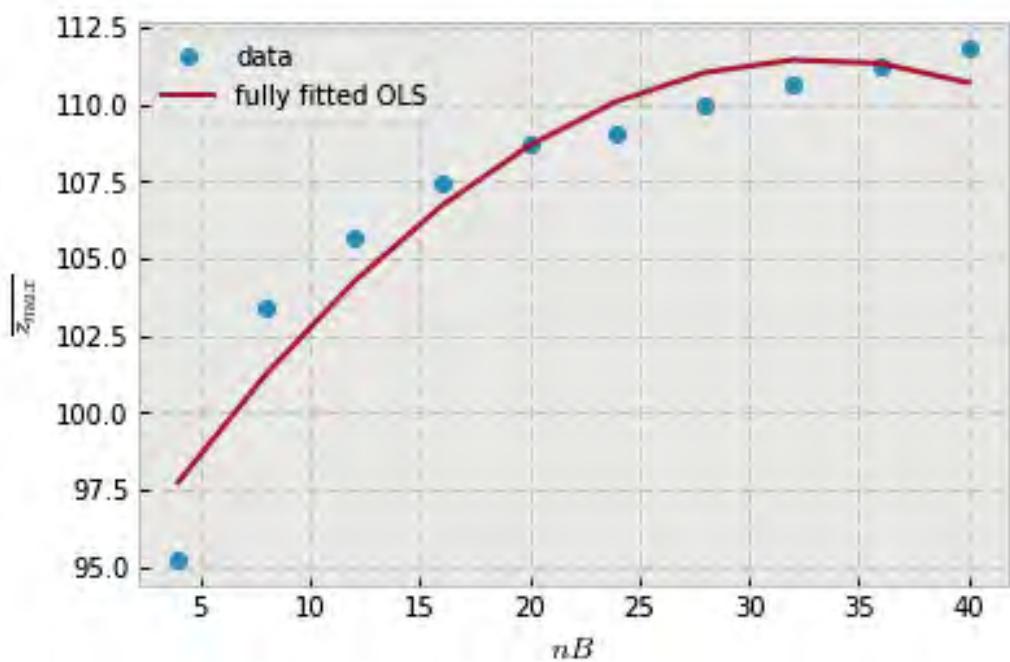
```
variable name: theta  
model: poly2  
r-squared: 0.9990843892116165  
explained variance: 0.9990957781741907
```



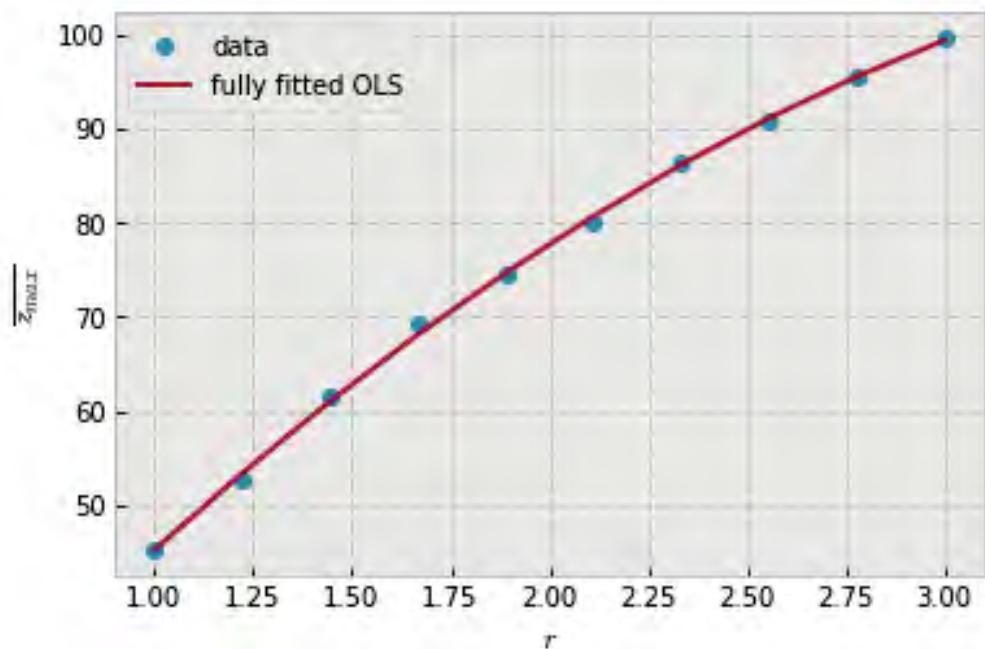
```

variable name: nB
model: poly2
r-squared: 0.684972351750482
explained variance: 0.6862501252820528

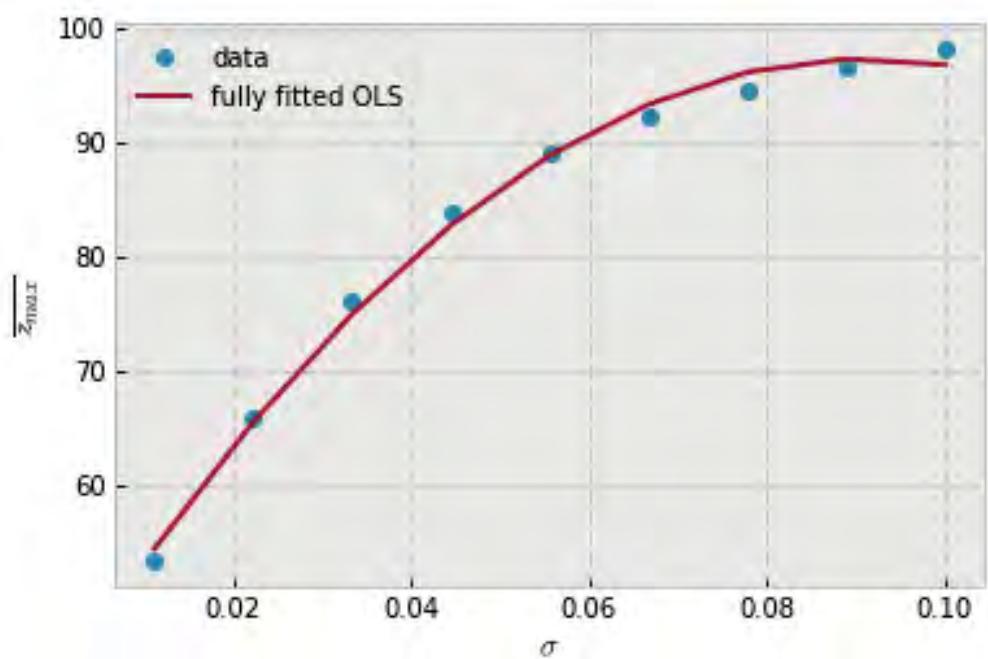
```



```
variable name: r
model: poly2
r-squared: 0.998522699383428
explained variance: 0.998523764819633
```



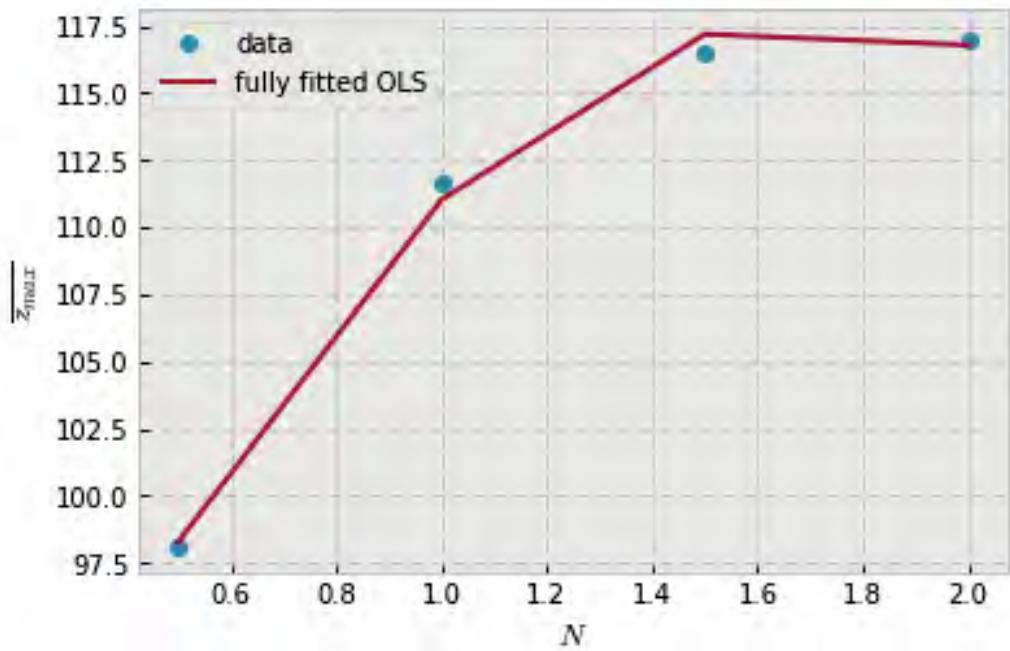
```
variable name: sigma
model: poly2
r-squared: 0.9790504019281303
explained variance: 0.9790634755552131
```



```

variable name: N
model: poly2
r-squared: 0.8196779526594431
explained variance: 0.8196779526594431

```

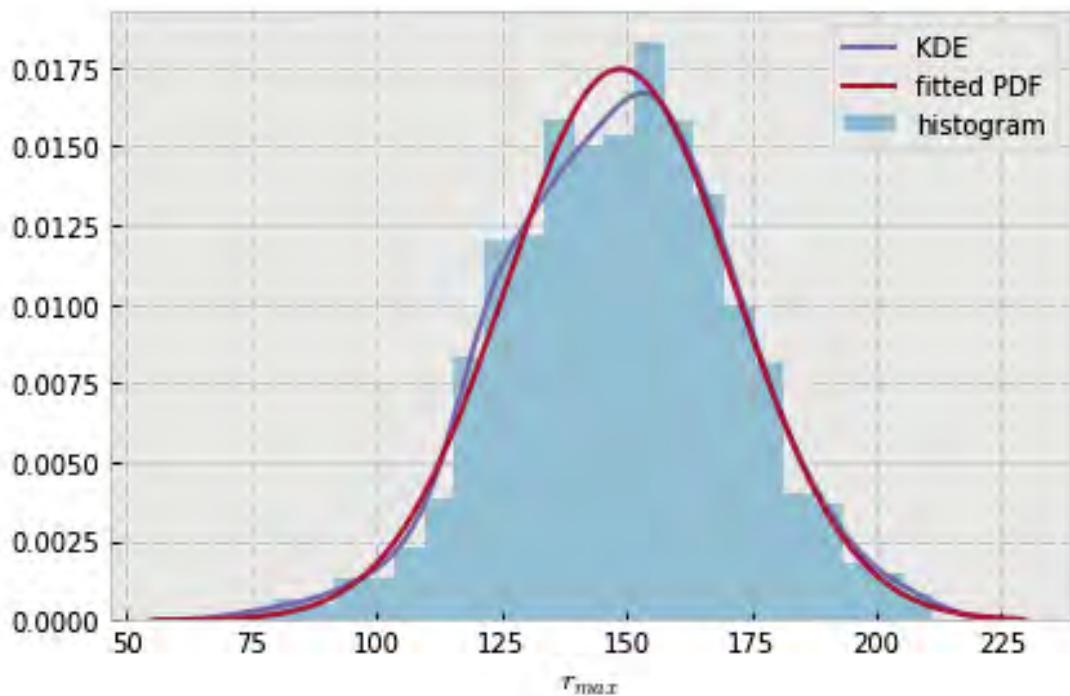


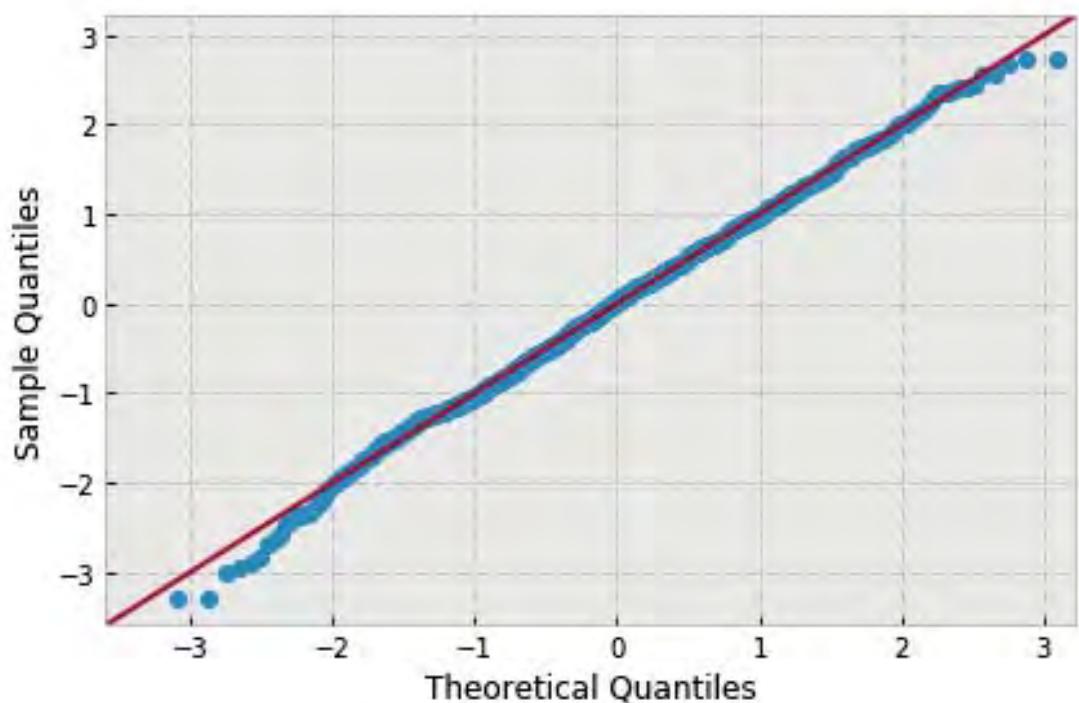
3 Maximal horizontal spread

3.1 Probability distributions

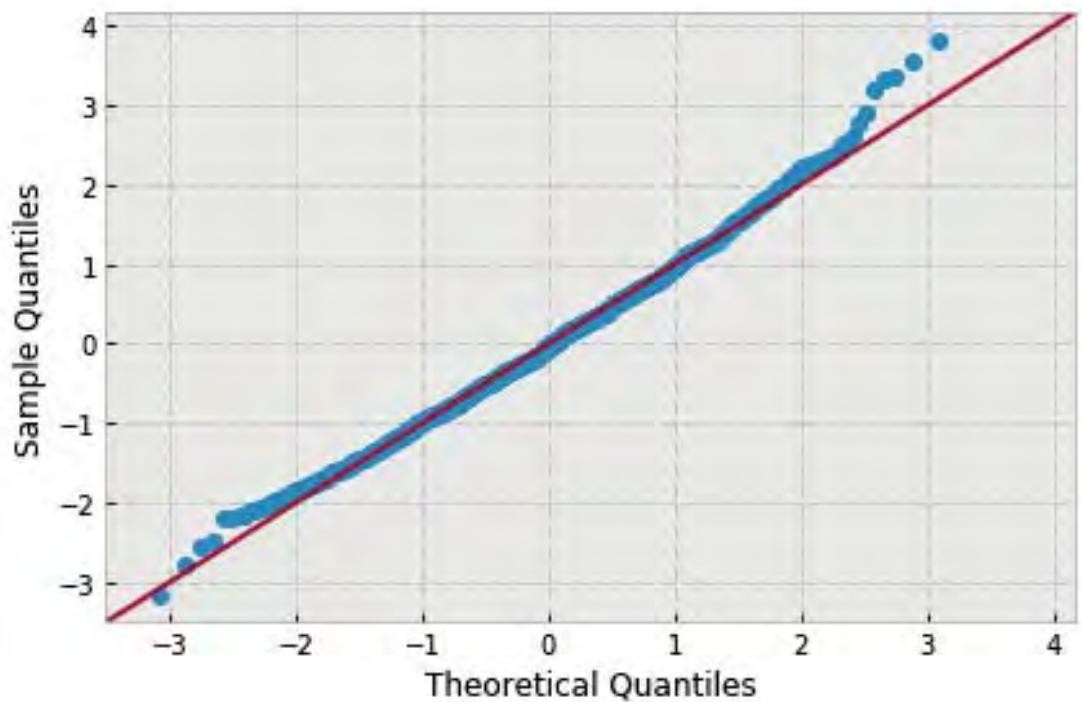
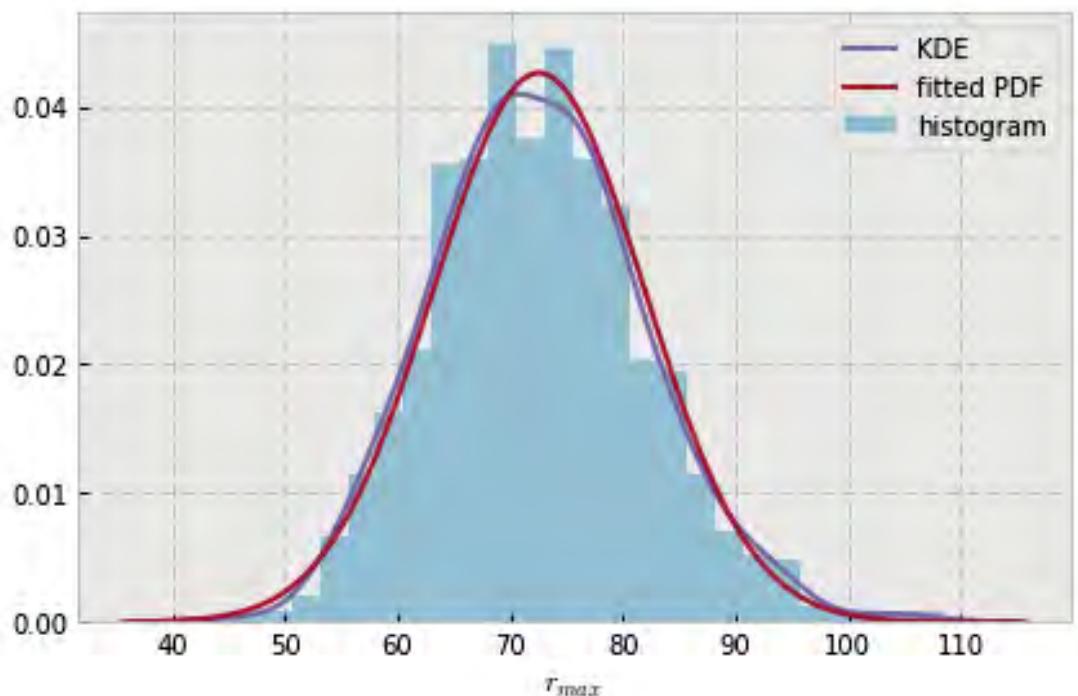
```
In [11]: plot_dist('maximal_horizontal_spread')

variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=148.7670340811633, scale=22.868543671466135)
DescribeResult(nobs=1000, minmax=(73.49364497532612, 211.53158736979407)
               mean=148.7670340811633, variance=523.49378343719115,
               skewness=-0.076536902793879, kurtosis=0.01832299861350517)
```

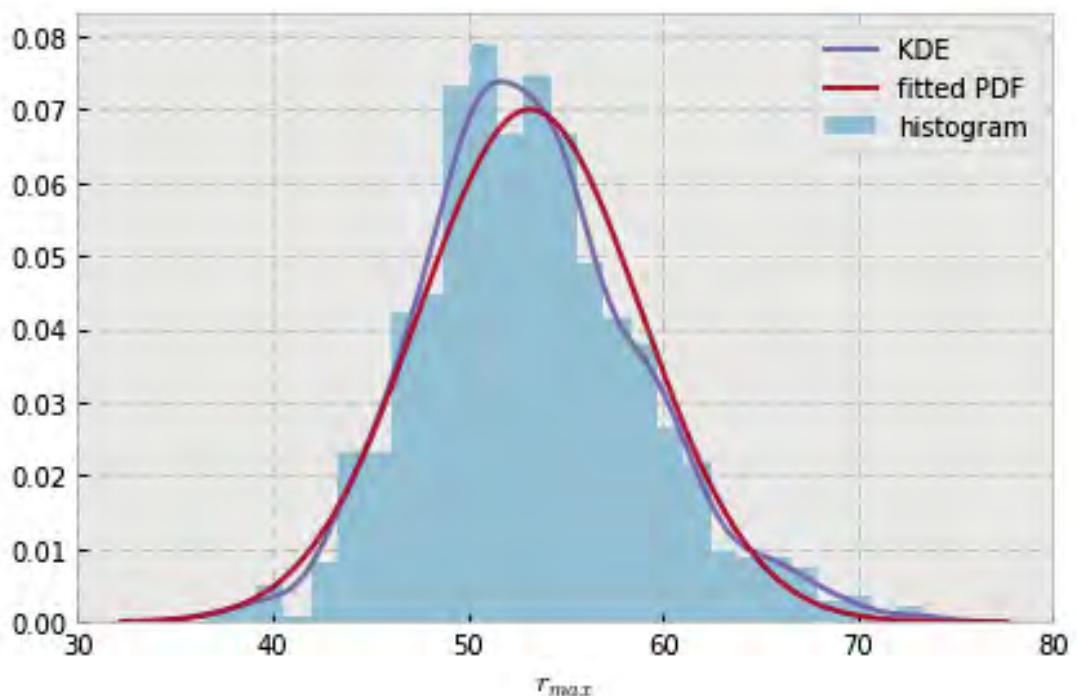


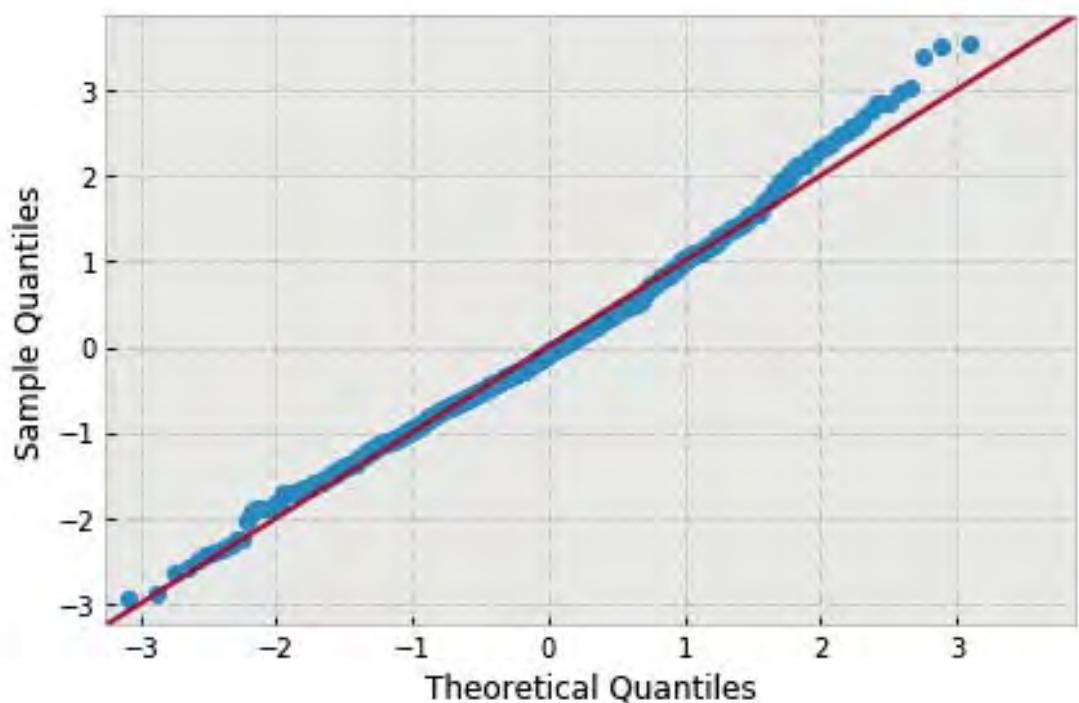


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=72.5526522785993, scale=9.359055594877024)
DescribeResult(nobs=1000, minmax=(43.006608147751479, 108.26387995255632)
               mean=72.552652278599297, variance=87.679601229228169,
               skewness=0.28418739234318724, kurtosis=0.25375593502875216)
```

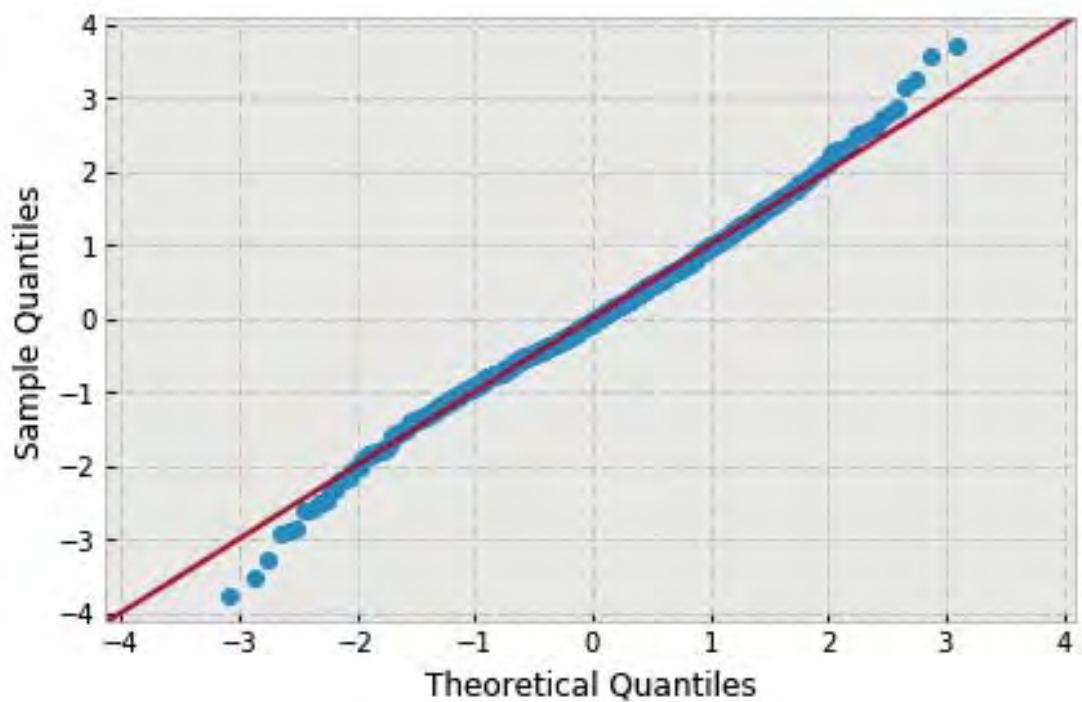
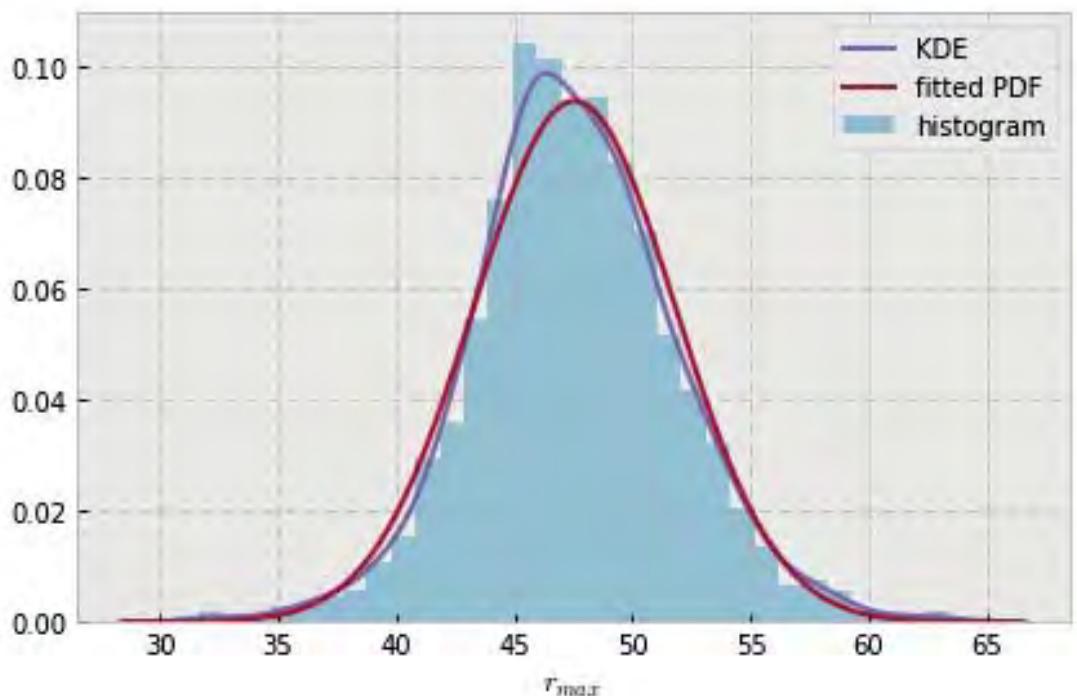


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=53.20058703721677, scale=5.6984750082593845)
DescribeResult(nobs=1000, minmax=(36.508482005477305, 73.386107246960222)
               mean=53.200587037216771, variance=32.505122542299098,
               skewness=0.38098171980612555, kurtosis=0.40224268574793376)
```

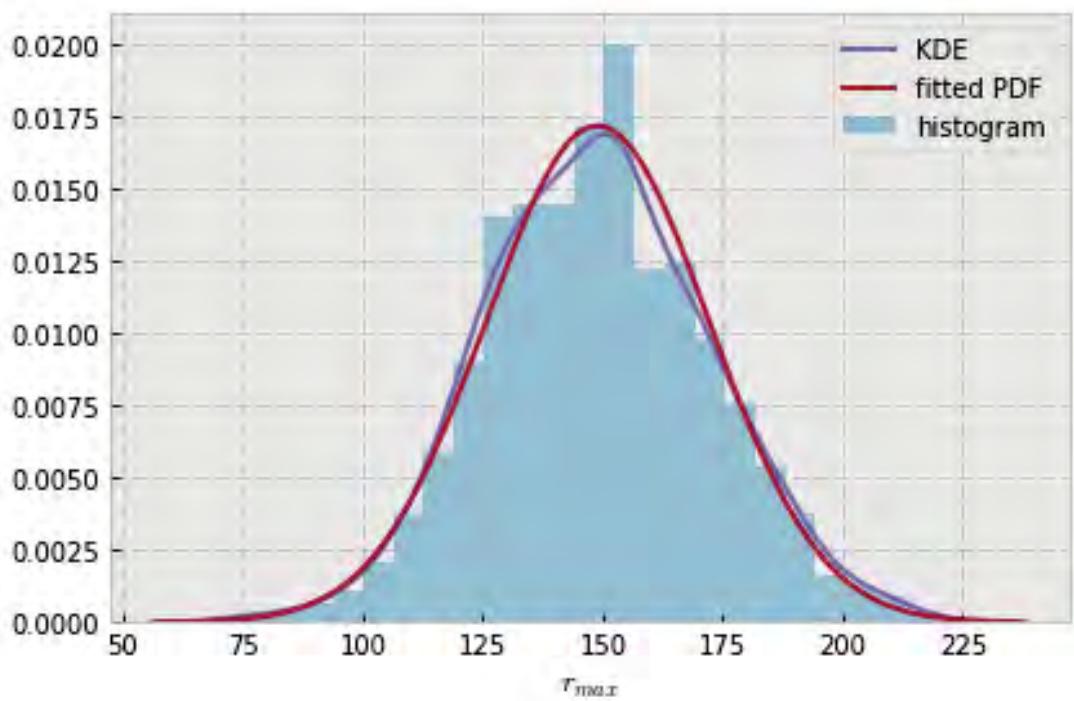


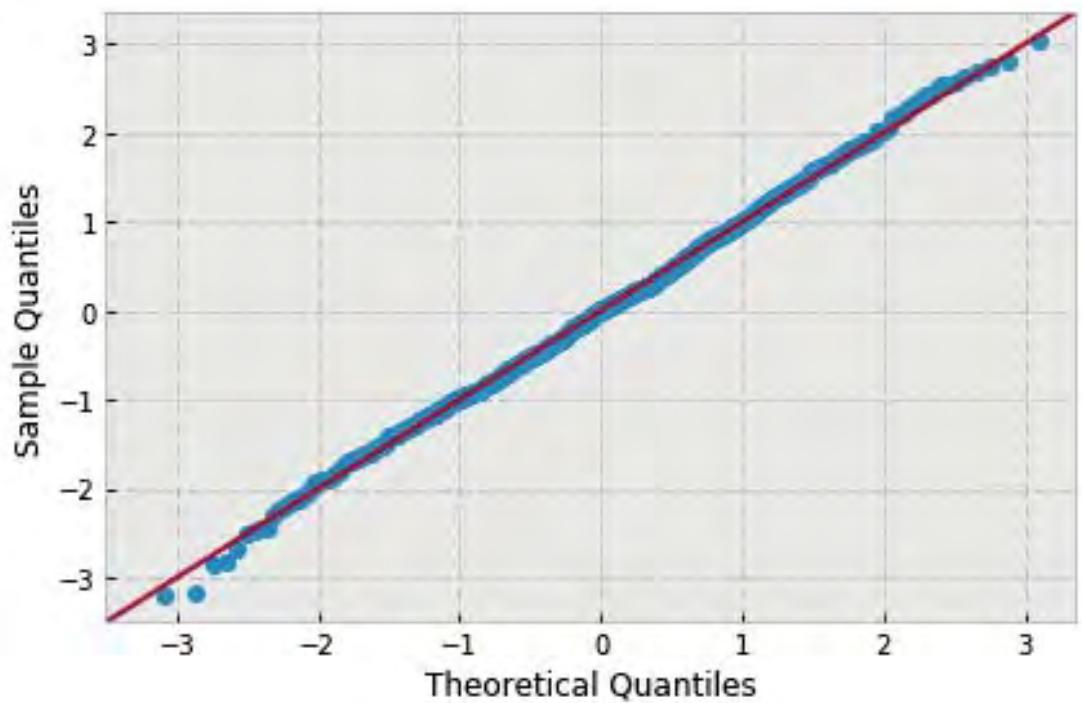


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=47.55665739148845, scale=4.251142903429944)
DescribeResult(nobs=1000, minmax=(31.59117740483428, 63.35385033970006)
               mean=47.55665739148845, variance=18.090306291674452,
               skewness=0.10320251173641505, kurtosis=0.70189997929121)
```

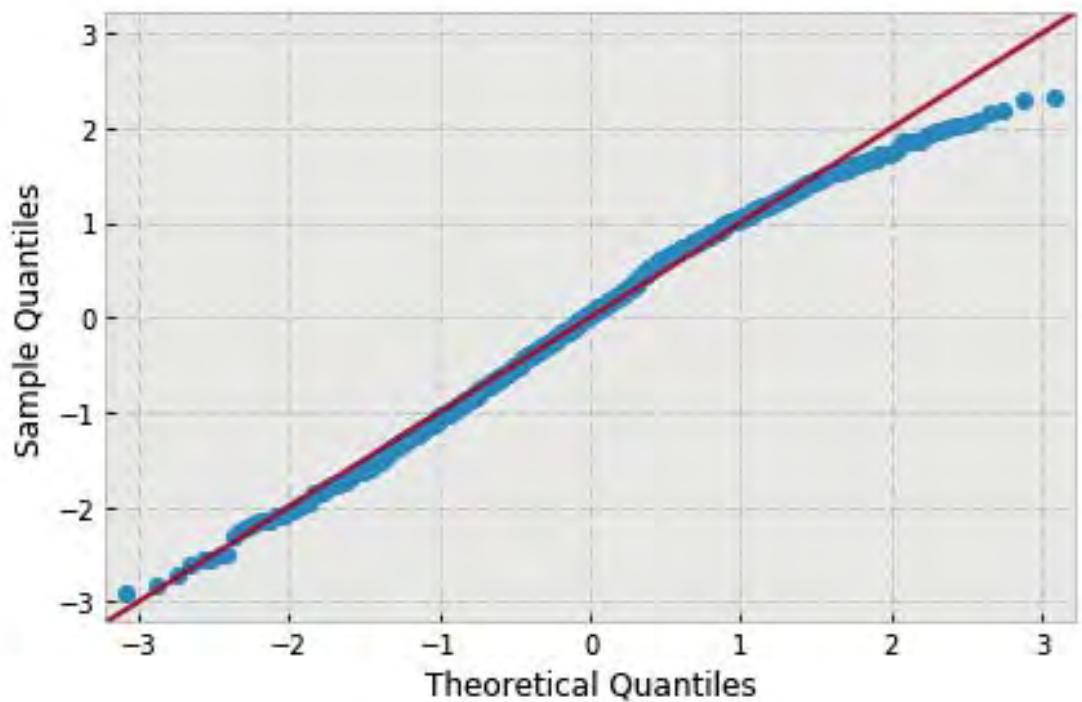
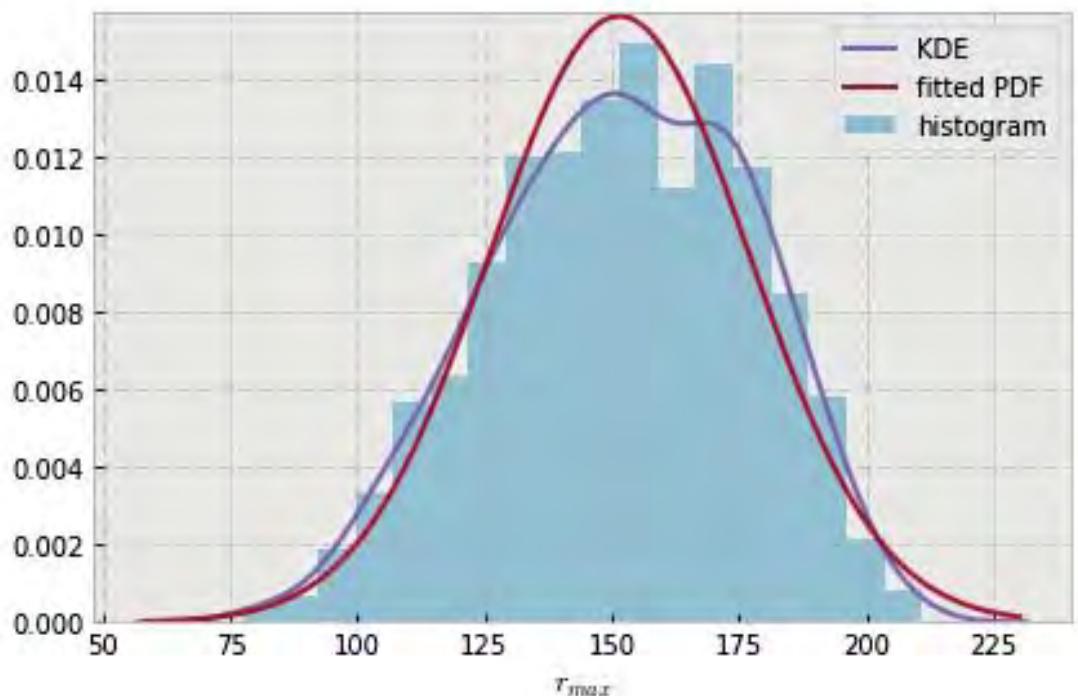


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=148.90296673943087, scale=23.239785323289333)
DescribeResult(nobs=1000, minmax=(74.766450162712147, 219.46380253224368)
               mean=148.90296673943087, variance=540.62825012269707,
               skewness=0.07574756874372819, kurtosis=-0.034949240933307646)
```

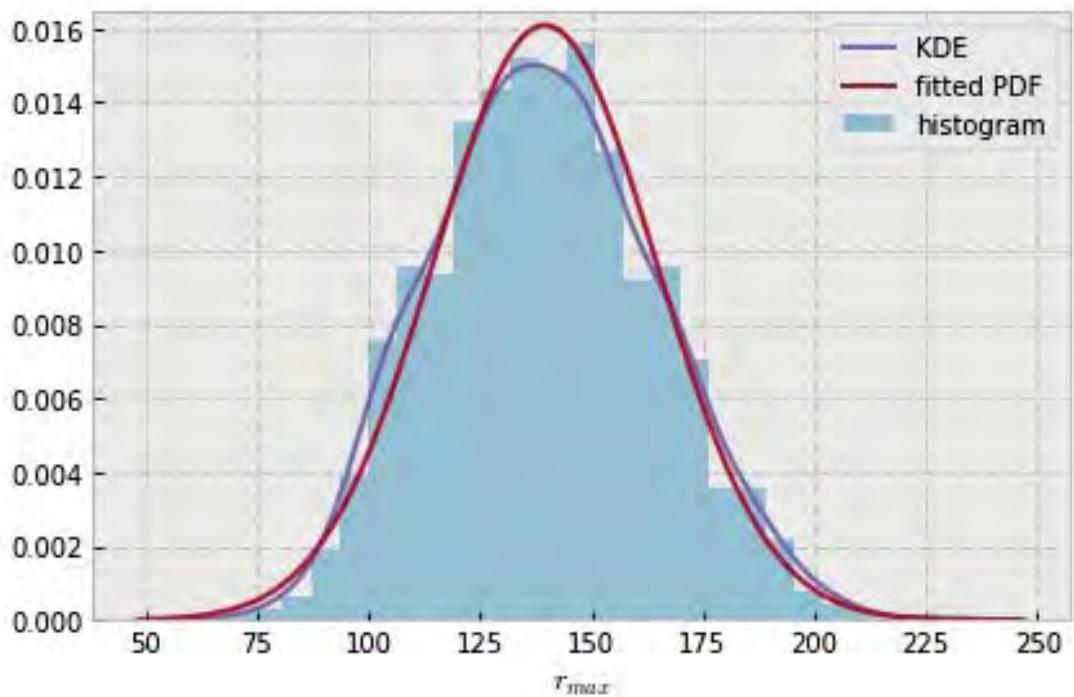


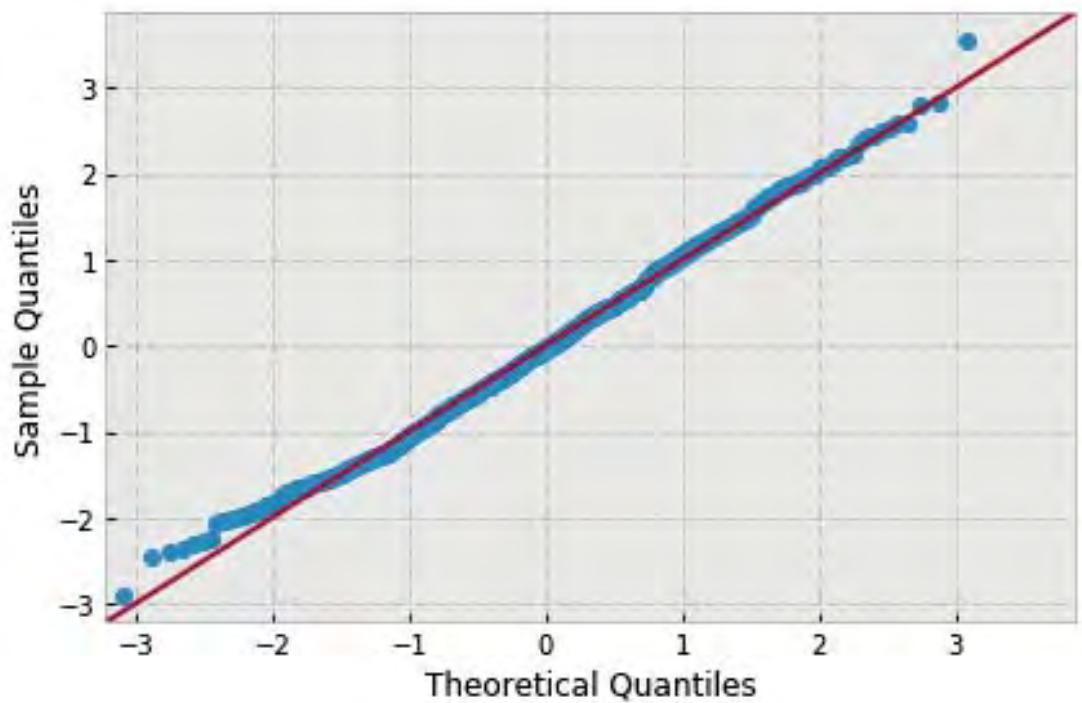


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=151.44659026586172, scale=25.526977121218657)
DescribeResult(nobs=1000, minmax=(77.261252758788117, 210.84727231243824)
               mean=151.44659026586172, variance=652.27883978700777,
               skewness=-0.21274152489448162, kurtosis=-0.5353679563091762)
```

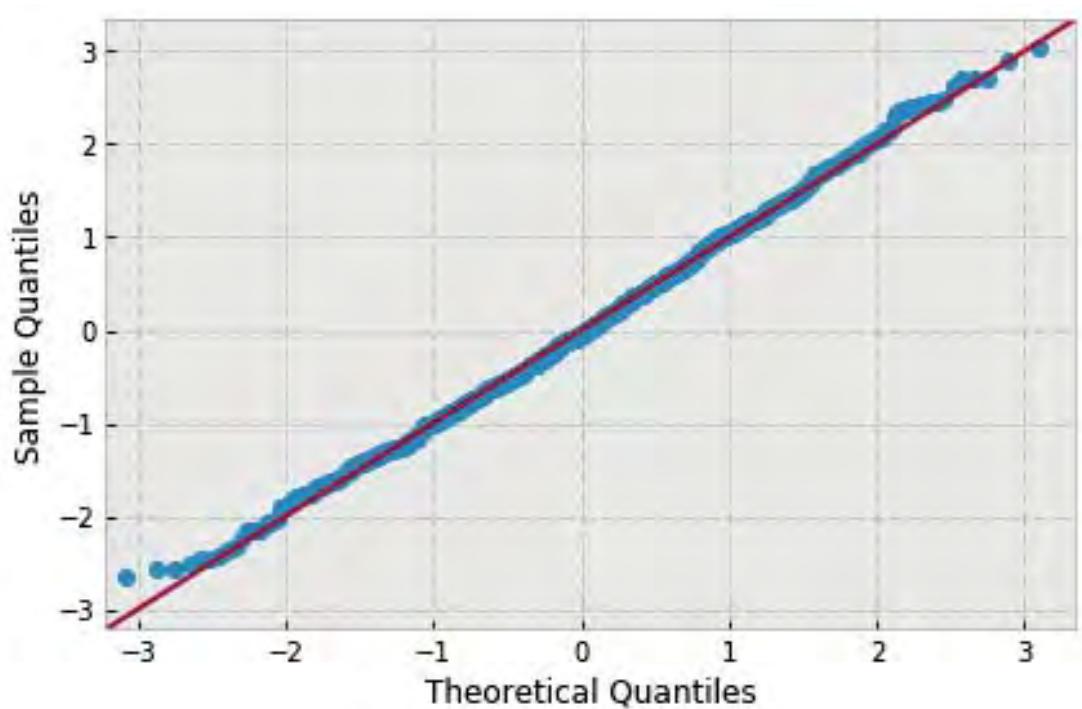
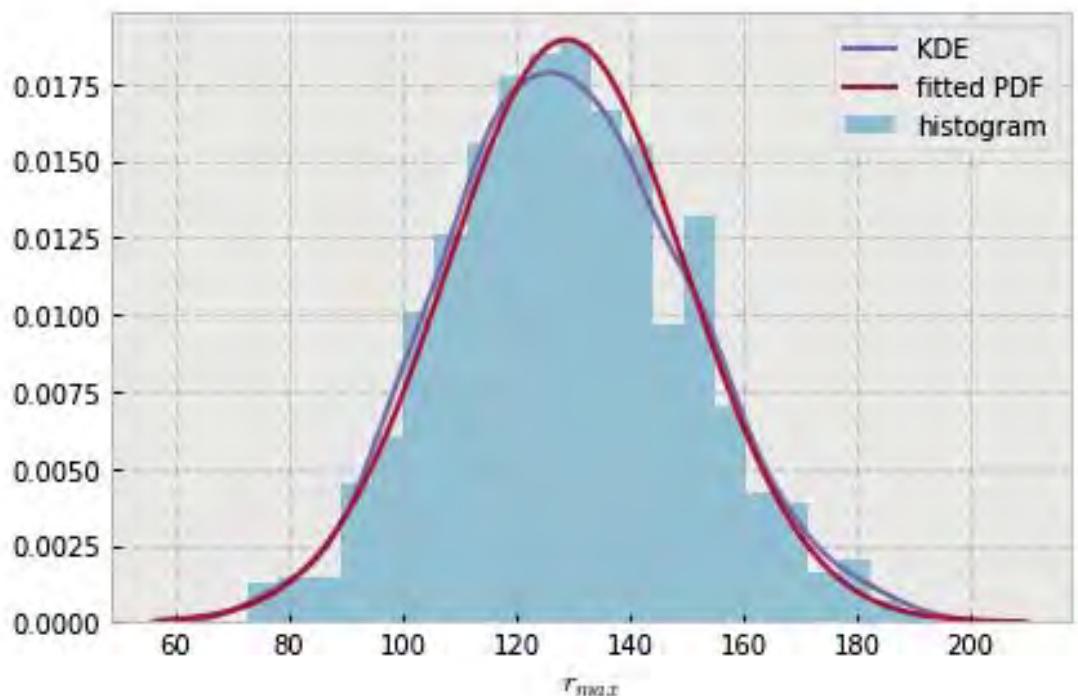


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=139.48395803187313, scale=24.79959005529943)
DescribeResult(nobs=1000, minmax=(67.743594021528878, 227.40073952783086)
               mean=139.48395803187313, variance=615.63530221311942,
               skewness=0.17161379436290974, kurtosis=-0.2889590670121662)
```

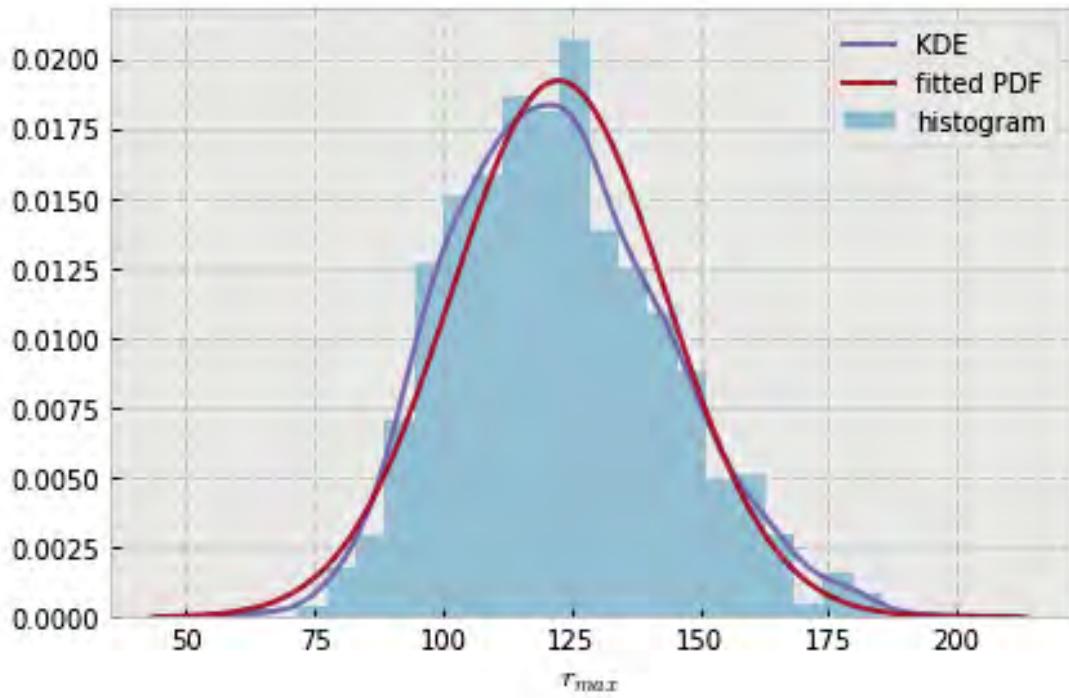


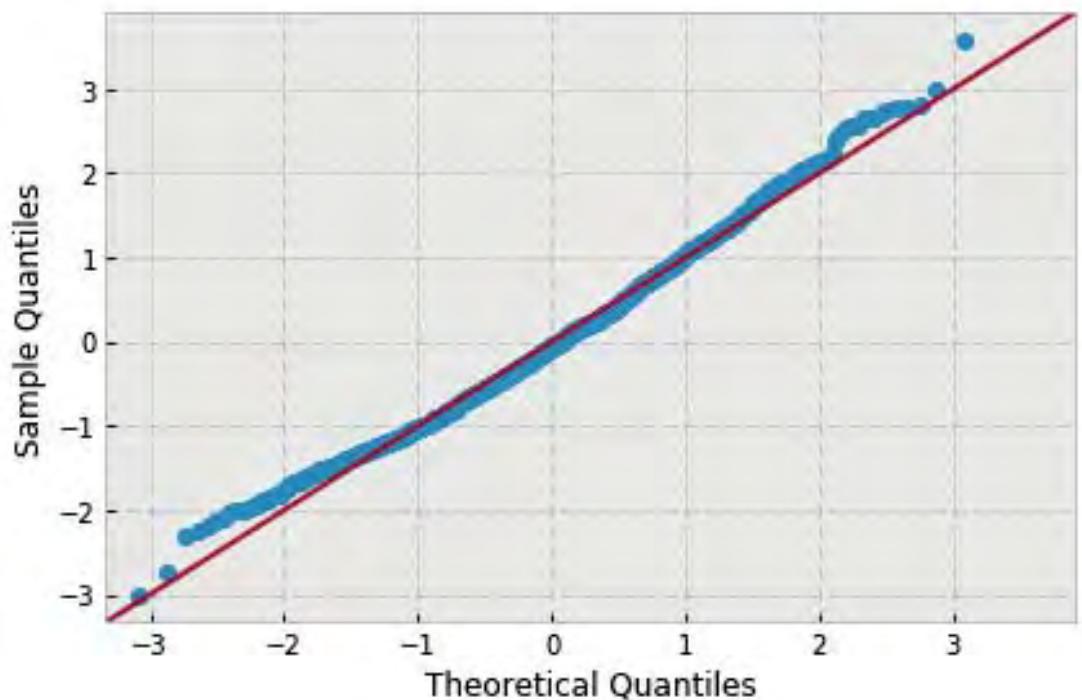


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=128.86320307741187, scale=21.062512218903645)
DescribeResult(nobs=1000, minmax=(72.948648134499948, 193.07022386438365)
               mean=128.86320307741187, variance=444.07349446593122,
               skewness=0.13736131209561597, kurtosis=-0.18562919333712635)
```

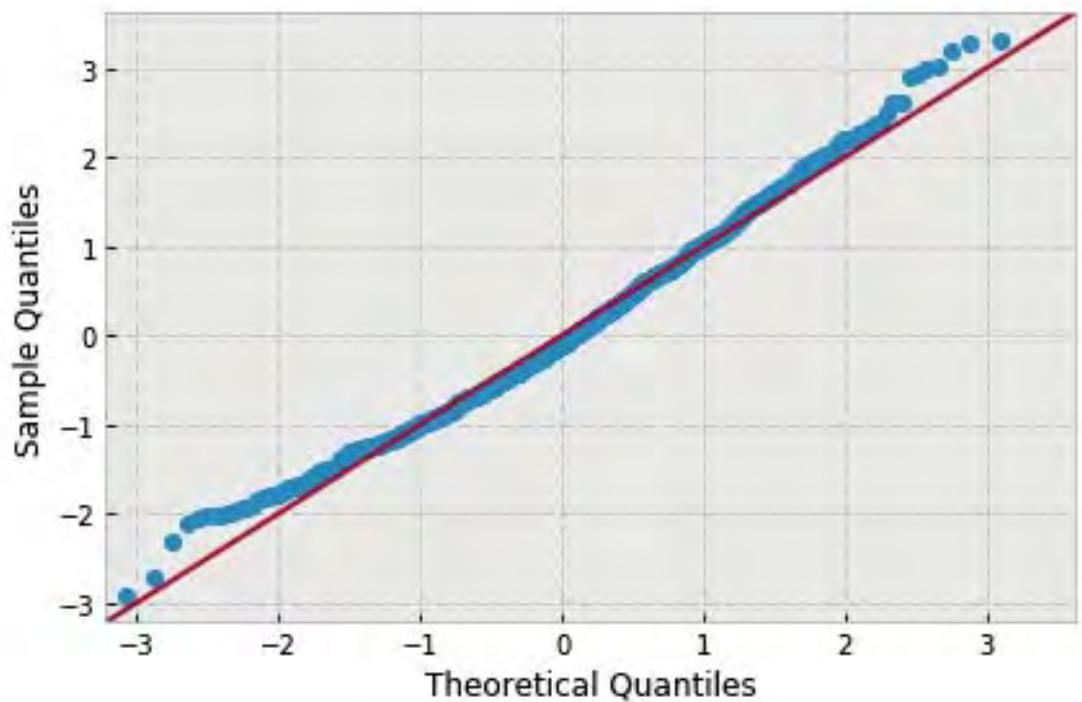
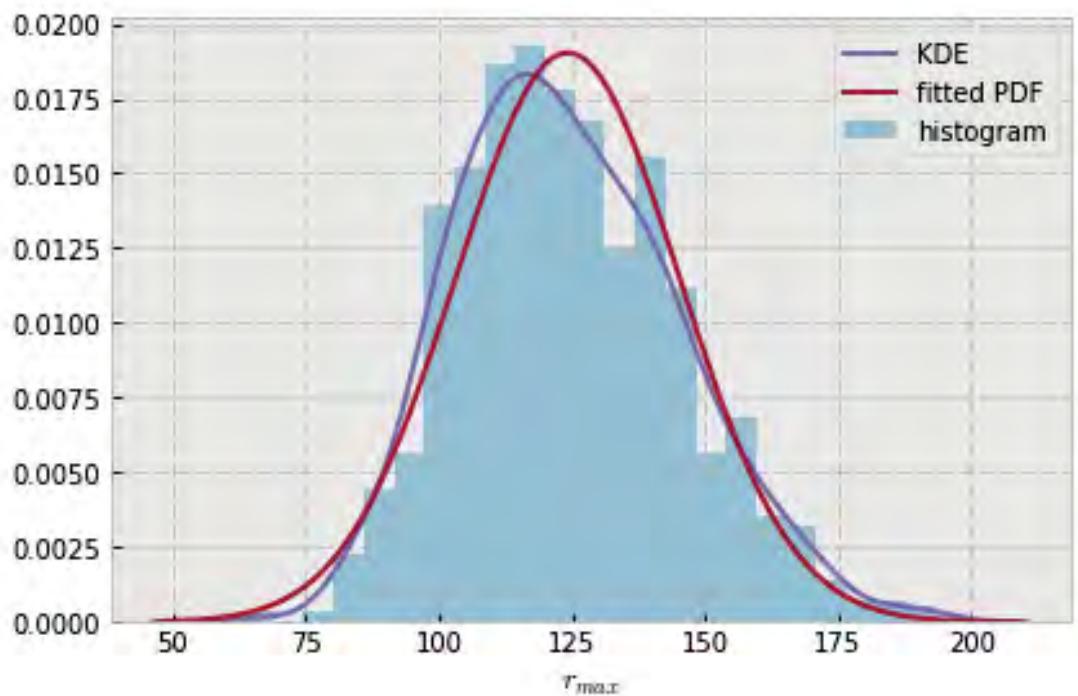


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=122.47198732664005, scale=20.775041701721)
DescribeResult(nobs=1000, minmax=(60.118334755838234, 196.73815101324948)
               mean=122.47198732664005, variance=432.03439210034702,
               skewness=0.3477712374816198, kurtosis=-0.06646915419758903)
```

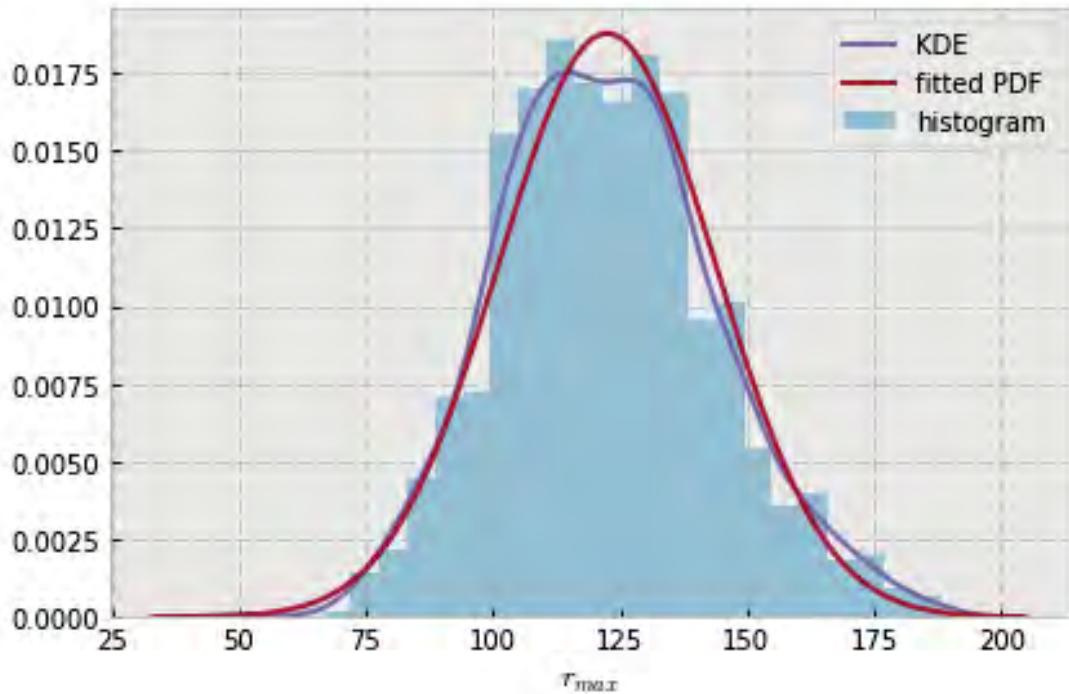


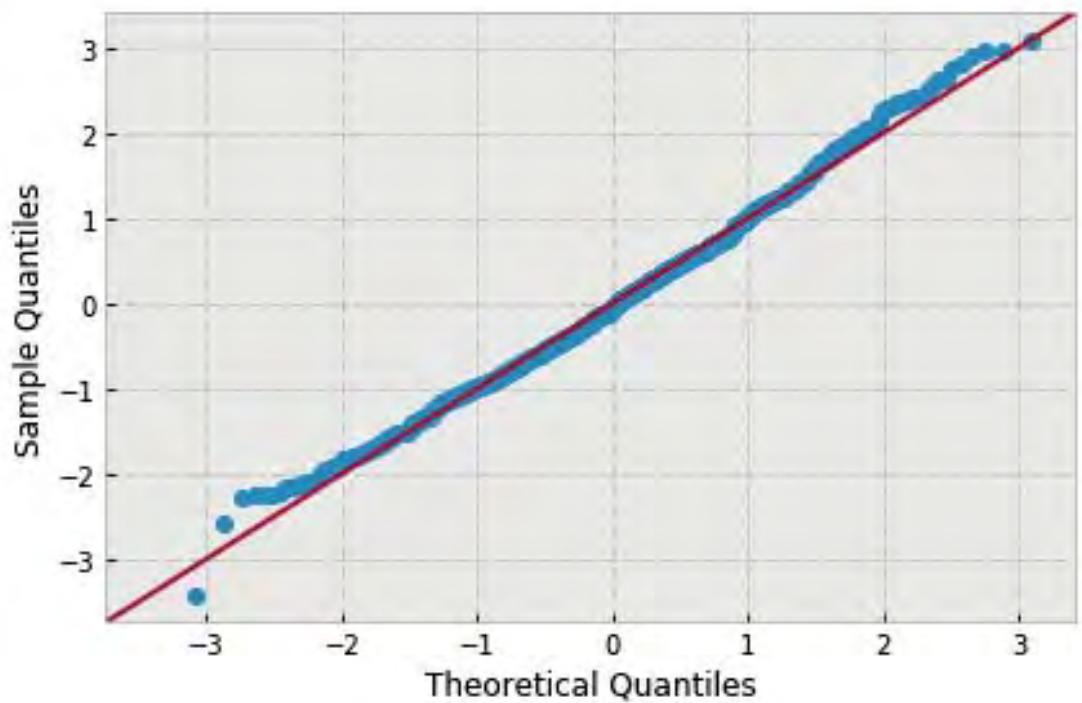


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=124.19589766478032, scale=20.959319971701934)
DescribeResult(nobs=1000, minmax=(63.230887299876436, 193.56052202848676)
               mean=124.19589766478032, variance=439.73282650268629,
               skewness=0.3820344649694134, kurtosis=-0.04581117173943827)
```

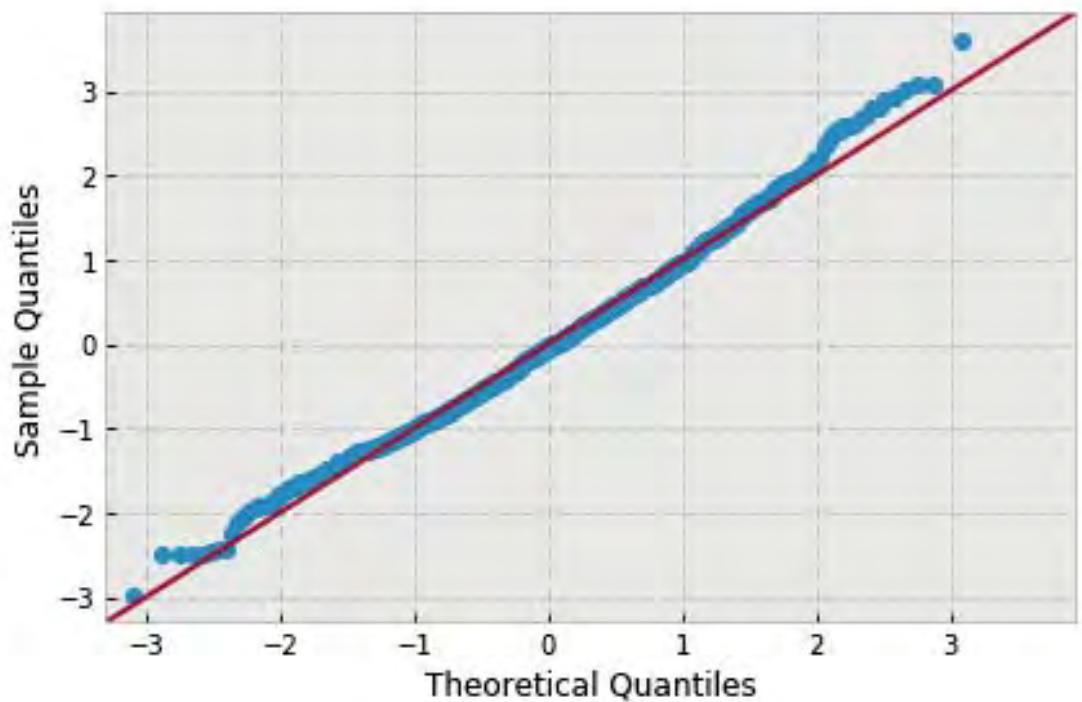
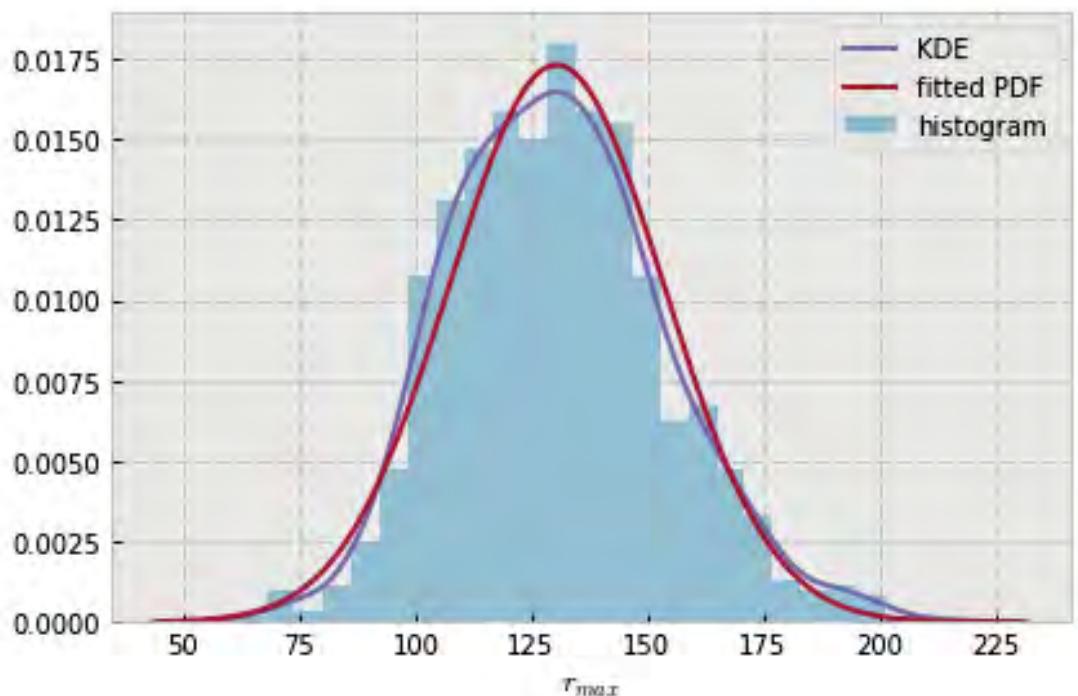


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=122.47812753547664, scale=21.277207624850245)
DescribeResult(nobs=1000, minmax=(49.722915373416171, 188.15142238921288)
               mean=122.47812753547664, variance=453.17273704803347,
               skewness=0.25049113033841386, kurtosis=-0.004921000140198917)
```

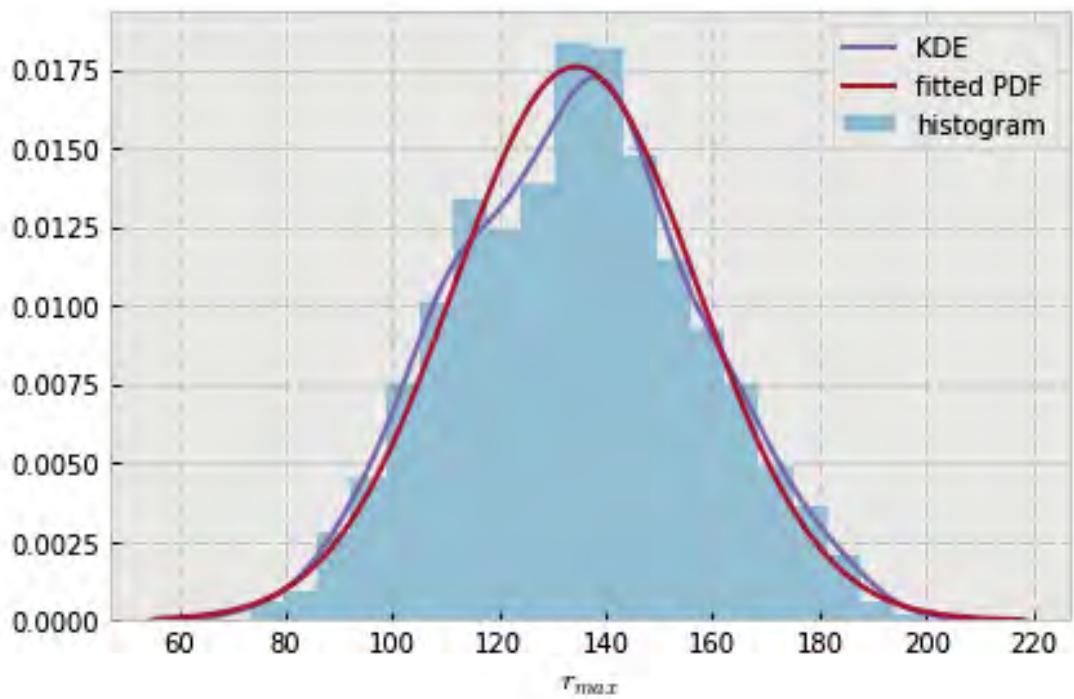


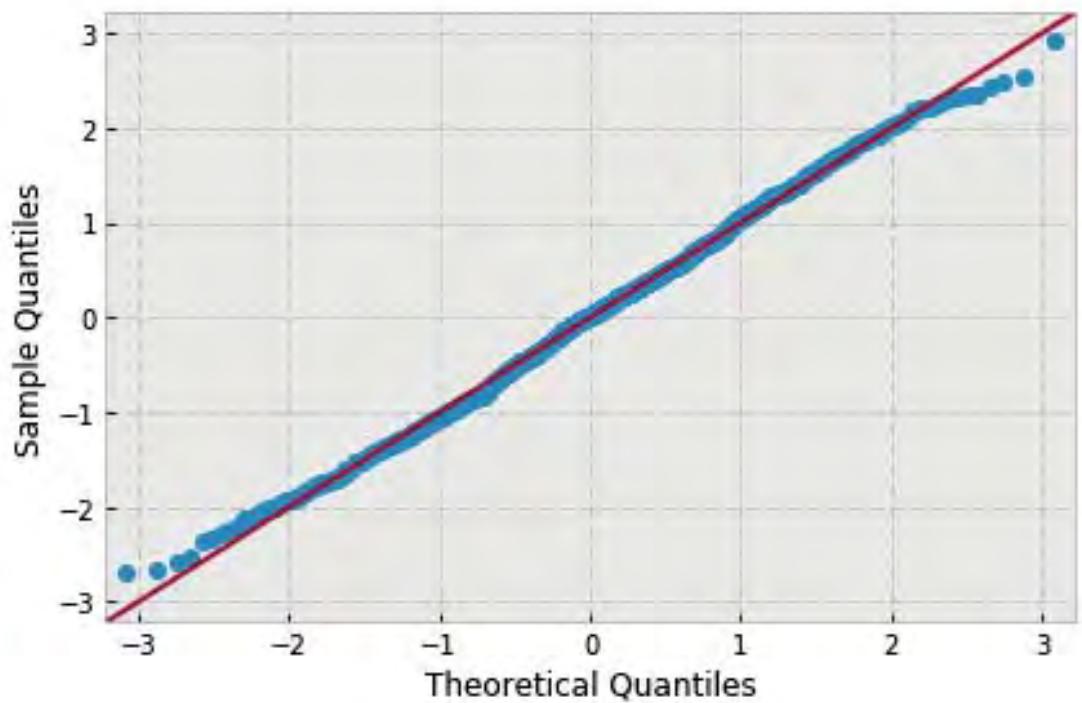


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=130.4412528354438, scale=23.0524547099134)
DescribeResult(nobs=1000, minmax=(62.037973703180533, 213.28019544436609)
               mean=130.44125283544381, variance=531.94761576837686,
               skewness=0.33594149279257324, kurtosis=0.13939549305427024)
```

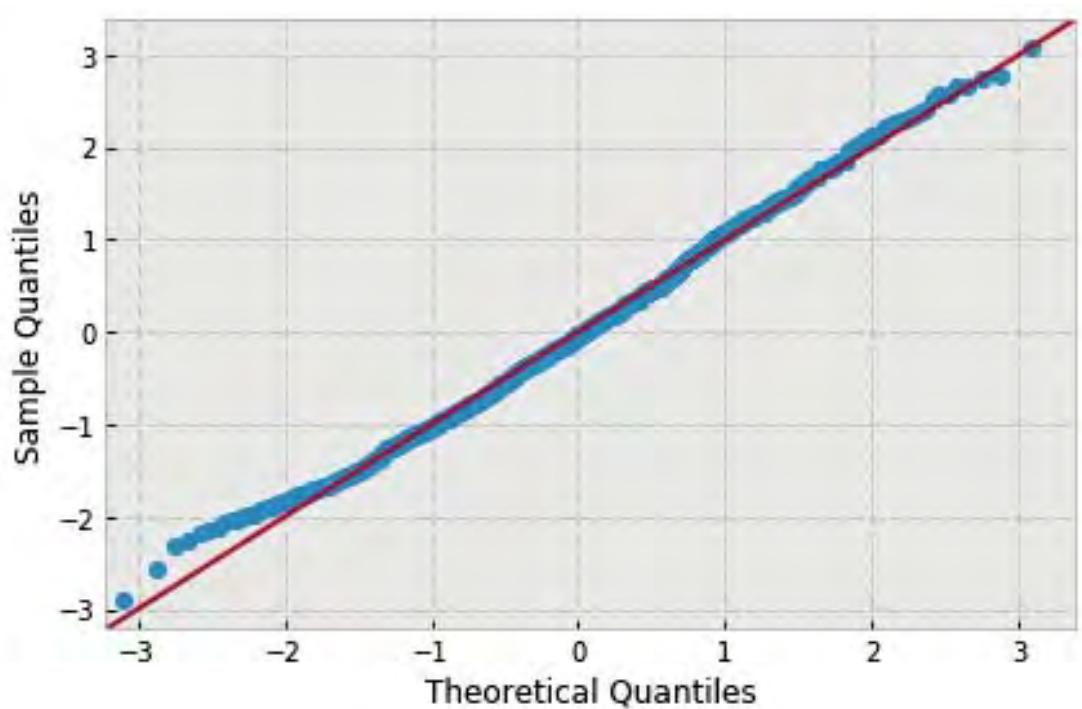
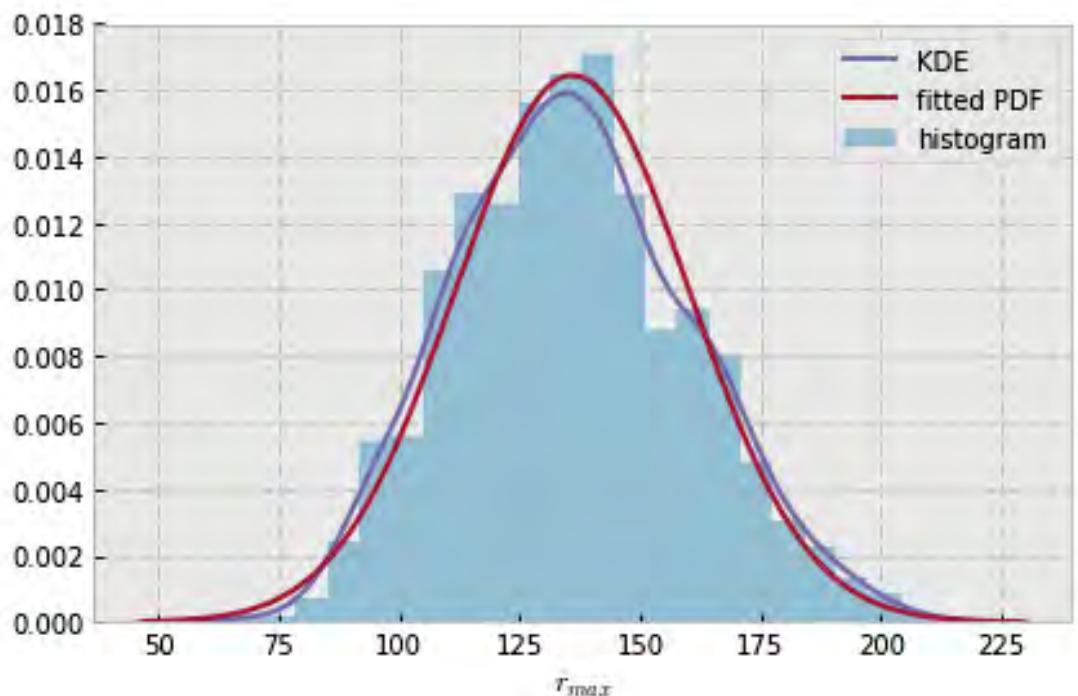


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=134.38406973468983, scale=22.699088131238156)
DescribeResult(nobs=1000, minmax=(73.243829866960226, 200.48783848871031)
               mean=134.38406973468983, variance=515.76436635607297,
               skewness=0.04455250226264711, kurtosis=-0.3783570402579728)
```

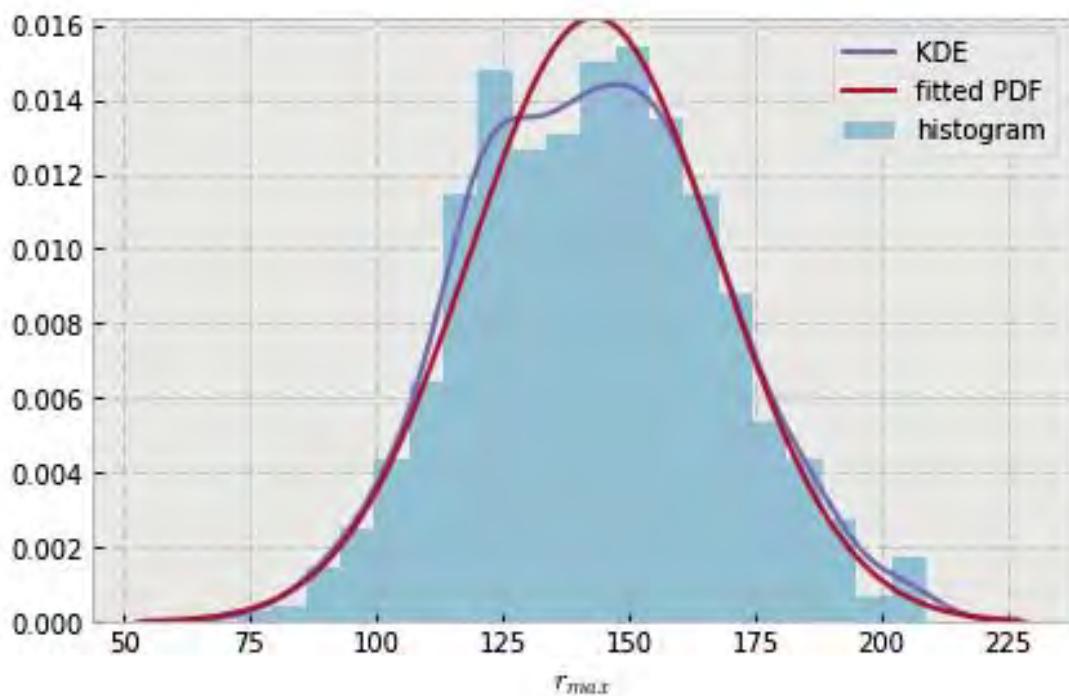


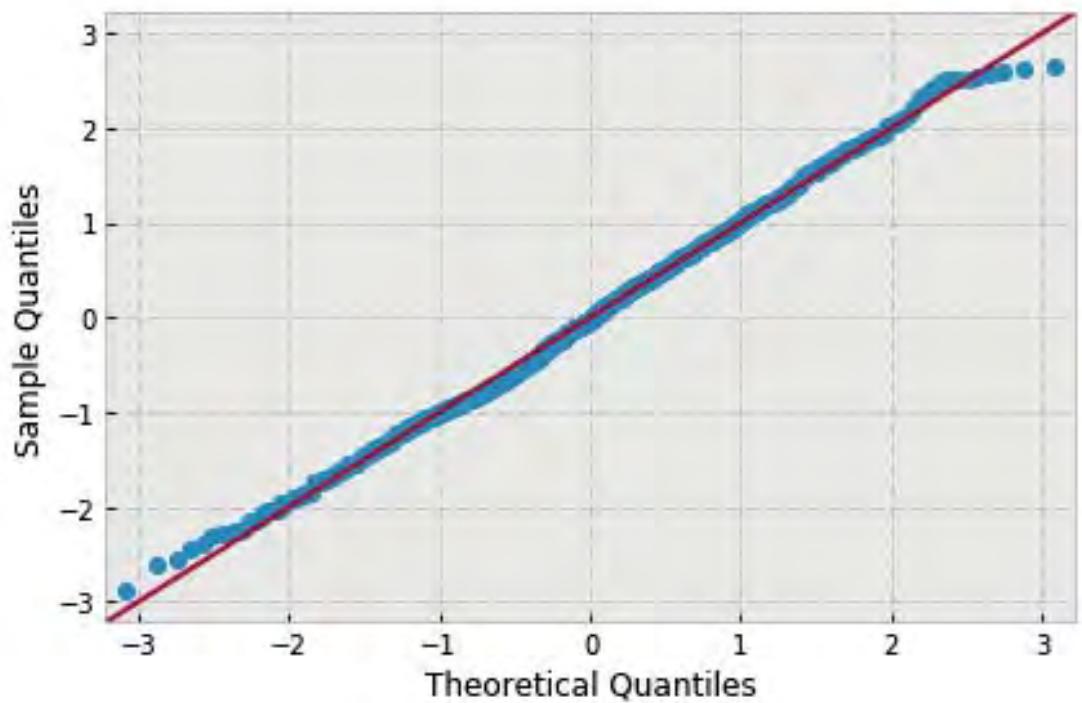


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=135.92808521935845, scale=24.268542886686955)
DescribeResult(nobs=1000, minmax=(65.268714607813237, 210.55450727388578)
               mean=135.92808521935845, variance=589.55172556853256,
               skewness=0.19640785357324397, kurtosis=-0.3014706413684656)
```

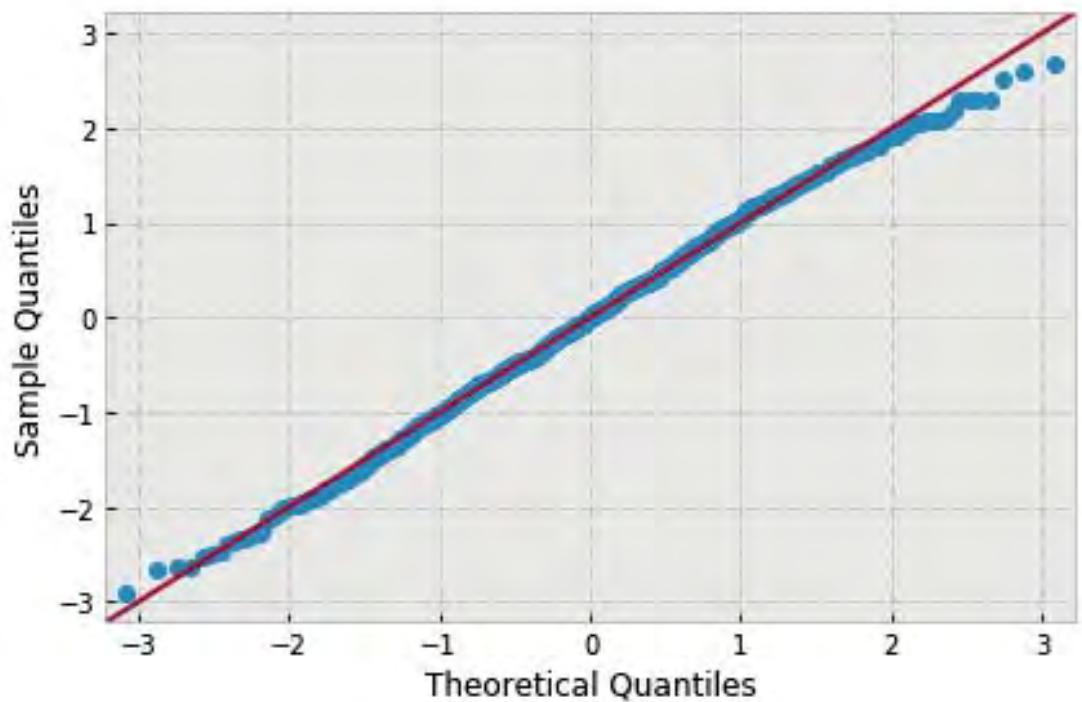
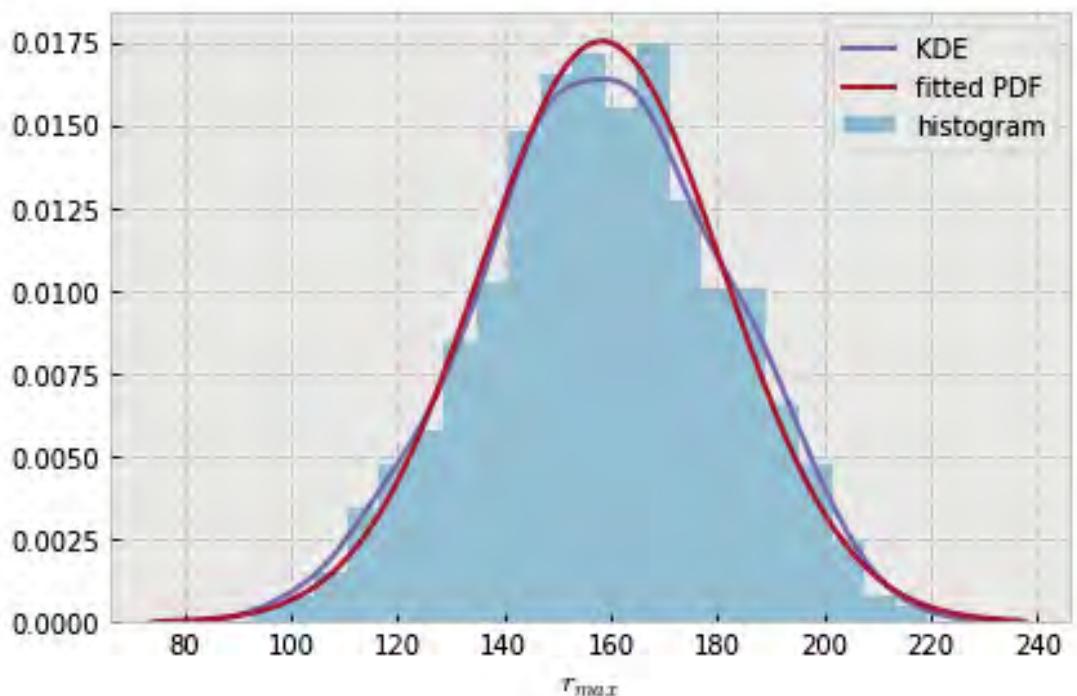


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=143.096839234708, scale=24.62769492667748)
DescribeResult(nobs=1000, minmax=(72.382540440009521, 208.60283614481276)
               mean=143.096839234708, variance=607.13048788938522,
               skewness=0.09546727600565144, kurtosis=-0.3248290683376567)
```

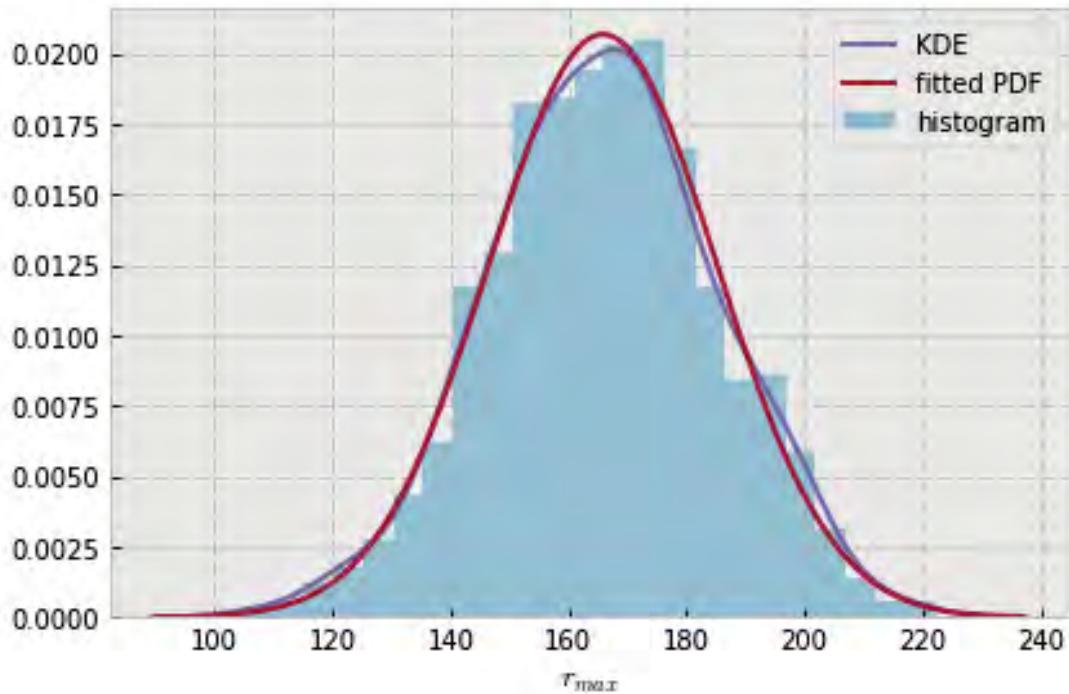


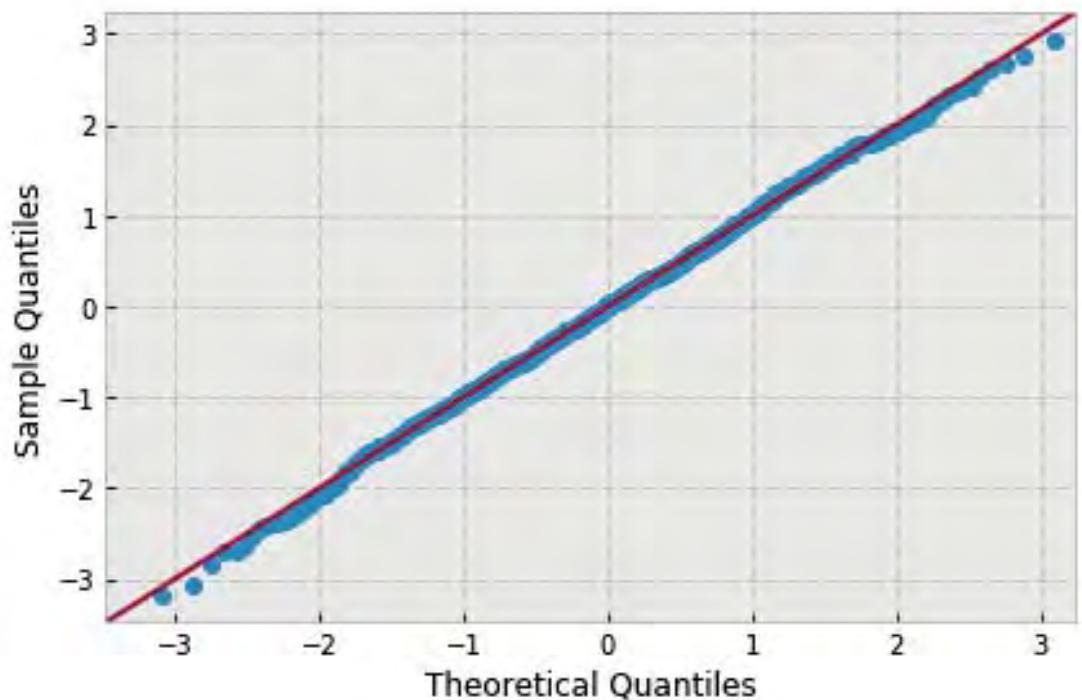


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=158.35921551415035, scale=22.76209826982621)
DescribeResult(nobs=1000, minmax=(92.326699586234668, 219.55000757930097)
               mean=158.35921551415035, variance=518.63174939461999,
               skewness=-0.08193869230673463, kurtosis=-0.33815498132559085)
```

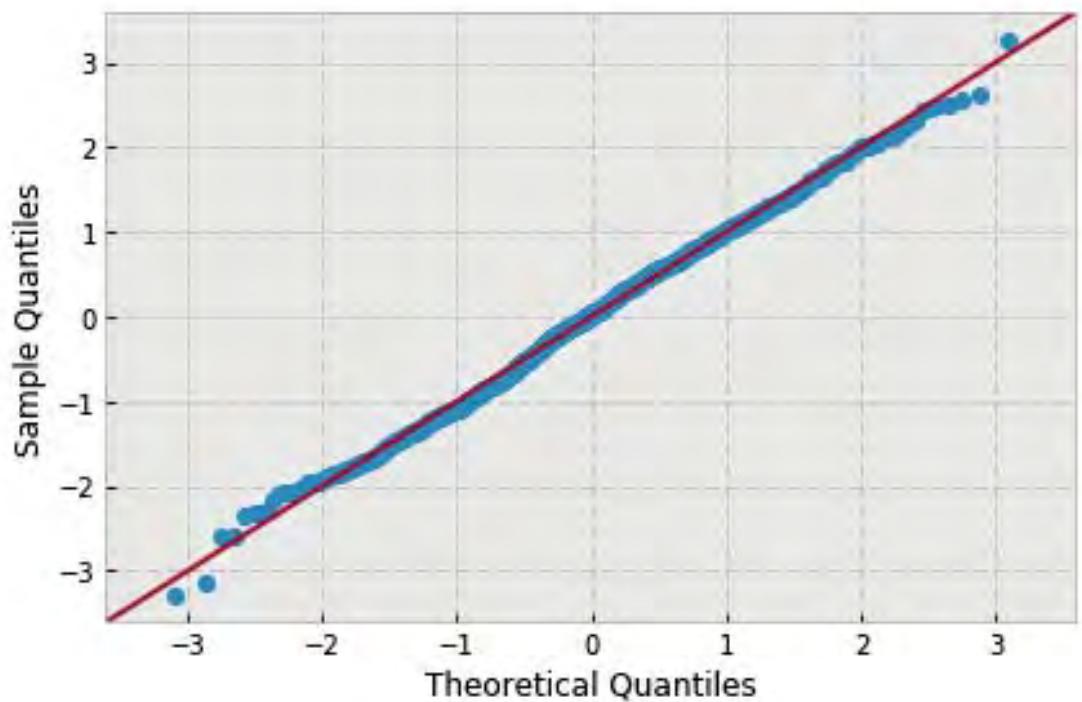
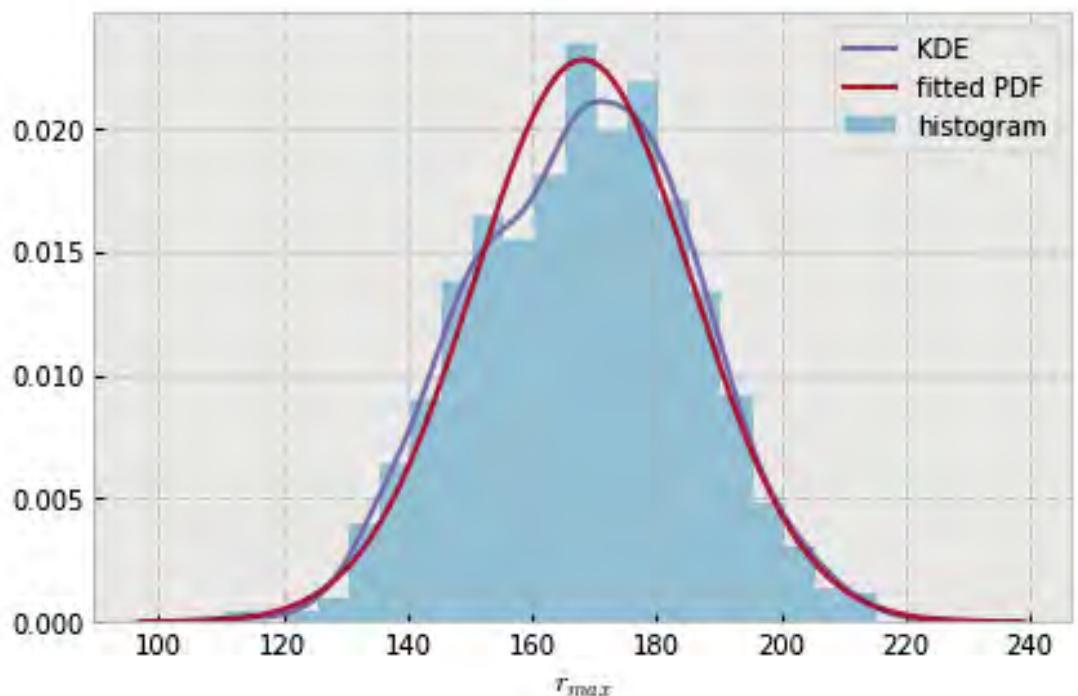


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=165.85033481333045, scale=19.301123588640476)
DescribeResult(nobs=1000, minmax=(104.69061004813781, 222.28537197736881)
               mean=165.85033481333045, variance=372.90627806203588,
               skewness=-0.04144107618787465, kurtosis=-0.08574129405267739)
```

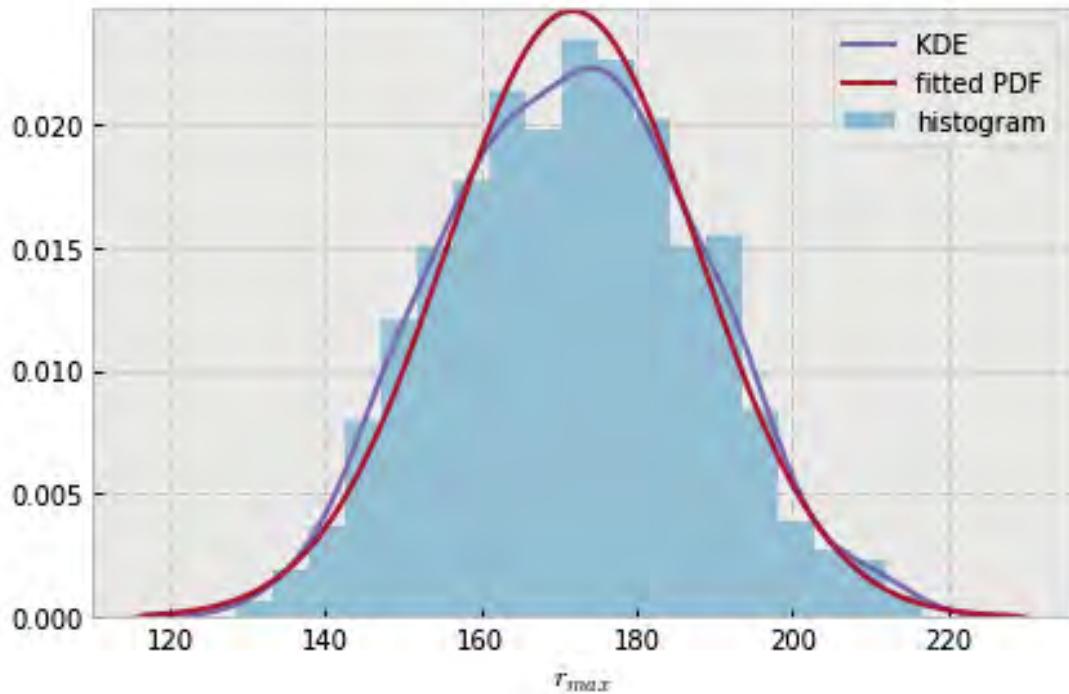


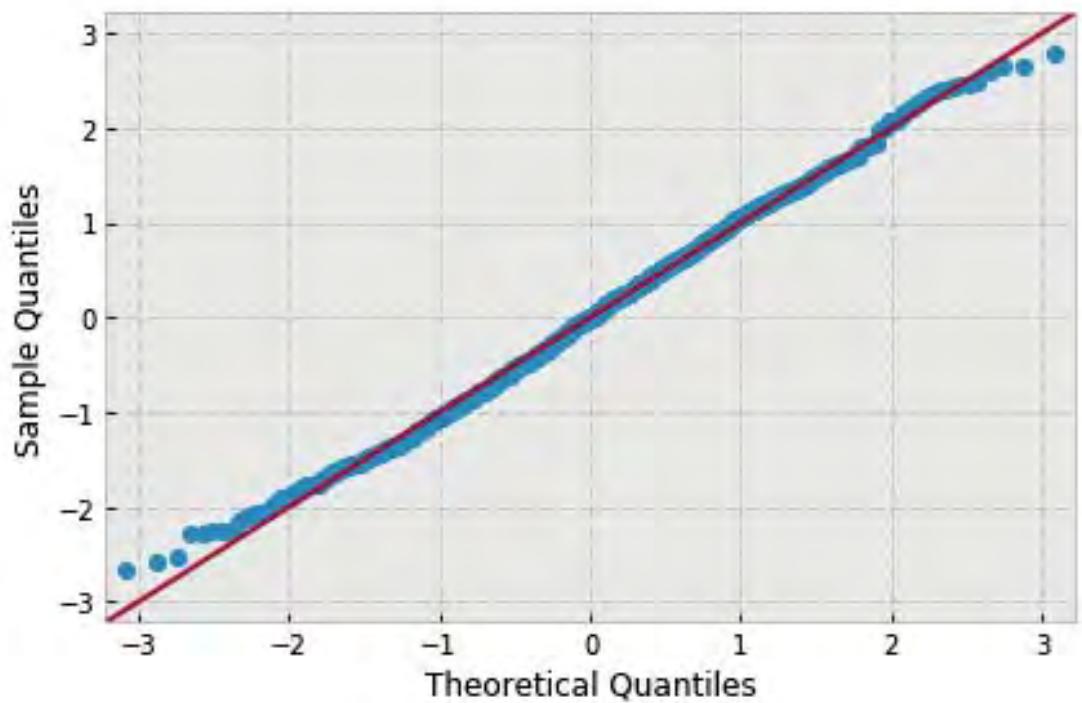


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=168.20172370878421, scale=17.536419545073496)
DescribeResult(nobs=1000, minmax=(110.65114352328639, 225.33414439630224)
               mean=168.20172370878421, variance=307.83384430514081,
               skewness=-0.045974547227893105, kurtosis=-0.3013072201052682)
```

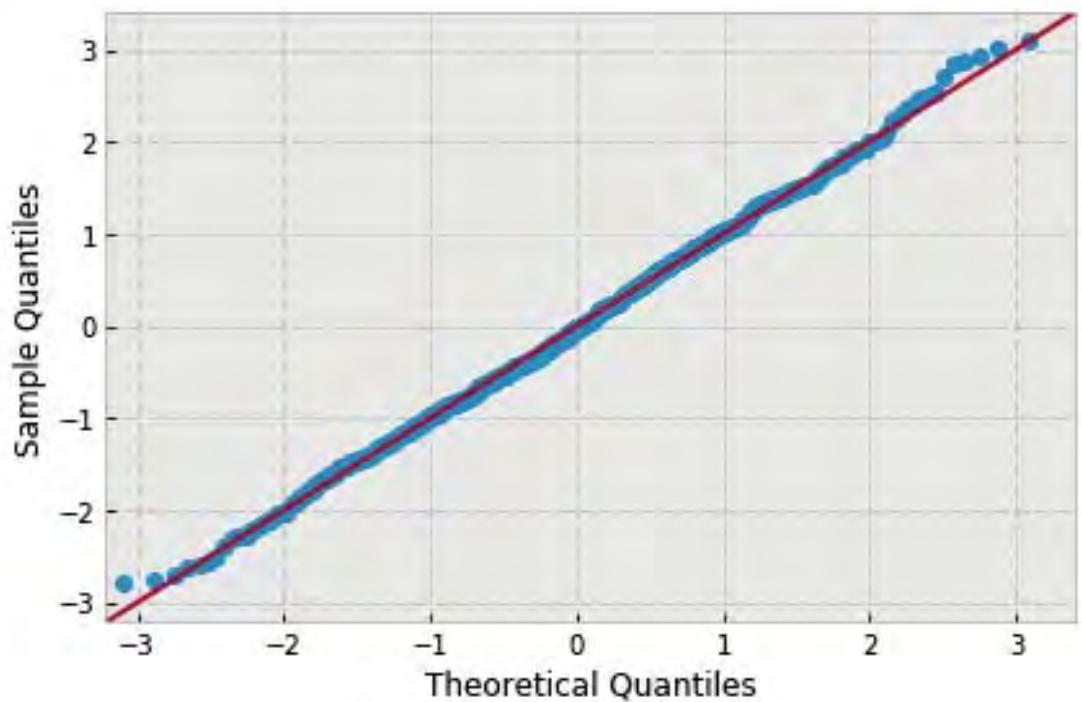
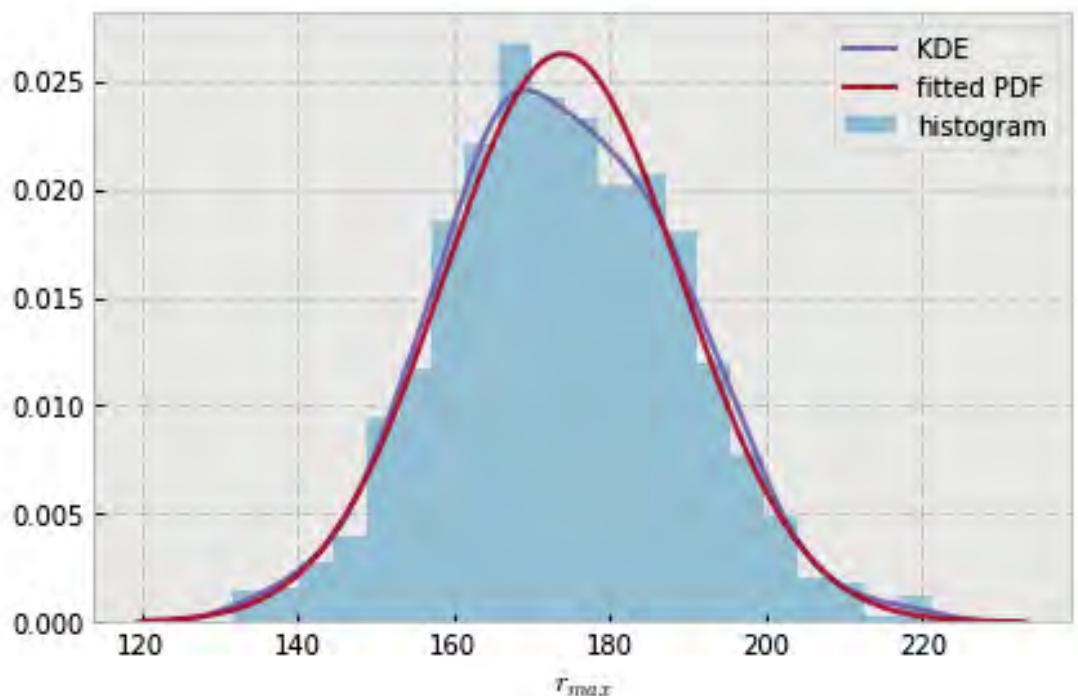


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=171.73792594513904, scale=16.220855319245523)
DescribeResult(nobs=1000, minmax=(128.73544334285515, 216.91617221426043)
               mean=171.73792594513904, variance=263.37952681471052,
               skewness=0.0707239462713022, kurtosis=-0.41180942540334664)
```

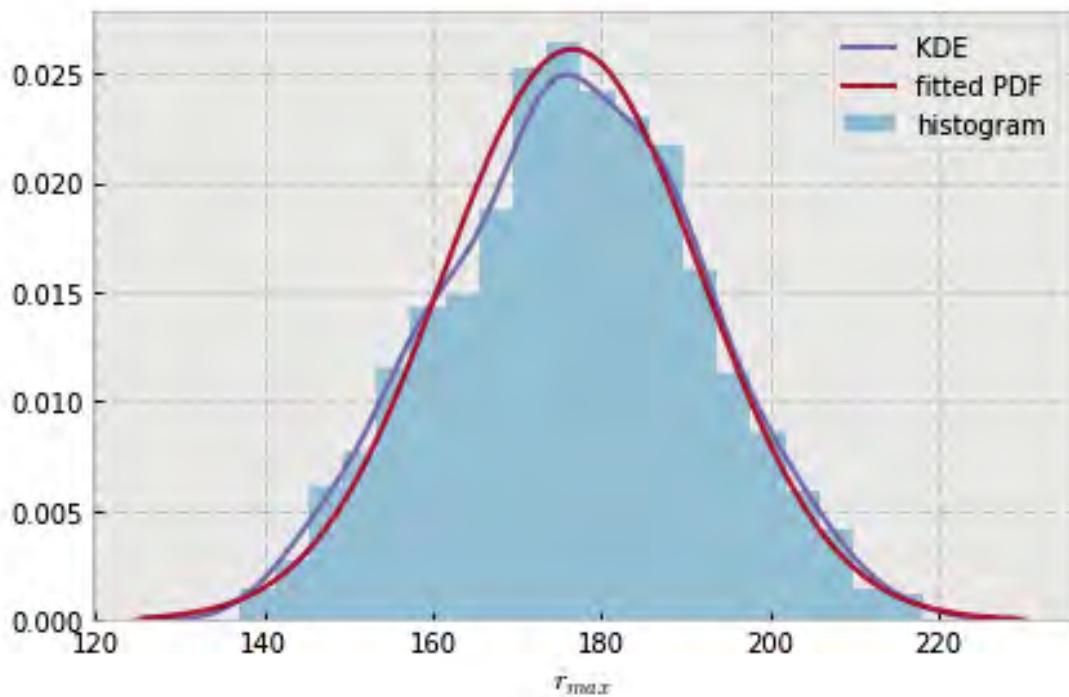


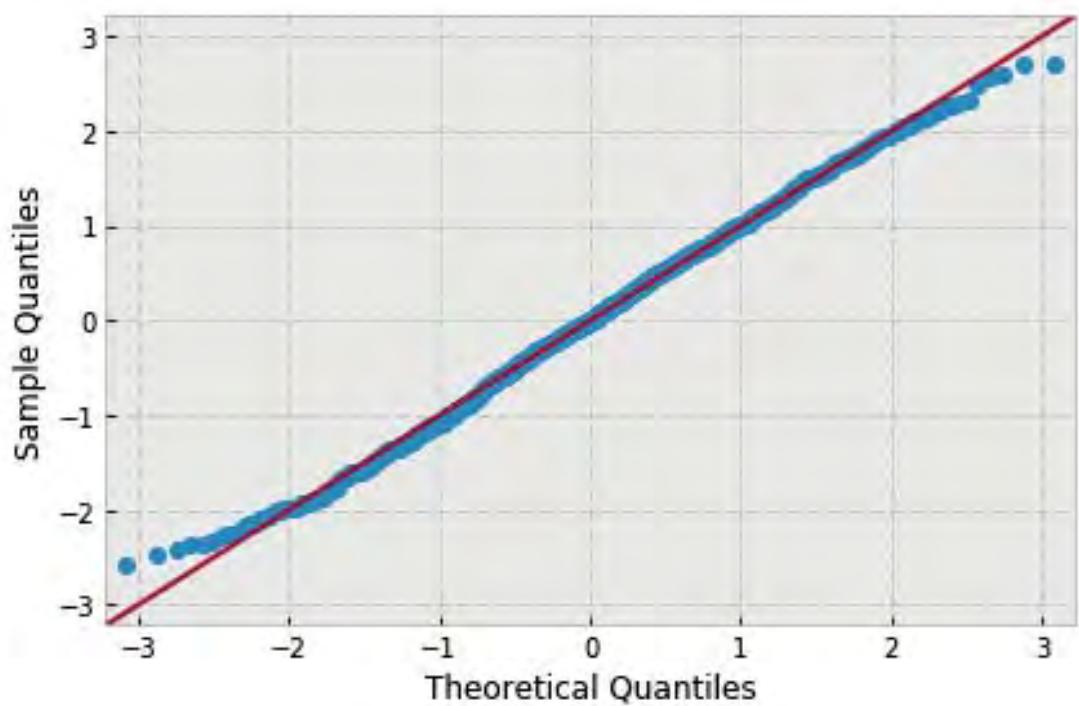


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=173.896039710697, scale=15.198563134085)
DescribeResult(nobs=1000, minmax=(131.69123738296818, 221.04983353714553)
               mean=173.89603971069701, variance=231.2275488896573,
               skewness=0.07910436829414269, kurtosis=-0.12247183297206332)
```

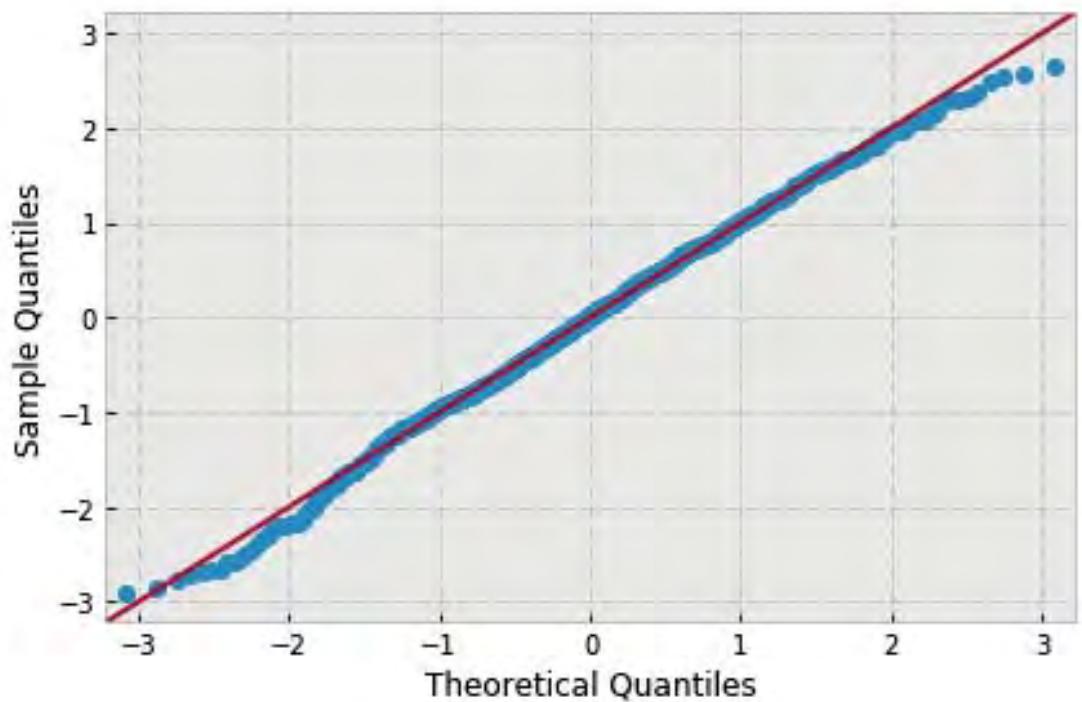
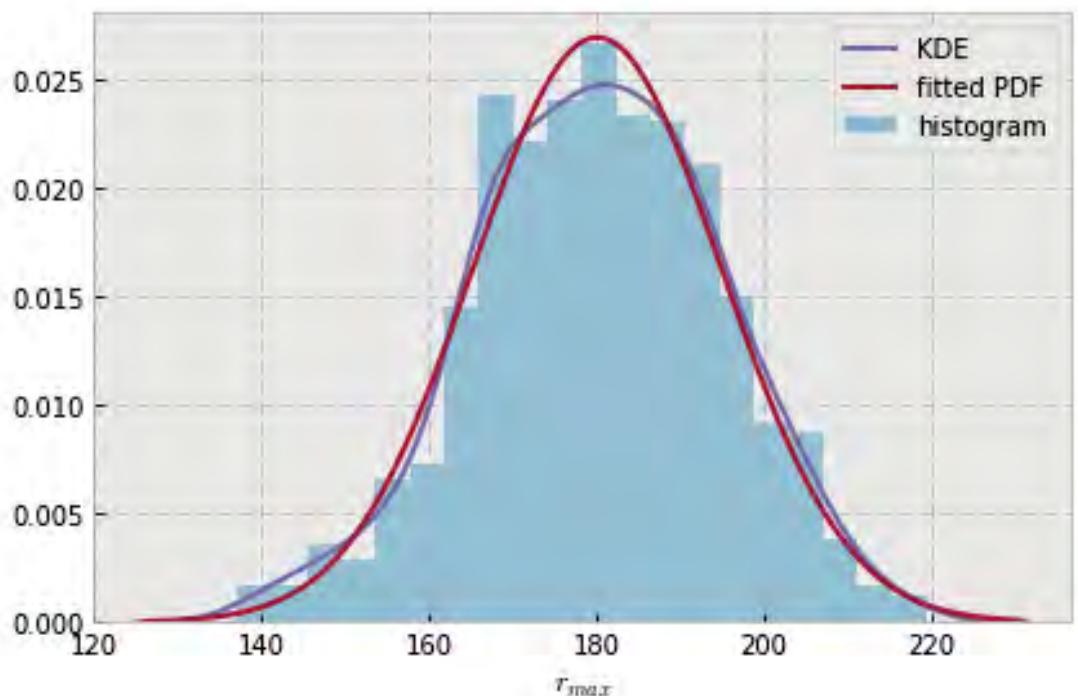


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=176.52010616457824, scale=15.291244750143395)
DescribeResult(nobs=1000, minmax=(137.09891616397442, 217.96043589313044)
               mean=176.52010616457824, variance=234.05622223101898,
               skewness=-0.021925573310730643, kurtosis=-0.3941728228820818)
```

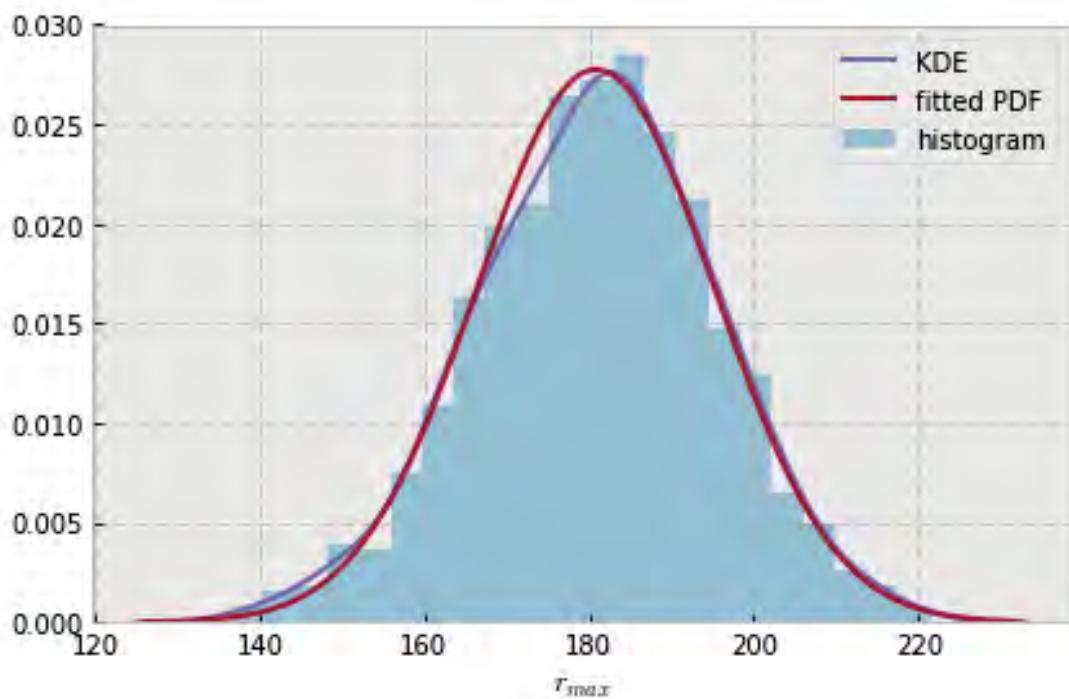


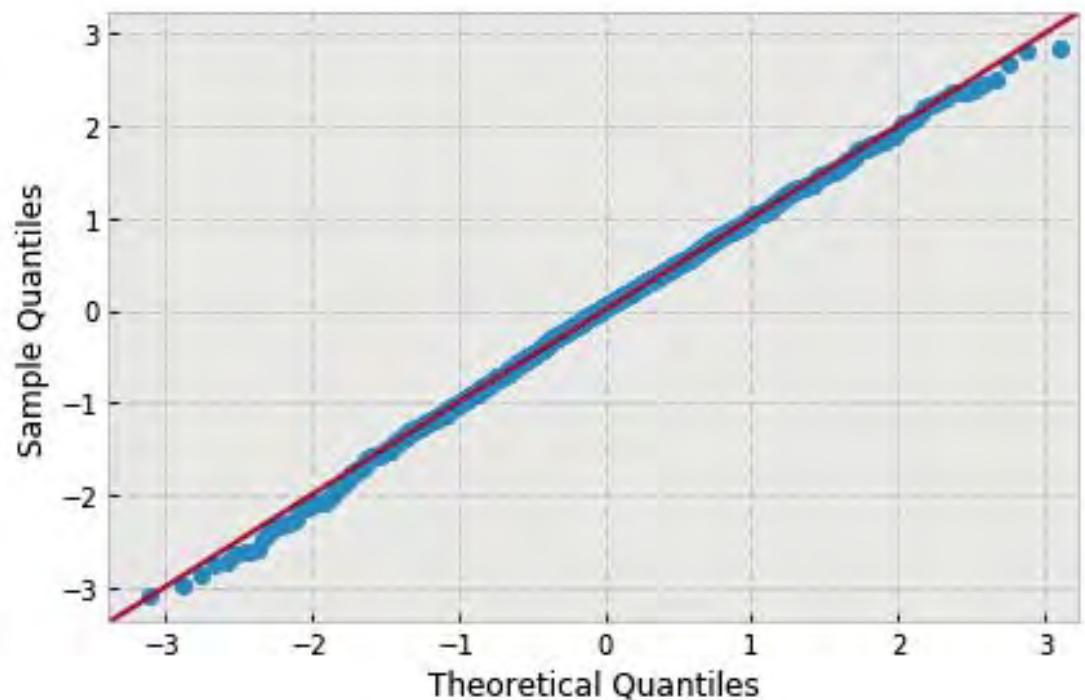


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=180.07906213313203, scale=14.816885436213513)
DescribeResult(nobs=1000, minmax=(137.05614314776574, 219.42212602755725)
               mean=180.07906213313203, variance=219.75985388375983,
               skewness=-0.12859925357460236, kurtosis=-0.1438821700406585)
```

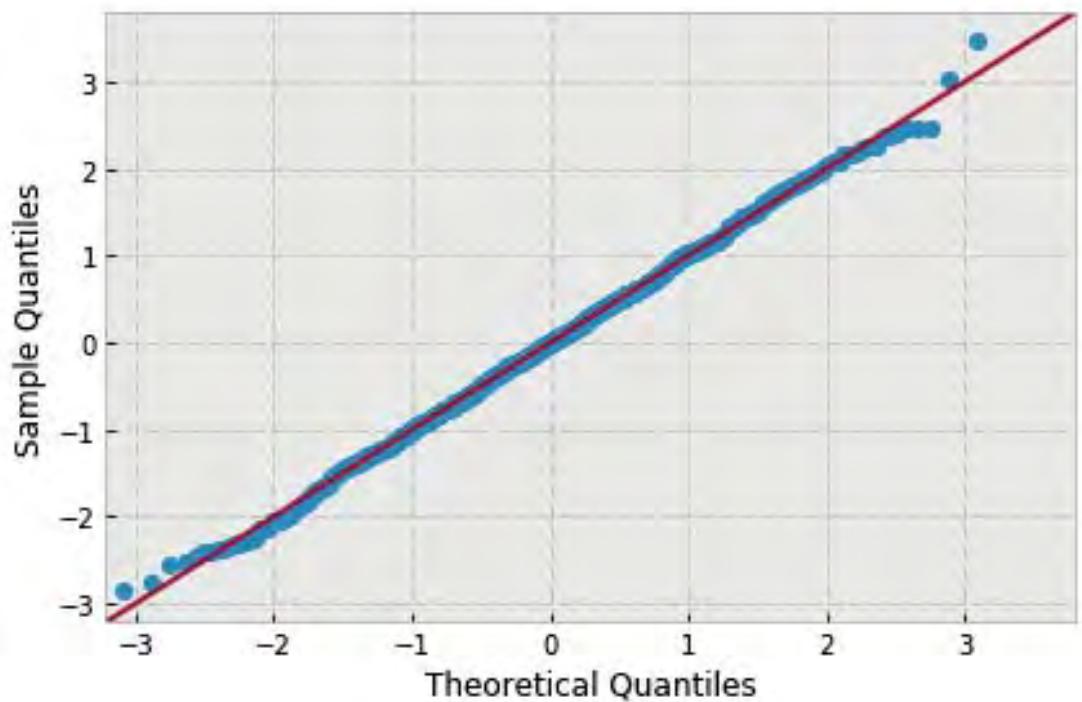
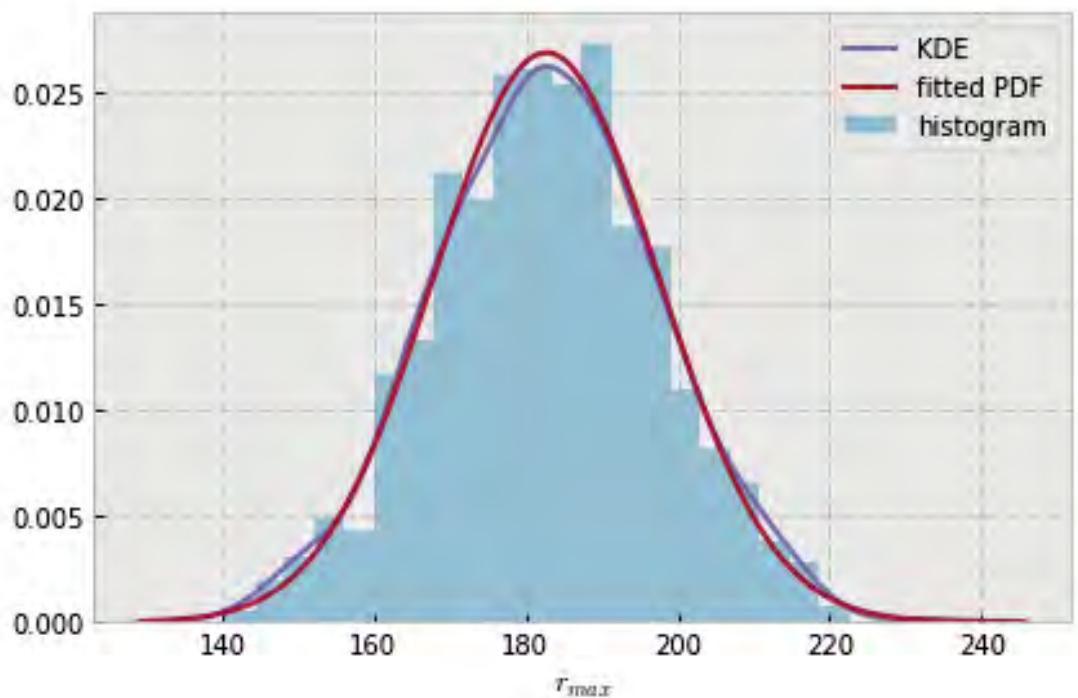


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=180.79751697228116, scale=14.3850408711608)
DescribeResult(nobs=1000, minmax=(136.53995035941253, 221.41153698654293)
               mean=180.79751697228116, variance=207.13653740236904,
               skewness=-0.12201875080490893, kurtosis=-0.06349968230064773)
```

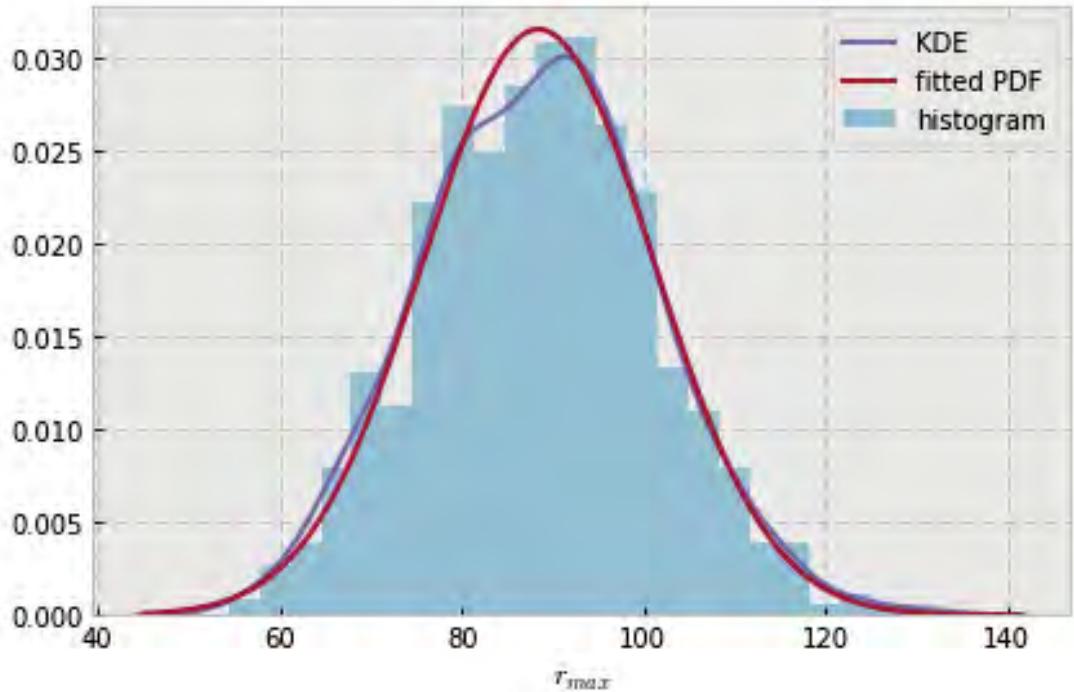


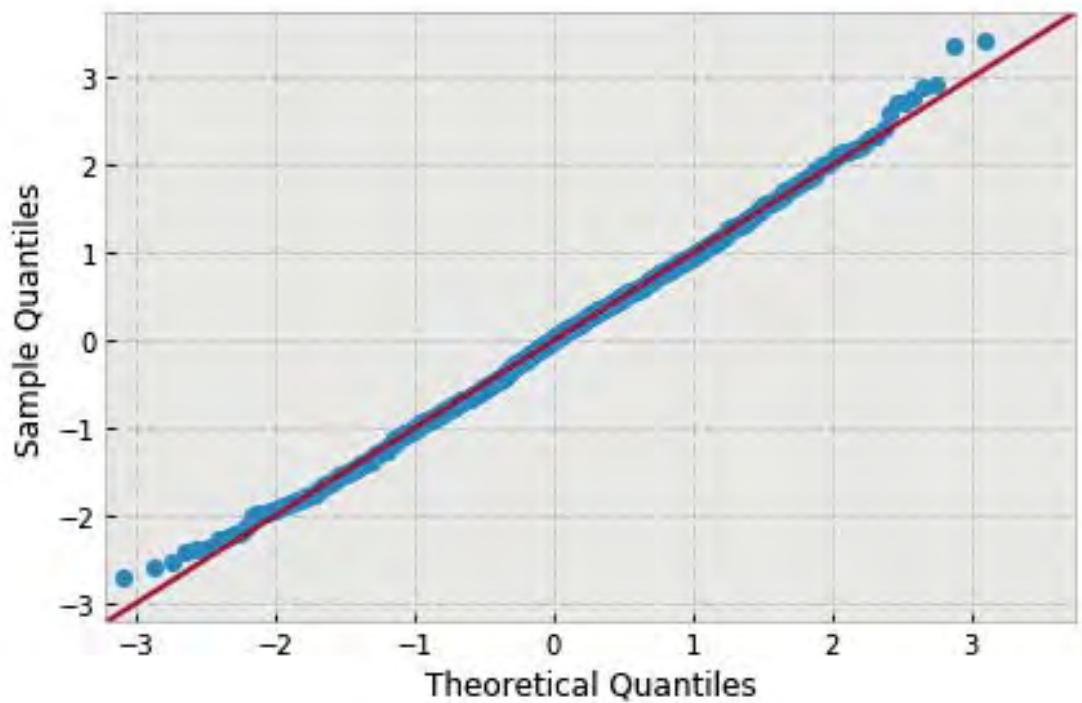


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=182.7122688141931, scale=14.817809980281883)
DescribeResult(nobs=1000, minmax=(140.55036368634666, 234.20369977619674)
               mean=182.7122688141931, variance=219.78727989163298,
               skewness=0.01606110483132167, kurtosis=-0.11626360728297414)
```

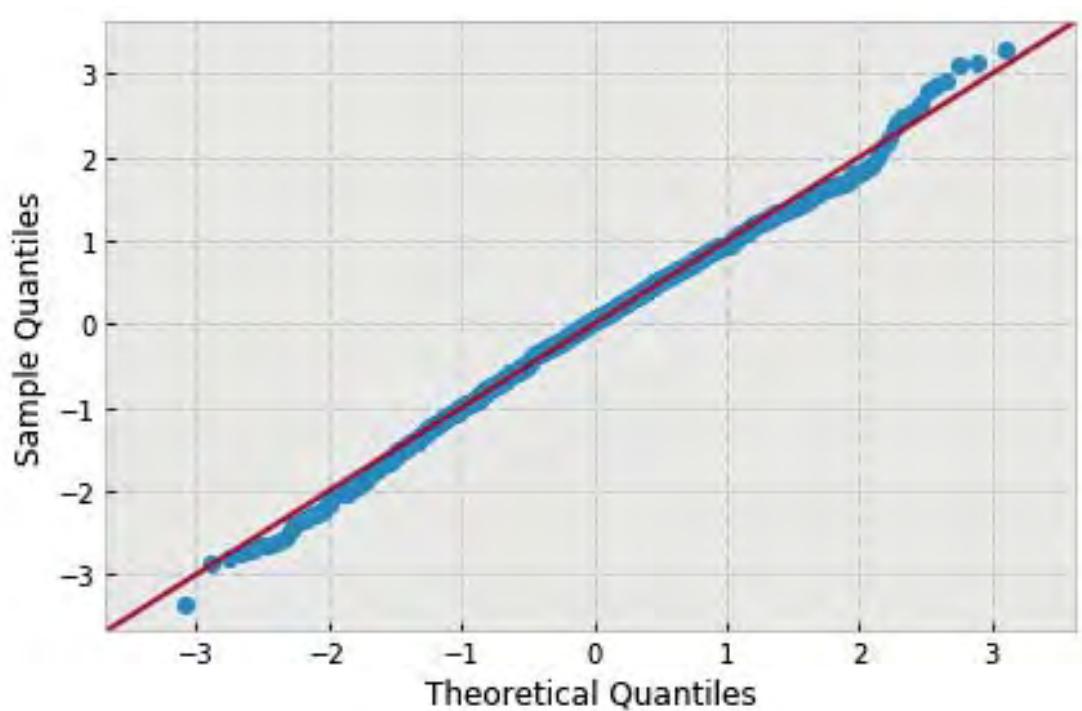
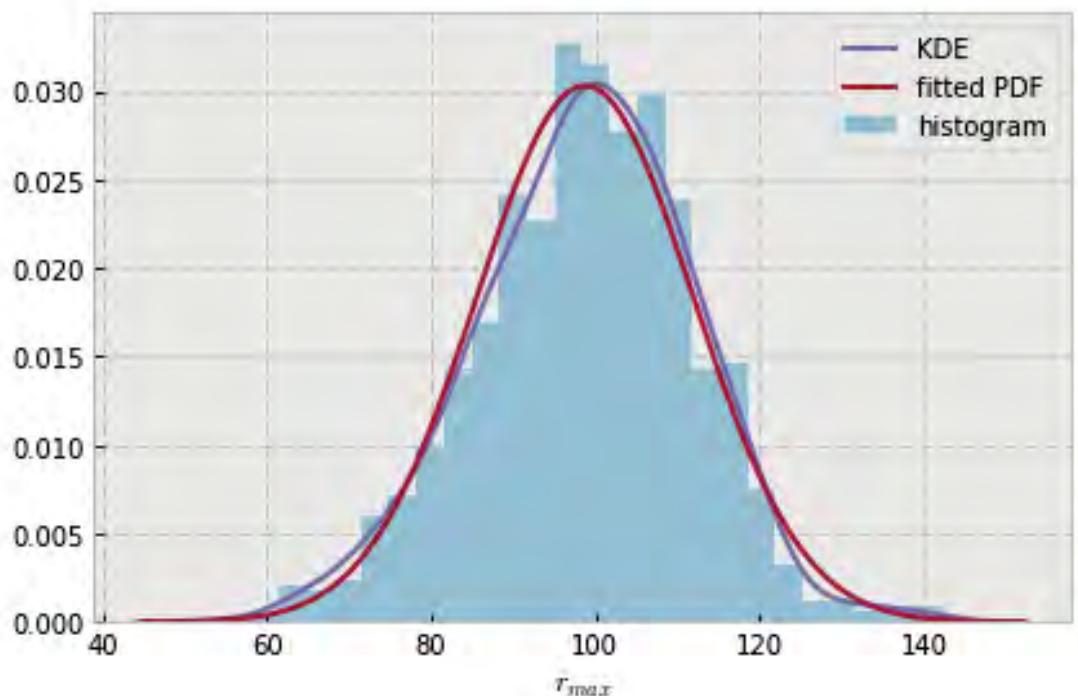


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=88.45013404746486, scale=12.634414118917265)
DescribeResult(nobs=1000, minmax=(54.412895439480025, 131.79424698378924)
               mean=88.450134047464857, variance=159.78820833663258,
               skewness=0.11064125049702866, kurtosis=-0.07273639665424803)
```

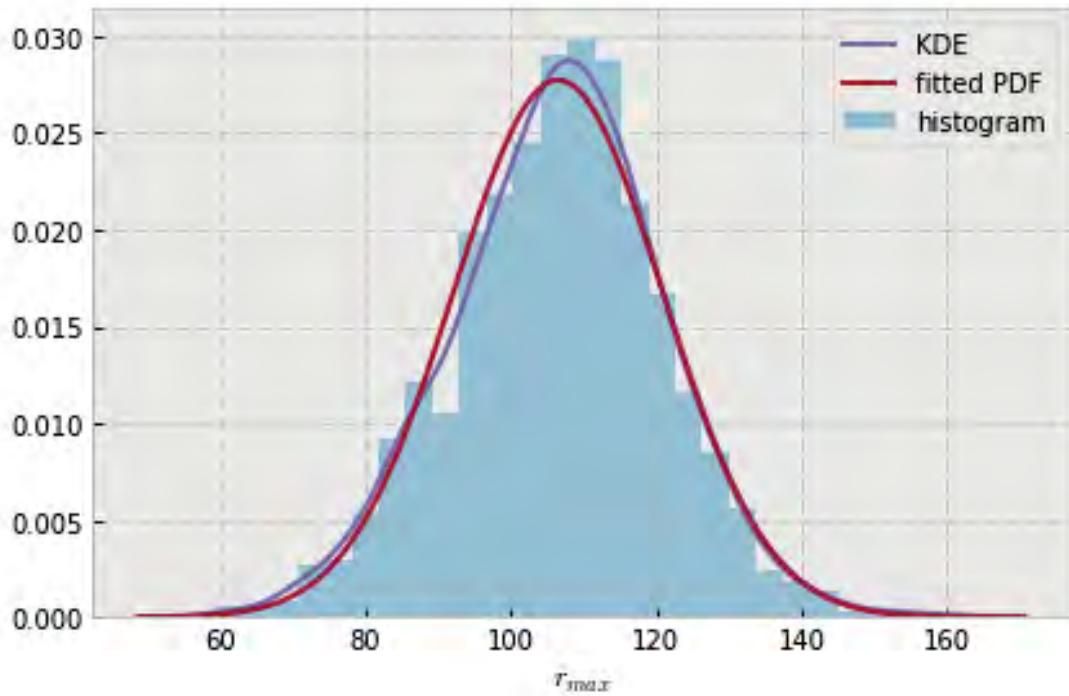


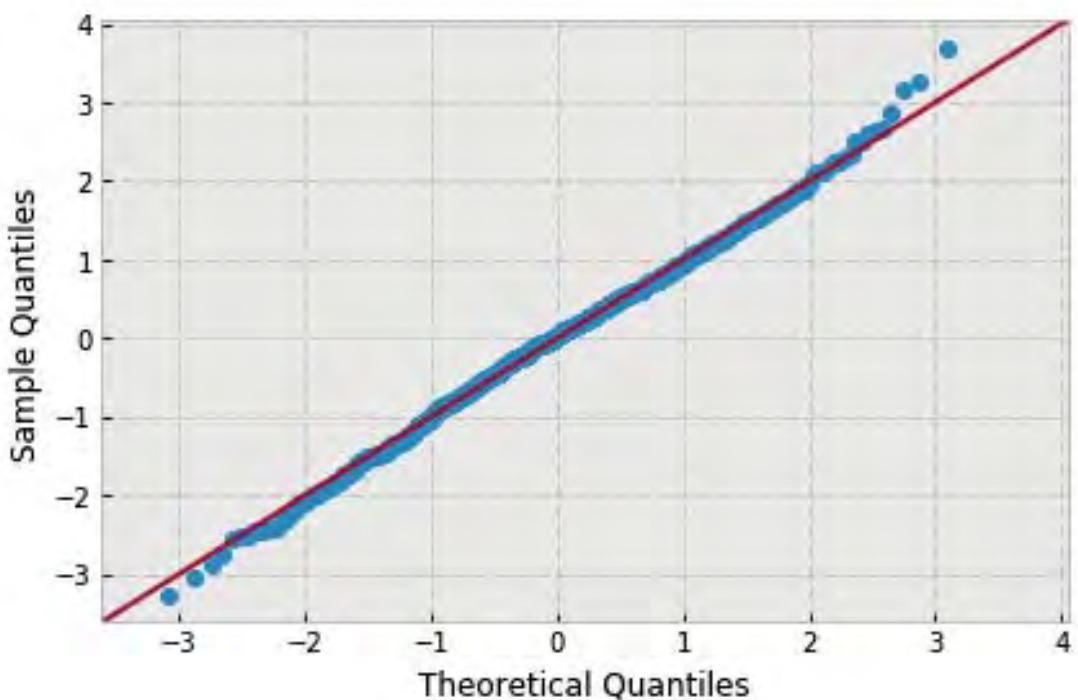


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(), loc=98.72812475358317, scale=13.169909893790619)
DescribeResult(nobs=1000, minmax=(54.675365624739634, 141.99855721806688)
               mean=98.728124753583174, variance=173.62014675732135,
               skewness=-0.13926691401210062, kurtosis=0.17641981657564276)
```

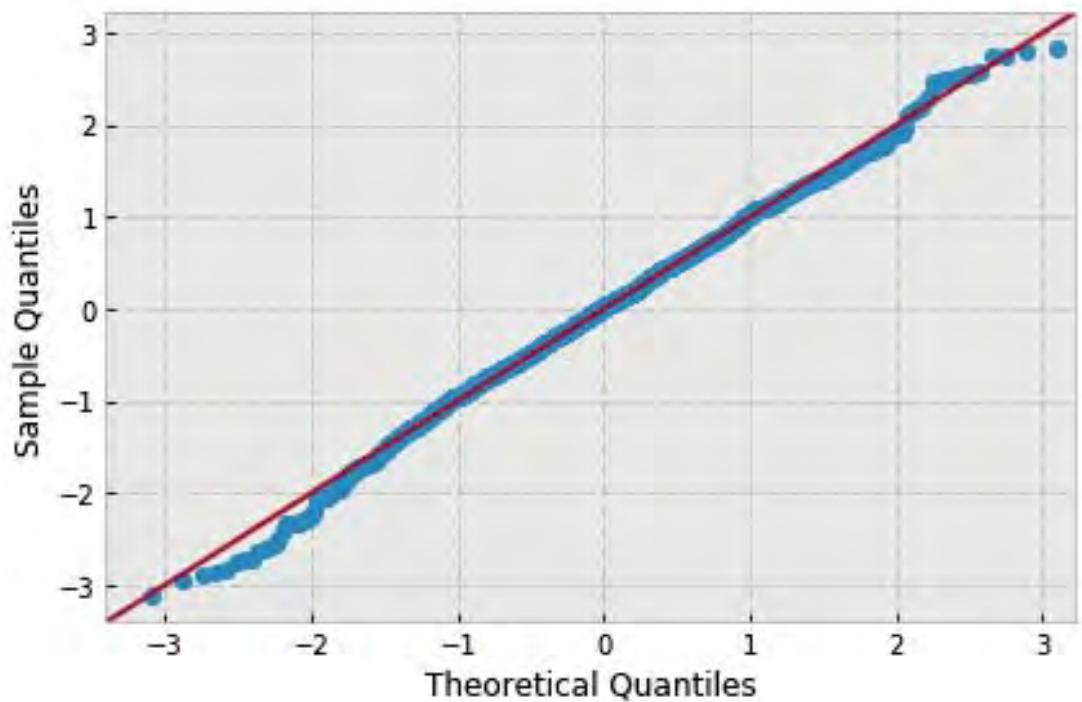
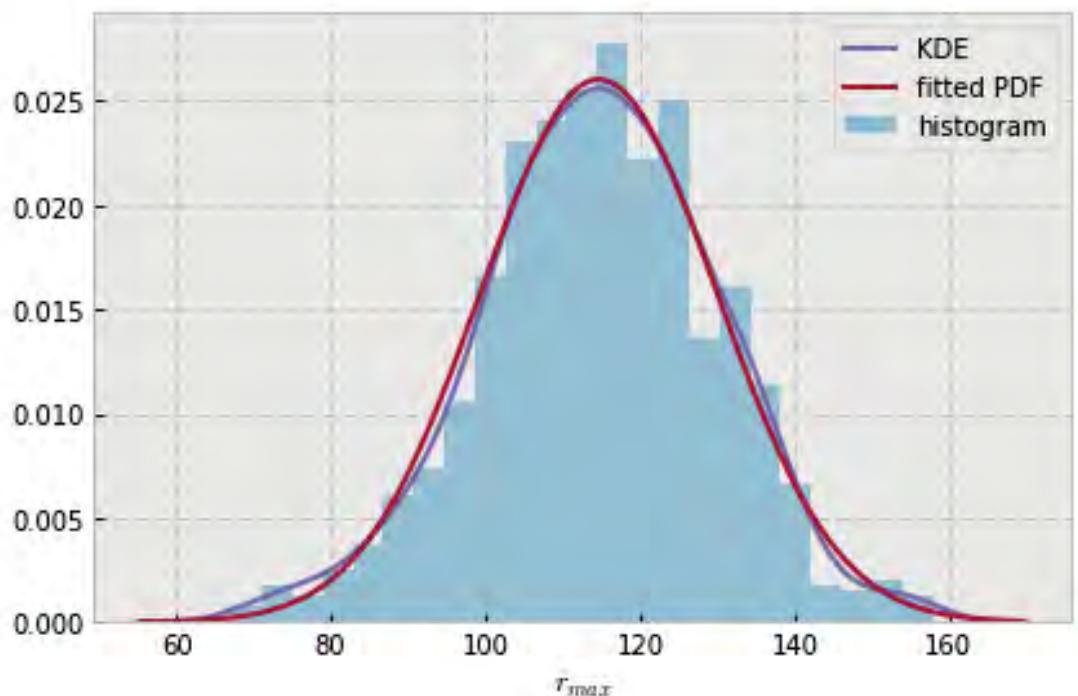


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=106.47812226117752, scale=14.392227816427084)
DescribeResult(nobs=1000, minmax=(59.465894381812689, 159.73678753488289)
               mean=106.47812226117752, variance=207.34356508502253,
               skewness=-0.05984384369323599, kurtosis=0.2328935393355165)
```

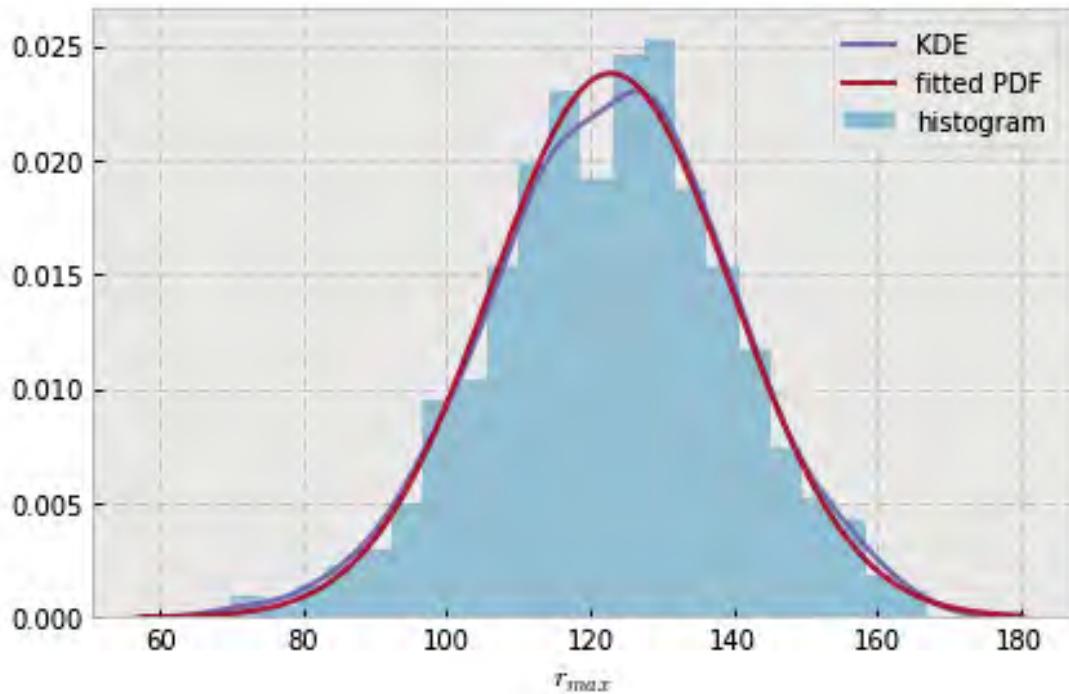


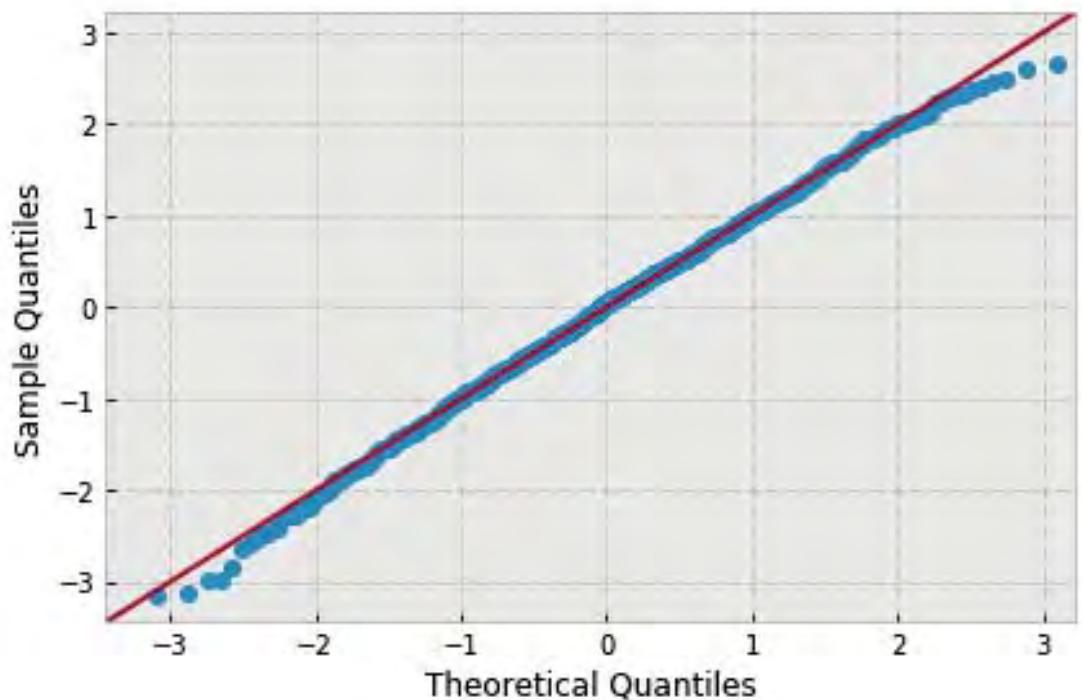


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=114.66443725344944, scale=15.330812442108211)
DescribeResult(nobs=1000, minmax=(67.003195889599709, 157.96535225434997)
               mean=114.66443725344944, variance=235.26907921431425,
               skewness=-0.1373038767239501, kurtosis=0.13947072464252708)
```

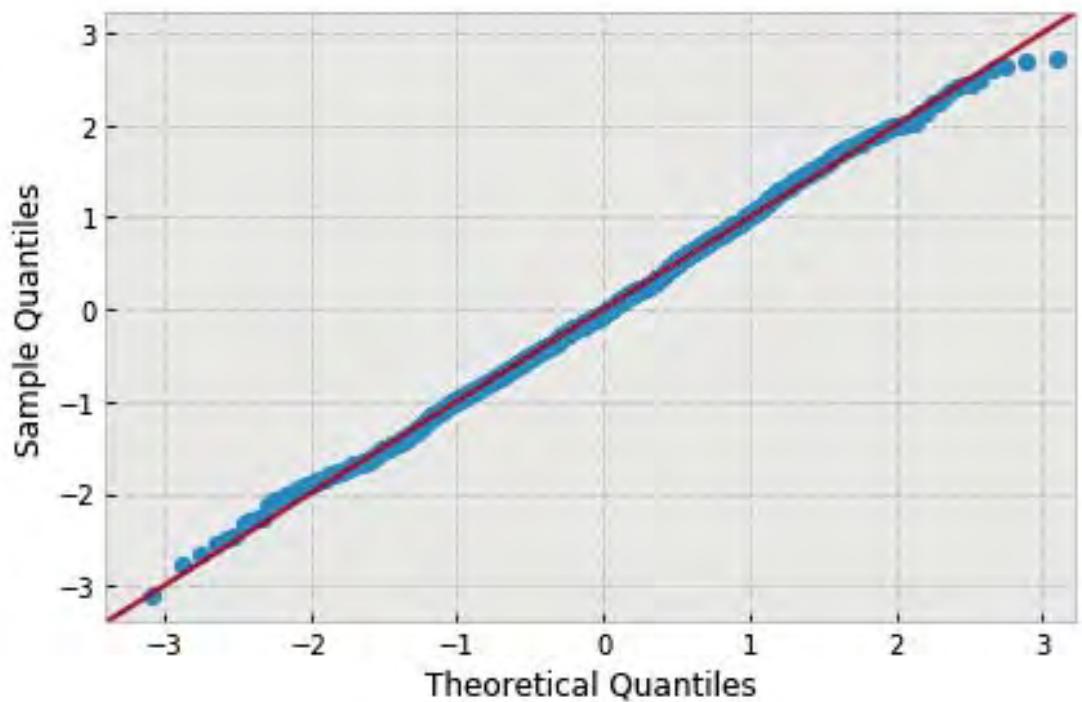
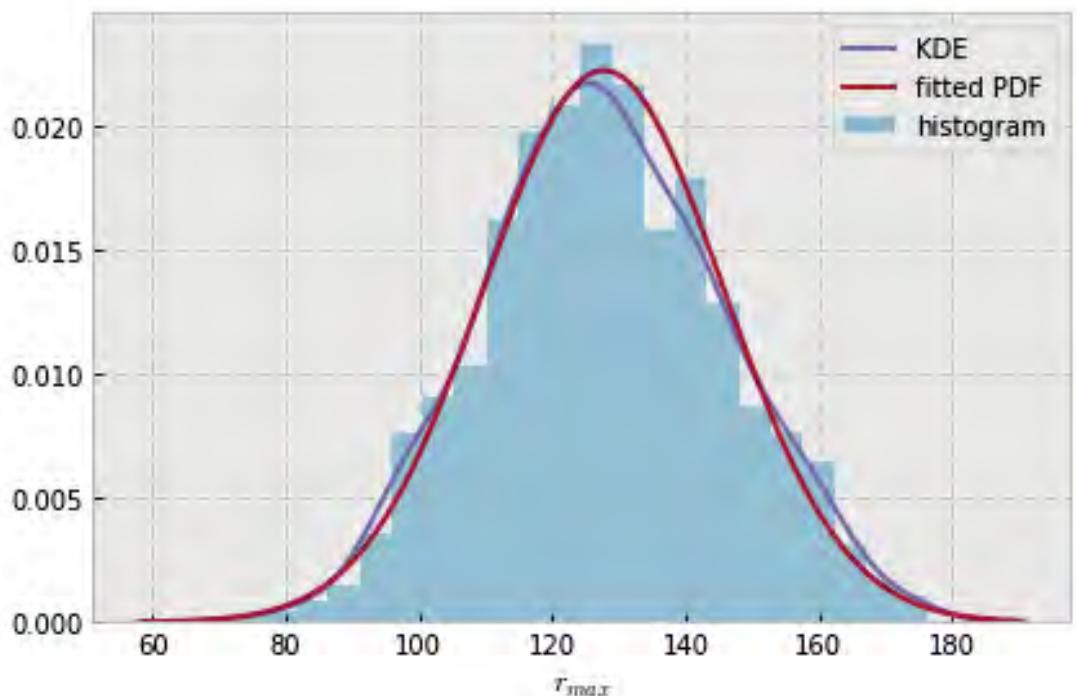


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=122.92601659093837, scale=16.76295910991927)
DescribeResult(nobs=1000, minmax=(70.088367062293372, 167.31642852554907)
               mean=122.92601659093837, variance=281.27807619702241,
               skewness=-0.12715037149858957, kurtosis=-0.06384209180502998)
```

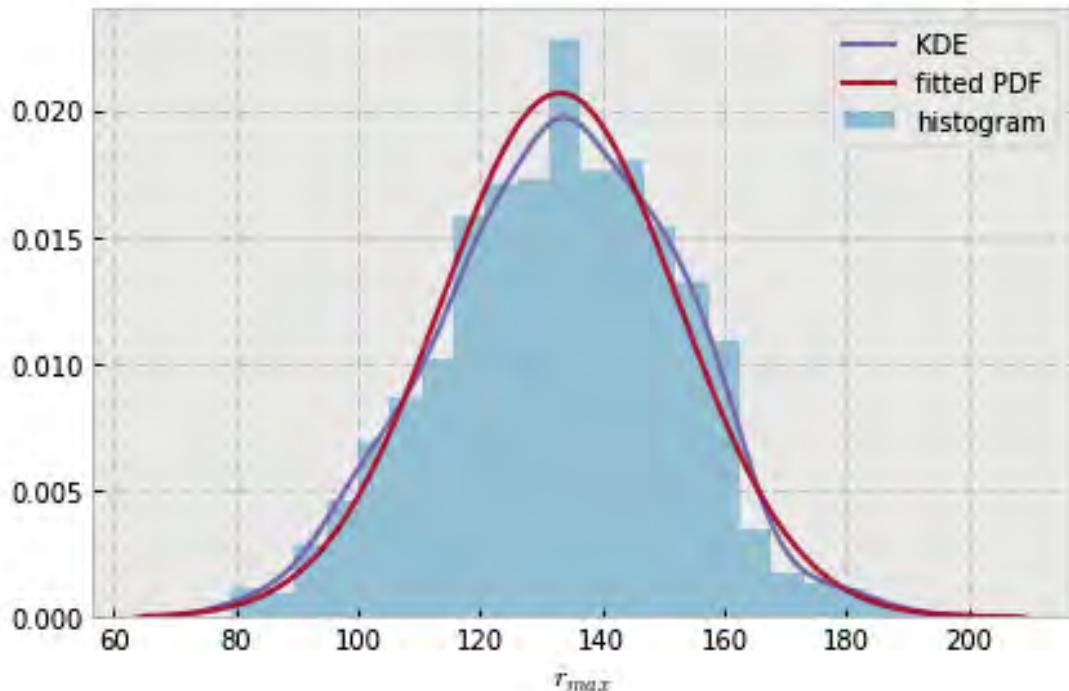


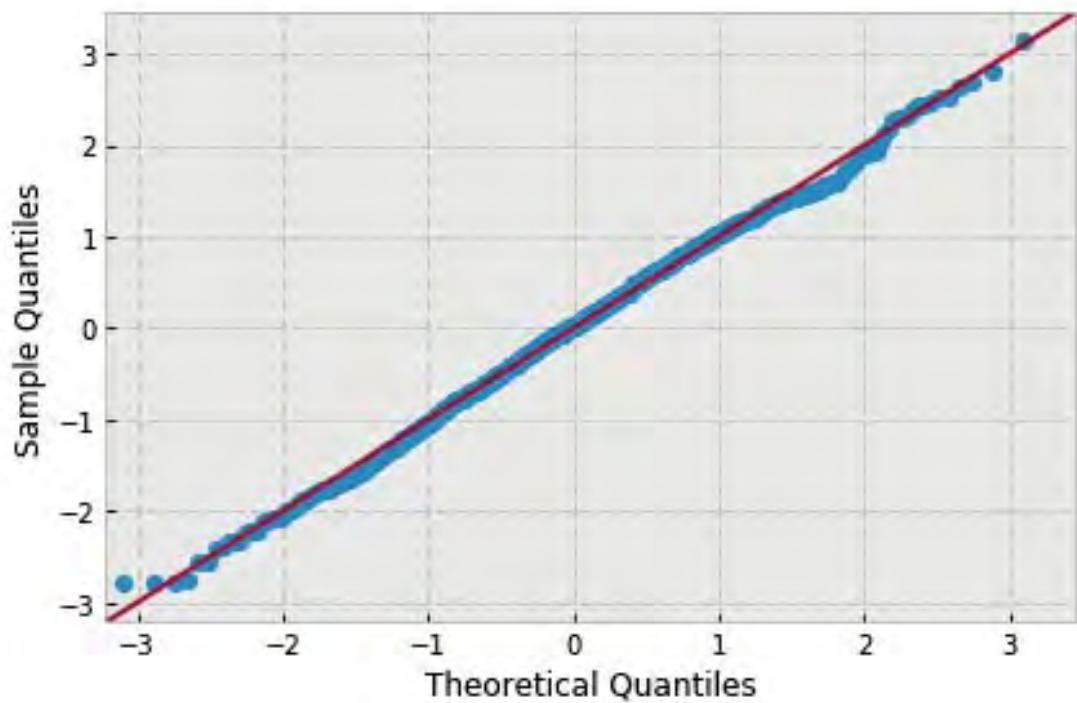


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(), loc=127.76104503355424, scale=17.949857767341893)
DescribeResult(nobs=1000, minmax=(72.072845915542857, 176.59308566534582)
               mean=127.76104503355424, variance=322.51991378158573,
               skewness=0.06533587829104441, kurtosis=-0.2715229717671712)
```

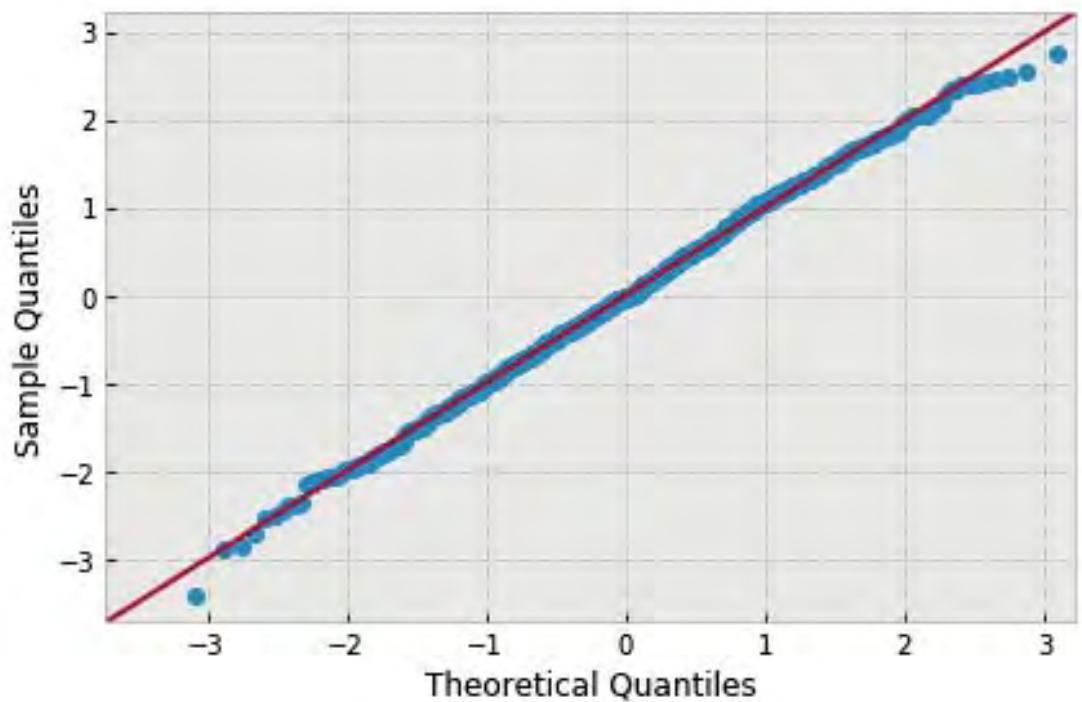
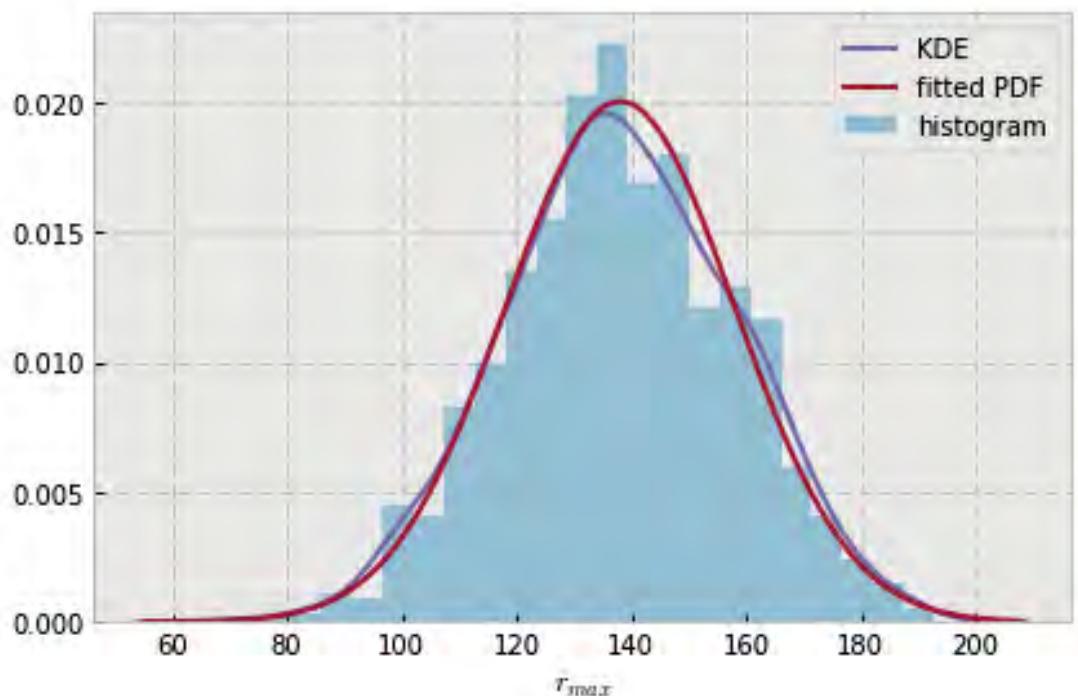


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=133.06869304796285, scale=19.29560781723754)
DescribeResult(nobs=1000, minmax=(79.185155068106269, 193.78039131438365)
               mean=133.06869304796285, variance=372.69317421084924,
               skewness=-0.09992785862432038, kurtosis=-0.20651980078089327)
```

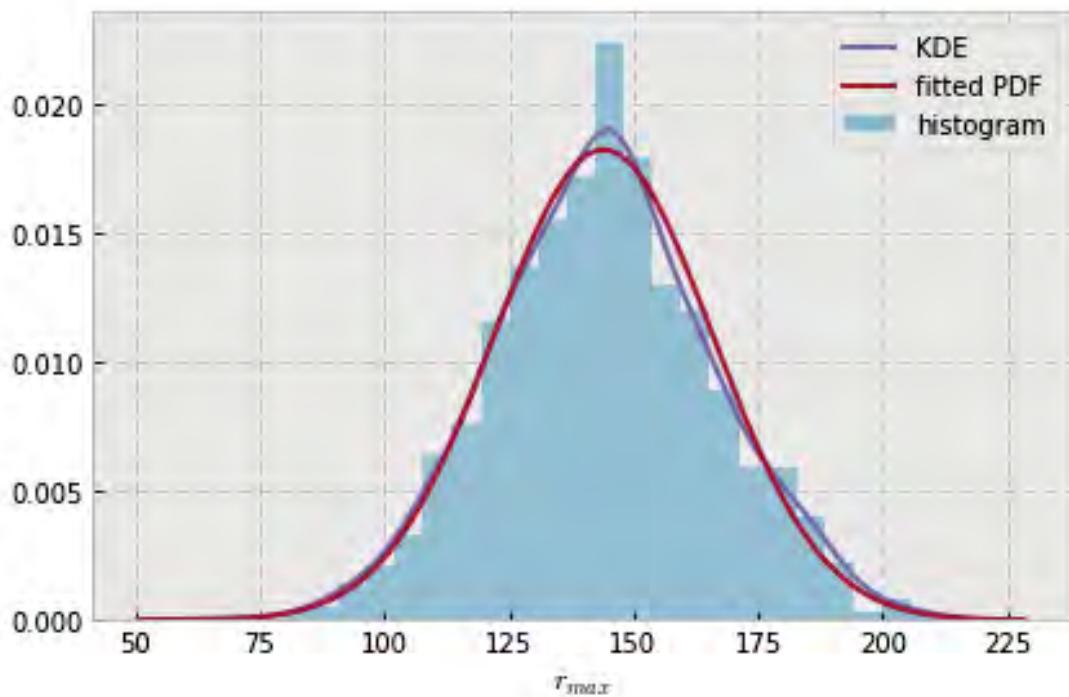


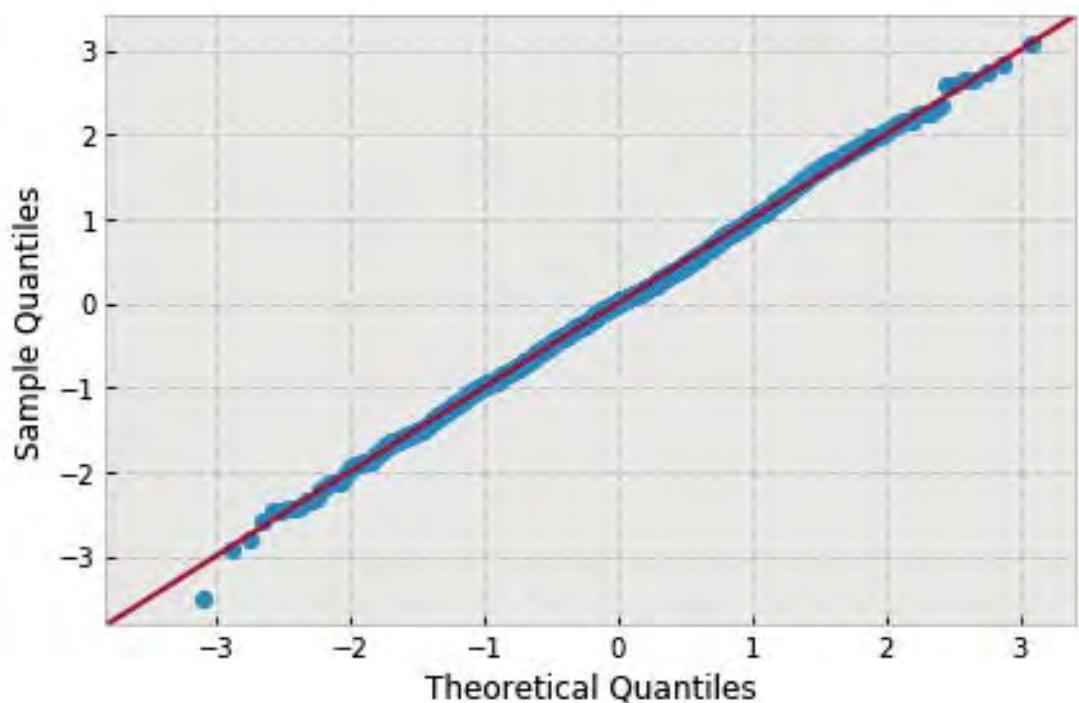


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=138.0739922987366, scale=19.930980529494565)
DescribeResult(nobs=1000, minmax=(70.016350481144528, 192.76726166582748)
               mean=138.07399229873661, variance=397.64162649358497,
               skewness=-0.04986544486799727, kurtosis=-0.21425343218380455)
```

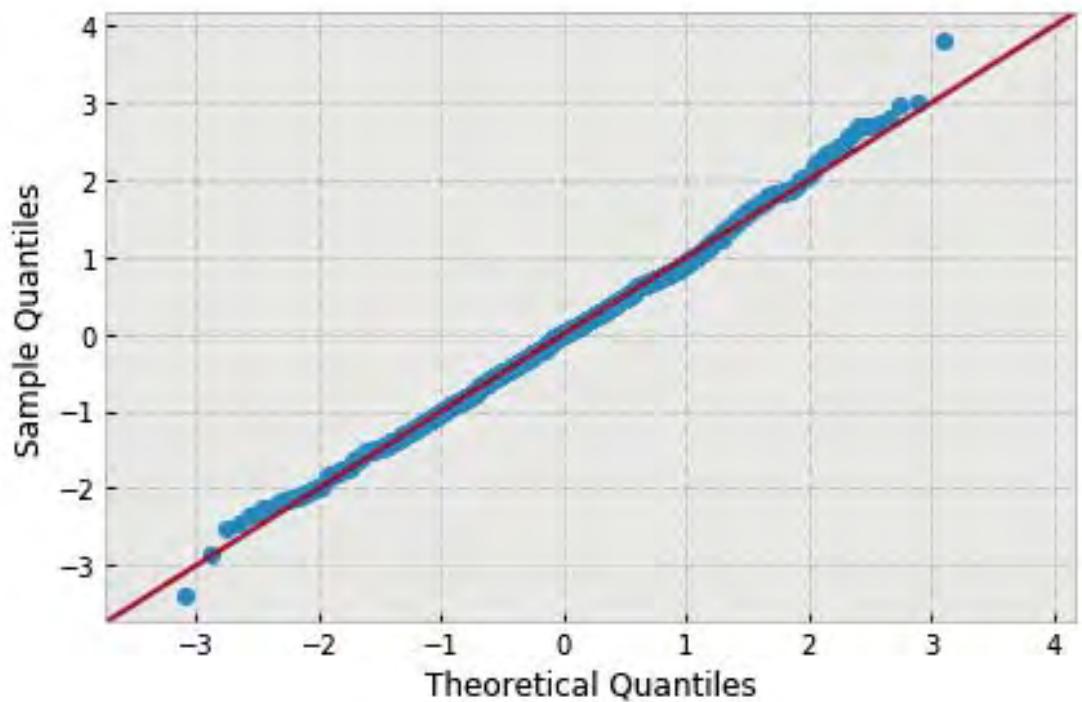
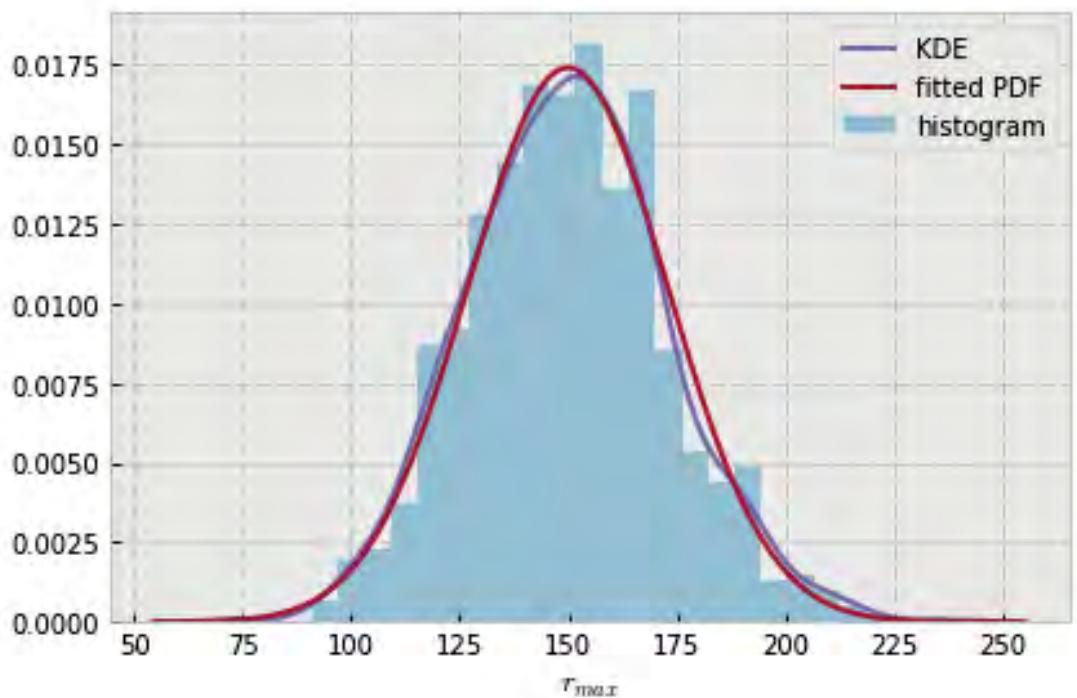


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=143.97732612075882, scale=21.884844065306282)
DescribeResult(nobs=1000, minmax=(67.566600175302526, 211.28328720464529)
               mean=143.97732612075882, variance=479.42582558835994,
               skewness=0.03544201763149416, kurtosis=-0.015230560927653958)
```

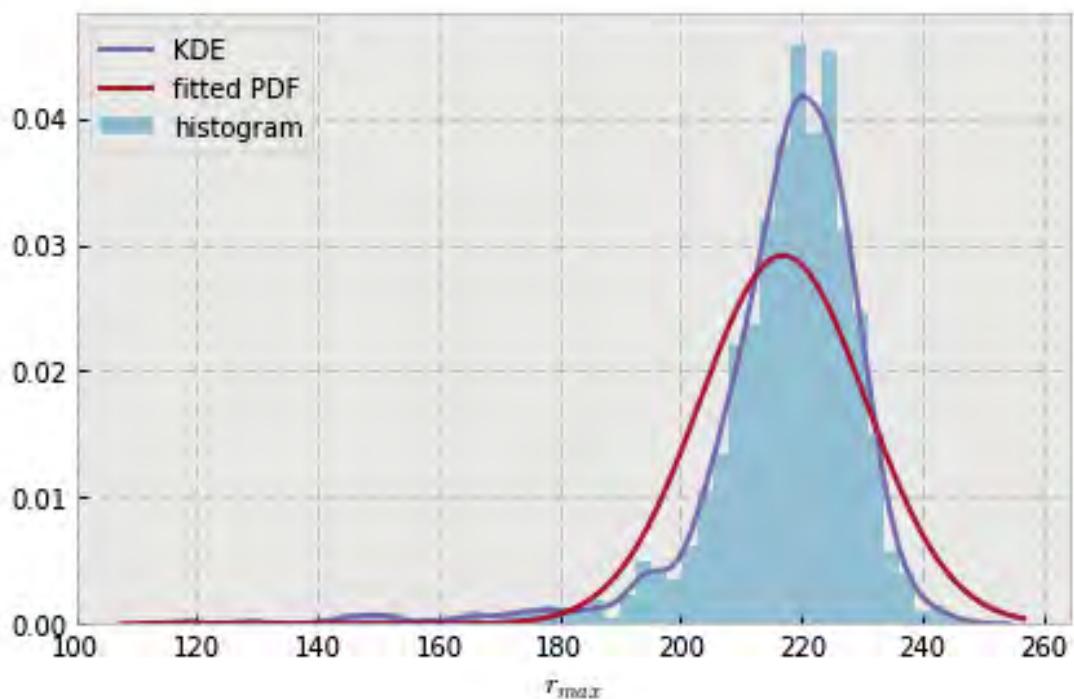


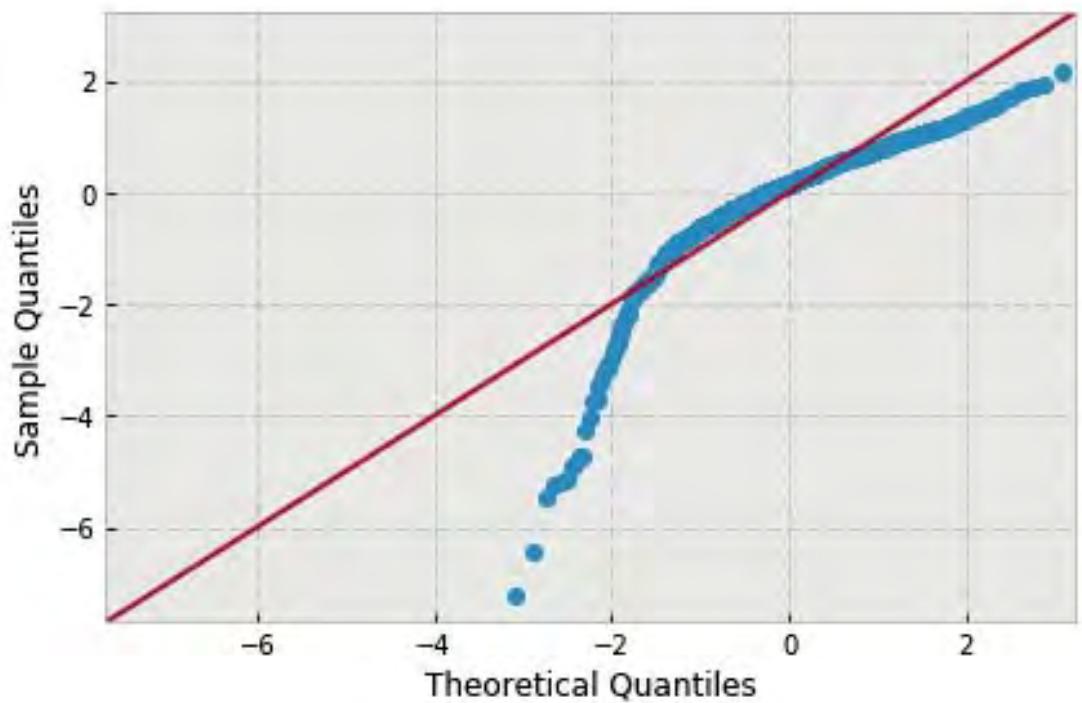


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=149.8509684564523, scale=22.908956603679382)
DescribeResult(nobs=1000, minmax=(72.466055248371163, 236.986802788695)
               mean=149.85096845645231, variance=525.3456383075727,
               skewness=0.1609327492946264, kurtosis=0.1390139516471991)
```

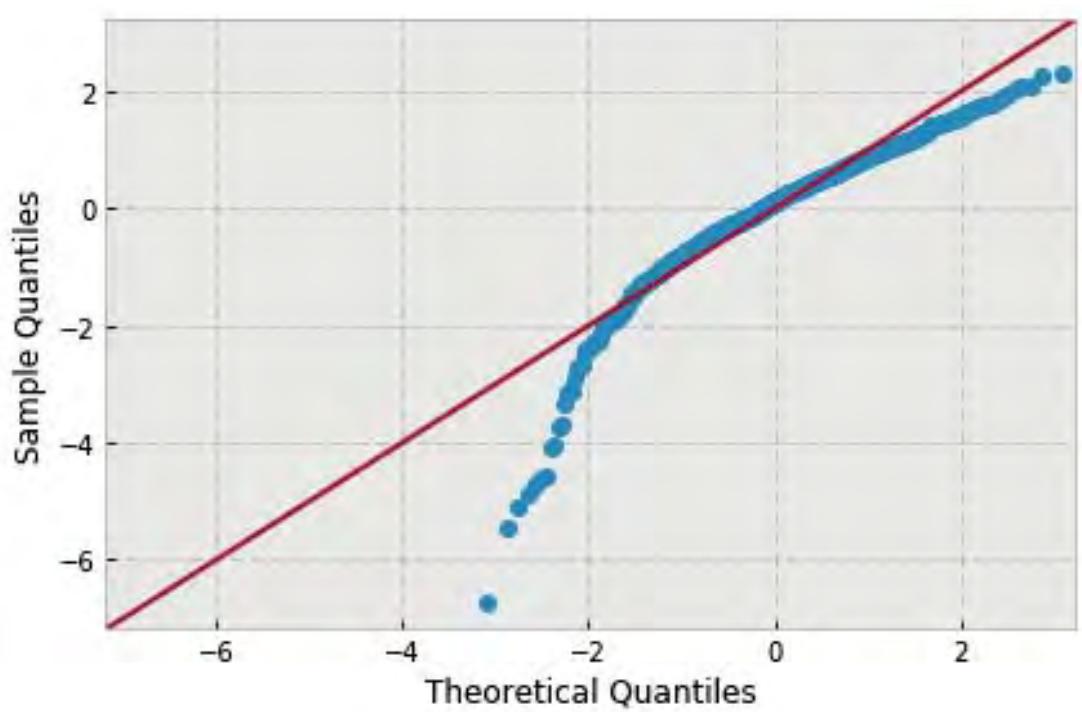
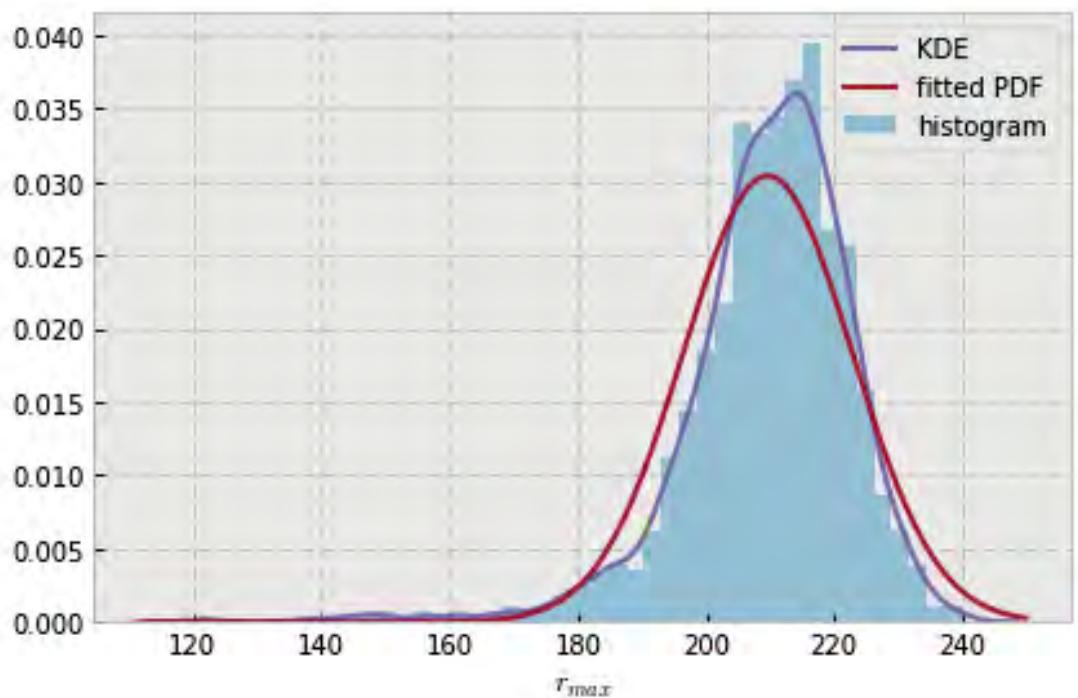


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=216.92030551530544, scale=13.69112952493616)
DescribeResult(nobs=1000, minmax=(117.9165848806994, 246.38164168464965)
               mean=216.92030551530544, variance=187.63466233090955,
               skewness=-2.37397193760475, kurtosis=9.81373206296626)
```

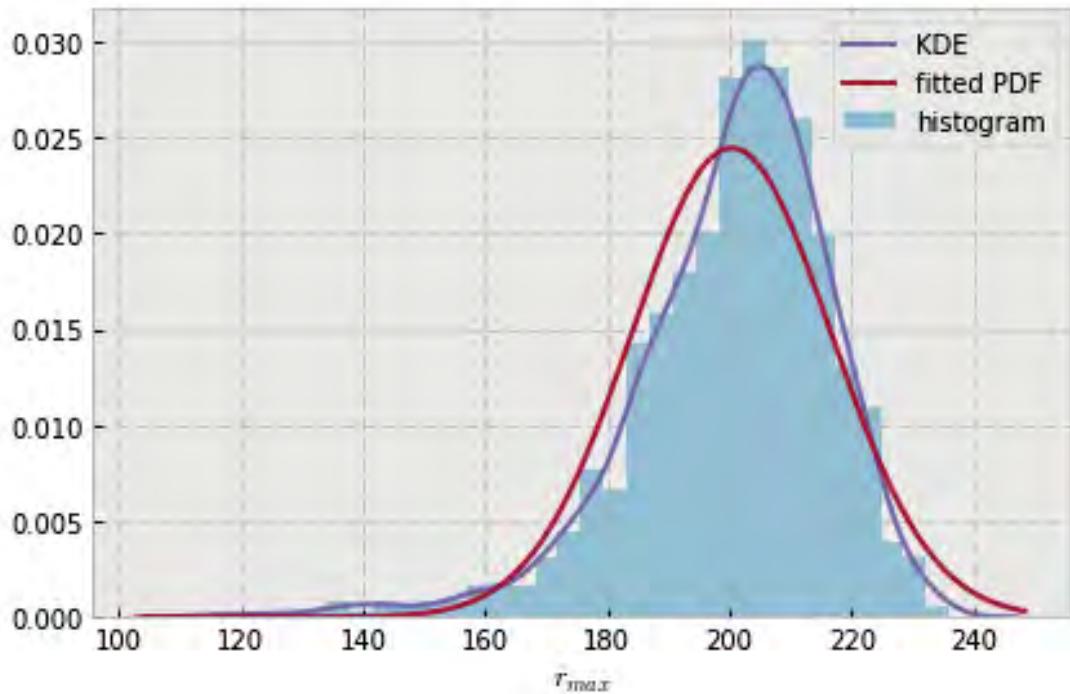


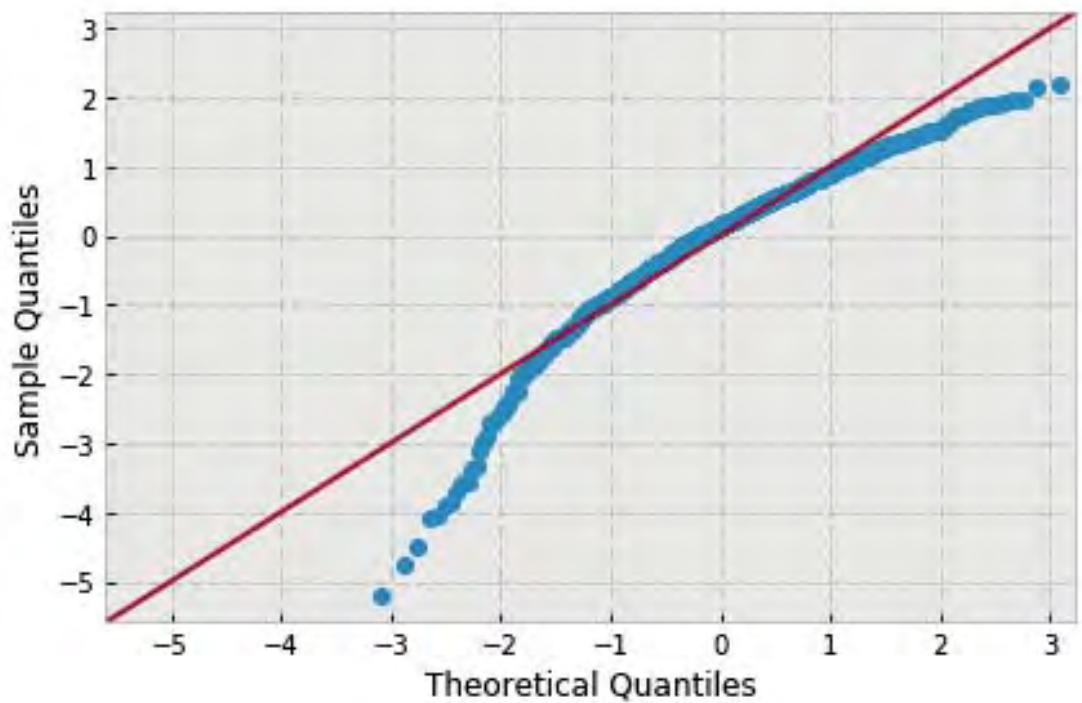


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=209.62501579396073, scale=13.118023180542133)
DescribeResult(nobs=1000, minmax=(121.32844660230974, 239.94634862656957)
               mean=209.62501579396073, variance=172.25478695219292,
               skewness=-1.52521879143638, kurtosis=5.559423841669057)
```

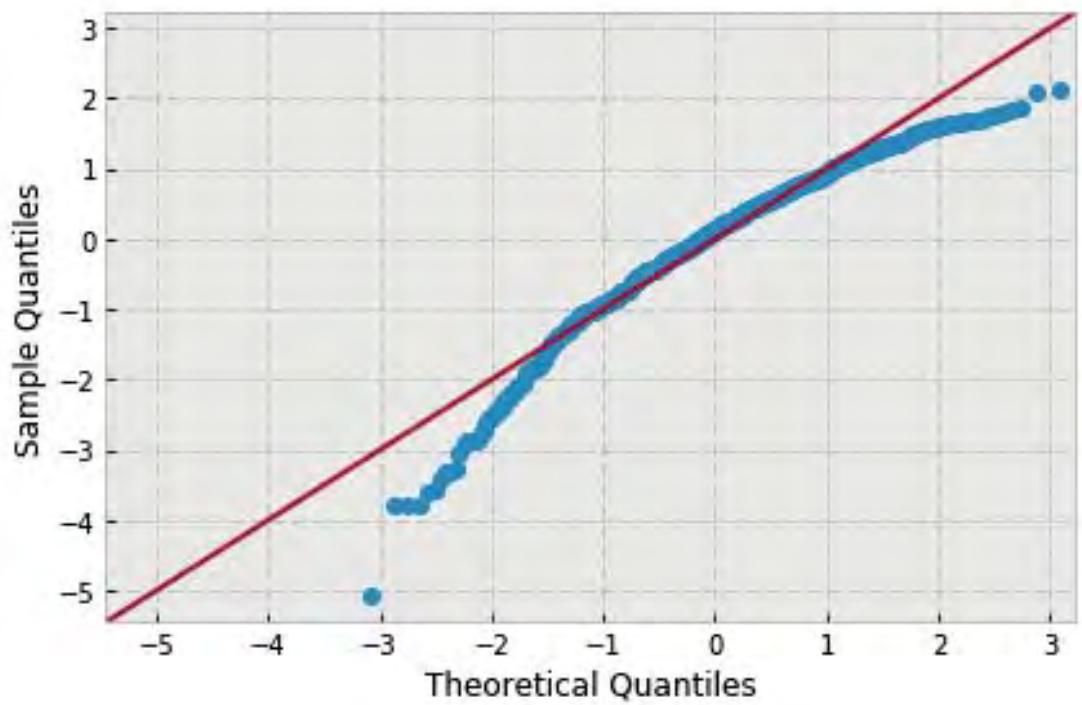
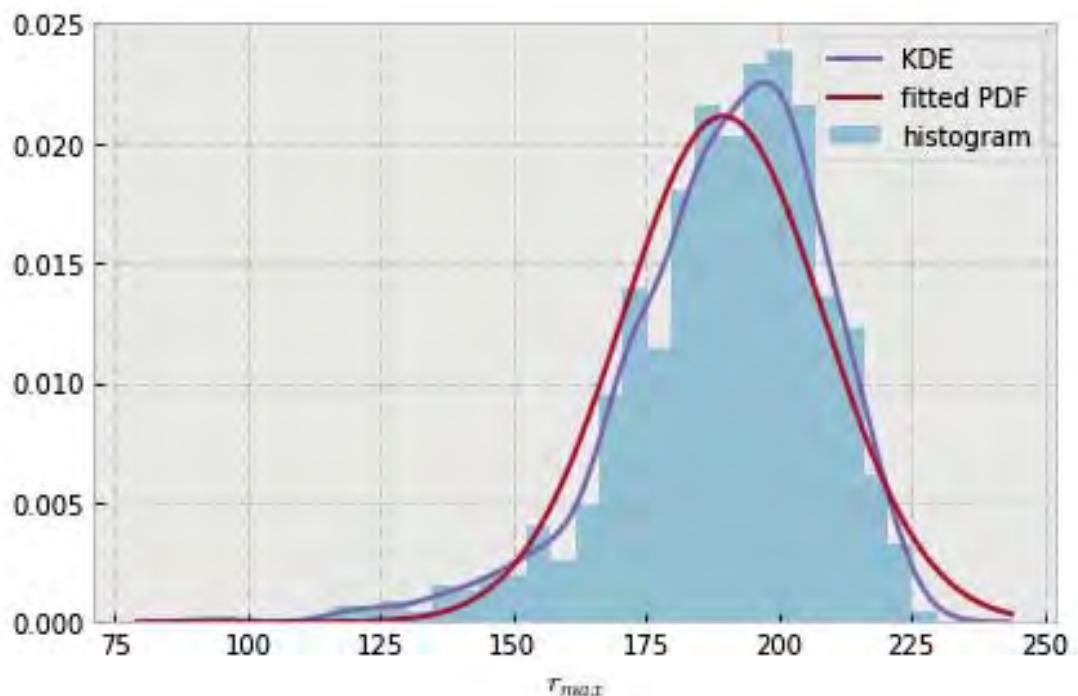


```
variable name: sigma
variable value: 0.03333333333333333
distribution: normal(shape=(), loc=200.32299877169407, scale=16.30906444237312)
DescribeResult(nobs=1000, minmax=(115.22546382455286, 236.05255786715892)
               mean=200.32299877169407, variance=266.25183482029956,
               skewness=-1.1435268559540674, kurtosis=2.66291351248608)
```

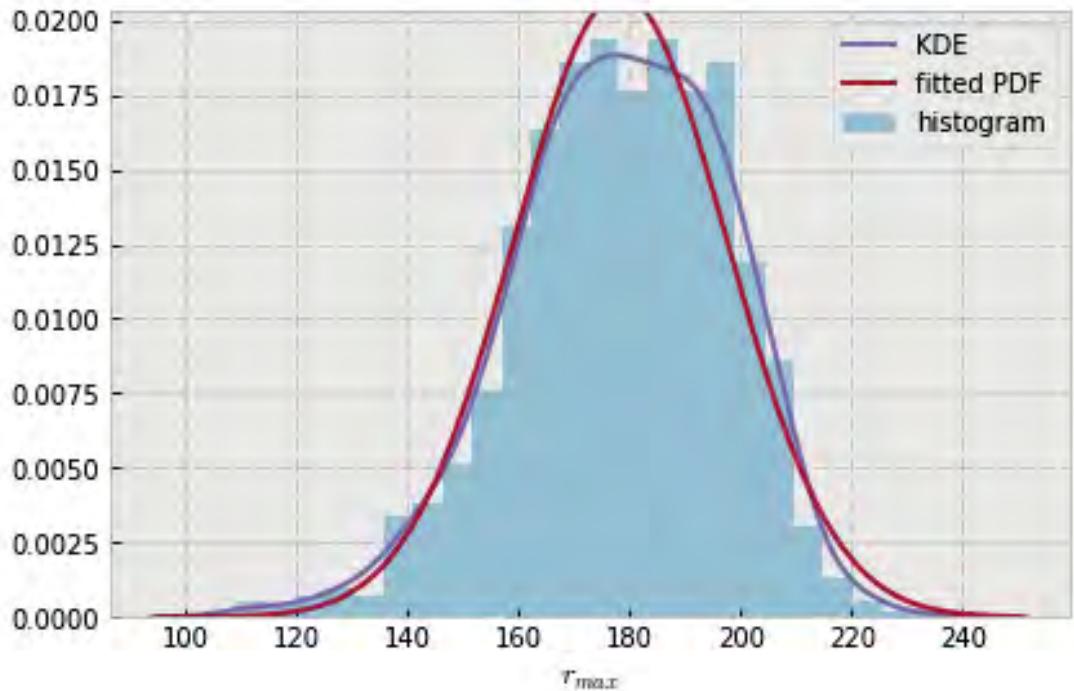


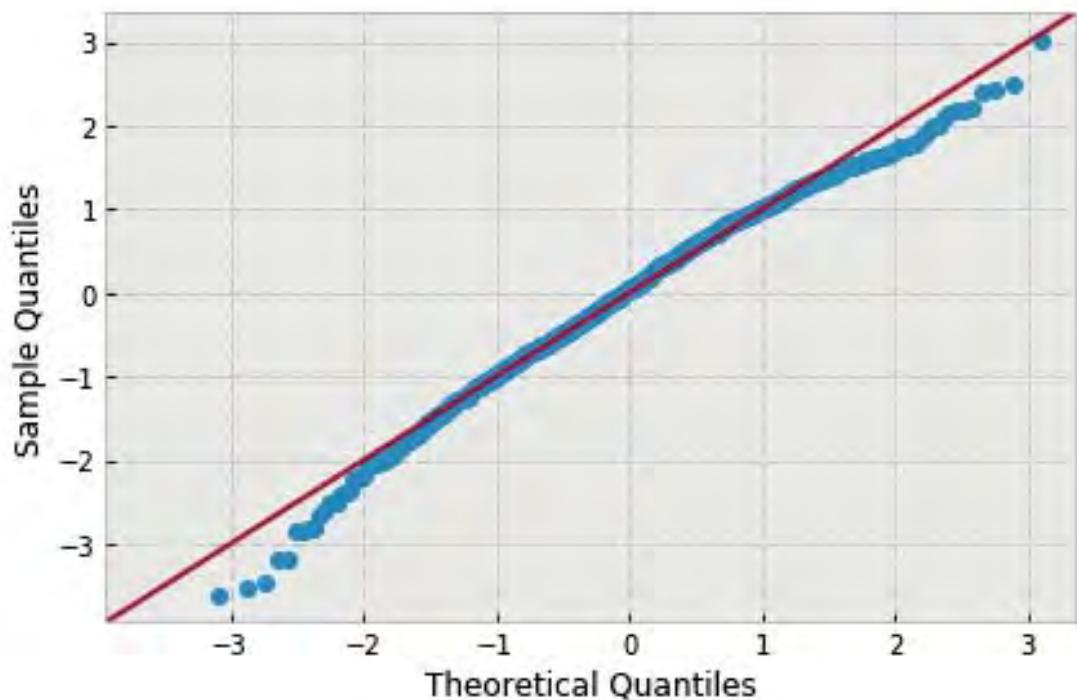


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=189.5703129117257, scale=18.87918396210126)
DescribeResult(nobs=1000, minmax=(93.495399697815429, 229.58722454843149)
               mean=189.57031291172569, variance=356.78036744230366,
               skewness=-0.9199188645873662, kurtosis=1.4814986275394766)
```

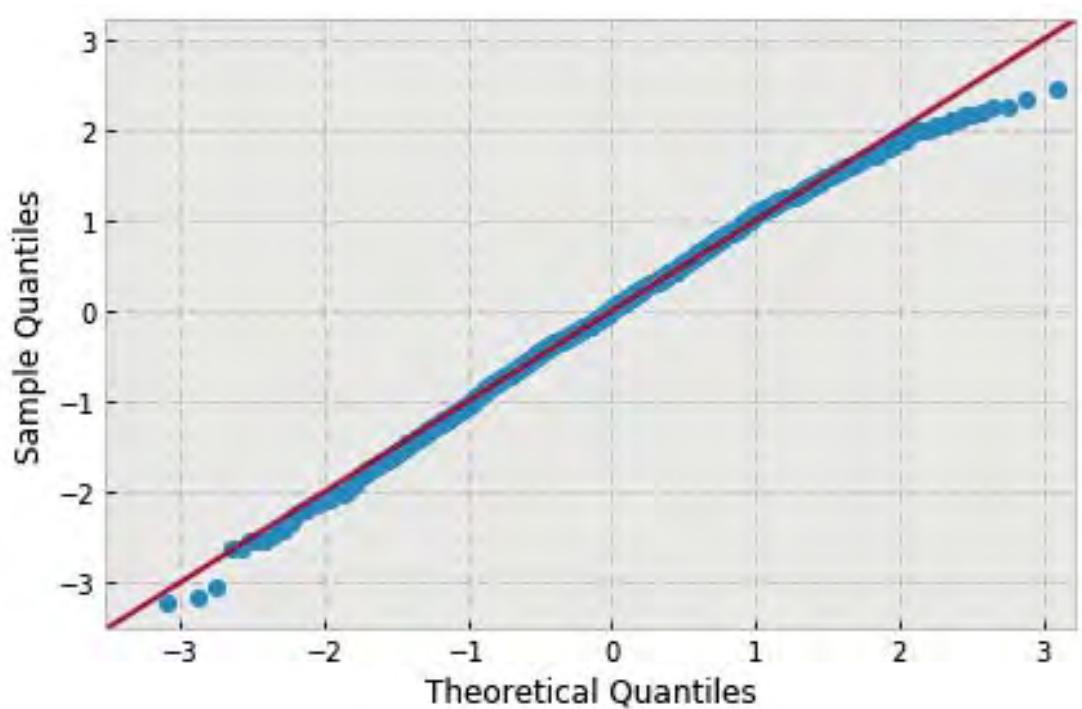
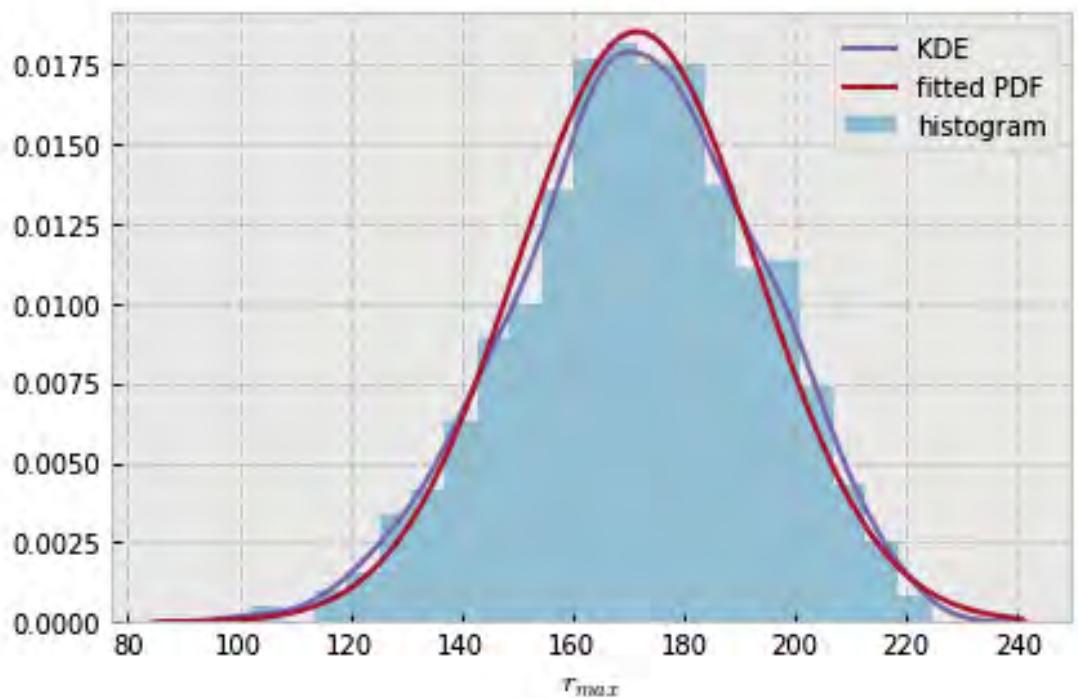


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=178.40672496823242, scale=19.113233678519073)
DescribeResult(nobs=1000, minmax=(109.49991857300593, 235.99505905191623)
               mean=178.40672496823242, variance=365.68138303270842,
               skewness=-0.372124384001116, kurtosis=0.137660928813093)
```

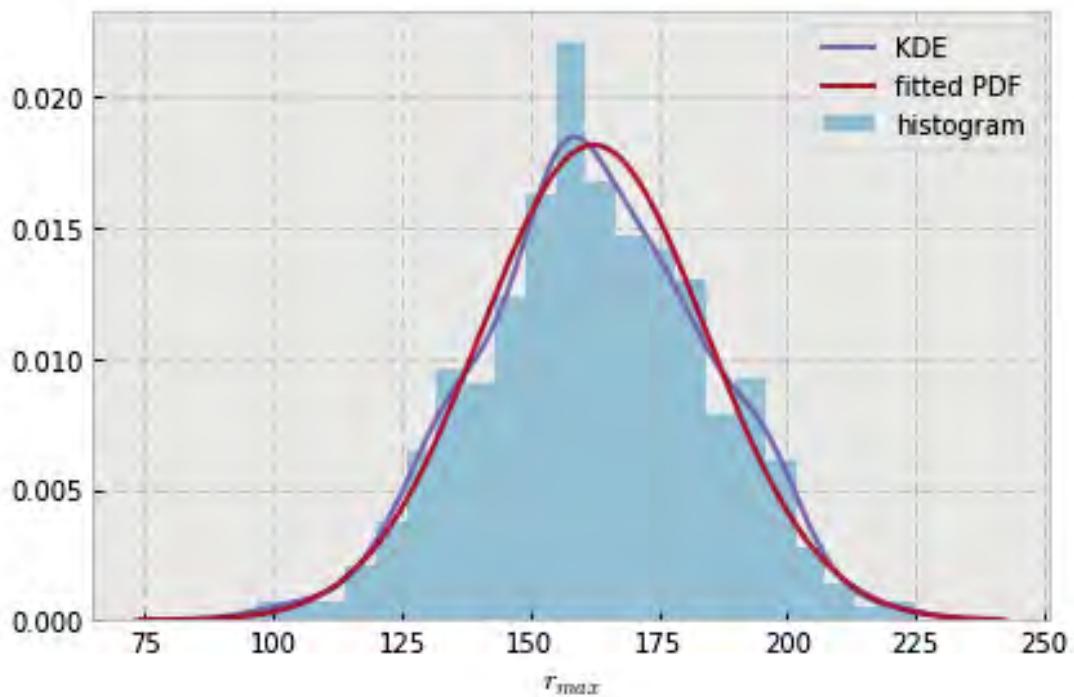


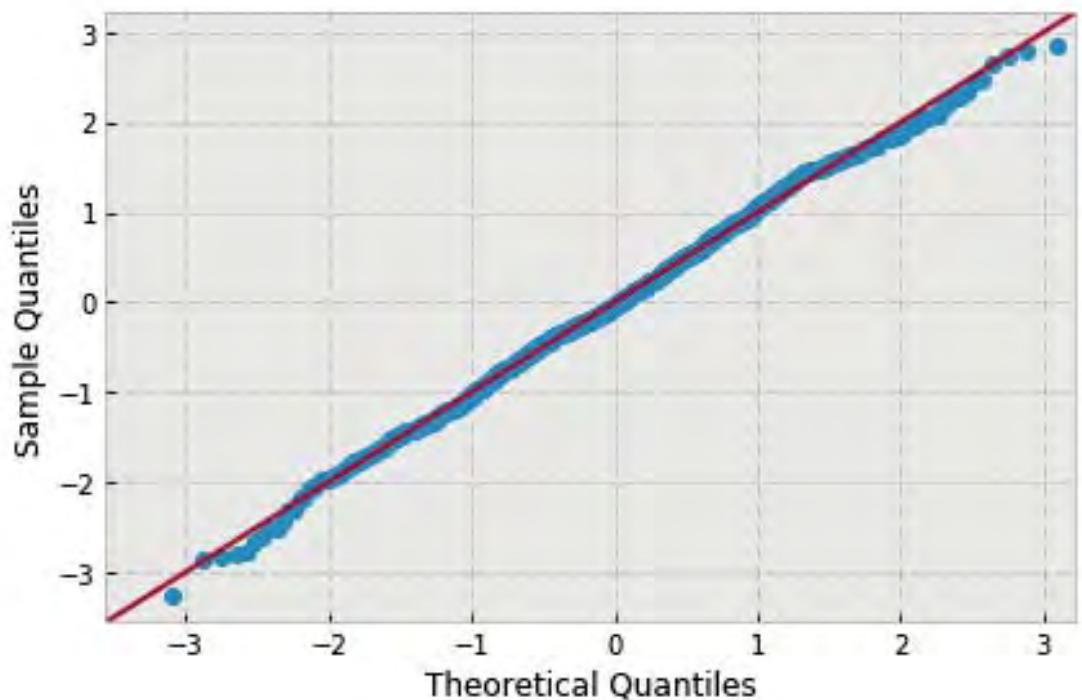


```
variable name: sigma
variable value: 0.066666666666666667
distribution: normal(shape=(), loc=171.56029008385835, scale=21.557343505579194)
DescribeResult(nobs=1000, minmax=(101.89700133471638, 224.35923132561763)
               mean=171.56029008385835, variance=465.18424326079821,
               skewness=-0.1957723507844222, kurtosis=-0.23004491256159687)
```

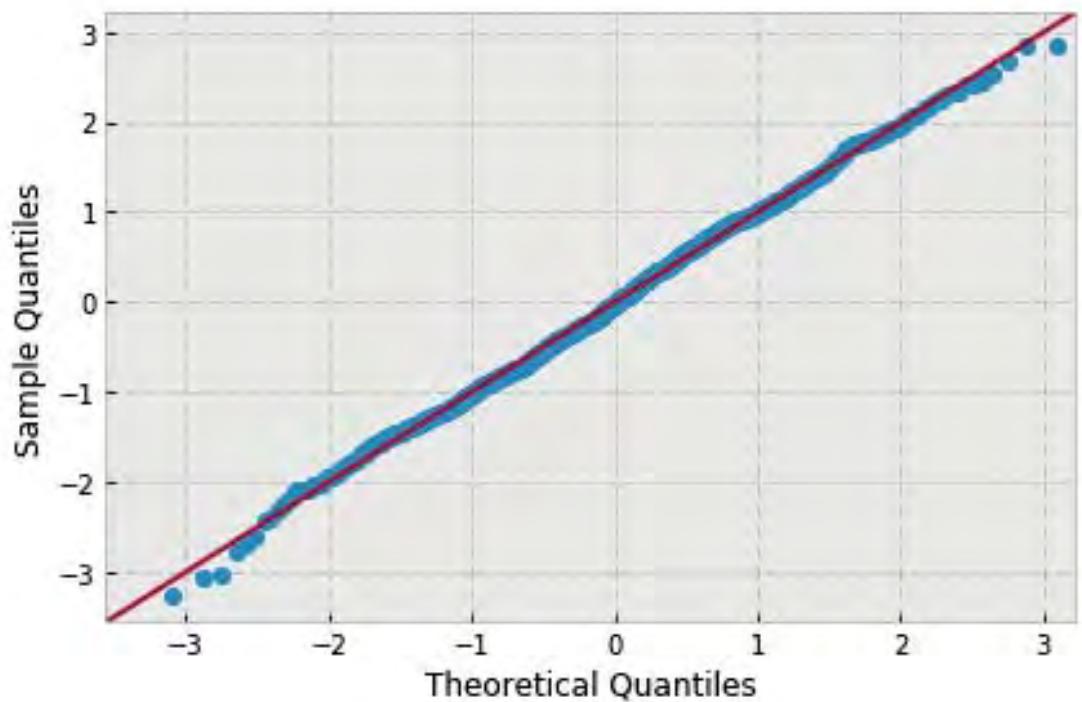
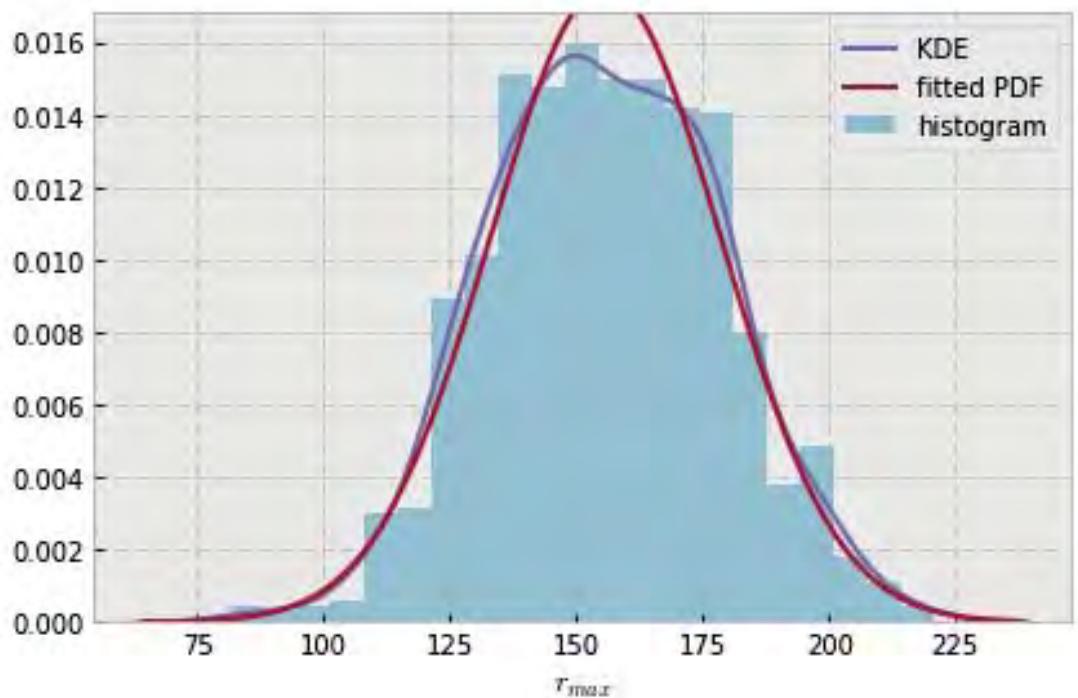


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=162.32963699765298, scale=21.974303765895975)
DescribeResult(nobs=1000, minmax=(90.831720946641468, 224.92479773713458)
               mean=162.32963699765298, variance=483.35337937524537,
               skewness=-0.03902629039064222, kurtosis=-0.18029408913063039)
```

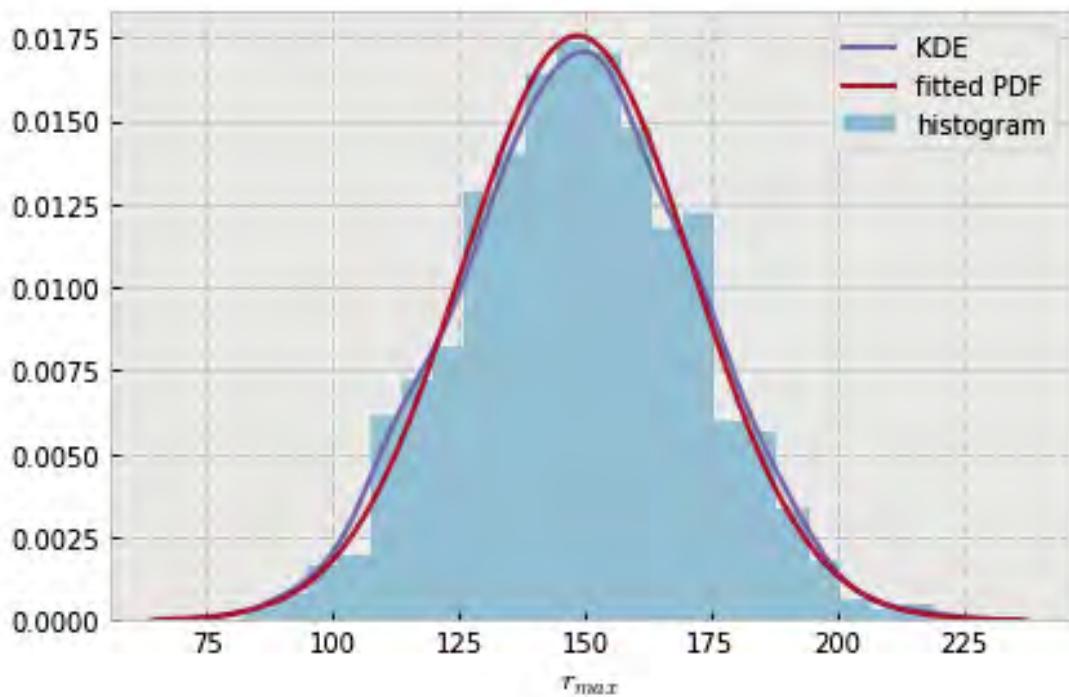


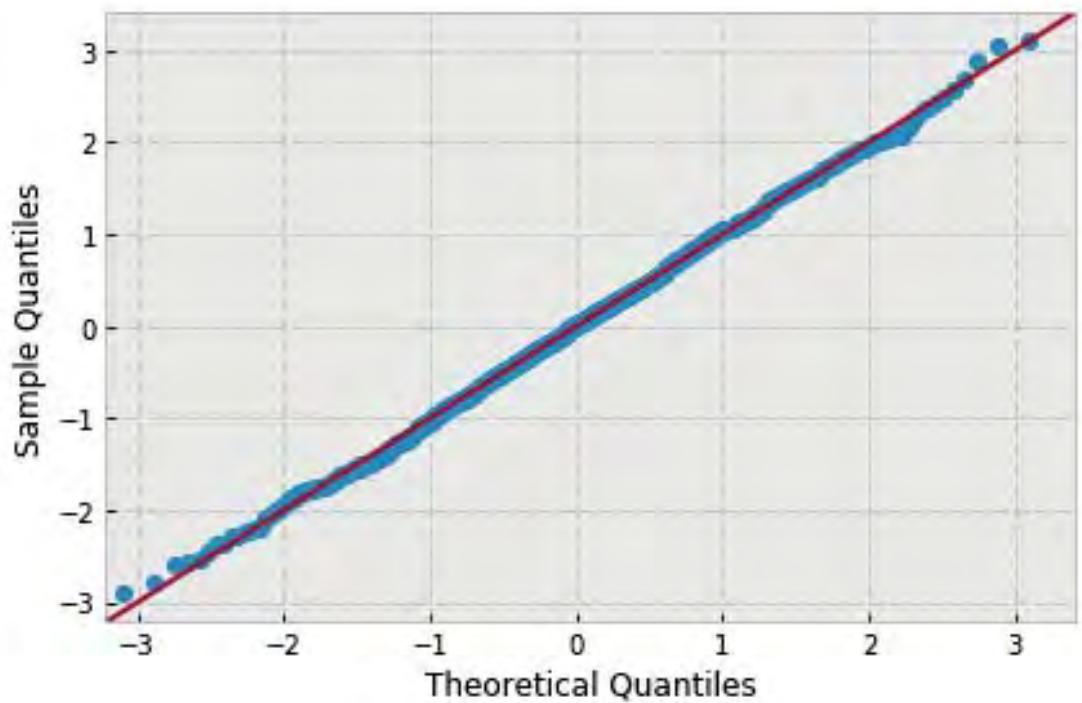


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=155.79673281226843, scale=22.809837779121732)
DescribeResult(nobs=1000, minmax=(81.537291166236585, 220.67130263844427)
               mean=155.79673281226843, variance=520.80950901886797,
               skewness=0.0029767192063026634, kurtosis=-0.2191217194629469)
```

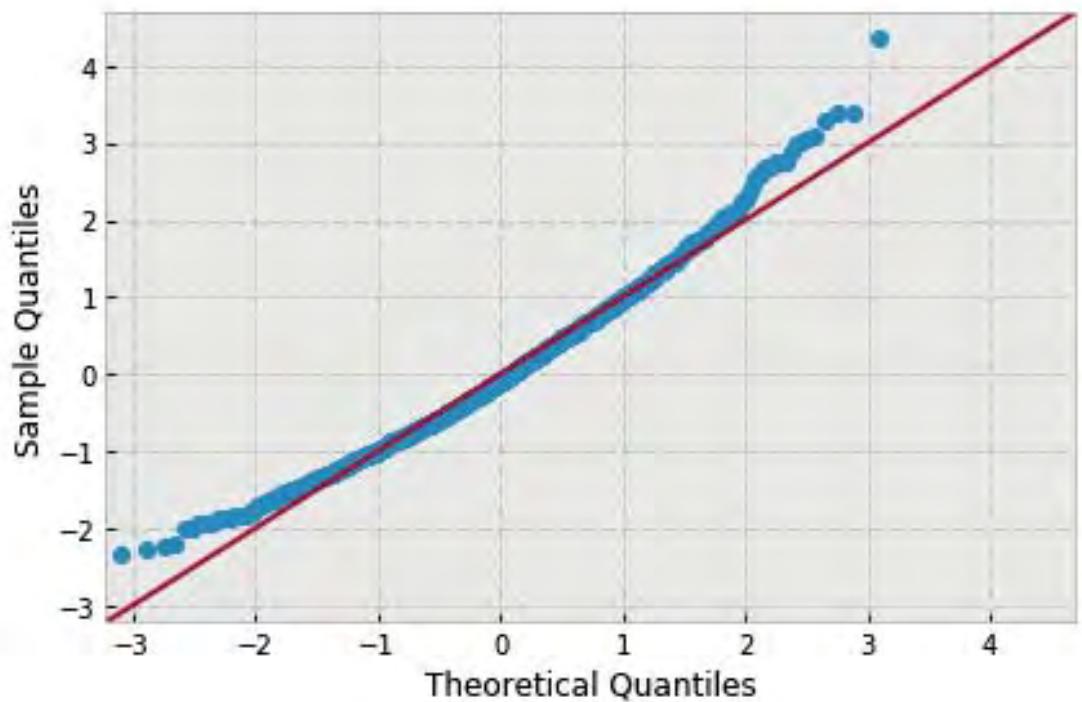
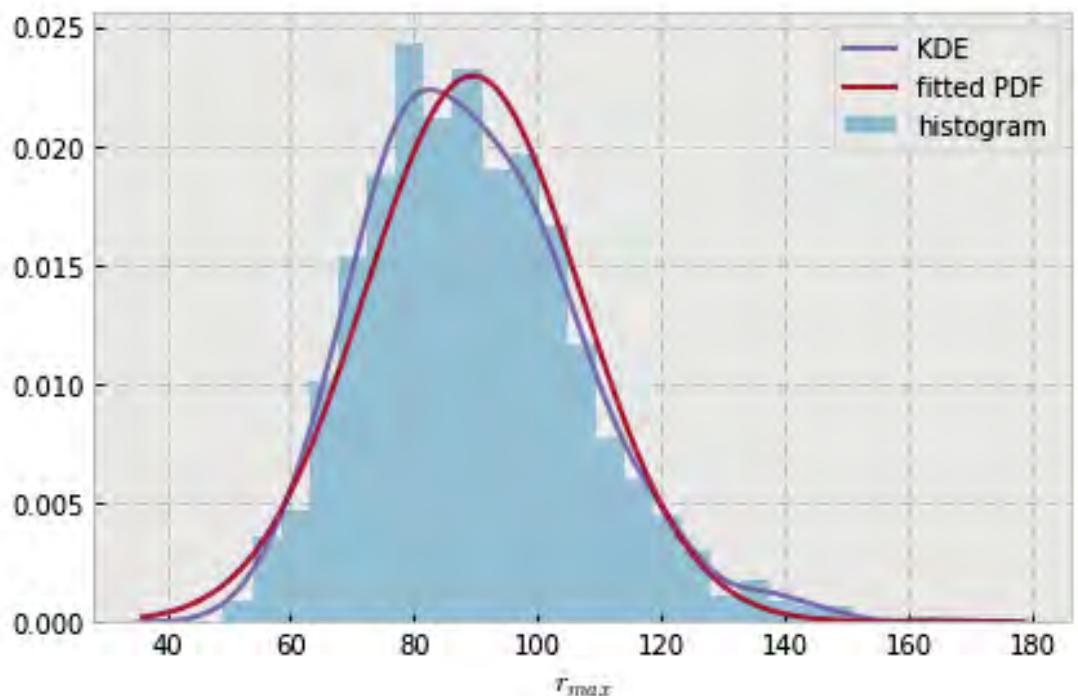


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=148.50357707872163, scale=22.734809401150915)
DescribeResult(nobs=1000, minmax=(82.484830702125493, 219.10079463733251)
               mean=148.50357707872163, variance=517.38894745411415,
               skewness=0.0060766259955762365, kurtosis=-0.20852446721850226)
```

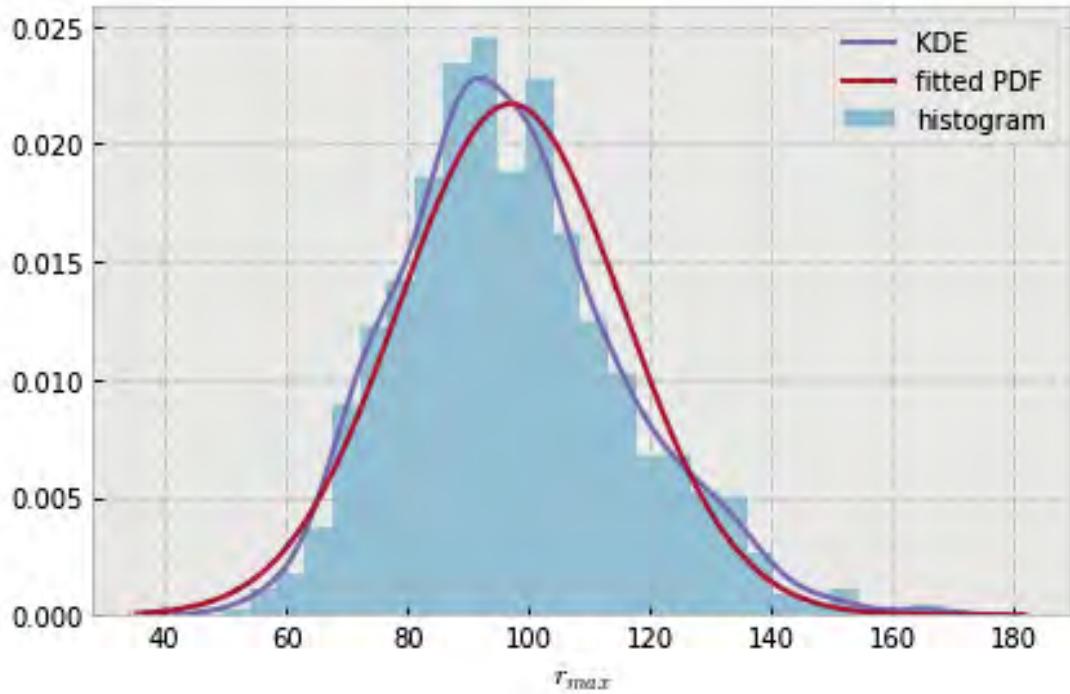


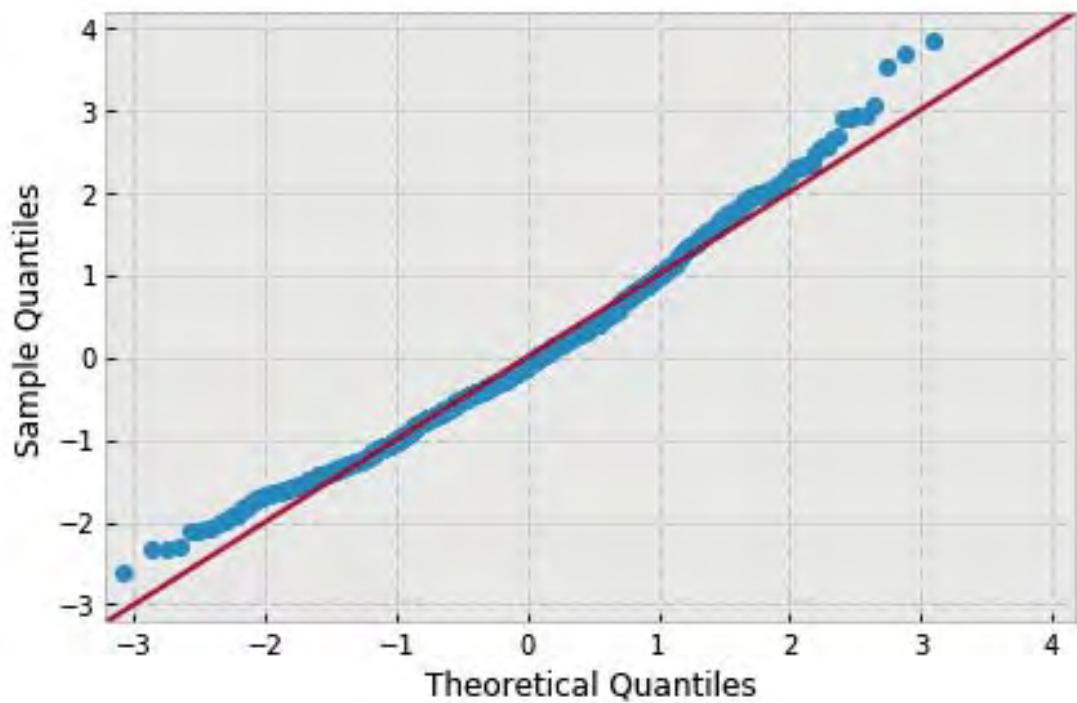


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=89.50500897834814, scale=17.390592275741632)
DescribeResult(nobs=1000, minmax=(49.157876901299176, 165.16032175104638)
               mean=89.505008978348144, variance=302.73543513622081,
               skewness=0.5575036328013092, kurtosis=0.43910038556231257)
```

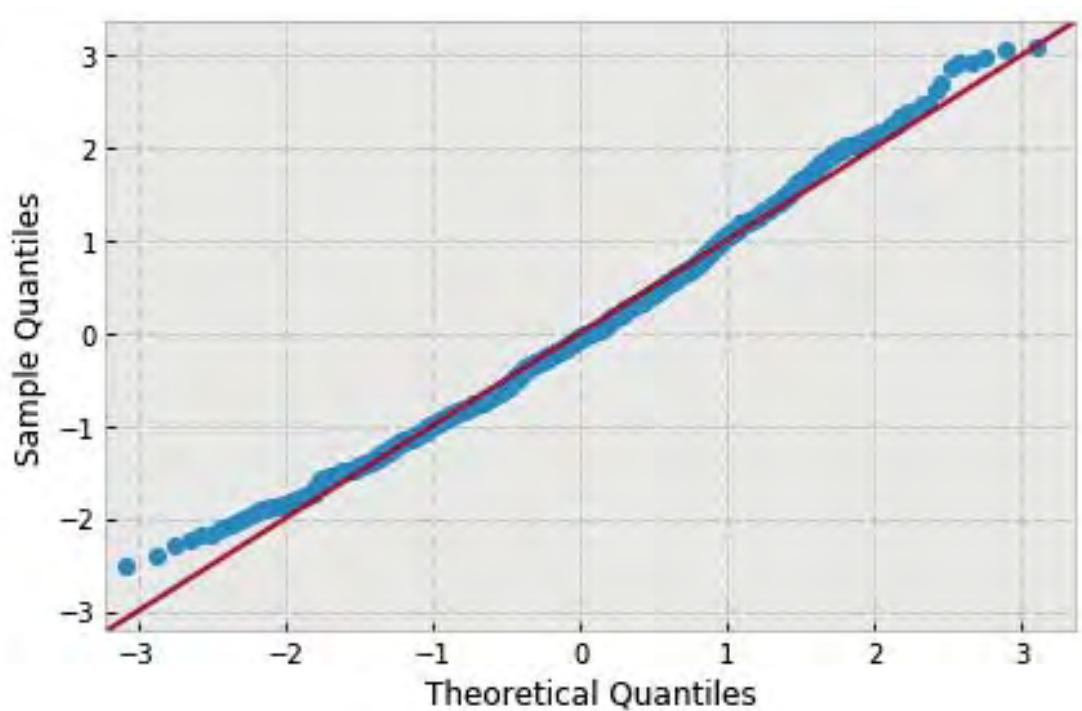
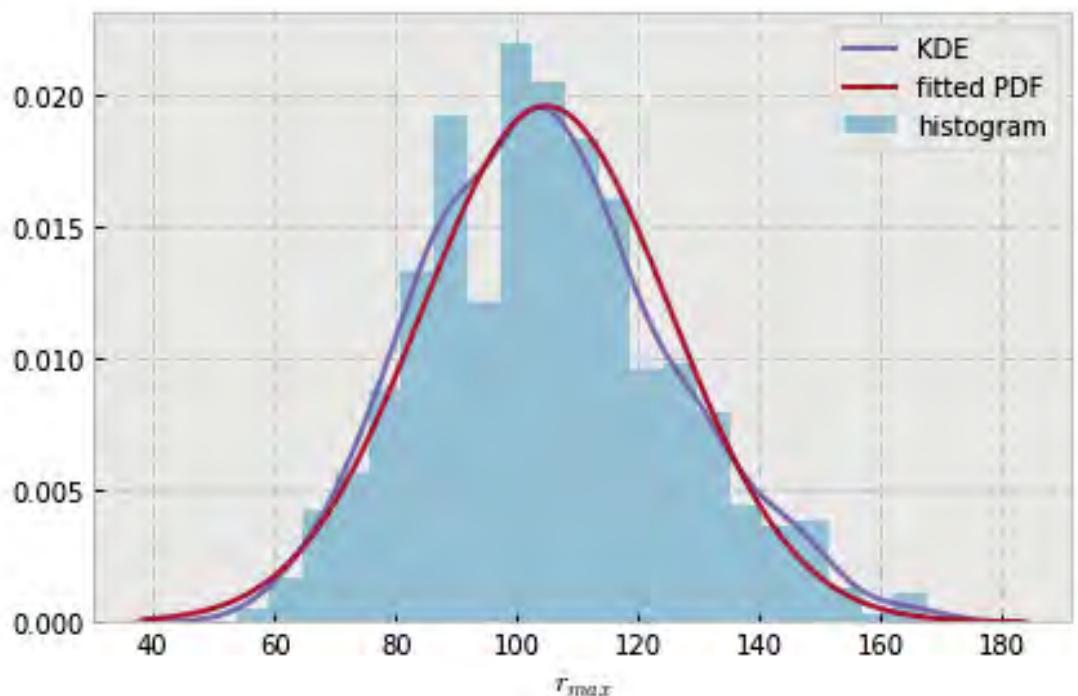


```
variable name: theta
variable value: 0.78539816333974483
distribution: normal(shape=(), loc=97.0769900936782, scale=18.391228087010845)
DescribeResult(nobs=1000, minmax=(49.345639862903639, 167.85148114986728)
               mean=97.076990093678205, variance=338.57584639485145,
               skewness=0.49364835712446986, kurtosis=0.2551187827231085)
```

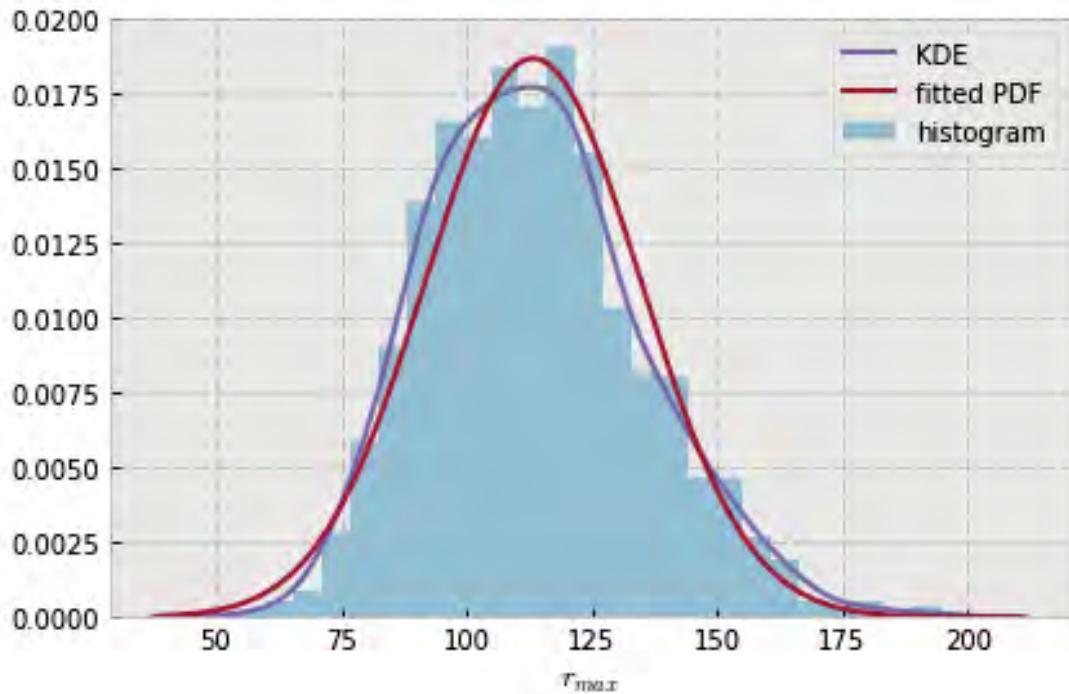


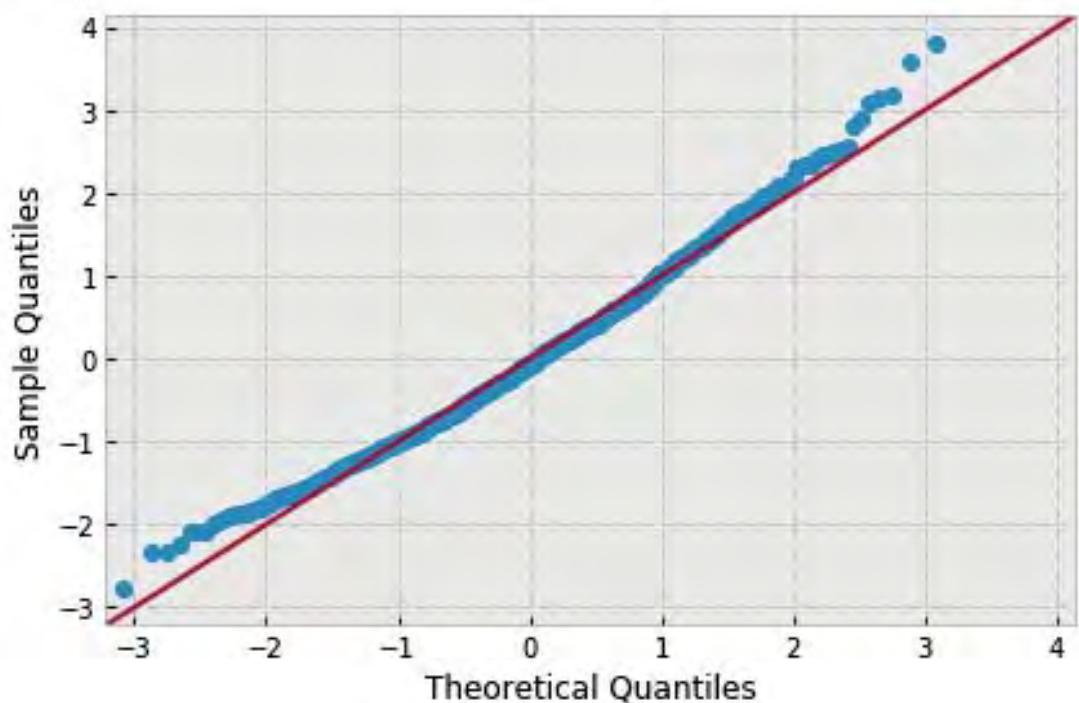


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=104.84655174527103, scale=20.402980397593414)
DescribeResult(nobs=1000, minmax=(53.846687968938959, 167.56669563734263)
               mean=104.84655174527103, variance=416.69830741199303,
               skewness=0.3240125222597742, kurtosis=-0.1320580695257294)
```

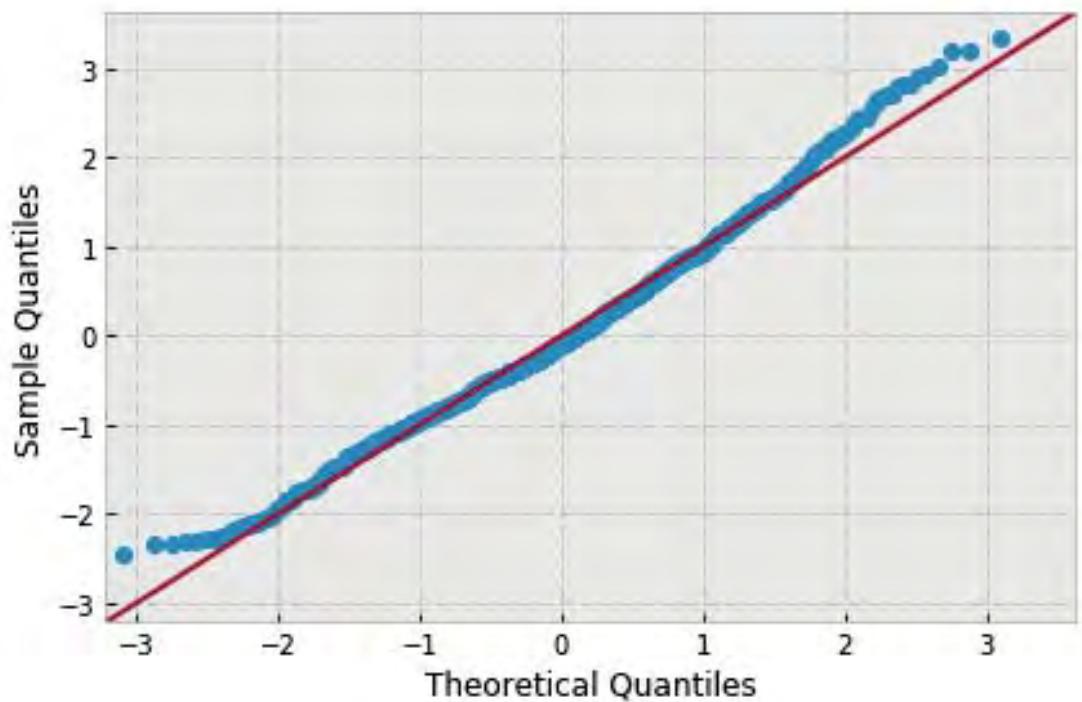
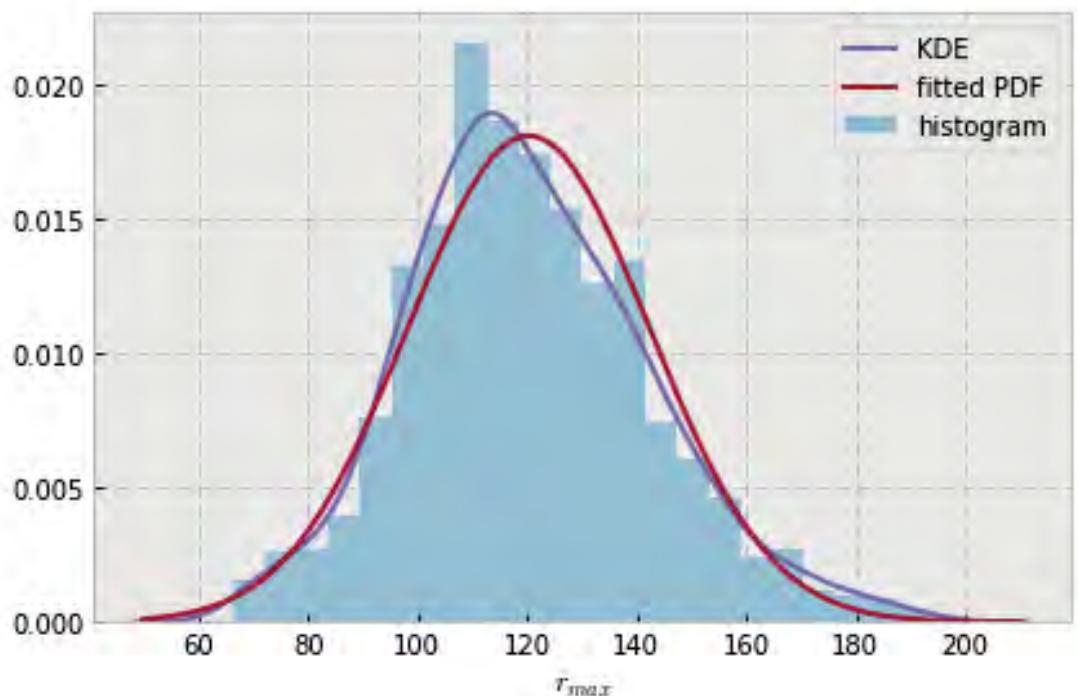


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=113.29507808494814, scale=21.336769236046536)
DescribeResult(nobs=1000, minmax=(54.425689234355282, 194.48093596538408)
               mean=113.29507808494814, variance=455.71343486716904,
               skewness=0.42379780174048287, kurtosis=0.10554195061152072)
```

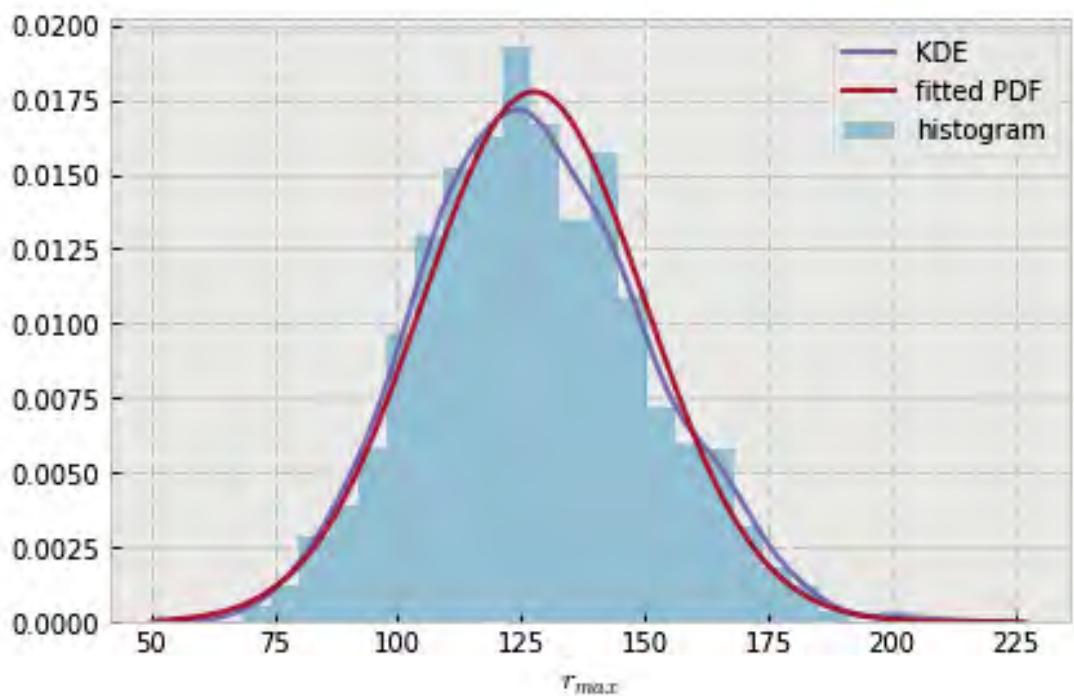


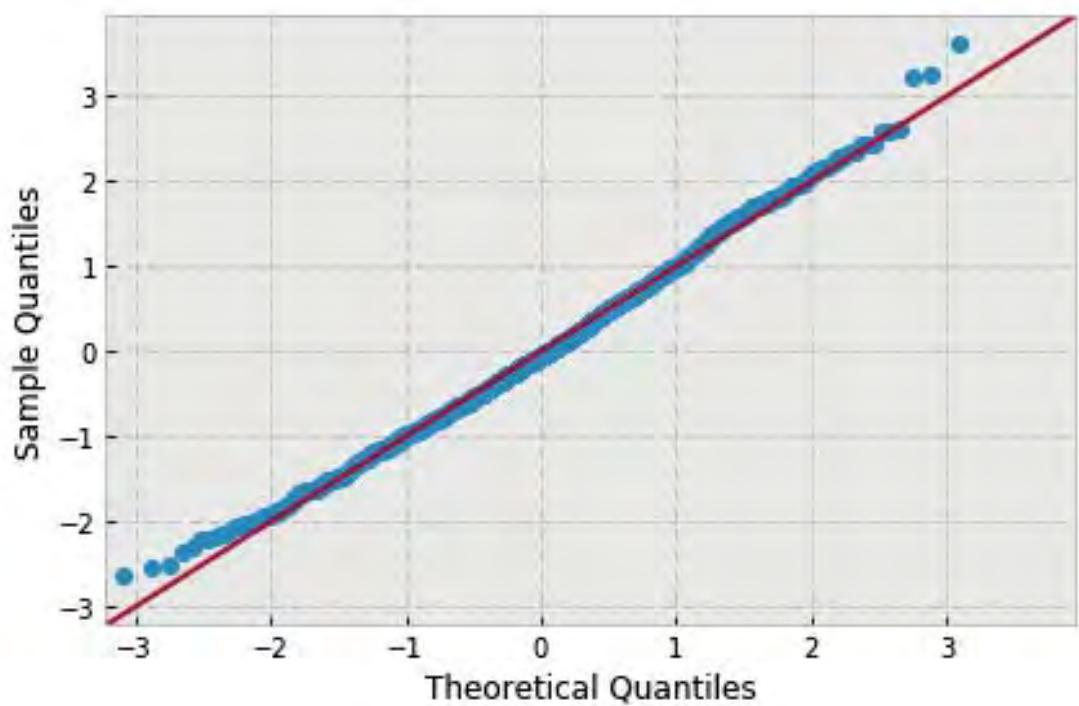


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=120.25416435587485, scale=22.013790762121534)
DescribeResult(nobs=1000, minmax=(66.360570805998478, 193.56934527366829)
               mean=120.25416435587485, variance=485.09207579426169,
               skewness=0.3814690311858938, kurtosis=0.2119930897079234)
```

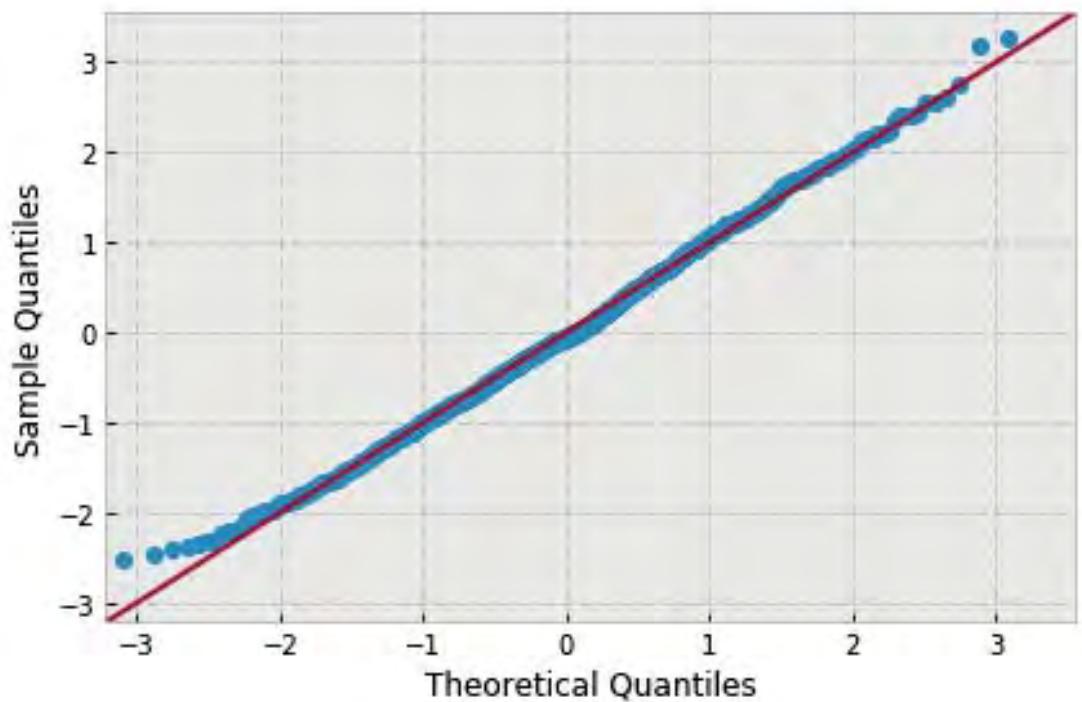
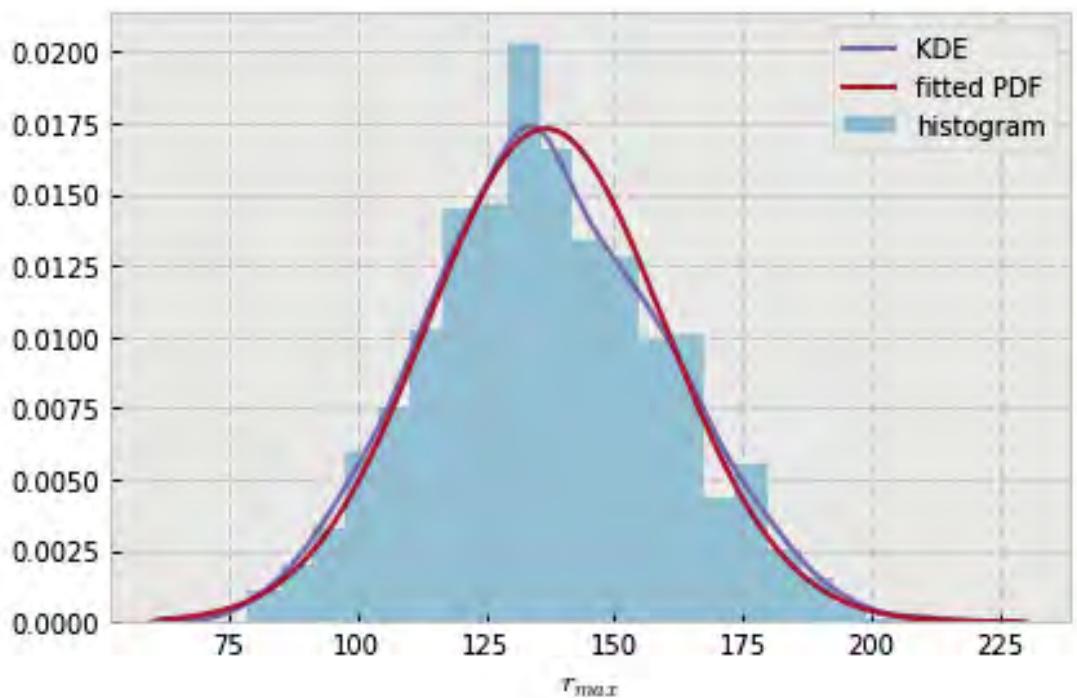


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=127.48200844442529, scale=22.45955636175725)
DescribeResult(nobs=1000, minmax=(68.360738204347072, 208.98386955068534)
               mean=127.48200844442529, variance=504.93660857552618,
               skewness=0.23131302224287373, kurtosis=-0.11865049635915392)
```

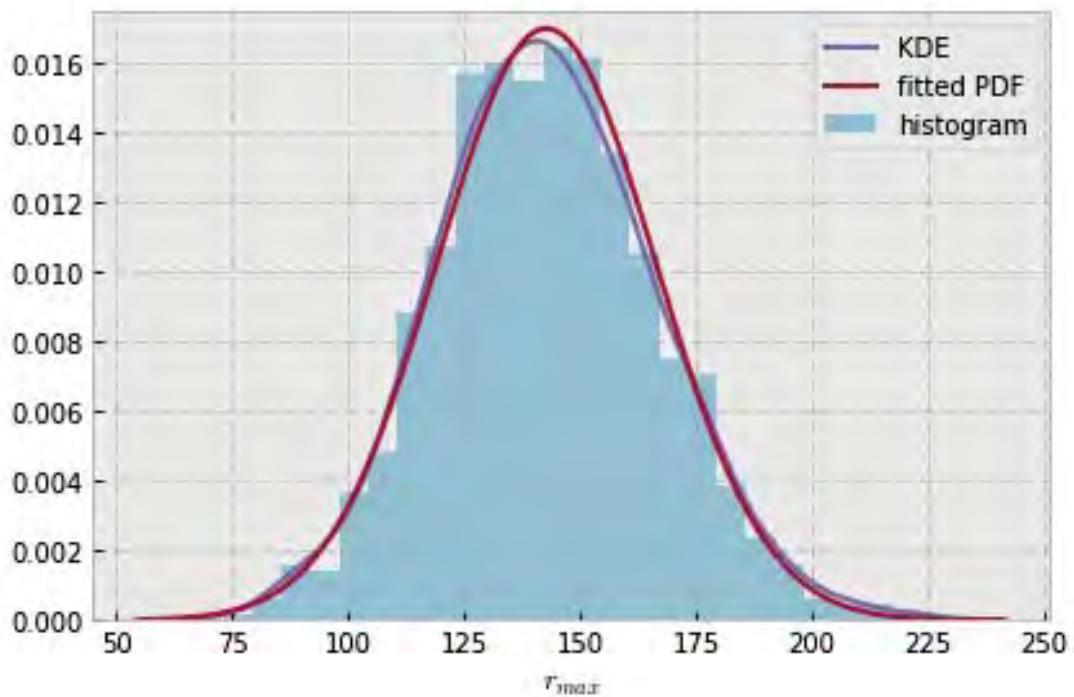


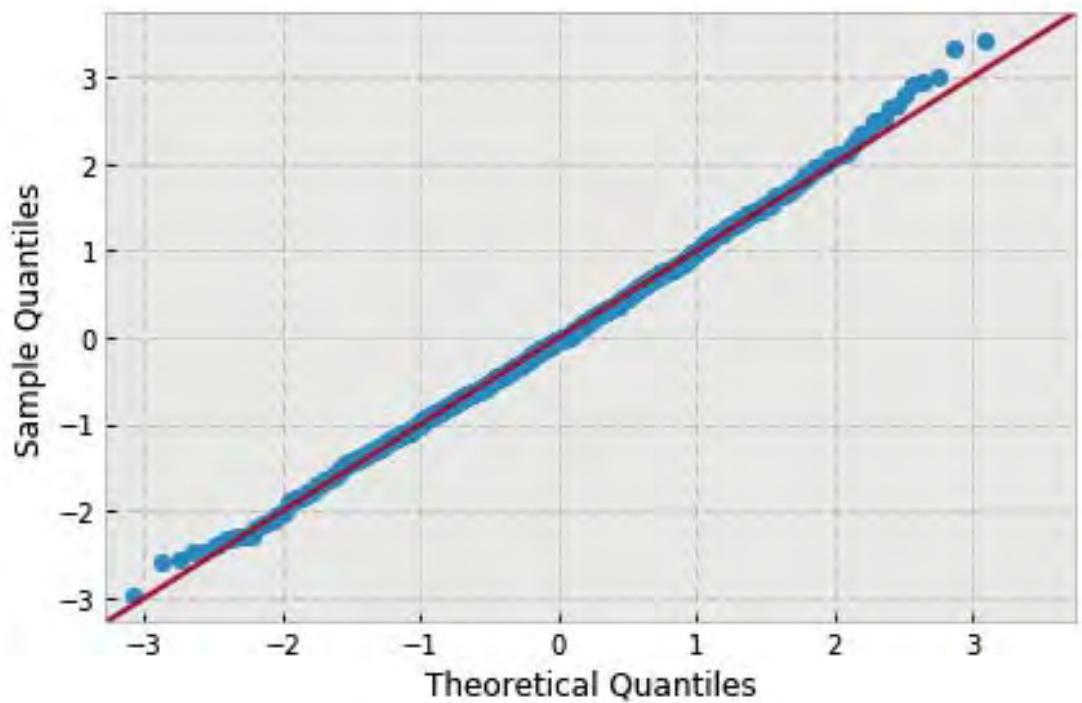


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=136.69013495230598, scale=23.0460404695473)
DescribeResult(nobs=1000, minmax=(78.545777154705462, 211.69275209322763)
               mean=136.69013495230598, variance=531.65163295696891,
               skewness=0.1465037839420444, kurtosis=-0.24230417370819346)
```

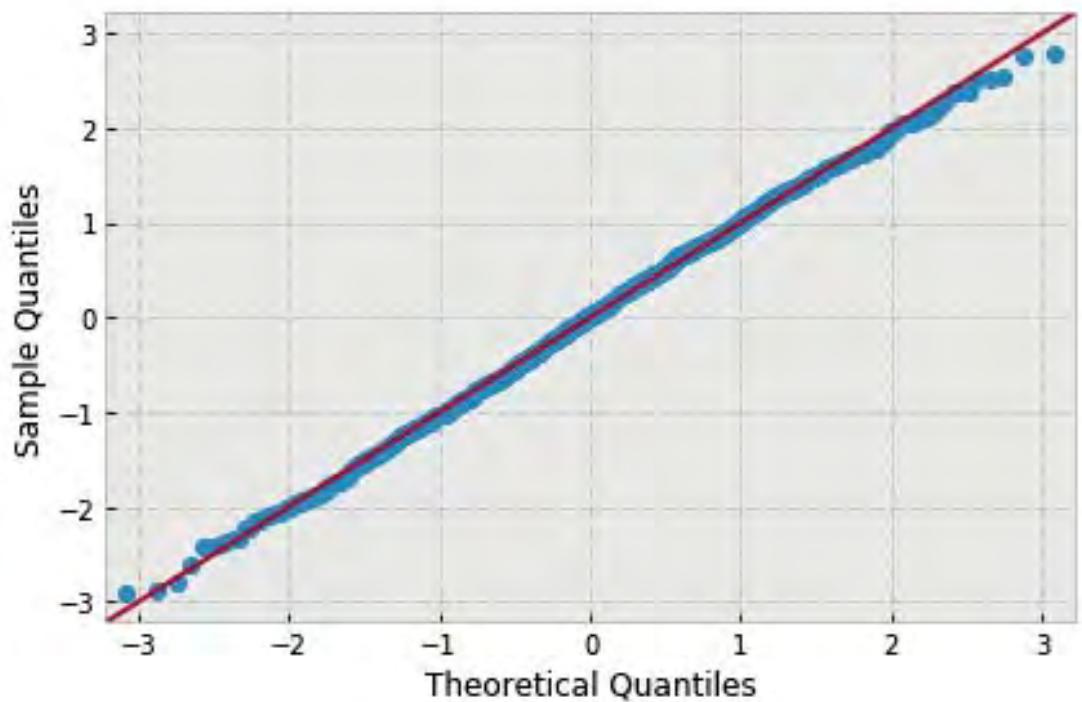
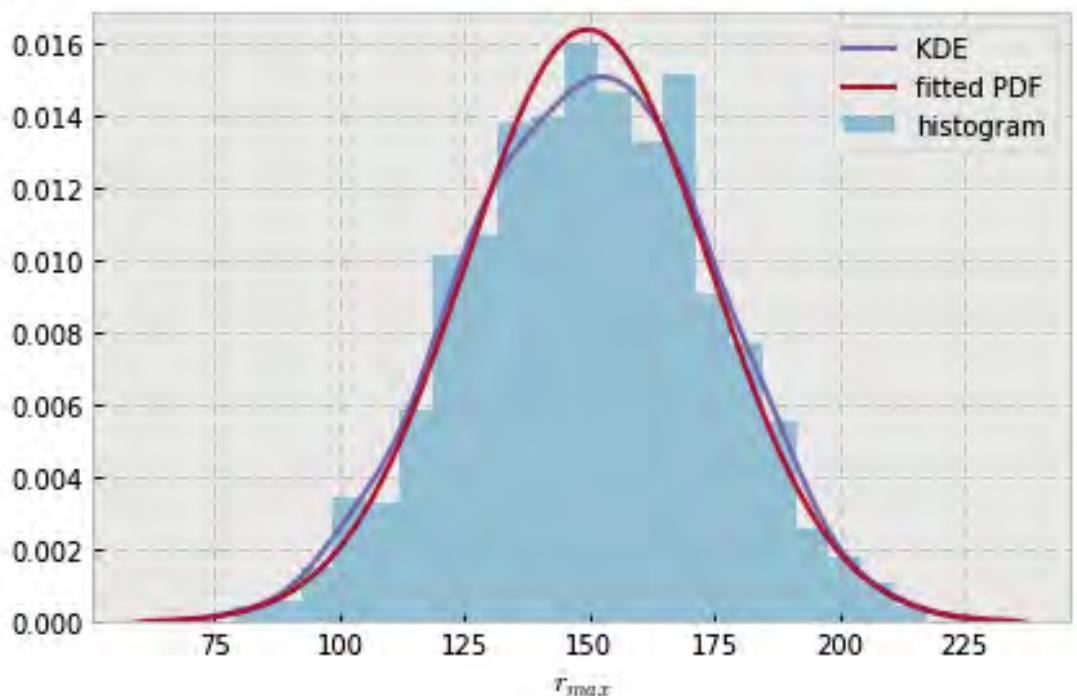


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=142.7353507742947, scale=23.527996587436594)
DescribeResult(nobs=1000, minmax=(73.035932491125763, 223.26710490710201)
               mean=142.73535077429469, variance=554.12074416259054,
               skewness=0.16507227211302966, kurtosis=0.07252355837766622)
```

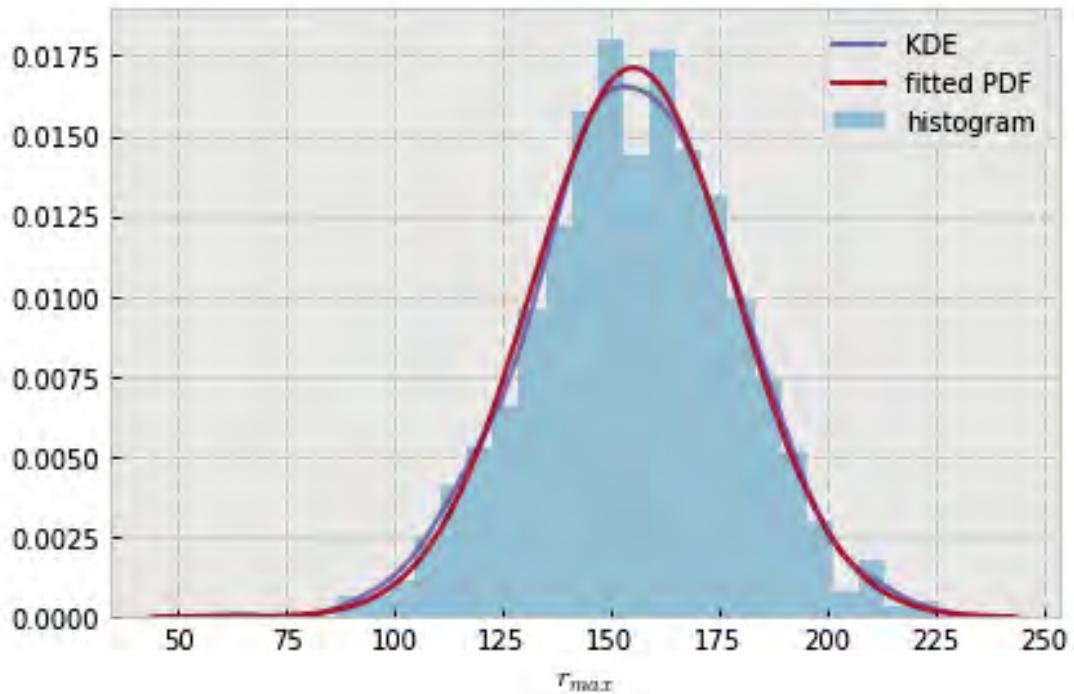


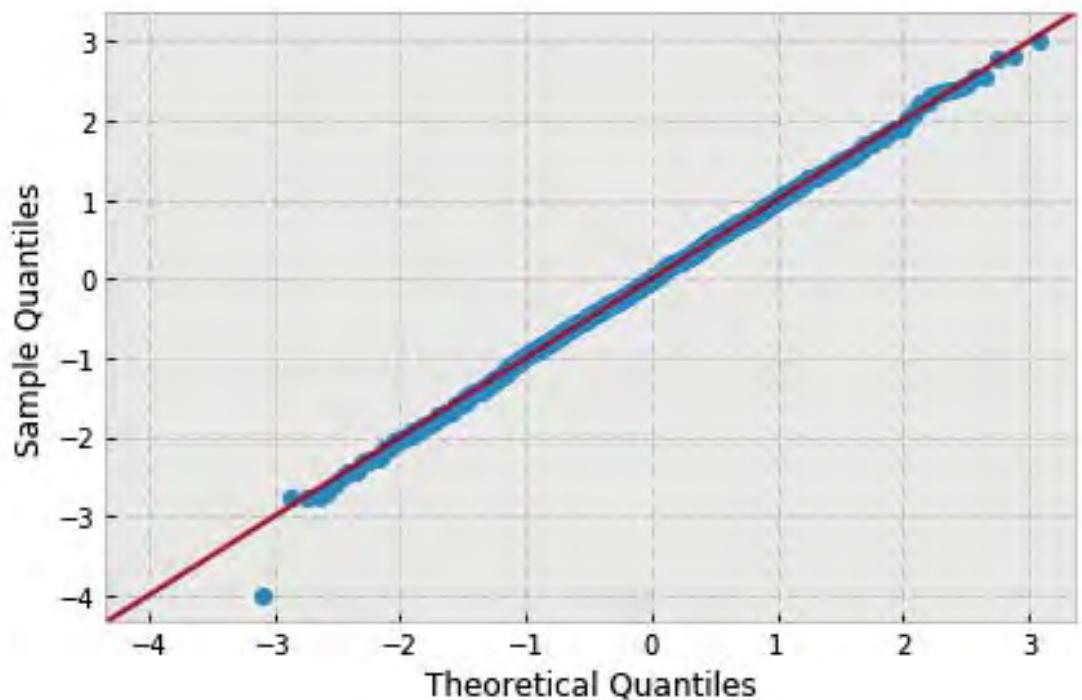


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=149.62154650913376, scale=24.320632691486722)
DescribeResult(nobs=1000, minmax=(78.961385011195659, 217.60255718055666)
               mean=149.62154650913376, variance=592.08525977398665,
               skewness=-0.05203836733032086, kurtosis=-0.3095679010941108)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=155.37052239221526, scale=23.302150684049227)
DescribeResult(nobs=1000, minmax=(62.447561454192353, 225.54396874804226)
               mean=155.37052239221526, variance=543.53376026239823,
               skewness=-0.08501699580941136, kurtosis=0.06445531527982151)
```

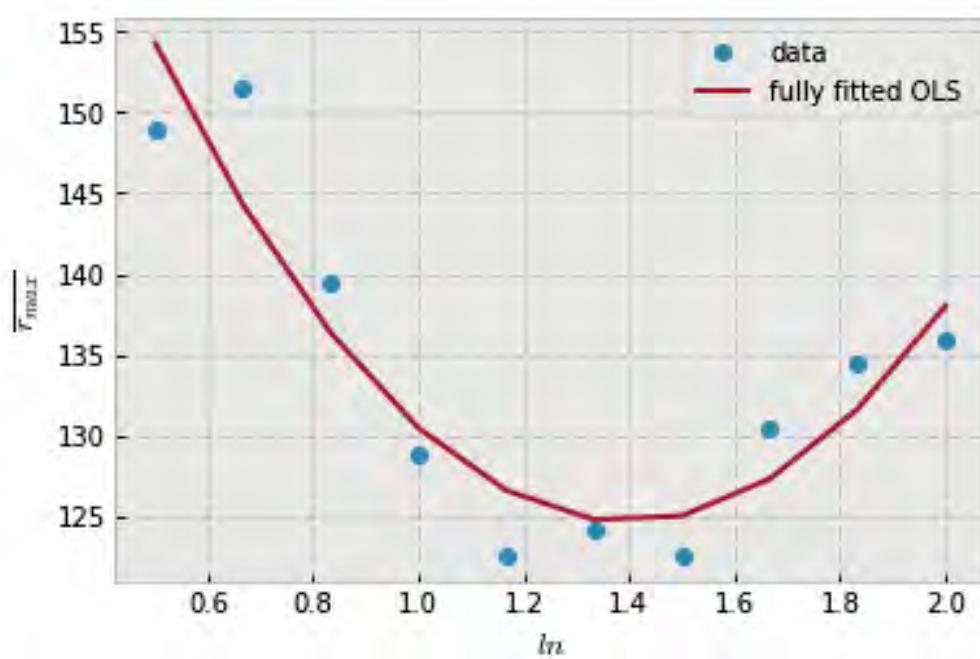




3.2 Parameter regression

```
In [12]: var_name = 'maximal_horizontal_spread'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly1')
mean_reg('nB', var_name, 'poly2')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly2')
mean_reg('N', var_name, 'poly2')
```

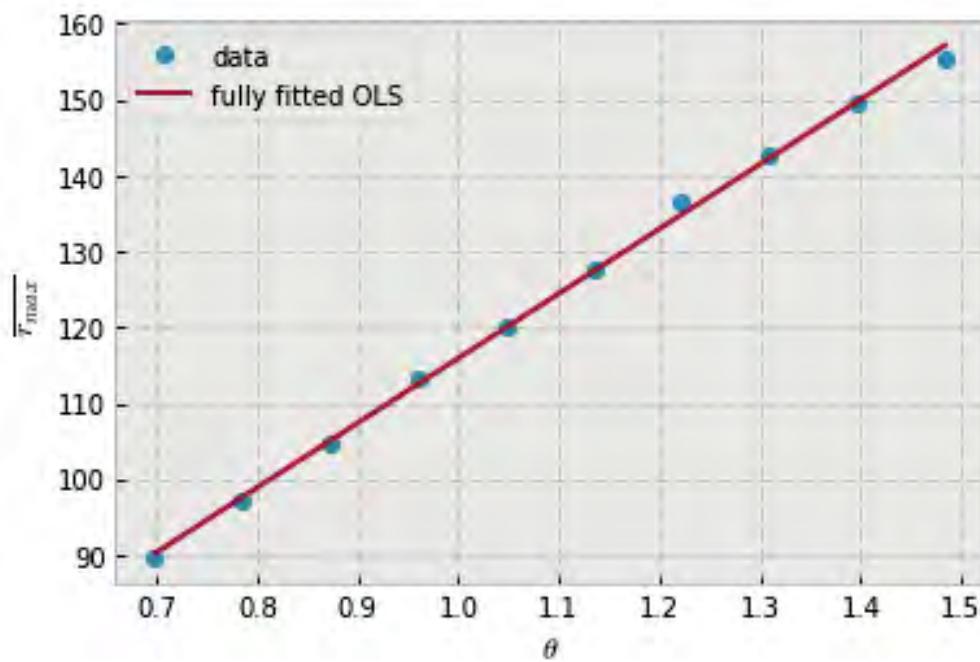
```
variable name: ln
model: poly2
r-squared: 0.5703655389383422
explained variance: 0.5790566613786311
```



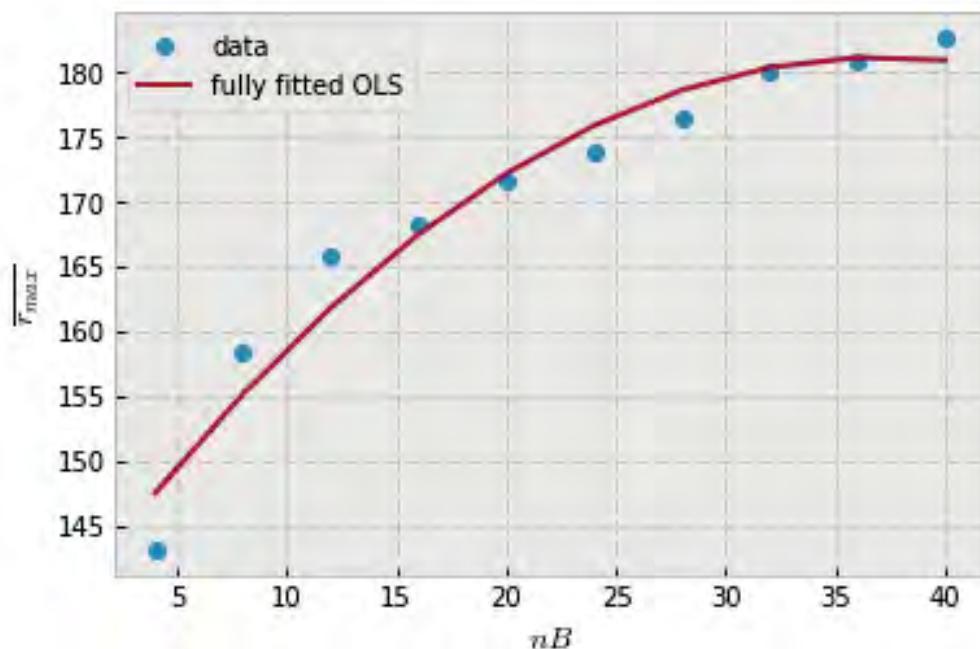
```

variable name: theta
model: poly1
r-squared: 0.9967204506218774
explained variance: 0.99674581917928

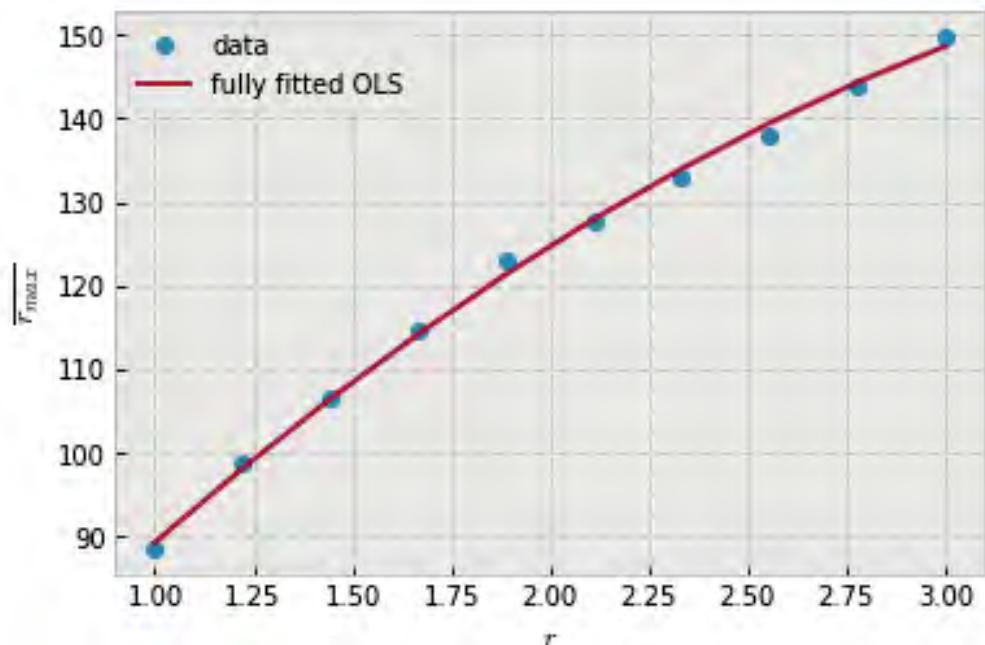
```



```
variable name: nB
model: poly2
r-squared: 0.8358873850289894
explained variance: 0.8367903090840743
```



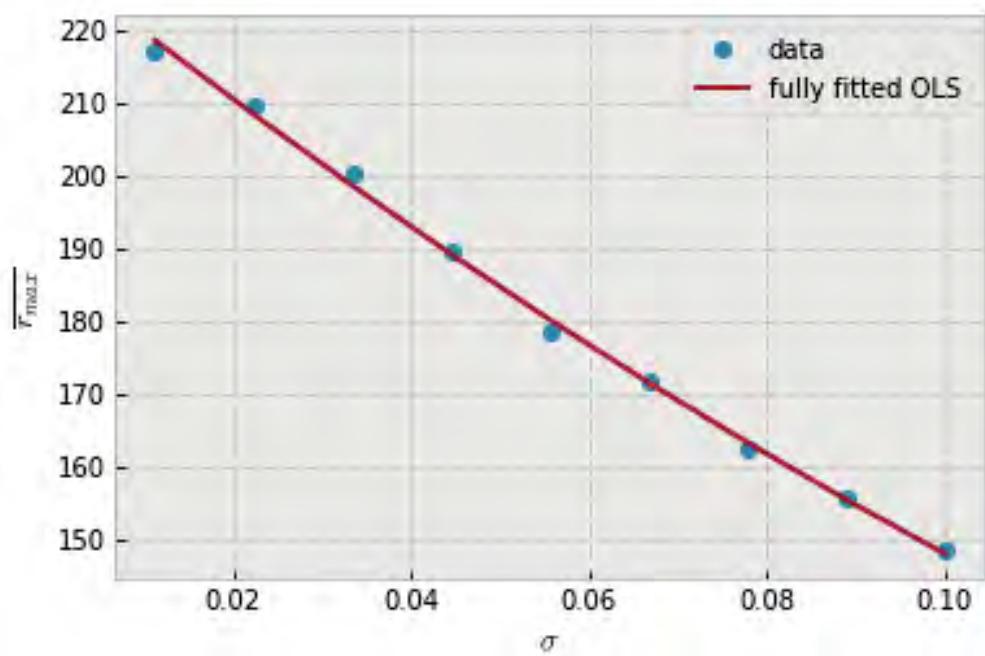
```
variable name: r
model: poly2
r-squared: 0.9932210684801767
explained variance: 0.9932308023322288
```



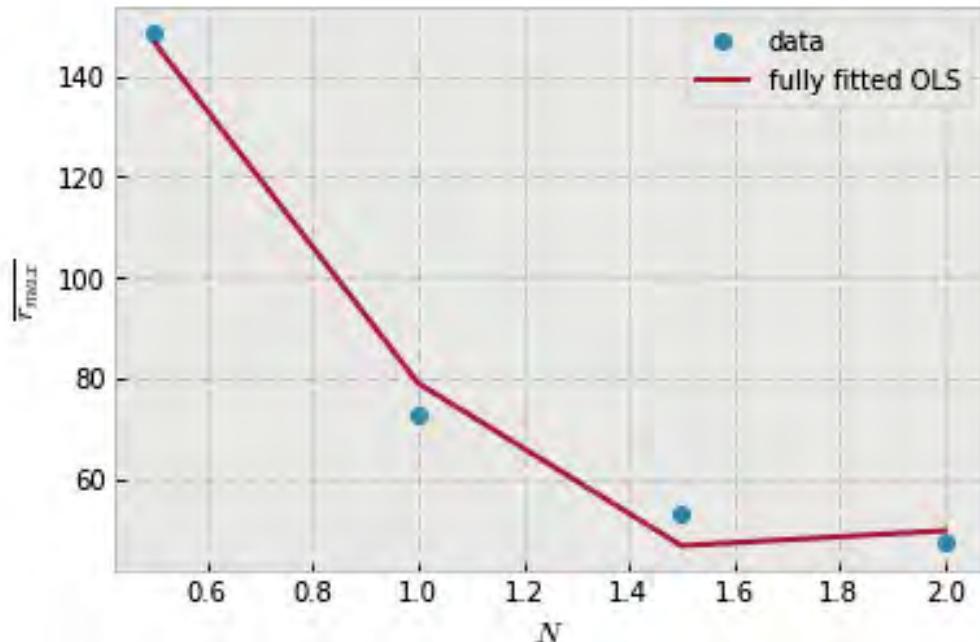
```

variable name: sigma
model: poly2
r-squared: 0.9913245648220791
explained variance: 0.9914048624864077

```



```
variable name: N
model: poly2
r-squared: 0.3685742882848685
explained variance: 0.3685742882848685
```

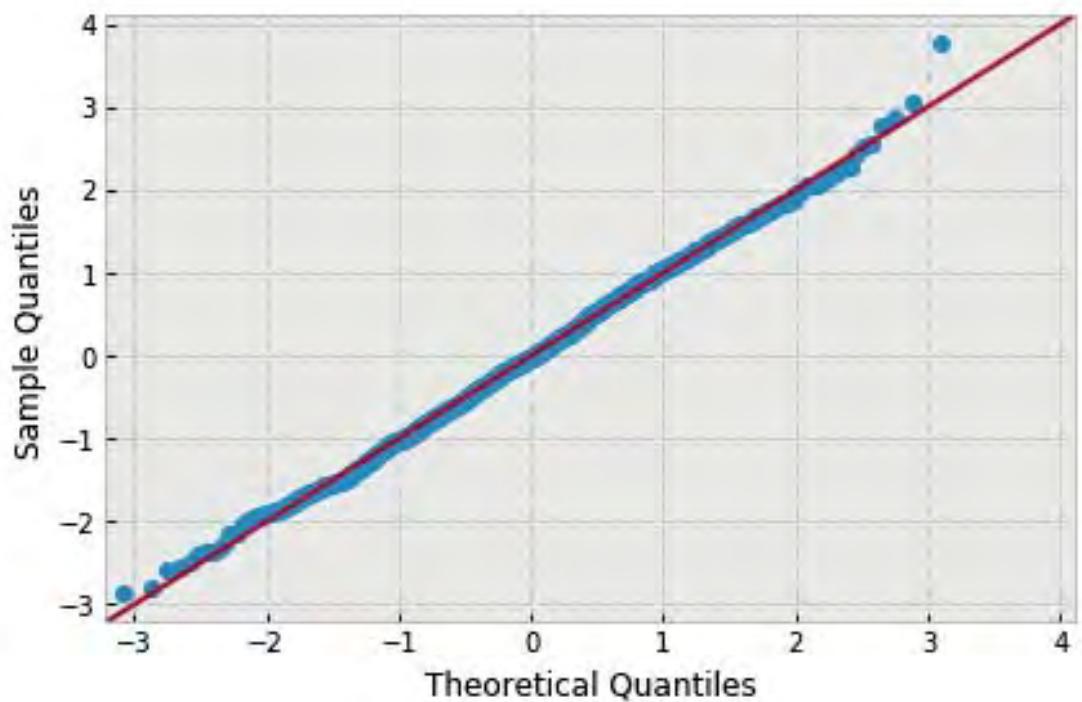
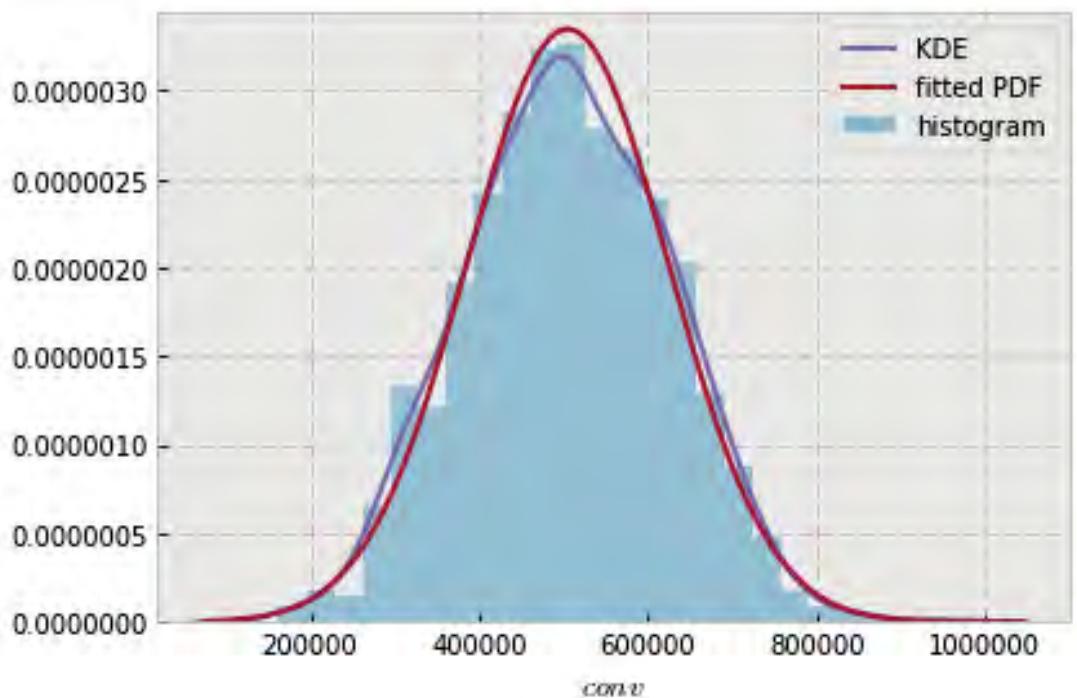


4 Volume of the convex hull

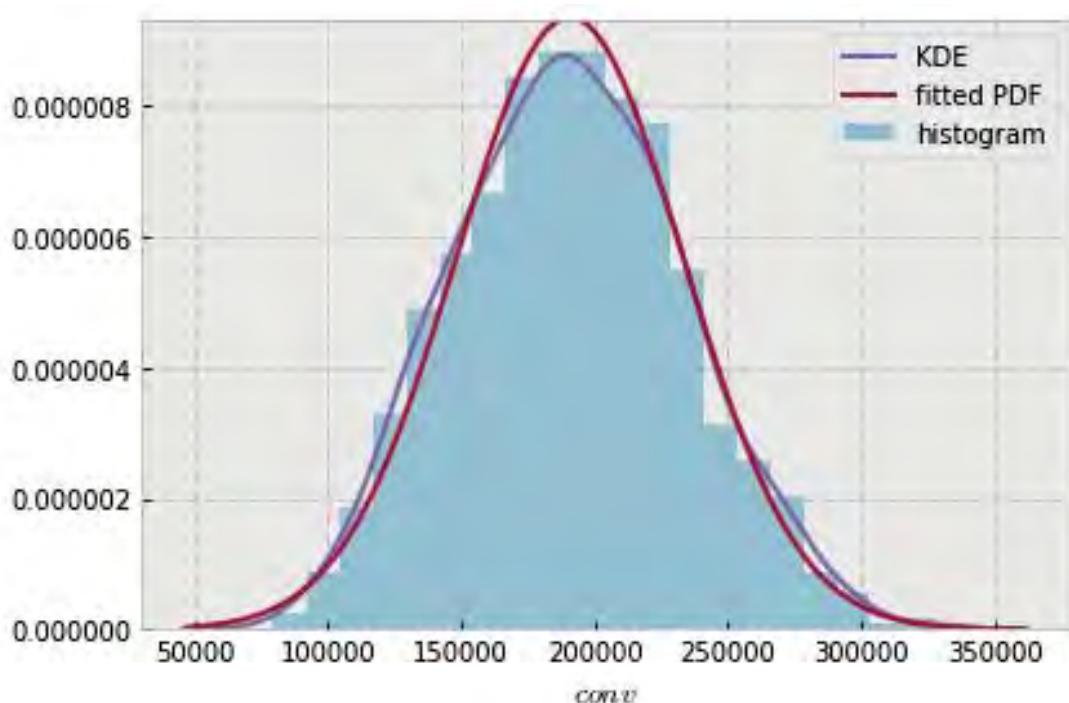
4.1 Probability distributions

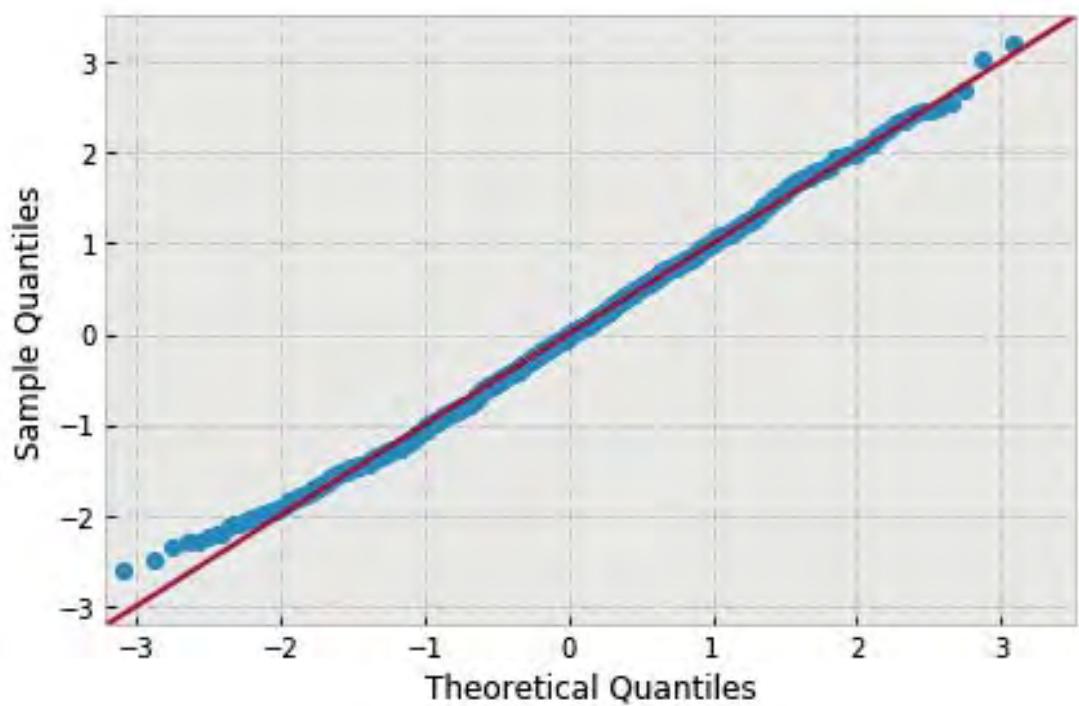
```
In [13]: plot_dist('convex_hull_volume')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=504890.9797993, scale=119154.77130099032)
DescribeResult(nobs=1000, minmax=(162948.9911999999, 954513.82539999997)
               mean=504890.97979930002, variance=14212071595.386694,
               skewness=0.04025480510545997, kurtosis=-0.19149133717461853)
```

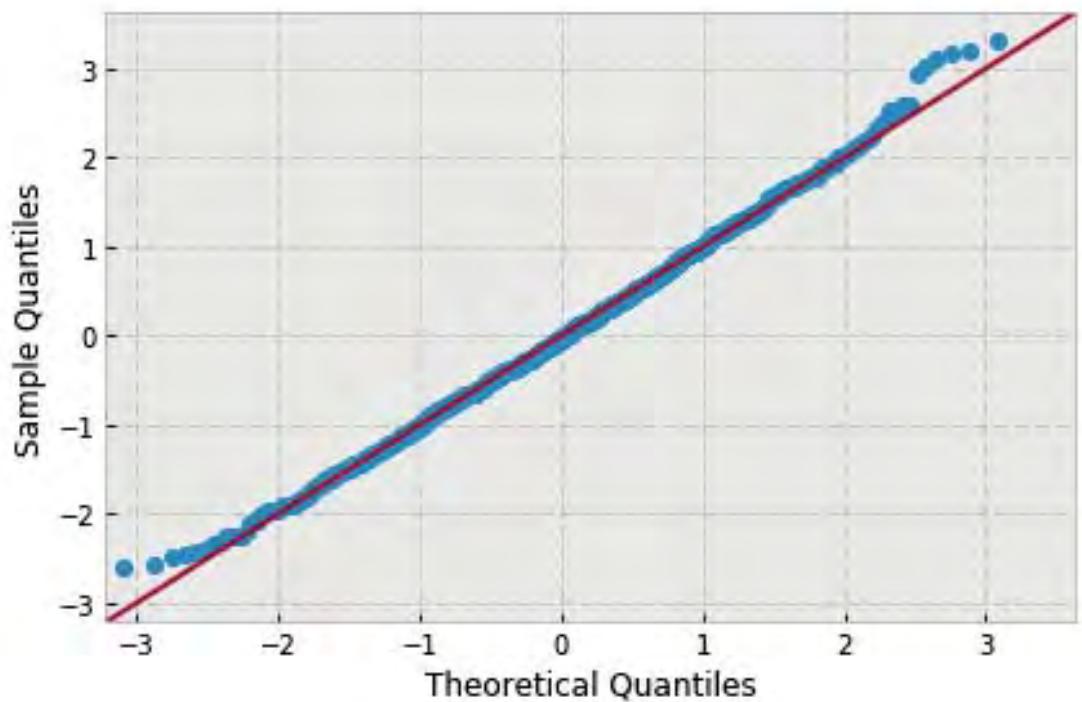
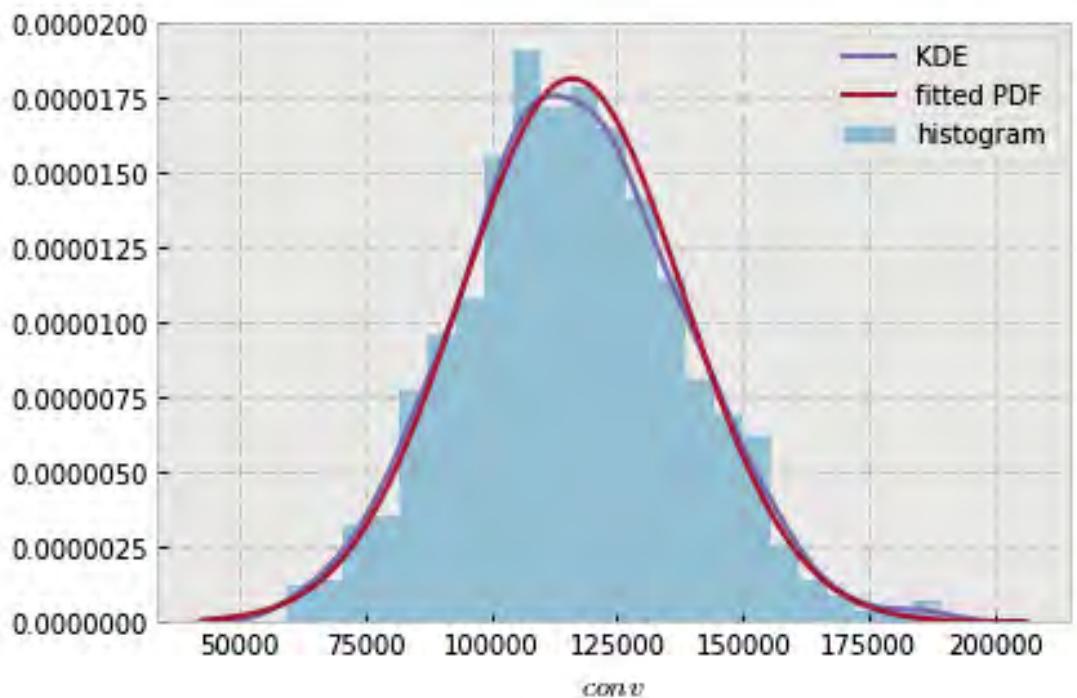


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=190796.34017243, scale=42651.64594323637)
DescribeResult(nobs=1000, minmax=(79754.783720000007, 328133.67469999997)
               mean=190796.34017243001, variance=1820983885.5527439,
               skewness=0.12717073592123052, kurtosis=-0.3176677340881753)
```

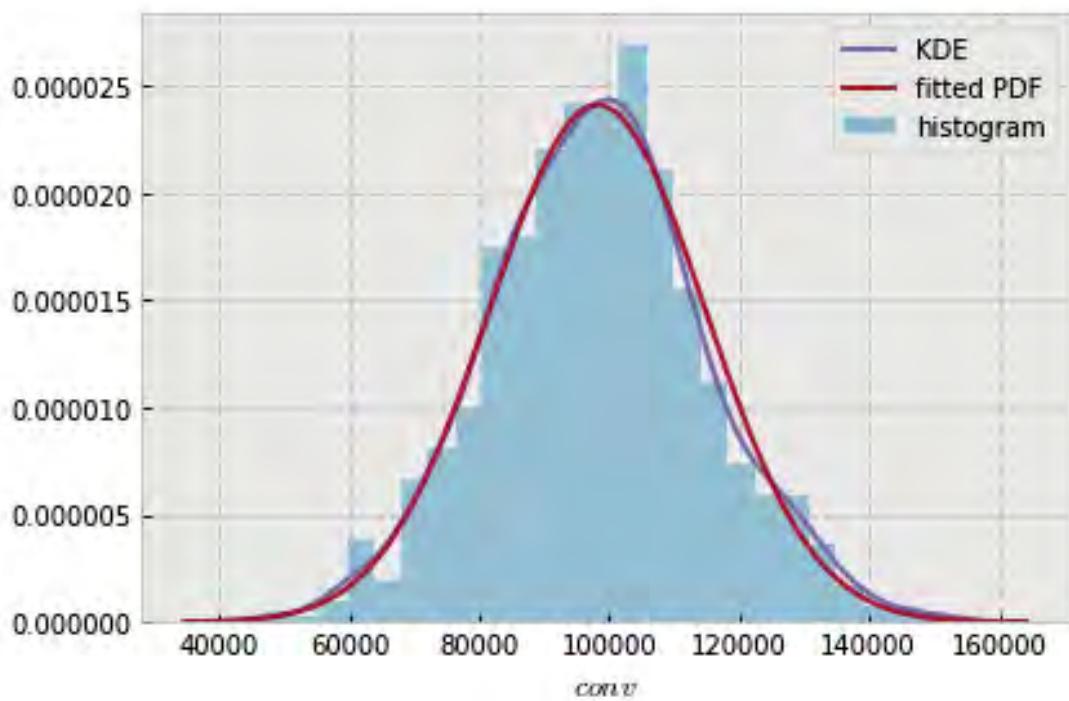


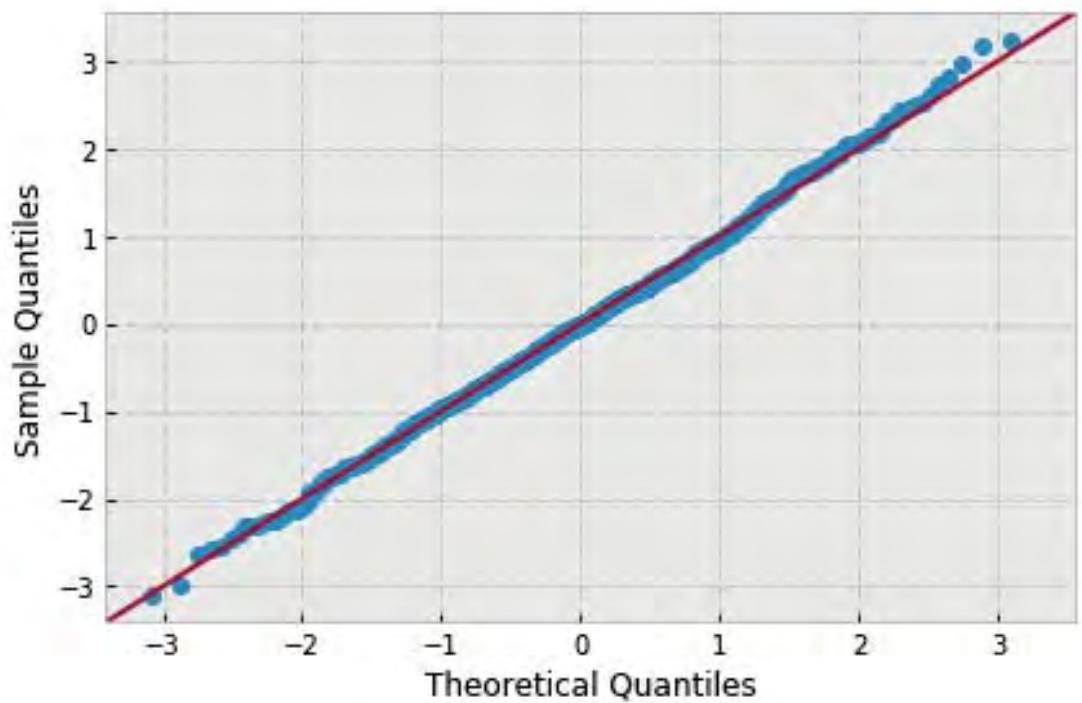


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=116116.57567993, scale=21966.93052871548)
DescribeResult(nobs=1000, minmax=(59282.98060999999, 189205.02299999999)
               mean=116116.57567993, variance=483029065.91933149,
               skewness=0.16411676973676737, kurtosis=-0.012440850202235954)
```

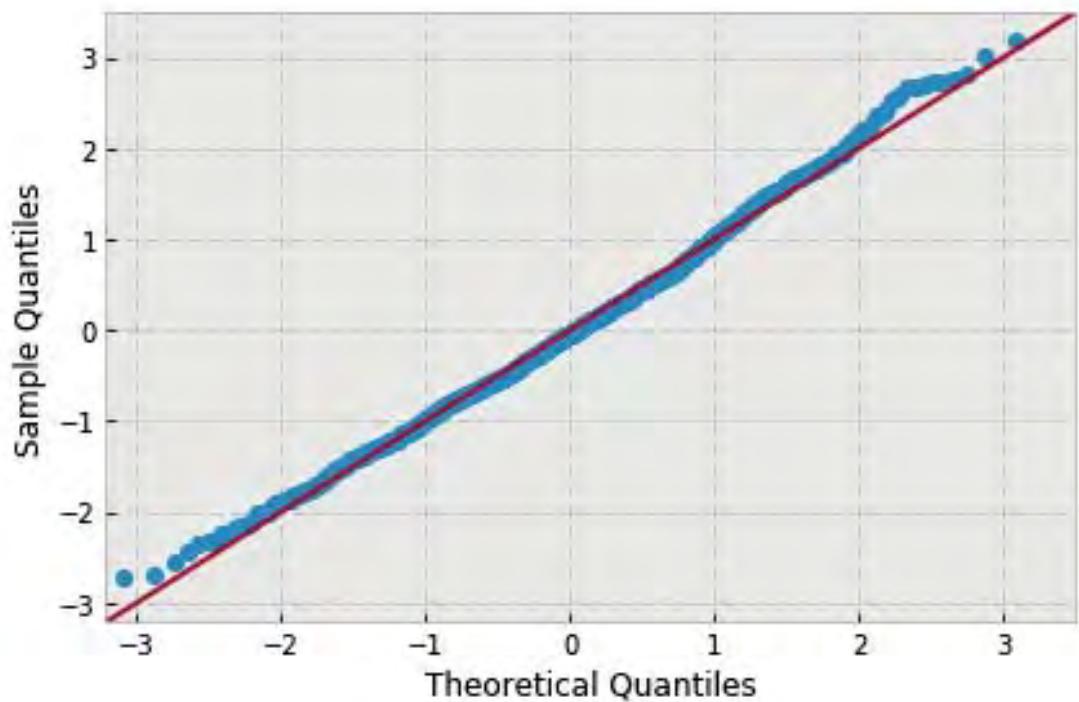
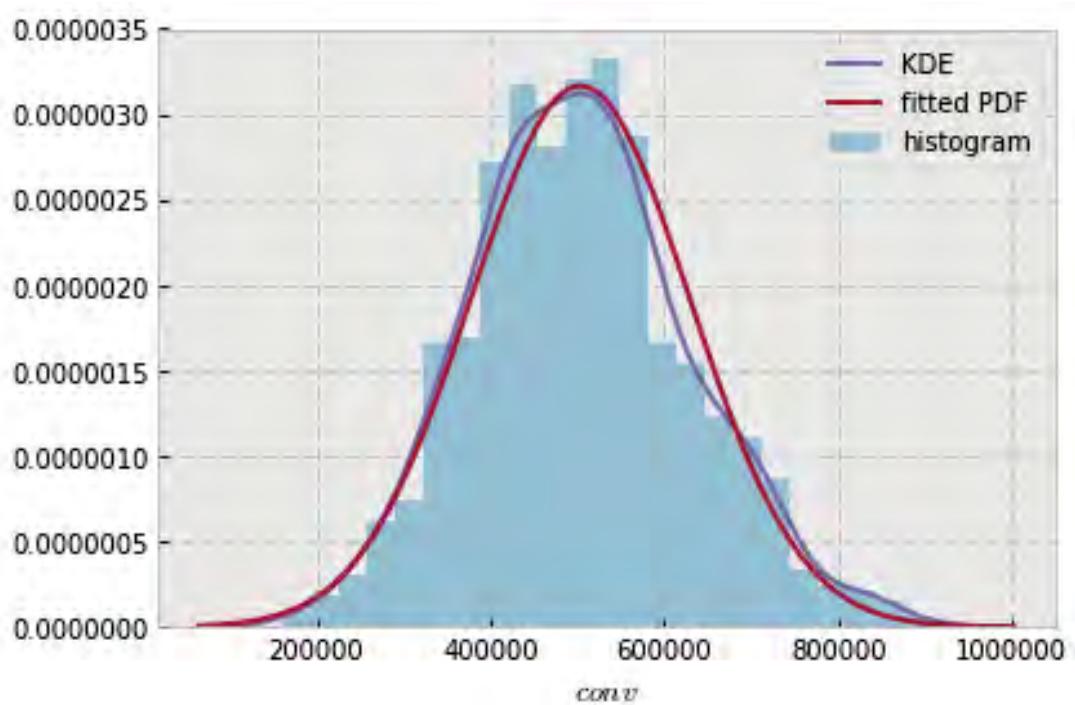


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=98174.67485847, scale=16549.48459837525)
DescribeResult(nobs=1000, minmax=(46989.55984999999, 151651.8168)
               mean=98174.674858469996, variance=274159600.07193148,
               skewness=0.09662930695731461, kurtosis=0.08135984899798654)
```

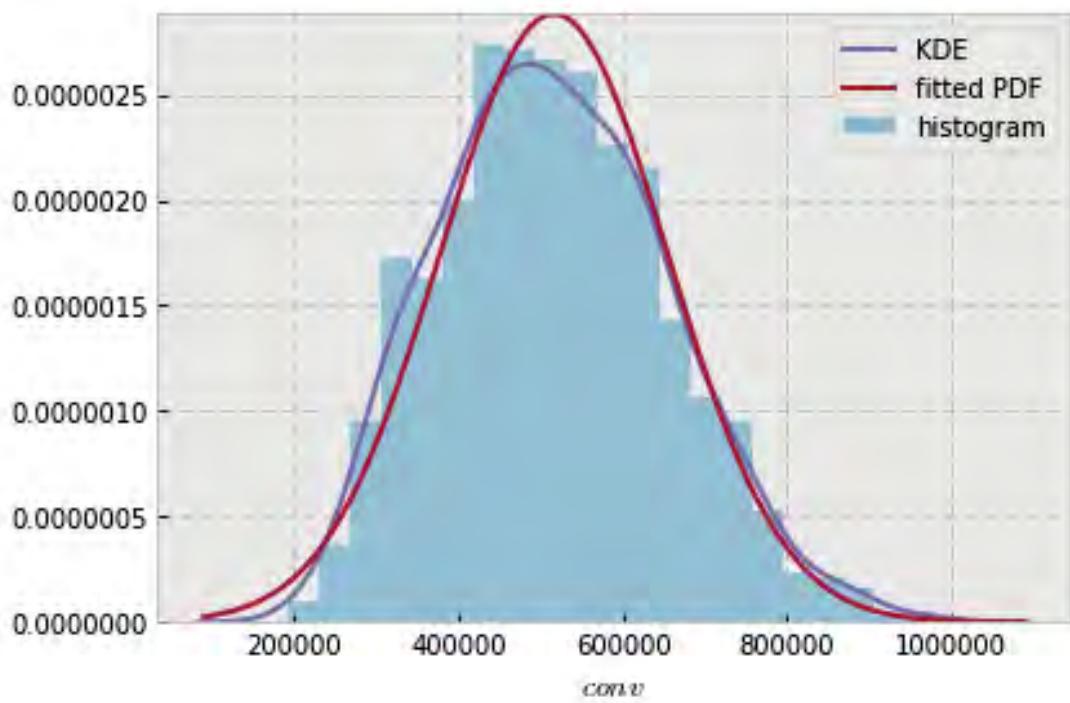


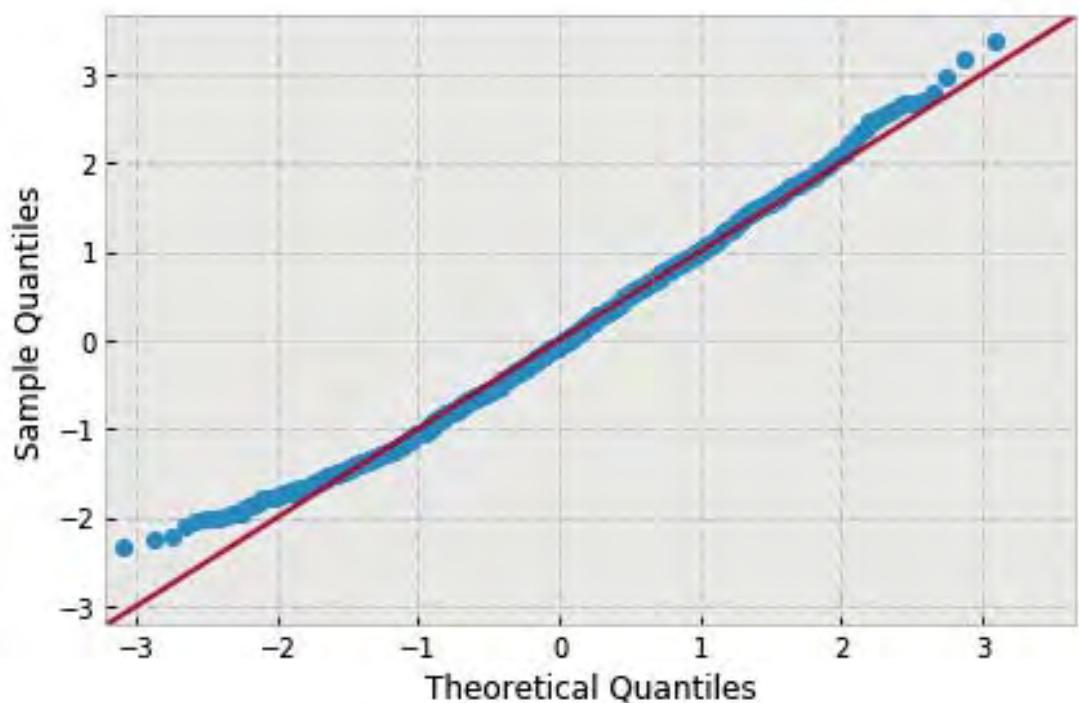


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=503673.4695341, scale=126089.99686070206)
DescribeResult(nobs=1000, minmax=(160880.7386000001, 906451.18229999999)
               mean=503673.46953409998, variance=15914601910.242096,
               skewness=0.2425691971486958, kurtosis=-0.02108312386823563)
```

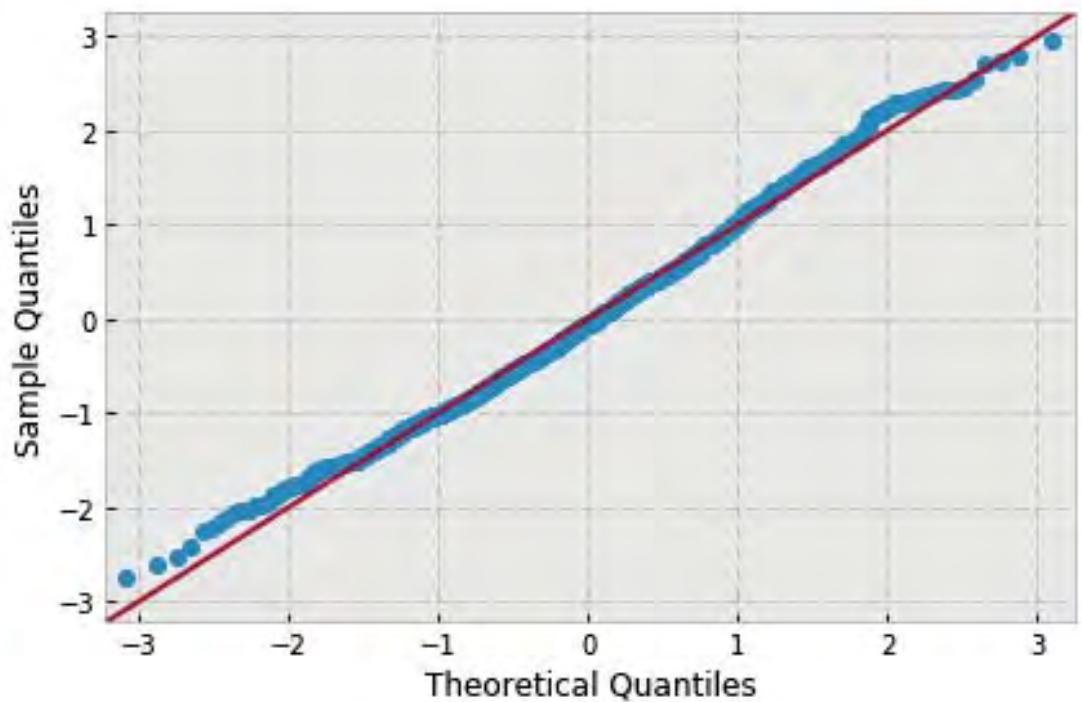
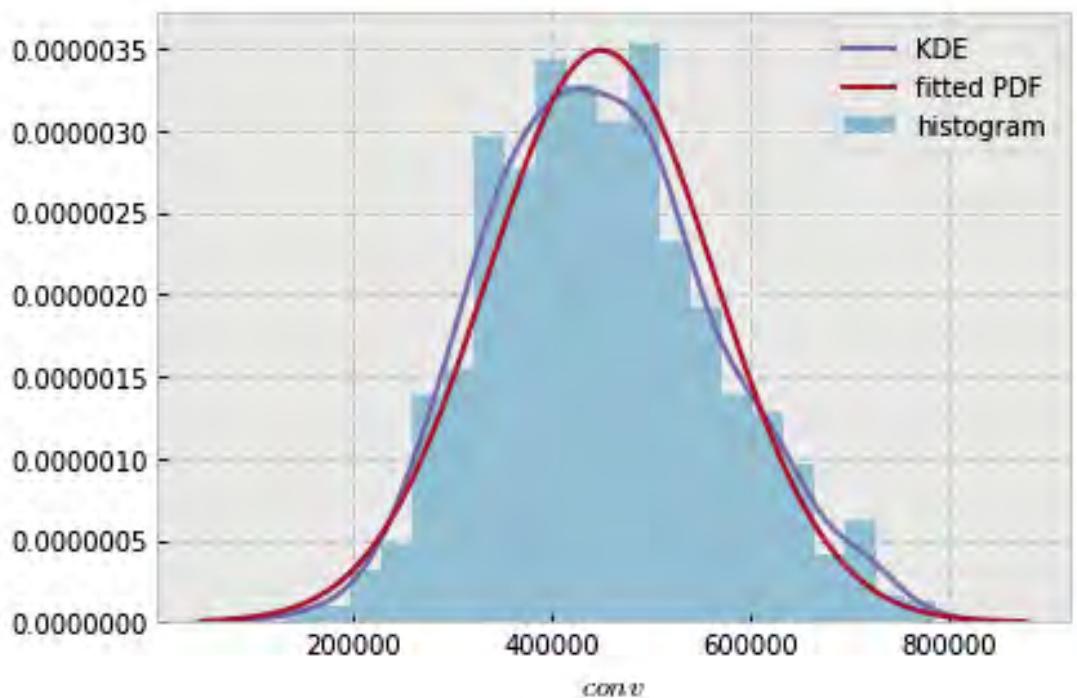


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=515898.30276329996, scale=138420.63455963708)
DescribeResult(nobs=1000, minmax=(193882.8063, 982139.9277)
    mean=515898.30276329996, variance=19179451523.416008,
    skewness=0.288200380759445, kurtosis=-0.2502412728473815)
```

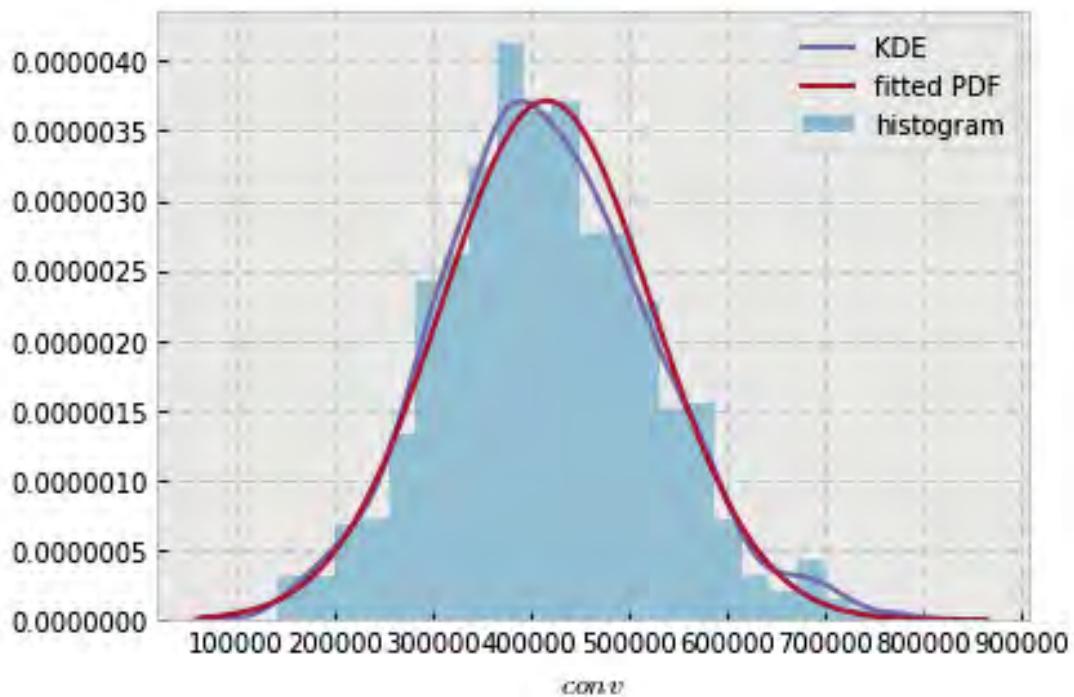


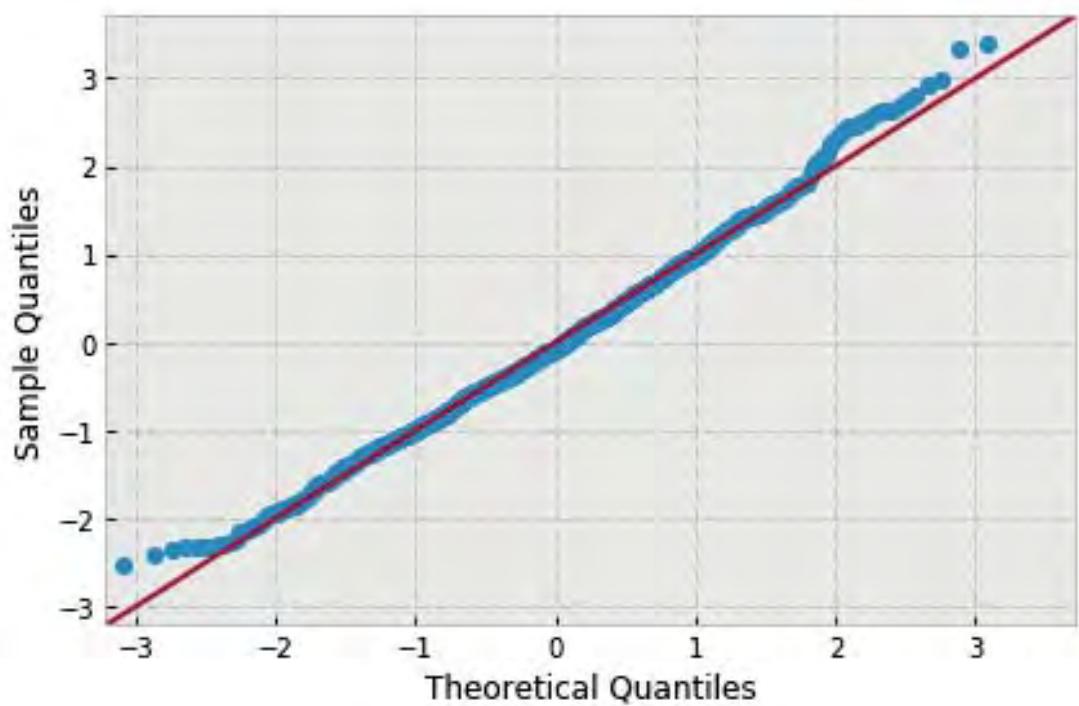


```
variable name: ln
variable value: 0.833333333333333
distribution: normal(shape=(), loc=450548.74619140005, scale=114276.44329359241)
DescribeResult(nobs=1000, minmax=(136238.0100000001, 788679.19889999996)
               mean=450548.74619140005, variance=13072177669.503143,
               skewness=0.2636808281207275, kurtosis=-0.24830182547694735)
```

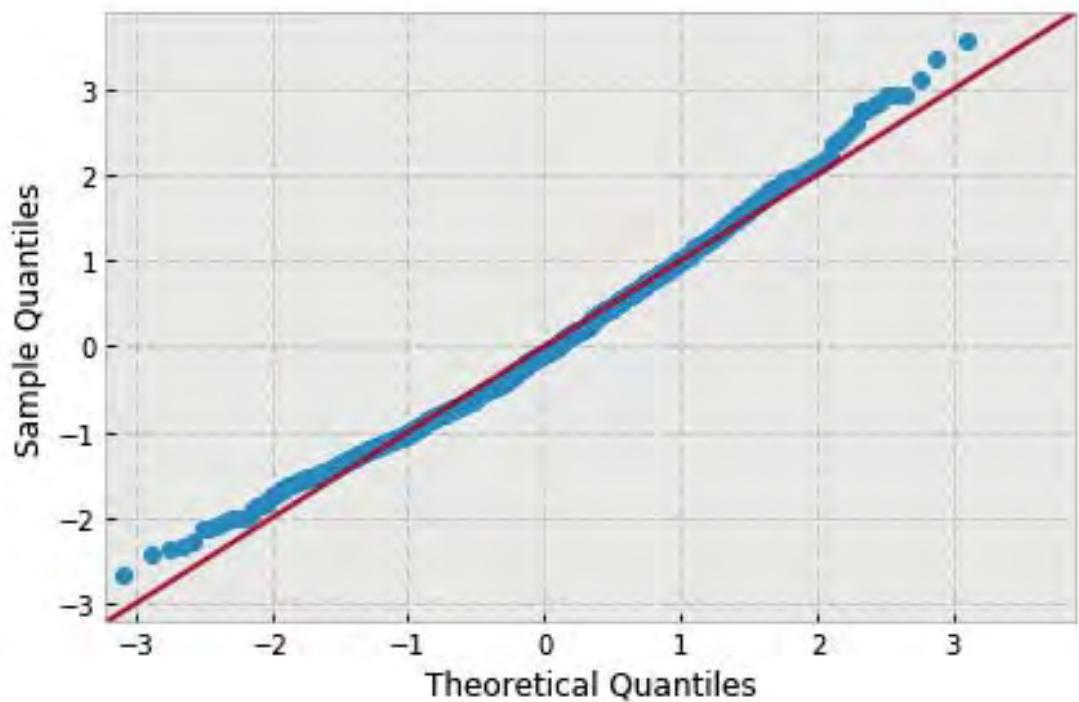
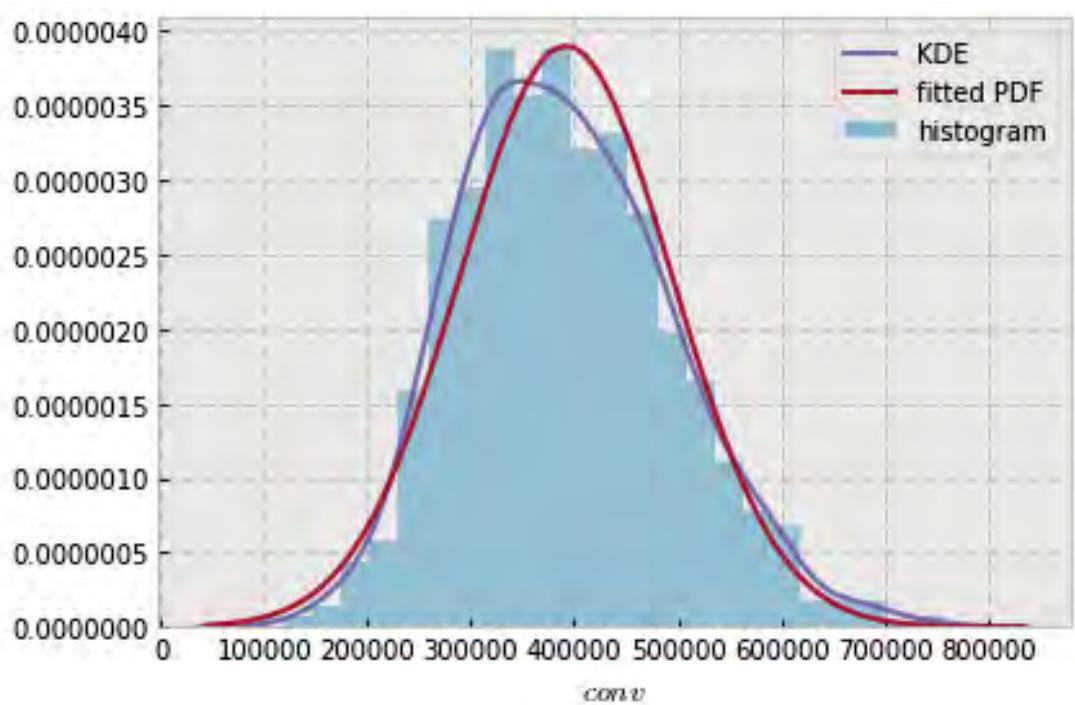


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=417104.8582146, scale=107373.10018362661)
DescribeResult(nobs=1000, minmax=(144323.296, 783000.0860000001)
               mean=417104.85821460001, variance=11540523166.209324,
               skewness=0.2684073482256936, kurtosis=0.08931849008600246)
```

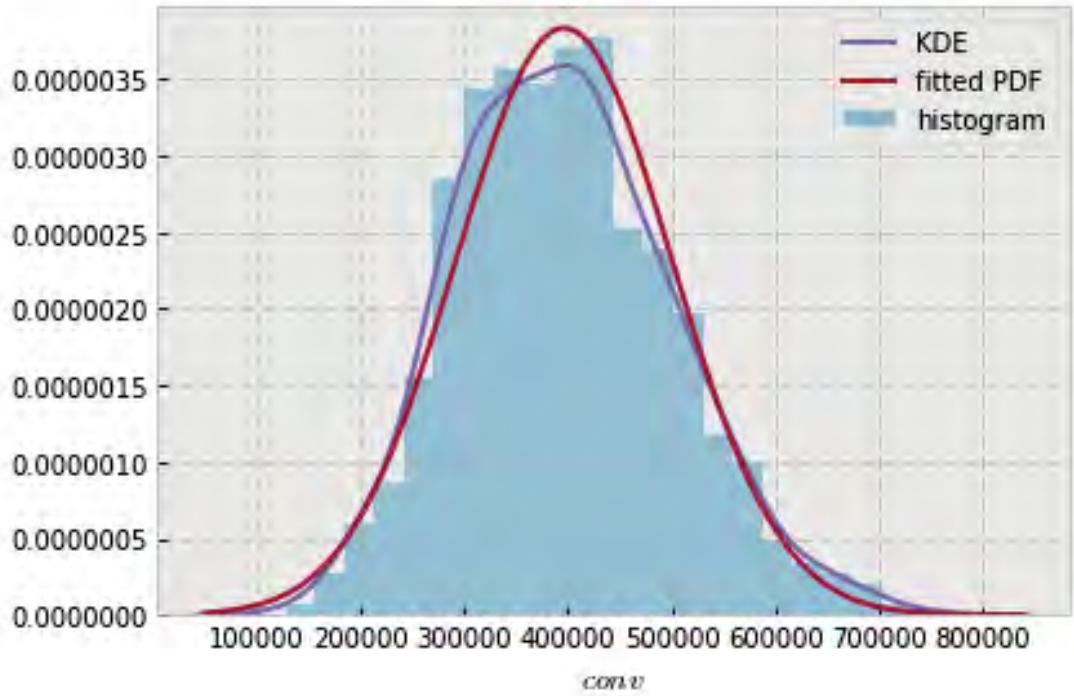


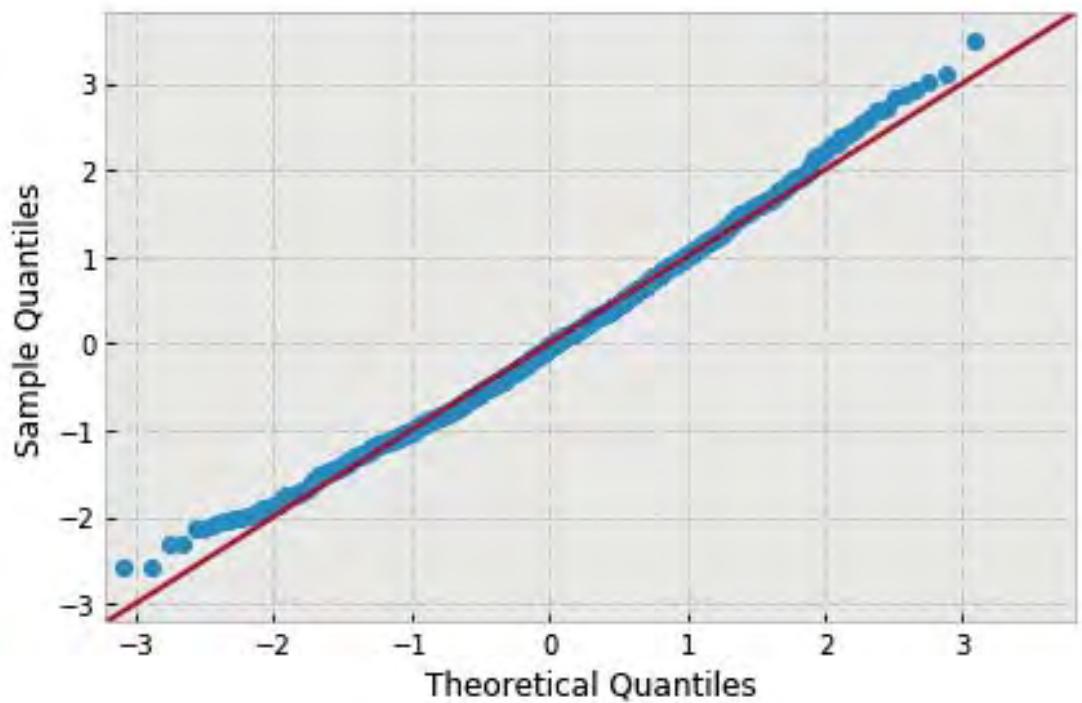


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=392001.6336975, scale=102313.01051772699)
DescribeResult(nobs=1000, minmax=(119508.0107, 757578.2249999998)
               mean=392001.63369749999, variance=10478430551.752266,
               skewness=0.3916979367958242, kurtosis=0.011273116407439776)
```

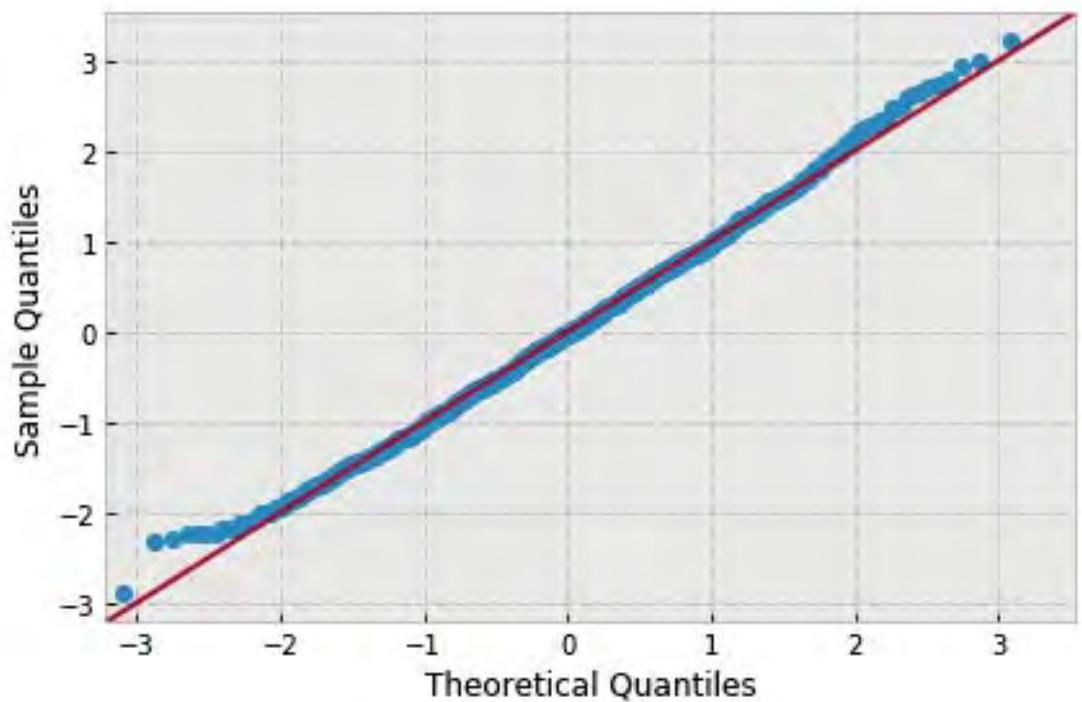
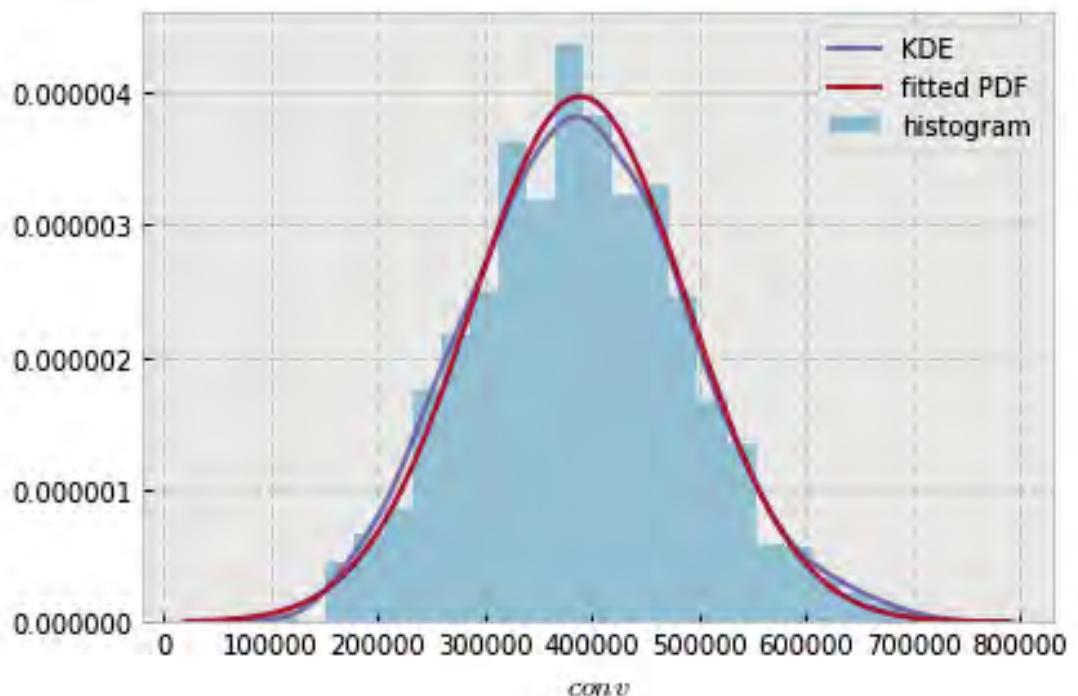


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=396205.54994389997, scale=104061.5064887984)
DescribeResult(nobs=1000, minmax=(126500.8039, 760721.6352000002)
               mean=396205.54994389997, variance=10839636769.487719,
               skewness=0.3241096915345769, kurtosis=-0.04075245757331114)
```

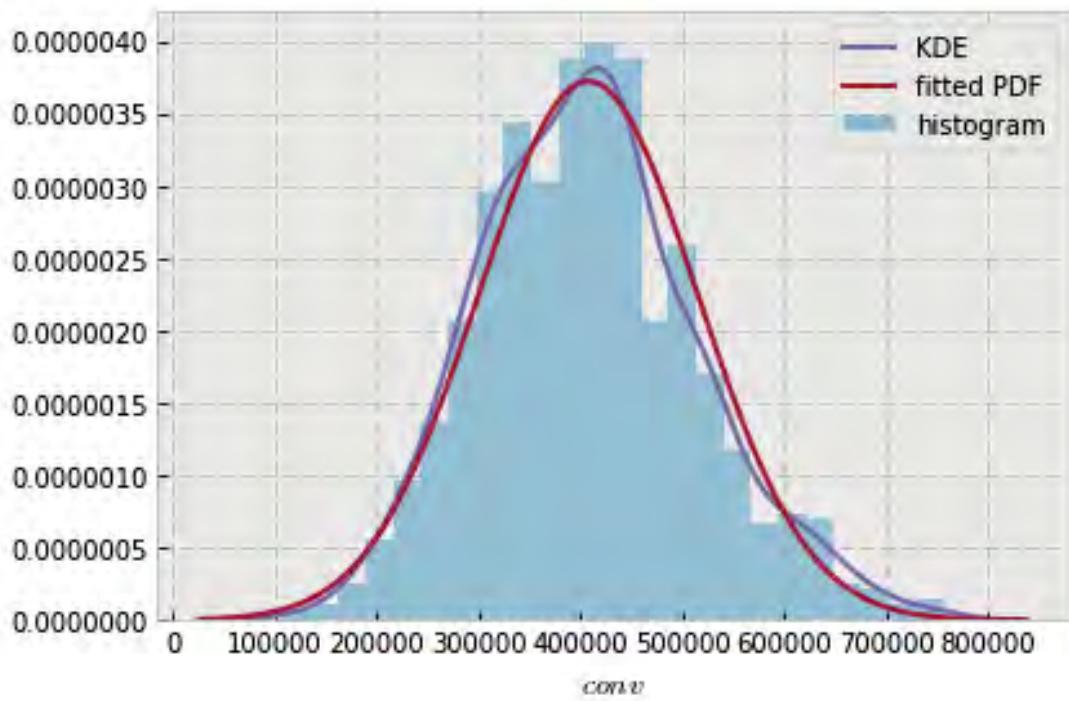


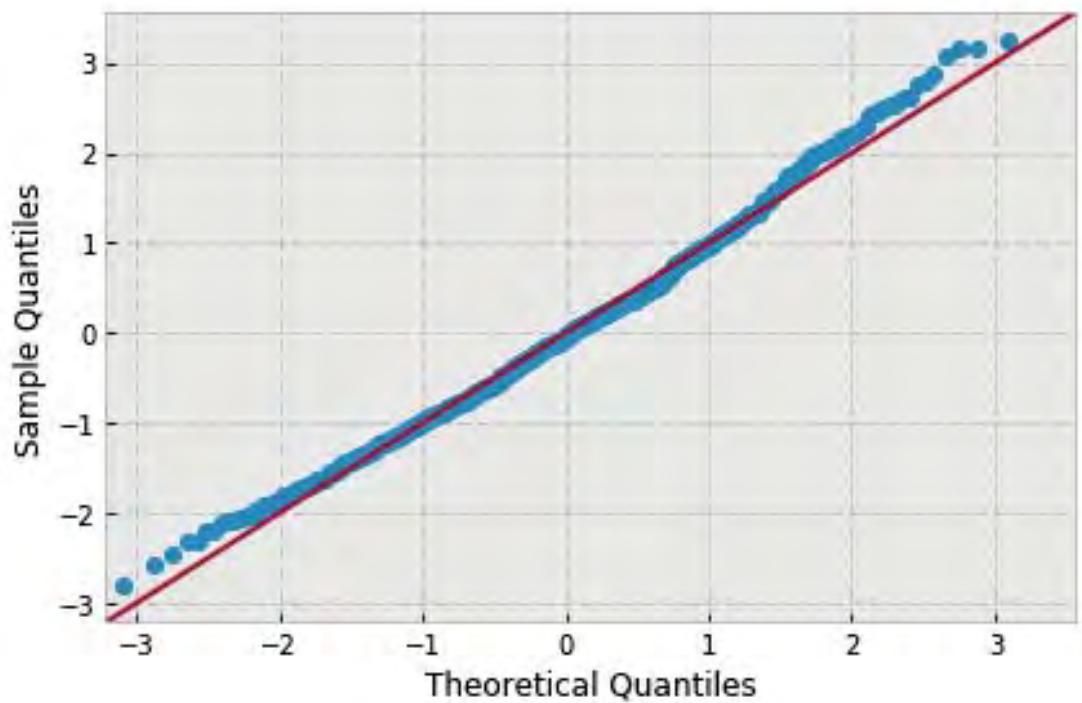


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=389371.82484827, scale=100491.01449238499)
DescribeResult(nobs=1000, minmax=(98916.171069999997, 713210.14980000001)
               mean=389371.82484826999, variance=10108552546.254984,
               skewness=0.1716632561568022, kurtosis=-0.12113763945525147)
```

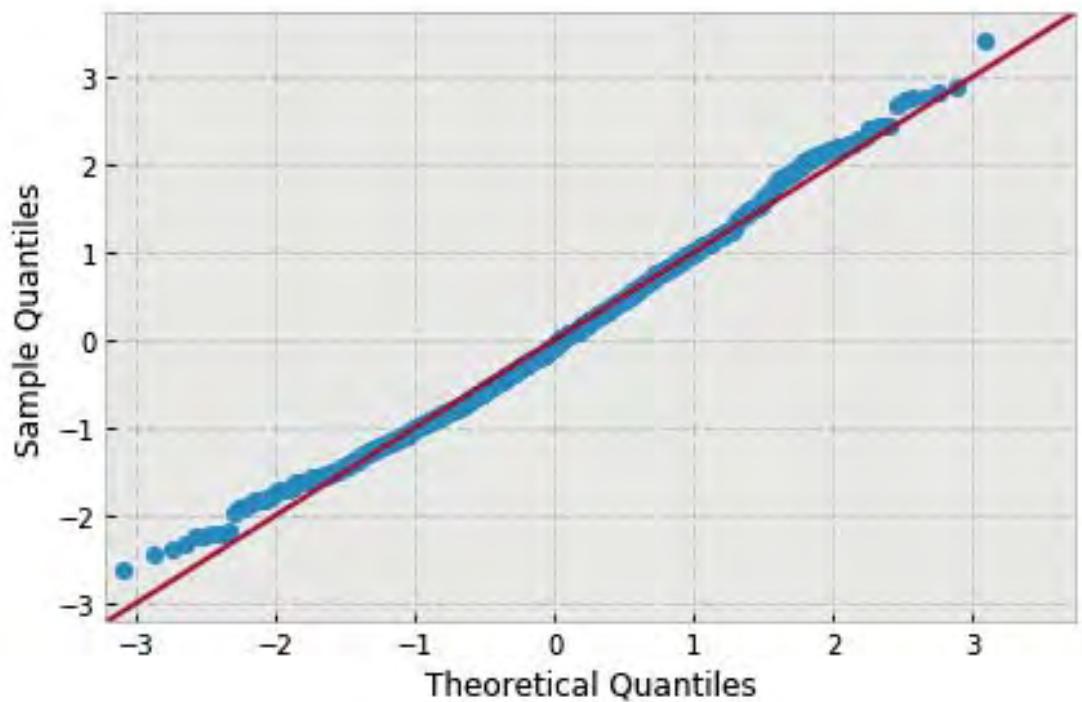
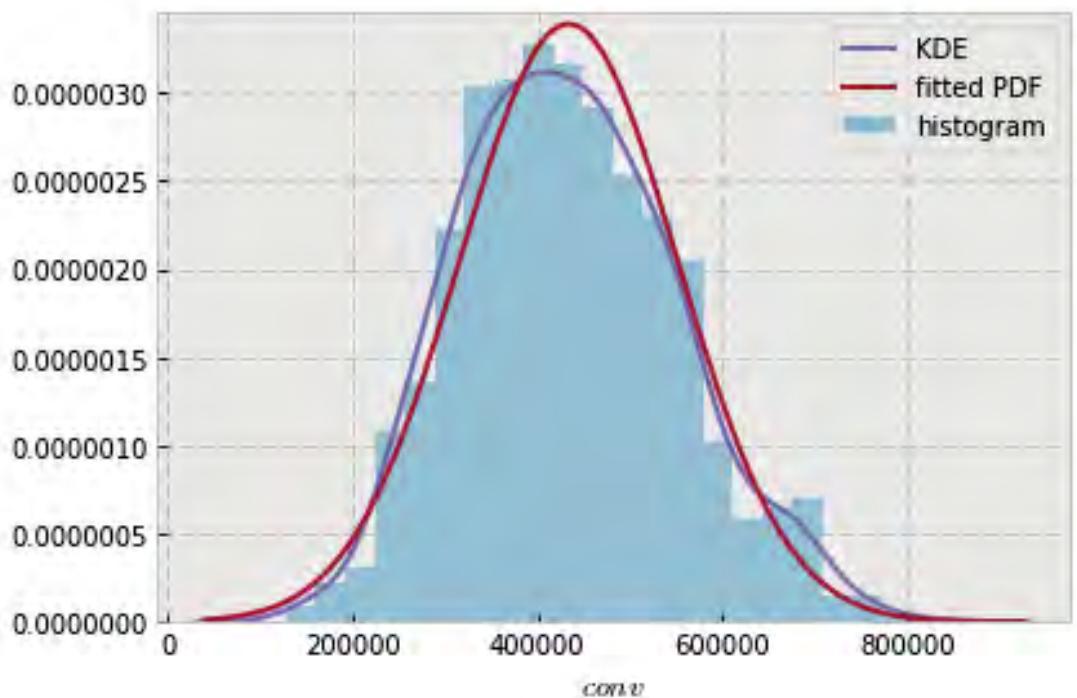


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=407289.73211240006, scale=107192.65716517481)
DescribeResult(nobs=1000, minmax=(107686.5505, 755768.67700000003)
               mean=407289.73211240006, variance=11501767517.648352,
               skewness=0.33106515981685425, kurtosis=0.0857063192236911)
```

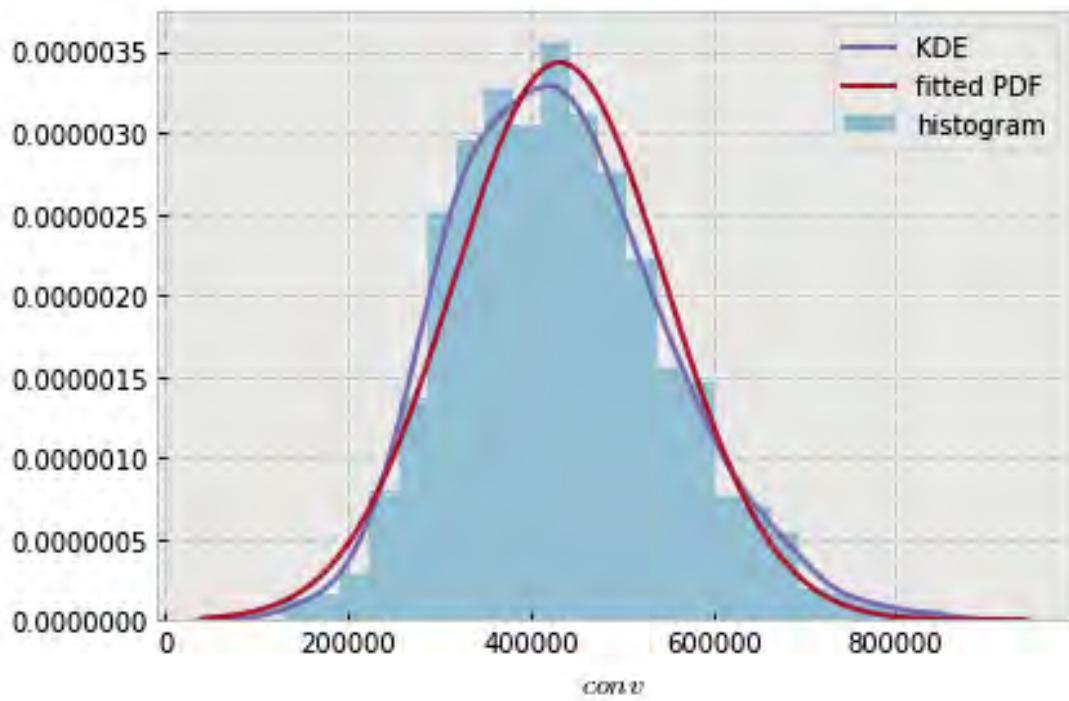


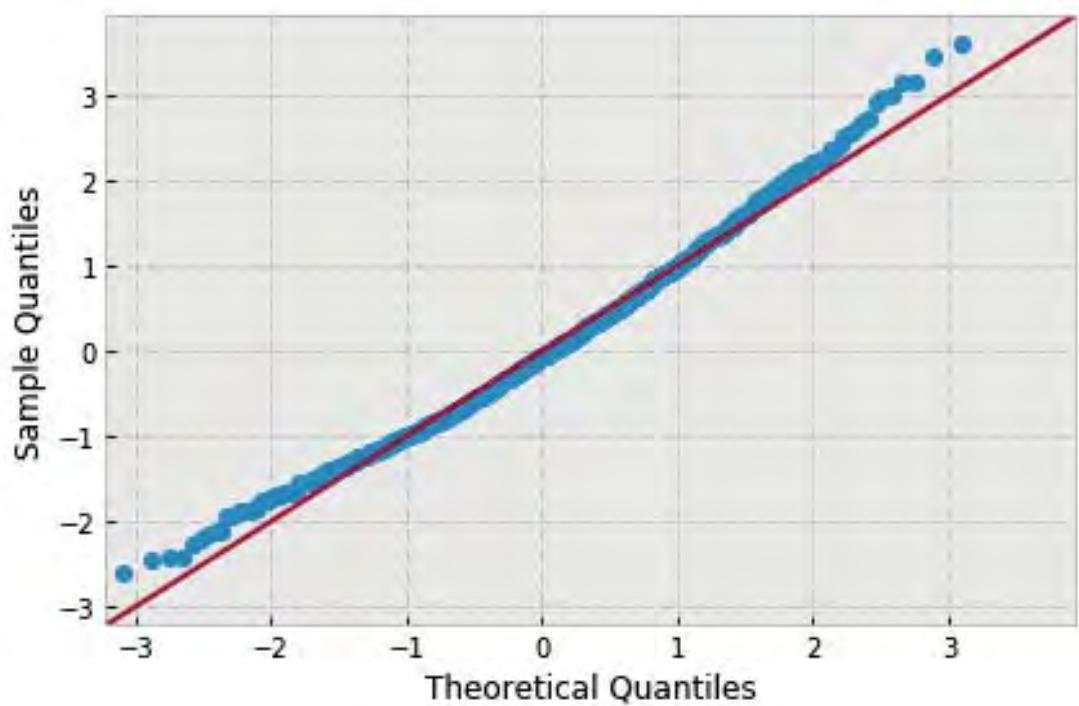


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=433713.60826209997, scale=117722.31849987003)
DescribeResult(nobs=1000, minmax=(127050.341, 836748.81779999996)
               mean=433713.60826209997, variance=13872416689.674515,
               skewness=0.3056543992784221, kurtosis=-0.1780757939979445)
```

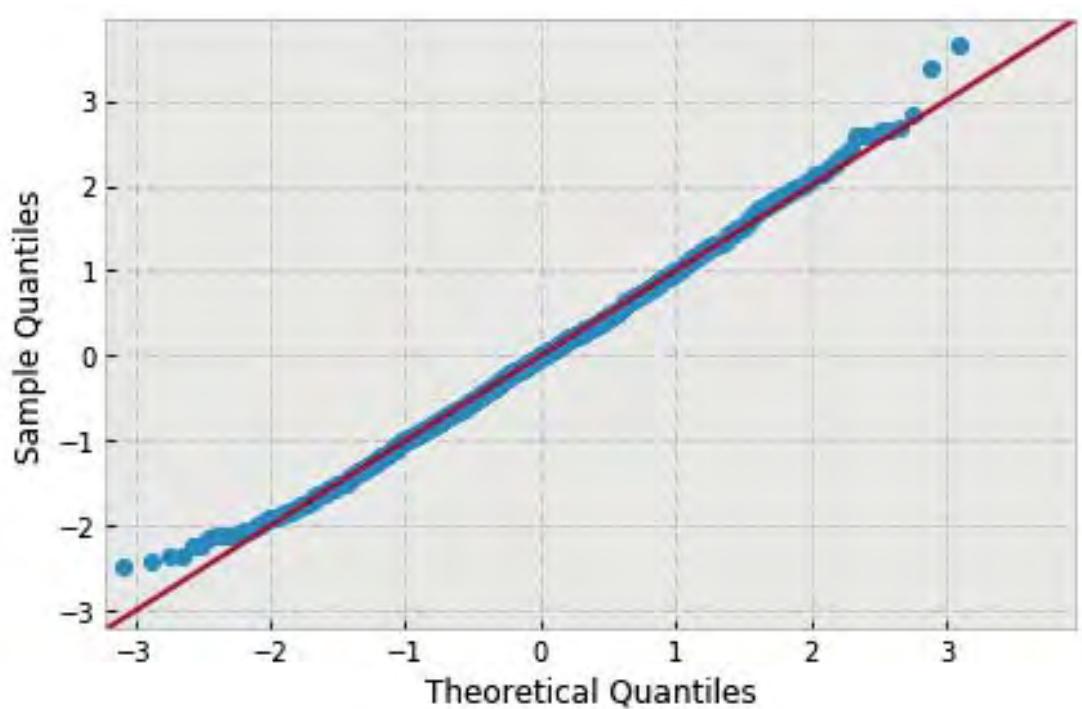
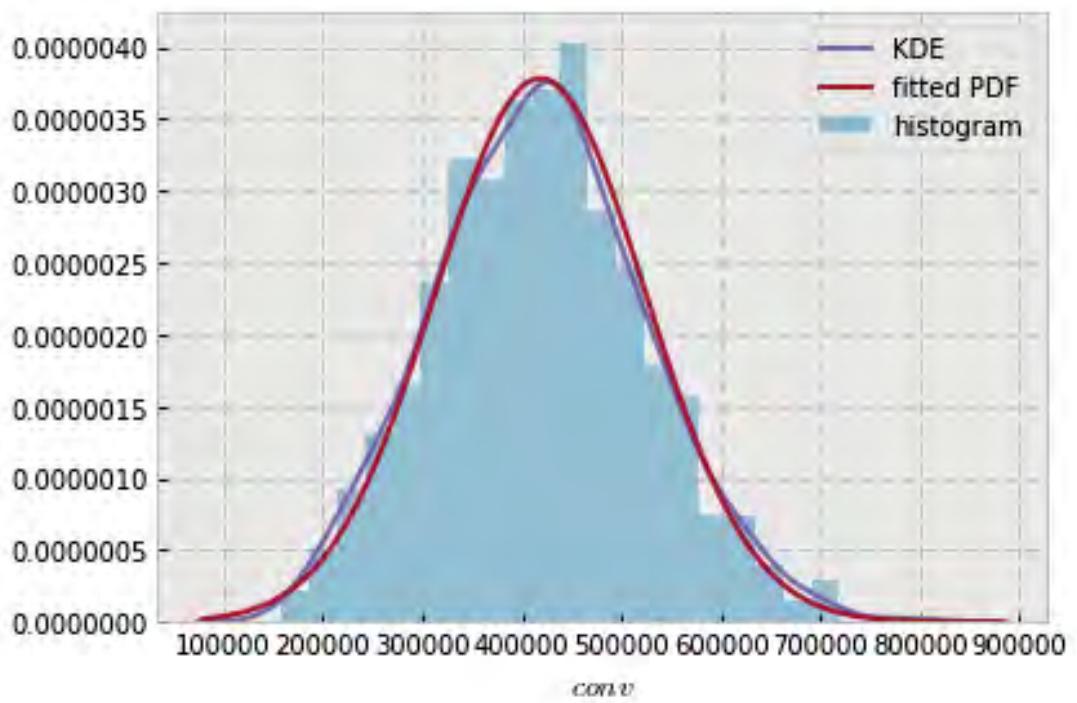


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=431883.4697952, scale=116244.52652265171)
DescribeResult(nobs=1000, minmax=(129213.7752999999, 851959.4647000001)
               mean=431883.4697951998, variance=13526316262.738214,
               skewness=0.42444180182667923, kurtosis=0.093509690887974)
```

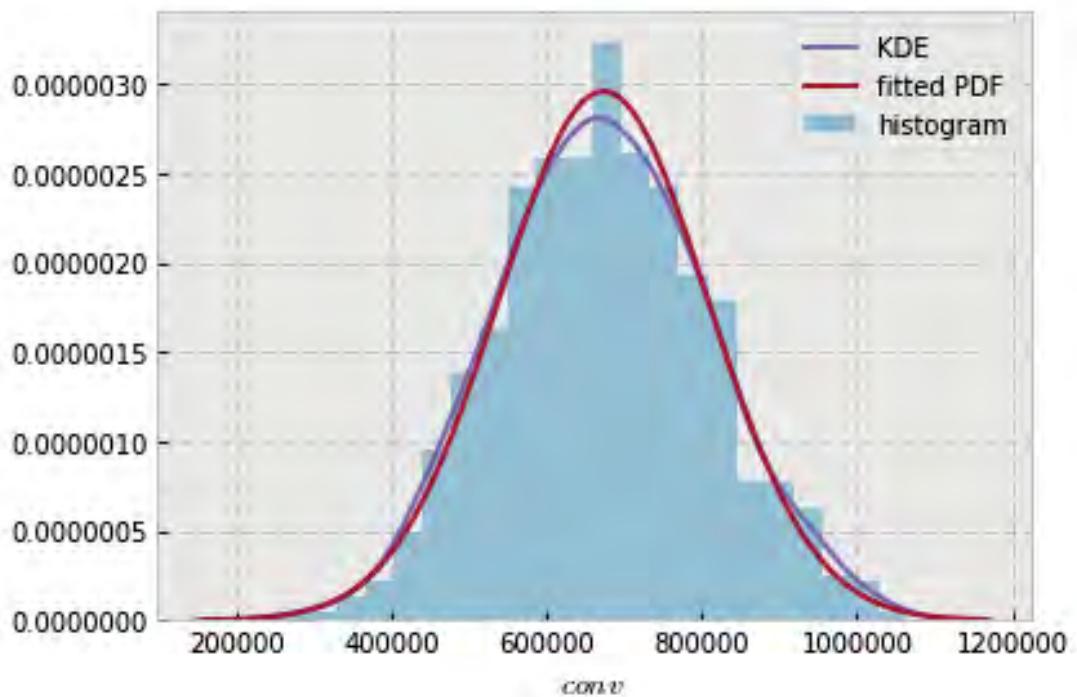


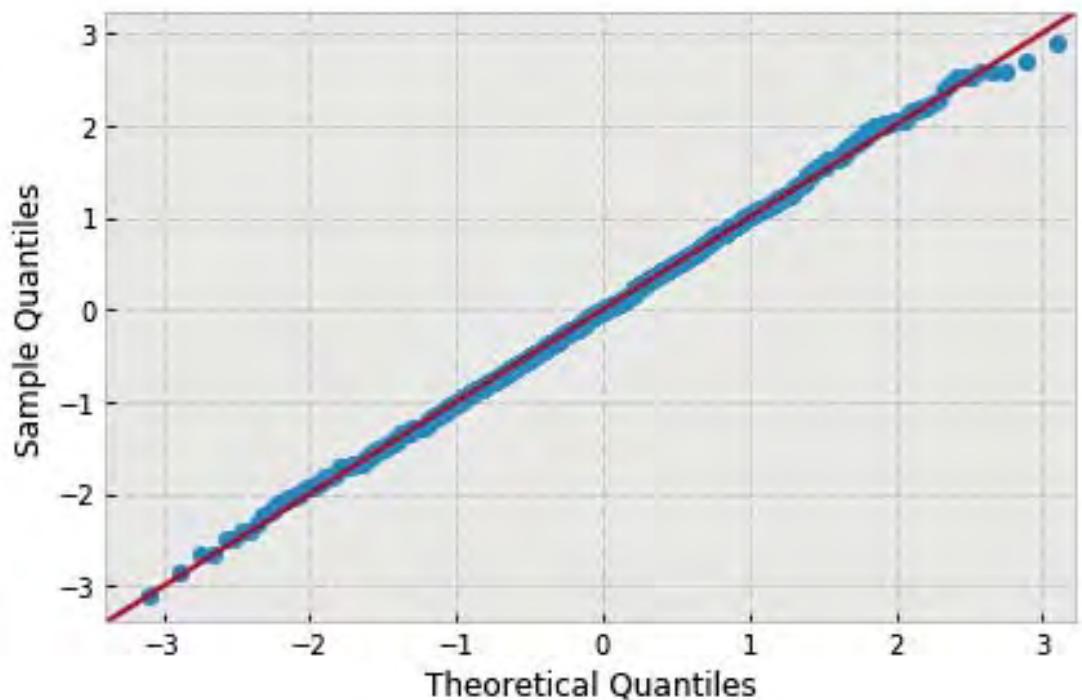


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=418634.4643035001, scale=105475.59351796337)
DescribeResult(nobs=1000, minmax=(158426.1406999999, 802599.42319999996)
               mean=418634.4643035008, variance=11136237065.031668,
               skewness=0.18451584950856026, kurtosis=-0.07006918421029784)
```

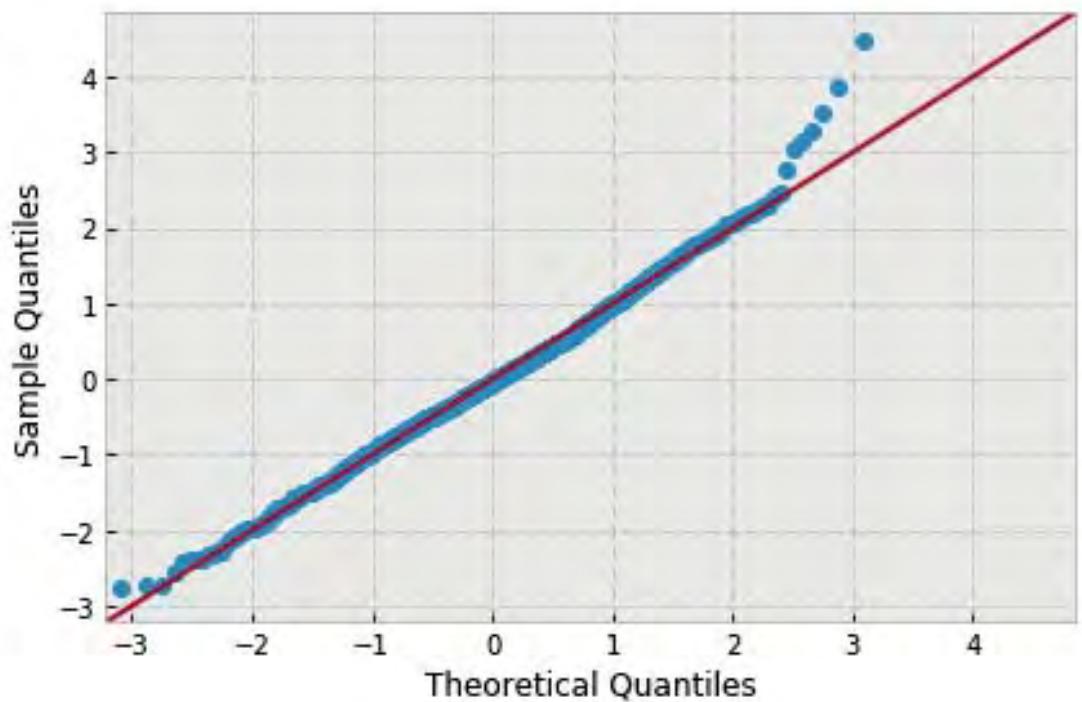
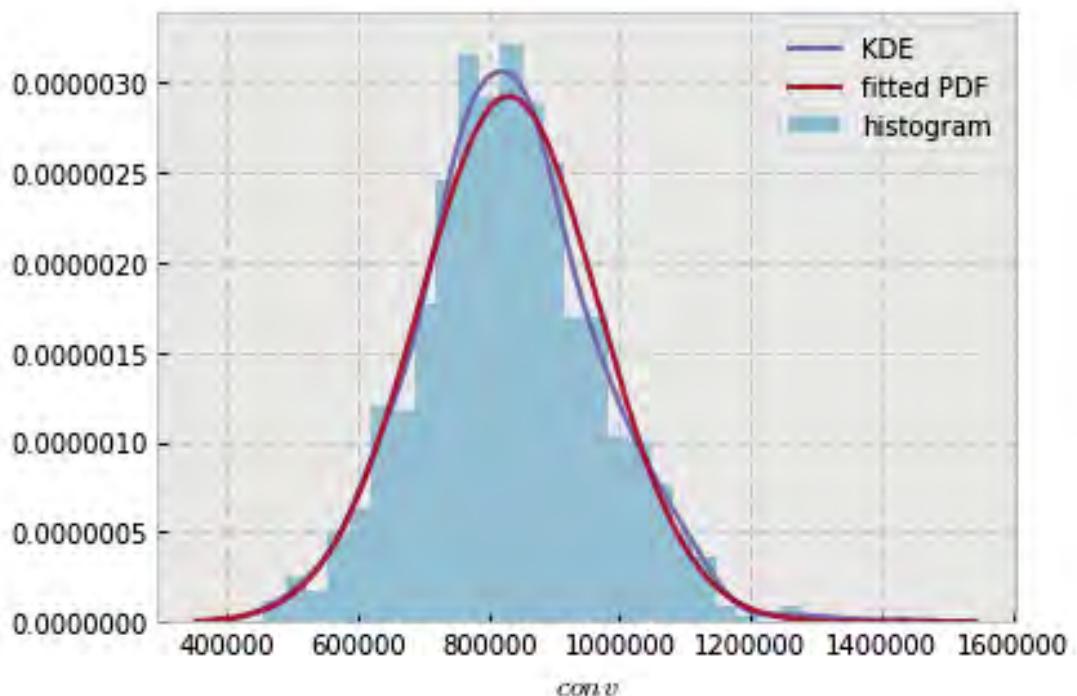


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=673217.2827369001, scale=134787.10983327928)
DescribeResult(nobs=1000, minmax=(256850.1686, 1064546.773)
               mean=673217.28273690015, variance=18185750727.936428,
               skewness=0.06305142906665993, kurtosis=-0.1944775395930698)
```

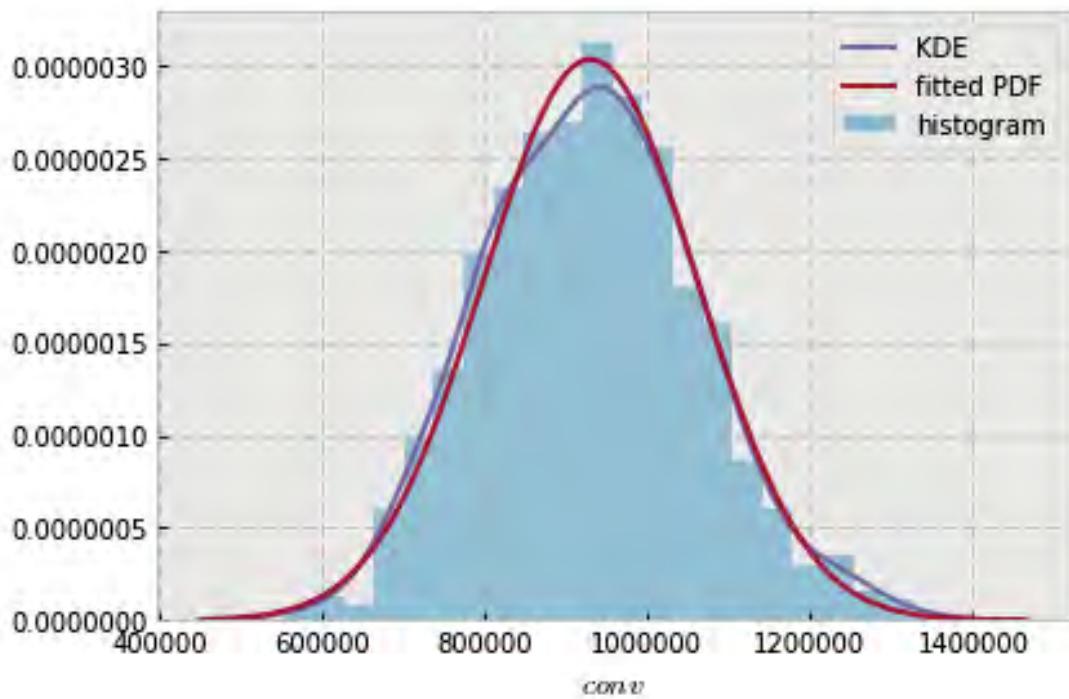


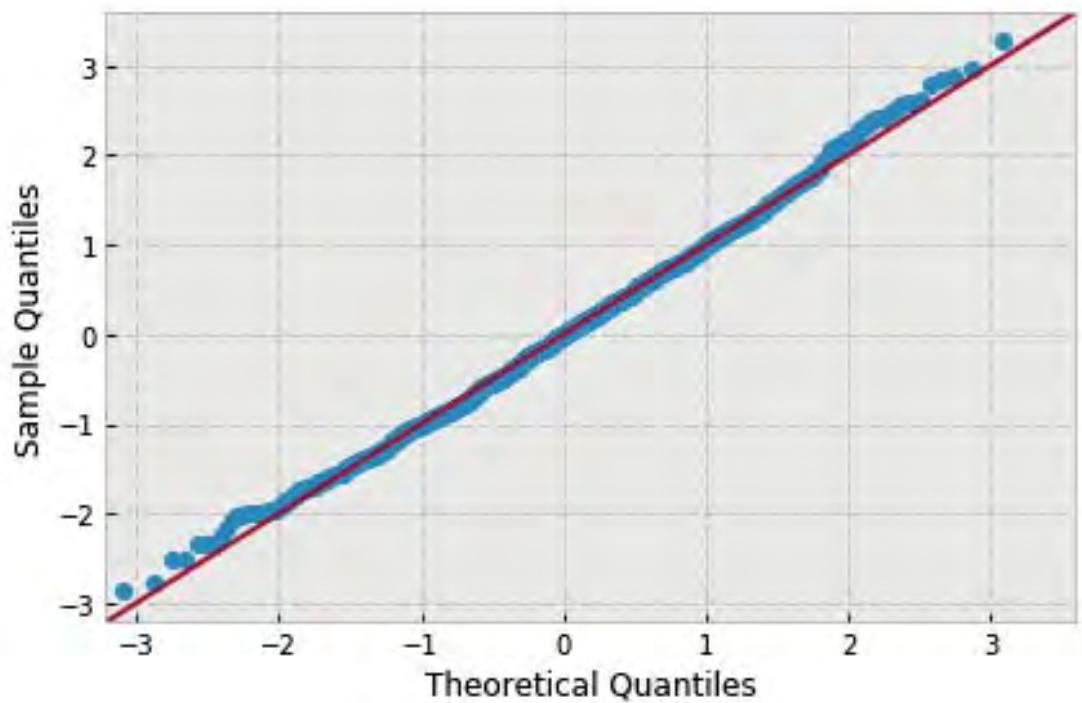


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=830972.0054825001, scale=136449.3689947833)
DescribeResult(nobs=1000, minmax=(456283.9907000002, 1442641.7549999999)
               mean=830972.0054825007, variance=18637067366.440964,
               skewness=0.27027871328610165, kurtosis=0.5432195137097606)
```

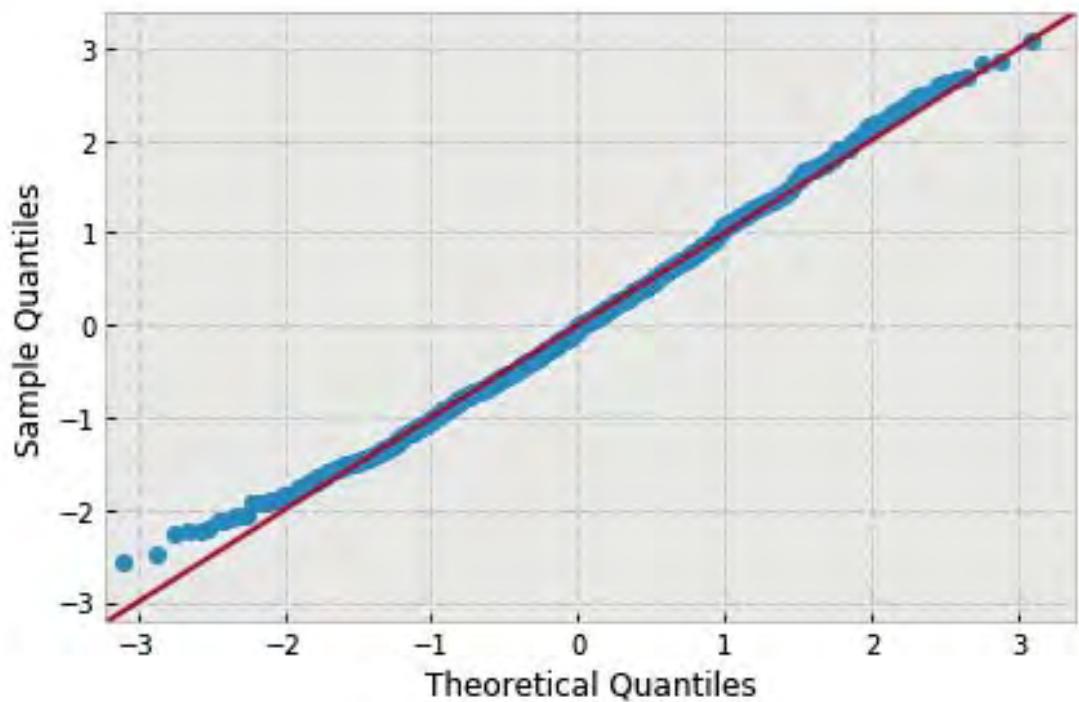
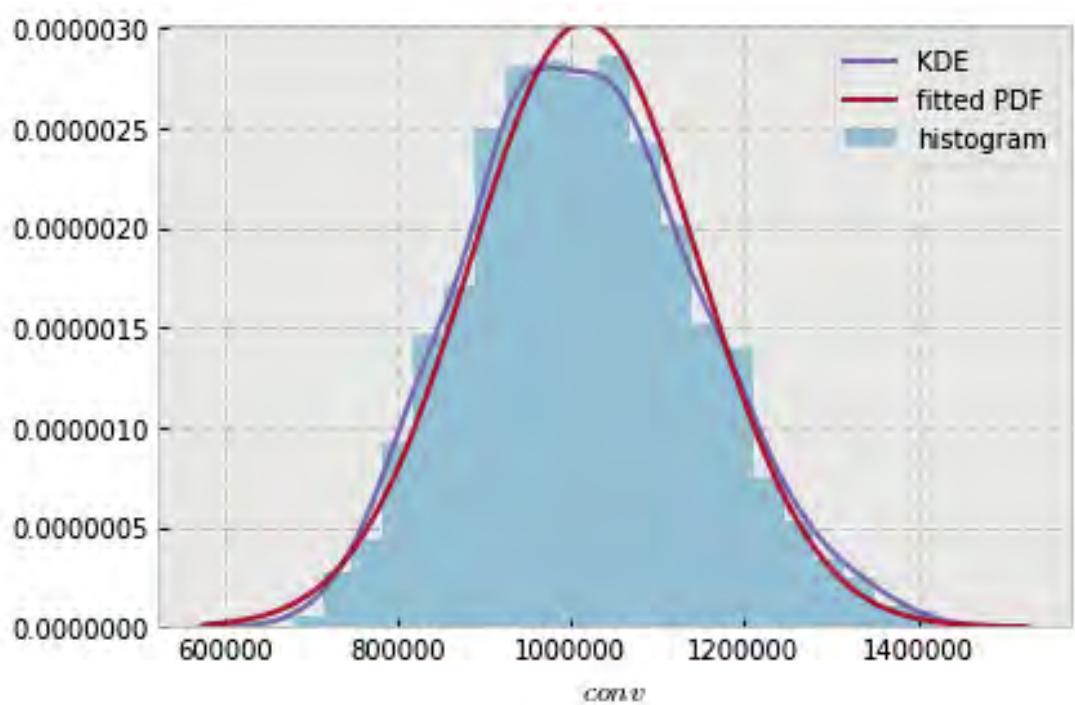


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=929958.864188, scale=131573.10656357638)
DescribeResult(nobs=1000, minmax=(553605.5233999995, 1361742.426999999)
               mean=929958.8641879998, variance=17328811181.972202,
               skewness=0.16626964136508887, kurtosis=-0.0909816971214572)
```

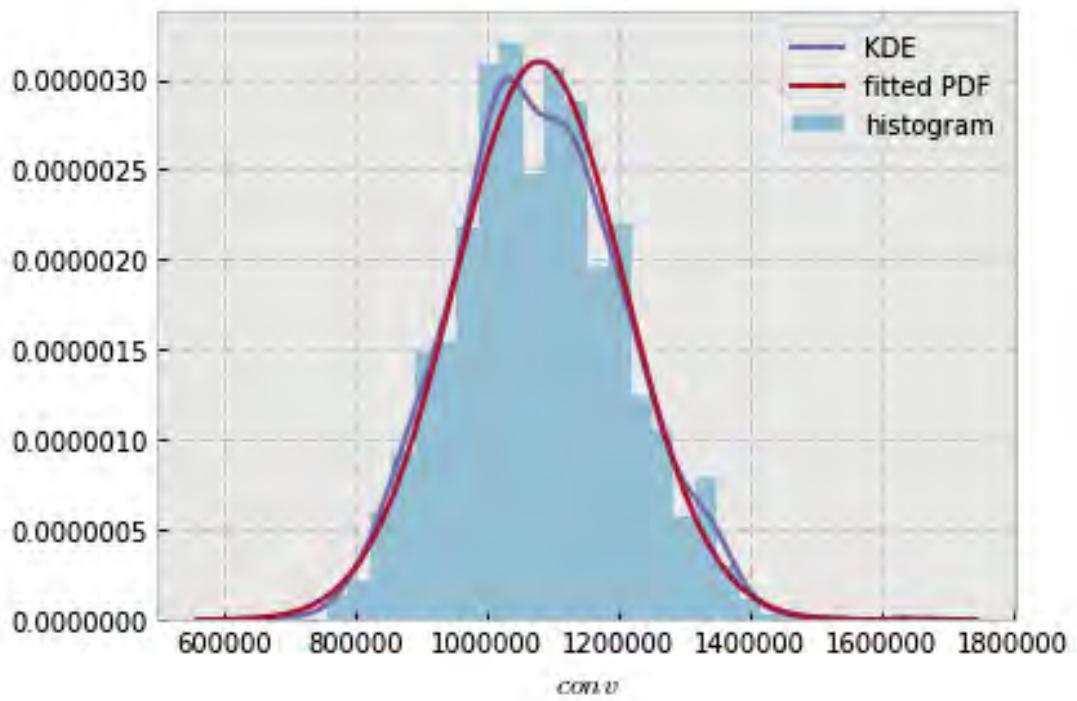


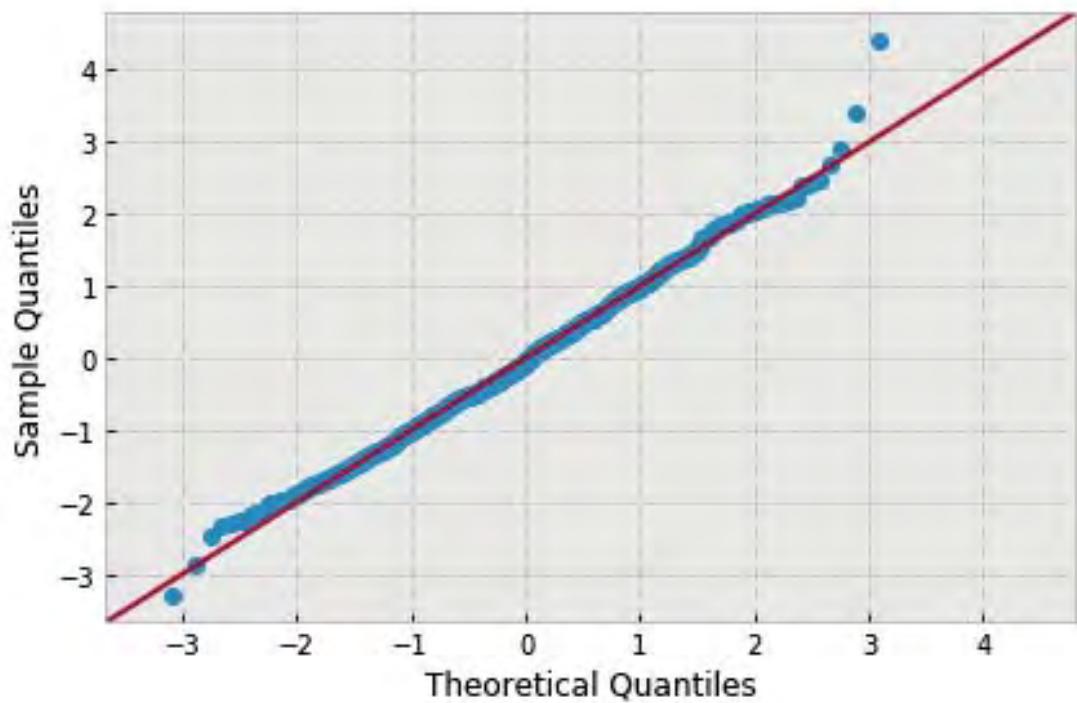


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=1015523.4947795001, scale=131665.62652654198)
DescribeResult(nobs=1000, minmax=(676500.5145999999, 1422924.818)
               mean=1015523.4947795001, variance=17353190399.025856,
               skewness=0.2307113192709296, kurtosis=-0.2608331314635377)
```

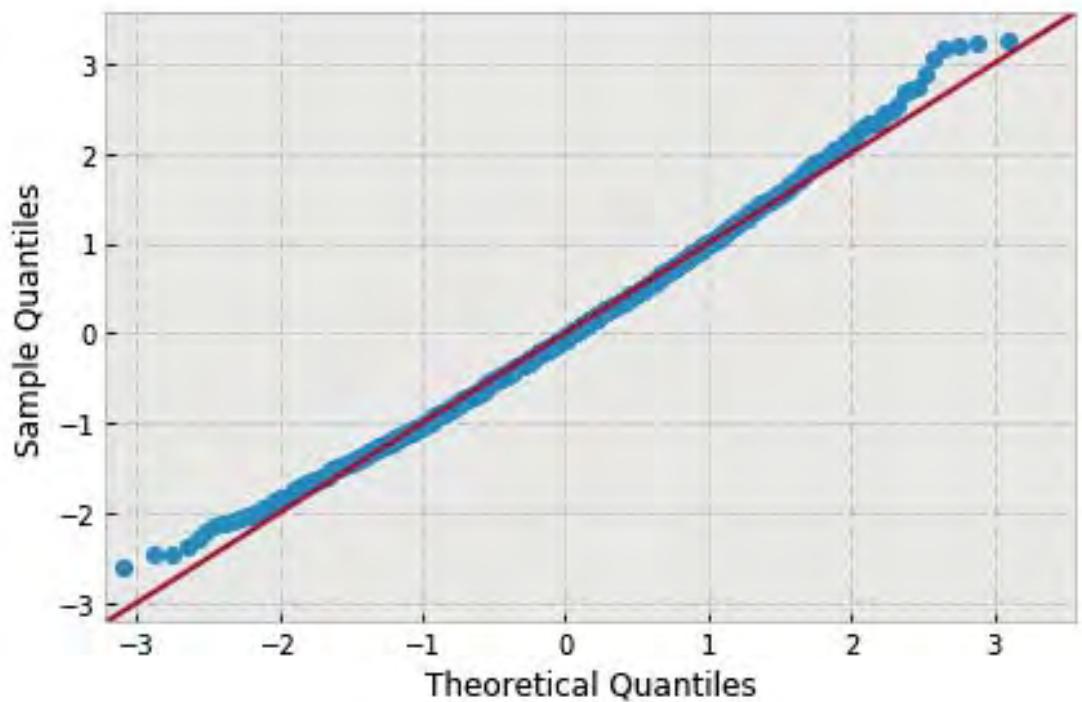
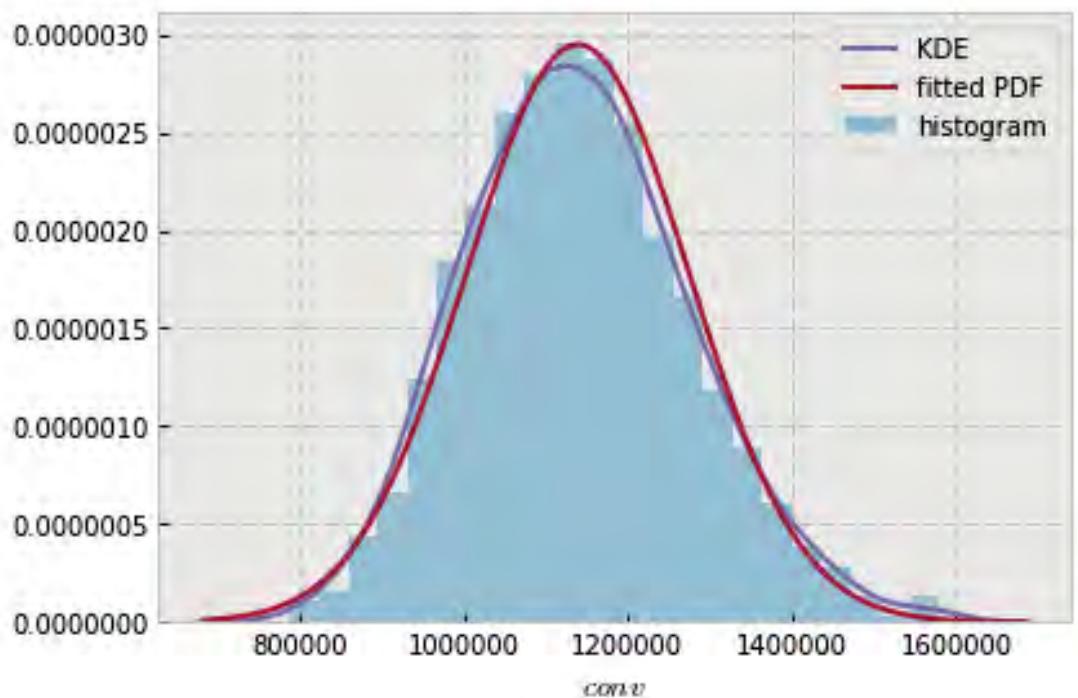


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=1080650.7421215, scale=128696.45735385464)
DescribeResult(nobs=1000, minmax=(658217.4264, 1647913.000999999)
               mean=1080650.7421215, variance=16579357492.925453,
               skewness=0.1924786002638292, kurtosis=0.07846495742118265)
```

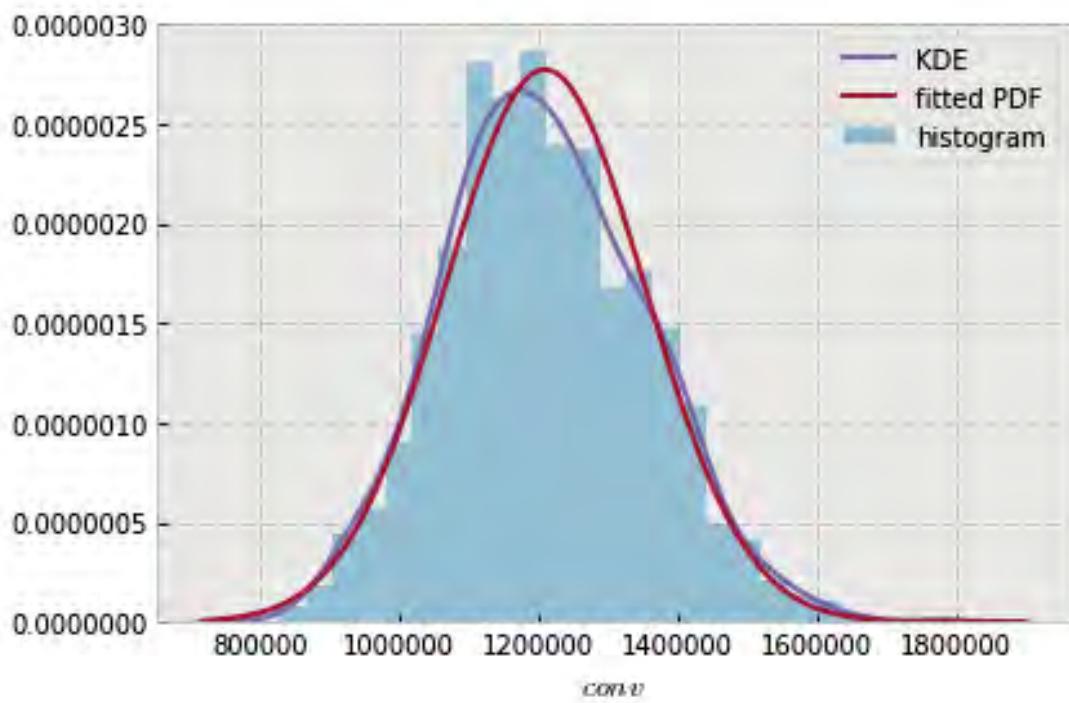


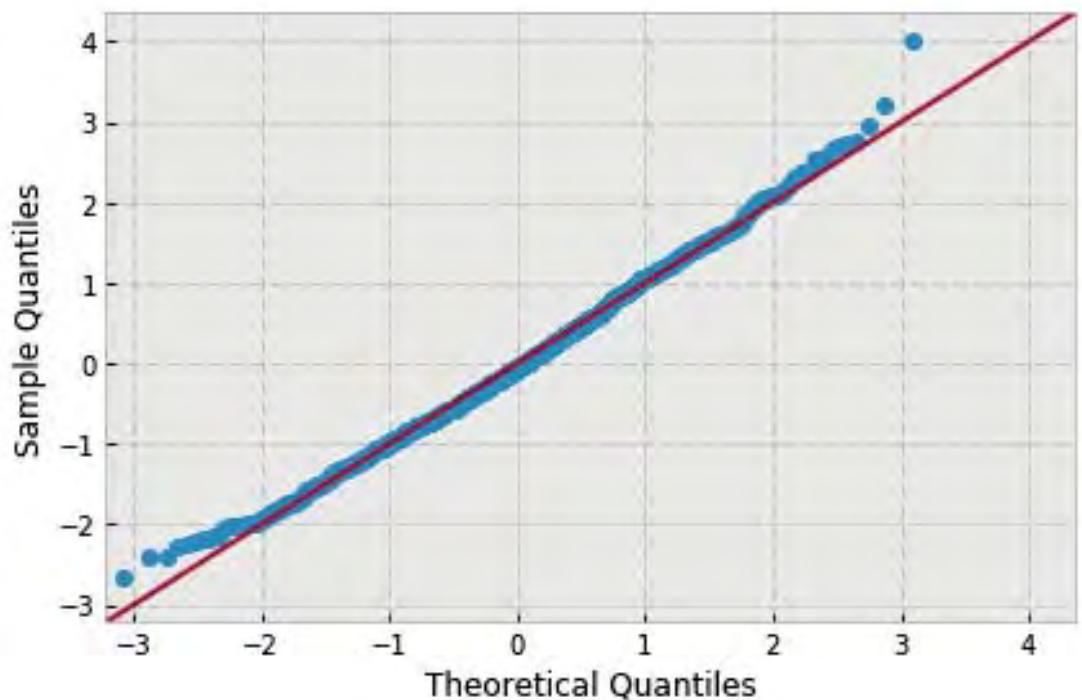


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=1138581.4527681002, scale=135275.2992819908)
DescribeResult(nobs=1000, minmax=(785340.3828000002, 1580333.548)
               mean=1138581.4527681002, variance=18317724320.152328,
               skewness=0.2979683177505183, kurtosis=0.007297104140776334)
```

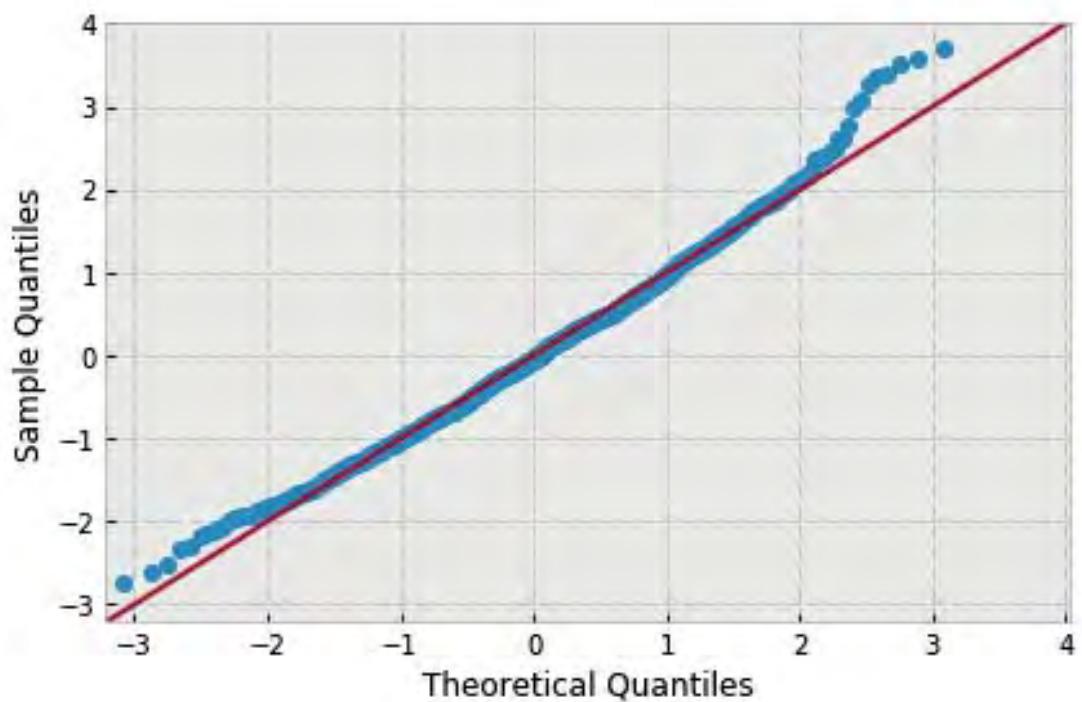
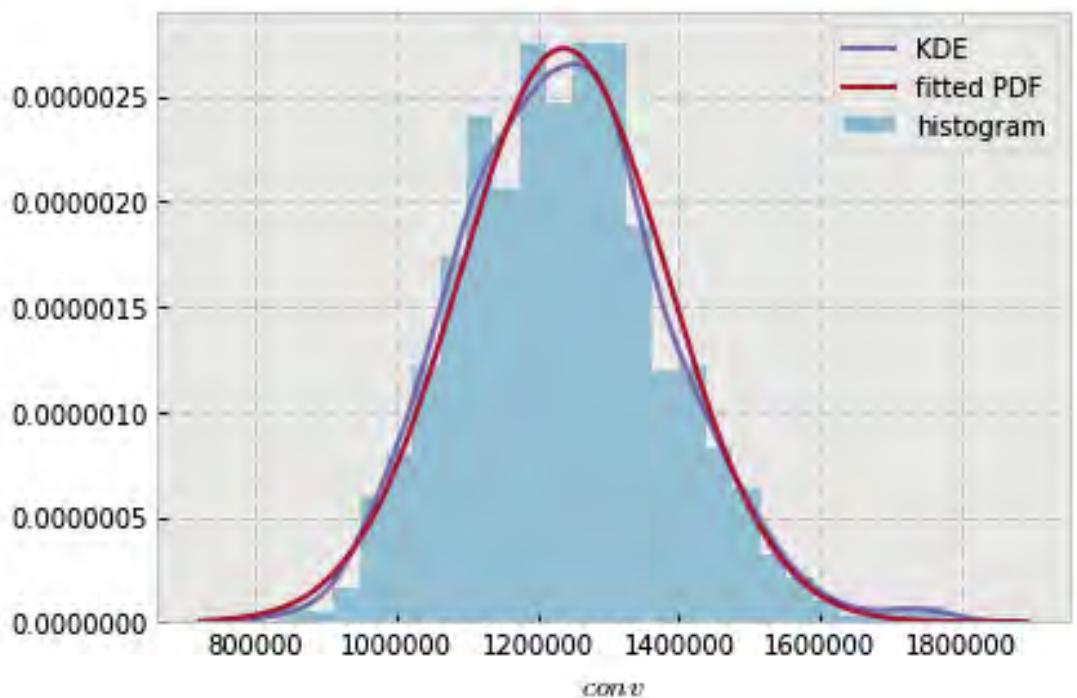


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=1209545.9841411, scale=143850.3852602835)
DescribeResult(nobs=1000, minmax=(827041.8993999999, 1787739.993)
               mean=1209545.9841411, variance=20713646986.518509,
               skewness=0.24314803845305522, kurtosis=-0.04346023012953193)
```

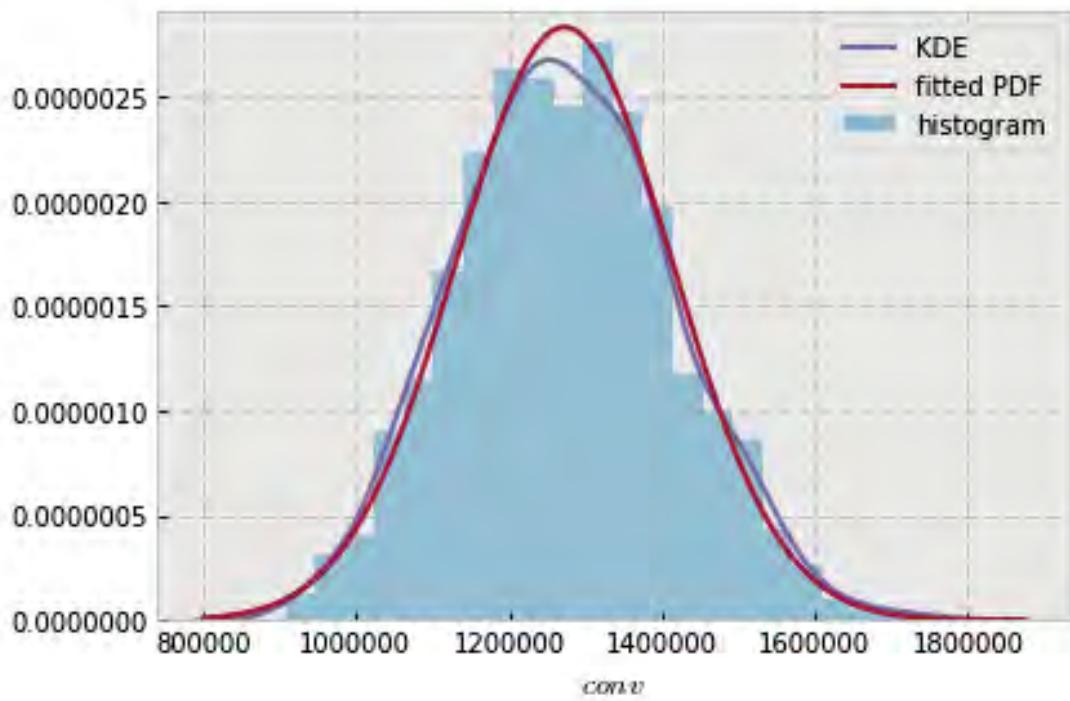


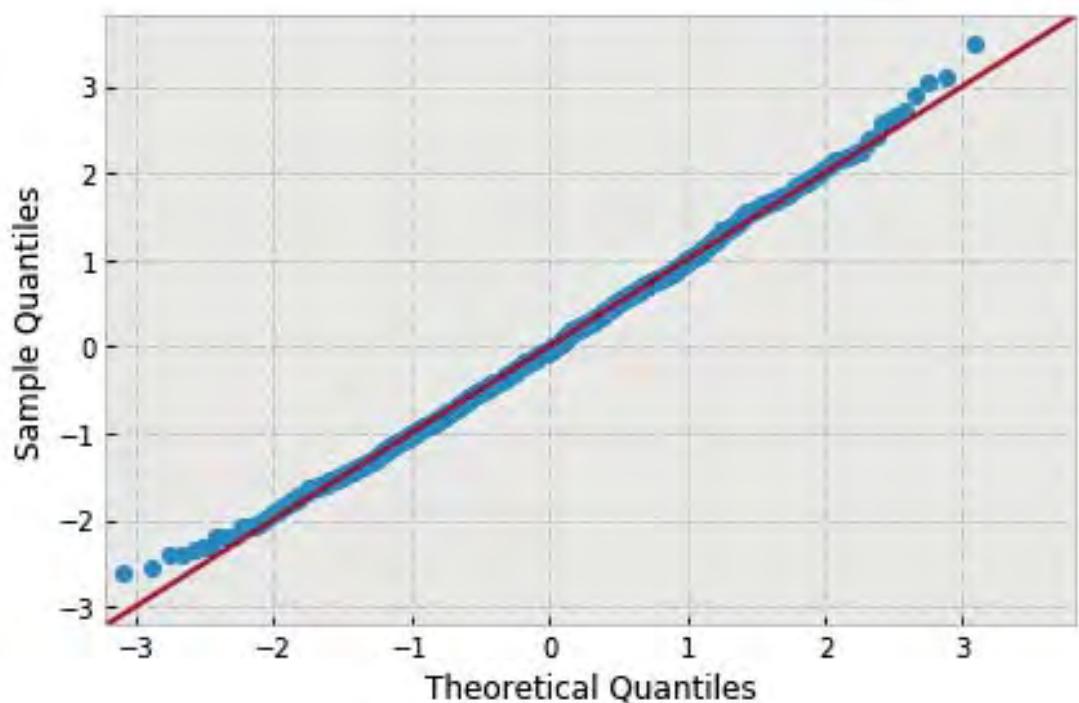


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=1236887.0064407, scale=146355.89116826386)
DescribeResult(nobs=1000, minmax=(835198.58089999994, 1778444.6869999999)
               mean=1236887.0064407, variance=21441488368.024719,
               skewness=0.34914806358093503, kurtosis=0.3192321014646149)
```

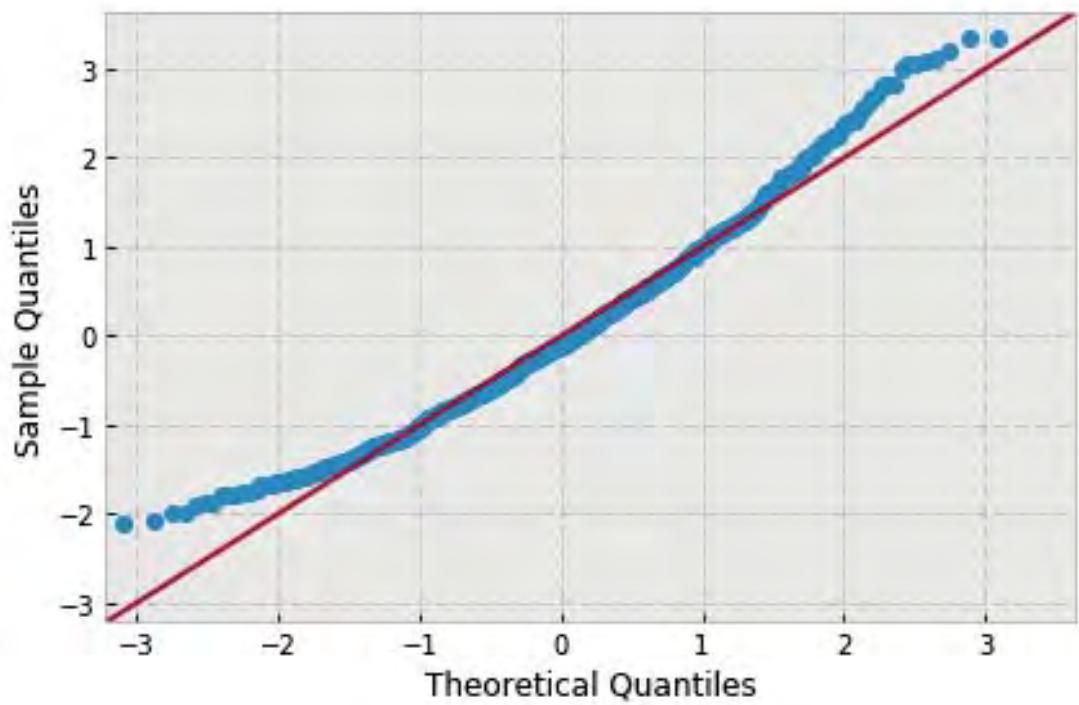
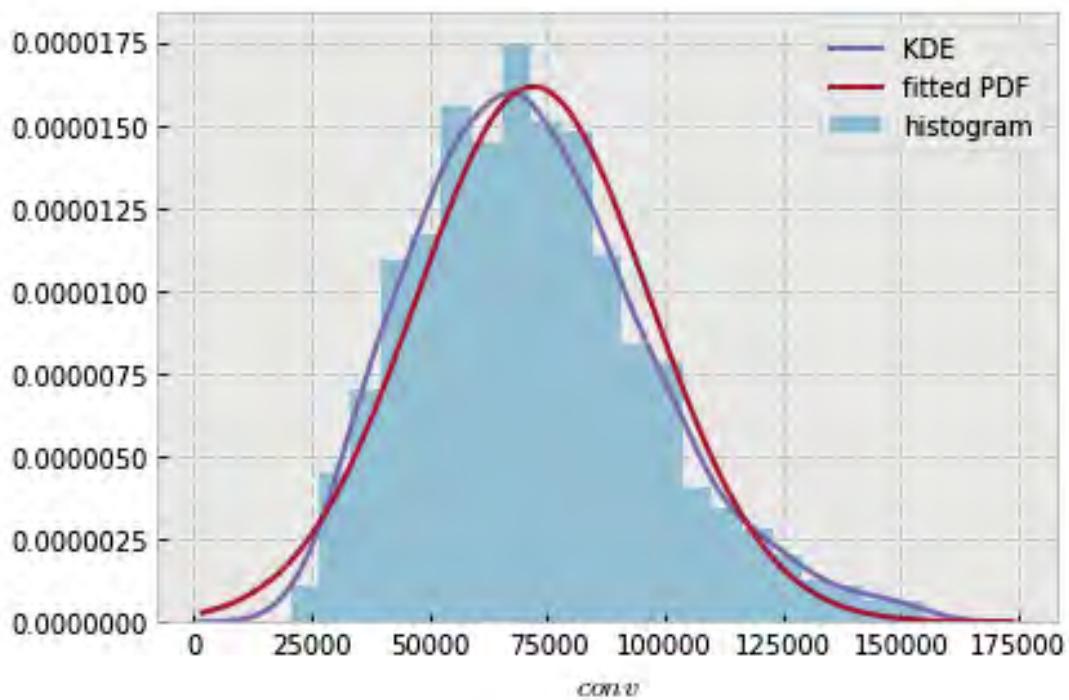


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=1272471.2107255, scale=140851.03513770364)
DescribeResult(nobs=1000, minmax=(906773.96990000003, 1765829.29)
               mean=1272471.2107255, variance=19858872972.334965,
               skewness=0.16374302251022493, kurtosis=-0.14422378805198832)
```

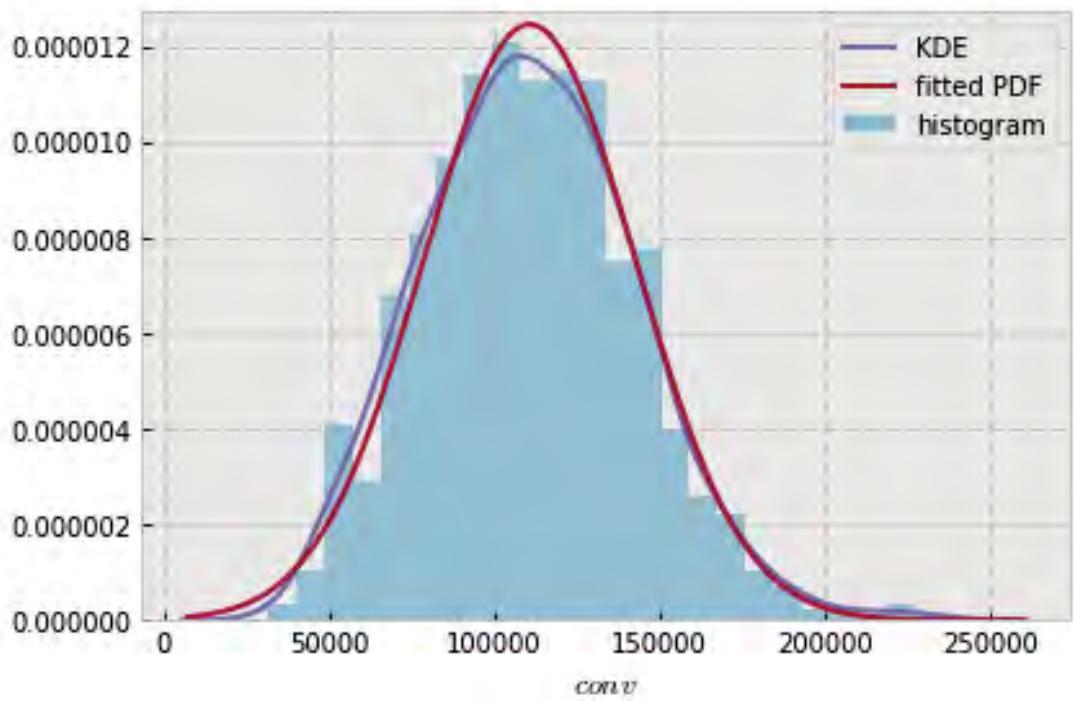


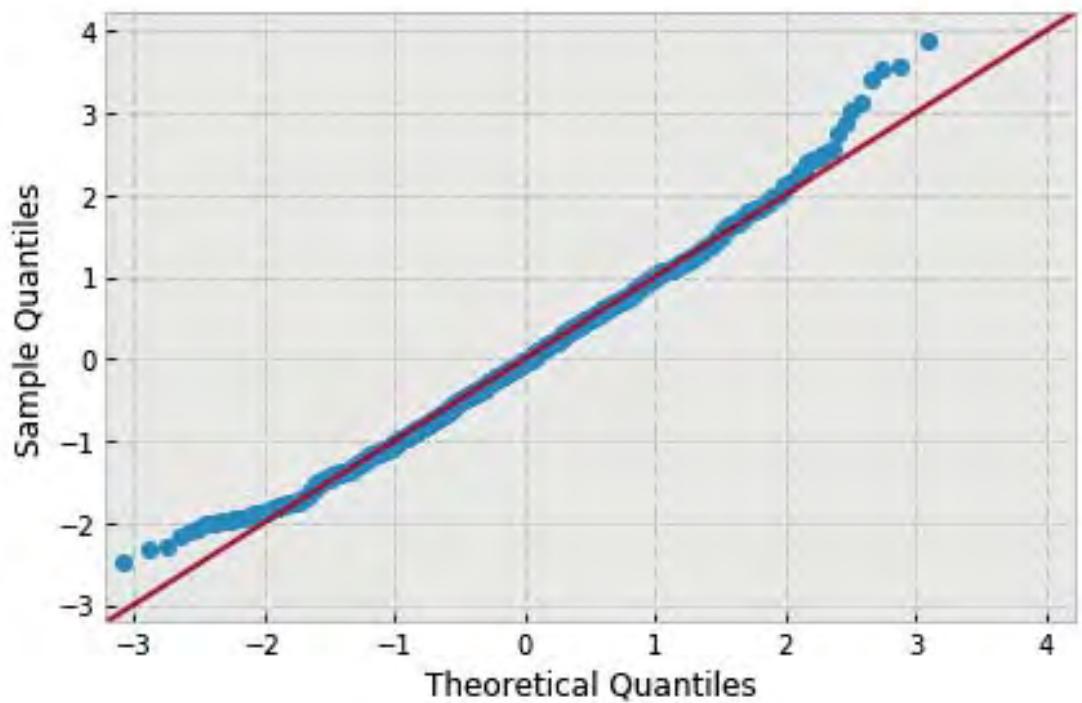


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=71953.96866771001, scale=24686.739728622724)
DescribeResult(nobs=1000, minmax=(20378.284810000001, 154768.9191)
               mean=71953.968667710011, variance=610045163.59235191,
               skewness=0.5643227169879185, kurtosis=0.19009478671950264)
```

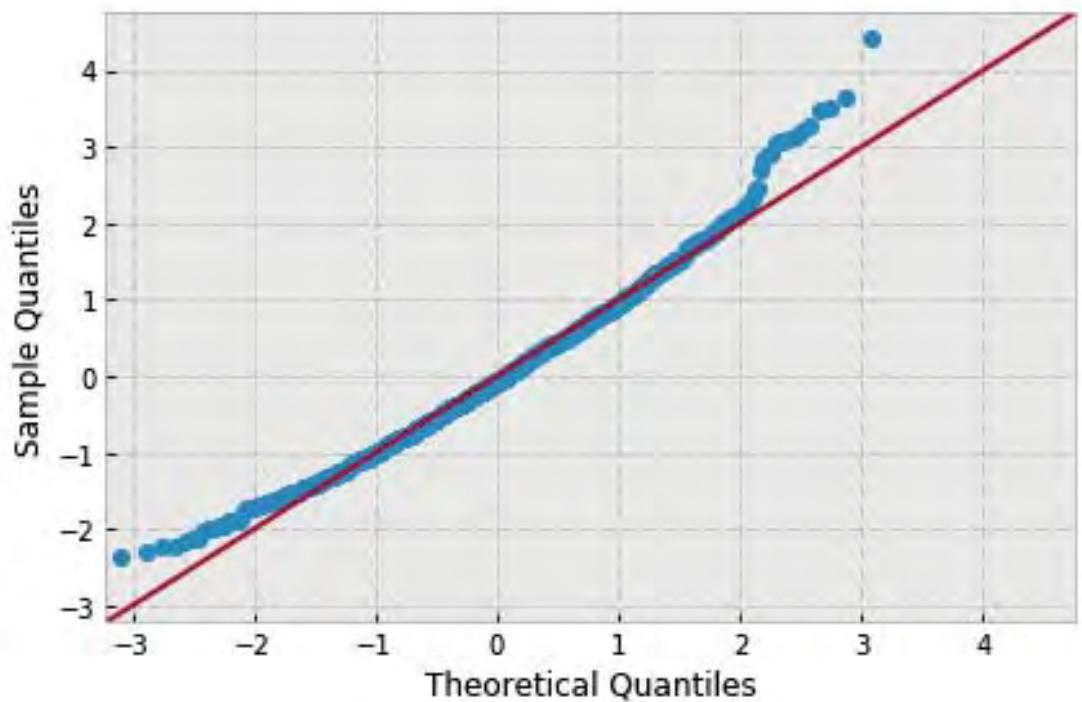
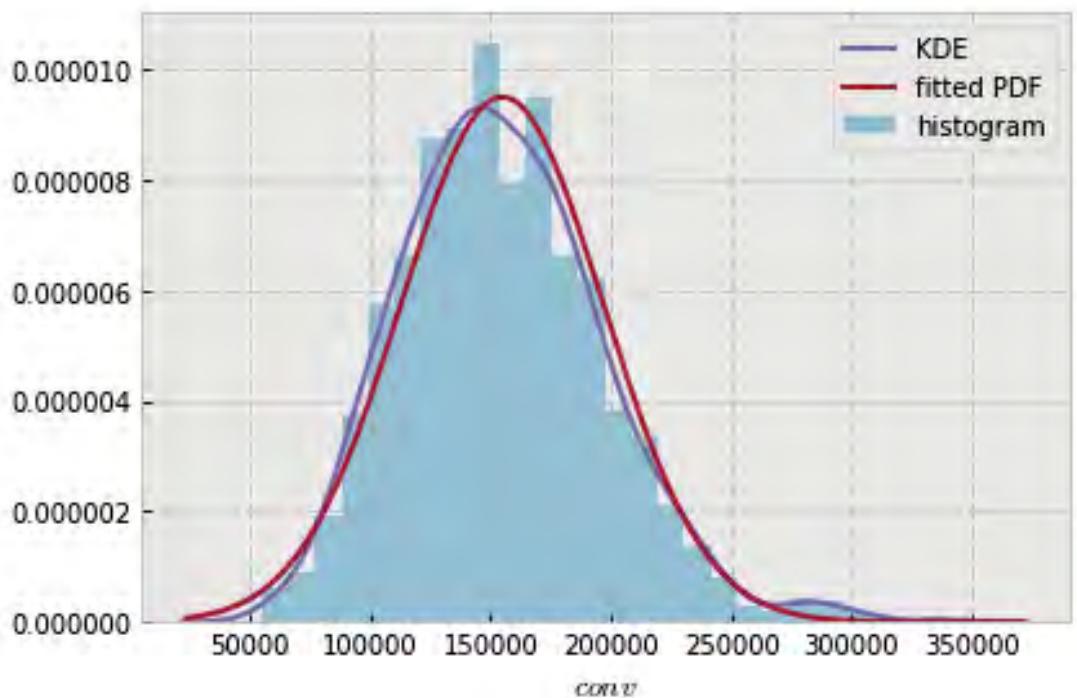


```
variable name: r
variable value: 1.222222222222223
distribution: normal(shape=(), loc=110677.27159736001, scale=32034.84393583683)
DescribeResult(nobs=1000, minmax=(31382.269090000002, 235601.8665)
               mean=110677.27159736001, variance=1027258484.4778996,
               skewness=0.30245145451960903, kurtosis=0.15870271906577793)
```

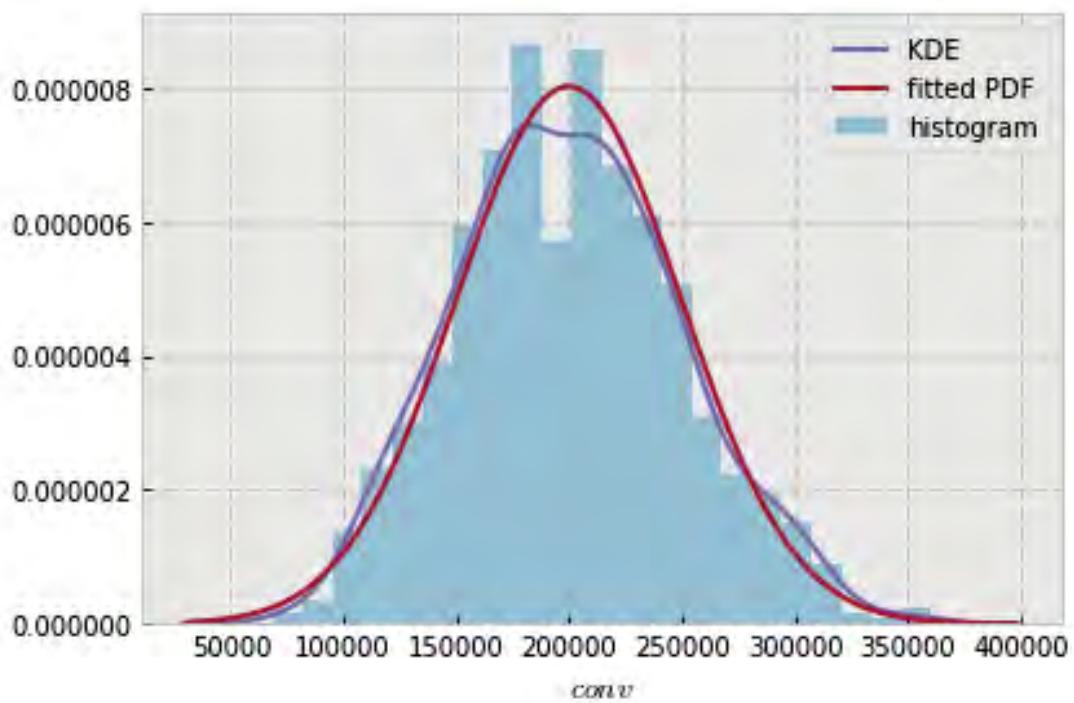


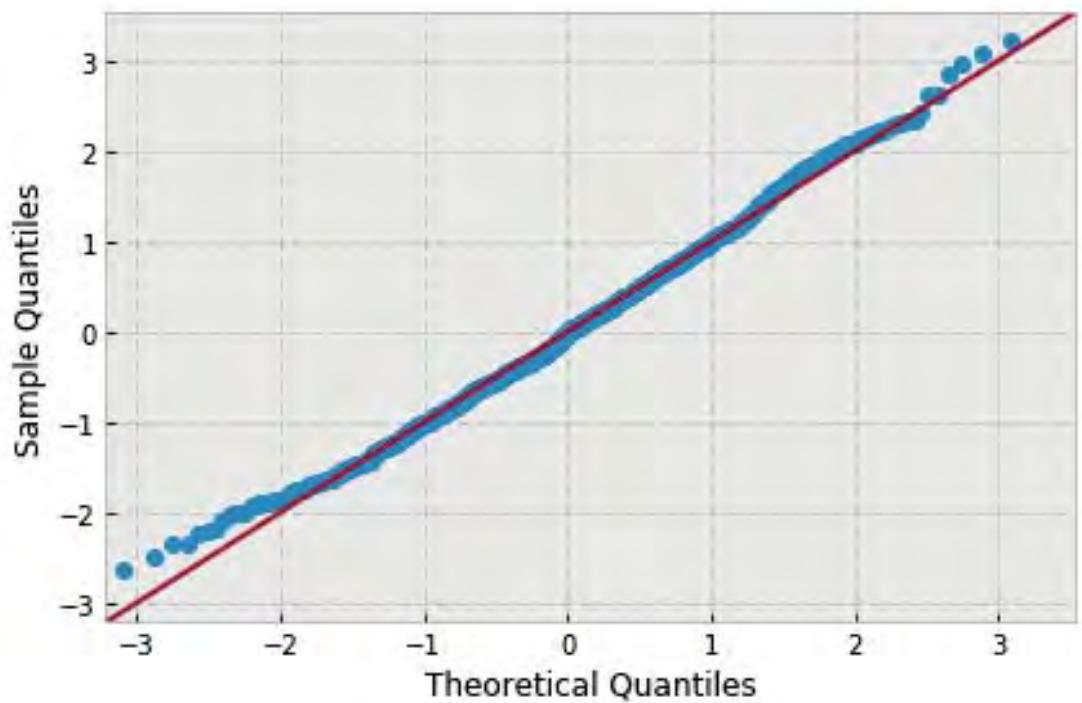


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=154740.98993698, scale=42025.20351681824)
DescribeResult(nobs=1000, minmax=(54858.68063999999, 340075.59590000001)
               mean=154740.98993698001, variance=1767885616.2462385,
               skewness=0.5329125666339823, kurtosis=0.633425463759755)
```

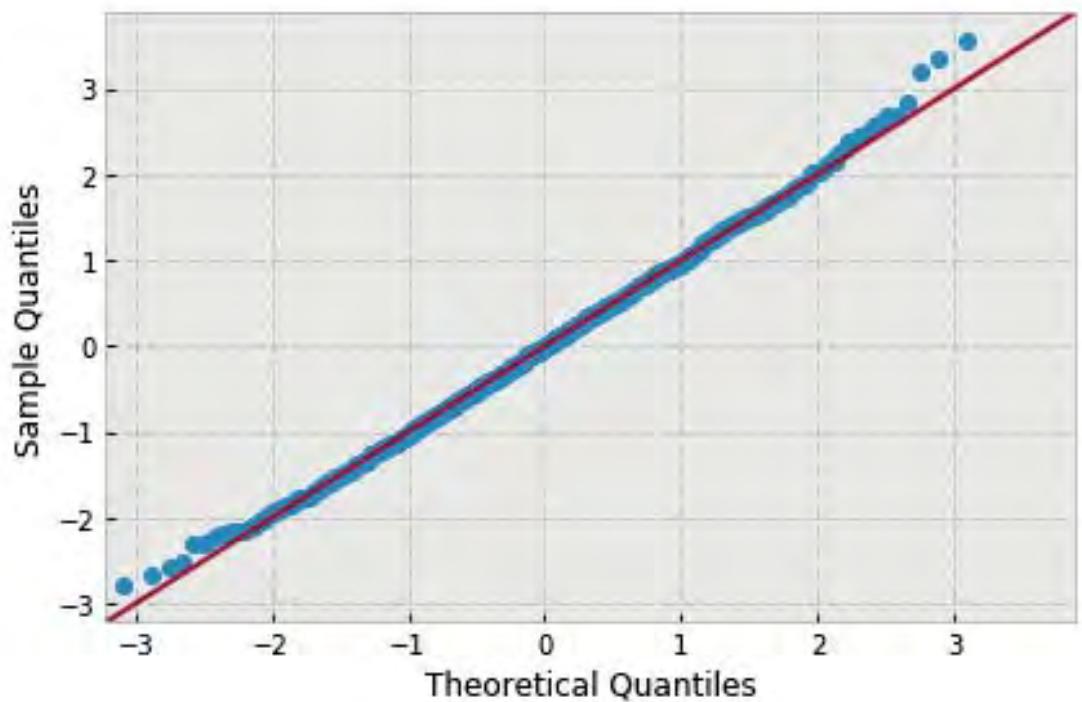
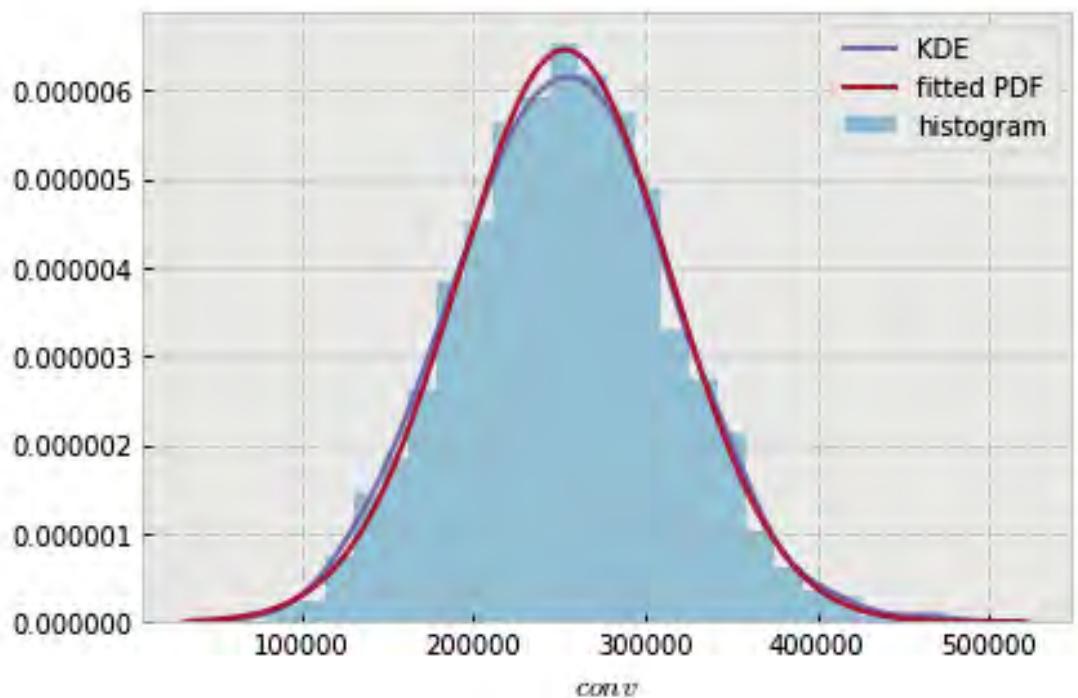


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=199799.08903824002, scale=49696.35358252225)
DescribeResult(nobs=1000, minmax=(68585.520990000005, 360333.60609999998)
               mean=199799.08903824002, variance=2472199759.1582298,
               skewness=0.2084562765772152, kurtosis=-0.2291917116856279)
```

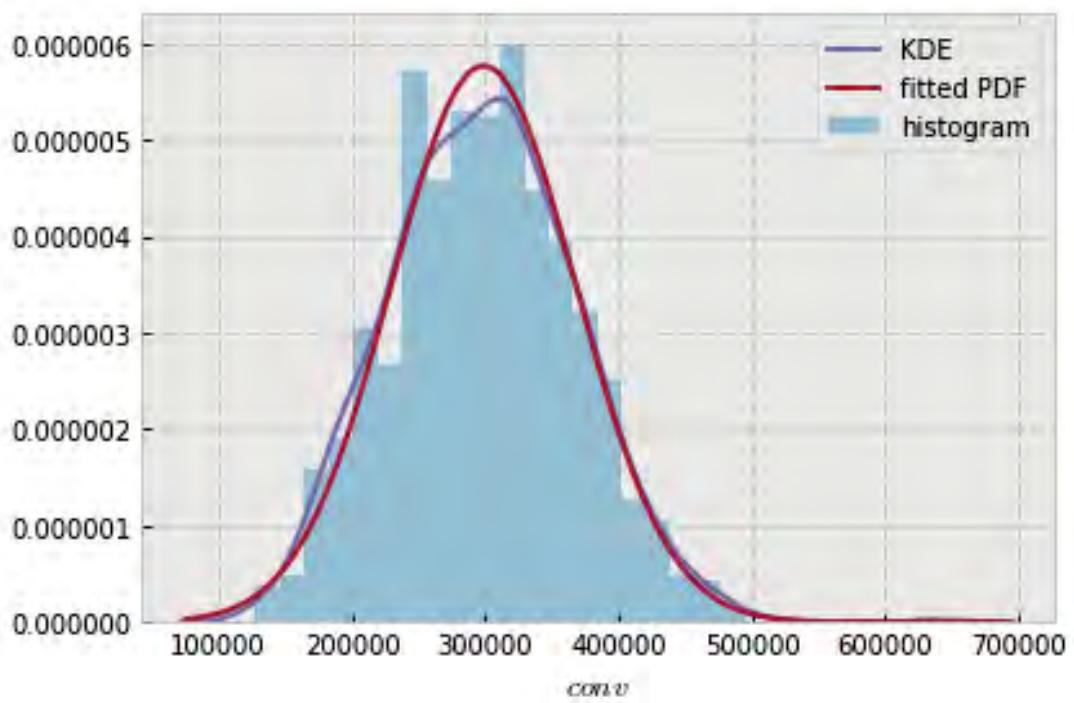


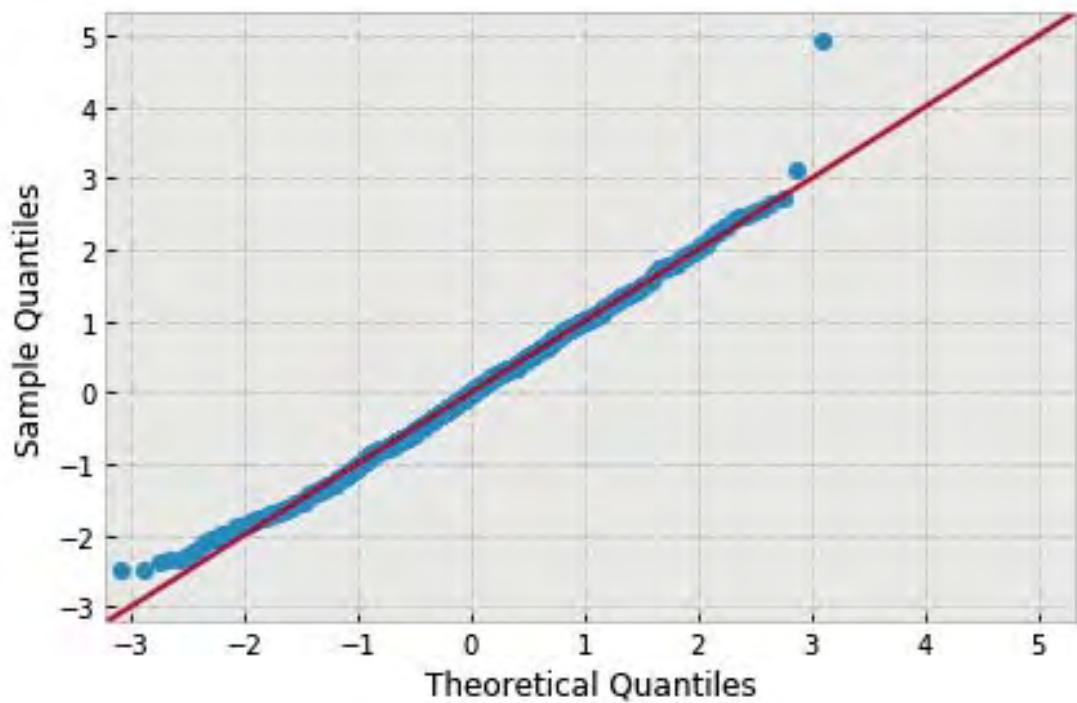


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=253147.82558320998, scale=61779.66280428897)
DescribeResult(nobs=1000, minmax=(80766.88826000007, 473191.98700000002)
               mean=253147.82558320998, variance=3820547283.495141,
               skewness=0.11914893259991571, kurtosis=-0.03768702976584937)
```

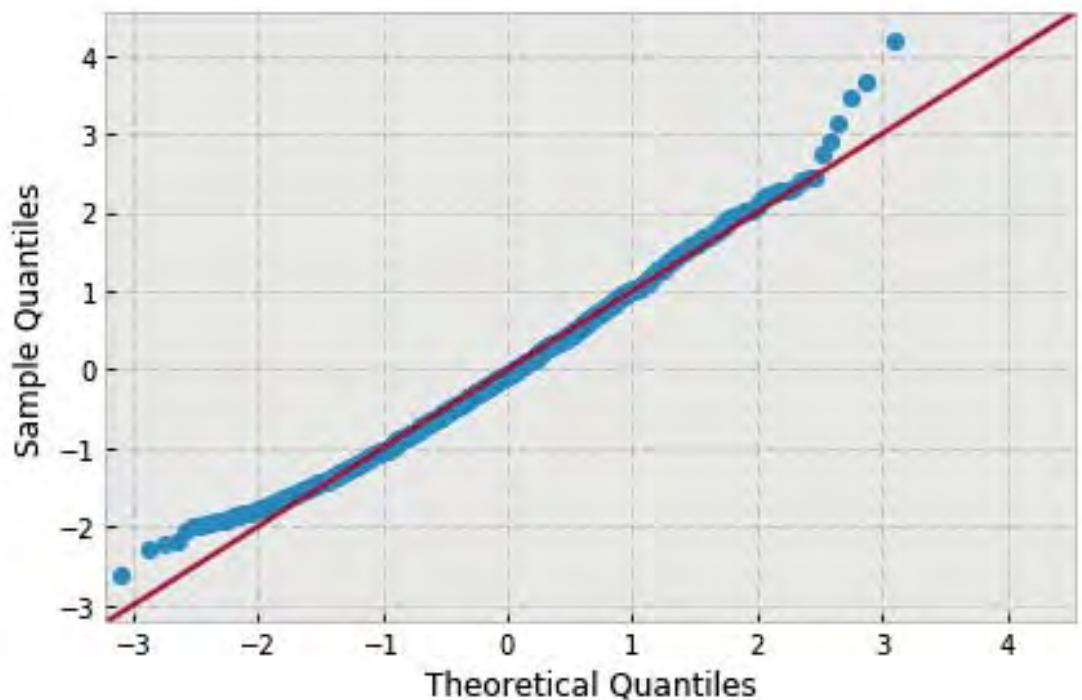
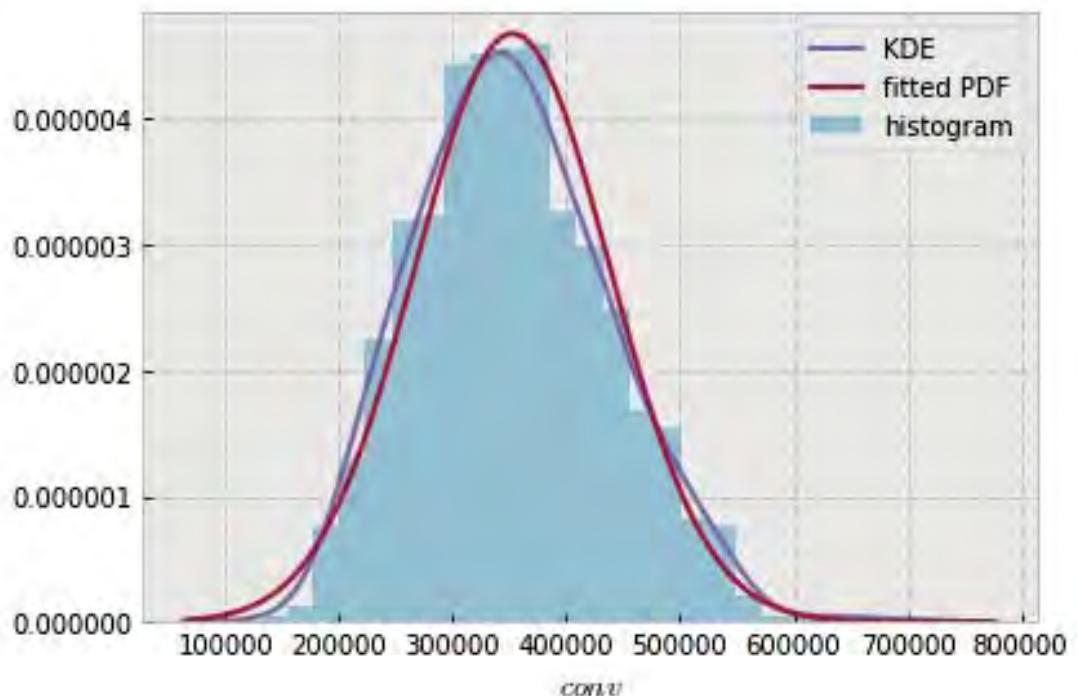


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=298528.80682490004, scale=69188.92658019639)
DescribeResult(nobs=1000, minmax=(127736.73910000001, 640770.04550000001)
               mean=298528.80682490004, variance=4791899460.7805862,
               skewness=0.2089285125102461, kurtosis=0.15026728963883418)
```

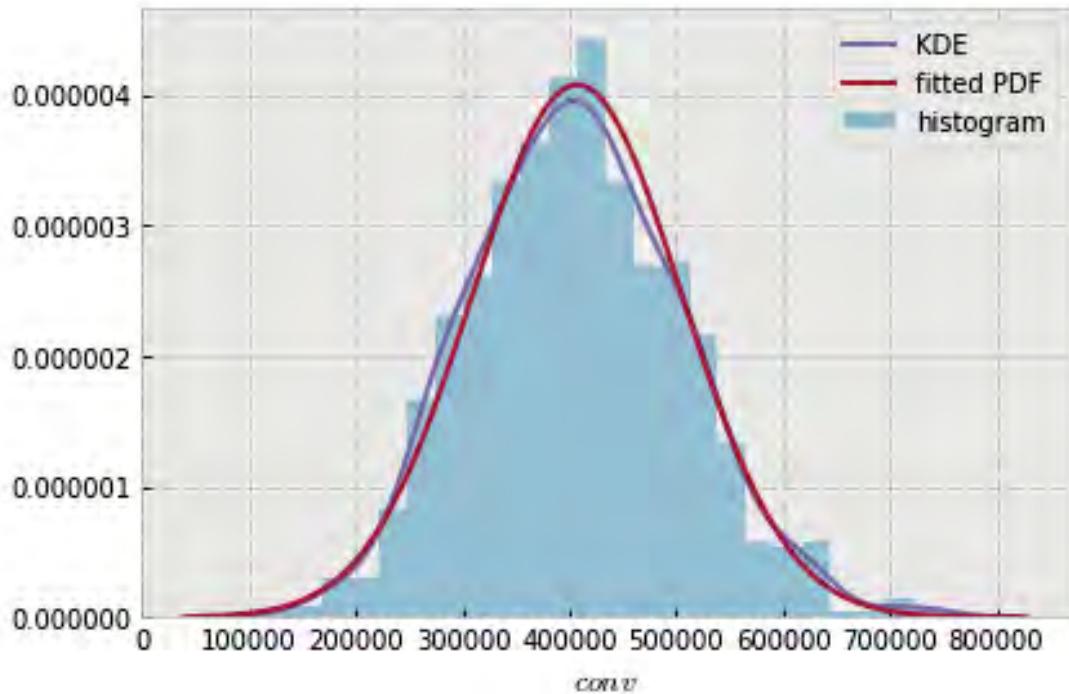


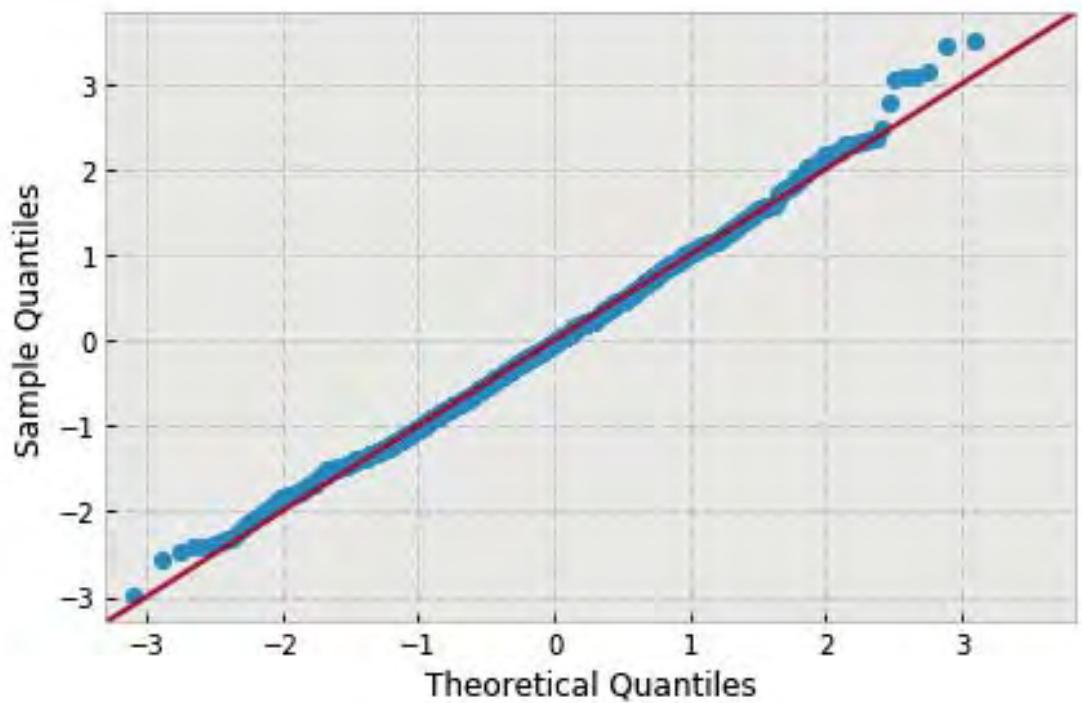


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=352424.3462058, scale=85297.97206474436)
DescribeResult(nobs=1000, minmax=(130148.2464999999, 710537.1824000005)
               mean=352424.3462058002, variance=7283027065.4233313,
               skewness=0.36760204451717987, kurtosis=0.07146622035924421)
```

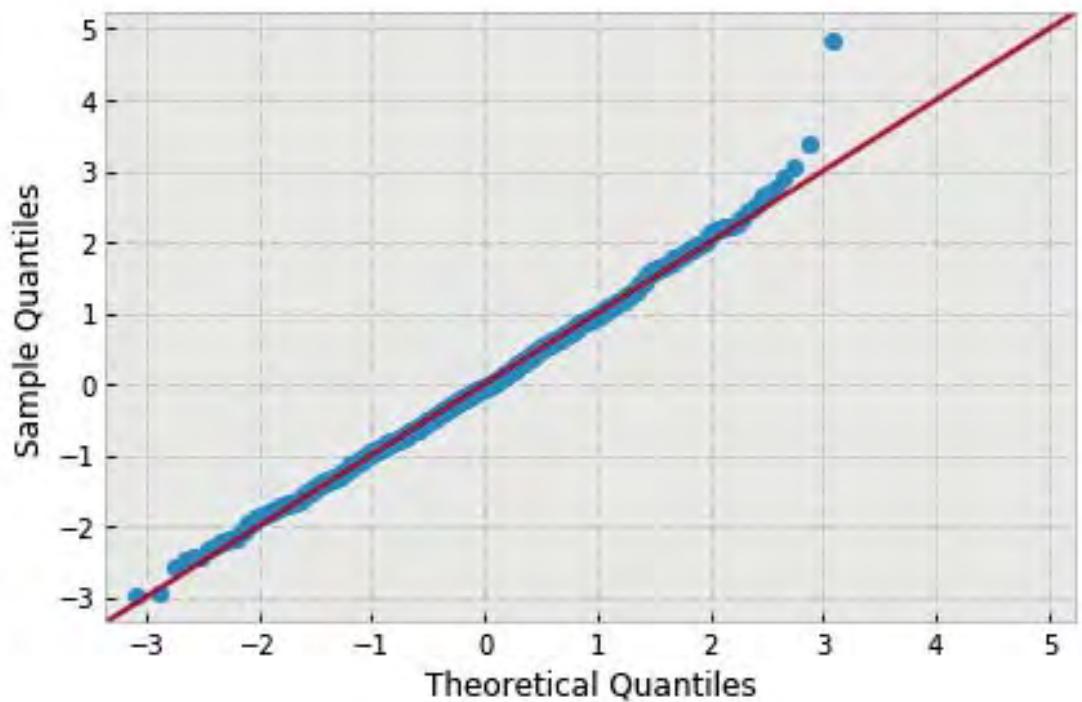
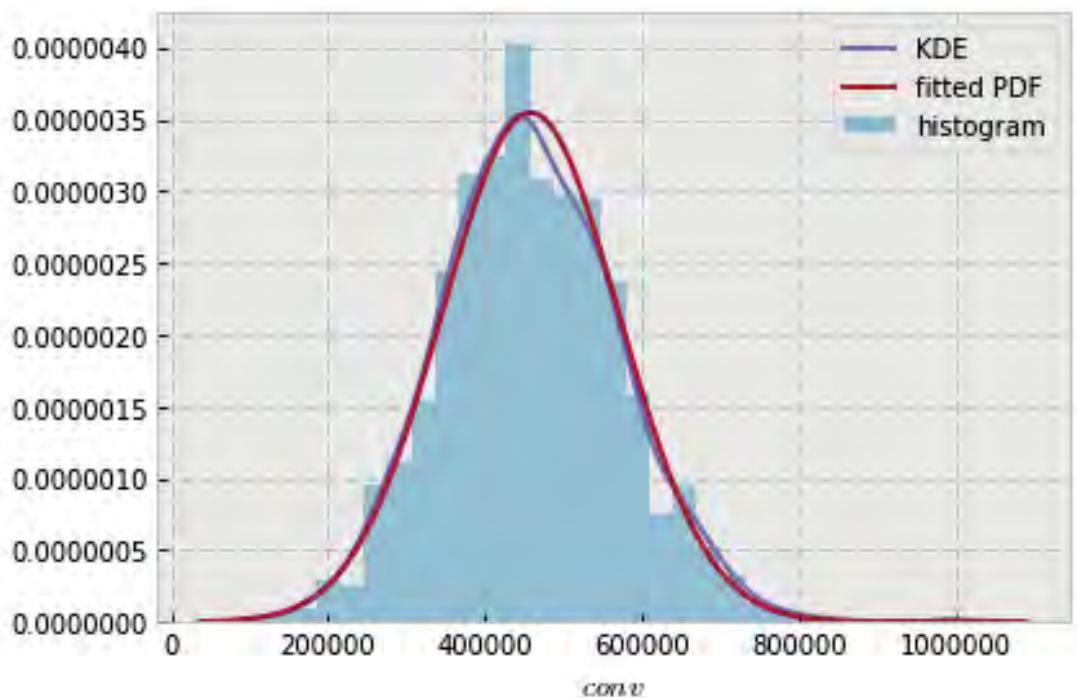


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=407294.7136827, scale=97887.05788278453)
DescribeResult(nobs=1000, minmax=(116504.94, 750024.1001999999)
               mean=407294.7136827001, variance=9591467568.5161247,
               skewness=0.21878148510466197, kurtosis=0.04746503754658038)
```

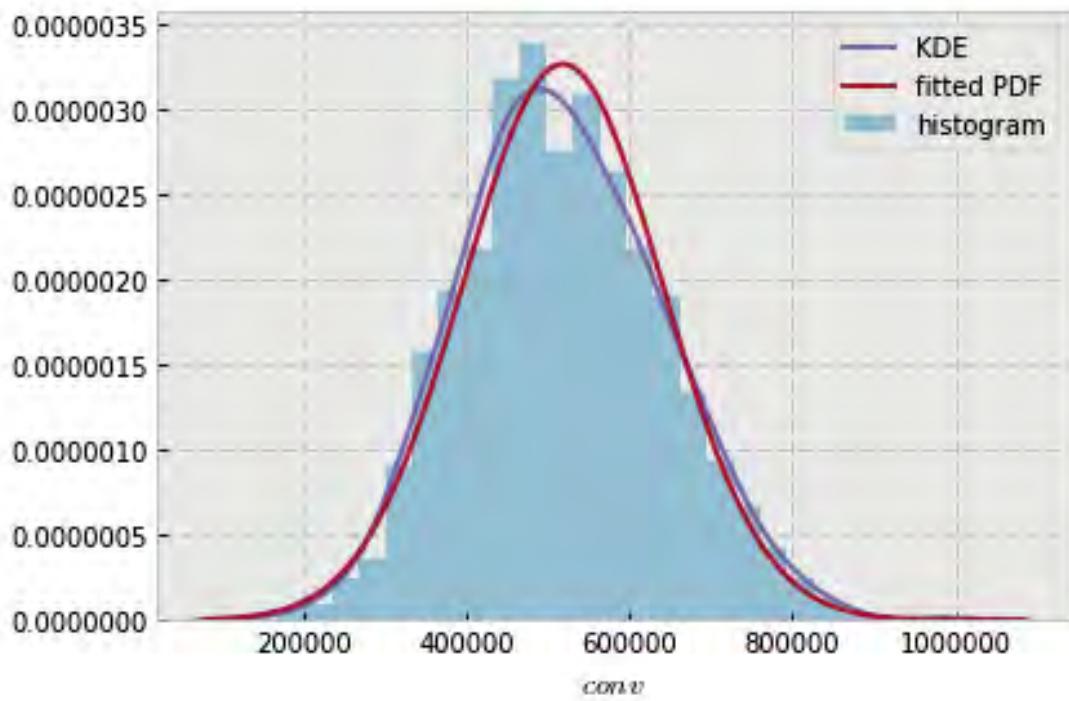


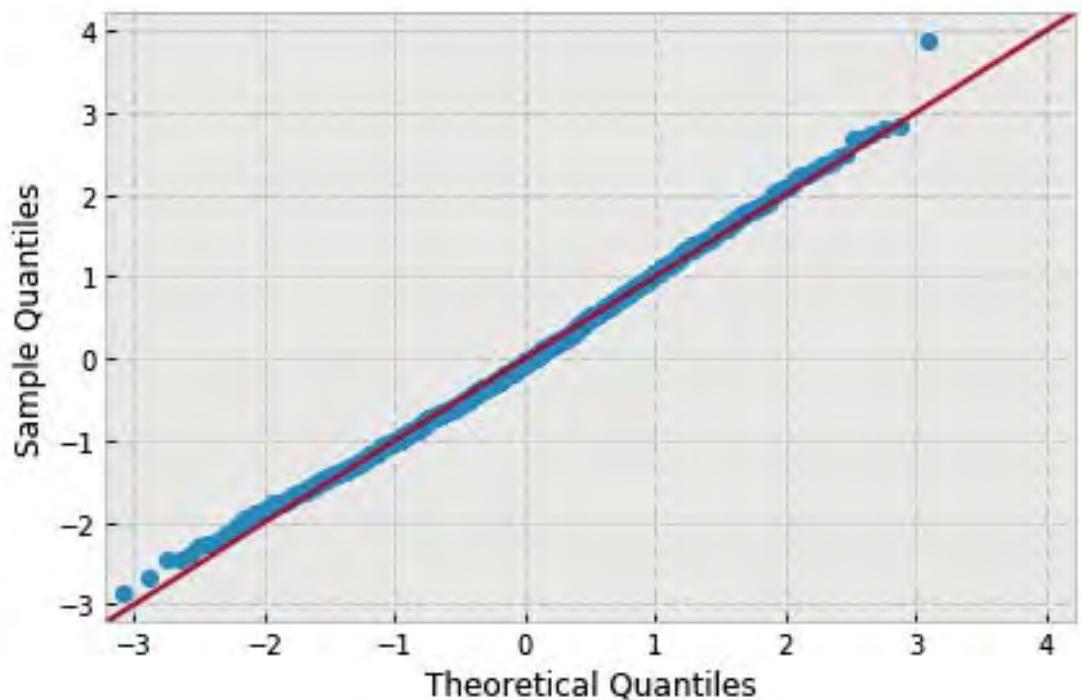


```
variable name: r
variable value: 2.7777777777777777
distribution: normal(shape=(), loc=457472.8951913, scale=112473.37744279904)
DescribeResult(nobs=1000, minmax=(124498.3317, 1000879.329)
               mean=457472.89519130002, variance=12662923556.947285,
               skewness=0.23557302978873548, kurtosis=0.39077027106980866)
```

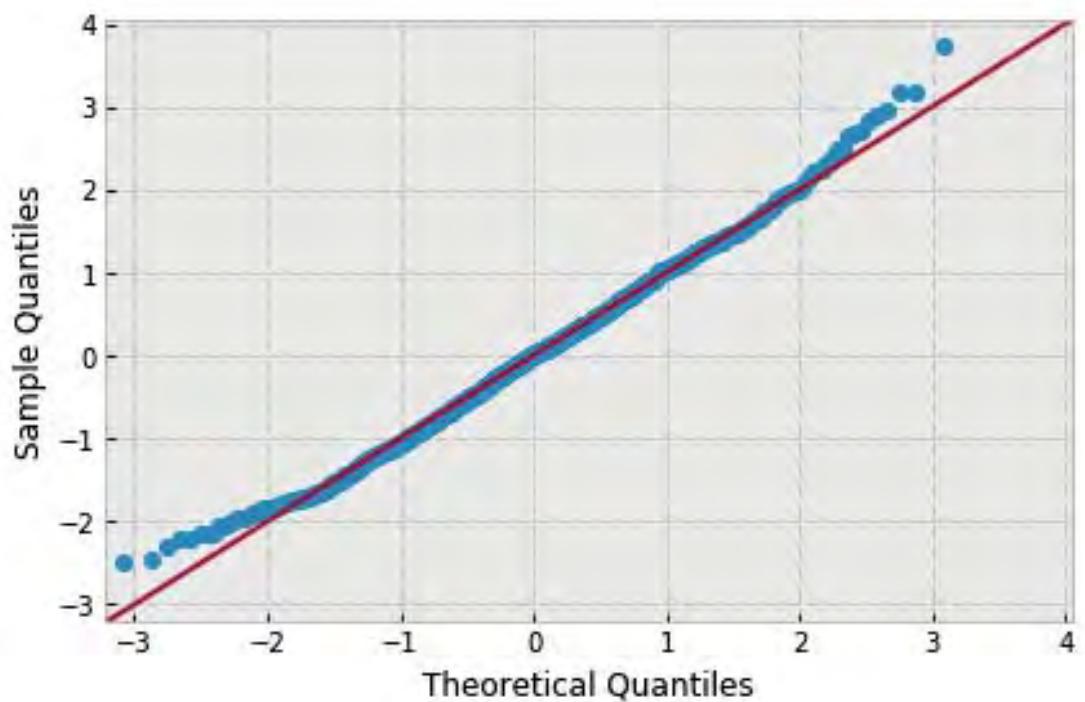
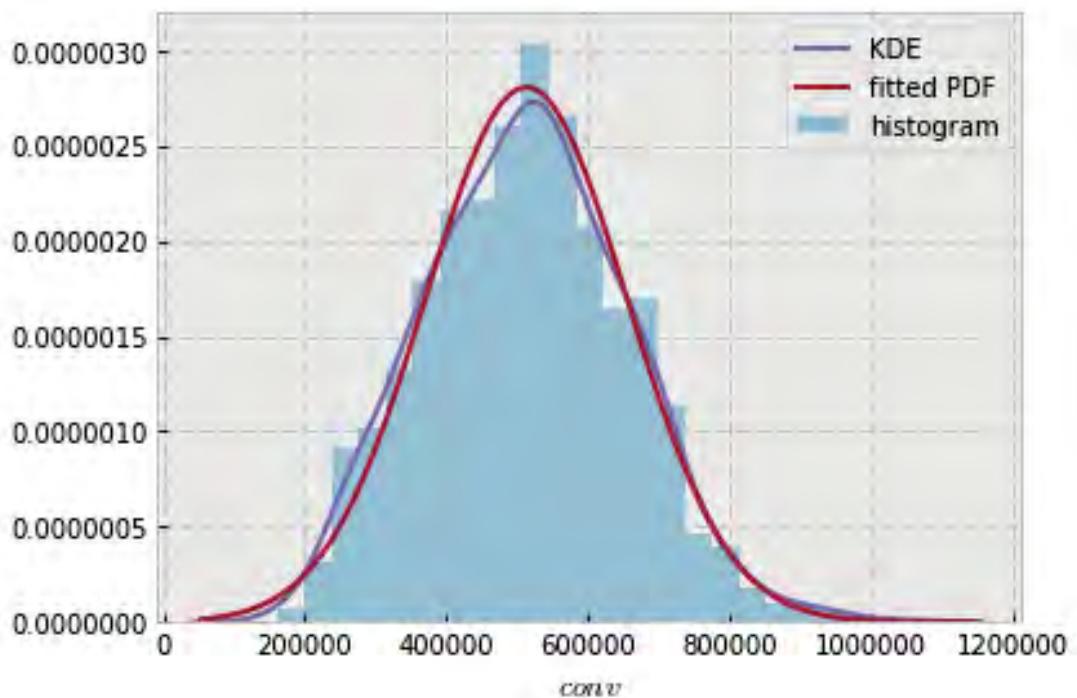


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=518031.91771280003, scale=122092.07069640115)
DescribeResult(nobs=1000, minmax=(168411.99189999999, 991325.05559999996)
               mean=518031.9177128003, variance=14921395122.057074,
               skewness=0.2122082586362381, kurtosis=-0.12248451890562384)
```

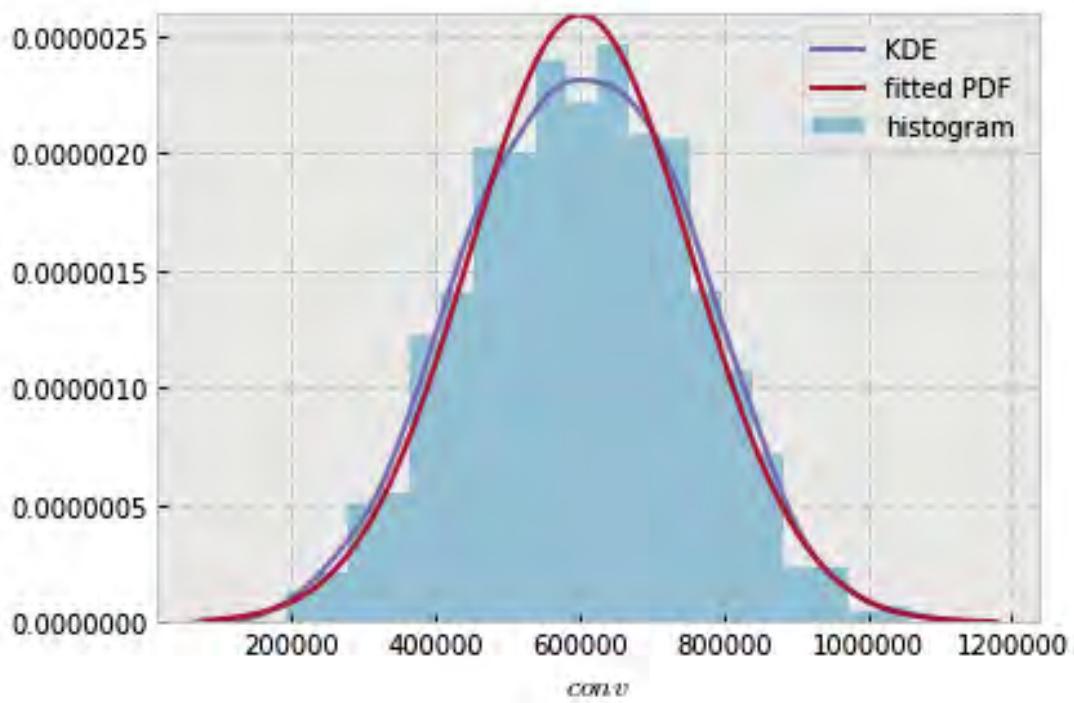


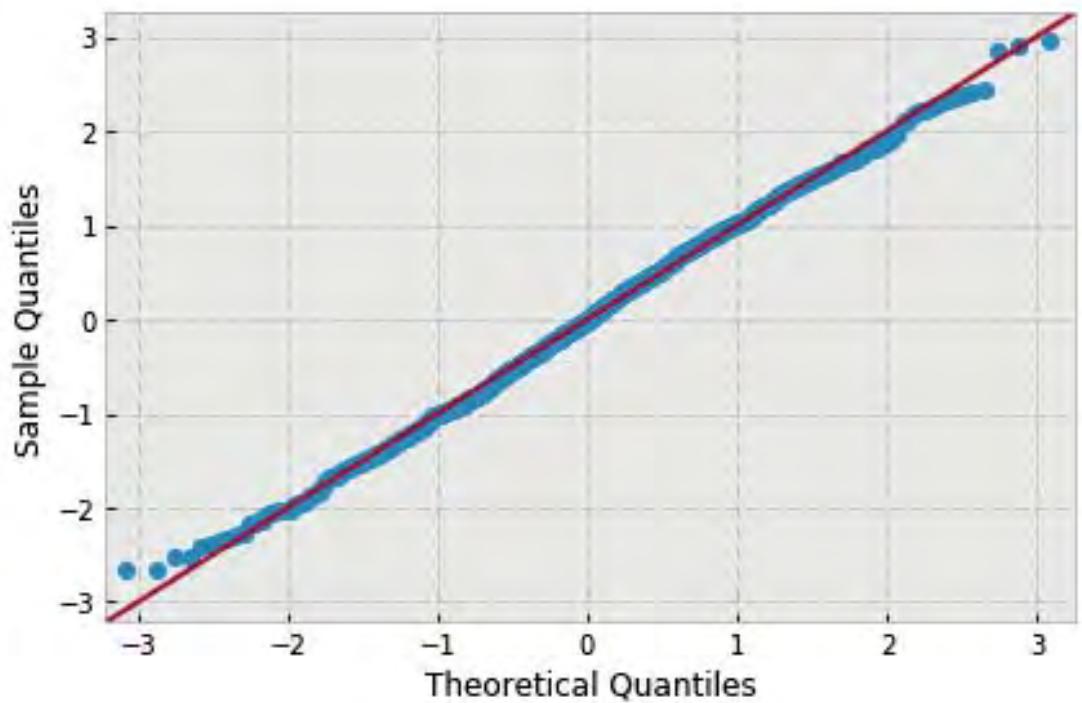


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=512766.83339419996, scale=142215.66931879084)
DescribeResult(nobs=1000, minmax=(160078.6185999999, 1044569.102)
               mean=512766.83339419996, variance=20245542141.933598,
               skewness=0.18458266585157482, kurtosis=-0.10055771947382519)
```

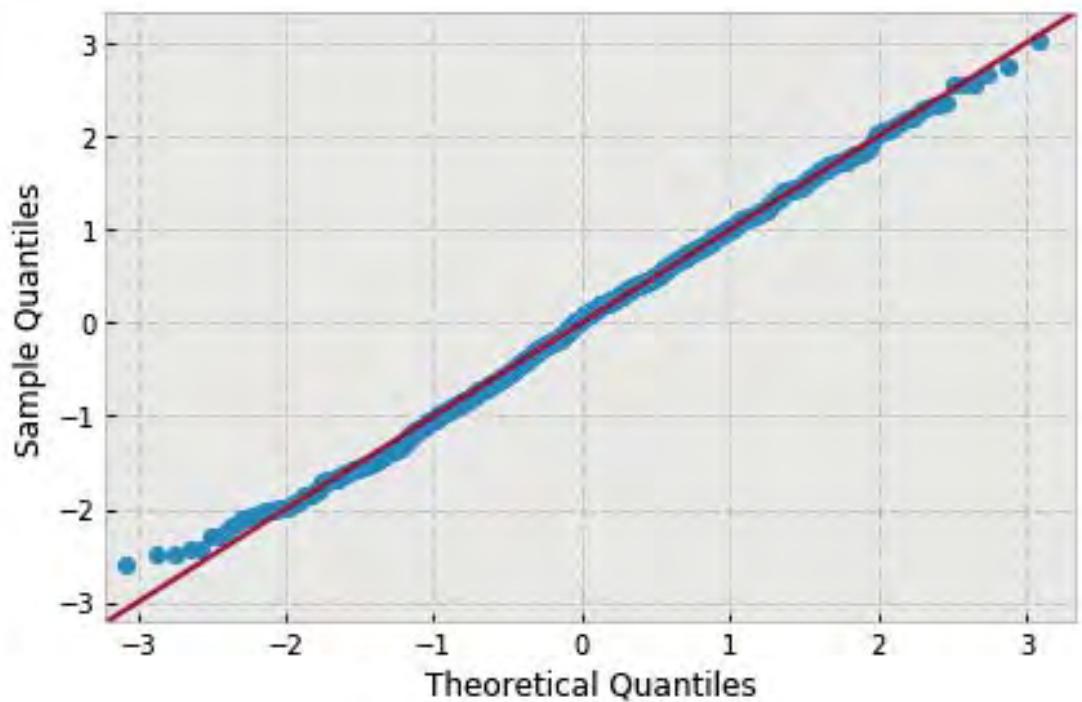
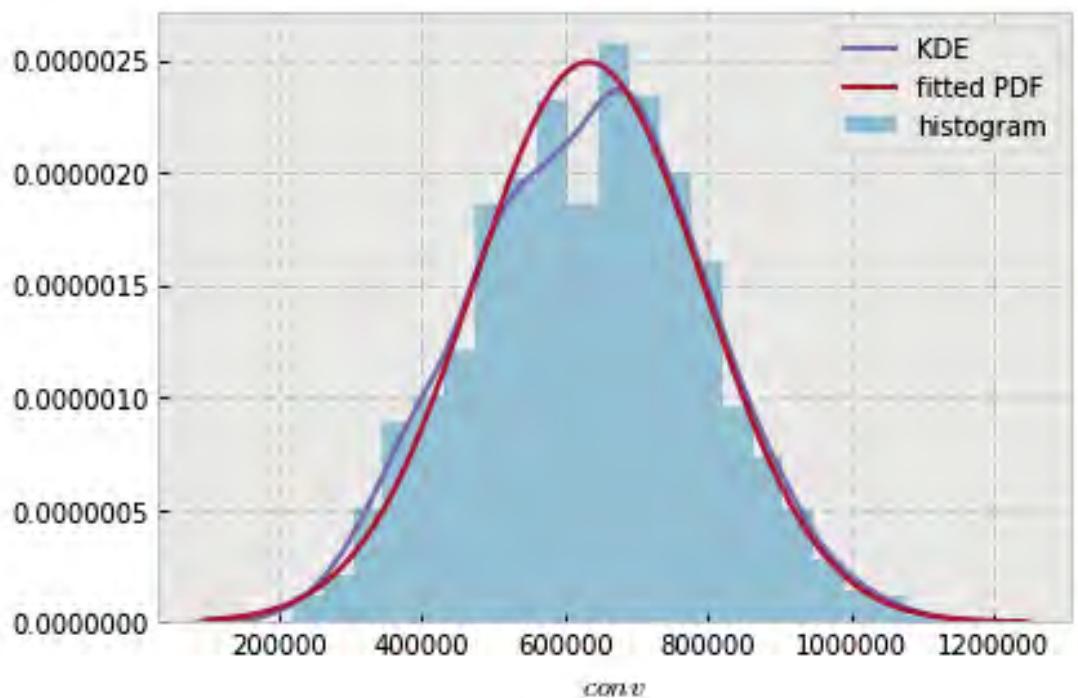


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=601297.1546831, scale=153860.41245579987)
DescribeResult(nobs=1000, minmax=(192433.2677, 1058373.013)
    mean=601297.15468309994, variance=23696723244.313171,
    skewness=-0.010546926145286305, kurtosis=-0.36187442953224735)
```

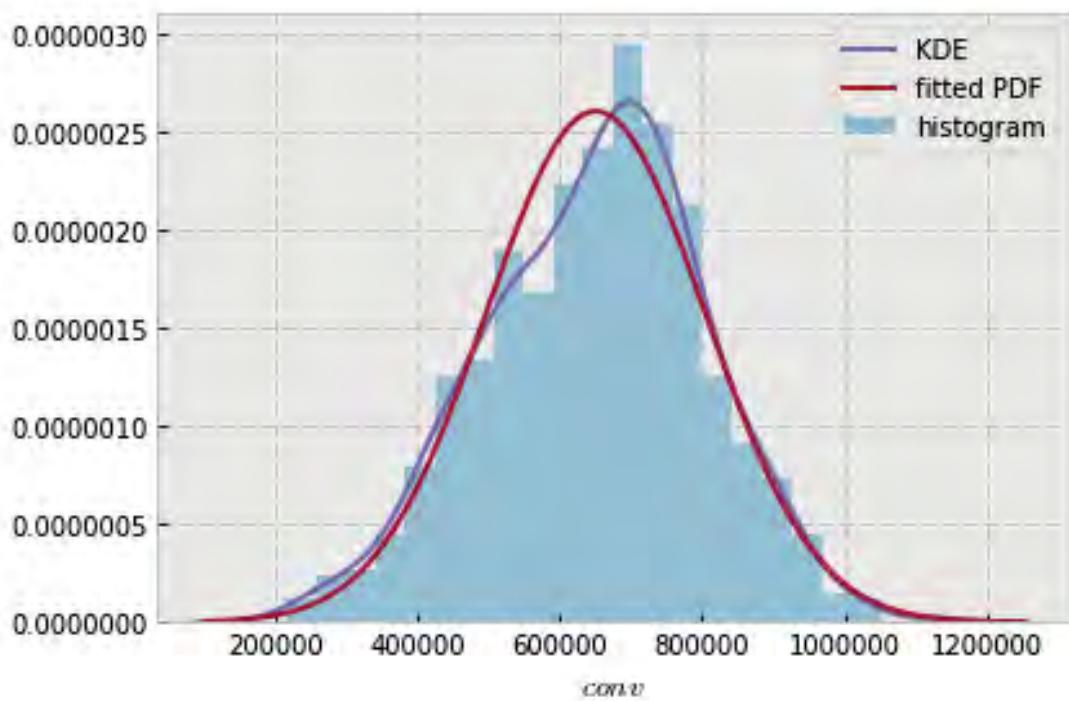


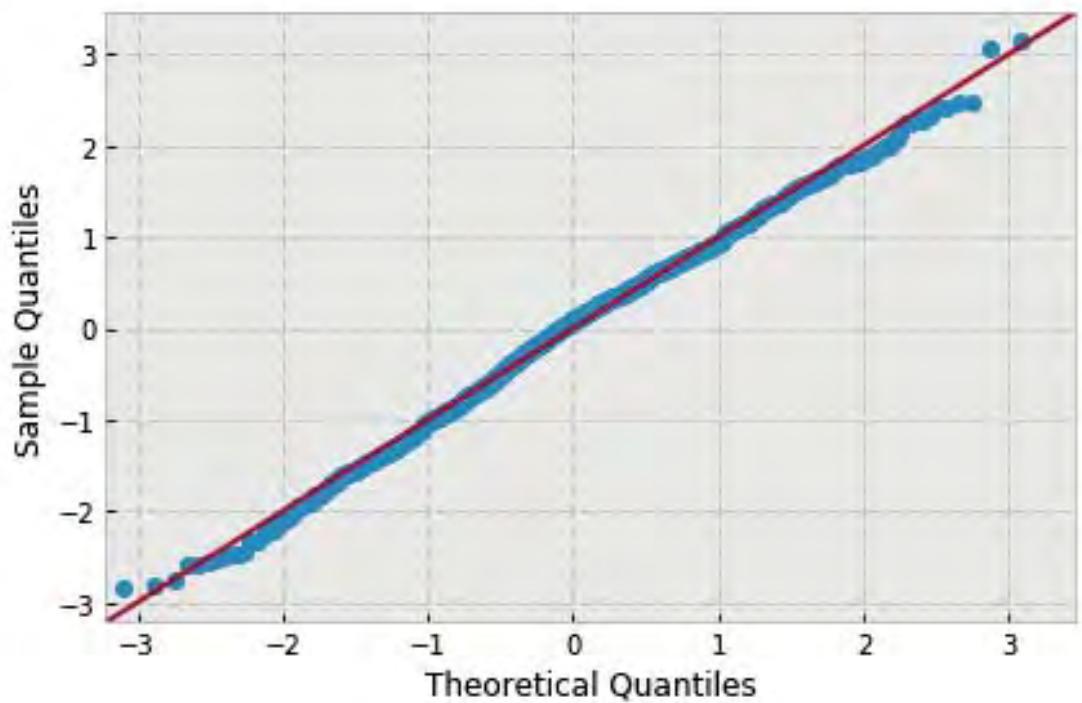


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(), loc=633580.6953345999, scale=160125.32064901068)
DescribeResult(nobs=1000, minmax=(217638.155, 1120990.199)
               mean=633580.6953345994, variance=25665784097.045528,
               skewness=0.014226381327634994, kurtosis=-0.35734135573376147)
```

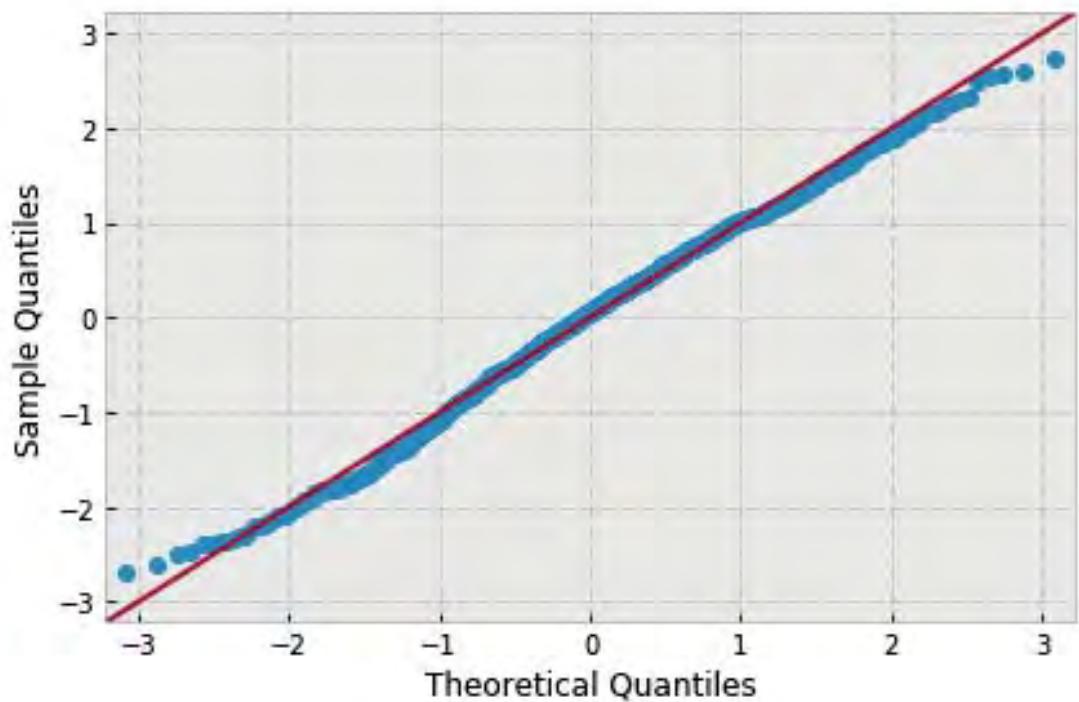
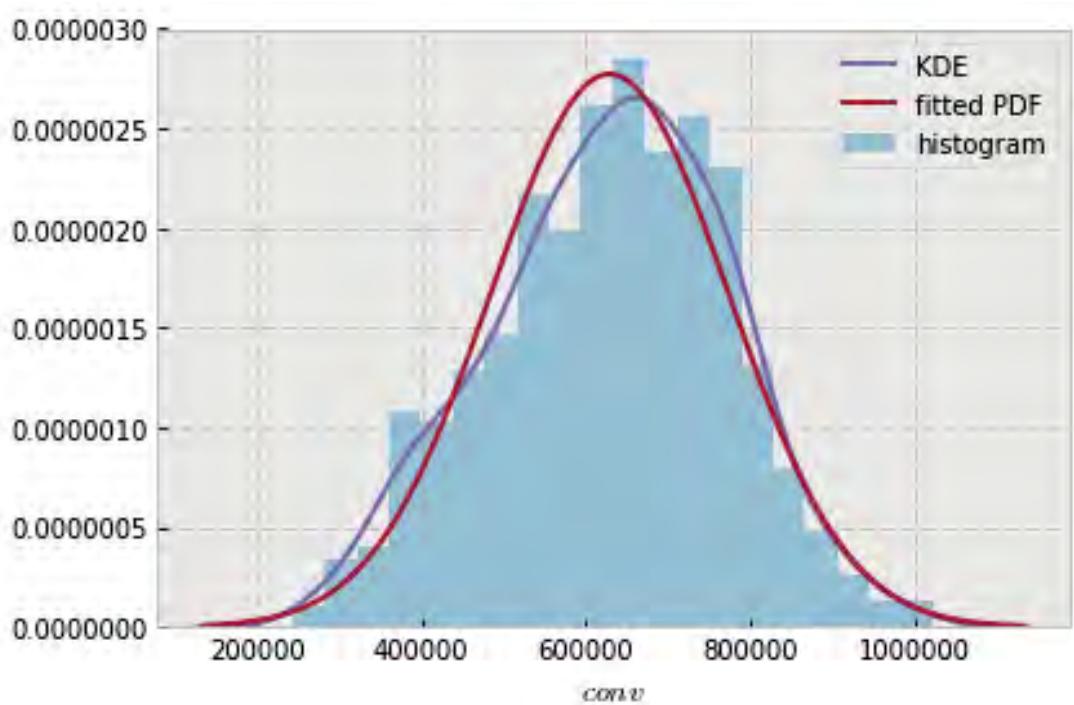


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=650616.7296519999, scale=153359.53469997967)
DescribeResult(nobs=1000, minmax=(216123.0013, 1133795.733)
    mean=650616.72965199989, variance=23542689572.967239,
    skewness=-0.15738114054966287, kurtosis=-0.16555456814853864)
```

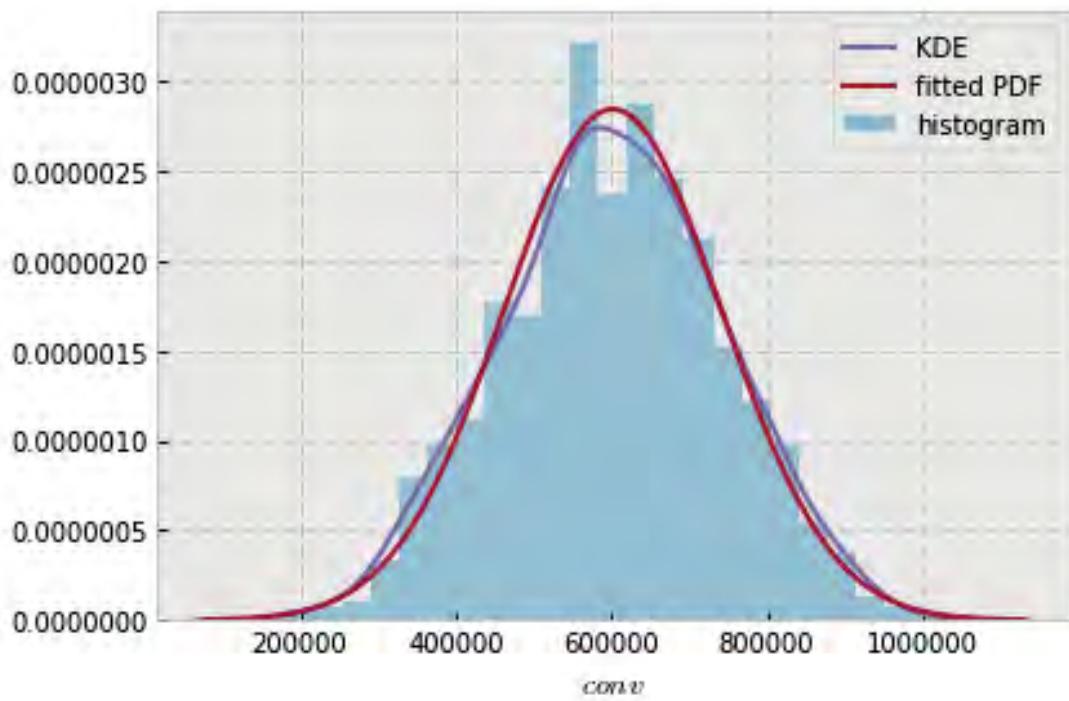


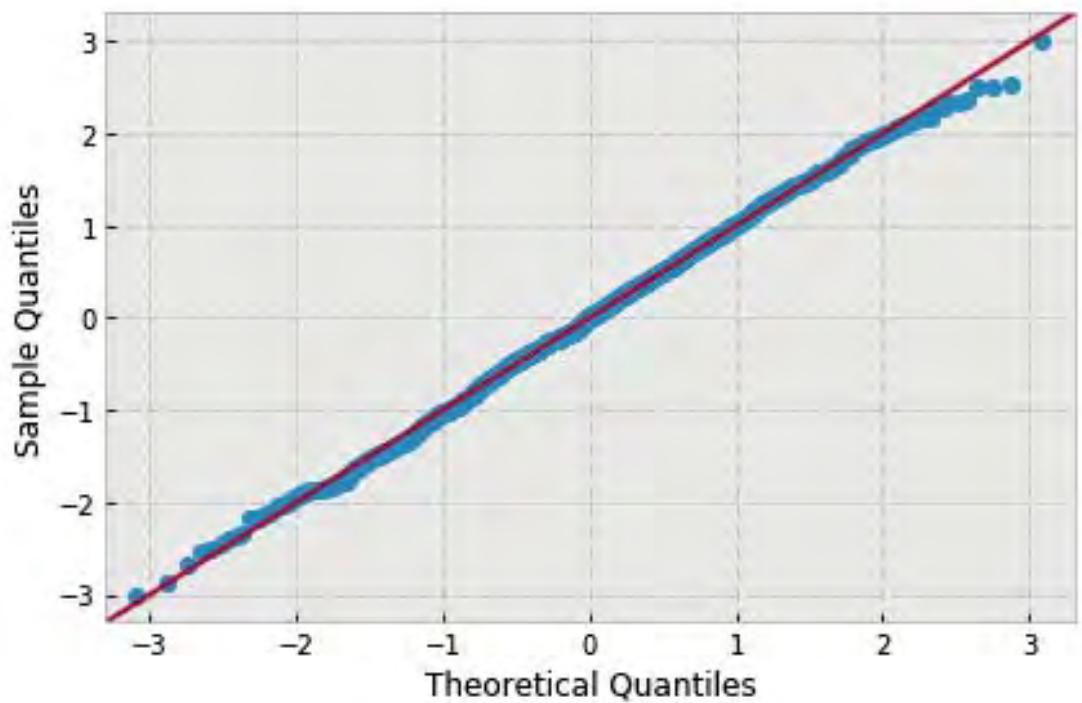


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=628690.1118337, scale=143979.4436223398)
DescribeResult(nobs=1000, minmax=(243577.2874, 1022190.179)
               mean=628690.11183369998, variance=20750831016.815342,
               skewness=-0.16498634021035208, kurtosis=-0.3575422414484546)
```

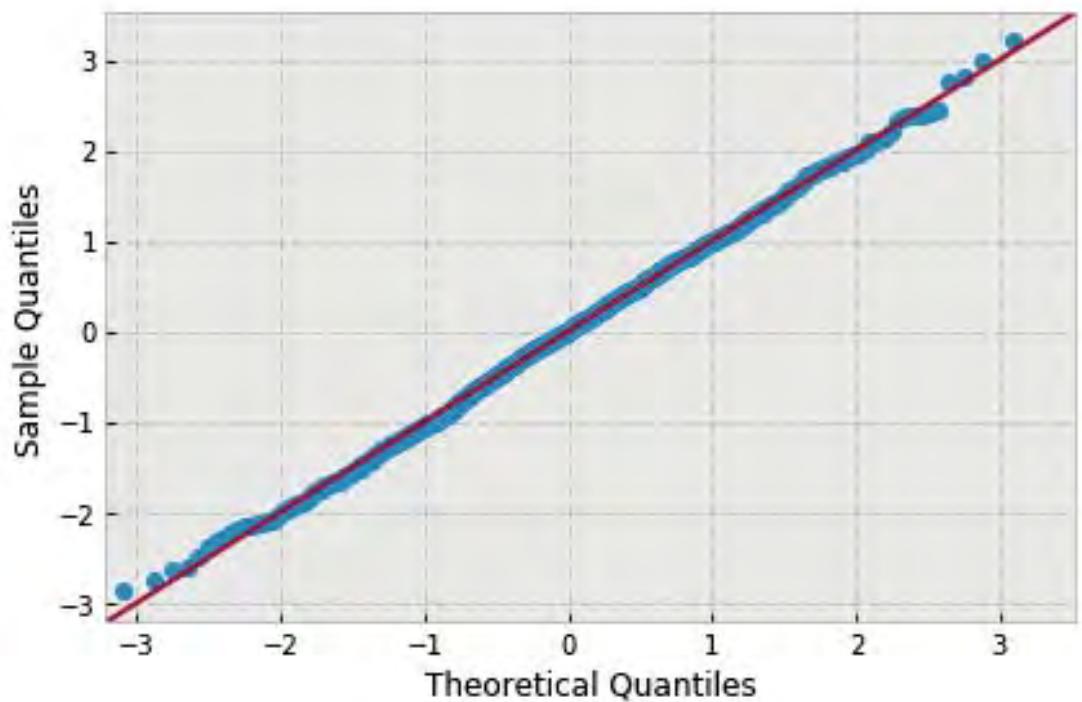
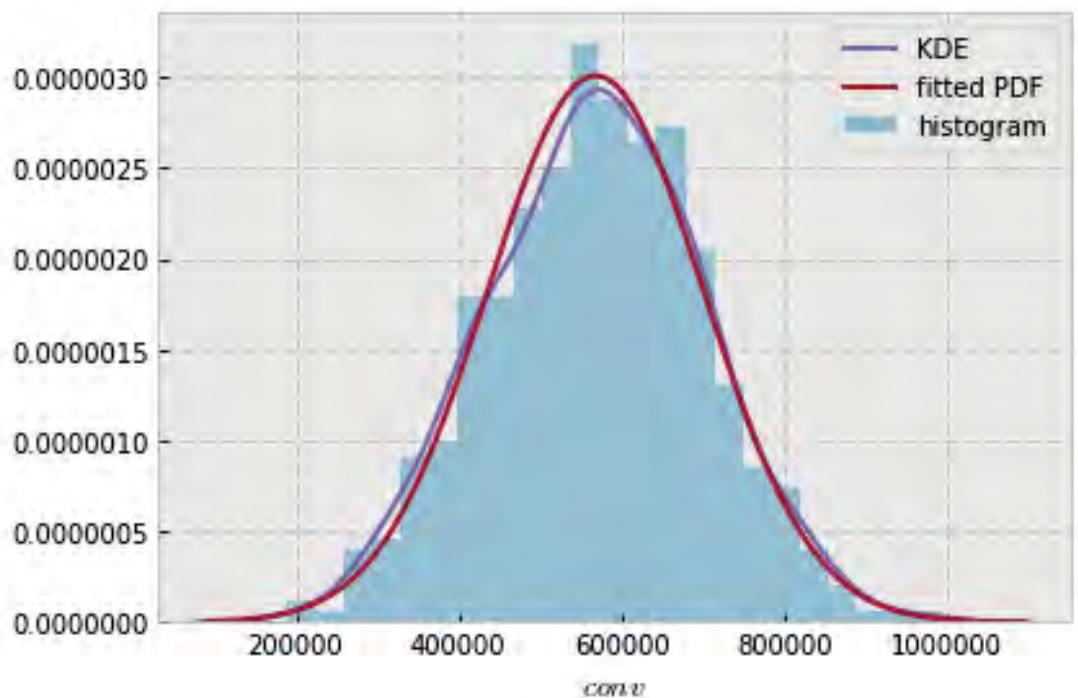


```
variable name: sigma
variable value: 0.066666666666666667
distribution: normal(shape=(), loc=601718.151852, scale=140098.06036857425)
DescribeResult(nobs=1000, minmax=(181756.03690000001, 1022764.11)
    mean=601718.15185200004, variance=19647113632.66935,
    skewness=-0.03289885587670868, kurtosis=-0.2861596011809855)
```

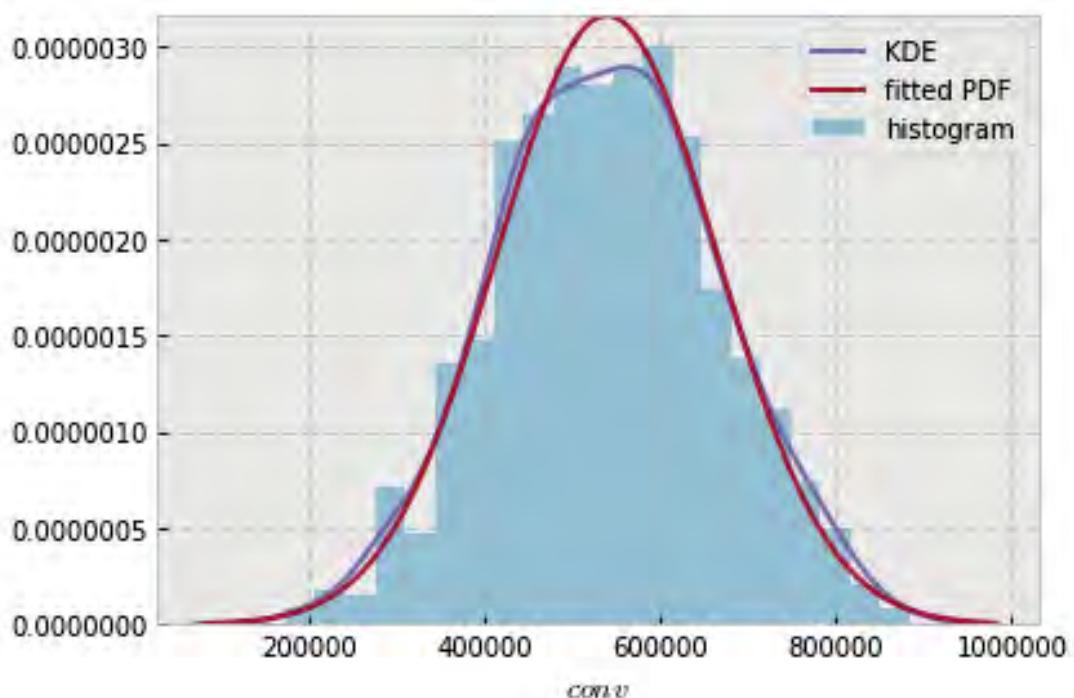


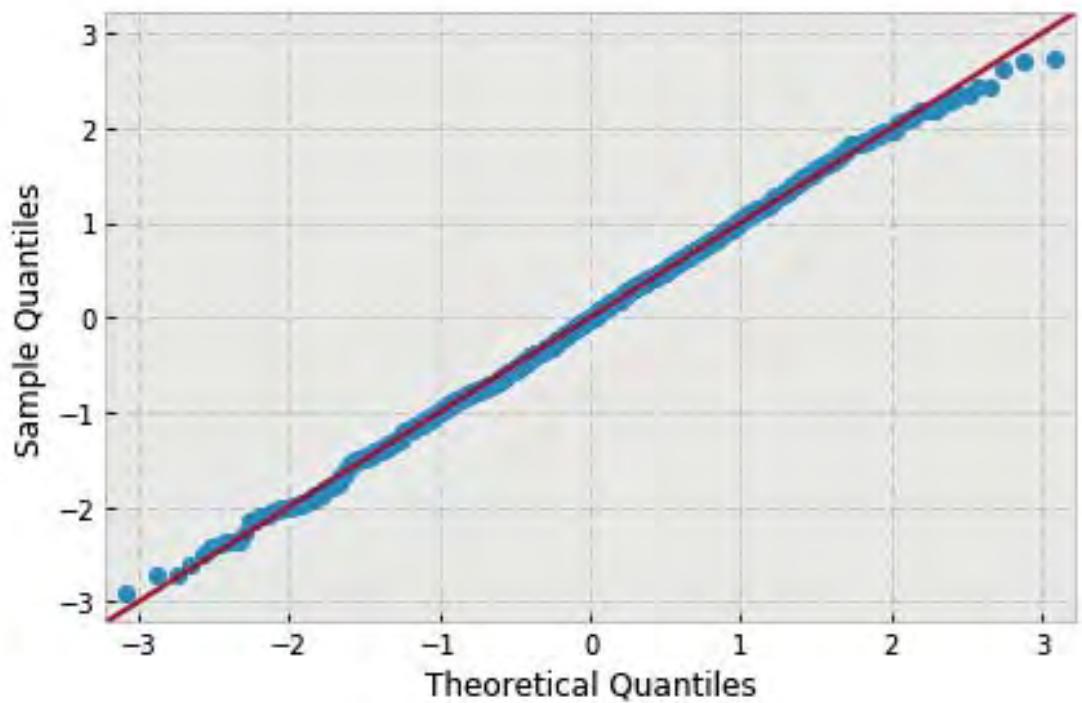


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=568411.1205014999, scale=132629.99025931334)
DescribeResult(nobs=1000, minmax=(187699.0333000001, 994481.2582000004)
               mean=568411.12050149986, variance=17608322638.824371,
               skewness=-0.005543336584638611, kurtosis=-0.18682856941844062)
```

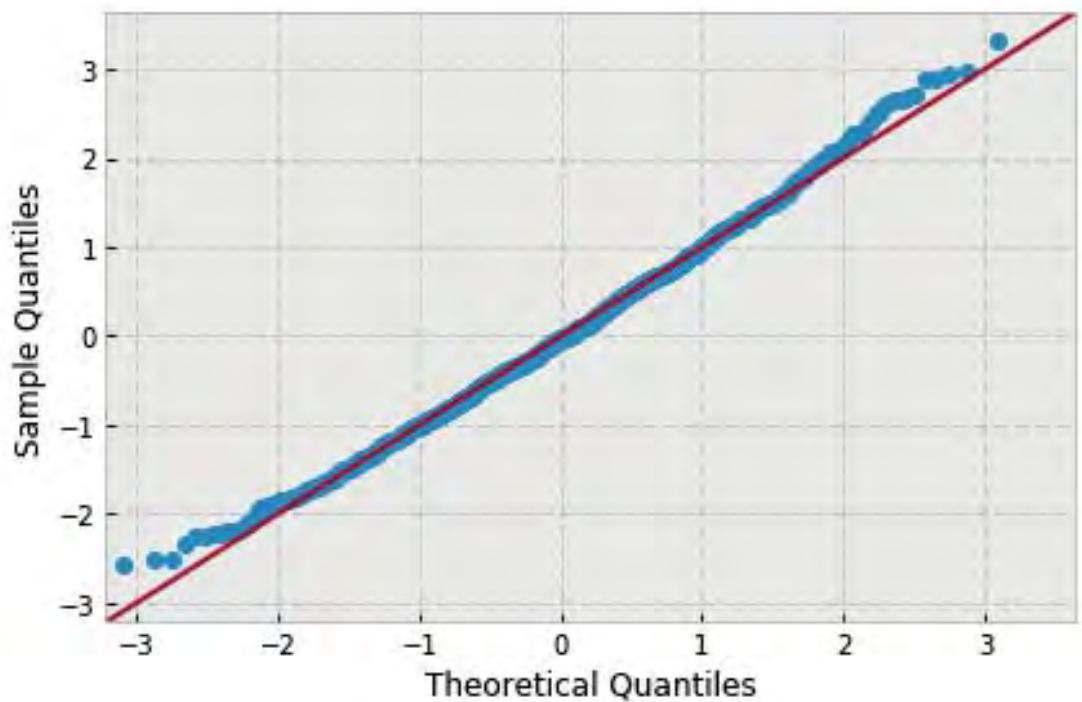
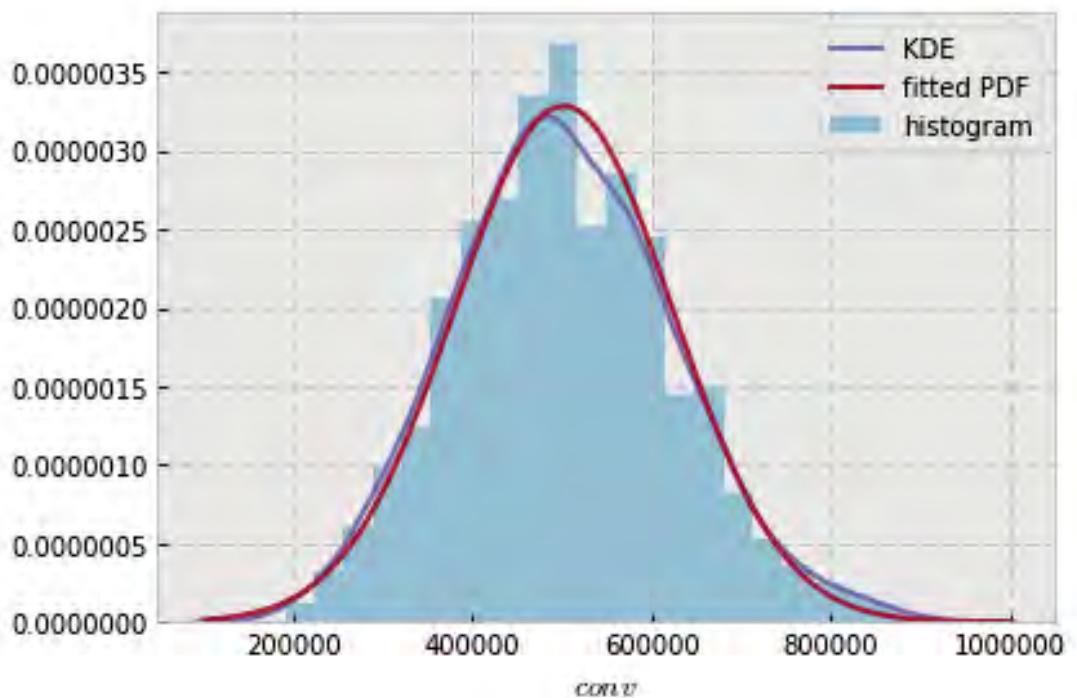


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=538497.6524252, scale=126118.46823446287)
DescribeResult(nobs=1000, minmax=(172191.1951999999, 883964.7798000002)
               mean=538497.6524251998, variance=15921789819.626846,
               skewness=0.012015047164622403, kurtosis=-0.27701168992351466)
```

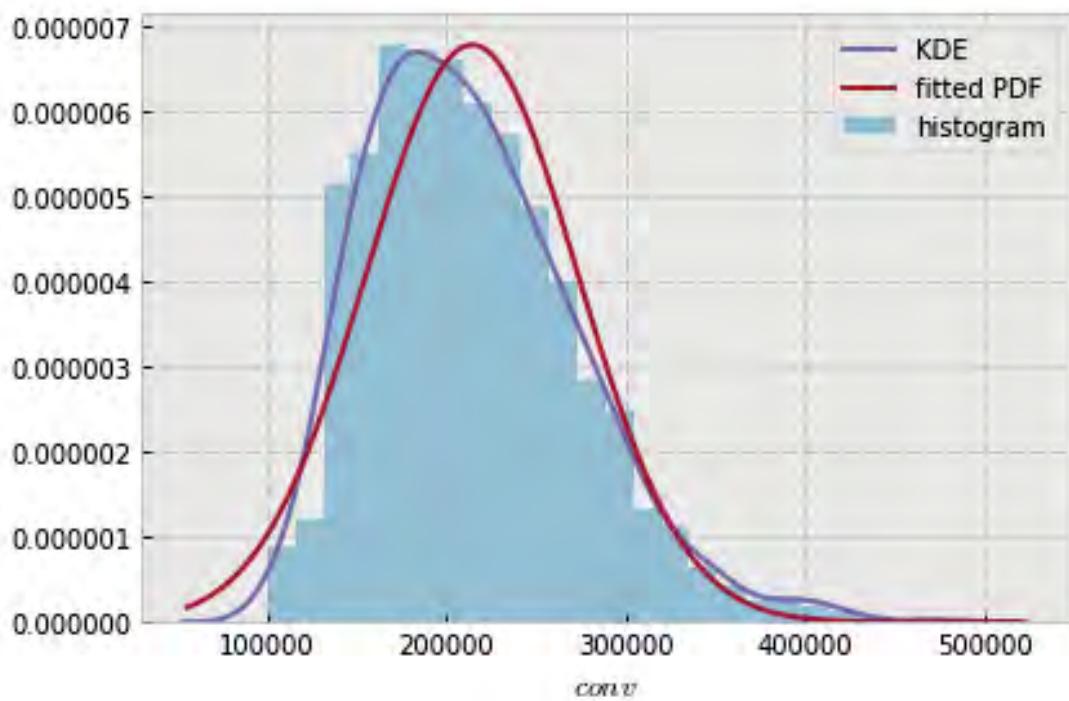


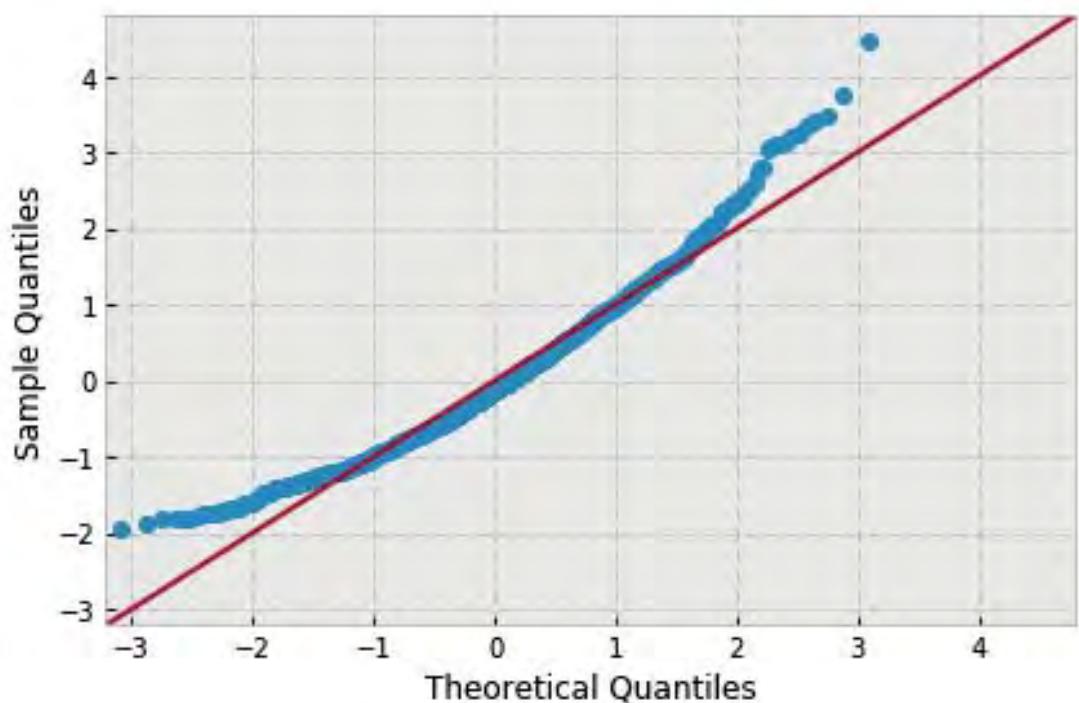


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=502052.04192870005, scale=121504.74227330228)
DescribeResult(nobs=1000, minmax=(191700.4602, 907213.37699999998)
               mean=502052.04192870005, variance=14778180575.477089,
               skewness=0.21871445767274342, kurtosis=-0.05468264478629514)
```

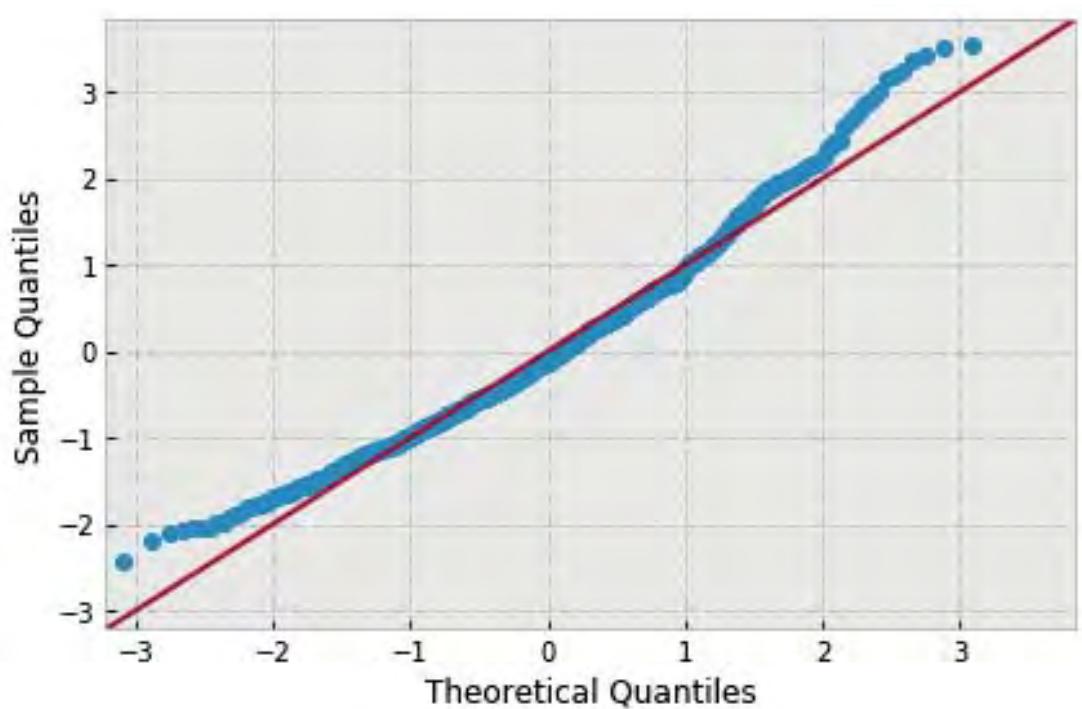
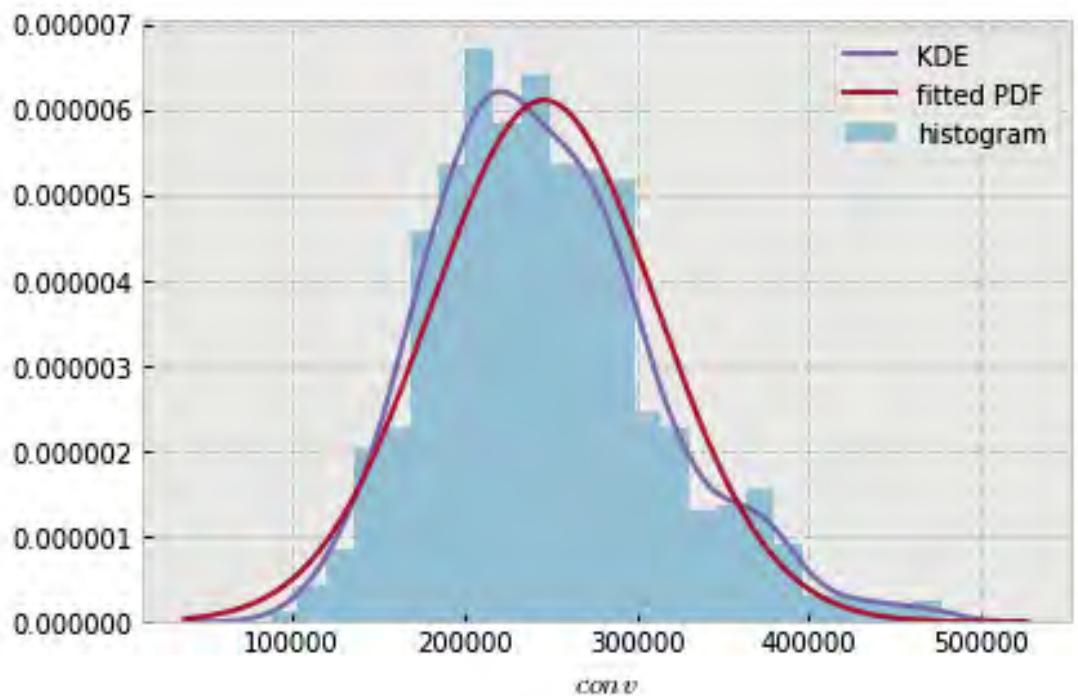


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=214142.67314539, scale=58760.23283084854)
DescribeResult(nobs=1000, minmax=(99718.223790000004, 476803.505999999999)
               mean=214142.67314539, variance=3456221183.5190496,
               skewness=0.759025883693774, kurtosis=0.6808478674189975)
```

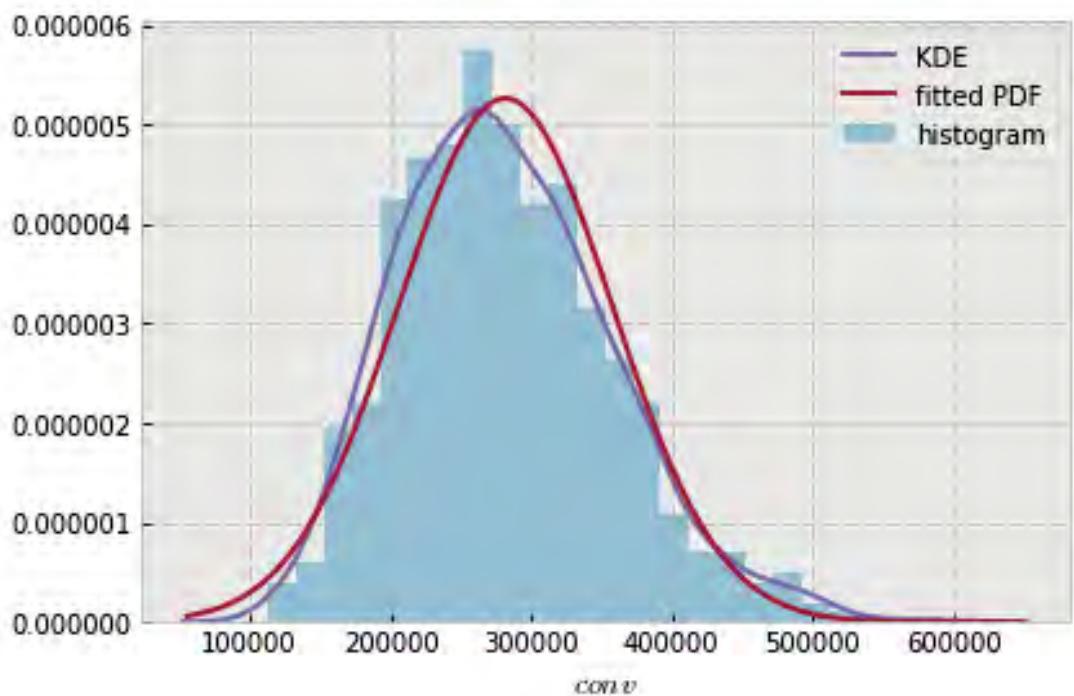


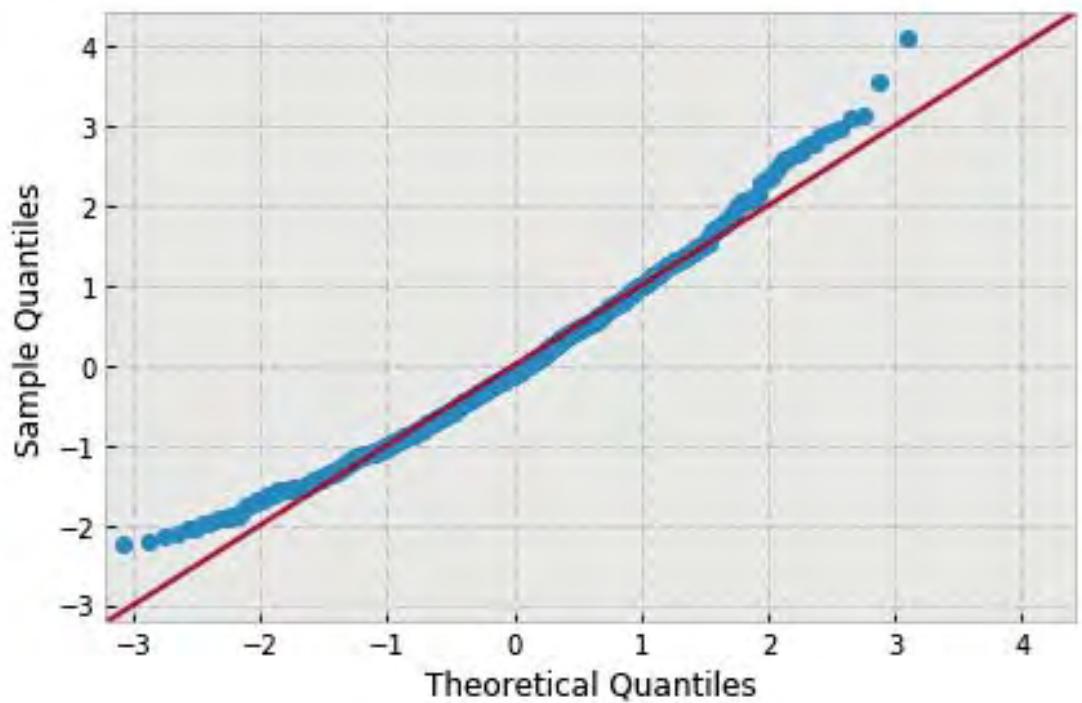


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=246251.6763355, scale=65345.67301162494)
DescribeResult(nobs=1000, minmax=(87226.03010000004, 476918.64230000001)
               mean=246251.6763355, variance=4274331312.6548629,
               skewness=0.6008531628820079, kurtosis=0.40649637779731407)
```

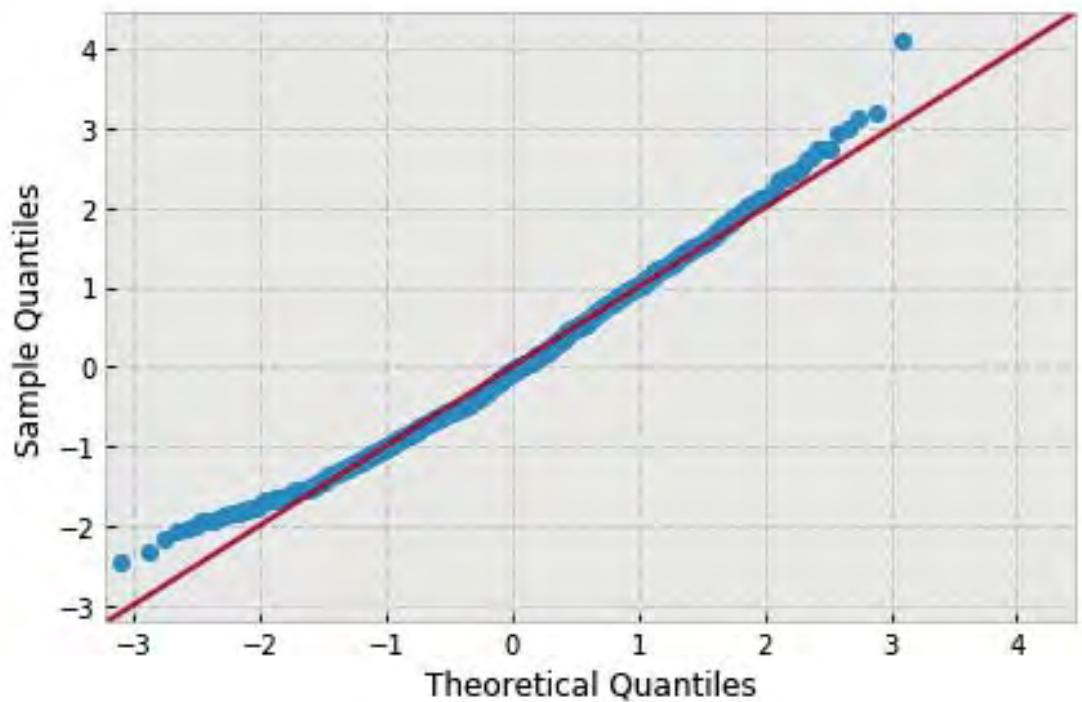
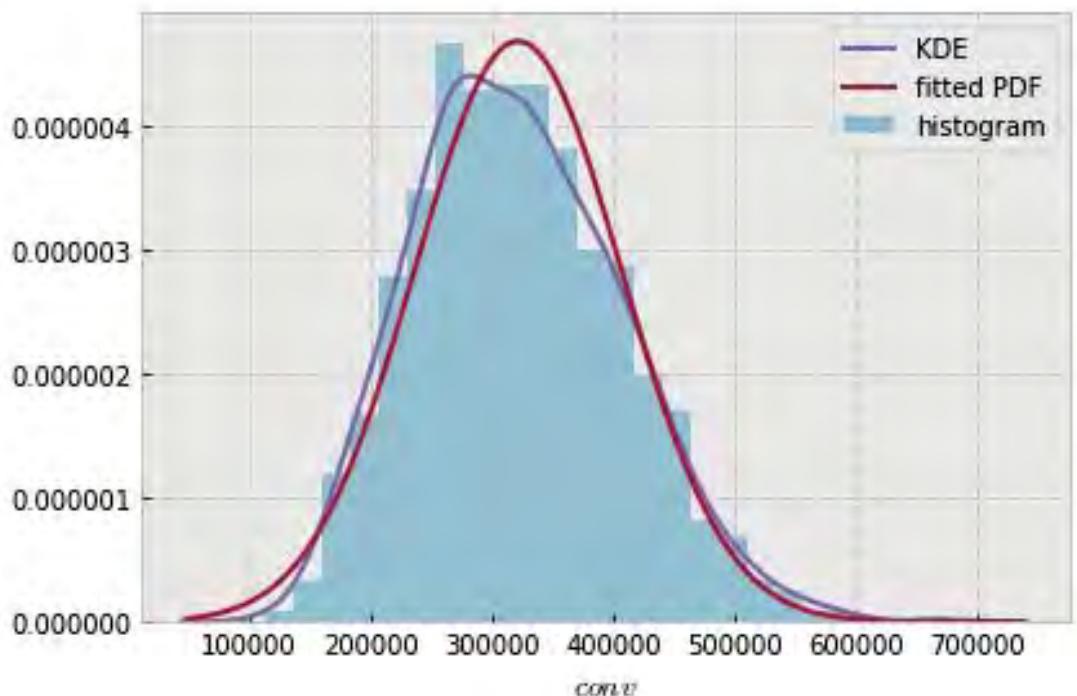


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=281708.87613530003, scale=75846.79110012914)
DescribeResult(nobs=1000, minmax=(112724.5307, 592217.05599999998)
               mean=281708.87613530003, variance=5758494214.4010277,
               skewness=0.5236058637063428, kurtosis=0.27164025214962306)
```

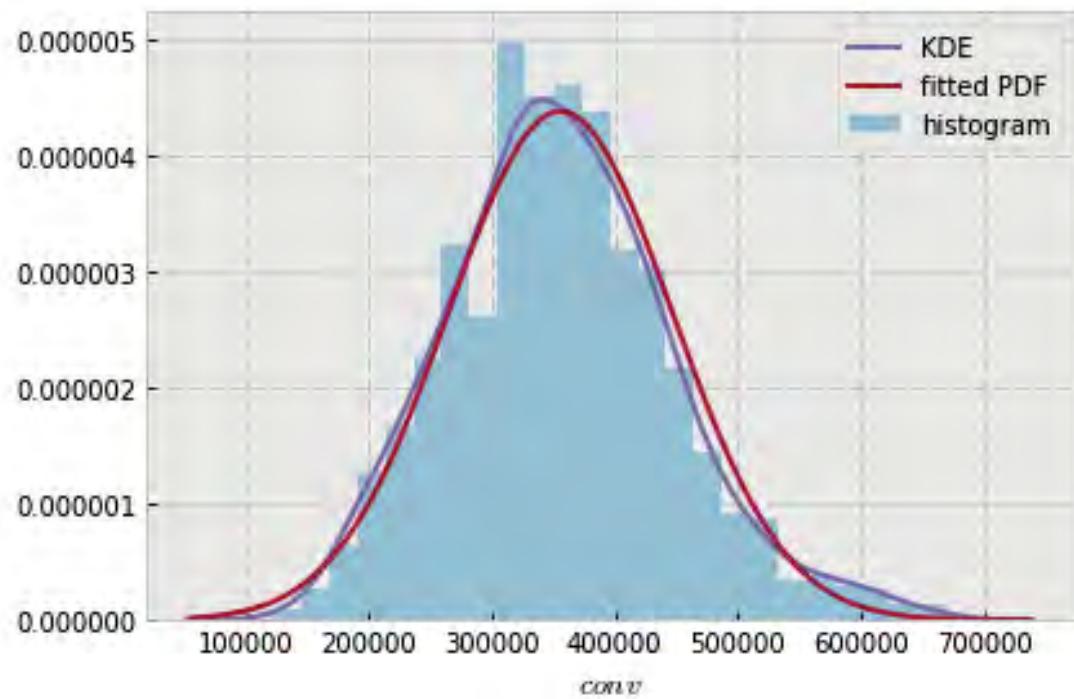


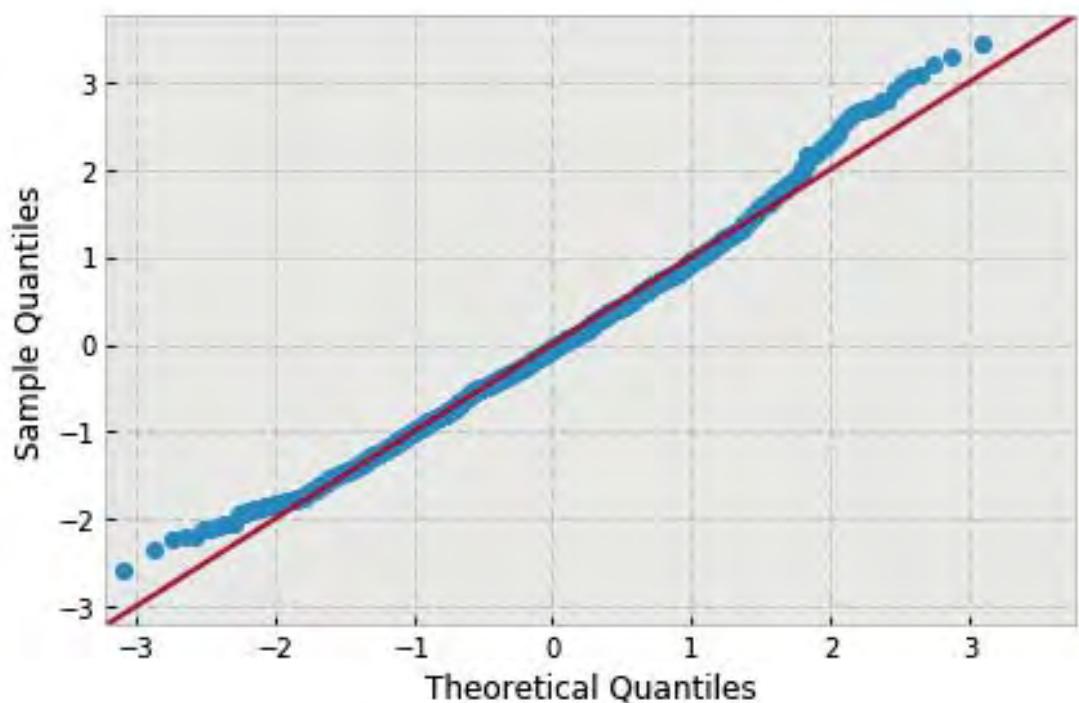


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=321351.8899207, scale=85057.62919188368)
DescribeResult(nobs=1000, minmax=(113544.219, 672129.84389999998)
               mean=321351.88992069999, variance=7242042326.0700521,
               skewness=0.3928924956665979, kurtosis=-0.02329470689271007)
```

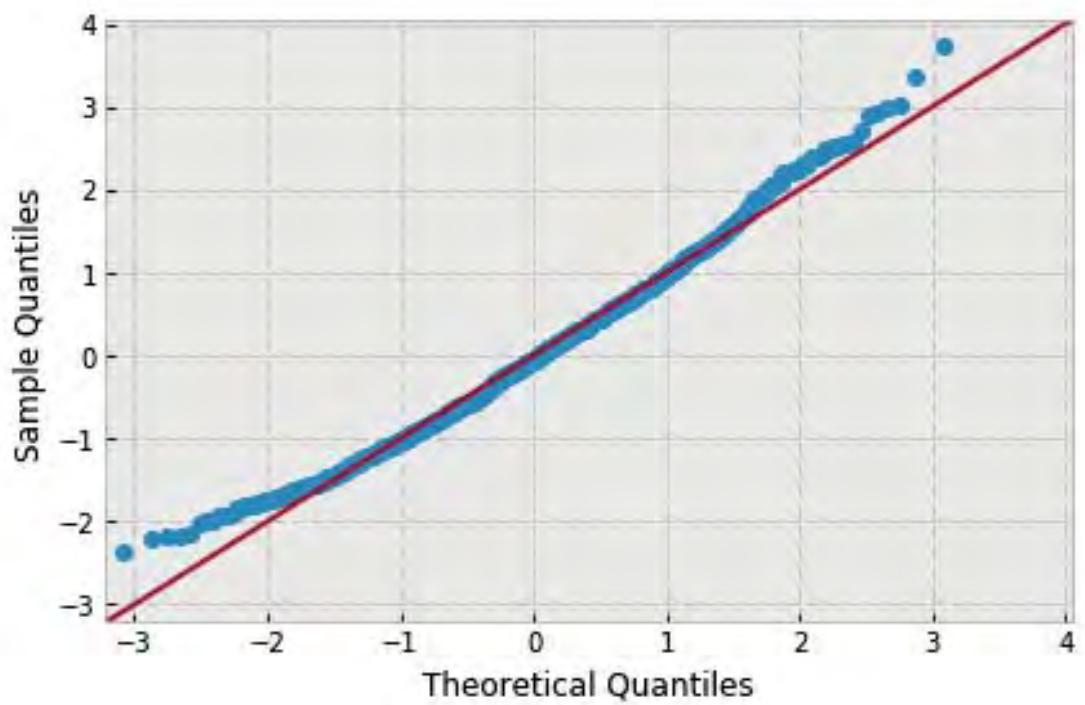
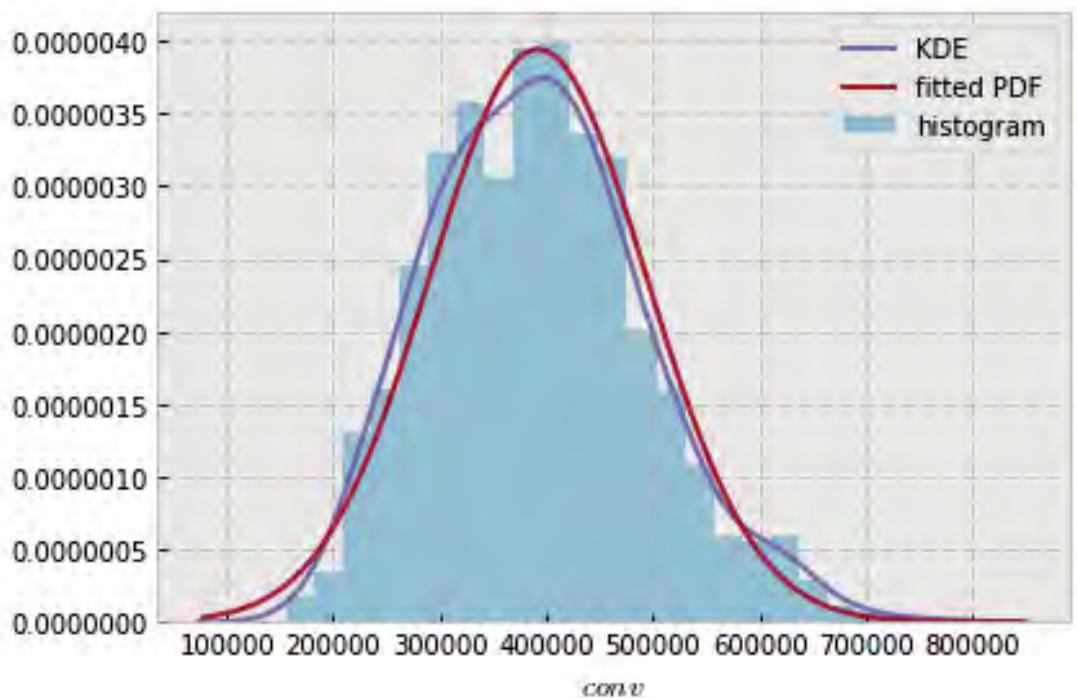


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=355634.3666453, scale=90889.52680538228)
DescribeResult(nobs=1000, minmax=(122019.4255, 669656.1299000006)
               mean=355634.3666453, variance=8269175258.1644678,
               skewness=0.3871333081514963, kurtosis=0.27171959553035574)
```

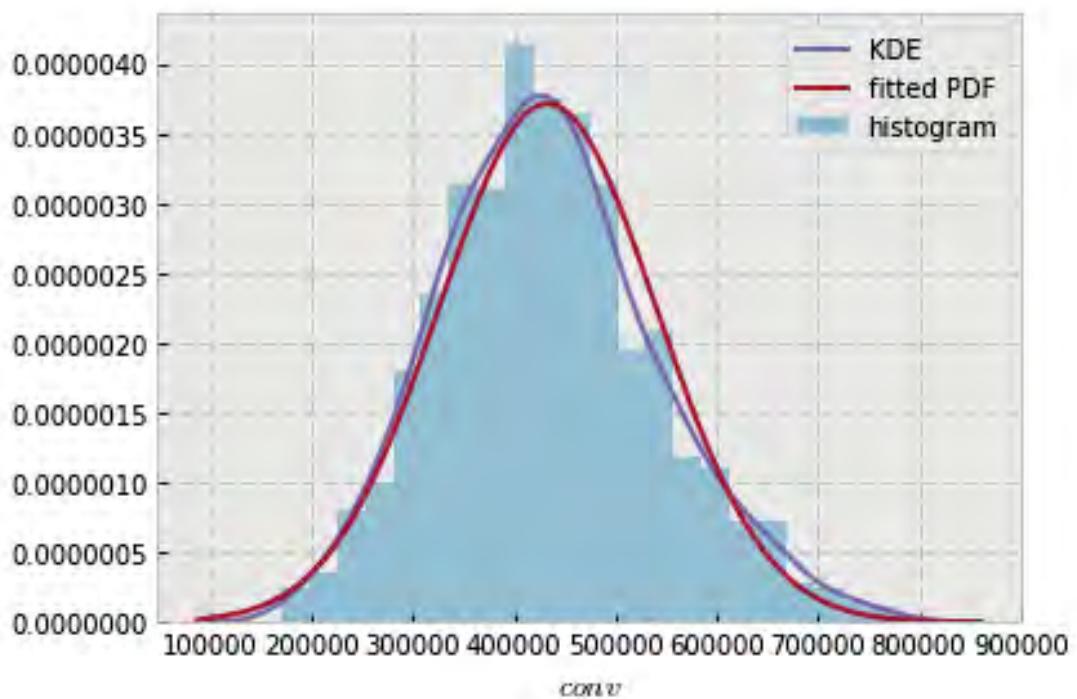


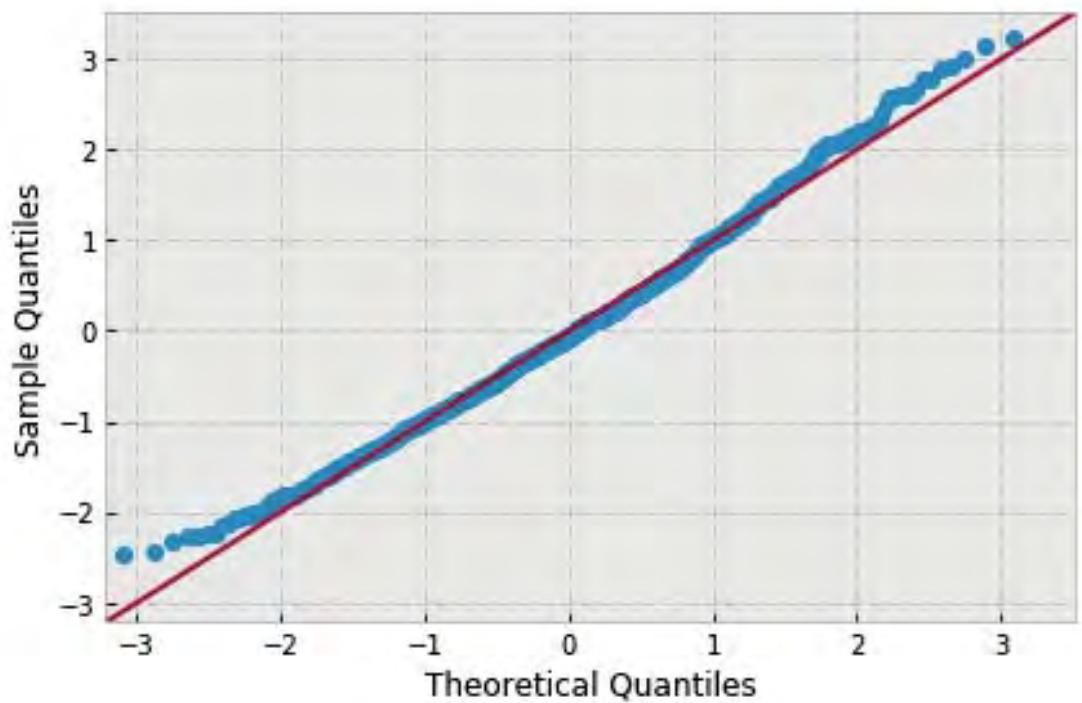


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=392101.0012248, scale=101159.20084219502)
DescribeResult(nobs=1000, minmax=(154287.7784999999, 770681.1787000005)
               mean=392101.00122480001, variance=10243427342.373922,
               skewness=0.40066269827409334, kurtosis=0.030575682832897577)
```

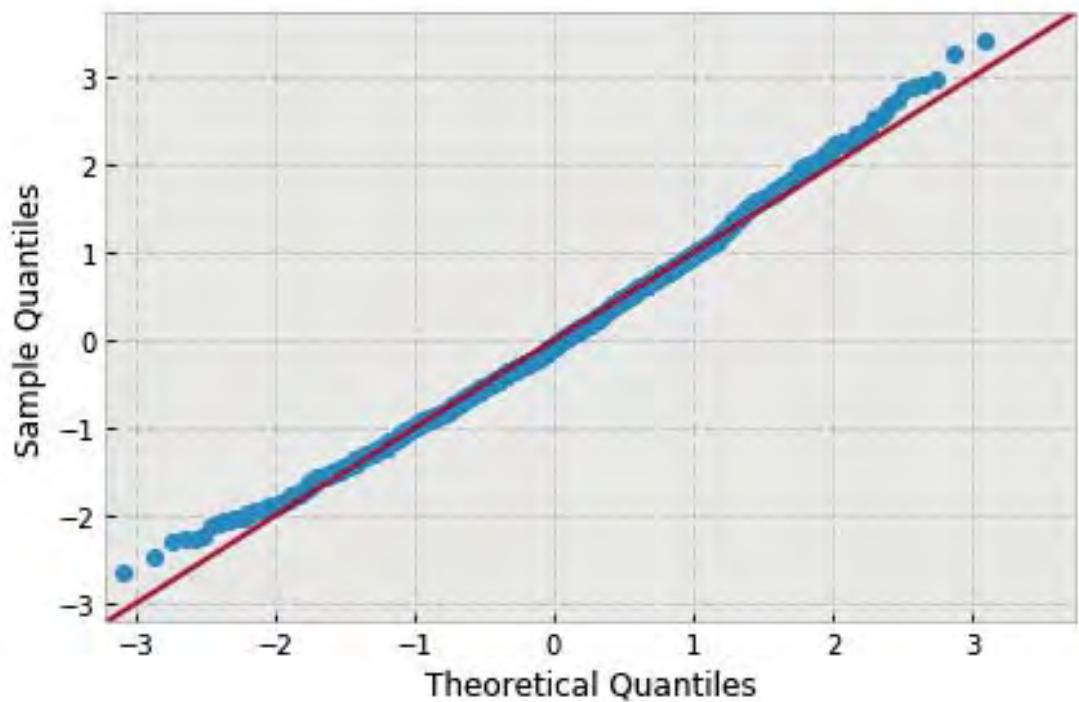
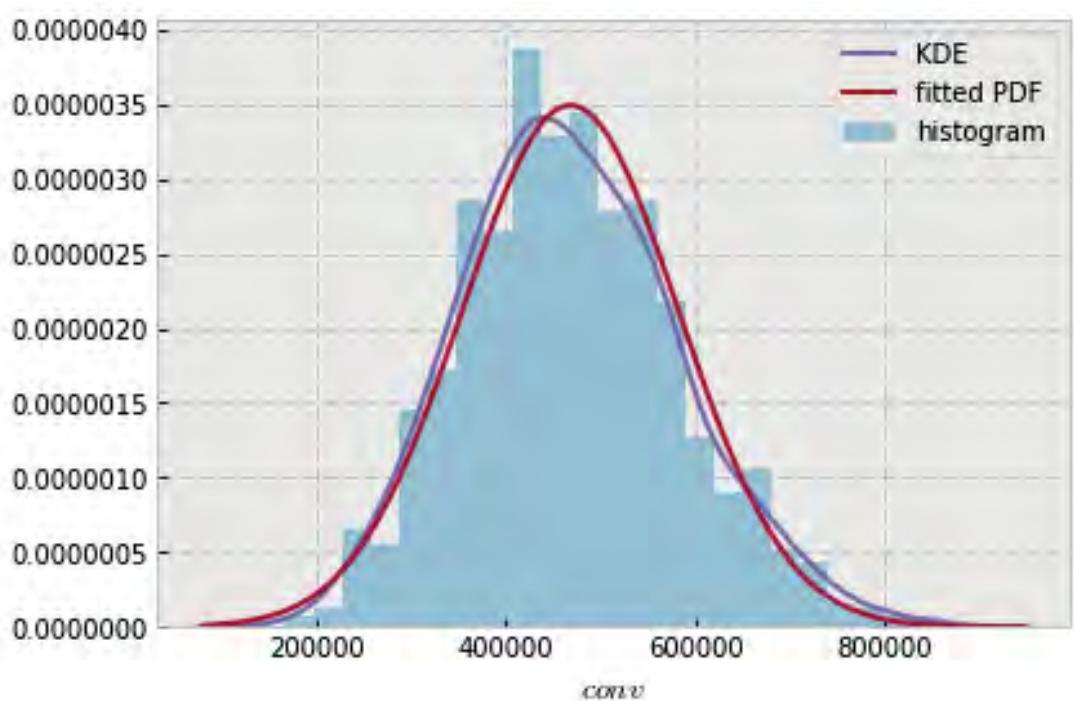


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=432650.6826091, scale=107439.02060220322)
DescribeResult(nobs=1000, minmax=(168988.1995999999, 778974.1925)
               mean=432650.68260910001, variance=11554697845.806456,
               skewness=0.3330155717450263, kurtosis=0.014940025898865006)
```

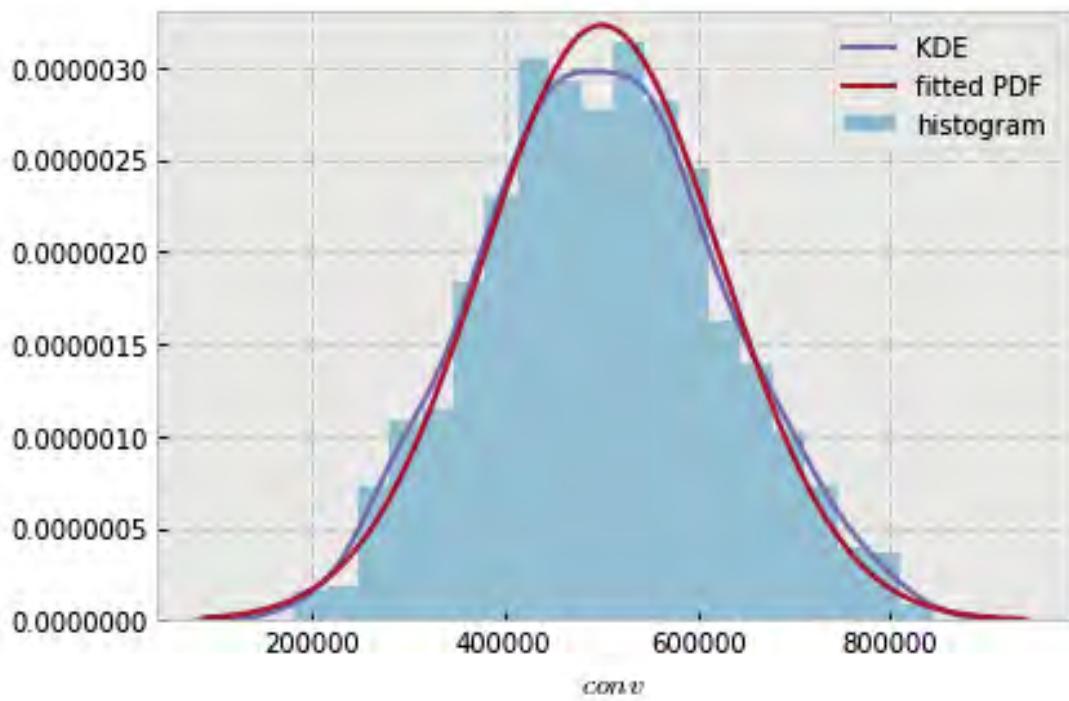


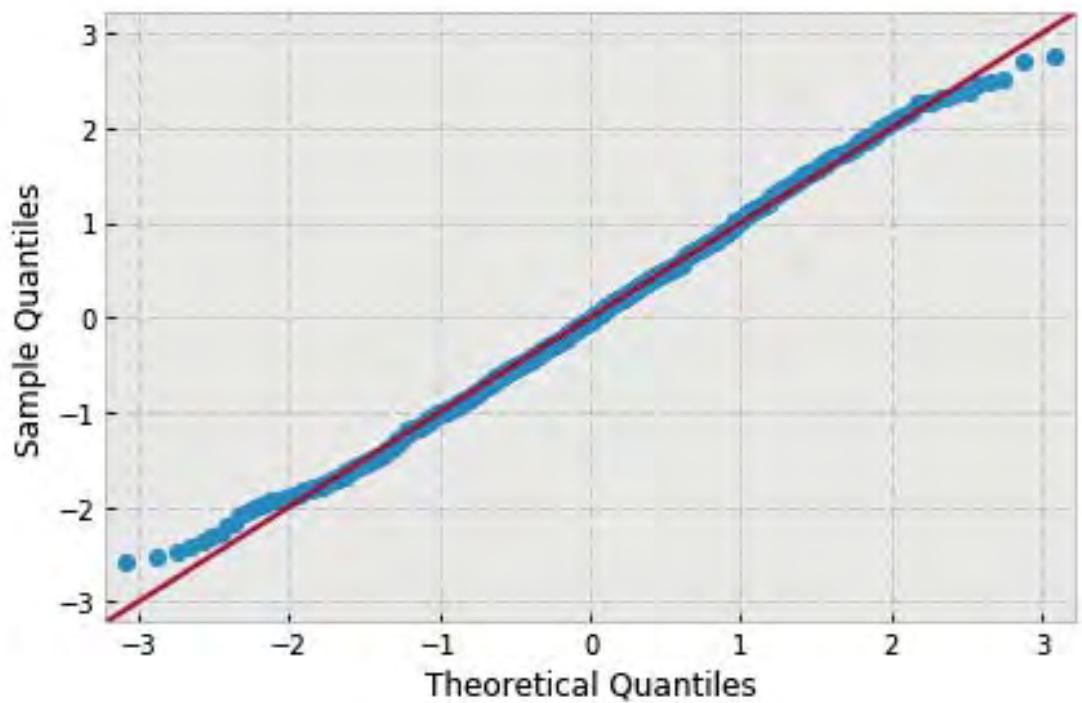


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=468398.275182, scale=114283.83046188799)
DescribeResult(nobs=1000, minmax=(166503.7758, 860508.53650000005)
               mean=468398.27518200001, variance=13073867772.814371,
               skewness=0.3108572595807076, kurtosis=-0.0161516244254849)
```

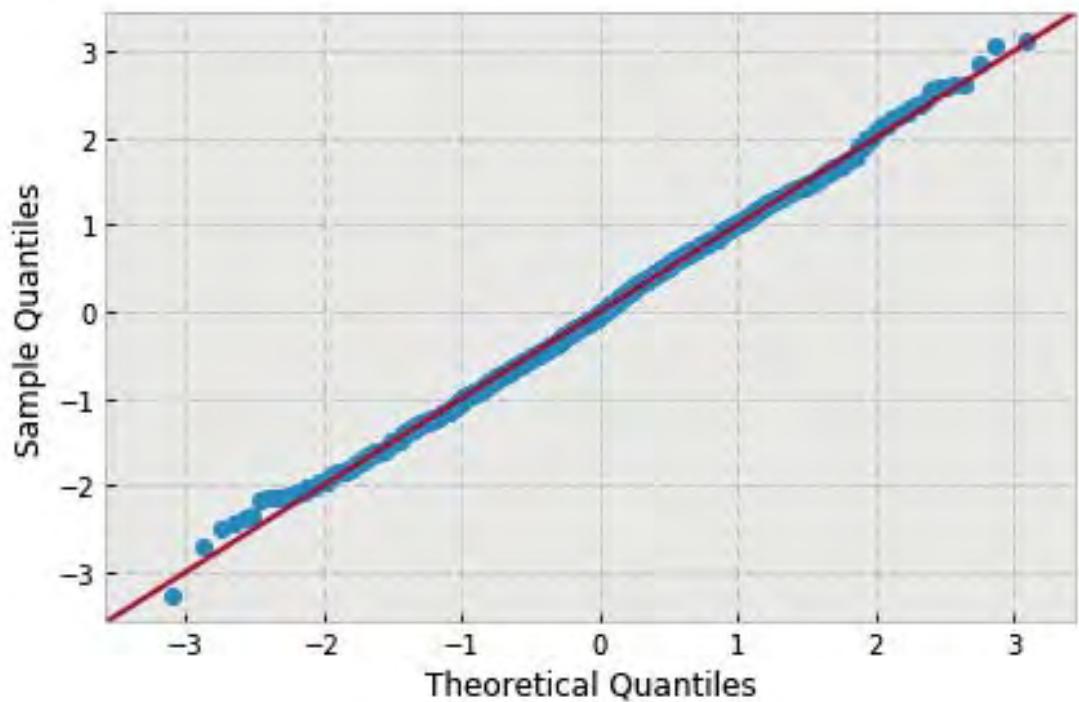
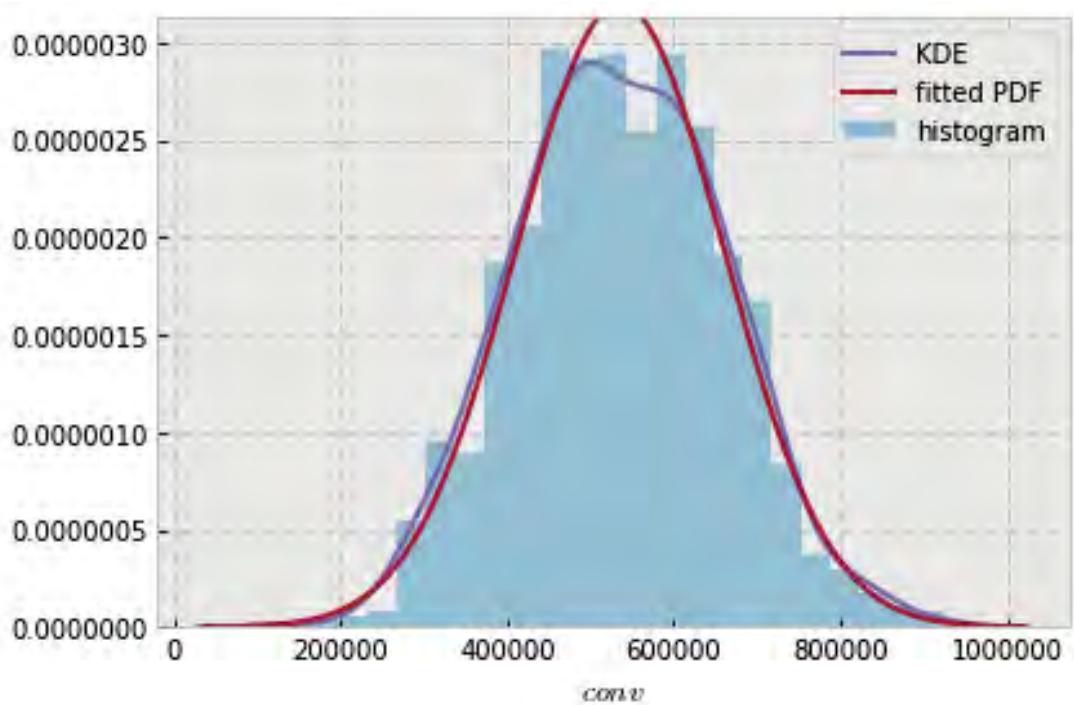


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=500954.6194148, scale=123478.3705690747)
DescribeResult(nobs=1000, minmax=(181782.4604999999, 843188.1448000001)
               mean=500954.6194148002, variance=15262170168.562296,
               skewness=0.09784035236321317, kurtosis=-0.36924626797699966)
```





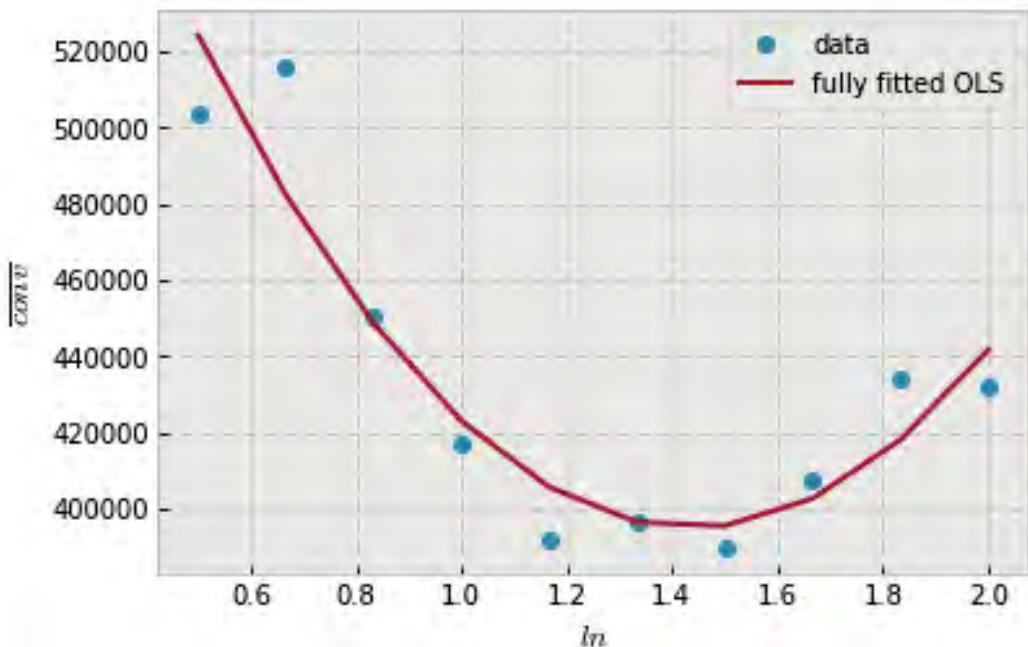
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=535398.1395644001, scale=124733.44338422752)
DescribeResult(nobs=1000, minmax=(129205.5124000001, 924951.0814000002)
               mean=535398.13956440007, variance=15574005904.39068,
               skewness=0.06756287513672024, kurtosis=-0.20984422330416308)
```



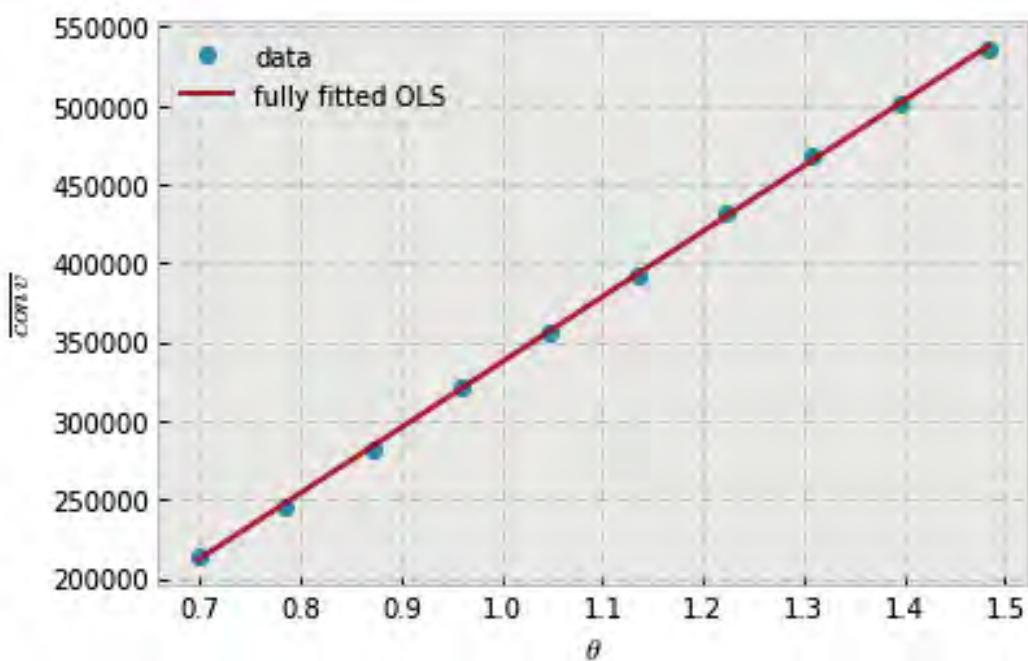
4.2 Parameter regression

```
In [14]: var_name = 'convex_hull_volume'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly1')  
        mean_reg('nB', var_name, 'poly2')  
        mean_reg('r', var_name, 'poly1')  
        mean_reg('sigma', var_name, 'poly1')  
        mean_reg('N', var_name, 'poly1')
```

variable name: ln
model: poly2
r-squared: 0.6298836281717898
explained variance: 0.6368751435413317



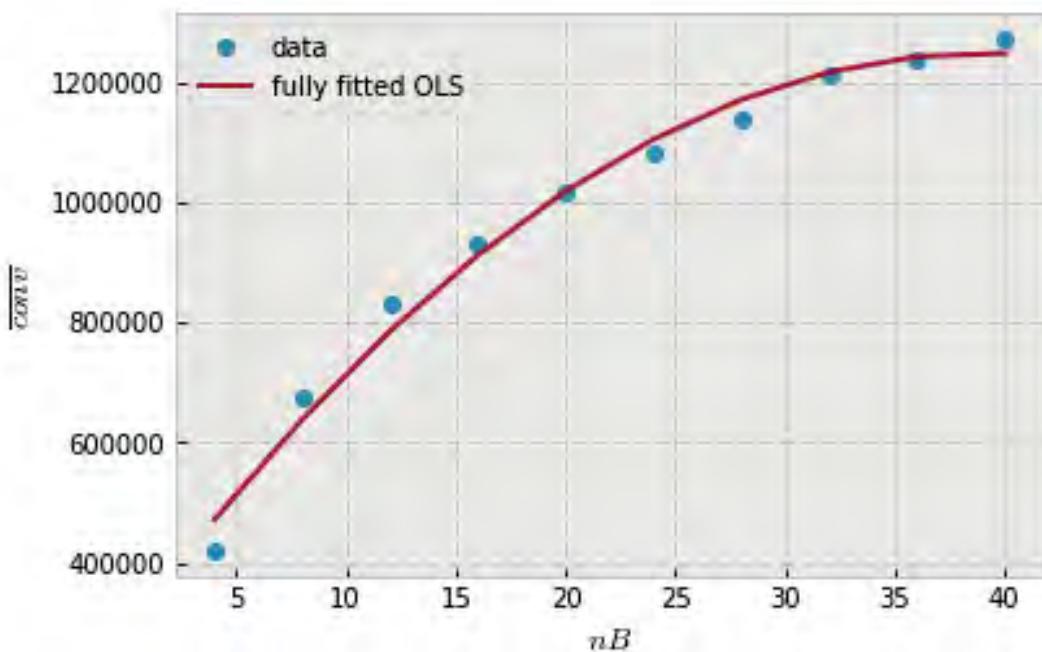
```
variable name: theta  
model: poly1  
r-squared: 0.9993036610422747  
explained variance: 0.9993038434190464
```



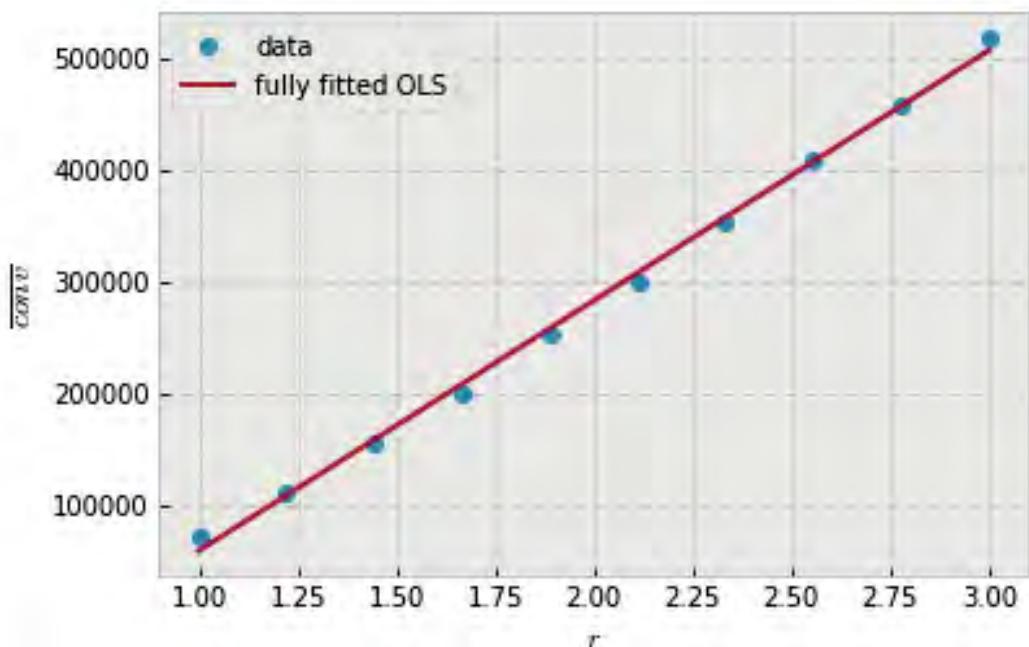
```

variable name: nB
model: poly2
r-squared: 0.9520481161903154
explained variance: 0.9522426897960712

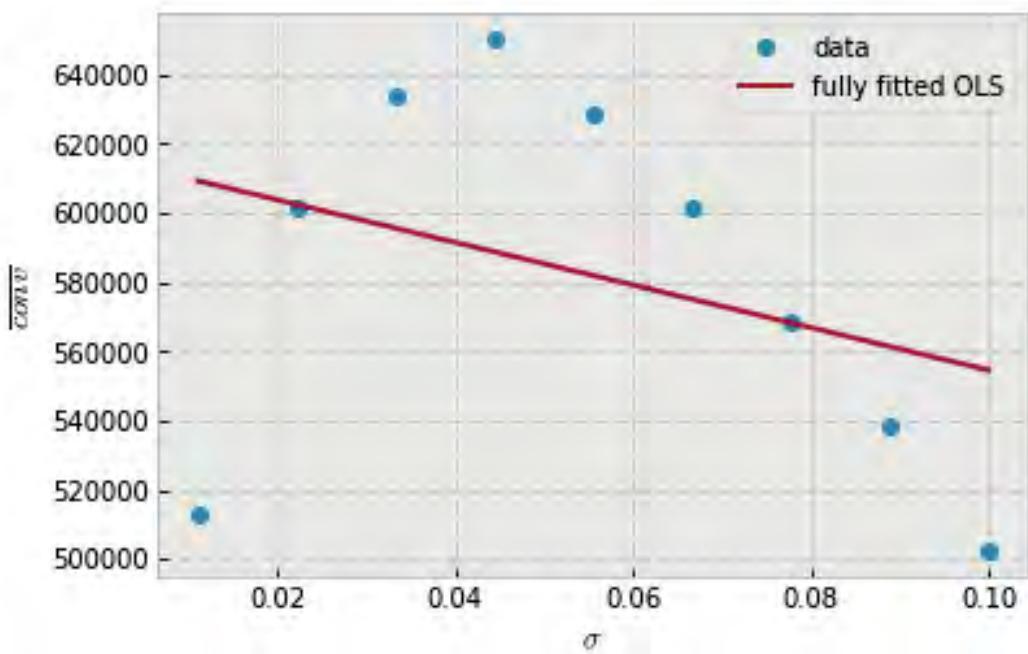
```



```
variable name: r
model: poly1
r-squared: 0.9951076557668824
explained variance: 0.9951610086934837
```



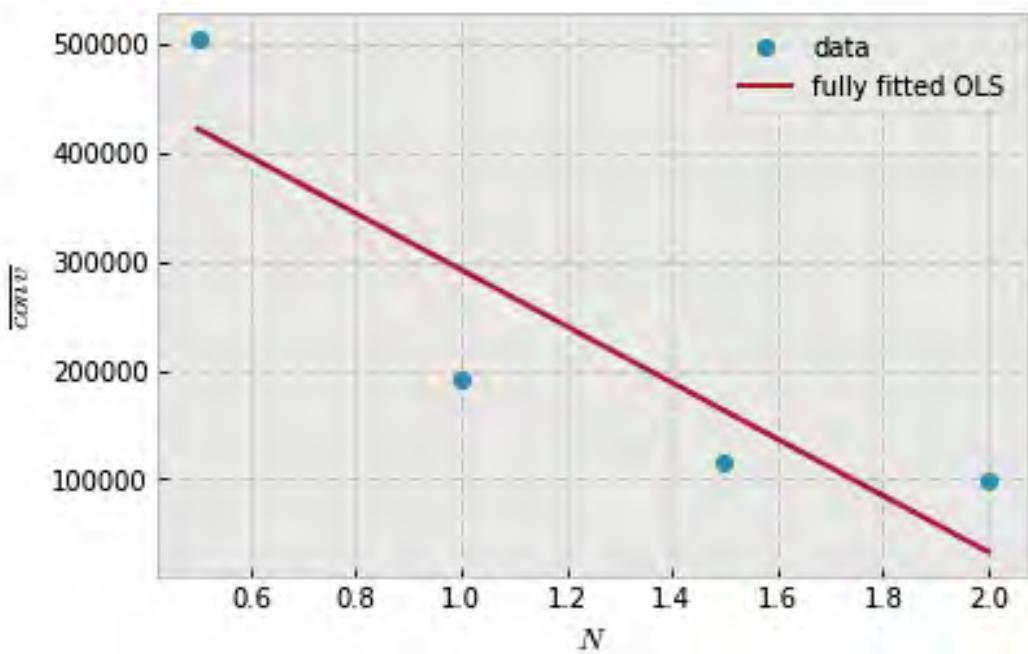
```
variable name: sigma
model: poly1
r-squared: -0.8252206212184423
explained variance: -0.8007694238703627
```



```

variable name: N
model: poly1
r-squared: -0.38802227249209964
explained variance: -0.20288634608959488

```

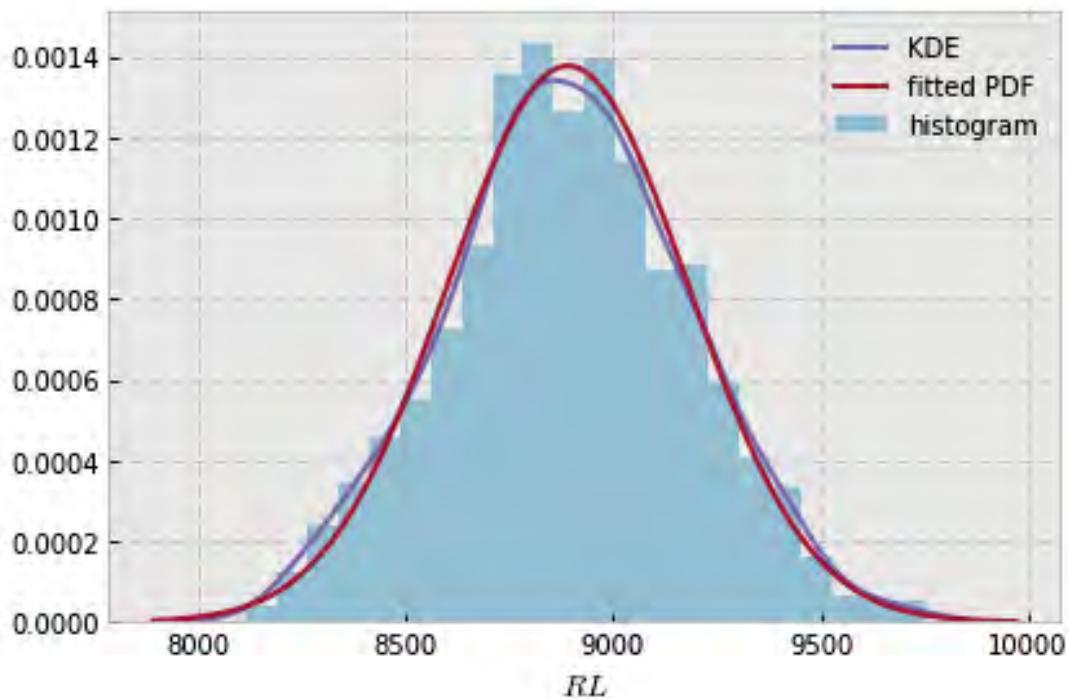


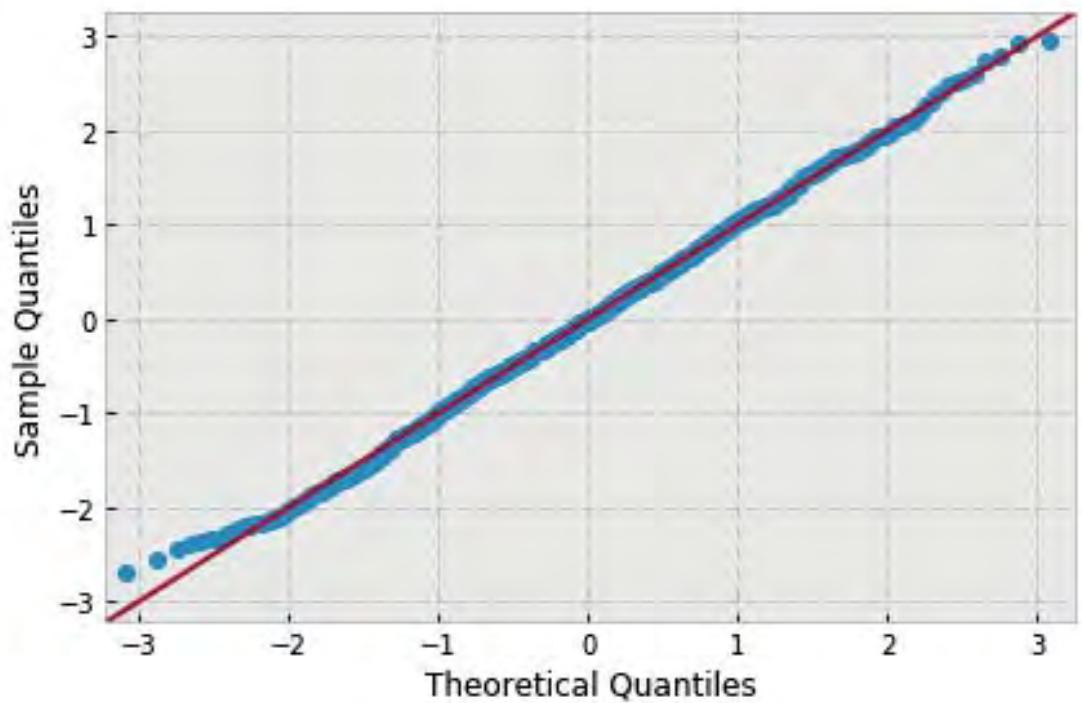
5 Total root length

5.1 Probability distributions

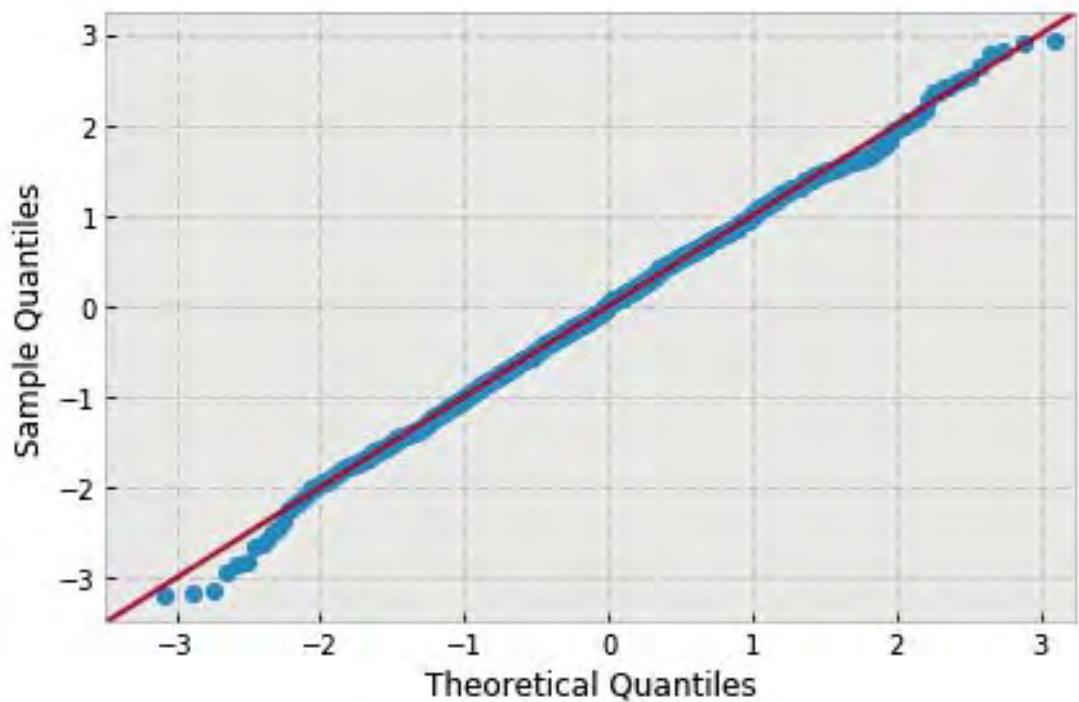
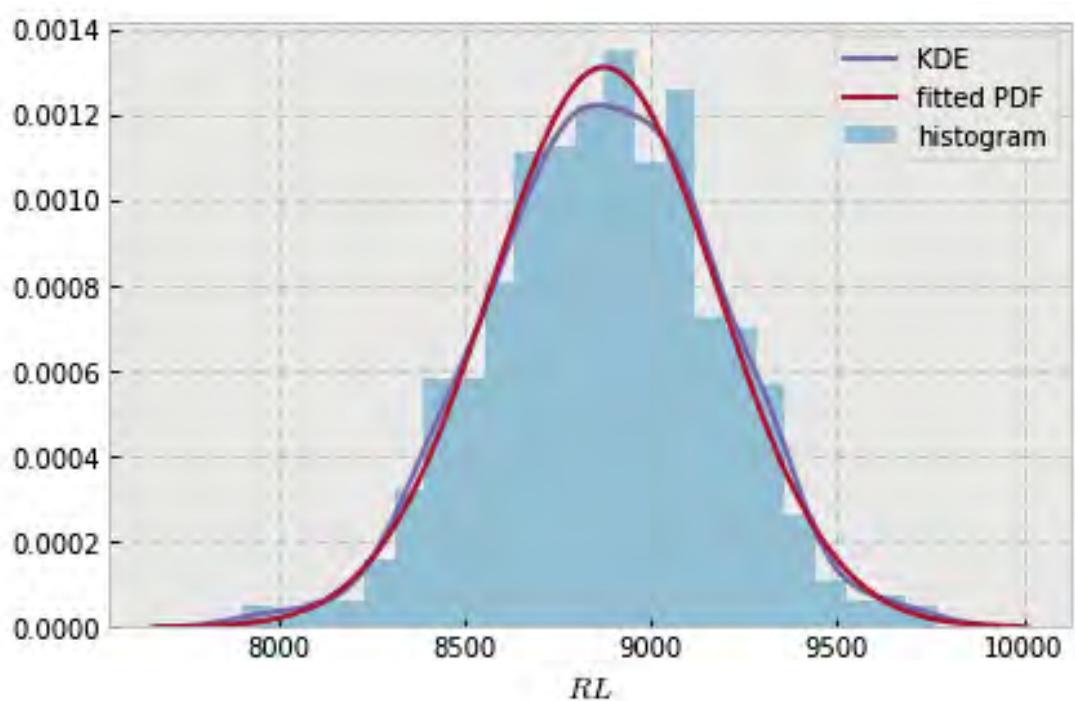
```
In [15]: plot_dist('total_root_length')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=8890.529435828374, scale=289.84949503462246)
DescribeResult(nobs=1000, minmax=(8112.5984027331015, 9750.0770228082965)
               mean=8890.5294358283736, variance=84096.826598424072,
               skewness=0.031793353772924894, kurtosis=-0.20400781064700801)
```

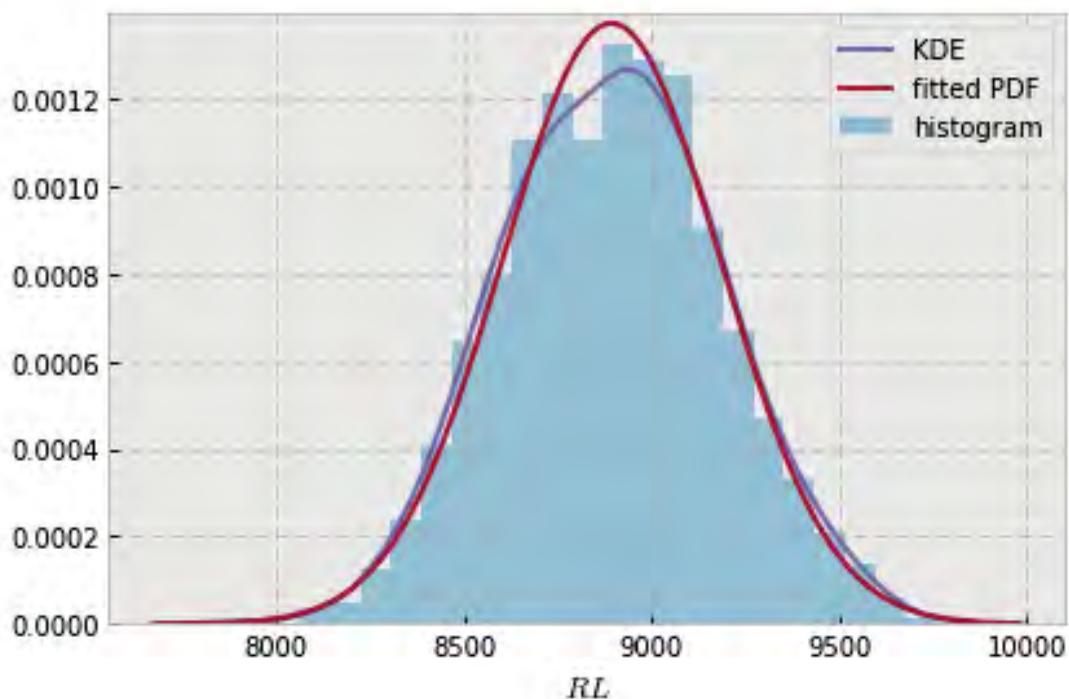


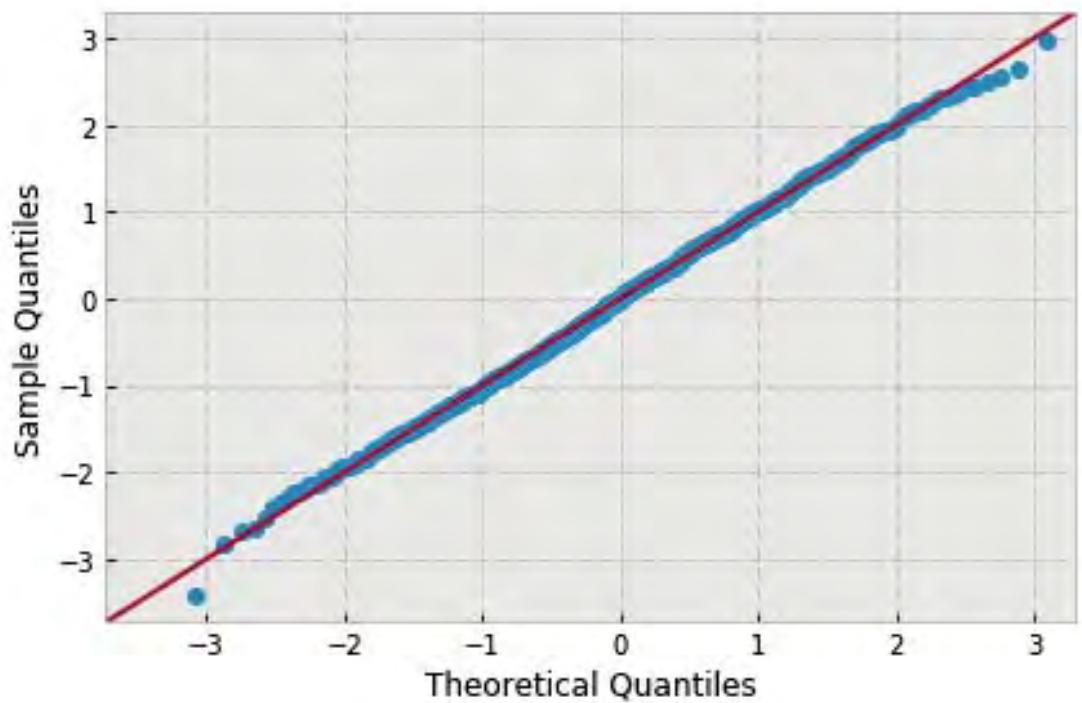


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=8873.204074936704, scale=304.4019958243945)
DescribeResult(nobs=1000, minmax=(7905.0238376431189, 9764.8300570286683)
               mean=8873.2040749367043, variance=92753.328390264956,
               skewness=-0.07983531310620573, kurtosis=-0.011499039756156293)
```

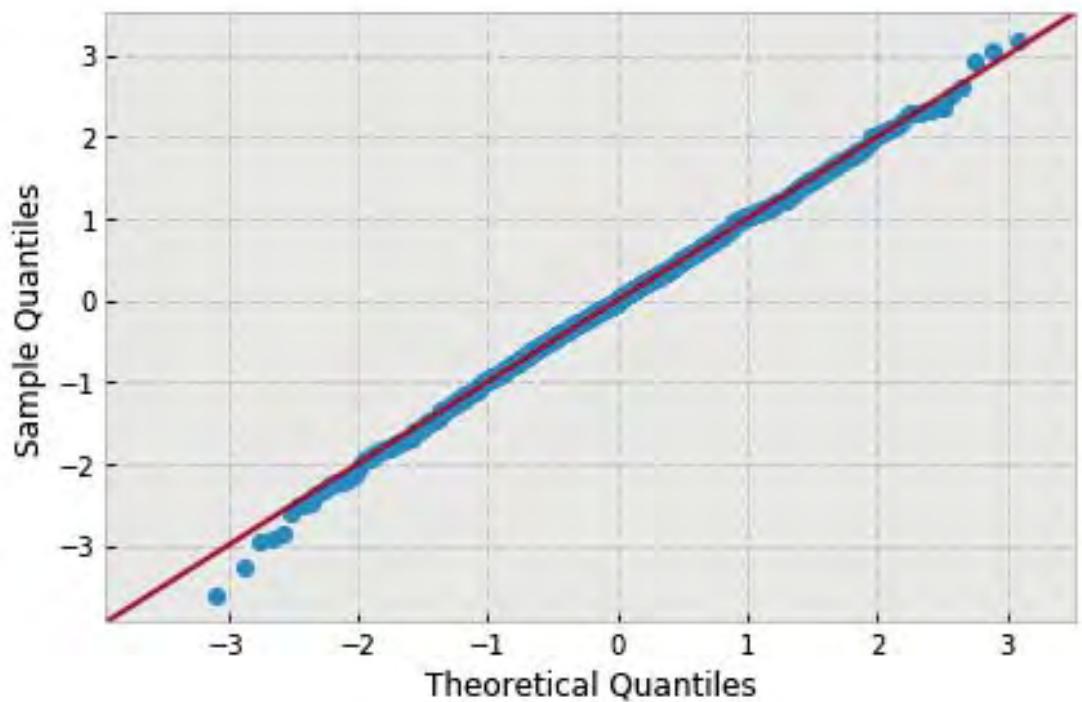
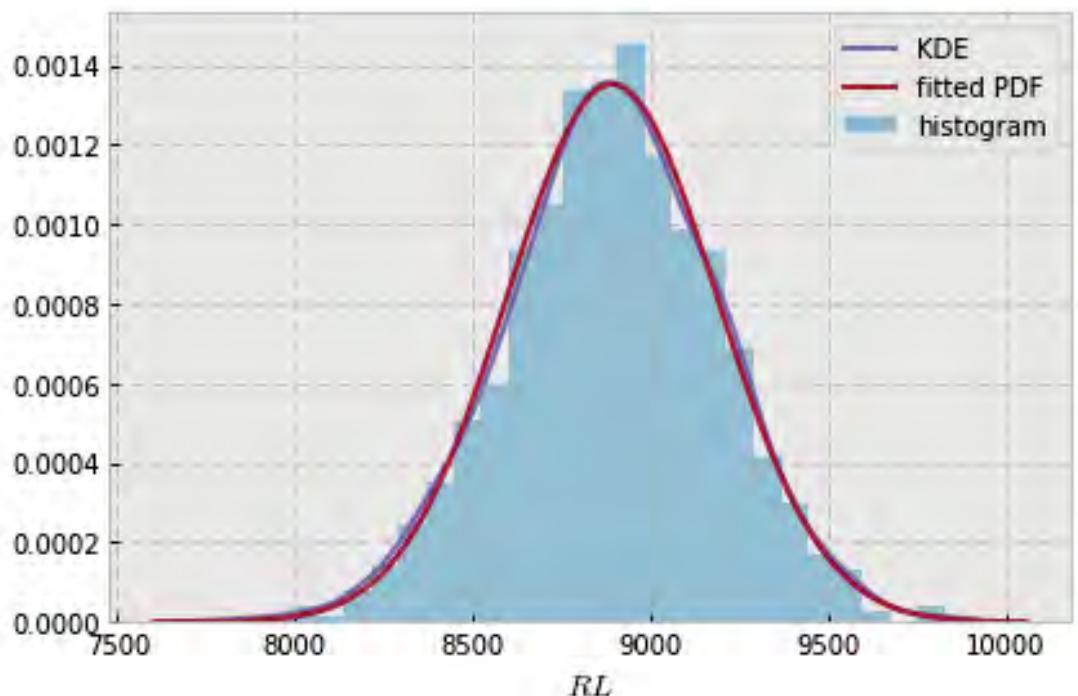


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=8892.405371161714, scale=290.35831355187025)
DescribeResult(nobs=1000, minmax=(7902.4172017548881, 9754.9720688726557)
               mean=8892.4053711617144, variance=84392.342591277469,
               skewness=0.012046332878024873, kurtosis=-0.2456007996134537)
```

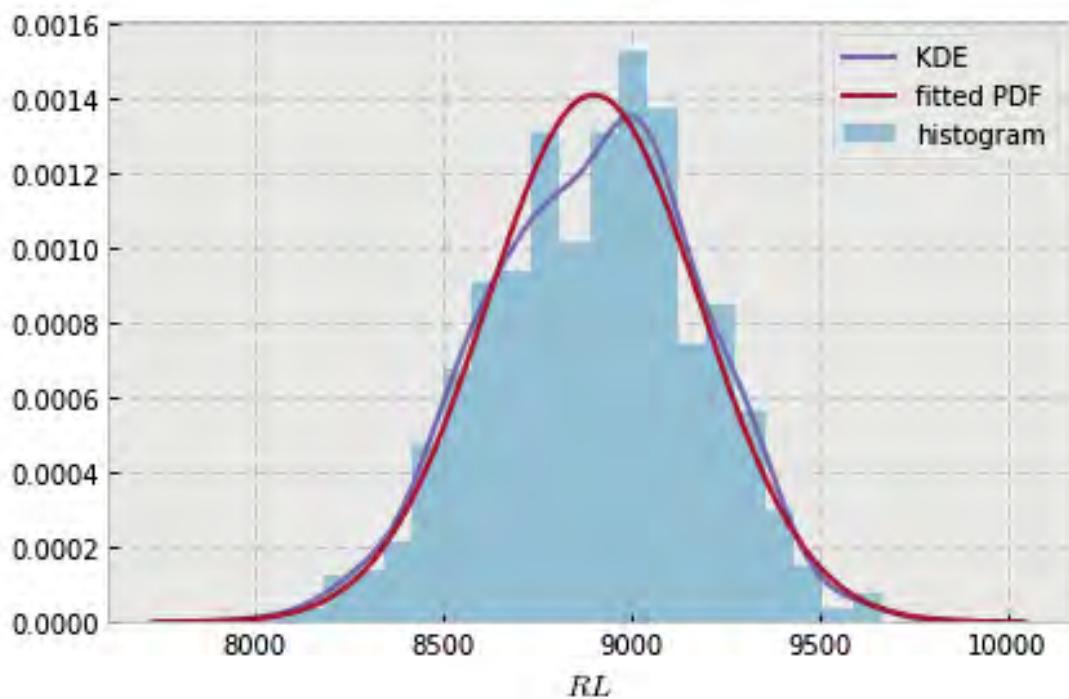


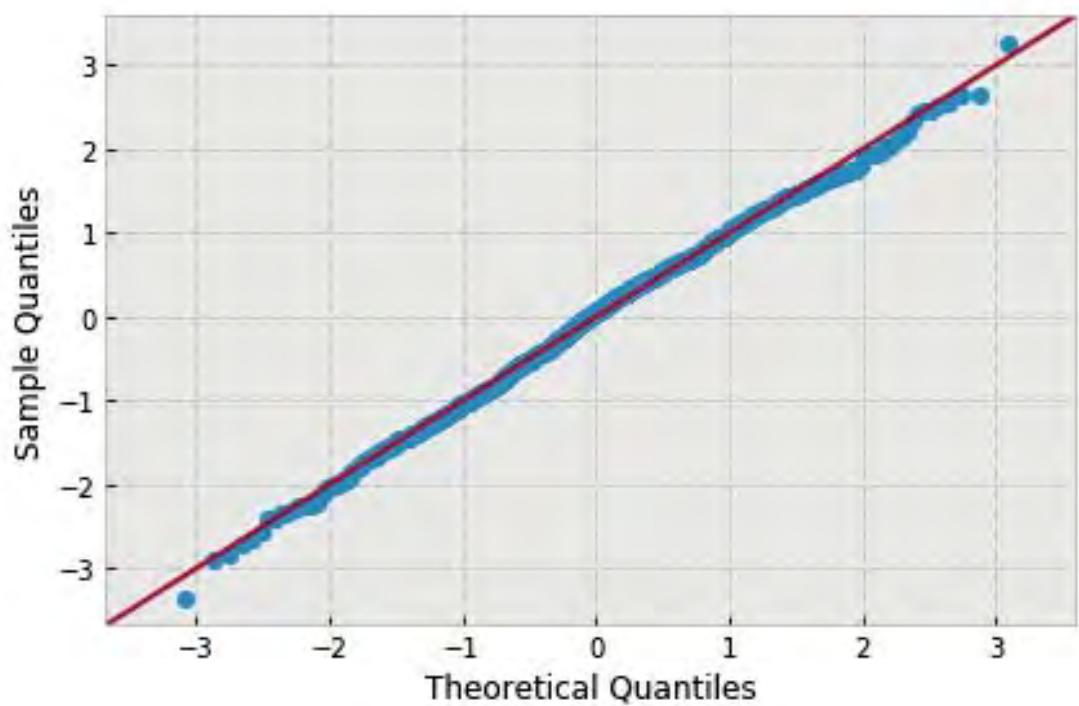


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=8890.936801042883, scale=294.8469442908724)
DescribeResult(nobs=1000, minmax=(7830.2964079502053, 9826.9353065293544)
               mean=8890.9368010428825, variance=87021.742299964768,
               skewness=-0.09043646275551268, kurtosis=0.11323556761954556)
```

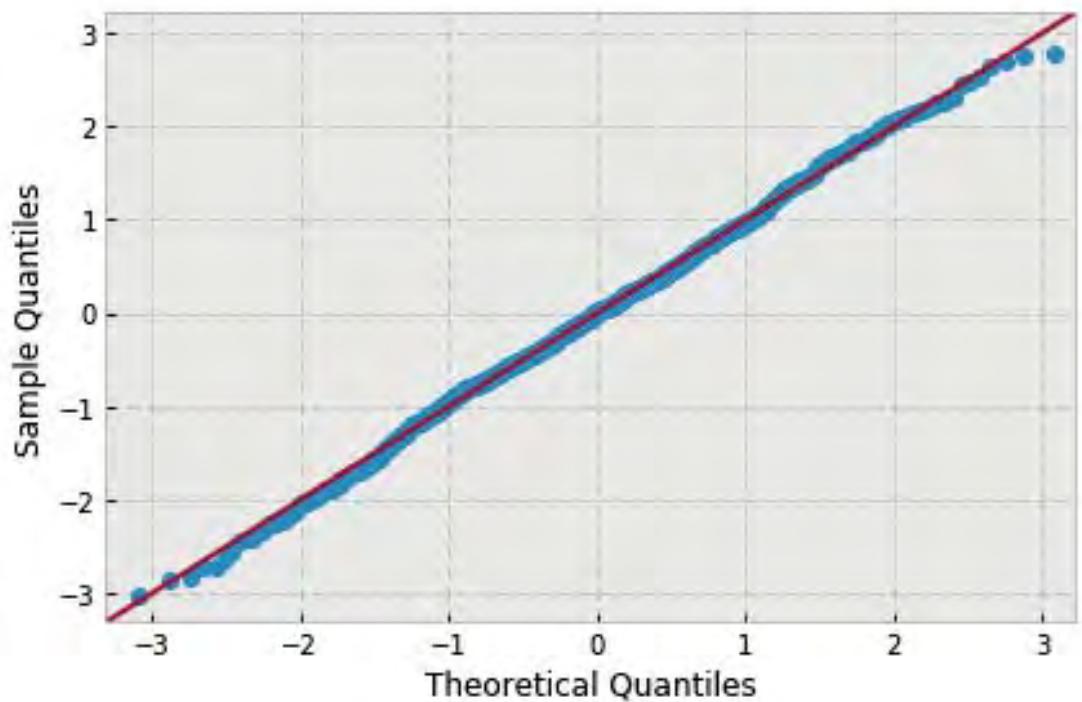
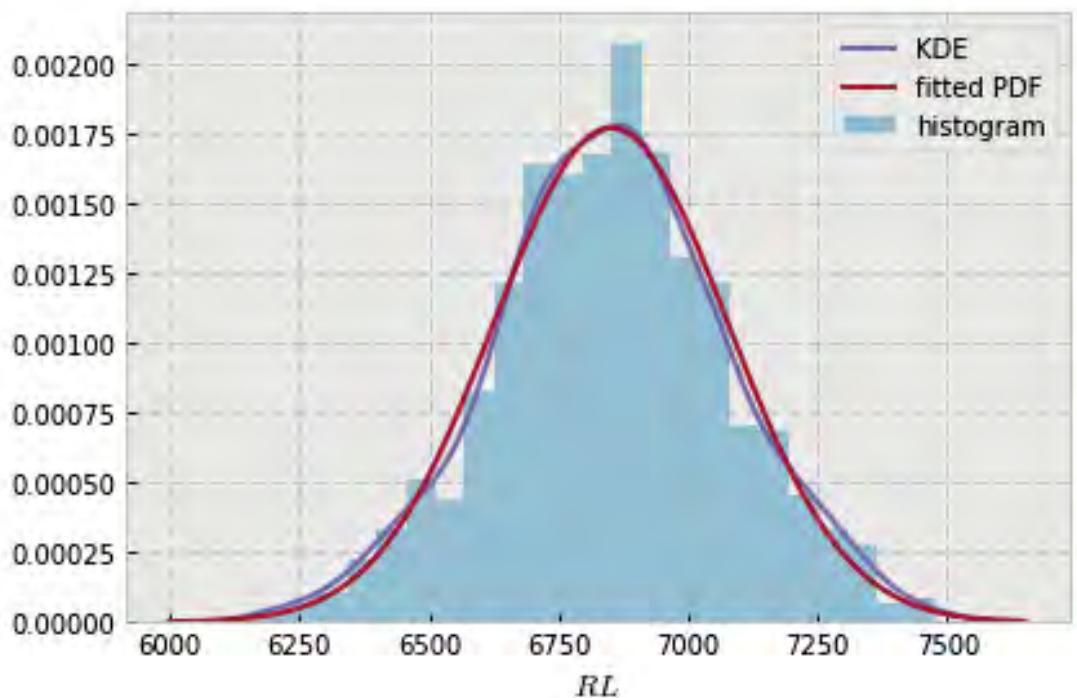


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=8900.776052858922, scale=282.6044073466456)
DescribeResult(nobs=1000, minmax=(7955.1925778830719, 9820.9575209055874)
               mean=8900.7760528589224, variance=79945.196247996777,
               skewness=-0.11533371946890708, kurtosis=-0.2035079698058544)
```

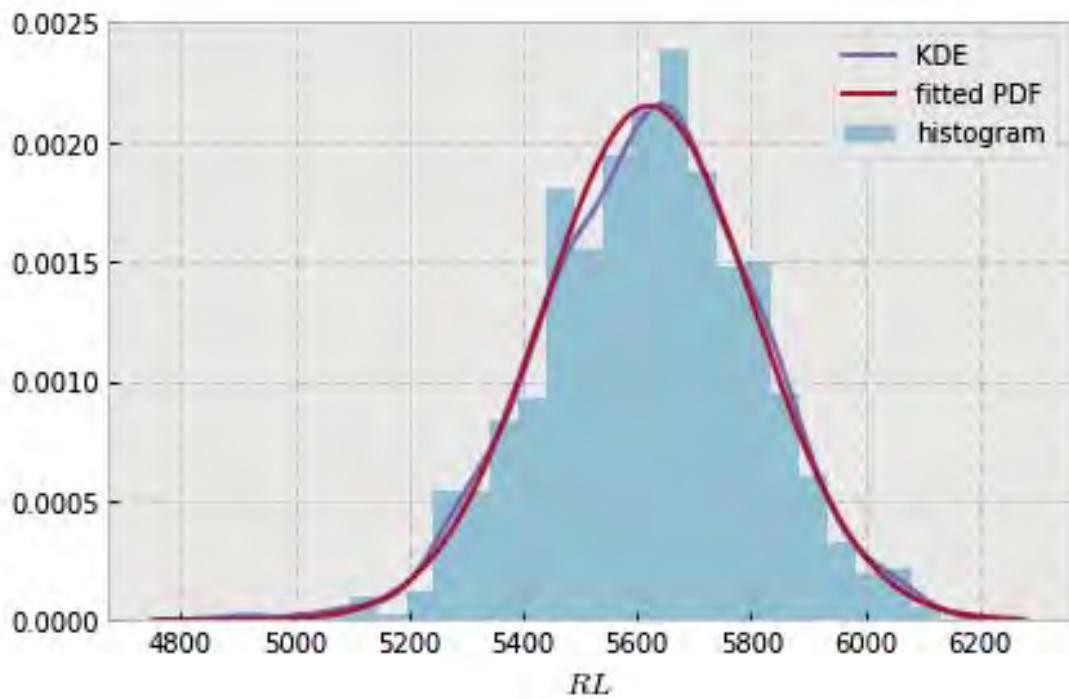


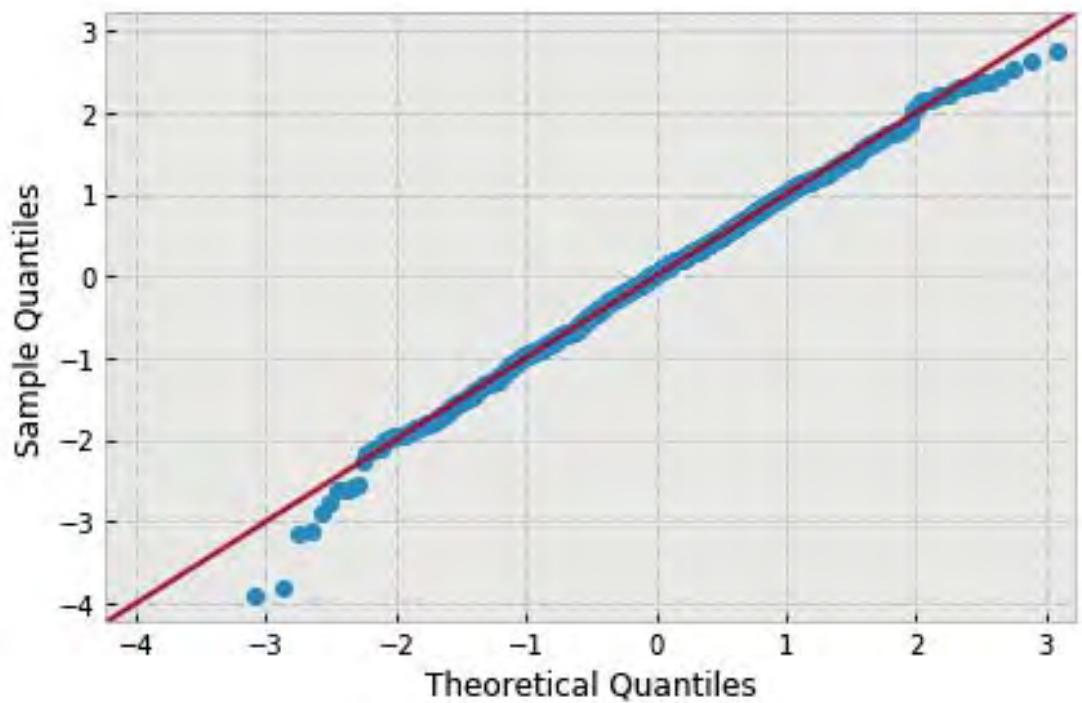


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=6850.932396277722, scale=225.2918586774509)
DescribeResult(nobs=1000, minmax=(6171.1624871924187, 7477.4676692632702)
               mean=6850.9323962777216, variance=50807.228815155657,
               skewness=-0.020611569816929326, kurtosis=-0.014867059102519065)
```

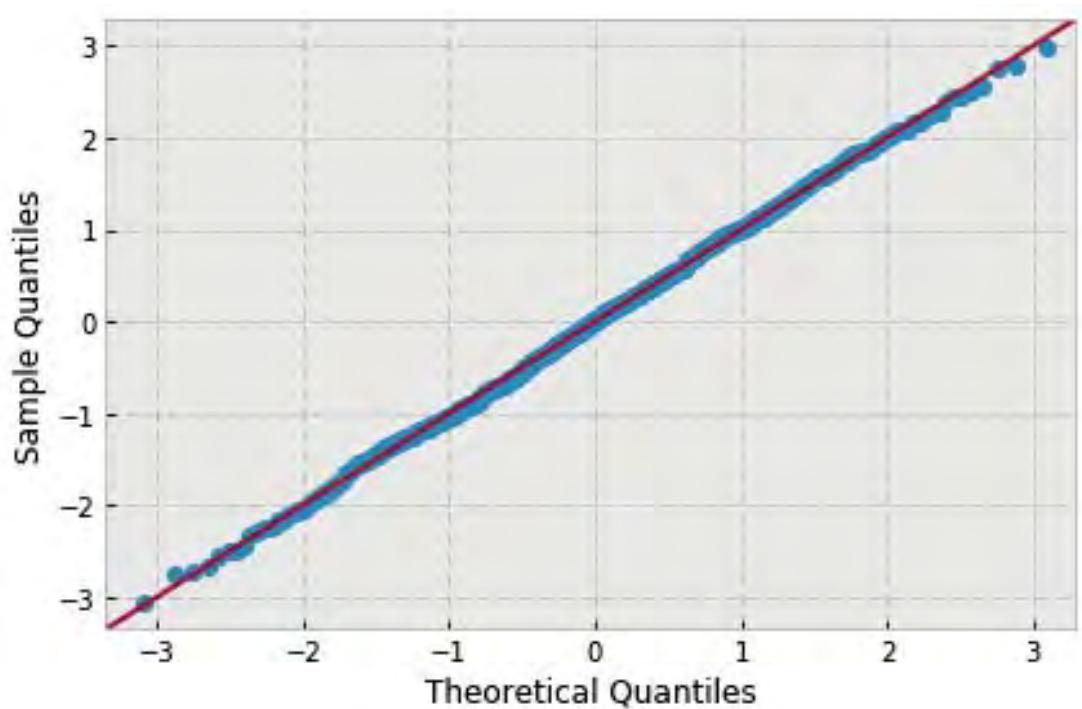
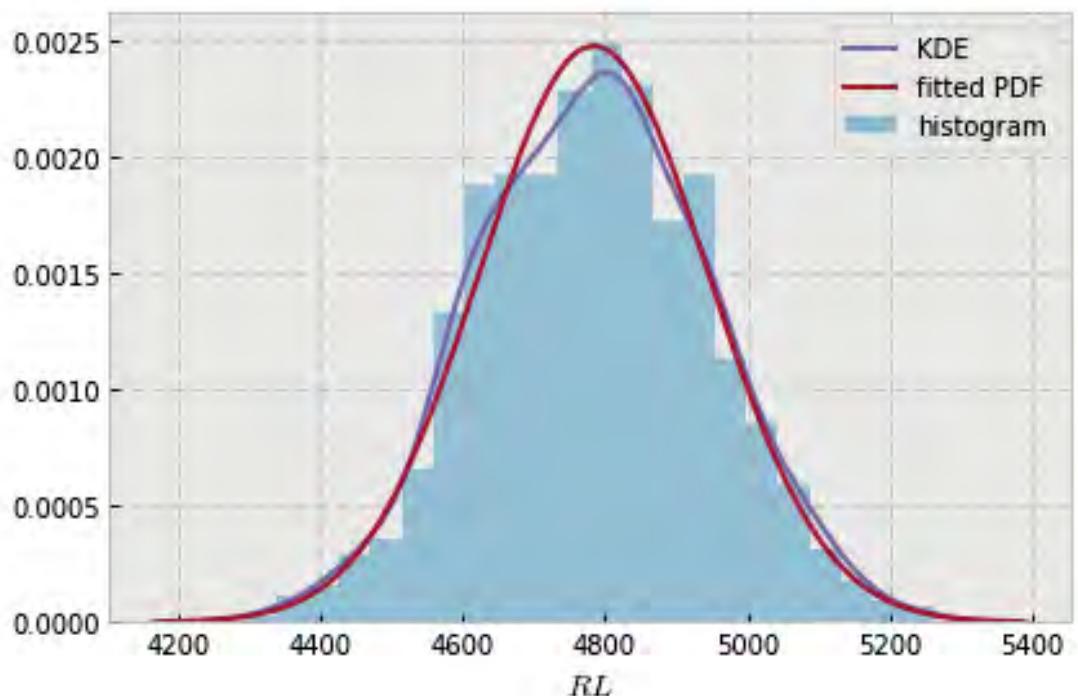


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=5620.805627685591, scale=185.50625961667004)
DescribeResult(nobs=1000, minmax=(4897.7482895044286, 6133.0176440698897)
               mean=5620.8056276855914, variance=34447.019376343735,
               skewness=-0.17651174512214668, kurtosis=0.17163382480163758)
```

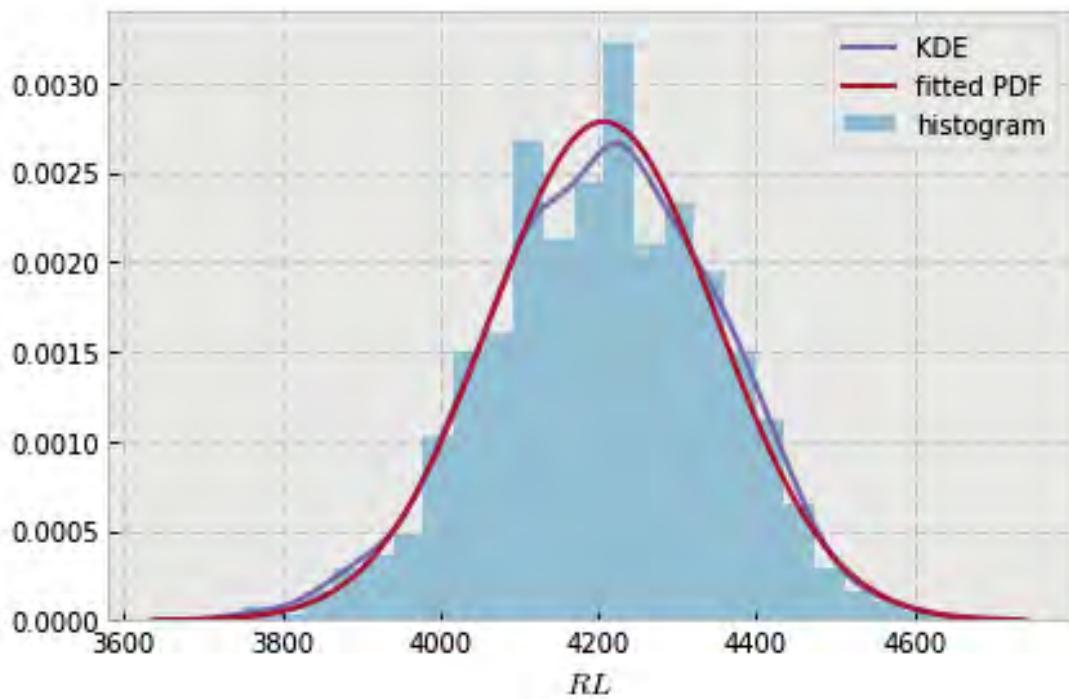


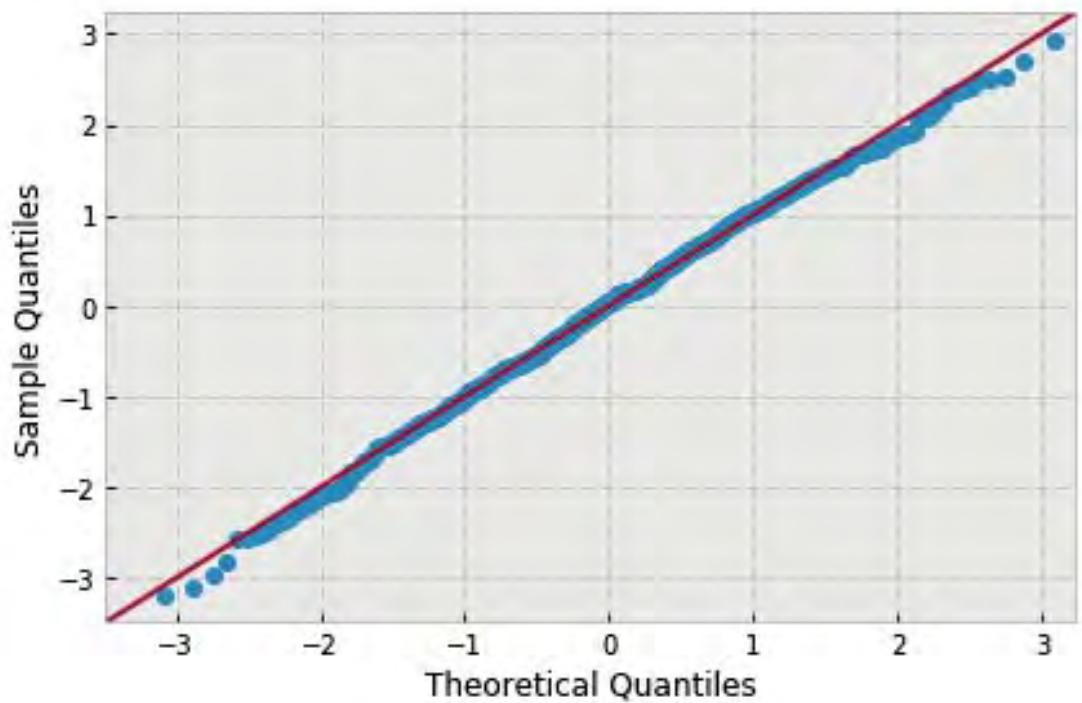


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=4784.389372724816, scale=160.7516435428016)
DescribeResult(nobs=1000, minmax=(4294.5114110423237, 5263.1649763019759)
               mean=4784.389372724816, variance=25866.957859571514,
               skewness=0.012379054961225256, kurtosis=-0.2097827580149758)
```

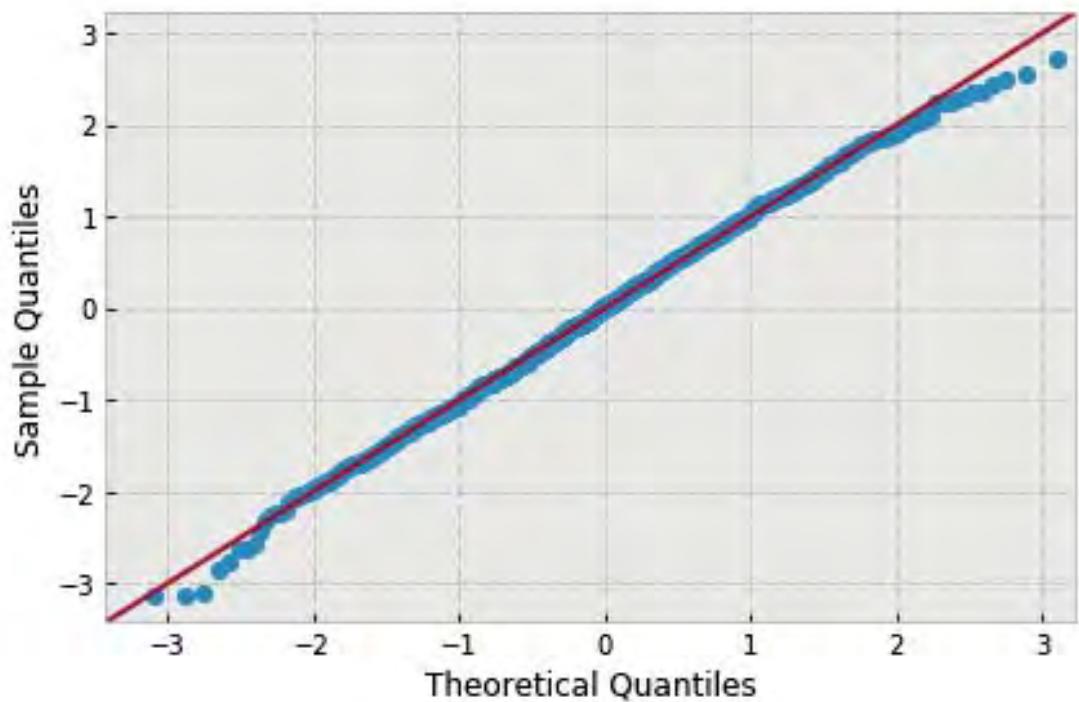
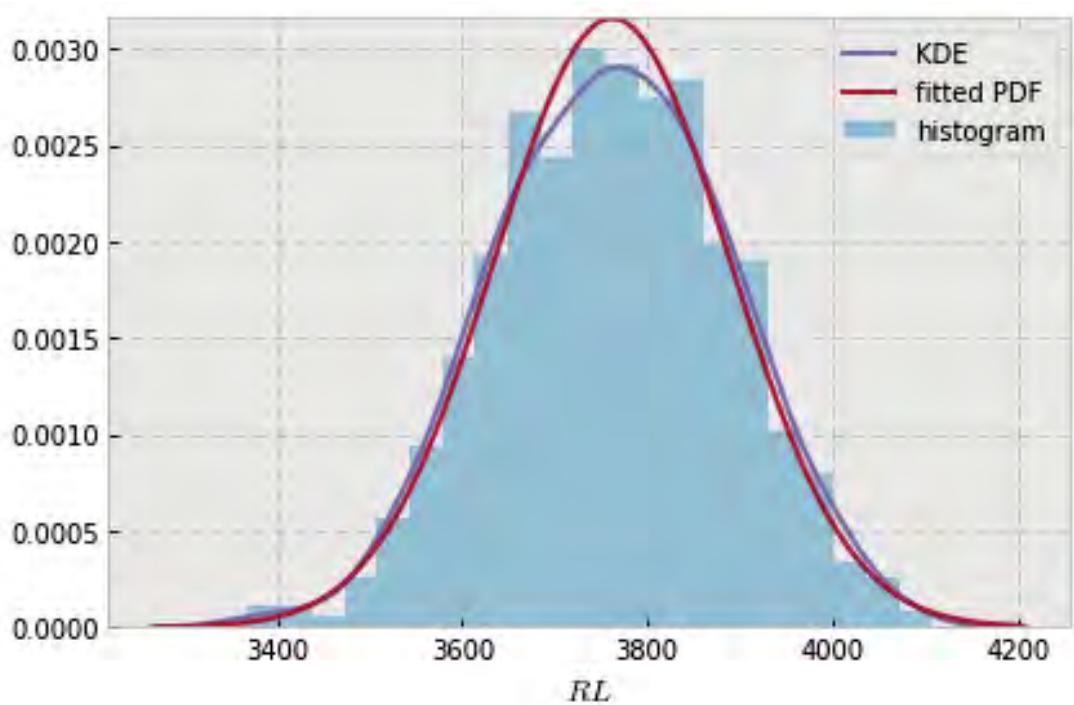


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=4206.746806533822, scale=143.3159677812821)
DescribeResult(nobs=1000, minmax=(3750.8179848574632, 4625.5541288561899)
               mean=4206.7468065338217, variance=20560.026647733222,
               skewness=-0.13398904242060464, kurtosis=-0.14836545976938575)
```

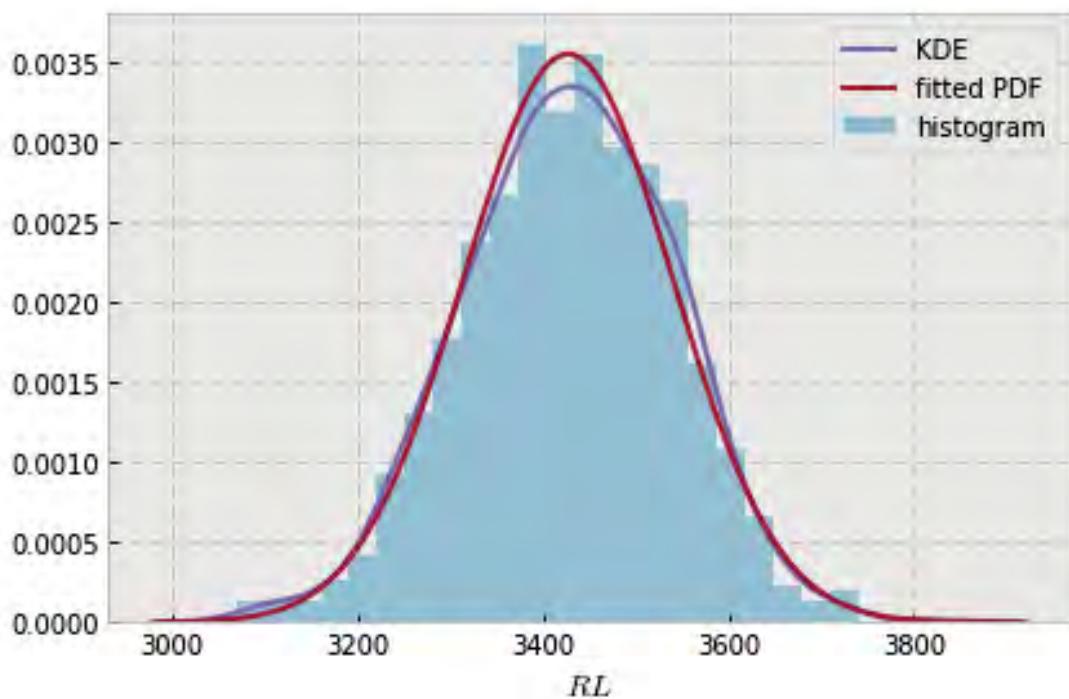


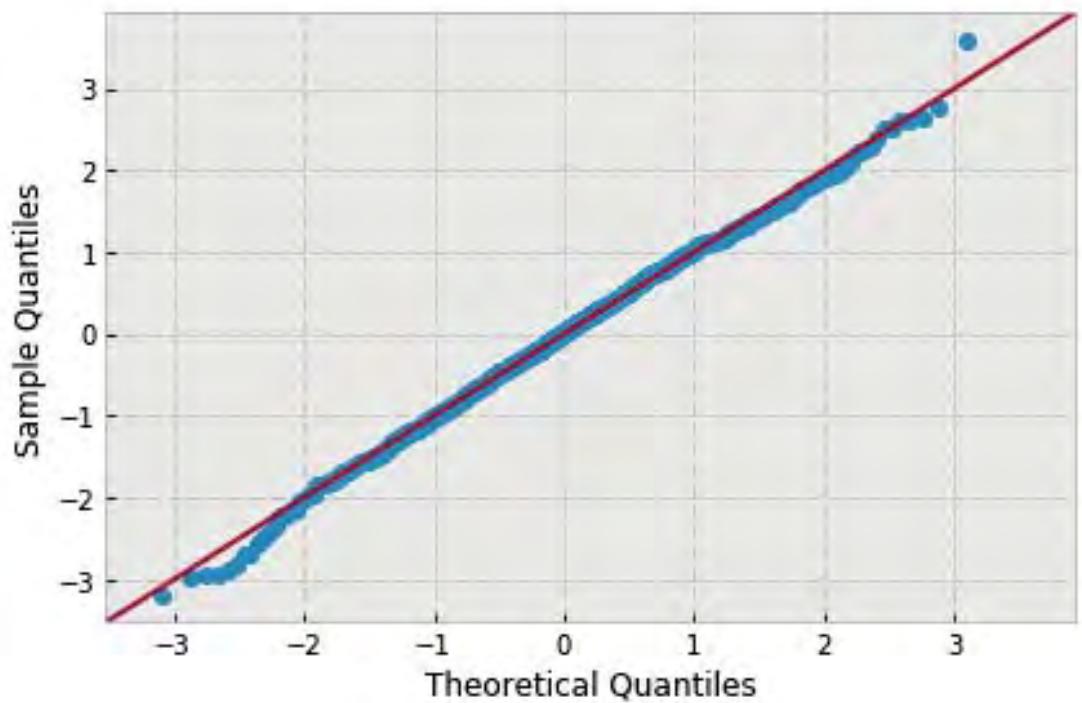


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=3761.9597386000996, scale=126.35956674860543)
DescribeResult(nobs=1000, minmax=(3366.8678093598078, 4106.9079484135336)
               mean=3761.9597386000996, variance=15982.722831726998,
               skewness=-0.0827710348967881, kurtosis=-0.22525111461016722)
```

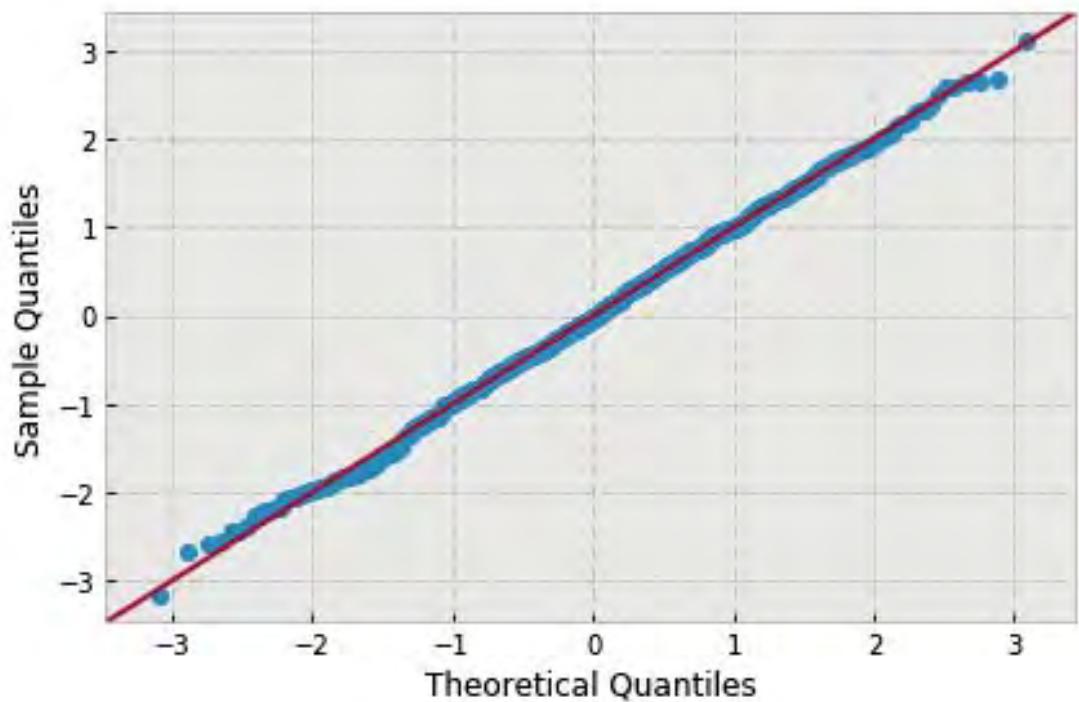
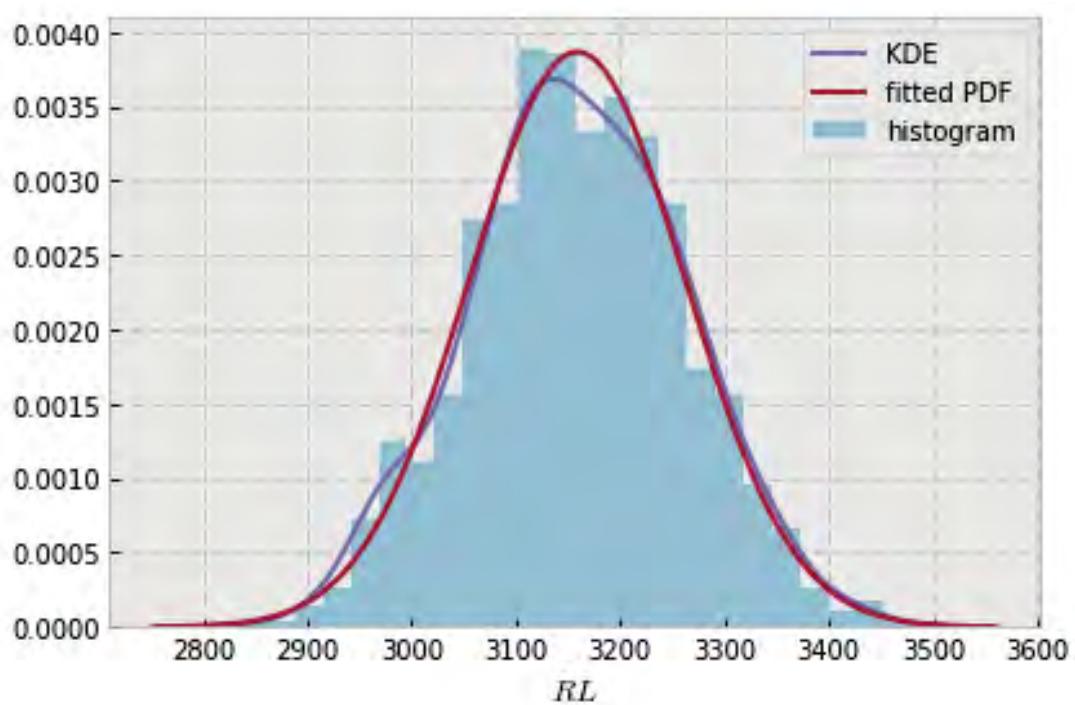


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=3427.360485073408, scale=112.52590688544345)
DescribeResult(nobs=1000, minmax=(3069.3605578012694, 3830.2740888522012)
               mean=3427.3604850734082, variance=12674.754474866357,
               skewness=-0.11342266240425428, kurtosis=-0.00517966282550697)
```

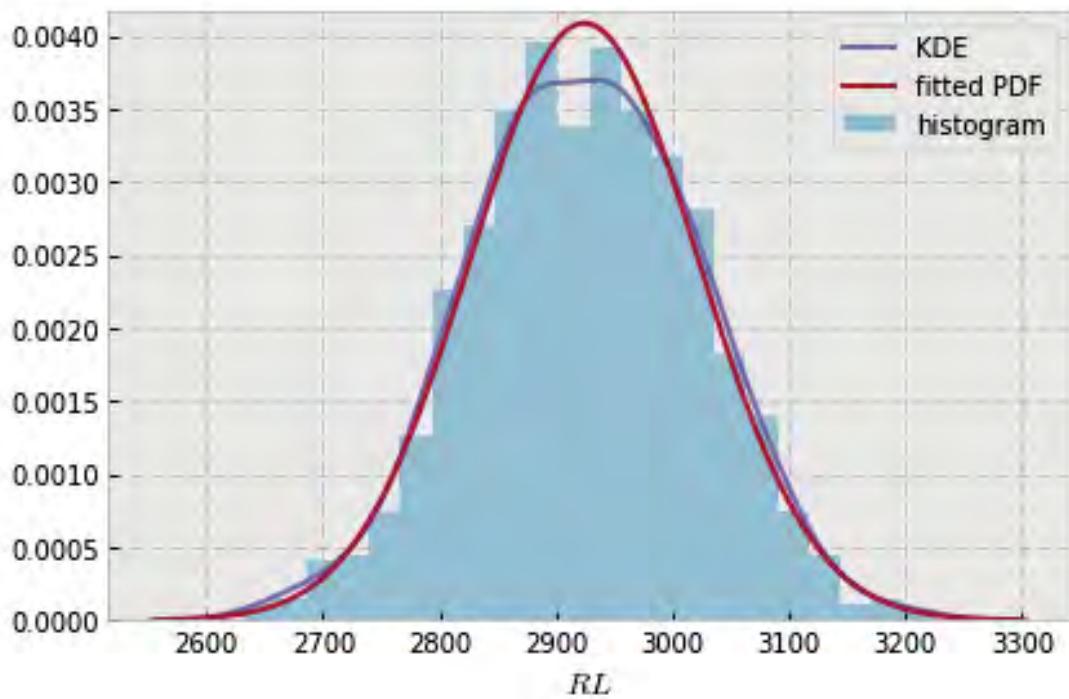


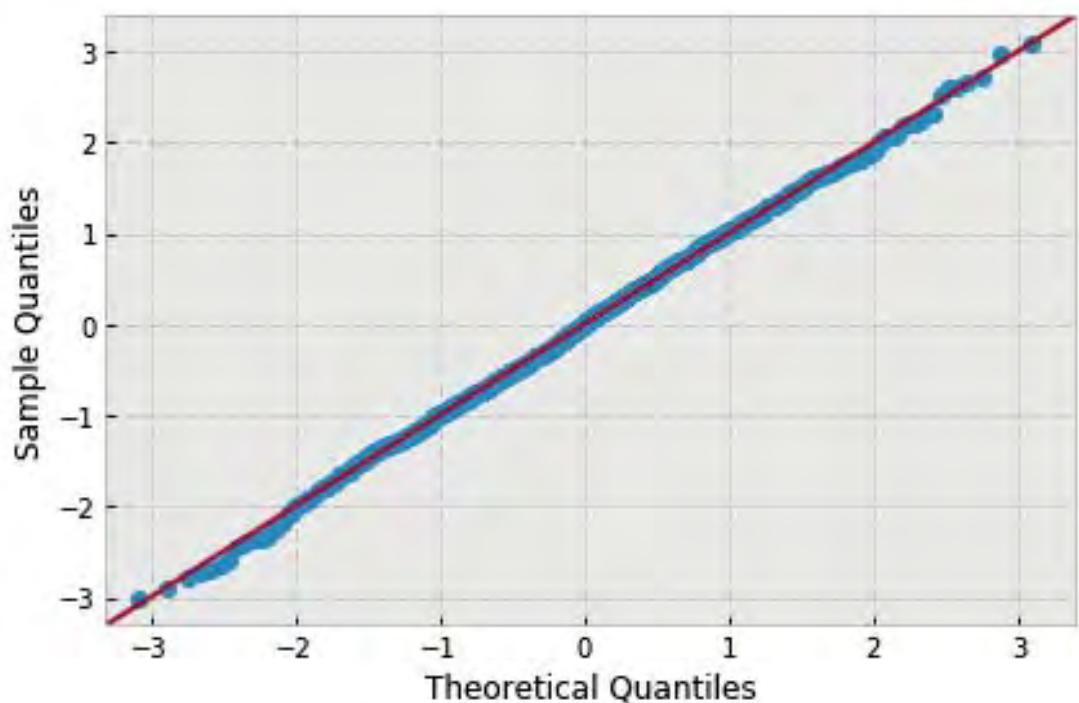


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=3158.666475600519, scale=103.15864875983623)
DescribeResult(nobs=1000, minmax=(2833.4478495650496, 3479.6269142749657)
               mean=3158.6664756005189, variance=10652.359173128387,
               skewness=-0.010504683125537847, kurtosis=-0.20147794697938215)
```

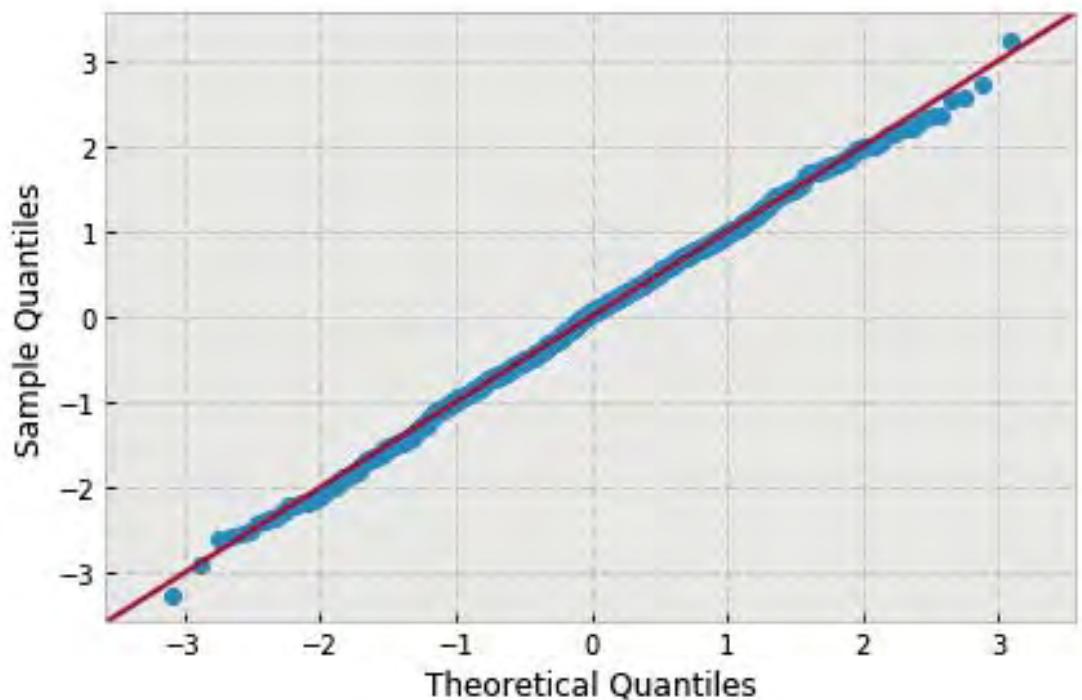
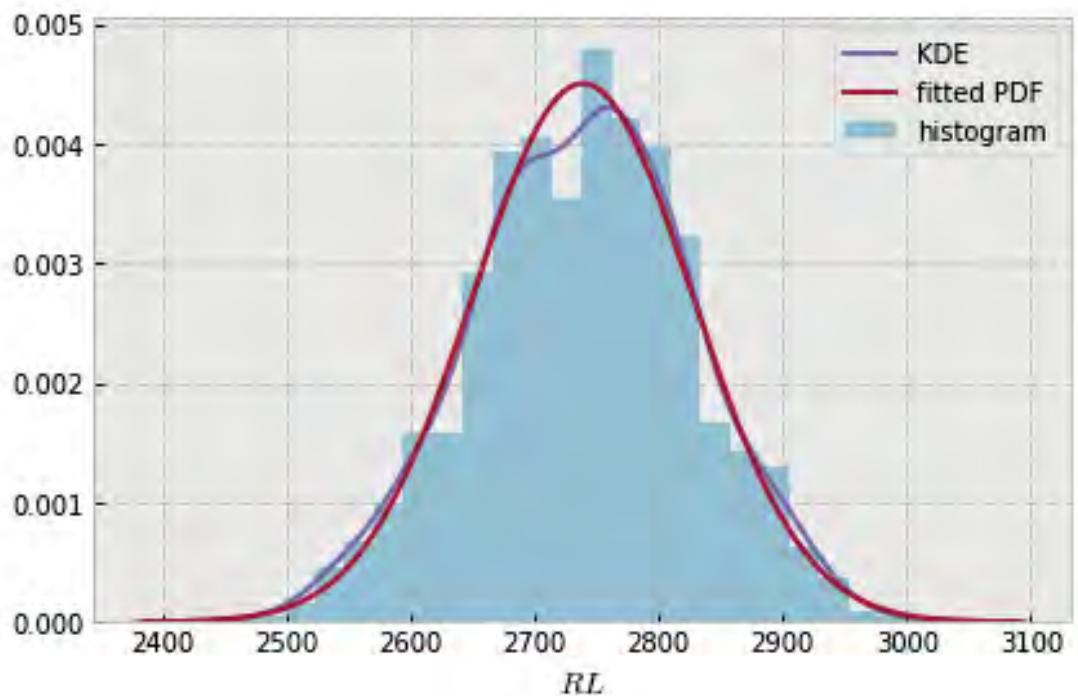


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=2924.502294457333, scale=97.59089803025766)
DescribeResult(nobs=1000, minmax=(2631.5967404984822, 3225.2199607539574)
               mean=2924.5022944573329, variance=9533.5168952473978,
               skewness=-0.02870972391914144, kurtosis=-0.1379215911565077)
```

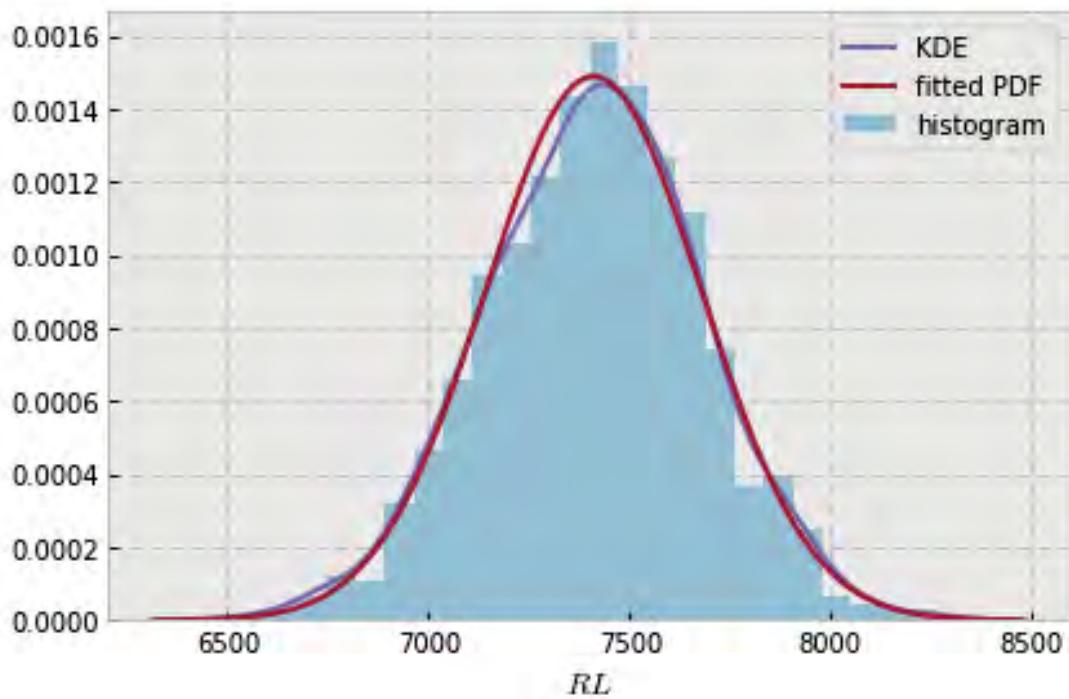


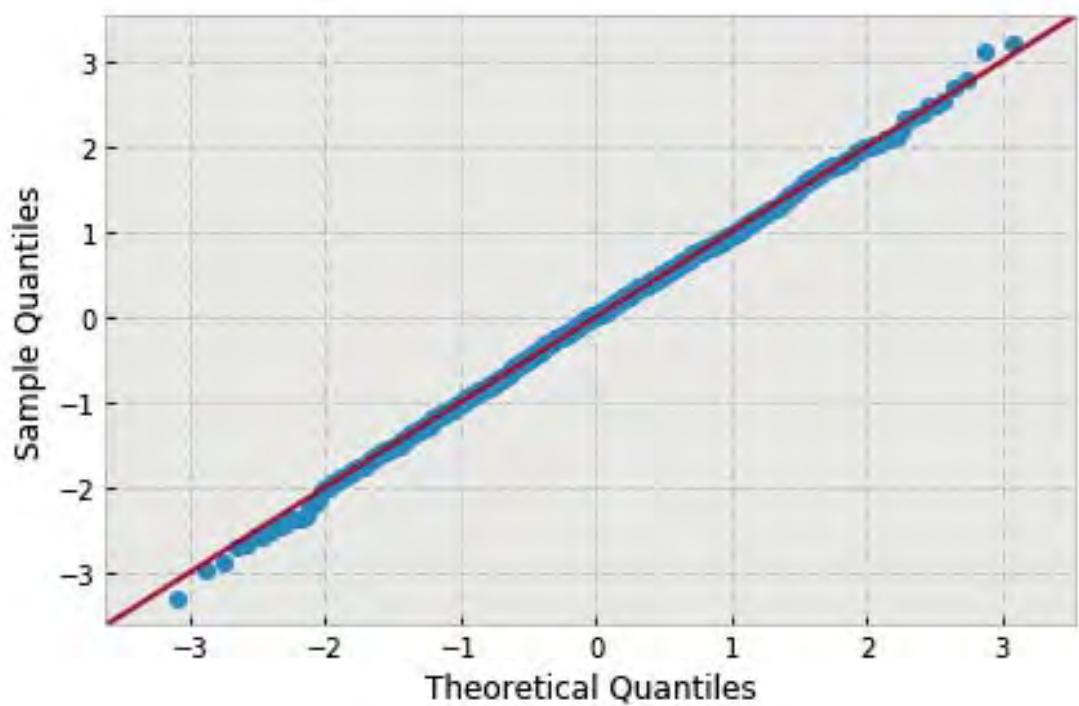


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=2738.7636936870203, scale=88.63310823777364)
DescribeResult(nobs=1000, minmax=(2450.7794475778883, 3025.0895344693217)
               mean=2738.7636936870203, variance=7863.6915674563534,
               skewness=-0.0672496364558661, kurtosis=-0.14962943412561058)
```

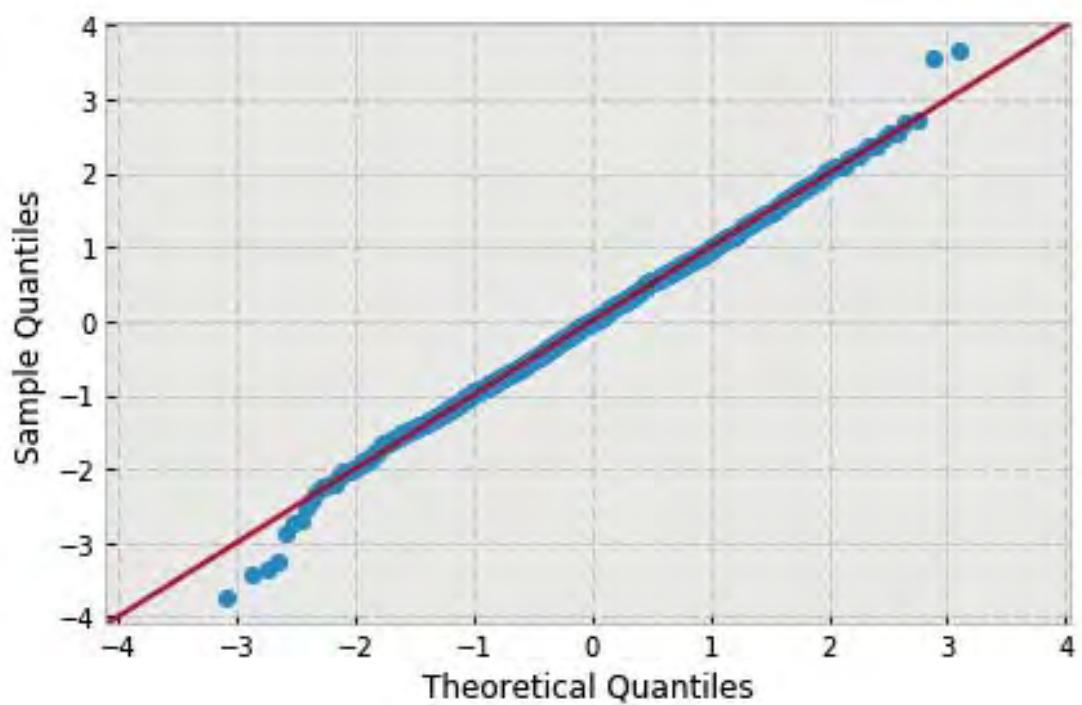
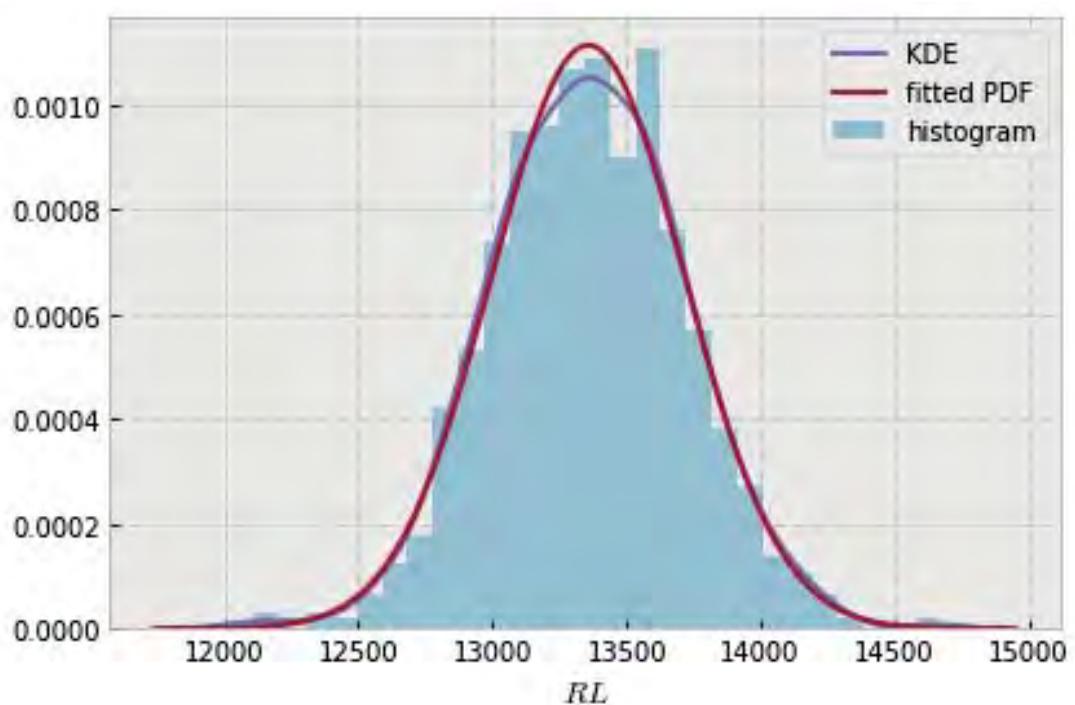


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=7409.690513553469, scale=267.9966521765638)
DescribeResult(nobs=1000, minmax=(6527.7502679954532, 8269.0018235244679)
               mean=7409.6905135534689, variance=71894.099677523656,
               skewness=-0.06277769497652826, kurtosis=0.02547766388910766)
```

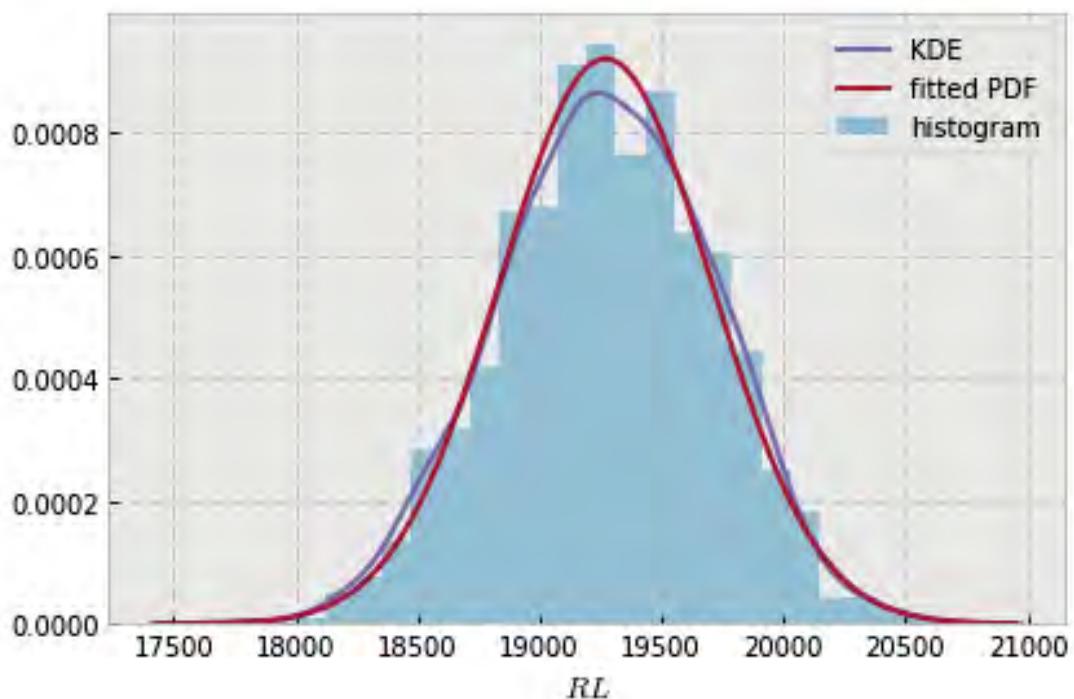


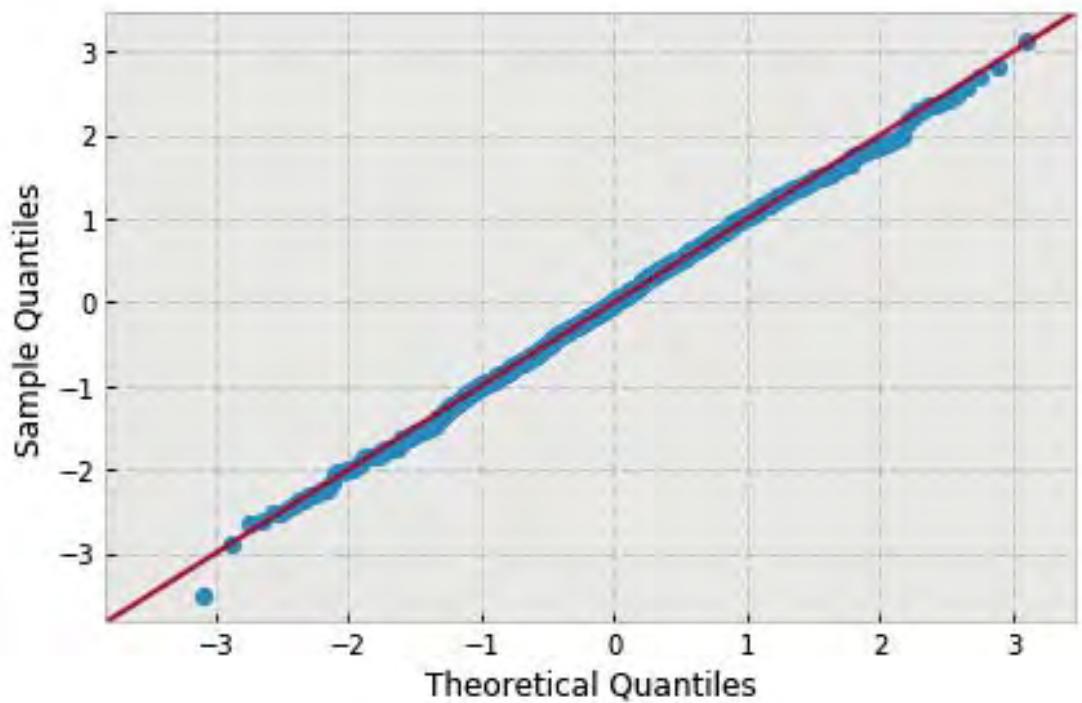


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=13356.337228294986, scale=358.13075222657824)
DescribeResult(nobs=1000, minmax=(12019.876944371916, 14668.869846186924)
               mean=13356.337228294986, variance=128386.02171208685,
               skewness=-0.022808967433929173, kurtosis=0.3117518132110906)
```

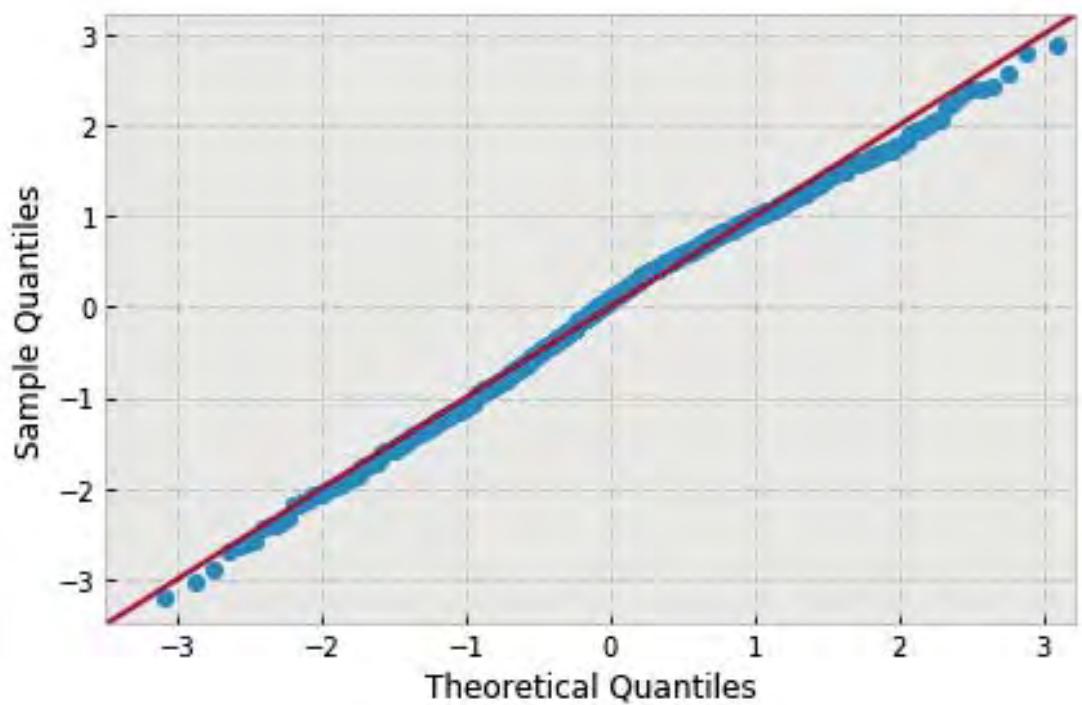
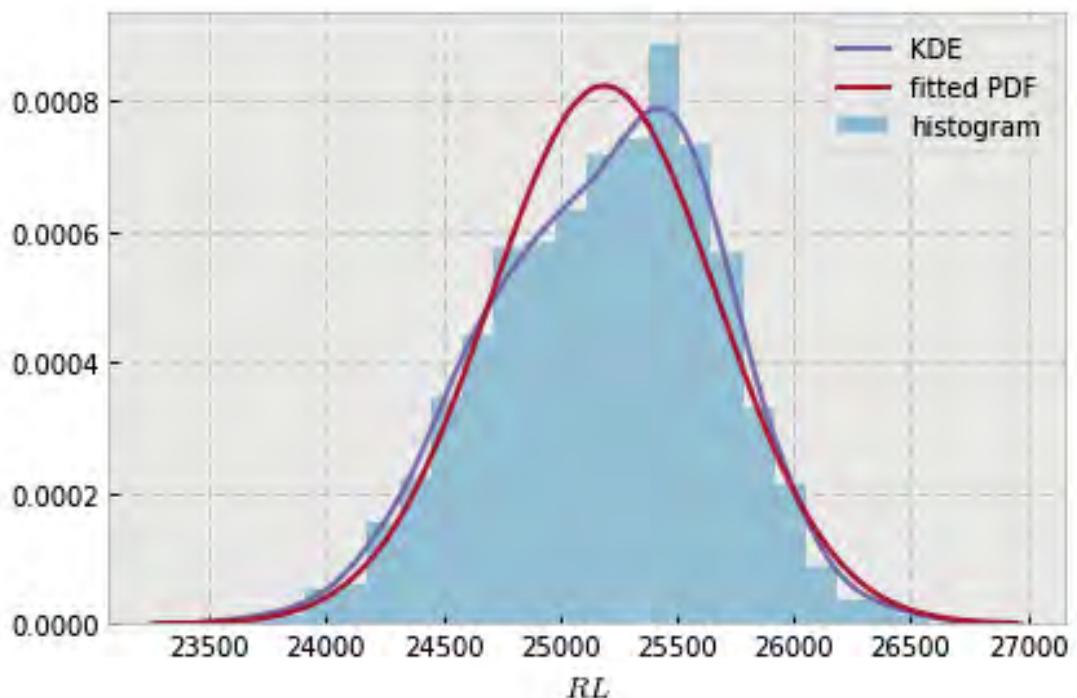


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=19270.670531146, scale=433.5323001902279)
DescribeResult(nobs=1000, minmax=(17757.663620377603, 20626.321432418146)
               mean=19270.670531145999, variance=188138.39370193181,
               skewness=-0.08725808785972644, kurtosis=-0.17795475201211497)
```

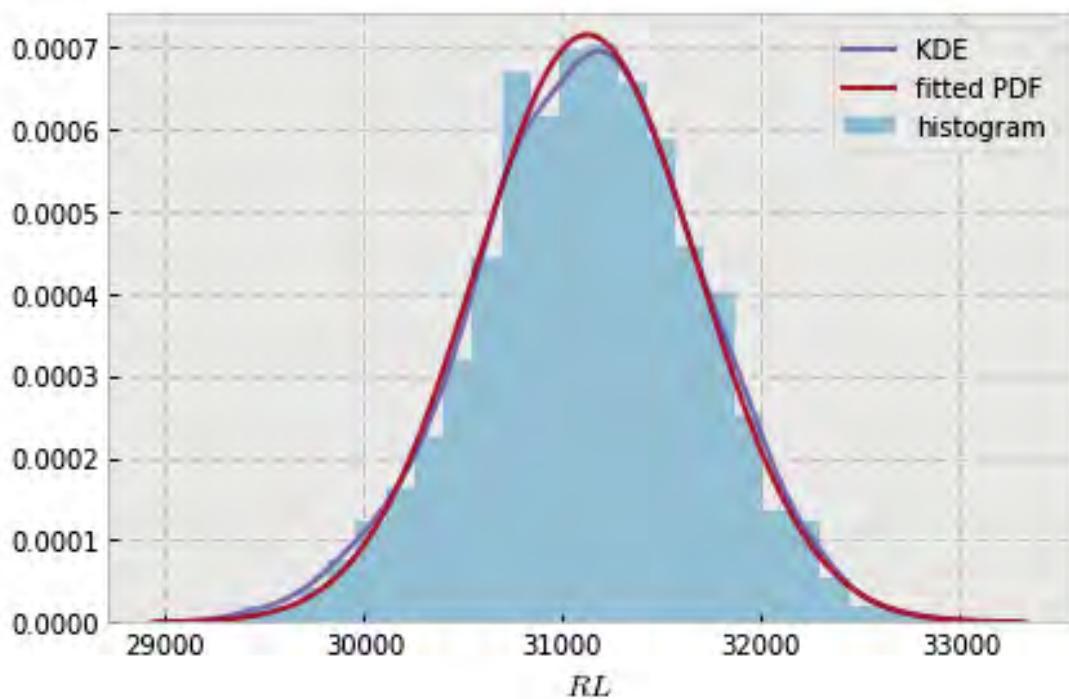


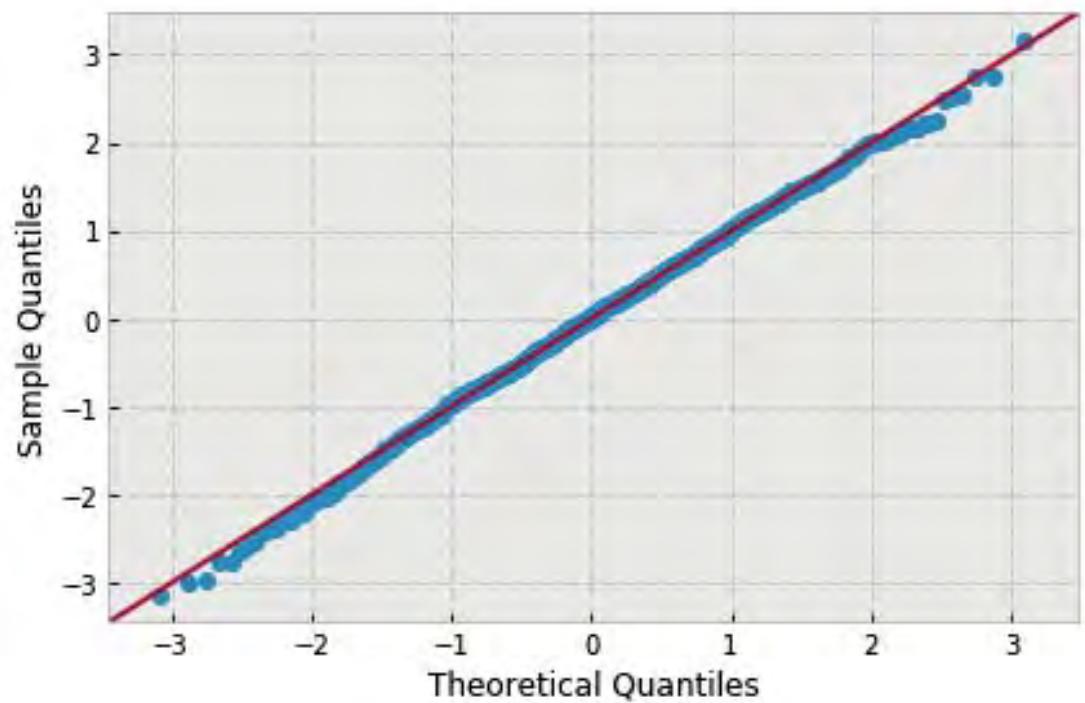


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=25189.69380010707, scale=484.8663667831329)
DescribeResult(nobs=1000, minmax=(23642.123697203271, 26585.403340460733)
               mean=25189.69380010707, variance=235330.72436183738,
               skewness=-0.21723937374568827, kurtosis=-0.2751994383044343)
```

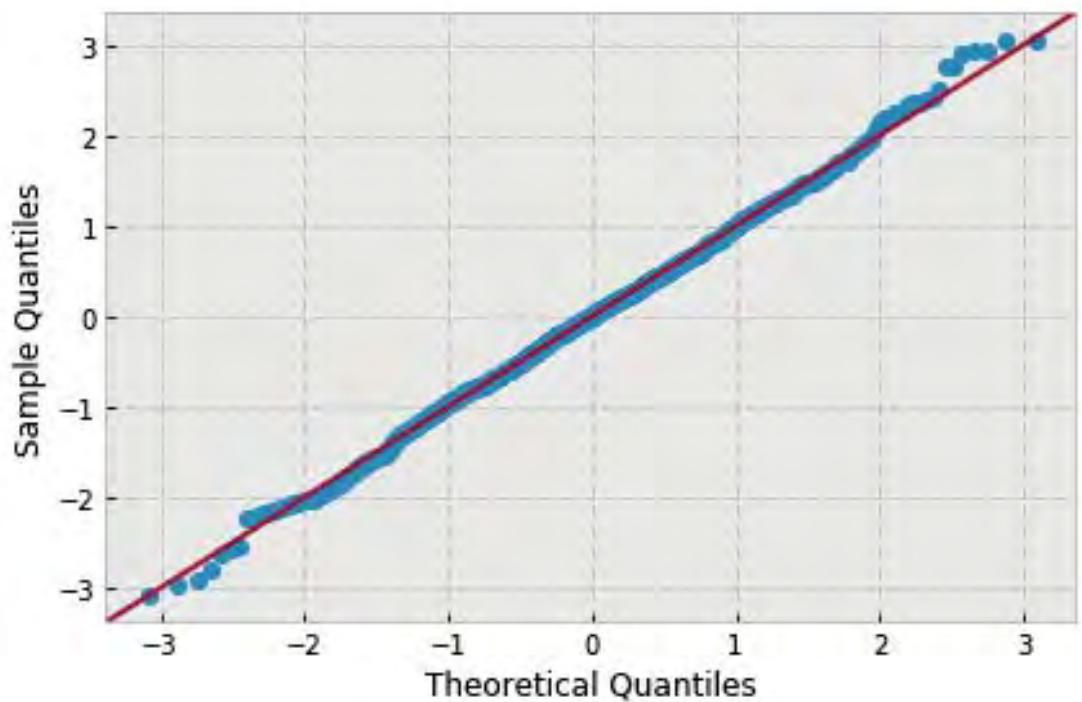
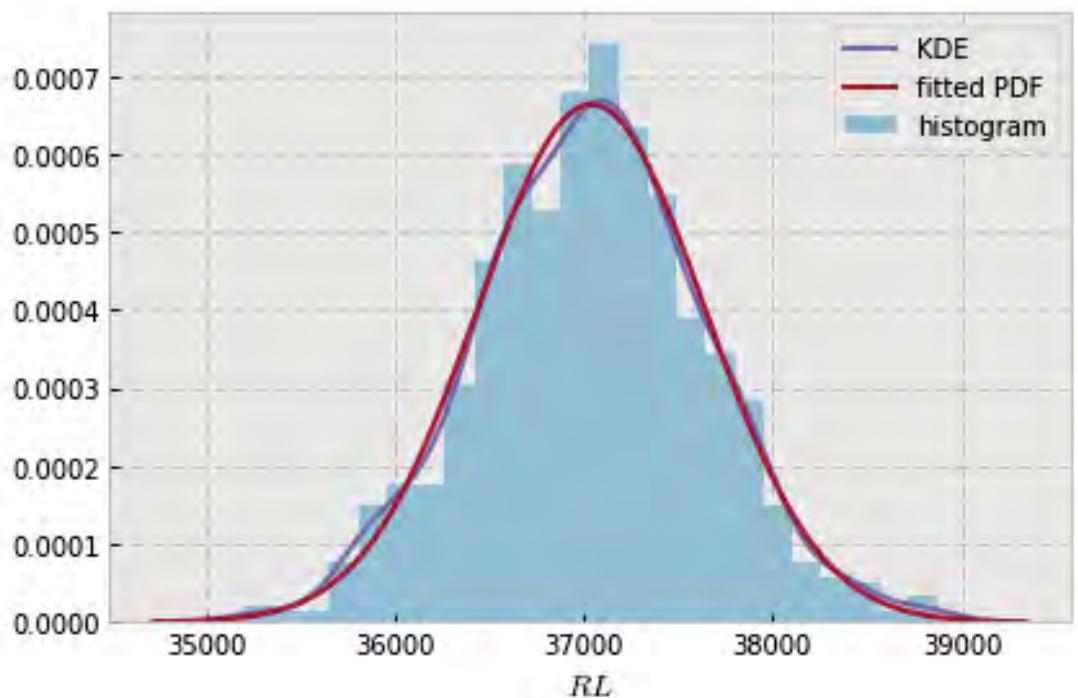


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=31122.74460758081, scale=557.9087603374098)
DescribeResult(nobs=1000, minmax=(29376.923701765532, 32883.471664190751)
               mean=31122.744607580811, variance=311573.7586198452,
               skewness=-0.10826706326997004, kurtosis=-0.03600778490692402)
```

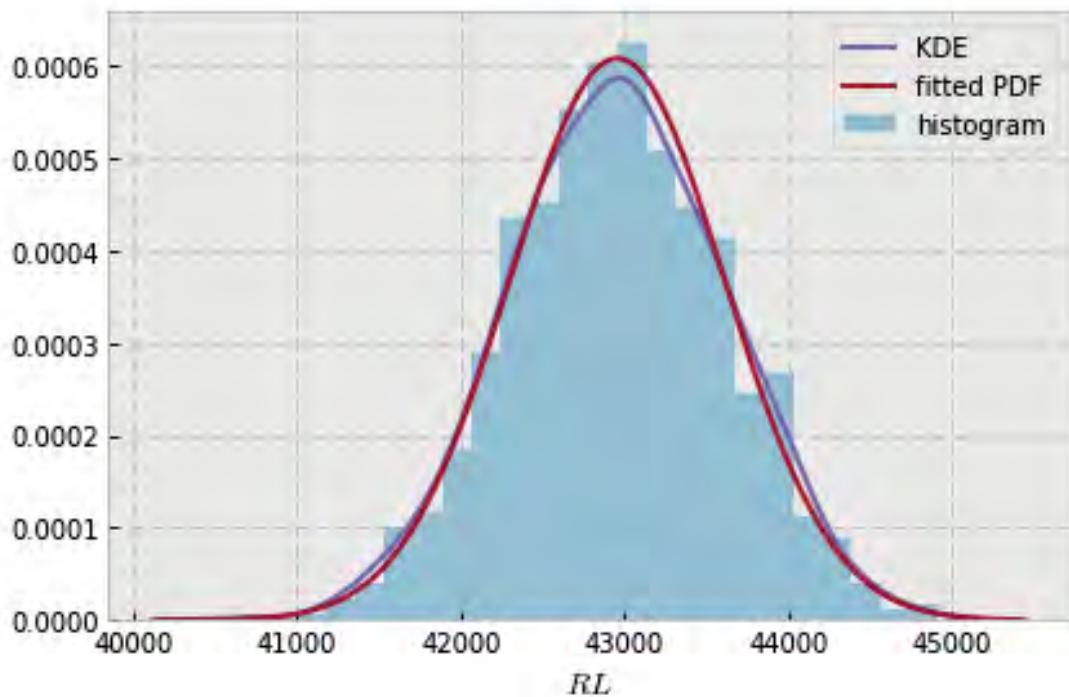


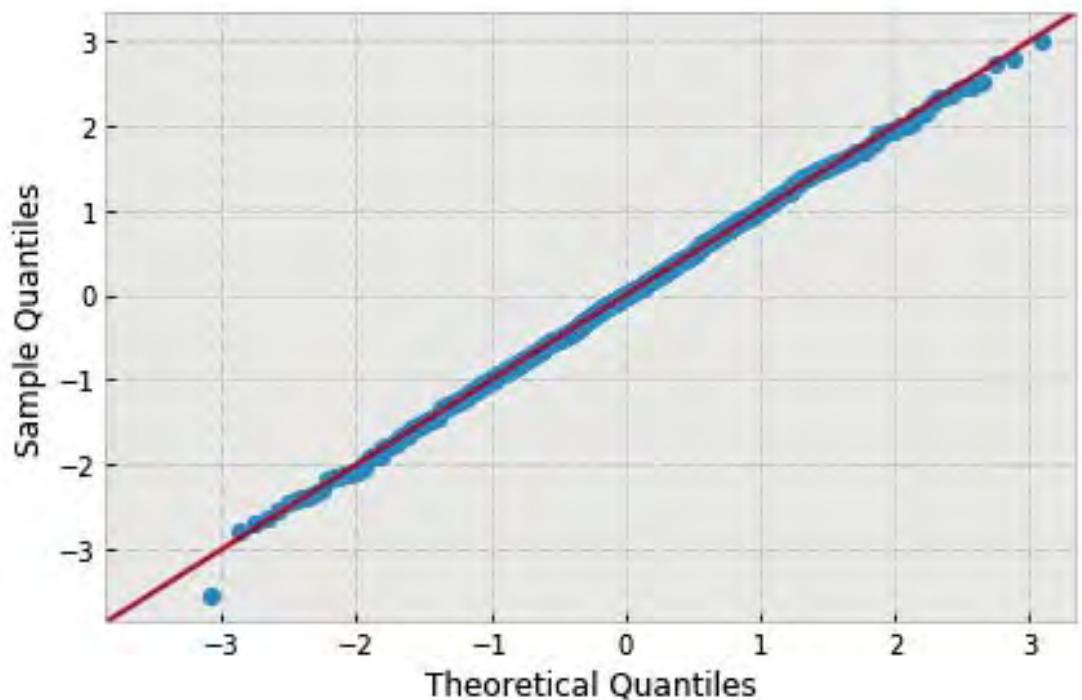


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=37041.66625173225, scale=600.8707504969034)
DescribeResult(nobs=1000, minmax=(35194.048422962645, 38872.299519513501)
               mean=37041.666251732247, variance=361407.06586858042,
               skewness=0.024907311890289788, kurtosis=0.08505813625470404)
```

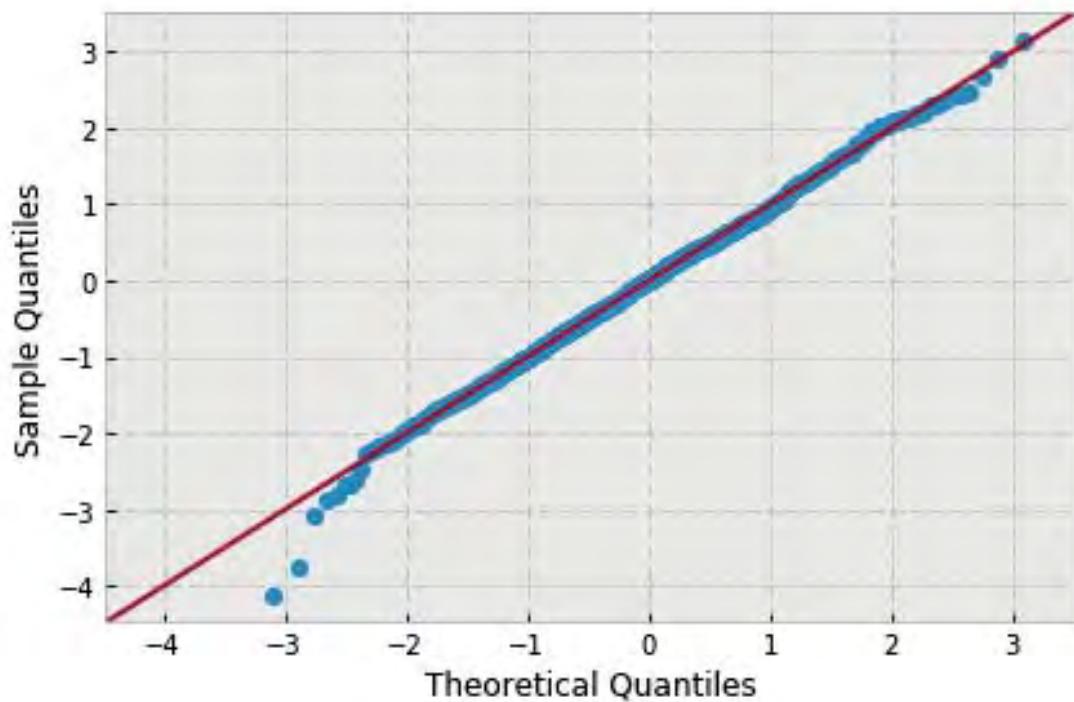
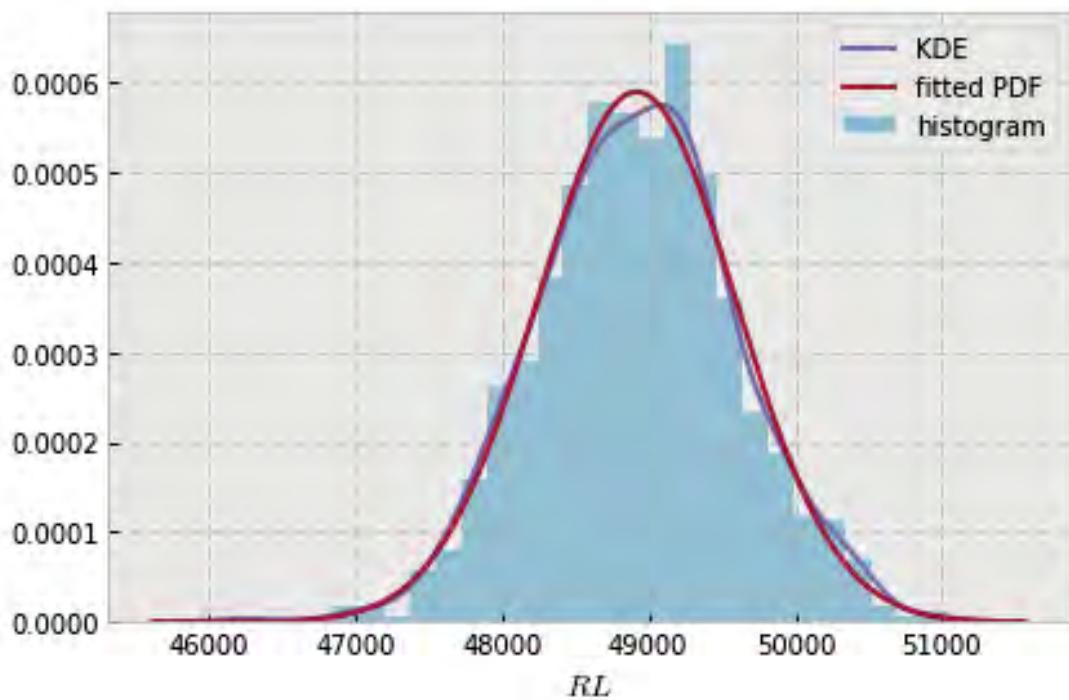


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=42955.26548529134, scale=655.4102831588602)
DescribeResult(nobs=1000, minmax=(40639.661102610131, 44923.415747877159)
               mean=42955.26548529134, variance=429992.63190227957,
               skewness=-0.03161101473936712, kurtosis=-0.1409510396537983)
```

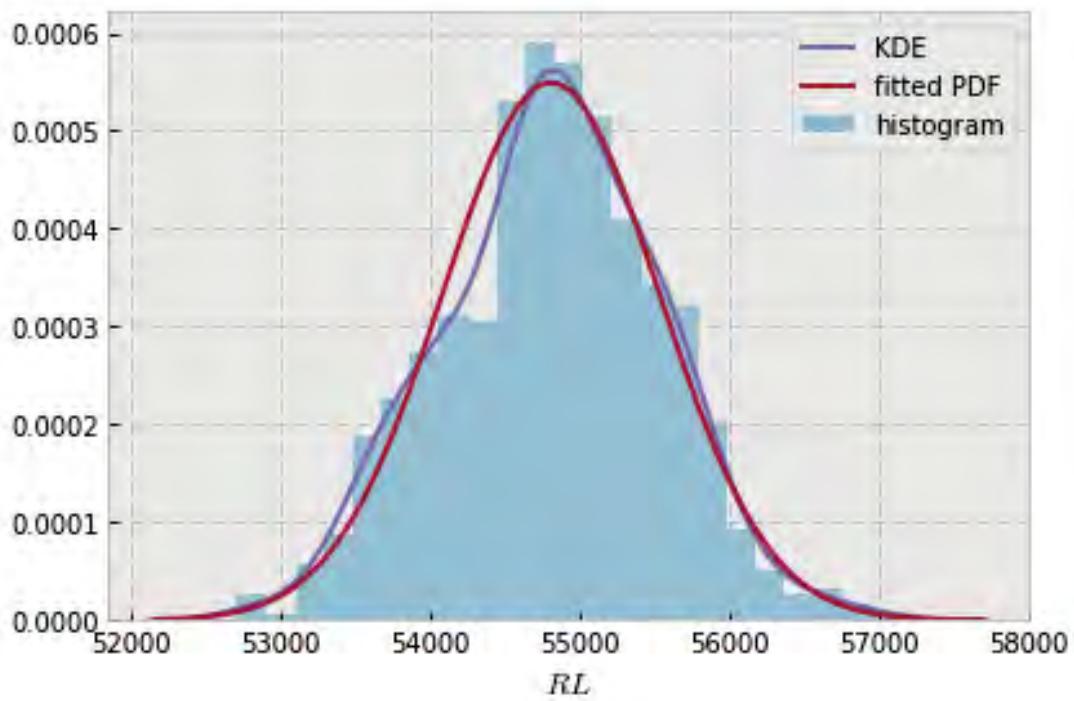


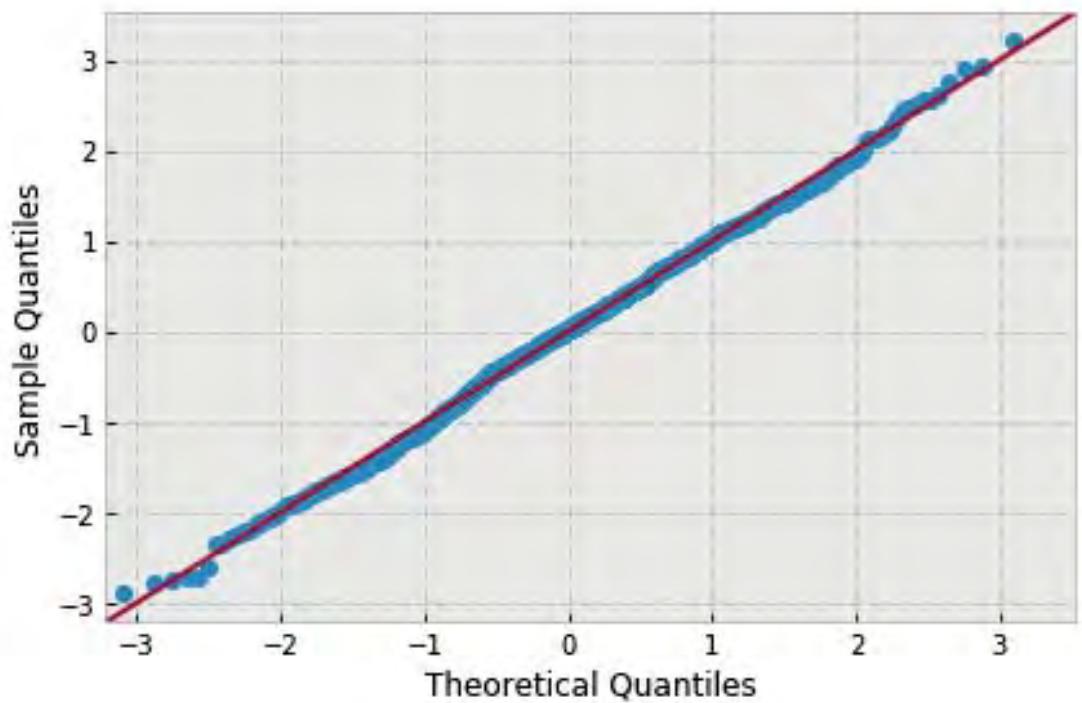


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=48919.24073746399, scale=676.4539293676997)
DescribeResult(nobs=1000, minmax=(46139.778203215894, 51045.402847824676)
               mean=48919.240737463988, variance=458047.96652352437,
               skewness=-0.0735254144546601, kurtosis=0.23913725706287492)
```

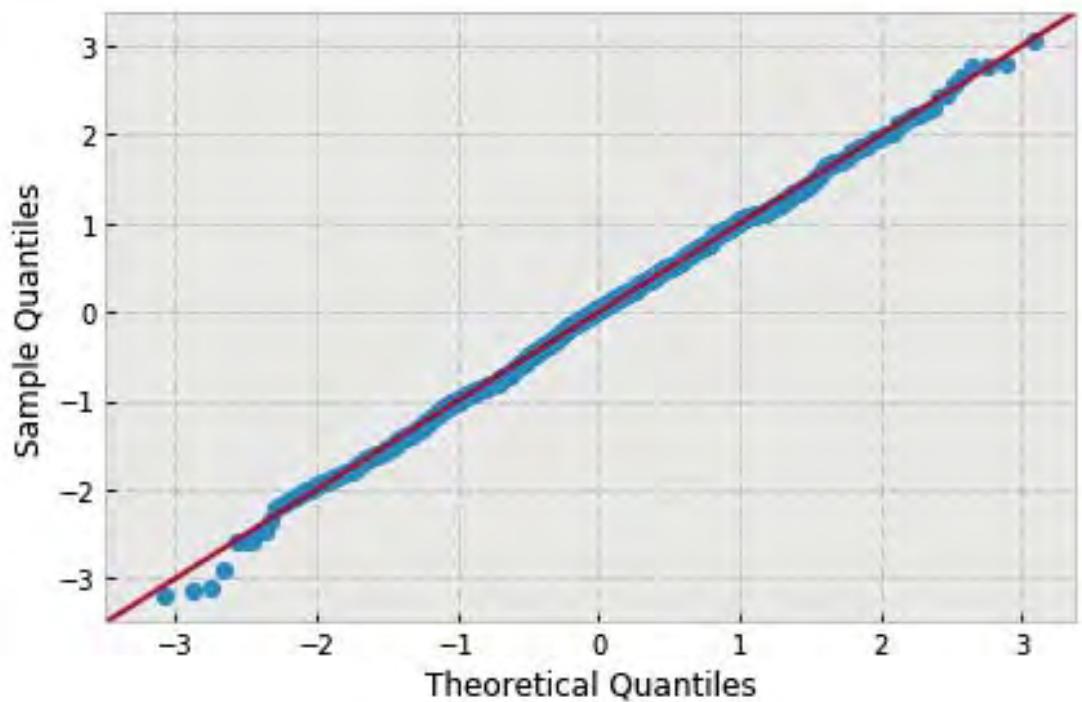
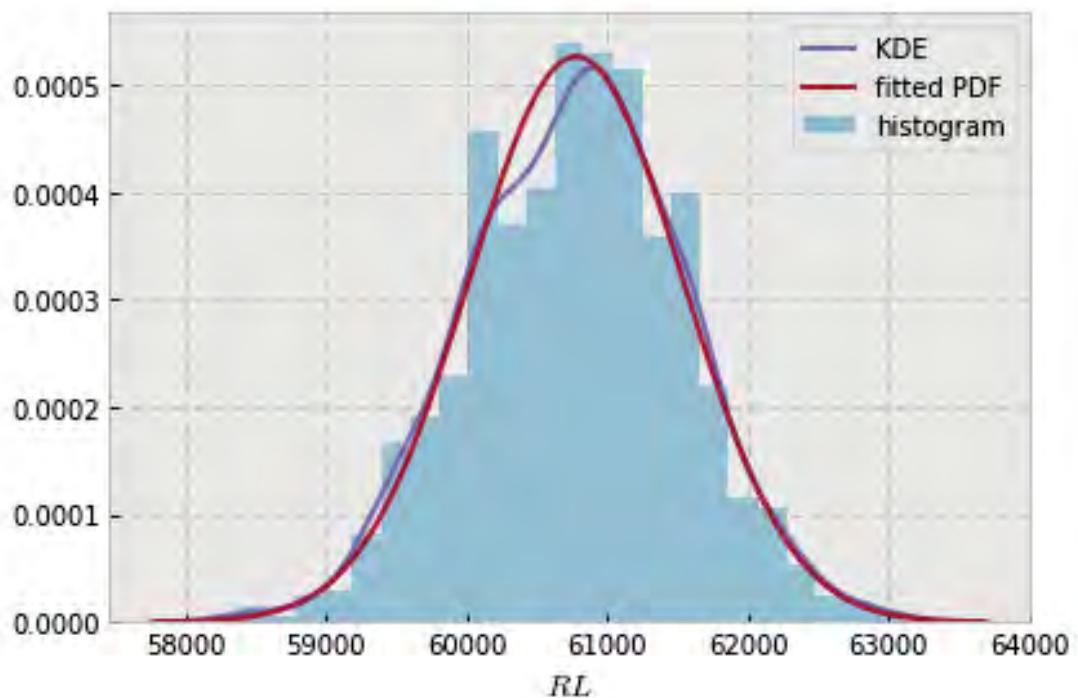


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=54807.84717718455, scale=726.3313436316798)
DescribeResult(nobs=1000, minmax=(52710.247649438039, 57140.913107282635)
               mean=54807.847177184551, variance=528085.30604784912,
               skewness=-0.05654143450283809, kurtosis=-0.1134884808851182)
```

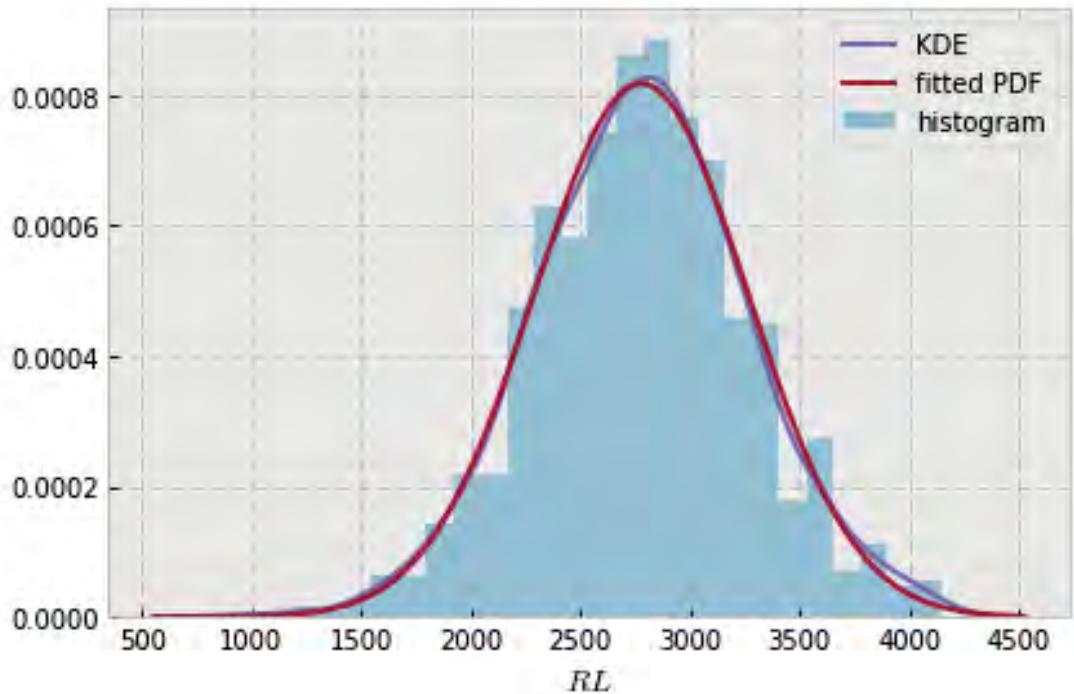


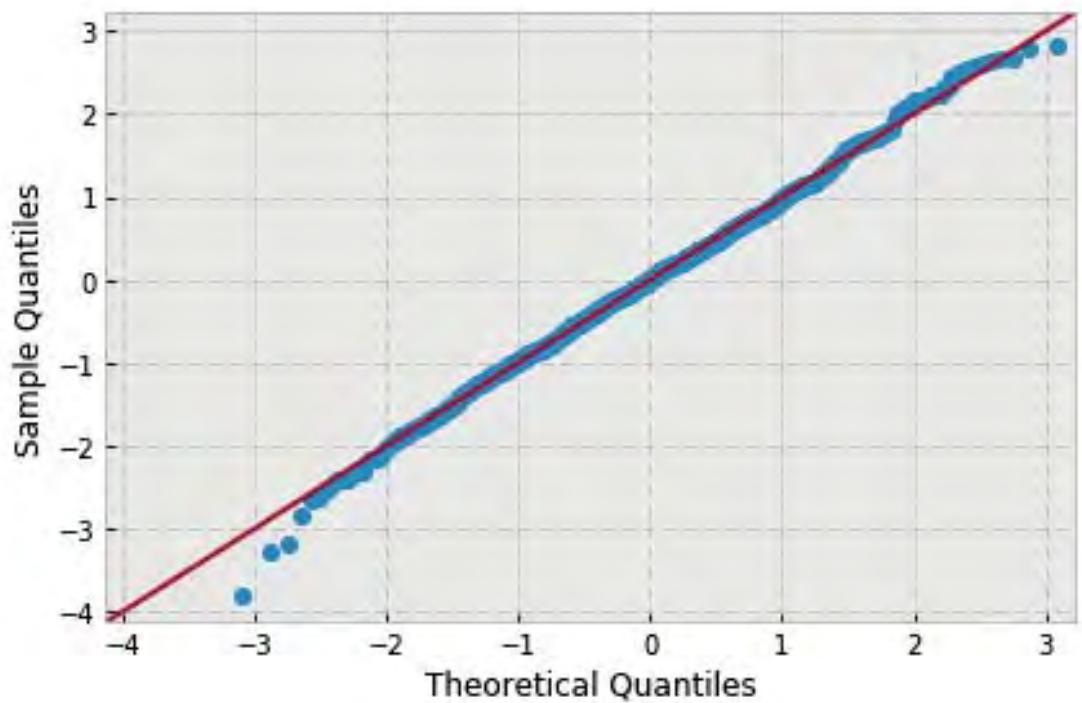


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=60778.814121159005, scale=758.3118108488464)
DescribeResult(nobs=1000, minmax=(58365.73295189596, 63098.115897691336)
               mean=60778.814121159005, variance=575612.41488774447,
               skewness=-0.05240603149469387, kurtosis=-0.07820523650411237)
```

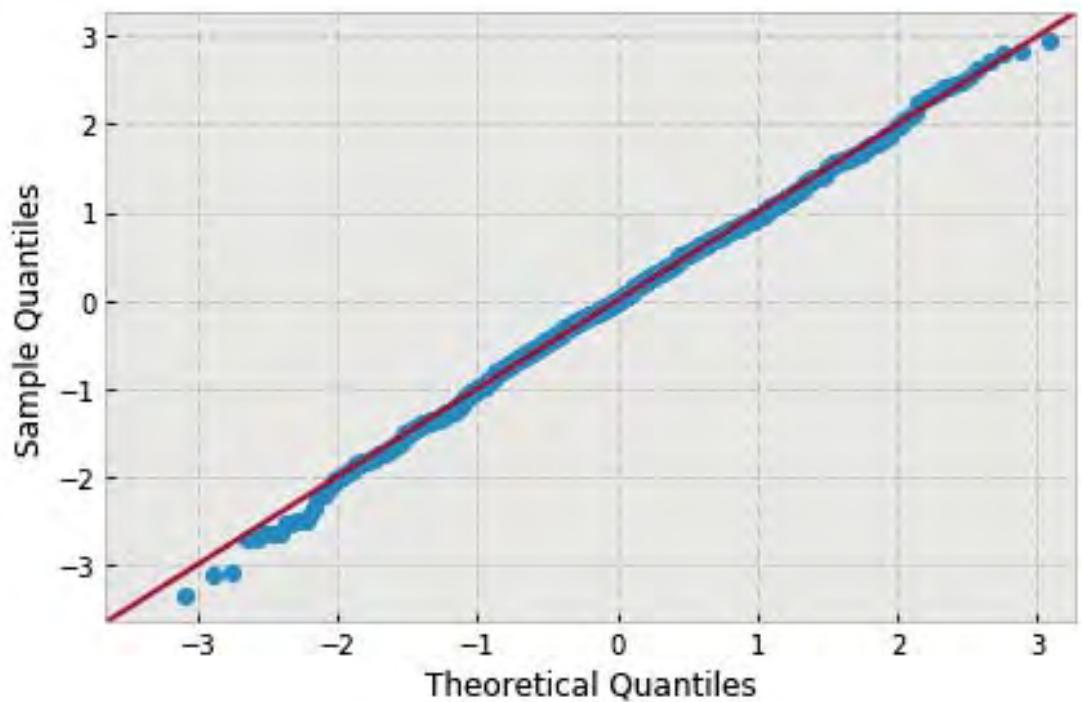
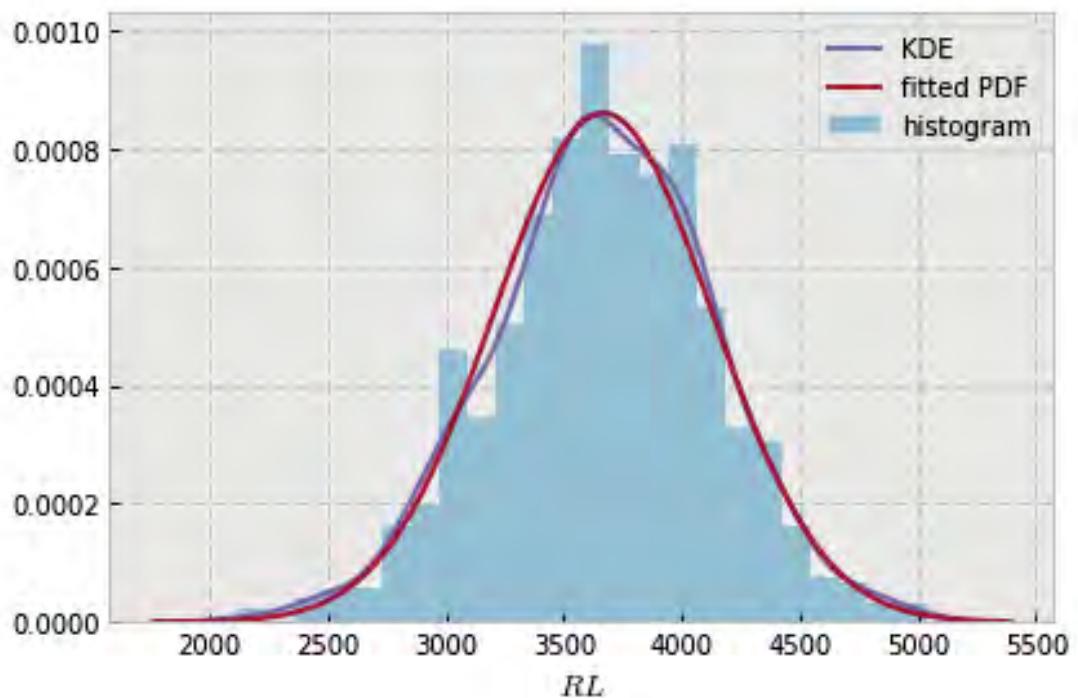


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=2778.8305555319703, scale=487.6168625823738)
DescribeResult(nobs=1000, minmax=(928.97709919566614, 4149.963508197432)
               mean=2778.8305555319703, variance=238008.2128875652,
               skewness=-0.034434738782295006, kurtosis=0.20391834649899332)
```

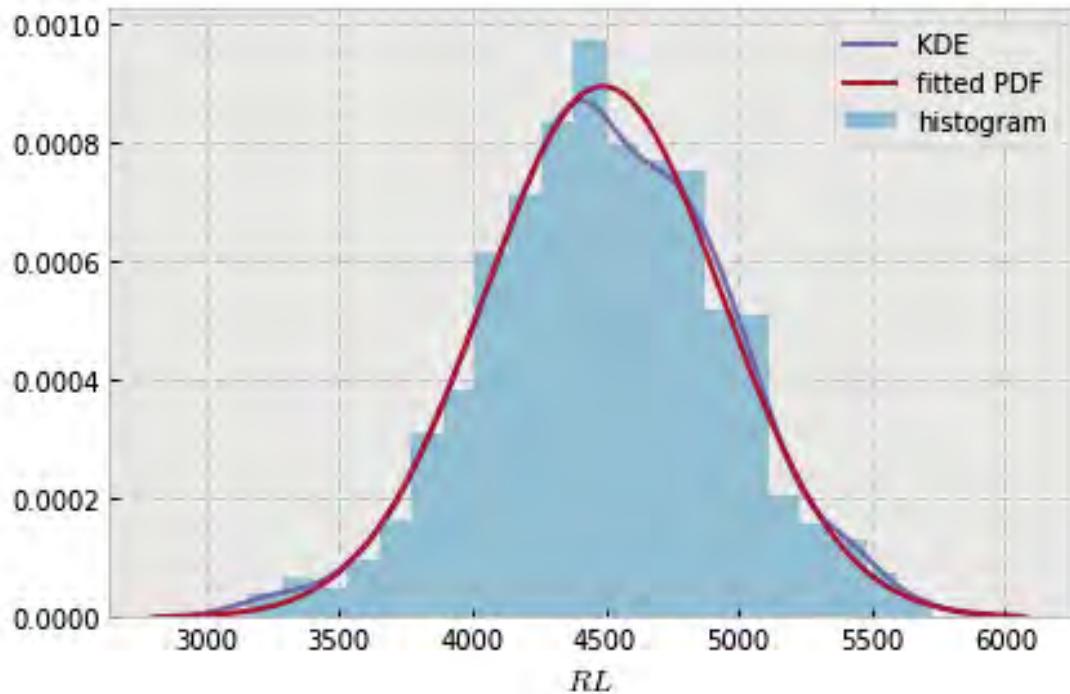


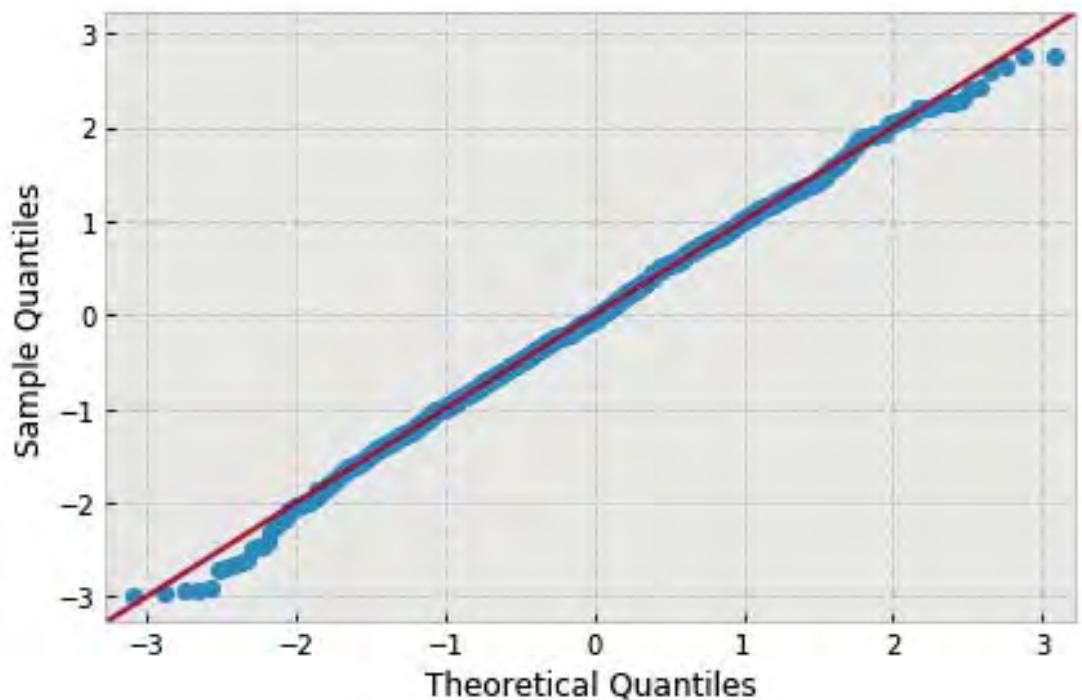


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(), loc=3666.6413680982487, scale=463.8199694028768)
DescribeResult(nobs=1000, minmax=(2119.3972371087125, 5033.3713420090271)
               mean=3666.6413680982487, variance=215344.30832521073,
               skewness=-0.1125076691042469, kurtosis=0.09181807008577048)
```

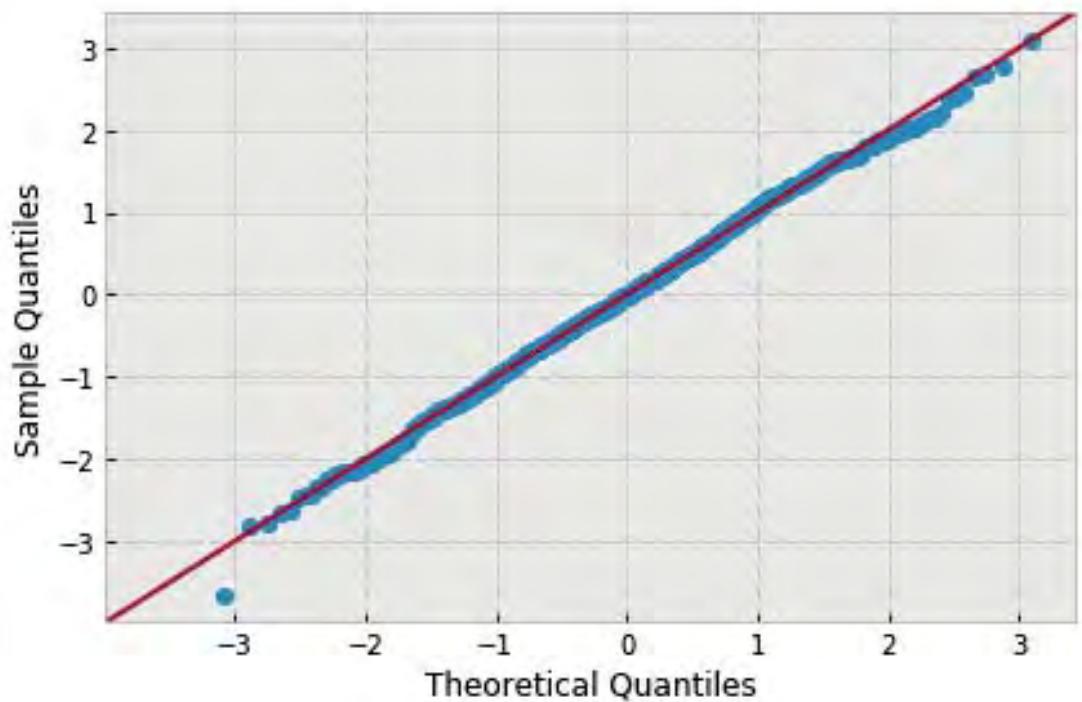
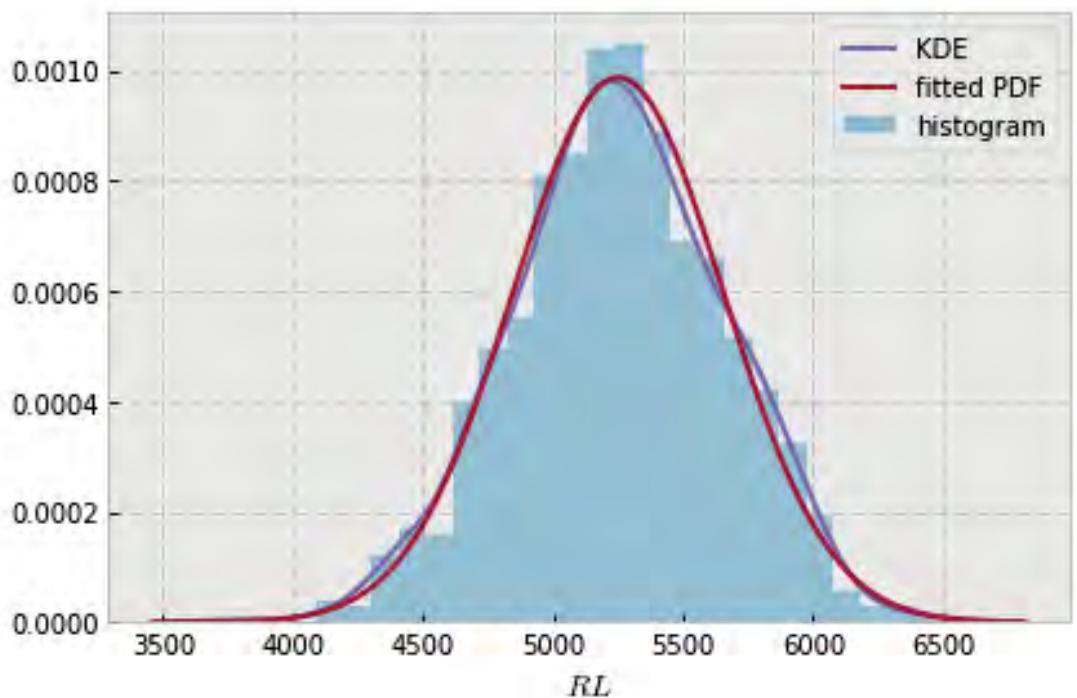


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=4491.64579534022, scale=446.73503950449265)
DescribeResult(nobs=1000, minmax=(3158.8515030722101, 5723.8451121044909)
               mean=4491.6457953402196, variance=199771.96748856918,
               skewness=-0.08378288559119029, kurtosis=-0.03482076741237616)
```

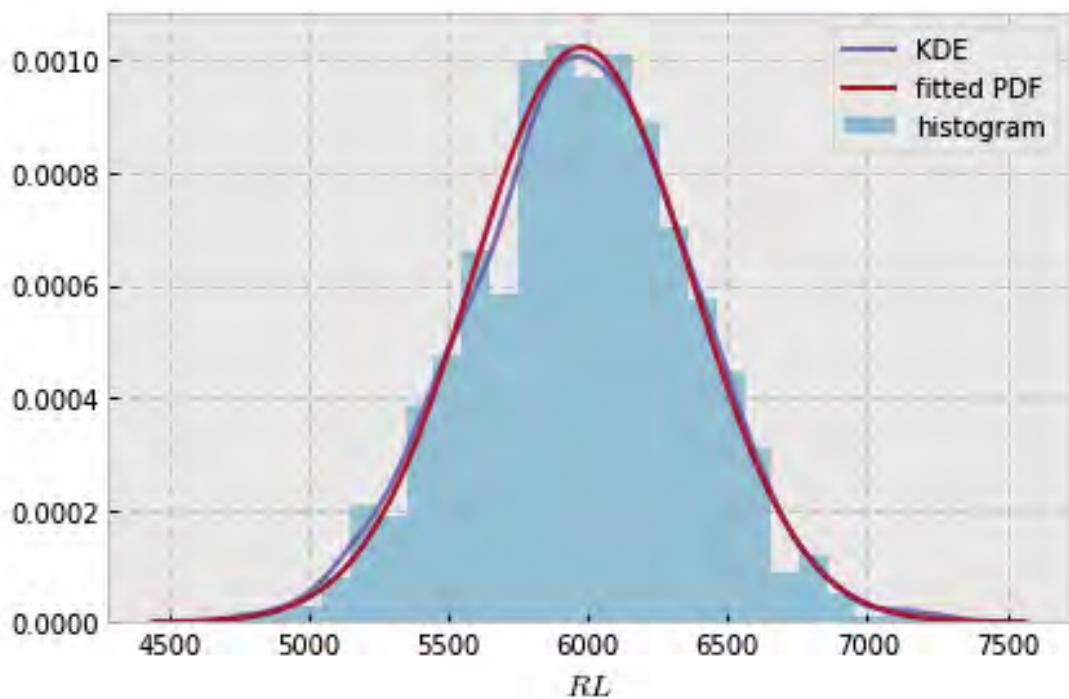


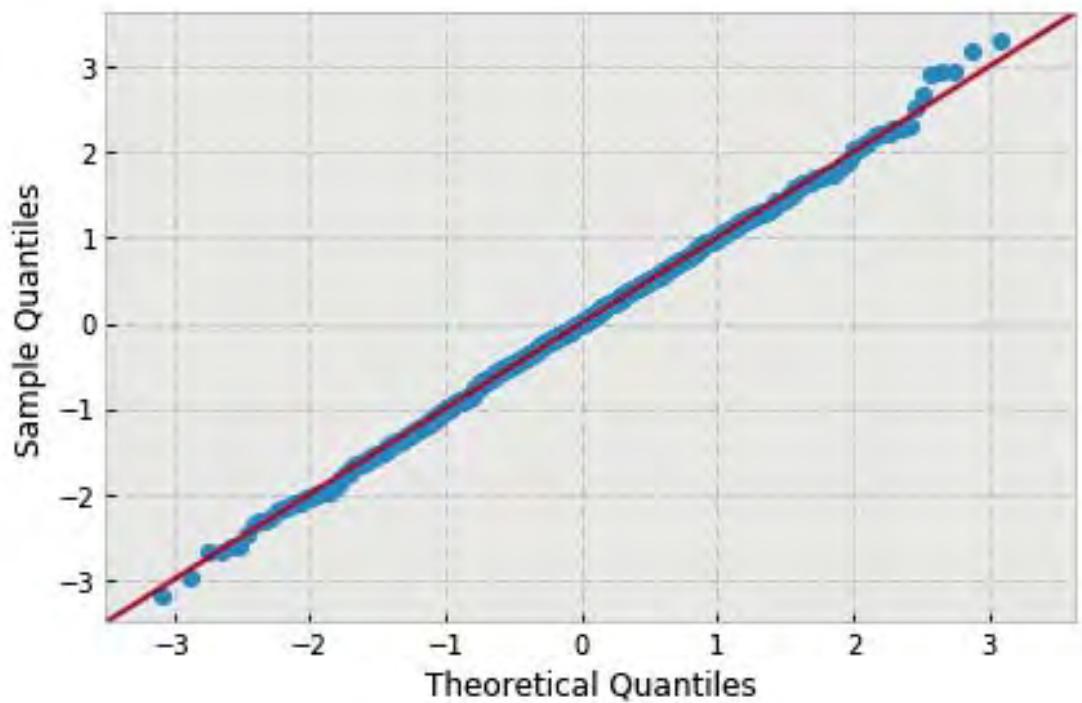


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=5250.218678257399, scale=404.11051375494884)
DescribeResult(nobs=1000, minmax=(3776.7481691455723, 6497.0921181139975)
               mean=5250.218678257399, variance=163468.77610339207,
               skewness=-0.07038202238047254, kurtosis=-0.12379853671581698)
```

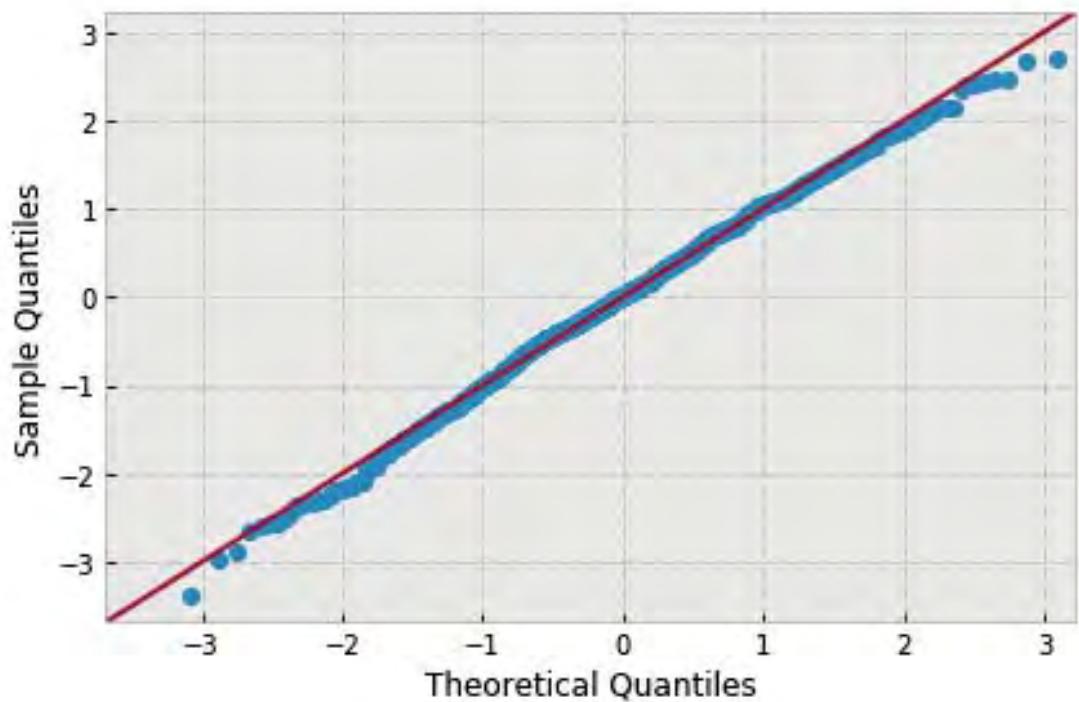
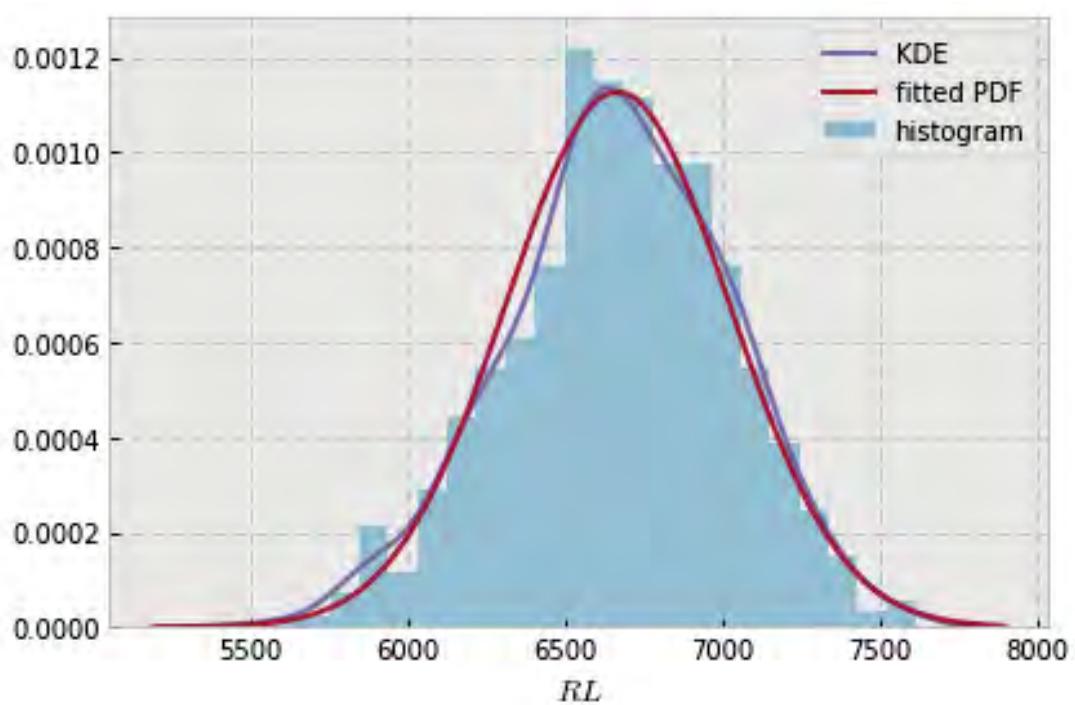


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=5976.549011430559, scale=389.7891706211633)
DescribeResult(nobs=1000, minmax=(4740.4744459980029, 7263.2498175016035)
               mean=5976.5490114305594, variance=152087.68521875309,
               skewness=-0.021929565544741465, kurtosis=0.005469709109163112)
```

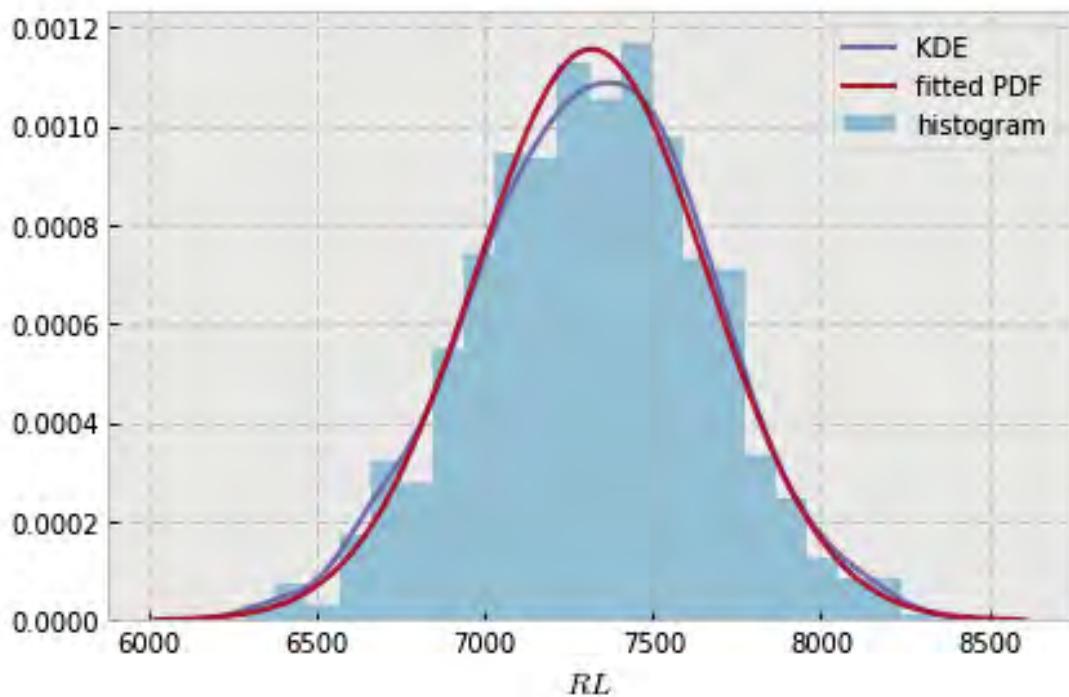


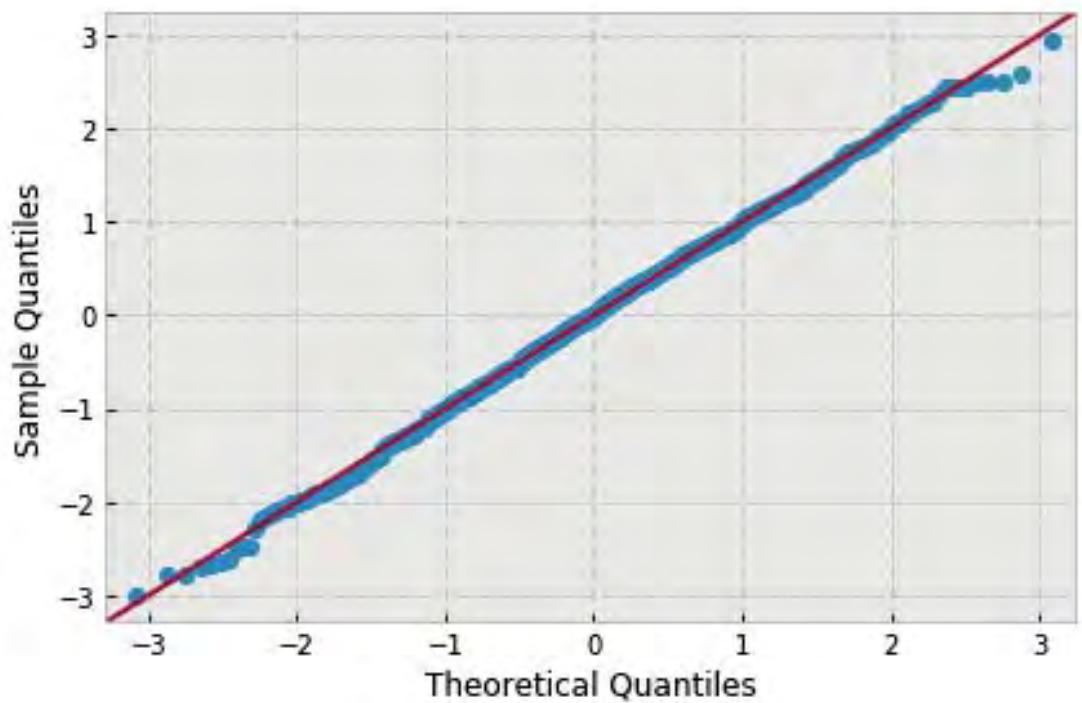


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(), loc=6668.262734627992, scale=353.9170371959584)
DescribeResult(nobs=1000, minmax=(5472.5546506290393, 7618.8417434054209)
               mean=6668.2627346279924, variance=125382.65186943483,
               skewness=-0.18346564276431093, kurtosis=-0.09314313218330961)
```

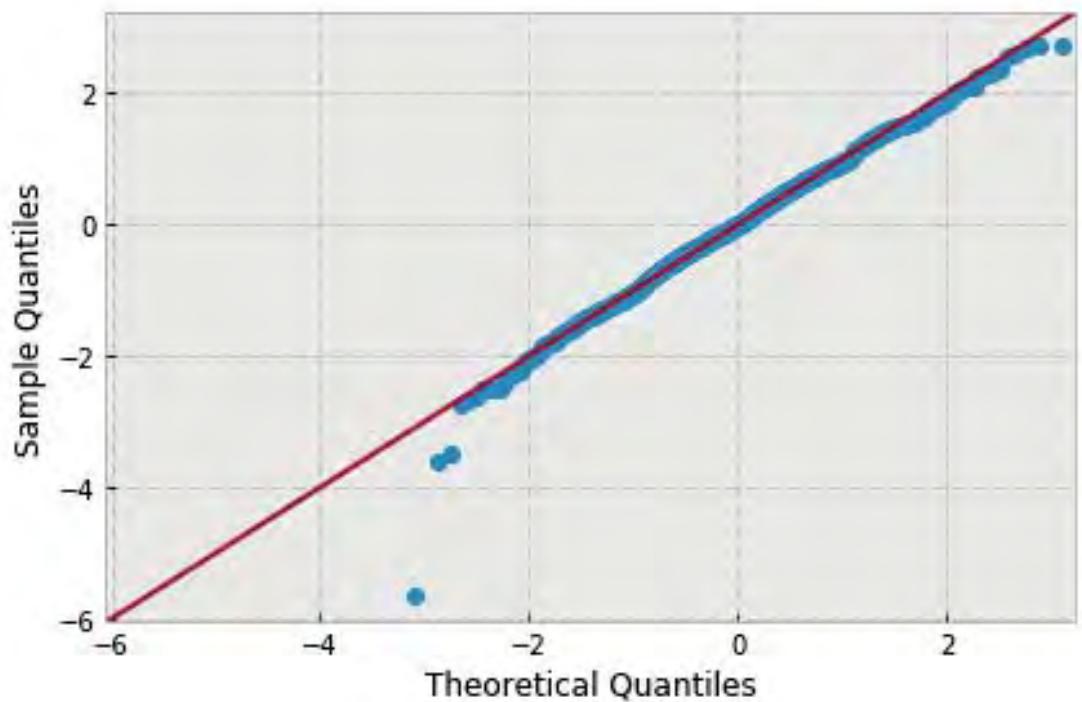
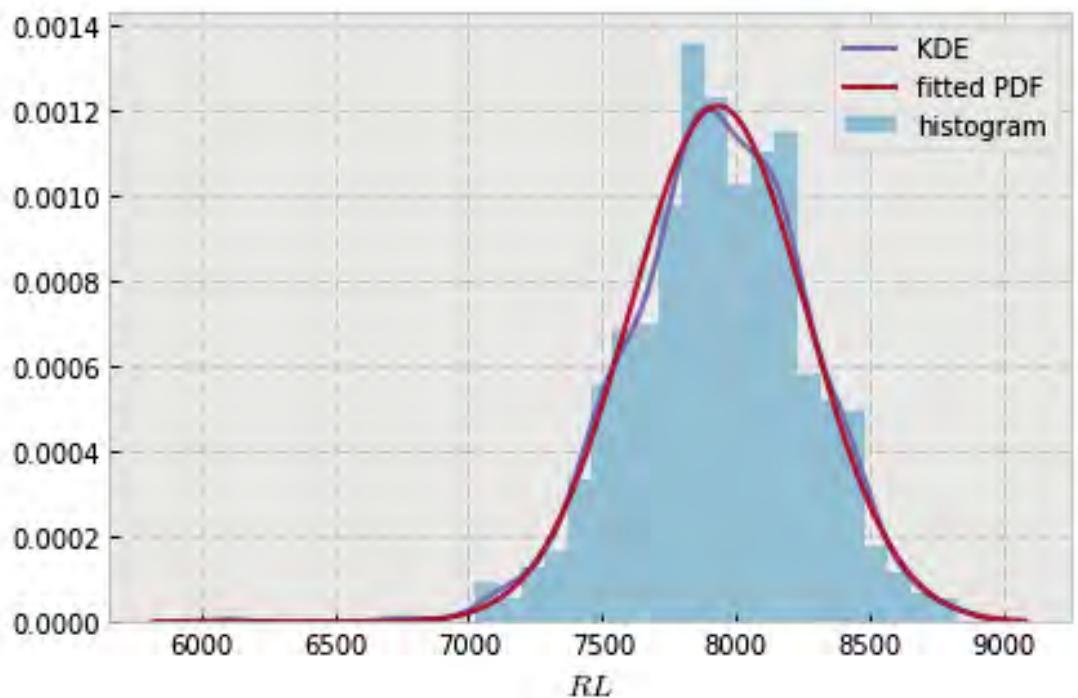


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=7318.090957984401, scale=345.1540074769156)
DescribeResult(nobs=1000, minmax=(6286.8219968220983, 8332.6005903332189)
               mean=7318.090957984401, variance=119250.53941679146,
               skewness=-0.07211706638044871, kurtosis=-0.16118421762596213)
```

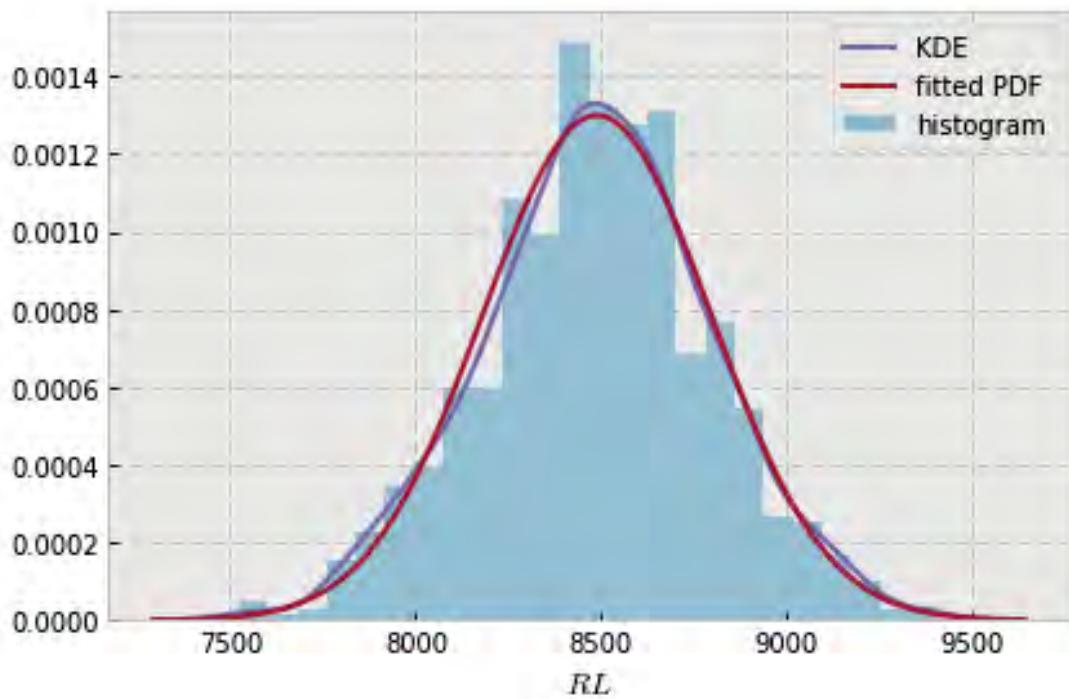


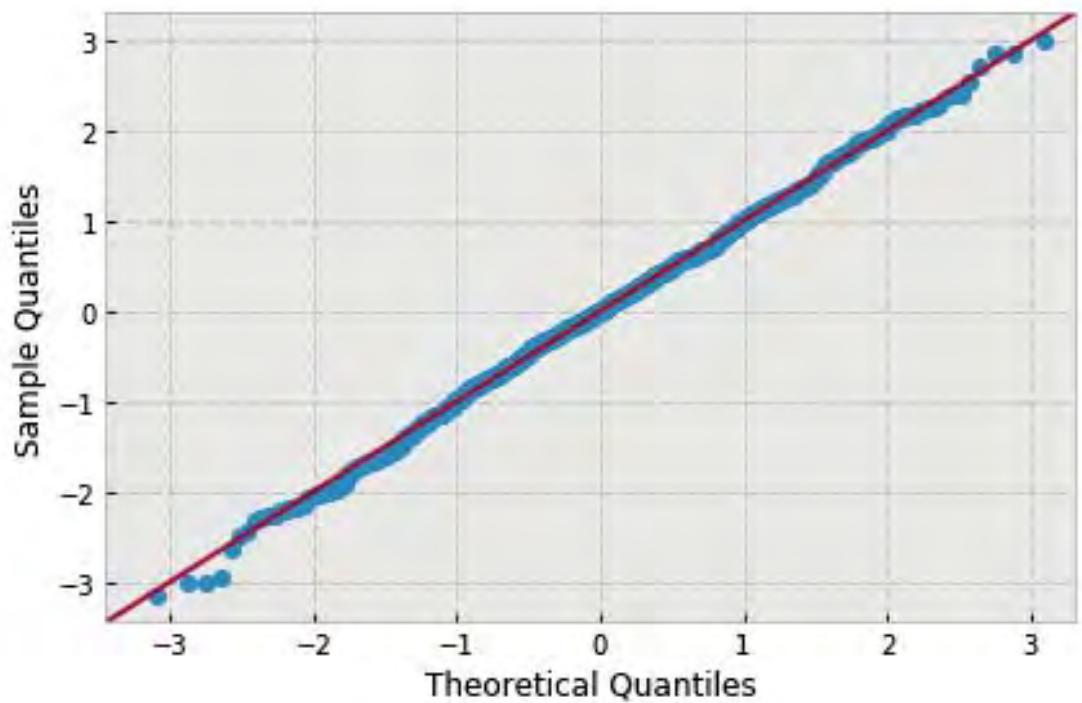


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=7933.120002365586, scale=329.16994540378676)
DescribeResult(nobs=1000, minmax=(6078.8521077481319, 8828.7090118494998)
               mean=7933.1200023655856, variance=108461.31427140336,
               skewness=-0.30507684617641323, kurtosis=0.8414012375527364)
```

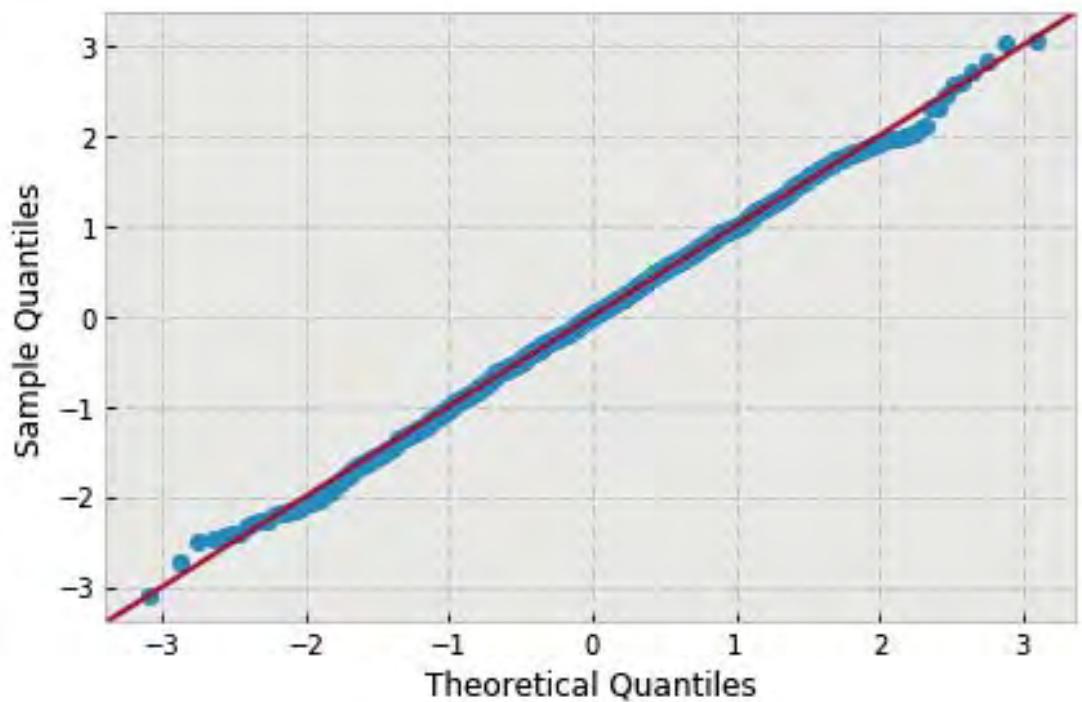
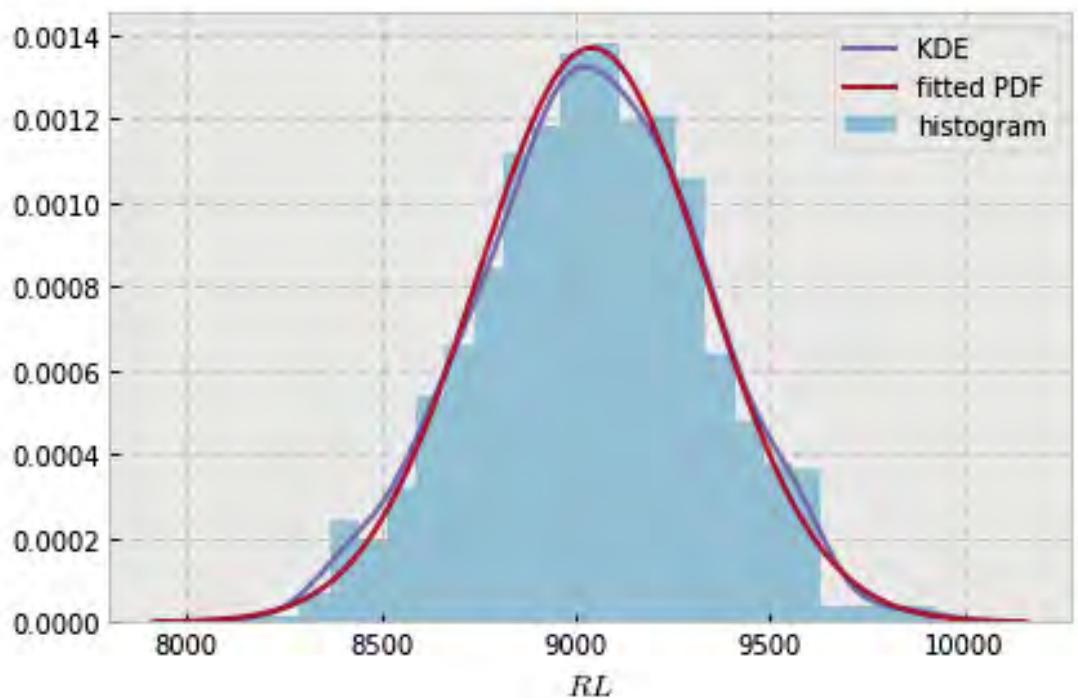


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=8487.809292537817, scale=307.0370953340848)
DescribeResult(nobs=1000, minmax=(7524.1072107198643, 9408.117144972146)
               mean=8487.8092925378169, variance=94366.144055247132,
               skewness=-0.059315394191519114, kurtosis=0.013568290924173887)
```

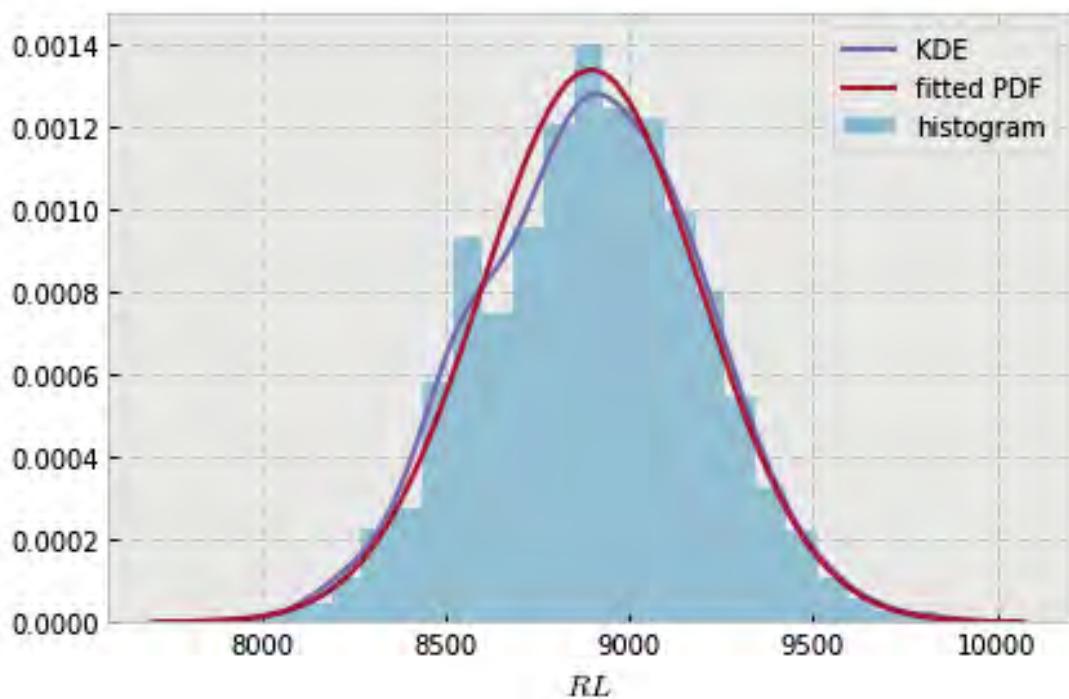


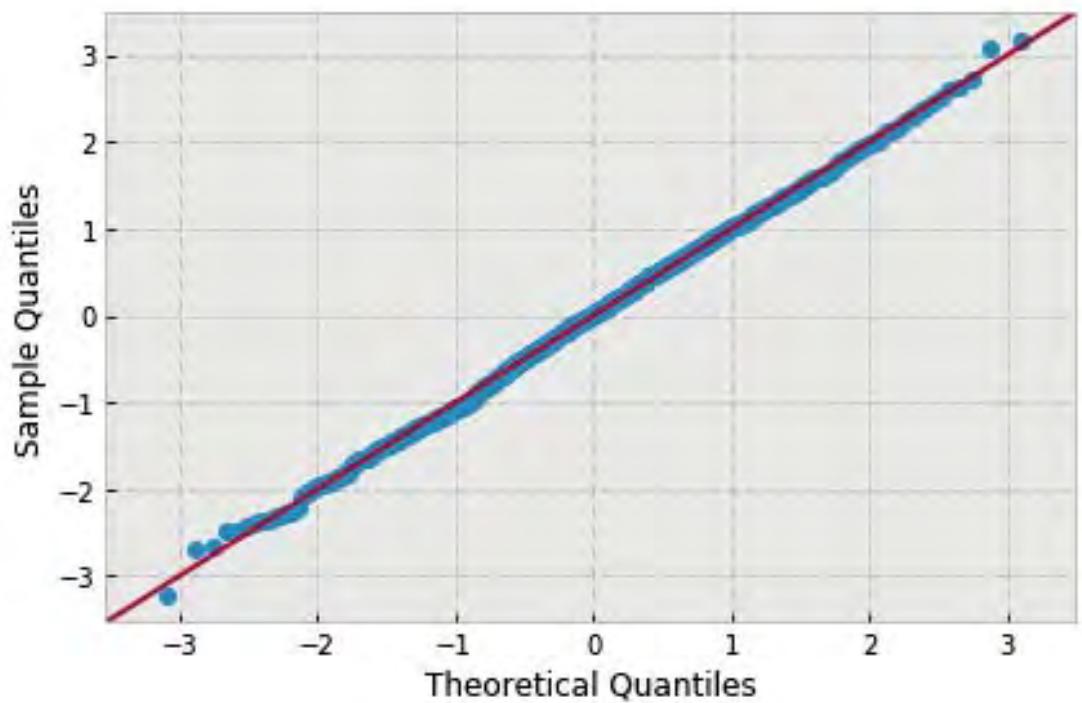


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=9041.529155753151, scale=291.2178935897016)
DescribeResult(nobs=1000, minmax=(8143.1705465589612, 9929.9588385041916)
               mean=9041.5291557531509, variance=84892.754301123889,
               skewness=-0.04039113819656151, kurtosis=-0.17886253192973012)
```

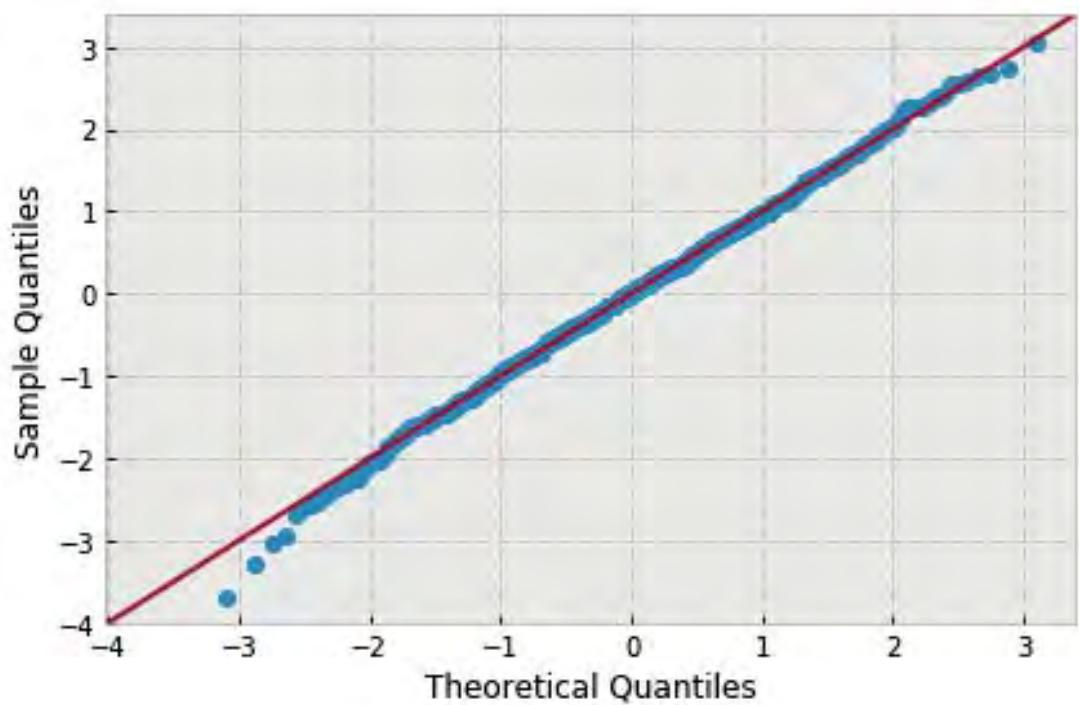
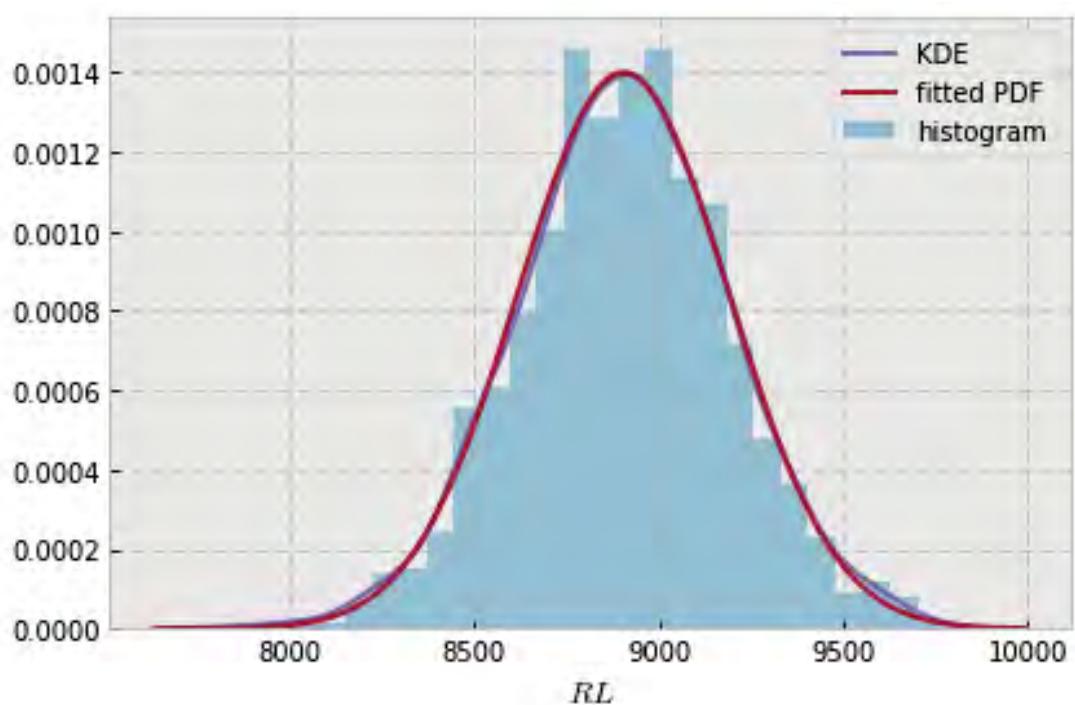


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=8895.769522462751, scale=298.2551213003227)
DescribeResult(nobs=1000, minmax=(7938.1826643519853, 9839.9339456579582)
               mean=8895.7695224627514, variance=89045.162544414619,
               skewness=-0.023366441845583726, kurtosis=-0.18422205431911776)
```

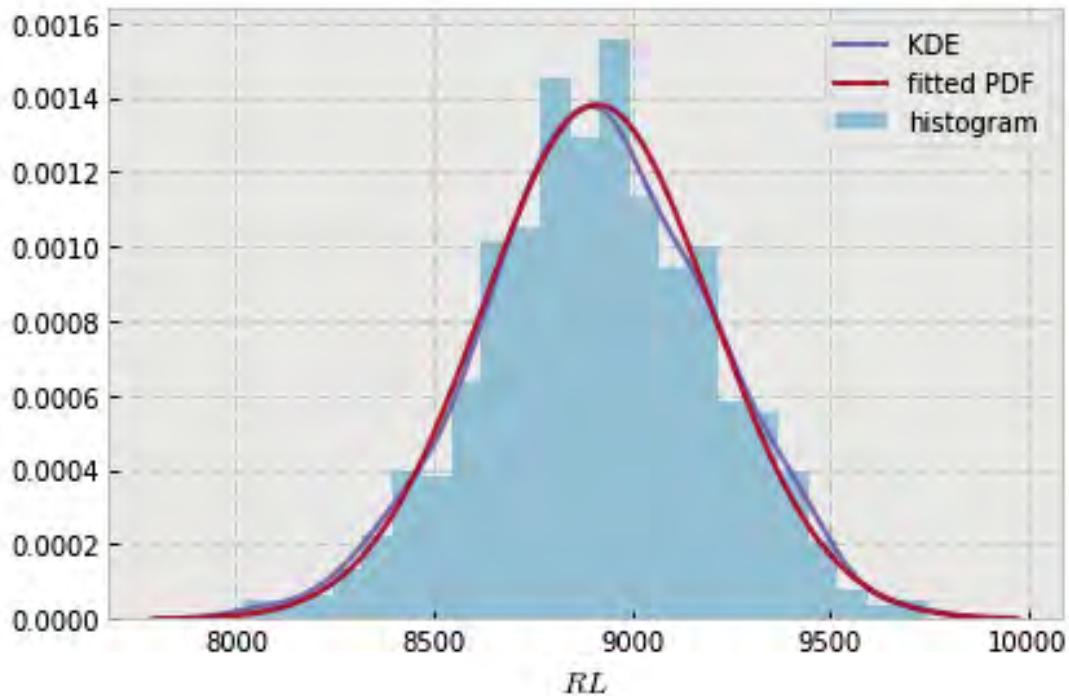


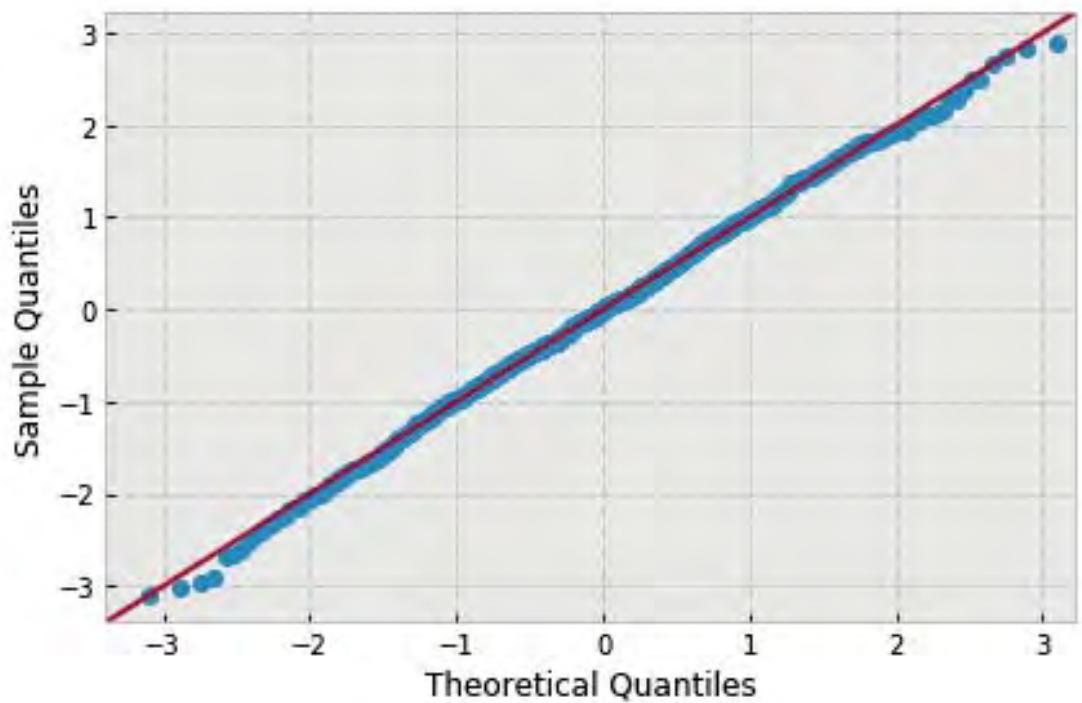


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=8904.767236624006, scale=284.9716443735661)
DescribeResult(nobs=1000, minmax=(7855.5230875669258, 9774.2197959192035)
               mean=8904.7672366240058, variance=81290.128225199427,
               skewness=-0.06356555924718664, kurtosis=0.15351952388324452)
```

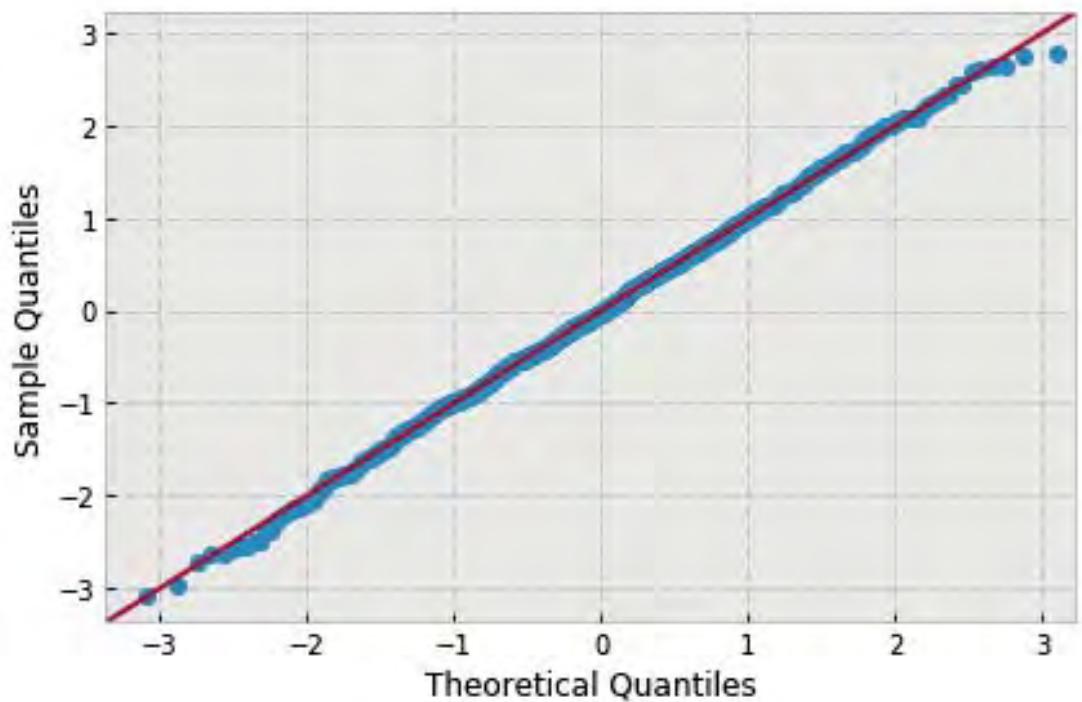
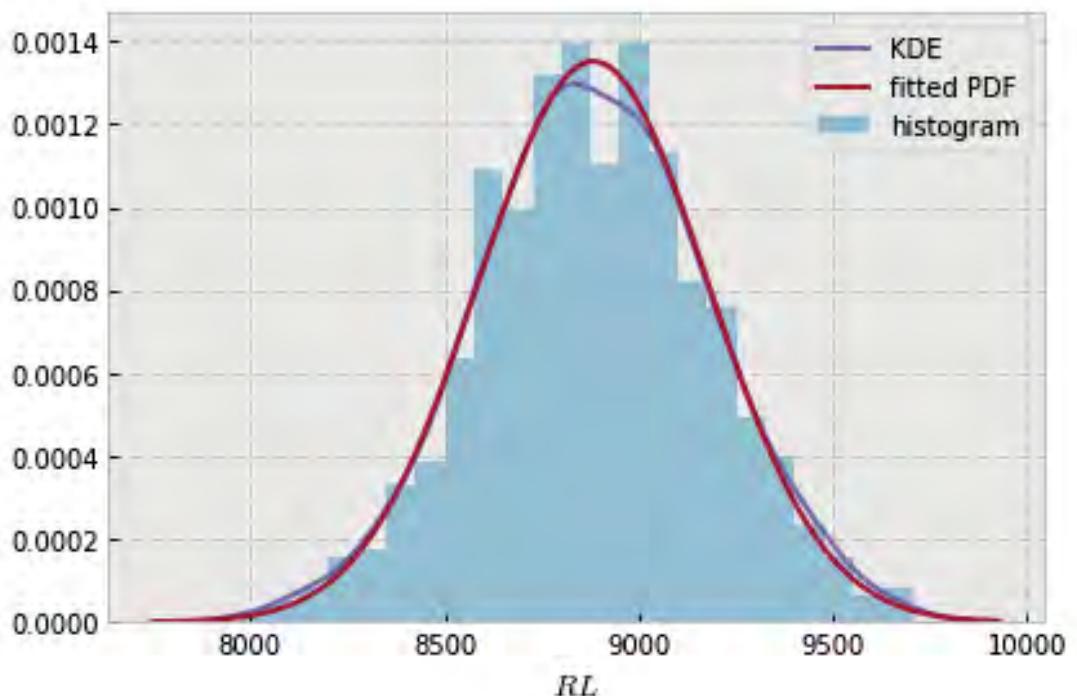


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(), loc=8912.05678466684, scale=288.70975385299636)
DescribeResult(nobs=1000, minmax=(8019.6749037870786, 9744.5332035191204)
               mean=8912.05678466684, variance=83436.758728586341,
               skewness=-0.06184953107086665, kurtosis=-0.07463624118297885)
```

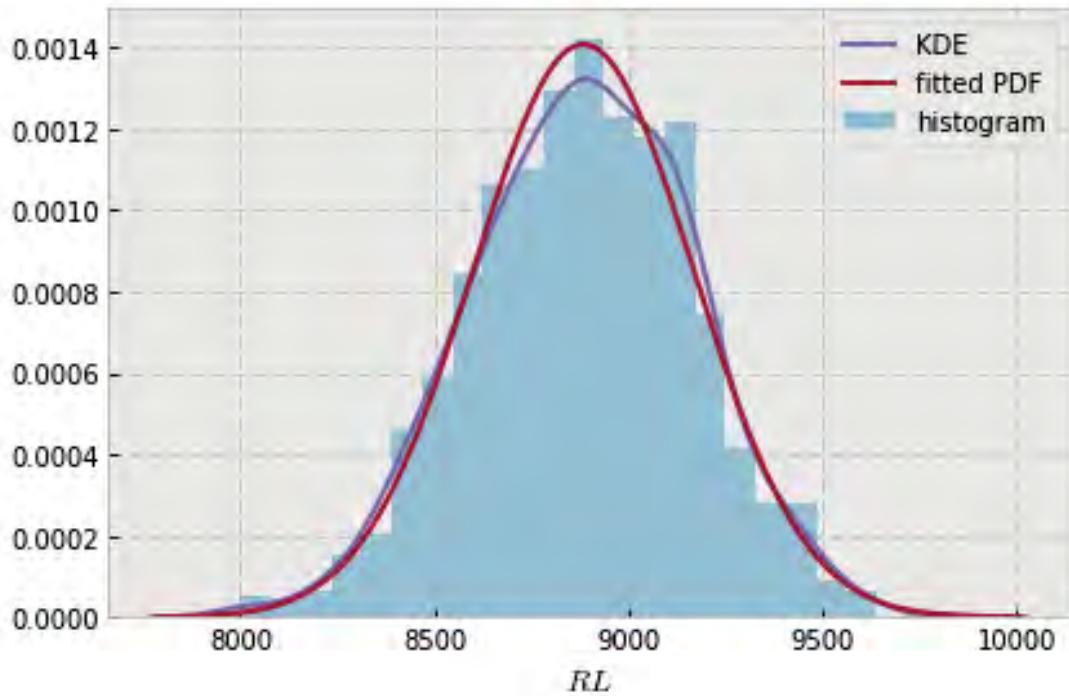


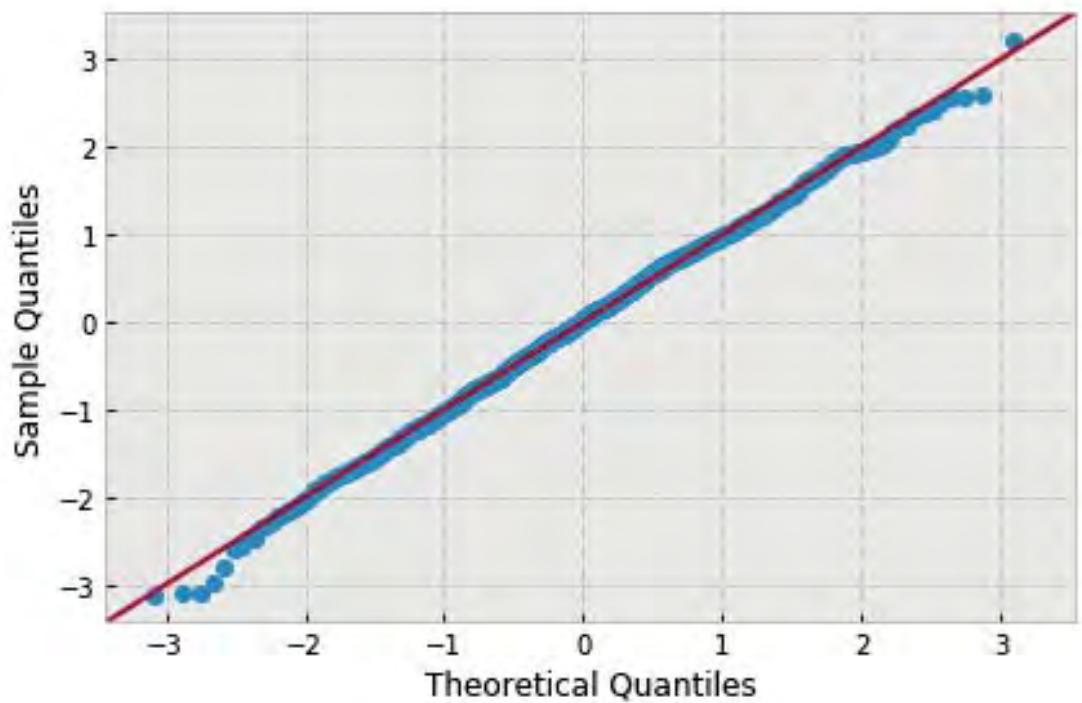


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=8882.388591785026, scale=295.2185321624677)
DescribeResult(nobs=1000, minmax=(7974.1056391051961, 9702.1130222822503)
               mean=8882.3885917850257, variance=87241.222955117075,
               skewness=-0.019151181455849705, kurtosis=-0.08671630676229825)
```

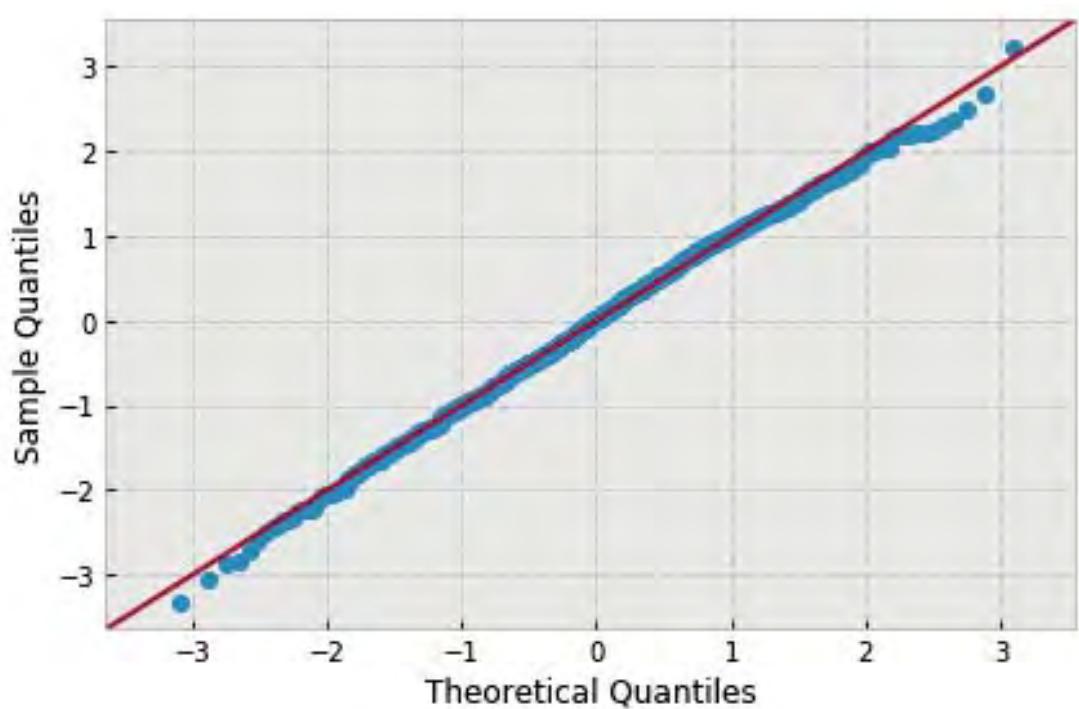
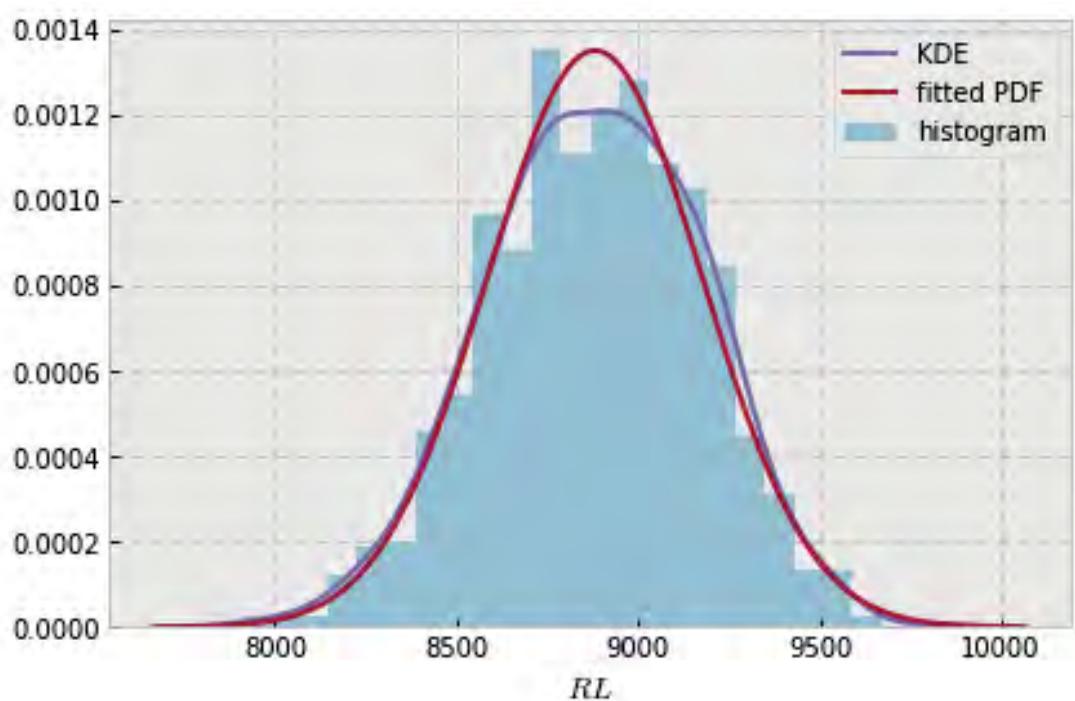


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=8883.078759286605, scale=283.60051139482044)
DescribeResult(nobs=1000, minmax=(7999.9007281057811, 9795.0109674953801)
               mean=8883.0787592866054, variance=80509.759823226908,
               skewness=-0.10452390330558342, kurtosis=-0.08960350482126511)
```

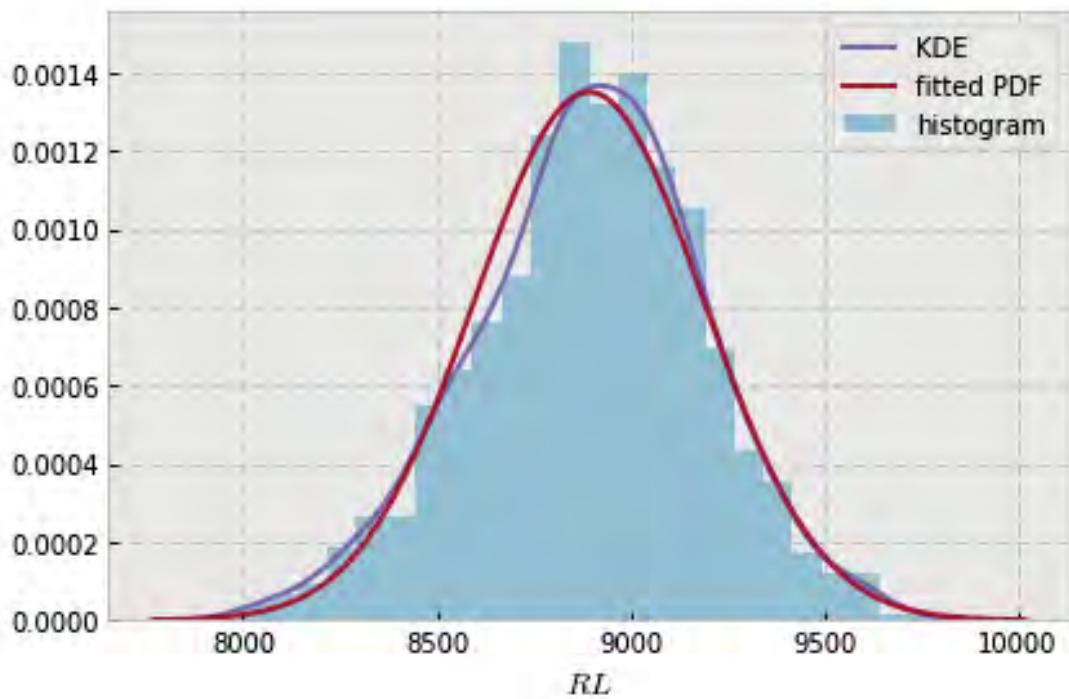


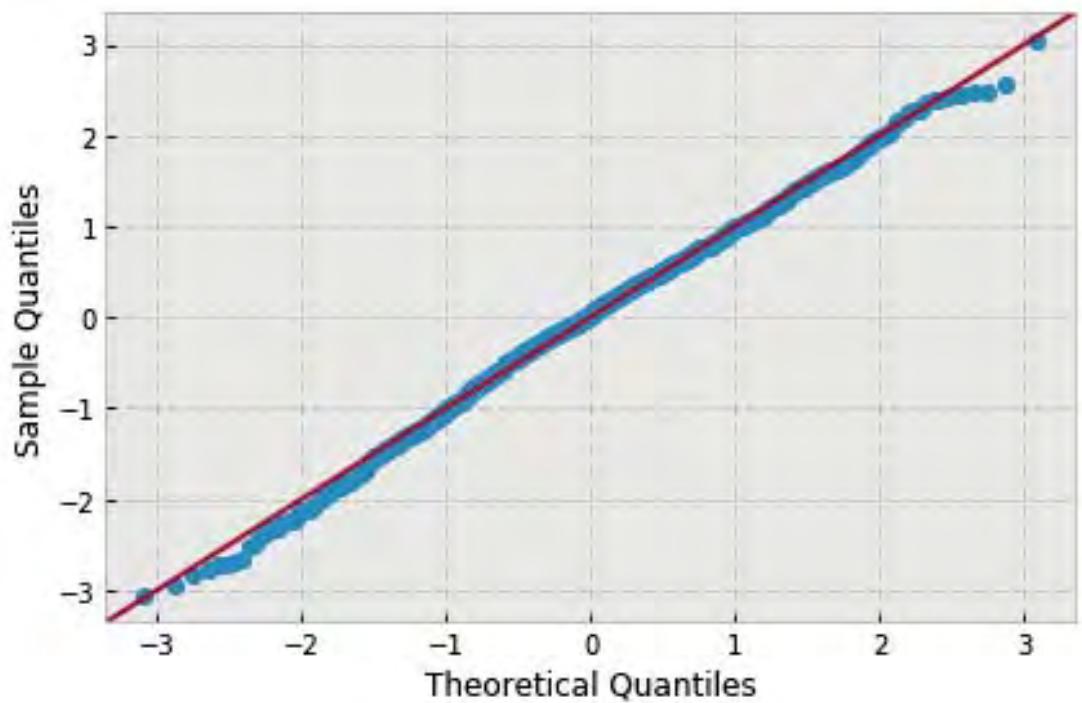


```
variable name: sigma
variable value: 0.066666666666666667
distribution: normal(shape=(), loc=8880.946242795184, scale=295.4858738721719)
DescribeResult(nobs=1000, minmax=(7902.0075517346449, 9832.492235231879)
               mean=8880.9462427951839, variance=87399.300958960055,
               skewness=-0.12394269070725283, kurtosis=-0.1844533895835032)
```

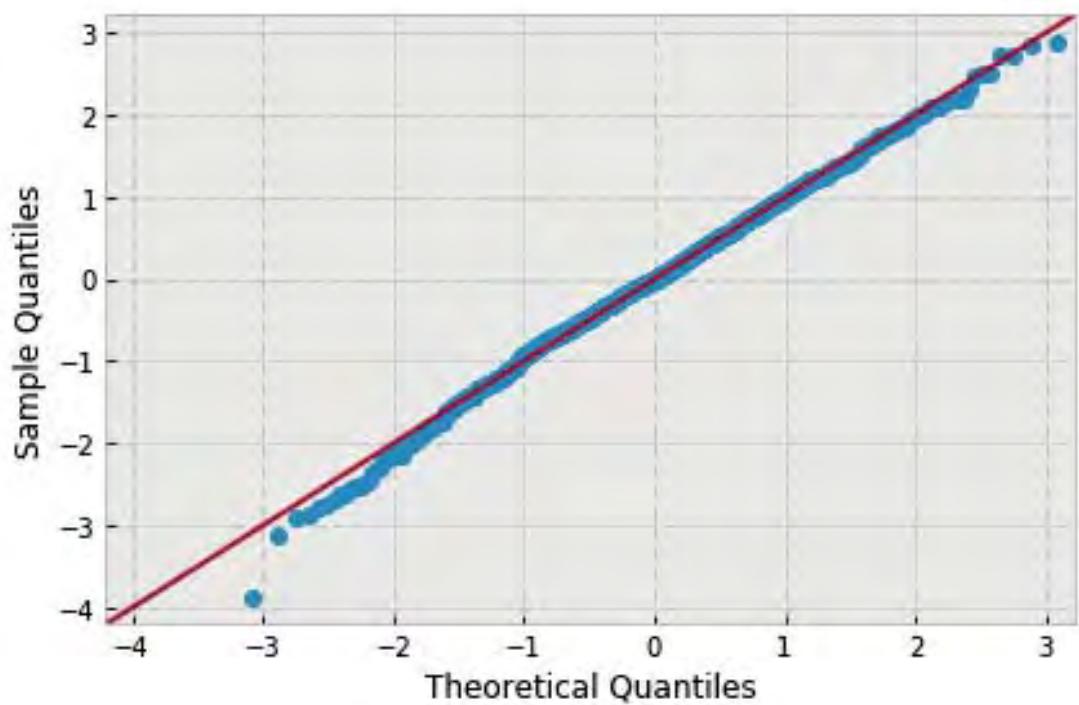
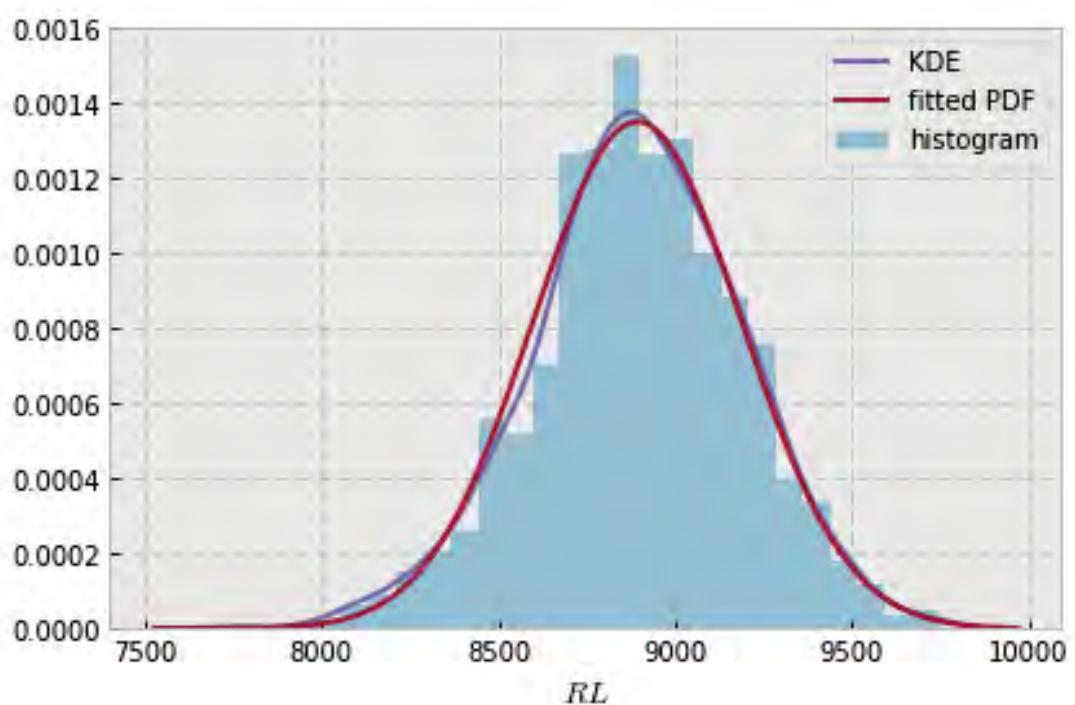


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=8892.846635460086, scale=295.3353972893993)
DescribeResult(nobs=1000, minmax=(7993.0336899914473, 9791.5603214926687)
               mean=8892.846635460086, variance=87310.307199286603,
               skewness=-0.17274084783082305, kurtosis=0.01072024969397889)
```

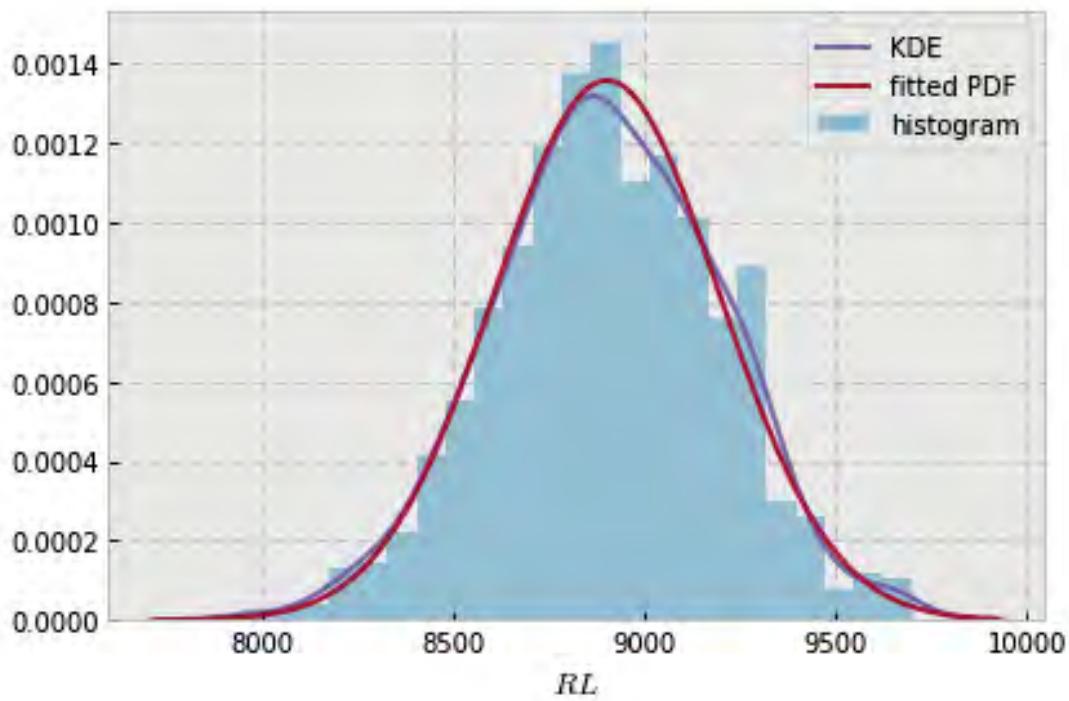


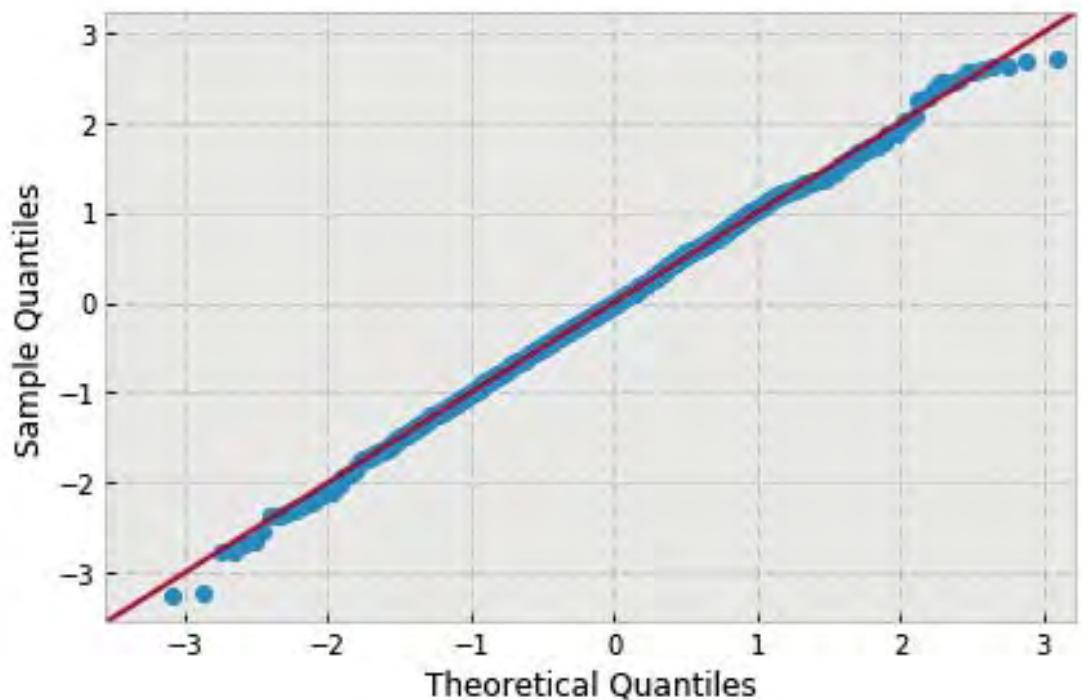


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=8892.68520866906, scale=295.0987883293162)
DescribeResult(nobs=1000, minmax=(7752.1433839712427, 9742.1728012617223)
               mean=8892.6852086690596, variance=87170.465338769354,
               skewness=-0.17399448884873608, kurtosis=0.20011292625464927)
```

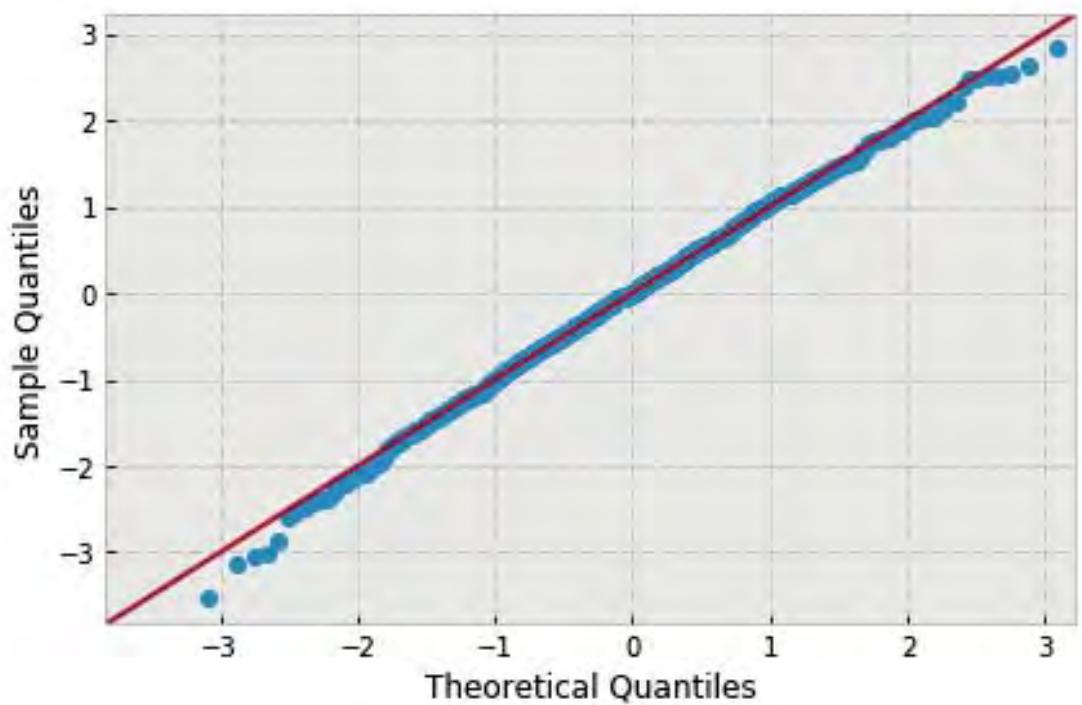
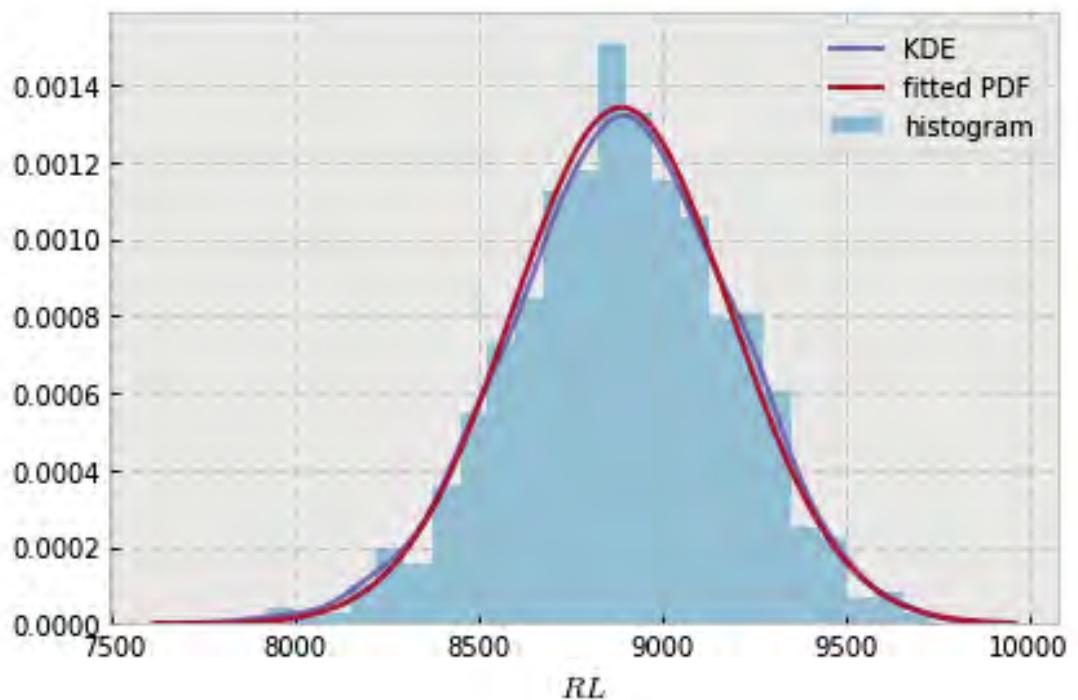


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=8902.022643208613, scale=293.780166286381)
DescribeResult(nobs=1000, minmax=(7947.4223047121595, 9699.2606528527358)
               mean=8902.0226432086129, variance=86393.179282536221,
               skewness=-0.06843267545565648, kurtosis=-0.0600662055564154)
```

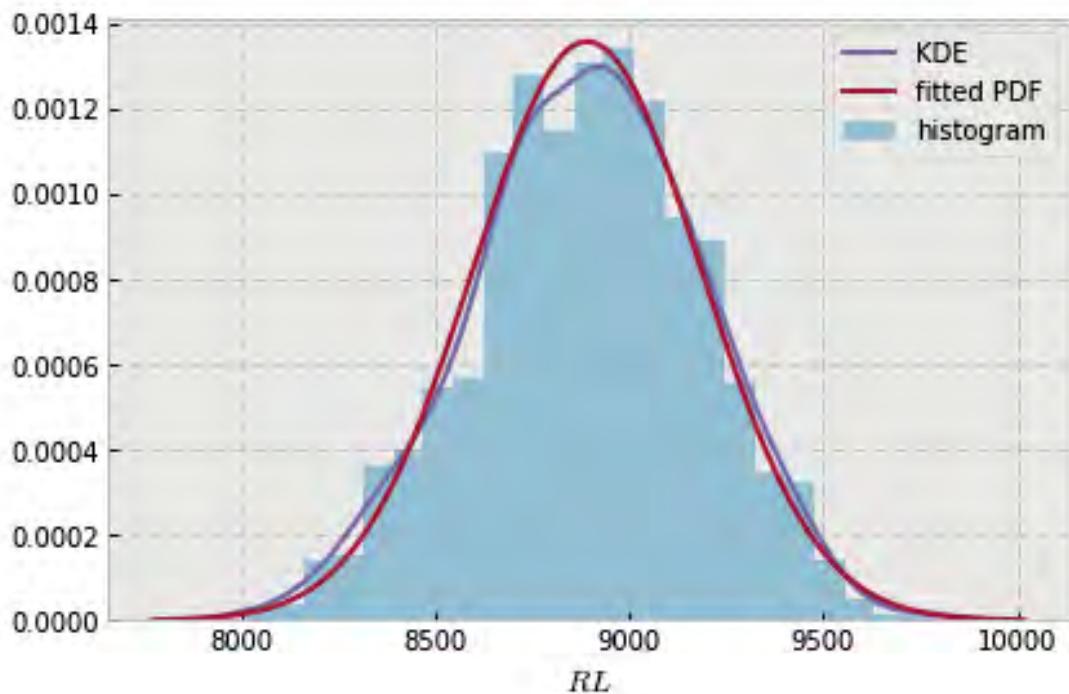


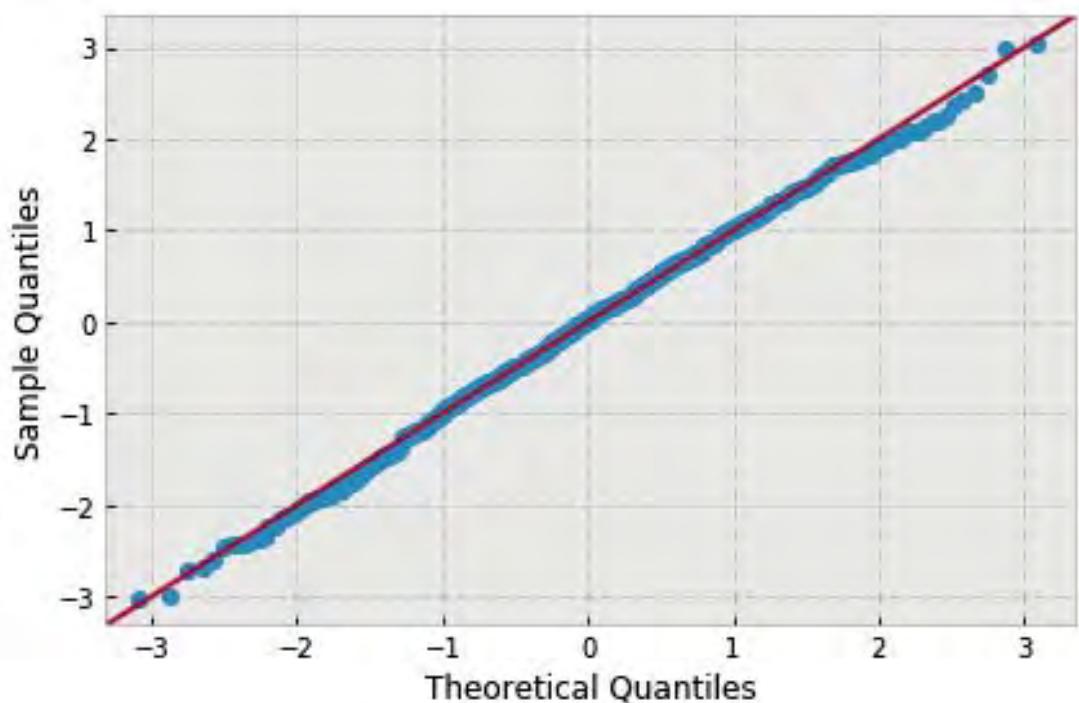


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=8887.539035040167, scale=297.4444911597158)
DescribeResult(nobs=1000, minmax=(7842.4921710504022, 9729.0927400968085)
               mean=8887.5390350401667, variance=88561.787108370627,
               skewness=-0.14674352216049869, kurtosis=-0.01448045150698185)
```

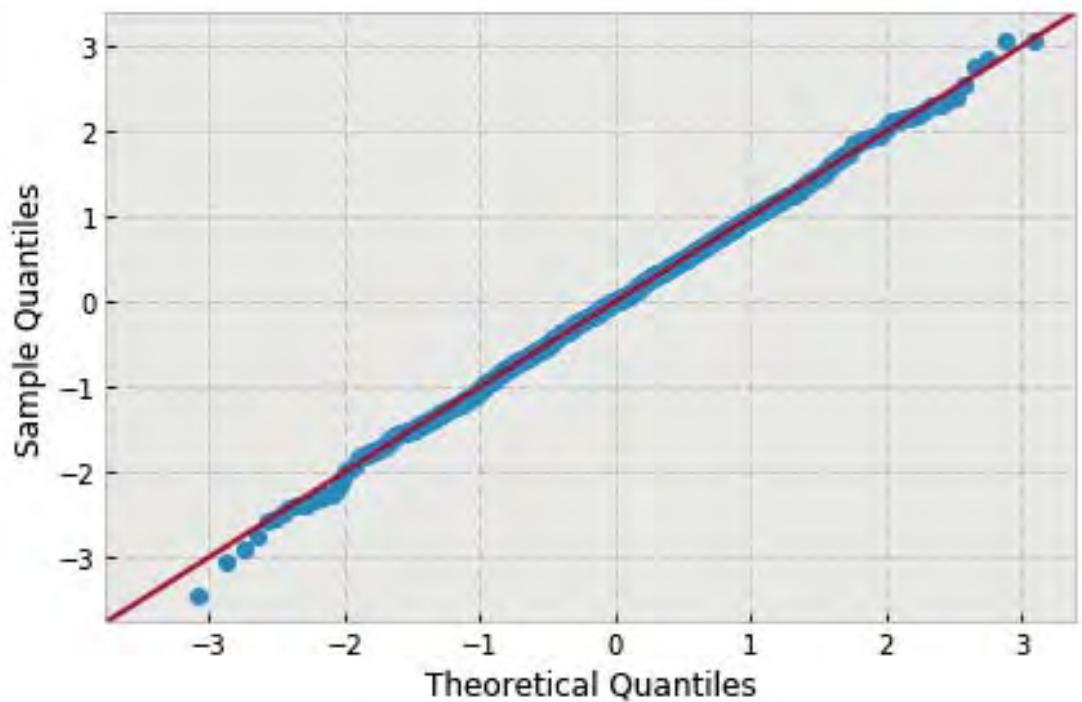
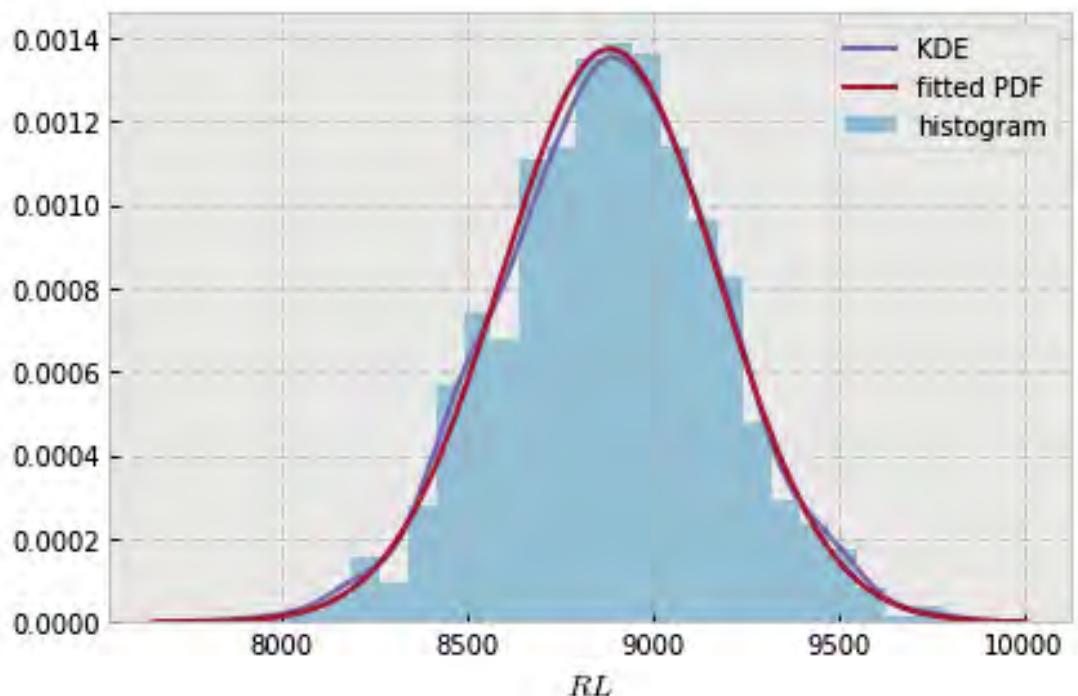


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=8892.267021434629, scale=294.2293252162429)
DescribeResult(nobs=1000, minmax=(8006.5779603882538, 9787.0355191459639)
               mean=8892.267021434629, variance=86657.553370576192,
               skewness=-0.10718694159862996, kurtosis=-0.15514707727498944)
```

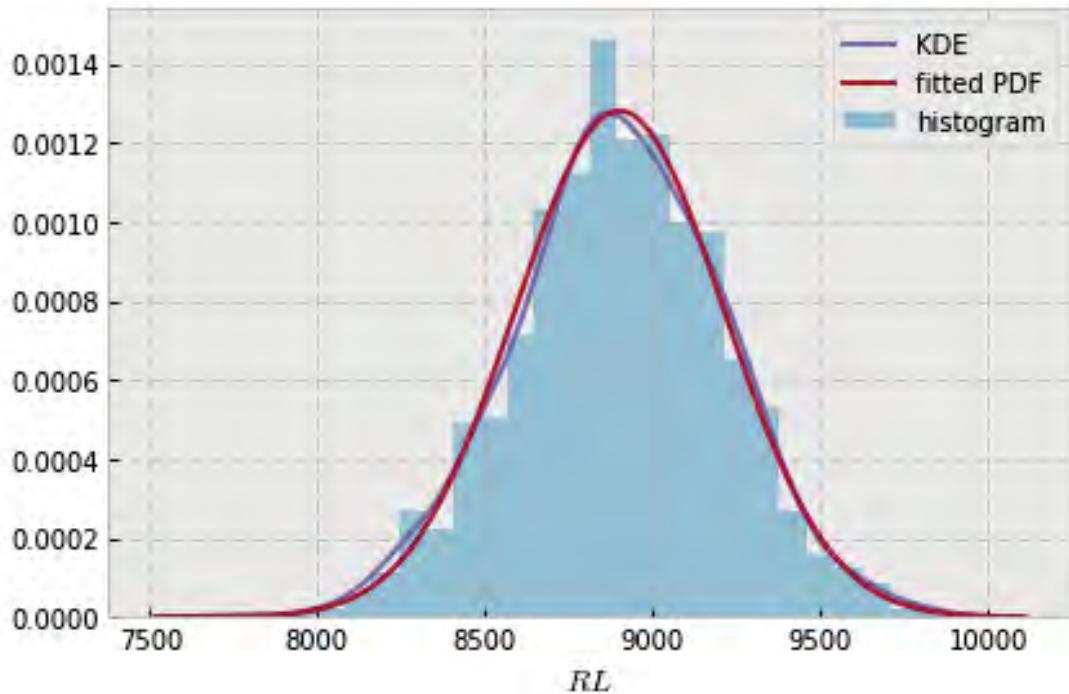


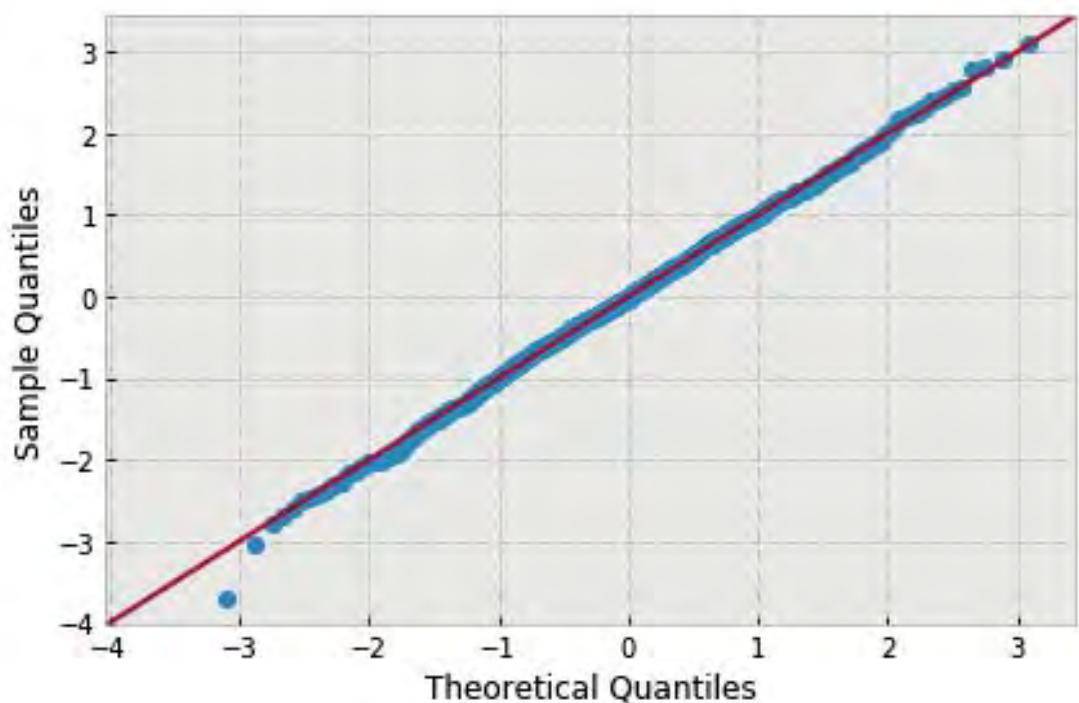


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=8884.07524885335, scale=290.71440392177175)
DescribeResult(nobs=1000, minmax=(7885.2456133858568, 9775.3011578971727)
               mean=8884.0752488533508, variance=84599.464111702764,
               skewness=-0.03902436714050272, kurtosis=0.014378776910580537)
```

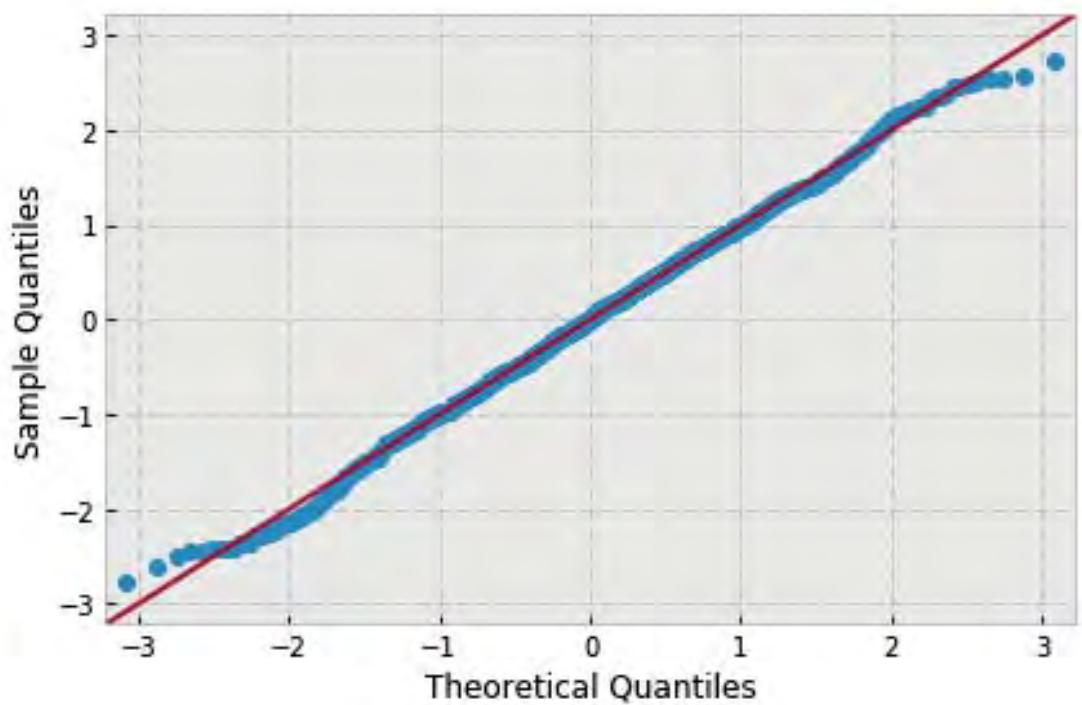
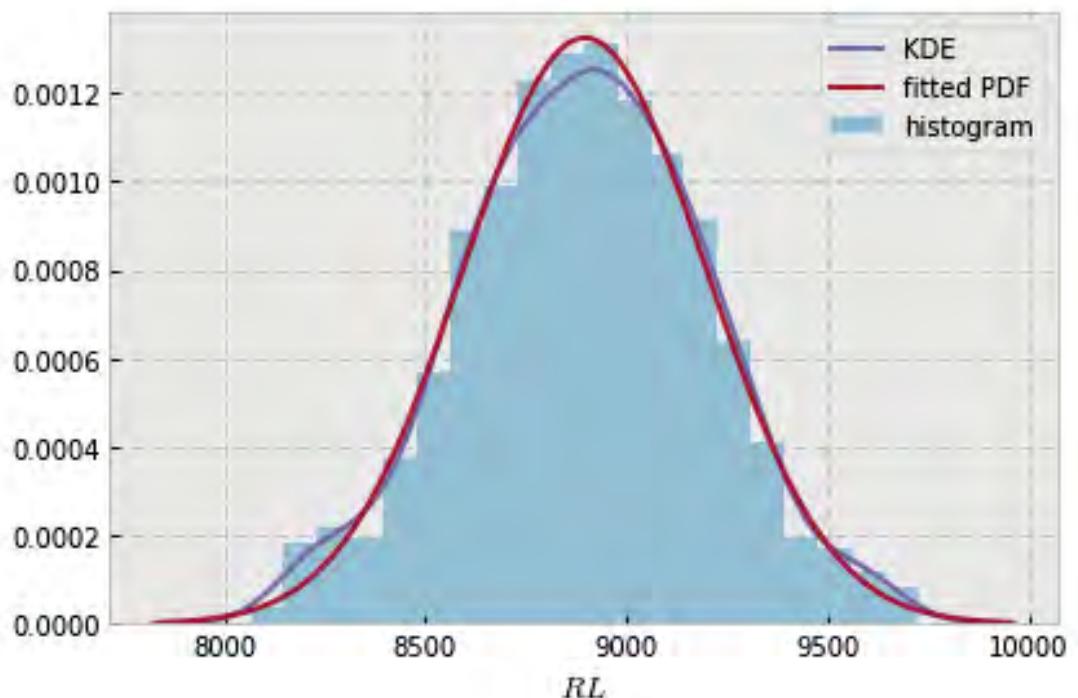


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=8901.436779045735, scale=311.6398215719581)
DescribeResult(nobs=1000, minmax=(7753.4187457500011, 9866.1348001363749)
               mean=8901.4367790457345, variance=97216.594984386291,
               skewness=-0.06784939812456103, kurtosis=0.05491467644058057)
```

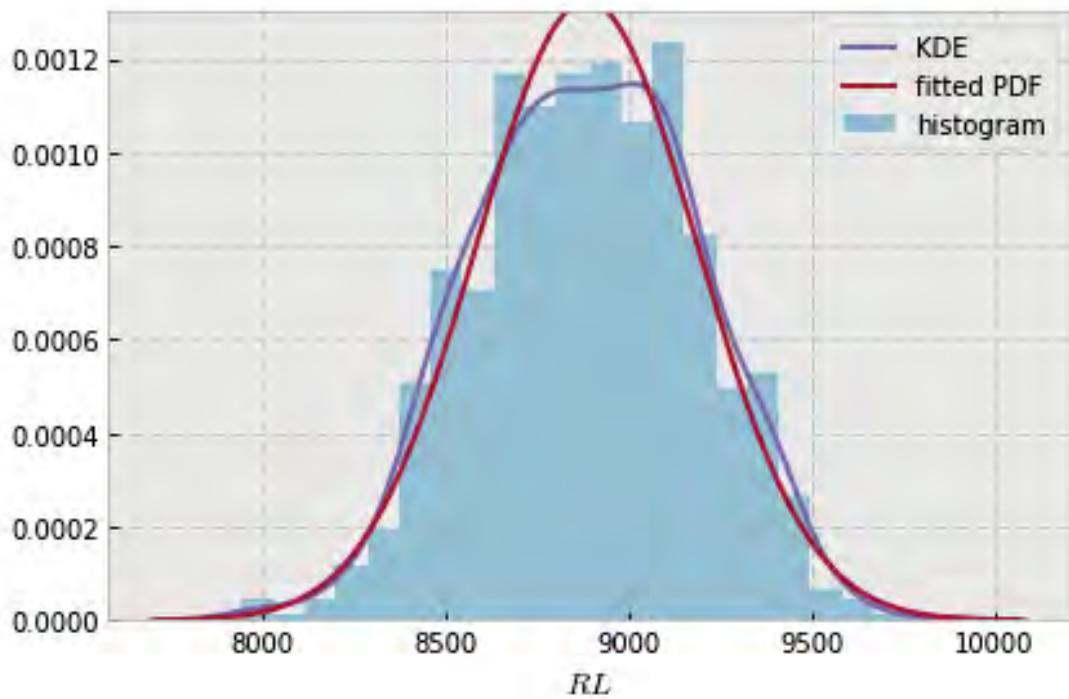


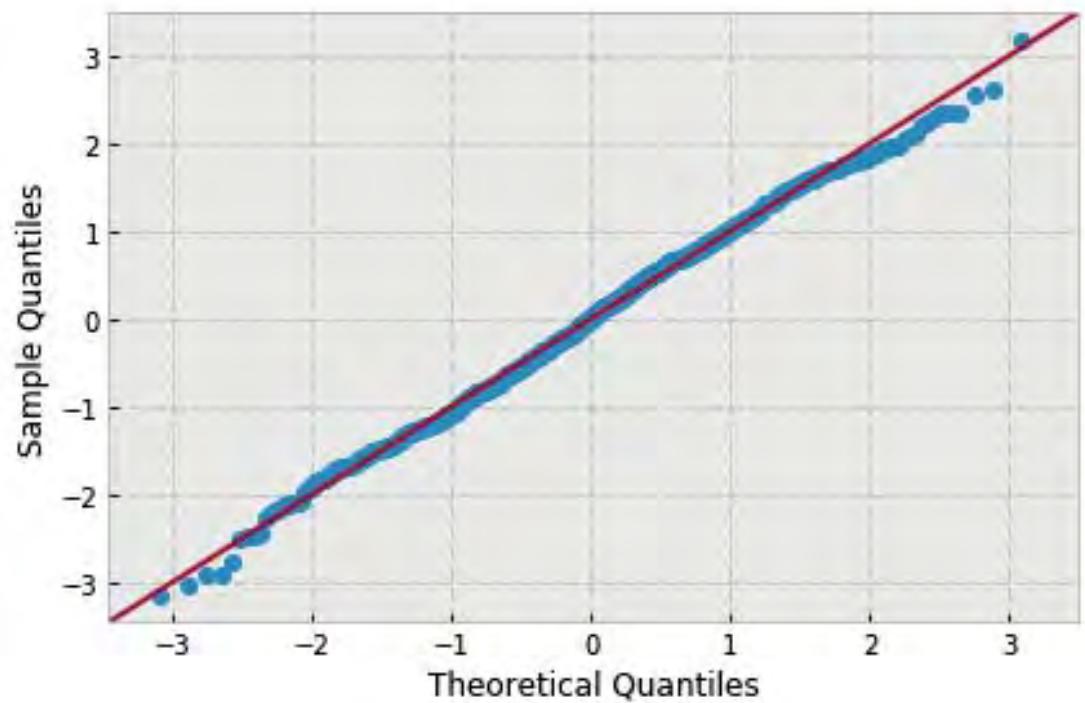


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=8895.568954602593, scale=301.3678006006078)
DescribeResult(nobs=1000, minmax=(8064.5719637805933, 9723.6400576445521)
               mean=8895.5689546025933, variance=90913.464703551261,
               skewness=-0.04328960005626475, kurtosis=-0.16322488515177502)
```

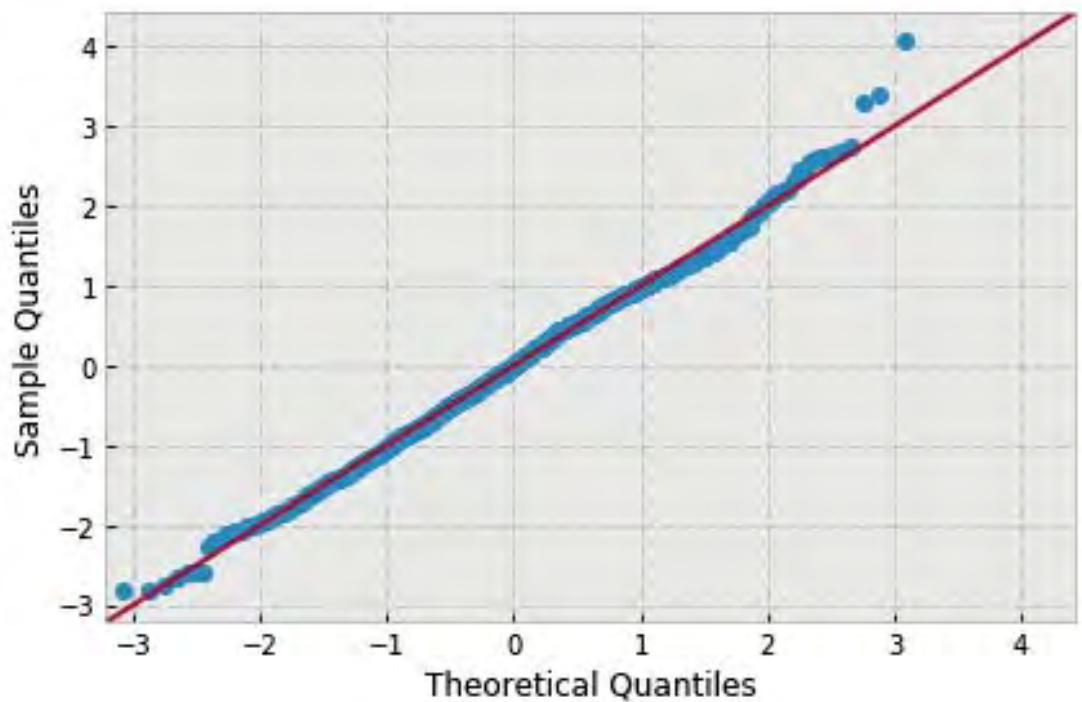
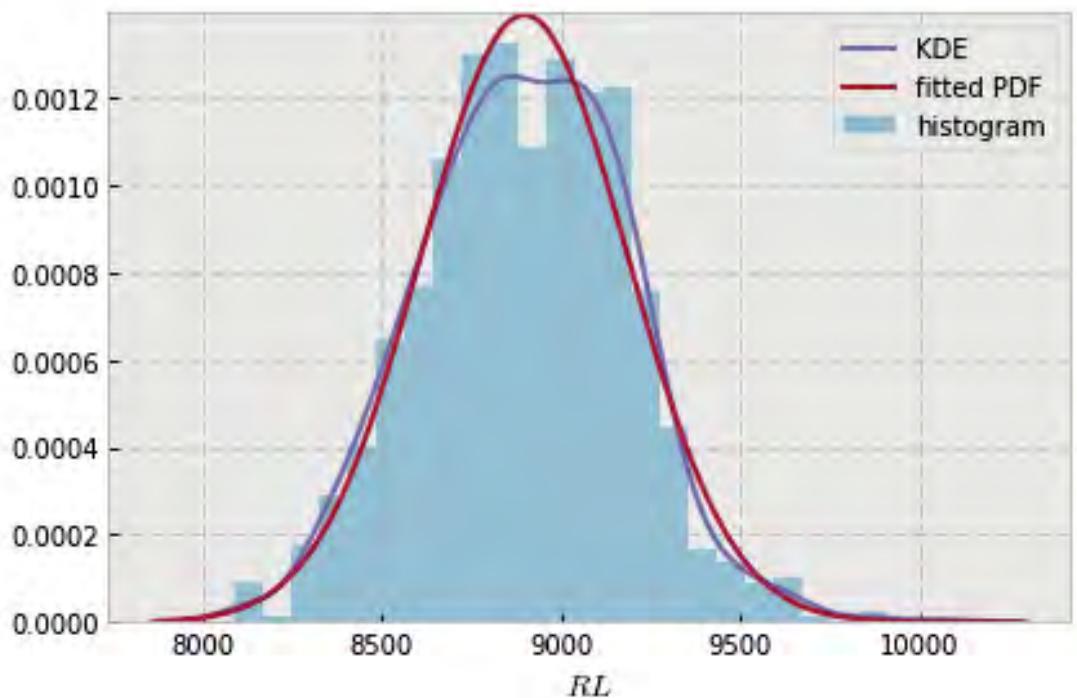


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=8885.237795545188, scale=301.3774584913905)
DescribeResult(nobs=1000, minmax=(7940.0224167996748, 9841.3184400439131)
               mean=8885.2377955451884, variance=90919.291778508312,
               skewness=-0.05827576953576213, kurtosis=-0.32636682108884907)
```

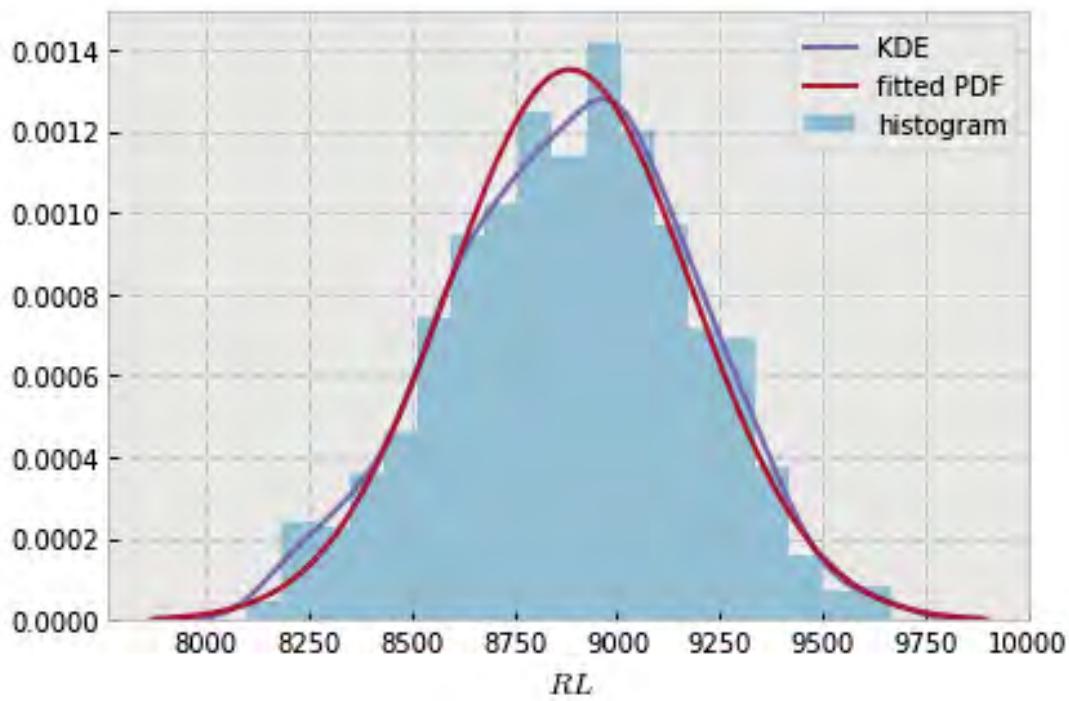


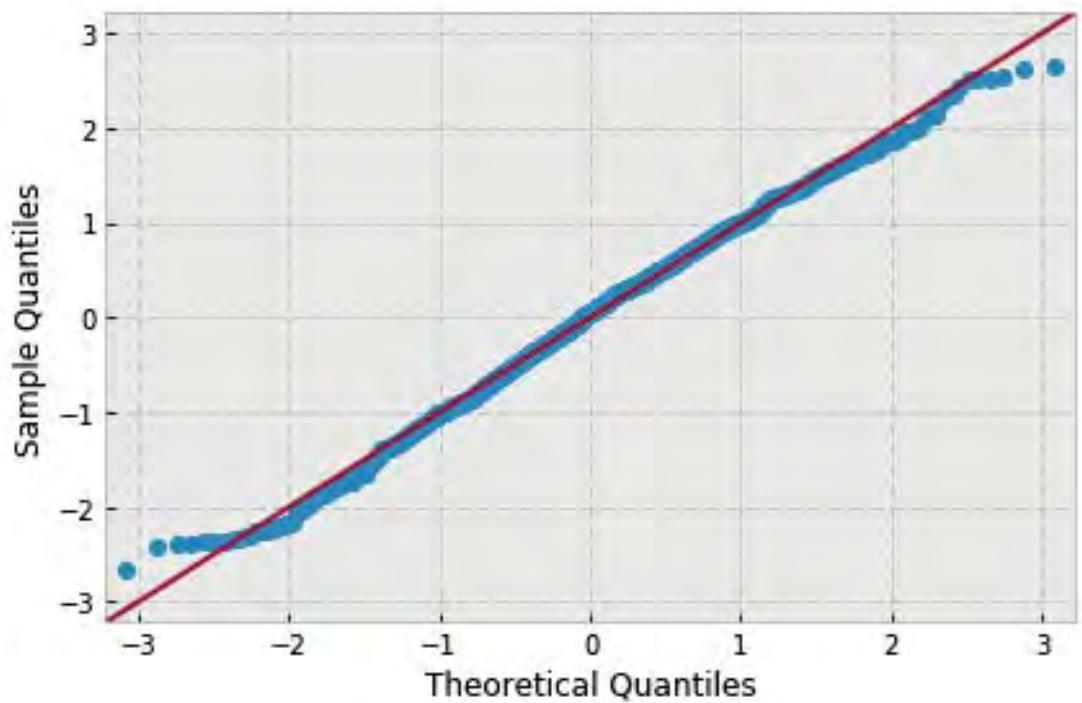


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=8896.608017686714, scale=287.14739898191385)
DescribeResult(nobs=1000, minmax=(8088.0252981169169, 10065.960188501833)
               mean=8896.6080176867144, variance=82536.164906985388,
               skewness=0.060182073482801954, kurtosis=0.10510345267401933)
```

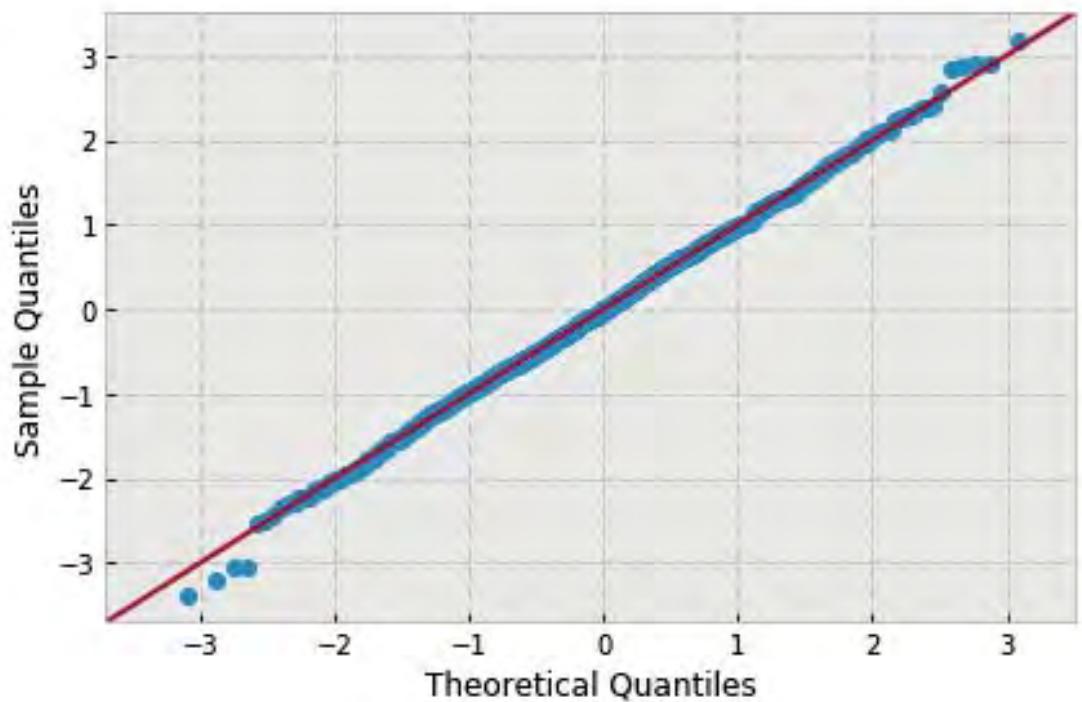
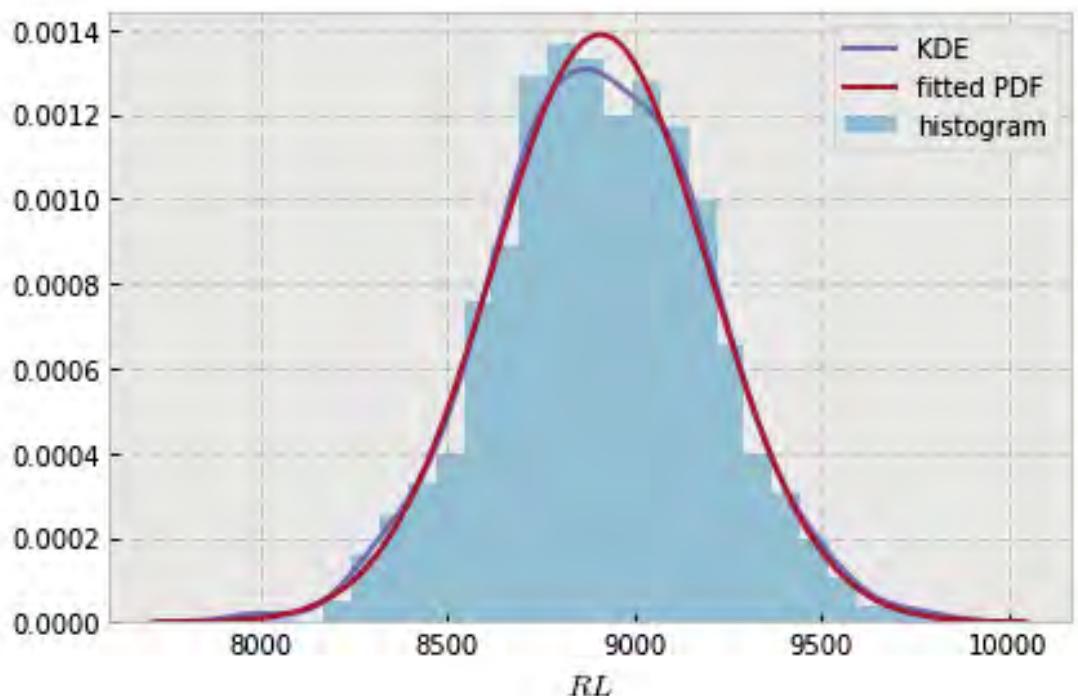


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=8884.981863815598, scale=295.3469140469768)
DescribeResult(nobs=1000, minmax=(8101.3833447968173, 9665.9848721160561)
               mean=8884.9818638155975, variance=87317.116753826136,
               skewness=-0.11497487302266225, kurtosis=-0.3432571250917271)
```

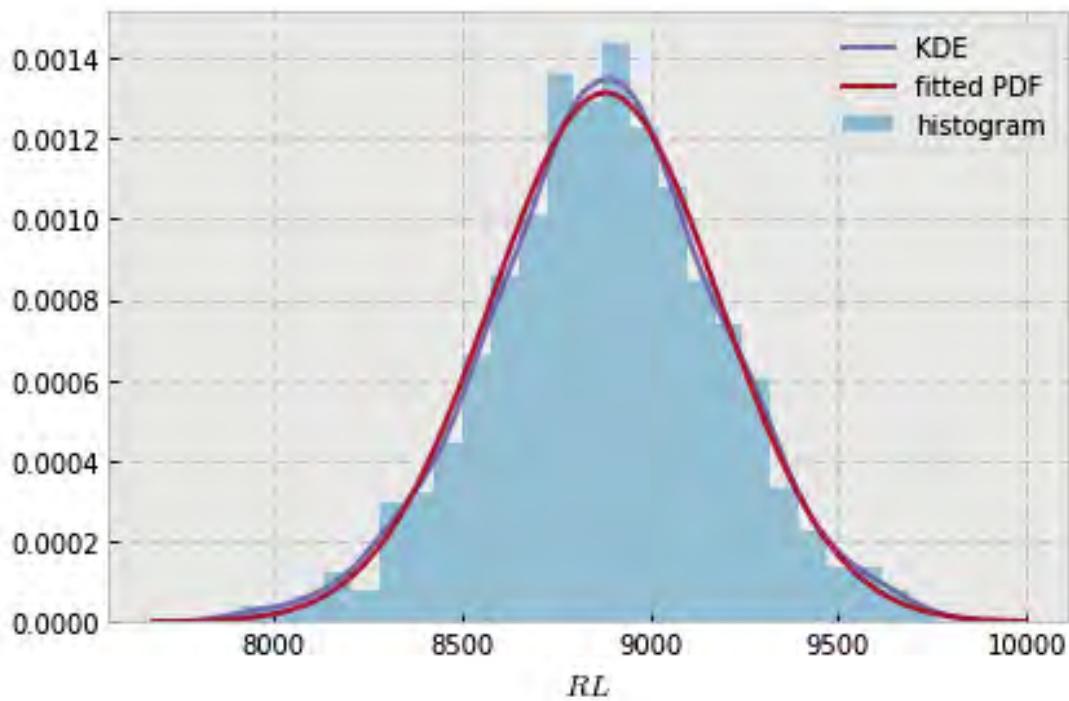


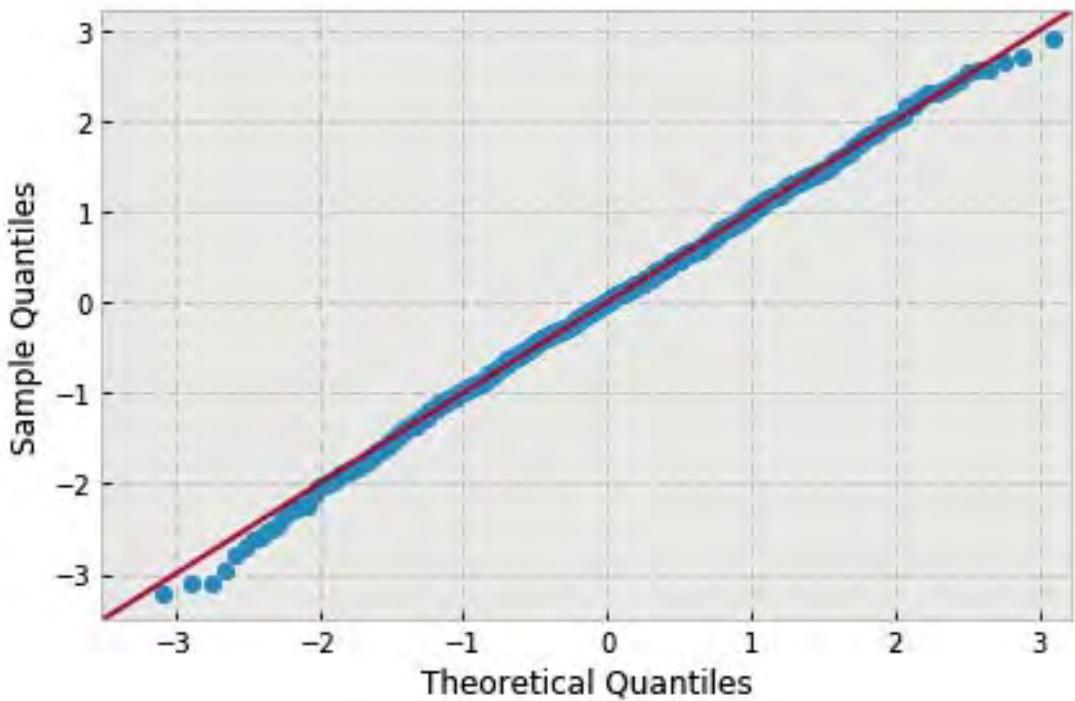


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=8909.889963416197, scale=287.3499877560367)
DescribeResult(nobs=1000, minmax=(7941.7428632239989, 9821.0408548768064)
               mean=8909.8899634161971, variance=82652.668131525948,
               skewness=-0.014273990063063373, kurtosis=0.056166833053407306)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=8881.587970969511, scale=303.7754744405431)
DescribeResult(nobs=1000, minmax=(7909.30493754894, 9765.0451107403551)
               mean=8881.5879709695109, variance=92371.910782359424,
               skewness=-0.08187450849267093, kurtosis=0.09133626571536091)
```

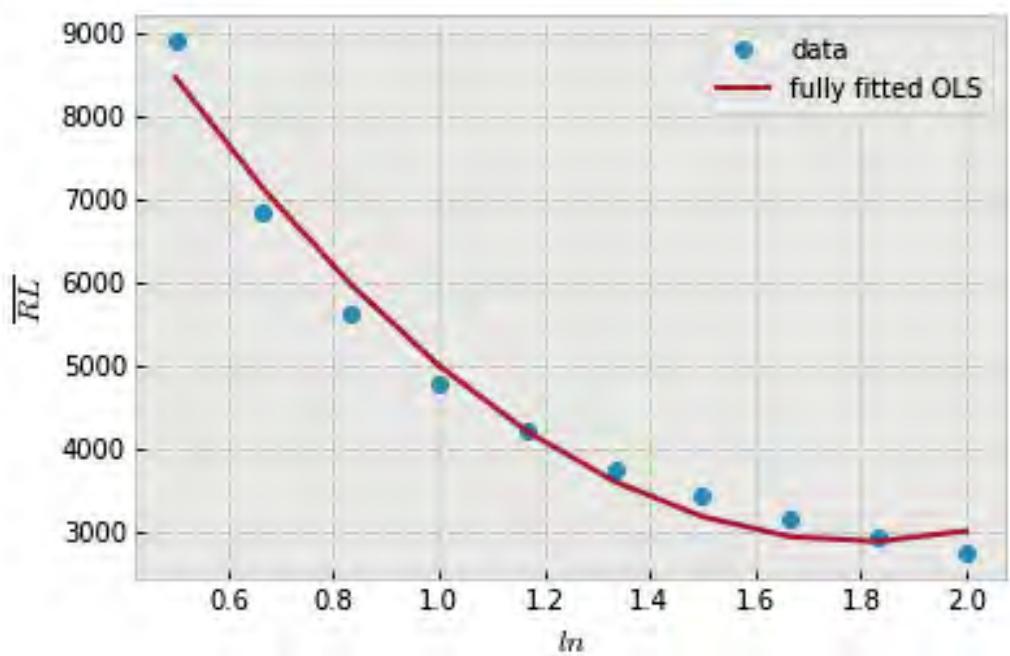




5.2 Parameter regression

```
In [16]: var_name = 'total_root_length'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly1')  
        mean_reg('nB', var_name, 'poly1')  
        mean_reg('r', var_name, 'poly1')  
        mean_reg('sigma', var_name, 'poly1')  
        mean_reg('N', var_name, 'poly1')
```

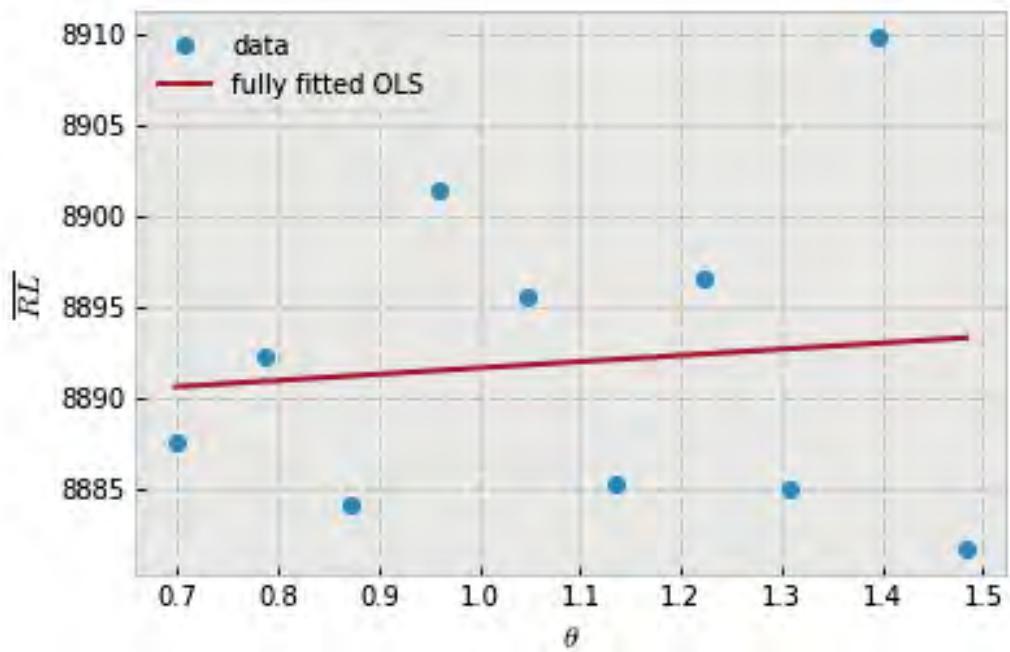
```
variable name: ln  
model: poly2  
r-squared: 0.9297134976359074  
explained variance: 0.9298455993259996
```



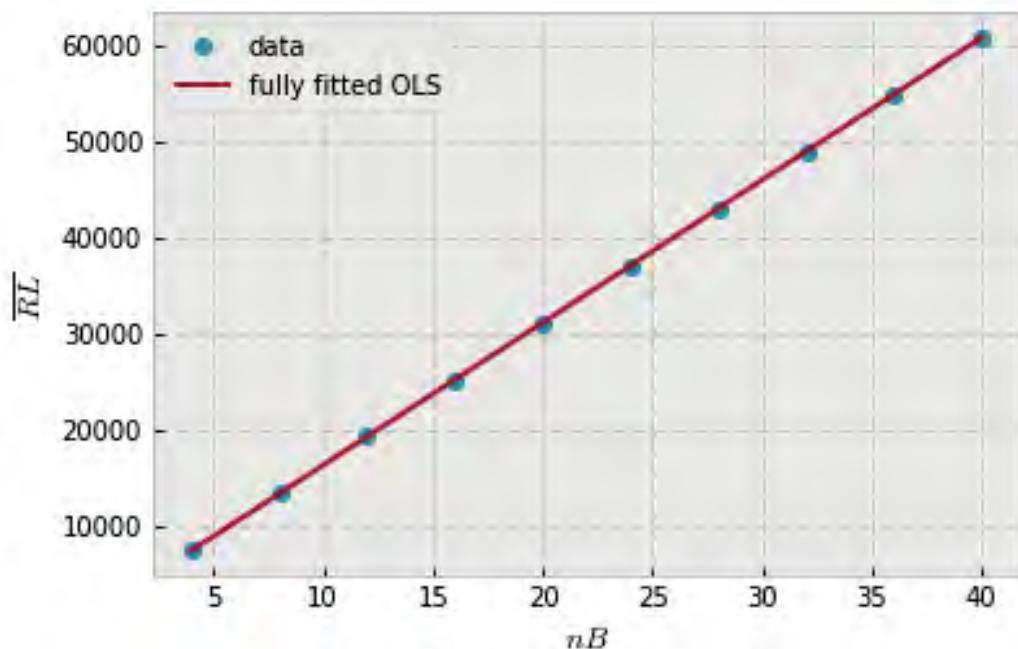
```

variable name: theta
model: poly1
r-squared: -0.7024425628746567
explained variance: -0.7010939118978465

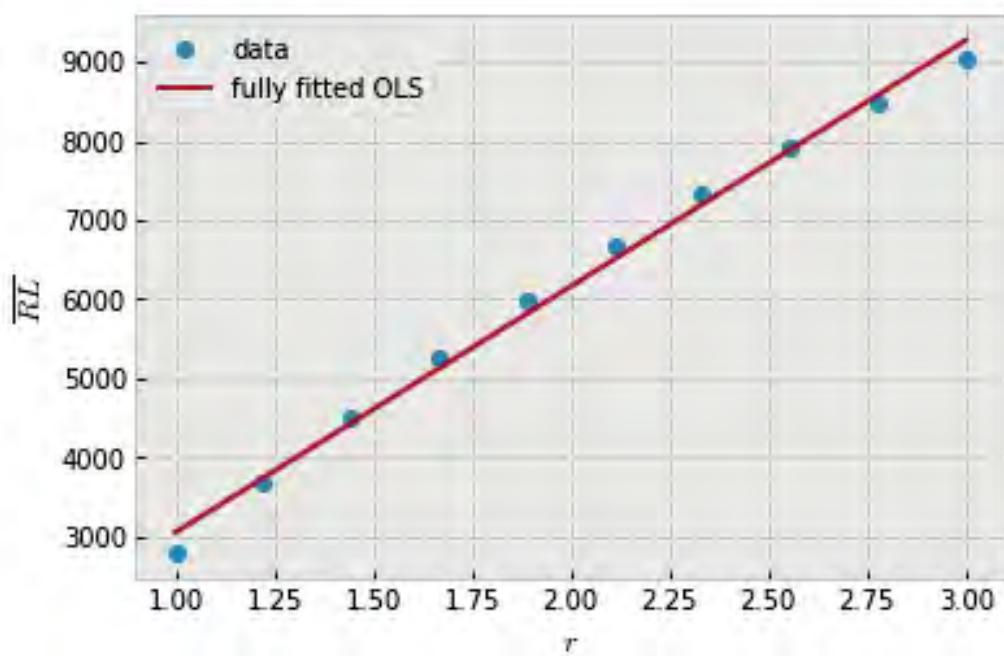
```



```
variable name: nB
model: poly1
r-squared: 0.9999987826703696
explained variance: 0.9999987844942784
```



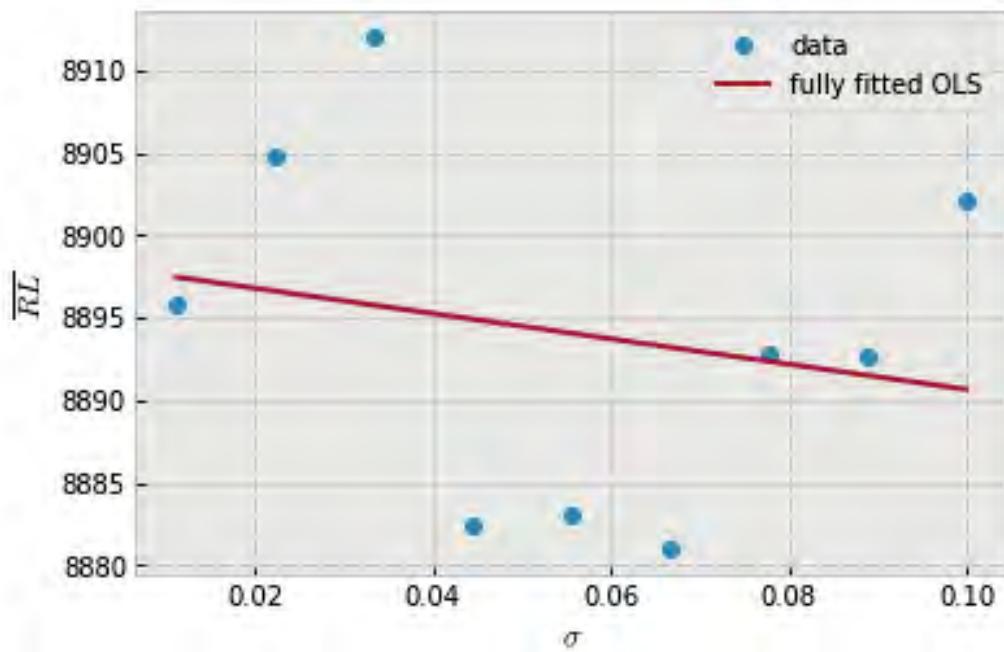
```
variable name: r
model: poly1
r-squared: 0.9891012756025228
explained variance: 0.989229692864304
```



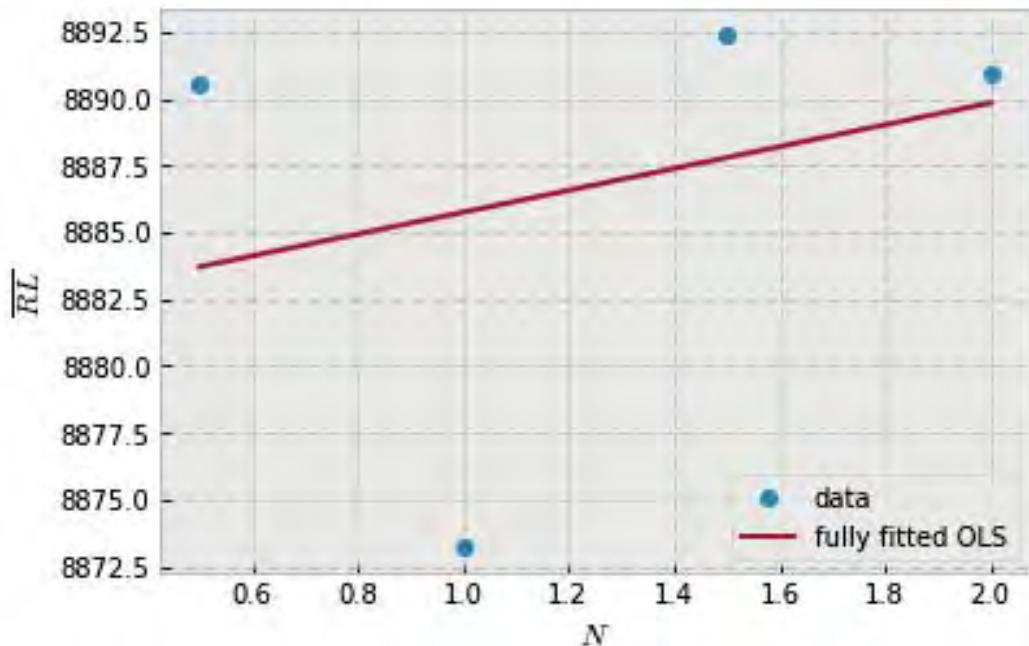
```

variable name: sigma
model: poly1
r-squared: -0.5100679957527106
explained variance: -0.5028739755407698

```



```
variable name: N
model: poly1
r-squared: -2.621483202587284
explained variance: -2.3909199112887927
```

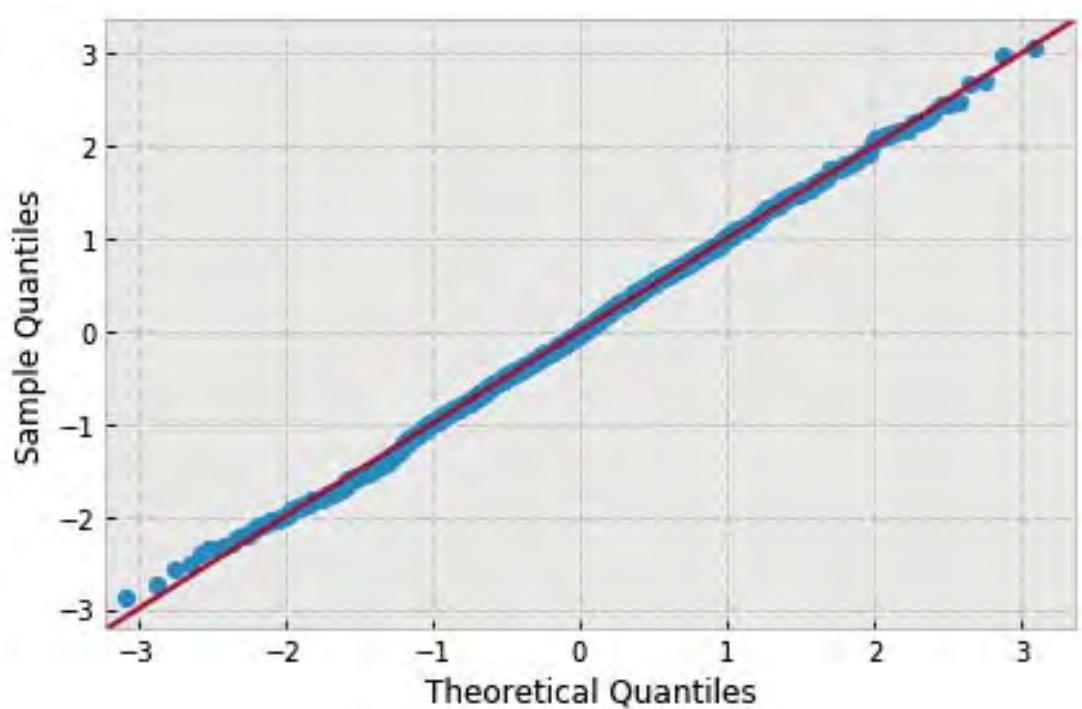
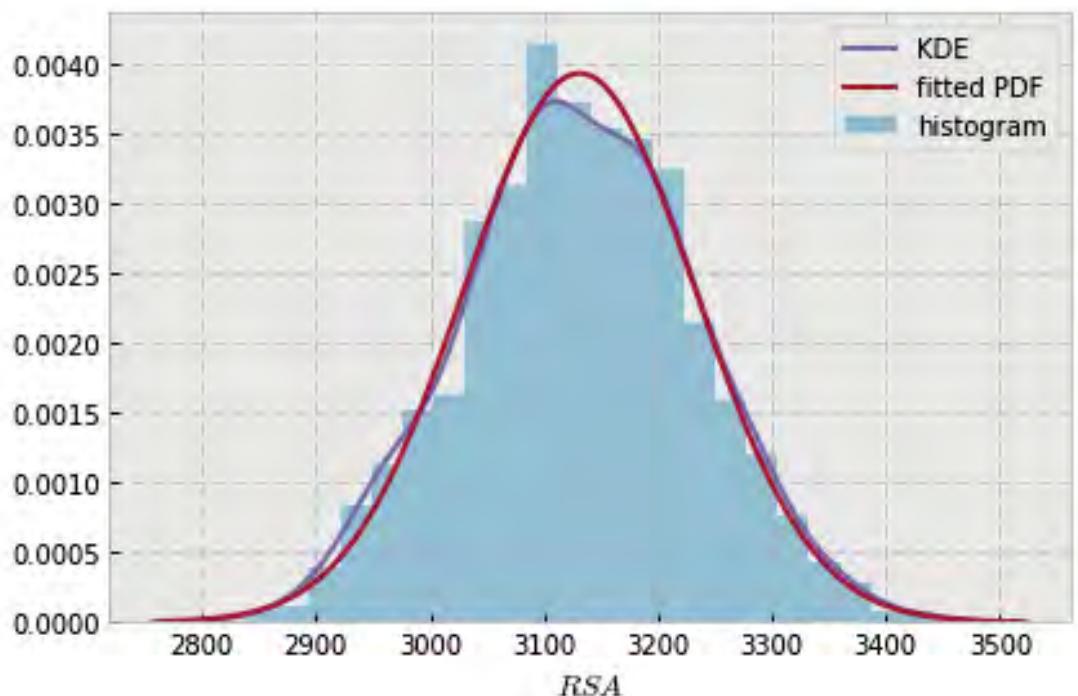


6 Total root surface area

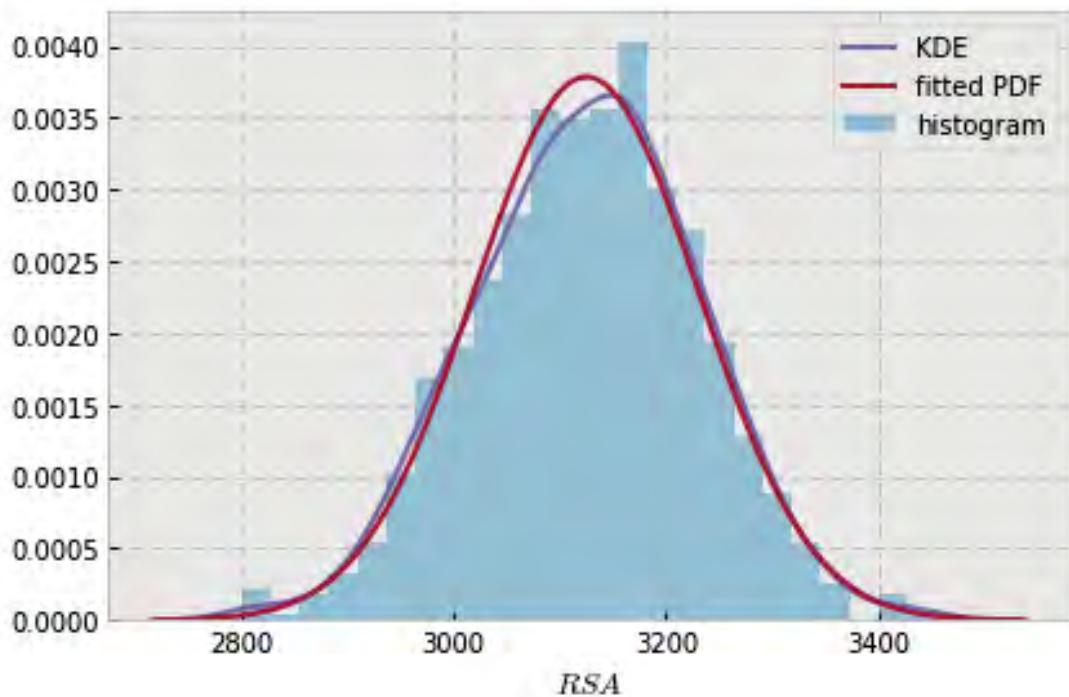
6.1 Probability distributions

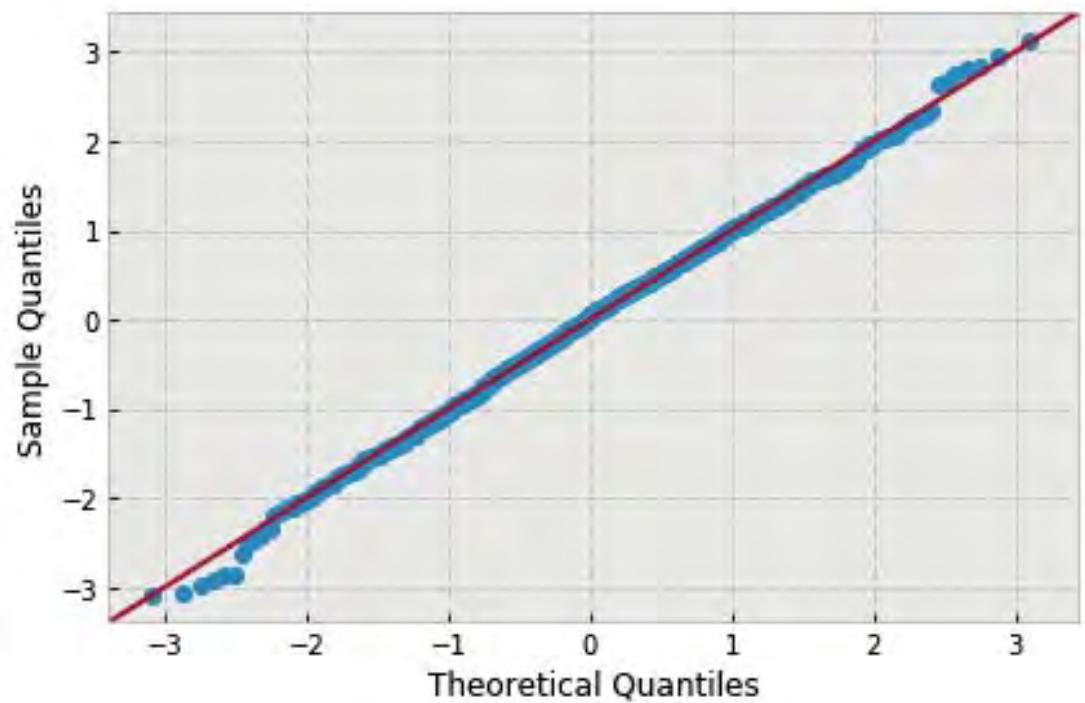
```
In [17]: plot_dist('total_root_surface_area')

variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=3131.2876212139995, scale=101.50440008313855)
DescribeResult(nobs=1000, minmax=(2838.4246830000002, 3442.2302709999999)
               mean=3131.2876212139995, variance=10313.456692930788,
               skewness=0.01080016267018221, kurtosis=-0.22351992728685843)
```

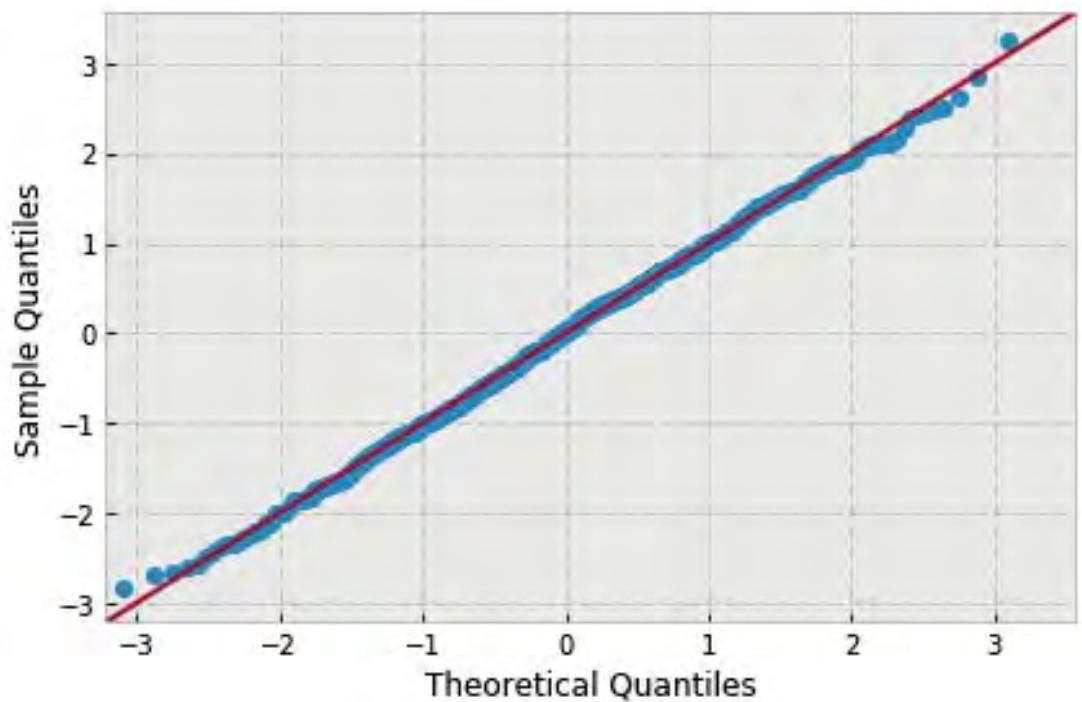
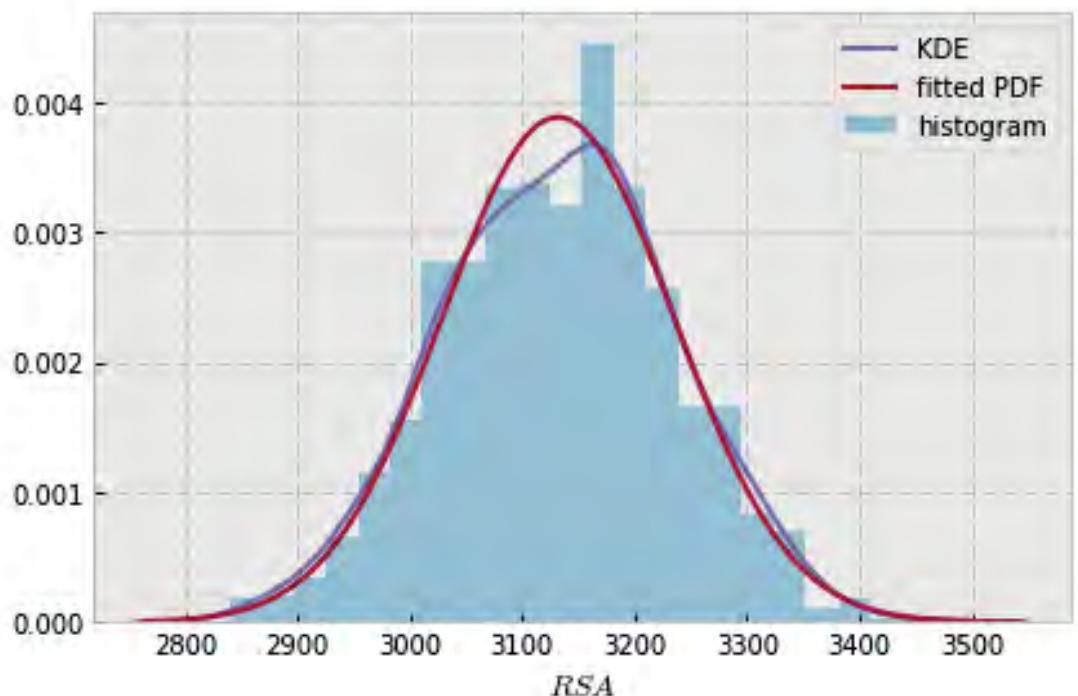


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=3125.196250888, scale=105.40504298983605)
DescribeResult(nobs=1000, minmax=(2800.373325999999, 3454.705645)
    mean=3125.1962508880001, variance=11121.344432121305,
    skewness=-0.076617523185813, kurtosis=-0.01339880264033022)
```

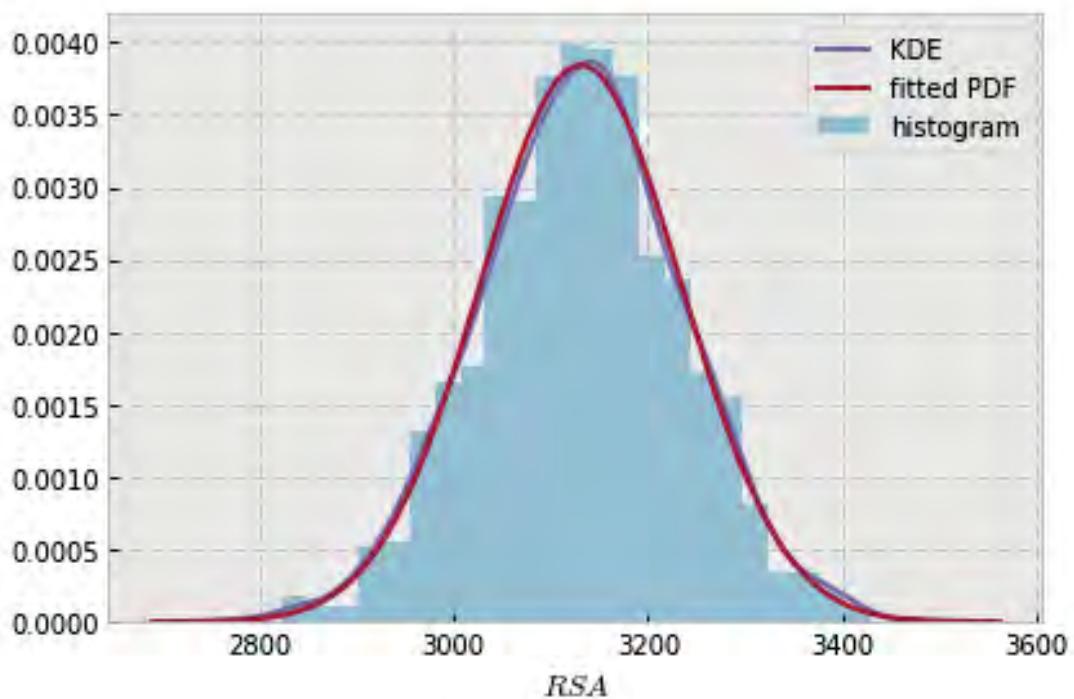


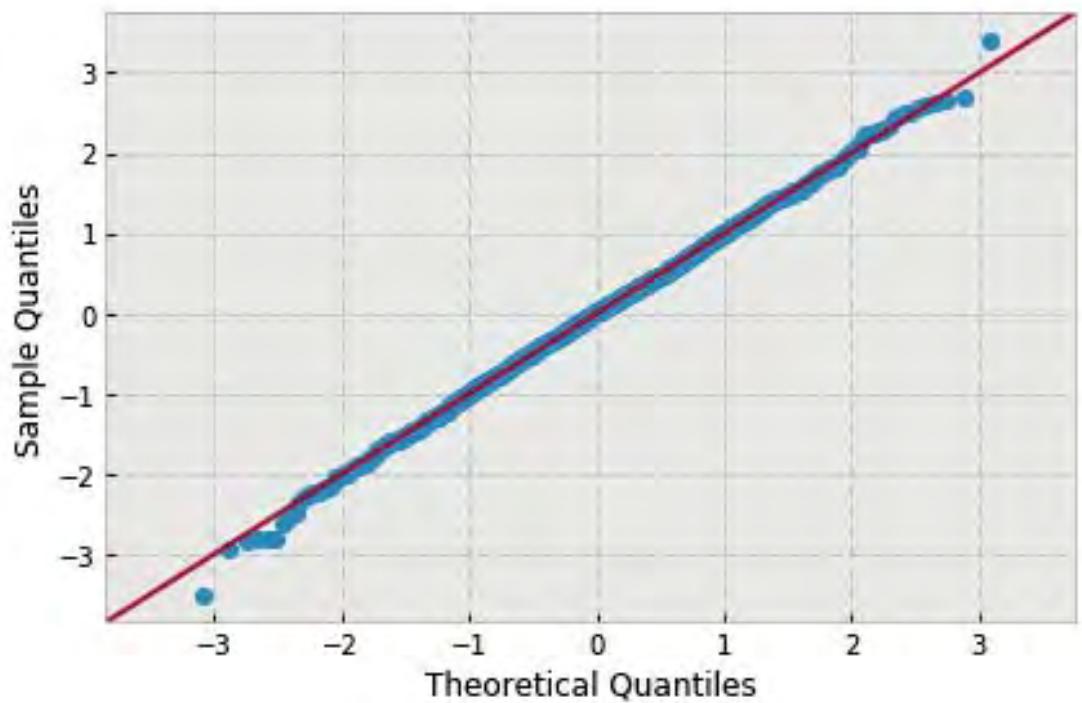


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=3131.853244065, scale=102.60712761733893)
DescribeResult(nobs=1000, minmax=(2839.986269, 3465.6309110000002)
               mean=3131.8532440650001, variance=10538.761399280156,
               skewness=-0.025240995162718834, kurtosis=-0.234375106316715)
```

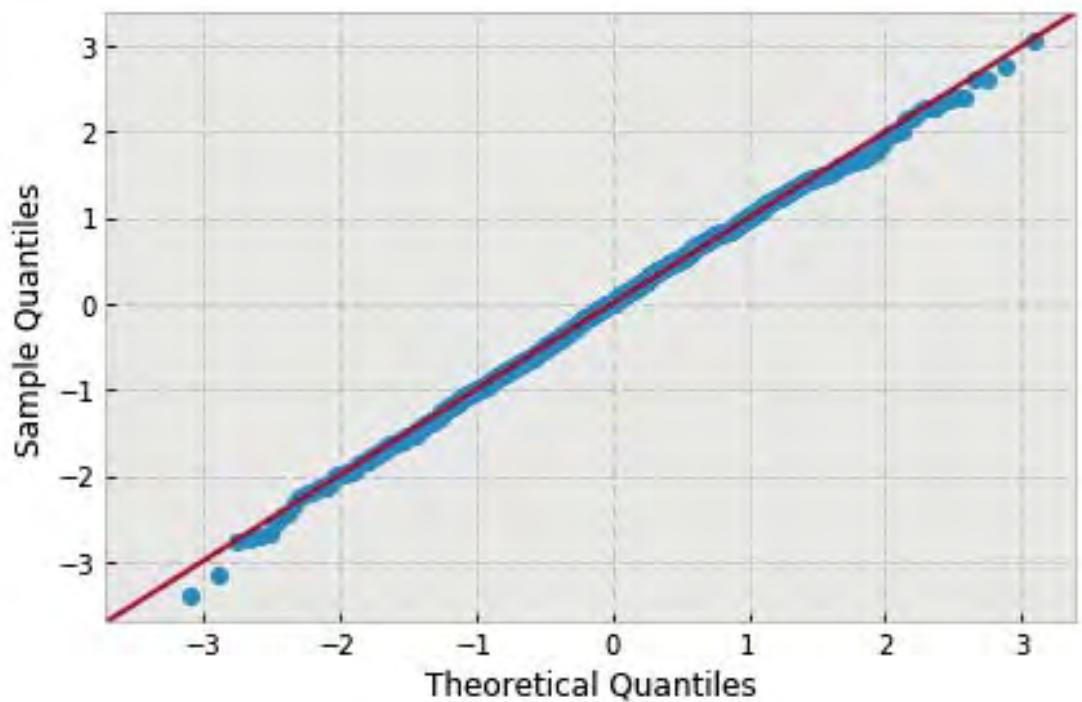
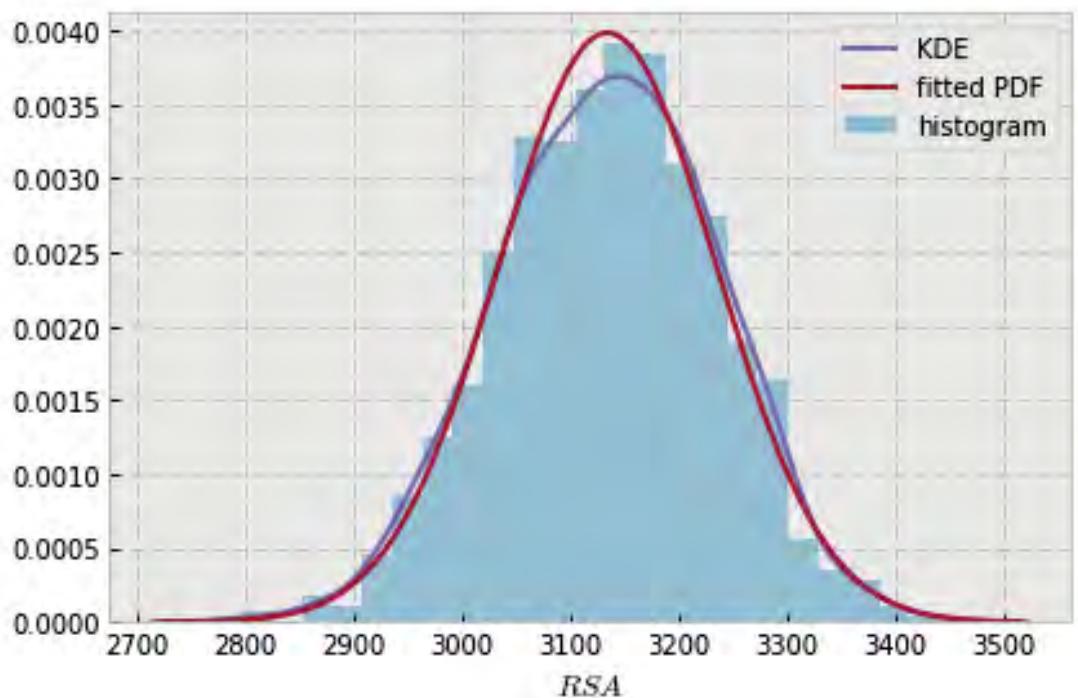


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=3130.7434038300003, scale=103.89043137847723)
DescribeResult(nobs=1000, minmax=(2767.162245, 3483.72487)
    mean=3130.743403830003, variance=10804.025757763849,
    skewness=-0.03994483074551858, kurtosis=0.07698597154542997)
```

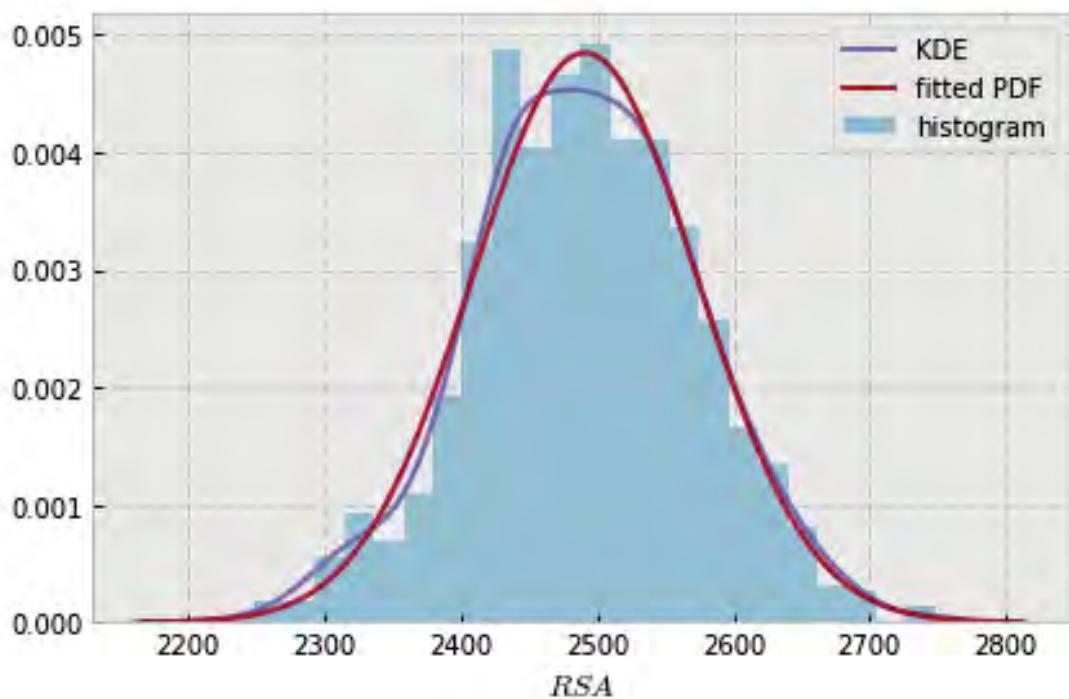


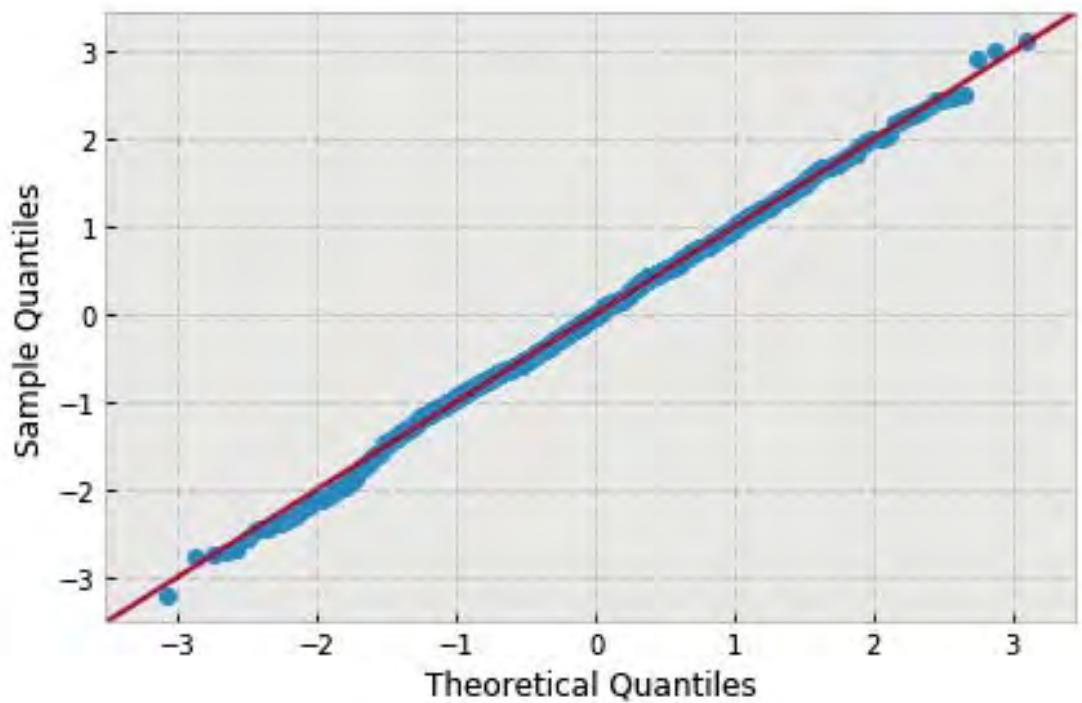


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=3133.9899223879997, scale=100.13462029867279)
DescribeResult(nobs=1000, minmax=(2795.214829999999, 3440.3626840000002)
               mean=3133.9899223879997, variance=10036.979161520892,
               skewness=-0.10388945608652751, kurtosis=-0.155107707478094)
```

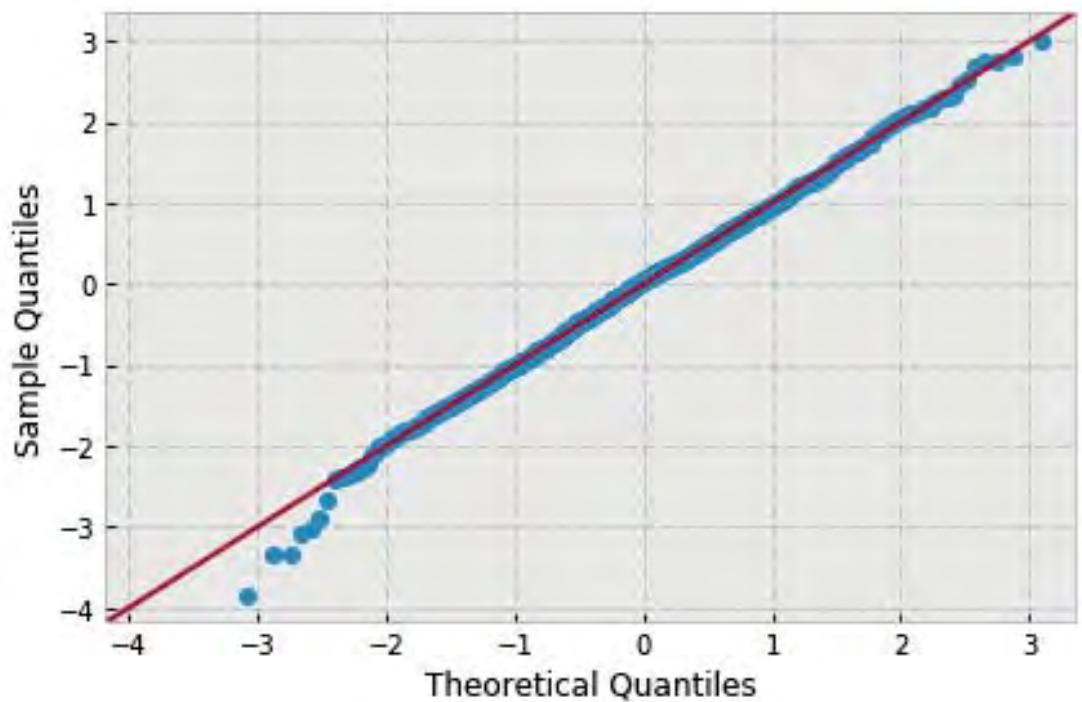
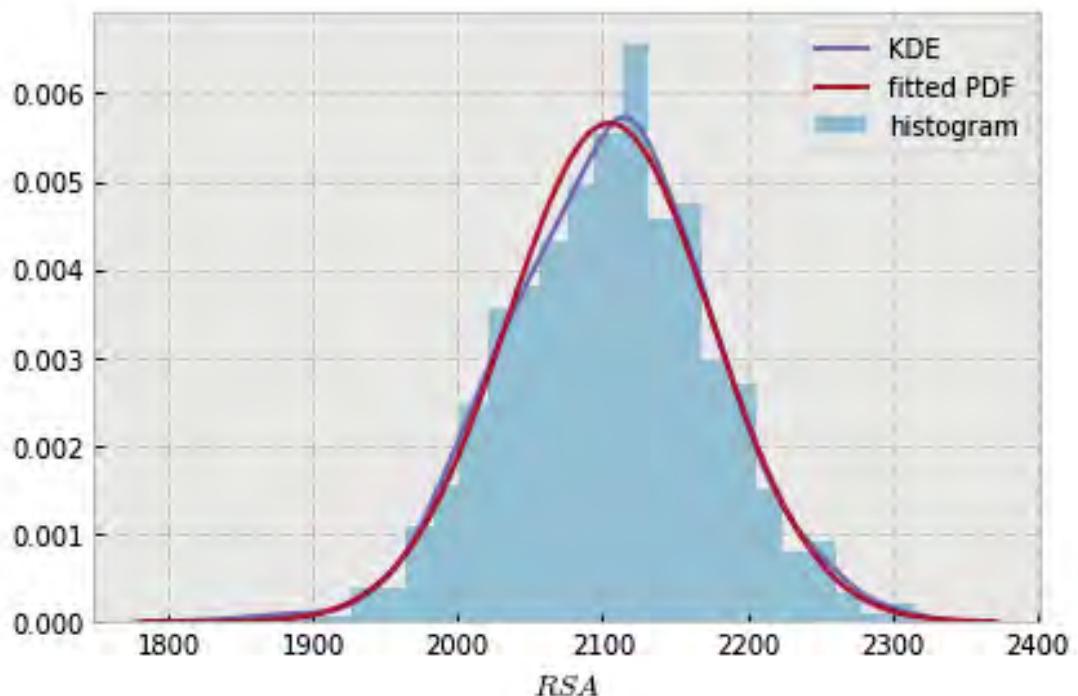


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=2490.9434045989997, scale=82.31769768753115)
DescribeResult(nobs=1000, minmax=(2227.36409, 2748.452072999999)
               mean=2490.9434045989997, variance=6782.9863389146858,
               skewness=-0.042516860382876435, kurtosis=0.010071239513129271)
```

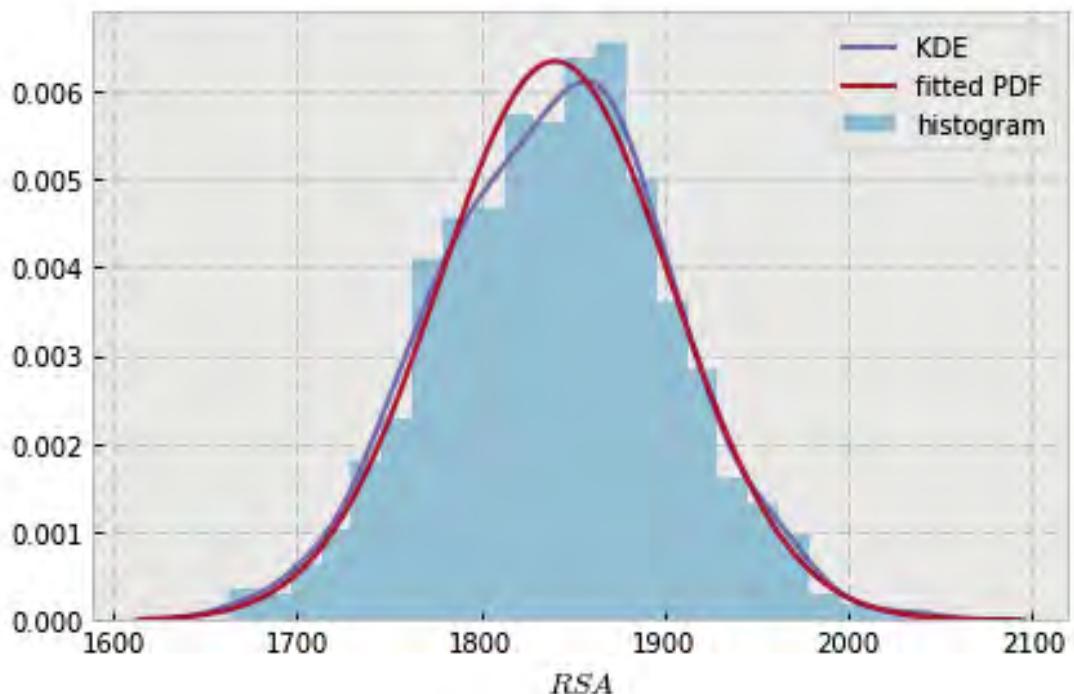


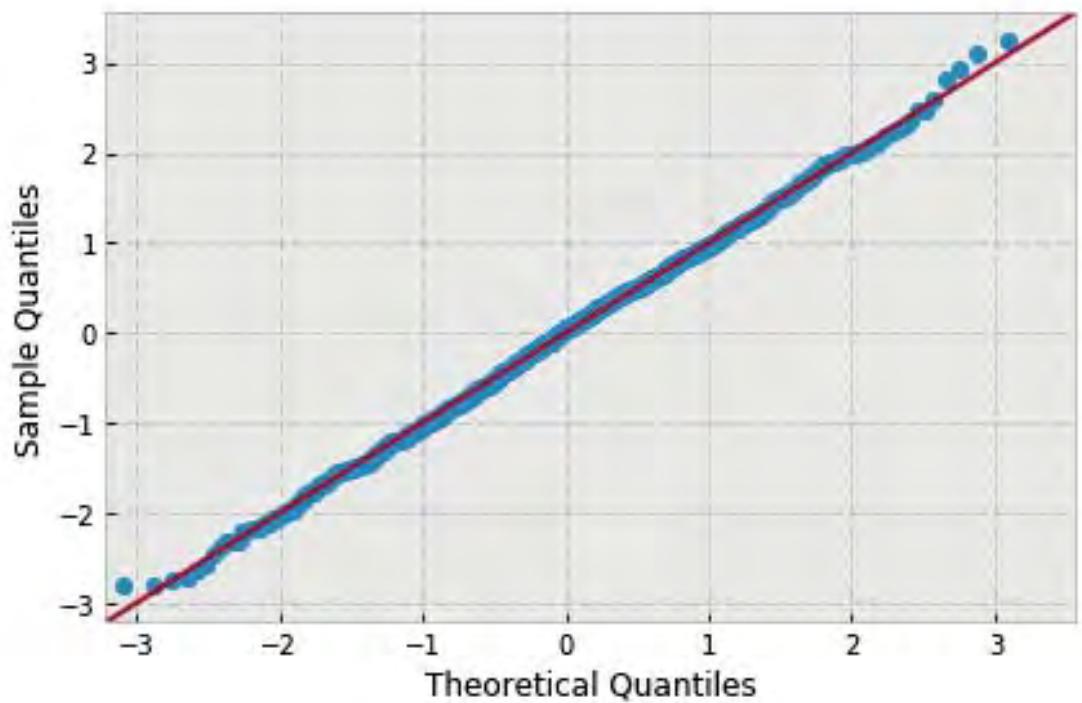


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=2104.969479507, scale=70.49775362464702)
DescribeResult(nobs=1000, minmax=(1834.588649999999, 2316.758503999999)
               mean=2104.969479507, variance=4974.9081742957278,
               skewness=-0.10426518367370975, kurtosis=0.19657049335478005)
```

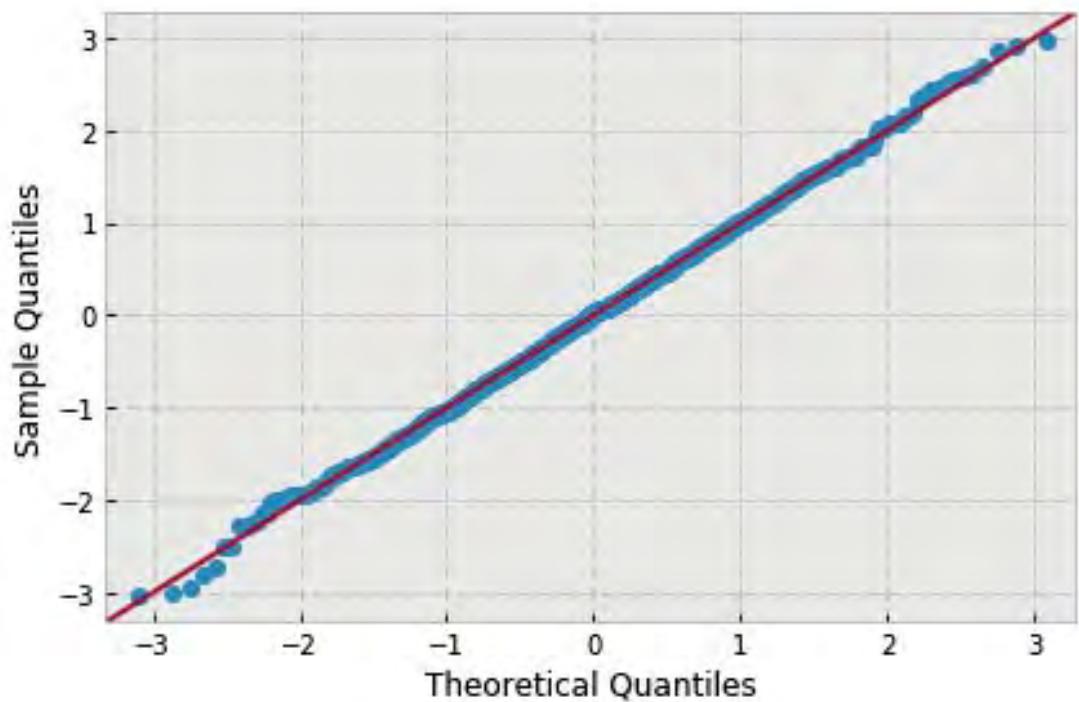
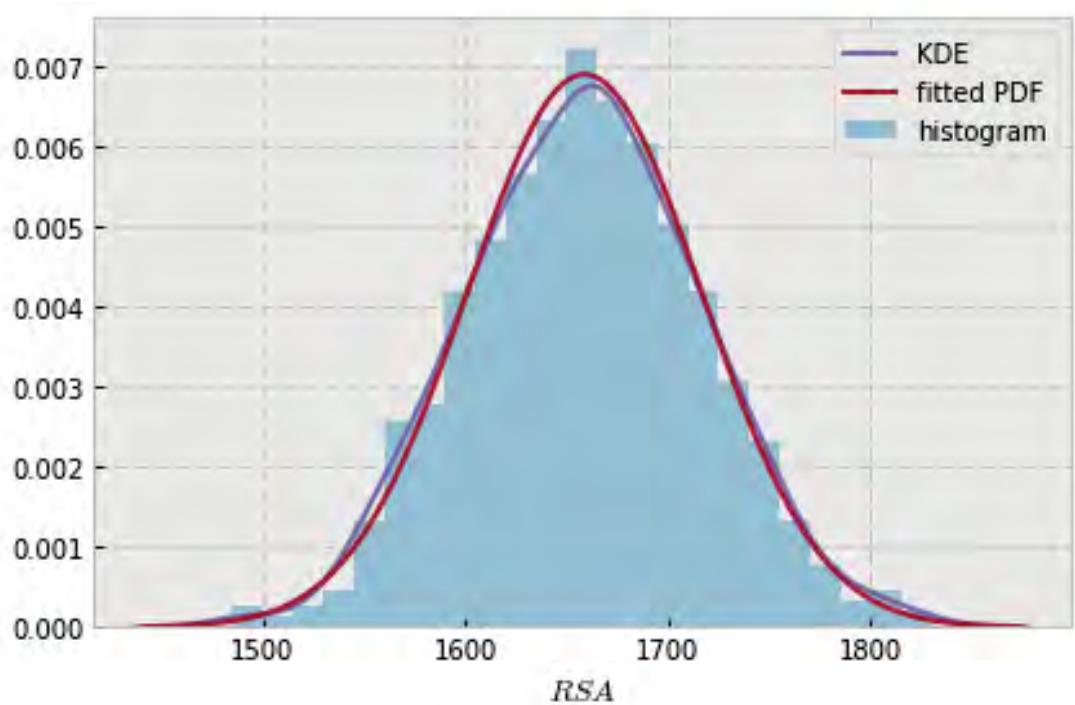


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=1840.3293523240002, scale=62.999218684689936)
DescribeResult(nobs=1000, minmax=(1663.020845999999, 2045.096994)
               mean=1840.3293523240002, variance=3972.8744293106965,
               skewness=-0.006961758432932275, kurtosis=-0.11511598872258588)
```

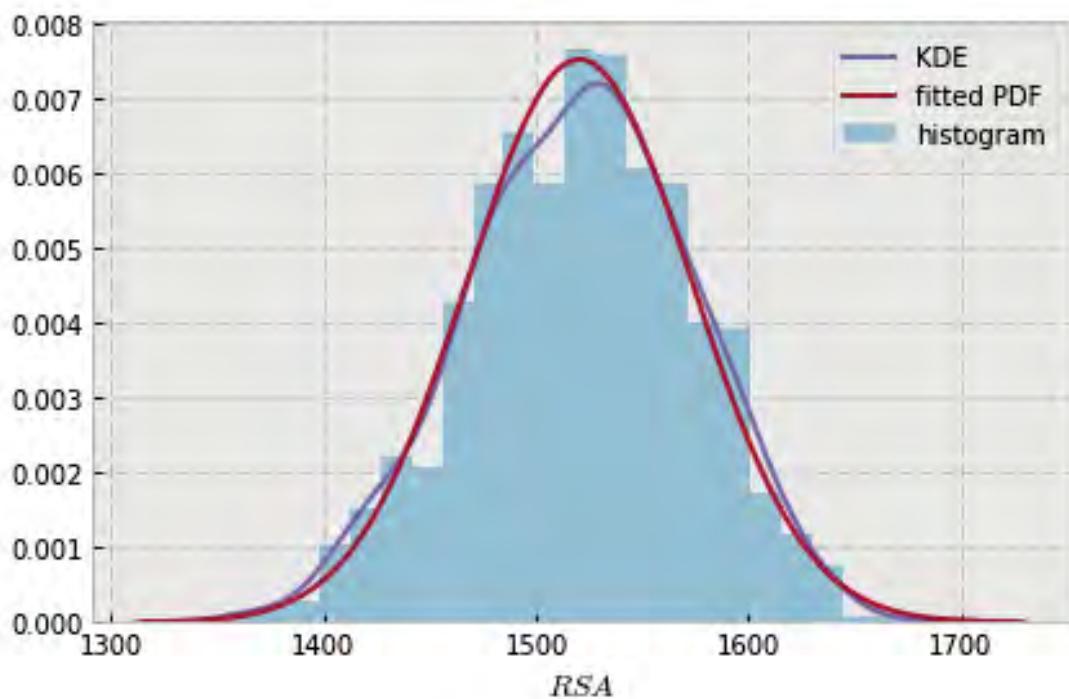


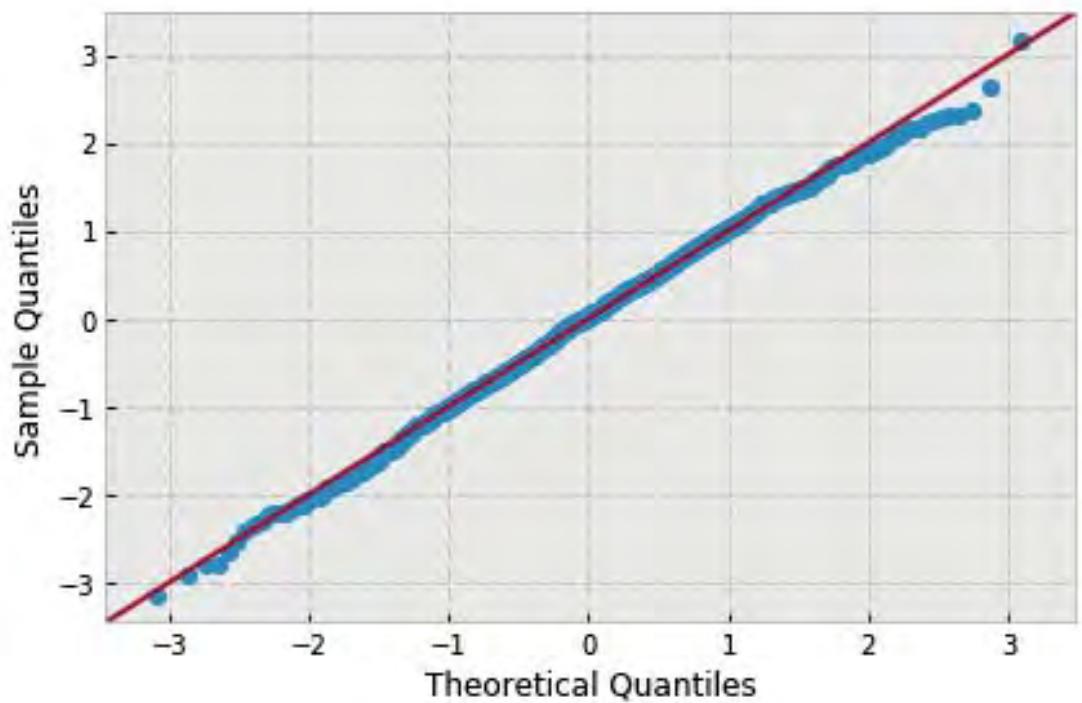


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=1658.671955247, scale=57.815129710446165)
DescribeResult(nobs=1000, minmax=(1484.124573000001, 1830.353959)
               mean=1658.6719552469999, variance=3345.9351585943095,
               skewness=0.02831859064367777, kurtosis=-0.09470756332564934)
```

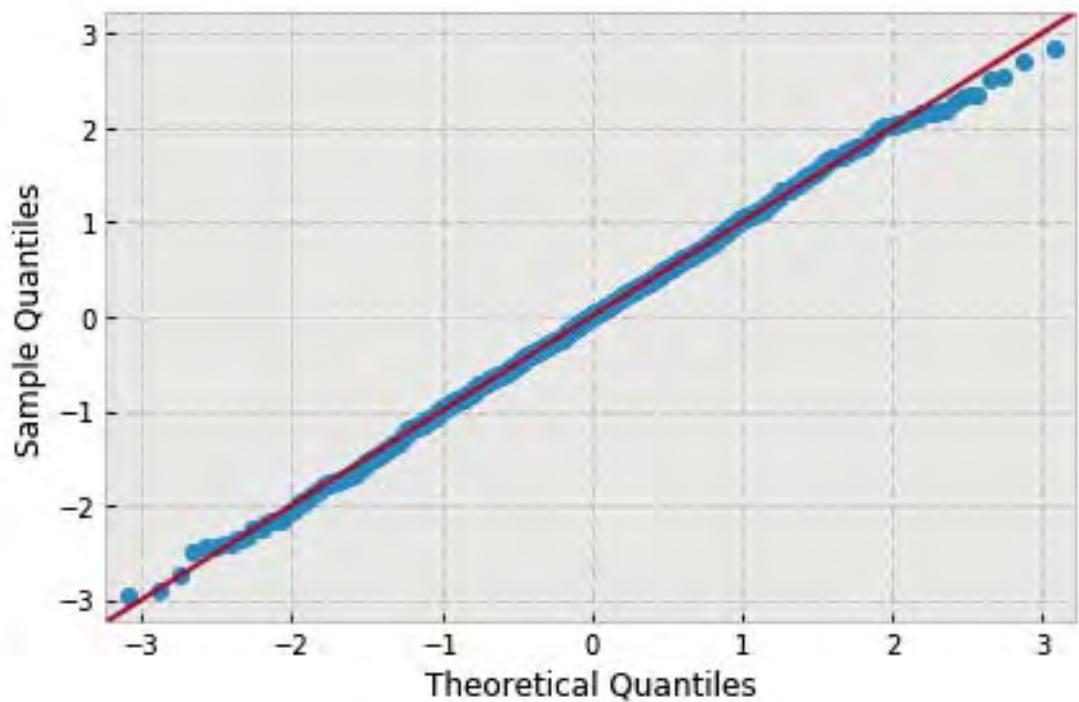
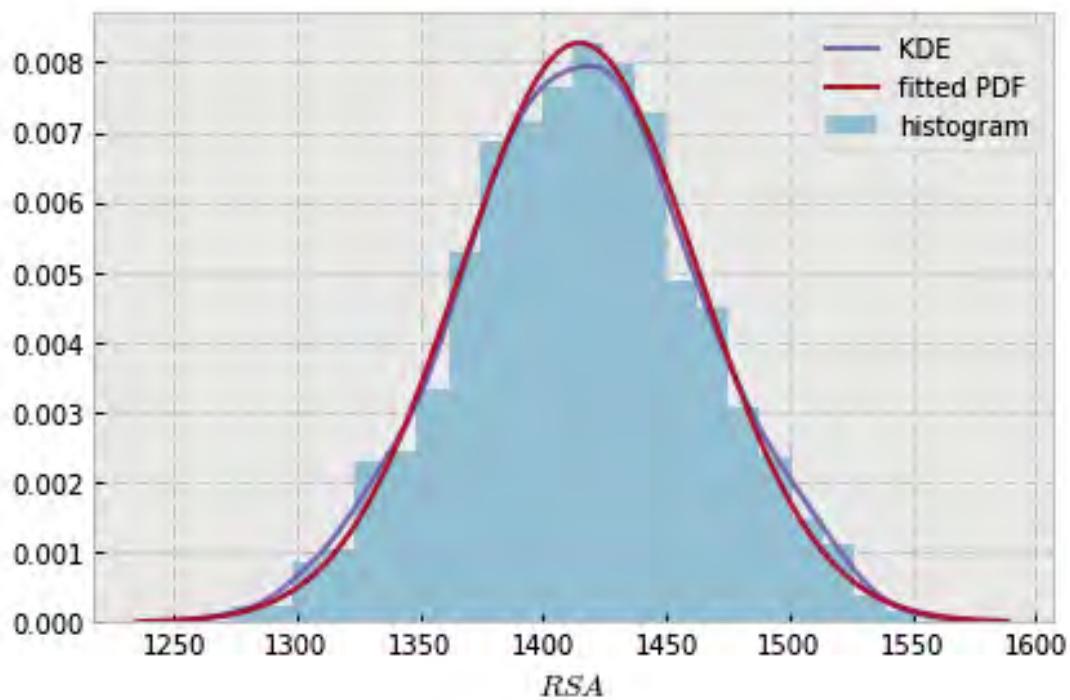


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=1520.334157062, scale=52.9501413775849)
DescribeResult(nobs=1000, minmax=(1354.175606, 1687.626906)
    mean=1520.334157062, variance=2806.5239959021301,
    skewness=-0.12884564501093296, kurtosis=-0.22822779407861704)
```

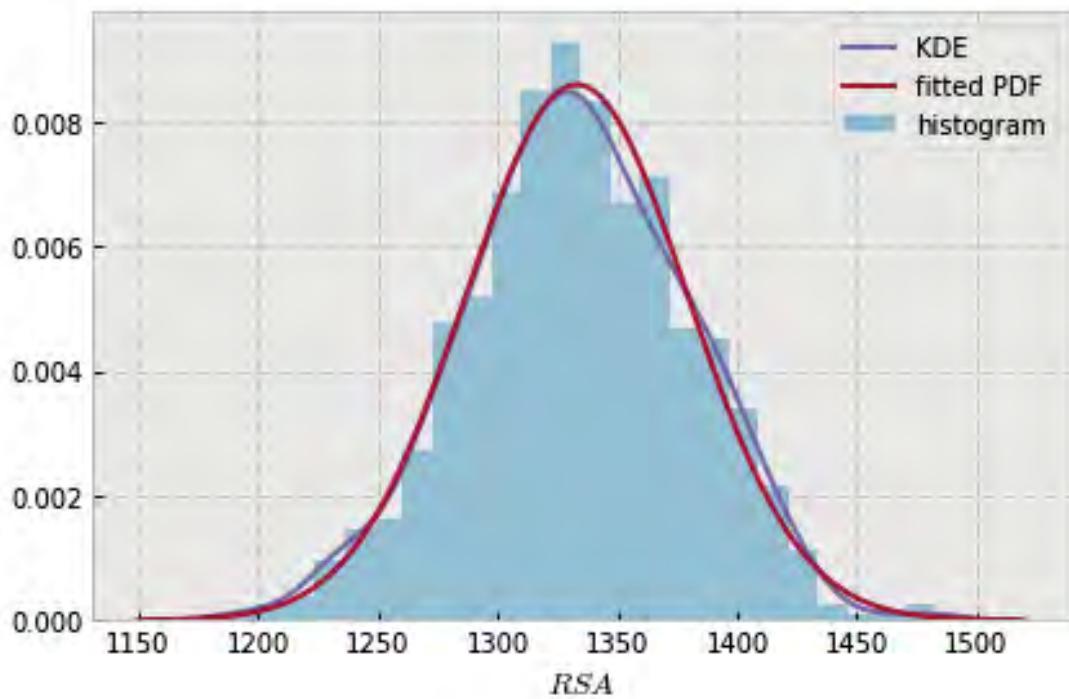


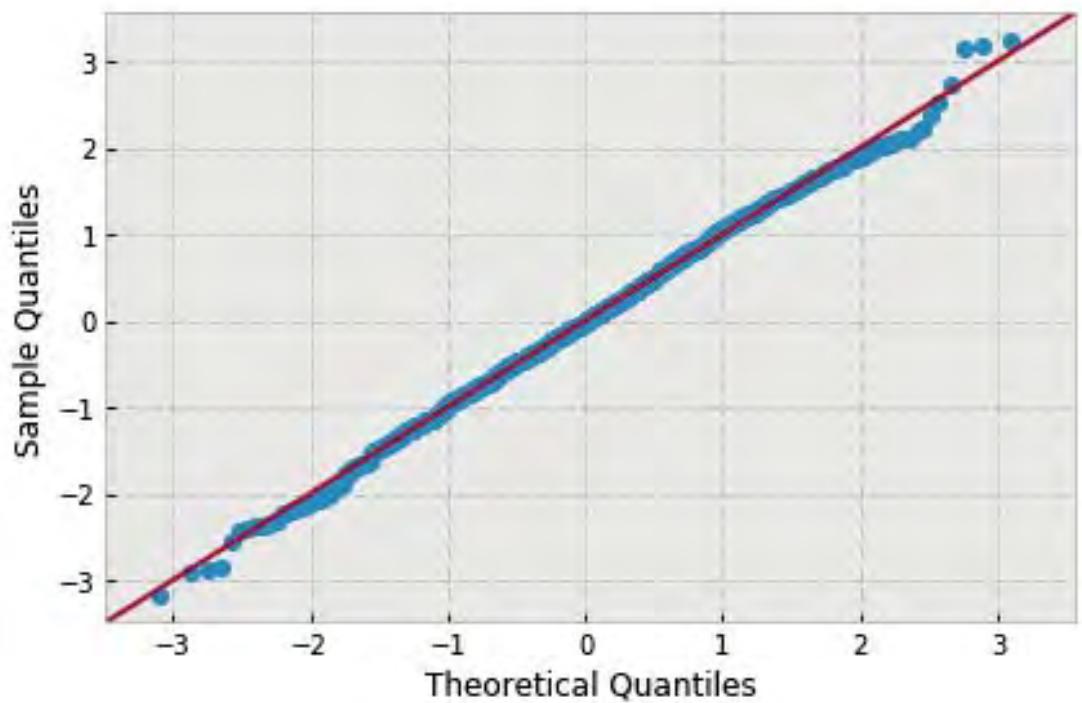


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=1414.7847791610002, scale=48.199346781365016)
DescribeResult(nobs=1000, minmax=(1272.7391990000001, 1551.4320769999999)
               mean=1414.7847791610002, variance=2325.502532682965,
               skewness=-0.0242390478690617, kurtosis=-0.2066254739906559)
```

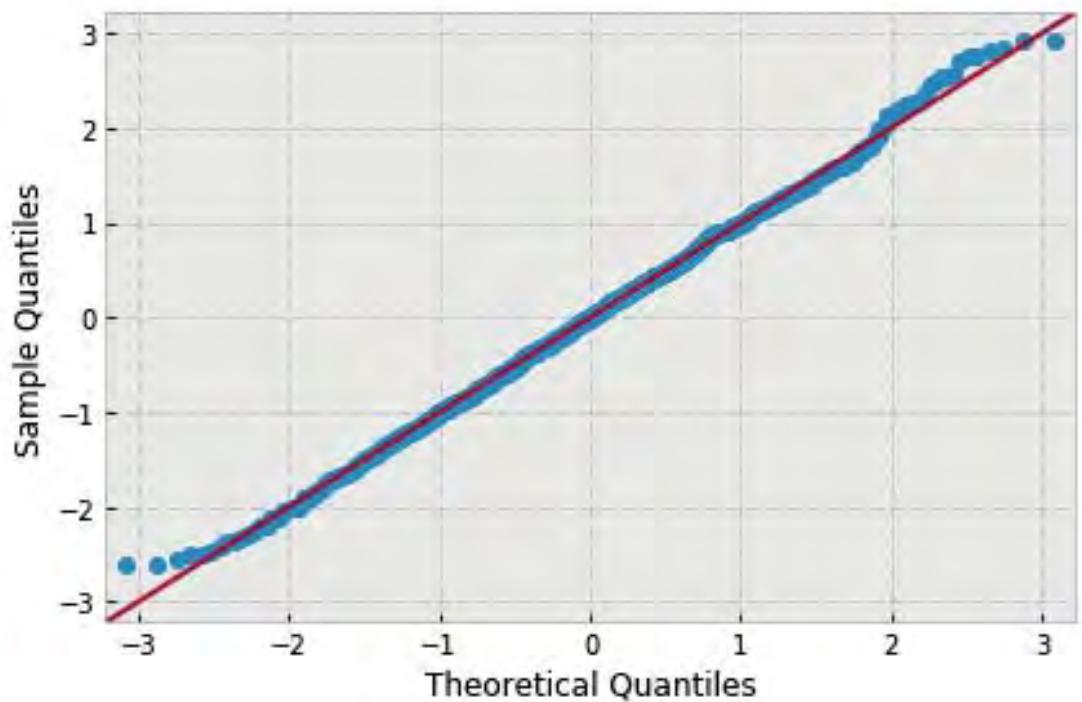
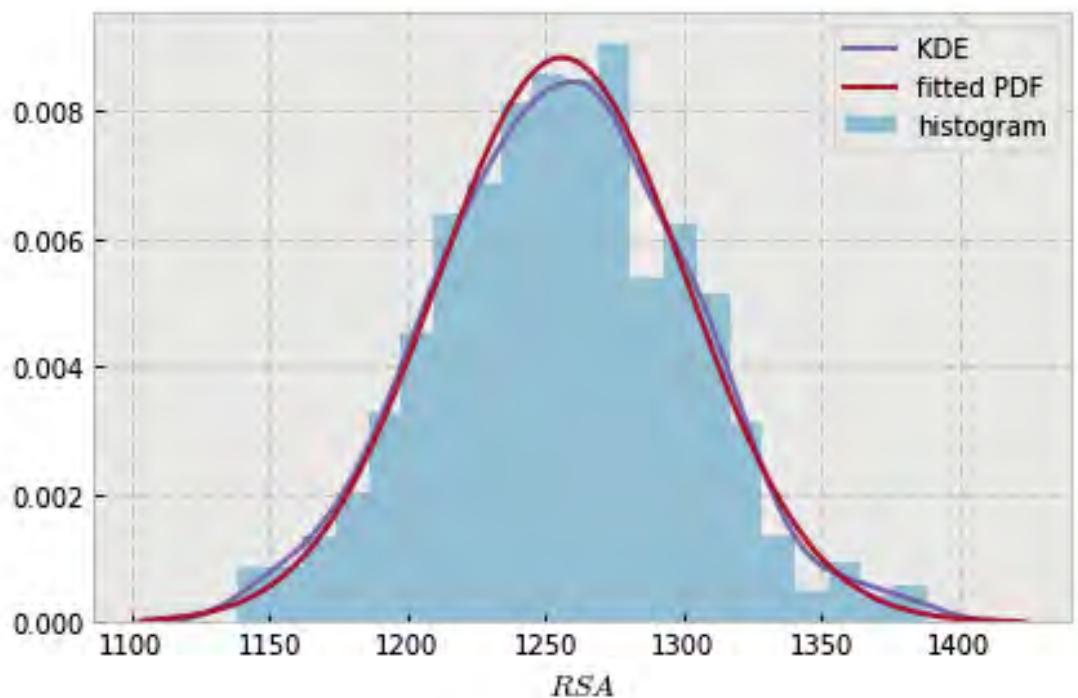


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=1332.9750984749999, scale=46.39189454210657)
DescribeResult(nobs=1000, minmax=(1186.203759, 1483.031291)
    mean=1332.9750984749999, variance=2154.3622414473848,
    skewness=-0.0471367329783479, kurtosis=-0.0645531273429314)
```

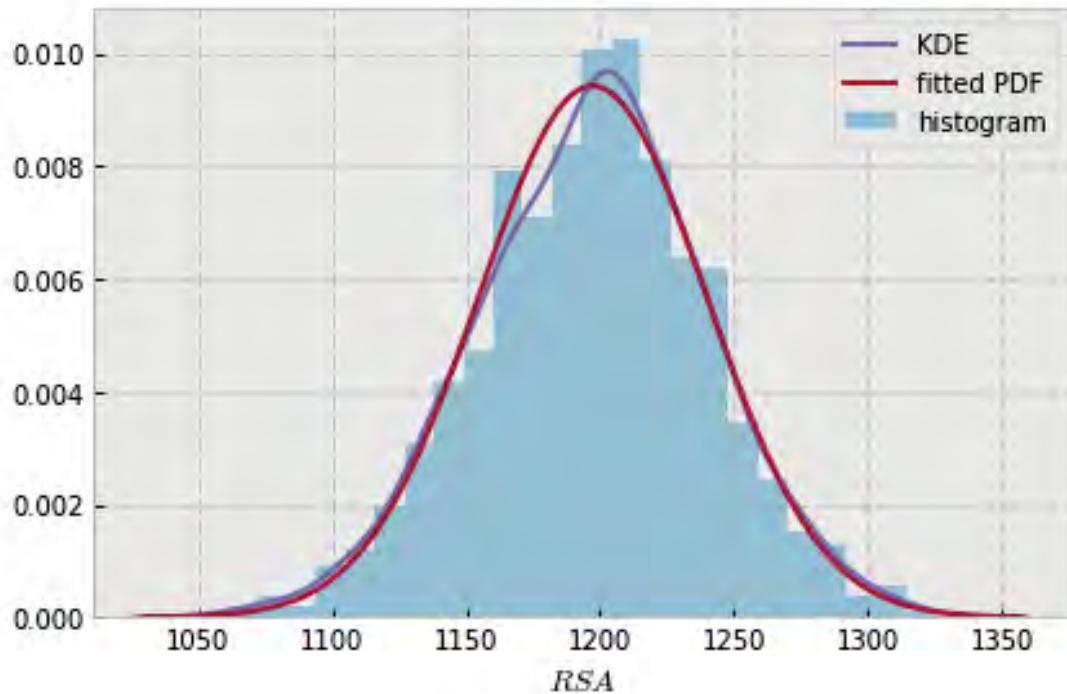


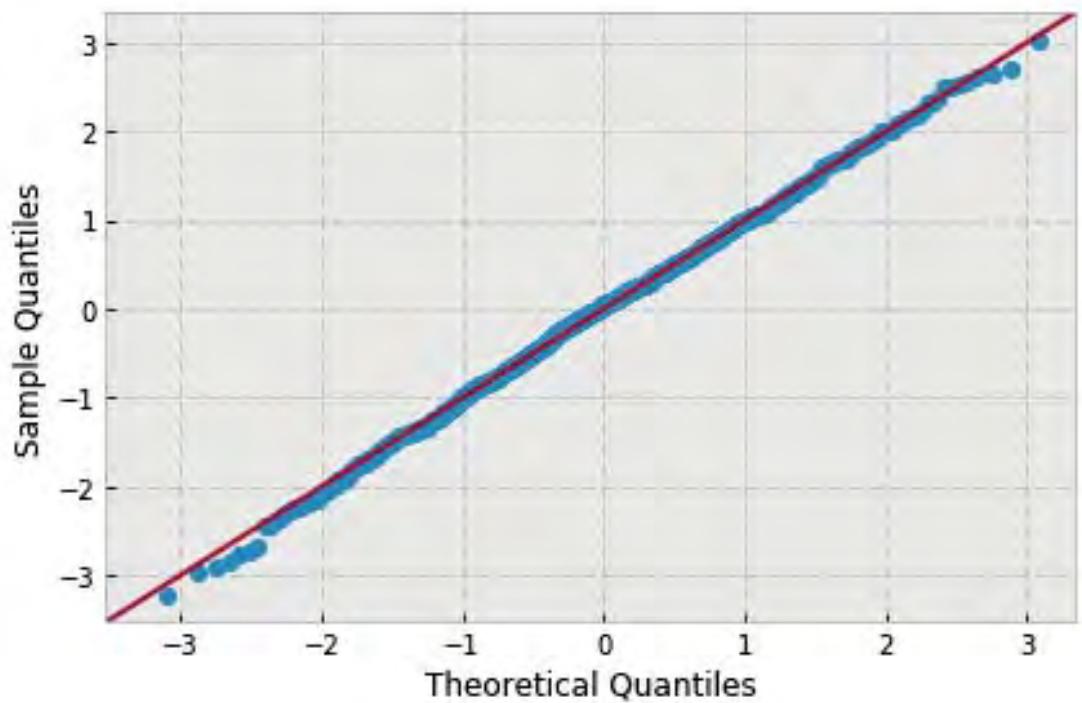


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=1255.9900557659998, scale=45.12357134418236)
DescribeResult(nobs=1000, minmax=(1138.294861000001, 1387.992162999999)
               mean=1255.9900557659998, variance=2038.1748657192345,
               skewness=0.048980941342156116, kurtosis=-0.098616451124105)
```

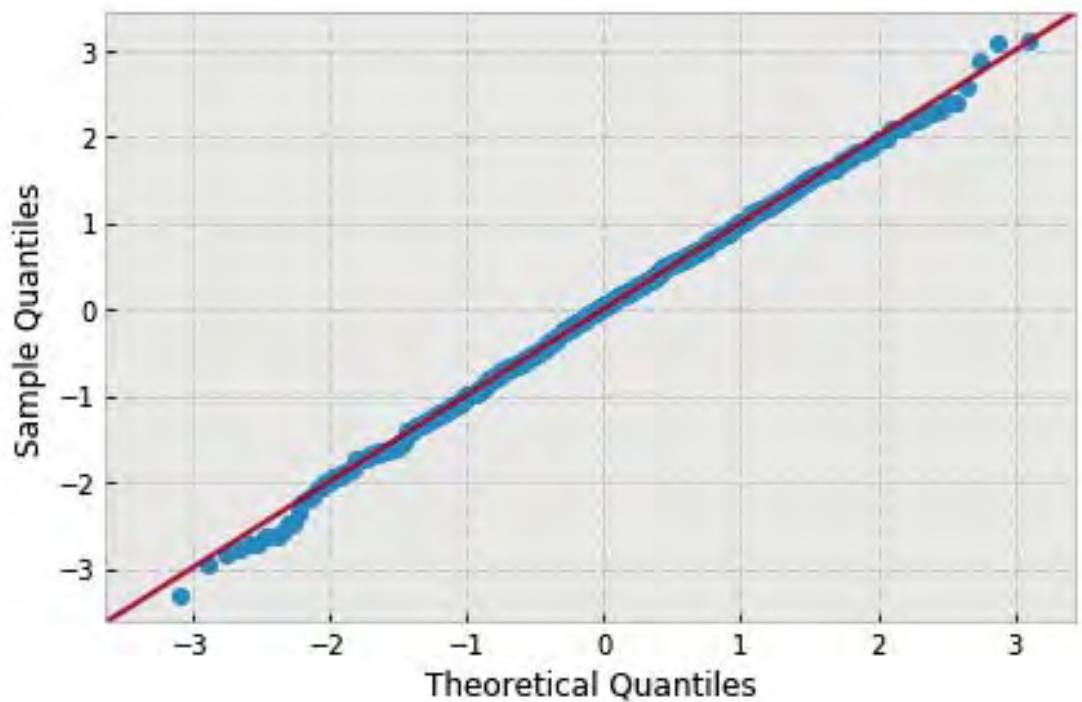
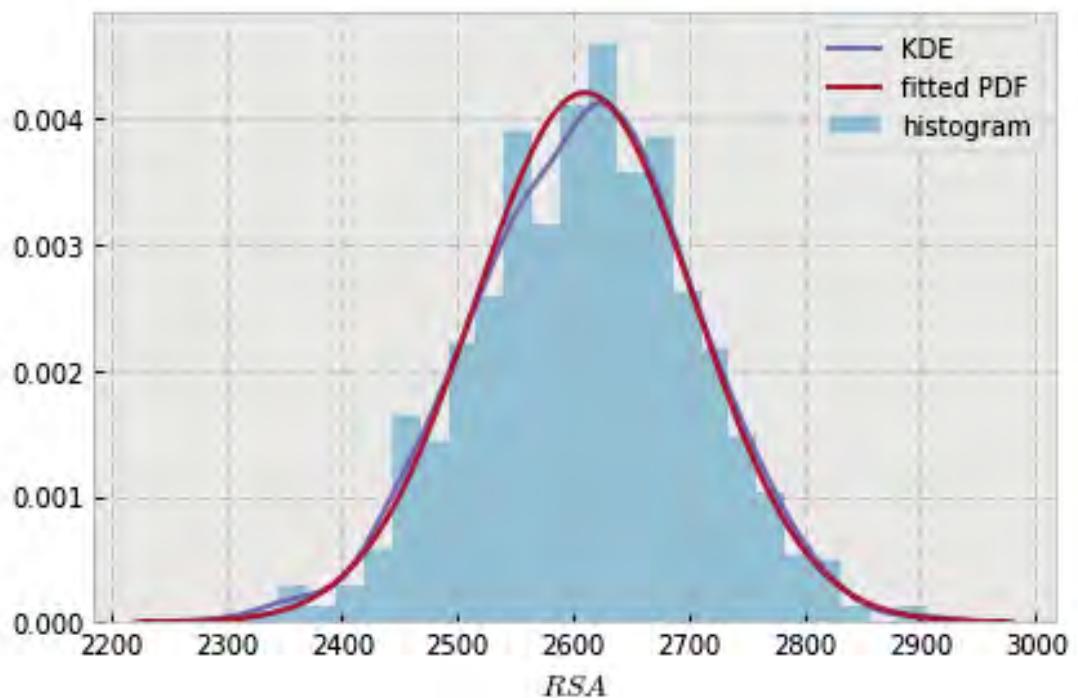


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=1197.226307884, scale=42.3453275523533)
DescribeResult(nobs=1000, minmax=(1061.0753380000001, 1325.1668219999999)
               mean=1197.2263078840001, variance=1794.9216872032944,
               skewness=-0.08458251958564857, kurtosis=-0.006273407190194202)
```

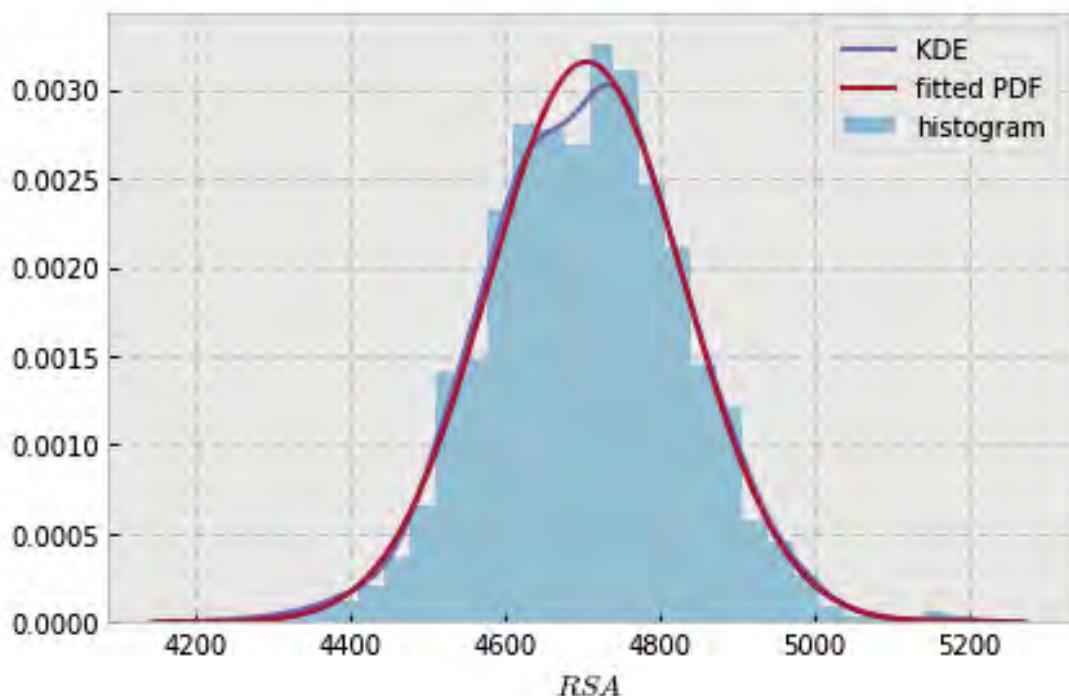


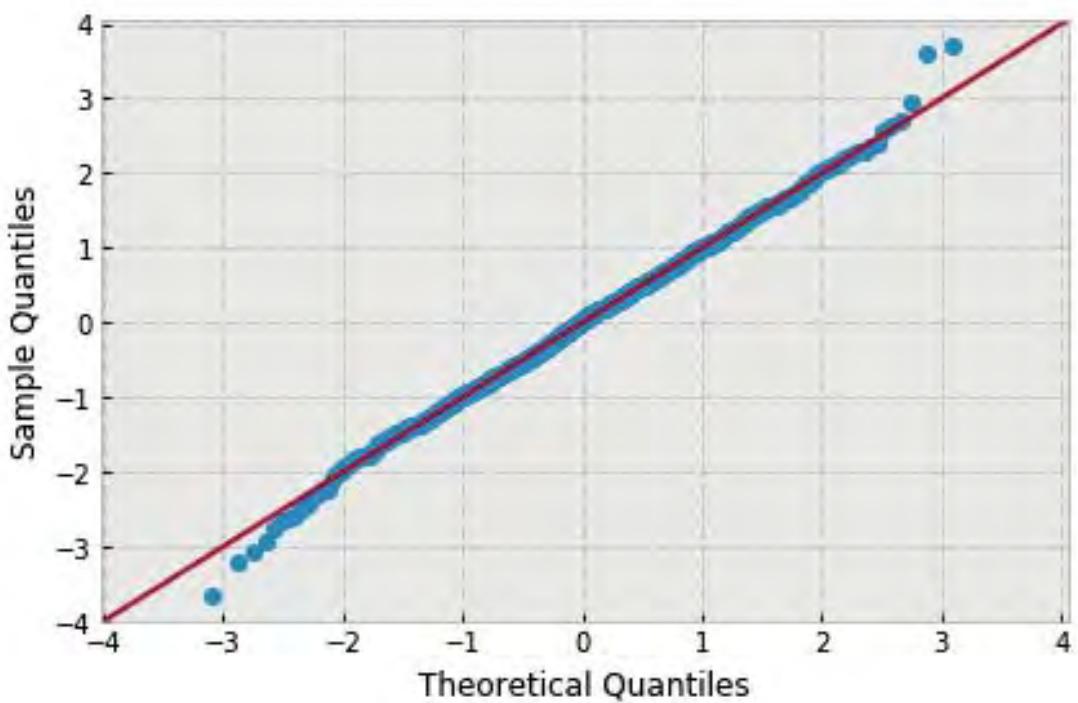


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=2610.3565048230003, scale=94.79205914802228)
DescribeResult(nobs=1000, minmax=(2297.462156, 2904.9002770000002)
               mean=2610.3565048230003, variance=8994.5290065286827,
               skewness=-0.09082572839061173, kurtosis=-0.039911396007745736)
```

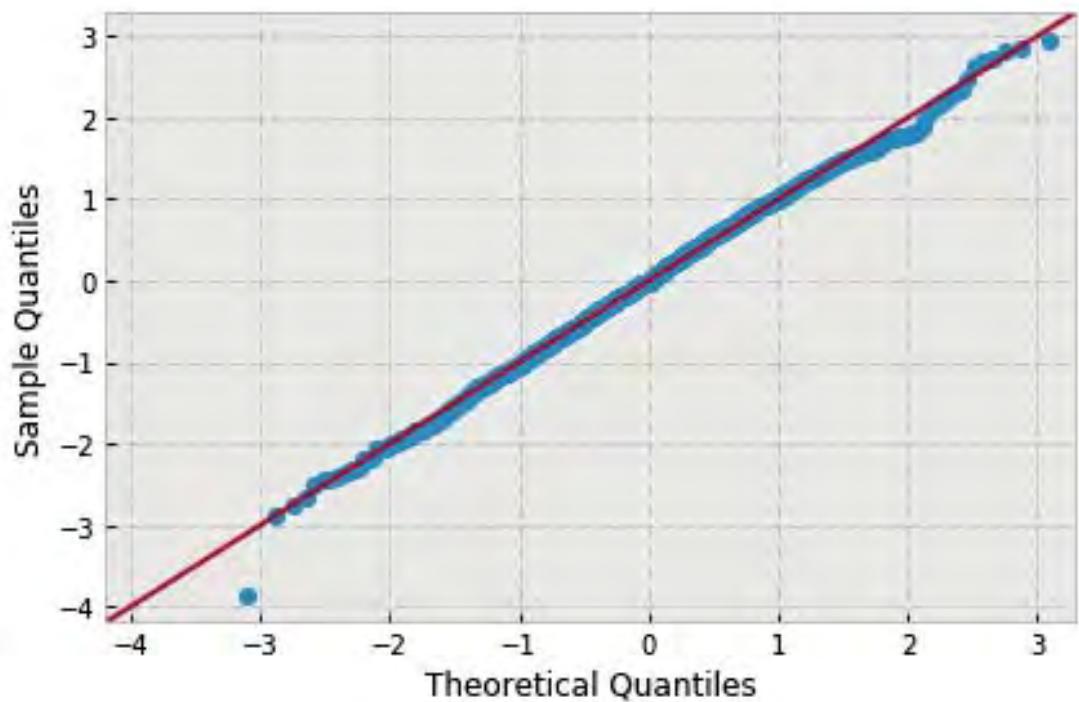
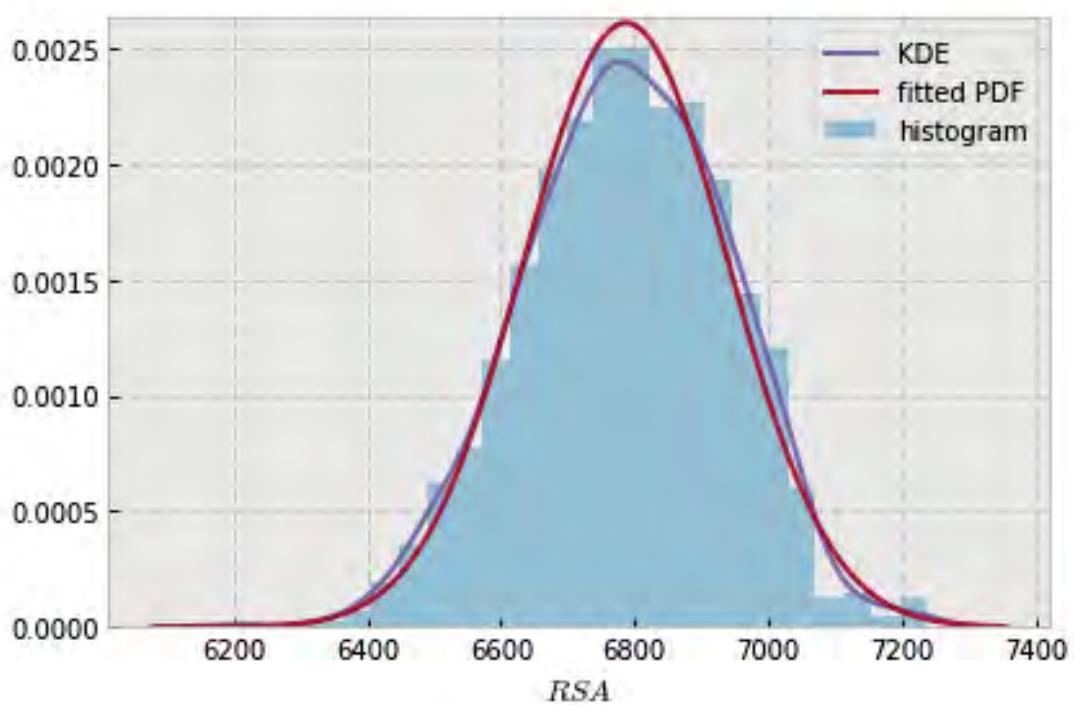


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=4704.548169934, scale=126.40174688358954)
DescribeResult(nobs=1000, minmax=(4243.886731999999, 5171.118051000004)
               mean=4704.5481699339998, variance=15993.395010233271,
               skewness=-0.006814727099776592, kurtosis=0.23352207800067726)
```

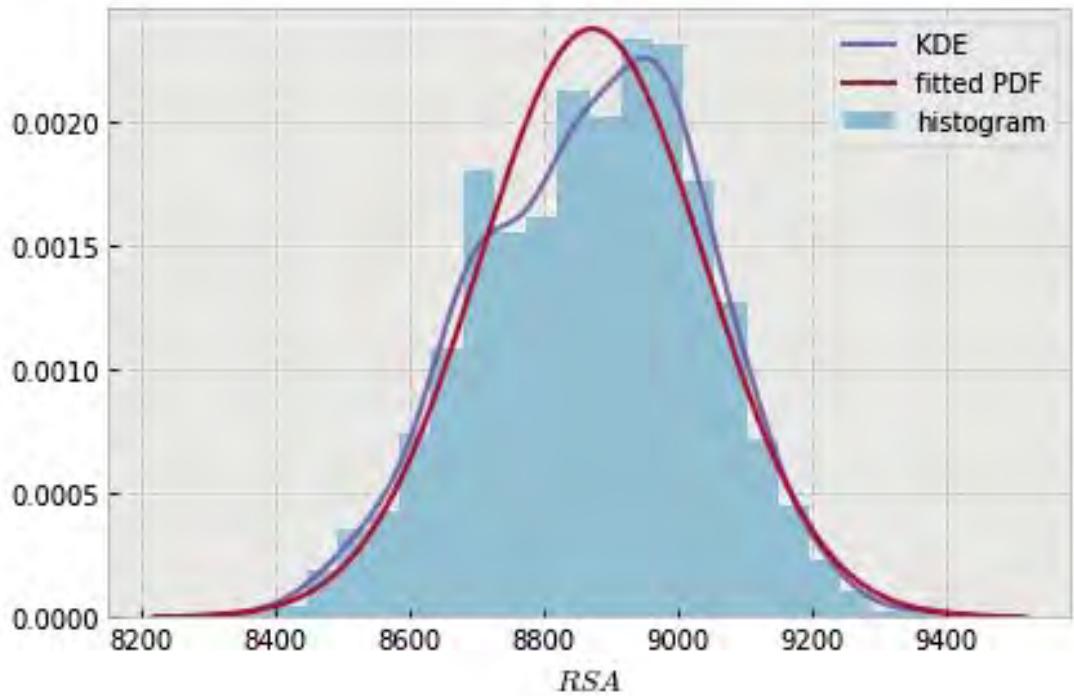


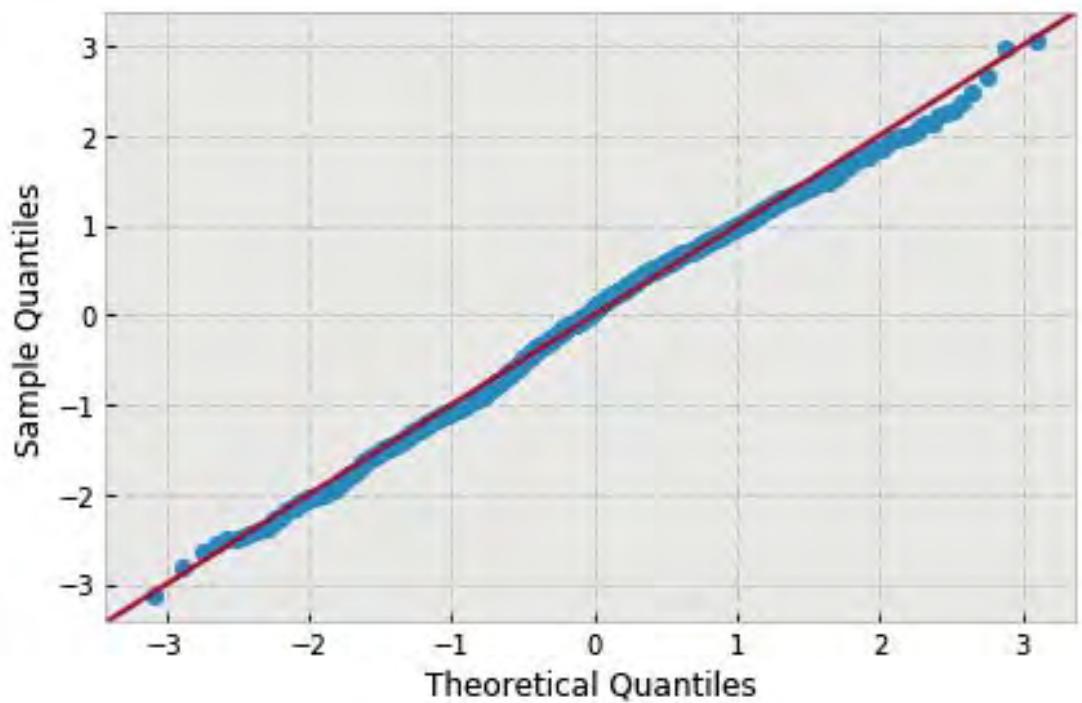


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=6787.010499125, scale=152.8268676558993)
DescribeResult(nobs=1000, minmax=(6199.105507999999, 7236.660941000001)
               mean=6787.010499125, variance=23379.430908422182,
               skewness=-0.11511507212845526, kurtosis=-0.10294388576845703)
```

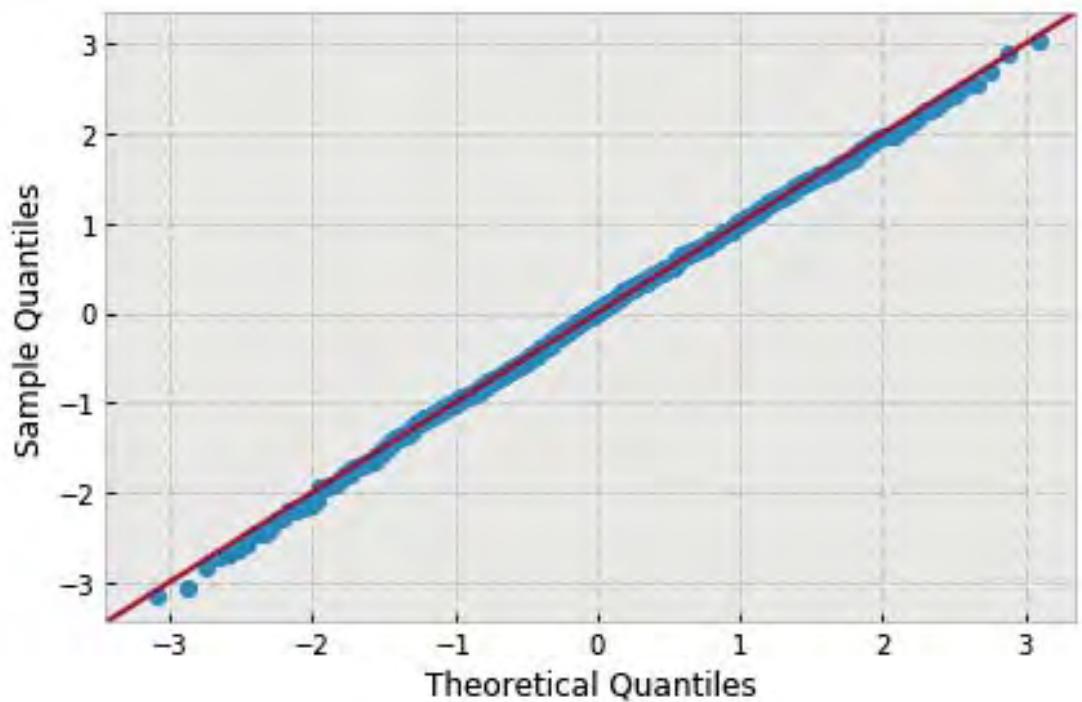
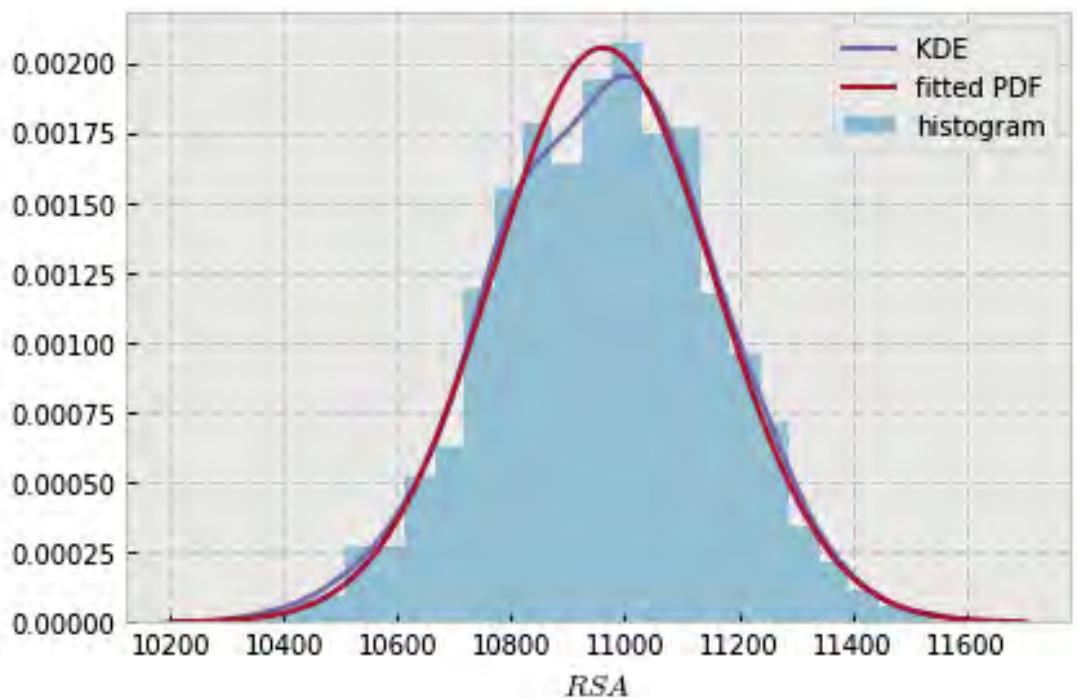


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=8872.185561866001, scale=167.7405939446958)
DescribeResult(nobs=1000, minmax=(8350.2240770000008, 9384.0257220000003)
               mean=8872.1855618660011, variance=28165.071928848167,
               skewness=-0.1642882175820818, kurtosis=-0.3139346832917336)
```

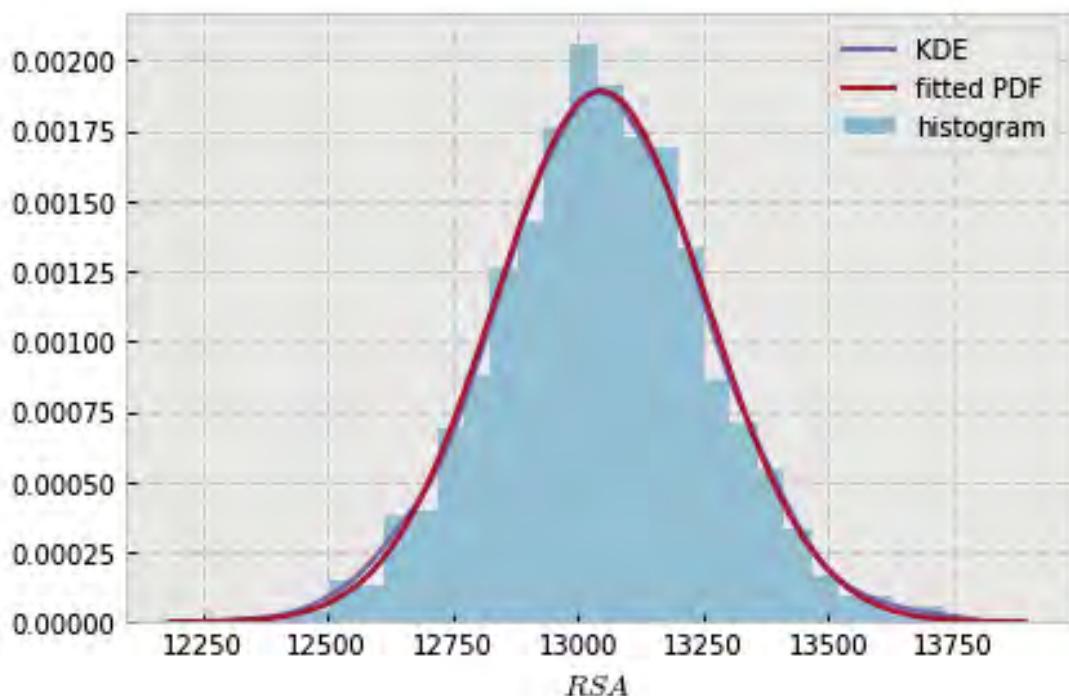


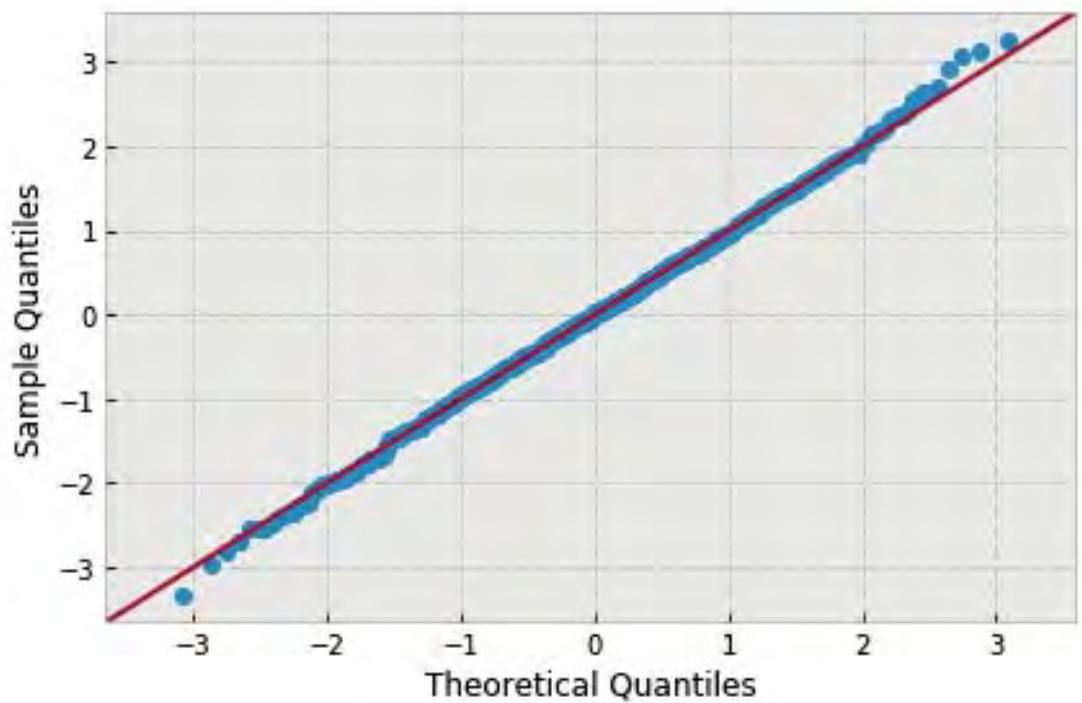


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=10961.02237656, scale=194.32737134396638)
DescribeResult(nobs=1000, minmax=(10351.98674, 11550.253049999999)
               mean=10961.02237656, variance=37800.928181637442,
               skewness=-0.08941247285642467, kurtosis=-0.1040135914175222)
```

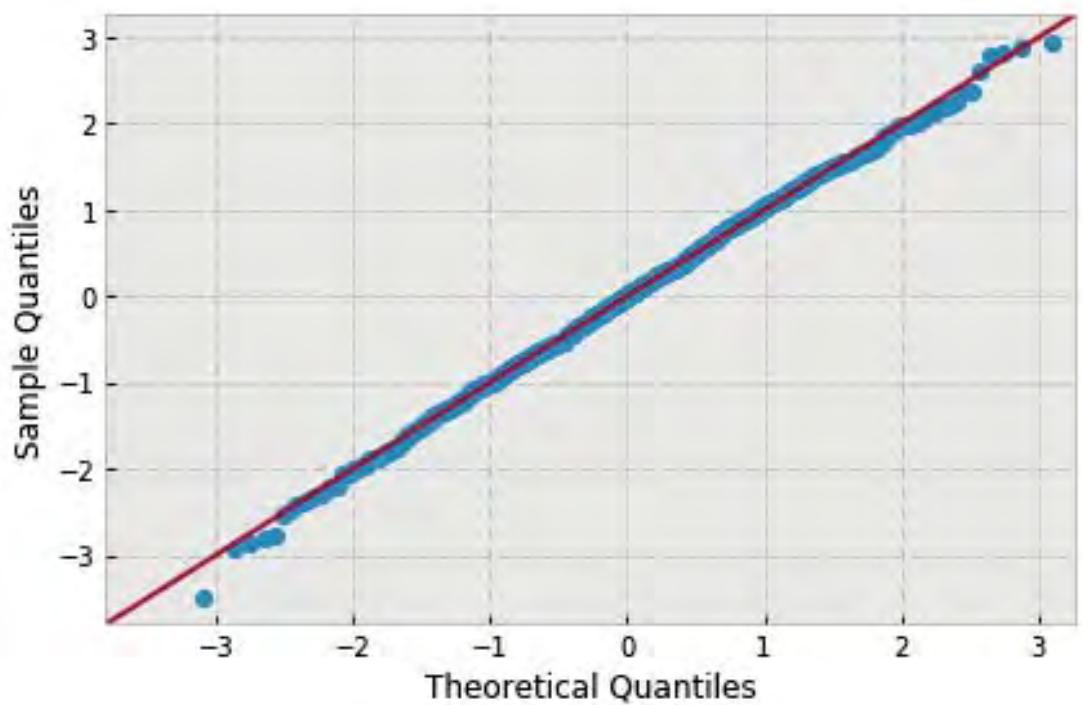
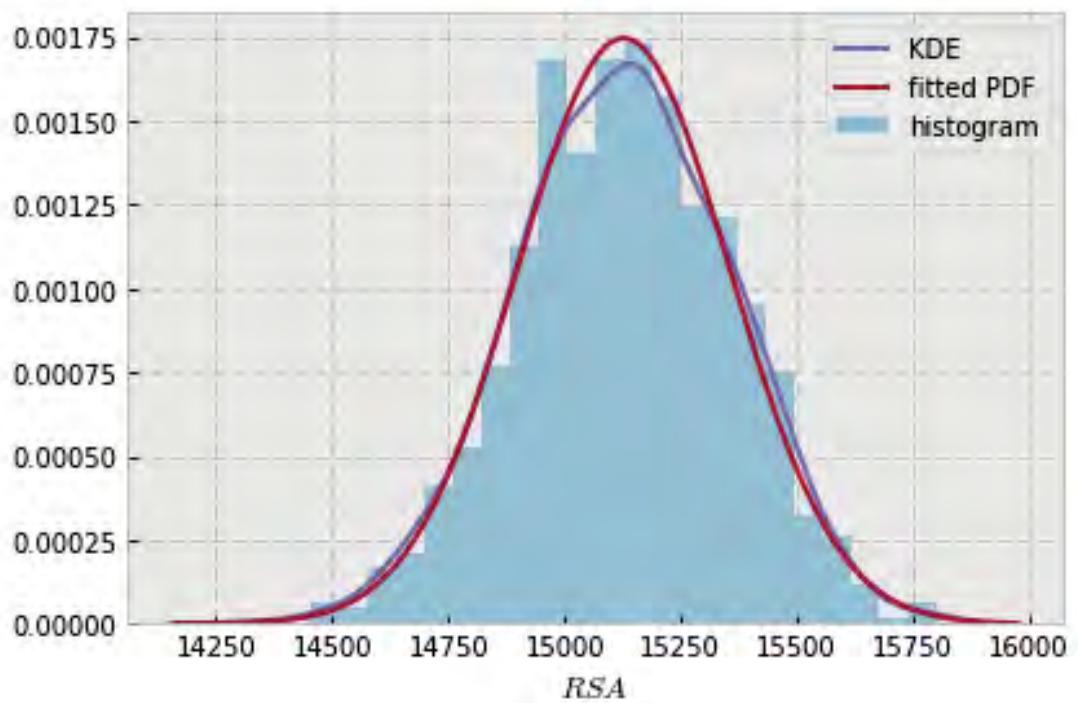


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=13045.92600808, scale=210.89529635550429)
DescribeResult(nobs=1000, minmax=(12345.145210000001, 13731.93764)
               mean=13045.92600808, variance=44521.347372248223,
               skewness=0.013902165220700917, kurtosis=0.12788028446997002)
```

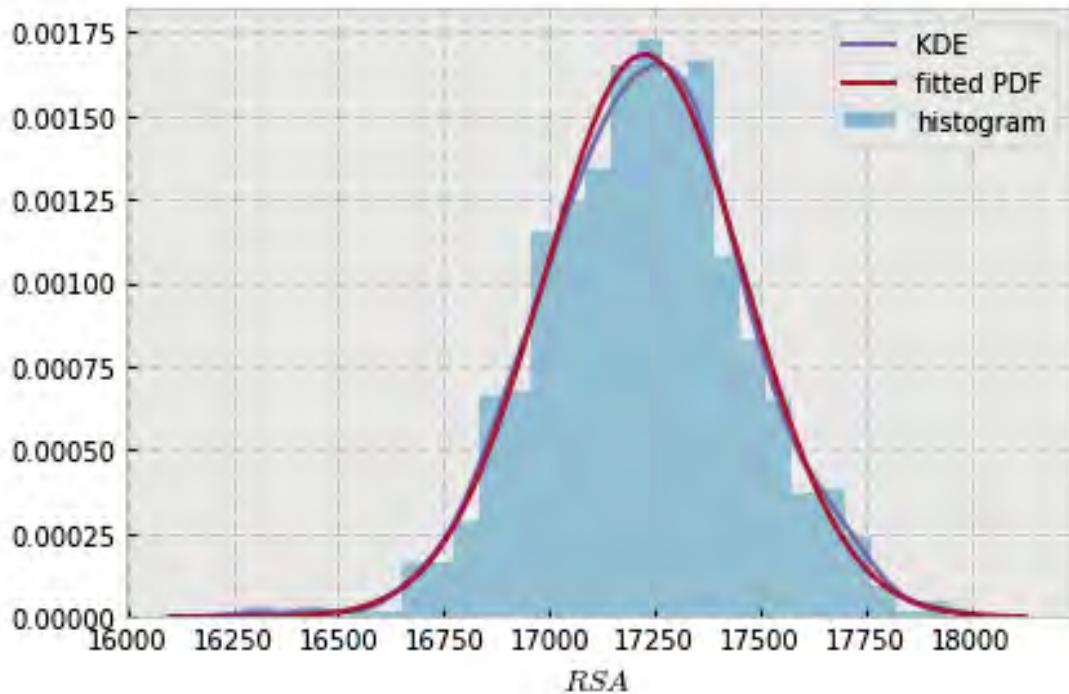


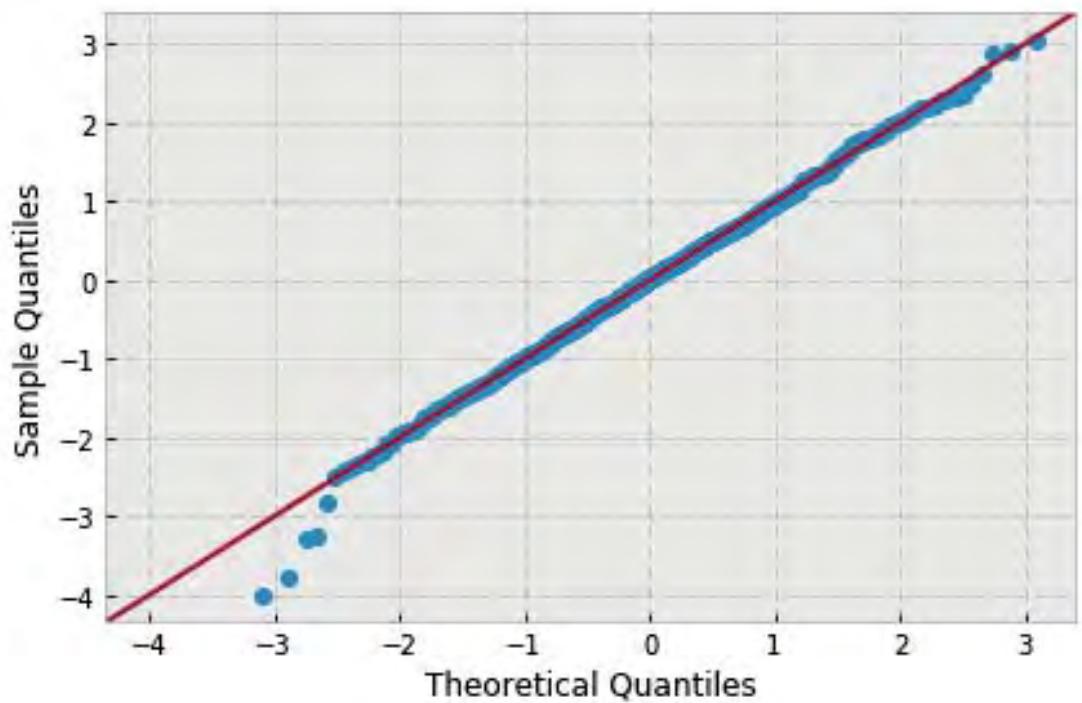


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=15127.00496831, scale=228.4676219682814)
DescribeResult(nobs=1000, minmax=(14331.67024999999, 15798.45904999999)
               mean=15127.004968310001, variance=52249.703991833376,
               skewness=-0.06596048896386075, kurtosis=-0.09664912273809989)
```

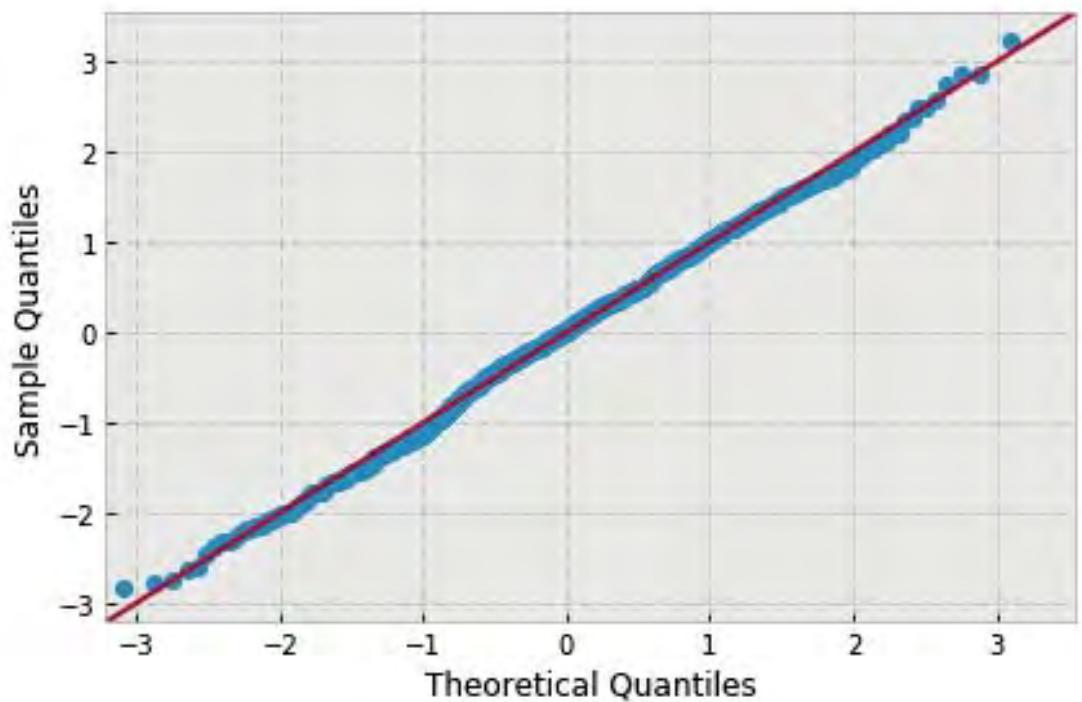
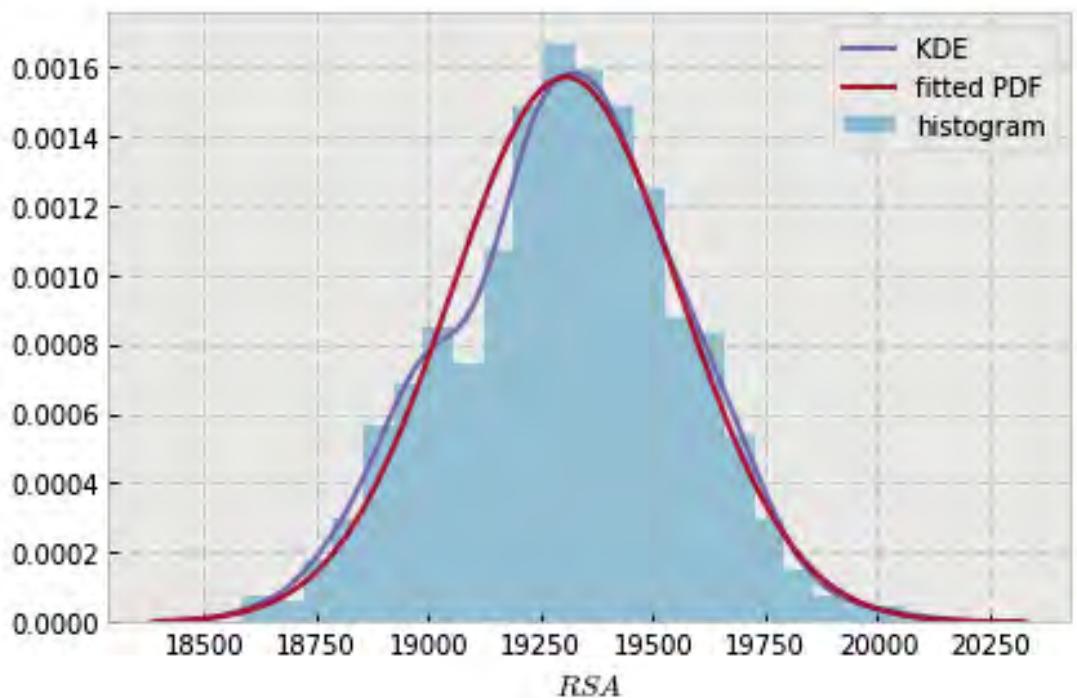


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=17225.55199468, scale=237.02449013147708)
DescribeResult(nobs=1000, minmax=(16279.35621, 17946.61204)
               mean=17225.551994680001, variance=56236.845767854531,
               skewness=-0.08959536573074788, kurtosis=0.2507542087755521)
```

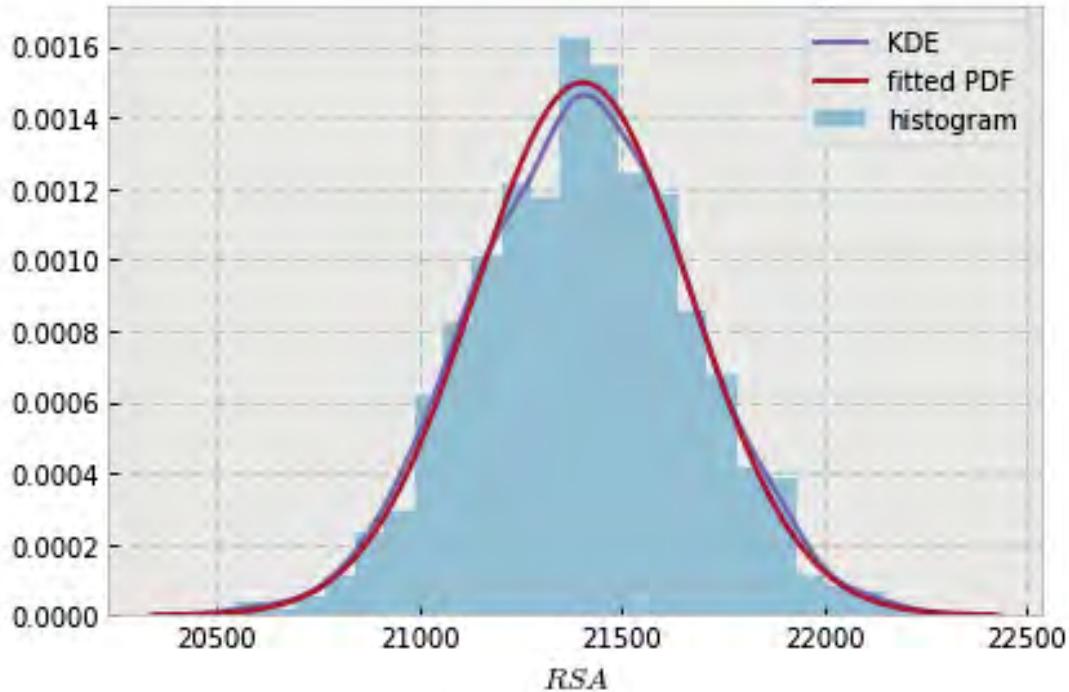


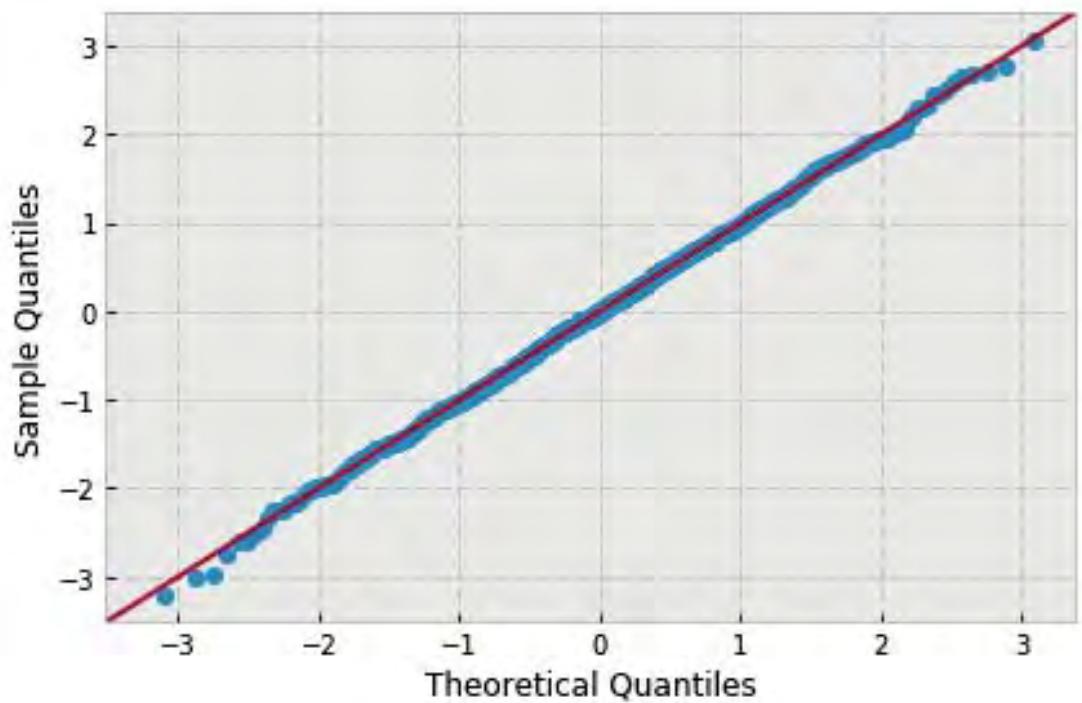


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=19305.08306065, scale=254.194602882026)
DescribeResult(nobs=1000, minmax=(18585.70178999999, 20128.191309999998)
               mean=19305.083060649999, variance=64679.57571006097,
               skewness=-0.07892359868384538, kurtosis=-0.187384196335588)
```

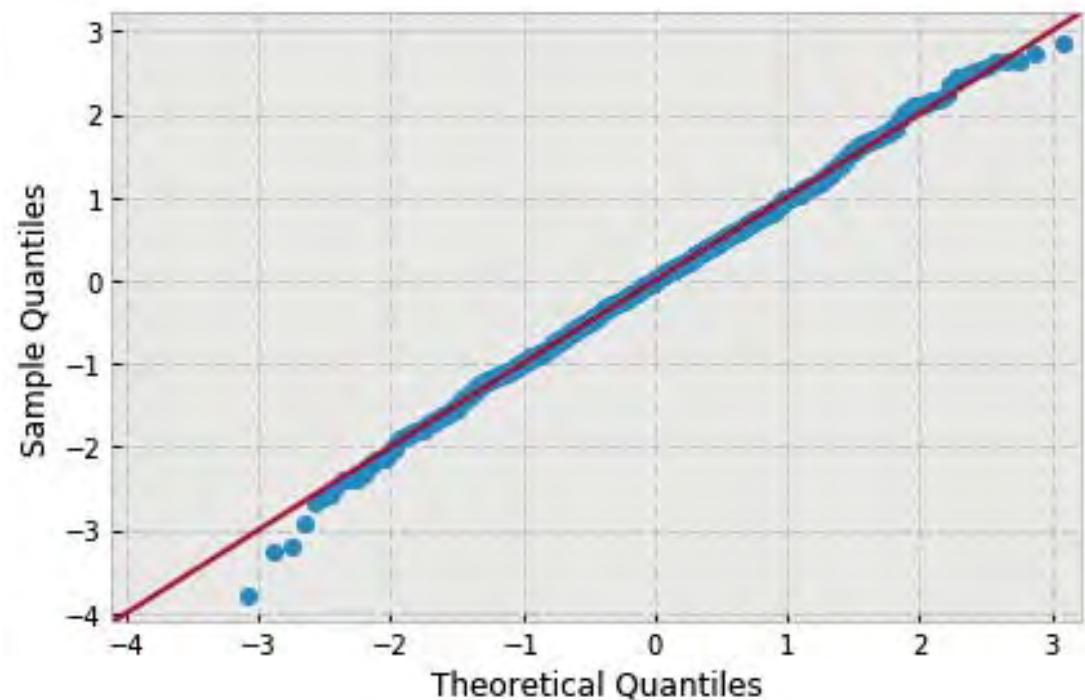
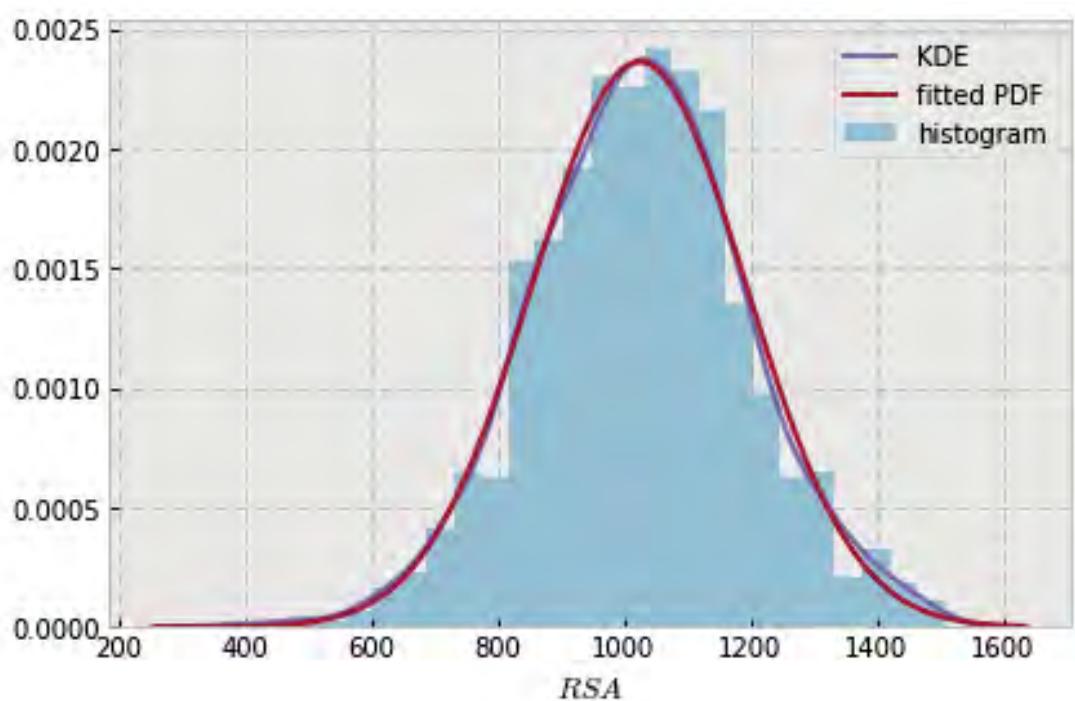


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=21404.03708287, scale=266.15666264964904)
DescribeResult(nobs=1000, minmax=(20552.2081, 22217.951110000002)
               mean=21404.03708287, variance=70910.279352151236,
               skewness=-0.01671377513545639, kurtosis=-0.09943647266825506)
```

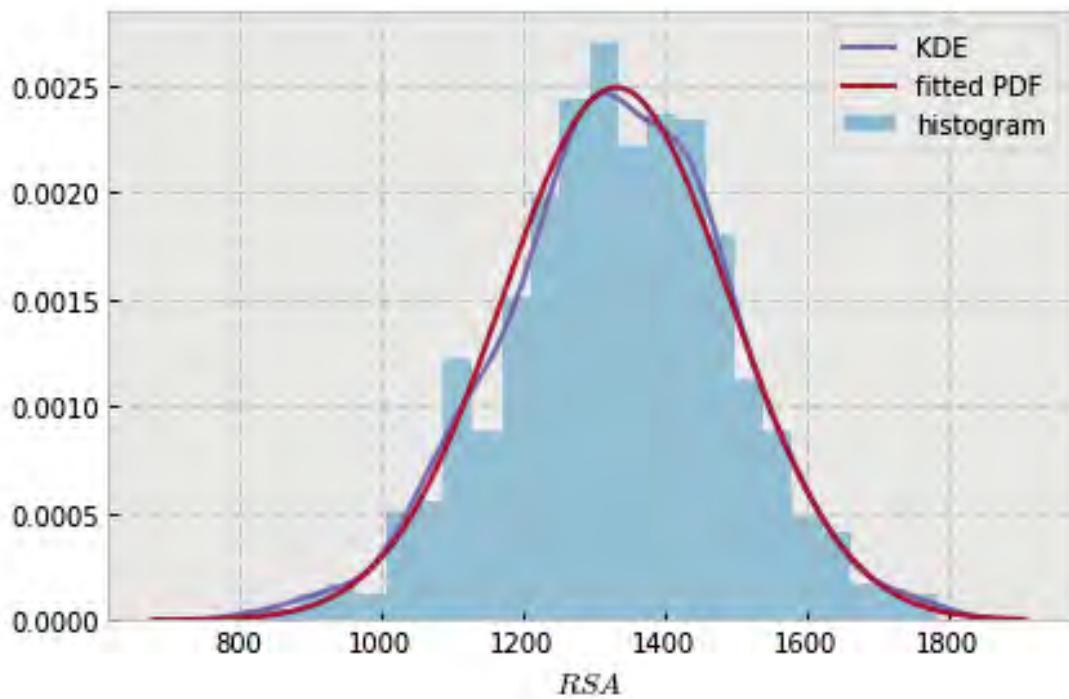


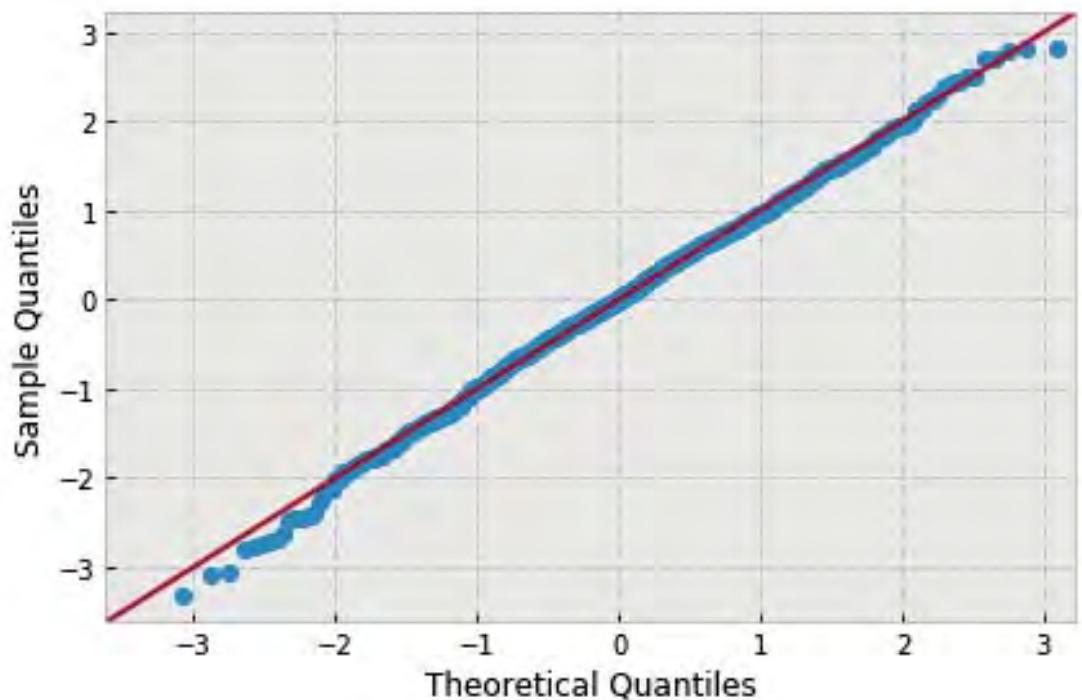


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=1024.3574376793, scale=168.81592523306577)
DescribeResult(nobs=1000, minmax=(386.9268644, 1505.707535)
               mean=1024.3574376792999, variance=28527.343956252305,
               skewness=-0.040293074667804595, kurtosis=0.1950175771307059)
```

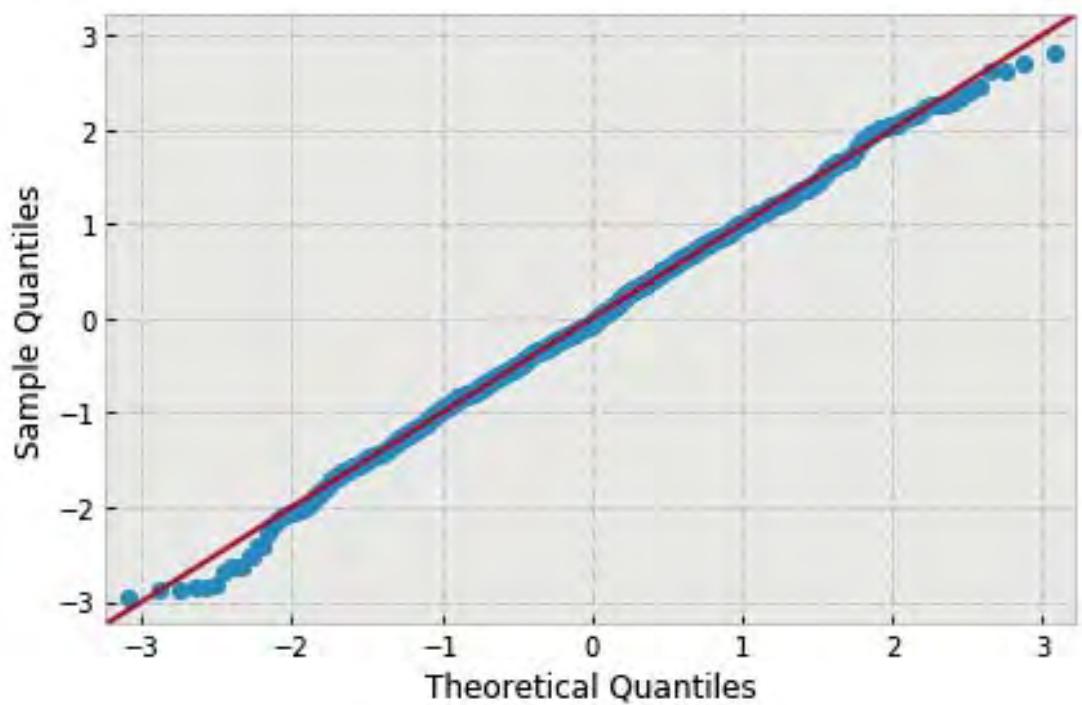
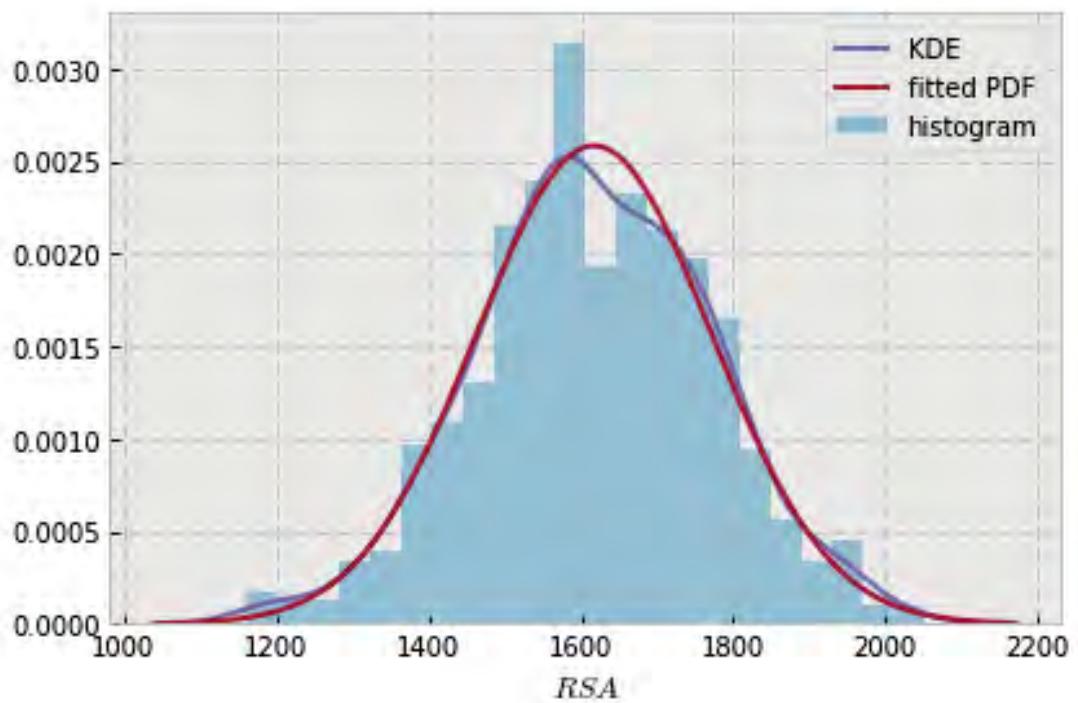


```
variable name: r
variable value: 1.222222222222223
distribution: normal(shape=(), loc=1332.0712591418, scale=160.05047655724445)
DescribeResult(nobs=1000, minmax=(801.8350789999995, 1785.017061)
               mean=1332.0712591418001, variance=25641.796843044103,
               skewness=-0.1234857026577768, kurtosis=0.09774267107312662)
```

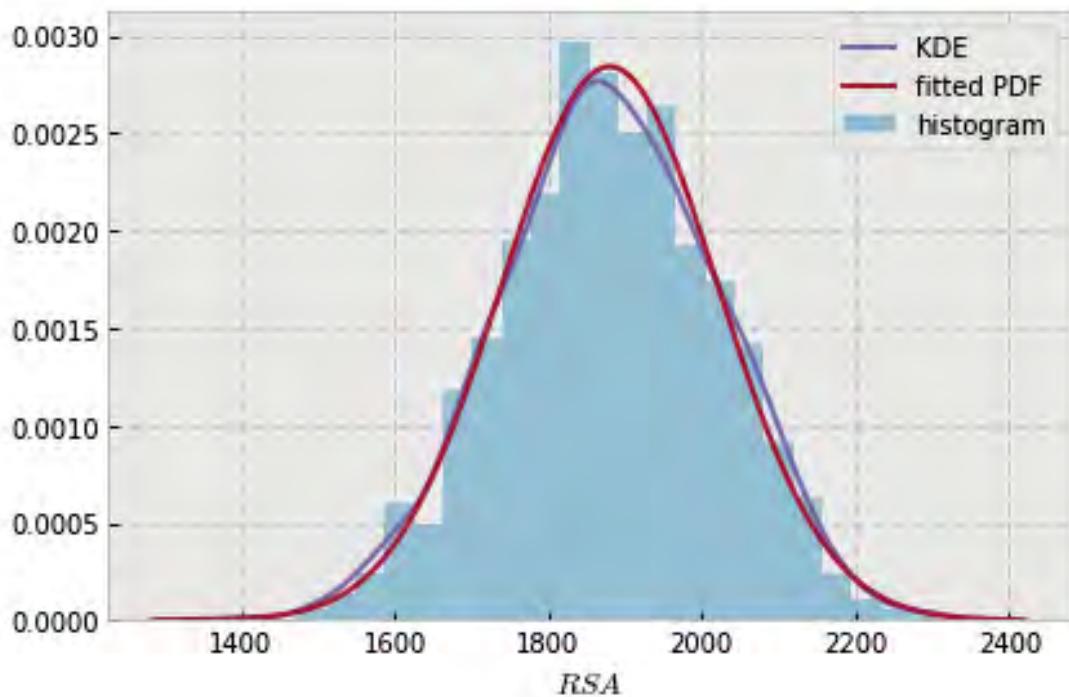


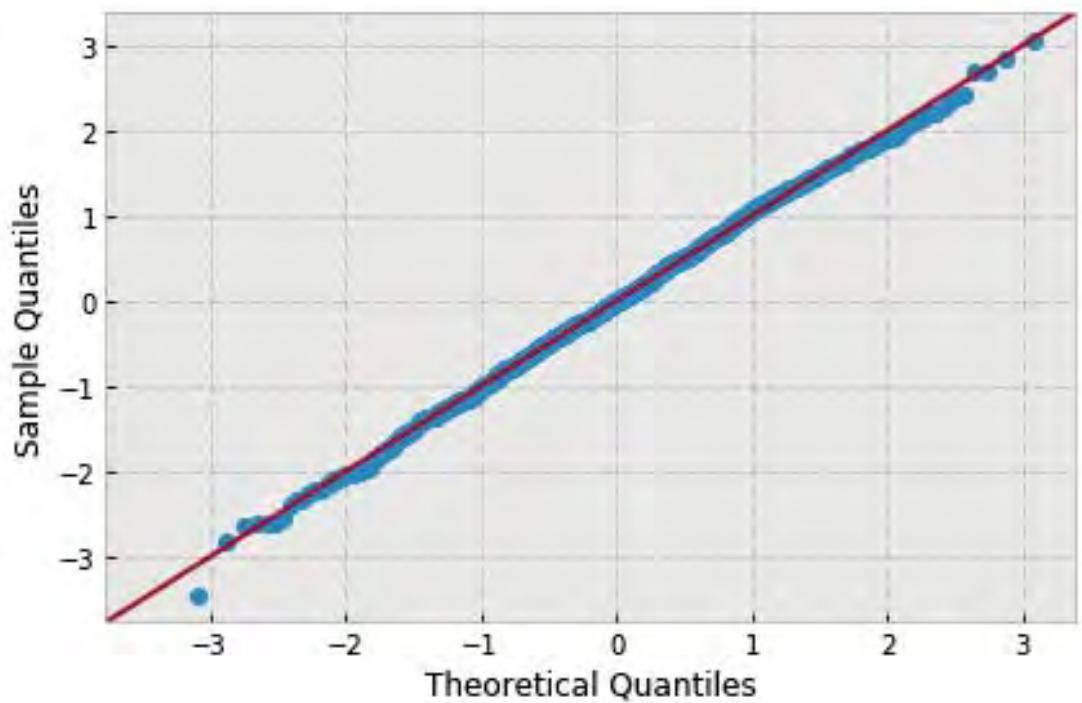


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=1617.132068707, scale=154.42690014158183)
DescribeResult(nobs=1000, minmax=(1161.5365400000001, 2050.5941379999999)
               mean=1617.1320687069999, variance=23871.539026364451,
               skewness=-0.06901387998540189, kurtosis=-0.03489512925374472)
```

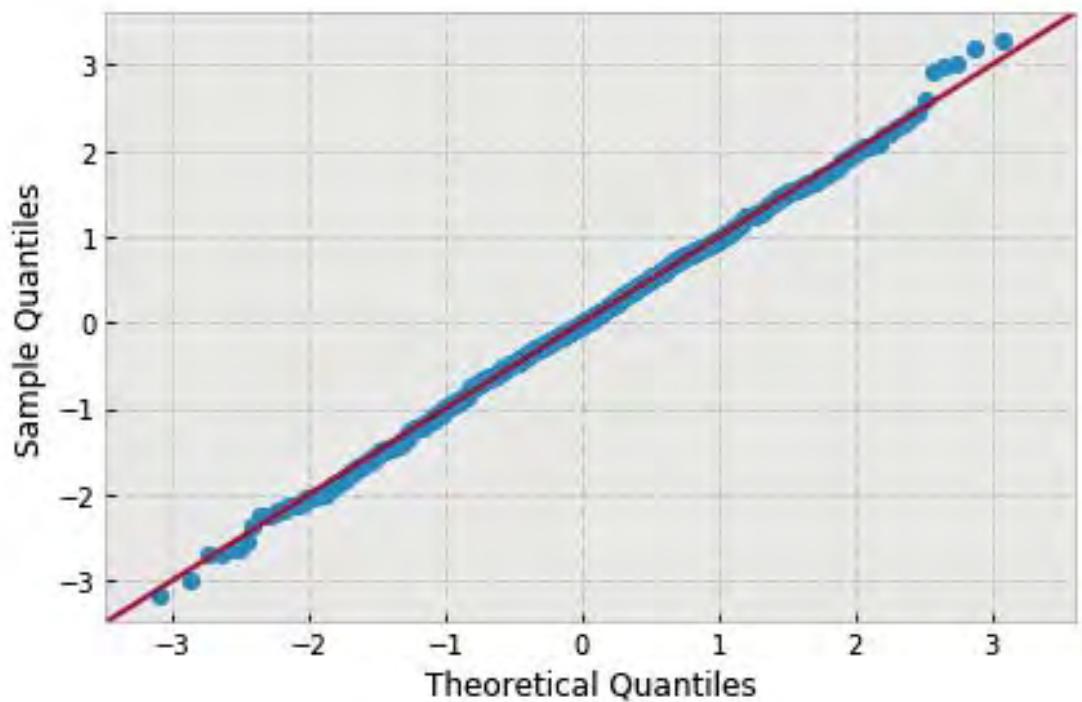
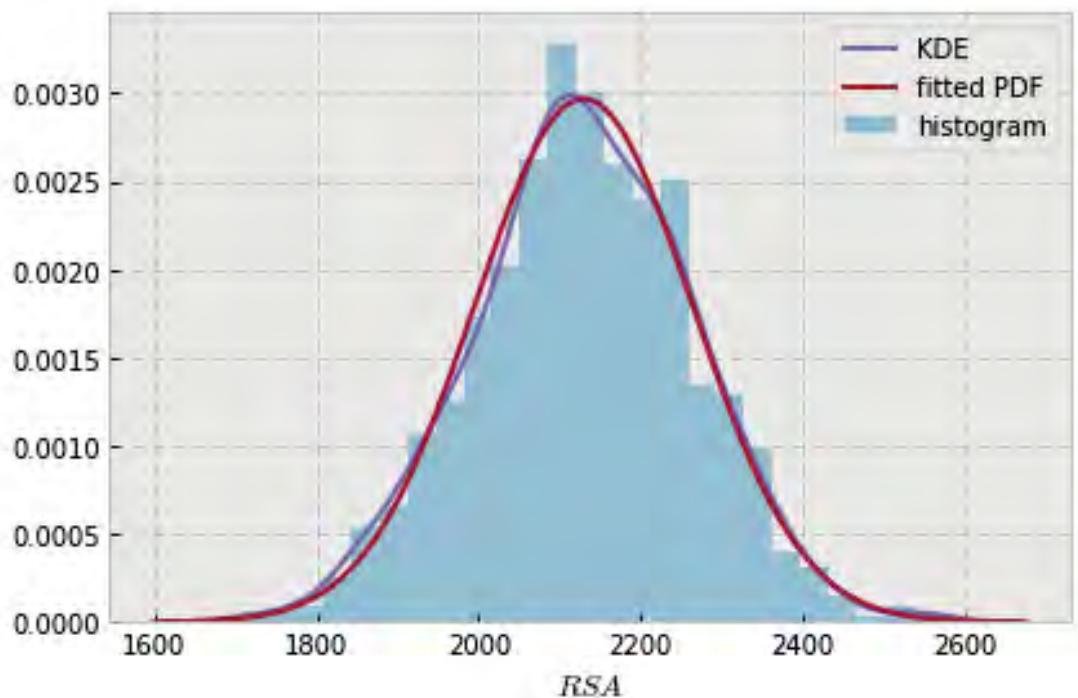


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=1879.8836284400002, scale=140.44224066147925)
DescribeResult(nobs=1000, minmax=(1396.595378, 2309.1776100000002)
               mean=1879.8836284400002, variance=19743.766728745602,
               skewness=-0.061215389164794055, kurtosis=-0.16225347202693952)
```

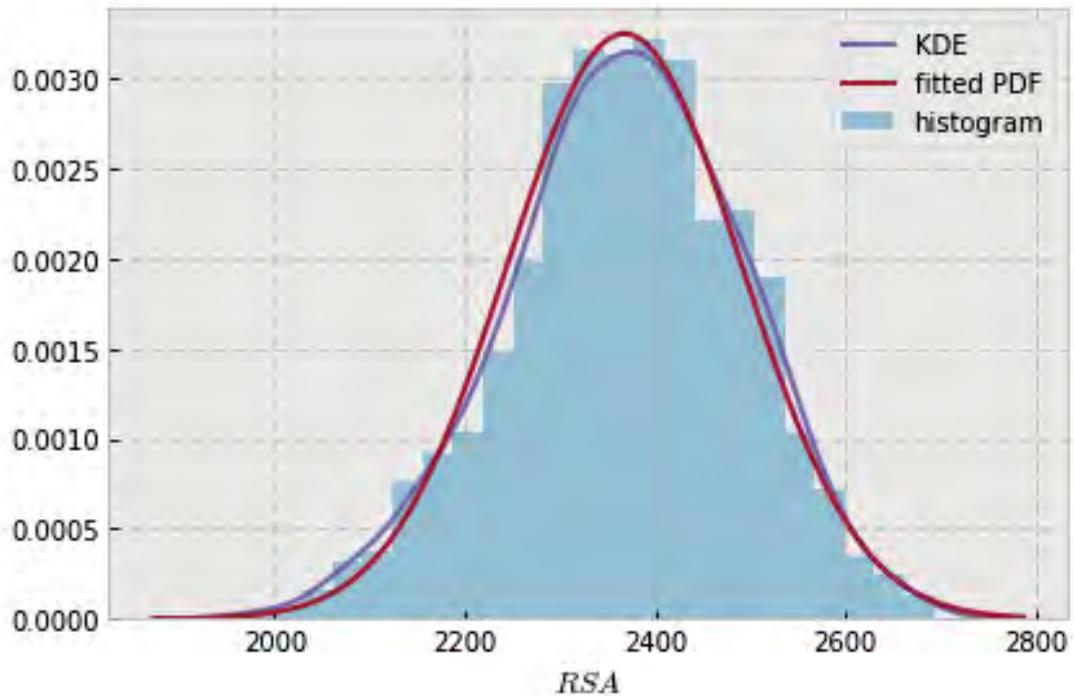


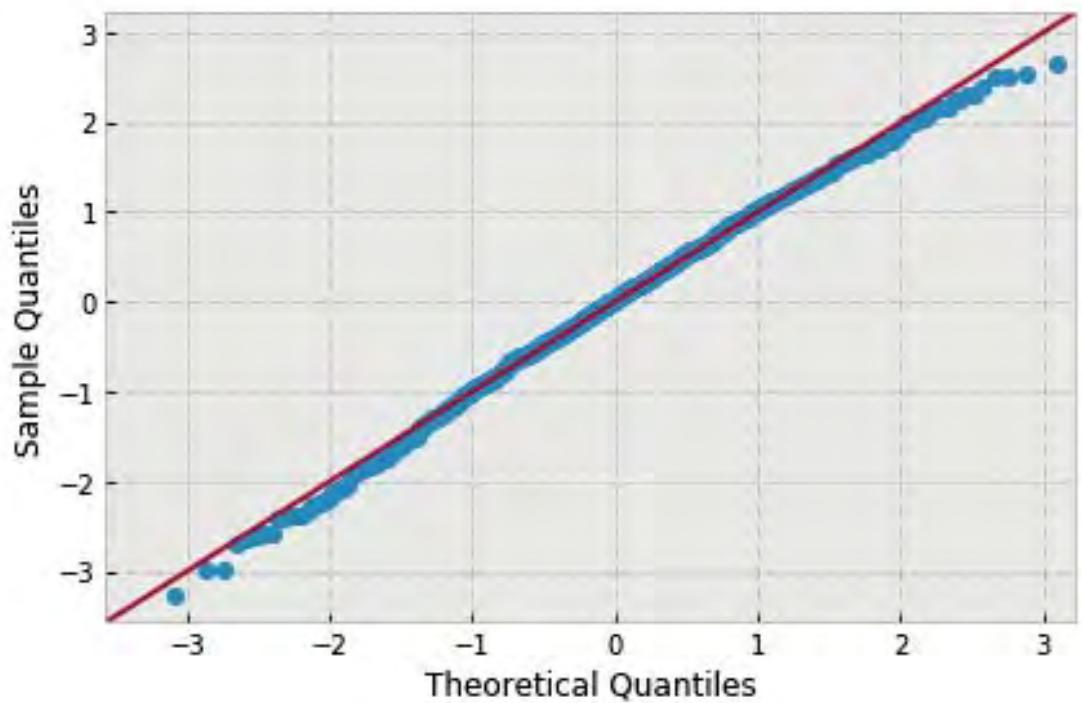


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=2129.551411247, scale=134.37568053059414)
DescribeResult(nobs=1000, minmax=(1704.269158000001, 2570.518203000001)
               mean=2129.551411247, variance=18074.898416476775,
               skewness=-0.026068149235202825, kurtosis=0.03321642863989327)
```

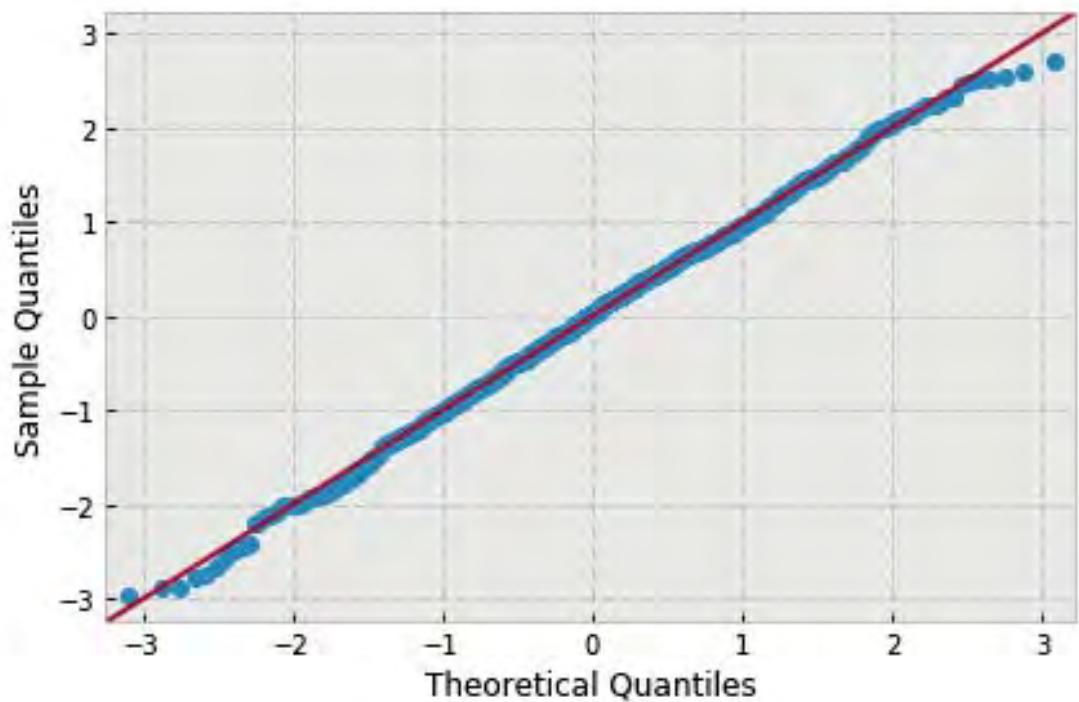
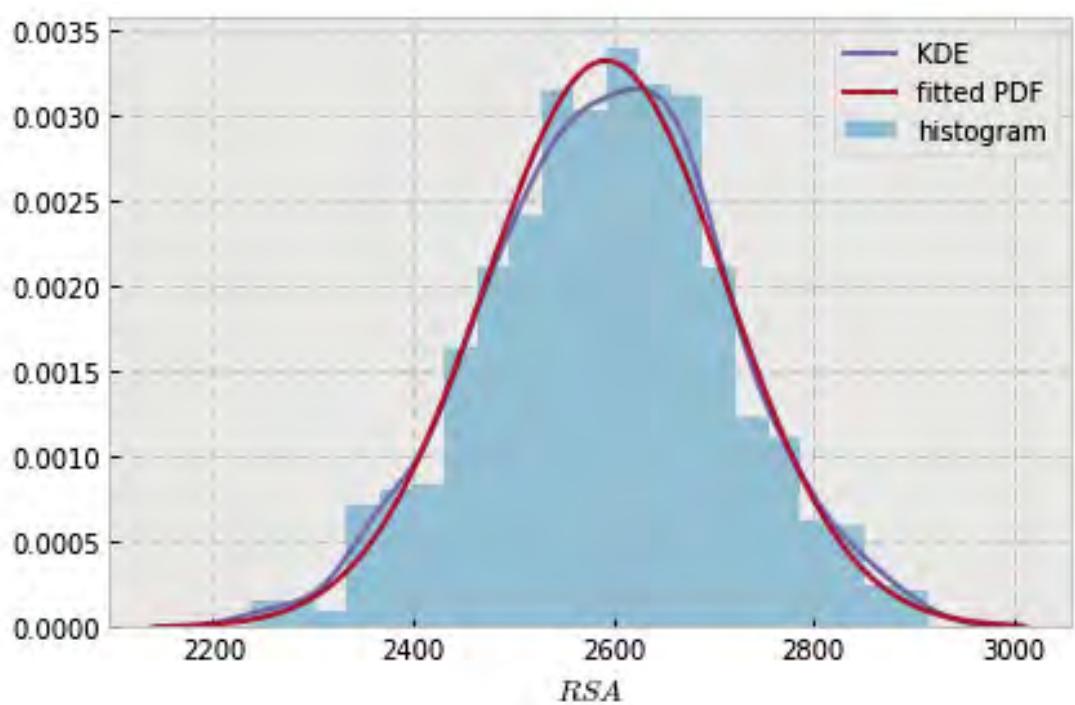


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=2367.786662375, scale=122.76800805361428)
DescribeResult(nobs=1000, minmax=(1966.205443999999, 2693.873091999998)
               mean=2367.786662375001, variance=15087.070872324626,
               skewness=-0.19626632746709033, kurtosis=-0.10694694572270746)
```

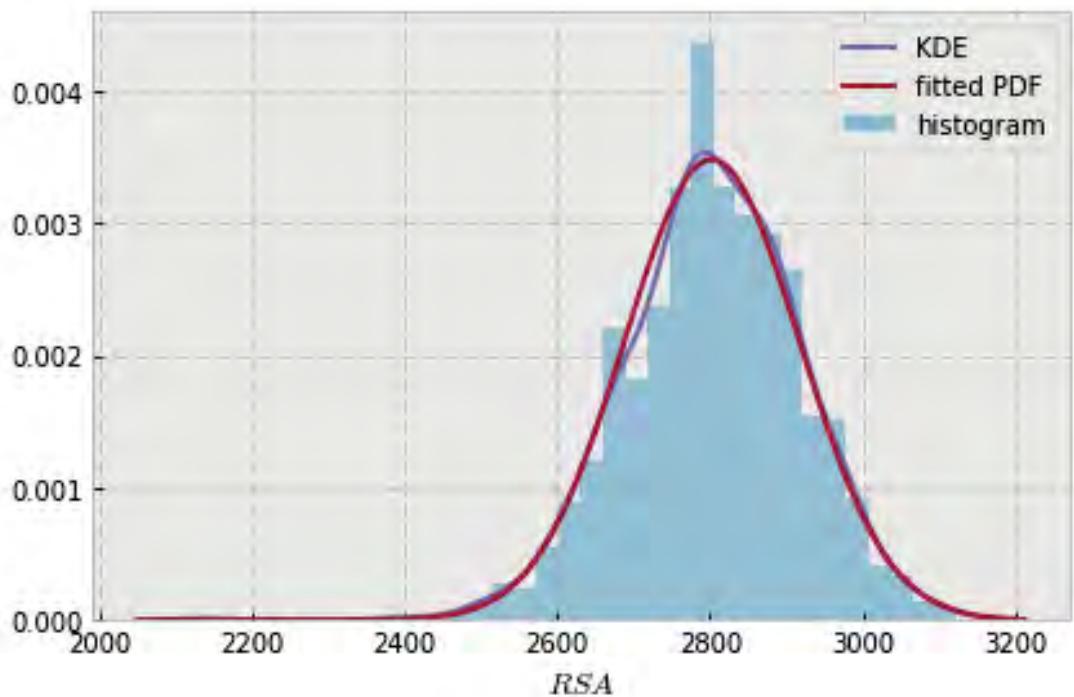


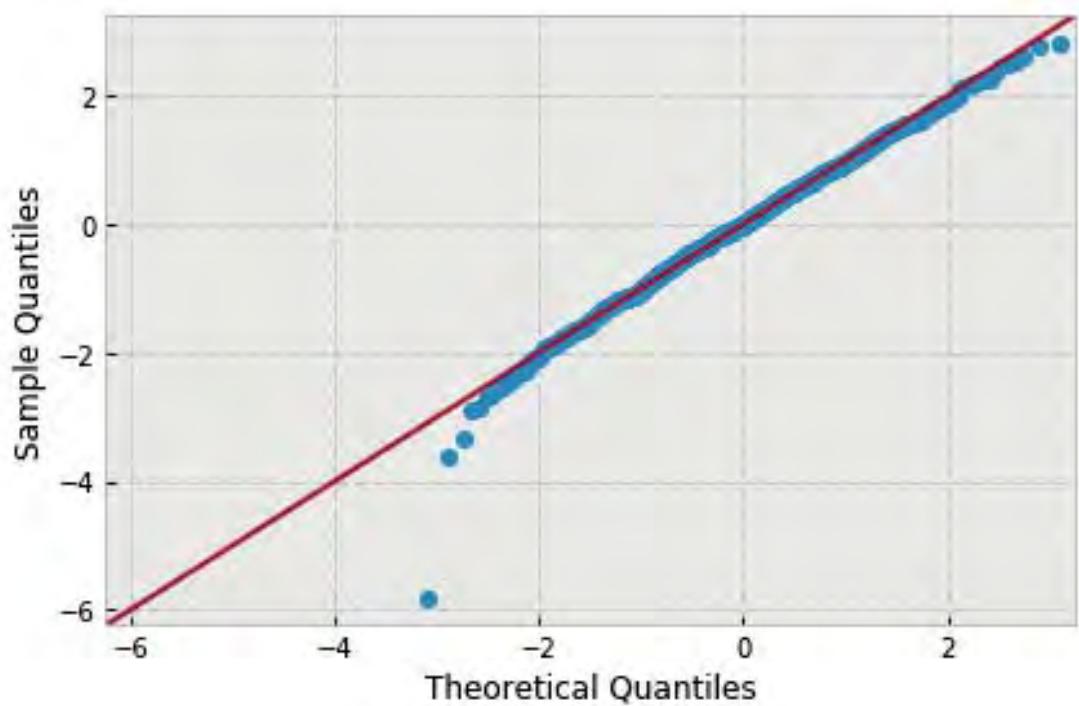


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=2592.352942994, scale=120.20420414047769)
DescribeResult(nobs=1000, minmax=(2236.6155520000002, 2916.2809219999999)
               mean=2592.3529429939999, variance=14463.514207252885,
               skewness=-0.07268717108422972, kurtosis=-0.12652192317465616)
```

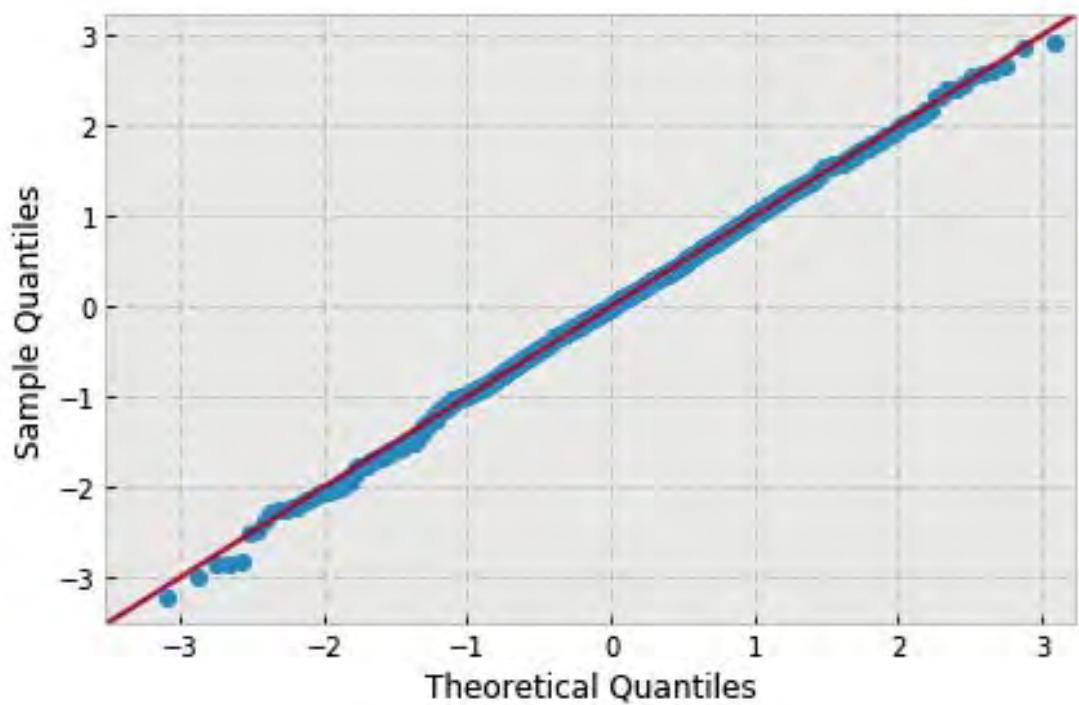
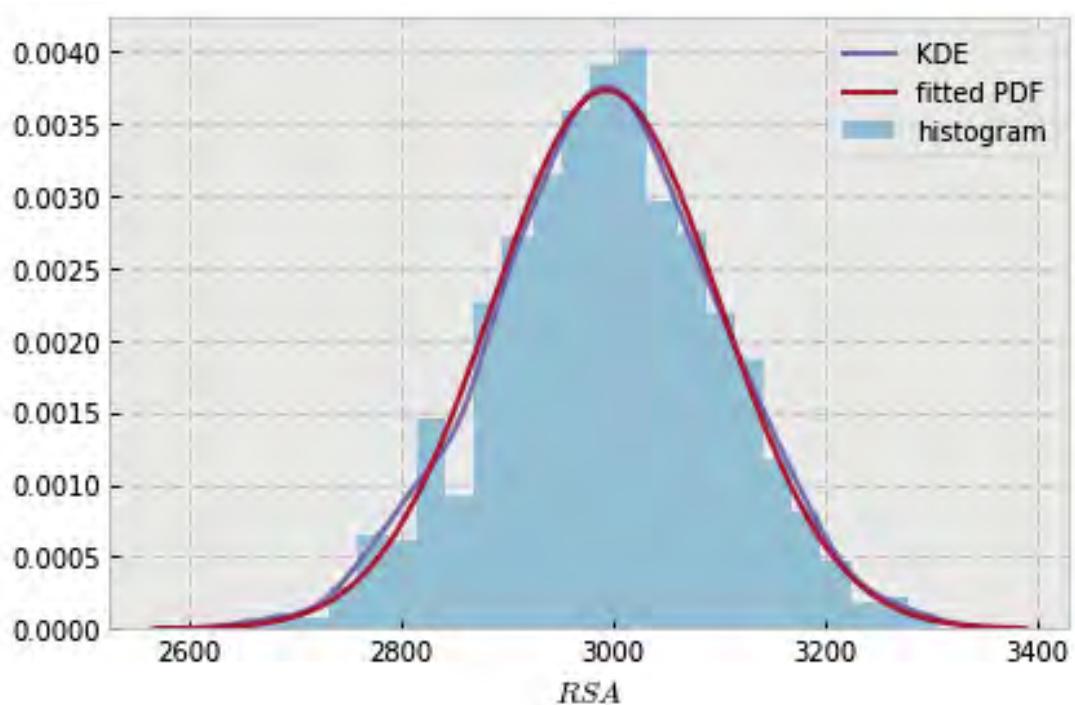


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=2802.68245541, scale=114.74032984894401)
DescribeResult(nobs=1000, minmax=(2136.6289550000001, 3124.4061839999999)
               mean=2802.682455409999, variance=13178.521815660131,
               skewness=-0.3212252875771477, kurtosis=1.0105798118627094)
```

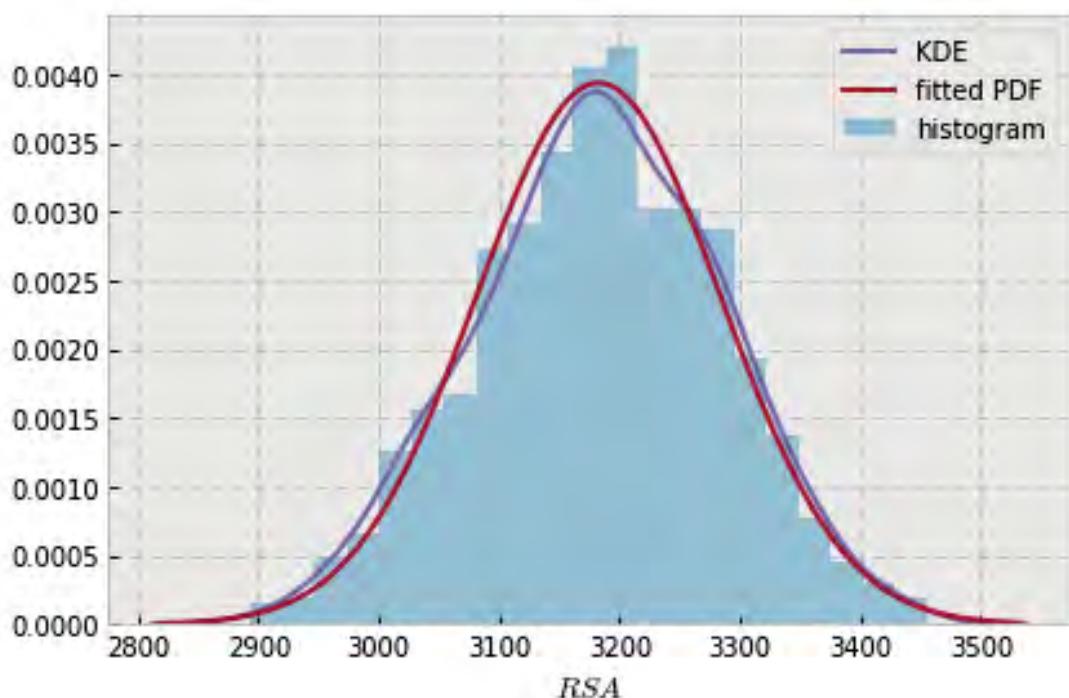


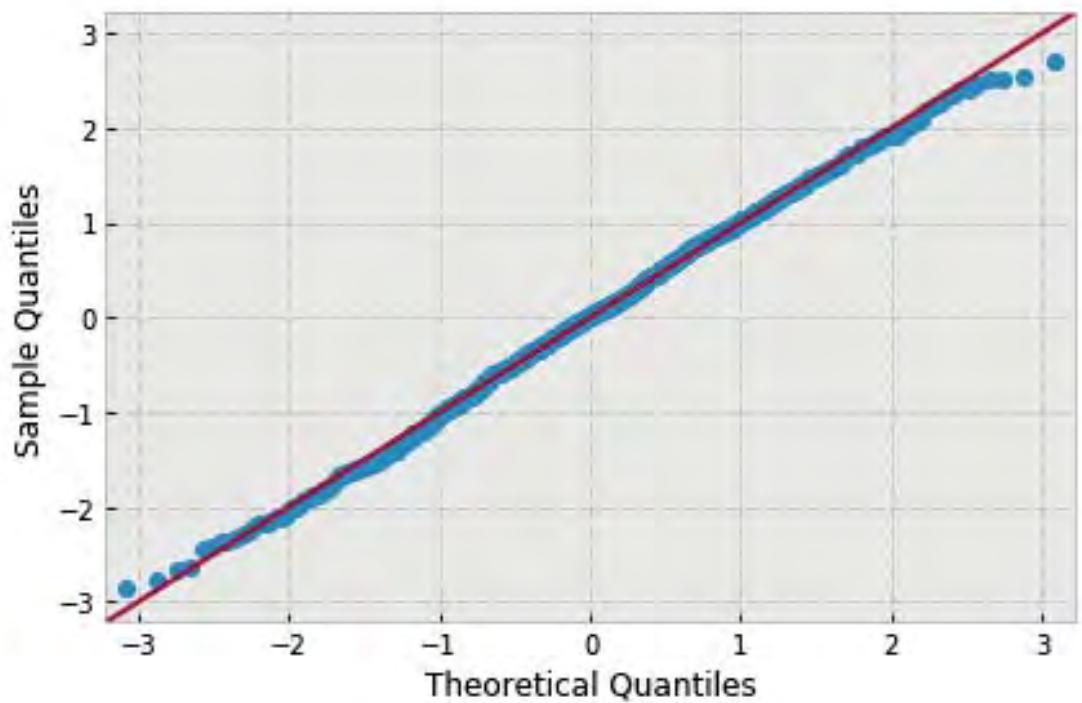


```
variable name: r
variable value: 2.7777777777777777
distribution: normal(shape=(), loc=2993.39682701, scale=106.97190904660643)
DescribeResult(nobs=1000, minmax=(2650.163395, 3305.489986)
               mean=2993.396827009999, variance=11454.443768844285,
               skewness=-0.05740855710702652, kurtosis=-0.044039418865050806)
```

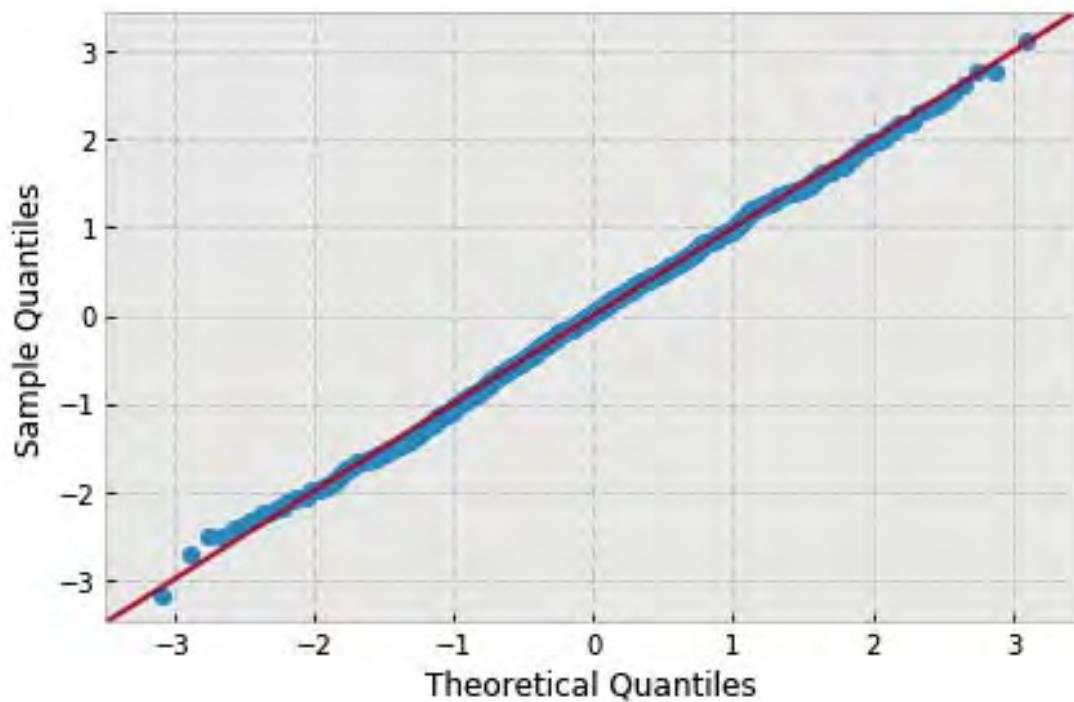
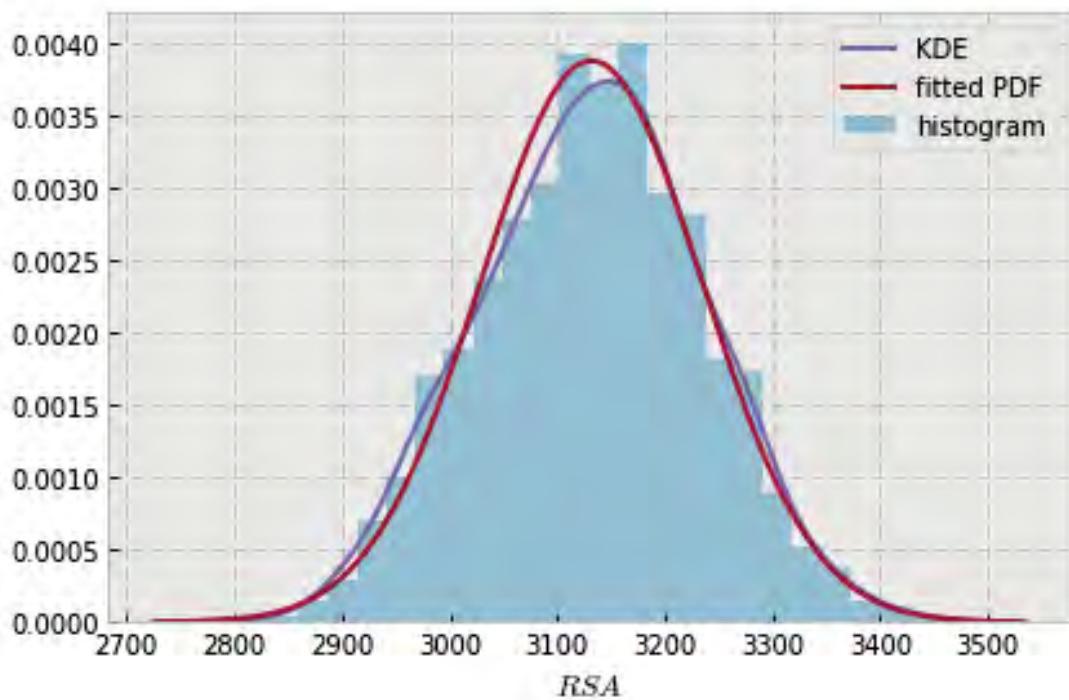


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=3182.486474712, scale=101.34504517217748)
DescribeResult(nobs=1000, minmax=(2892.7434250000001, 3456.2147810000001)
               mean=3182.486474712, variance=10281.099280230925,
               skewness=-0.08071190992711148, kurtosis=-0.2919981758034922)
```

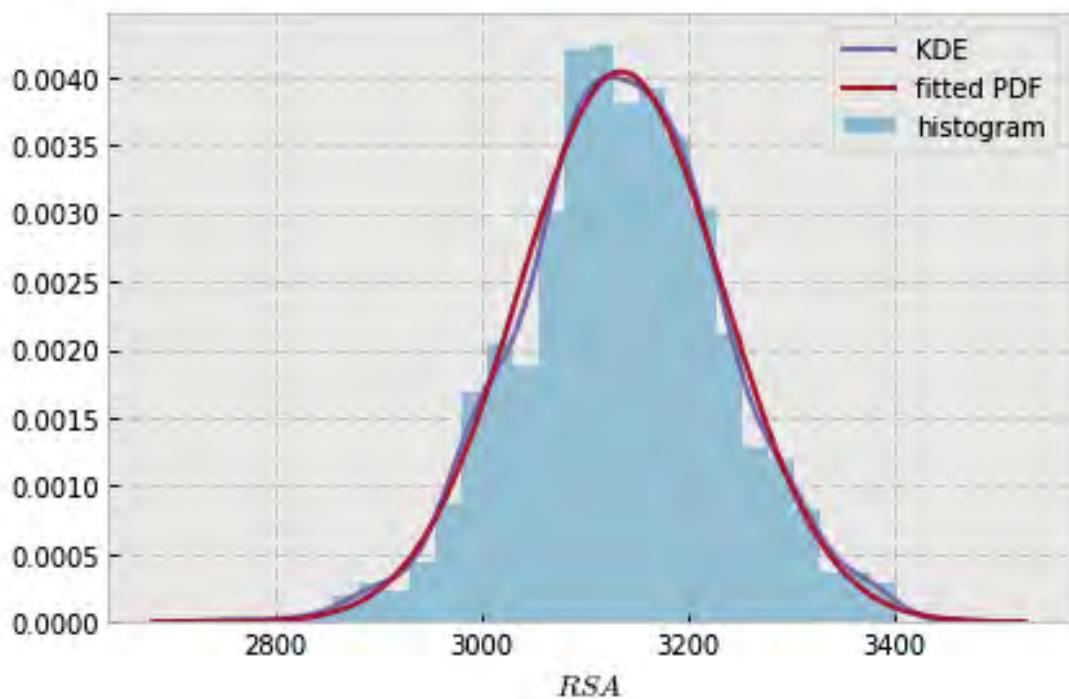


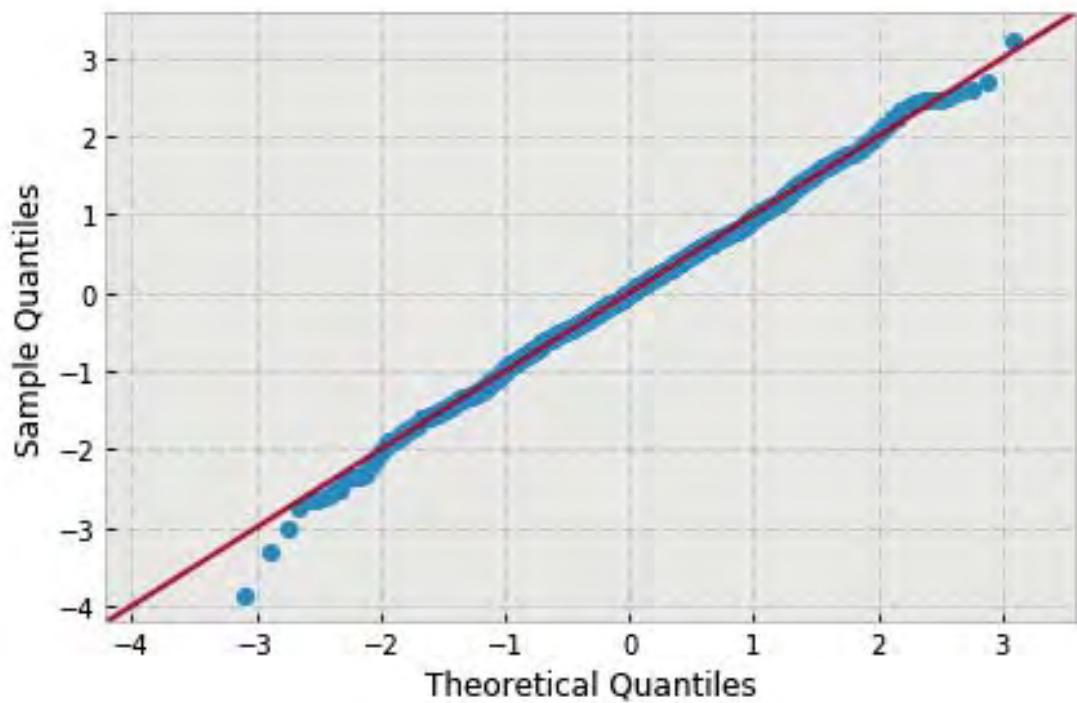


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=3131.6742278399997, scale=102.8848516584566)
DescribeResult(nobs=1000, minmax=(2805.783715, 3452.8384639999999)
               mean=3131.6742278399997, variance=10595.888589371989,
               skewness=-0.03637774804533017, kurtosis=-0.24899669593619533)
```

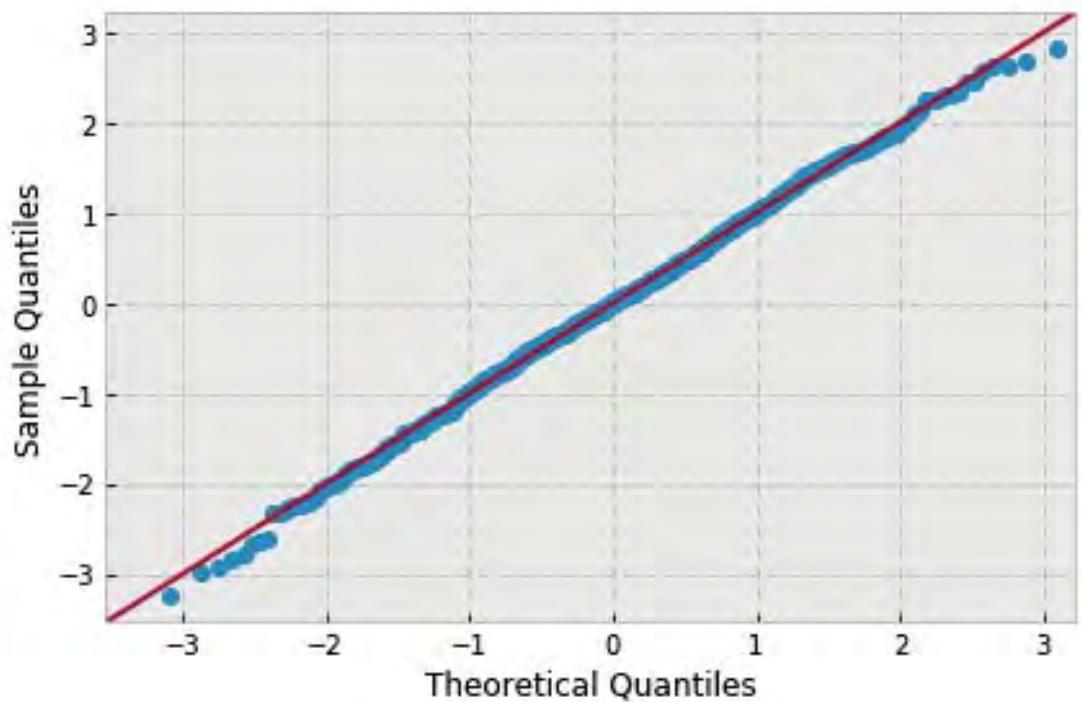
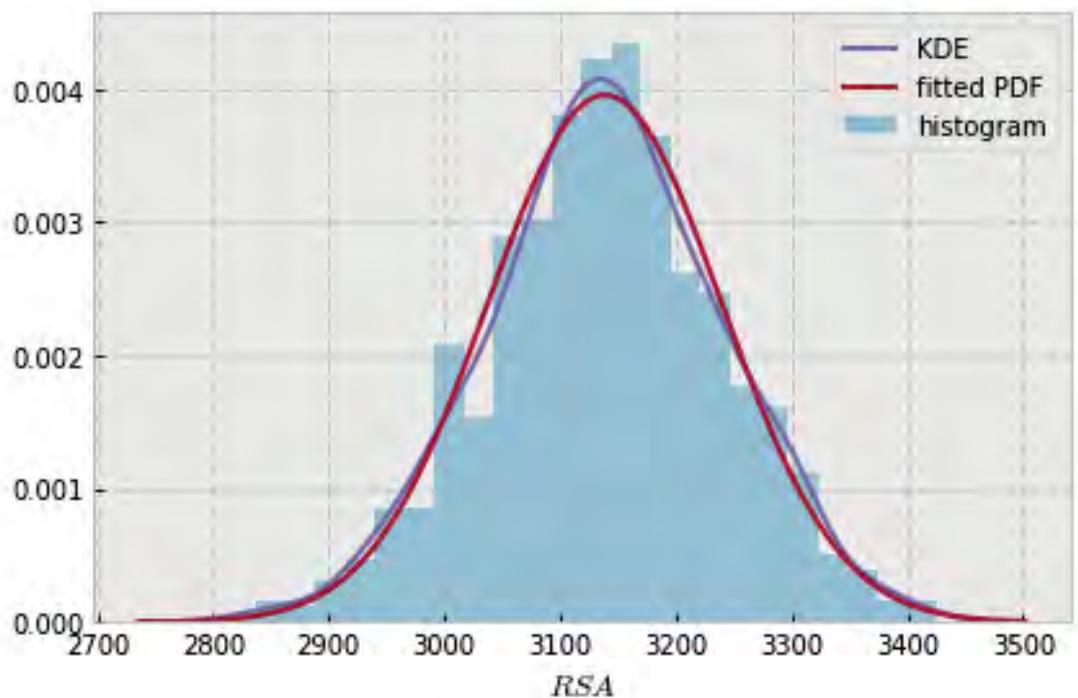


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=3135.180332738, scale=98.73353826641761)
DescribeResult(nobs=1000, minmax=(2754.8087270000001, 3453.2319859999998)
               mean=3135.1803327379998, variance=9758.0696482544045,
               skewness=-0.06222954670862398, kurtosis=0.20072995217901424)
```

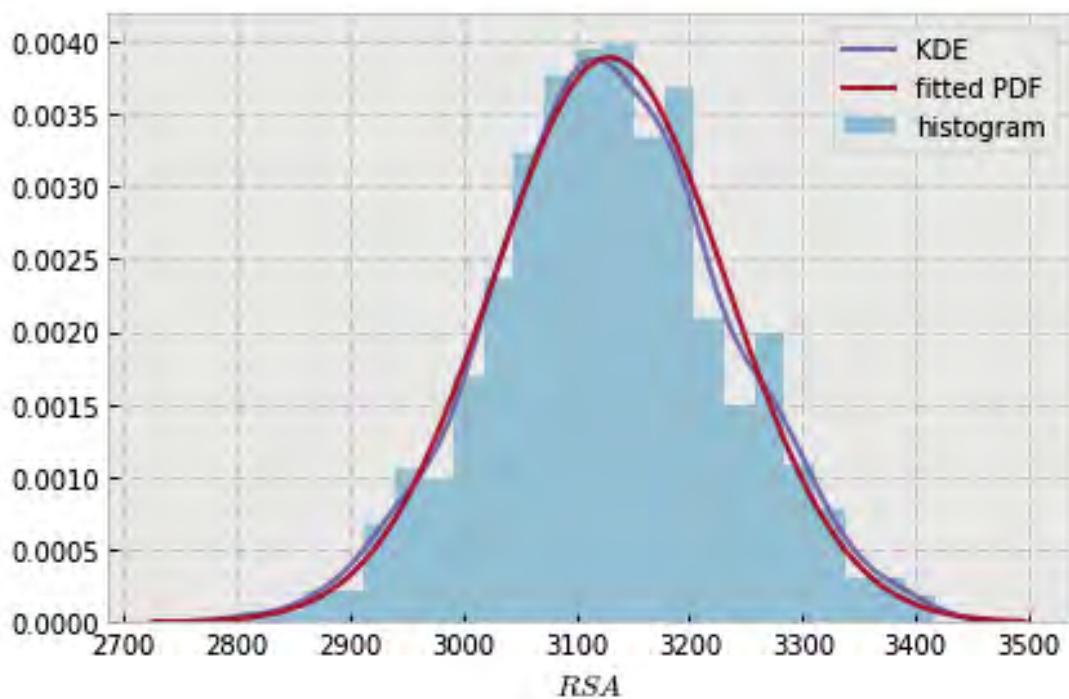


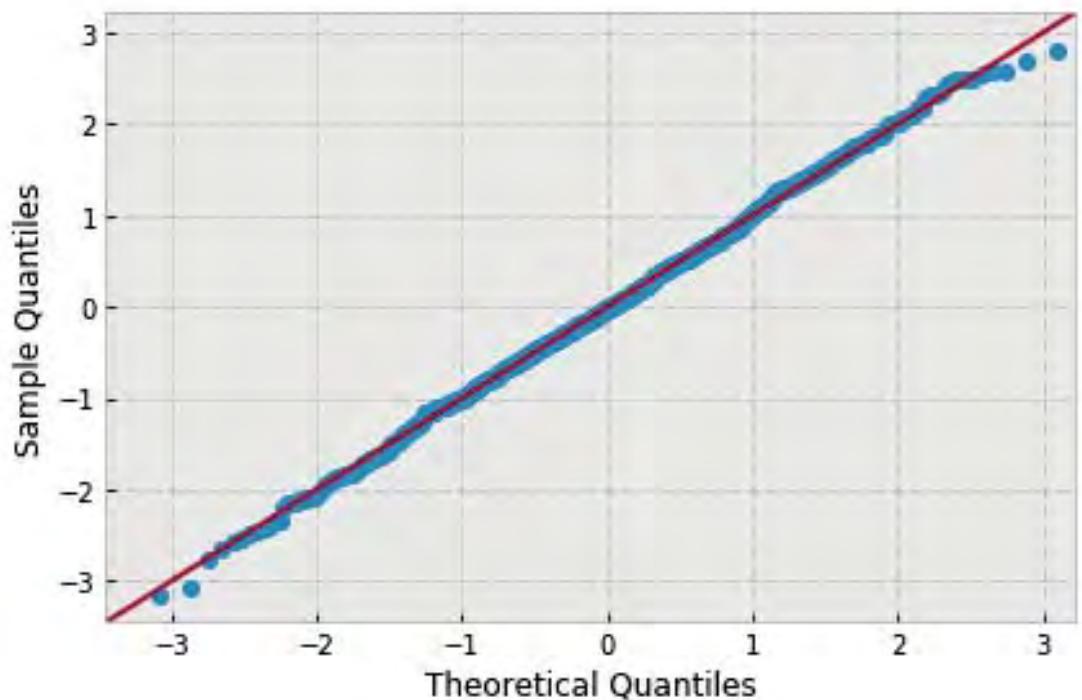


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(), loc=3138.4312522640002, scale=100.90761640080488)
DescribeResult(nobs=1000, minmax=(2812.3734060000002, 3424.6216639999998)
               mean=3138.4312522640002, variance=10192.539587279265,
               skewness=-0.06944095027188492, kurtosis=-0.04591695347800551)
```

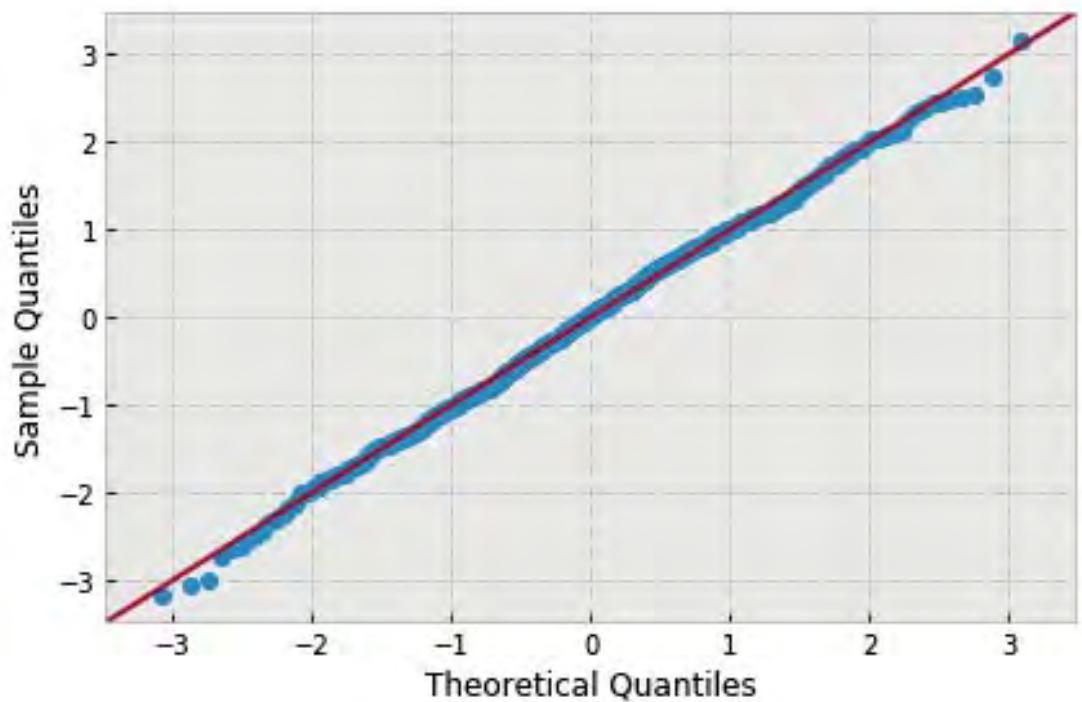
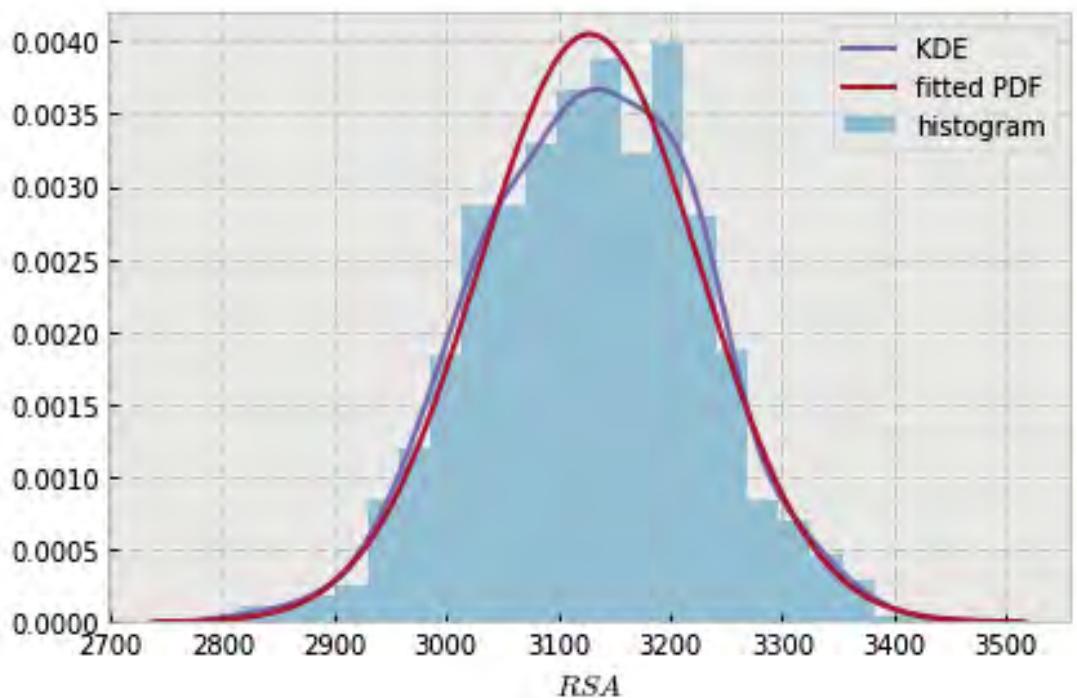


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=3129.4717979650004, scale=102.70841872548829)
DescribeResult(nobs=1000, minmax=(2805.444555, 3417.3452200000002)
               mean=3129.471797965004, variance=10559.57885594618,
               skewness=0.003885077446856966, kurtosis=-0.06123539107783538)
```

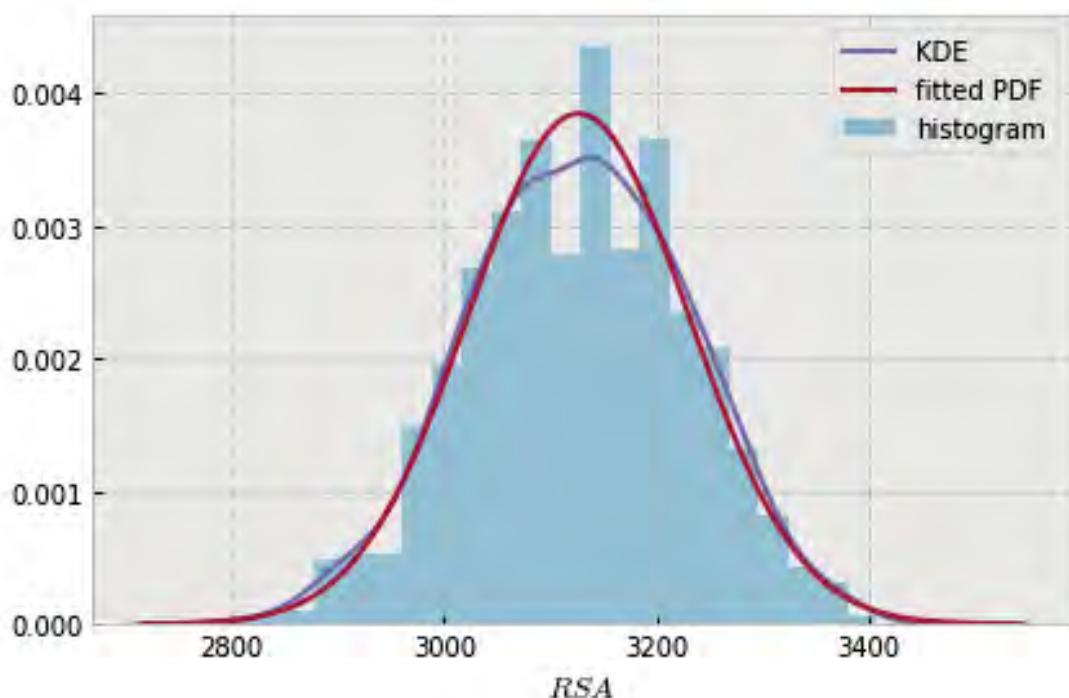


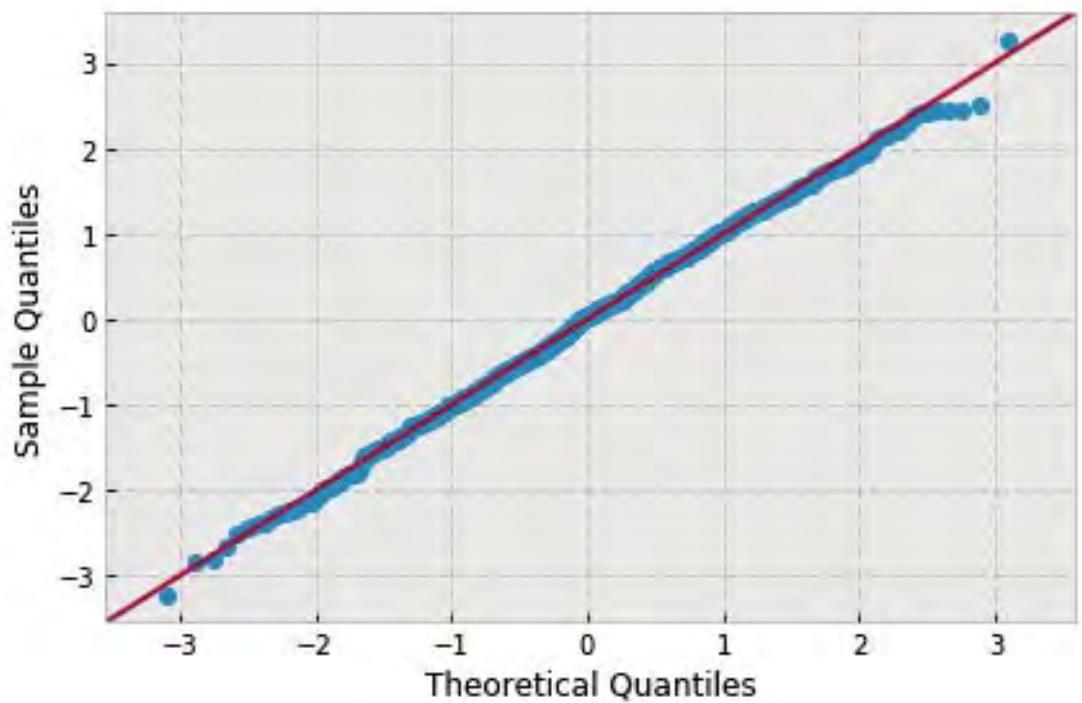


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=3128.2603472779997, scale=98.48809012727699)
DescribeResult(nobs=1000, minmax=(2816.495969999999, 3438.7202160000002)
               mean=3128.2603472779997, variance=9709.6135104290643,
               skewness=-0.07102087682309616, kurtosis=-0.16965625172688892)
```

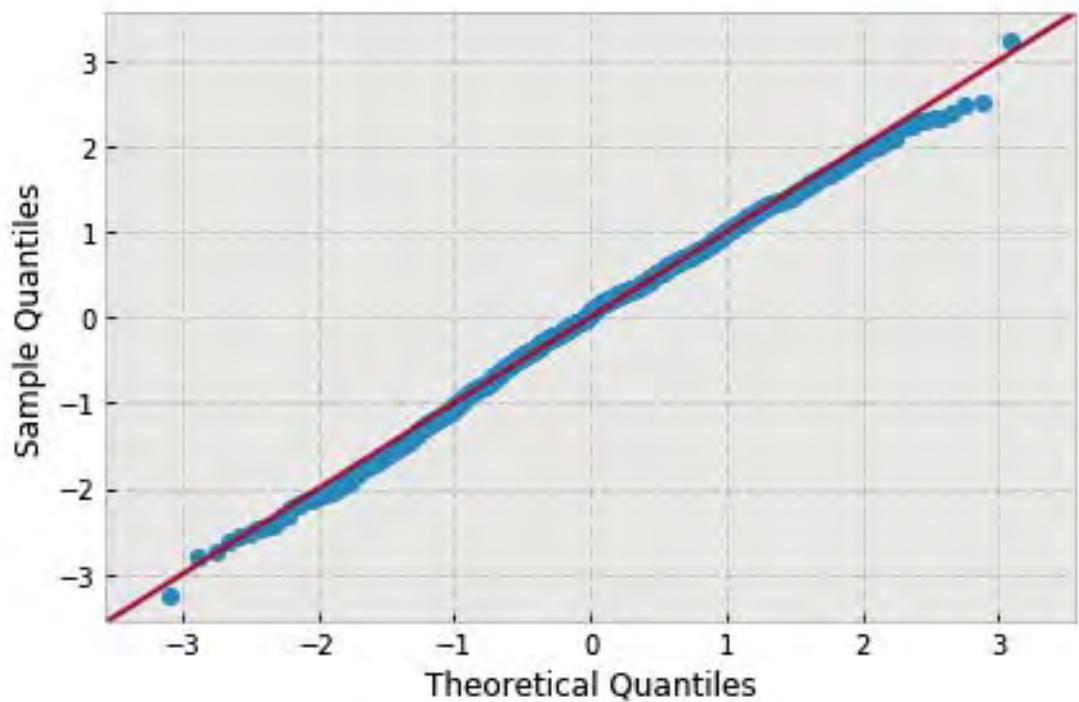
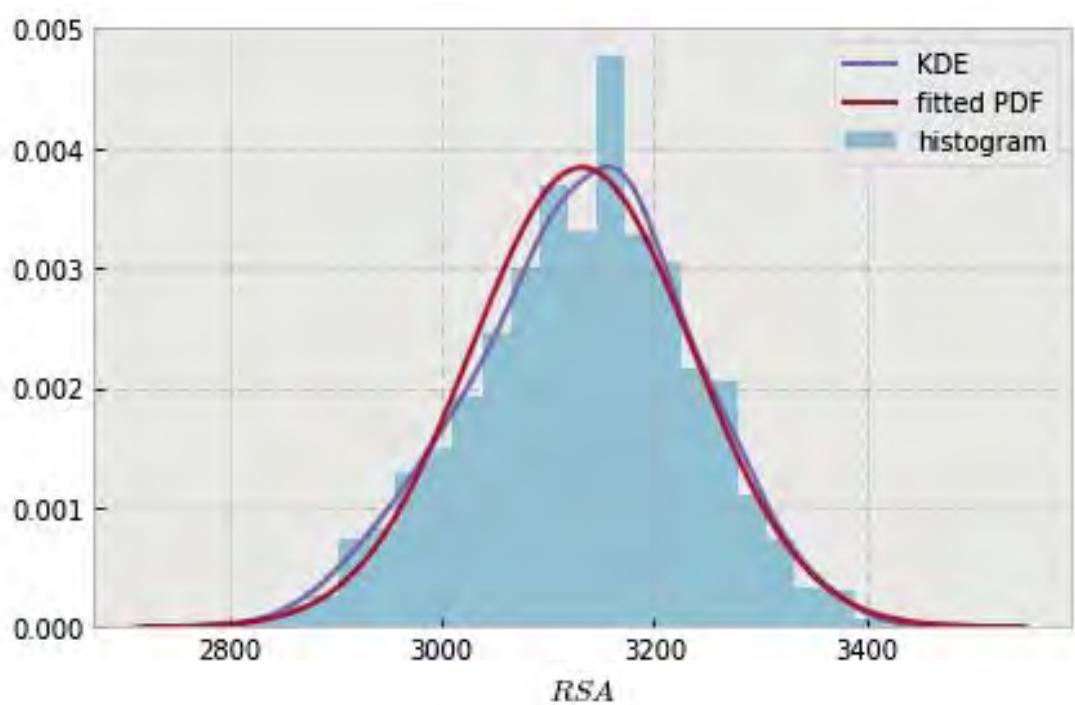


```
variable name: sigma
variable value: 0.066666666666666667
distribution: normal(shape=(), loc=3126.866145847, scale=103.69282646154554)
DescribeResult(nobs=1000, minmax=(2793.532193, 3464.4767350000002)
    mean=3126.8661458470001, variance=10762.965224809006,
    skewness=-0.062031733294812534, kurtosis=-0.21739579066584414)
```

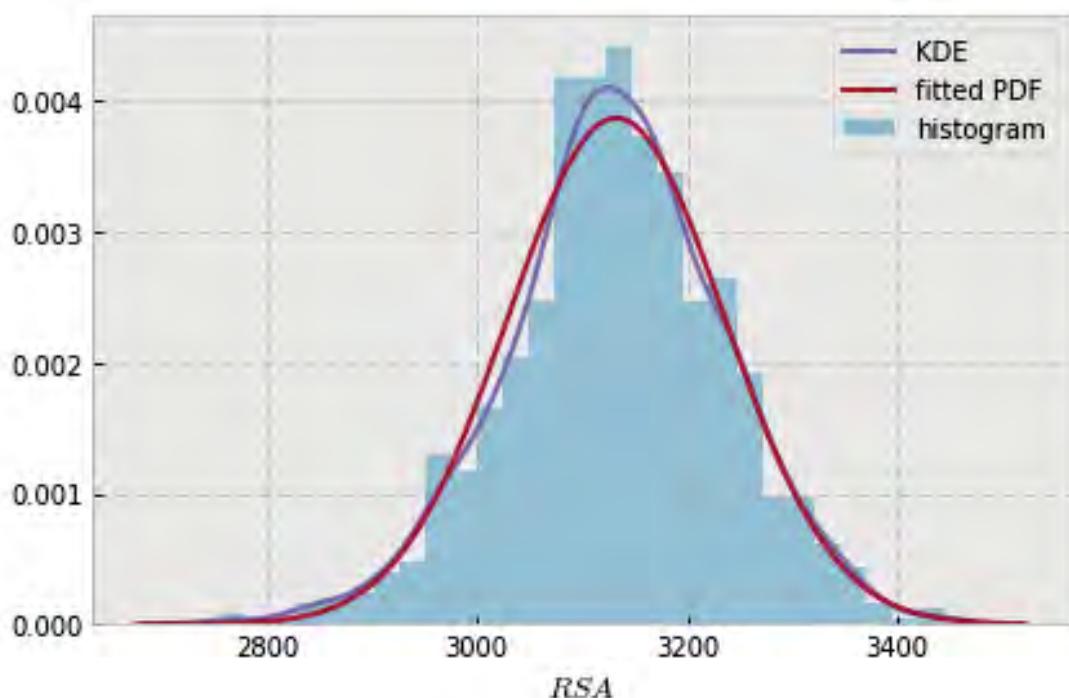


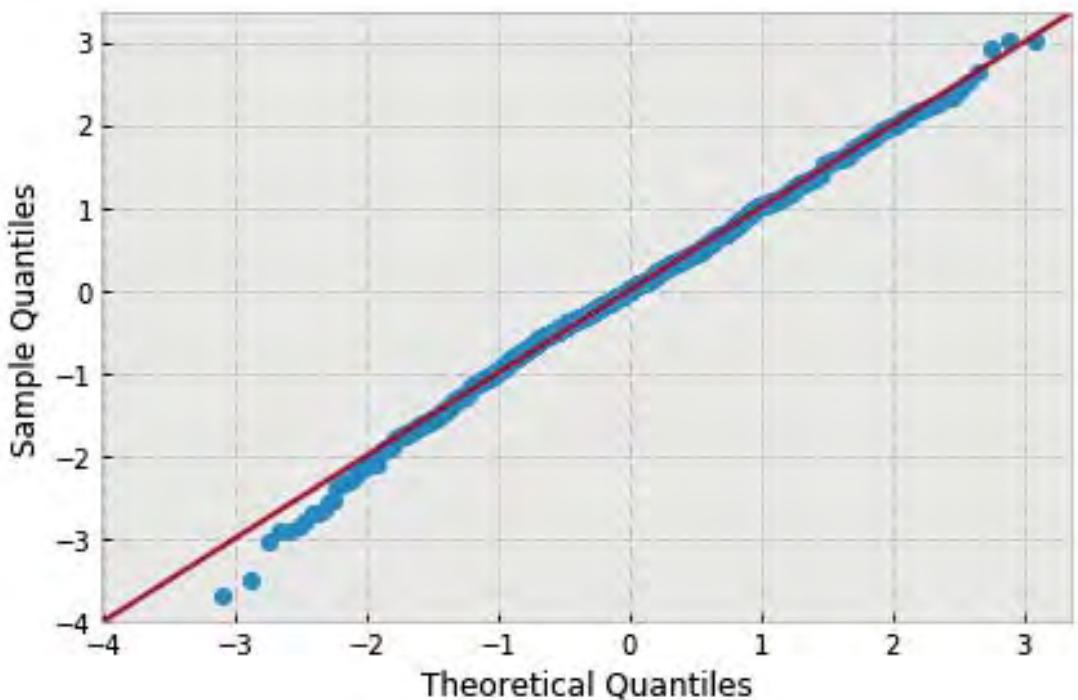


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=3132.2611929940003, scale=103.90601629996502)
DescribeResult(nobs=1000, minmax=(2796.0598450000002, 3467.87698)
               mean=3132.2611929940003, variance=10807.267490819413,
               skewness=-0.16775403640233016, kurtosis=-0.1544603597203018)
```

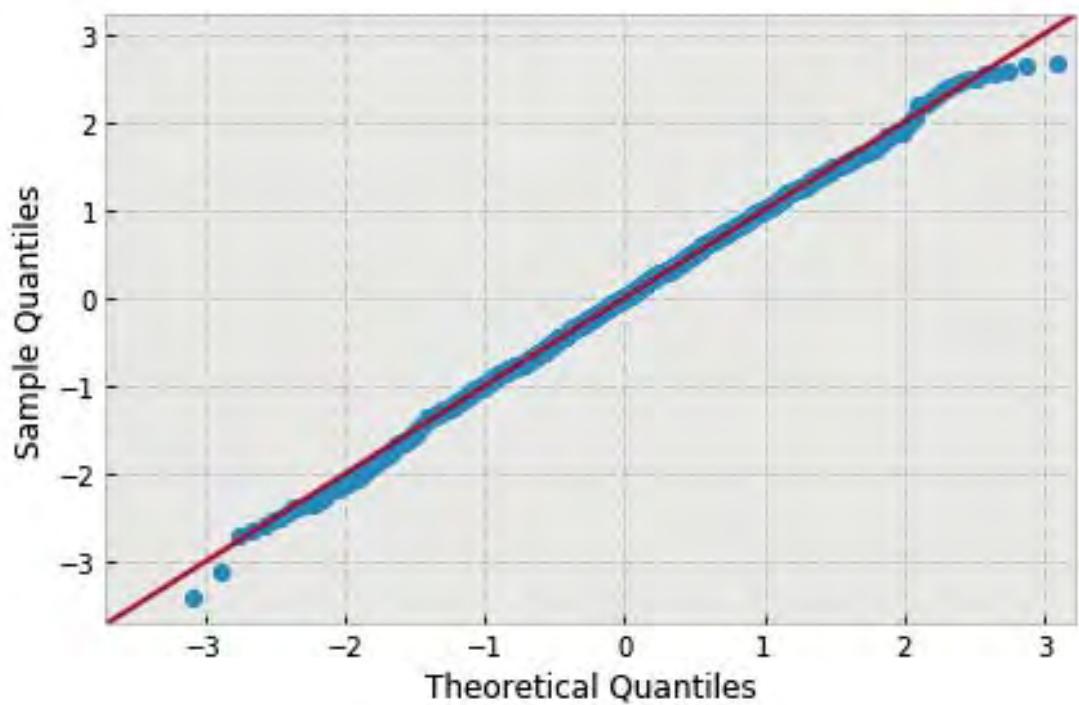
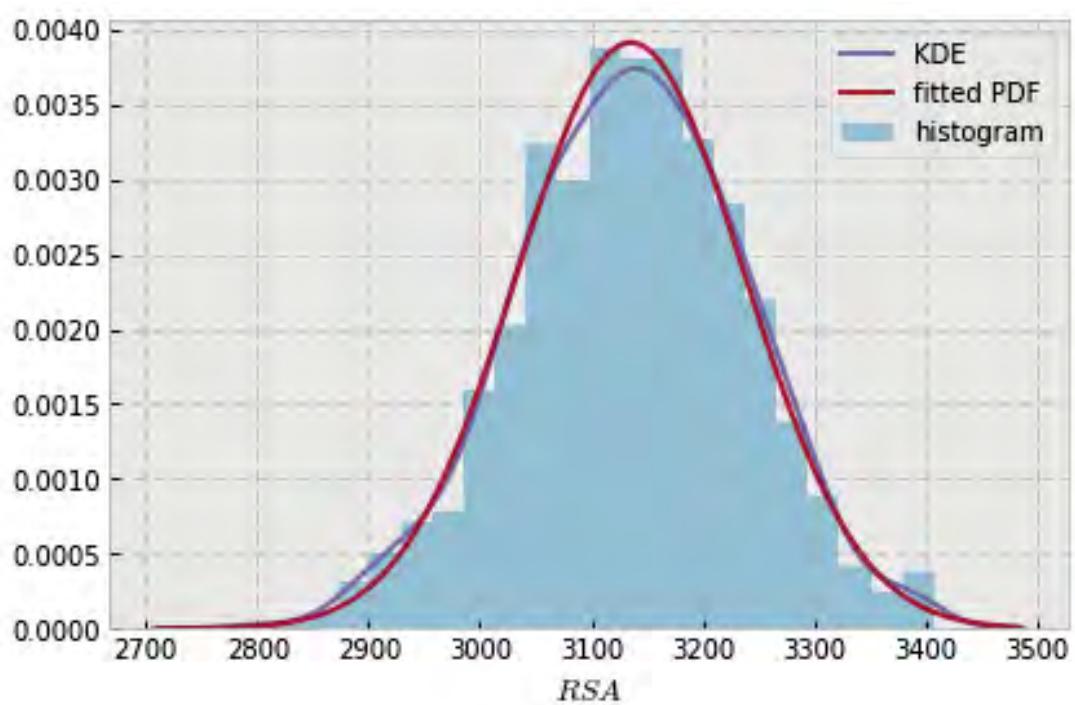


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=3132.474594838, scale=103.20292174979019)
DescribeResult(nobs=1000, minmax=(2753.552737, 3443.5904369999998)
               mean=3132.474594838, variance=10661.504562255574,
               skewness=-0.15332866392599062, kurtosis=0.30317313502586574)
```

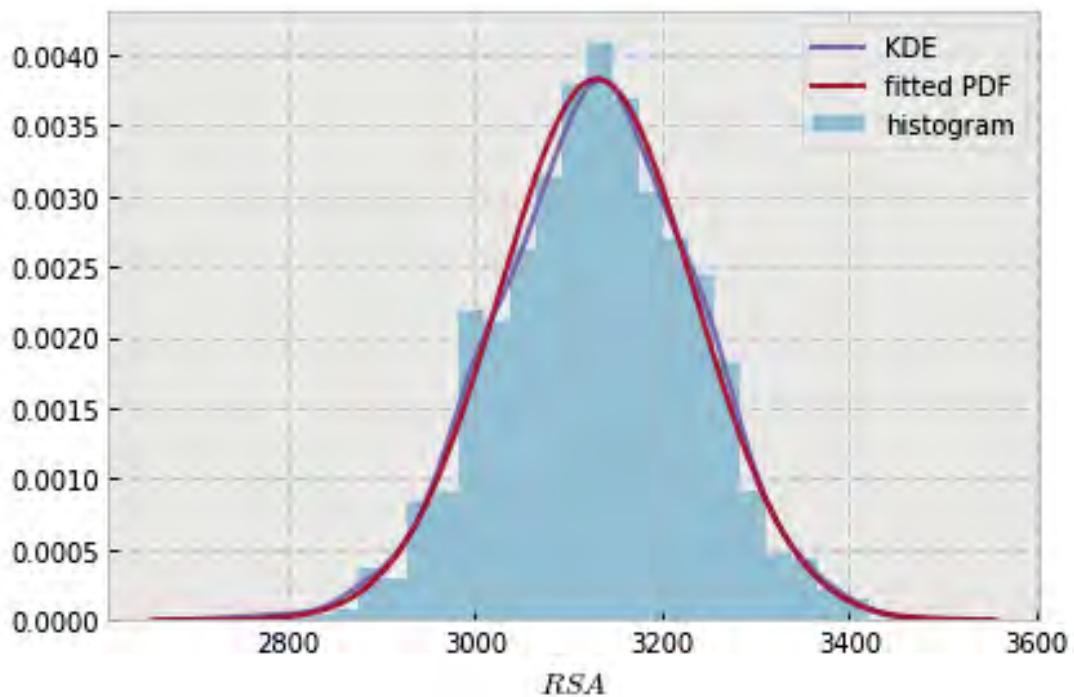


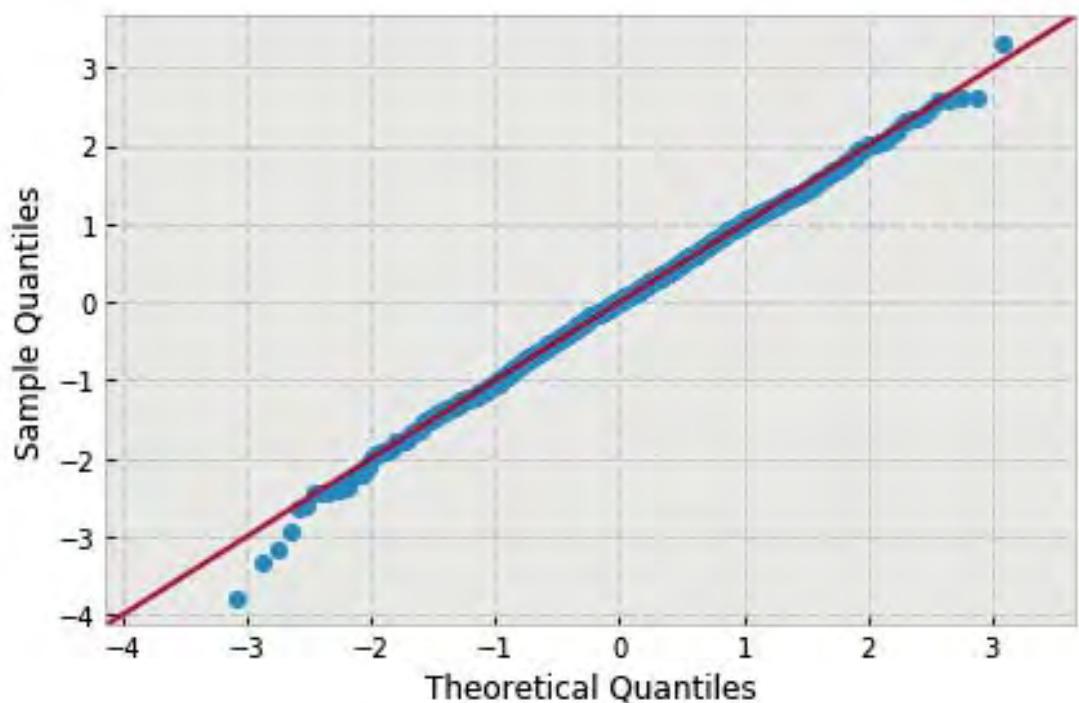


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=3135.335721614, scale=101.98304423576943)
DescribeResult(nobs=1000, minmax=(2788.494126999999, 3406.829417999998)
               mean=3135.335721613998, variance=10410.952263858764,
               skewness=-0.09346452339326895, kurtosis=-0.061822405569606165)
```

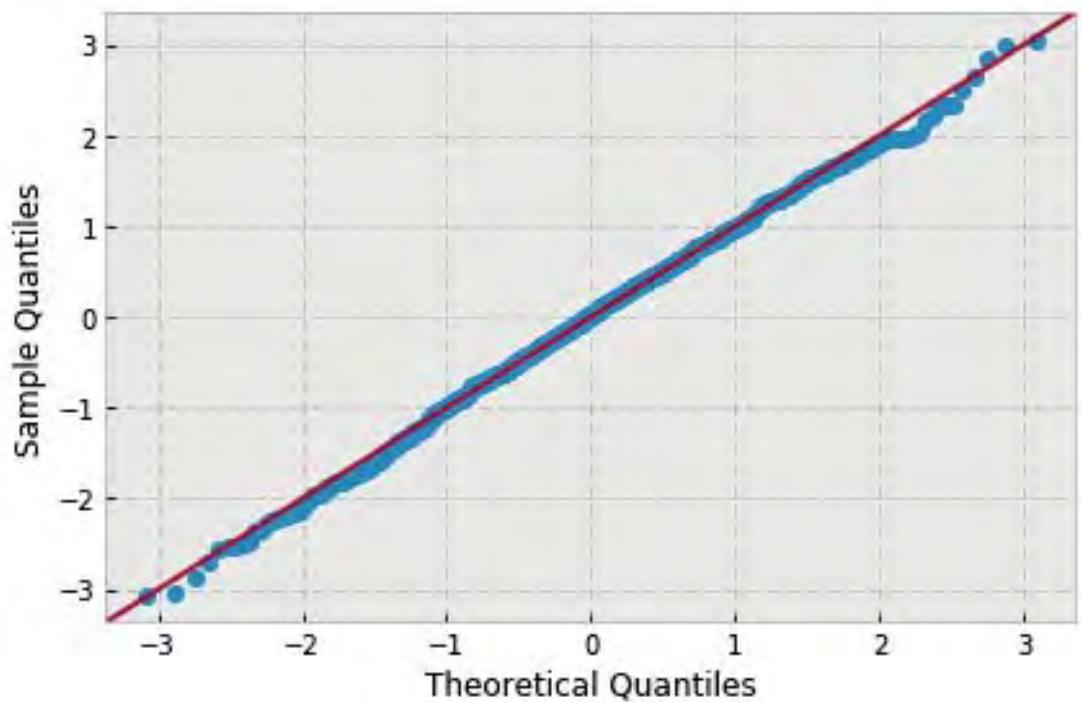
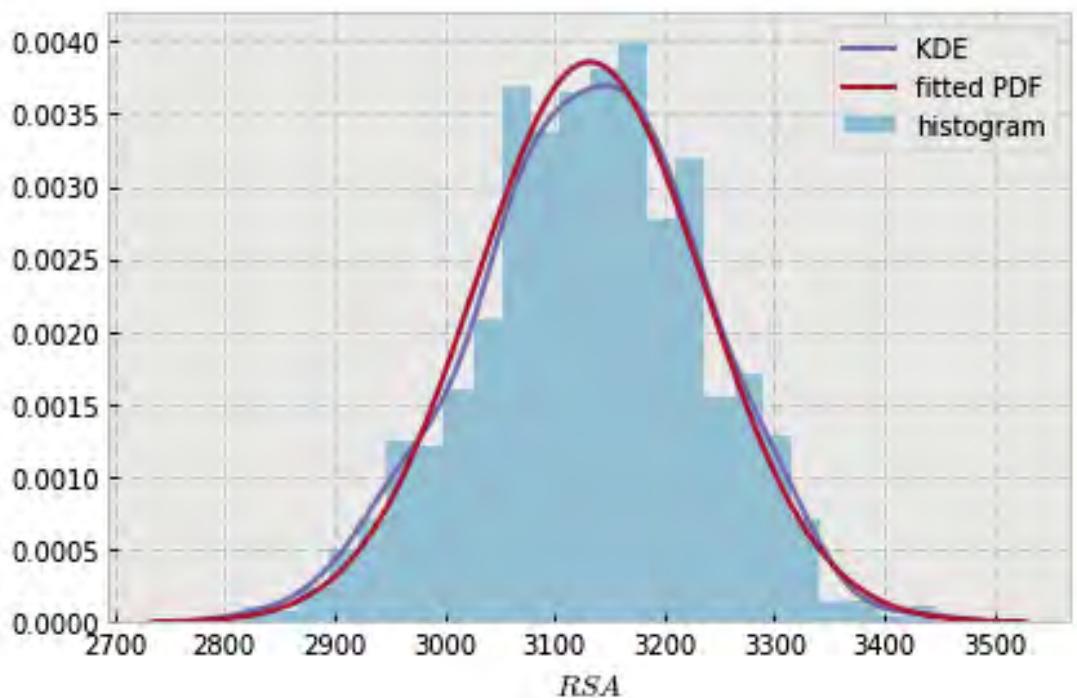


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=3130.6240909289995, scale=104.10676482653479)
DescribeResult(nobs=1000, minmax=(2737.131617, 3474.7532120000001)
               mean=3130.6240909289995, variance=10849.067550197618,
               skewness=-0.11598763273079538, kurtosis=0.08916224845270992)
```

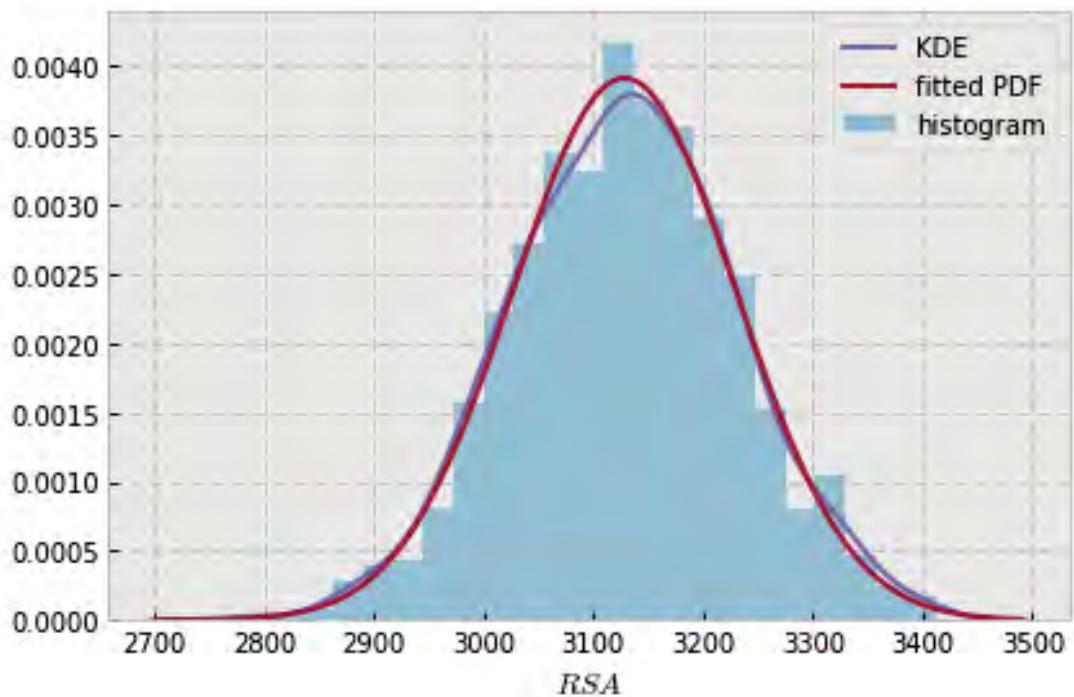


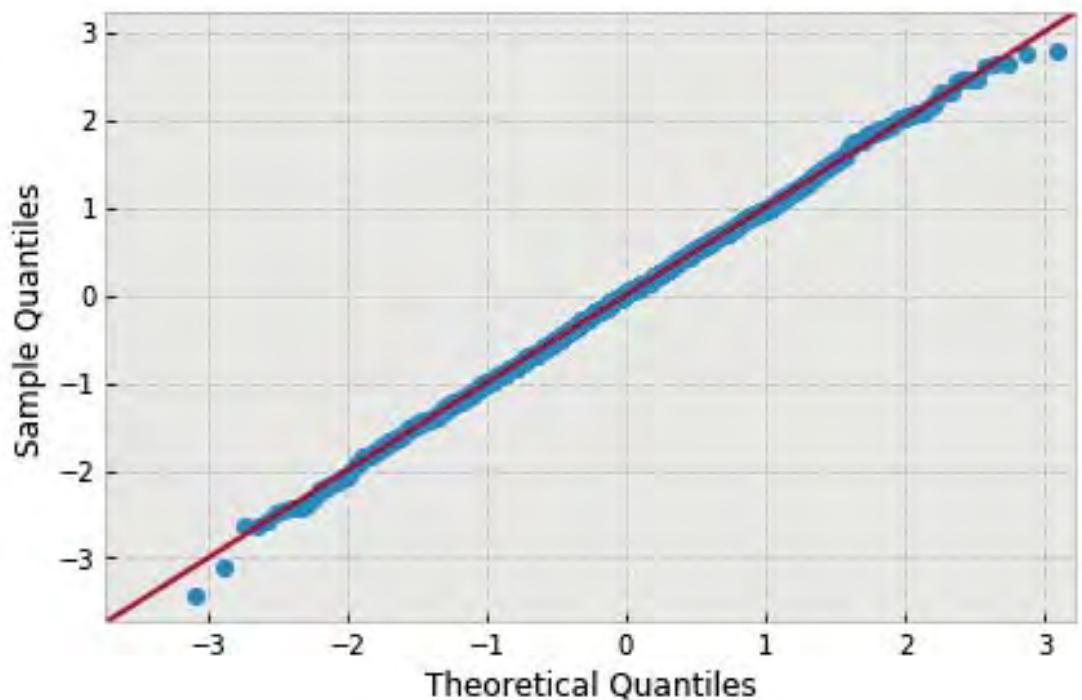


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=3131.941092627, scale=103.43043559279894)
DescribeResult(nobs=1000, minmax=(2815.8531119999998, 3446.803296)
               mean=3131.9410926270002, variance=10708.563570486616,
               skewness=-0.12002354255325746, kurtosis=-0.11017003273435932)
```

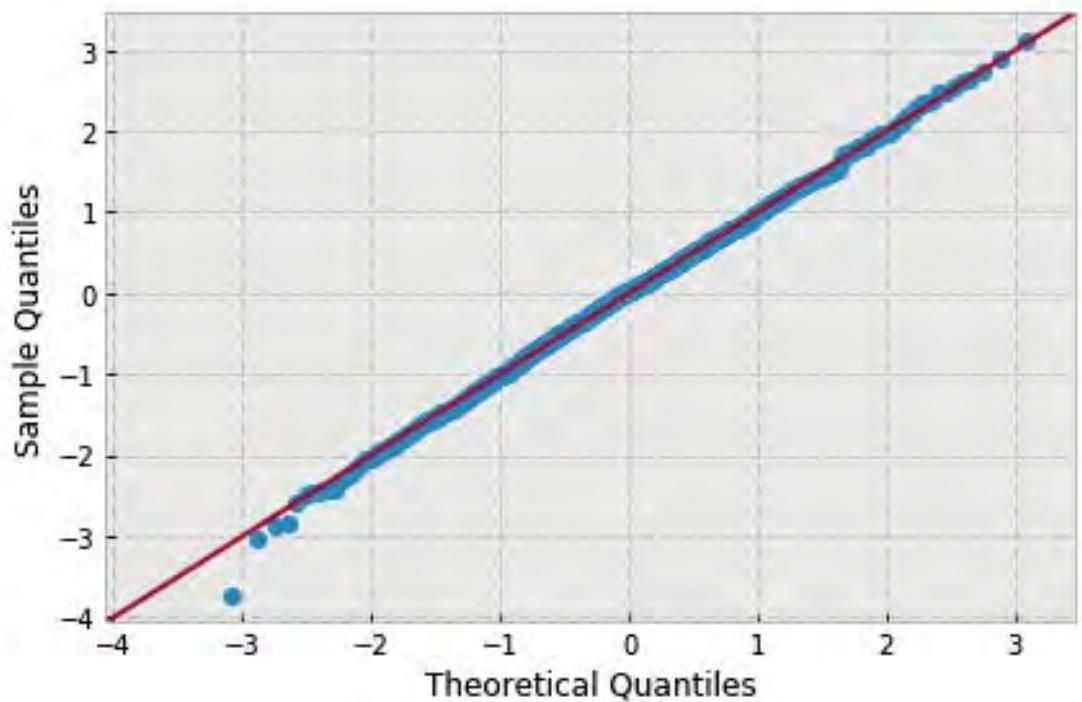
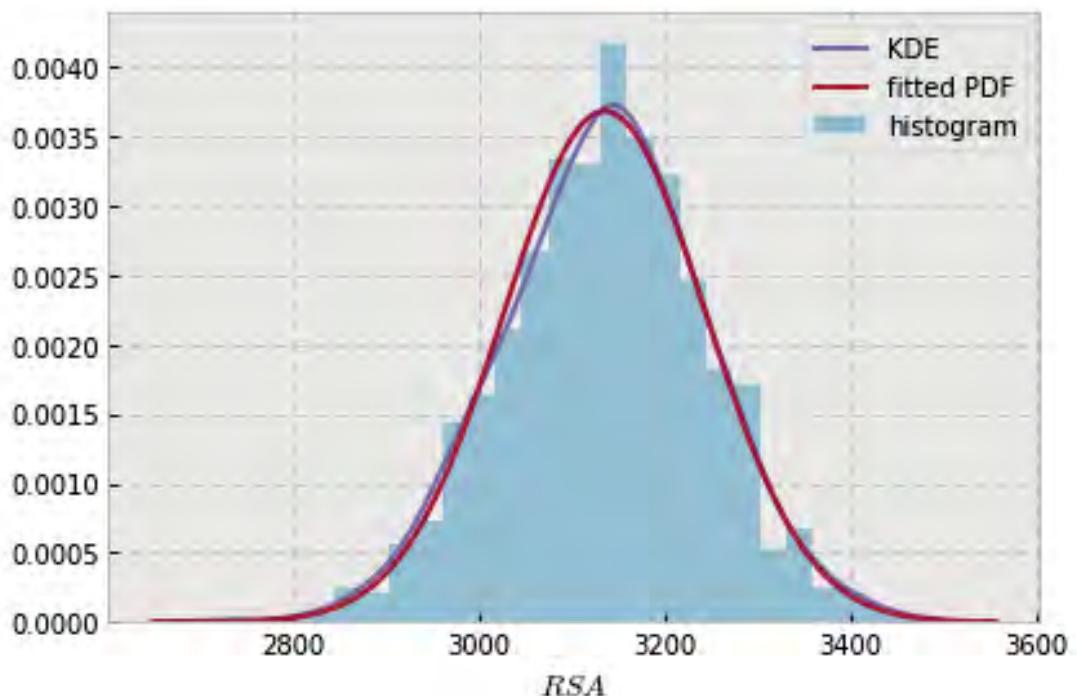


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=3128.374348625, scale=101.9010432695996)
DescribeResult(nobs=1000, minmax=(2779.839767999998, 3412.170098999999)
               mean=3128.374348625, variance=10394.216836269079,
               skewness=-0.01824470972938809, kurtosis=-0.07458069159333602)
```

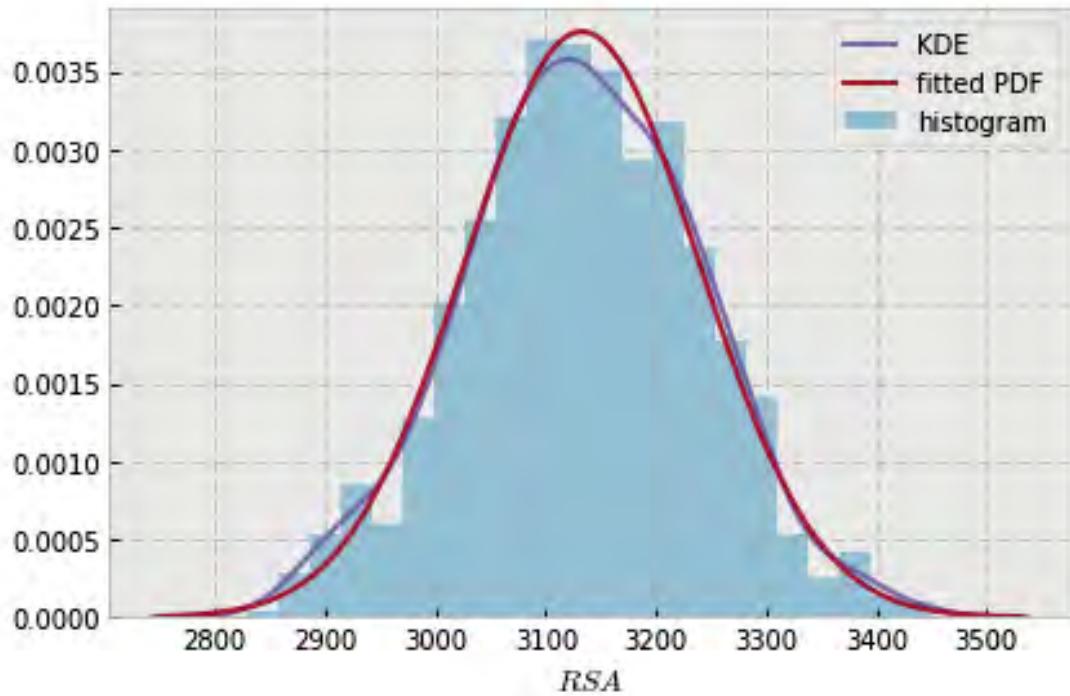


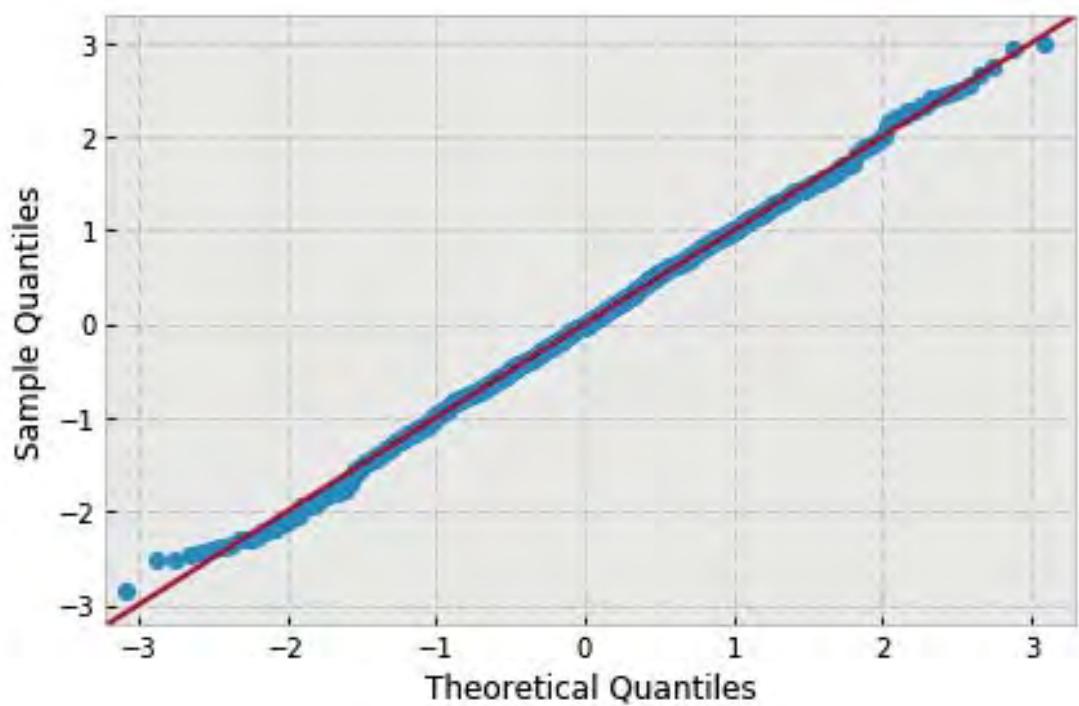


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=3135.771148513, scale=108.41537302195535)
DescribeResult(nobs=1000, minmax=(2732.7028100000002, 3473.4138269999999)
               mean=3135.7711485129998, variance=11765.65876625598,
               skewness=-0.07790252261790732, kurtosis=0.061350853811289685)
```

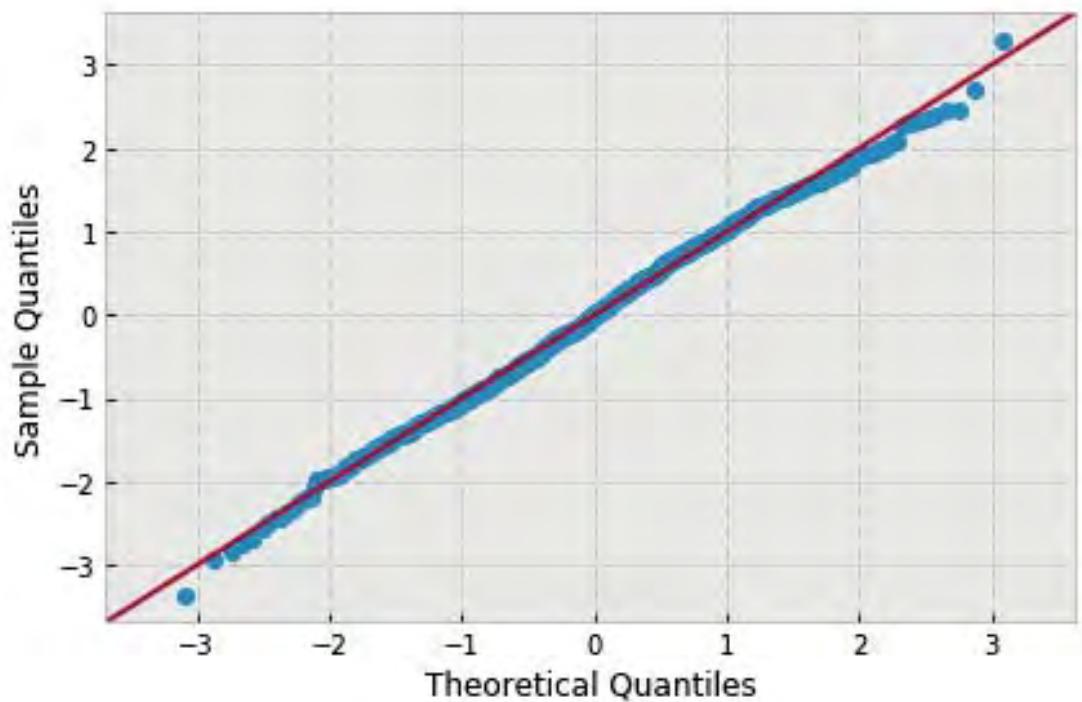
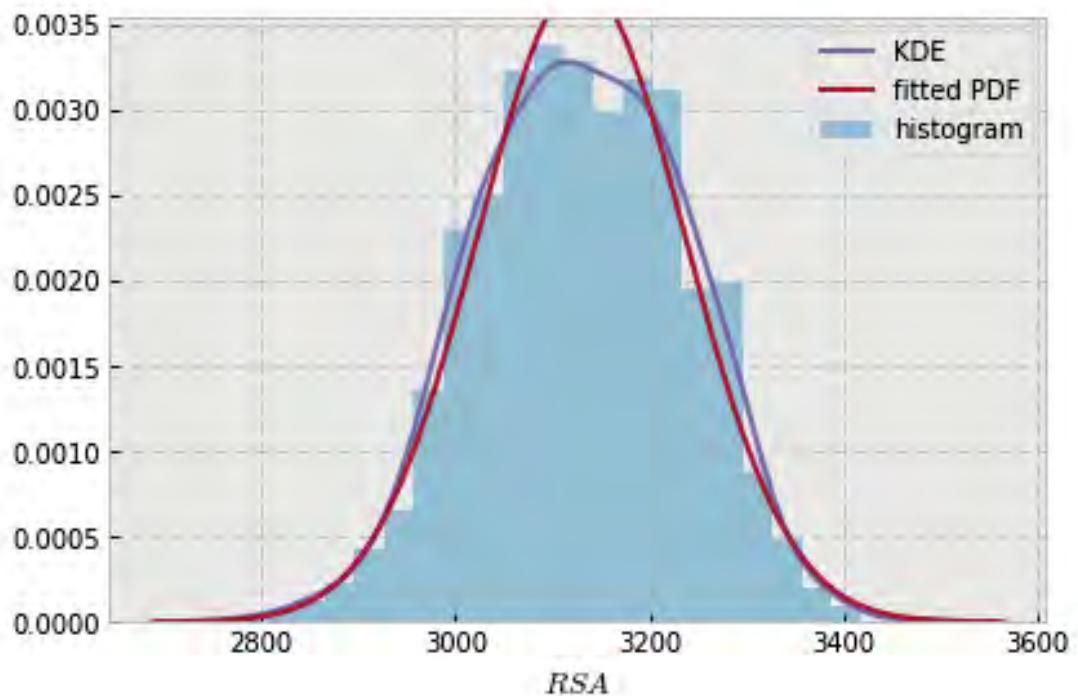


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=3132.9198656630006, scale=106.23586816174215)
DescribeResult(nobs=1000, minmax=(2829.1171639999998, 3451.1239770000002)
               mean=3132.9198656630006, variance=11297.357041120178,
               skewness=-0.014891754496721398, kurtosis=-0.13728432423407444)
```

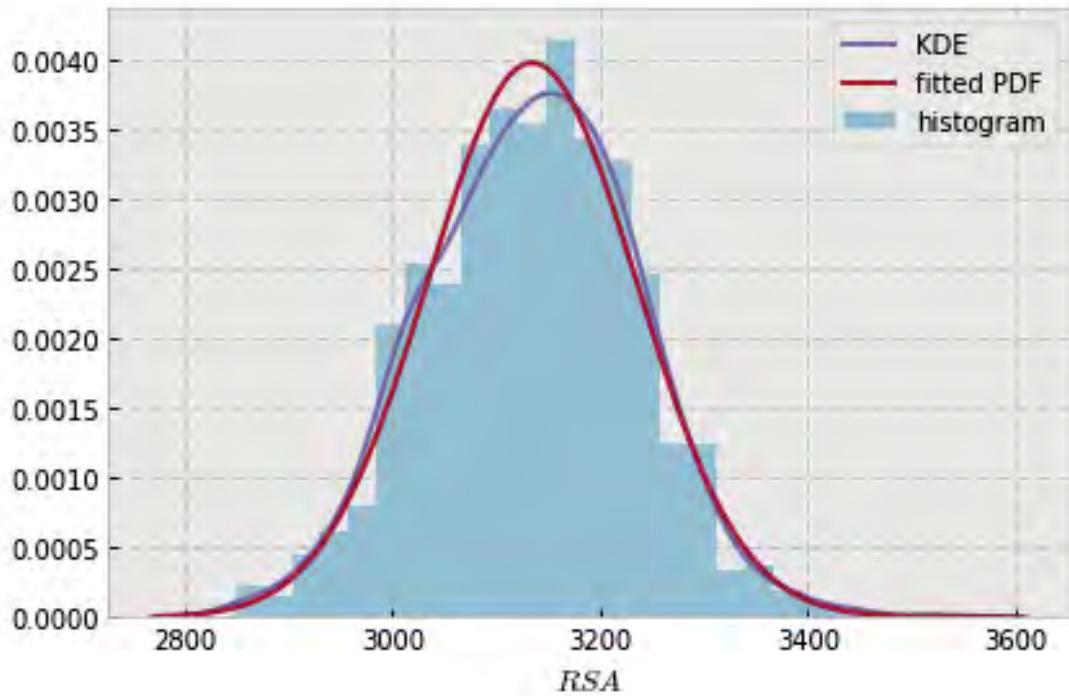


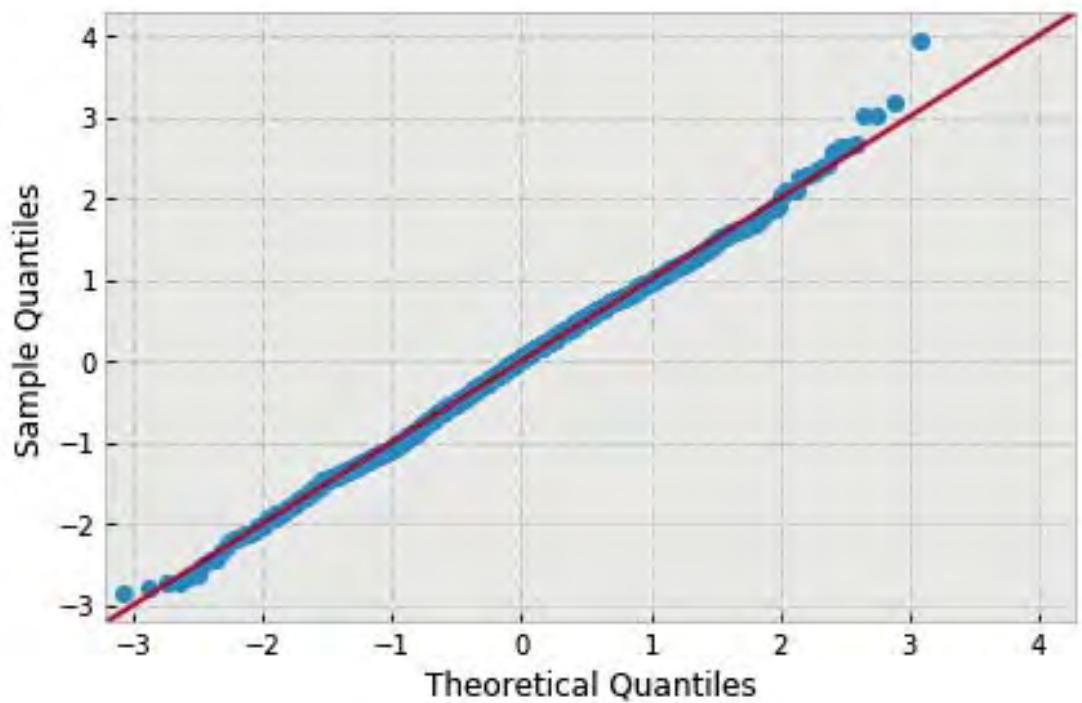


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=3129.806037346, scale=106.62148683207414)
DescribeResult(nobs=1000, minmax=(2772.3326480000001, 3480.5504550000001)
               mean=3129.8060373459998, variance=11379.520975257416,
               skewness=-0.06651873952300266, kurtosis=-0.28730683229675735)
```

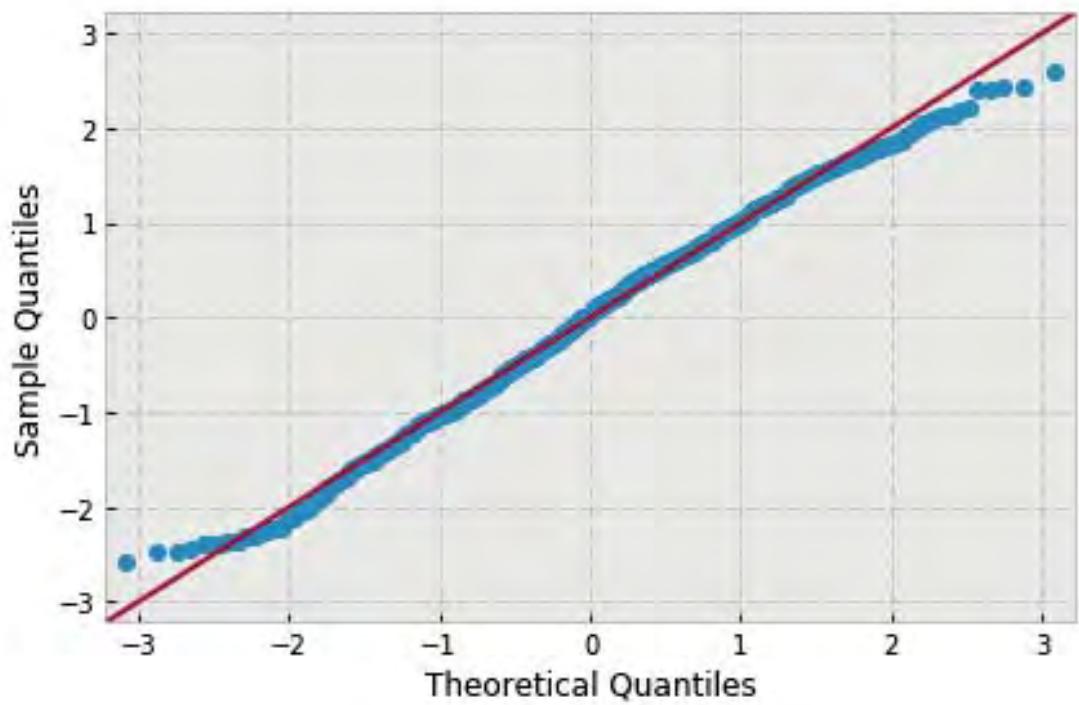
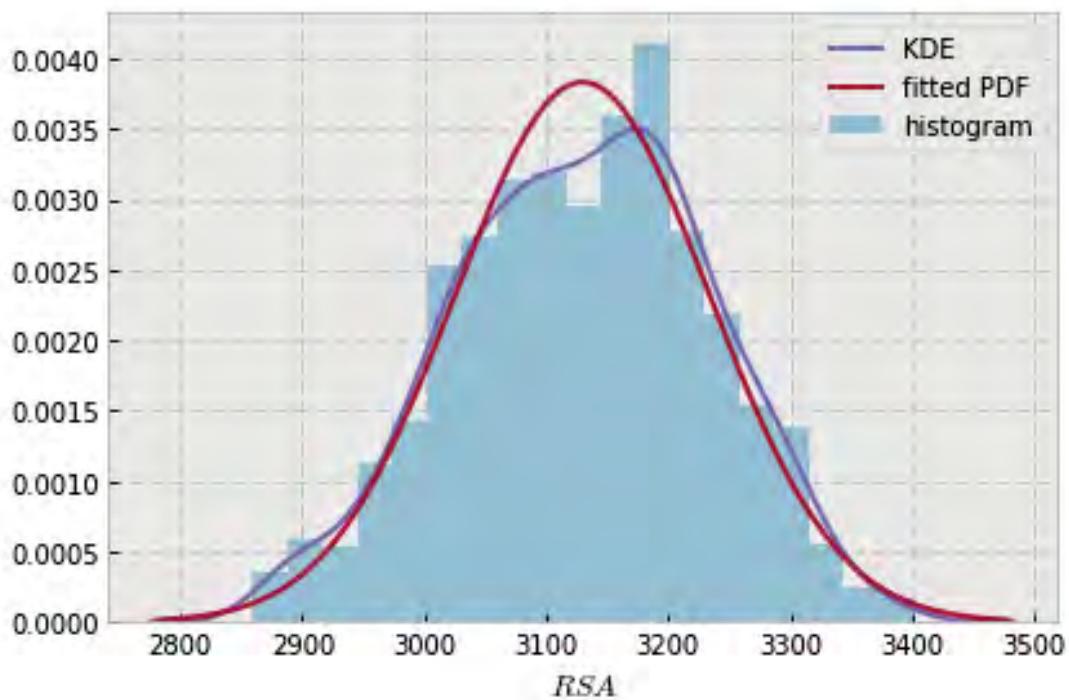


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=3134.4767507969996, scale=100.31516207703177)
DescribeResult(nobs=1000, minmax=(2849.116935, 3529.180042)
    mean=3134.4767507969996, variance=10073.204947488641,
    skewness=0.018713248503409278, kurtosis=0.05257816959205375)
```

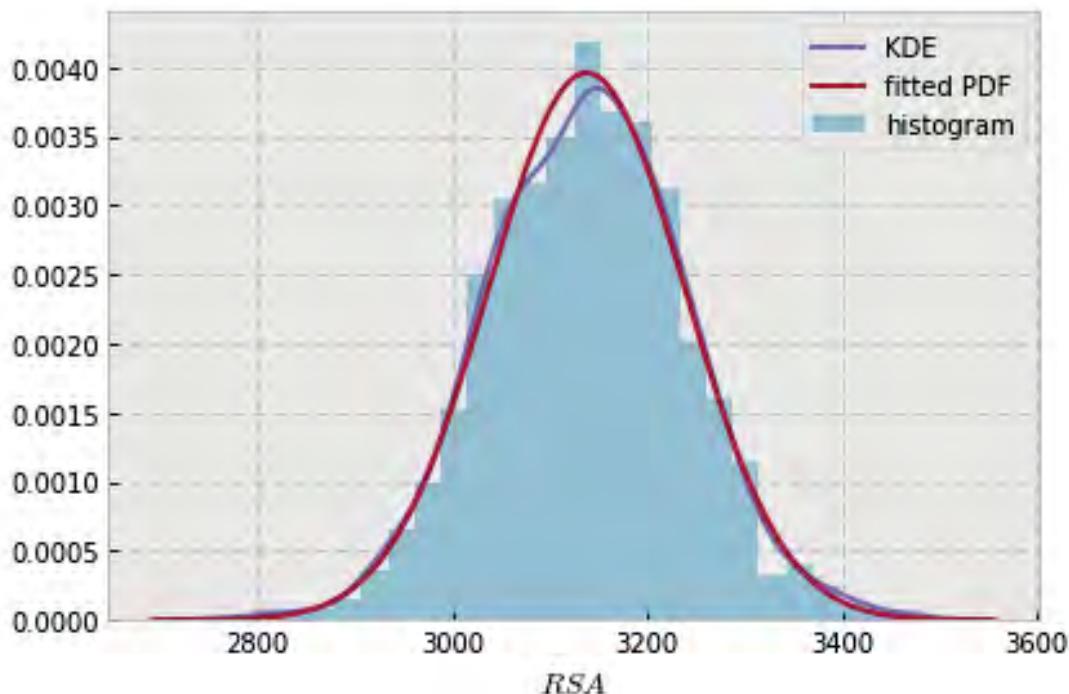


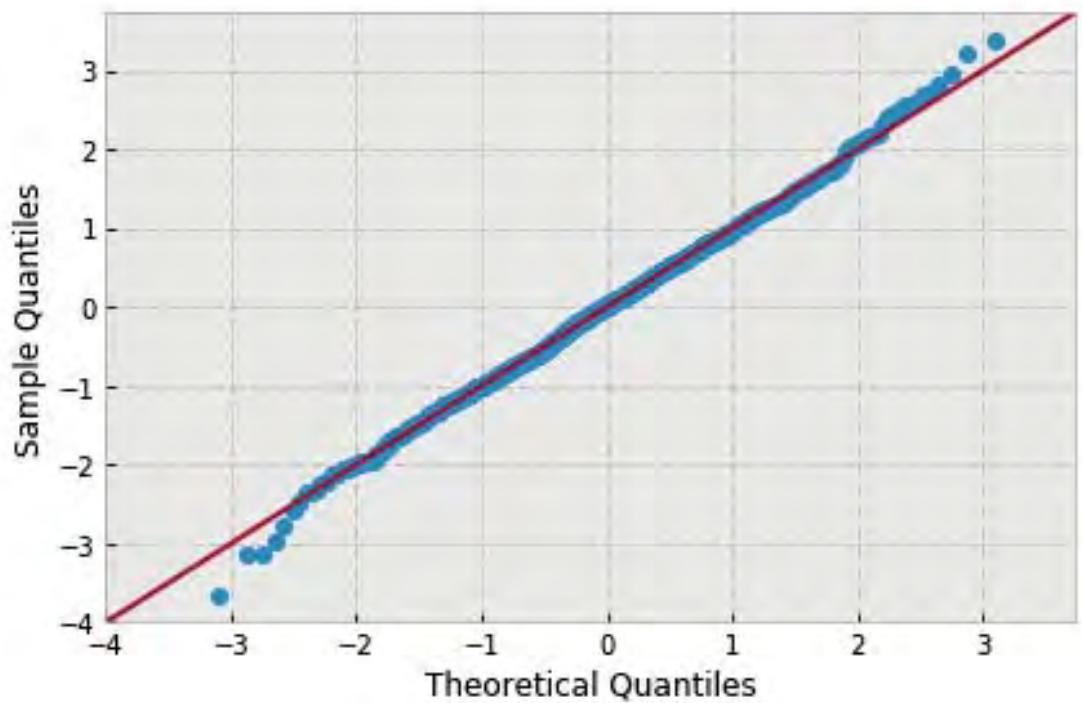


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=3129.619333926, scale=104.13567843400001)
DescribeResult(nobs=1000, minmax=(2860.0776040000001, 3400.2378440000002)
               mean=3129.6193339259999, variance=10855.094617526982,
               skewness=-0.12636398754521208, kurtosis=-0.4371318854534154)
```

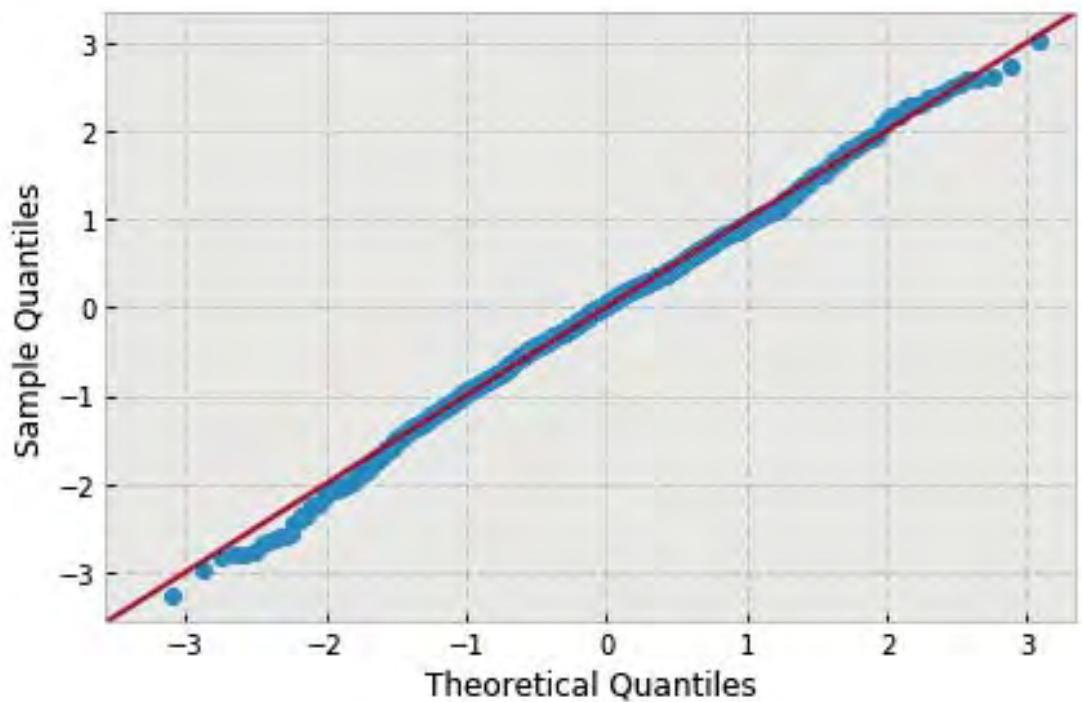
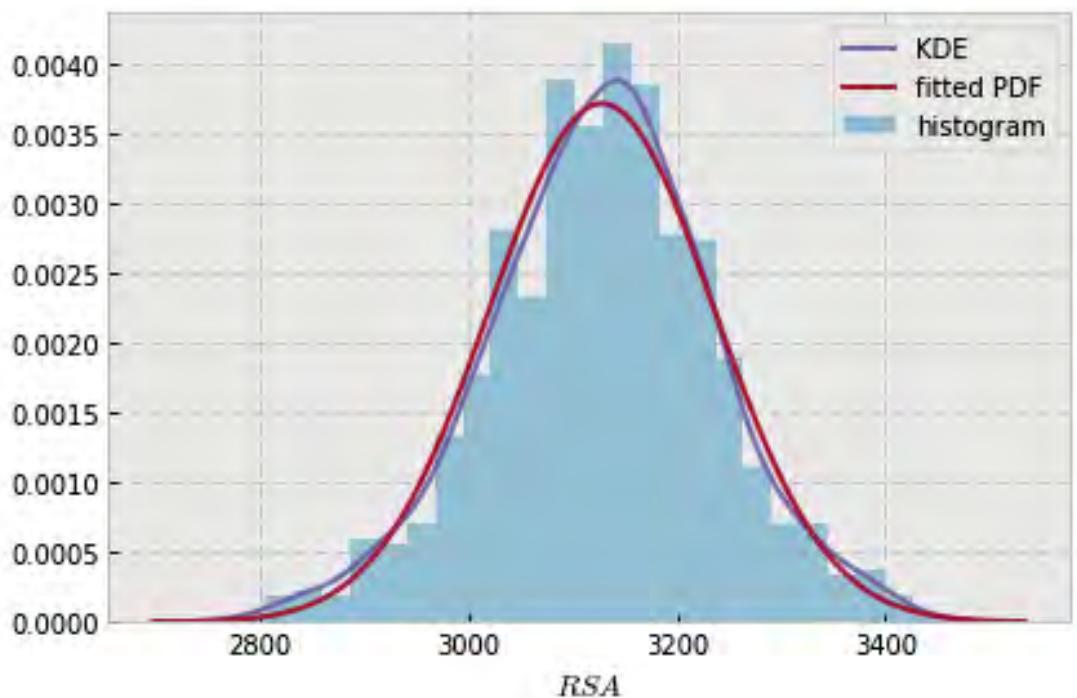


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=3137.7558778820003, scale=100.75007694386433)
DescribeResult(nobs=1000, minmax=(2770.4740190000002, 3477.819105)
               mean=3137.7558778820003, variance=10160.738742937521,
               skewness=0.0025182411502170728, kurtosis=0.1939114435931364)
```





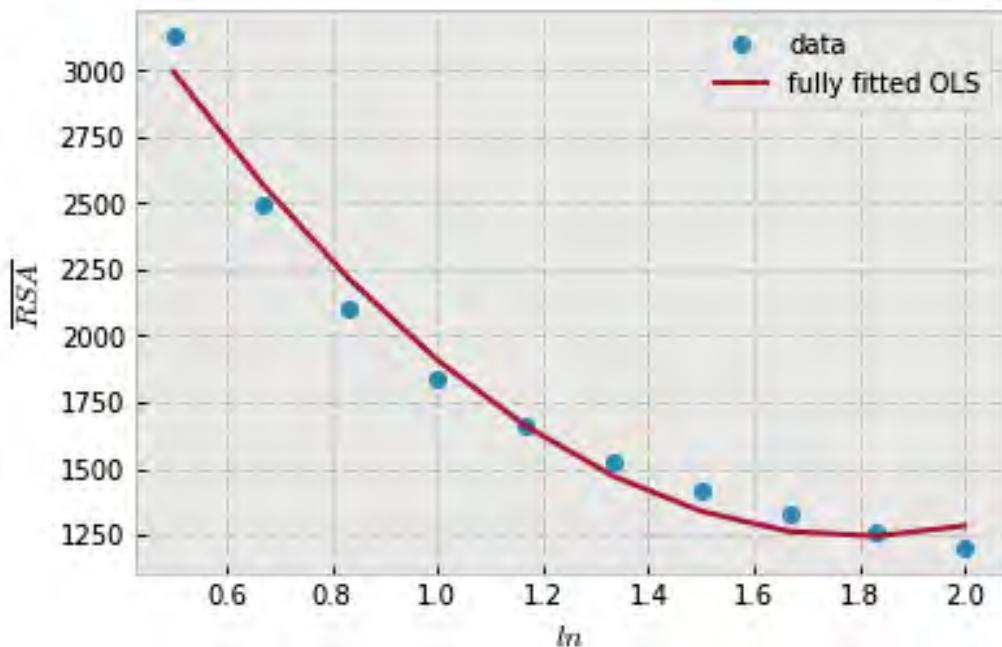
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=3128.128093959, scale=107.51595255469948)
DescribeResult(nobs=1000, minmax=(2778.378048999999, 3452.965385999999)
               mean=3128.1280939590001, variance=11571.25130504944,
               skewness=-0.10673511722115682, kurtosis=0.20392335432762598)
```



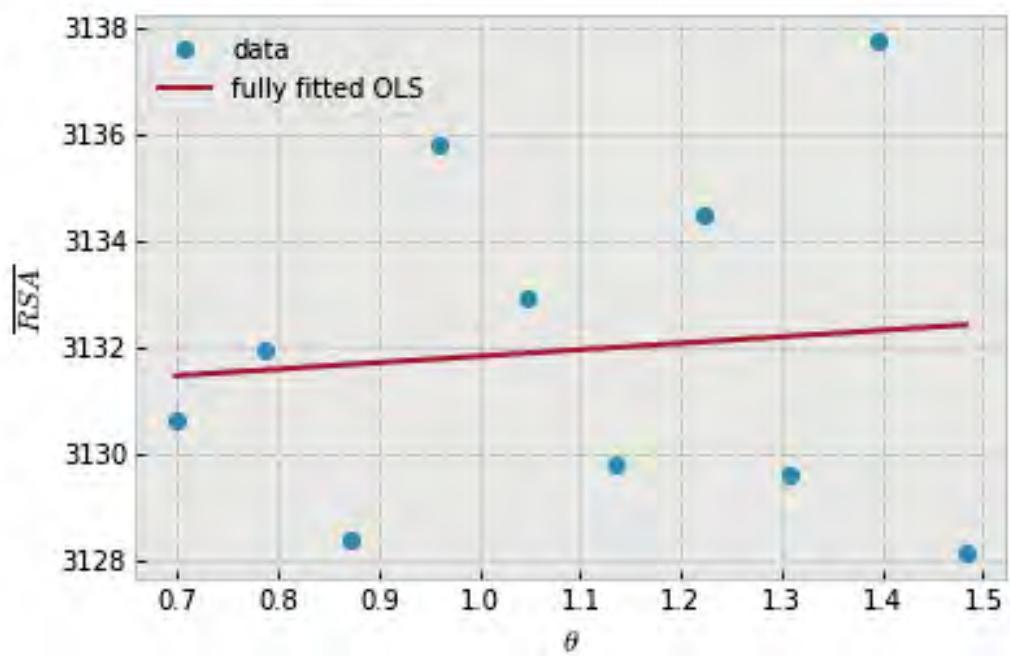
6.2 Parameter regression

```
In [18]: var_name = 'total_root_surface_area'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly1')  
        mean_reg('nB', var_name, 'poly1')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly1')  
        mean_reg('N', var_name, 'poly1')
```

```
variable name: ln  
model: poly2  
r-squared: 0.9295032719230995  
explained variance: 0.9296302758409627
```



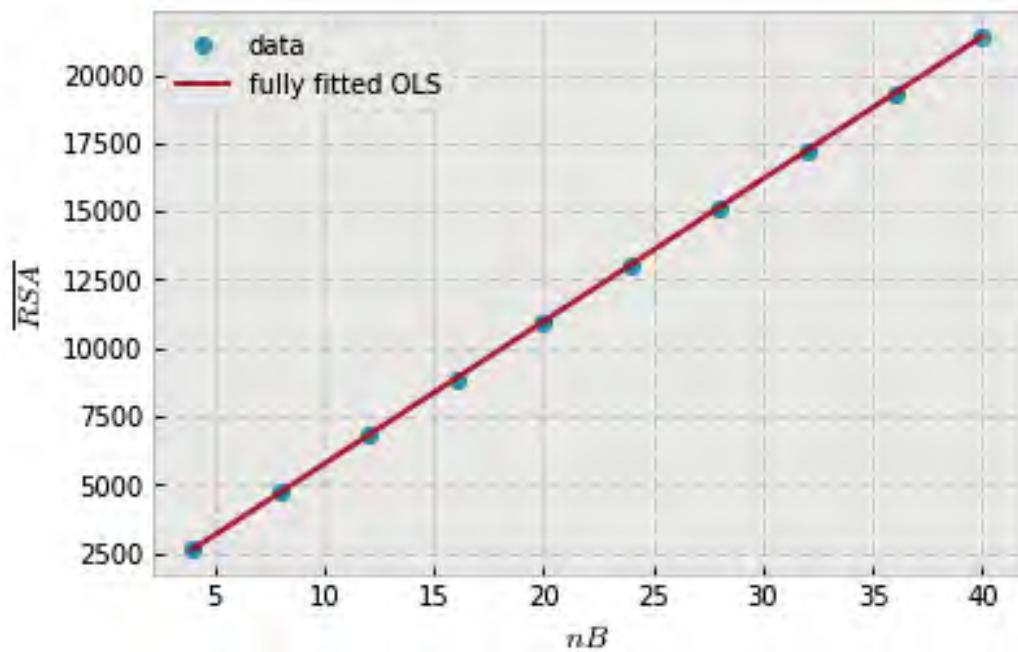
```
variable name: theta  
model: poly1  
r-squared: -0.6748710714199542  
explained variance: -0.673232271334455
```



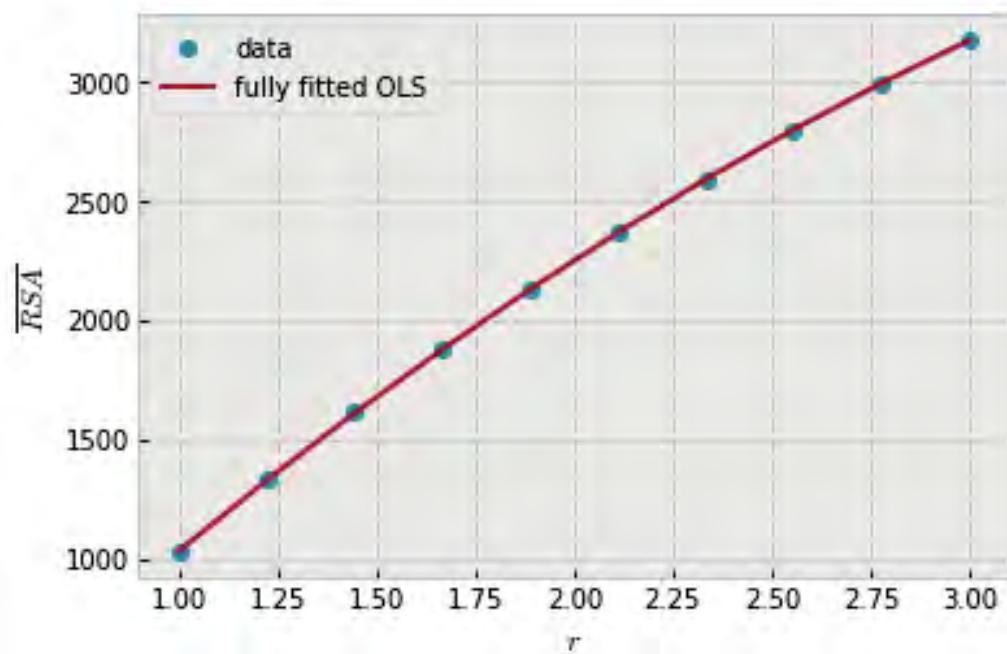
```

variable name: nB
model: poly1
r-squared: 0.9999990874295286
explained variance: 0.9999990895408468

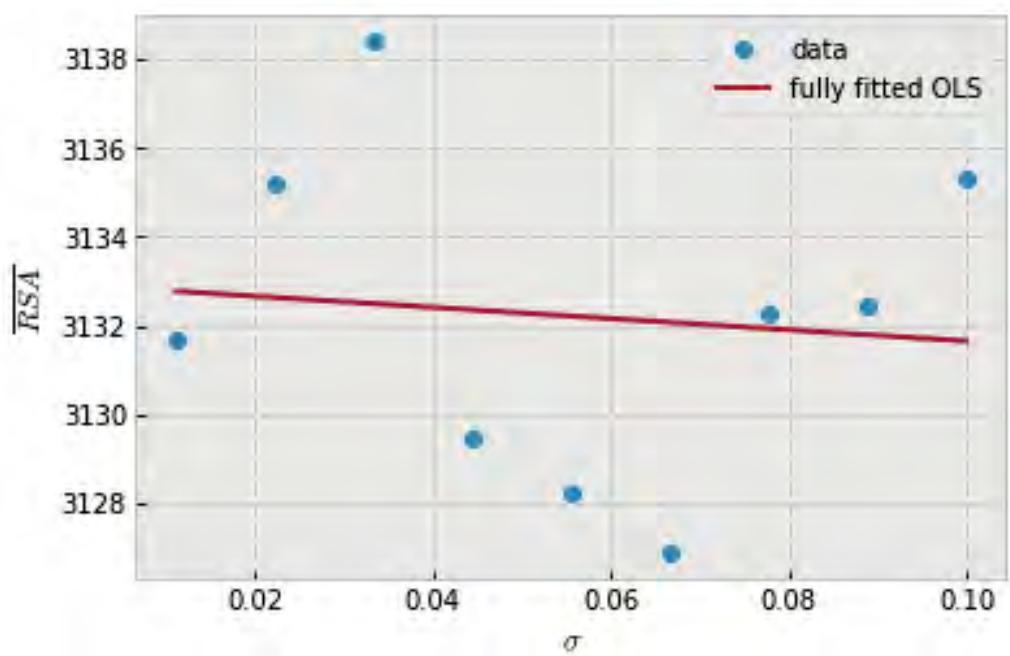
```



```
variable name: r
model: poly2
r-squared: 0.9998700274611575
explained variance: 0.9998701785865394
```



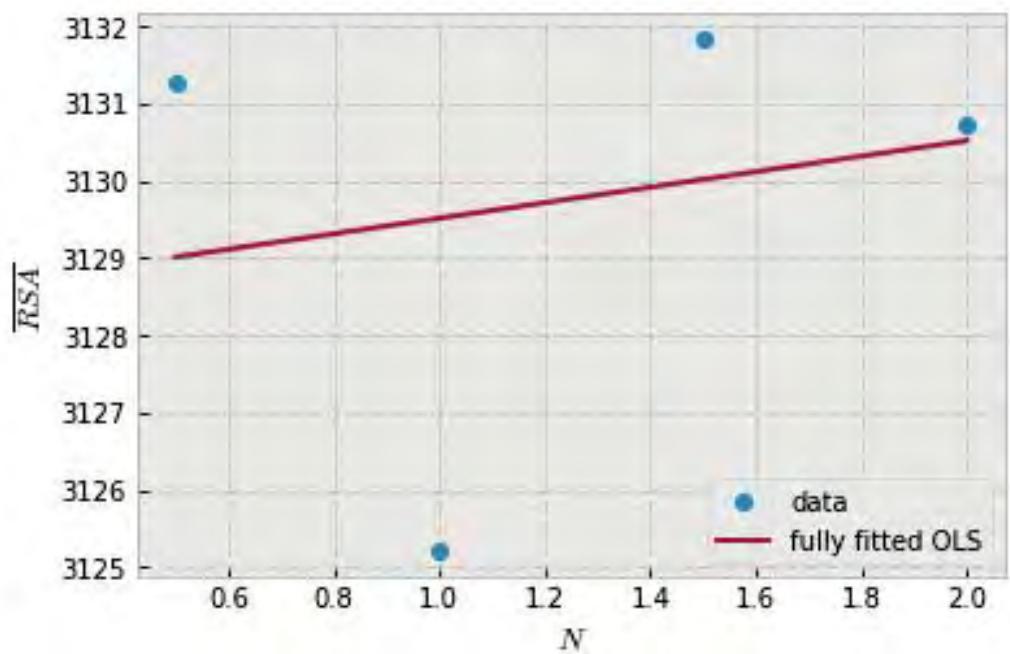
```
variable name: sigma
model: poly1
r-squared: -0.5504862995175284
explained variance: -0.5447578928753471
```



```

variable name: N
model: poly1
r-squared: -2.6068389717609137
explained variance: -2.409463526183814

```

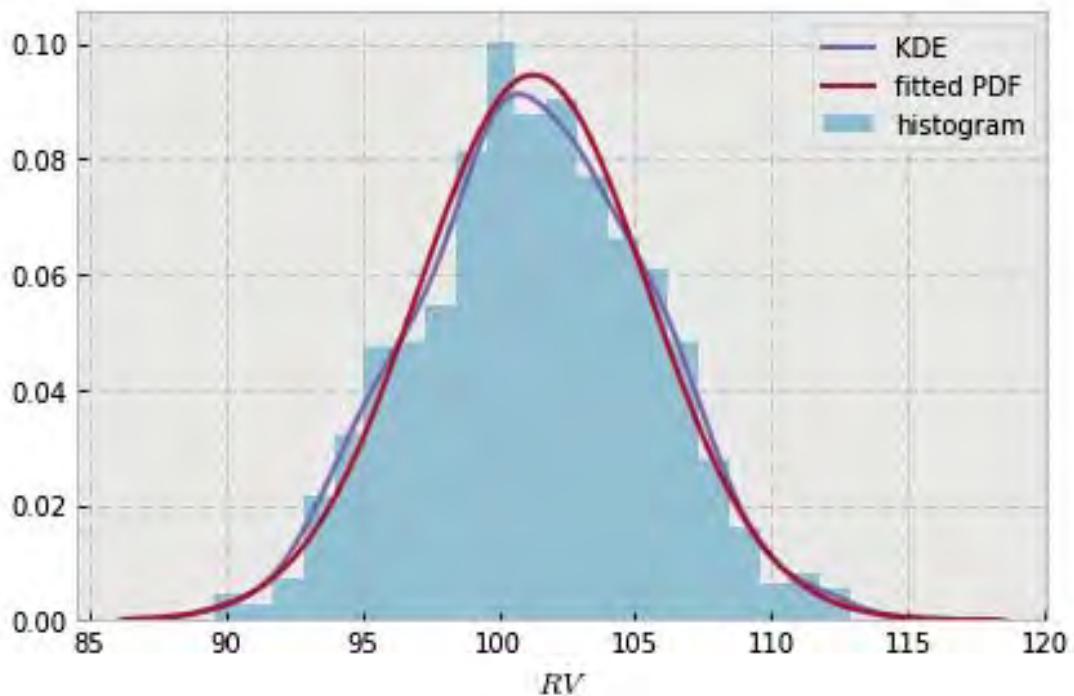


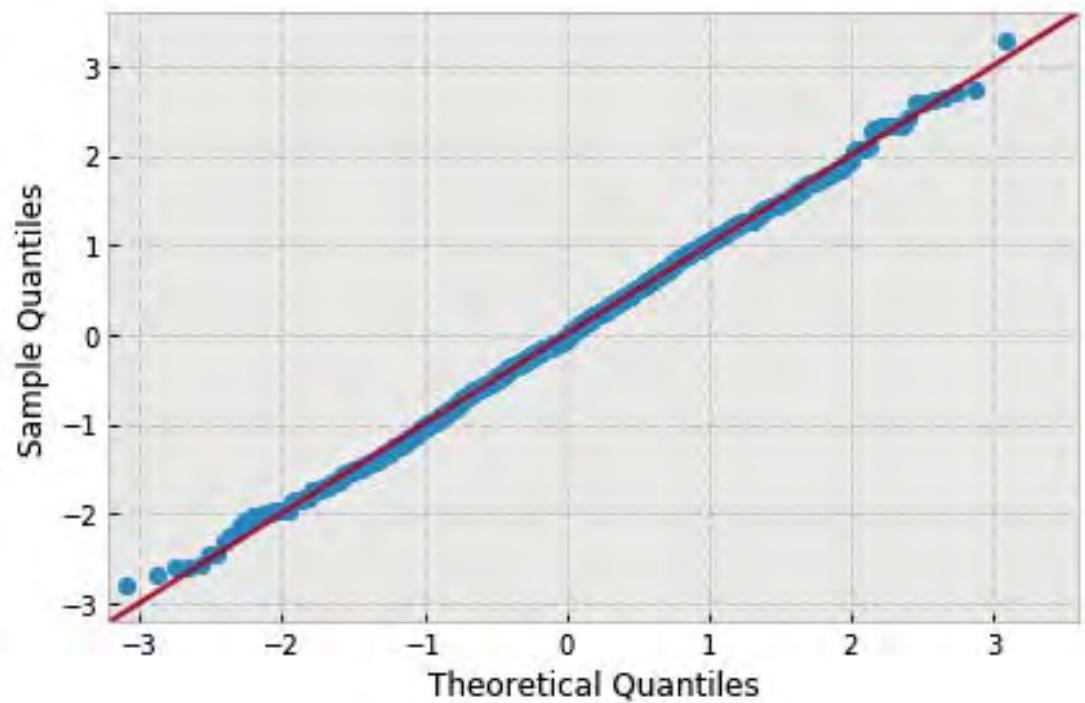
7 Total root volume

7.1 Probability distributions

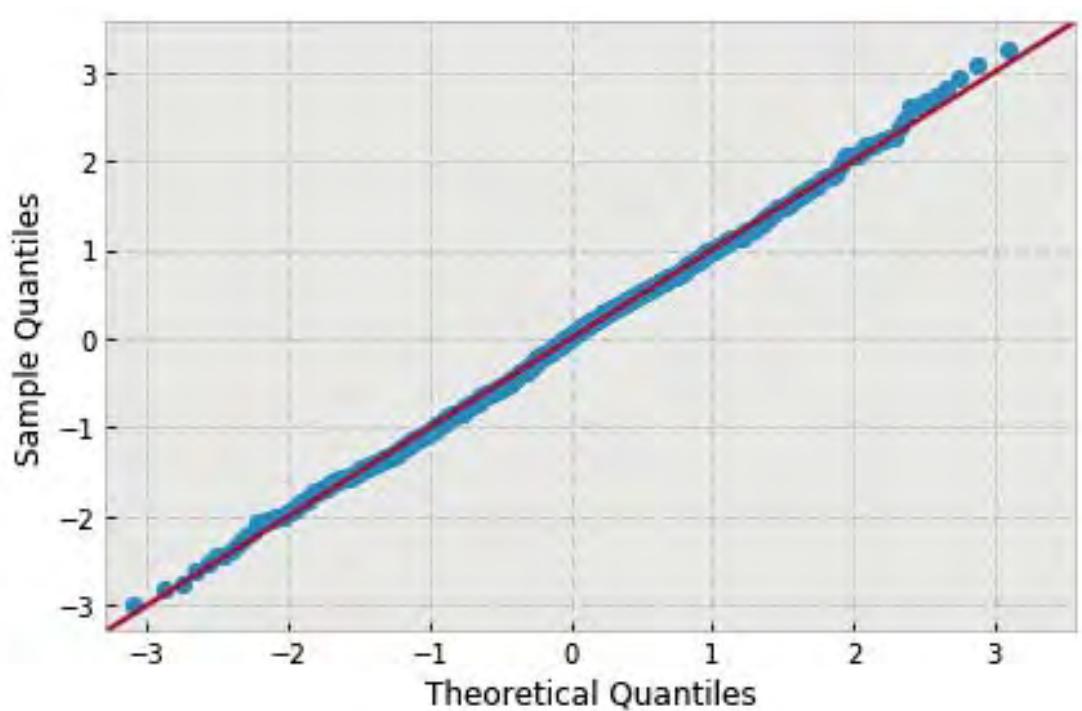
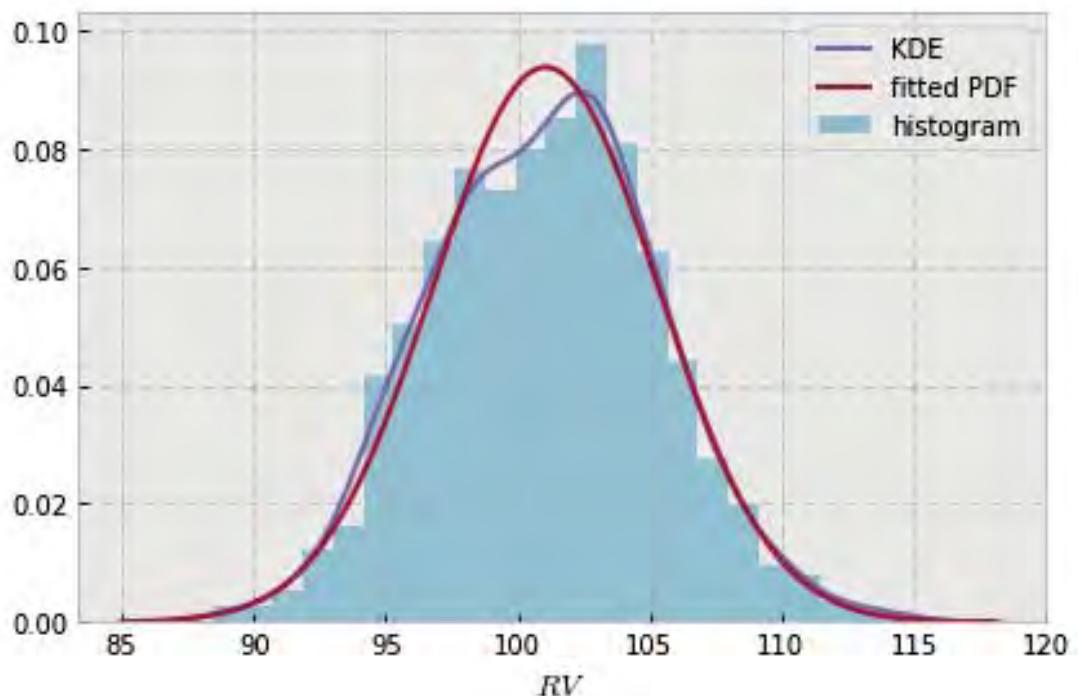
```
In [19]: plot_dist('total_root_volume')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=101.26882389756608, scale=4.219858325785858)
DescribeResult(nobs=1000, minmax=(89.487767541940372, 115.14035880159976)
               mean=101.26882389756608, variance=17.825029319023251,
               skewness=0.02672889812458017, kurtosis=-0.22150820637437807)
```

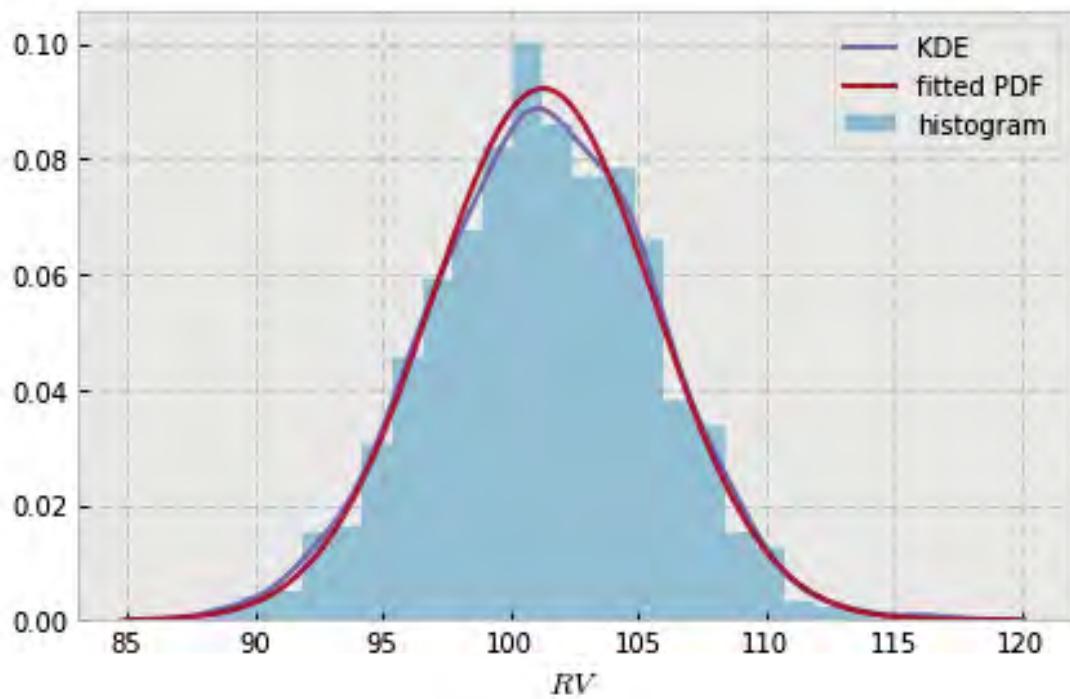


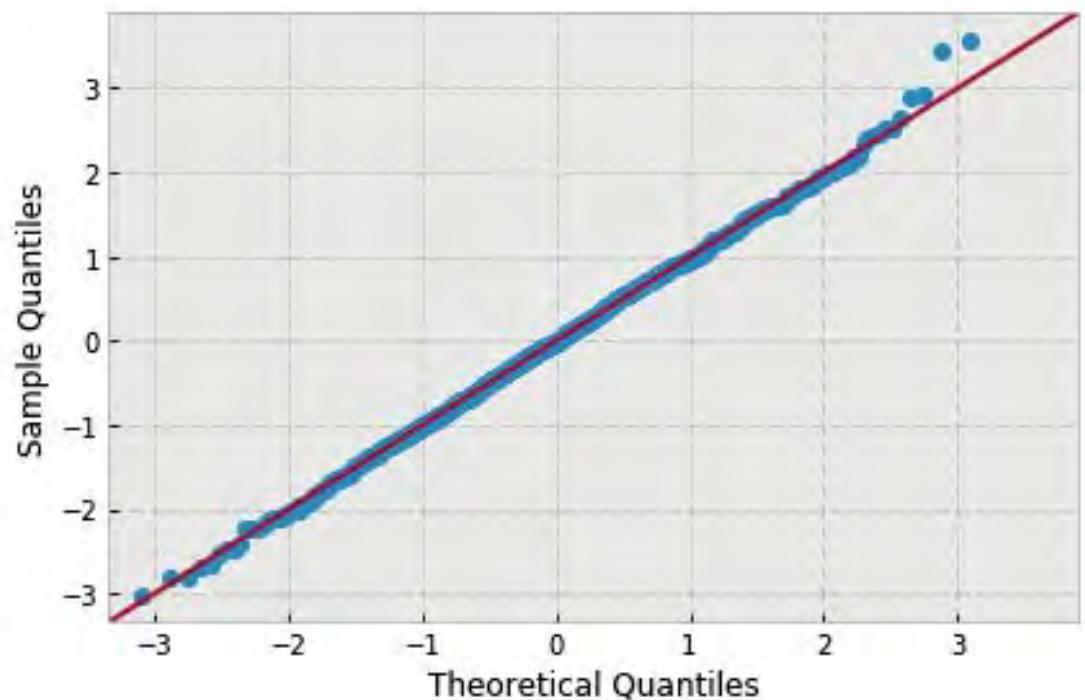


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=101.05618038869707, scale=4.2559158573617175)
DescribeResult(nobs=1000, minmax=(88.373667758270031, 114.89968798675888)
               mean=101.05618038869707, variance=18.130950735678603,
               skewness=0.05693329832595995, kurtosis=-0.11121314979498109)
```

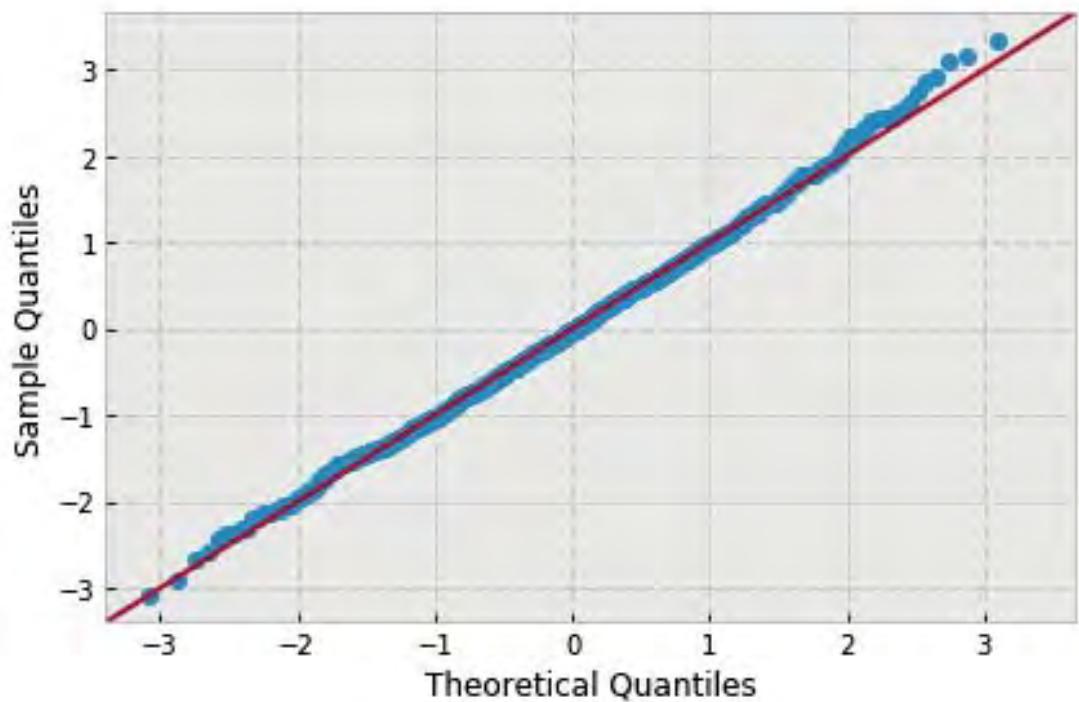
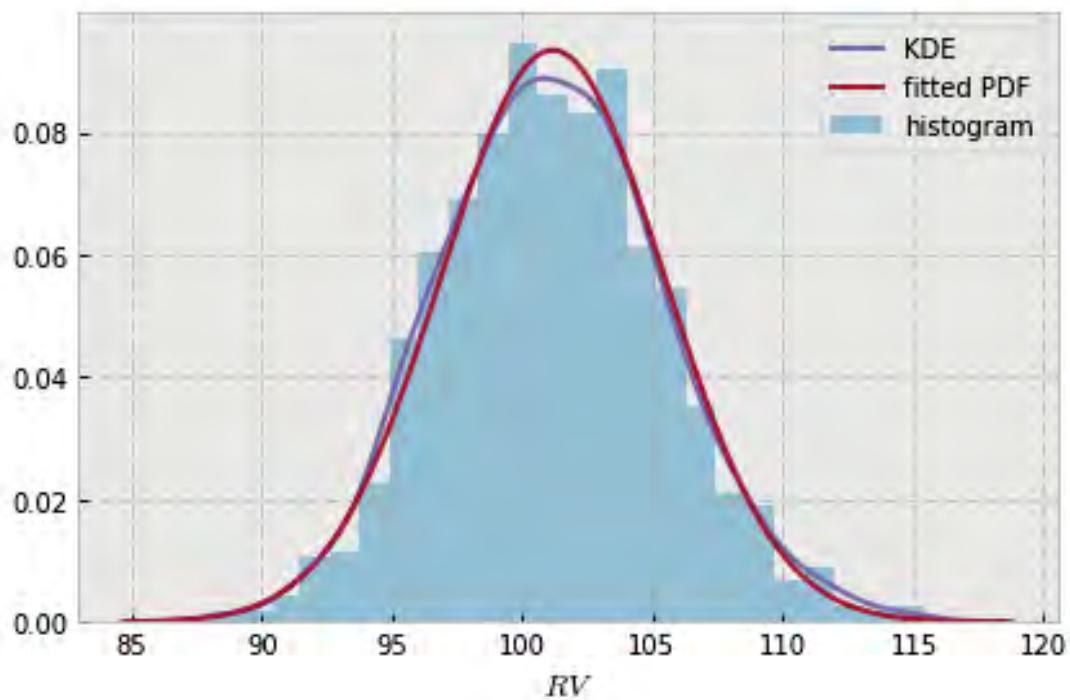


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=101.29189455885032, scale=4.325639741071396)
DescribeResult(nobs=1000, minmax=(88.274251596622463, 116.70402397771248)
               mean=101.29189455885032, variance=18.729889058594811,
               skewness=0.010518147178201457, kurtosis=0.016520041257692242)
```

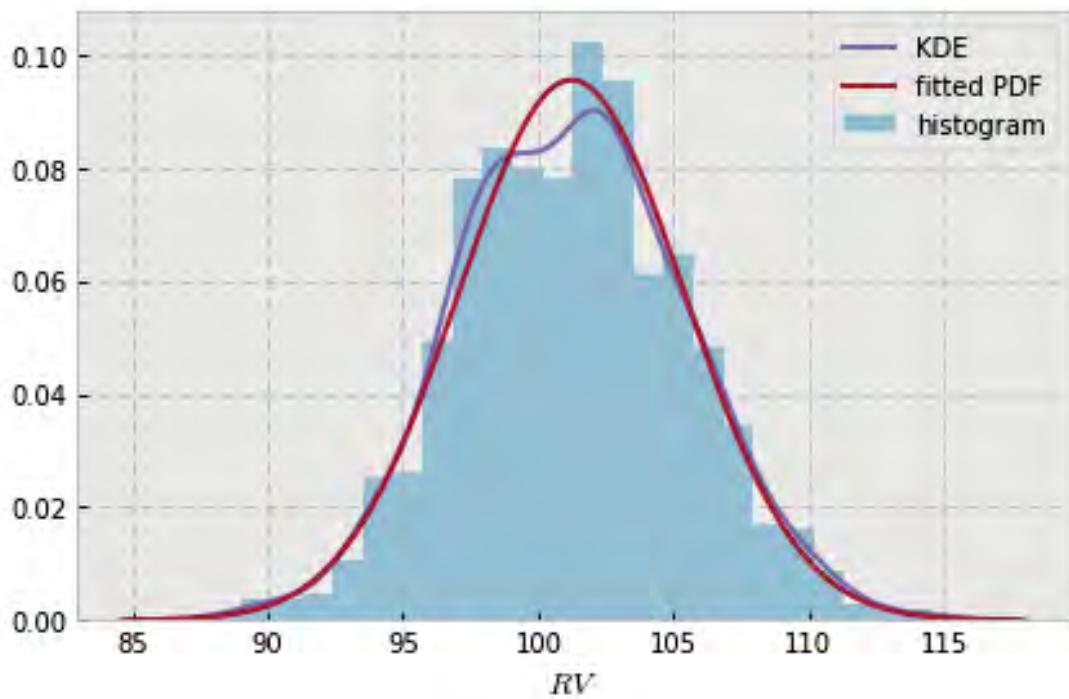


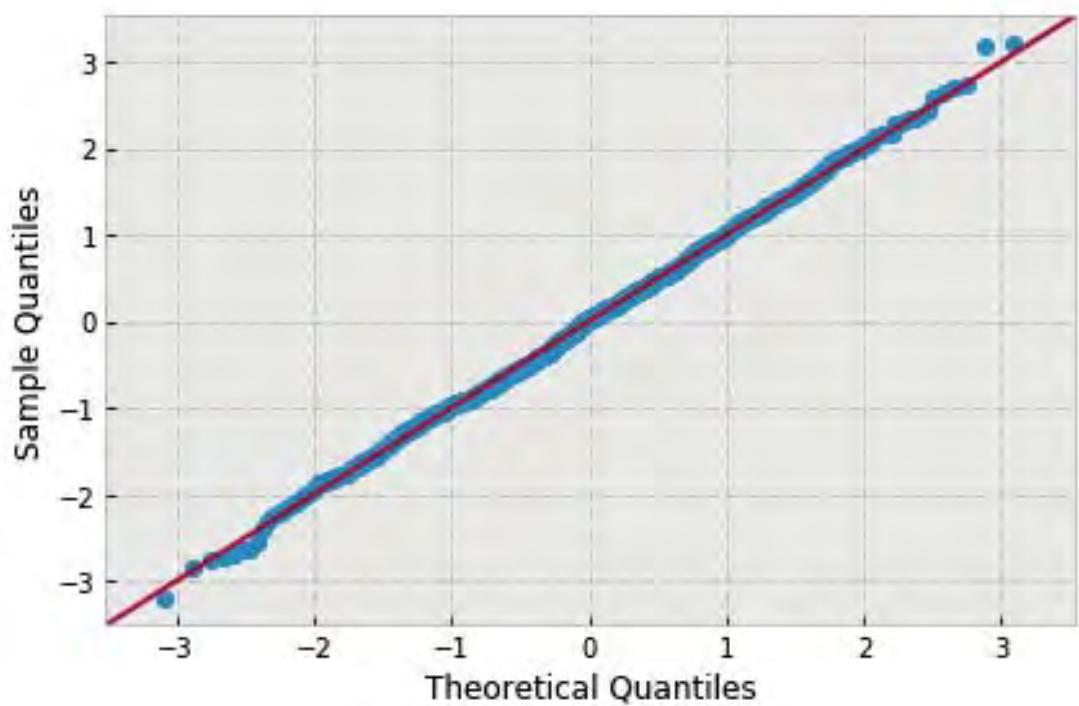


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=101.17013187249015, scale=4.268051129516685)
DescribeResult(nobs=1000, minmax=(88.051216980926327, 115.39809198977892)
               mean=101.17013187249015, variance=18.234494939107762,
               skewness=0.15397499680099977, kurtosis=0.03924819750492148)
```

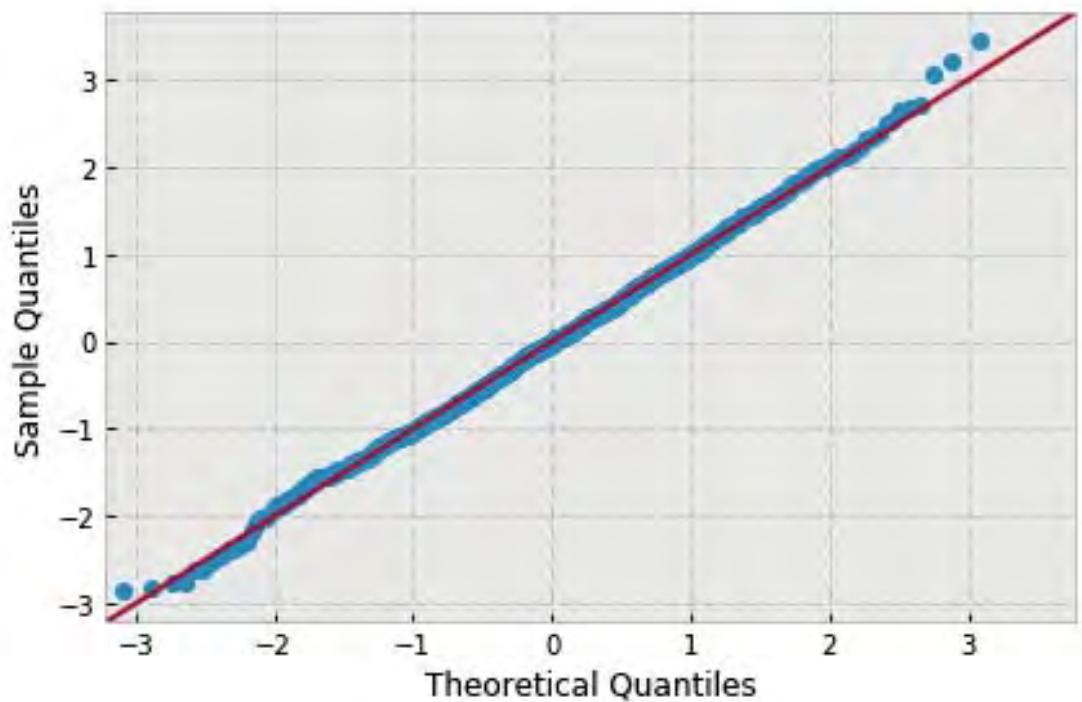
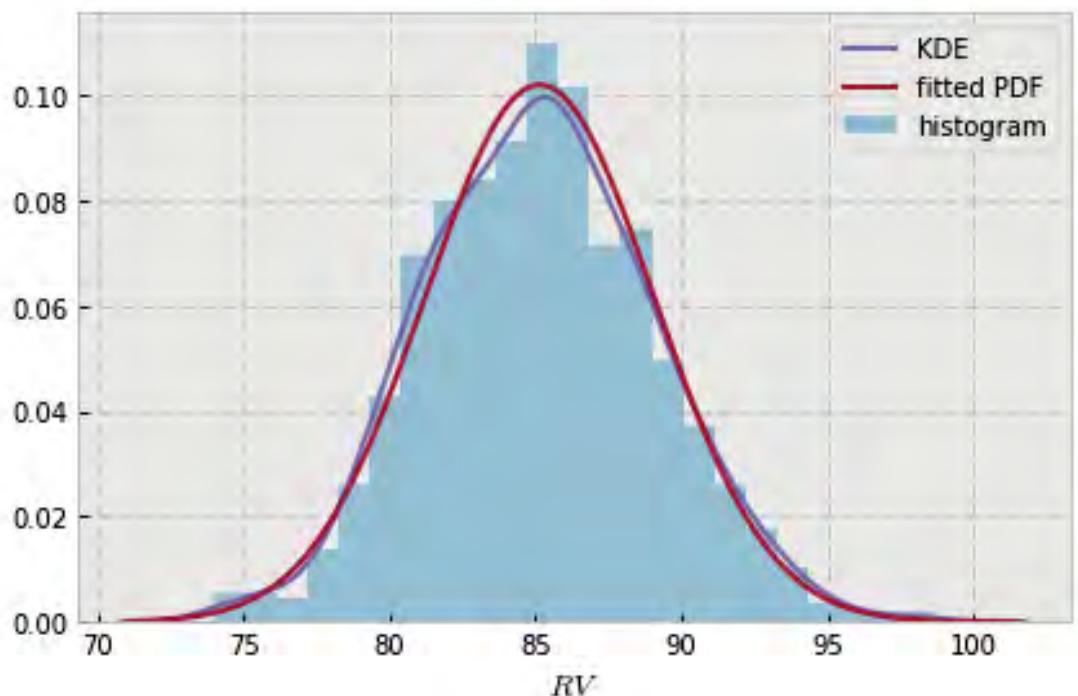


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=101.23454911831331, scale=4.1769410656327866)
DescribeResult(nobs=1000, minmax=(87.91588336932972, 114.63439649906908)
               mean=101.23454911831331, variance=17.464300966736296,
               skewness=0.05570545389390994, kurtosis=-0.06999844753951168)
```

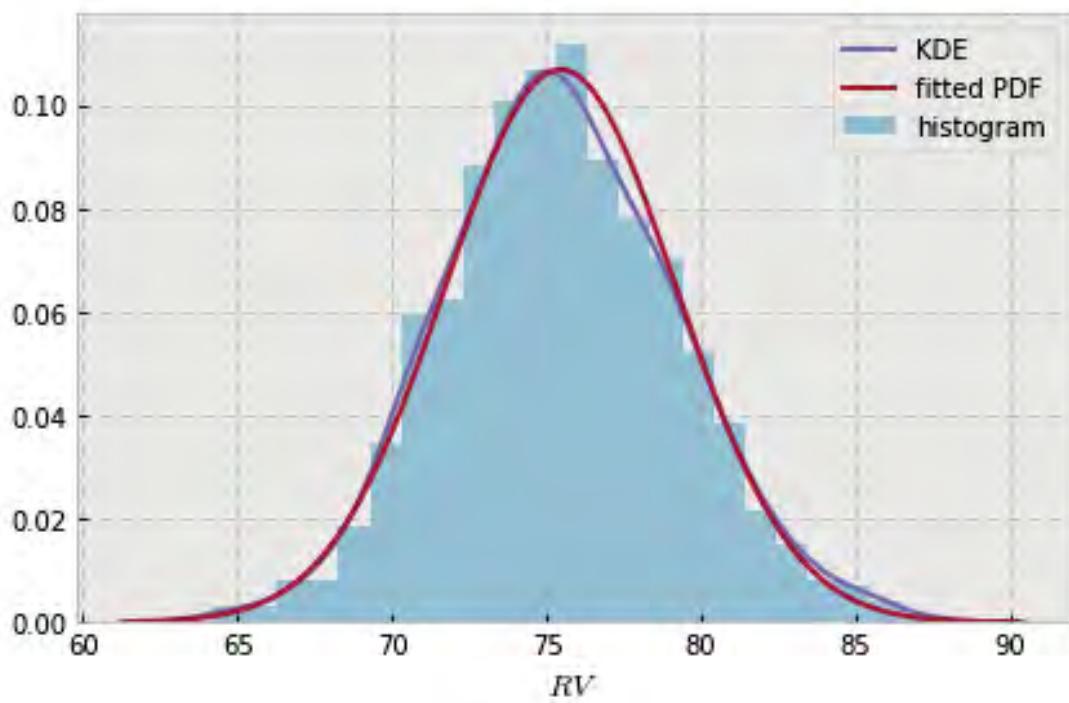


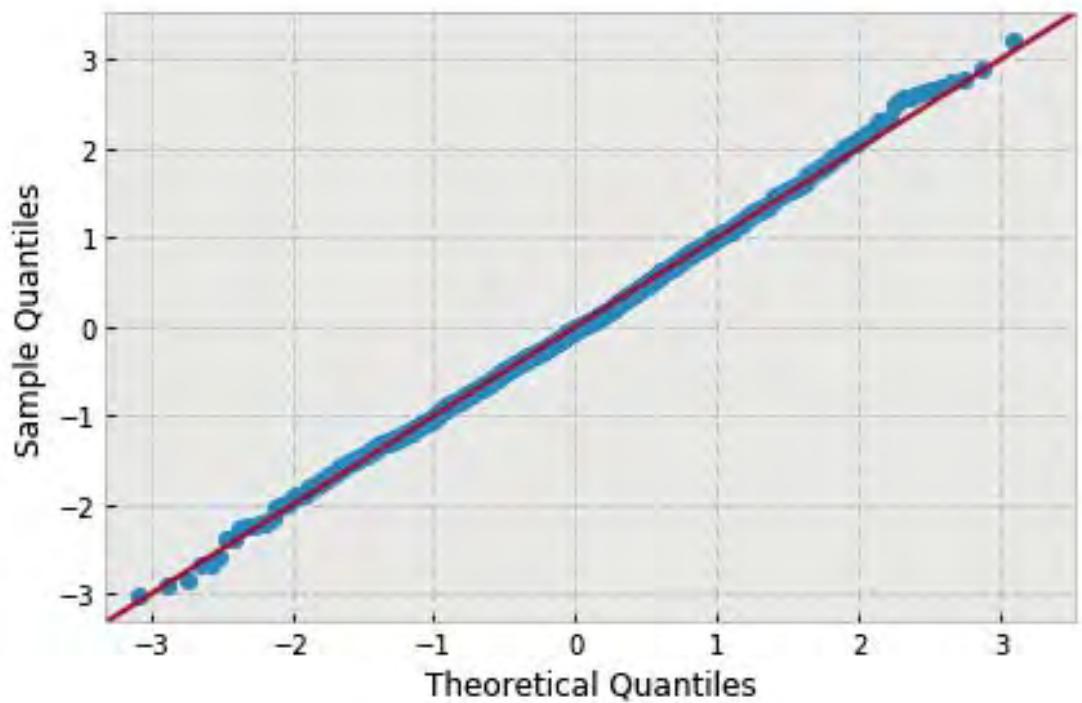


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=85.14206008992265, scale=3.9128237410223004)
DescribeResult(nobs=1000, minmax=(73.98026765863753, 98.638283161269428)
               mean=85.142060089922651, variance=15.3255151434512,
               skewness=0.10033925911010178, kurtosis=-0.015549373193721738)
```

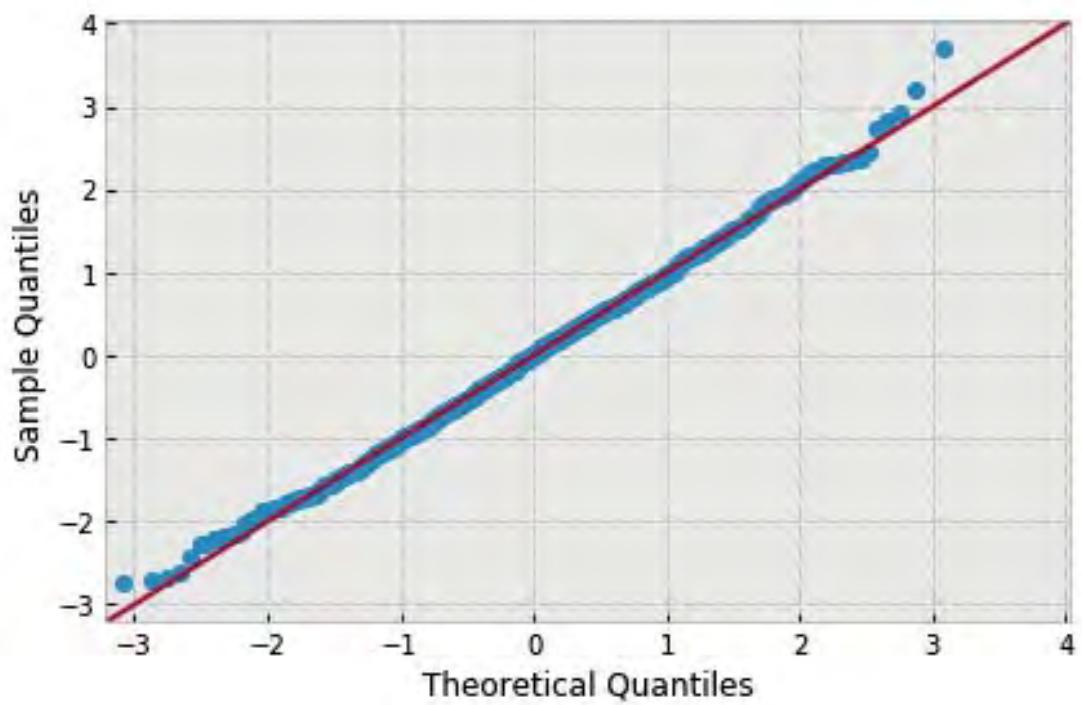
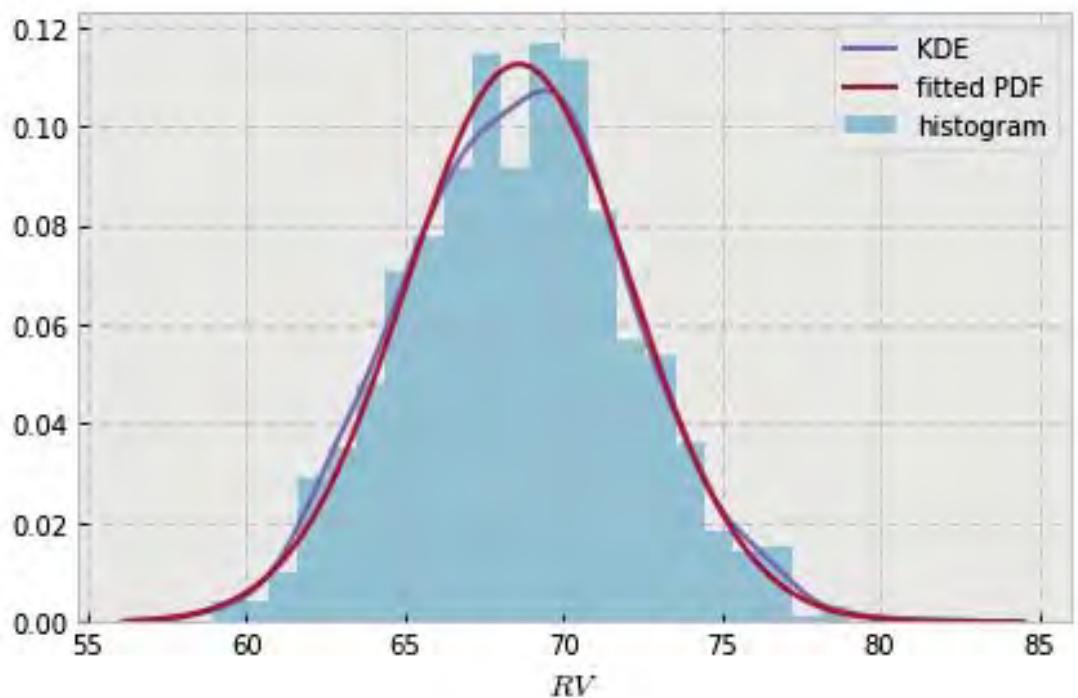


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=75.45082095817237, scale=3.7344118296274273)
DescribeResult(nobs=1000, minmax=(64.233386042354653, 87.44660102604422)
               mean=75.450820958172372, variance=13.959791504766036,
               skewness=0.12206324654615237, kurtosis=0.0007907445530834778)
```

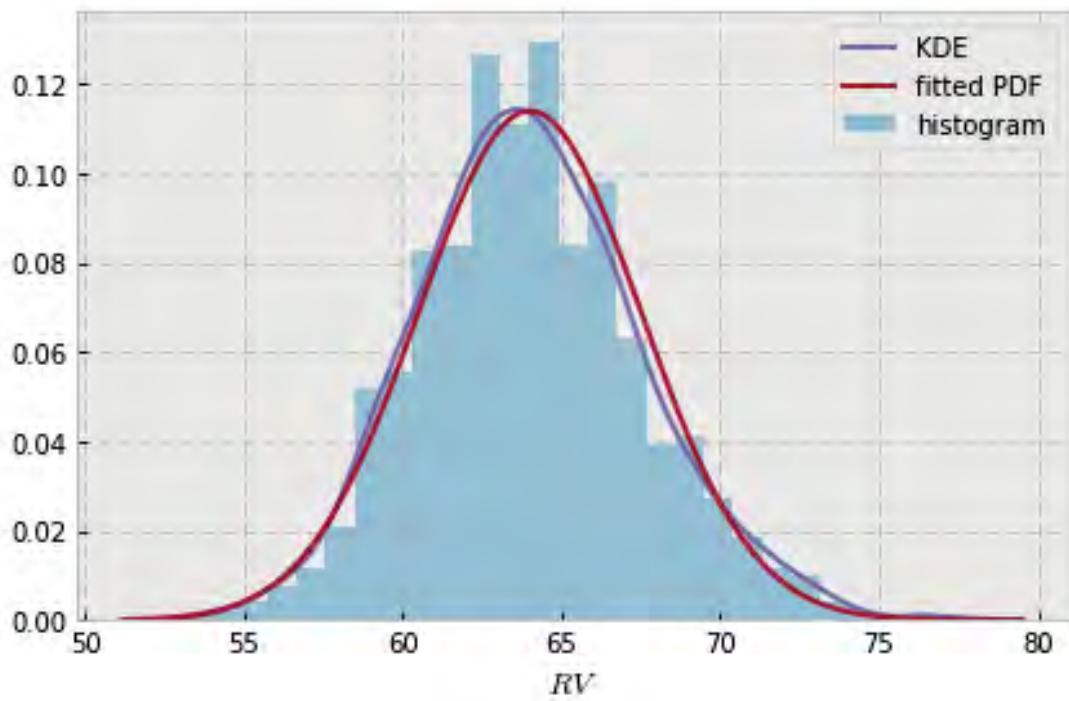


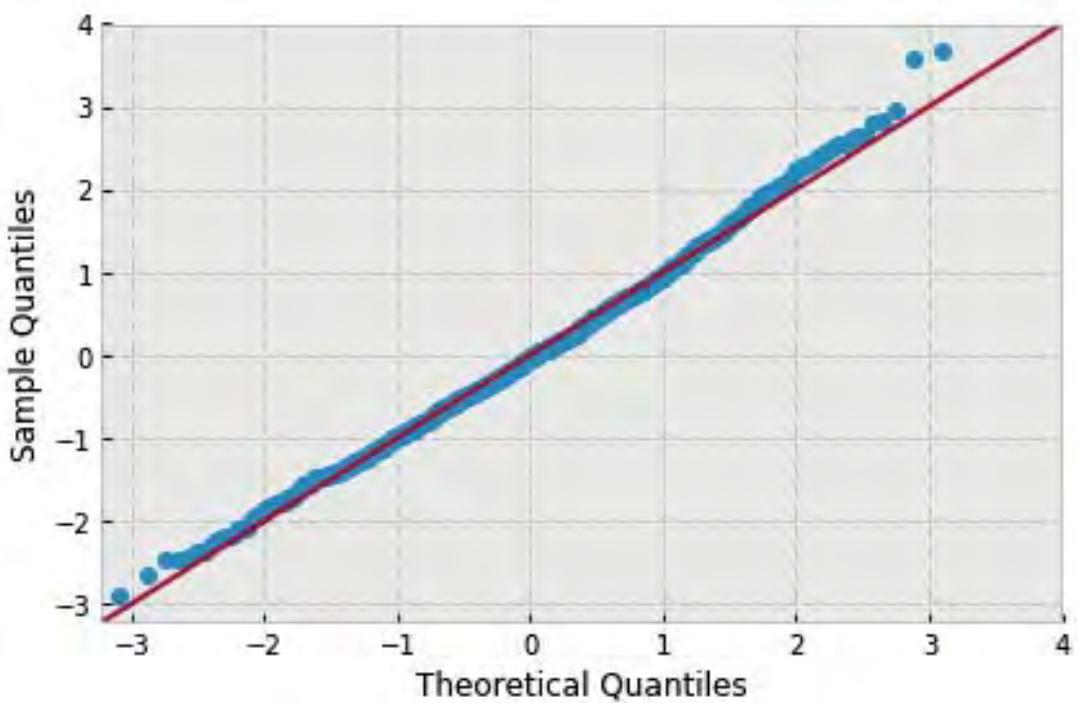


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=68.59772086519894, scale=3.5475194562700856)
DescribeResult(nobs=1000, minmax=(58.888116177346966, 81.767687540593315)
               mean=68.597720865198937, variance=12.597491784399201,
               skewness=0.11676249684898082, kurtosis=-0.07856581270998264)
```

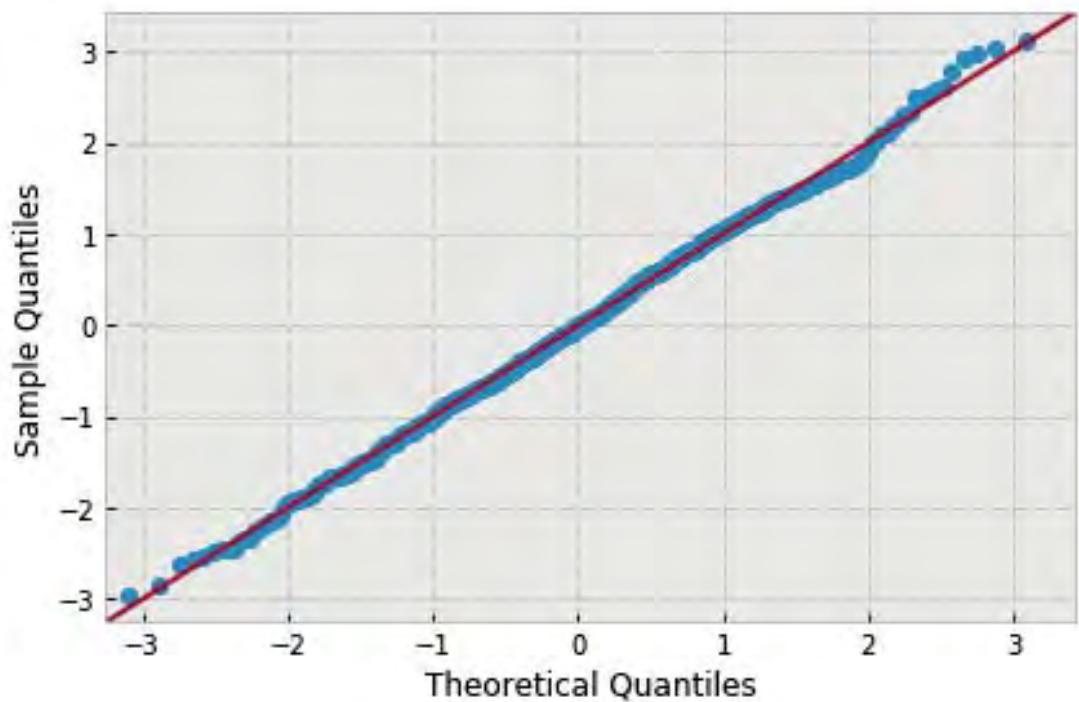
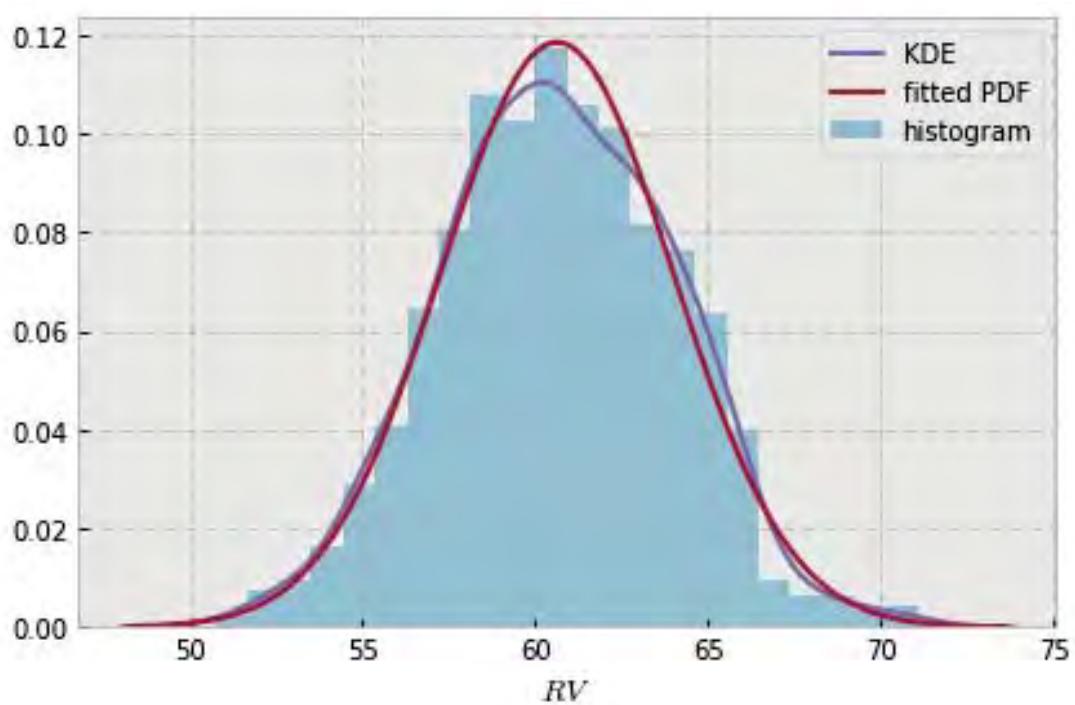


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=63.99753941769221, scale=3.499202205019487)
DescribeResult(nobs=1000, minmax=(53.880148755922797, 76.823733528455037)
               mean=63.997539417692209, variance=12.256672744357596,
               skewness=0.26792509563517464, kurtosis=0.14502141984922012)
```

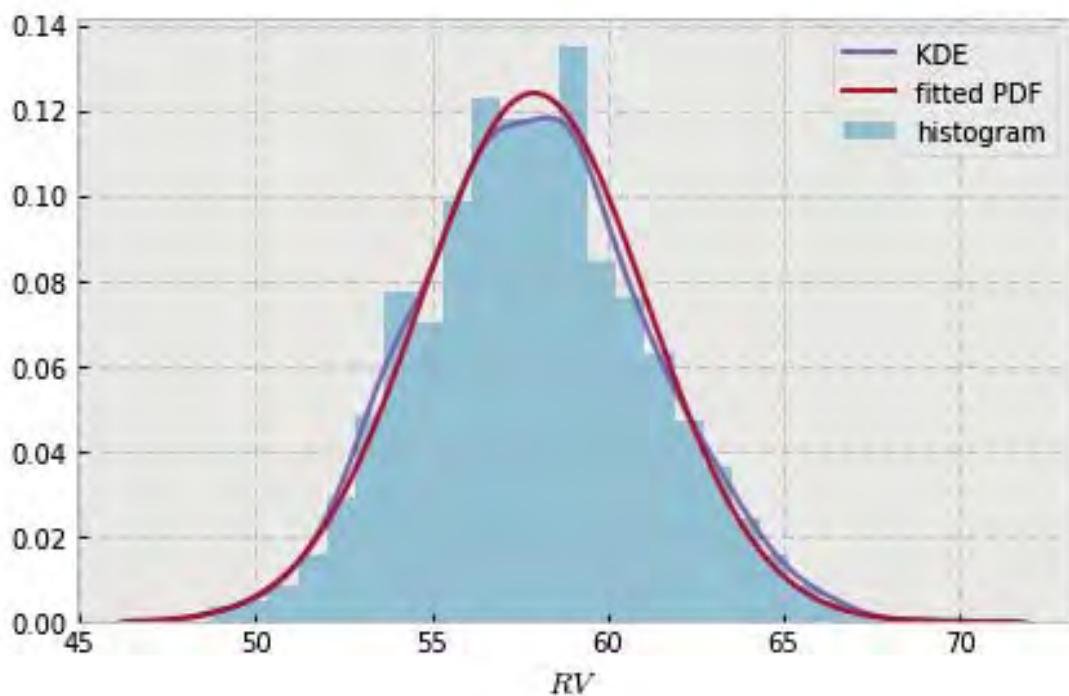


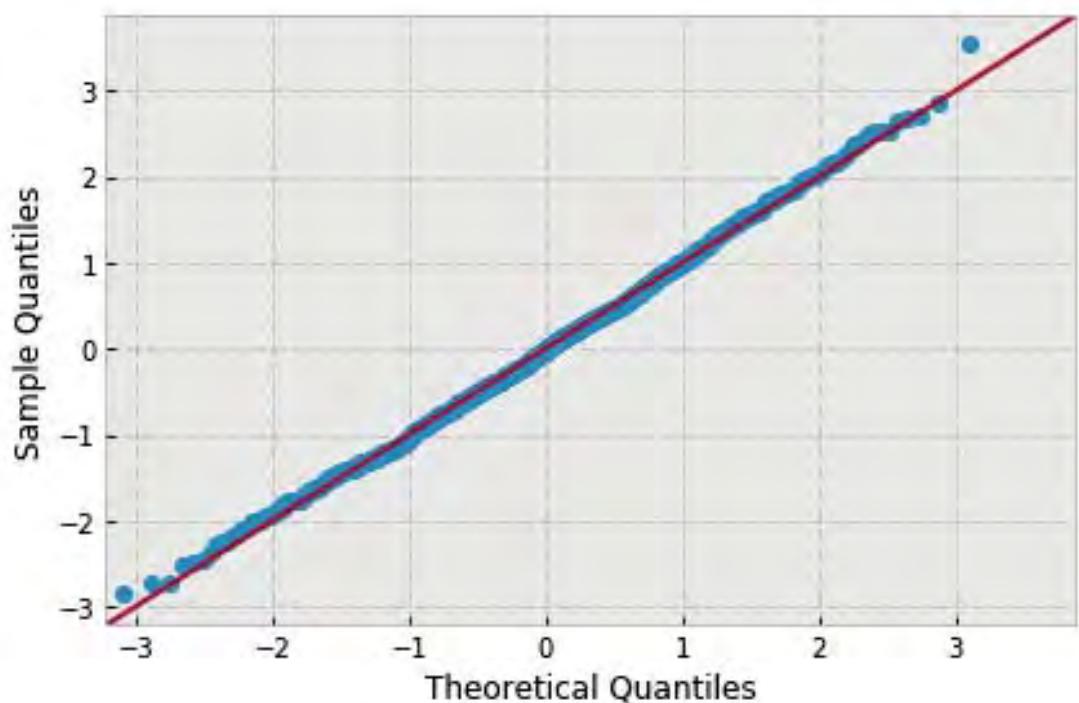


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=60.62771026498317, scale=3.3637252710235814)
DescribeResult(nobs=1000, minmax=(50.713748598381407, 71.102543164625246)
               mean=60.627710264983172, variance=11.325973672595262,
               skewness=0.011427520962545975, kurtosis=-0.13545922377180997)
```

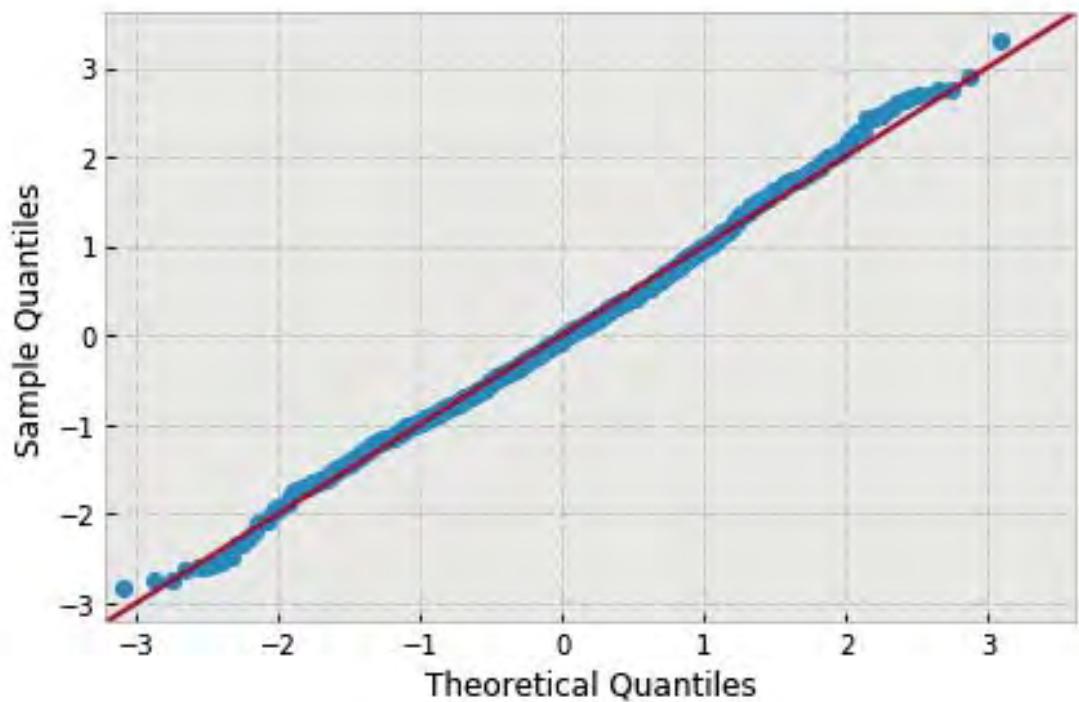
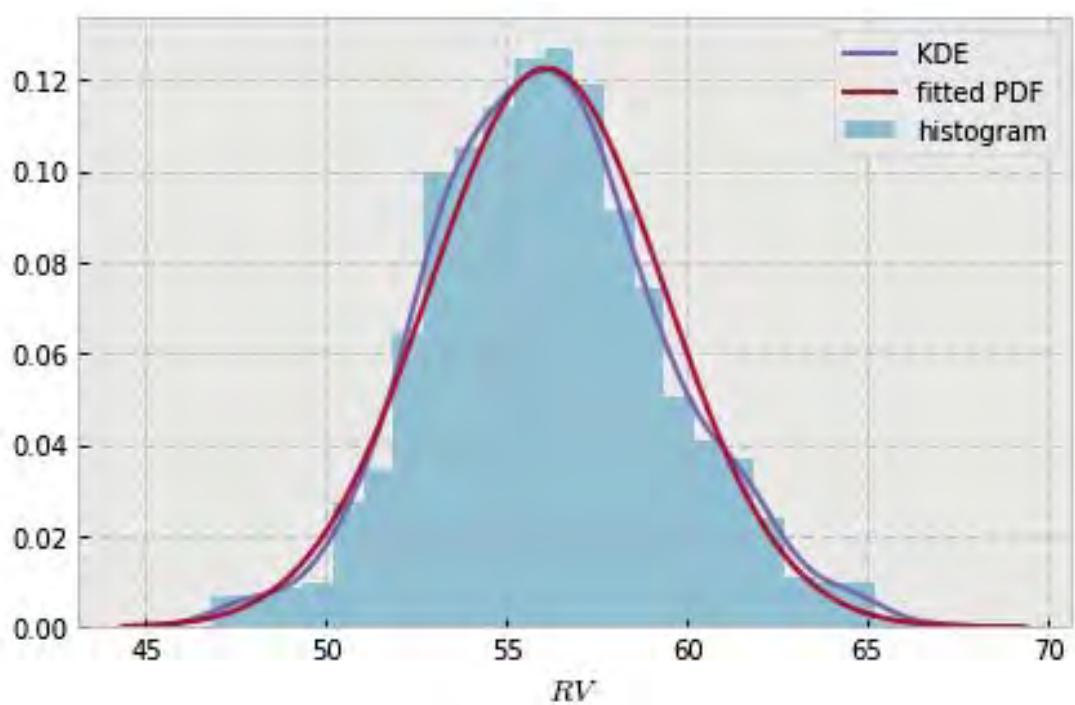


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=57.90760661129035, scale=3.2161963809467786)
DescribeResult(nobs=1000, minmax=(48.724930509218495, 69.307489337033374)
               mean=57.907606611290348, variance=10.354273434249405,
               skewness=0.13497036706076487, kurtosis=-0.12503692806395827)
```

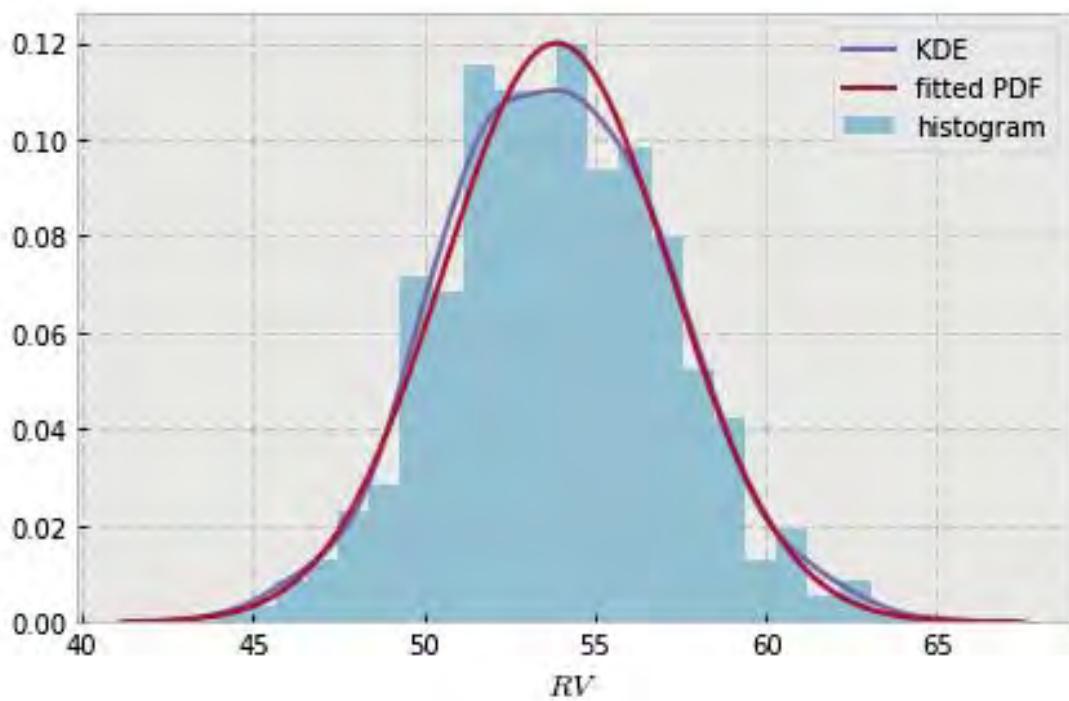


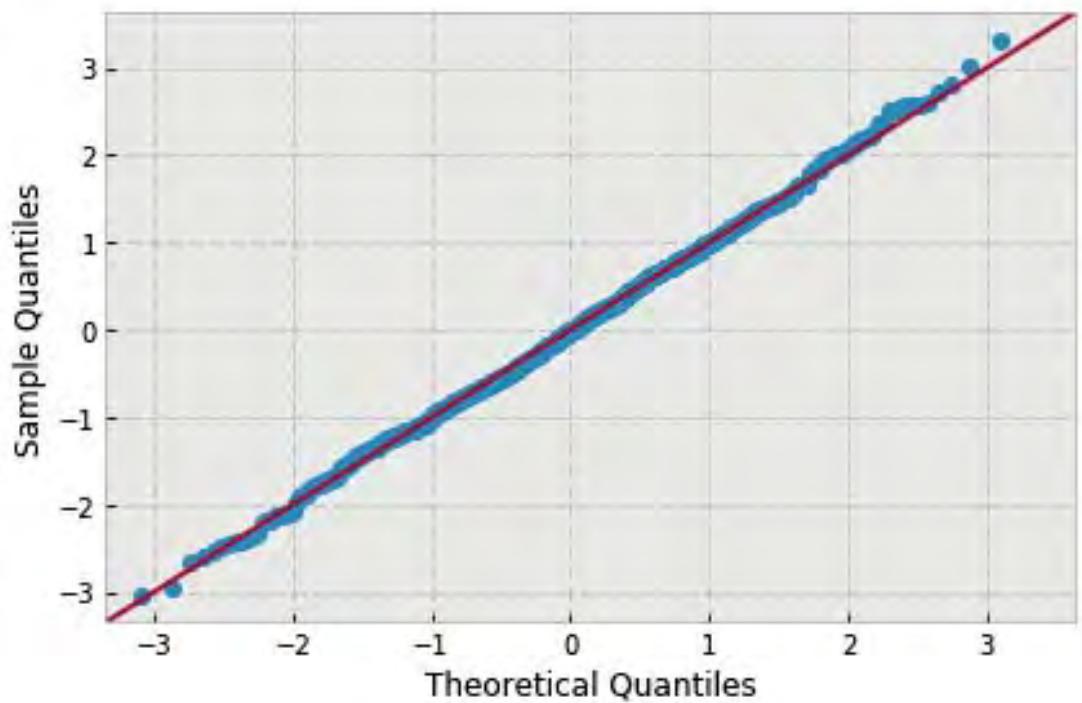


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=56.10954999199473, scale=3.2546764575848512)
DescribeResult(nobs=1000, minmax=(46.857209903028995, 66.863021393210062)
               mean=56.109549991994733, variance=10.603522365922998,
               skewness=0.18941176166957863, kurtosis=0.09371484993653789)
```

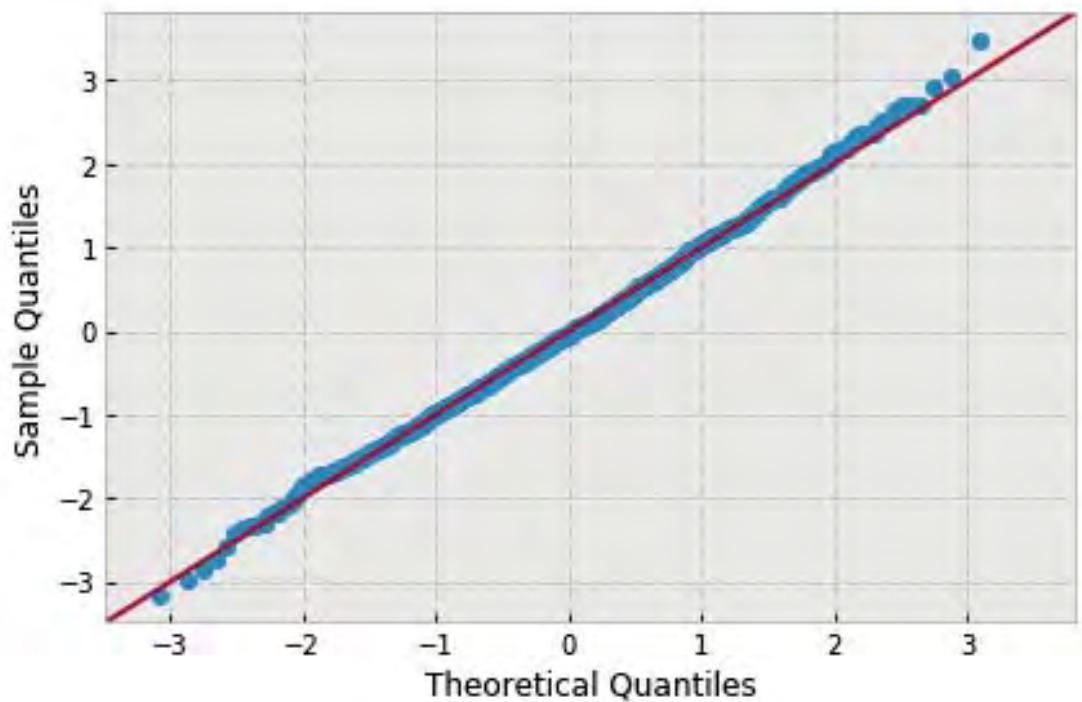
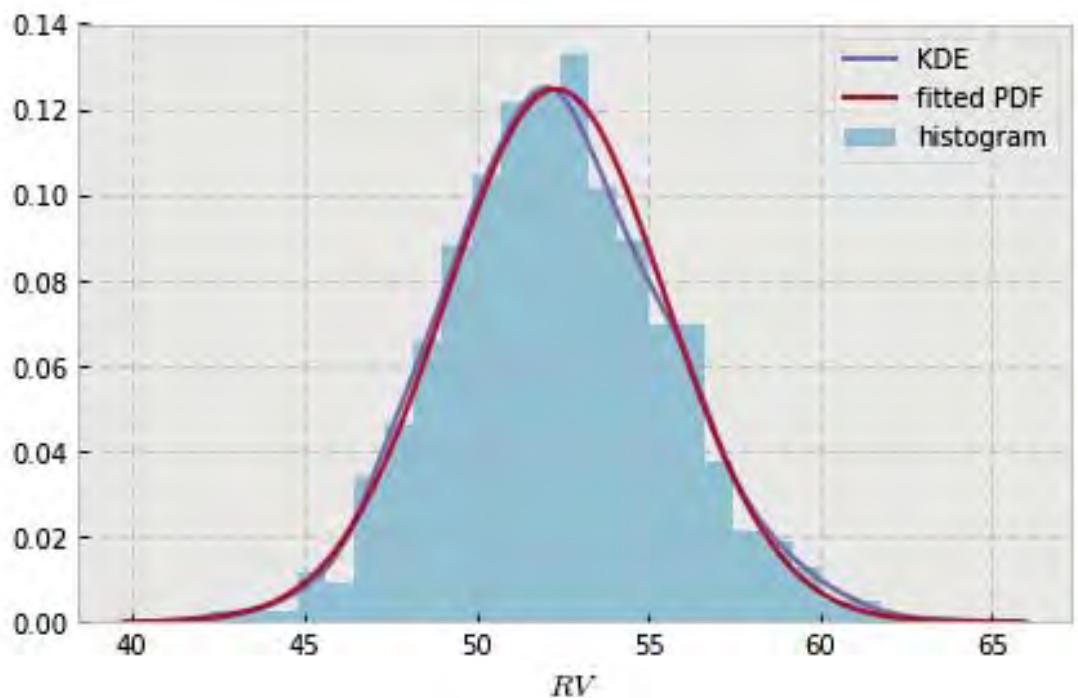


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=53.878520087658416, scale=3.325124535223977)
DescribeResult(nobs=1000, minmax=(43.802498520536041, 64.889751474520935)
               mean=53.878520087658416, variance=11.06752069544391,
               skewness=0.08634570806433597, kurtosis=-0.028637478127238936)
```

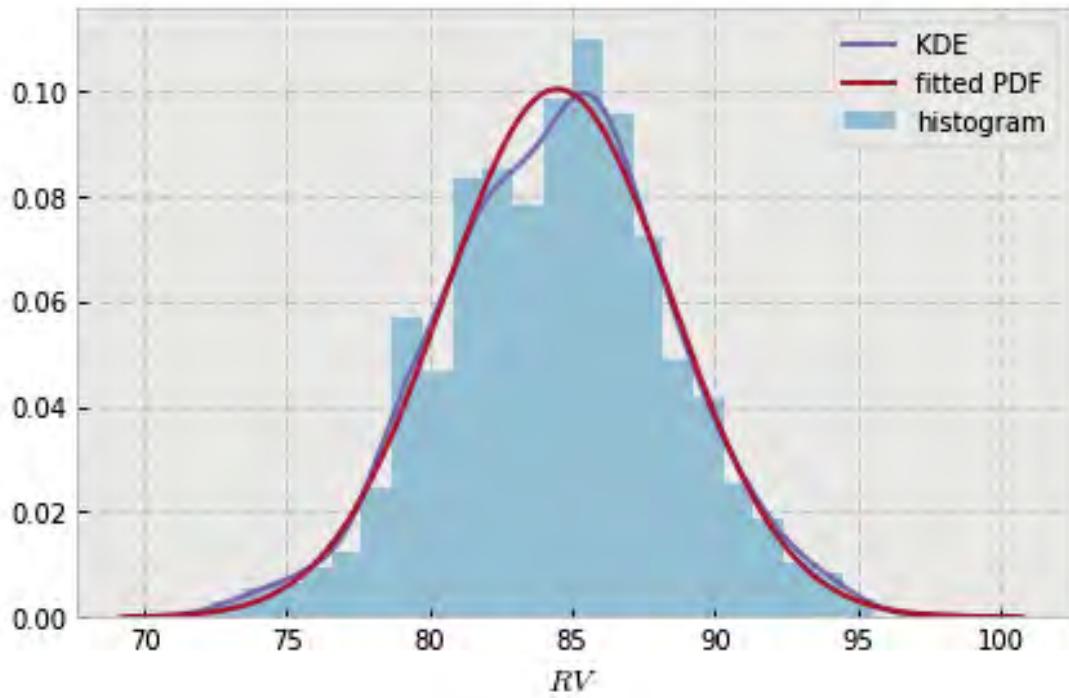


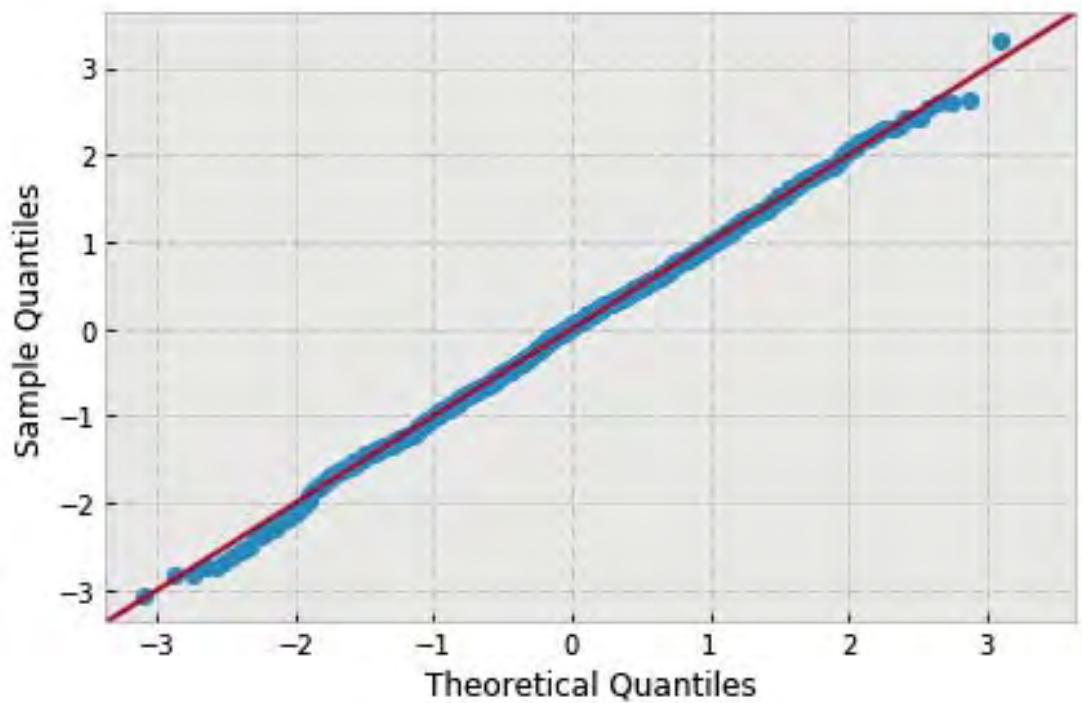


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=52.314897302048685, scale=3.2007856693261445)
DescribeResult(nobs=1000, minmax=(42.228940064152155, 63.436839060894627)
               mean=52.314897302048685, variance=10.255284185148763,
               skewness=0.14596570621139093, kurtosis=0.0367653421704639)
```

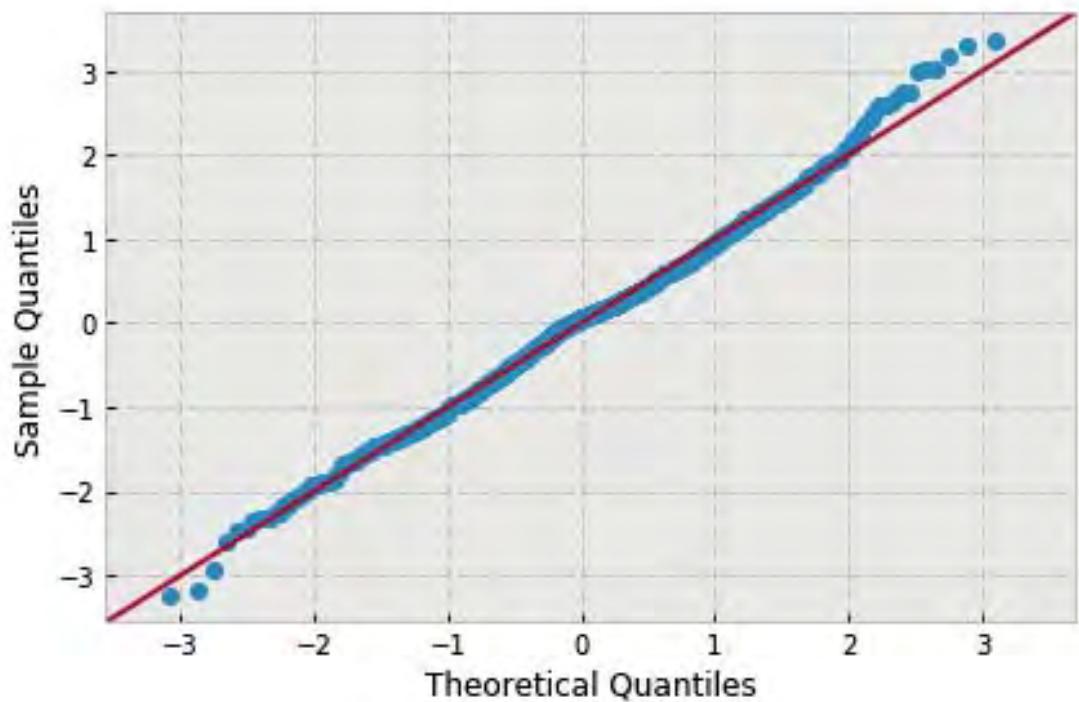
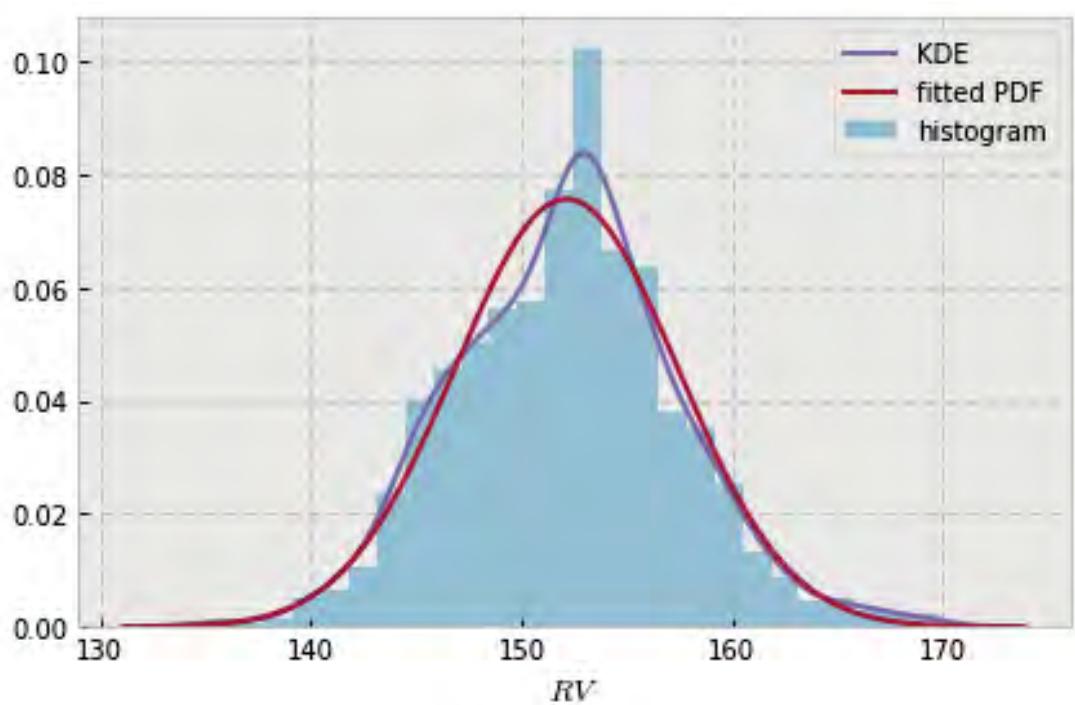


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=84.48483078826804, scale=3.9810279994424045)
DescribeResult(nobs=1000, minmax=(72.342342547688801, 97.67668488101036)
               mean=84.484830788268042, variance=15.864448380725118,
               skewness=-0.03567716418897212, kurtosis=-0.026116667443468966)
```

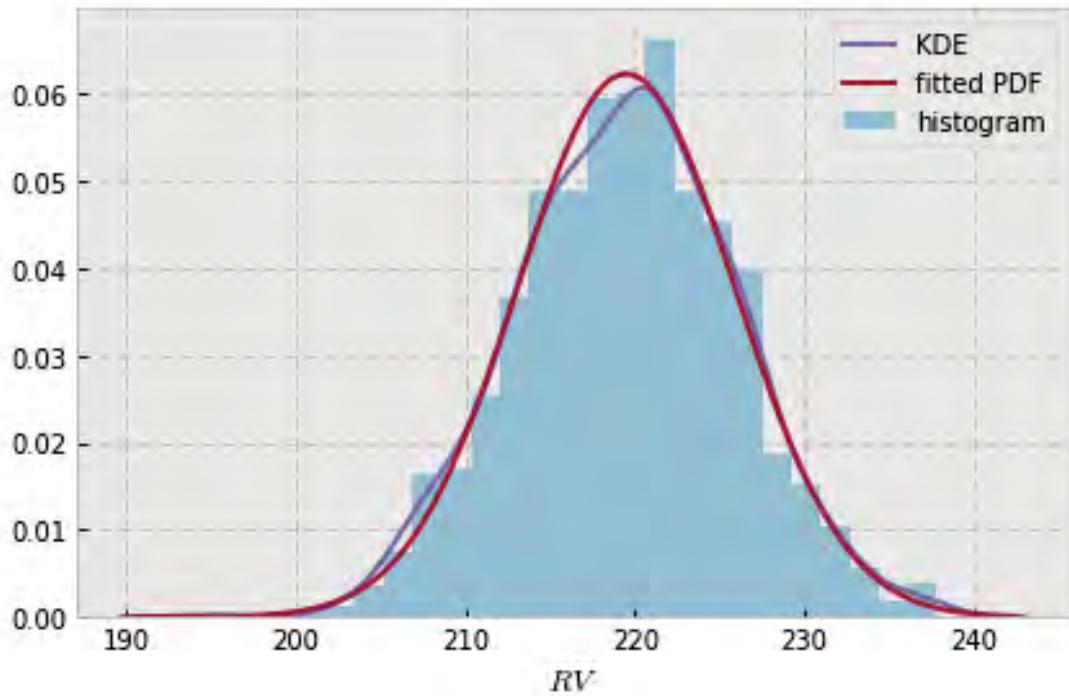


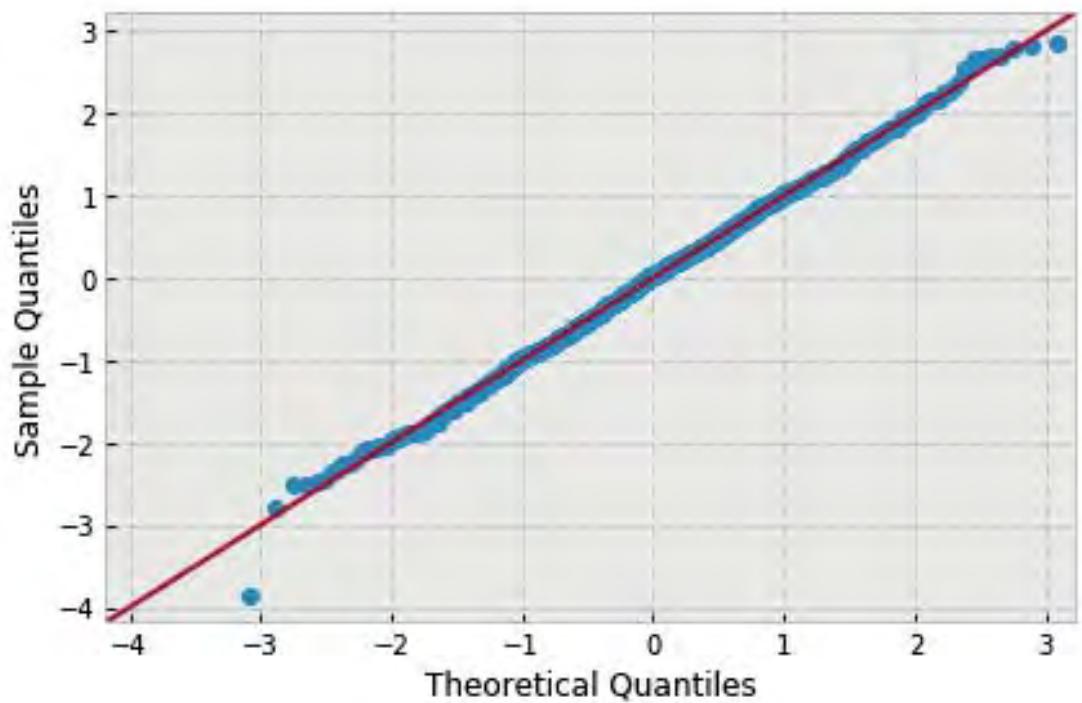


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=152.14408166747202, scale=5.2715157906555445)
DescribeResult(nobs=1000, minmax=(135.11486908677719, 169.85388209466294)
               mean=152.14408166747202, variance=27.816695426557303,
               skewness=0.11981062033707697, kurtosis=0.2763904874710037)
```

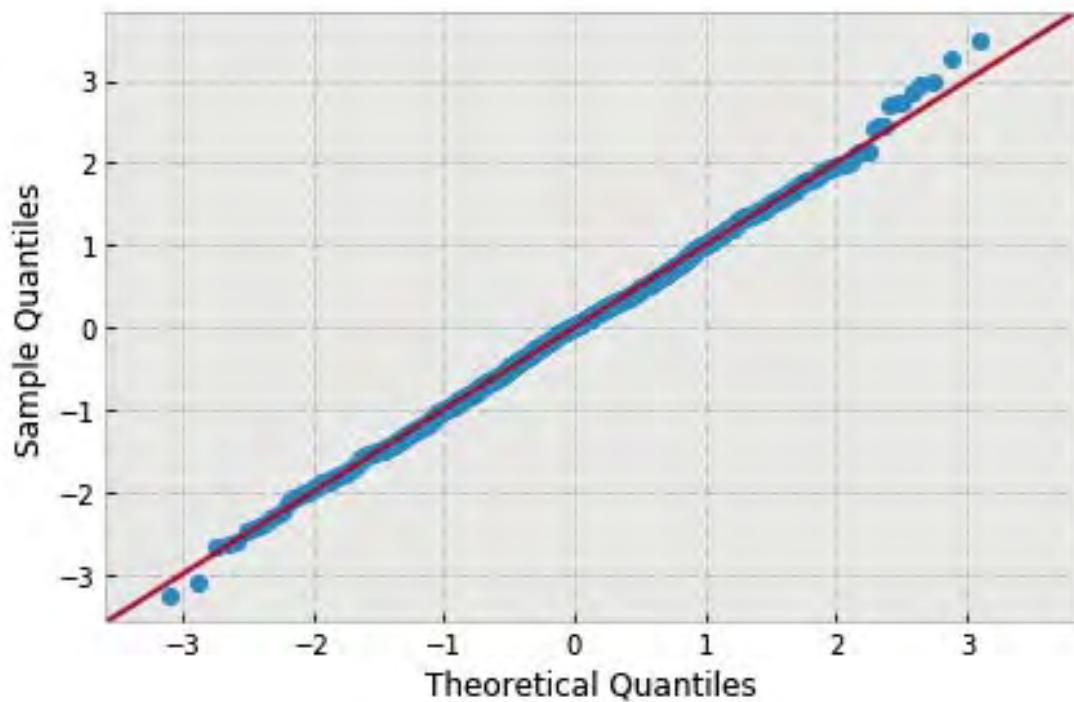
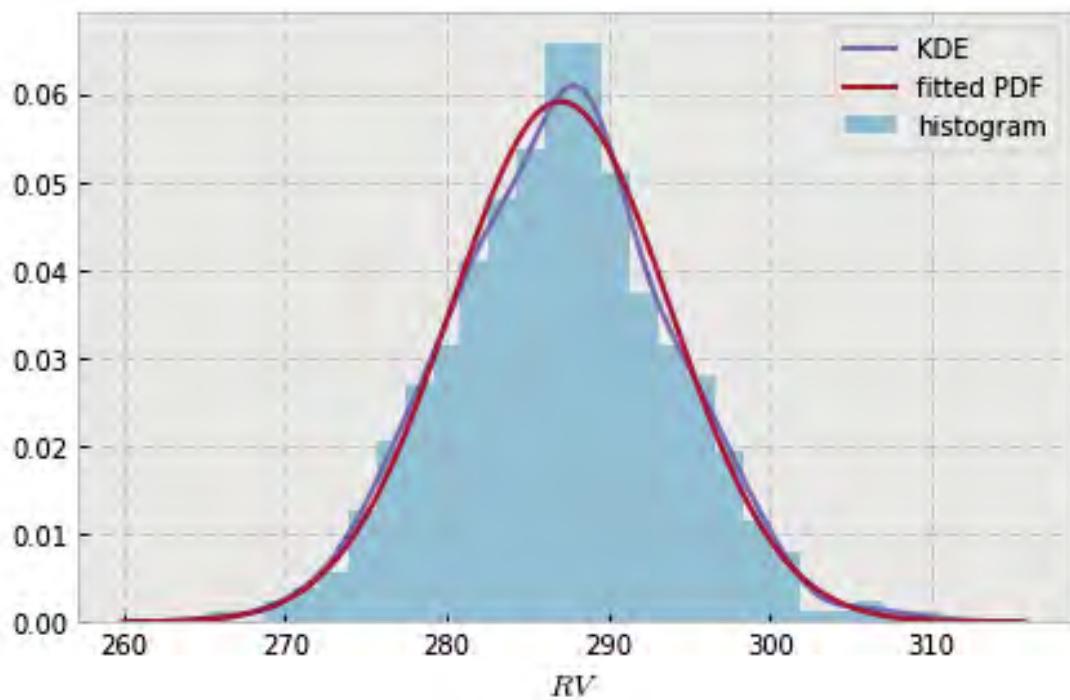


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=219.49760066171962, scale=6.409232328978432)
DescribeResult(nobs=1000, minmax=(194.86413709431392, 237.81933196379939)
               mean=219.49760066171962, variance=41.119378425247554,
               skewness=-0.020719827584291976, kurtosis=-0.05350857438202583)
```

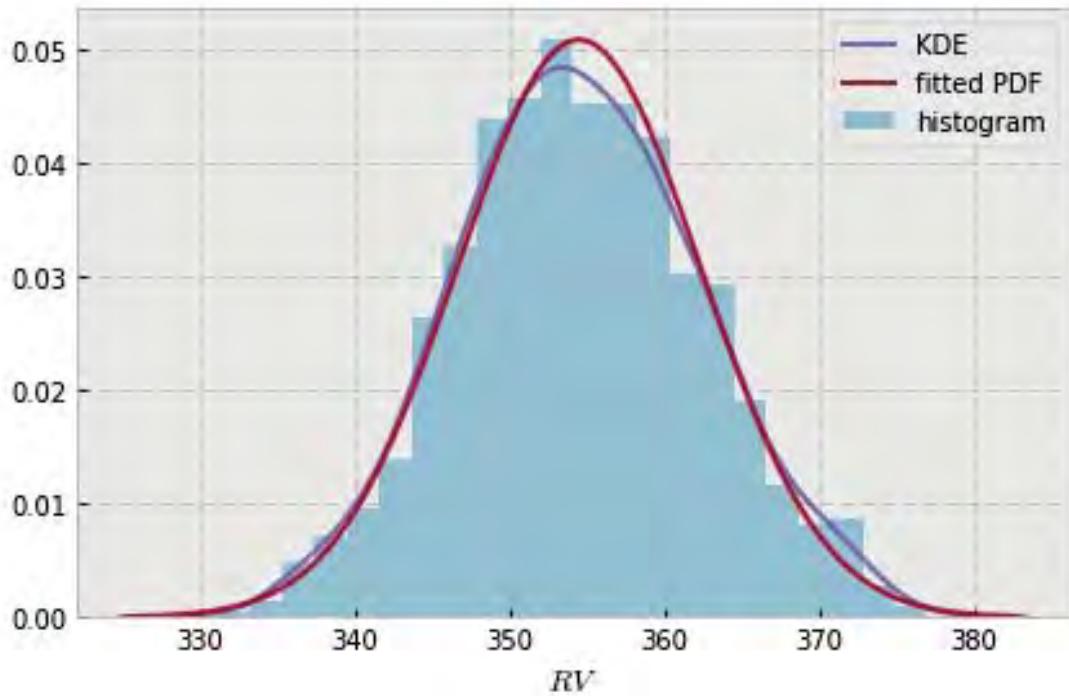


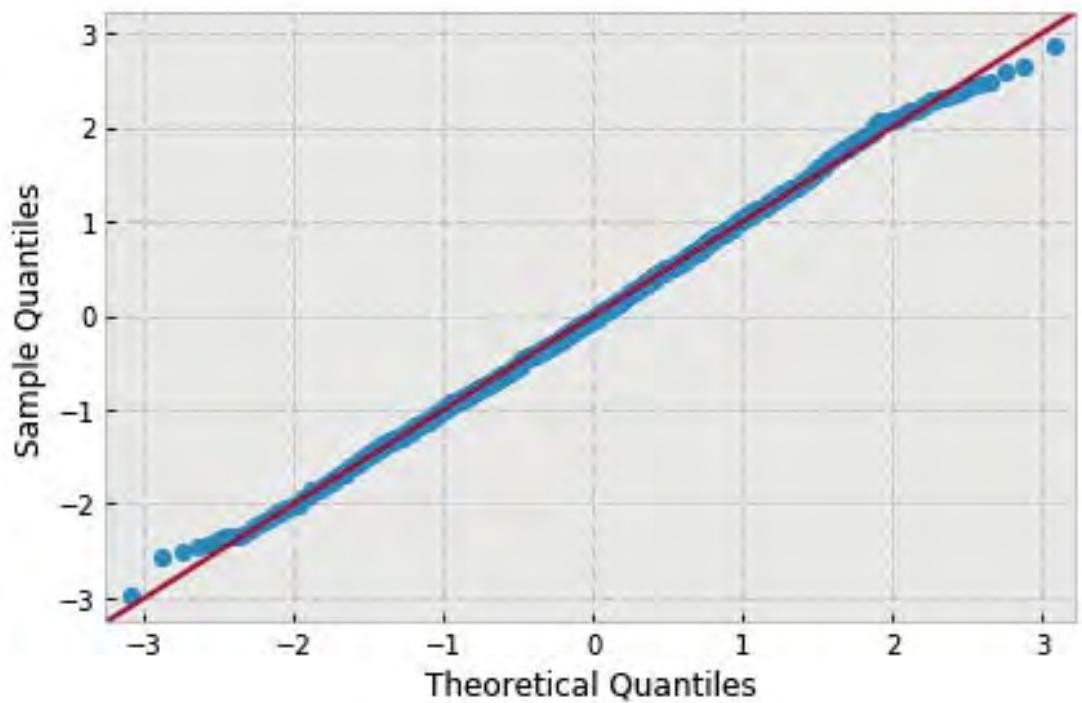


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=287.05617268542807, scale=6.746134812581202)
DescribeResult(nobs=1000, minmax=(265.16972365508201, 310.55484582539015)
               mean=287.05617268542807, variance=45.555890800320327,
               skewness=0.04765019063548426, kurtosis=0.07090046933749639)
```

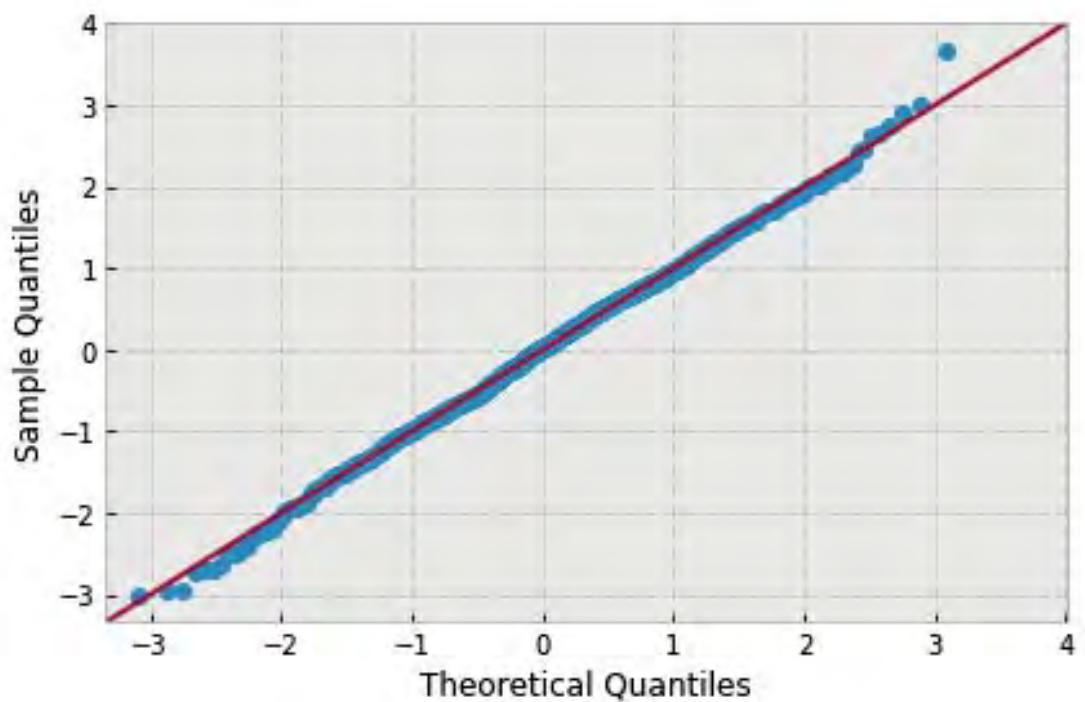
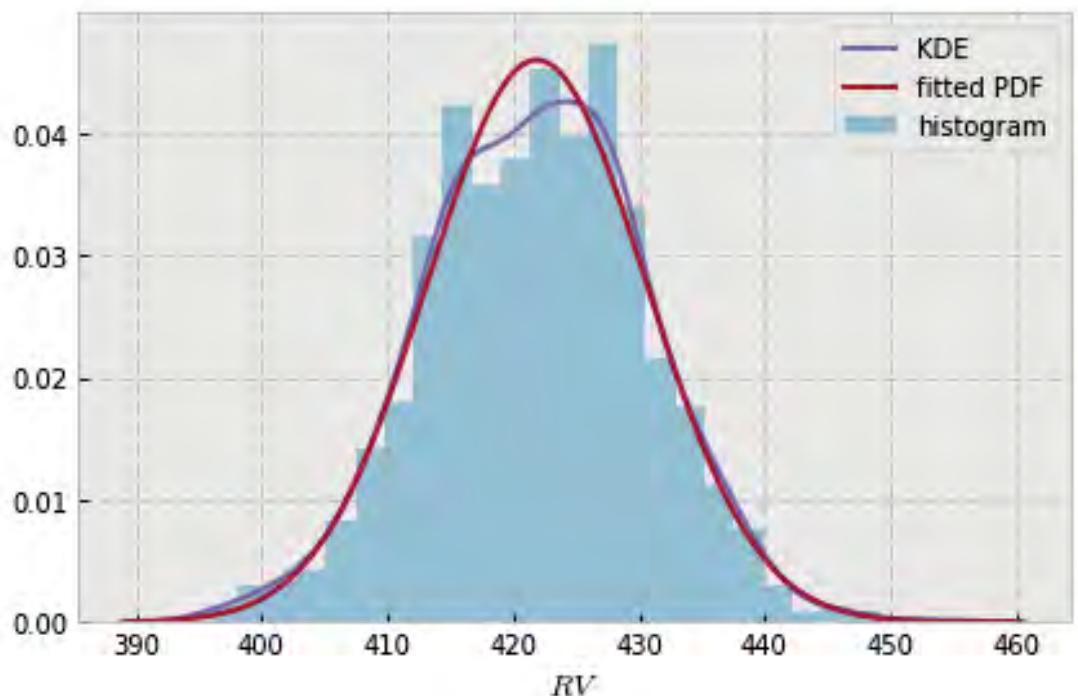


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=354.473927899195, scale=7.852935119847973)
DescribeResult(nobs=1000, minmax=(331.17403106105837, 376.94793039691353)
               mean=354.47392789919502, variance=61.730320316858545,
               skewness=0.06700782786716879, kurtosis=-0.23328490161528315)
```

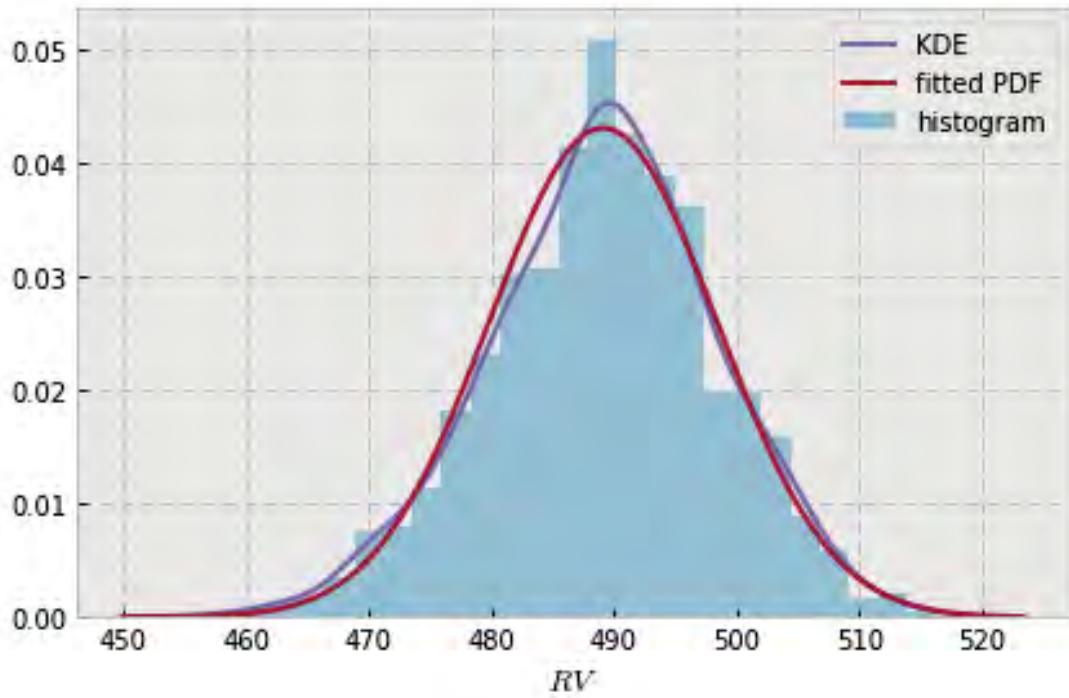


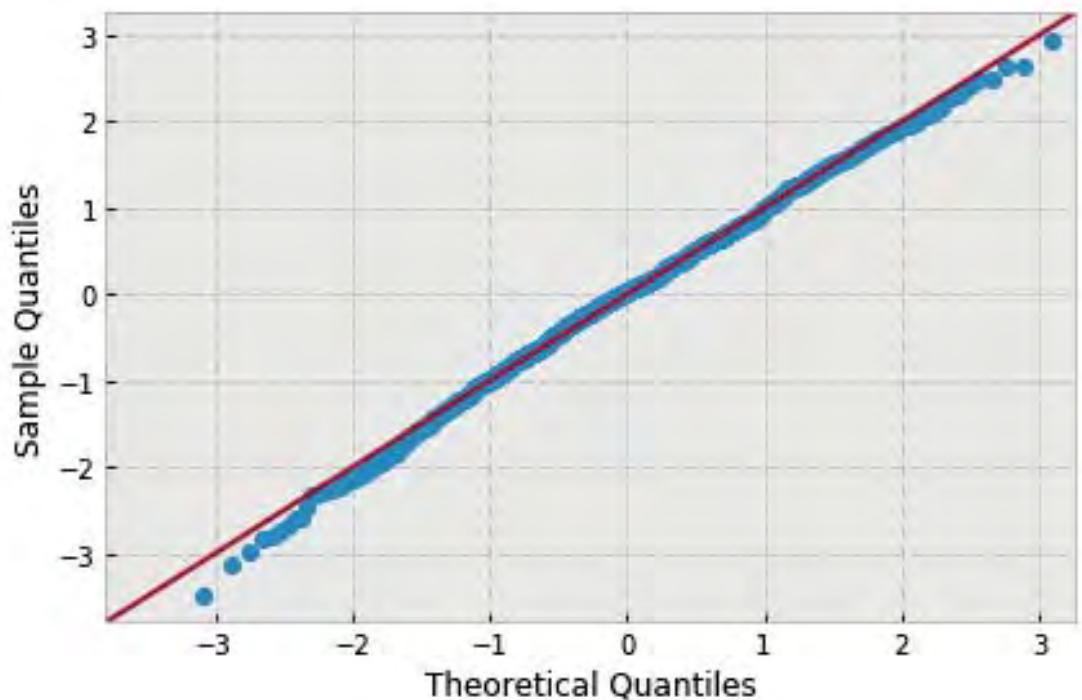


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=421.8606937951904, scale=8.669687374686895)
DescribeResult(nobs=1000, minmax=(395.82613124252867, 453.75793895391382)
               mean=421.8606937951904, variance=75.238717892698034,
               skewness=-0.03377201852960518, kurtosis=0.019386529388161655)
```

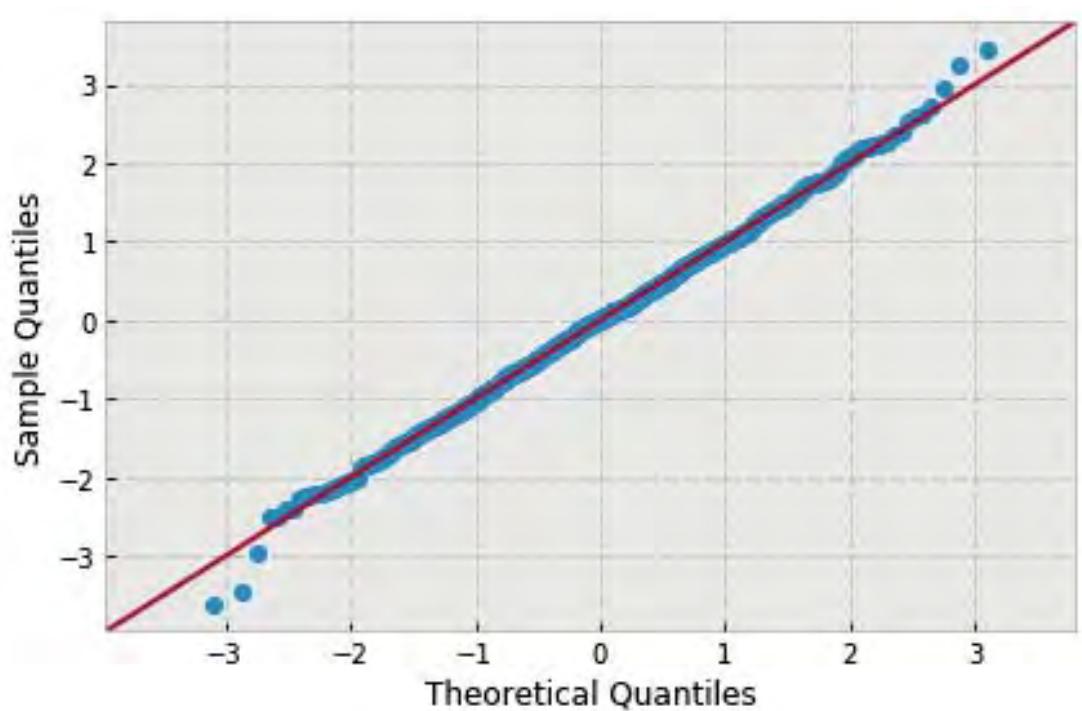
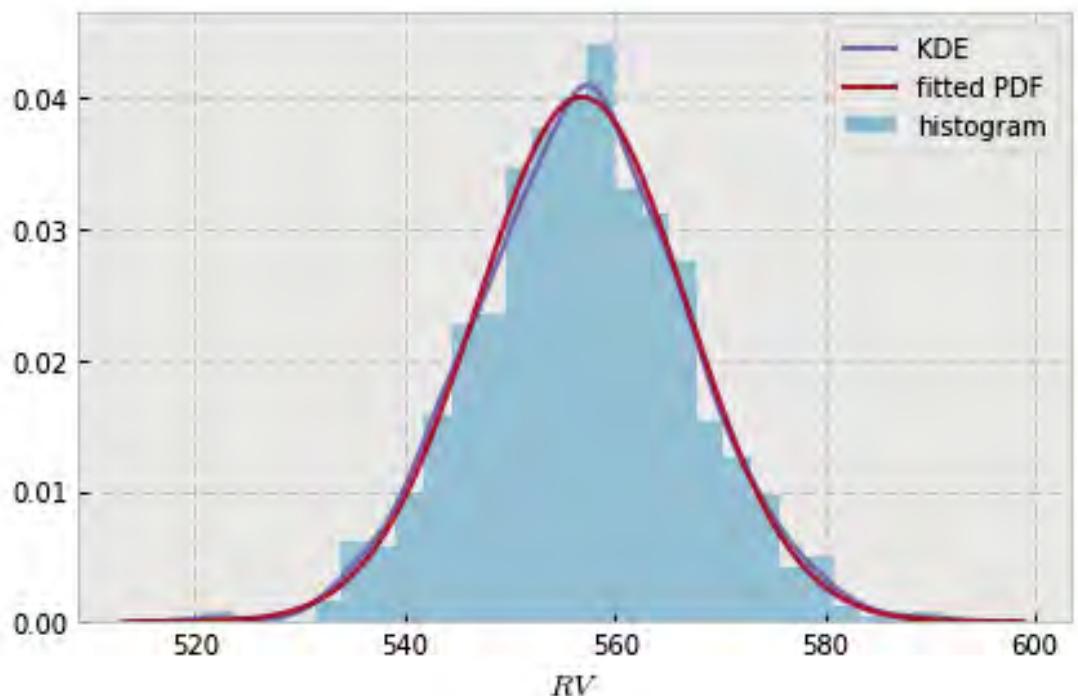


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=489.0782015028997, scale=9.25366841083901)
DescribeResult(nobs=1000, minmax=(456.96696144554312, 516.23227578162744)
               mean=489.07820150289967, variance=85.7160951529127,
               skewness=-0.17143712196111155, kurtosis=0.056592692519986)
```

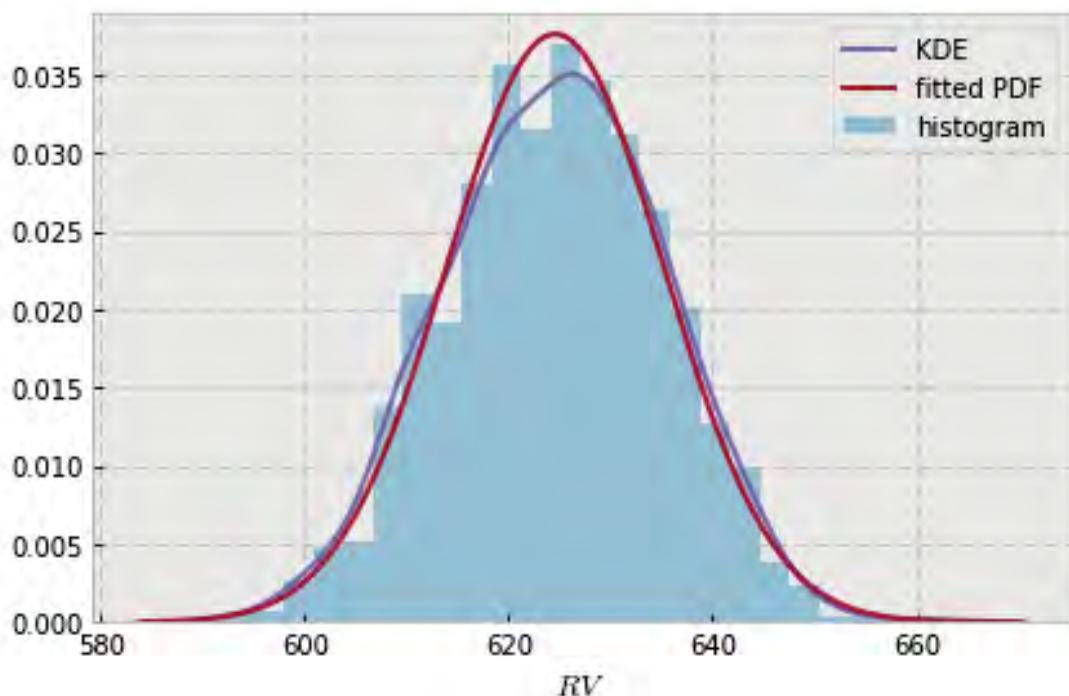


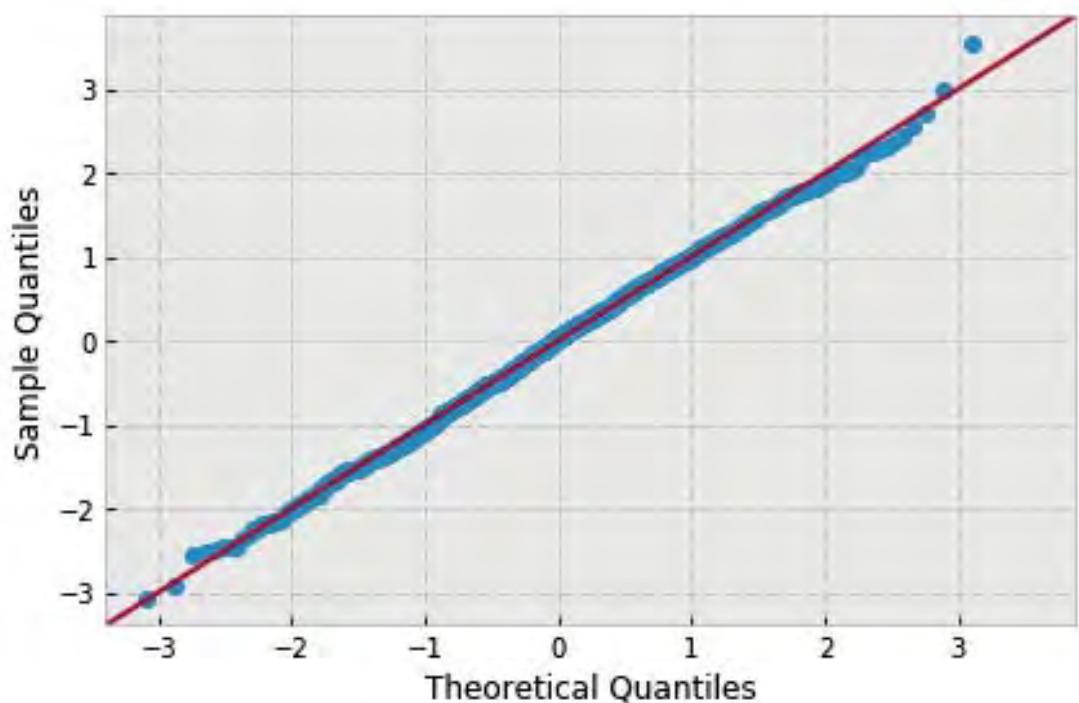


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=556.8661373654461, scale=9.956978204150074)
DescribeResult(nobs=1000, minmax=(520.98916369840629, 591.13974314677273)
               mean=556.86613736544609, variance=99.240655613533178,
               skewness=0.01303834268092583, kurtosis=0.13248871018993214)
```

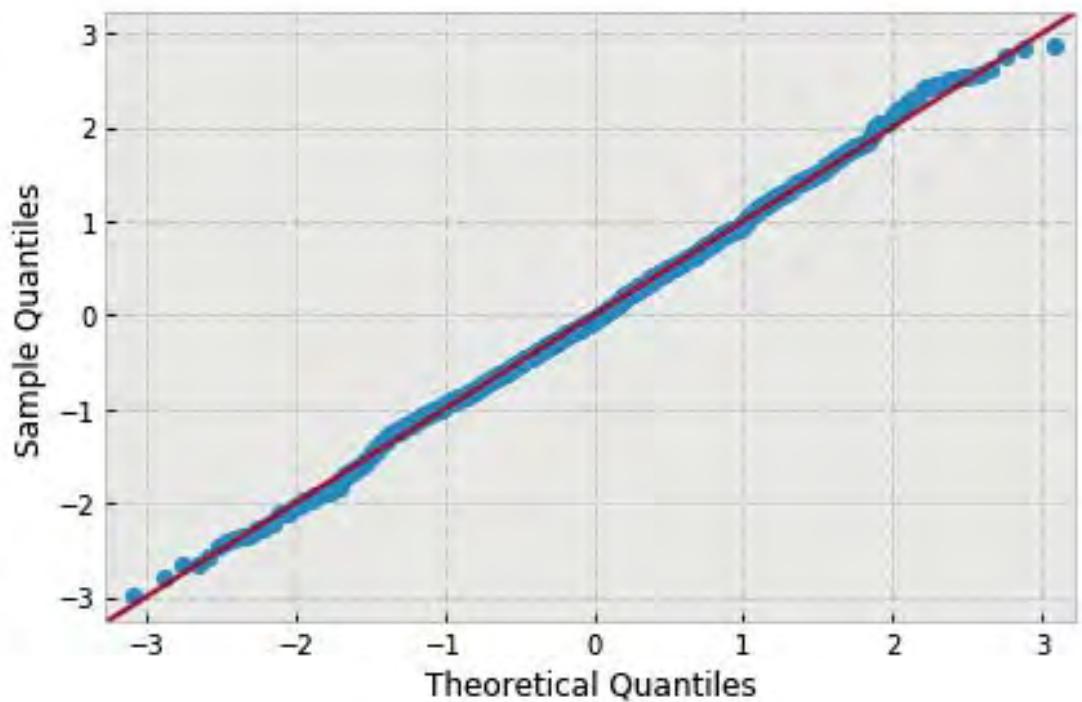
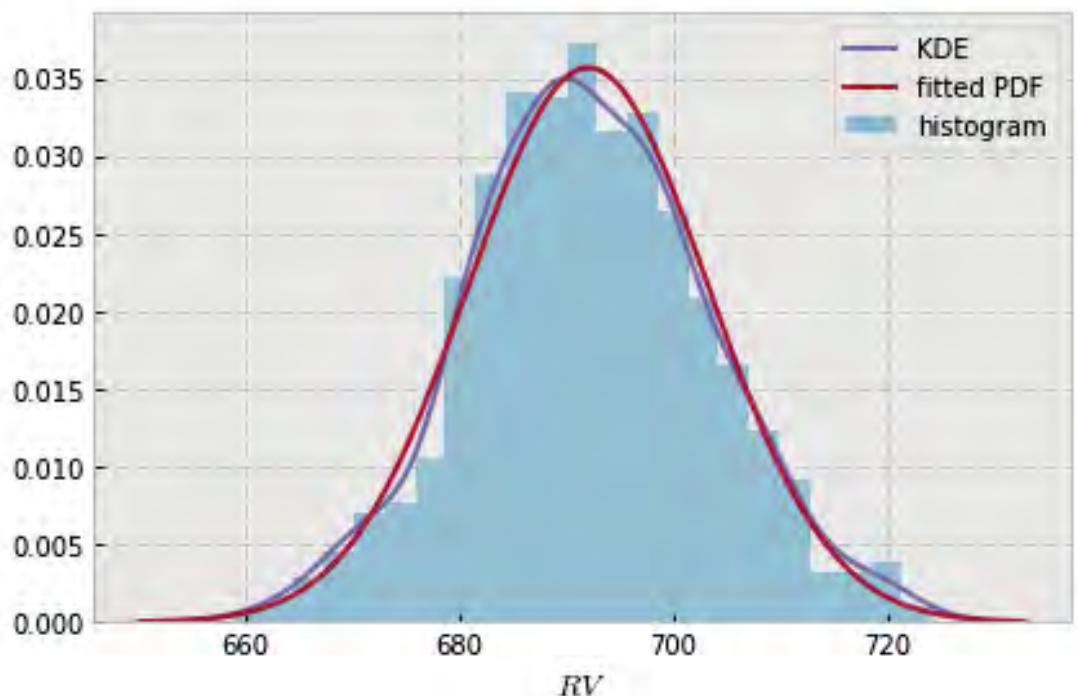


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=624.5594660596901, scale=10.616667637878615)
DescribeResult(nobs=1000, minmax=(592.05193818114356, 662.10162502629441)
               mean=624.55946605969007, variance=112.82645819137046,
               skewness=-0.03923896621861167, kurtosis=-0.2159071285701053)
```

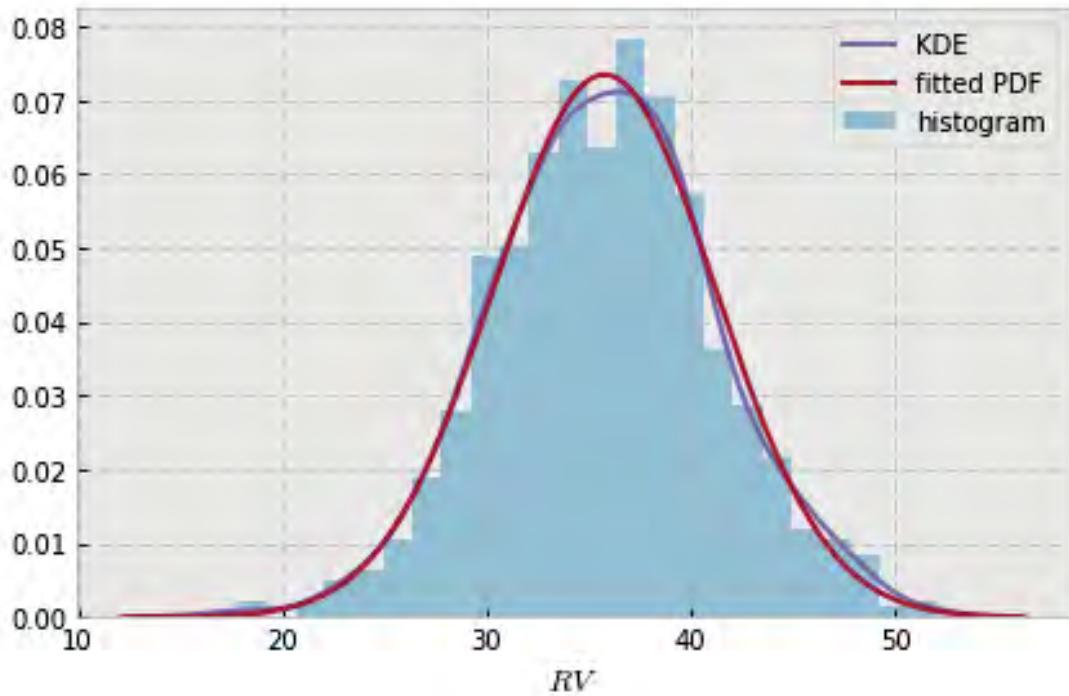


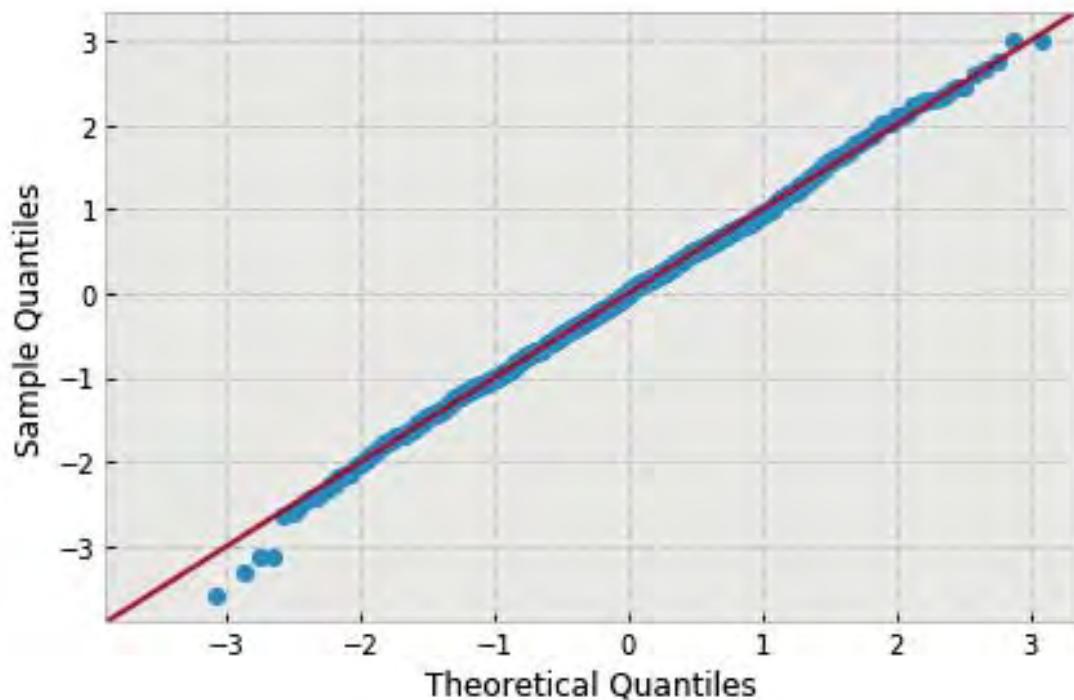


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=692.0501399017521, scale=11.185537061208304)
DescribeResult(nobs=1000, minmax=(658.77847156892597, 724.10714153511503)
               mean=692.05013990175212, variance=125.241480828493,
               skewness=0.06843054369092691, kurtosis=-0.02546065468532177)
```

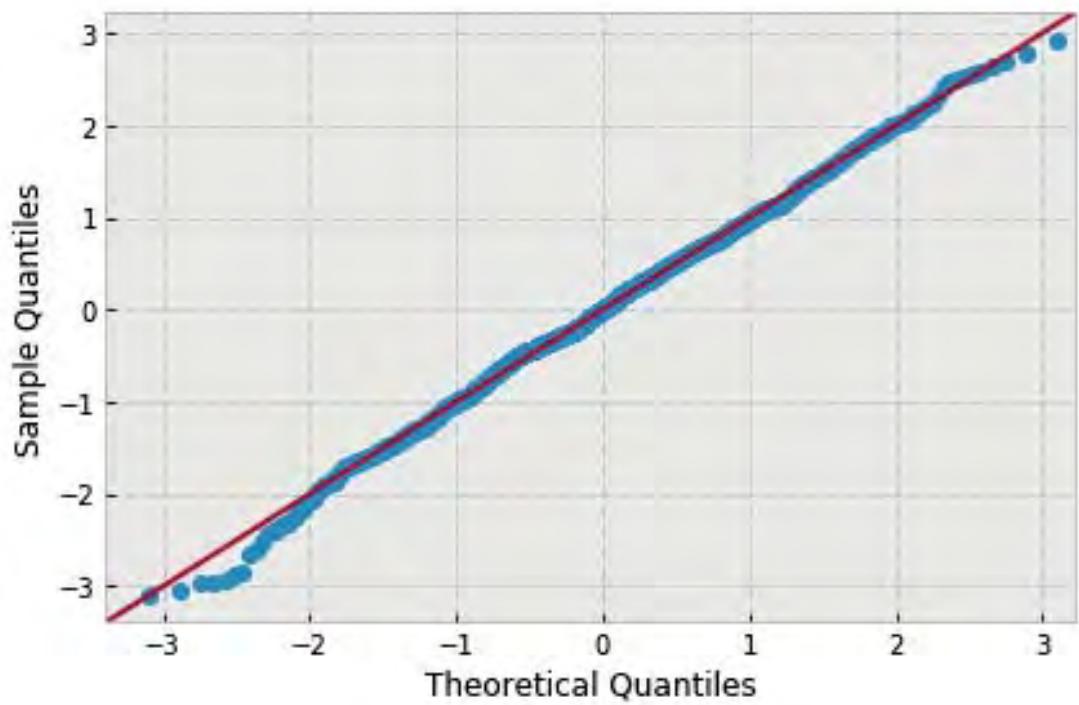
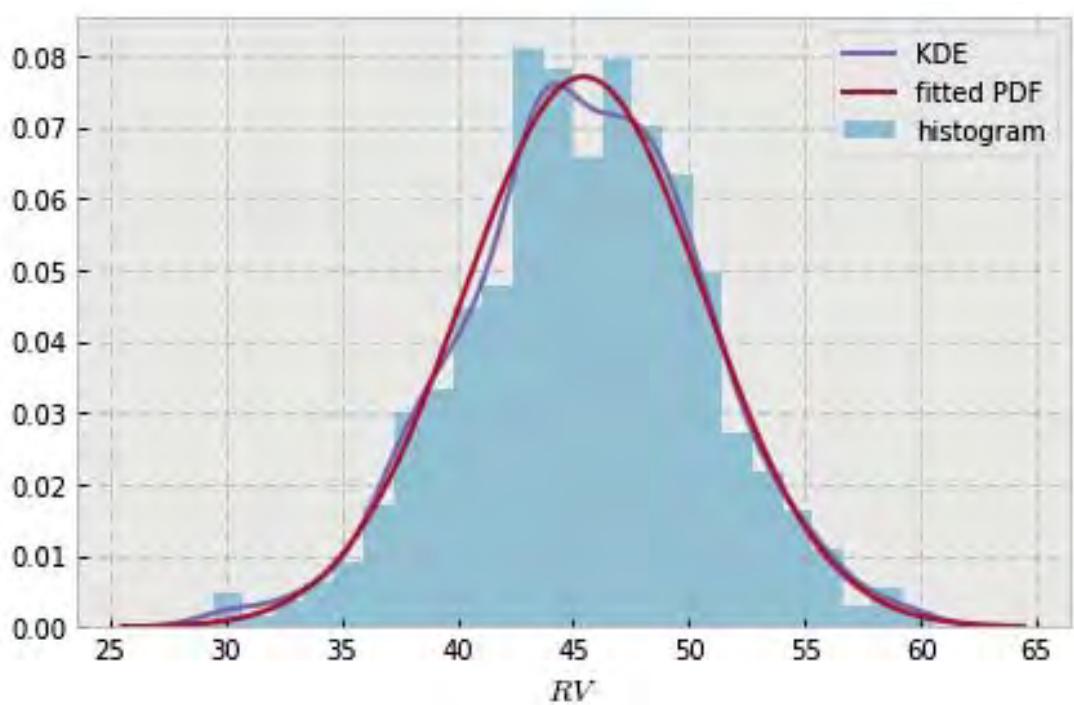


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=35.7814677128654, scale=5.432232152230148)
DescribeResult(nobs=1000, minmax=(16.361828550315362, 52.07559786853664)
    mean=35.7814677128654, variance=29.538684840563551,
    skewness=-0.014759006877380547, kurtosis=0.15199633482234676)
```

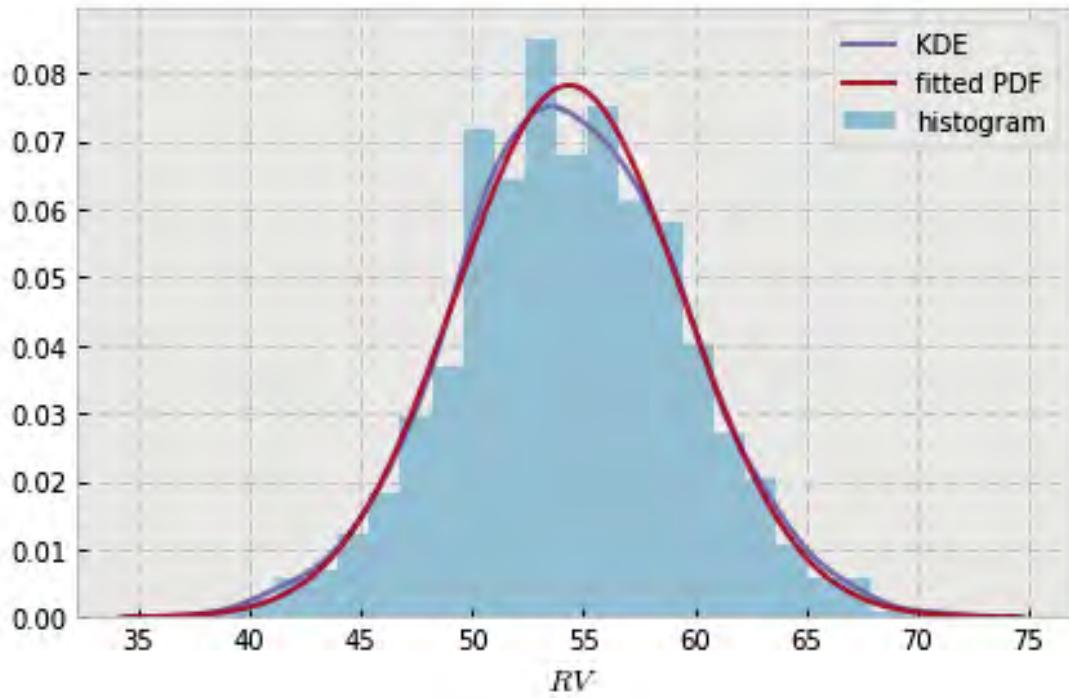


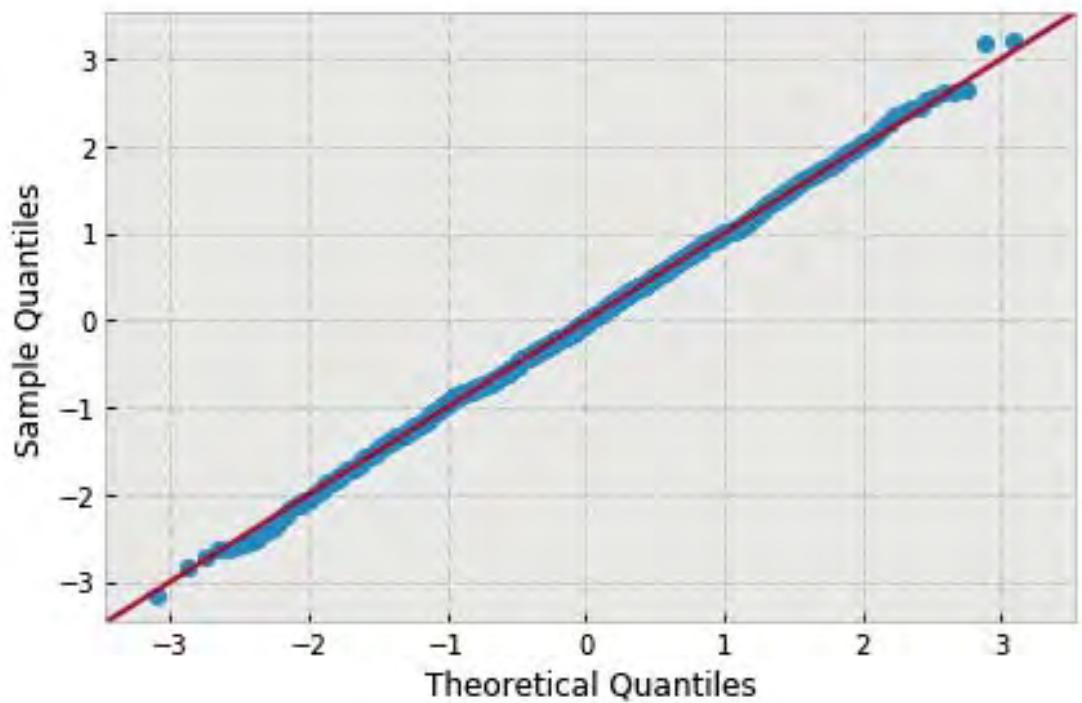


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(), loc=45.45380934715949, scale=5.177241552141578)
DescribeResult(nobs=1000, minmax=(29.461517124278895, 60.494128592071839)
               mean=45.453809347159492, variance=26.830660749971308,
               skewness=-0.09330966607748555, kurtosis=0.07646068878738355)
```

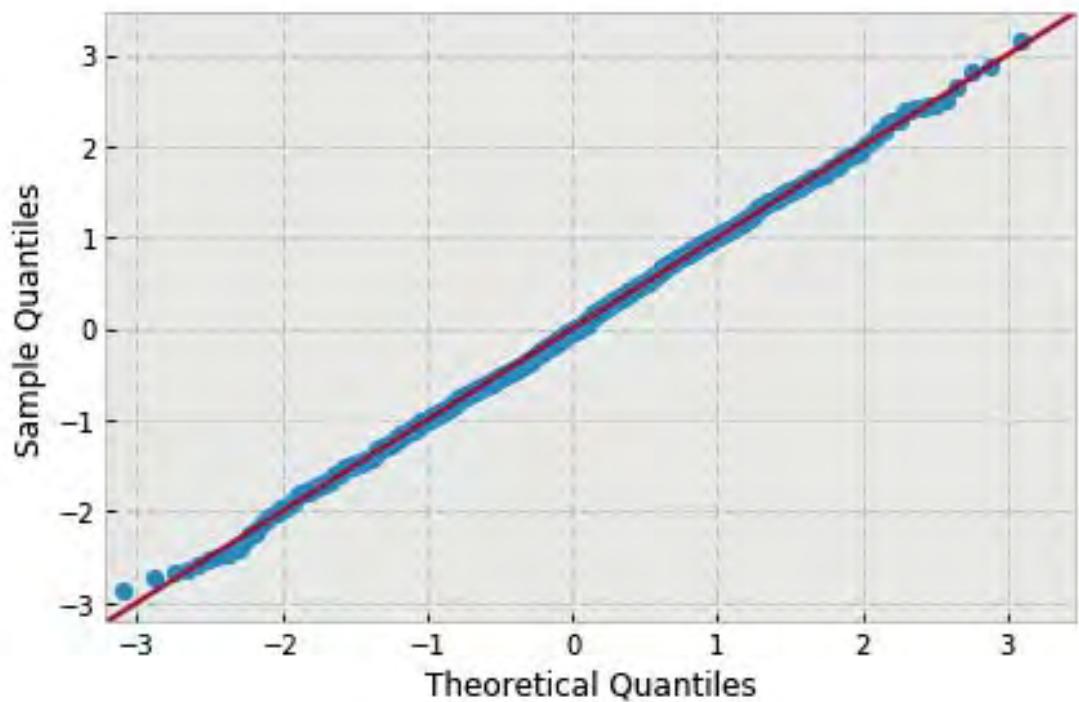
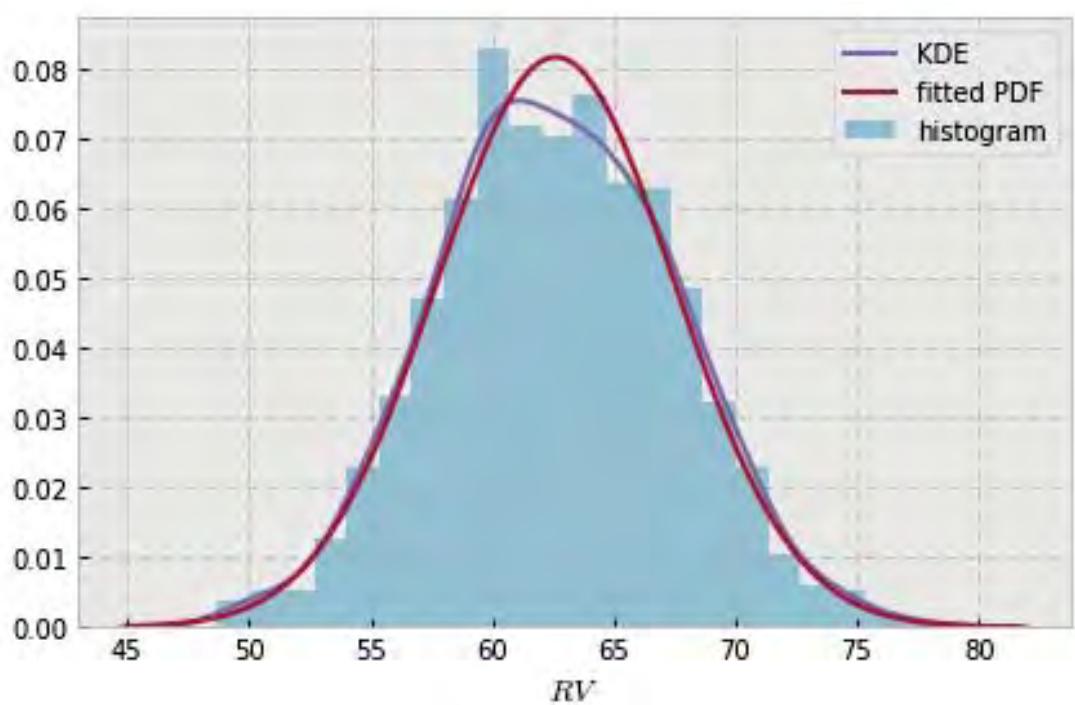


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=54.355700001195636, scale=5.099001011046041)
DescribeResult(nobs=1000, minmax=(38.320642199259296, 70.71815527442935)
               mean=54.355700001195636, variance=26.025837147796341,
               skewness=0.024630935324187194, kurtosis=0.011135811712001509)
```

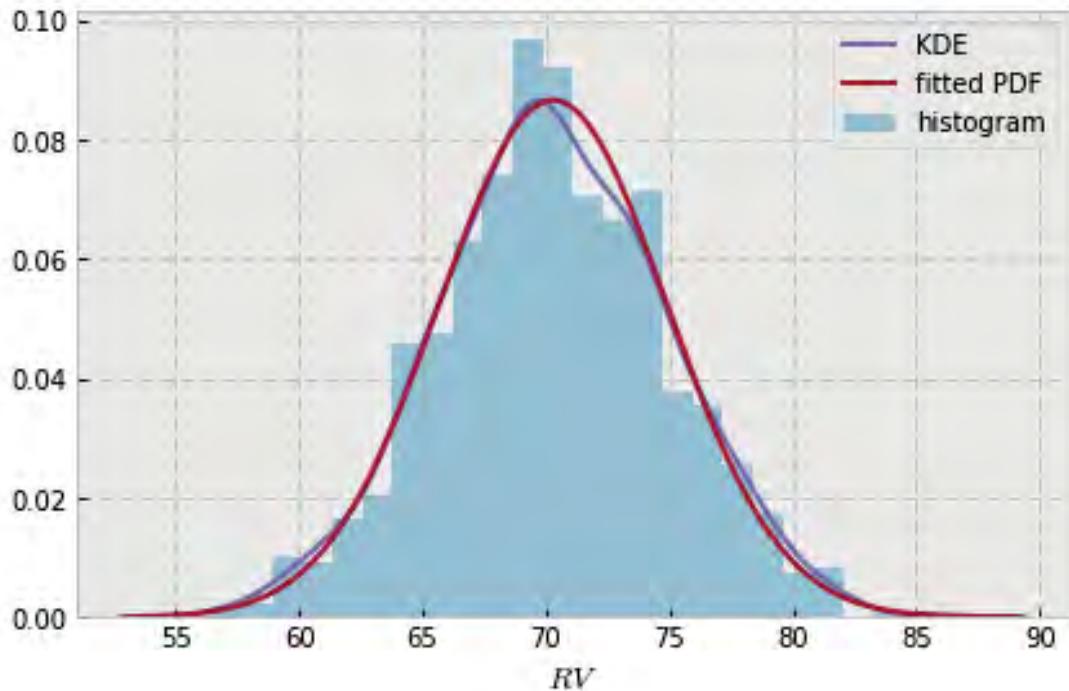


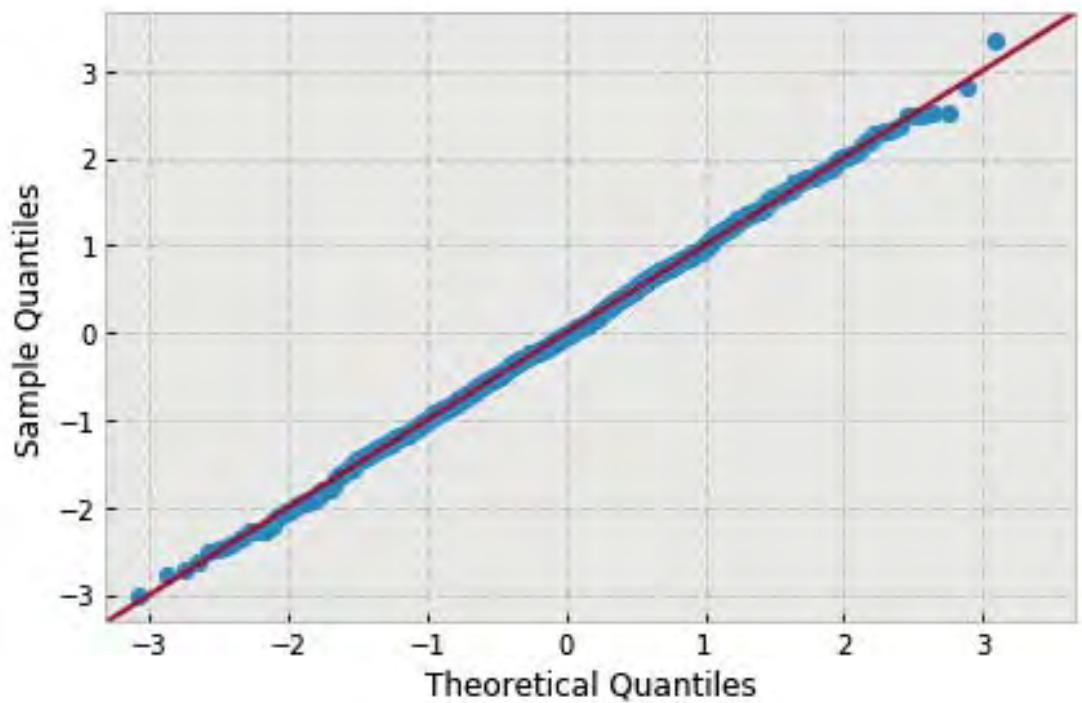


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=62.613439923823535, scale=4.873966167590694)
DescribeResult(nobs=1000, minmax=(48.688686409957022, 78.00989549153671)
               mean=62.613439923823535, variance=23.779325528347059,
               skewness=0.03645723479838222, kurtosis=-0.1769424524492189)
```

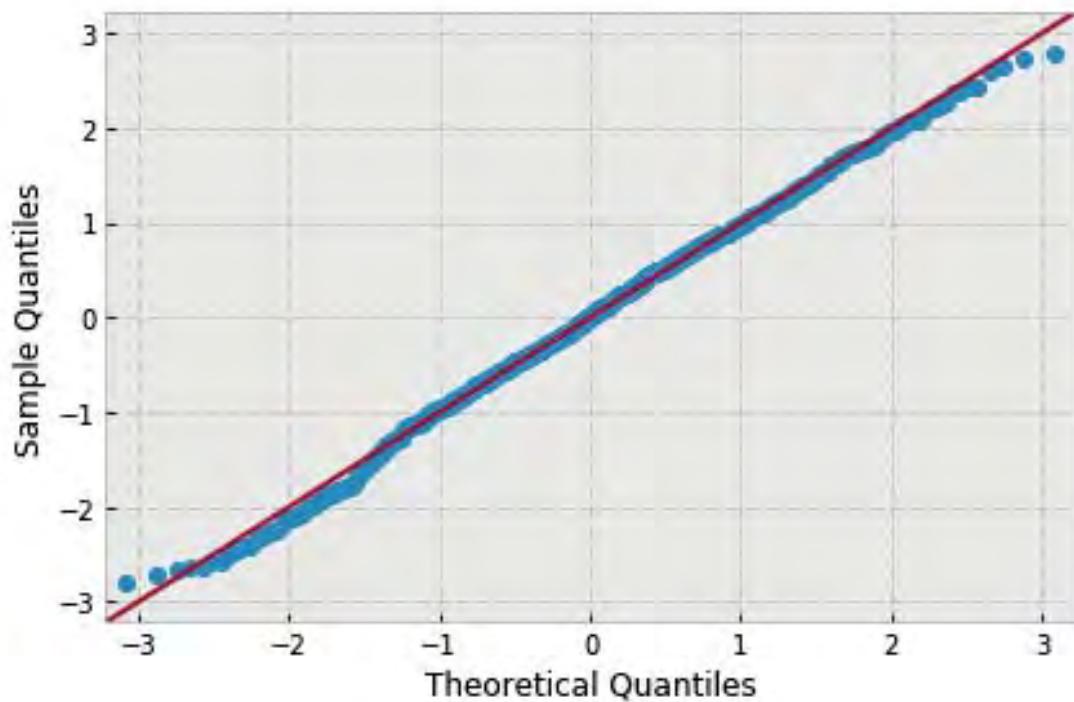
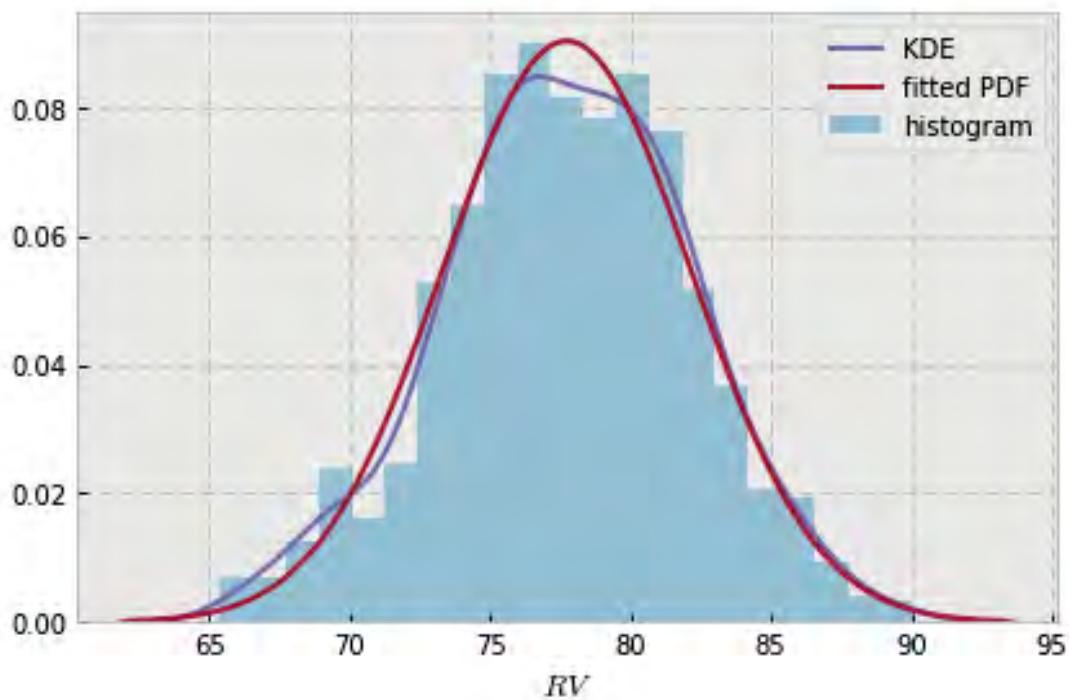


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=70.28158918342174, scale=4.605318935512461)
DescribeResult(nobs=1000, minmax=(56.472150131445055, 85.680418878658685)
               mean=70.281589183421744, variance=21.230192690480102,
               skewness=0.015555349127356084, kurtosis=-0.10806168329142496)
```

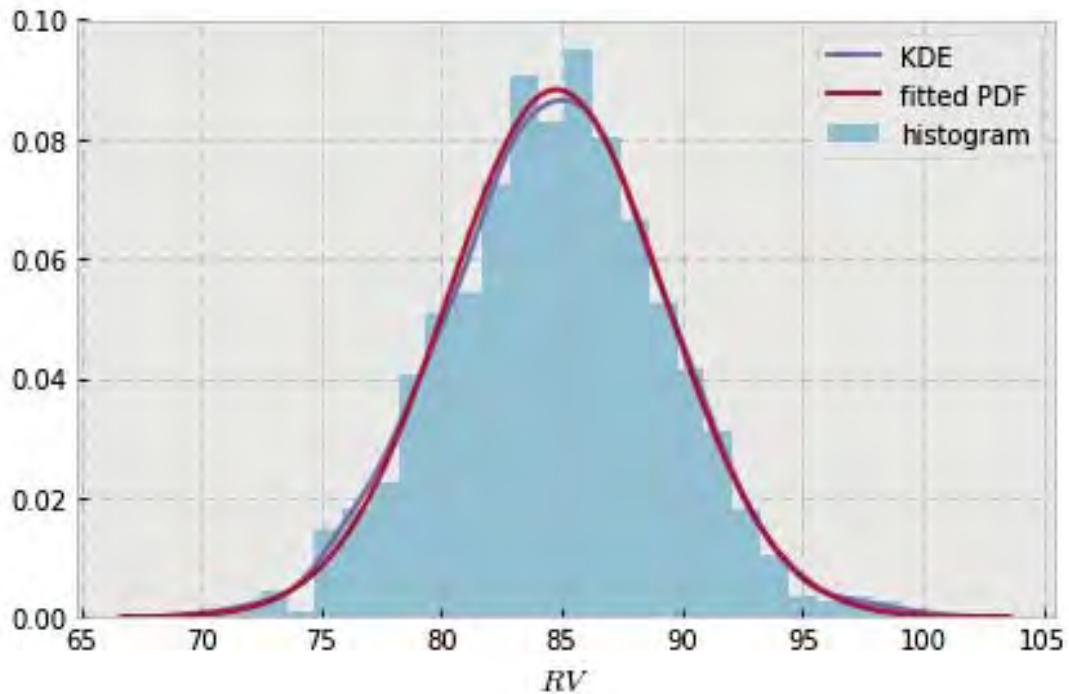


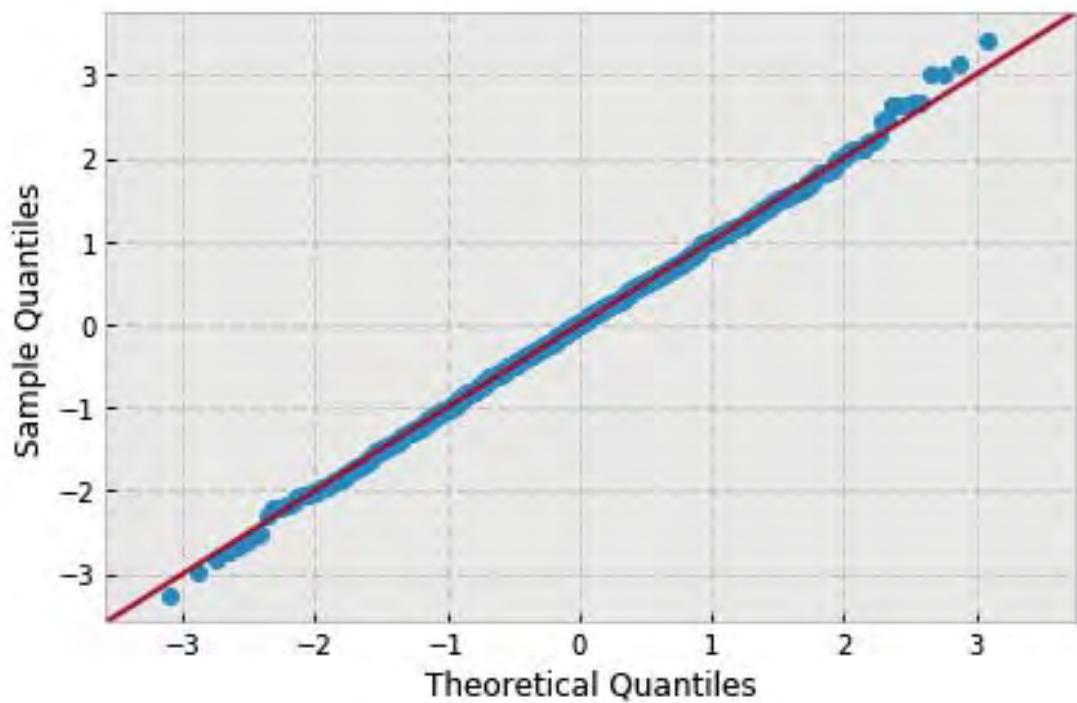


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(), loc=77.72022936241277, scale=4.421312541232051)
DescribeResult(nobs=1000, minmax=(65.35550597927714, 90.072358520659634)
               mean=77.720229362412766, variance=19.567572159415231,
               skewness=-0.1214190666418257, kurtosis=-0.11258691412347321)
```

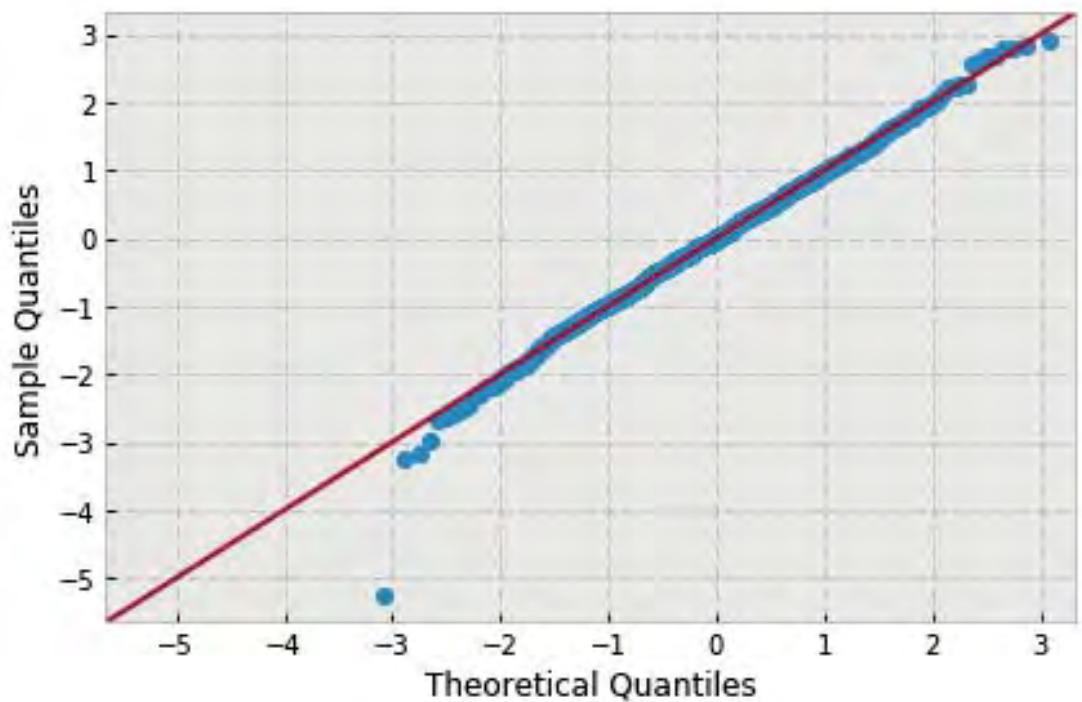
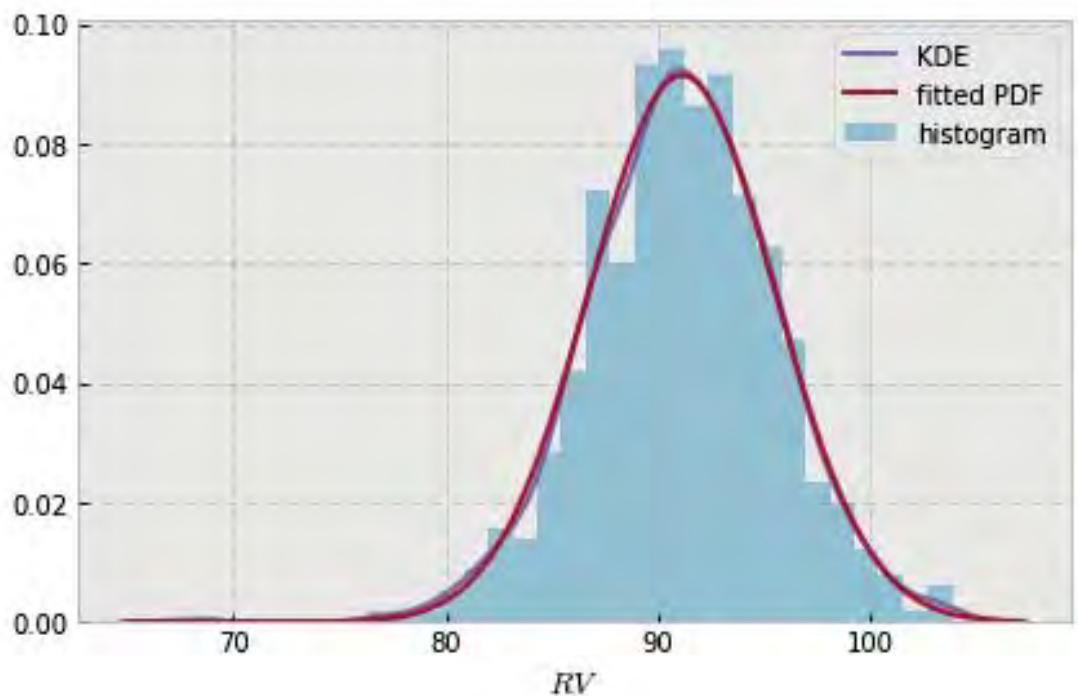


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=84.77397328399014, scale=4.52005557904689)
DescribeResult(nobs=1000, minmax=(70.10808535482316, 100.18209128685776)
               mean=84.773973283990145, variance=20.451353791464381,
               skewness=0.01426528060830885, kurtosis=0.08292079462216151)
```

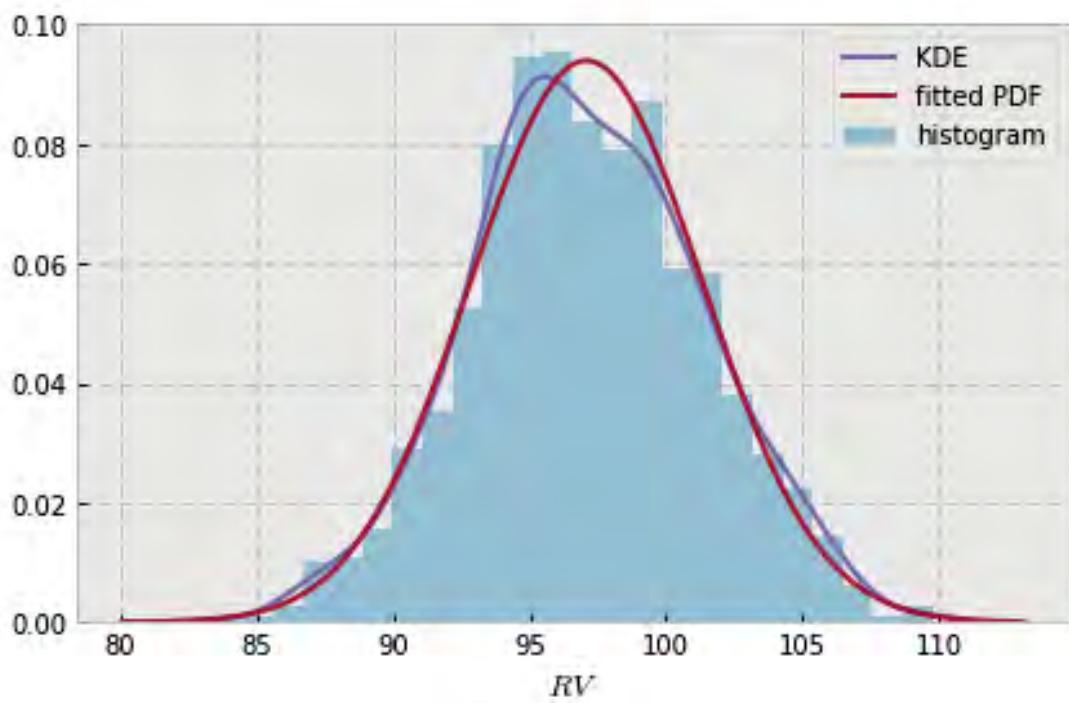


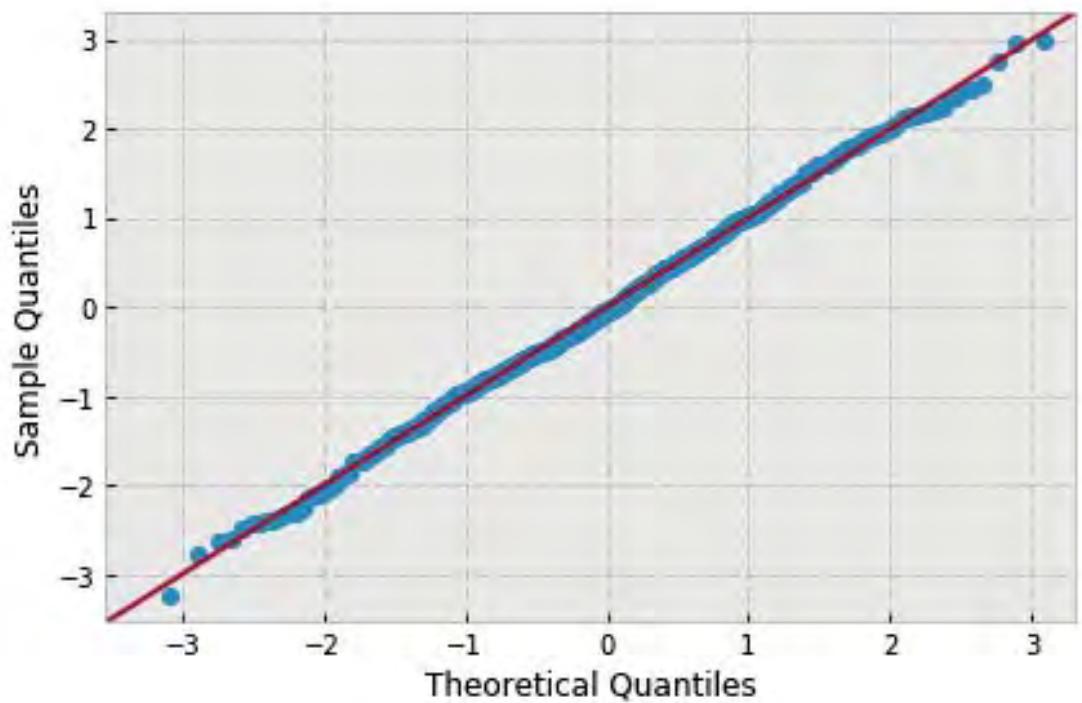


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=91.12692732433489, scale=4.365276620164248)
DescribeResult(nobs=1000, minmax=(68.239621267233574, 103.80859752080968)
               mean=91.126927324334886, variance=19.07471468523784,
               skewness=-0.17891452797984672, kurtosis=0.7279076491405982)
```

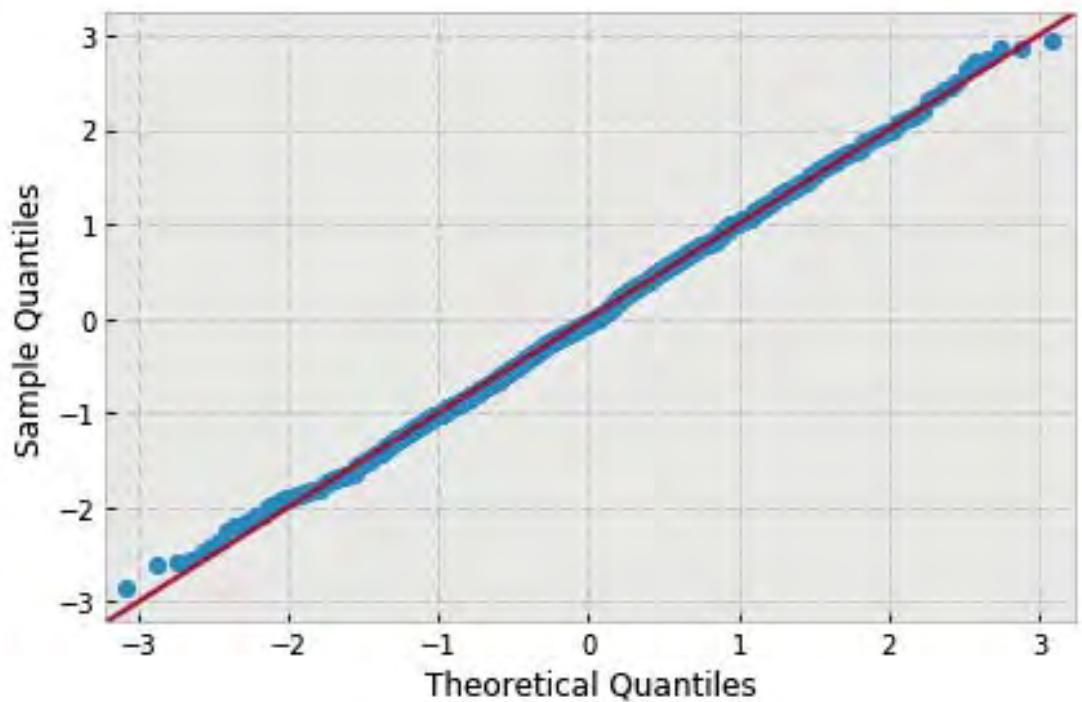
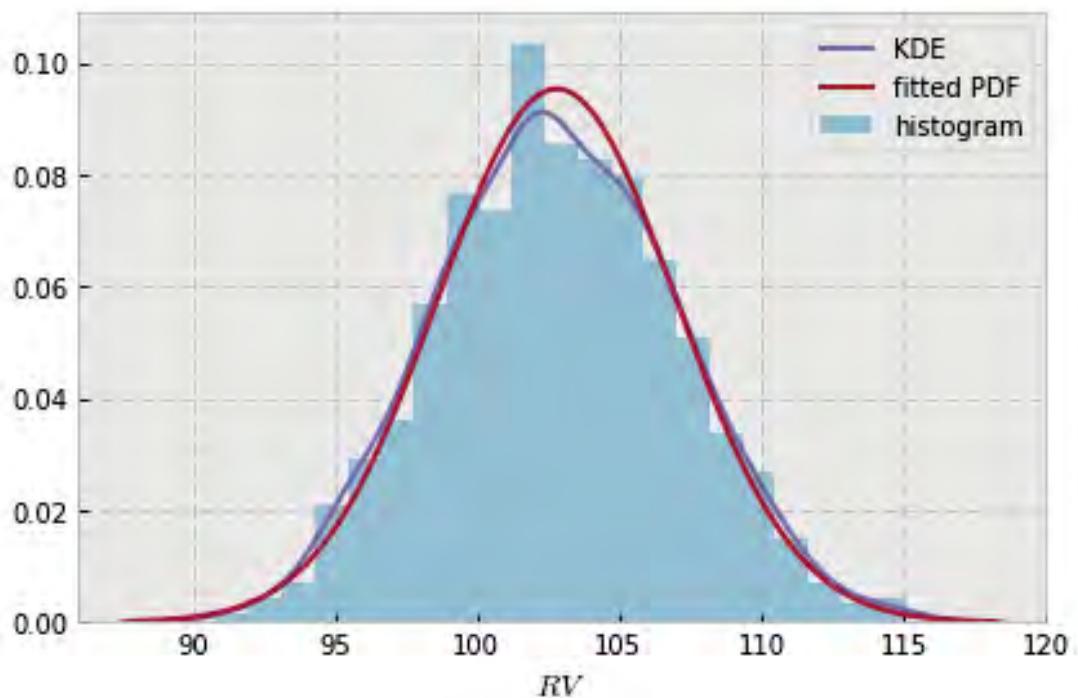


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=97.07809915915477, scale=4.245575589699727)
DescribeResult(nobs=1000, minmax=(83.394140256351321, 109.75180155509851)
               mean=97.078099159154775, variance=18.042955042897081,
               skewness=0.040131009370359516, kurtosis=-0.1331137587744493)
```

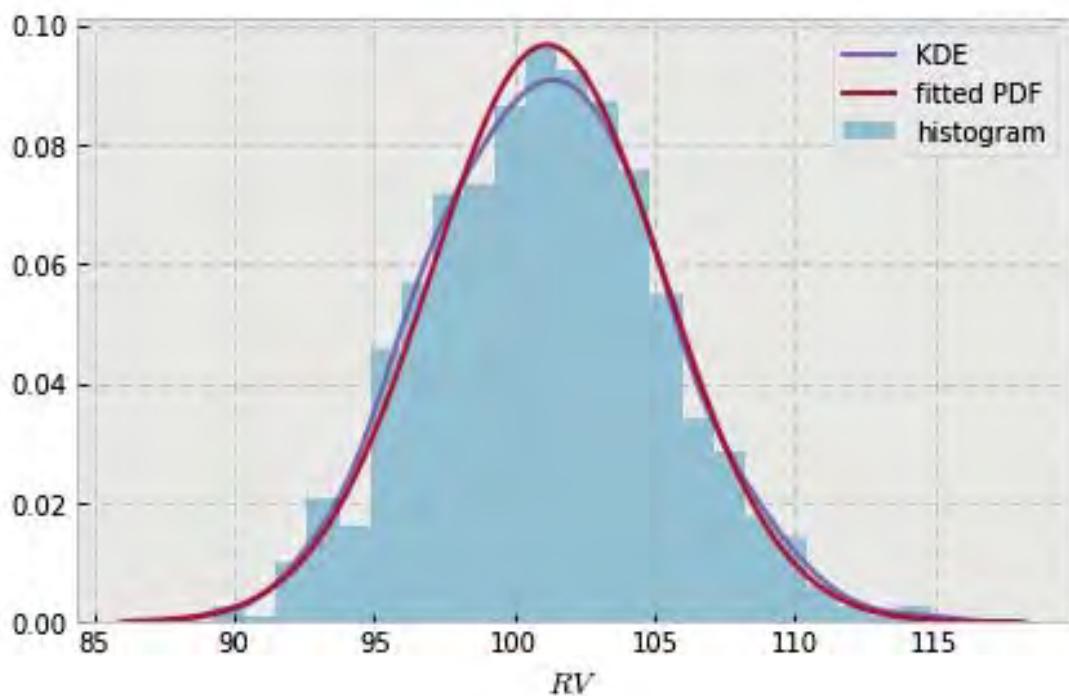


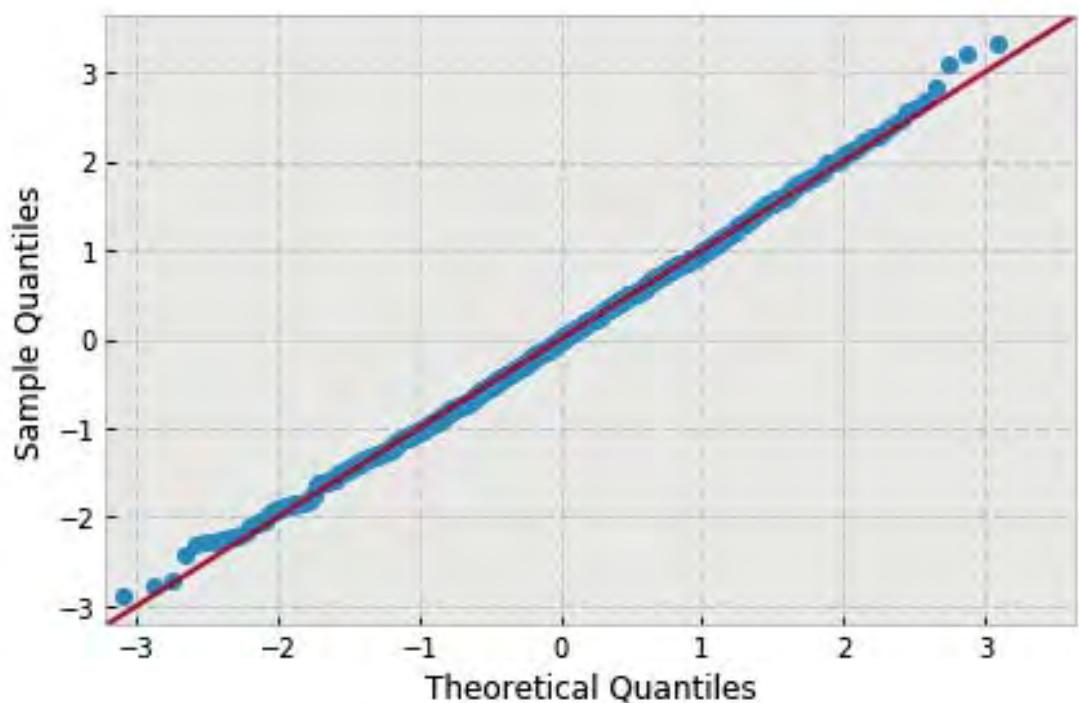


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=102.80632920641915, scale=4.186404137277915)
DescribeResult(nobs=1000, minmax=(90.822516020447438, 115.13578344714658)
               mean=102.80632920641915, variance=17.54352312374138,
               skewness=0.07095836949651567, kurtosis=-0.2393216862098848)
```

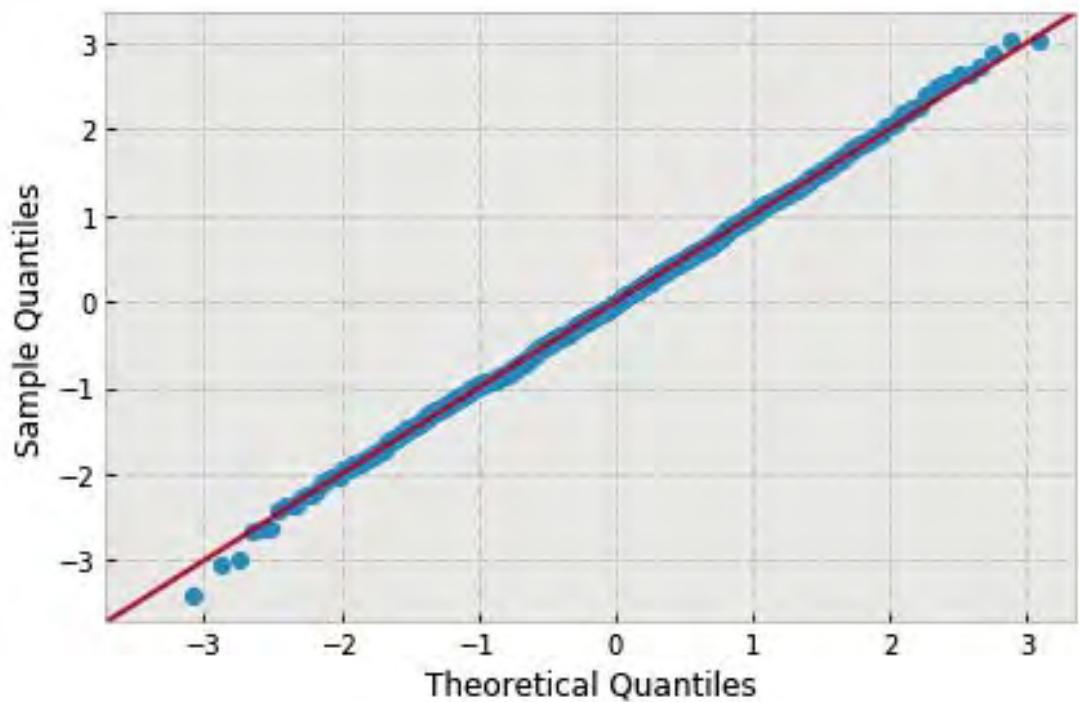
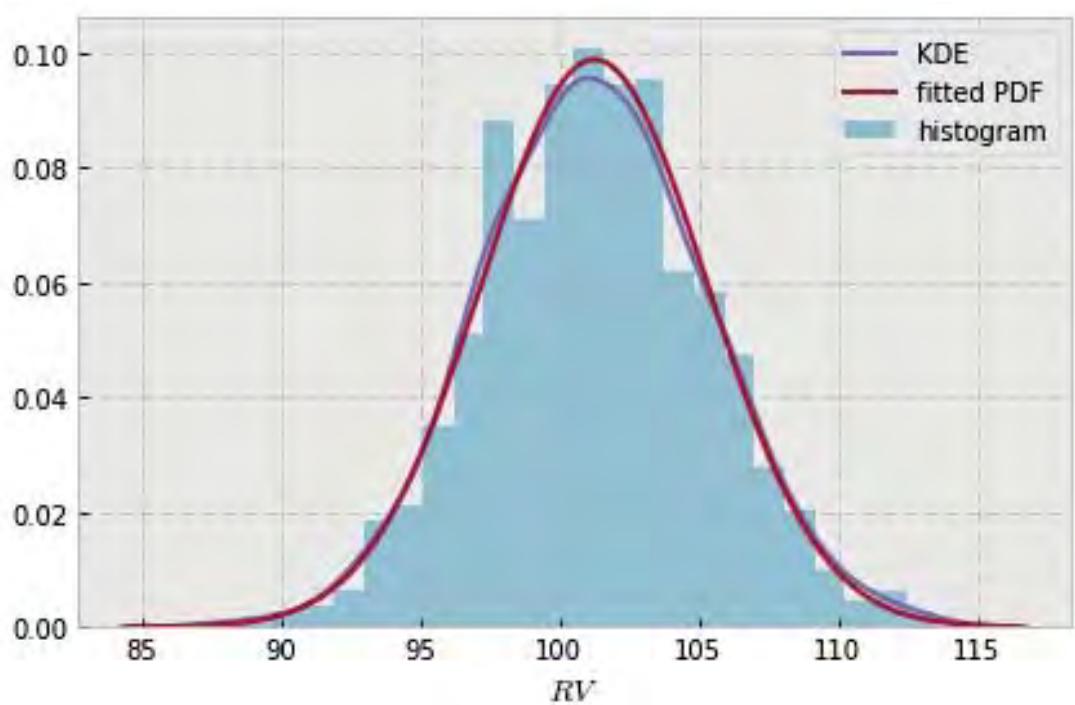


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=101.16707411529069, scale=4.129002823051369)
DescribeResult(nobs=1000, minmax=(89.276193464270492, 114.88518797794389)
               mean=101.16707411529069, variance=17.065730042808983,
               skewness=0.11886525372948281, kurtosis=-0.1057825455327821)
```

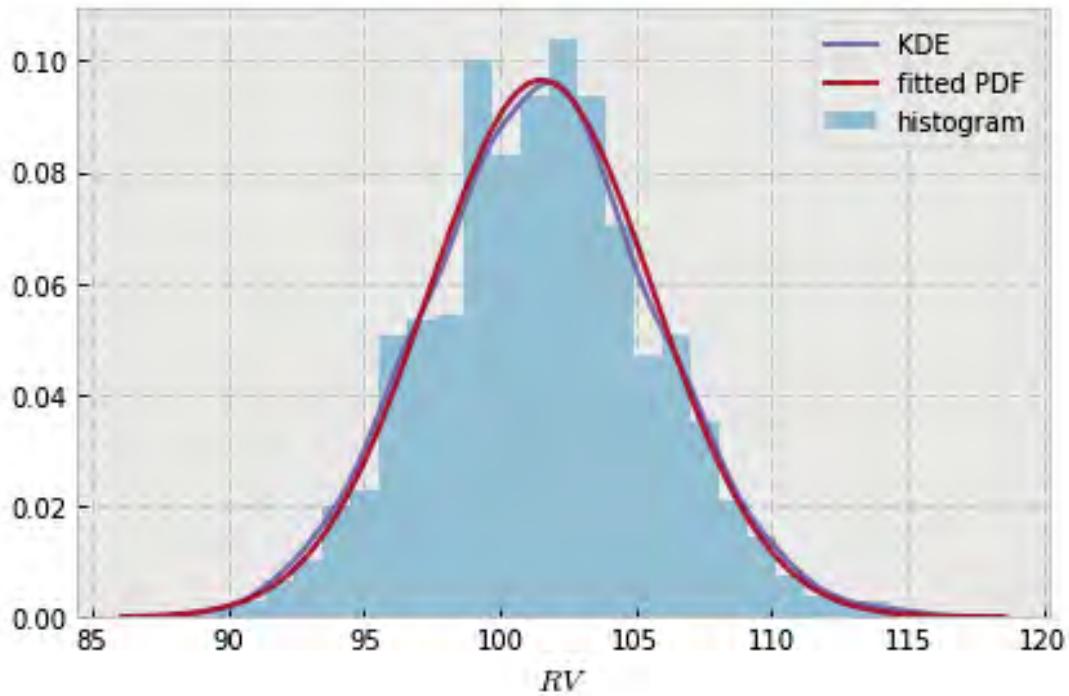


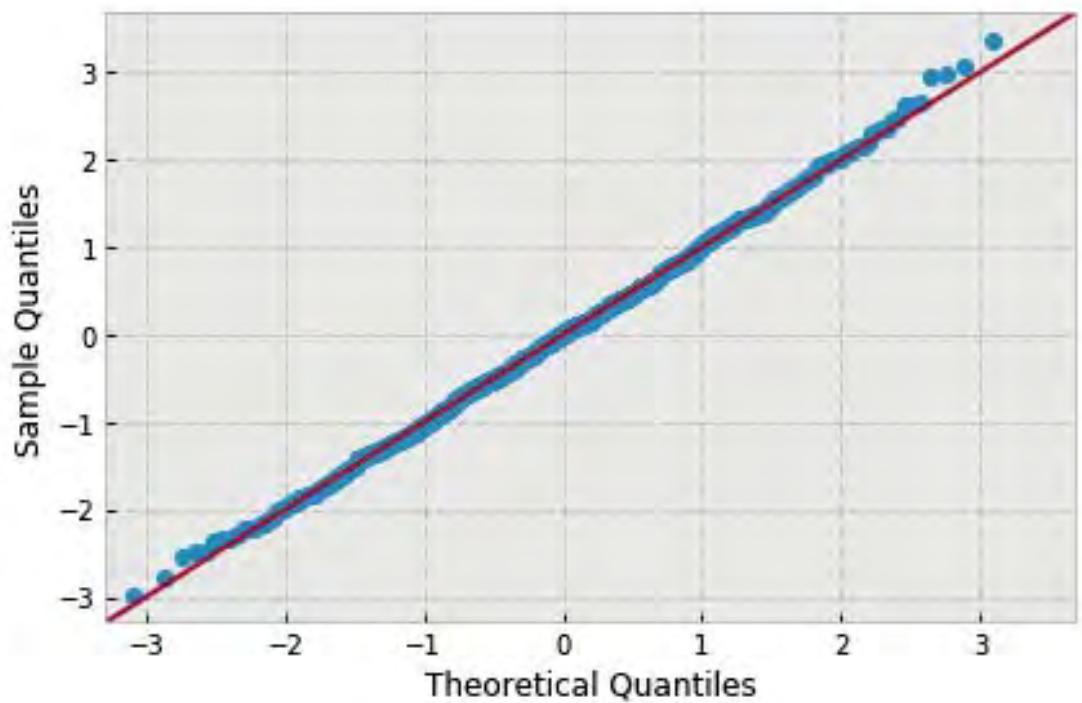


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=101.25382650648379, scale=4.03899753302194)
DescribeResult(nobs=1000, minmax=(87.548071989906518, 113.46675036822299)
               mean=101.25382650648379, variance=16.329830902659971,
               skewness=0.04001655257963342, kurtosis=0.020255162540871563)
```

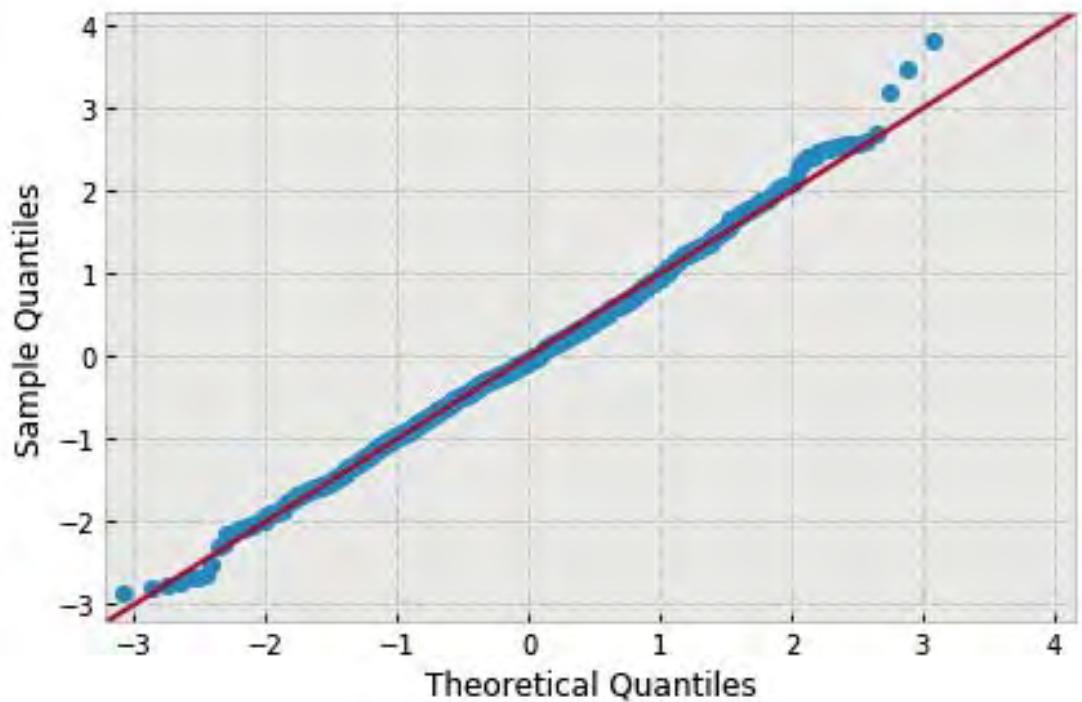
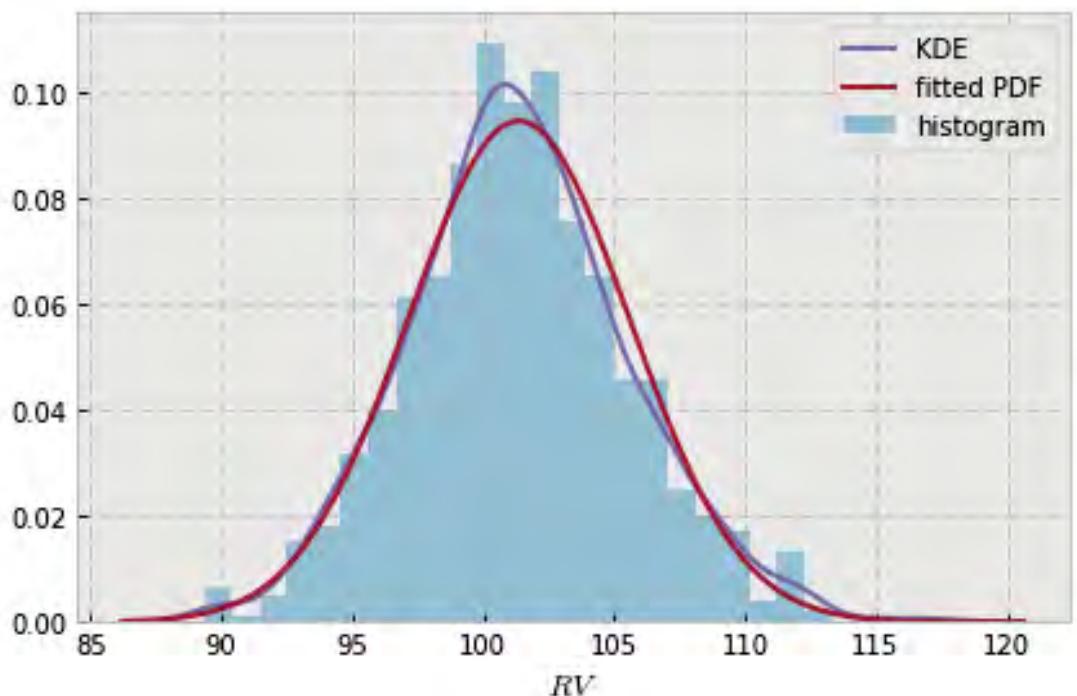


```
variable name: sigma
variable value: 0.03333333333333333
distribution: normal(shape=(), loc=101.50937664401867, scale=4.135014813406515)
DescribeResult(nobs=1000, minmax=(89.220189382315198, 115.40909541573504)
               mean=101.50937664401867, variance=17.115462970061383,
               skewness=0.07998571874613579, kurtosis=-0.047882623631088084)
```

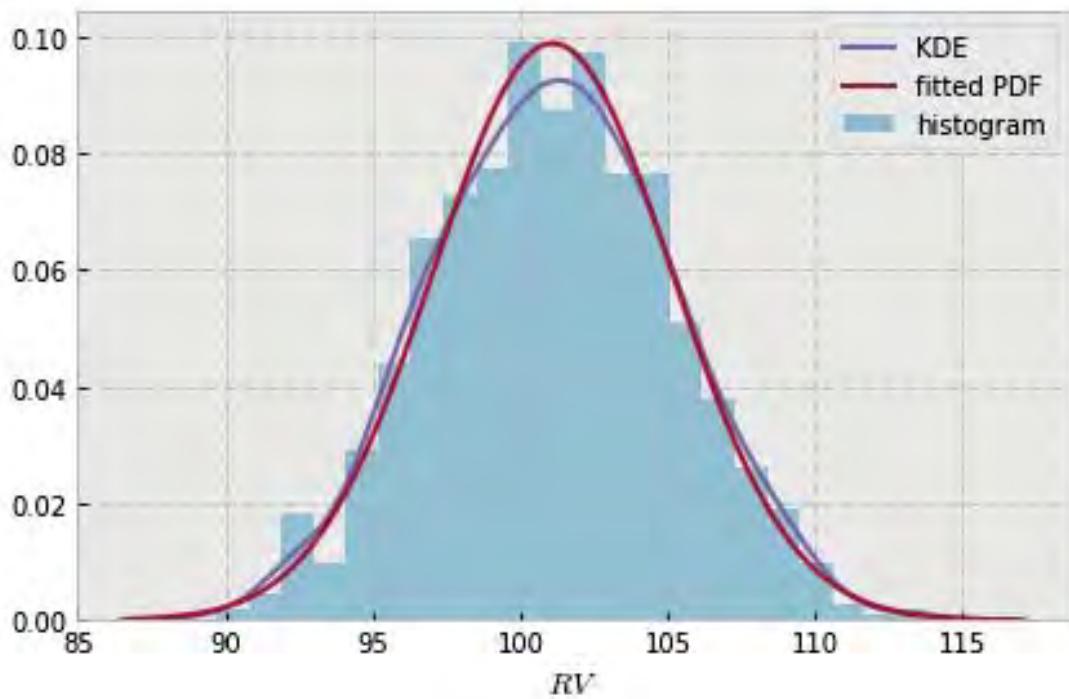


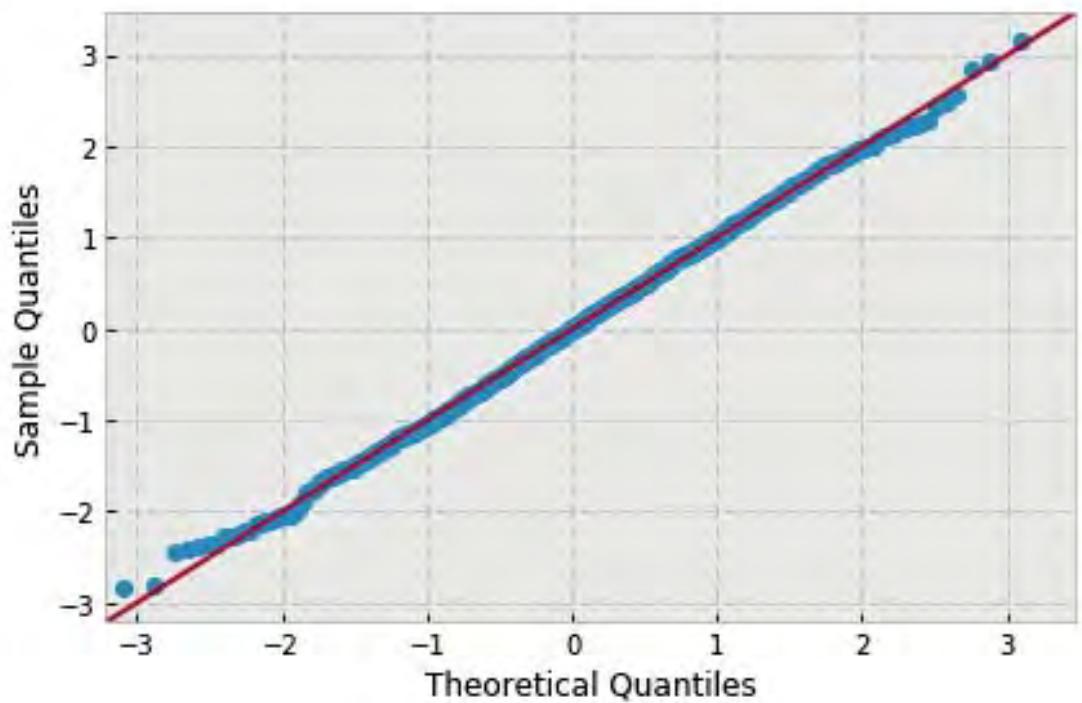


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=101.36042499860415, scale=4.216248093195603)
DescribeResult(nobs=1000, minmax=(89.362093876270606, 117.46755642666183)
               mean=101.36042499860415, variance=17.794542525901456,
               skewness=0.19486114752527411, kurtosis=0.2864522672647576)
```

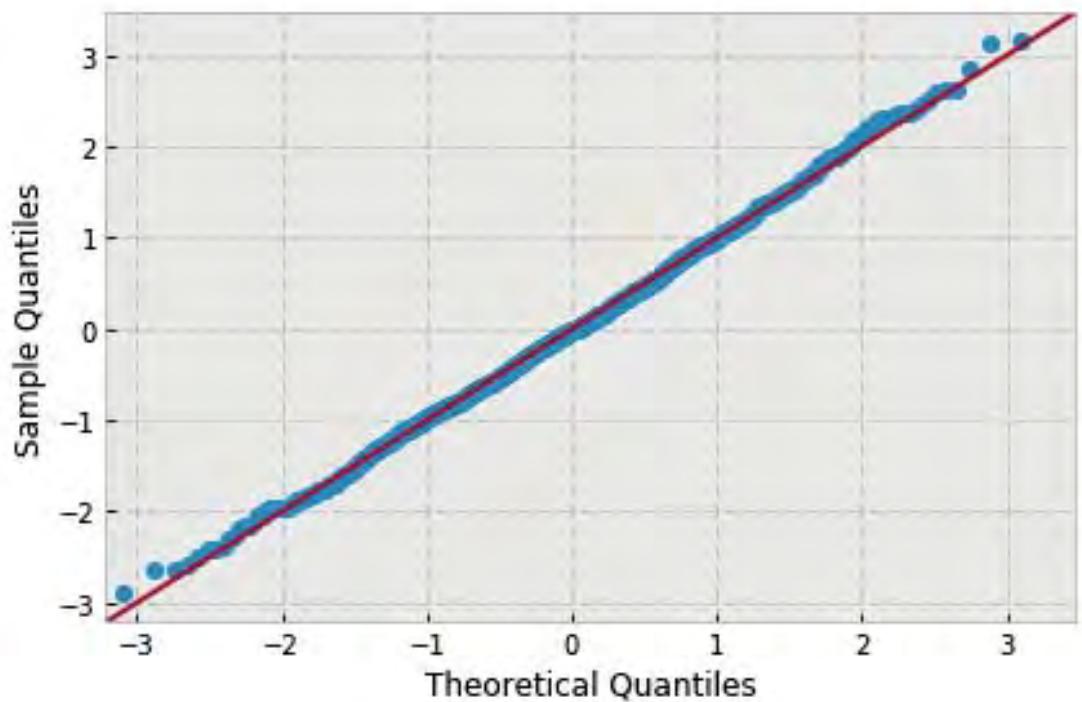
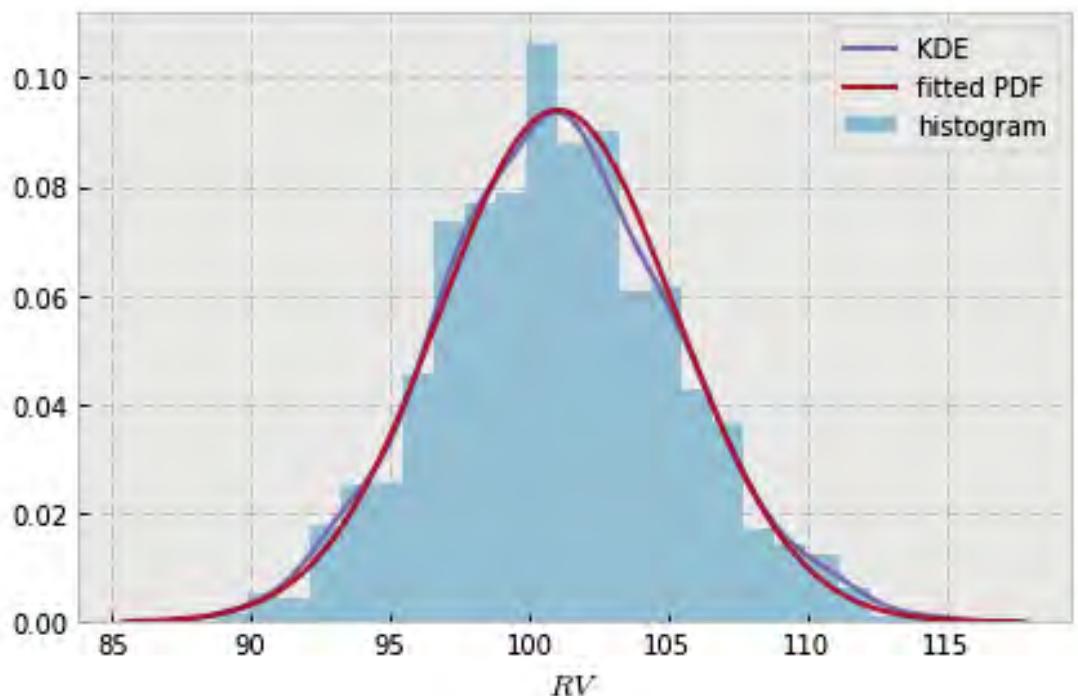


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=101.10475492847465, scale=4.036458862612048)
DescribeResult(nobs=1000, minmax=(89.70034542787877, 113.86671651106936)
               mean=101.10475492847465, variance=16.309309459018369,
               skewness=0.025815977181383388, kurtosis=-0.26617151692700336)
```

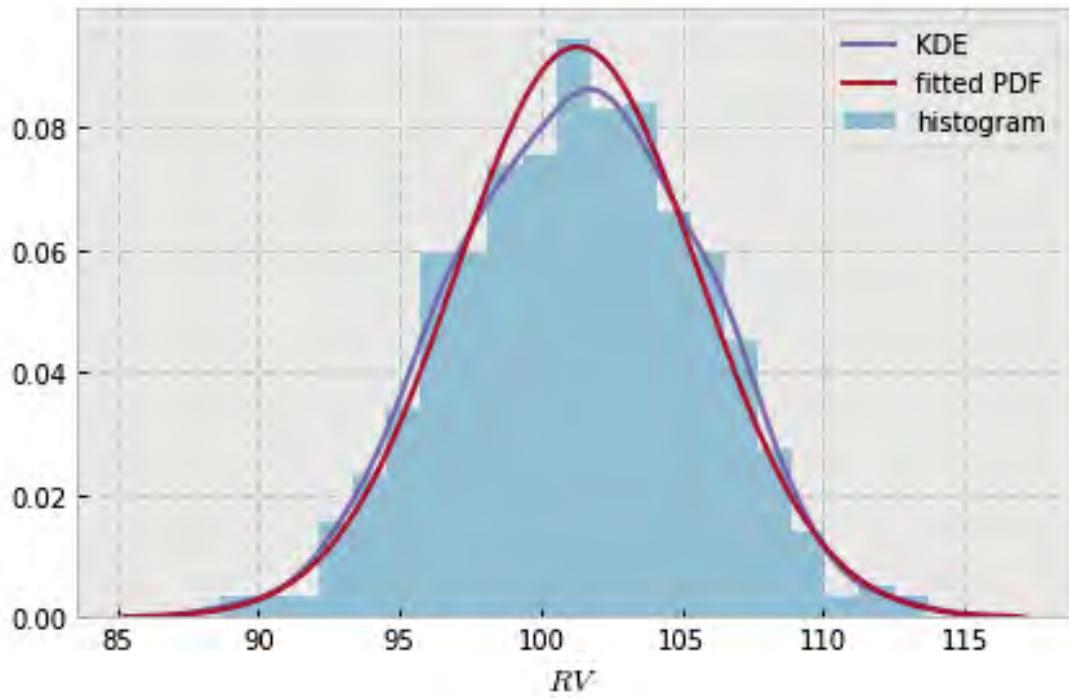


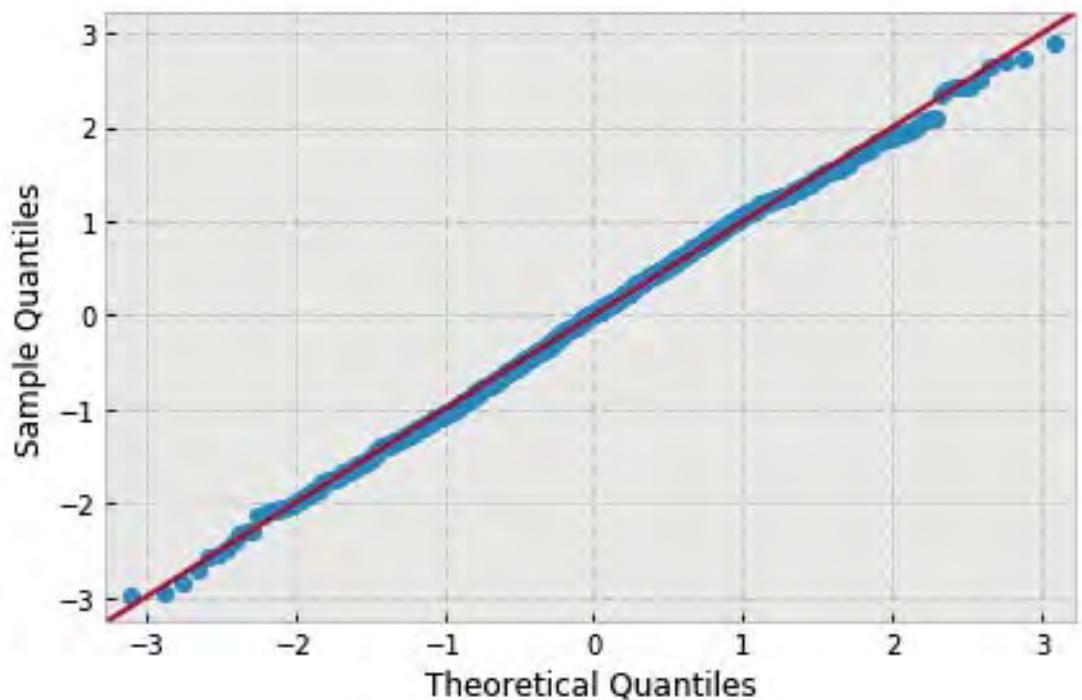


```
variable name: sigma
variable value: 0.06666666666666667
distribution: normal(shape=(), loc=101.04374162580568, scale=4.240211480936028)
DescribeResult(nobs=1000, minmax=(88.764859185756364, 114.45022652565935)
               mean=101.04374162580568, variance=17.997390793855558,
               skewness=0.11229308556448422, kurtosis=-0.06549810375534326)
```

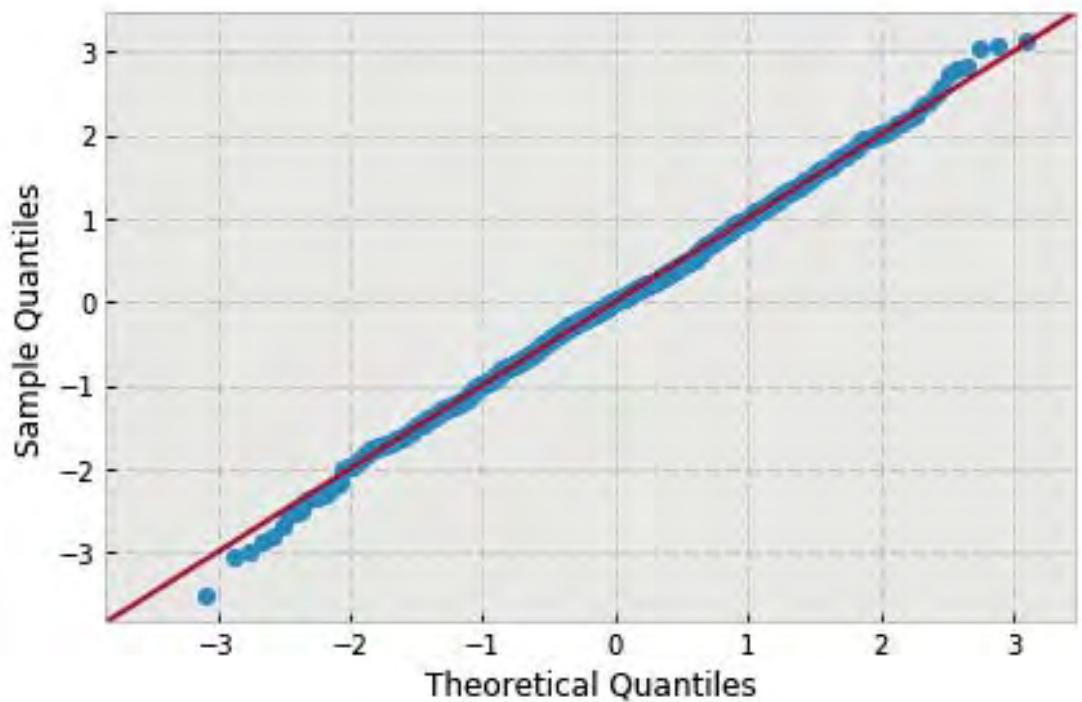
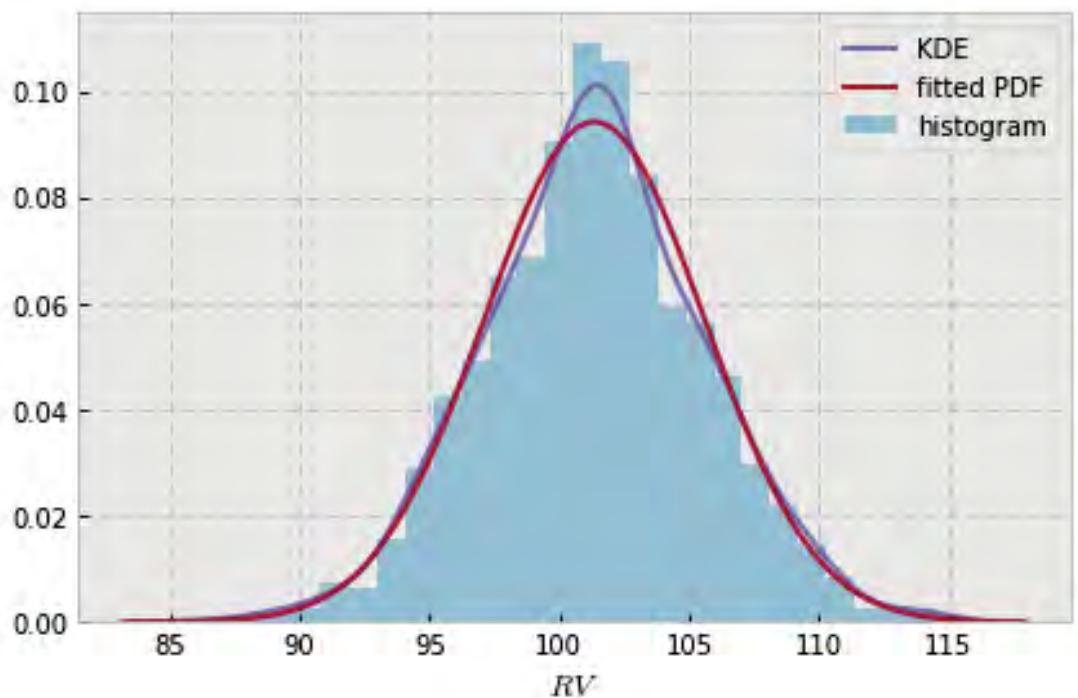


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=101.30625903436065, scale=4.283434899322953)
DescribeResult(nobs=1000, minmax=(88.581668779665293, 113.67605820933456)
               mean=101.30625903436065, variance=18.366180717455286,
               skewness=-0.05666330864672879, kurtosis=-0.29819600908548693)
```

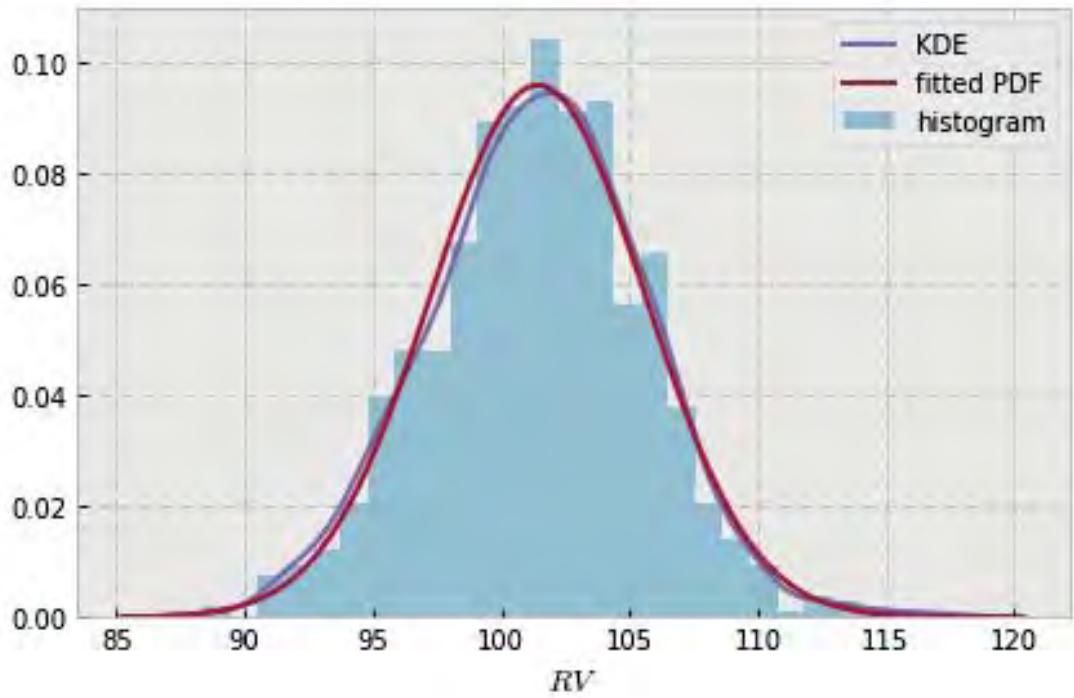


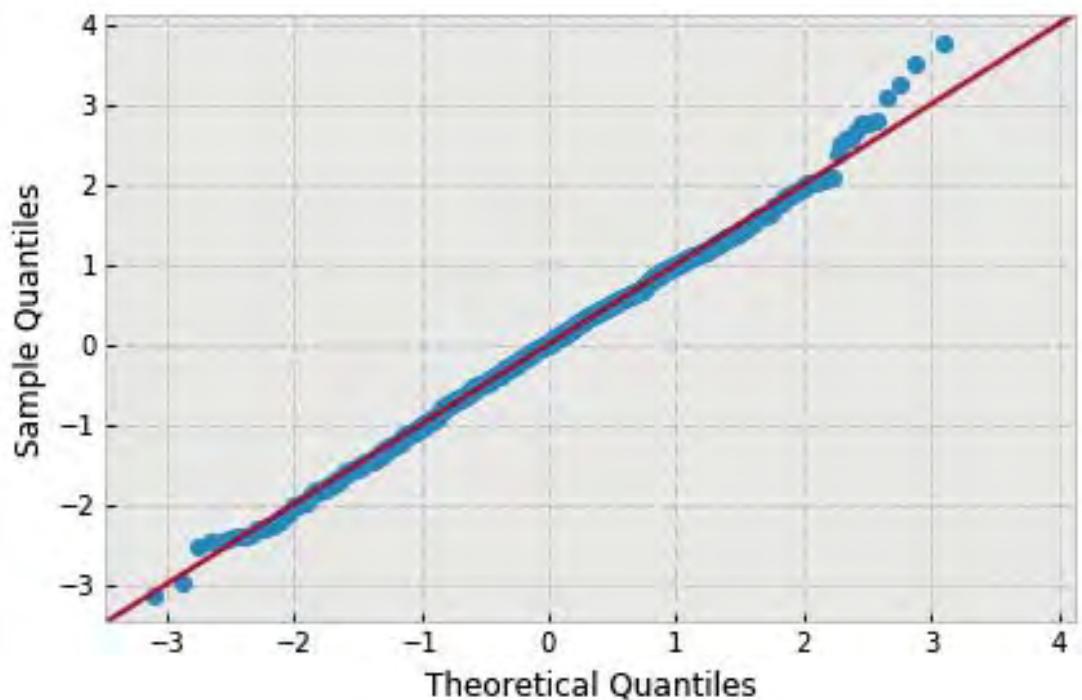


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=101.3469278489566, scale=4.235837715108321)
DescribeResult(nobs=1000, minmax=(86.496020630764406, 114.59832602558187)
               mean=101.3469278489566, variance=17.960281430164244,
               skewness=-0.0022627737350483287, kurtosis=0.16610093657567937)
```

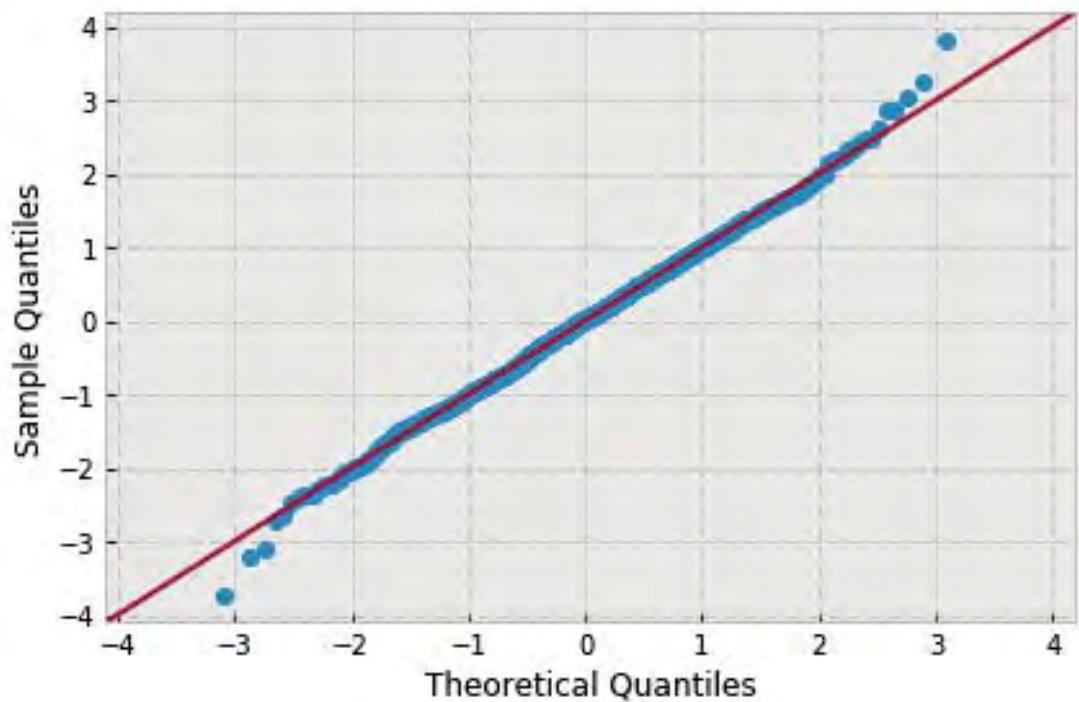
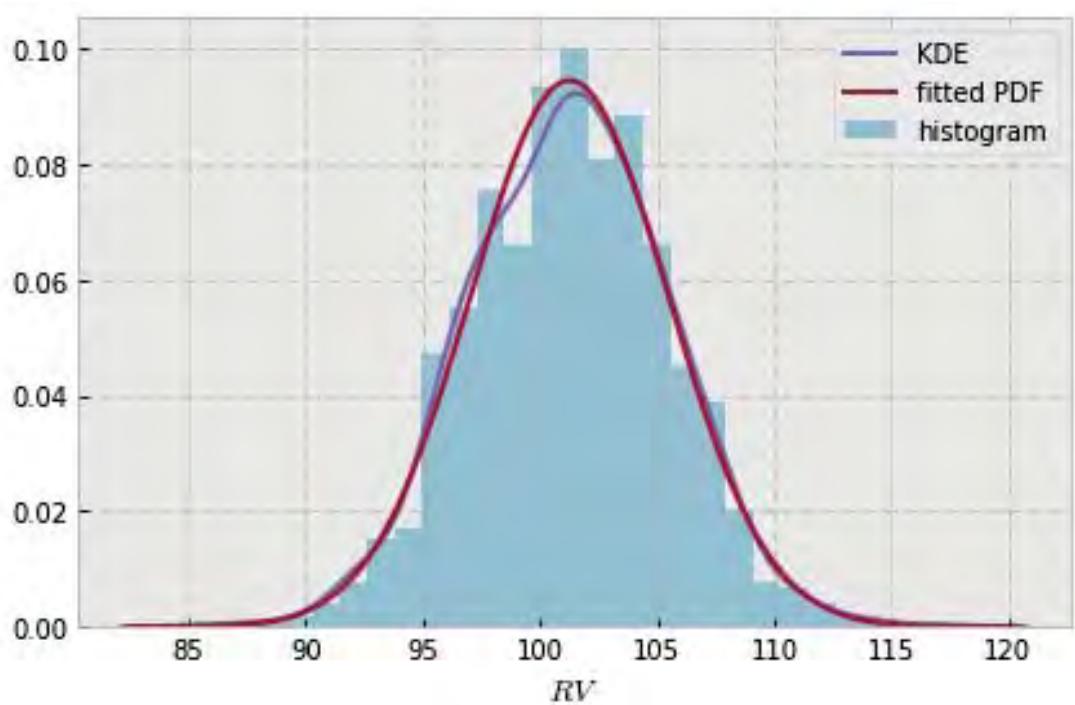


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=101.4306885448615, scale=4.166512139702129)
DescribeResult(nobs=1000, minmax=(88.404103473282419, 117.16442534304852)
               mean=101.4306885448615, variance=17.377200610896107,
               skewness=0.018898887632125792, kurtosis=0.20353049672607648)
```

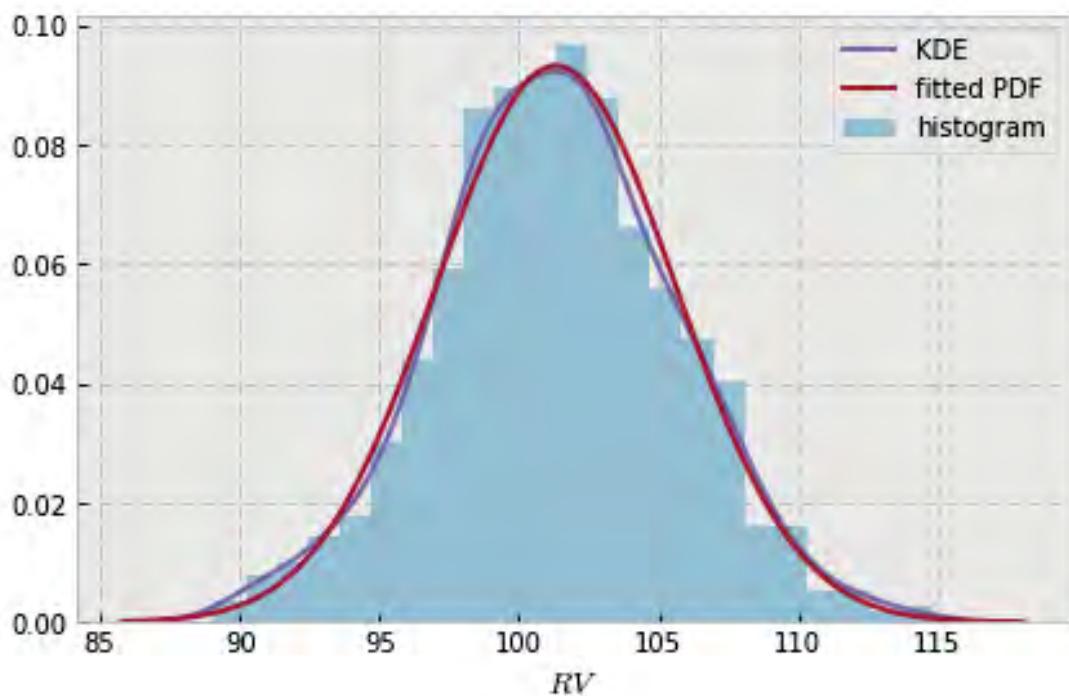


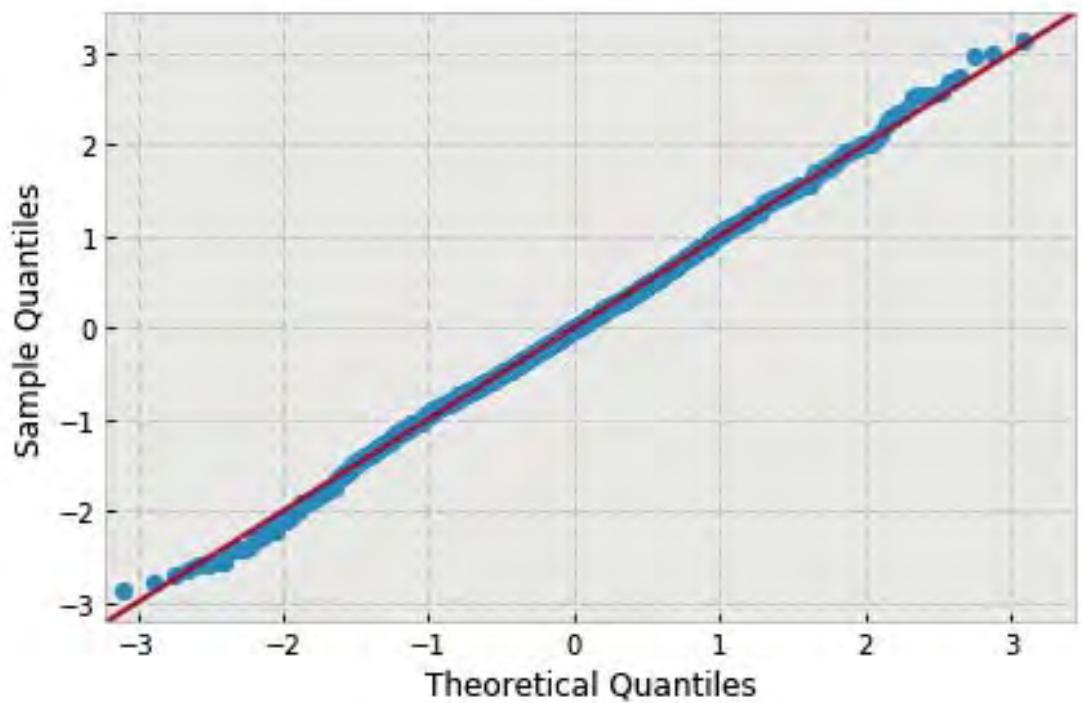


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=101.26700483058498, scale=4.220533390856076)
DescribeResult(nobs=1000, minmax=(85.559027579456242, 117.32641967356341)
               mean=101.26700483058498, variance=17.830732836167254,
               skewness=-0.0004824218991789193, kurtosis=0.19975426033022048)
```

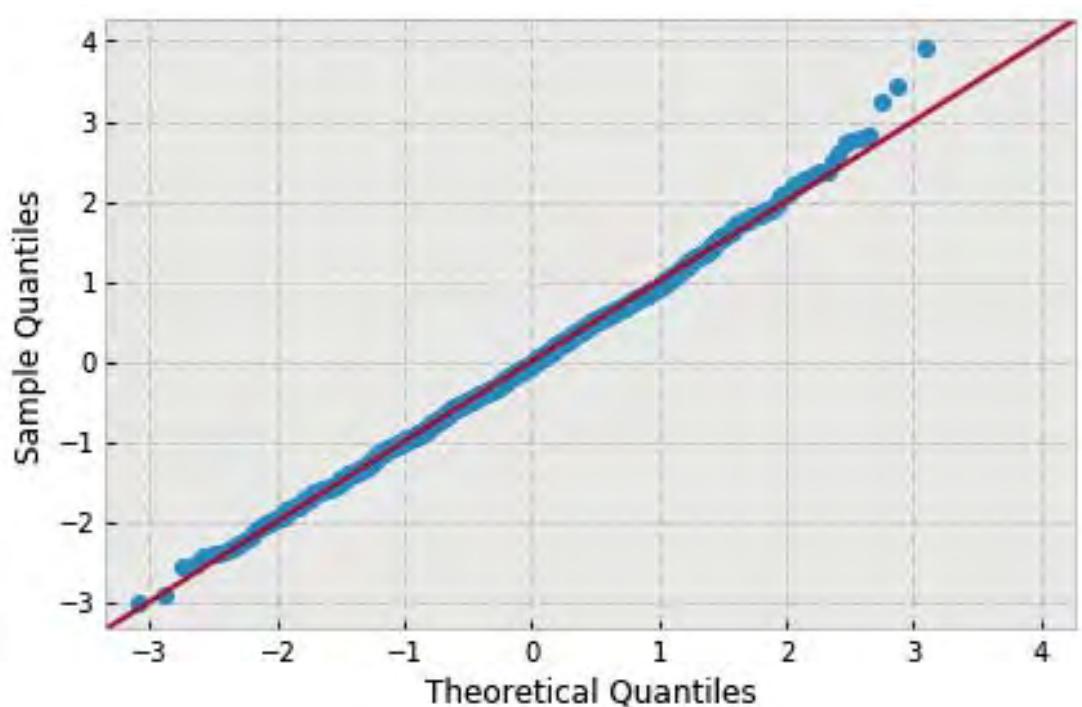
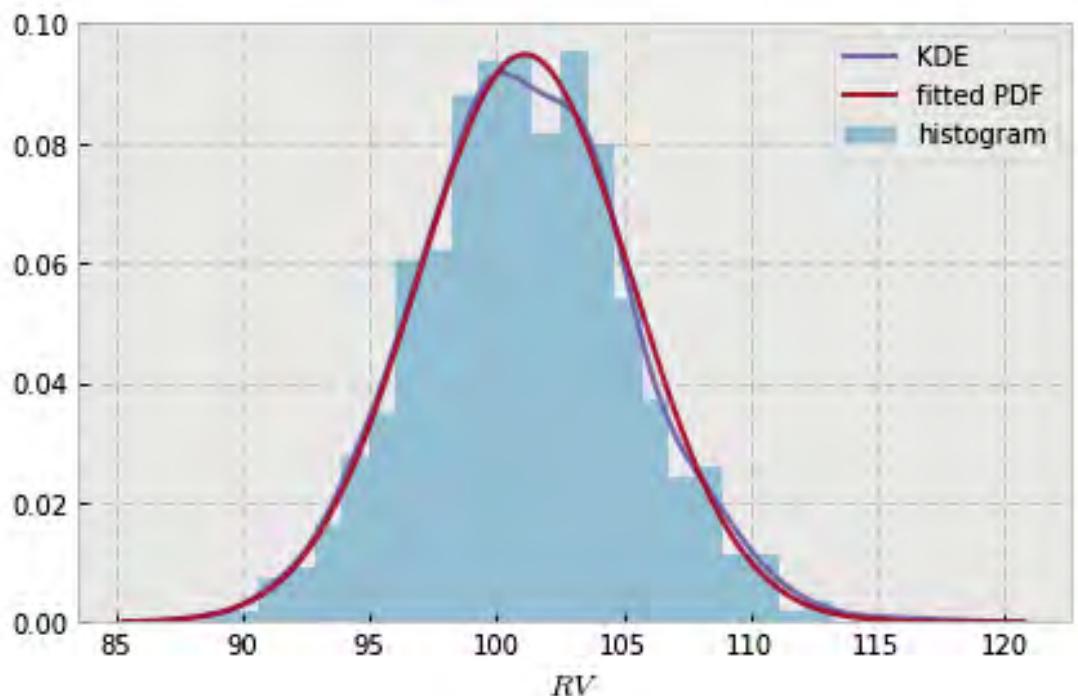


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=101.31719607488004, scale=4.279947104668575)
DescribeResult(nobs=1000, minmax=(89.099129471082122, 114.73472667518823)
               mean=101.31719607488004, variance=18.336283502263186,
               skewness=0.023578016683336506, kurtosis=0.05961327422201146)
```

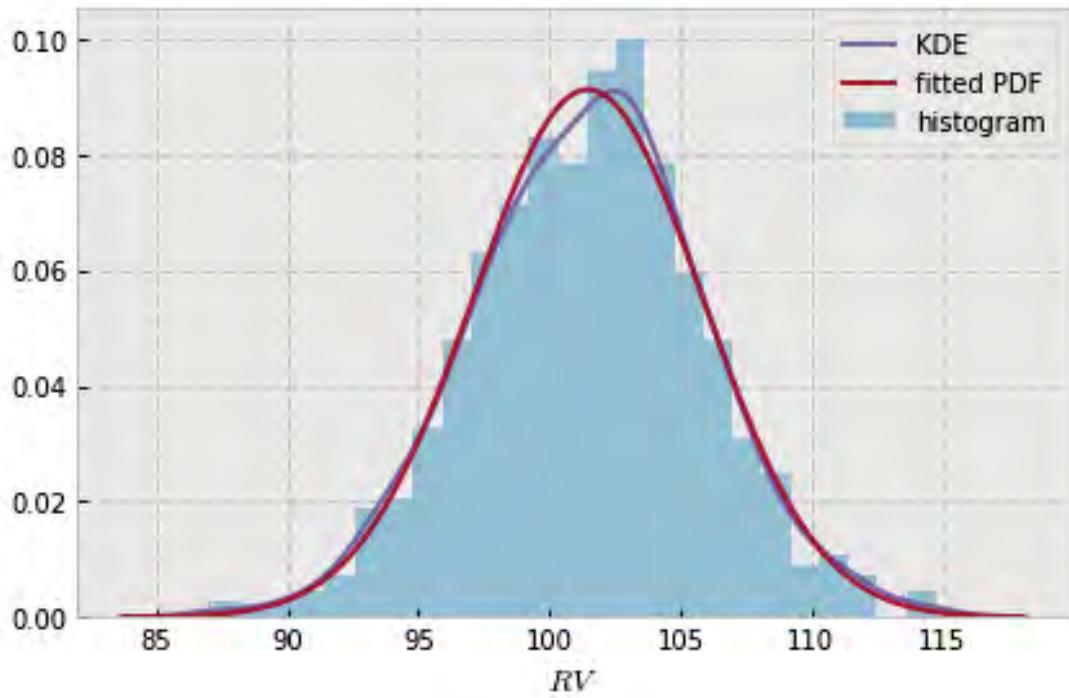


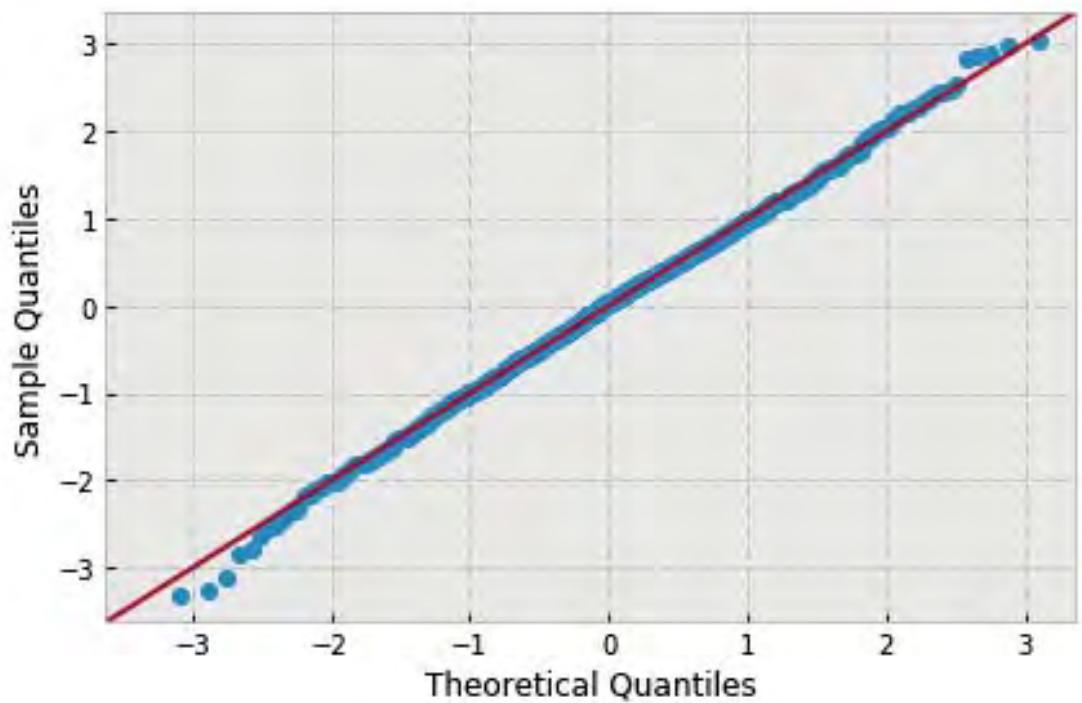


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=101.11109817830997, scale=4.20587812064204)
DescribeResult(nobs=1000, minmax=(88.494915815230939, 117.57777636392383)
               mean=101.11109817830997, variance=17.707117883578995,
               skewness=0.16360724552260164, kurtosis=0.18969079403273303)
```

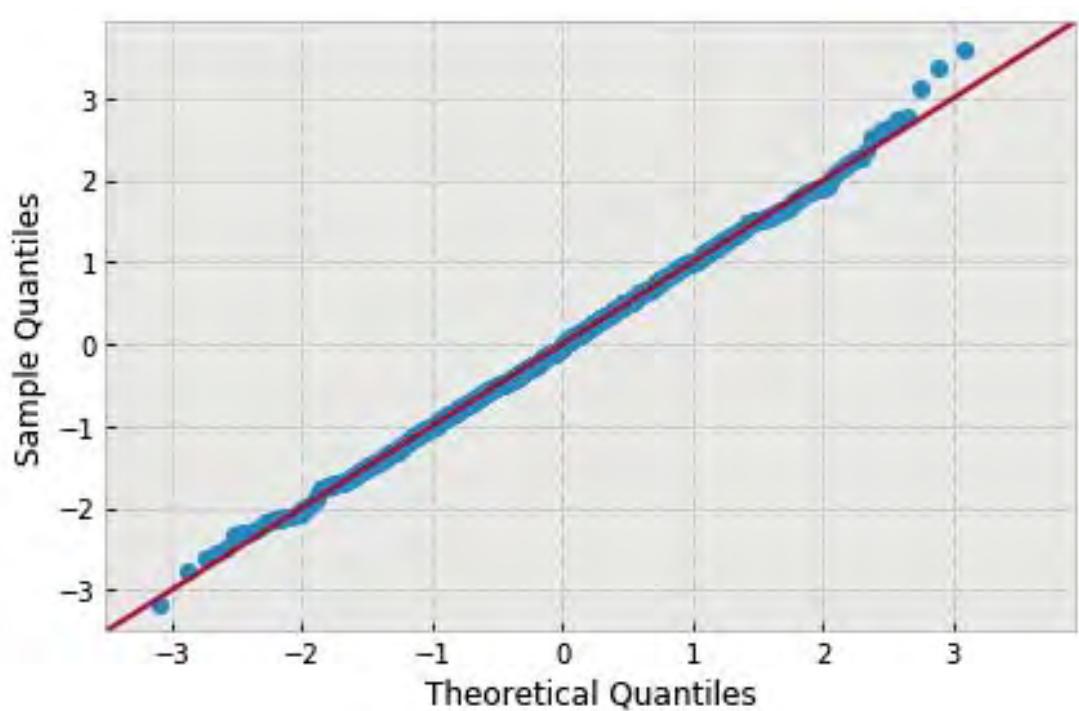
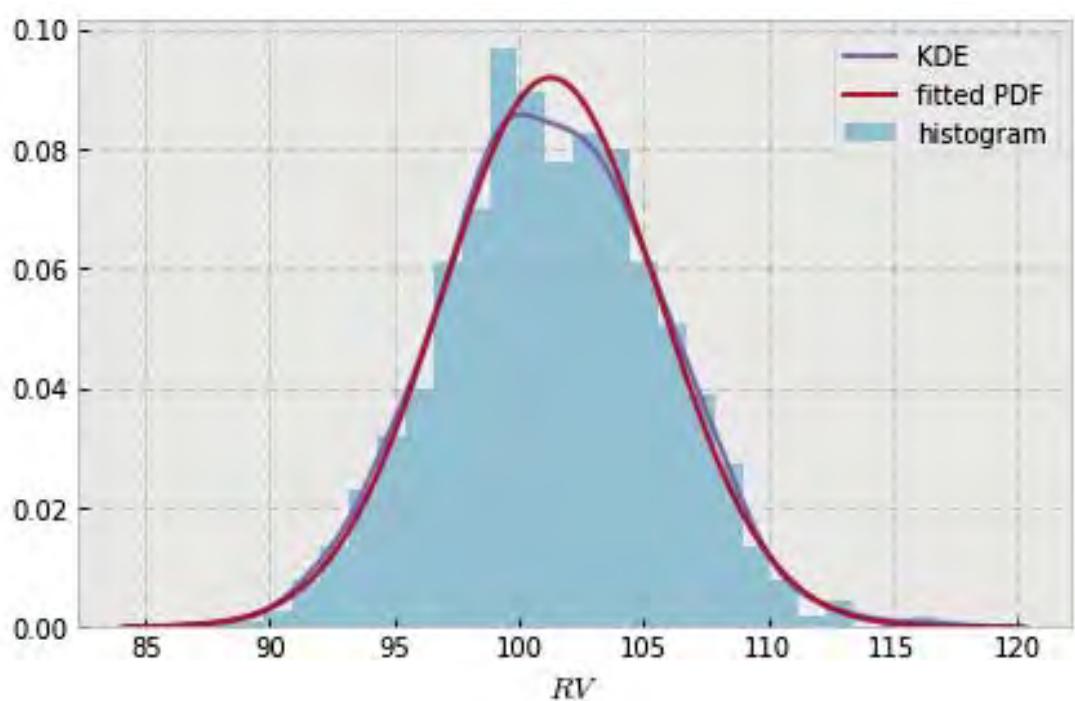


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=101.46877491199076, scale=4.373403981438741)
DescribeResult(nobs=1000, minmax=(87.025177575444474, 114.73930717480692)
               mean=101.46877491199076, variance=19.145808193057288,
               skewness=-0.04895075429293916, kurtosis=0.12665858615513503)
```

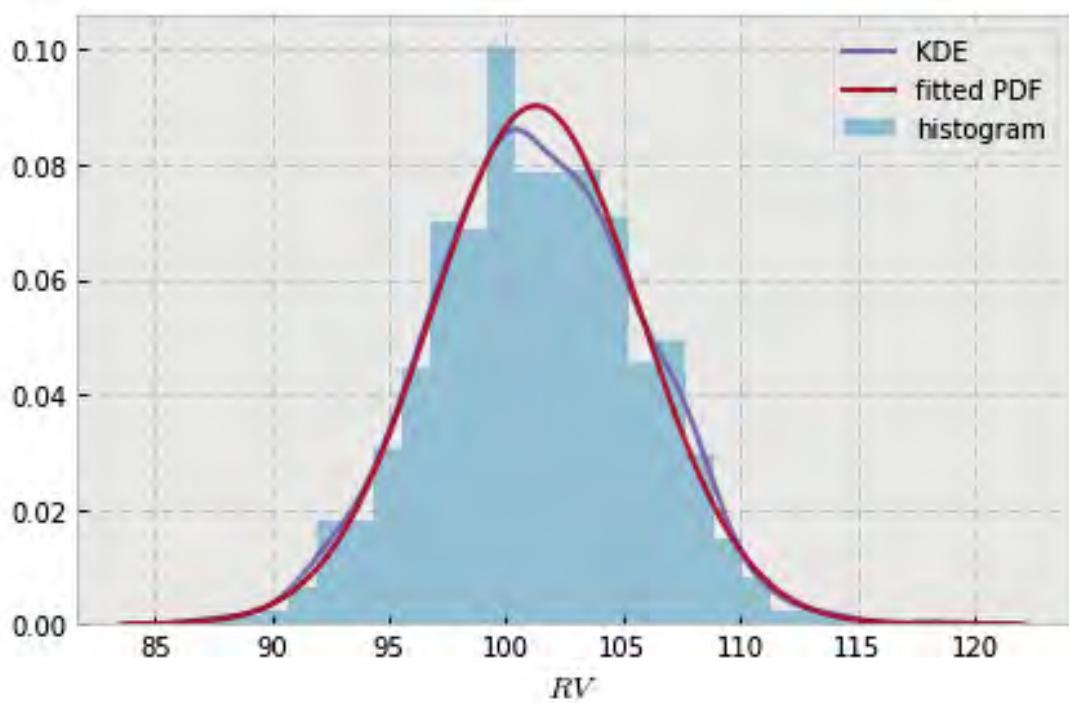


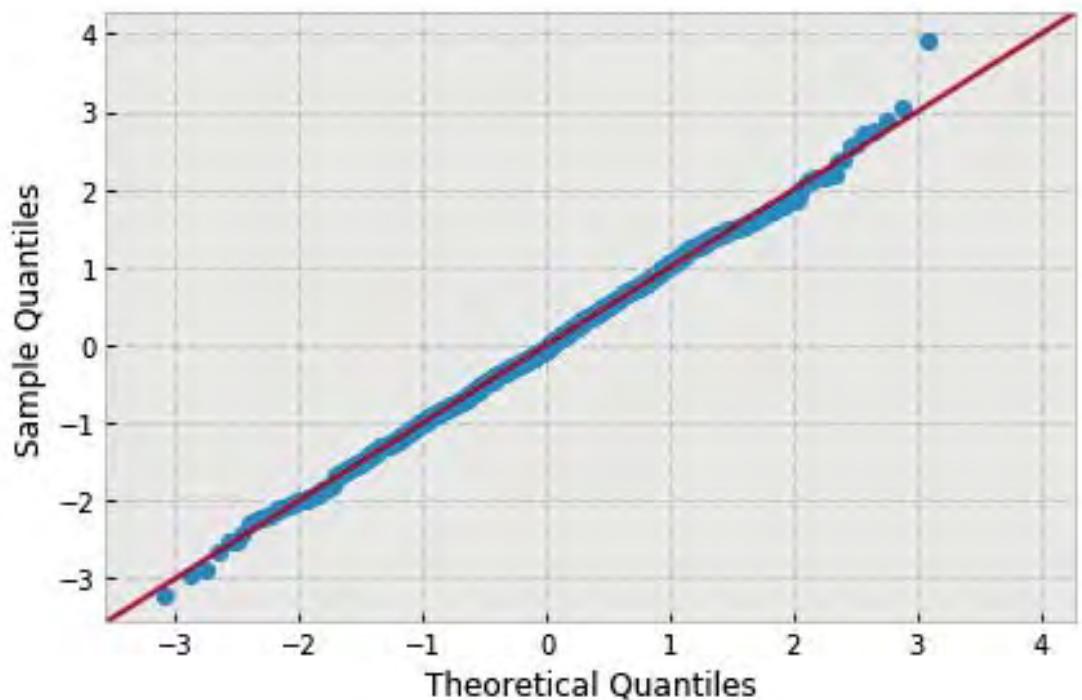


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=101.26713668490784, scale=4.341668139744876)
DescribeResult(nobs=1000, minmax=(87.529079044703707, 116.85791013392898)
               mean=101.26713668490784, variance=18.868951186862599,
               skewness=0.07734547651421207, kurtosis=-0.03297634785440229)
```

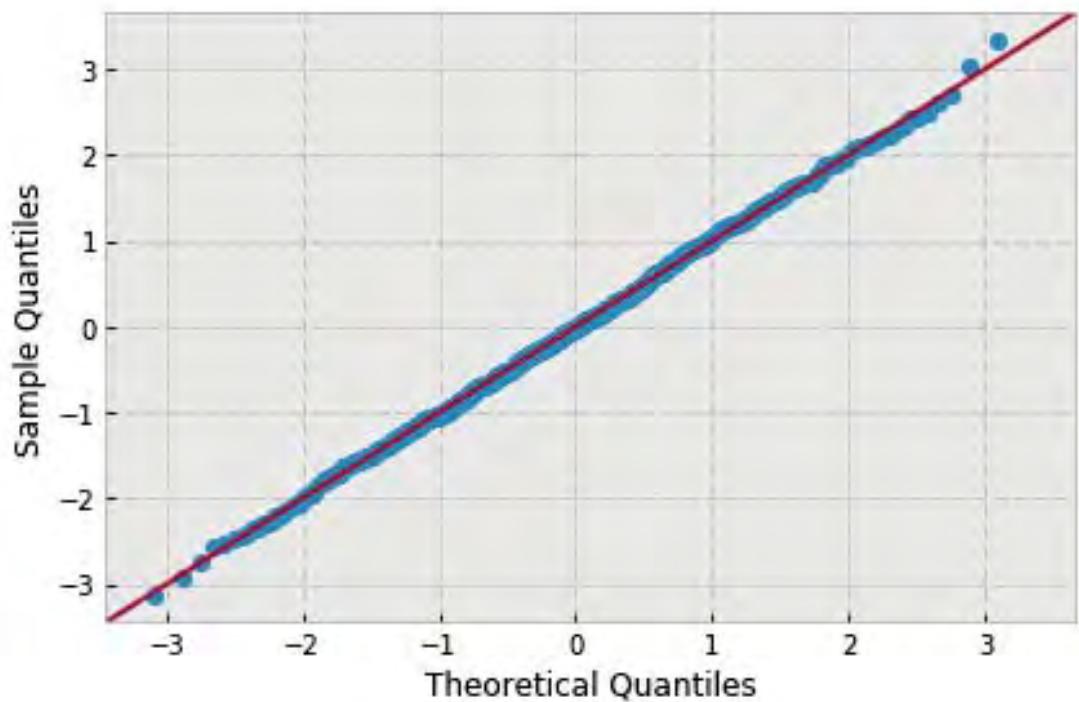
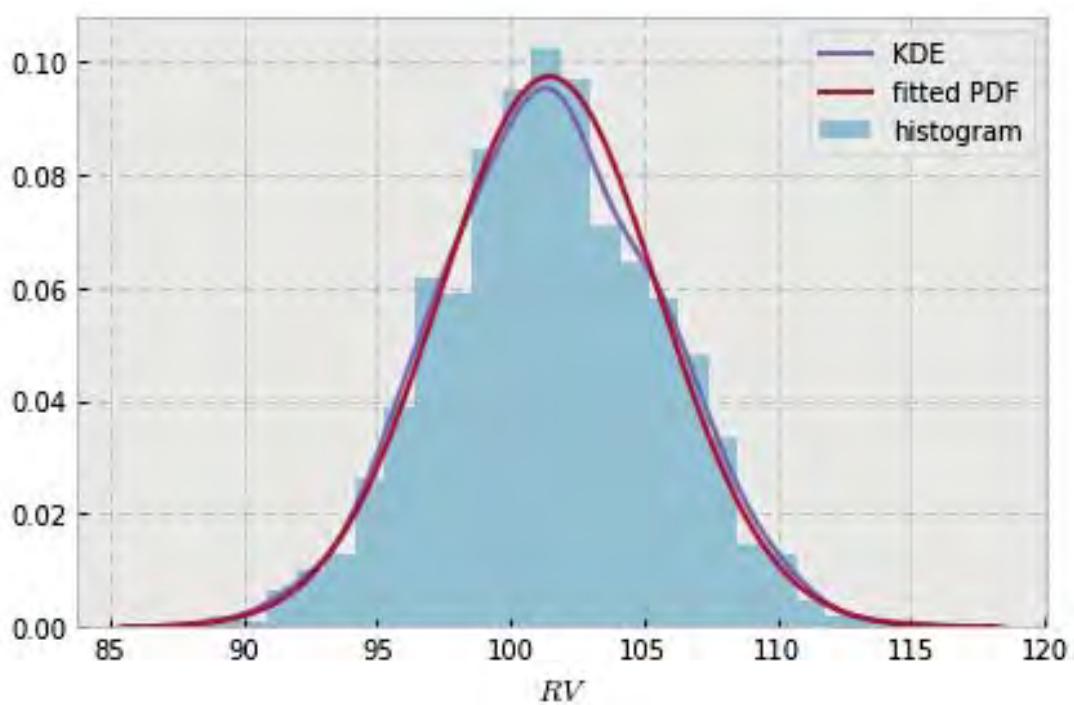


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=101.26882378436302, scale=4.421768385919707)
DescribeResult(nobs=1000, minmax=(87.099568889349541, 118.57088716470518)
               mean=101.26882378436302, variance=19.571607265984952,
               skewness=0.029202262480166837, kurtosis=-0.019401716852288864)
```

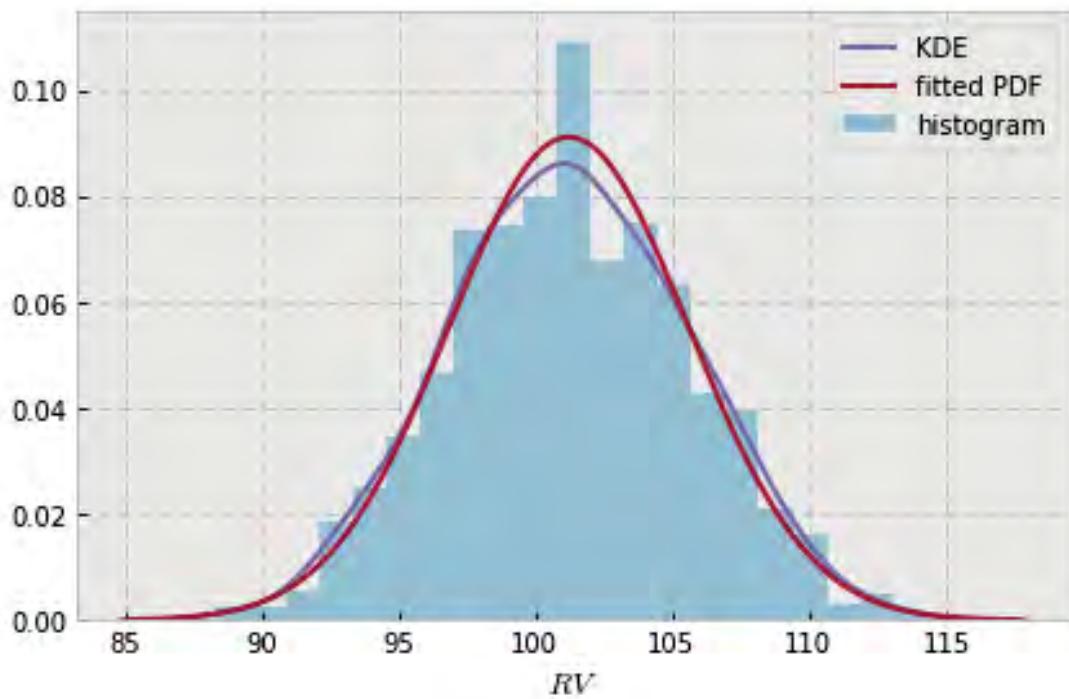


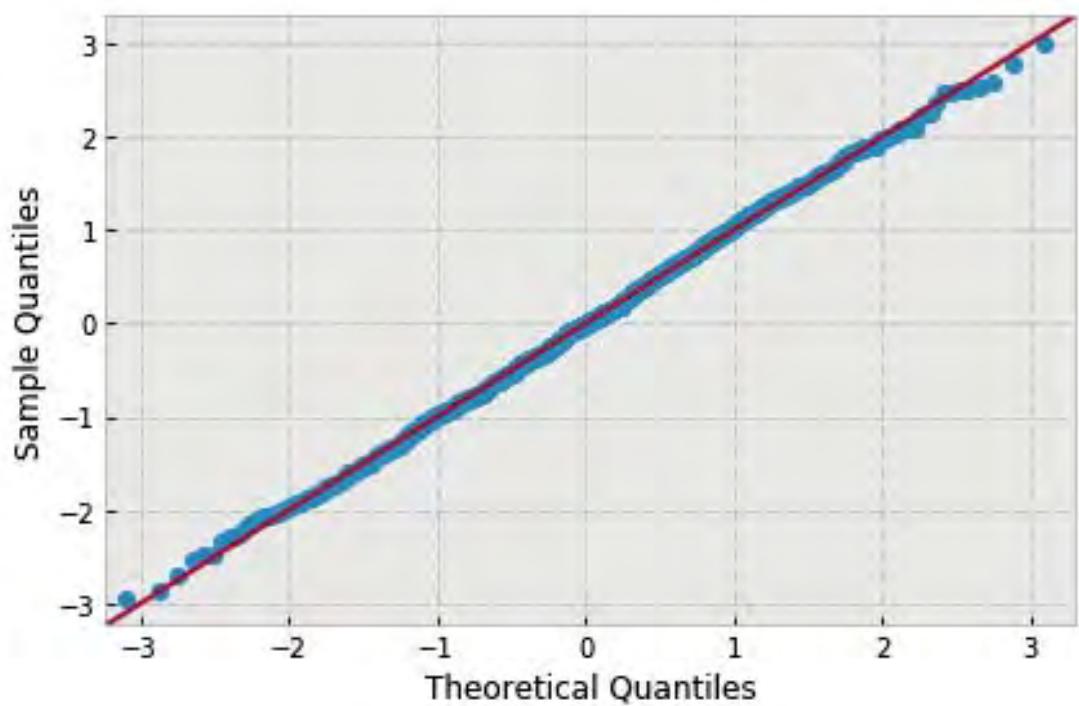


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=101.45986107843298, scale=4.101883603390274)
DescribeResult(nobs=1000, minmax=(88.658835936885609, 115.1229900260665)
               mean=101.45986107843298, variance=16.842291387149128,
               skewness=0.02739713245525337, kurtosis=-0.1427706011975216)
```

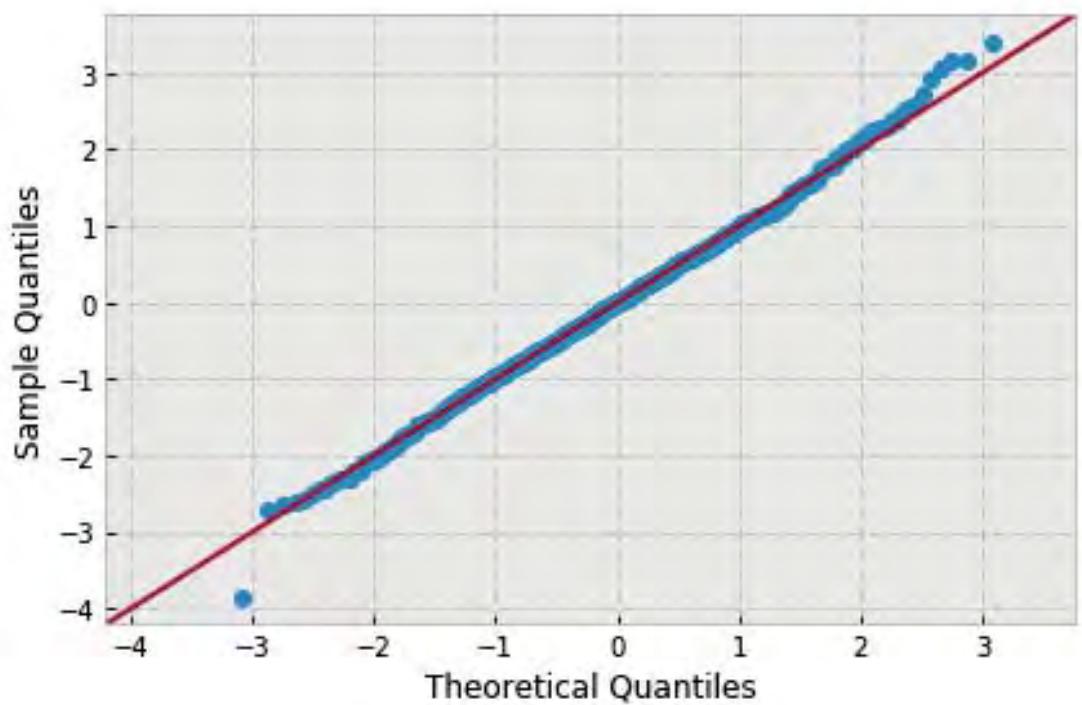
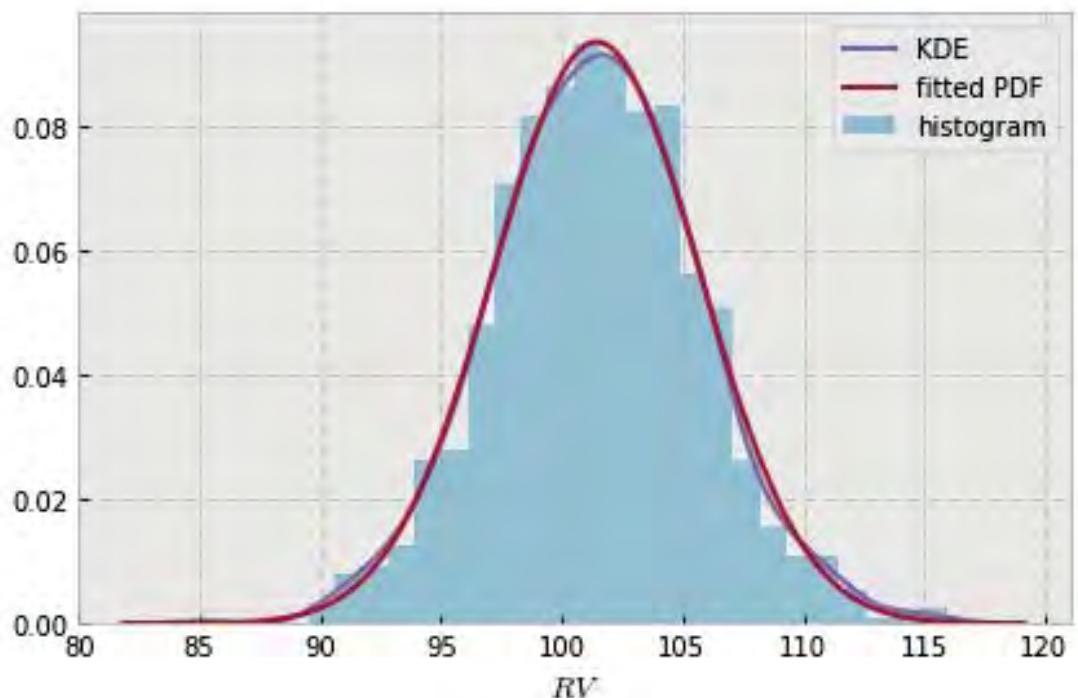


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=101.22468672827206, scale=4.3798707509707455)
DescribeResult(nobs=1000, minmax=(88.381106613273914, 114.3456593868941)
               mean=101.22468672827206, variance=19.202470265474521,
               skewness=0.017228907875422847, kurtosis=-0.27317982269604935)
```

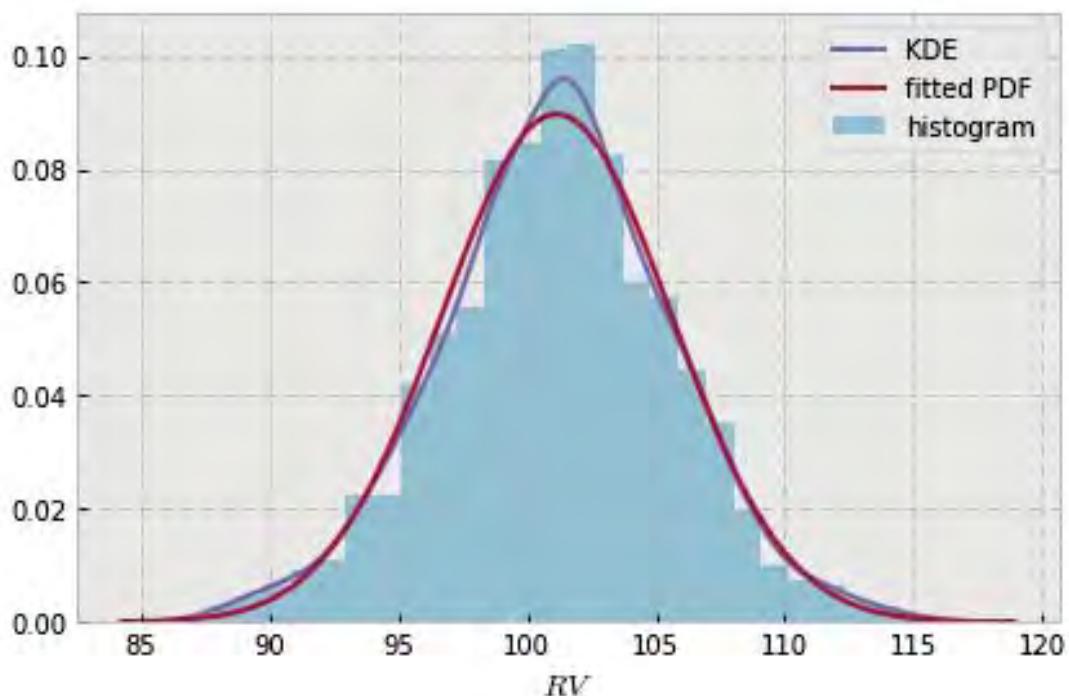


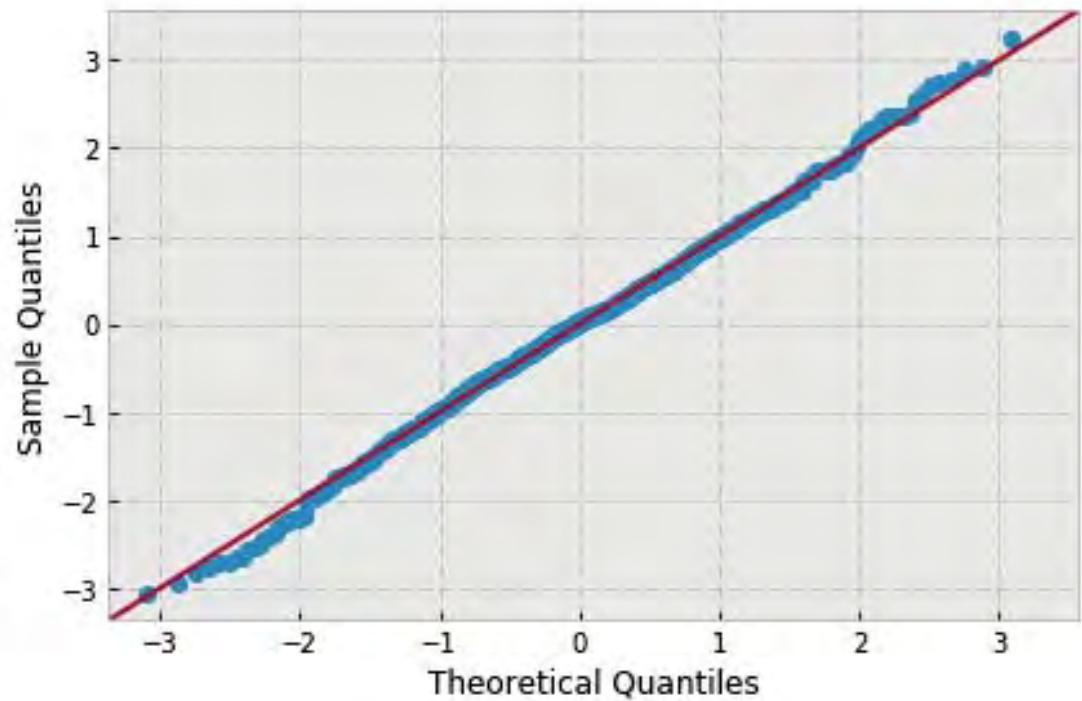


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=101.44034016581446, scale=4.261105842786701)
DescribeResult(nobs=1000, minmax=(85.076870460427472, 115.89494082527349)
               mean=101.44034016581446, variance=18.175198201632593,
               skewness=0.05543401983927169, kurtosis=0.2529277754504835)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=101.13529450620068, scale=4.447145818722859)
DescribeResult(nobs=1000, minmax=(87.581090903341618, 115.55236257119448)
               mean=101.13529450620068, variance=19.796902835820028,
               skewness=-0.04768215305289694, kurtosis=0.2032617143713189)
```

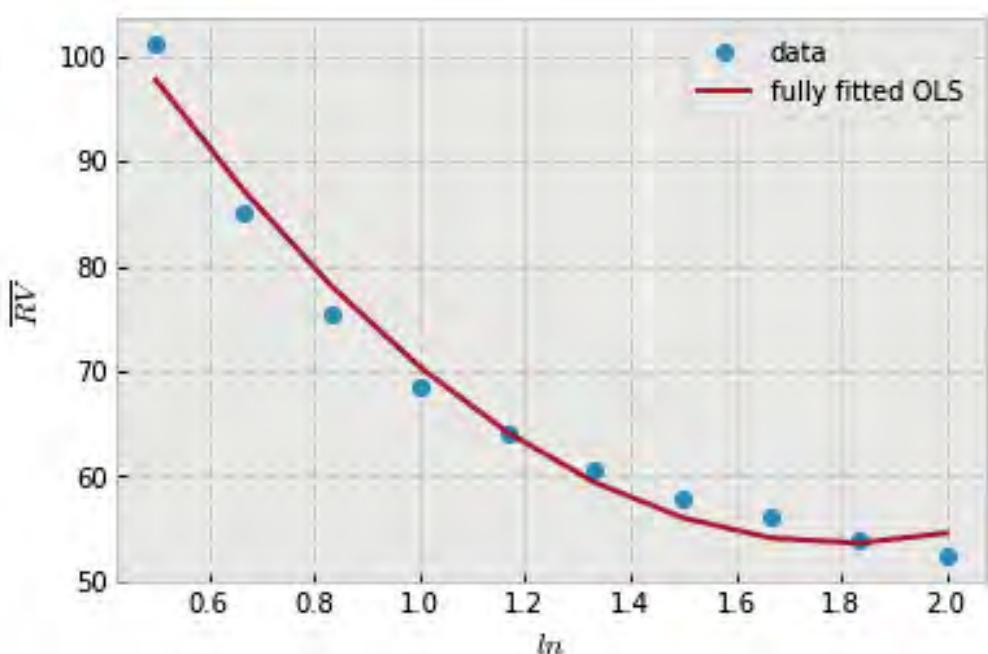




7.2 Parameter regression

```
In [20]: var_name = 'total_root_volume'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly1')  
        mean_reg('nB', var_name, 'poly1')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly1')  
        mean_reg('N', var_name, 'poly1')
```

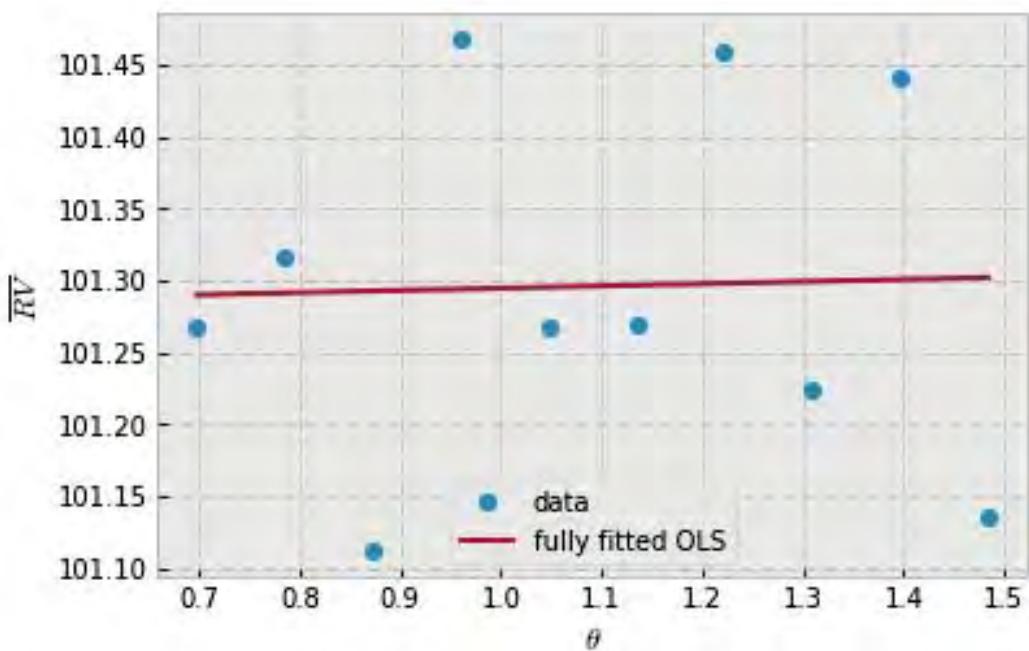
```
variable name: ln  
model: poly2  
r-squared: 0.9290524592320429  
explained variance: 0.9291569558885409
```



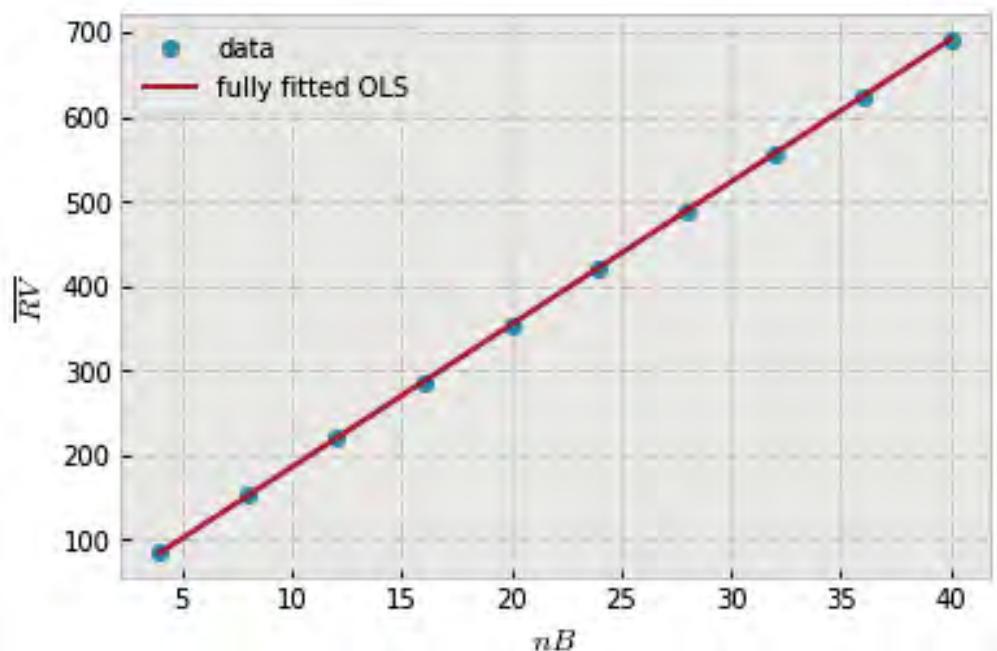
```

variable name: theta
model: poly1
r-squared: -0.6152367594290664
explained variance: -0.6129705364384297

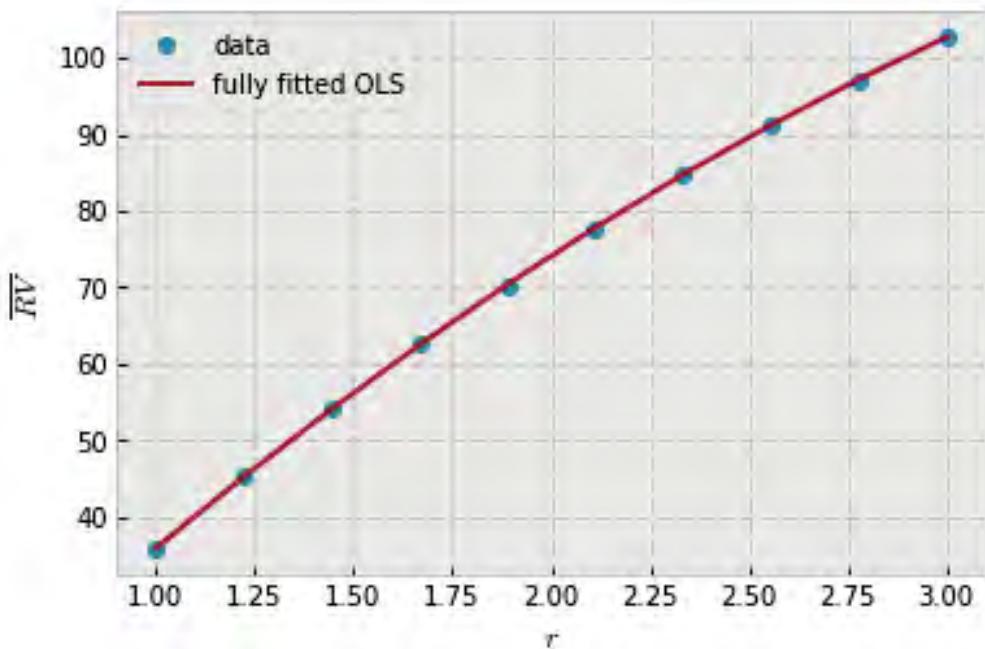
```



```
variable name: nB
model: poly1
r-squared: 0.9999991382596531
explained variance: 0.9999991408489118
```



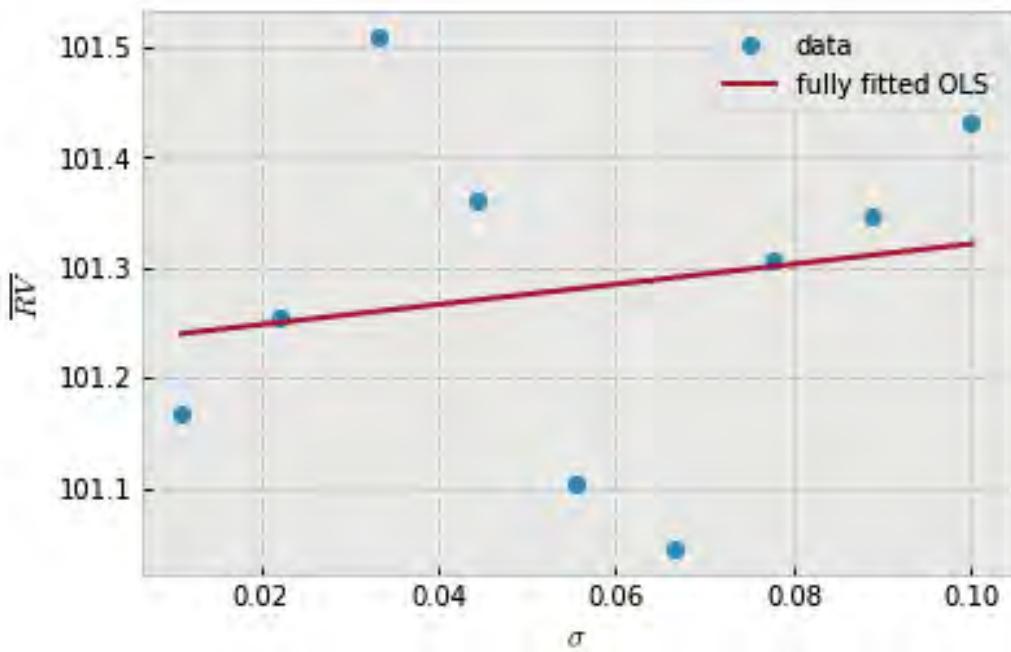
```
variable name: r
model: poly2
r-squared: 0.9998644129208107
explained variance: 0.99986469258683
```



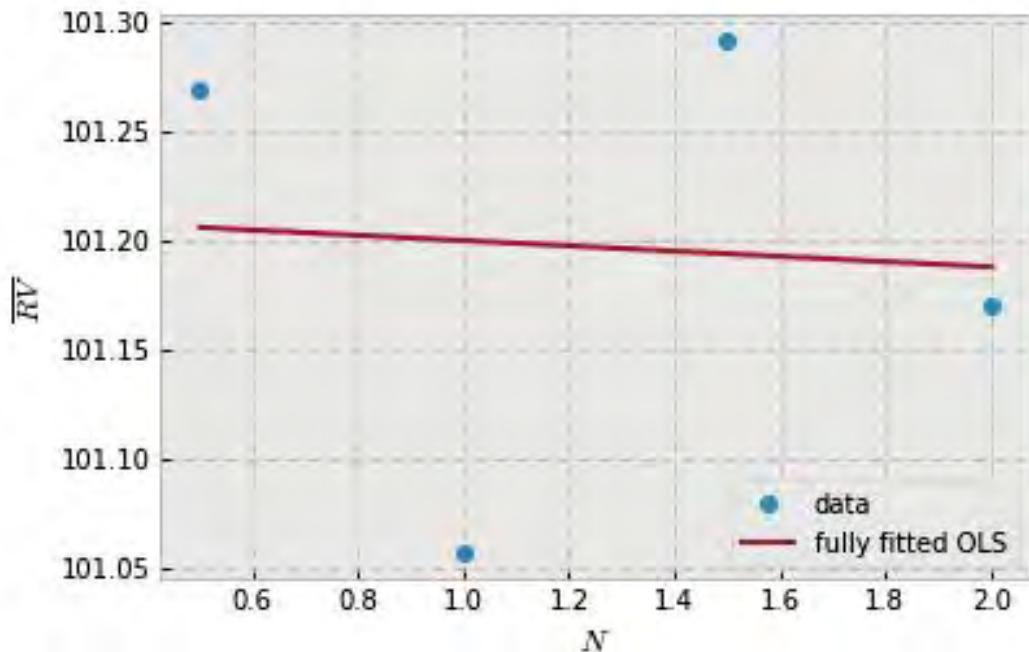
```

variable name: sigma
model: poly1
r-squared: -0.4384121481657073
explained variance: -0.43710351286484106

```



```
variable name: N
model: poly1
r-squared: -2.148171967324859
explained variance: -2.094223040445508
```

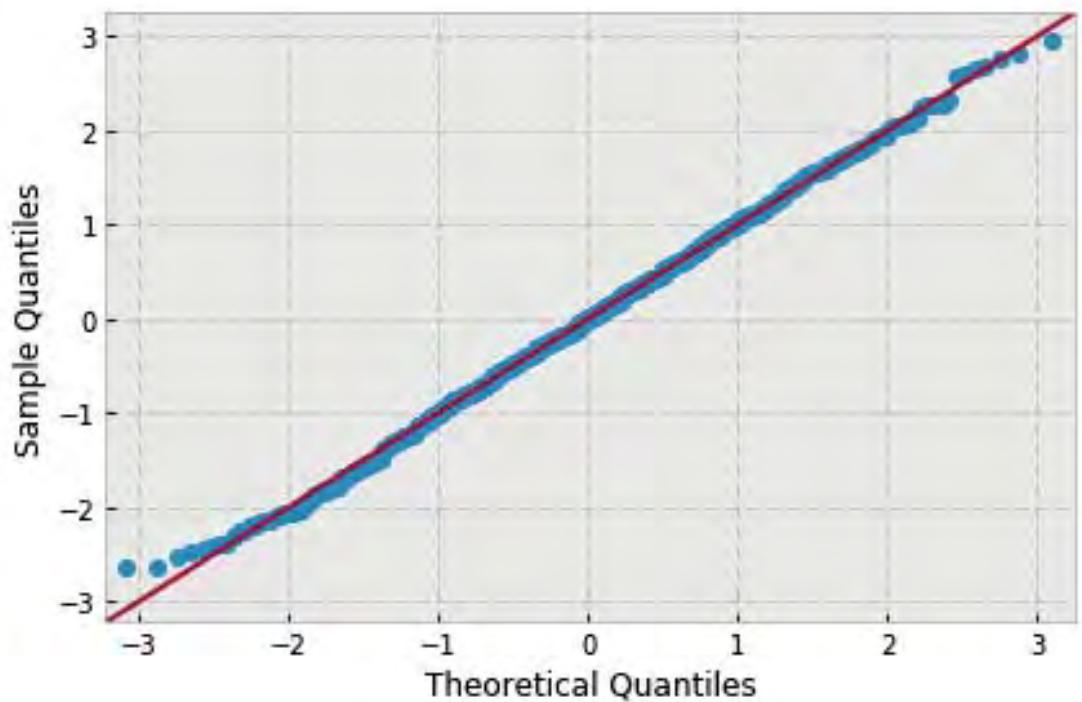
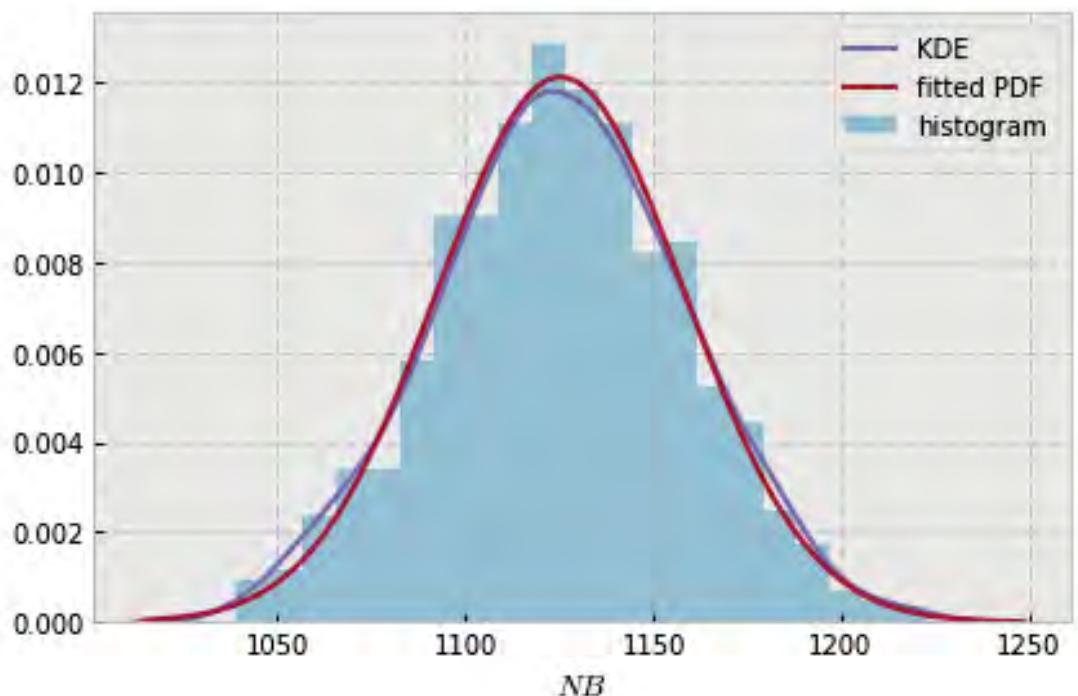


8 Number of root branches

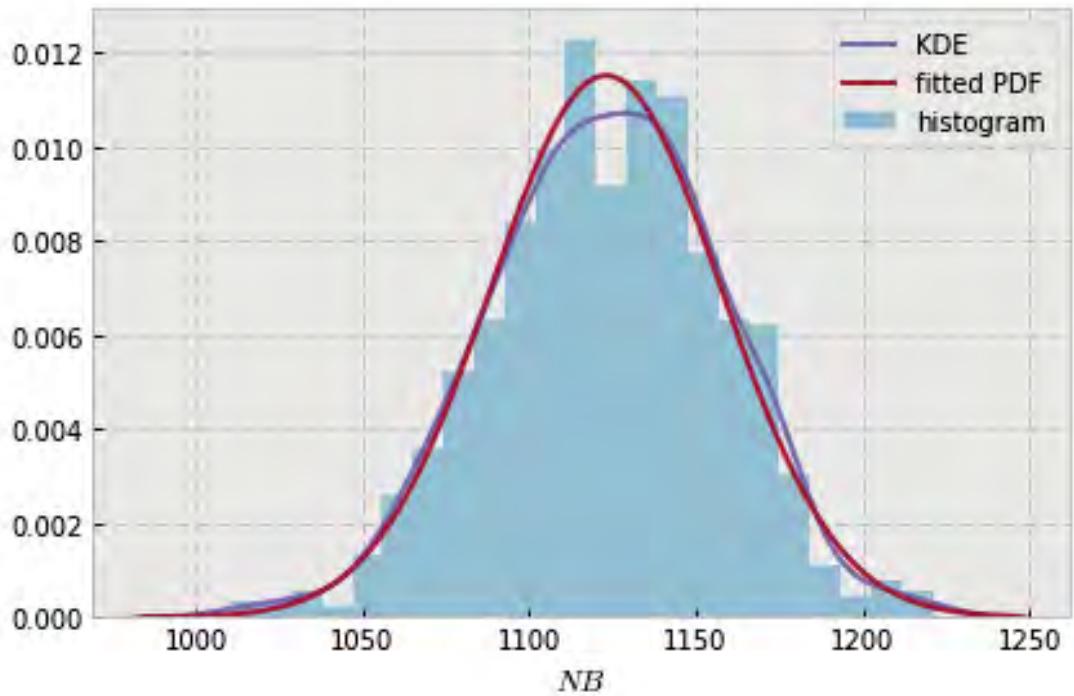
8.1 Probability distributions

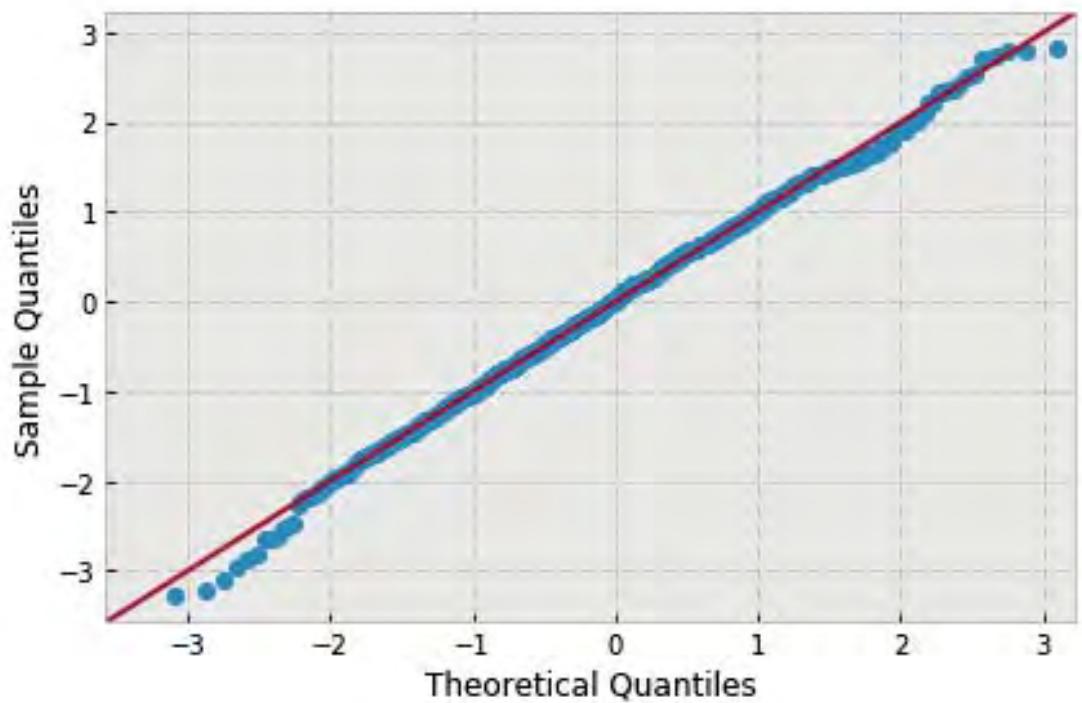
```
In [21]: plot_dist('number_of_root_branches')

variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=1125.35, scale=32.922294877483864)
DescribeResult(nobs=1000, minmax=(1039.0, 1223.0)
    mean=1125.349999999999, variance=1084.9624624624626,
    skewness=0.004017565743661127, kurtosis=-0.20101022395219248)
```

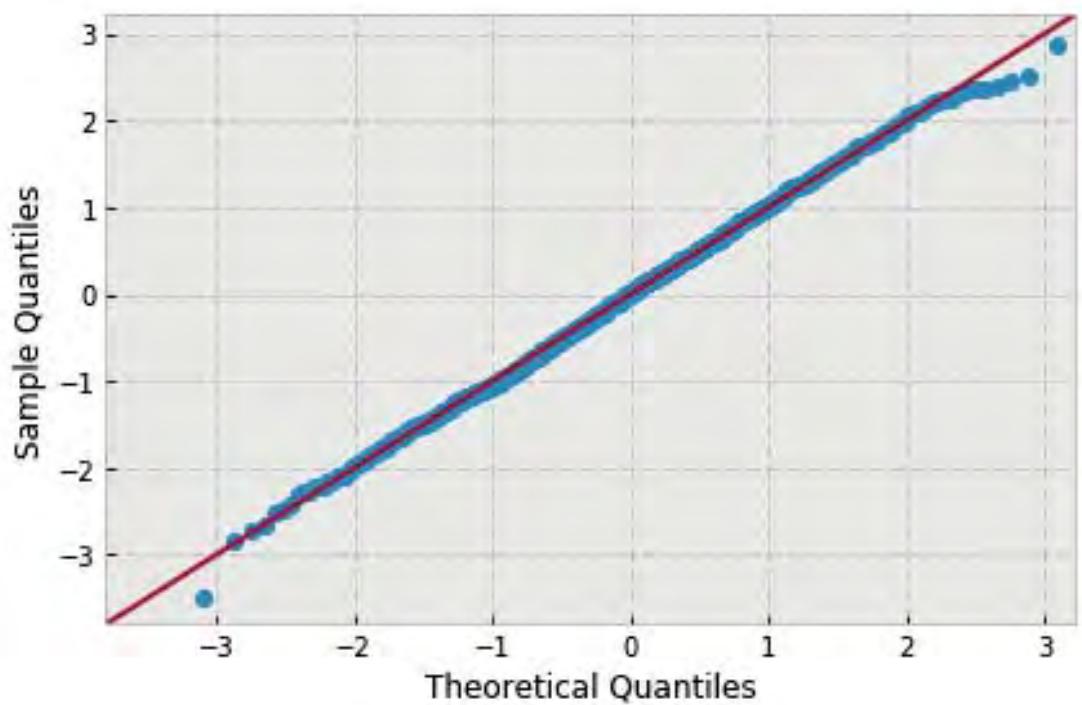
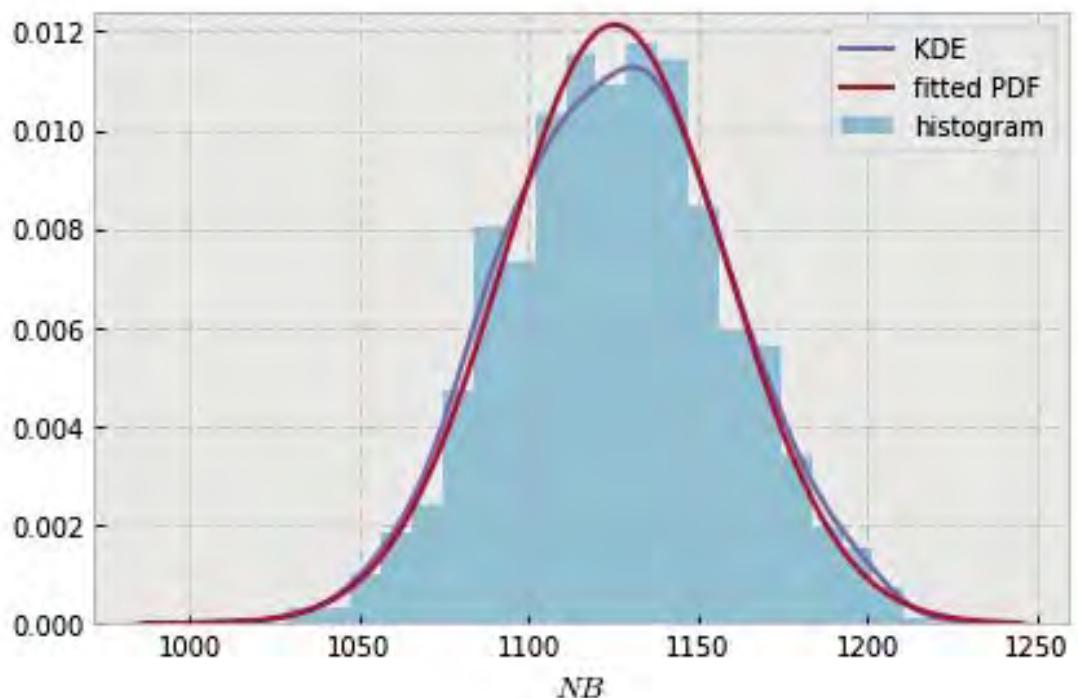


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=1123.051, scale=34.62560322940237)
DescribeResult(nobs=1000, minmax=(1010.0, 1221.0)
    mean=1123.050999999999, variance=1200.1325315315312,
    skewness=-0.11720220180575858, kurtosis=-0.011838820362719993)
```

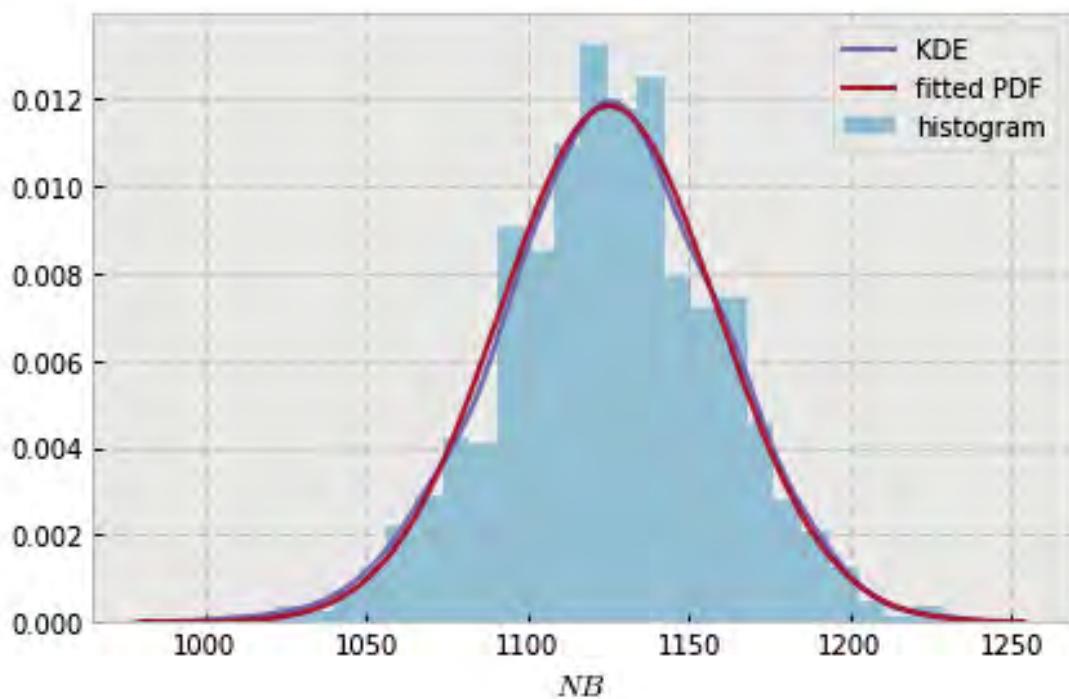


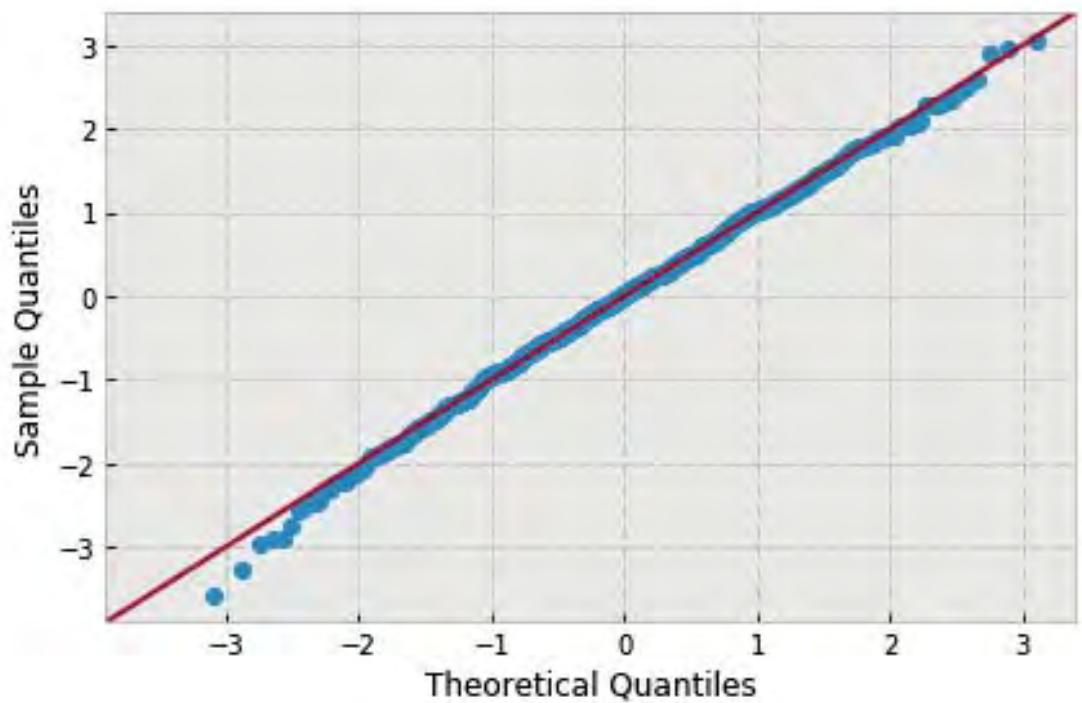


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=1125.494, scale=32.883977314187526)
DescribeResult(nobs=1000, minmax=(1011.0, 1220.0)
               mean=1125.493999999999, variance=1082.4384024024023,
               skewness=-0.01881562277316979, kurtosis=-0.24260505717075365)
```

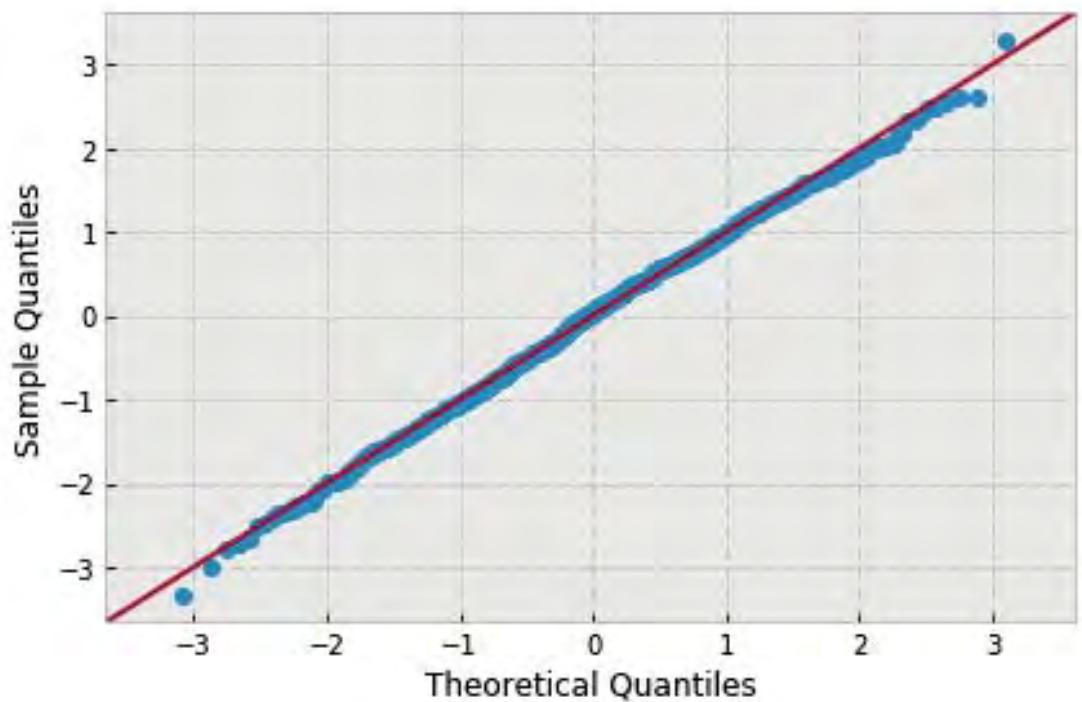
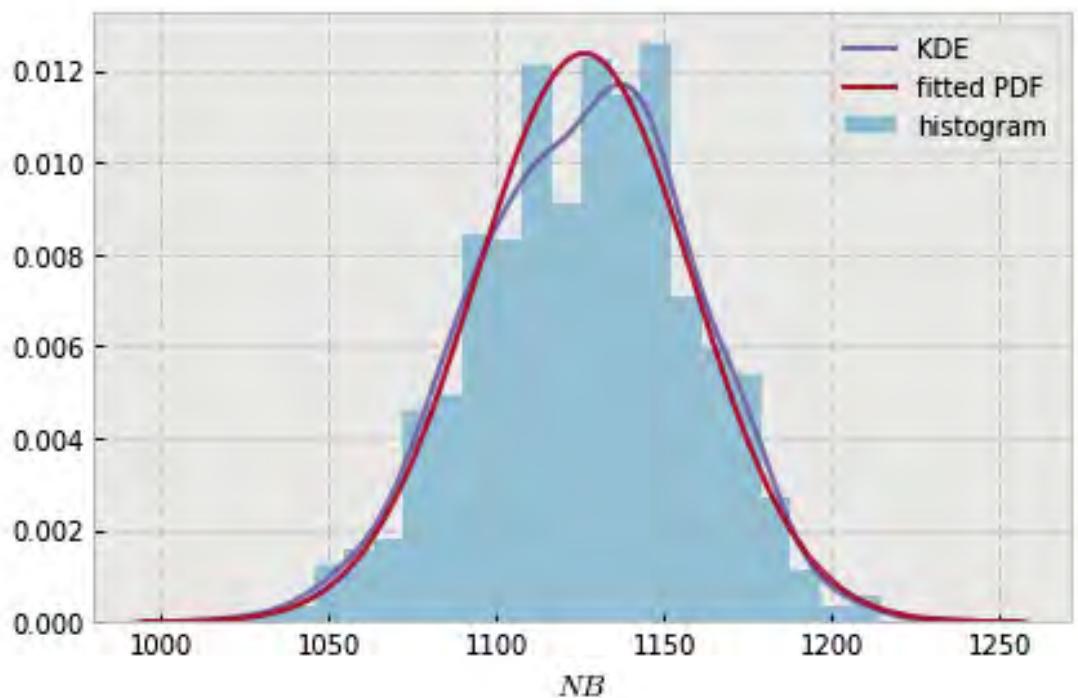


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=1125.217, scale=33.674350936580794)
DescribeResult(nobs=1000, minmax=(1005.0, 1228.0)
    mean=1125.2170000000001, variance=1135.0970080080078,
    skewness=-0.11923946548790126, kurtosis=0.10179567372187925)
```

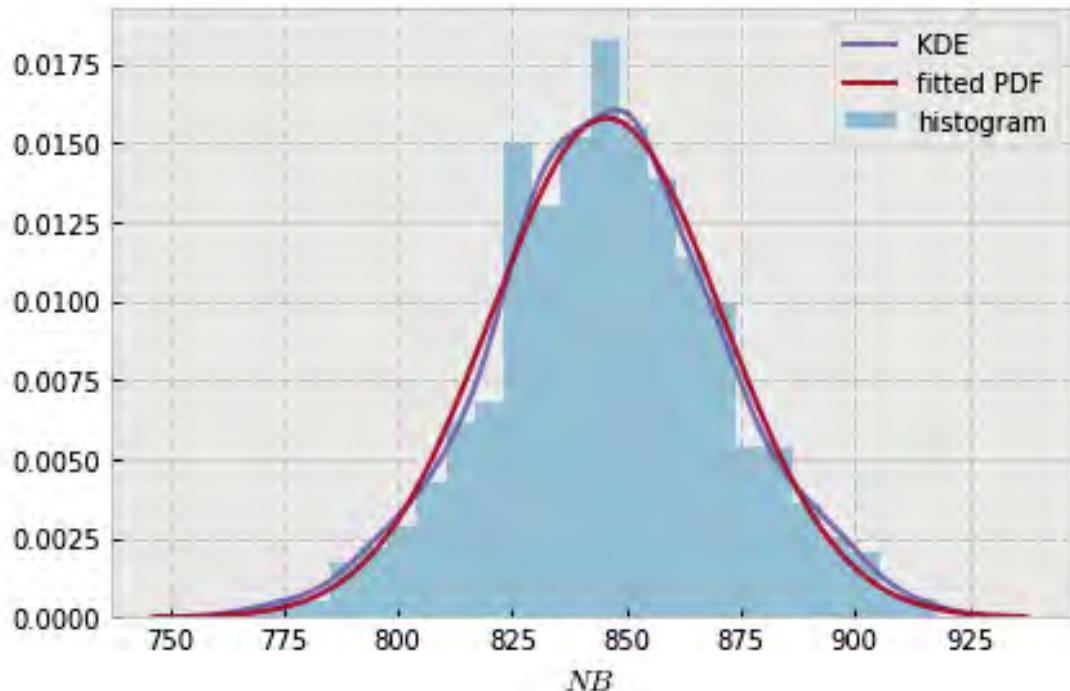


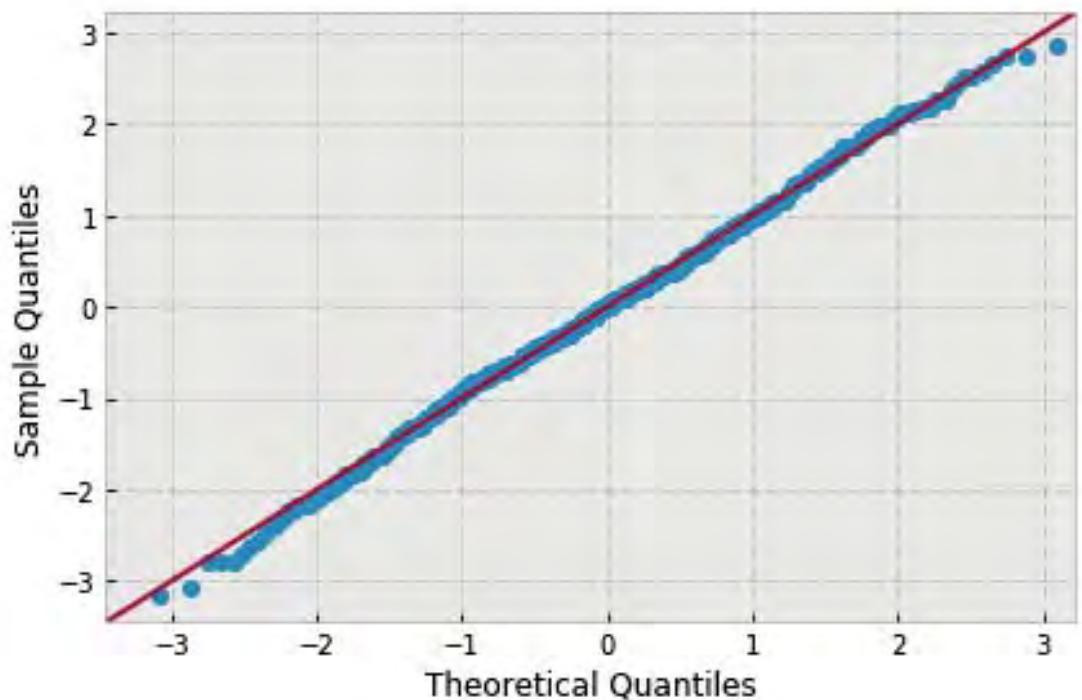


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=1126.213, scale=32.23668145141494)
DescribeResult(nobs=1000, minmax=(1019.0, 1232.0)
               mean=1126.213, variance=1040.243874874875,
               skewness=-0.12385709780118952, kurtosis=-0.1956372245266862)
```

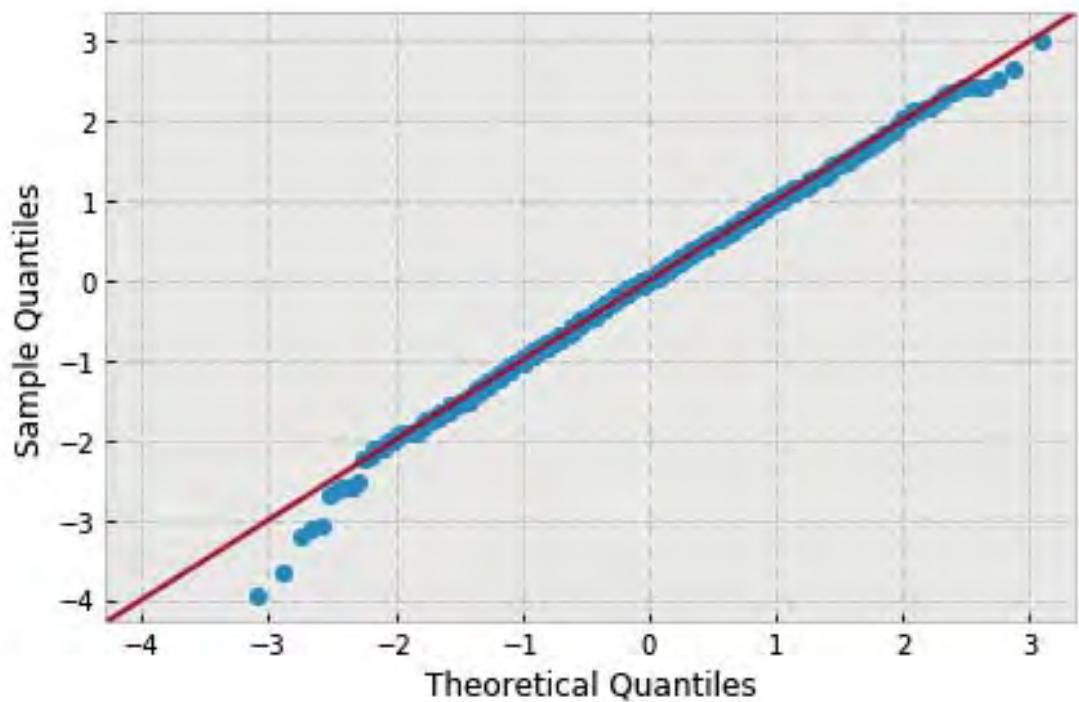
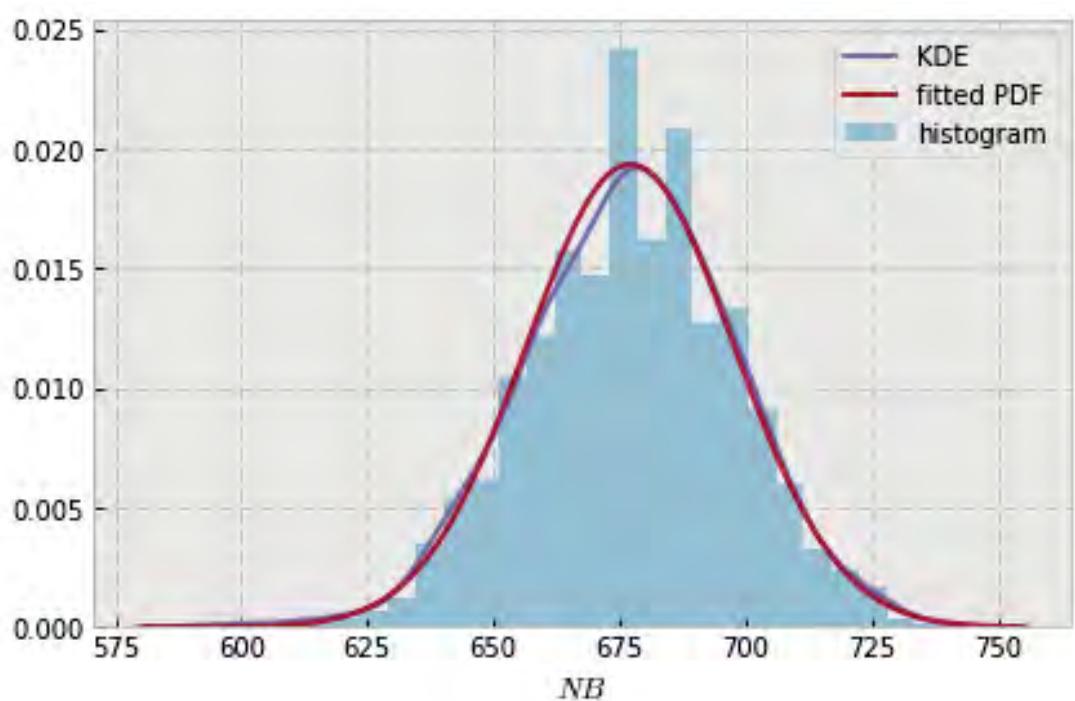


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=845.649, scale=25.273143828973872)
DescribeResult(nobs=1000, minmax=(766.0, 918.0)
               mean=845.649, variance=639.37117017017022,
               skewness=-0.026153096234814068, kurtosis=0.08519074217099876)
```

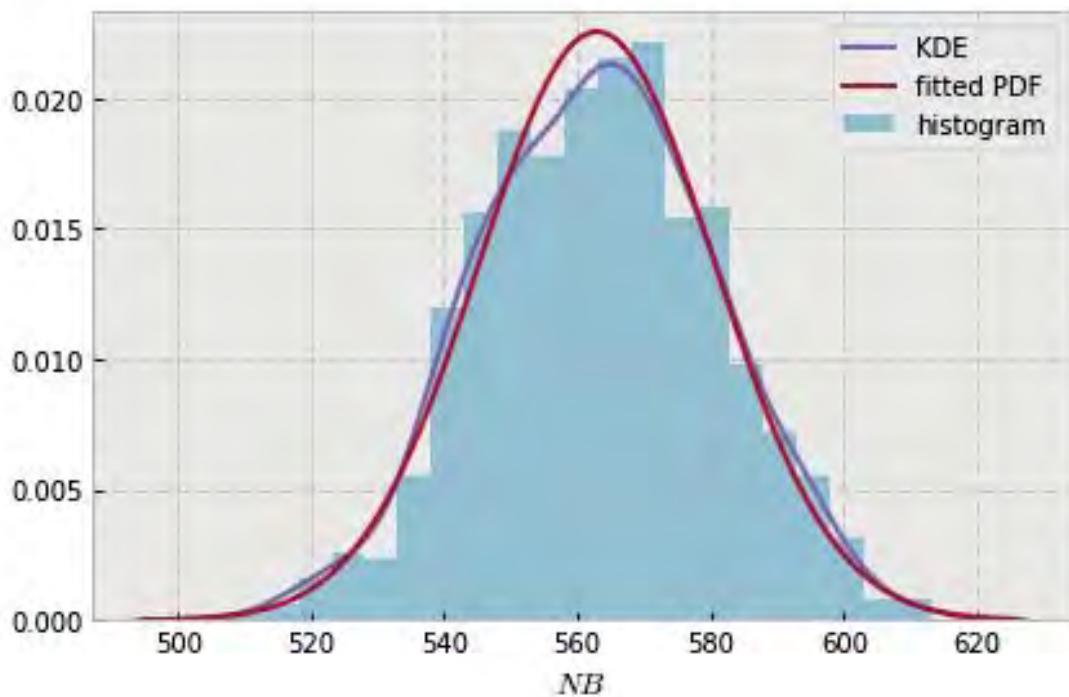


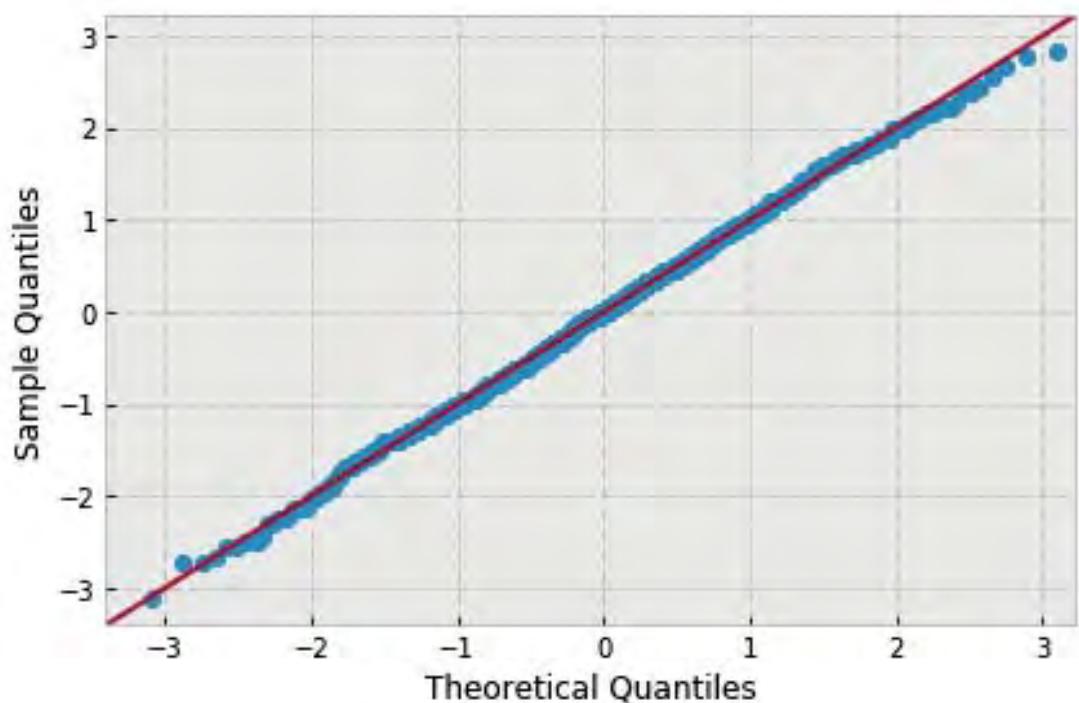


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=677.086, scale=20.61777398265875)
DescribeResult(nobs=1000, minmax=(596.0, 739.0)
               mean=677.0860000000001, variance=425.51812212212218,
               skewness=-0.16457009287128668, kurtosis=0.185175006453556)
```

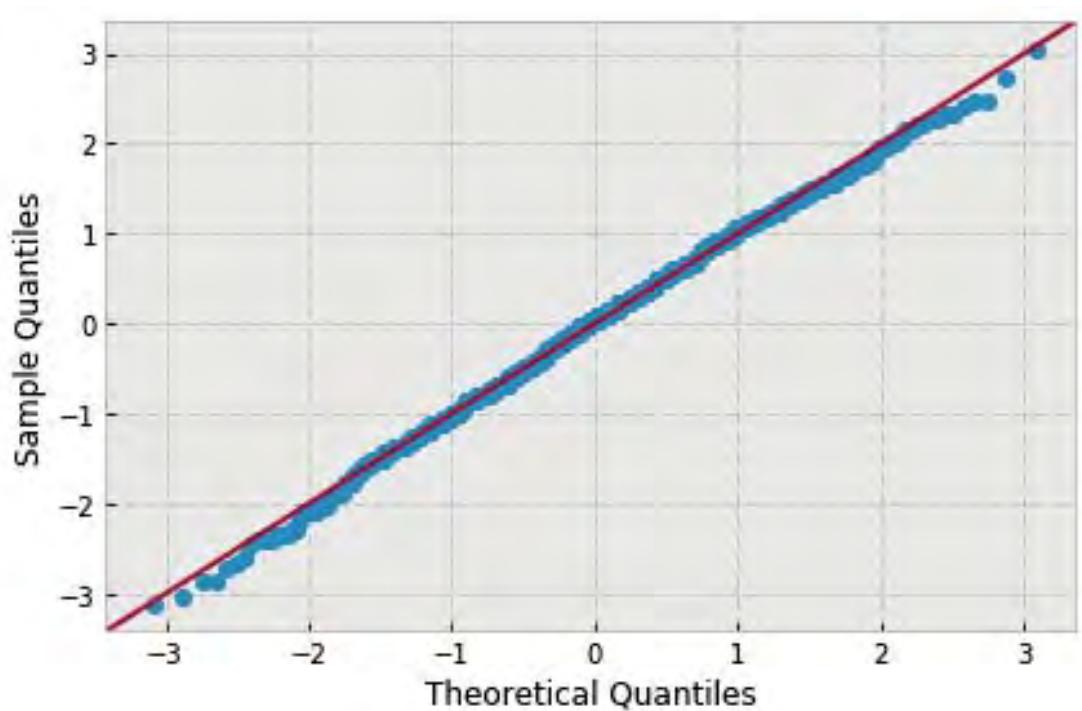
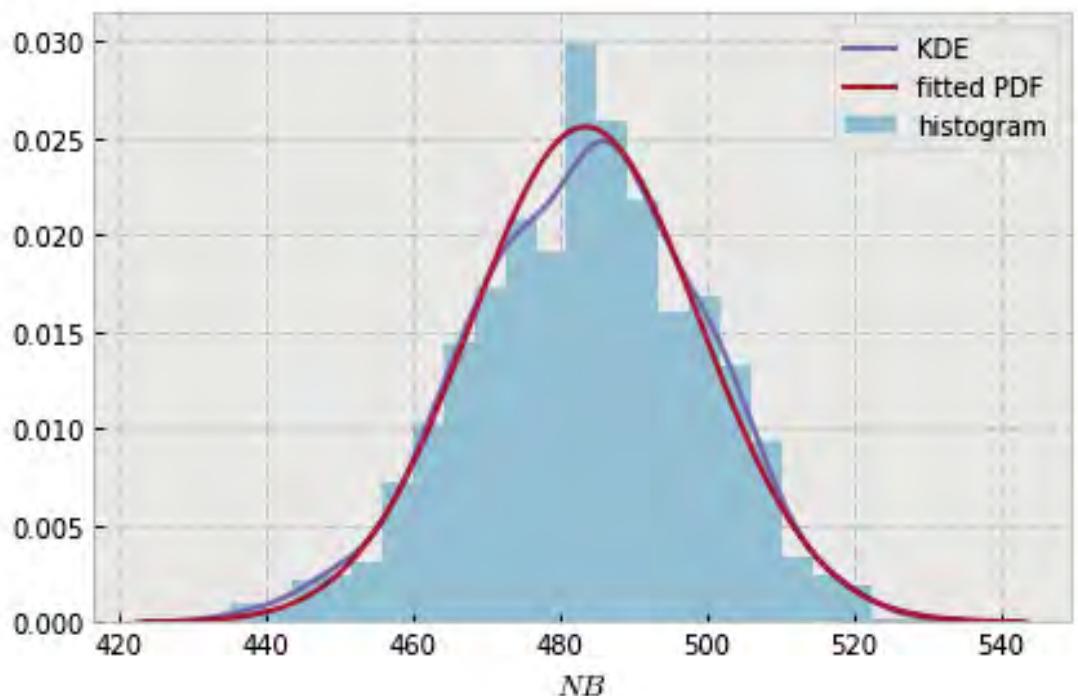


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=562.914, scale=17.68515207737836)
DescribeResult(nobs=1000, minmax=(508.0, 613.0)
    mean=562.9139999999999, variance=313.07768168168172,
    skewness=-0.026932254005413655, kurtosis=-0.23322213920994495)
```

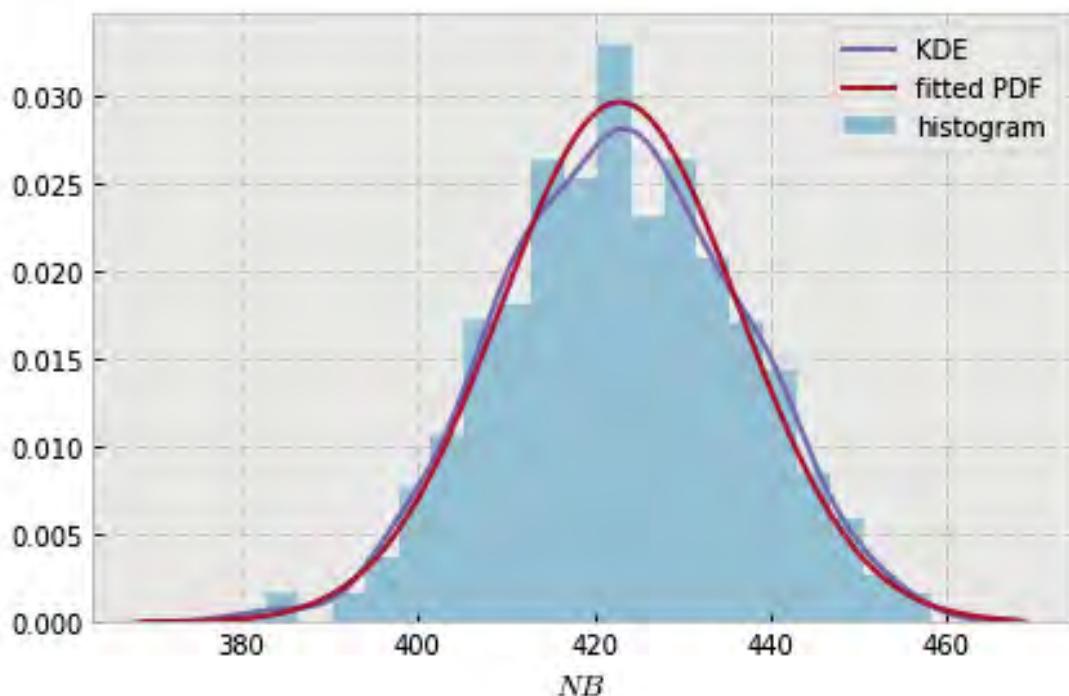


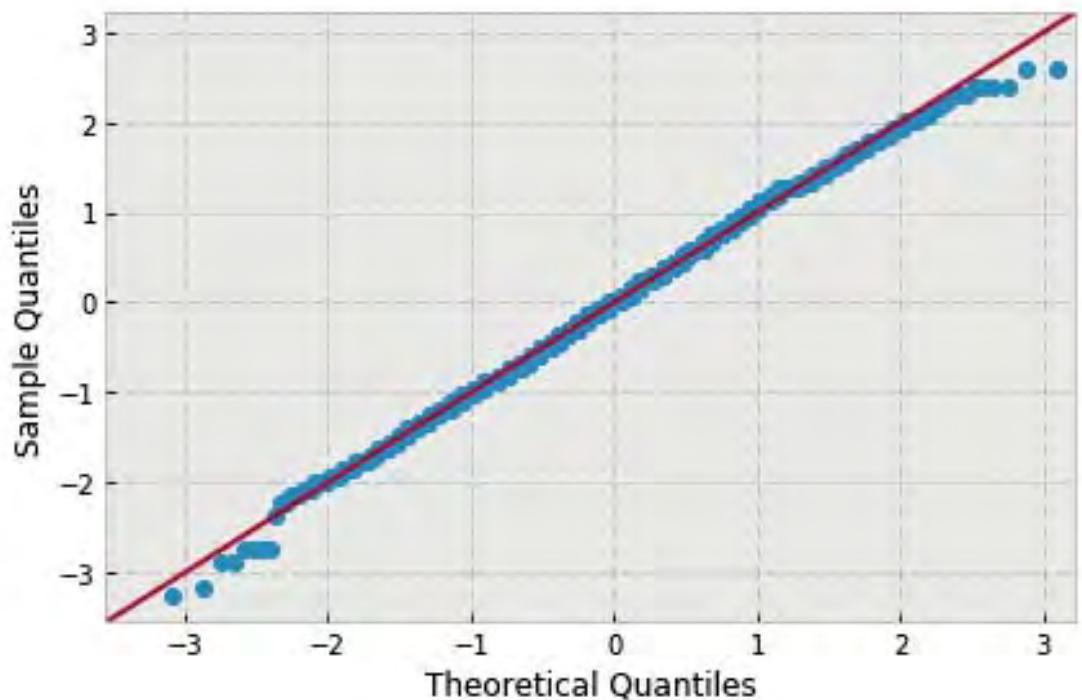


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=483.521, scale=15.598575543939901)
DescribeResult(nobs=1000, minmax=(435.0, 531.0)
               mean=483.5210000000002, variance=243.55911811811814,
               skewness=-0.13335032481599696, kurtosis=-0.13101918862707107)
```

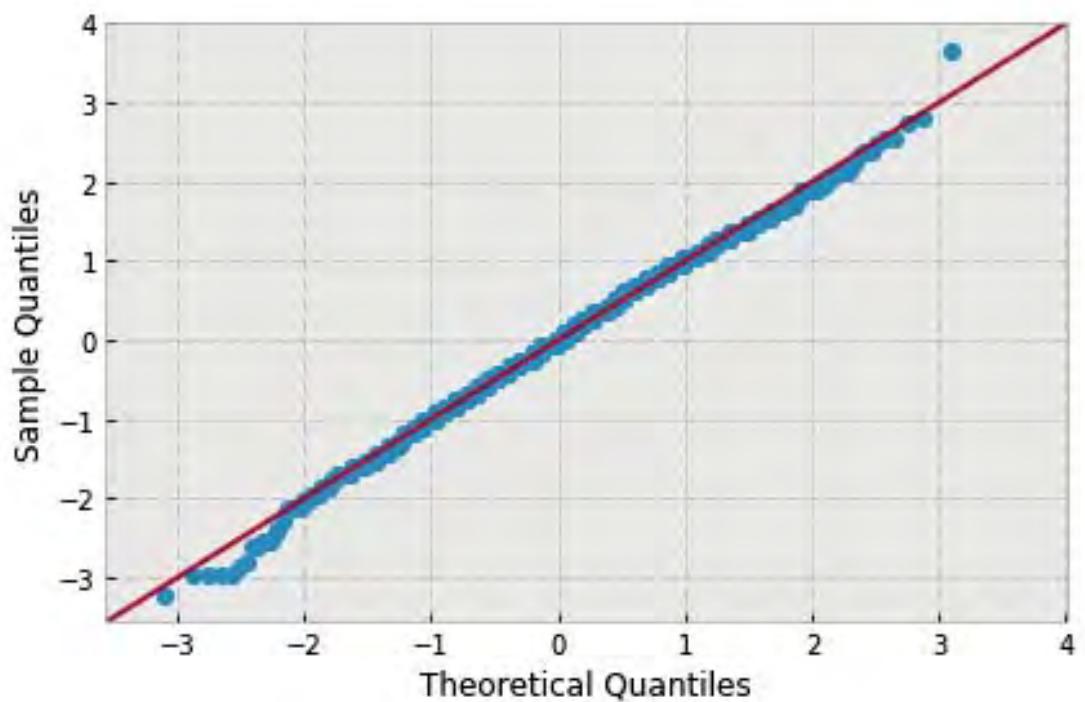
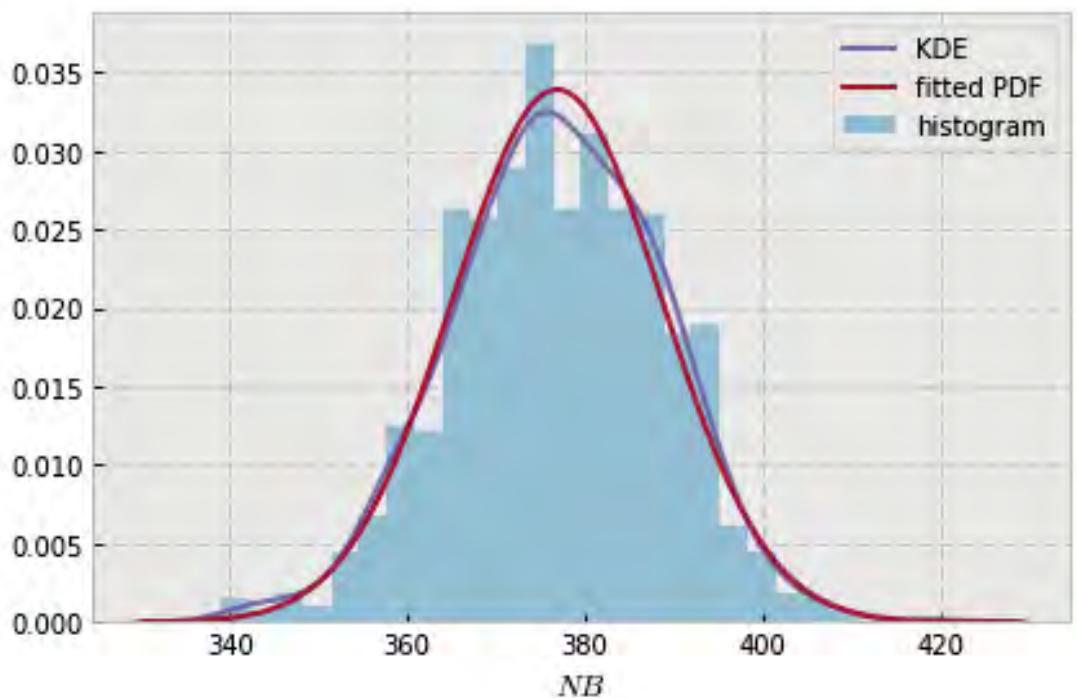


```
variable name: ln
variable value: 1.3333333333333333
distribution: normal(shape=(), loc=422.898, scale=13.477299284352188)
DescribeResult(nobs=1000, minmax=(379.0, 458.0)
    mean=422.89800000000002, variance=181.81941541541545,
    skewness=-0.07306004315916424, kurtosis=-0.2138575404554275)
```

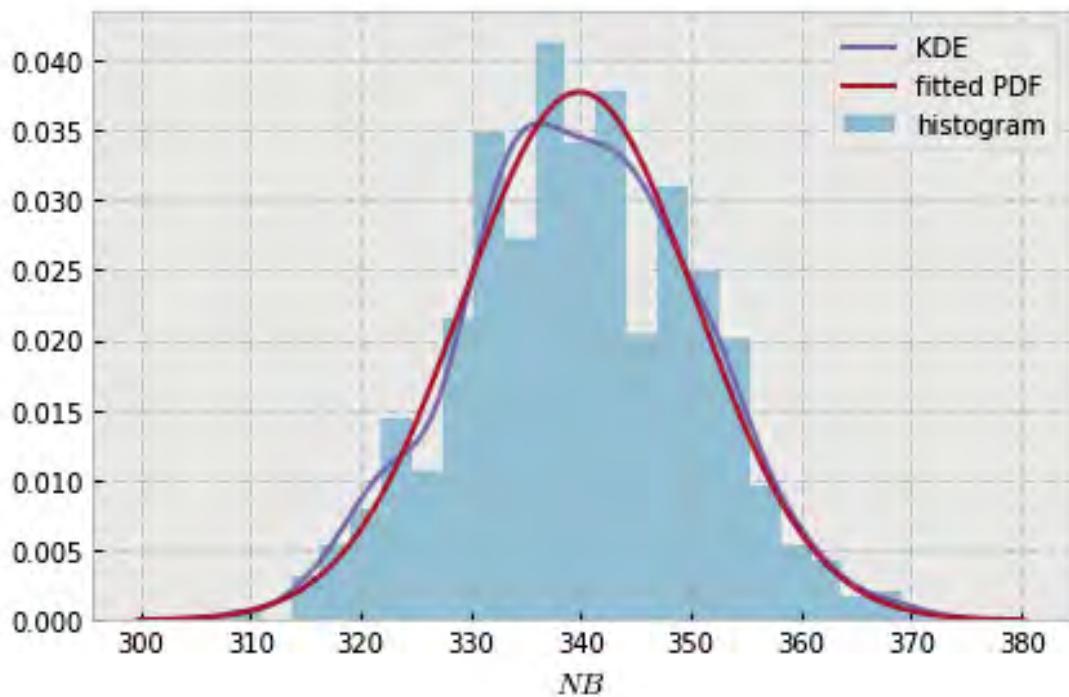


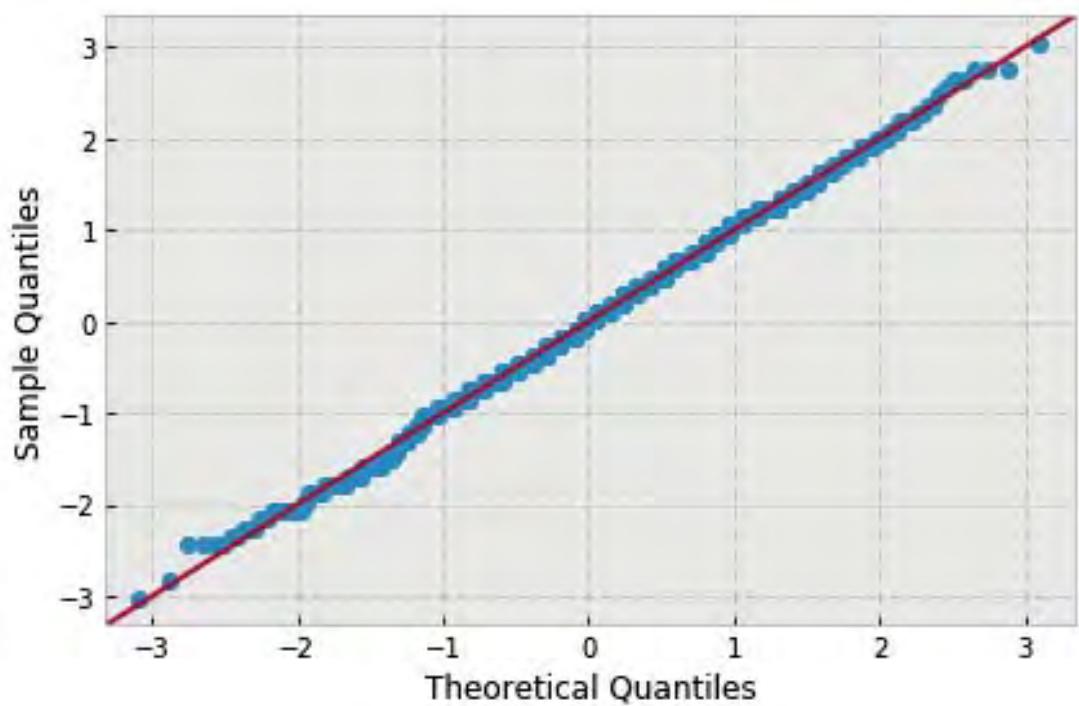


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=376.834, scale=11.770150551288628)
DescribeResult(nobs=1000, minmax=(339.0, 420.0)
               mean=376.834, variance=138.6751191191191,
               skewness=-0.1276480682047909, kurtosis=0.05875491930131593)
```

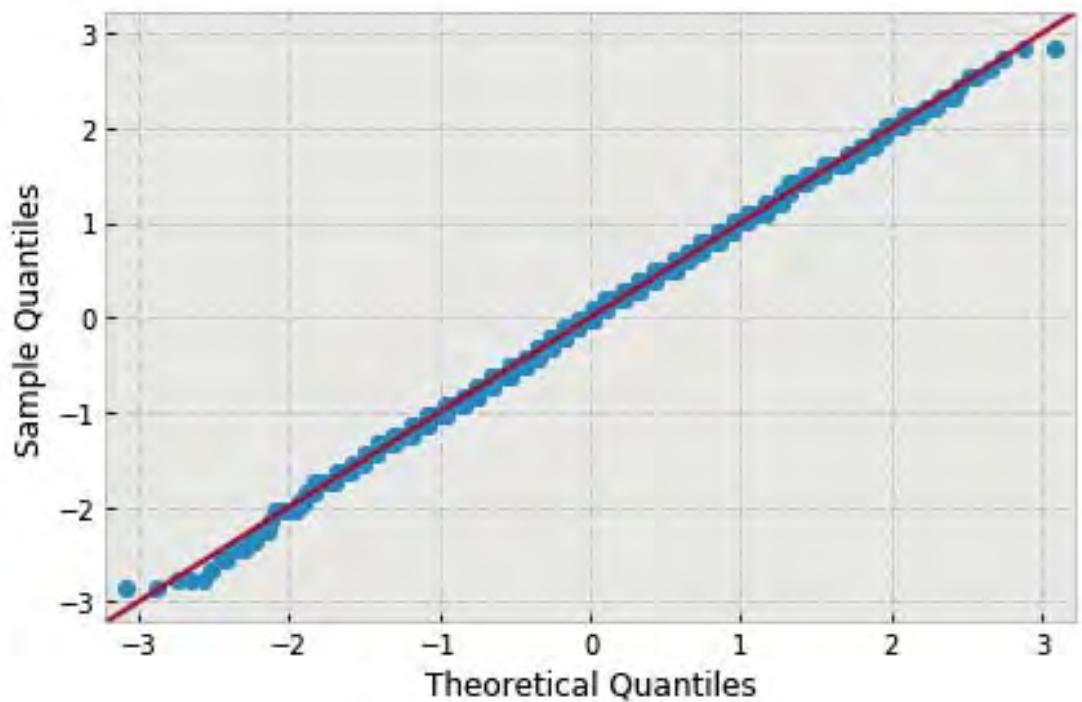
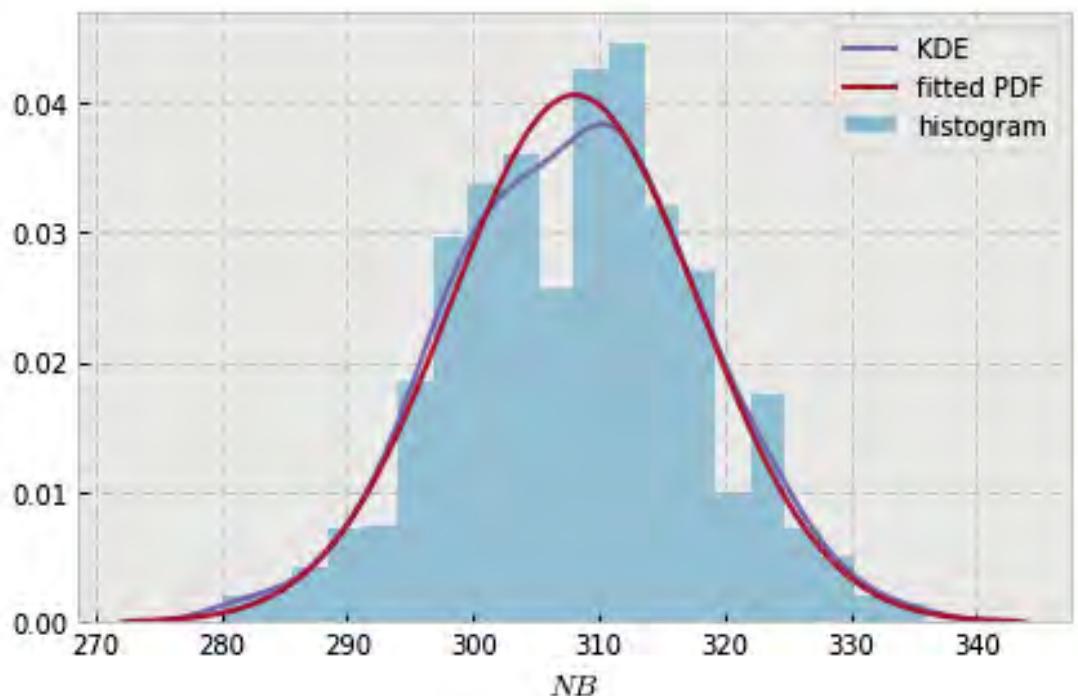


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=339.878, scale=10.58419179720398)
DescribeResult(nobs=1000, minmax=(308.0, 372.0)
    mean=339.87799999999999, variance=112.13725325325326,
    skewness=0.007775191650277858, kurtosis=-0.17709829535942578)
```

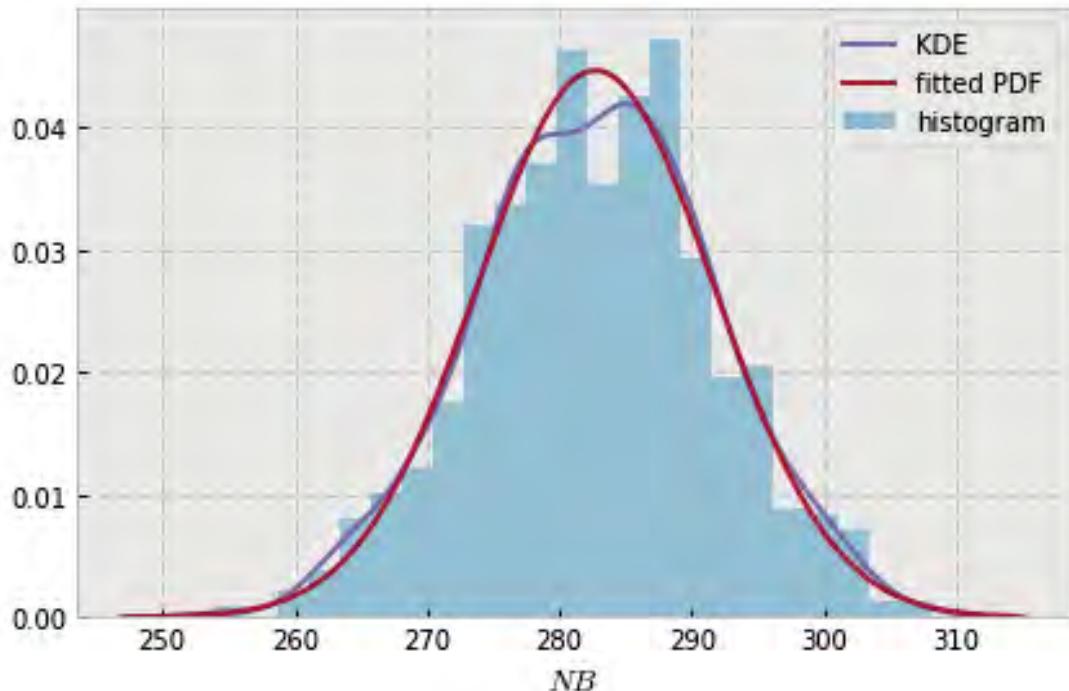


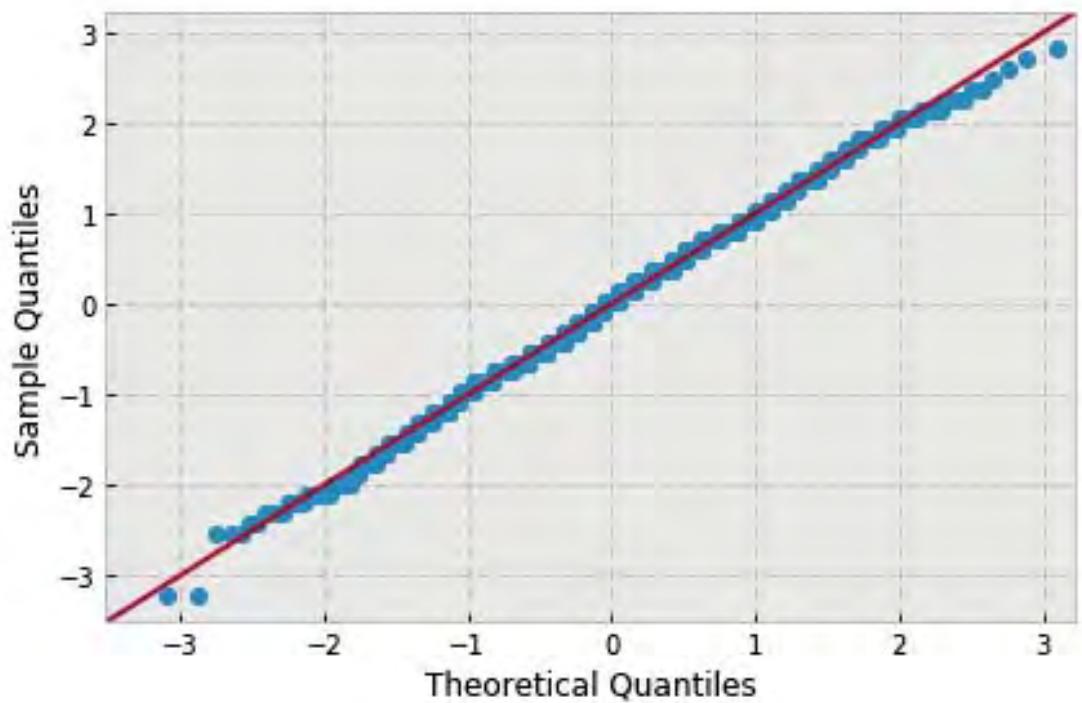


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=308.104, scale=9.833167546625045)
DescribeResult(nobs=1000, minmax=(280.0, 336.0)
               mean=308.1039999999998, variance=96.787971971971984,
               skewness=-0.03143814027898271, kurtosis=-0.15737602860204358)
```

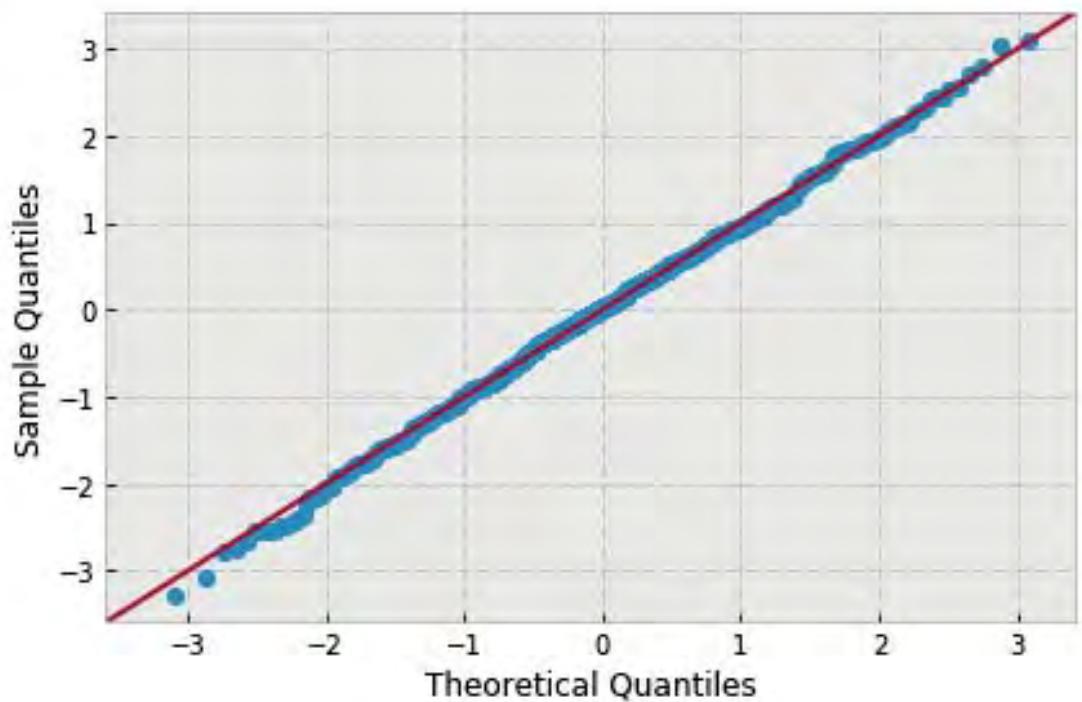
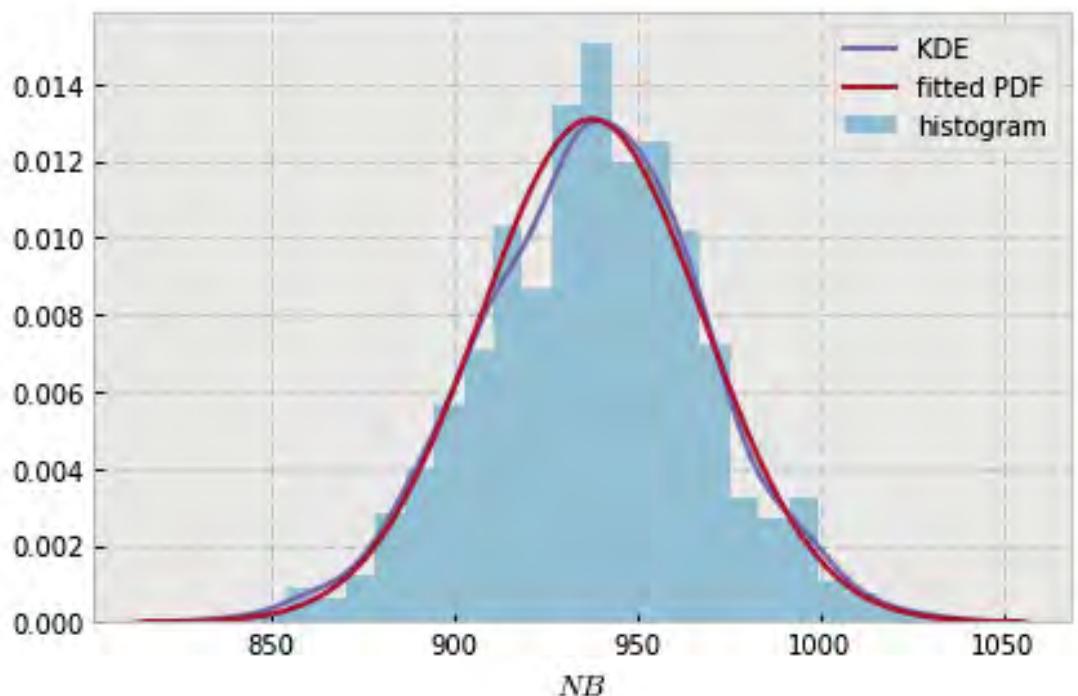


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=282.737, scale=8.938894282851768)
DescribeResult(nobs=1000, minmax=(254.0, 308.0)
               mean=282.73700000000002, variance=79.983814814814821,
               skewness=-0.06645930661160593, kurtosis=-0.16105646384122174)
```

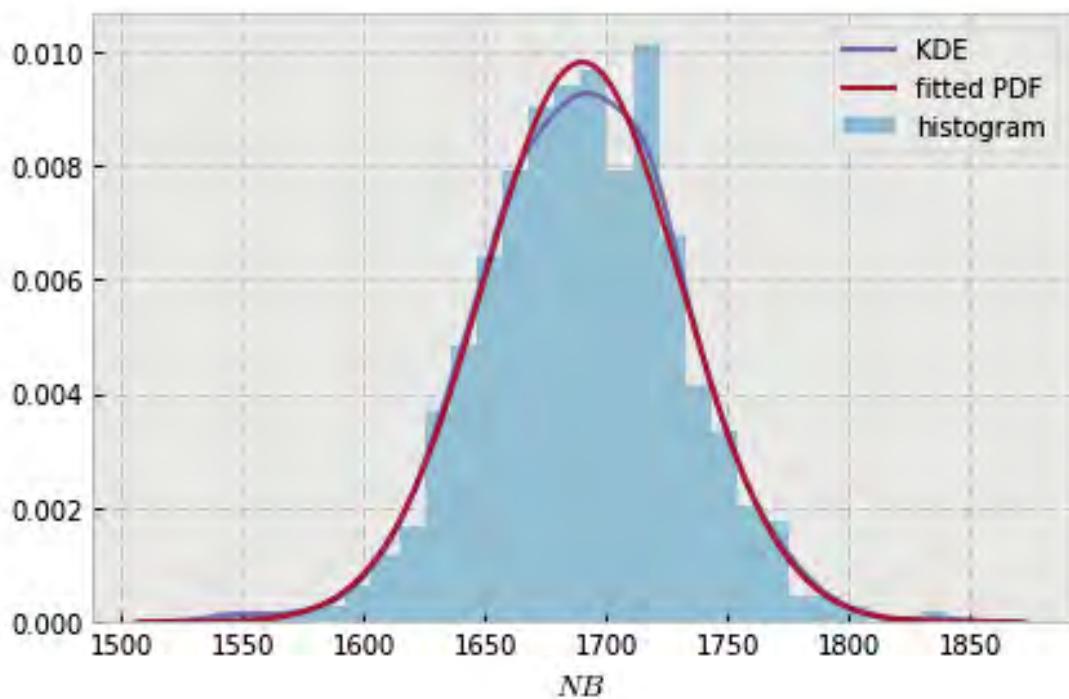


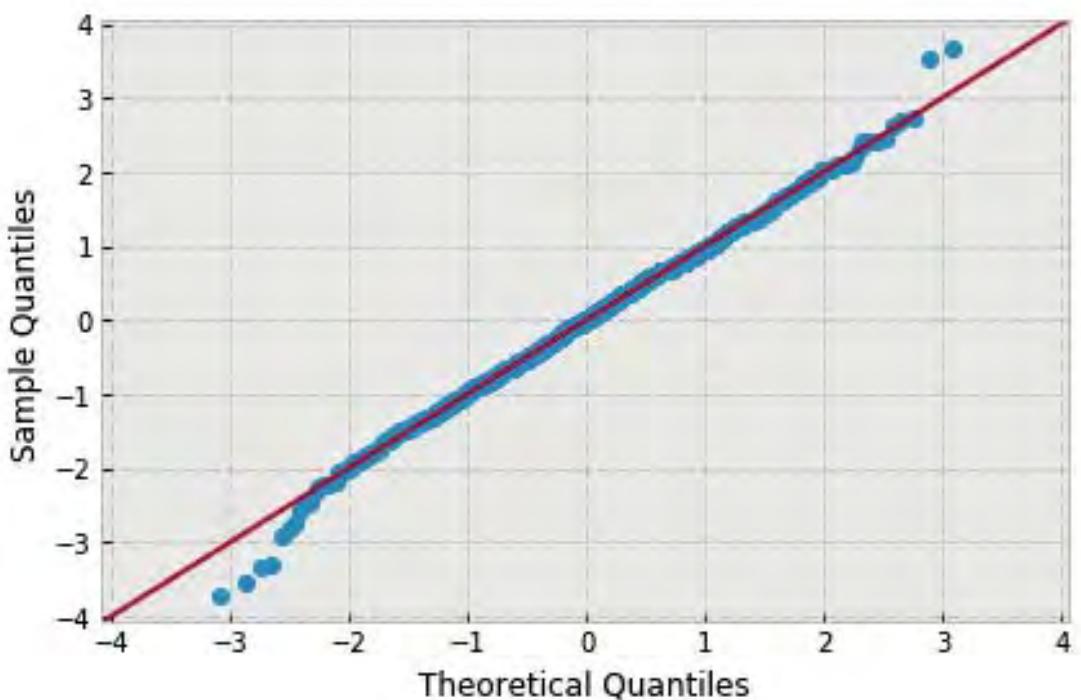


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=937.742, scale=30.478507771870984)
DescribeResult(nobs=1000, minmax=(838.0, 1032.0)
               mean=937.7419999999996, variance=929.86930530530526,
               skewness=-0.07192809513497425, kurtosis=0.047342989496574894)
```

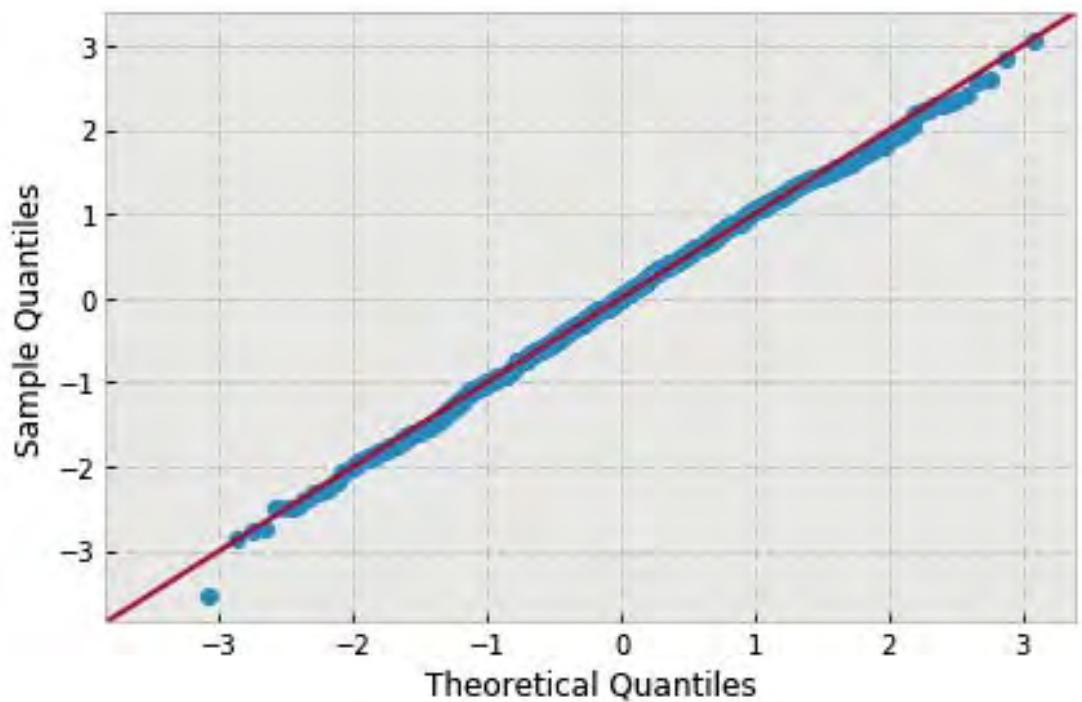
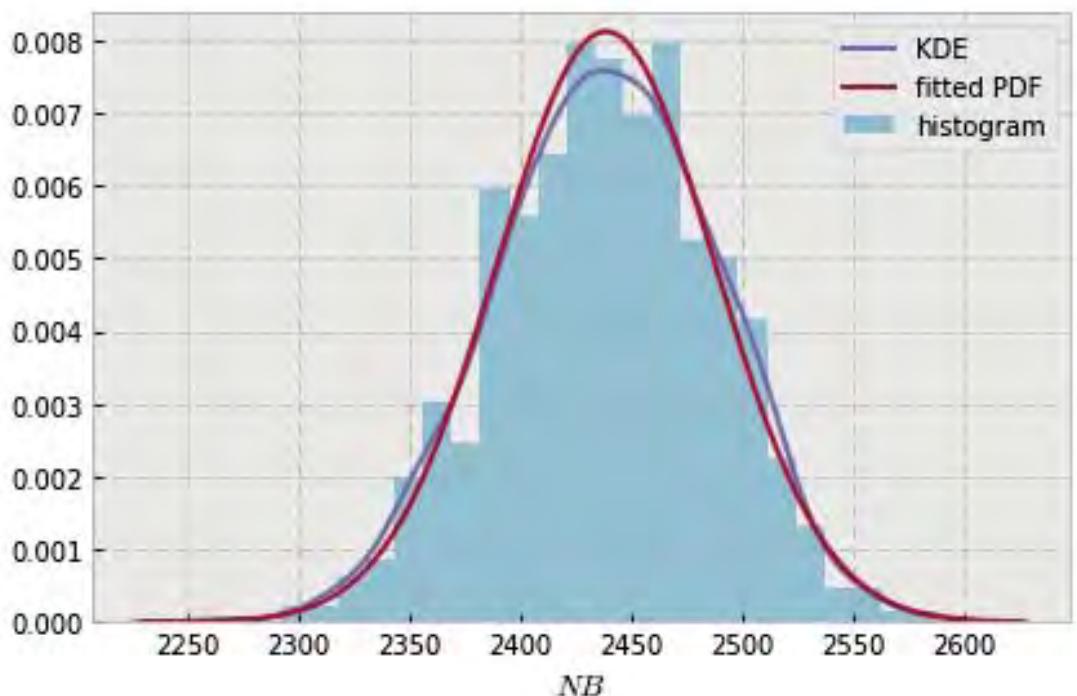


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=1690.117, scale=40.68485358213791)
DescribeResult(nobs=1000, minmax=(1539.0, 1840.0)
               mean=1690.117, variance=1656.9142252252252,
               skewness=-0.06781641435944913, kurtosis=0.3563007867942418)
```

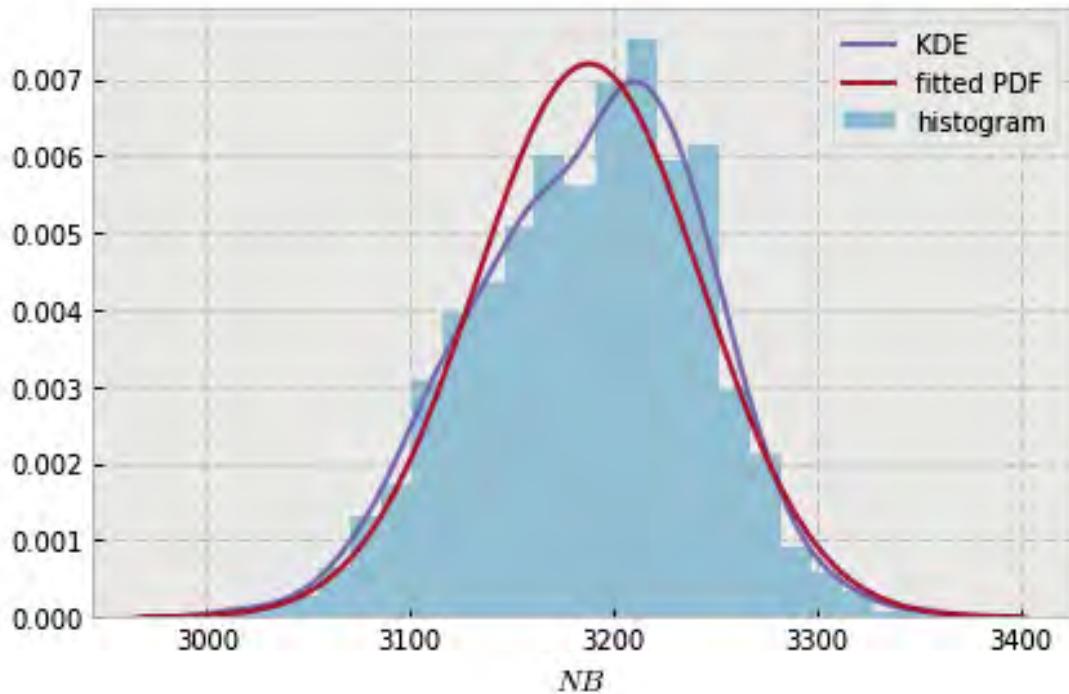


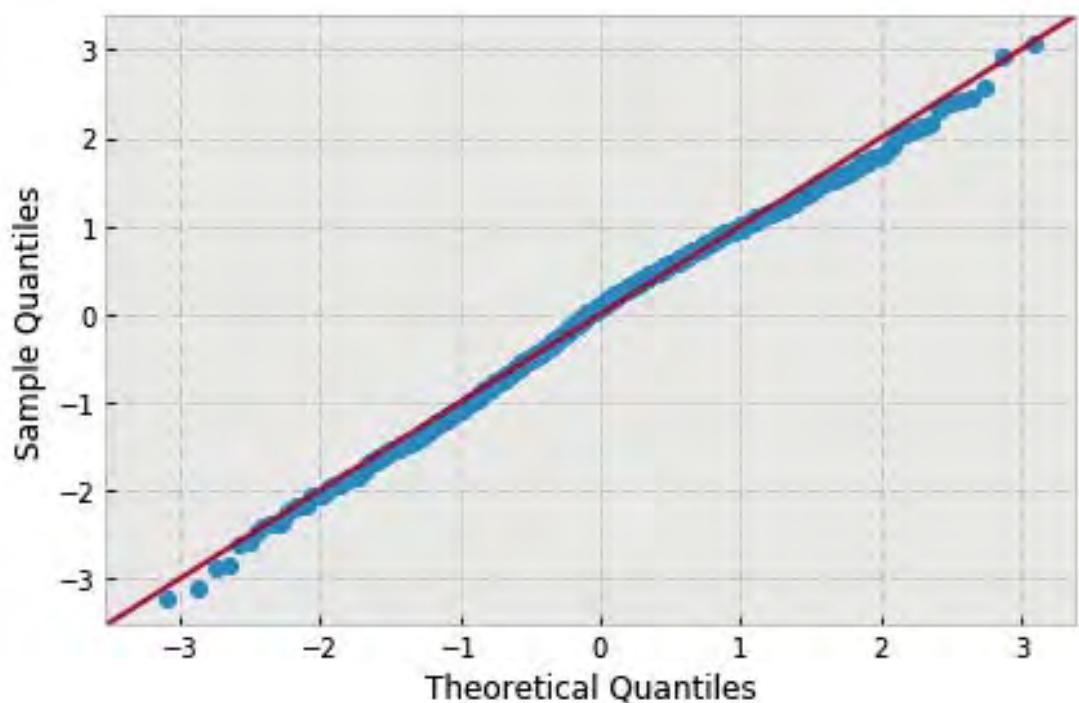


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=2438.902, scale=49.14275527481136)
DescribeResult(nobs=1000, minmax=(2266.0, 2589.0)
               mean=2438.902, variance=2417.4278238238235,
               skewness=-0.10688761114207036, kurtosis=-0.17412822892679847)
```

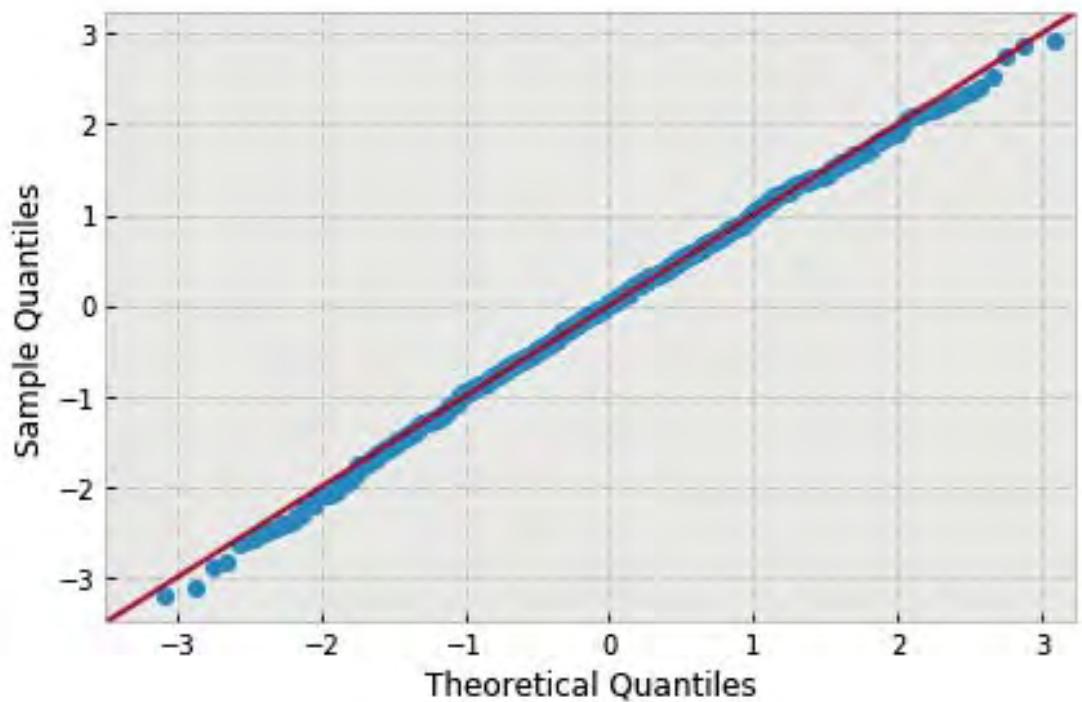
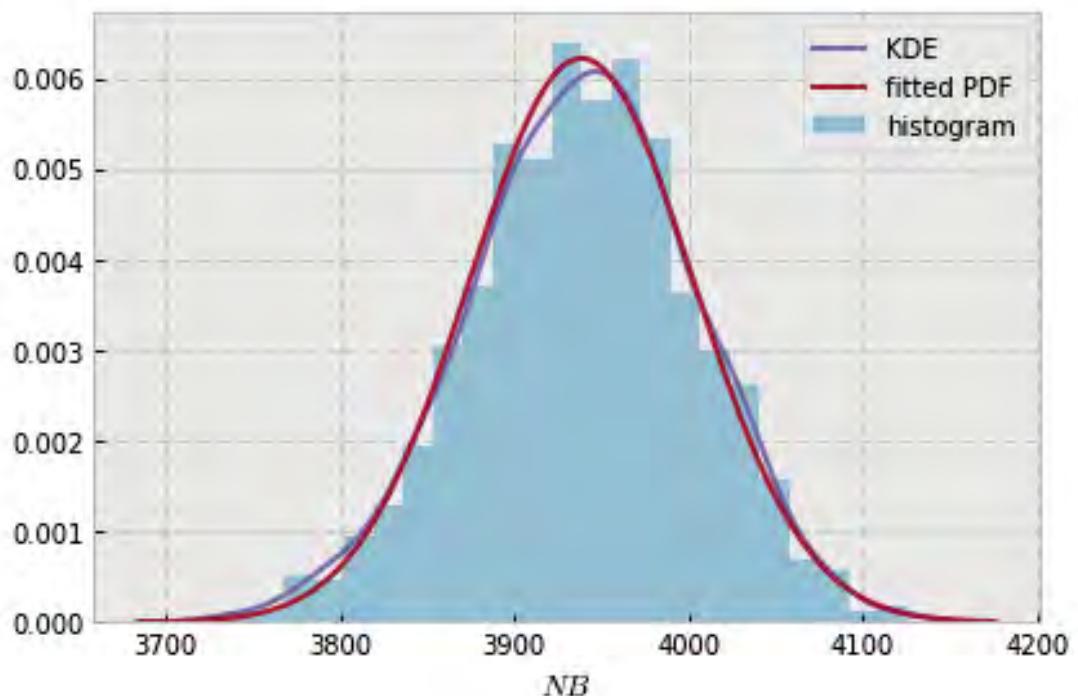


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=3188.05, scale=55.411709051427025)
DescribeResult(nobs=1000, minmax=(3010.0, 3358.0)
    mean=3188.050000000002, variance=3073.5310310310306,
    skewness=-0.20771568496649212, kurtosis=-0.2243180124111177)
```

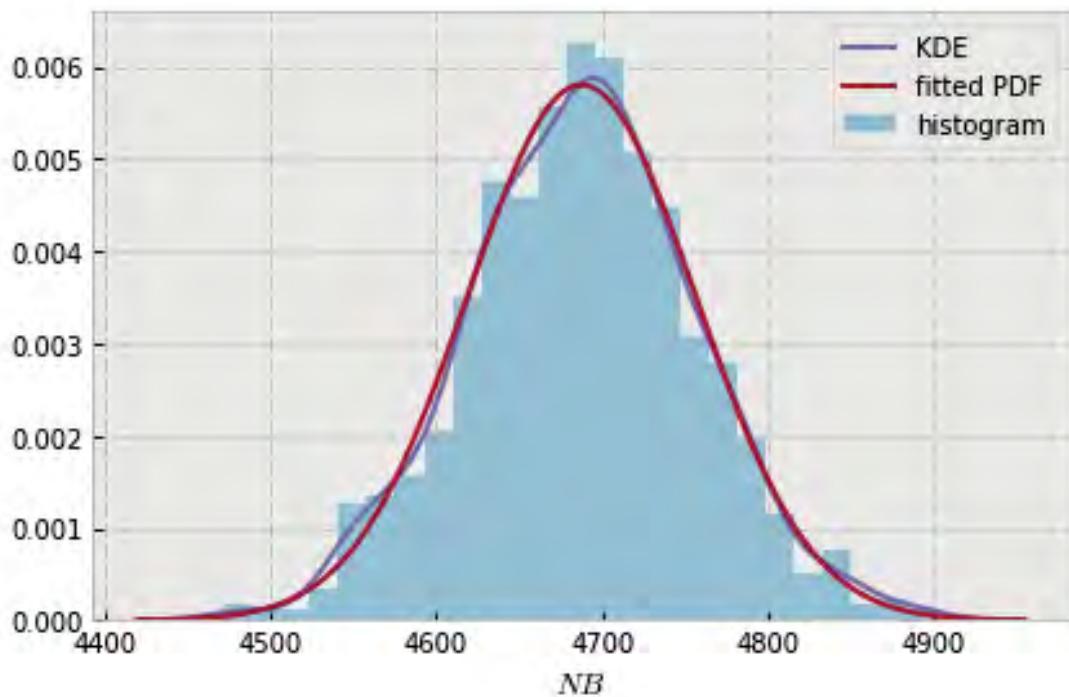


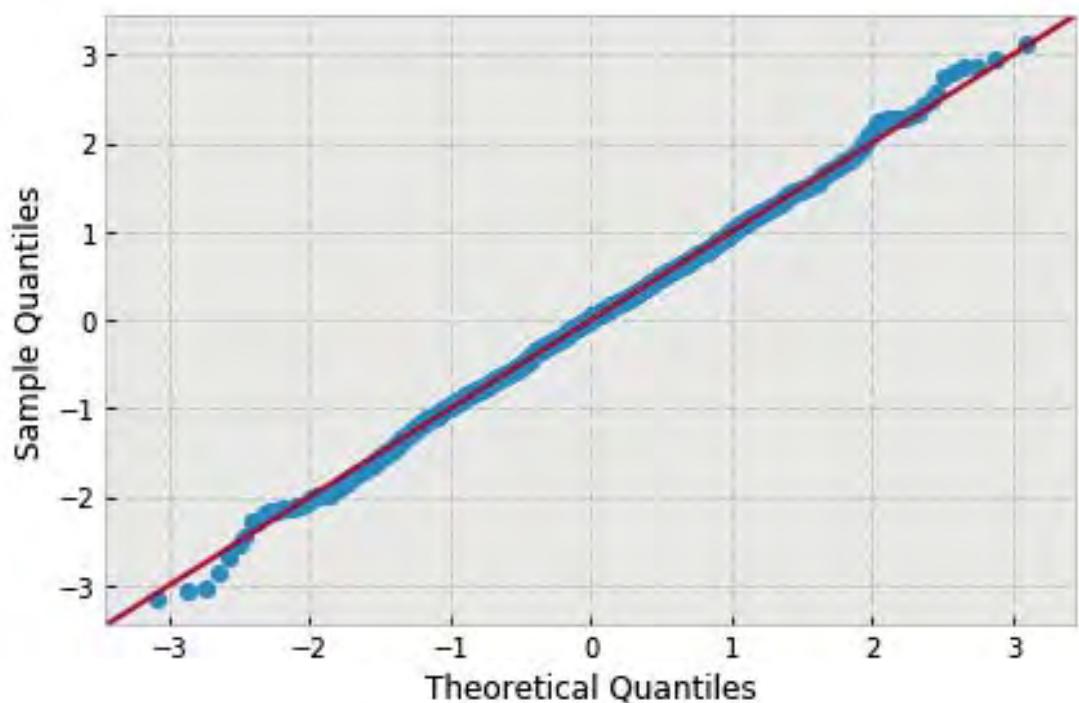


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=3938.519, scale=64.1689460642763)
DescribeResult(nobs=1000, minmax=(3734.0, 4126.0)
               mean=3938.518999999998, variance=4121.7754144144146,
               skewness=-0.12990029447688103, kurtosis=-0.04942373928773325)
```

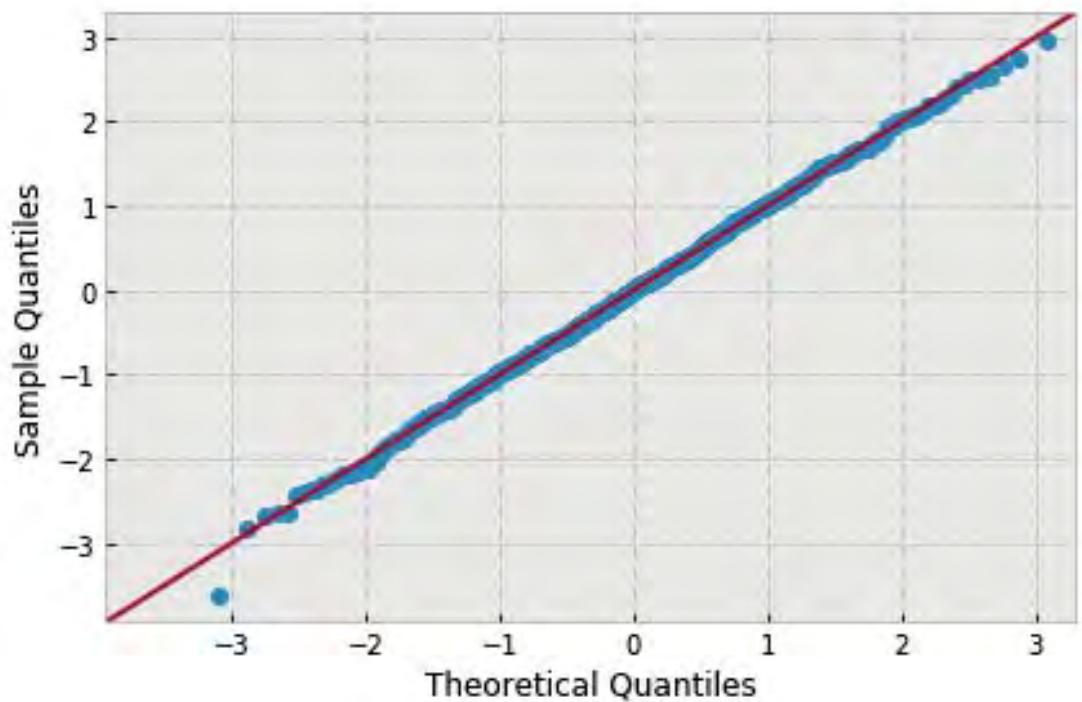
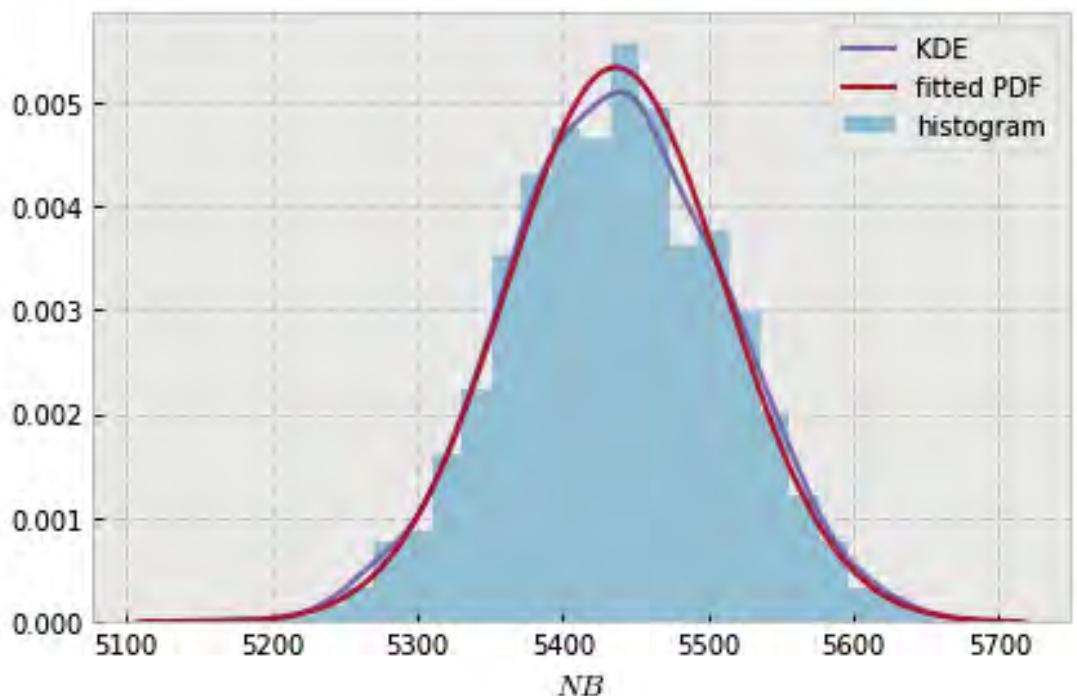


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=4687.547, scale=68.72833324764976)
DescribeResult(nobs=1000, minmax=(4472.0, 4902.0)
    mean=4687.546999999996, variance=4728.3121031031033,
    skewness=0.006059228212327086, kurtosis=0.088866771895181)
```

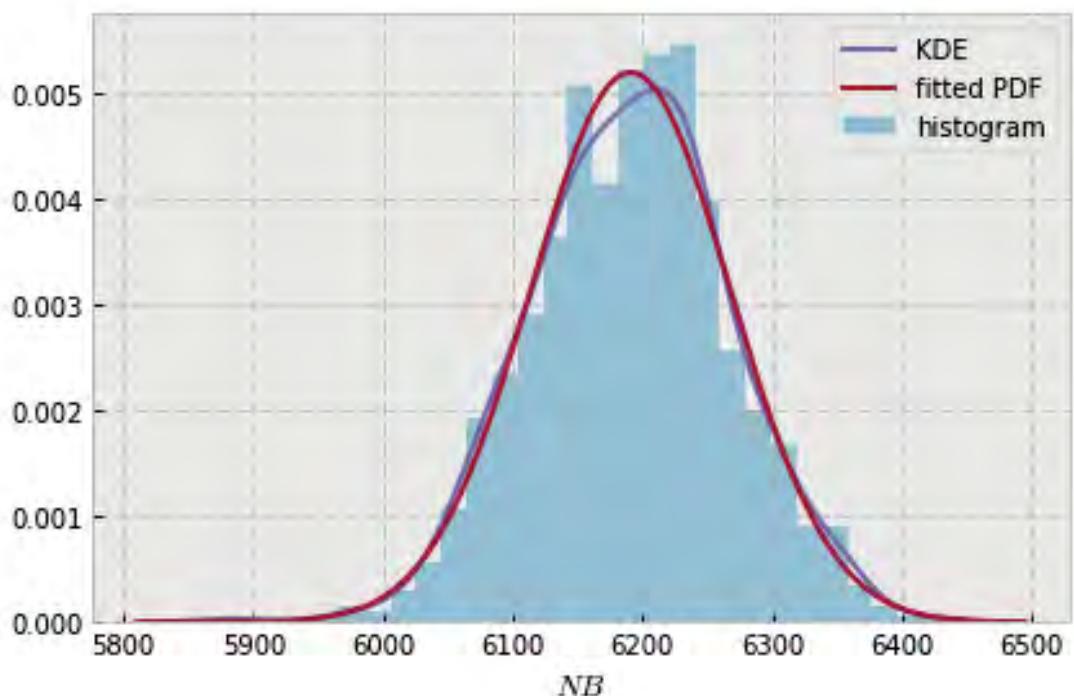


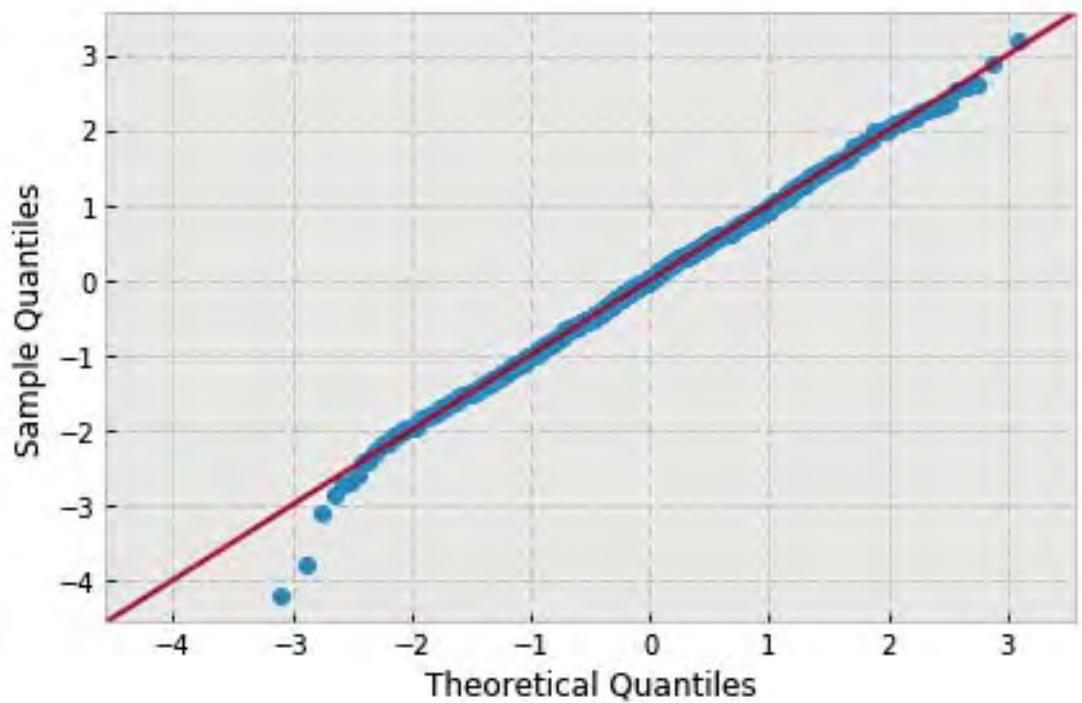


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=5436.656, scale=74.85241254629005)
DescribeResult(nobs=1000, minmax=(5167.0, 5658.0)
               mean=5436.655999999999, variance=5608.4921561561569,
               skewness=-0.027567631581954754, kurtosis=-0.11874901949210148)
```

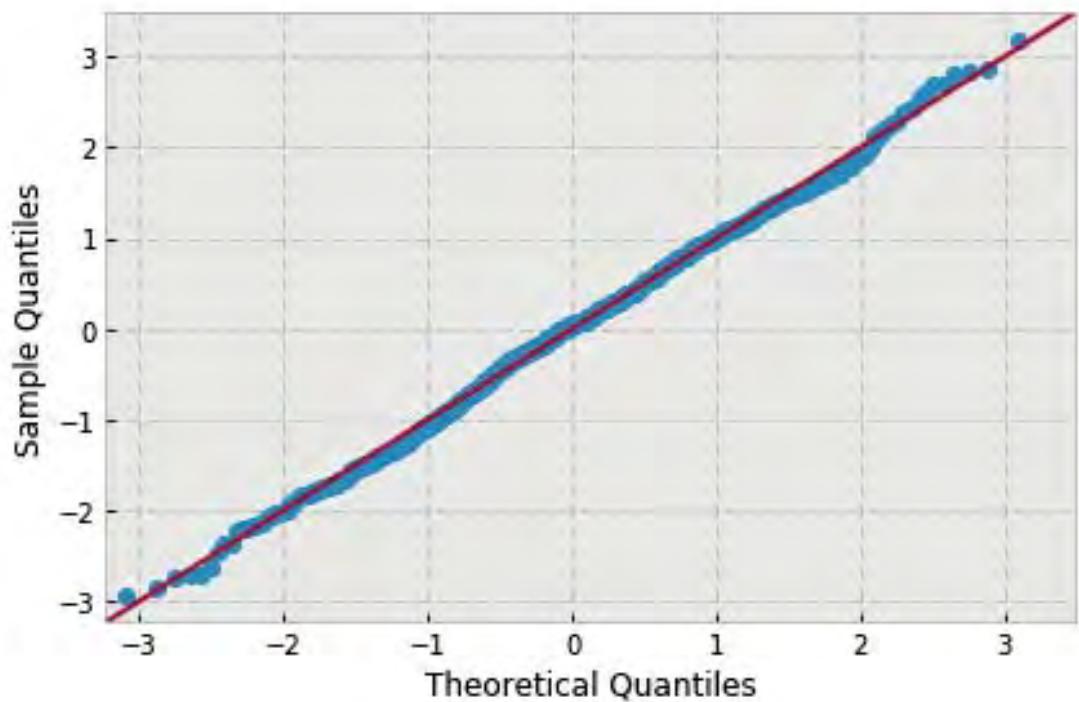
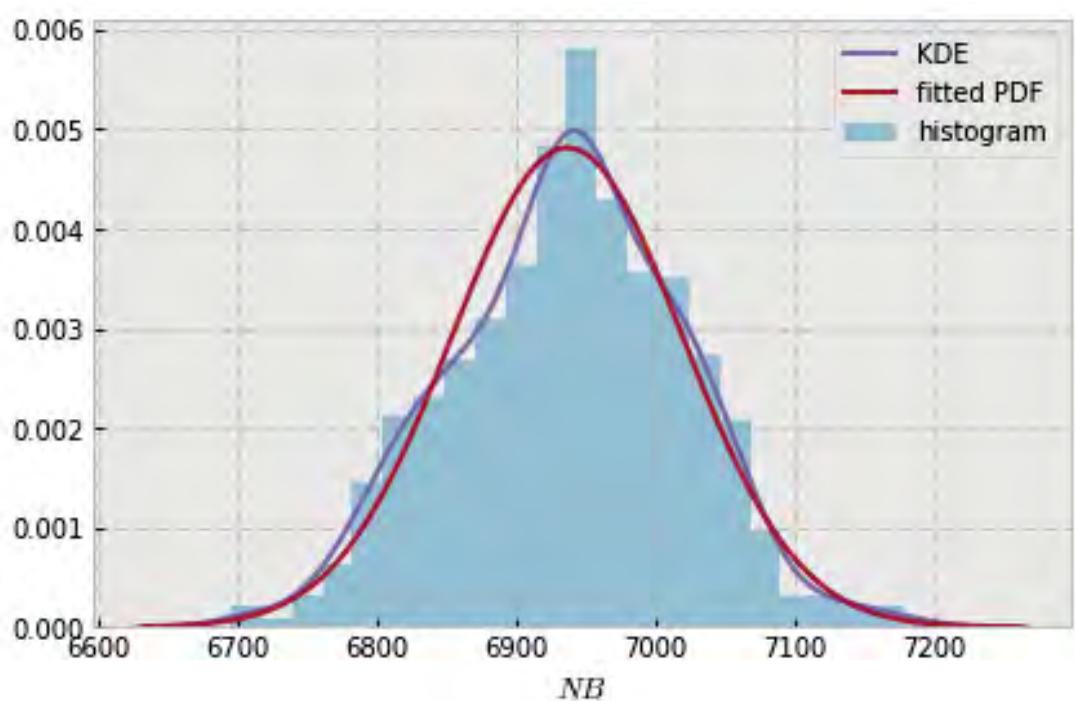


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=6190.992, scale=76.79893186757222)
DescribeResult(nobs=1000, minmax=(5869.0, 6436.0)
    mean=6190.992000000000, variance=5903.9799159159165,
    skewness=-0.0919954222535947, kurtosis=0.21438002338780526)
```

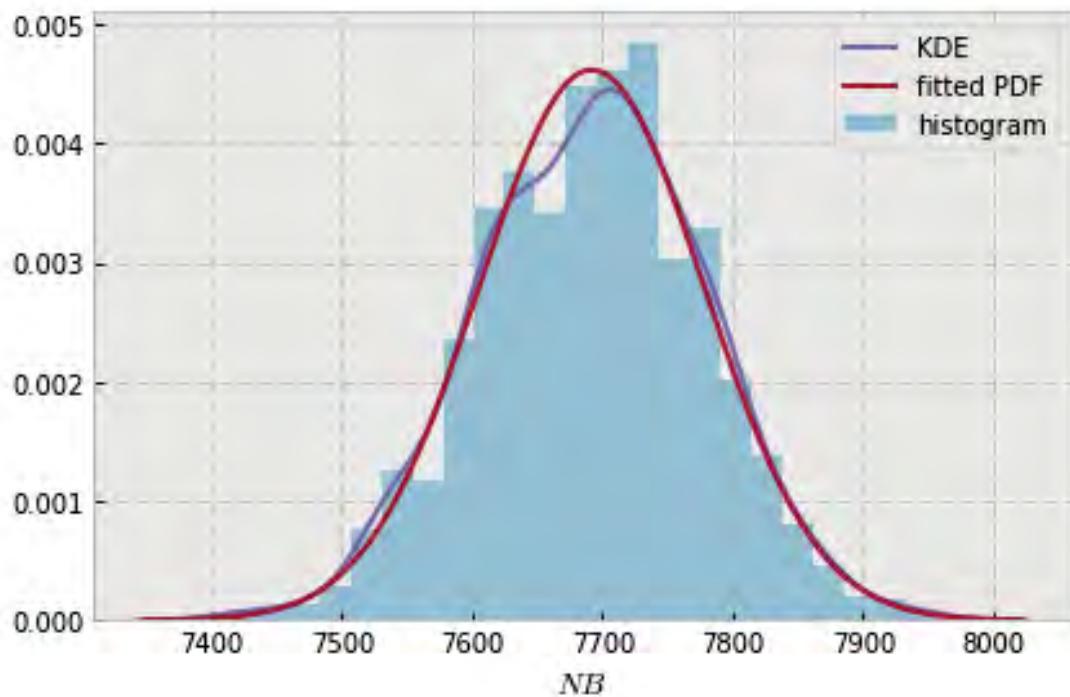


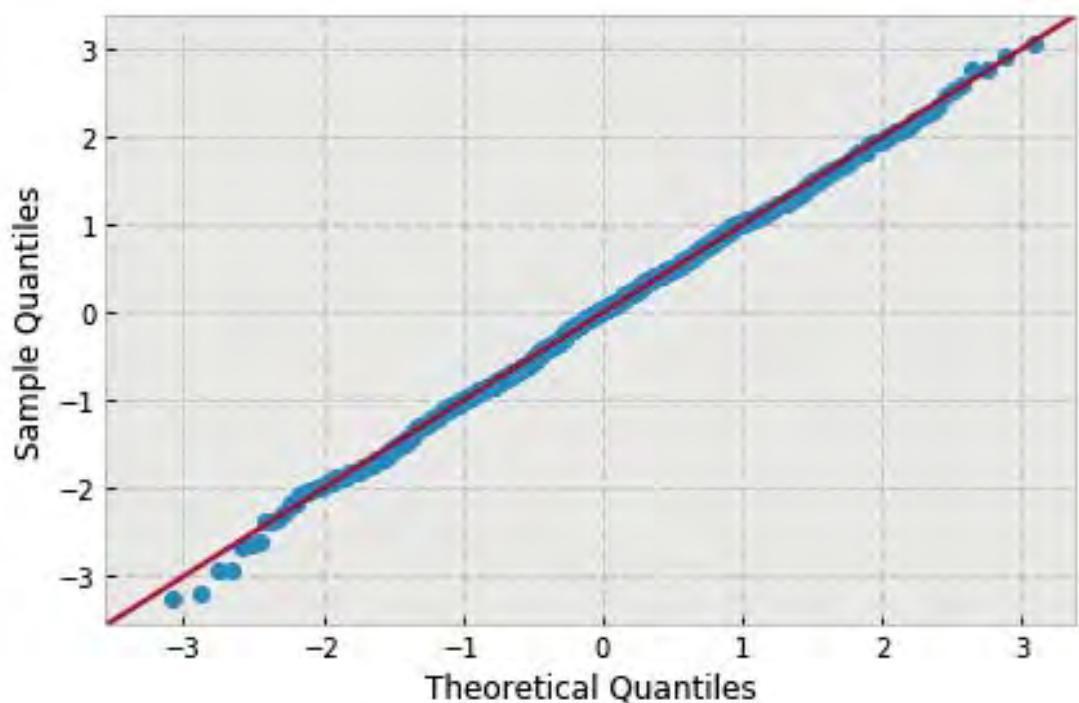


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=6936.651, scale=82.9174722178625)
DescribeResult(nobs=1000, minmax=(6694.0, 7200.0)
               mean=6936.650999999998, variance=6882.1893883883886,
               skewness=-0.05876604345319681, kurtosis=-0.11185173642756352)
```

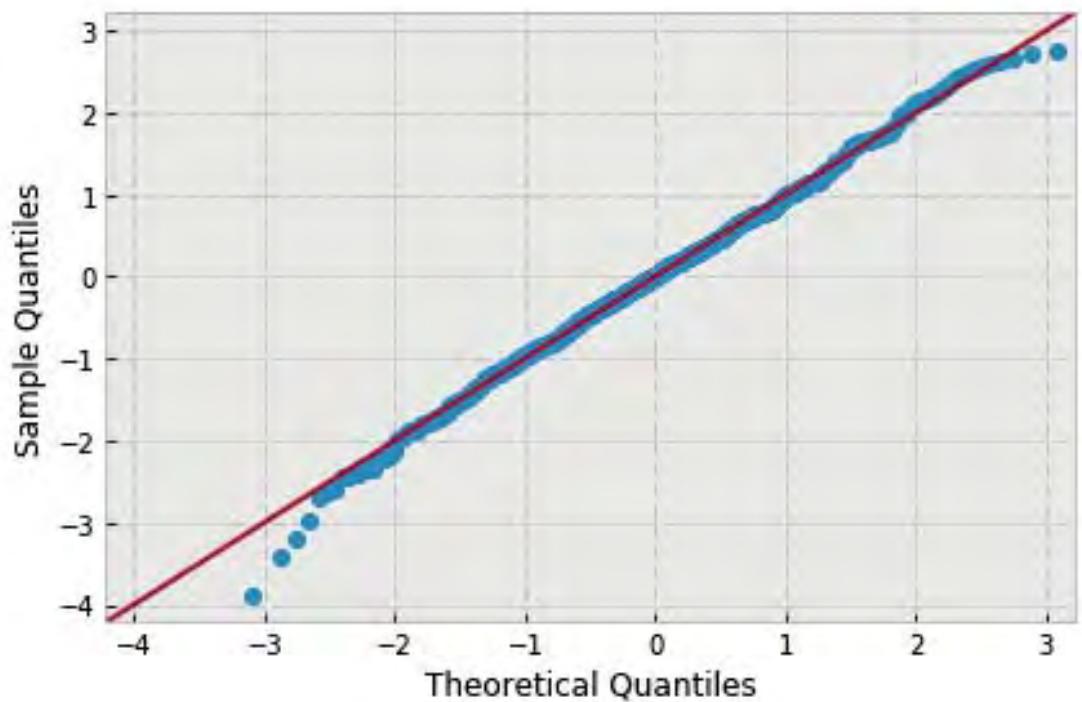
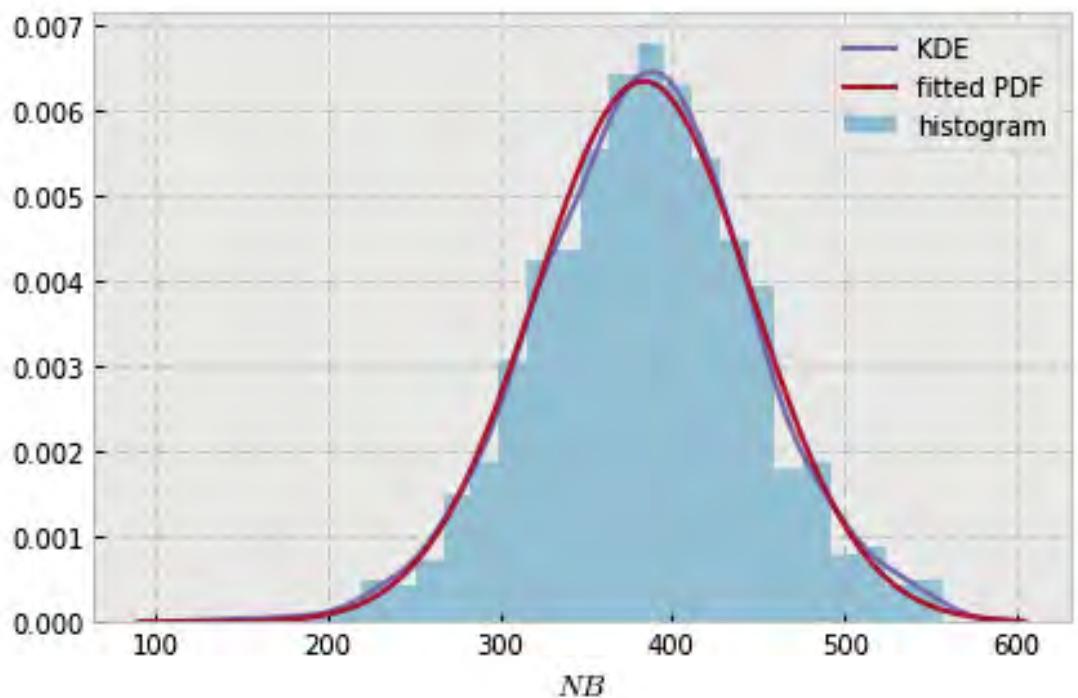


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=7691.652, scale=86.352086807442)
DescribeResult(nobs=1000, minmax=(7412.0, 7956.0)
    mean=7691.652, variance=7464.1470430430427,
    skewness=-0.06574630912998834, kurtosis=-0.08442355811346003)
```

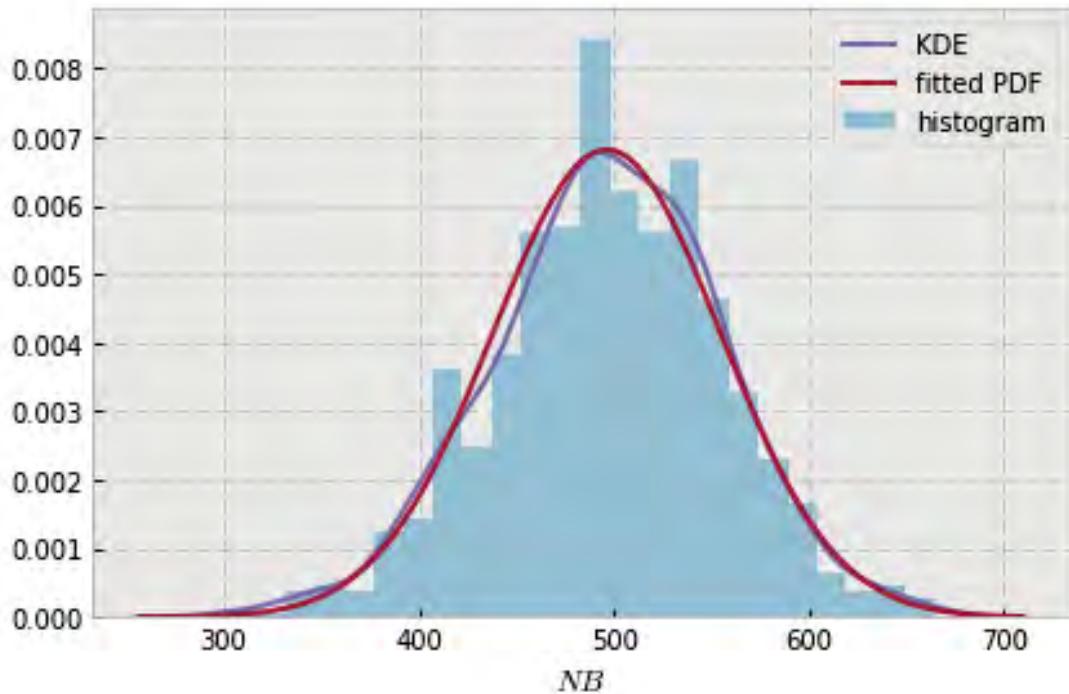


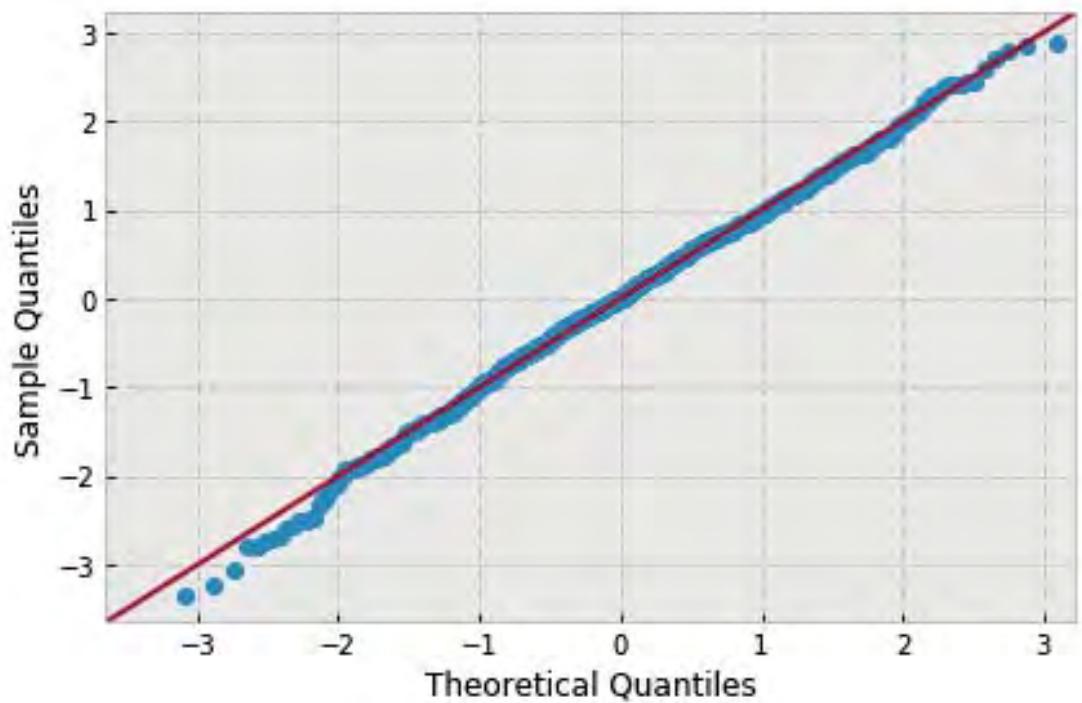


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=383.343, scale=62.93259370946028)
DescribeResult(nobs=1000, minmax=(139.0, 556.0)
               mean=383.3430000000002, variance=3964.4758268268265,
               skewness=-0.08783031725367633, kurtosis=0.25029828426630907)
```

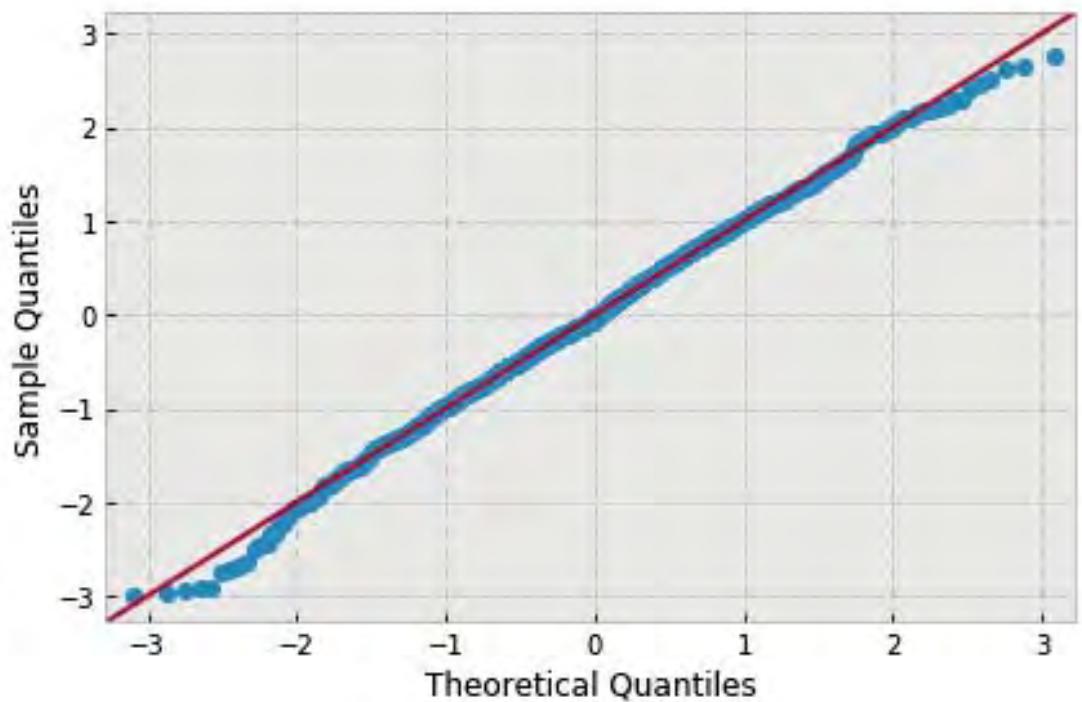
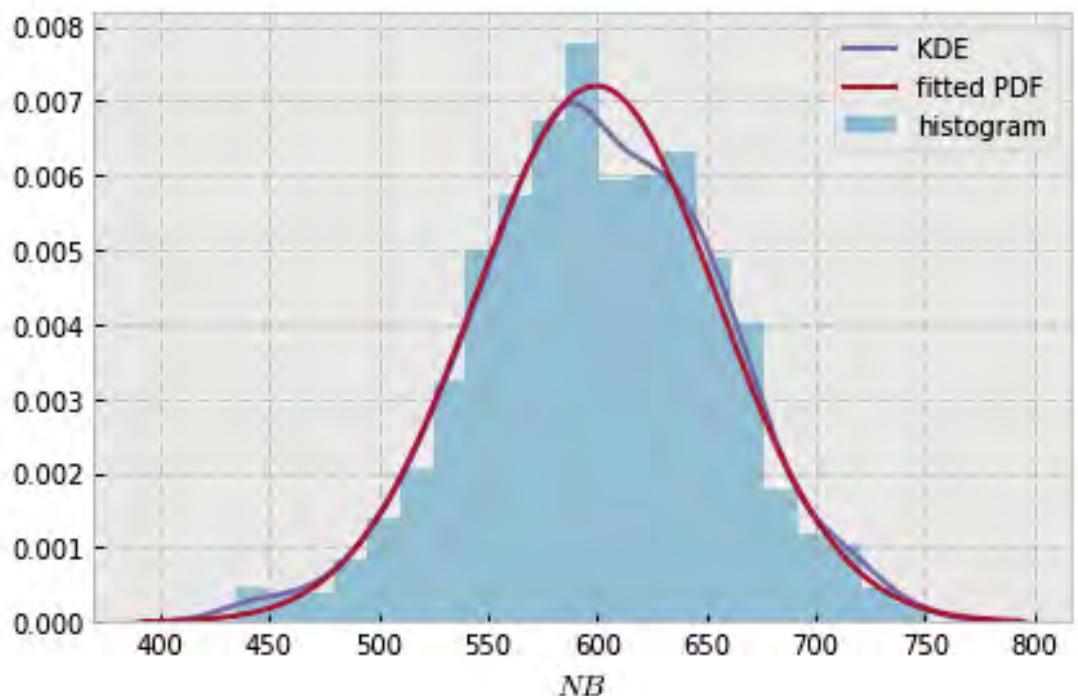


```
variable name: r
variable value: 1.222222222222223
distribution: normal(shape=(), loc=496.363, scale=58.56400969025259)
DescribeResult(nobs=1000, minmax=(301.0, 665.0)
    mean=496.363, variance=3433.1764074074072,
    skewness=-0.1531167148634229, kurtosis=0.1199001046742163)
```

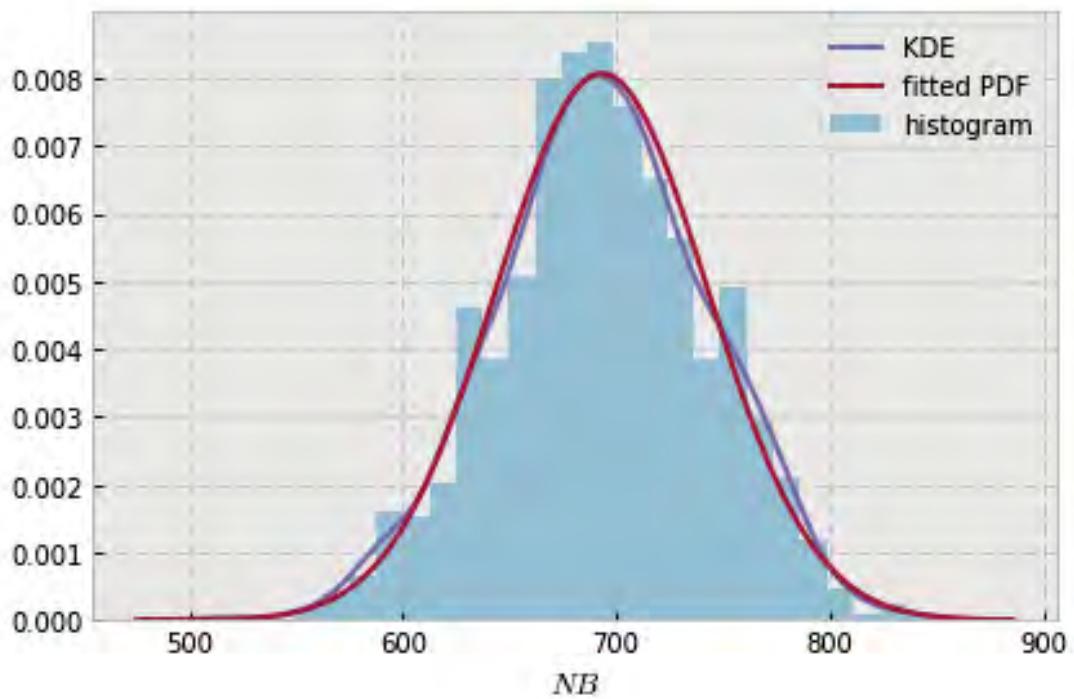


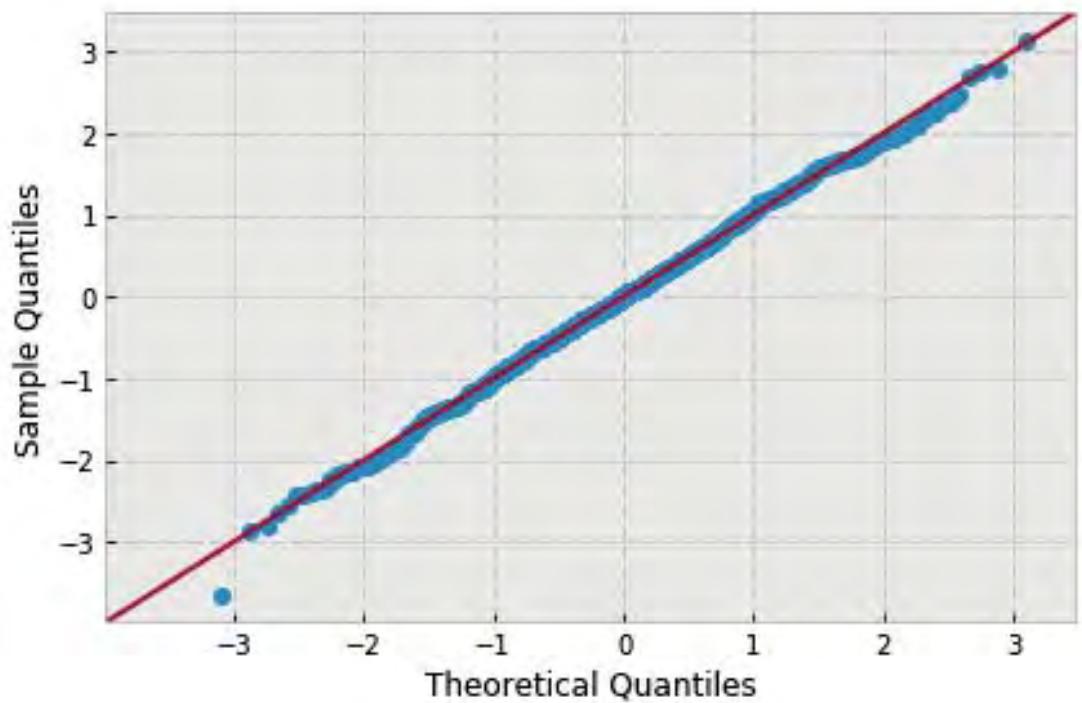


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=599.644, scale=55.37753392848042)
DescribeResult(nobs=1000, minmax=(434.0, 752.0)
               mean=599.6440000000001, variance=3069.7410050050048,
               skewness=-0.11206276161203048, kurtosis=-0.034758066601593196)
```

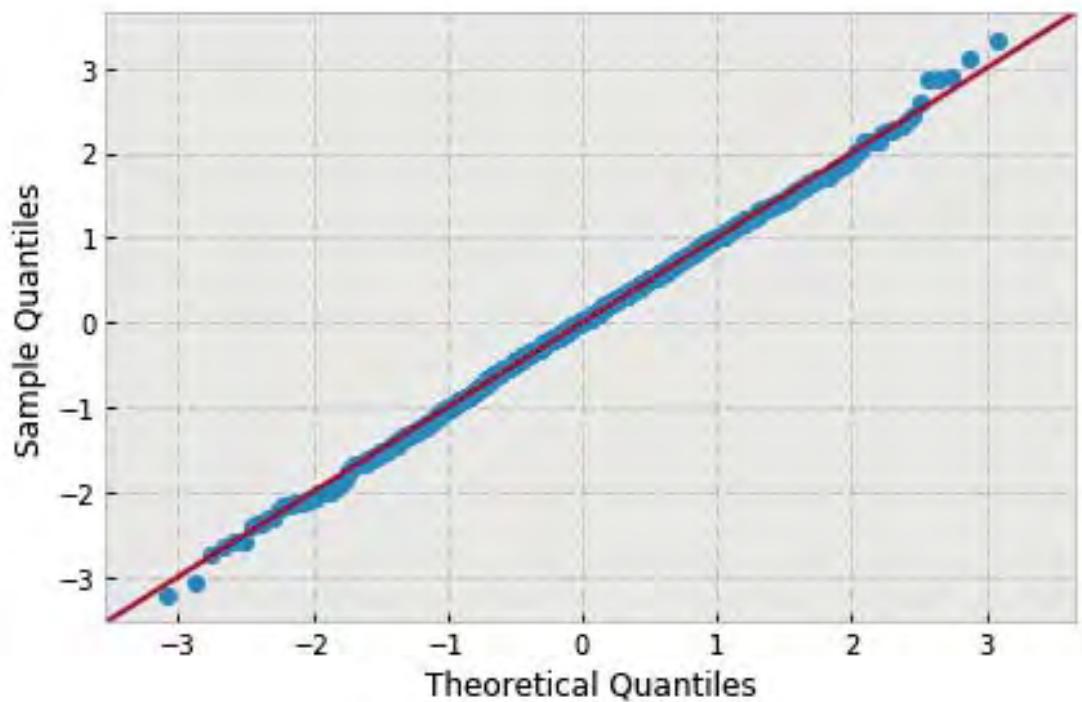
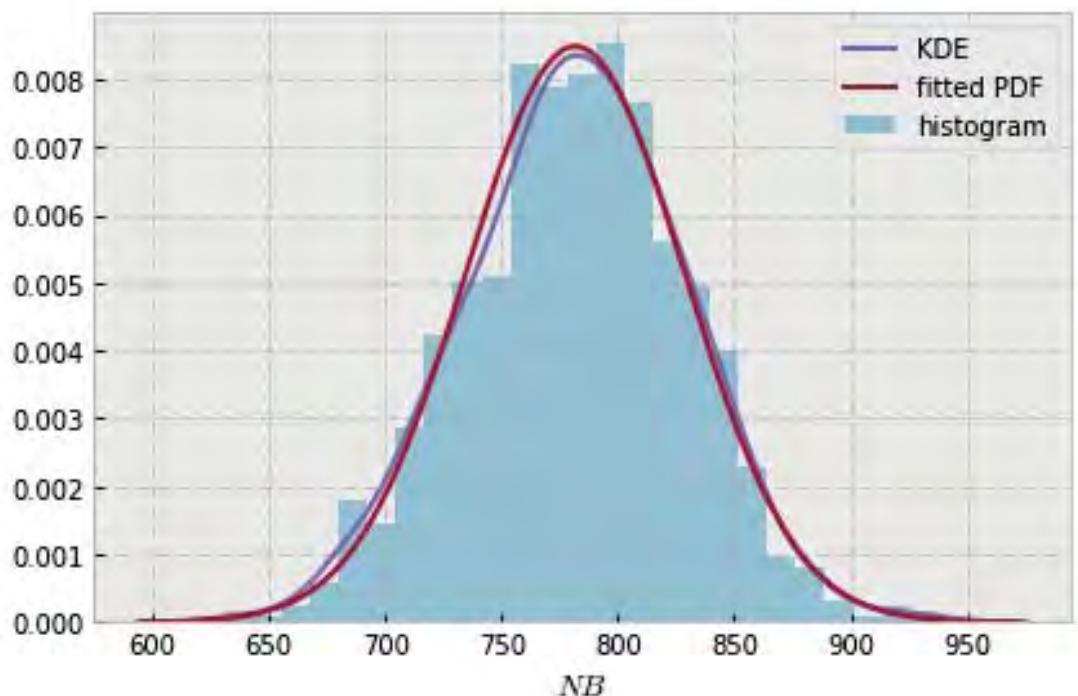


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=693.246, scale=49.46525532128587)
DescribeResult(nobs=1000, minmax=(513.0, 848.0)
    mean=693.24599999999998, variance=2449.2607447447449,
    skewness=-0.07285452182703331, kurtosis=-0.09355322698702917)
```

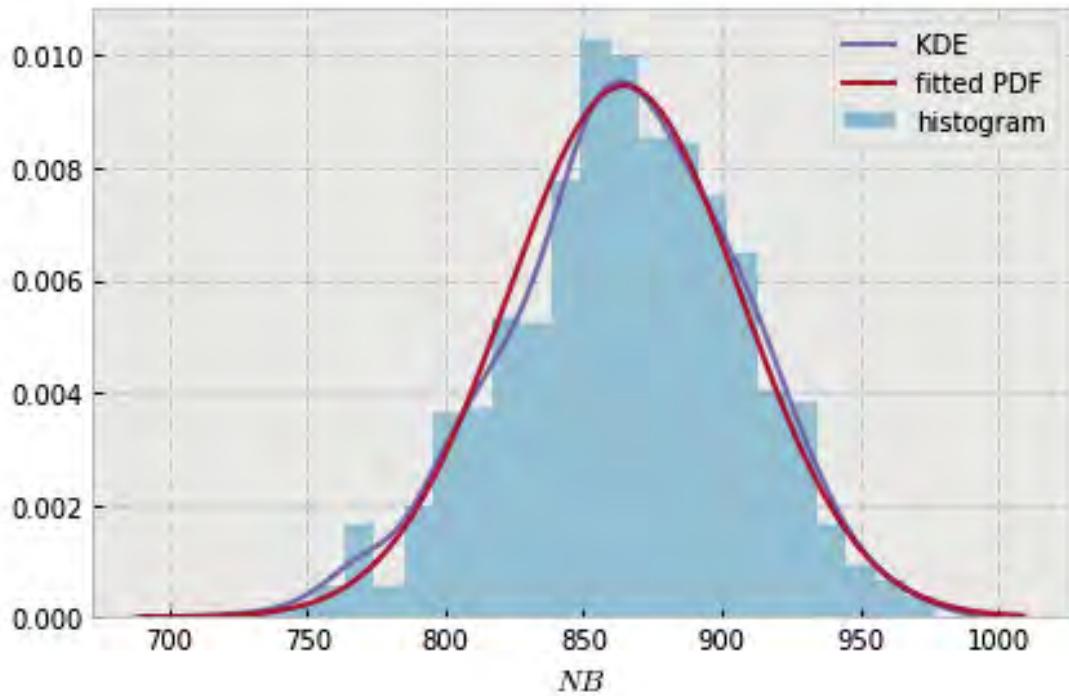


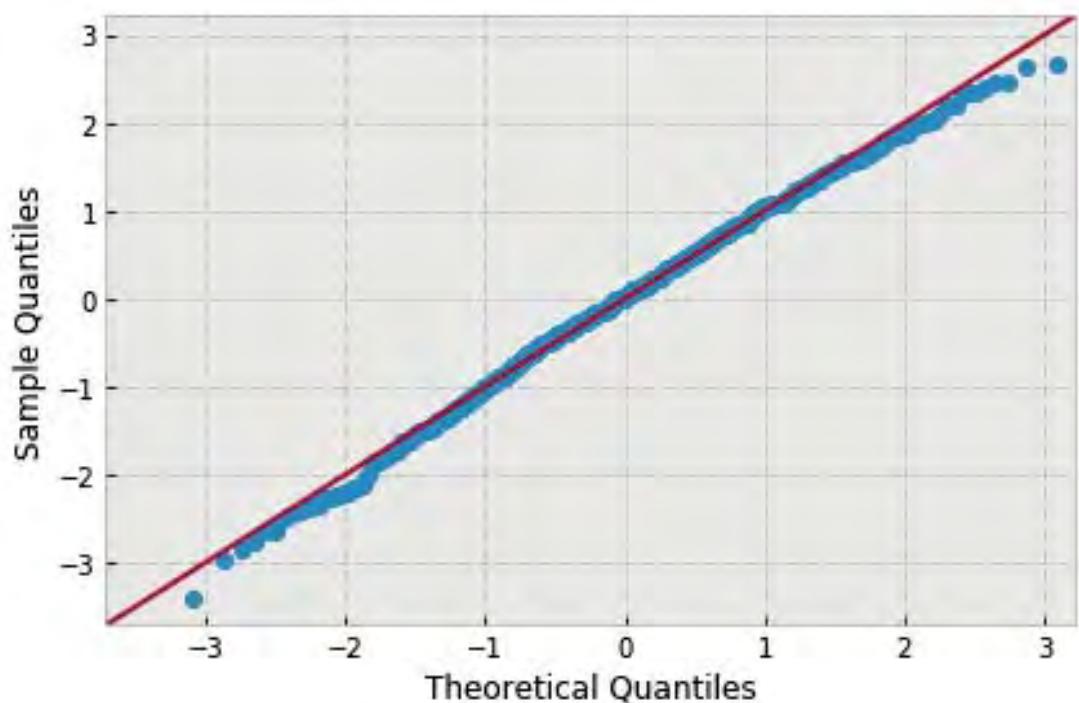


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=781.738, scale=47.00418445202512)
DescribeResult(nobs=1000, minmax=(631.0, 938.0)
               mean=781.7380000000006, variance=2211.6049609609613,
               skewness=-0.04671110838746178, kurtosis=-0.008189824907279153)
```

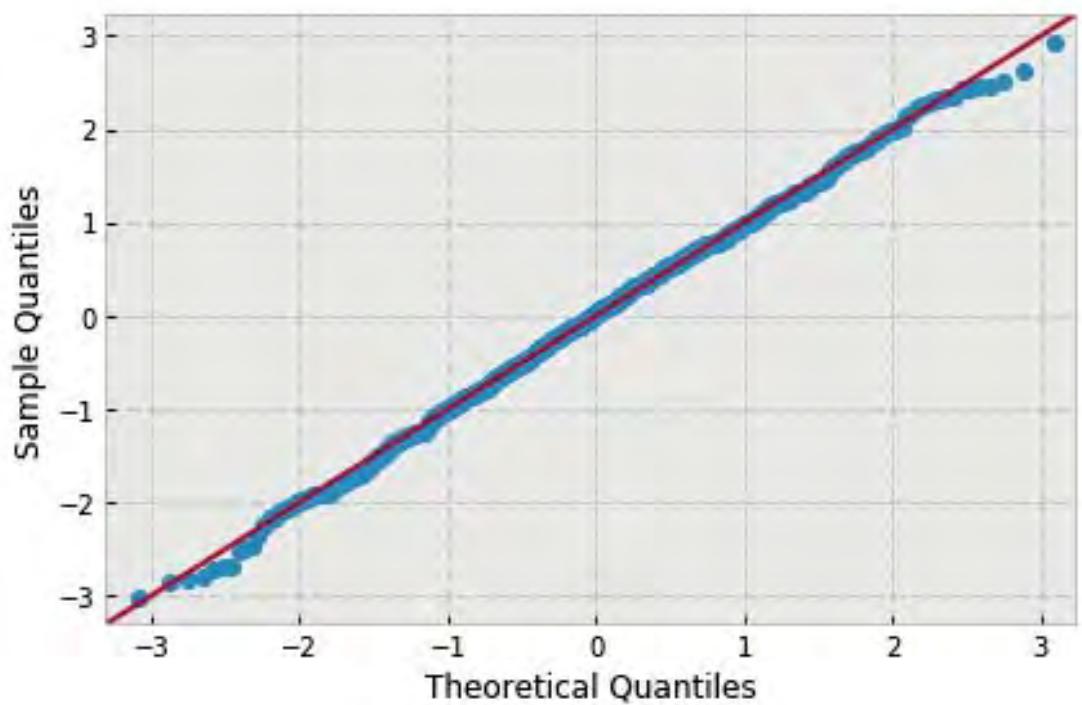
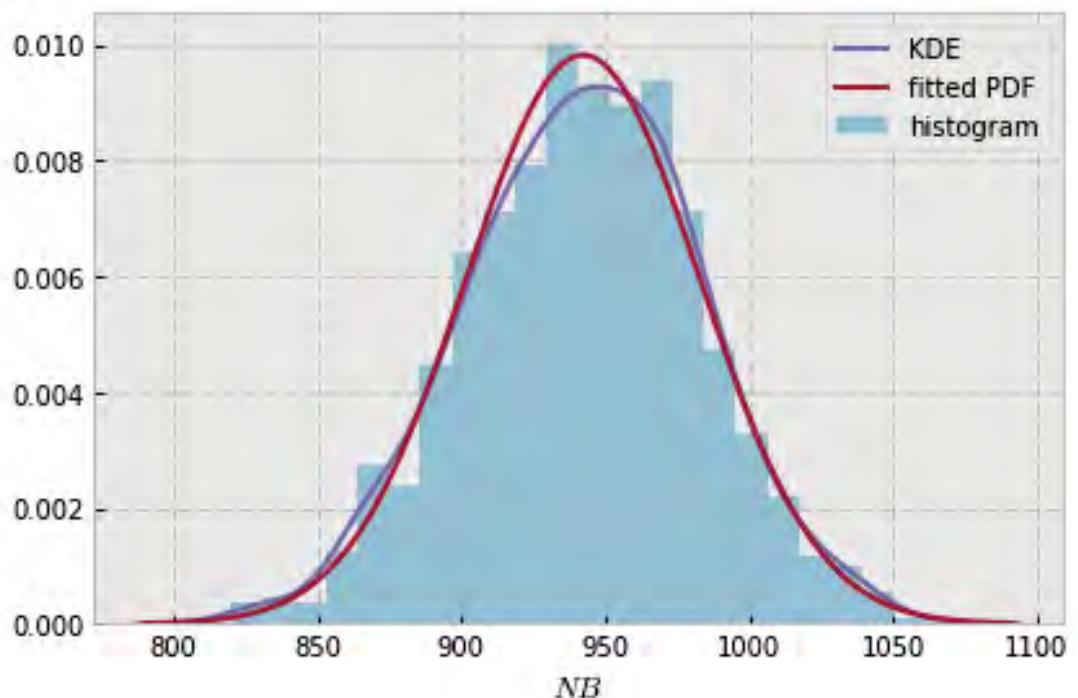


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=864.792, scale=42.15911213486356)
DescribeResult(nobs=1000, minmax=(721.0, 977.0)
    mean=864.7920000000003, variance=1779.169905905906,
    skewness=-0.2044947772622384, kurtosis=-0.08557391822357419)
```

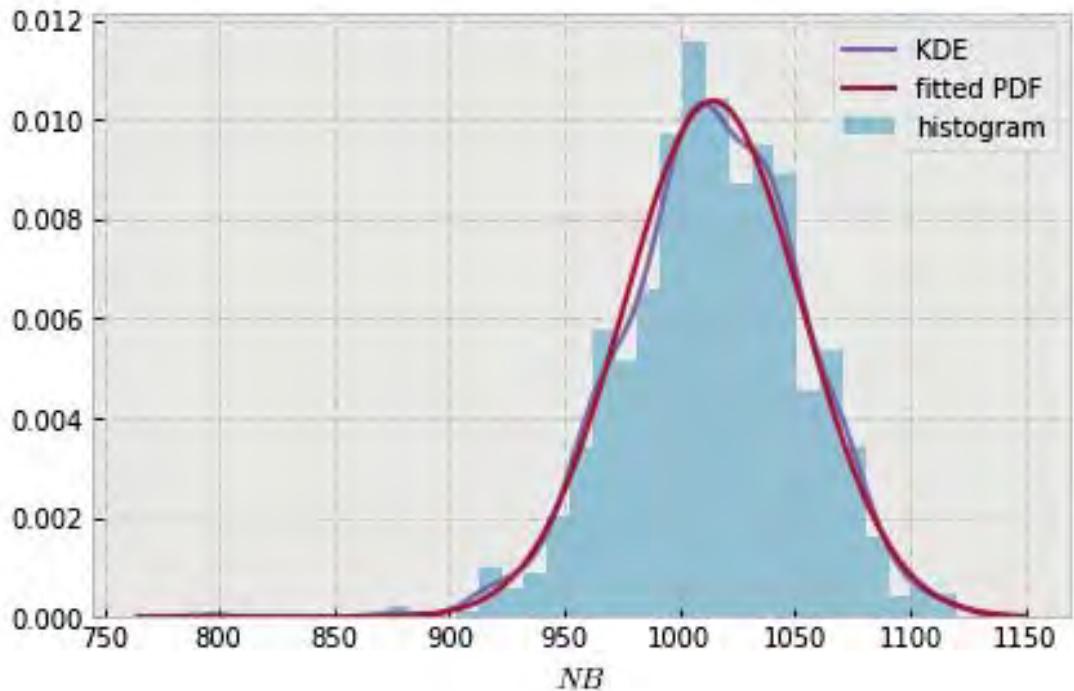


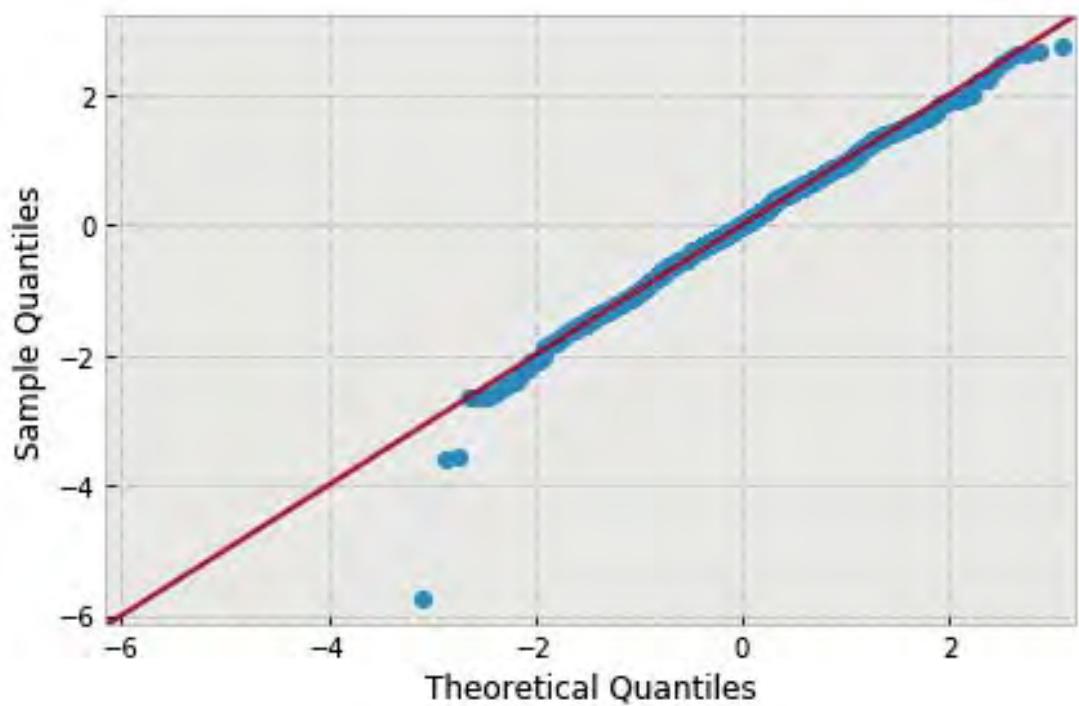


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=942.21, scale=40.61612856981817)
DescribeResult(nobs=1000, minmax=(820.0, 1061.0)
               mean=942.2100000000004, variance=1651.3212212212211,
               skewness=-0.09269976444565865, kurtosis=-0.12501356055381008)
```

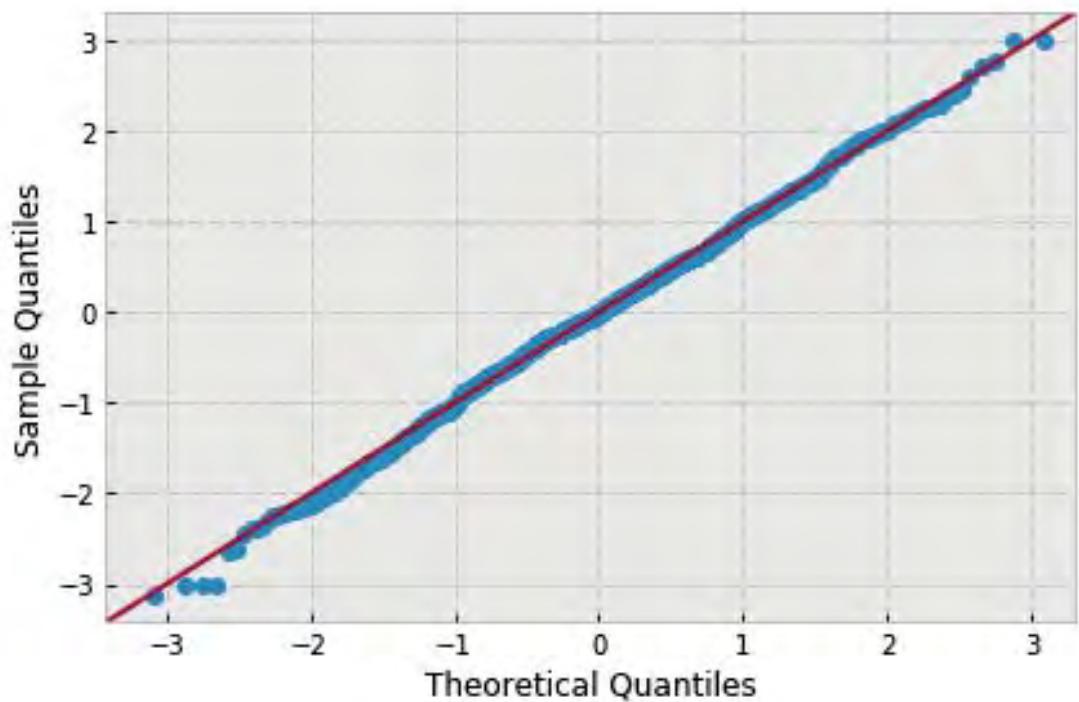
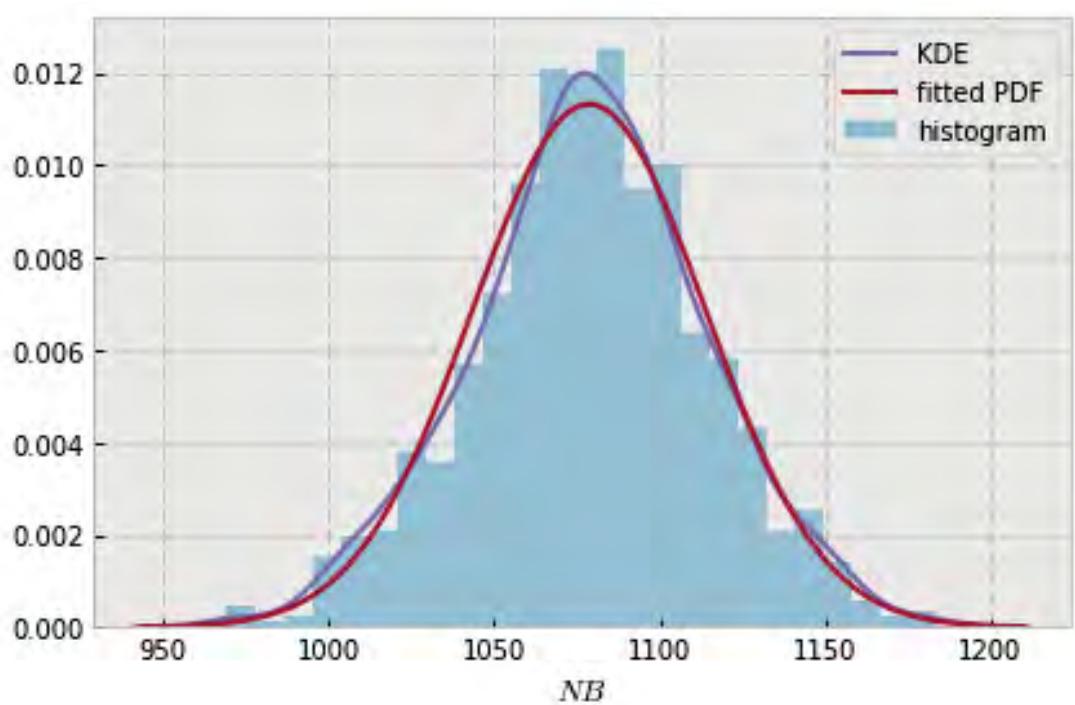


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=1014.355, scale=38.479851545971435)
DescribeResult(nobs=1000, minmax=(794.0, 1120.0)
    mean=1014.355, variance=1482.1811561561562,
    skewness=-0.32732736064385143, kurtosis=0.9188522852271306)
```

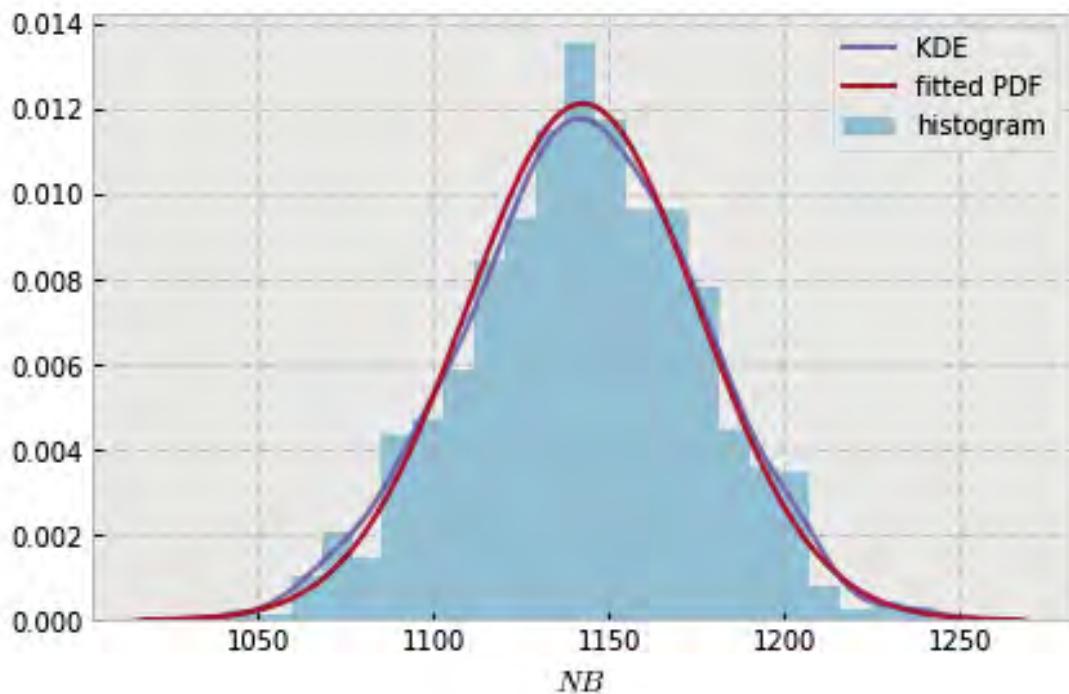


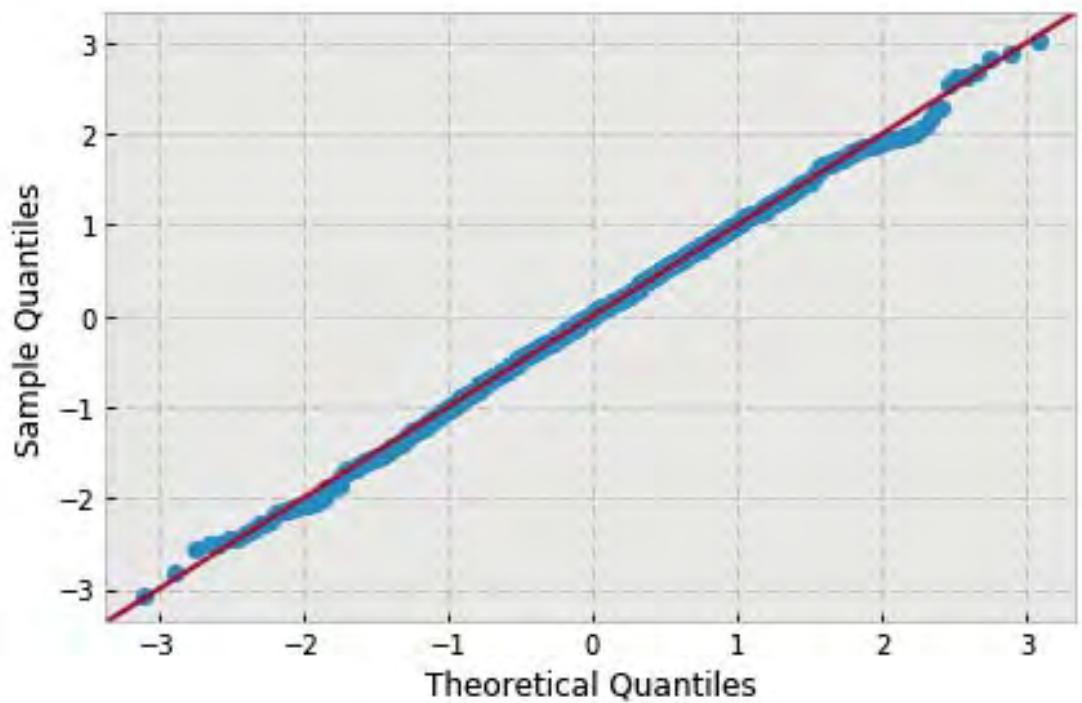


```
variable name: r
variable value: 2.7777777777777777
distribution: normal(shape=(), loc=1078.689, scale=35.23055888004049)
DescribeResult(nobs=1000, minmax=(969.0, 1184.0)
               mean=1078.689000000001, variance=1242.4347137137138,
               skewness=-0.06904495035345062, kurtosis=0.06237211002773835)
```

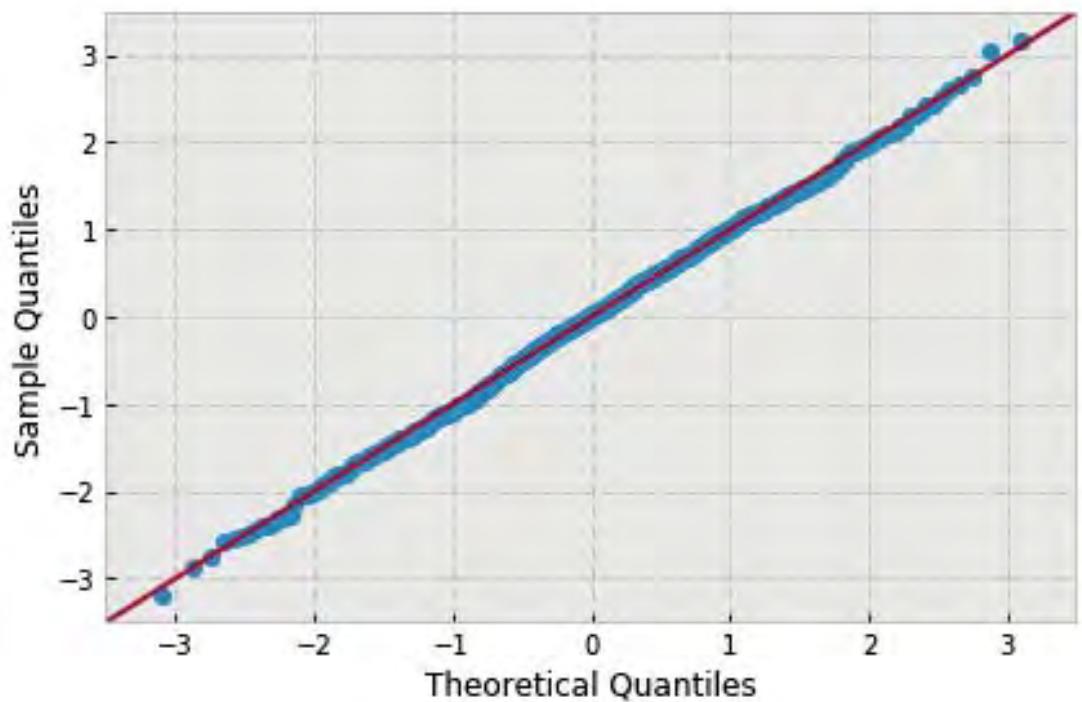
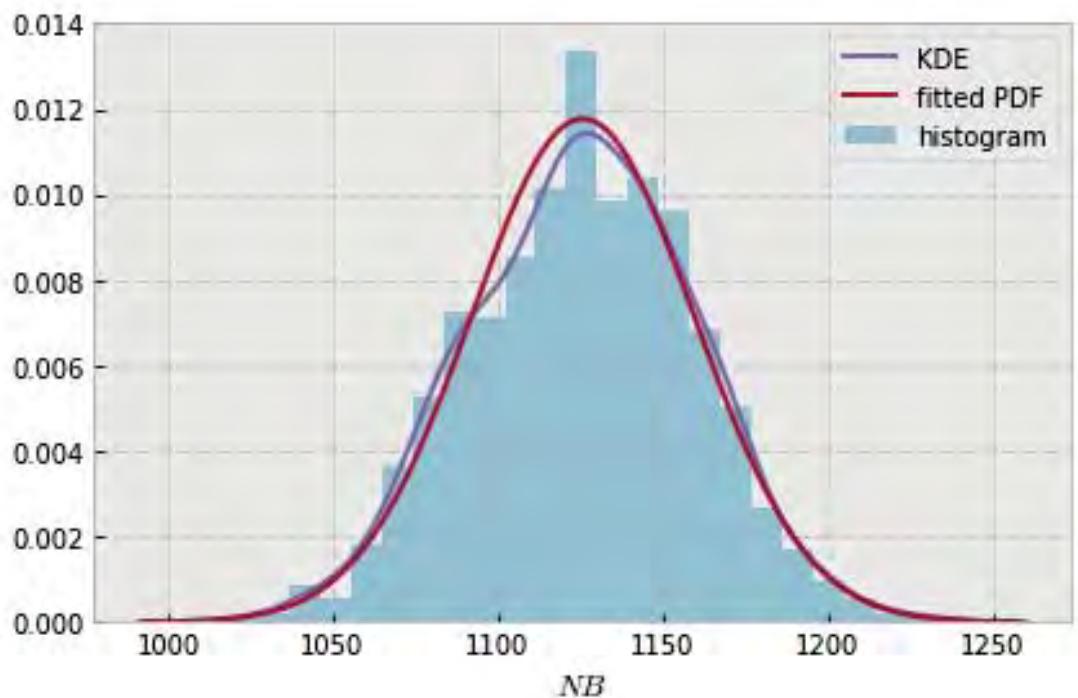


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=1142.544, scale=32.92521927034048)
DescribeResult(nobs=1000, minmax=(1042.0, 1242.0)
    mean=1142.544000000001, variance=1085.1552192192191,
    skewness=-0.05579009020026107, kurtosis=-0.17150475882089466)
```

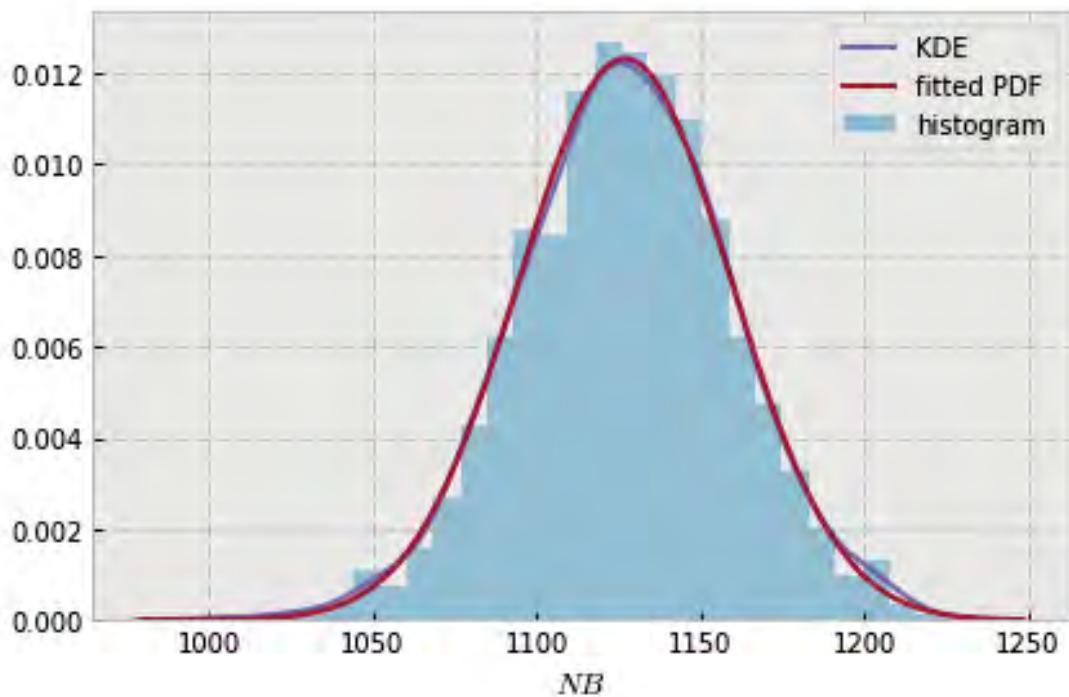


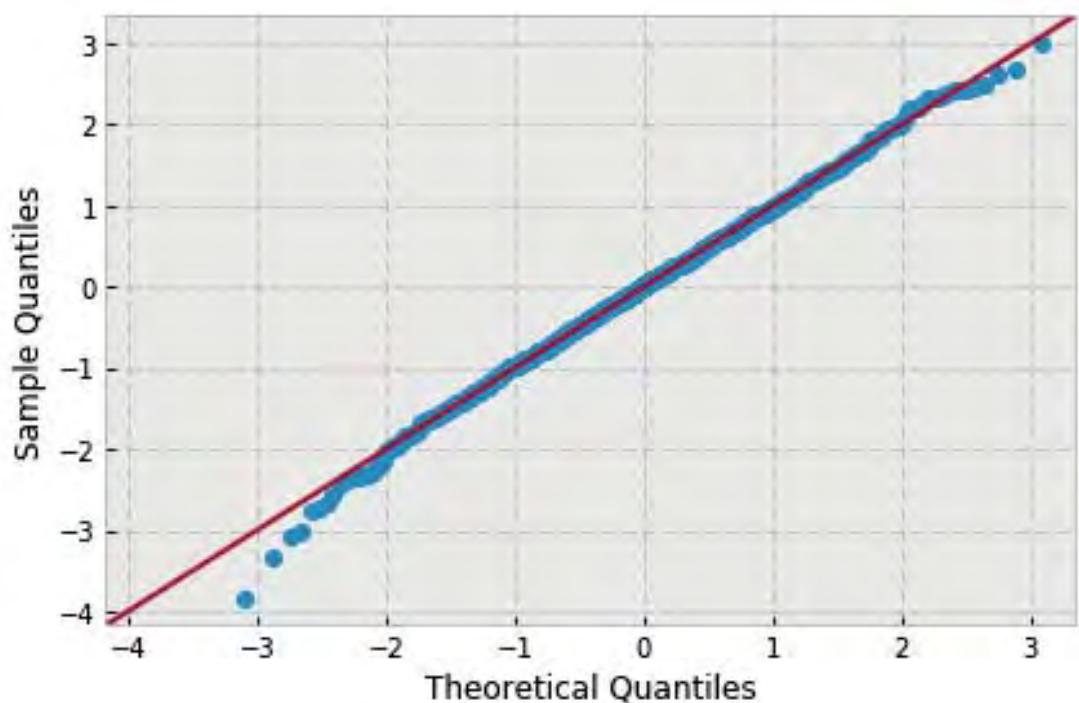


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=1125.81, scale=33.887282275213515)
DescribeResult(nobs=1000, minmax=(1018.0, 1233.0)
               mean=1125.809999999999, variance=1149.4973973973974,
               skewness=-0.040734876247970744, kurtosis=-0.1545700909627823)
```

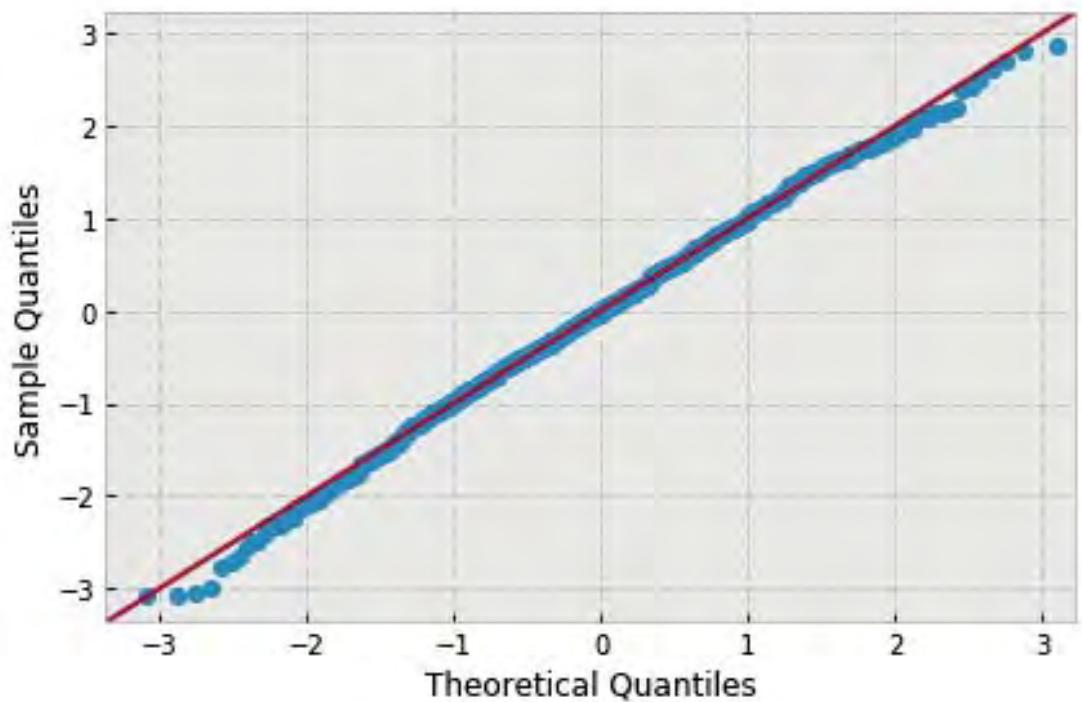
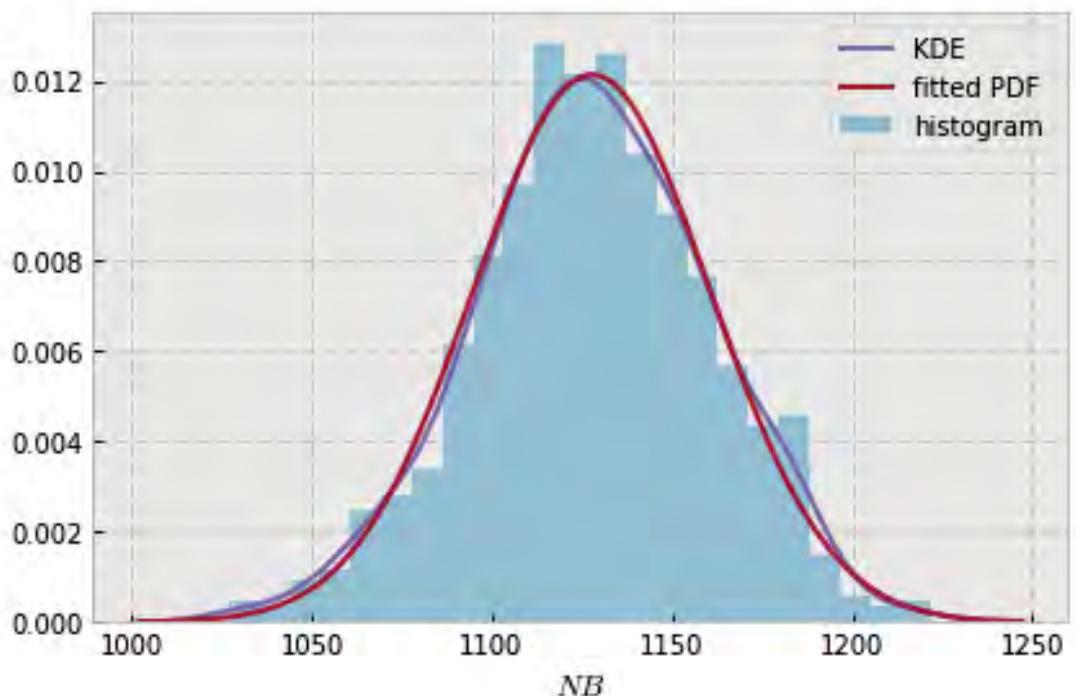


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=1126.968, scale=32.369877602487165)
DescribeResult(nobs=1000, minmax=(1003.0, 1224.0)
    mean=1126.9680000000001, variance=1048.8578338338339,
    skewness=-0.09439785318895536, kurtosis=0.1829447817911256)
```

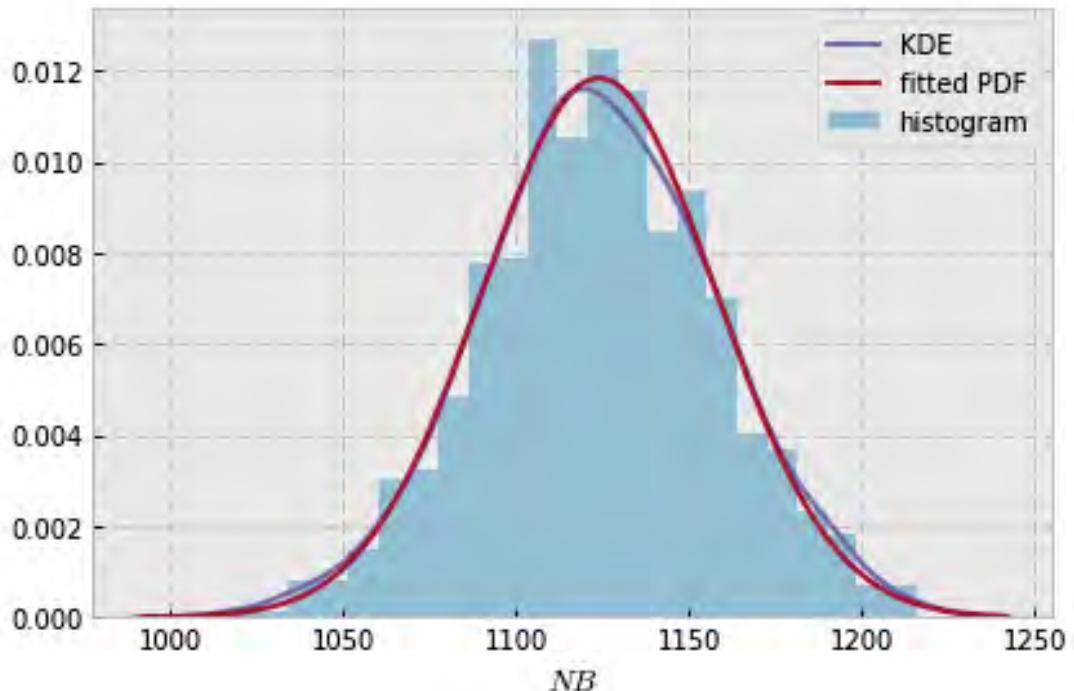


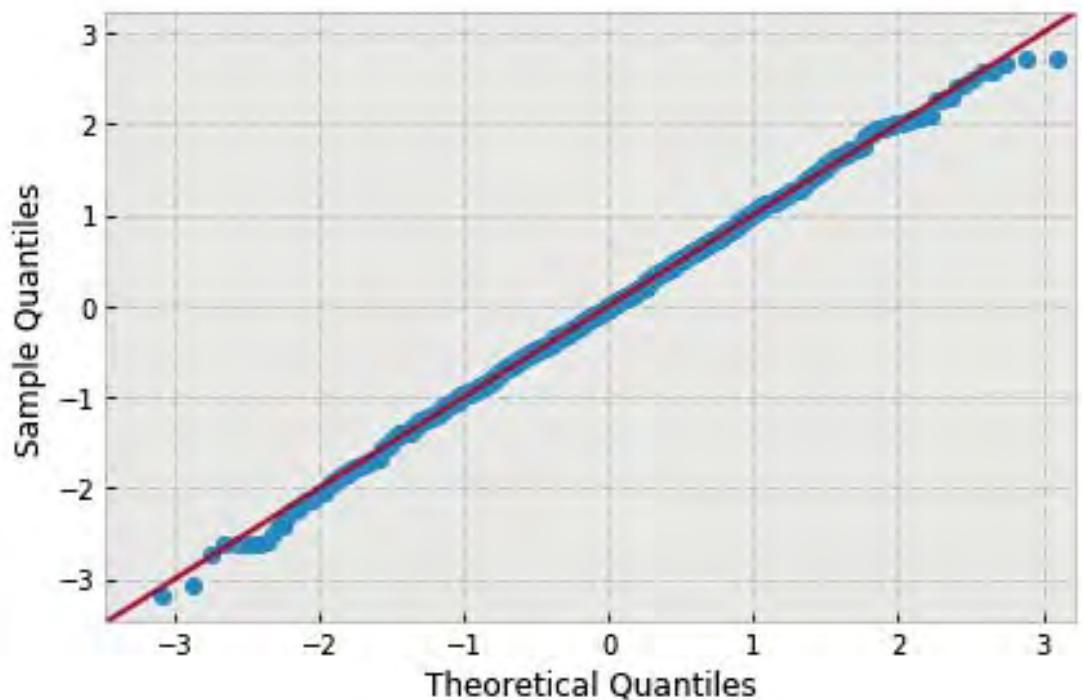


```
variable name: sigma
variable value: 0.03333333333333333
distribution: normal(shape=(), loc=1127.744, scale=32.87607738158553)
DescribeResult(nobs=1000, minmax=(1027.0, 1222.0)
               mean=1127.743999999999, variance=1081.9183823823823,
               skewness=-0.11655800281377422, kurtosis=-0.031236712500033903)
```

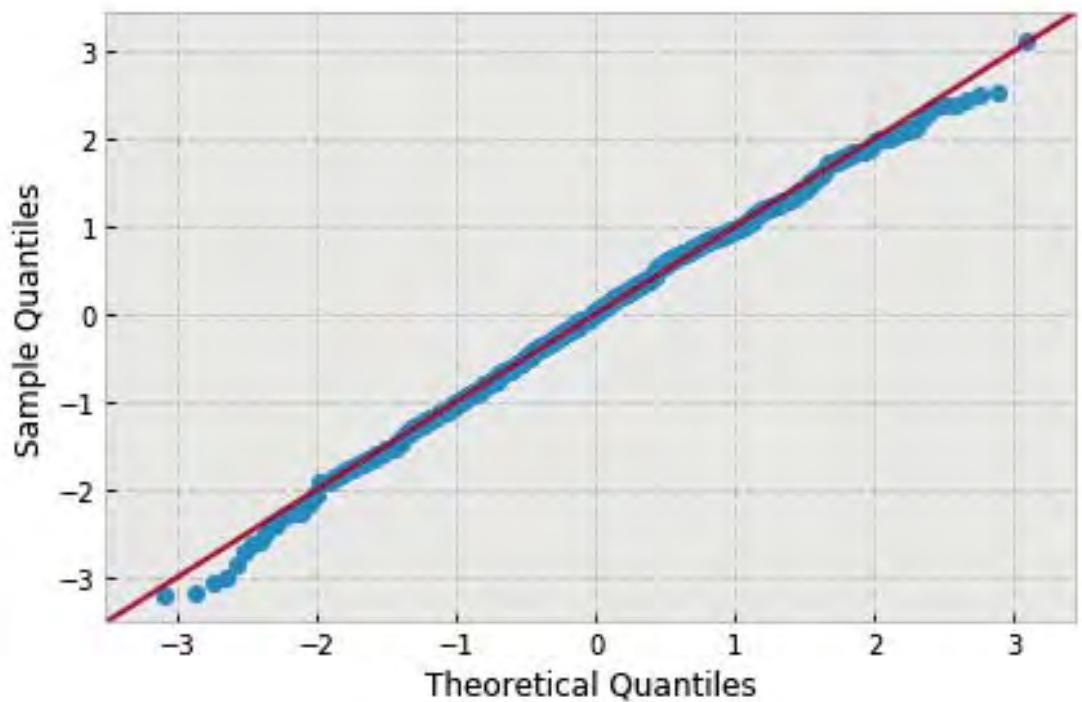
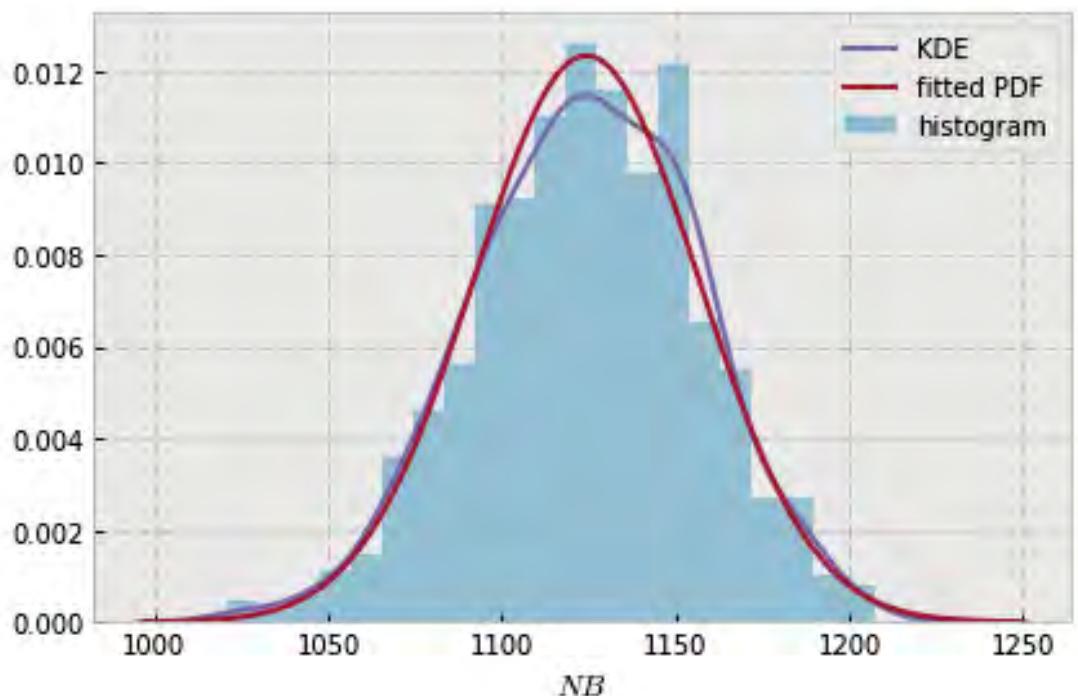


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=1124.12, scale=33.71079352373658)
DescribeResult(nobs=1000, minmax=(1017.0, 1216.0)
    mean=1124.119999999999, variance=1137.5551551551553,
    skewness=-0.04018597198919402, kurtosis=-0.06693167662379951)
```

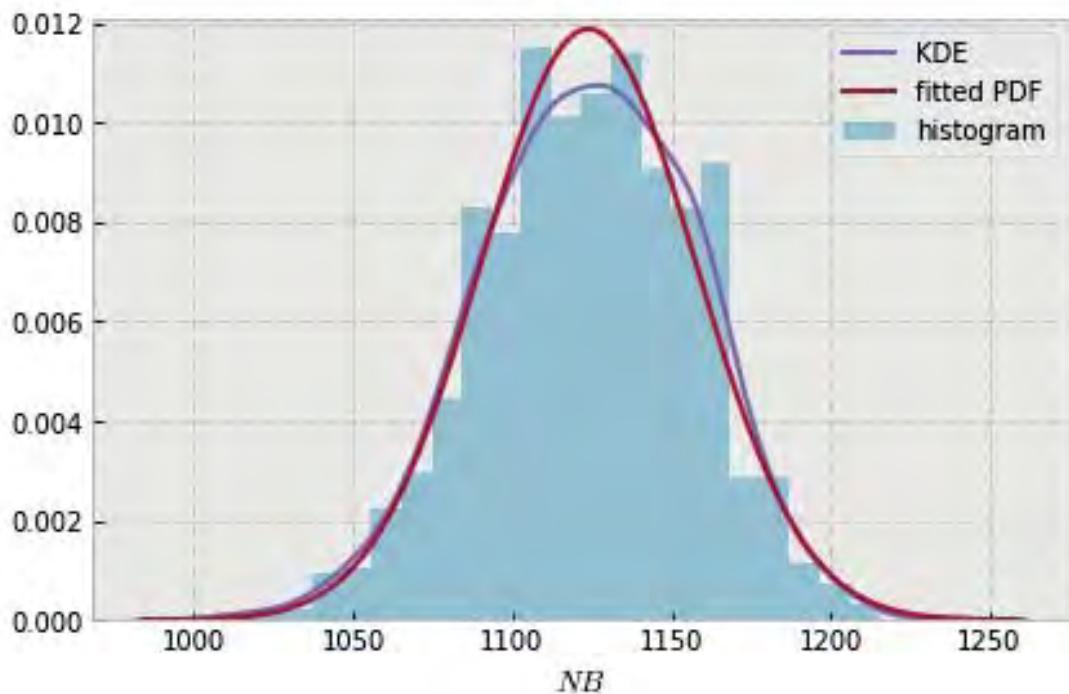


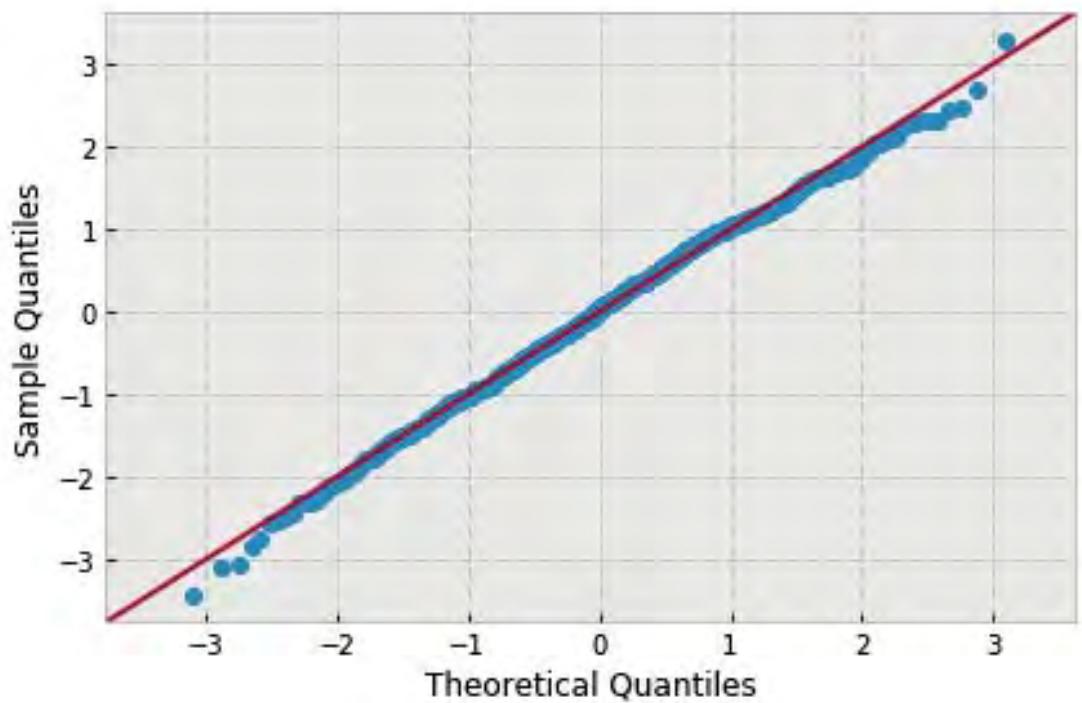


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=1124.321, scale=32.34130422540192)
DescribeResult(nobs=1000, minmax=(1021.0, 1225.0)
               mean=1124.320999999999, variance=1047.0069659659659,
               skewness=-0.1446382751410086, kurtosis=-0.09347261337961354)
```

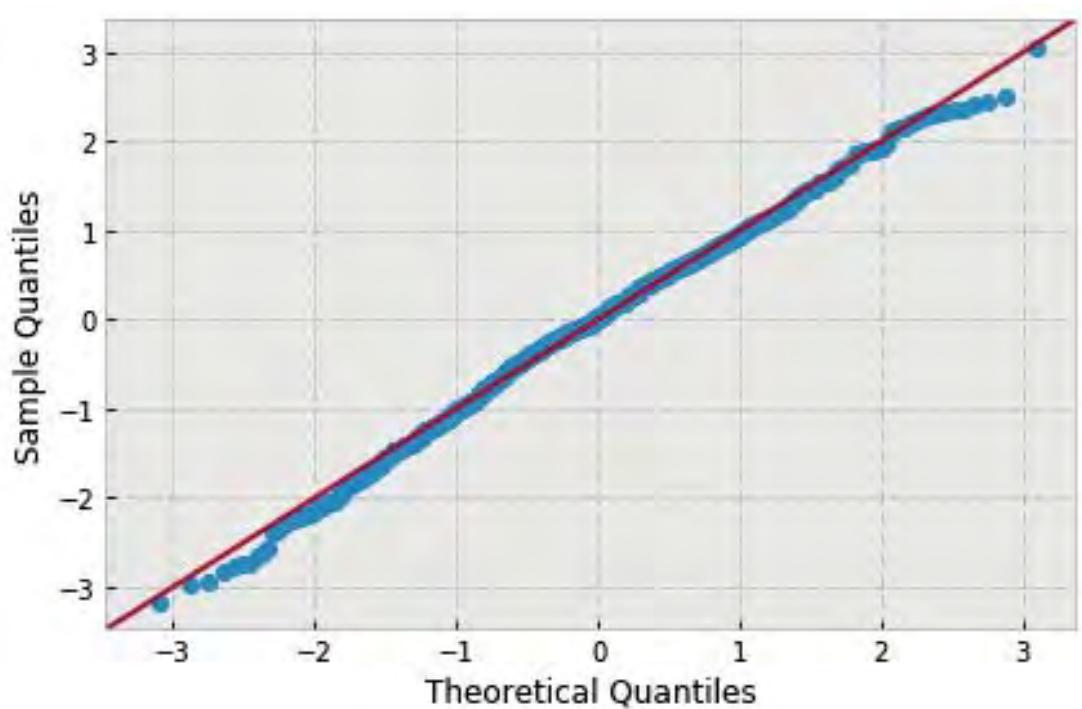
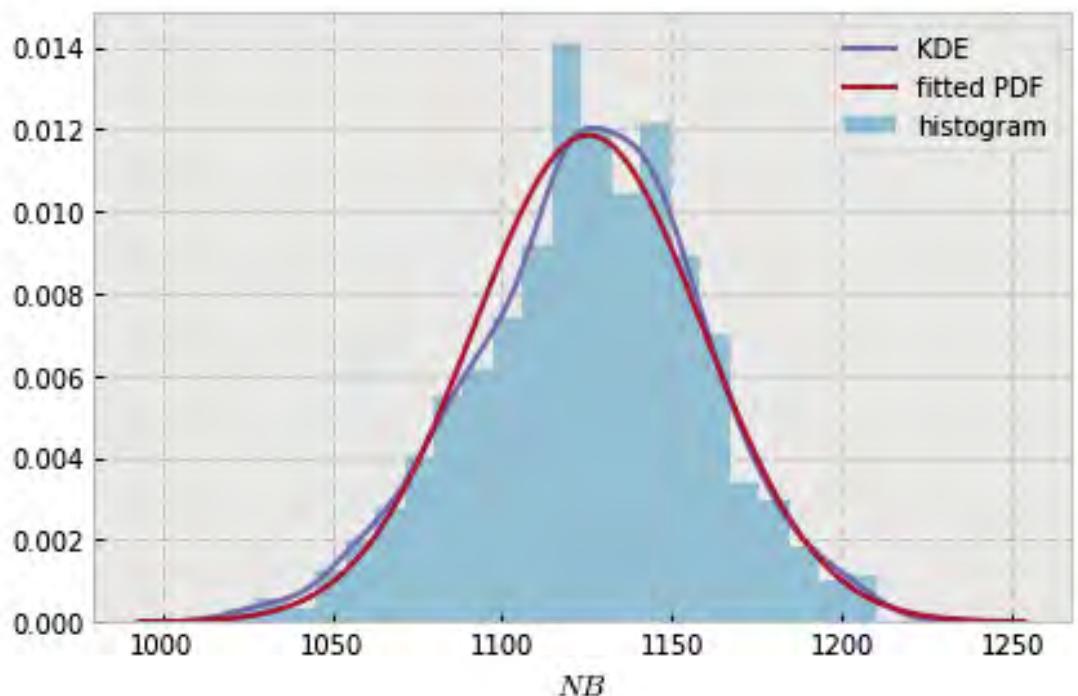


```
variable name: sigma
variable value: 0.066666666666666667
distribution: normal(shape=(), loc=1123.887, scale=33.59265144343328)
DescribeResult(nobs=1000, minmax=(1009.0, 1234.0)
    mean=1123.886999999999, variance=1129.5958268268271,
    skewness=-0.14428918900665513, kurtosis=-0.13127712639115652)
```

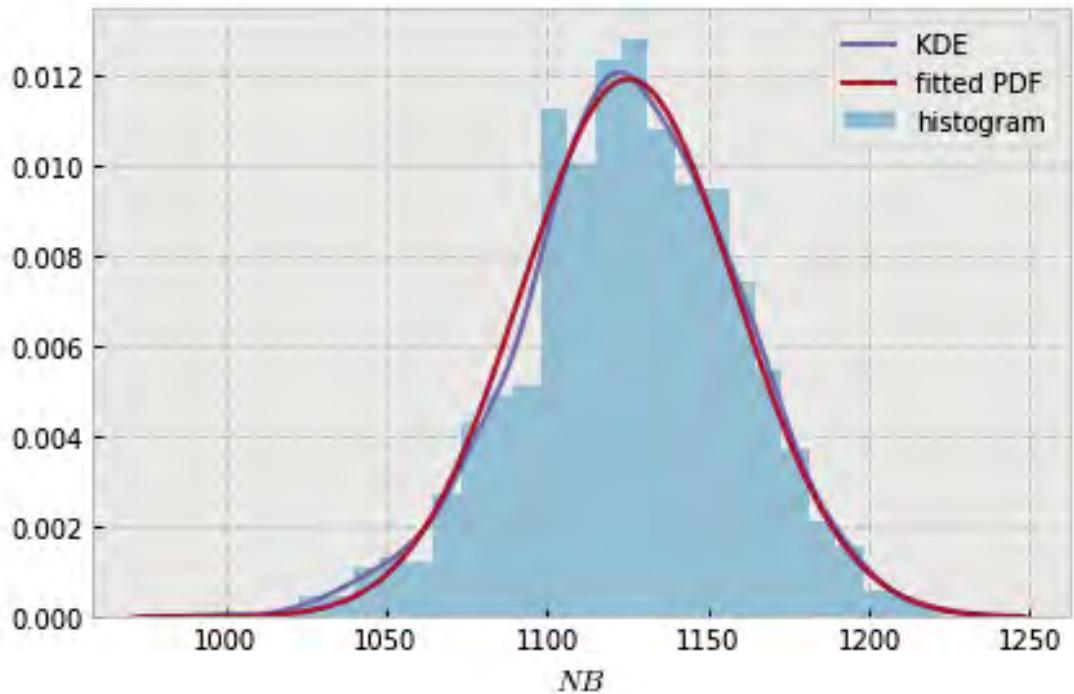


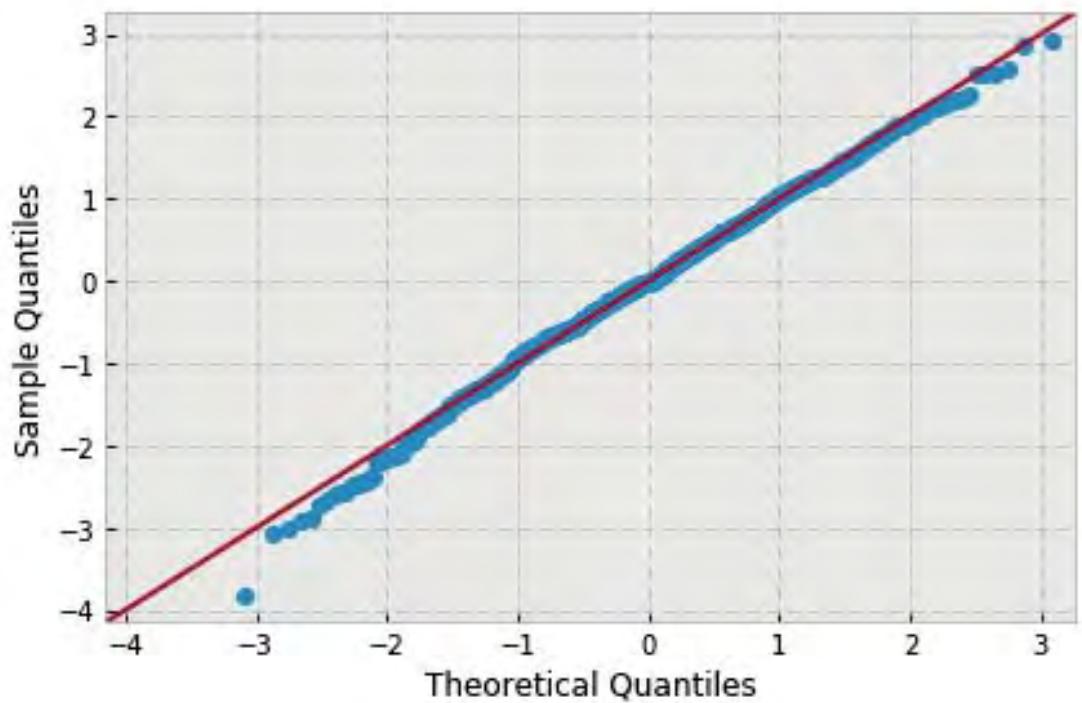


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=1125.403, scale=33.642244143338594)
DescribeResult(nobs=1000, minmax=(1019.0, 1228.0)
               mean=1125.403, variance=1132.9335245245245,
               skewness=-0.19099742125188365, kurtosis=0.0259460957522597)
```

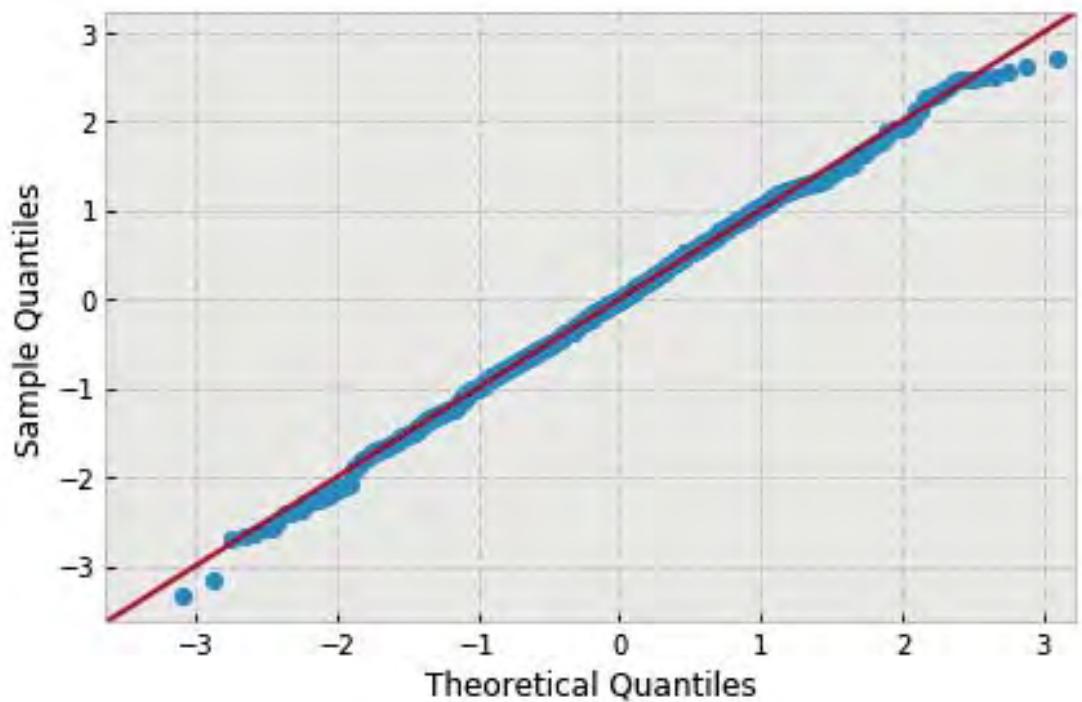
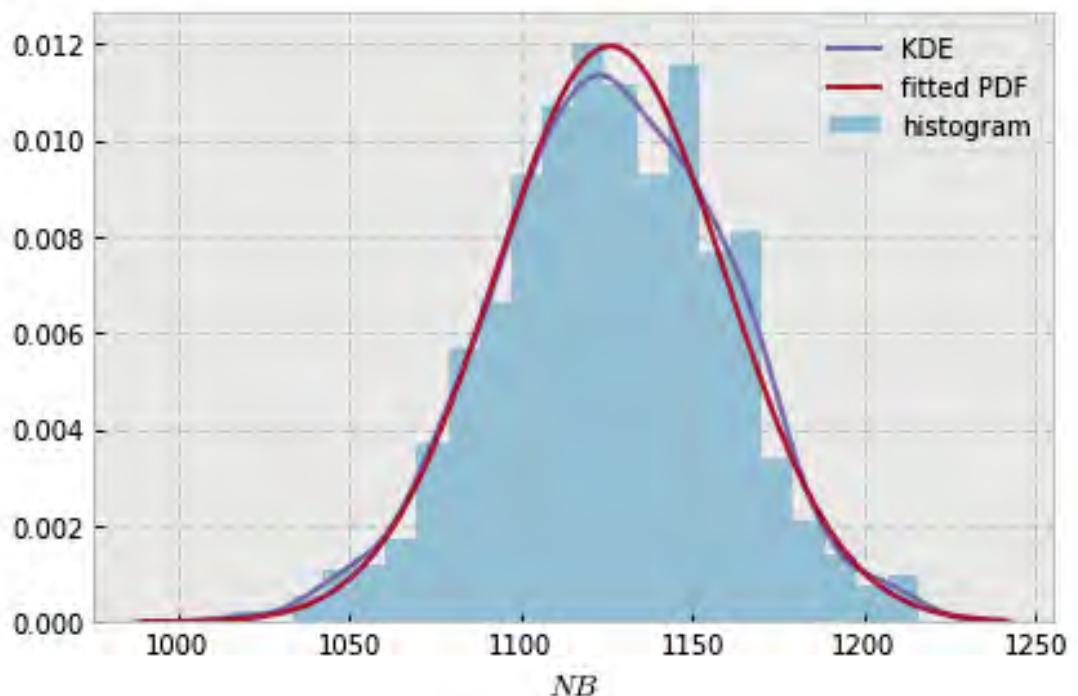


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=1125.407, scale=33.43222025232545)
DescribeResult(nobs=1000, minmax=(998.0, 1223.0)
    mean=1125.4069999999999, variance=1118.8321831831831,
    skewness=-0.1947796711059768, kurtosis=0.1687175173532509)
```

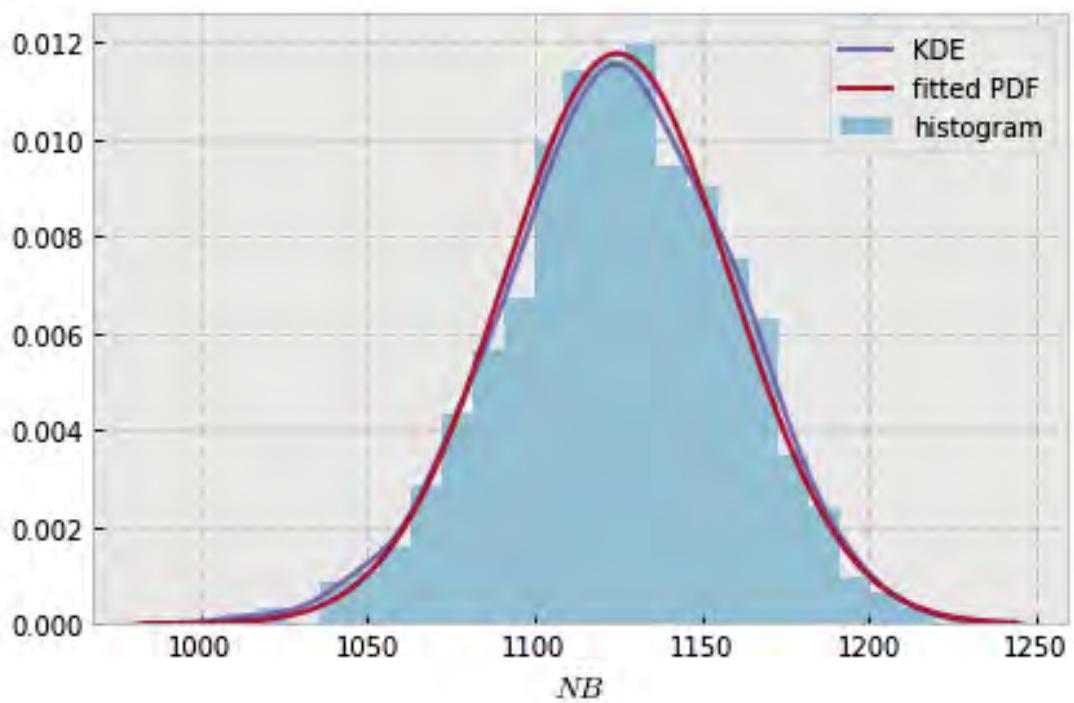


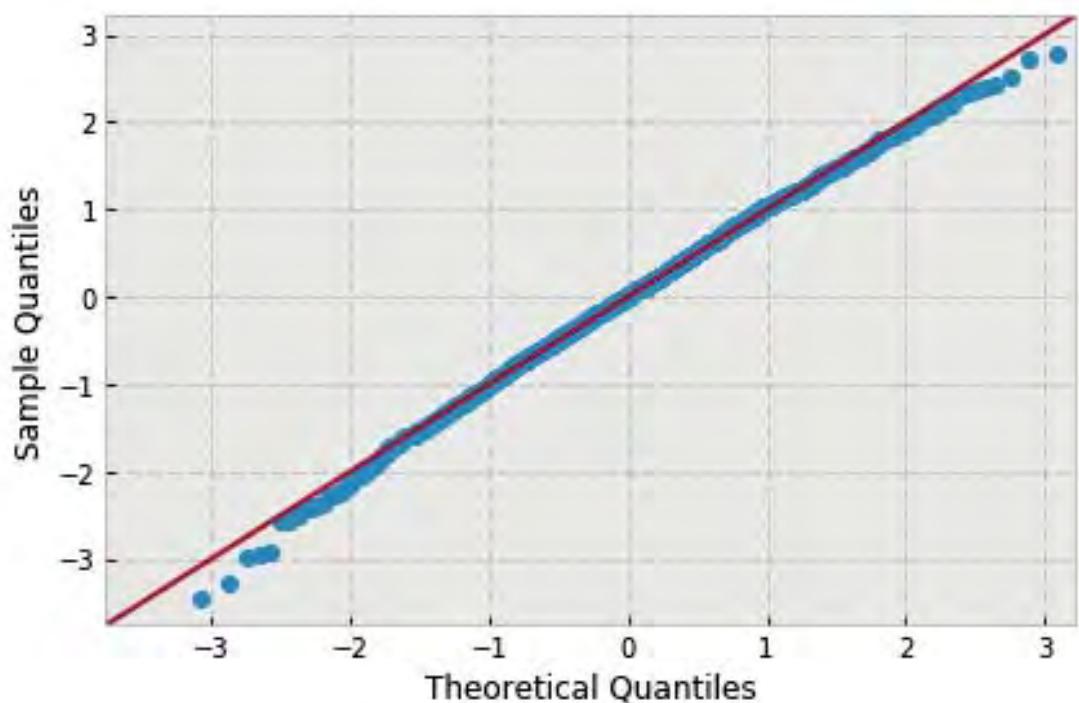


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=1126.156, scale=33.367254367118676)
DescribeResult(nobs=1000, minmax=(1015.0, 1216.0)
               mean=1126.155999999999, variance=1114.4881521521525,
               skewness=-0.09512224845291055, kurtosis=-0.10404193744004253)
```

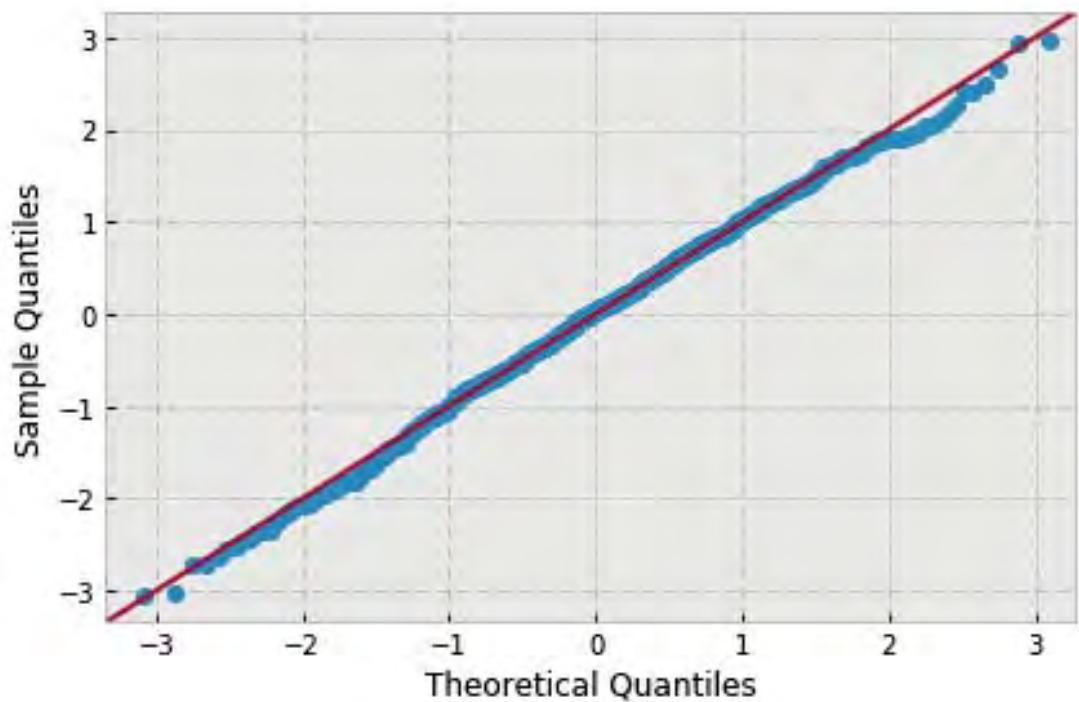
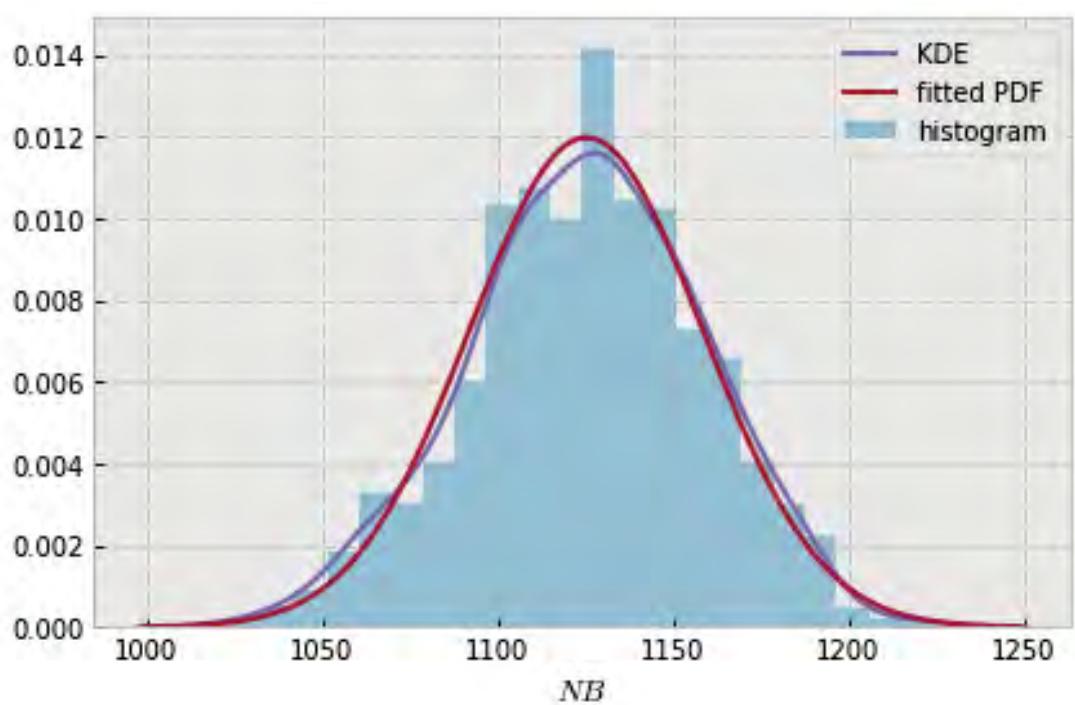


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=1124.84, scale=33.87078977526211)
DescribeResult(nobs=1000, minmax=(1008.0, 1219.0)
    mean=1124.839999999999, variance=1148.378778778779,
    skewness=-0.1646716565367034, kurtosis=-0.030850689612061277)
```

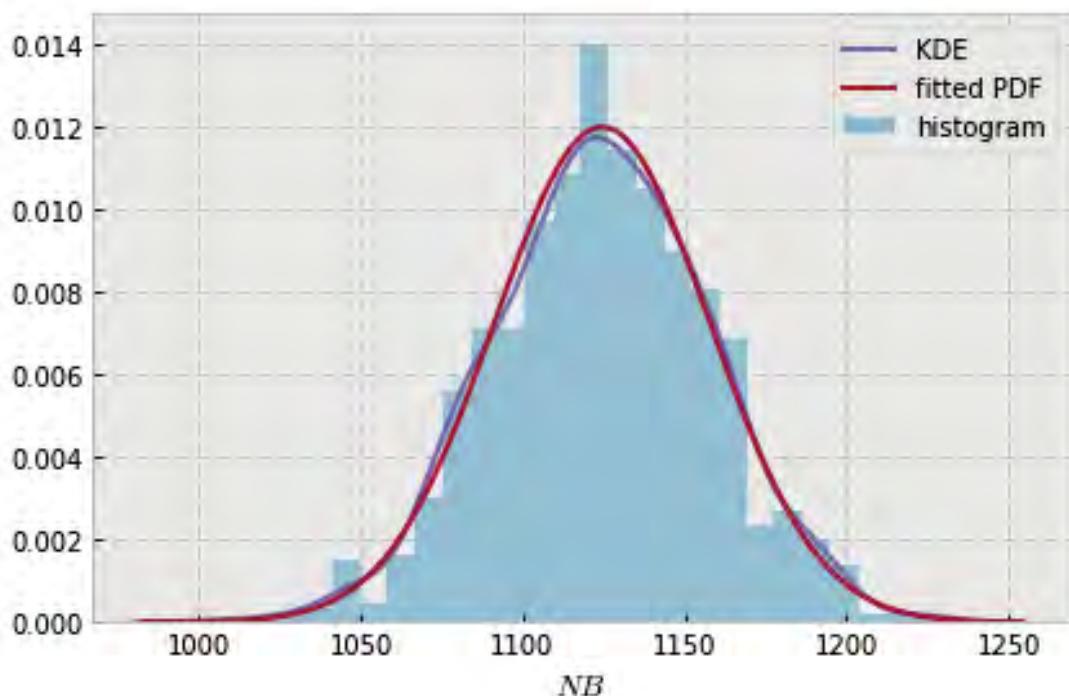


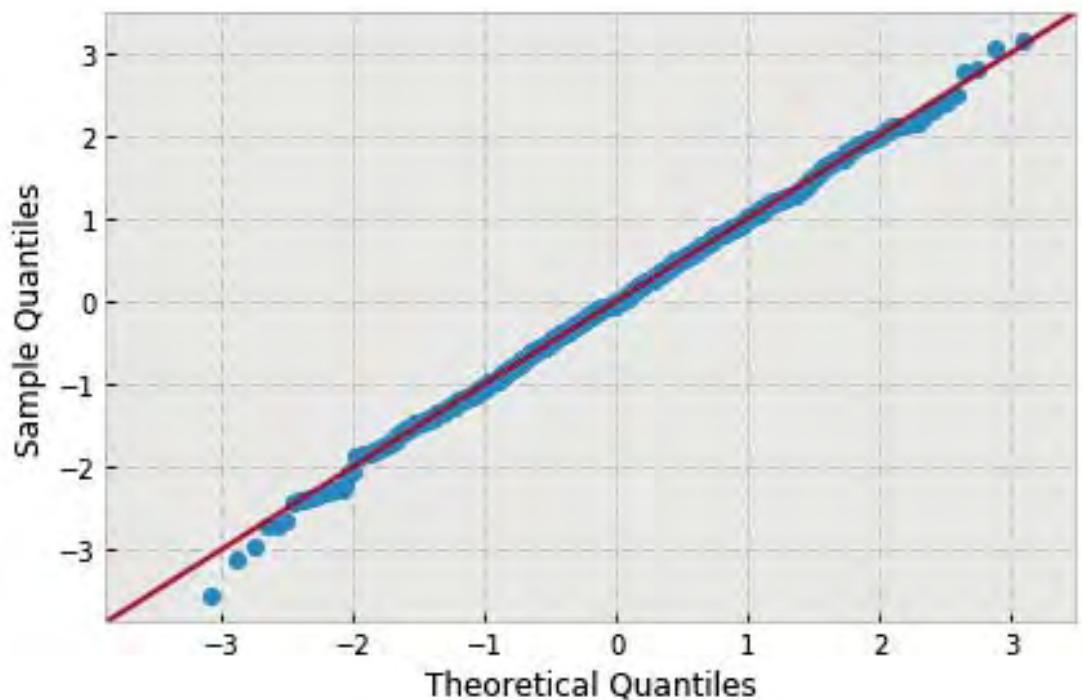


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=1125.233, scale=33.312650915230385)
DescribeResult(nobs=1000, minmax=(1024.0, 1224.0)
               mean=1125.232999999999, variance=1110.8435545545544,
               skewness=-0.1330176477832919, kurtosis=-0.15418822339204485)
```

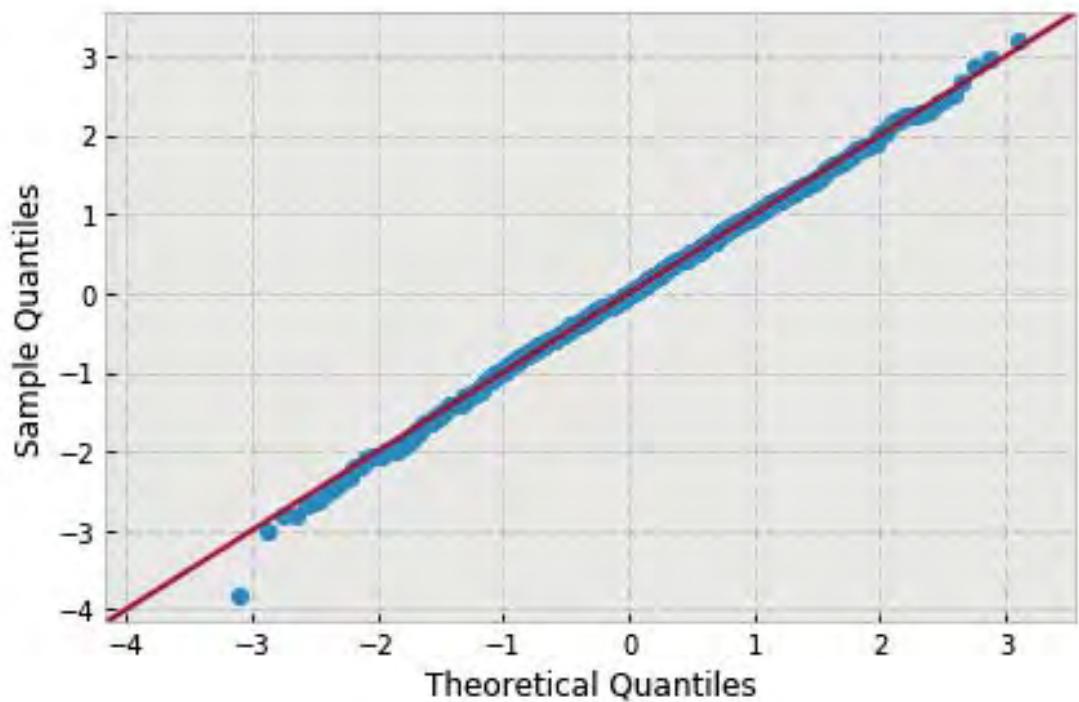
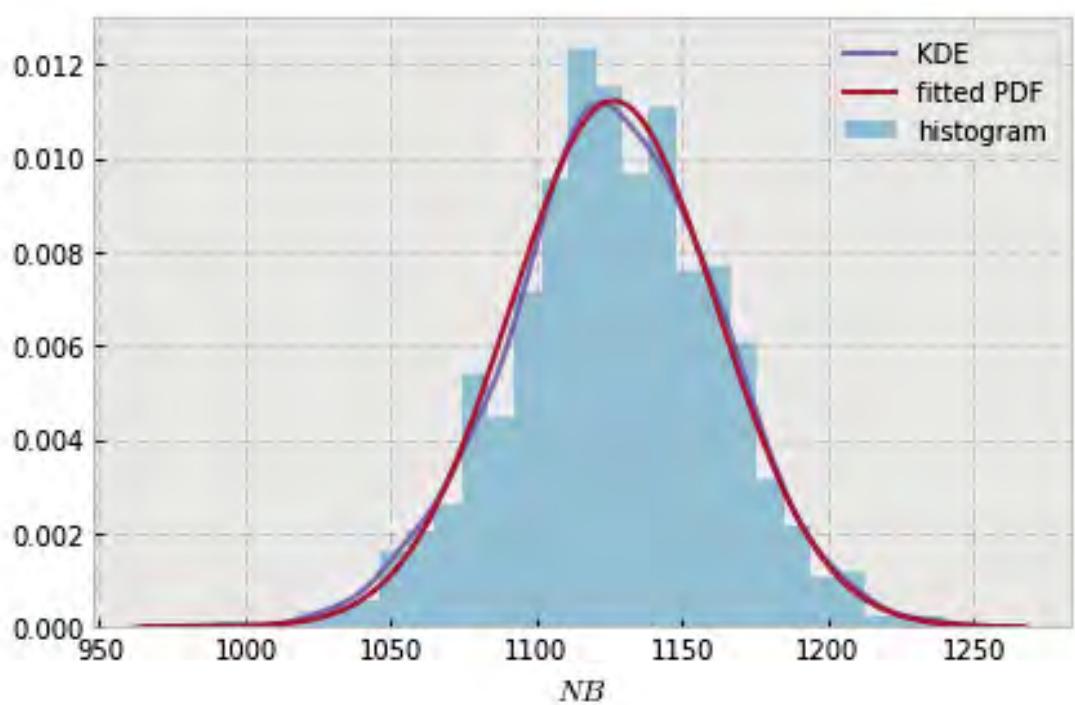


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=1124.592, scale=33.199962891545525)
DescribeResult(nobs=1000, minmax=(1007.0, 1229.0)
    mean=1124.5920000000001, variance=1103.3408768768768,
    skewness=-0.049291557297831363, kurtosis=0.014989542627259933)
```

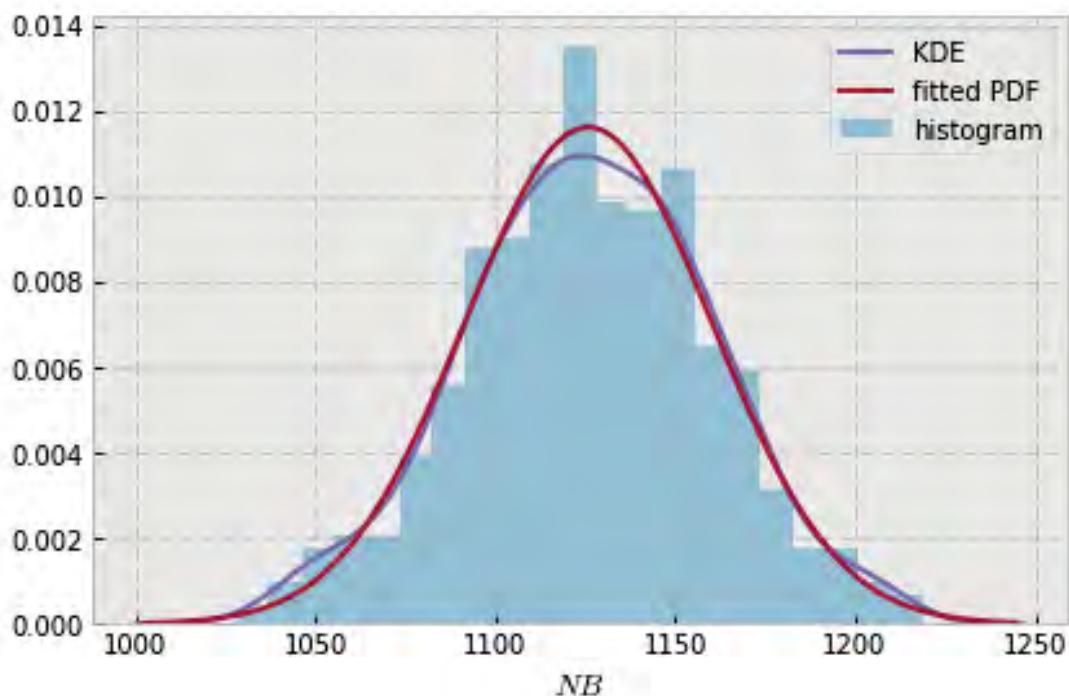


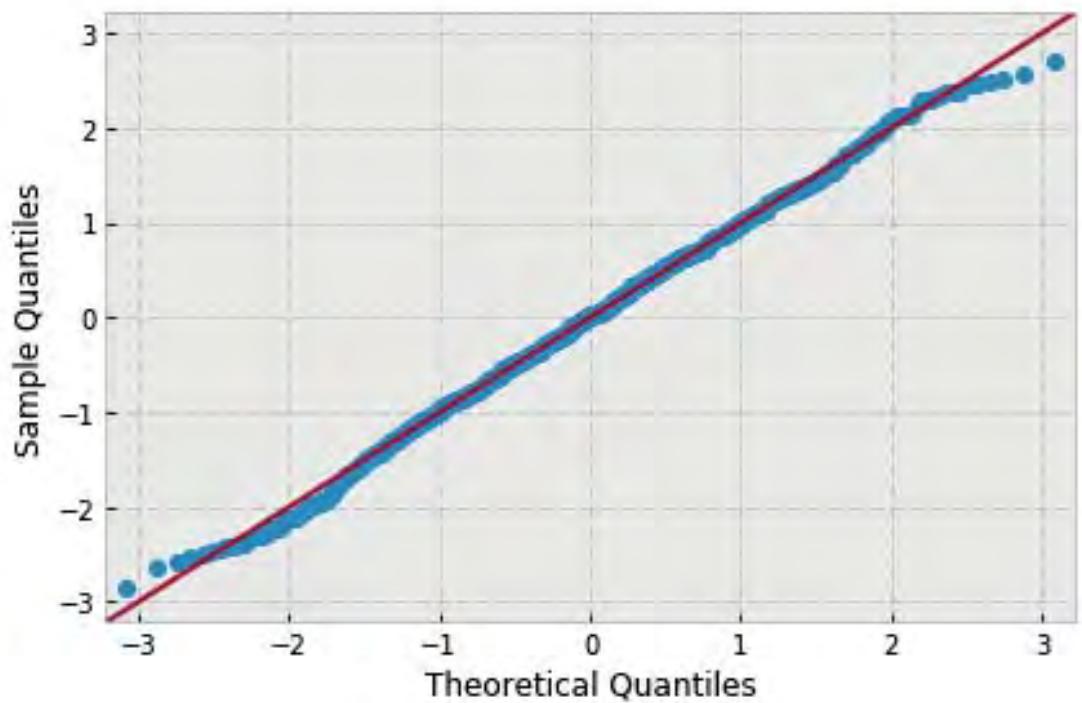


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=1126.455, scale=35.56501054407266)
DescribeResult(nobs=1000, minmax=(991.0, 1240.0)
               mean=1126.454999999999, variance=1266.136111111109,
               skewness=-0.09400306760277793, kurtosis=0.11482109663821705)
```

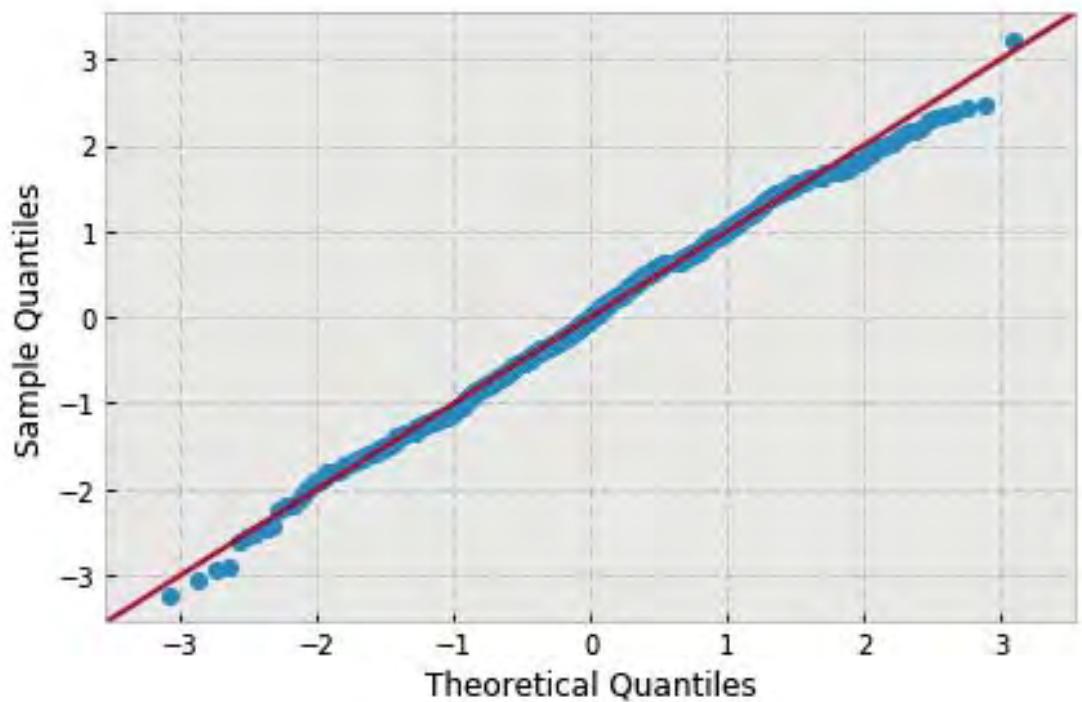
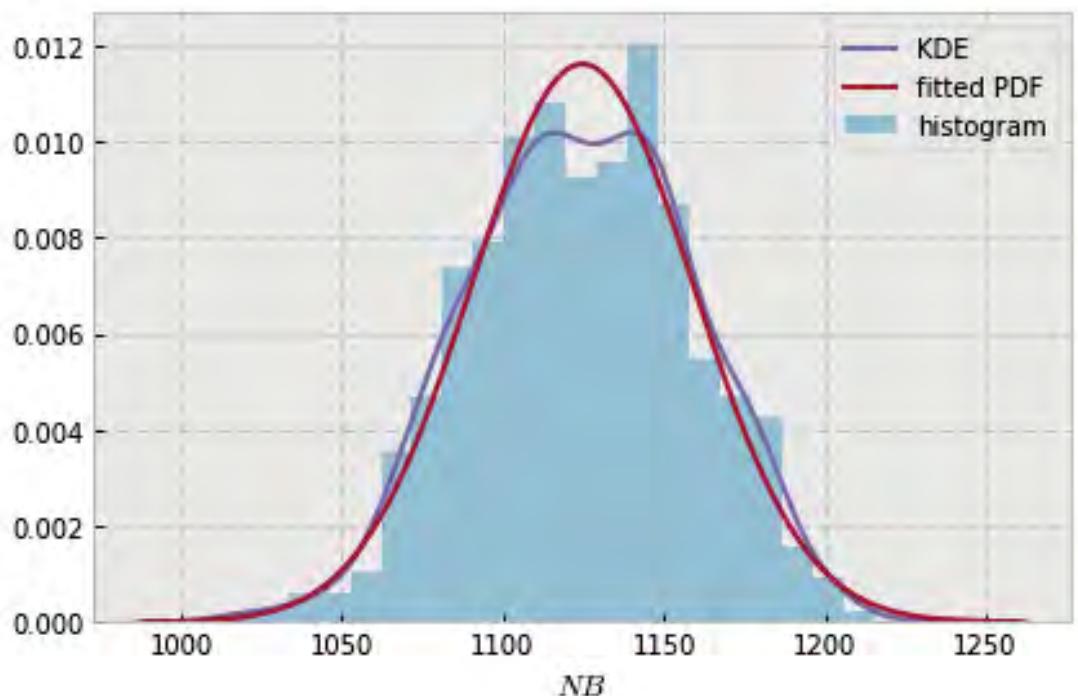


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=1125.864, scale=34.356360459163895)
DescribeResult(nobs=1000, minmax=(1028.0, 1219.0)
    mean=1125.864, variance=1181.5410450450452,
    skewness=-0.06482603268710212, kurtosis=-0.15223939461831426)
```

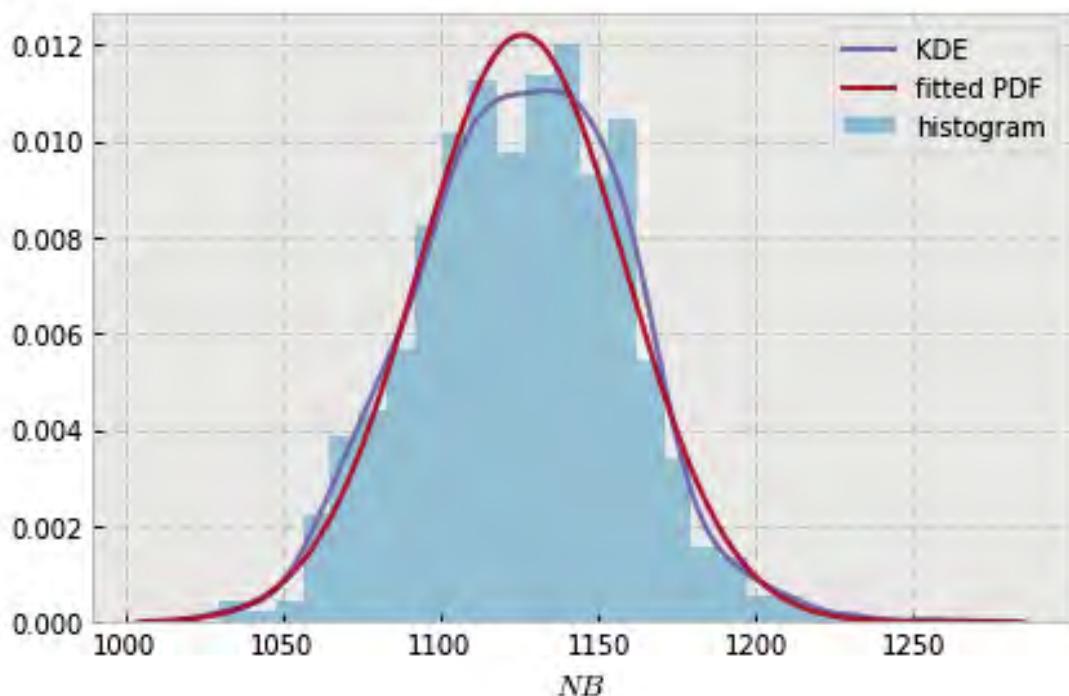


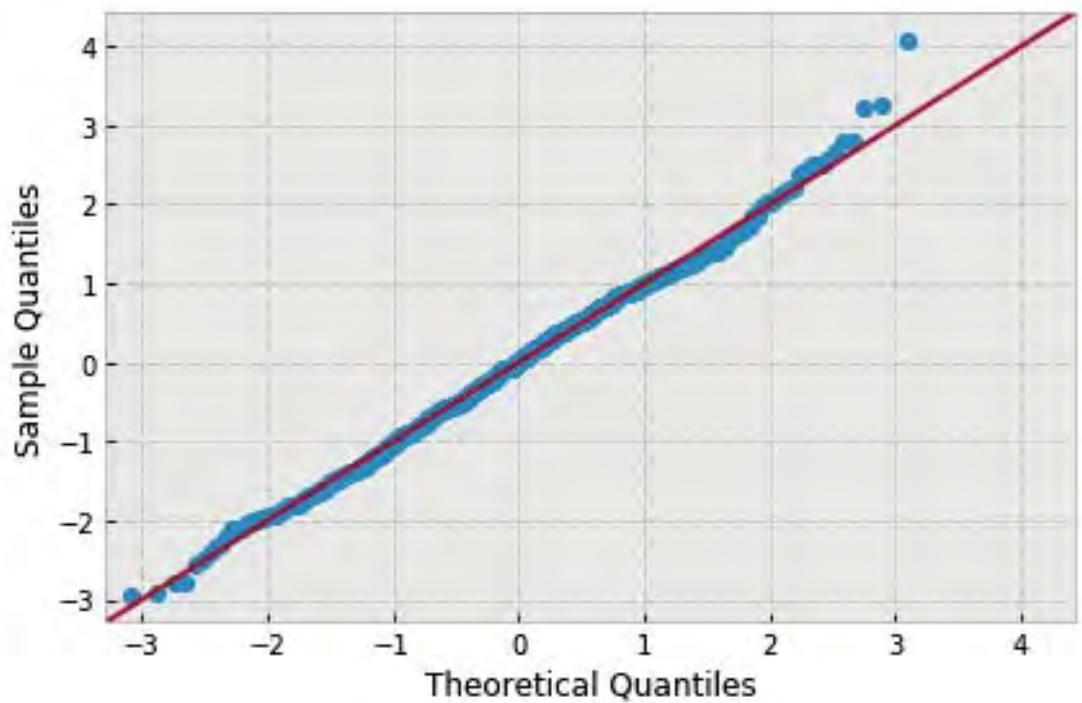


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=1124.705, scale=34.34009282165673)
DescribeResult(nobs=1000, minmax=(1014.0, 1235.0)
               mean=1124.704999999999, variance=1180.4223973973974,
               skewness=-0.0745899834064309, kurtosis=-0.30335655444578125)
```

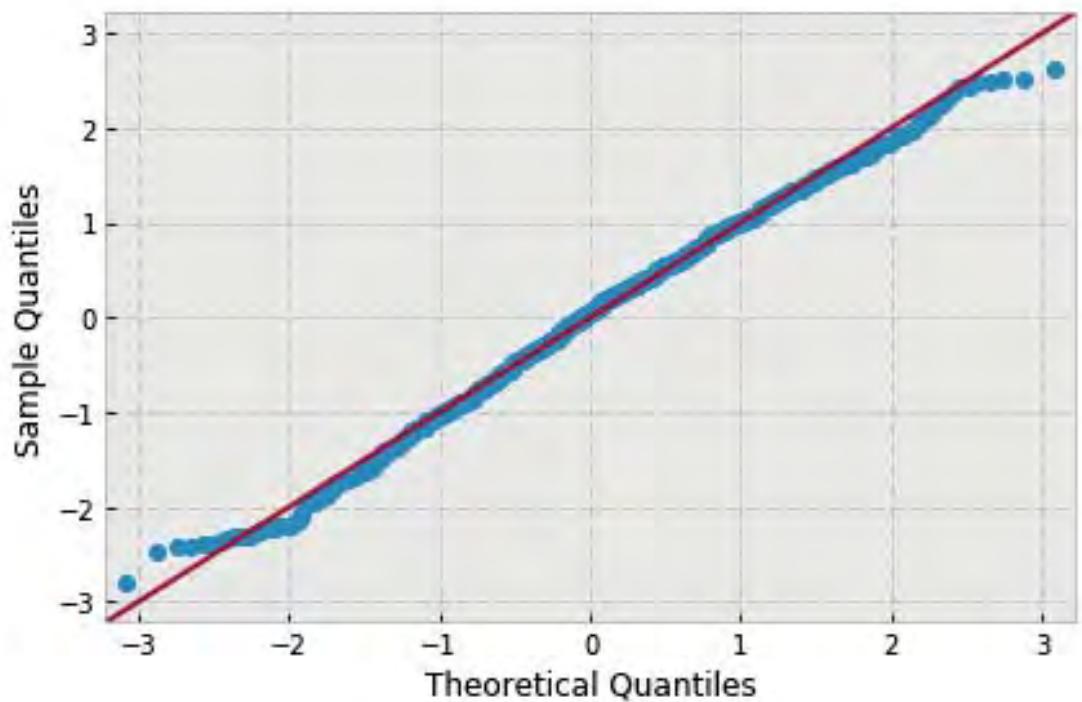
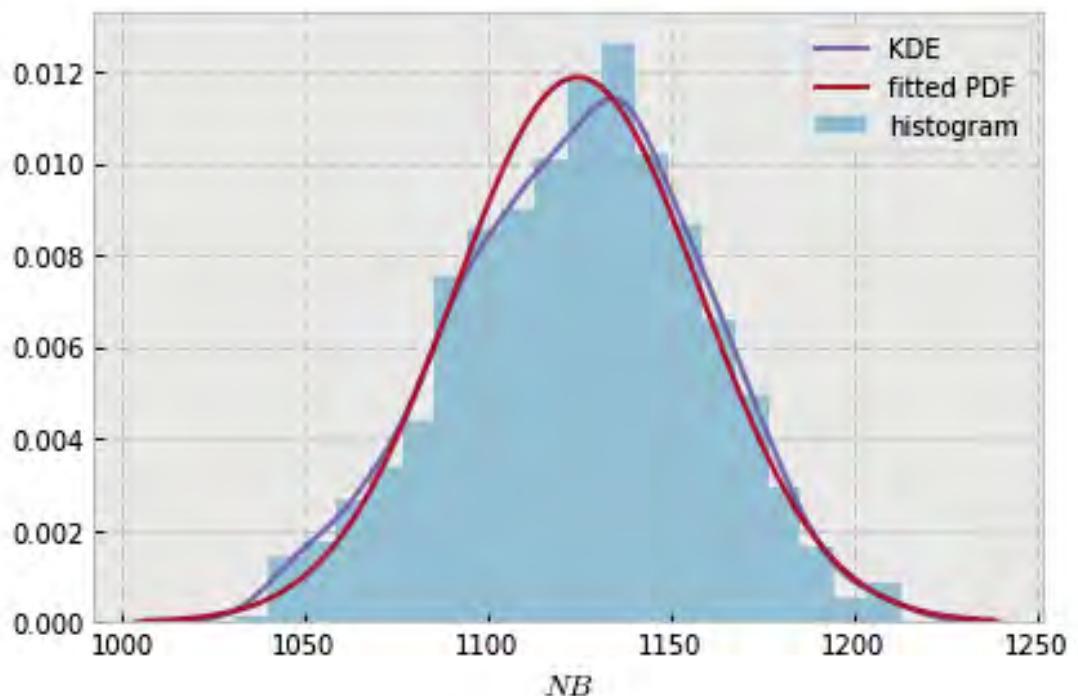


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=1125.945, scale=32.69278169565875)
DescribeResult(nobs=1000, minmax=(1030.0, 1259.0)
    mean=1125.944999999999, variance=1069.8878628628629,
    skewness=0.03609539504448593, kurtosis=0.08337080499981164)
```

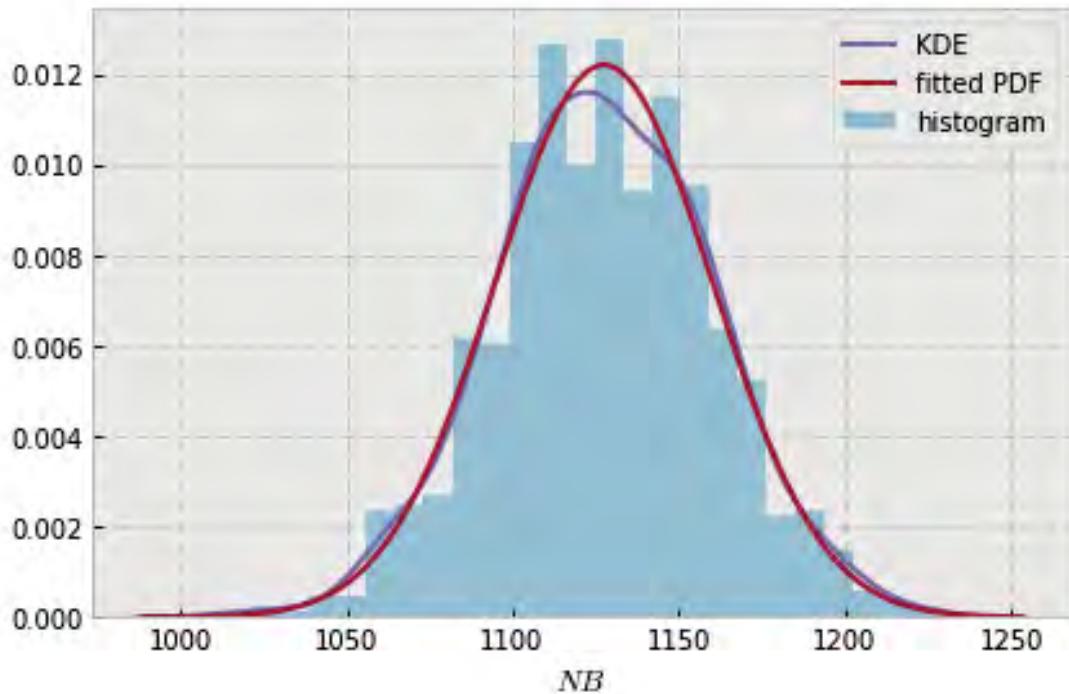


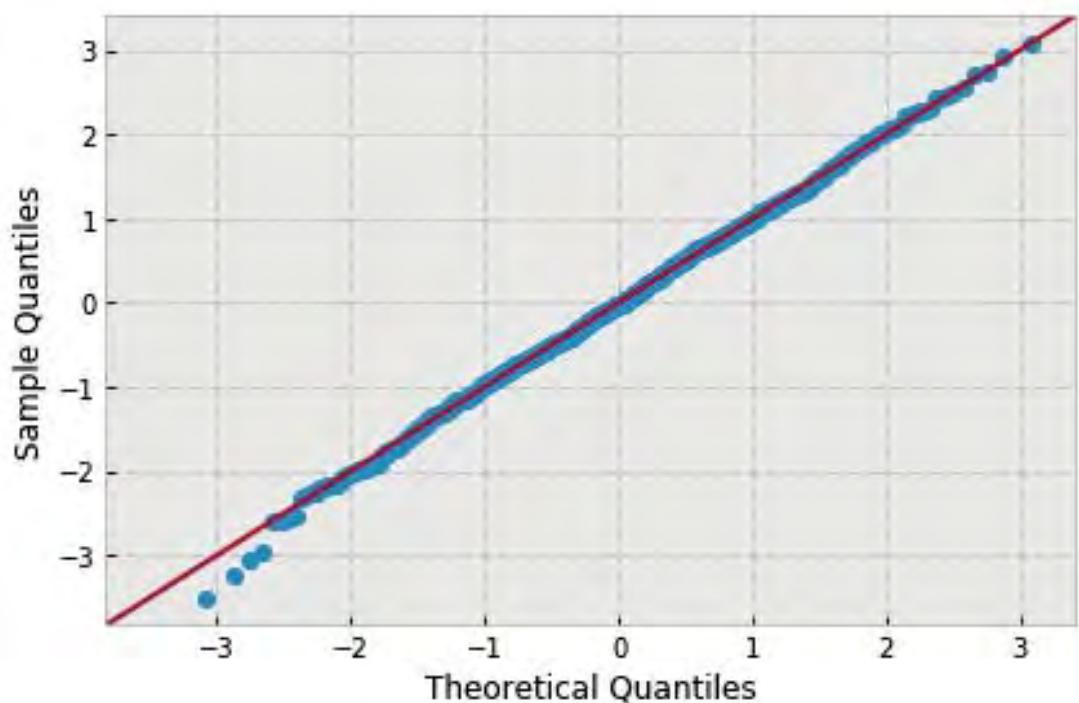


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=1124.604, scale=33.606326547244045)
DescribeResult(nobs=1000, minmax=(1031.0, 1213.0)
               mean=1124.604, variance=1130.5156996996996,
               skewness=-0.13722826137320196, kurtosis=-0.34163839340394064)
```

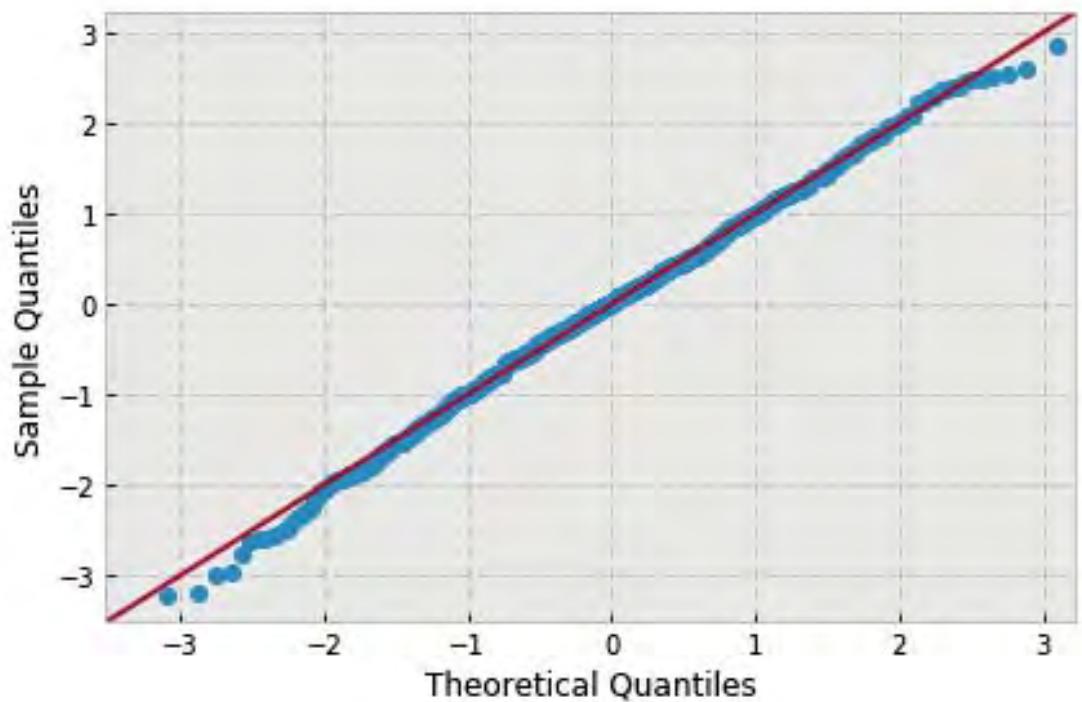
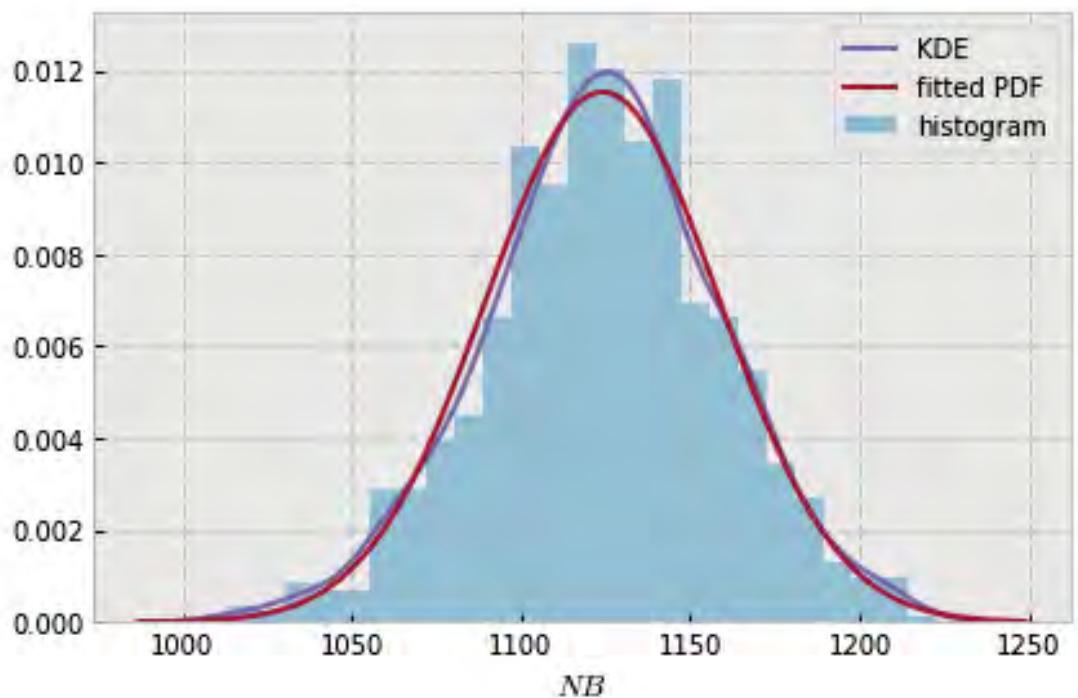


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=1127.431, scale=32.71380807854689)
DescribeResult(nobs=1000, minmax=(1013.0, 1228.0)
    mean=1127.431, variance=1071.2645035035036,
    skewness=-0.037223917139381696, kurtosis=0.04656044739166498)
```





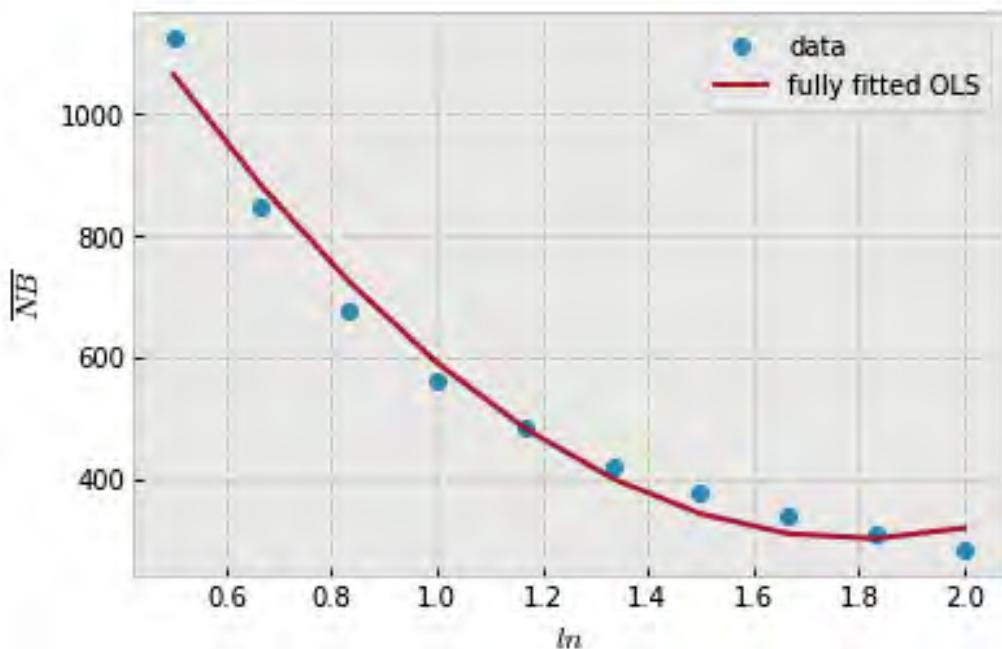
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=1124.136, scale=34.57969785871473)
DescribeResult(nobs=1000, minmax=(1013.0, 1223.0)
               mean=1124.136, variance=1196.9524564564565,
               skewness=-0.11867892564585361, kurtosis=0.0887417851092911)
```



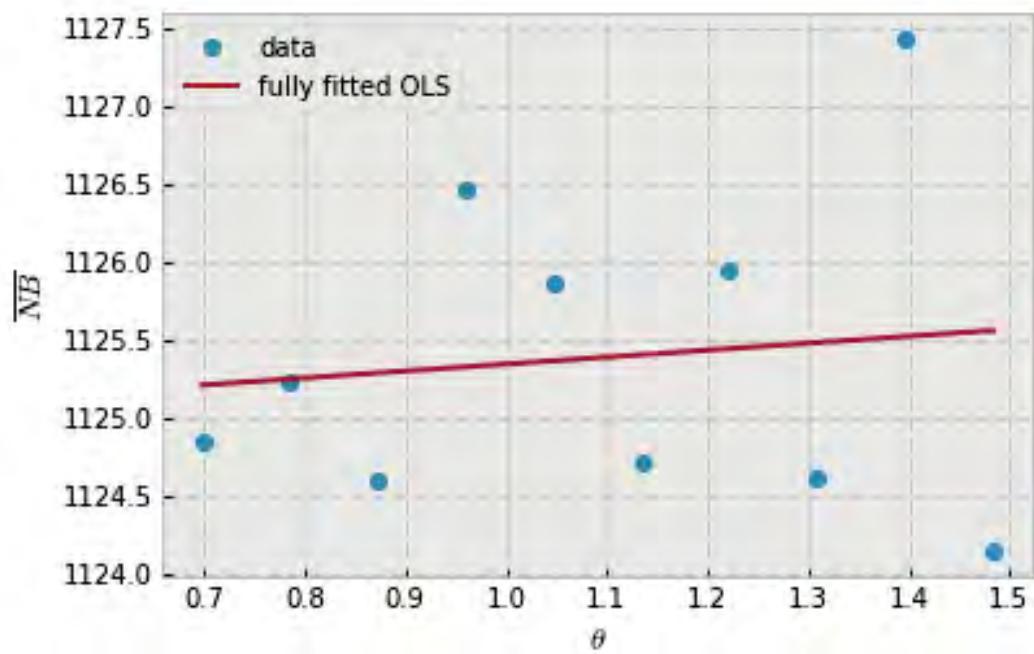
8.2 Parameter regression

```
In [22]: var_name = 'number_of_root_branches'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly1')  
        mean_reg('nB', var_name, 'poly1')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly1')  
        mean_reg('N', var_name, 'poly1')
```

```
variable name: ln  
model: poly2  
r-squared: 0.9298722998961  
explained variance: 0.9300052503727683
```



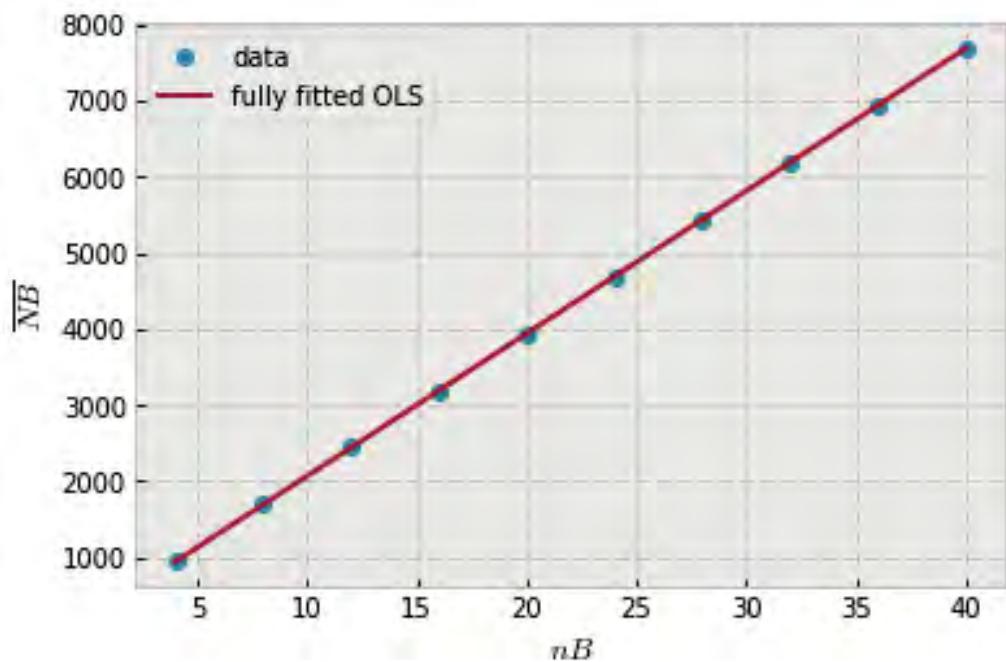
```
variable name: theta  
model: poly1  
r-squared: -0.7204747521799859  
explained variance: -0.7185227403913084
```



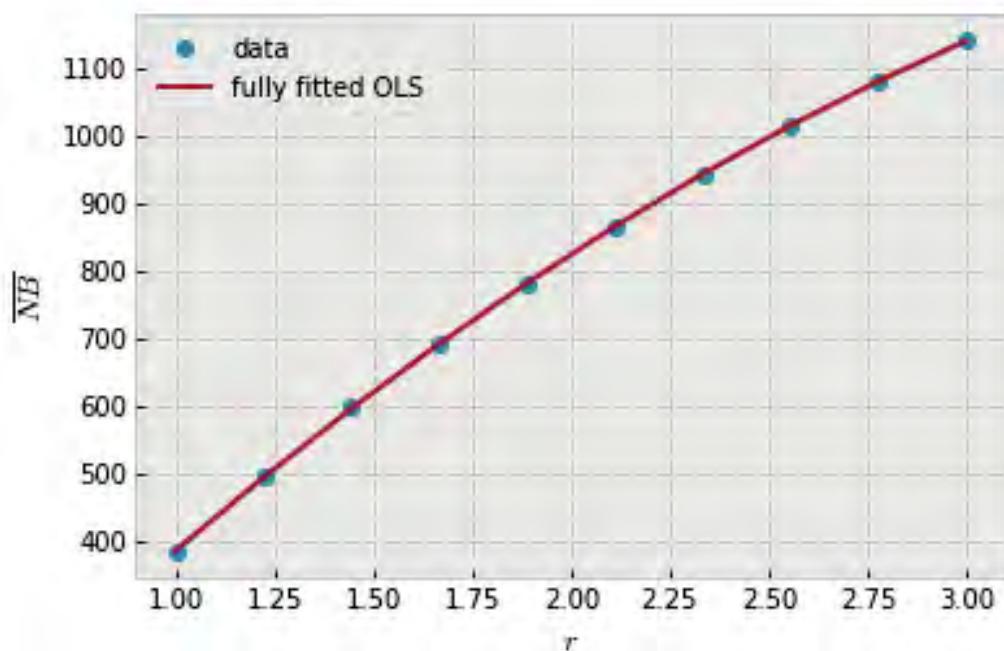
```

variable name: nB
model: poly1
r-squared: 0.9999990103244277
explained variance: 0.9999990119736687

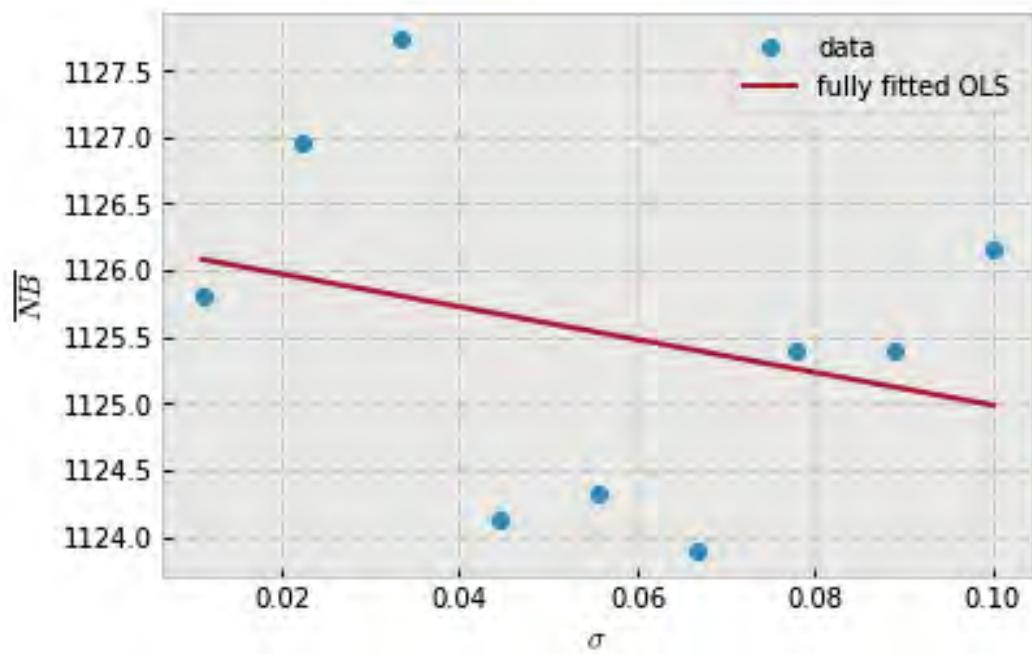
```



```
variable name: r
model: poly2
r-squared: 0.9997413980506127
explained variance: 0.9997415858540416
```



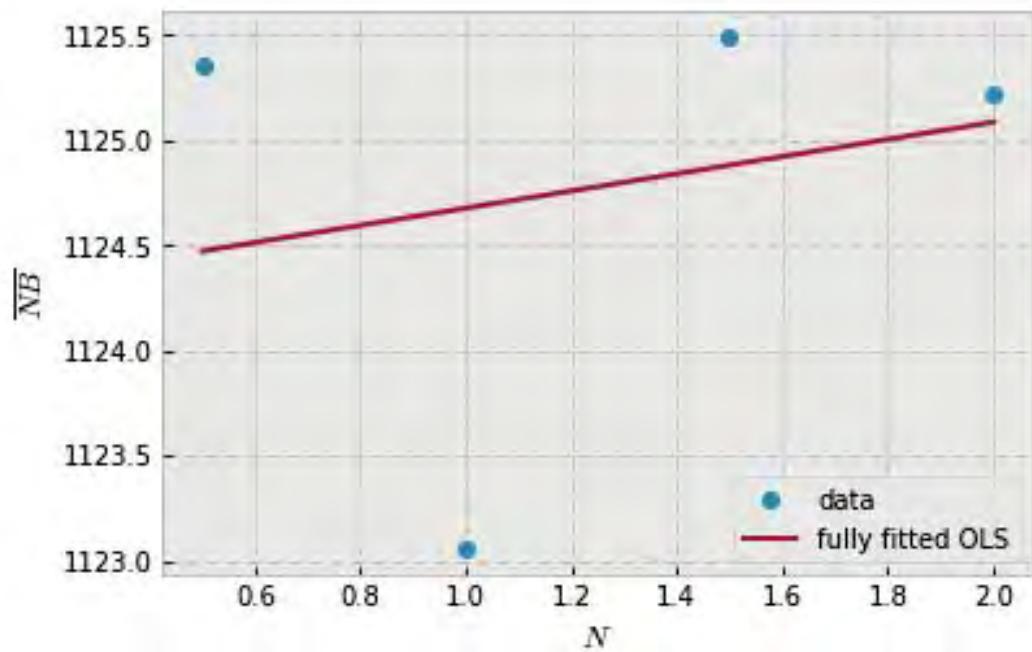
```
variable name: sigma
model: poly1
r-squared: -0.4229704136433907
explained variance: -0.41702817206359466
```



```

variable name: N
model: poly1
r-squared: -2.7180850660386775
explained variance: -2.4871826759176514

```

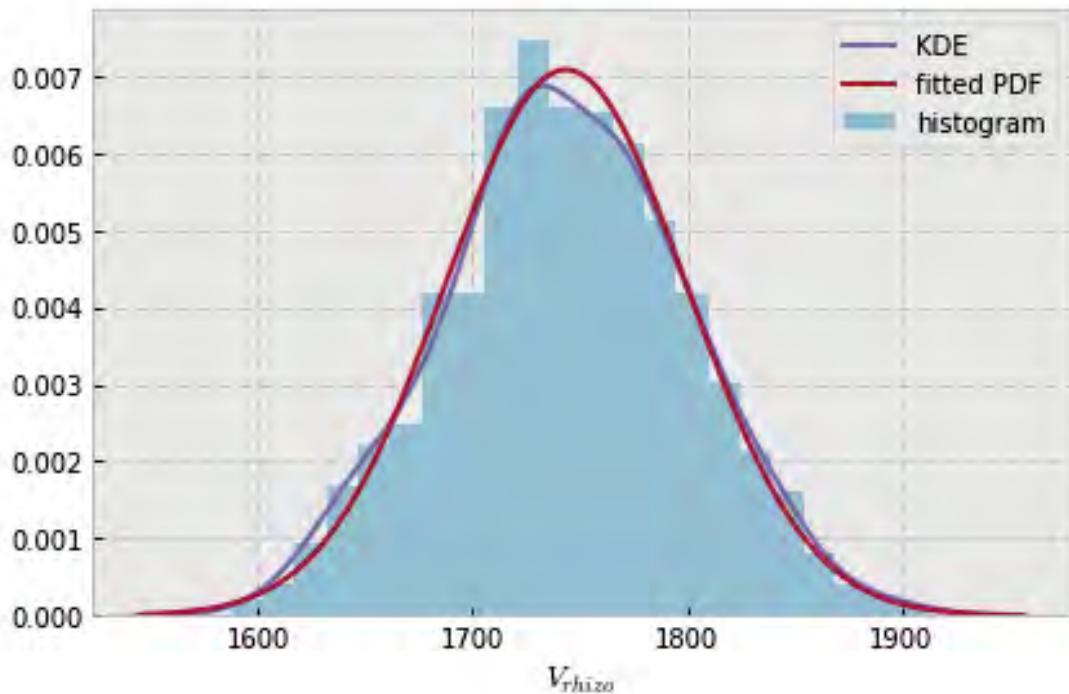


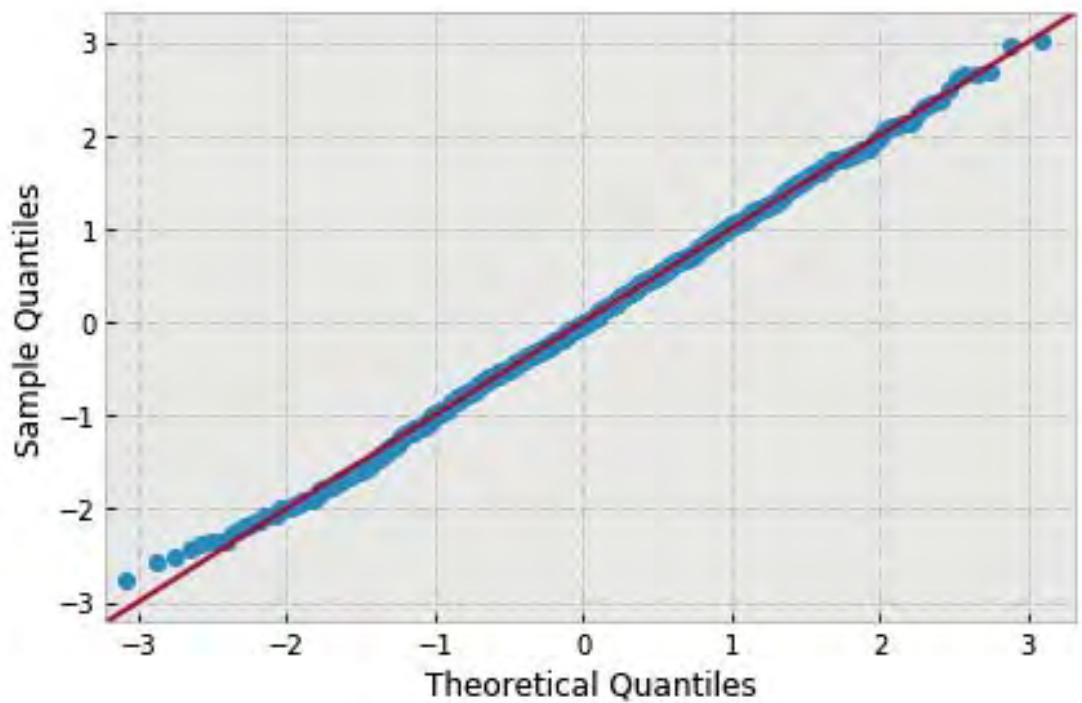
9 Volume of the rhizosphere: phosphate (0.2cm)

9.1 Probability distributions

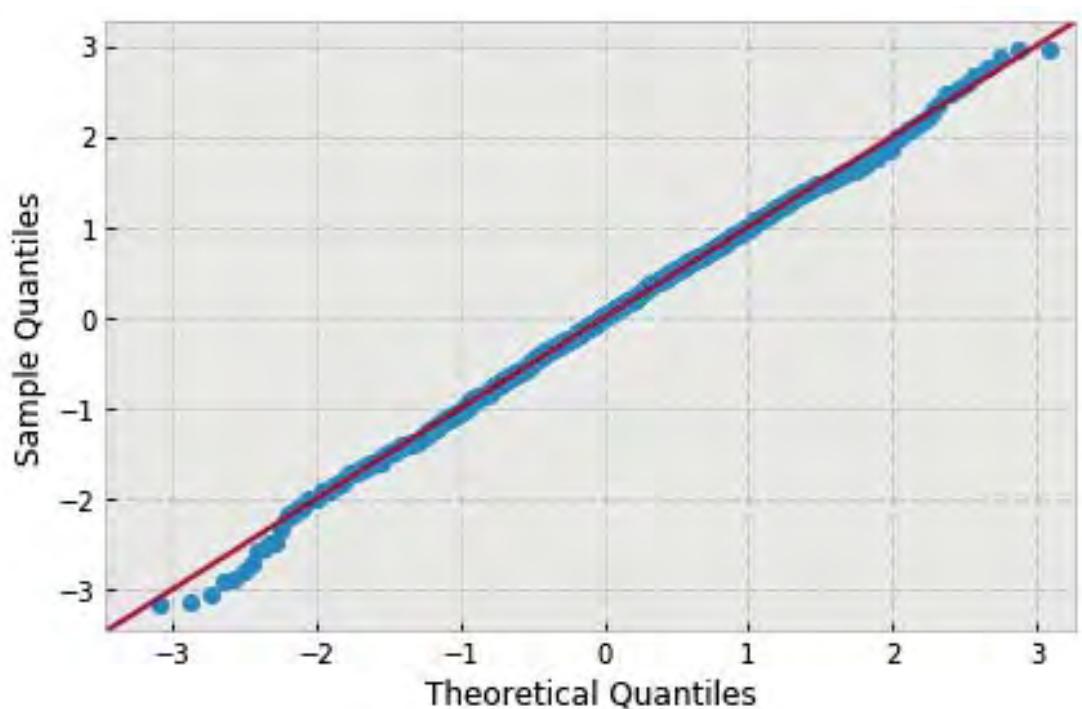
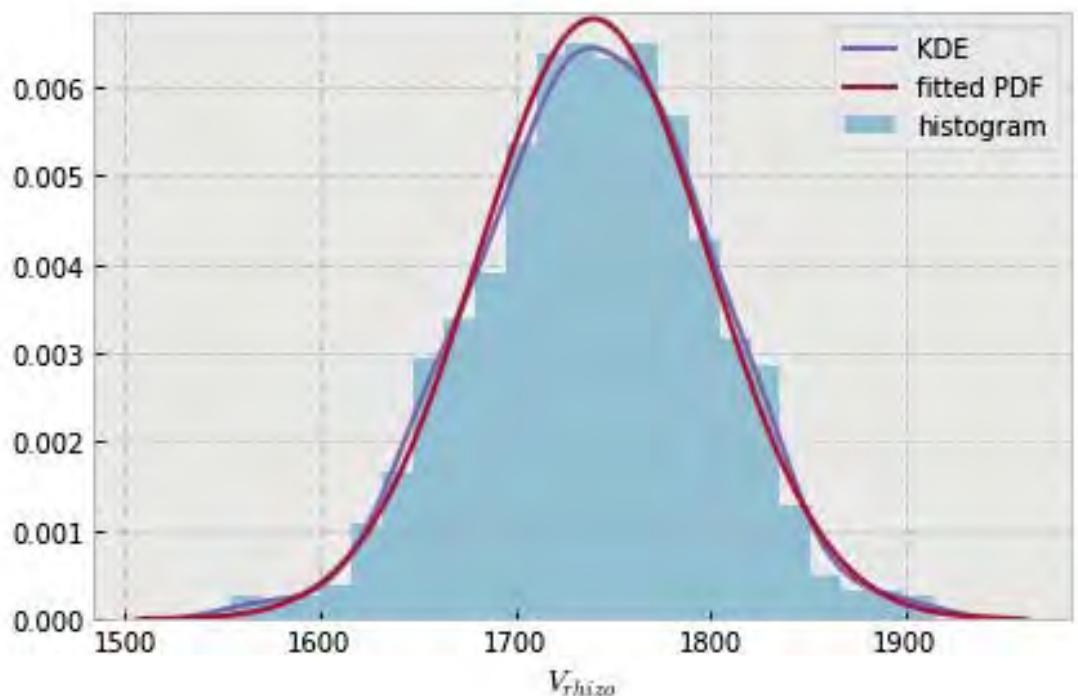
```
In [23]: plot_dist('rhizosphere_volume_phosphate')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=1743.4741617982802, scale=56.34885530654683)
DescribeResult(nobs=1000, minmax=(1587.1453283196986, 1913.6607466333917)
               mean=1743.4741617982802, variance=3178.371866224375,
               skewness=0.022062274213444245, kurtosis=-0.21362384647677146)
```

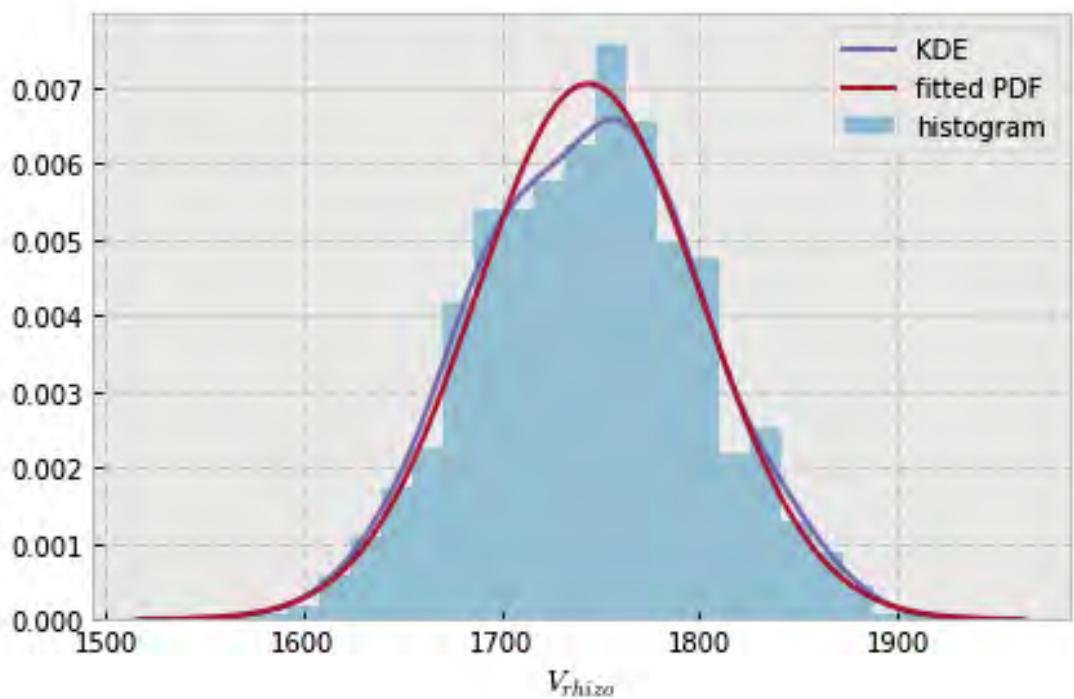


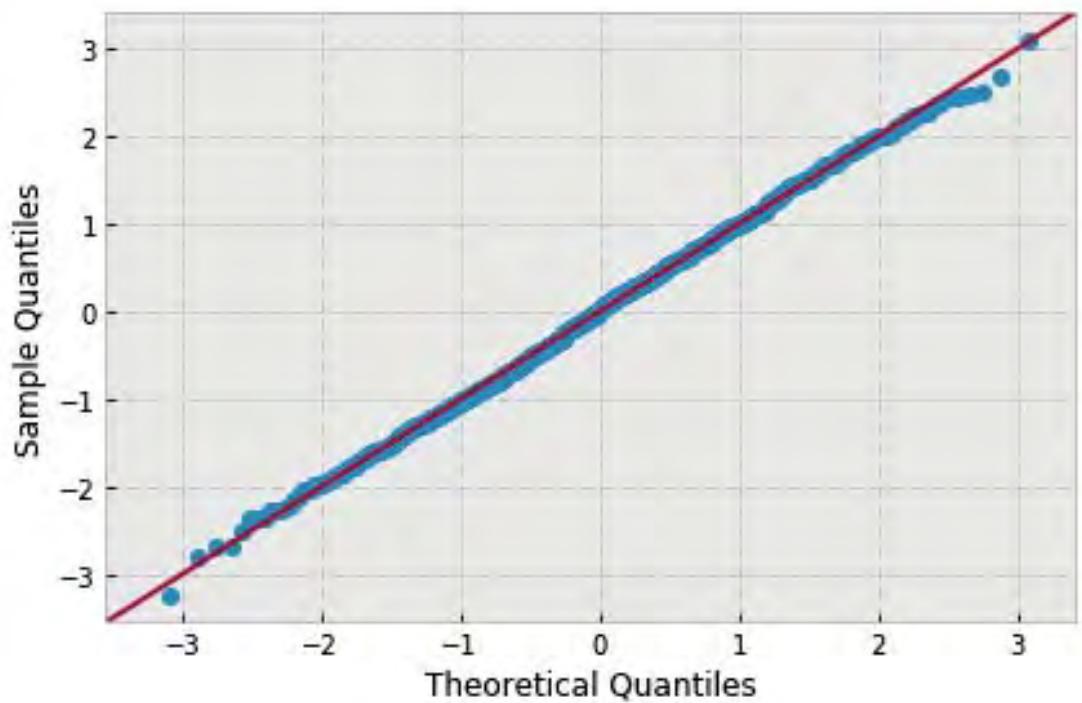


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=1740.0786446364023, scale=58.95035475191497)
DescribeResult(nobs=1000, minmax=(1553.730716150292, 1914.4069569206029)
               mean=1740.0786446364023, variance=3478.622948324949,
               skewness=-0.08497535908975812, kurtosis=-0.0083629574555113)
```

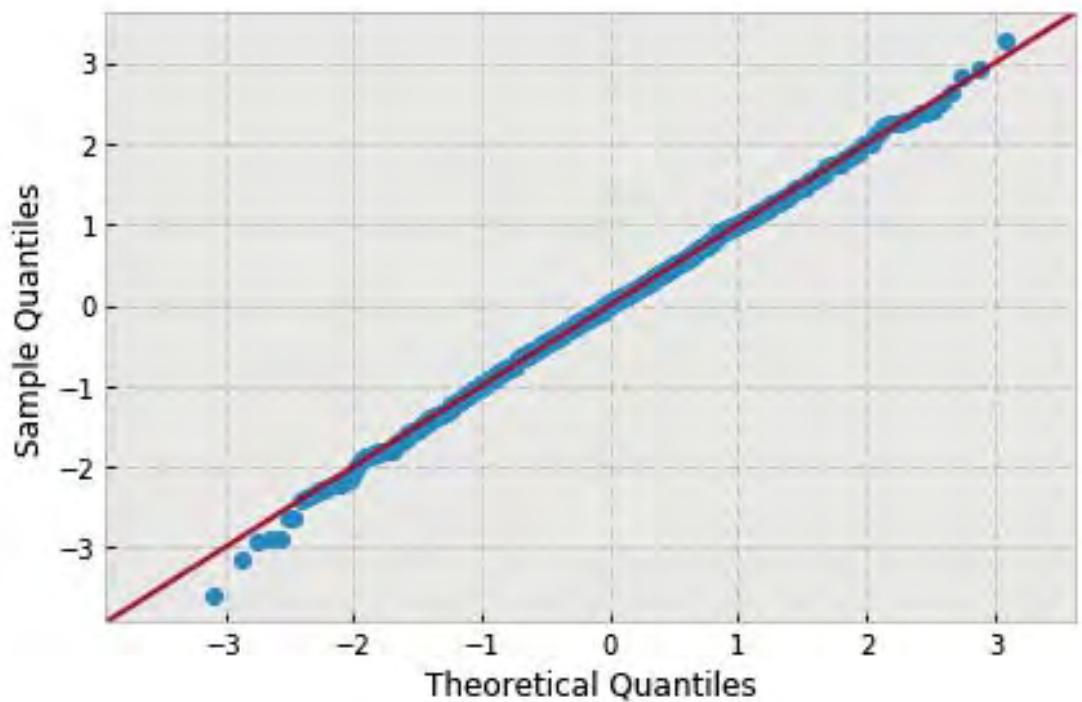
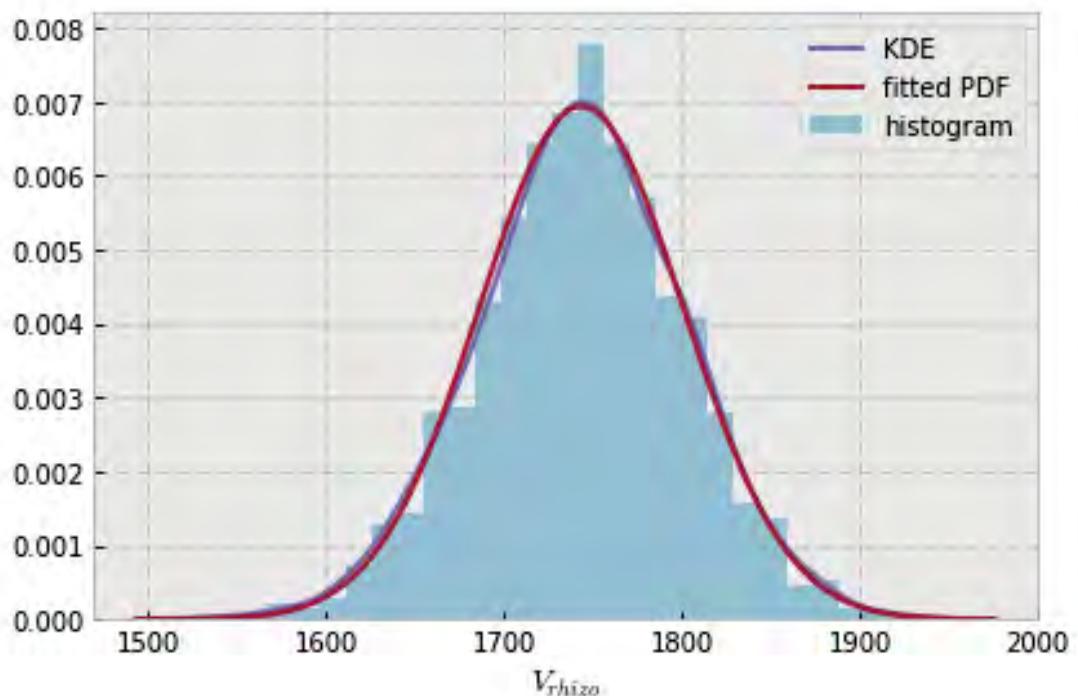


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=1743.8235139448395, scale=56.62980892592853)
DescribeResult(nobs=1000, minmax=(1561.0375285437965, 1918.9619370335054)
               mean=1743.8235139448395, variance=3210.145404391566,
               skewness=-0.0027353403151204126, kurtosis=-0.2579884682537892)
```

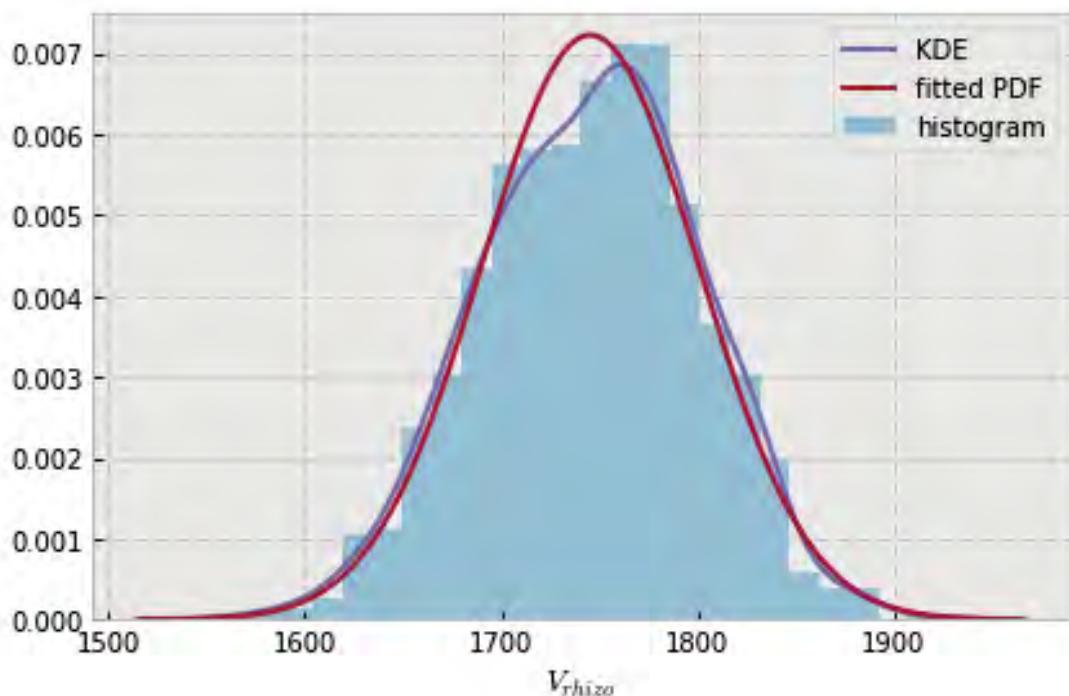


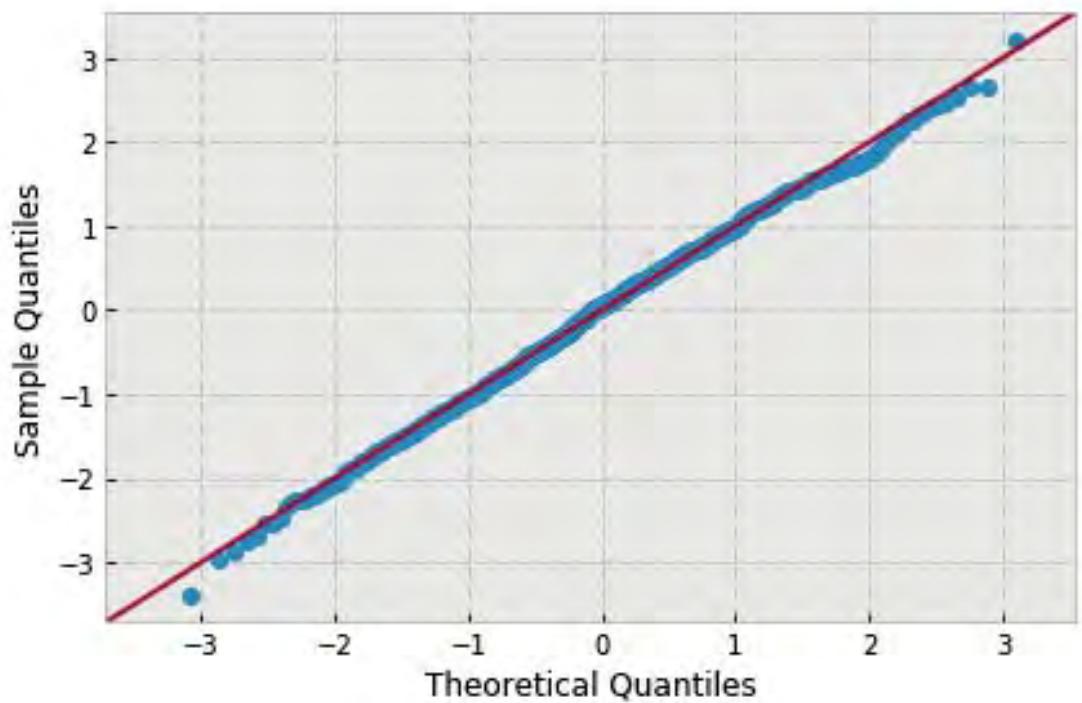


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=1743.4163974878481, scale=57.48835017818454)
DescribeResult(nobs=1000, minmax=(1537.4109722920443, 1931.6269148734352)
               mean=1743.4163974878481, variance=3308.2186248344046,
               skewness=-0.07657739239300747, kurtosis=0.10053855352414143)
```

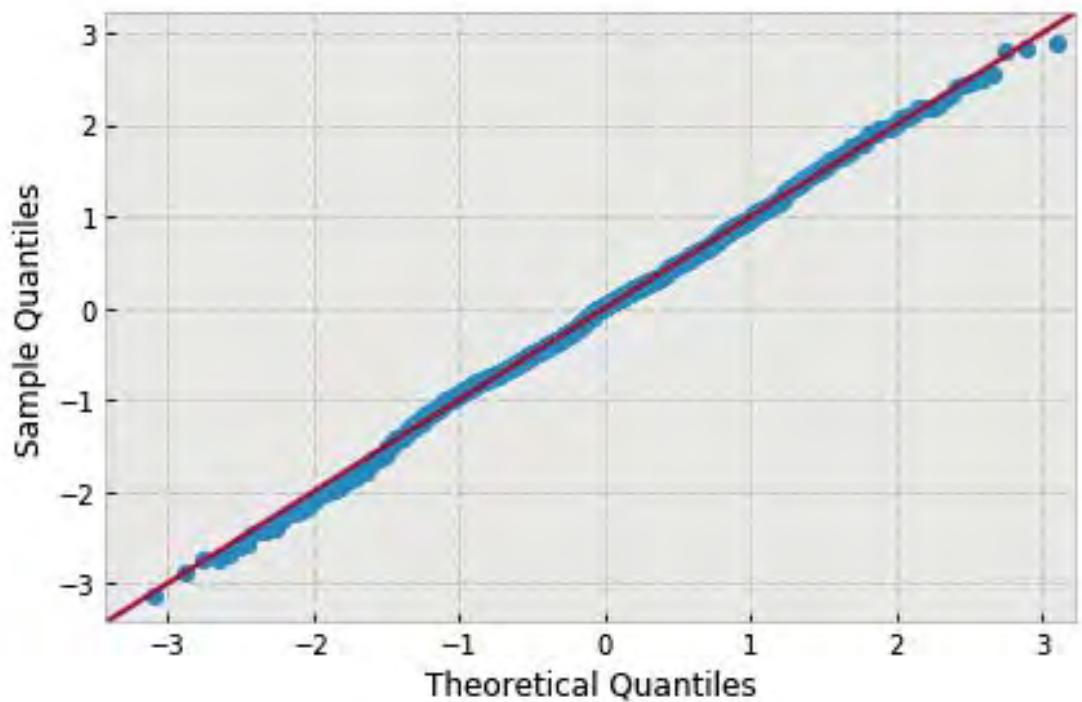
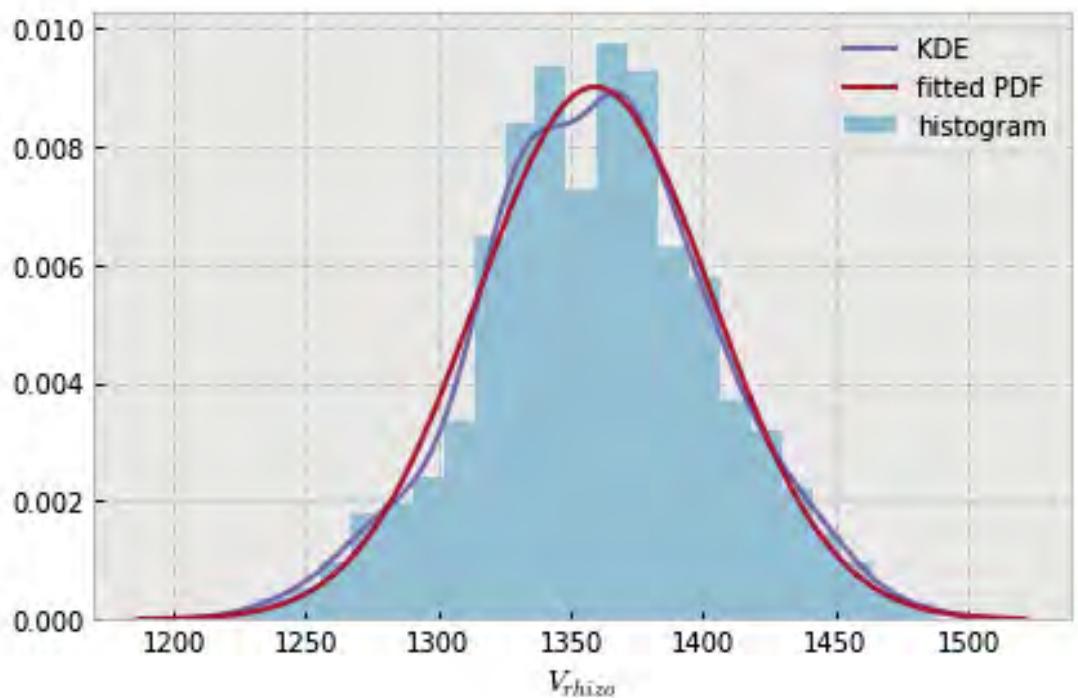


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=1745.3025263915774, scale=55.18759184786652)
DescribeResult(nobs=1000, minmax=(1558.7245484534405, 1922.214826422746)
               mean=1745.3025263915774, variance=3048.7190129796832,
               skewness=-0.11731129396596043, kurtosis=-0.18910121418171277)
```

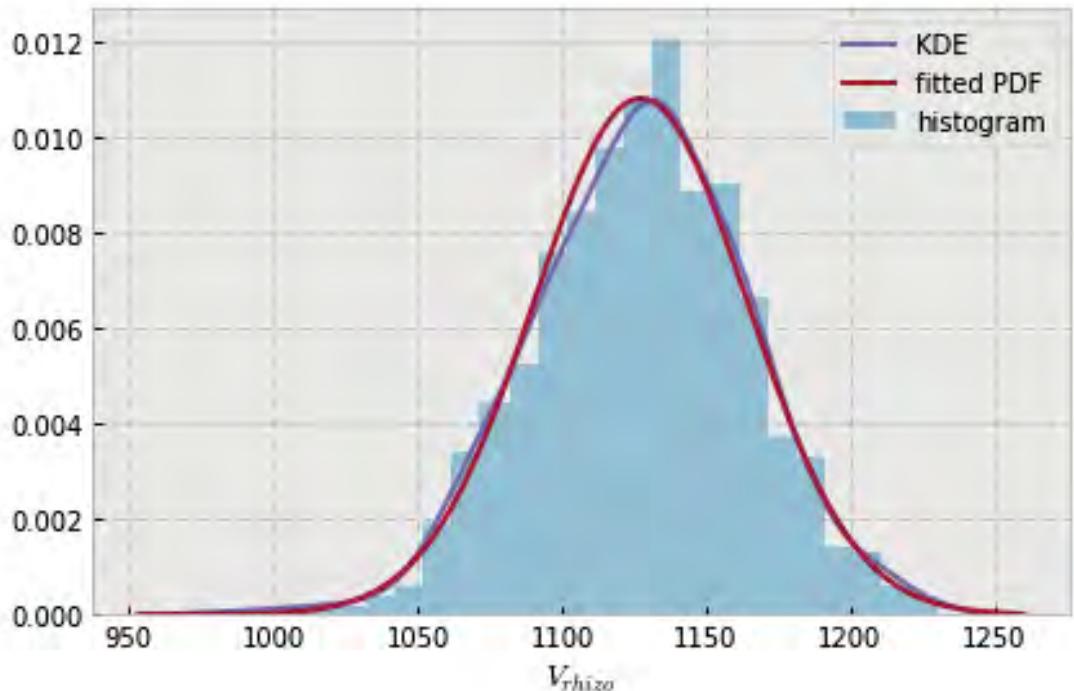


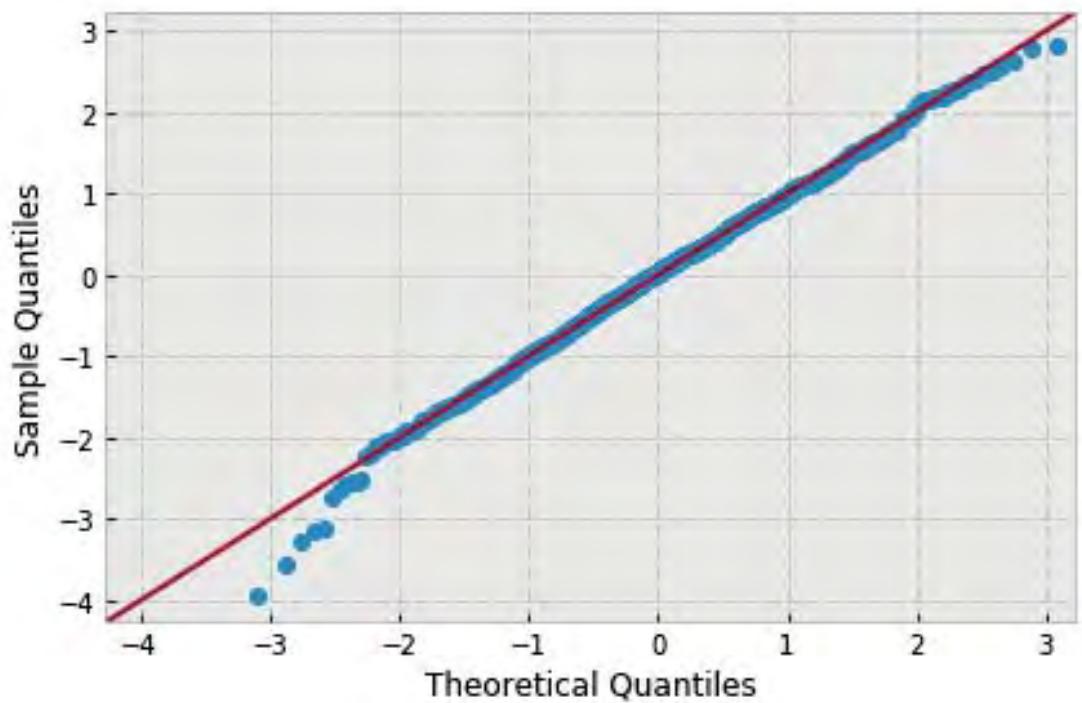


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=1359.1023075777214, scale=44.3142541782257)
DescribeResult(nobs=1000, minmax=(1220.9659697147761, 1487.2281065014752)
               mean=1359.1023075777214, variance=1965.7188422146087,
               skewness=-0.03595848369353028, kurtosis=0.0027399819645710366)
```

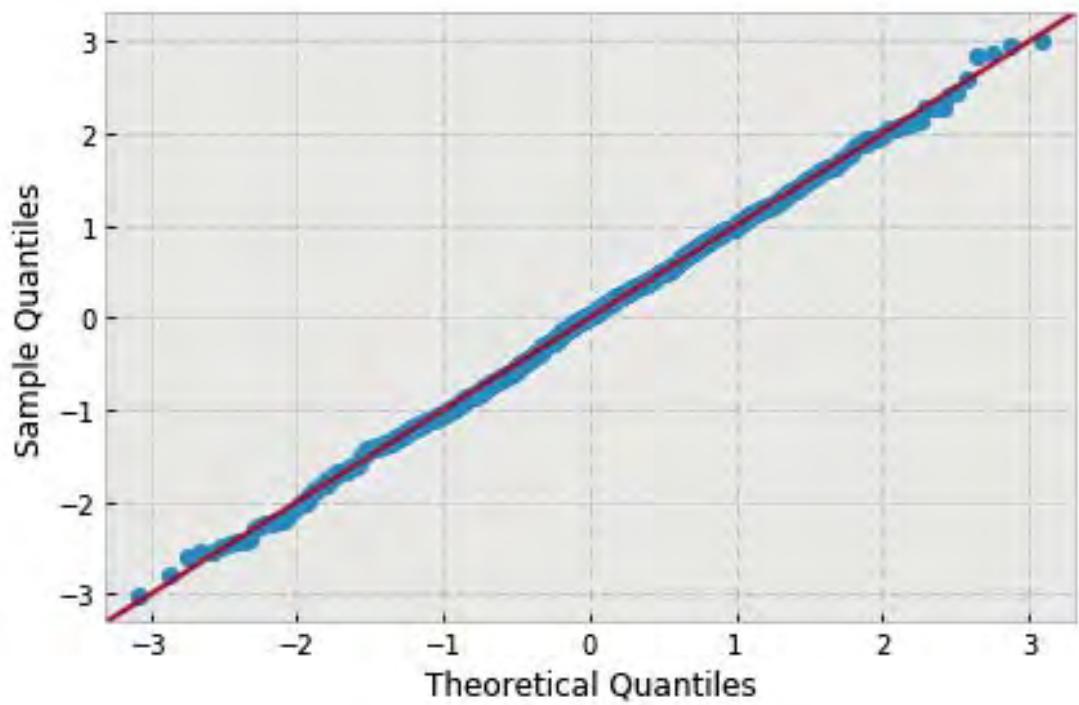
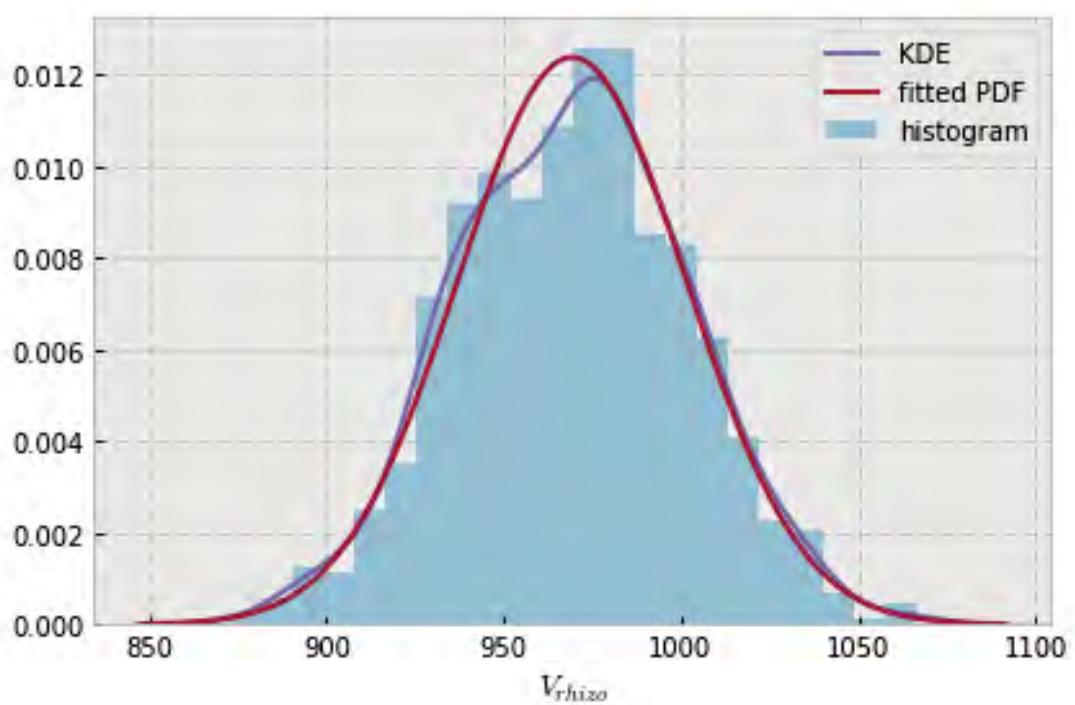


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=1127.325765668571, scale=36.912233380766175)
DescribeResult(nobs=1000, minmax=(982.38200330585539, 1230.829527818905)
               mean=1127.325765668571, variance=1363.8768500061547,
               skewness=-0.1556825439518013, kurtosis=0.19739135553949172)
```

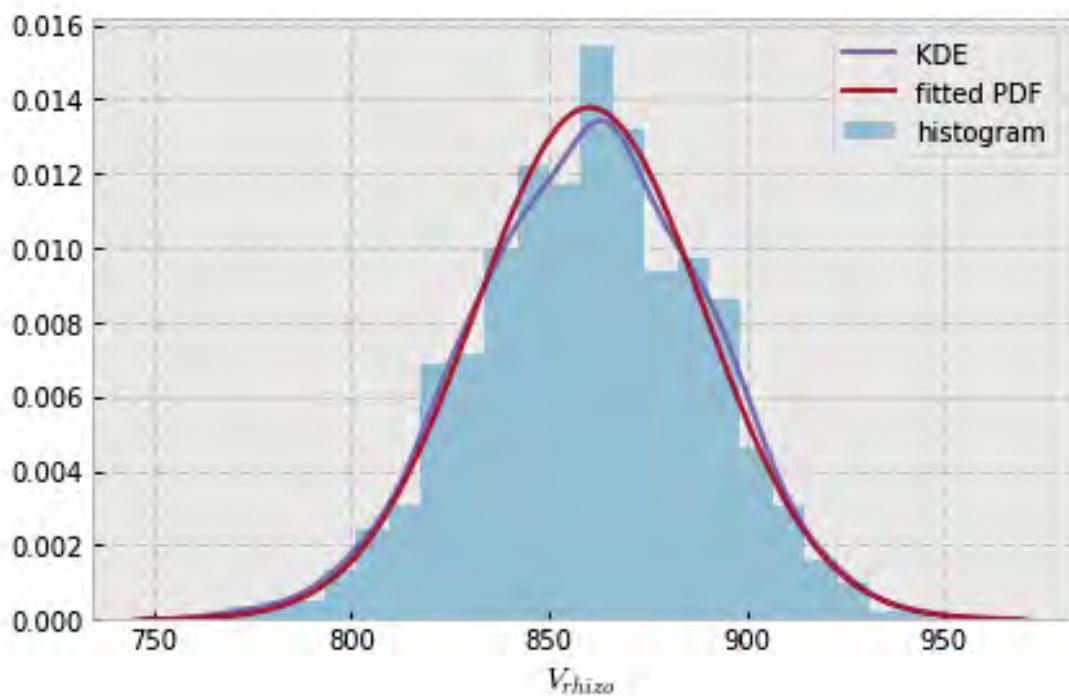


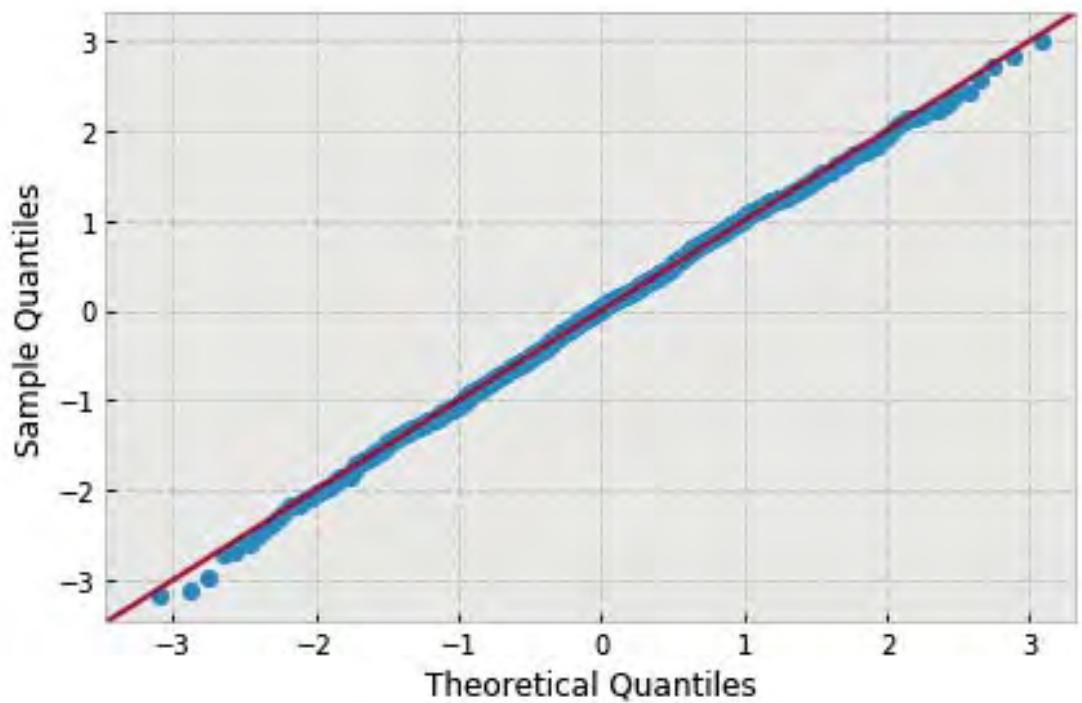


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=969.2904029374874, scale=32.24133049953211)
DescribeResult(nobs=1000, minmax=(872.48456839252538, 1066.3152246489285)
               mean=969.29040293748744, variance=1040.5439363163759,
               skewness=-0.0006985976129427895, kurtosis=-0.1809348631726002)
```

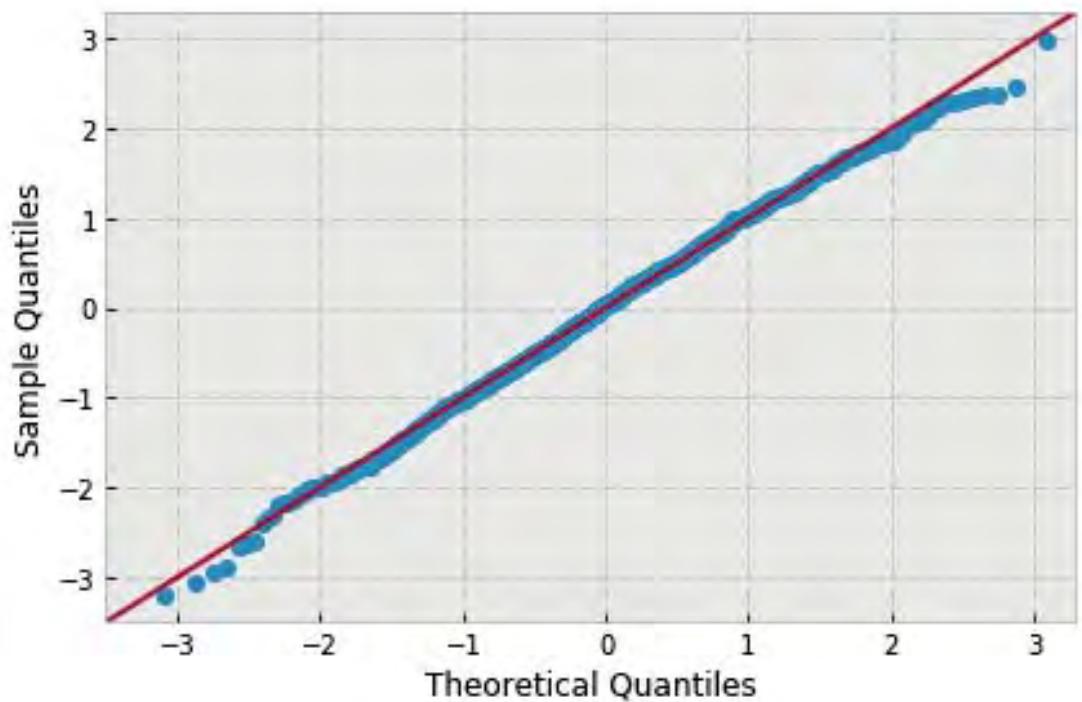
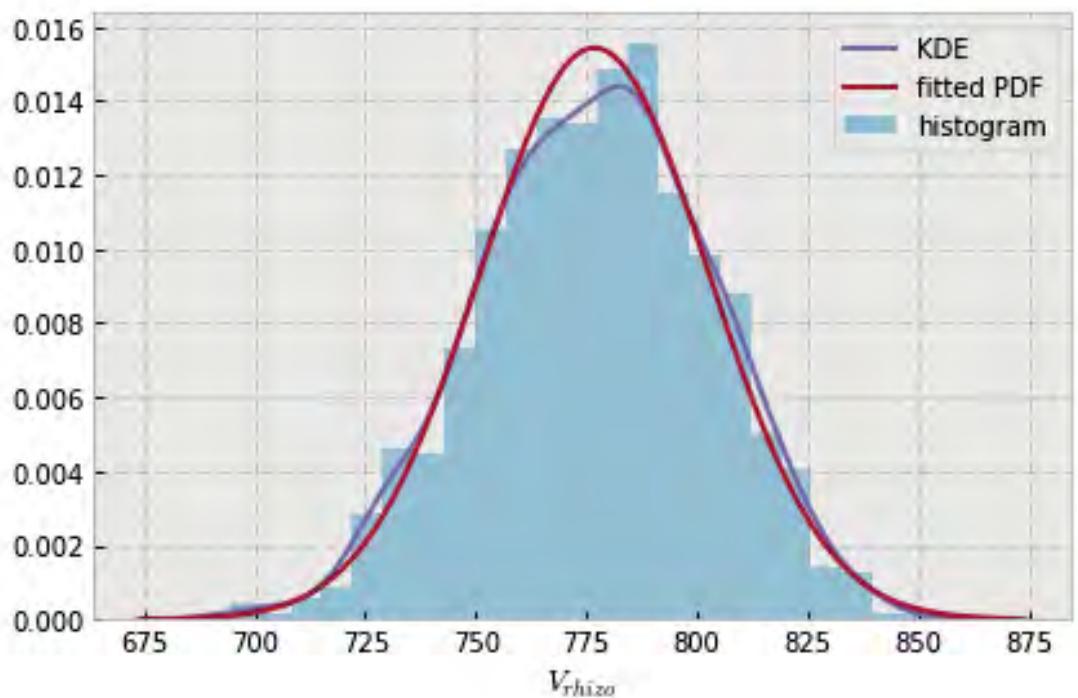


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=860.3694050202039, scale=28.95991888285074)
DescribeResult(nobs=1000, minmax=(768.89766672913163, 947.33966452144489)
               mean=860.3694050202039, variance=839.5164181194142,
               skewness=-0.08366165321630135, kurtosis=-0.1346401936921775)
```

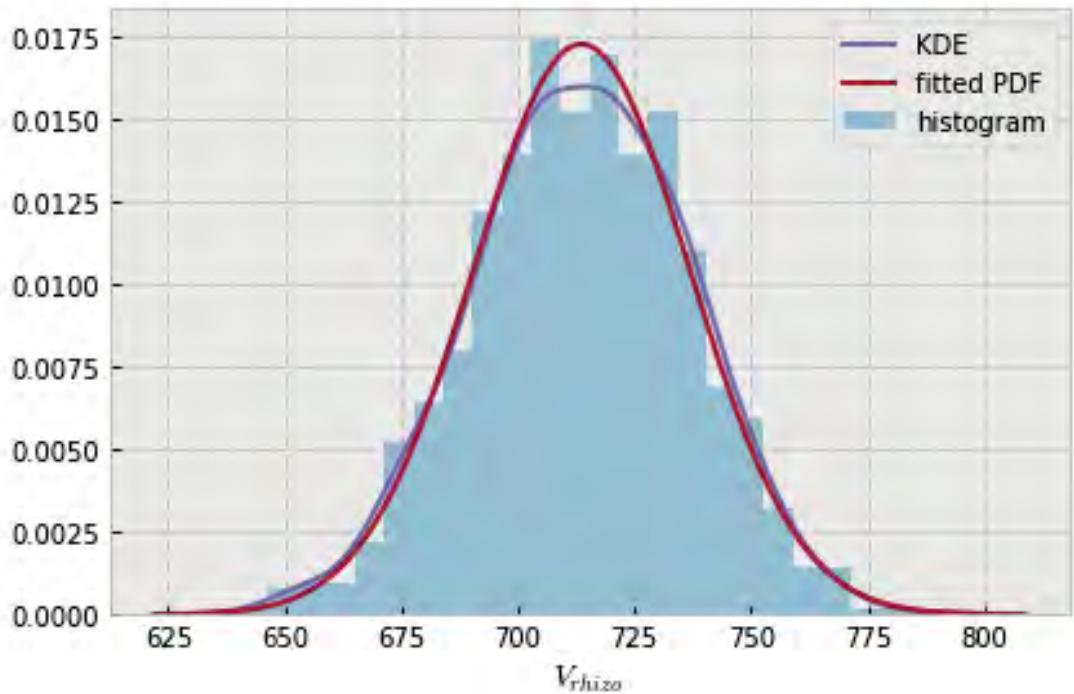


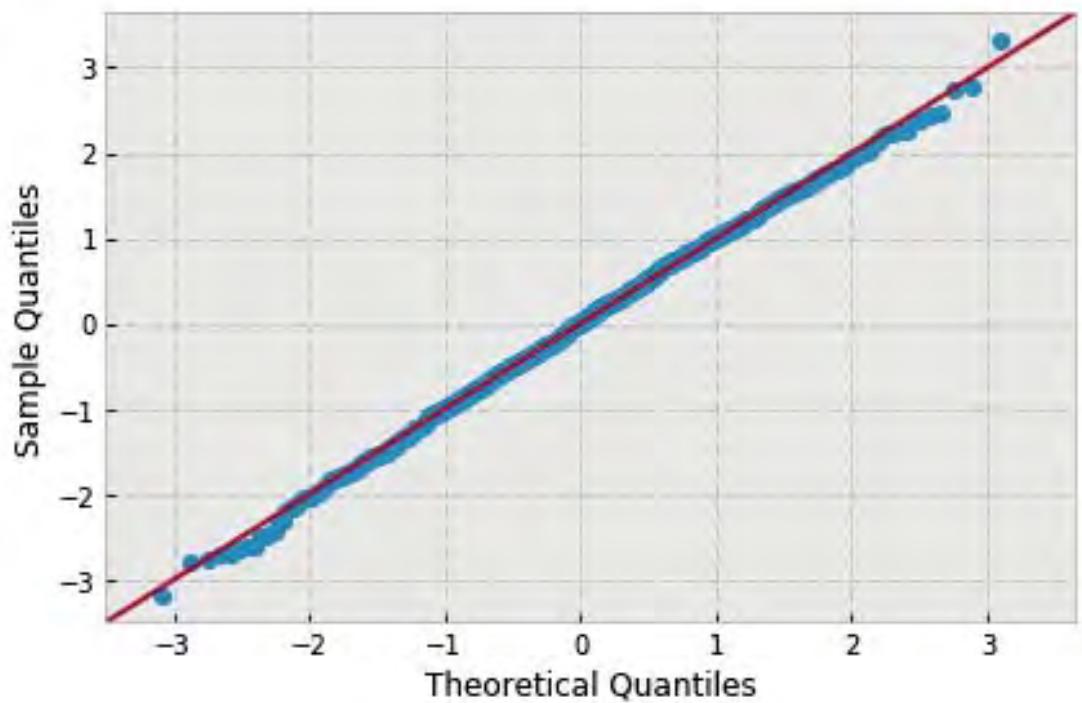


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=776.8083050213021, scale=25.851368458406093)
DescribeResult(nobs=1000, minmax=(694.29776658748312, 853.62564735869512)
               mean=776.80830502130209, variance=668.96221338565908,
               skewness=-0.11181693791090777, kurtosis=-0.24227672837564818)
```

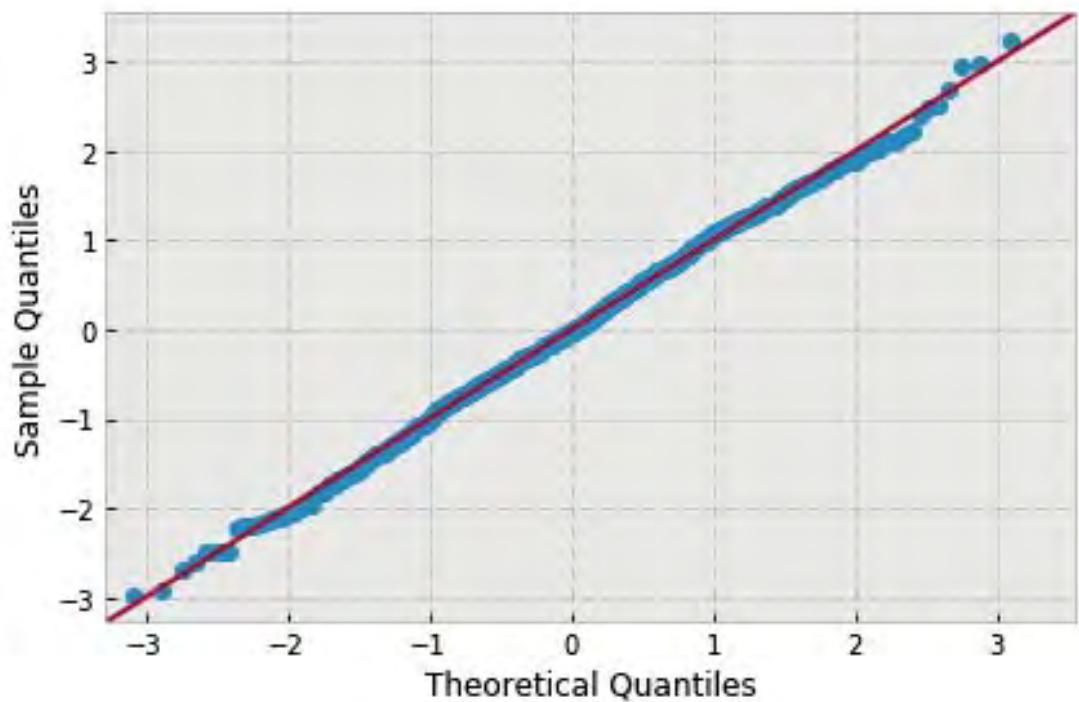
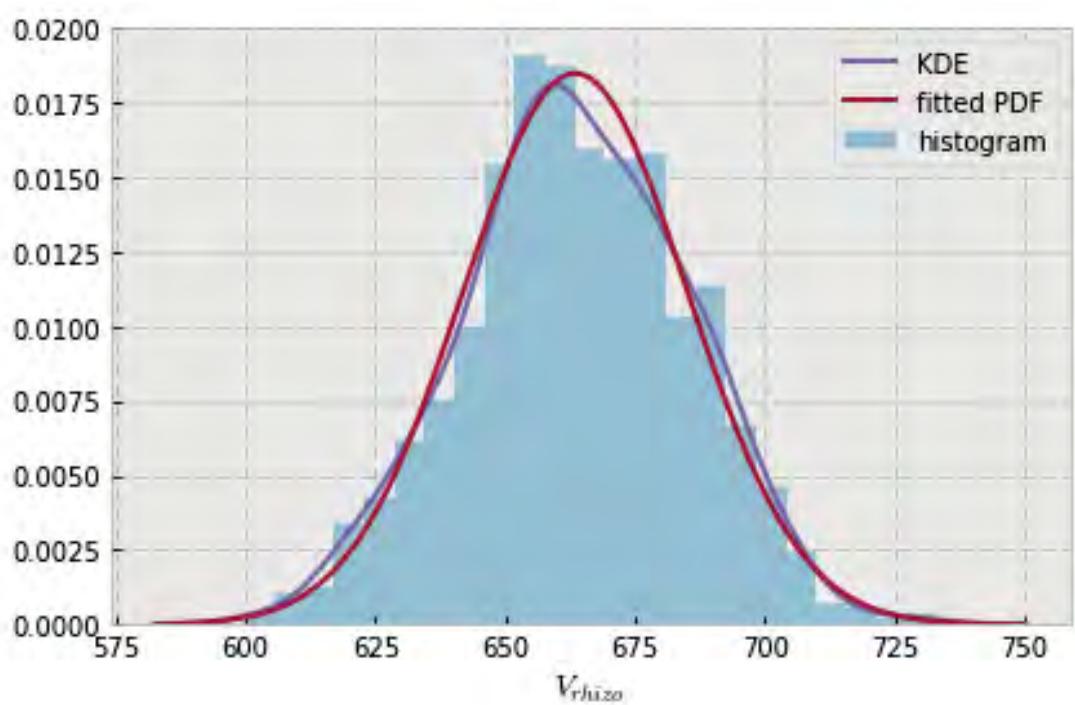


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=713.6518044950167, scale=23.13310737042945)
DescribeResult(nobs=1000, minmax=(640.24533335280034, 790.30783354688288)
               mean=713.65180449501668, variance=535.67633294476207,
               skewness=-0.08706452415170406, kurtosis=-0.11827624565716821)
```

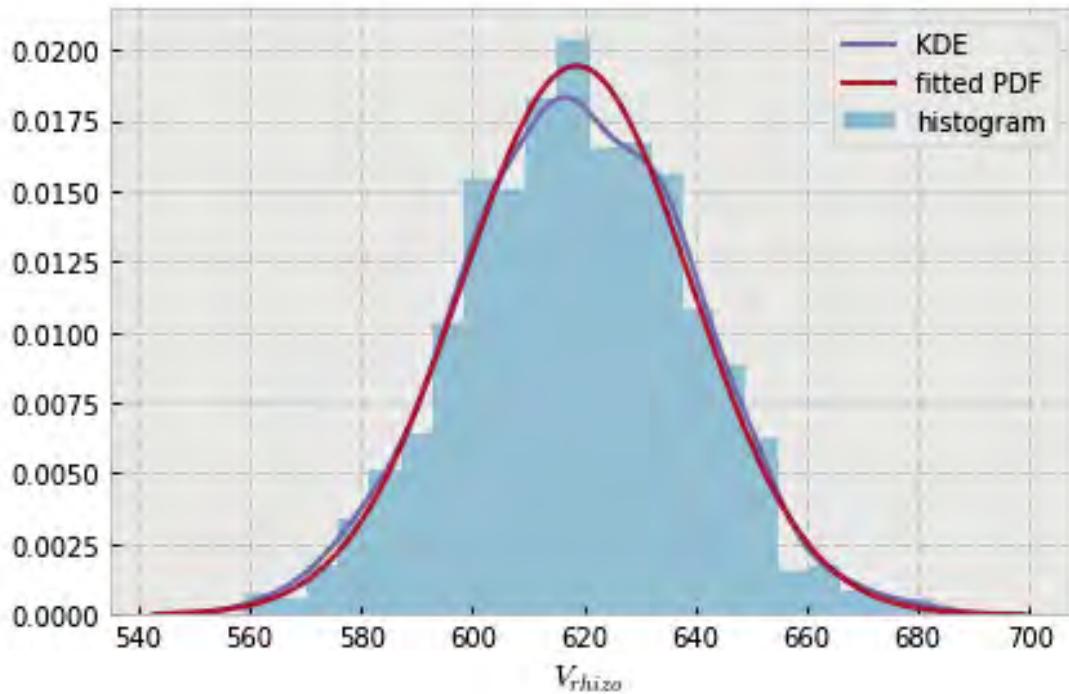


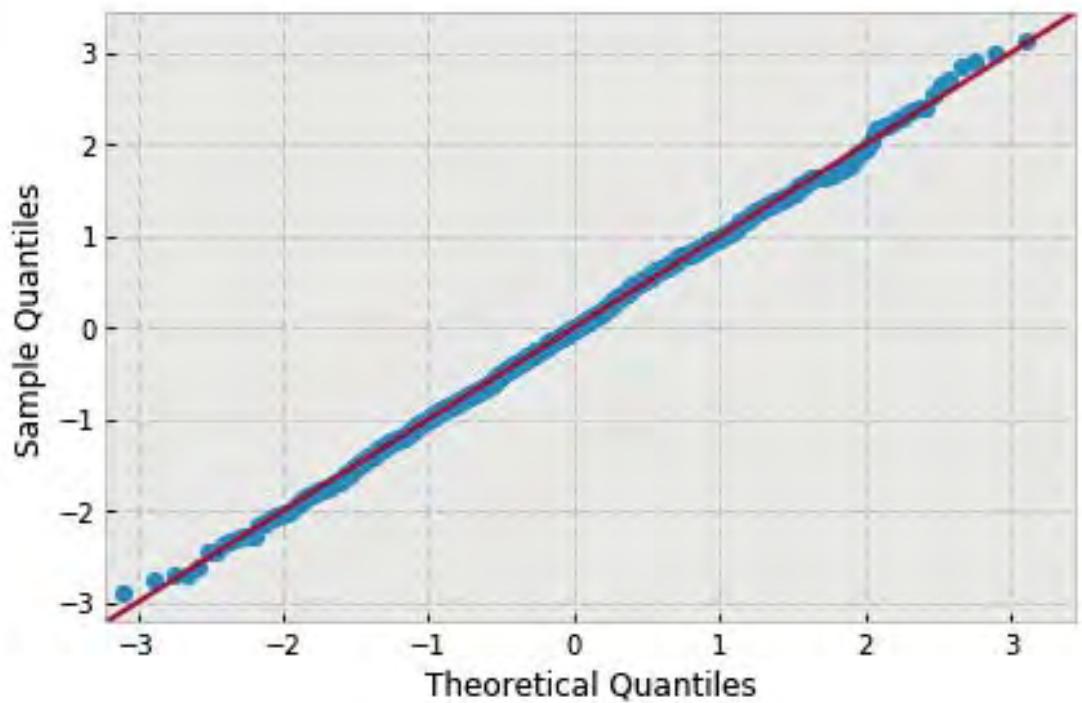


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=663.5243731438107, scale=21.569489612170766)
DescribeResult(nobs=1000, minmax=(599.54382555309894, 733.21312480490565)
               mean=663.52437314381075, variance=465.70859072026292,
               skewness=-0.044737882403939576, kurtosis=-0.16553682249537482)
```

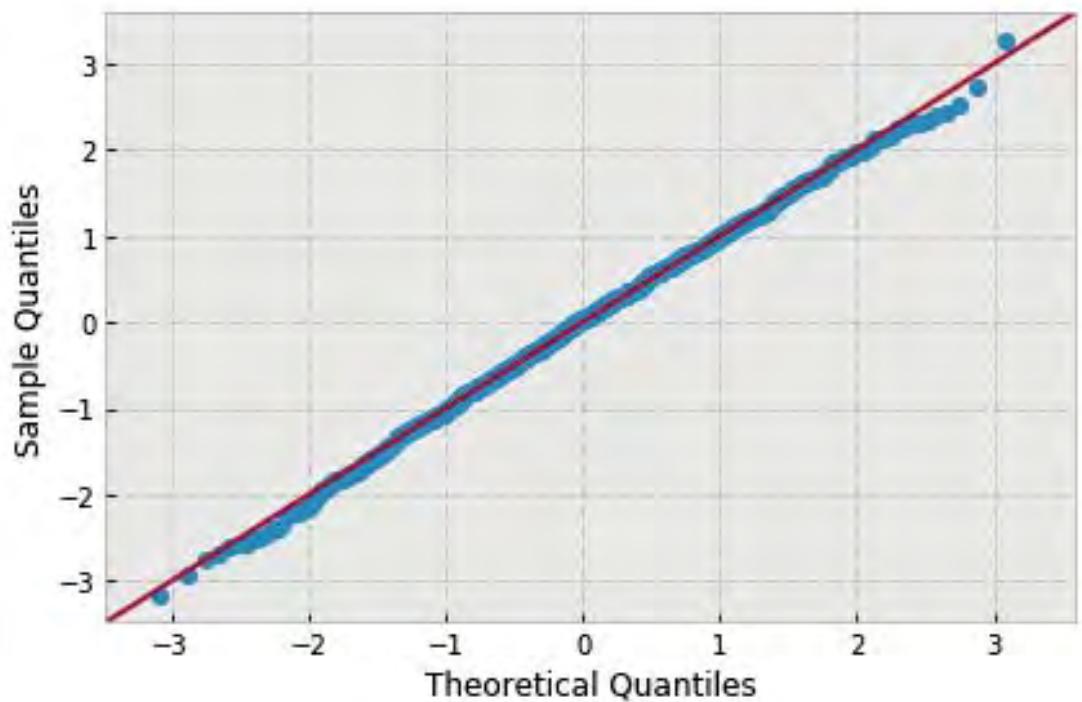
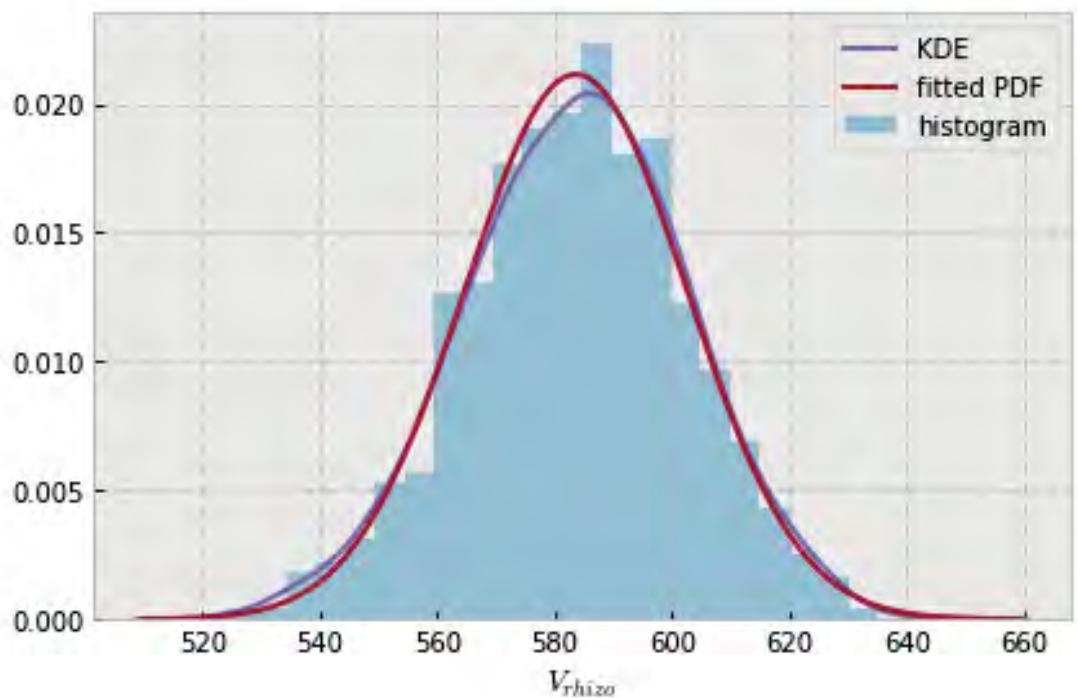


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=618.7014398691354, scale=20.52914960483281)
DescribeResult(nobs=1000, minmax=(558.99778236435952, 682.89014232025681)
               mean=618.70143986913536, variance=421.86785134895615,
               skewness=0.004077681134218601, kurtosis=-0.11604790909890594)
```

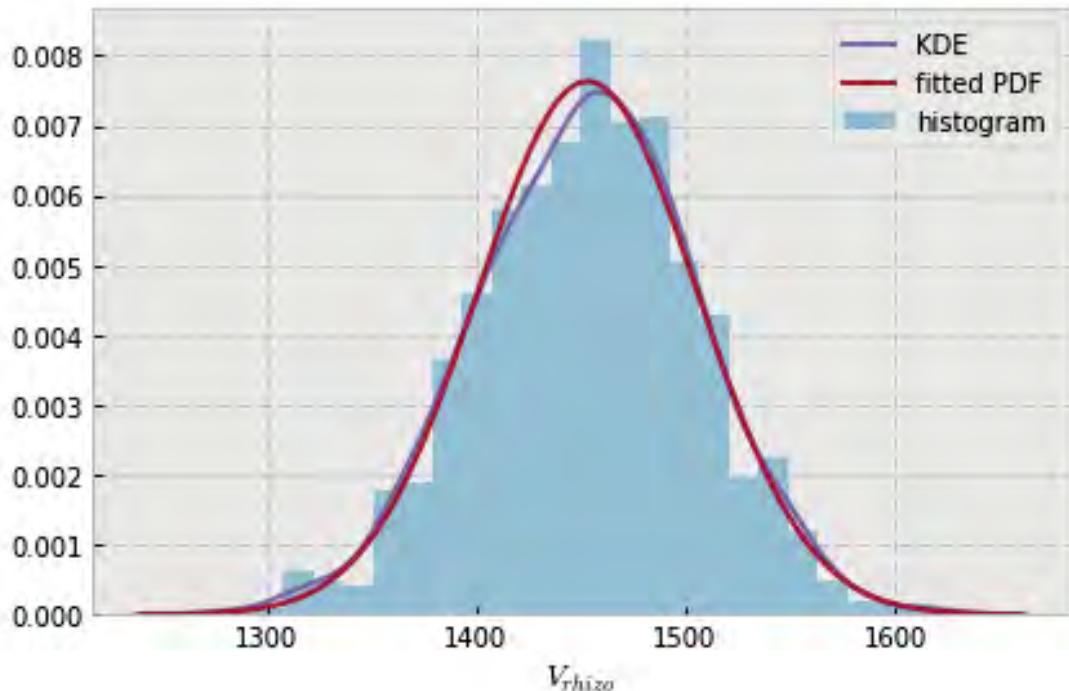


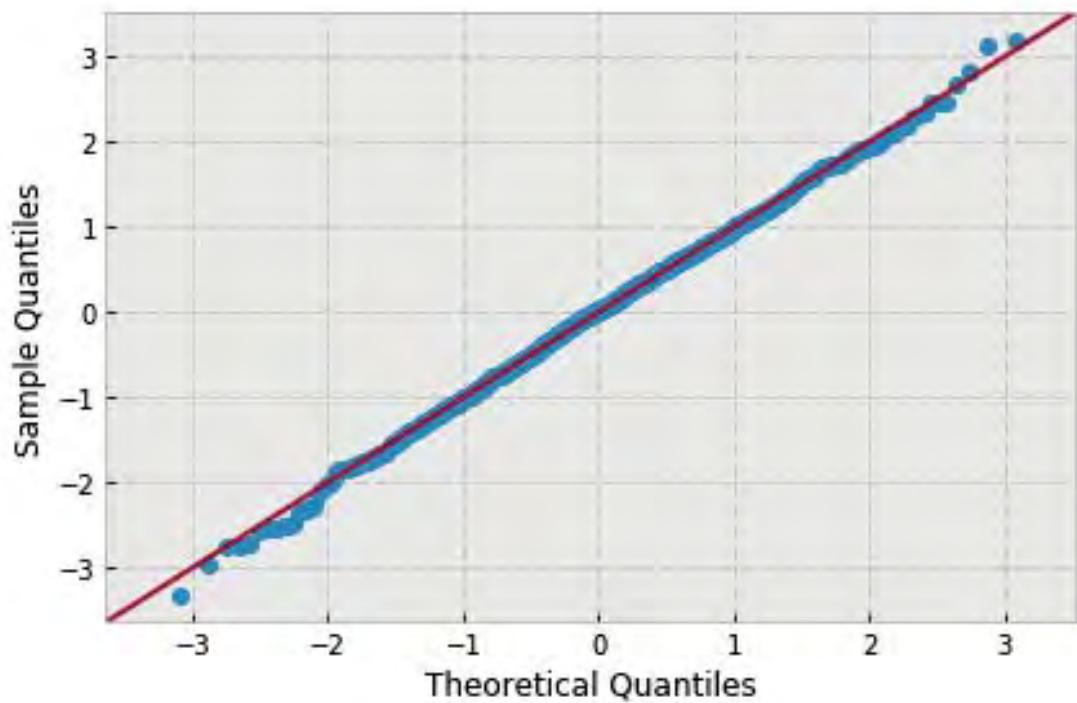


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=583.6085688254342, scale=18.868468503559978)
DescribeResult(nobs=1000, minmax=(524.0977992400974, 645.17288604673274)
               mean=583.60856882543419, variance=356.37547914898391,
               skewness=-0.10448371797412752, kurtosis=-0.08800487880415675)
```

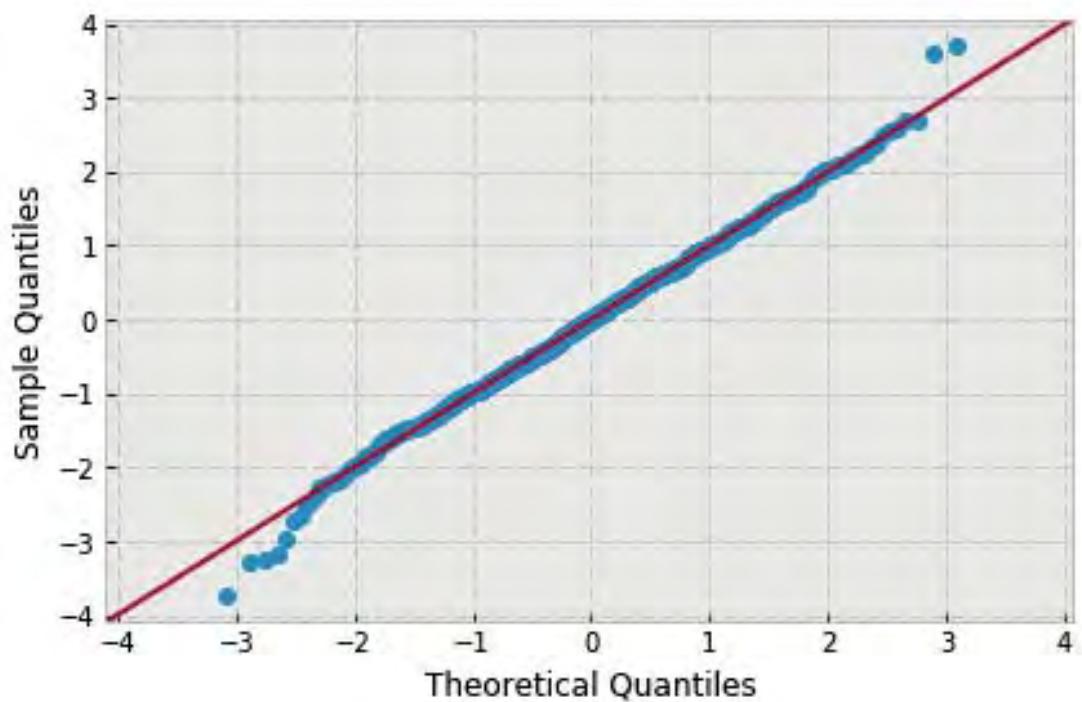
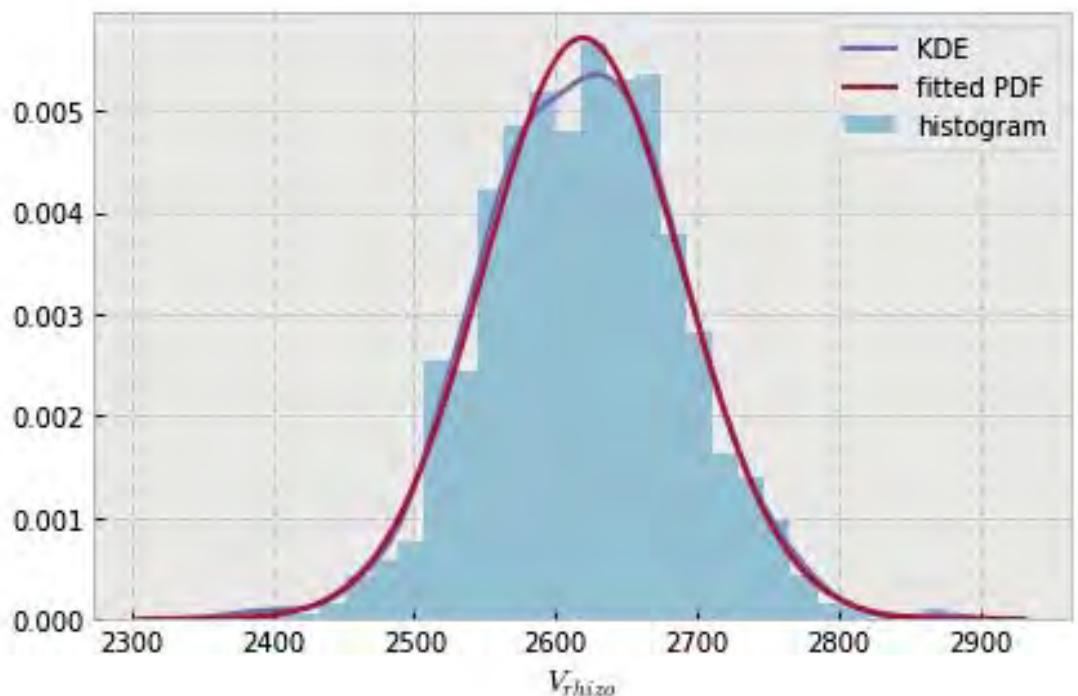


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=1453.2000252399907, scale=52.291292775562155)
DescribeResult(nobs=1000, minmax=(1279.7861901139552, 1620.0927712190121)
               mean=1453.2000252399907, variance=2737.1164165561149,
               skewness=-0.07494477495340478, kurtosis=-0.0020462324231185214)
```

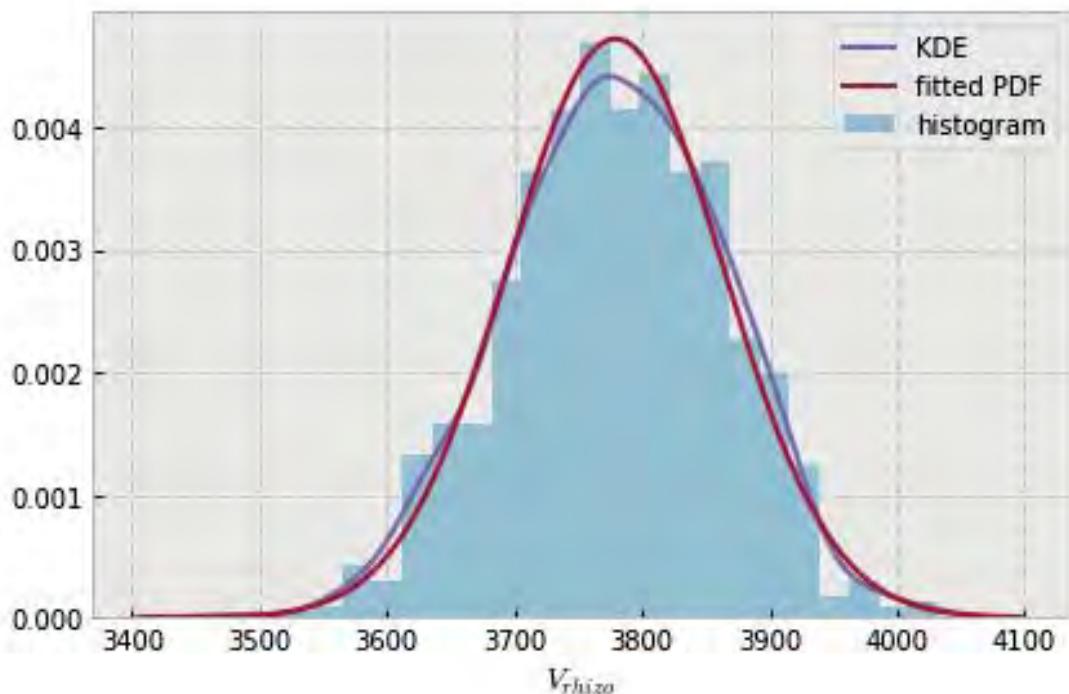


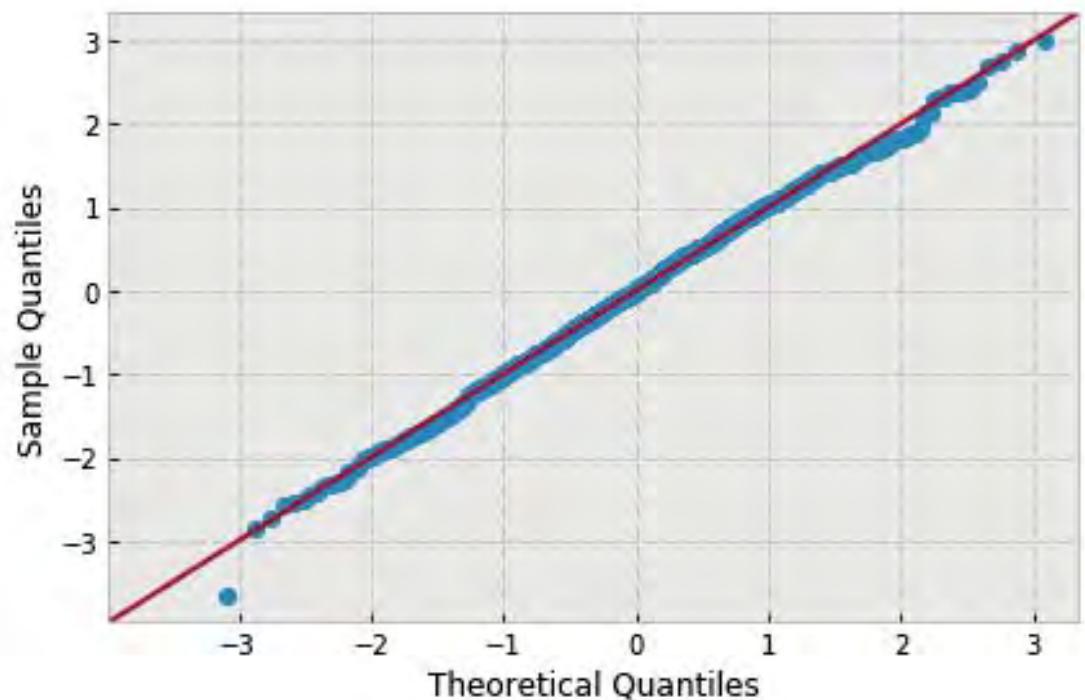


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=2619.3161444390225, scale=69.83402074958904)
DescribeResult(nobs=1000, minmax=(2359.2462103718108, 2877.5493371573675)
               mean=2619.3161444390225, variance=4881.6721261802149,
               skewness=-0.01806337145342376, kurtosis=0.27764515788961663)
```

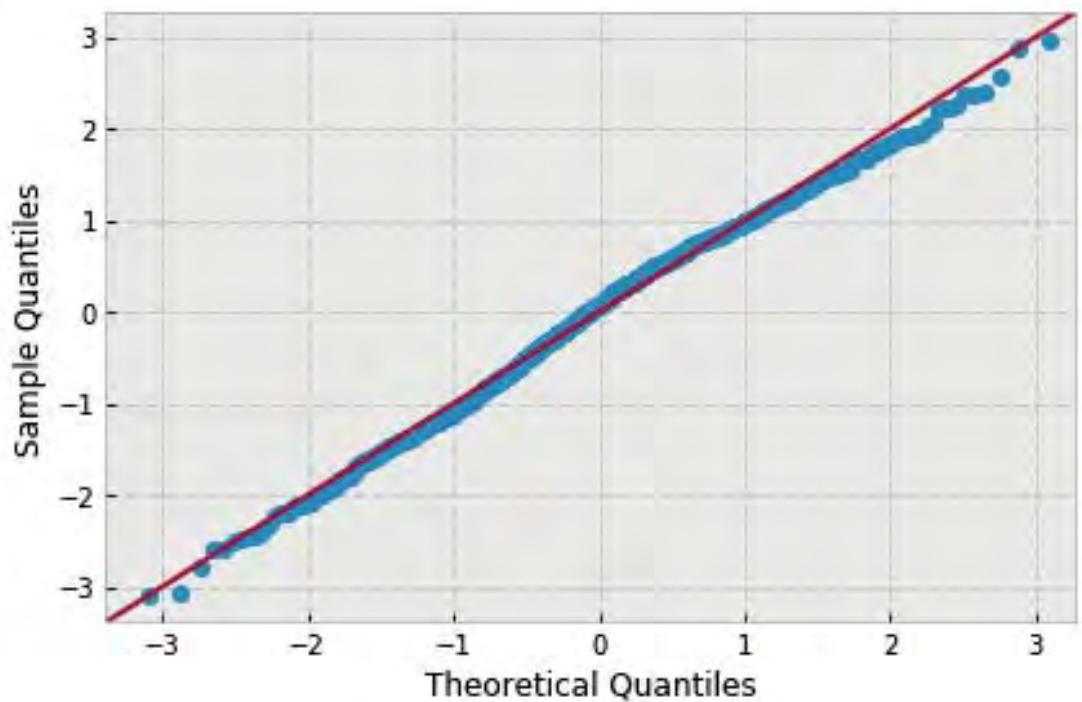
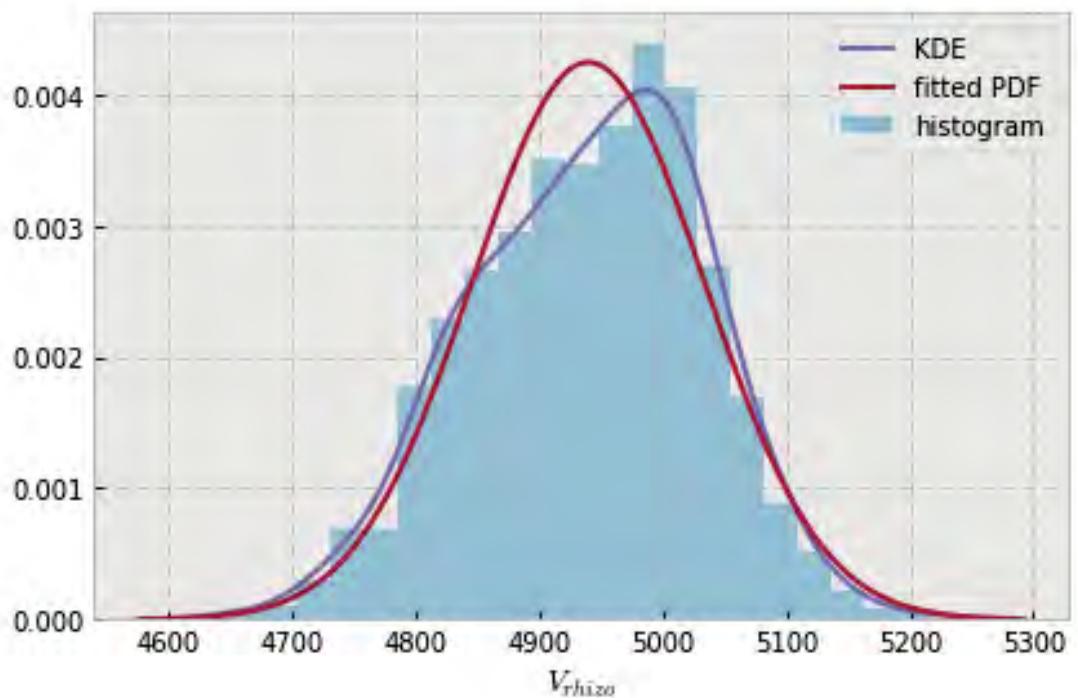


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=3779.025635818468, scale=84.47973176434652)
DescribeResult(nobs=1000, minmax=(3471.2889210383782, 4032.4431490596708)
               mean=3779.0256358184679, variance=7143.9690480239624,
               skewness=-0.10144830352750757, kurtosis=-0.1530162622386988)
```

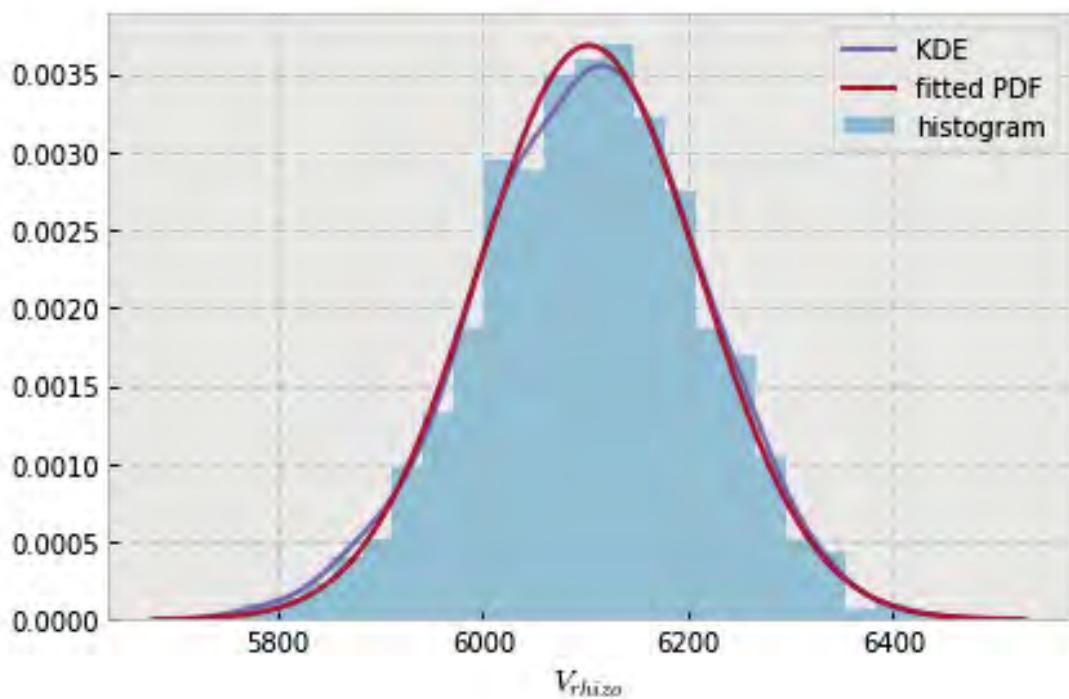


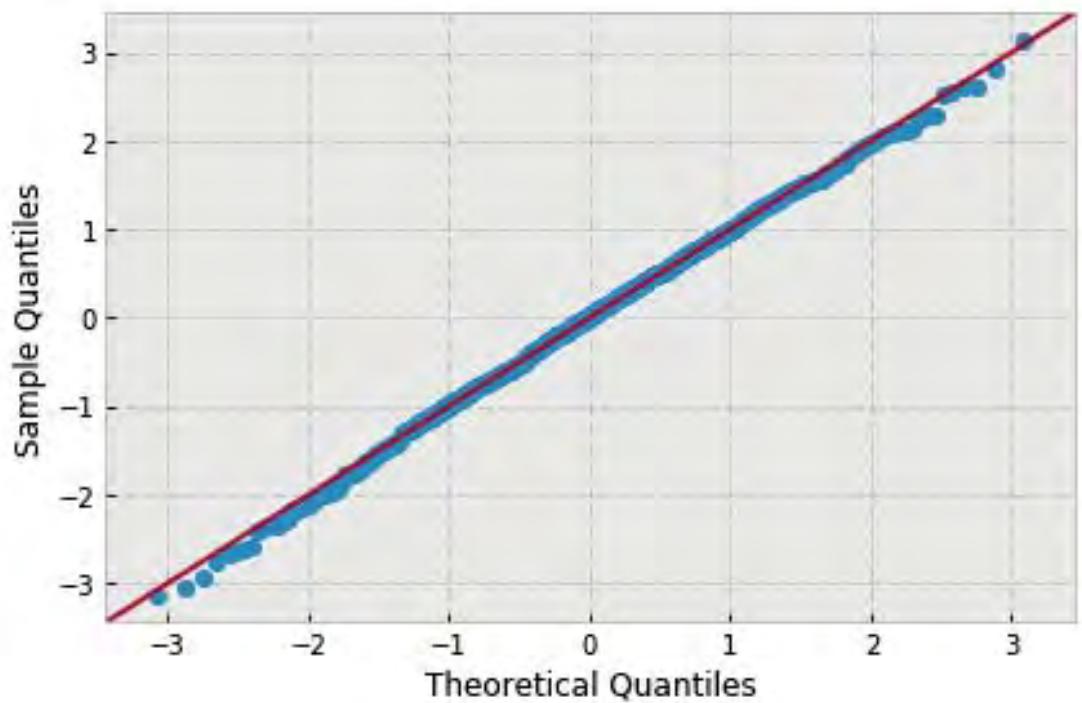


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=4939.86810783088, scale=93.8759518209005)
DescribeResult(nobs=1000, minmax=(4650.2534185531113, 5217.6119452653566)
               mean=4939.86810783088, variance=8821.5158461261562,
               skewness=-0.20408375480087557, kurtosis=-0.3066819118551116)
```

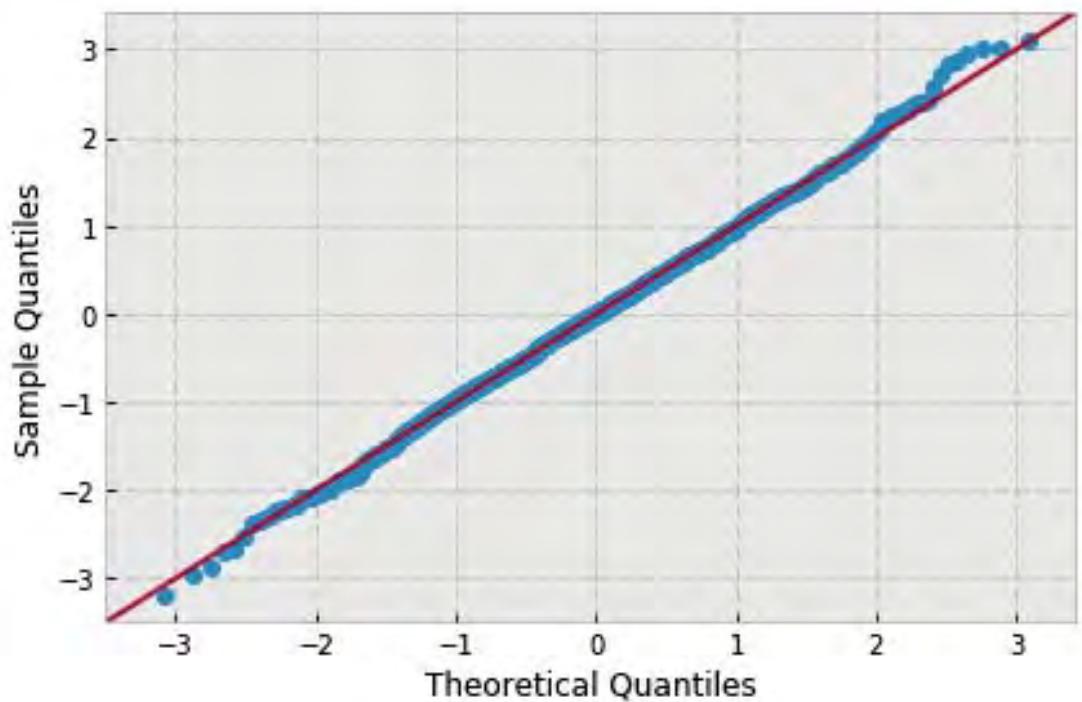
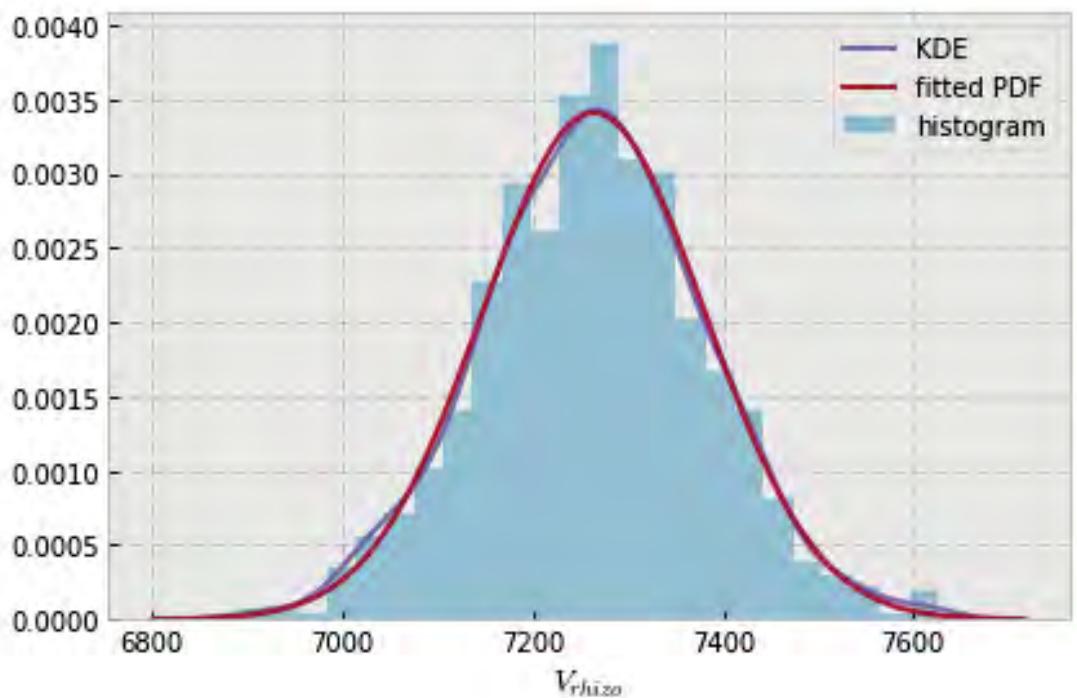


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=6103.204632546824, scale=108.33474421442249)
DescribeResult(nobs=1000, minmax=(5764.3417117346589, 6442.2836692424999)
               mean=6103.2046325468236, variance=11748.164968973319,
               skewness=-0.10615545566631834, kurtosis=-0.05271192226347354)
```

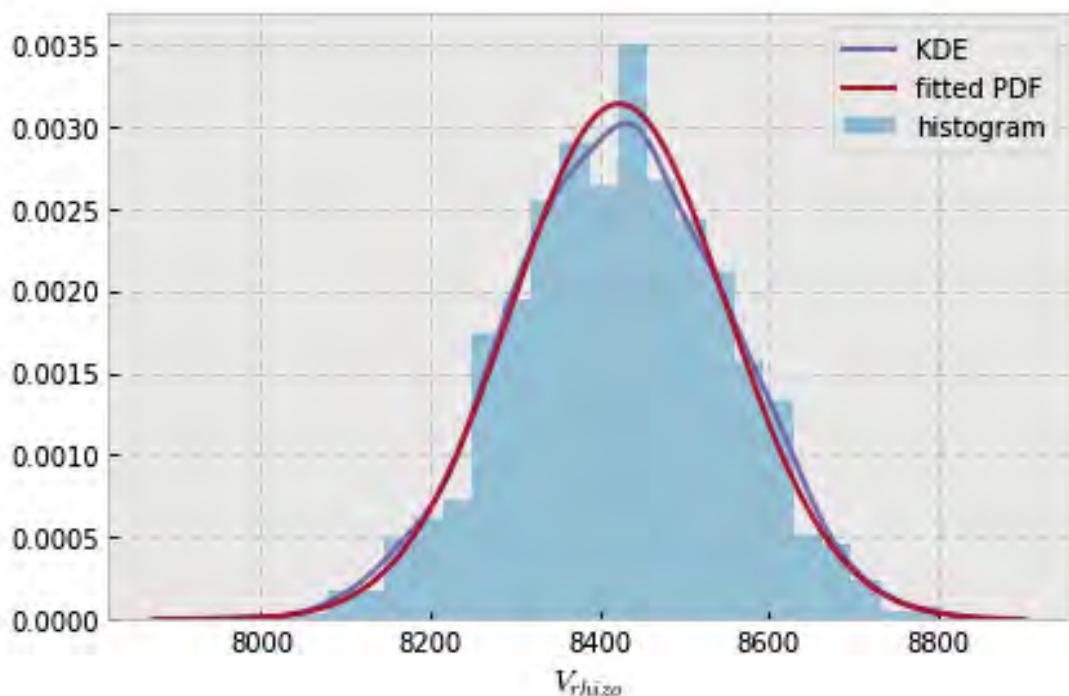


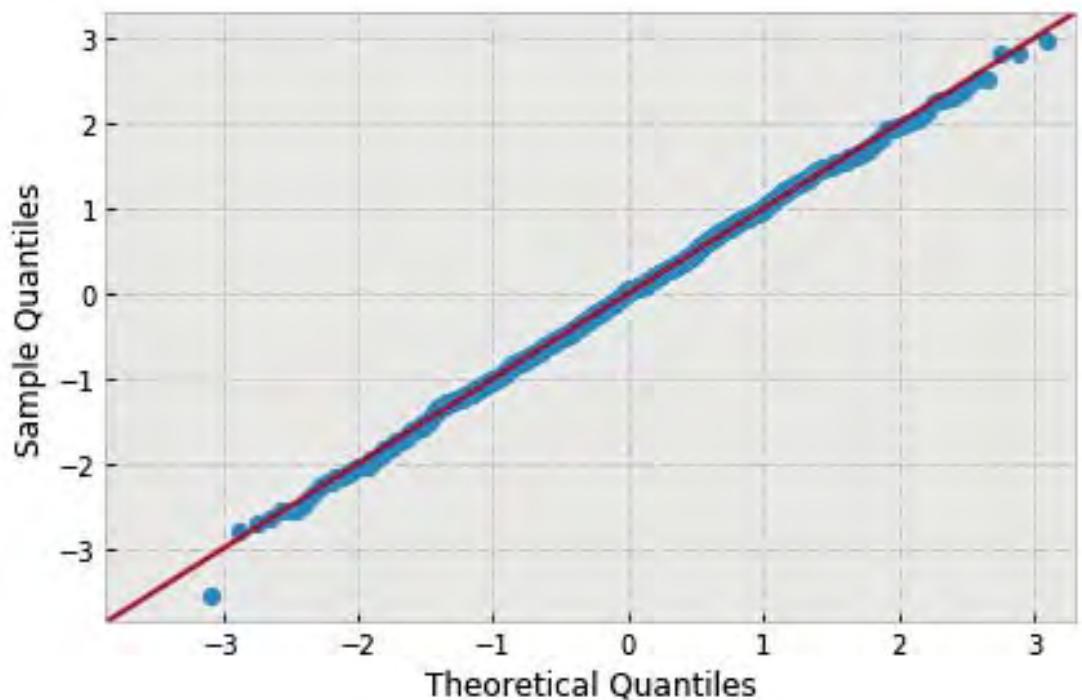


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=7263.977198564112, scale=116.9420867009029)
DescribeResult(nobs=1000, minmax=(6891.6594080339719, 7626.3200068976248)
               mean=7263.9771985641119, variance=13689.140782744236,
               skewness=0.022920329721936857, kurtosis=0.10608726887044417)
```

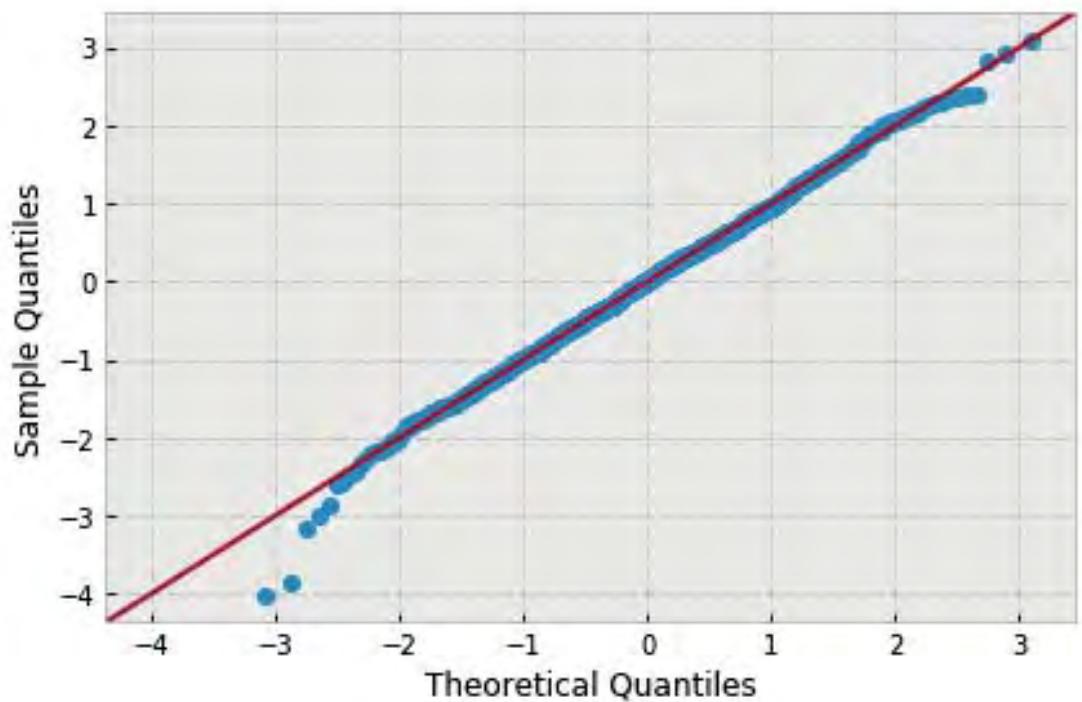
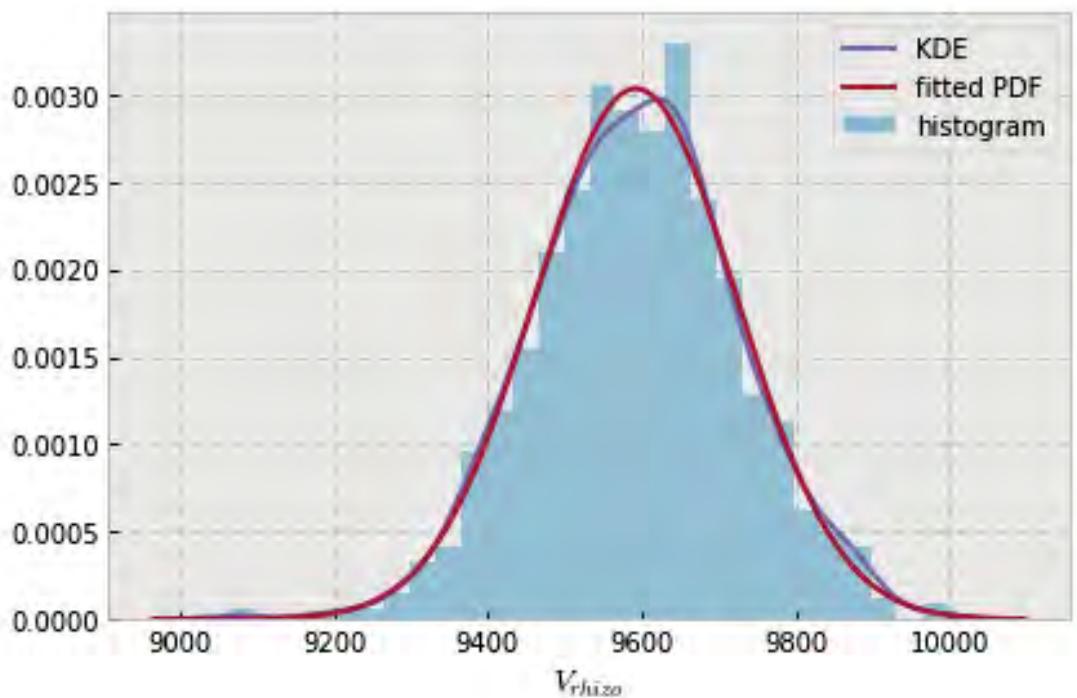


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=8423.317702499939, scale=127.276689179703)
DescribeResult(nobs=1000, minmax=(7973.2233053348828, 8801.7943204923358)
               mean=8423.3177024999386, variance=16215.571179726452,
               skewness=-0.044137073625663686, kurtosis=-0.1318161752535727)
```

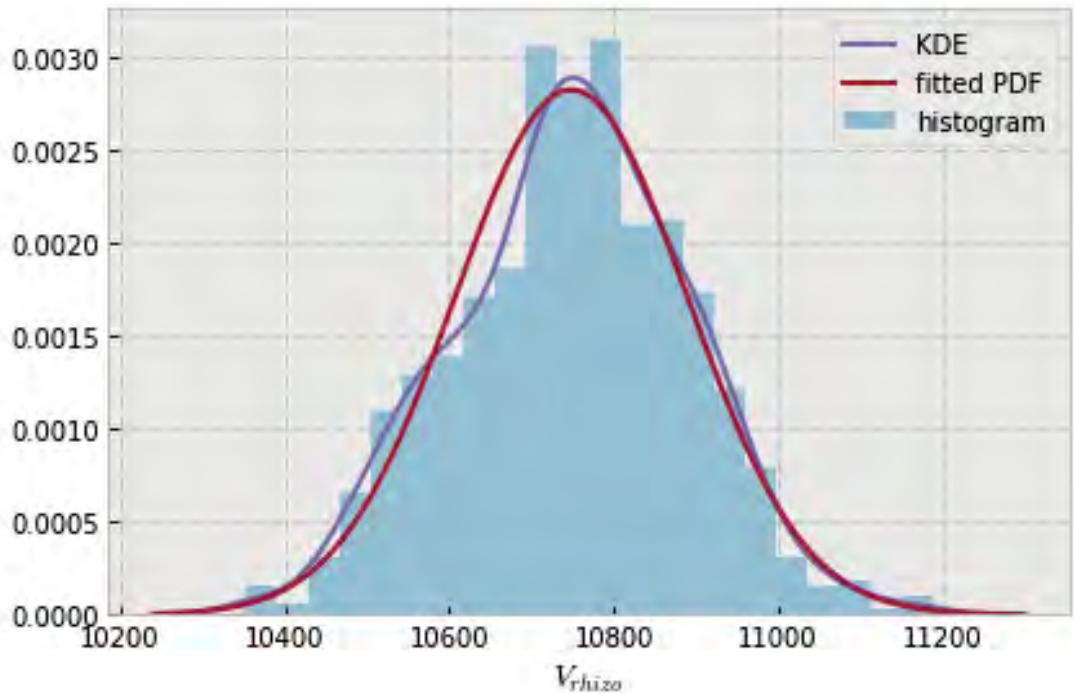


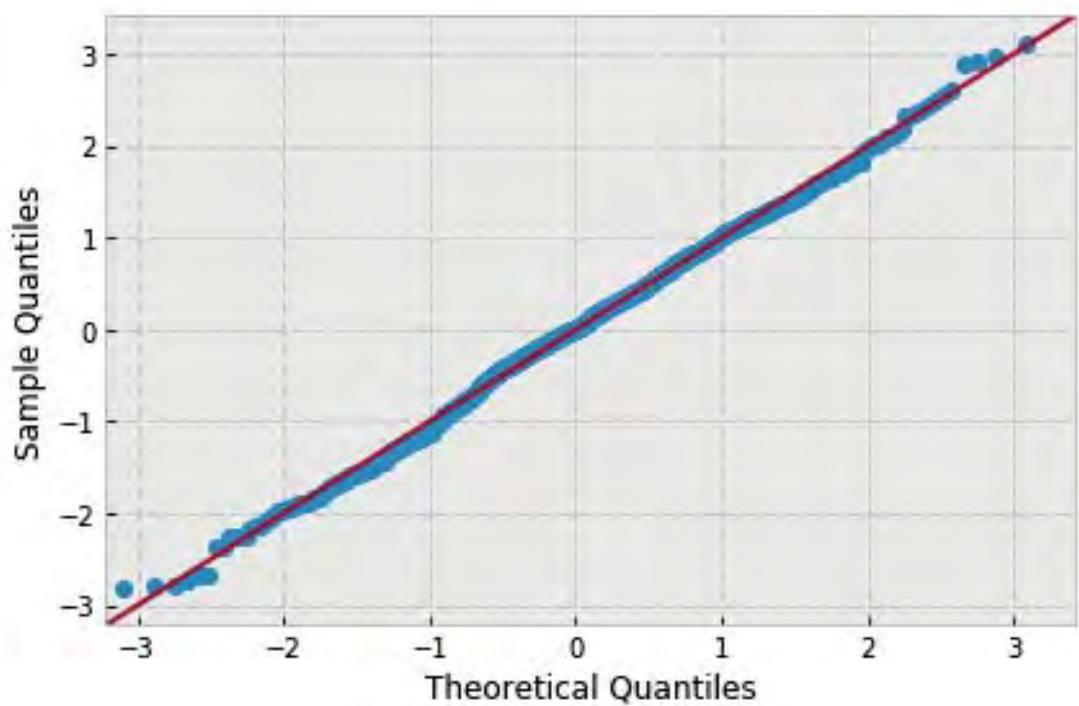


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=9592.481543127351, scale=131.51341247791228)
DescribeResult(nobs=1000, minmax=(9064.6013404512687, 9998.103298443013)
               mean=9592.481543127351, variance=17313.090752337834,
               skewness=-0.08190263385696116, kurtosis=0.24929818807648418)
```

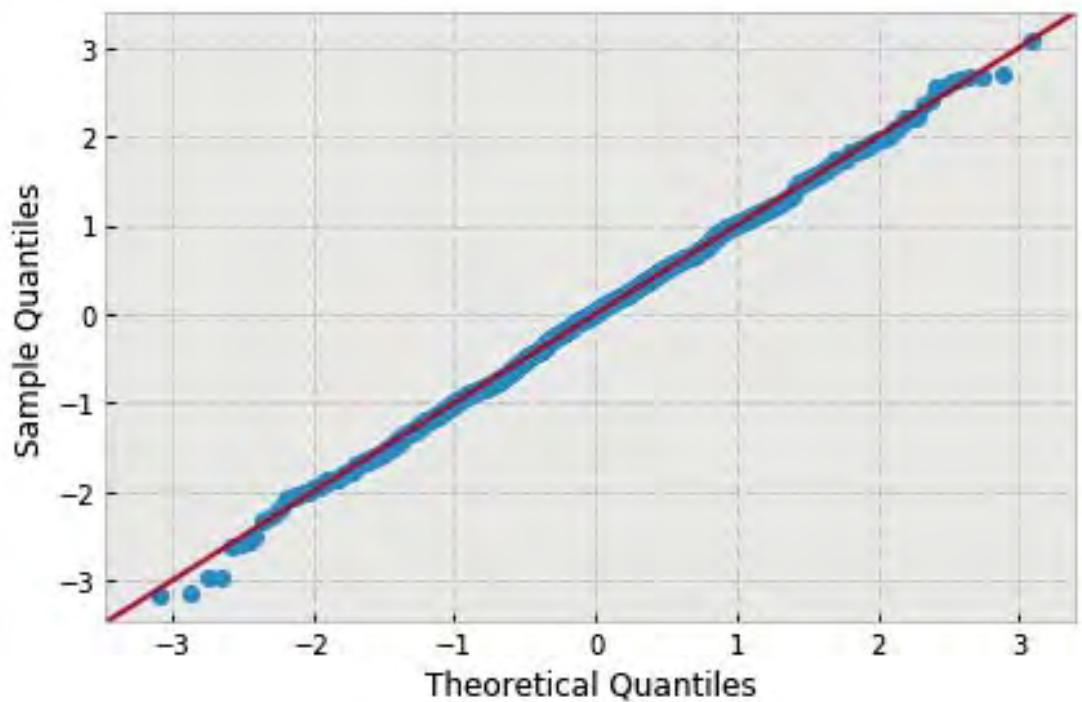
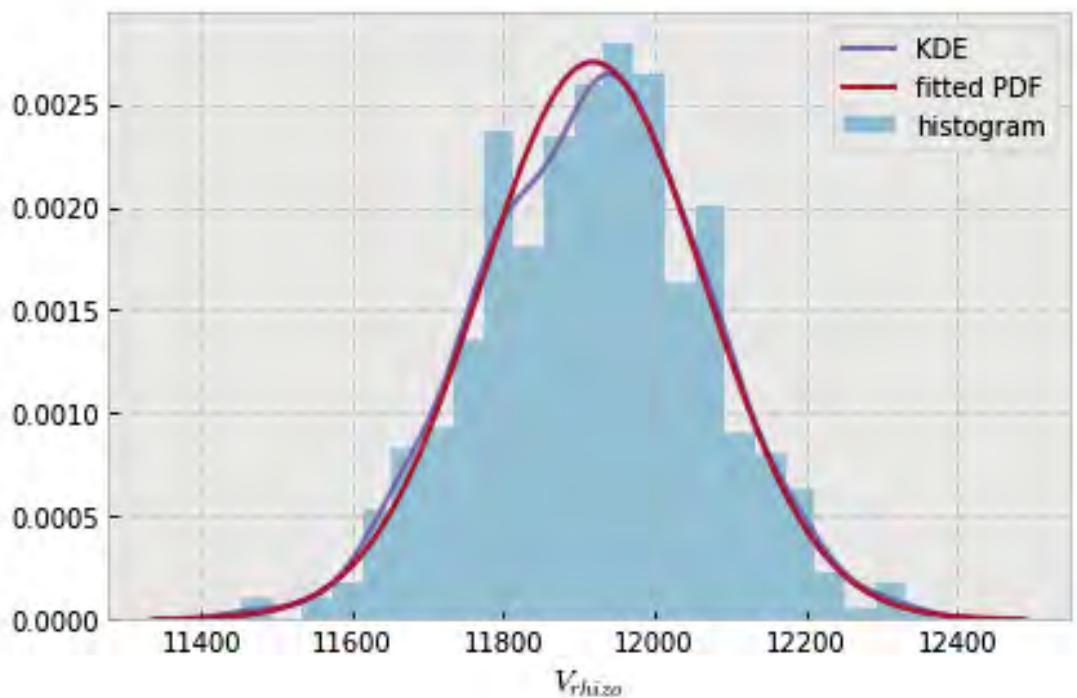


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=10748.373674957764, scale=141.12787398039495)
DescribeResult(nobs=1000, minmax=(10351.995344607487, 11187.862268502406)
               mean=10748.373674957764, variance=19937.0138280543,
               skewness=-0.06626703510759054, kurtosis=-0.1394440396658574)
```

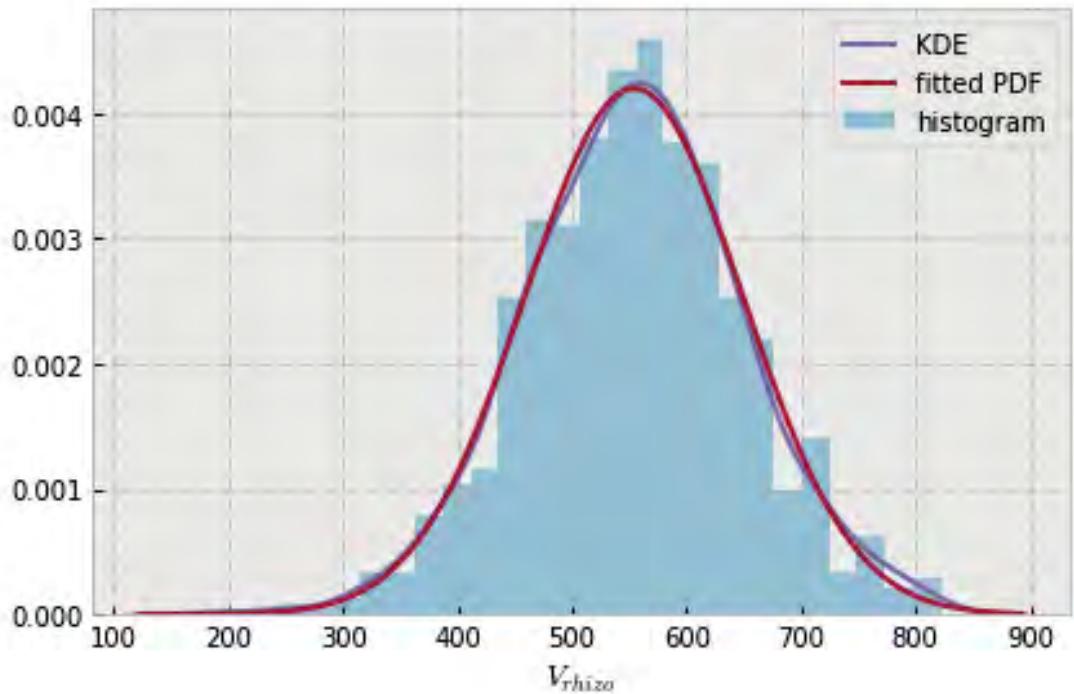


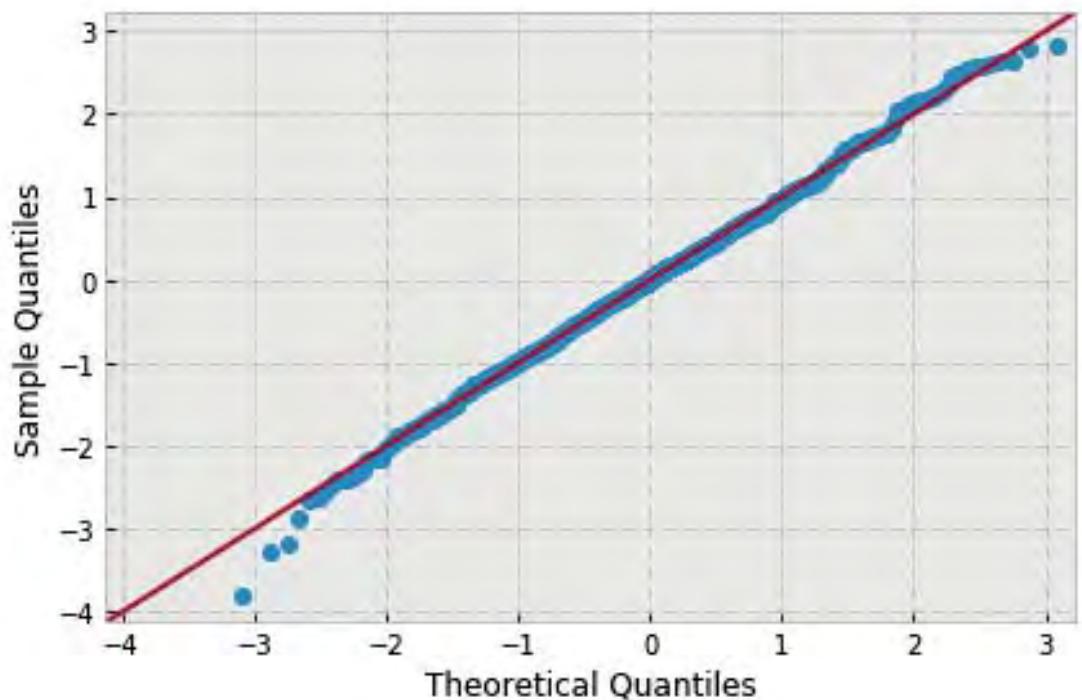


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=11918.500080630009, scale=147.51511086240794)
DescribeResult(nobs=1000, minmax=(11453.104213308425, 12372.691625170364)
               mean=11918.500080630009, variance=21782.490423171676,
               skewness=-0.039993092148023296, kurtosis=-0.08976101321260277)
```

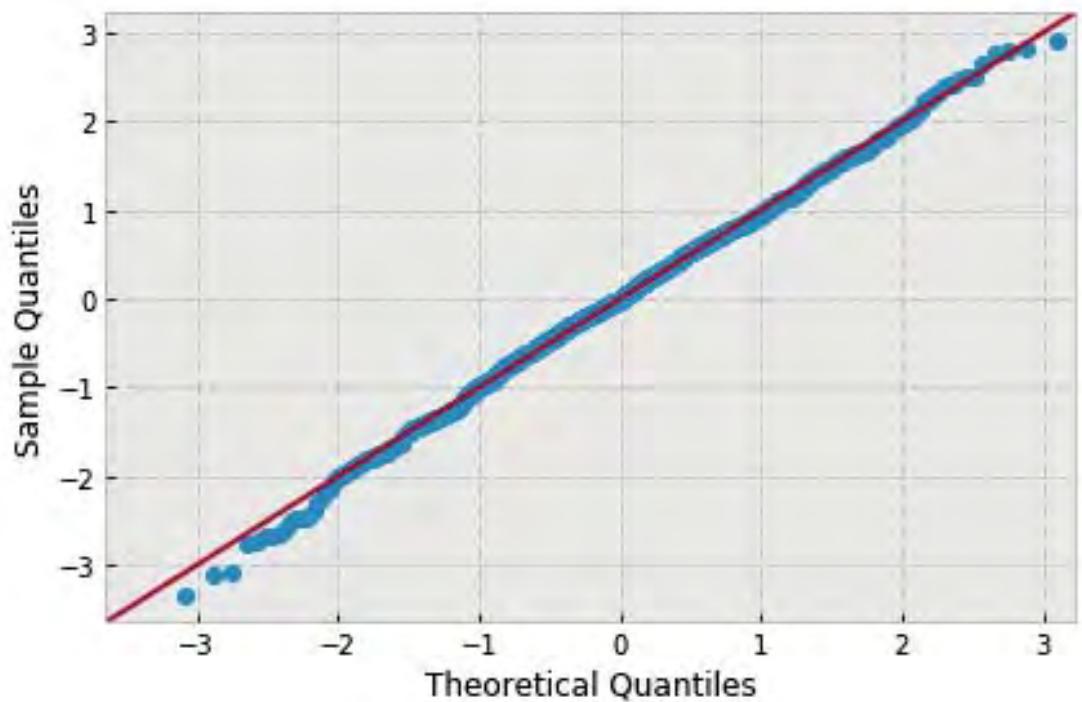
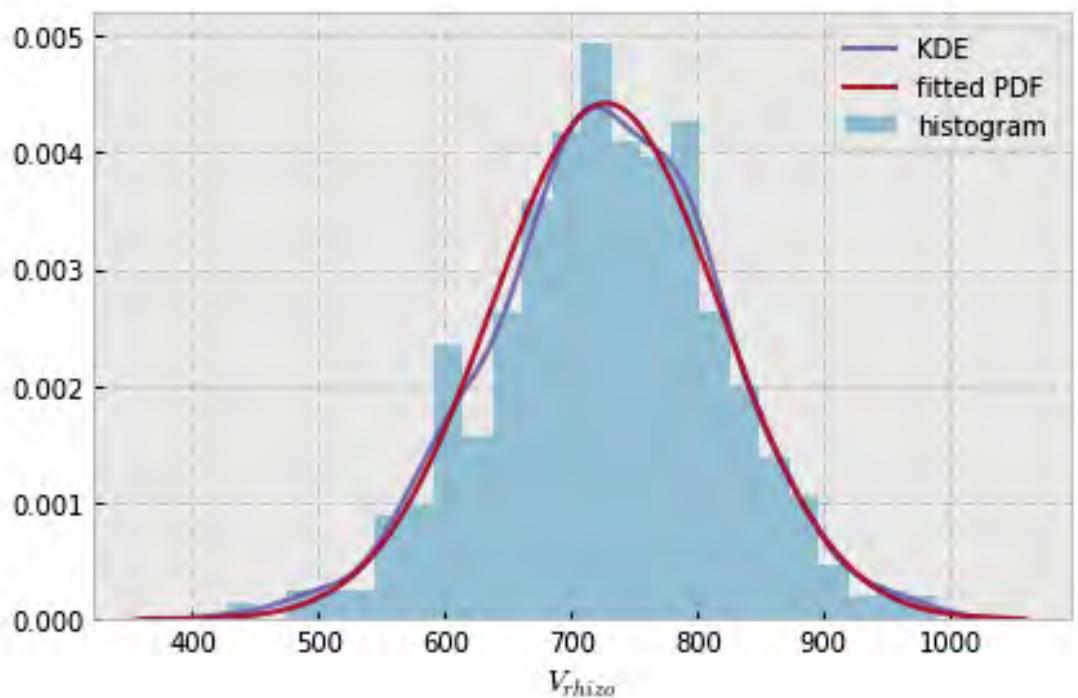


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=554.0695184939487, scale=94.98980447552621)
DescribeResult(nobs=1000, minmax=(194.12243994791757, 821.08722486193494)
               mean=554.06951849394875, variance=9032.0950493480486,
               skewness=-0.037257818690103646, kurtosis=0.20132820932186002)
```

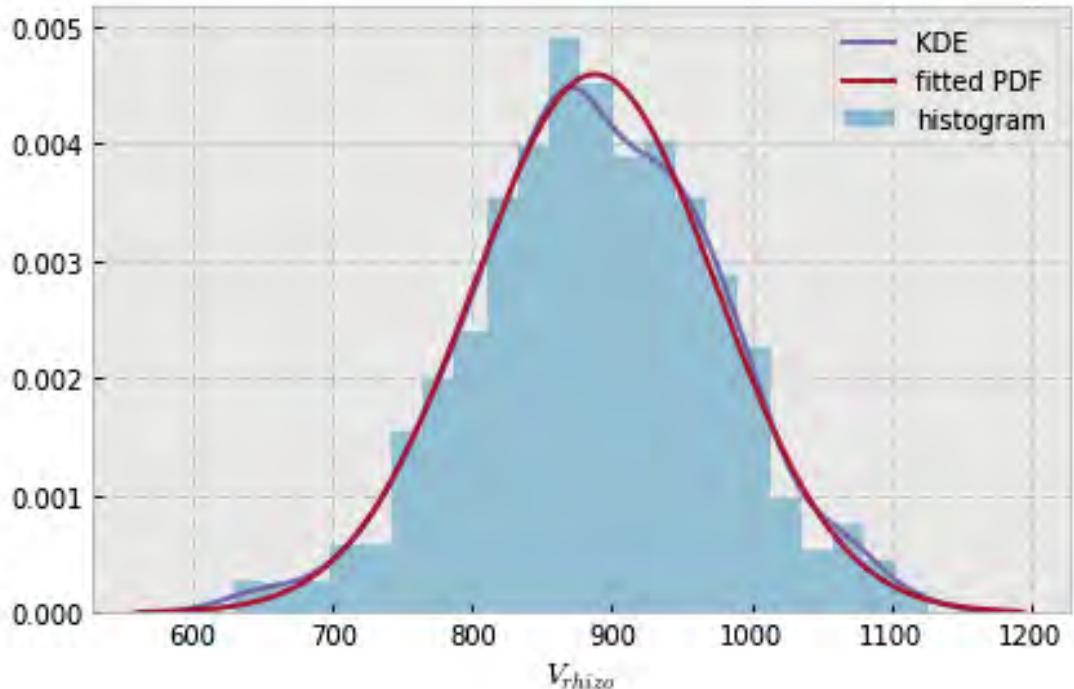


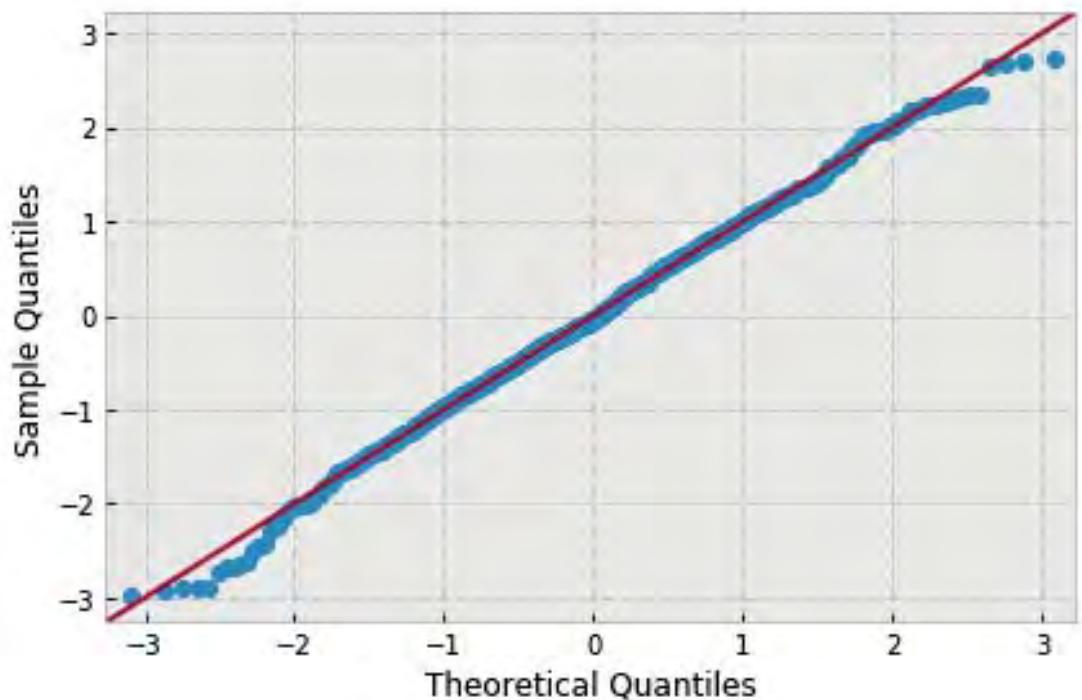


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(), loc=727.1782391383954, scale=90.22201733567485)
DescribeResult(nobs=1000, minmax=(426.70468346696657, 989.52769108779614)
               mean=727.17823913839538, variance=8148.160572691505,
               skewness=-0.11763545118187402, kurtosis=0.09436765762173183)
```

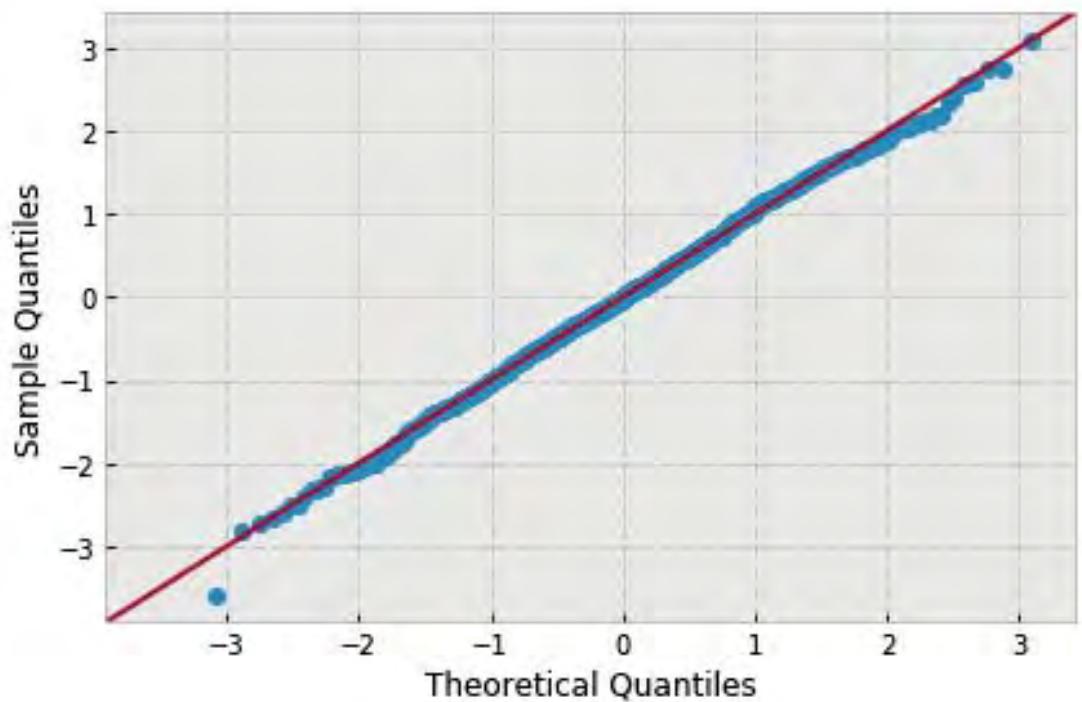
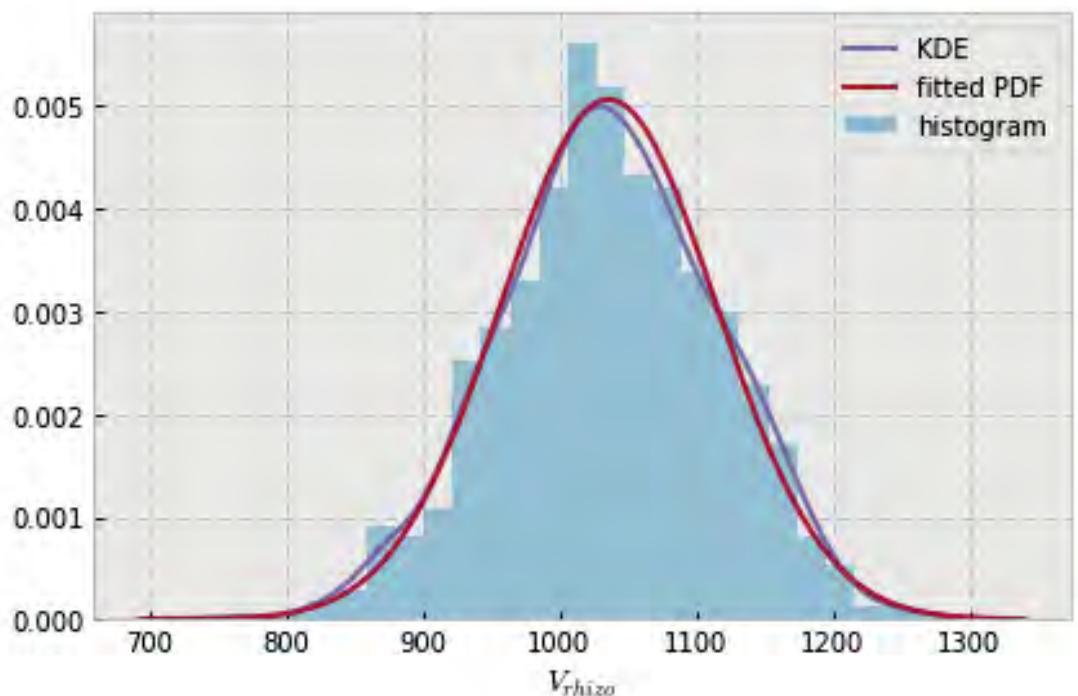


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=887.8635223277263, scale=86.92725355544732)
DescribeResult(nobs=1000, minmax=(629.26180817424824, 1125.9216764474302)
               mean=887.86352232772629, variance=7563.9113220150439,
               skewness=-0.08002316912995791, kurtosis=-0.035673105615332545)
```

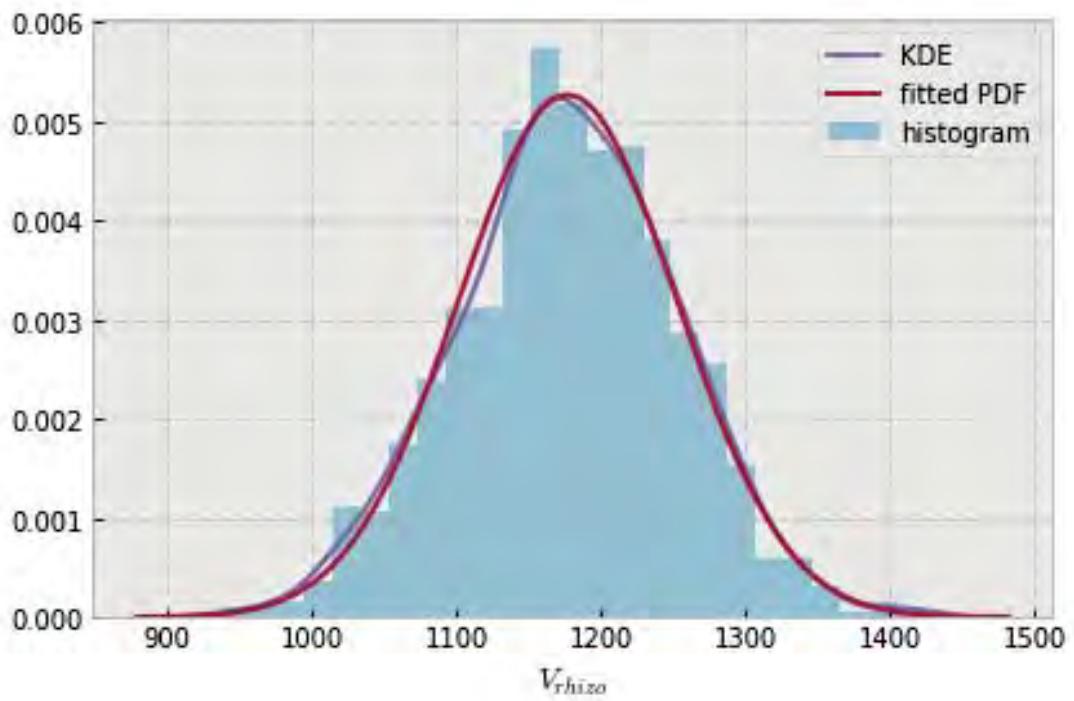


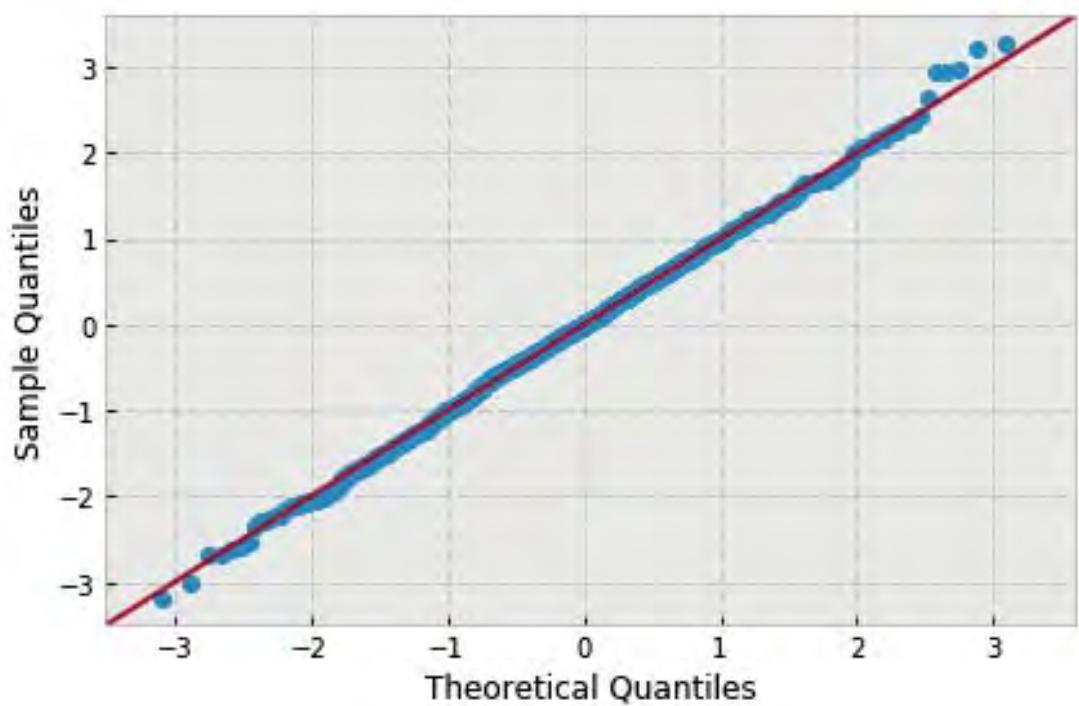


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=1035.7385124384205, scale=78.73259094504611)
DescribeResult(nobs=1000, minmax=(753.92546358301684, 1278.2784393481159)
               mean=1035.7385124384205, variance=6205.0259028227792,
               skewness=-0.06943449567652406, kurtosis=-0.13960000116426663)
```

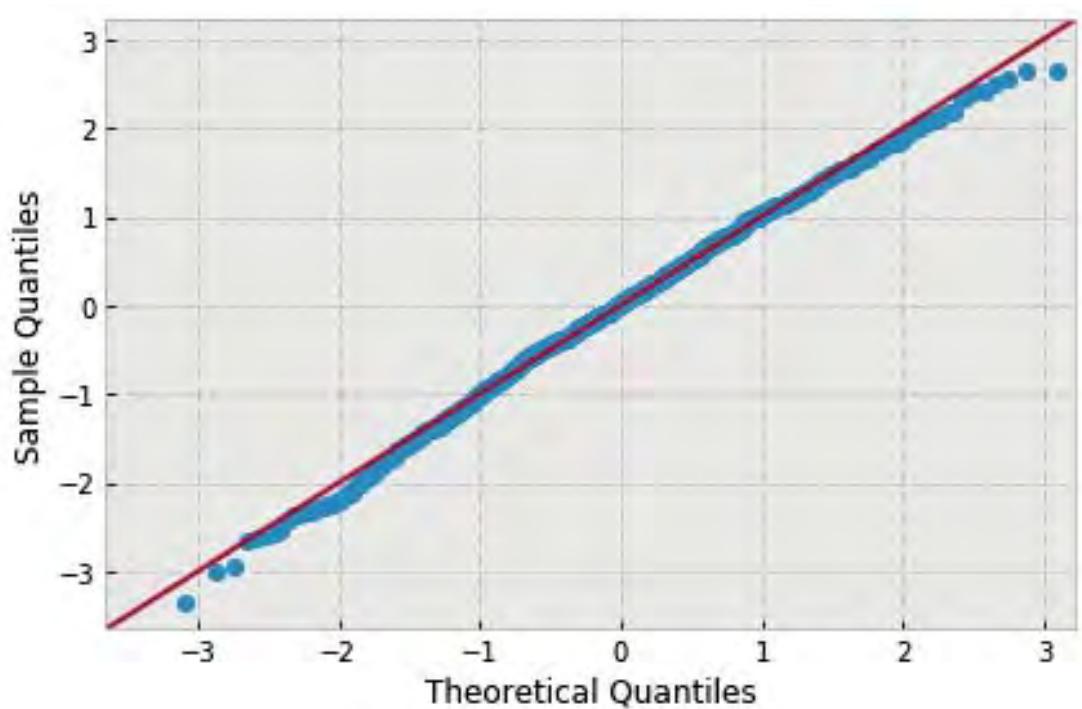
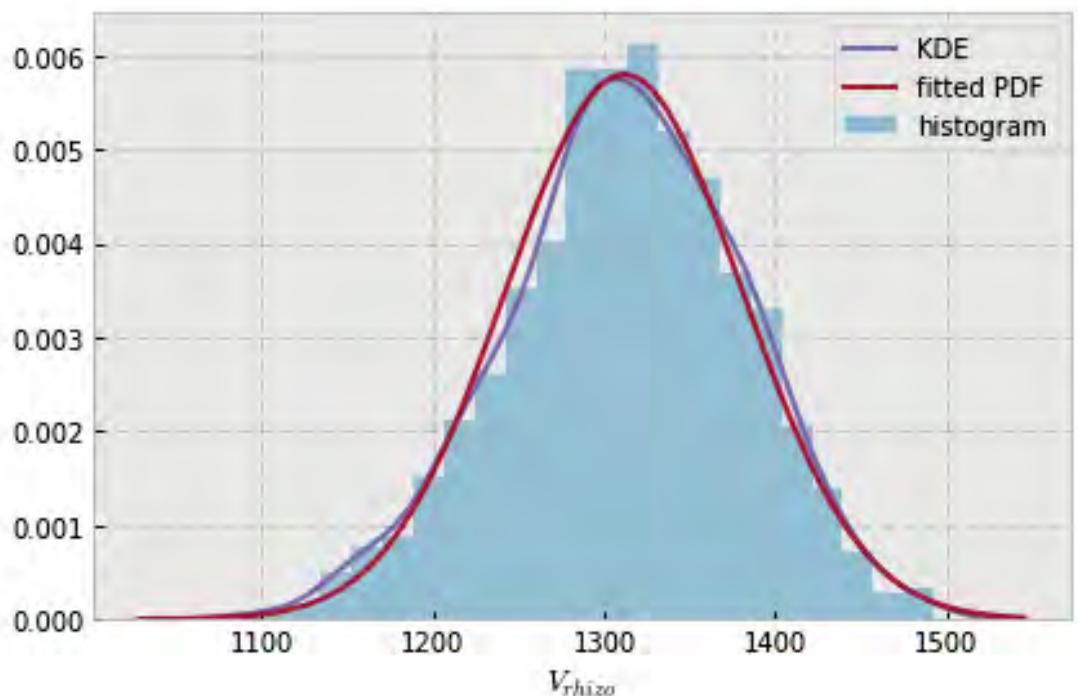


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=1176.945623231736, scale=75.69637548997625)
DescribeResult(nobs=1000, minmax=(936.5618226403418, 1424.2304460941016)
               mean=1176.9456232317359, variance=5735.6769392587339,
               skewness=-0.02425823983324145, kurtosis=0.020030255057090773)
```

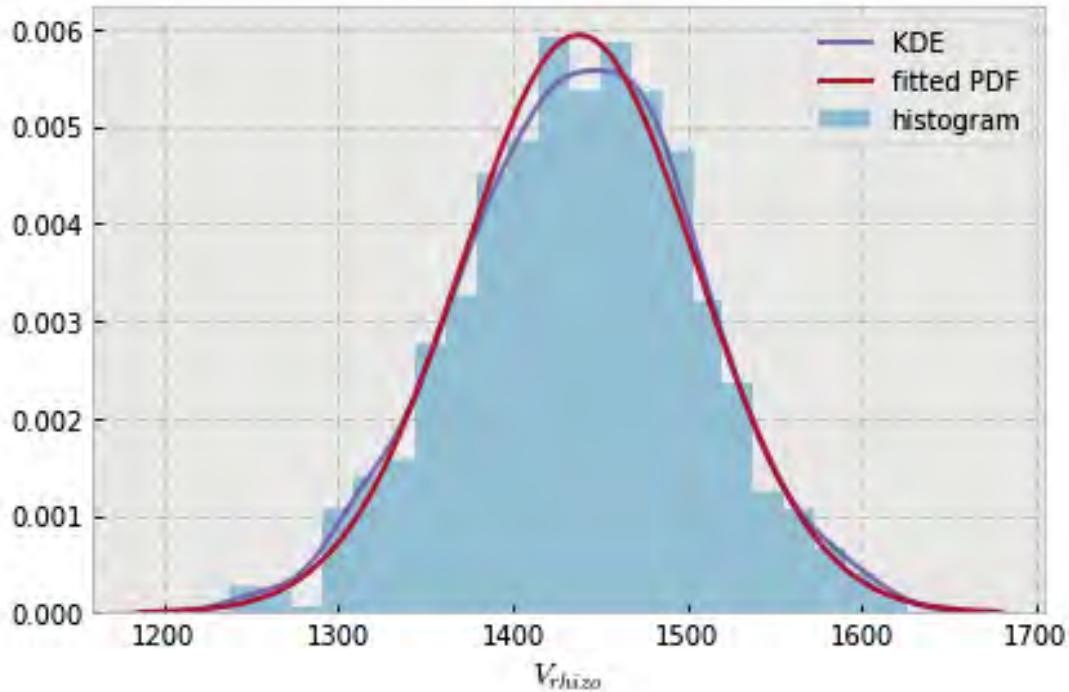


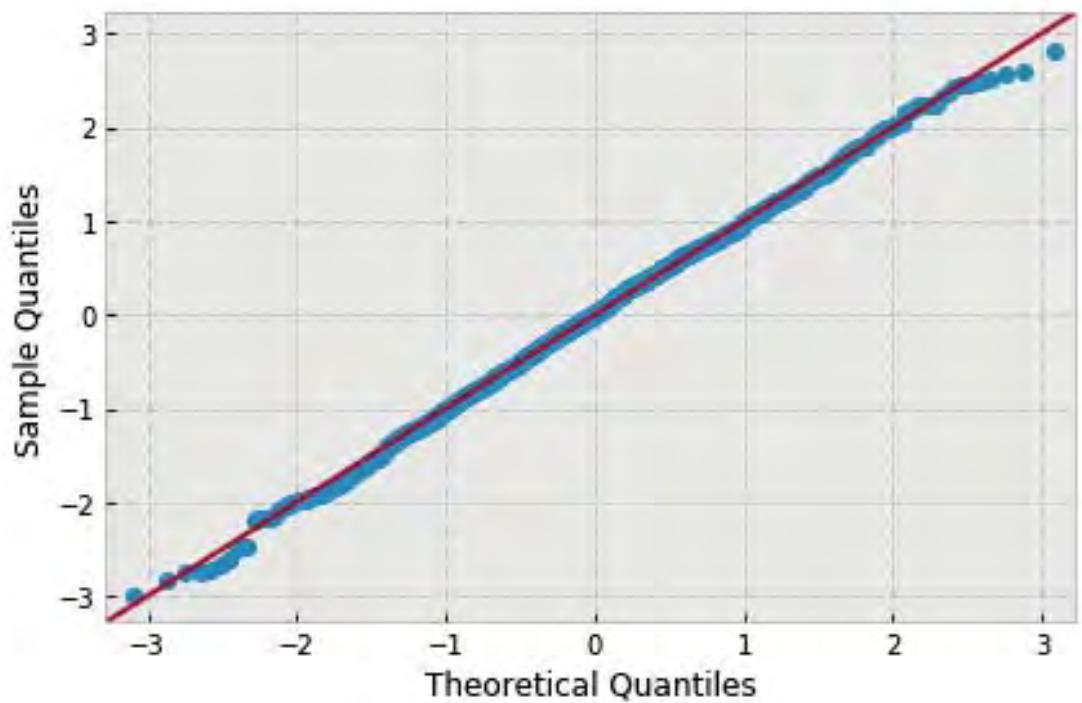


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(), loc=1311.5159194490443, scale=68.83404745560766)
DescribeResult(nobs=1000, minmax=(1080.9210140874004, 1492.8067560118589)
               mean=1311.5159194490443, variance=4742.868958078926,
               skewness=-0.1899052283283912, kurtosis=-0.09678024166003052)
```

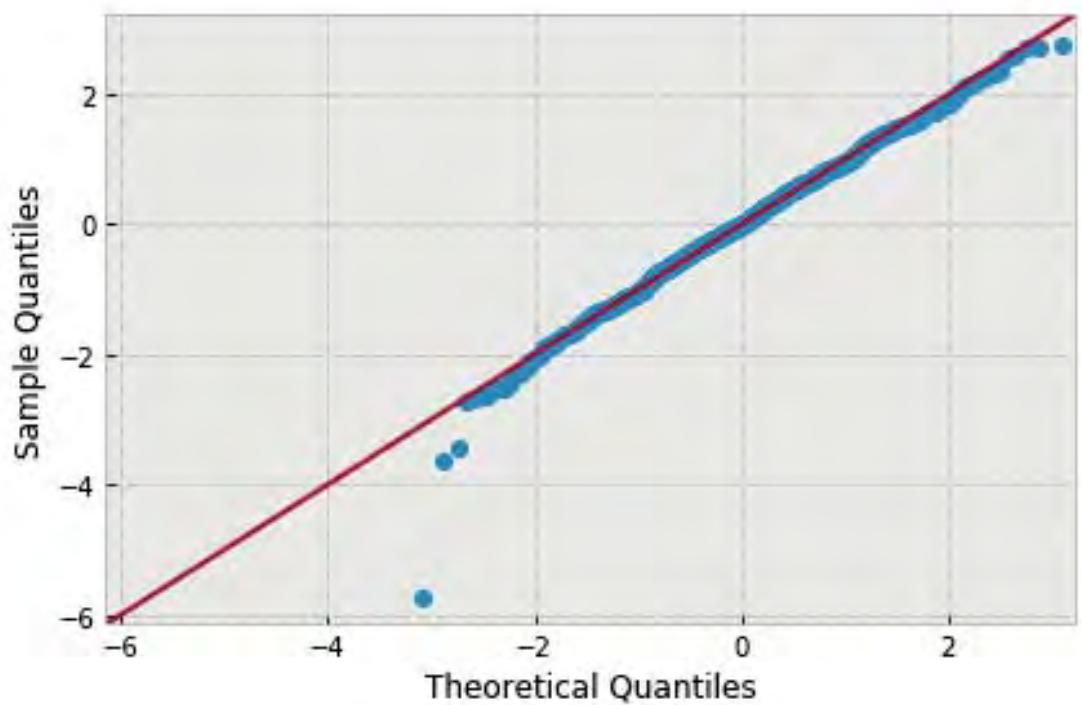
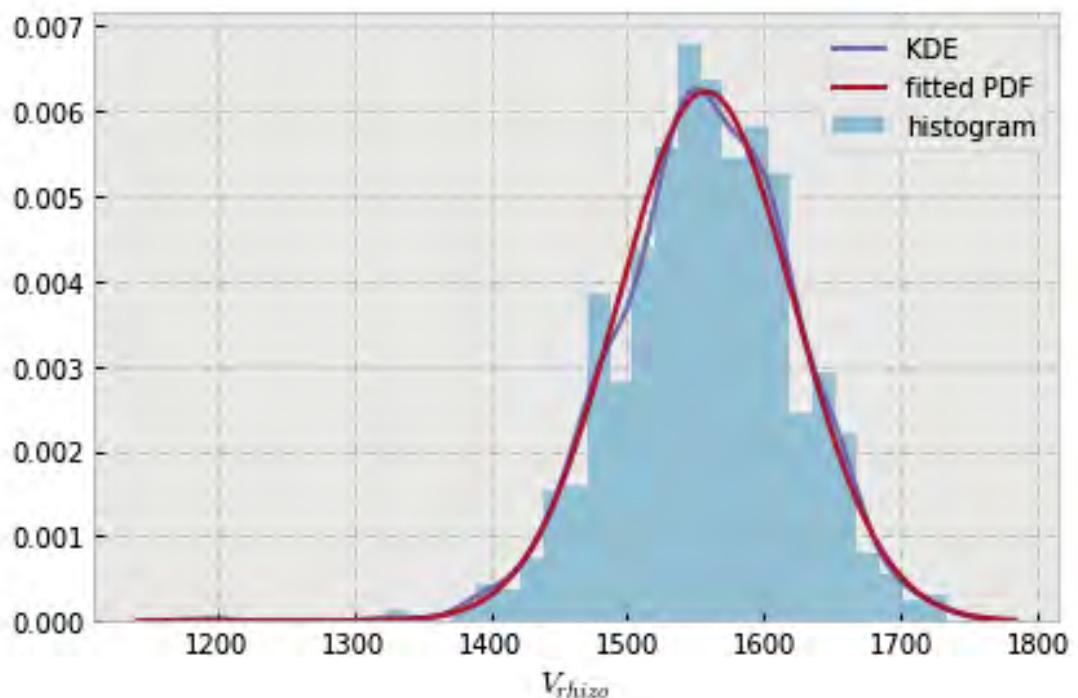


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=1438.089150227518, scale=67.16611768823748)
DescribeResult(nobs=1000, minmax=(1237.3394537521651, 1626.8638261592102)
               mean=1438.0891502275181, variance=4515.8031684786465,
               skewness=-0.07439554285302573, kurtosis=-0.14928771747911895)
```

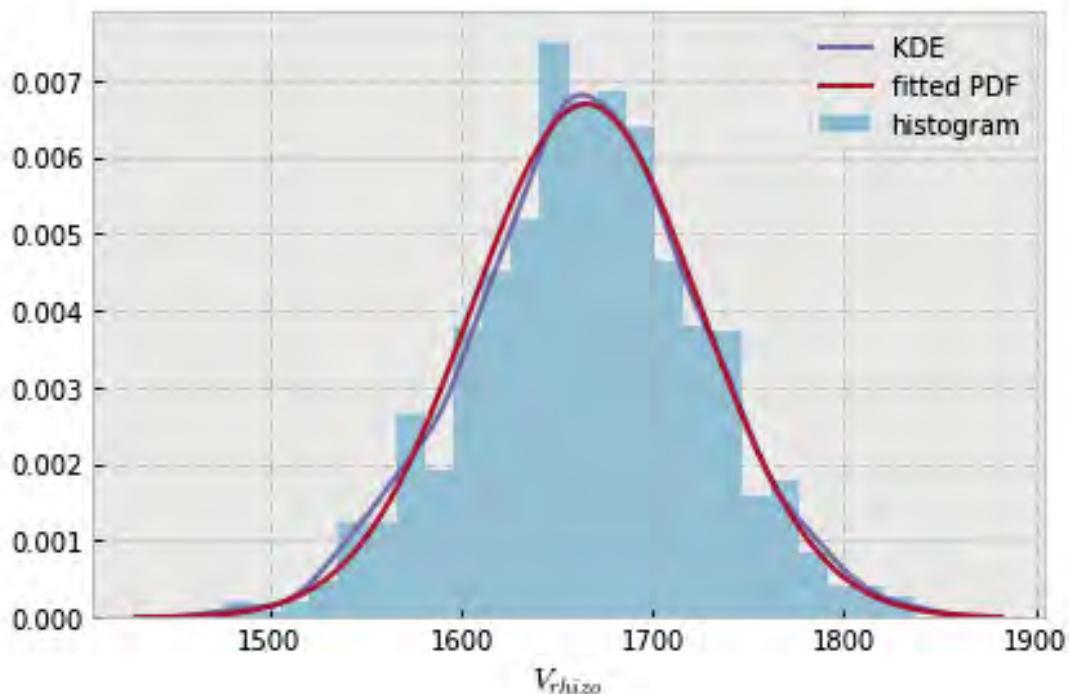


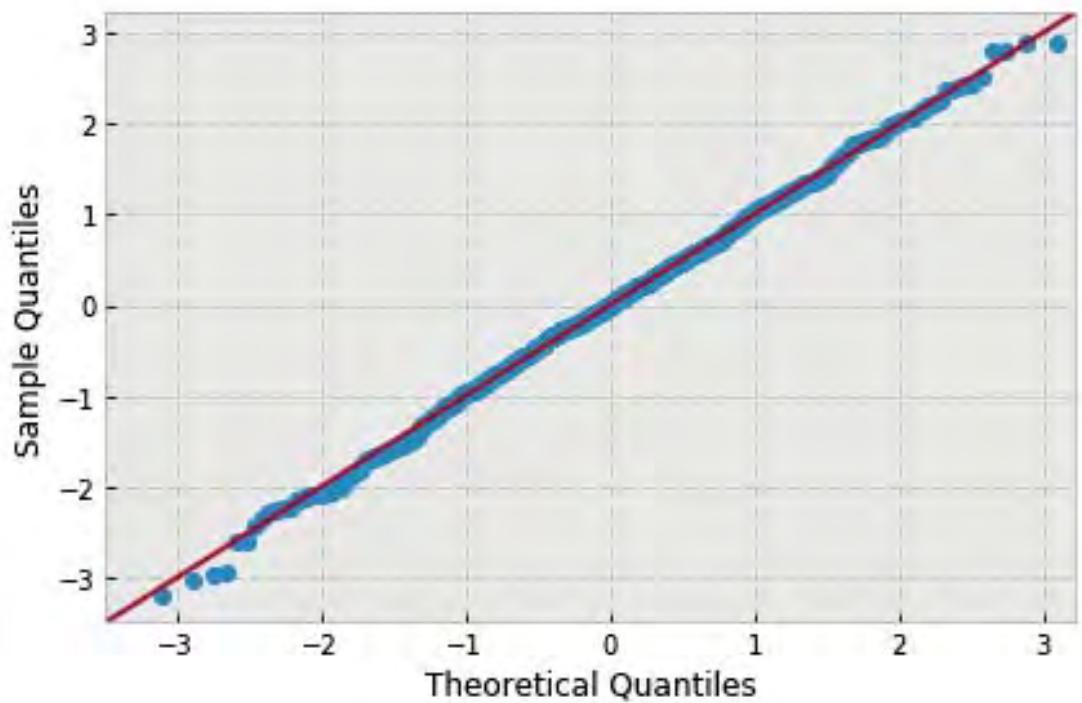


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=1557.4419973547194, scale=64.04623605704194)
DescribeResult(nobs=1000, minmax=(1191.2110900659579, 1734.0632746113849)
               mean=1557.4419973547194, variance=4106.0263794537923,
               skewness=-0.3157220973298956, kurtosis=0.9181155786926878)
```

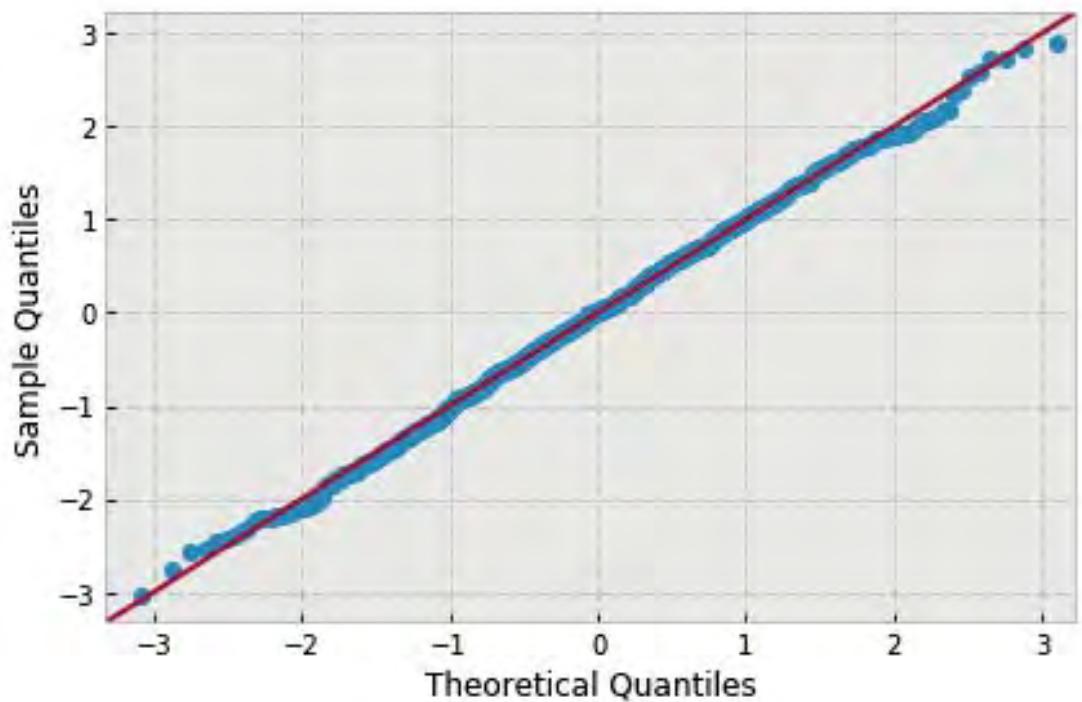
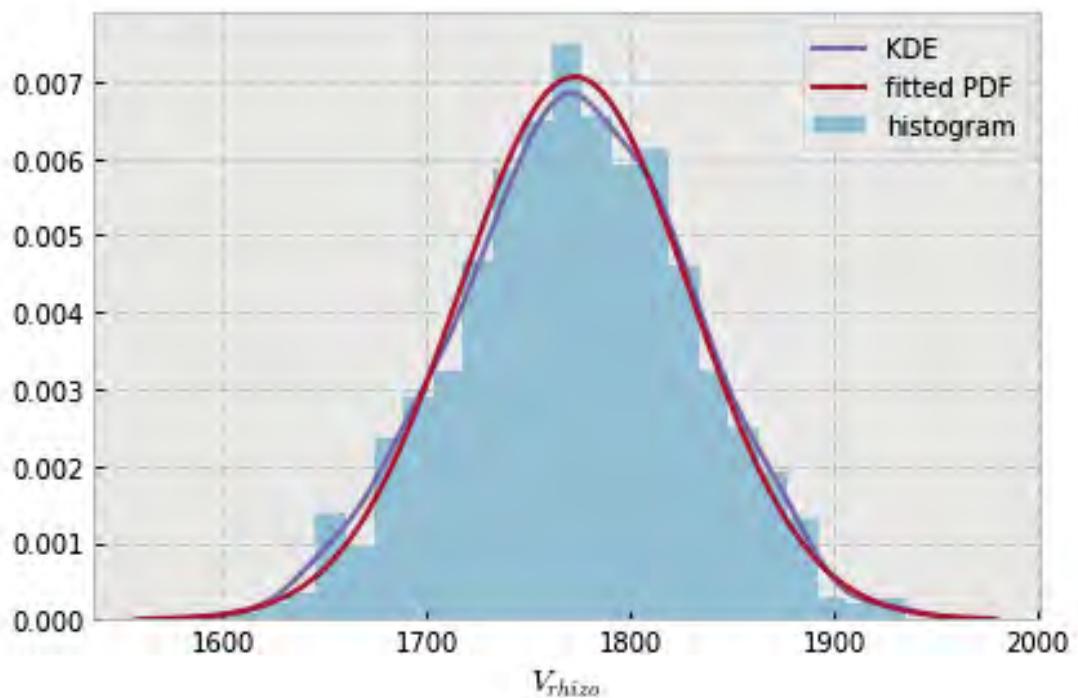


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=1665.2889584676113, scale=59.653828534556204)
DescribeResult(nobs=1000, minmax=(1475.5193323468764, 1836.6618343016617)
               mean=1665.2889584676113, variance=3562.1414002304632,
               skewness=-0.0617862553401327, kurtosis=-0.00720709554466481)
```

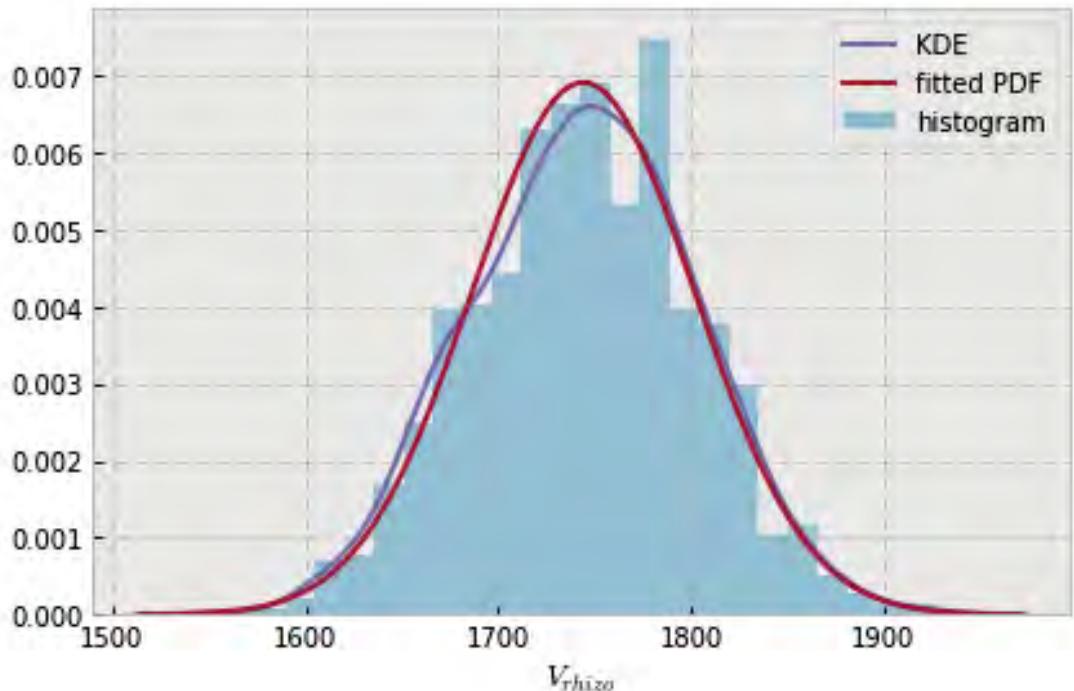


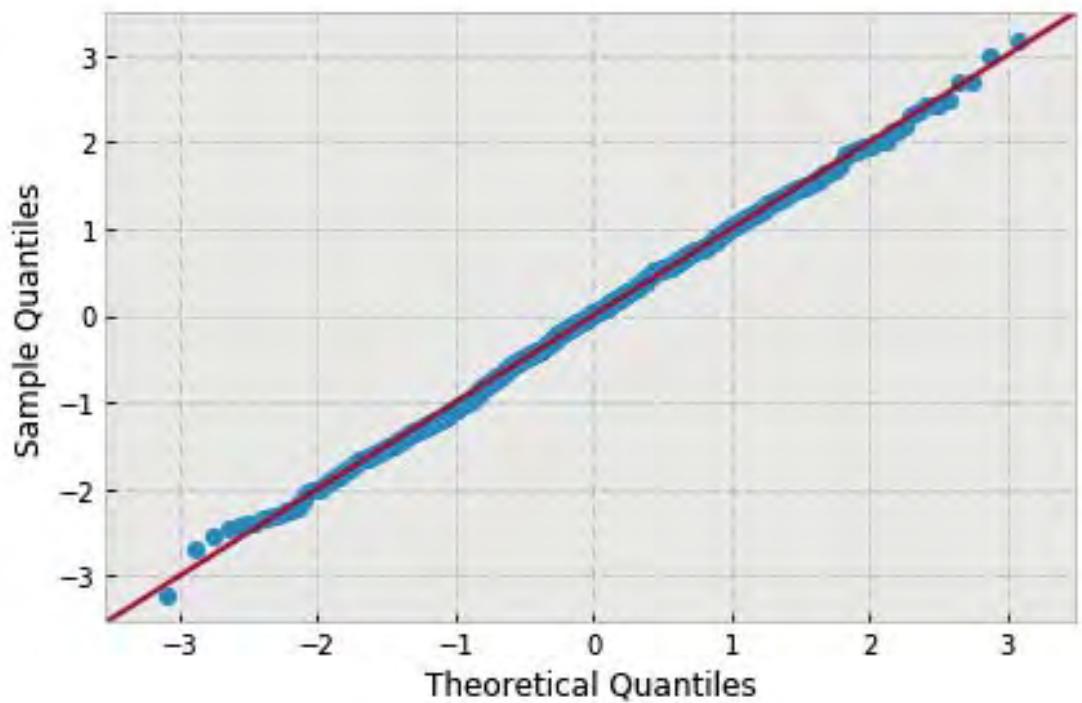


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=1772.68950017063, scale=56.47637714789638)
DescribeResult(nobs=1000, minmax=(1601.8244768790551, 1935.3467862183136)
               mean=1772.68950017063, variance=3192.773949701133,
               skewness=-0.062006993297589595, kurtosis=-0.22945001613591076)
```

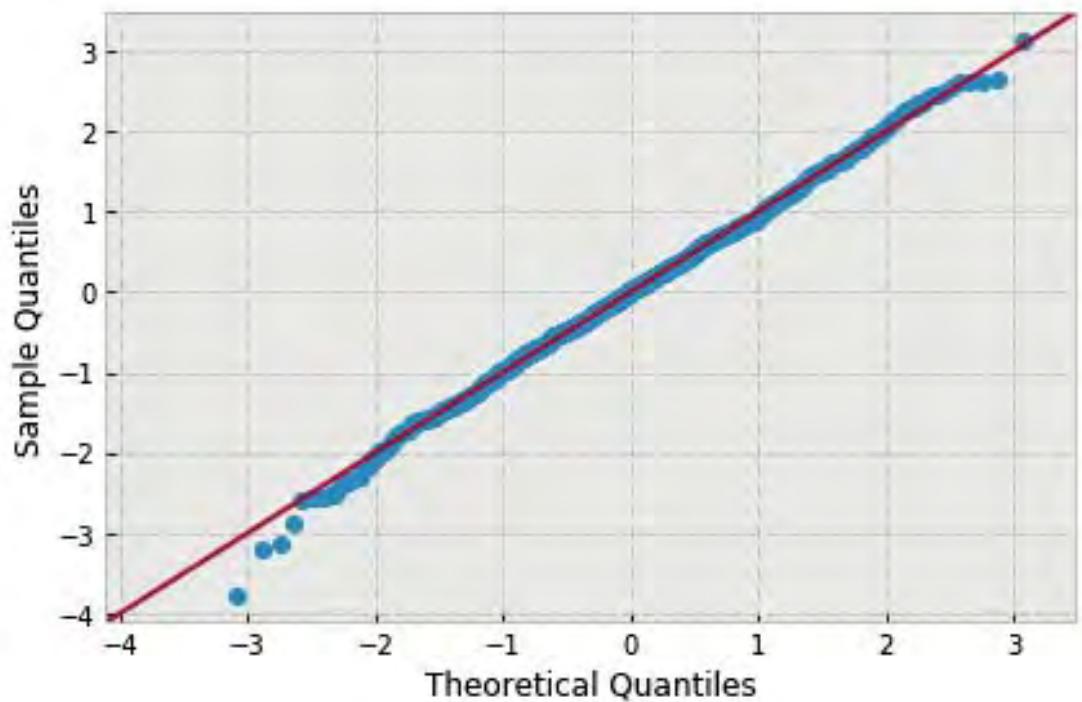
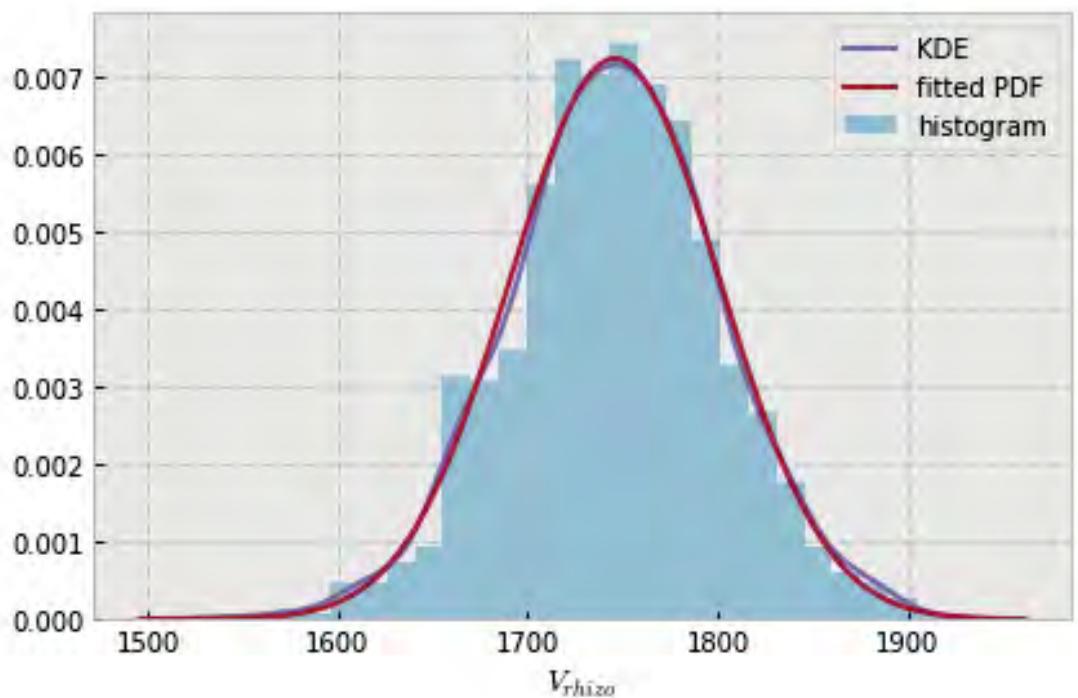


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=1744.2108820054136, scale=57.70396996020353)
DescribeResult(nobs=1000, minmax=(1558.691764756273, 1927.0850650814998)
               mean=1744.2108820054136, variance=3333.0812303984694,
               skewness=-0.031492115091371536, kurtosis=-0.21037848853803576)
```

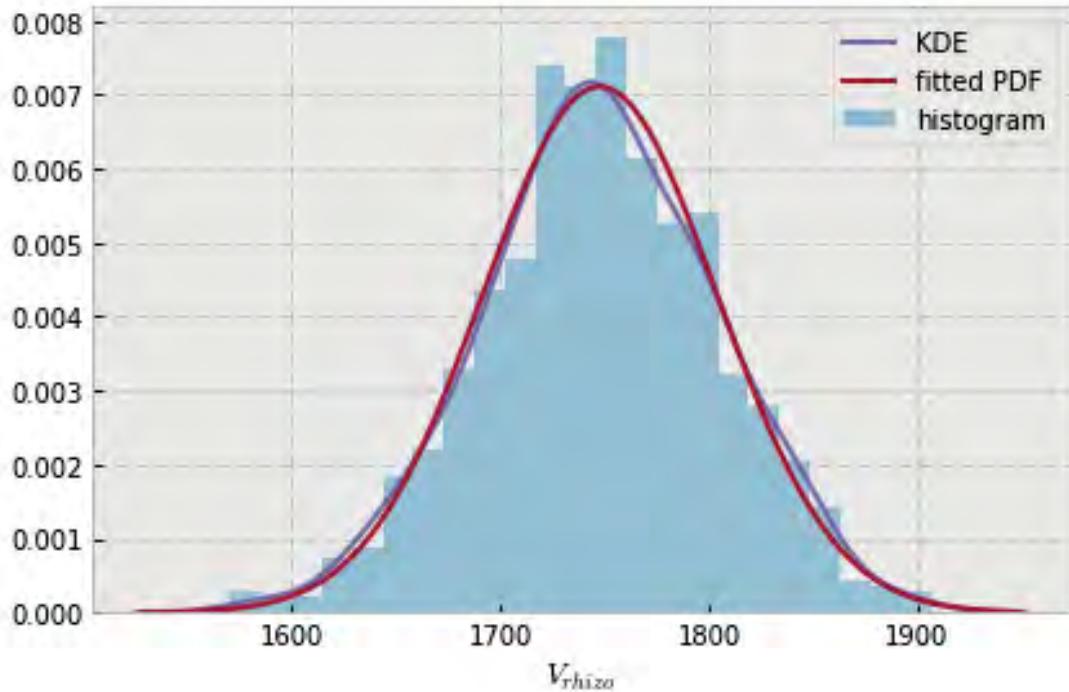


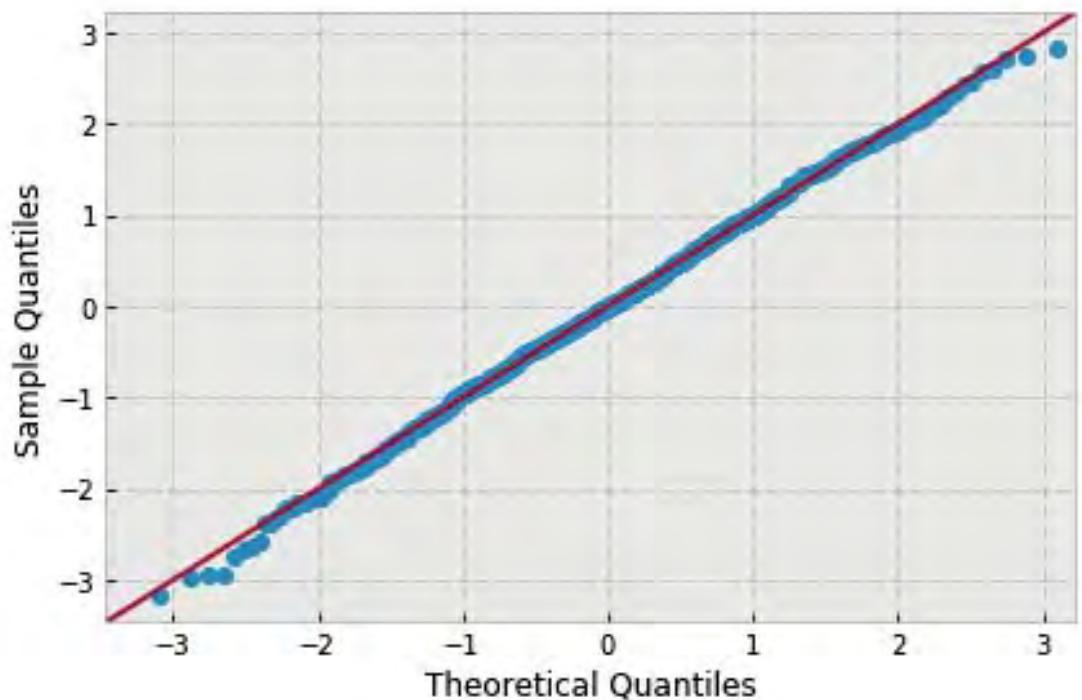


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=1746.0426533807897, scale=55.19429982399885)
DescribeResult(nobs=1000, minmax=(1538.1178128202328, 1918.9124494557502)
               mean=1746.0426533807897, variance=3049.4601932547339,
               skewness=-0.06533692330910552, kurtosis=0.1792366871931348)
```

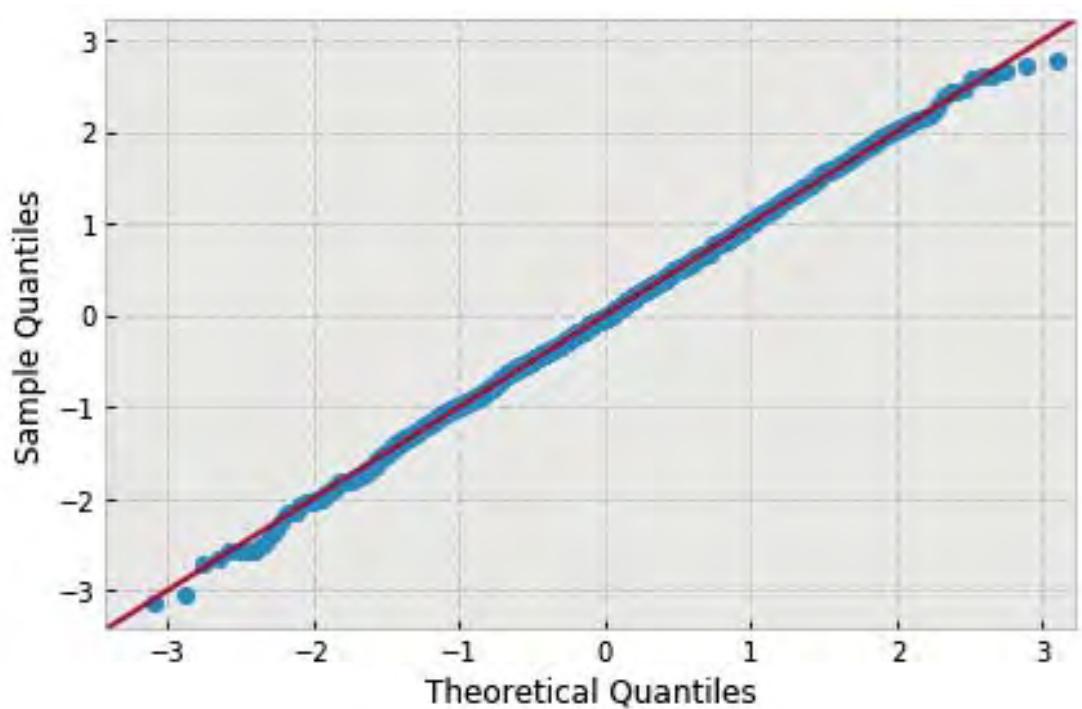
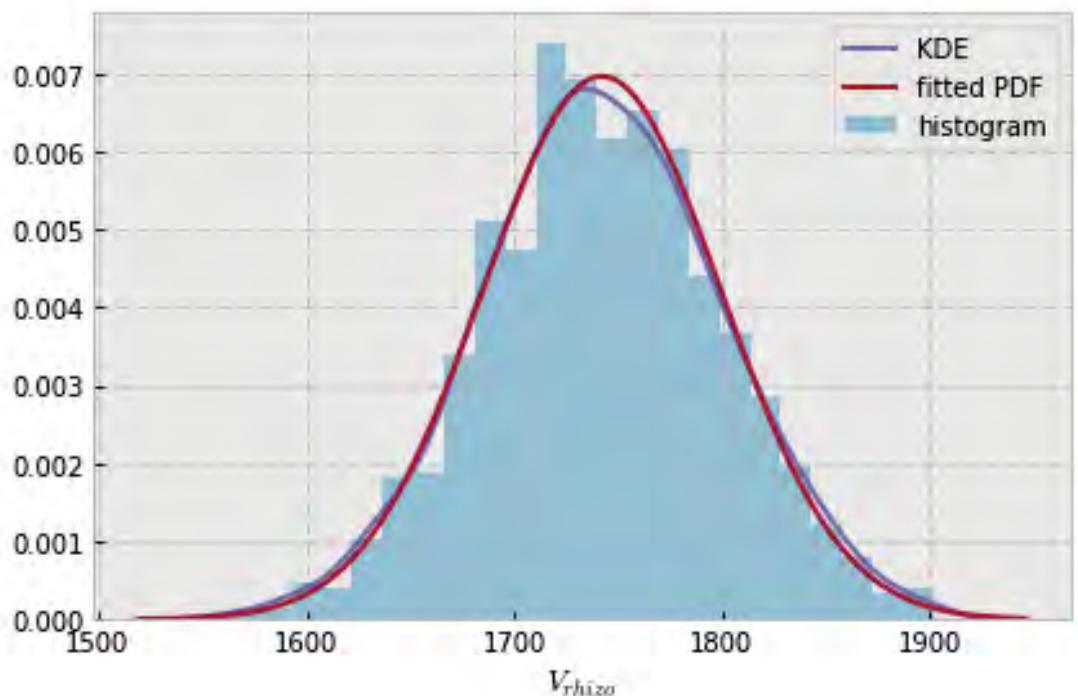


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(), loc=1747.6084551095917, scale=56.11784501026188)
DescribeResult(nobs=1000, minmax=(1570.2571051565981, 1906.5698809959454)
               mean=1747.6084551095917, variance=3152.3648934892635,
               skewness=-0.06903507436831137, kurtosis=-0.05960499140335784)
```

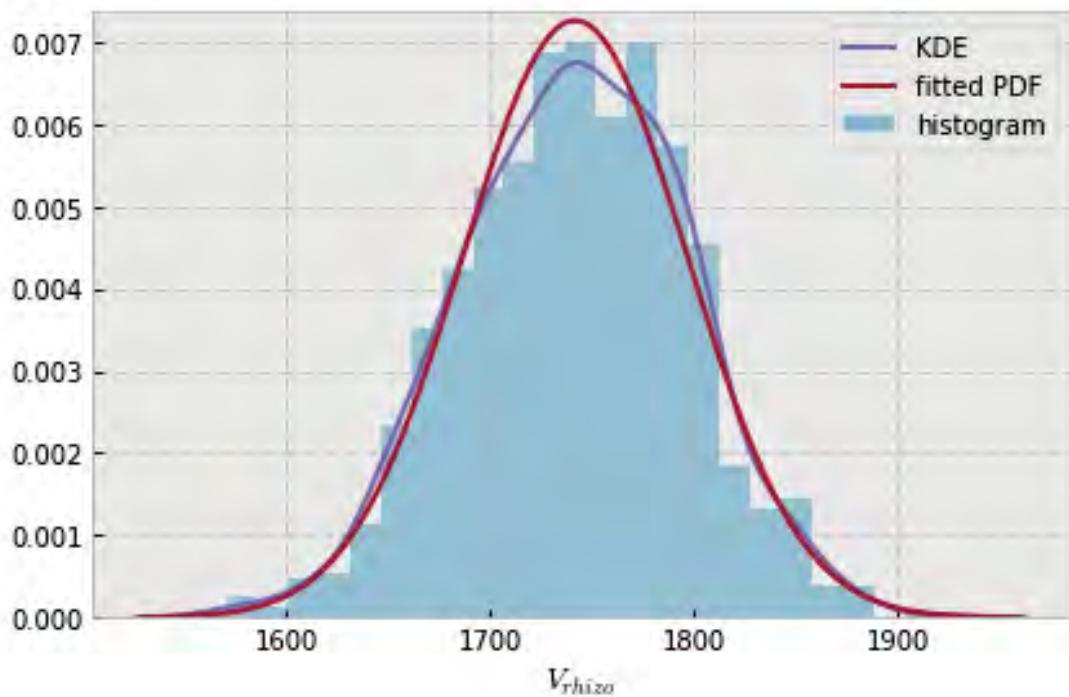


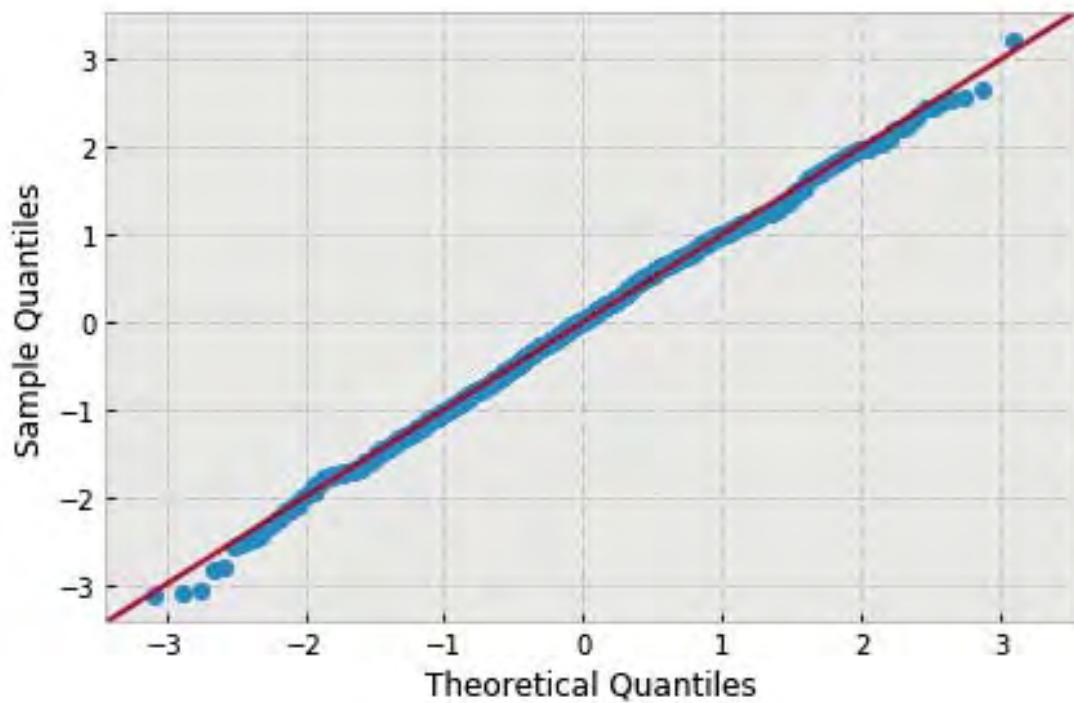


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=1742.087840461061, scale=57.28261386952508)
DescribeResult(nobs=1000, minmax=(1563.1300849866154, 1901.7357083019683)
               mean=1742.087840461061, variance=3284.5824341592665,
               skewness=-0.014021947323285, kurtosis=-0.077672556082645)
```

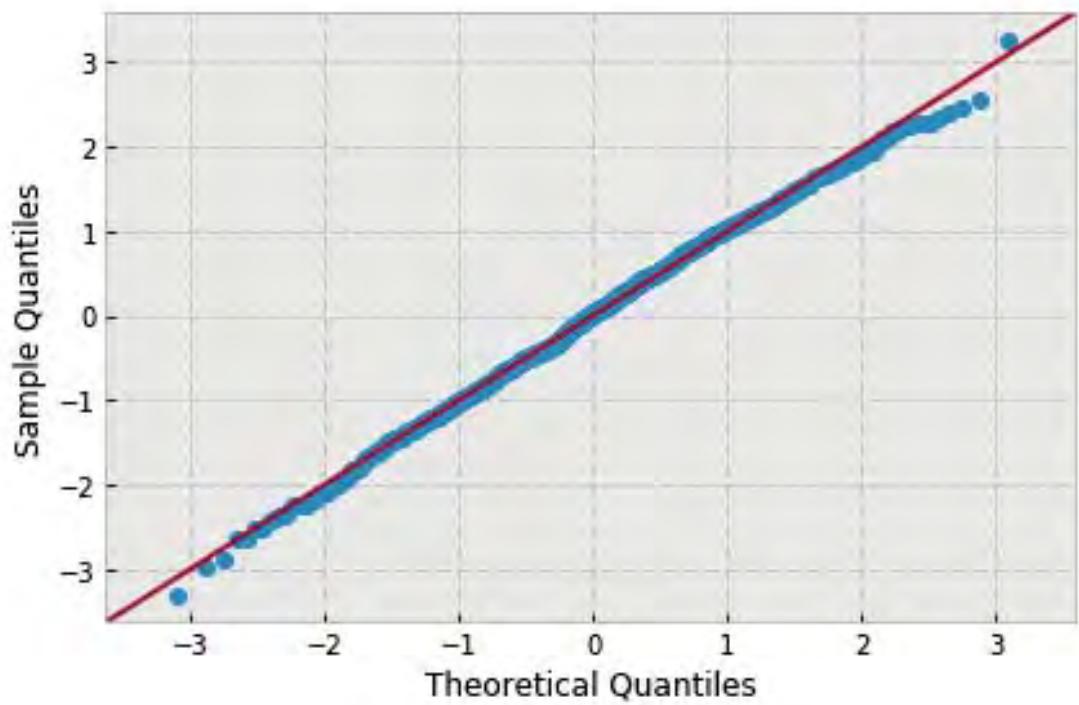
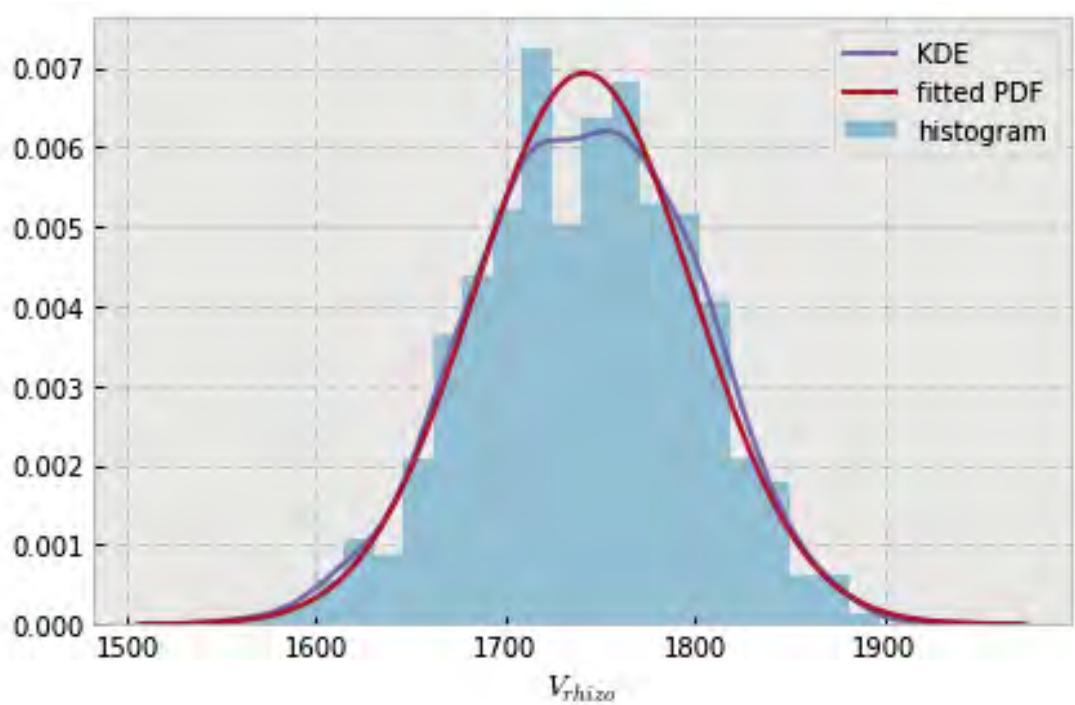


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=1741.9328821604927, scale=54.96546380375735)
DescribeResult(nobs=1000, minmax=(1570.8133804575491, 1918.6135807601186)
               mean=1741.9328821604927, variance=3024.2264375997593,
               skewness=-0.09284348680364068, kurtosis=-0.12152102494177974)
```

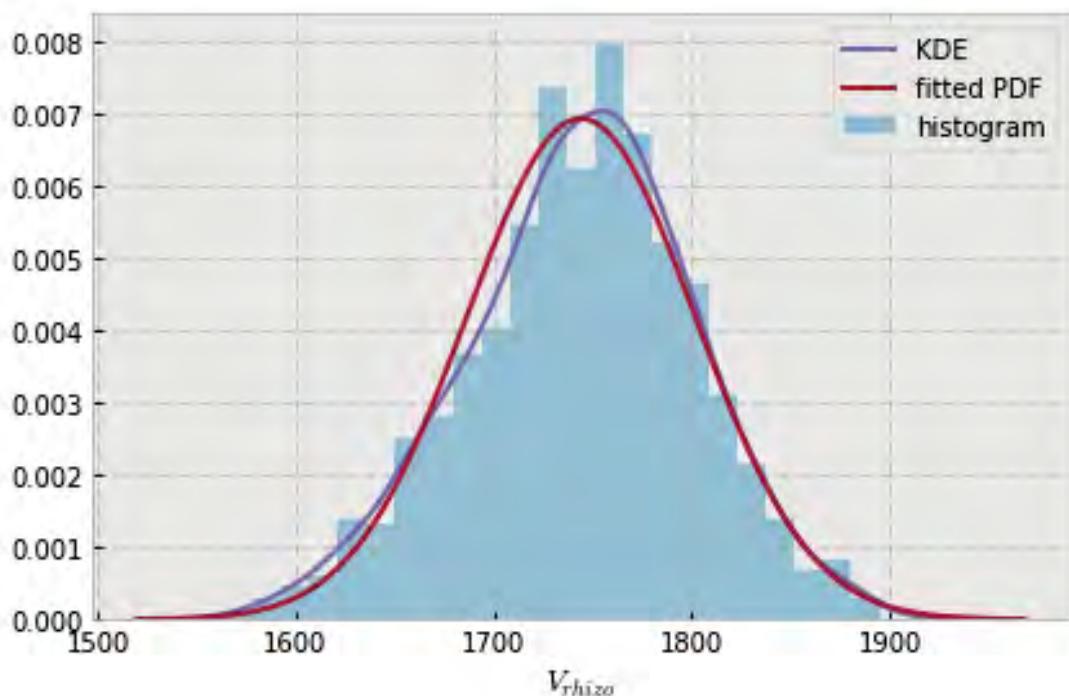


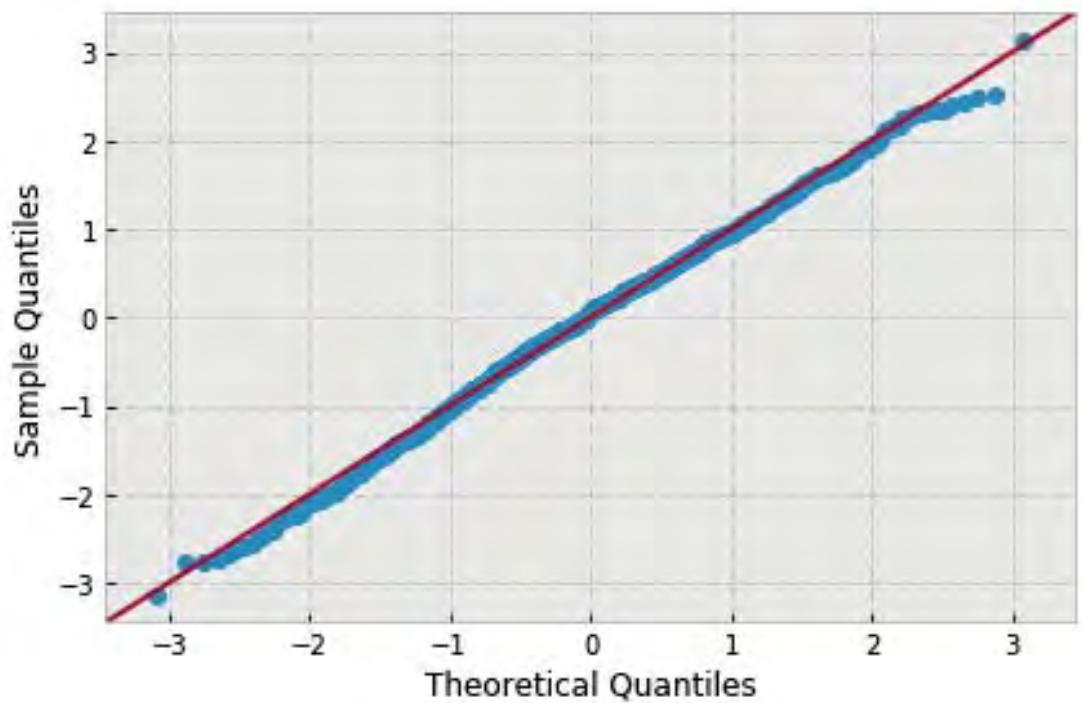


```
variable name: sigma
variable value: 0.066666666666666667
distribution: normal(shape=(), loc=1741.3861522819511, scale=57.521409595423314)
DescribeResult(nobs=1000, minmax=(1551.6954185739587, 1928.4792559350749)
               mean=1741.3861522819511, variance=3312.0245864308877,
               skewness=-0.1040910288376349, kurtosis=-0.20479351068945606)
```

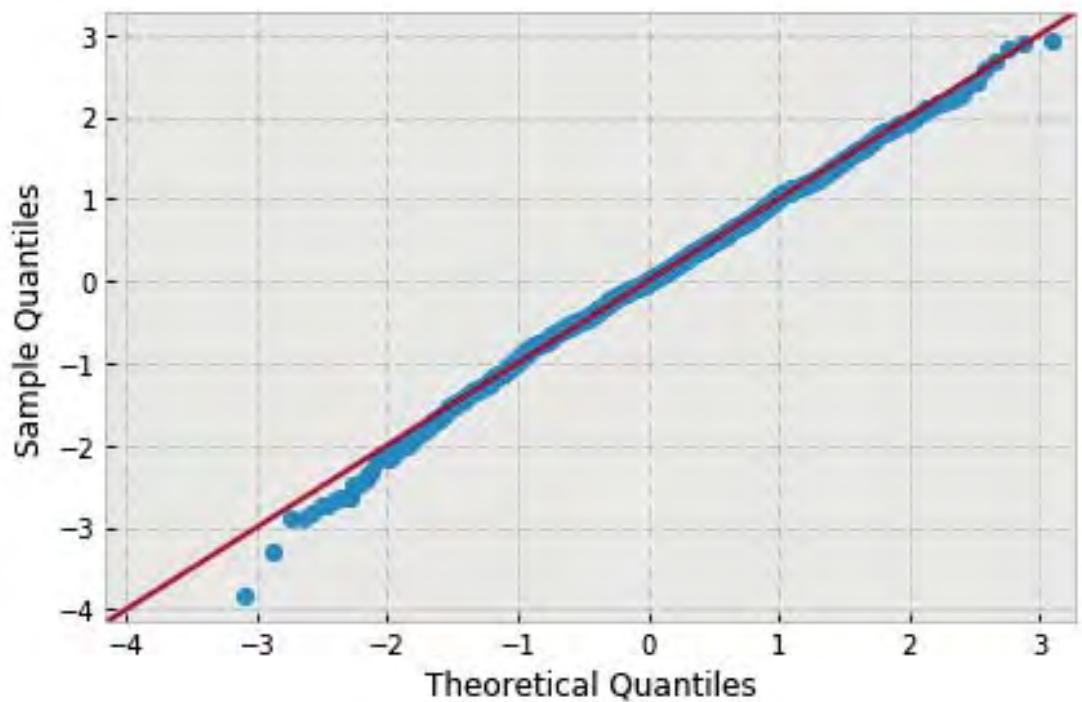
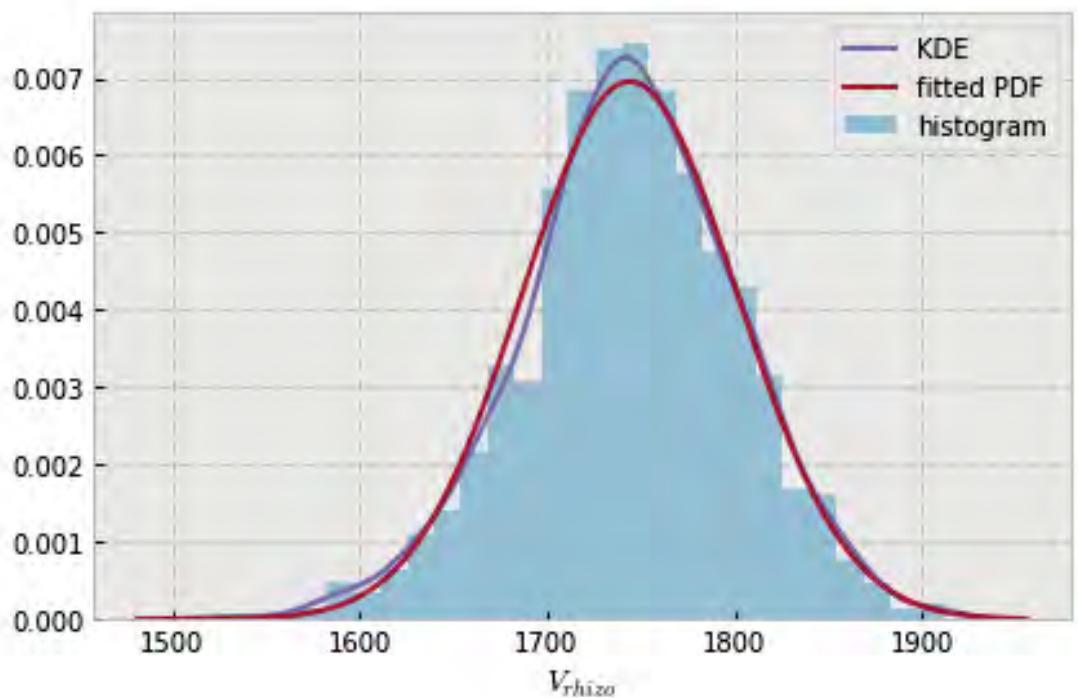


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=1743.9607649104921, scale=57.542377276285016)
DescribeResult(nobs=1000, minmax=(1563.6601466396251, 1924.0035528820385)
               mean=1743.9607649104921, variance=3314.4396222285513,
               skewness=-0.17368890980882, kurtosis=-0.0499684052135283)
```

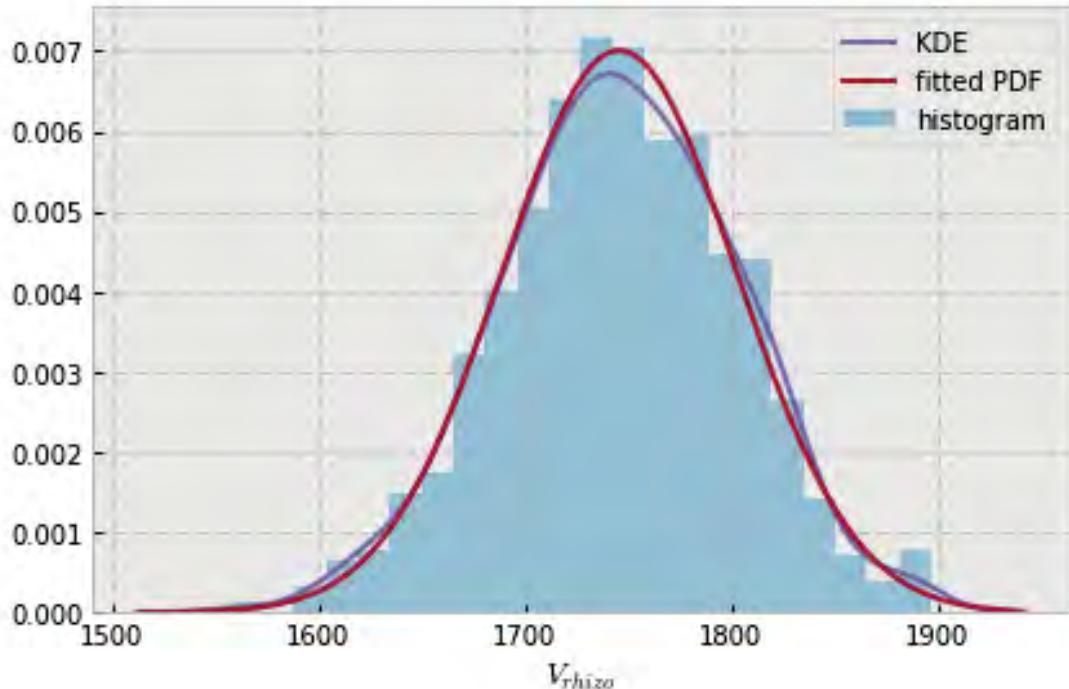


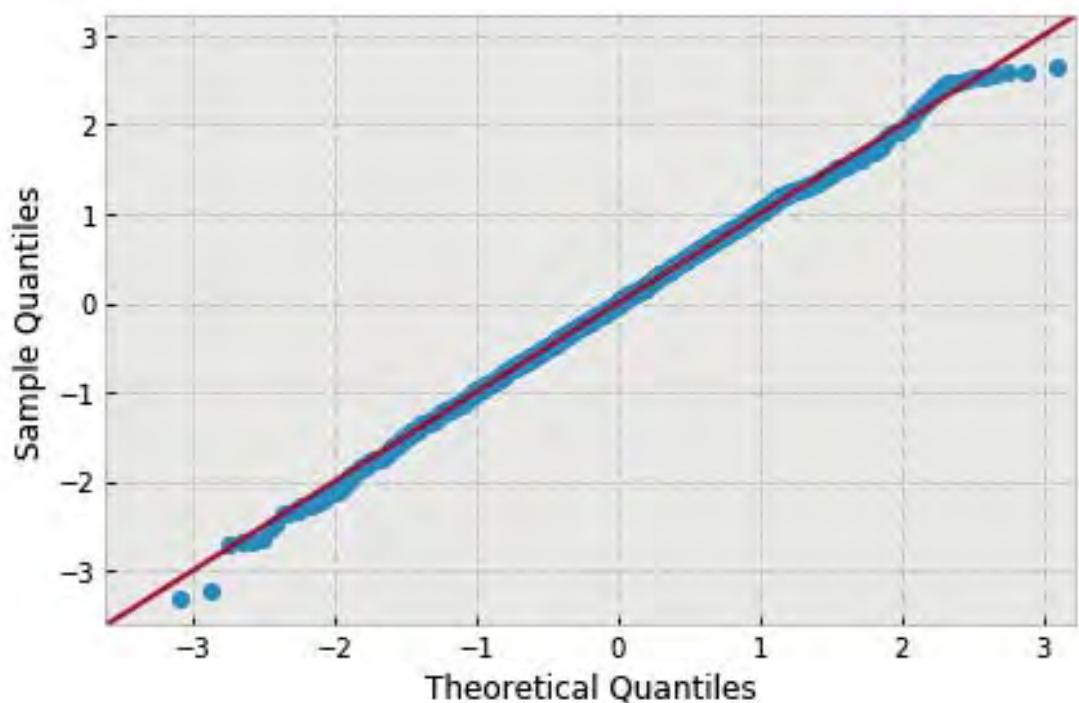


```
variable name: sigma
variable value: 0.0888888888888888
distribution: normal(shape=(), loc=1743.982687591328, scale=57.359784048451914)
DescribeResult(nobs=1000, minmax=(1524.8855431830896, 1911.8627601077524)
               mean=1743.9826875913279, variance=3293.4382643493877,
               skewness=-0.1716017694600497, kurtosis=0.24917139525782206)
```

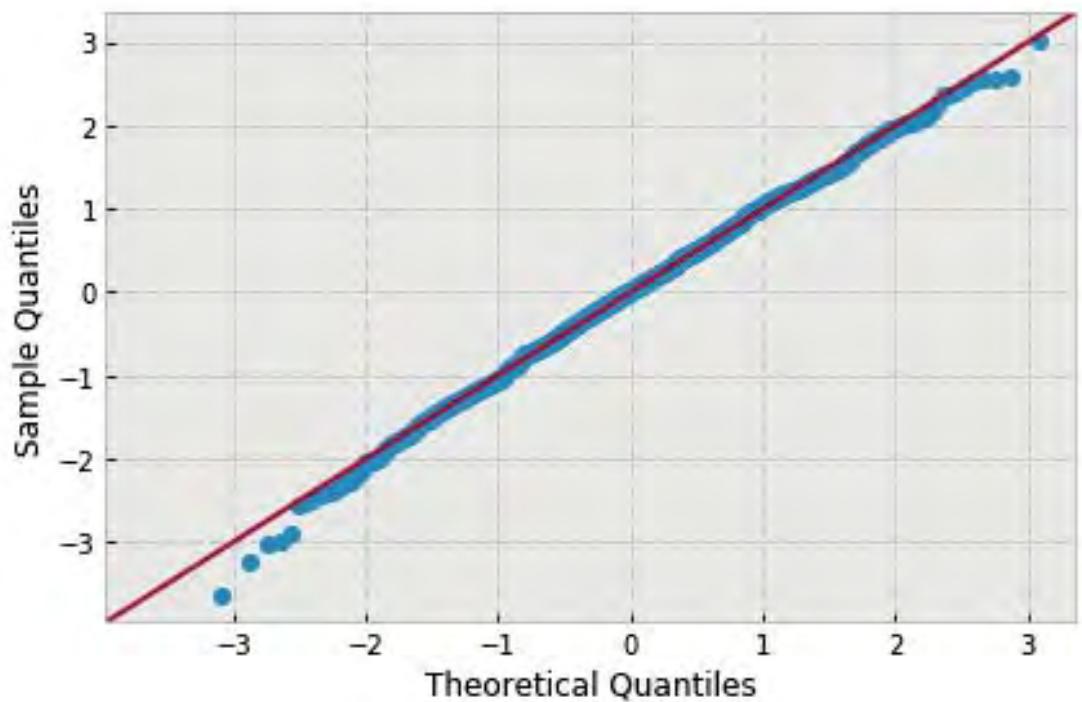
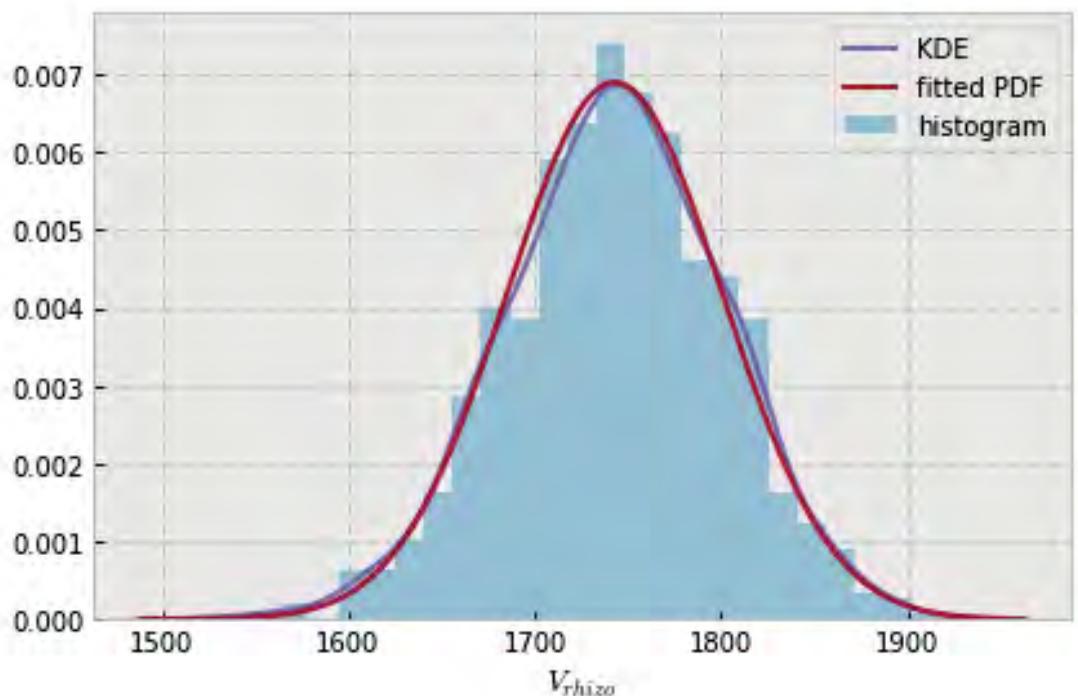


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=1745.7282285736208, scale=56.95740817133087)
DescribeResult(nobs=1000, minmax=(1557.1266830708048, 1896.9230789752203)
               mean=1745.7282285736208, variance=3247.3937393349229,
               skewness=-0.08066433910277883, kurtosis=-0.06552602129533325)
```

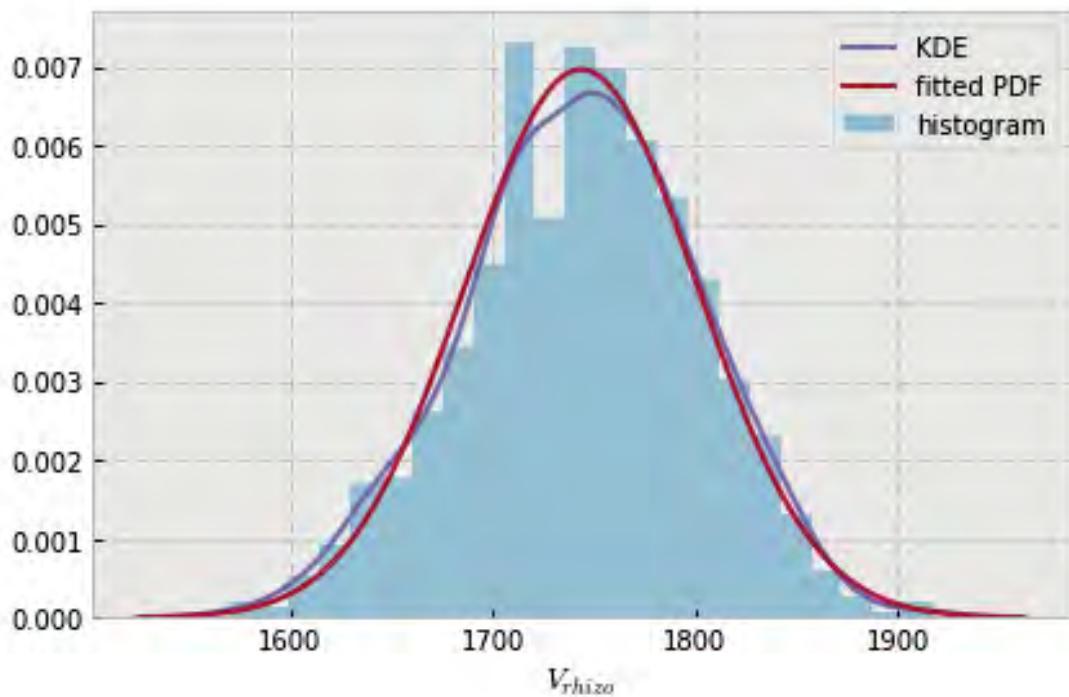


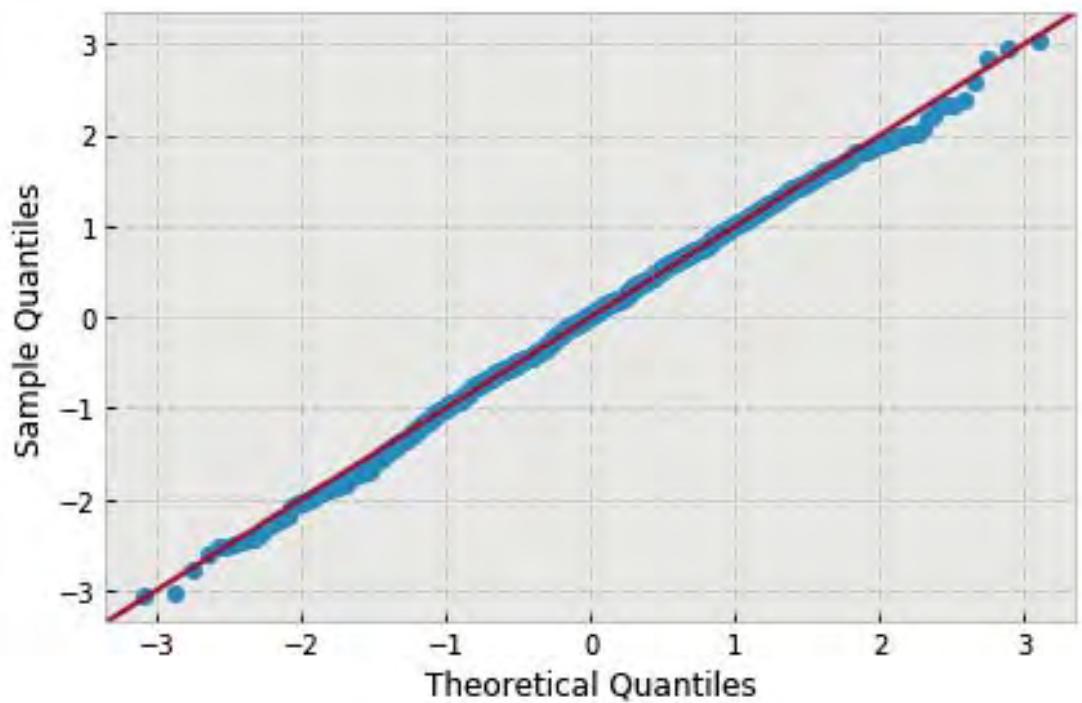


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=1742.965955572017, scale=57.87412610898681)
DescribeResult(nobs=1000, minmax=(1532.9526957172764, 1917.5427645518707)
               mean=1742.965955572017, variance=3352.767240119028,
               skewness=-0.13853078721777315, kurtosis=0.025819778355304912)
```

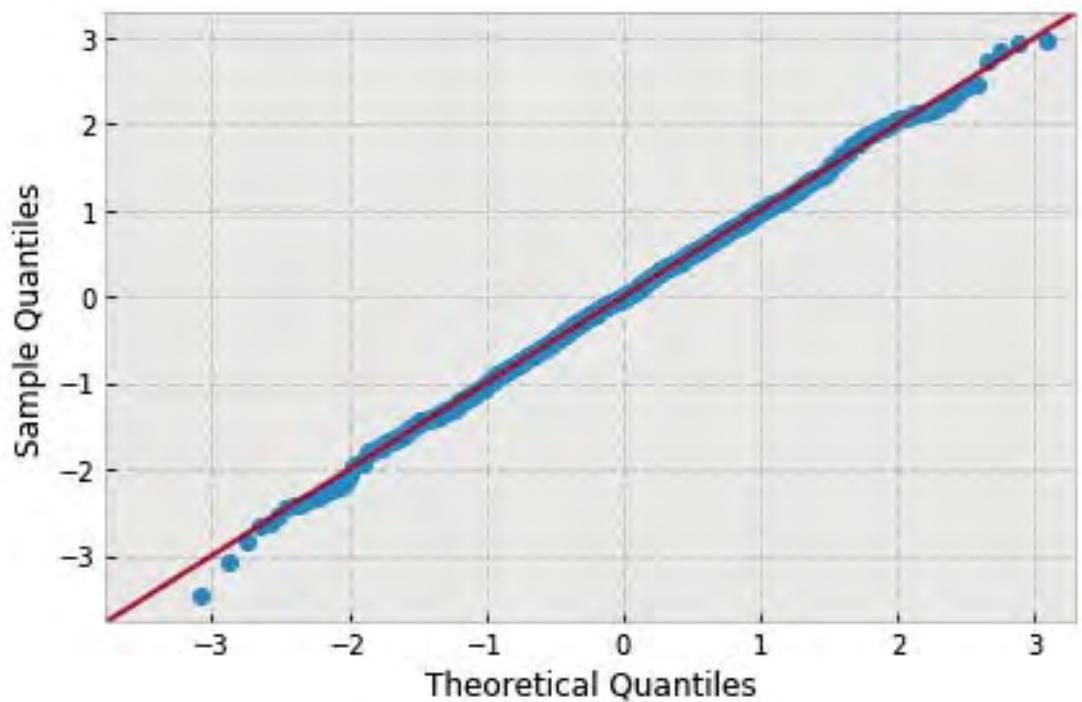
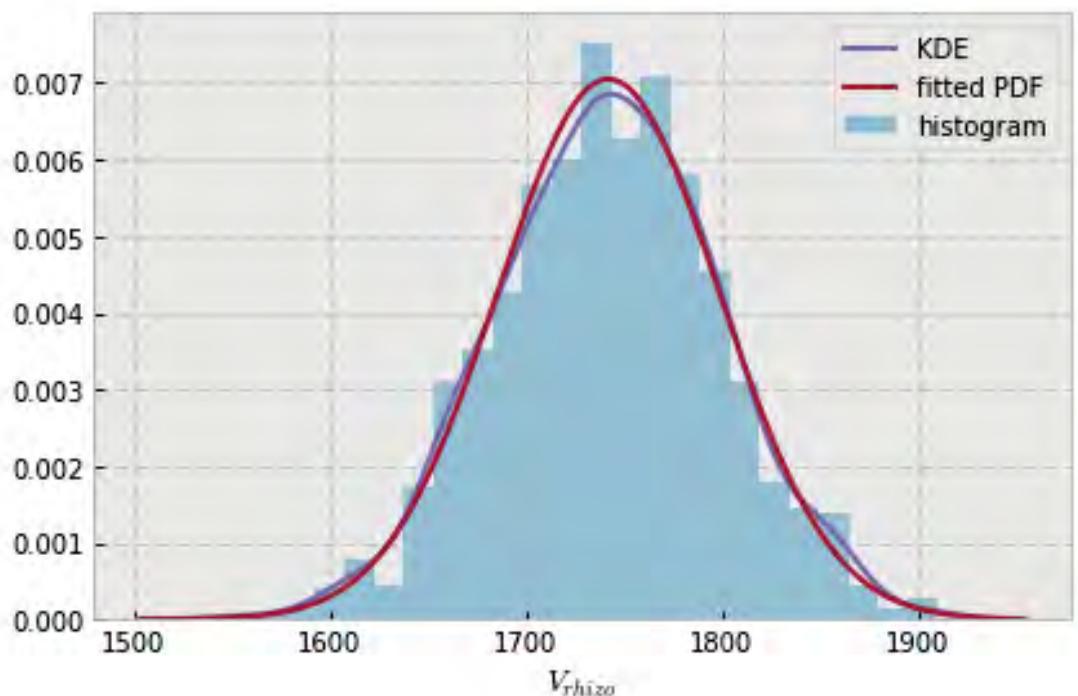


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=1743.8235194761226, scale=57.305009276598646)
DescribeResult(nobs=1000, minmax=(1569.3163550084253, 1917.8579980533721)
               mean=1743.8235194761226, variance=3287.1512394304877,
               skewness=-0.11596607520832979, kurtosis=-0.1432344518727766)
```

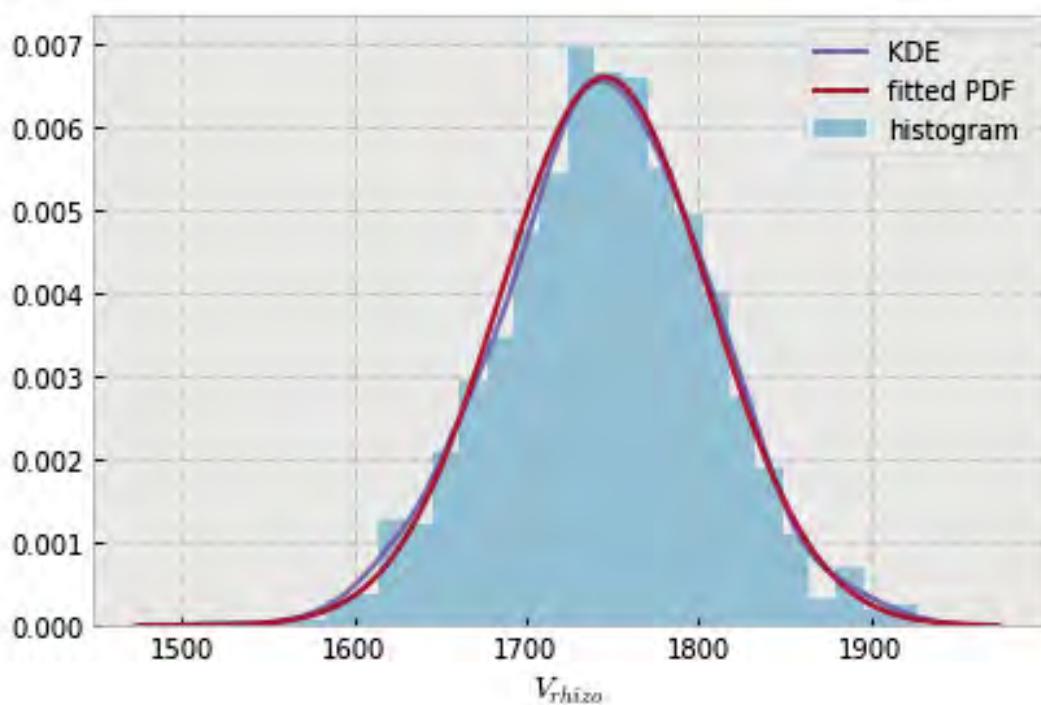


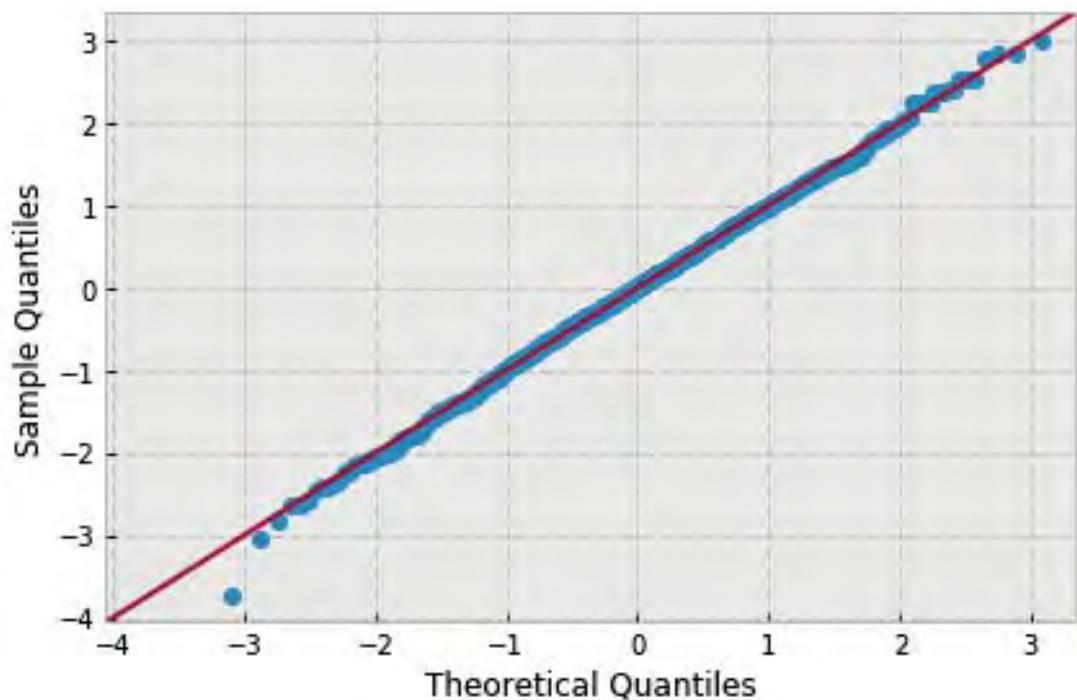


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=1742.0808233807402, scale=56.56624621314898)
DescribeResult(nobs=1000, minmax=(1546.8761167060591, 1909.9587787465348)
               mean=1742.0808233807402, variance=3202.9431538003914,
               skewness=-0.036844760009423584, kurtosis=-0.026591821922131054)
```

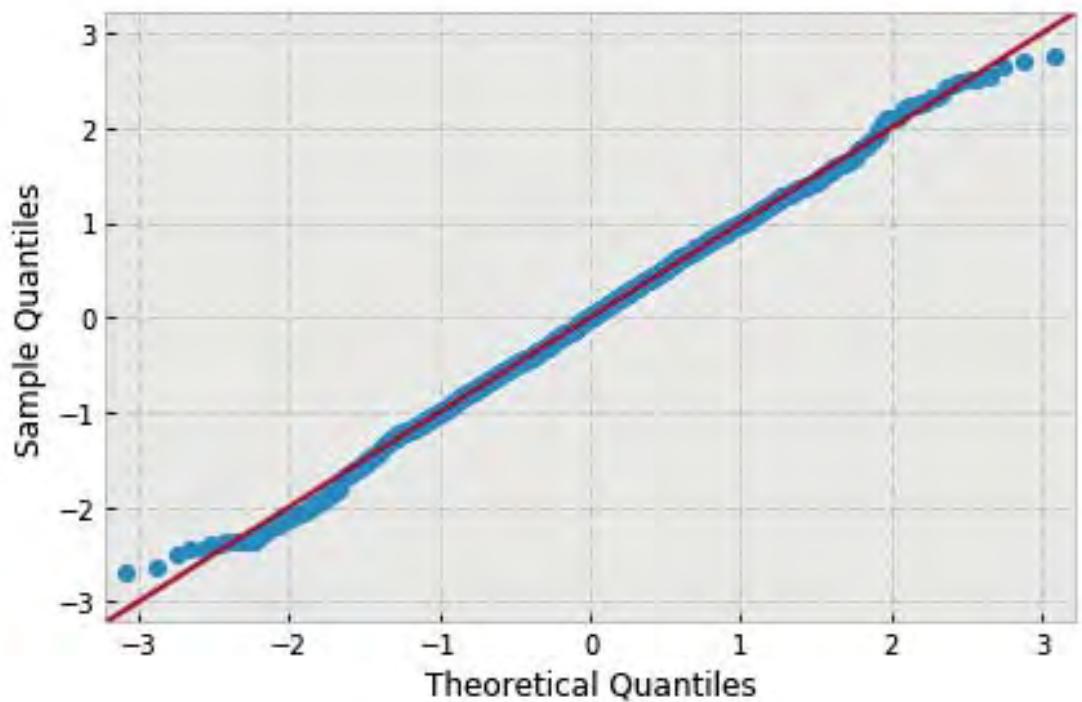
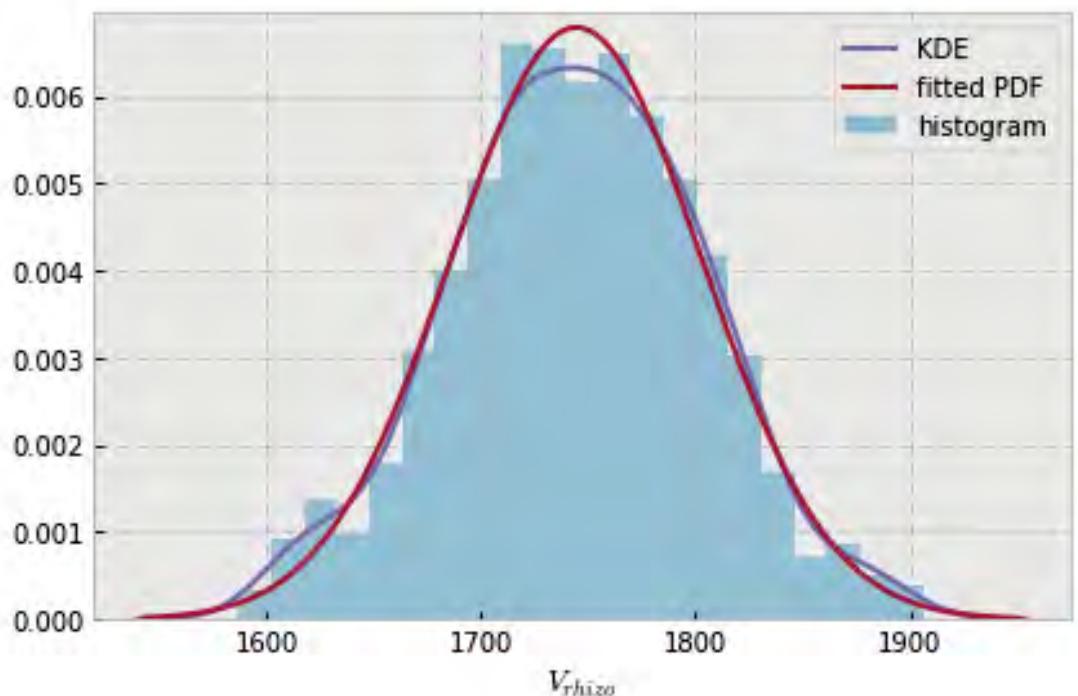


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=1745.7412747596597, scale=60.48555649923886)
DescribeResult(nobs=1000, minmax=(1520.8687949743012, 1926.8980440813959)
               mean=1745.7412747596597, variance=3662.1647097323485,
               skewness=-0.073137032710753, kurtosis=0.055694989872196565)
```

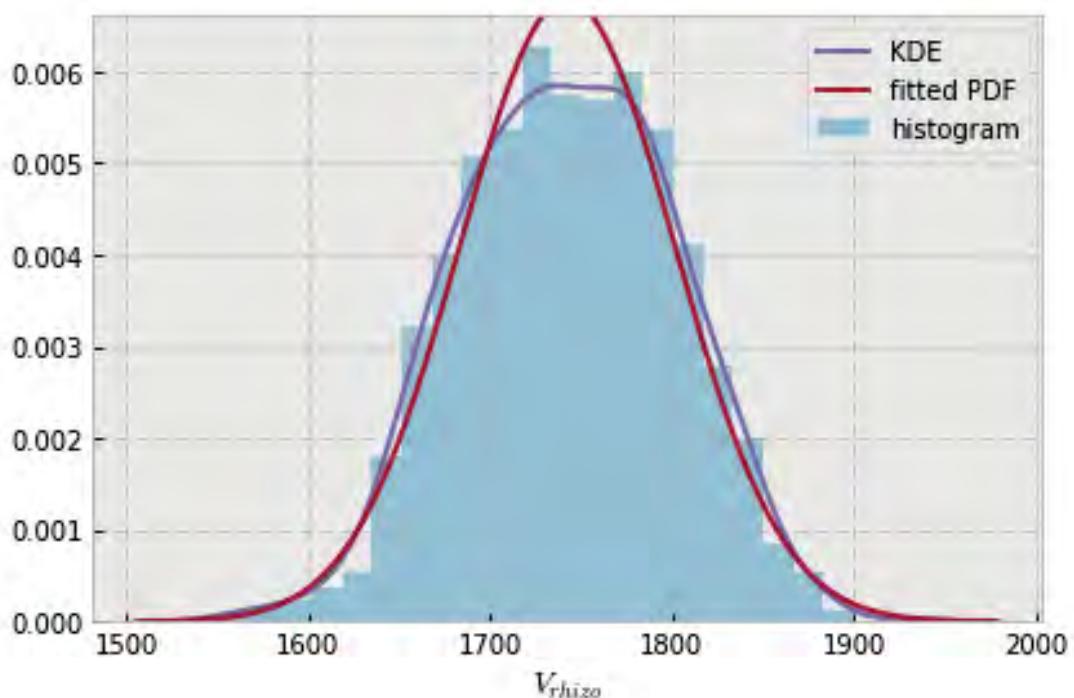


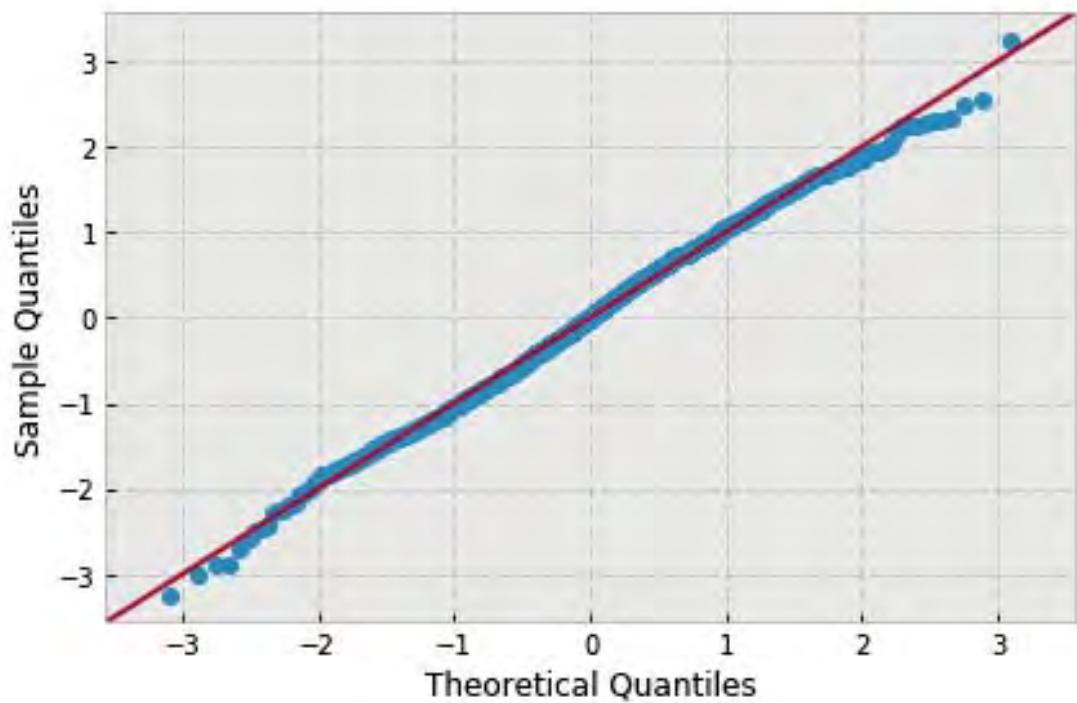


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=1744.434538669596, scale=58.780993384022885)
DescribeResult(nobs=1000, minmax=(1586.6810983487533, 1907.0806987020533)
               mean=1744.4345386695959, variance=3458.6638470596022,
               skewness=-0.03360574753858457, kurtosis=-0.15614779091588638)
```

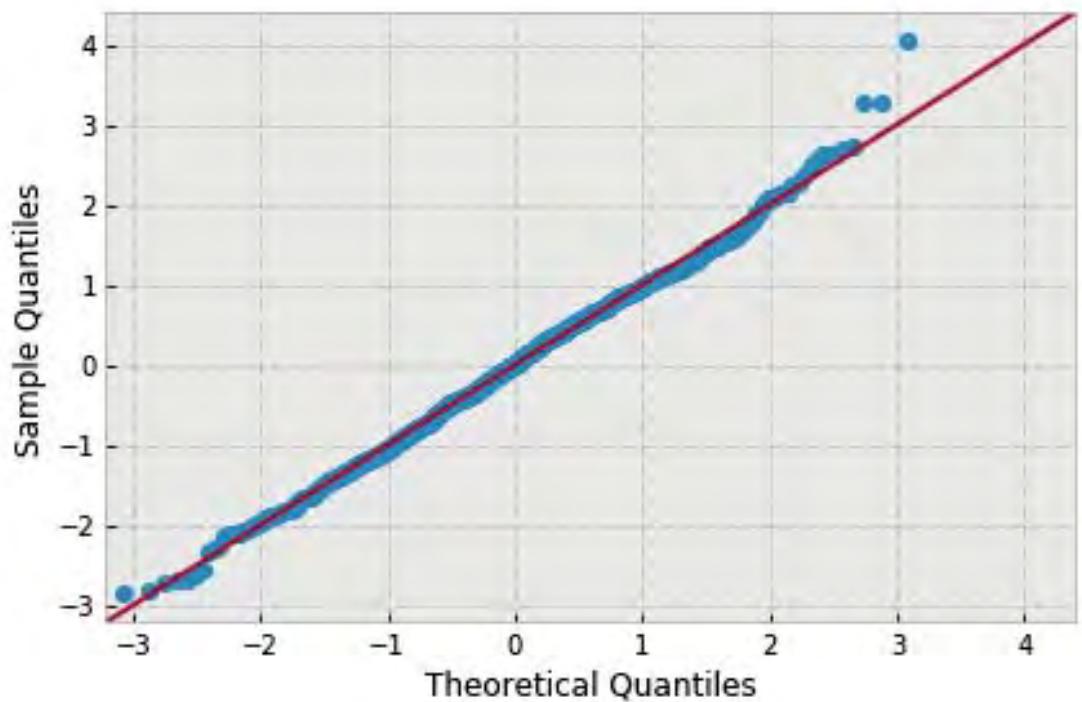
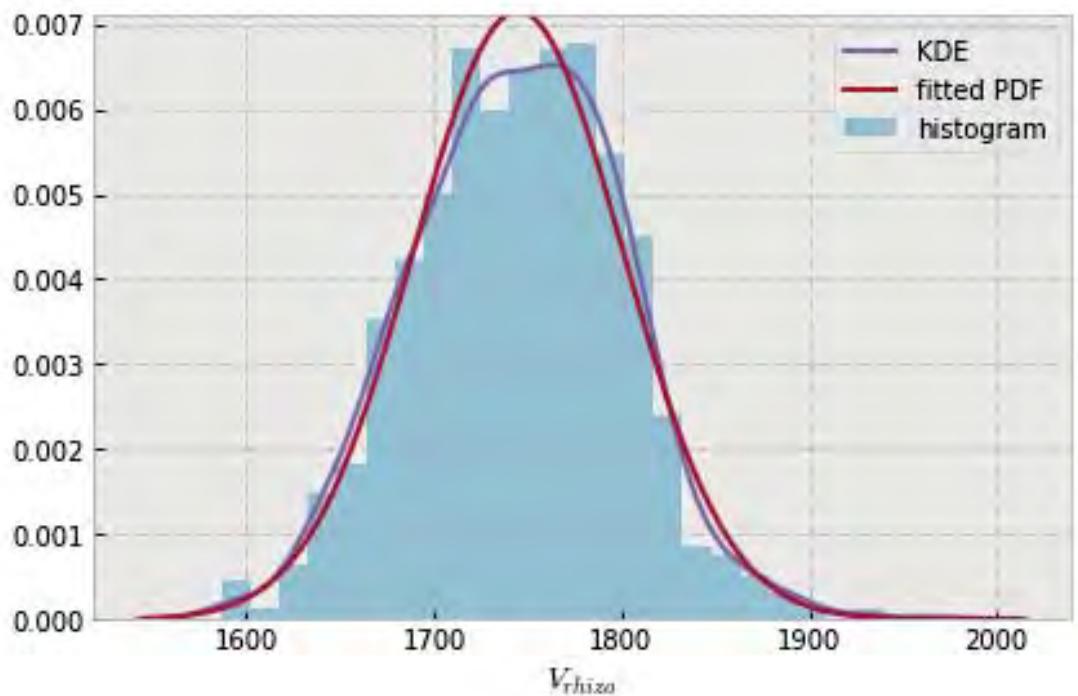


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=1742.5132220136336, scale=58.84043423105184)
DescribeResult(nobs=1000, minmax=(1552.2468716082542, 1932.7940041302741)
               mean=1742.5132220136336, variance=3465.6623628615985,
               skewness=-0.06337265159395698, kurtosis=-0.3205022157584376)
```

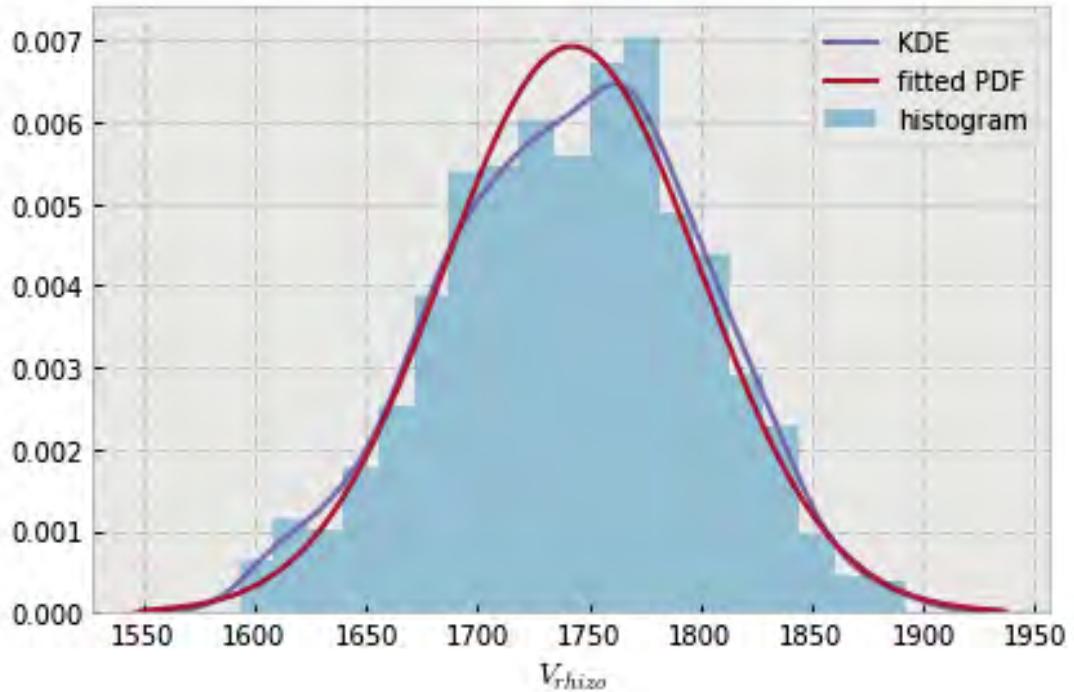


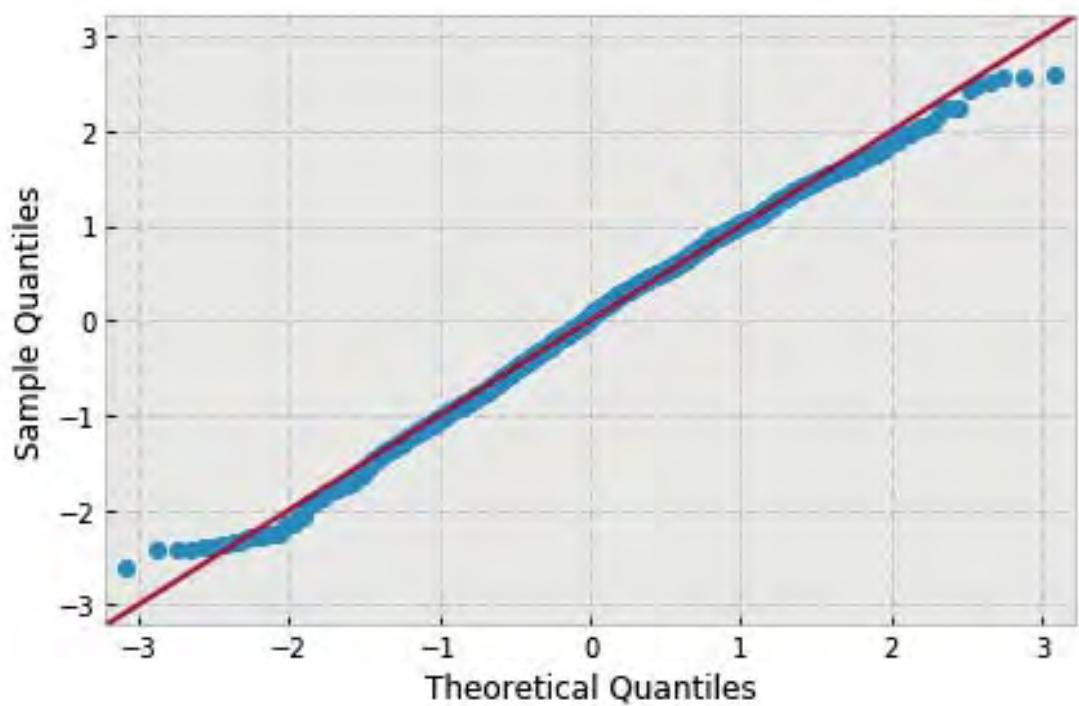


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=1744.8754330560132, scale=55.79615977685549)
DescribeResult(nobs=1000, minmax=(1586.6628339254091, 1970.7702733814087)
               mean=1744.8754330560132, variance=3116.3277736180048,
               skewness=0.042486310347704954, kurtosis=0.09269533445619693)
```

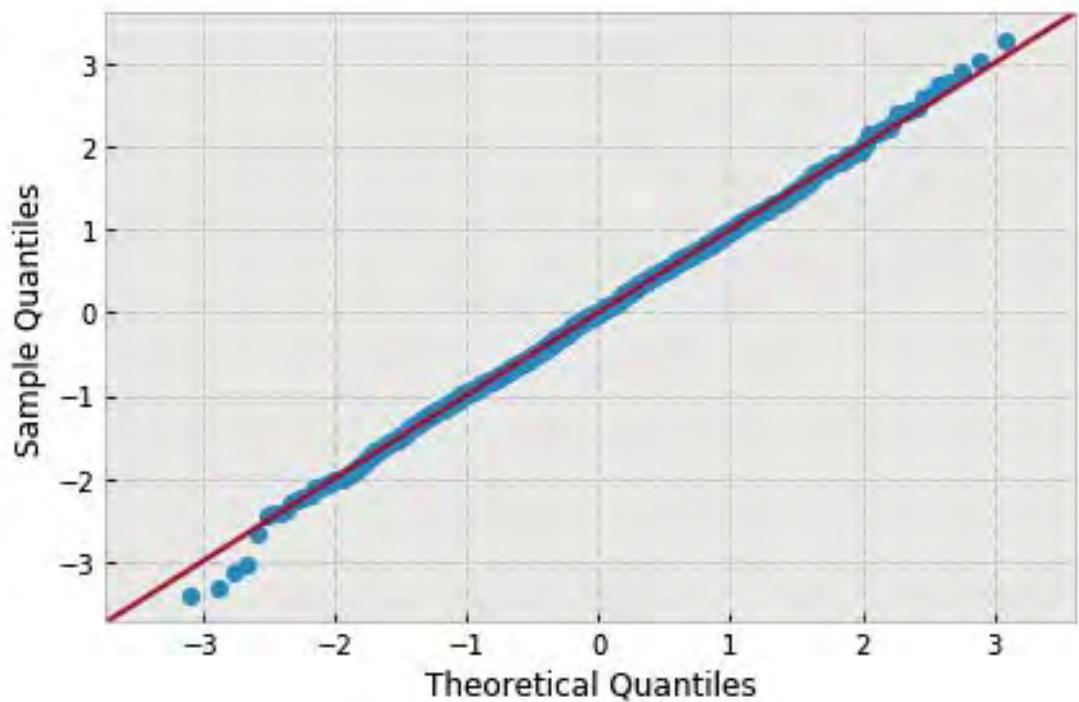
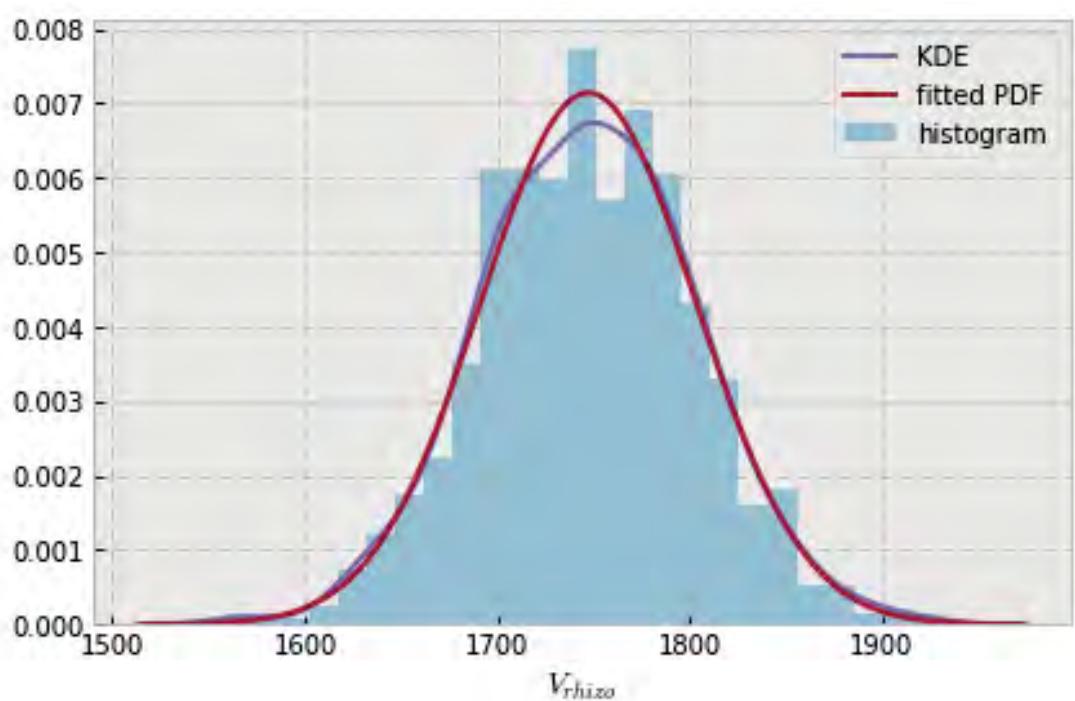


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=1742.443833939682, scale=57.570369388524426)
DescribeResult(nobs=1000, minmax=(1592.7268693962944, 1891.5589600895025)
               mean=1742.443833939682, variance=3317.665096627778,
               skewness=-0.1231138292683463, kurtosis=-0.38316748179687954)
```

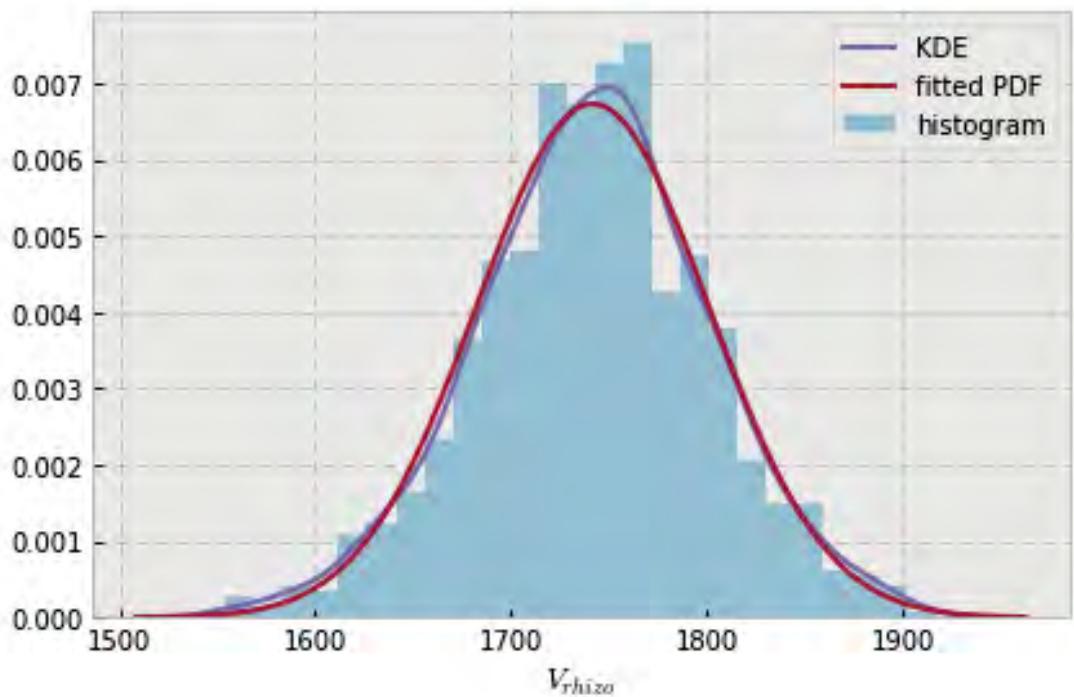


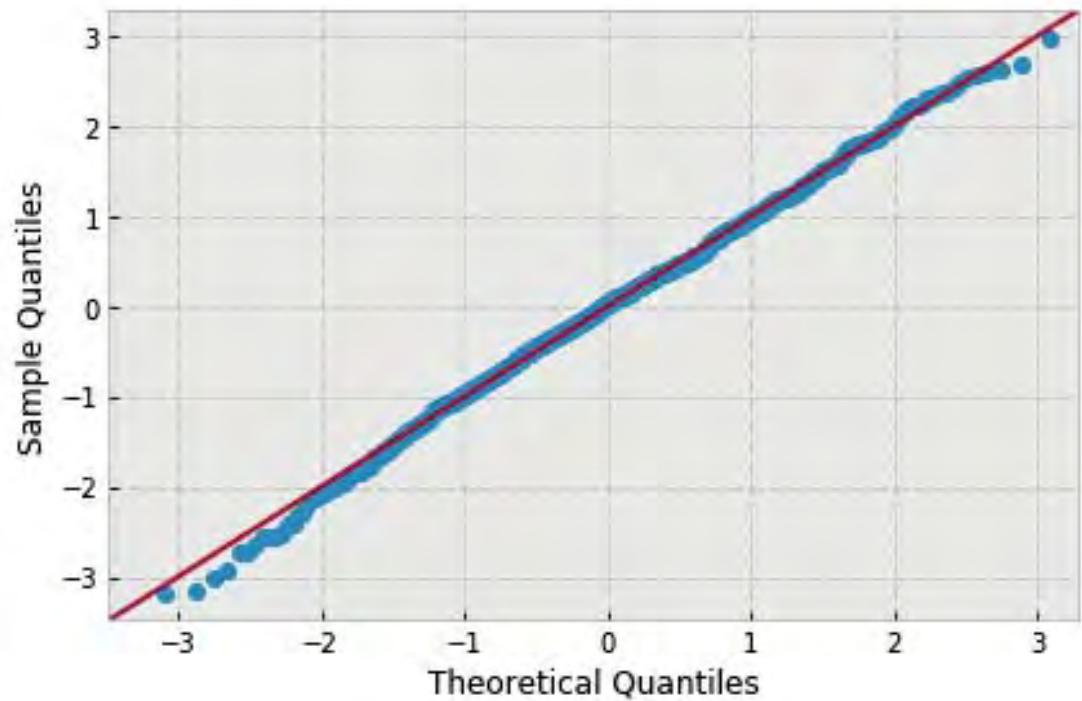


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=1747.2015201219872, scale=55.84339812425714)
DescribeResult(nobs=1000, minmax=(1557.7791520841324, 1929.6929540674025)
               mean=1747.2015201219872, variance=3121.6067207850715,
               skewness=-0.007707661056550271, kurtosis=0.10830544613616988)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=1741.7189251327056, scale=59.318664938988114)
DescribeResult(nobs=1000, minmax=(1553.4937564911336, 1917.703726661108)
               mean=1741.7189251327056, variance=3522.2262363803179,
               skewness=-0.09338369956779327, kurtosis=0.13575480375463922)
```

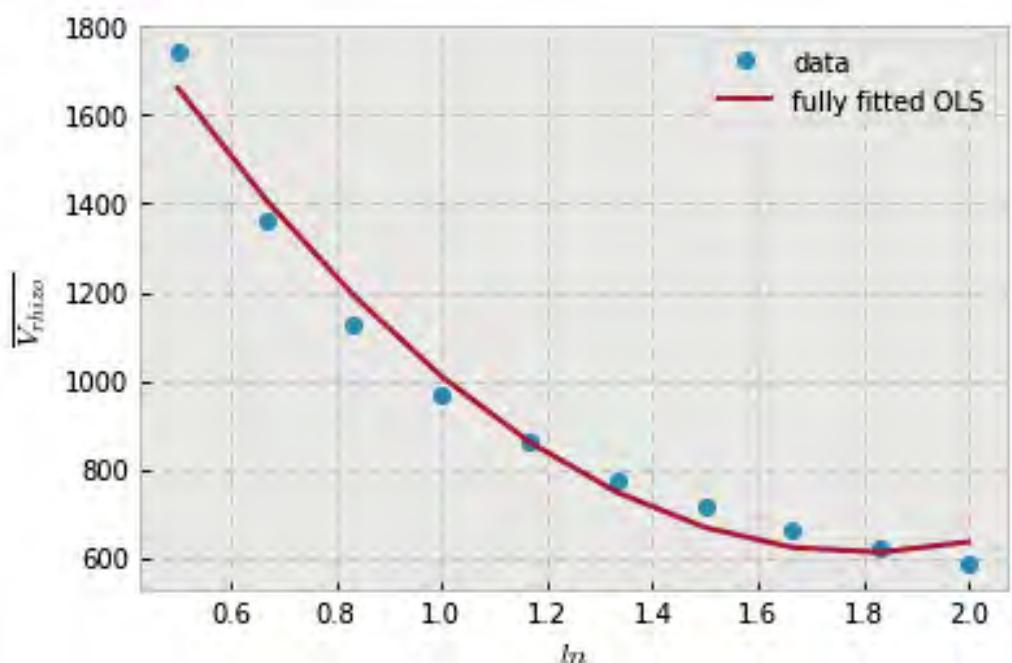




9.2 Parameter regression

```
In [24]: var_name = 'rhizosphere_volume_phosphate'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly1')
mean_reg('nB', var_name, 'poly1')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly1')
mean_reg('N', var_name, 'poly1')
```

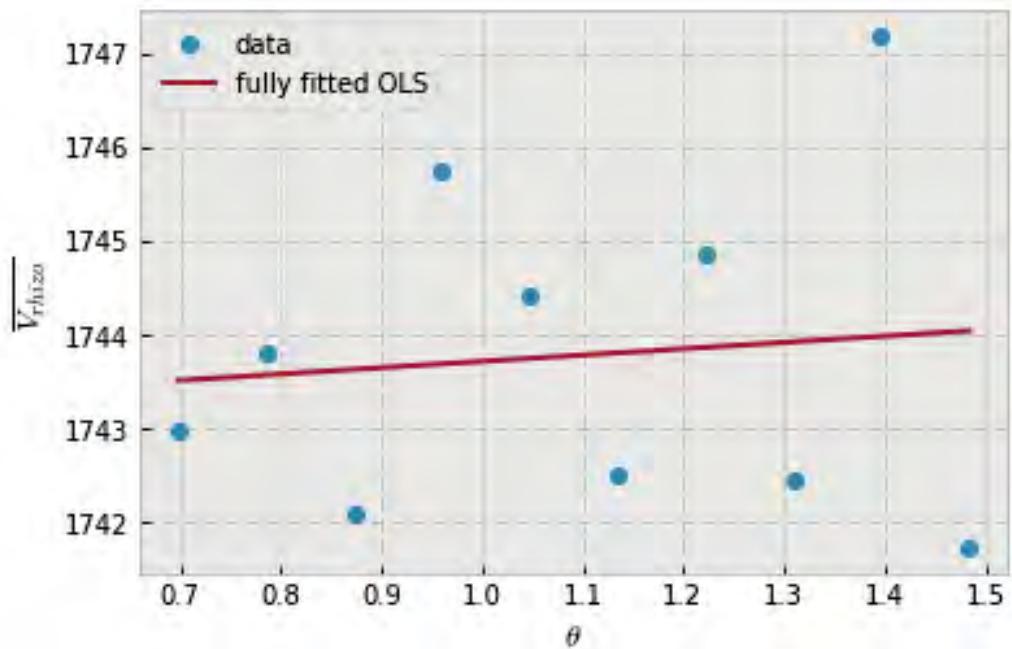
```
variable name: ln
model: poly2
r-squared: 0.9296452549540998
explained variance: 0.9297756460580475
```



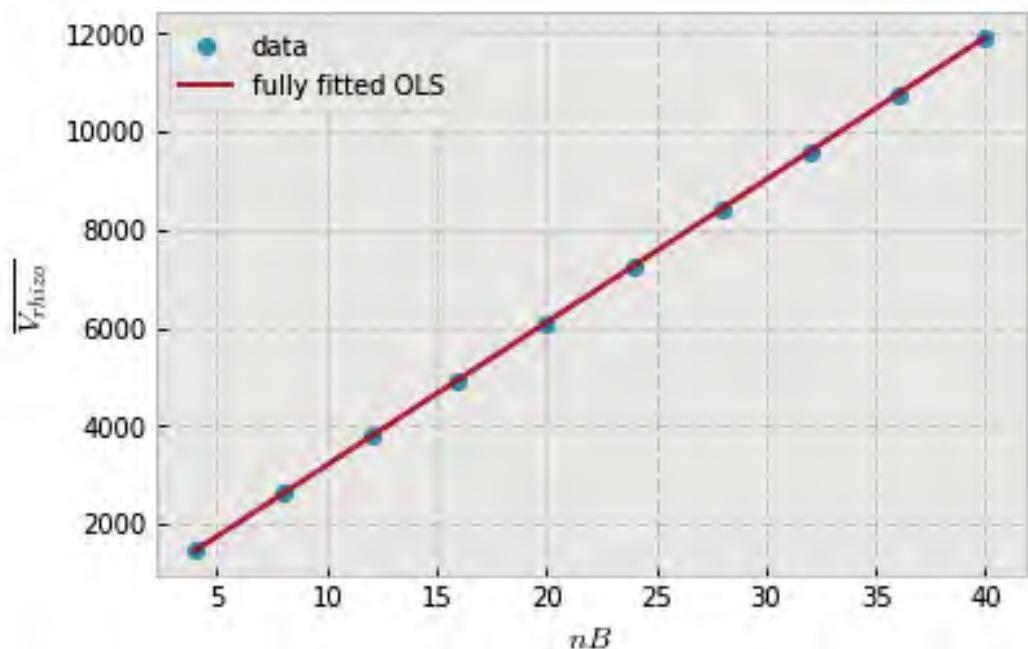
```

variable name: theta
model: poly1
r-squared: -0.6935905174218402
explained variance: -0.6921320396744748

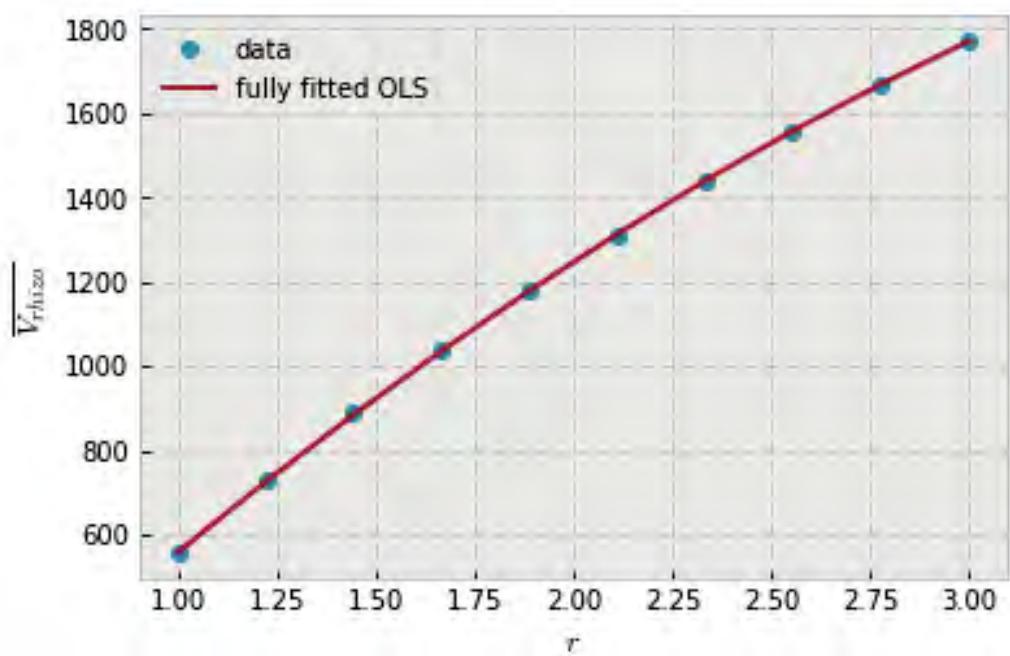
```



```
variable name: nB
model: poly1
r-squared: 0.9999989115960336
explained variance: 0.9999989135220785
```



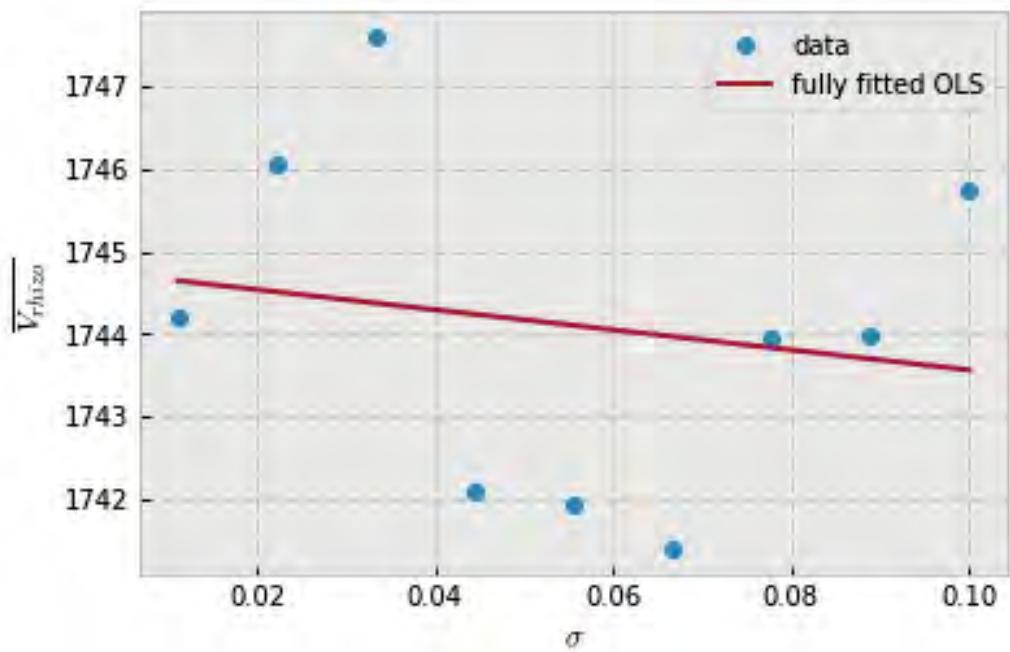
```
variable name: r
model: poly2
r-squared: 0.9998723695196802
explained variance: 0.9998724993039633
```



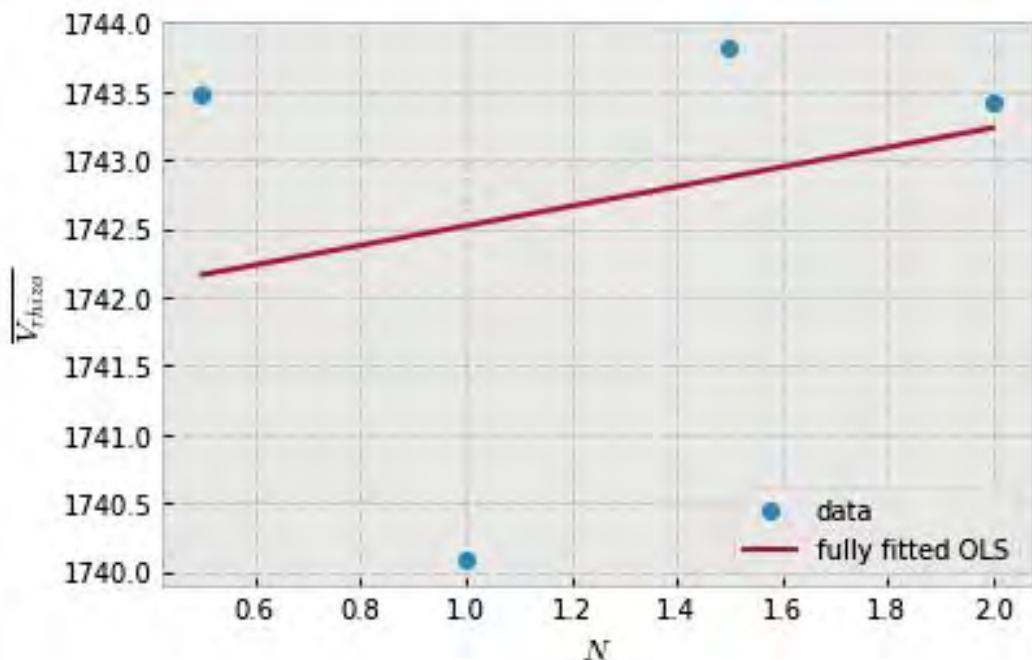
```

variable name: sigma
model: poly1
r-squared: -0.5291843415548669
explained variance: -0.52245448652538

```



```
variable name: N
model: poly1
r-squared: -2.6205565151214705
explained variance: -2.4015435303753105
```



10 Root tip density

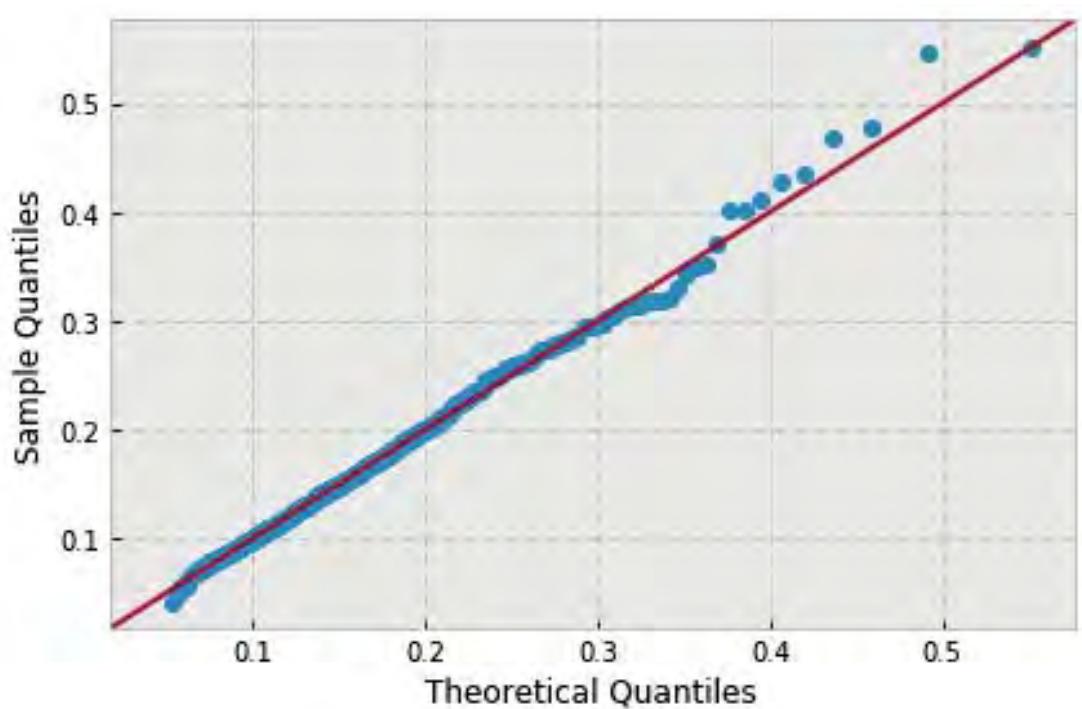
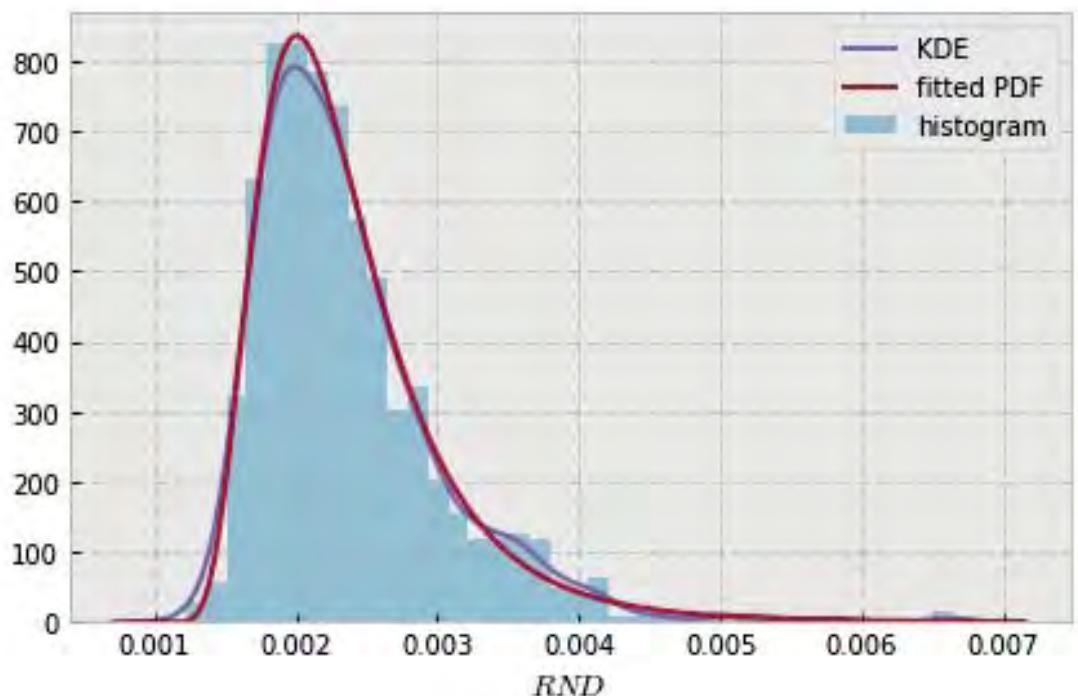
10.1 Probability distributions

```
In [25]: plot_dist('root_tip_density', 'inverse-gamma')
```

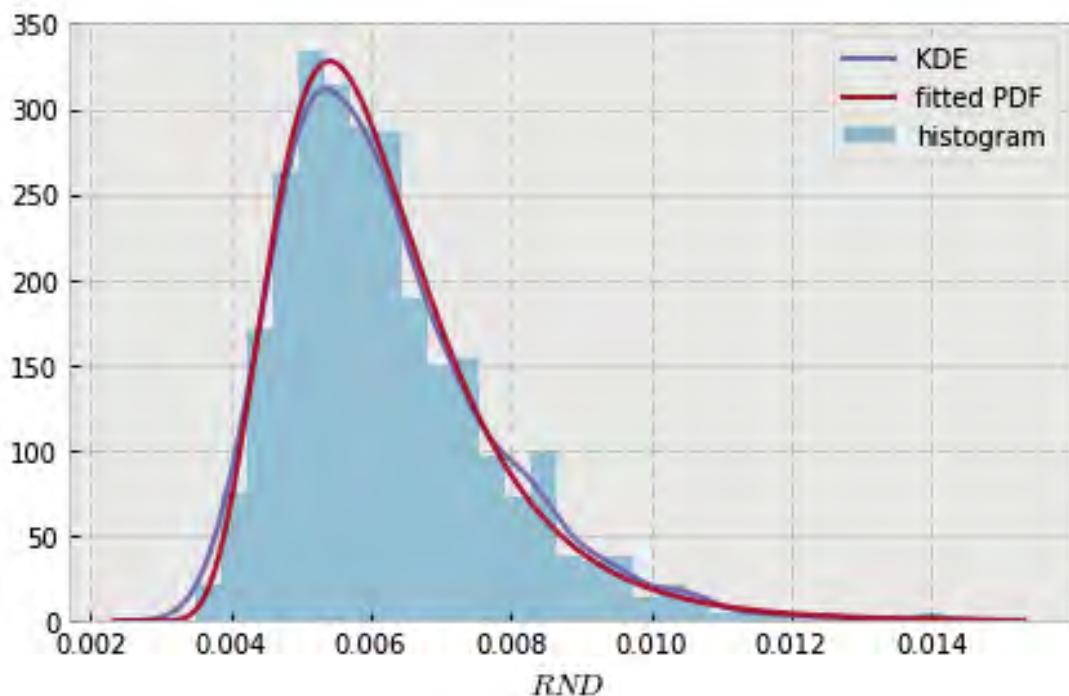
```
/home/lpetrich/.local/share/anaconda3/envs/Master/lib/python3.6/site-packages/scipy/stats/_distn_infra.py
```

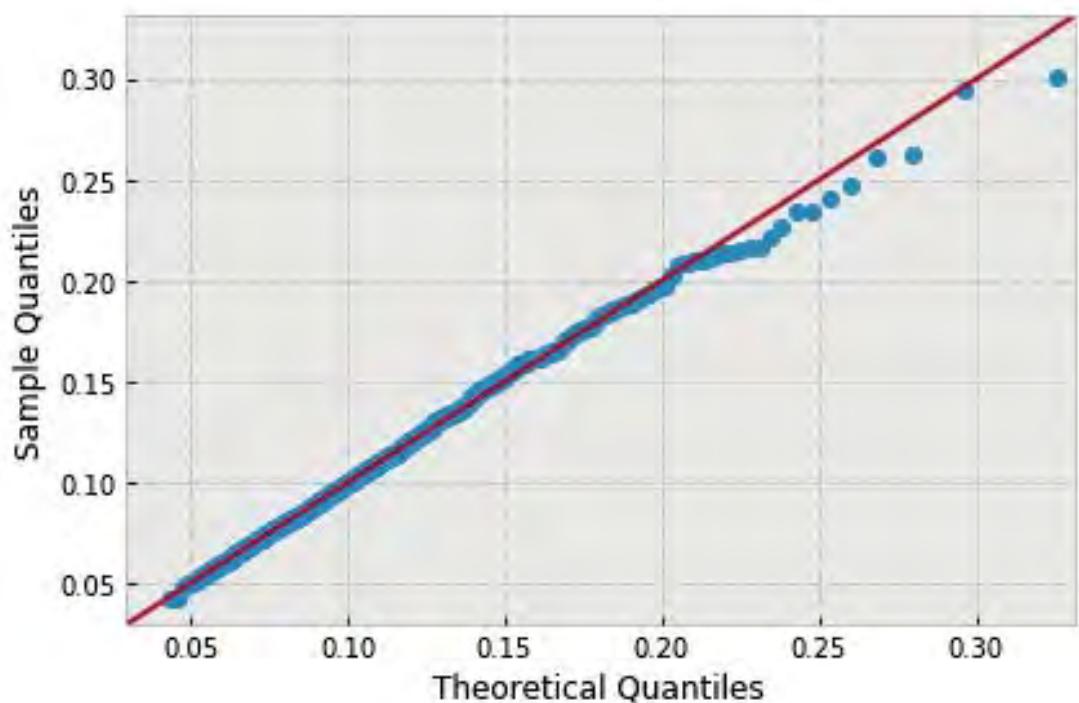
```
Lhat = muhat - Shat*mu
```

```
variable name: N
variable value: 0.5
distribution: inverse-gamma(shape=(7.6597523723327594,), loc=0.0007738103072321231,
    scale=0.01065879756073549)
DescribeResult(nobs=1000, minmax=(0.0012194689788932209, 0.0066462516400040165)
    mean=0.0023745120625213167, variance=4.4927062113585908e-07,
    skewness=1.7987372939645103, kurtosis=5.540434645730894)
```

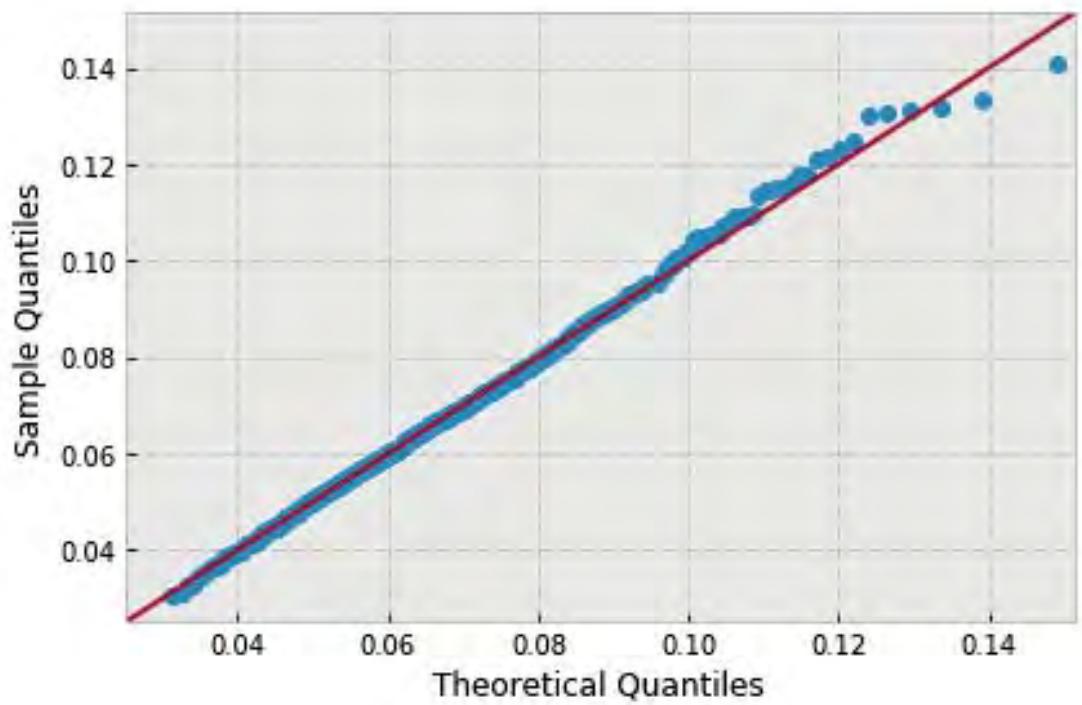
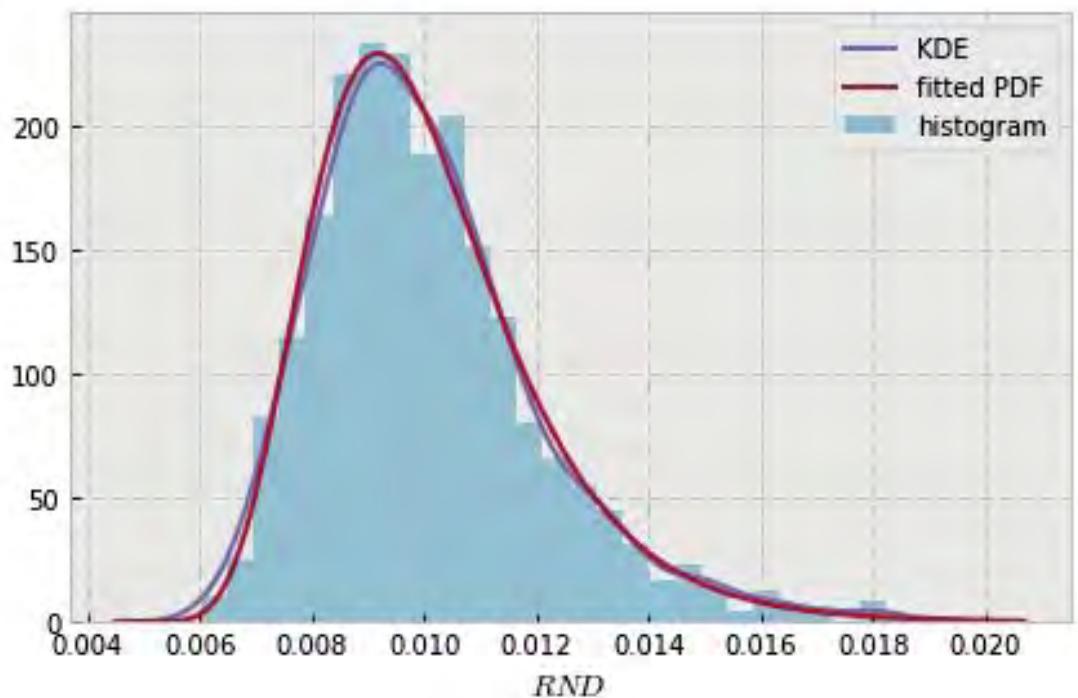


```
variable name: N
variable value: 1.0
distribution: inverse-gamma(shape=(10.217187108181434,), loc=0.001727217434534203,
    scale=0.04132255899860701)
DescribeResult(nobs=1000, minmax=(0.003477241404873098, 0.014168429118523599)
    mean=0.006207526806693309, variance=2.3353382924956831e-06,
    skewness=1.2165848594478115, kurtosis=2.136160019340233)
```

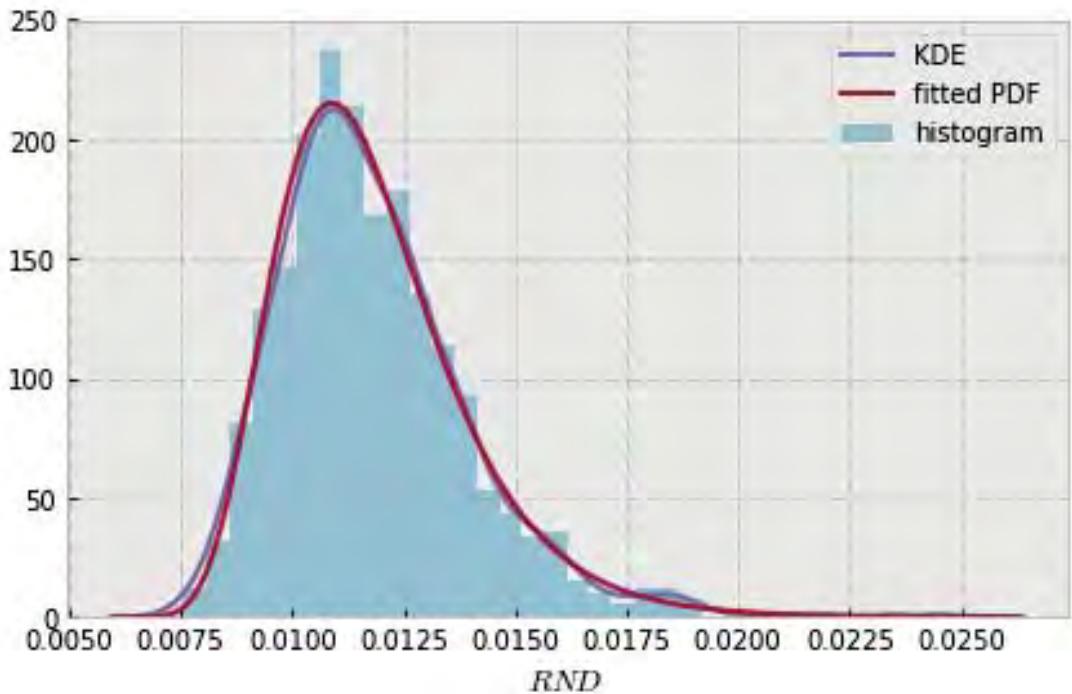


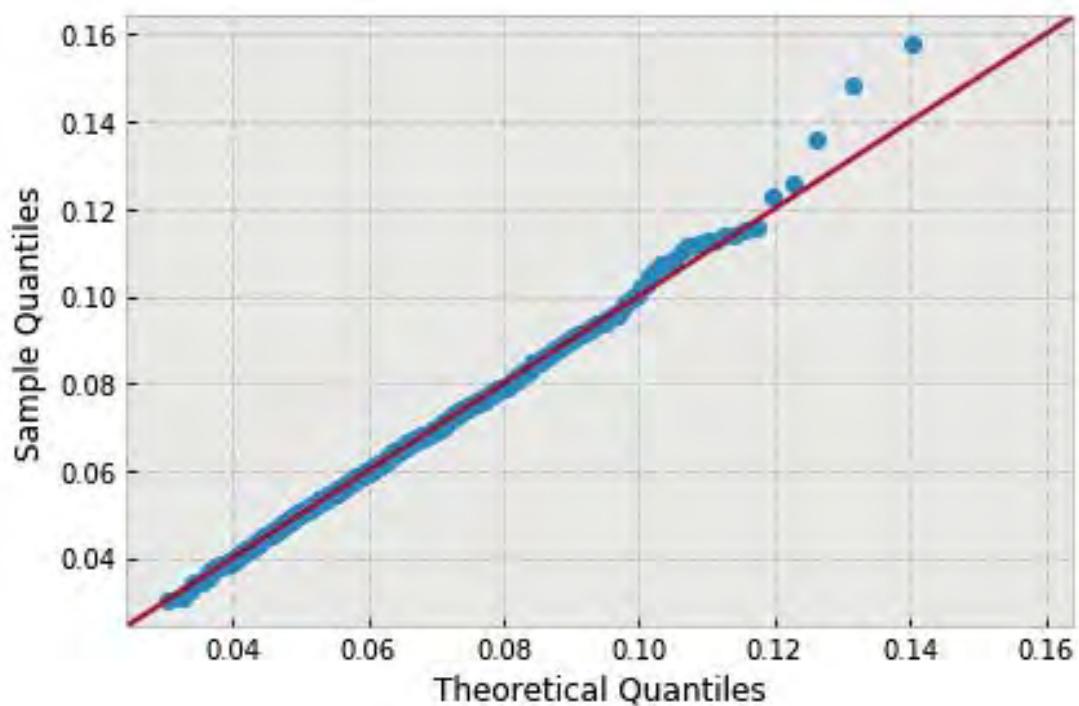


```
variable name: N
variable value: 1.5
distribution: inverse-gamma(shape=(16.497825997435555,), loc=0.0023547979153463808,
scale=0.11935333315368521)
DescribeResult(nobs=1000, minmax=(0.0060070844987077255, 0.019162329361833347)
mean=0.010056338844834688, variance=4.0965361380441957e-06,
skewness=1.077201066808754, kurtosis=1.739863332699911)
```

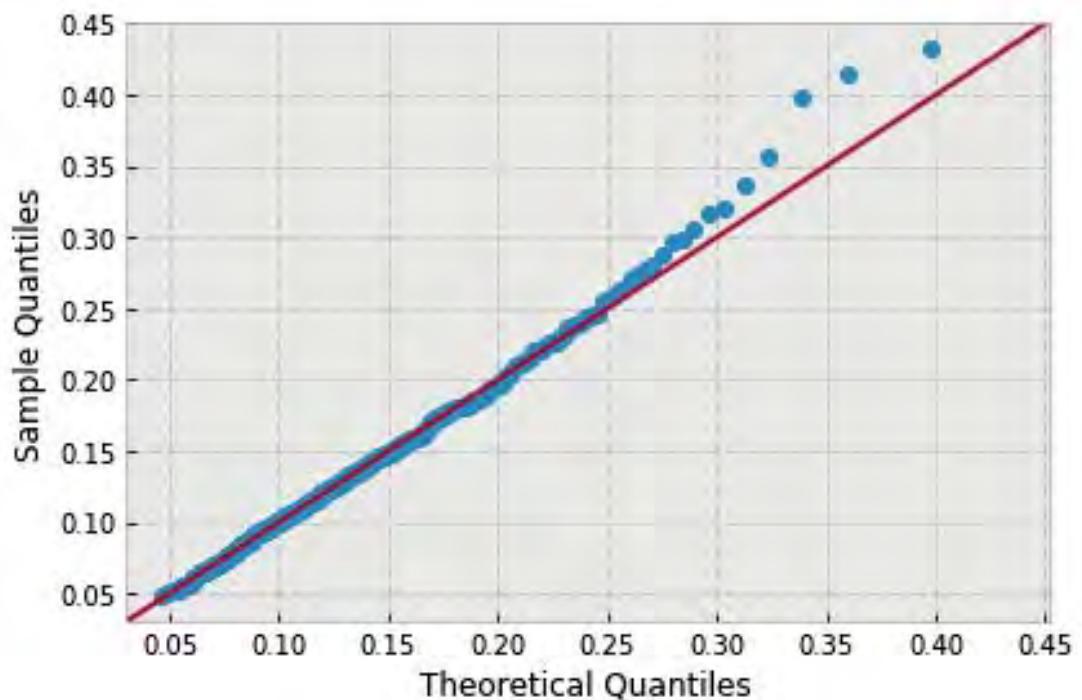
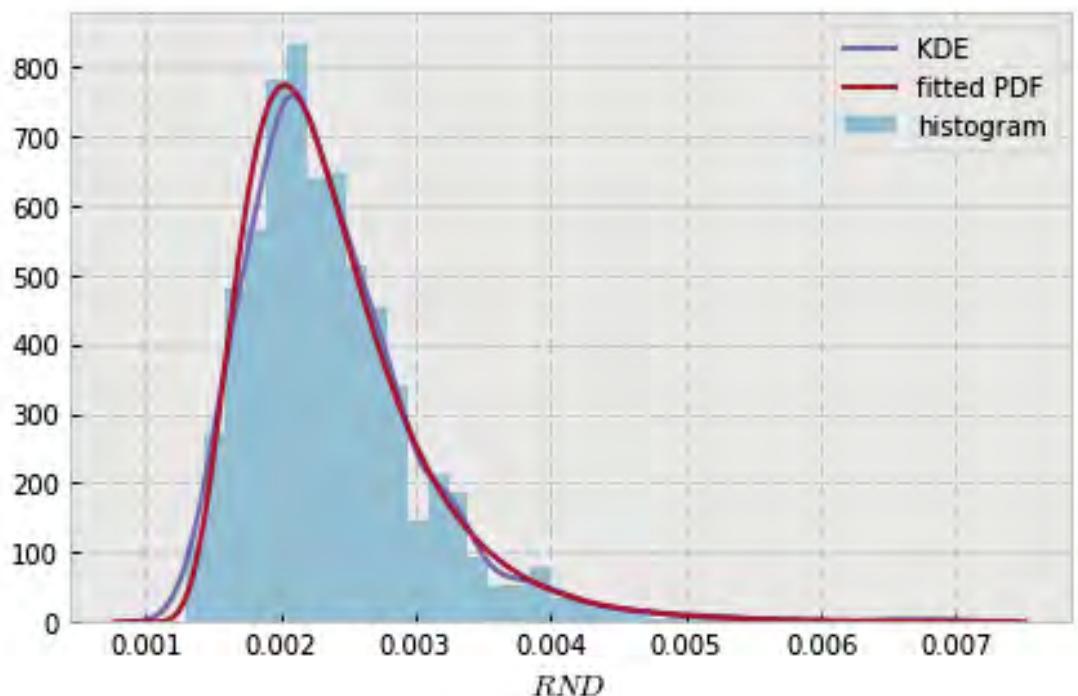


```
variable name: N
variable value: 2.0
distribution: inverse-gamma(shape=(17.142062306770406,), loc=0.0034535697946684206,
    scale=0.13482703759467068)
DescribeResult(nobs=1000, minmax=(0.0075749480225817574, 0.024724846892995197)
    mean=0.011807156498800863, variance=4.6775603941161935e-06,
    skewness=1.206583363116419, kurtosis=2.9682170906933987)
```

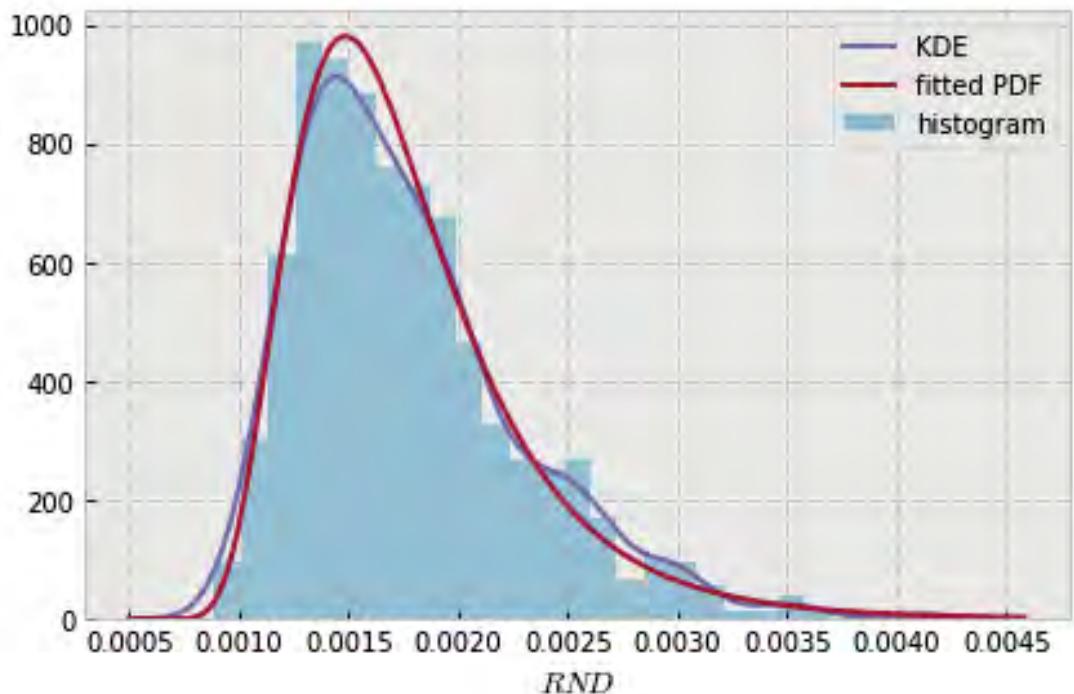


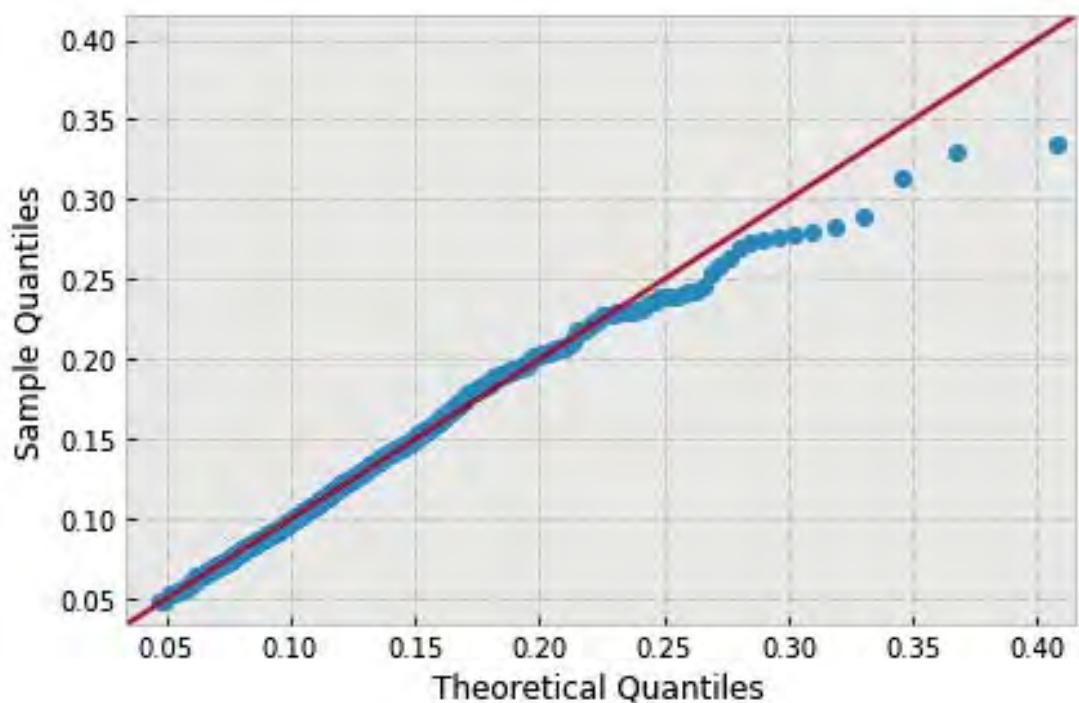


```
variable name: ln
variable value: 0.5
distribution: inverse-gamma(shape=(9.1210016782027523,), loc=0.0005645128220015788,
scale=0.014852167471725516)
DescribeResult(nobs=1000, minmax=(0.0012940575542343799, 0.0069827658368684679)
mean=0.002394167679528496, variance=4.8250983542434575e-07,
skewness=1.850199652612085, kurtosis=6.297771527742082)
```



```
variable name: ln
variable value: 0.6666666666666666
distribution: inverse-gamma(shape=(9.0037156855751572,), loc=0.0003345351123713474,
                           scale=0.011497806740135622)
DescribeResult(nobs=1000, minmax=(0.00088785719367103985, 0.0041880970030089767)
               mean=0.0017690232568291877, variance=2.6977151491494418e-07,
               skewness=1.1355191590879237, kurtosis=1.5202045296035873)
```

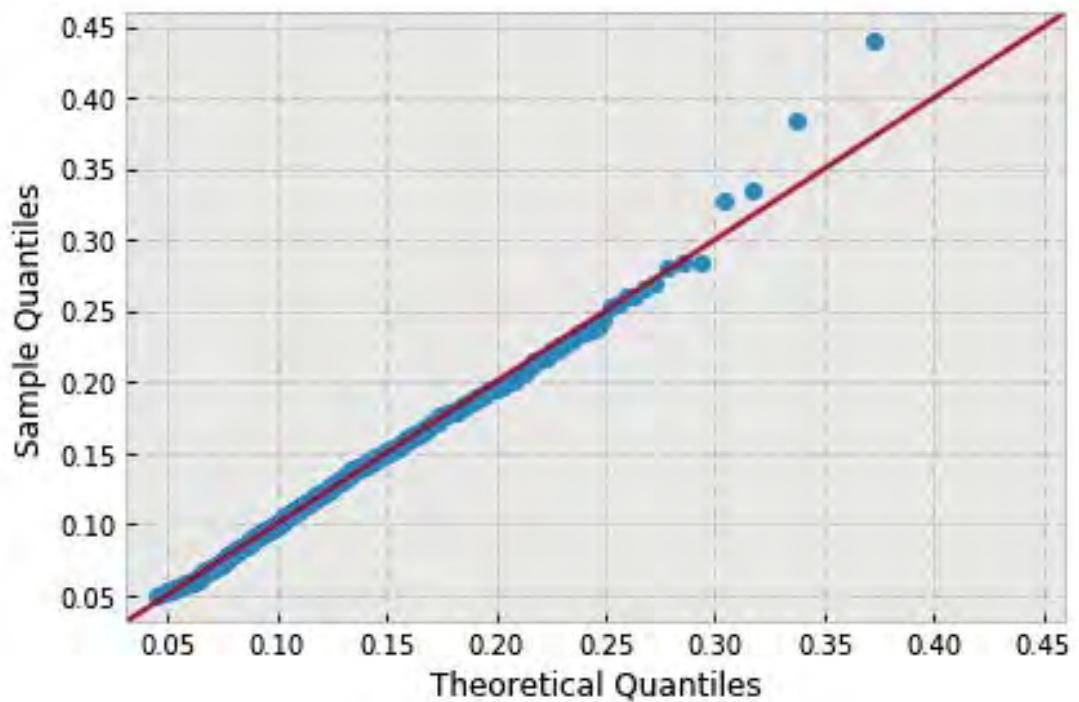
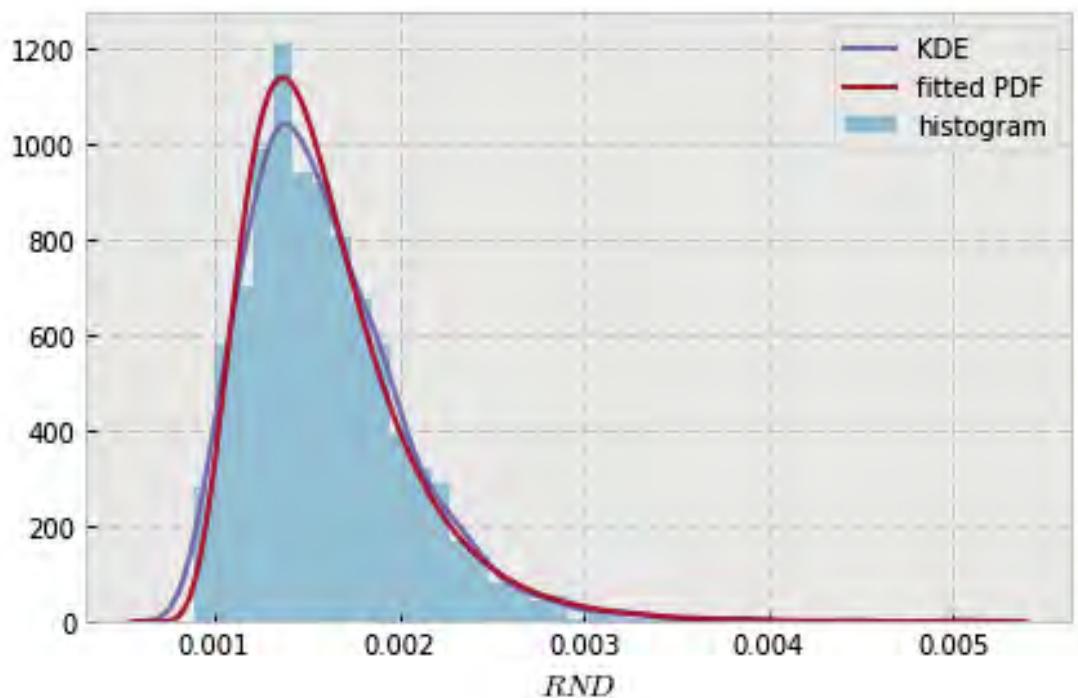




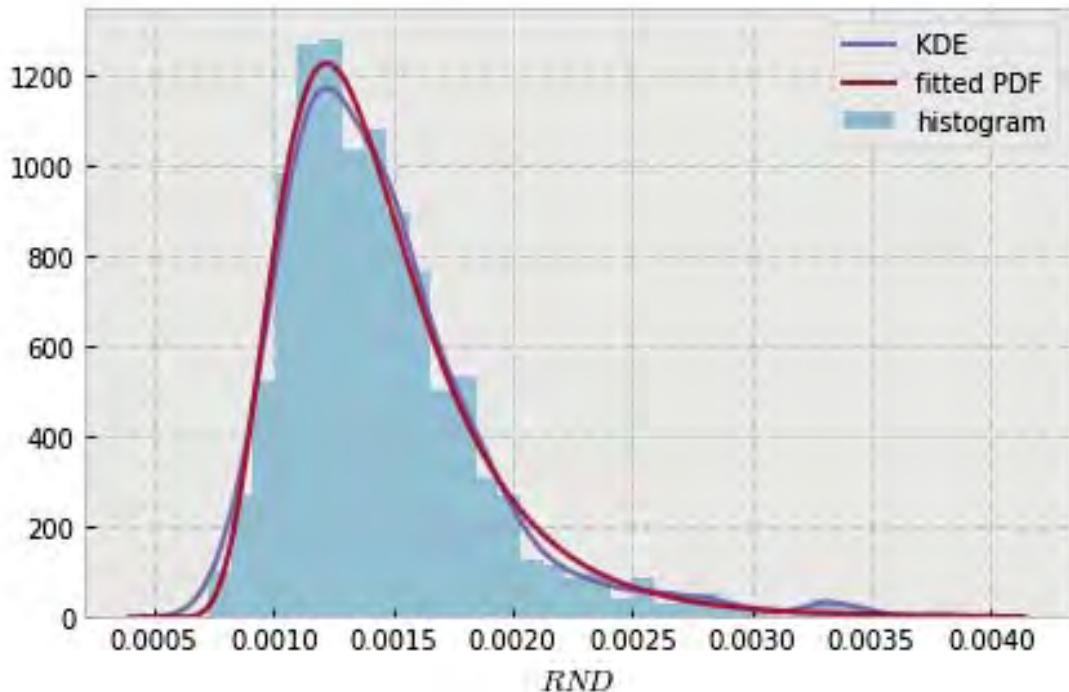
```

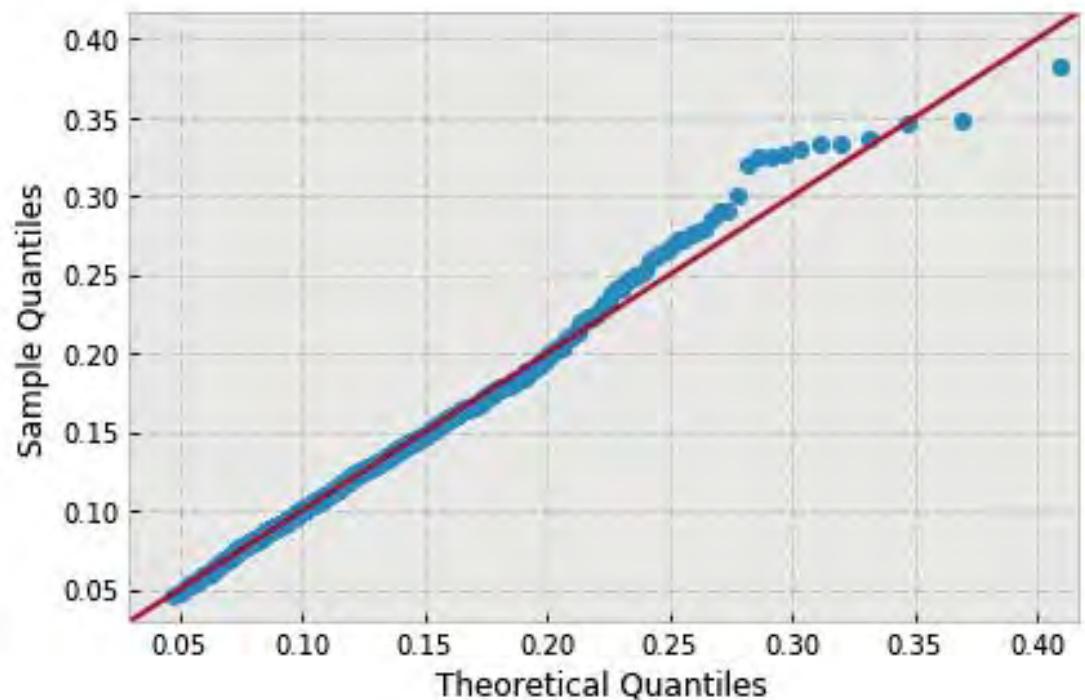
variable name: ln
variable value: 0.8333333333333333
distribution: inverse-gamma(shape=(9.4729965252635804,), loc=0.0003506674008919872,
                           scale=0.010666476356011582)
DescribeResult(nobs=1000, minmax=(0.00088882780346902853, 0.0050426455876741005)
               mean=0.001609482843270437, variance=2.1003323272646491e-07,
               skewness=1.6610518050490881, kurtosis=5.979772731131083)

```

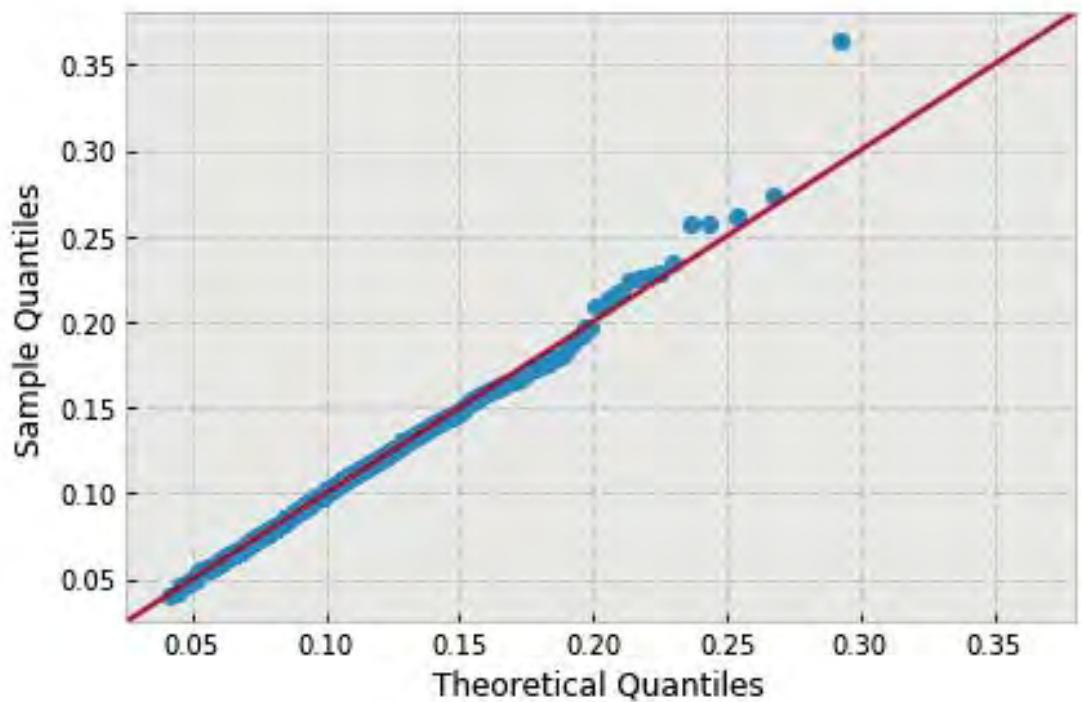
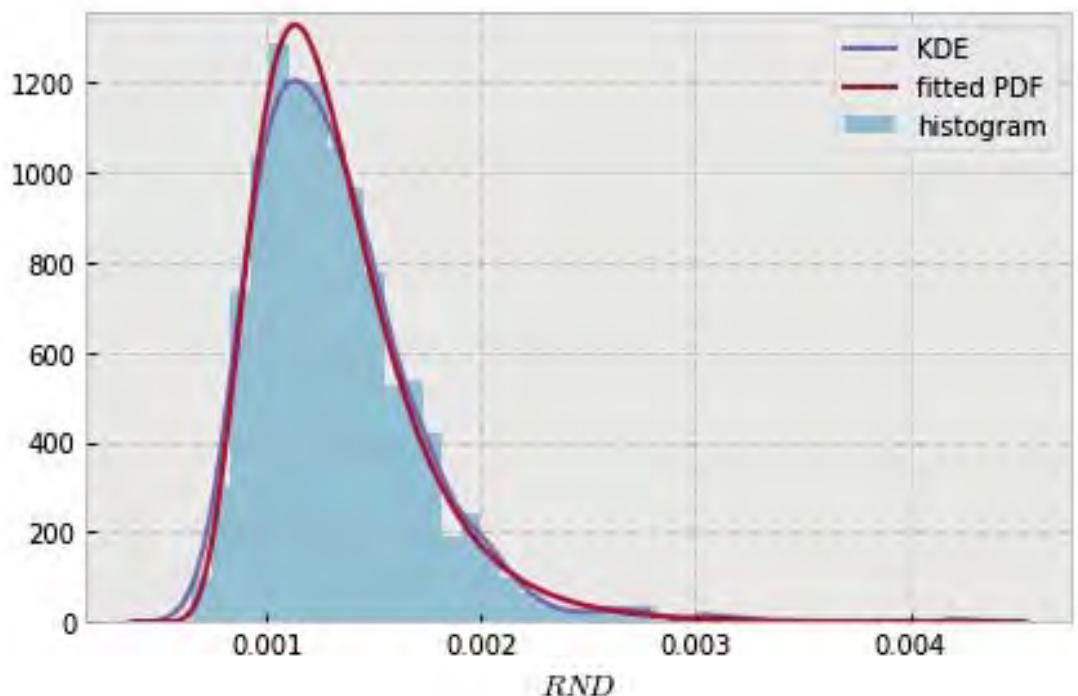


```
variable name: ln
variable value: 1.0
distribution: inverse-gamma(shape=(8.986494423448729,), loc=0.00030557152979978285,
                           scale=0.009152262826795408)
DescribeResult(nobs=1000, minmax=(0.00072521826947069665, 0.0038108885761589037)
               mean=0.0014521234386859203, variance=1.9152153667660772e-07,
               skewness=1.6776156320081734, kurtosis=4.239590077553565)
```

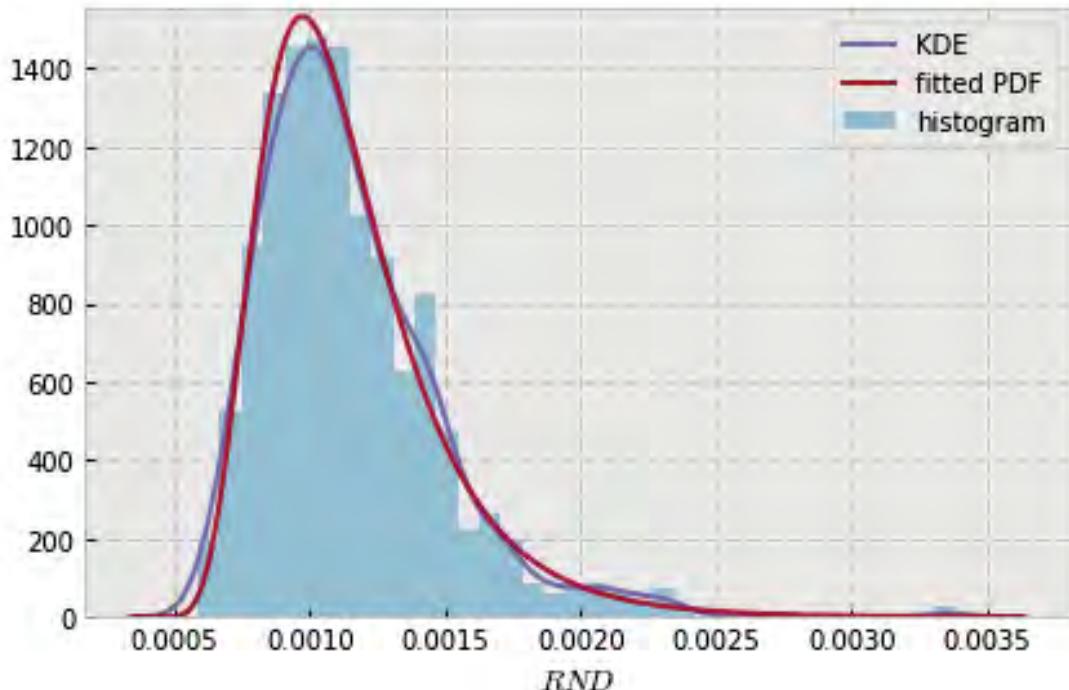


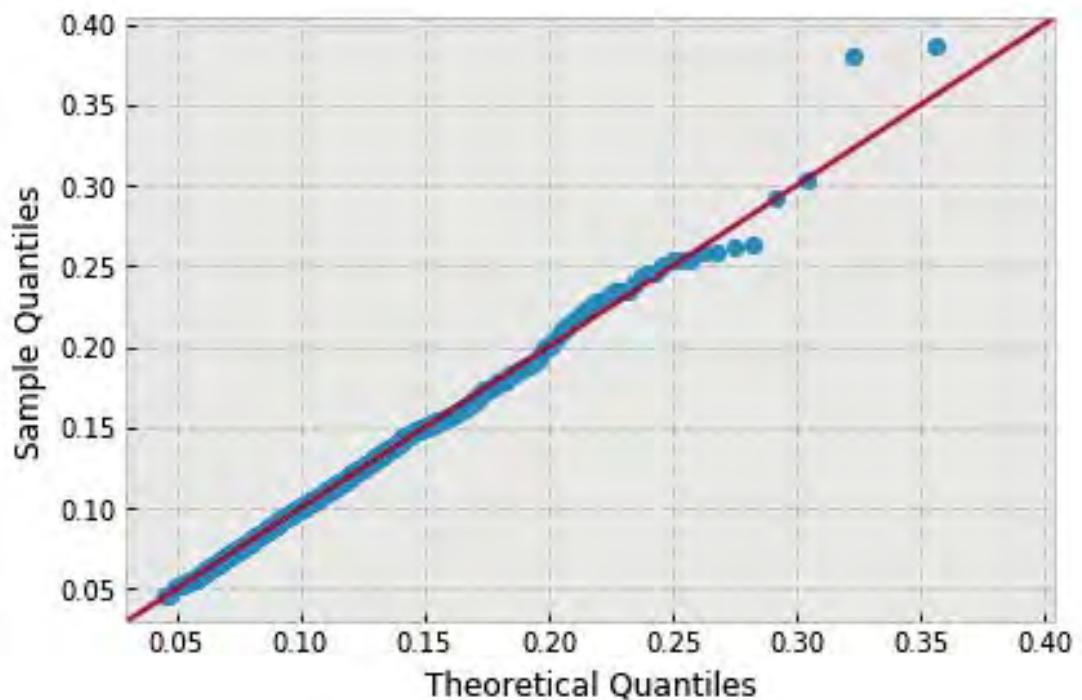


```
variable name: ln
variable value: 1.1666666666666665
distribution: inverse-gamma(shape=(10.857750316544212,), loc=0.0001926111208456953,
                           scale=0.011154629383812655)
DescribeResult(nobs=1000, minmax=(0.00065137940302353544, 0.0042507610747134624)
               mean=0.0013241123623787202, variance=1.4504561703332024e-07,
               skewness=1.5971218305009955, kurtosis=5.583879891518713)
```

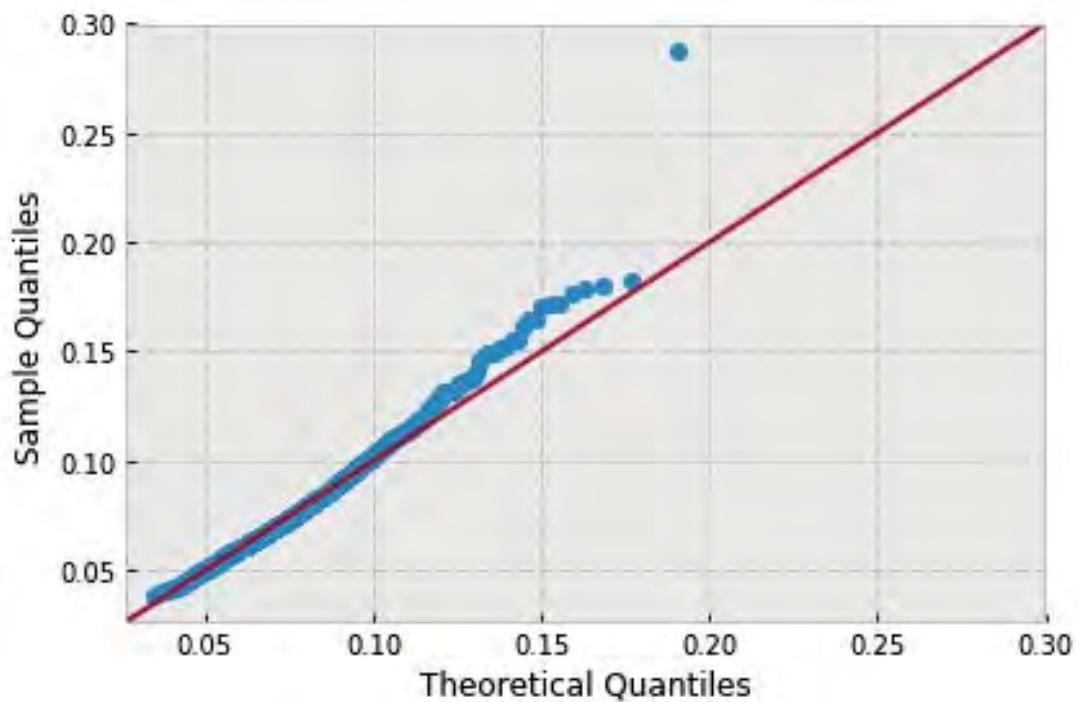
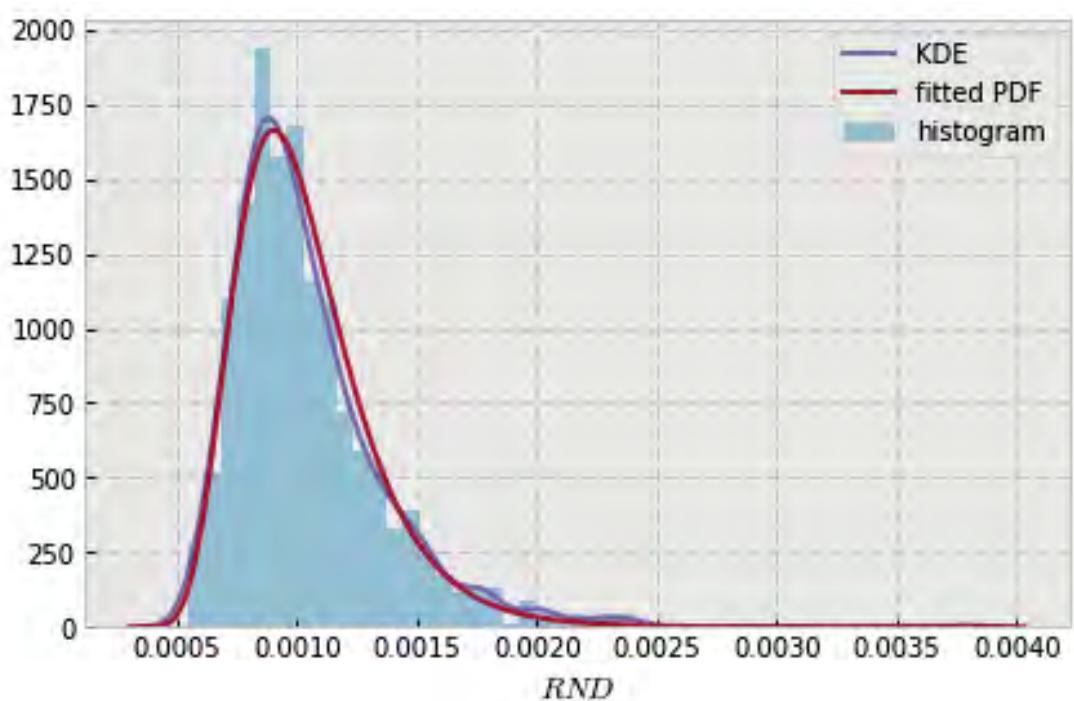


```
variable name: ln
variable value: 1.333333333333333
distribution: inverse-gamma(shape=(9.7100570488957345,), loc=0.00020683093233207073,
                           scale=0.008203076921327499)
DescribeResult(nobs=1000, minmax=(0.00058759995682583683, 0.0033833777083214253)
               mean=0.0011484932467901645, variance=1.14330855487305e-07,
               skewness=1.5955571520748002, kurtosis=4.857350030483503)
```

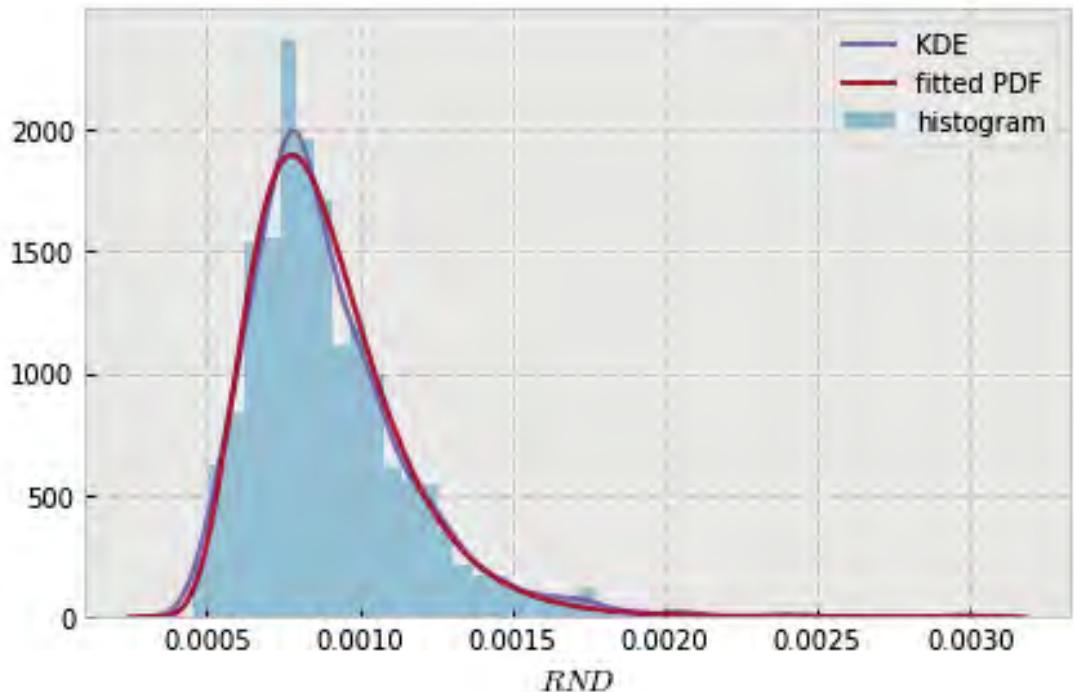


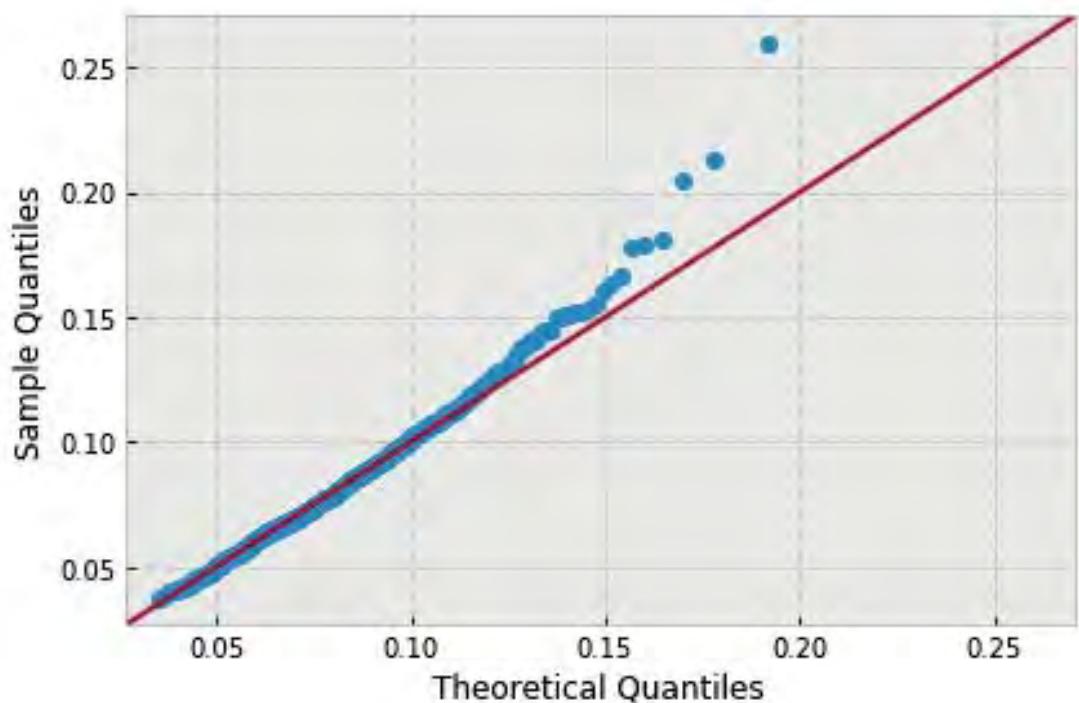


```
variable name: ln
variable value: 1.5
distribution: inverse-gamma(shape=(14.098366676974599,), loc=3.836548239606208e-05,
                           scale=0.013066955778810436)
DescribeResult(nobs=1000, minmax=(0.00054336439665528908, 0.0037910889184602971)
               mean=0.0010431340922836911, variance=1.0248459099133162e-07,
               skewness=1.8591718652867188, kurtosis=7.172608168152507)
```

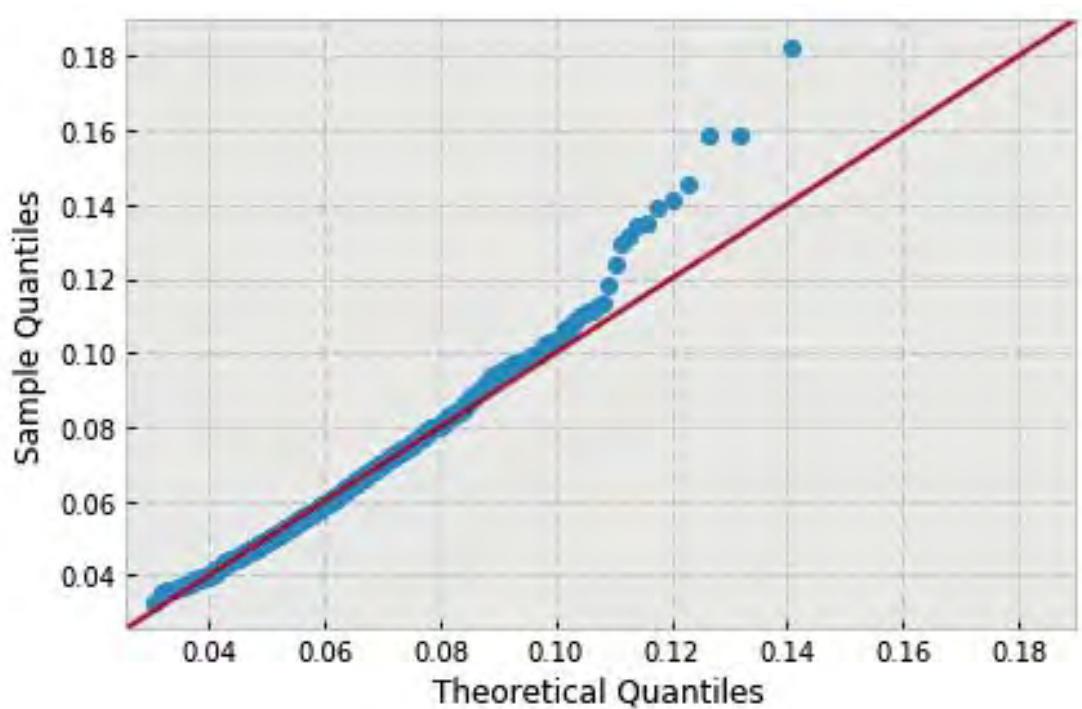
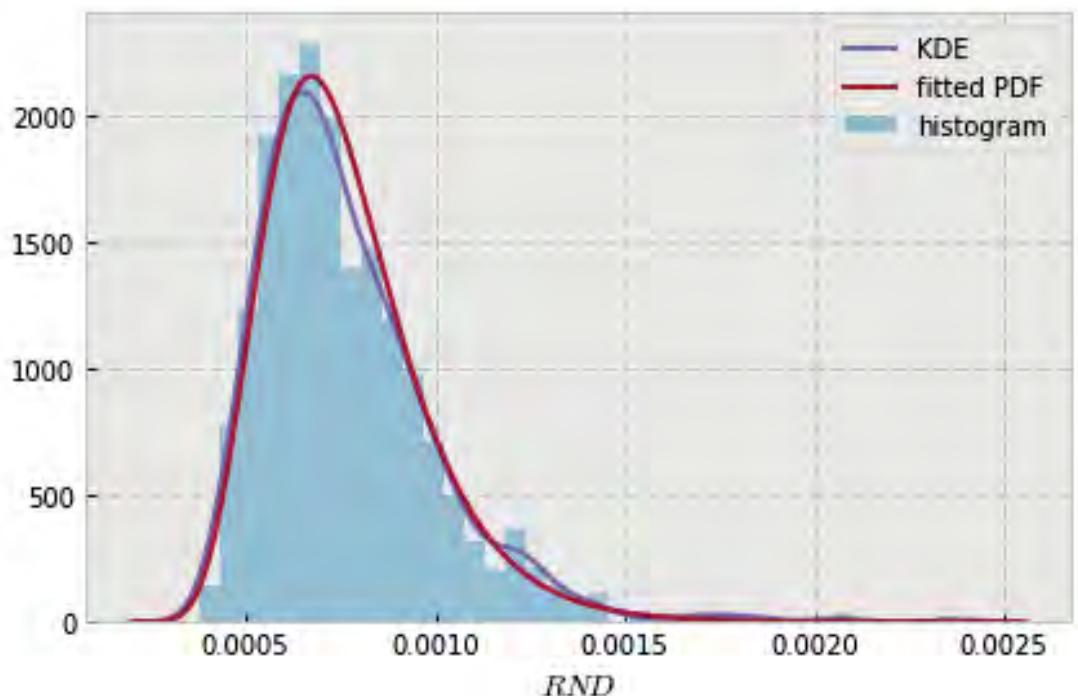


```
variable name: ln
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(14.021036219483733,), loc=2.1948851641173348e-05,
                           scale=0.011373809841013586)
DescribeResult(nobs=1000, minmax=(0.0004518266129493394, 0.0029715874314313745)
               mean=0.0008997161646819025, variance=7.4528321677554258e-08,
               skewness=1.7611585970234545, kurtosis=6.127348651580023)
```

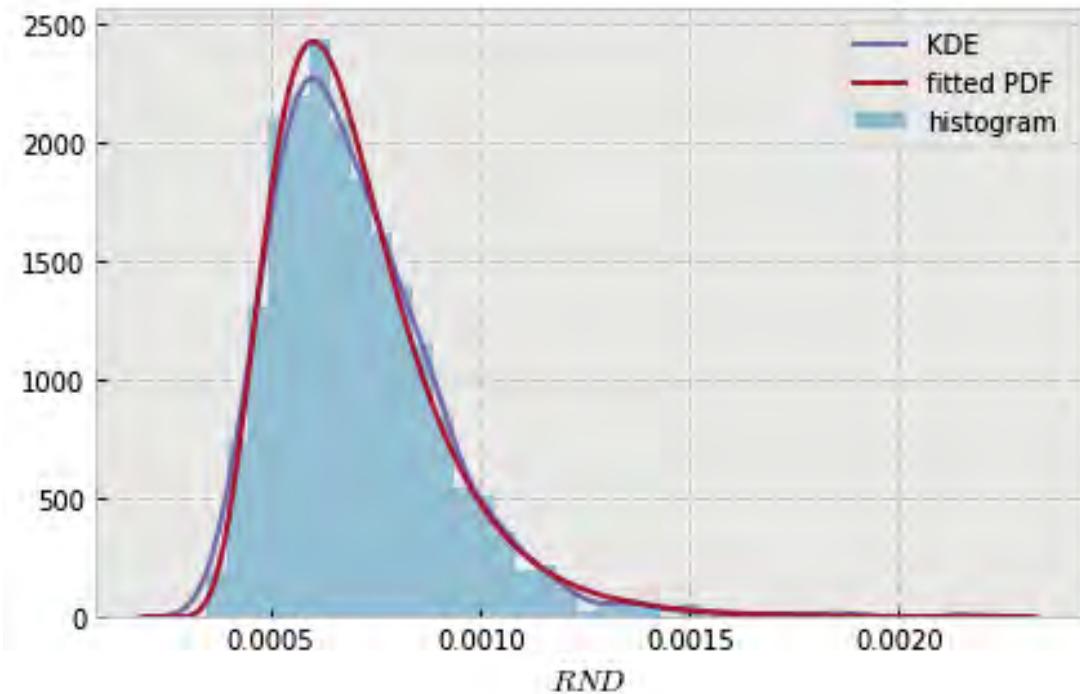


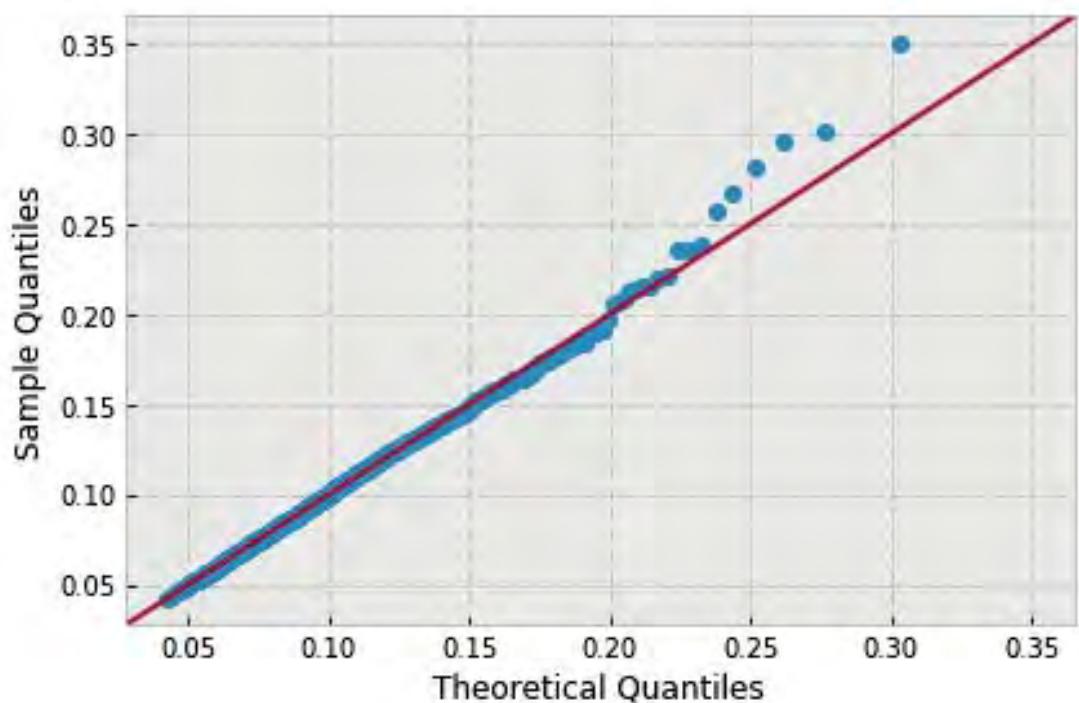


```
variable name: ln
variable value: 1.833333333333333
distribution: inverse-gamma(shape=(17.12109962975849,), loc=-6.819839462147471e-05,
                           scale=0.01341966552563333)
DescribeResult(nobs=1000, minmax=(0.00037406685655433261, 0.0023770105426163318)
               mean=0.00076884734564426778, variance=5.6203072210380379e-08,
               skewness=1.6707675280213647, kurtosis=5.212219530564875)
```

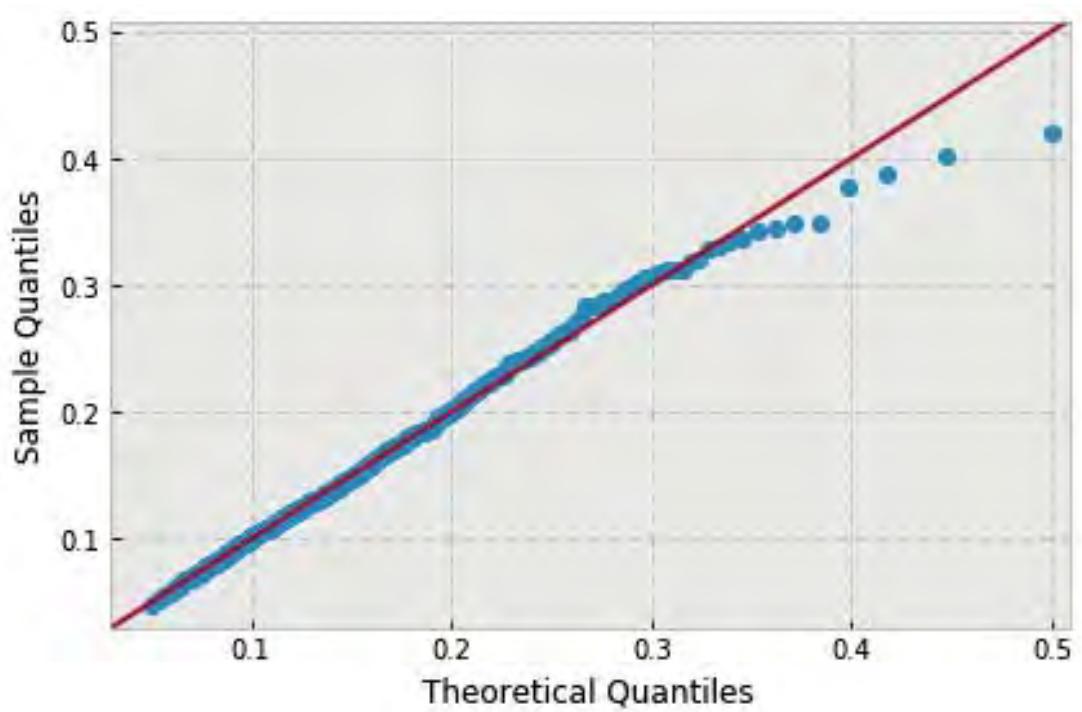
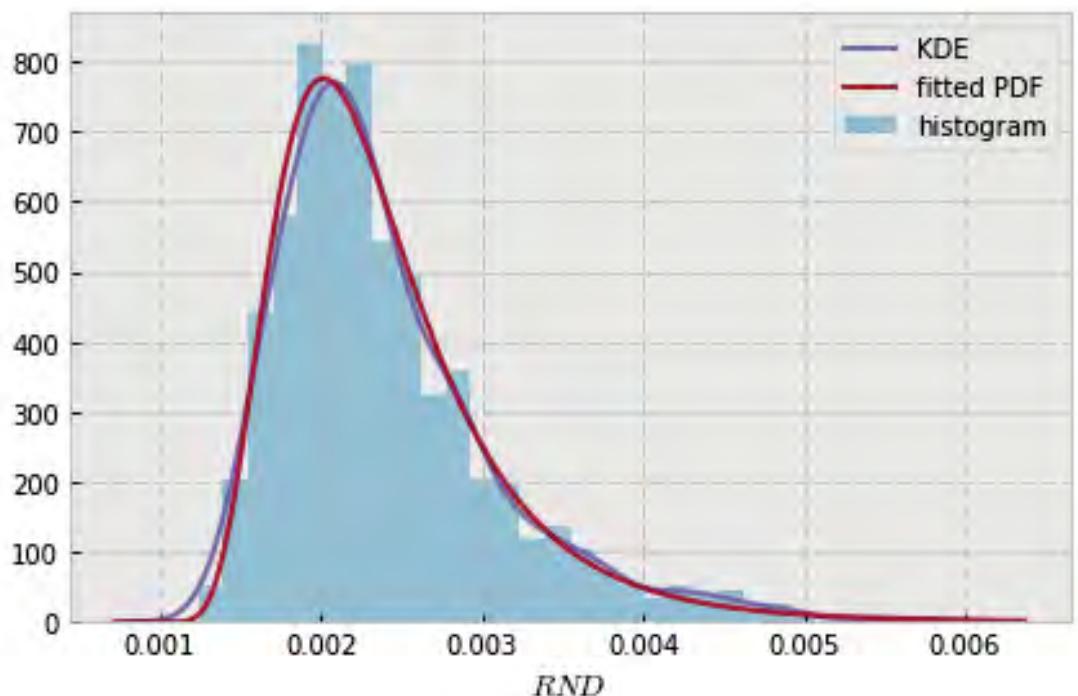


```
variable name: ln
variable value: 2.0
distribution: inverse-gamma(shape=(10.64234612532821,), loc=9.034227007348637e-05,
    scale=0.005936701809966423)
DescribeResult(nobs=1000, minmax=(0.00034508684060869732, 0.0021669516222238266)
    mean=0.00070605342863764686, variance=4.4230207094265221e-08,
    skewness=1.6243558127645155, kurtosis=5.386548184033272)
```

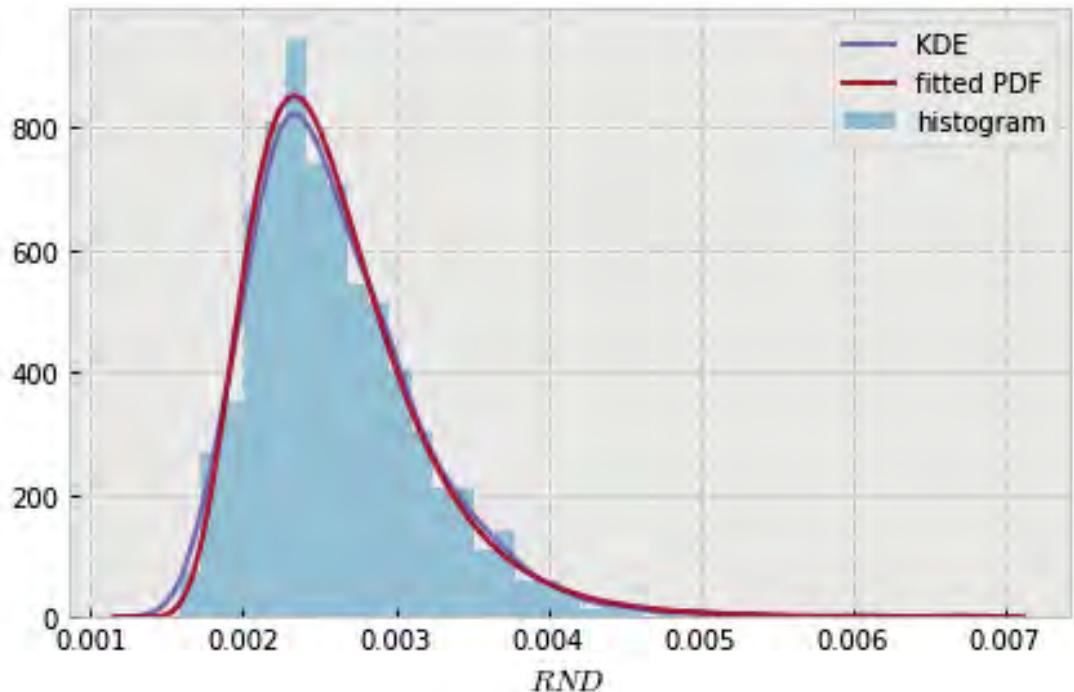


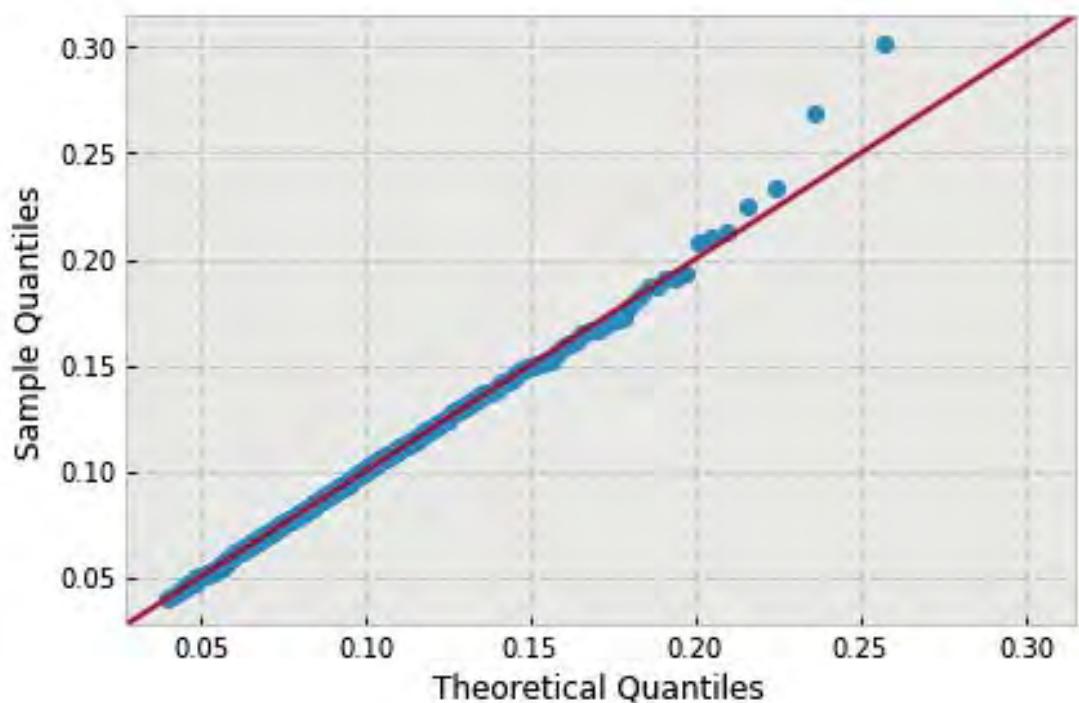


```
variable name: nB
variable value: 4.0
distribution: inverse-gamma(shape=(8.0680377593288775,), loc=0.0006525093027268251,
                           scale=0.012369468665566919)
DescribeResult(nobs=1000, minmax=(0.0012422137011074792, 0.0058441863016140275)
               mean=0.0024015159751704629, variance=4.8187199064089205e-07,
               skewness=1.4356865924776931, kurtosis=2.6514091875881833)
```

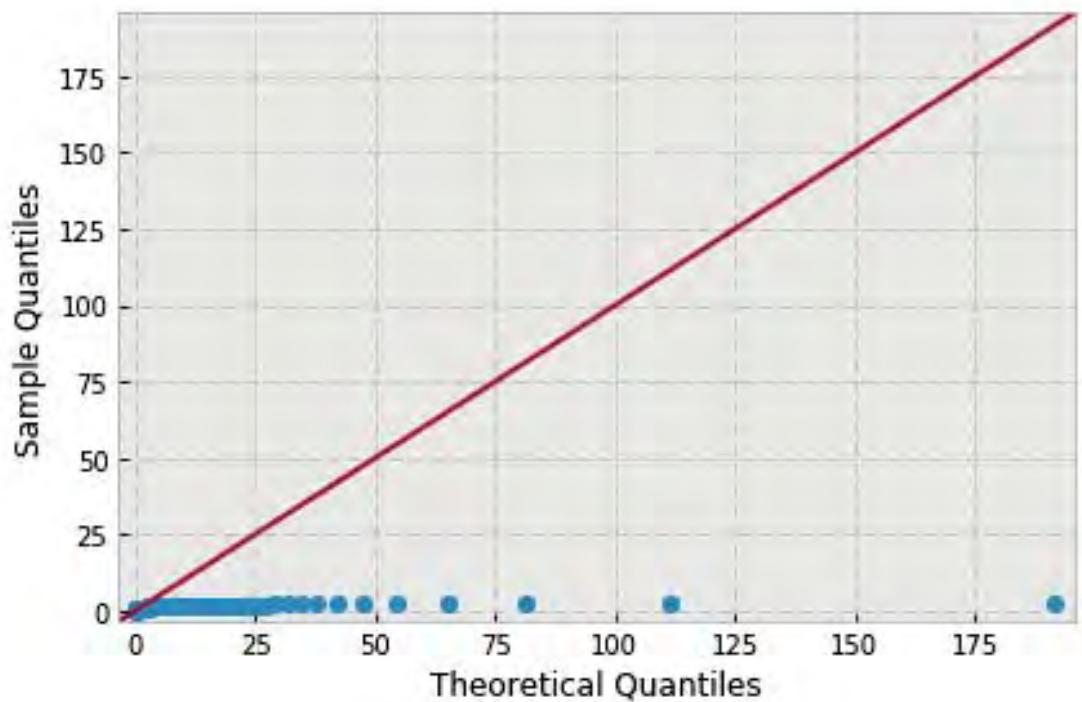
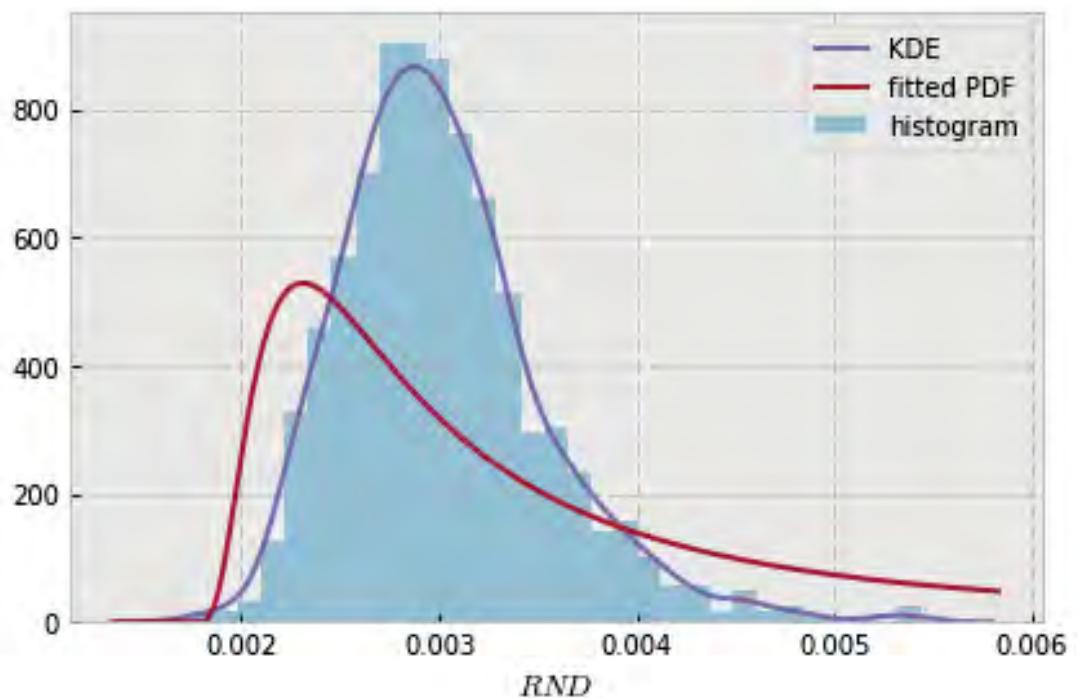


```
variable name: nB
variable value: 8.0
distribution: inverse-gamma(shape=(11.719993724603537,), loc=0.000804064215589948,
scale=0.019474885650699433)
DescribeResult(nobs=1000, minmax=(0.0015903481584270416, 0.006677044478295896)
mean=0.0026207204931535232, variance=3.4046715421678768e-07,
skewness=1.4722877243809247, kurtosis=4.540812050335441)
```

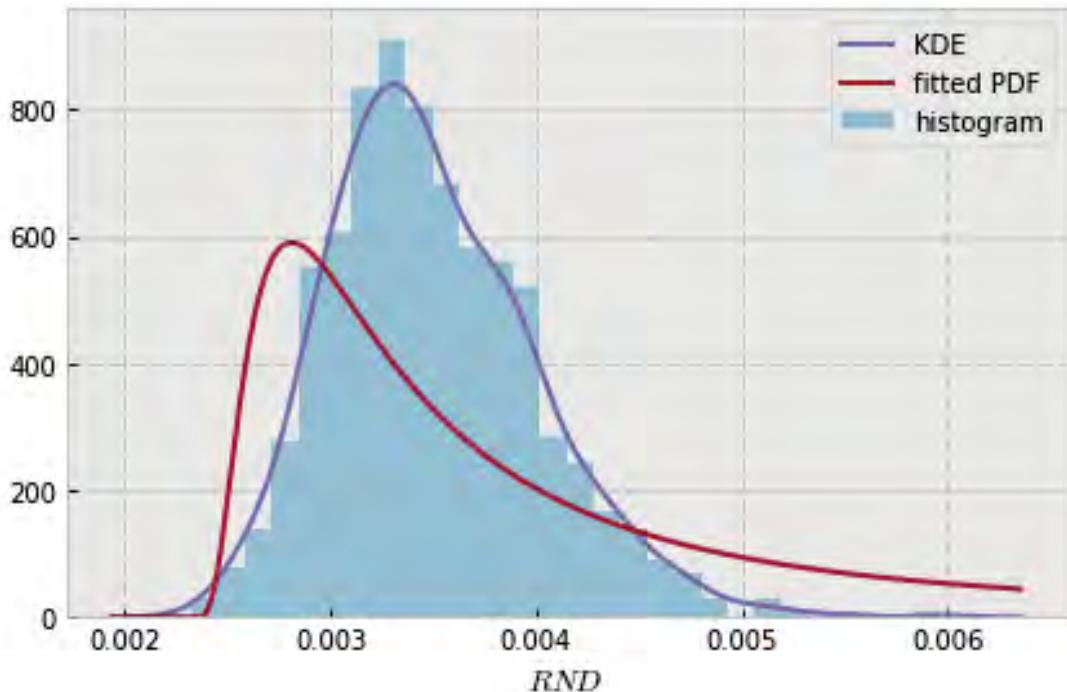


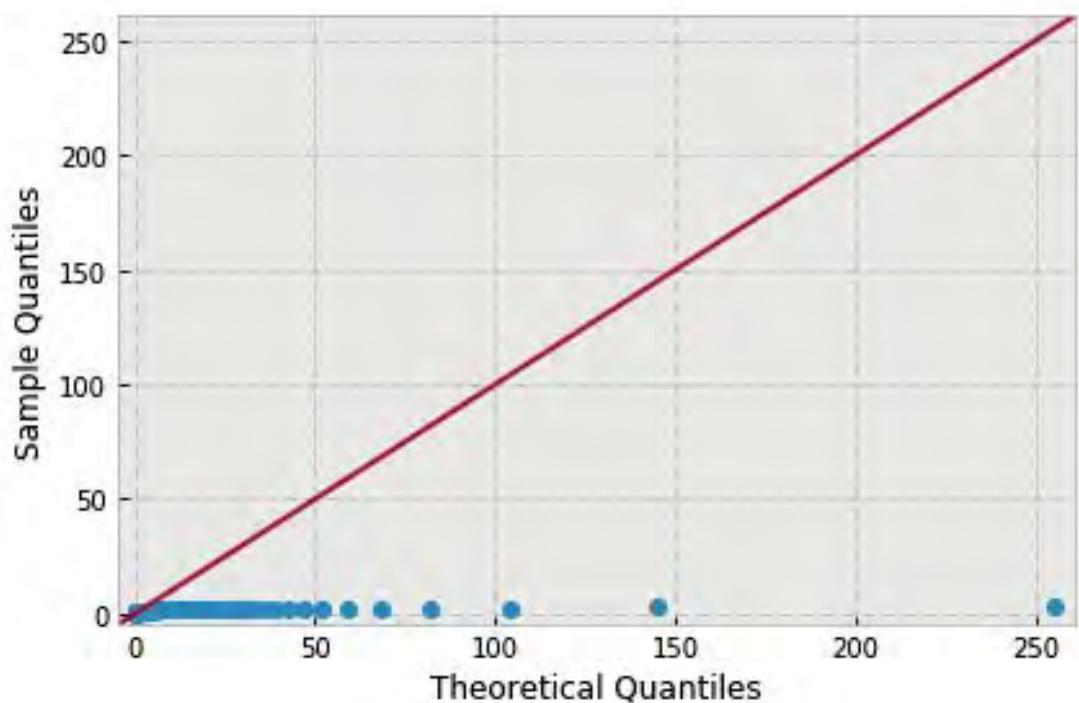


```
variable name: nB
variable value: 12.0
distribution: inverse-gamma(shape=(1.2859230018078964,), loc=0.0016924177451570351,
                           scale=0.0014151146787063336)
DescribeResult(nobs=1000, minmax=(0.0017398636850075092, 0.0054417863668430478)
               mean=0.0030160065724048825, variance=2.6487395463057792e-07,
               skewness=0.9715156047772535, kurtosis=1.826275161584931)
```

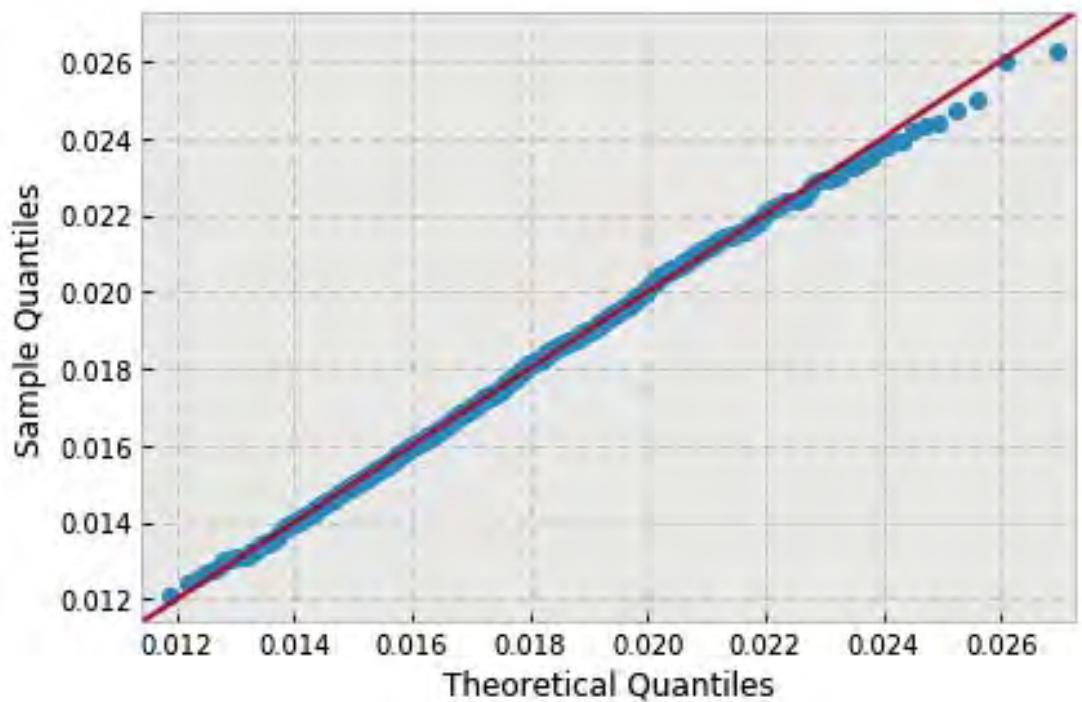
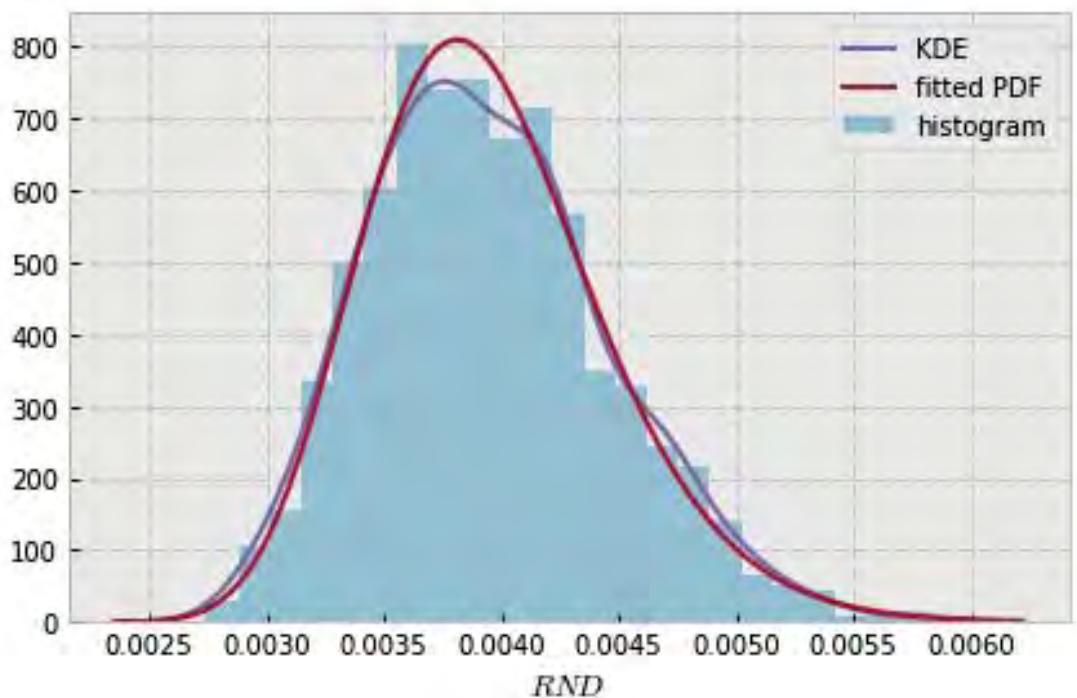


```
variable name: nB
variable value: 16.0
distribution: inverse-gamma(shape=(1.2257949664607755,), loc=0.0022769165196521407,
                           scale=0.001189916734824022)
DescribeResult(nobs=1000, minmax=(0.0023220250300683334, 0.0059717612275542557)
               mean=0.0034974190553540463, variance=2.5337359894042704e-07,
               skewness=0.6843780390901987, kurtosis=0.7943908199035423)
```

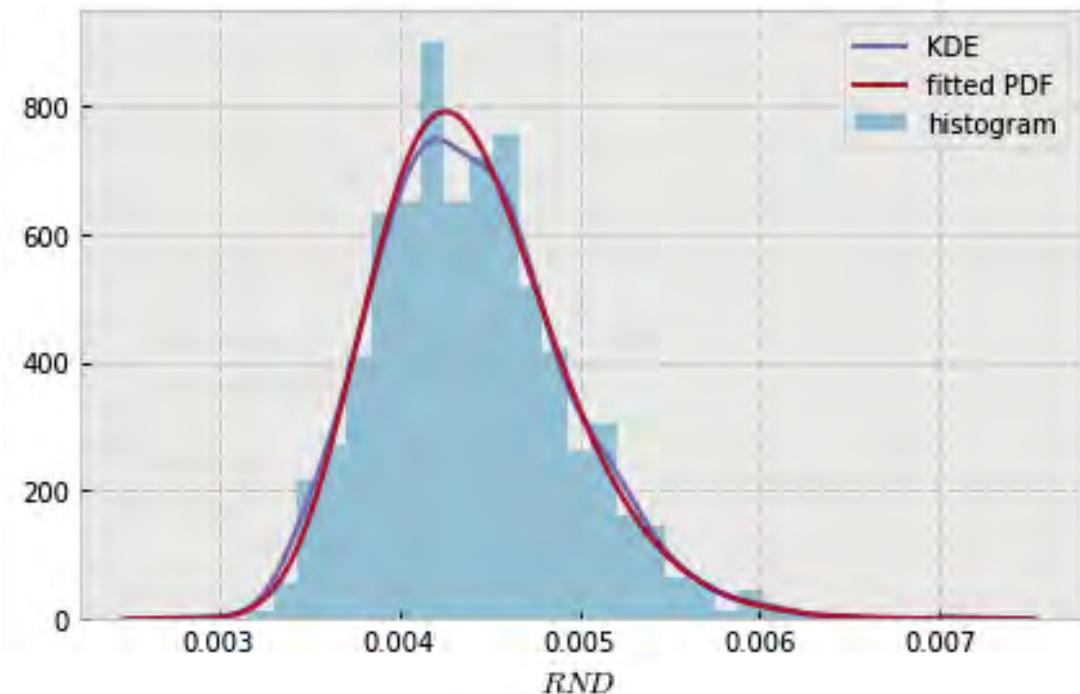


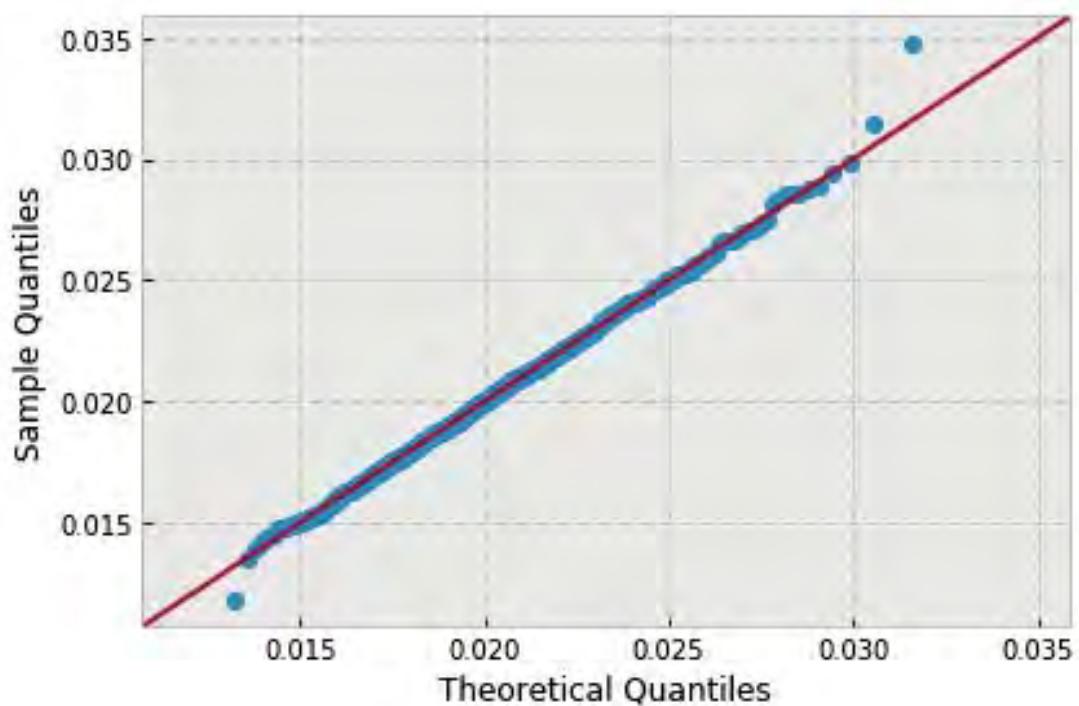


```
variable name: nB
variable value: 20.0
distribution: inverse-gamma(shape=(57.796215371583102,), loc=0.00010503080289526307,
                           scale=0.2179970947027995)
DescribeResult(nobs=1000, minmax=(0.0027457529382975453, 0.005825568369789382)
               mean=0.0039431782604635252, variance=2.6223384416891871e-07,
               skewness=0.44992481882679264, kurtosis=0.03687918177954641)
```

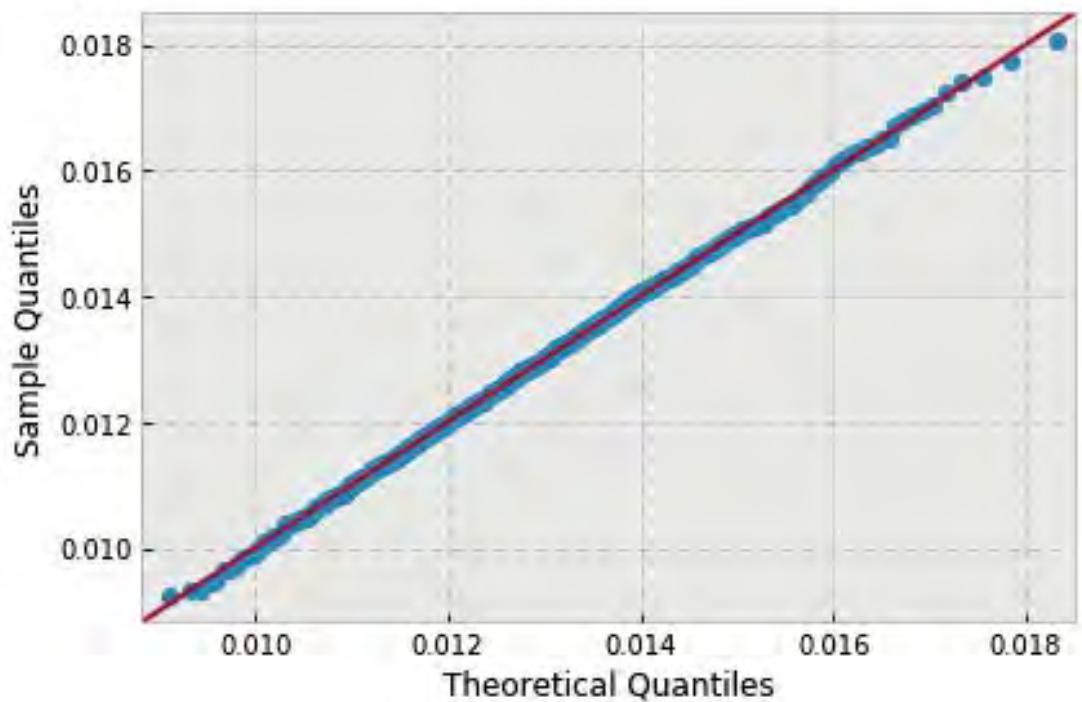
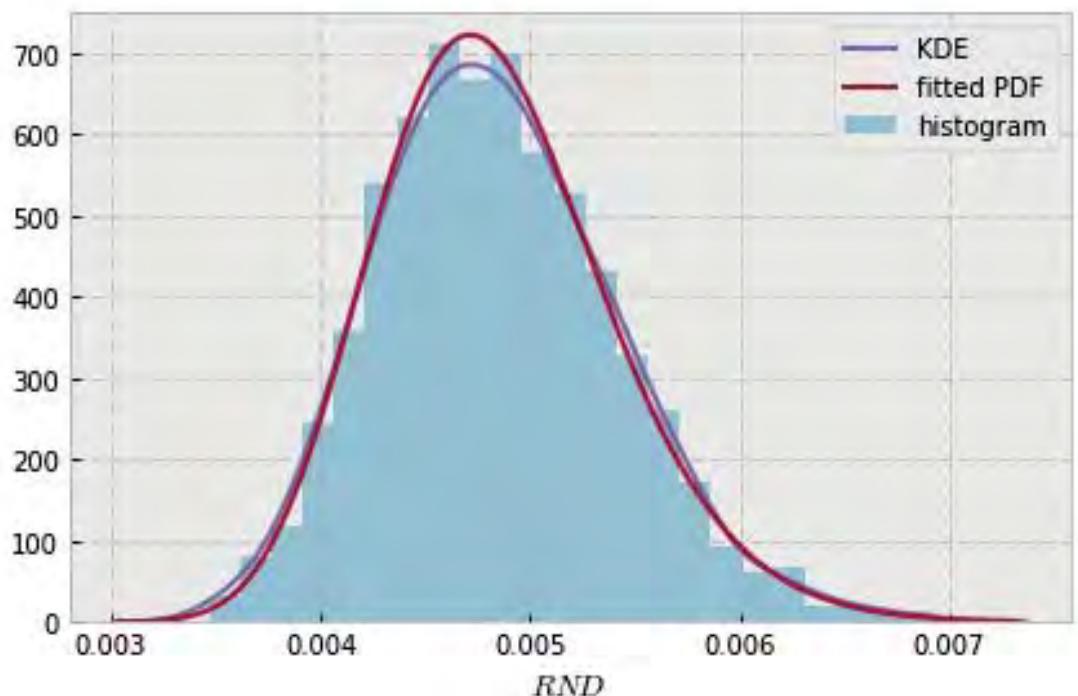


```
variable name: nB
variable value: 24.0
distribution: inverse-gamma(shape=(50.890331848430115,), loc=0.0006988354743477779,
                           scale=0.18461578907087908)
DescribeResult(nobs=1000, minmax=(0.0028866815160225805, 0.0071131510838407059)
               mean=0.0043992852832388622, variance=2.8078836259060537e-07,
               skewness=0.6038646689792617, kurtosis=0.8073825115272273)
```

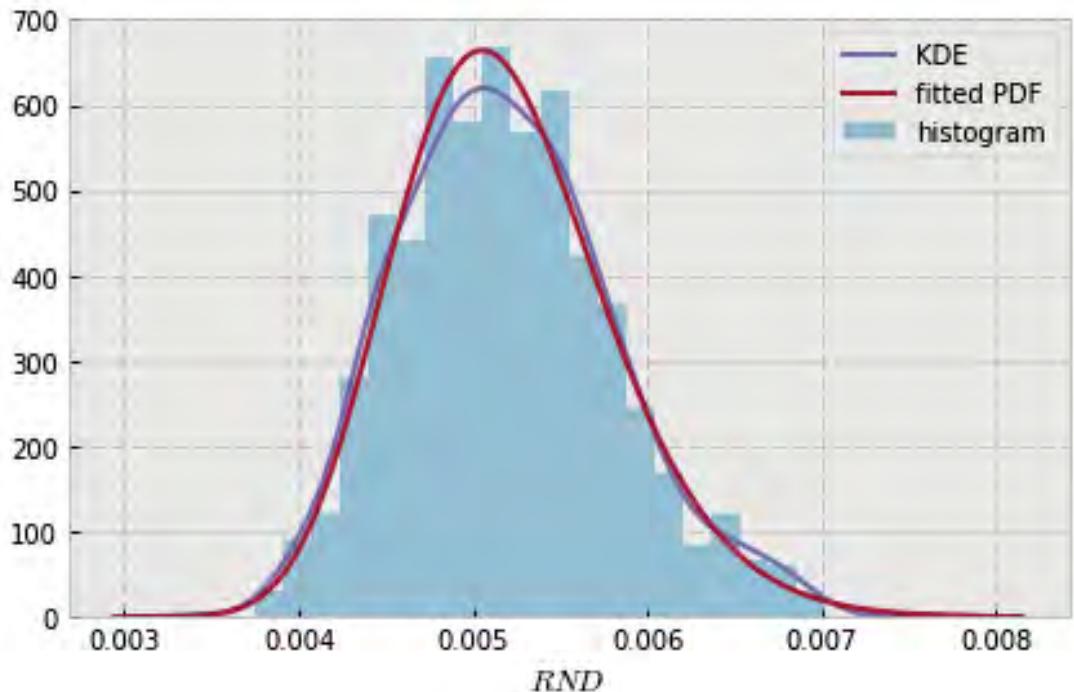


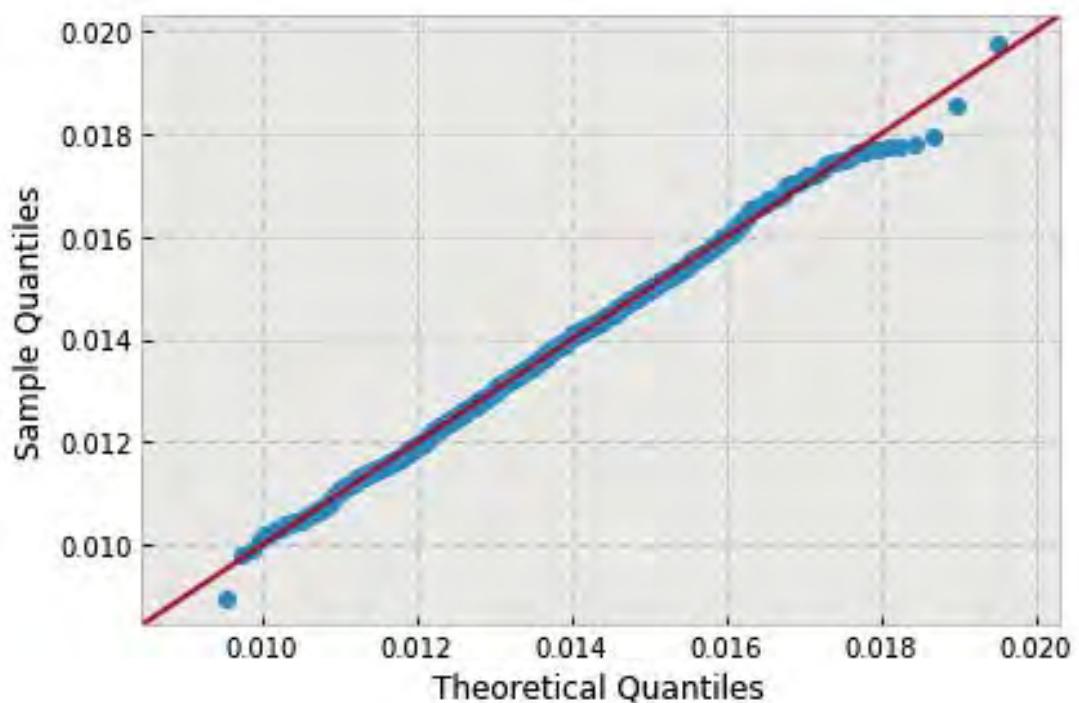


```
variable name: nB
variable value: 28.0
distribution: inverse-gamma(shape=(79.31449891635512,), loc=-0.00016586391732324008,
                           scale=0.3919306364290197)
DescribeResult(nobs=1000, minmax=(0.0034638257264915065, 0.0069116527290337118)
               mean=0.0048408908224029813, variance=3.2155837527474251e-07,
               skewness=0.3997857409663429, kurtosis=0.15855803835067306)
```

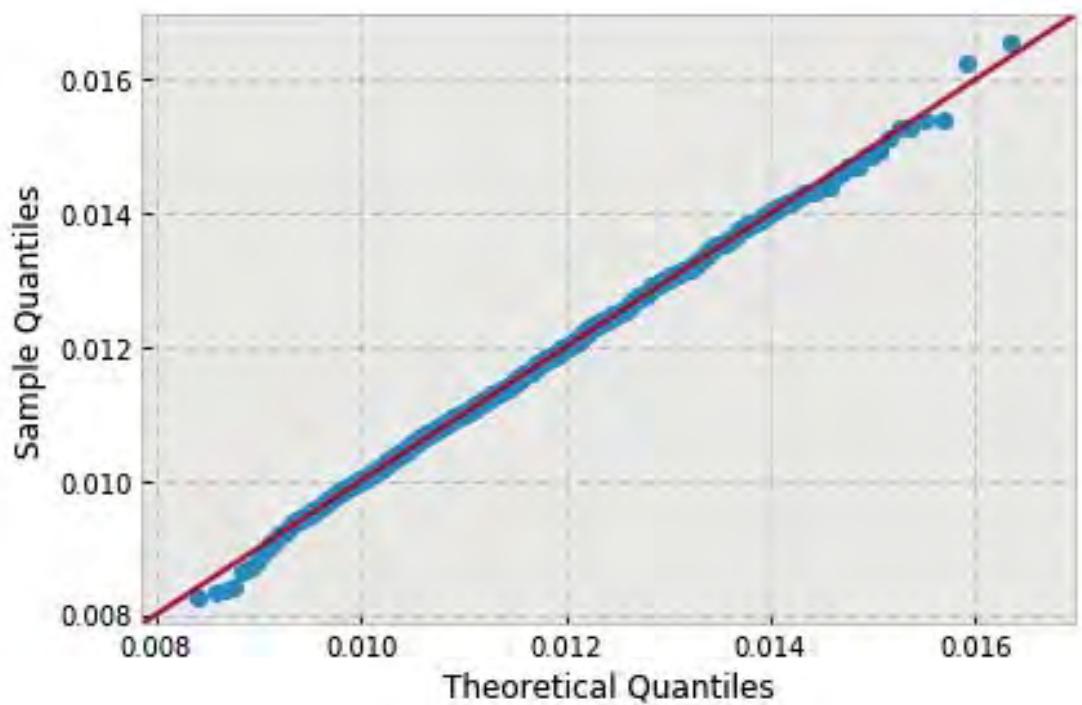
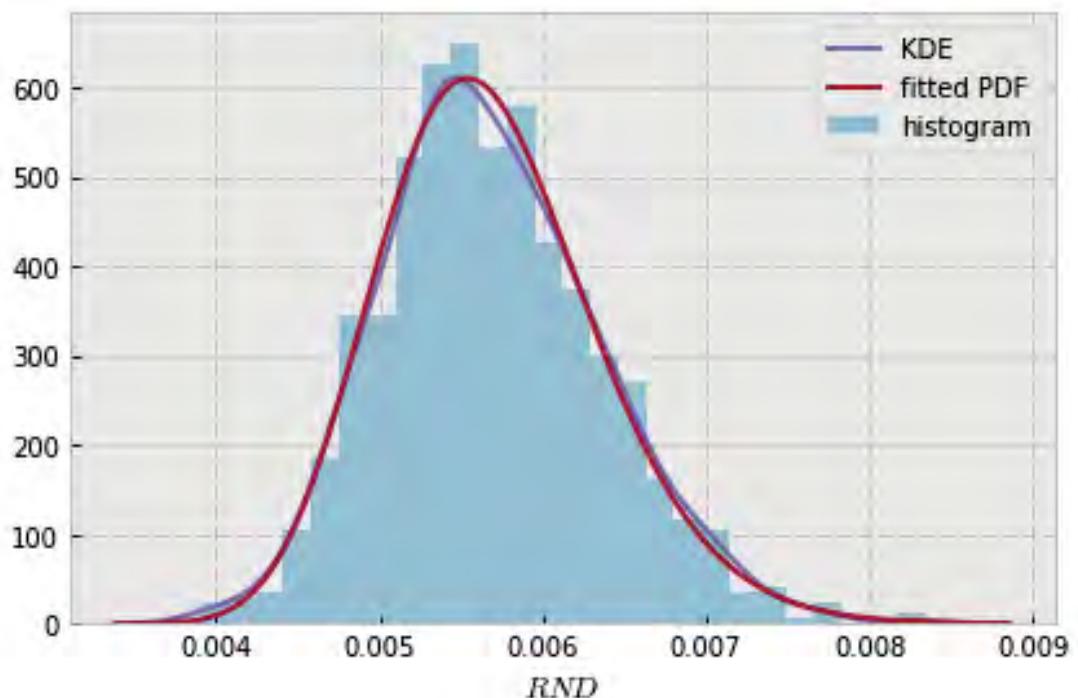


```
variable name: nB
variable value: 32.0
distribution: inverse-gamma(shape=(75.27825192519677,), loc=-0.00012212589349136927,
    scale=0.39465374210862325)
DescribeResult(nobs=1000, minmax=(0.0034216385066908328, 0.0076719208598780217)
    mean=0.0051907213583006241, variance=3.8367247518687185e-07,
    skewness=0.4215067182976278, kurtosis=0.12788993739953147)
```

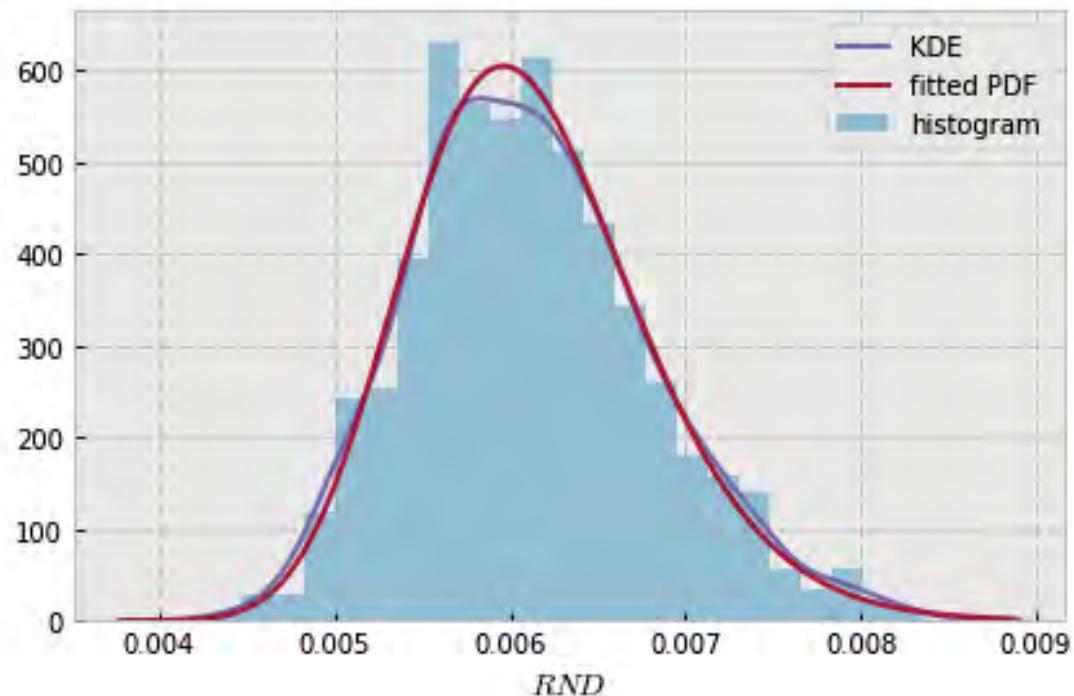


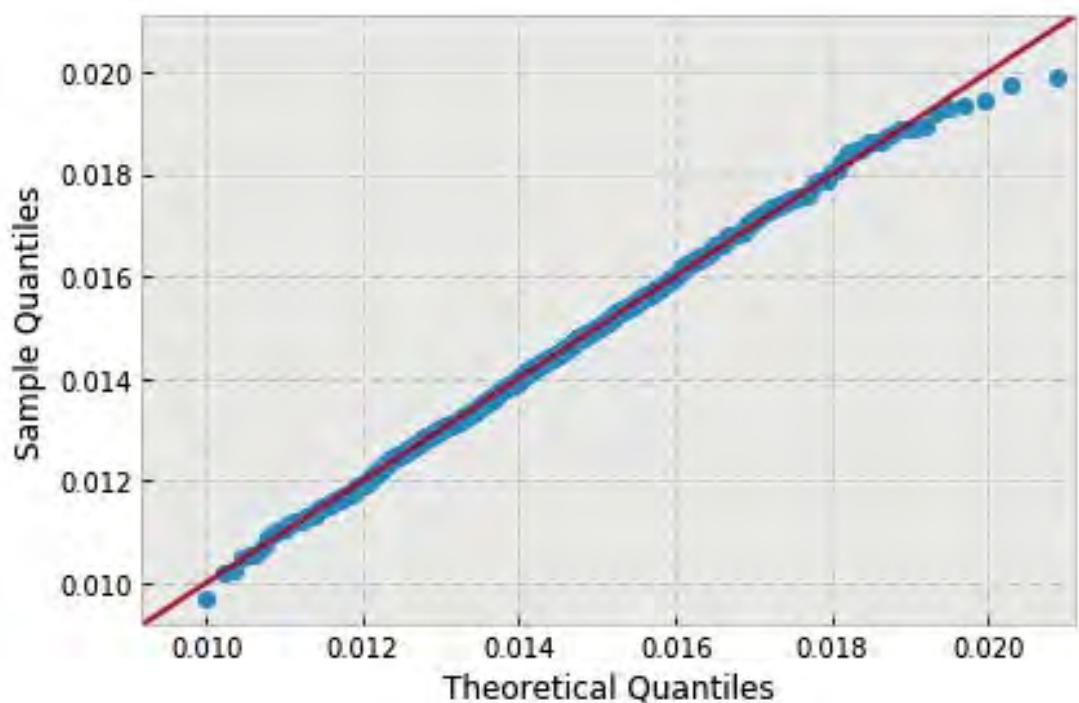


```
variable name: nB
variable value: 36.0
distribution: inverse-gamma(shape=(87.265150547346281,), loc=-0.0005308410275796911,
                           scale=0.5359027985886033)
DescribeResult(nobs=1000, minmax=(0.0039000369540309466, 0.0083393340928508281)
               mean=0.0056856833219737111, variance=4.4666940428670374e-07,
               skewness=0.3790783749300781, kurtosis=0.236107664686096)
```

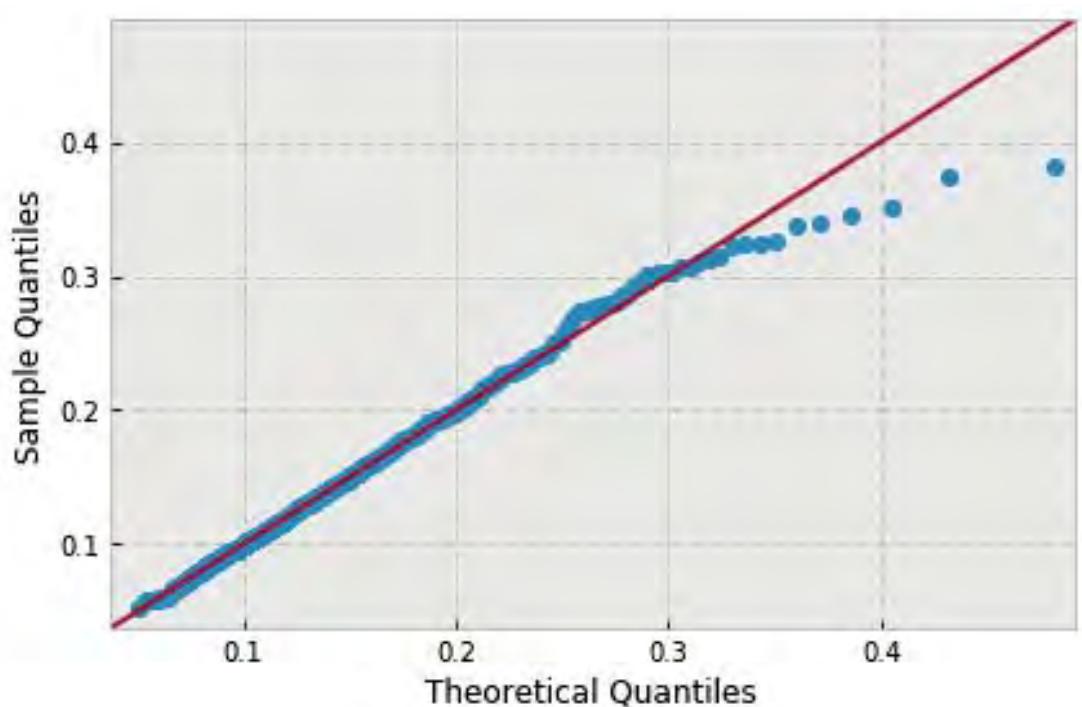
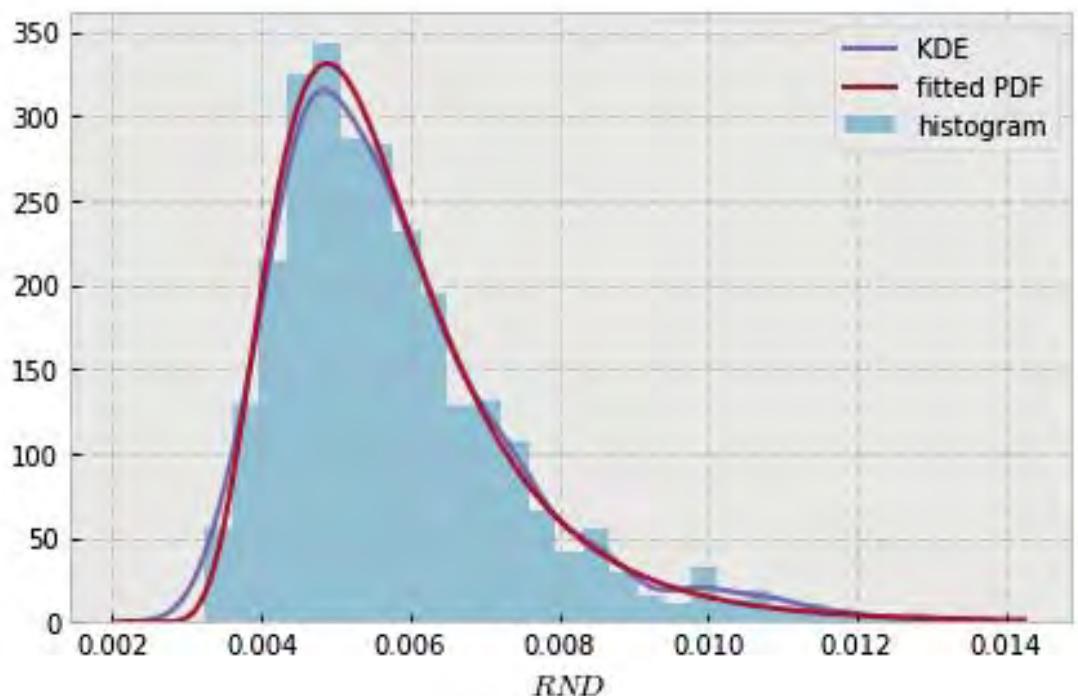


```
variable name: nB
variable value: 40.0
distribution: inverse-gamma(shape=(71.156385983807411,), loc=0.0004433205310128627,
                           scale=0.3981763445863866)
DescribeResult(nobs=1000, minmax=(0.0043010952661228083, 0.0083700615215994197)
               mean=0.0061188149876124425, variance=4.6356930234884657e-07,
               skewness=0.425117281075665, kurtosis=0.05639620510888799)
```

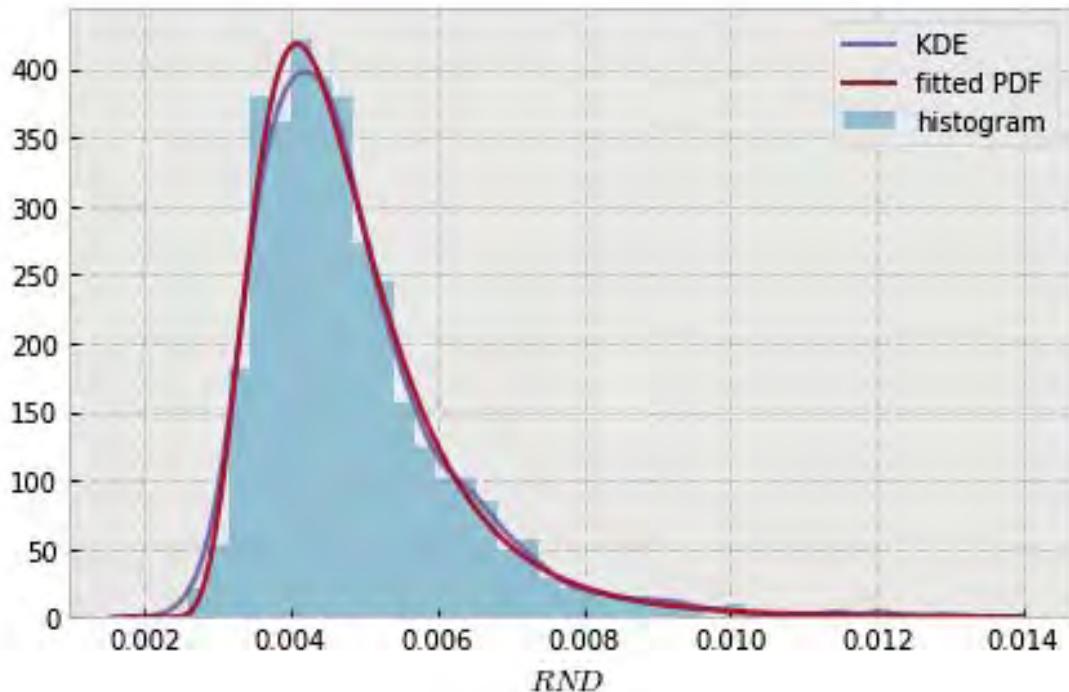


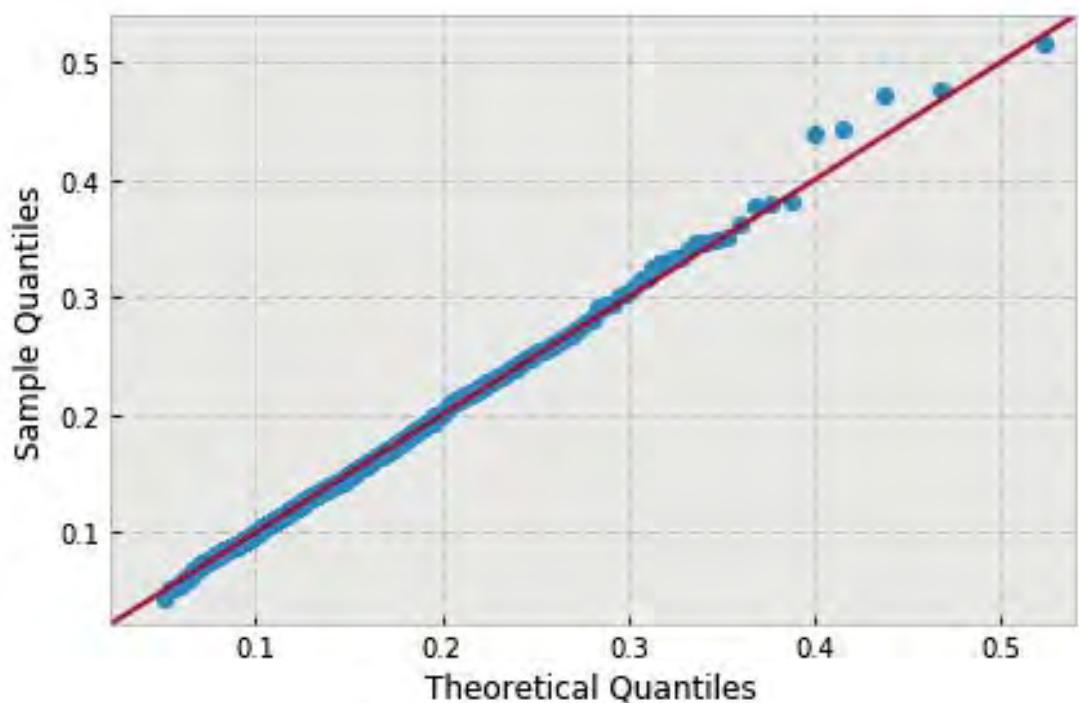


```
variable name: r
variable value: 1.0
distribution: inverse-gamma(shape=(8.2194931300272351,), loc=0.001650856878440292,
scale=0.029801015219340858)
DescribeResult(nobs=1000, minmax=(0.0032352343628683764, 0.013051754908406684)
mean=0.0057741820232263662, variance=2.5592932869891839e-06,
skewness=1.3271777986379898, kurtosis=2.1509965496938452)
```

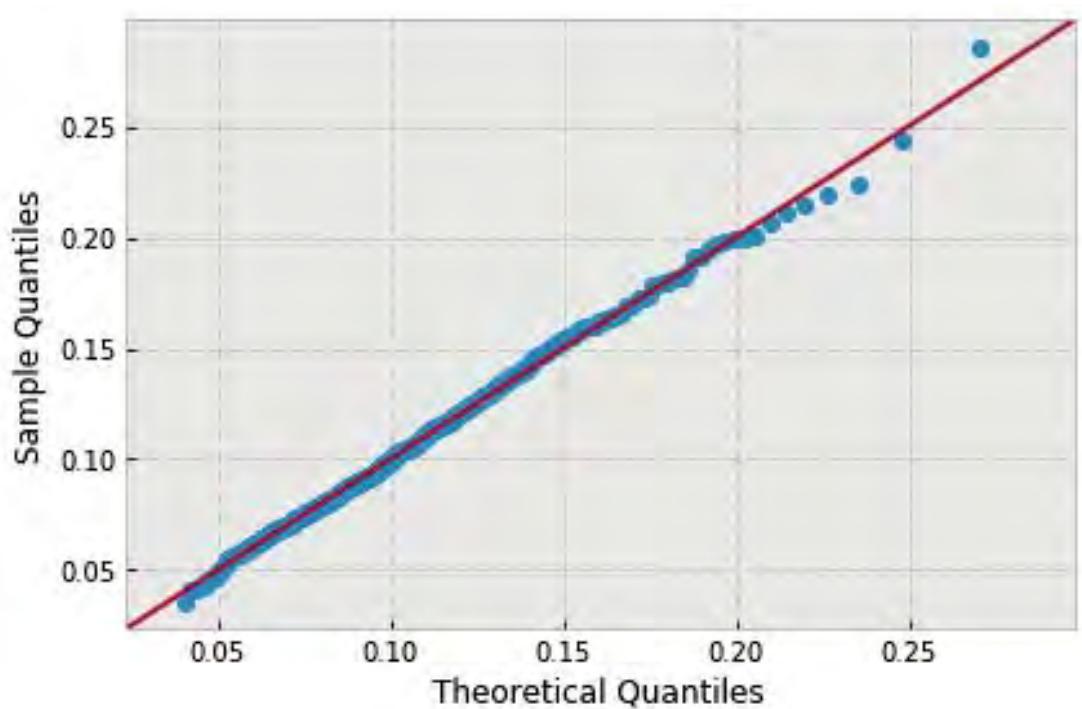
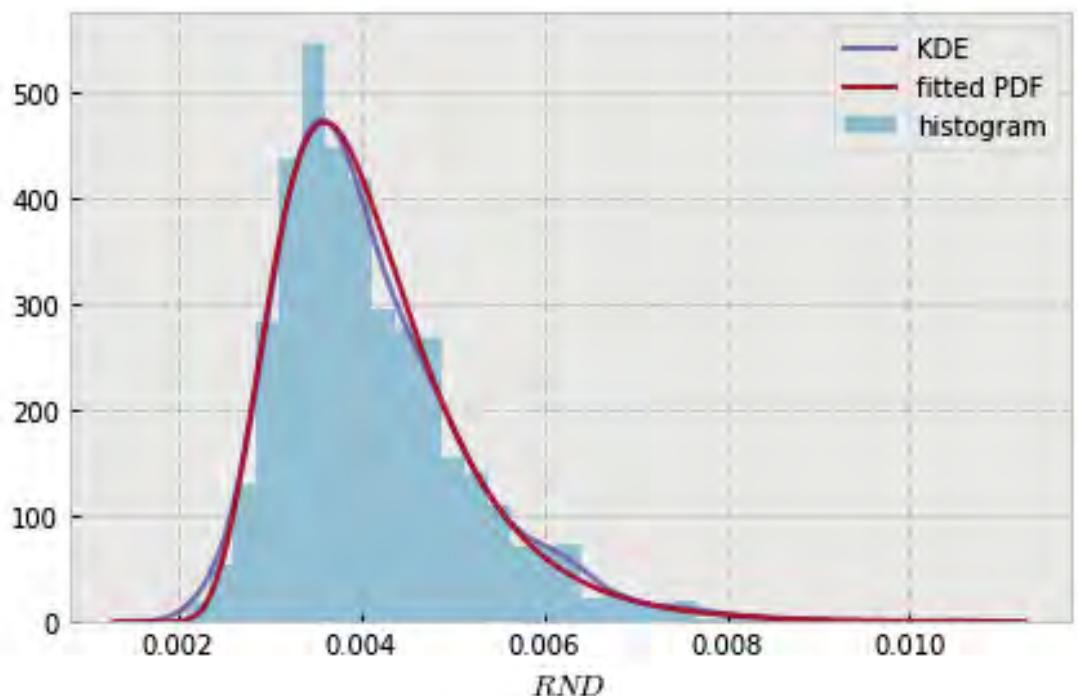


```
variable name: r
variable value: 1.2222222222222223
distribution: inverse-gamma(shape=(7.8703938643099836,), loc=0.0015868223783364684,
    scale=0.022137326101279164)
DescribeResult(nobs=1000, minmax=(0.002581876043244035, 0.013000971944696302)
    mean=0.0048095789917068973, variance=1.7620195825527052e-06,
    skewness=1.75597429123646, kurtosis=5.016949253524199)
```

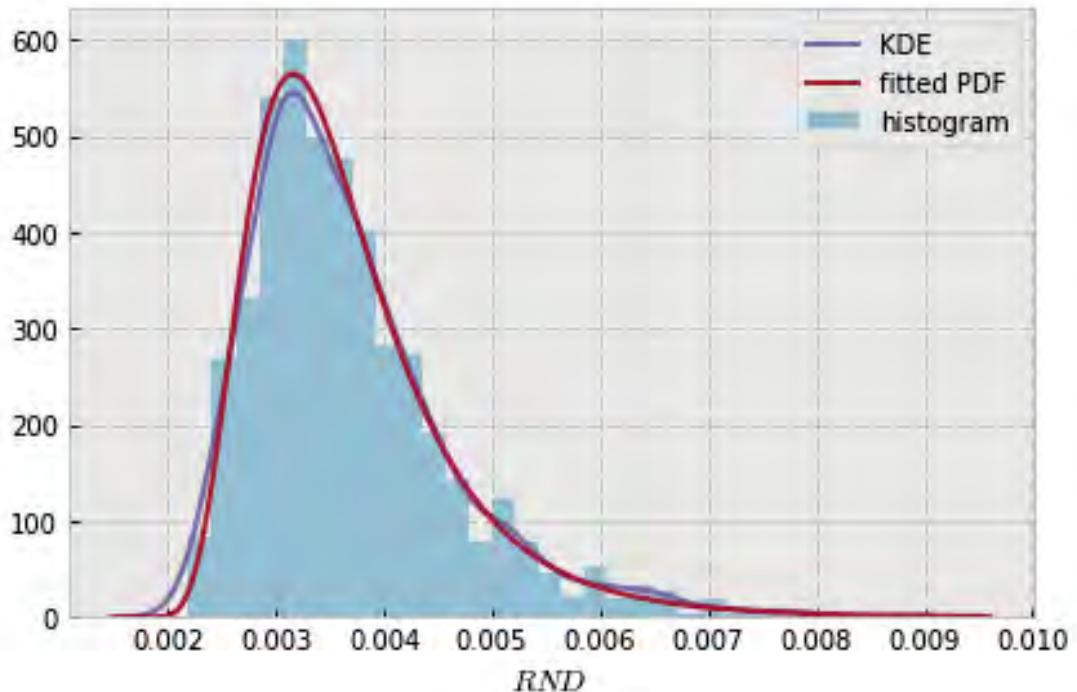


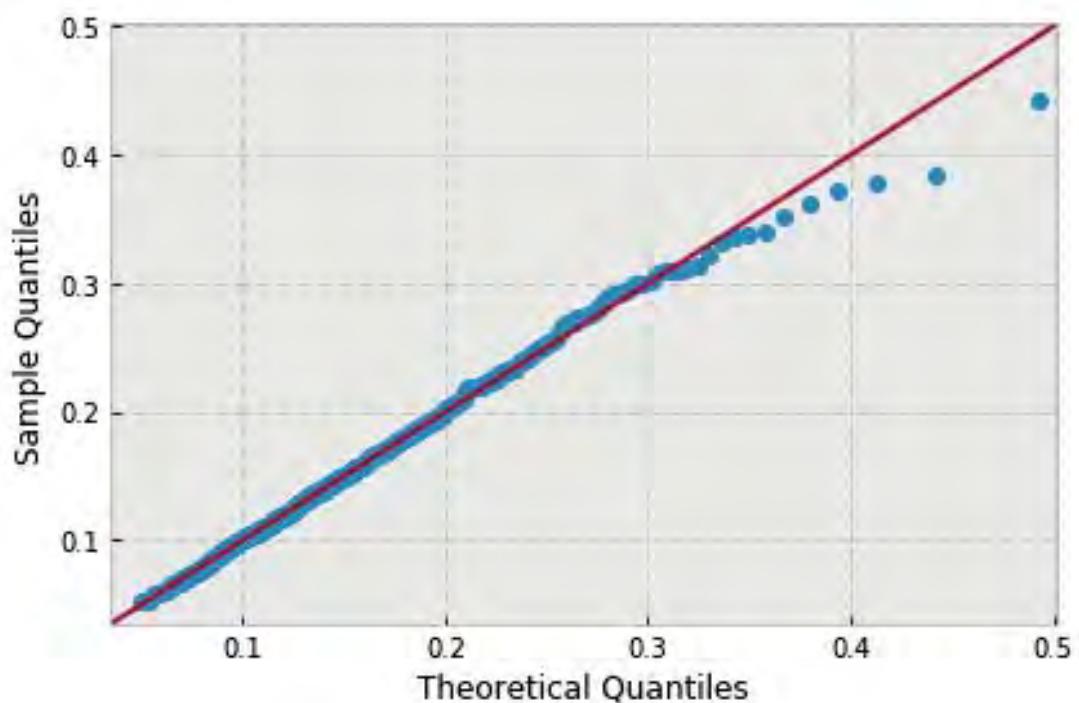


```
variable name: r
variable value: 1.444444444444444
distribution: inverse-gamma(shape=(11.374506397215765,), loc=0.0008911885312249372,
                           scale=0.03354474720225793)
DescribeResult(nobs=1000, minmax=(0.0020965926652662817, 0.010463548797291855)
               mean=0.0041245795081579877, variance=1.1064794444818589e-06,
               skewness=1.2962712673607448, kurtosis=2.6439559107667376)
```



```
variable name: r
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(8.122302596582081,), loc=0.001271714501771777,
    scale=0.017175192945557076)
DescribeResult(nobs=1000, minmax=(0.0021914936792302238, 0.0088860377634421683)
    mean=0.003680854535311555, variance=8.9598342361238596e-07,
    skewness=1.4227841502791032, kurtosis=2.8210368209107077)
```

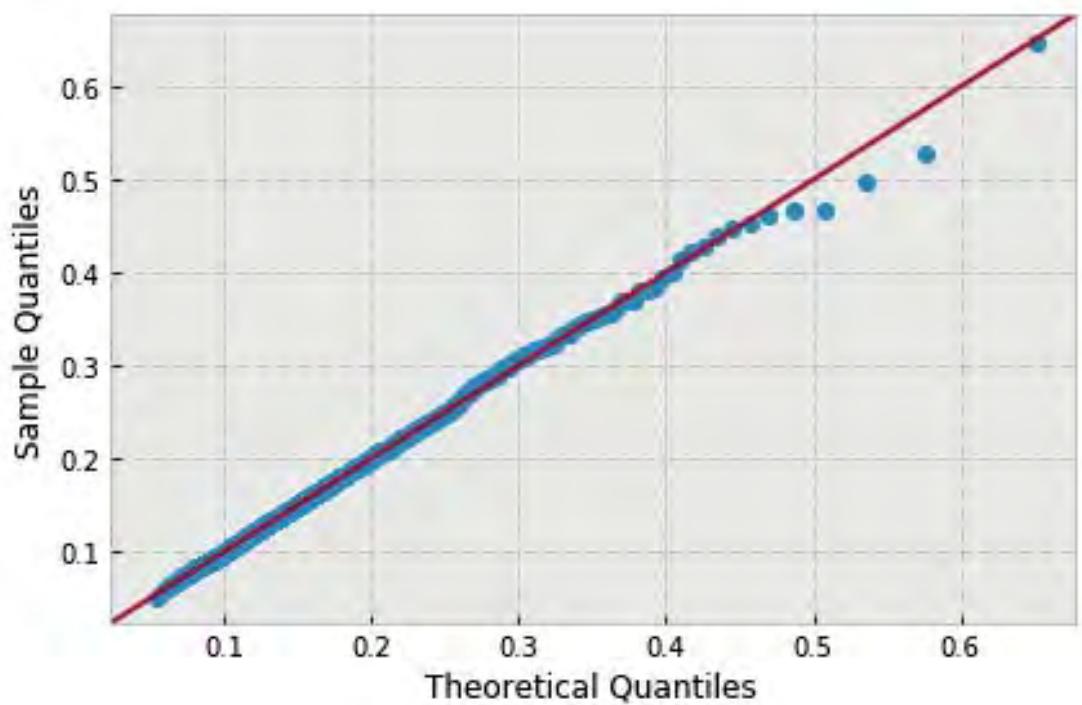
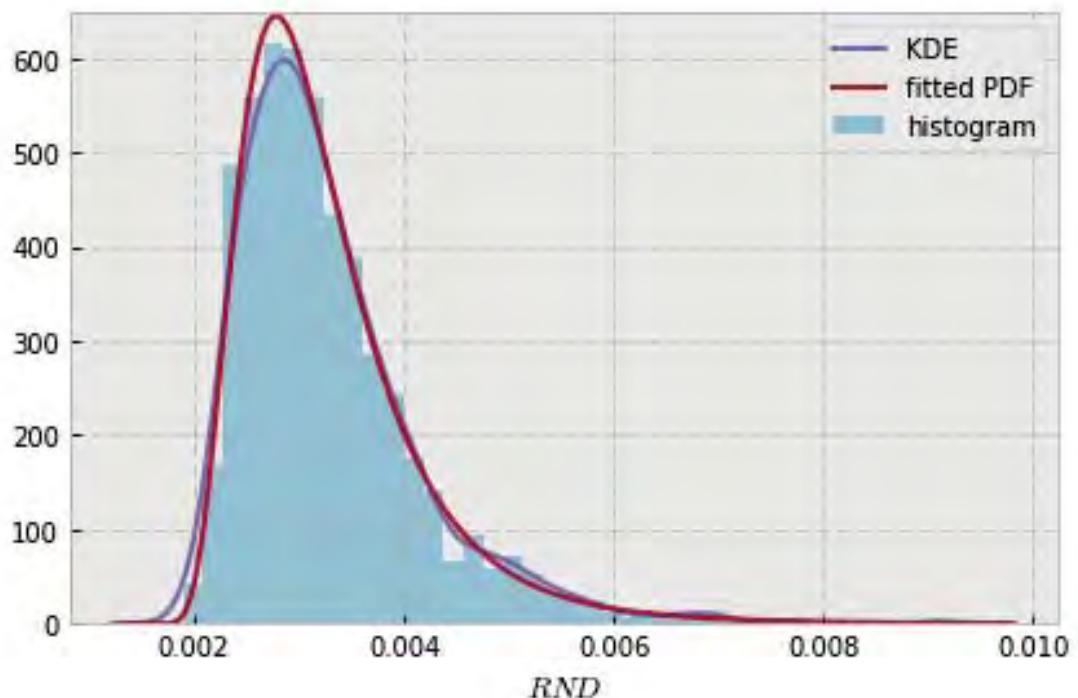




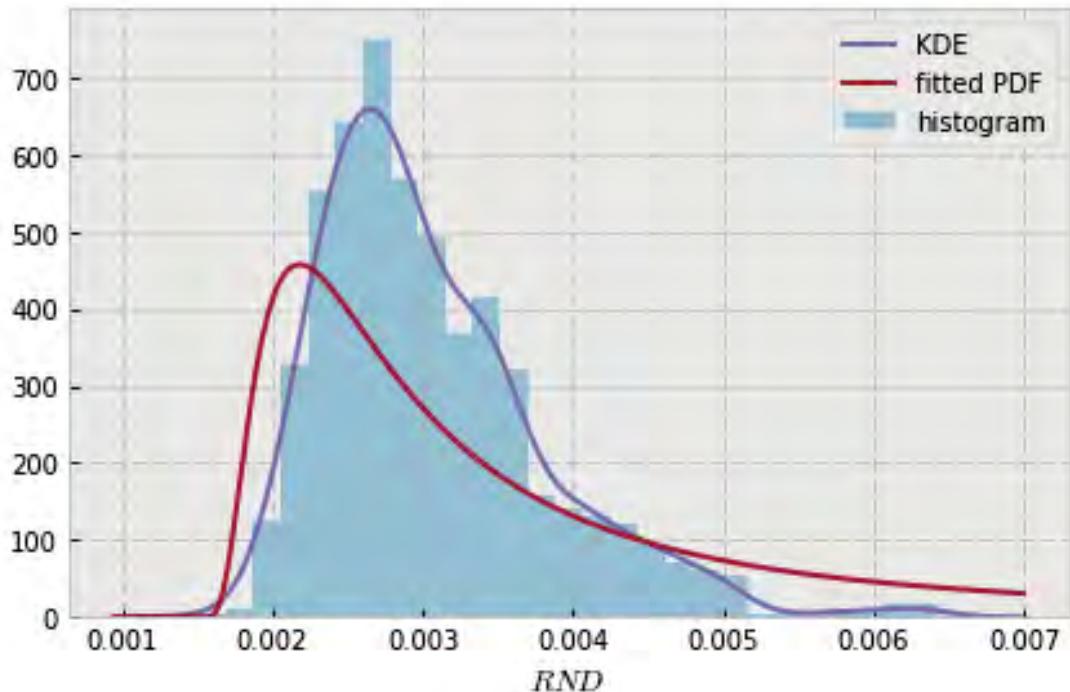
```

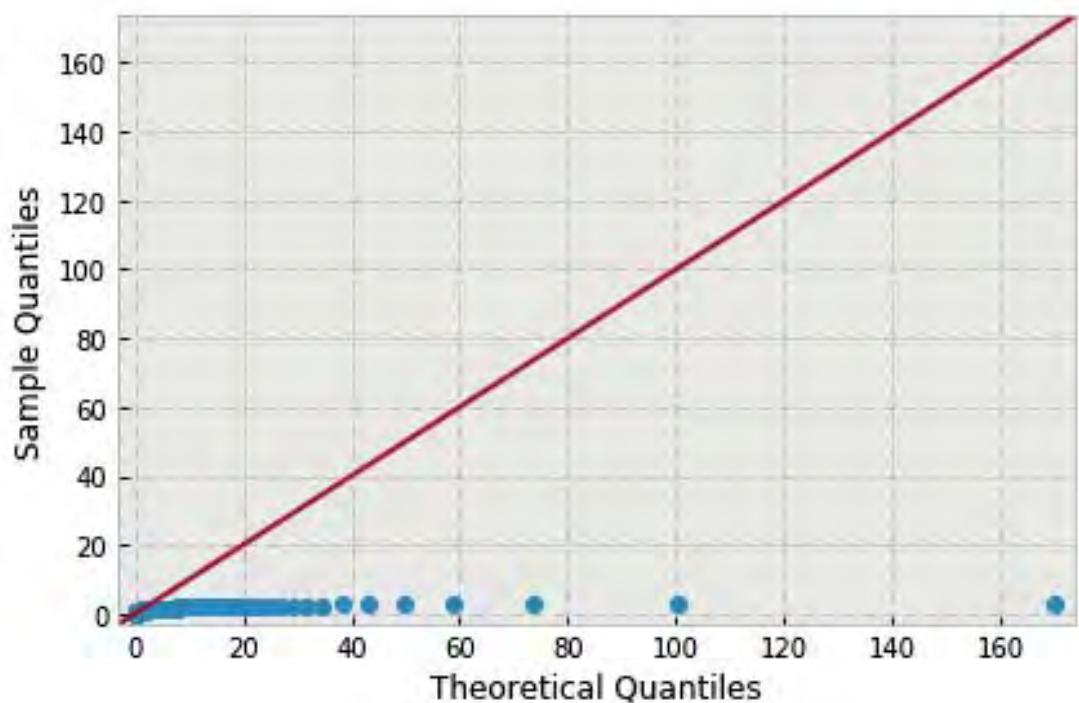
variable name: r
variable value: 1.888888888888888
distribution: inverse-gamma(shape=(7.0350544853897627,), loc=0.001260134768520139,
                           scale=0.012209546688660516)
DescribeResult(nobs=1000, minmax=(0.0018935231885065711, 0.0091621704877107024)
               mean=0.0032811473377947083, variance=7.6797103842598215e-07,
               skewness=1.6891047873353375, kurtosis=4.5729693904967395)

```

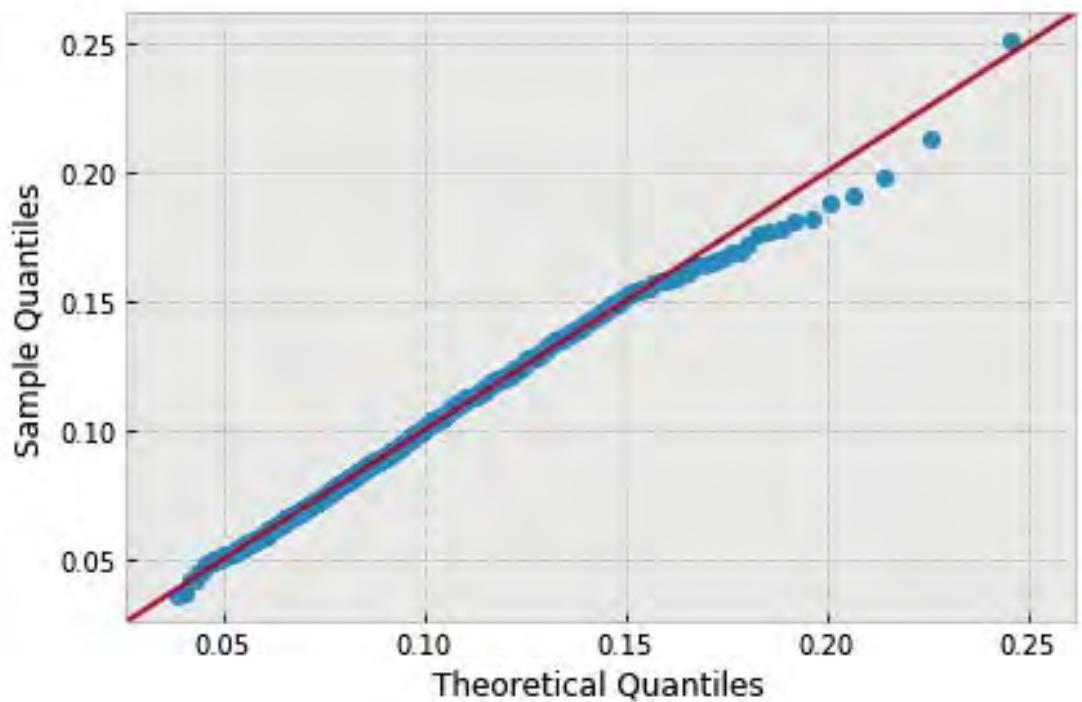
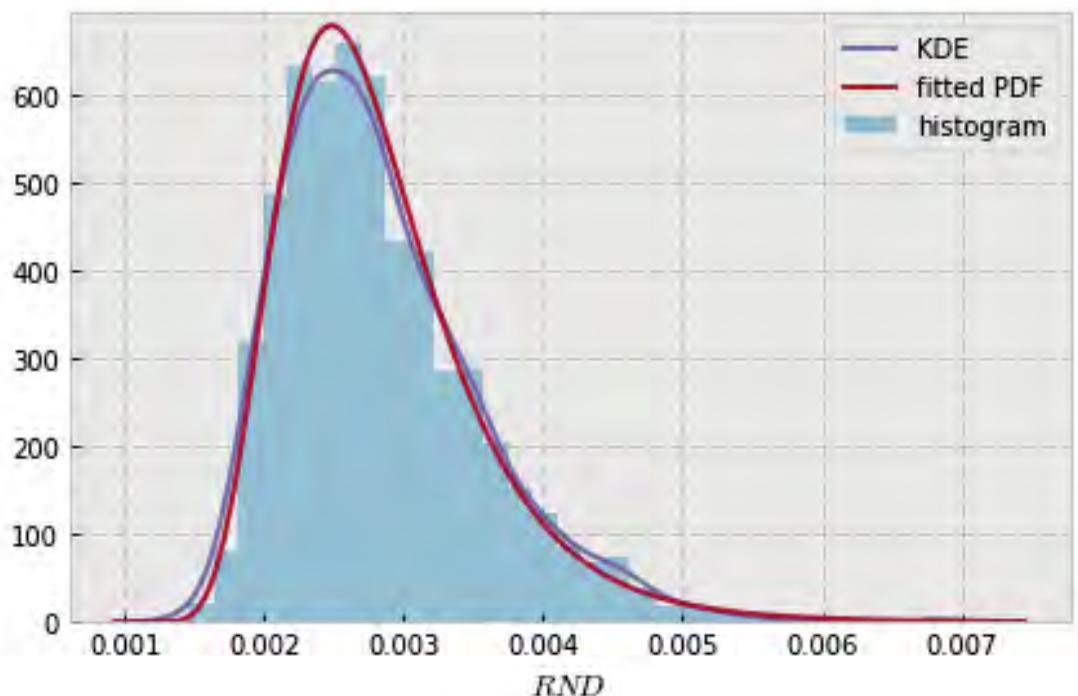


```
variable name: r
variable value: 2.11111111111111
distribution: inverse-gamma(shape=(1.3129419166321838,), loc=0.0014535968434145397,
                           scale=0.001680227340676252)
DescribeResult(nobs=1000, minmax=(0.0015075611083633278, 0.0064314030240019016)
               mean=0.0030582350931018069, variance=5.6520180306817787e-07,
               skewness=1.2146953119658233, kurtosis=1.9238148820582097)
```

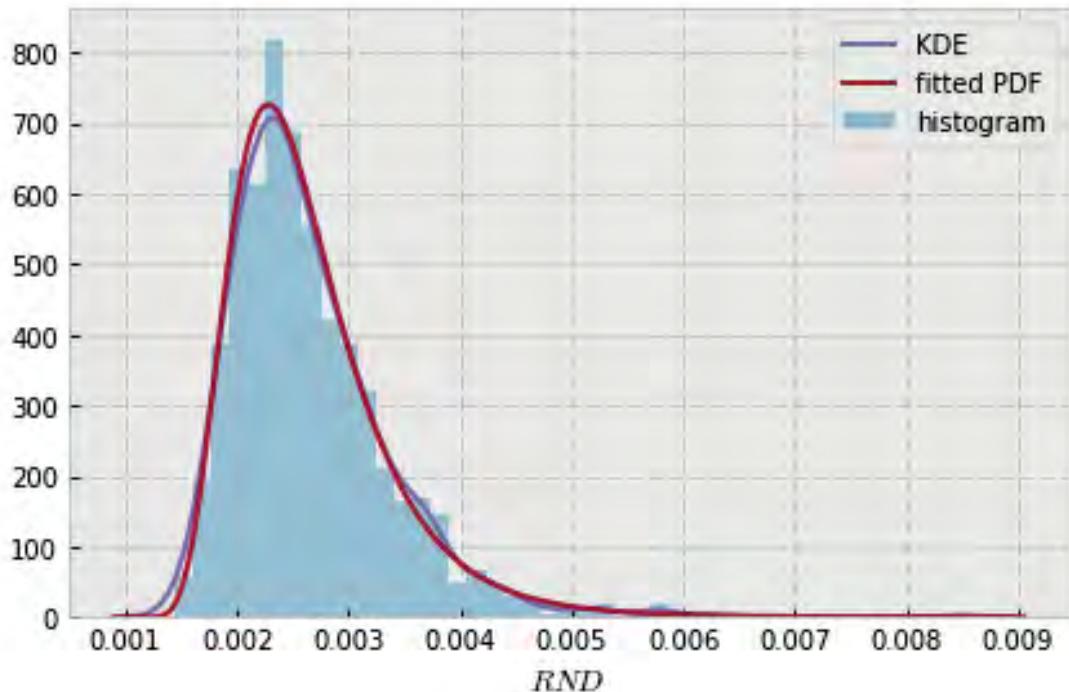


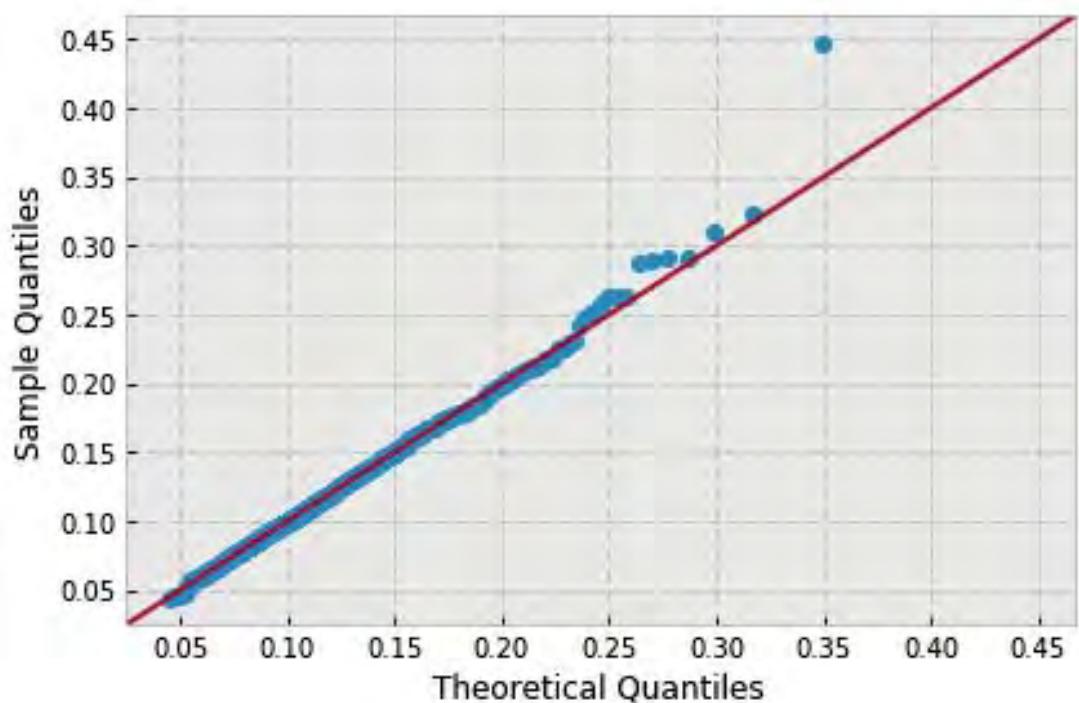


```
variable name: r
variable value: 2.333333333333333
distribution: inverse-gamma(shape=(12.058980290250858,), loc=0.0005313381013989833,
                           scale=0.025469672210419708)
DescribeResult(nobs=1000, minmax=(0.0014608917387780565, 0.006915191131676139)
               mean=0.0028332152674186302, variance=5.0593607458837639e-07,
               skewness=1.0876827445523898, kurtosis=1.773156194012329)
```



```
variable name: r
variable value: 2.5555555555555554
distribution: inverse-gamma(shape=(9.8109105602453575,), loc=0.0006533735958187208,
                           scale=0.01757681514244481)
DescribeResult(nobs=1000, minmax=(0.0014332872766533002, 0.0085060770813666781)
               mean=0.0026487938386748388, variance=5.1925167859330232e-07,
               skewness=1.772470651611544, kurtosis=6.682015102507471)
```

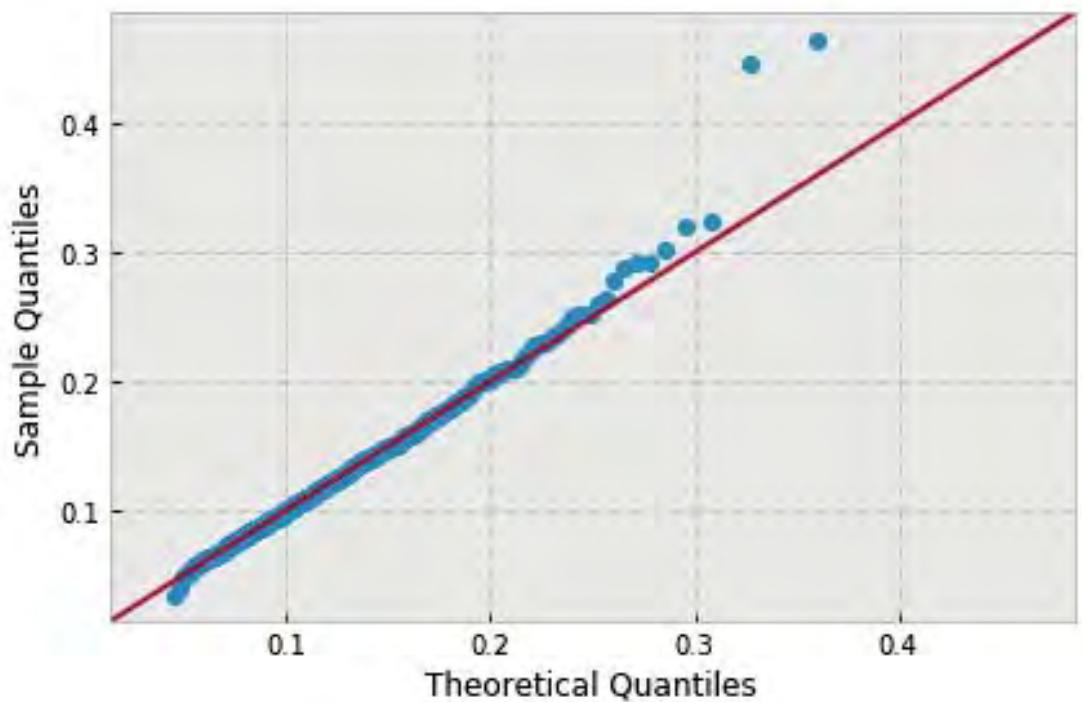
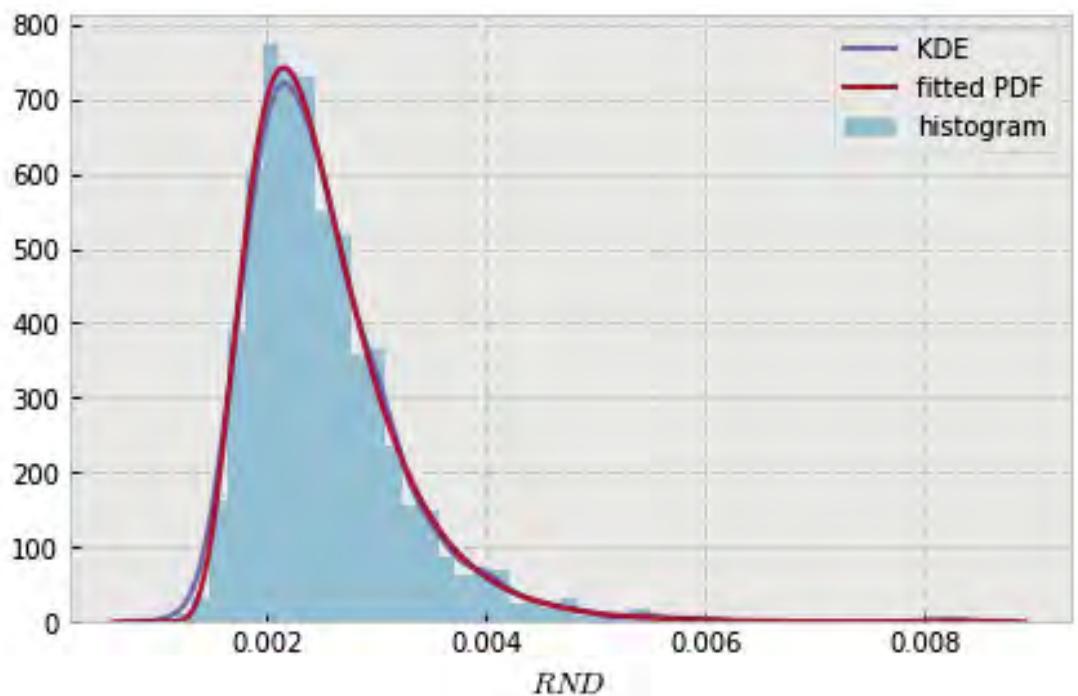




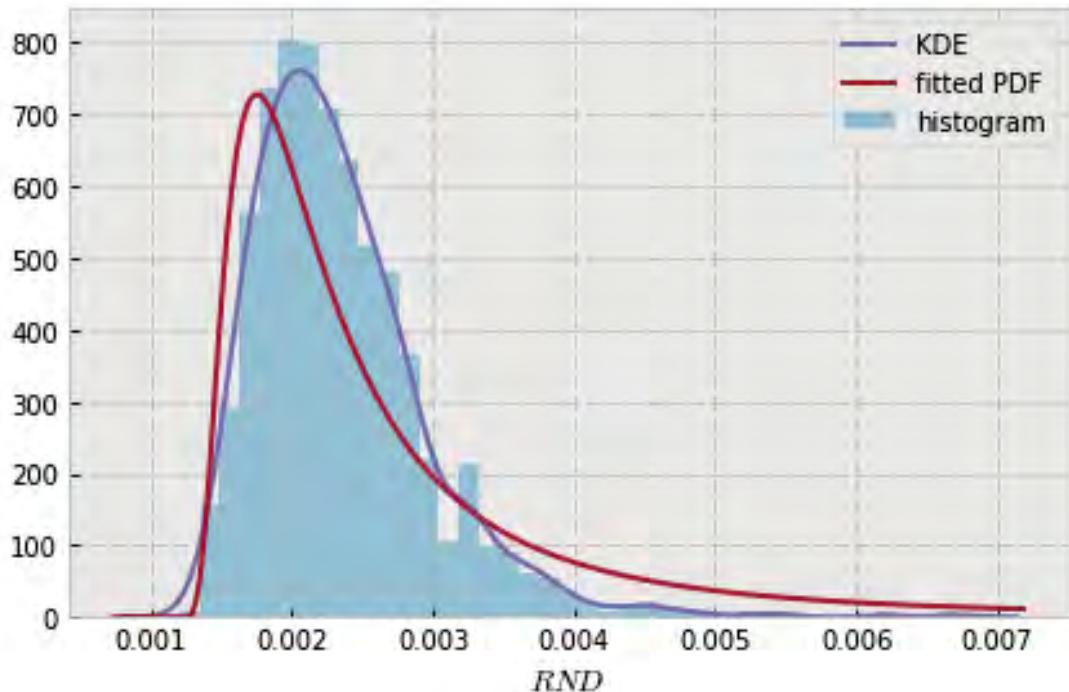
```

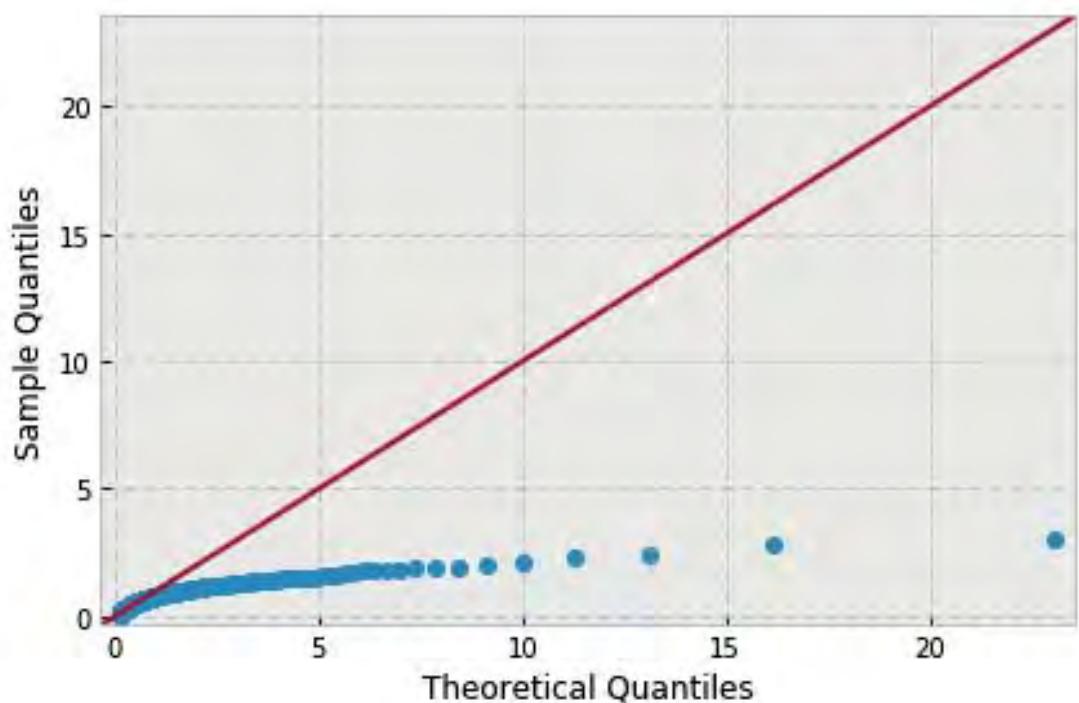
variable name: r
variable value: 2.7777777777777777
distribution: inverse-gamma(shape=(9.6377047571794527,), loc=0.0005761964149588306,
                           scale=0.01676902430428682)
DescribeResult(nobs=1000, minmax=(0.0011579817530630607, 0.008362853345104726)
               mean=0.0025199244912648879, variance=5.3020197844960996e-07,
               skewness=2.0970849095140904, kurtosis=9.208075759175248)

```

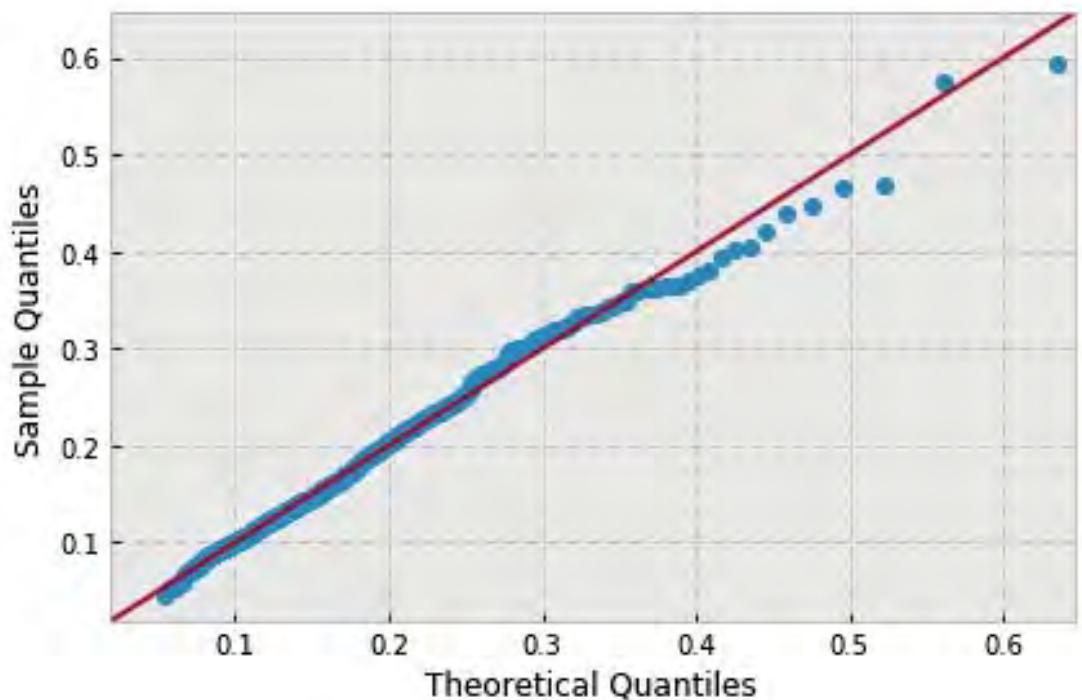
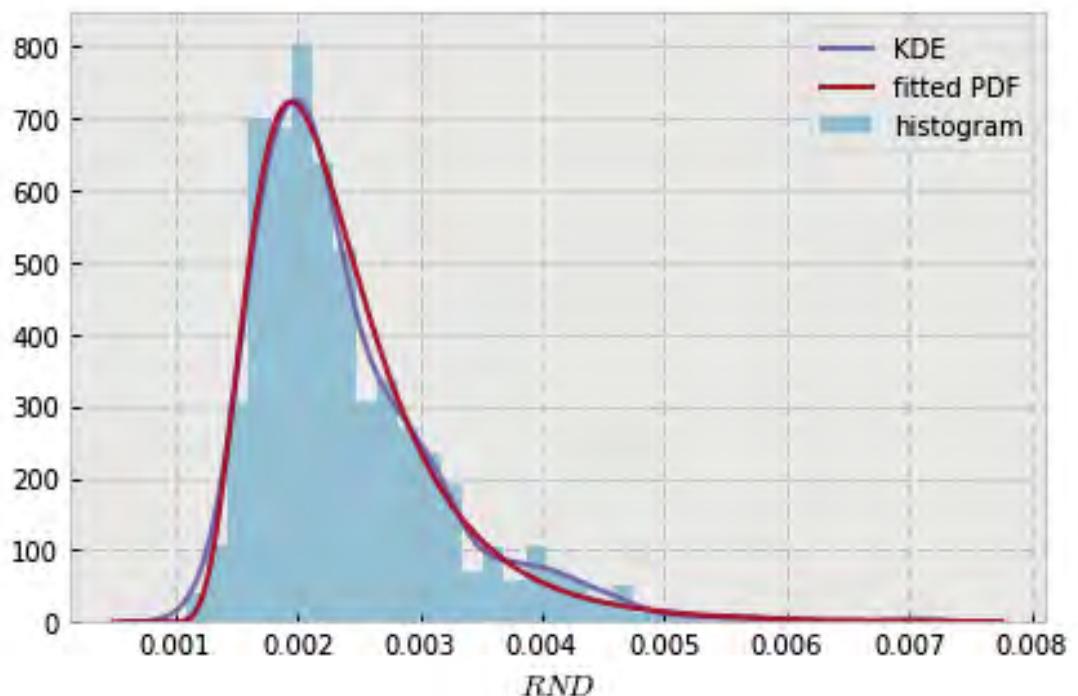


```
variable name: r
variable value: 3.0
distribution: inverse-gamma(shape=(1.97728366113635,), loc=0.0011418742166770372,
    scale=0.0018172887402797574)
DescribeResult(nobs=1000, minmax=(0.0012034397731205697, 0.0067156737904481732)
    mean=0.0023406502949119318, variance=3.8638752640996922e-07,
    skewness=1.6515949318953824, kurtosis=5.501816572180807)
```

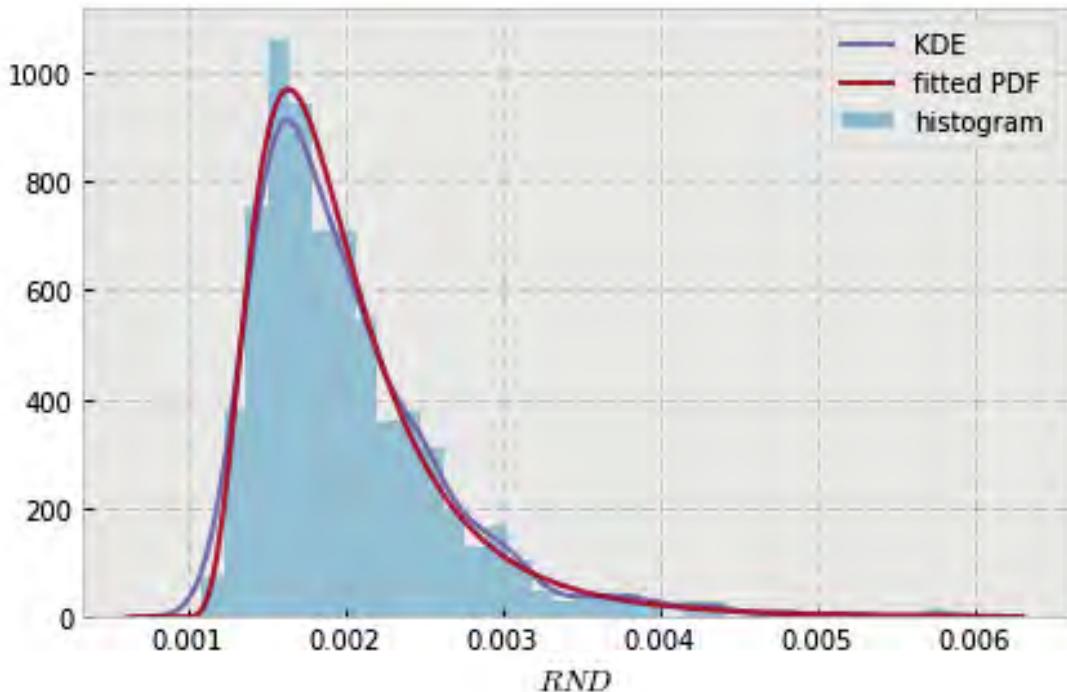


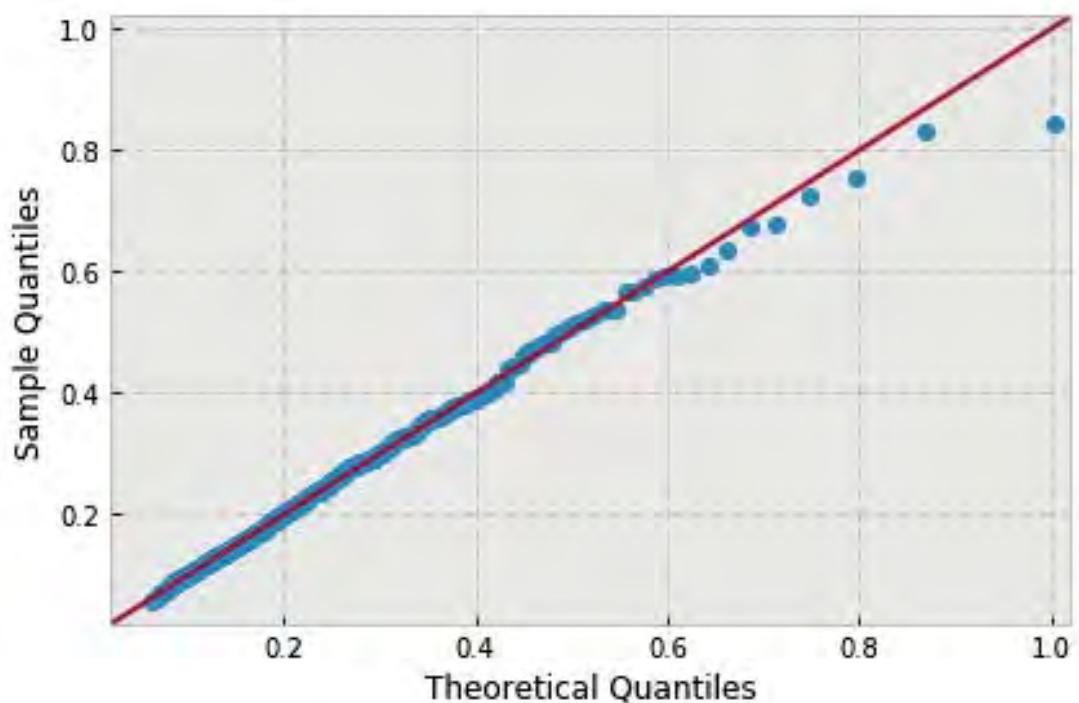


```
variable name: sigma
variable value: 0.01111111111111112
distribution: inverse-gamma(shape=(7.1294325718250864,), loc=0.0005831380926948795,
                           scale=0.011101379523133937)
DescribeResult(nobs=1000, minmax=(0.0010865724420020227, 0.0071652292481739349)
               mean=0.0023934841101512588, variance=6.1017347318825891e-07,
               skewness=1.606657353299011, kurtosis=3.8417525963303936)
```

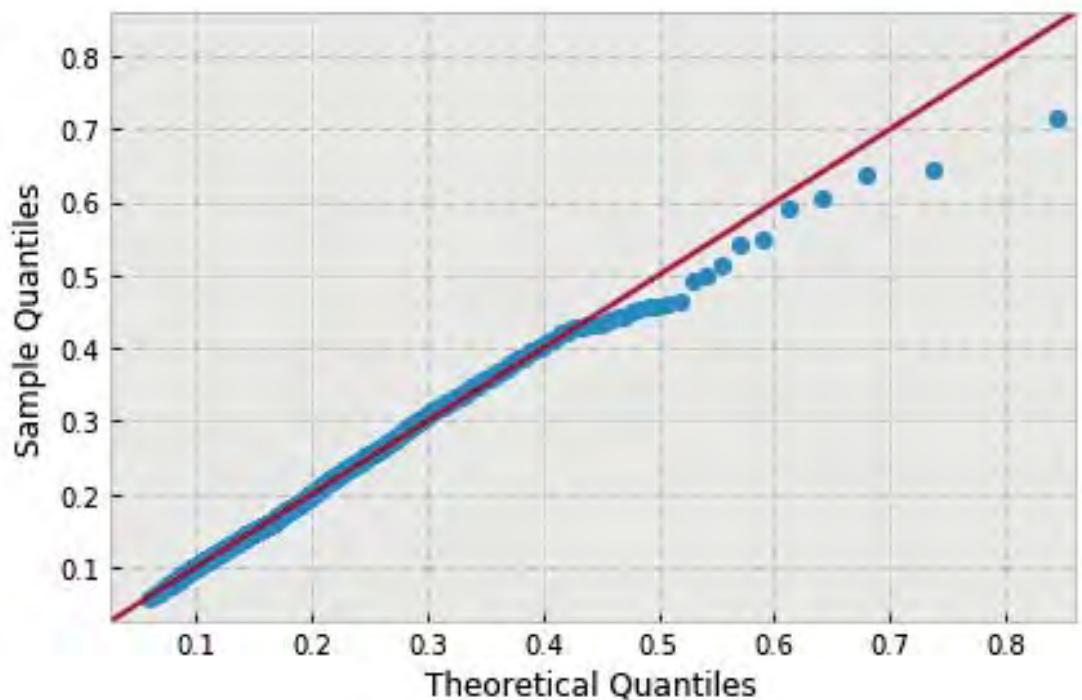
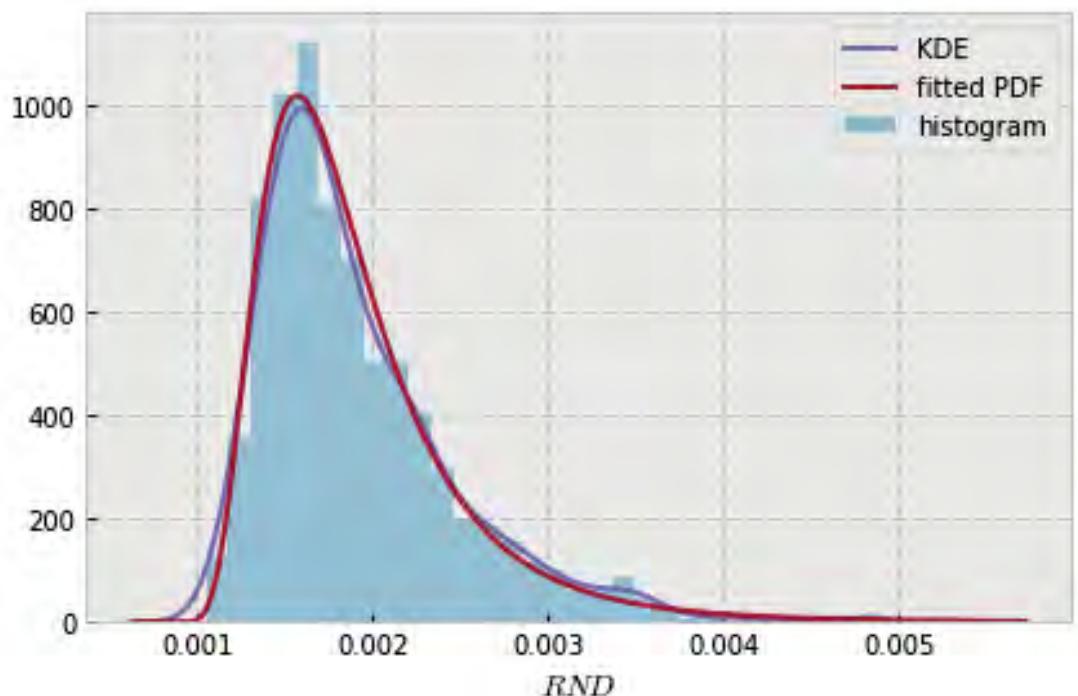


```
variable name: sigma
variable value: 0.02222222222222223
distribution: inverse-gamma(shape=(5.7184141132688131,), loc=0.0007465505796944842,
    scale=0.006029968013753788)
DescribeResult(nobs=1000, minmax=(0.001092438282800187, 0.0058289831285405014)
    mean=0.0020219938655322935, variance=3.9902627249406751e-07,
    skewness=1.8687563039192818, kurtosis=5.299805870877238)
```

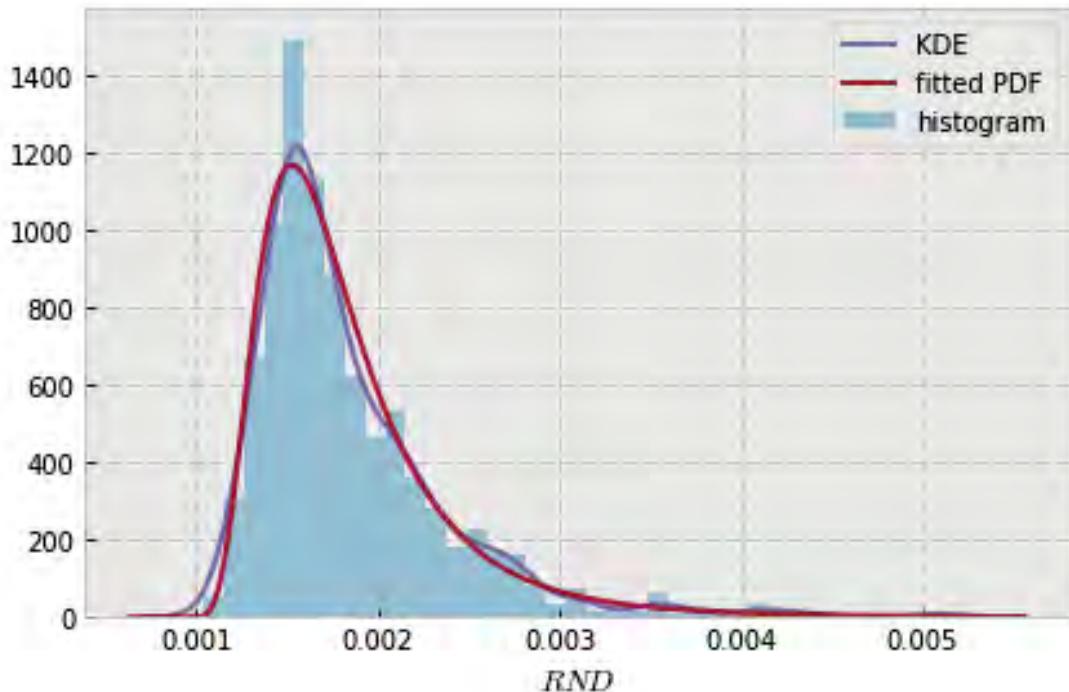


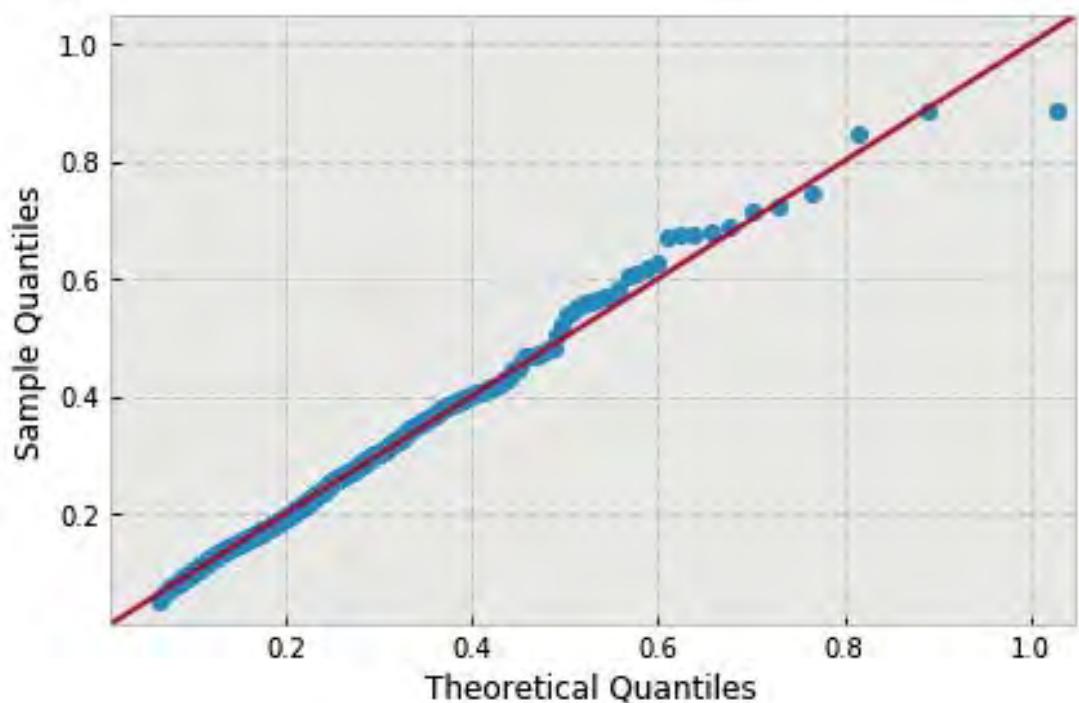


```
variable name: sigma
variable value: 0.0333333333333333
distribution: inverse-gamma(shape=(6.1987715411233886,), loc=0.0006773822543704434,
                           scale=0.006442055774027032)
DescribeResult(nobs=1000, minmax=(0.0010490725084385862, 0.0052885947319301621)
               mean=0.0019139670785234192, variance=3.2922851894993815e-07,
               skewness=1.6248083948632845, kurtosis=3.8069299543733903)
```

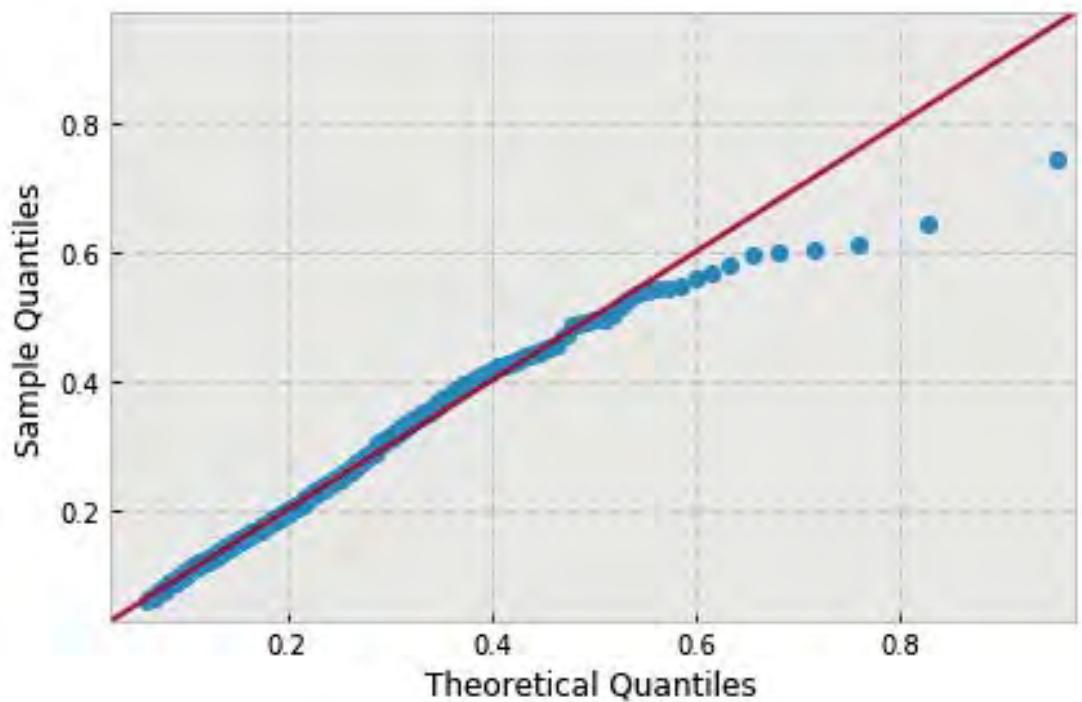
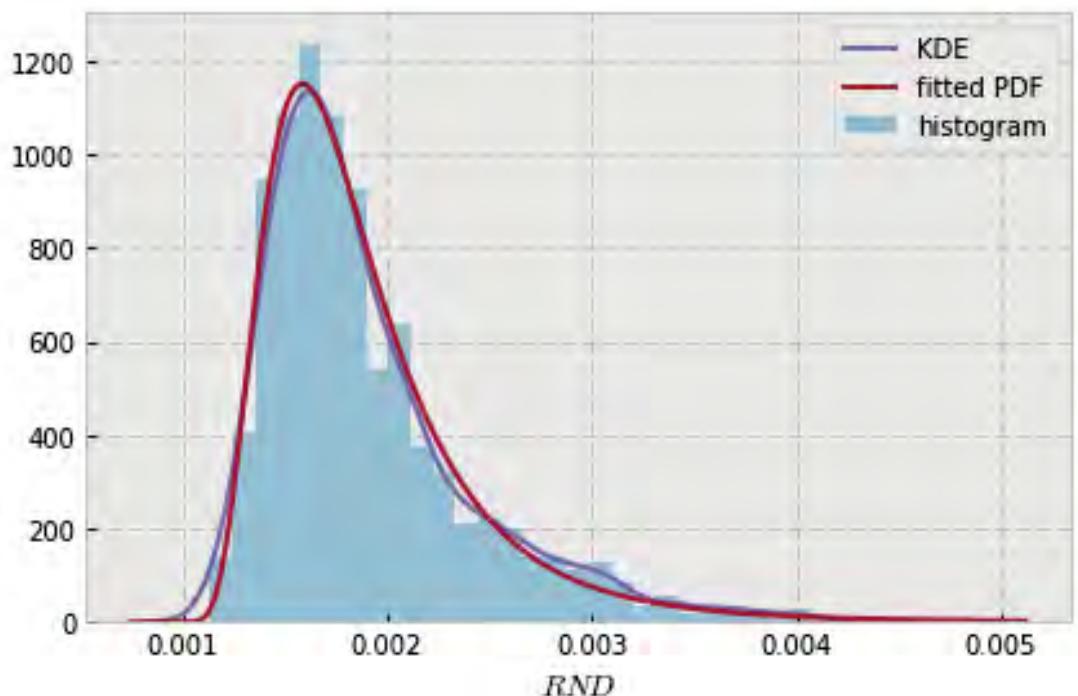


```
variable name: sigma
variable value: 0.04444444444444446
distribution: inverse-gamma(shape=(5.6513501930300549,), loc=0.0007878752380146173,
                           scale=0.004916231934573344)
DescribeResult(nobs=1000, minmax=(0.0010407518441419288, 0.0051476445036652838)
               mean=0.0018451963342749264, variance=2.9677450423618964e-07,
               skewness=2.082884751775205, kurtosis=6.425756523096034)
```

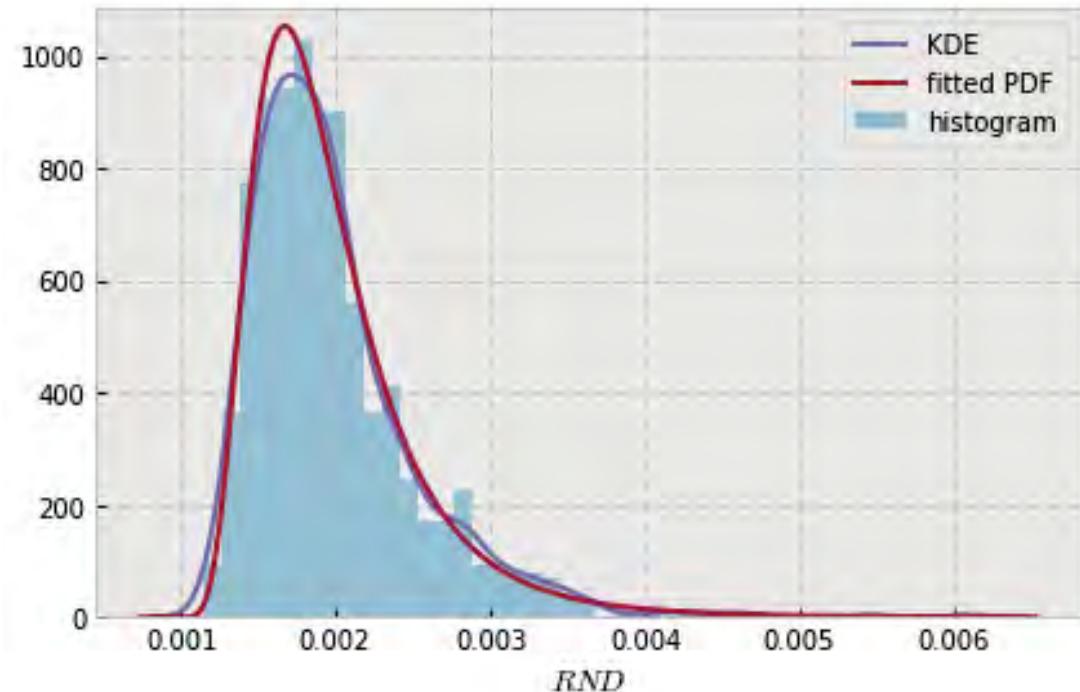


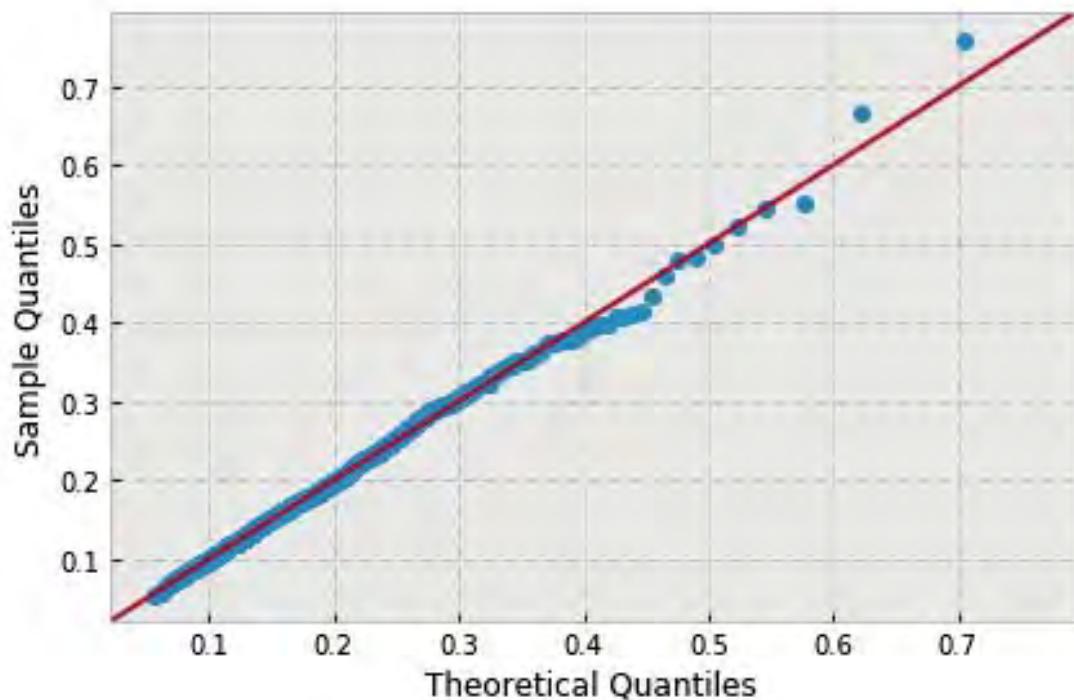


```
variable name: sigma
variable value: 0.05555555555555556
distribution: inverse-gamma(shape=(5.8553801168932162,), loc=0.0008217342823178373,
                           scale=0.005244442354400233)
DescribeResult(nobs=1000, minmax=(0.0011357964729575042, 0.0047253995324688883)
               mean=0.0018995172899466768, variance=2.6849807087637718e-07,
               skewness=1.567305608643505, kurtosis=2.954440443426229)
```

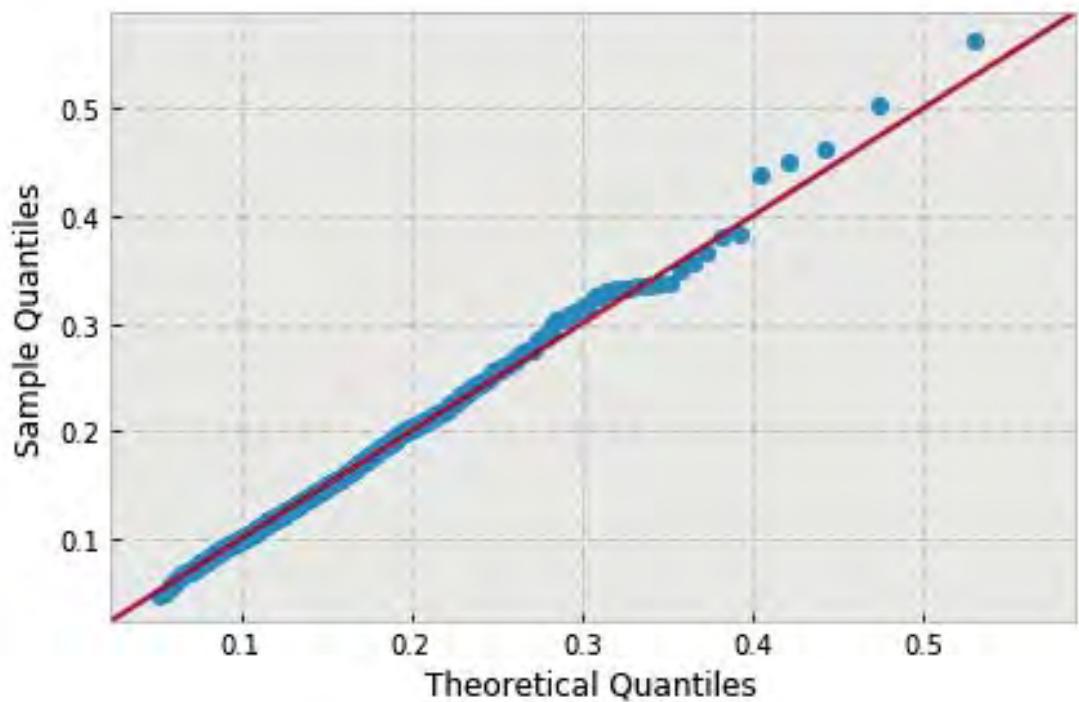
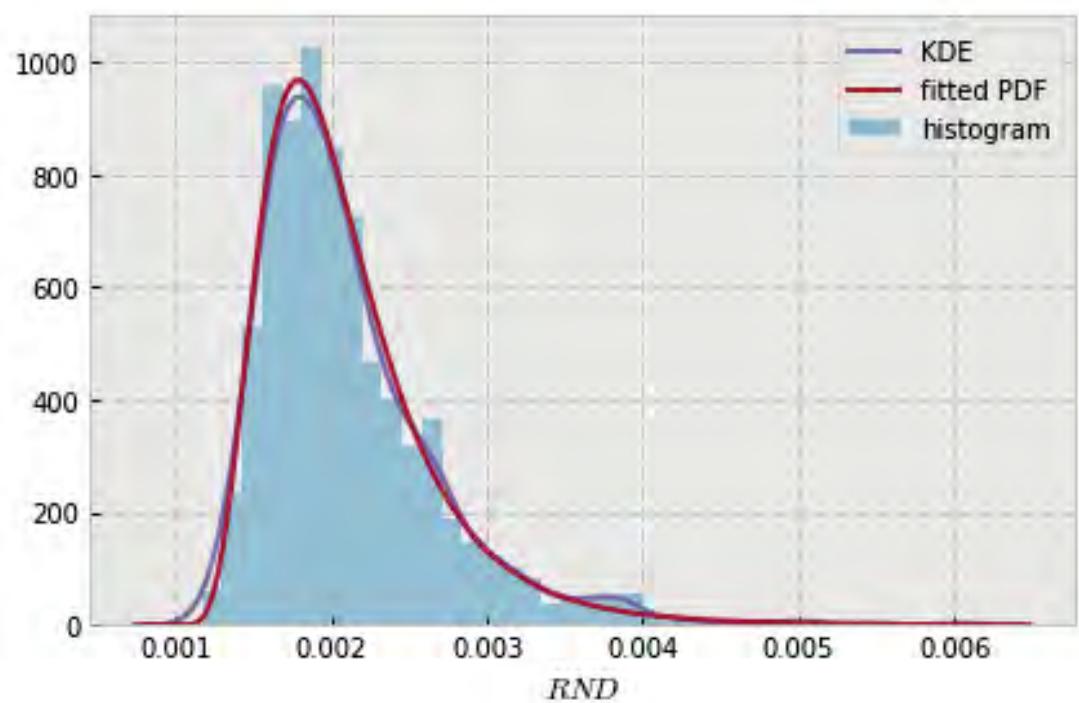


```
variable name: sigma
variable value: 0.06666666666666667
distribution: inverse-gamma(shape=(6.7613000035280049,), loc=0.0007621510254981579,
    scale=0.007054477688366865)
DescribeResult(nobs=1000, minmax=(0.001149481007397167, 0.0061070873844520975)
    mean=0.0019857160783252402, variance=3.0259266304244608e-07,
    skewness=1.909236304197017, kurtosis=6.581794183496292)
```

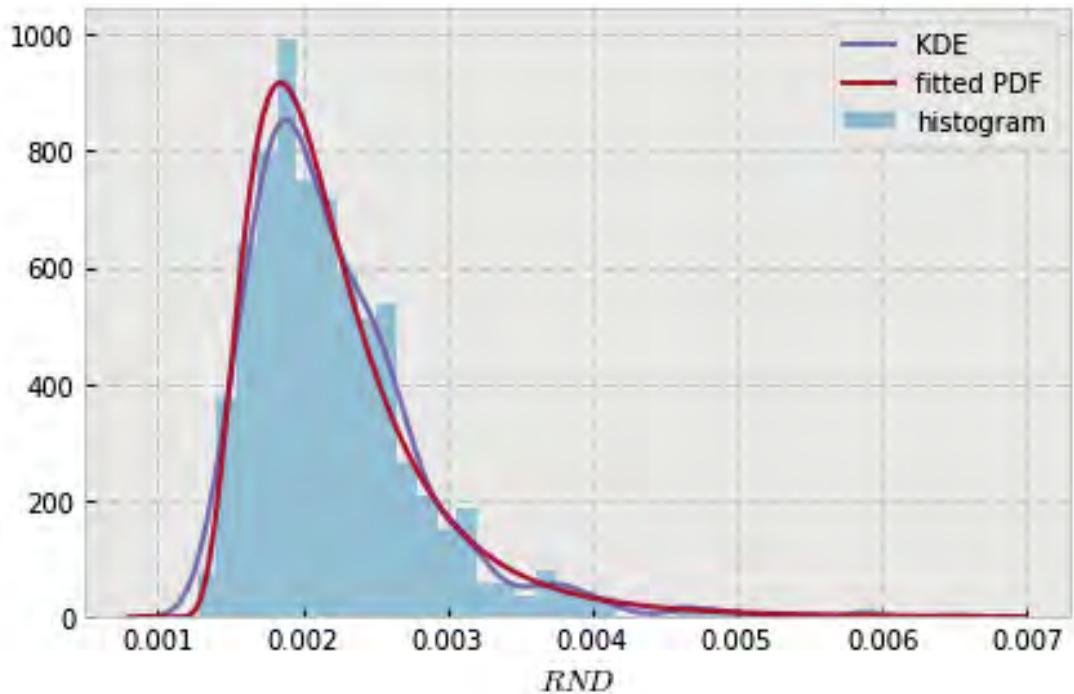


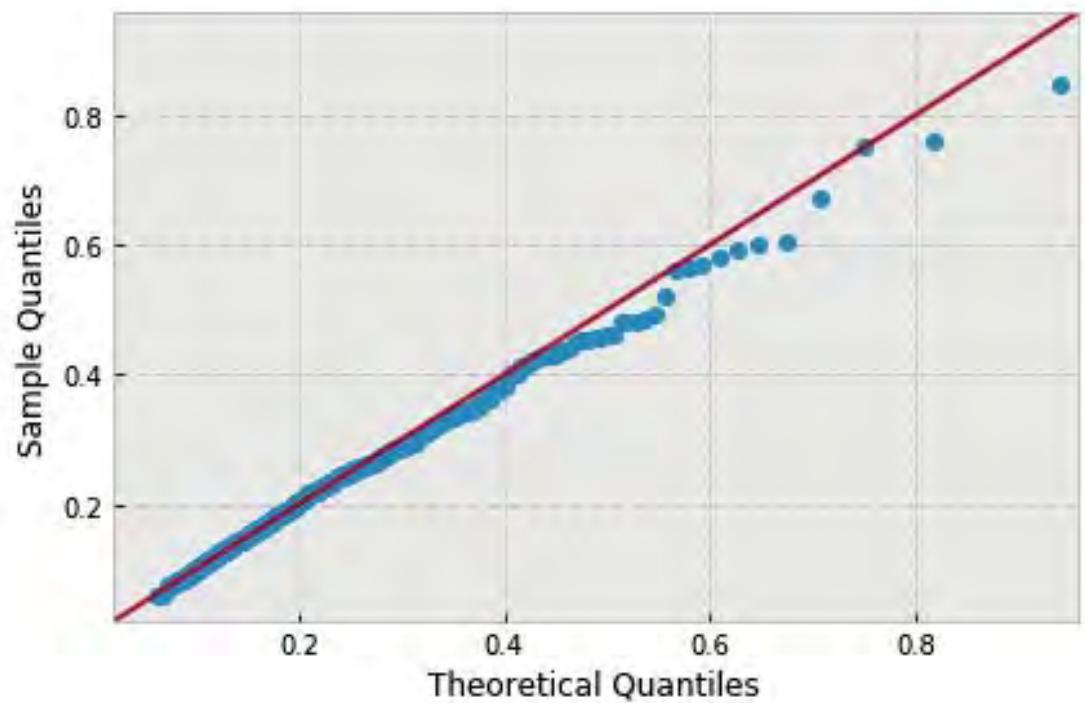


```
variable name: sigma
variable value: 0.07777777777777778
distribution: inverse-gamma(shape=(7.81275815202169,), loc=0.0007126825642379032,
                           scale=0.009468047480453)
DescribeResult(nobs=1000, minmax=(0.0011714650129341171, 0.006041586789568191)
               mean=0.0021057131517739263, variance=3.4123536989635804e-07,
               skewness=1.7931297343555197, kurtosis=5.326455249382628)
```

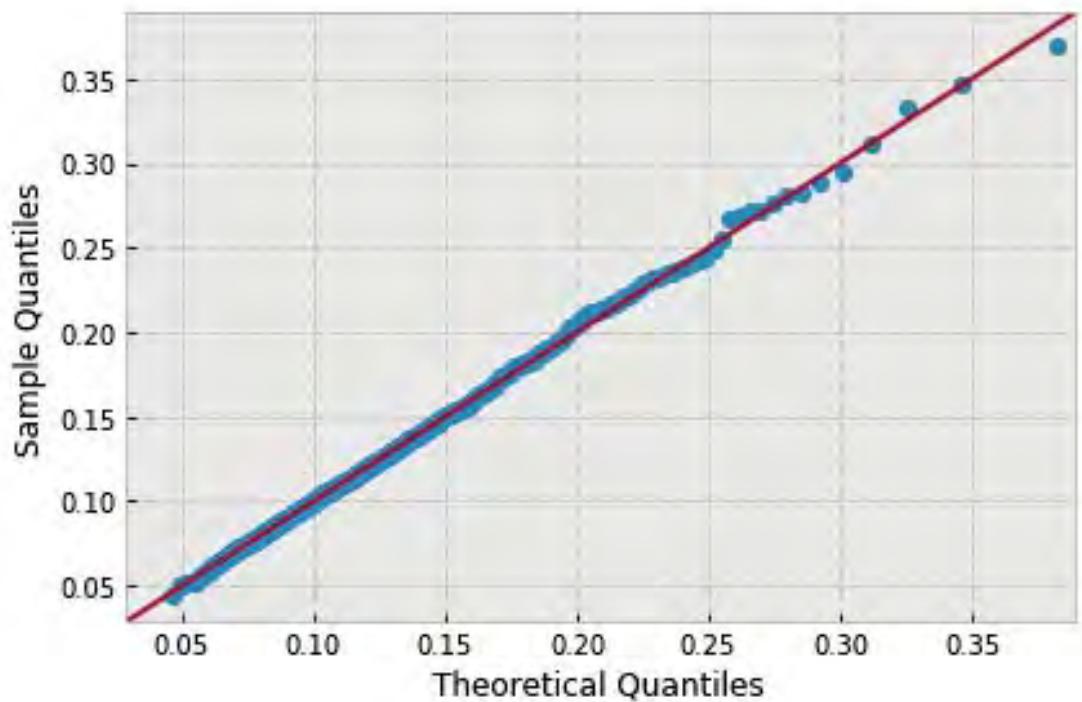
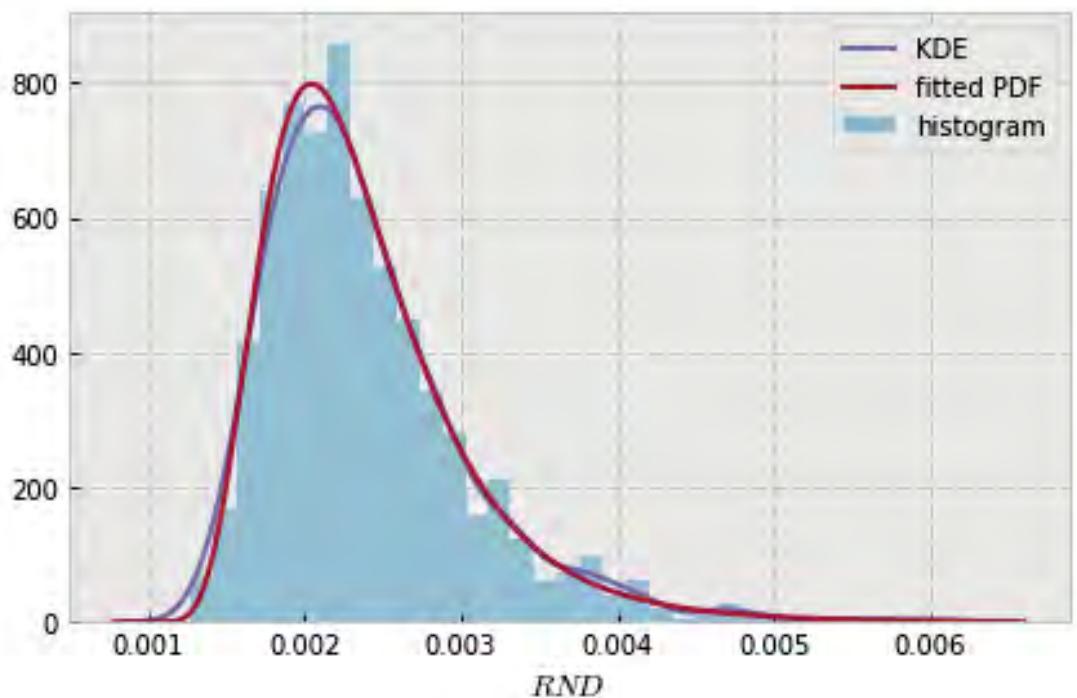


```
variable name: sigma
variable value: 0.08888888888888889
distribution: inverse-gamma(shape=(5.8931516085316105,), loc=0.0008810944300765879,
                           scale=0.006651751041633177)
DescribeResult(nobs=1000, minmax=(0.0012692813397541204, 0.0065102051164576628)
               mean=0.0022241555437955726, variance=3.8949087207058788e-07,
               skewness=1.9088010401973008, kurtosis=6.3994059679225685)
```

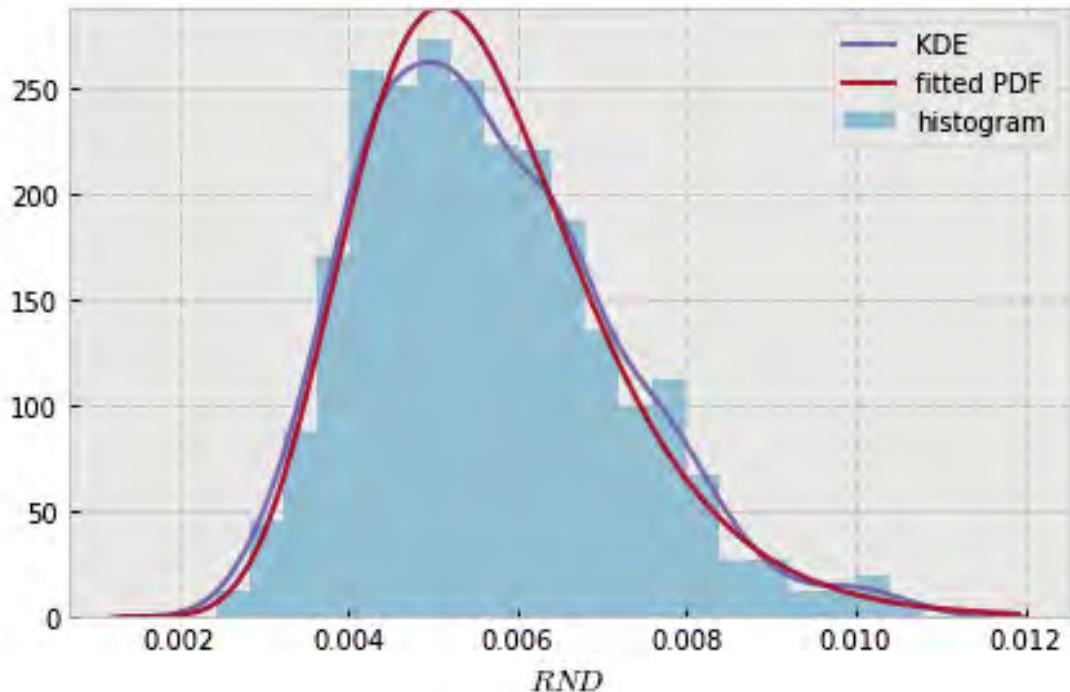


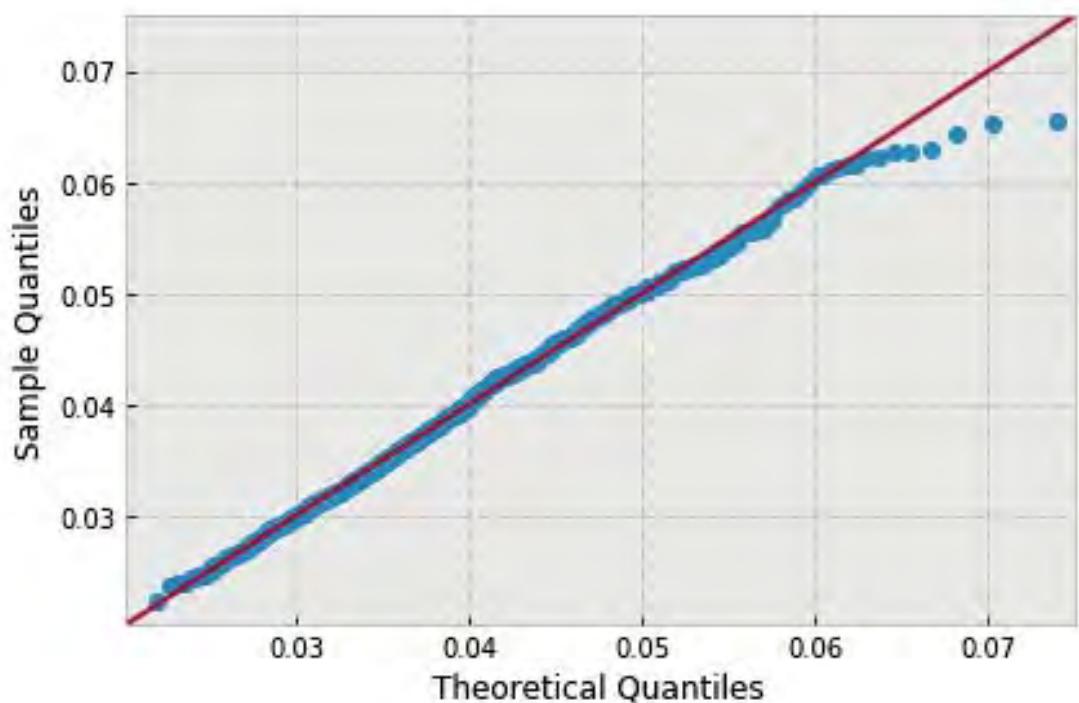


```
variable name: sigma
variable value: 0.1
distribution: inverse-gamma(shape=(9.3307500819871567,), loc=0.0006061780118854752,
                           scale=0.014856765004509482)
DescribeResult(nobs=1000, minmax=(0.0012709248223530109, 0.0061032717333038512)
               mean=0.0023891162320771289, variance=4.2518839798470721e-07,
               skewness=1.4643648567993106, kurtosis=3.366662762704647)
```

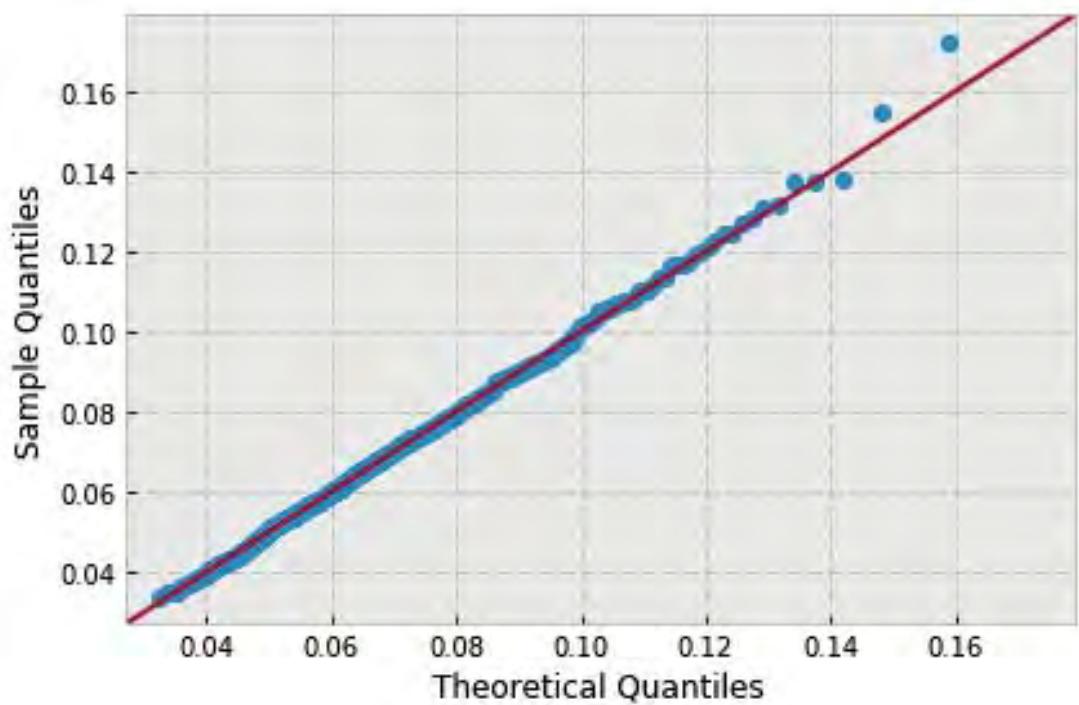
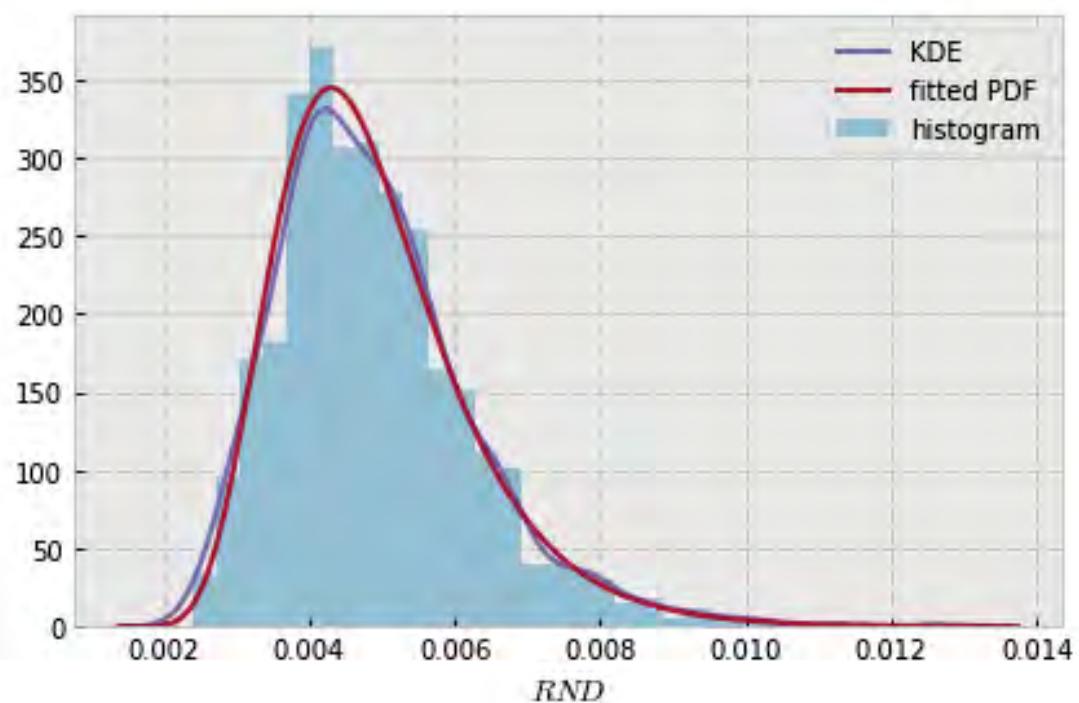


```
variable name: theta
variable value: 0.6981317007977318
distribution: inverse-gamma(shape=(26.713933111899252,), loc=-0.0019103165880605334,
                           scale=0.19412394283811374)
DescribeResult(nobs=1000, minmax=(0.0024181869166037548, 0.010784604980312279)
               mean=0.0056379442872697078, variance=2.2527427220307903e-06,
               skewness=0.6225823684465879, kurtosis=0.19145337638262783)
```

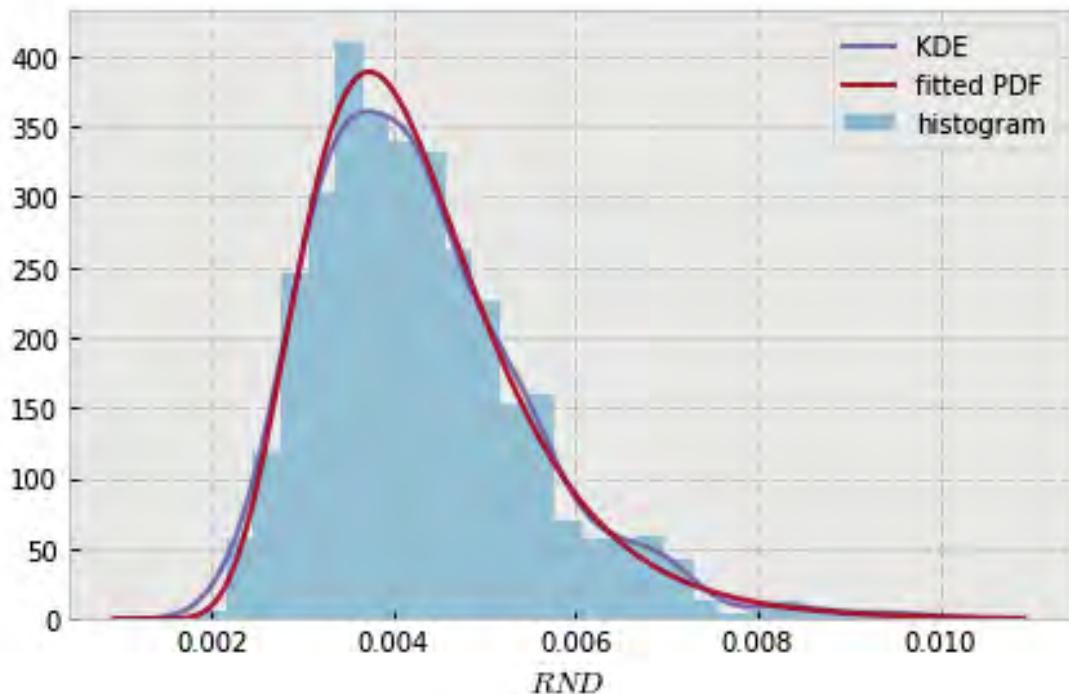


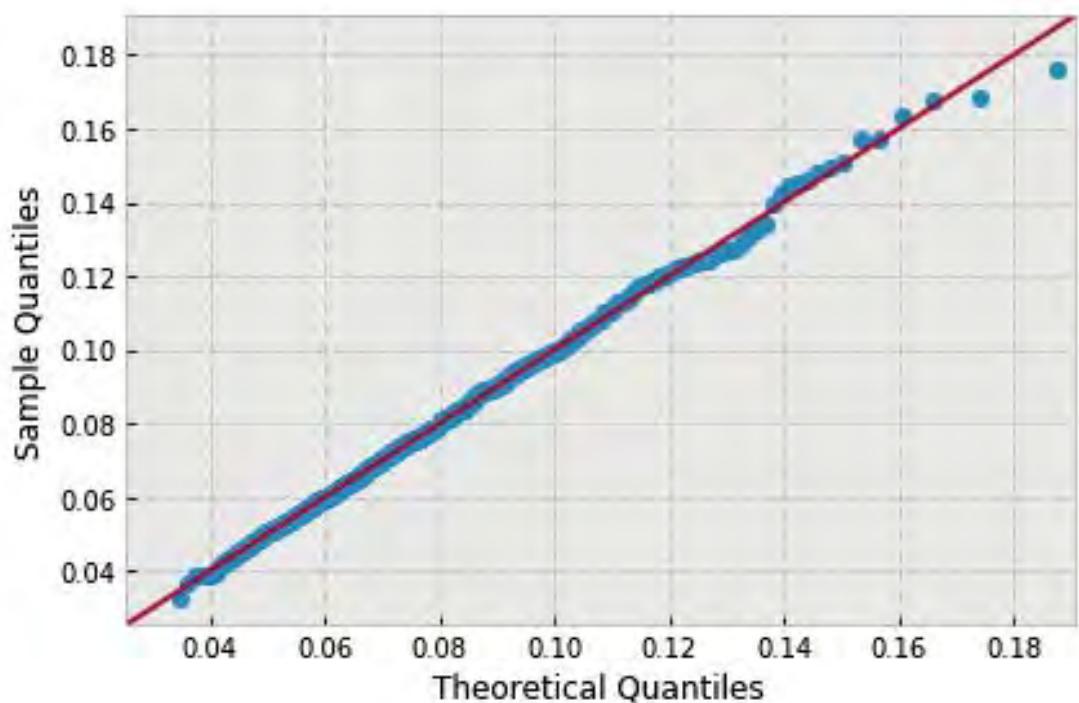


```
variable name: theta
variable value: 0.7853981633974483
distribution: inverse-gamma(shape=(15.828857201088734,), loc=-0.0001440462335362261,
                           scale=0.07481061676828527)
DescribeResult(nobs=1000, minmax=(0.0023951589065198536, 0.012725559087435757)
               mean=0.0049006001612865828, variance=1.8331766969831501e-06,
               skewness=1.1223557727944928, kurtosis=2.3183211448915824)
```



```
variable name: theta
variable value: 0.8726646259971648
distribution: inverse-gamma(shape=(14.23285417970331,), loc=1.0596465353663373e-05,
    scale=0.05670258498271216)
DescribeResult(nobs=1000, minmax=(0.0018523613747456812, 0.0099929372413421931)
    mean=0.0042948176803821328, variance=1.4753655937822054e-06,
    skewness=1.0800849318329897, kurtosis=1.7452633307748702)
```

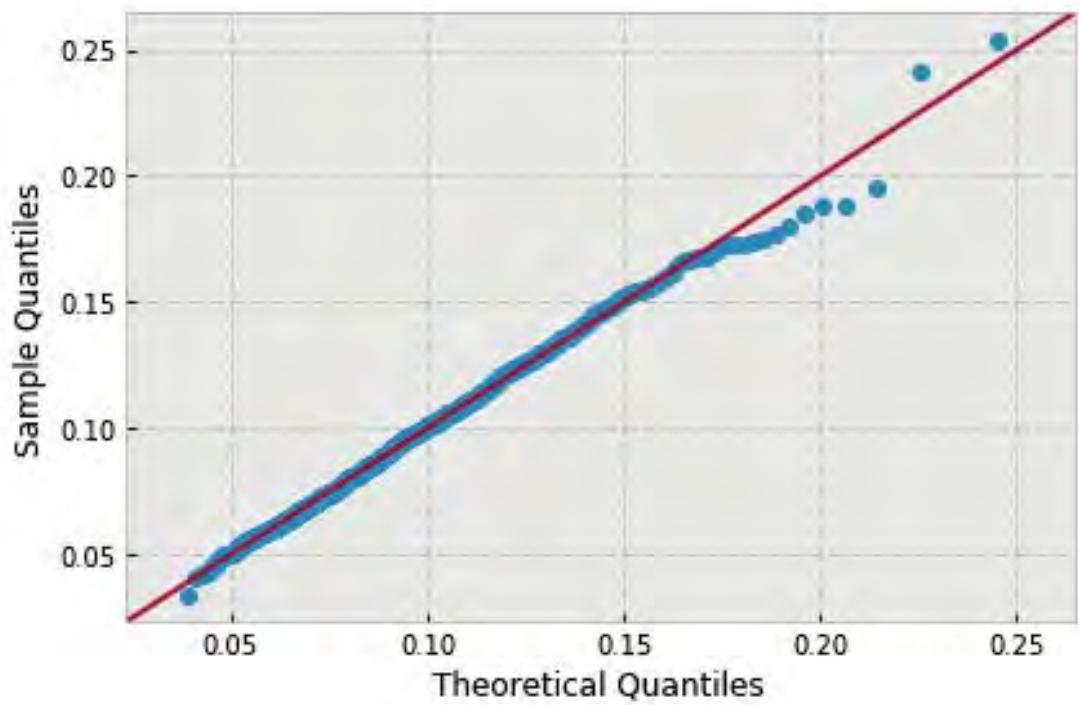
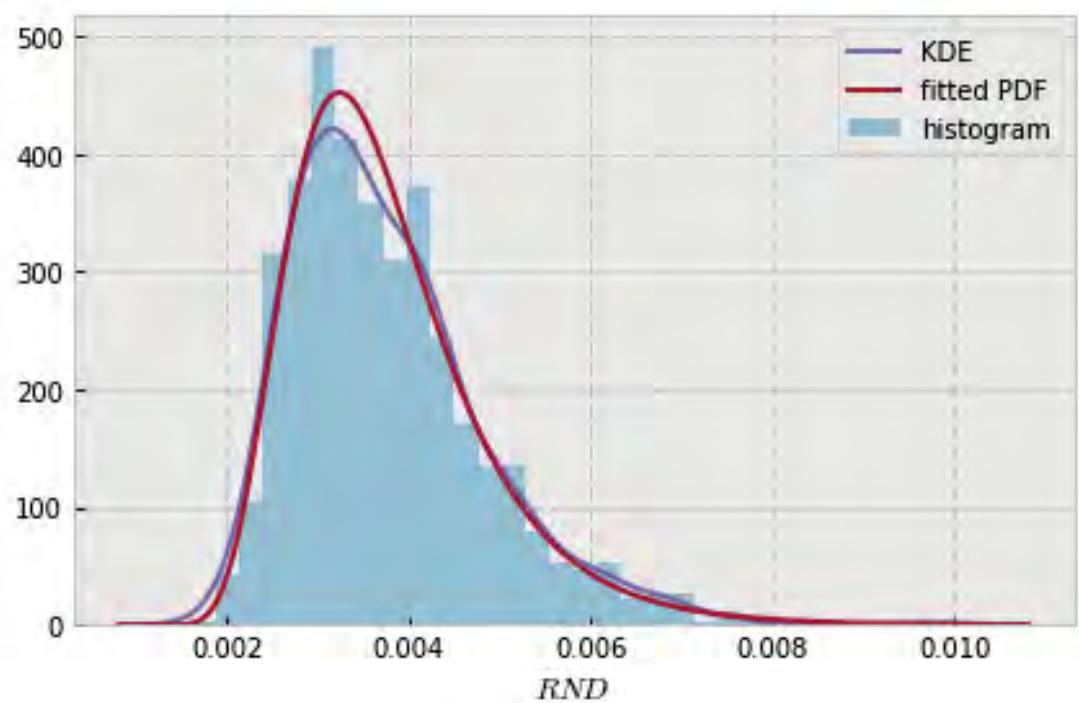




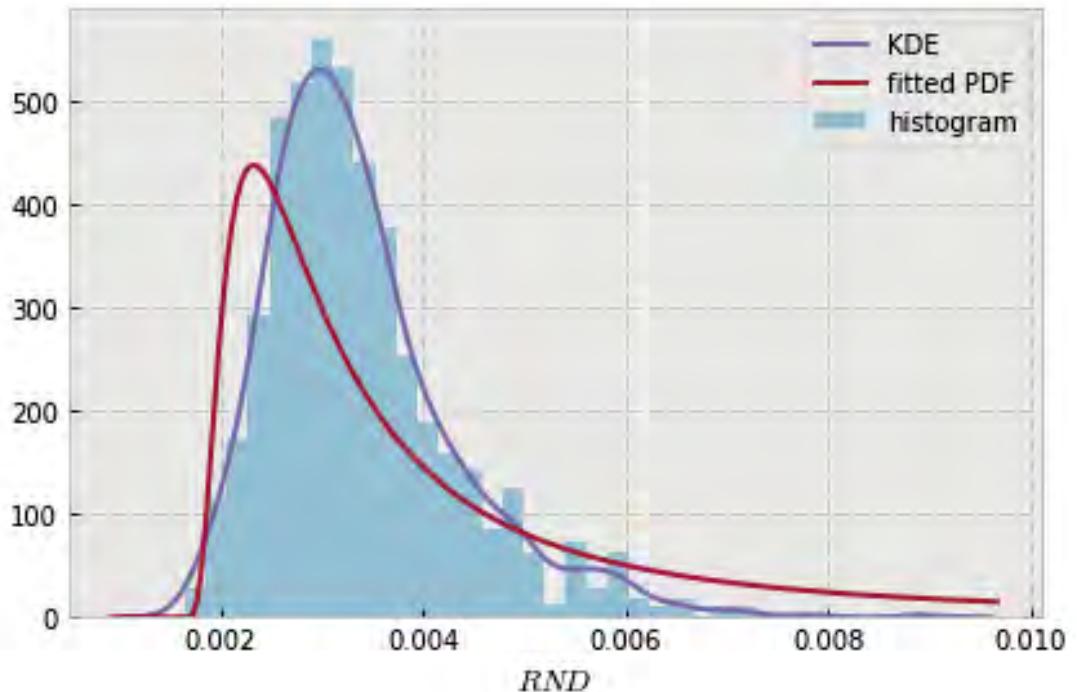
```

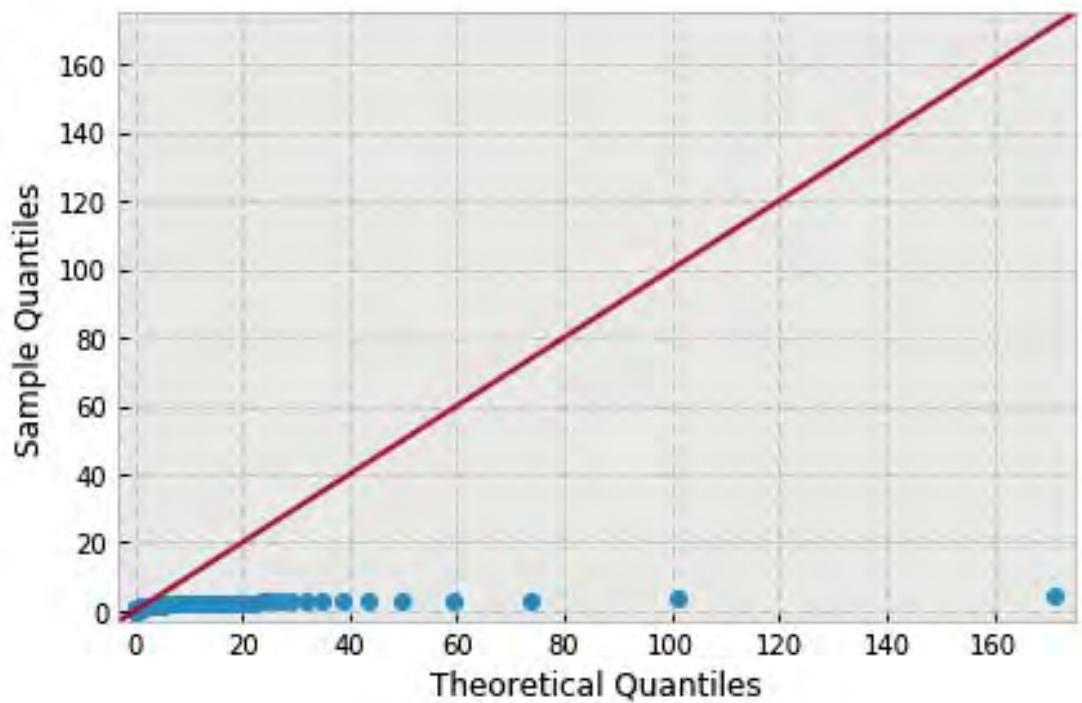
variable name: theta
variable value: 0.9599310885968813
distribution: inverse-gamma(shape=(12.059791939604422,), loc=0.00031513818644757566,
                           scale=0.03821243856171727)
DescribeResult(nobs=1000, minmax=(0.0016202226547197066, 0.010004912711584197)
               mean=0.0037690424208182182, variance=1.1540437021955503e-06,
               skewness=1.1747169935079307, kurtosis=2.3030263771807844)

```

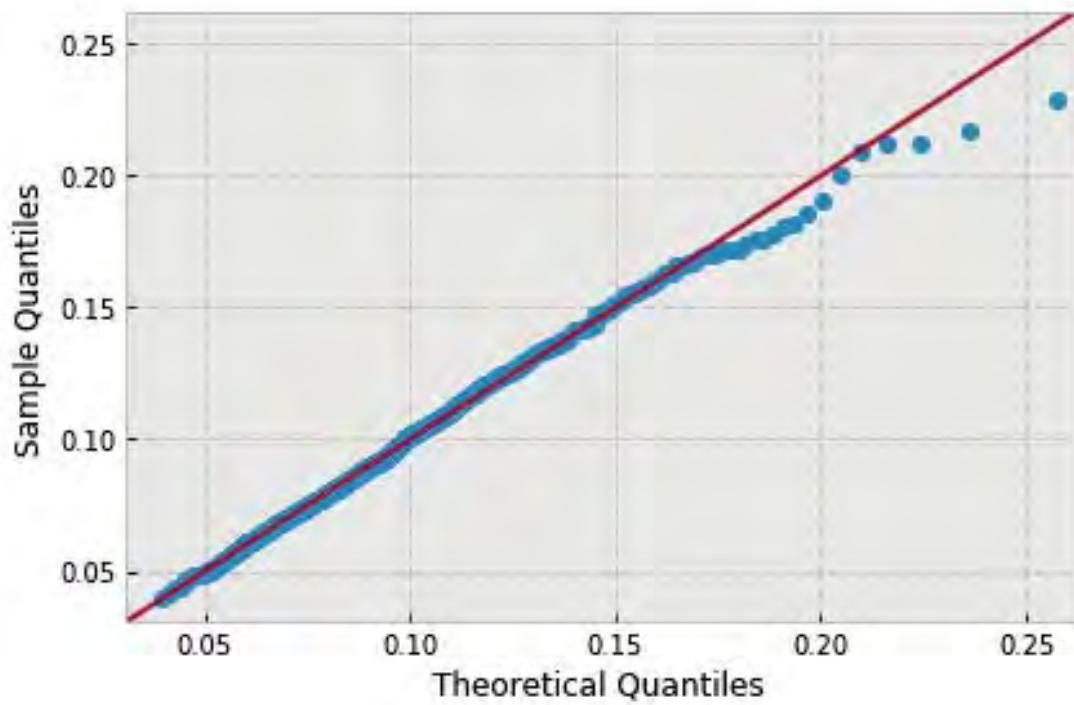
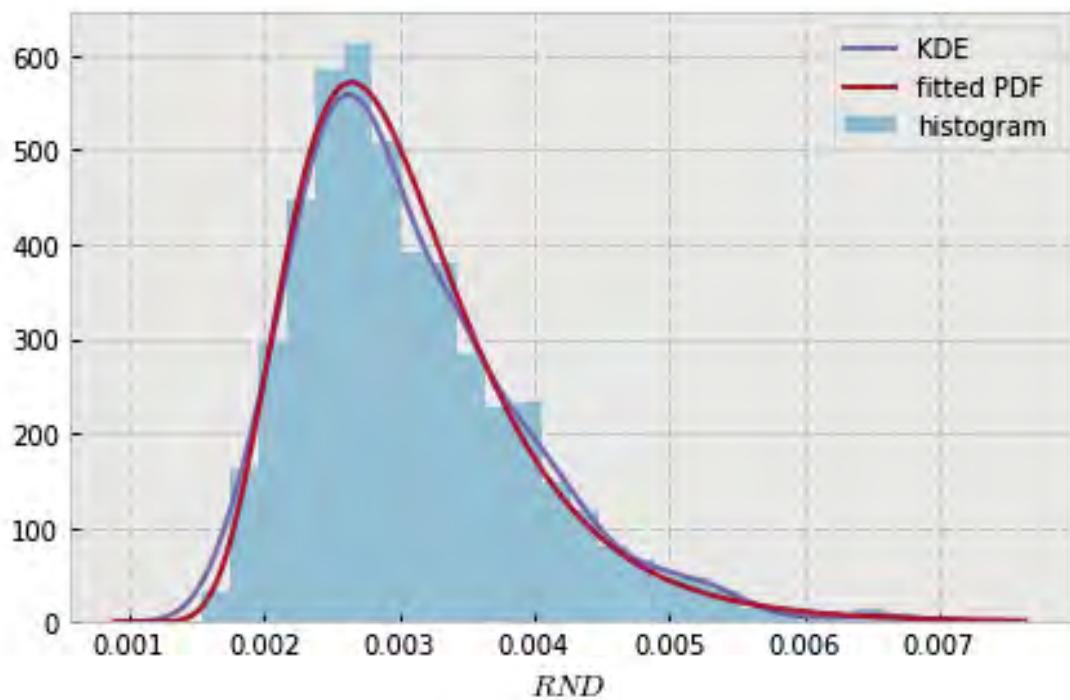


```
variable name: theta
variable value: 1.0471975511965979
distribution: inverse-gamma(shape=(1.3110443858986005,), loc=0.0015742771513559866,
    scale=0.001750682473879836)
DescribeResult(nobs=1000, minmax=(0.0016530872347353986, 0.0089493947010920814)
    mean=0.0033901211261146385, variance=9.0358620254277982e-07,
    skewness=1.3107465510484149, kurtosis=2.5948595562643426)
```

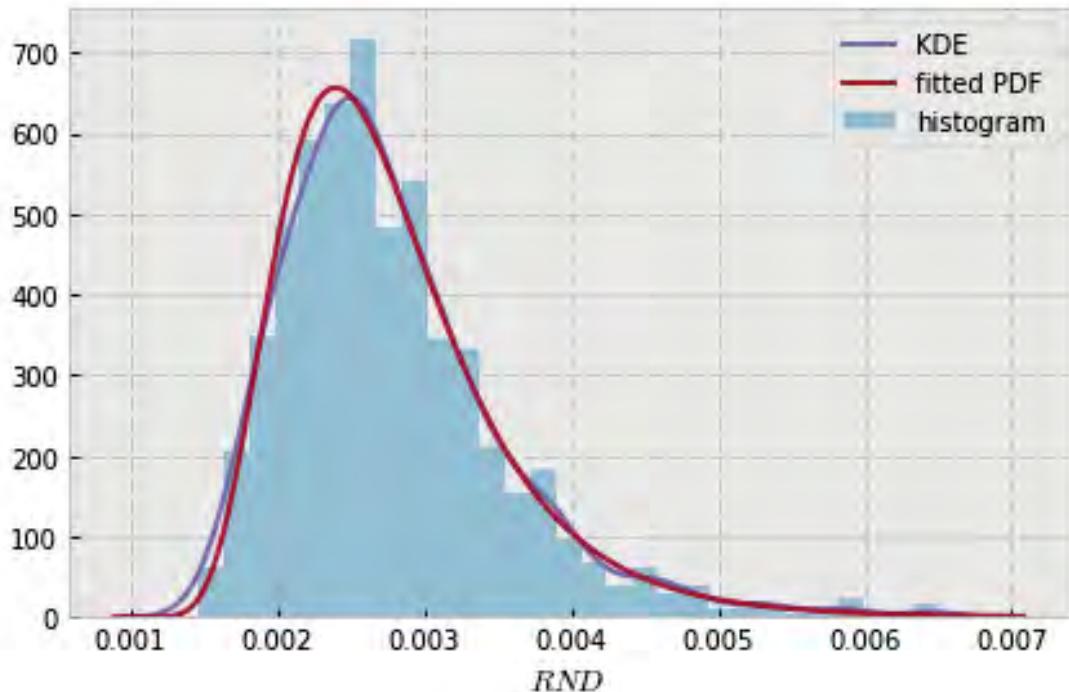


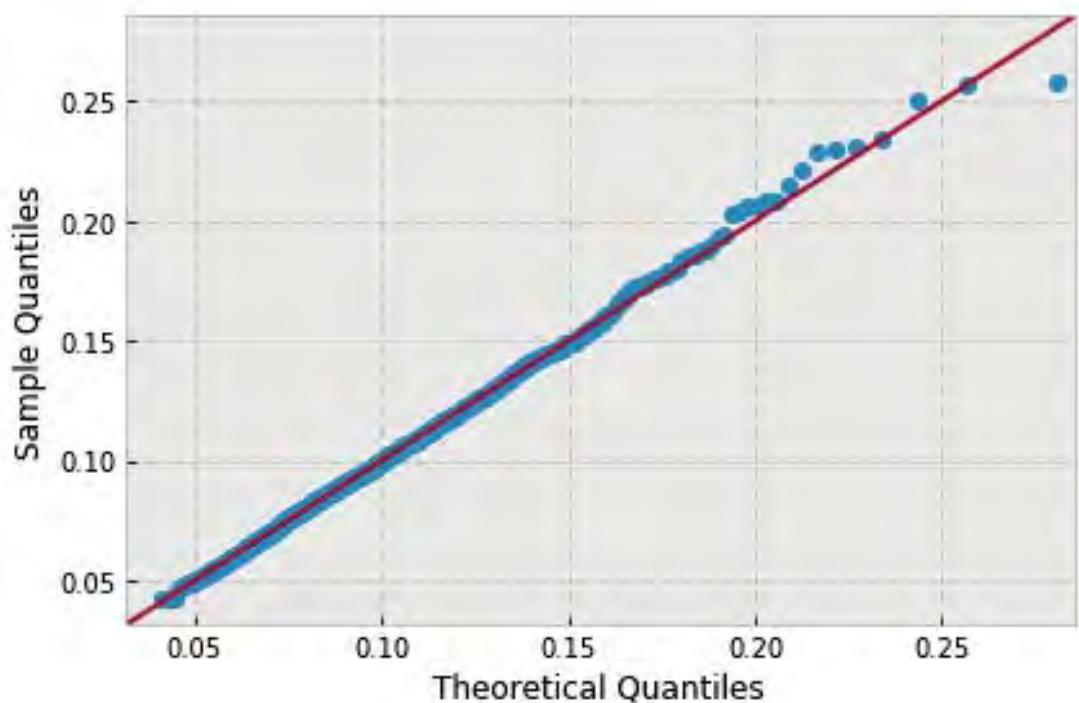


```
variable name: theta
variable value: 1.1344640137963142
distribution: inverse-gamma(shape=(11.721253650949407,), loc=0.0003692505372555832,
                           scale=0.02899497484363682)
DescribeResult(nobs=1000, minmax=(0.0015311130368986653, 0.0069934249523205113)
               mean=0.0030721290279496472, variance=7.1884169498416139e-07,
               skewness=1.0706043996156314, kurtosis=1.4598090239513741)
```

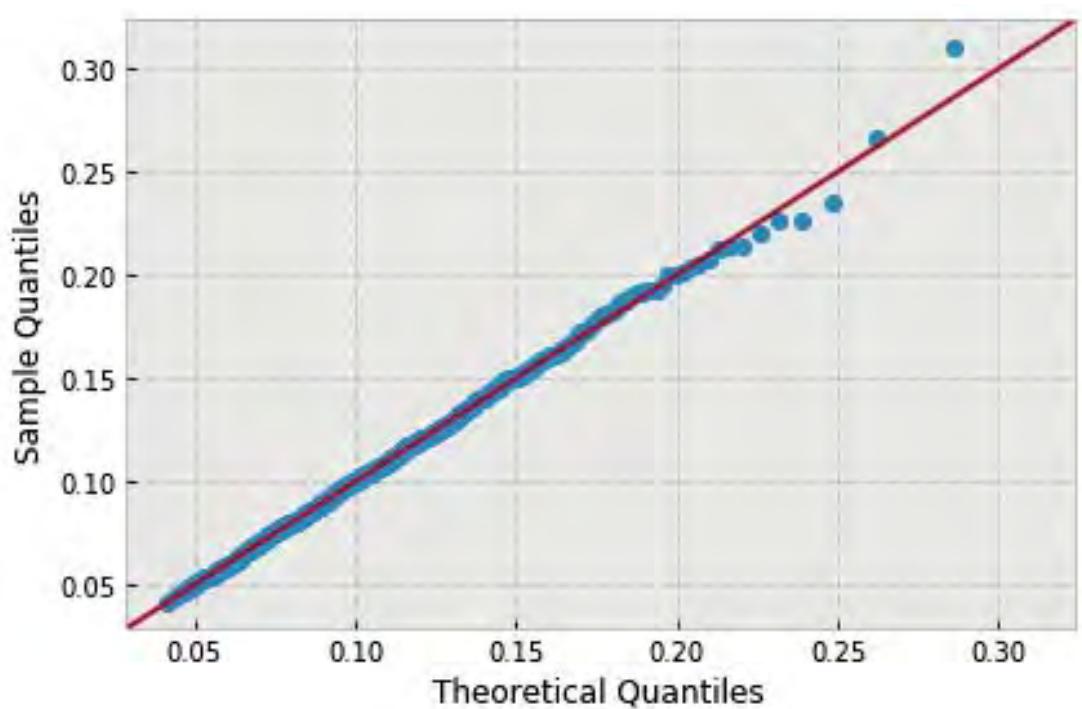
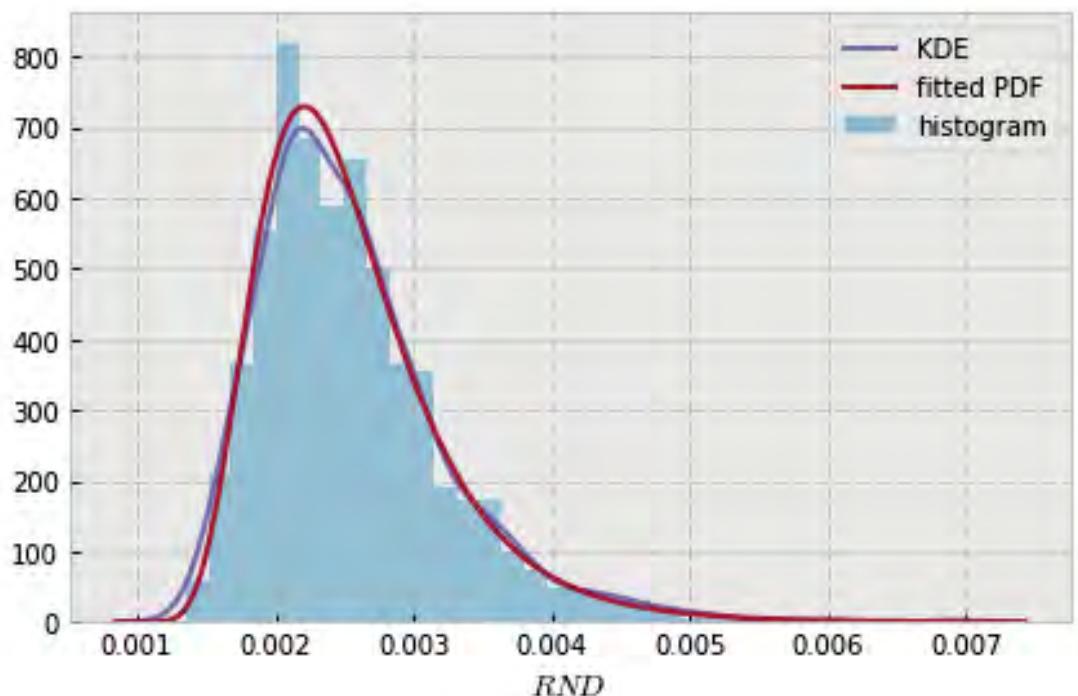


```
variable name: theta
variable value: 1.2217304763960306
distribution: inverse-gamma(shape=(11.13491250658209,), loc=0.000467767337694939,
                           scale=0.023408076219350364)
DescribeResult(nobs=1000, minmax=(0.0014570444193502598, 0.0065063839915163509)
               mean=0.0027771135373555948, variance=5.7749213948061332e-07,
               skewness=1.3383413989379485, kurtosis=2.8085051300562993)
```

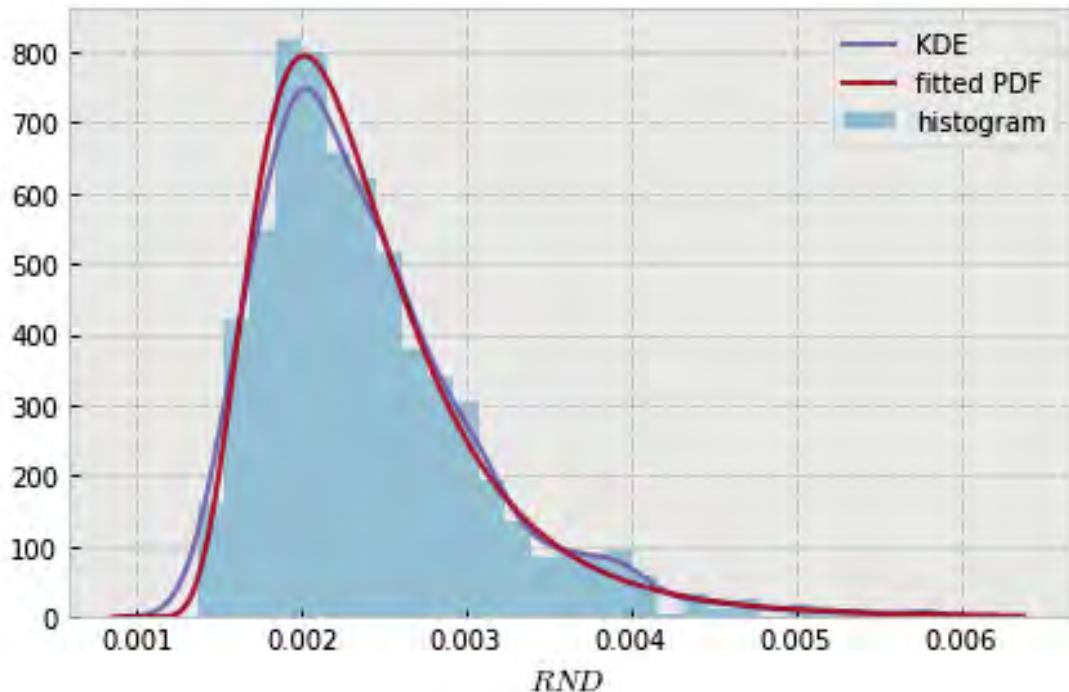


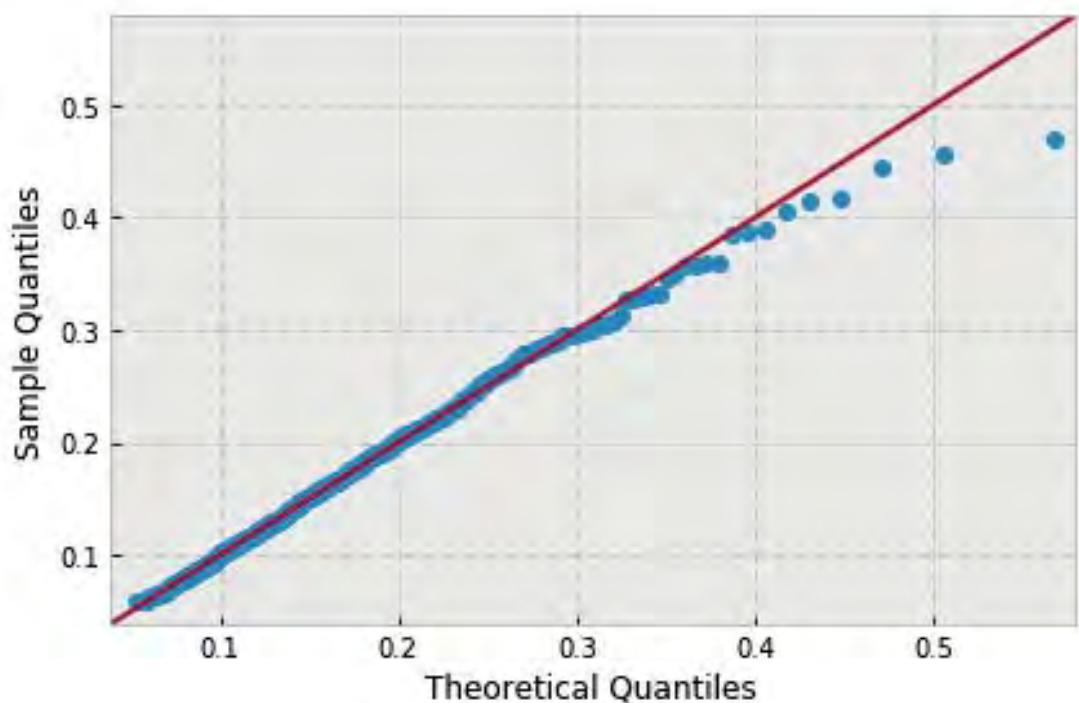


```
variable name: theta
variable value: 1.3089969389957472
distribution: inverse-gamma(shape=(11.003228169564551,), loc=0.00048624097993710334,
                           scale=0.020706051886592518)
DescribeResult(nobs=1000, minmax=(0.0013526884982854553, 0.0069067502792330071)
               mean=0.0025556880800398966, variance=4.6791106201495336e-07,
               skewness=1.3472785036617763, kurtosis=3.180295240120439)
```

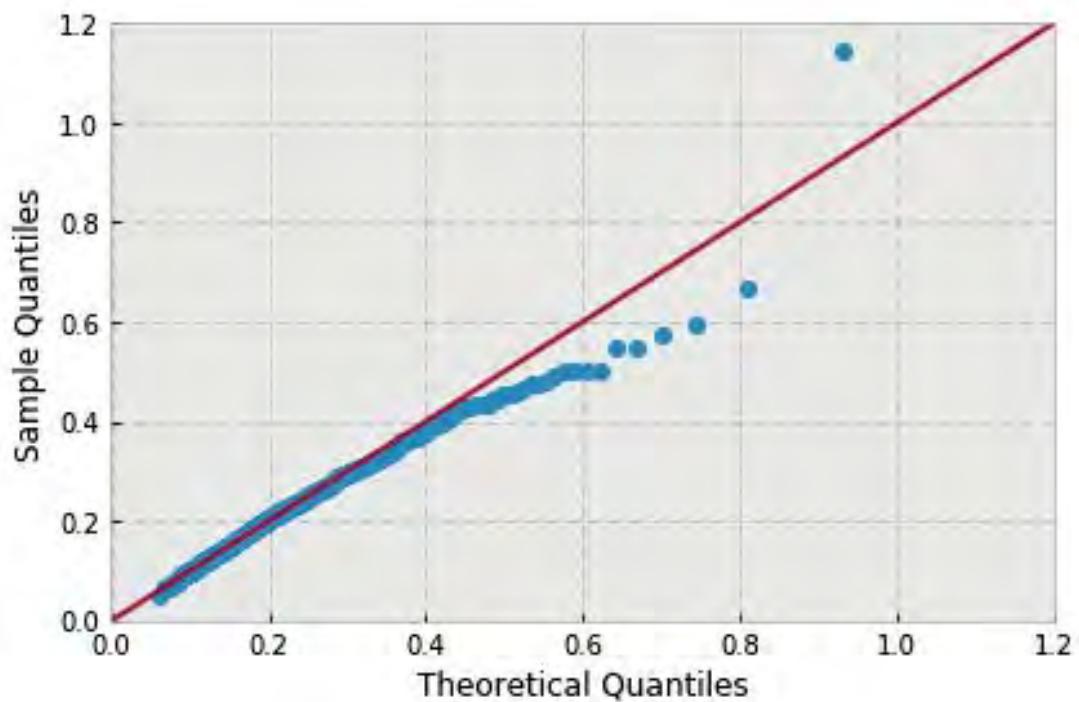
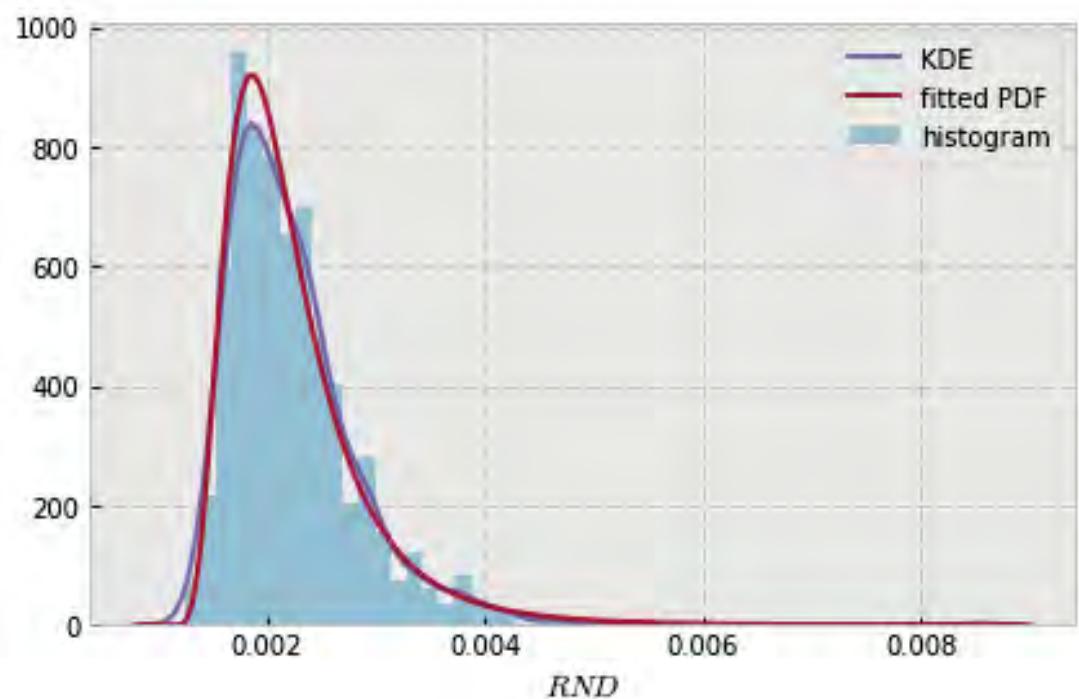


```
variable name: theta
variable value: 1.3962634015954638
distribution: inverse-gamma(shape=(7.5380958473641417,), loc=0.0007352666592390434,
                           scale=0.010948327943730485)
DescribeResult(nobs=1000, minmax=(0.0013736957558775464, 0.0058668524927937751)
               mean=0.0024071157285678033, variance=4.6546926831608823e-07,
               skewness=1.4313808780030552, kurtosis=2.8981562904327793)
```





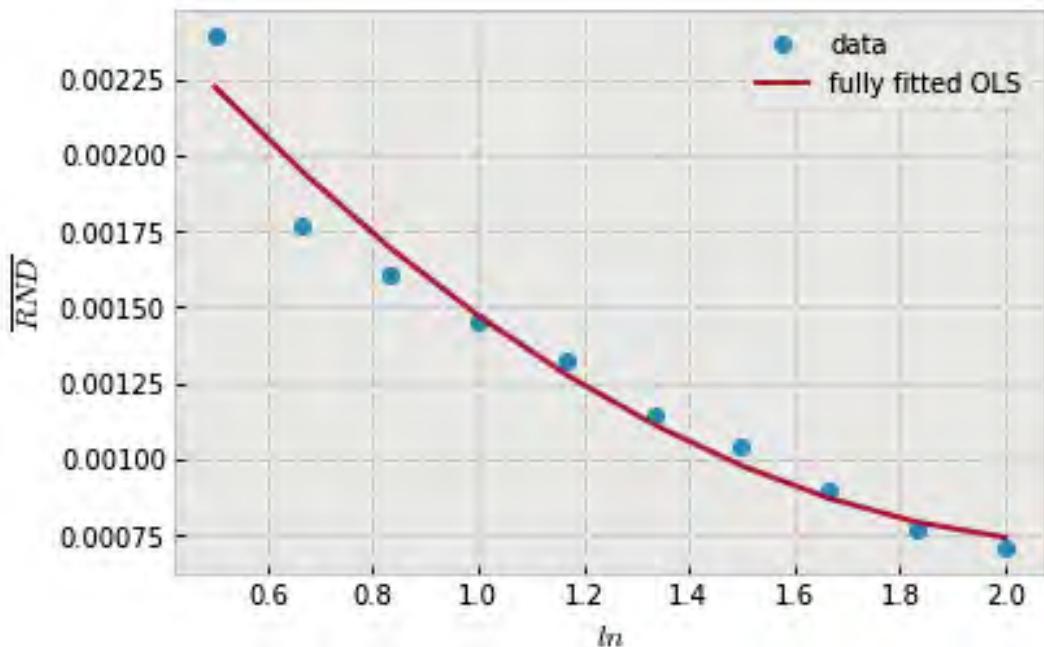
```
variable name: theta
variable value: 1.4835298641951802
distribution: inverse-gamma(shape=(5.9107895068292047,), loc=0.0008935974353300442,
                           scale=0.006660533529226161)
DescribeResult(nobs=1000, minmax=(0.0012357410278065327, 0.0085367874753306573)
               mean=0.0022298817559736881, variance=3.7436240712260079e-07,
               skewness=2.2031338172374855, kurtosis=12.653845078342659)
```



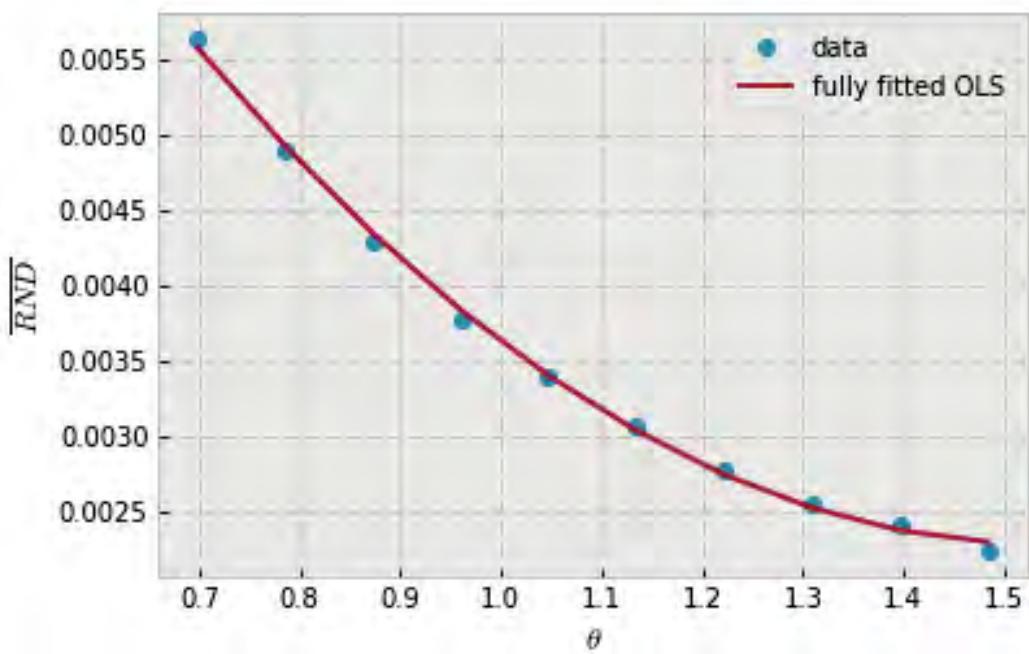
10.2 Parameter regression

```
In [26]: var_name = 'root_tip_density'  
        mean_reg('ln', var_name, 'poly2')  
        mean_reg('theta', var_name, 'poly2')  
        mean_reg('nB', var_name, 'poly1')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly2')  
        mean_reg('N', var_name, 'poly1')
```

variable name: ln
model: poly2
r-squared: 0.8790453328713239
explained variance: 0.8800809921099311



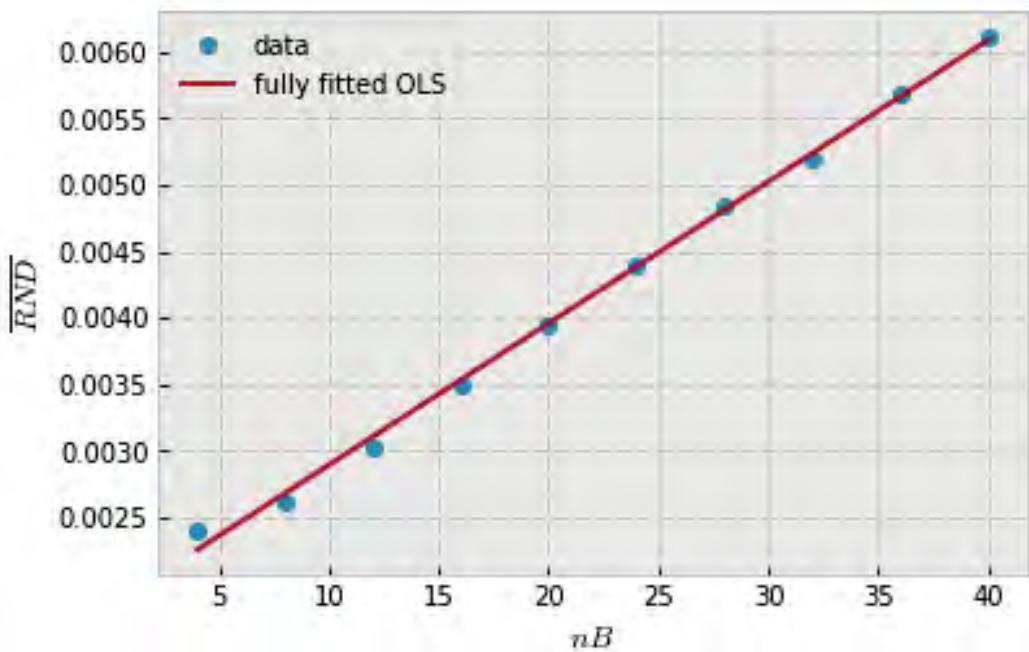
```
variable name: theta  
model: poly2  
r-squared: 0.9934182090244179  
explained variance: 0.9934182363856531
```



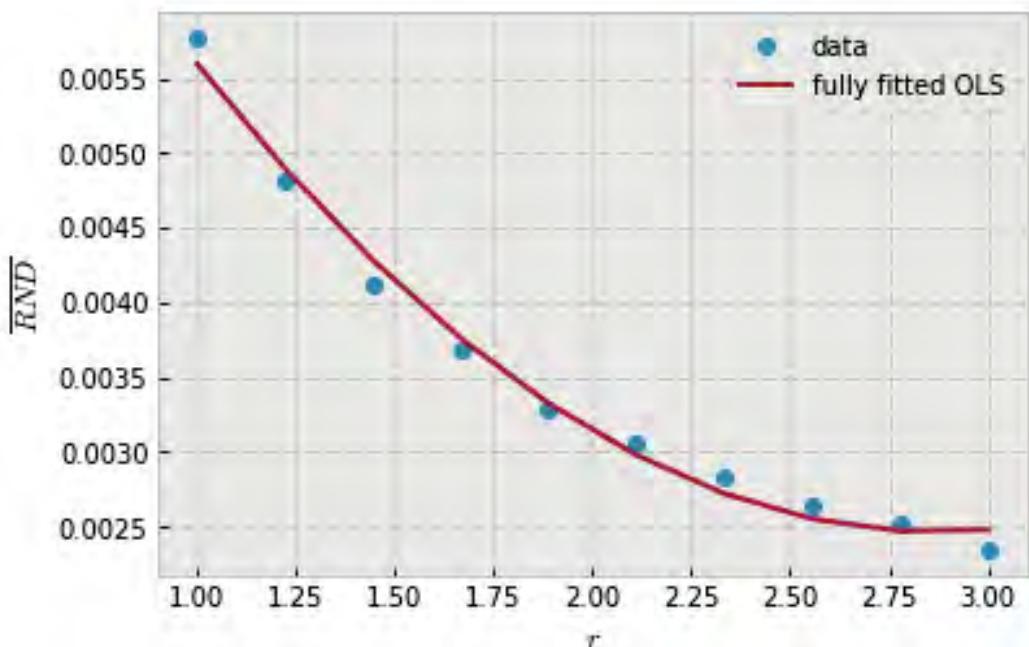
```

variable name: nB
model: poly1
r-squared: 0.994778851400622
explained variance: 0.9947967989214536

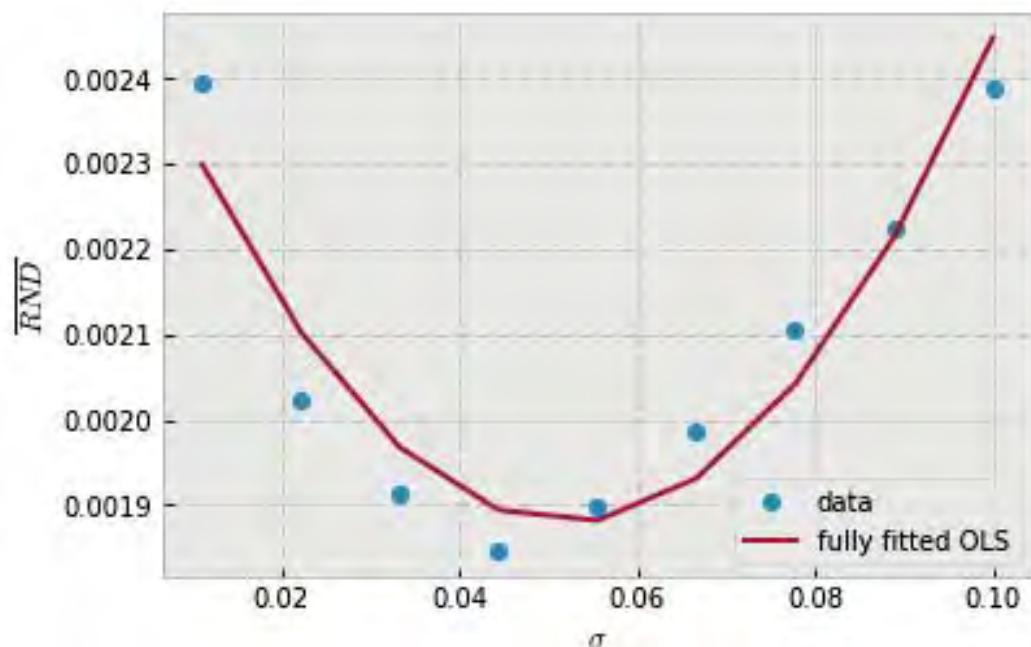
```



```
variable name: r
model: poly2
r-squared: 0.9574765352127268
explained variance: 0.9575002782739763
```



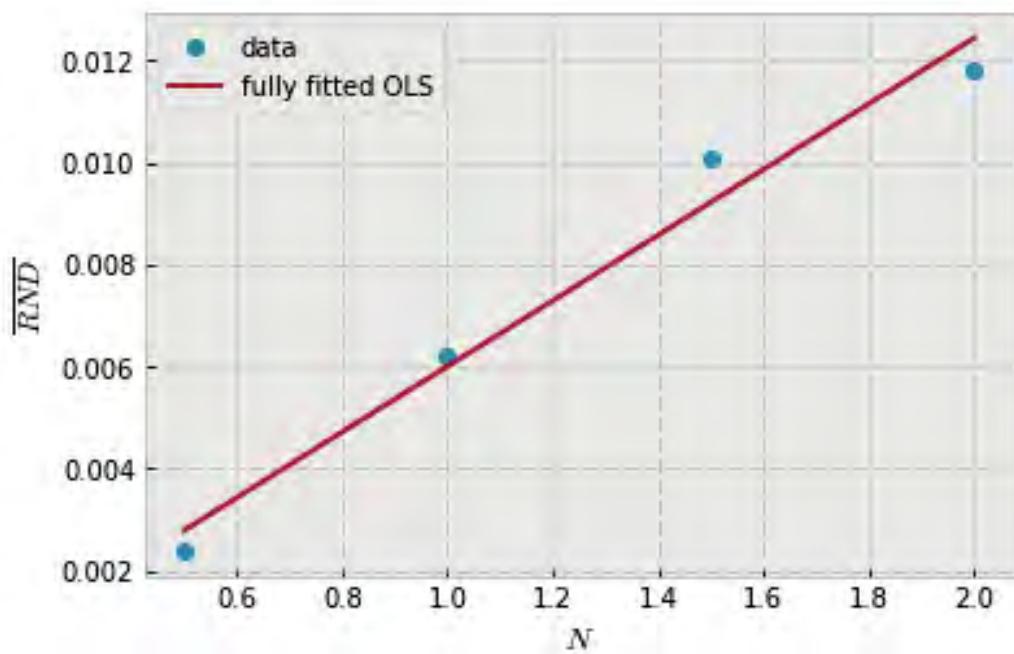
```
variable name: sigma
model: poly2
r-squared: 0.5901729182904394
explained variance: 0.5911316030940146
```



```

variable name: N
model: poly1
r-squared: 0.8530301495817244
explained variance: 0.8715872407350764

```



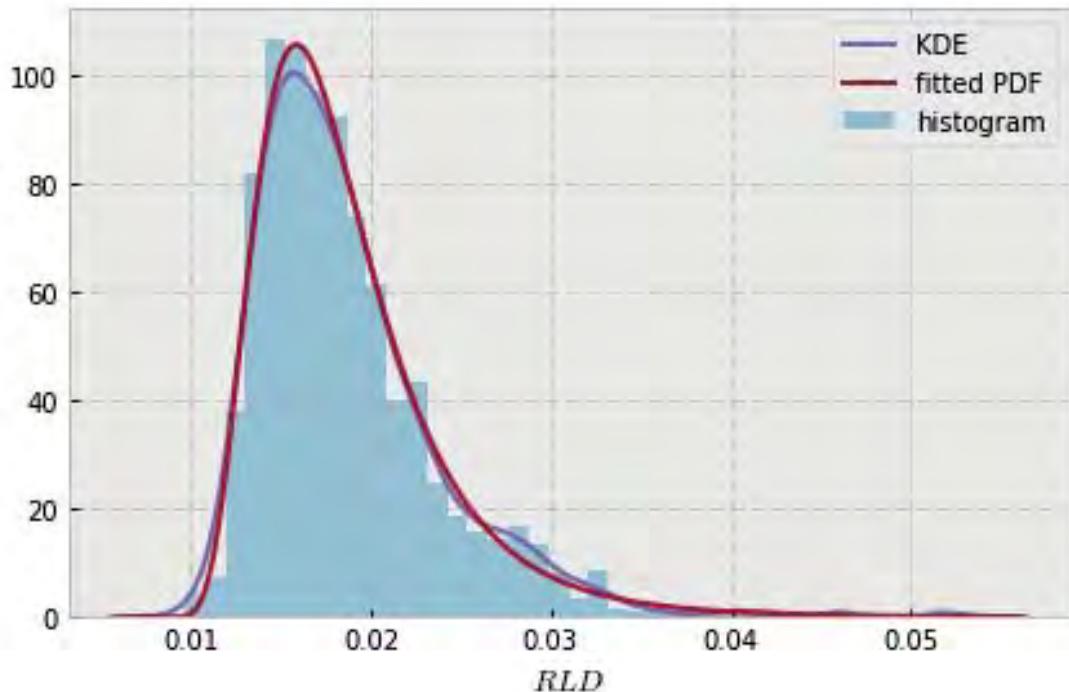
11 Root length relative to convex hull

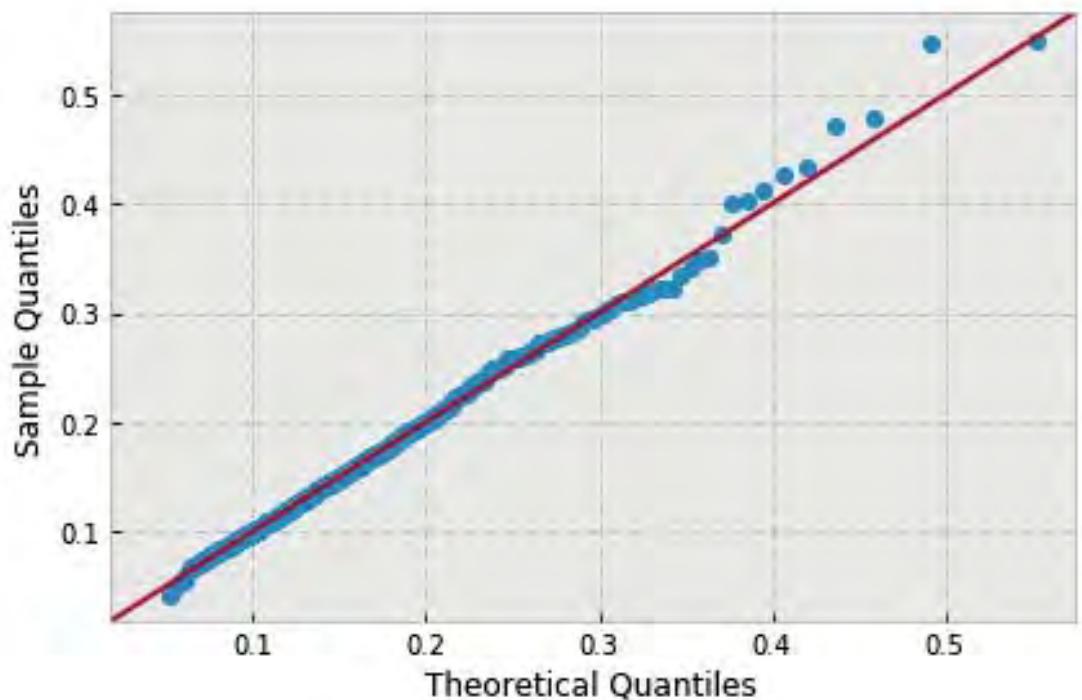
11.1 Probability distributions

```
In [27]: plot_dist('total_root_length_per_convex_hull_volume', 'inverse-gamma')

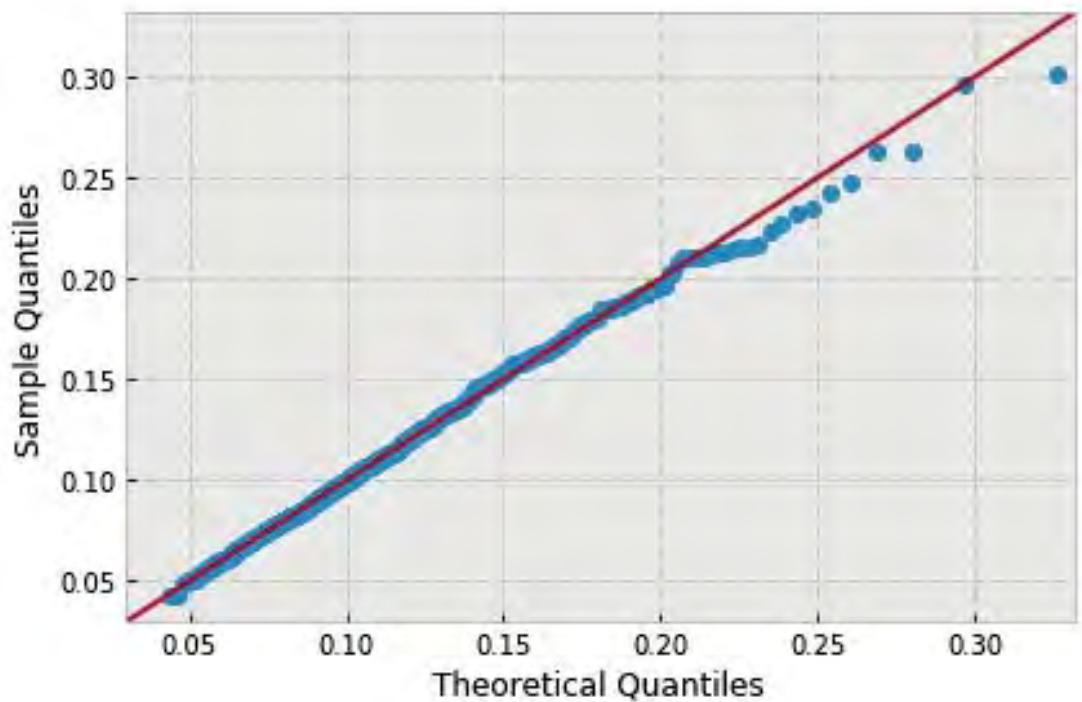
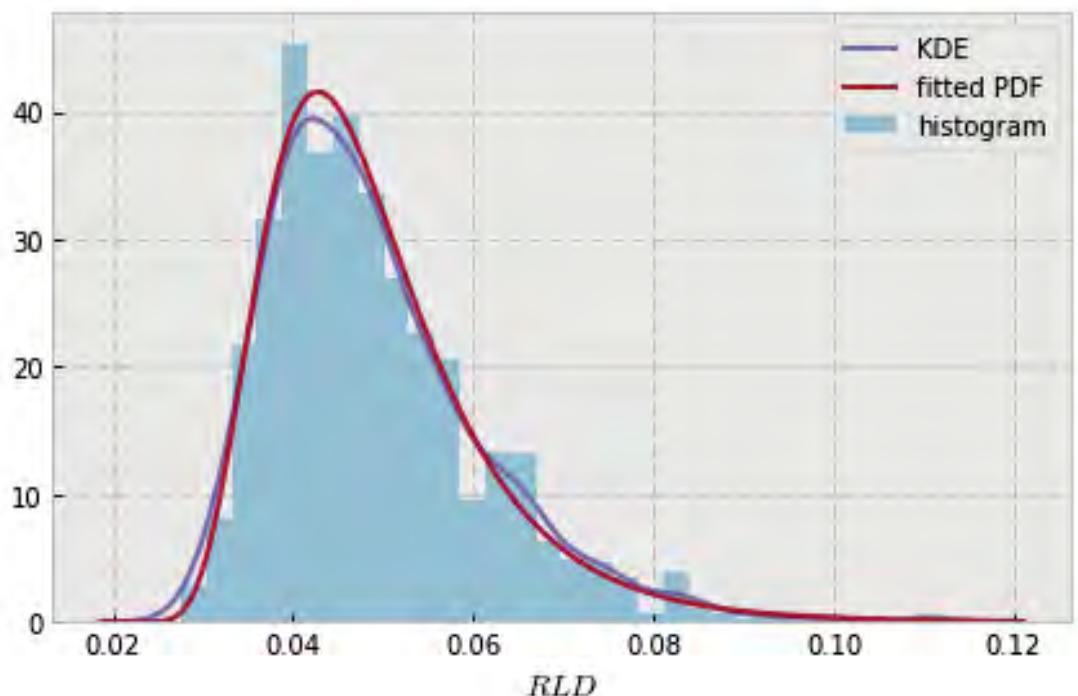
/home/lpetrich/.local/share/anaconda3/envs/Master/lib/python3.6/site-packages/scipy/stats/_distn_infra.py:1405: UserWarning: The distribution inverse-gamma is not supported by the R interface. It is recommended to use rgamma instead.
  warnings.warn("The distribution %s is not supported by the R interface." % name)

variable name: N
variable value: 0.5
distribution: inverse-gamma(shape=(7.6518480402278515,), loc=0.006121216511868739,
    scale=0.08405172276712261)
DescribeResult(nobs=1000, minmax=(0.009662484871099249, 0.052279764334324588)
    mean=0.018757069990123023, variance=2.8036194059545509e-05,
    skewness=1.7988100982753636, kurtosis=5.5147390189864876)
```

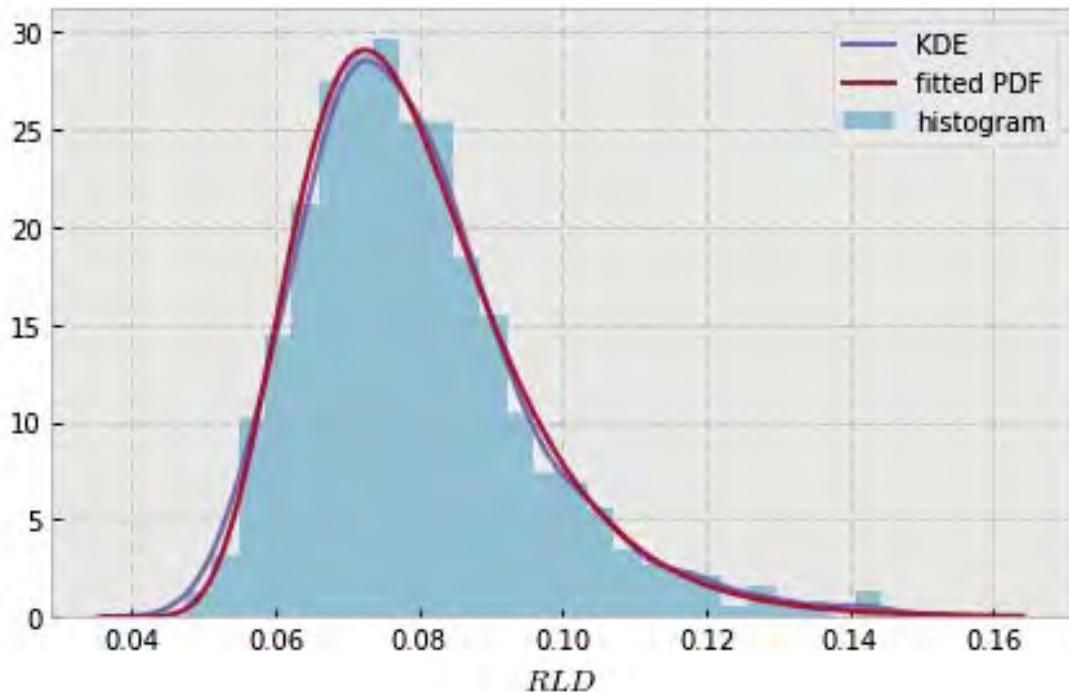


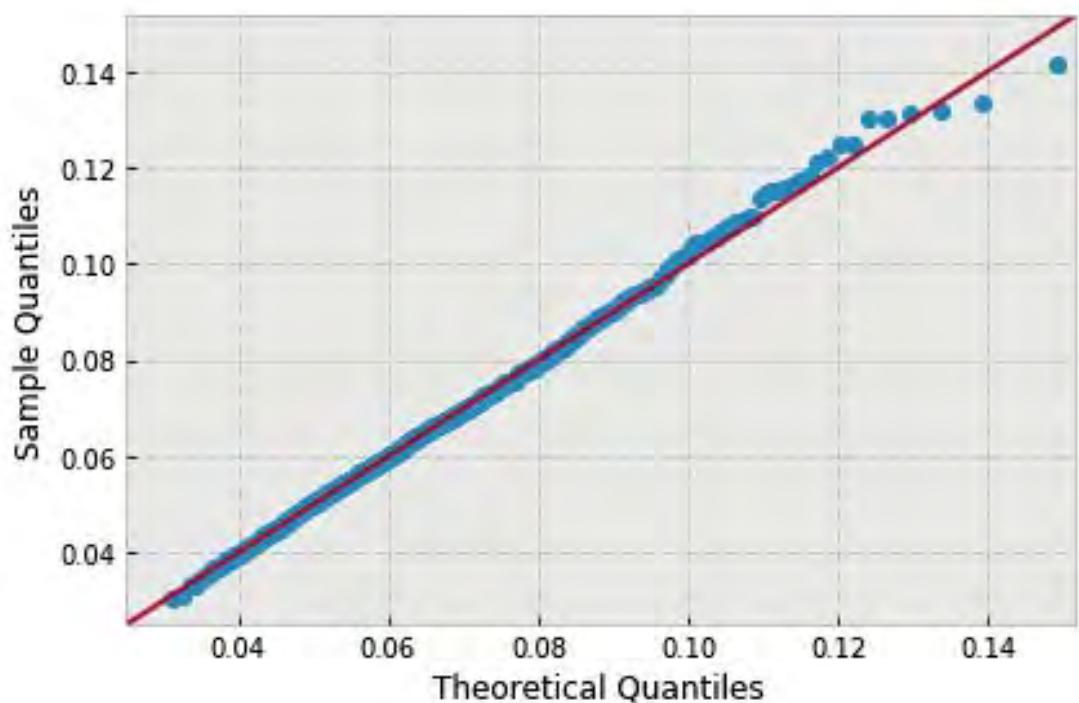


```
variable name: N
variable value: 1.0
distribution: inverse-gamma(shape=(10.203010749065053,), loc=0.013707658578072884,
scale=0.32534931530364386)
DescribeResult(nobs=1000, minmax=(0.027544827710664993, 0.11202021207302901)
mean=0.049037917799901833, variance=0.00014562424930398182,
skewness=1.2241468392845467, kurtosis=2.1793870550837644)
```

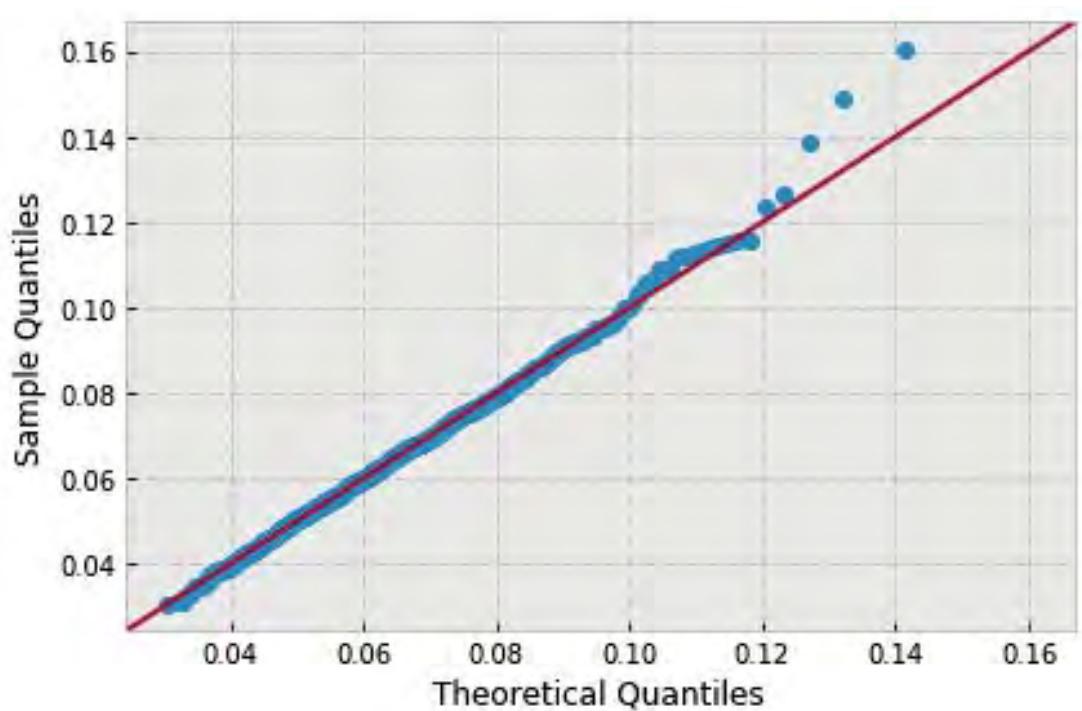
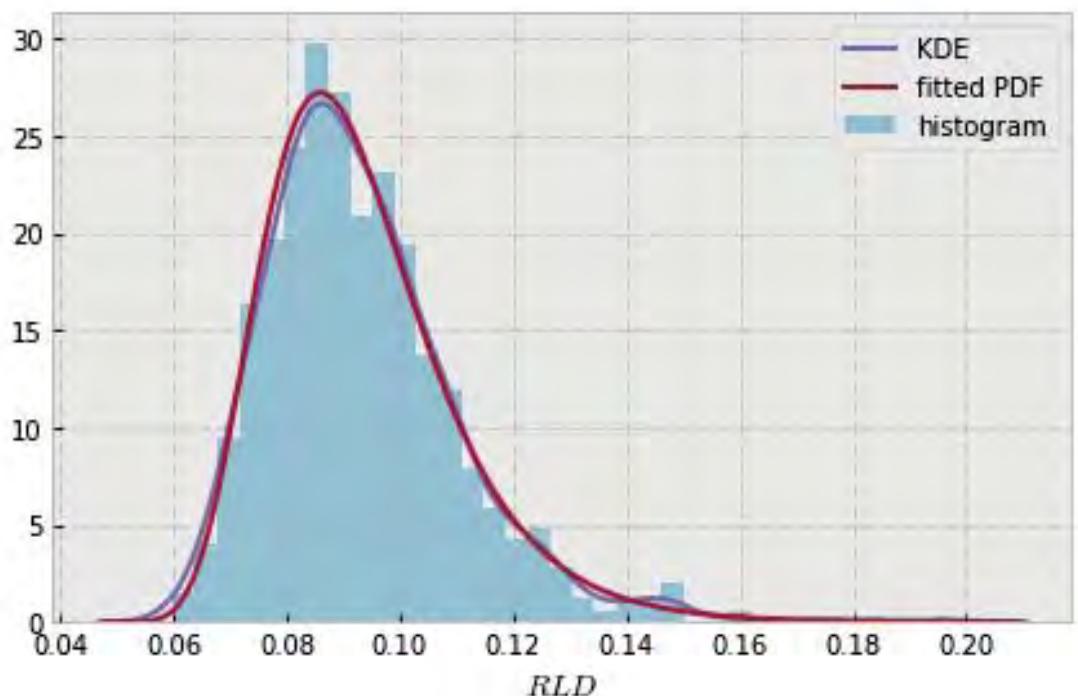


```
variable name: N
variable value: 1.5
distribution: inverse-gamma(shape=(16.487371145906451,), loc=0.018689014940395217,
                           scale=0.9408795627604083)
DescribeResult(nobs=1000, minmax=(0.047400724200290484, 0.15200886697524249)
               mean=0.079442722363798457, variance=0.00025523994985104373,
               skewness=1.0808304230113916, kurtosis=1.7555379141930878)
```

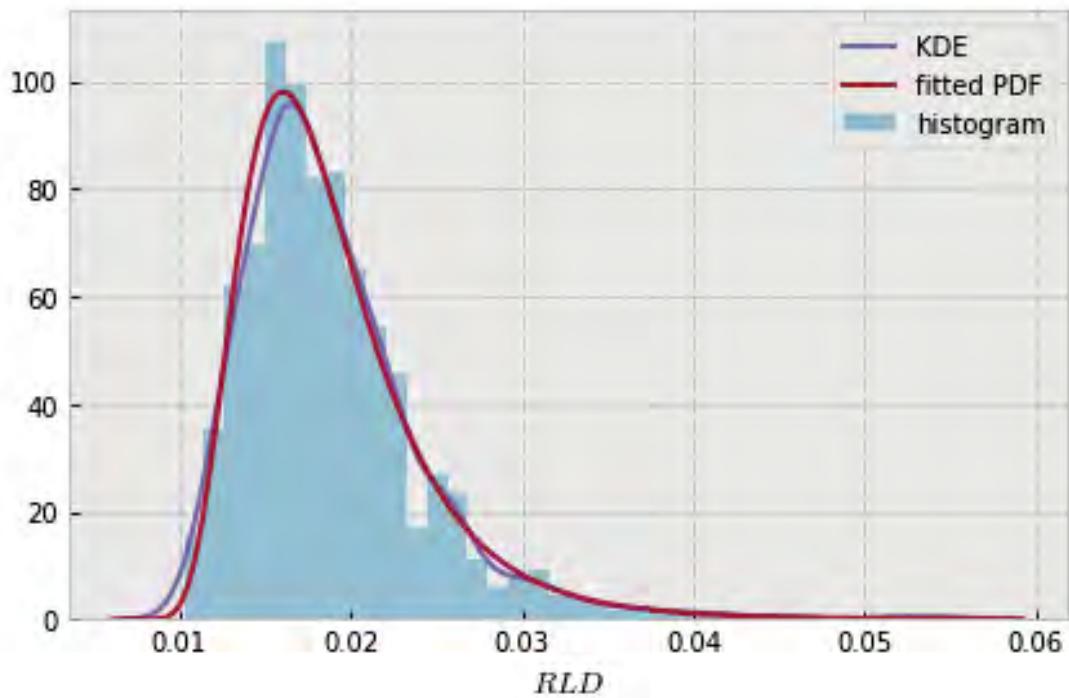


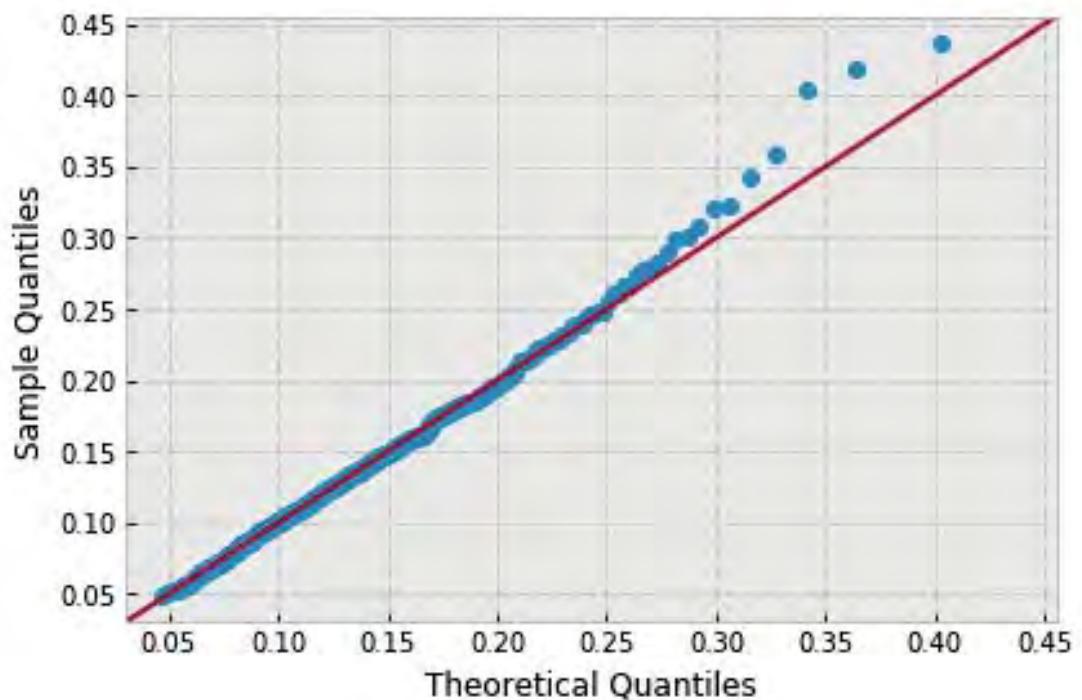


```
variable name: N
variable value: 2.0
distribution: inverse-gamma(shape=(17.075375170461868,), loc=0.027386483918896753,
scale=1.0592347256339063)
DescribeResult(nobs=1000, minmax=(0.059844527844415164, 0.19722867015622744)
mean=0.093287143120182645, variance=0.00029271241190512057,
skewness=1.2218224615447102, kurtosis=3.0919744339336486)
```

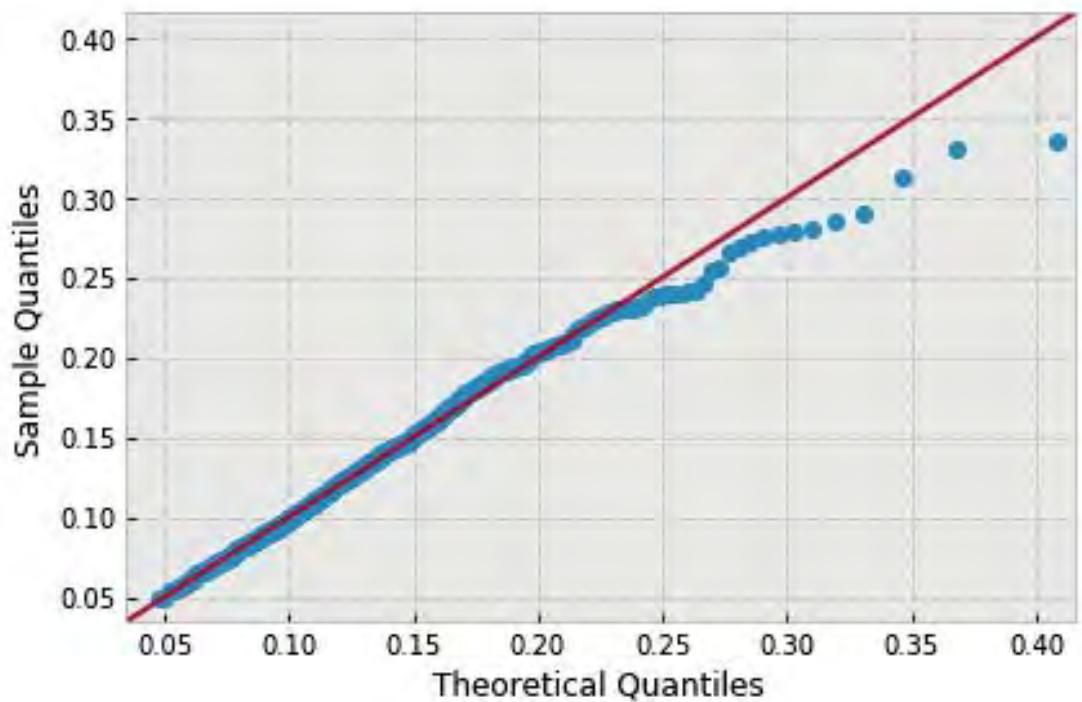
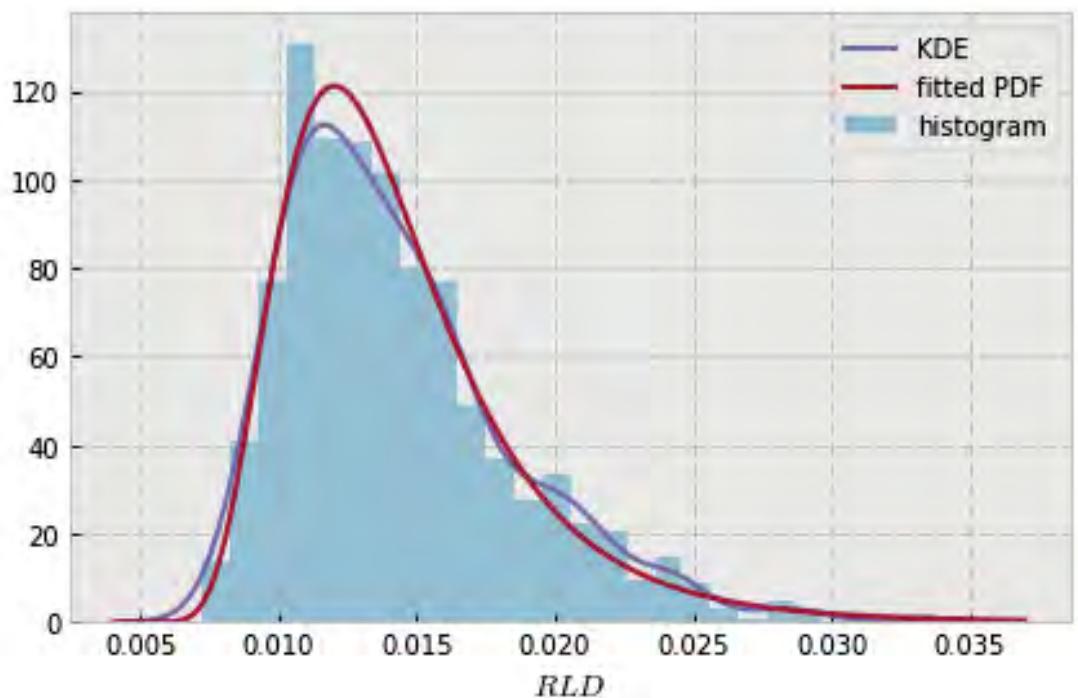


```
variable name: ln
variable value: 0.5
distribution: inverse-gamma(shape=(9.067090635144524,), loc=0.004523068667862116,
    scale=0.11607973530129564)
DescribeResult(nobs=1000, minmax=(0.01027839895125247, 0.055113833568200425)
    mean=0.018918595079620276, variance=3.0090361855107915e-05,
    skewness=1.8574896387724635, kurtosis=6.34888642962858)
```

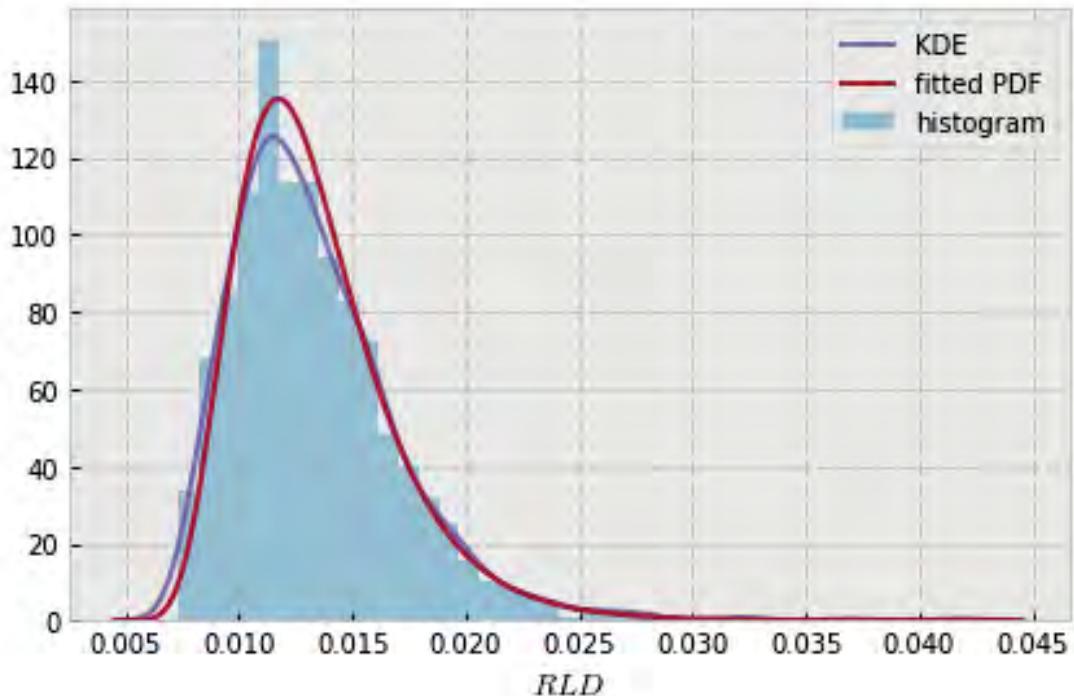


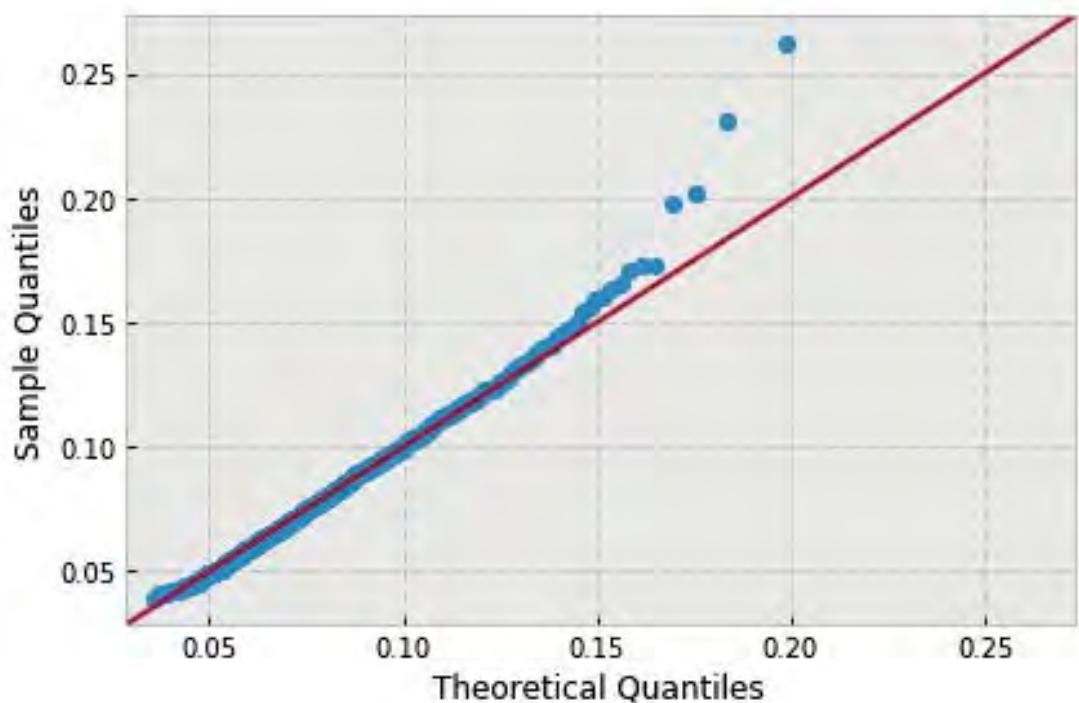


```
variable name: ln
variable value: 0.6666666666666666
distribution: inverse-gamma(shape=(8.9978185315812667,), loc=0.0027334134210720033,
                           scale=0.0928677365697815)
DescribeResult(nobs=1000, minmax=(0.0072383286619386346, 0.033817842464274939)
               mean=0.014328023656774831, variance=1.7618458918352365e-05,
               skewness=1.1309546653470595, kurtosis=1.4986232242821753)
```

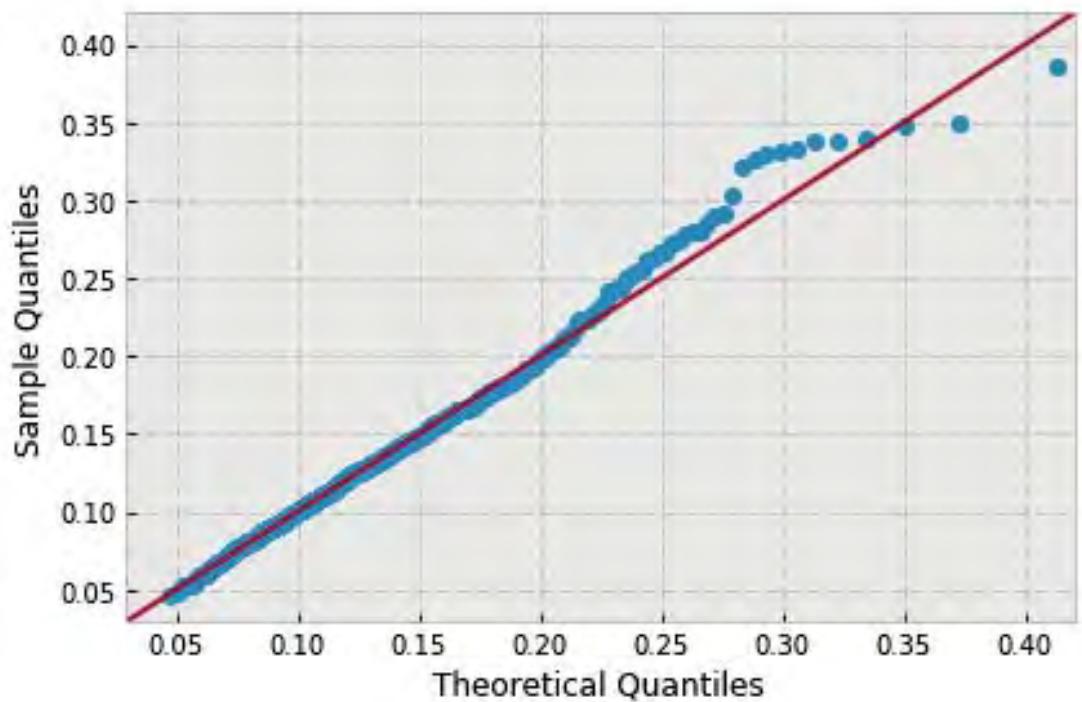
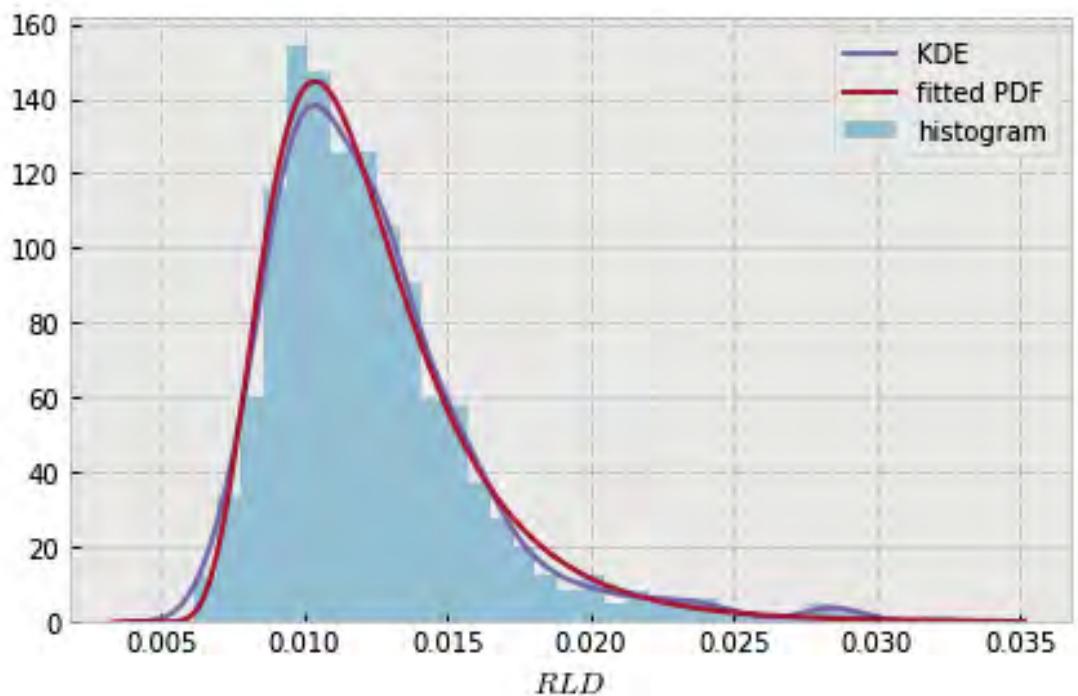


```
variable name: ln
variable value: 0.8333333333333333
distribution: inverse-gamma(shape=(13.735372852221587,), loc=0.001232715093535152,
                           scale=0.15473388039309355)
DescribeResult(nobs=1000, minmax=(0.0073530336810886856, 0.041750366291065606)
               mean=0.013359179305783399, variance=1.4443895773584995e-05,
               skewness=1.658890992818328, kurtosis=5.9537091384226475)
```

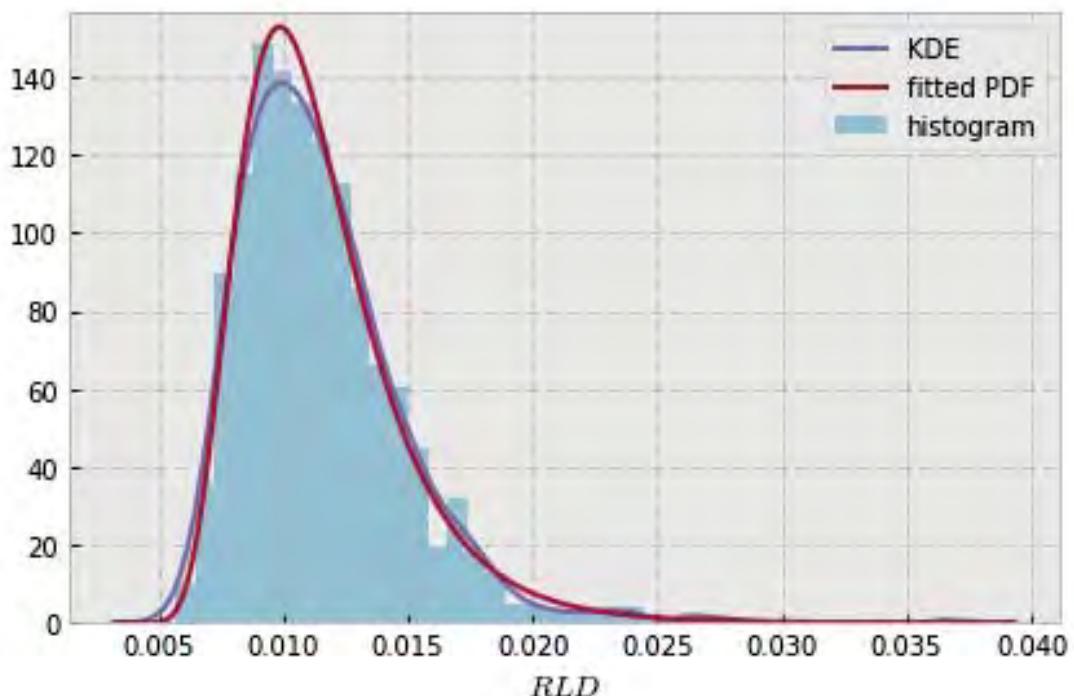


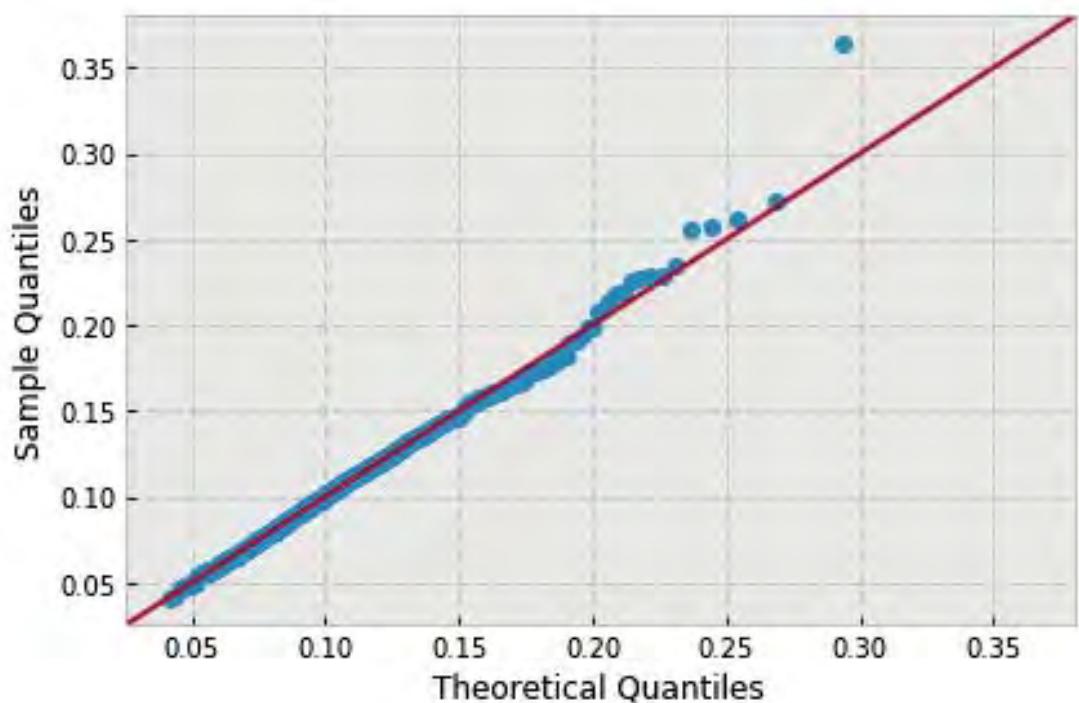


```
variable name: ln
variable value: 1.0
distribution: inverse-gamma(shape=(8.9400679694773864,), loc=0.0026274326783980607,
                           scale=0.07708672721409959)
DescribeResult(nobs=1000, minmax=(0.0061845049904421046, 0.03235391033141443)
               mean=0.012340746654129389, variance=1.3821562899332335e-05,
               skewness=1.6786244713449485, kurtosis=4.2453684974623505)
```

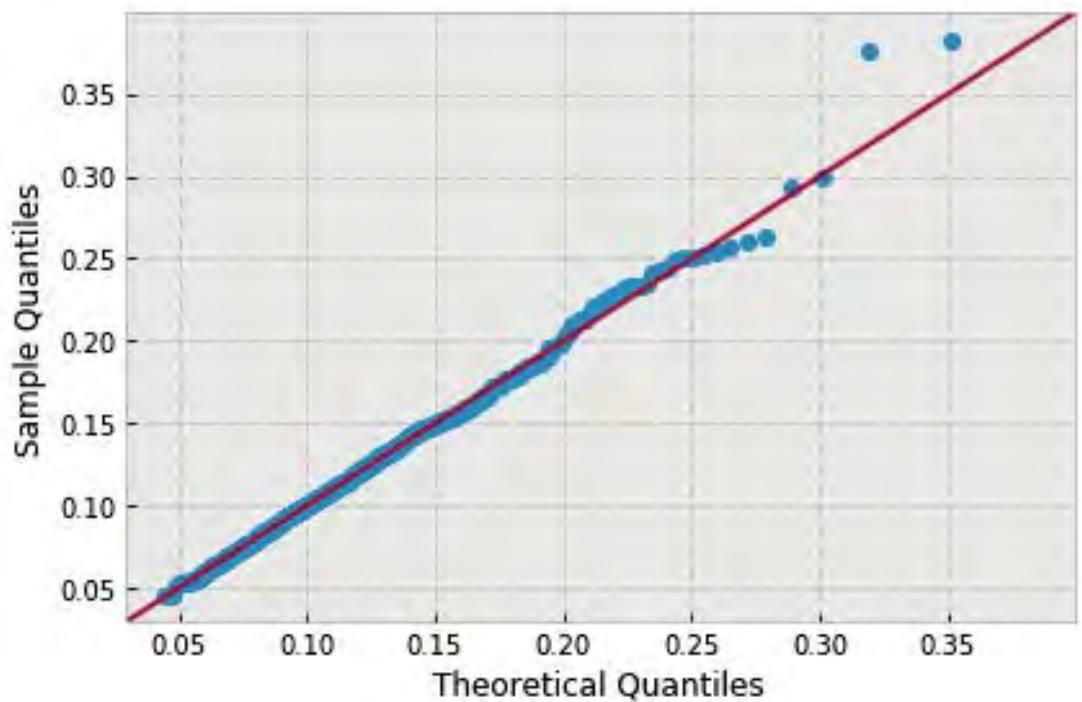
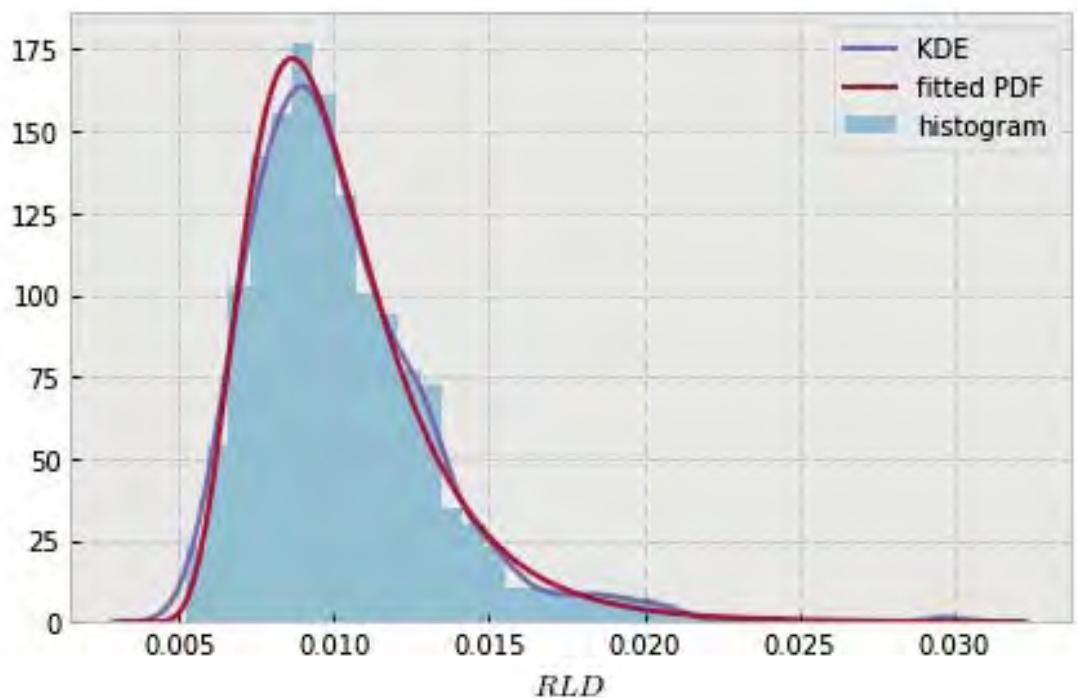


```
variable name: ln
variable value: 1.1666666666666665
distribution: inverse-gamma(shape=(10.832277490379902,), loc=0.0016978197922584513,
                           scale=0.09656537475100972)
DescribeResult(nobs=1000, minmax=(0.0056847187648307951, 0.036855574646734612)
               mean=0.01151840612734603, variance=1.0944674133806671e-05,
               skewness=1.5899038805728234, kurtosis=5.510056346616068)
```

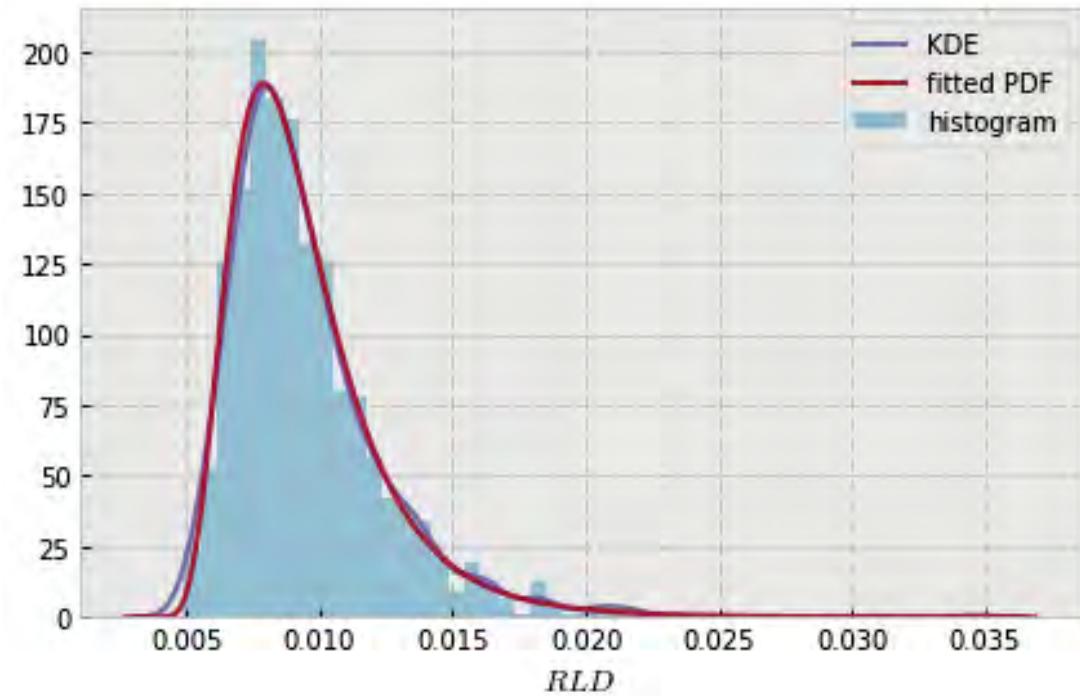


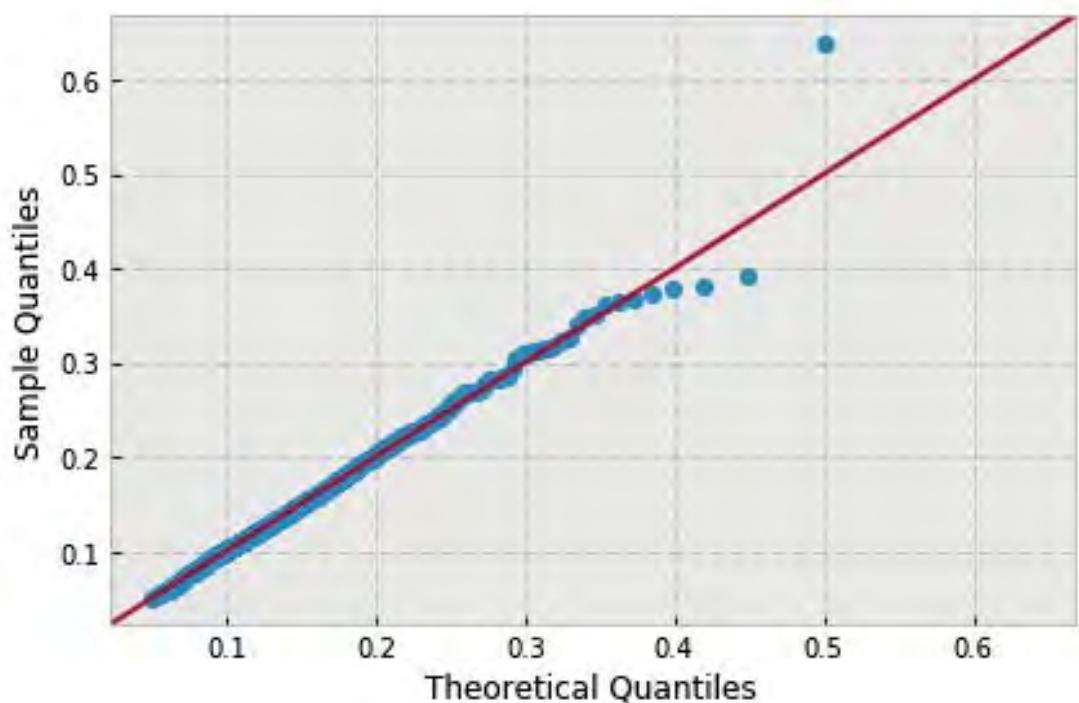


```
variable name: ln
variable value: 1.333333333333333
distribution: inverse-gamma(shape=(9.7754290369978865,), loc=0.0018158563018805525,
                           scale=0.07371964153320226)
DescribeResult(nobs=1000, minmax=(0.0052140969033877757, 0.029992160716120958)
               mean=0.010215604800915851, variance=9.0343429643139177e-06,
               skewness=1.5950229928242936, kurtosis=4.8421876168626135)
```

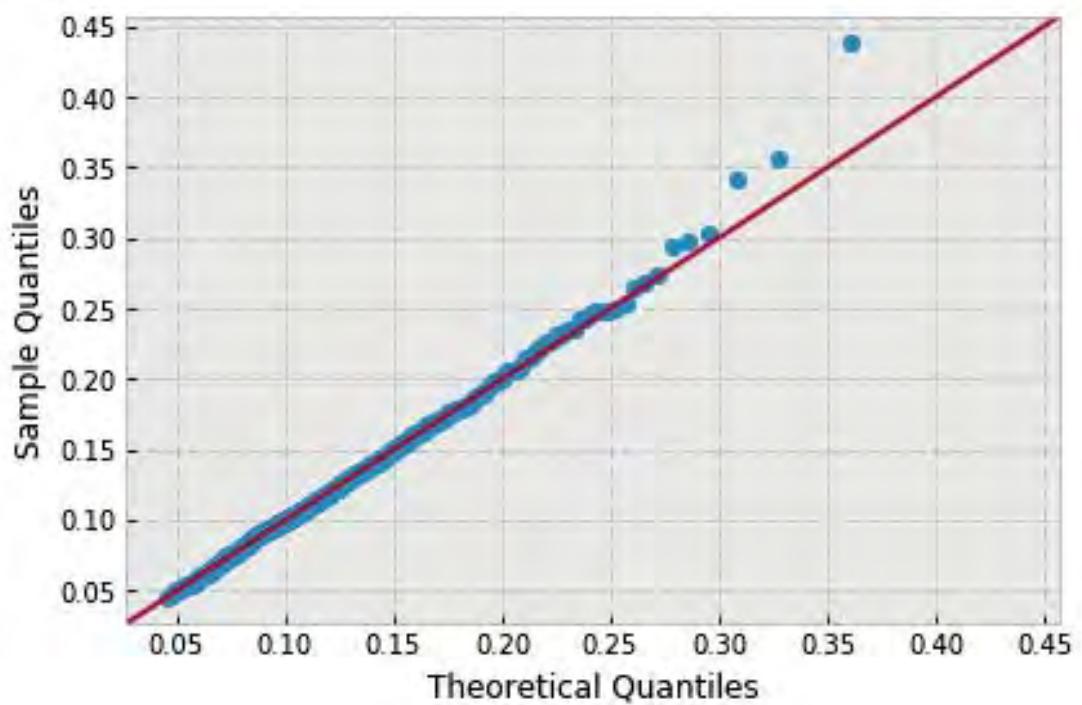
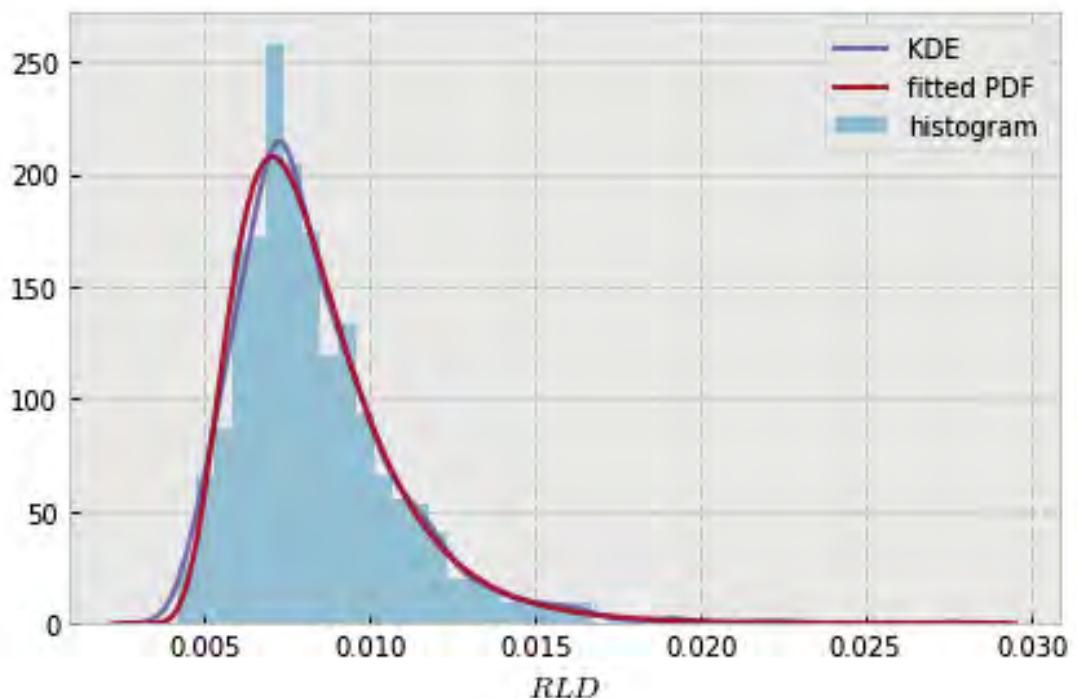


```
variable name: ln
variable value: 1.5
distribution: inverse-gamma(shape=(8.0548304453443631,), loc=0.0022996687881148992,
                           scale=0.05071130612564276)
DescribeResult(nobs=1000, minmax=(0.0048960332076242868, 0.034647388991293472)
               mean=0.0094876854721283408, variance=8.5004692748217146e-06,
               skewness=1.869434377423541, kurtosis=7.289415099938886)
```

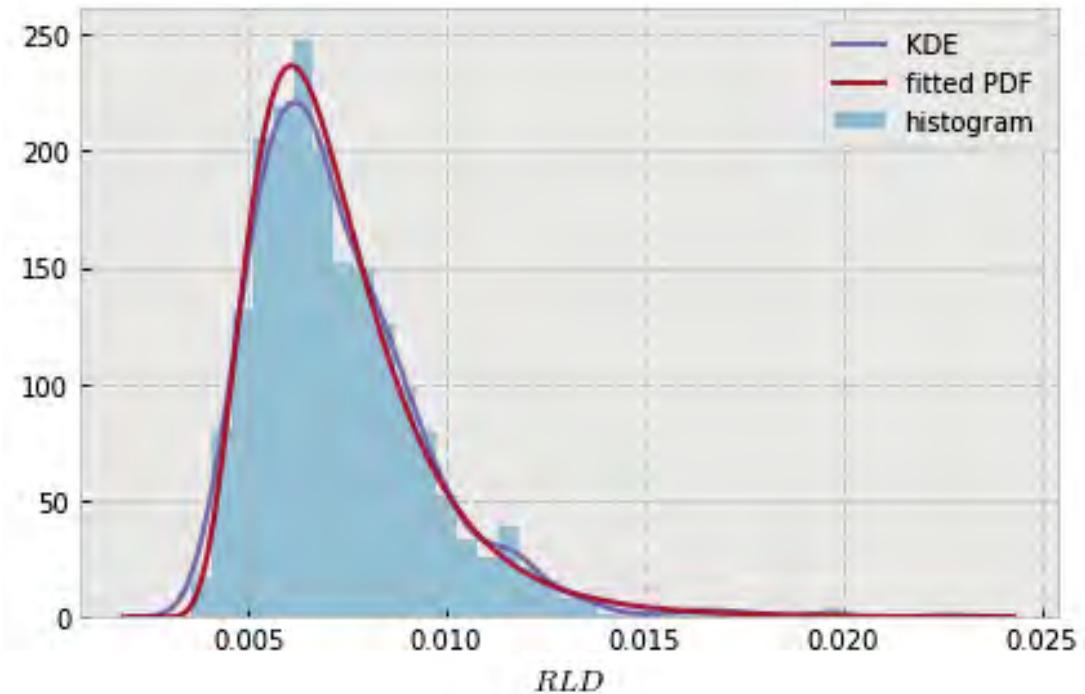


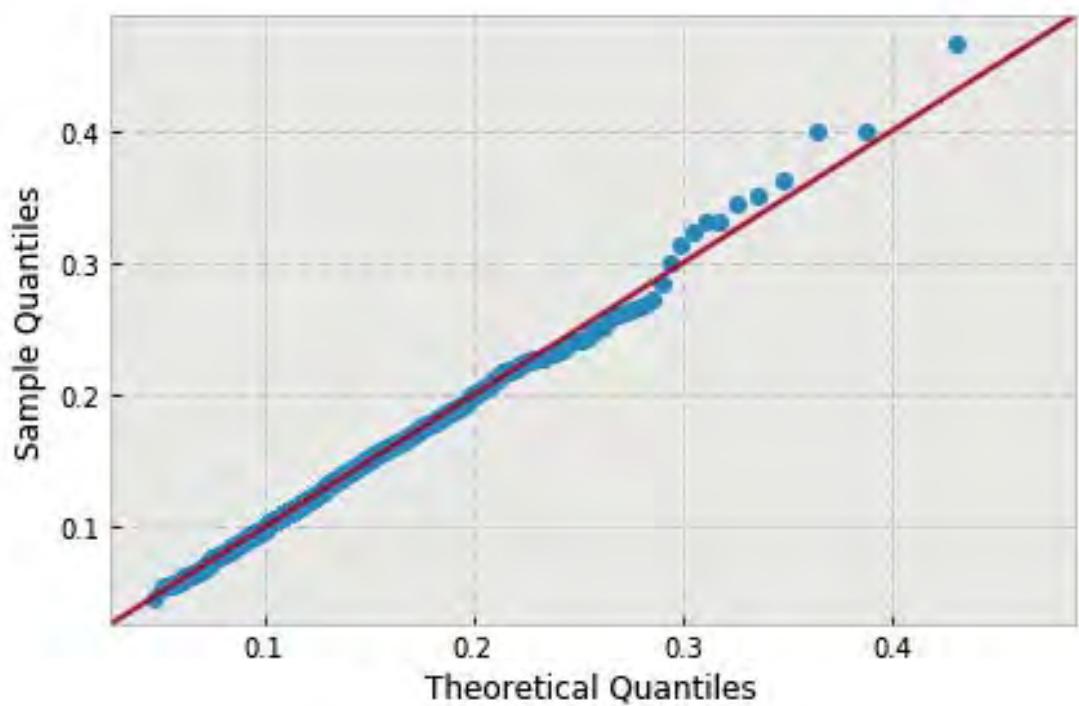


```
variable name: ln
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(9.6295263019626436,), loc=0.0014249408494441354,
                           scale=0.05983472466990065)
DescribeResult(nobs=1000, minmax=(0.0041673310377313231, 0.02760877517211692)
               mean=0.0083608916611420791, variance=6.432864246099307e-06,
               skewness=1.763407308553682, kurtosis=6.141610305895961)
```

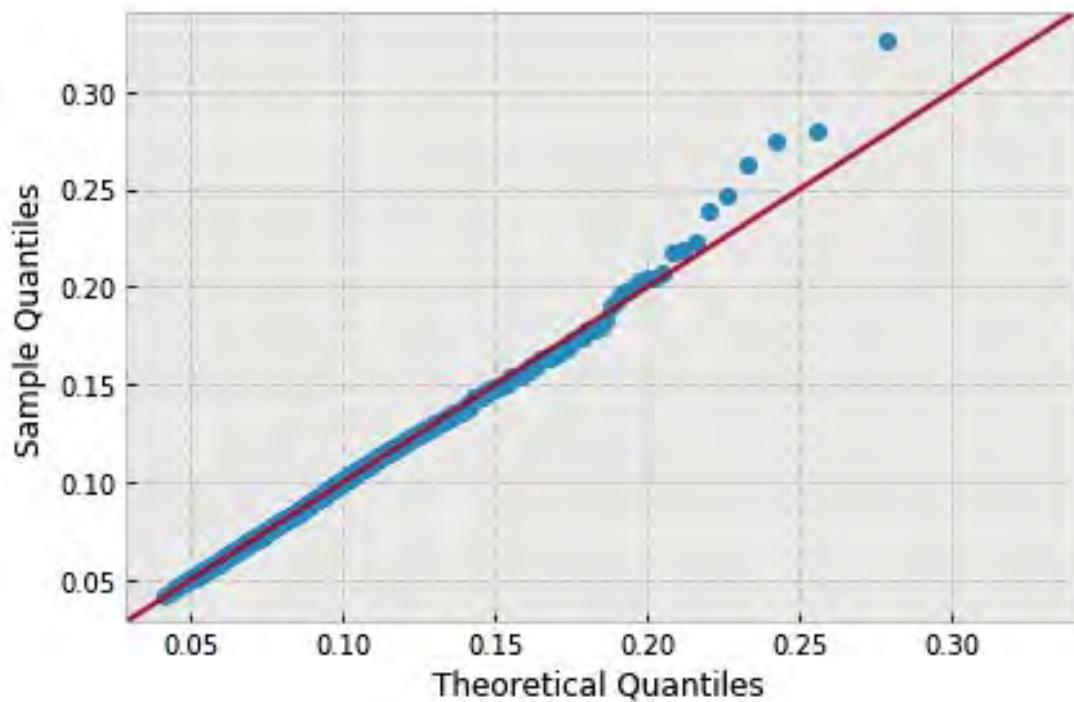
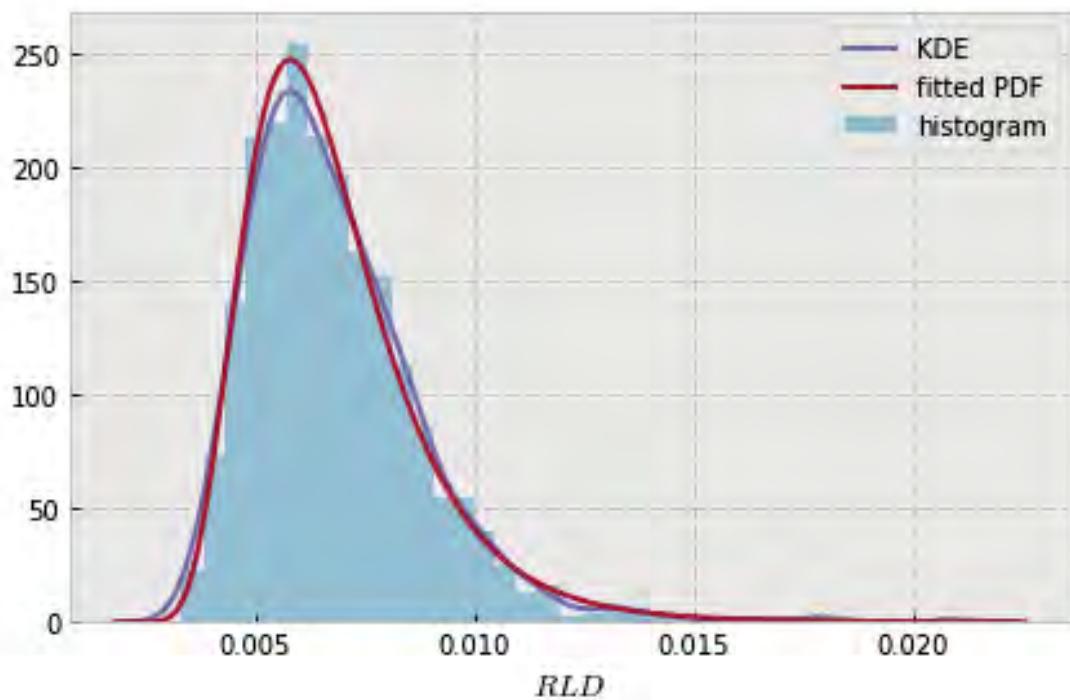


```
variable name: ln
variable value: 1.833333333333333
distribution: inverse-gamma(shape=(8.730551379470139,), loc=0.0014132463709173762,
                           scale=0.04550216521126606)
DescribeResult(nobs=1000, minmax=(0.0035555158388325256, 0.022616376617418794)
               mean=0.0072969695695494375, variance=5.0535217478446979e-06,
               skewness=1.6755805164044861, kurtosis=5.278246558590807)
```

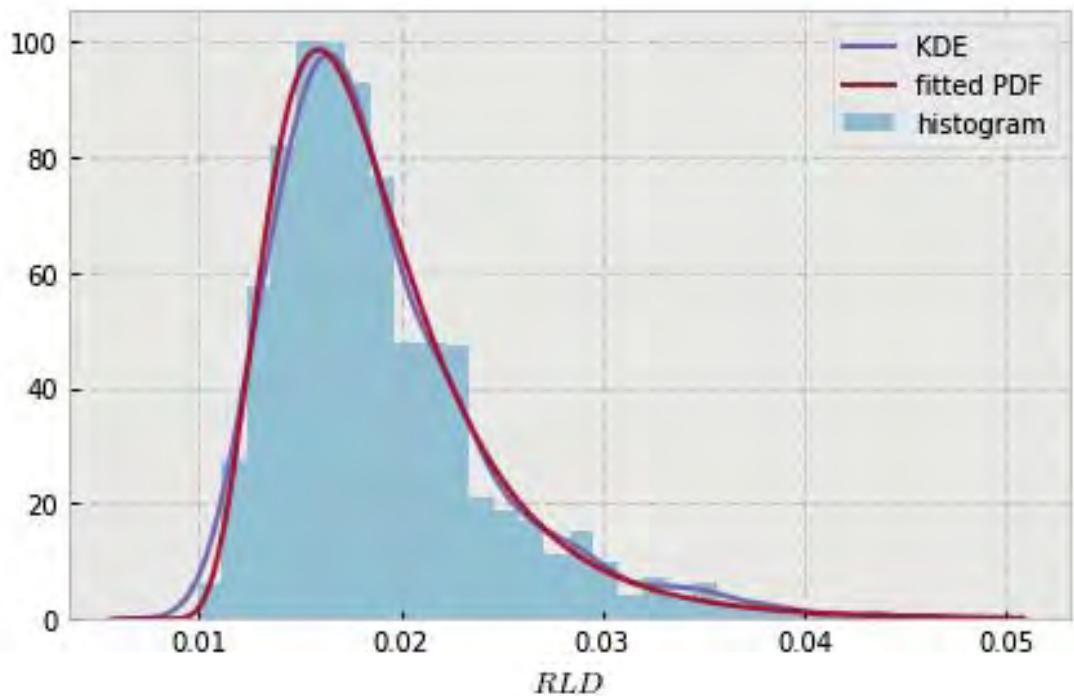


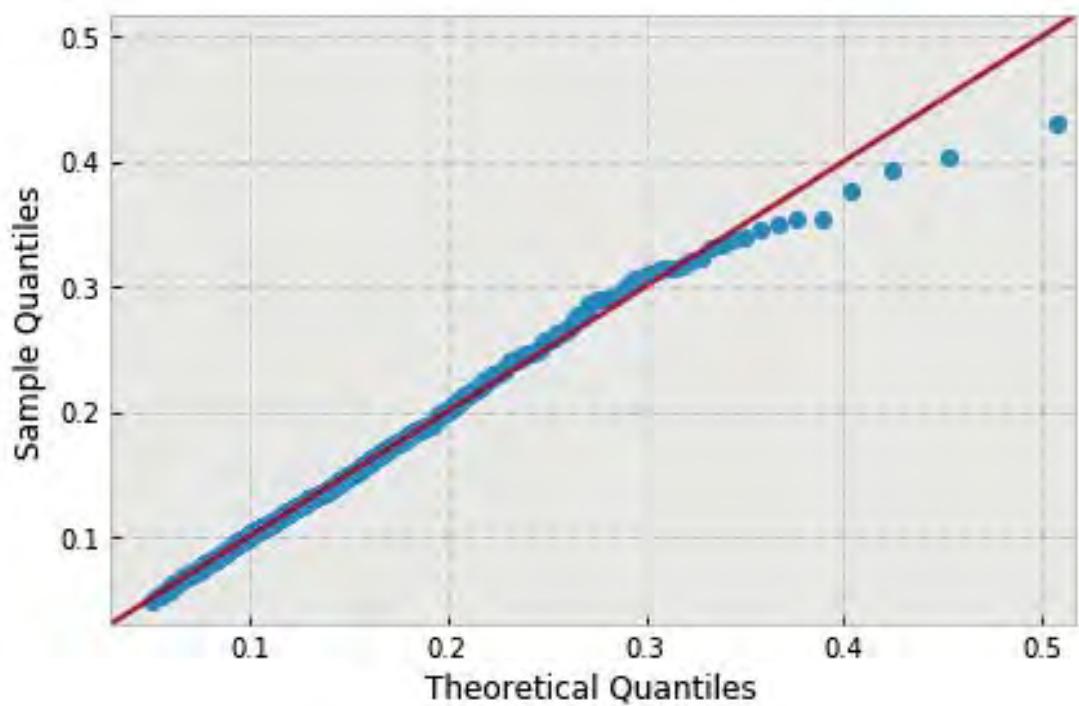


```
variable name: ln
variable value: 2.0
distribution: inverse-gamma(shape=(11.1739821932306,), loc=0.000691938928395688,
scale=0.062350931986946884)
DescribeResult(nobs=1000, minmax=(0.003345573404739391, 0.021016079680042789)
mean=0.0068383412951932886, variance=4.1364114506719687e-06,
skewness=1.6252837932926059, kurtosis=5.410389946263745)
```

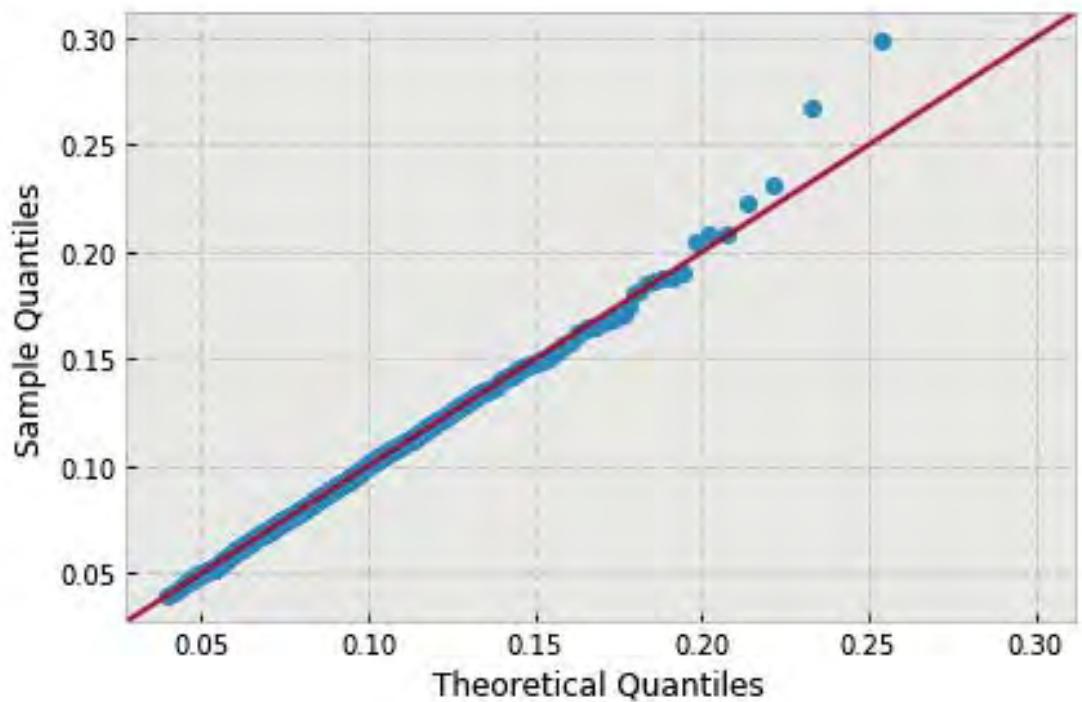
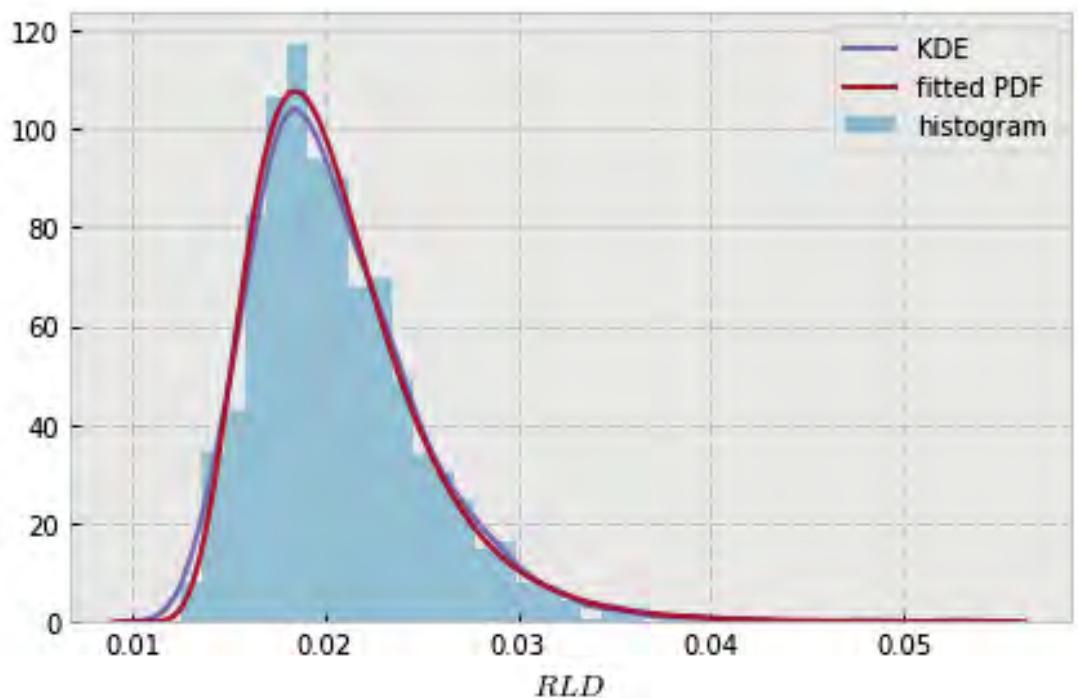


```
variable name: nB
variable value: 4.0
distribution: inverse-gamma(shape=(8.0036279530913248,), loc=0.00523311797060554,
    scale=0.09628019255136169)
DescribeResult(nobs=1000, minmax=(0.0099026856795272501, 0.04671254058030435)
    mean=0.018971290732750206, variance=2.997792109154598e-05,
    skewness=1.4362054745126767, kurtosis=2.6574767120141036)
```

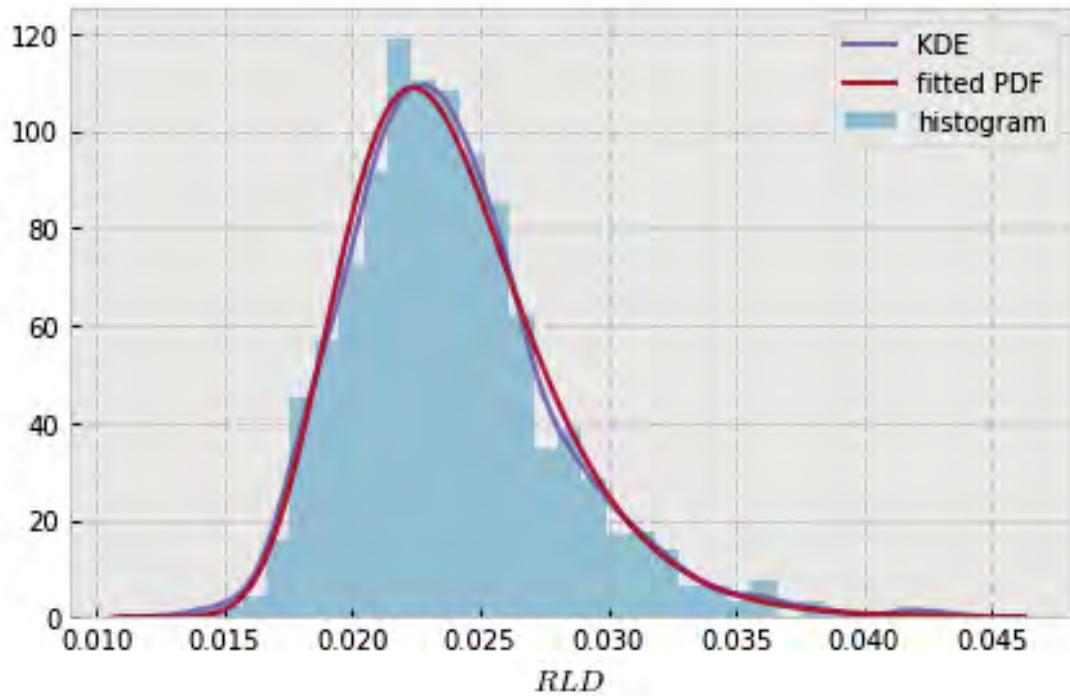


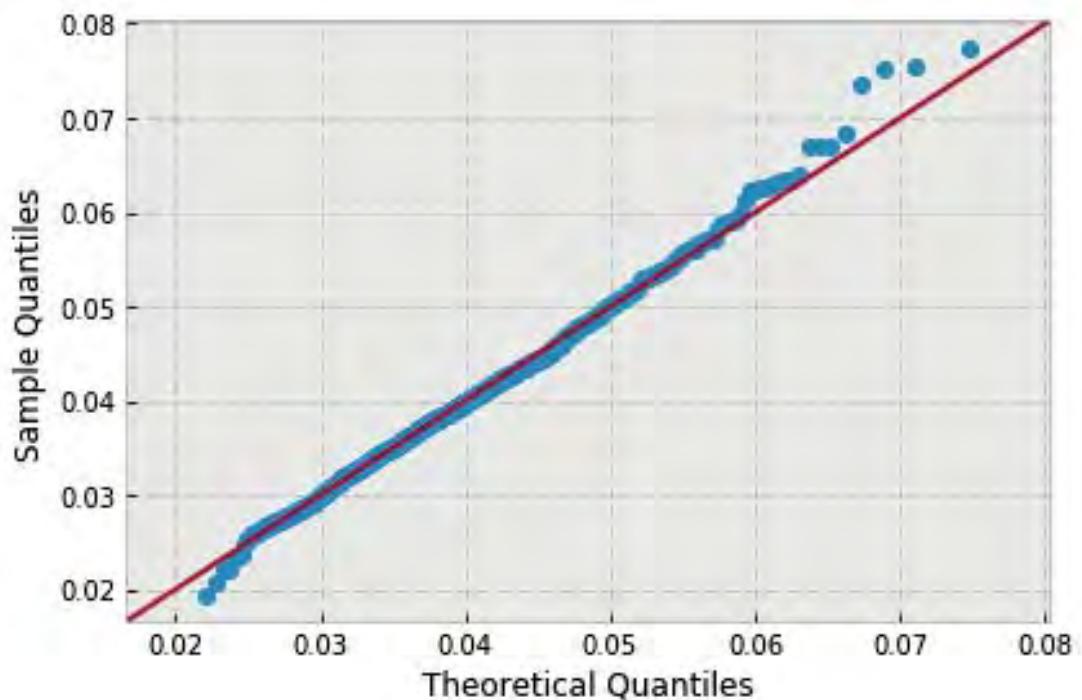


```
variable name: nB
variable value: 8.0
distribution: inverse-gamma(shape=(11.81870955225591,), loc=0.006312035521577535,
                           scale=0.15574333135779905)
DescribeResult(nobs=1000, minmax=(0.012561616283442488, 0.052824723730203624)
               mean=0.020707763239050884, variance=2.1192411458345449e-05,
               skewness=1.47546620328036, kurtosis=4.601212895123473)
```

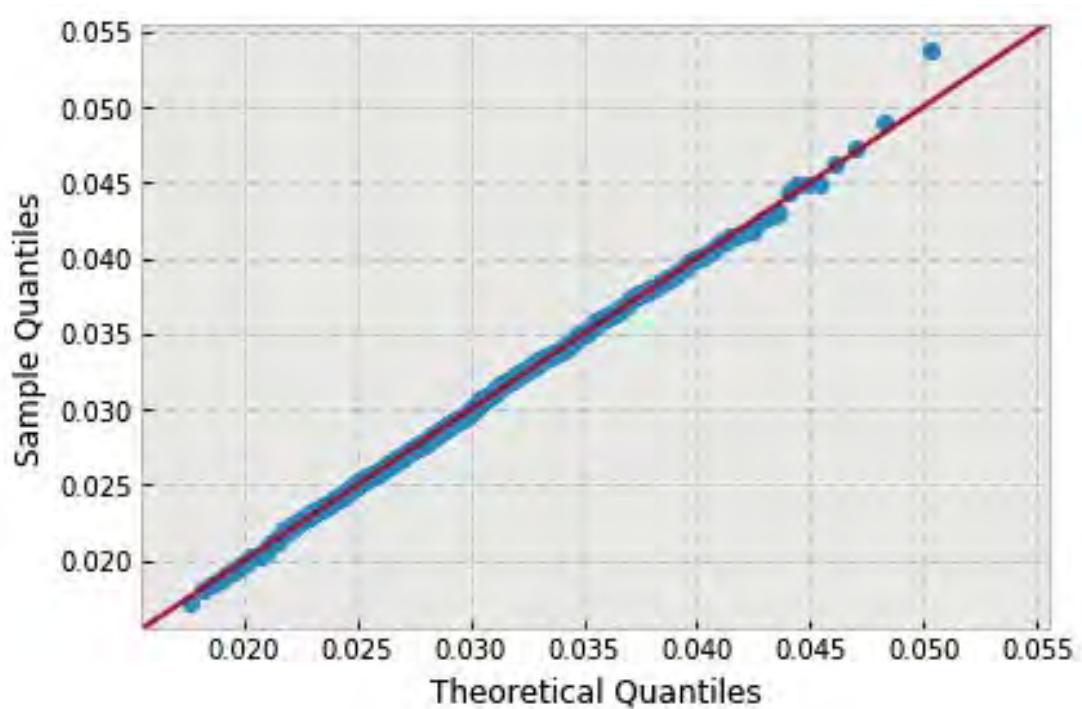
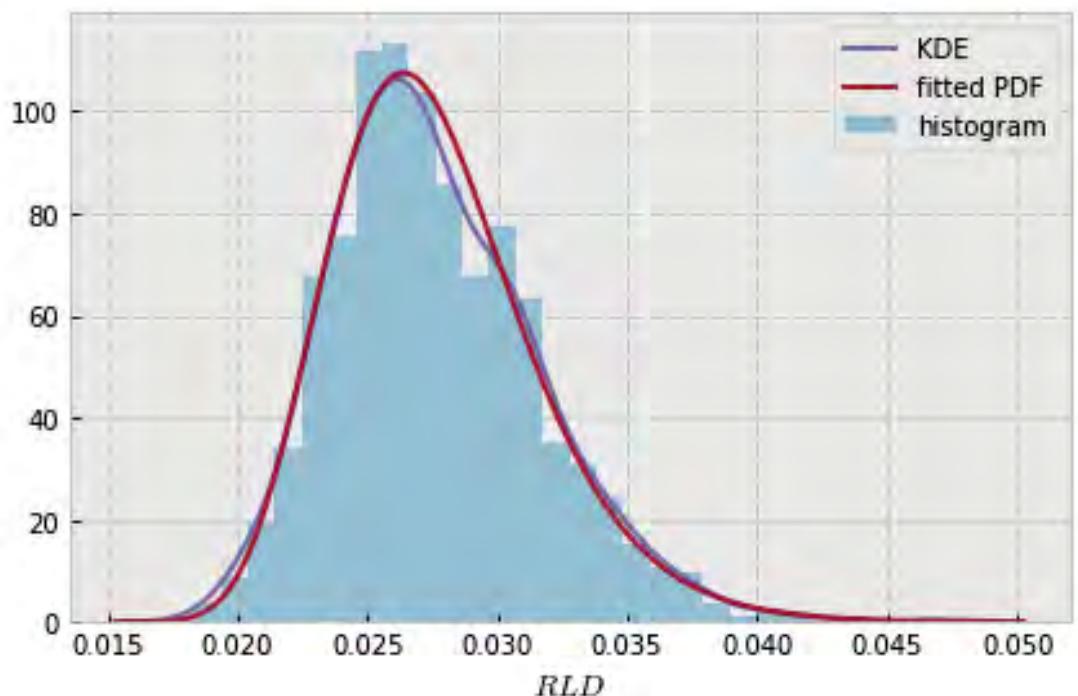


```
variable name: nB
variable value: 12.0
distribution: inverse-gamma(shape=(26.505094786339903,), loc=0.003911555854224657,
                           scale=0.5079272751806045)
DescribeResult(nobs=1000, minmax=(0.013777507958225008, 0.043212377052028648)
               mean=0.023828806233171618, variance=1.6508885898339098e-05,
               skewness=0.9754351194817755, kurtosis=1.8561146164403173)
```

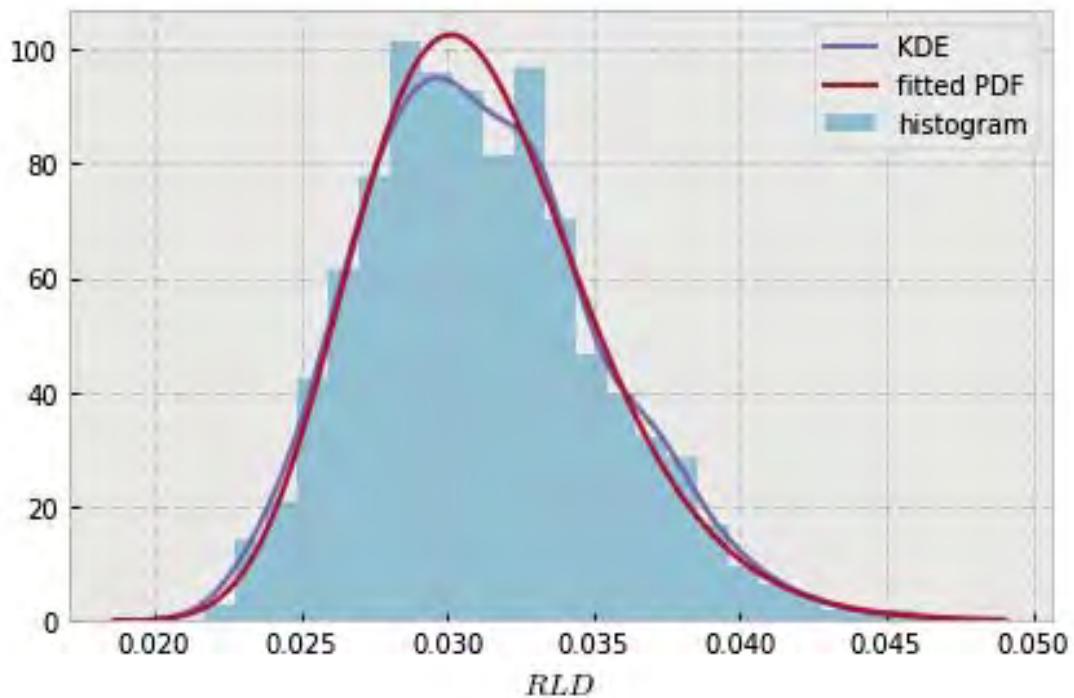


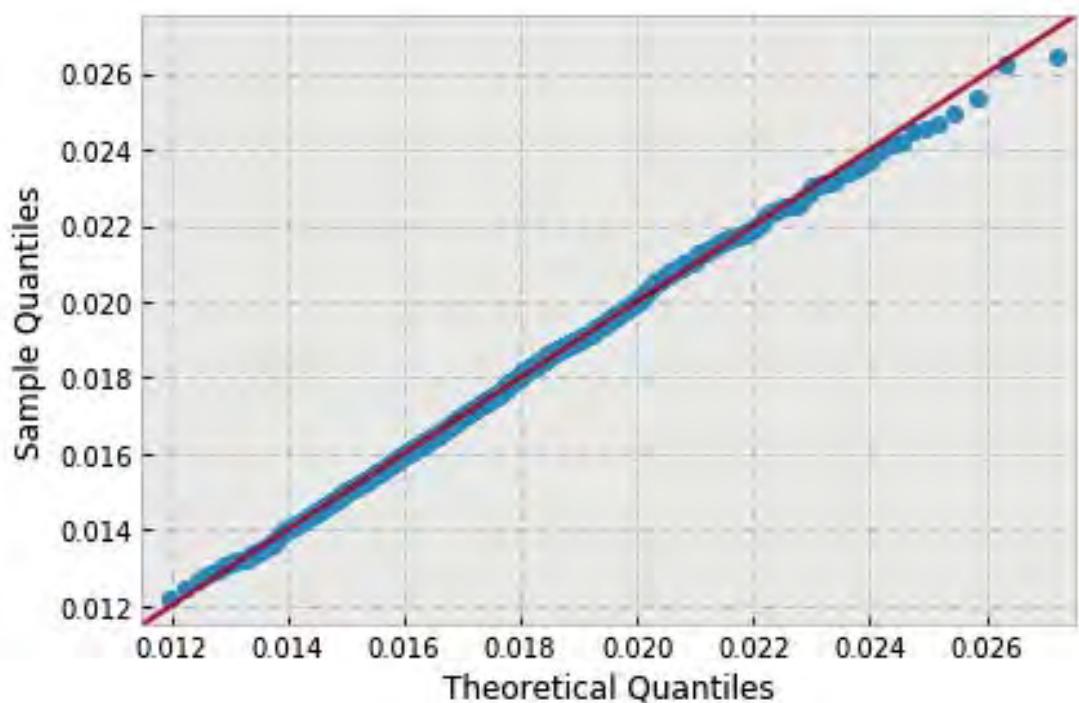


```
variable name: nB
variable value: 16.0
distribution: inverse-gamma(shape=(35.459702967376366,), loc=0.0046295167258445106,
                           scale=0.7926915760114338)
DescribeResult(nobs=1000, minmax=(0.018338109086942587, 0.04720042166166627)
               mean=0.02763273757358313, variance=1.5795318500857504e-05,
               skewness=0.6874534395827142, kurtosis=0.8125279912480785)
```



```
variable name: nB
variable value: 20.0
distribution: inverse-gamma(shape=(57.346621616607791,), loc=0.0009520923362754206,
    scale=1.7020755561823124)
DescribeResult(nobs=1000, minmax=(0.021727176992377729, 0.045957106746036501)
    mean=0.031158648230398756, variance=1.6374795607148748e-05,
    skewness=0.45296374024177766, kurtosis=0.04656612594356524)
```

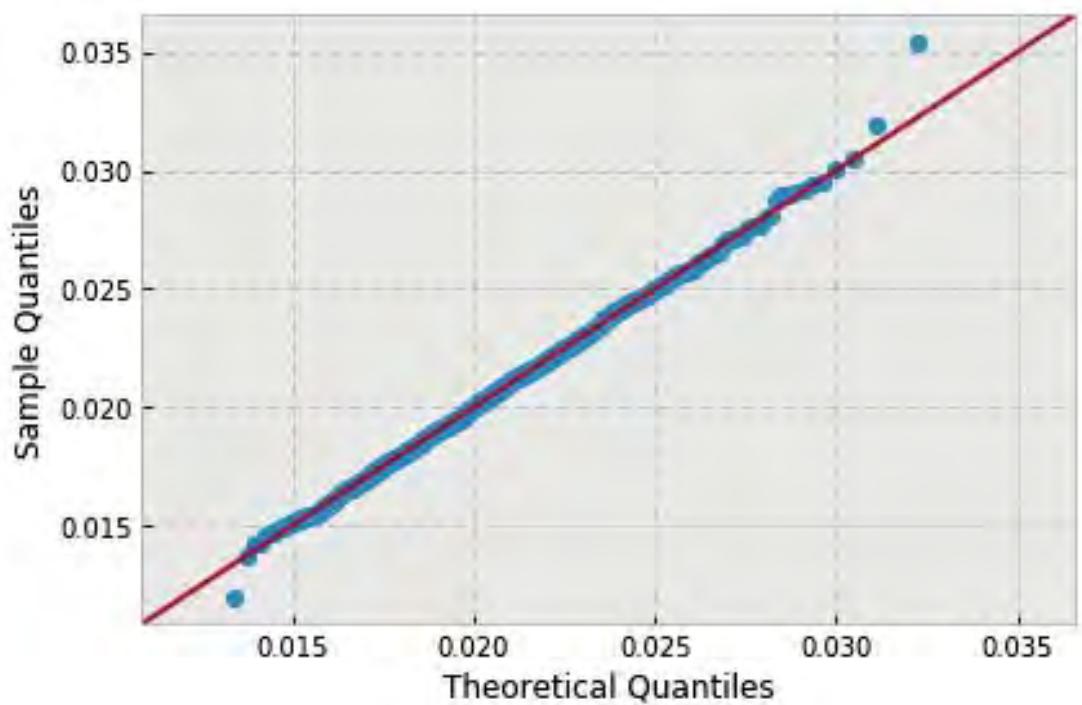
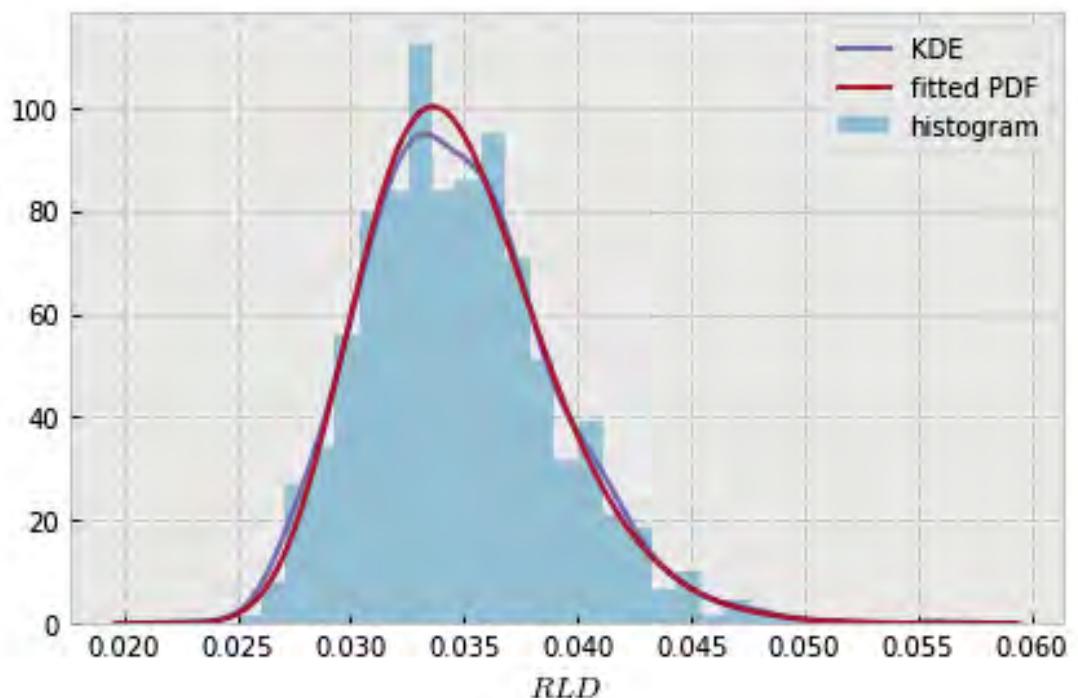




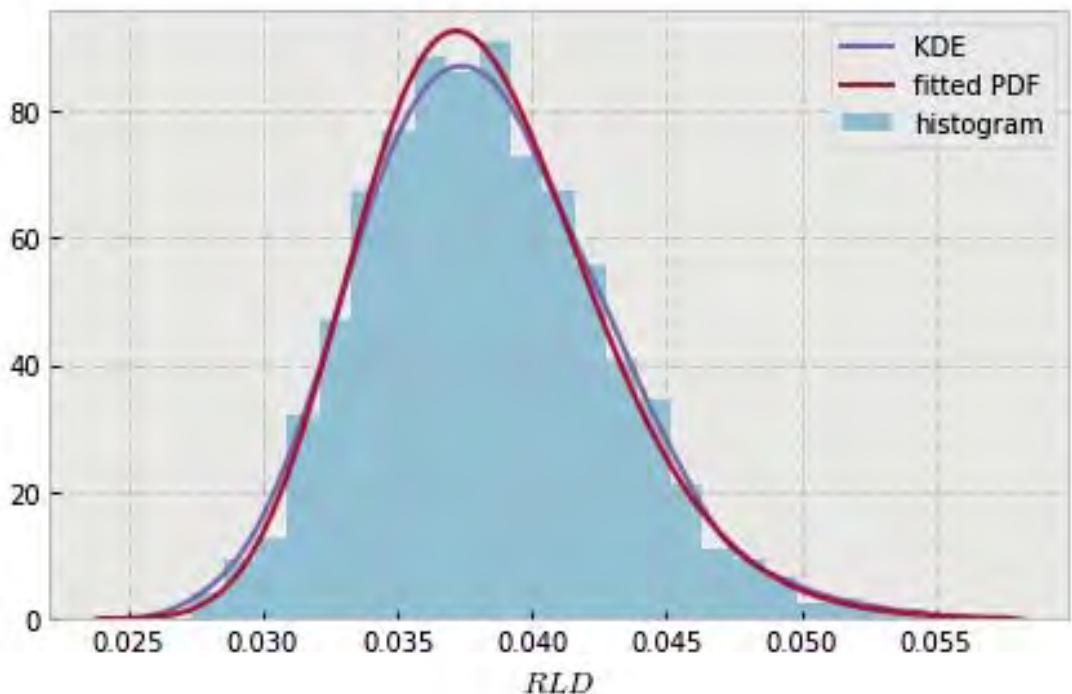
```

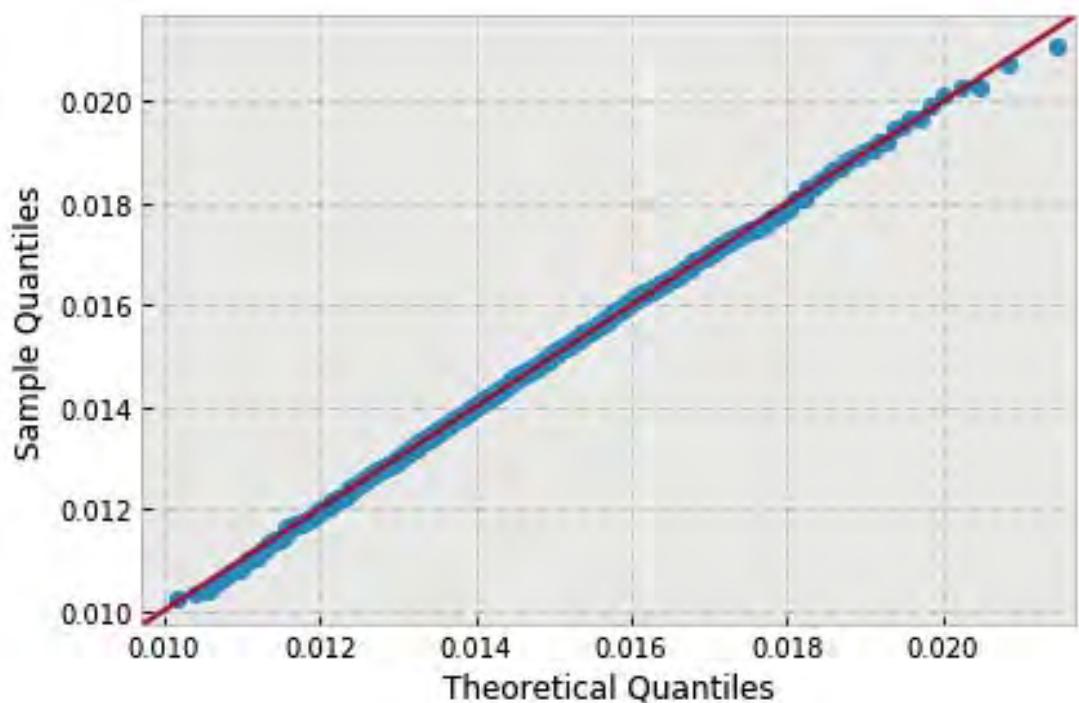
variable name: nB
variable value: 24.0
distribution: inverse-gamma(shape=(50.100032354820705,), loc=0.005750808479756456,
                           scale=1.4244922874741652)
DescribeResult(nobs=1000, minmax=(0.02284458955053768, 0.056161994361394213)
               mean=0.034763003650078637, variance=1.75436056101198e-05,
               skewness=0.6087597611835883, kurtosis=0.813540901567579)

```

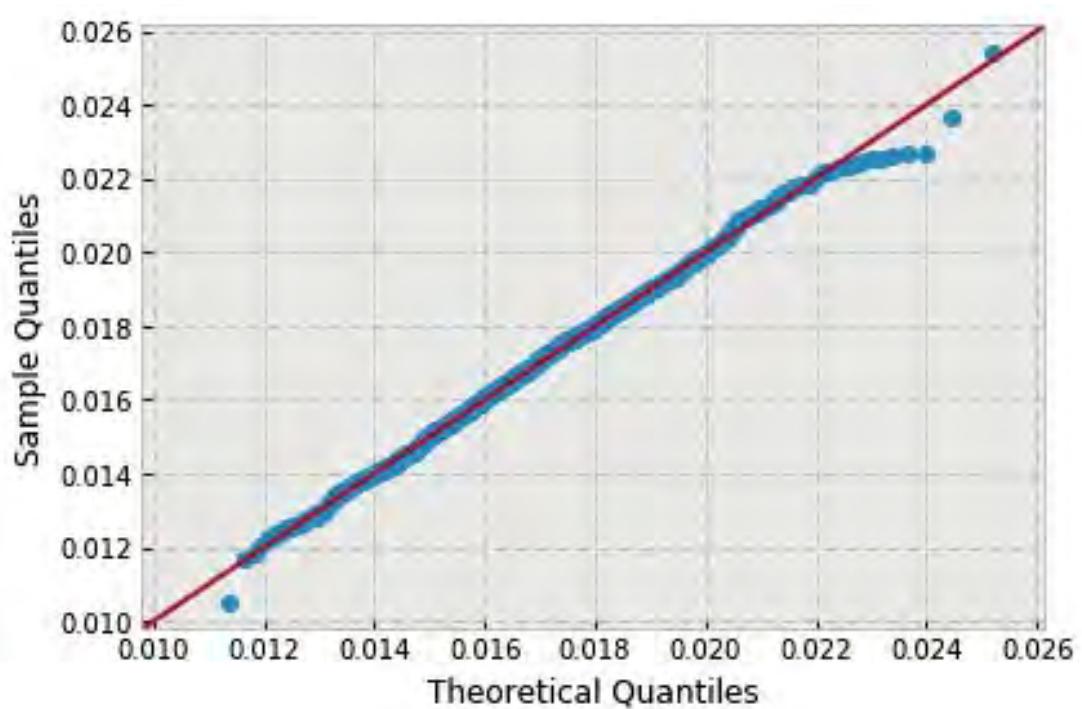
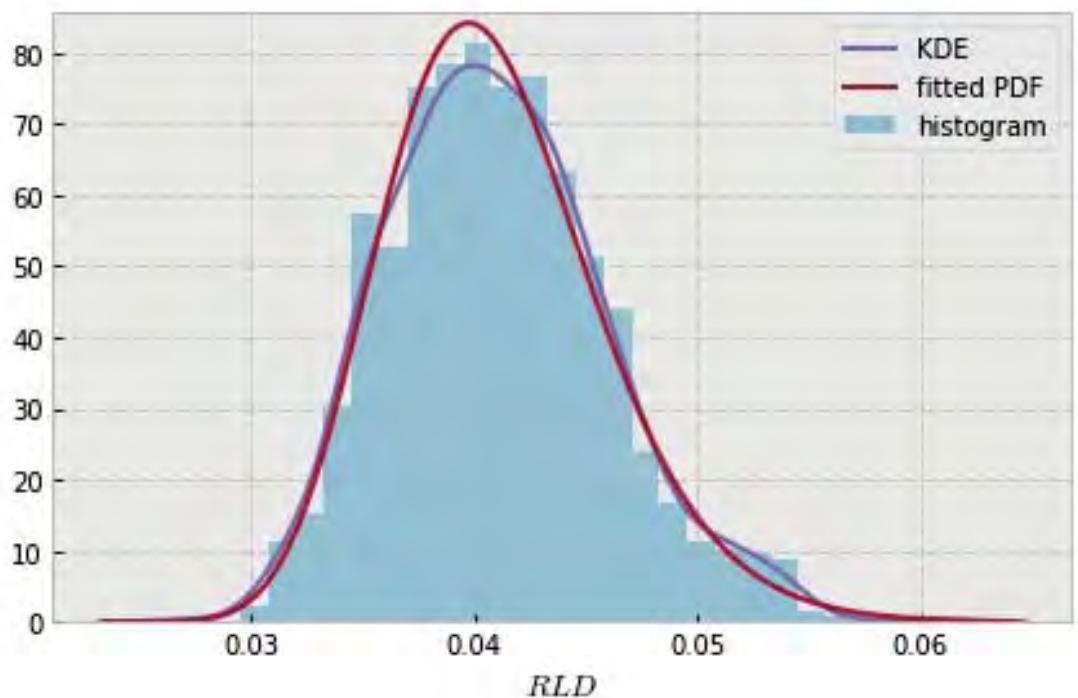


```
variable name: nB
variable value: 28.0
distribution: inverse-gamma(shape=(69.579423413675386,), loc=0.0014741559736194607,
                           scale=2.5201378509209356)
DescribeResult(nobs=1000, minmax=(0.027331419449752117, 0.054627553850148093)
               mean=0.038246504508748995, variance=2.0018258052666299e-05,
               skewness=0.4037254615970546, kurtosis=0.17075349767530845)
```

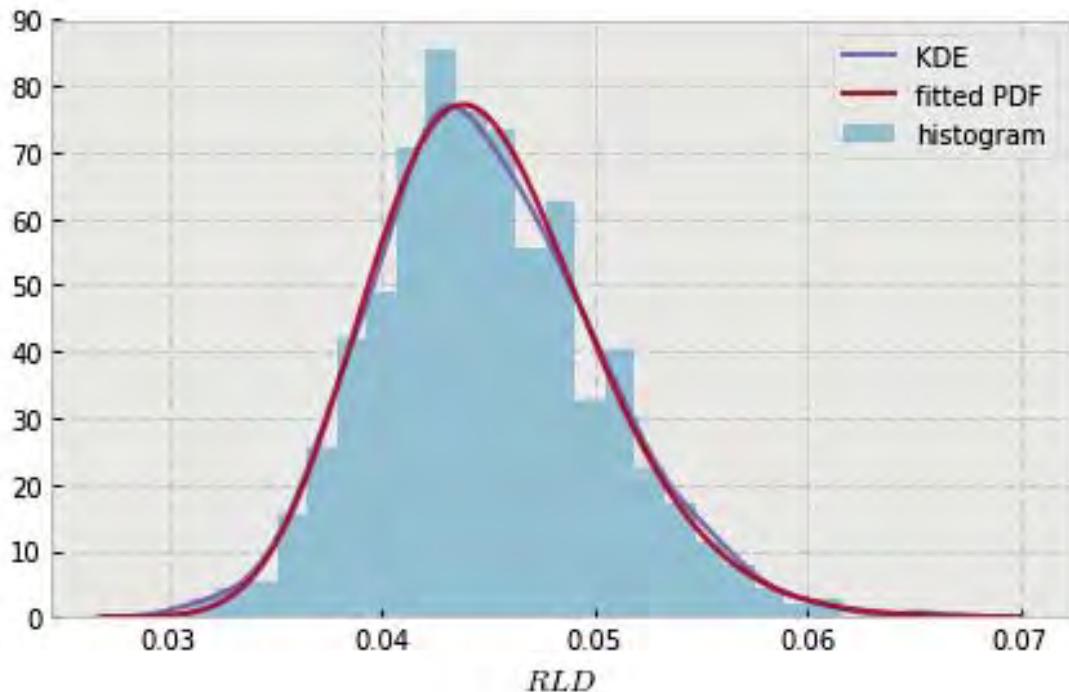


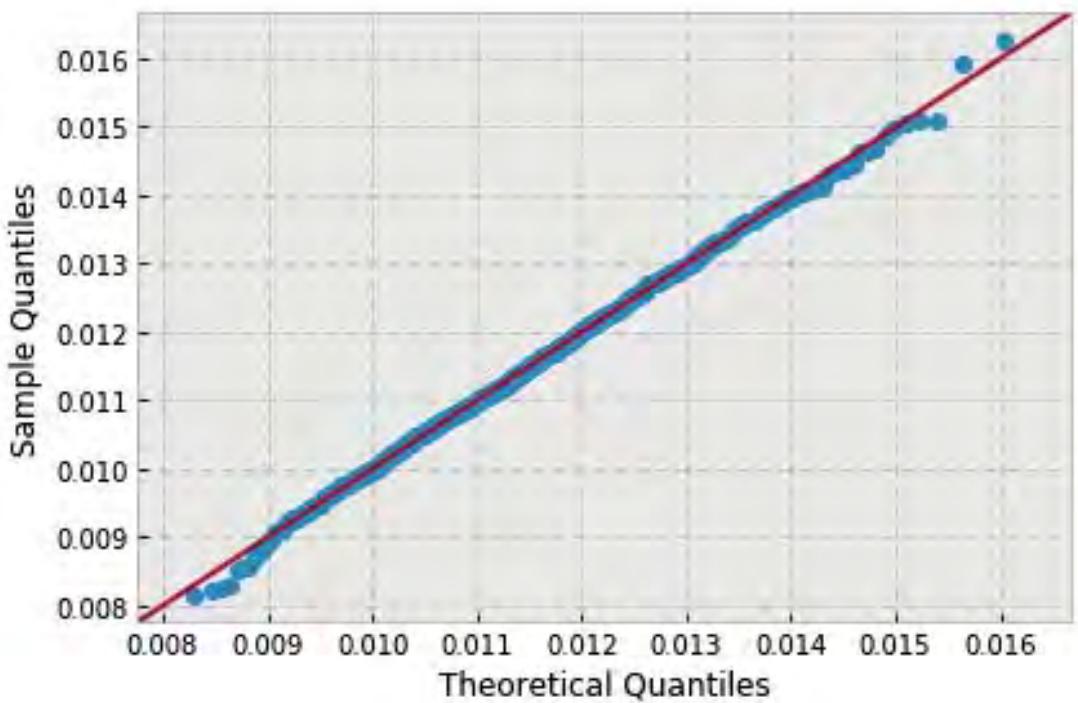


```
variable name: nB
variable value: 32.0
distribution: inverse-gamma(shape=(60.981773878629056,), loc=0.003085992550640848,
scale=2.2721970044923623)
DescribeResult(nobs=1000, minmax=(0.027011426773548782, 0.060806095211313985)
mean=0.041014545768920899, variance=2.3944126823533659e-05,
skewness=0.4213200367565847, kurtosis=0.13275081054198745)
```

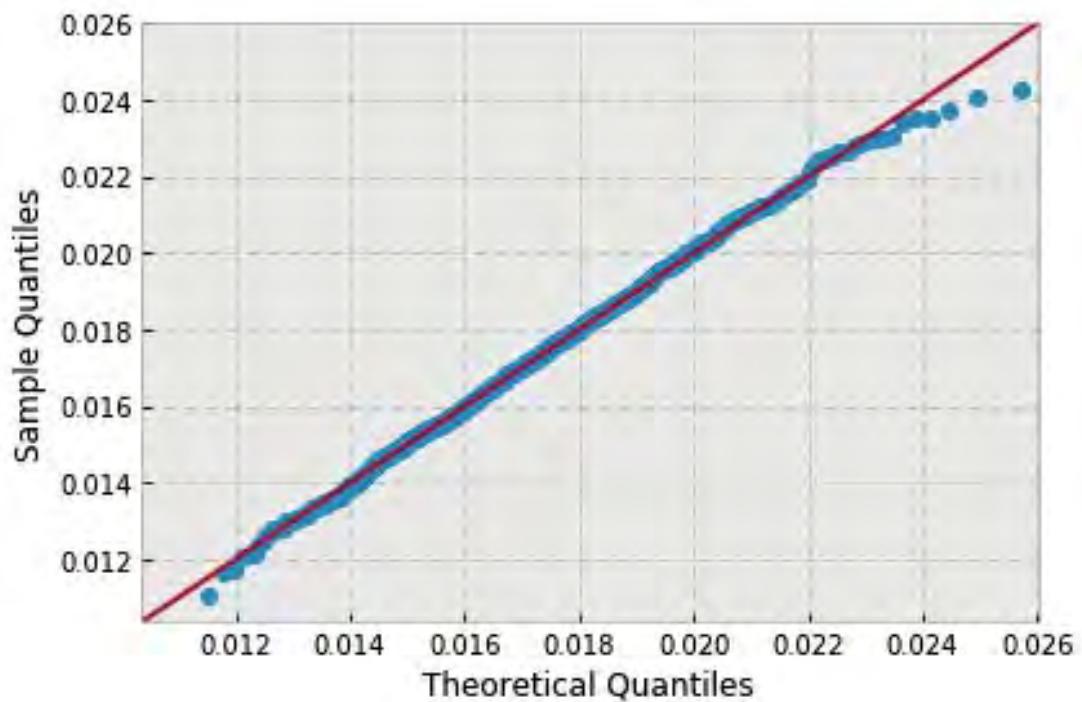
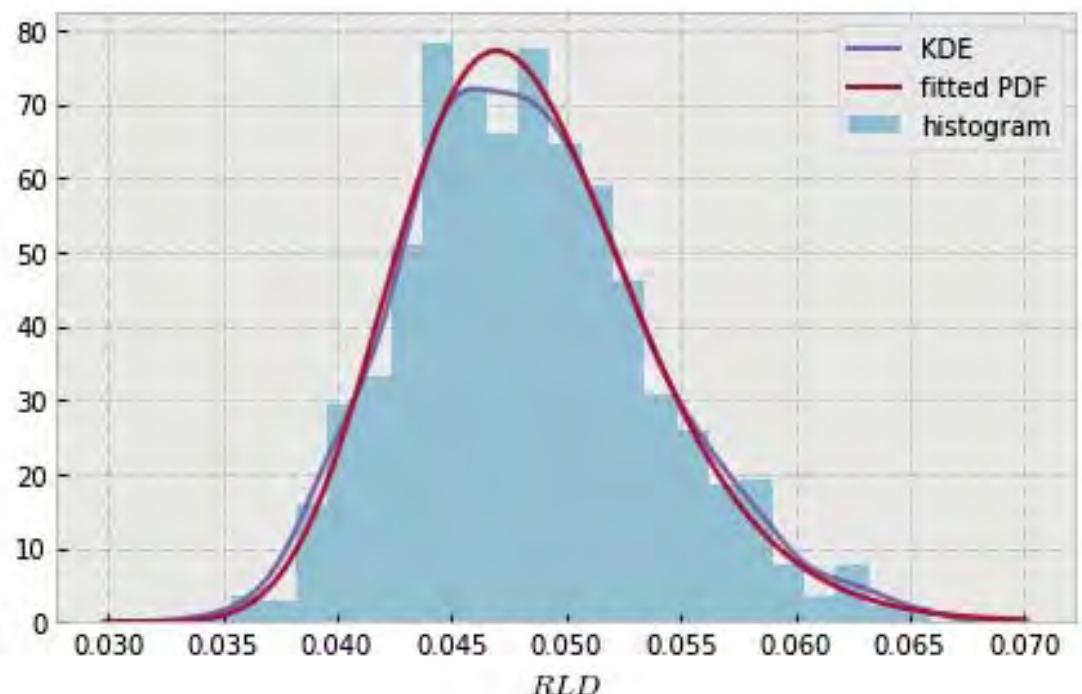


```
variable name: nB
variable value: 36.0
distribution: inverse-gamma(shape=(88.61129249384777,), loc=-0.004516627932886272,
                           scale=4.332316002897279)
DescribeResult(nobs=1000, minmax=(0.030855162076599333, 0.065924631249535959)
               mean=0.044922881333185428, variance=2.7877274682257962e-05,
               skewness=0.37933974182901237, kurtosis=0.2342638330887743)
```

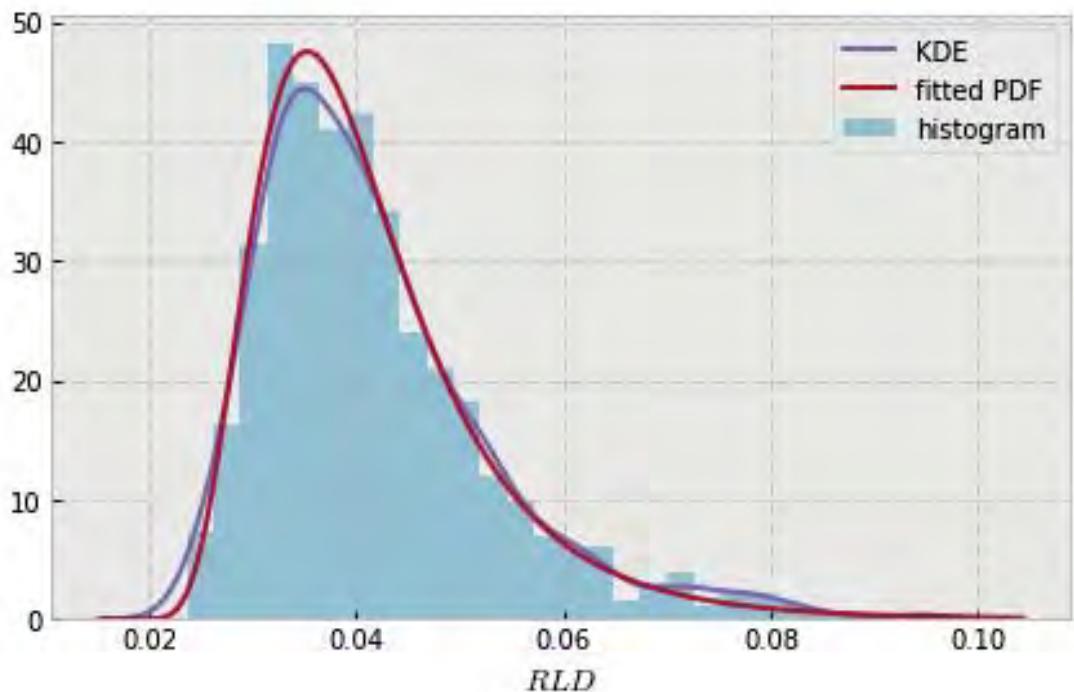


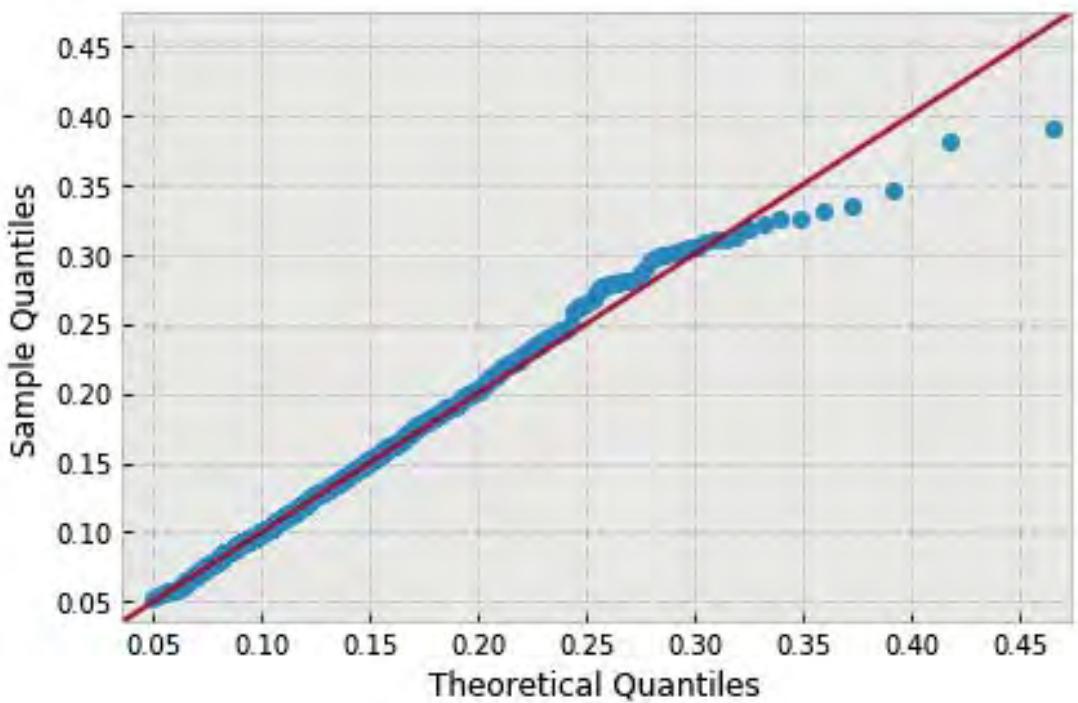


```
variable name: nB
variable value: 40.0
distribution: inverse-gamma(shape=(59.997015442661024,), loc=0.007339250967365372,
                           scale=2.4176501887070154)
DescribeResult(nobs=1000, minmax=(0.033976188398847543, 0.065954766128591233)
               mean=0.048349071592438432, variance=2.8879386798858455e-05,
               skewness=0.42181961910128357, kurtosis=0.04788498469086999)
```



```
variable name: r
variable value: 1.0
distribution: inverse-gamma(shape=(8.3820531649231889,), loc=0.012500860145287954,
    scale=0.21361789158573324)
DescribeResult(nobs=1000, minmax=(0.023652680443488706, 0.095748848144371776)
    mean=0.041719976411923175, variance=0.0001291460627467851,
    skewness=1.3513533344525228, kurtosis=2.2560842122734366)
```

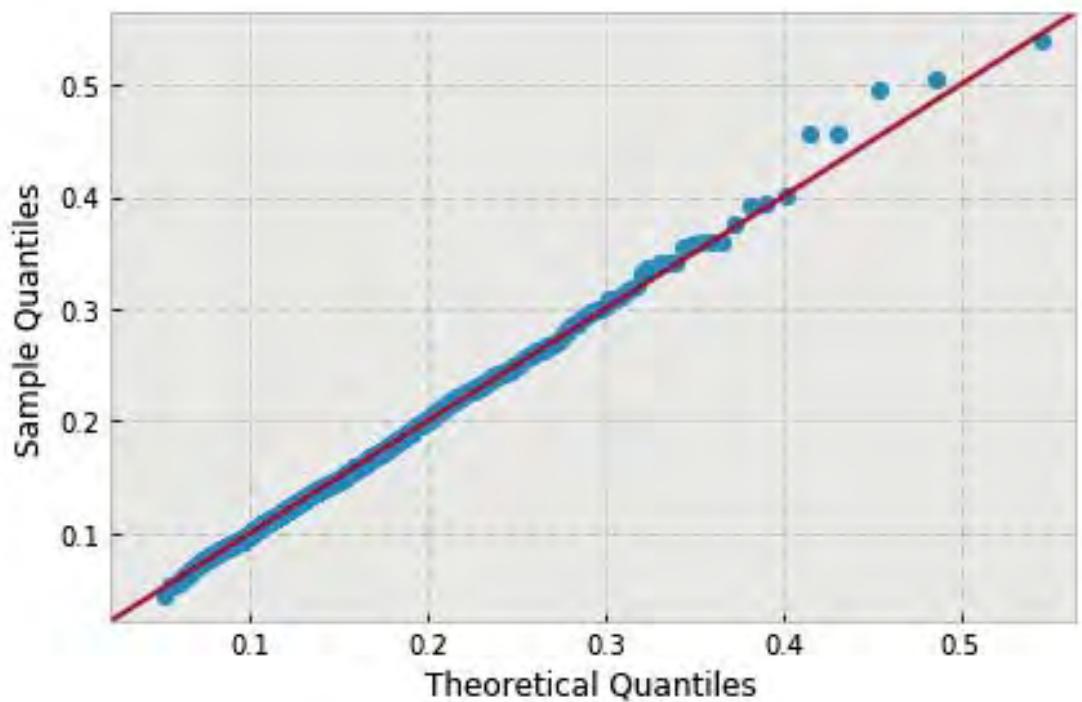
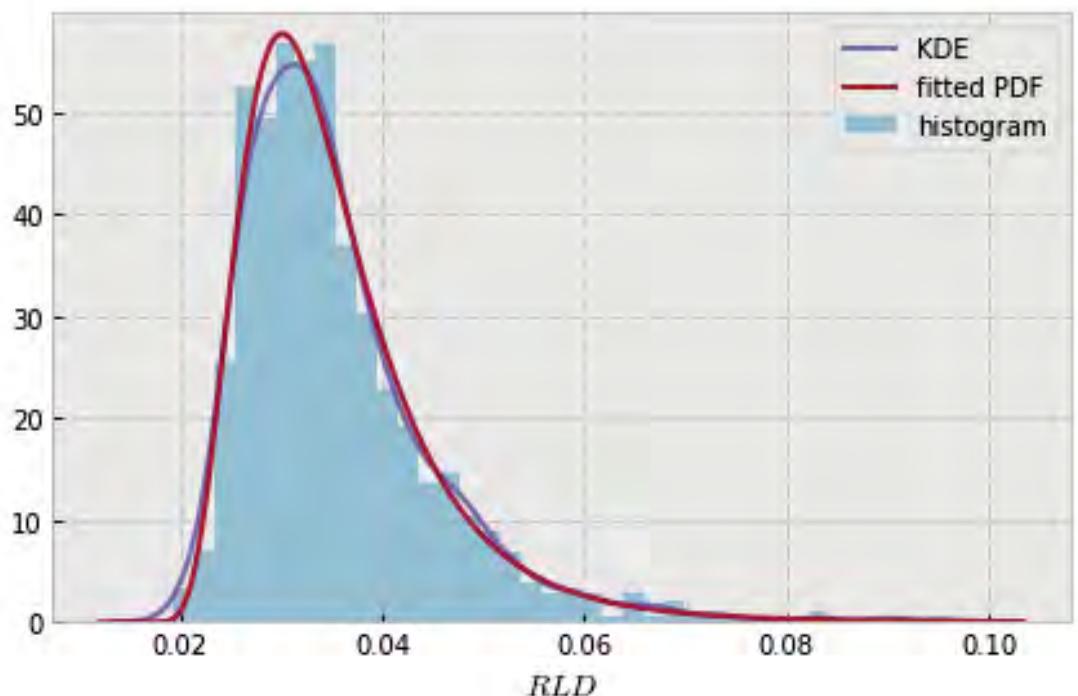




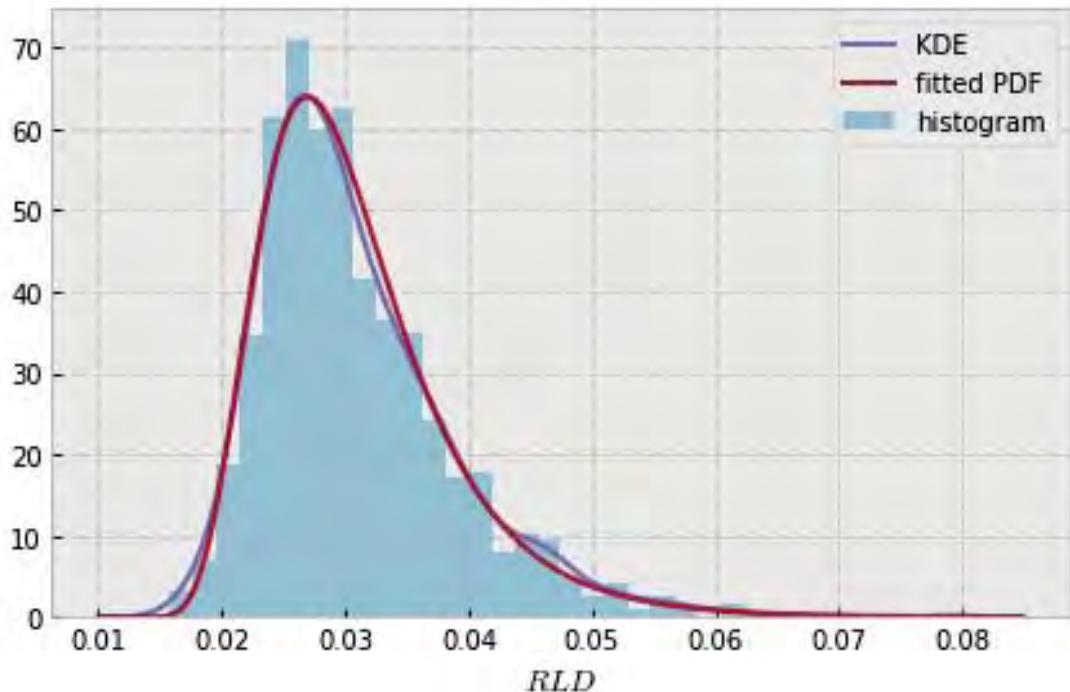
```

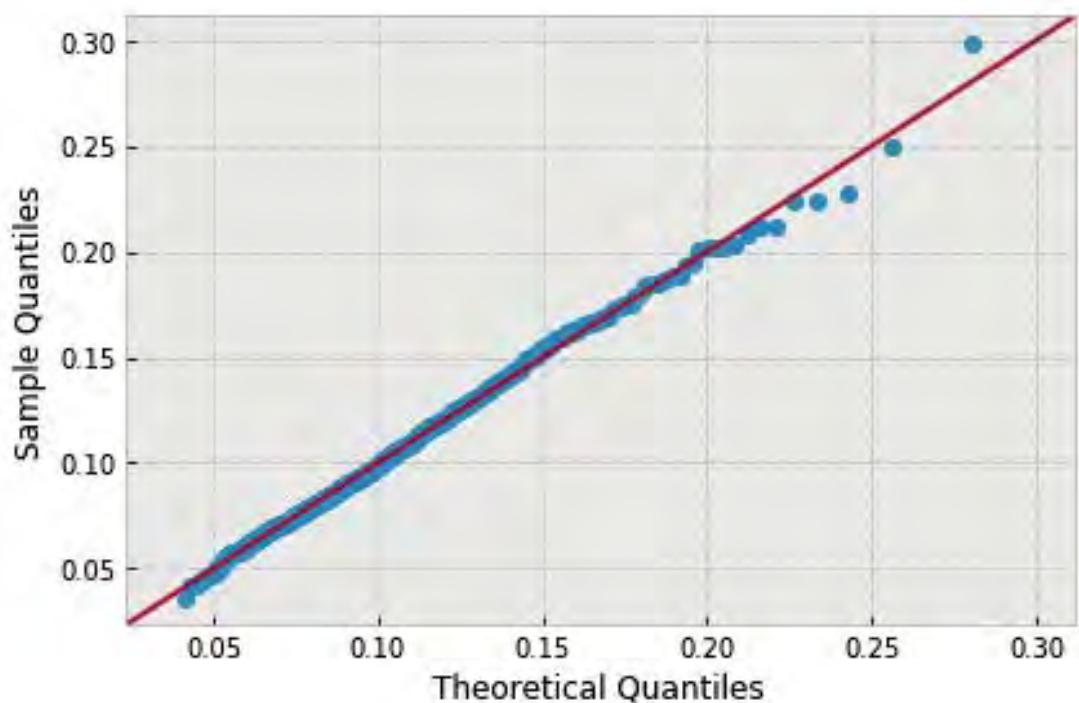
variable name: r
variable value: 1.222222222222223
distribution: inverse-gamma(shape=(7.705338926635017,), loc=0.012241787751296713,
                           scale=0.15569937504772913)
DescribeResult(nobs=1000, minmax=(0.019321490311306363, 0.096181203422360223)
               mean=0.035465943932726438, variance=9.4129475516480121e-05,
               skewness=1.7989149911465385, kurtosis=5.336814305415972)

```

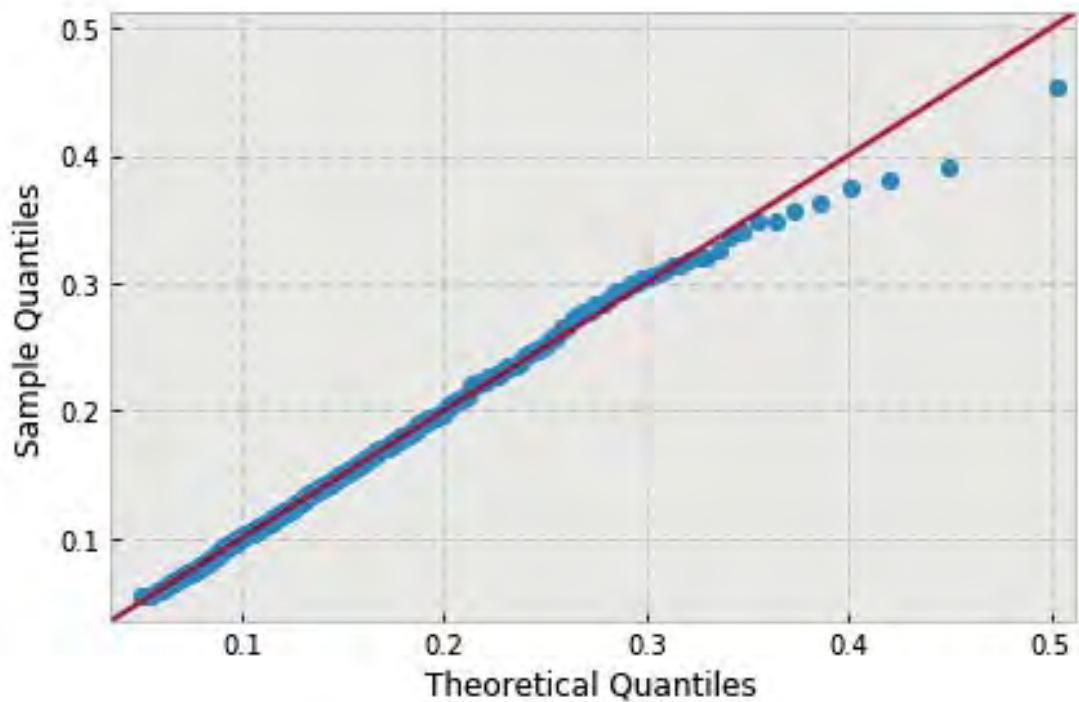
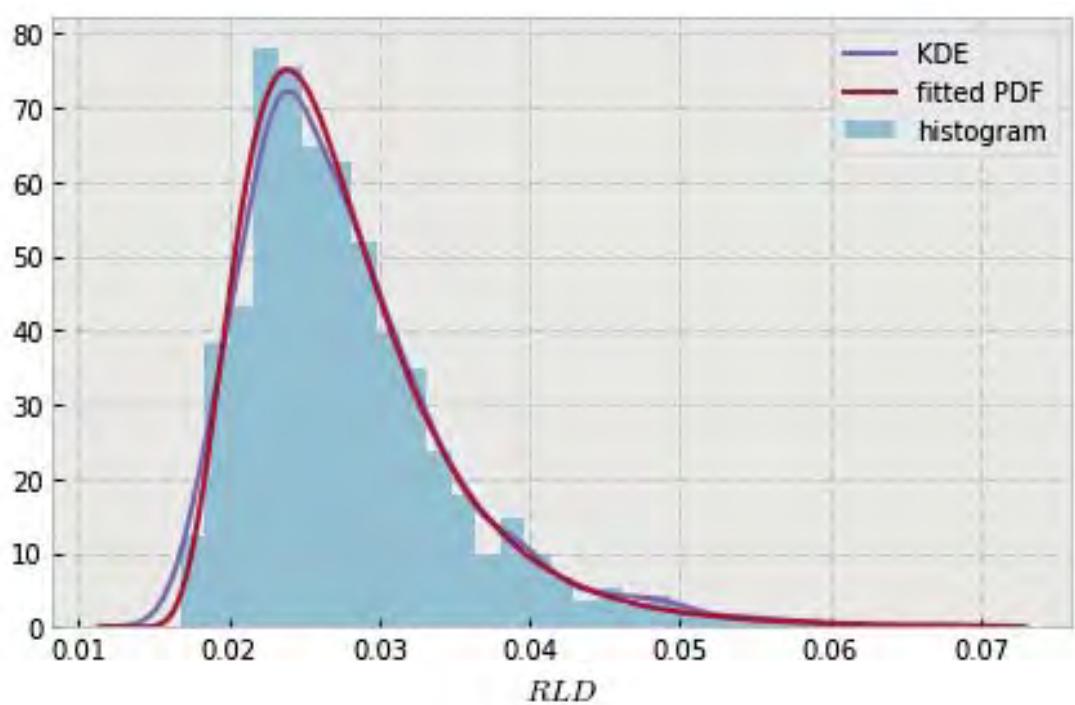


```
variable name: r
variable value: 1.444444444444444
distribution: inverse-gamma(shape=(11.147938040739501,), loc=0.007116623955728103,
                           scale=0.2409454208221891)
DescribeResult(nobs=1000, minmax=(0.015873761285682797, 0.078996682364022117)
               mean=0.030859247988892214, variance=6.1005347072948658e-05,
               skewness=1.3053427531508088, kurtosis=2.7087332559861927)
```

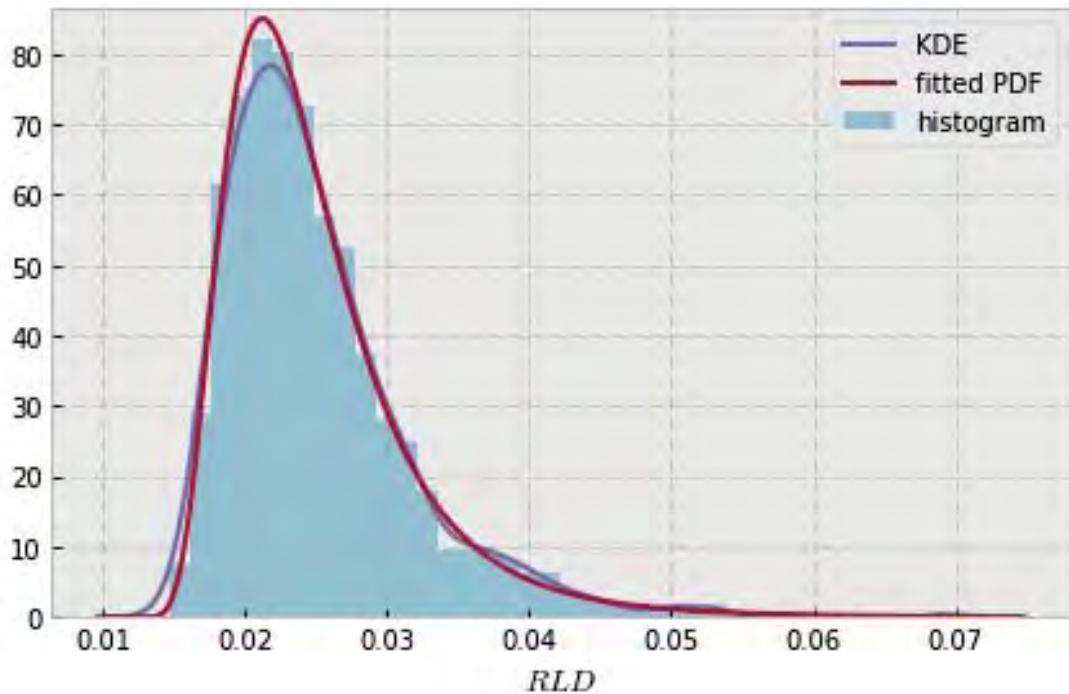


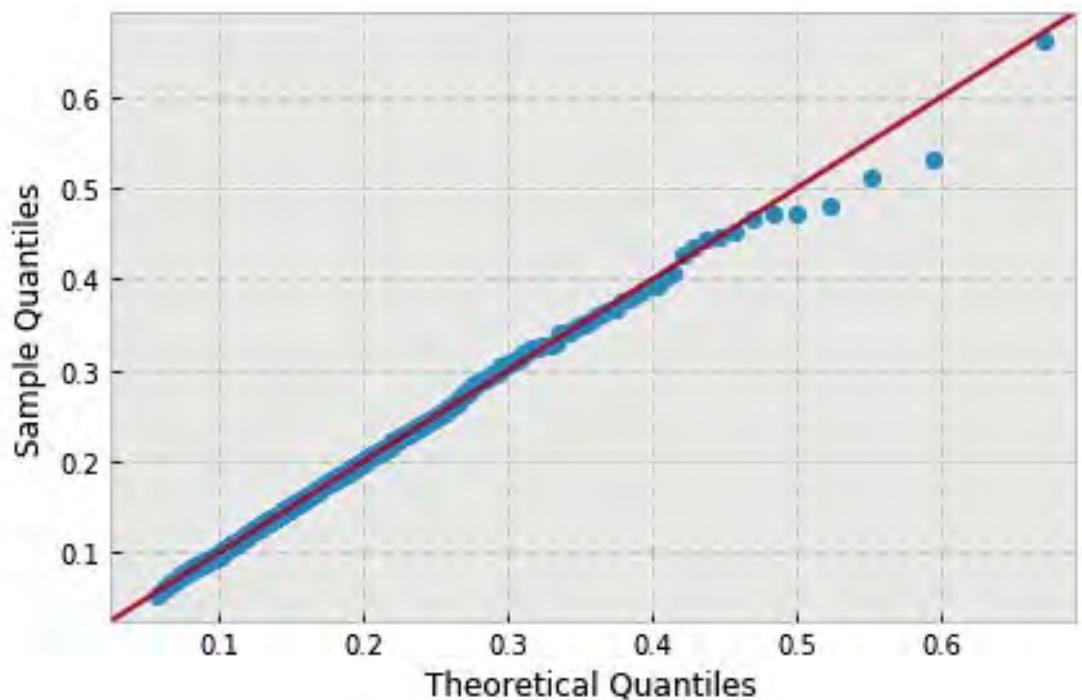


```
variable name: r
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(8.042247802453776,), loc=0.00980967709382725,
                           scale=0.12722198641683047)
DescribeResult(nobs=1000, minmax=(0.016730339402314793, 0.067492196642389757)
               mean=0.027857191878979812, variance=5.0882400791852531e-05,
               skewness=1.4297804872081608, kurtosis=2.848512352889845)
```

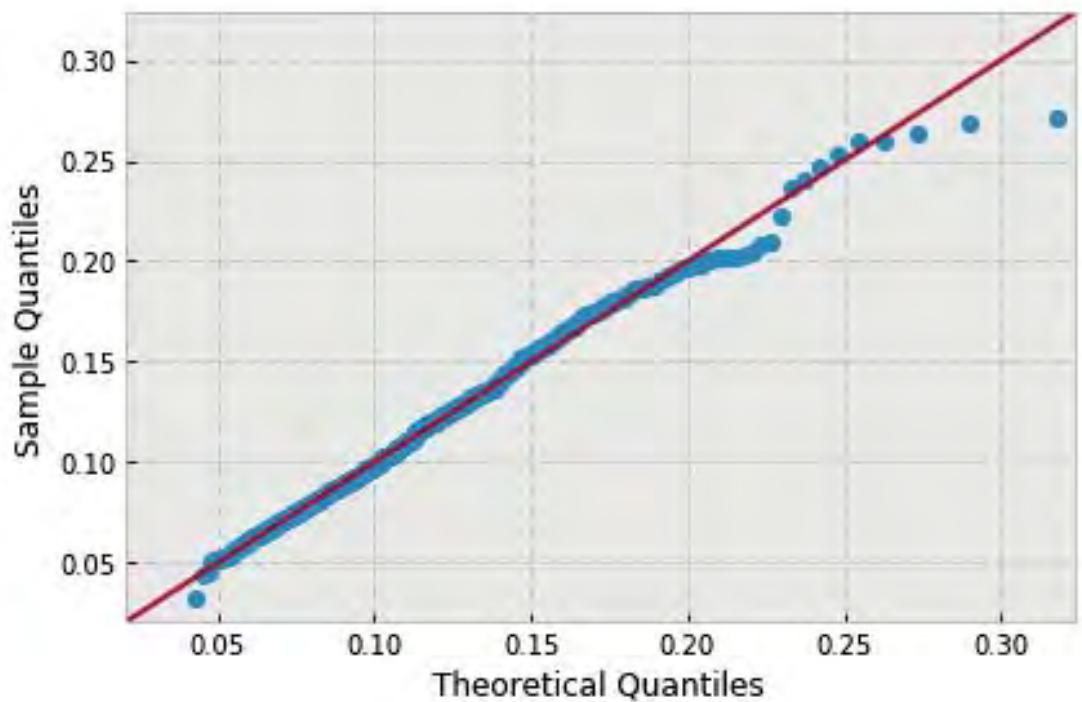
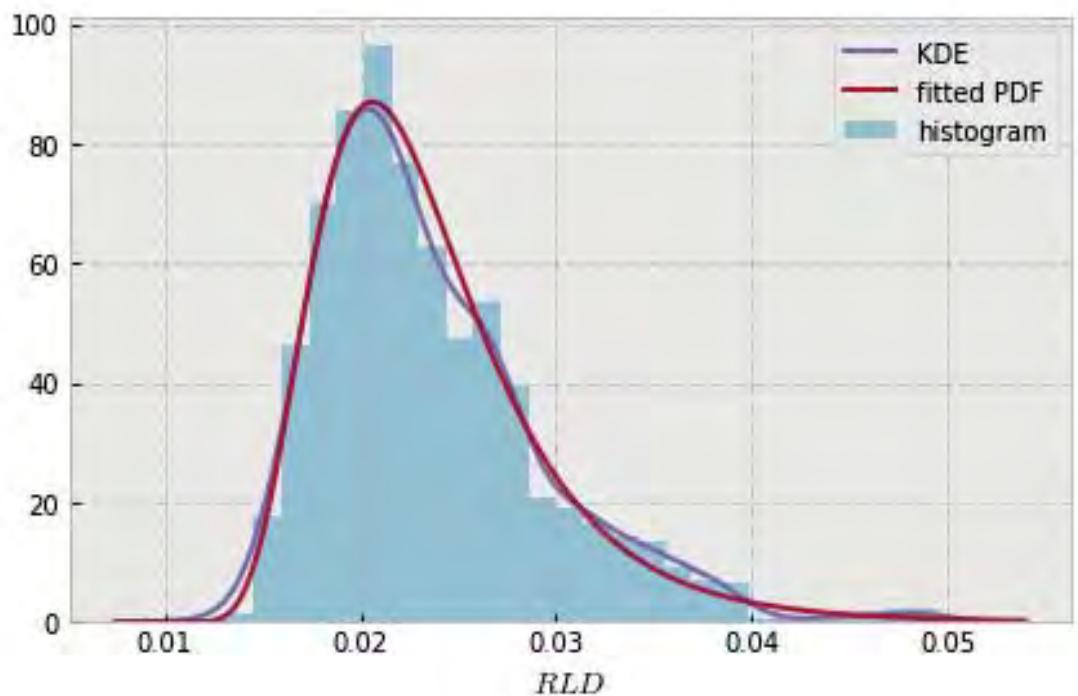


```
variable name: r
variable value: 1.888888888888888
distribution: inverse-gamma(shape=(6.9245933951232193,), loc=0.009819066790327183,
                           scale=0.09046673015738944)
DescribeResult(nobs=1000, minmax=(0.01470676753888421, 0.069683417543478621)
               mean=0.025070422338451238, variance=4.4432081615407945e-05,
               skewness=1.6845014538184304, kurtosis=4.519348813689556)
```

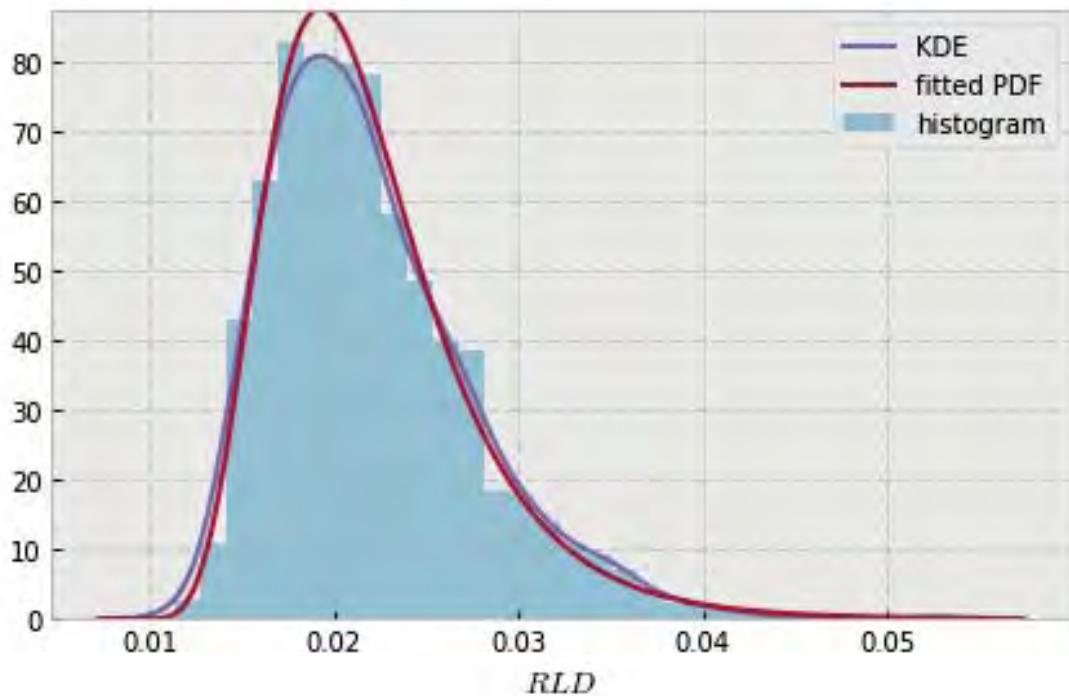


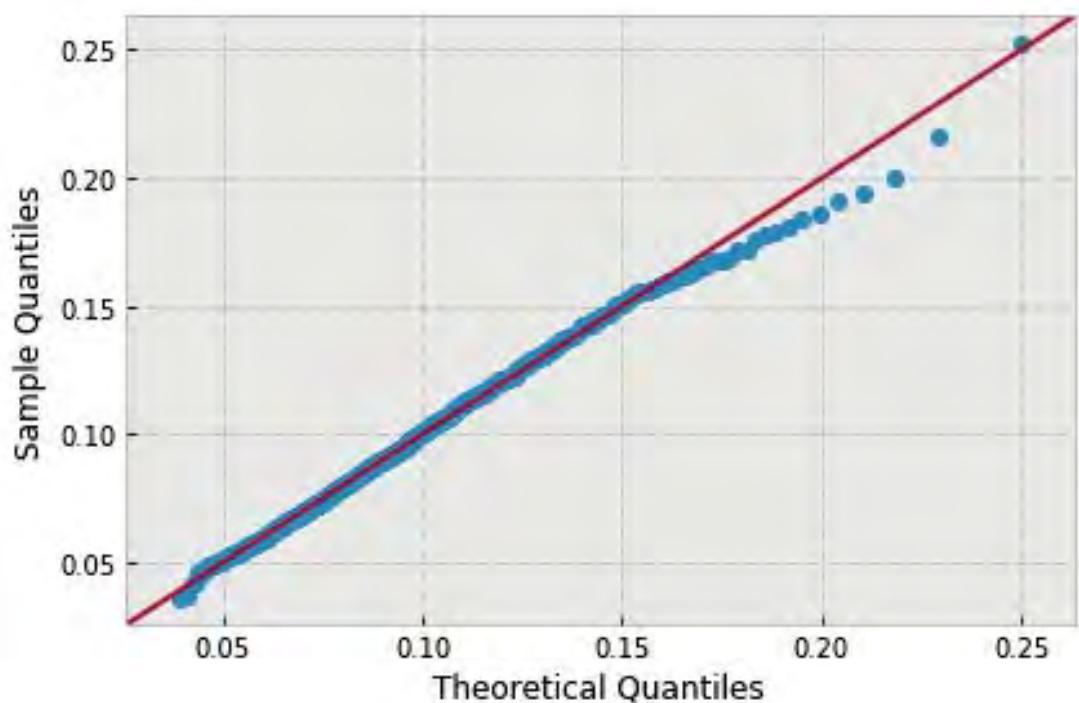


```
variable name: r
variable value: 2.111111111111111
distribution: inverse-gamma(shape=(10.35539452874475,), loc=0.006596717579518836,
                           scale=0.15886654398545835)
DescribeResult(nobs=1000, minmax=(0.011756425827545072, 0.04958857338670572)
               mean=0.023572390557845269, variance=3.3380101975725419e-05,
               skewness=1.216474072364544, kurtosis=1.9385645860718368)
```

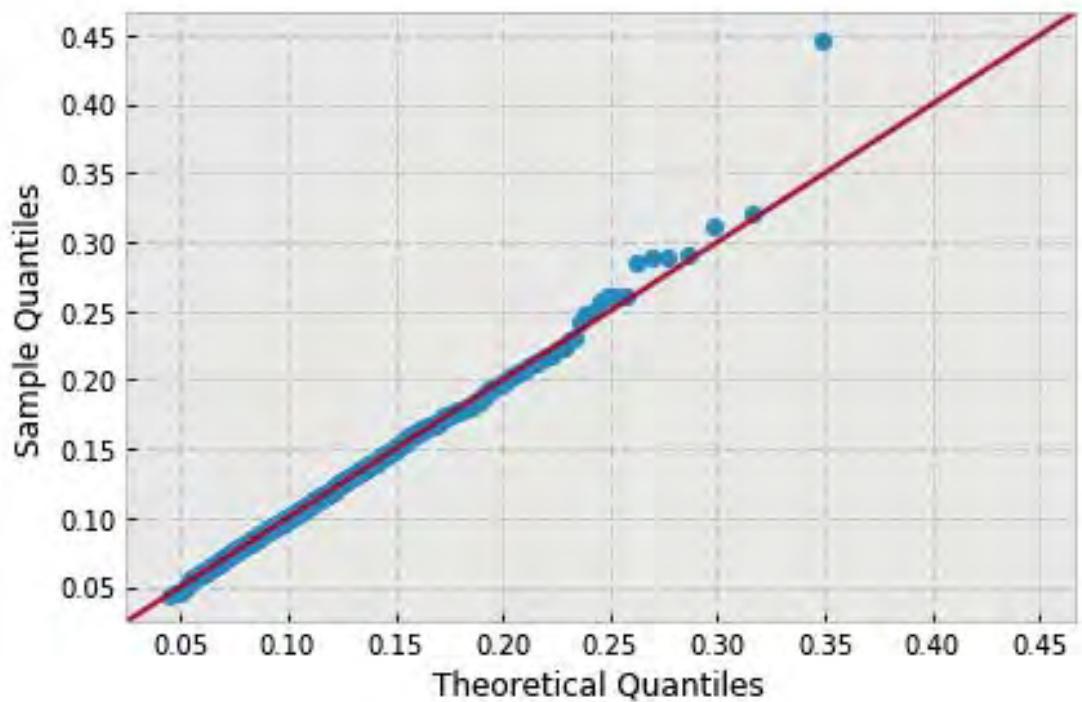
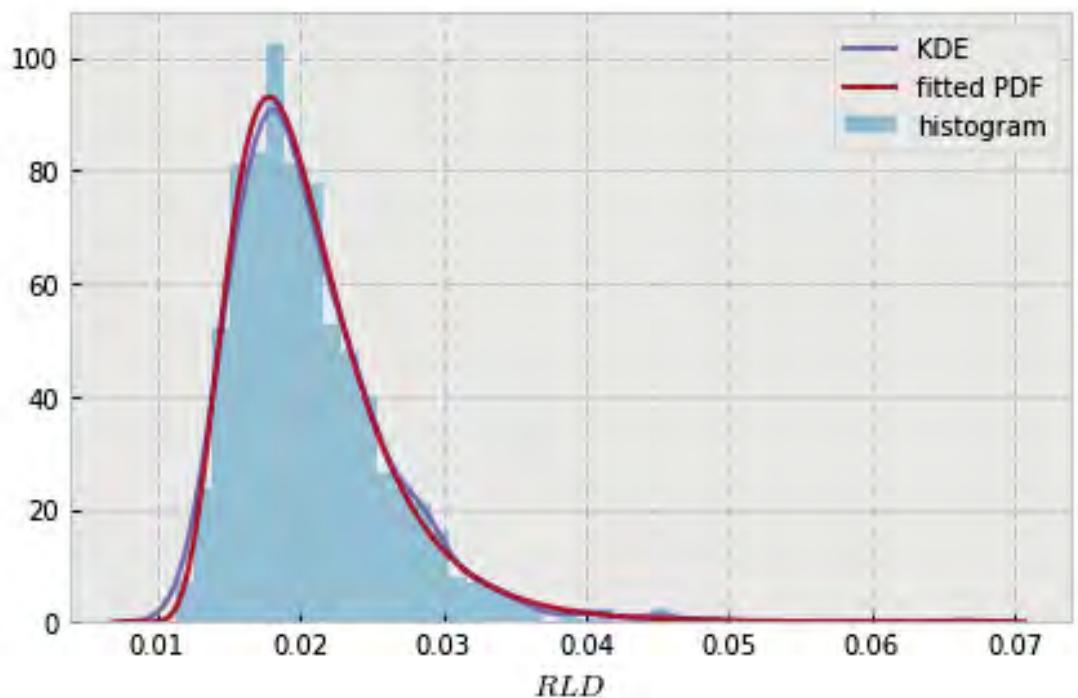


```
variable name: r
variable value: 2.333333333333333
distribution: inverse-gamma(shape=(11.933498426620837,), loc=0.0042667537981464125,
    scale=0.1939714483329934)
DescribeResult(nobs=1000, minmax=(0.011349941844817578, 0.053206492799994905)
    mean=0.021997937874538662, variance=3.0331085900628036e-05,
    skewness=1.0828407917475362, kurtosis=1.7243881793004663)
```

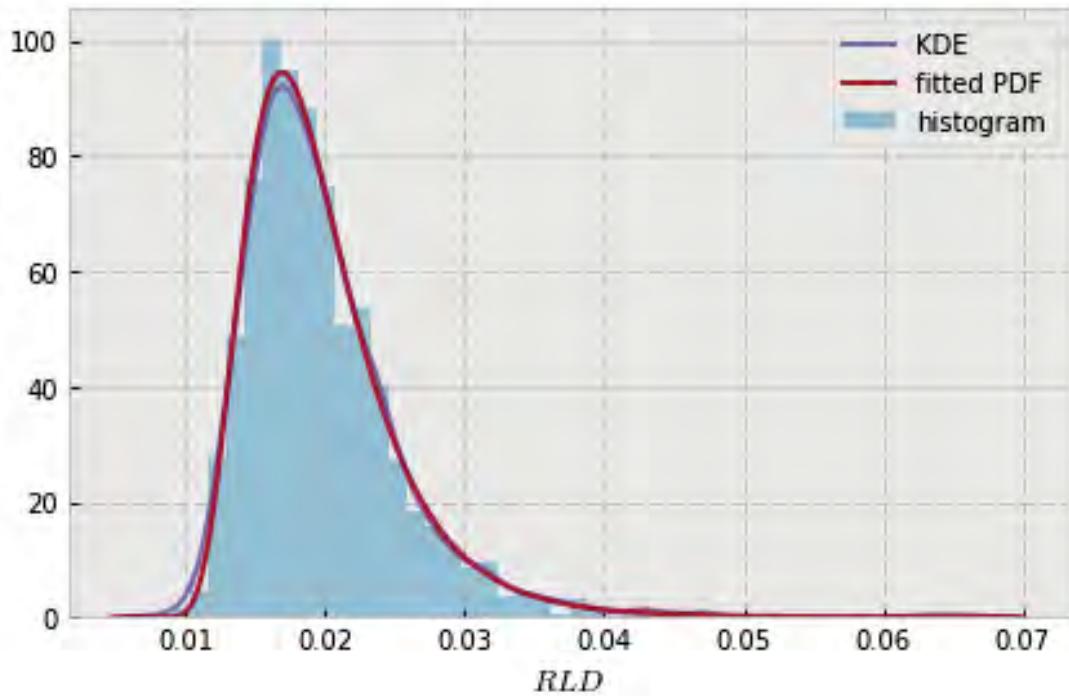


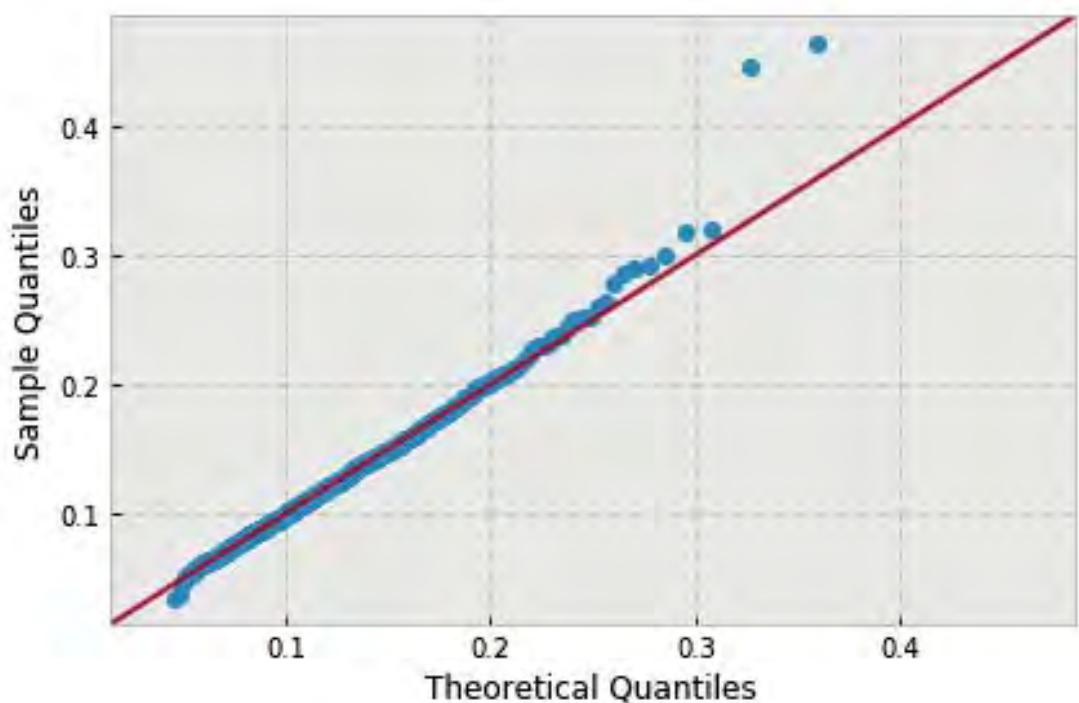


```
variable name: r
variable value: 2.5555555555555554
distribution: inverse-gamma(shape=(9.8290000708388732,), loc=0.005120271146880185,
                           scale=0.13761041388291867)
DescribeResult(nobs=1000, minmax=(0.011238532556233078, 0.066483765881497972)
               mean=0.020710387253990218, variance=3.1610074381752423e-05,
               skewness=1.770983106793816, kurtosis=6.694485036105739)
```

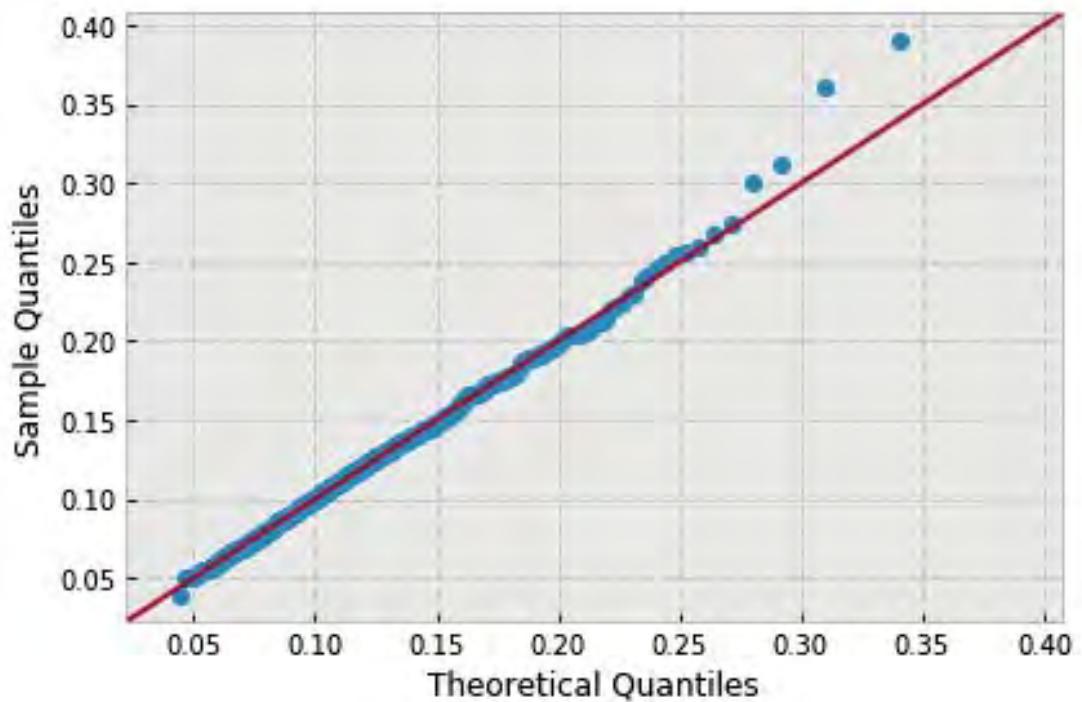
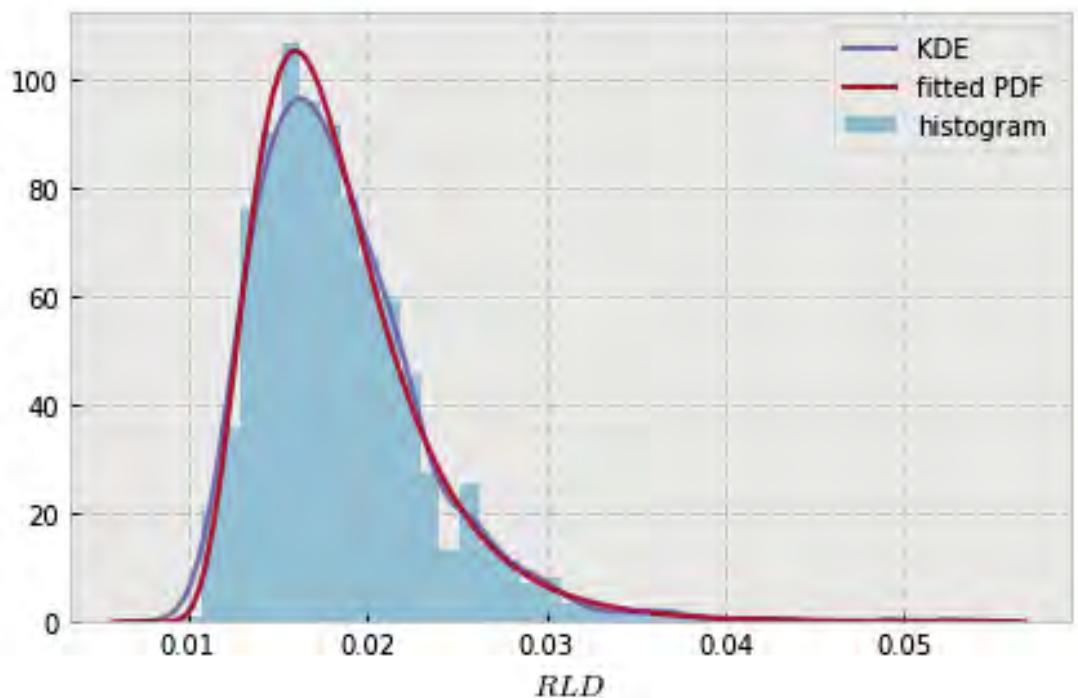


```
variable name: r
variable value: 2.777777777777777
distribution: inverse-gamma(shape=(9.6487670877842469,), loc=0.004554344839258315,
    scale=0.13189721097047358)
DescribeResult(nobs=1000, minmax=(0.0091755718552603855, 0.06571422388555756)
    mean=0.01982289903813313, variance=3.2631691902018094e-05,
    skewness=2.088794971729858, kurtosis=9.148018004980427)
```

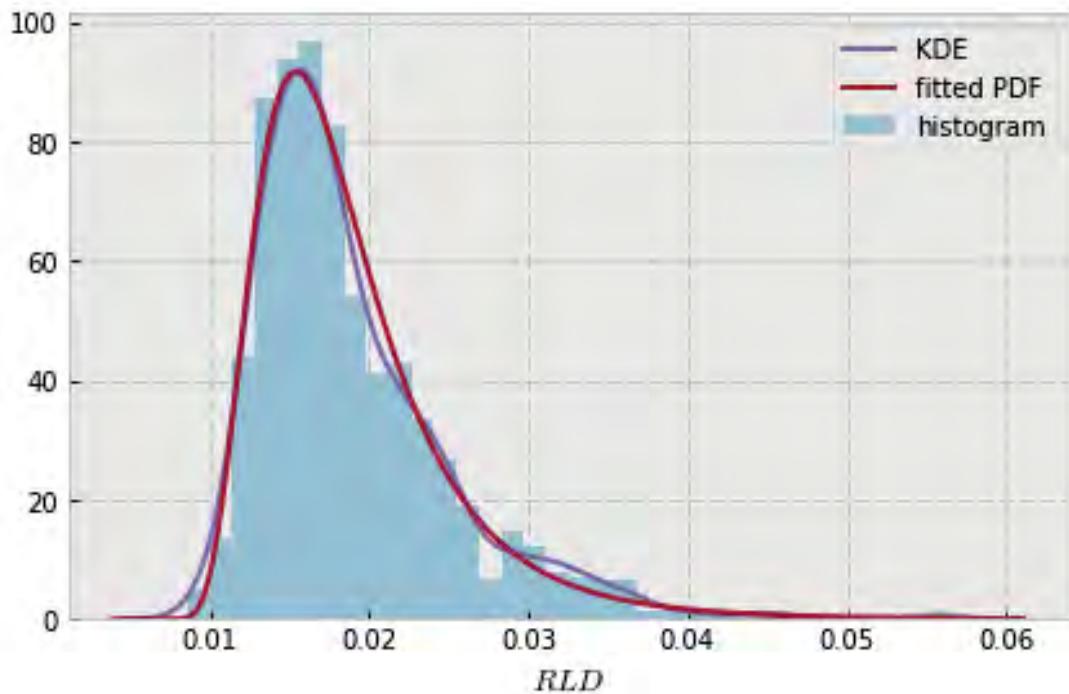


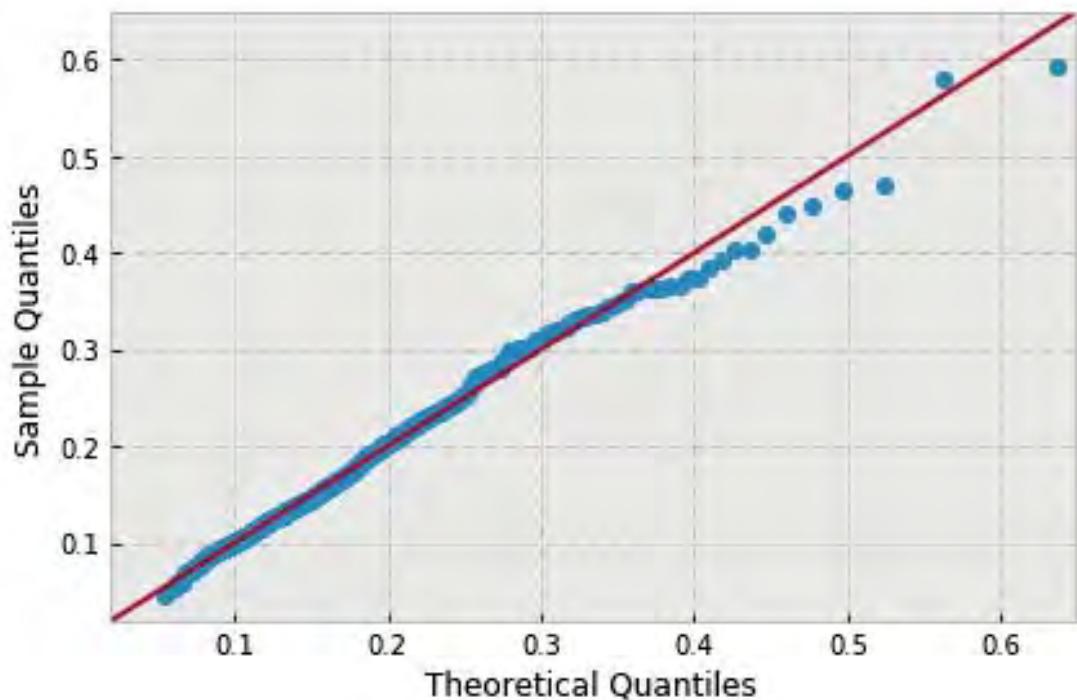


```
variable name: r
variable value: 3.0
distribution: inverse-gamma(shape=(9.9525182548618609,), loc=0.00468434564489378,
scale=0.12386246201084007)
DescribeResult(nobs=1000, minmax=(0.0095848023187109499, 0.053002462534387229)
mean=0.018519432536541931, variance=2.4141986425180587e-05,
skewness=1.6571378598316135, kurtosis=5.542362808642213)
```

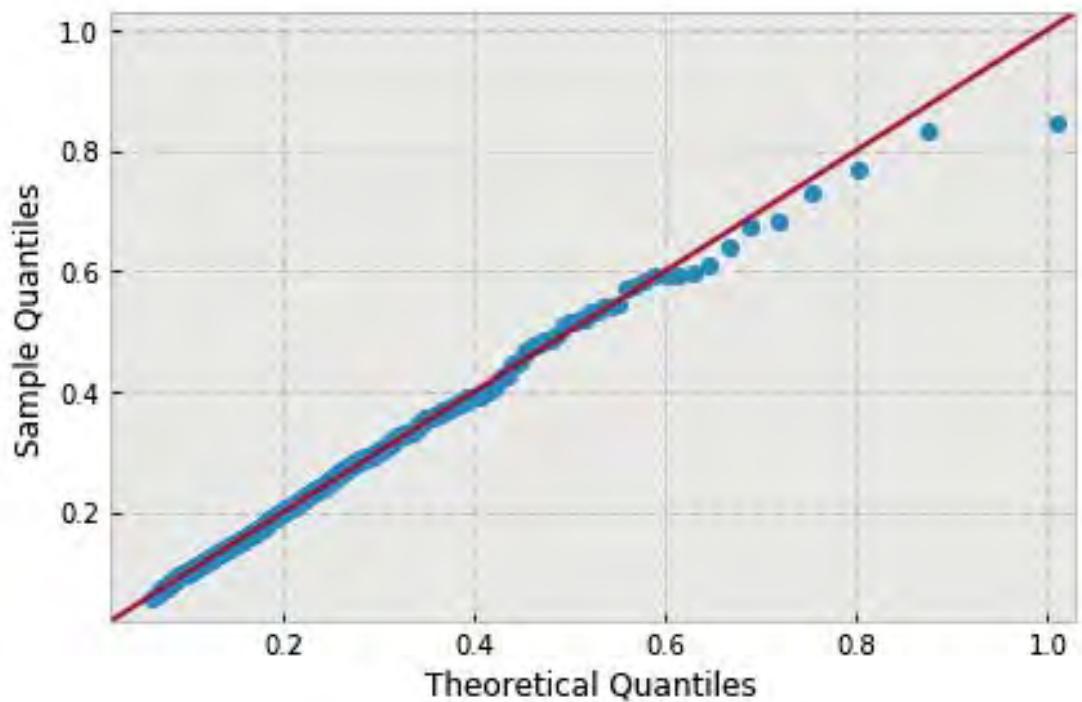
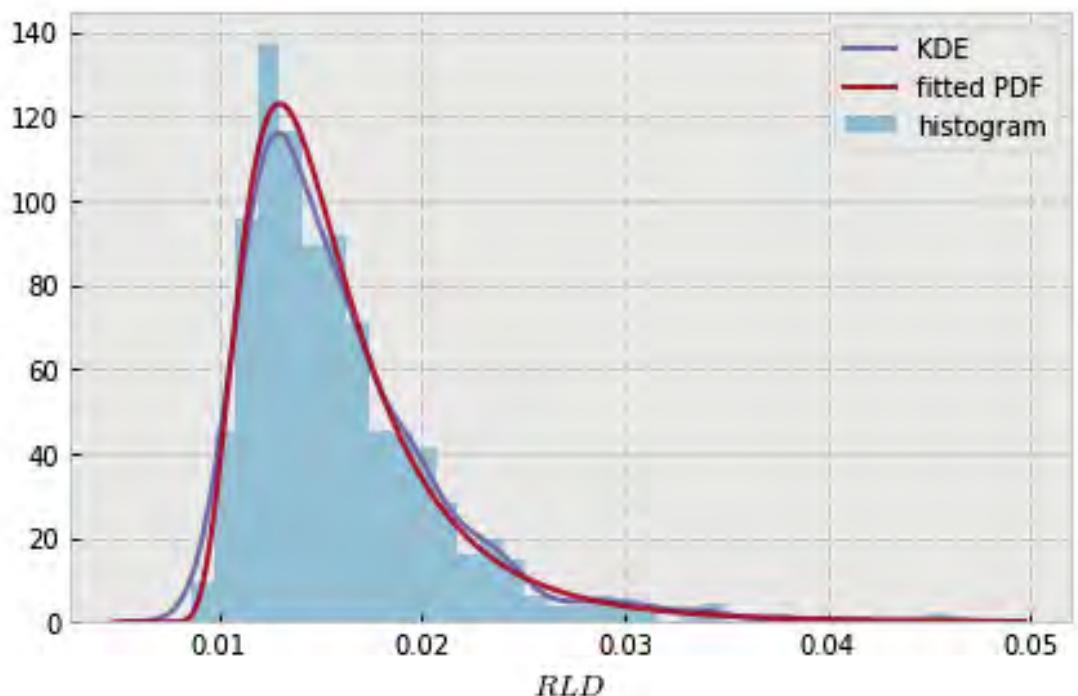


```
variable name: sigma
variable value: 0.01111111111111112
distribution: inverse-gamma(shape=(7.1199631756920958,), loc=0.004640431234000775,
    scale=0.08735471233744396)
DescribeResult(nobs=1000, minmax=(0.0086014250182118116, 0.056426225140124374)
    mean=0.018907528974831053, variance=3.7937865890037595e-05,
    skewness=1.6056126679066198, kurtosis=3.8452192887515135)
```

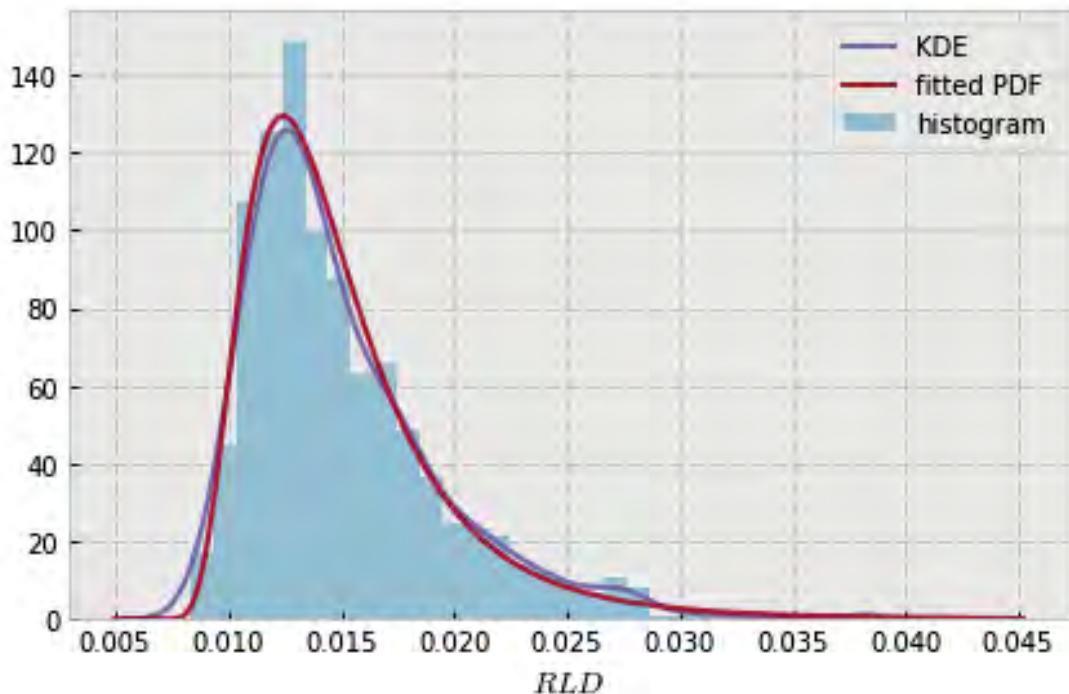


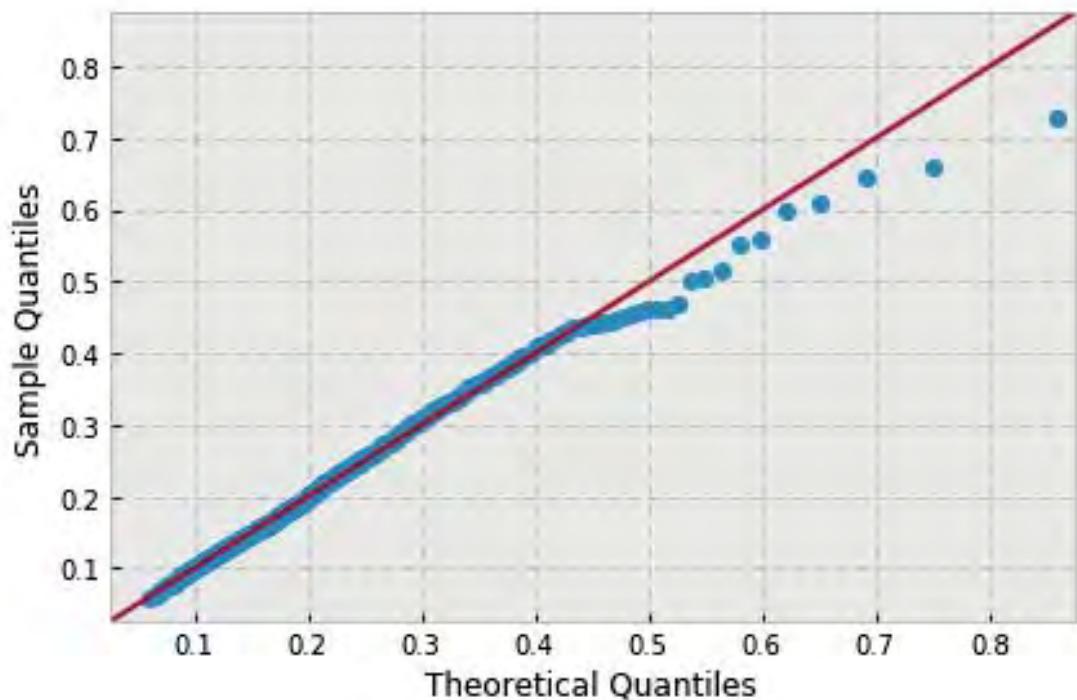


```
variable name: sigma
variable value: 0.02222222222222223
distribution: inverse-gamma(shape=(5.7001208498156721,), loc=0.0059358384822591394,
                           scale=0.04726895325342498)
DescribeResult(nobs=1000, minmax=(0.0086177404485816844, 0.045946738812753574)
               mean=0.015973393459432737, variance=2.4848825368540315e-05,
               skewness=1.8730956399782146, kurtosis=5.314236202310266)
```

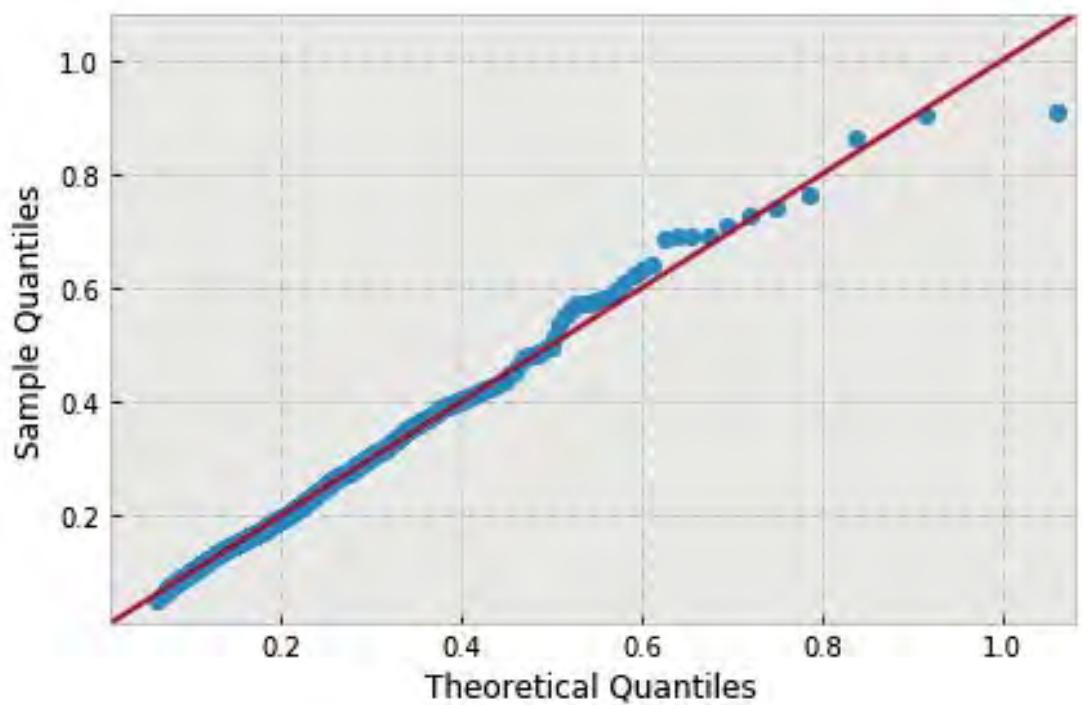
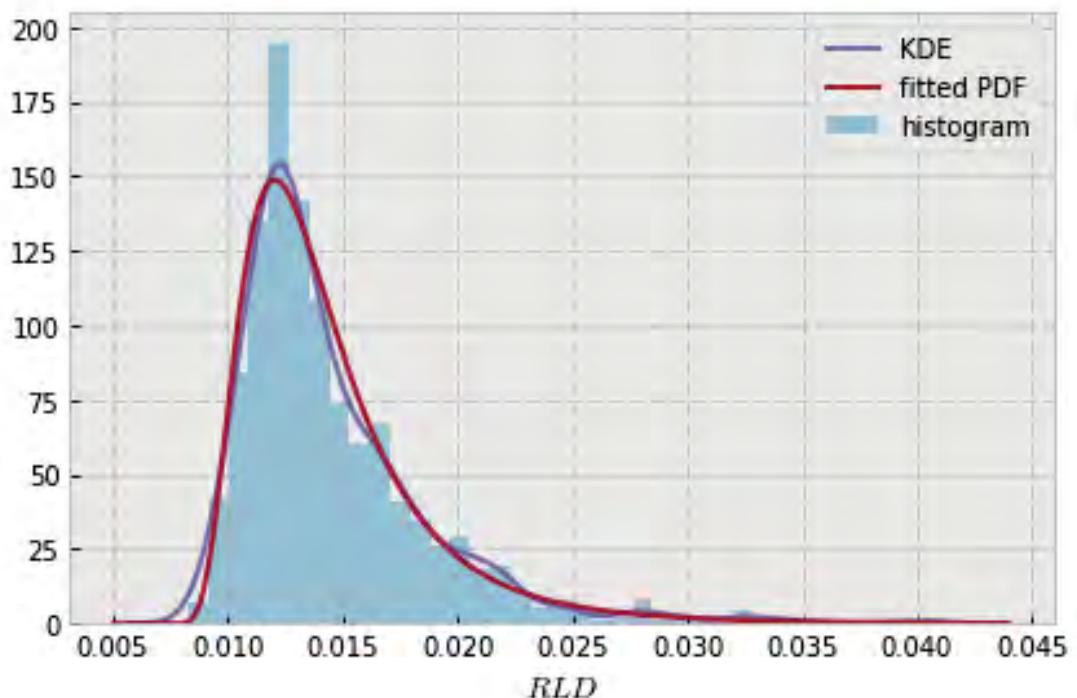


```
variable name: sigma
variable value: 0.03333333333333333
distribution: inverse-gamma(shape=(6.1464510115453486,), loc=0.005409391865409506,
    scale=0.05009458991335952)
DescribeResult(nobs=1000, minmax=(0.008336150464532158, 0.041811794786943288)
    mean=0.015122350075917217, variance=2.0517790375292094e-05,
    skewness=1.6293832082356412, kurtosis=3.8293680350769987)
```

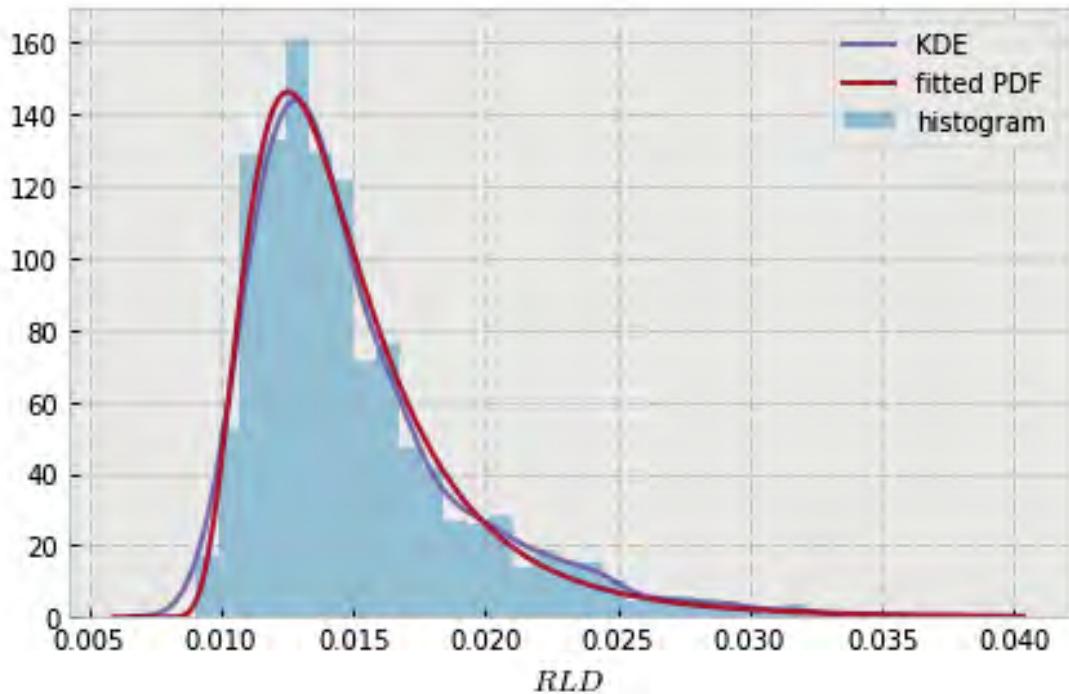


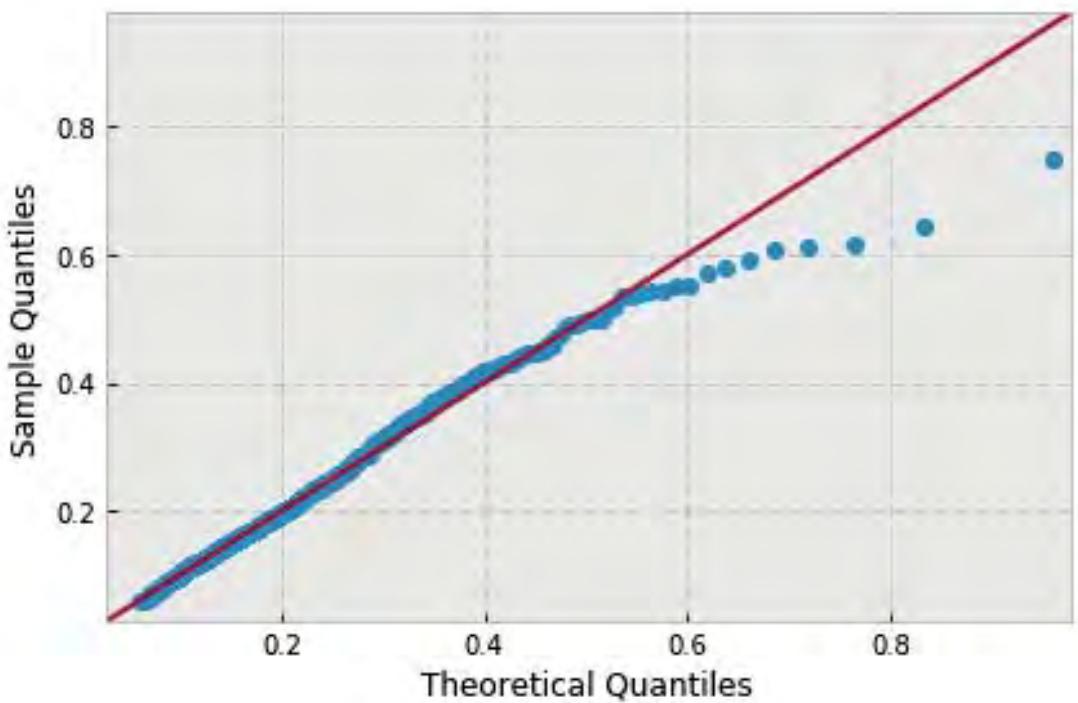


```
variable name: sigma
variable value: 0.04444444444444446
distribution: inverse-gamma(shape=(5.5721967664992835,), loc=0.006305161019191196,
                           scale=0.03781061536717012)
DescribeResult(nobs=1000, minmax=(0.0082815394857386759, 0.040740713128223083)
               mean=0.014576712213284425, variance=1.8466651551738495e-05,
               skewness=2.0874078980810395, kurtosis=6.437849658560738)
```

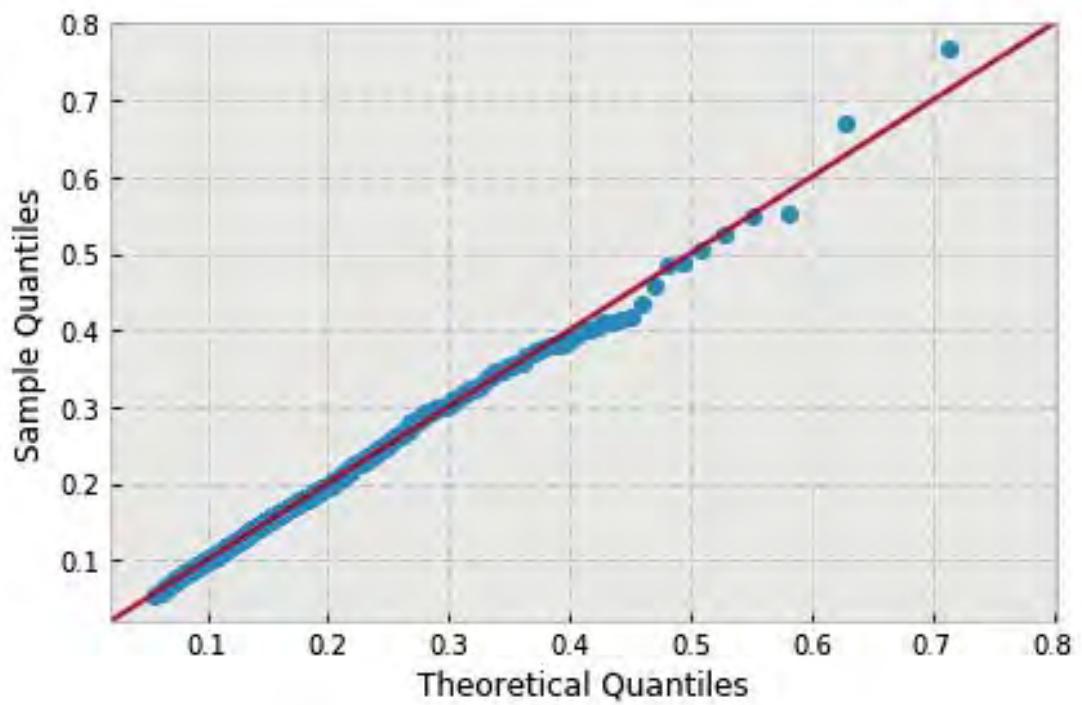
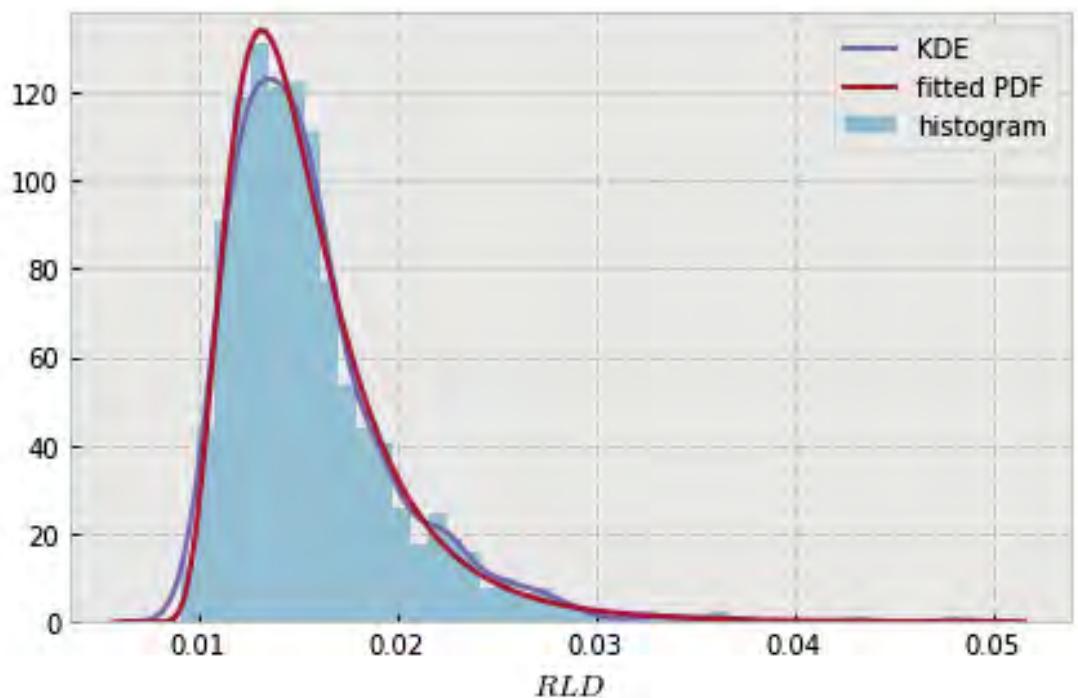


```
variable name: sigma
variable value: 0.05555555555555555
distribution: inverse-gamma(shape=(5.834139136630505,), loc=0.006525400178599936,
                           scale=0.04107988164030046)
DescribeResult(nobs=1000, minmax=(0.0090226193139567104, 0.037264661335753145)
               mean=0.015004407328231042, variance=1.668606650383748e-05,
               skewness=1.5685854986187258, kurtosis=2.9569918023096395)
```

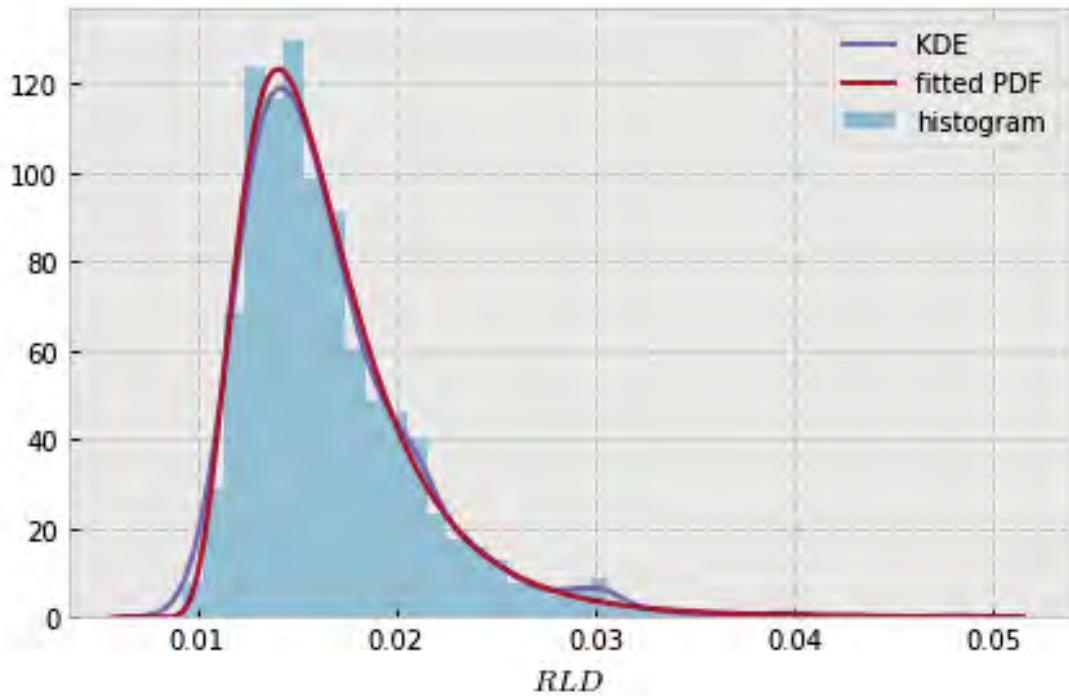


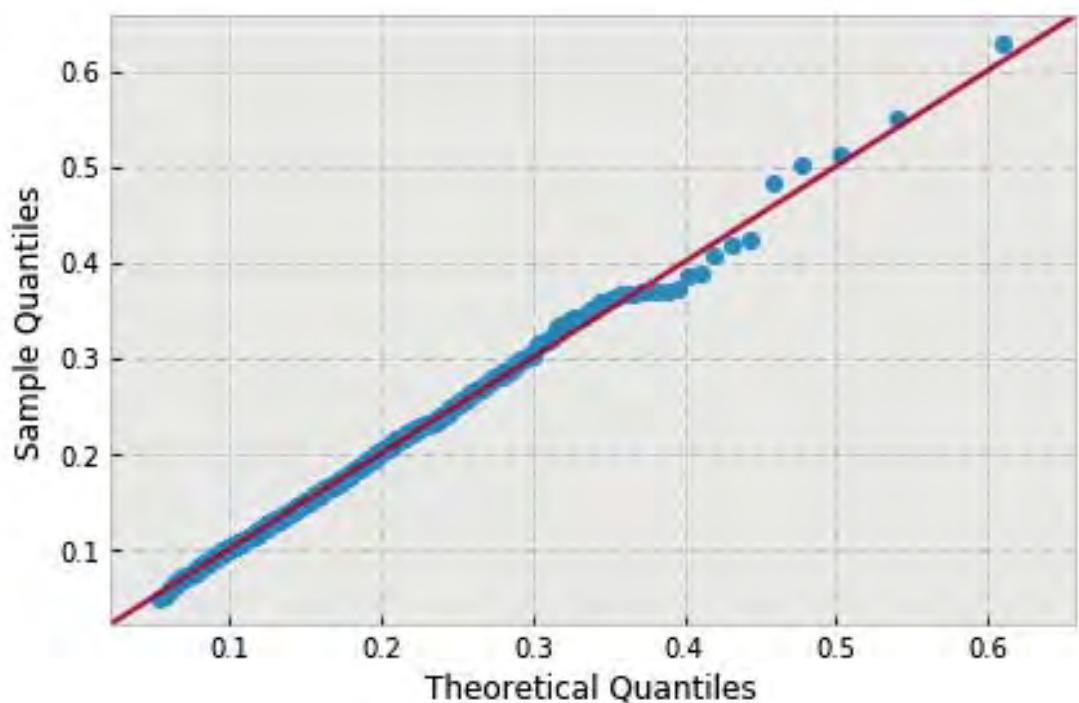


```
variable name: sigma
variable value: 0.06666666666666667
distribution: inverse-gamma(shape=(6.7276211835629169,), loc=0.006071364506935142,
                           scale=0.055116588716440686)
DescribeResult(nobs=1000, minmax=(0.0090801829343274305, 0.048321176240643006)
               mean=0.015687355922797446, variance=1.8817899667853744e-05,
               skewness=1.913245373642265, kurtosis=6.600784310473152)
```

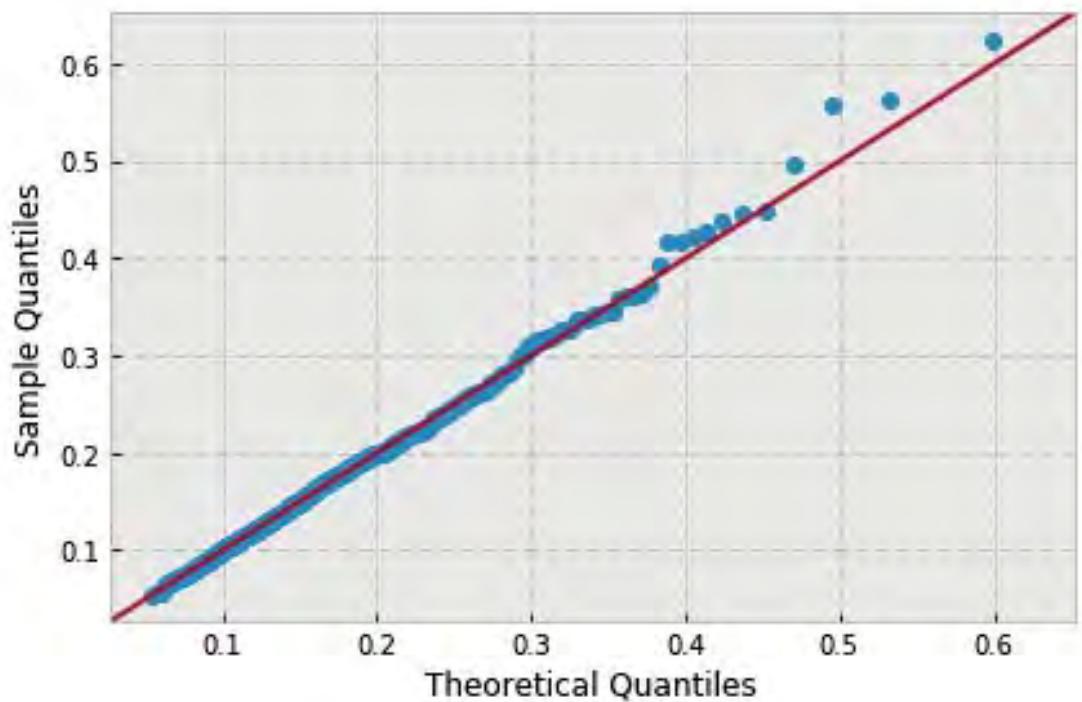
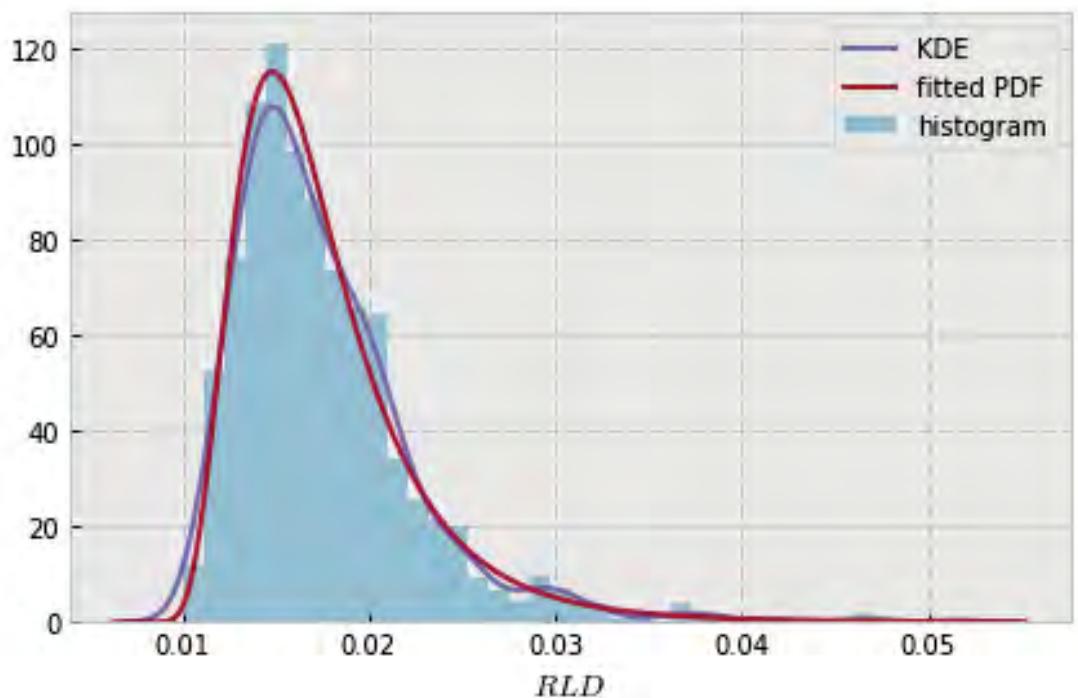


```
variable name: sigma
variable value: 0.07777777777777778
distribution: inverse-gamma(shape=(7.2738406784464669,), loc=0.005945260602309613,
                           scale=0.06707806355379364)
DescribeResult(nobs=1000, minmax=(0.009251217312919939, 0.048074074661452859)
               mean=0.016636177514124099, variance=2.1270929905547223e-05,
               skewness=1.8032411374778397, kurtosis=5.401658994636554)
```

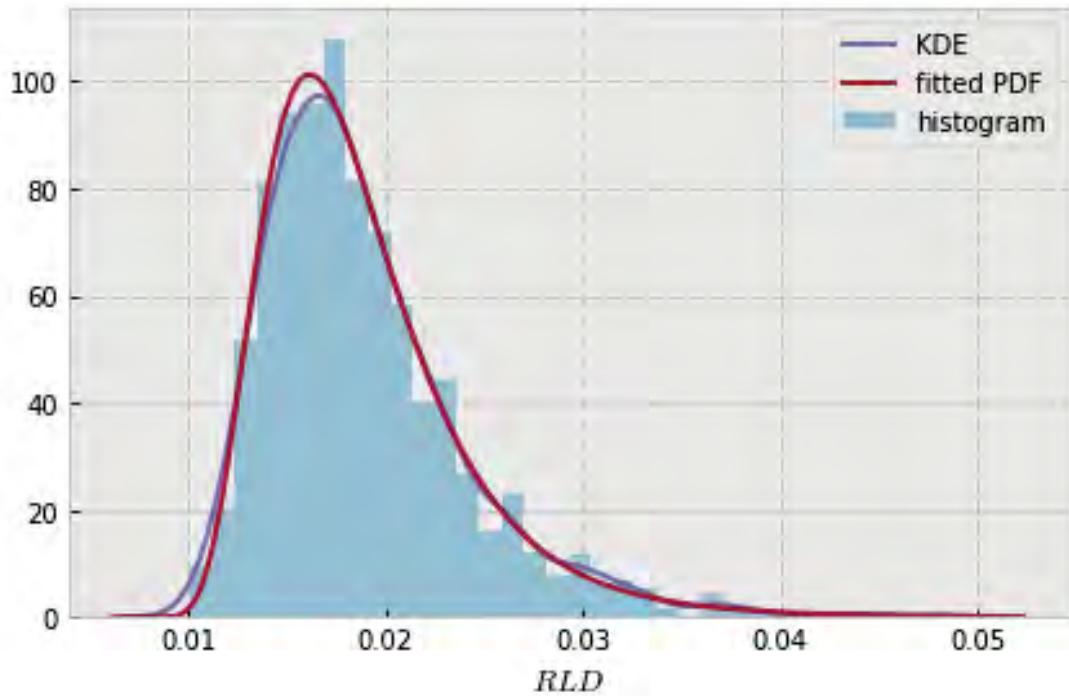


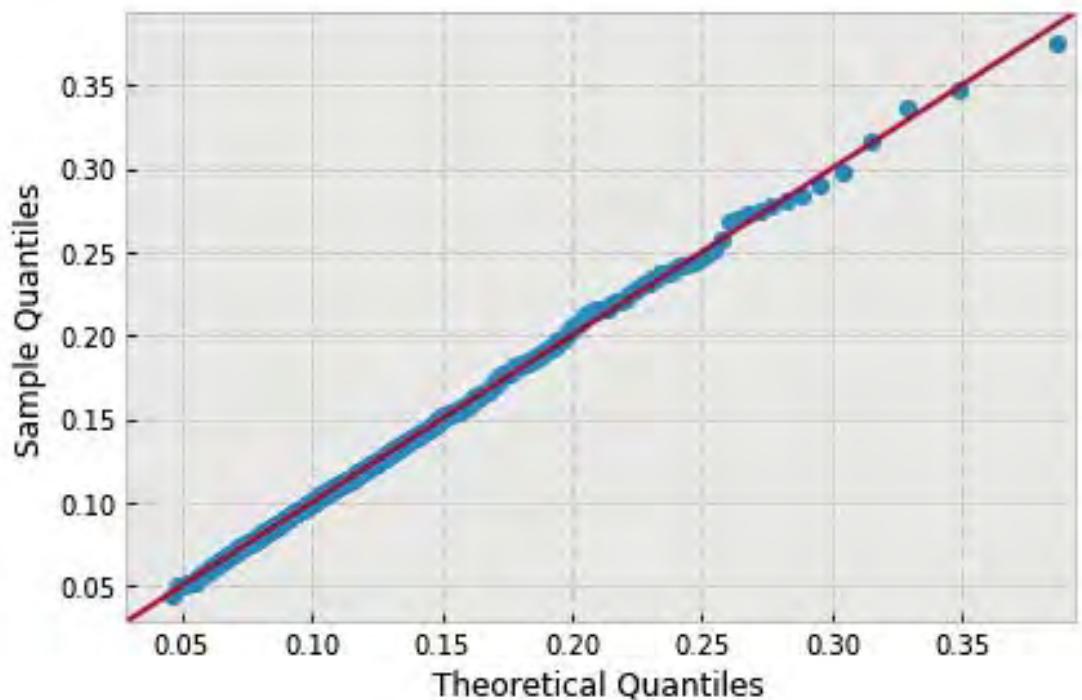


```
variable name: sigma
variable value: 0.08888888888888889
distribution: inverse-gamma(shape=(7.3415975778503899,), loc=0.00610120625500524,
    scale=0.07276262885722554)
DescribeResult(nobs=1000, minmax=(0.010081540555912752, 0.051465018191489298)
    mean=0.017572086627795074, variance=2.4307597949119932e-05,
    skewness=1.9207202338609026, kurtosis=6.499432950658321)
```

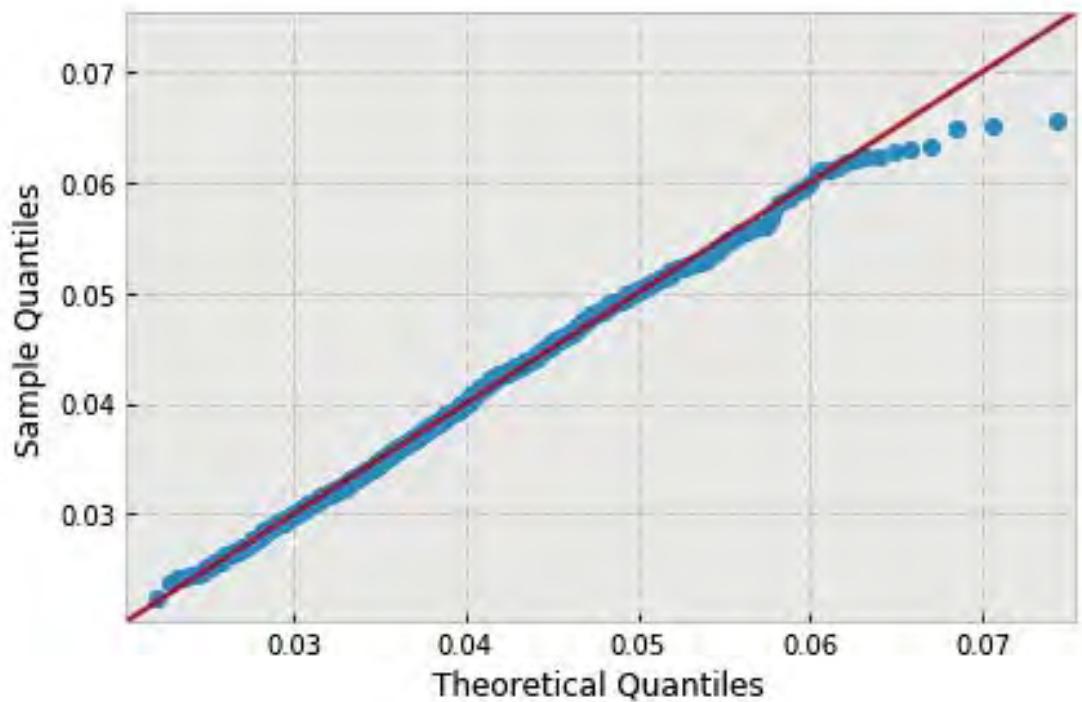
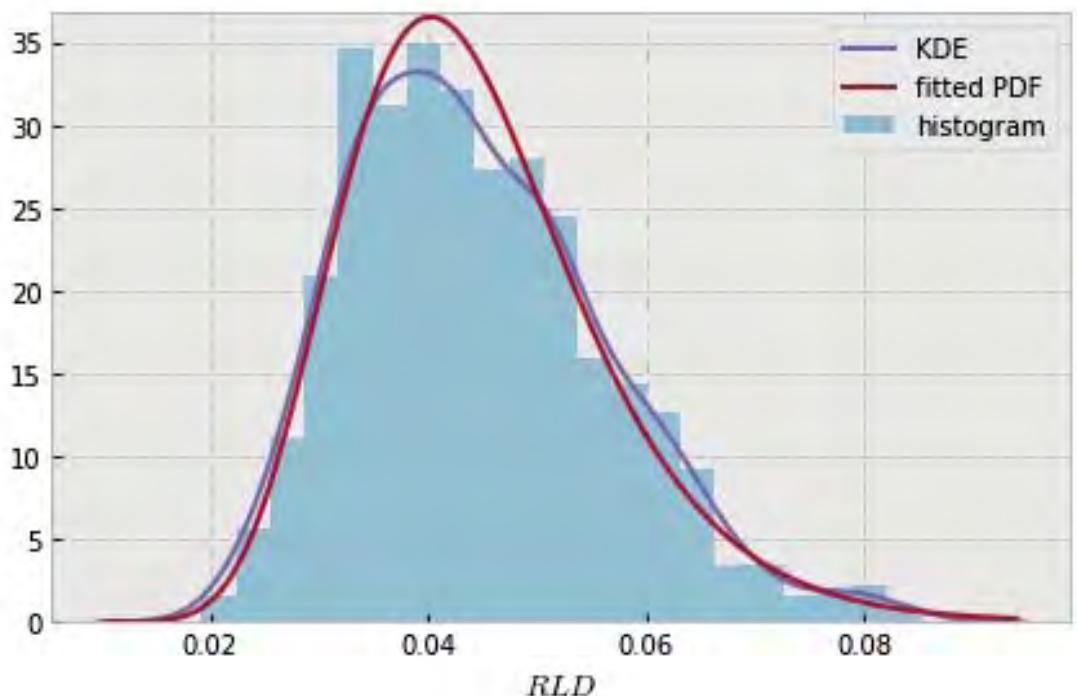


```
variable name: sigma
variable value: 0.1
distribution: inverse-gamma(shape=(9.2768513487201822,), loc=0.004846906446694442,
    scale=0.116198797391913)
DescribeResult(nobs=1000, minmax=(0.010085246382423779, 0.048467103472549171)
    mean=0.018882291622132556, variance=2.6515849178750486e-05,
    skewness=1.466064781182595, kurtosis=3.376317114699087)
```

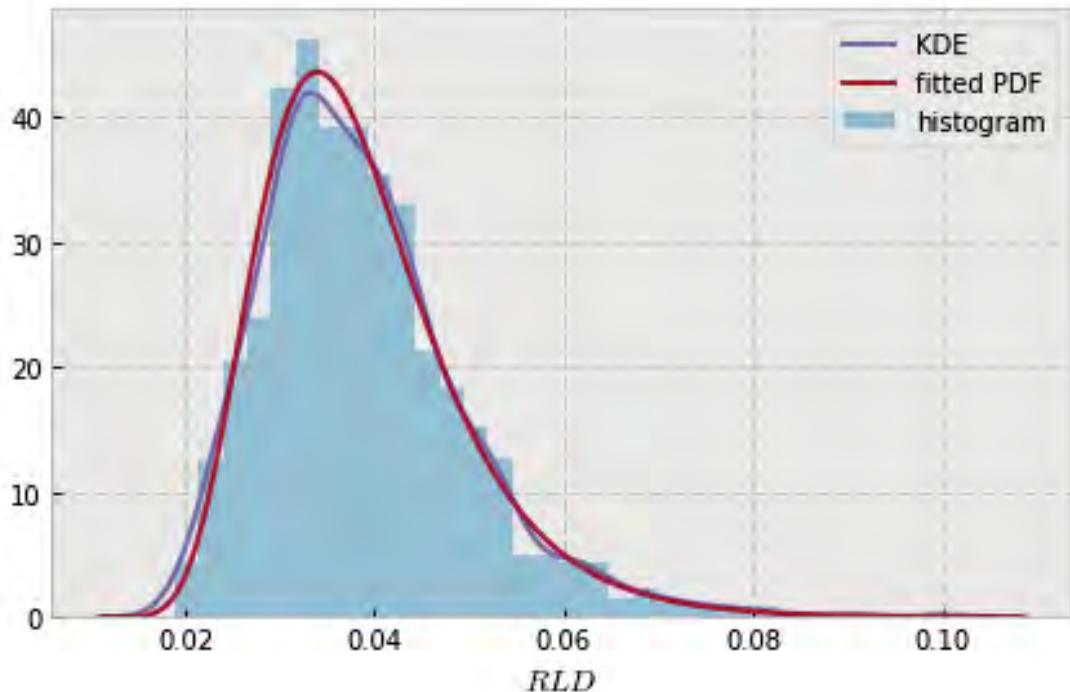


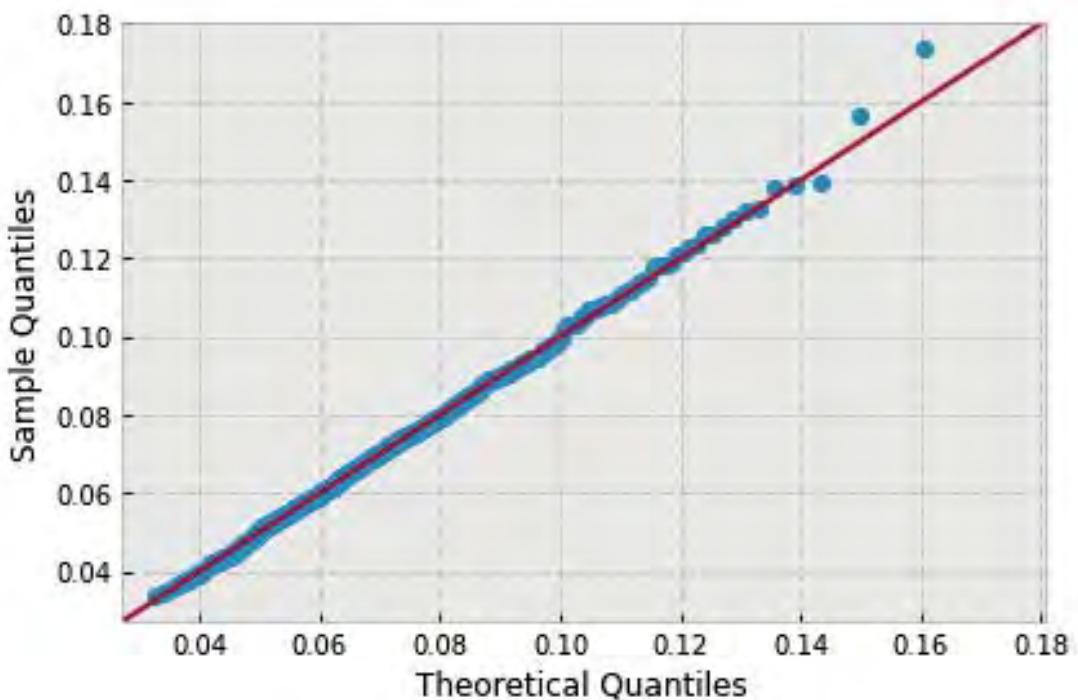


```
variable name: theta
variable value: 0.6981317007977318
distribution: inverse-gamma(shape=(26.616527180443711,), loc=-0.014913048900563487,
                           scale=1.5231485677558534)
DescribeResult(nobs=1000, minmax=(0.019177194841051681, 0.08518105115112011)
               mean=0.044537701375638505, variance=0.00014025640722225487,
               skewness=0.6221685561422626, kurtosis=0.1859801204788134)
```

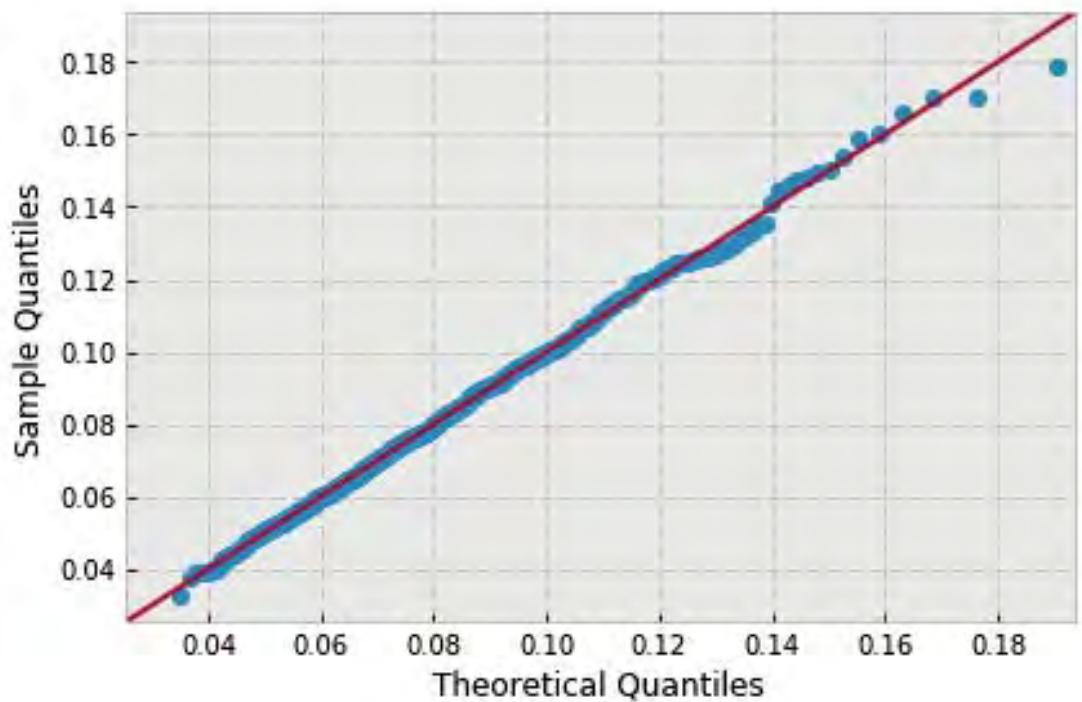
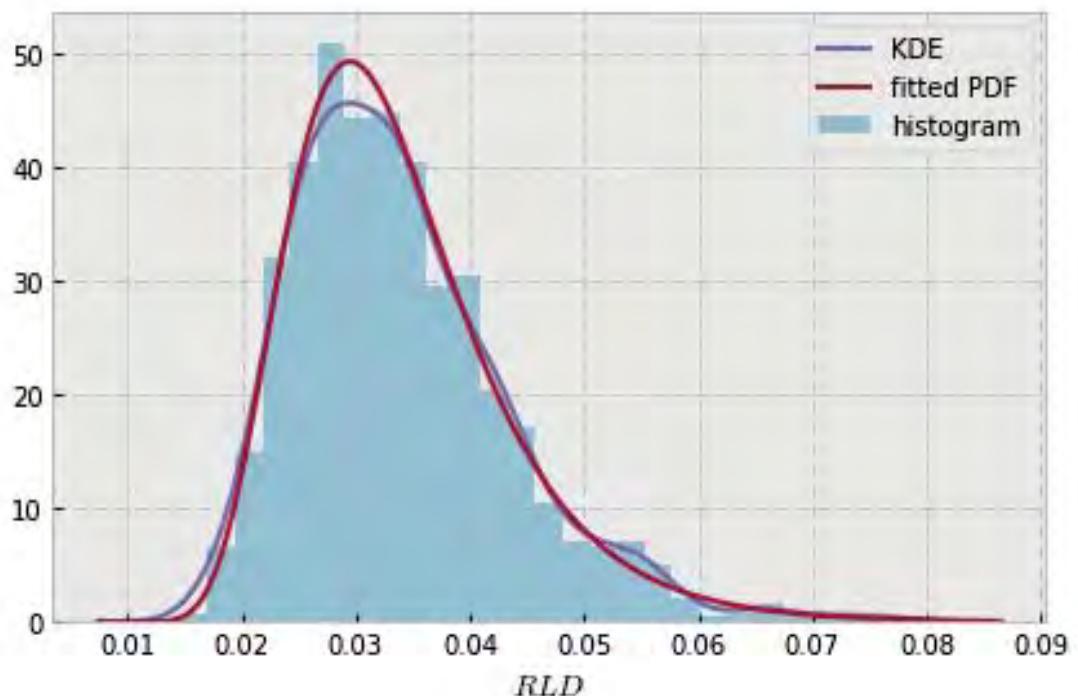


```
variable name: theta
variable value: 0.7853981633974483
distribution: inverse-gamma(shape=(15.705561784644878,), loc=-0.0009713023155058773,
                           scale=0.5837590497404348)
DescribeResult(nobs=1000, minmax=(0.018888408209669898, 0.10035799078646927)
               mean=0.038722692574654168, variance=0.00011446324636621822,
               skewness=1.1241662865988848, kurtosis=2.315186901963072)
```

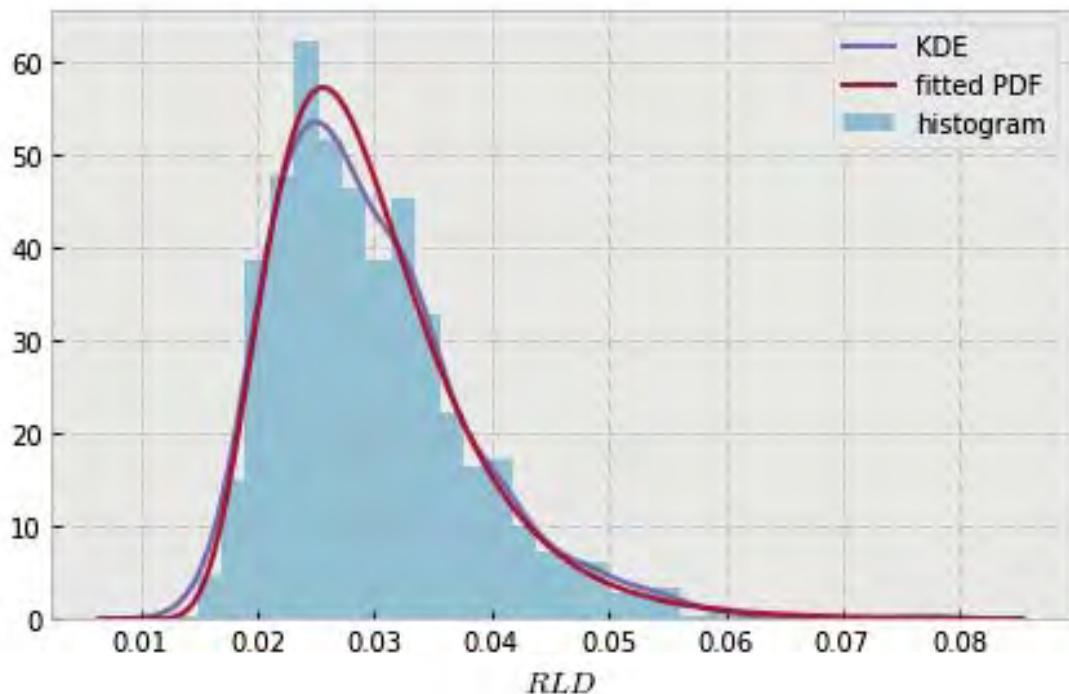


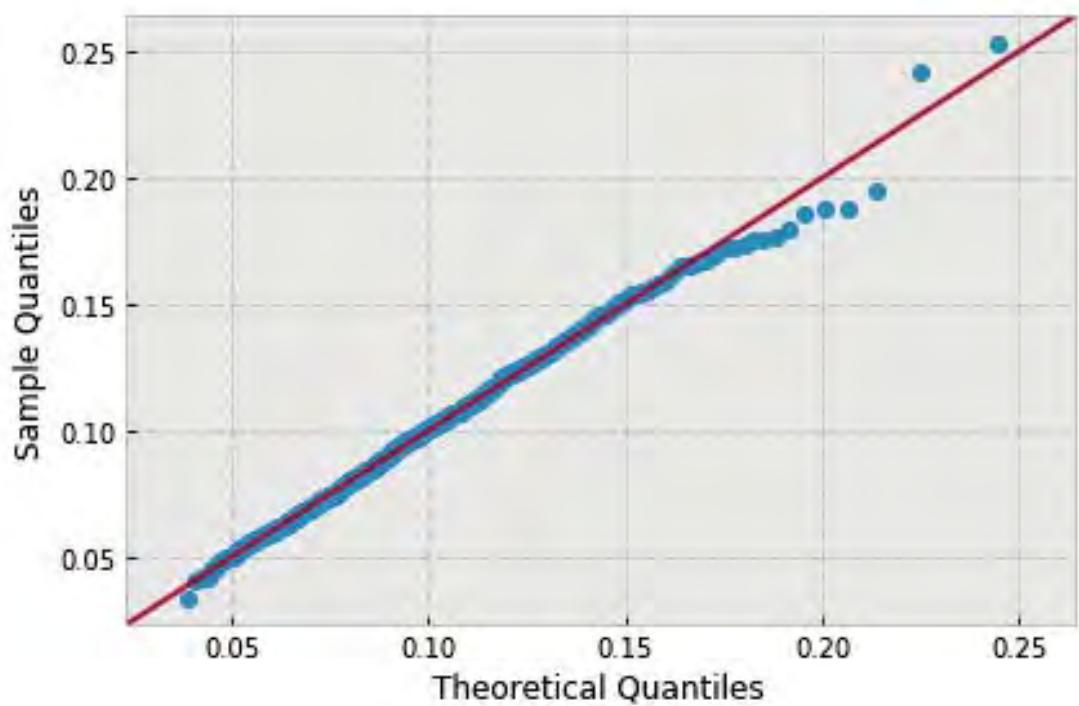


```
variable name: theta
variable value: 0.8726646259971648
distribution: inverse-gamma(shape=(14.094234353110028,), loc=0.00028978003010023655,
                           scale=0.44047272044897634)
DescribeResult(nobs=1000, minmax=(0.014657252324051925, 0.079208907324887959)
               mean=0.033922299081465108, variance=9.1933317316299541e-05,
               skewness=1.0853268597755297, kurtosis=1.7610050568804514)
```

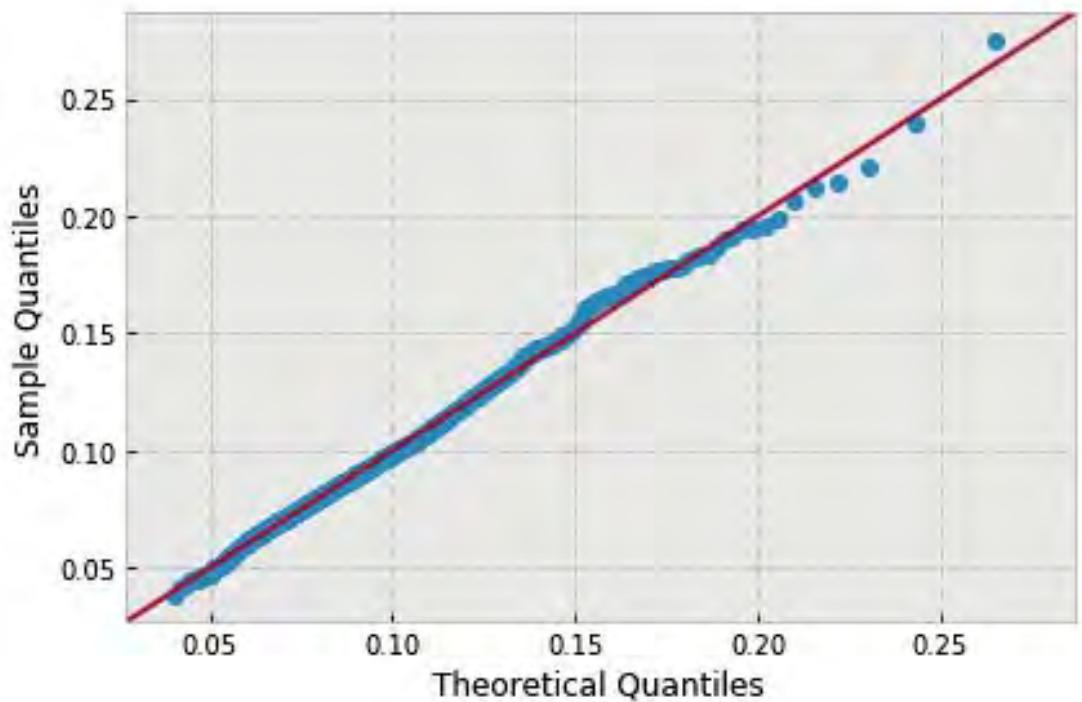
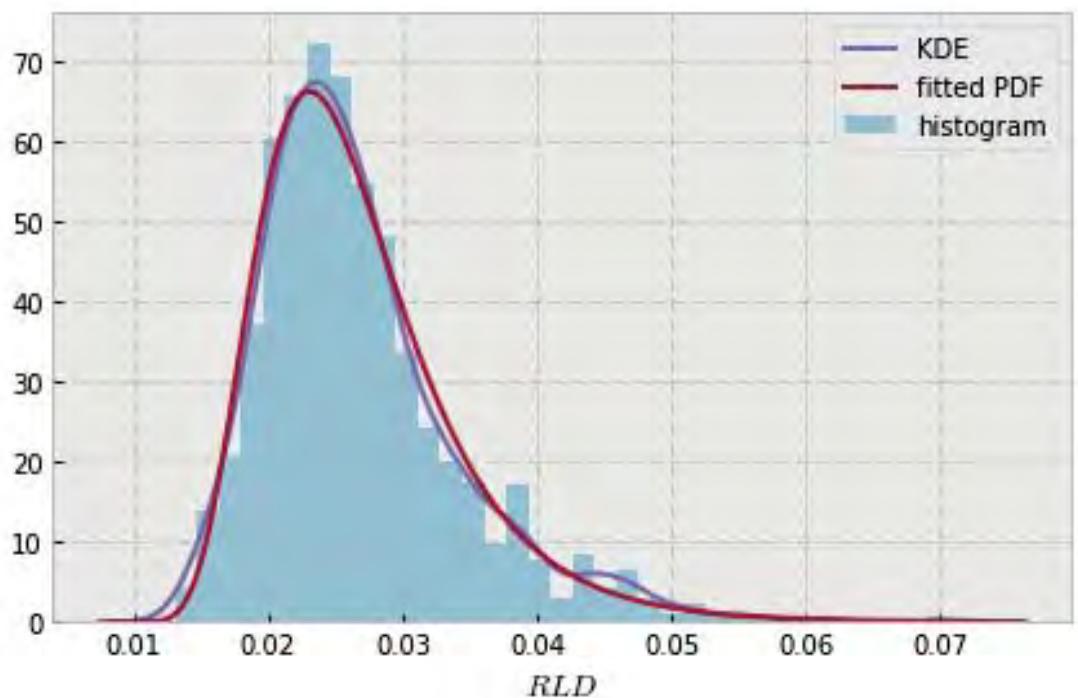


```
variable name: theta
variable value: 0.9599310885968813
distribution: inverse-gamma(shape=(12.076003758497453,), loc=0.002496062127979575,
    scale=0.30227853198960064)
DescribeResult(nobs=1000, minmax=(0.012789822317704296, 0.078979441517859031)
    mean=0.029778397109200504, variance=7.1944391755180854e-05,
    skewness=1.1779086283003184, kurtosis=2.323093204666435)
```

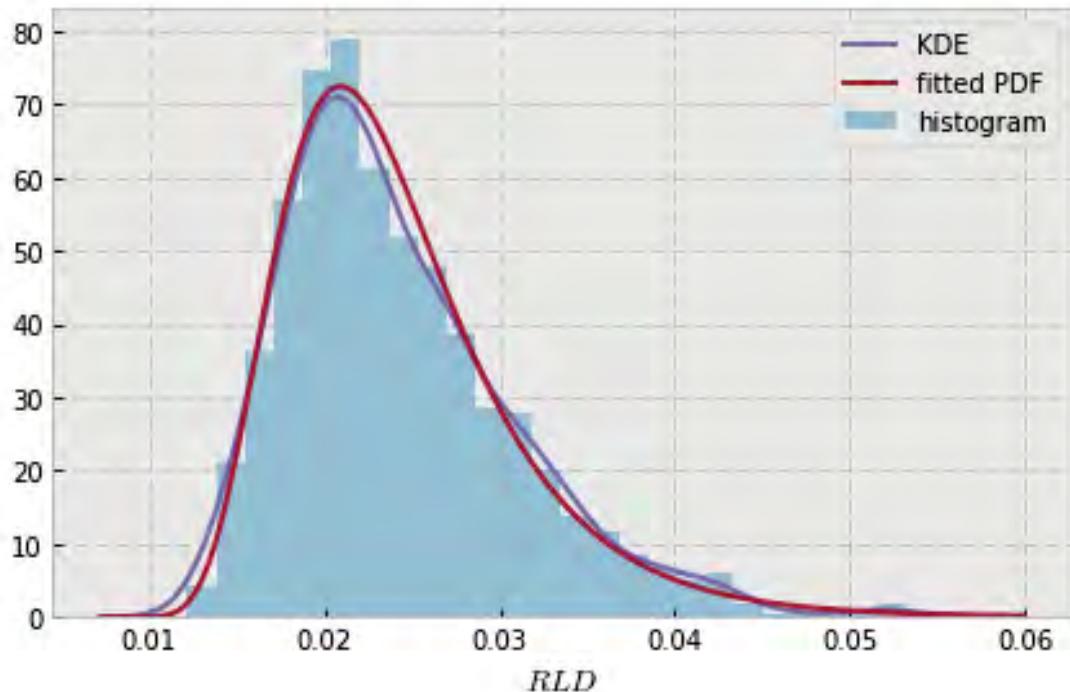


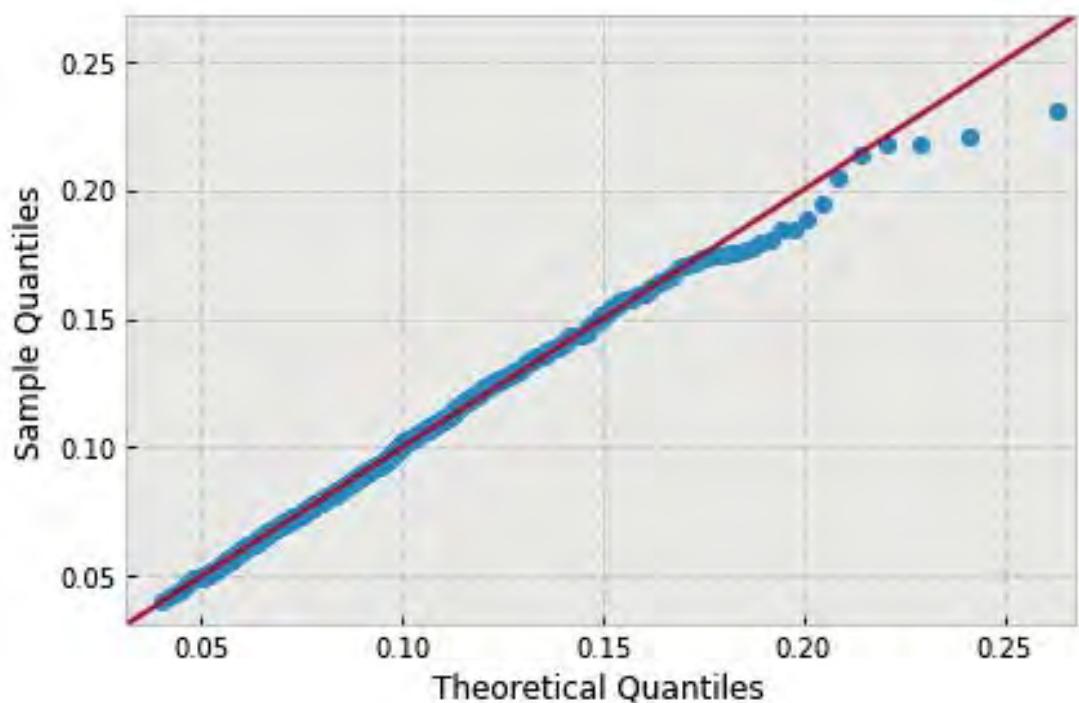


```
variable name: theta
variable value: 1.0471975511965979
distribution: inverse-gamma(shape=(11.520294395154988,), loc=0.0036133065675875344,
                           scale=0.2437059081433237)
DescribeResult(nobs=1000, minmax=(0.013062704557660215, 0.070564612974755997)
               mean=0.026780451295168983, variance=5.6275731561339643e-05,
               skewness=1.3132294893620362, kurtosis=2.599089897532158)
```

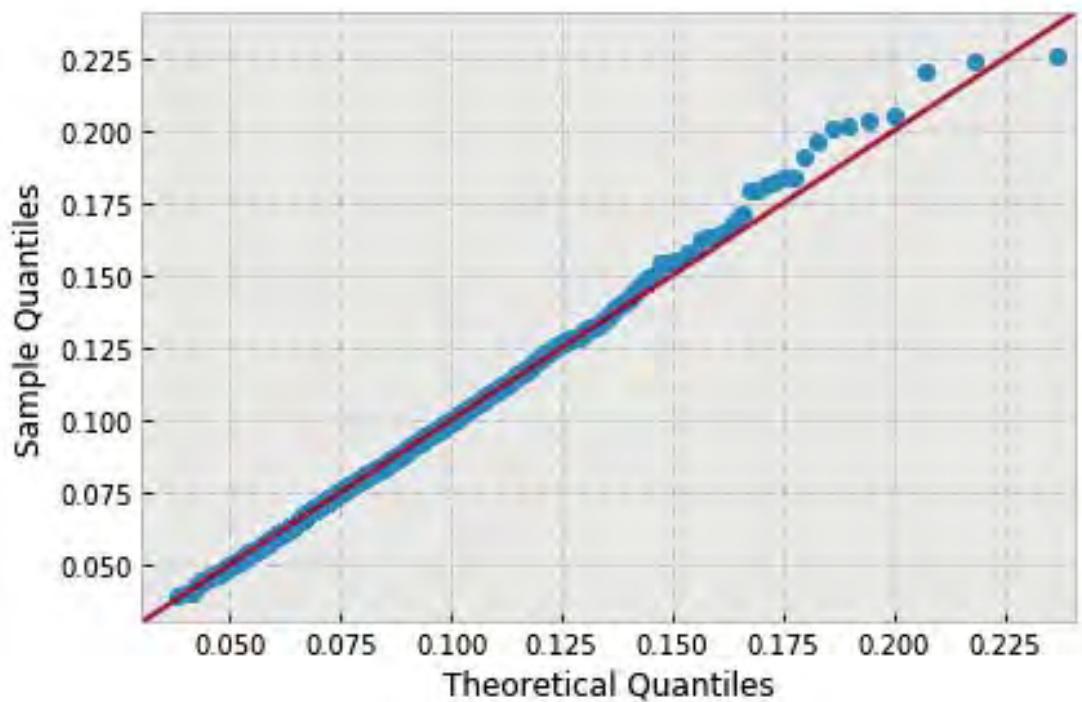
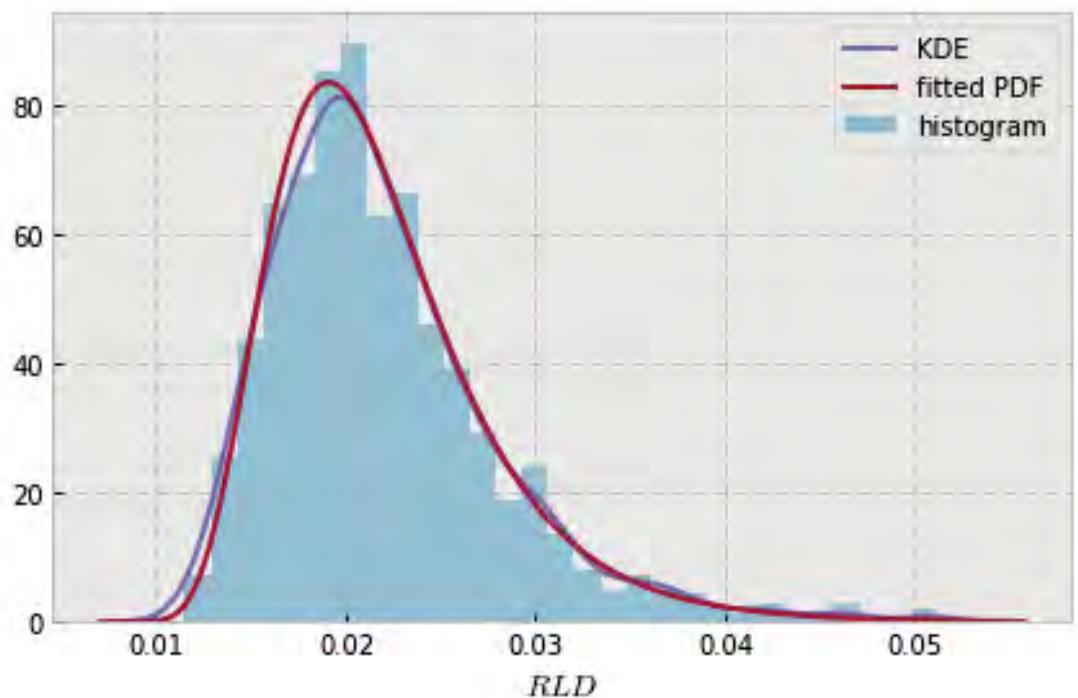


```
variable name: theta
variable value: 1.1344640137963142
distribution: inverse-gamma(shape=(11.571046003777646,), loc=0.0030679062939218565,
    scale=0.22422771869152996)
DescribeResult(nobs=1000, minmax=(0.012159194789600265, 0.054889032965544975)
    mean=0.024266761295634487, variance=4.487049745365464e-05,
    skewness=1.0773354244410906, kurtosis=1.477081360664564)
```

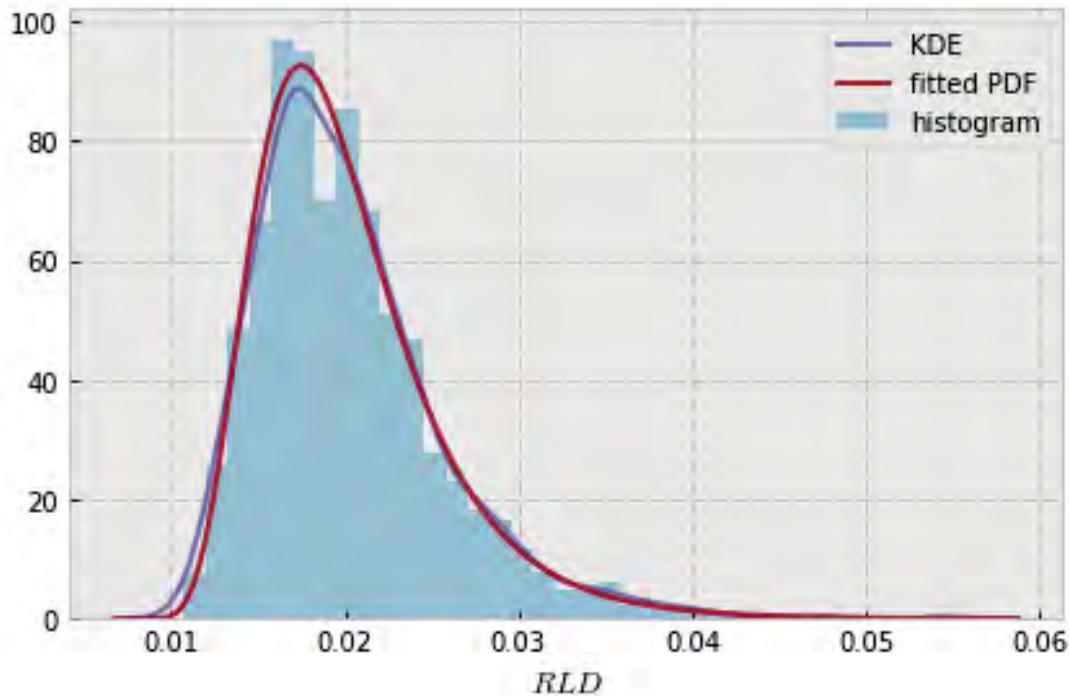


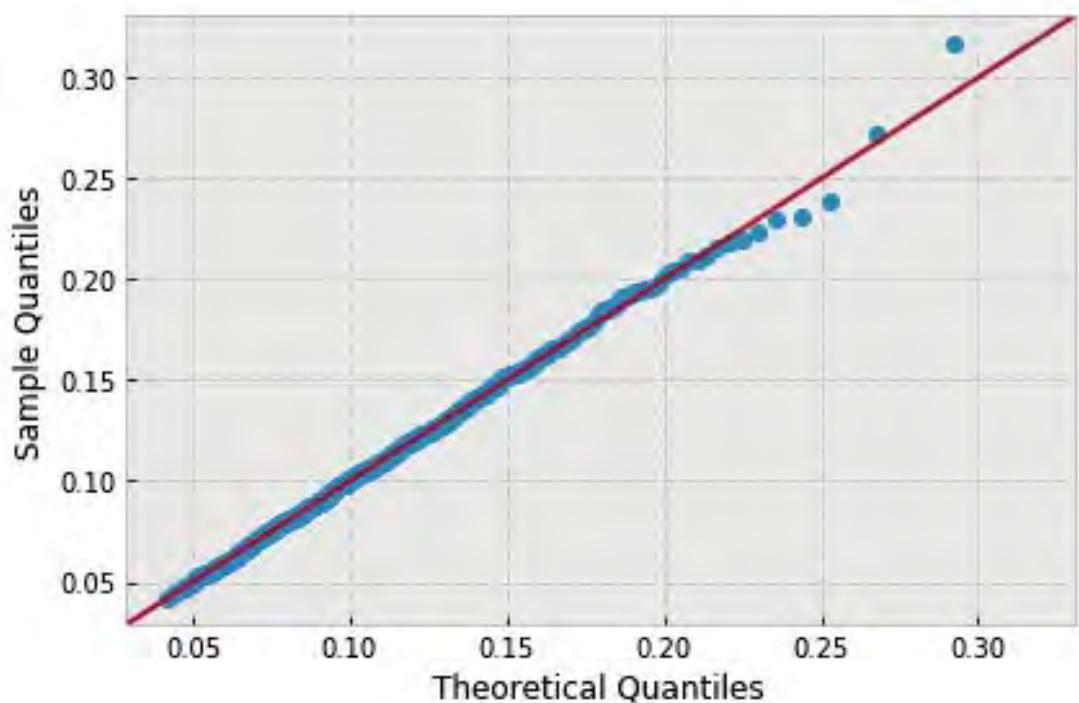


```
variable name: theta
variable value: 1.2217304763960306
distribution: inverse-gamma(shape=(12.327656010363837,), loc=0.003071780475877169,
                           scale=0.21338643269215107)
DescribeResult(nobs=1000, minmax=(0.011503466359629119, 0.051218782555127337)
               mean=0.0219406238664678, variance=3.6018958238112163e-05,
               skewness=1.3351699336488274, kurtosis=2.781752728371999)
```

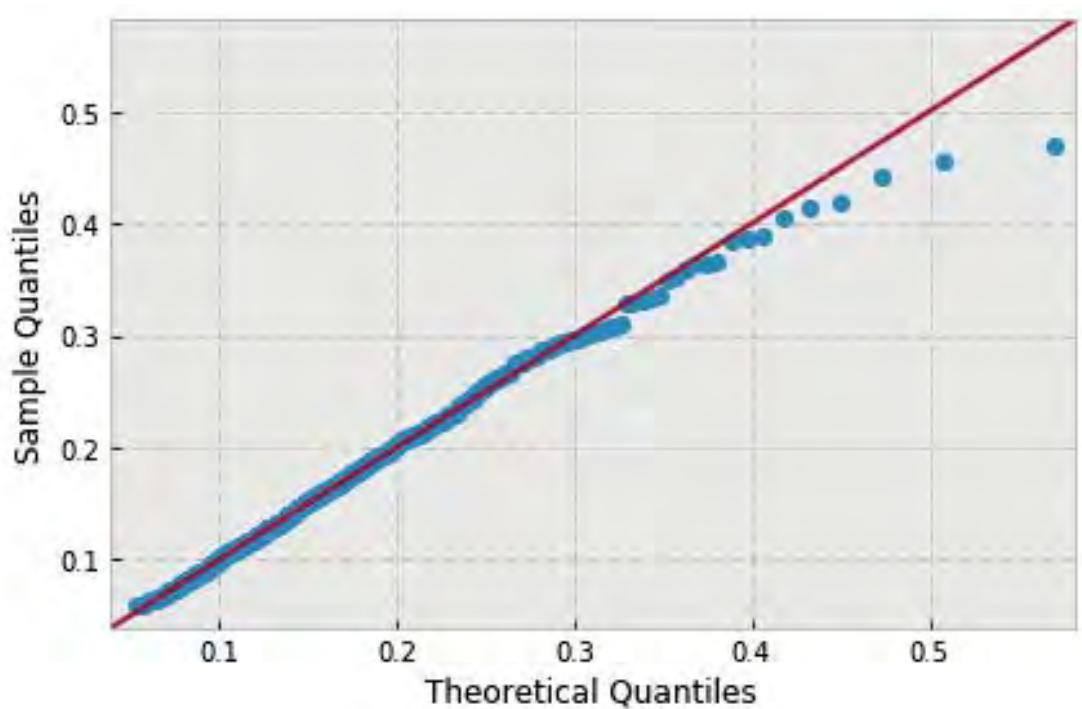
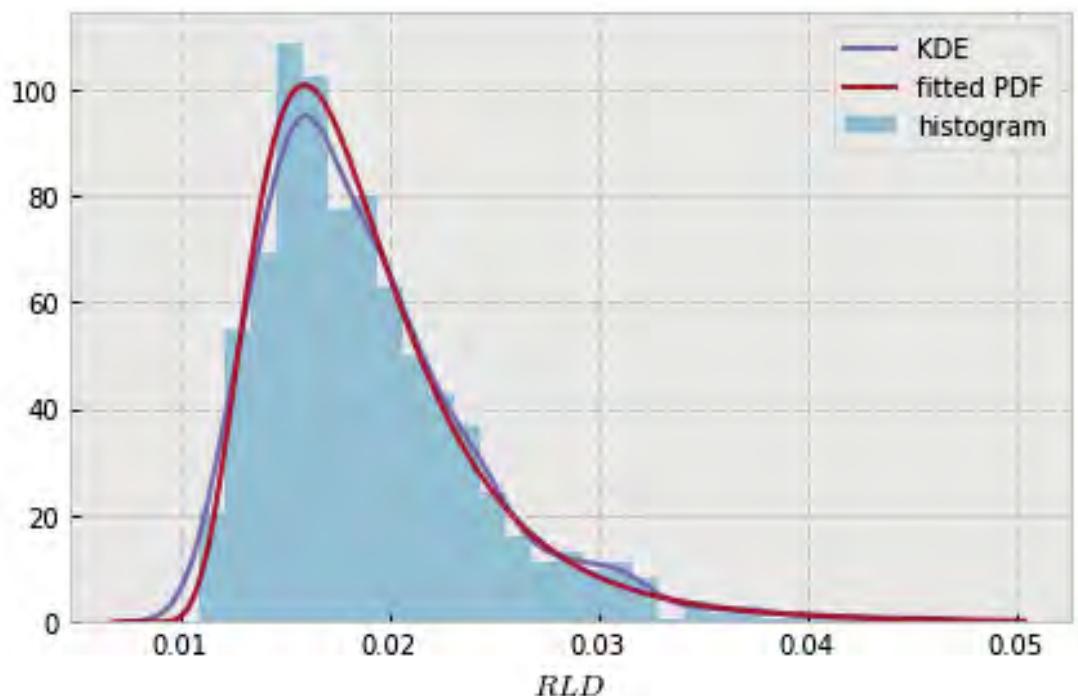


```
variable name: theta
variable value: 1.3089969389957472
distribution: inverse-gamma(shape=(10.865103047938064,), loc=0.003961070193092312,
                           scale=0.16012432602071344)
DescribeResult(nobs=1000, minmax=(0.010732754770822788, 0.054686738650277927)
               mean=0.020188264947368736, variance=2.918755073400164e-05,
               skewness=1.355159609435606, kurtosis=3.217103538141223)
```

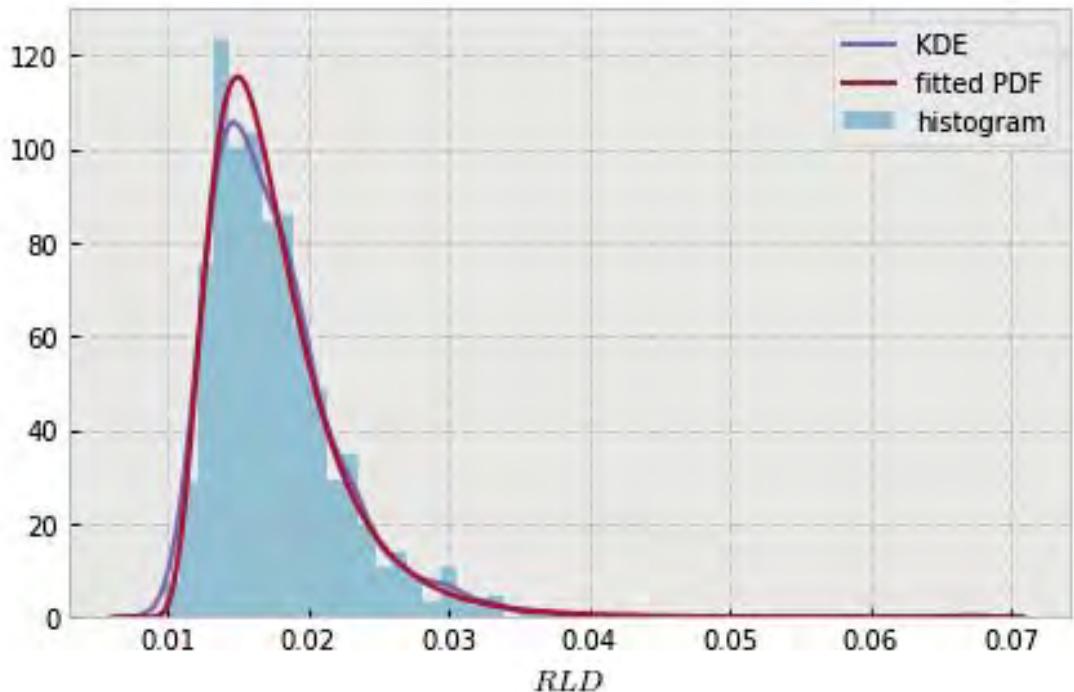


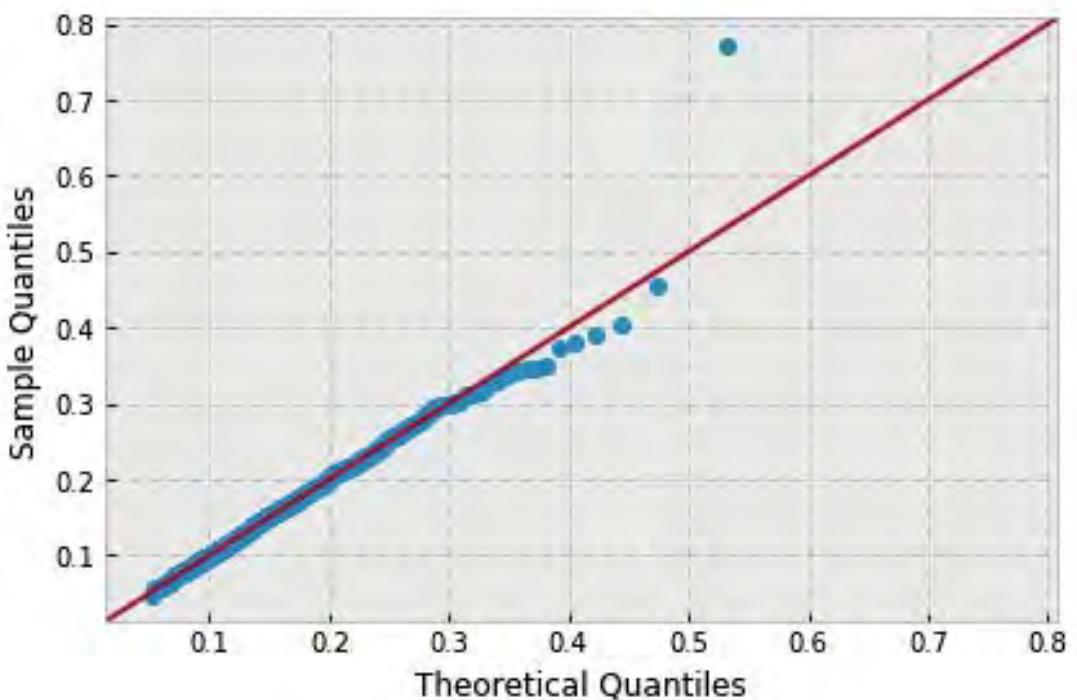


```
variable name: theta
variable value: 1.3962634015954638
distribution: inverse-gamma(shape=(7.5221983701573727,), loc=0.005837613408451714,
                           scale=0.08611679394849688)
DescribeResult(nobs=1000, minmax=(0.01089793134798423, 0.046359358295632043)
               mean=0.019019629259162847, variance=2.8985796919617022e-05,
               skewness=1.4276443404509866, kurtosis=2.8667054254102267)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: inverse-gamma(shape=(7.8039549017268932,), loc=0.0059671043156568175,
    scale=0.07927665647783516)
DescribeResult(nobs=1000, minmax=(0.0097921197884409845, 0.067214019103406533)
    mean=0.017614011606907672, variance=2.3251602557532216e-05,
    skewness=2.1898875954541466, kurtosis=12.518481040243122)
```

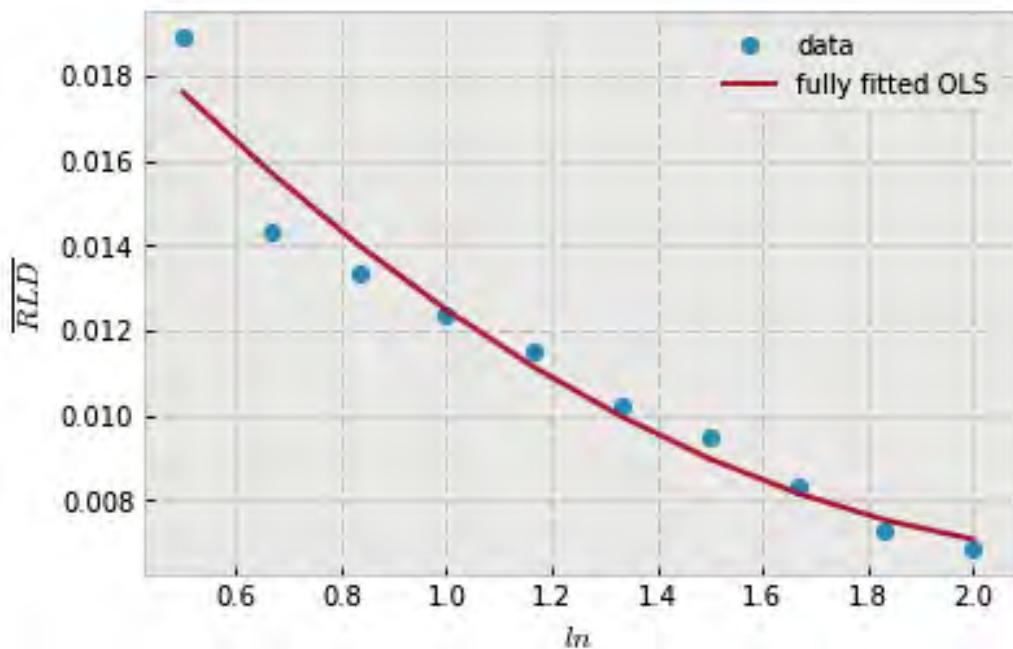




11.2 Parameter regression

```
In [28]: var_name = 'total_root_length_per_convex_hull_volume'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly2')
mean_reg('nB', var_name, 'poly1')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly2')
mean_reg('N', var_name, 'poly1')

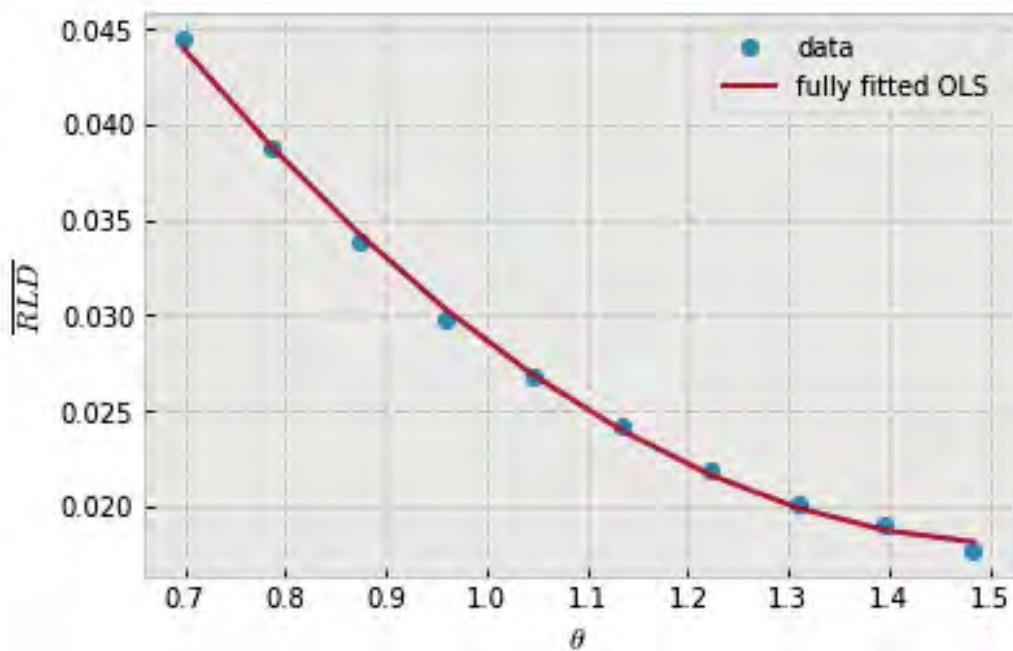
variable name: ln
model: poly2
r-squared: 0.8550551376625488
explained variance: 0.8564073755303313
```



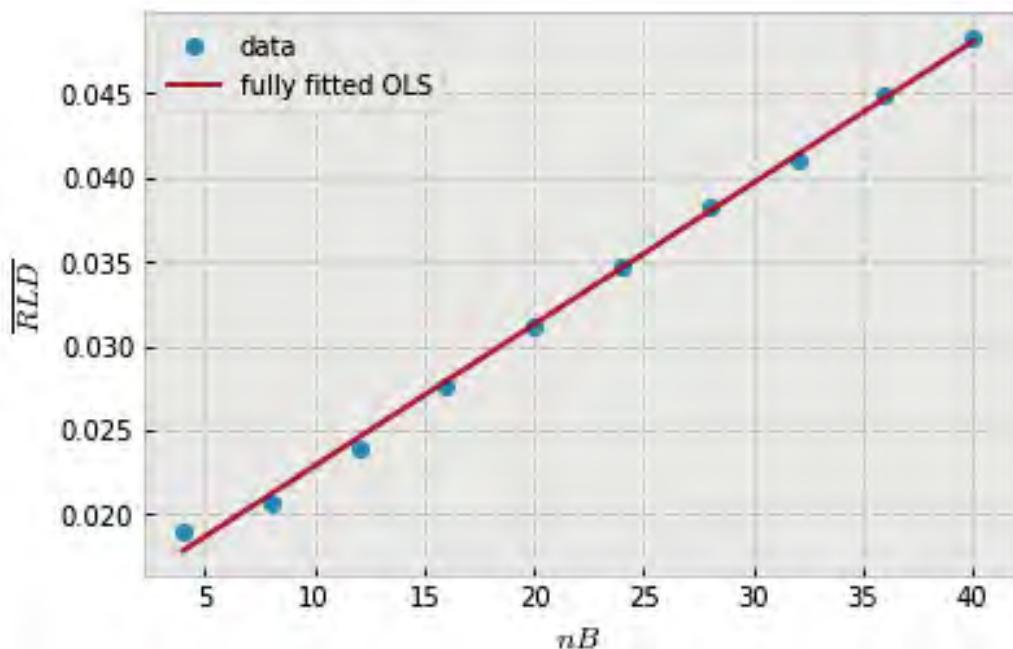
```

variable name: theta
model: poly2
r-squared: 0.9934248759598359
explained variance: 0.9934248870313761

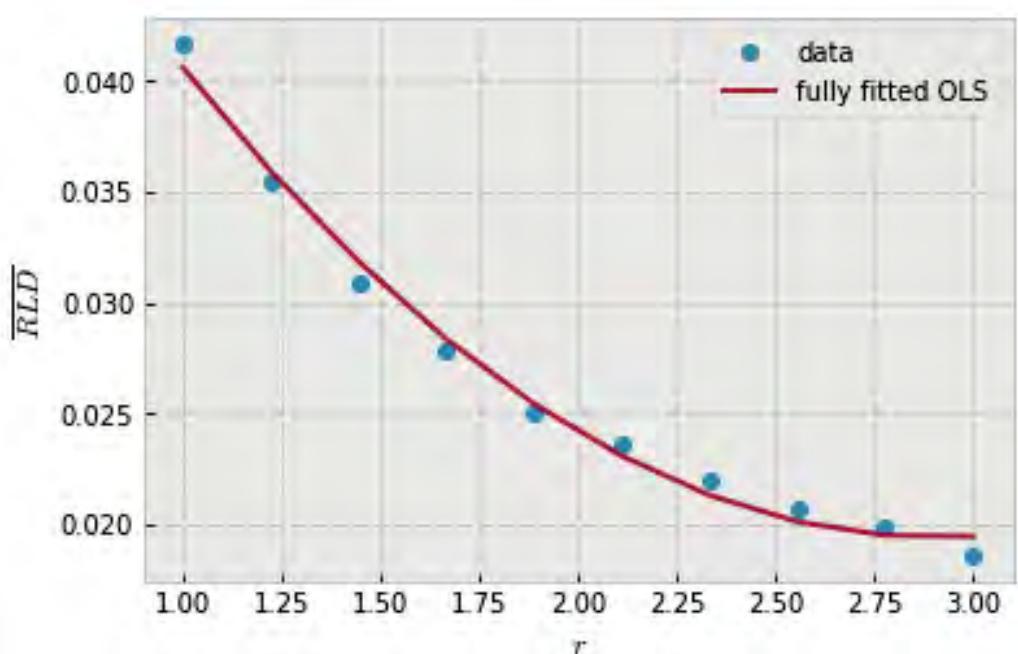
```



```
variable name: nB
model: poly1
r-squared: 0.9947968726804679
explained variance: 0.9948147585803564
```



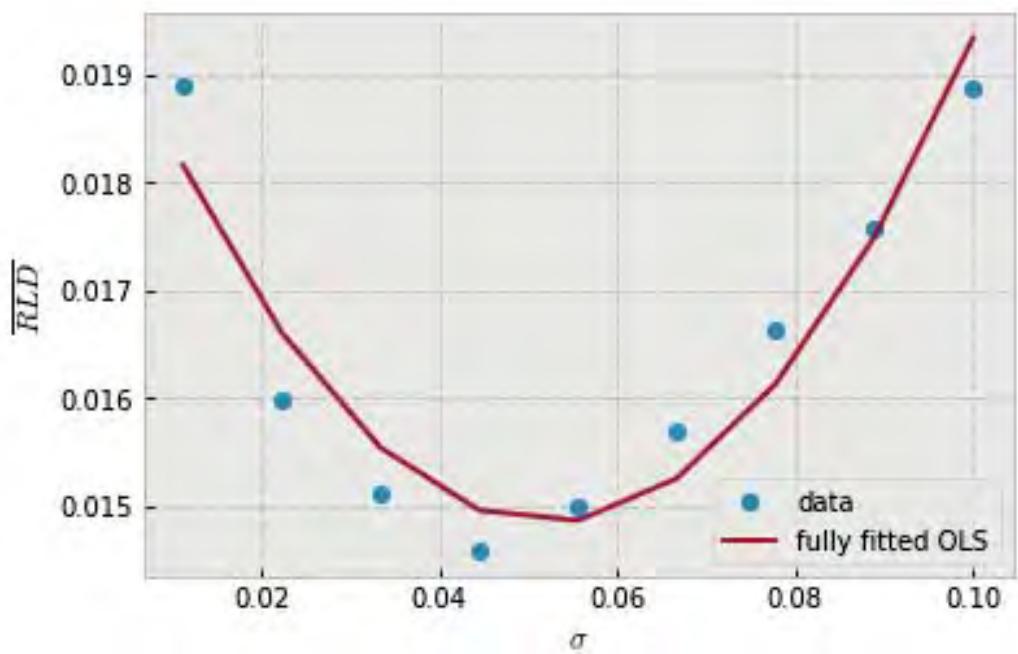
```
variable name: r
model: poly2
r-squared: 0.9629106057593357
explained variance: 0.9629225439663144
```



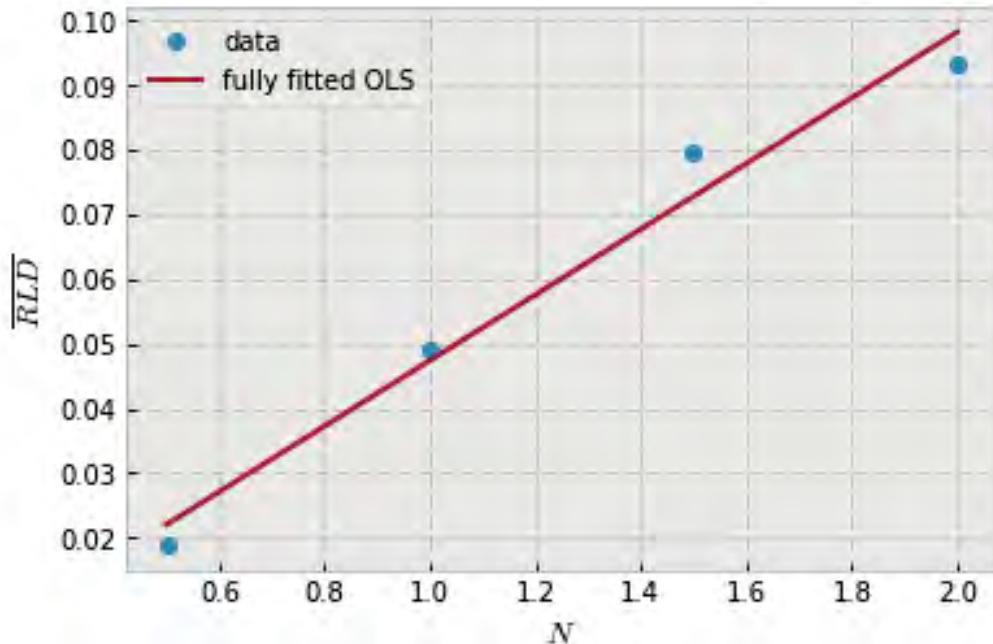
```

variable name: sigma
model: poly2
r-squared: 0.5930821389234098
explained variance: 0.5940425481095104

```



```
variable name: N
model: poly1
r-squared: 0.8532977932160298
explained variance: 0.8718213697893067
```

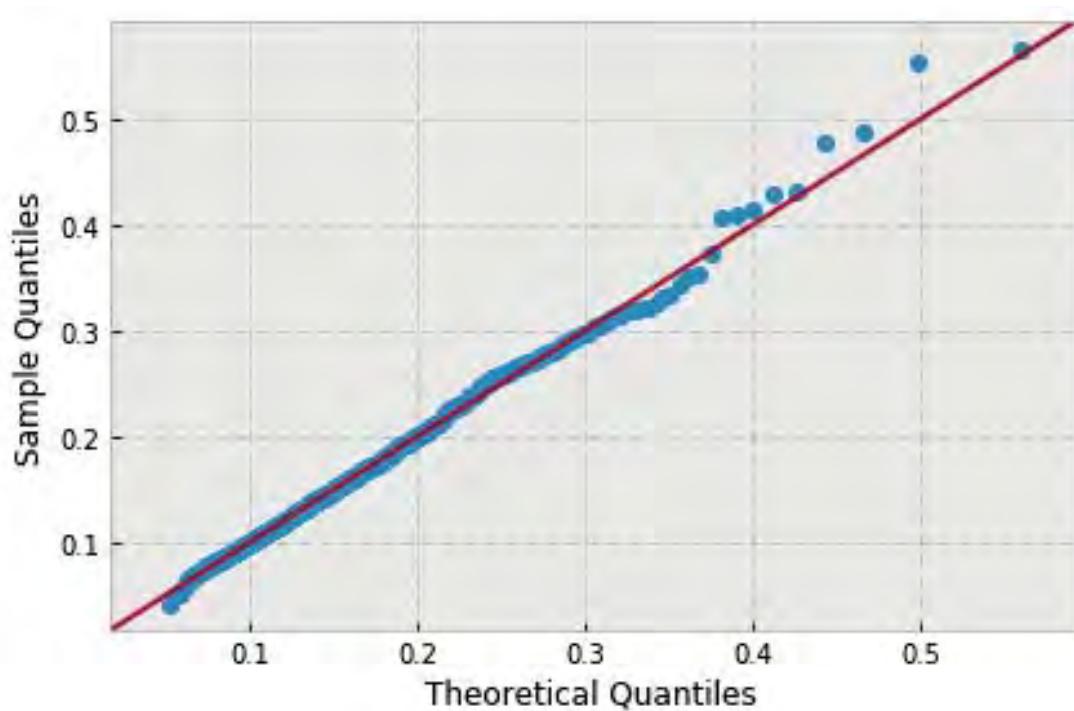
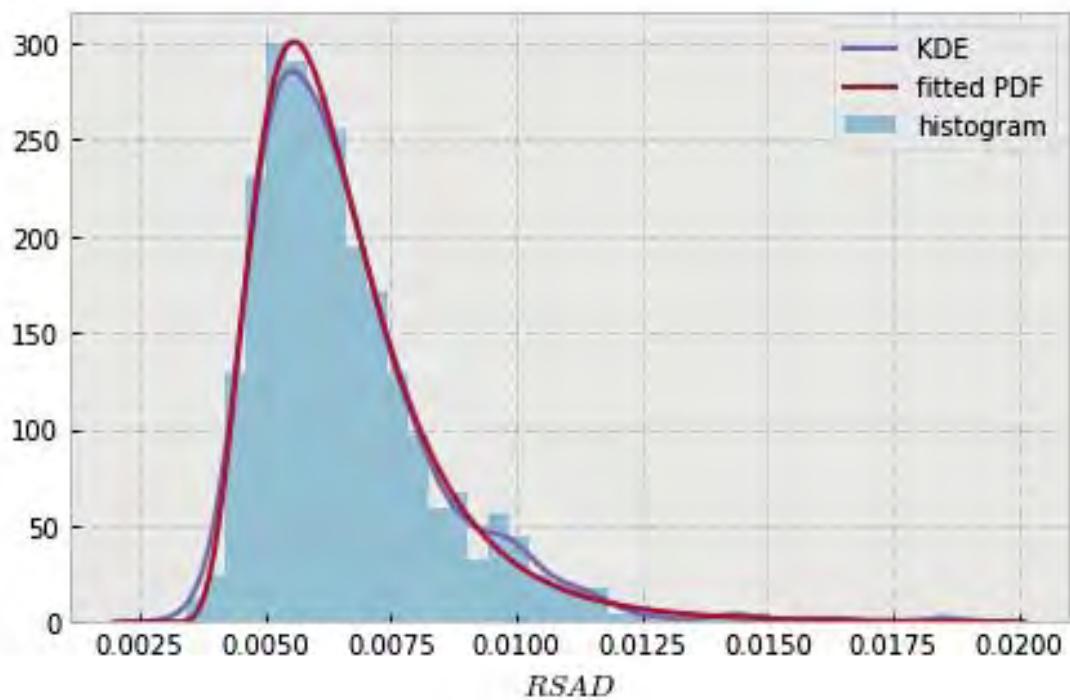


12 Root surface area relative to convex hull

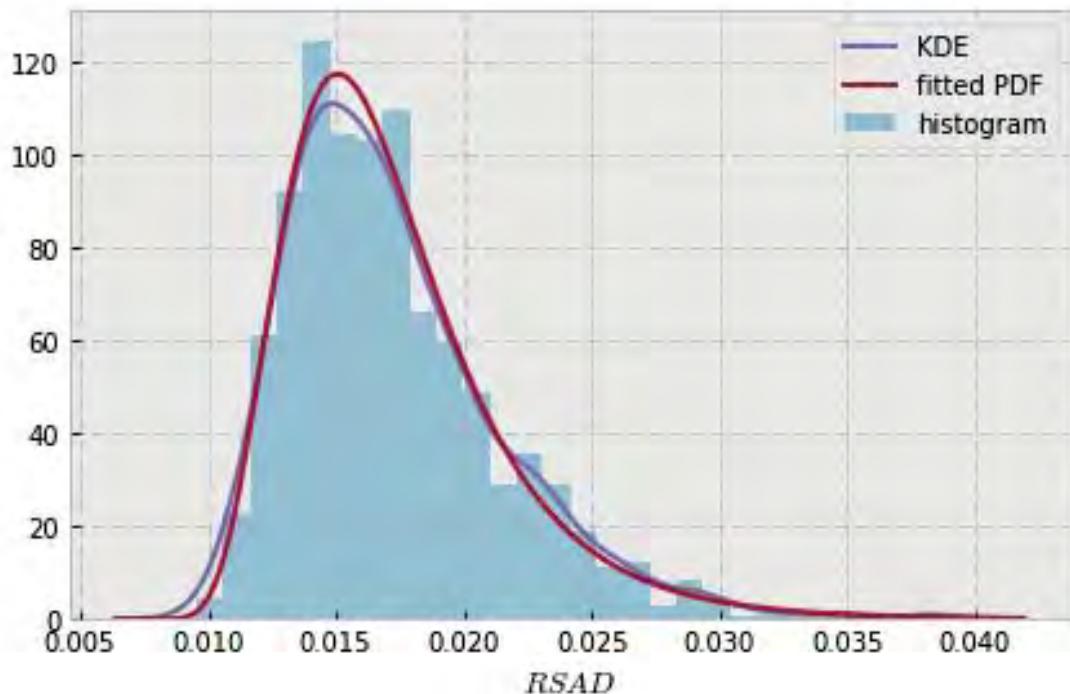
12.1 Probability distributions

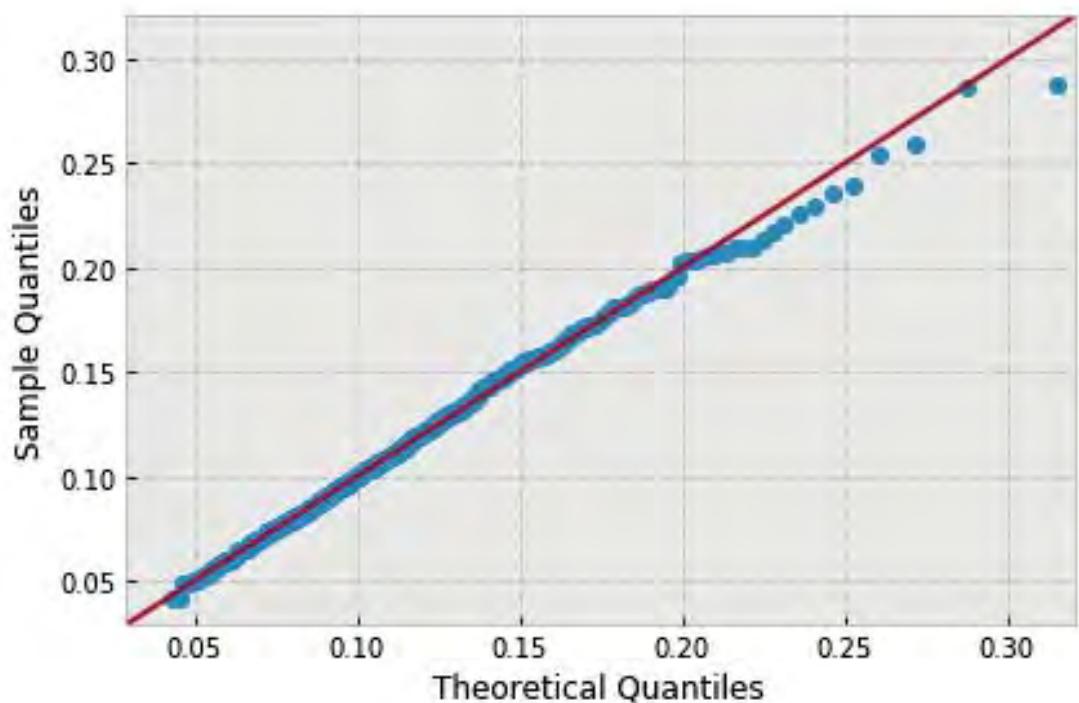
```
In [29]: plot_dist('total_root_surface_area_per_convex_hull_volume', 'inverse-gamma')
/home/lpetrich/.local/share/anaconda3/envs/Master/lib/python3.6/site-packages/scipy/stats/_distn_infra.py:1430: RuntimeWarning: invalid value encountered in double_scalars
  Lhat = muhat - Shat*mu

variable name: N
variable value: 0.5
distribution: inverse-gamma(shape=(7.5944667445317533,), loc=0.0021739215906456566,
scale=0.029226709738631527)
DescribeResult(nobs=1000, minmax=(0.003412130517475897, 0.018704416532773233)
               mean=0.0066064594153740436, variance=3.4819364471861154e-06,
               skewness=1.8093577644695733, kurtosis=5.631618268975794)
```

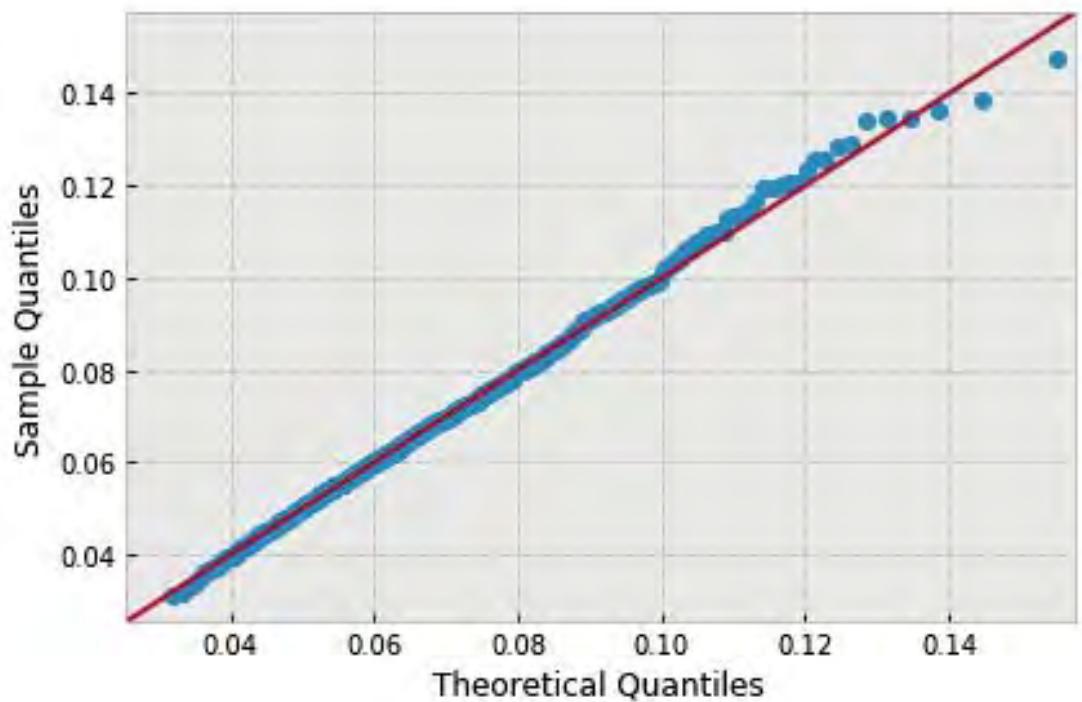
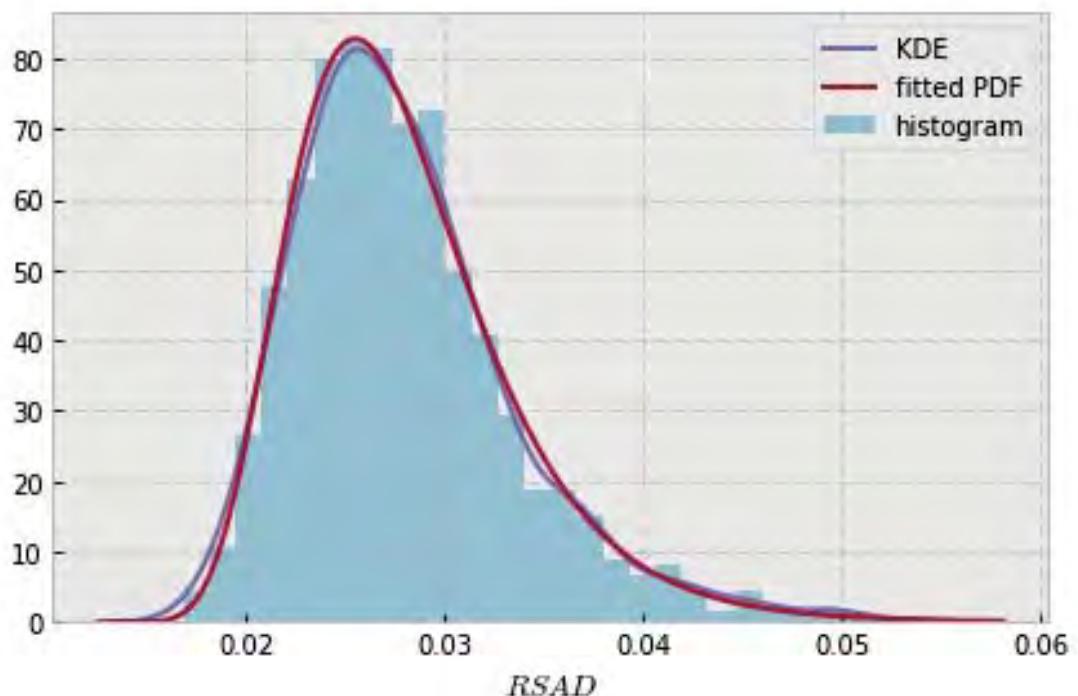


```
variable name: N
variable value: 1.0
distribution: inverse-gamma(shape=(10.407256918491104,), loc=0.004682771759653872,
    scale=0.11850719748290317)
DescribeResult(nobs=1000, minmax=(0.0095502871317810264, 0.038726326947902852)
    mean=0.017272654783304, variance=1.8061621988564113e-05,
    skewness=1.204905063692676, kurtosis=2.0761032599397824)
```

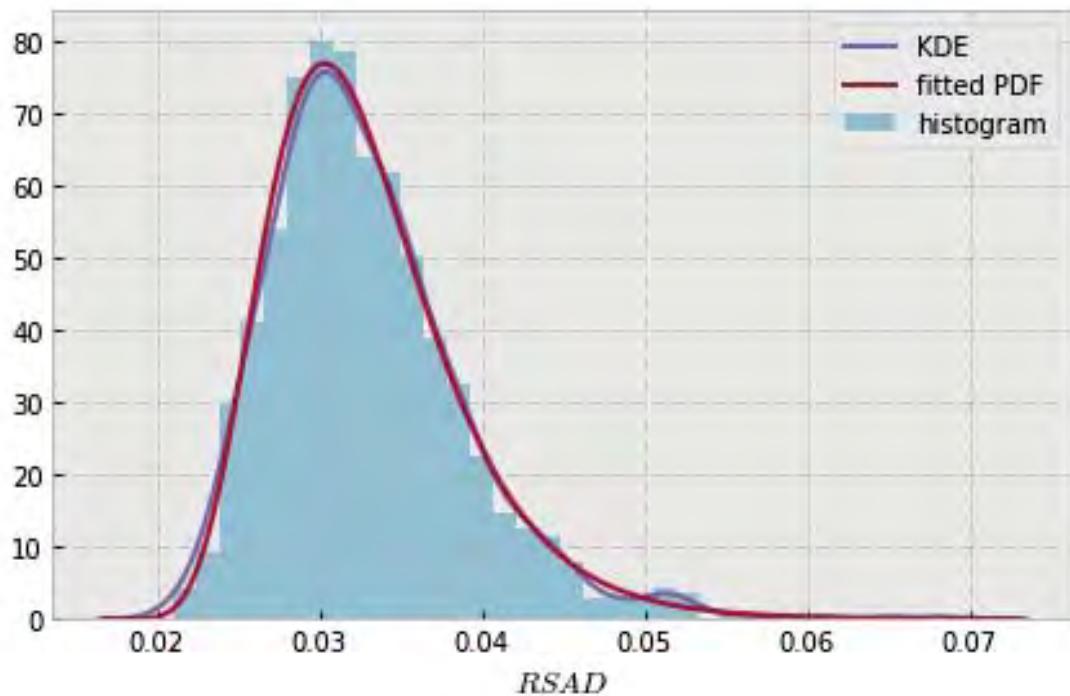


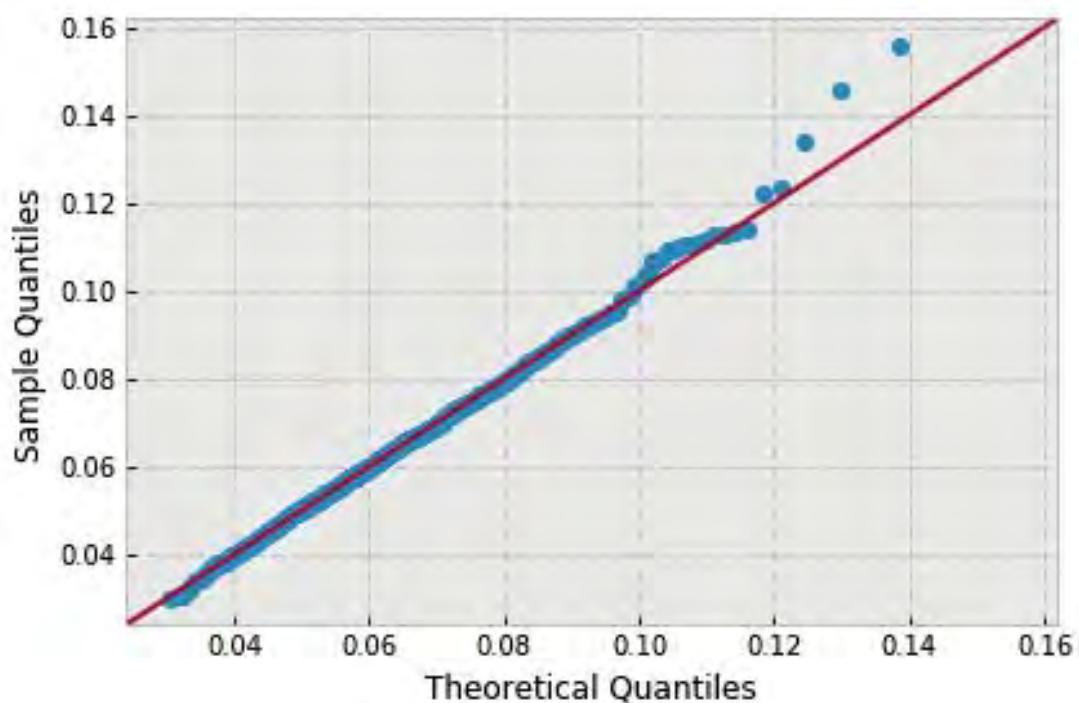


```
variable name: N
variable value: 1.5
distribution: inverse-gamma(shape=(16.082175986437353,), loc=0.006851710007435539,
scale=0.31862905687161025)
DescribeResult(nobs=1000, minmax=(0.016798626041433451, 0.053862611159962055)
mean=0.0279801949287223, variance=3.177691659994143e-05,
skewness=1.0923619375455855, kurtosis=1.7712562176911302)
```



```
variable name: N
variable value: 2.0
distribution: inverse-gamma(shape=(17.30086561704757,), loc=0.009446148331349671,
                           scale=0.3814597428115636)
DescribeResult(nobs=1000, minmax=(0.020942848447169697, 0.068808486629182208)
               mean=0.032850303352511601, variance=3.6334160741766985e-05,
               skewness=1.1993558160670863, kurtosis=2.9313469681182074)
```

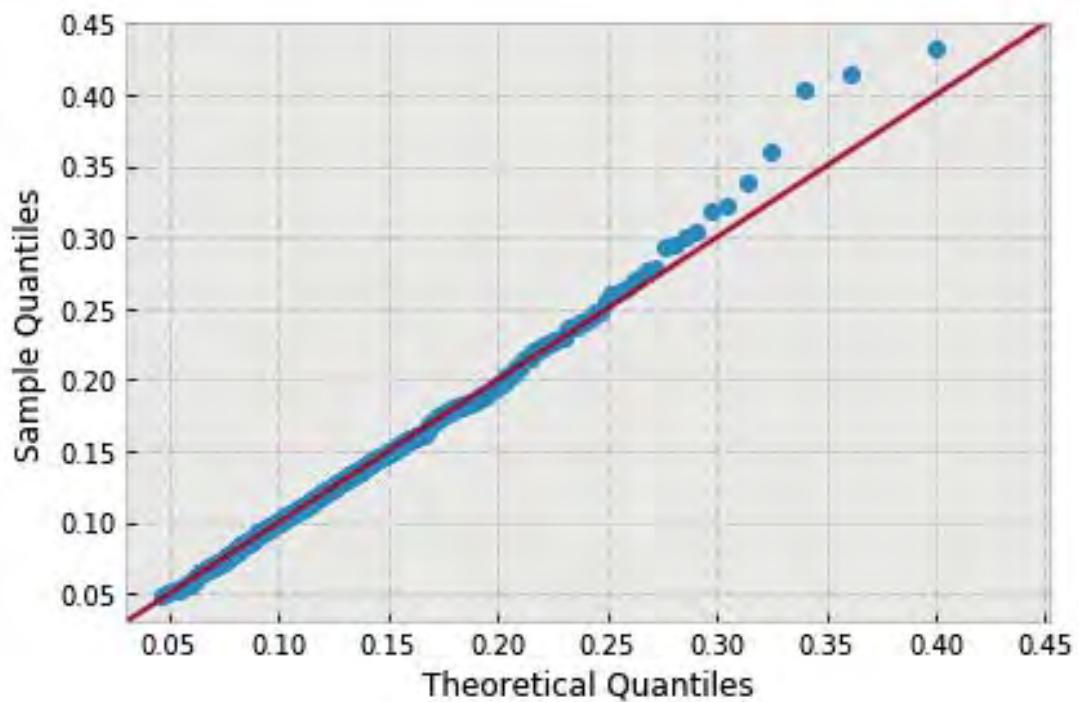
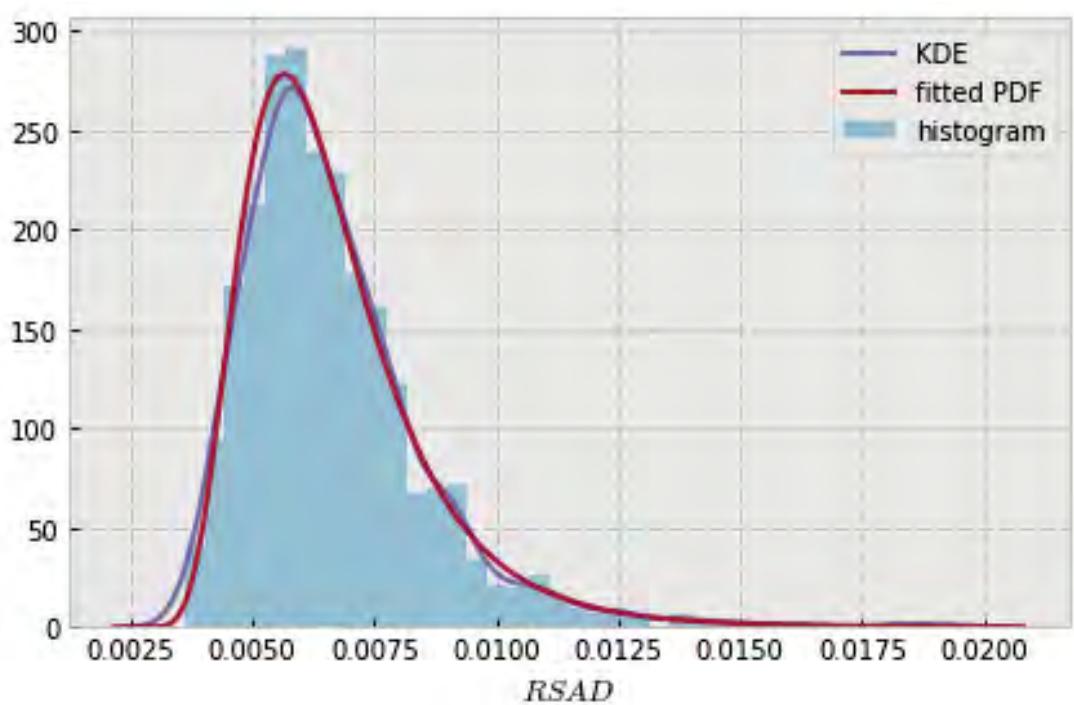




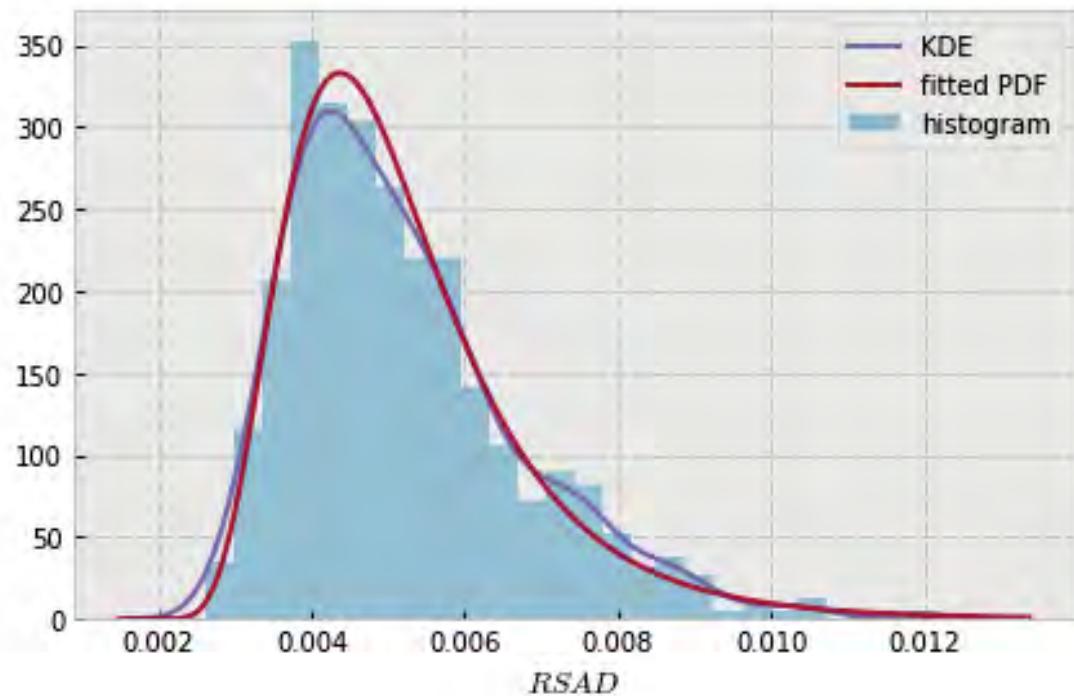
```

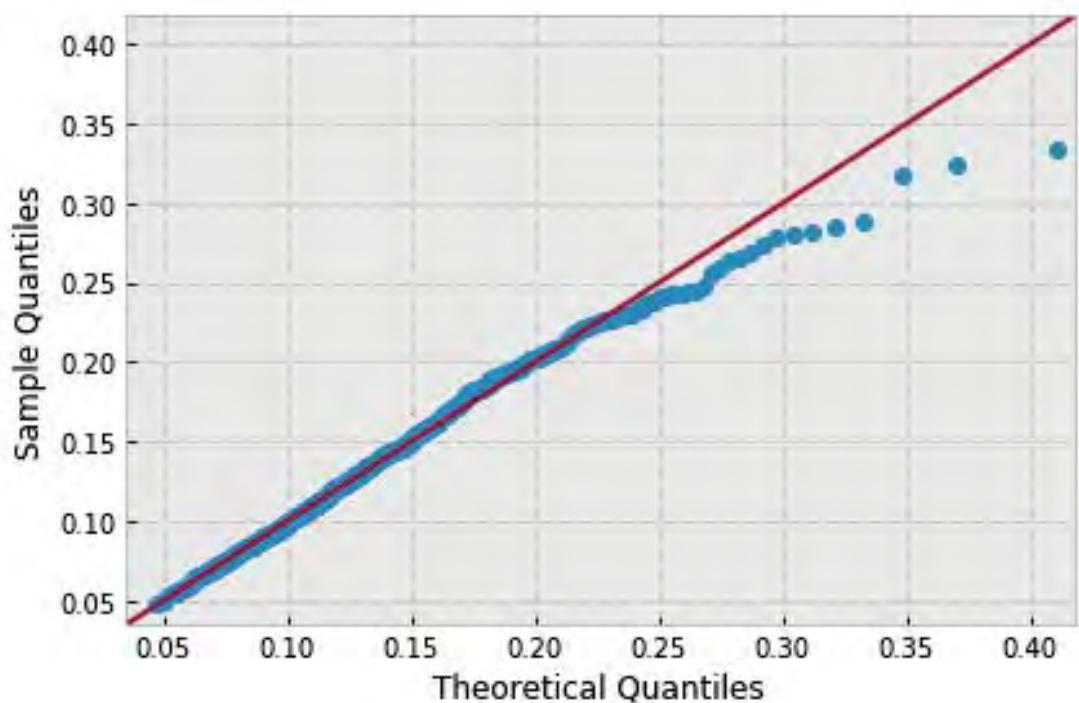
variable name: ln
variable value: 0.5
distribution: inverse-gamma(shape=(9.0949326049983519,), loc=0.0015837226295882952,
                           scale=0.04108460758548205)
DescribeResult(nobs=1000, minmax=(0.0036051847477412682, 0.01935417921448411)
               mean=0.0066611563524463652, variance=3.7235721508831977e-06,
               skewness=1.845186638725284, kurtosis=6.276479585117663)

```

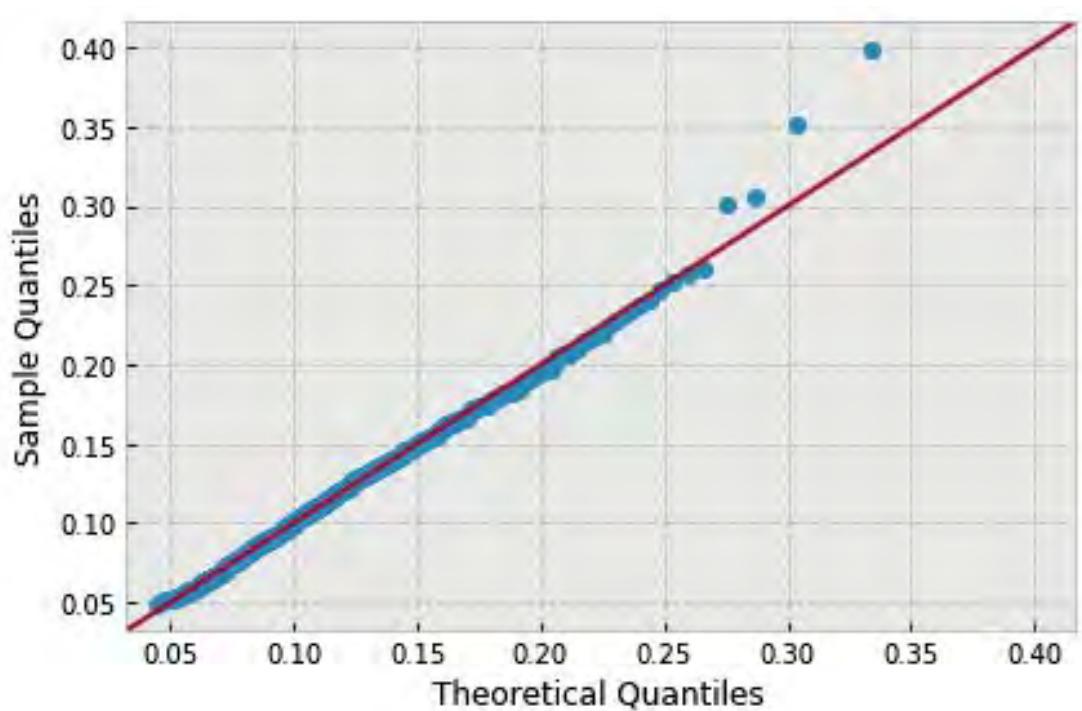
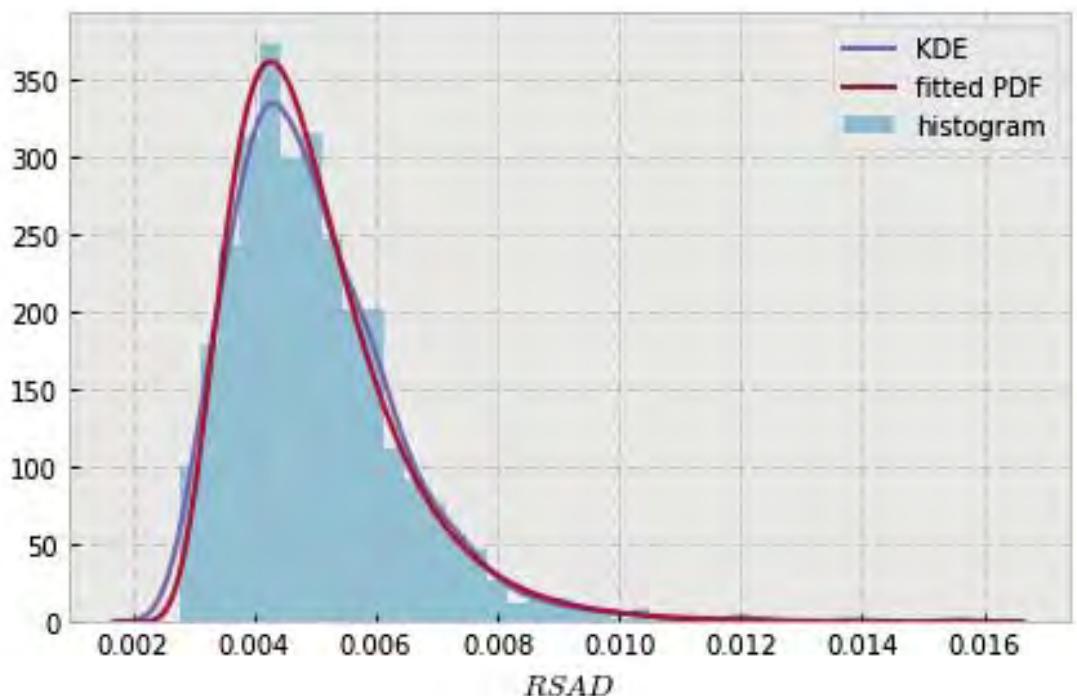


```
variable name: ln
variable value: 0.6666666666666666
distribution: inverse-gamma(shape=(8.9695865465901079,), loc=0.00099533575979842,
                           scale=0.03363934893744365)
DescribeResult(nobs=1000, minmax=(0.002632968590387958, 0.012206543205992373)
               mean=0.0052098831984670347, variance=2.3306156593561044e-06,
               skewness=1.1180059582823503, kurtosis=1.4231085000294028)
```

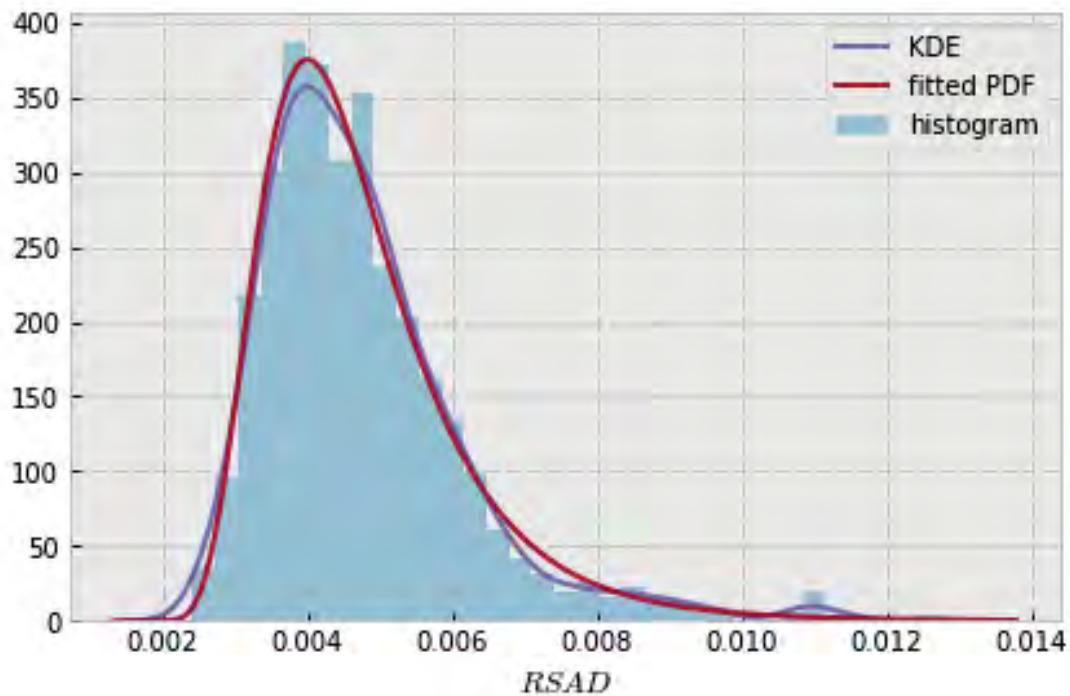


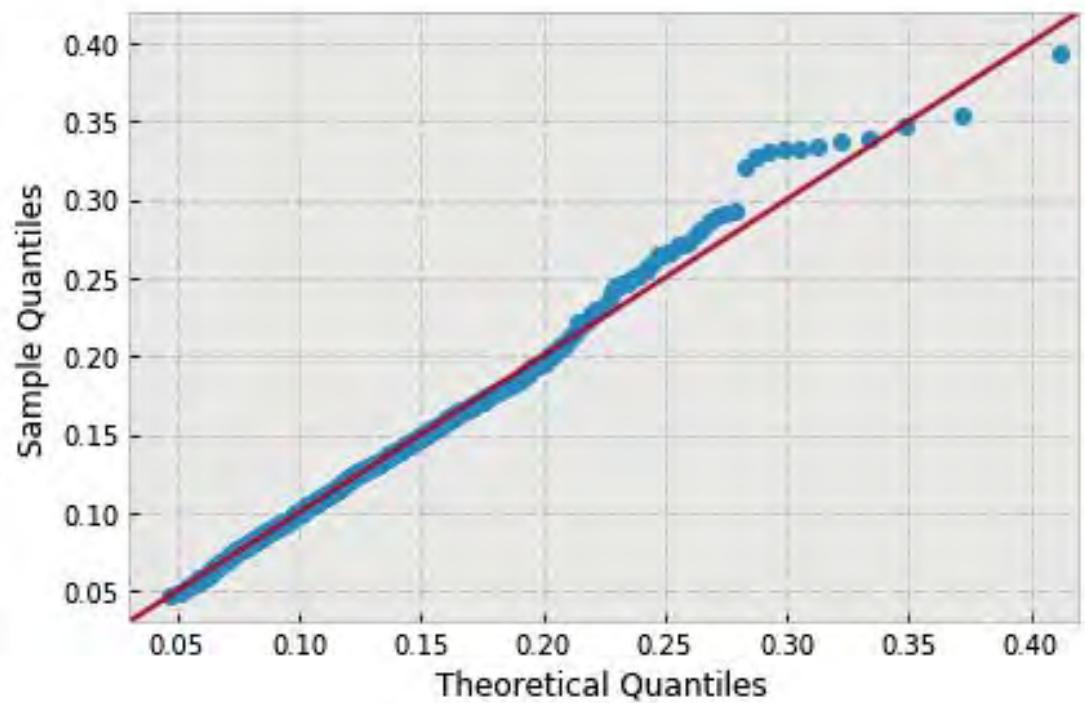


```
variable name: ln
variable value: 0.8333333333333333
distribution: inverse-gamma(shape=(10.062242981213792,), loc=0.0009544271668441174,
                           scale=0.03669630458709848)
DescribeResult(nobs=1000, minmax=(0.0027640078036797085, 0.015581220629984245)
               mean=0.0050025959714742217, variance=2.0169908920689899e-06,
               skewness=1.6456955073396902, kurtosis=5.935123366374722)
```

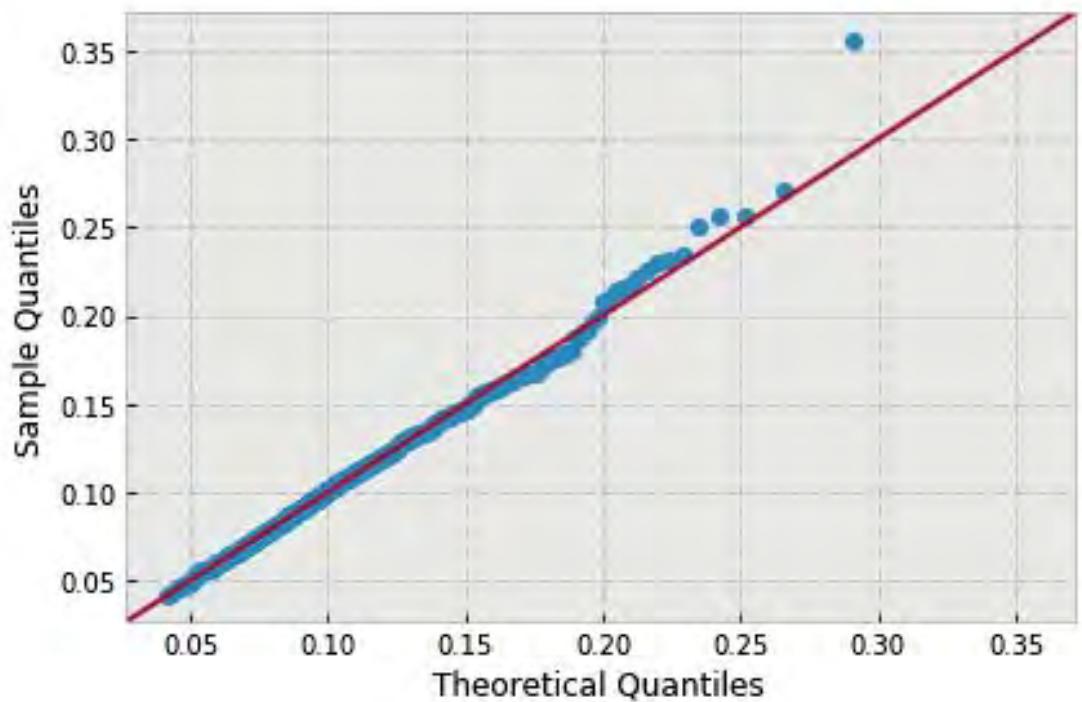
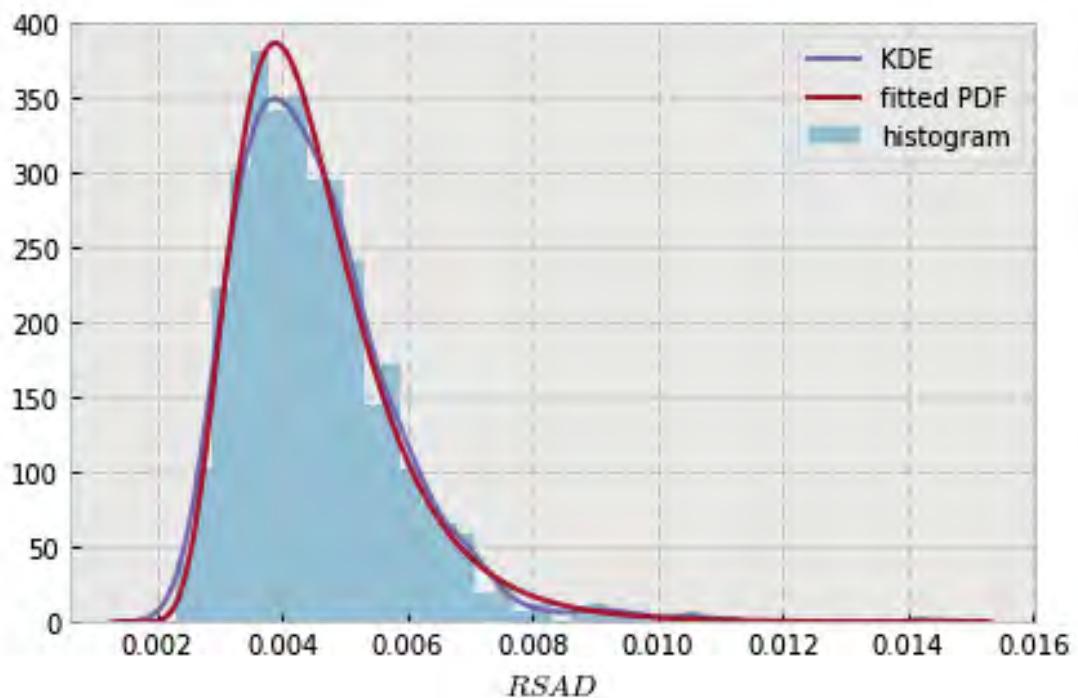


```
variable name: ln
variable value: 1.0
distribution: inverse-gamma(shape=(8.9514893344889099,), loc=0.0009998440305081261,
                           scale=0.02978570171837977)
DescribeResult(nobs=1000, minmax=(0.002407397715168901, 0.012707102545662482)
               mean=0.0047475673177228674, variance=2.0550385353085347e-06,
               skewness=1.6882037296542065, kurtosis=4.354142999987451)
```

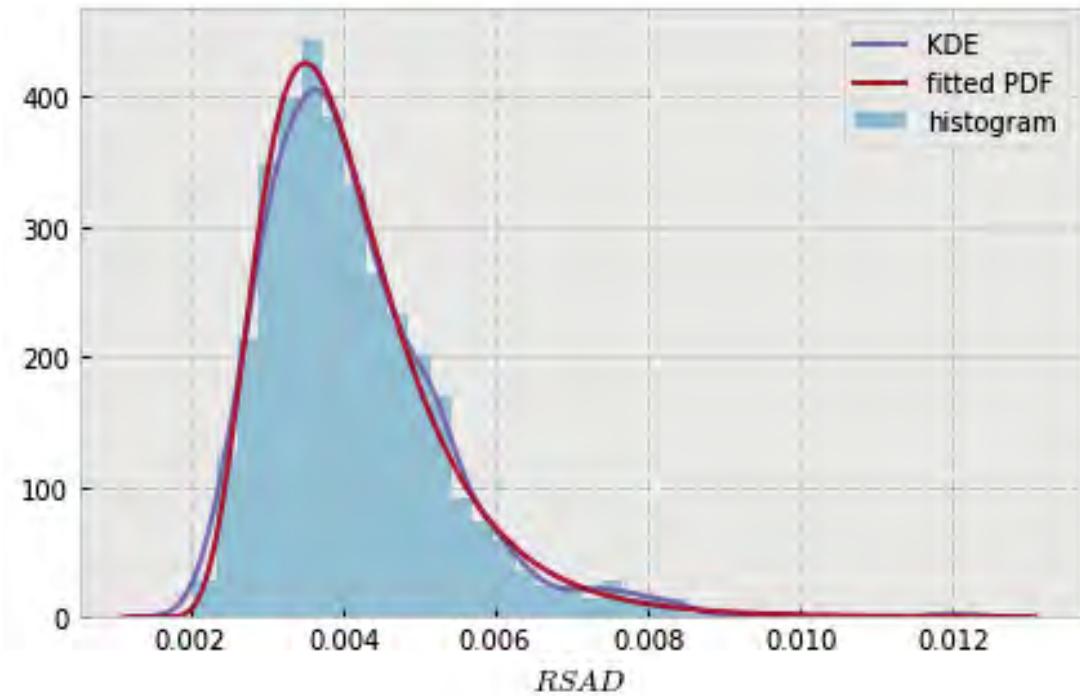


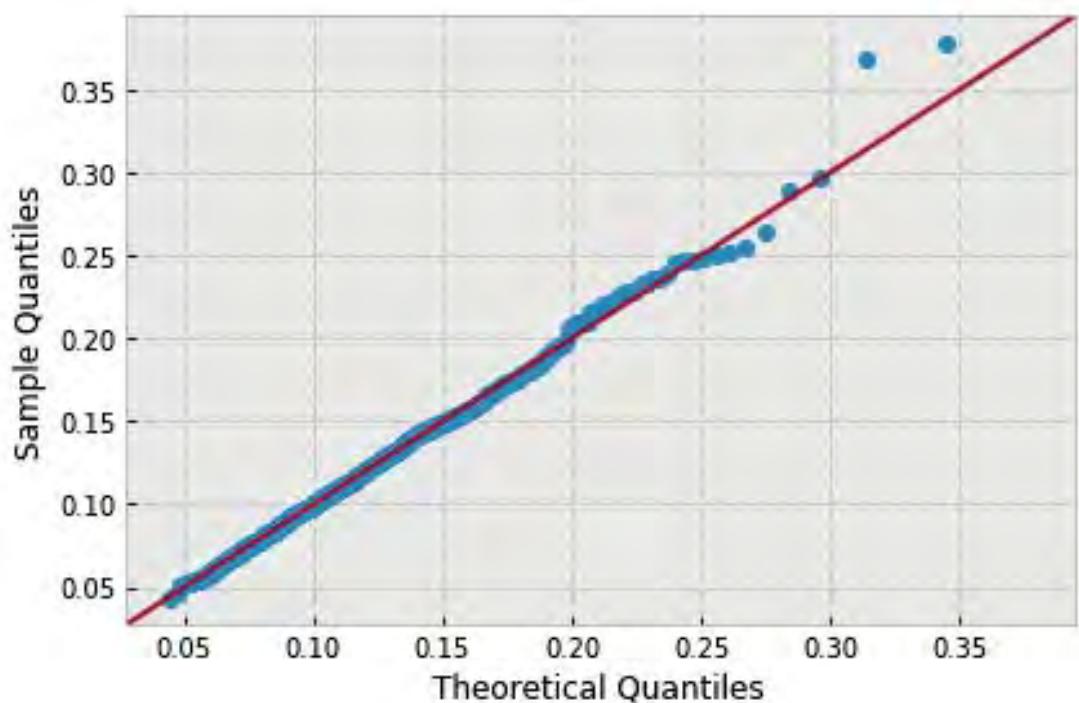


```
variable name: ln
variable value: 1.1666666666666665
distribution: inverse-gamma(shape=(10.899204276400397,), loc=0.0006549438632698617,
                           scale=0.0384773513696315)
DescribeResult(nobs=1000, minmax=(0.0022772122931898494, 0.01433430185111432)
               mean=0.0045414586483363394, variance=1.6977599465584493e-06,
               skewness=1.567588185659292, kurtosis=5.293544858372476)
```

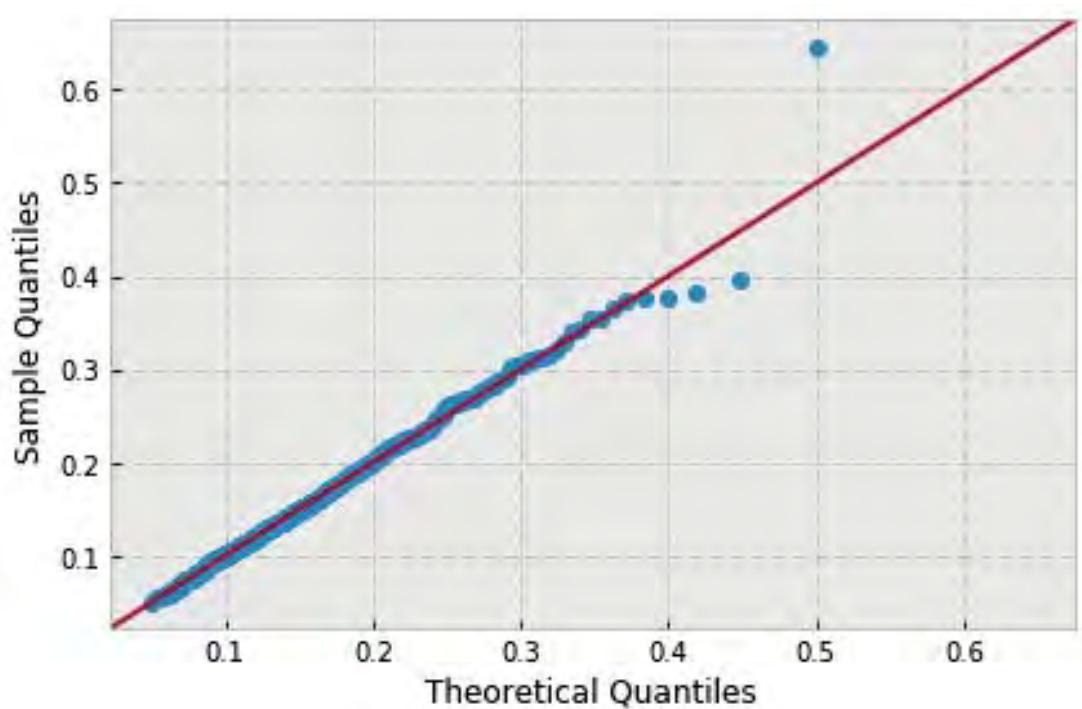
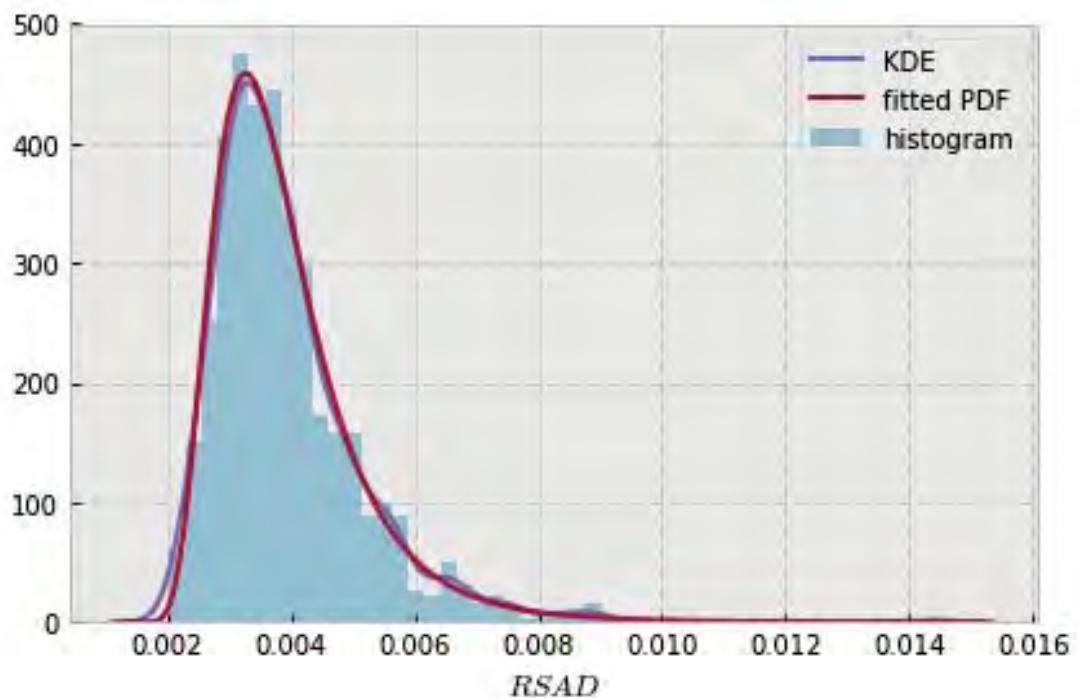


```
variable name: ln
variable value: 1.333333333333333
distribution: inverse-gamma(shape=(9.8794816088259605,), loc=0.0007143517926419068,
                           scale=0.030317087288220895)
DescribeResult(nobs=1000, minmax=(0.0020429453207169673, 0.012161444936082341)
               mean=0.0041283936977009788, variance=1.4757275746303977e-06,
               skewness=1.591627517914968, kurtosis=4.822847871798885)
```

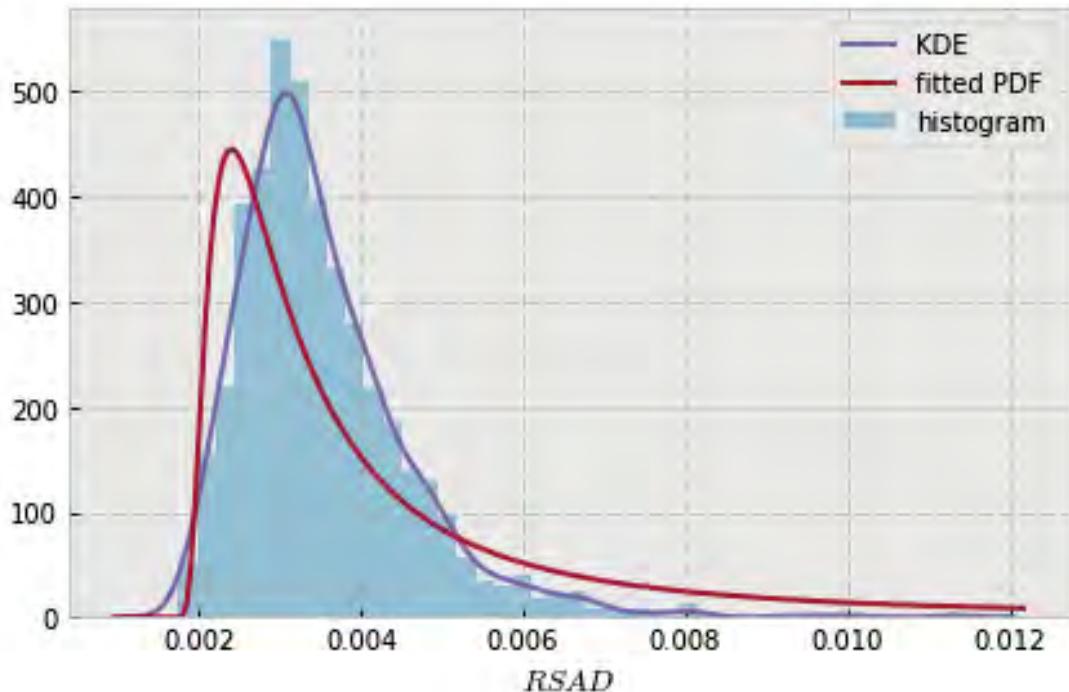


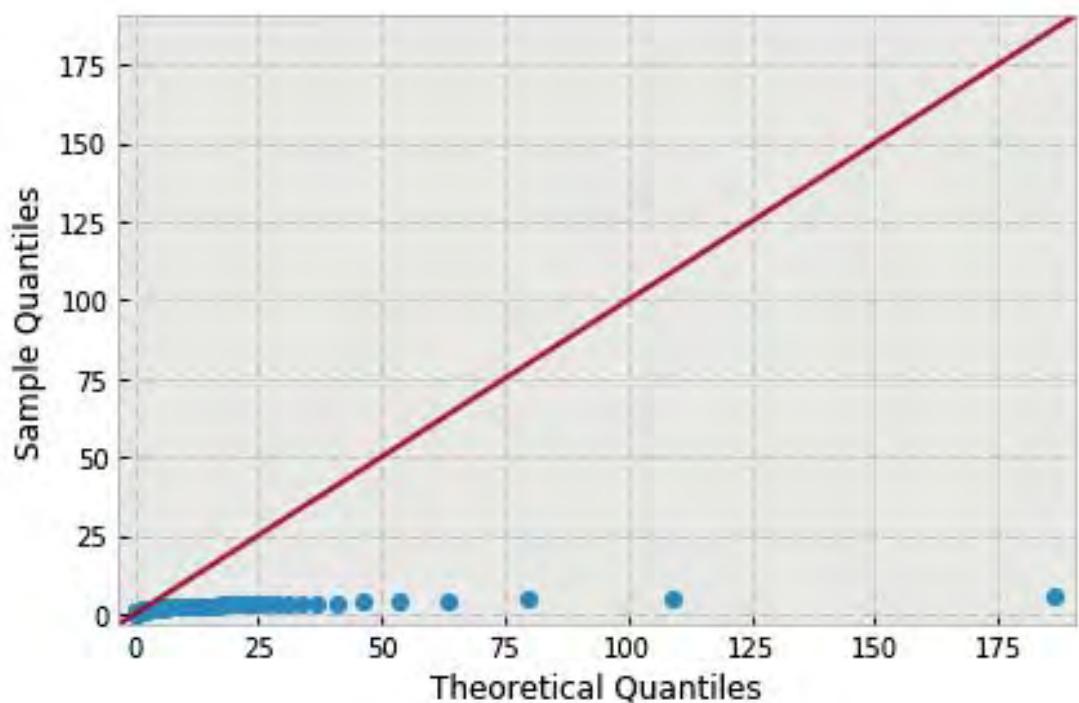


```
variable name: ln
variable value: 1.5
distribution: inverse-gamma(shape=(8.060484852469429,), loc=0.0009539098161319032,
                           scale=0.0209118198165128)
DescribeResult(nobs=1000, minmax=(0.0020192689438537528, 0.014439760026590767)
               mean=0.0039157750787993393, variance=1.4458799562287537e-06,
               skewness=1.896501870394956, kurtosis=7.612681358882897)
```

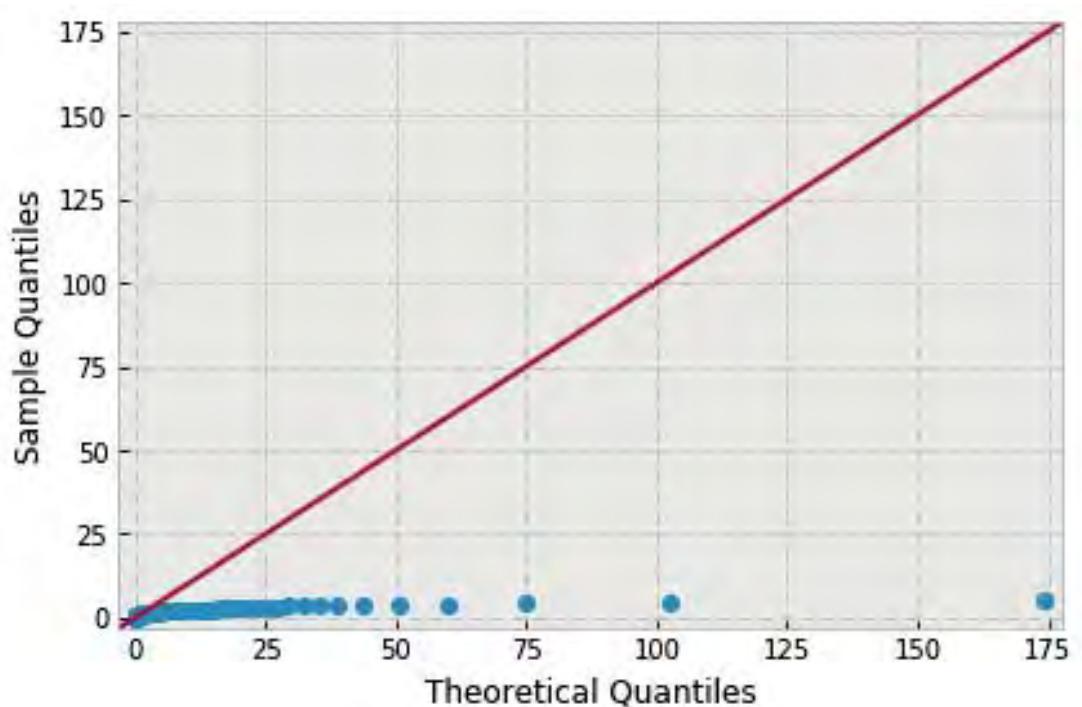
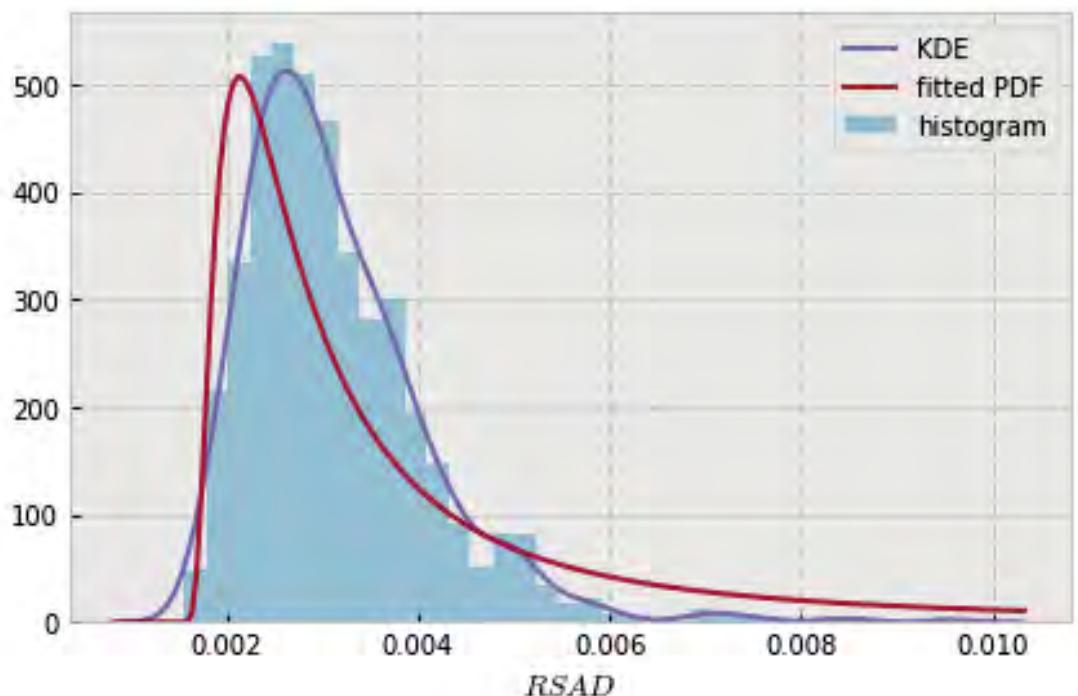


```
variable name: ln
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(1.2919307968307938,), loc=0.001670413656155391,
    scale=0.0016891451367500641)
DescribeResult(nobs=1000, minmax=(0.0017567794539327436, 0.011361430079422965)
    mean=0.0035274480928405212, variance=1.137514833575441e-06,
    skewness=1.7517802858270775, kurtosis=6.06130893397259)
```

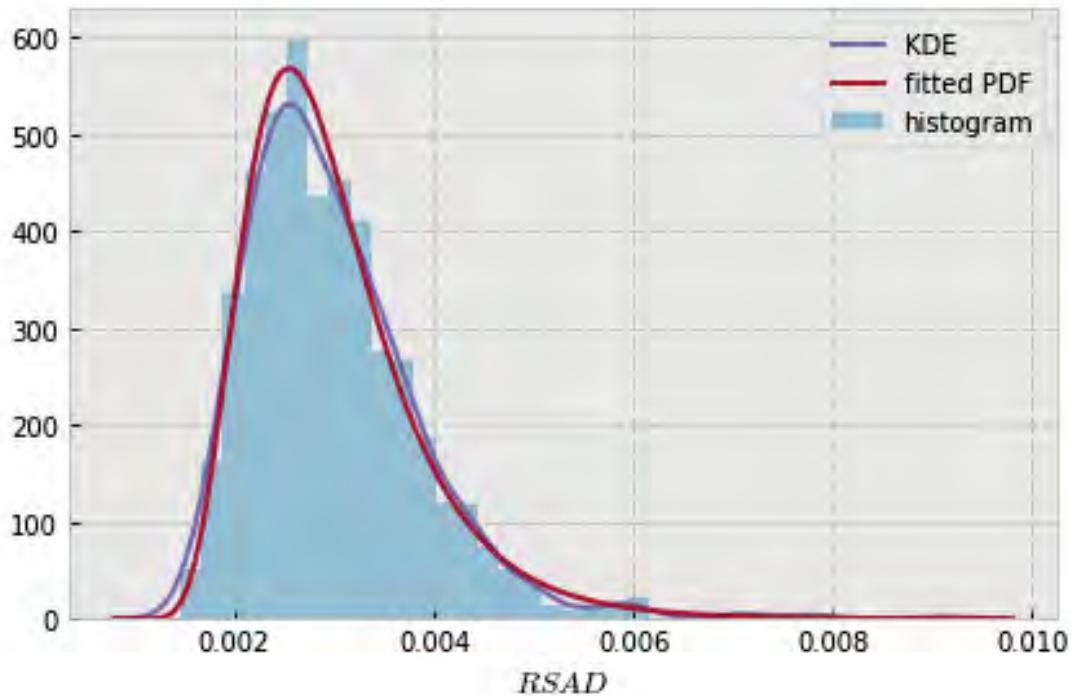


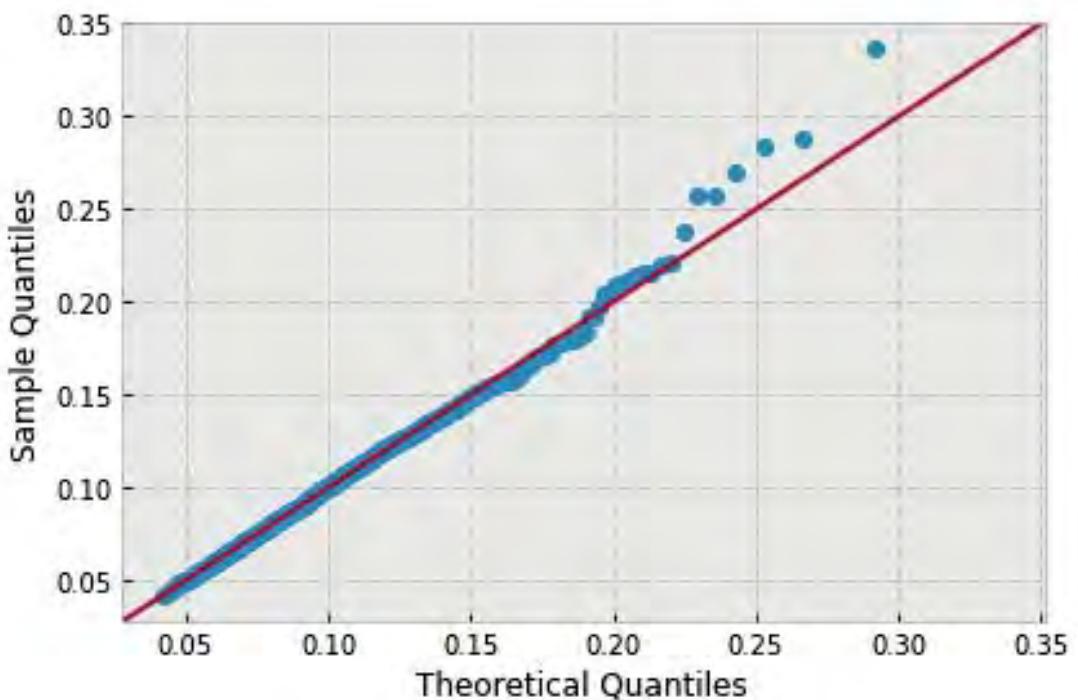


```
variable name: ln
variable value: 1.833333333333333
distribution: inverse-gamma(shape=(1.3074624568179734,), loc=0.0014798090407235273,
    scale=0.0015074495780952211)
DescribeResult(nobs=1000, minmax=(0.001552233228622794, 0.0096070095632407627)
    mean=0.003133966601054289, variance=9.3236107175269252e-07,
    skewness=1.6477585742616307, kurtosis=5.093013633034245)
```

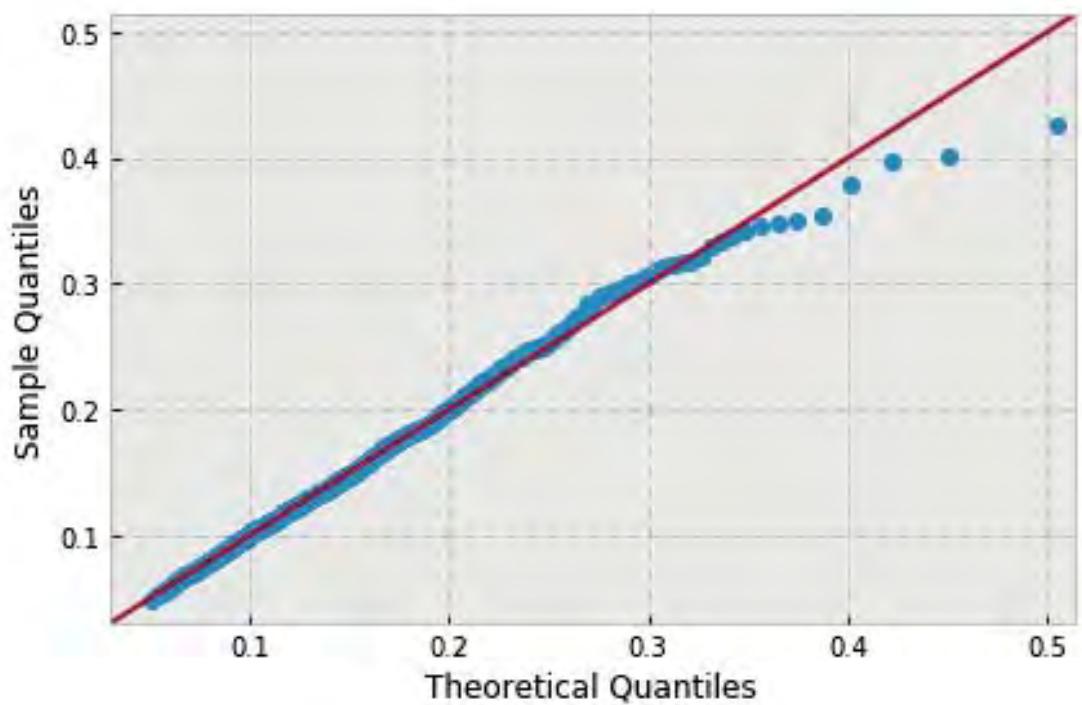
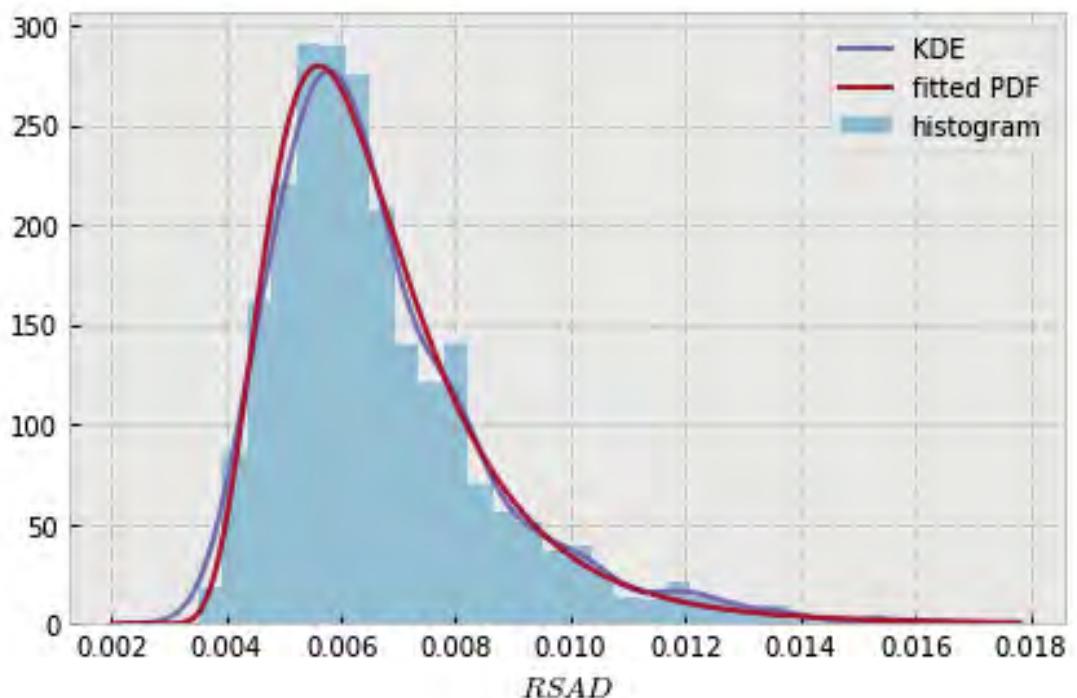


```
variable name: ln
variable value: 2.0
distribution: inverse-gamma(shape=(10.882776603546372,), loc=0.000344928159316251,
    scale=0.026138473162592872)
DescribeResult(nobs=1000, minmax=(0.0014516691530308201, 0.0091366279737513396)
    mean=0.0029899760157405622, variance=7.963827302273691e-07,
    skewness=1.6149583905202263, kurtosis=5.3019248766101725)
```

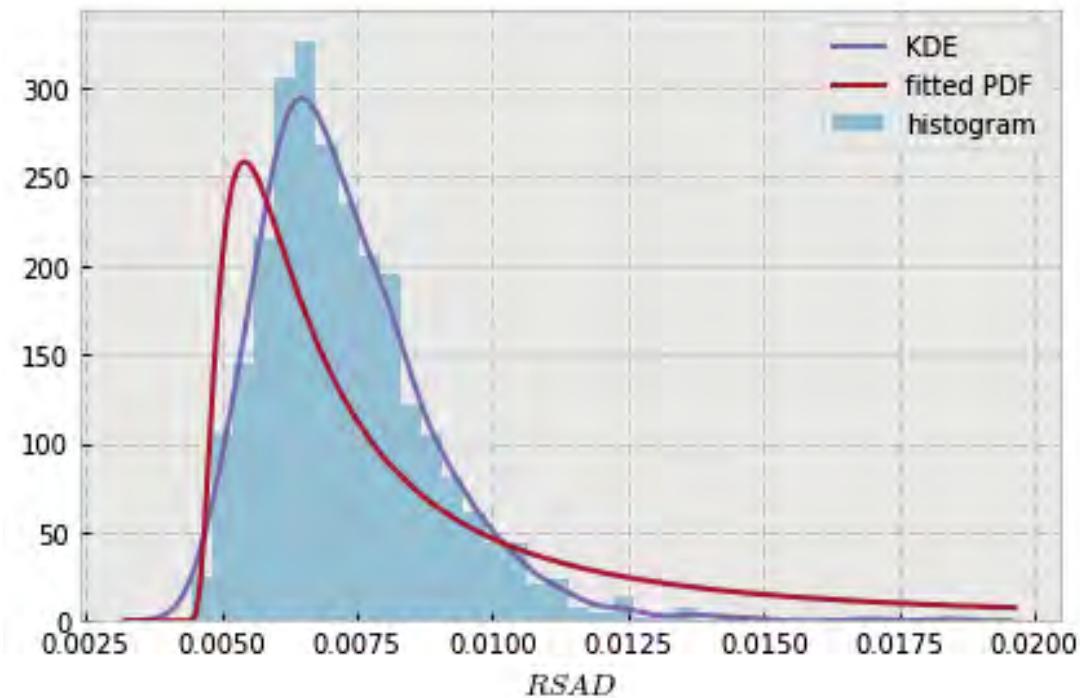


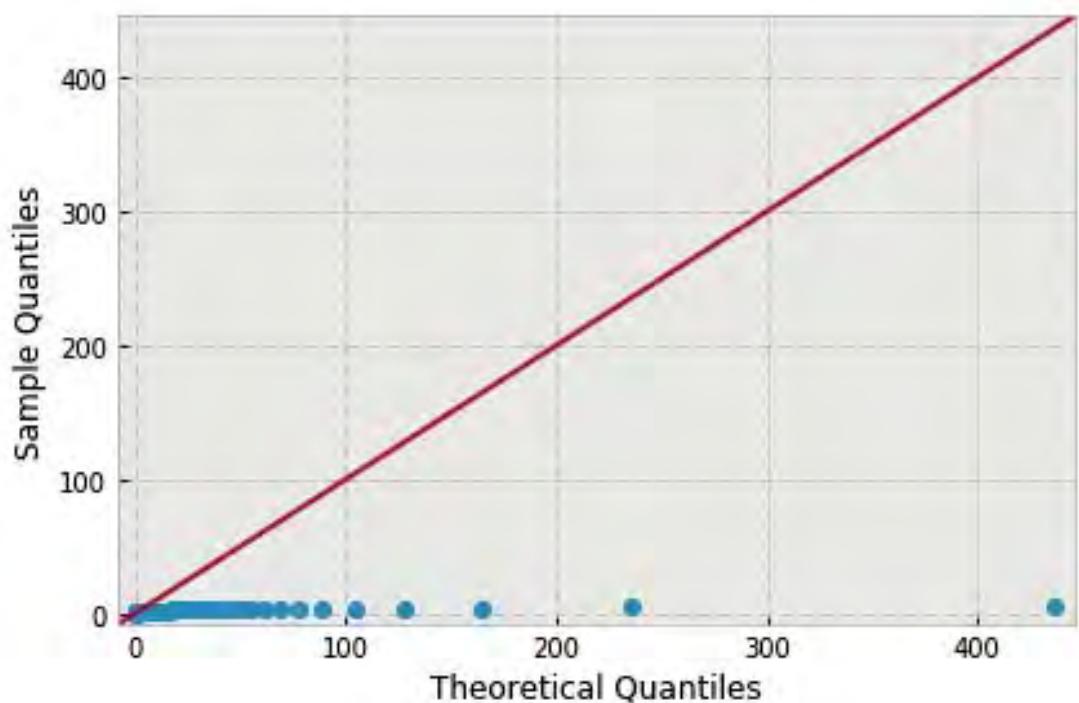


```
variable name: nB
variable value: 4.0
distribution: inverse-gamma(shape=(8.02742507322988,), loc=0.0018368707077292198,
scale=0.03407951073819514)
DescribeResult(nobs=1000, minmax=(0.0034990703815895789, 0.016365747718763601)
mean=0.0066832776063180223, variance=3.7205020616408888e-06,
skewness=1.4415189110679856, kurtosis=2.6959310639385077)
```

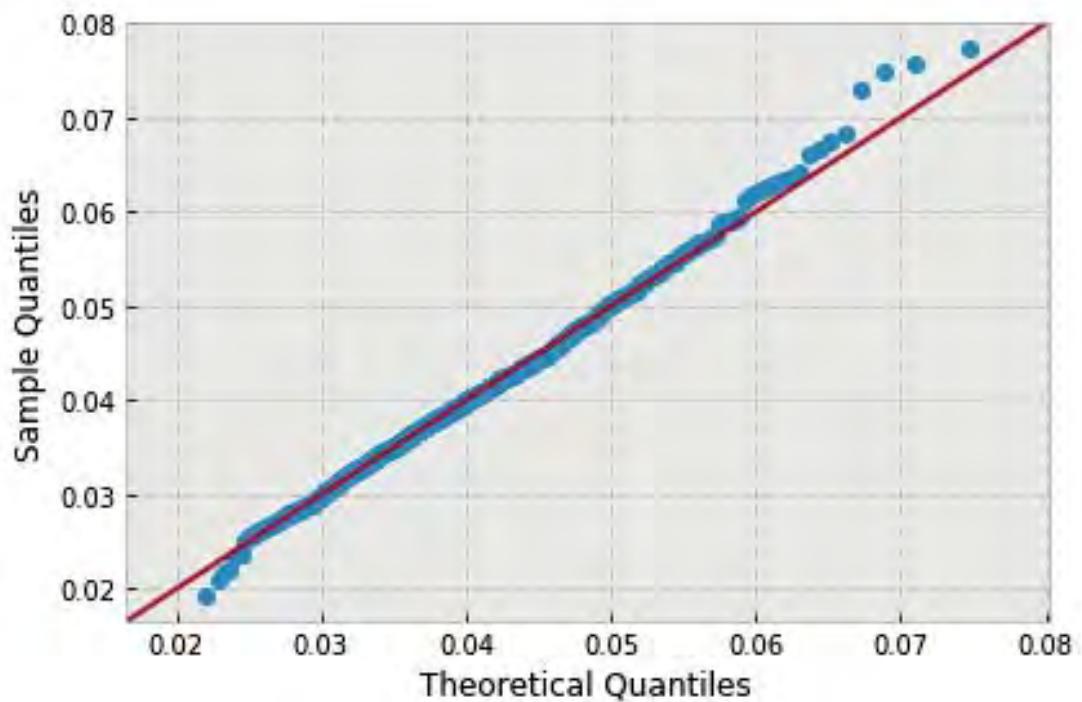
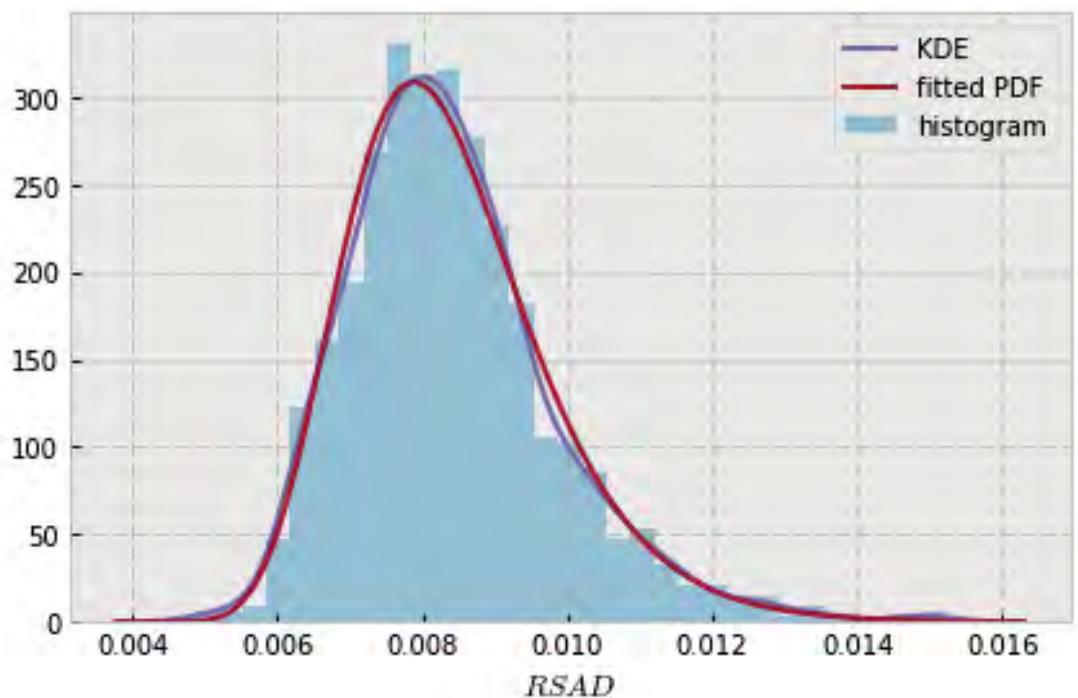


```
variable name: nB
variable value: 8.0
distribution: inverse-gamma(shape=(1.1264462147973191,), loc=0.004278426322785342,
                           scale=0.0024379321092378144)
DescribeResult(nobs=1000, minmax=(0.0044277644924108931, 0.018415062757330812)
               mean=0.0072939874805593285, variance=2.6259555571748564e-06,
               skewness=1.4568708147584148, kurtosis=4.4501167941910404)
```

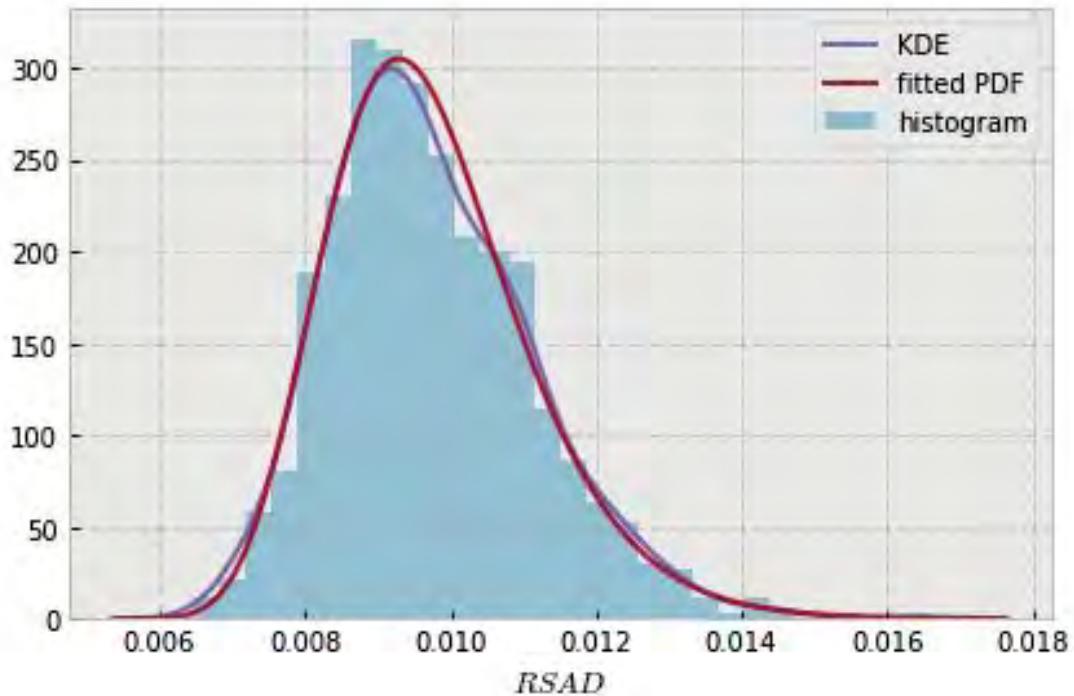


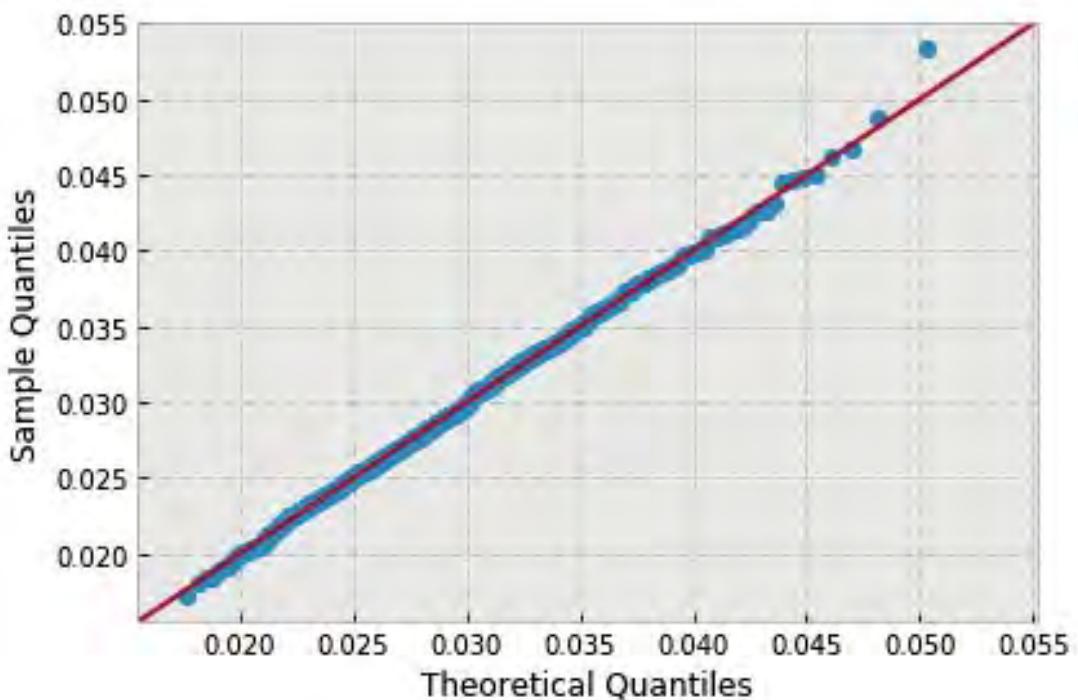


```
variable name: nB
variable value: 12.0
distribution: inverse-gamma(shape=(26.513916118713126,), loc=0.0013612175018064801,
                           scale=0.17938189096902368)
DescribeResult(nobs=1000, minmax=(0.0048346555191728802, 0.015224129657372175)
               mean=0.0083928542174252323, variance=2.0569700156423505e-06,
               skewness=0.9748934141925566, kurtosis=1.8491181548794424)
```

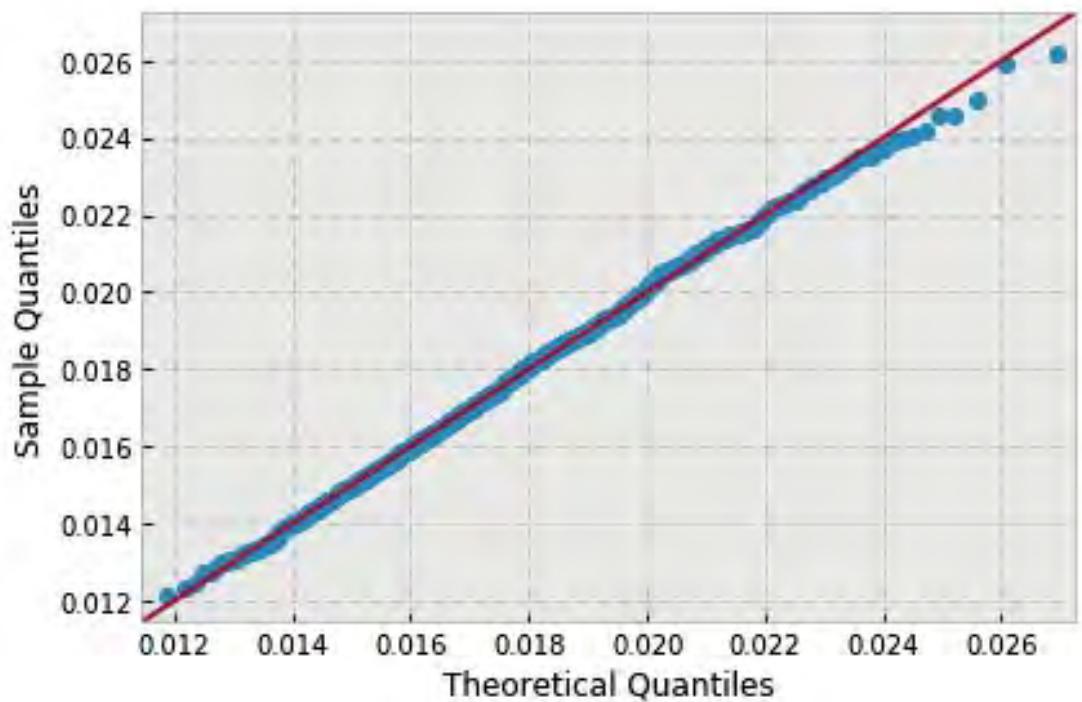
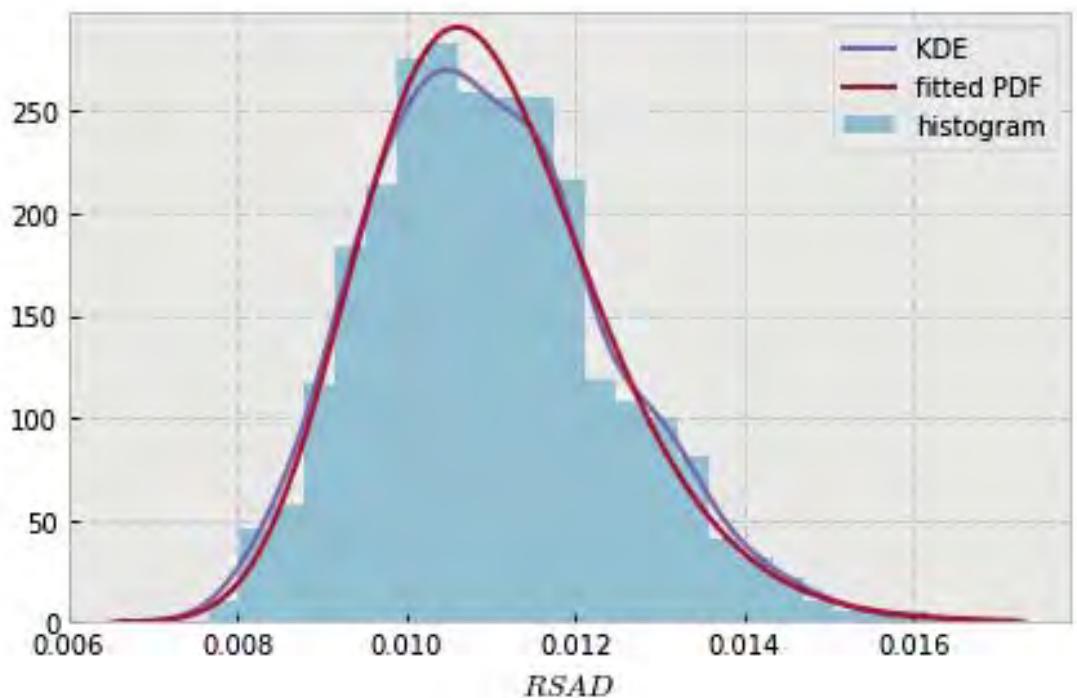


```
variable name: nB
variable value: 16.0
distribution: inverse-gamma(shape=(35.50730214464663,), loc=0.0016190516067704948,
                           scale=0.2799886250407331)
DescribeResult(nobs=1000, minmax=(0.0064575259987842033, 0.016554085415759784)
               mean=0.0097328652005849257, variance=1.9619933716316191e-06,
               skewness=0.6839296808410613, kurtosis=0.7856091639347378)
```

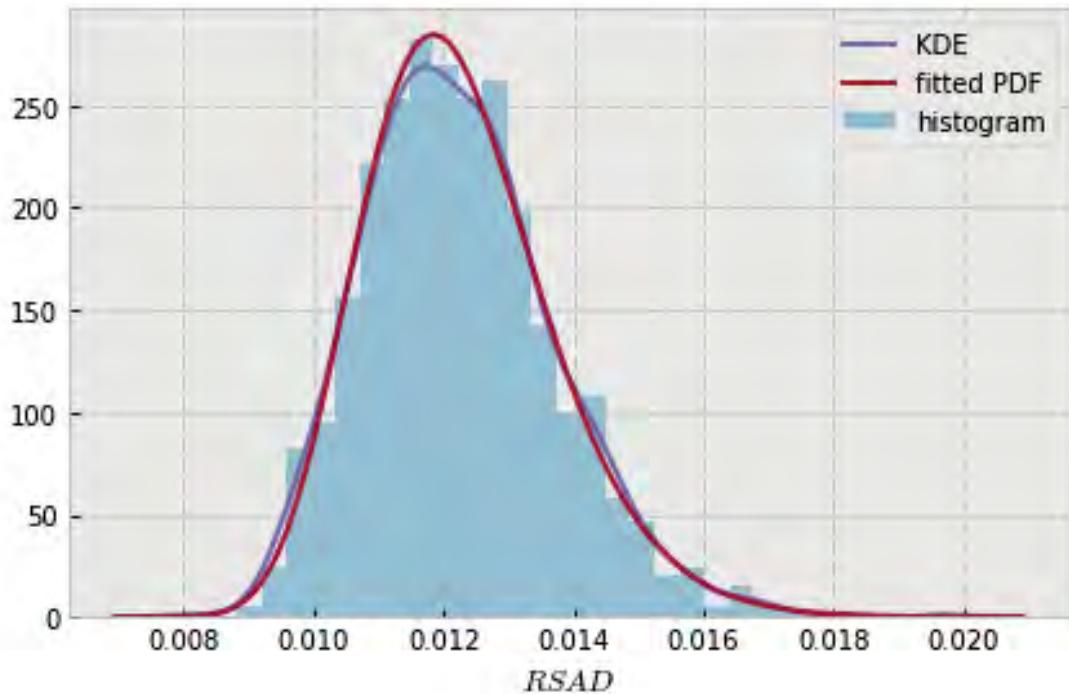


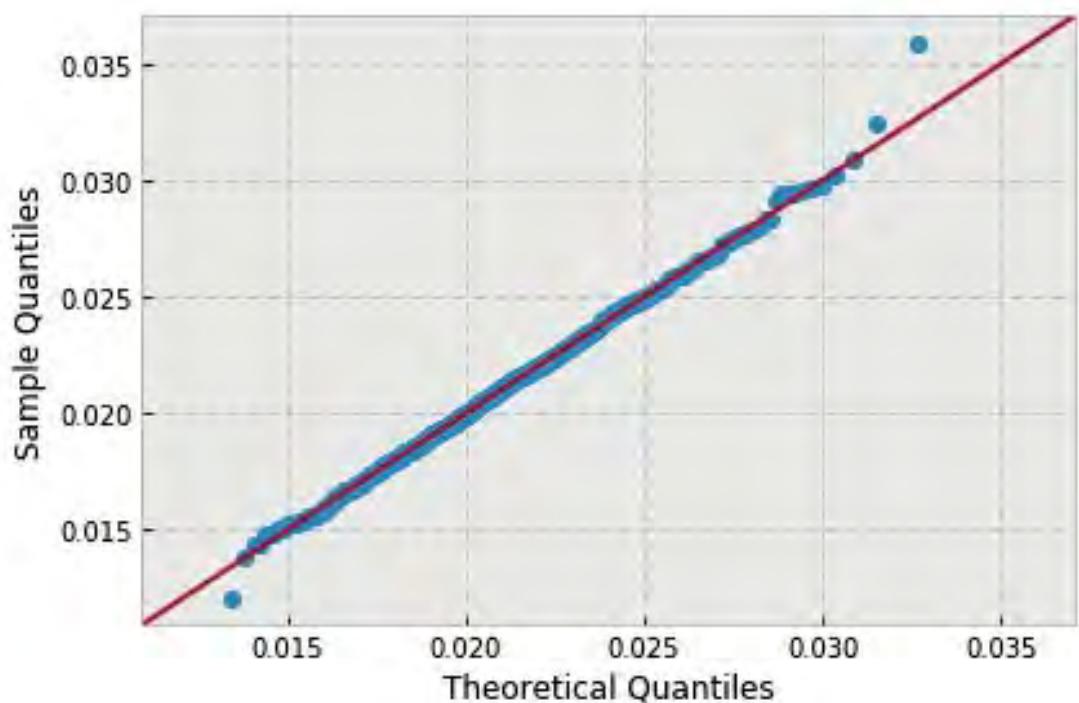


```
variable name: nB
variable value: 20.0
distribution: inverse-gamma(shape=(57.825043765196327,), loc=0.00029110611753321717,
                           scale=0.607052053743629)
DescribeResult(nobs=1000, minmax=(0.0076656679551990224, 0.016177976518571002)
               mean=0.010973699463210484, variance=2.0306015914343628e-06,
               skewness=0.4507710152318971, kurtosis=0.0371763165406227)
```

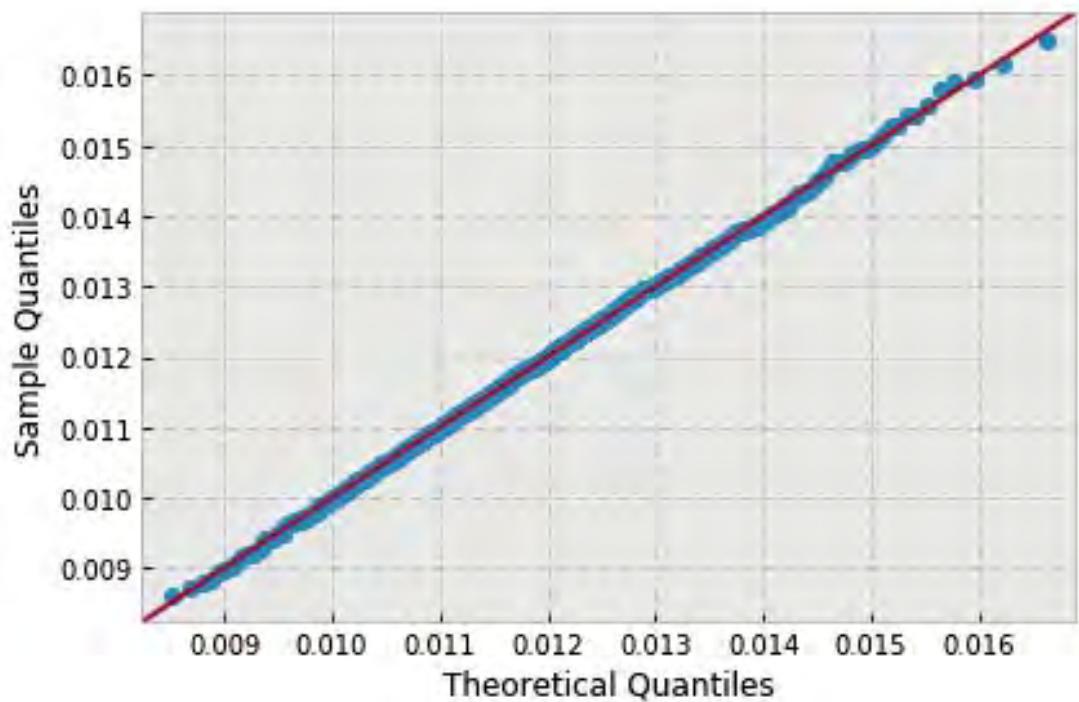
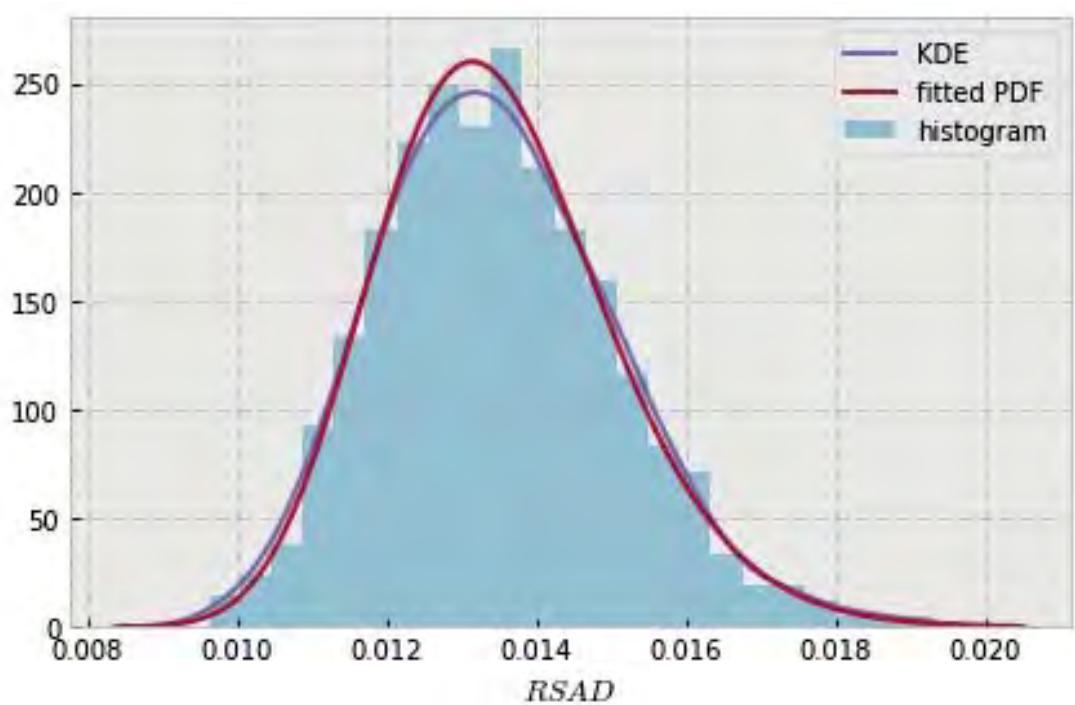


```
variable name: nB
variable value: 24.0
distribution: inverse-gamma(shape=(49.59517474339097,), loc=0.0020807673650686457,
                           scale=0.4938501761509275)
DescribeResult(nobs=1000, minmax=(0.008059889540248855, 0.019786749556650753)
               mean=0.012243347947342557, variance=2.1751047074292555e-06,
               skewness=0.6104547786844097, kurtosis=0.8160312317035308)
```

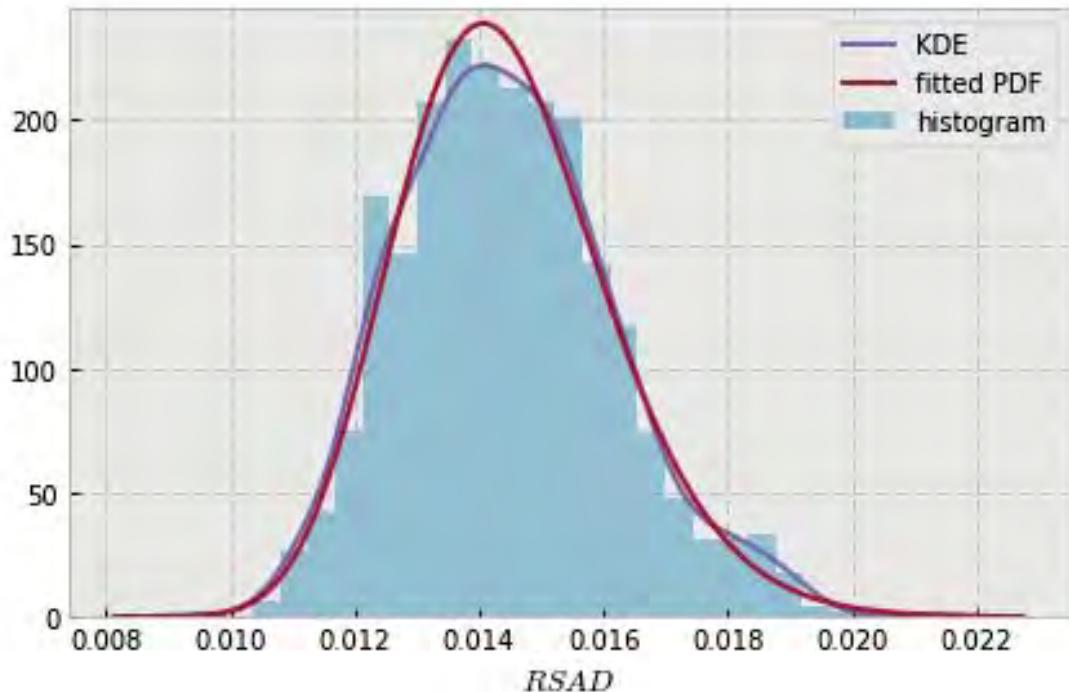


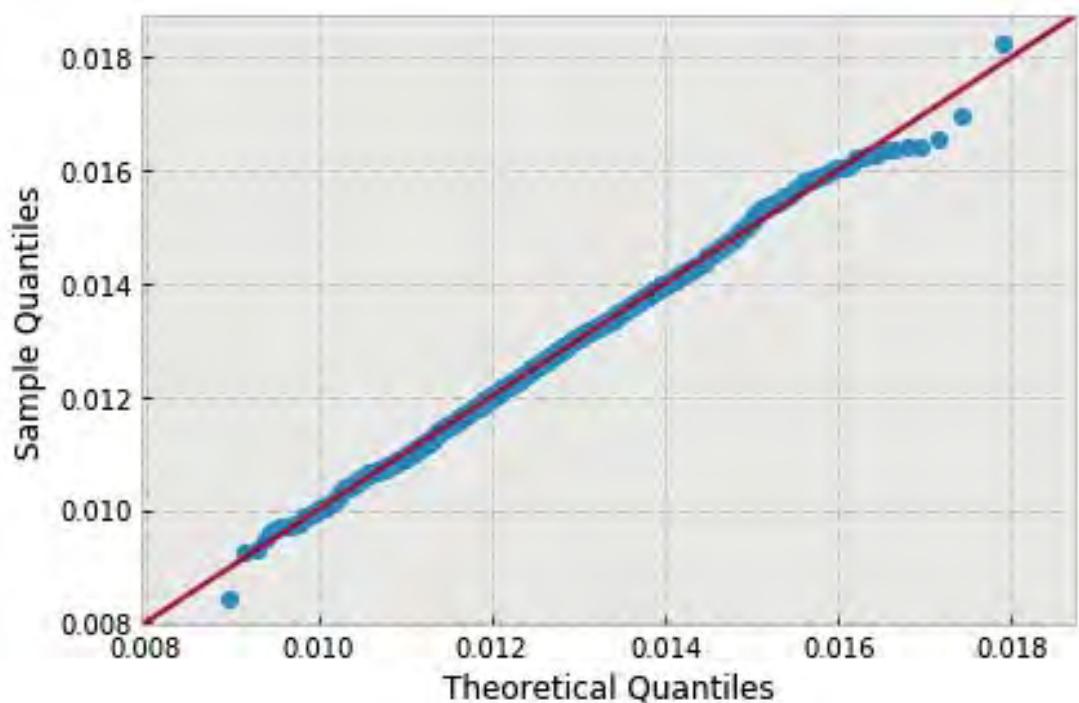


```
variable name: nB
variable value: 28.0
distribution: inverse-gamma(shape=(85.96845676396299,), loc=-0.0009969350610204812,
                           scale=1.2291774446450114)
DescribeResult(nobs=1000, minmax=(0.0096073077605766301, 0.019268831746620838)
               mean=0.013469102496744764, variance=2.4897968041065446e-06,
               skewness=0.39907025674812785, kurtosis=0.16171211679277064)
```

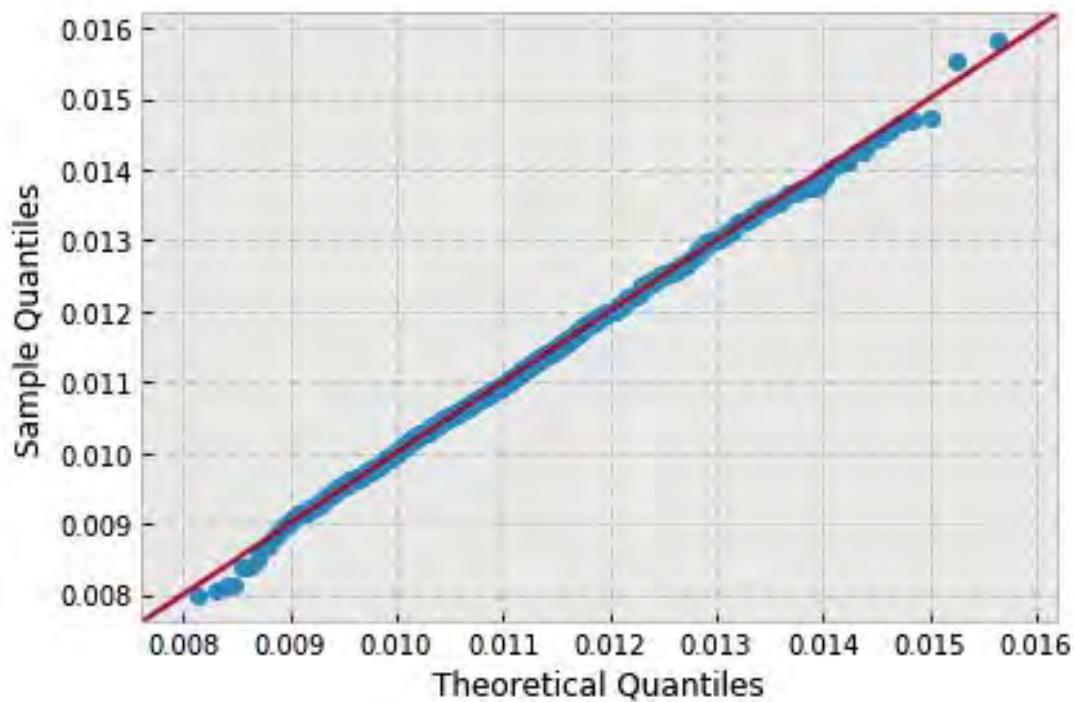
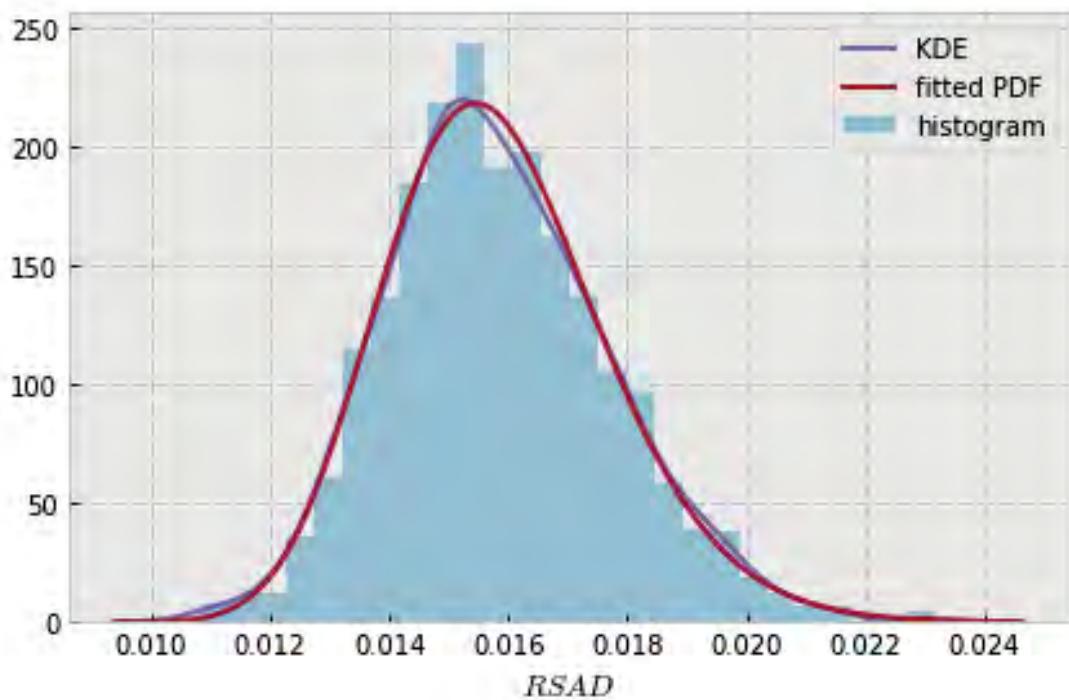


```
variable name: nB
variable value: 32.0
distribution: inverse-gamma(shape=(80.852973226652409,), loc=-0.0008550708480592451,
                           scale=1.2215216802319078)
DescribeResult(nobs=1000, minmax=(0.0094658805510091874, 0.021413989077032725)
               mean=0.014441980730009947, variance=2.9627273215951875e-06,
               skewness=0.41647357363225324, kurtosis=0.13067074344071328)
```

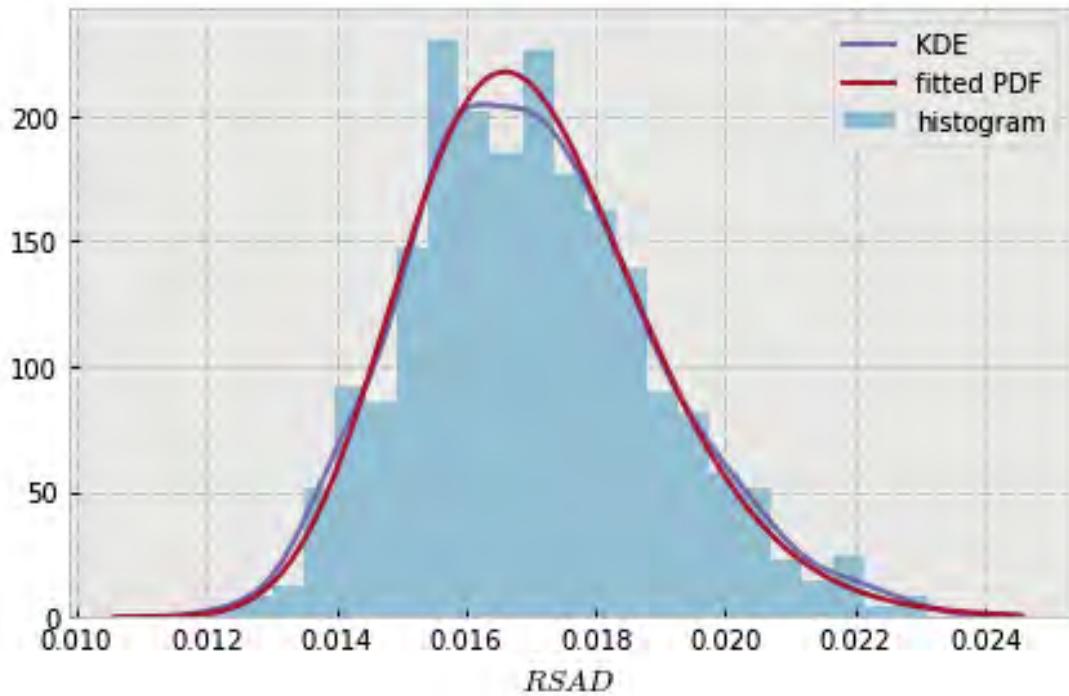


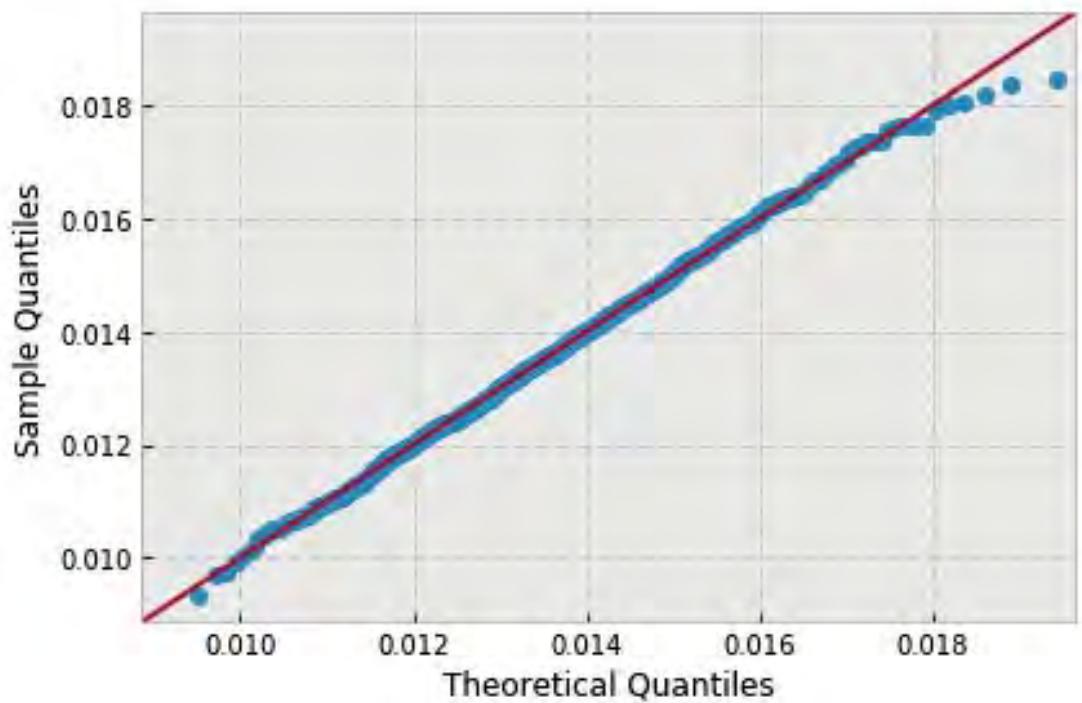


```
variable name: nB
variable value: 36.0
distribution: inverse-gamma(shape=(90.557856496758404,), loc=-0.001864843462089565,
                           scale=1.5850339948762162)
DescribeResult(nobs=1000, minmax=(0.010822357636810775, 0.02320483589557306)
               mean=0.015823446142574131, variance=3.4646732488544428e-06,
               skewness=0.38419386775340314, kurtosis=0.2371120554631876)
```

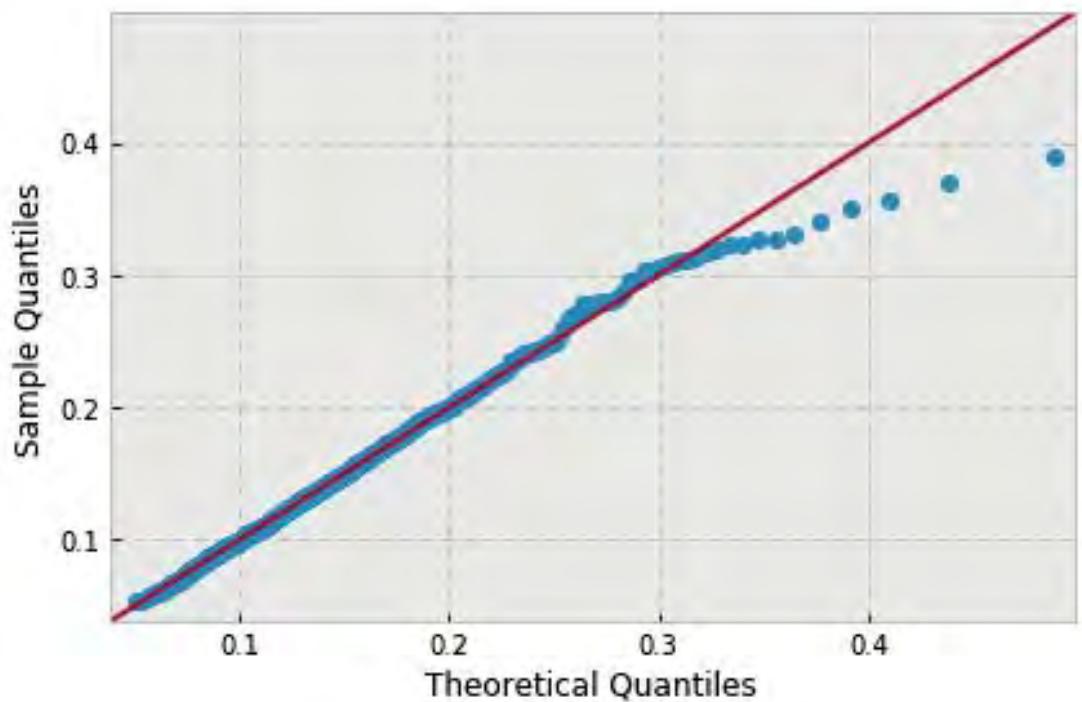
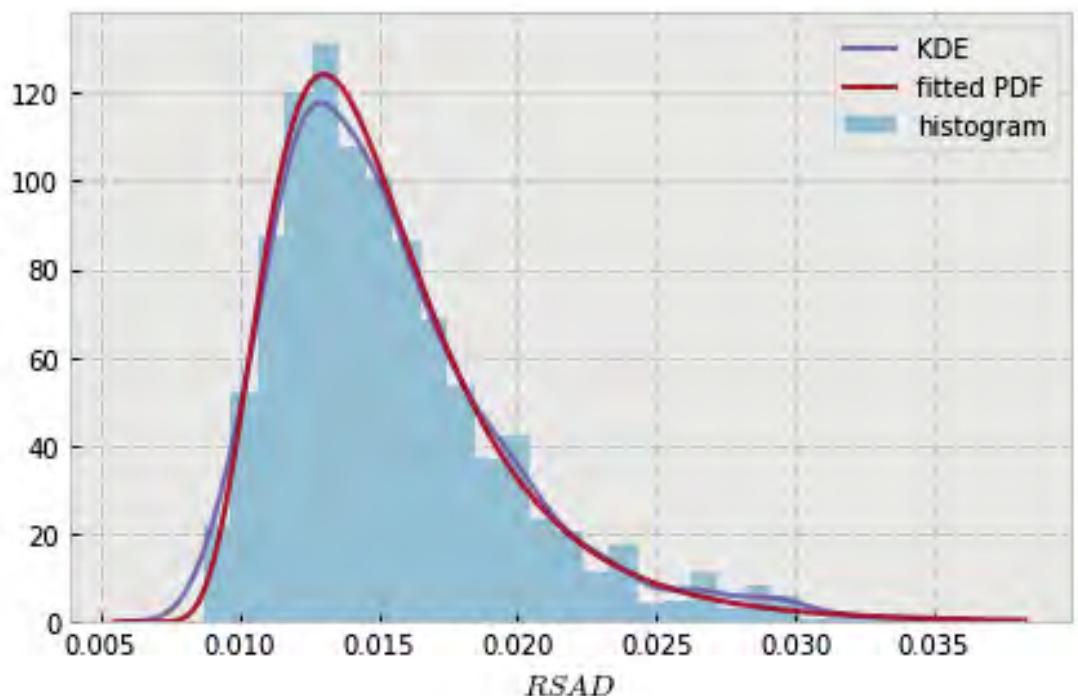


```
variable name: nB
variable value: 40.0
distribution: inverse-gamma(shape=(75.478820490434572,), loc=0.0007955338250840059,
                           scale=1.2088799781444237)
DescribeResult(nobs=1000, minmax=(0.012057133405007683, 0.023105287618328533)
               mean=0.017026504052727268, variance=3.570724019025469e-06,
               skewness=0.41208659699791766, kurtosis=0.020266406316899577)
```

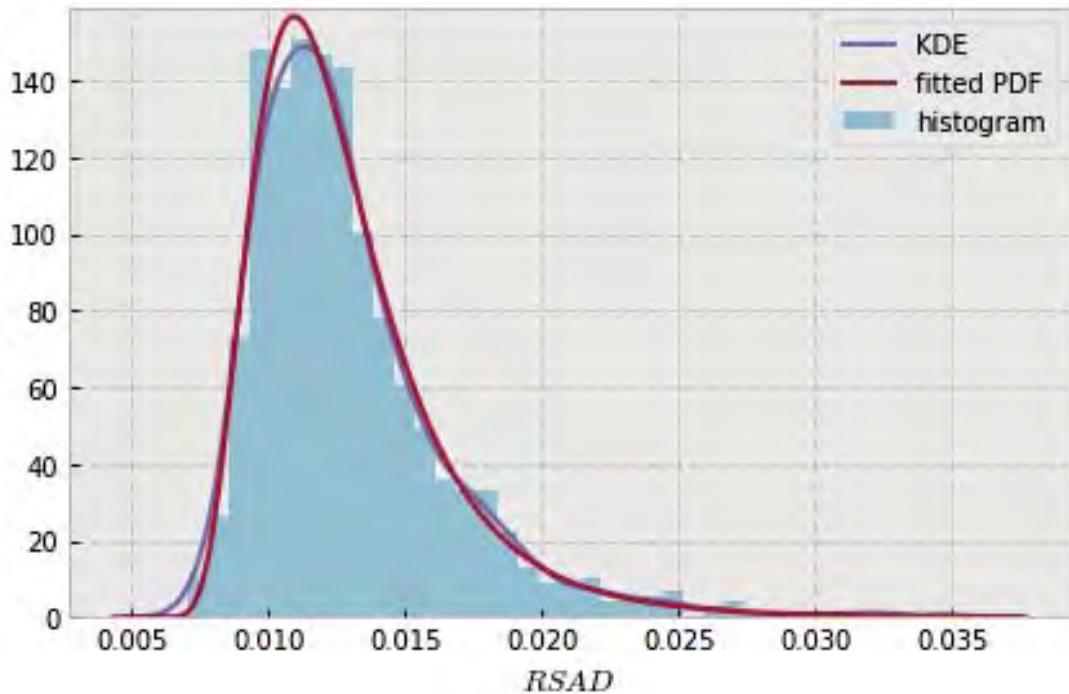


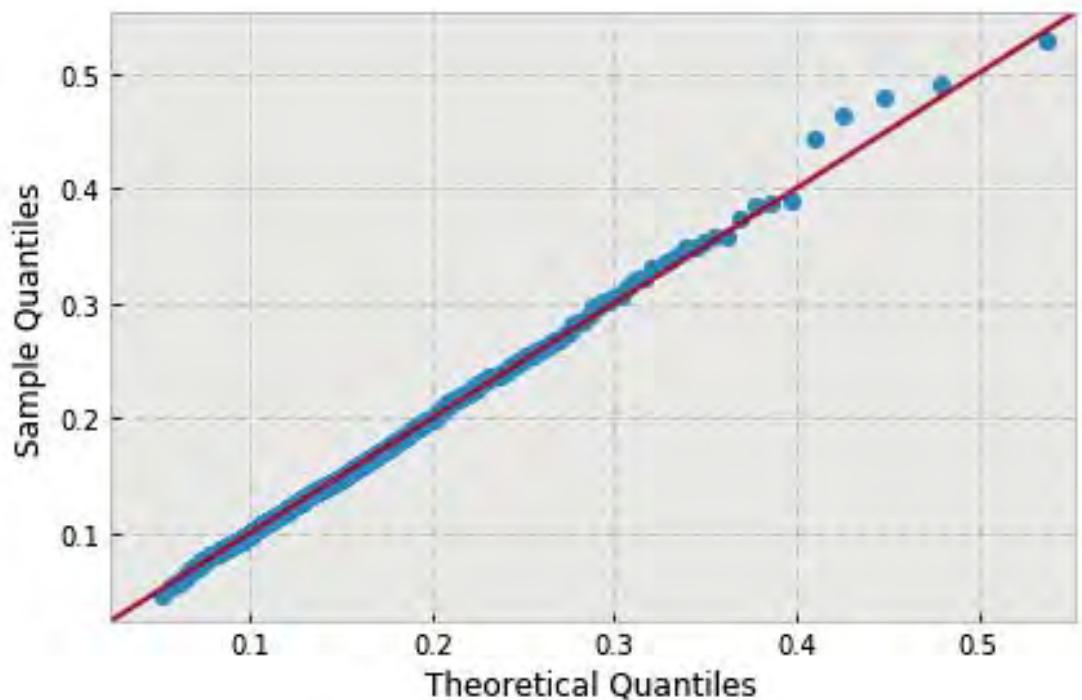


```
variable name: r
variable value: 1.0
distribution: inverse-gamma(shape=(8.1563366351904349,), loc=0.004457959798718492,
scale=0.07860300062231473)
DescribeResult(nobs=1000, minmax=(0.0087339803245598761, 0.03502693387496876)
mean=0.015428158397178376, variance=1.8193898960348825e-05,
skewness=1.3097990081876463, kurtosis=2.0686283379007264)
```

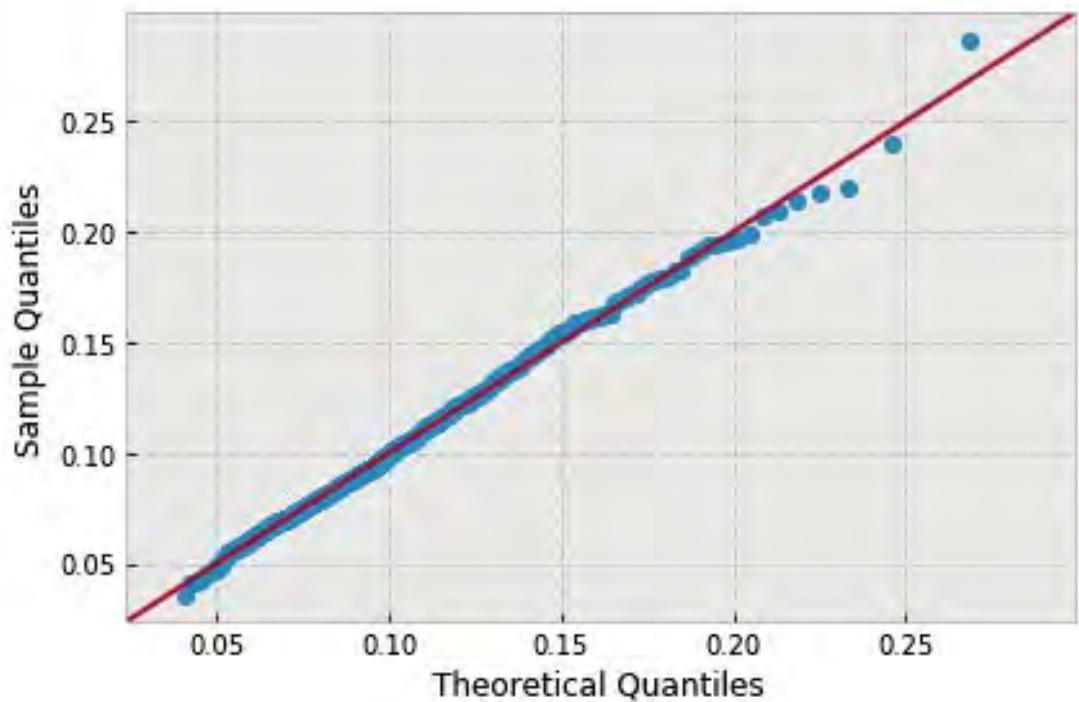
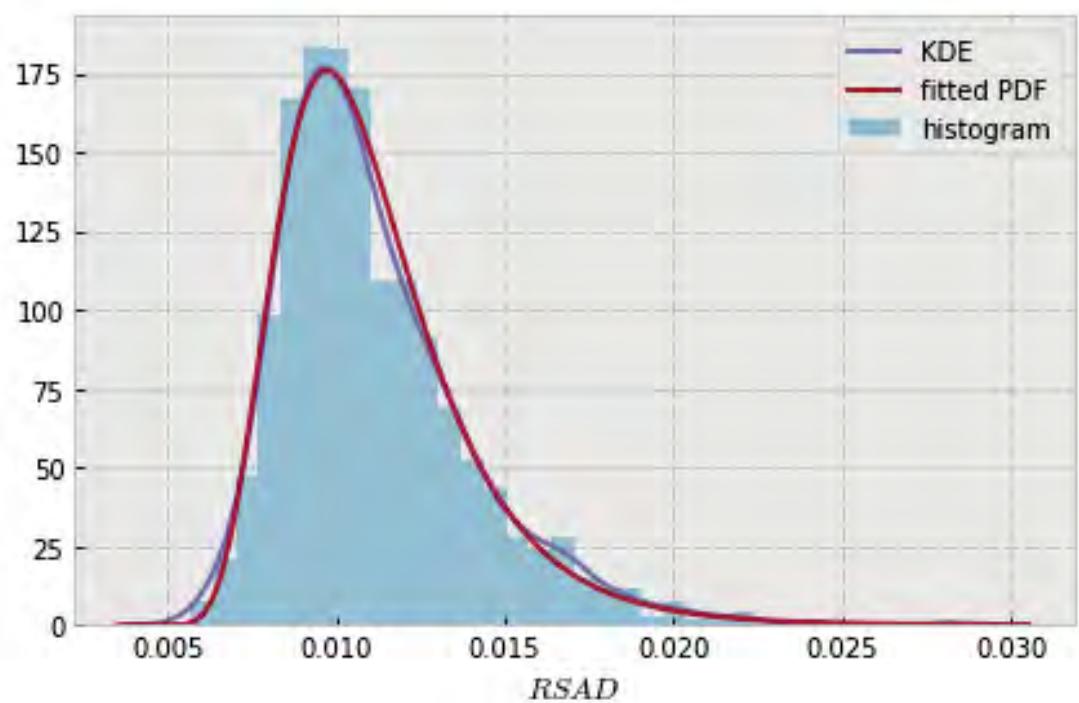


```
variable name: r
variable value: 1.222222222222223
distribution: inverse-gamma(shape=(7.7610241077300595,), loc=0.004333302170693927,
    scale=0.05793152582119196)
DescribeResult(nobs=1000, minmax=(0.0070225889223939214, 0.034949522000928067)
    mean=0.012902845483428198, variance=1.2663127070609843e-05,
    skewness=1.7684297452852247, kurtosis=5.096435124957951)
```

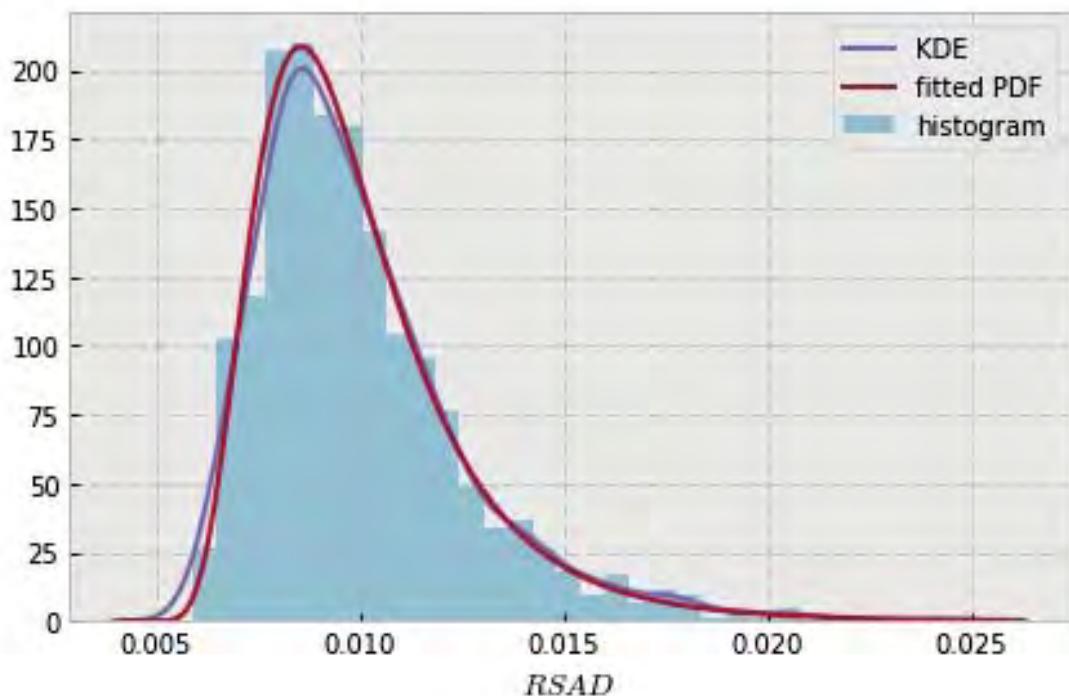


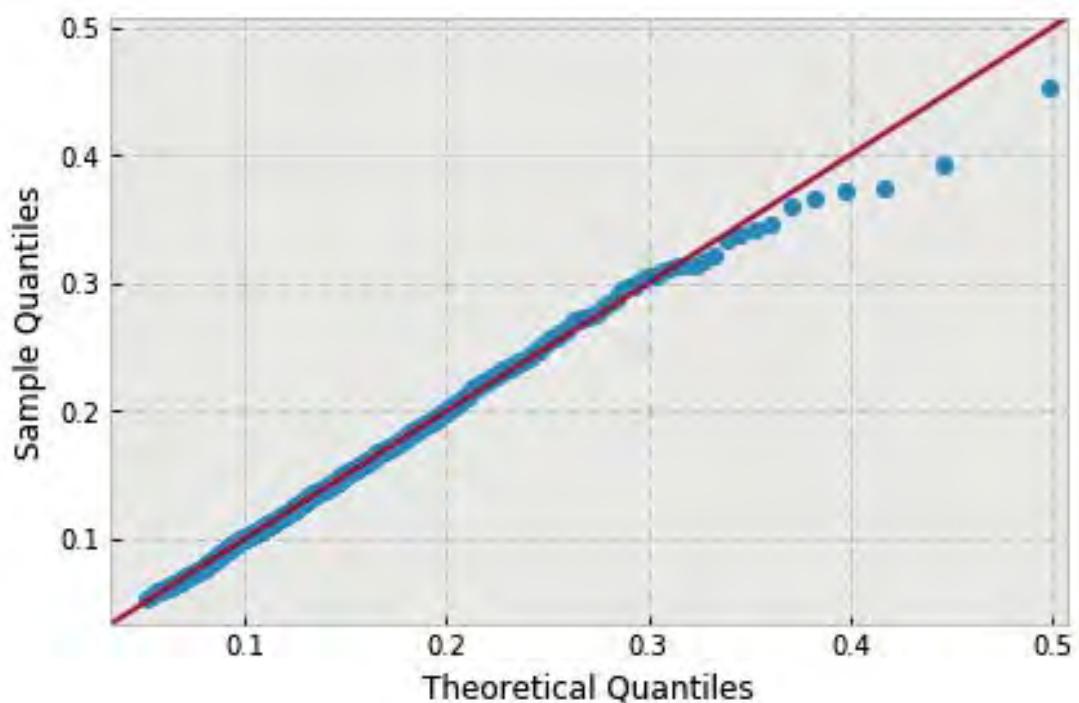


```
variable name: r
variable value: 1.444444444444444
distribution: inverse-gamma(shape=(11.424378901478057,), loc=0.0024168642108182616,
                           scale=0.09069720548613952)
DescribeResult(nobs=1000, minmax=(0.0056736300083330971, 0.028399665439922249)
               mean=0.01111768585239872, variance=7.9863459566852043e-06,
               skewness=1.3012244966091975, kurtosis=2.6704496573515595)
```



```
variable name: r
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(8.0780933085338091,), loc=0.0034731543827904374,
    scale=0.04609012868448316)
DescribeResult(nobs=1000, minmax=(0.005901868174266584, 0.024323138404621372)
    mean=0.0099784072910387104, variance=6.5773760684445872e-06,
    skewness=1.4315676821710208, kurtosis=2.8868062731769877)
```

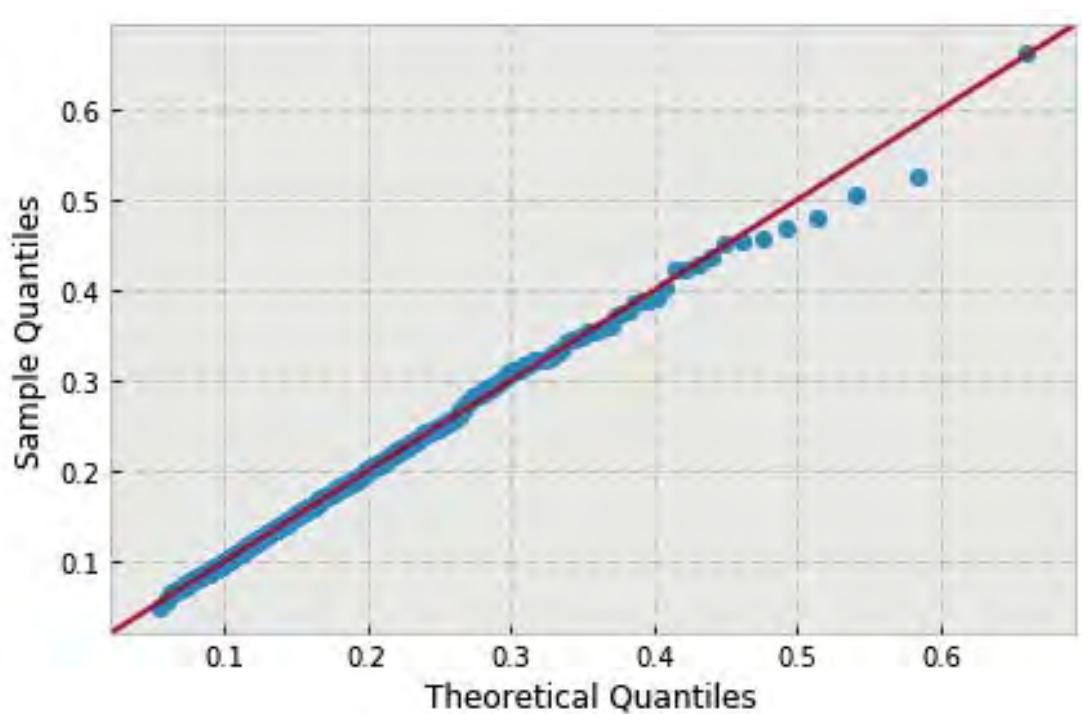
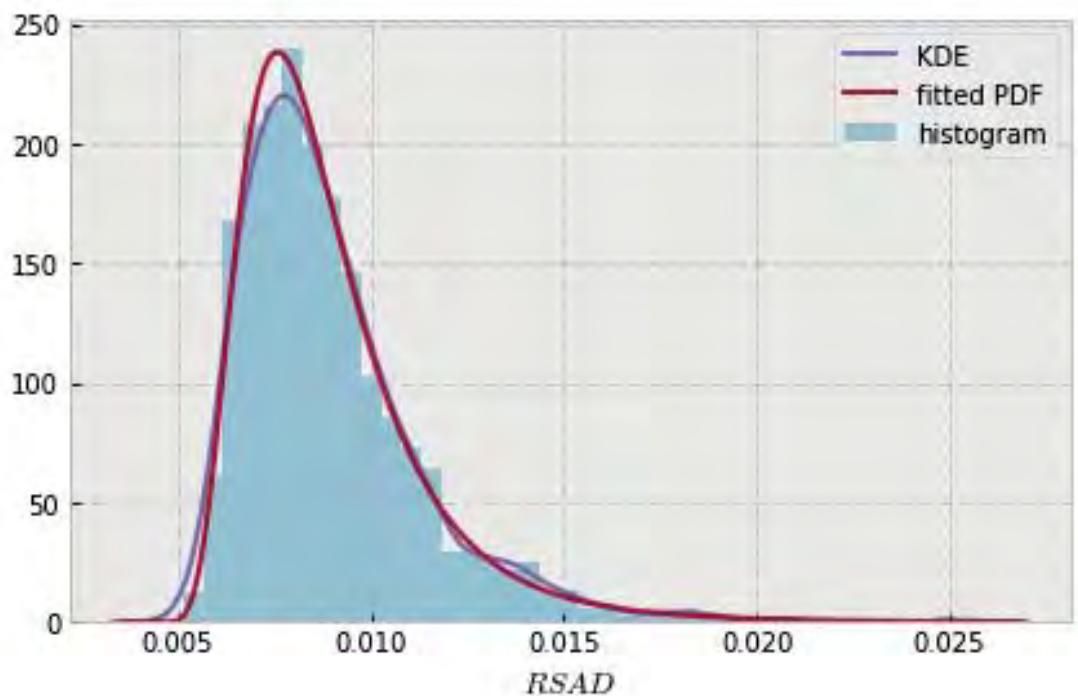




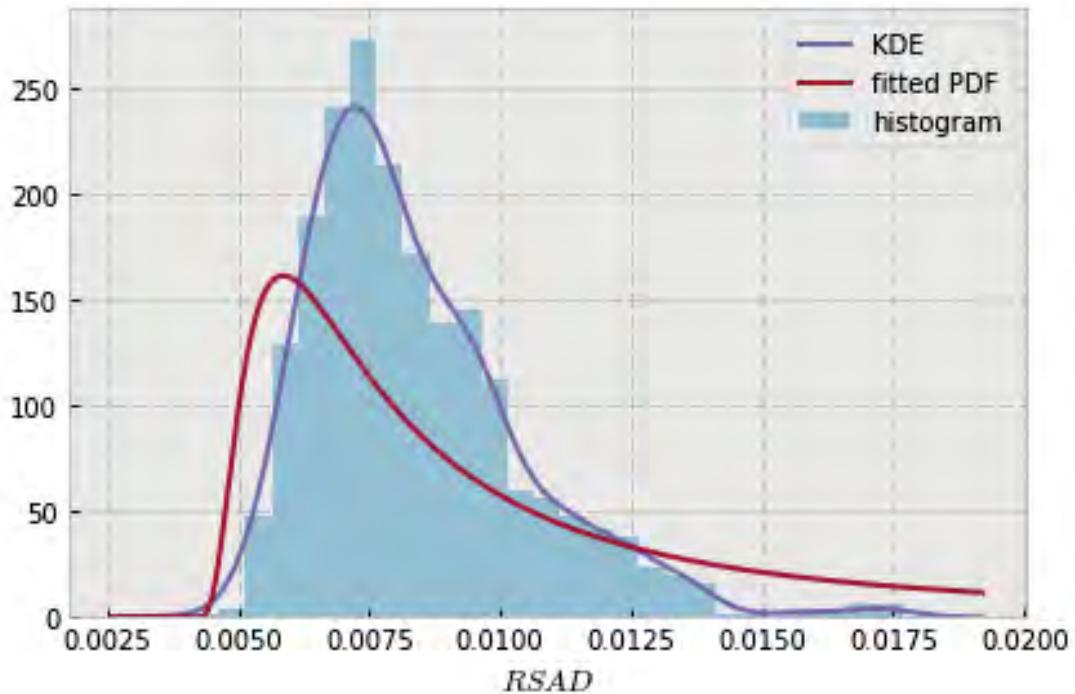
```

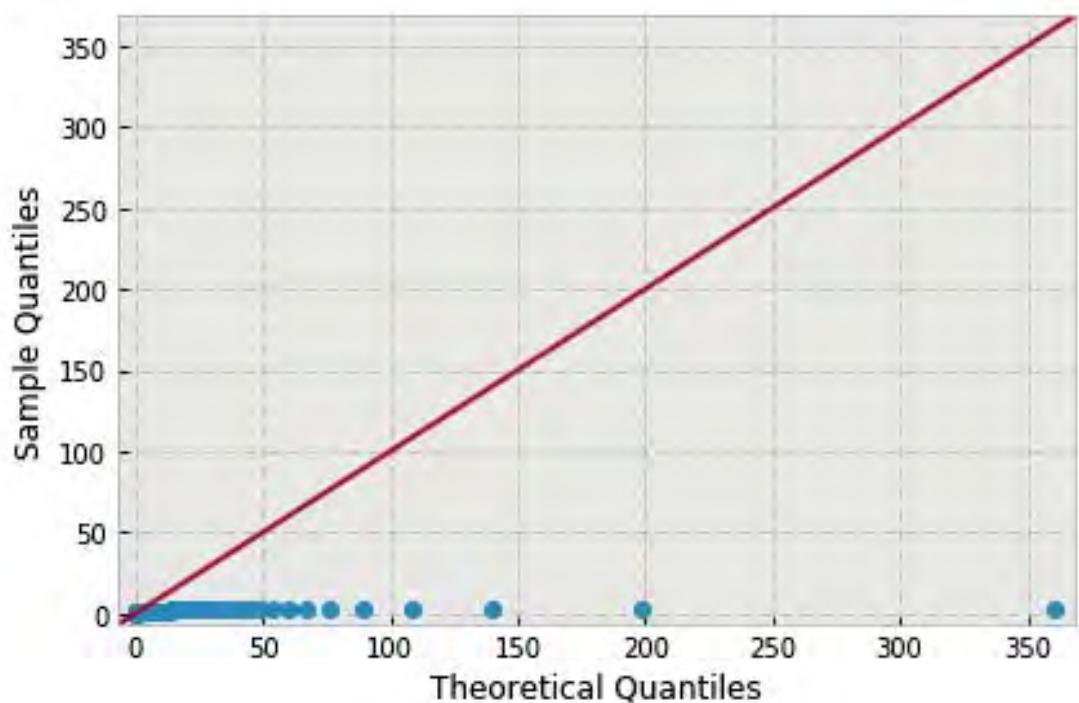
variable name: r
variable value: 1.888888888888888
distribution: inverse-gamma(shape=(6.9916147564269089,), loc=0.0034736644777170625,
                           scale=0.03275327660191675)
DescribeResult(nobs=1000, minmax=(0.0051405279502334429, 0.025154894149935891)
               mean=0.0089342012140586208, variance=5.6416951705792034e-06,
               skewness=1.6912978441614175, kurtosis=4.633155447558597)

```

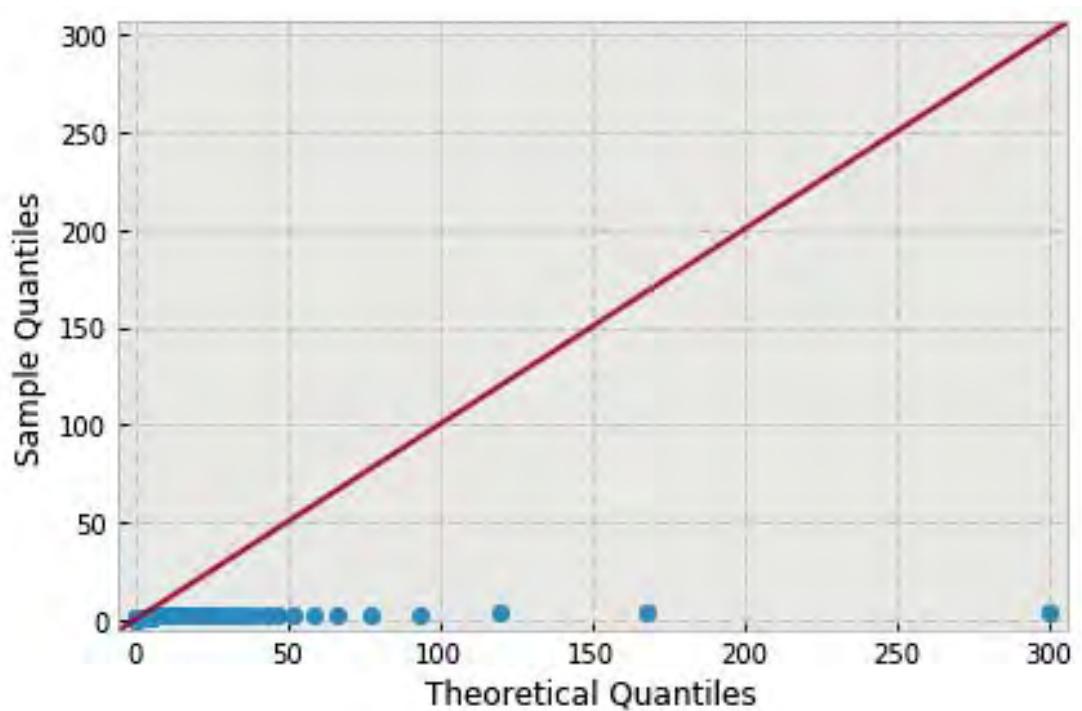
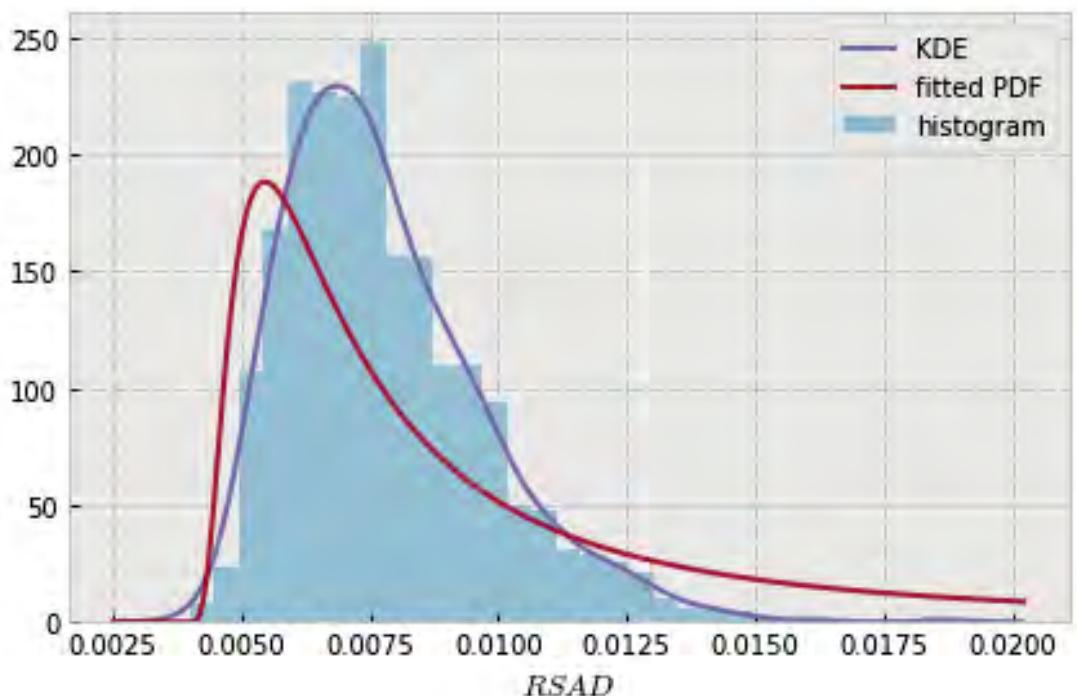


```
variable name: r
variable value: 2.111111111111111
distribution: inverse-gamma(shape=(1.1600502899275682,), loc=0.003983902481026335,
                           scale=0.0040560157719311055)
DescribeResult(nobs=1000, minmax=(0.0041363696830300717, 0.017656586920700226)
               mean=0.0083709138090768392, variance=4.2123313634018187e-06,
               skewness=1.220263963066044, kurtosis=1.999298532495012)
```

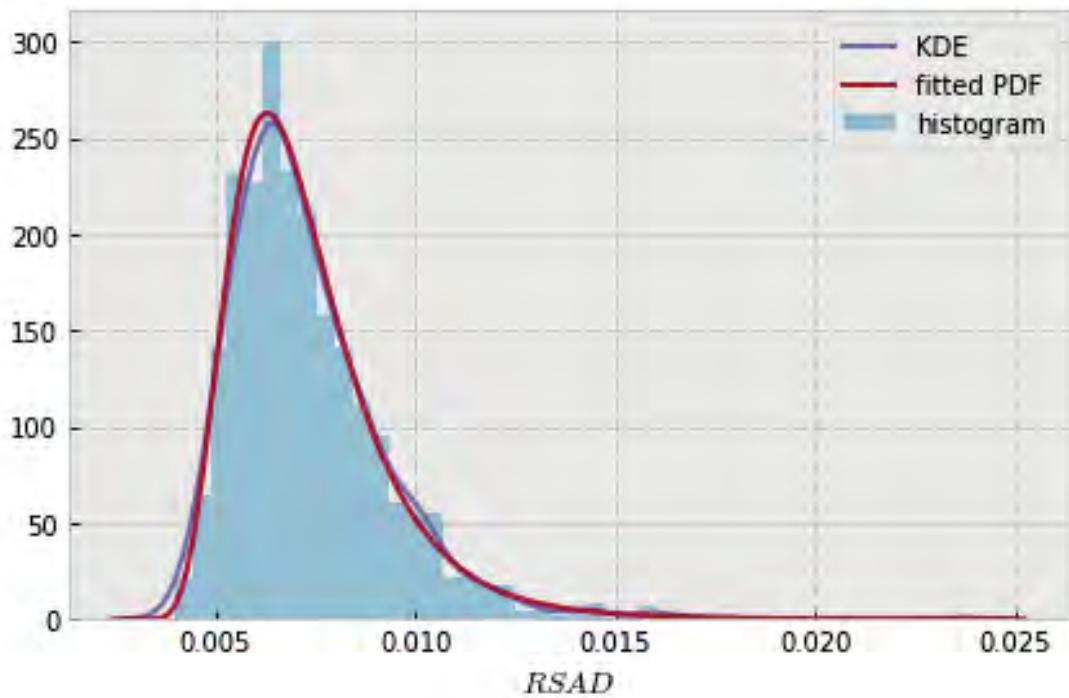


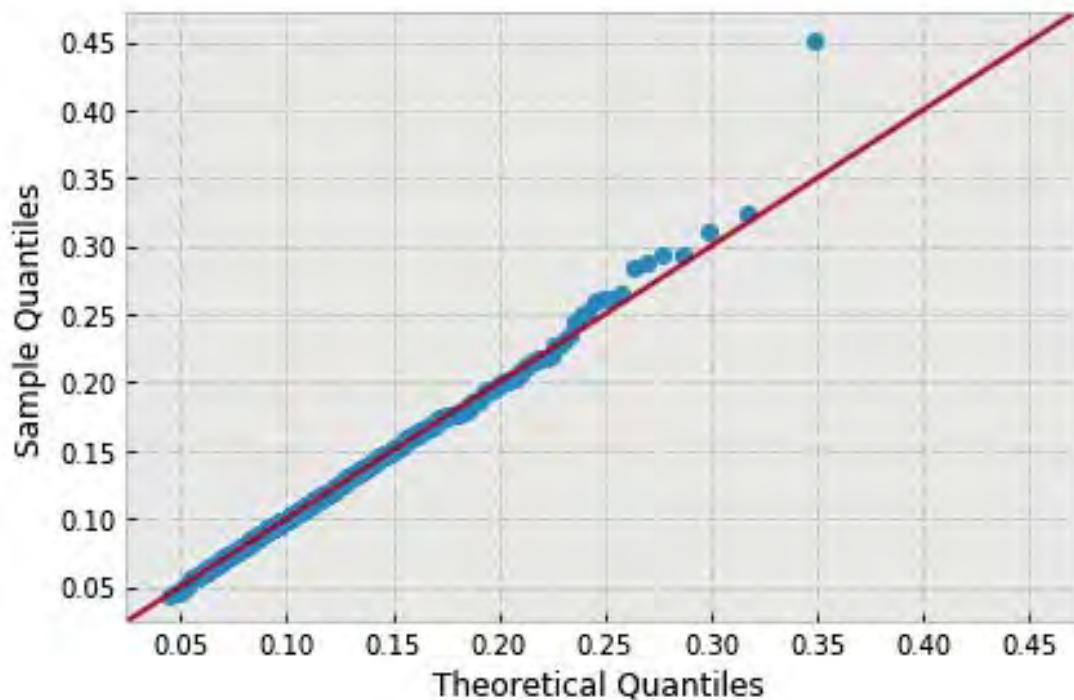


```
variable name: r
variable value: 2.333333333333333
distribution: inverse-gamma(shape=(1.1942624383031737,), loc=0.0038176042393212235,
                           scale=0.003607971062753953)
DescribeResult(nobs=1000, minmax=(0.0039926804228801833, 0.018738868225934954)
               mean=0.0077935931716035271, variance=3.8136235206448161e-06,
               skewness=1.0660683323359006, kurtosis=1.6290930638738725)
```

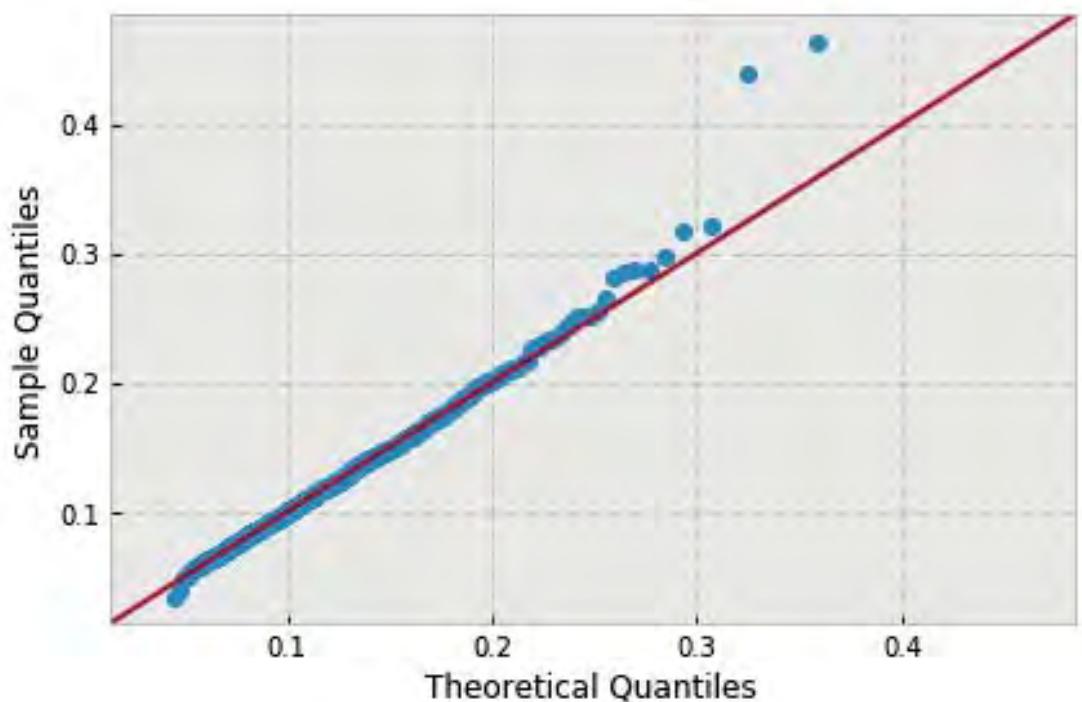
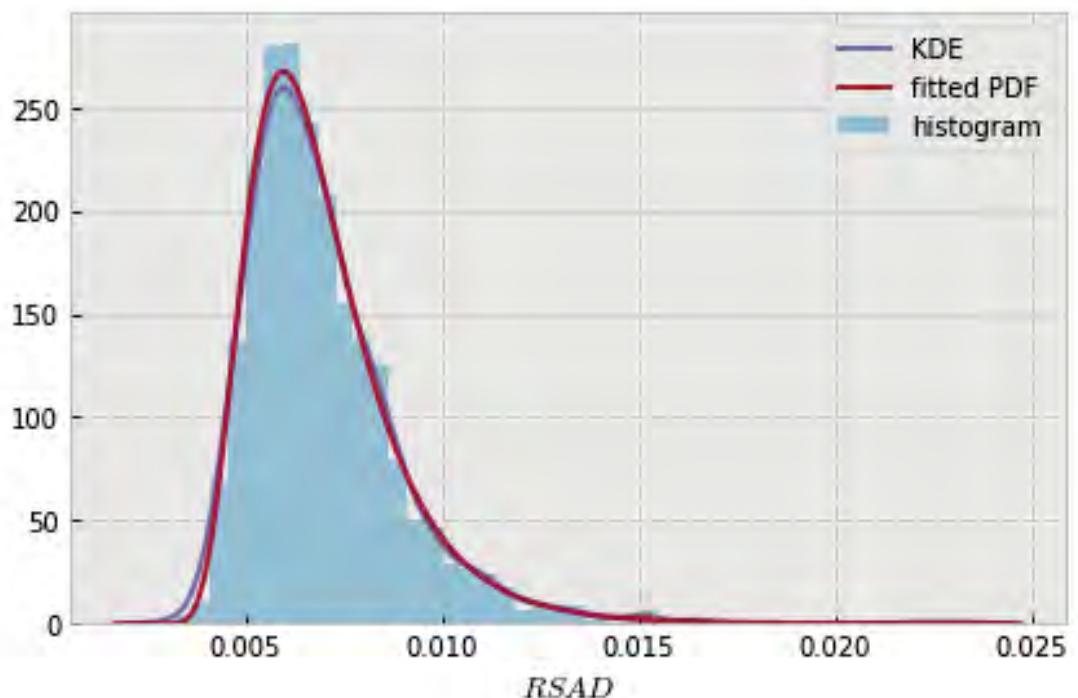


```
variable name: r
variable value: 2.5555555555555554
distribution: inverse-gamma(shape=(9.8168343844151273,), loc=0.0018066084553869008,
                           scale=0.04857580482203368)
DescribeResult(nobs=1000, minmax=(0.0039644356537438105, 0.023702439604706891)
               mean=0.0073177149086470095, variance=3.9705097300222463e-06,
               skewness=1.8008885766207356, kurtosis=6.94872495843007)
```

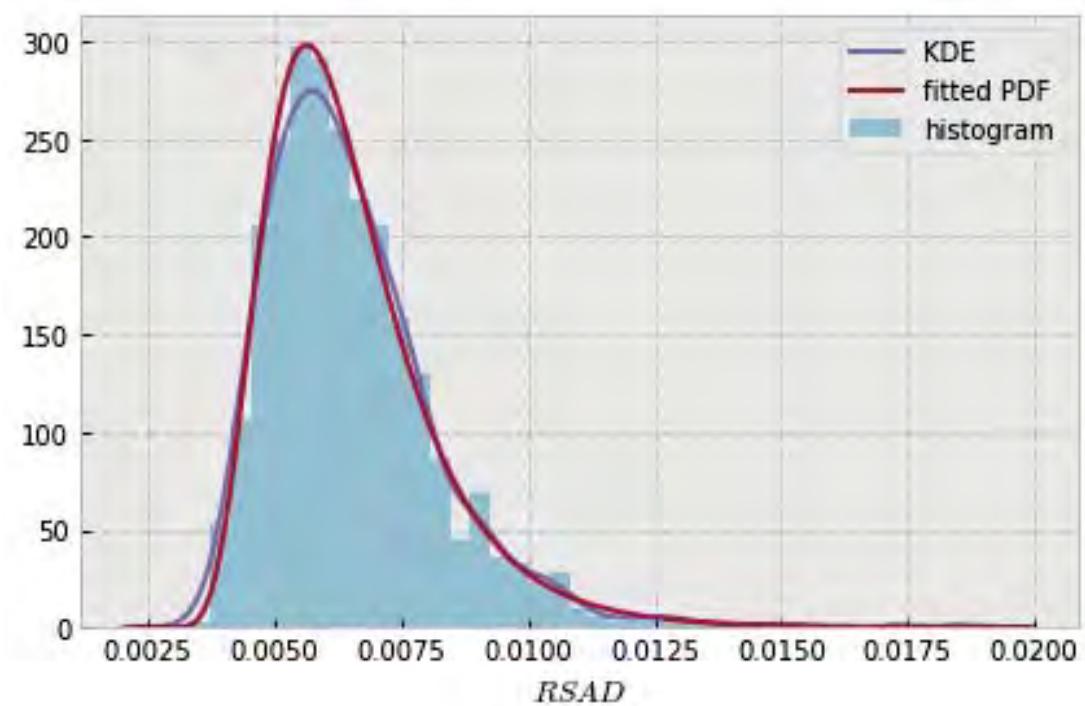


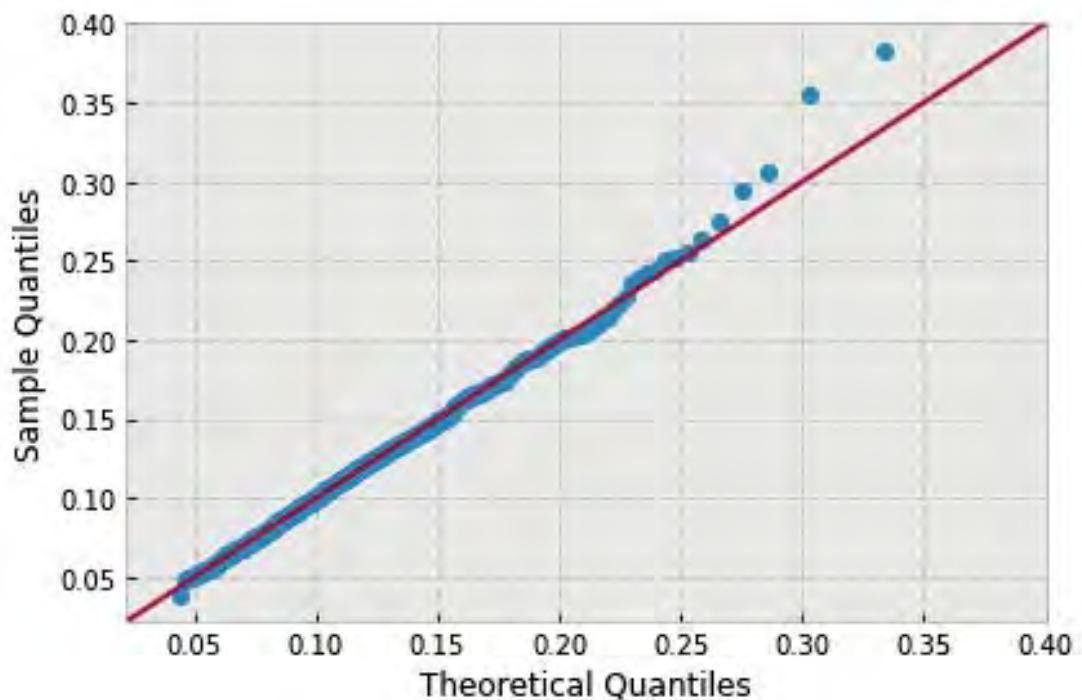


```
variable name: r
variable value: 2.7777777777777777
distribution: inverse-gamma(shape=(9.6681604495039046,), loc=0.001594096813728357,
                           scale=0.04673090681206908)
DescribeResult(nobs=1000, minmax=(0.0032071272919654832, 0.023217522592267868)
               mean=0.0069914014039176983, variance=4.0591180036187449e-06,
               skewness=2.074933377114107, kurtosis=9.037283833058103)
```

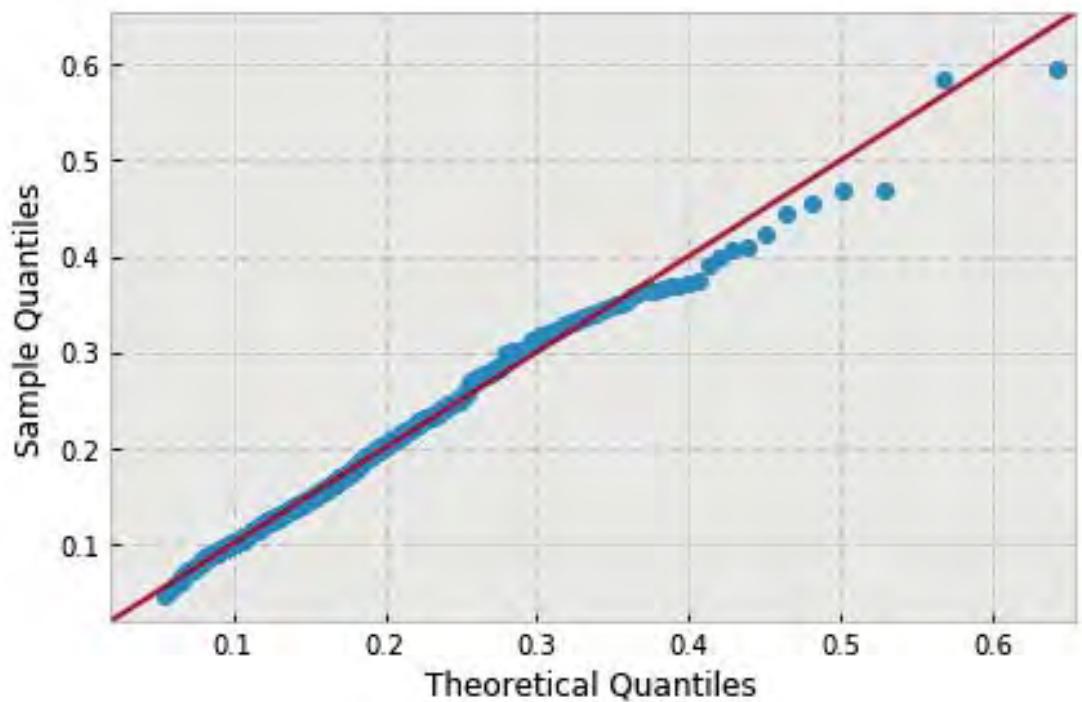
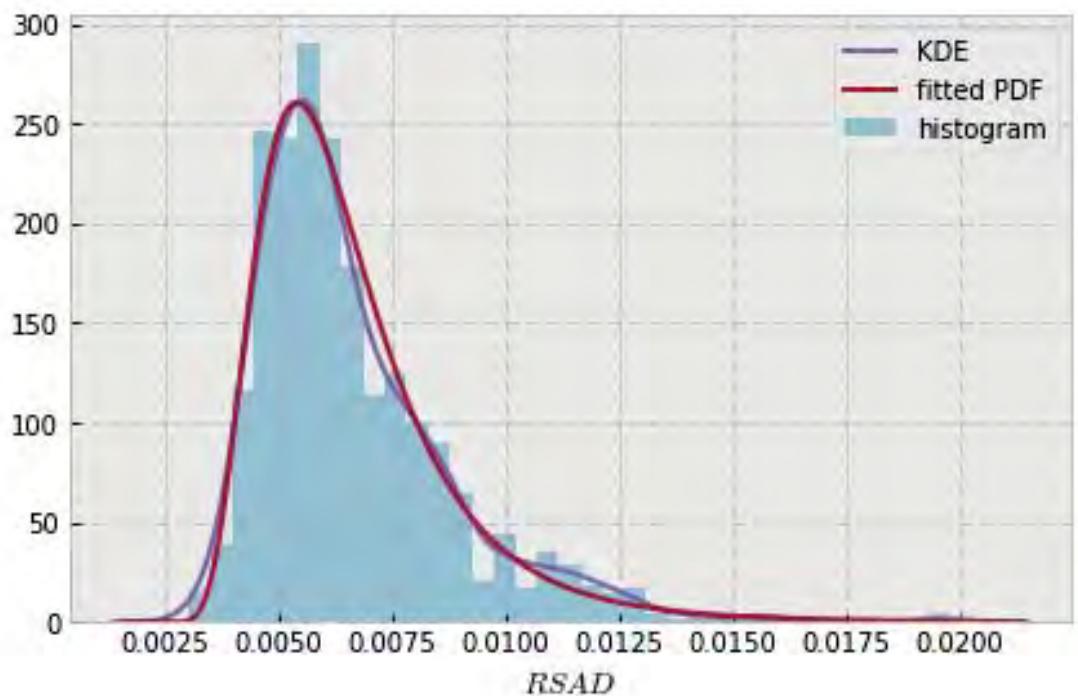


```
variable name: r
variable value: 3.0
distribution: inverse-gamma(shape=(10.072881967754455,), loc=0.0016132988282657559,
                           scale=0.0445090719647472)
DescribeResult(nobs=1000, minmax=(0.0033496172211548007, 0.018671024263326227)
               mean=0.0065191569392732858, variance=2.9995121204153563e-06,
               skewness=1.6593599862442925, kurtosis=5.565330412934774)
```

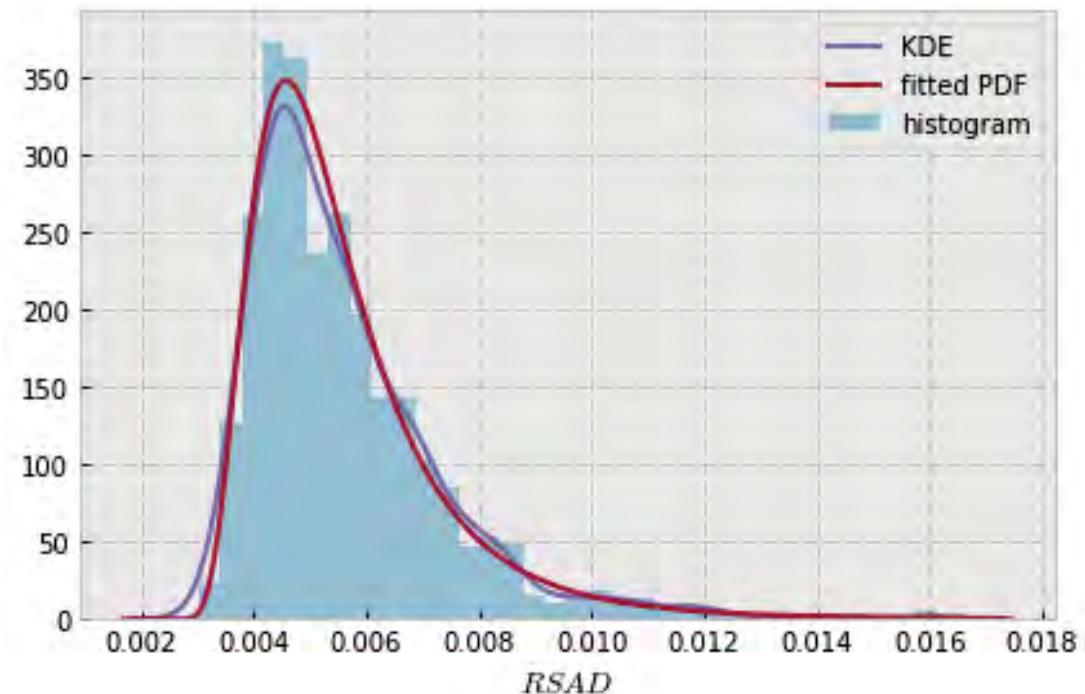


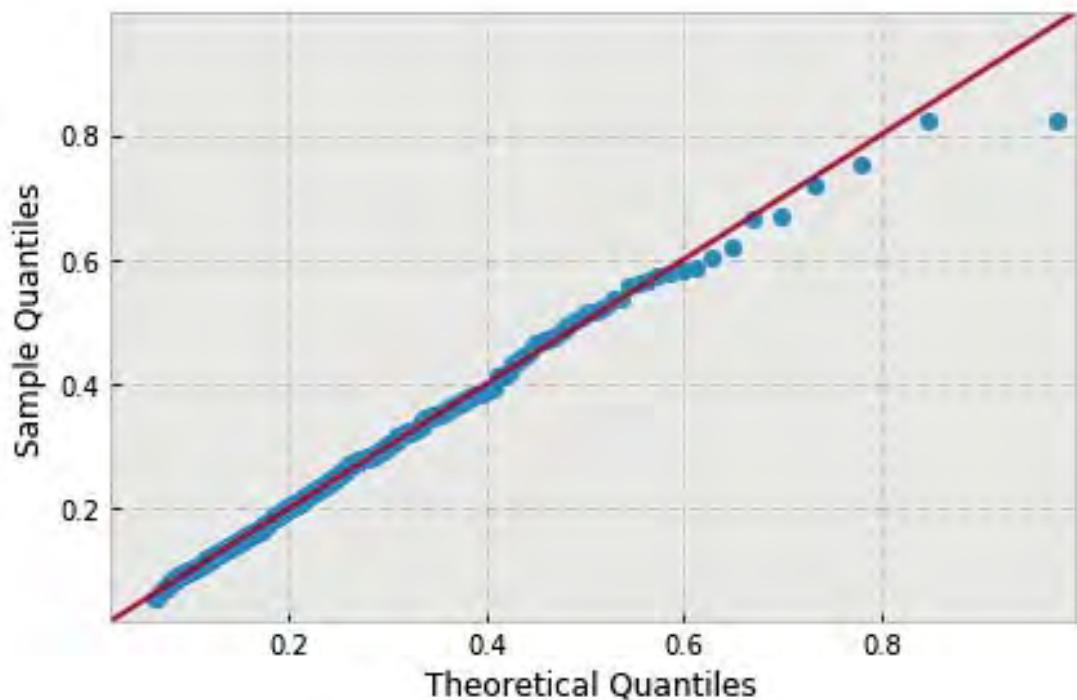


```
variable name: sigma
variable value: 0.01111111111111112
distribution: inverse-gamma(shape=(7.0887957726797417,), loc=0.001643935175597782,
                           scale=0.03053575005315546)
DescribeResult(nobs=1000, minmax=(0.003042136029024531, 0.01980850242669448)
               mean=0.0066564561036142099, variance=4.7050292000106014e-06,
               skewness=1.6082347945709212, kurtosis=3.852450766011435)
```

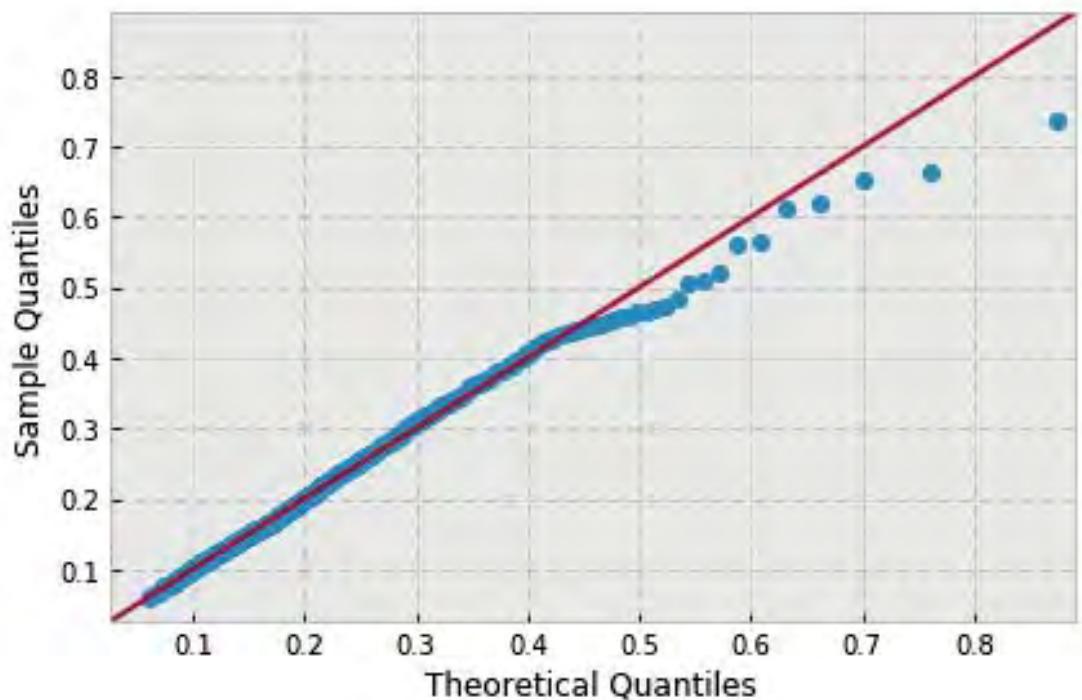
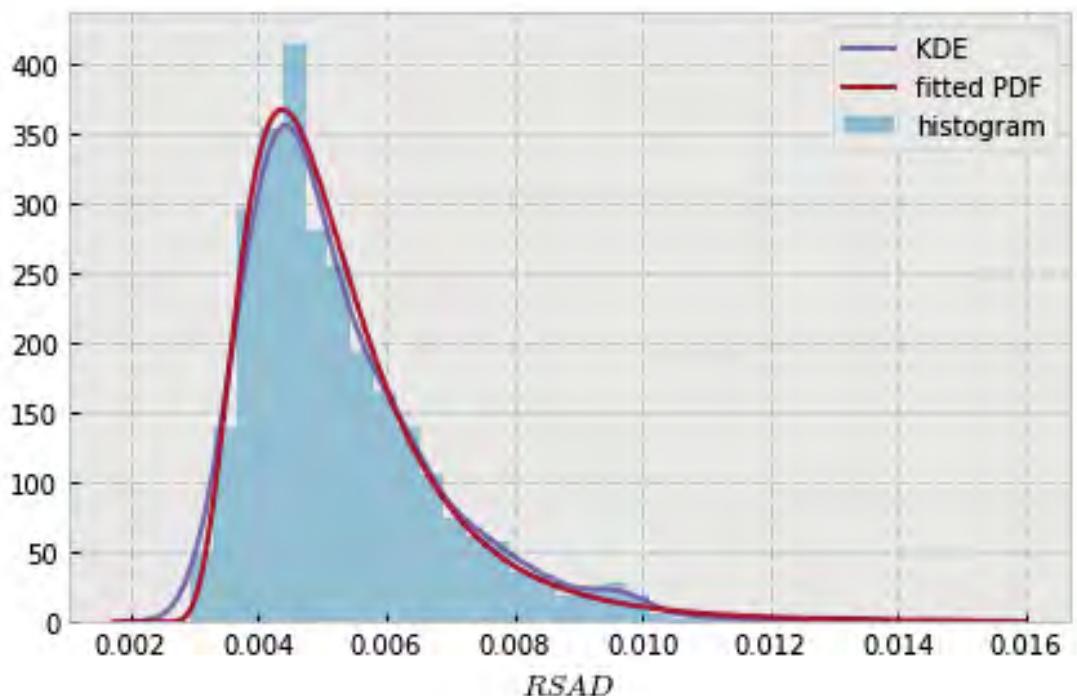


```
variable name: sigma
variable value: 0.02222222222222223
distribution: inverse-gamma(shape=(5.7843701323652397,), loc=0.002063030381956211,
                           scale=0.017066061888444423)
DescribeResult(nobs=1000, minmax=(0.003016238388258563, 0.01611088422424117)
               mean=0.005624147442104899, variance=3.0837872952404496e-06,
               skewness=1.8801218445717018, kurtosis=5.38507127858375)
```

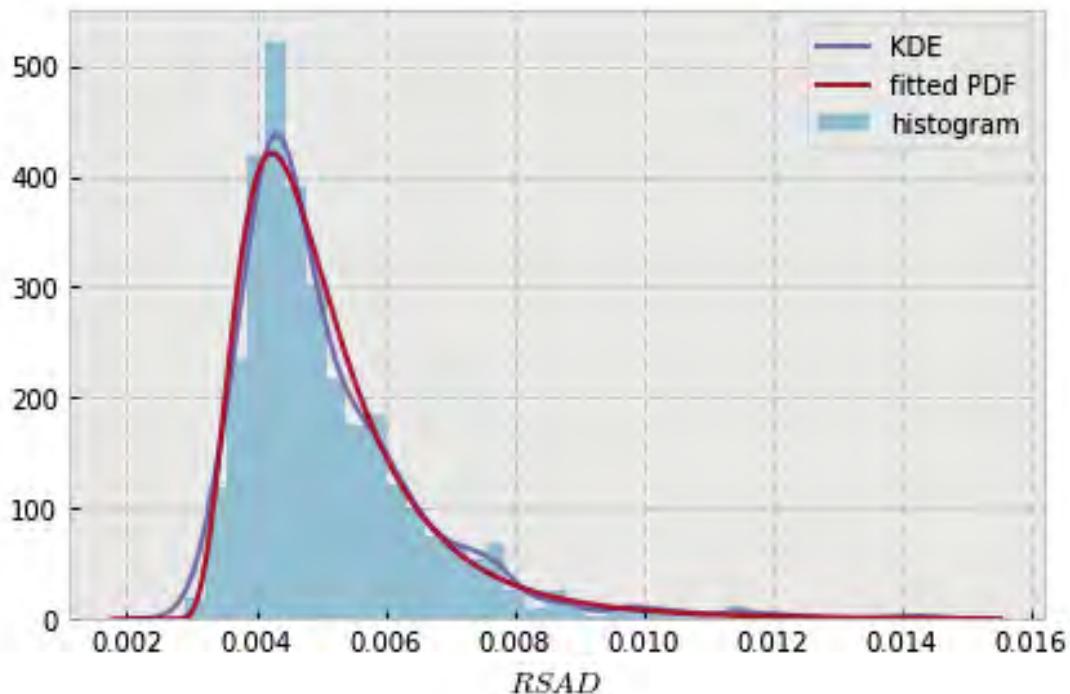


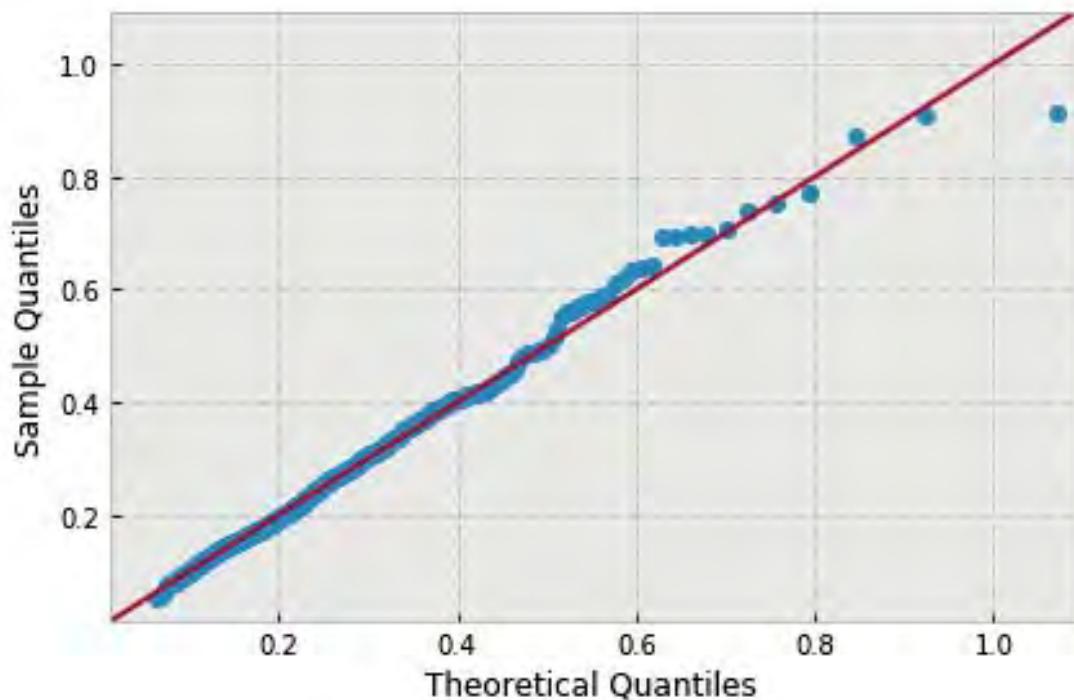


```
variable name: sigma
variable value: 0.0333333333333333
distribution: inverse-gamma(shape=(6.0974150057864245,), loc=0.0019188330819076968,
                           scale=0.017404221436246155)
DescribeResult(nobs=1000, minmax=(0.0029420345110439277, 0.014752830821415482)
               mean=0.005325601329502062, variance=2.5487102556837363e-06,
               skewness=1.6350227164441382, kurtosis=3.8492718386612292)
```

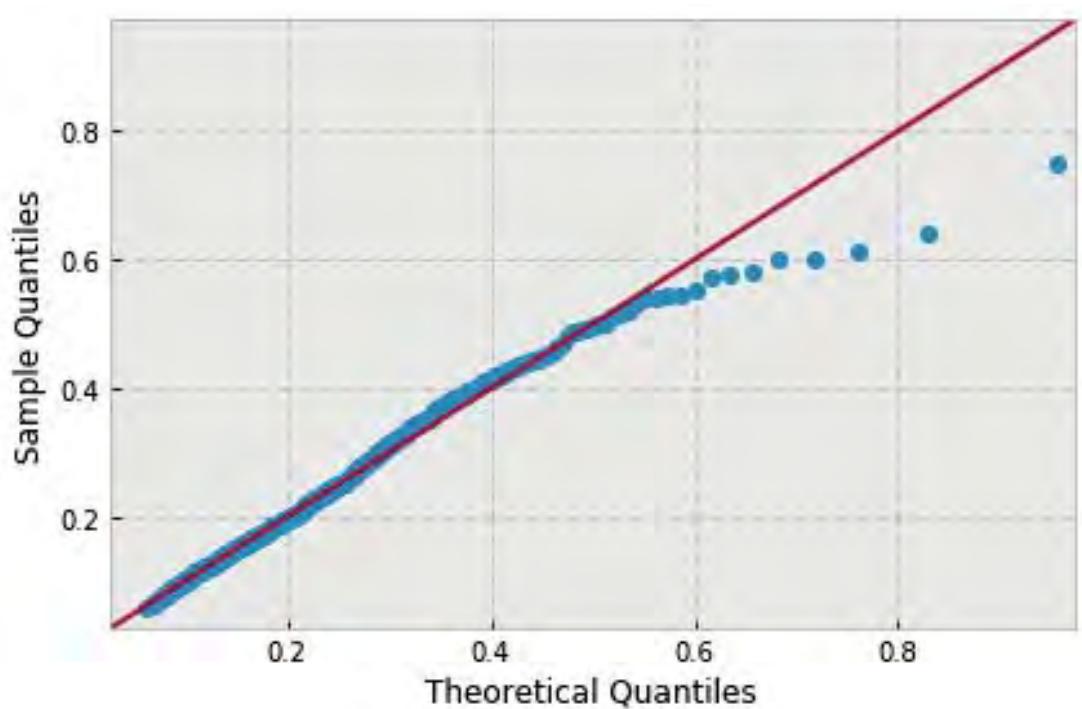
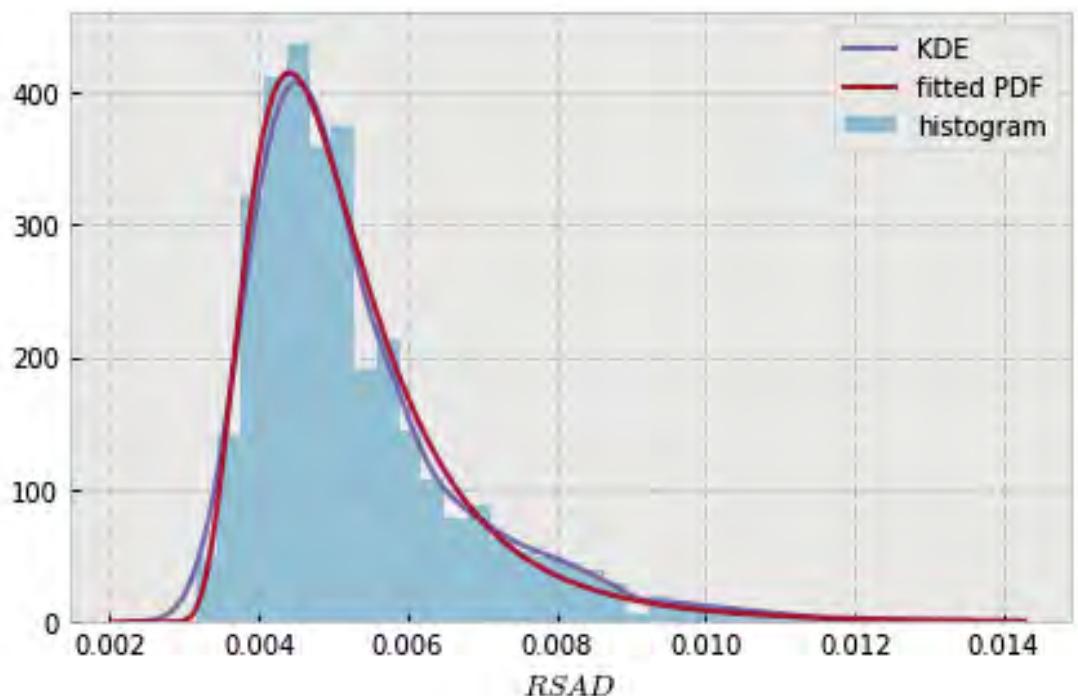


```
variable name: sigma
variable value: 0.04444444444444446
distribution: inverse-gamma(shape=(5.5477978614673304,), loc=0.0022176951858443386,
                           scale=0.0132745770230605)
DescribeResult(nobs=1000, minmax=(0.0029186591752678607, 0.014373906226720429)
               mean=0.0051371926863357674, variance=2.3144868655021382e-06,
               skewness=2.09474305826677, kurtosis=6.487239852944198)
```

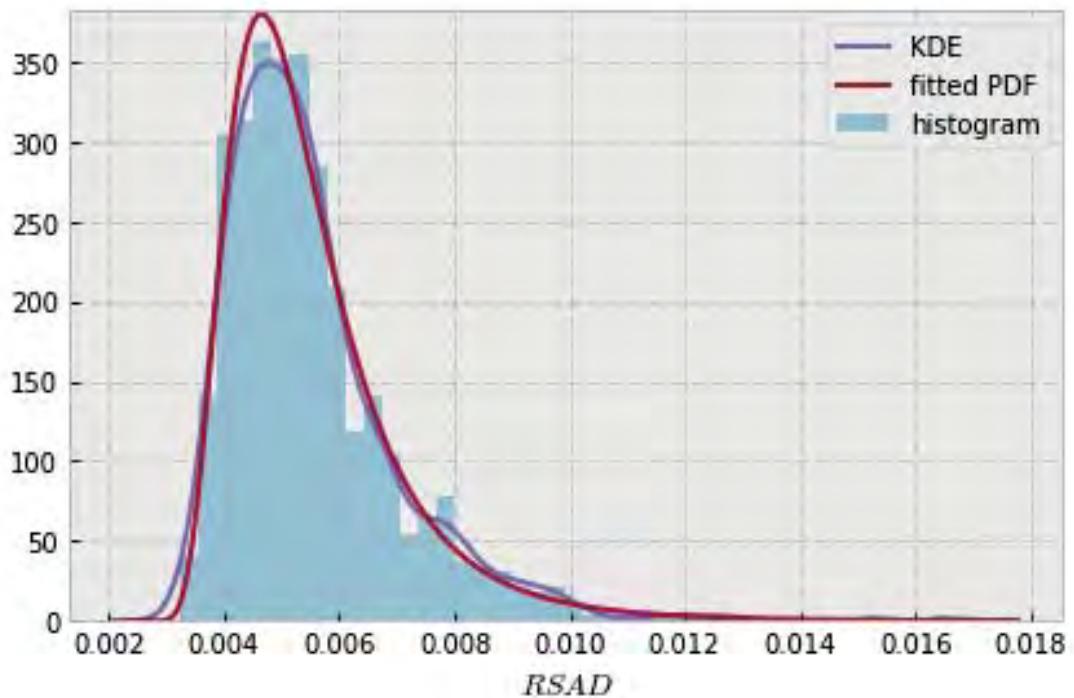


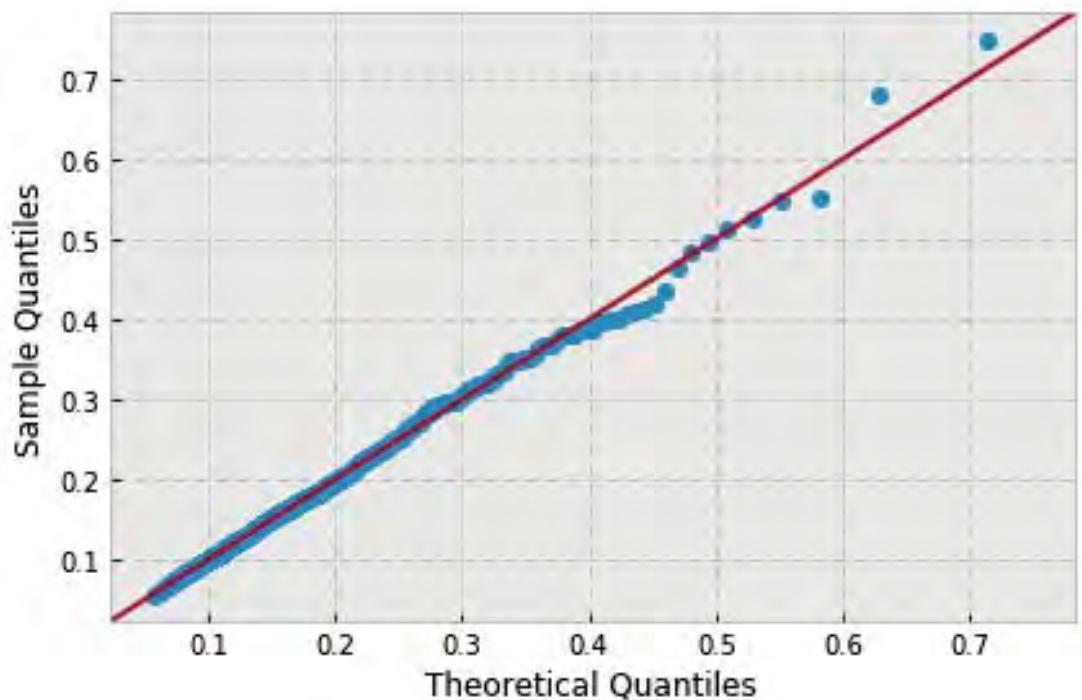


```
variable name: sigma
variable value: 0.05555555555555555
distribution: inverse-gamma(shape=(5.8468865587919554,), loc=0.0022878898711165448,
                           scale=0.01455743926357259)
DescribeResult(nobs=1000, minmax=(0.0031535290796158833, 0.013190813036371797)
               mean=0.0052847005521469096, variance=2.0770530429401927e-06,
               skewness=1.5610774849514775, kurtosis=2.9151397817826705)
```



```
variable name: sigma
variable value: 0.066666666666666667
distribution: inverse-gamma(shape=(6.7238517051182196,), loc=0.002135502061605183,
                           scale=0.01940752519033959)
DescribeResult(nobs=1000, minmax=(0.0032323809387484275, 0.016635078991425786)
               mean=0.0055235866855012153, variance=2.3337211624046604e-06,
               skewness=1.8936915647498185, kurtosis=6.355124270678214)
```

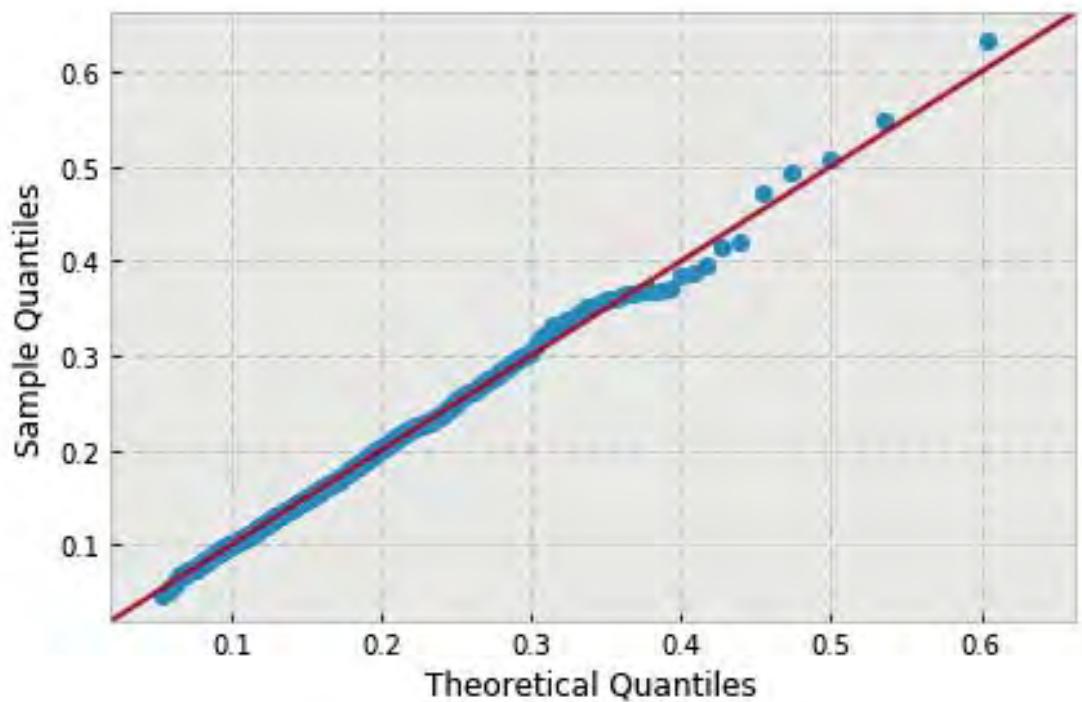
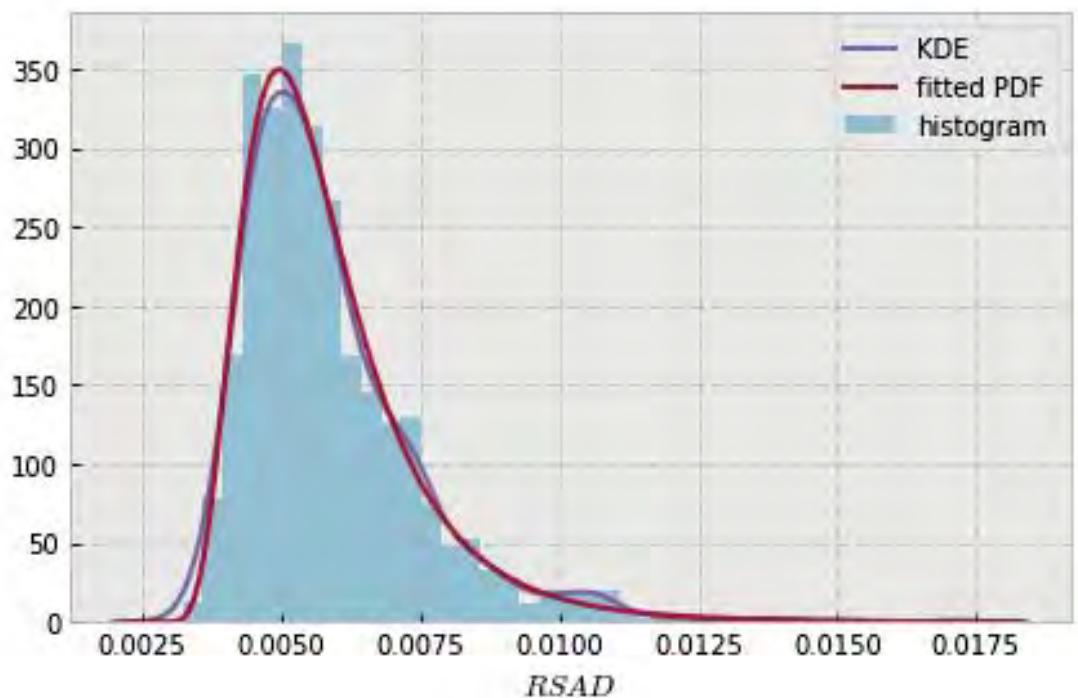




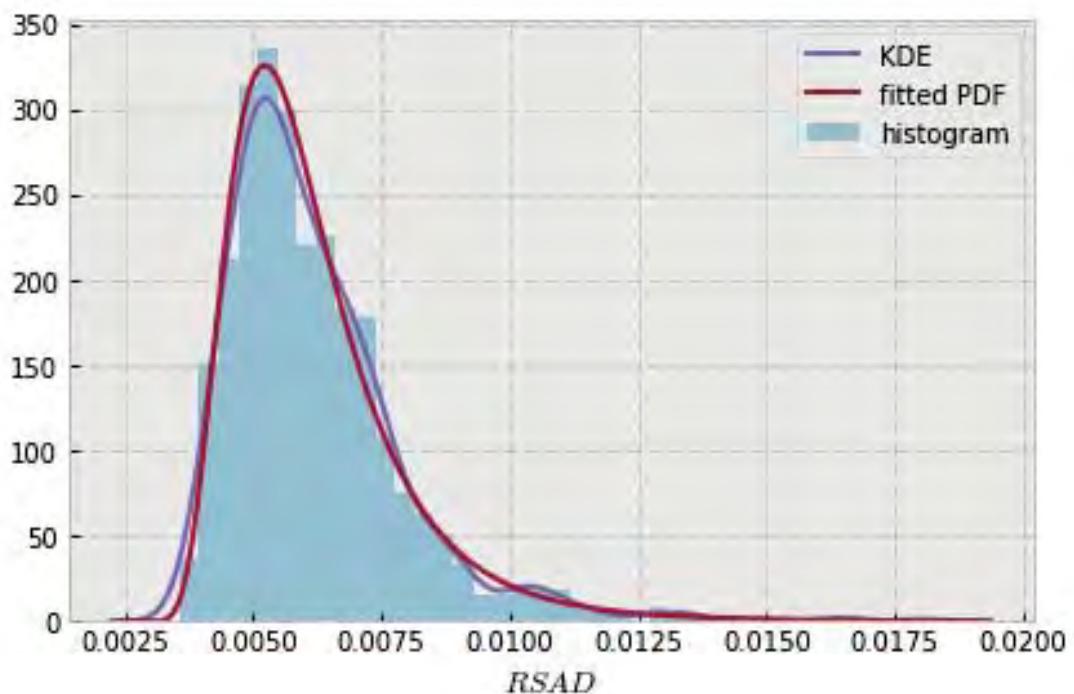
```

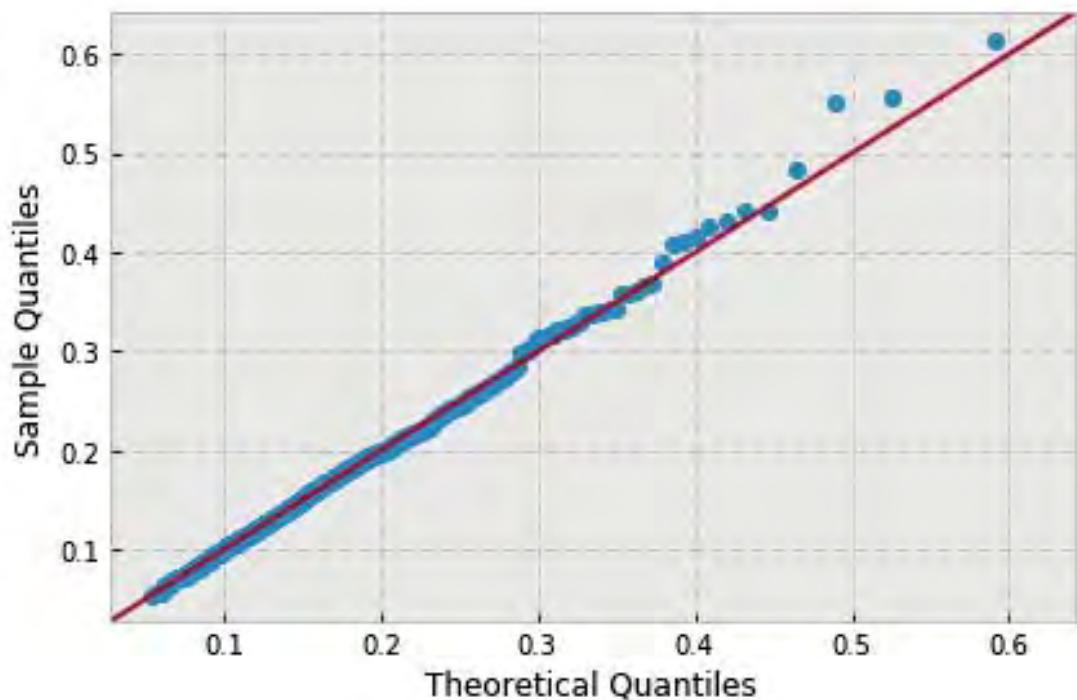
variable name: sigma
variable value: 0.07777777777777778
distribution: inverse-gamma(shape=(7.3098313826562986,), loc=0.0020873863701045504,
                           scale=0.023801302699451264)
DescribeResult(nobs=1000, minmax=(0.0032189430998368911, 0.017147385521457555)
               mean=0.0058594109474850329, variance=2.6363831838734408e-06,
               skewness=1.8121347656790445, kurtosis=5.5209521969883255)

```

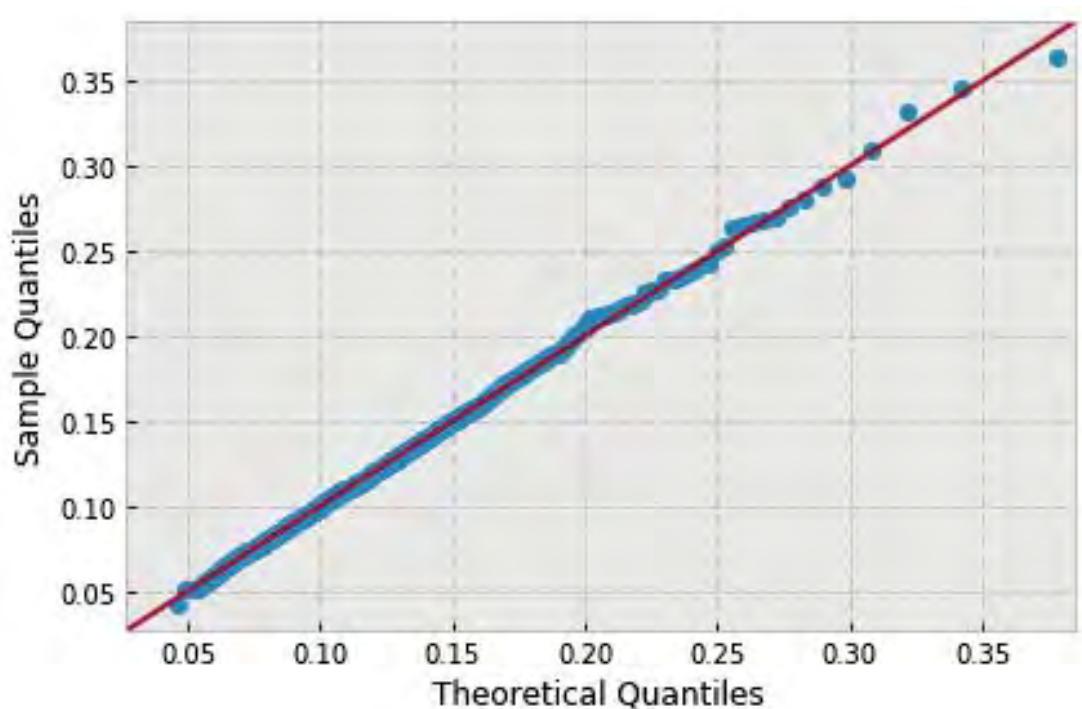
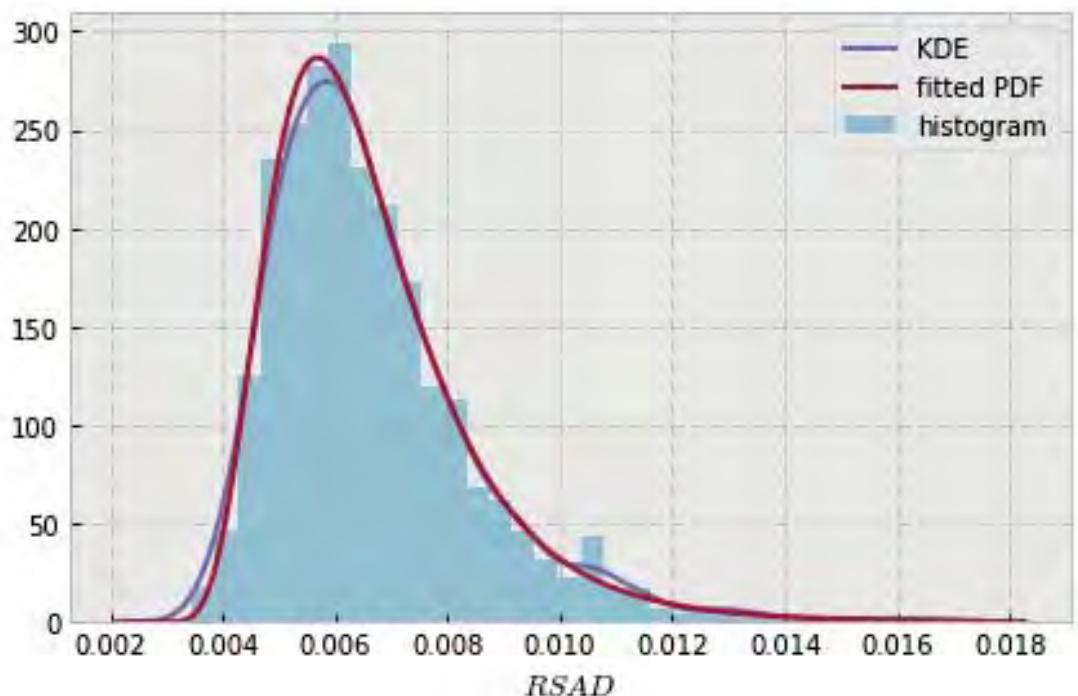


```
variable name: sigma
variable value: 0.08888888888888889
distribution: inverse-gamma(shape=(7.388058310132025,), loc=0.002129108429055588,
                           scale=0.0259503951626589)
DescribeResult(nobs=1000, minmax=(0.0035579876674629472, 0.018047286920742624)
               mean=0.0061903490999341837, variance=3.01911112419829e-06,
               skewness=1.903264787799752, kurtosis=6.349107923672509)
```

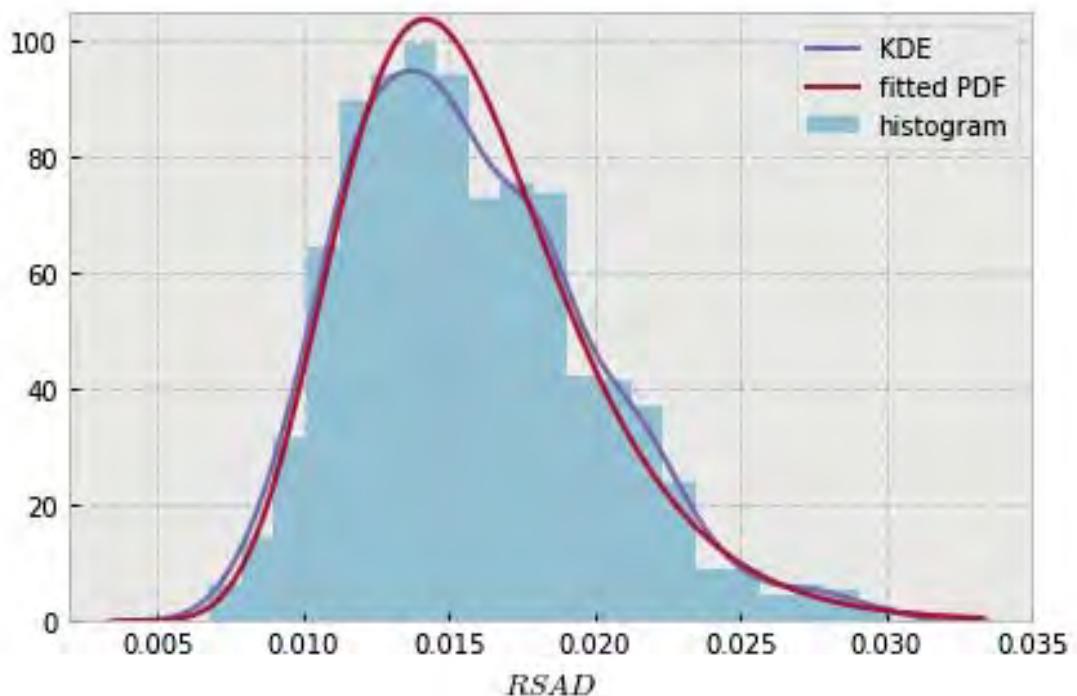


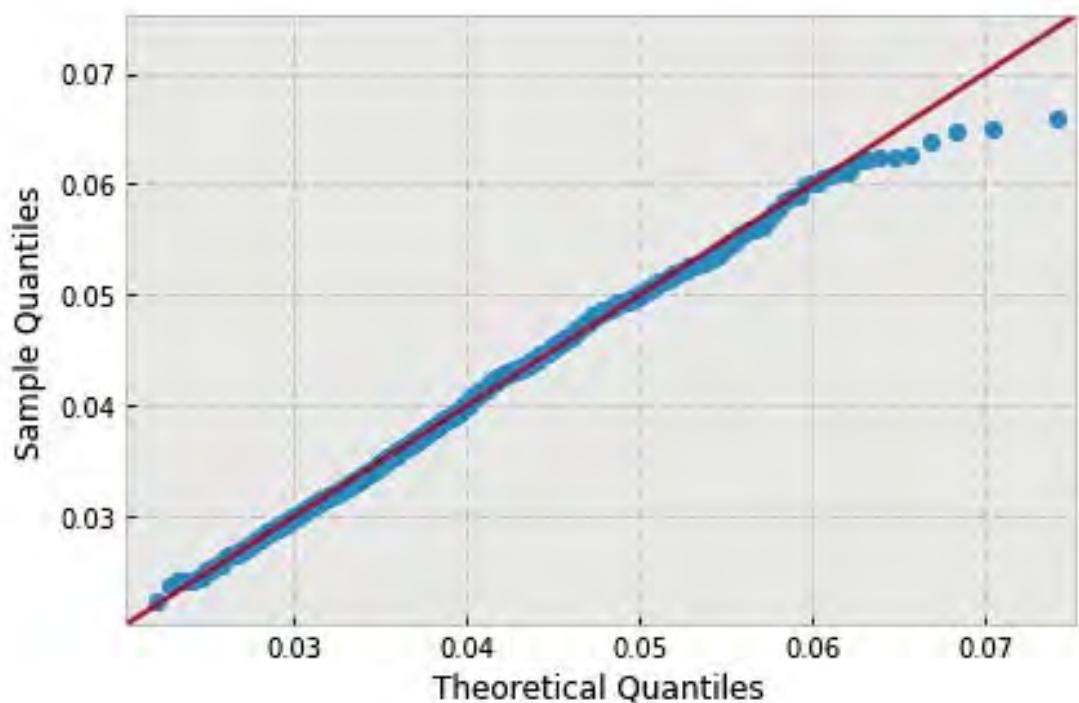


```
variable name: sigma
variable value: 0.1
distribution: inverse-gamma(shape=(9.3860105792156858,), loc=0.0016738831614609235,
                           scale=0.04174552284682534)
DescribeResult(nobs=1000, minmax=(0.0034492290362248485, 0.016894397481472503)
               mean=0.0066508934919163243, variance=3.2933475479889725e-06,
               skewness=1.4608156469988145, kurtosis=3.343935537007331)
```



```
variable name: theta
variable value: 0.6981317007977318
distribution: inverse-gamma(shape=(26.659799566730445,), loc=-0.0052839815140676444,
                           scale=0.5382475657468737)
DescribeResult(nobs=1000, minmax=(0.0067544318665307803, 0.030140918278170194)
               mean=0.015689195390684592, variance=1.7424639600882804e-05,
               skewness=0.620708847138992, kurtosis=0.17957977282253035)
```

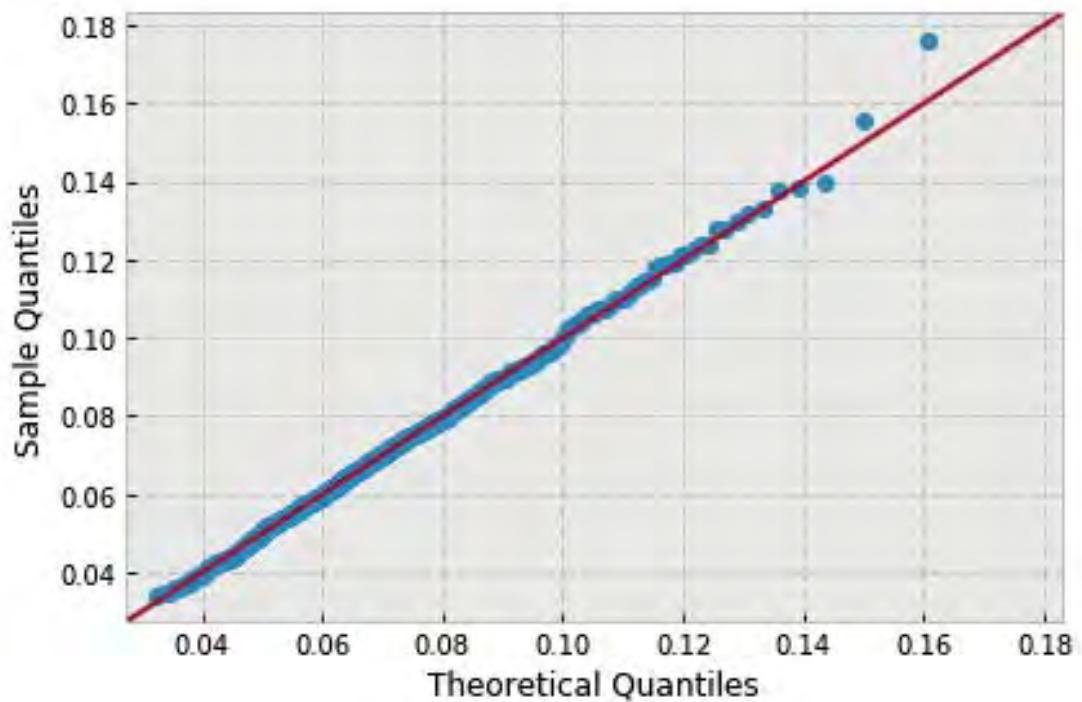
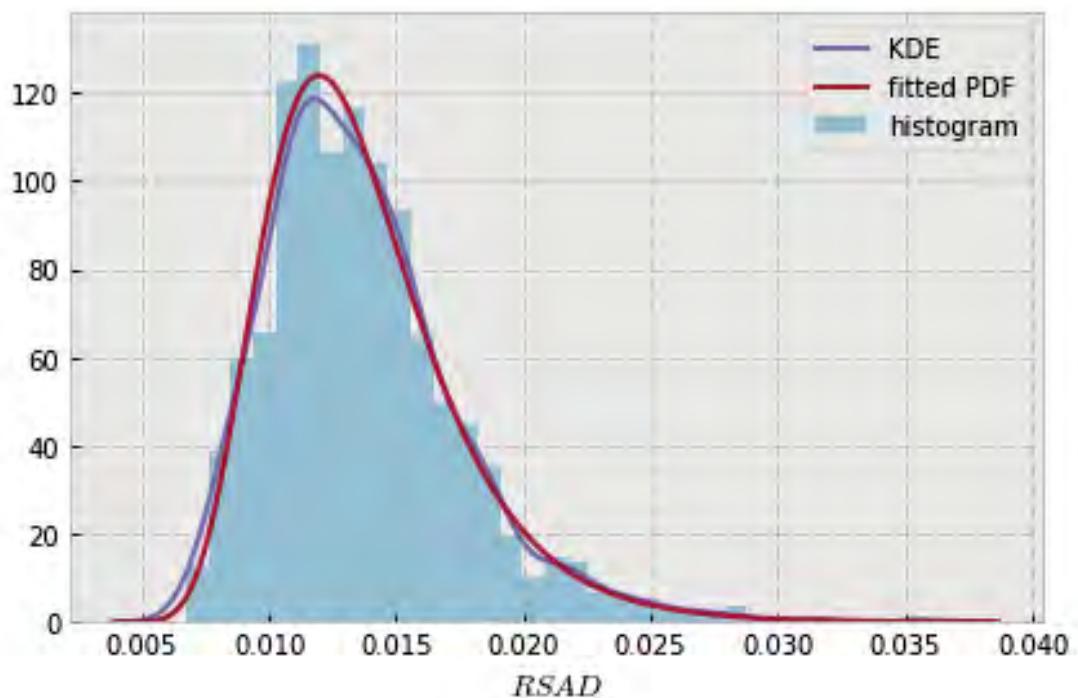




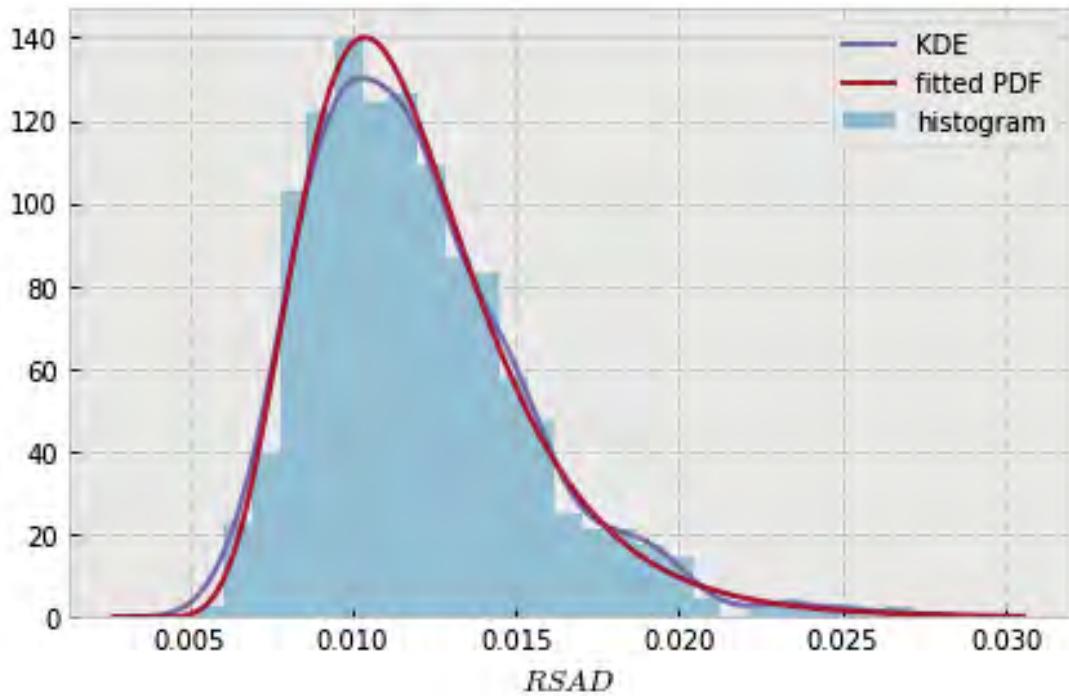
```

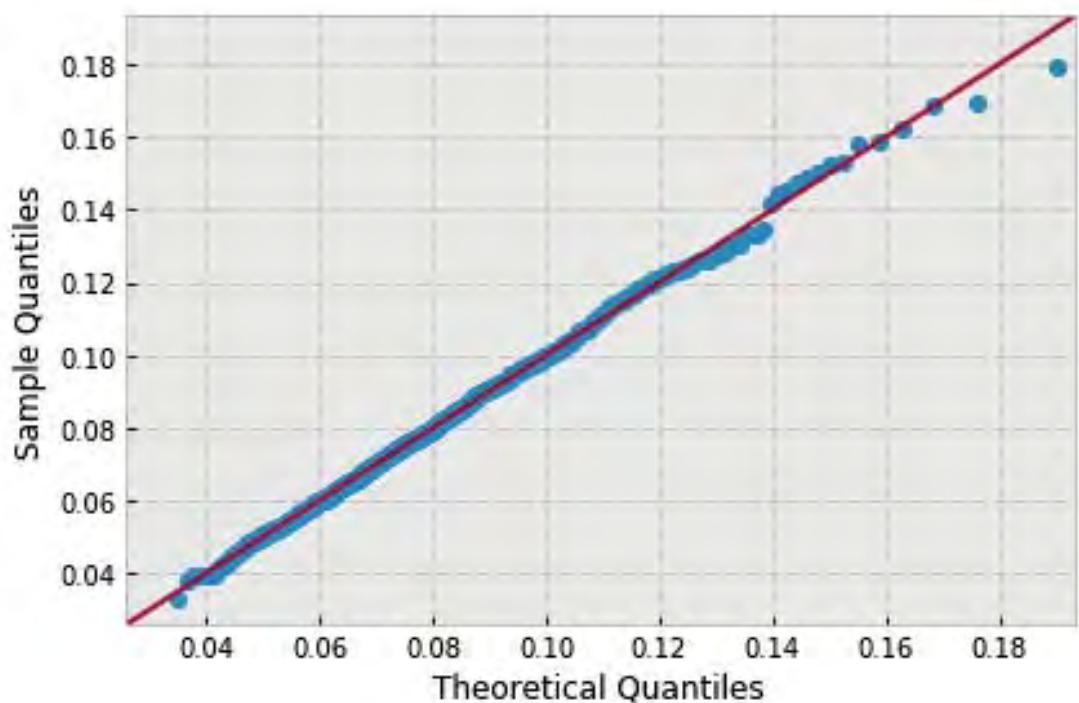
variable name: theta
variable value: 0.7853981633974483
distribution: inverse-gamma(shape=(15.702092302900443,), loc=-0.00033796722713517926,
                           scale=0.20549395915442448)
DescribeResult(nobs=1000, minmax=(0.0067235449391562204, 0.035835966286857299)
               mean=0.013638549179209914, variance=1.4215805421813491e-05,
               skewness=1.1370335006273948, kurtosis=2.406991333666908)

```

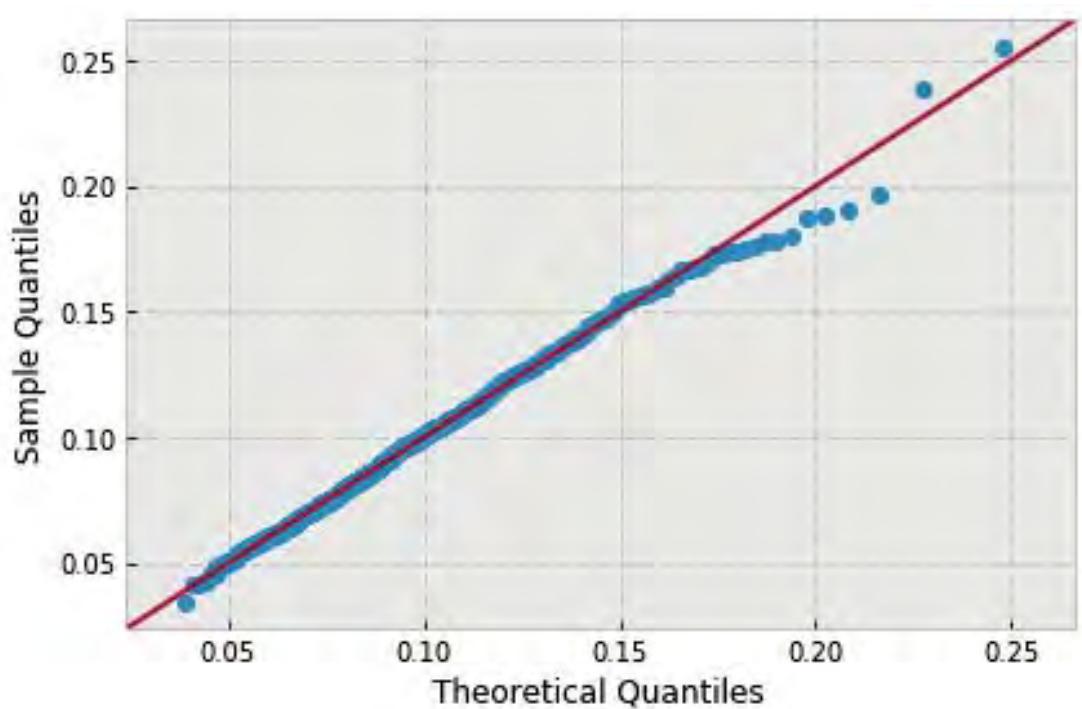
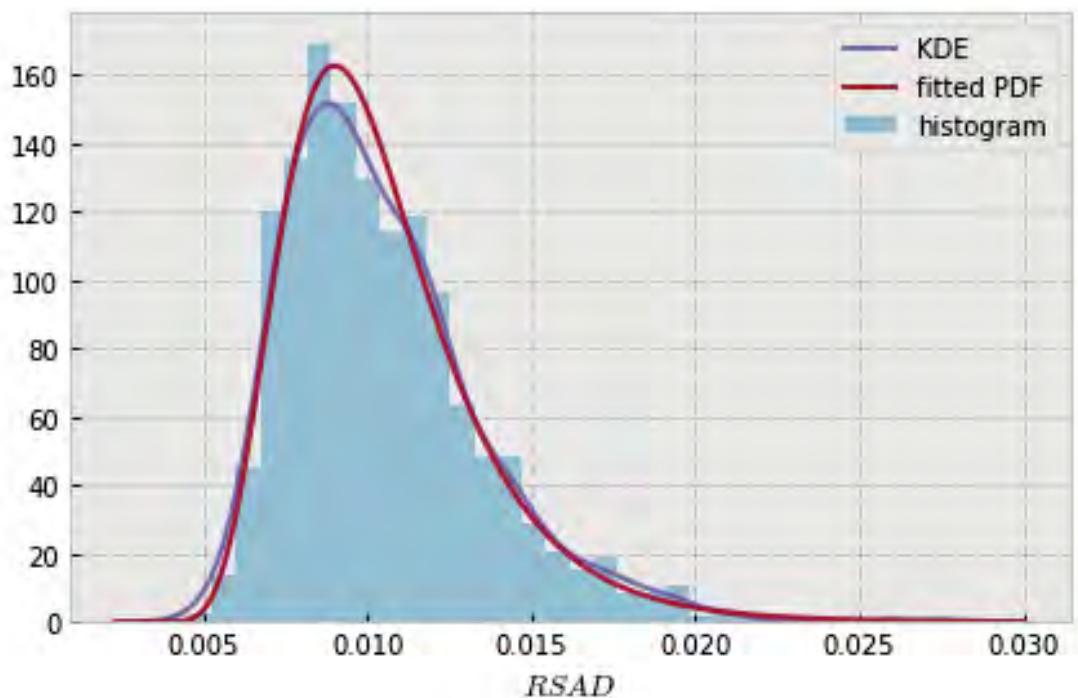


```
variable name: theta
variable value: 0.8726646259971648
distribution: inverse-gamma(shape=(14.108250414040032,), loc=9.541966273532998e-05,
    scale=0.15535897179556113)
DescribeResult(nobs=1000, minmax=(0.0052291155896732567, 0.028005439319069663)
    mean=0.011945011333992768, variance=1.1381045604624589e-05,
    skewness=1.0757415007010172, kurtosis=1.7182212421659173)
```

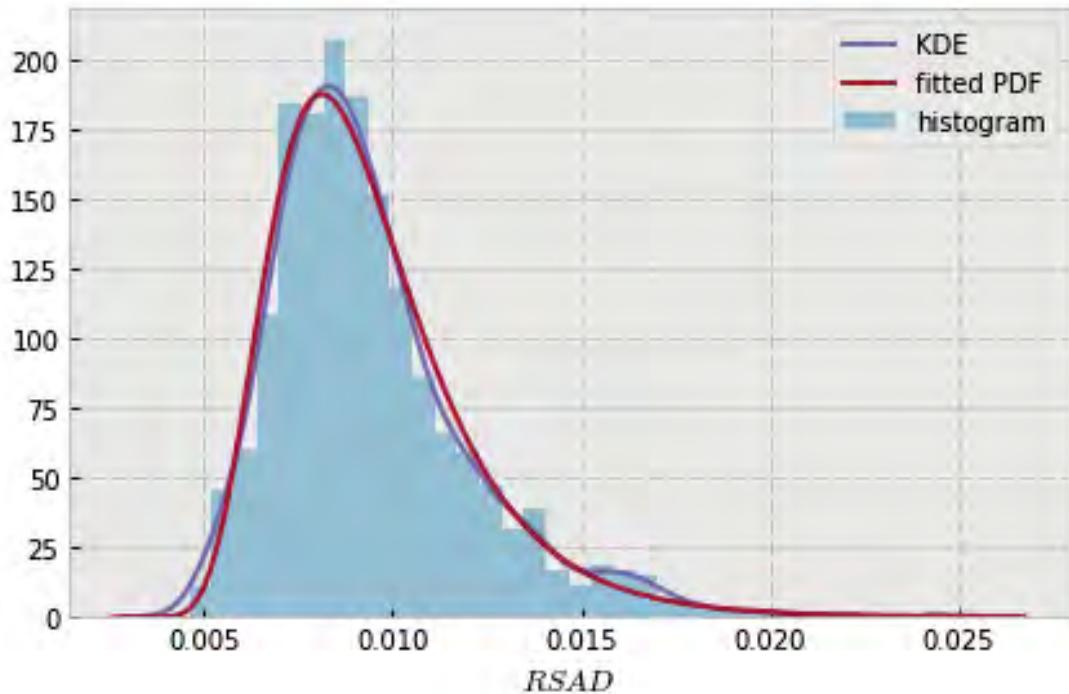


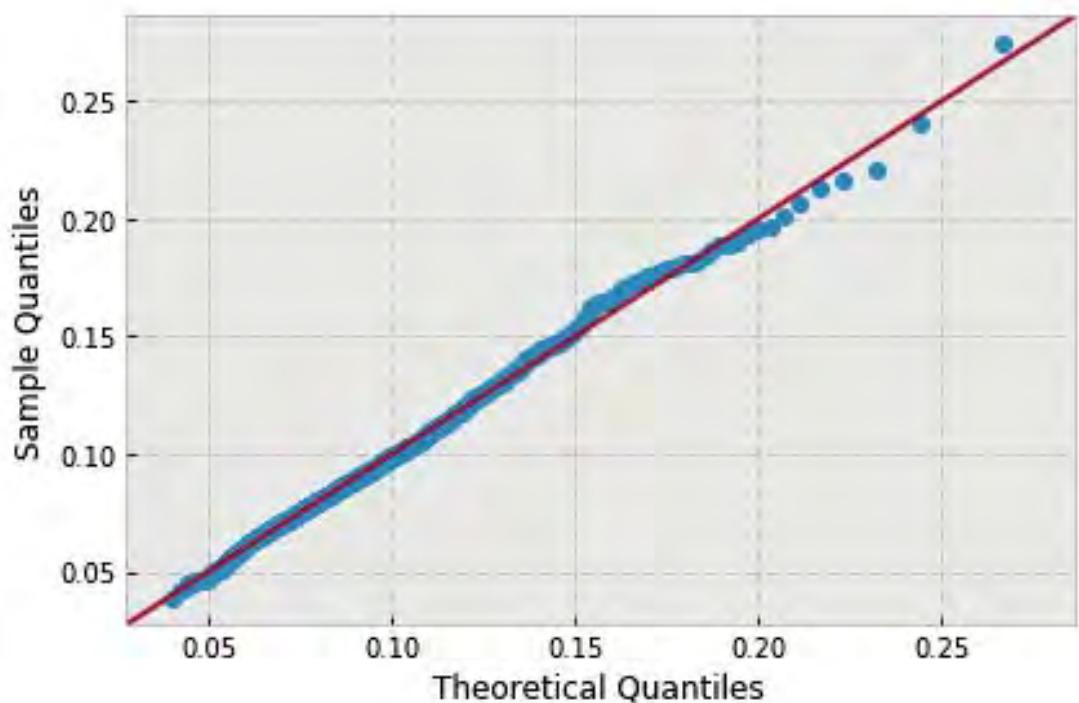


```
variable name: theta
variable value: 0.9599310885968813
distribution: inverse-gamma(shape=(11.993839008025148,), loc=0.000909372937979091,
                           scale=0.10537149339972254)
DescribeResult(nobs=1000, minmax=(0.0045630019286218458, 0.02778297963368791)
               mean=0.010490459642846157, variance=8.9225115708426385e-06,
               skewness=1.1661268590798095, kurtosis=2.2219373570467553)
```

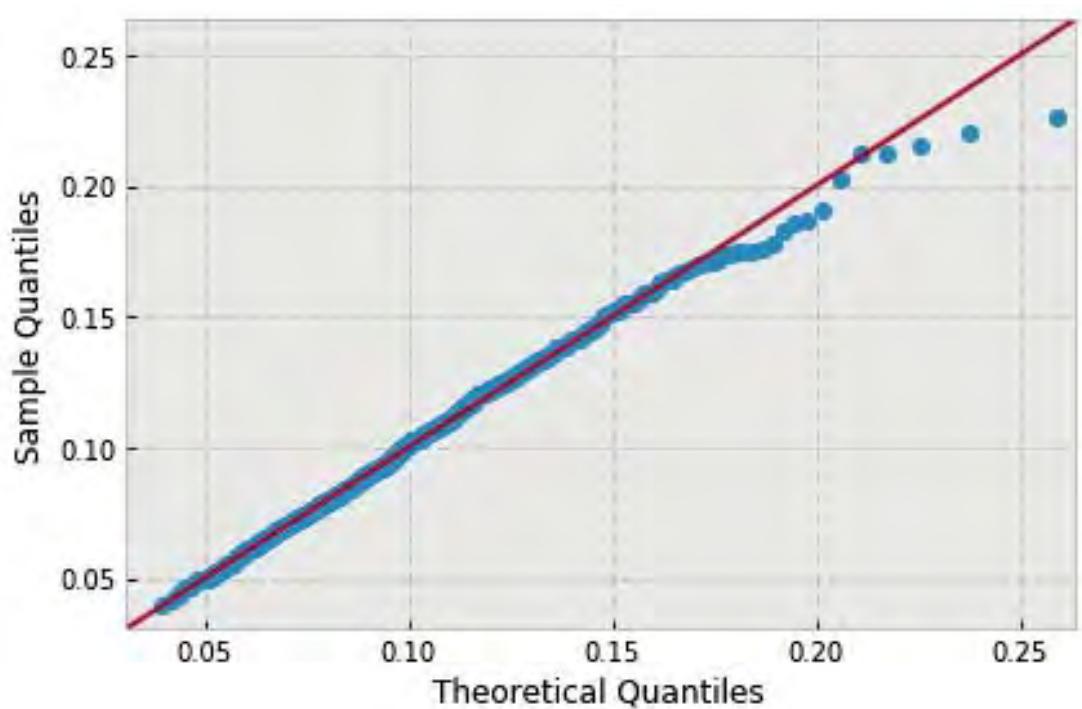
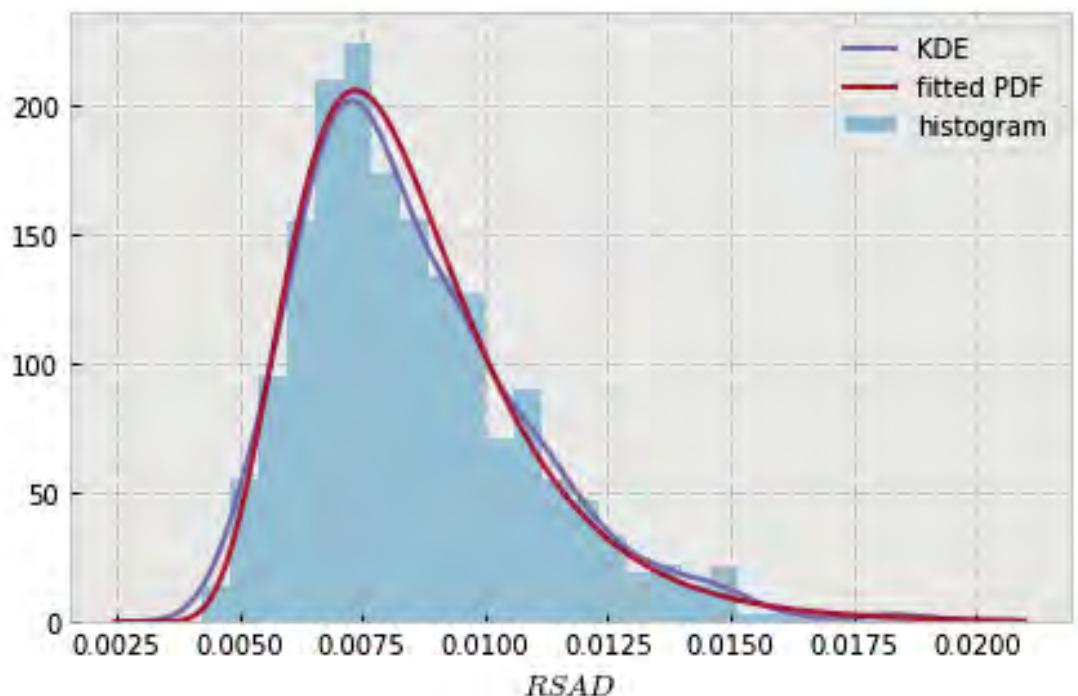


```
variable name: theta
variable value: 1.0471975511965979
distribution: inverse-gamma(shape=(11.476644756568321,), loc=0.0012767829397925003,
    scale=0.08544079808434434)
DescribeResult(nobs=1000, minmax=(0.0046214173287148757, 0.024709193955350987)
    mean=0.0094325802310628142, variance=6.9940873717700856e-06,
    skewness=1.3055761135355906, kurtosis=2.544325511494052)
```

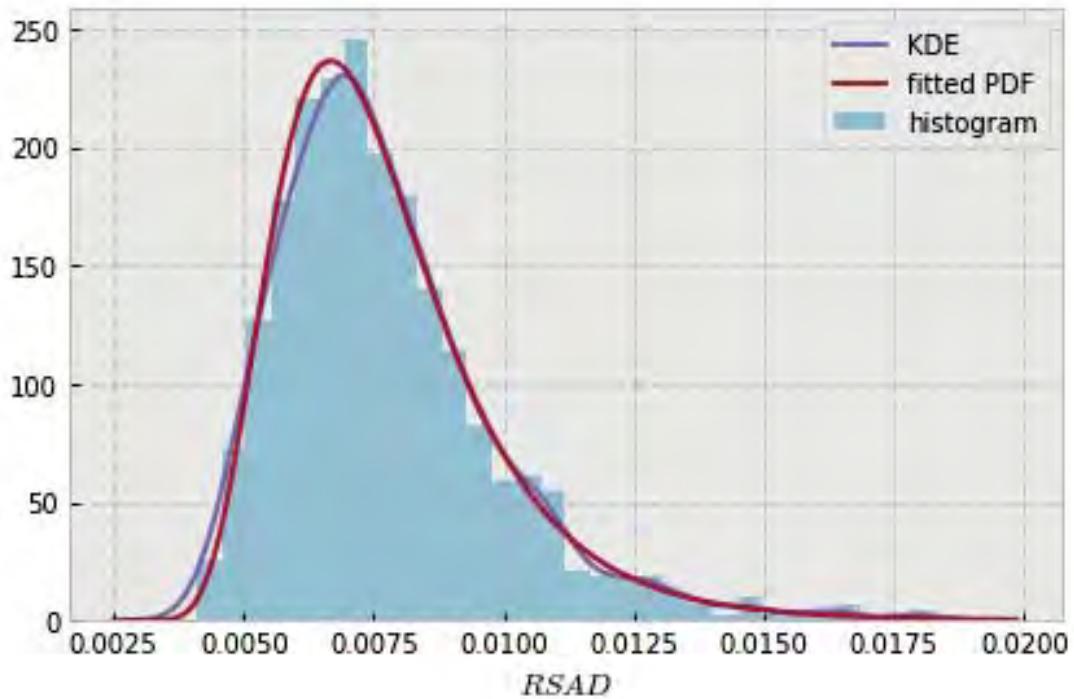


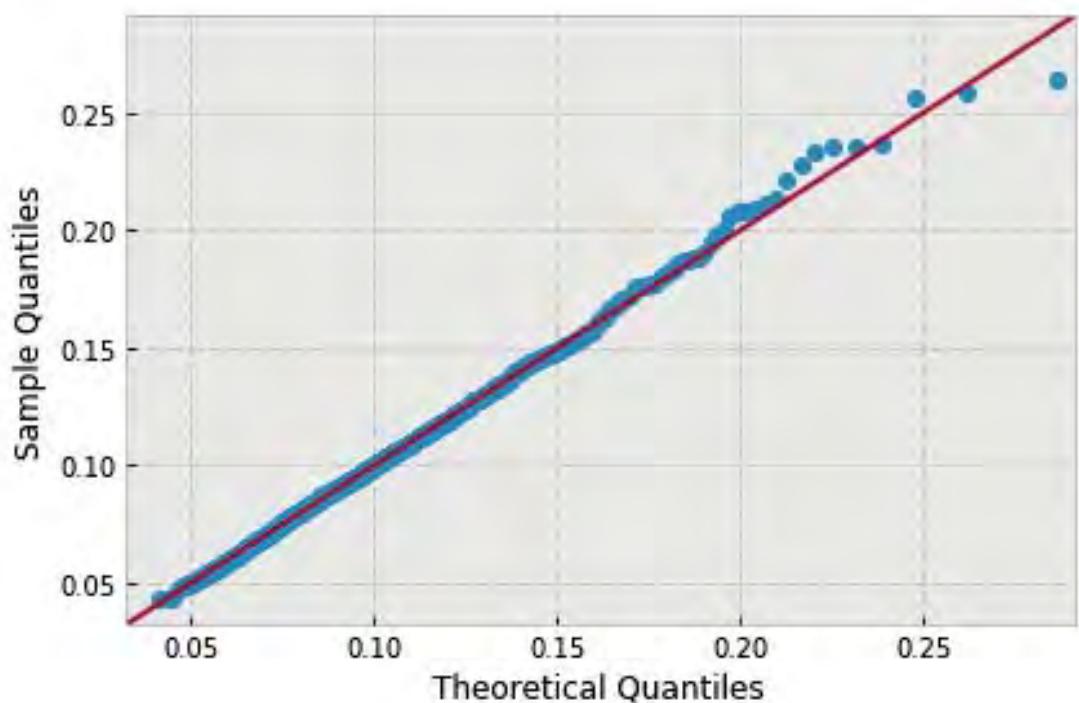


```
variable name: theta
variable value: 1.1344640137963142
distribution: inverse-gamma(shape=(11.679226954770002,), loc=0.00104894408118932,
                           scale=0.08012525536188847)
DescribeResult(nobs=1000, minmax=(0.0042484448297063363, 0.019196272872643638)
               mean=0.0085475164028002669, variance=5.5572611408637265e-06,
               skewness=1.0748463272261208, kurtosis=1.4755378528936562)
```

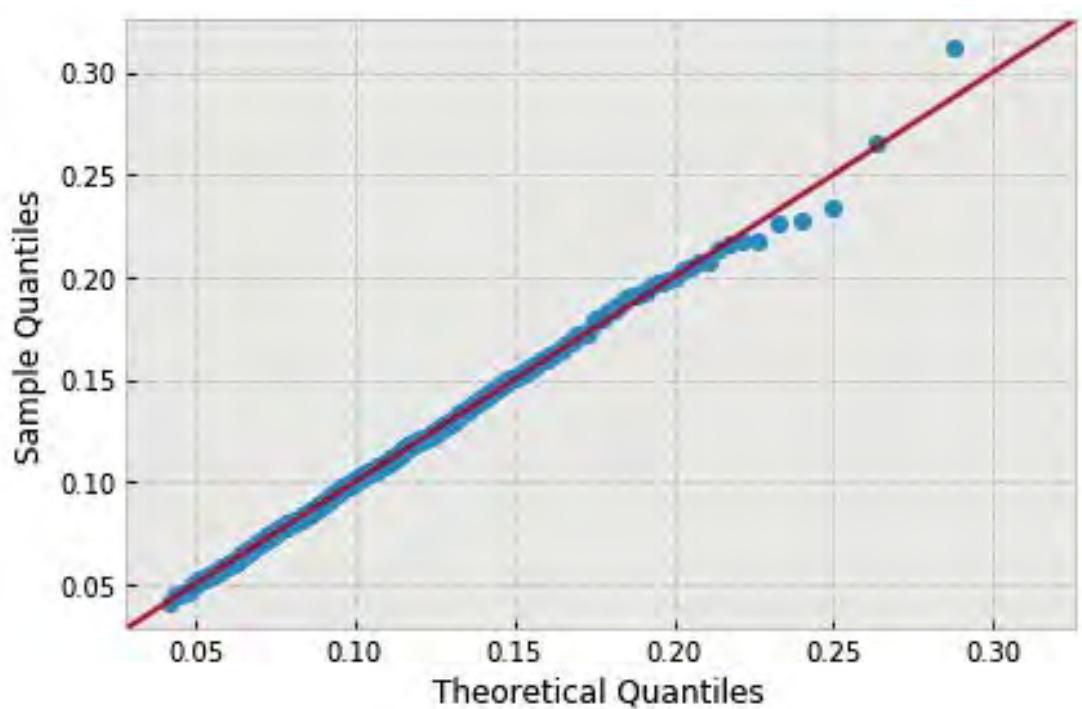
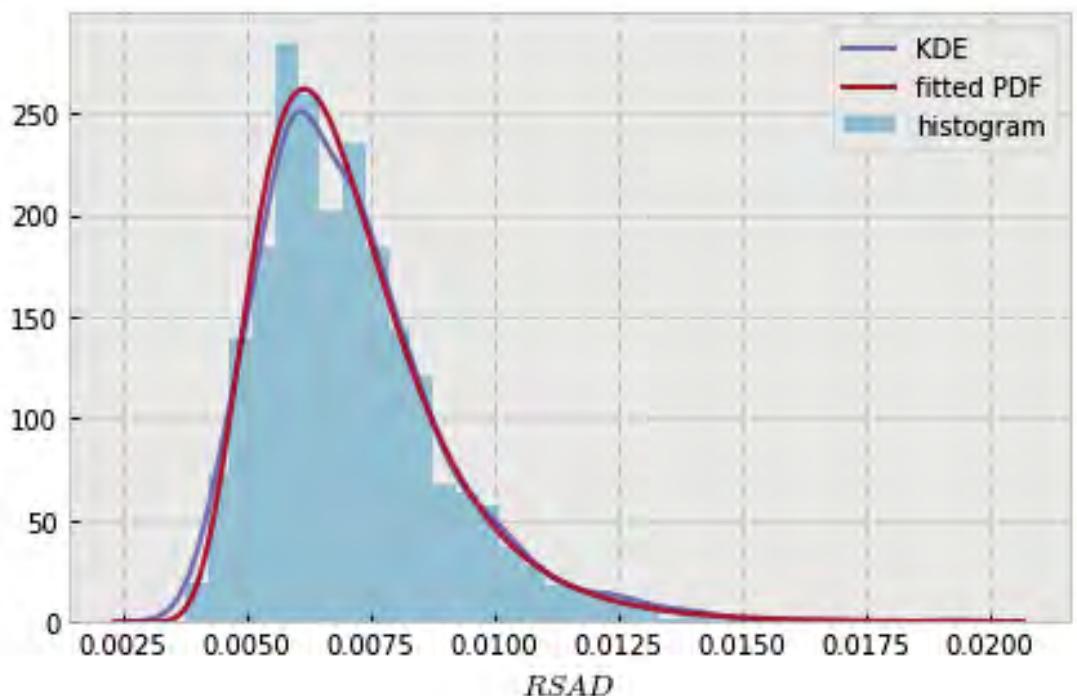


```
variable name: theta
variable value: 1.2217304763960306
distribution: inverse-gamma(shape=(11.001331782966279,), loc=0.0013436146047403883,
                           scale=0.06388514621553776)
DescribeResult(nobs=1000, minmax=(0.0040966503224696728, 0.018251851935865756)
               mean=0.0077302981031003261, variance=4.4773064689470104e-06,
               skewness=1.3467934758519475, kurtosis=2.853714791001252)
```

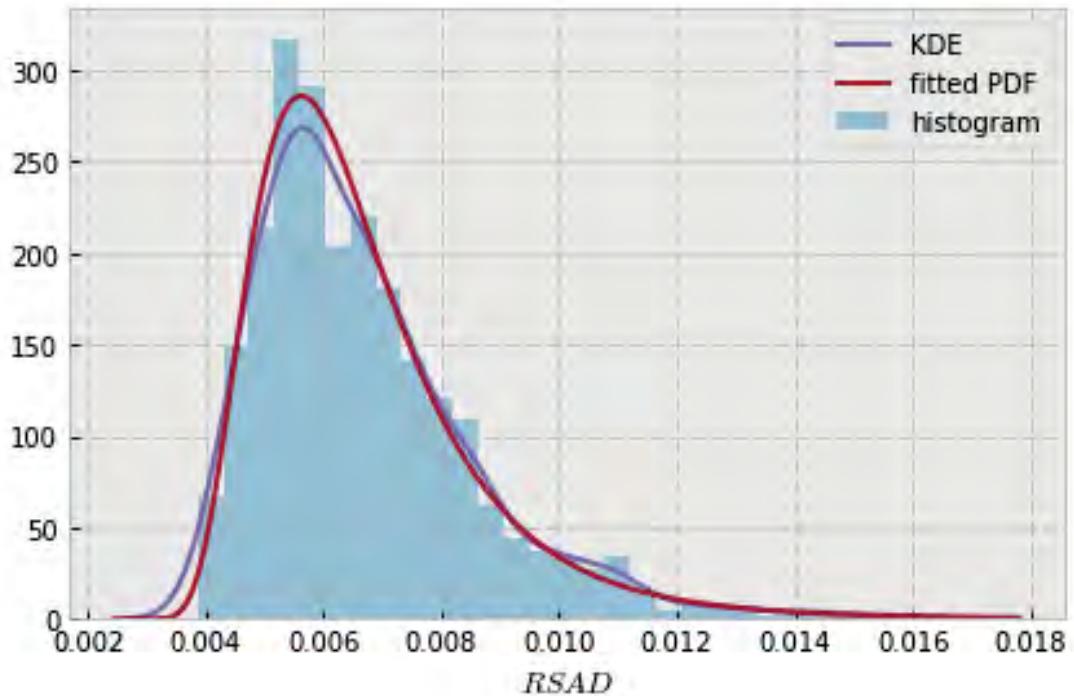


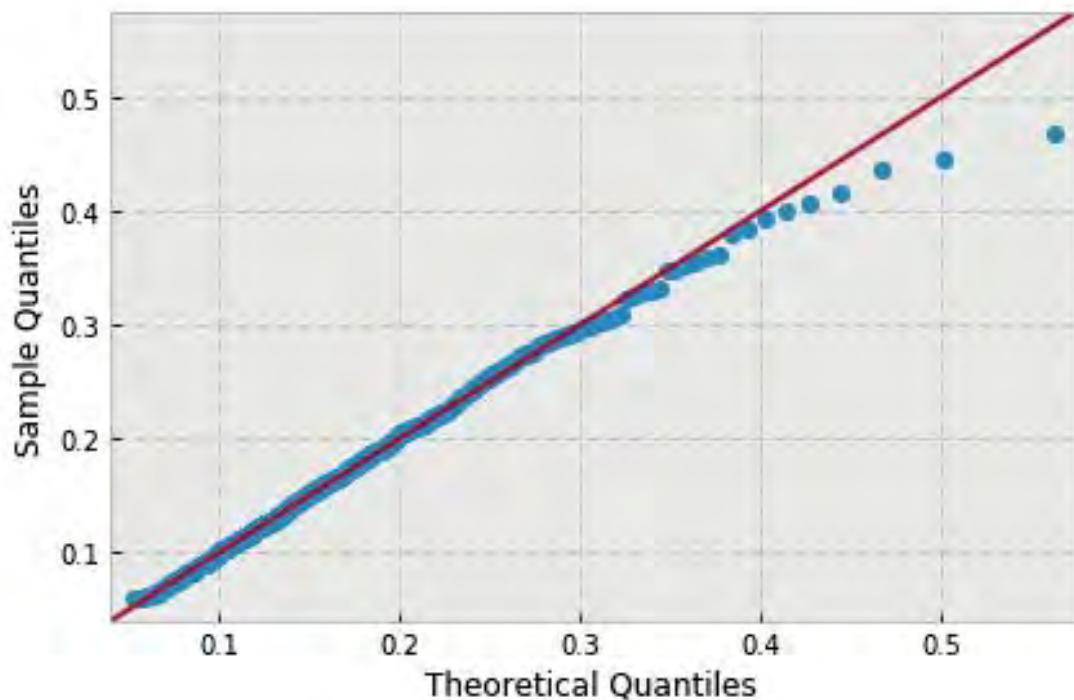


```
variable name: theta
variable value: 1.3089969389957472
distribution: inverse-gamma(shape=(10.968408980109459,), loc=0.0013537912488130962,
                           scale=0.05741478148194956)
DescribeResult(nobs=1000, minmax=(0.003739256508816749, 0.019259705142374316)
               mean=0.0071120620936239781, variance=3.6344209631949364e-06,
               skewness=1.343925705503349, kurtosis=3.146519474829028)
```

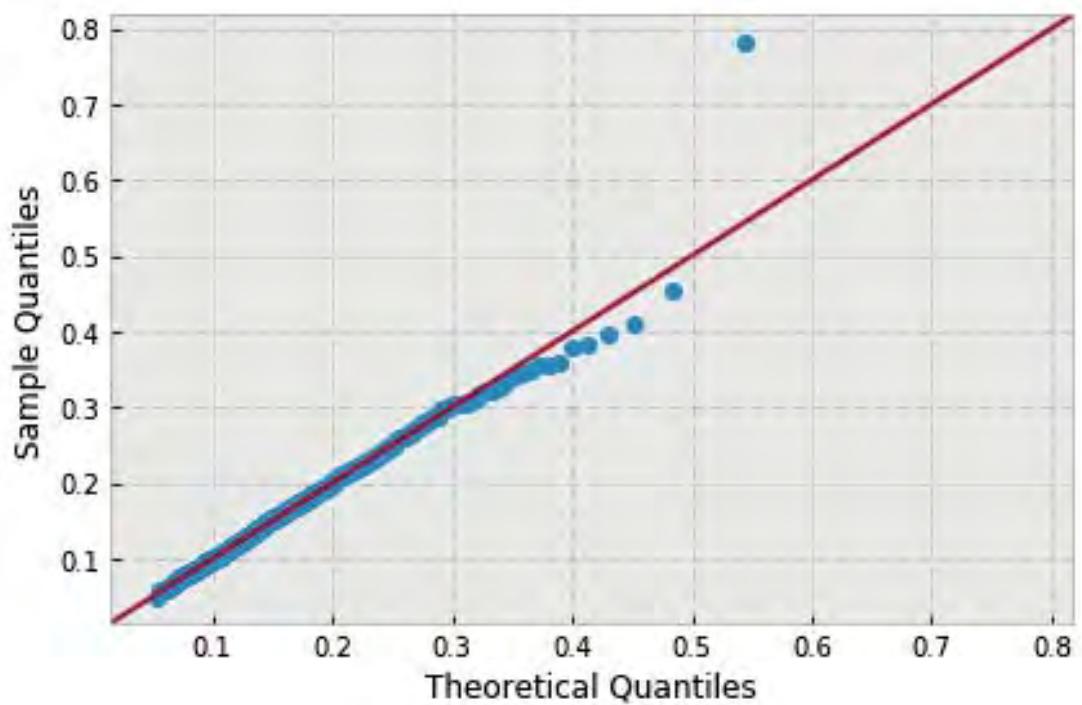
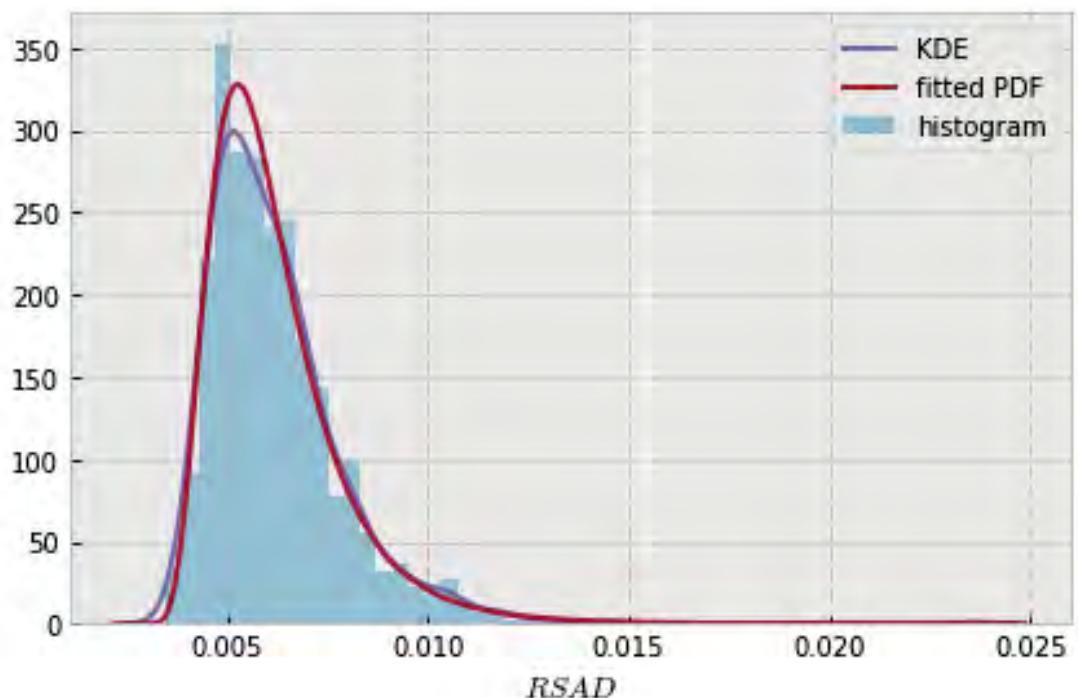


```
variable name: theta
variable value: 1.3962634015954638
distribution: inverse-gamma(shape=(7.5745299527271737,), loc=0.0020418687863107687,
                           scale=0.03065926236019024)
DescribeResult(nobs=1000, minmax=(0.0038629842414087123, 0.01638123120295383)
               mean=0.0066976825765582343, variance=3.5846544063849648e-06,
               skewness=1.4212442610945433, kurtosis=2.844975207721798)
```





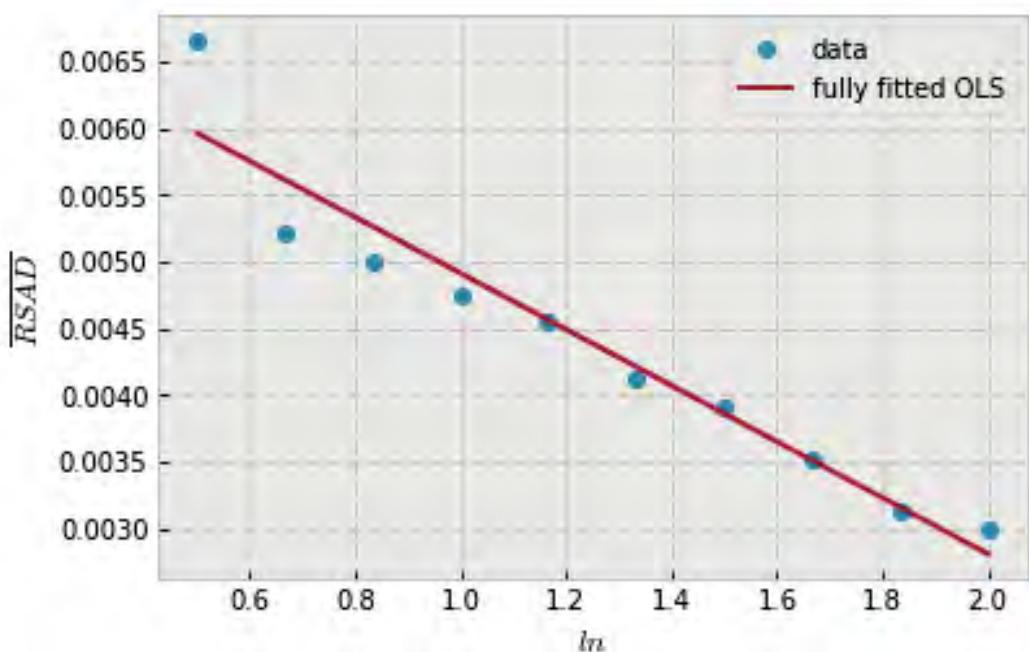
```
variable name: theta
variable value: 1.4835298641951802
distribution: inverse-gamma(shape=(7.721708819739785,), loc=0.00211685809148586,
                           scale=0.027488025118453302)
DescribeResult(nobs=1000, minmax=(0.0034902810763933661, 0.023586002155740842)
               mean=0.0062041183967371282, variance=2.8872301075910453e-06,
               skewness=2.1679236322591873, kurtosis=12.238967181359723)
```



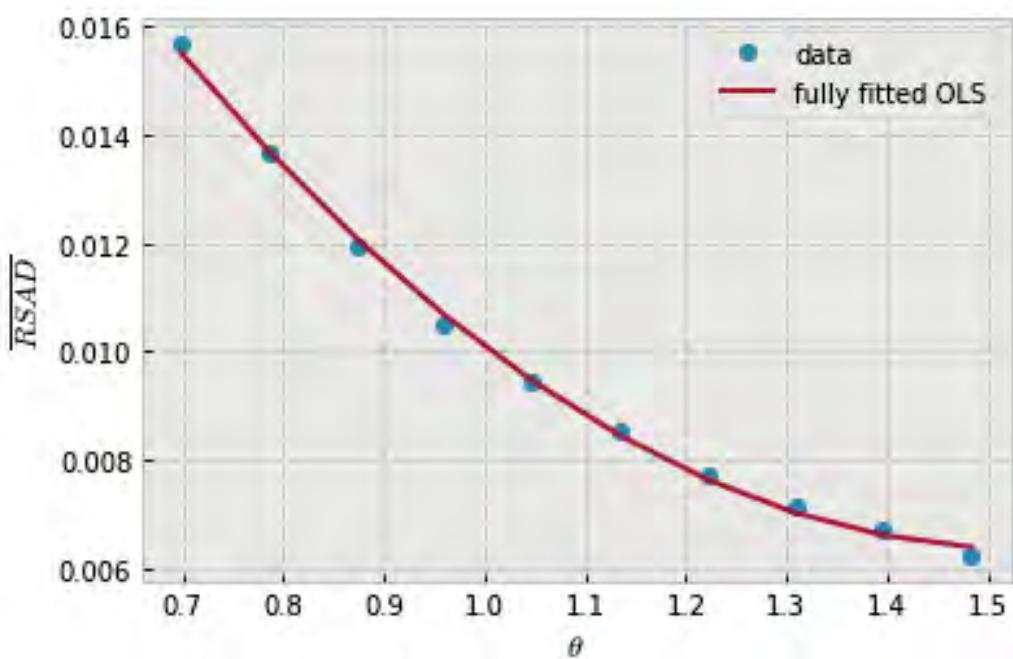
12.2 Parameter regression

```
In [30]: var_name = 'total_root_surface_area_per_convex_hull_volume'  
        mean_reg('ln', var_name, 'poly1')  
        mean_reg('theta', var_name, 'poly2')  
        mean_reg('nB', var_name, 'poly1')  
        mean_reg('r', var_name, 'poly2')  
        mean_reg('sigma', var_name, 'poly2')  
        mean_reg('N', var_name, 'poly1')
```

variable name: ln
model: poly1
r-squared: 0.8490694776015657
explained variance: 0.8496128777323296



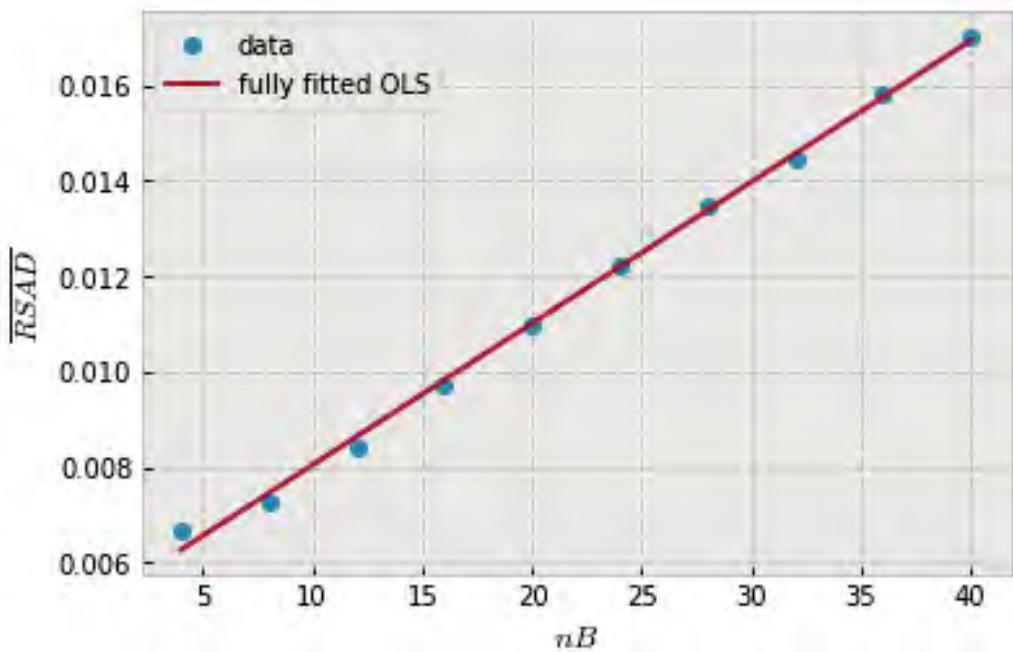
```
variable name: theta  
model: poly2  
r-squared: 0.9933615837450132  
explained variance: 0.9933616149133653
```



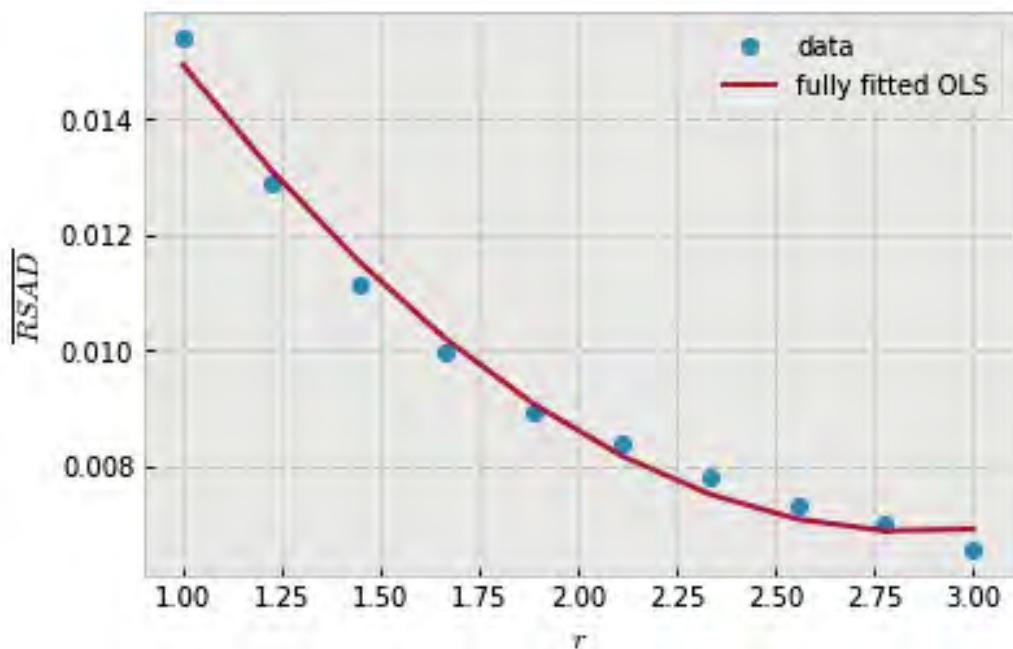
```

variable name: nB
model: poly1
r-squared: 0.9947747278427731
explained variance: 0.9947927122306905

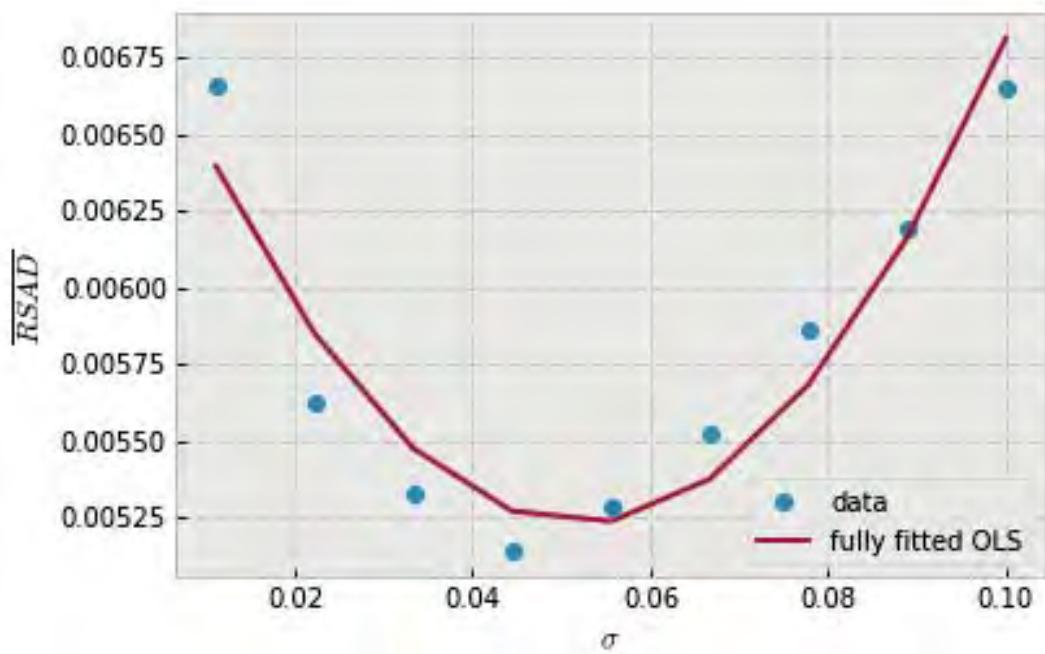
```



```
variable name: r
model: poly2
r-squared: 0.9550331540774908
explained variance: 0.9550578810043979
```



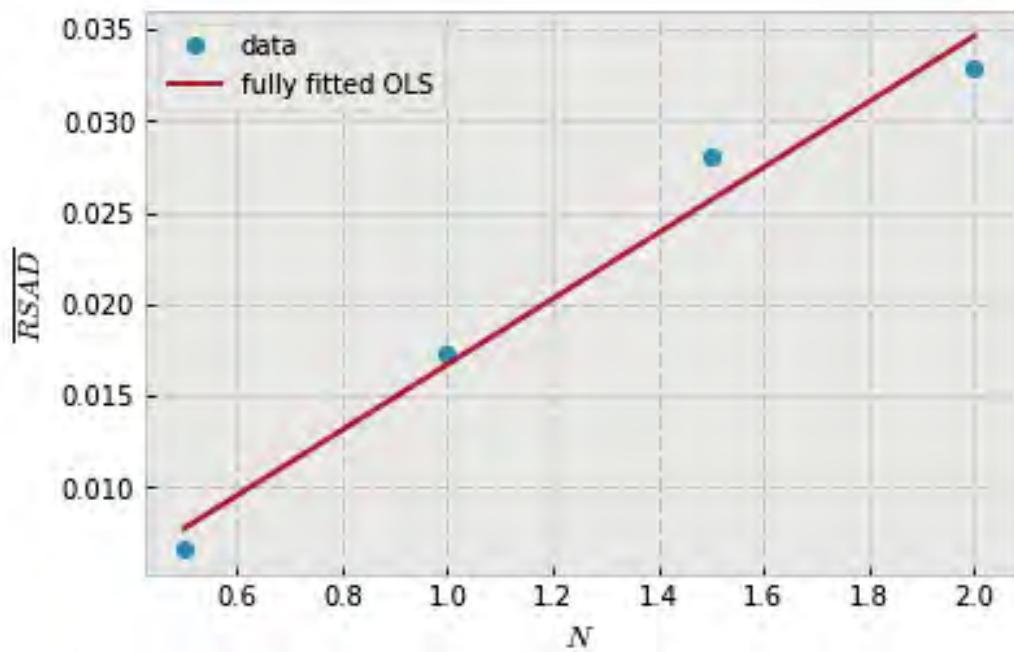
```
variable name: sigma
model: poly2
r-squared: 0.5943521464195418
explained variance: 0.5953124825961598
```



```

variable name: N
model: poly1
r-squared: 0.8529041364338753
explained variance: 0.8714799455587965

```



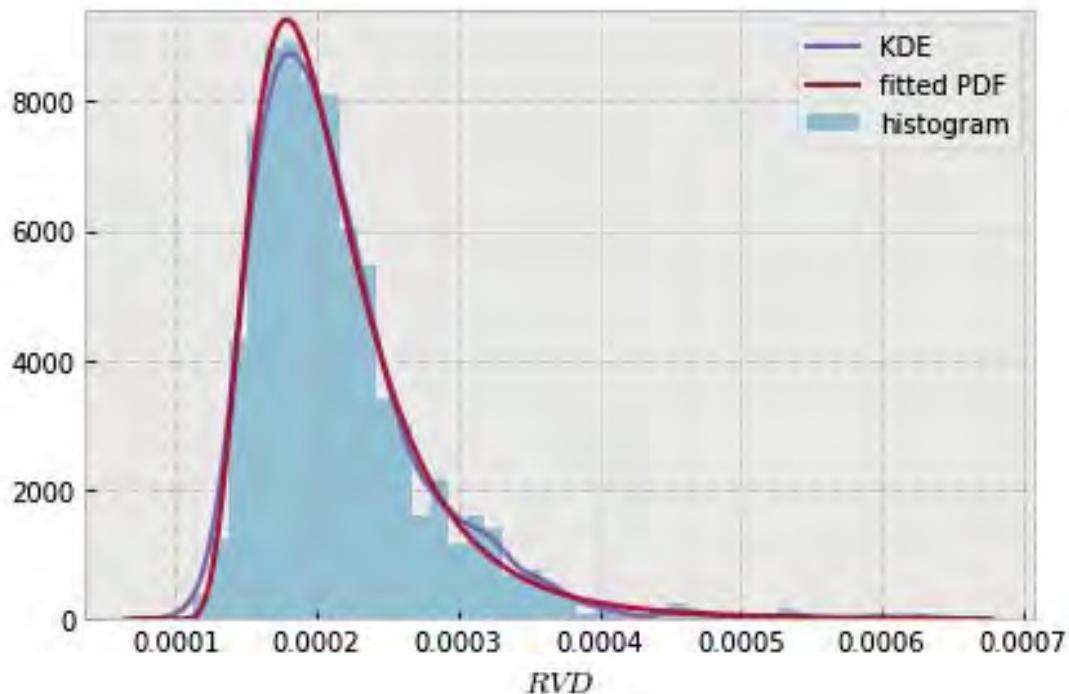
13 Root volume relative to convex hull

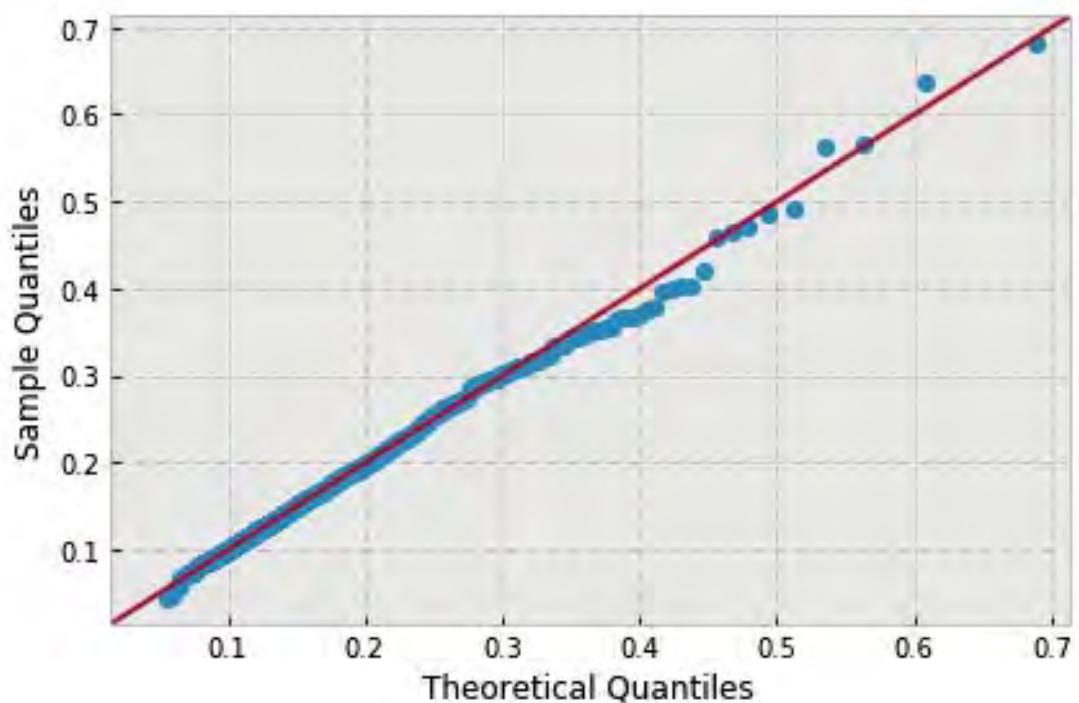
13.1 Probability distributions

```
In [31]: plot_dist('total_root_volume_per_convex_hull_volume', 'inverse-gamma')

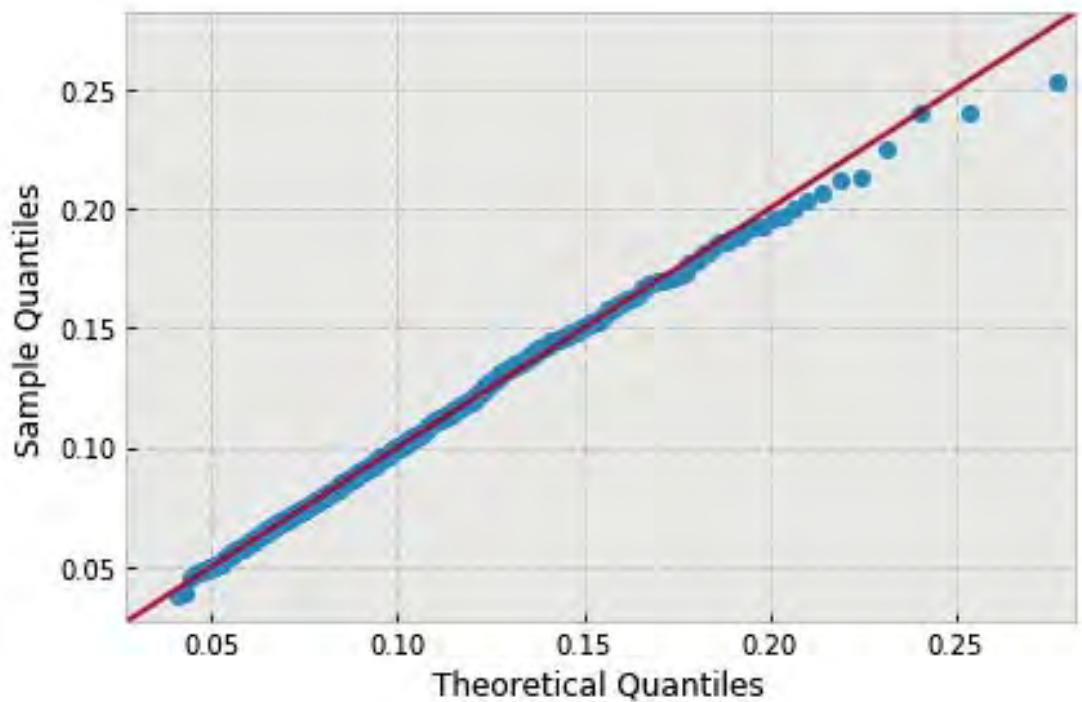
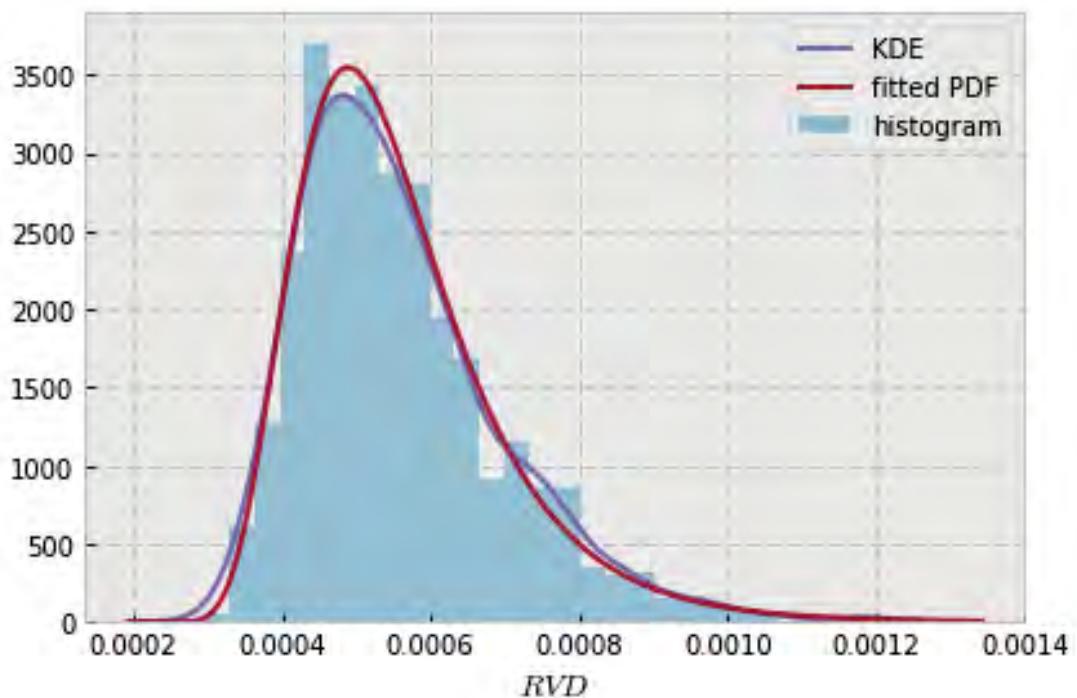
/home/lpetrich/.local/share/anaconda3/envs/Master/lib/python3.6/site-packages/scipy/stats/_distn_infra.py:1330: UserWarning: The distribution inverse-gamma is not supported by the R interface.
Lhat = muhat - Shat*mu

variable name: N
variable value: 0.5
distribution: inverse-gamma(shape=(6.8405266155592823,), loc=7.455192049209567e-05,
    scale=0.0008163057133623352)
DescribeResult(nobs=1000, minmax=(0.0001116955240372318, 0.00063046671666168452)
    mean=0.00021369454921087267, variance=3.7047017140764364e-09,
    skewness=1.8376426118988216, kurtosis=5.985918705967476)
```

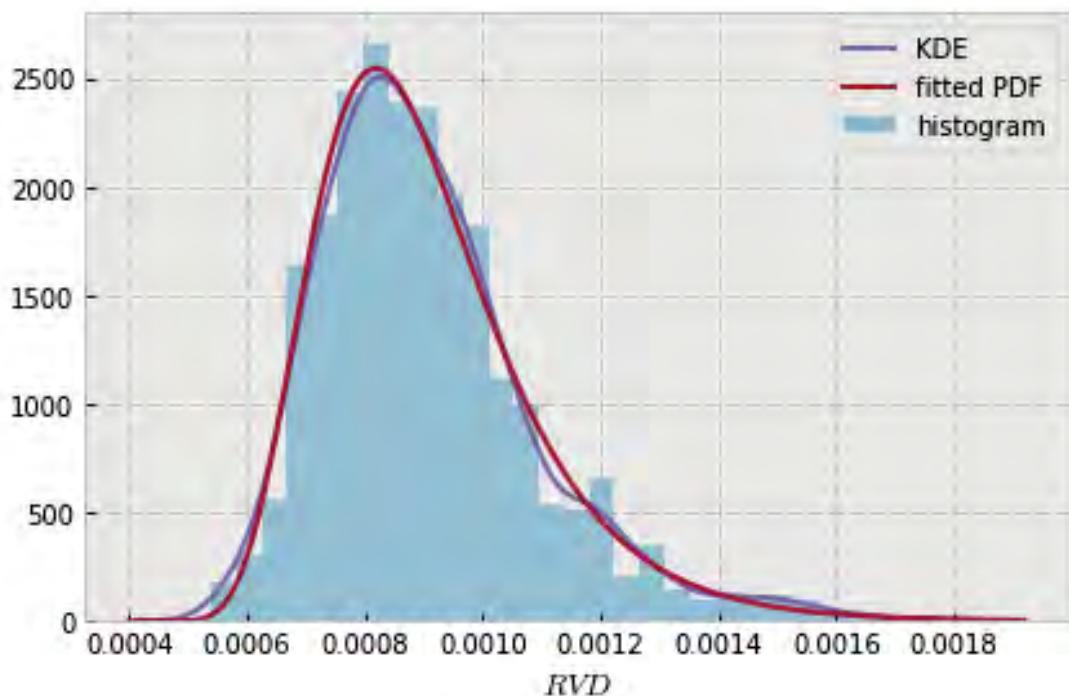


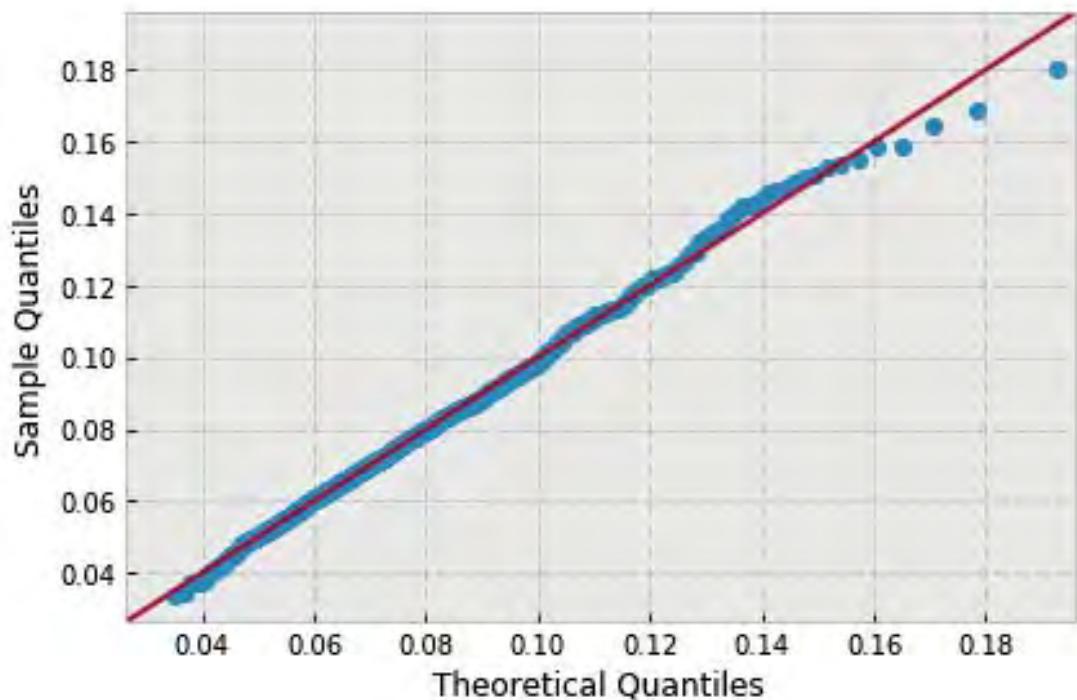


```
variable name: N
variable value: 1.0
distribution: inverse-gamma(shape=(11.219754949649939,), loc=0.00013046974558498608,
scale=0.004378112426616782)
DescribeResult(nobs=1000, minmax=(0.00029538674594706179, 0.0012384623509316248)
mean=0.00055866914912388959, variance=1.9173992709138562e-08,
skewness=1.1539672223580788, kurtosis=1.8291495284018708)
```

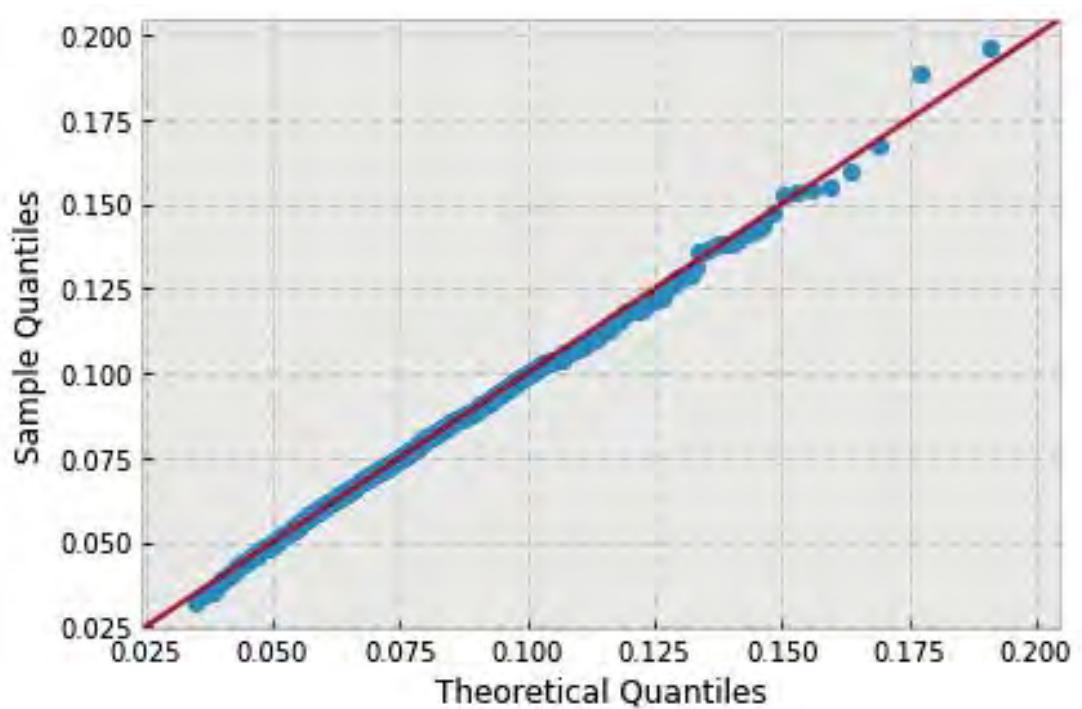
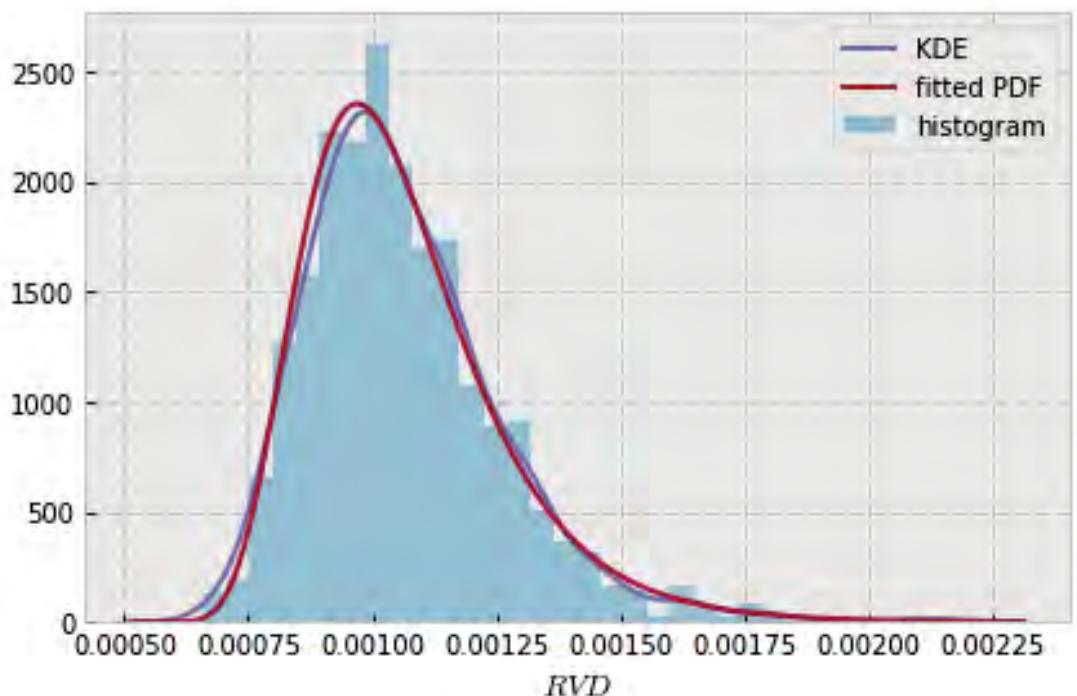


```
variable name: N
variable value: 1.5
distribution: inverse-gamma(shape=(13.99532300472983,), loc=0.00025559577060767257,
    scale=0.008447883525308593)
DescribeResult(nobs=1000, minmax=(0.00053970073671131841, 0.0017798228332375342)
    mean=0.00090507727035041708, variance=3.430686090483061e-08,
    skewness=1.1221243156050151, kurtosis=1.8389564455754988)
```

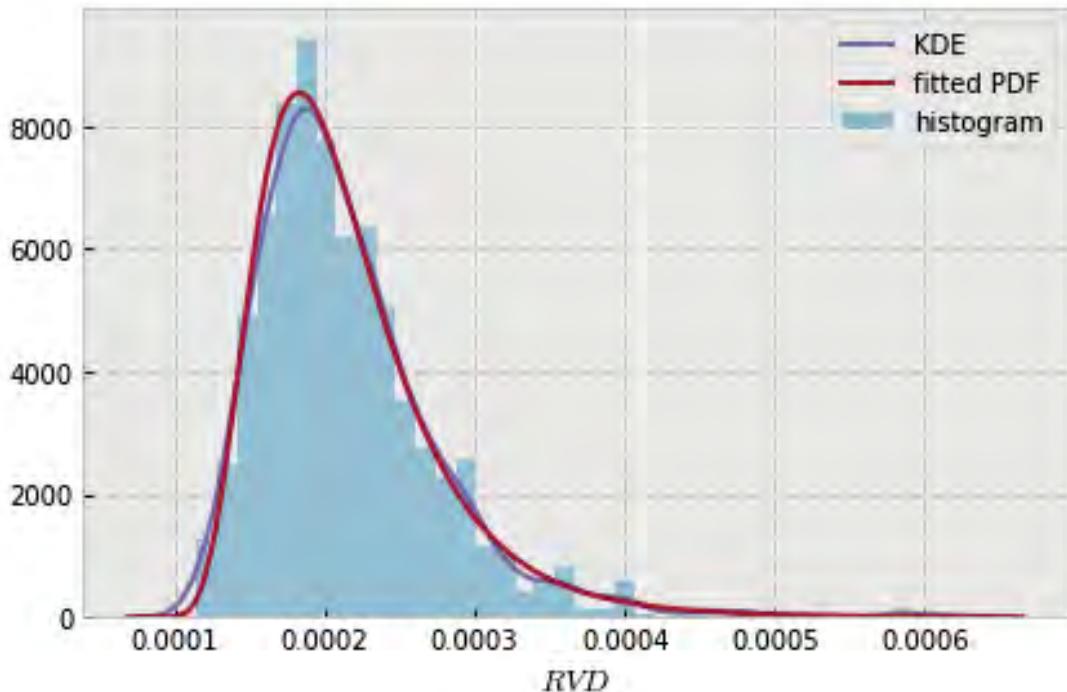


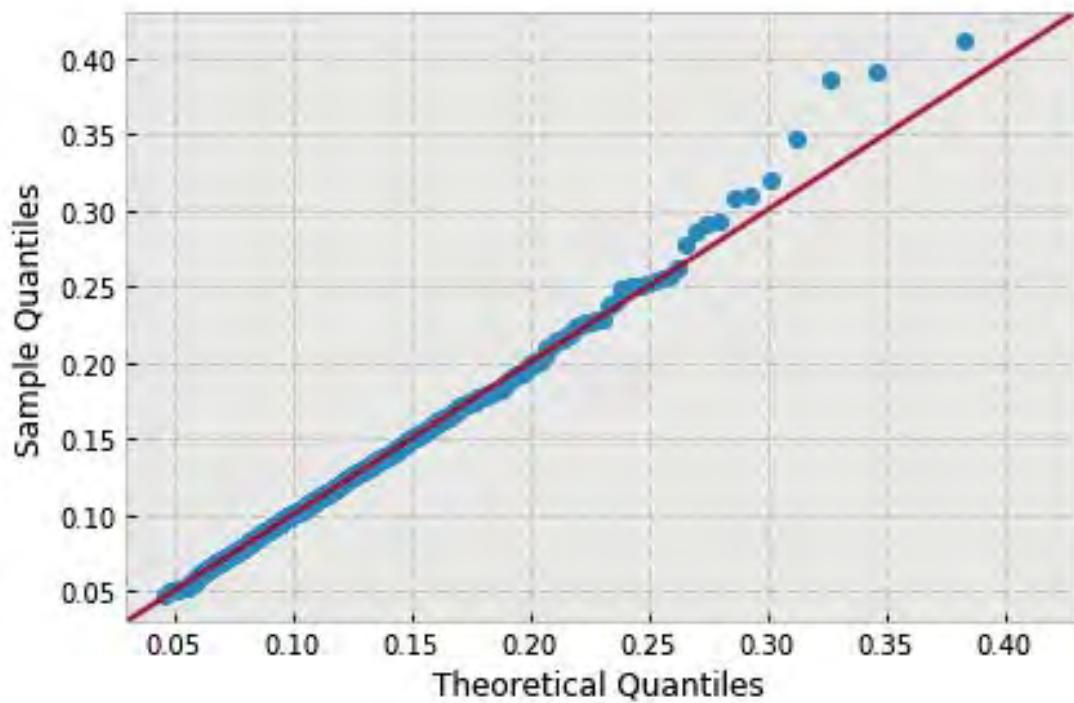


```
variable name: N
variable value: 2.0
distribution: inverse-gamma(shape=(14.071108899336183,), loc=0.00035846659985920684,
scale=0.009210923713918913)
DescribeResult(nobs=1000, minmax=(0.00065808838458192385, 0.0021654395670876346)
mean=0.0010615877386503916, variance=3.8648589781930844e-08,
skewness=1.1363791944028687, kurtosis=2.5363826321764176)
```



```
variable name: ln
variable value: 0.5
distribution: inverse-gamma(shape=(9.314884297759999,), loc=4.842049247220753e-05,
    scale=0.0013857296616906806)
DescribeResult(nobs=1000, minmax=(0.00011470526568819522, 0.00061836704723099481)
    mean=0.00021513592399024795, variance=3.8798464918276397e-09,
    skewness=1.7876493022142008, kurtosis=5.889301112758295)
```

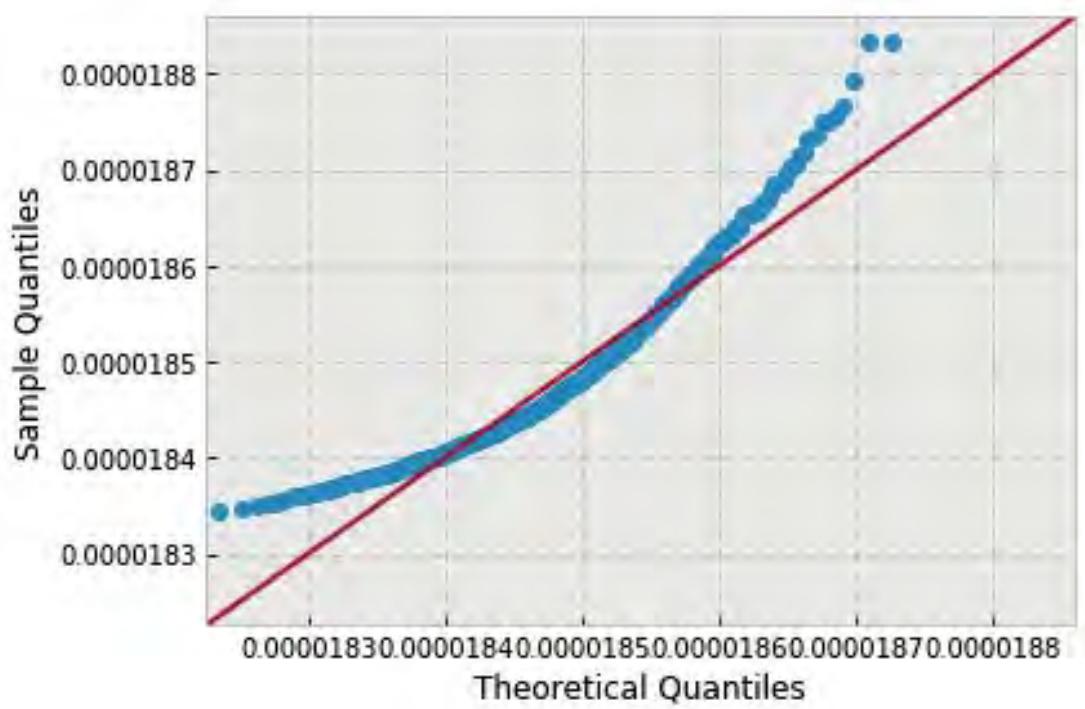
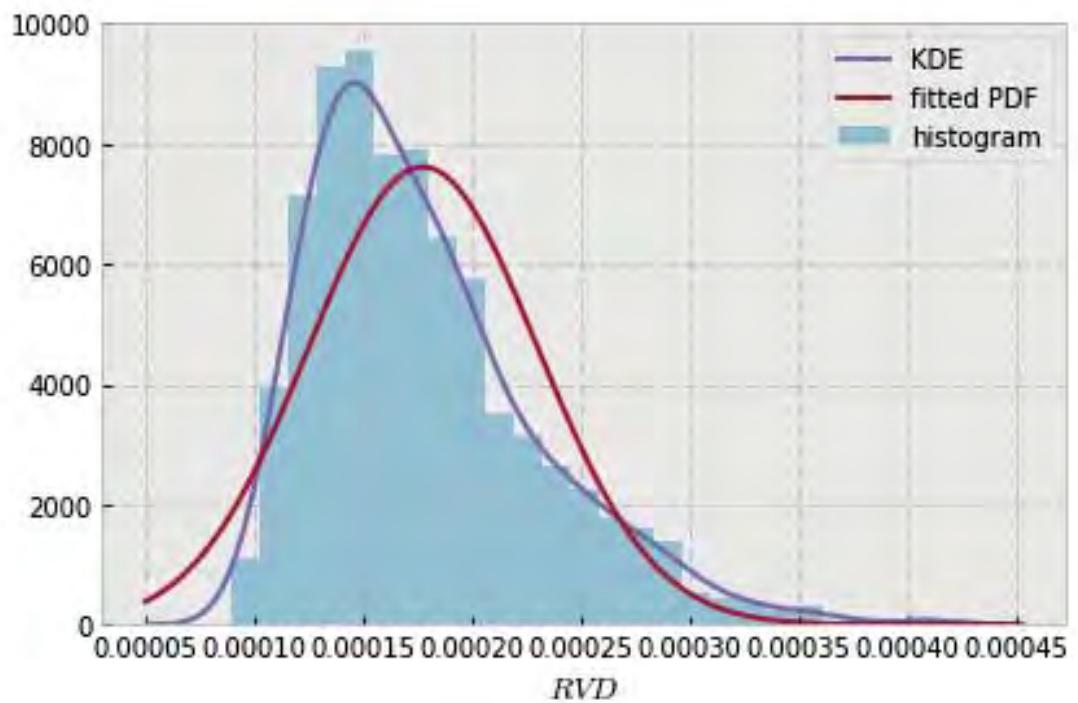




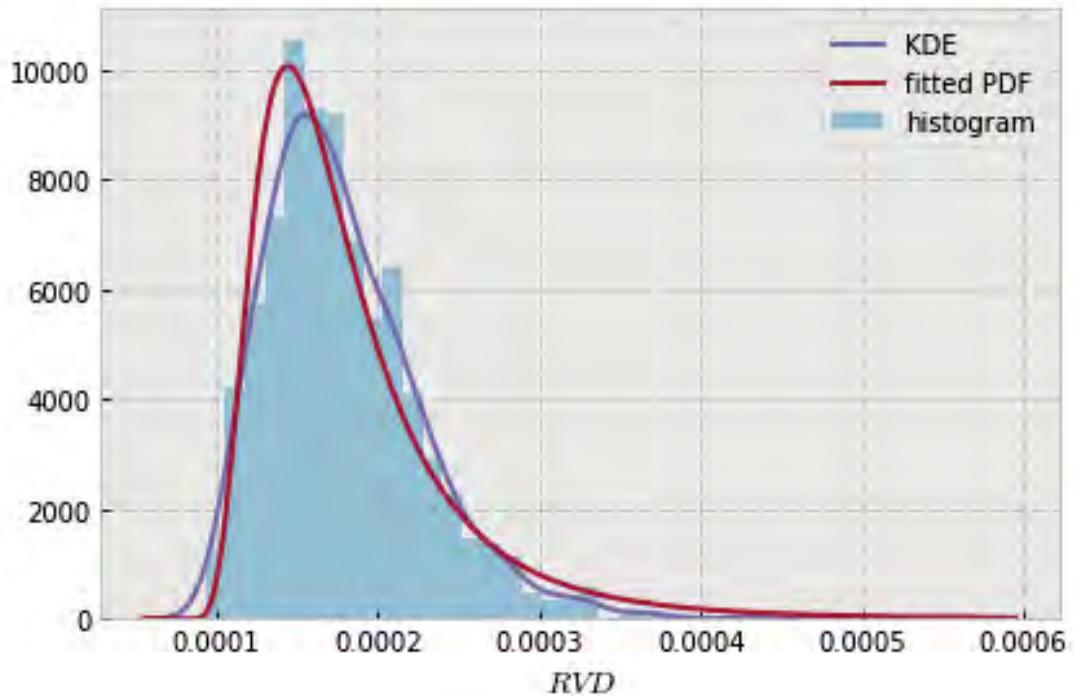
```

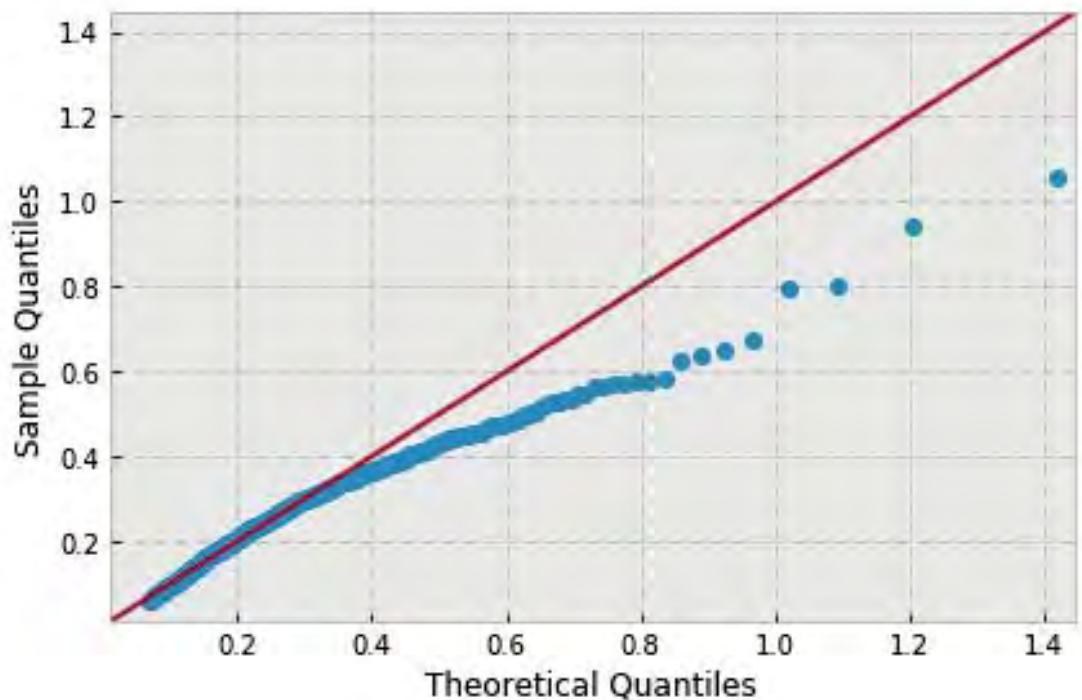
variable name: ln
variable value: 0.6666666666666666
distribution: inverse-gamma(shape=(54116.912553225819,), loc=-0.012007664375445894,
                           scale=659.4442908005371)
DescribeResult(nobs=1000, minmax=(9.0187736056421185e-05, 0.00041267087874957036)
               mean=0.00017810951770969496, variance=2.7637659029972354e-09,
               skewness=1.0889878445408354, kurtosis=1.2637265380535423)

```

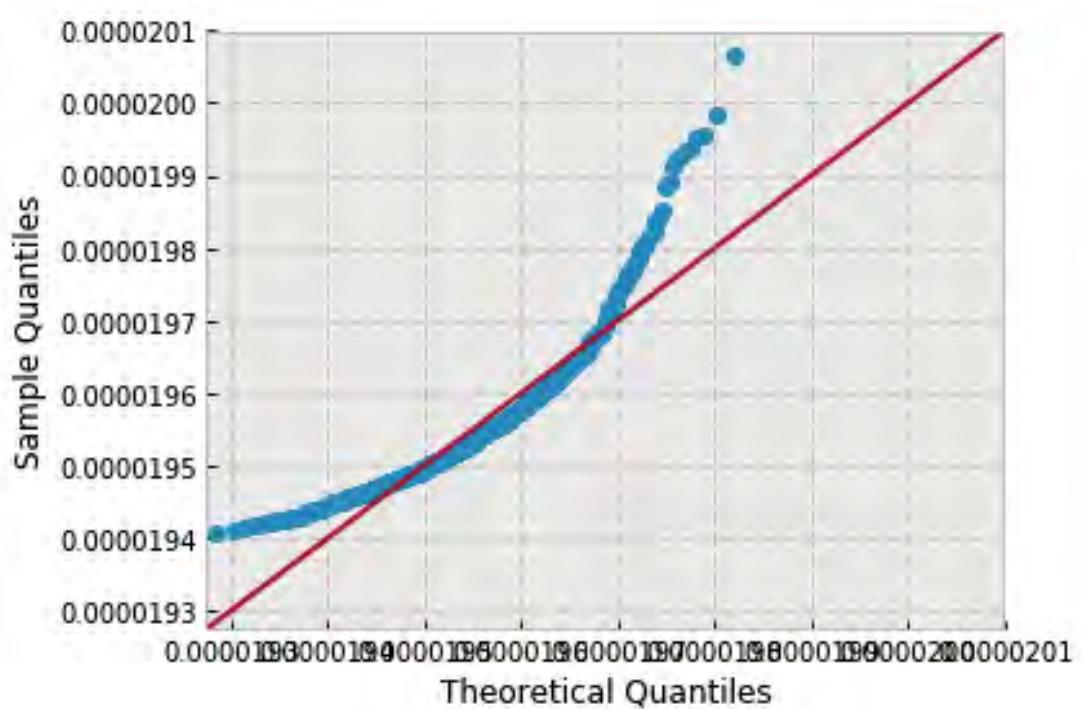
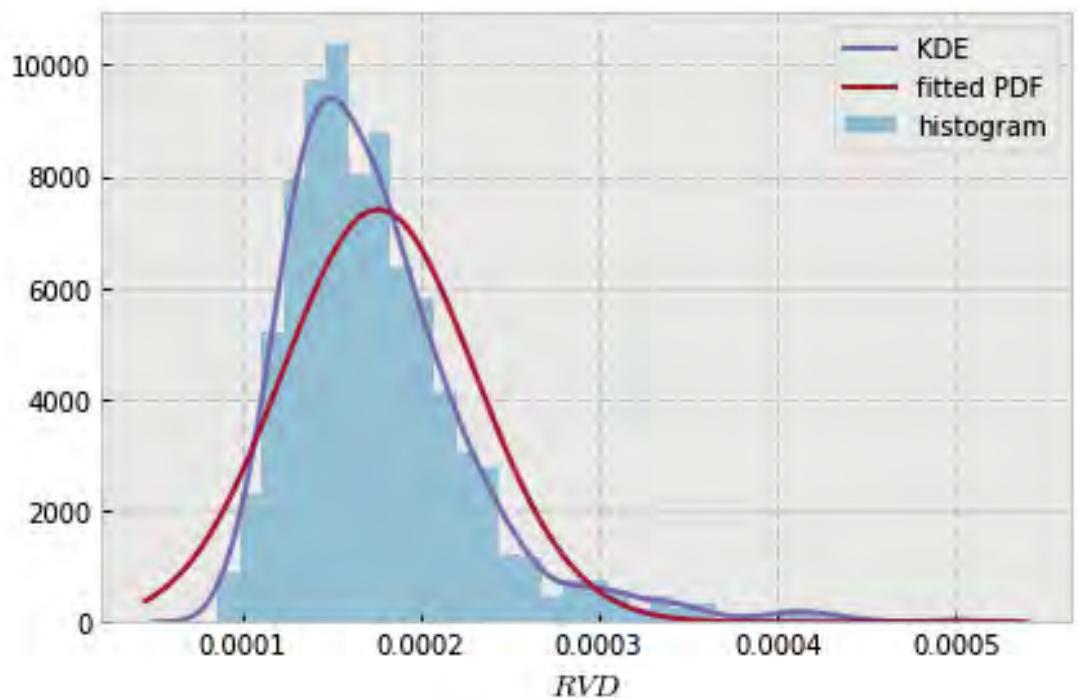


```
variable name: ln
variable value: 0.8333333333333333
distribution: inverse-gamma(shape=(4.9018923382726314,), loc=6.538771101004422e-05,
                           scale=0.00046461401853900716)
DescribeResult(nobs=1000, minmax=(9.3483497733016951e-05, 0.00055663778026833542)
               mean=0.00017925940876054478, variance=2.5983533482089845e-09,
               skewness=1.608394942200798, kurtosis=5.855516855035242)
```

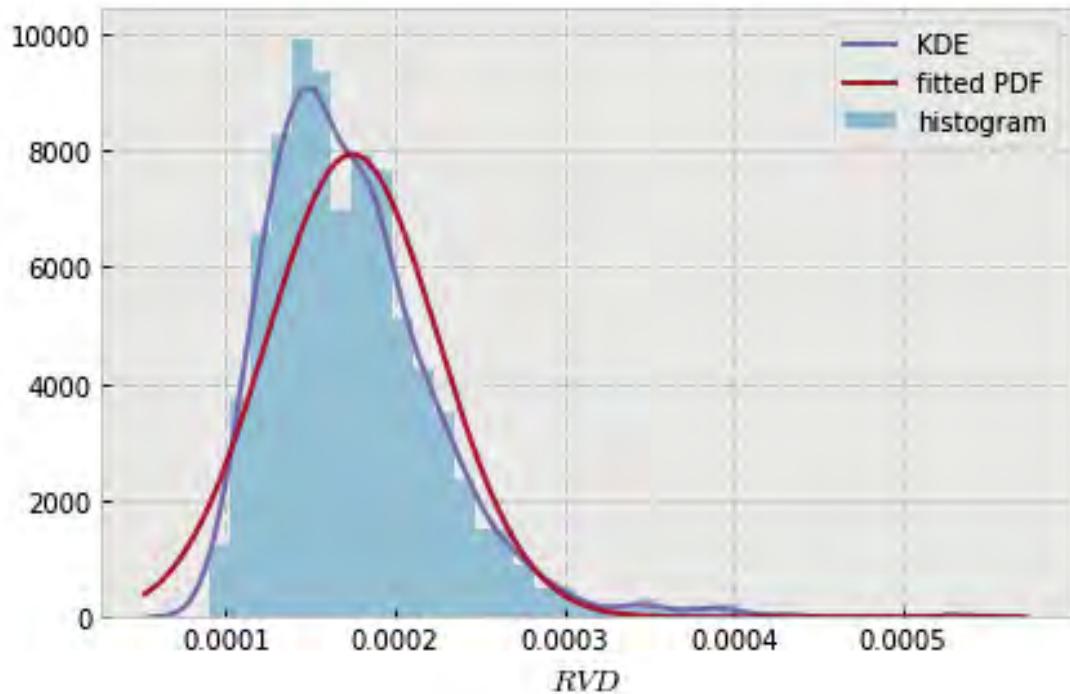


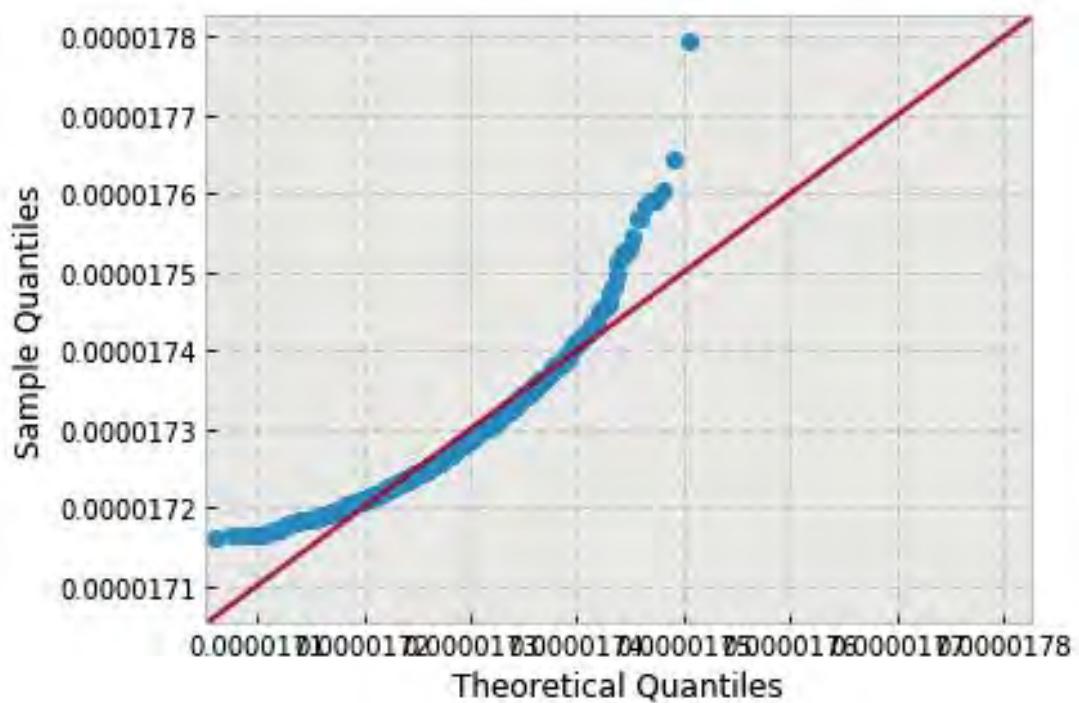


```
variable name: ln
variable value: 1.0
distribution: inverse-gamma(shape=(51150.388724660472,), loc=-0.012008736128258883,
    scale=623.2937318595409)
DescribeResult(nobs=1000, minmax=(8.6799885276605529e-05, 0.00049831537816011806)
    mean=0.00017701461396052665, variance=2.9350425641817136e-09,
    skewness=1.7068026217879684, kurtosis=4.623486973090805)
```



```
variable name: ln
variable value: 1.1666666666666665
distribution: inverse-gamma(shape=(57865.684914808604,), loc=-0.011924744150541775,
    scale=700.1542751887846)
DescribeResult(nobs=1000, minmax=(9.0555825457972405e-05, 0.00053431140477748214)
    mean=0.00017520259994152439, variance=2.5609596359554585e-09,
    skewness=1.49296104086403, kurtosis=4.618383511130817)
```

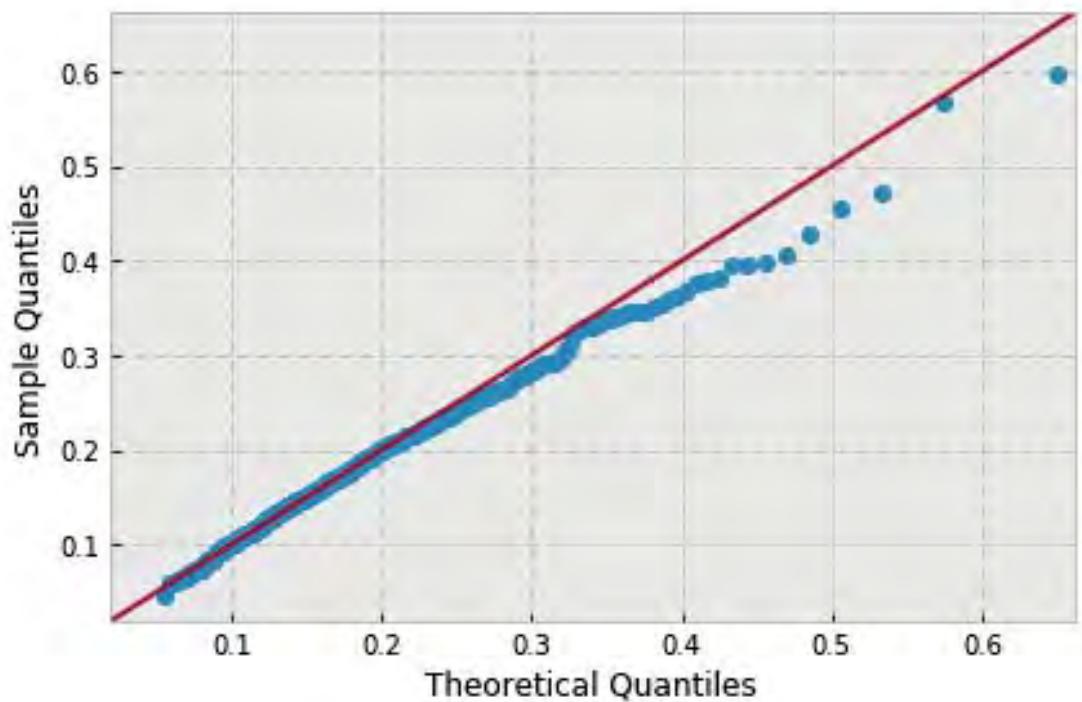
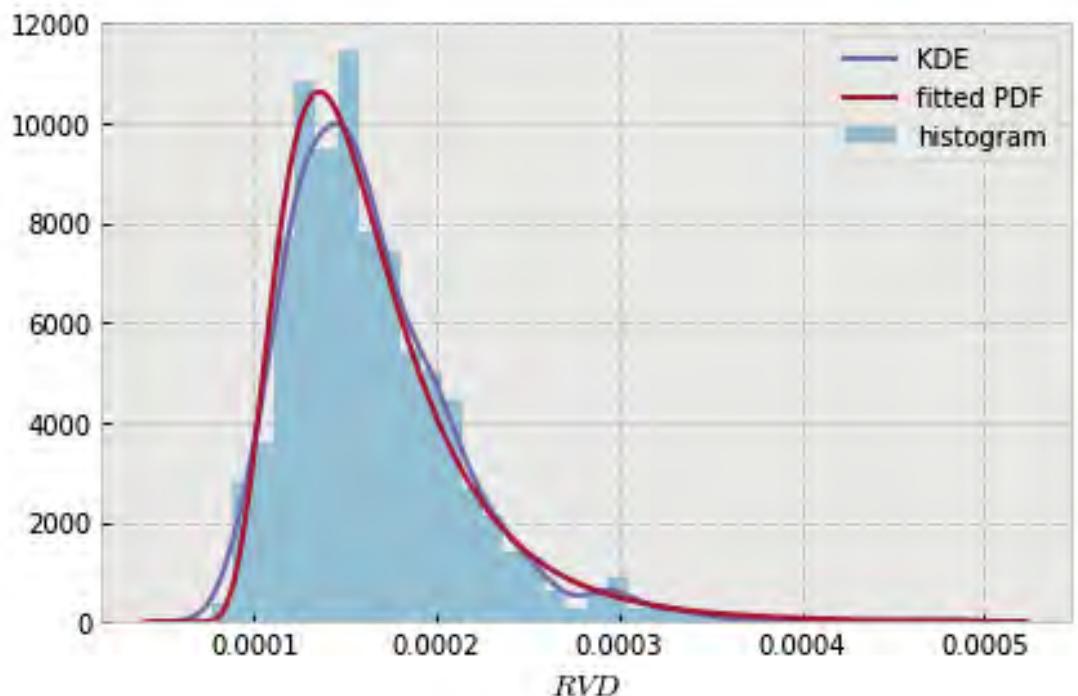




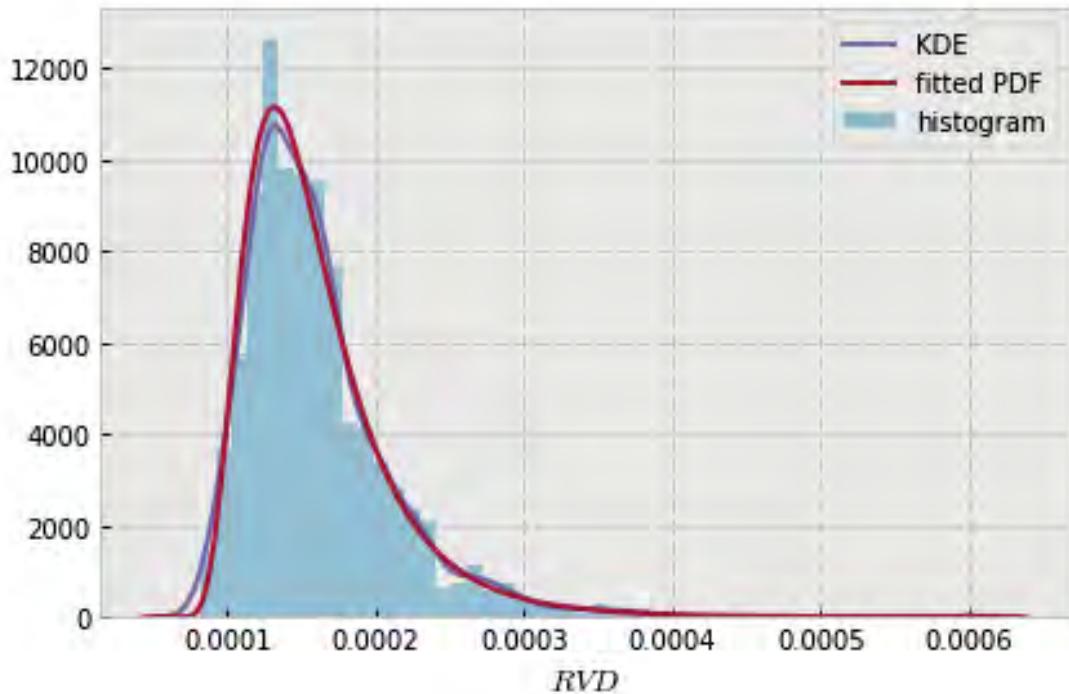
```

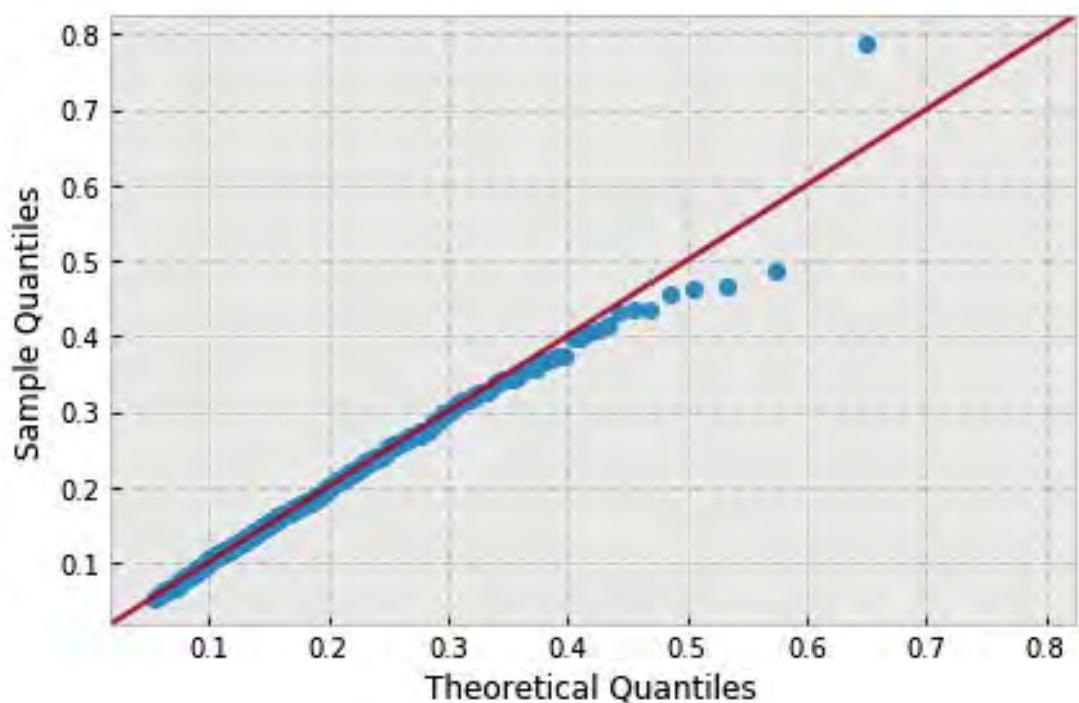
variable name: ln
variable value: 1.333333333333333
distribution: inverse-gamma(shape=(7.0455625096463201,), loc=4.3378623720665316e-05,
    scale=0.0007421225828159403)
DescribeResult(nobs=1000, minmax=(7.7311064916869656e-05, 0.00048613016625248398)
    mean=0.00016460806619096152, variance=2.3907077815150048e-09,
    skewness=1.5664092057402046, kurtosis=4.589895991054768)

```

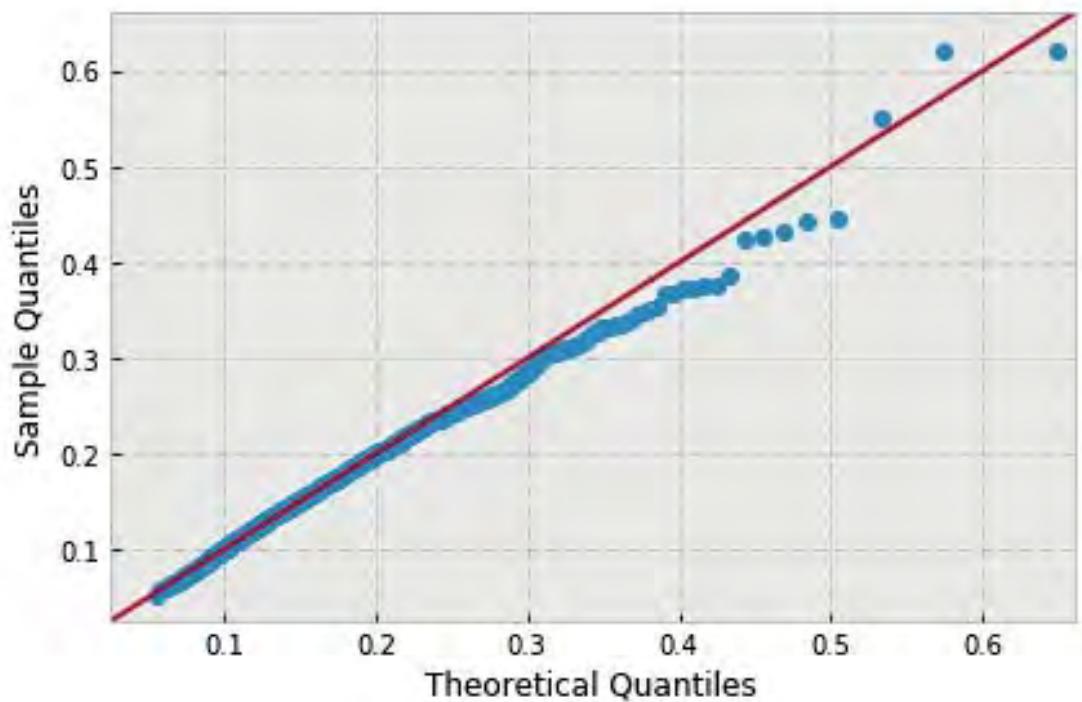
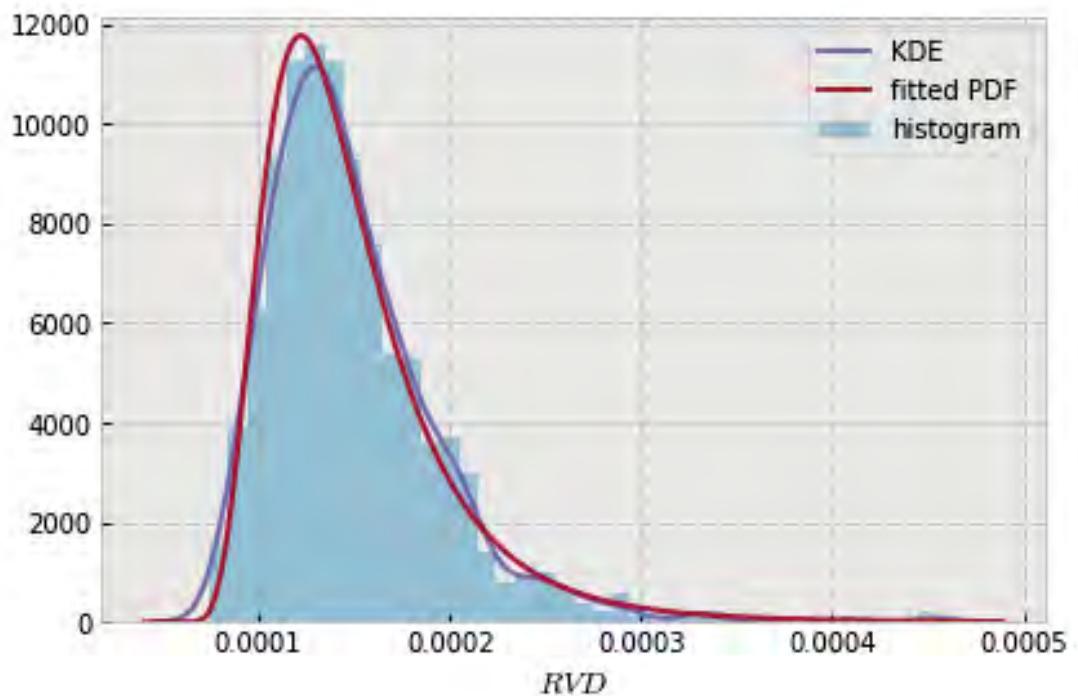


```
variable name: ln
variable value: 1.5
distribution: inverse-gamma(shape=(7.0451037650963997,), loc=4.389303288404551e-05,
    scale=0.0007071305541981683)
DescribeResult(nobs=1000, minmax=(8.1653982468577222e-05, 0.00060054936649852502)
    mean=0.00016021776776652558, variance=2.4612525529401303e-09,
    skewness=1.9274767779135005, kurtosis=8.019055803500375)
```

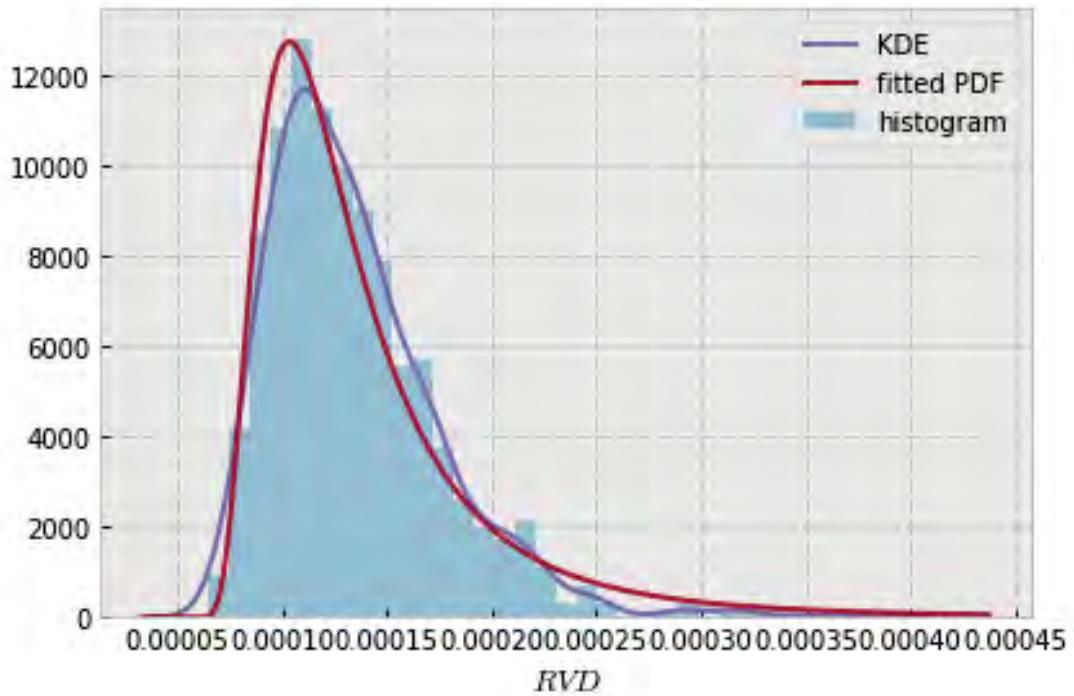


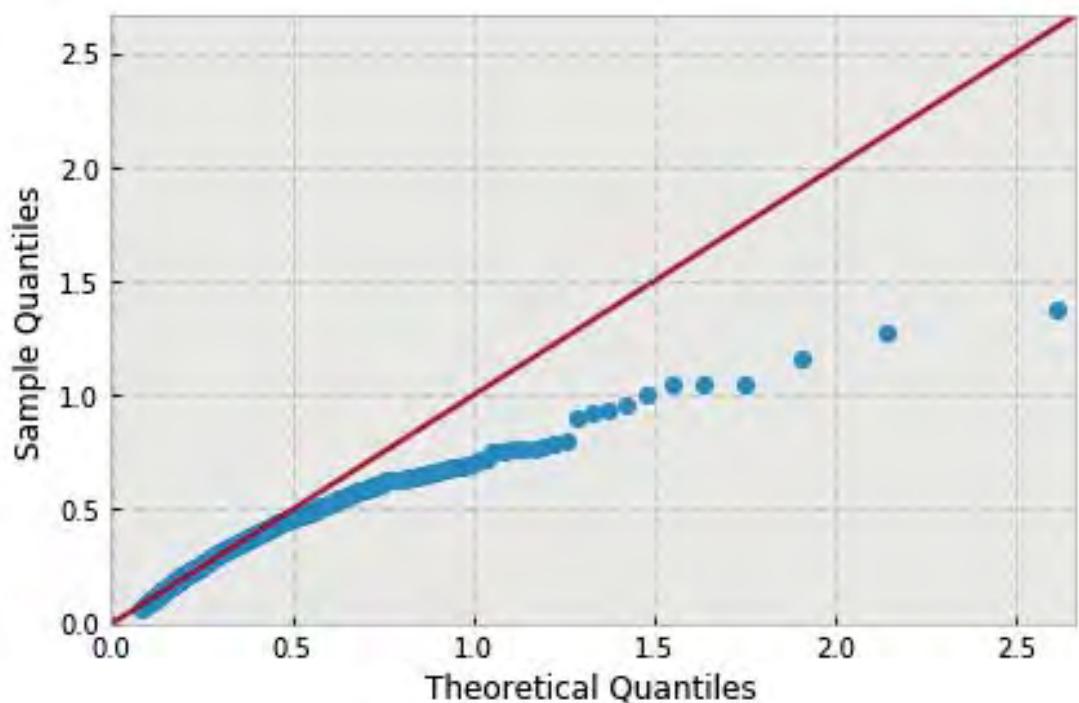


```
variable name: ln
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(7.052764185191517,), loc=3.861813013940955e-05,
                           scale=0.0006713607164125664)
DescribeResult(nobs=1000, minmax=(7.4129646148812564e-05, 0.00045490766151130271)
               mean=0.00014839705639972987, variance=2.0327069727015455e-09,
               skewness=1.7420930931227658, kurtosis=6.117227620500042)
```

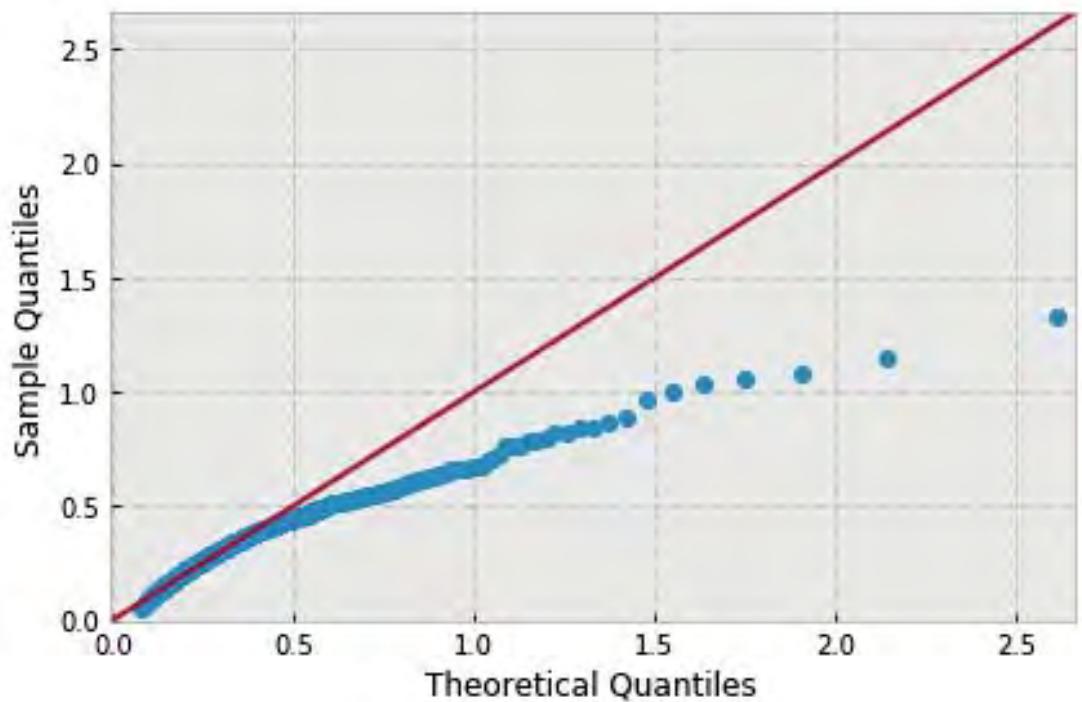
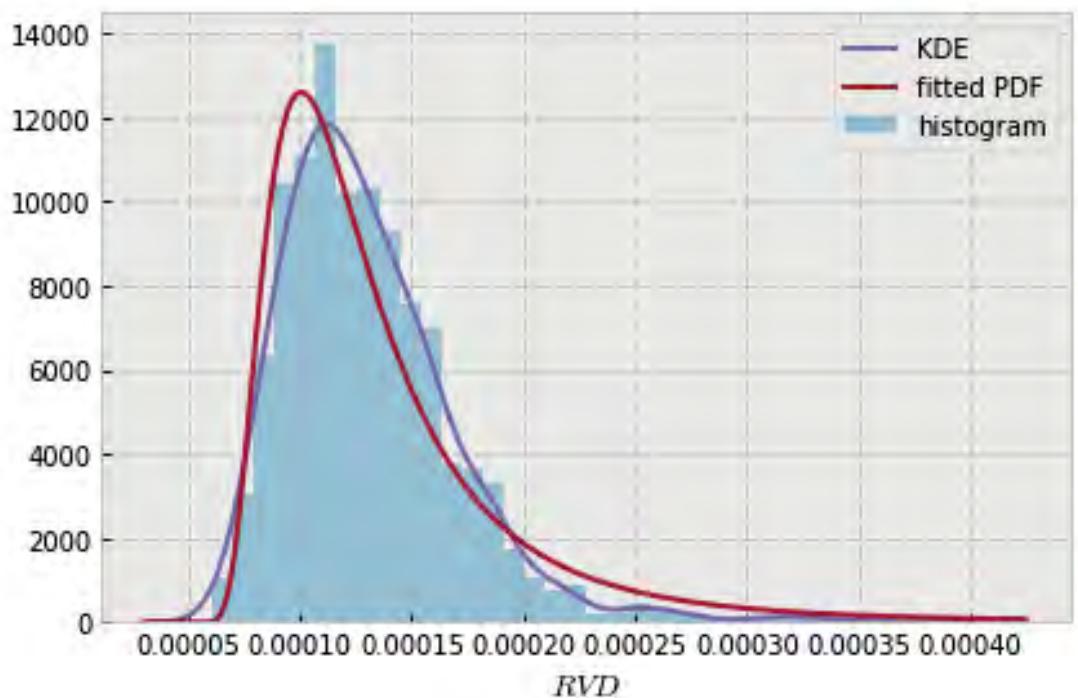


```
variable name: ln
variable value: 1.833333333333333
distribution: inverse-gamma(shape=(3.8336721983211057,), loc=4.985477764035136e-05,
                           scale=0.00025894641265116673)
DescribeResult(nobs=1000, minmax=(6.5106057764683712e-05, 0.00040581120958171294)
               mean=0.0001344565318028169, variance=1.7637751870101043e-09,
               skewness=1.5914841242596767, kurtosis=4.7138898129318)
```

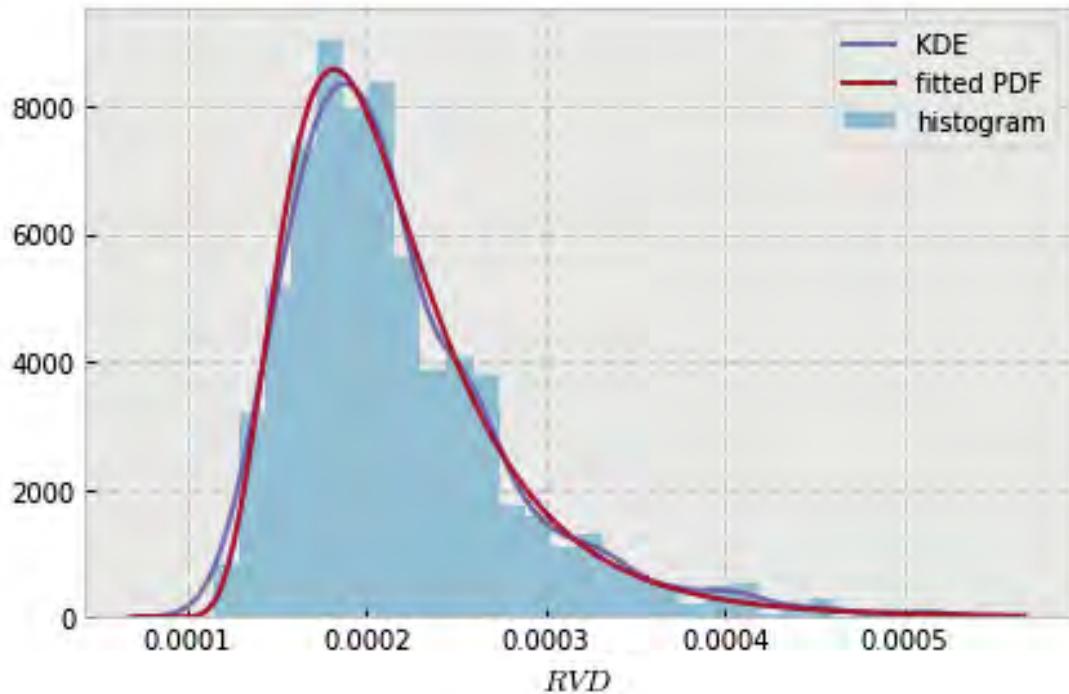


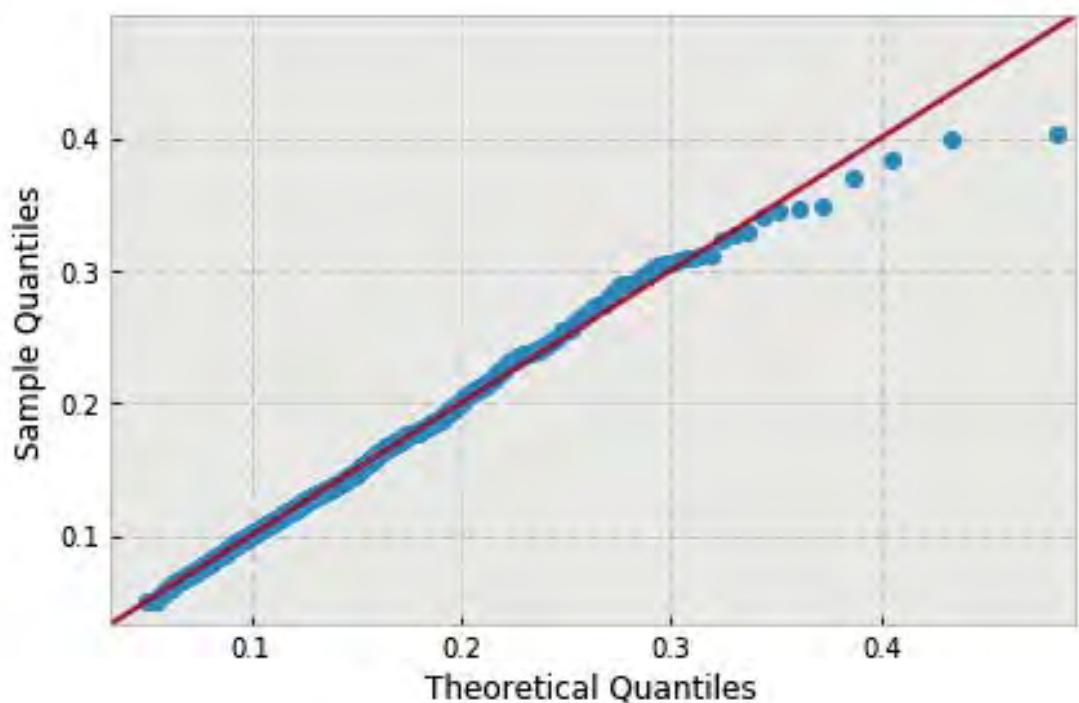


```
variable name: ln
variable value: 2.0
distribution: inverse-gamma(shape=(3.8346941887385517,), loc=4.637749340676849e-05,
                           scale=0.0002622598036809078)
DescribeResult(nobs=1000, minmax=(6.0910669050188601e-05, 0.00039498777941612882)
               mean=0.0001307011891577029, variance=1.5791345077224251e-09,
               skewness=1.5832841282111212, kurtosis=5.006256278793174)
```

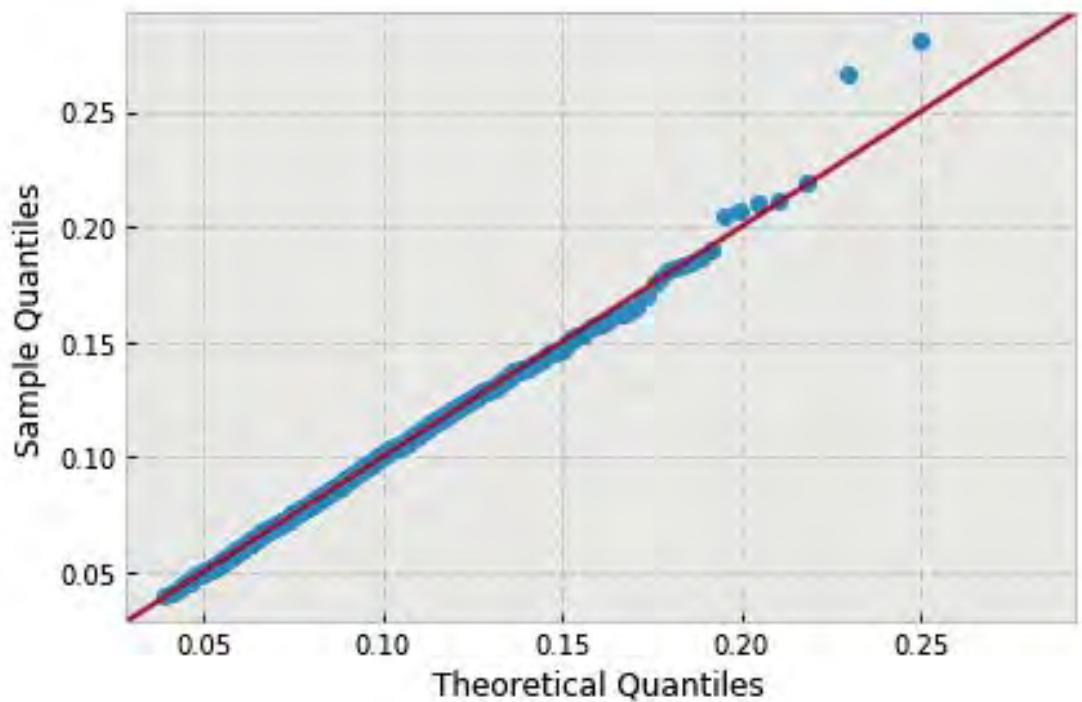
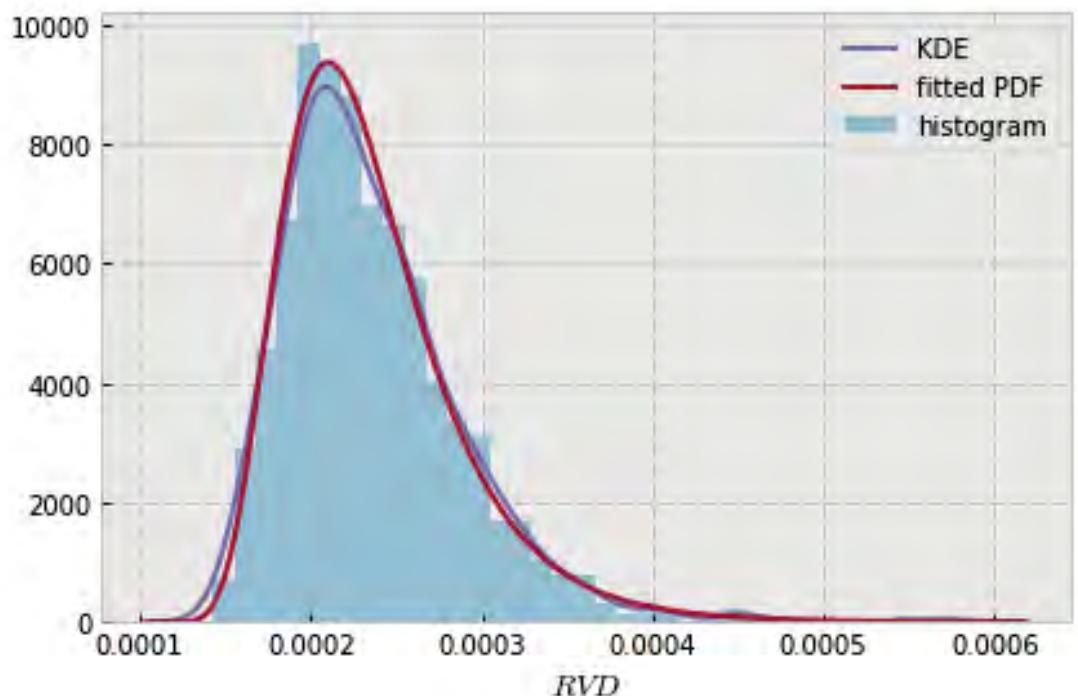


```
variable name: nB
variable value: 4.0
distribution: inverse-gamma(shape=(8.2117623335983723,), loc=5.69695876378831e-05,
                           scale=0.001149408180054608)
DescribeResult(nobs=1000, minmax=(0.00011454531095721588, 0.00051943887930988846)
               mean=0.00021626509864017685, variance=3.9240854104555613e-09,
               skewness=1.4500755837444654, kurtosis=2.7858749613437617)
```

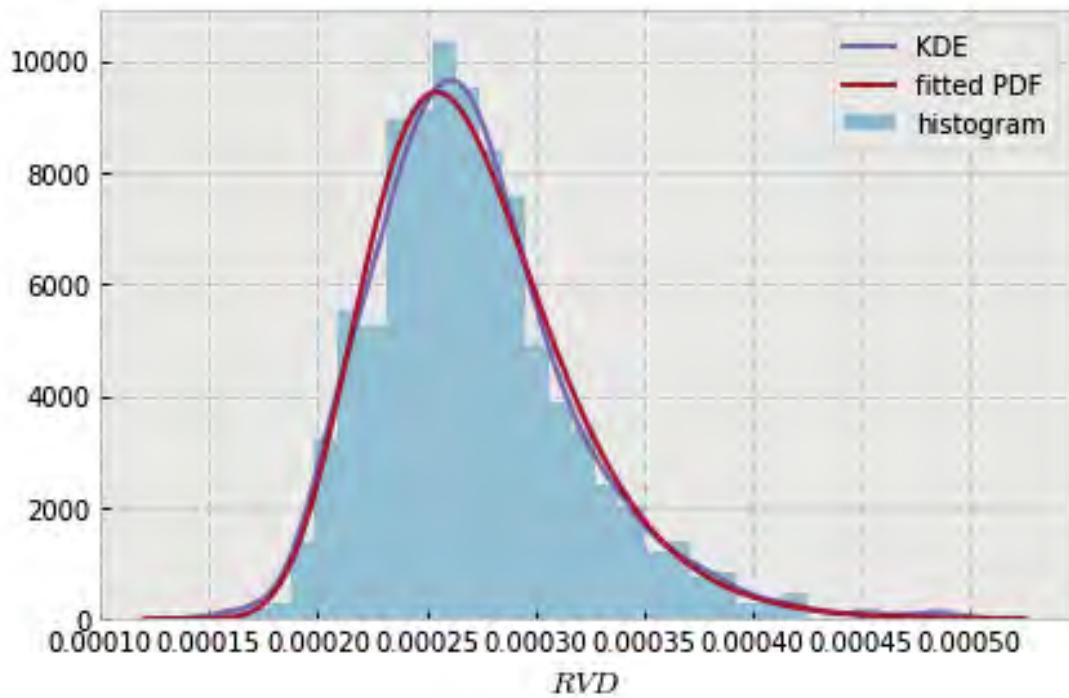


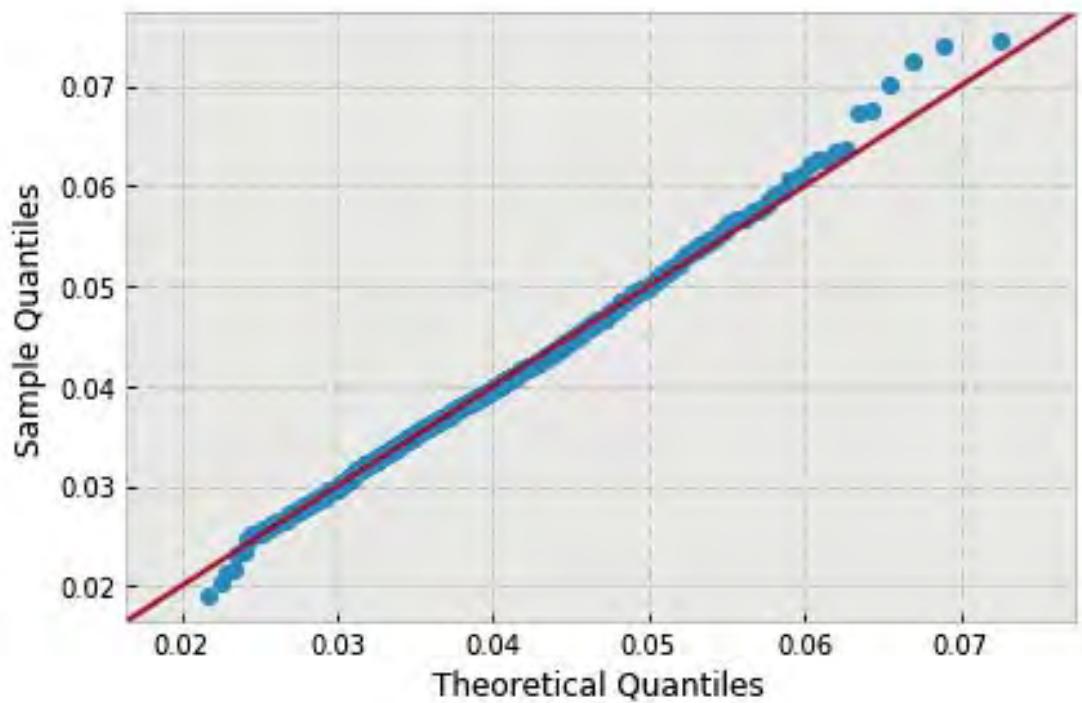


```
variable name: nB
variable value: 8.0
distribution: inverse-gamma(shape=(11.922202559890735,), loc=6.99682587338426e-05,
    scale=0.0018123666878056364)
DescribeResult(nobs=1000, minmax=(0.000142749089979044, 0.00057893471340346718)
    mean=0.00023588117148707843, variance=2.7609651353838198e-09,
    skewness=1.4013264757136084, kurtosis=4.028429750354097)
```

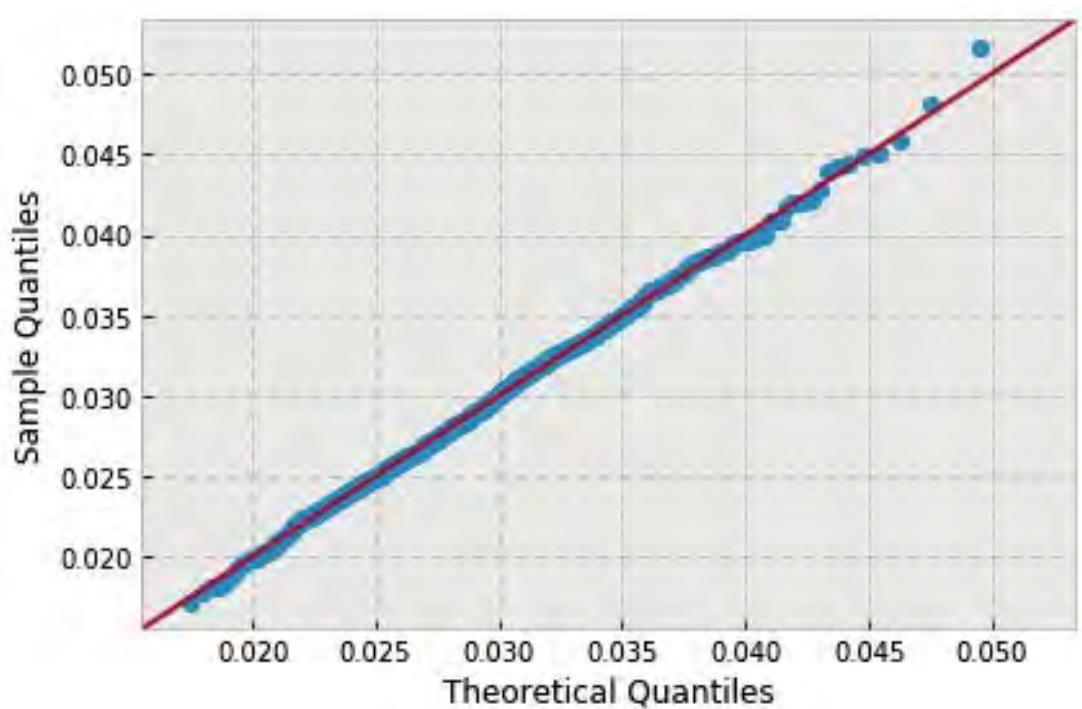
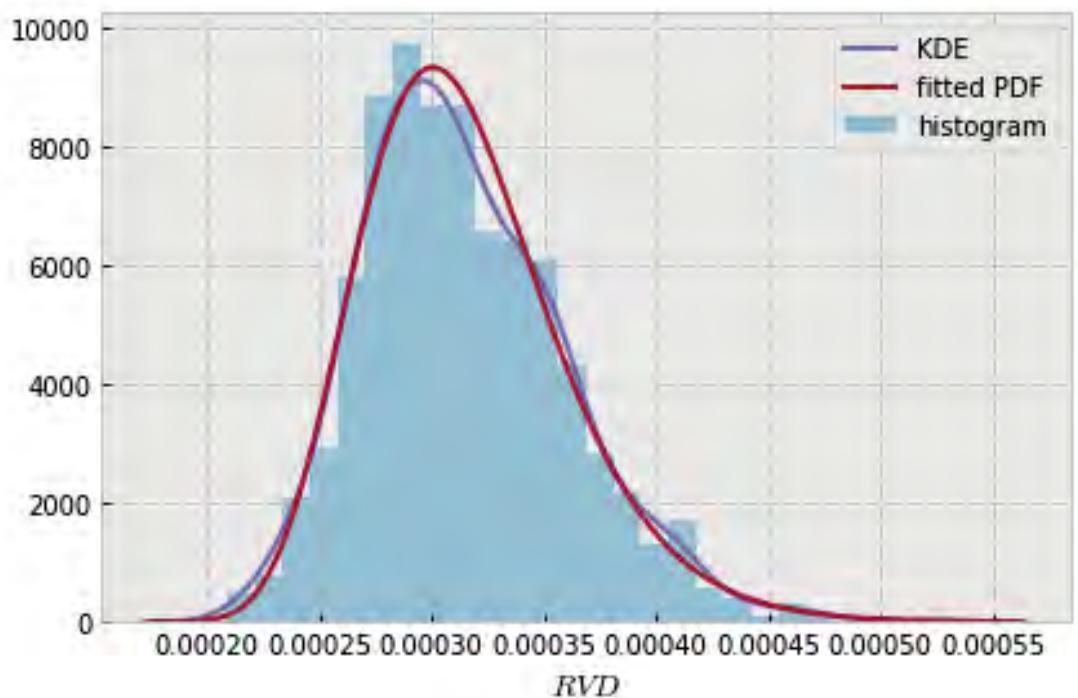


```
variable name: nB
variable value: 12.0
distribution: inverse-gamma(shape=(27.087111717652469,), loc=3.934281150260411e-05,
                           scale=0.006054379120095956)
DescribeResult(nobs=1000, minmax=(0.00015533081970933838, 0.00049032653034933749)
               mean=0.00027145407956680954, variance=2.1890682155080053e-09,
               skewness=0.9612646890422718, kurtosis=1.7928899125891302)
```

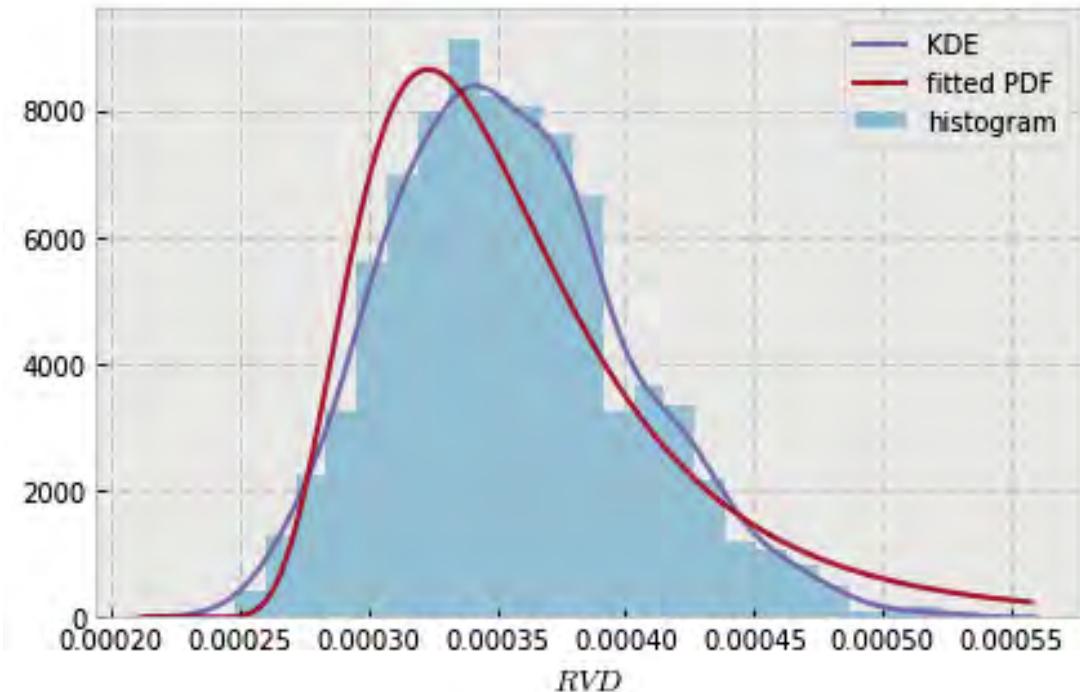


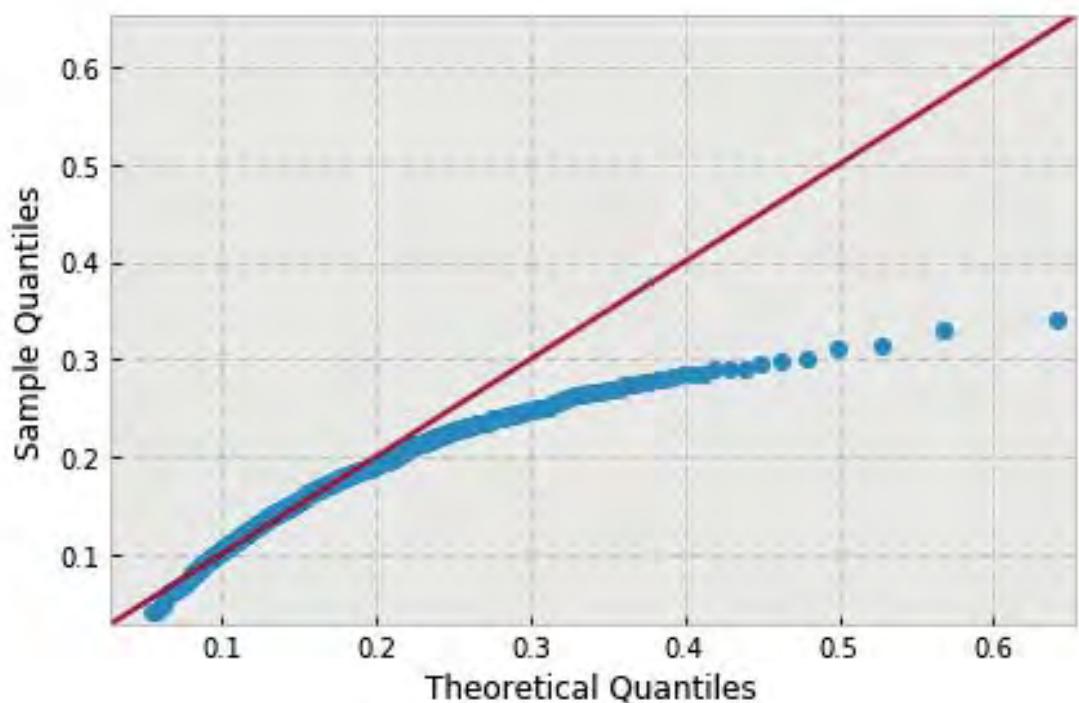


```
variable name: nB
variable value: 16.0
distribution: inverse-gamma(shape=(35.909001058078346,), loc=4.883062686496505e-05,
                           scale=0.009289066542023236)
DescribeResult(nobs=1000, minmax=(0.00020888486233235212, 0.00052815077608160449)
               mean=0.00031492110561088572, variance=2.0843856029507976e-09,
               skewness=0.6730292018536278, kurtosis=0.7105149535104021)
```

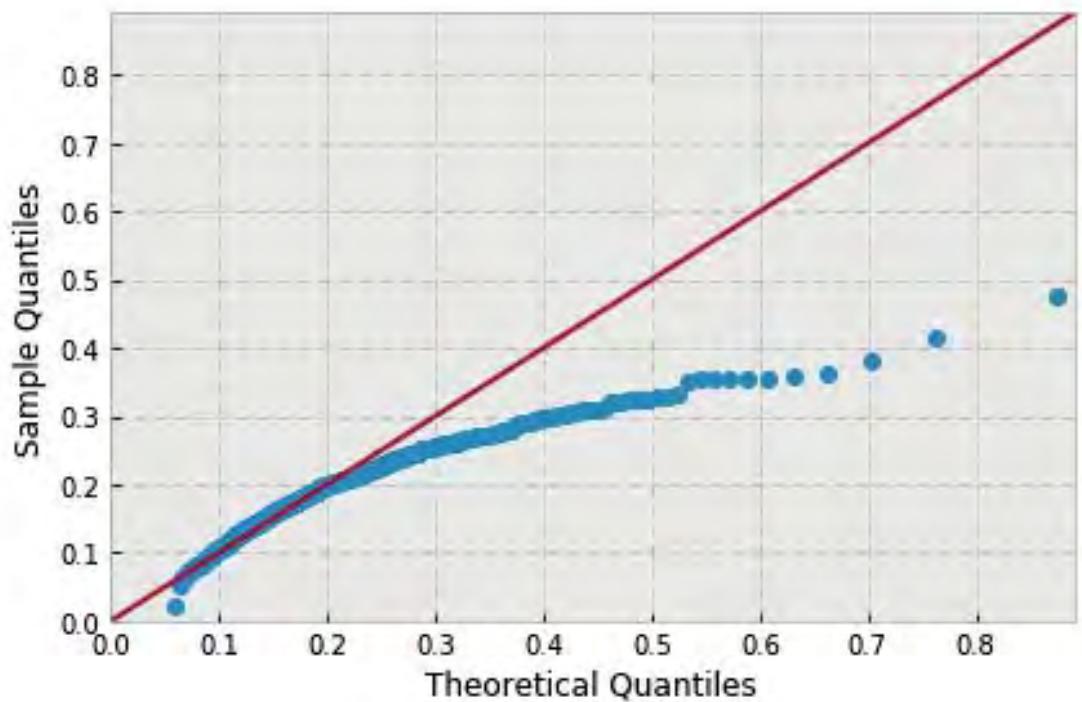
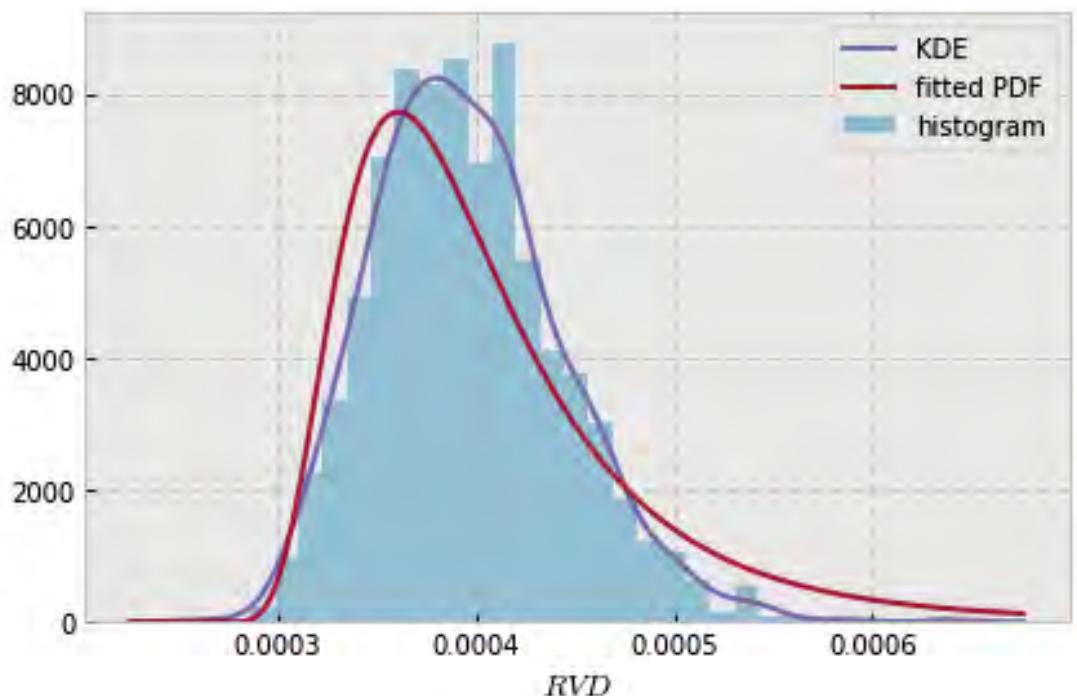


```
variable name: nB
variable value: 20.0
distribution: inverse-gamma(shape=(7.0949875634206112,), loc=0.00020934321959873518,
                           scale=0.0009228220242855653)
DescribeResult(nobs=1000, minmax=(0.00024795180091973917, 0.00052242896739253491)
               mean=0.0003548823577222328, variance=2.1442868407762261e-09,
               skewness=0.4473734729152729, kurtosis=0.031746320694193475)
```

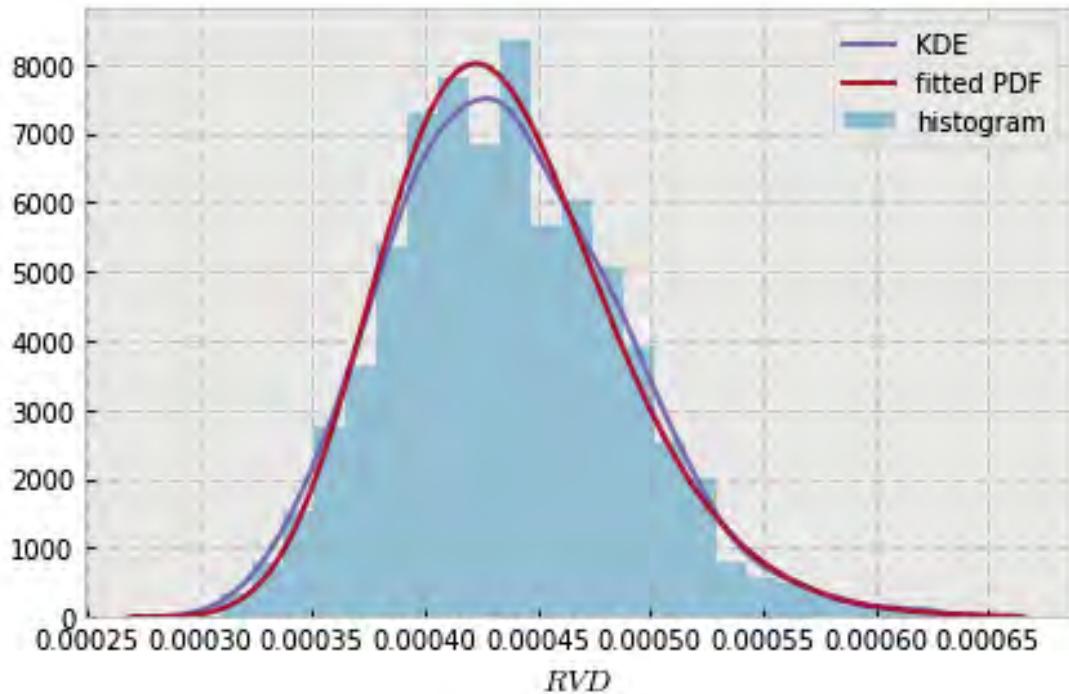


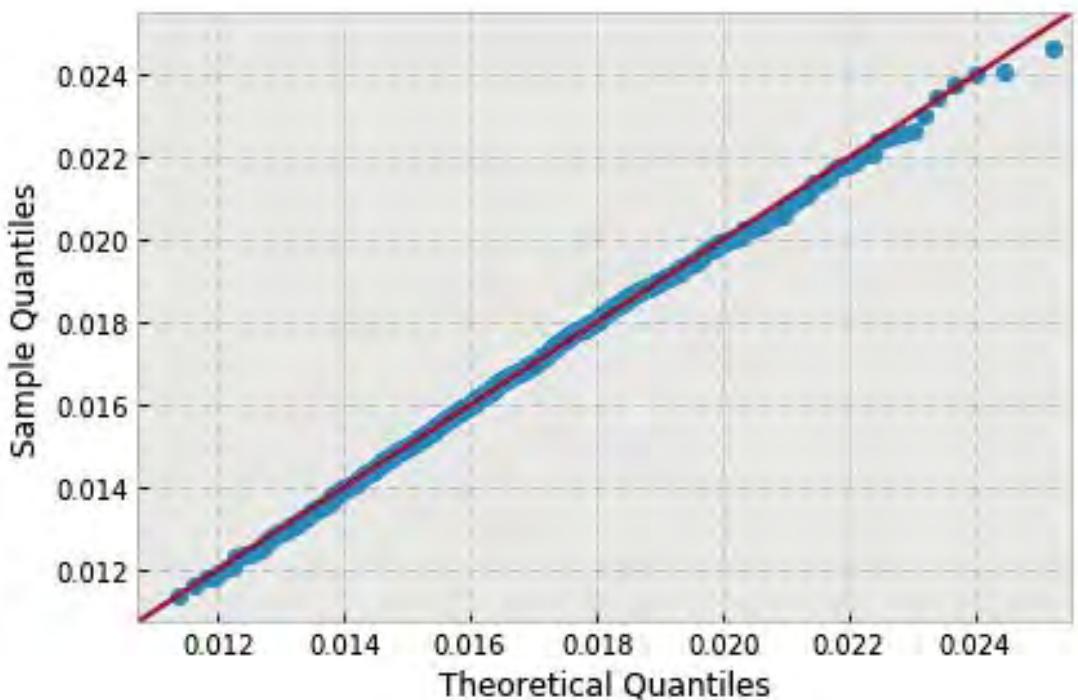


```
variable name: nB
variable value: 24.0
distribution: inverse-gamma(shape=(6.0942691877404425,), loc=0.00024406192721983423,
                           scale=0.0008291318509362074)
DescribeResult(nobs=1000, minmax=(0.00026176969017230729, 0.00063970313793276312)
               mean=0.00039589986390383401, variance=2.2948473918915552e-09,
               skewness=0.6146682336608799, kurtosis=0.7933941381927152)
```

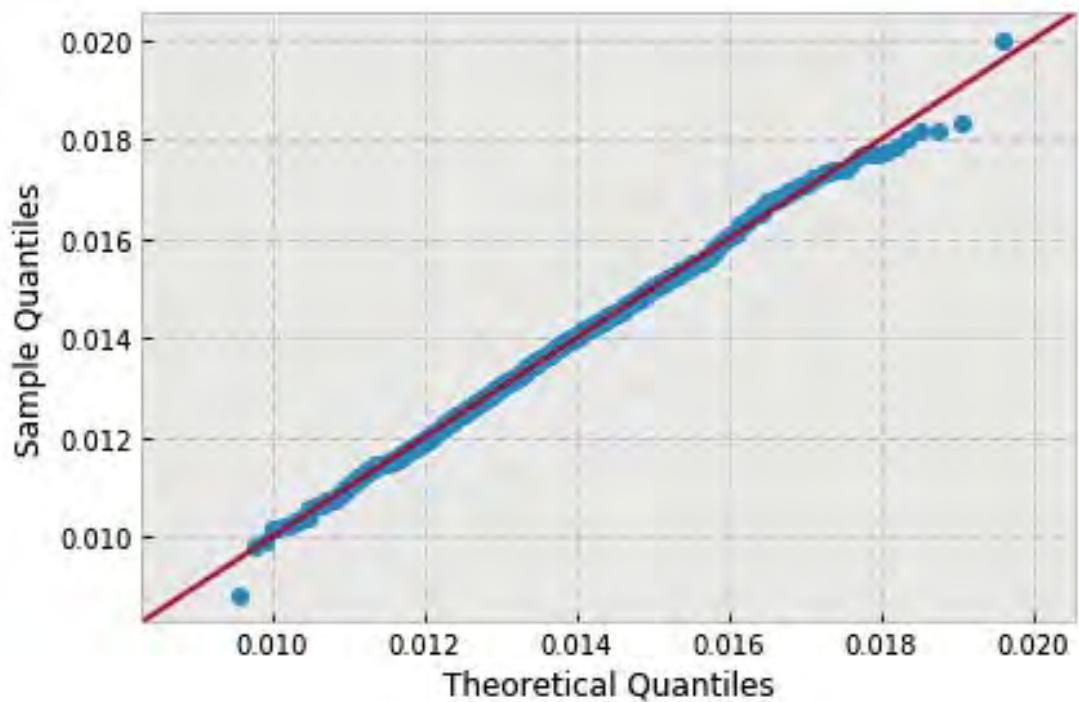
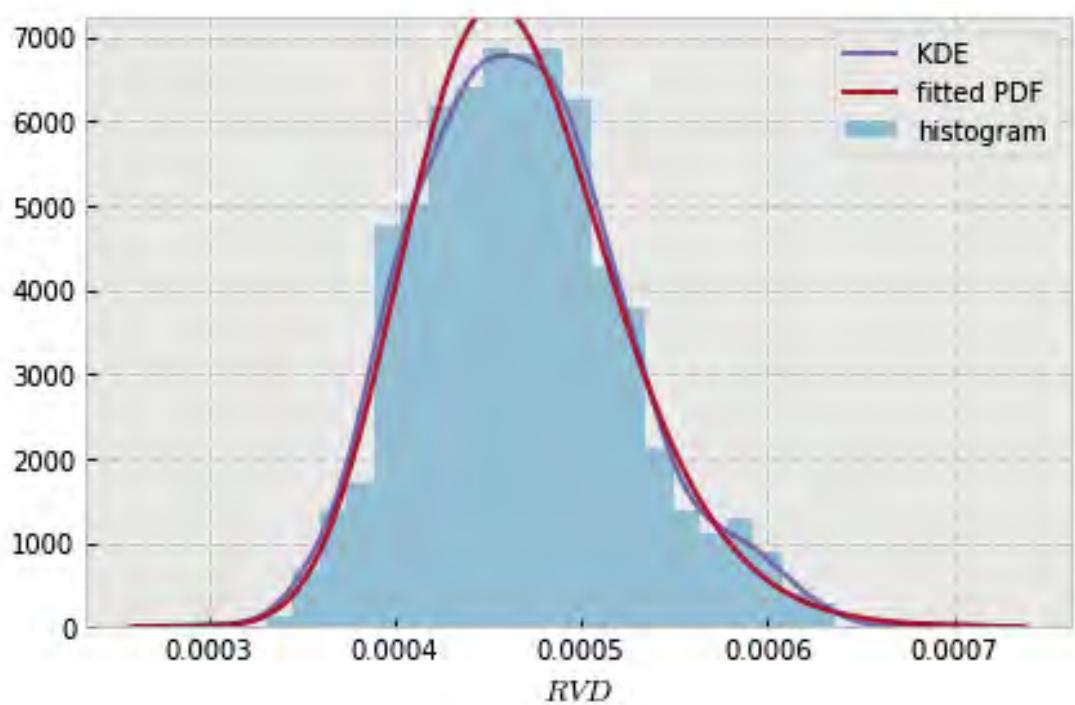


```
variable name: nB
variable value: 28.0
distribution: inverse-gamma(shape=(60.975483454265301,), loc=3.732390434785872e-05,
                           scale=0.023877938140426096)
DescribeResult(nobs=1000, minmax=(0.00030947365811300381, 0.00062517219474582083)
               mean=0.000435496207206845, variance=2.6430746073880413e-09,
               skewness=0.3832700139151923, kurtosis=0.12540694407615405)
```

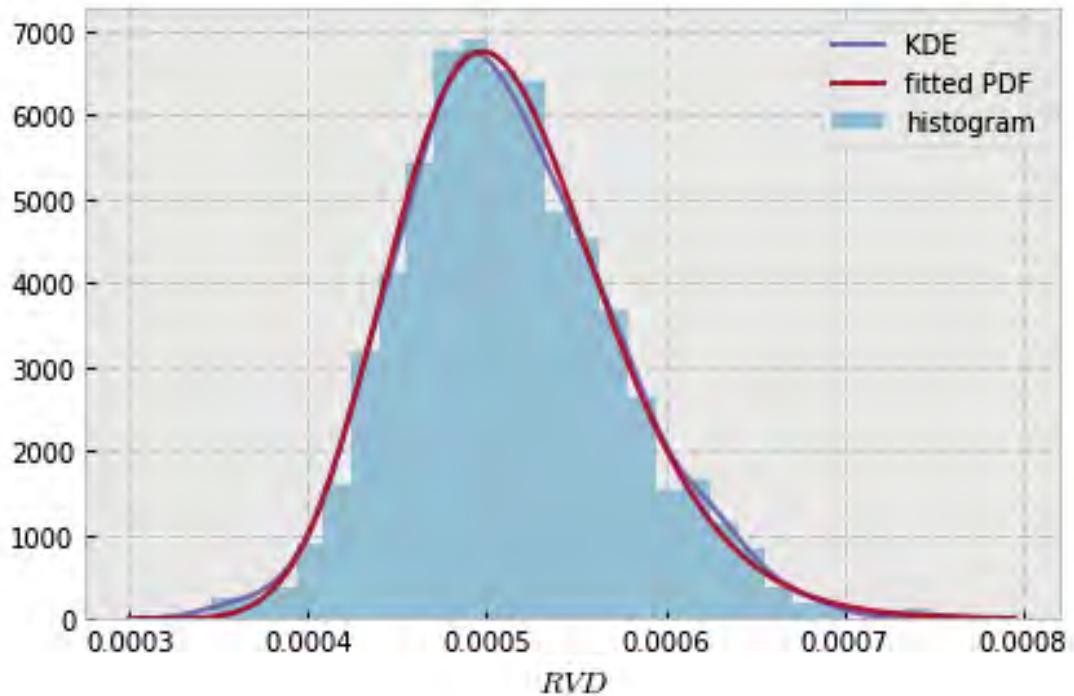


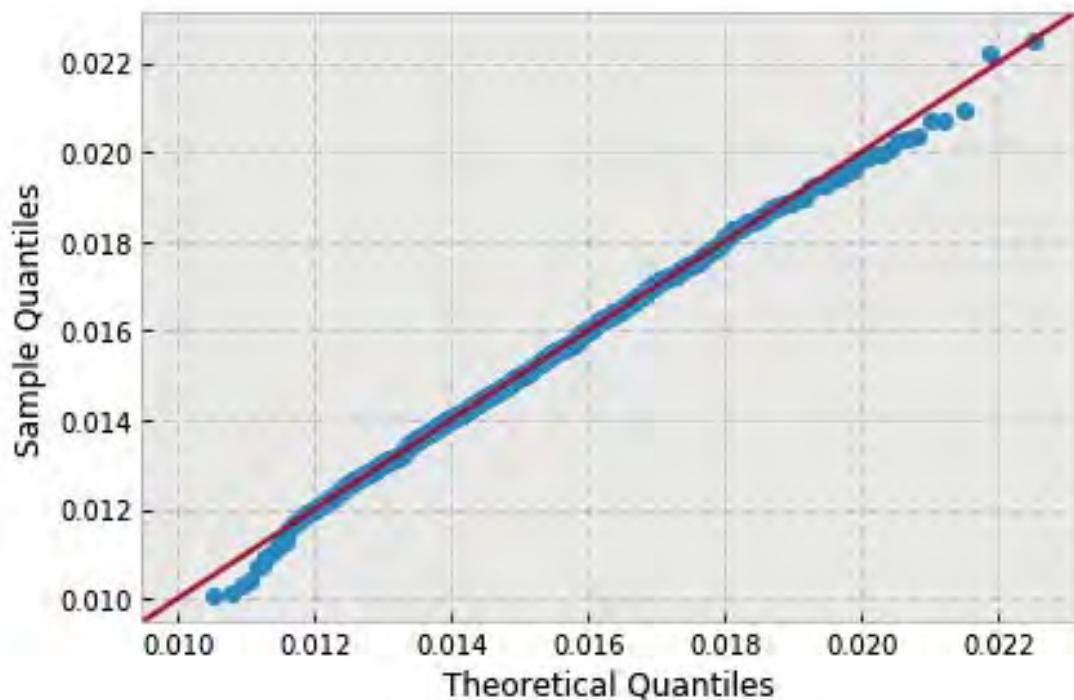


```
variable name: nB
variable value: 32.0
distribution: inverse-gamma(shape=(74.902244140803589,), loc=-1.0457350319738084e-05,
                           scale=0.03526173171251841)
DescribeResult(nobs=1000, minmax=(0.00030163477579445884, 0.00069434824770042865)
               mean=0.00046686712728952846, variance=3.1099726373685035e-09,
               skewness=0.4032394090844251, kurtosis=0.13241586237133118)
```

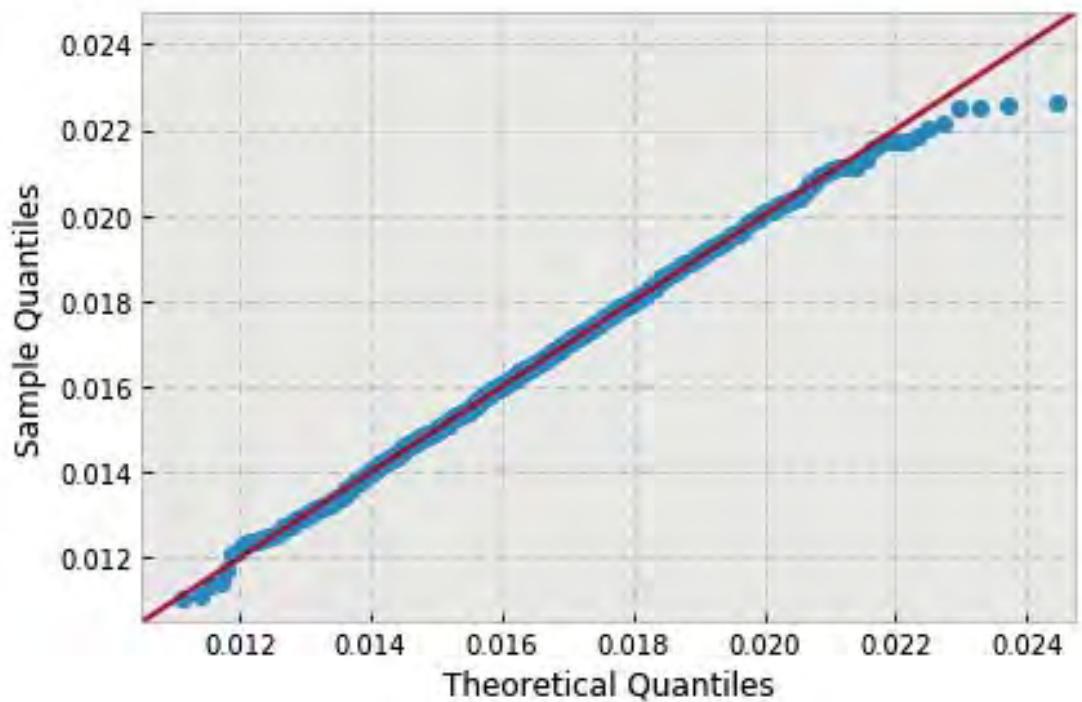
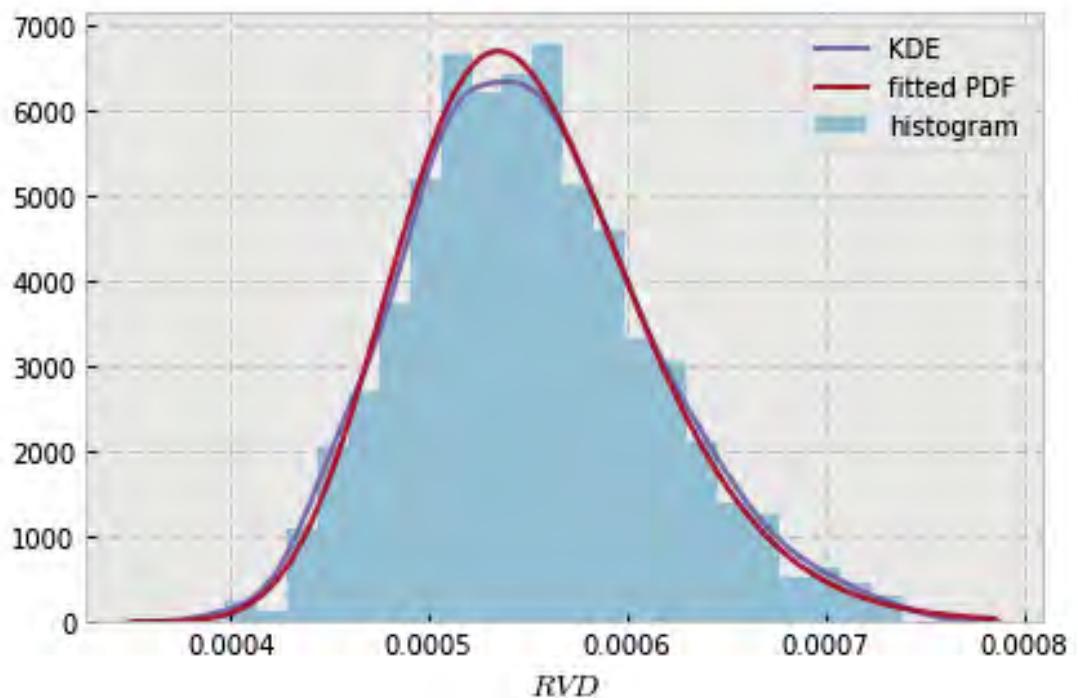


```
variable name: nB
variable value: 36.0
distribution: inverse-gamma(shape=(66.825645613956567,), loc=1.9129264019982878e-05,
scale=0.03245362281200867)
DescribeResult(nobs=1000, minmax=(0.00034653265005259046, 0.0007488398838177365)
mean=0.00051193335877610399, variance=3.6720691376199615e-09,
skewness=0.3937706103351912, kurtosis=0.23398203026844078)
```

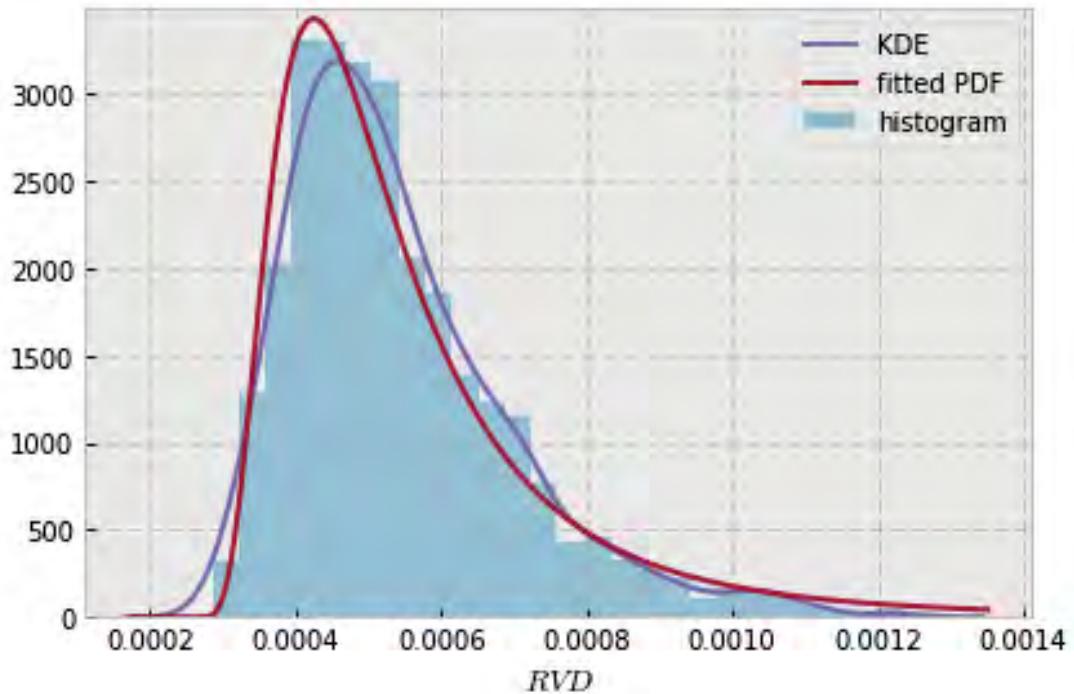


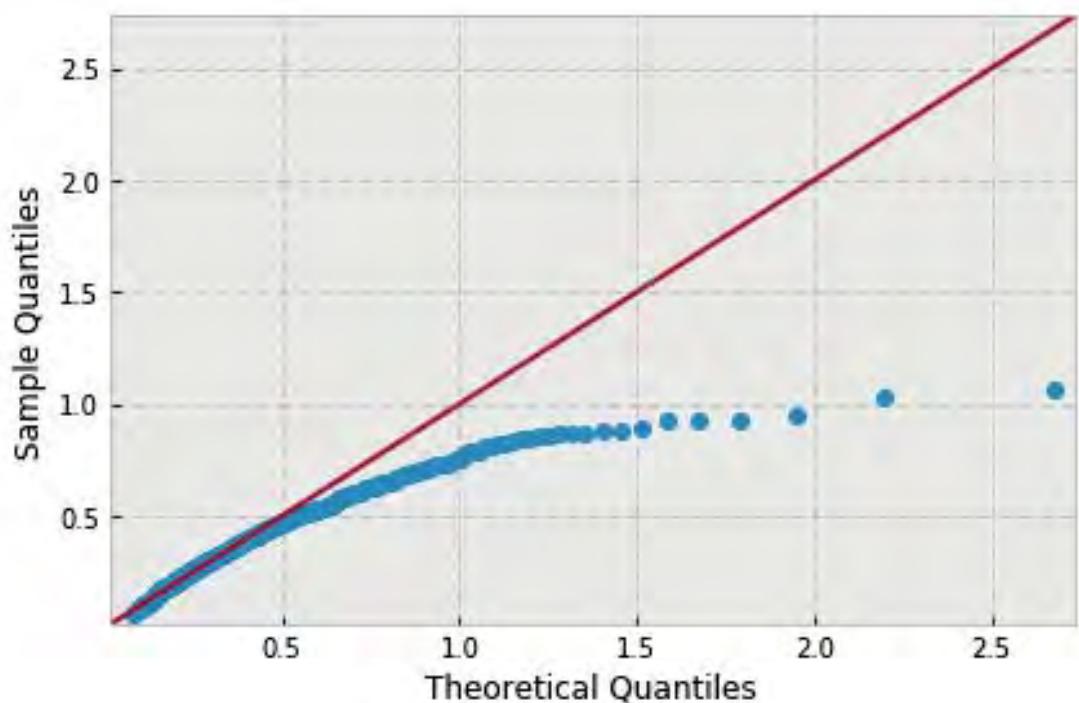


```
variable name: nB
variable value: 40.0
distribution: inverse-gamma(shape=(62.5020951112169,), loc=6.824086081344438e-05,
                           scale=0.029638644544459913)
DescribeResult(nobs=1000, minmax=(0.00039668462726307358, 0.00073950597666672504)
               mean=0.00055048412944633132, variance=3.7279230666969871e-09,
               skewness=0.3897037308260911, kurtosis=-0.03577646771205423)
```

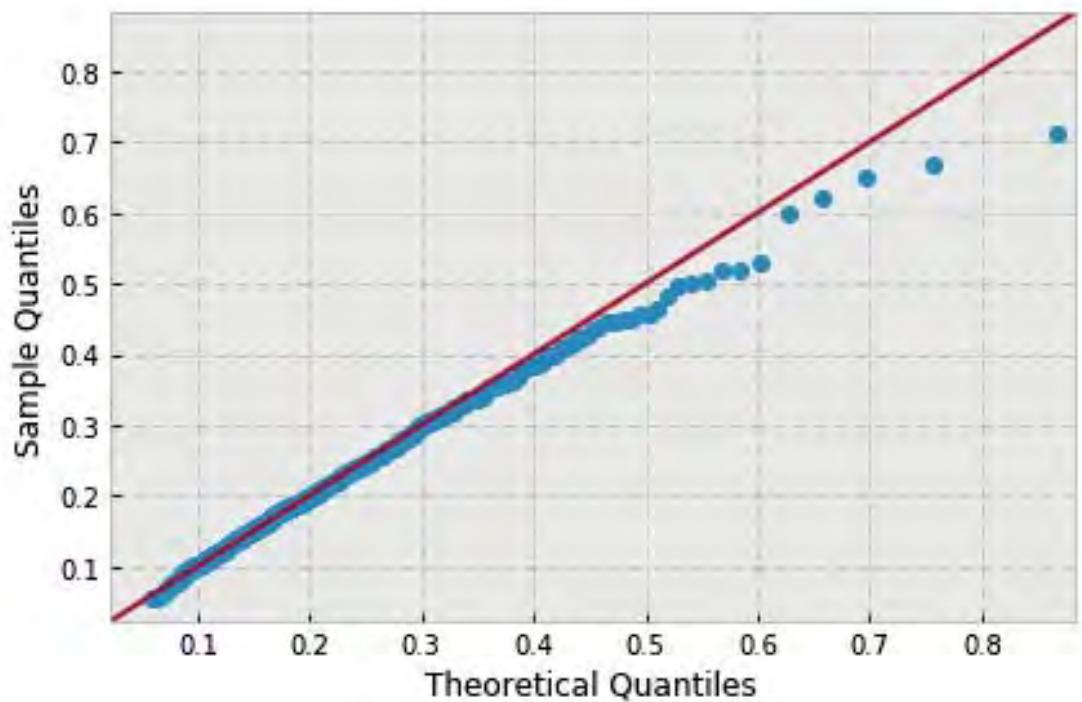
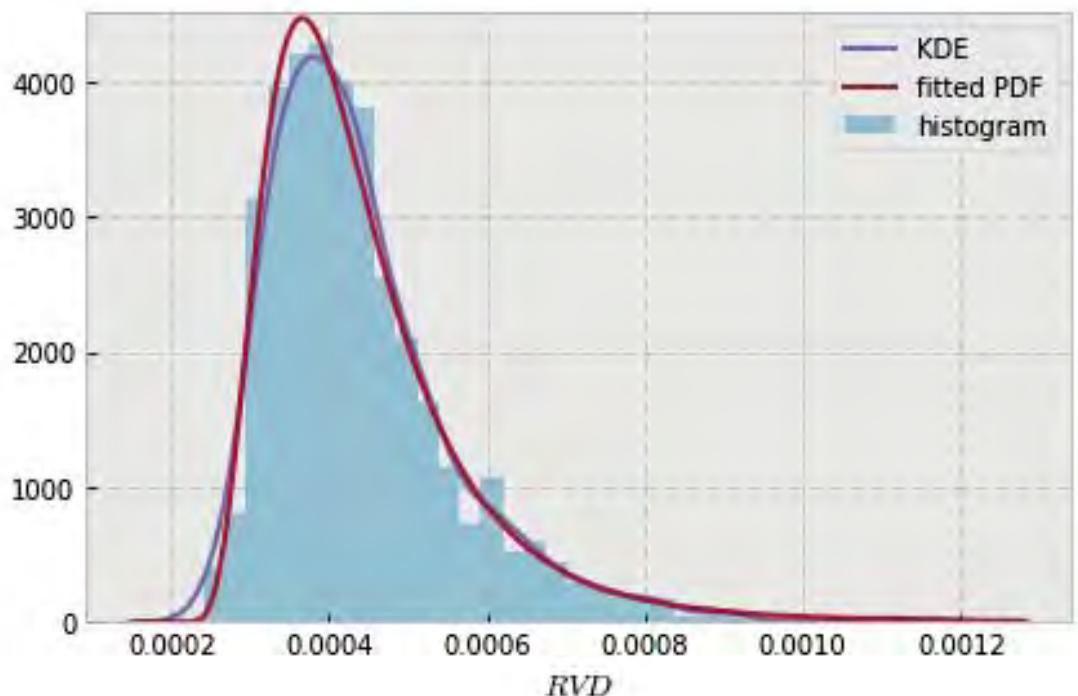


```
variable name: r
variable value: 1.0
distribution: inverse-gamma(shape=(3.7948289997073275,), loc=0.00022698401405493287,
    scale=0.000950240286392008)
DescribeResult(nobs=1000, minmax=(0.0002857391183987921, 0.0012341119478371406)
    mean=0.00054160122797479969, variance=2.3983061554804747e-08,
    skewness=1.245527660446143, kurtosis=1.8316774846098296)
```

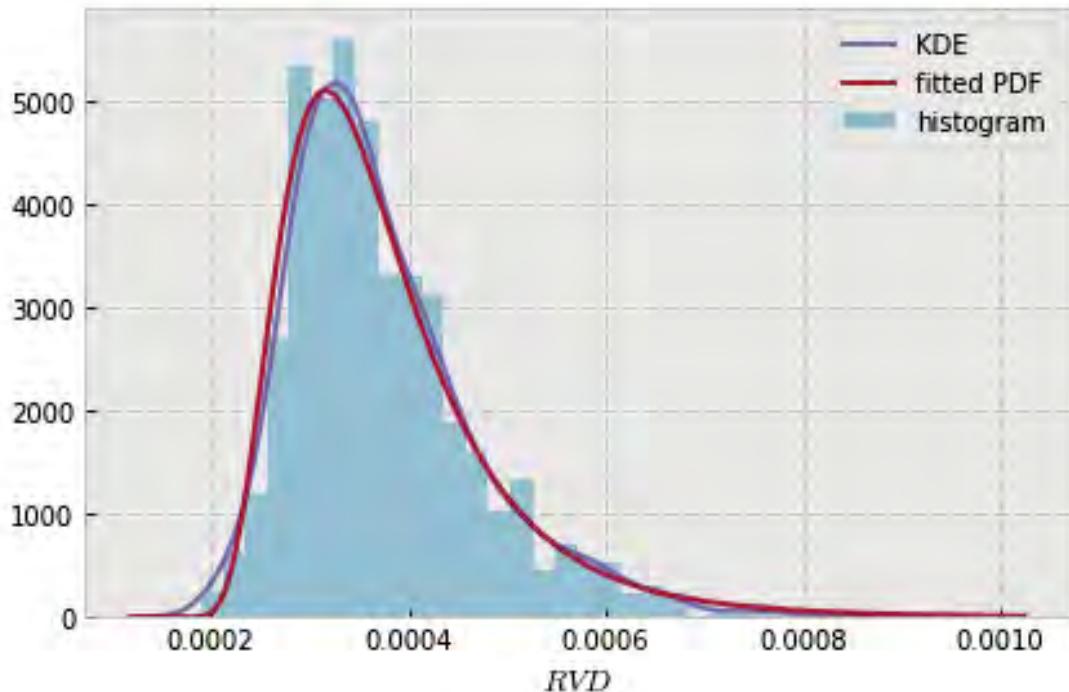


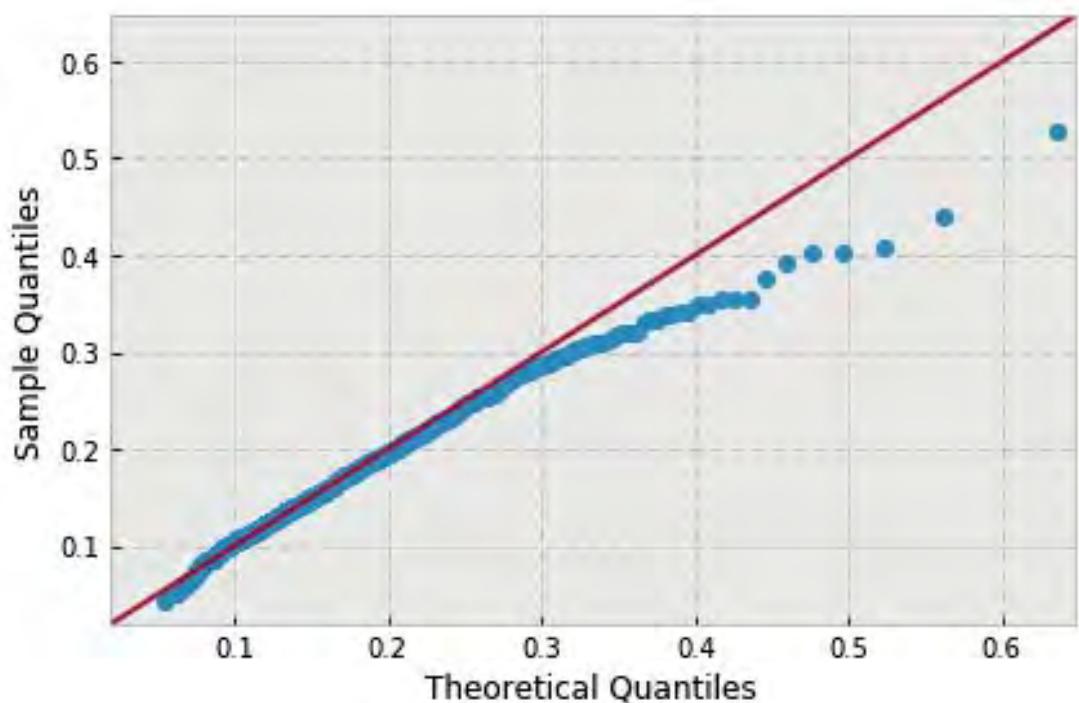


```
variable name: r
variable value: 1.222222222222223
distribution: inverse-gamma(shape=(6.1207938710357919,), loc=0.00016358356791977904,
                           scale=0.0014412717349521409)
DescribeResult(nobs=1000, minmax=(0.0002414275673415671, 0.0011878432506851665)
               mean=0.00044135553481712596, variance=1.5492475684927143e-08,
               skewness=1.7022526642619635, kurtosis=4.628391186970012)
```

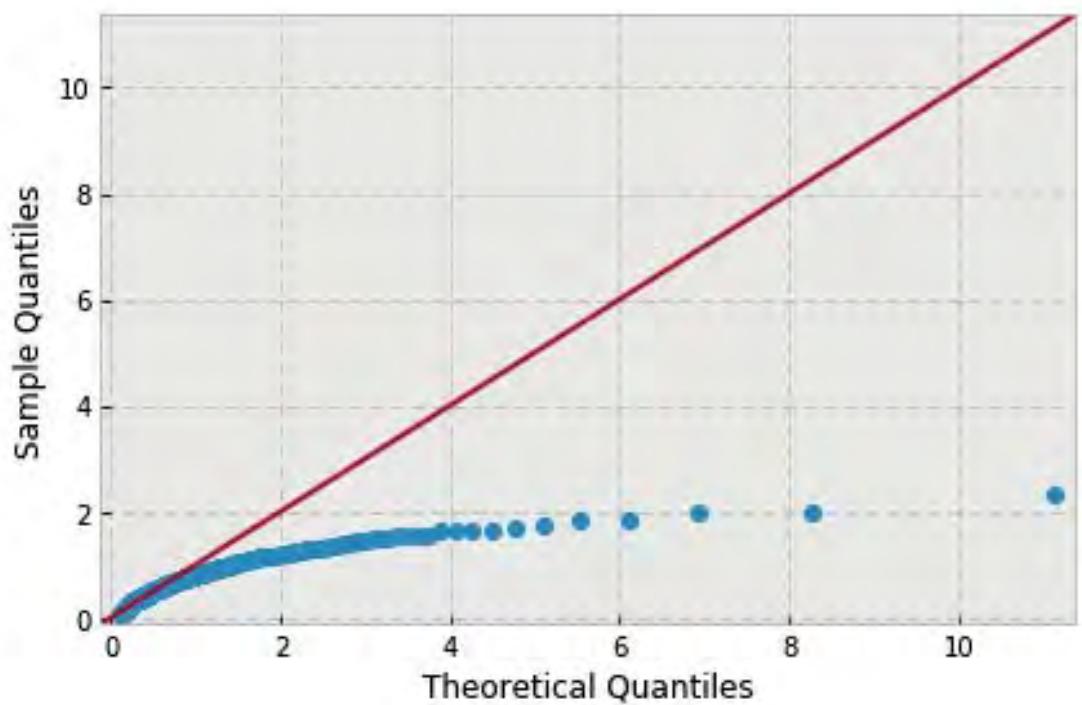
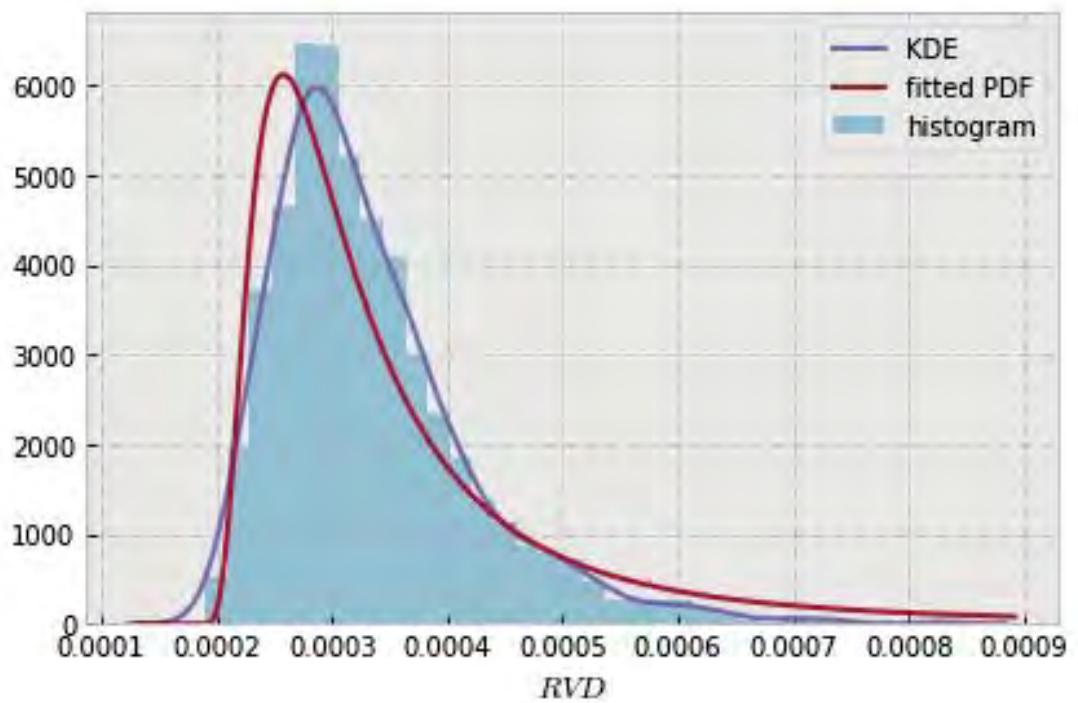


```
variable name: r
variable value: 1.444444444444444
distribution: inverse-gamma(shape=(7.125109420774668,), loc=0.00012268734867916083,
                           scale=0.001568373070429078)
DescribeResult(nobs=1000, minmax=(0.00018915435980505557, 0.00095127503598741077)
               mean=0.00037414628150419084, variance=9.3541982886288566e-09,
               skewness=1.2895130629604032, kurtosis=2.565746084018066)
```

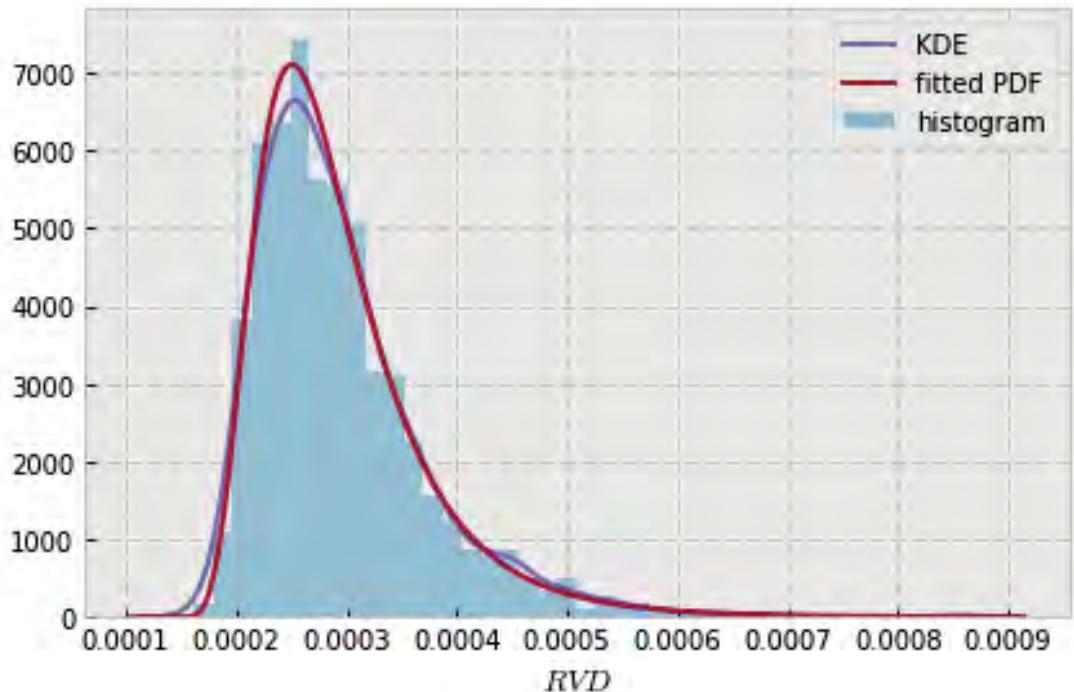


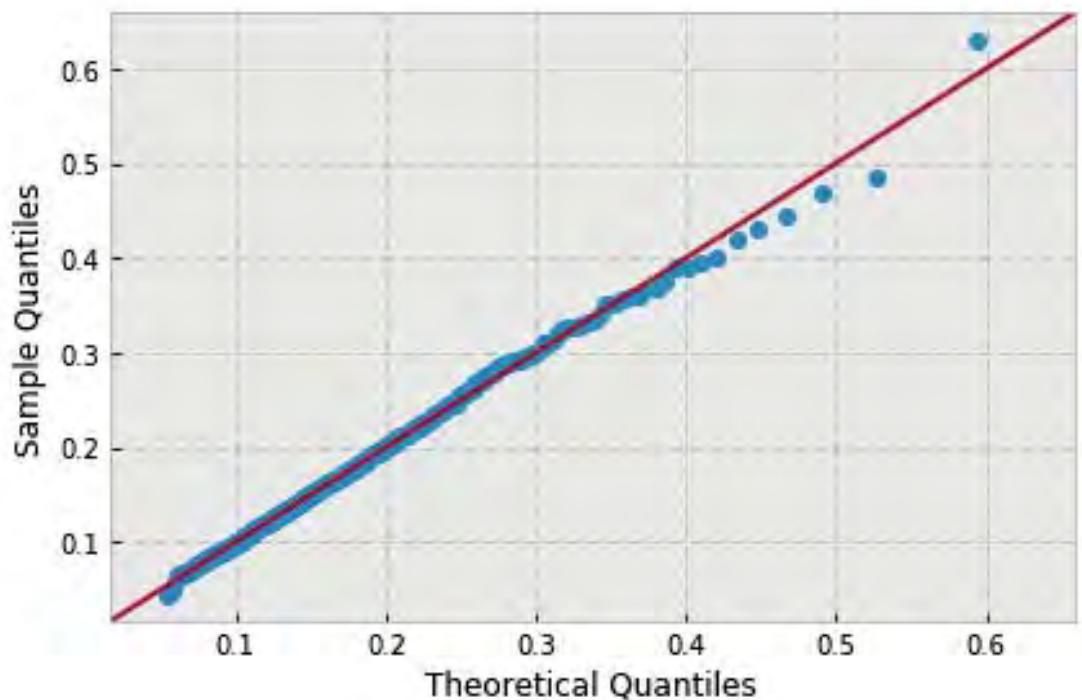


```
variable name: r
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(2.3904046859568107,), loc=0.00017497163304056021,
                           scale=0.0002800740719304409)
DescribeResult(nobs=1000, minmax=(0.00018994775009548365, 0.00082668510520774341)
               mean=0.00033258765994860907, variance=7.5543636052480684e-09,
               skewness=1.4462310072267583, kurtosis=3.039455959023279)
```

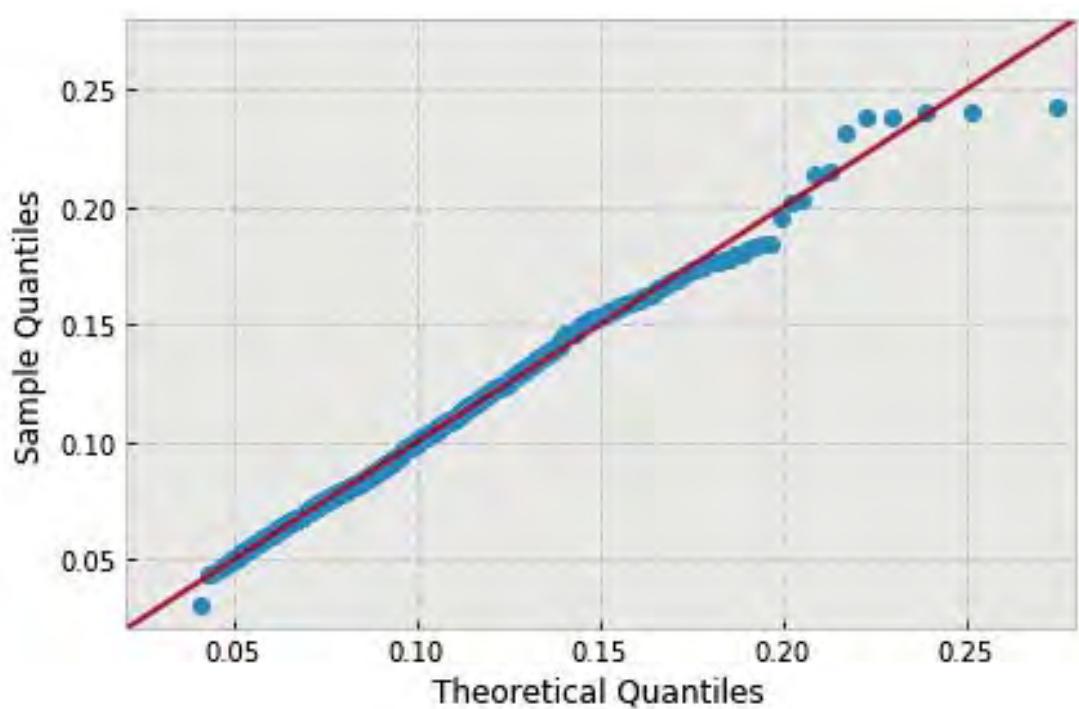
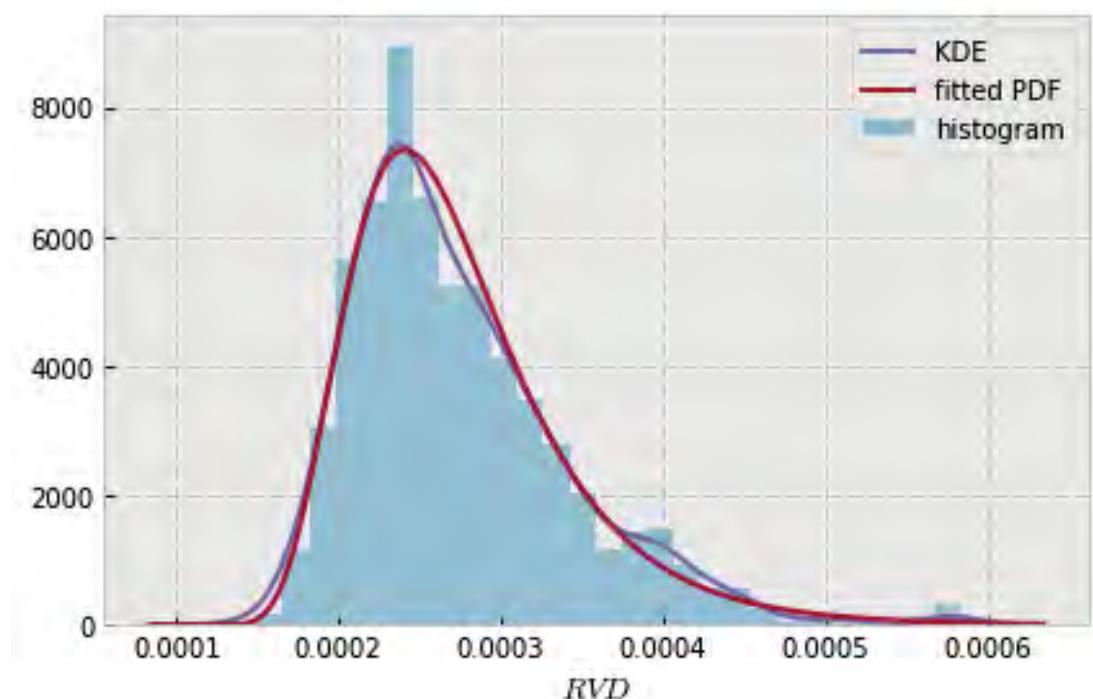


```
variable name: r
variable value: 1.888888888888888
distribution: inverse-gamma(shape=(7.373929083299545,), loc=0.00010894301129596825,
    scale=0.0011858615431974318)
DescribeResult(nobs=1000, minmax=(0.00016188500699774701, 0.00085509548224756409)
    mean=0.00029486991775246496, variance=6.1711423308878432e-09,
    skewness=1.6842913057119206, kurtosis=4.809081239926476)
```

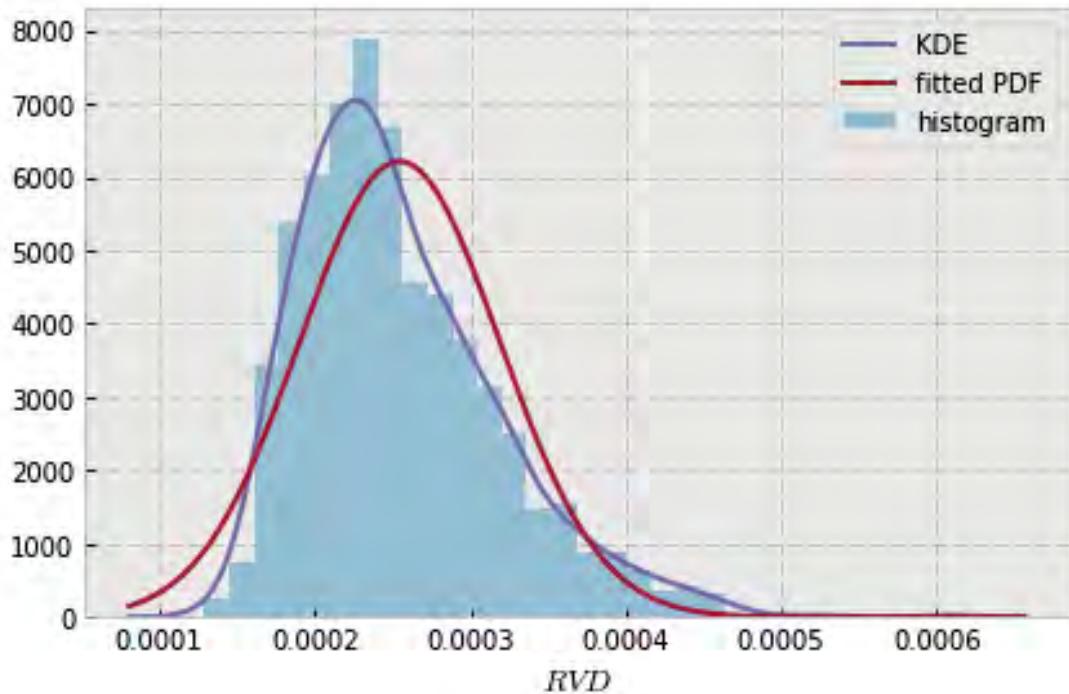


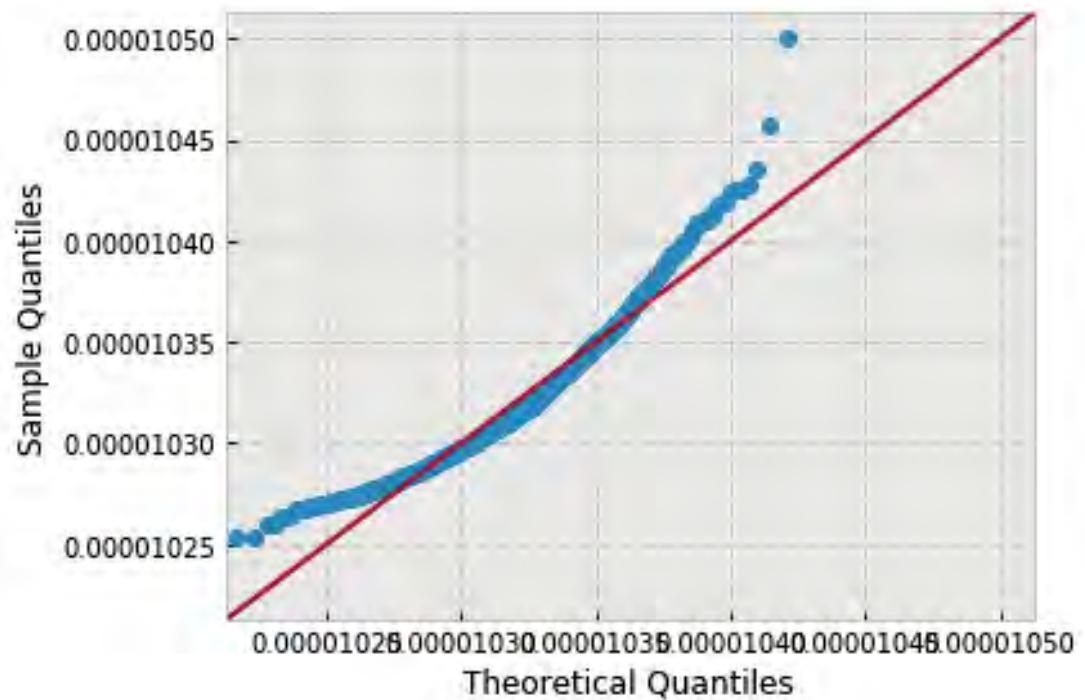


```
variable name: r
variable value: 2.111111111111111
distribution: inverse-gamma(shape=(11.275301138947592,), loc=6.702148922141604e-05,
                           scale=0.002134735168388798)
DescribeResult(nobs=1000, minmax=(0.00013325574007376275, 0.00058364350622420221)
               mean=0.00027475112039486638, variance=4.5708721737884842e-09,
               skewness=1.231905442222493, kurtosis=2.1794642515773512)
```



```
variable name: r
variable value: 2.333333333333333
distribution: inverse-gamma(shape=(96916.168152397673,), loc=-0.019731102971752908,
    scale=1936.9493716258692)
DescribeResult(nobs=1000, minmax=(0.00012893712027707986, 0.00060765248954171671)
    mean=0.00025492588952378479, variance=4.1435840327232571e-09,
    skewness=1.0232117843476143, kurtosis=1.4161036718148763)
```

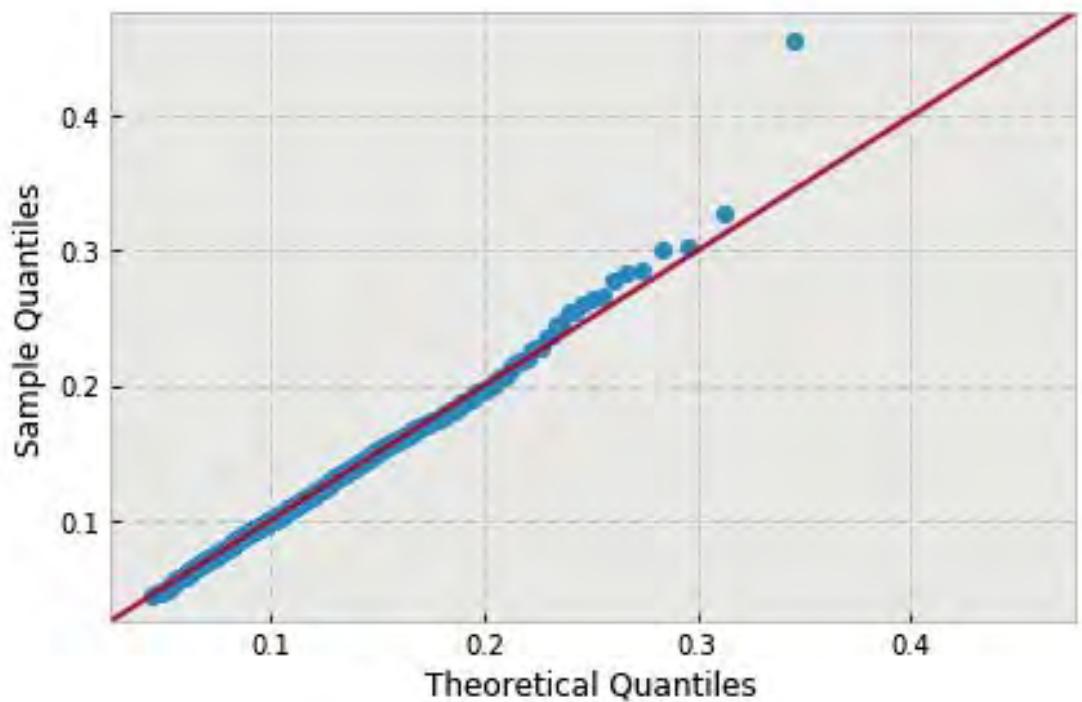
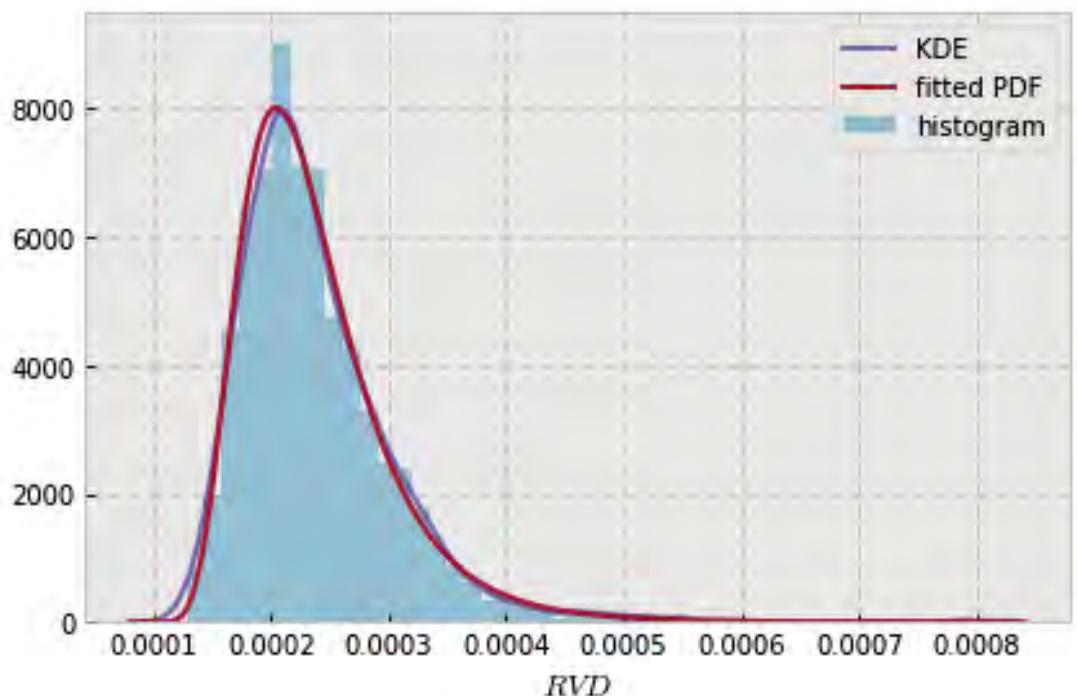




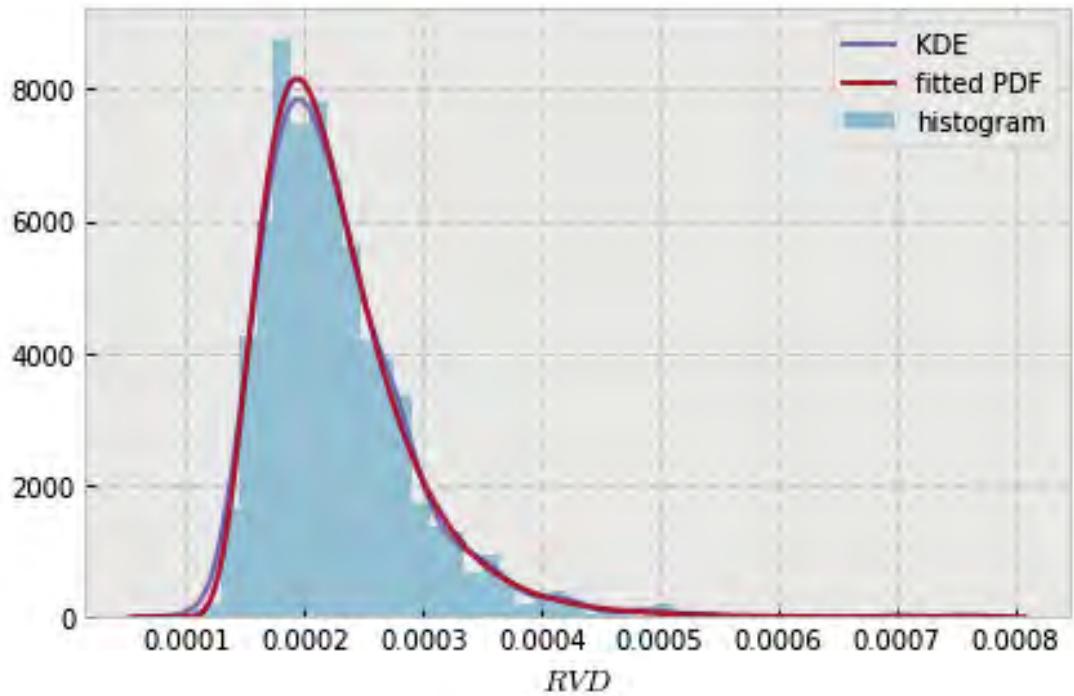
```

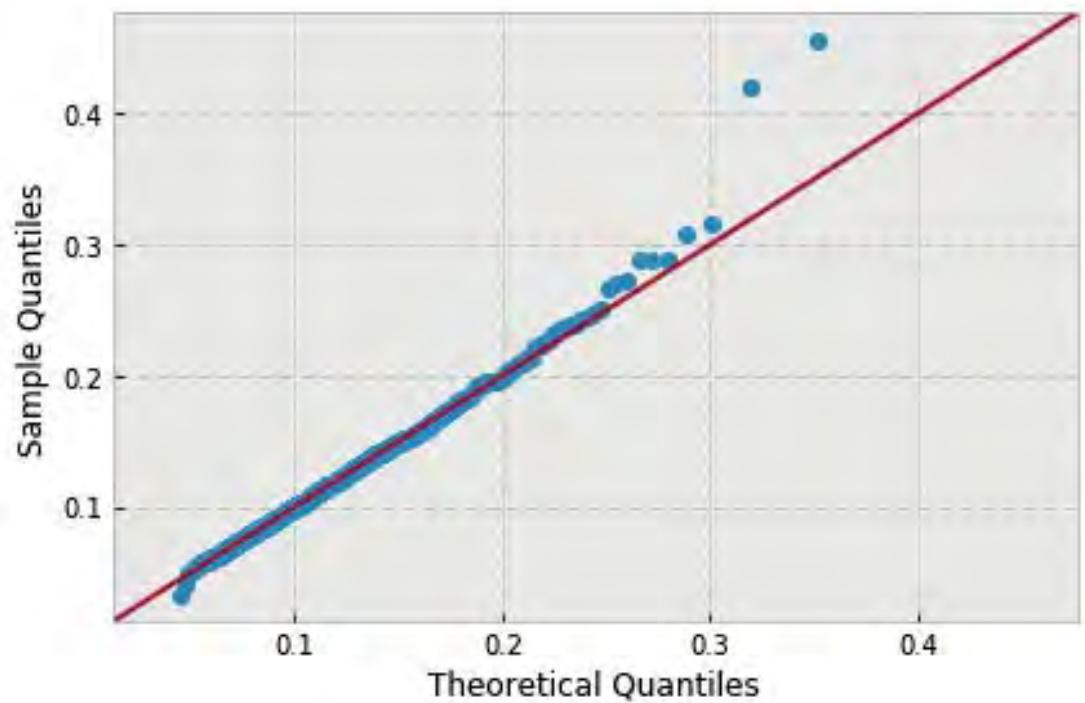
variable name: r
variable value: 2.5555555555555554
distribution: inverse-gamma(shape=(9.8861697508606738,), loc=5.670716280548041e-05,
                           scale=0.001610216514104523)
DescribeResult(nobs=1000, minmax=(0.00012874305258449688, 0.00079086325411914093)
               mean=0.00023798673901077934, variance=4.2933536921098776e-09,
               skewness=1.8622859509394, kurtosis=7.554463391452762)

```

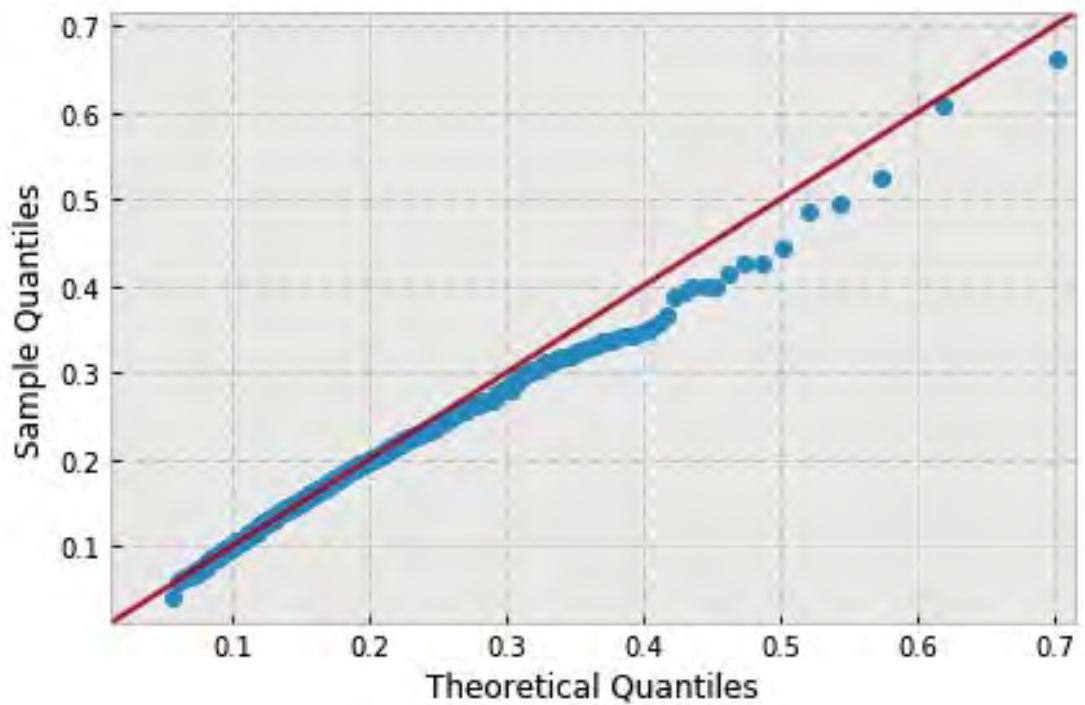
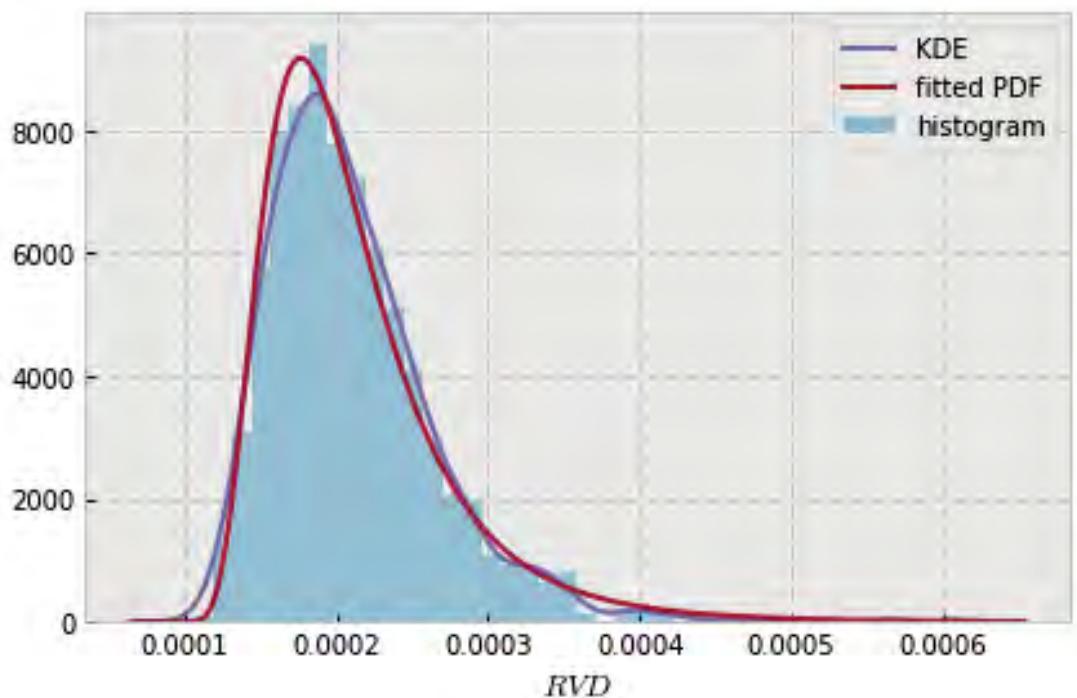


```
variable name: r
variable value: 2.777777777777777
distribution: inverse-gamma(shape=(9.7708047989539271,), loc=4.9198685379451556e-05,
    scale=0.0015558793410409015)
DescribeResult(nobs=1000, minmax=(0.00010090190426898683, 0.00075776601276126123)
    mean=0.00022677166742431556, variance=4.3061658495834299e-09,
    skewness=2.023282122855513, kurtosis=8.636358936344037)
```

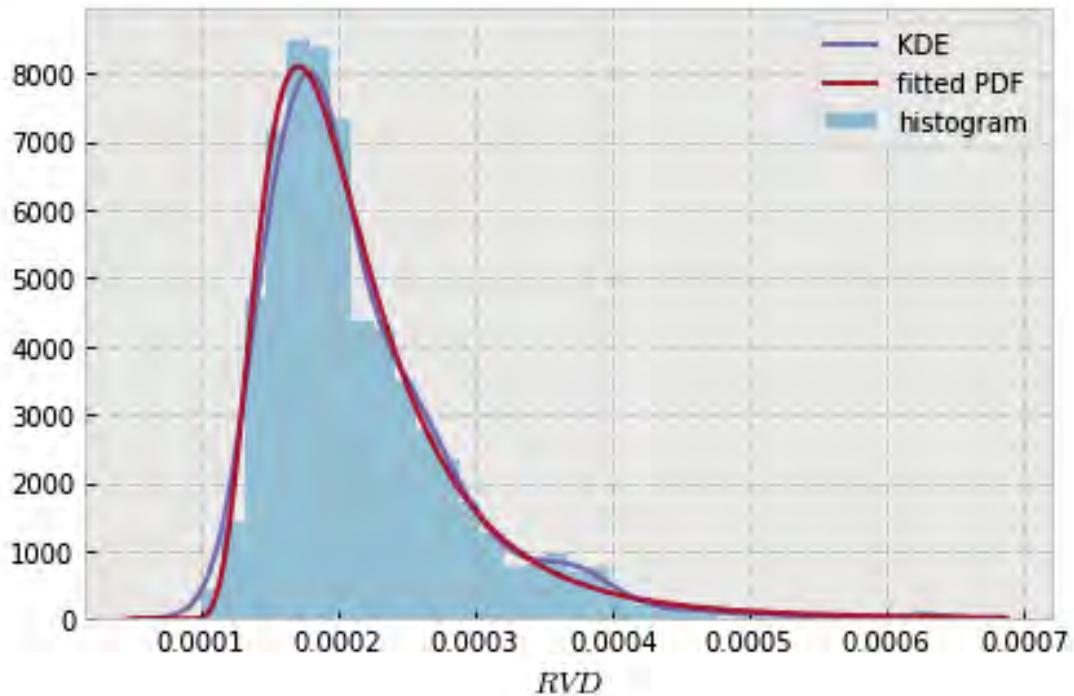


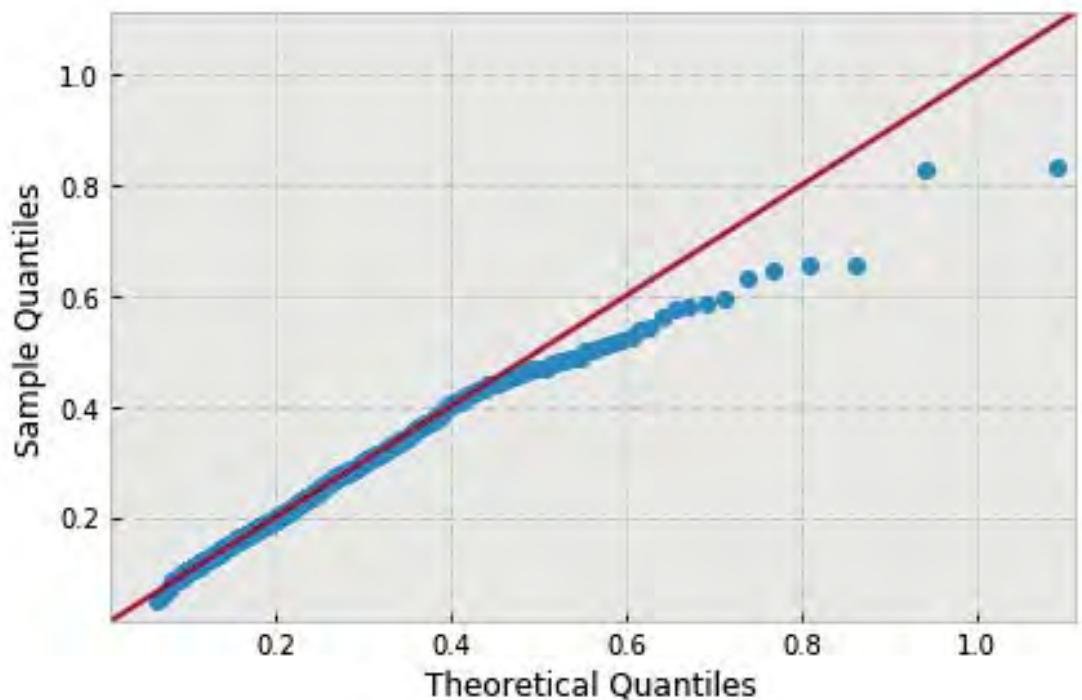


```
variable name: r
variable value: 3.0
distribution: inverse-gamma(shape=(6.7778543967318523,), loc=7.221423904499206e-05,
                           scale=0.0008127519045885931)
DescribeResult(nobs=1000, minmax=(0.00010603627332818829, 0.00061052120943536244)
               mean=0.00021065129458814979, variance=3.1944921715473631e-09,
               skewness=1.6705371941099623, kurtosis=5.688860493199279)
```

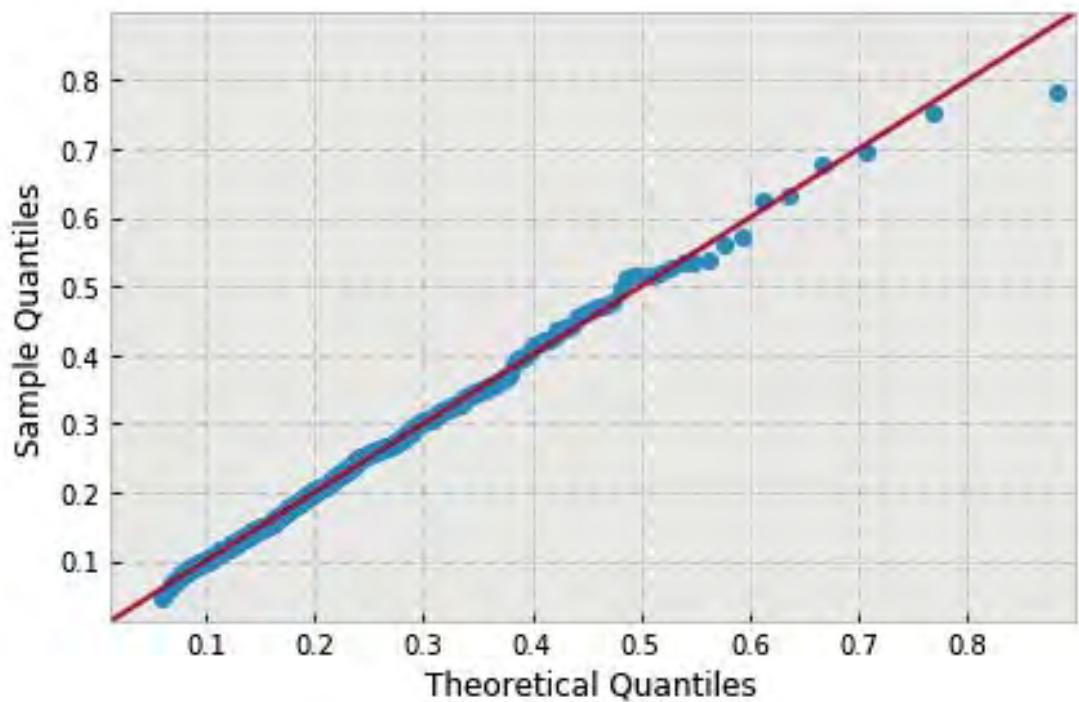
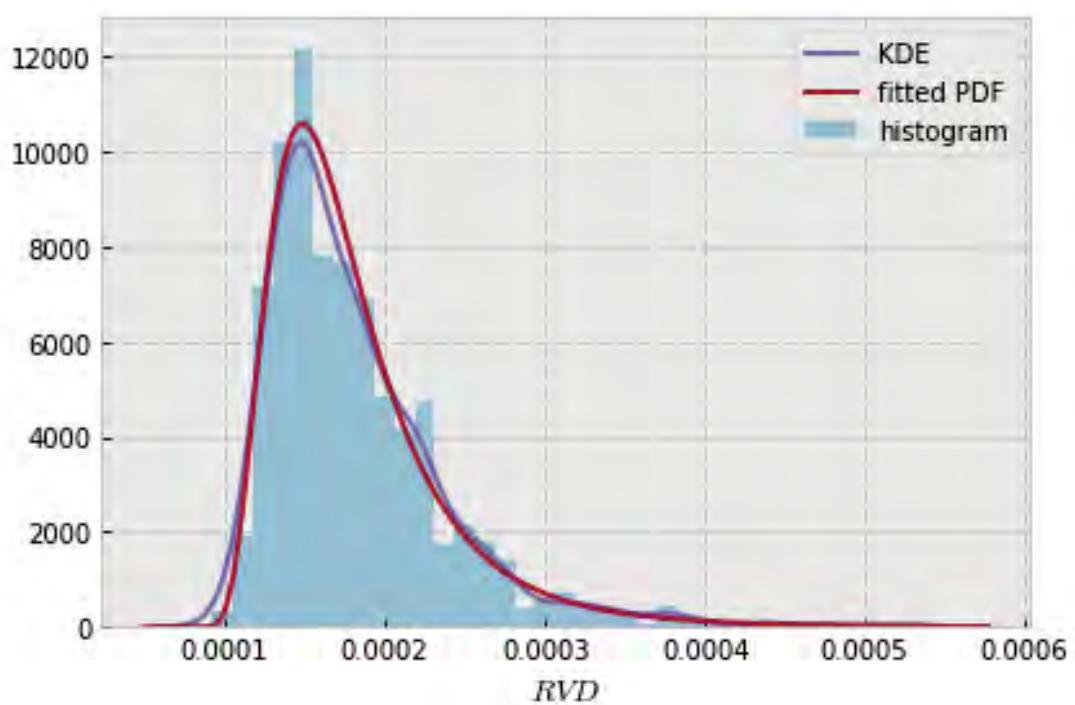


```
variable name: sigma
variable value: 0.01111111111111112
distribution: inverse-gamma(shape=(5.501245975752842,), loc=6.616676144875742e-05,
    scale=0.0006813907035488473)
DescribeResult(nobs=1000, minmax=(9.9690146005562515e-05, 0.00063462938062019275)
    mean=0.00021503920648848935, variance=4.9449265321587766e-09,
    skewness=1.6103109447063164, kurtosis=3.8564508040129306)
```

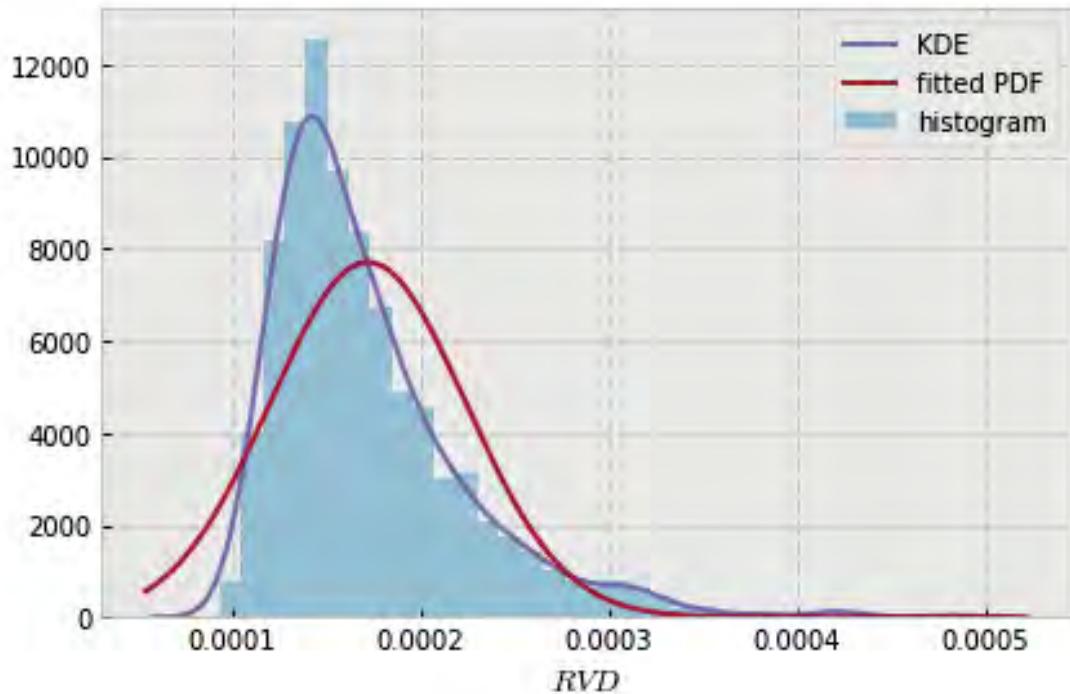


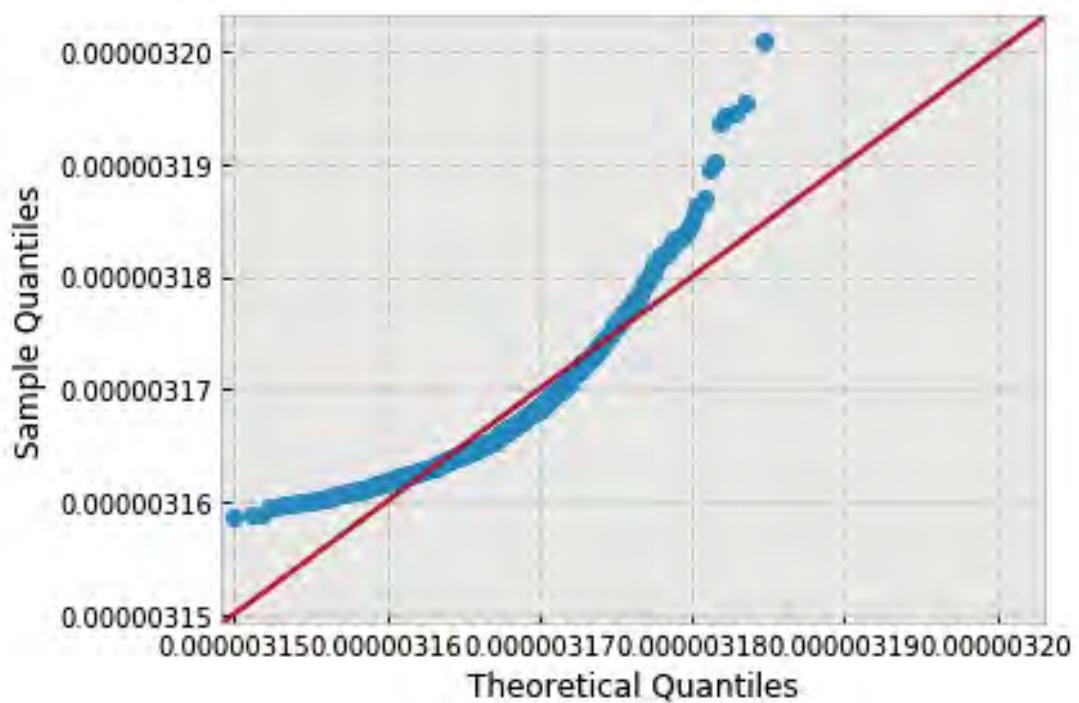


```
variable name: sigma
variable value: 0.02222222222222223
distribution: inverse-gamma(shape=(6.0703919321968307,), loc=6.315937094943387e-05,
    scale=0.0006014016118367795)
DescribeResult(nobs=1000, minmax=(9.2053423701236701e-05, 0.00053404334414177182)
    mean=0.00018166459368791865, variance=3.2543424387009611e-09,
    skewness=1.8906059310022776, kurtosis=5.525850576815367)
```

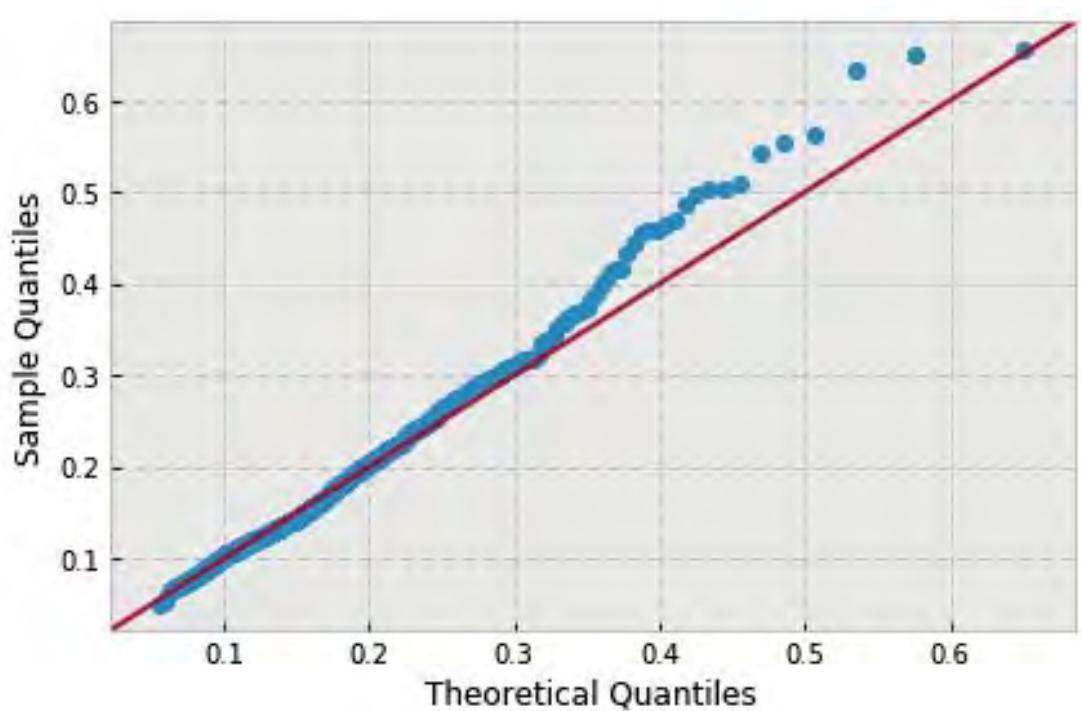
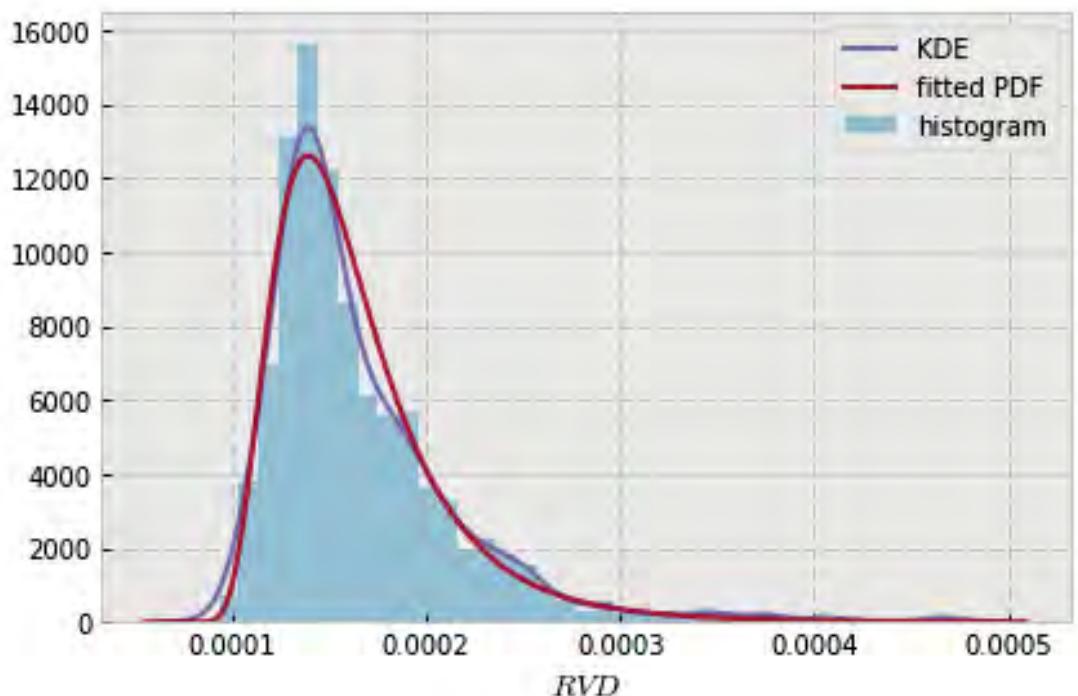


```
variable name: sigma
variable value: 0.03333333333333333
distribution: inverse-gamma(shape=(315733.2995607609,), loc=-0.028925699547413,
                           scale=9187.164530057067)
DescribeResult(nobs=1000, minmax=(9.3029343136950677e-05, 0.00048266886176622349)
               mean=0.00017225550321595207, variance=2.6948073415150883e-09,
               skewness=1.6483925864819957, kurtosis=3.922922266744181)
```

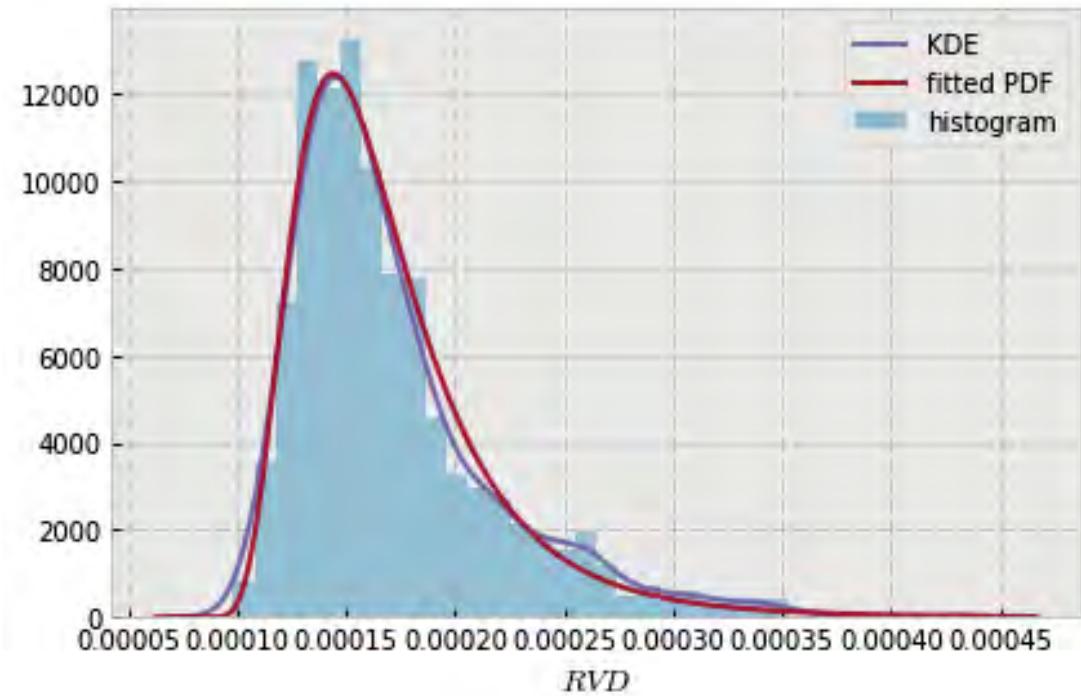


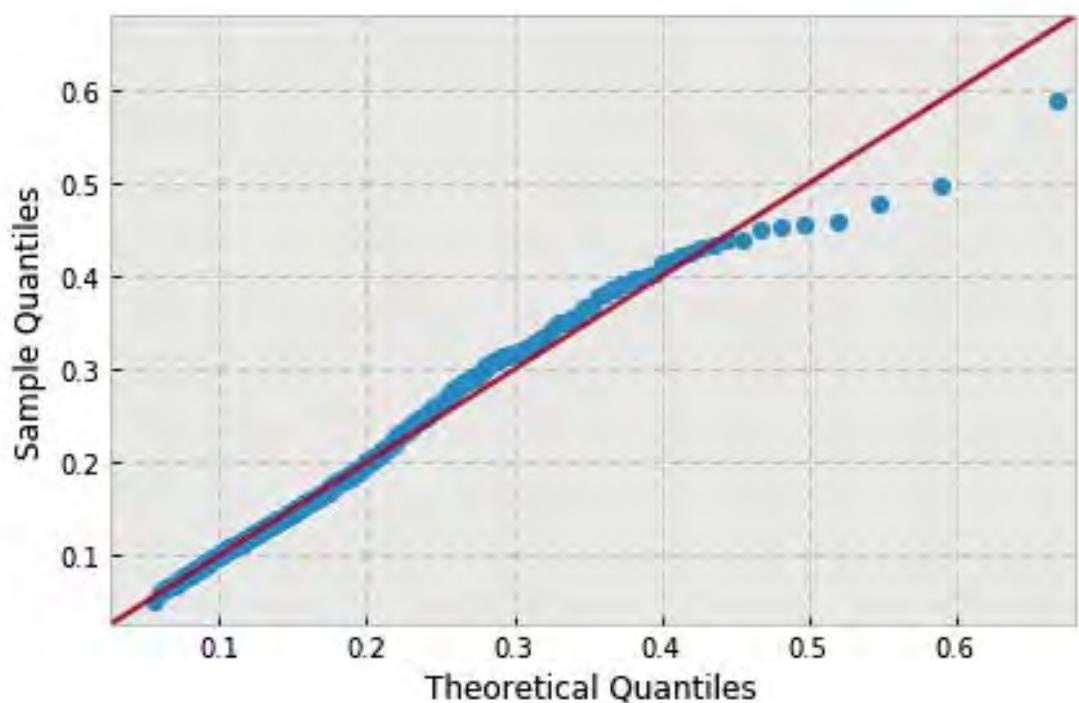


```
variable name: sigma
variable value: 0.04444444444444446
distribution: inverse-gamma(shape=(7.0391485828433851,), loc=6.179800231763817e-05,
    scale=0.0006248190876468347)
DescribeResult(nobs=1000, minmax=(9.3618585016859931e-05, 0.00047118200940351752)
    mean=0.00016651225800309196, variance=2.5149908783507719e-09,
    skewness=2.1188232736881116, kurtosis=6.6843787527406135)
```

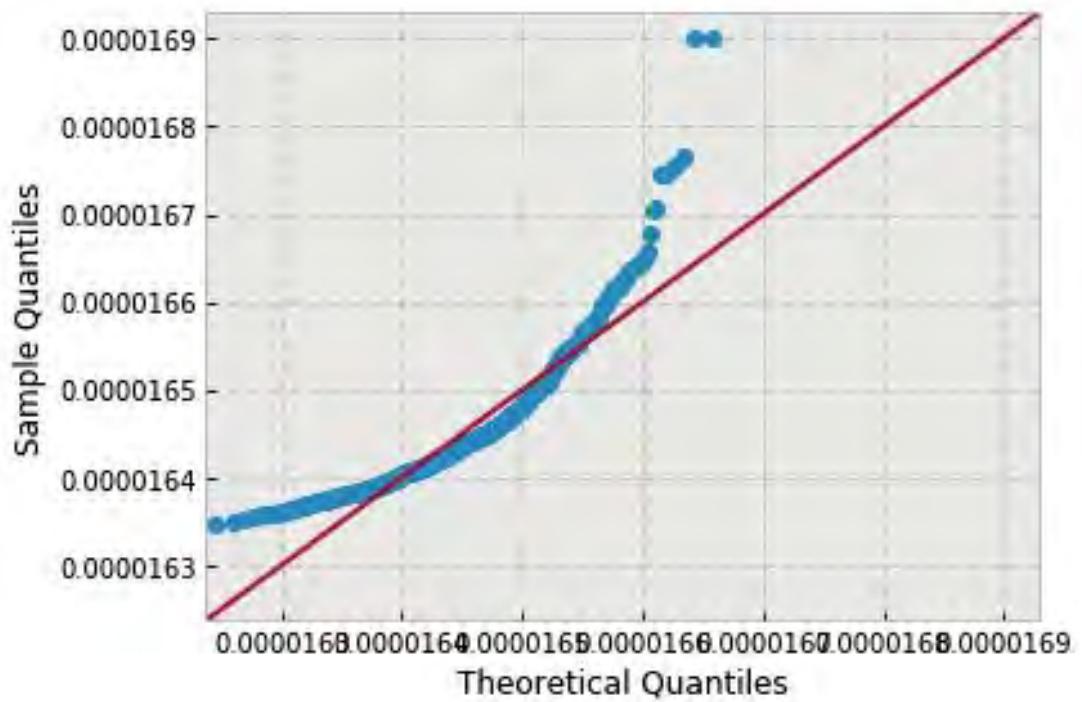
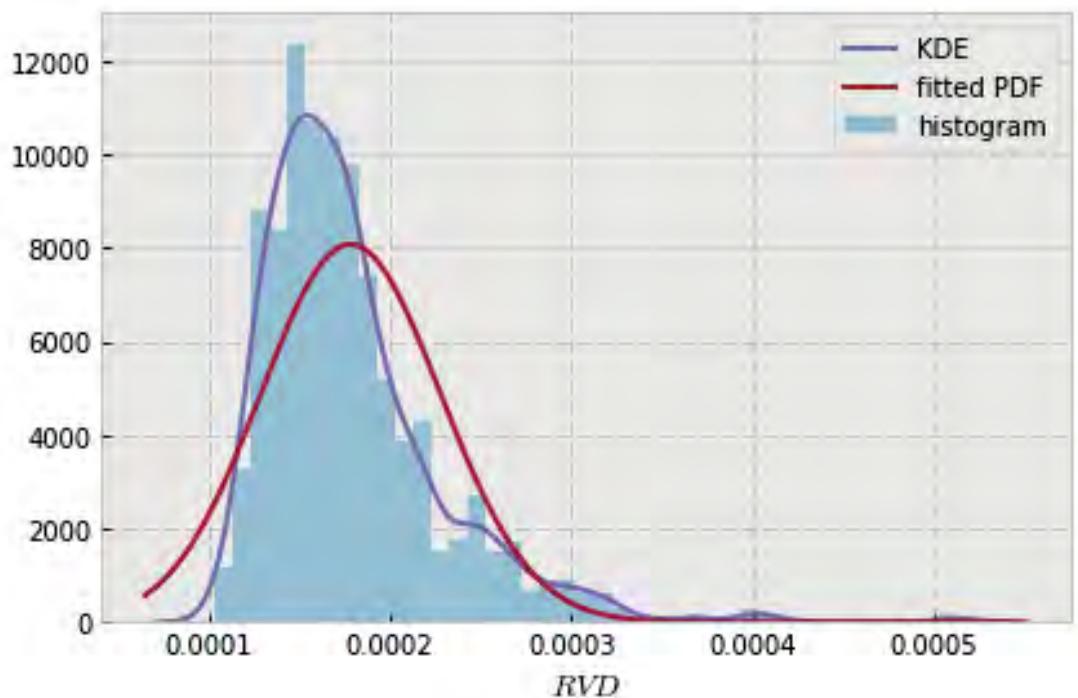


```
variable name: sigma
variable value: 0.05555555555555555
distribution: inverse-gamma(shape=(6.9460798872980583,), loc=6.60510610674006e-05,
                           scale=0.0006199894983115633)
DescribeResult(nobs=1000, minmax=(9.8124301329047337e-05, 0.00043097646357016632)
               mean=0.00017086599610996515, variance=2.2153039219331675e-09,
               skewness=1.5450799318172783, kurtosis=2.841183358585588)
```

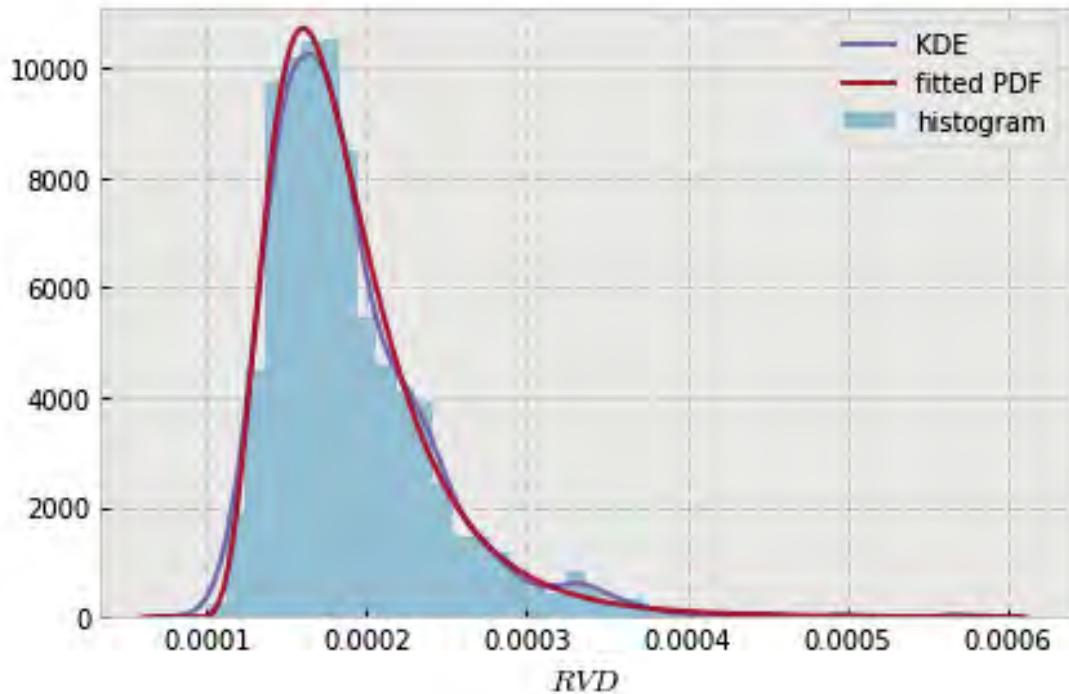


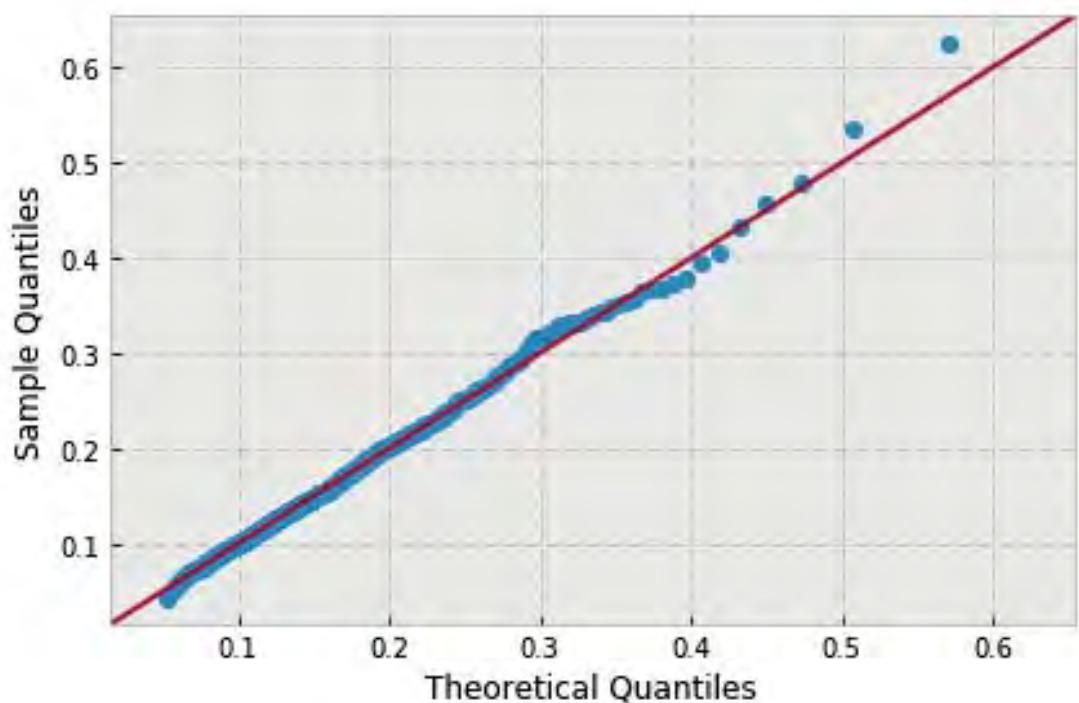


```
variable name: sigma
variable value: 0.06666666666666667
distribution: inverse-gamma(shape=(60791.248331857445,), loc=-0.012007356250877526,
    scale=740.782456708926)
DescribeResult(nobs=1000, minmax=(0.00010267002121520114, 0.00051292790436233702)
    mean=0.00017851972095312939, variance=2.469661946610575e-09,
    skewness=1.862363978747267, kurtosis=5.999162430302469)
```

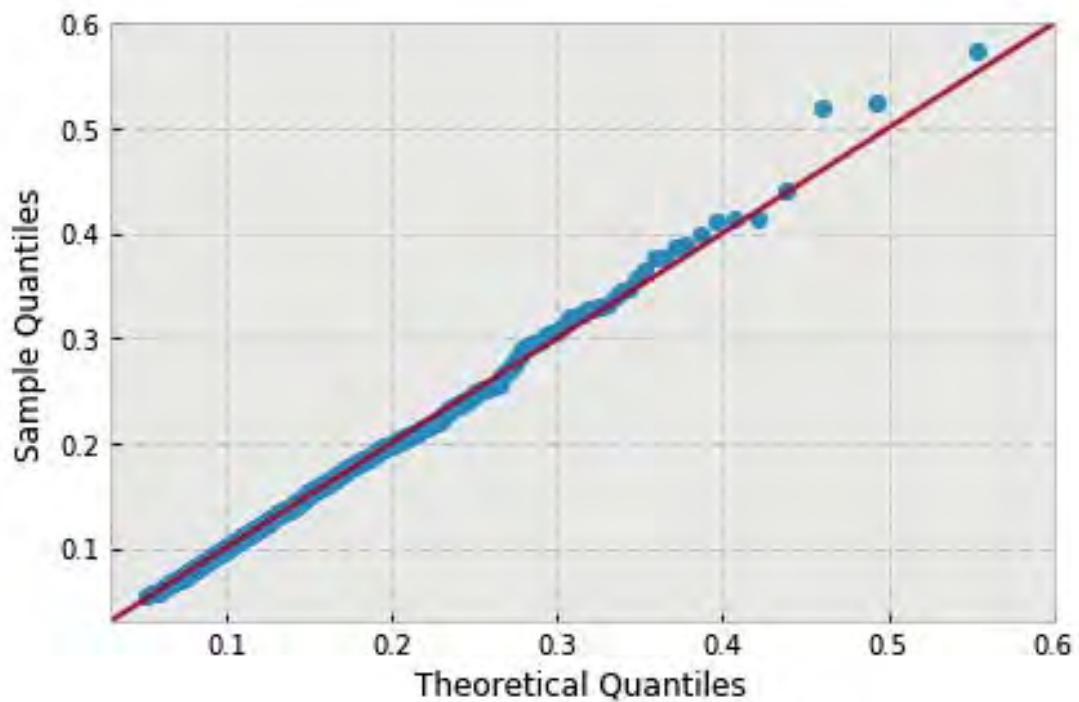
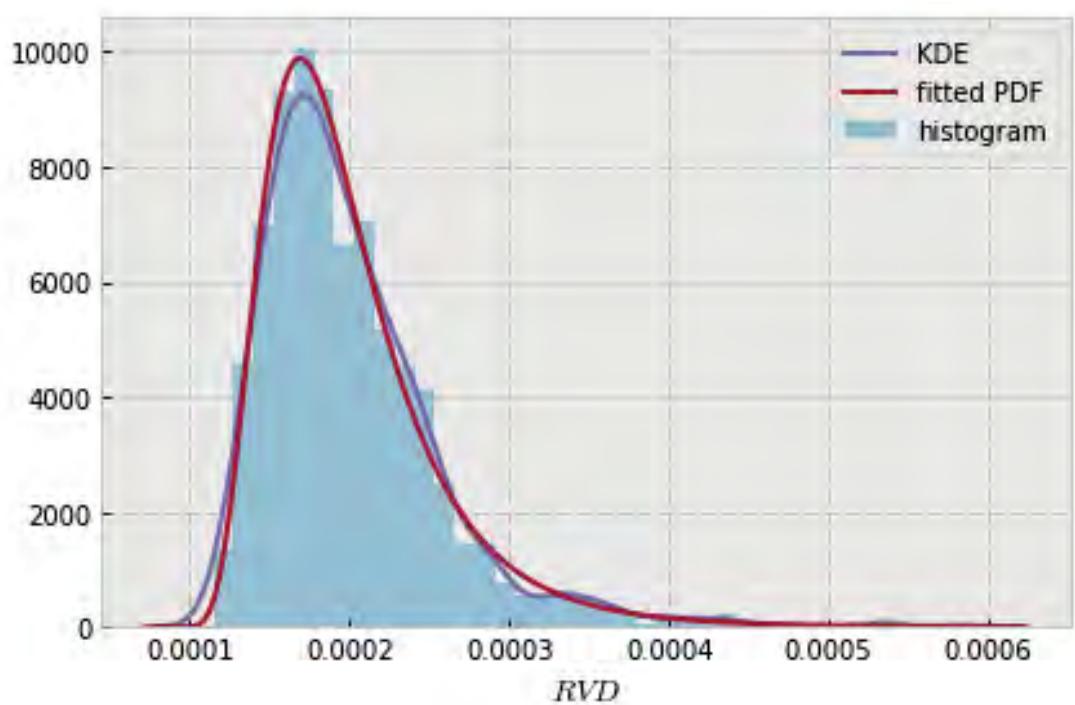


```
variable name: sigma
variable value: 0.07777777777777778
distribution: inverse-gamma(shape=(7.5274102637979592,), loc=6.534728190267806e-05,
    scale=0.0008100650549004886)
DescribeResult(nobs=1000, minmax=(0.0001007665664186771, 0.00057140717444815635)
    mean=0.00018946907434055498, variance=2.7675575733764984e-09,
    skewness=1.8308381514094156, kurtosis=5.845936088344624)
```

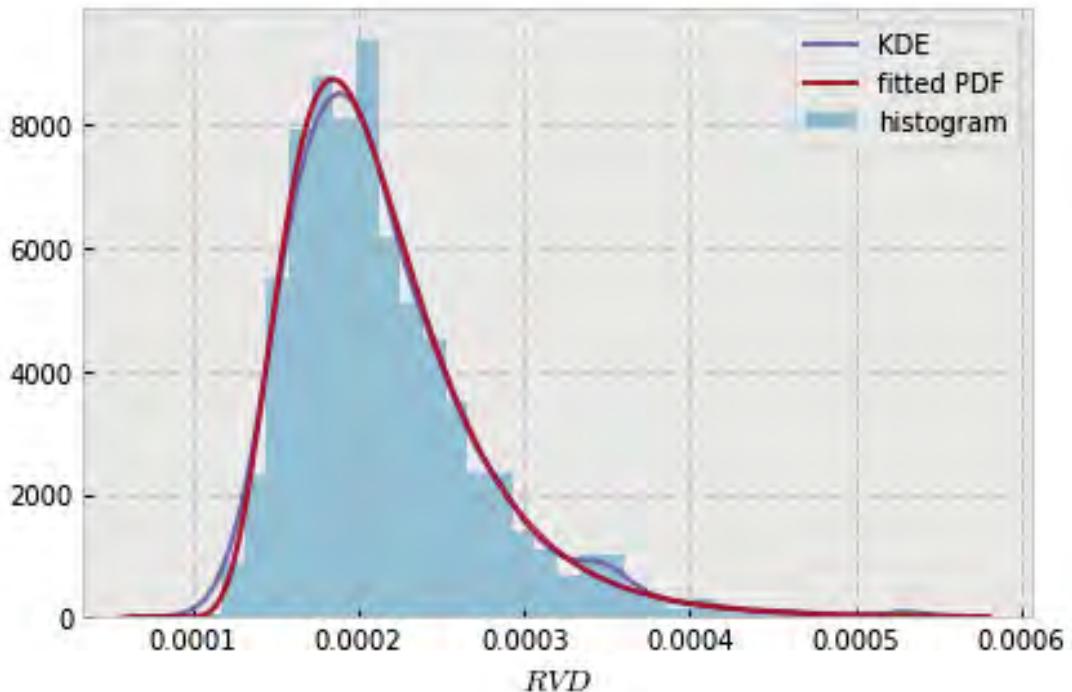


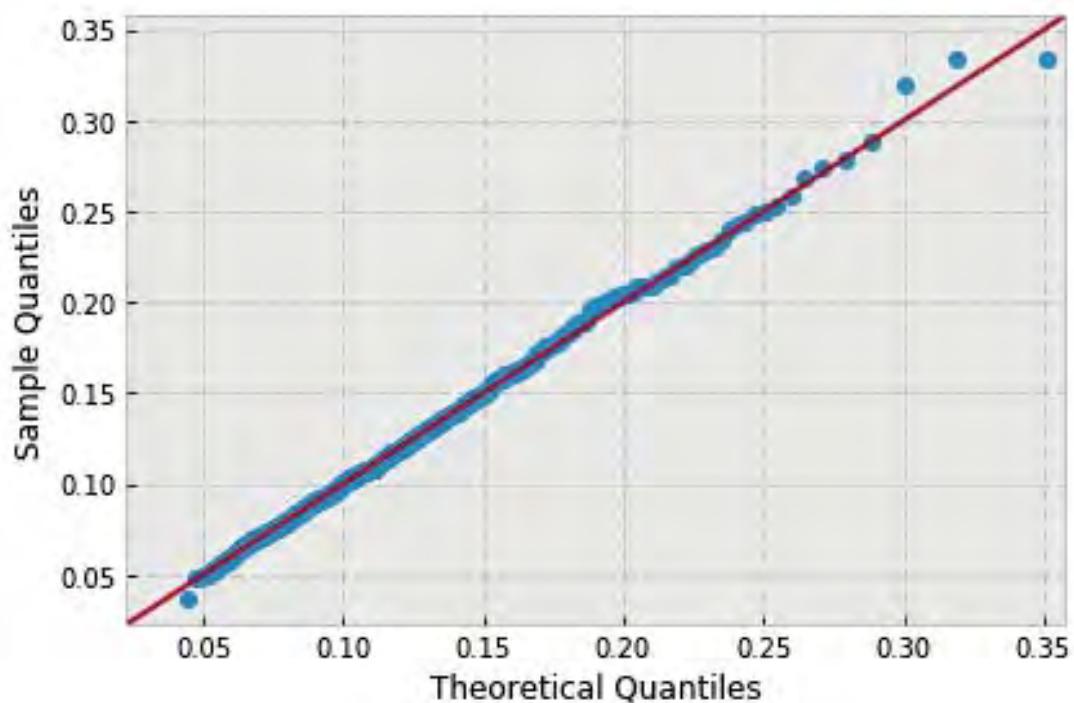


```
variable name: sigma
variable value: 0.08888888888888889
distribution: inverse-gamma(shape=(7.6412156267846605,), loc=6.519710388564133e-05,
                           scale=0.0008976064976212277)
DescribeResult(nobs=1000, minmax=(0.00011472069141484329, 0.00058088202821115868)
               mean=0.00020033012054966604, variance=3.2051178669333627e-09,
               skewness=1.8637747755261418, kurtosis=6.050014286060023)
```

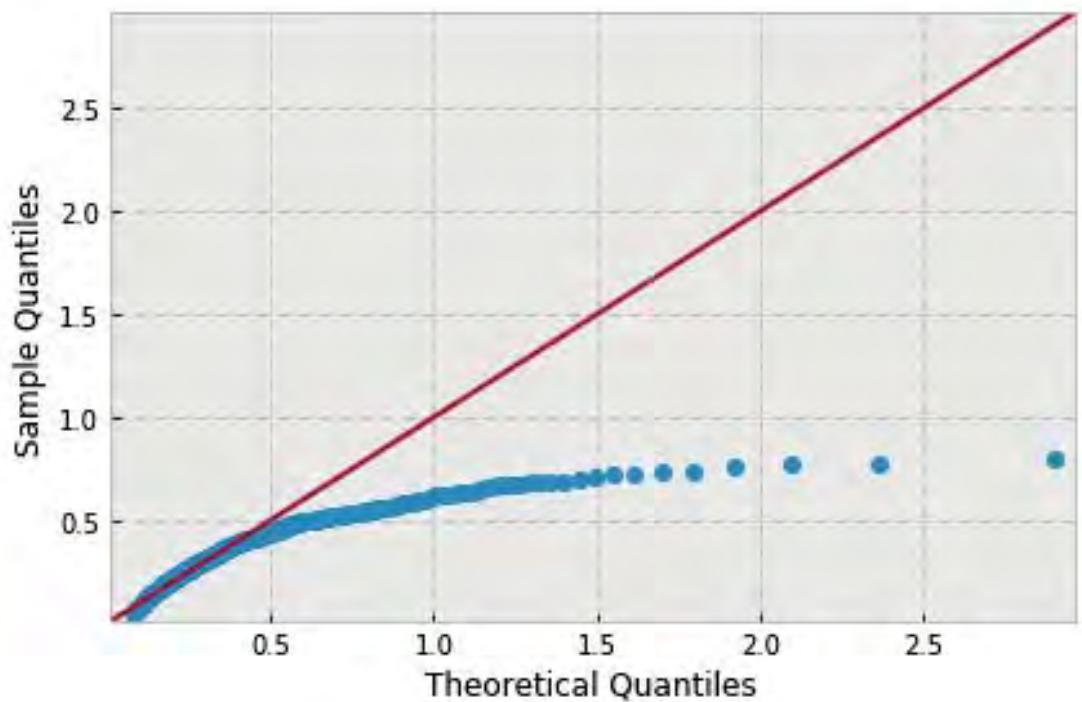
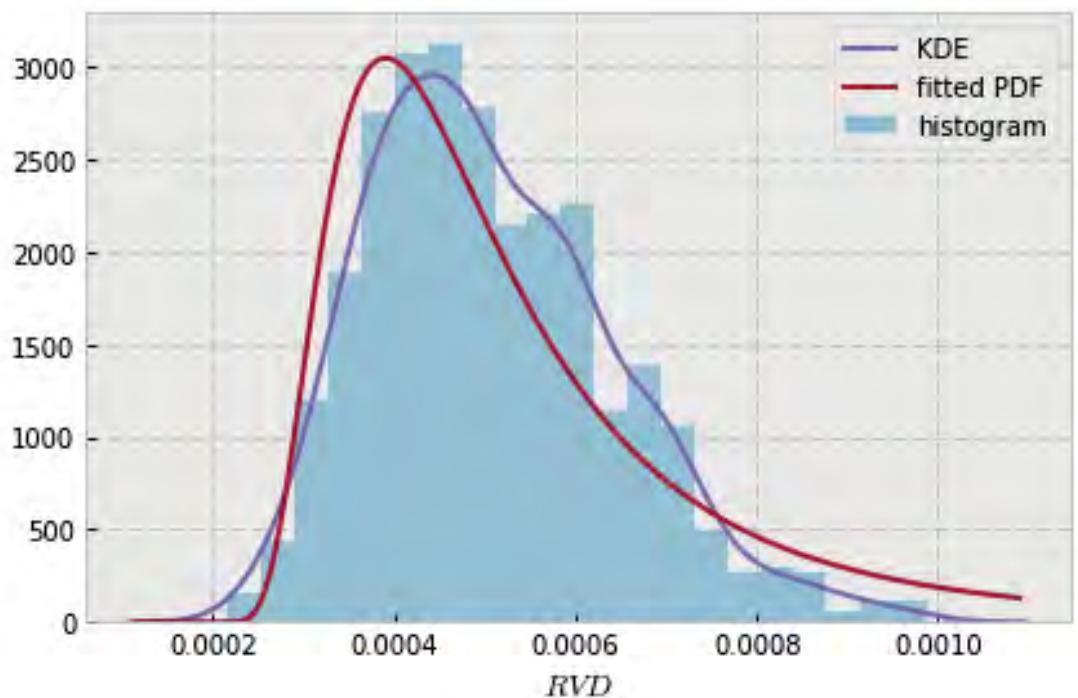


```
variable name: sigma
variable value: 0.1
distribution: inverse-gamma(shape=(9.780139944615781,), loc=4.9590362771853116e-05,
    scale=0.0014540414043000783)
DescribeResult(nobs=1000, minmax=(0.00010370076465254685, 0.00053561175537209221)
    mean=0.00021519916253889945, variance=3.497189543929493e-09,
    skewness=1.4549378715669543, kurtosis=3.325504140894343)
```

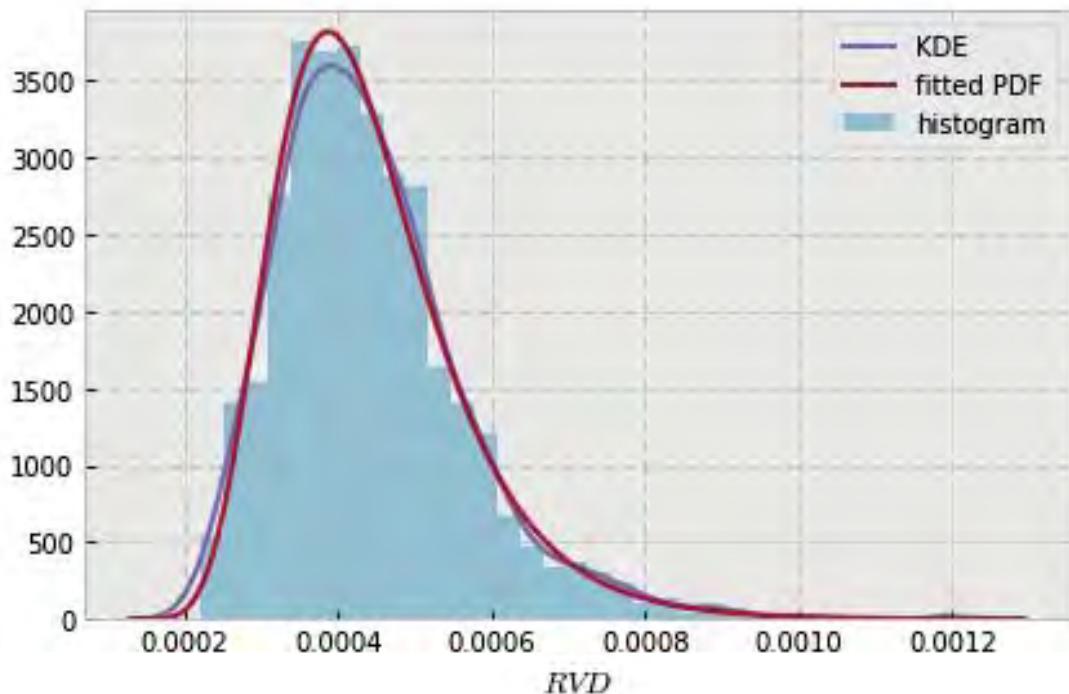


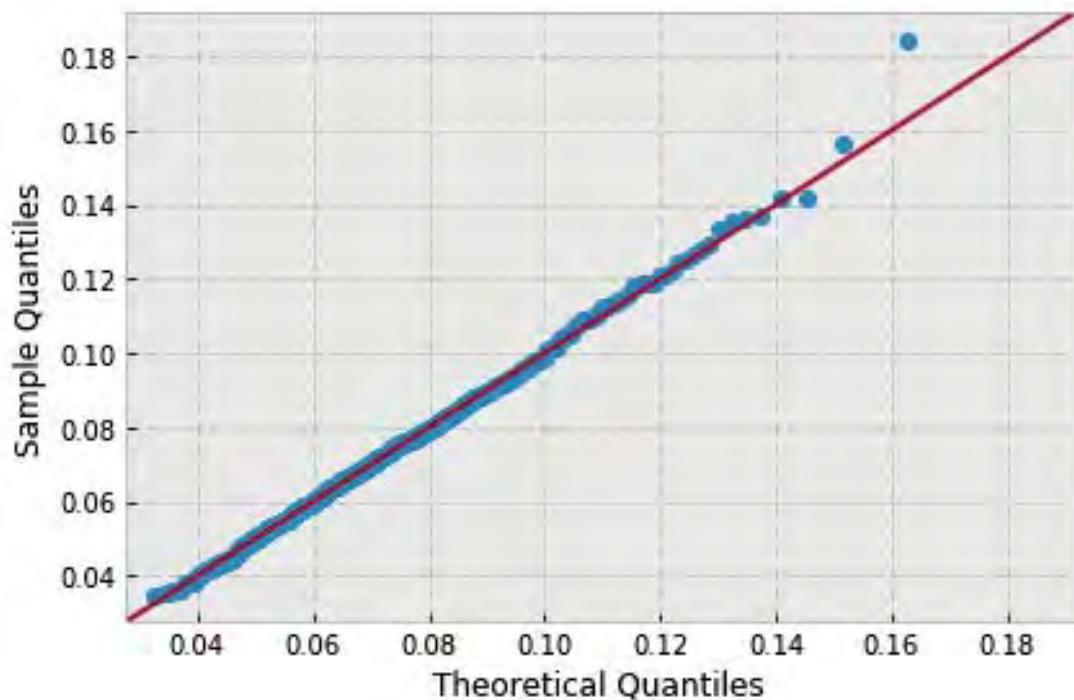


```
variable name: theta
variable value: 0.6981317007977318
distribution: inverse-gamma(shape=(3.6832481793337282,), loc=0.0001718873886398498,
                           scale=0.0010247242040539637)
DescribeResult(nobs=1000, minmax=(0.00021730313111118535, 0.00098787394604101441)
               mean=0.00050753049857437673, variance=1.8447855215452052e-08,
               skewness=0.6289199318327575, kurtosis=0.19031840127250854)
```

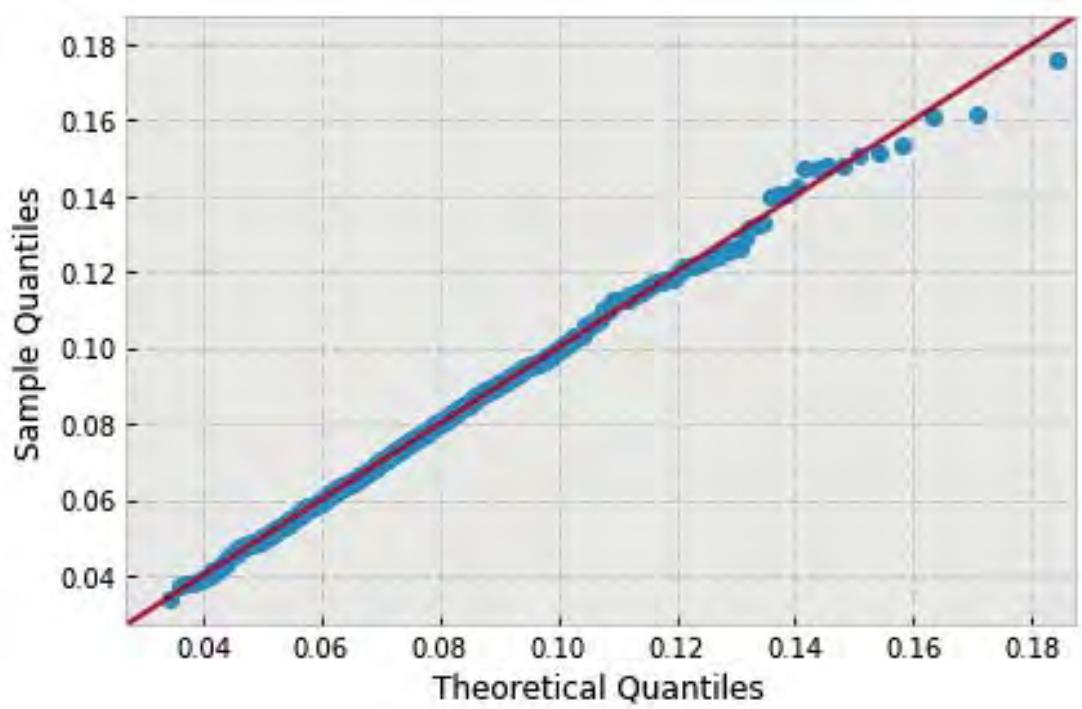
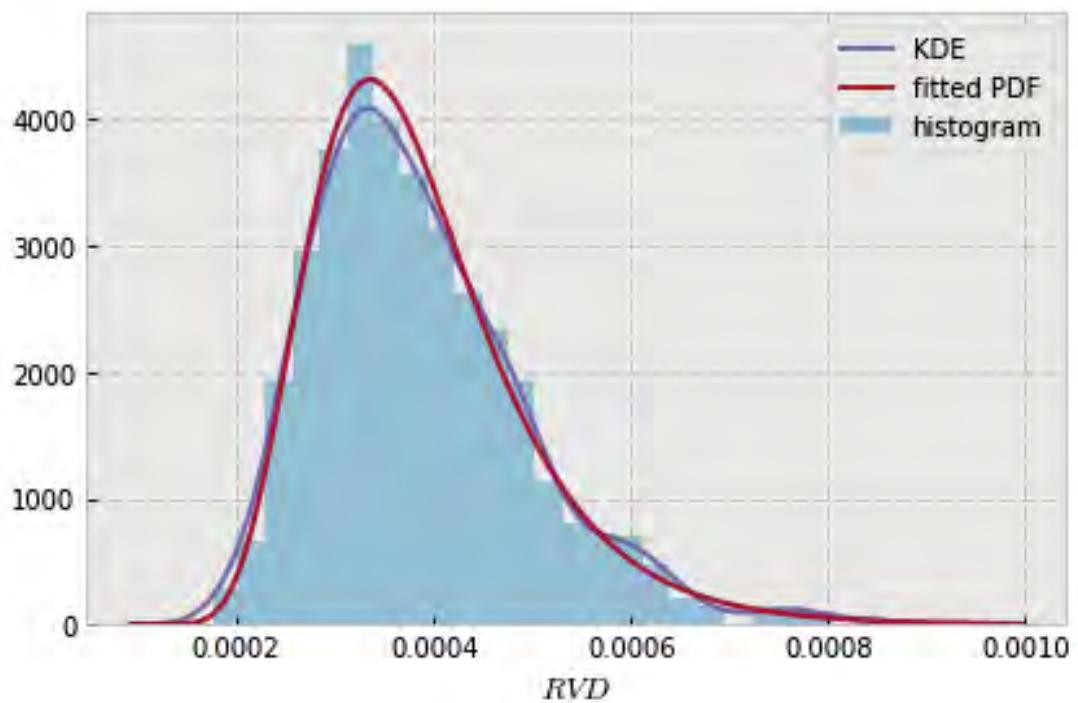


```
variable name: theta
variable value: 0.7853981633974483
distribution: inverse-gamma(shape=(15.56793939745911,), loc=-1.0814508628847151e-05,
    scale=0.006584990061455776)
DescribeResult(nobs=1000, minmax=(0.00021971921983182855, 0.0012013461391123725)
    mean=0.00044119509606584475, variance=1.5060375260271788e-08,
    skewness=1.1715821243645255, kurtosis=2.6603715181469534)
```

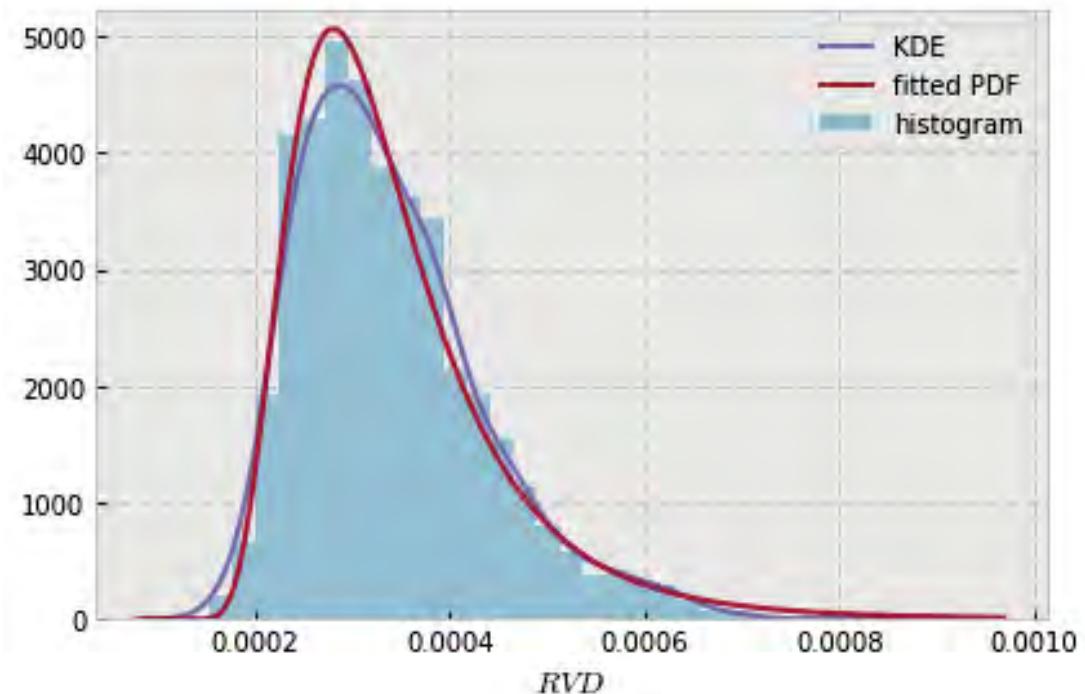


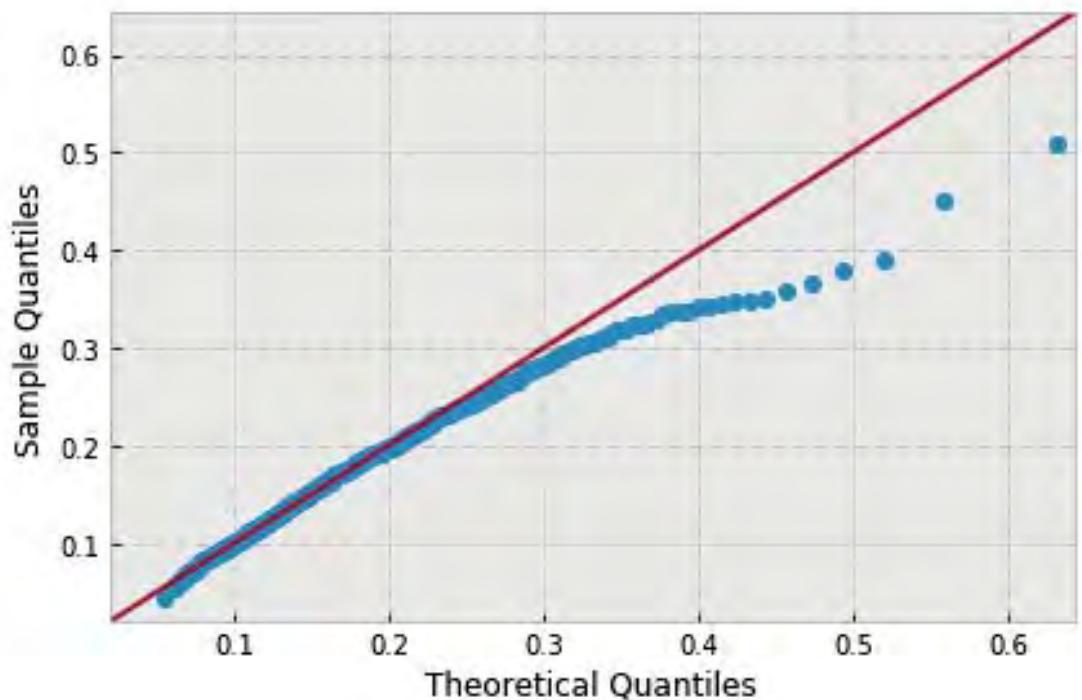


```
variable name: theta
variable value: 0.8726646259971648
distribution: inverse-gamma(shape=(14.397300950756049,), loc=-1.8758709138757787e-06,
                           scale=0.005197627591193225)
DescribeResult(nobs=1000, minmax=(0.00017487548463644408, 0.00091317504432959784)
               mean=0.00038599541595977317, variance=1.1881707970537097e-08,
               skewness=1.046505585014973, kurtosis=1.601312132046366)
```

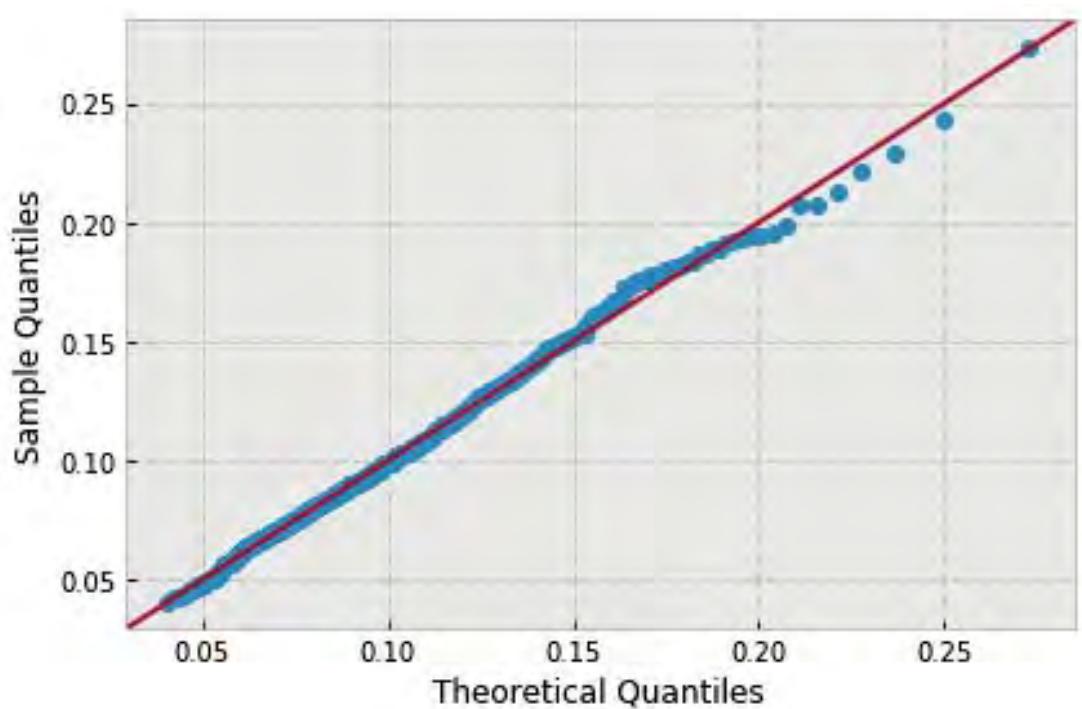
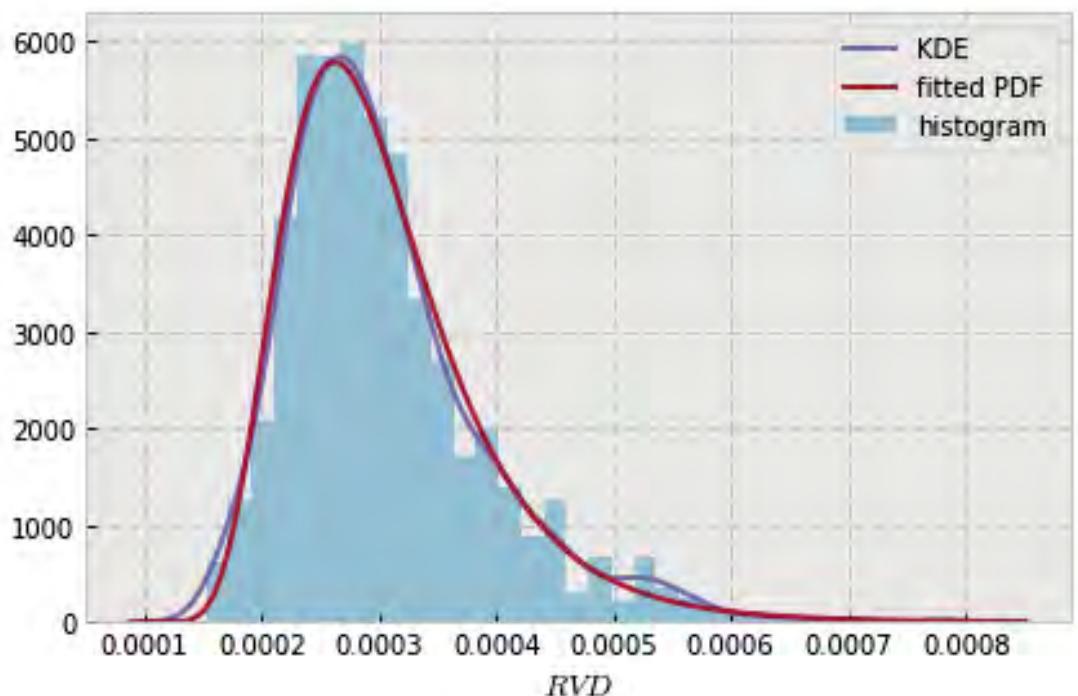


```
variable name: theta
variable value: 0.9599310885968813
distribution: inverse-gamma(shape=(7.1463041652359713,), loc=8.508153329143253e-05,
    scale=0.001589384550361997)
DescribeResult(nobs=1000, minmax=(0.00015297450258788058, 0.00089538272499553291)
    mean=0.00033946791309349327, variance=9.4159704815027297e-09,
    skewness=1.1378799729800801, kurtosis=1.99422058835414)
```

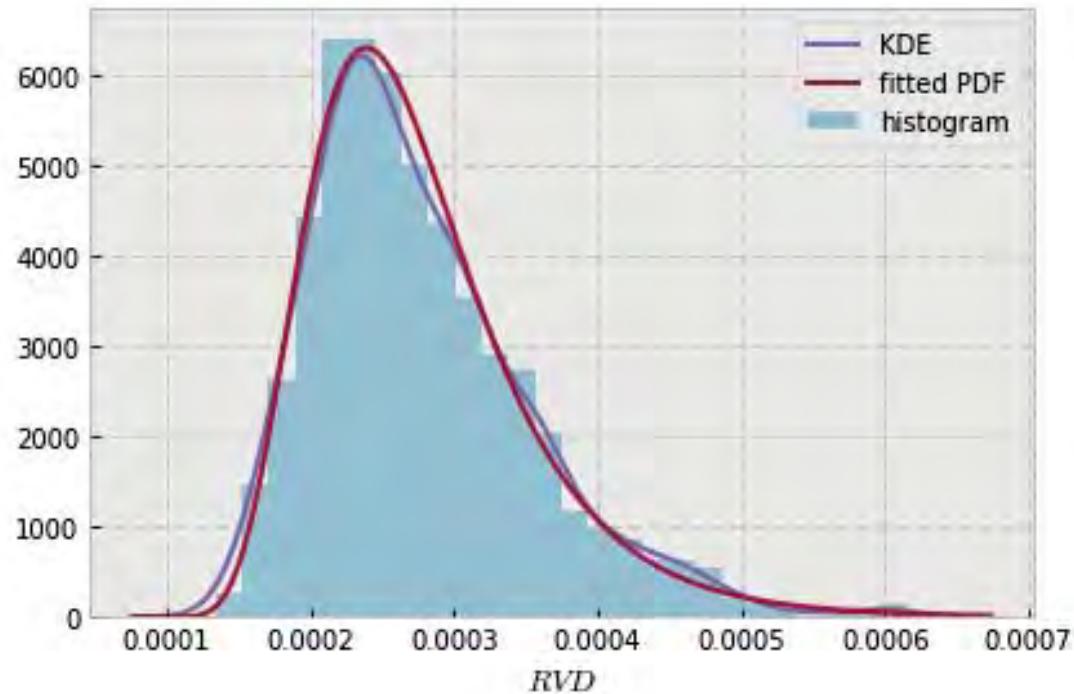


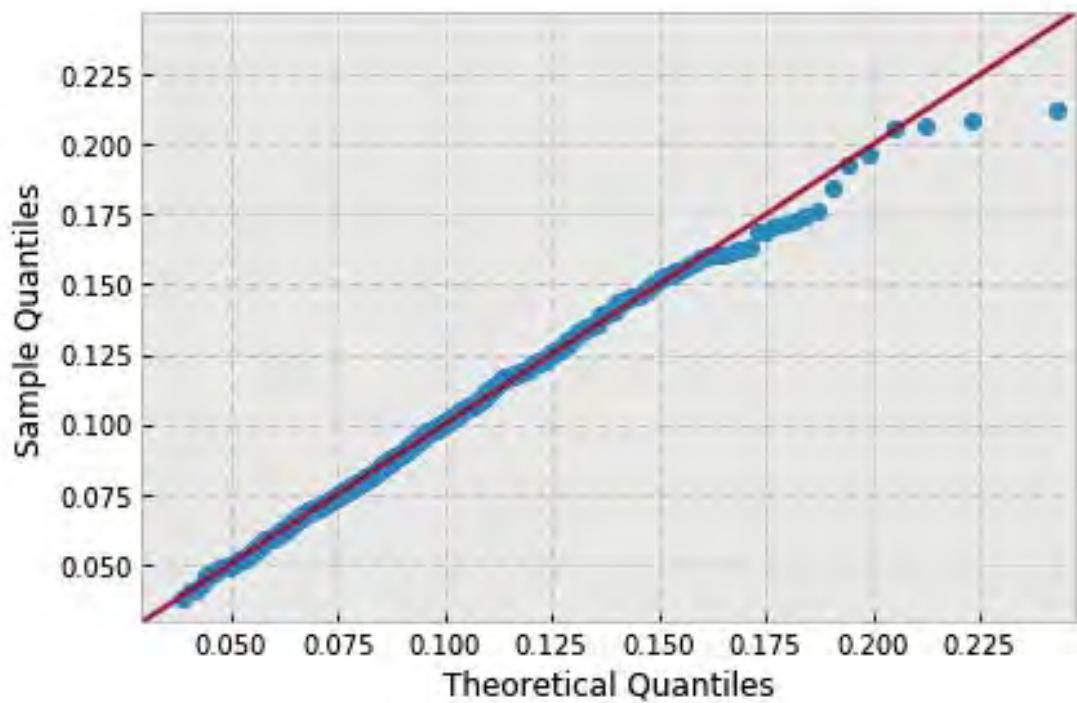


```
variable name: theta
variable value: 1.0471975511965979
distribution: inverse-gamma(shape=(11.331135722766131,), loc=4.114667855354841e-05,
                           scale=0.002725225071645478)
DescribeResult(nobs=1000, minmax=(0.00015249645291963329, 0.0007859921510821919)
               mean=0.00030493326788087418, variance=7.4000230714167897e-09,
               skewness=1.2908884292818252, kurtosis=2.4242650923175892)
```

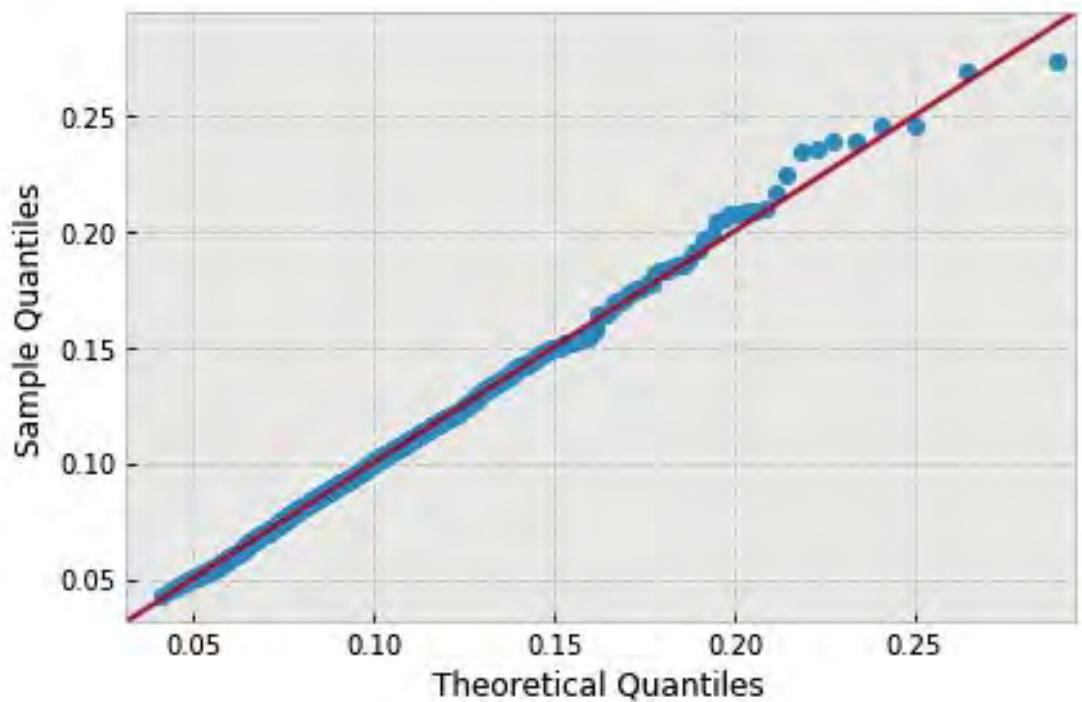
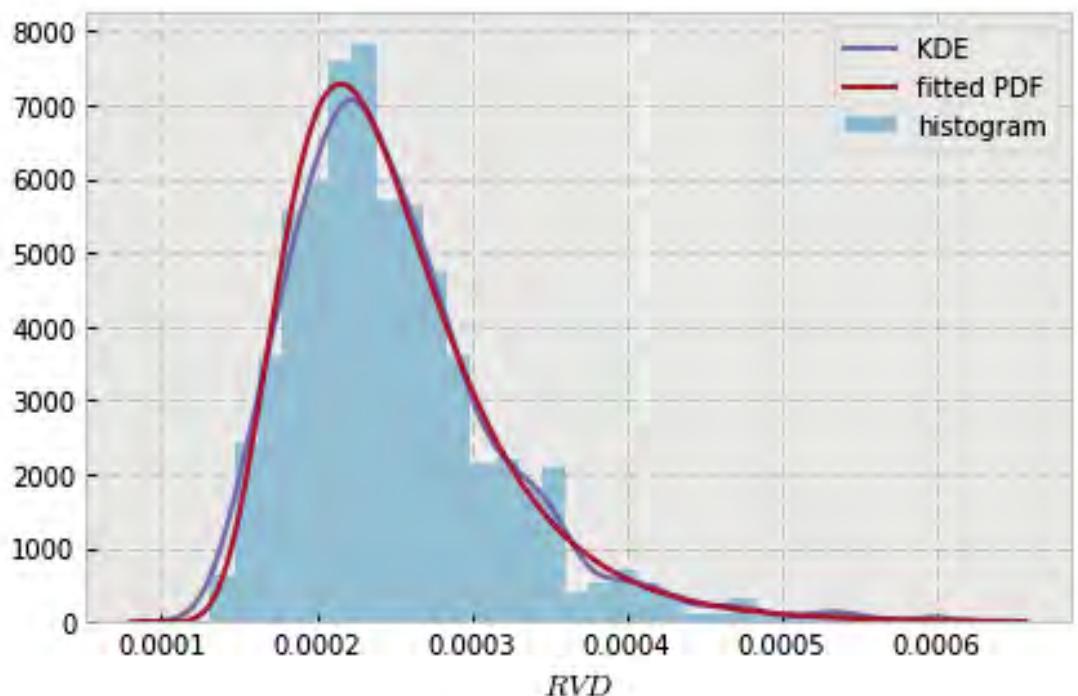


```
variable name: theta
variable value: 1.1344640137963142
distribution: inverse-gamma(shape=(12.134403758526876,), loc=2.841088555192341e-05,
                           scale=0.0027639019676780046)
DescribeResult(nobs=1000, minmax=(0.00013348633936523989, 0.00061583248819473385)
               mean=0.00027652289996693091, variance=5.8457911934422563e-09,
               skewness=1.0737668772535522, kurtosis=1.5022803573136532)
```

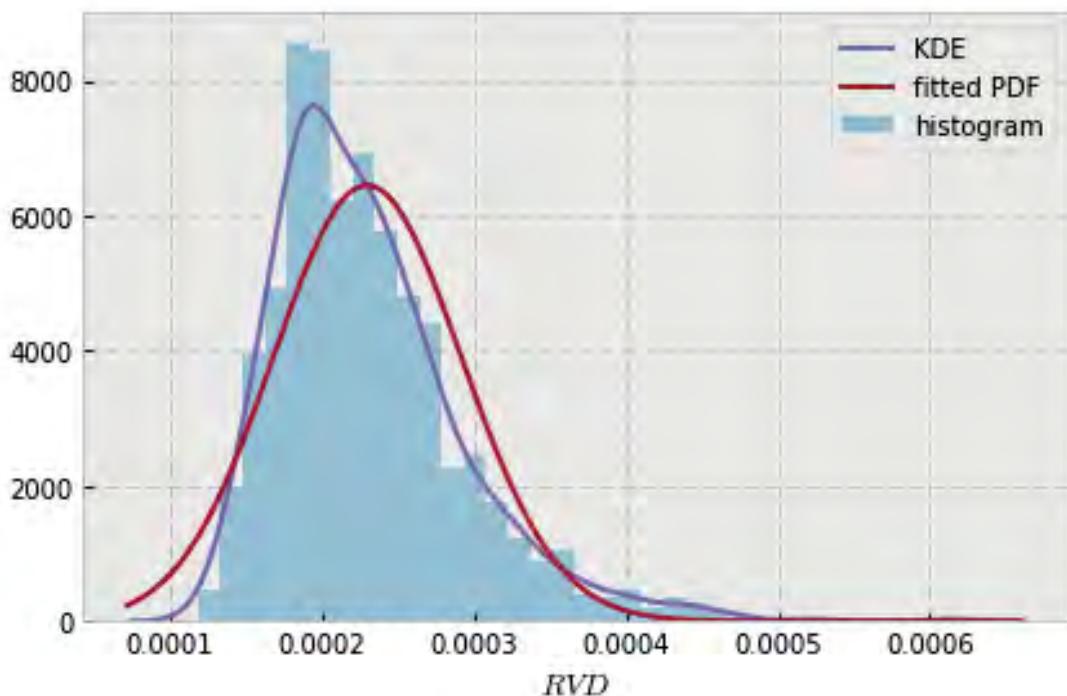


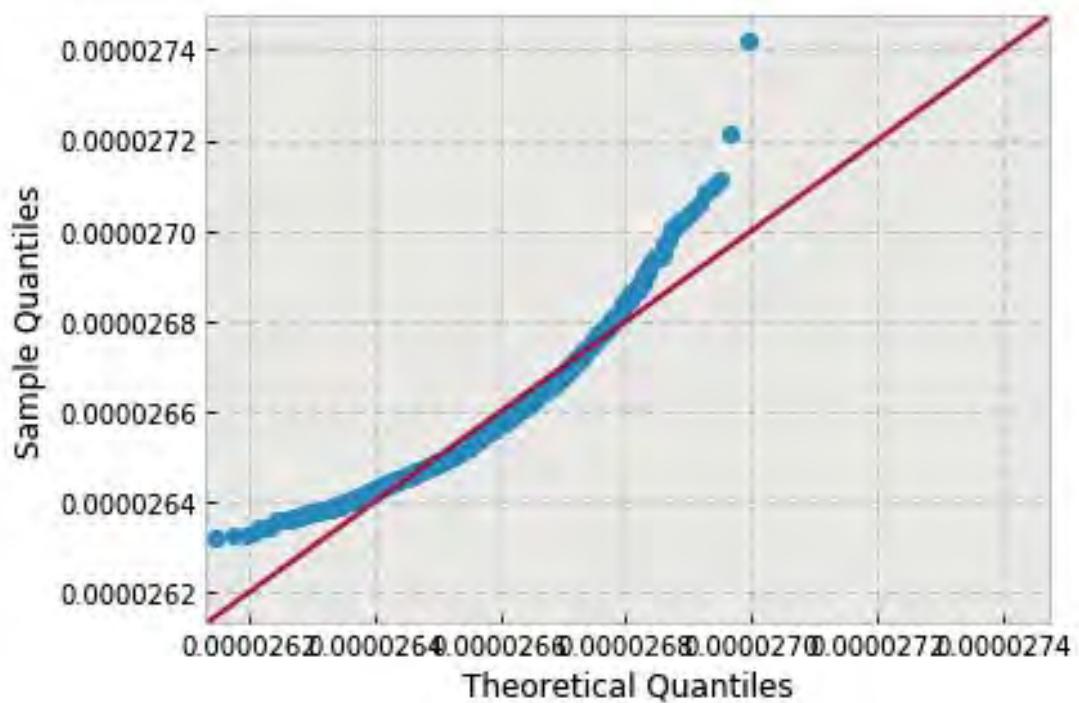


```
variable name: theta
variable value: 1.2217304763960306
distribution: inverse-gamma(shape=(10.940369122894209,), loc=4.3434979030799727e-05,
                           scale=0.0020556388720865742)
DescribeResult(nobs=1000, minmax=(0.00013147017164220871, 0.00060537369658281615)
               mean=0.00025020092483842444, variance=4.7302588692282077e-09,
               skewness=1.3690424848900427, kurtosis=3.0198320646743637)
```

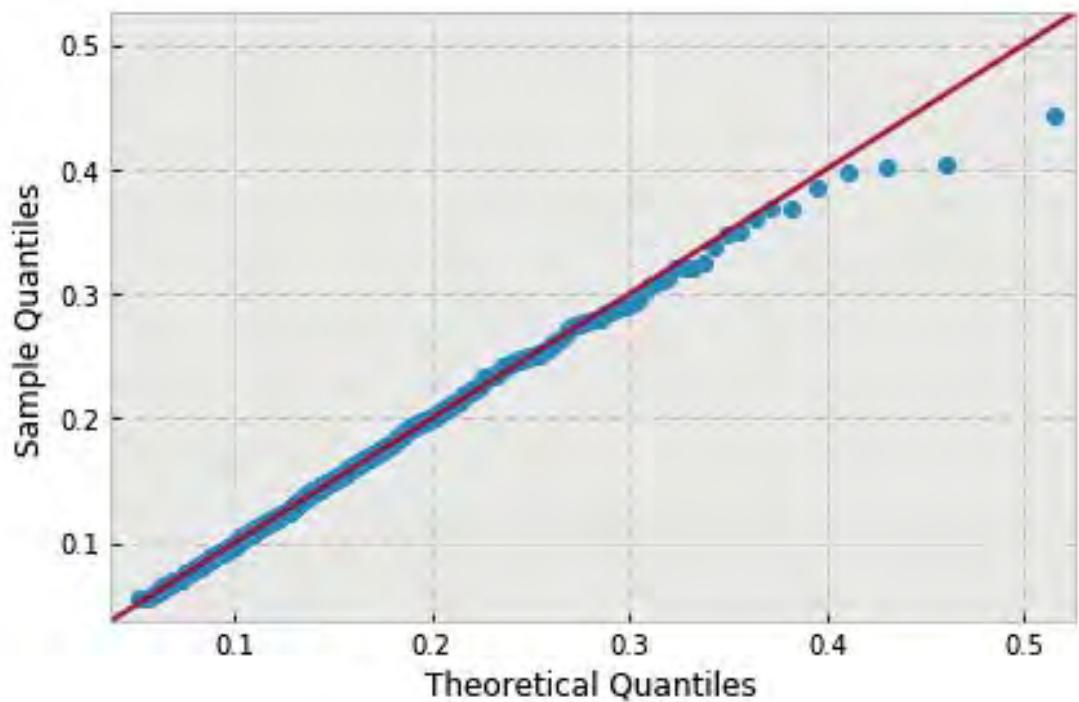
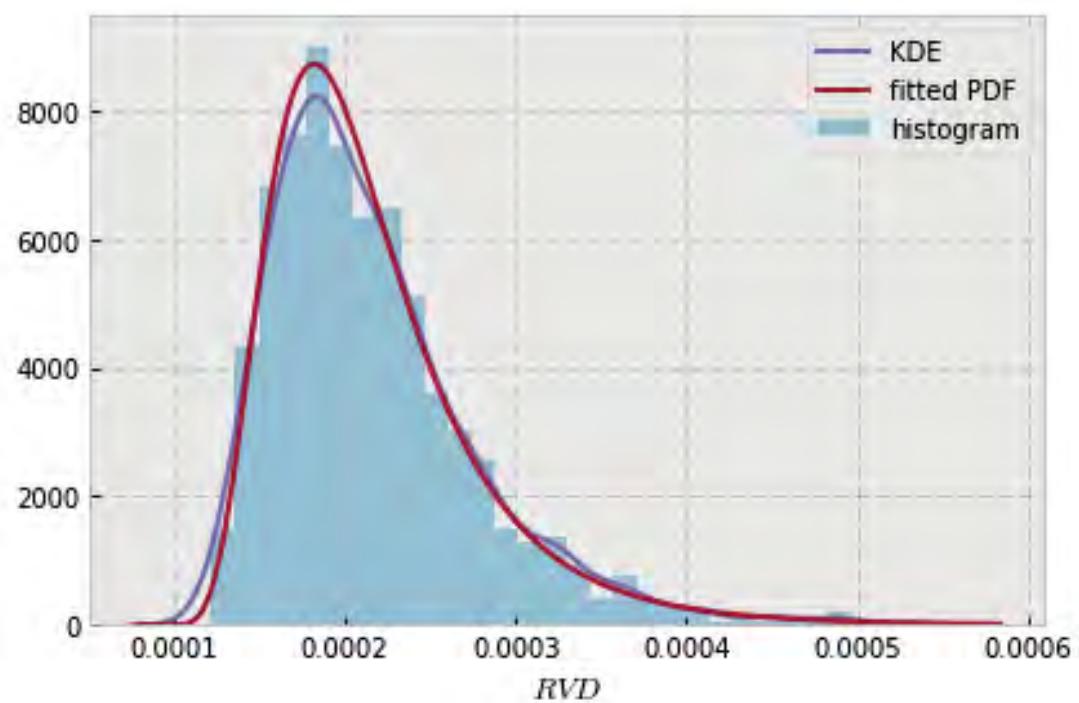


```
variable name: theta
variable value: 1.3089969389957472
distribution: inverse-gamma(shape=(37638.956017174845,), loc=-0.011767690780381423,
    scale=451.5677808463298)
DescribeResult(nobs=1000, minmax=(0.00011801763535591254, 0.00061385057903962212)
    mean=0.0002301119814498238, variance=3.8697199069262696e-09,
    skewness=1.291023045984961, kurtosis=2.7988406564370285)
```

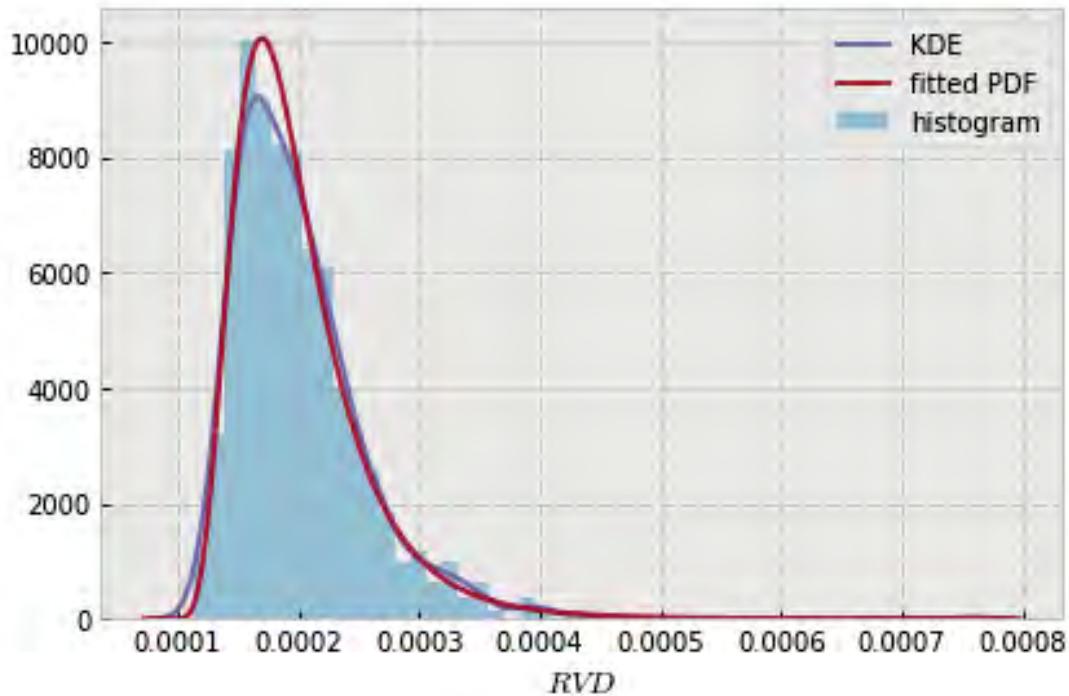


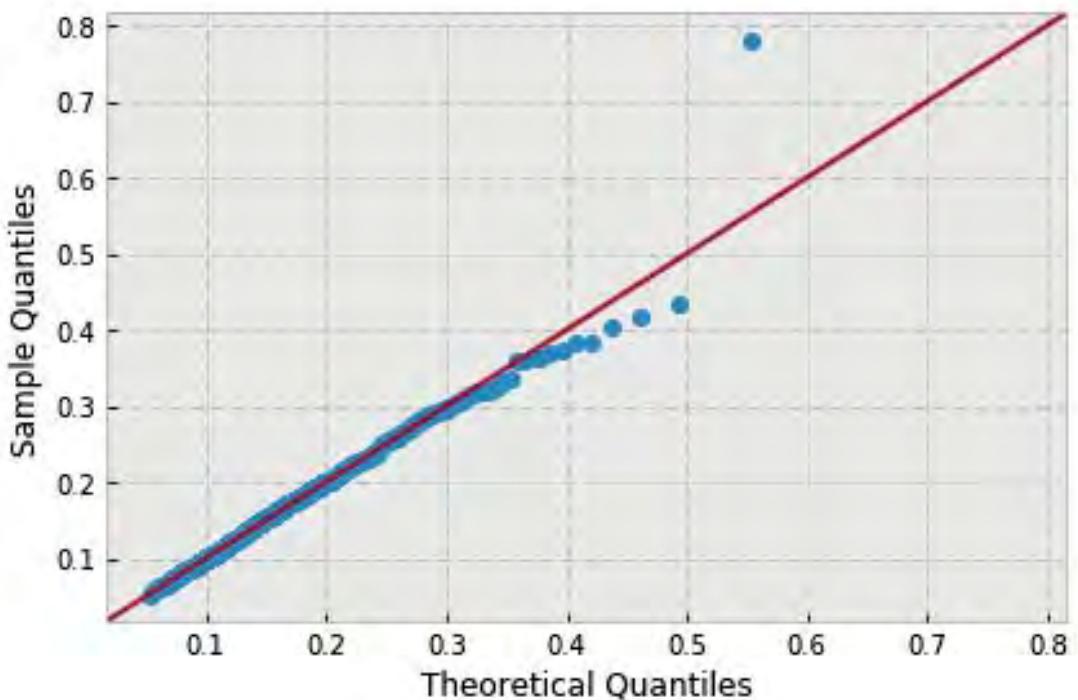


```
variable name: theta
variable value: 1.3962634015954638
distribution: inverse-gamma(shape=(7.9311956946770348,), loc=6.187805516106901e-05,
    scale=0.001073082946297212)
DescribeResult(nobs=1000, minmax=(0.0001217609468431391, 0.00053747040618674024)
    mean=0.00021648839412676928, variance=3.7521416186004547e-09,
    skewness=1.4037312303920533, kurtosis=2.8149572628112836)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: inverse-gamma(shape=(7.640998607856632,), loc=6.783082946632563e-05,
    scale=0.0008824922387013084)
DescribeResult(nobs=1000, minmax=(0.00011409481642380277, 0.0007556029972817205)
    mean=0.00020061244340085864, variance=3.0550886504342285e-09,
    skewness=2.099511480039028, kurtosis=11.37924072748378)
```

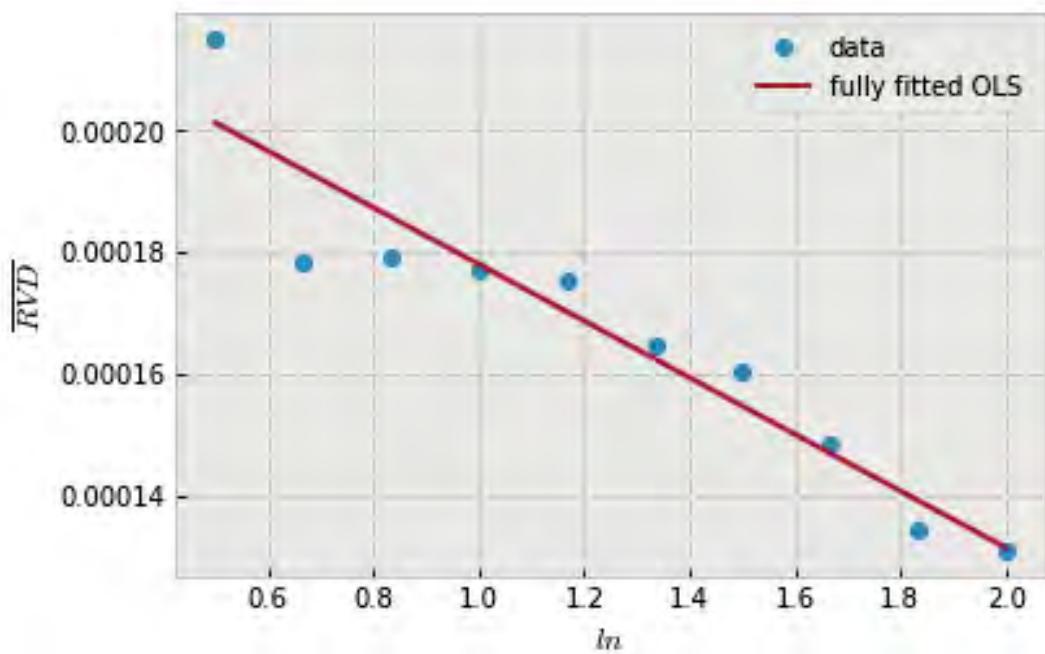




13.2 Parameter regression

```
In [32]: var_name = 'total_root_volume_per_convex_hull_volume'
mean_reg('ln', var_name, 'poly1')
mean_reg('theta', var_name, 'poly2')
mean_reg('nB', var_name, 'poly1')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly2')
mean_reg('N', var_name, 'poly1')

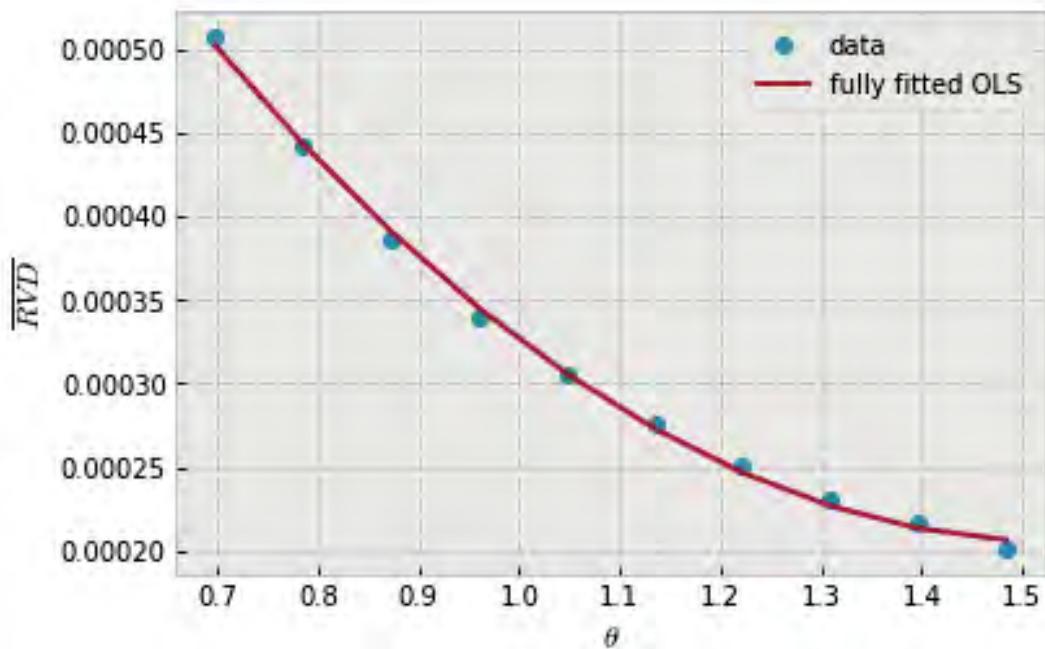
variable name: ln
model: poly1
r-squared: 0.809661400191044
explained variance: 0.8096725314070375
```



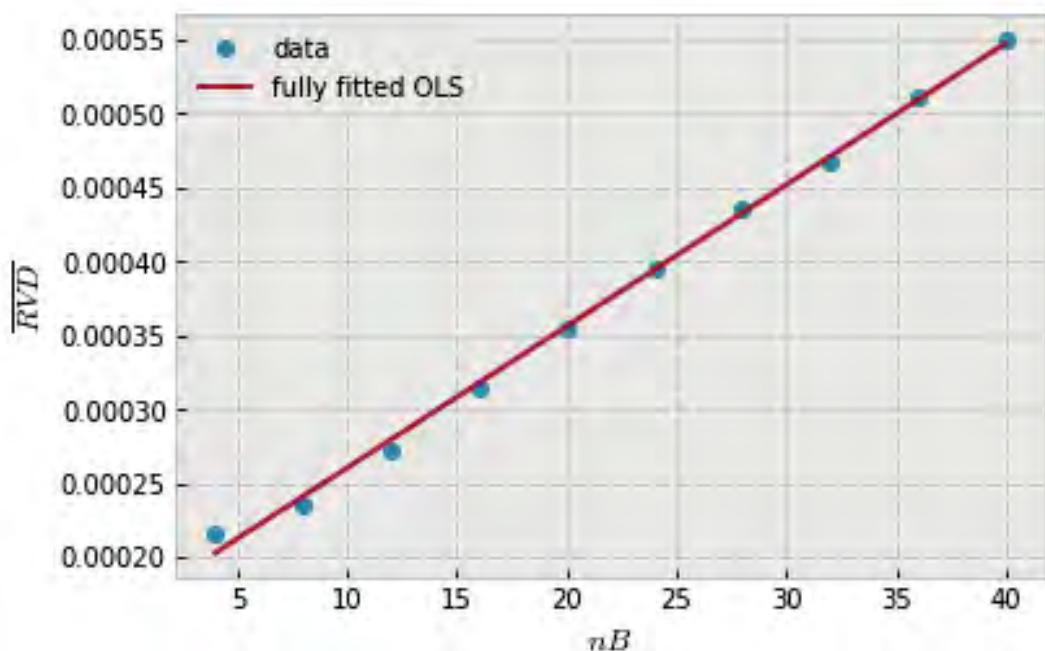
```

variable name: theta
model: poly2
r-squared: 0.9932210761499463
explained variance: 0.9932211511467576

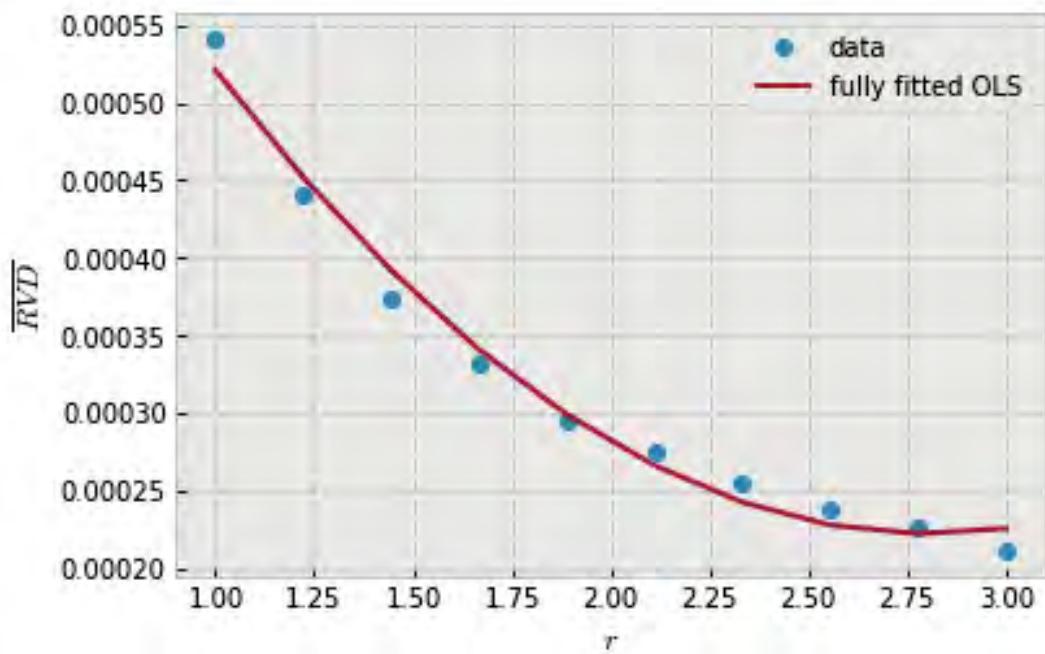
```



```
variable name: nB
model: poly1
r-squared: 0.9947209785931284
explained variance: 0.9947390963181044
```



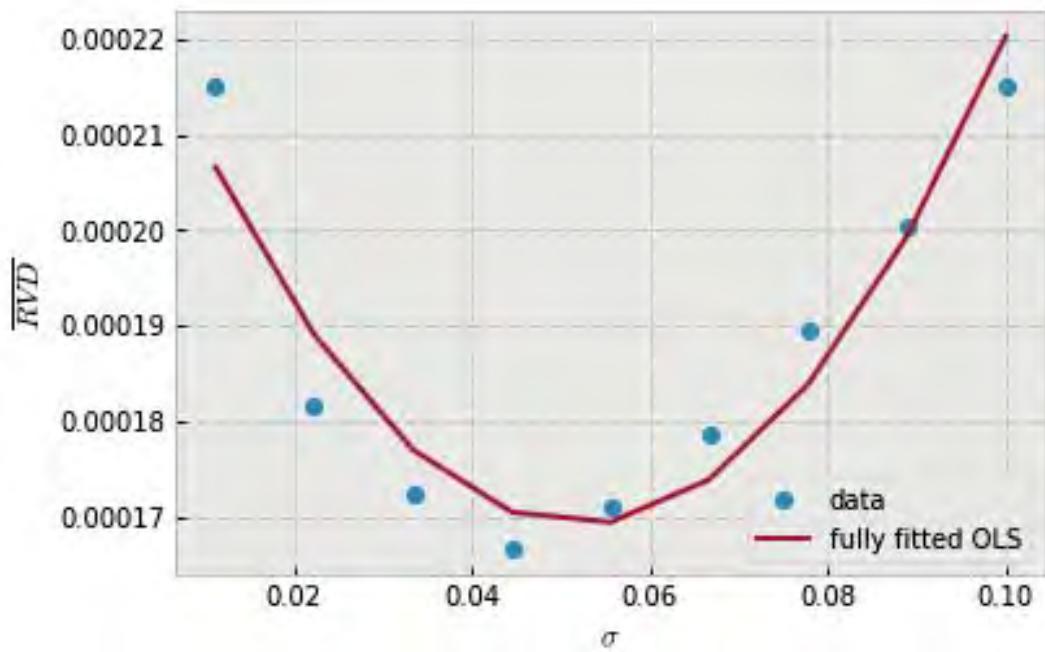
```
variable name: r
model: poly2
r-squared: 0.942364090591958
explained variance: 0.9424137457146897
```



```

variable name: sigma
model: poly2
r-squared: 0.5989946642199648
explained variance: 0.5999769888406399

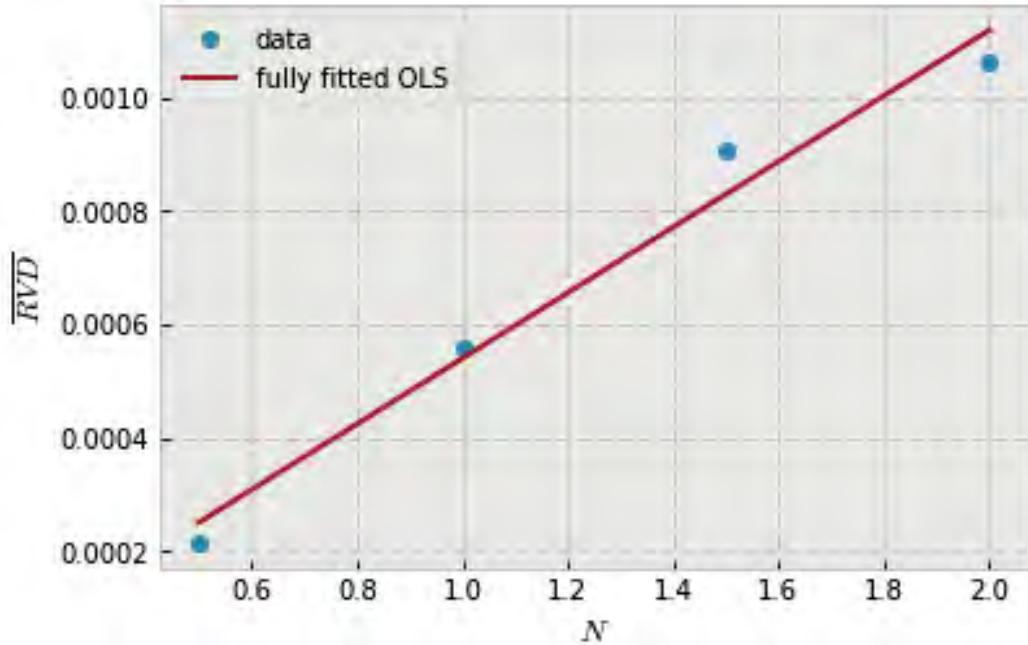
```



```

variable name: N
model: poly1
r-squared: 0.8510807106160571
explained variance: 0.869883824750038

```



14 Volume of the rhizosphere relative to convex hull: phosphate (0.2cm)

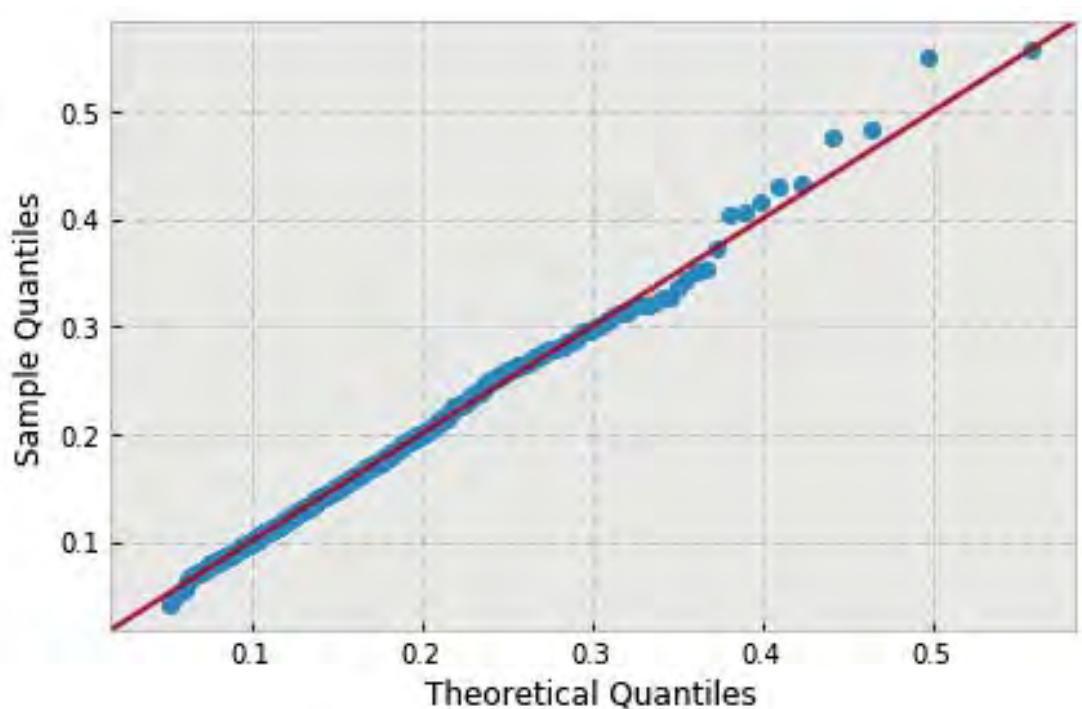
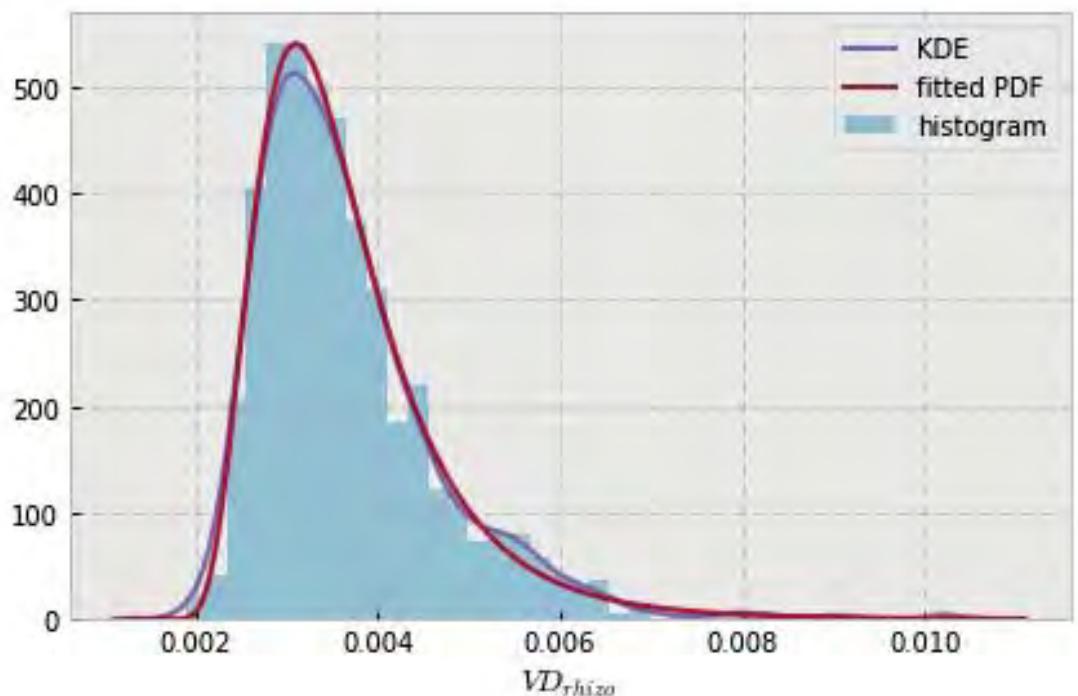
14.1 Probability distributions

```

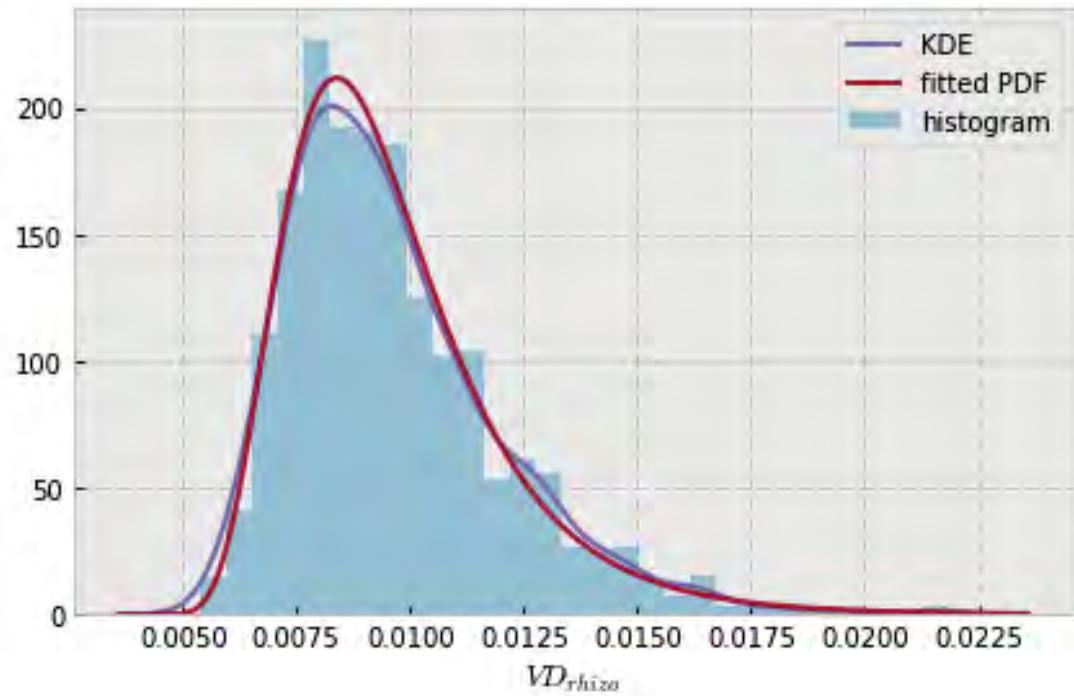
In [33]: plot_dist('rhizosphere_volume_phosphate_per_convex_hull_volume', 'inverse-gamma')
/home/lpetrich/.local/share/anaconda3/envs/Master/lib/python3.6/site-packages/scipy/stats/_distn_infra.py:1435: RuntimeWarning: invalid value encountered in double_scalars
  Lhat = muhat - Shat*mu

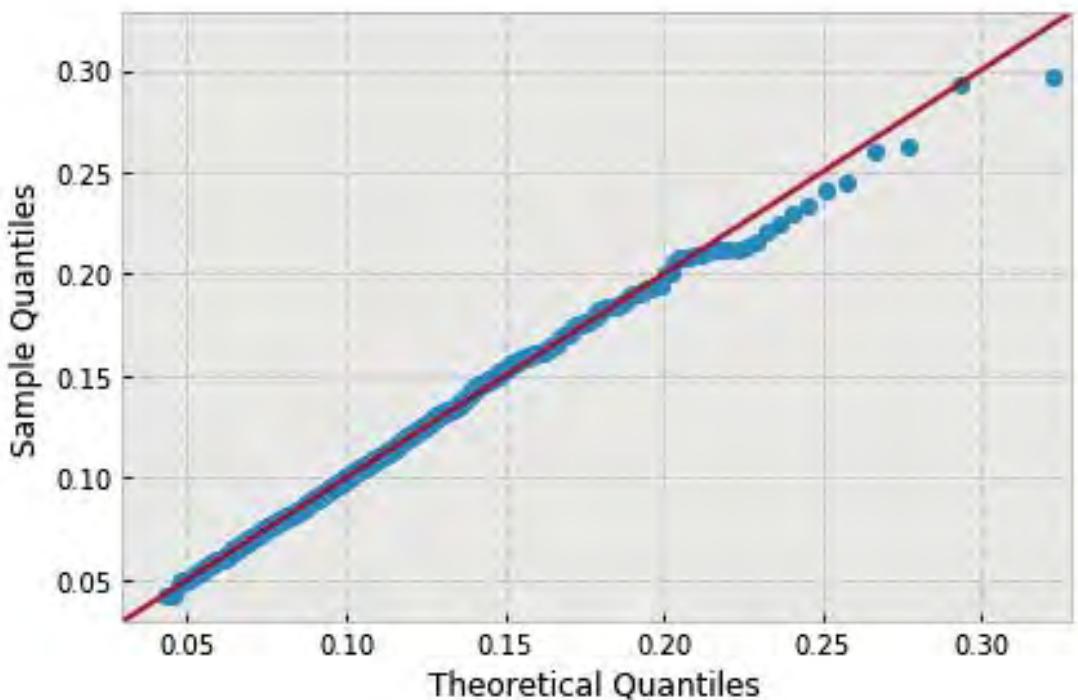
variable name: N
variable value: 0.5
distribution: inverse_gamma(shape=(7.6103011818812867,), loc=0.0012082430970182948,
                           scale=0.016326170992442254)
DescribeResult(nobs=1000, minmax=(0.0018966584652980495, 0.0103105436122346)
               mean=0.0036783740110576195, variance=1.0784352753835315e-06,
               skewness=1.8026677499285493, kurtosis=5.556231198388204)

```

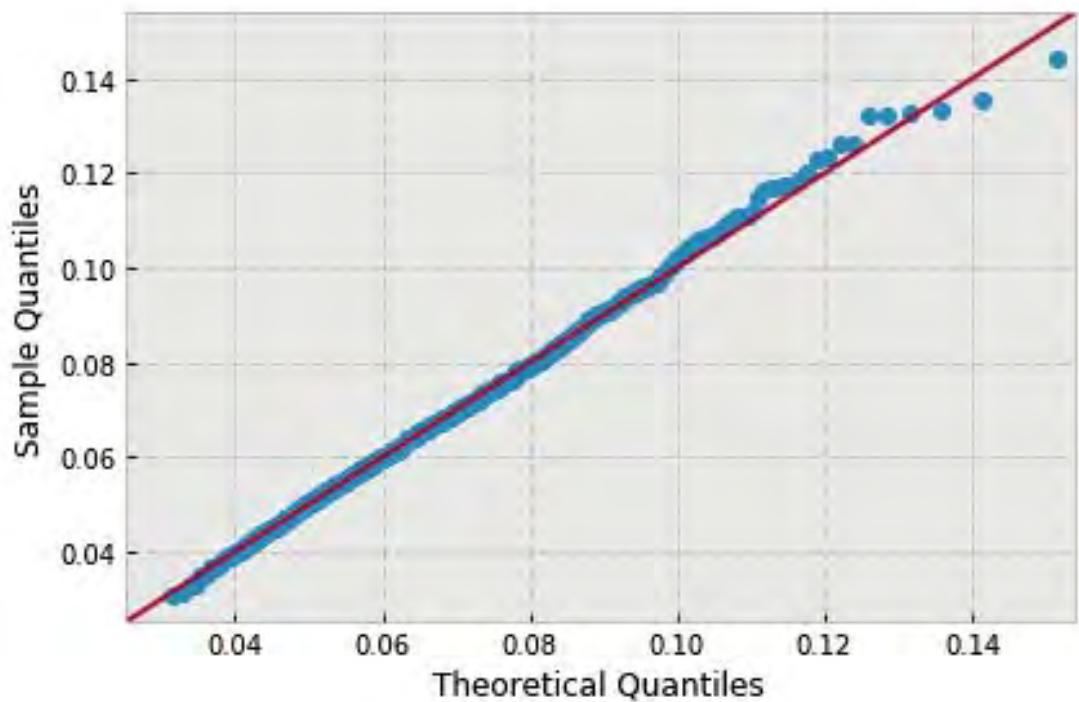
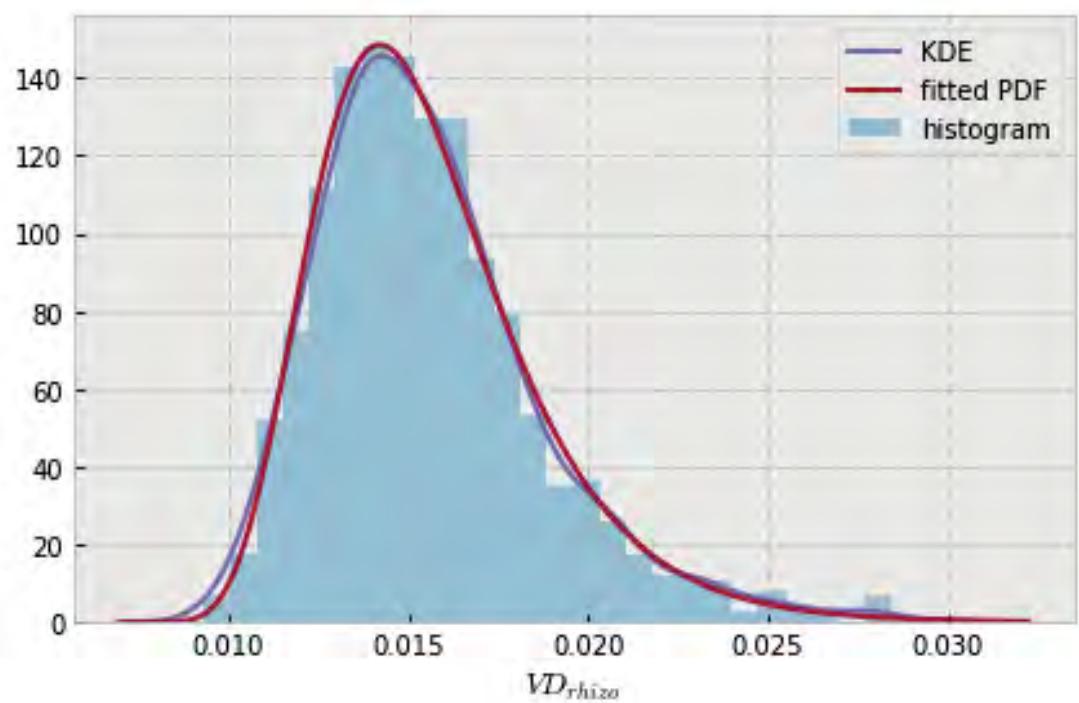


```
variable name: N
variable value: 1.0
distribution: inverse-gamma(shape=(10.270225995655117,), loc=0.0026621550771169754,
scale=0.06451127716622593)
DescribeResult(nobs=1000, minmax=(0.0053714243185457033, 0.021822017865906818)
mean=0.0096168155816905507, variance=5.5985078175138158e-06,
skewness=1.2172841858171801, kurtosis=2.1417758606490285)
```

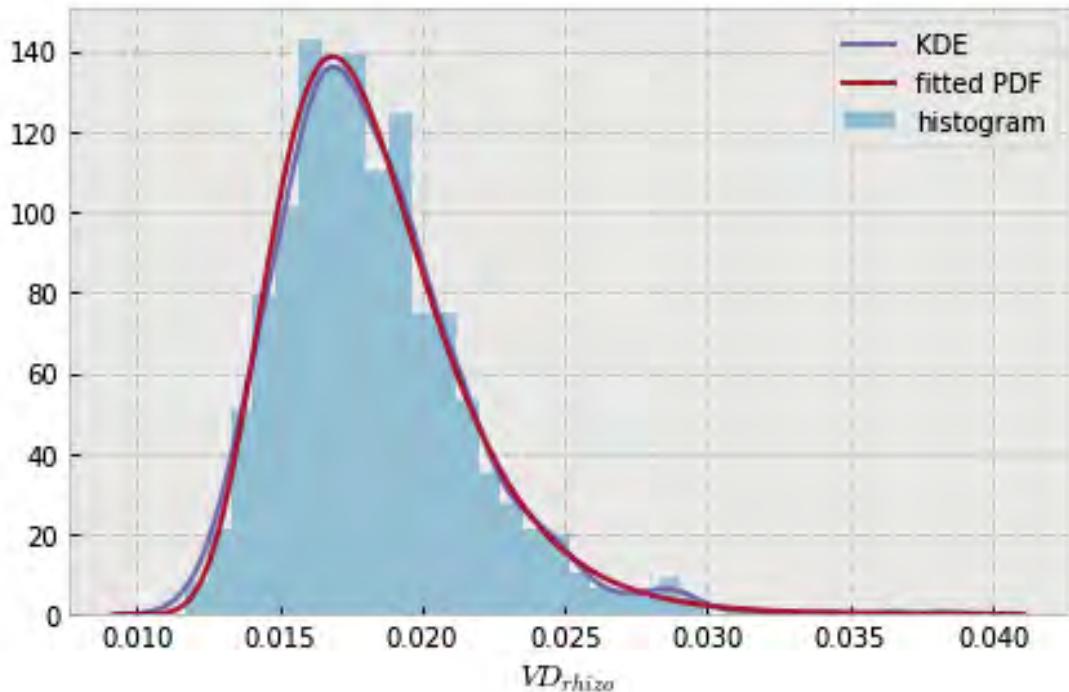


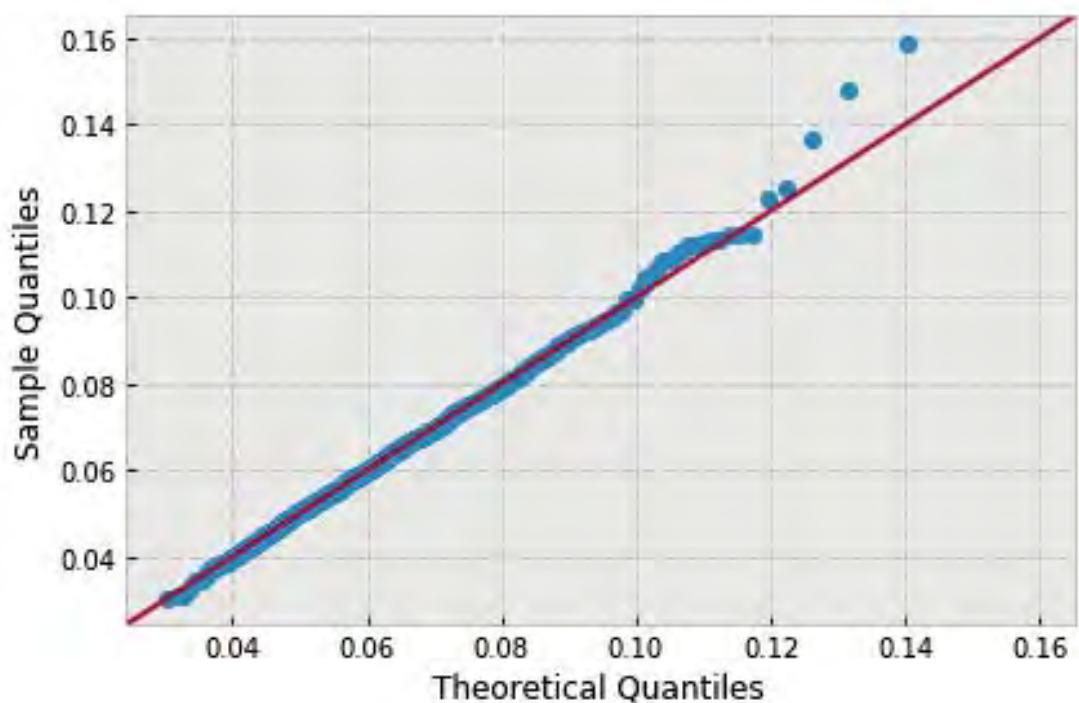


```
variable name: N
variable value: 1.5
distribution: inverse-gamma(shape=(16.318577455019689,), loc=0.0037277883495442153,
                           scale=0.18153856753479328)
DescribeResult(nobs=1000, minmax=(0.0093163927030555076, 0.029874584908317757)
               mean=0.015579108923111901, variance=9.8251000448028525e-06,
               skewness=1.085182041696817, kurtosis=1.7615652471551693)
```

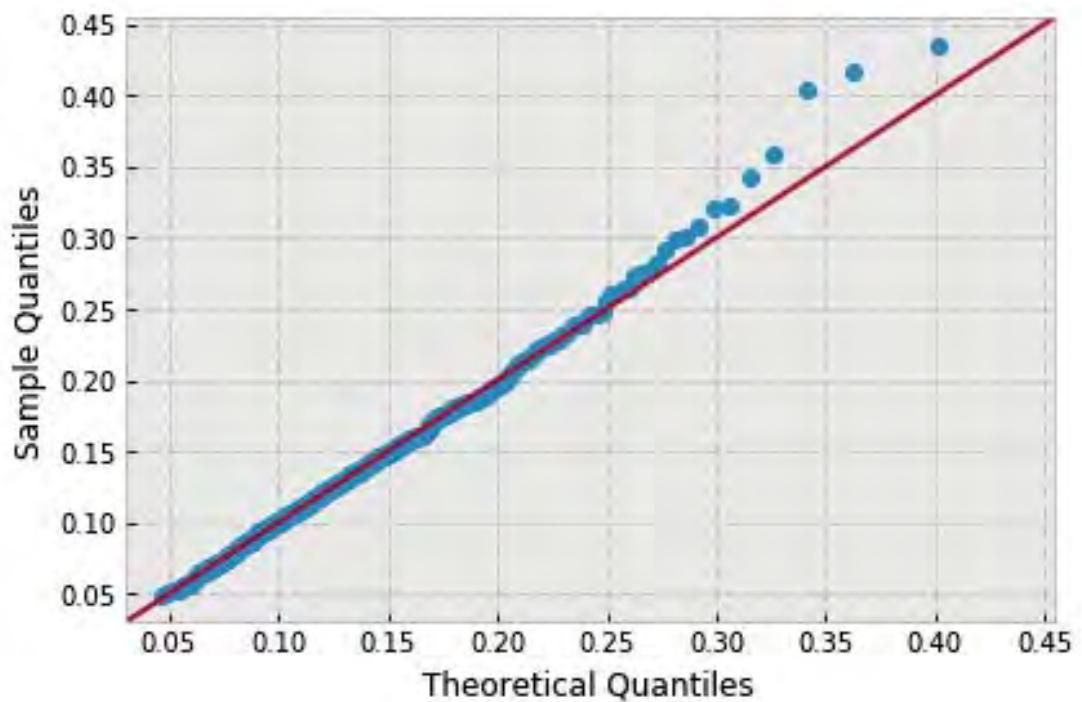
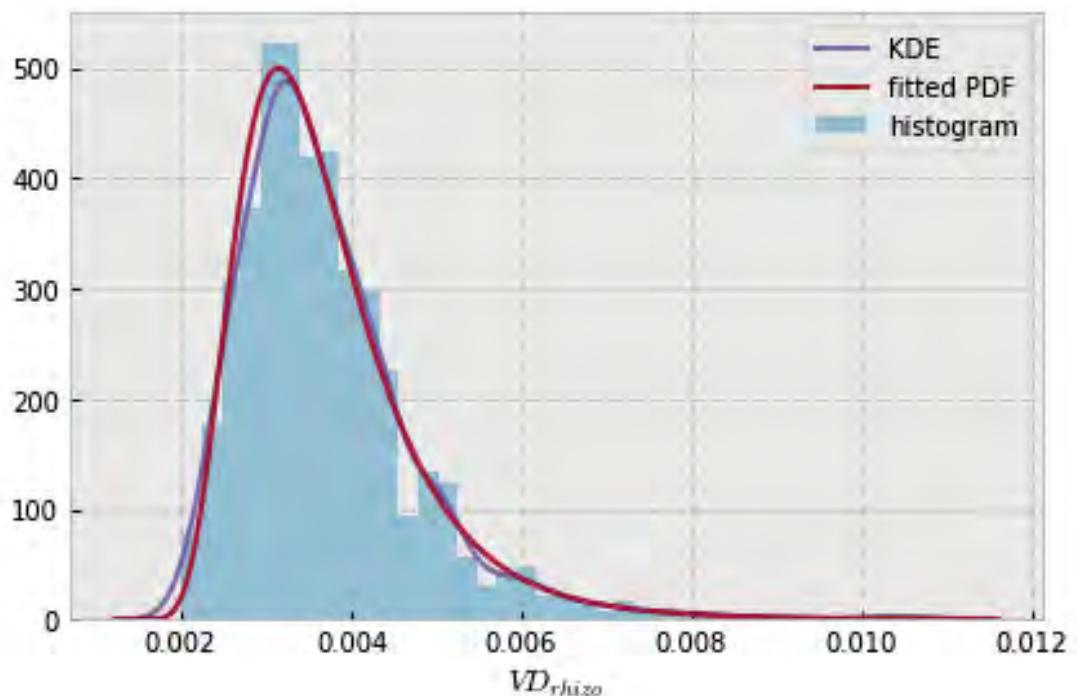


```
variable name: N
variable value: 2.0
distribution: inverse-gamma(shape=(17.15482858086034,), loc=0.005334030259684731,
    scale=0.20932012031988897)
DescribeResult(nobs=1000, minmax=(0.011722768791531862, 0.038545913074651848)
    mean=0.018292866452669032, variance=1.1254879057956187e-05,
    skewness=1.2138034968686122, kurtosis=3.0341408287344853)
```

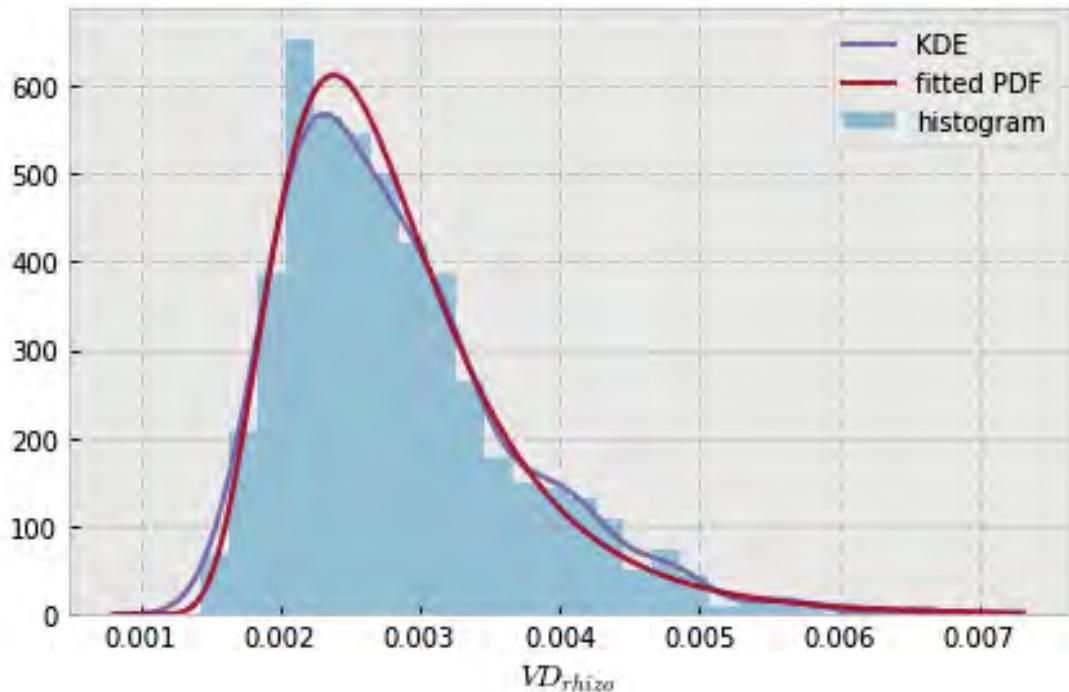


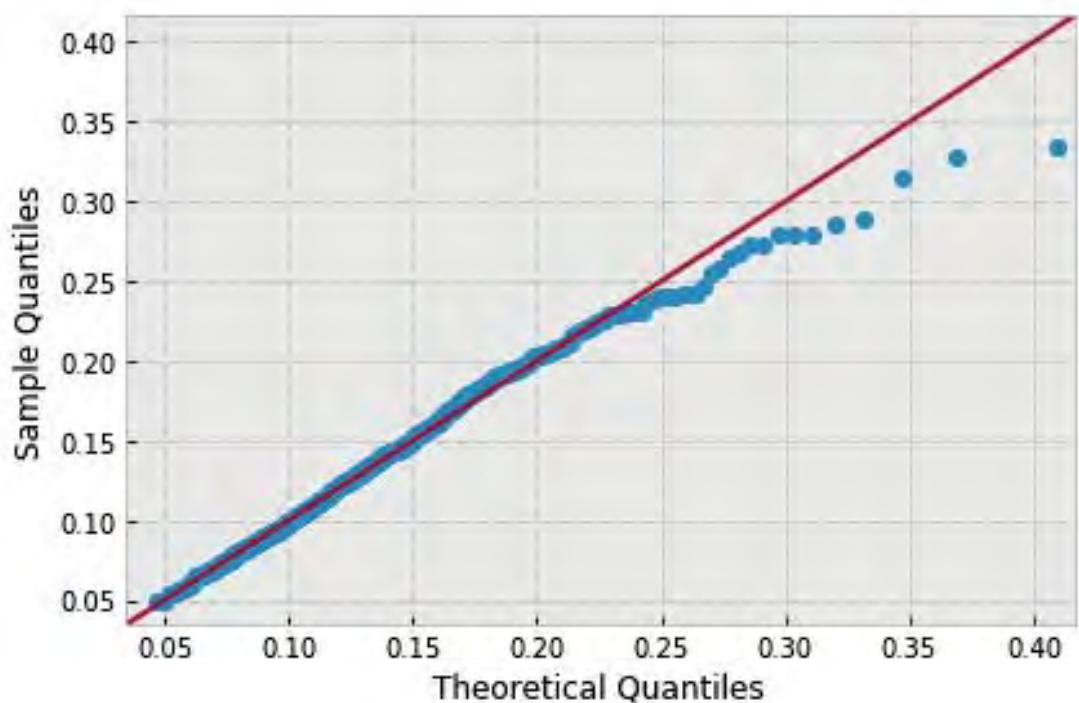


```
variable name: ln
variable value: 0.5
distribution: inverse-gamma(shape=(9.0751695159328918,), loc=0.0008857179942373535,
                           scale=0.02279373258455333)
DescribeResult(nobs=1000, minmax=(0.0020126917284518432, 0.01079668274823764)
               mean=0.0037096119855023523, variance=1.1559934797352866e-06,
               skewness=1.853290377527669, kurtosis=6.3242669375303056)
```

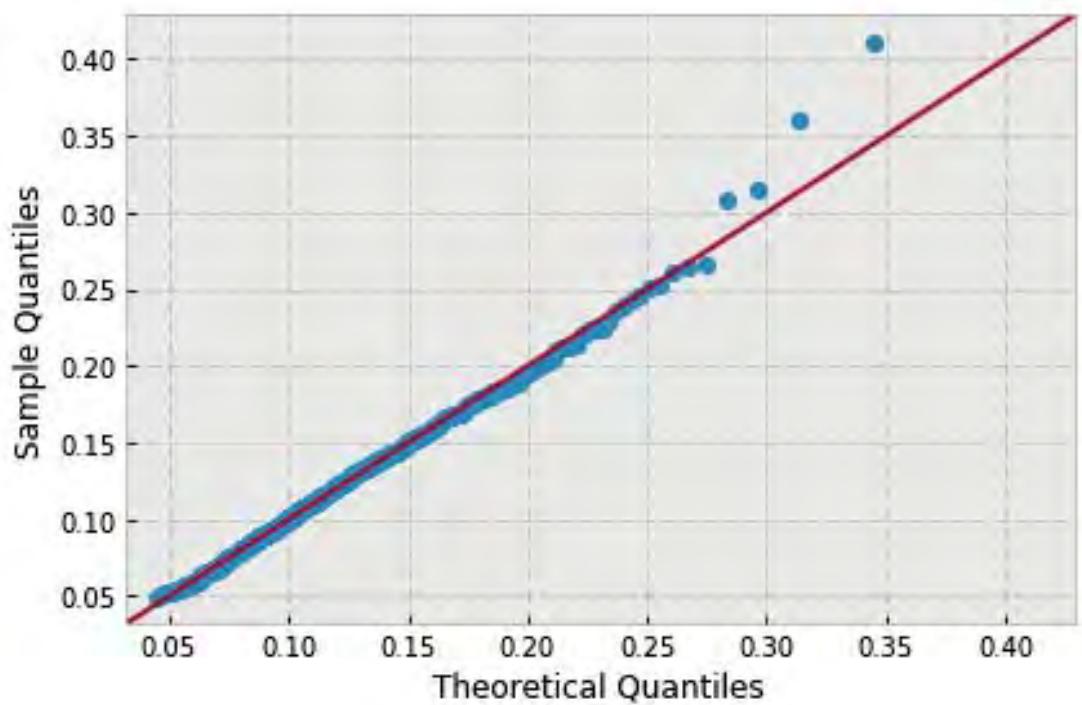
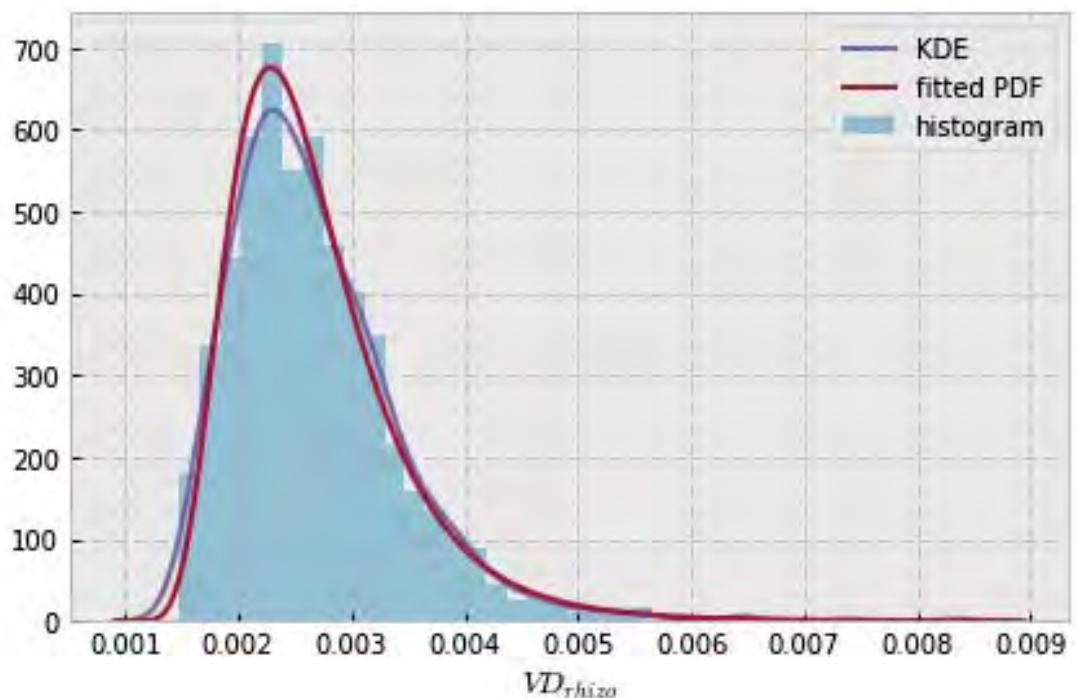


```
variable name: ln
variable value: 0.6666666666666666
distribution: inverse-gamma(shape=(8.9861145708073717,), loc=0.0005430487726158218,
                           scale=0.01839094592057149)
DescribeResult(nobs=1000, minmax=(0.0014362031464428516, 0.0066910493164055781)
               mean=0.0028424897778414663, variance=6.9334478038727693e-07,
               skewness=1.126070908175798, kurtosis=1.4697589656178636)
```

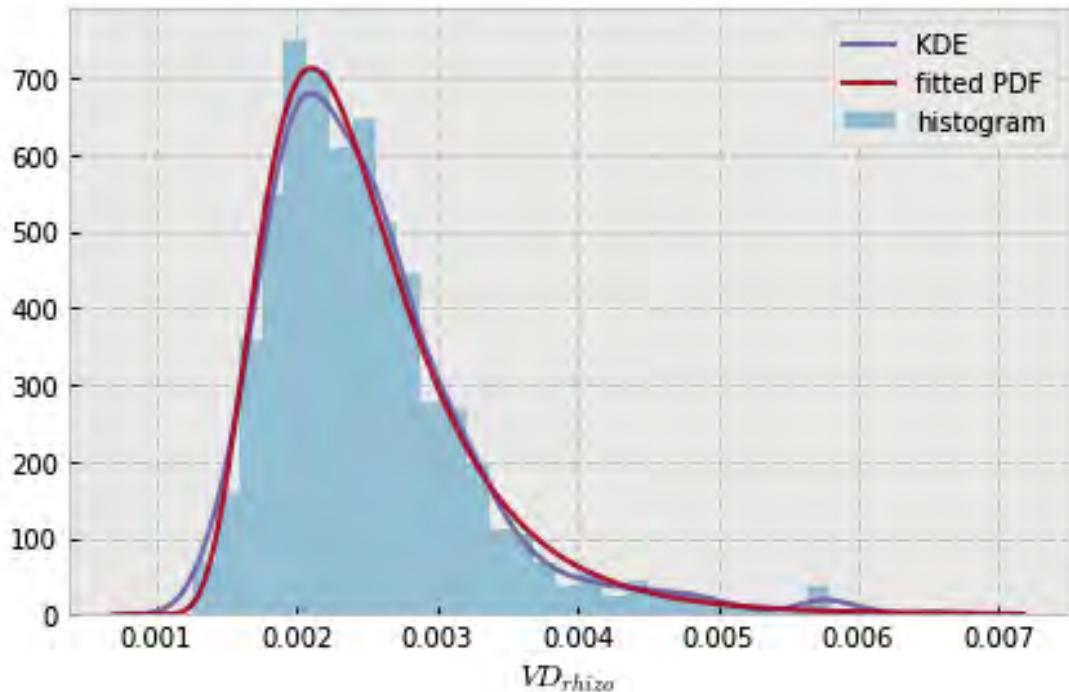


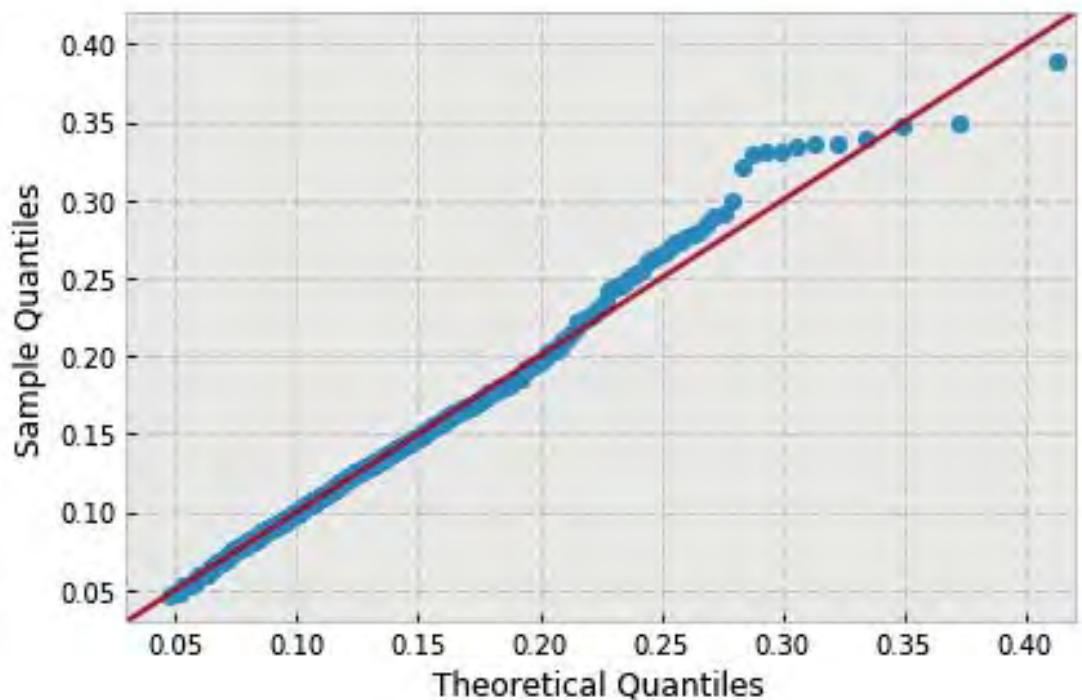


```
variable name: ln
variable value: 0.8333333333333333
distribution: inverse-gamma(shape=(9.8785674371762831,), loc=0.0005315980486835165,
                           scale=0.01907469164750128)
DescribeResult(nobs=1000, minmax=(0.0014805648371220544, 0.0083627264862101442)
               mean=0.0026792847956604113, variance=5.7985593310177553e-07,
               skewness=1.654312430145433, kurtosis=5.949861336987508)
```

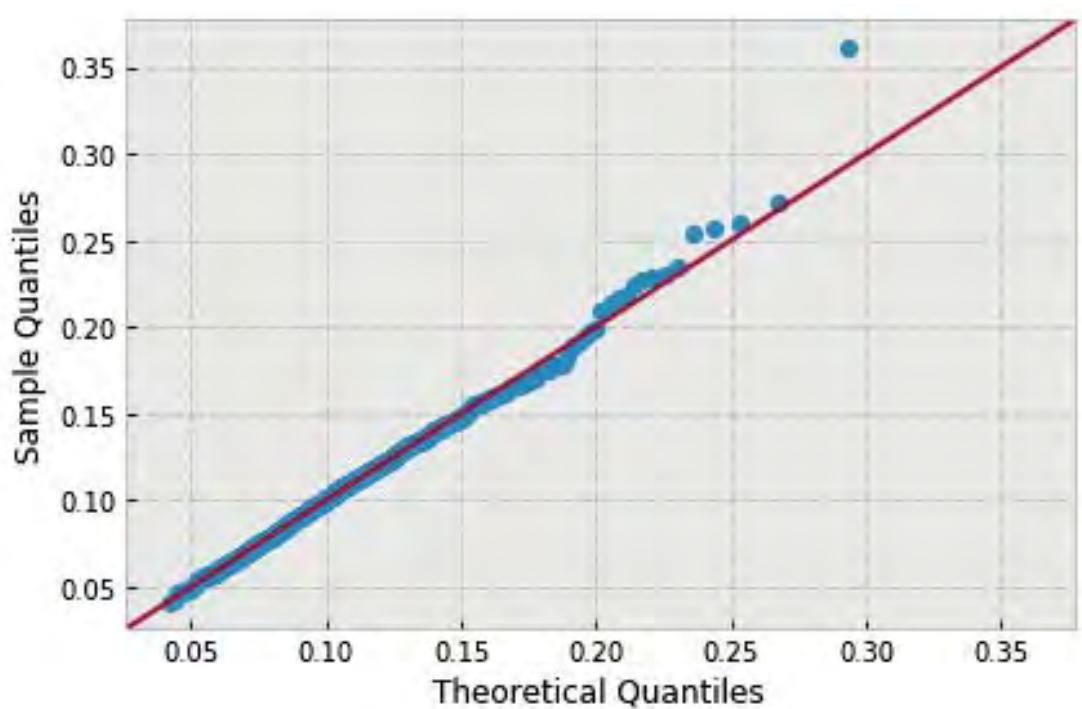
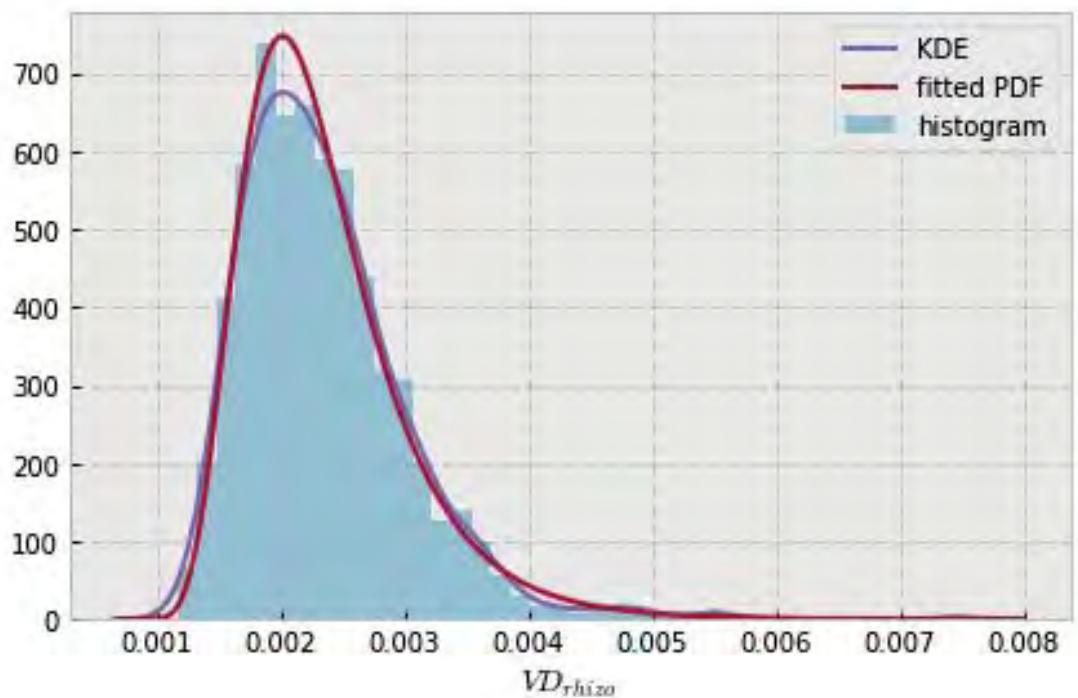


```
variable name: ln
variable value: 1.0
distribution: inverse-gamma(shape=(8.9426464479608132,), loc=0.0005308519679855053,
                           scale=0.0156350278661242)
DescribeResult(nobs=1000, minmax=(0.0012586663415426482, 0.0066069971037196109)
               mean=0.0025002986654465934, variance=5.6807578510000627e-07,
               skewness=1.6823158689081688, kurtosis=4.285803913782177)
```

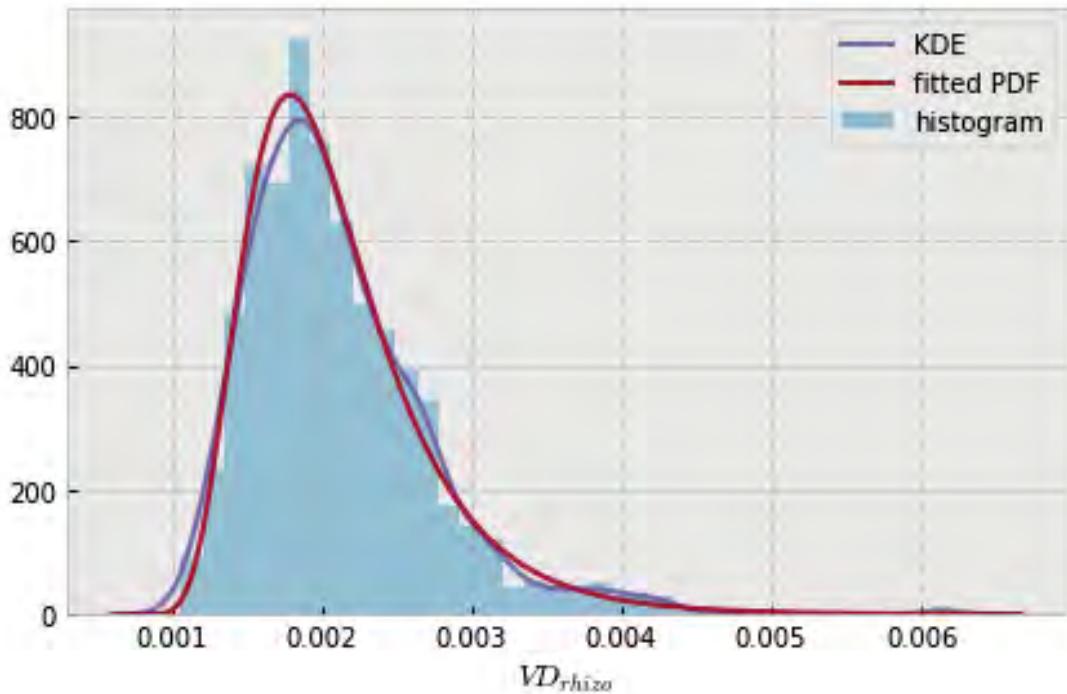


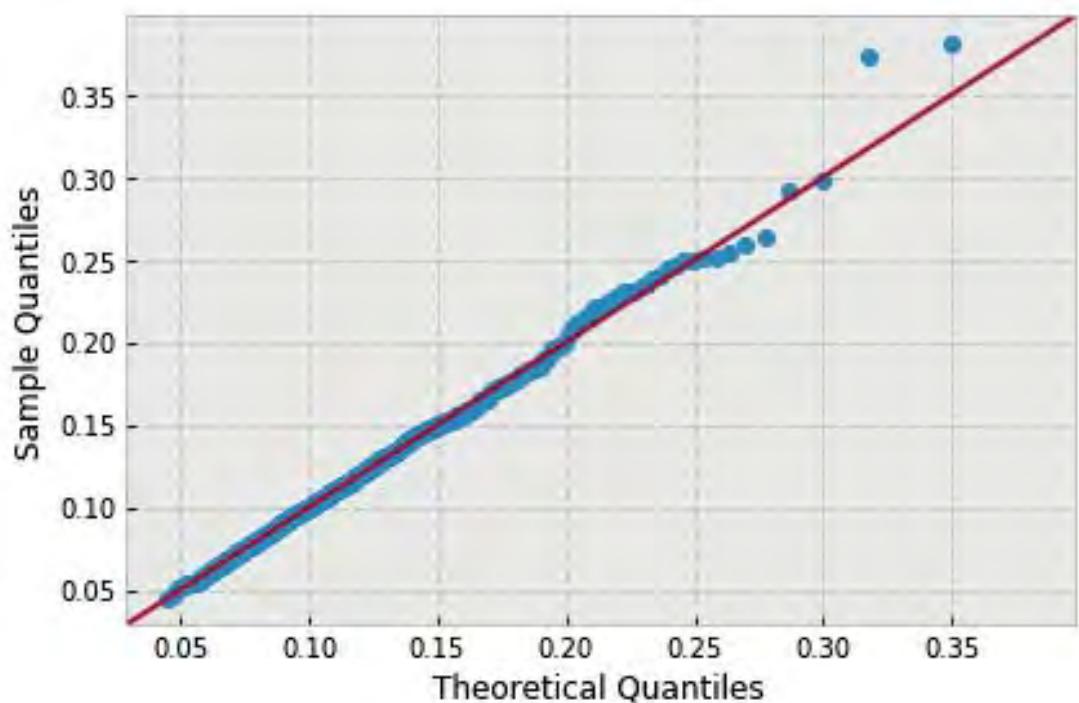


```
variable name: ln
variable value: 1.1666666666666665
distribution: inverse-gamma(shape=(10.852493036340688,), loc=0.00034555033175146226,
                           scale=0.019806967847642742)
DescribeResult(nobs=1000, minmax=(0.0011698390318036211, 0.0074981874522280188)
               mean=0.0023557356157343271, variance=4.5712370086504747e-07,
               skewness=1.581737872159245, kurtosis=5.430077058659025)
```

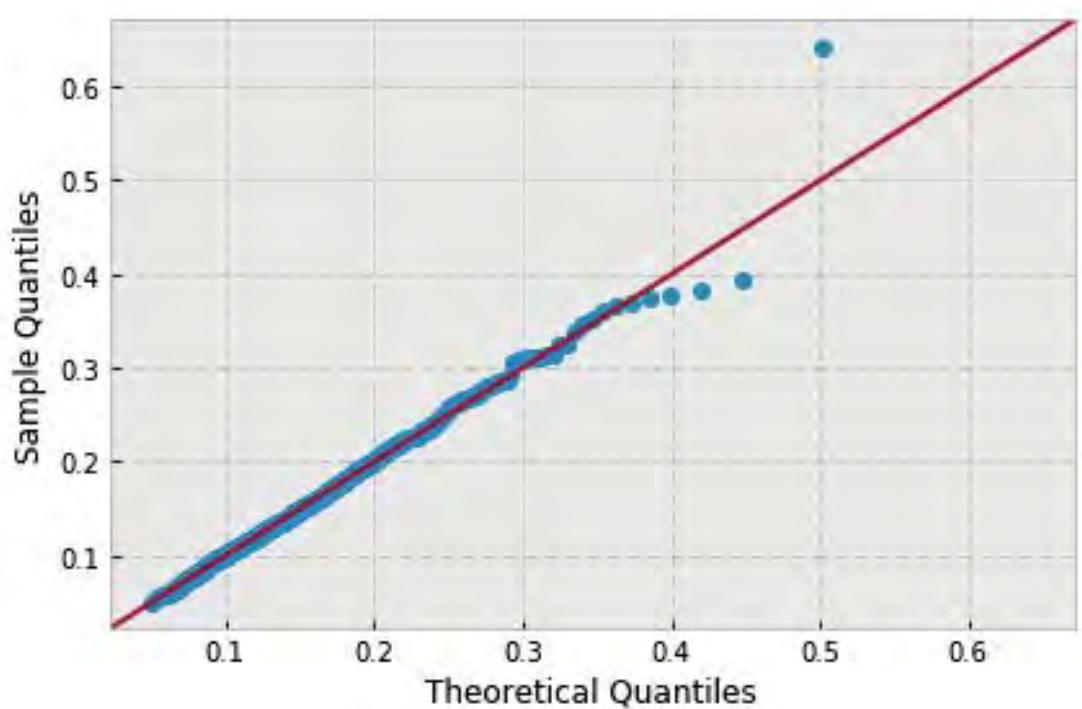
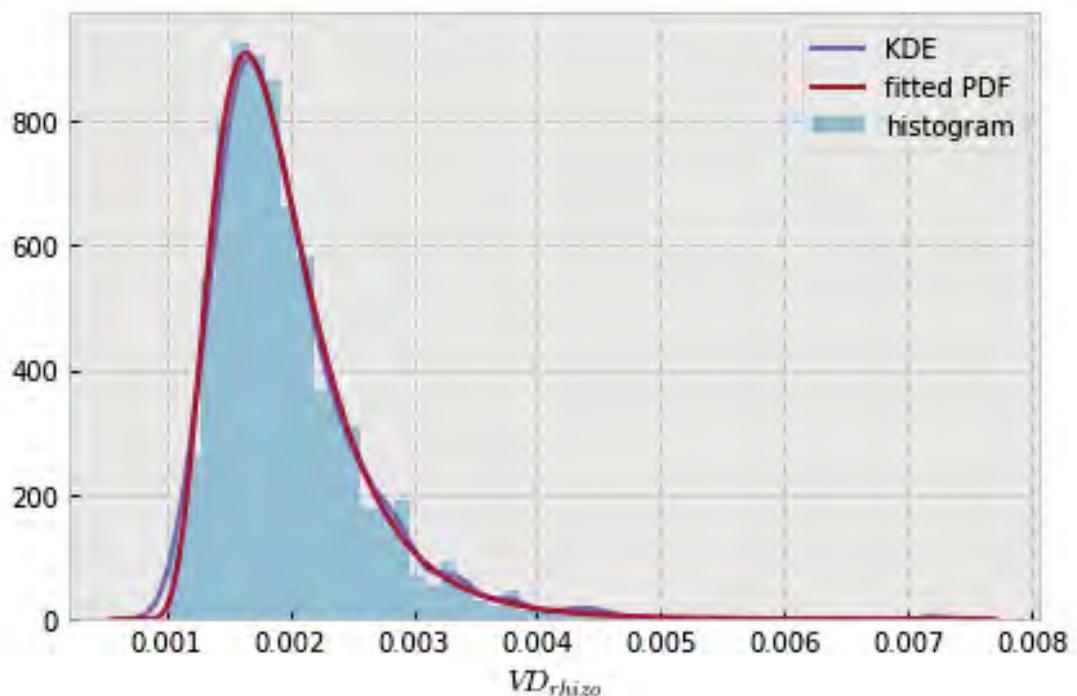


```
variable name: ln
variable value: 1.333333333333333
distribution: inverse-gamma(shape=(9.8099047144011315,), loc=0.000372255038013972,
                           scale=0.015305666190092456)
DescribeResult(nobs=1000, minmax=(0.0010647555451878778, 0.0062013628602897554)
               mean=0.0021094082191312302, variance=3.8497237120462655e-07,
               skewness=1.594291452957645, kurtosis=4.839290916315233)
```

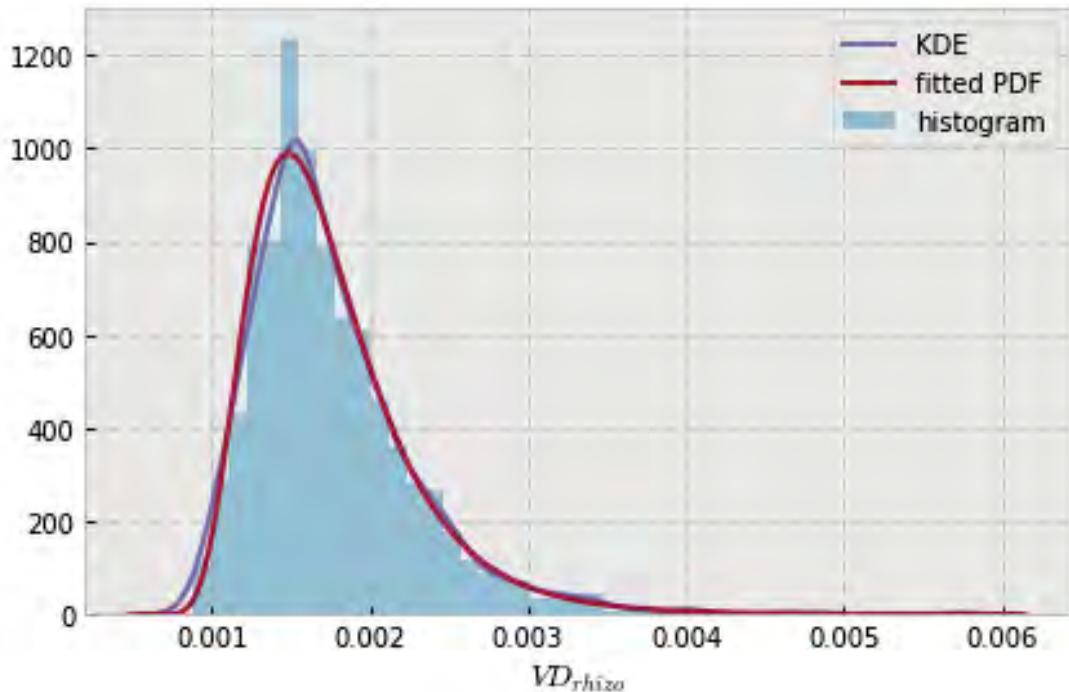


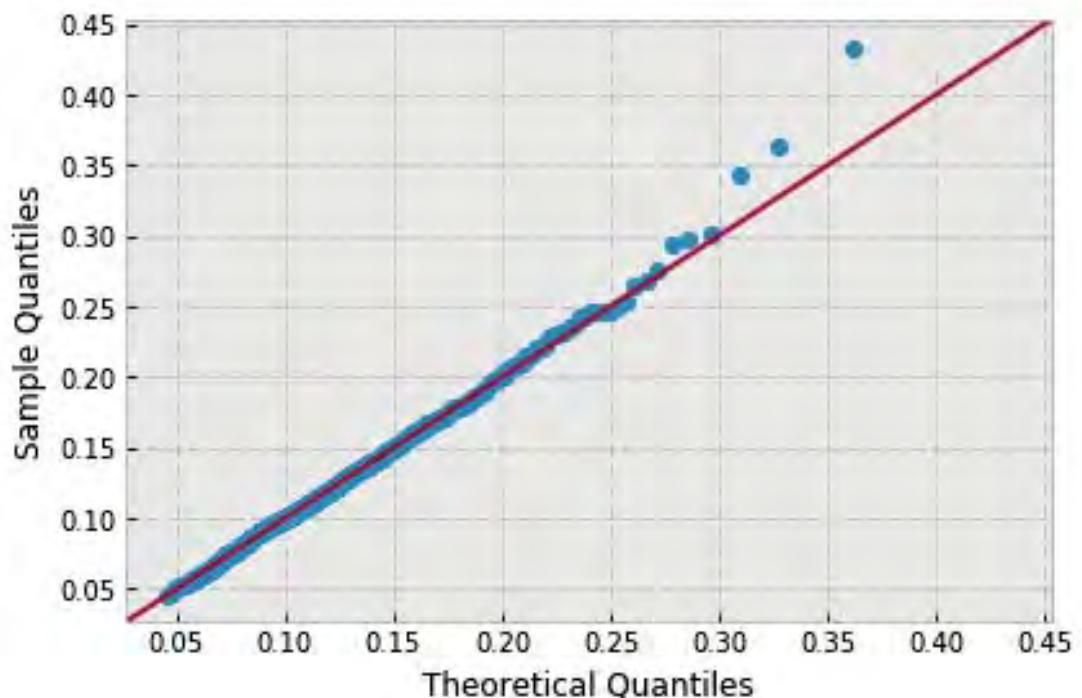


```
variable name: ln
variable value: 1.5
distribution: inverse-gamma(shape=(8.0534683426181388,), loc=0.00048073088196149143,
scale=0.010542734632277694)
DescribeResult(nobs=1000, minmax=(0.0010190874979945472, 0.0072416796741388567)
mean=0.0019754130070529347, variance=3.6803834023646658e-07,
skewness=1.8807870720668638, kurtosis=7.422311666638743)
```

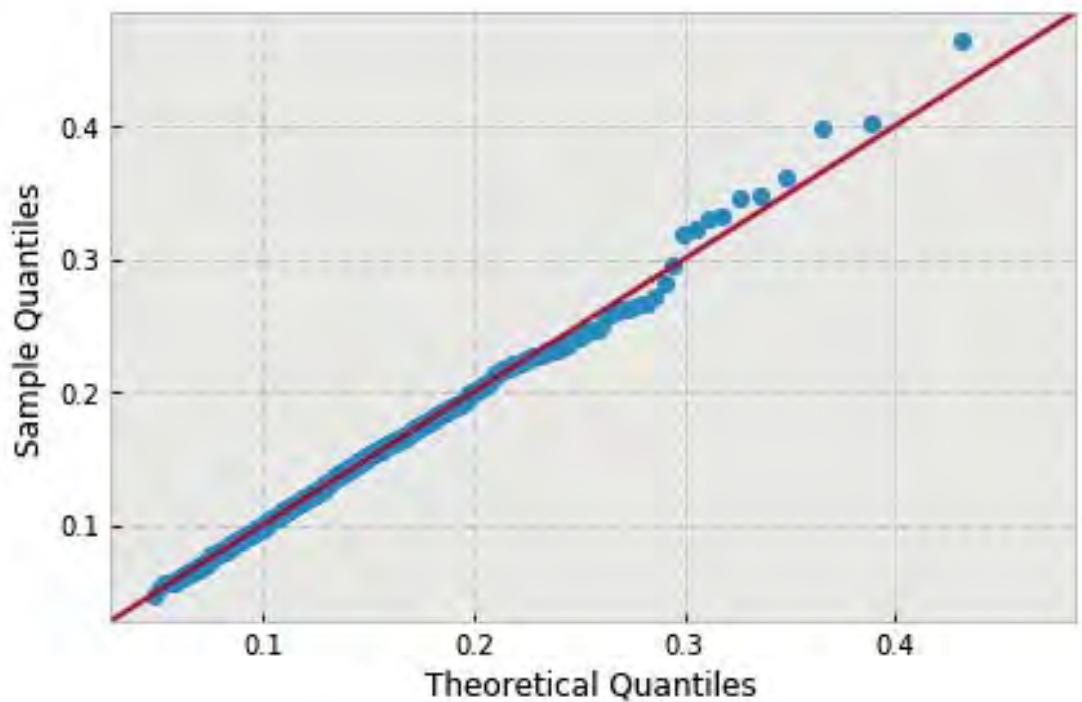
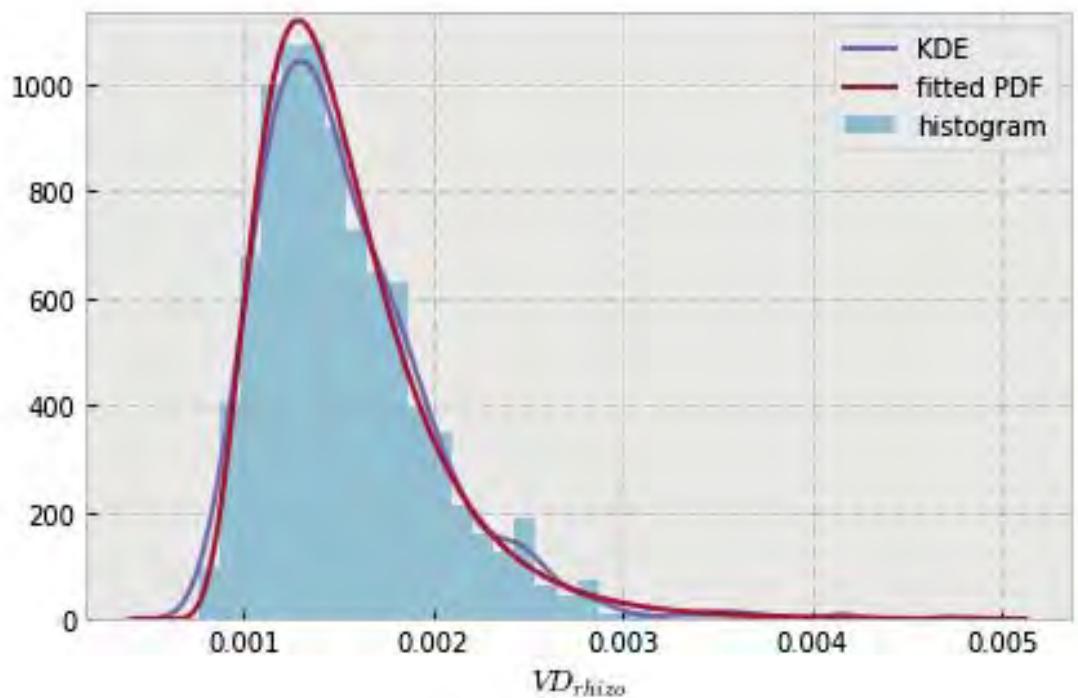


```
variable name: ln
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(9.6264963984703478,), loc=0.00030101017539544713,
                           scale=0.01254916087505683)
DescribeResult(nobs=1000, minmax=(0.00087504017859820675, 0.0057418750757117325)
               mean=0.0017561493741946107, variance=2.828197612983967e-07,
               skewness=1.7587021299110508, kurtosis=6.104545056051217)
```

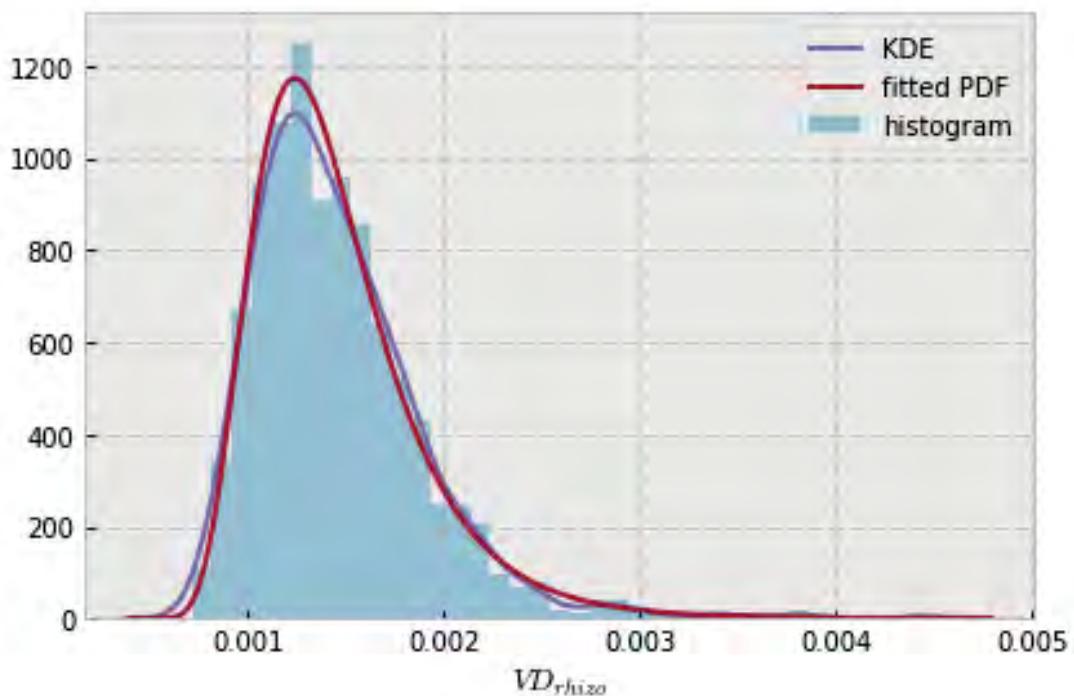


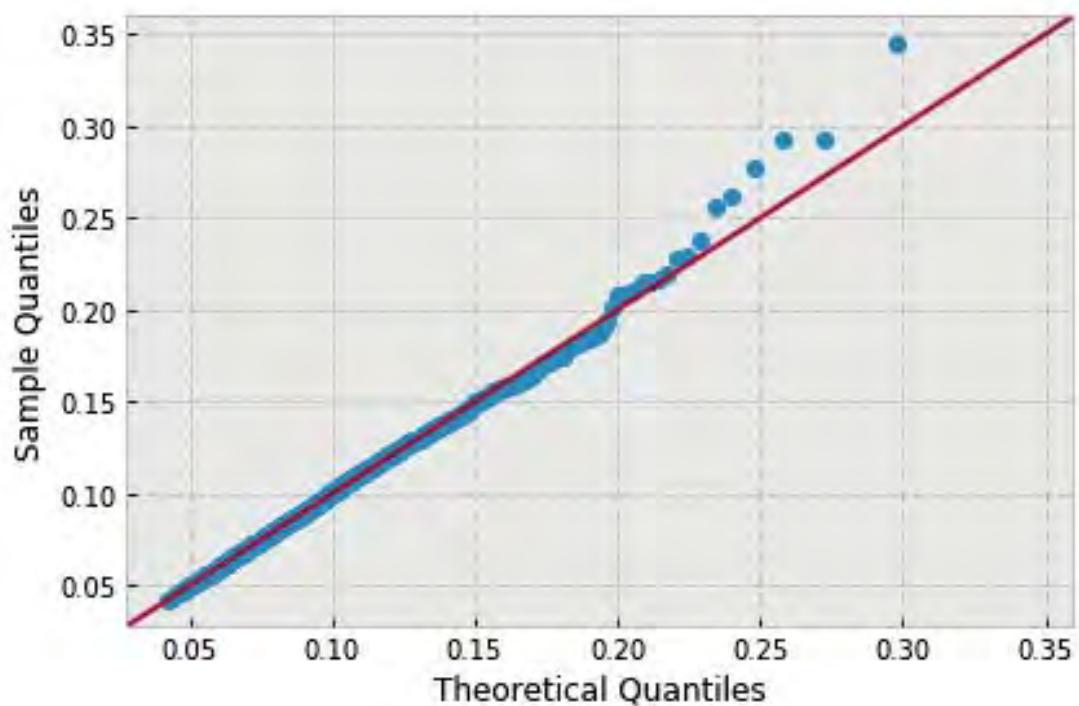


```
variable name: ln
variable value: 1.833333333333333
distribution: inverse-gamma(shape=(8.7239488404116976,), loc=0.0002989144894777813,
                           scale=0.009619207570862764)
DescribeResult(nobs=1000, minmax=(0.00075725769473970902, 0.0047634767881890652)
               mean=0.0015437568088381313, variance=2.2598112764729673e-07,
               skewness=1.6646591335134693, kurtosis=5.205766458029634)
```

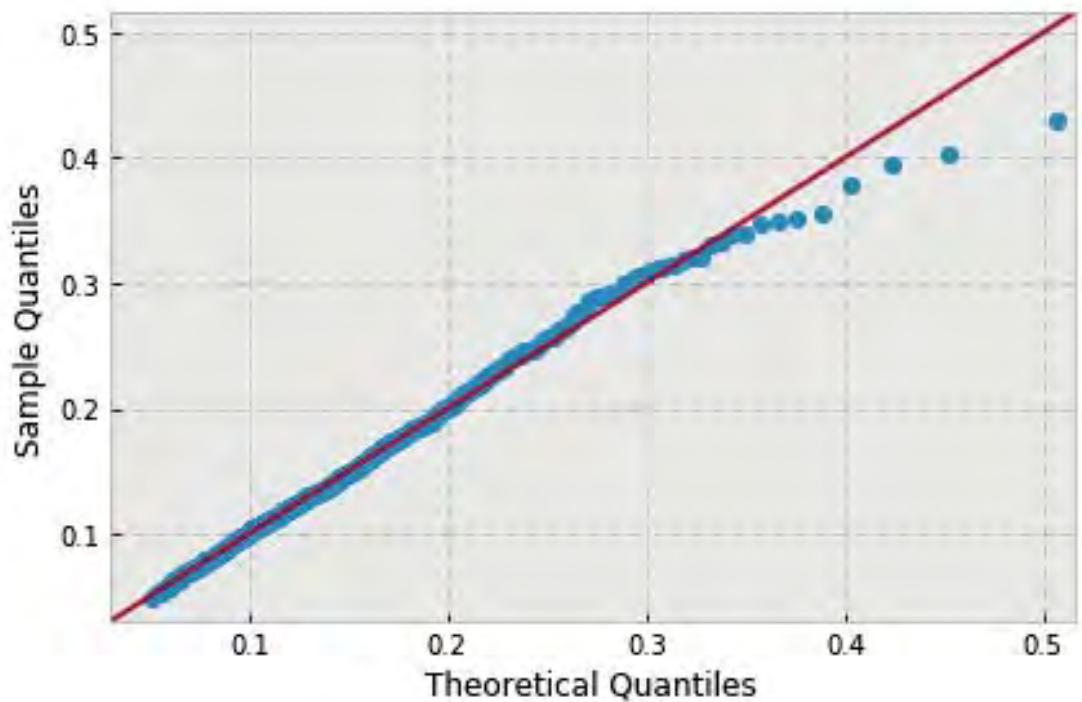
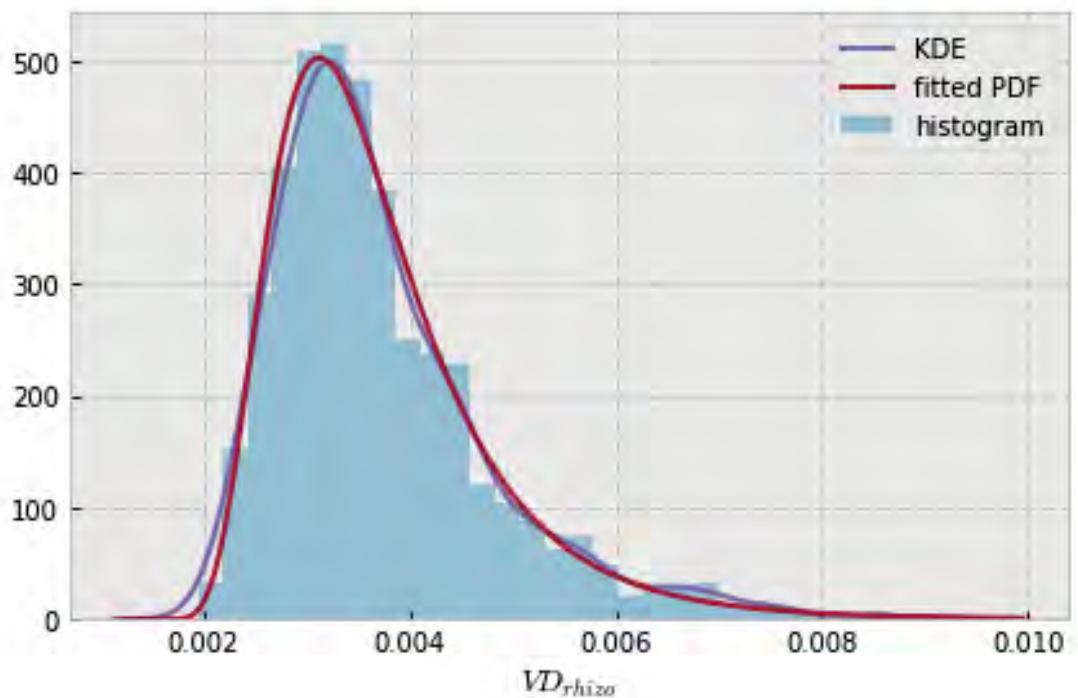


```
variable name: ln
variable value: 2.0
distribution: inverse-gamma(shape=(10.744565005180196,), loc=0.0001804332709162789,
    scale=0.012442186843862484)
DescribeResult(nobs=1000, minmax=(0.00071342416007436147, 0.0044682760687837948)
    mean=0.0014573270875595133, variance=1.8820950318122591e-07,
    skewness=1.621462398114542, kurtosis=5.369067406334759)
```

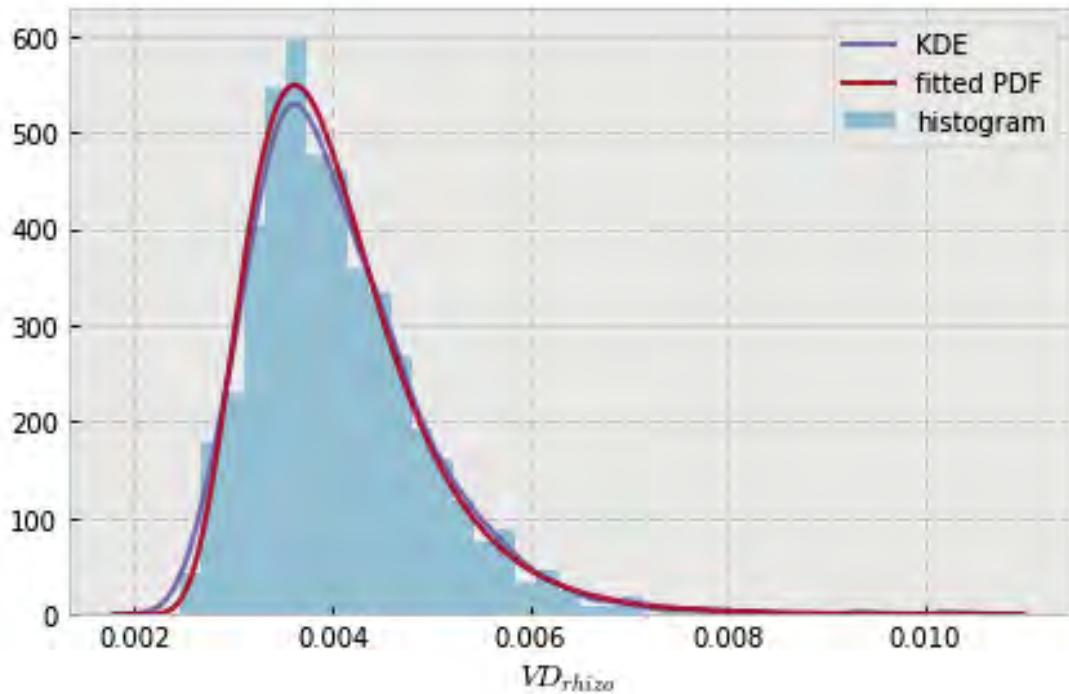


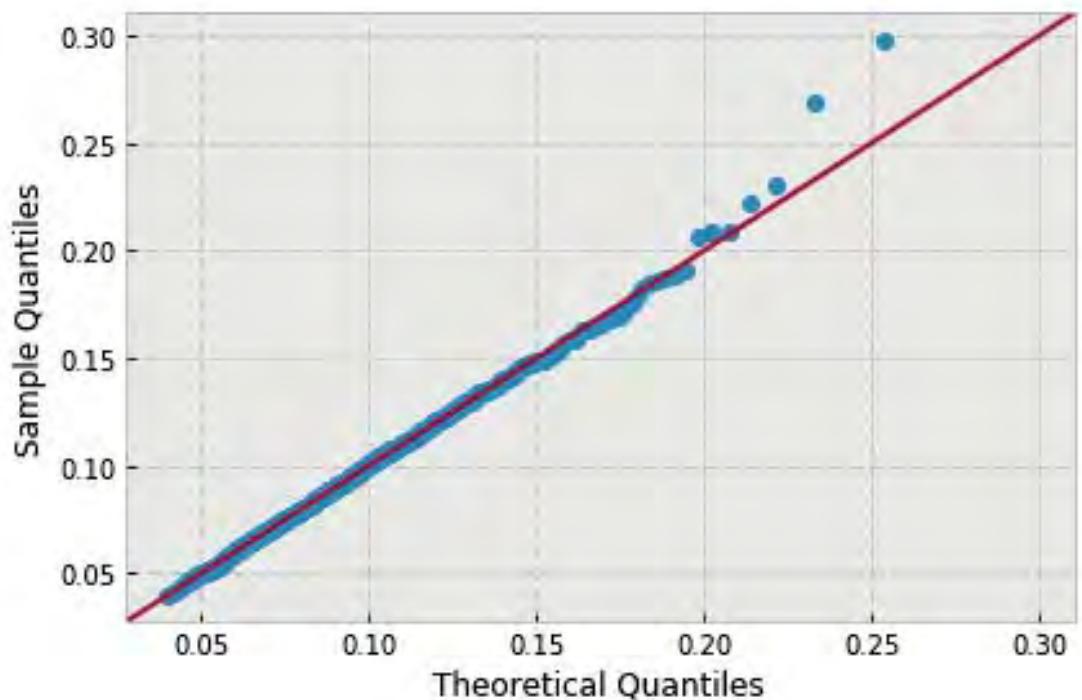


```
variable name: nB
variable value: 4.0
distribution: inverse-gamma(shape=(8.0104726207798809,), loc=0.0010256204576204494,
scale=0.018905776665849163)
DescribeResult(nobs=1000, minmax=(0.0019442132076032911, 0.0091433130748723914)
mean=0.0037206573322806614, variance=1.1528022519496199e-06,
skewness=1.4381717653790576, kurtosis=2.671209164554921)
```

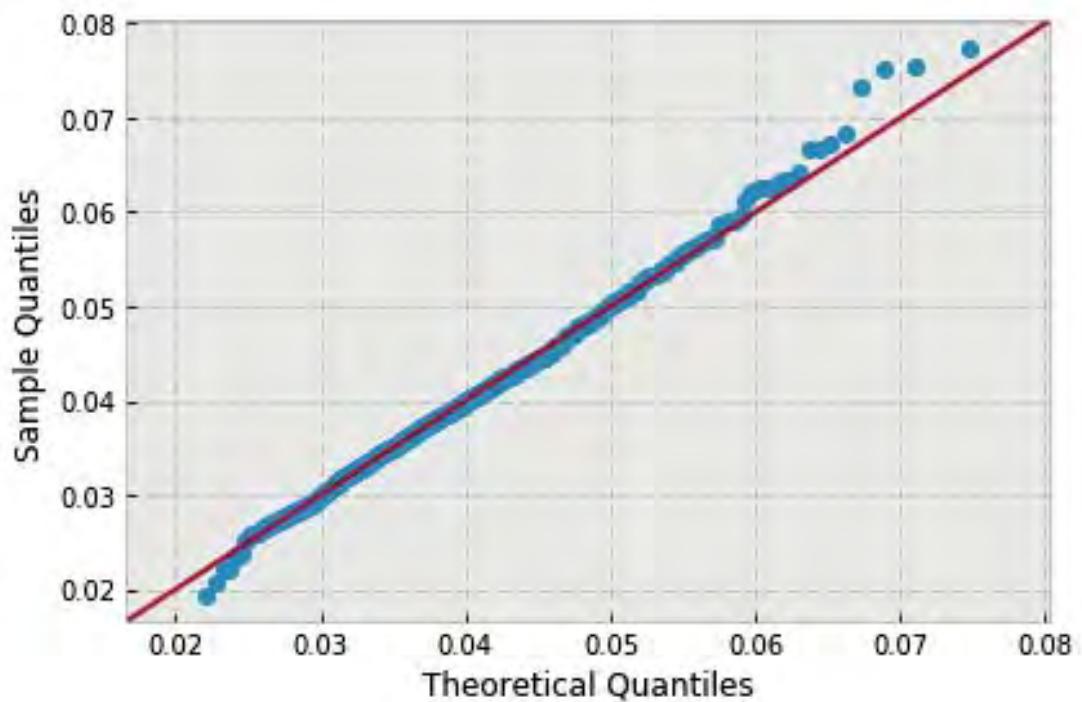
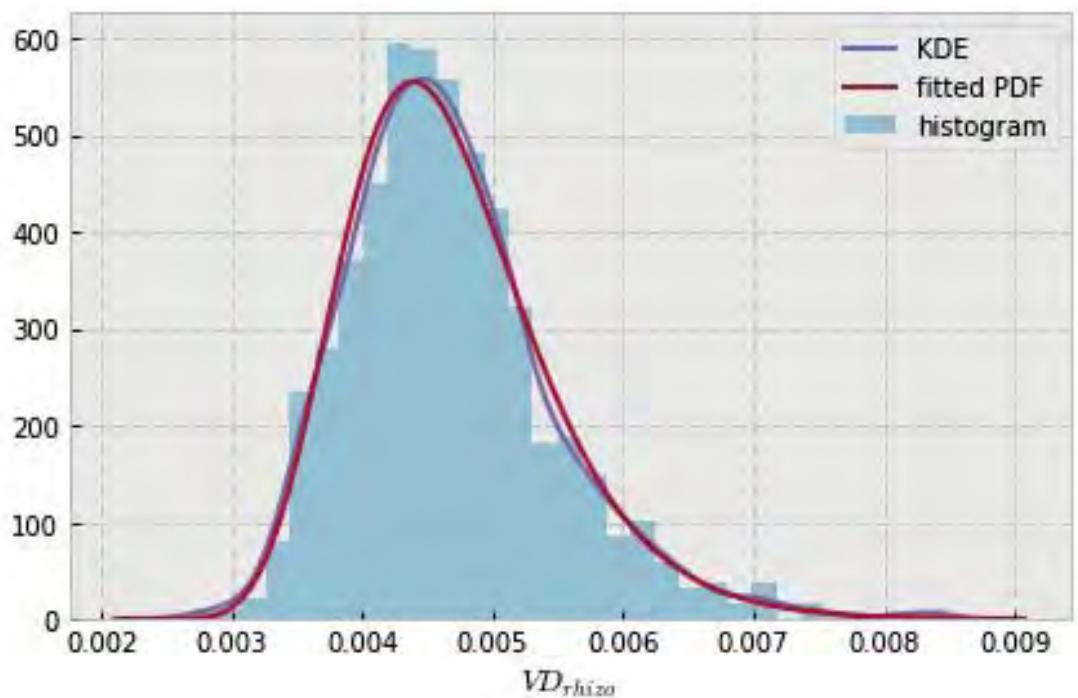


```
variable name: nB
variable value: 8.0
distribution: inverse-gamma(shape=(11.813771181388681,), loc=0.0012381184789665768,
                           scale=0.030526552060375512)
DescribeResult(nobs=1000, minmax=(0.0024640916836820323, 0.010321124080785775)
               mean=0.0040610110789647044, variance=8.145070821511065e-07,
               skewness=1.468890581489218, kurtosis=4.547192793281773)
```

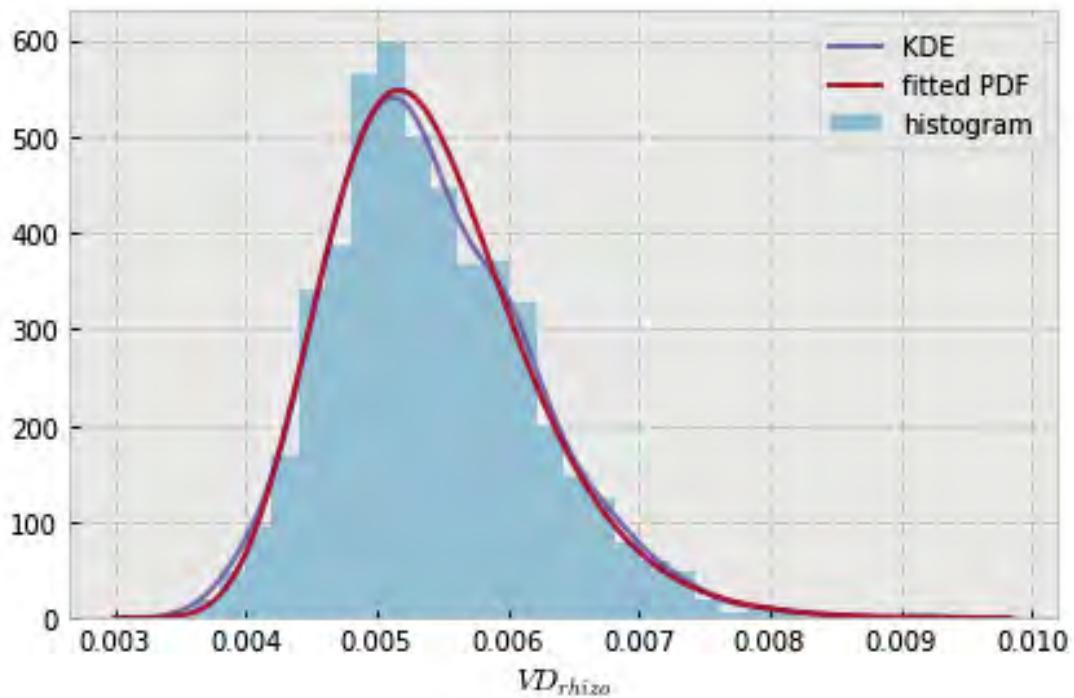


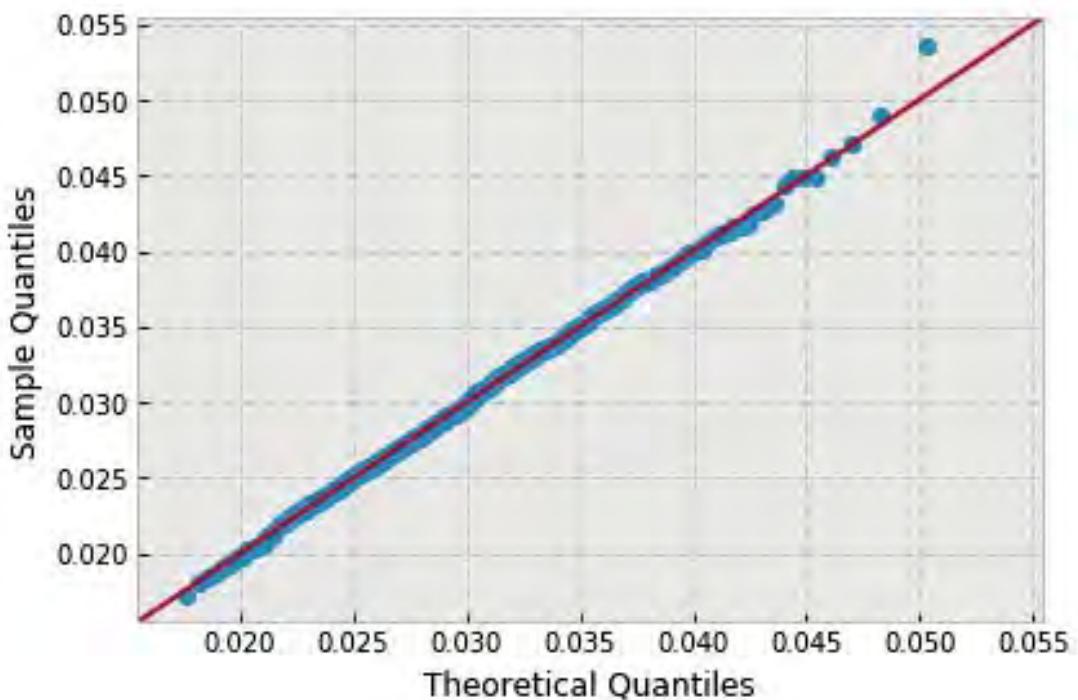


```
variable name: nB
variable value: 12.0
distribution: inverse-gamma(shape=(26.500672873412647,), loc=0.0007648497354108912,
                           scale=0.09964751163058158)
DescribeResult(nobs=1000, minmax=(0.0026982531104859579, 0.0084750270495297913)
               mean=0.0046729864416953024, variance=6.3574605845967557e-07,
               skewness=0.9753962673878616, kurtosis=1.854037612080461)
```

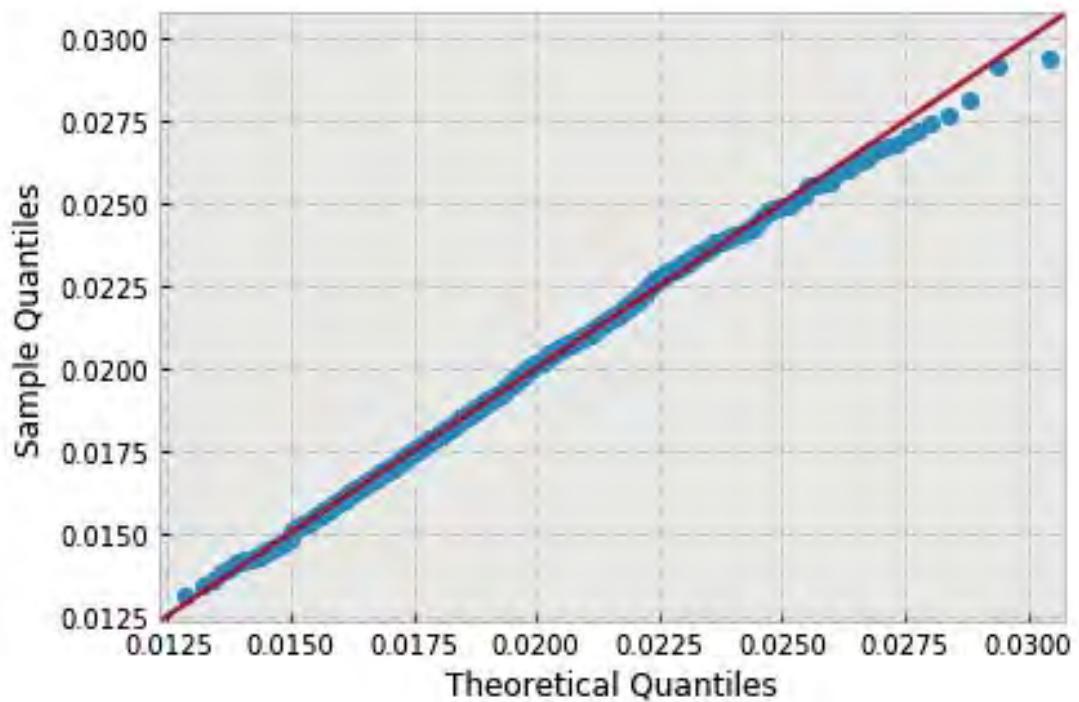
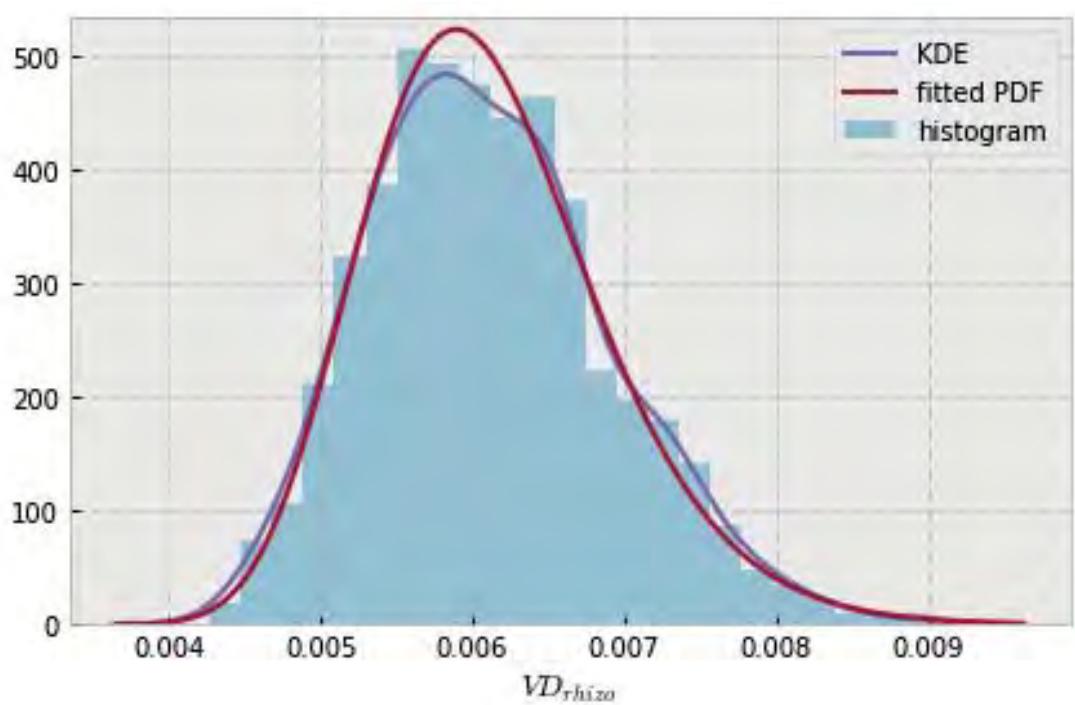


```
variable name: nB
variable value: 16.0
distribution: inverse-gamma(shape=(35.473672611088219,), loc=0.0009063066499440092,
                           scale=0.15557082075236567)
DescribeResult(nobs=1000, minmax=(0.0035959427377552223, 0.0092421799339248798)
               mean=0.0054190059915920543, variance=6.0759107970053346e-07,
               skewness=0.6862141189443981, kurtosis=0.8028792244046334)
```

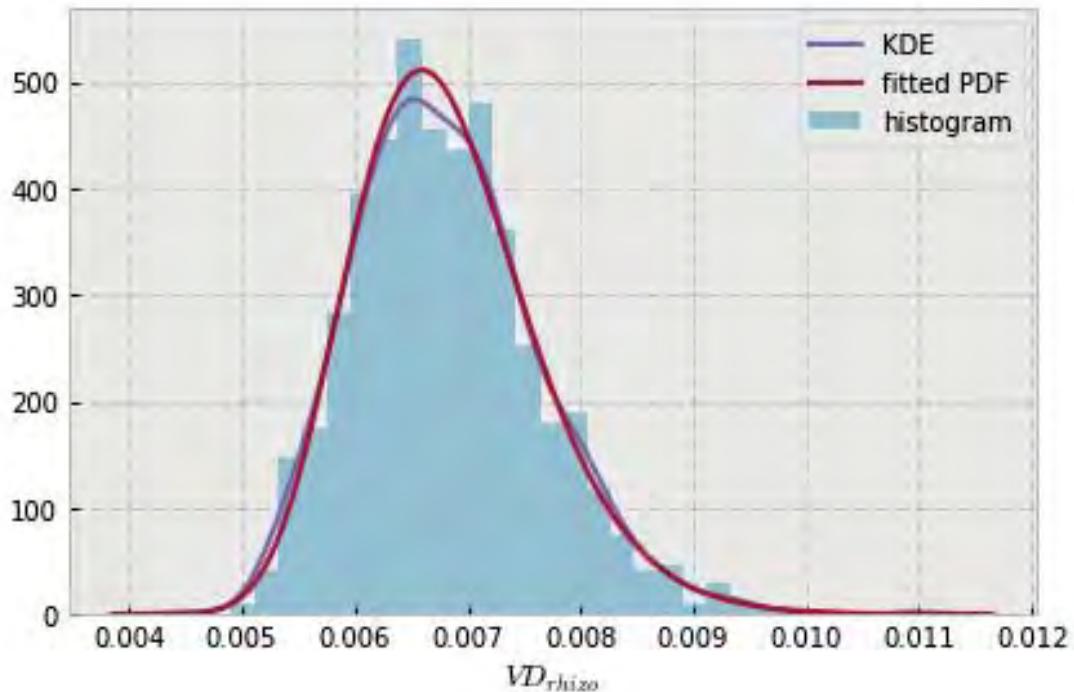


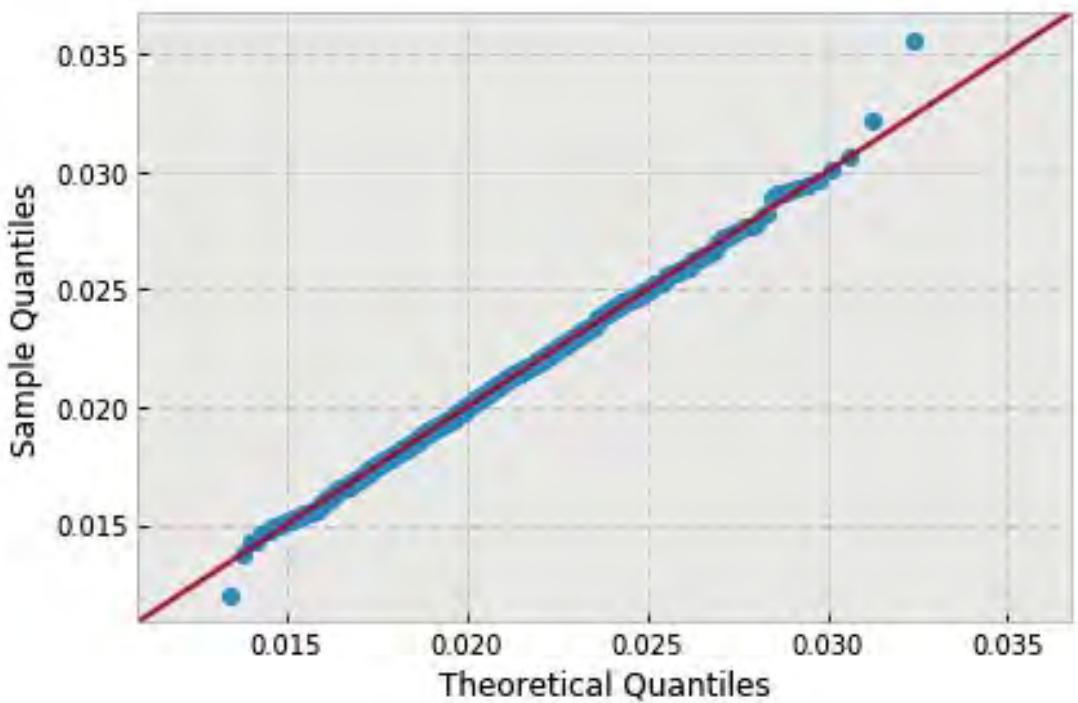


```
variable name: nB
variable value: 20.0
distribution: inverse-gamma(shape=(52.431264340085477,), loc=0.00044394658549417525,
                           scale=0.29118492285017317)
DescribeResult(nobs=1000, minmax=(0.0042634676366660931, 0.0090107428867594944)
               mean=0.0061102519108633385, variance=6.2951101082064256e-07,
               skewness=0.45213154934831573, kurtosis=0.04288118373058358)
```

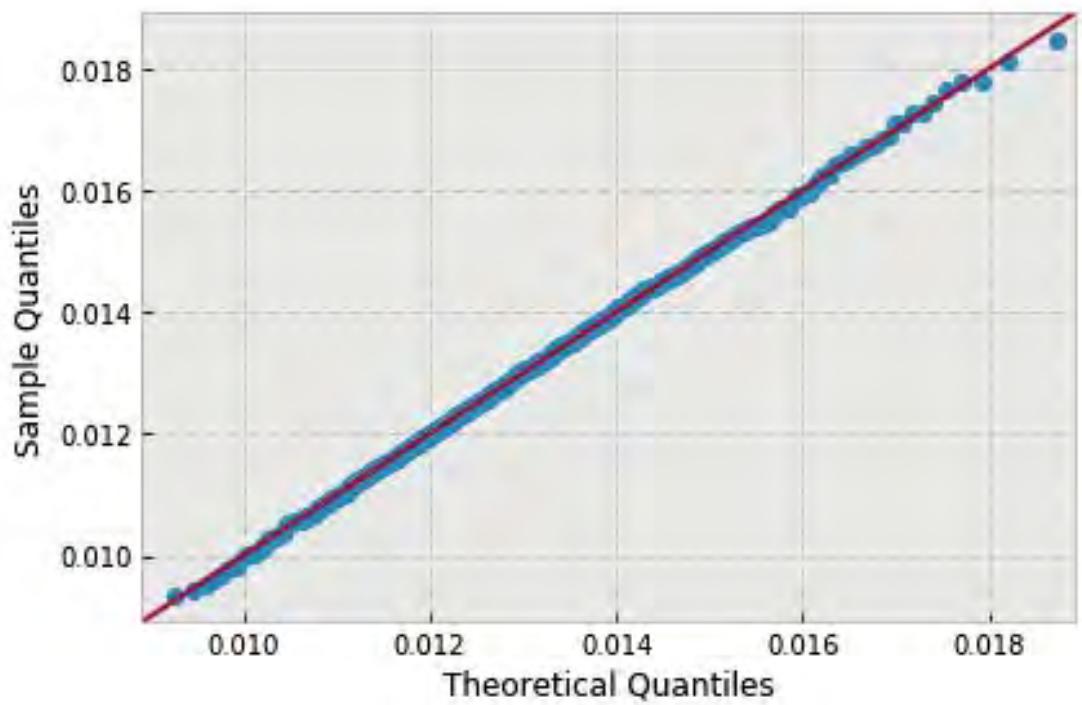
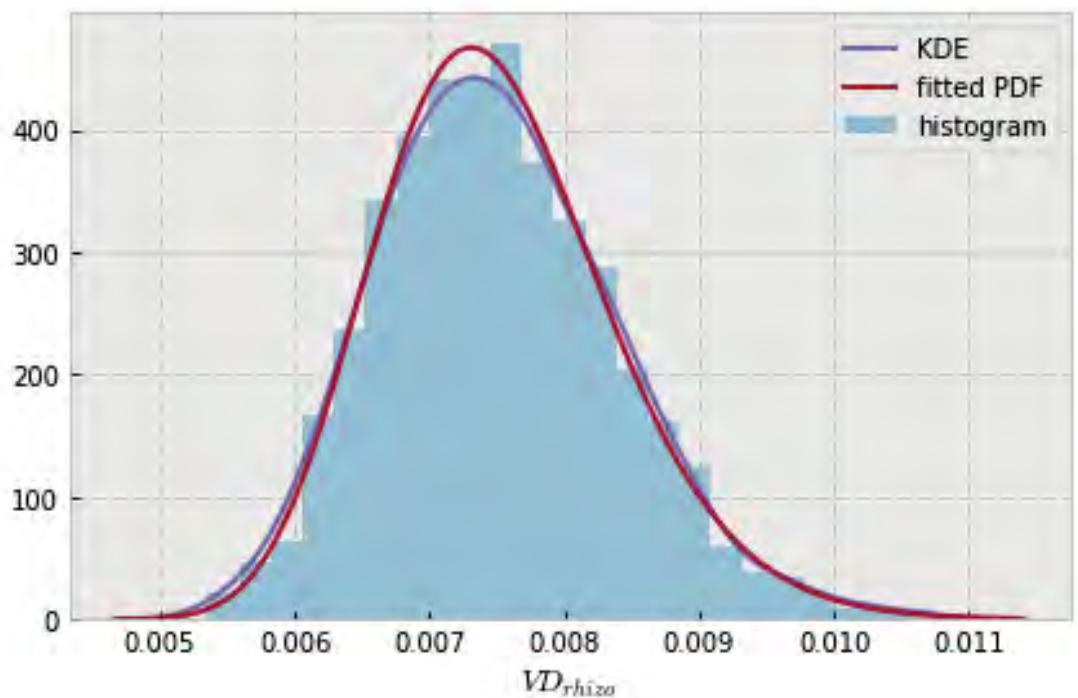


```
variable name: nB
variable value: 24.0
distribution: inverse-gamma(shape=(49.92545523369224,), loc=0.001139094253517458,
                           scale=0.27779848048376876)
DescribeResult(nobs=1000, minmax=(0.0044827267333018232, 0.011014849137745778)
               mean=0.006817116546687714, variance=6.7438979889019732e-07,
               skewness=0.6093968702604218, kurtosis=0.8148635511576097)
```

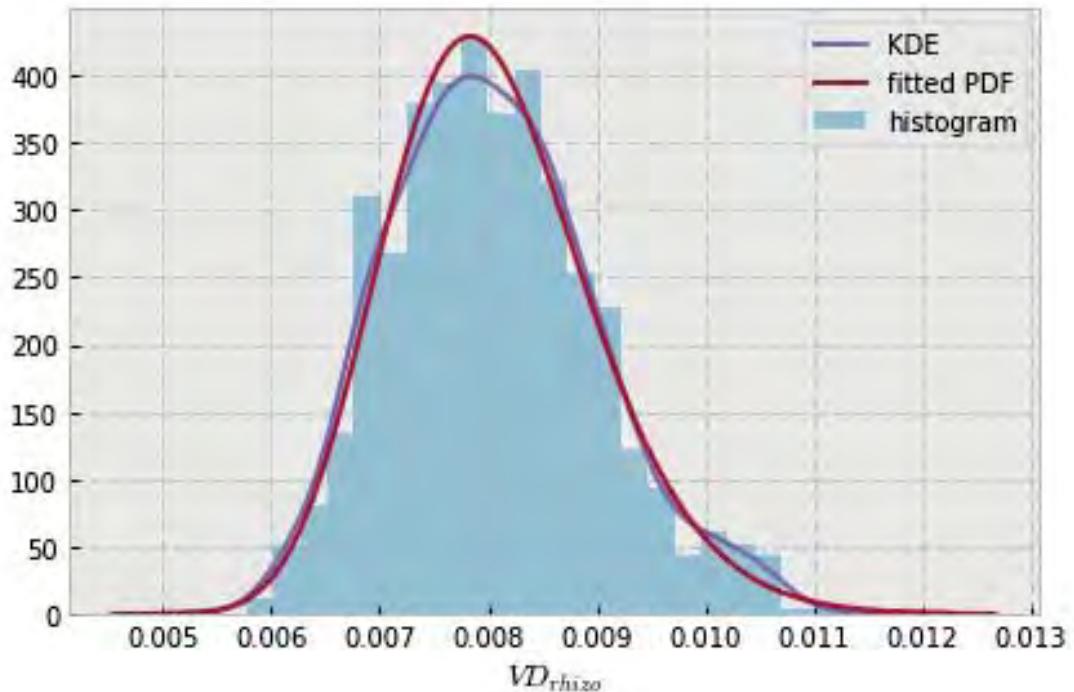


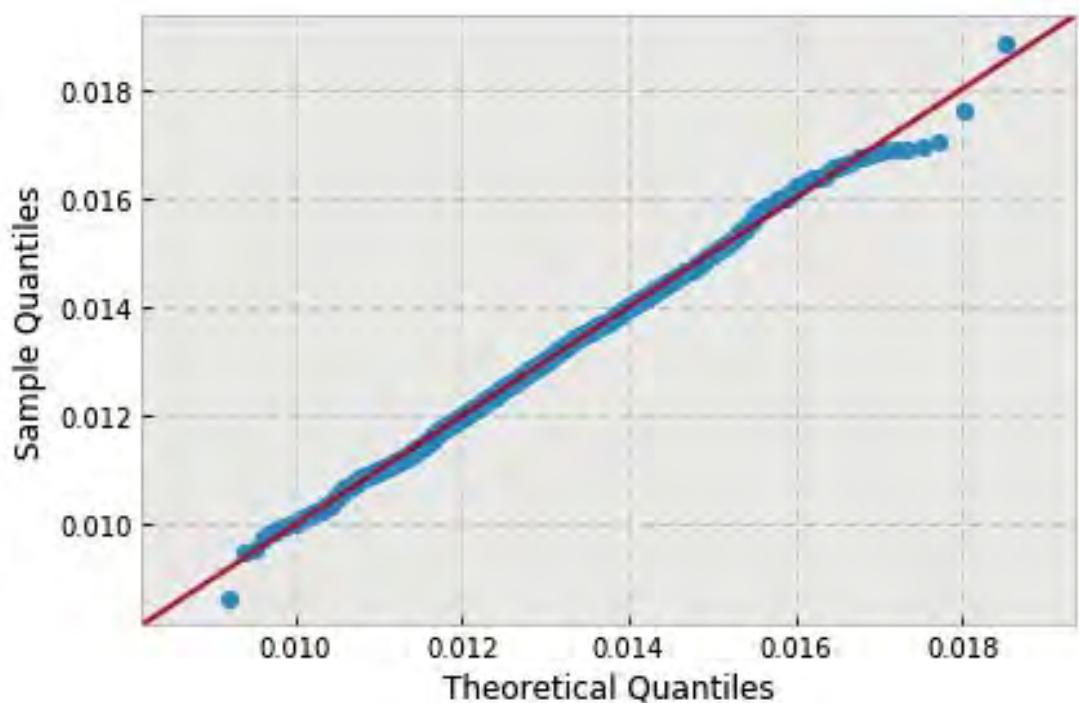


```
variable name: nB
variable value: 28.0
distribution: inverse-gamma(shape=(77.896404189639441,), loc=-0.0001698337821993993,
                           scale=0.5900333523908037)
DescribeResult(nobs=1000, minmax=(0.0053560251767997159, 0.01071843165880697)
               mean=0.0075000166845963787, variance=7.7040677748922811e-07,
               skewness=0.40199372345433193, kurtosis=0.16752426936218567)
```

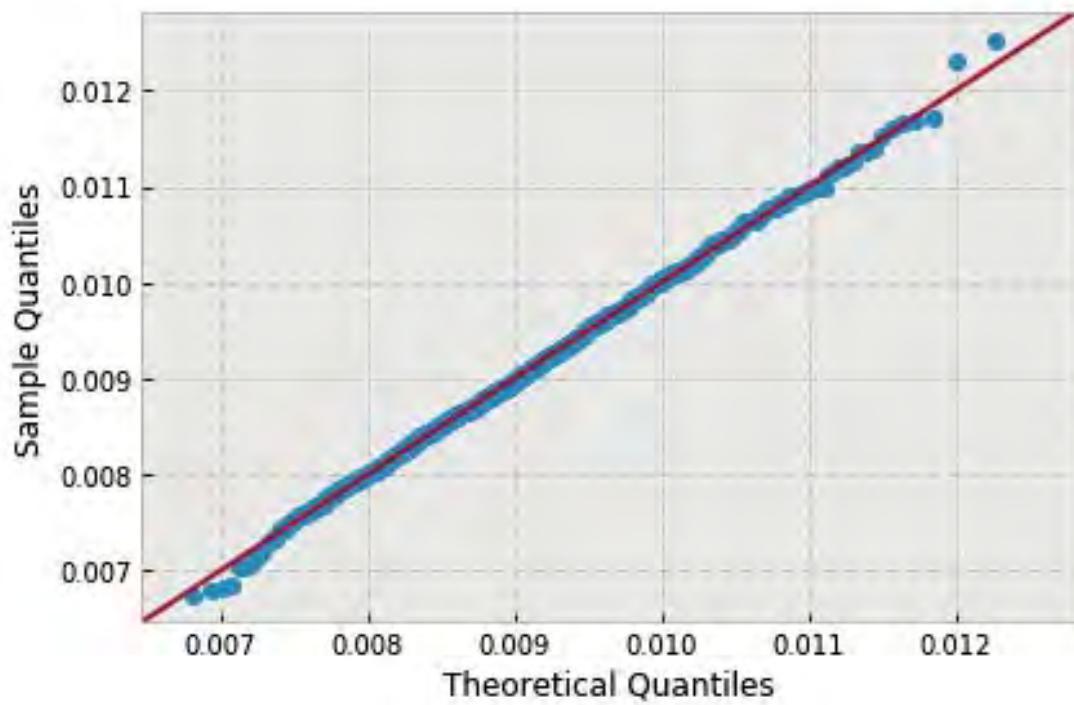
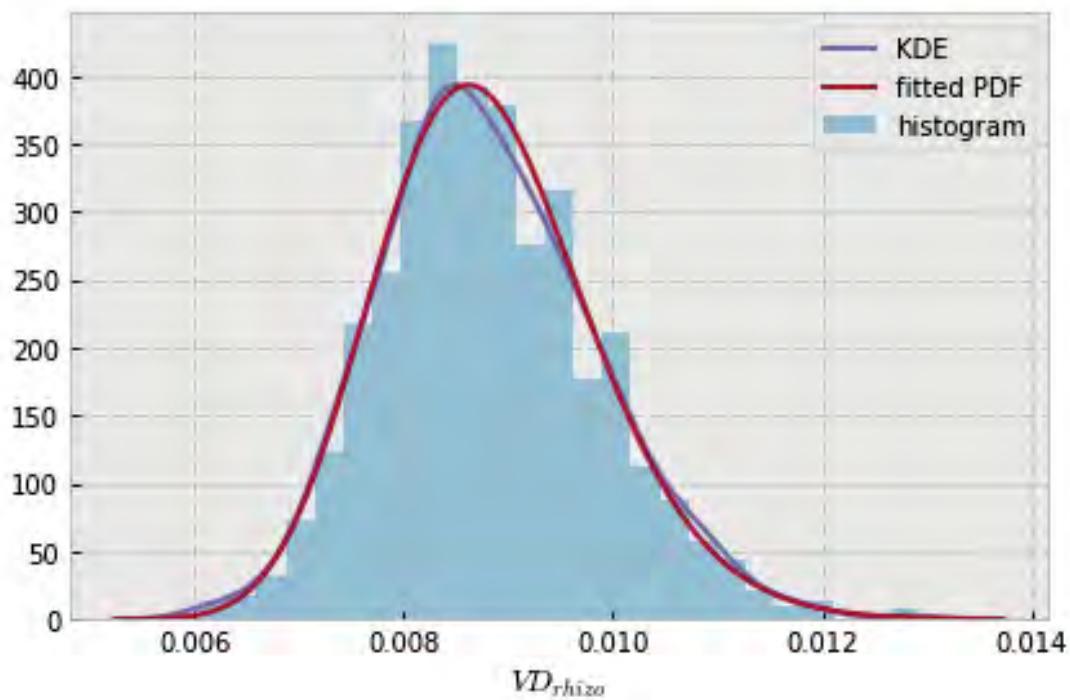


```
variable name: nB
variable value: 32.0
distribution: inverse-gamma(shape=(78.603216565503601,), loc=-0.00036295549584243426,
                           scale=0.6523188225012511)
DescribeResult(nobs=1000, minmax=(0.0052875250605549685, 0.011923982089949599)
               mean=0.0080424343532093864, variance=9.1981496058978906e-07,
               skewness=0.4195540882743349, kurtosis=0.13190397540692578)
```

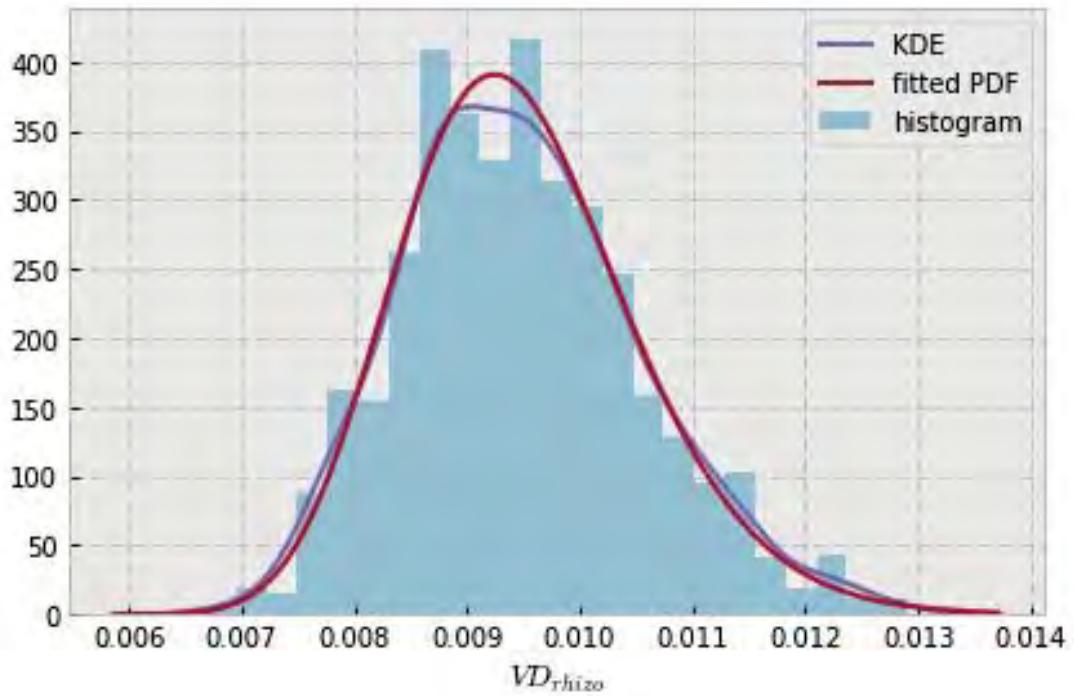


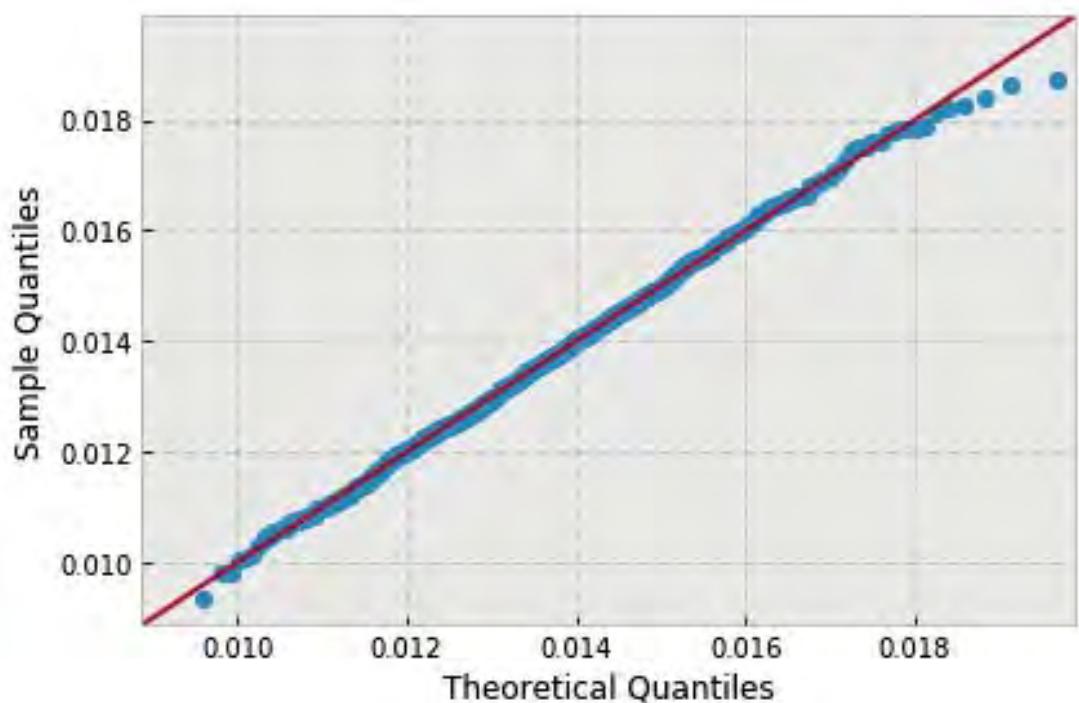


```
variable name: nB
variable value: 36.0
distribution: inverse-gamma(shape=(111.21135967126452,), loc=-0.0020217593780128863,
                           scale=1.193855142439943)
DescribeResult(nobs=1000, minmax=(0.0060418264499035321, 0.012925299871917815)
               mean=0.0088098648248777067, variance=1.0726152618374052e-06,
               skewness=0.38107176783736896, kurtosis=0.23540917368003544)
```

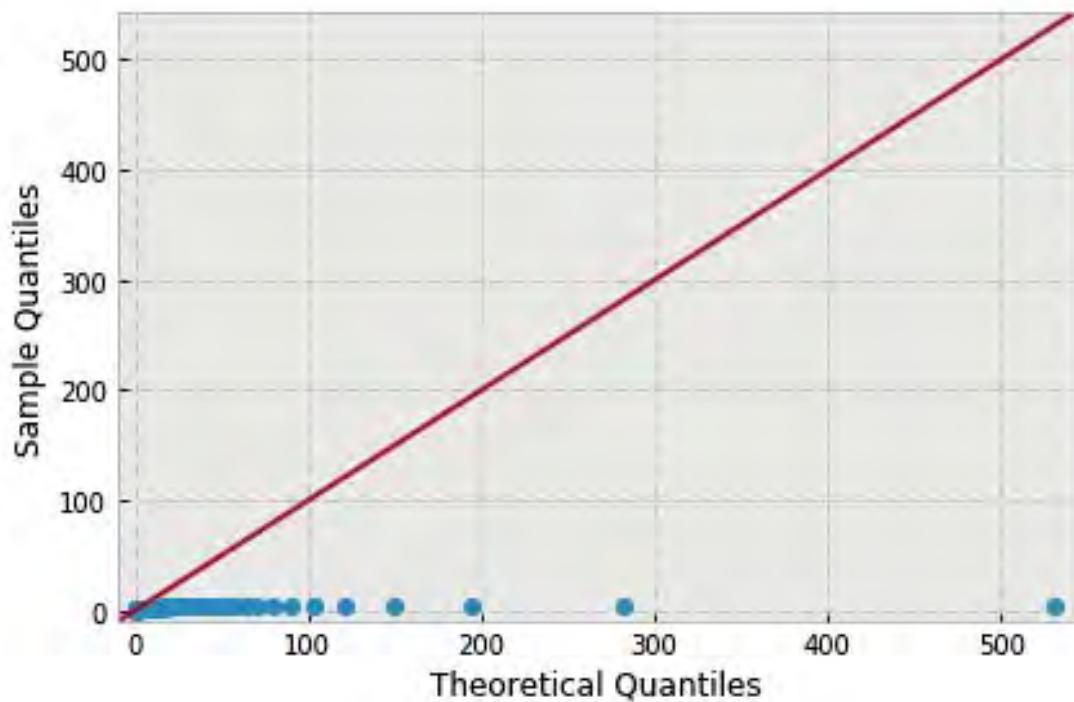
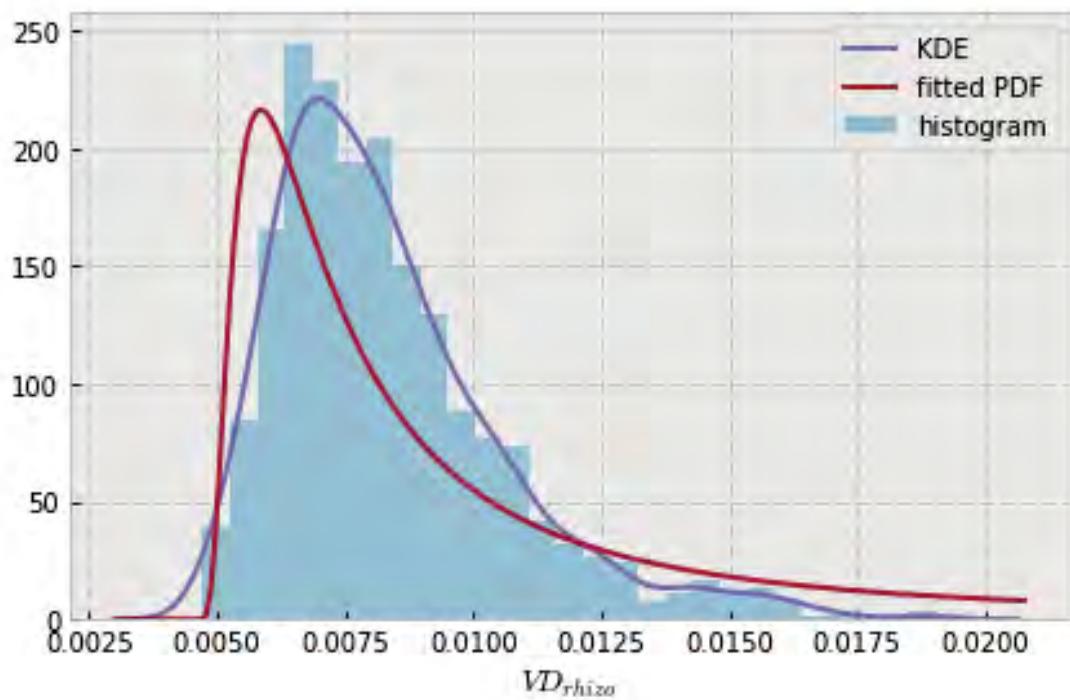


```
variable name: nB
variable value: 40.0
distribution: inverse-gamma(shape=(74.730739003254513,), loc=0.00048679860210071486,
                           scale=0.6631594196413808)
DescribeResult(nobs=1000, minmax=(0.006680978128720514, 0.012909129528221531)
               mean=0.0094810256909844248, variance=1.1091195372722806e-06,
               skewness=0.41829159540583105, kurtosis=0.03770779990903472)
```

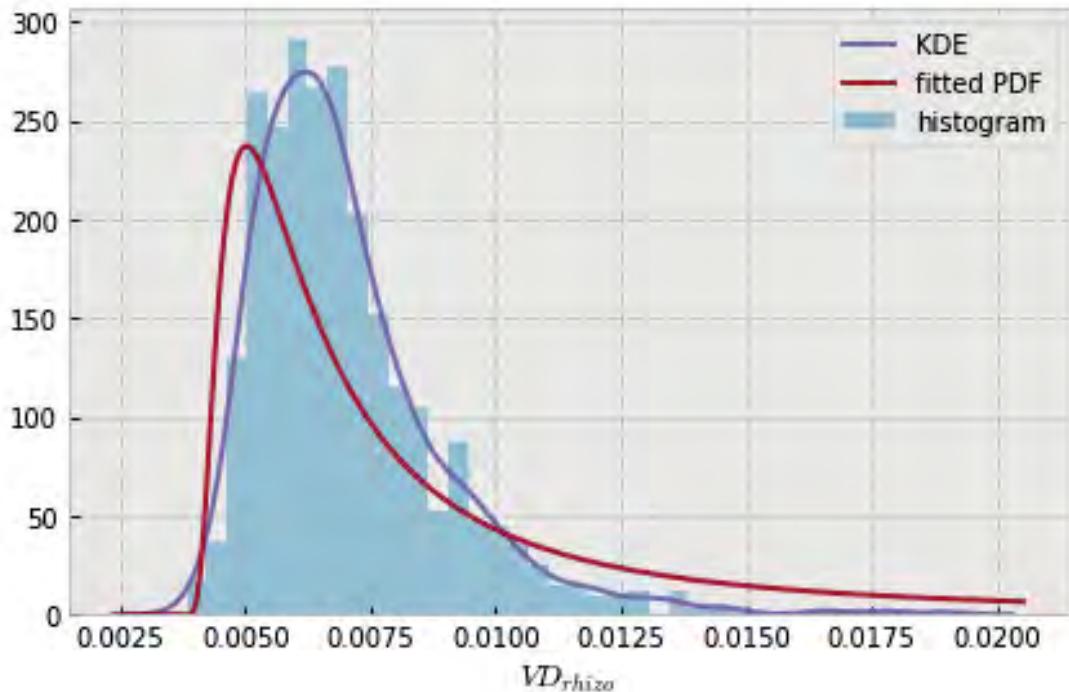


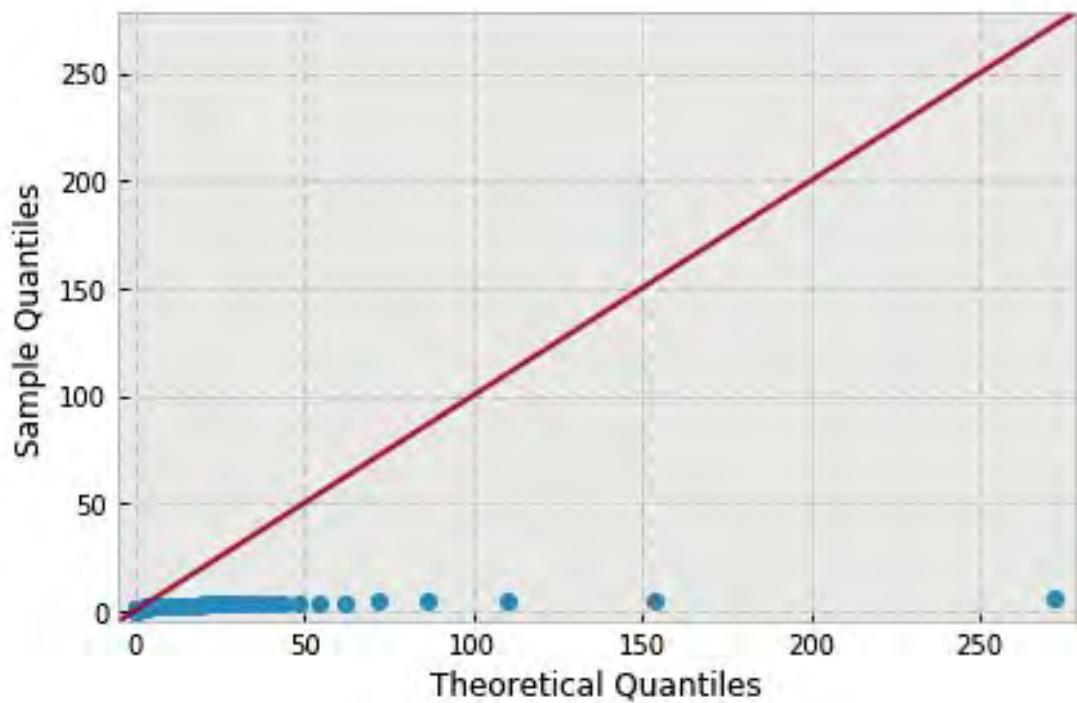


```
variable name: r
variable value: 1.0
distribution: inverse-gamma(shape=(1.0938643095302112,), loc=0.00451077861303528,
                           scale=0.0028045189775478866)
DescribeResult(nobs=1000, minmax=(0.0047190238817042705, 0.019037428911820985)
               mean=0.0083283160722629375, variance=5.1986464713326919e-06,
               skewness=1.3356445931500143, kurtosis=2.1838672114100435)
```

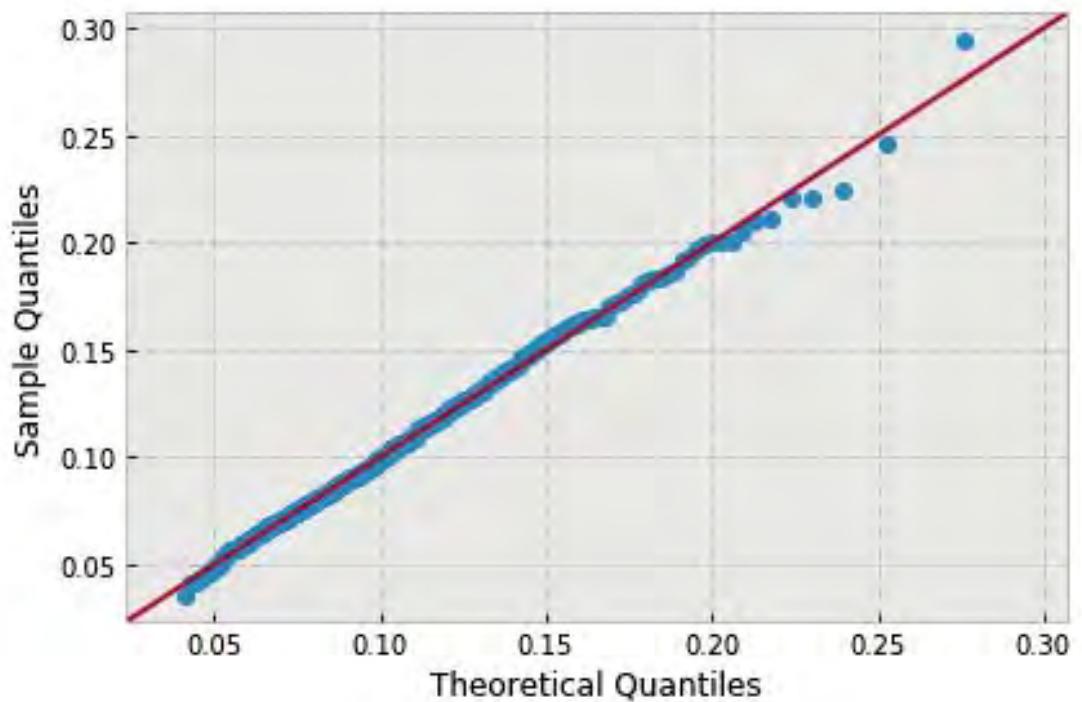
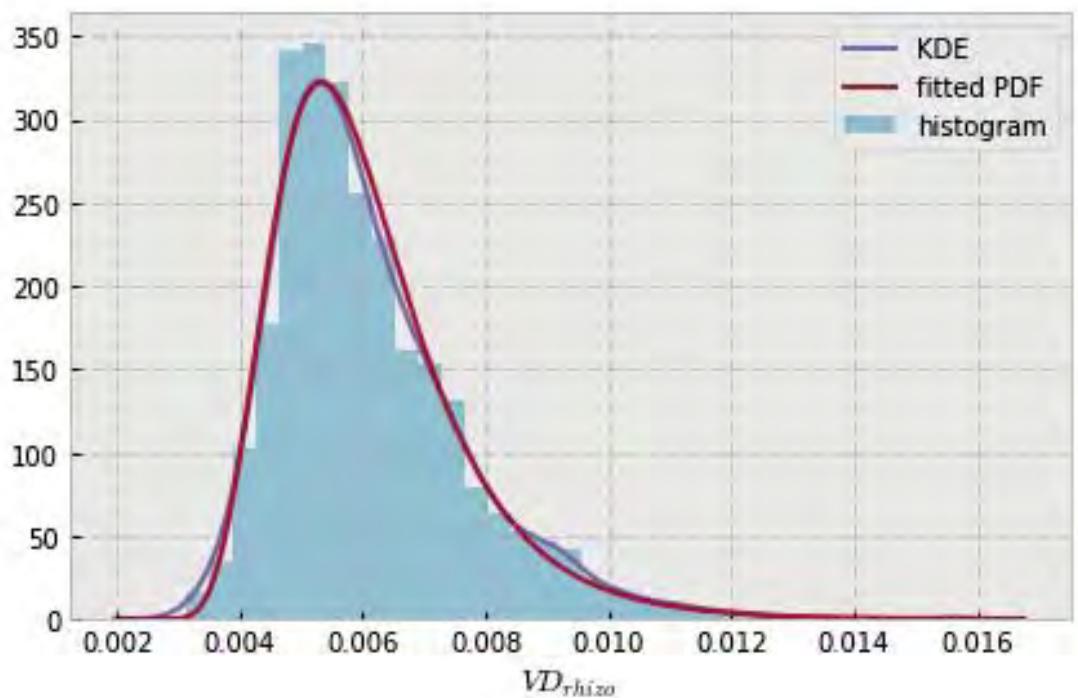


```
variable name: r
variable value: 1.222222222222223
distribution: inverse-gamma(shape=(1.2131204682921686,), loc=0.003704347731237405,
    scale=0.002926247981325655)
DescribeResult(nobs=1000, minmax=(0.0038324891131462915, 0.019076250606701369)
    mean=0.0070373539385138262, variance=3.7265249406810239e-06,
    skewness=1.7879974048080902, kurtosis=5.248486205301537)
```

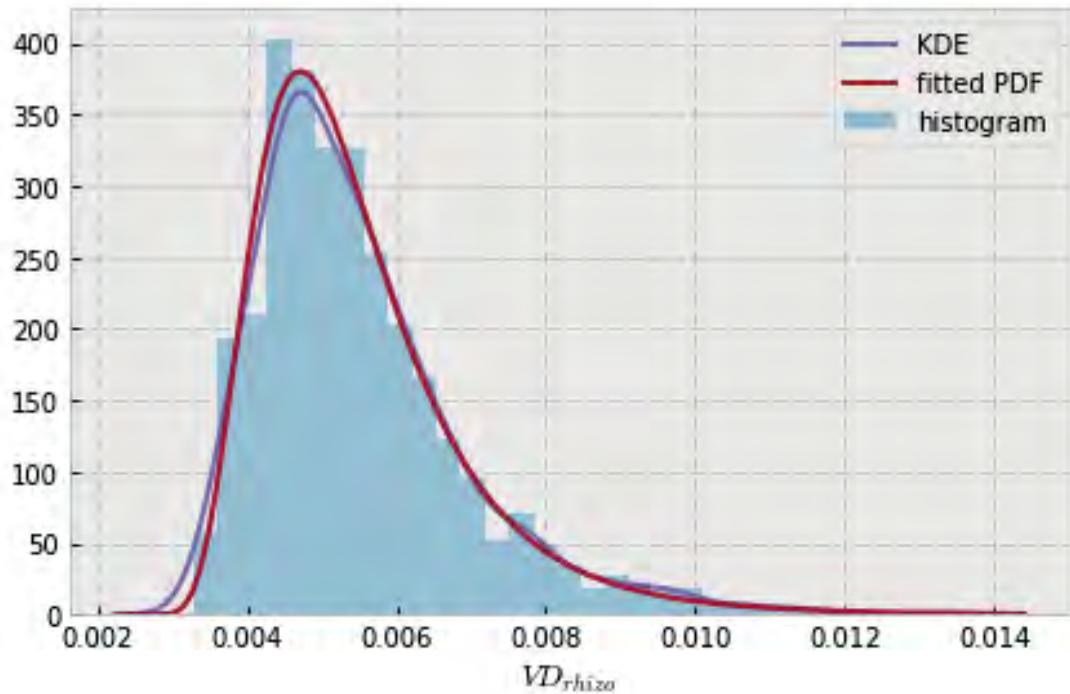


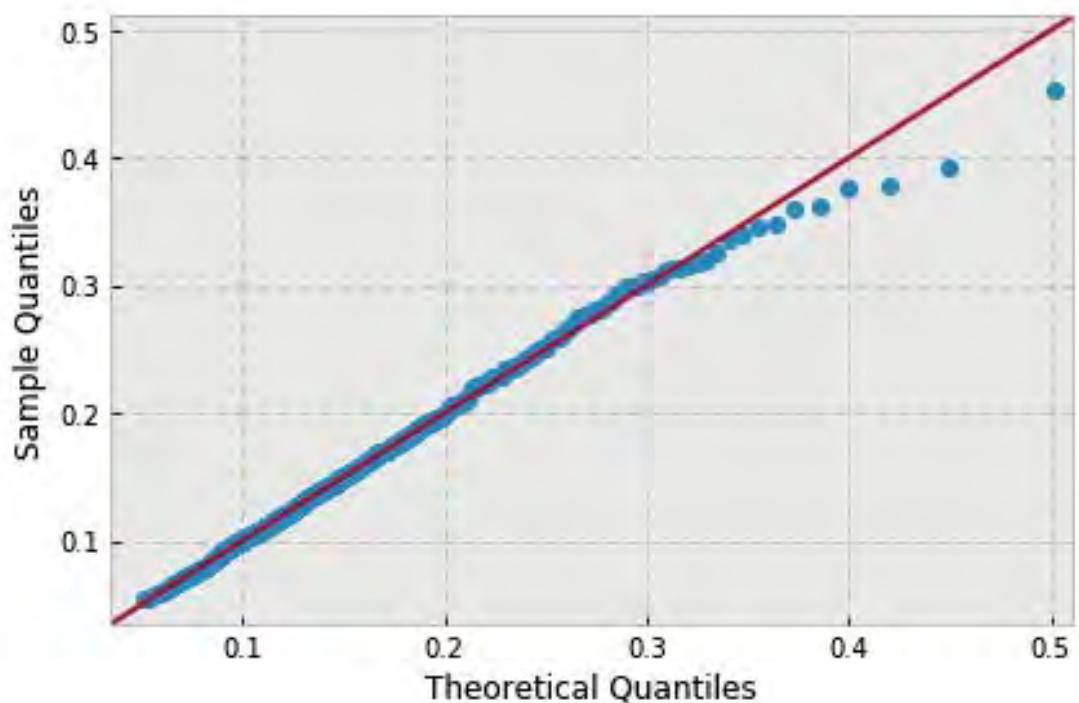


```
variable name: r
variable value: 1.444444444444444
distribution: inverse-gamma(shape=(11.247403361386162,), loc=0.0013790627502866105,
                           scale=0.04839205856361937)
DescribeResult(nobs=1000, minmax=(0.0031294580996540844, 0.015606680919136338)
               mean=0.0061014253250136277, variance=2.3913221456957216e-06,
               skewness=1.3040679674912916, kurtosis=2.695726343265669)
```

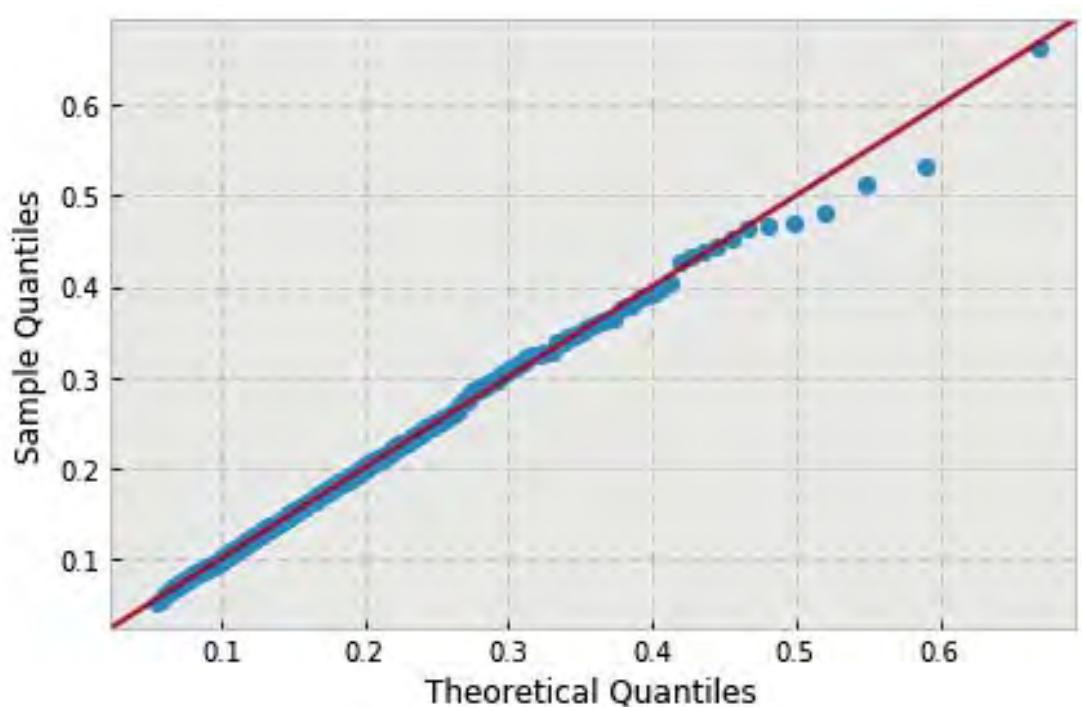
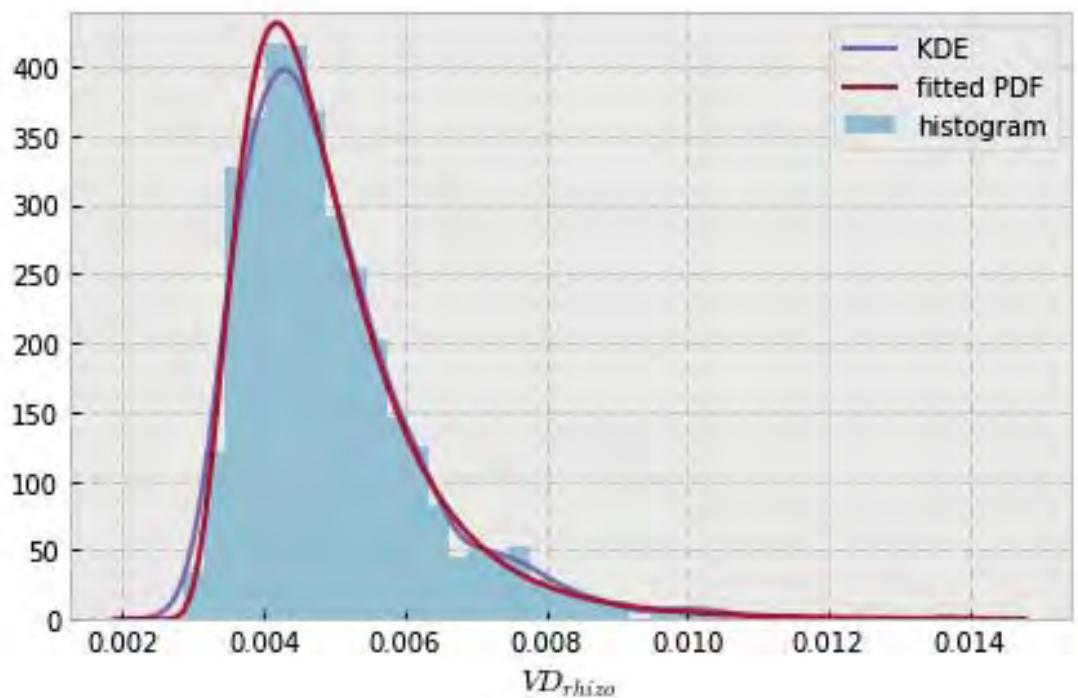


```
variable name: r
variable value: 1.6666666666666665
distribution: inverse-gamma(shape=(8.0481432009005793,), loc=0.0019296251886440547,
                           scale=0.025163692526551623)
DescribeResult(nobs=1000, minmax=(0.0032864345173158863, 0.013345832038683876)
               mean=0.0054963195634998756, variance=1.9854590264065984e-06,
               skewness=1.430428515243449, kurtosis=2.8614816560938747)
```

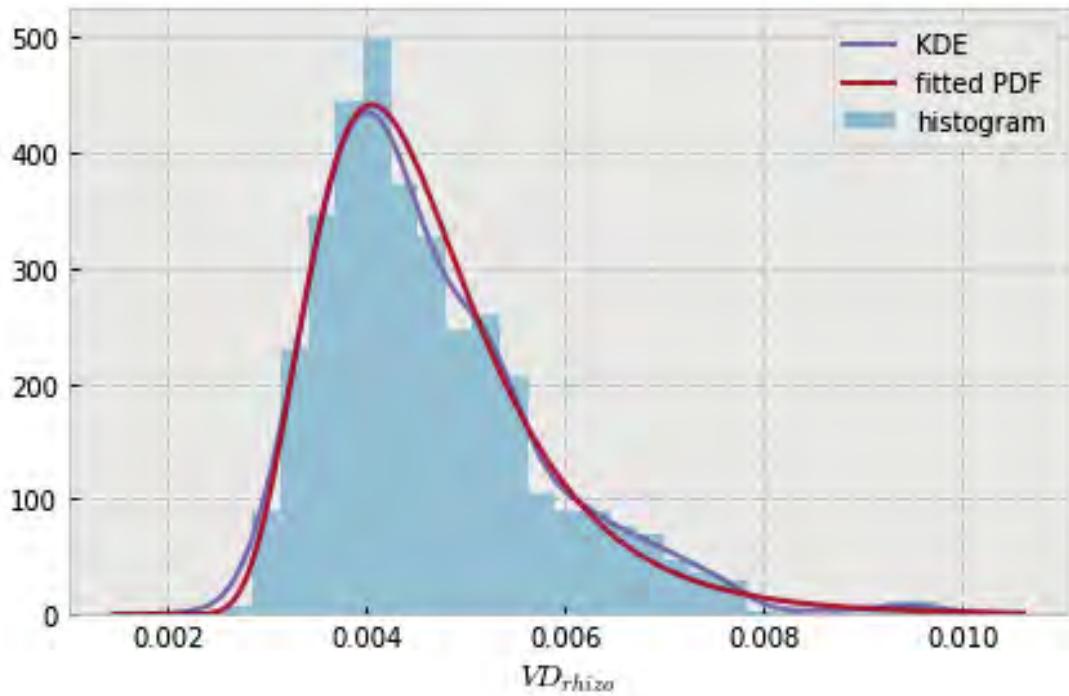


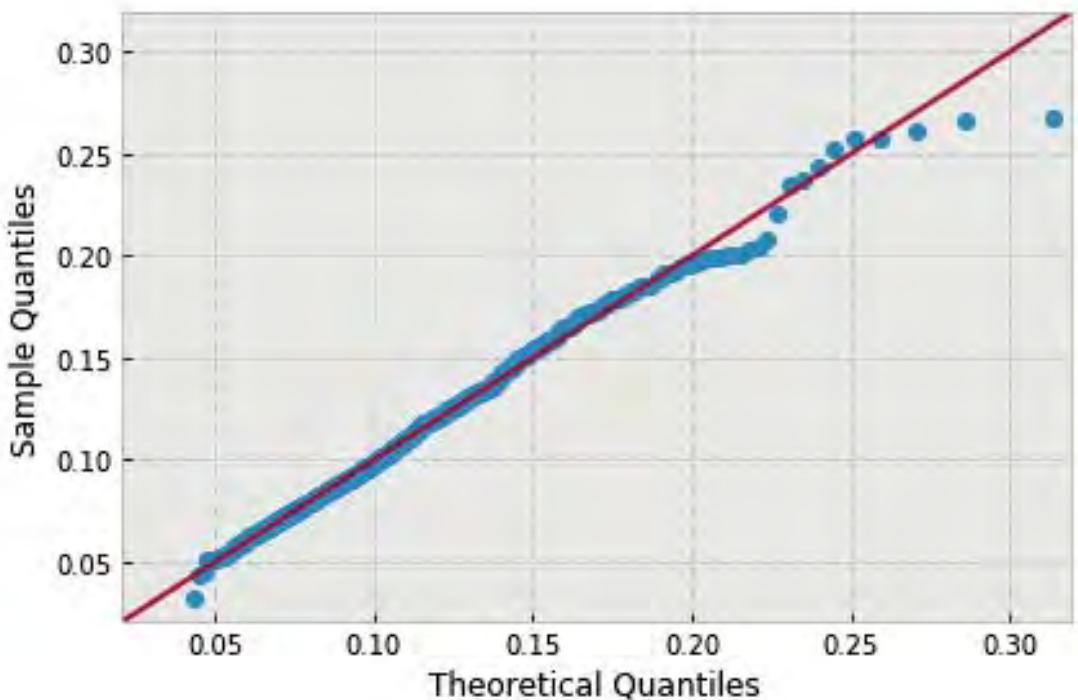


```
variable name: r
variable value: 1.888888888888888
distribution: inverse-gamma(shape=(6.9417803829681386,), loc=0.001930672665071207,
                           scale=0.017885566208050423)
DescribeResult(nobs=1000, minmax=(0.002876205929730894, 0.013787885739727954)
               mean=0.0049372833374327595, variance=1.7226624218826954e-06,
               skewness=1.6871520315716204, kurtosis=4.5607455666052035)
```

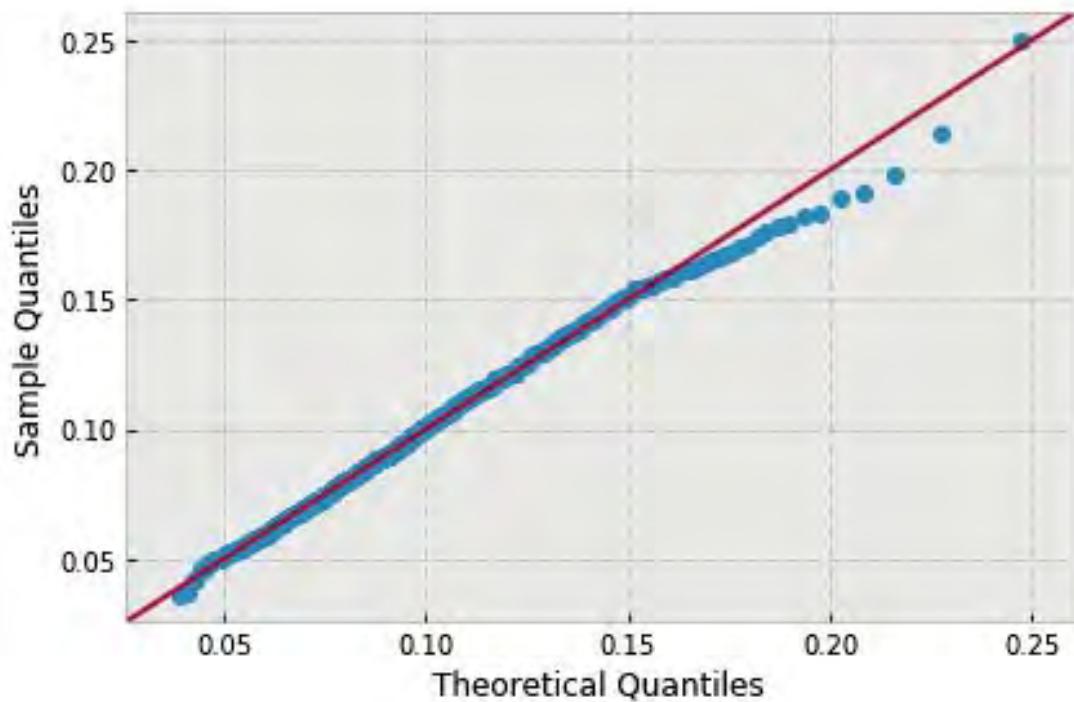
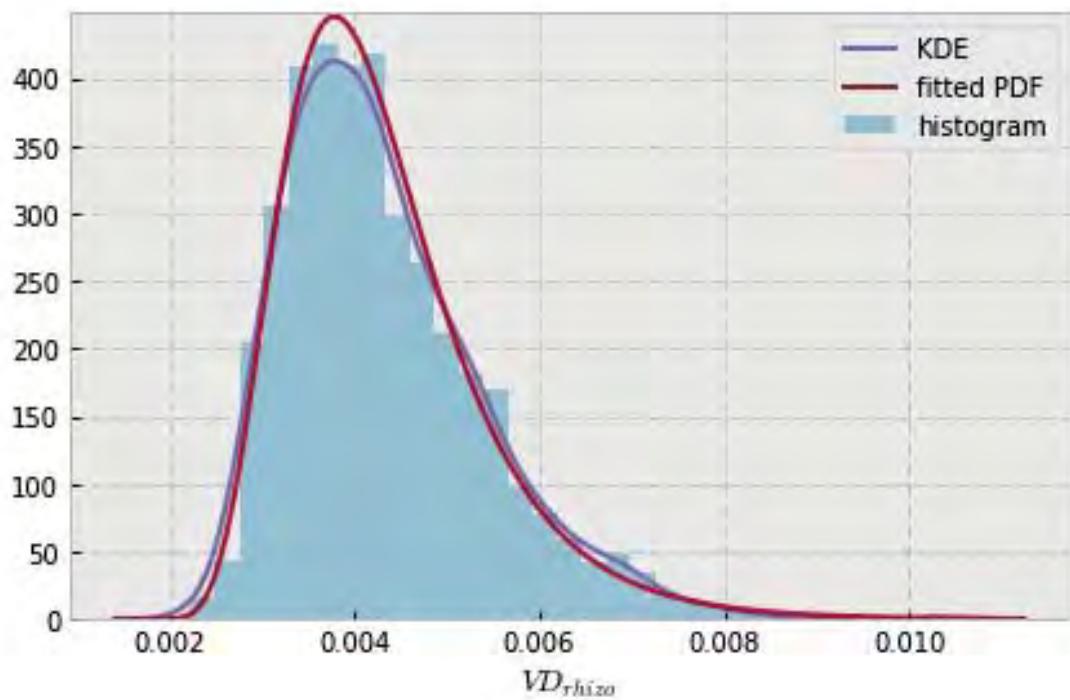


```
variable name: r
variable value: 2.11111111111111
distribution: inverse-gamma(shape=(10.436720462700194,), loc=0.0012839082615775013,
    scale=0.0316459851769063)
DescribeResult(nobs=1000, minmax=(0.00230461702103323, 0.0097627688087468089)
    mean=0.0046363767680091747, variance=1.2912886721616033e-06,
    skewness=1.2179470613823444, kurtosis=1.9605638106848575)
```





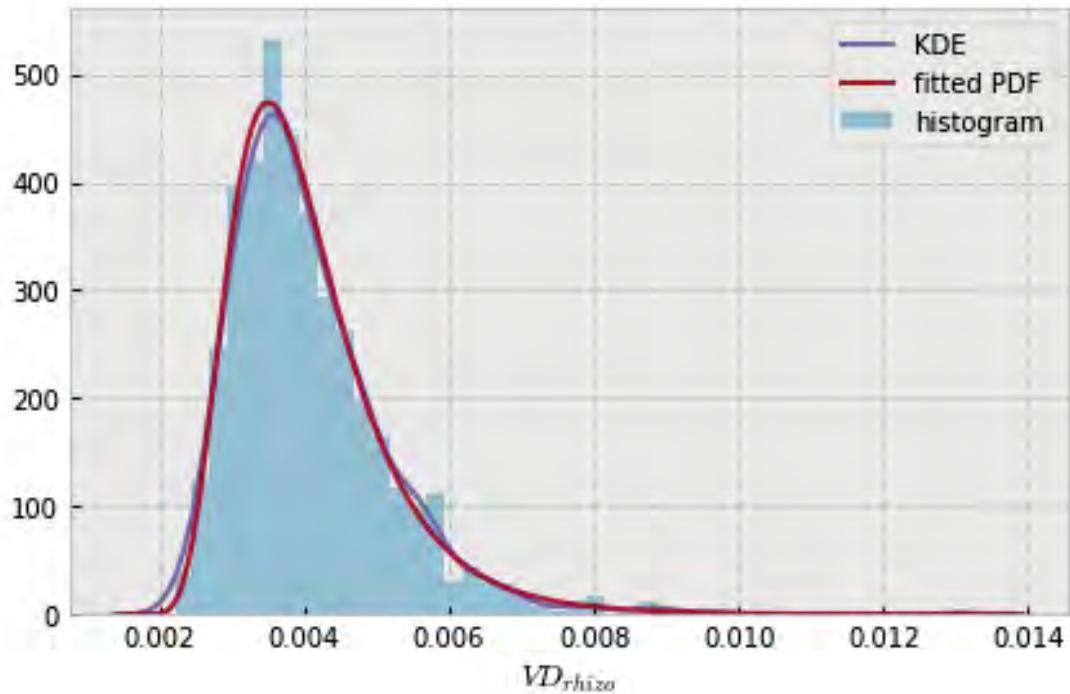
```
variable name: r
variable value: 2.333333333333333
distribution: inverse-gamma(shape=(12.000450121756892,), loc=0.0008264892873484918,
                           scale=0.038484962557652186)
DescribeResult(nobs=1000, minmax=(0.0022248245664479834, 0.01043387463253684)
               mean=0.0043230617324610705, variance=1.1718057654326208e-06,
               skewness=1.0768192797806933, kurtosis=1.6899487273565112)
```

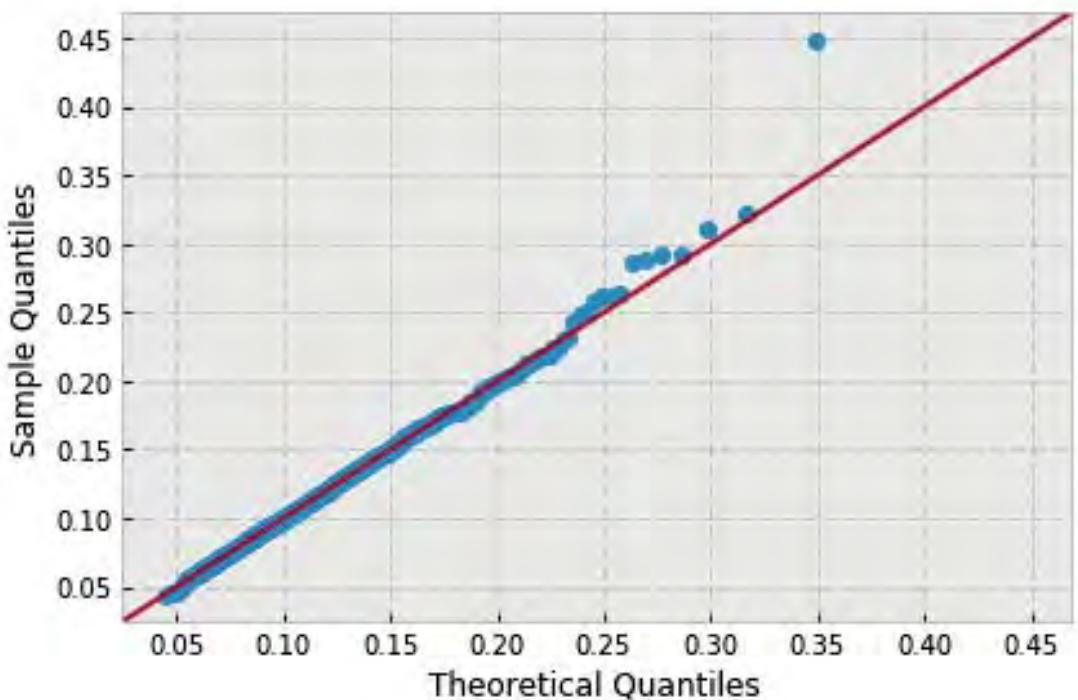


```

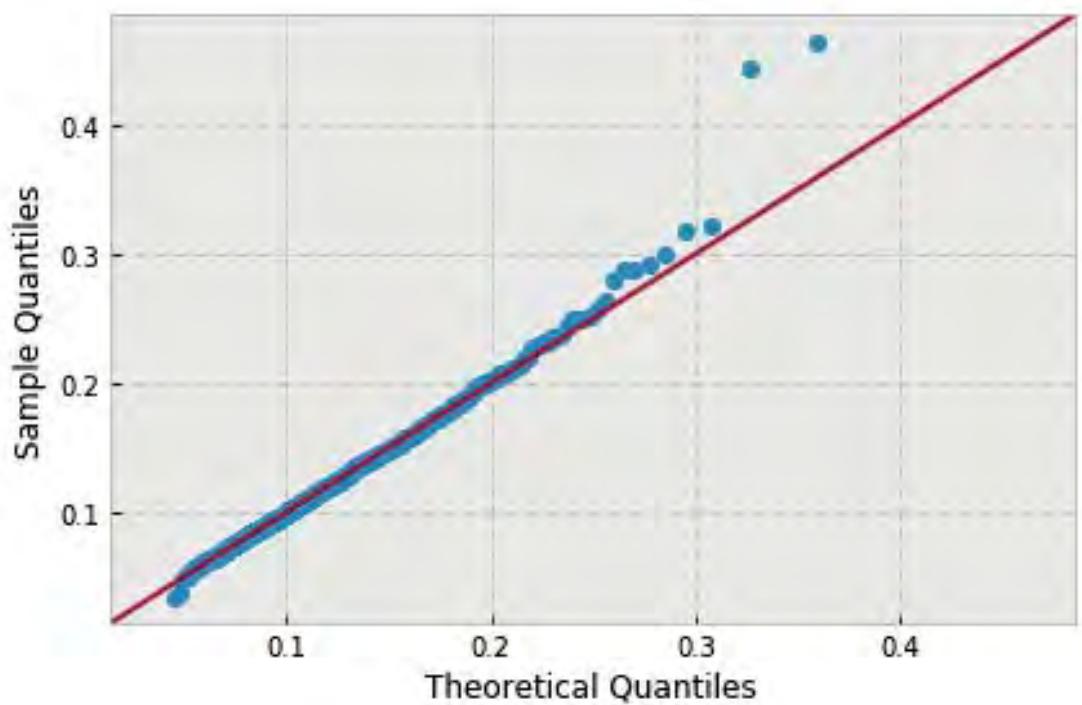
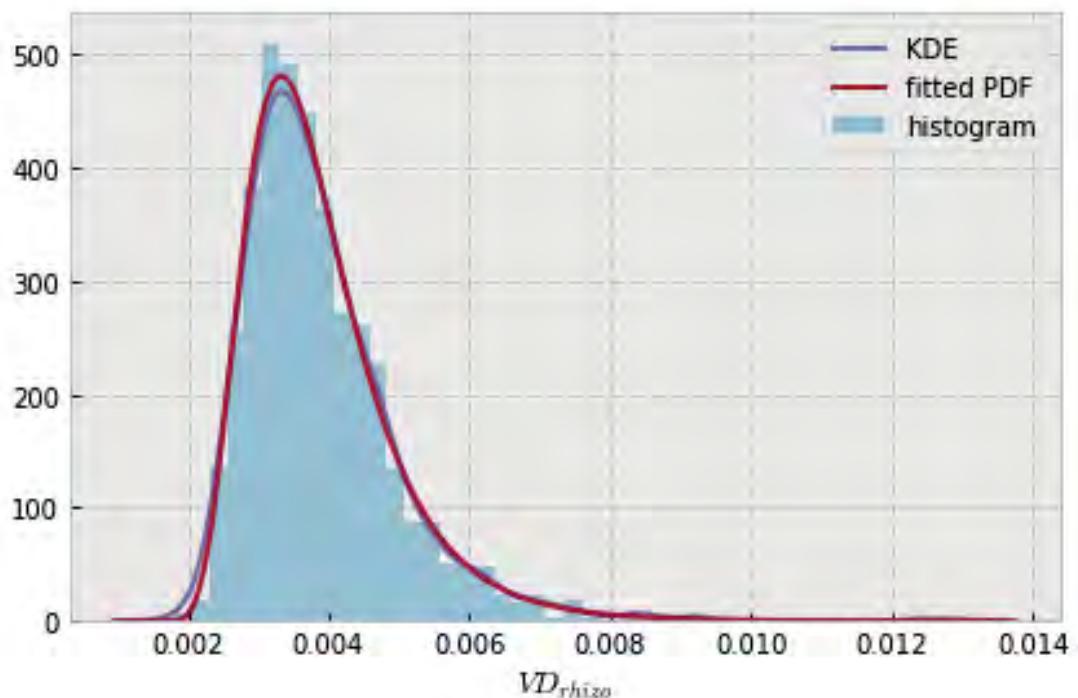
variable name: r
variable value: 2.5555555555555554
distribution: inverse-gamma(shape=(9.8229517690634616,), loc=0.0010053922507543111,
                           scale=0.02699707541933577)
DescribeResult(nobs=1000, minmax=(0.0022051818557029209, 0.013095108718633435)
               mean=0.0040660874426443625, variance=1.2208618804006043e-06,
               skewness=1.7819825191647027, kurtosis=6.787705979451719)

```

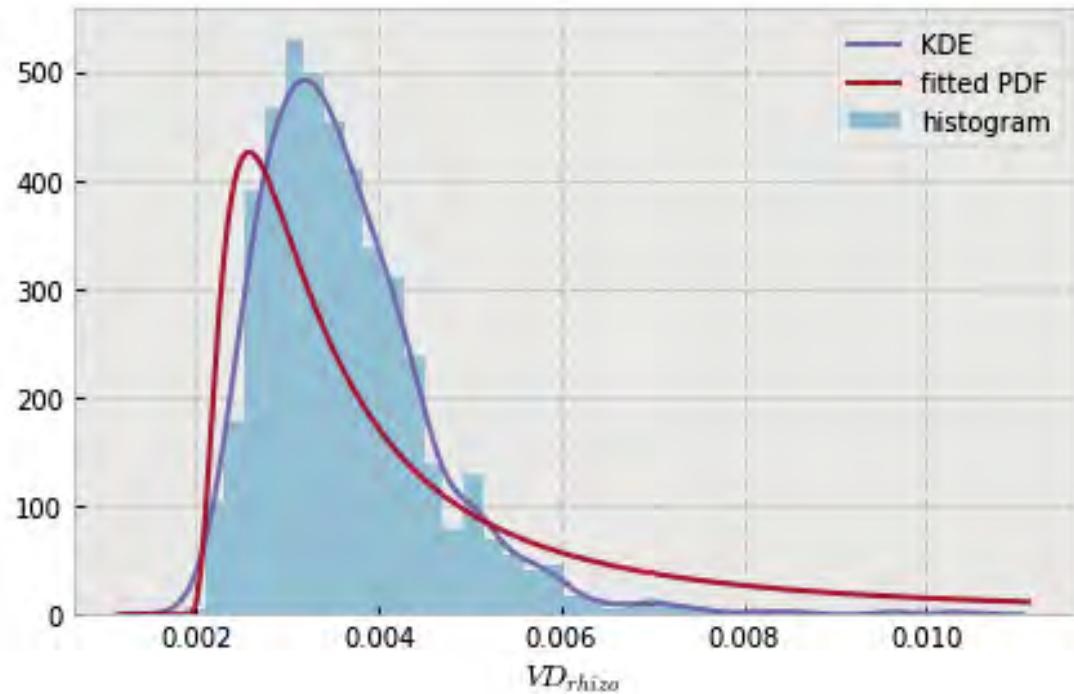


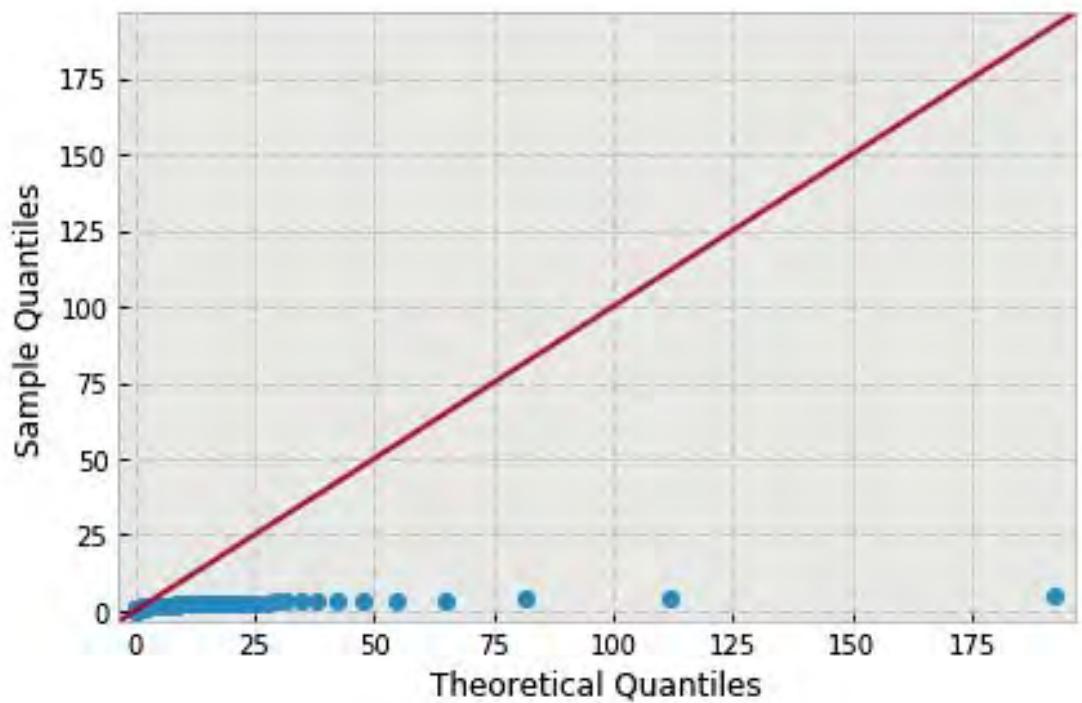


```
variable name: r
variable value: 2.7777777777777777
distribution: inverse-gamma(shape=(9.6555901126528489,), loc=0.000891501225237665,
                           scale=0.025917200894044923)
DescribeResult(nobs=1000, minmax=(0.0017944464552660856, 0.012901341555732979)
               mean=0.0038892988512592231, variance=1.2559293689837088e-06,
               skewness=2.0840174927837247, kurtosis=9.109903010072713)
```

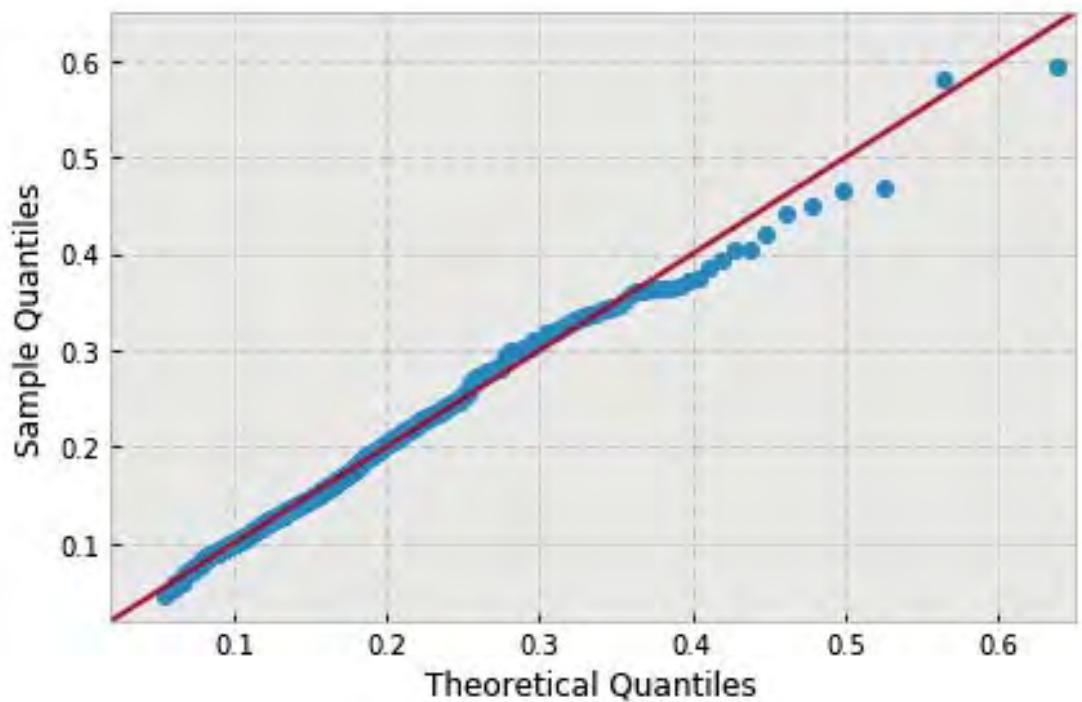
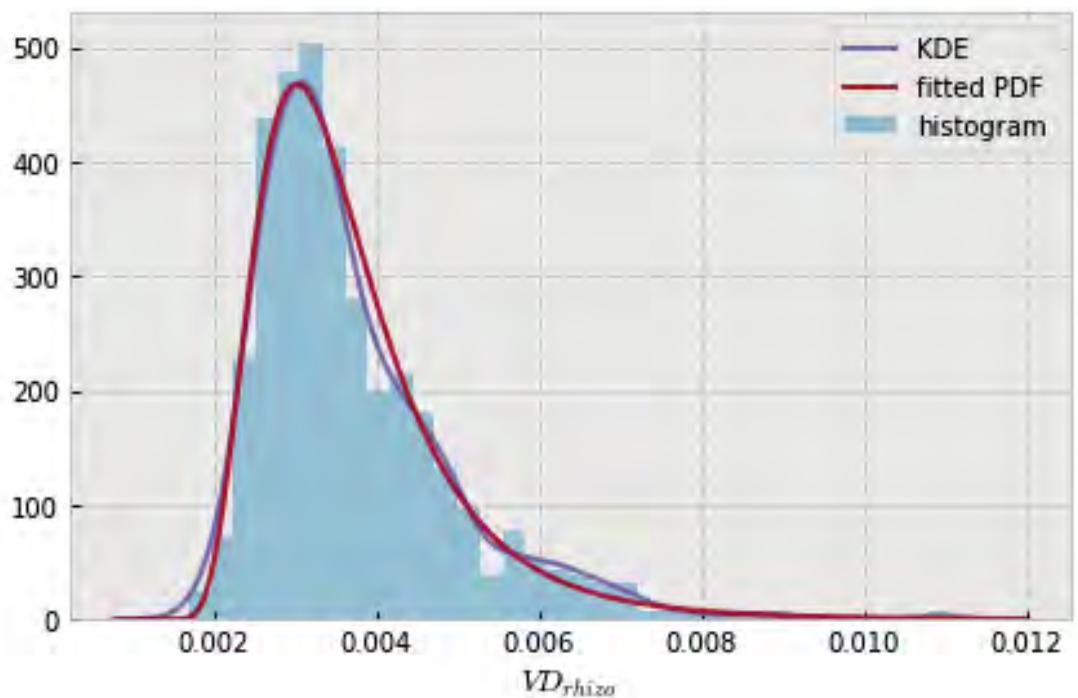


```
variable name: r
variable value: 3.0
distribution: inverse-gamma(shape=(1.2851854578637938,), loc=0.0018108733905792627,
    scale=0.0017553352520013911)
DescribeResult(nobs=1000, minmax=(0.0018743633965128113, 0.010394609499148427)
    mean=0.0036310522438424728, variance=9.287636368267442e-07,
    skewness=1.6581740344276792, kurtosis=5.552515488423943)
```

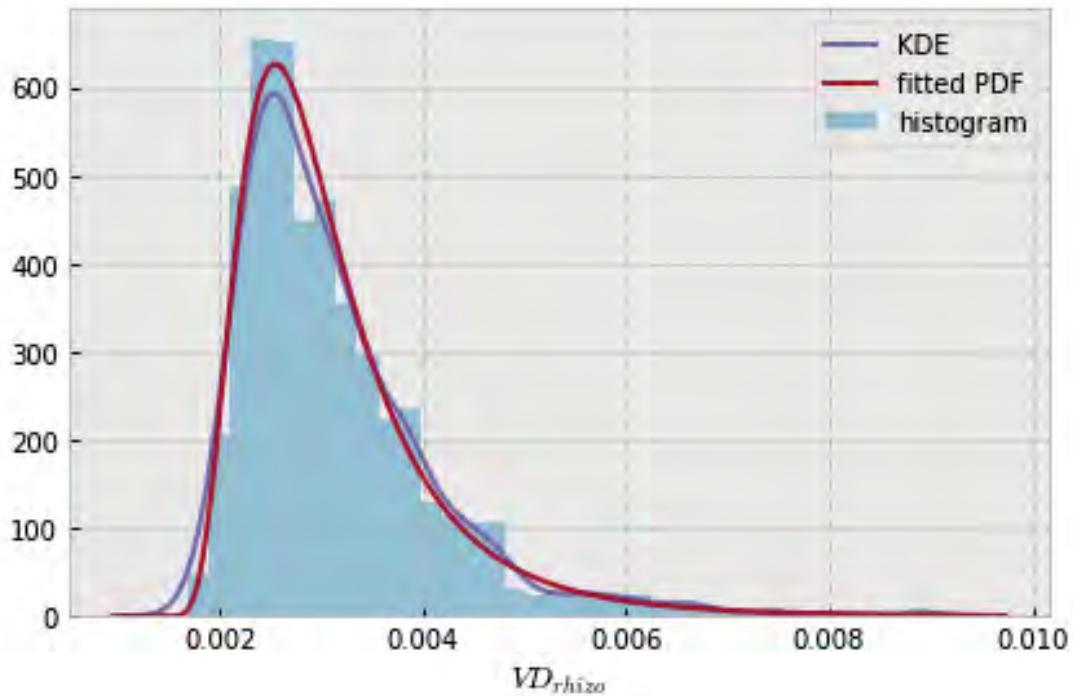


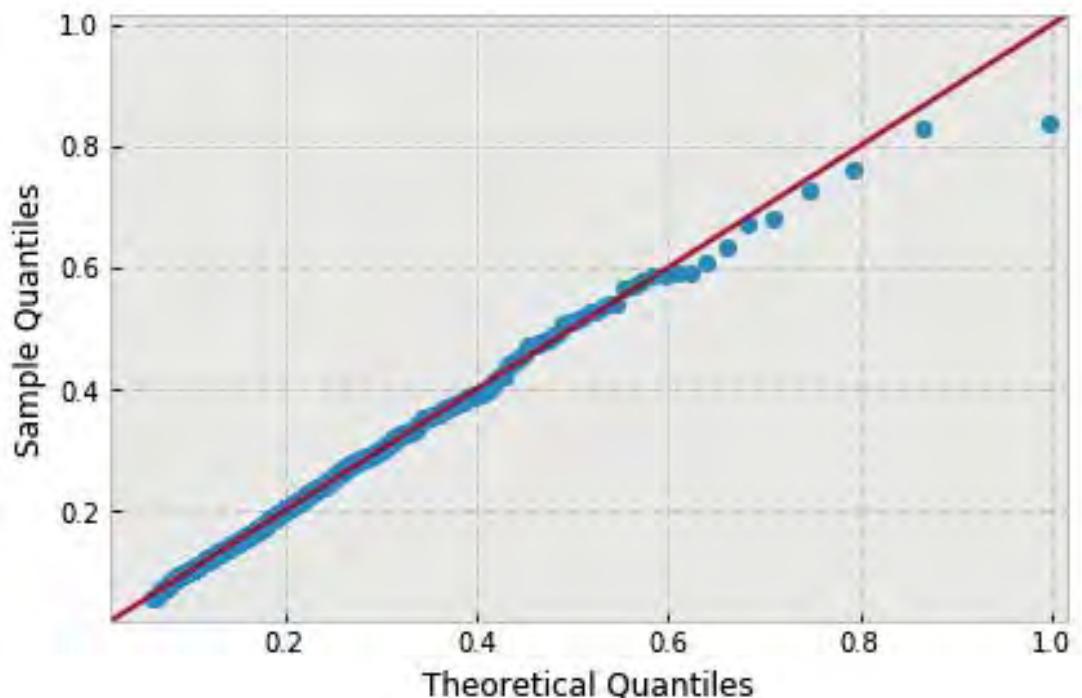


```
variable name: sigma
variable value: 0.01111111111111112
distribution: inverse-gamma(shape=(7.10810952543053,), loc=0.0009122908738682831,
                           scale=0.017080326580154605)
DescribeResult(nobs=1000, minmax=(0.0016893106205795067, 0.011052612670757129)
               mean=0.0037072825022714203, variance=1.4586533655188623e-06,
               skewness=1.606631942440314, kurtosis=3.8482798541493004)
```

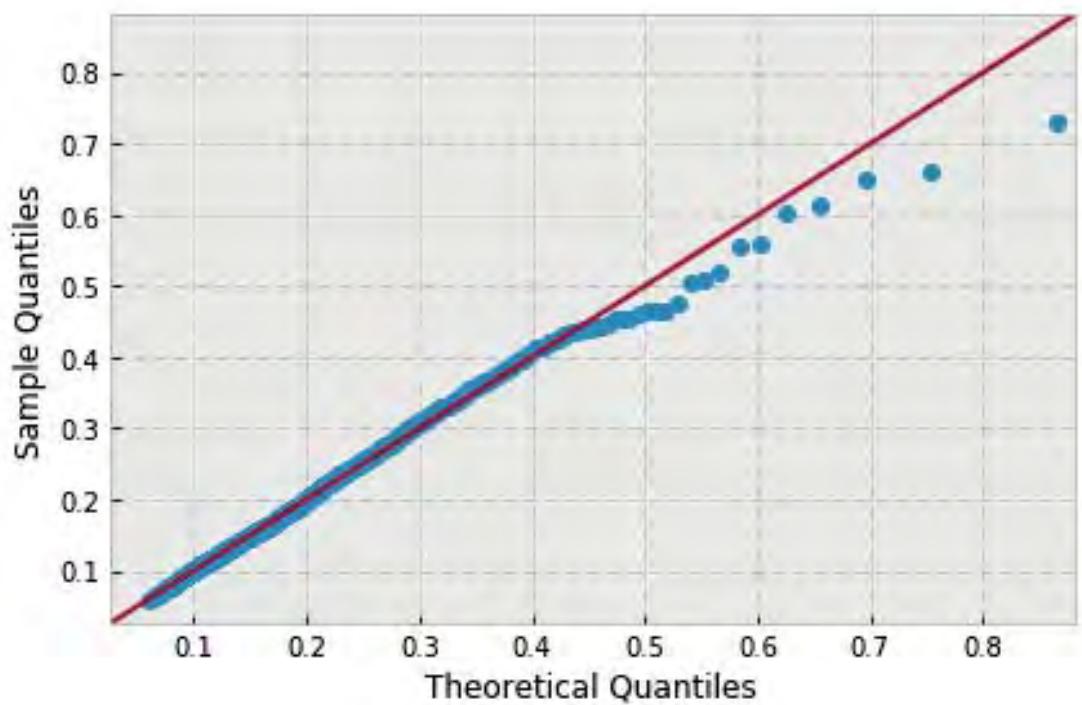
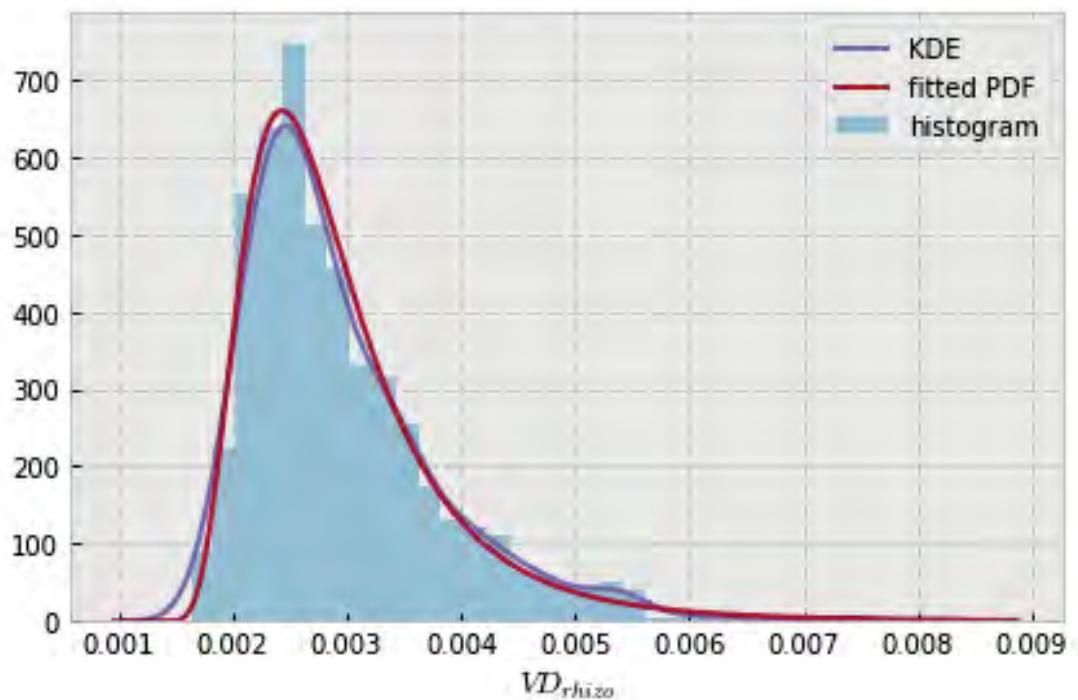


```
variable name: sigma
variable value: 0.02222222222222223
distribution: inverse-gamma(shape=(5.7275269825506481,), loc=0.0011592094309027614,
    scale=0.00934409699529453)
DescribeResult(nobs=1000, minmax=(0.0016861859743800916, 0.0089961431860861932)
    mean=0.0031321058218281078, variance=9.5561758401250544e-07,
    skewness=1.8757528907969778, kurtosis=5.3402569799214135)
```

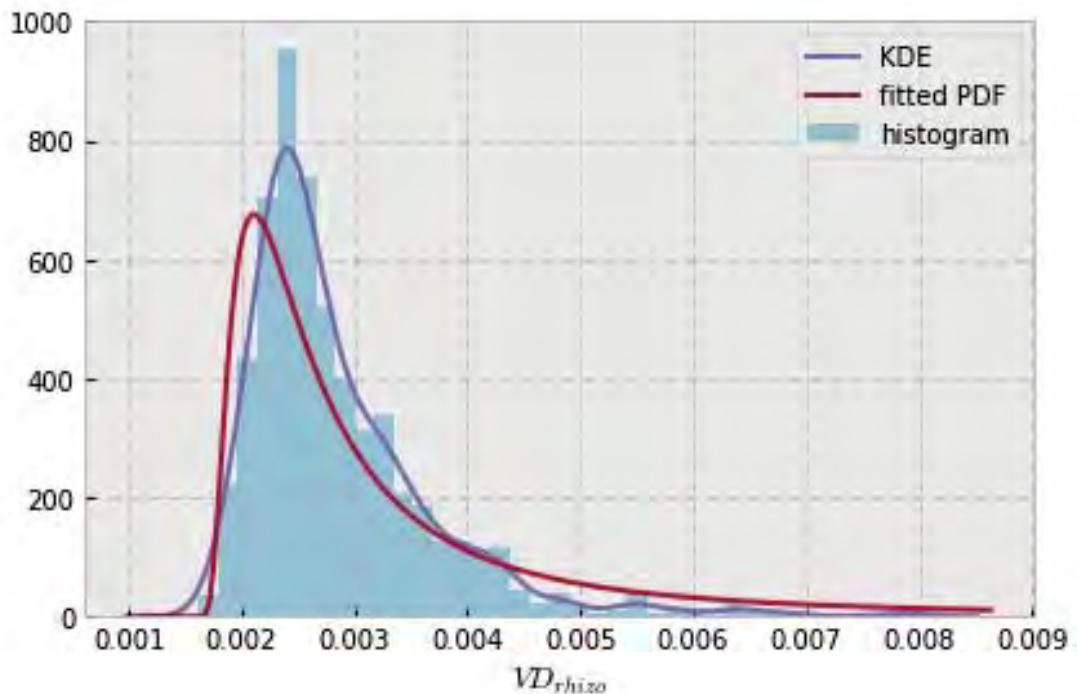


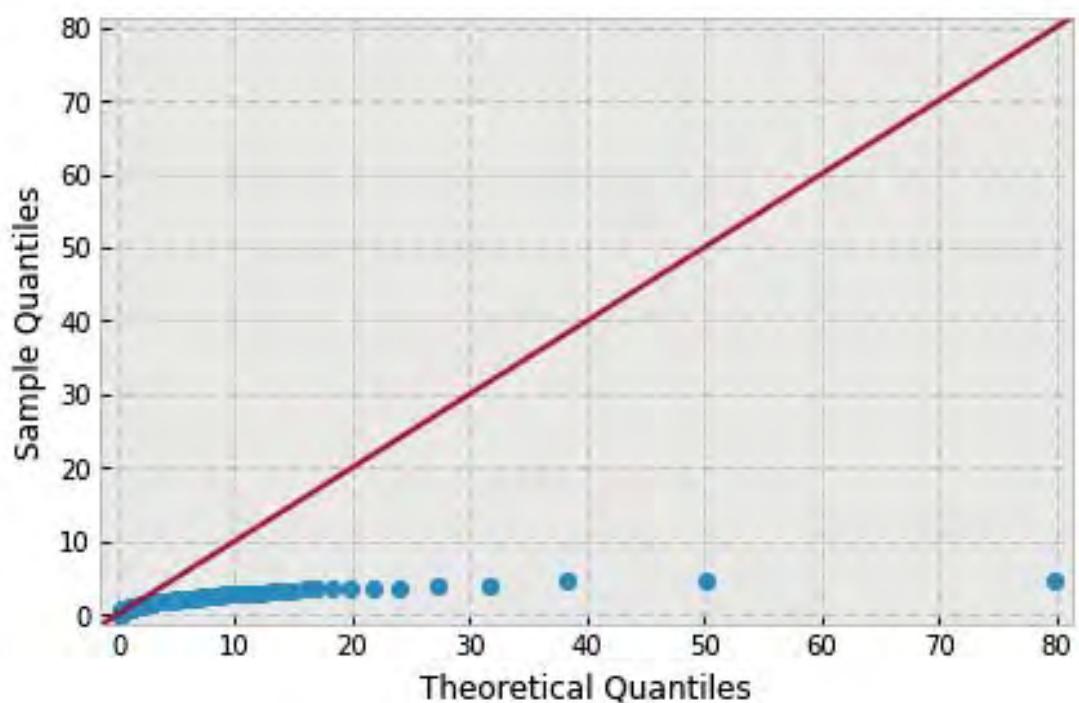


```
variable name: sigma
variable value: 0.03333333333333333
distribution: inverse-gamma(shape=(6.1266652585614523,), loc=0.0010640401613373962,
                           scale=0.009769102832142561)
DescribeResult(nobs=1000, minmax=(0.0016359404929755686, 0.0082047362163553943)
               mean=0.0029654508584627637, variance=7.8932494117826323e-07,
               skewness=1.6315430785405982, kurtosis=3.837135470469721)
```

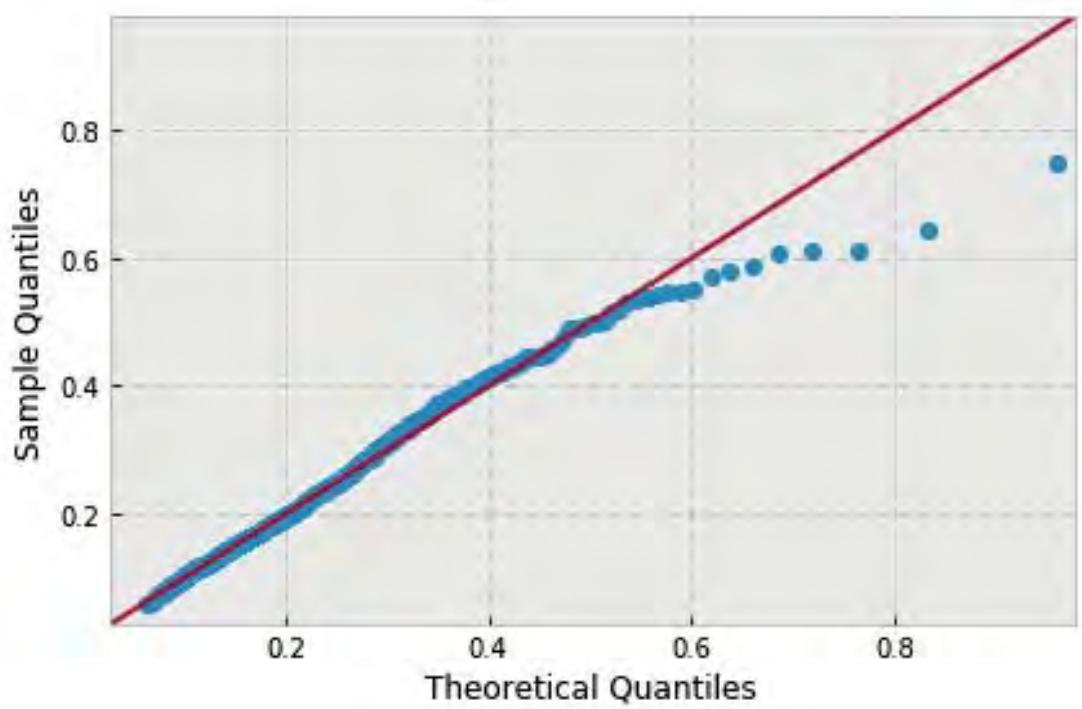
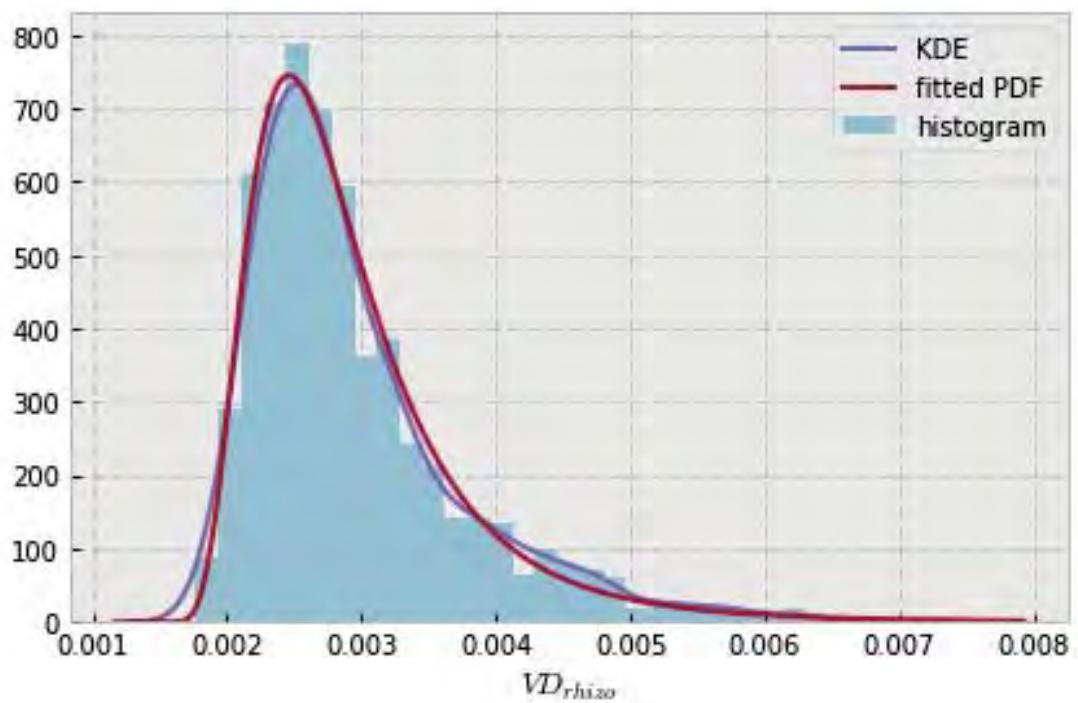


```
variable name: sigma
variable value: 0.04444444444444446
distribution: inverse-gamma(shape=(1.5093857845915739,), loc=0.0015649024624017362,
                           scale=0.0013658531474800045)
DescribeResult(nobs=1000, minmax=(0.0016244228472926511, 0.0079944248705396386)
               mean=0.0028592013965255234, variance=7.1269090678444115e-07,
               skewness=2.0902761697372676, kurtosis=6.456887074828524)
```

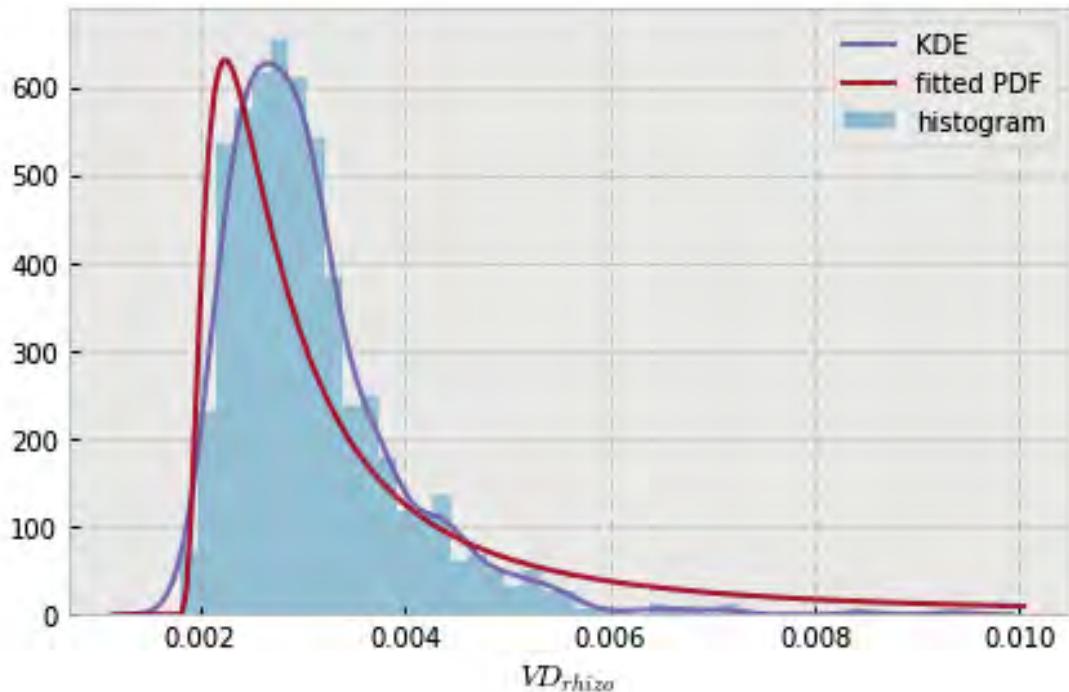


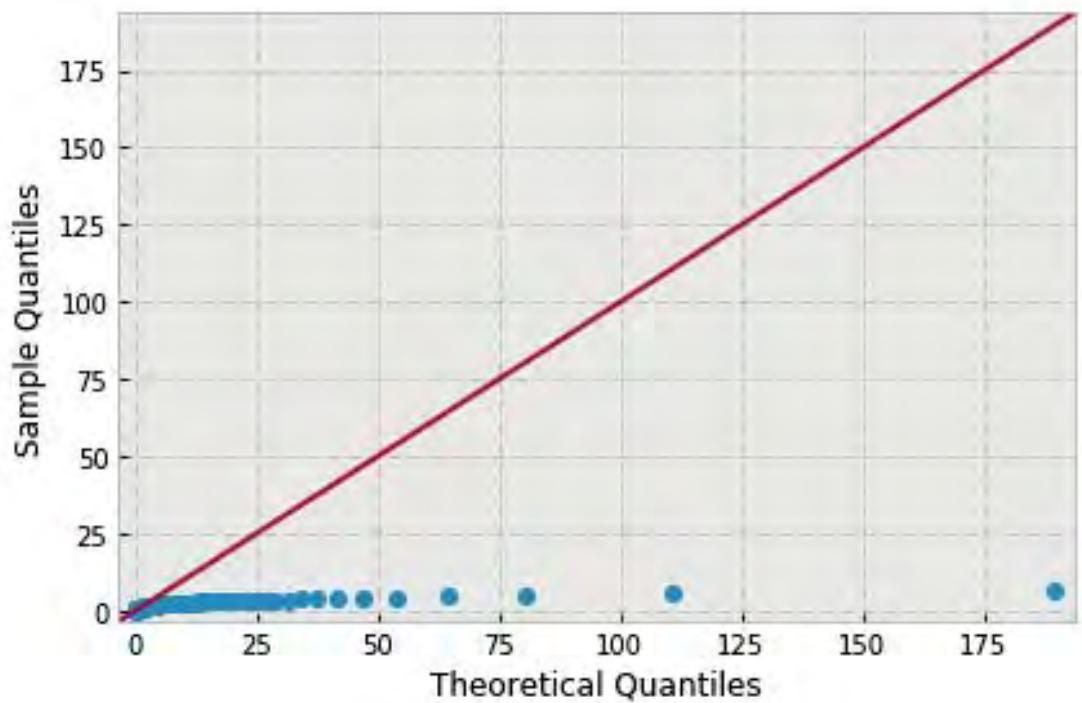


```
variable name: sigma
variable value: 0.05555555555555555
distribution: inverse-gamma(shape=(5.836822008338725,), loc=0.0012780889071907545,
                           scale=0.008068122197209373)
DescribeResult(nobs=1000, minmax=(0.0017674140994682153, 0.0073210570055445463)
               mean=0.0029424495022576077, variance=6.4236813858788419e-07,
               skewness=1.566006125389721, kurtosis=2.941864827520911)
```

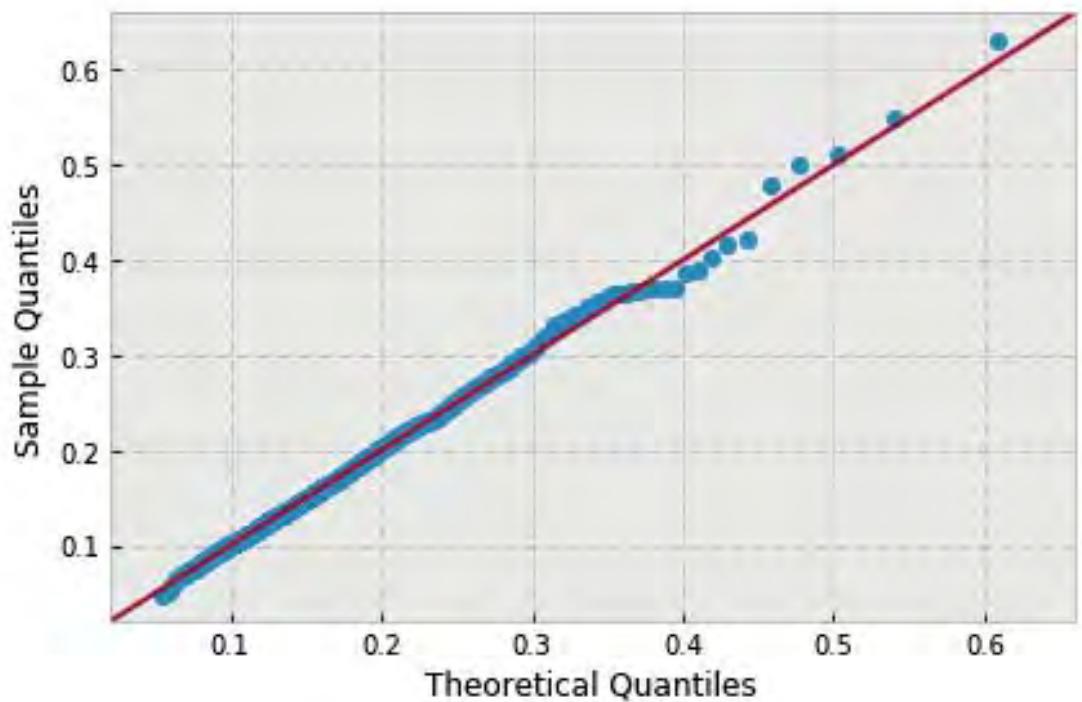
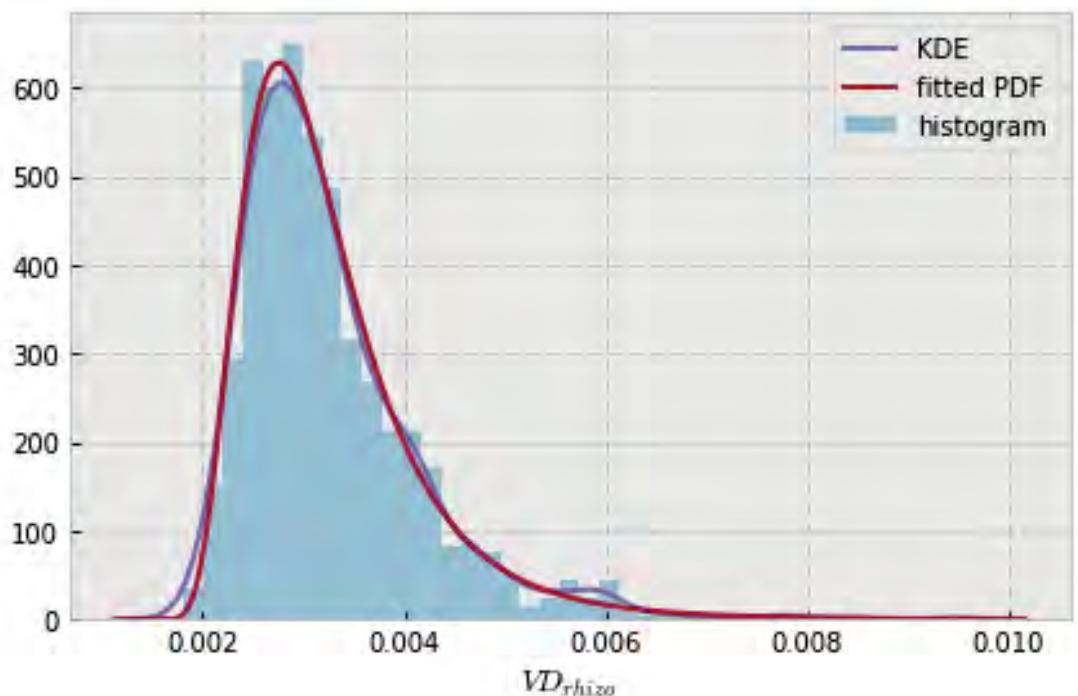


```
variable name: sigma
variable value: 0.06666666666666667
distribution: inverse-gamma(shape=(1.2884191339687763,), loc=0.0017229643640584826,
    scale=0.0011881786041281041)
DescribeResult(nobs=1000, minmax=(0.0017912037825022543, 0.0093993257187752269)
    mean=0.003076049192978081, variance=7.2347345035023666e-07,
    skewness=1.9061559901528455, kurtosis=6.50894388433411)
```

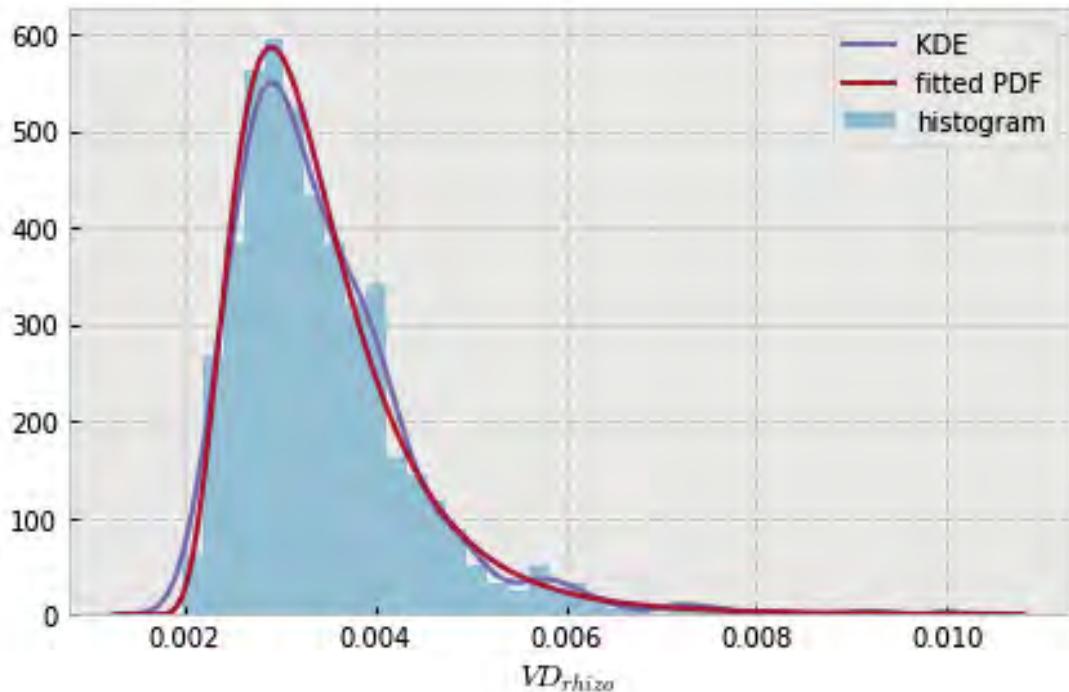


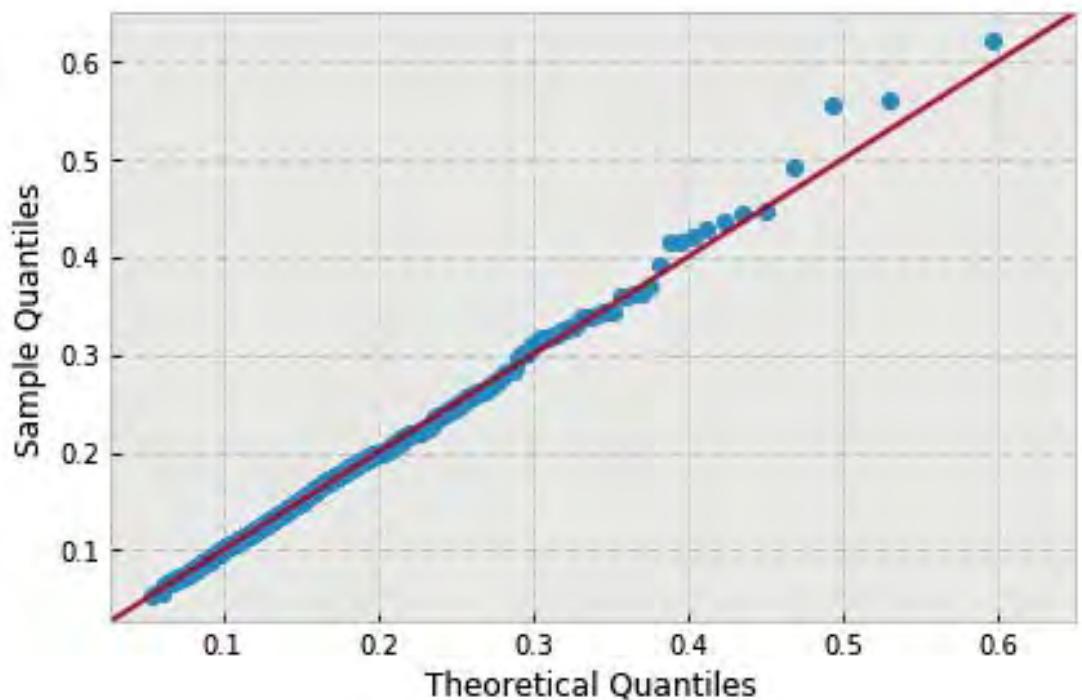


```
variable name: sigma
variable value: 0.07777777777777778
distribution: inverse-gamma(shape=(7.2832096844831096,), loc=0.001165358786954019,
                           scale=0.013177173800036843)
DescribeResult(nobs=1000, minmax=(0.0018063424991190927, 0.00947064883229164)
               mean=0.0032624470401502723, variance=8.1761652657359497e-07,
               skewness=1.806534916577904, kurtosis=5.444159123292465)
```

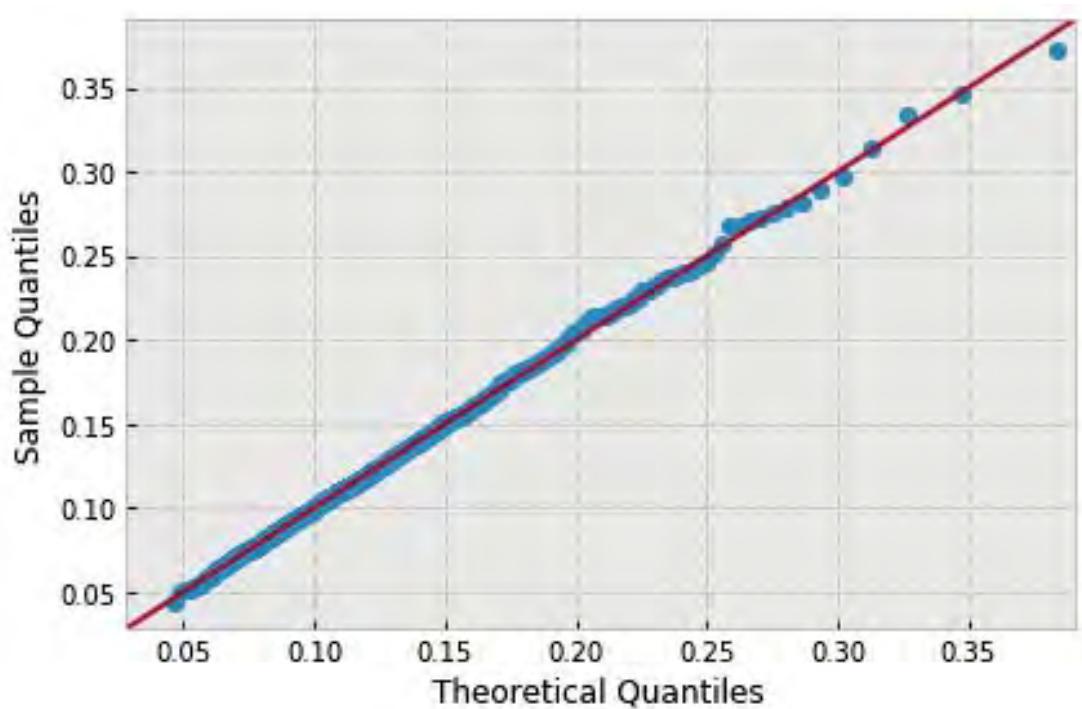
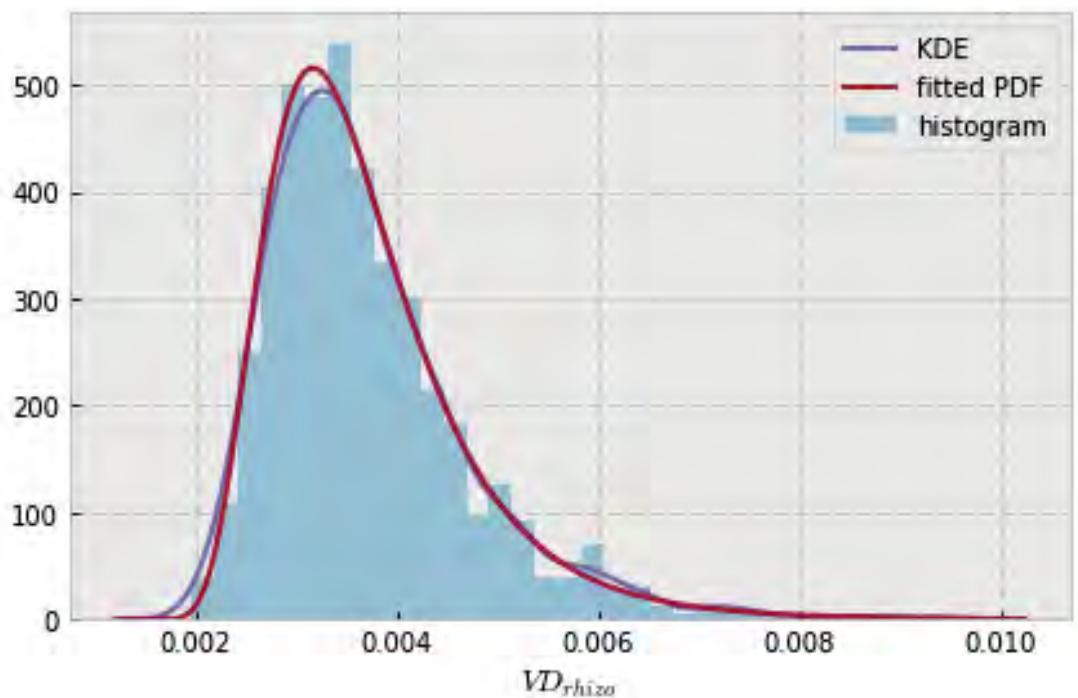


```
variable name: sigma
variable value: 0.08888888888888889
distribution: inverse-gamma(shape=(7.3561475784456487,), loc=0.0011931347516363493,
    scale=0.014324857738194578)
DescribeResult(nobs=1000, minmax=(0.0019784857293162561, 0.010076738159161236)
    mean=0.0034462432327736908, variance=9.3504823260342616e-07,
    skewness=1.9146816349124816, kurtosis=6.446734373199323)
```

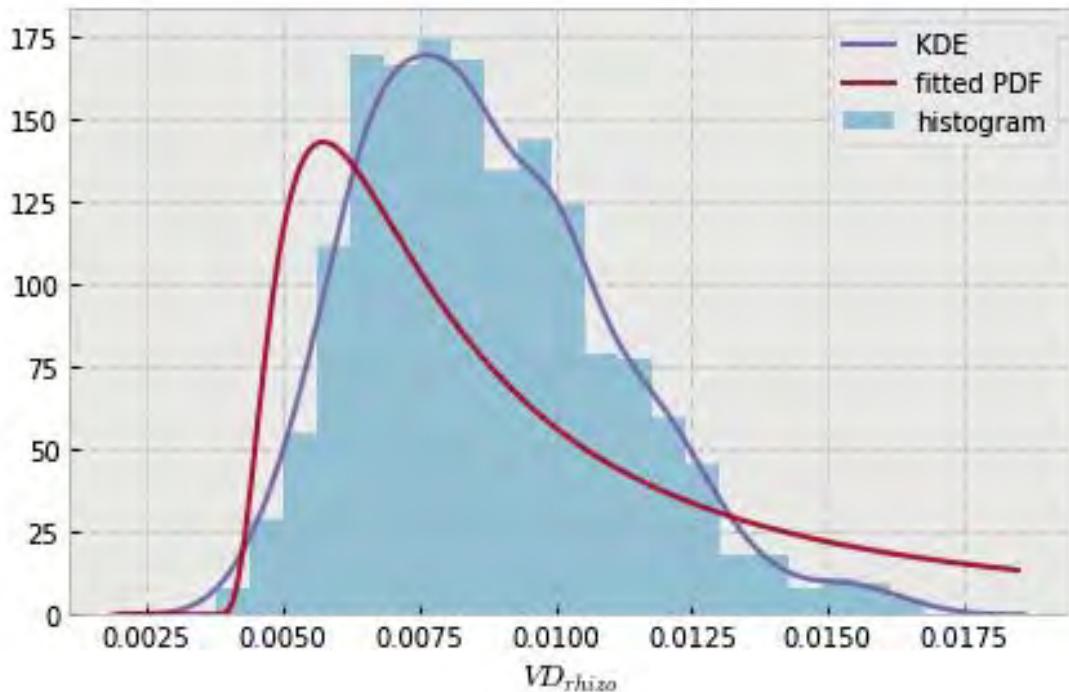


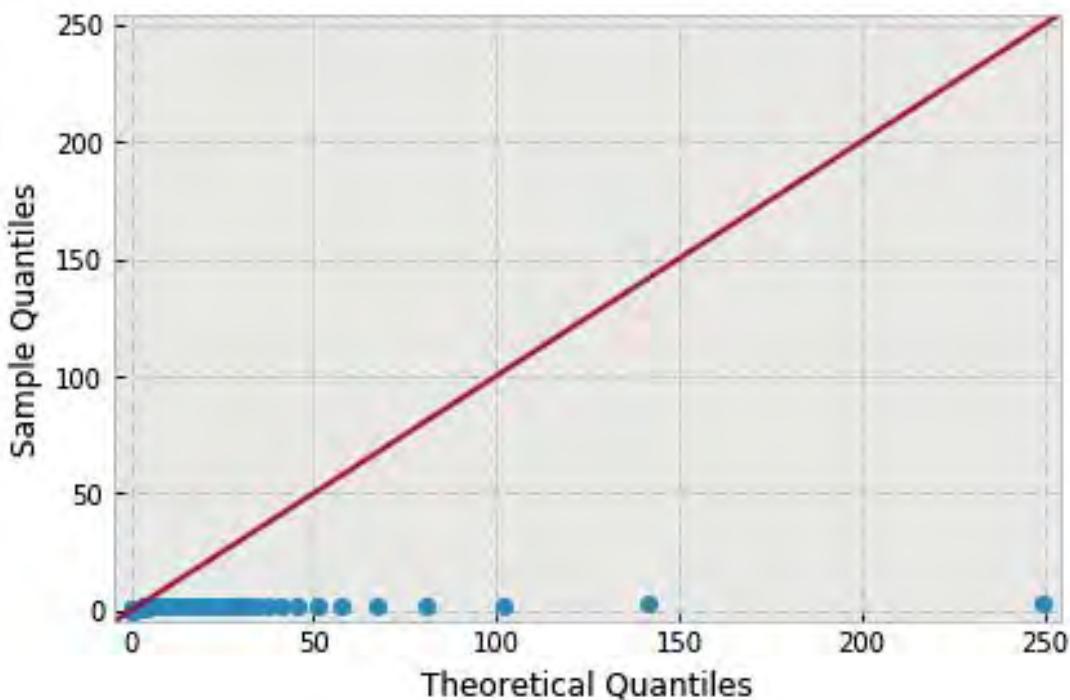


```
variable name: sigma
variable value: 0.1
distribution: inverse-gamma(shape=(9.3112893145902511,), loc=0.0009448625419084141,
                           scale=0.02292912194171268)
DescribeResult(nobs=1000, minmax=(0.0019572184805259152, 0.0094694656593463673)
               mean=0.0037029969437132495, variance=1.0199806576472797e-06,
               skewness=1.4642771440917934, kurtosis=3.3647734561245066)
```

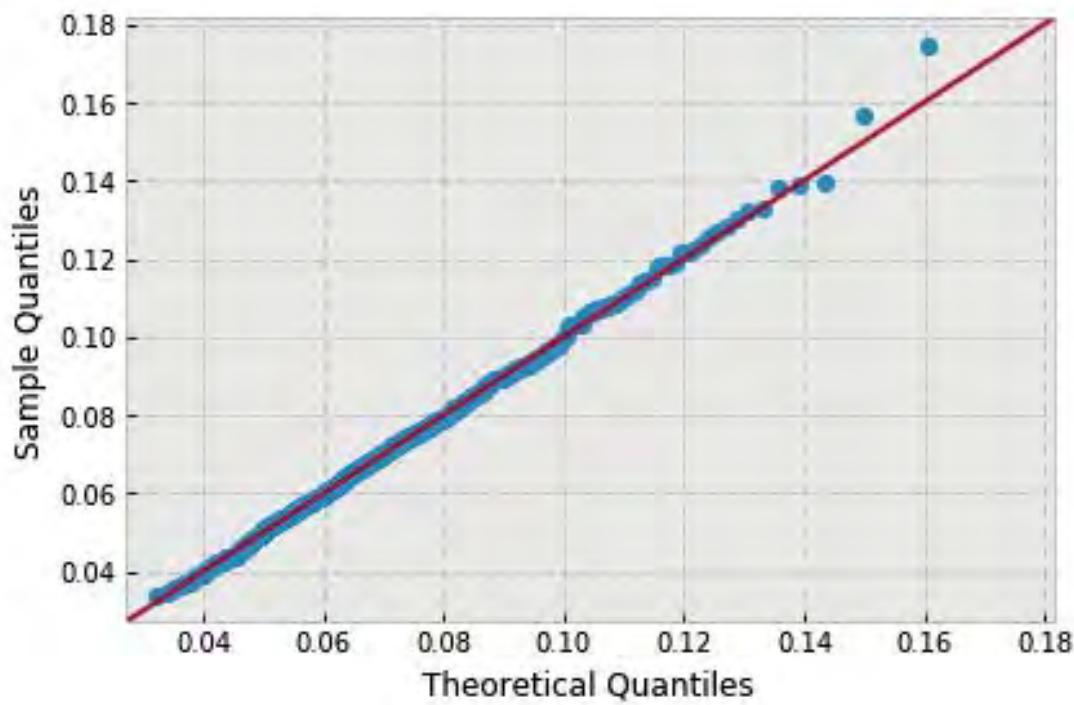
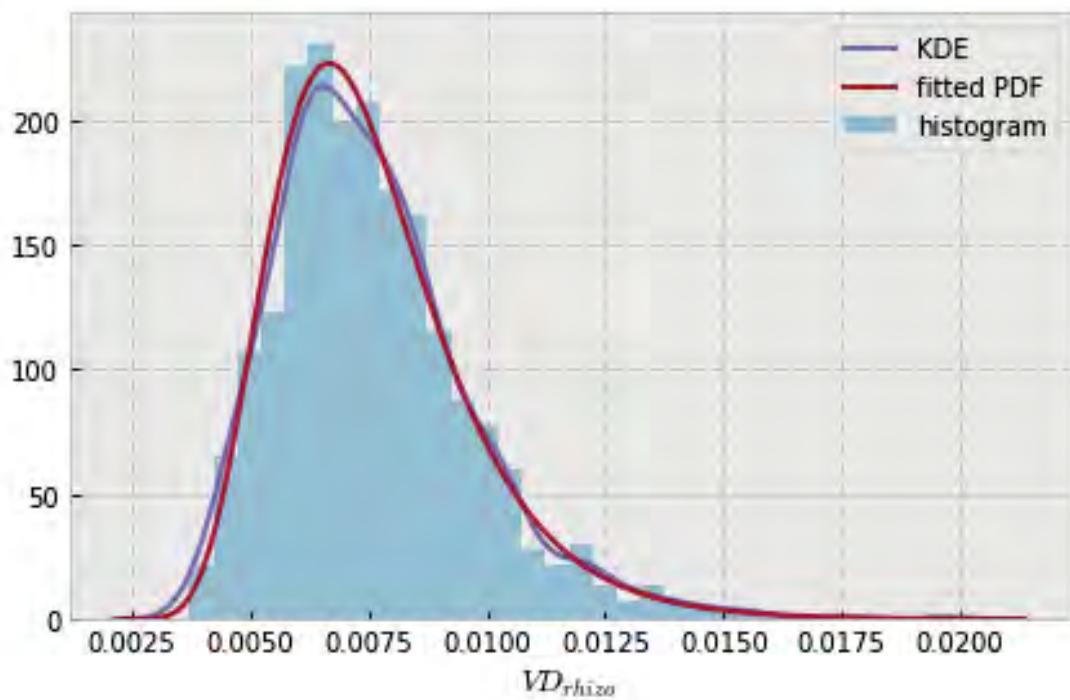


```
variable name: theta
variable value: 0.6981317007977318
distribution: inverse-gamma(shape=(1.2307663931687611,), loc=0.003532471225316033,
    scale=0.004931095891335732)
DescribeResult(nobs=1000, minmax=(0.0037608089453414549, 0.016732081358806706)
    mean=0.0087346114890267772, variance=5.3956976625590969e-06,
    skewness=0.6214777255318995, kurtosis=0.18310970257810855)
```

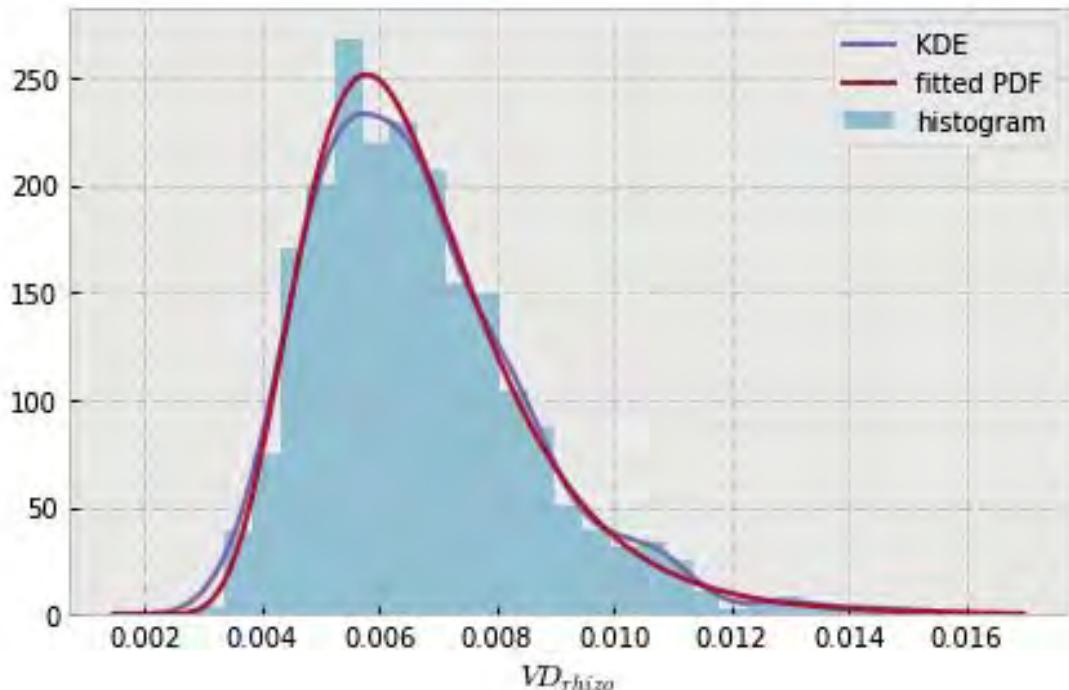


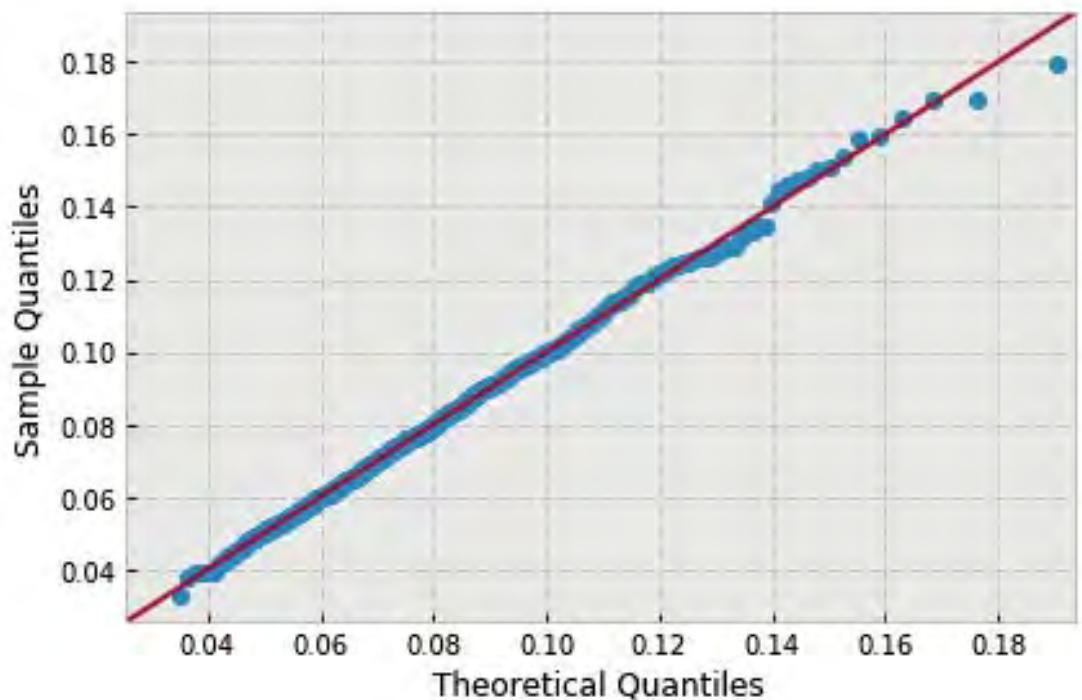


```
variable name: theta
variable value: 0.7853981633974483
distribution: inverse-gamma(shape=(15.70542855610255,), loc=-0.00018921553349609036,
                           scale=0.11445837519679658)
DescribeResult(nobs=1000, minmax=(0.003718300094256507, 0.0197783554744617)
               mean=0.0075937474293178413, variance=4.4029331088477259e-06,
               skewness=1.1287140456313176, kurtosis=2.347628227380257)
```

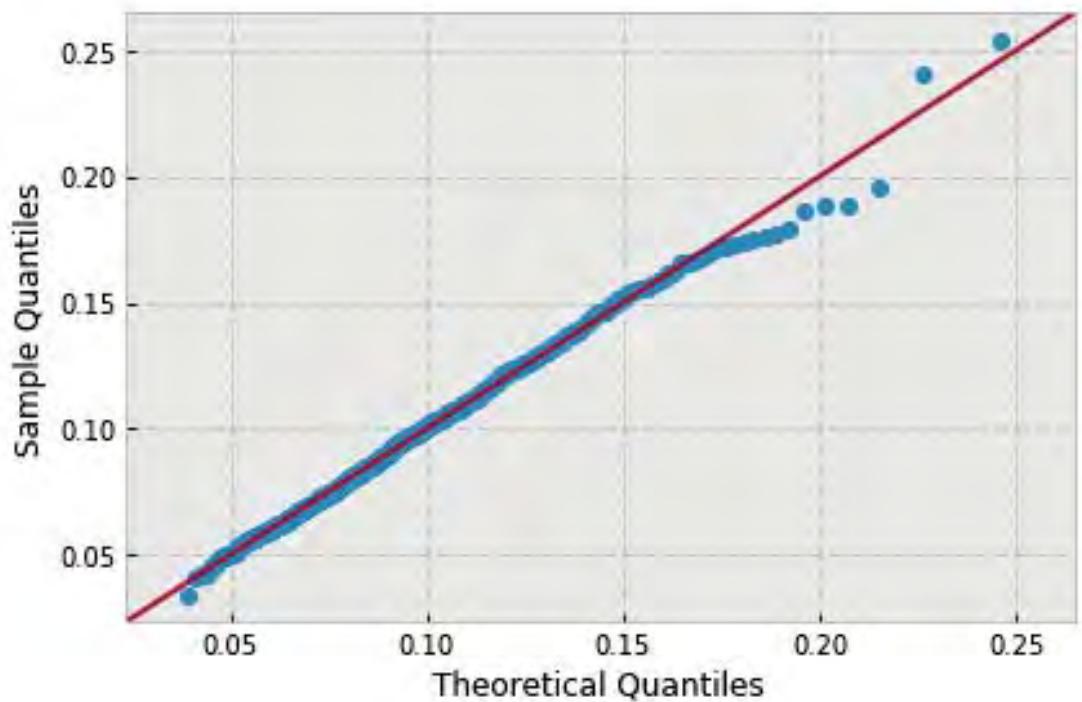
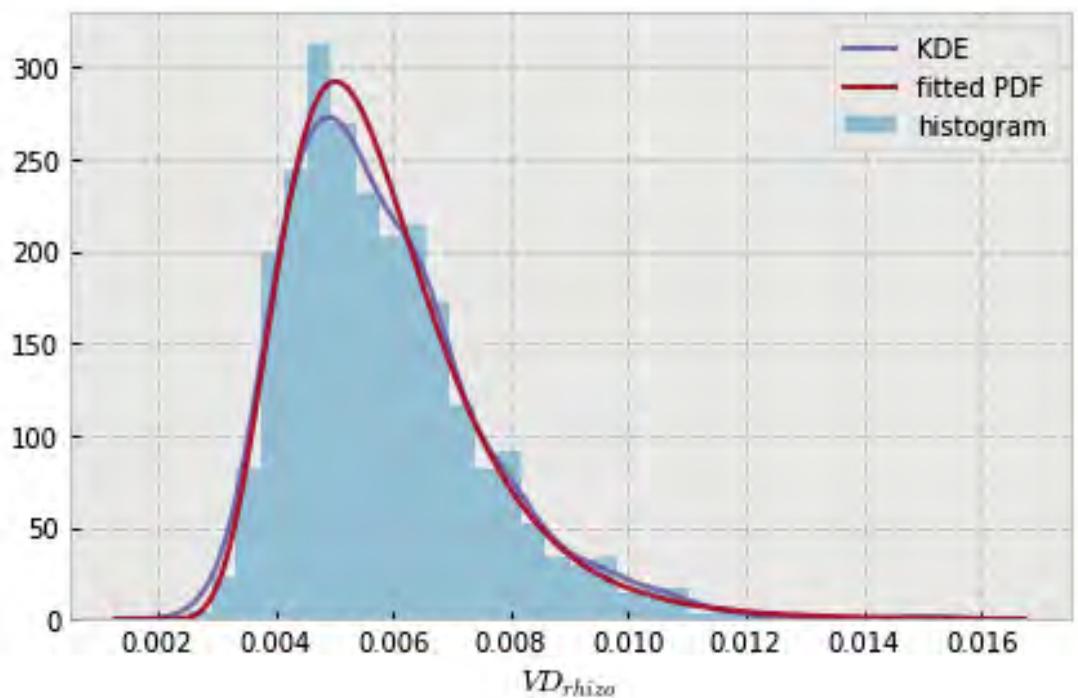


```
variable name: theta
variable value: 0.8726646259971648
distribution: inverse-gamma(shape=(14.09683369840938,), loc=5.662746913602786e-05,
    scale=0.0863923754359738)
DescribeResult(nobs=1000, minmax=(0.0028877270669843938, 0.015554770810298948)
    mean=0.006651804663040598, variance=3.532289867555285e-06,
    skewness=1.0818124263748519, kurtosis=1.7450721607948516)
```

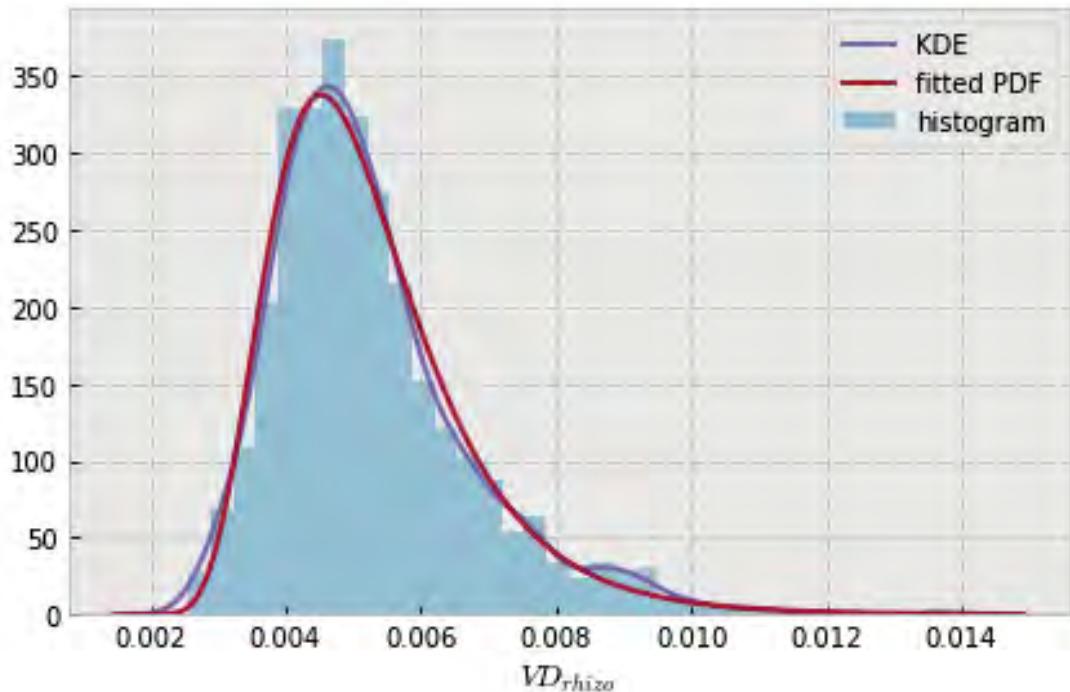


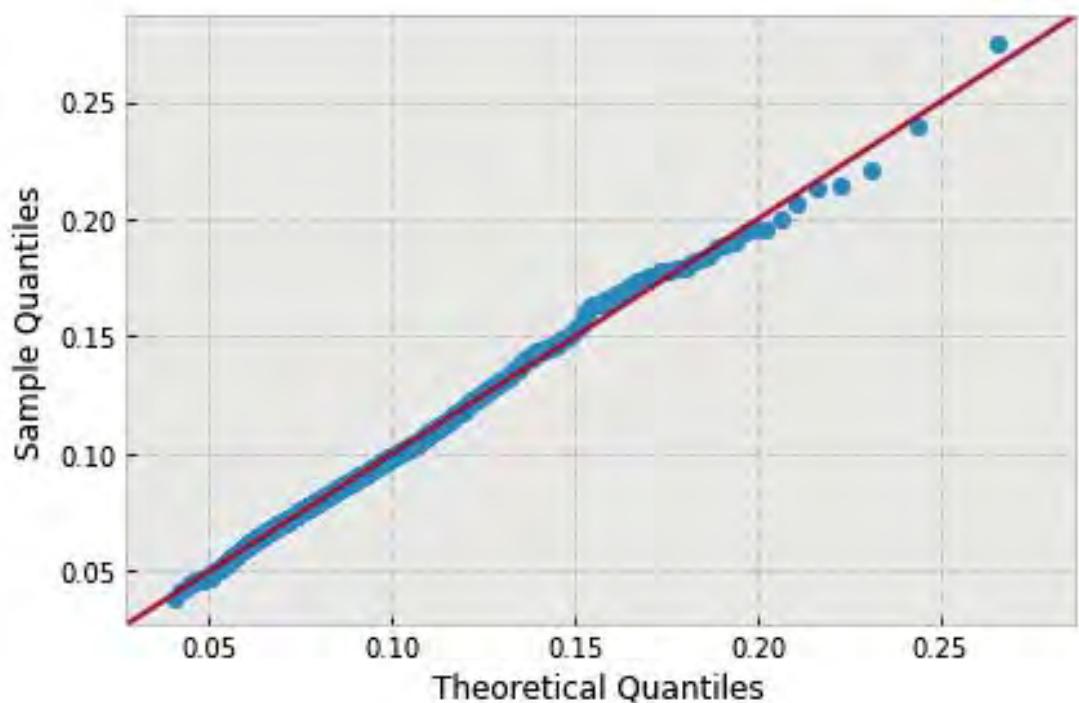


```
variable name: theta
variable value: 0.9599310885968813
distribution: inverse-gamma(shape=(12.046129961233337,), loc=0.0004961172240495228,
                           scale=0.0590511073370502)
DescribeResult(nobs=1000, minmax=(0.0025198359004930111, 0.015481515125034317)
               mean=0.0058401543644847724, variance=2.7659908653177378e-06,
               skewness=1.1735768659280152, kurtosis=2.285772043806145)
```

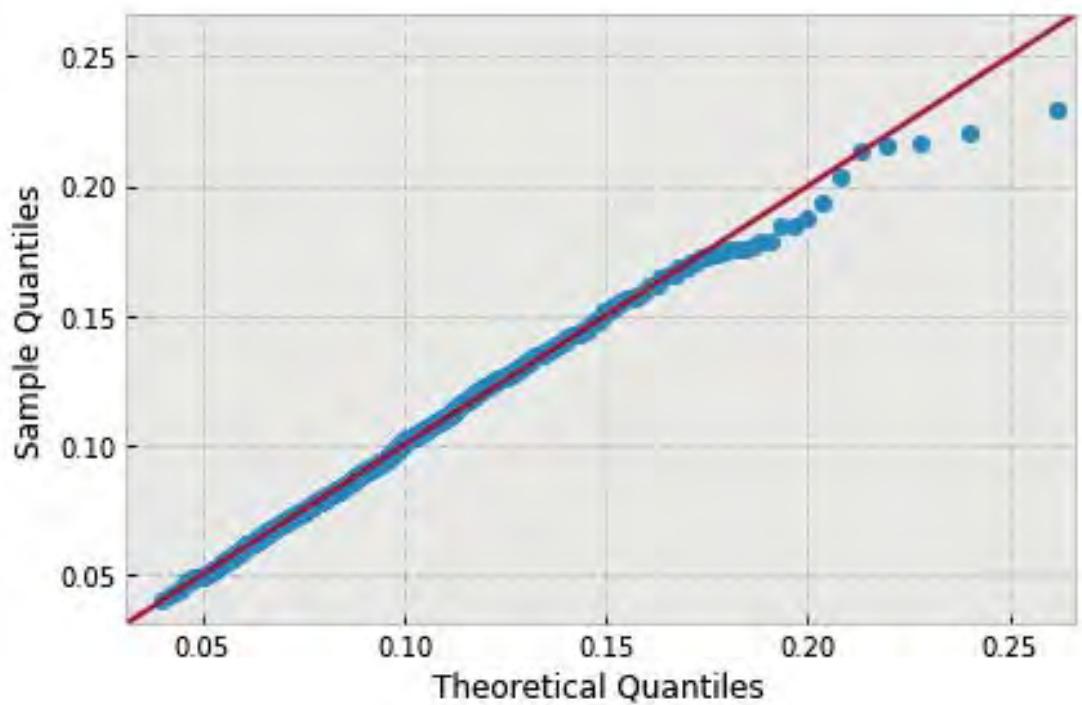
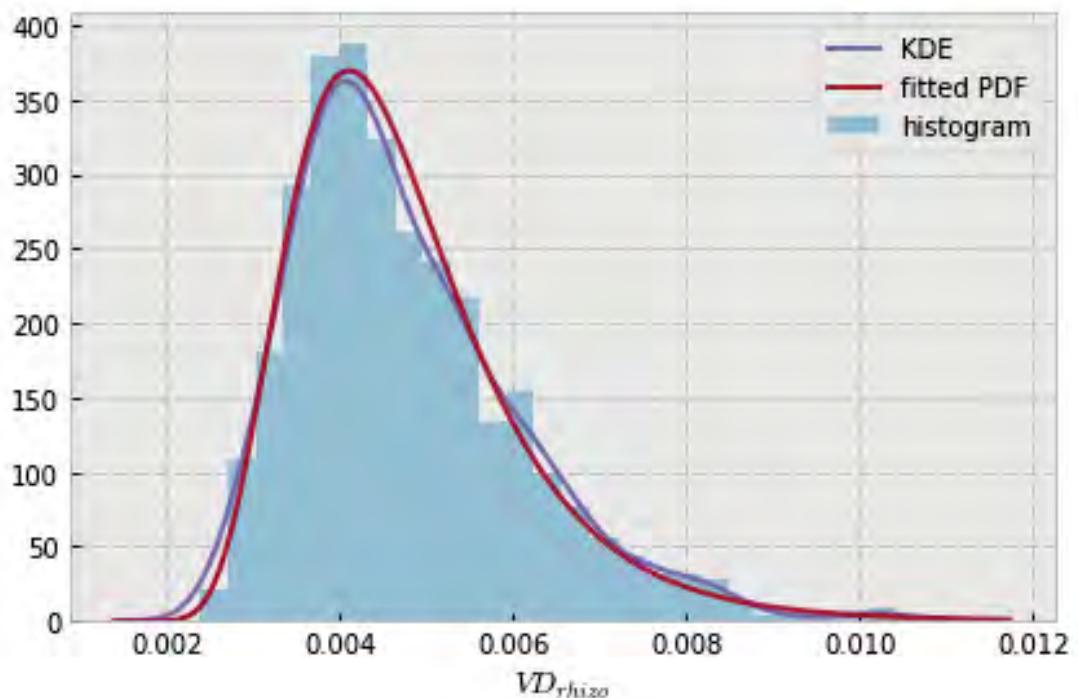


```
variable name: theta
variable value: 1.0471975511965979
distribution: inverse-gamma(shape=(11.503372102021705,), loc=0.0007101132231400918,
                           scale=0.04770018074855861)
DescribeResult(nobs=1000, minmax=(0.0025657769631552193, 0.013809295016725546)
               mean=0.0052518475843835654, variance=2.1652790298378616e-06,
               skewness=1.310523068057476, kurtosis=2.5793610804512994)
```

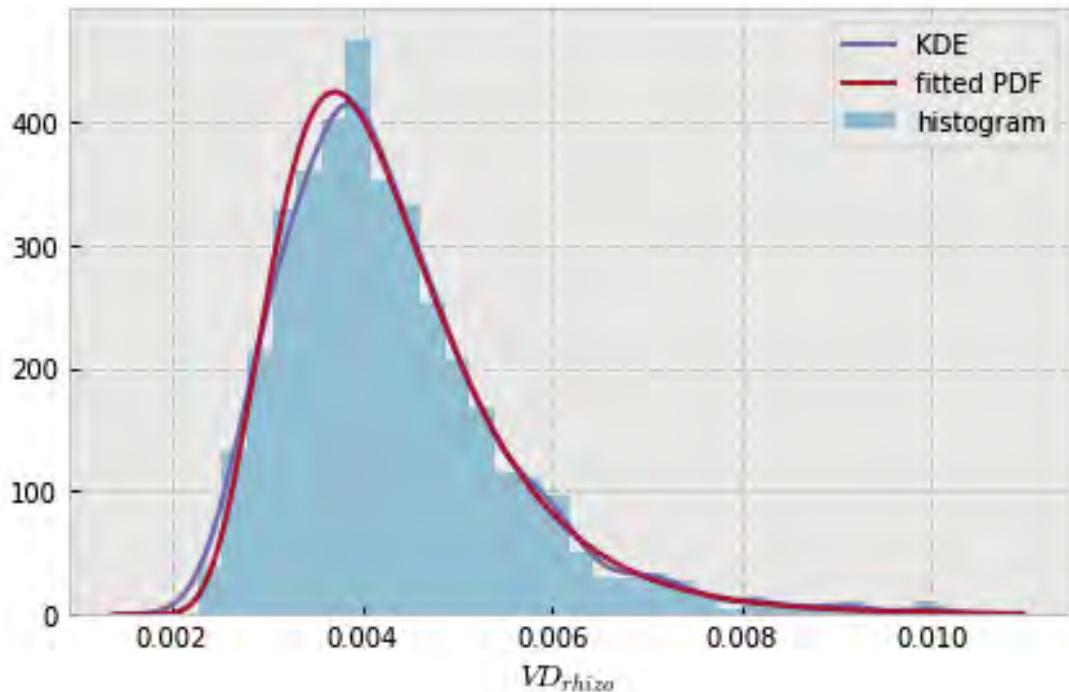


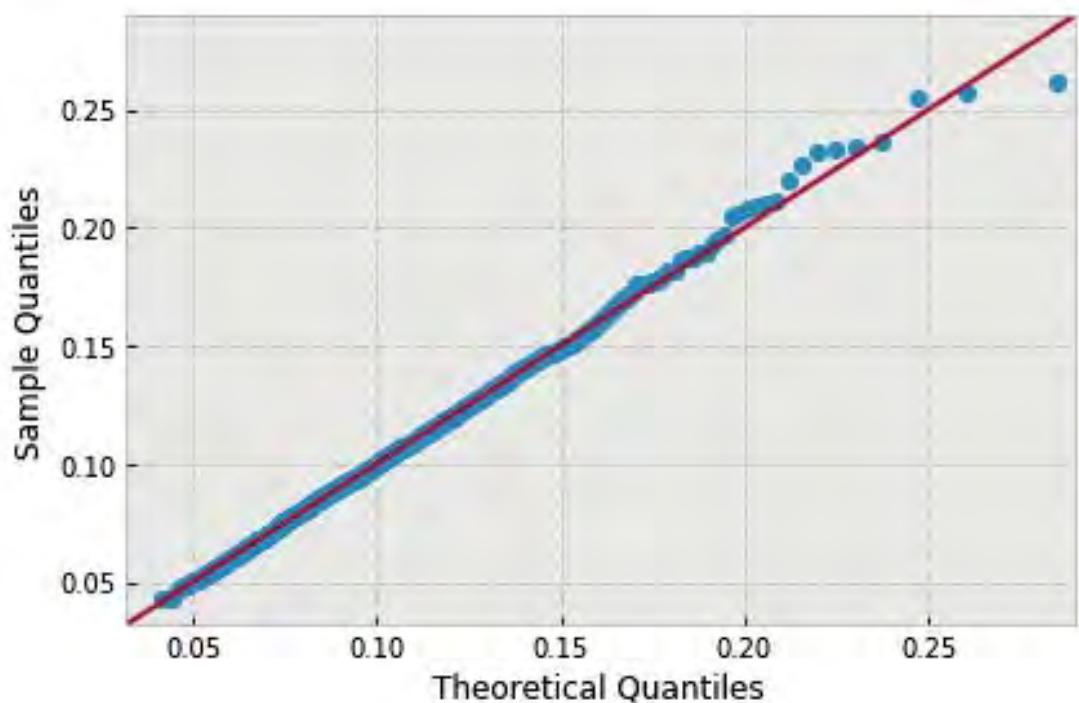


```
variable name: theta
variable value: 1.1344640137963142
distribution: inverse-gamma(shape=(11.607485850055793,), loc=0.0005961407585981307,
    scale=0.04418307390150973)
DescribeResult(nobs=1000, minmax=(0.0023776751529159942, 0.010736886526612959)
    mean=0.0047589547625148214, variance=1.7242662523797189e-06,
    skewness=1.0764211272510884, kurtosis=1.4762476976076568)
```

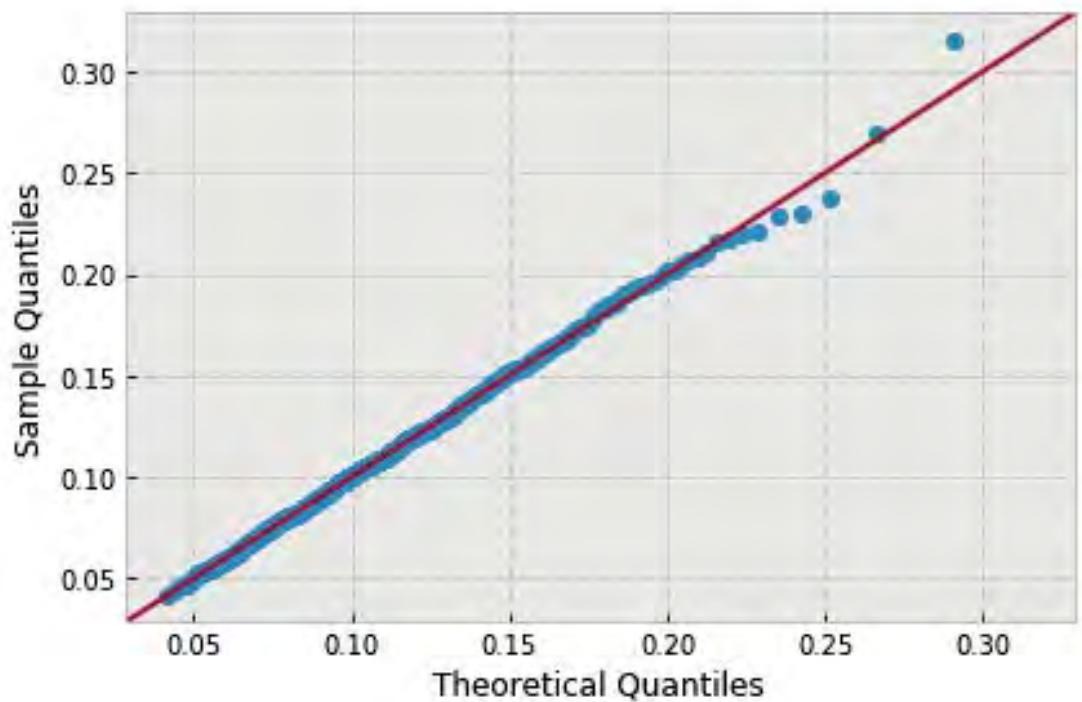
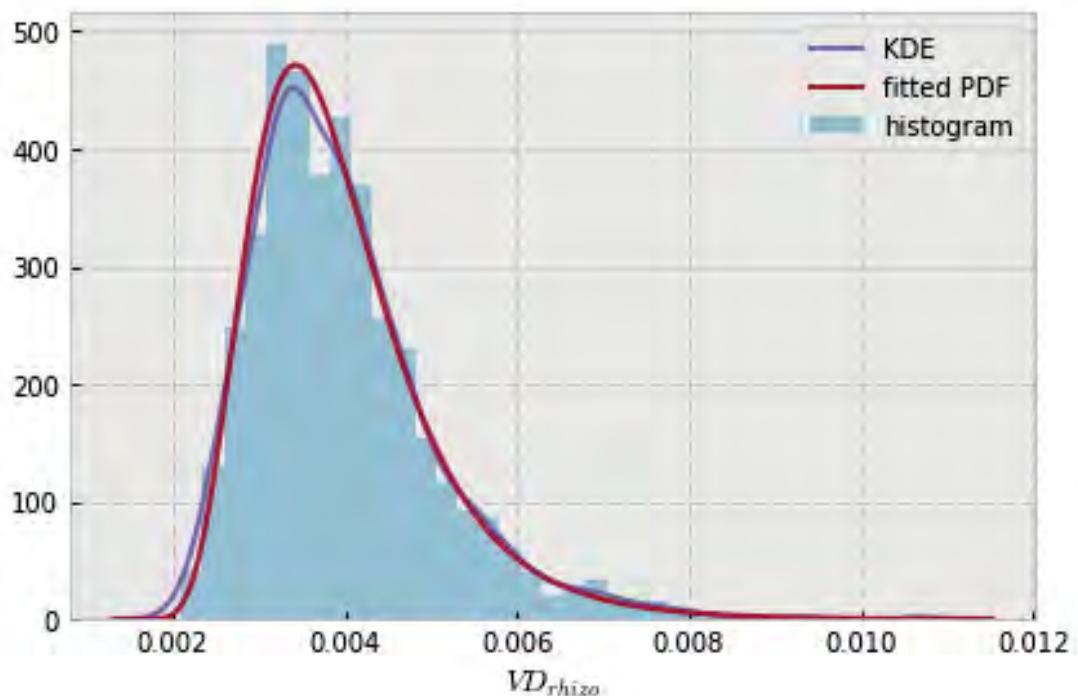


```
variable name: theta
variable value: 1.2217304763960306
distribution: inverse-gamma(shape=(11.038575635557656,), loc=0.0007419501596742319,
    scale=0.03575536422942654)
DescribeResult(nobs=1000, minmax=(0.0022681631248651249, 0.010086695934138466)
    mean=0.0043031980184460644, variance=1.3859476890423934e-06,
    skewness=1.3393720942210663, kurtosis=2.8073121235081713)
```

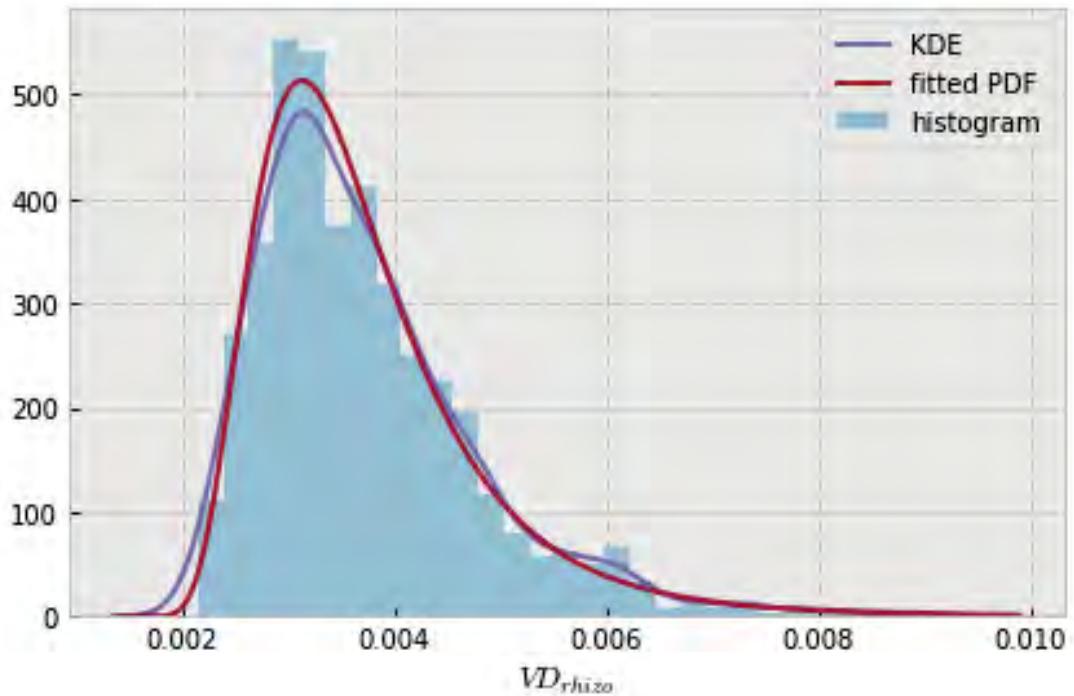


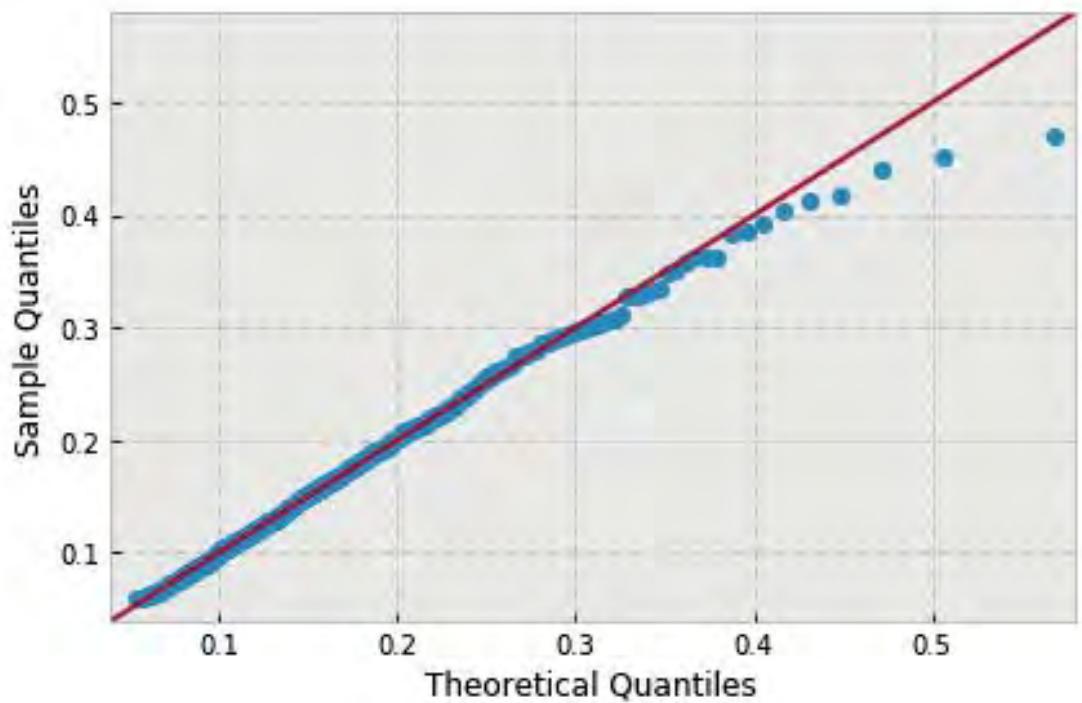


```
variable name: theta
variable value: 1.3089969389957472
distribution: inverse-gamma(shape=(10.898220679328542,), loc=0.0007694838713077968,
                           scale=0.03158193222794835)
DescribeResult(nobs=1000, minmax=(0.0020965931851537325, 0.010724158864890875)
               mean=0.0039593449978636329, variance=1.123772506035122e-06,
               skewness=1.3512424598595774, kurtosis=3.192166131335865)
```

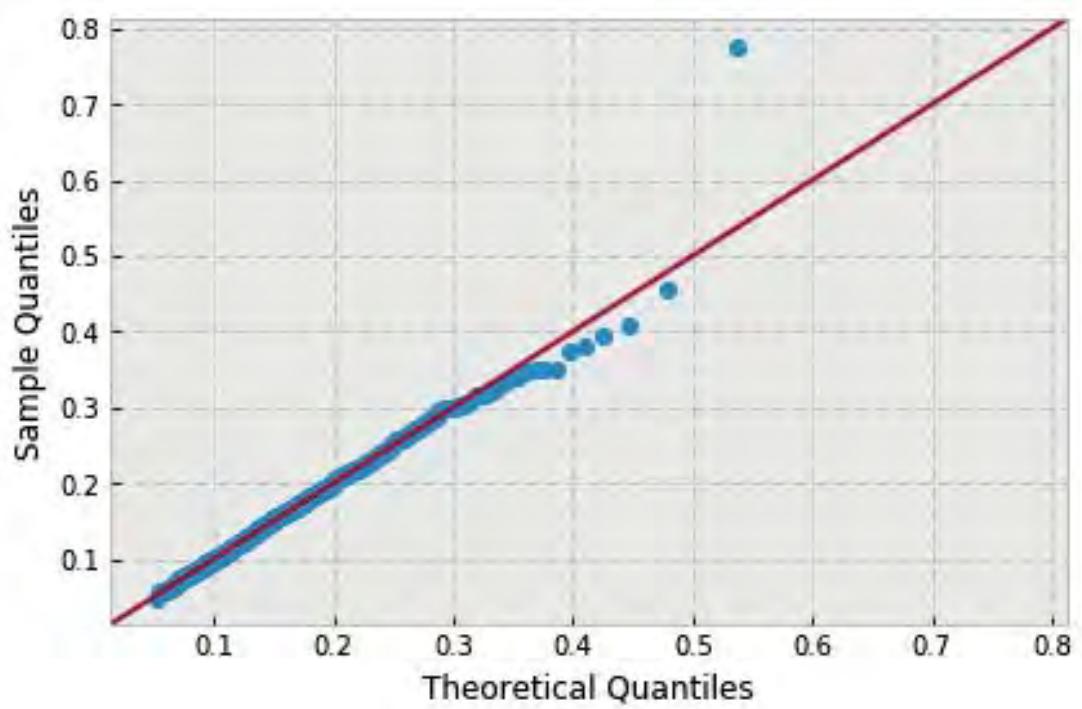
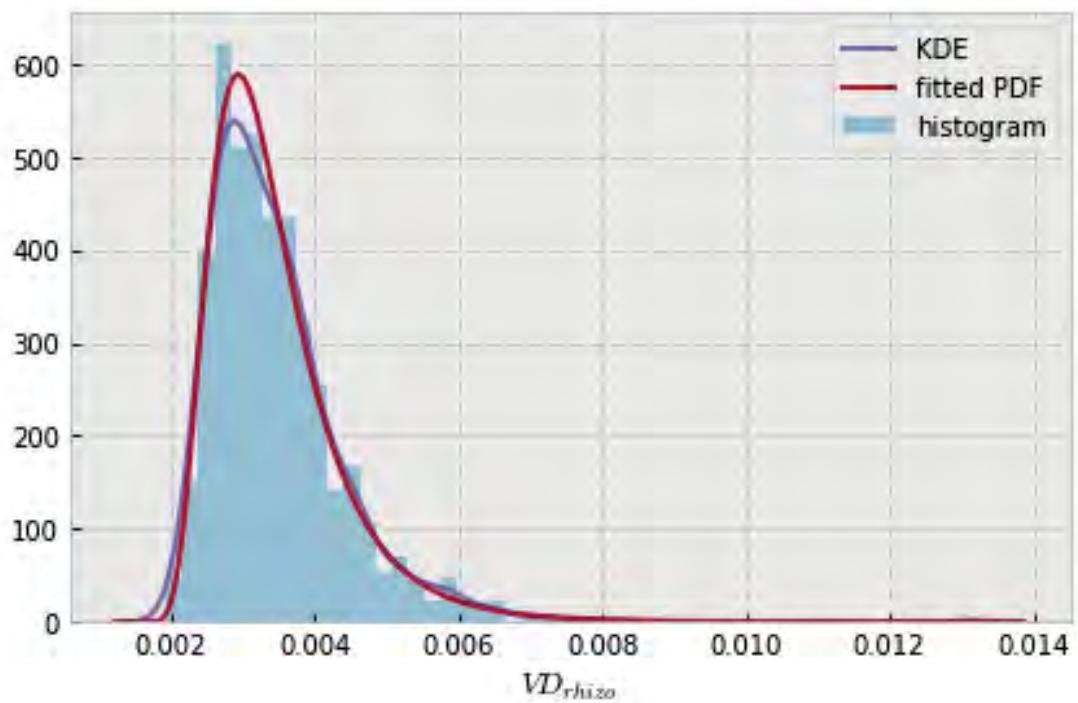


```
variable name: theta
variable value: 1.3962634015954638
distribution: inverse-gamma(shape=(7.5363503012667774,), loc=0.0011430464137846396,
                           scale=0.01693438222333094)
DescribeResult(nobs=1000, minmax=(0.002143360775687702, 0.0091020165919117602)
               mean=0.0037296145657502743, variance=1.1132588285082868e-06,
               skewness=1.4254576745224377, kurtosis=2.859211965663592)
```



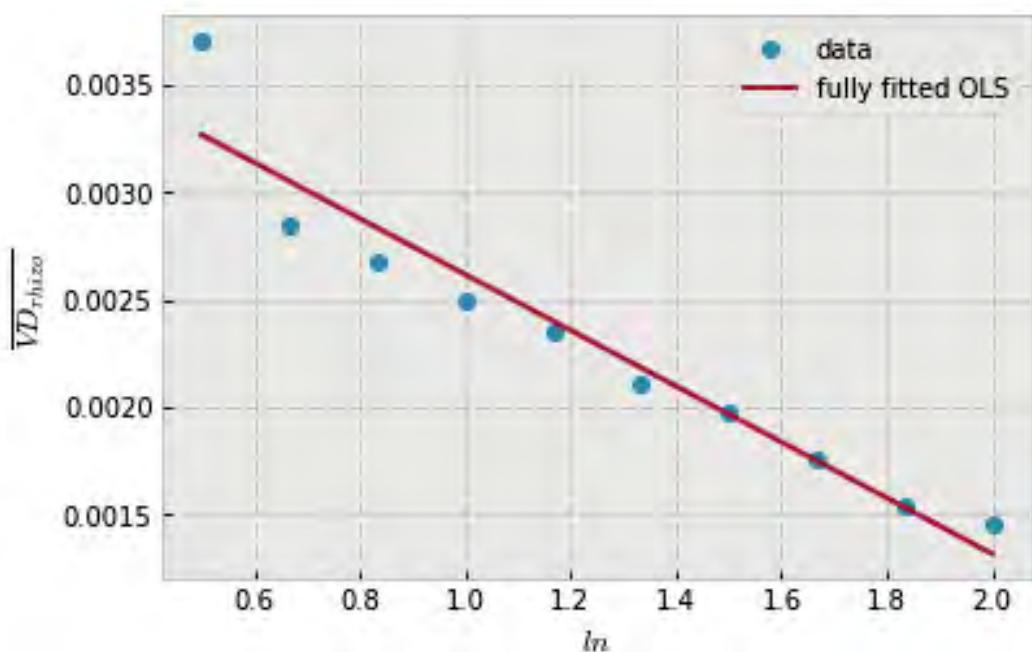


```
variable name: theta
variable value: 1.4835298641951802
distribution: inverse-gamma(shape=(7.7707112228133077,), loc=0.0011740394773986562,
                           scale=0.015445691660035524)
DescribeResult(nobs=1000, minmax=(0.0019285690458628849, 0.013163526825684602)
               mean=0.0034542655215307233, variance=8.9432717699323905e-07,
               skewness=2.1822673222614735, kurtosis=12.42133296982851)
```

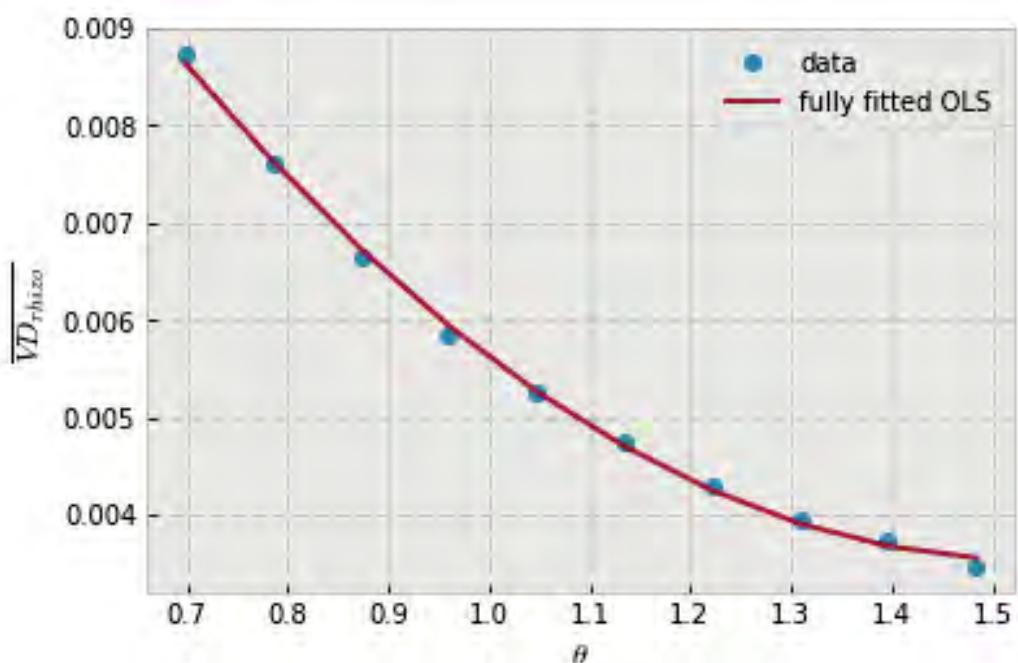


14.2 Parameter regression

```
In [34]: var_name = 'rhizosphere_volume_phosphate_per_convex_hull_volume'  
mean_reg('ln', var_name, 'poly1')  
mean_reg('theta', var_name, 'poly2')  
mean_reg('nB', var_name, 'poly1')  
mean_reg('r', var_name, 'poly2')  
mean_reg('sigma', var_name, 'poly2')  
mean_reg('N', var_name, 'poly1')  
  
variable name: ln  
model: poly1  
r-squared: 0.8469640536223517  
explained variance: 0.8477260568878546
```



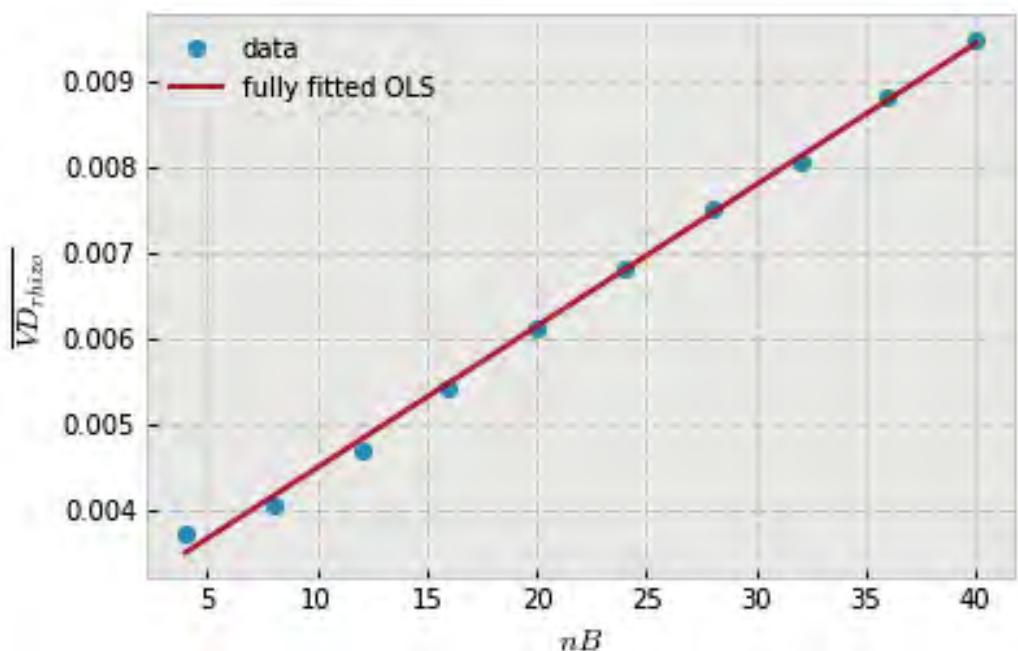
```
variable name: theta  
model: poly2  
r-squared: 0.9934022863281164  
explained variance: 0.9934023034342007
```



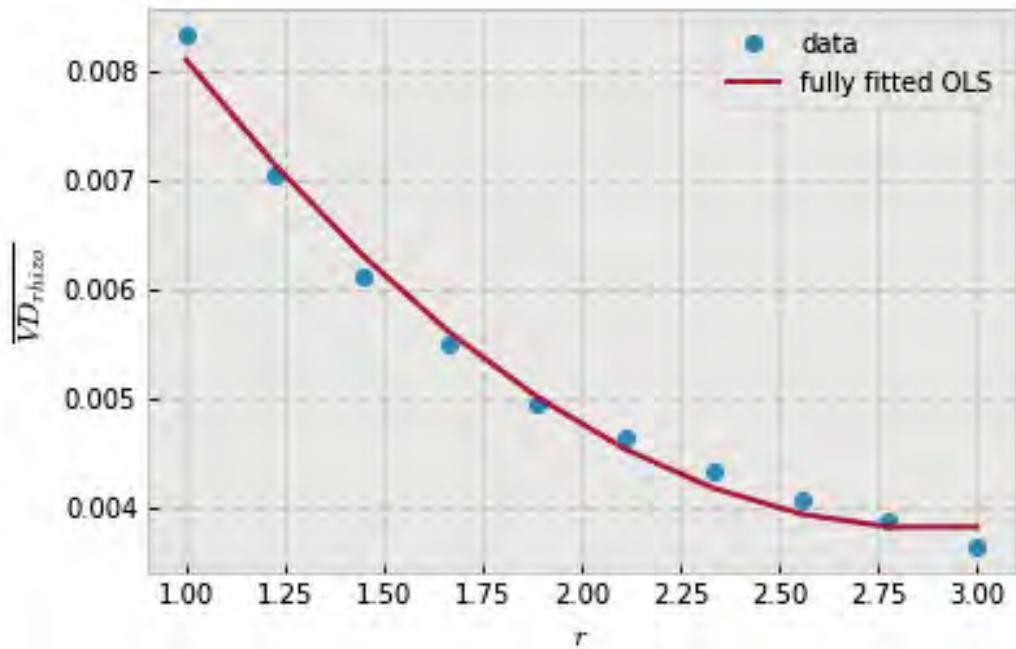
```

variable name: nB
model: poly1
r-squared: 0.9947889721294337
explained variance: 0.9948068934853409

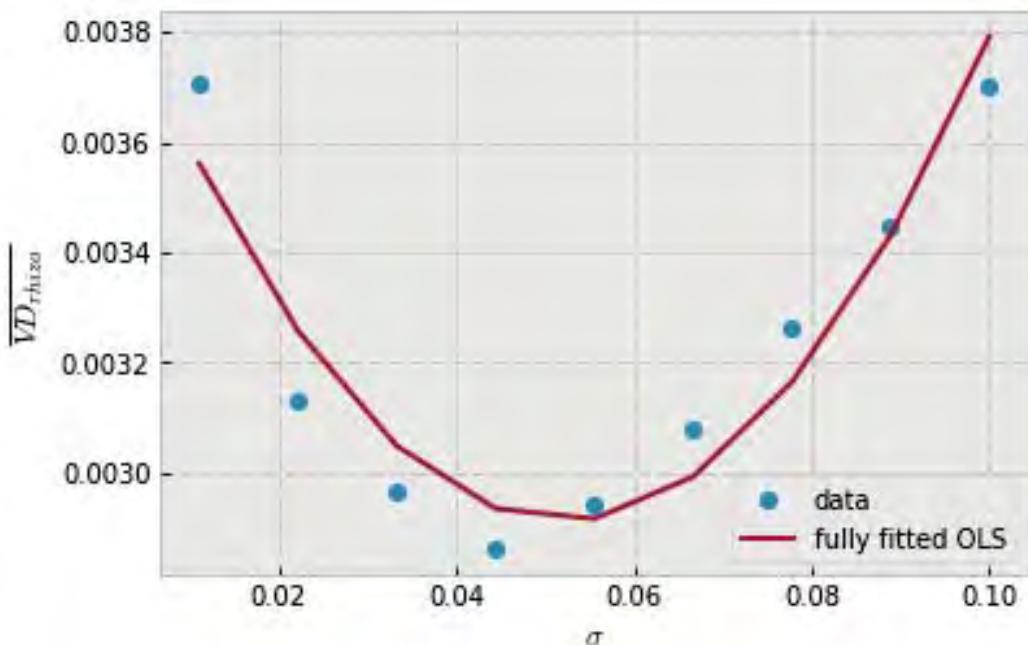
```



```
variable name: r
model: poly2
r-squared: 0.9600306650817723
explained variance: 0.9600469021315613
```



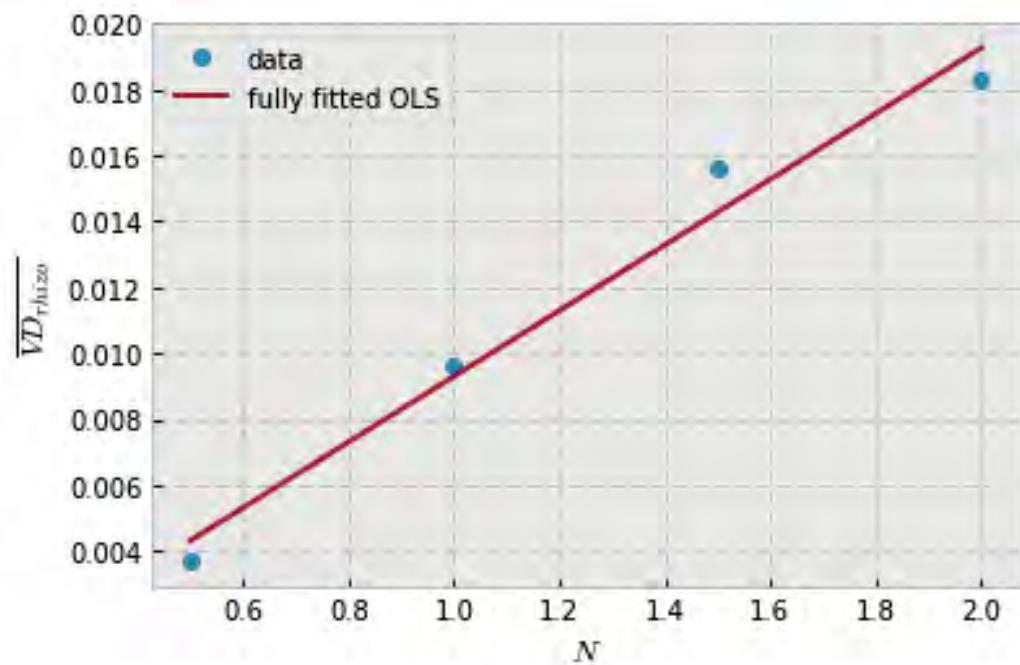
```
variable name: sigma
model: poly2
r-squared: 0.5935366076451363
explained variance: 0.5944969856533836
```



```

variable name: N
model: poly1
r-squared: 0.8531560016897918
explained variance: 0.8716983774172935

```

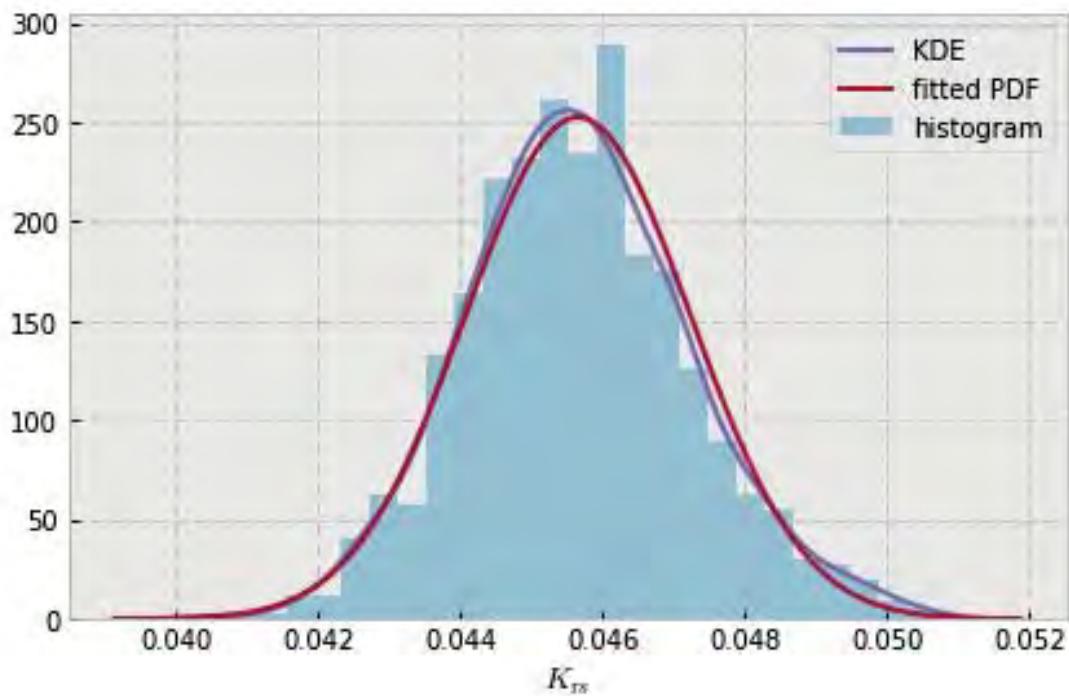


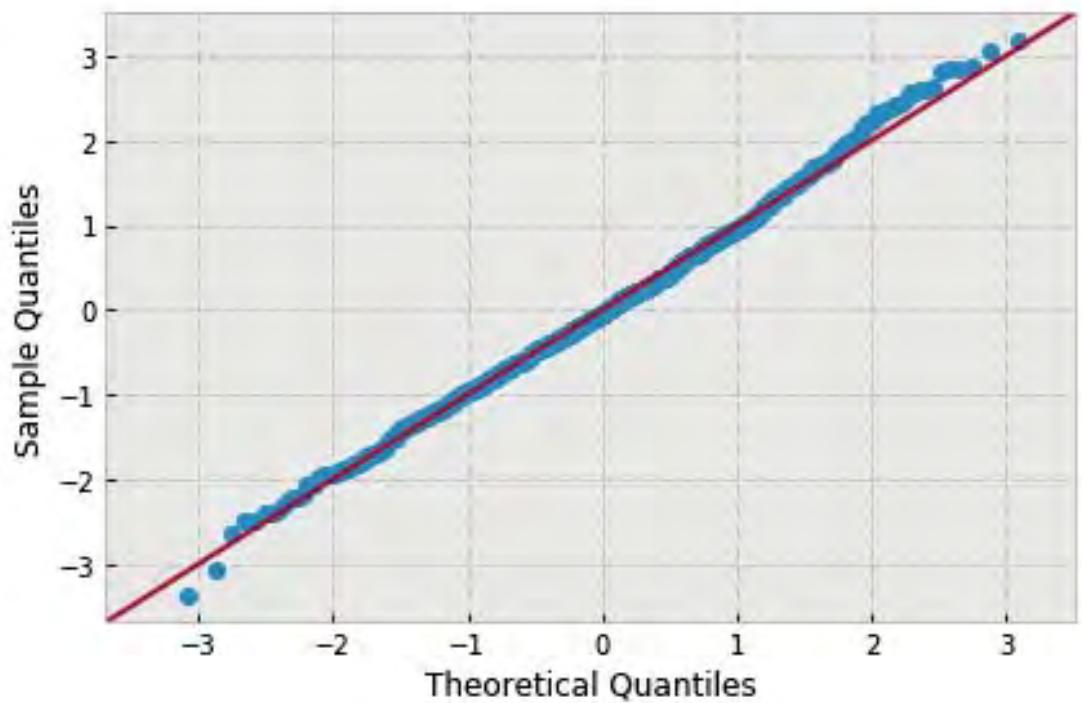
15 Root system total conductance

15.1 Probability distributions

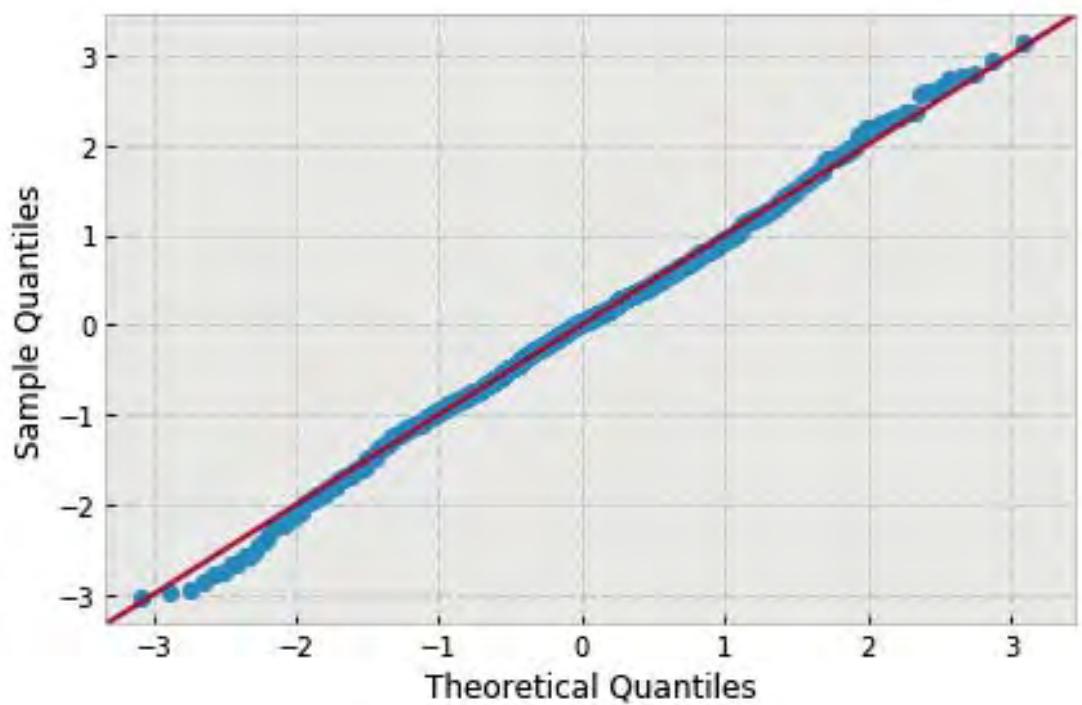
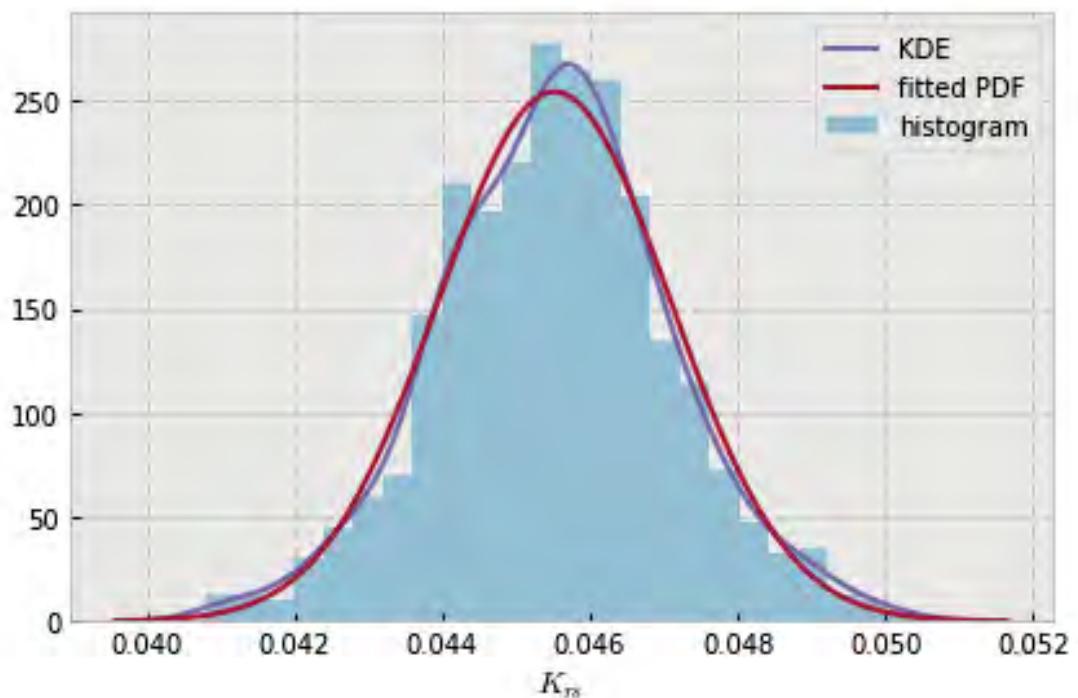
```
In [35]: plot_dist('root_system_total_conductance')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=0.04564901051487, scale=0.0015794061579788676)
DescribeResult(nobs=1000, minmax=(0.040342315970000001, 0.05066664191999998)
               mean=0.045649010514869998, variance=2.4970208326942622e-06,
               skewness=0.18277515883164833, kurtosis=0.17154175535701022)
```

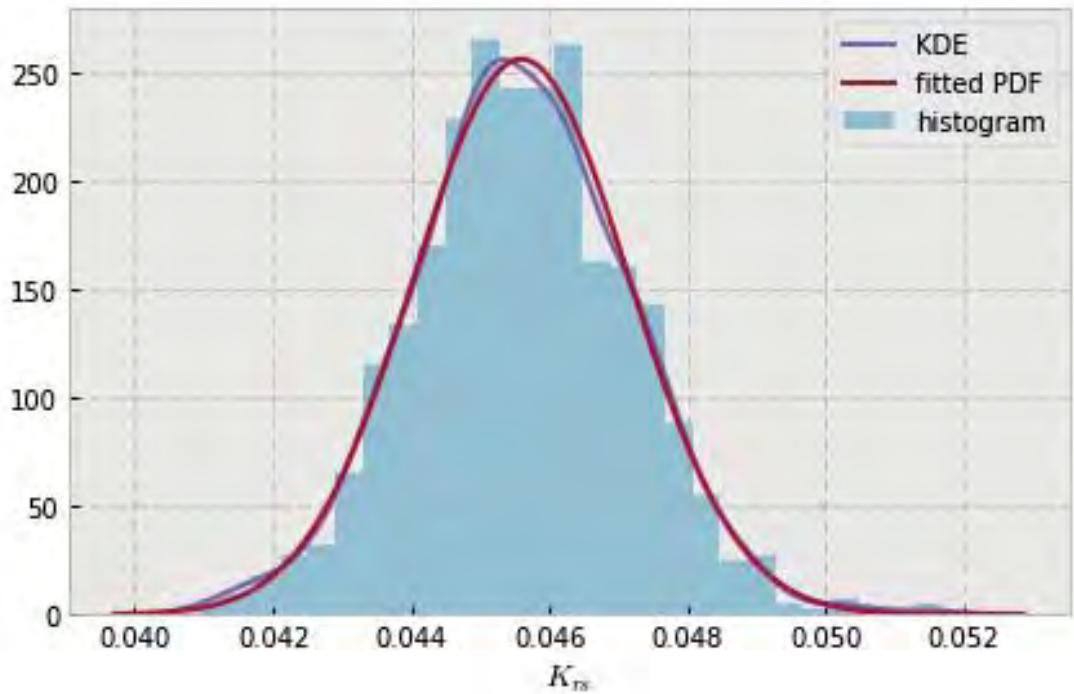


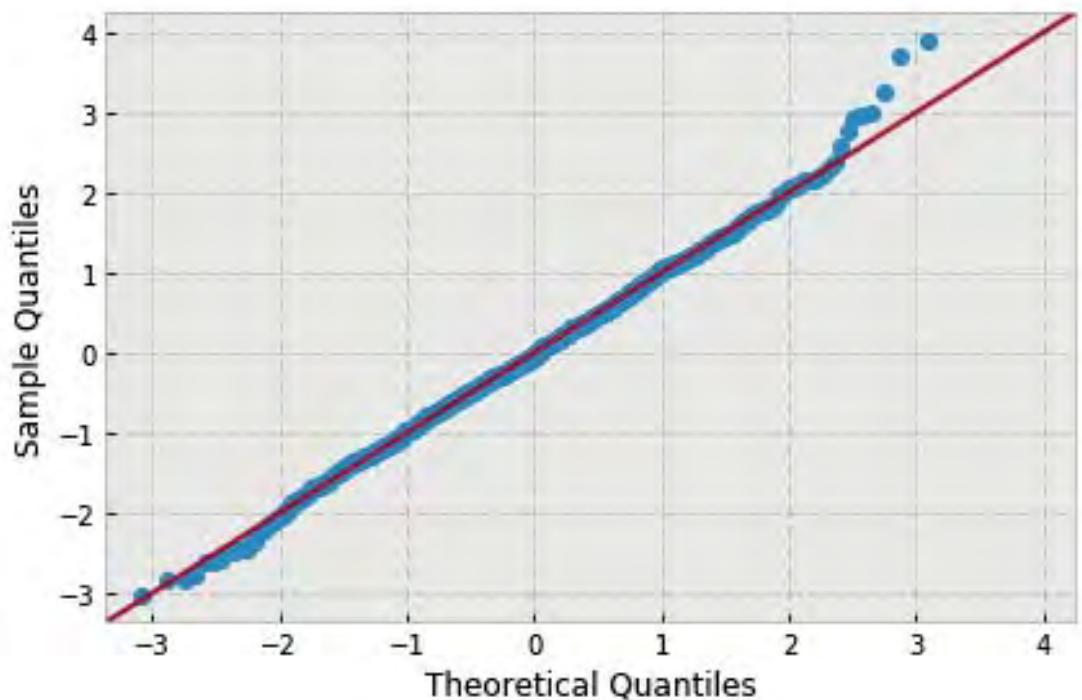


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=0.04551823857909, scale=0.0015691502512964934)
DescribeResult(nobs=1000, minmax=(0.04077490928, 0.05044209580000003)
               mean=0.04551823857909001, variance=2.4646972083522013e-06,
               skewness=-0.006880267557560633, kurtosis=0.23751006511327866)
```

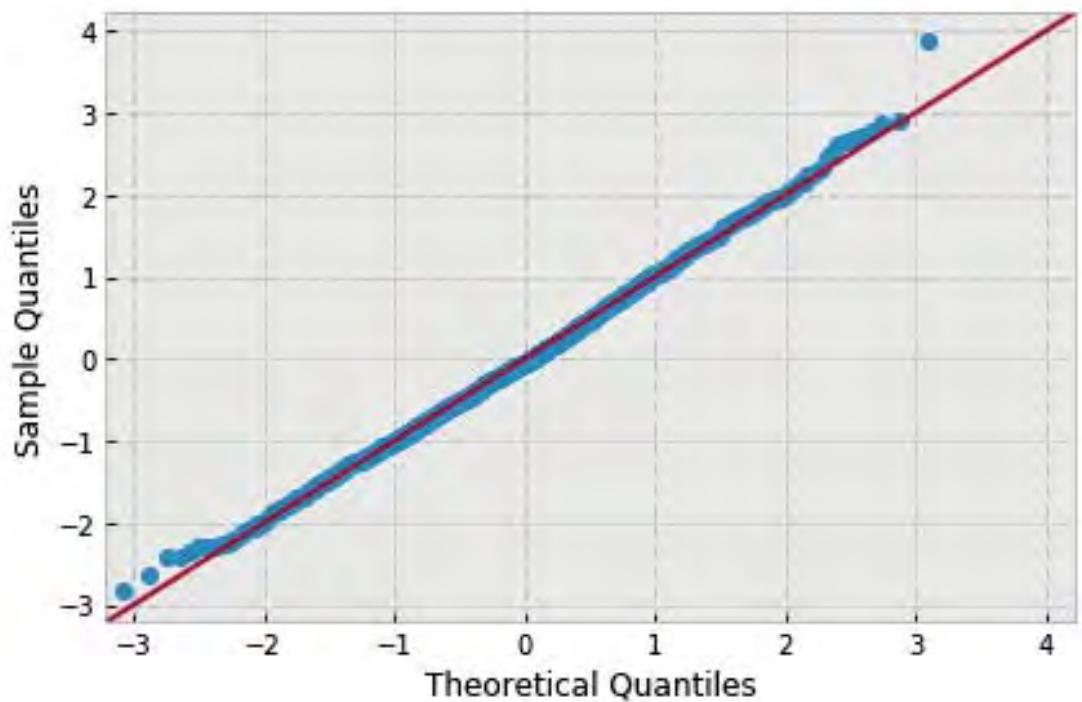
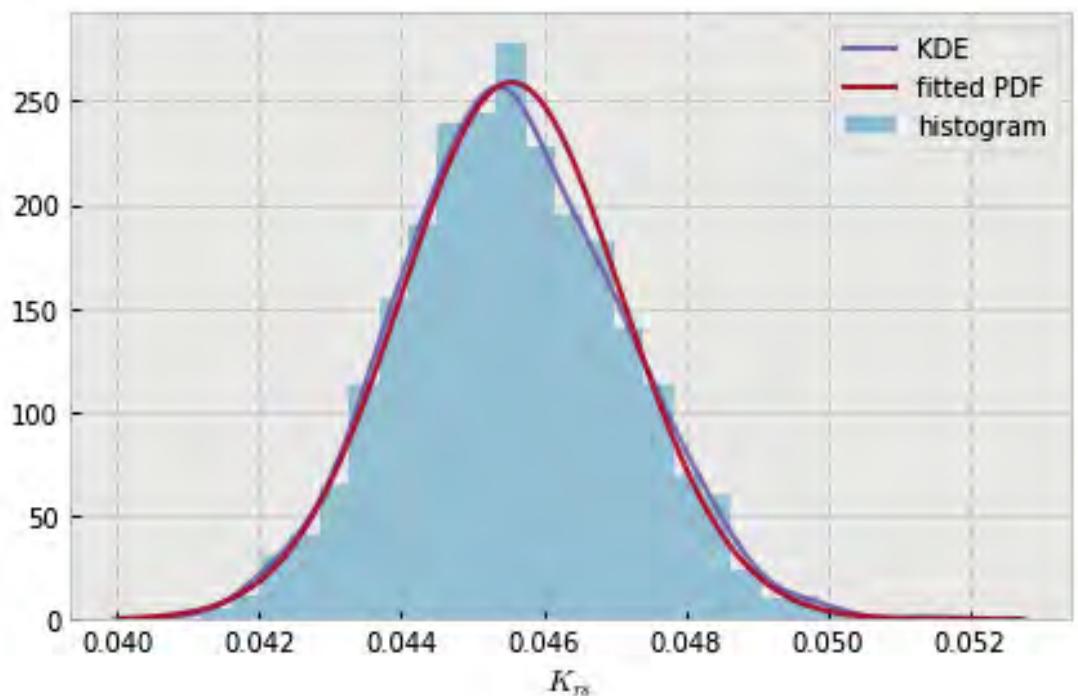


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=0.04559327550416001, scale=0.0015566813476997055)
DescribeResult(nobs=1000, minmax=(0.04089791986, 0.05165641925000002)
               mean=0.045593275504160008, variance=2.4256825007769481e-06,
               skewness=0.09091307187420154, kurtosis=0.3161975172025362)
```

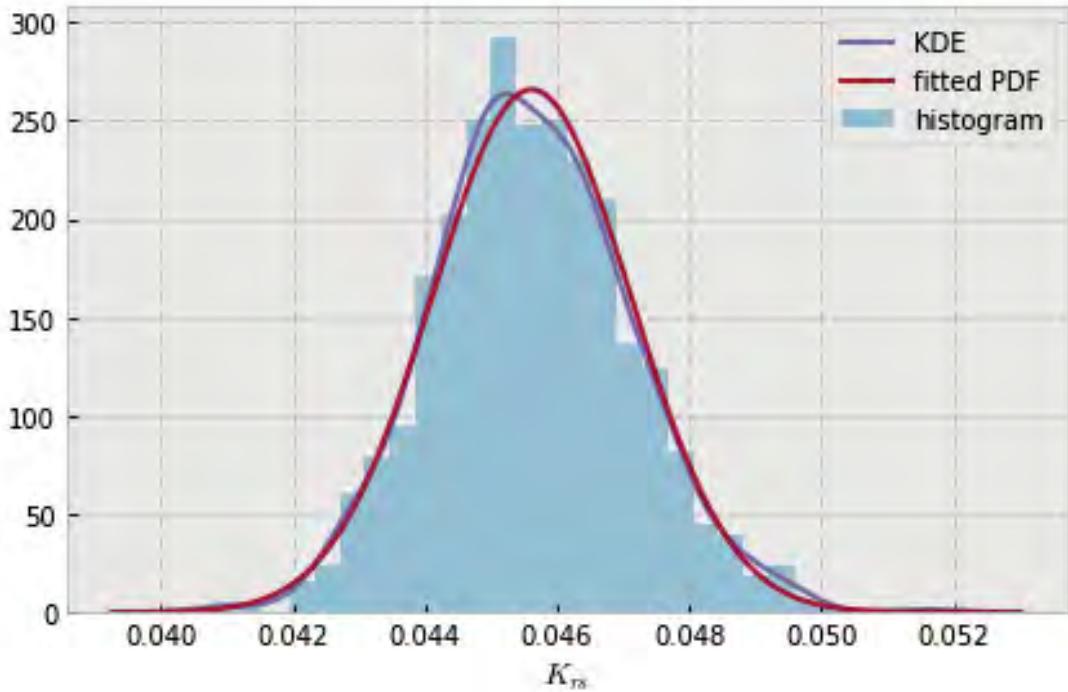


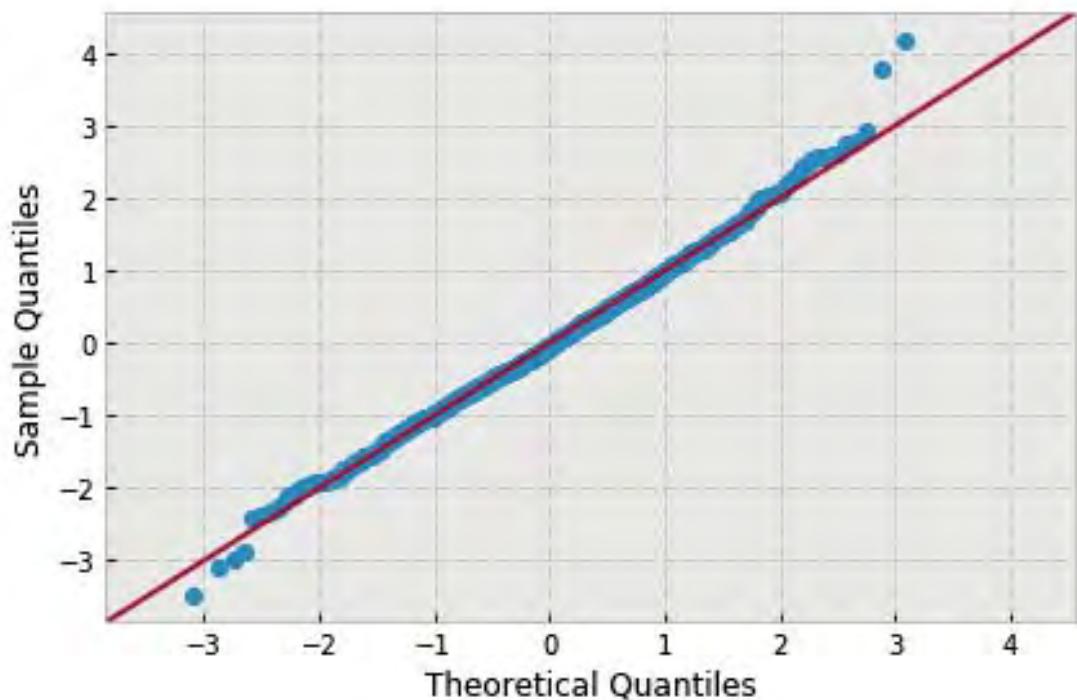


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=0.04555171280735, scale=0.0015405346637968448)
DescribeResult(nobs=1000, minmax=(0.04120486250000002, 0.05153108314999997)
               mean=0.04555171280735001, variance=2.3756226730326905e-06,
               skewness=0.16365100669709406, kurtosis=-0.042317883334735296)
```

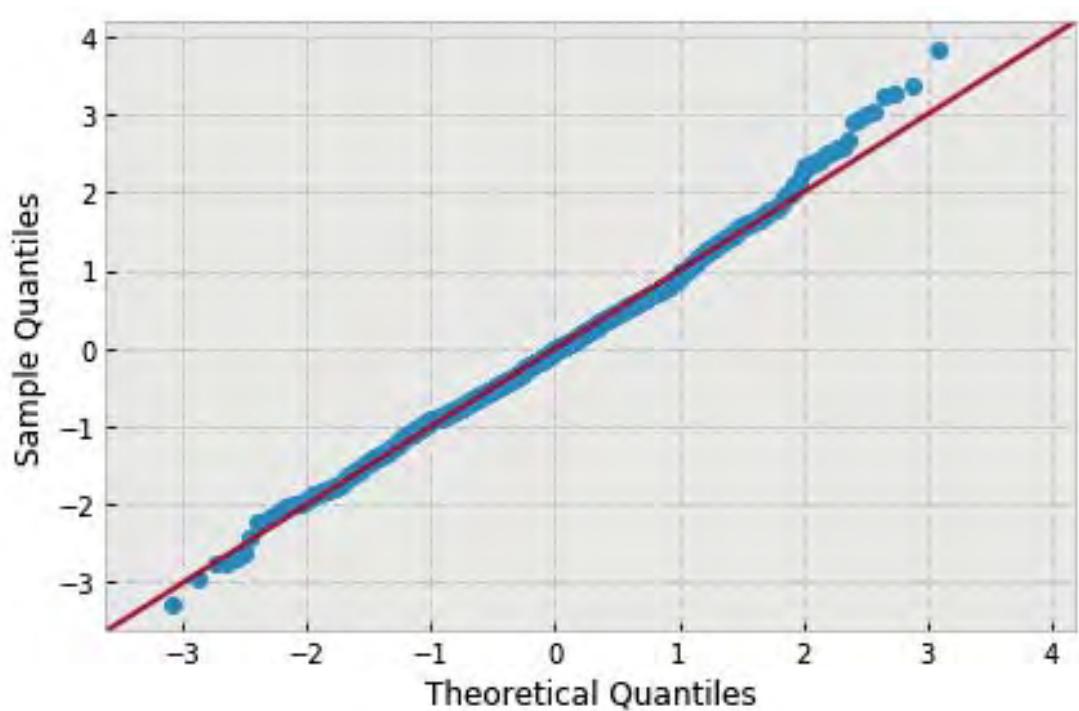
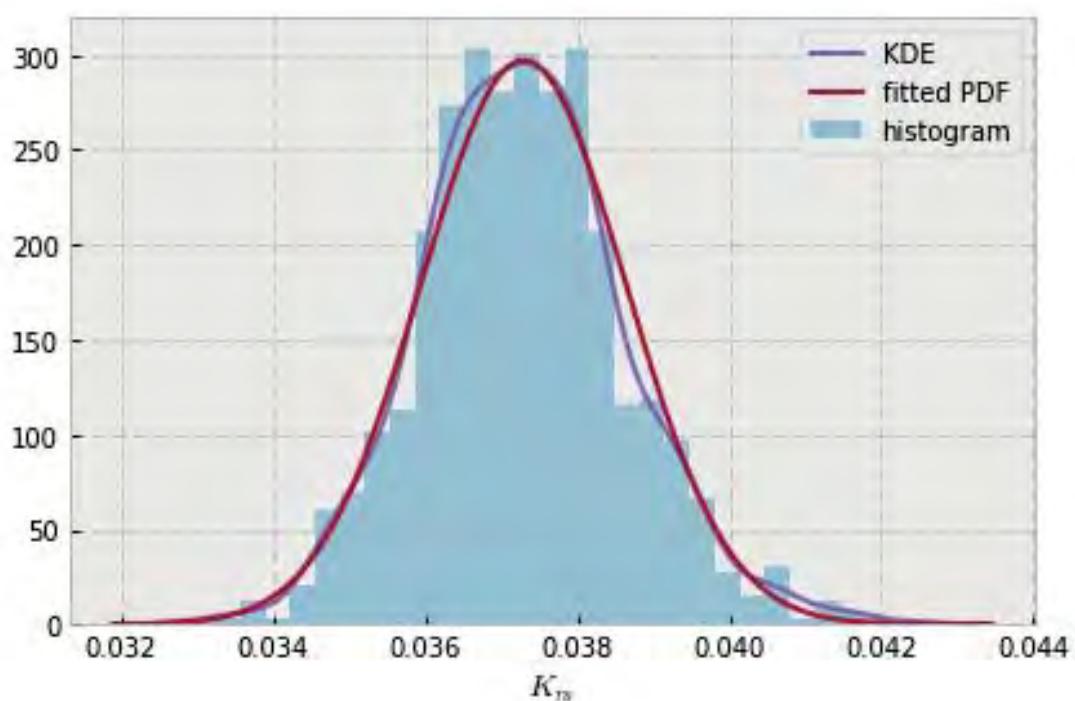


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=0.045609792761669994, scale=0.0015004481268919162)
DescribeResult(nobs=1000, minmax=(0.040409002200000002, 0.05187201781999998)
               mean=0.045609792761669994, variance=2.253598179673133e-06,
               skewness=0.18796517683555822, kurtosis=0.3944447067915968)
```

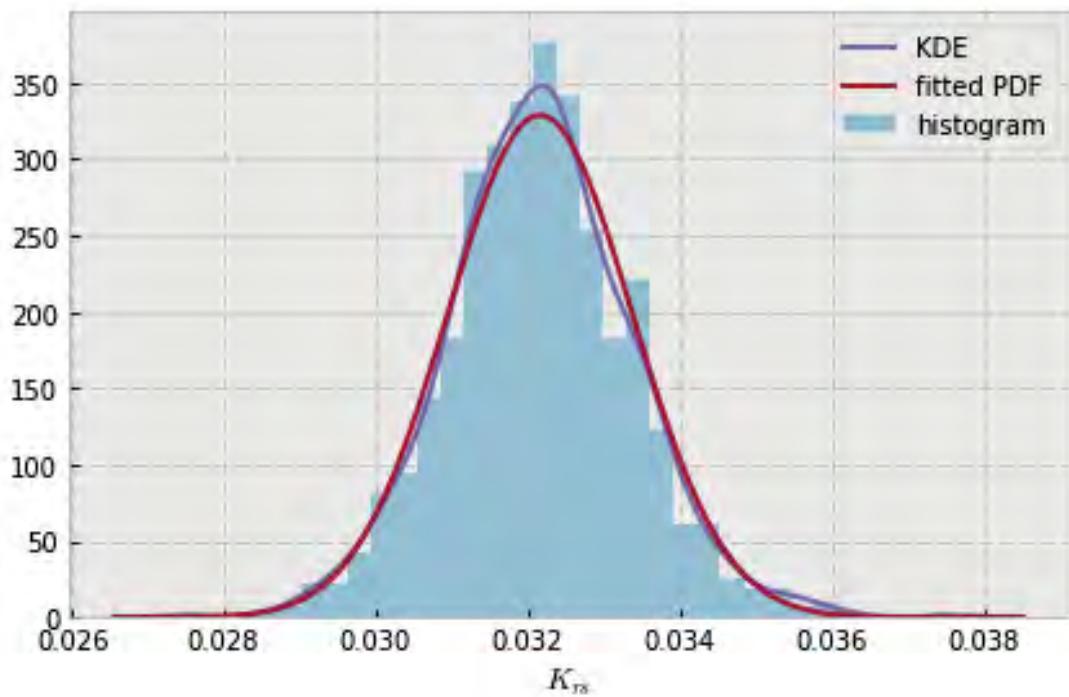


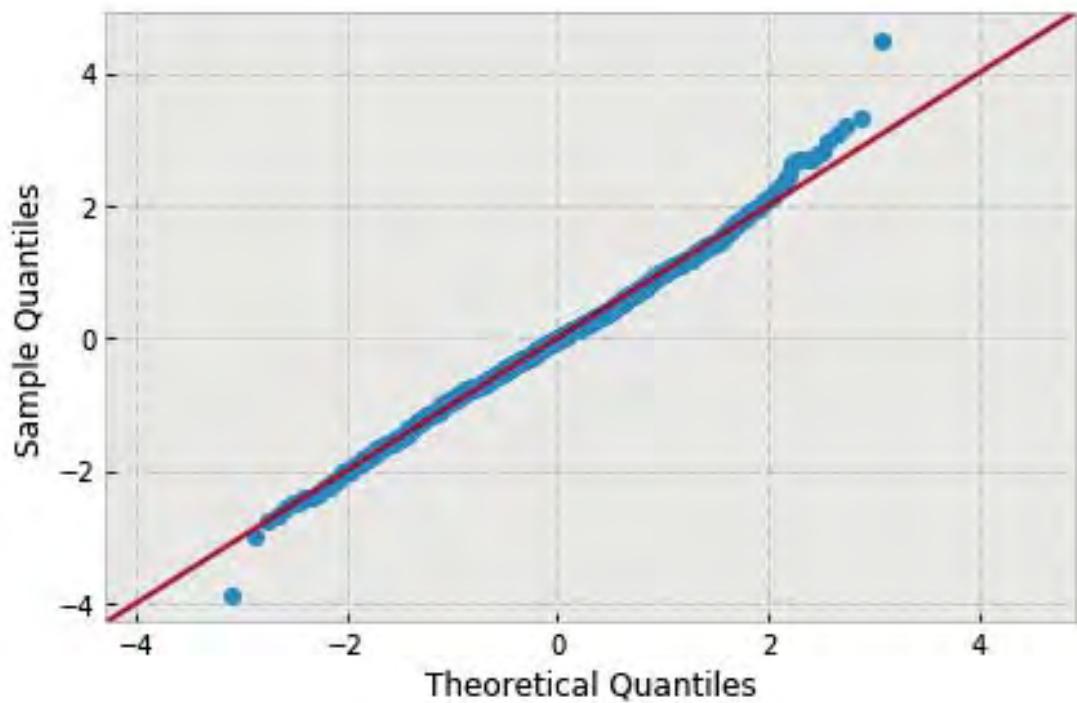


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=0.03729100269607, scale=0.0013420599428151856)
DescribeResult(nobs=1000, minmax=(0.03290402843000003, 0.04243511606)
               mean=0.037291002696069997, variance=1.8029278179270262e-06,
               skewness=0.24943143576515364, kurtosis=0.47372559036033124)
```

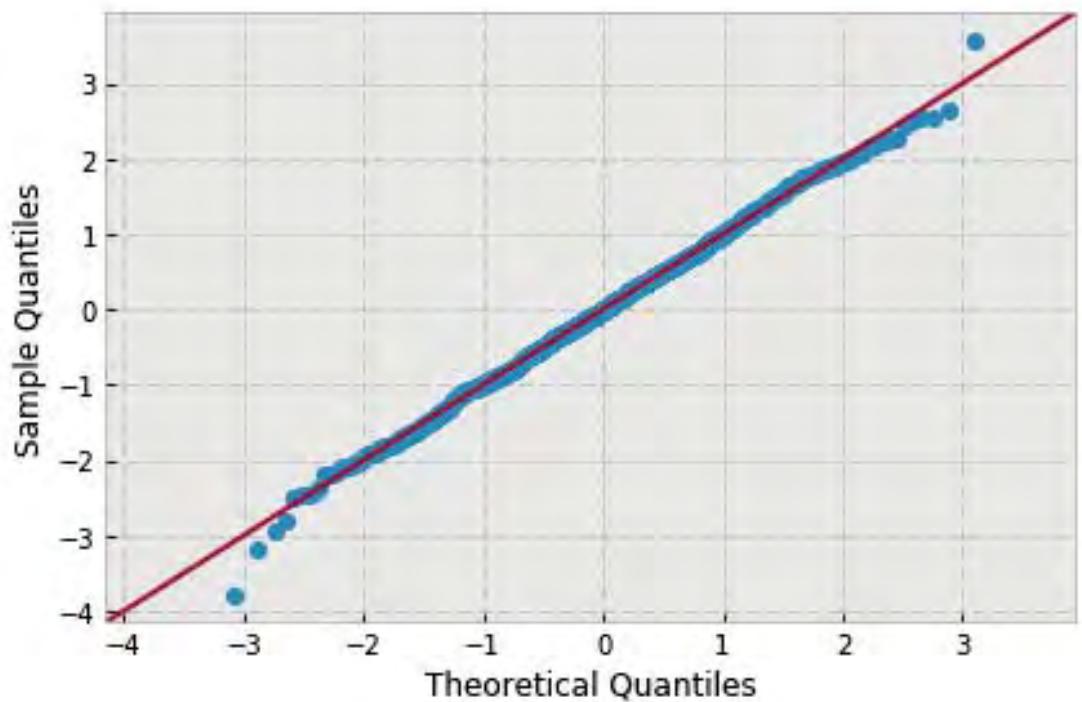
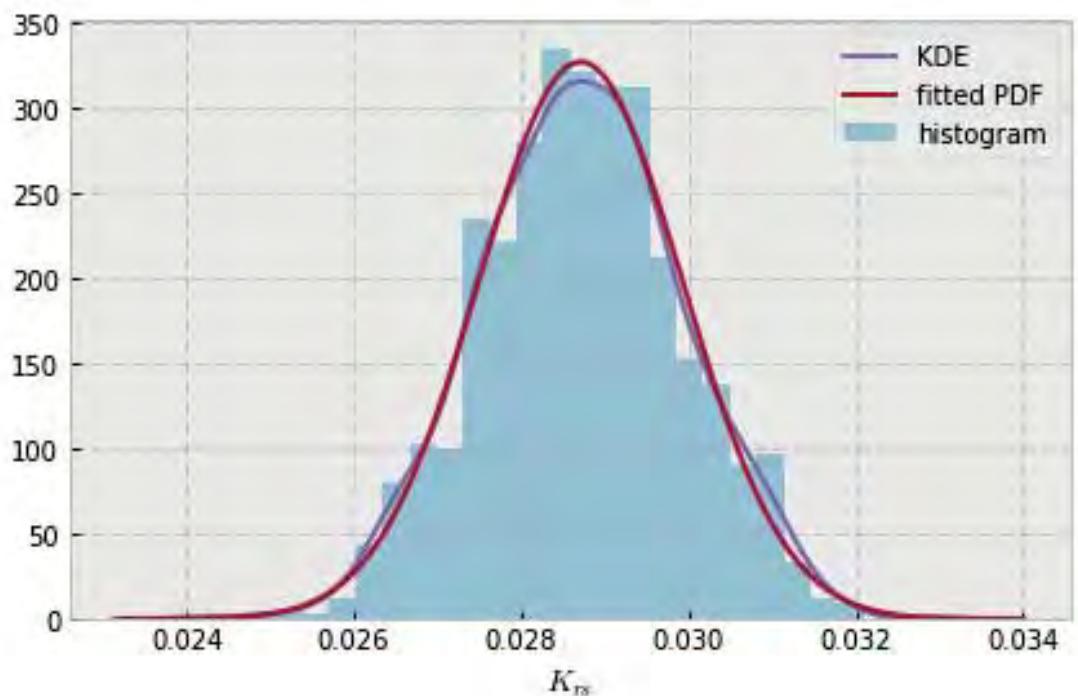


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=0.03216062897442, scale=0.0012135430961360482)
DescribeResult(nobs=1000, minmax=(0.02746550874999999, 0.037608858240000001)
               mean=0.03216062897442, variance=1.4741610071866525e-06,
               skewness=0.1608810197851539, kurtosis=0.7007935906816112)
```

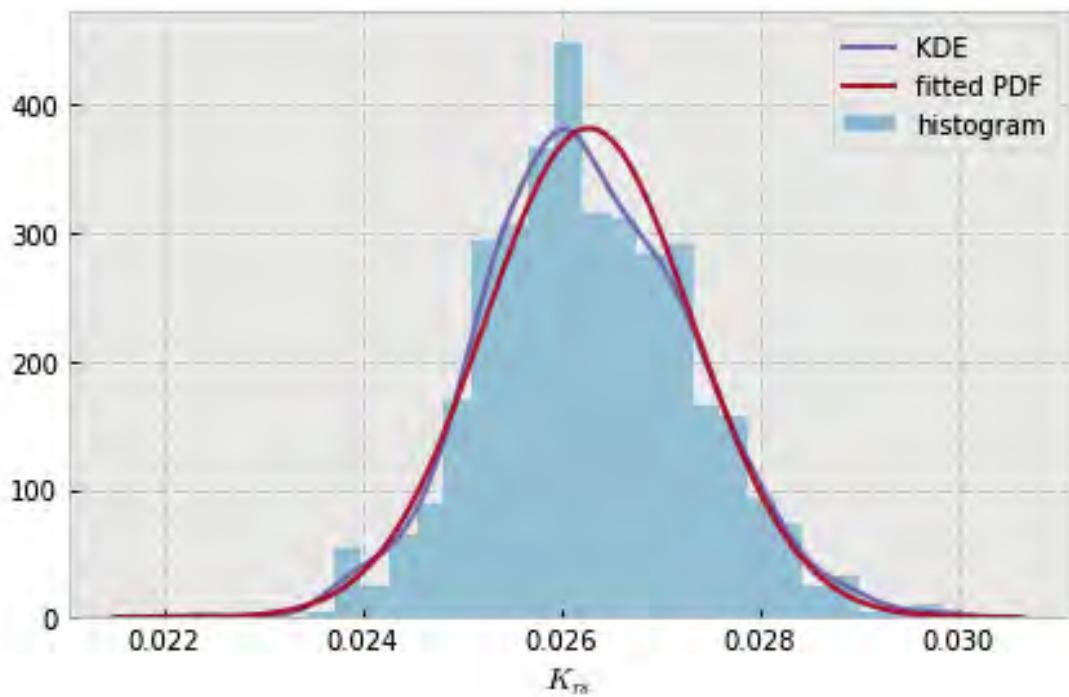


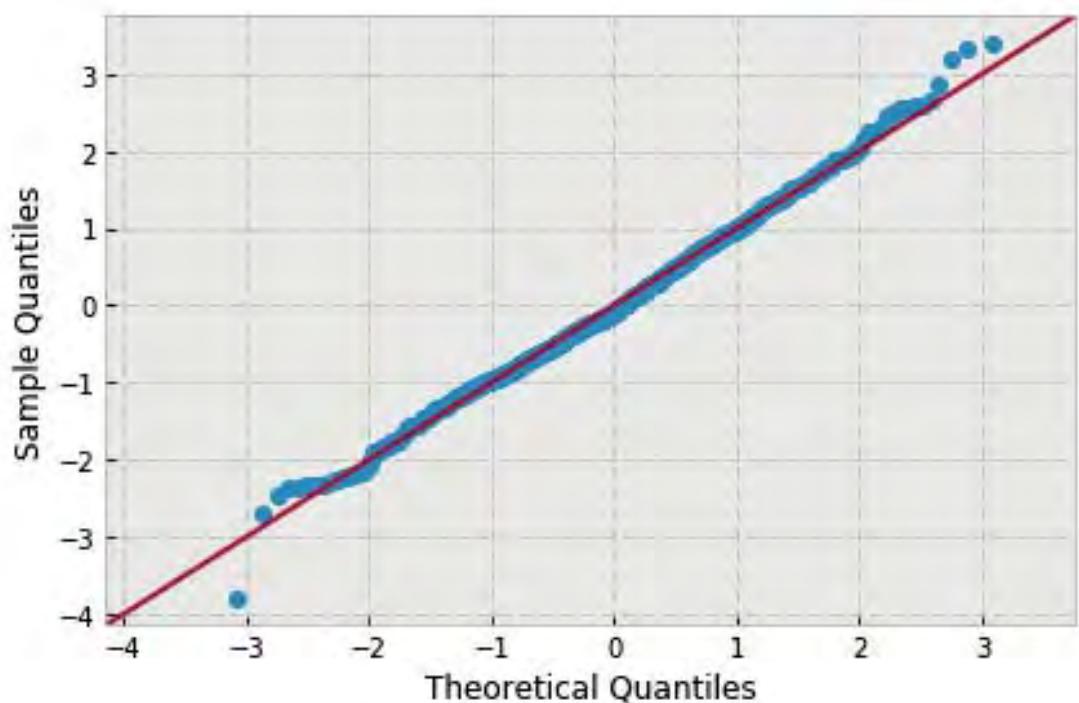


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=0.02871354146251, scale=0.0012179537027342513)
DescribeResult(nobs=1000, minmax=(0.024107460680000001, 0.03305298946000001)
               mean=0.028713541462510001, variance=1.4848961181221956e-06,
               skewness=-0.028638125643443296, kurtosis=0.0027326216798000402)
```

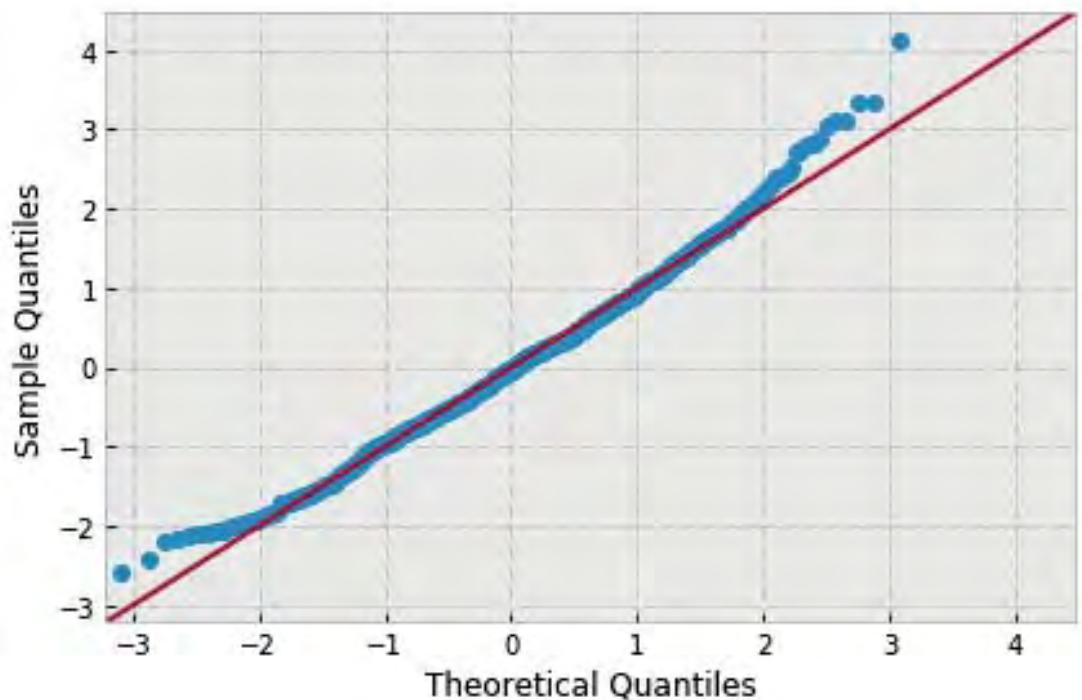
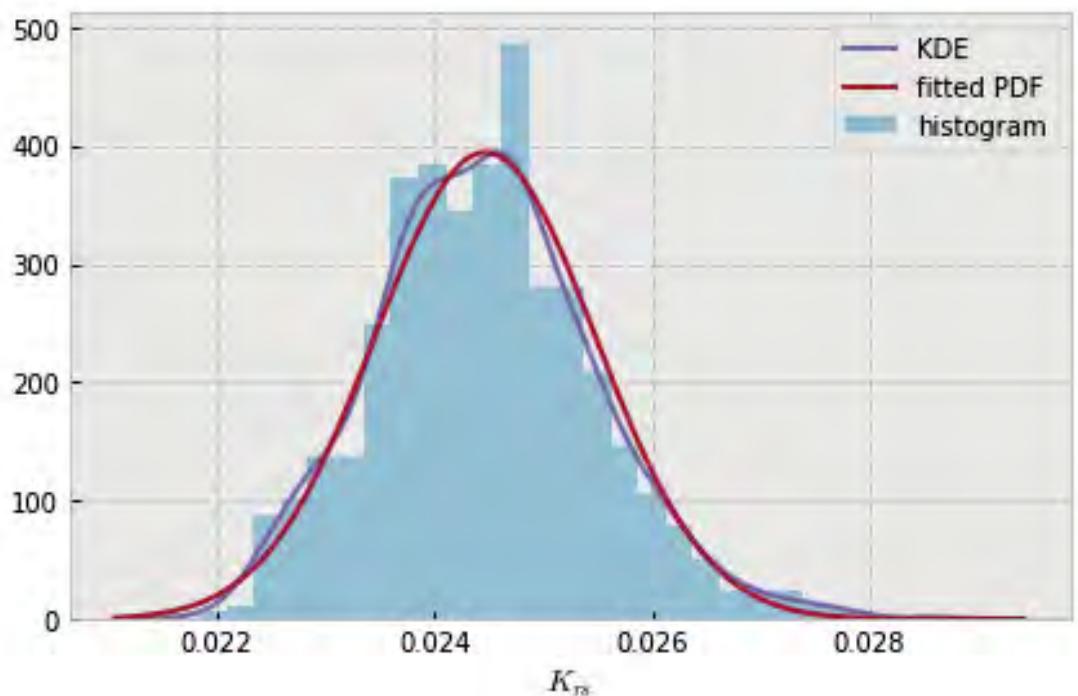


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=0.026275969500430003, scale=0.0010448558576742321)
DescribeResult(nobs=1000, minmax=(0.02232031647, 0.02982049904)
    mean=0.026275969500430003, variance=1.0928165798960512e-06,
    skewness=0.15380555297353268, kurtosis=0.14090933127990812)
```

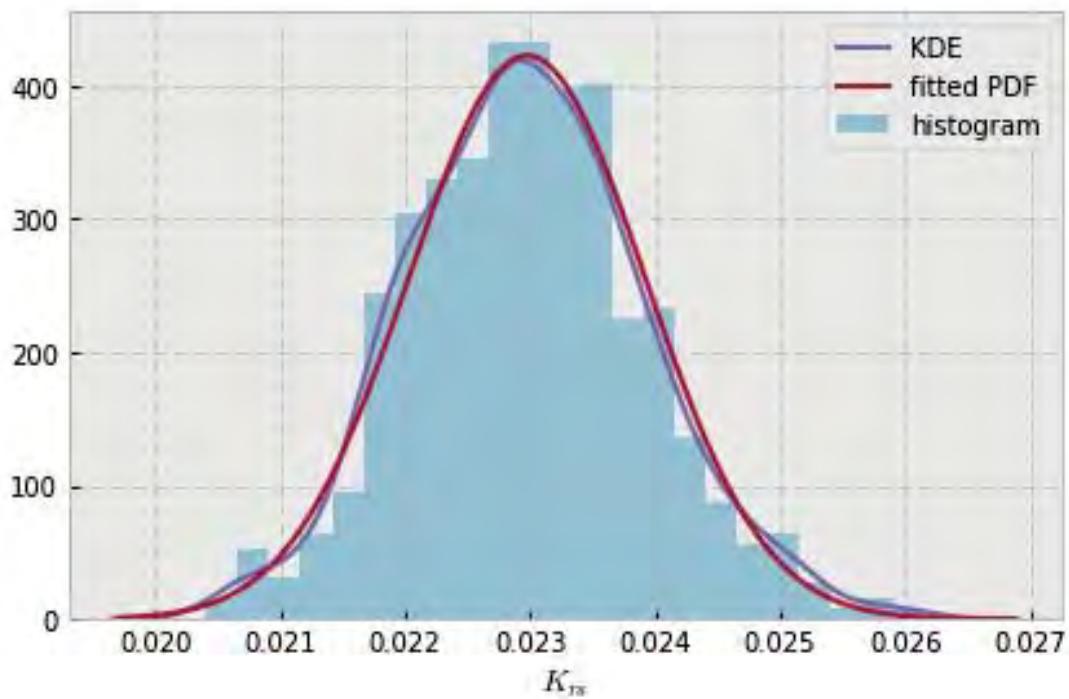


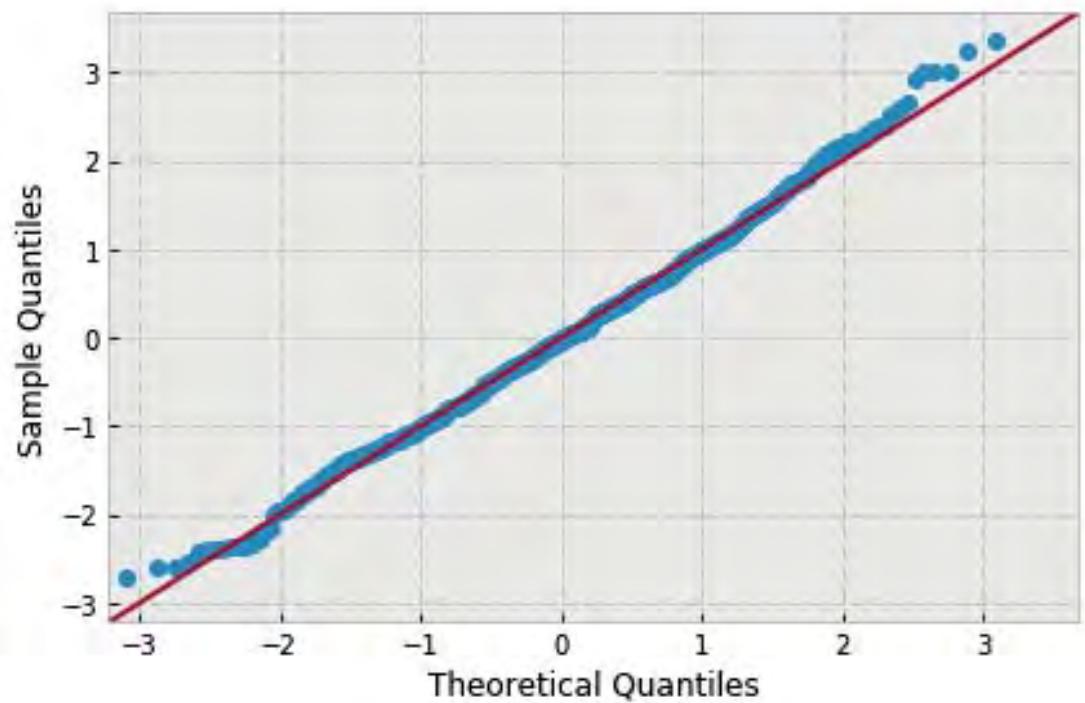


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=0.02446327367656, scale=0.0010124618973782924)
DescribeResult(nobs=1000, minmax=(0.02182957143, 0.02864572091999999)
               mean=0.02446327367656, variance=1.0261051988416937e-06,
               skewness=0.346522440042038, kurtosis=0.33275087934862846)
```

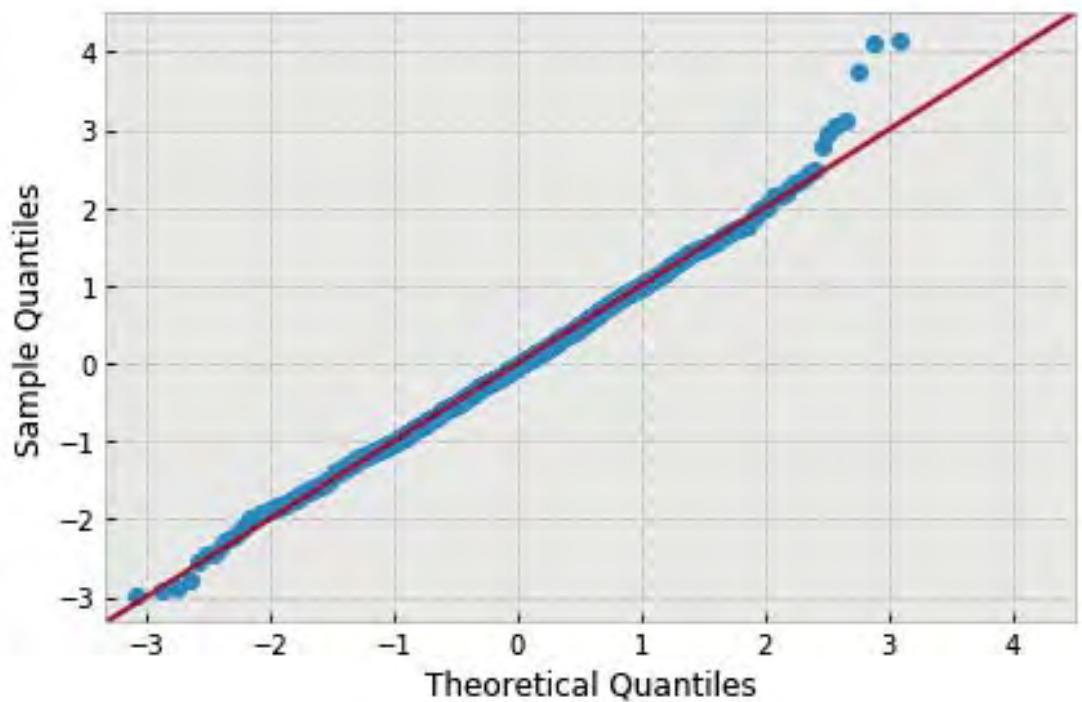
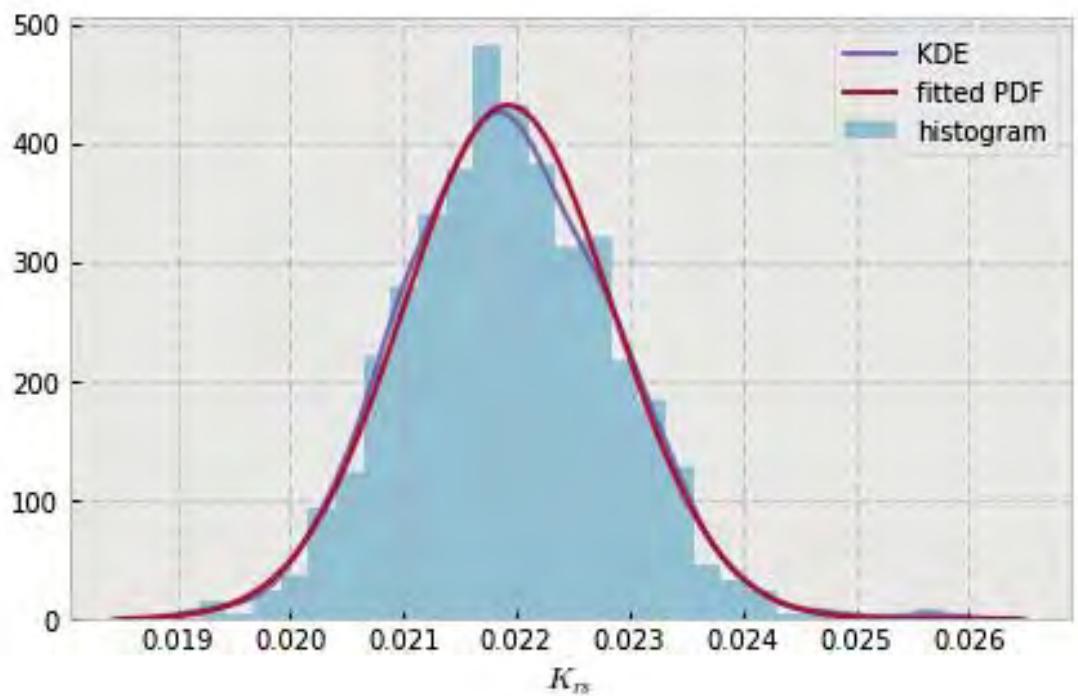


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=0.022974645377139998, scale=0.0009445958840763123)
DescribeResult(nobs=1000, minmax=(0.02041615102999999, 0.026150132940000001)
               mean=0.022974645377139998, variance=8.931545387526626e-07,
               skewness=0.19011527429066158, kurtosis=0.11463777699600453)
```

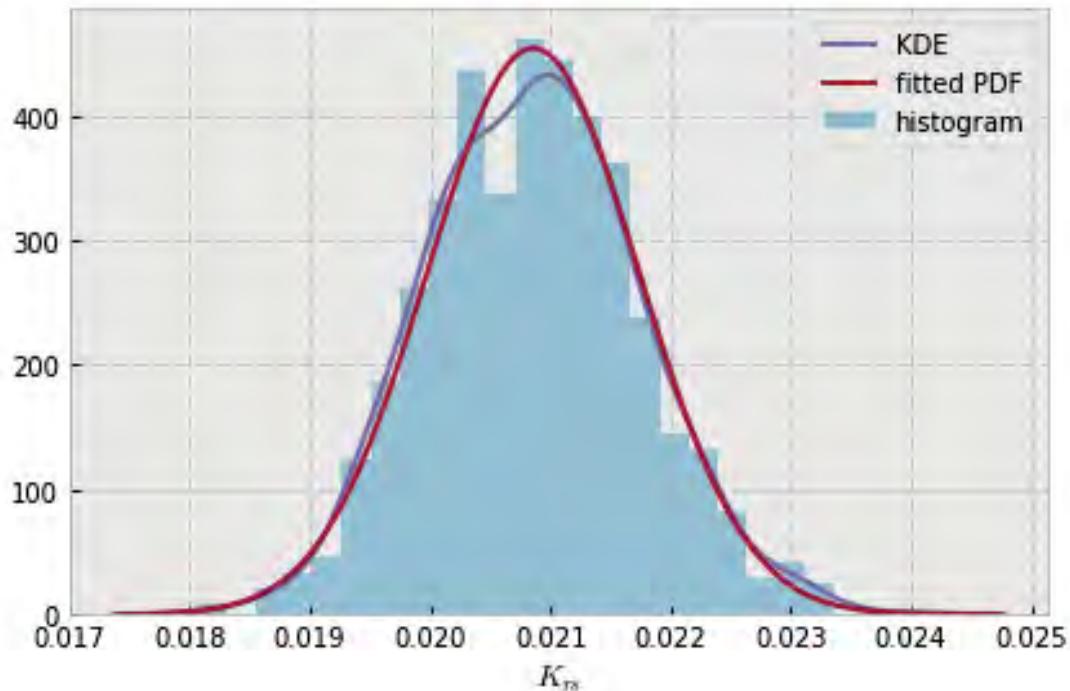


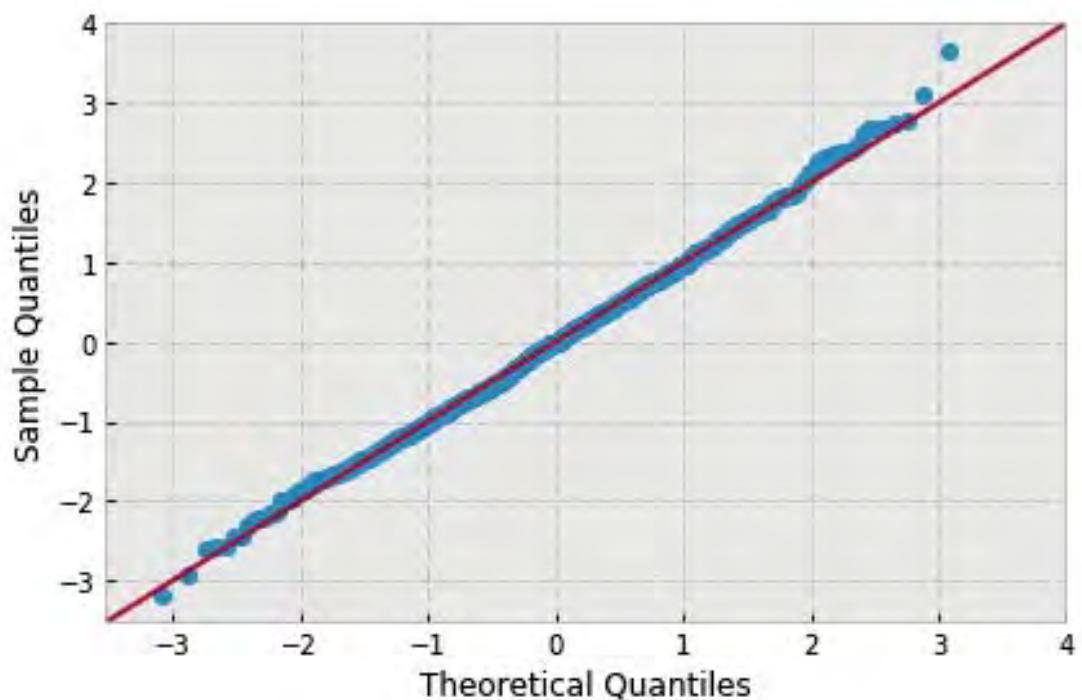


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=0.021925431591420002, scale=0.0009240593621583226)
DescribeResult(nobs=1000, minmax=(0.01918364314999999, 0.02575191365)
               mean=0.021925431591420002, variance=8.5474044523768371e-07,
               skewness=0.2159848011349591, kurtosis=0.42432628198503775)
```

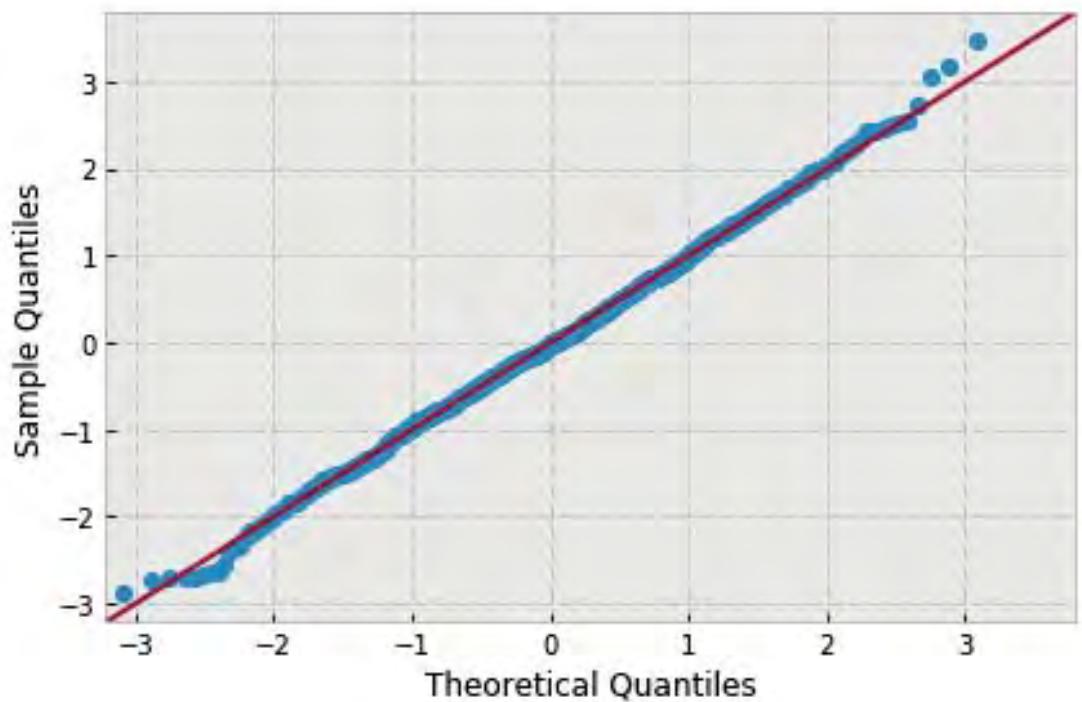
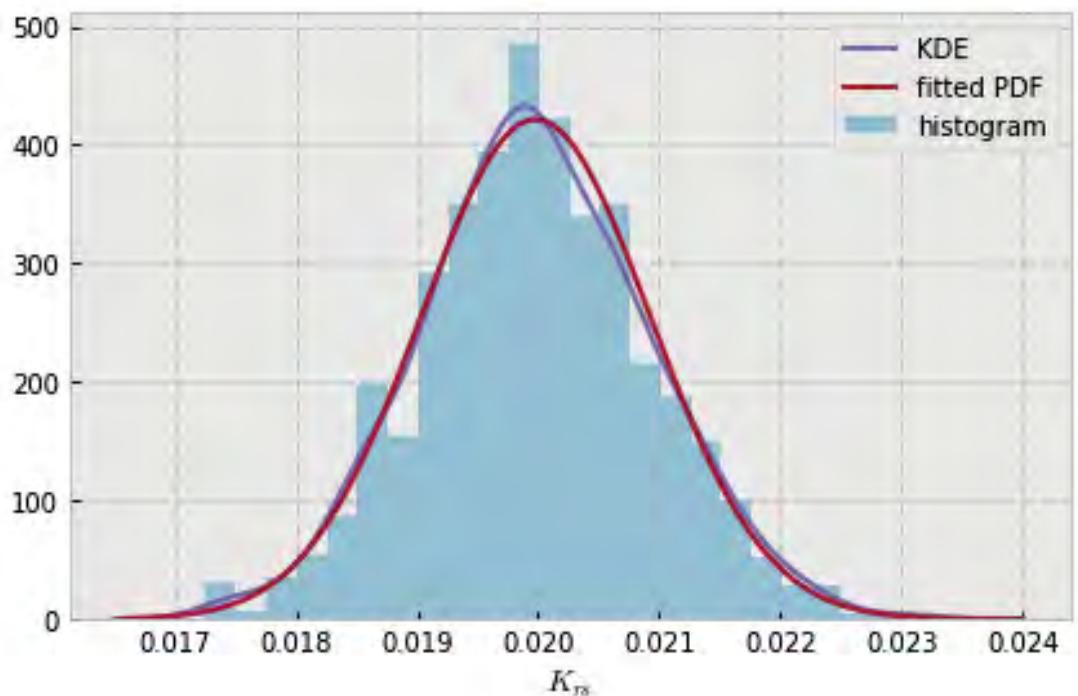


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=0.02085924896671, scale=0.0008786280470430236)
DescribeResult(nobs=1000, minmax=(0.01806812472999999, 0.02407139099)
               mean=0.020859248966710001, variance=7.7276000505569327e-07,
               skewness=0.1368835717550324, kurtosis=0.009893929444605654)
```

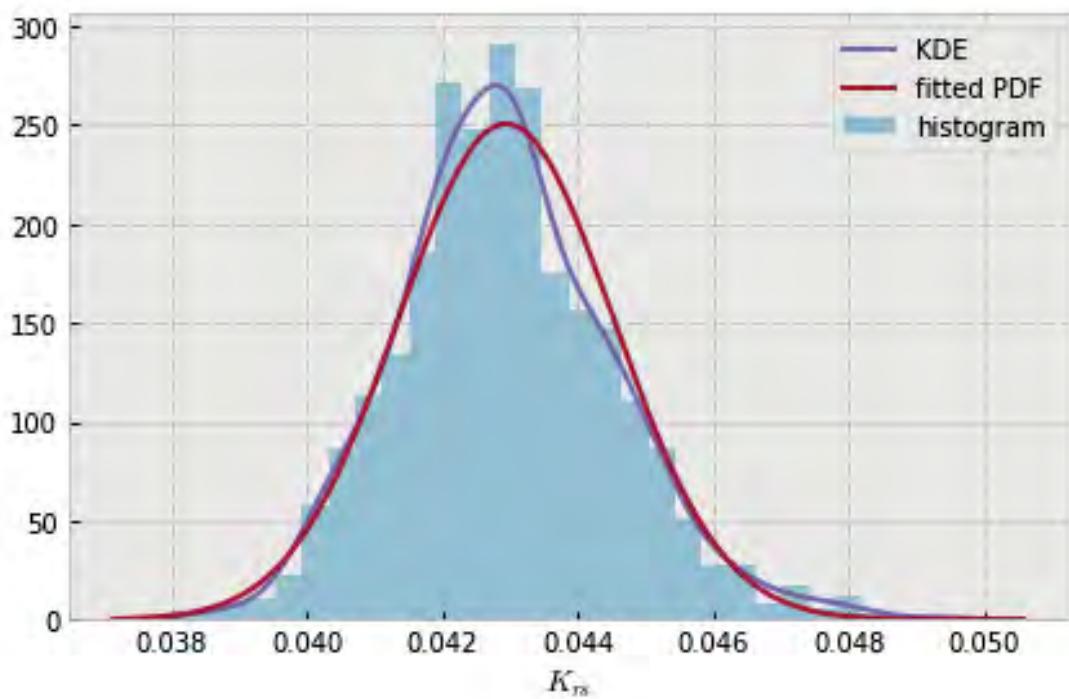


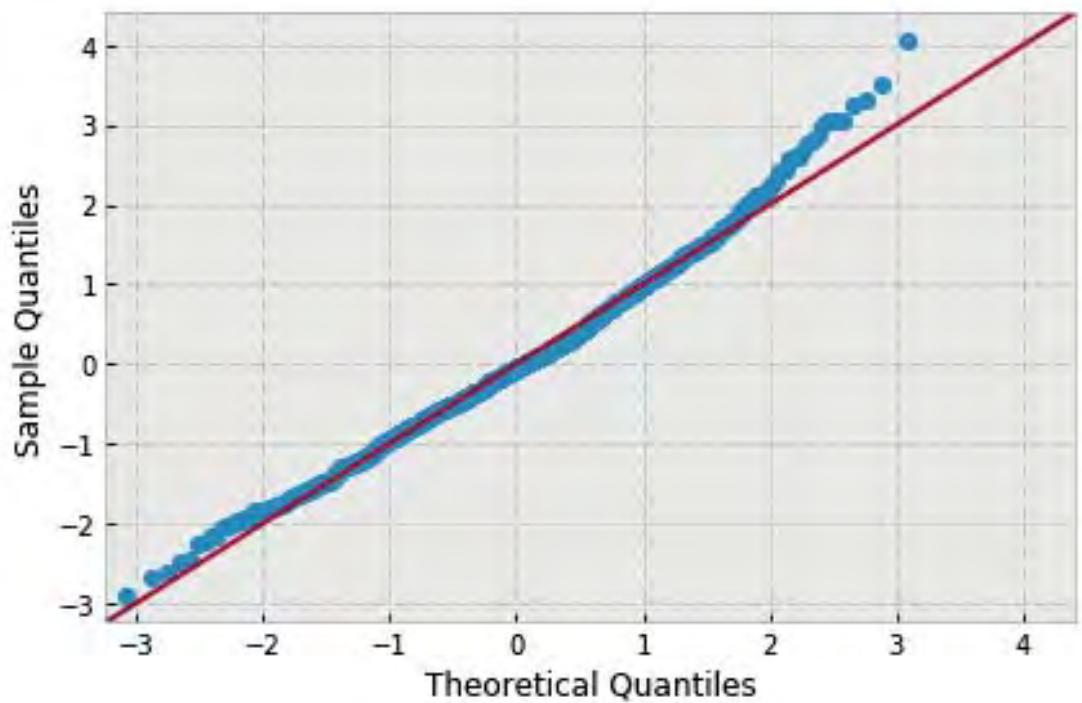


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=0.01997958996592, scale=0.000946427421668715)
DescribeResult(nobs=1000, minmax=(0.0172520821, 0.02326614743999999)
               mean=0.019979589965919999, variance=8.9662148597246426e-07,
               skewness=0.07049710334387325, kurtosis=0.06913438640563552)
```

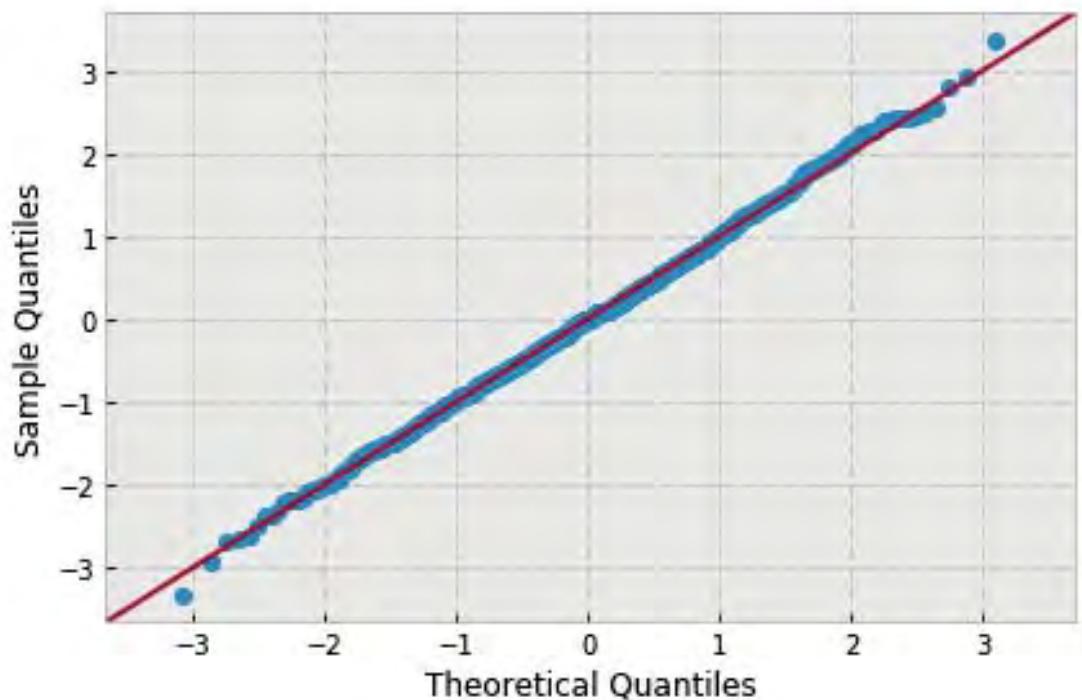
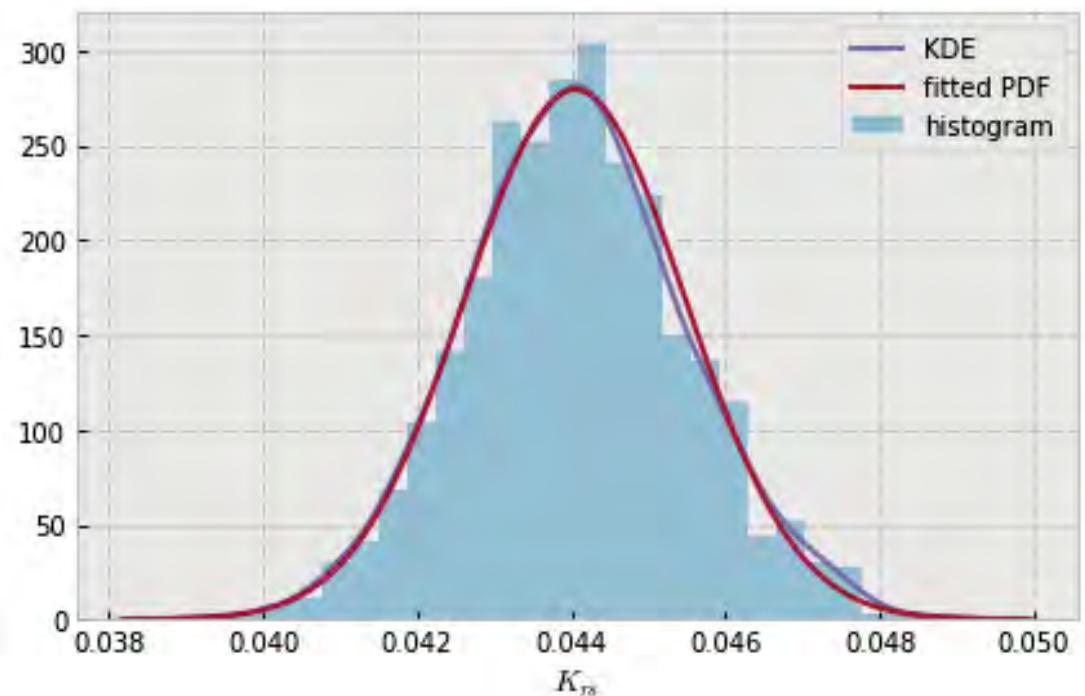


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=0.04293955460643, scale=0.0015912697029740479)
DescribeResult(nobs=1000, minmax=(0.038344034620000002, 0.049386237180000001)
               mean=0.042939554606430001, variance=2.534673941544659e-06,
               skewness=0.3892484102197144, kurtosis=0.4942360245607422)
```

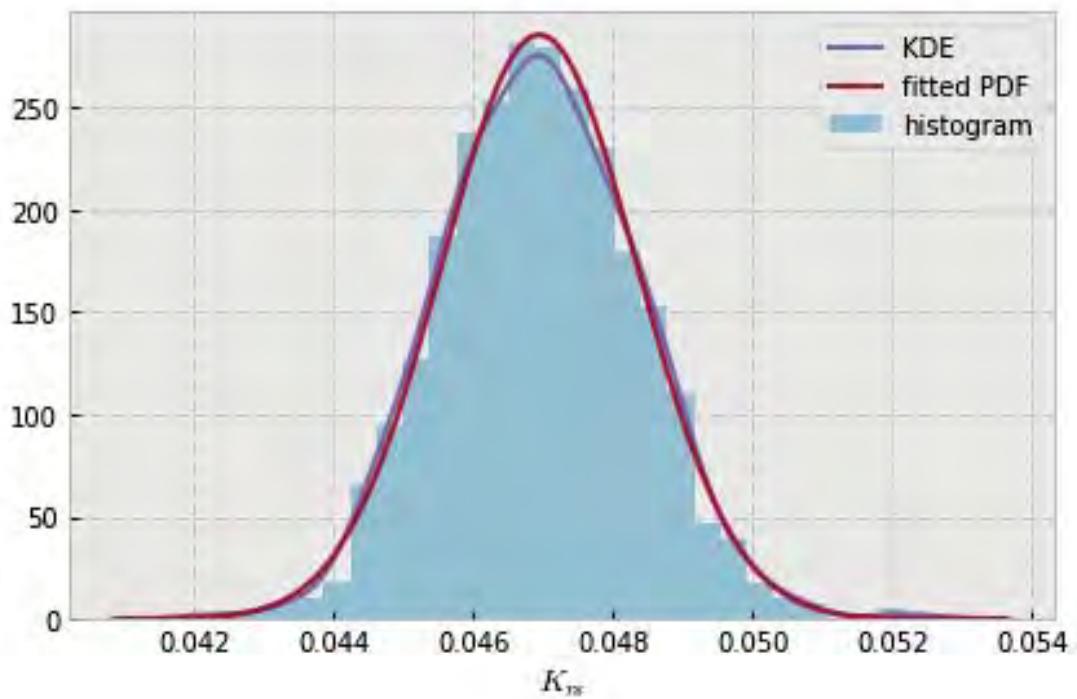


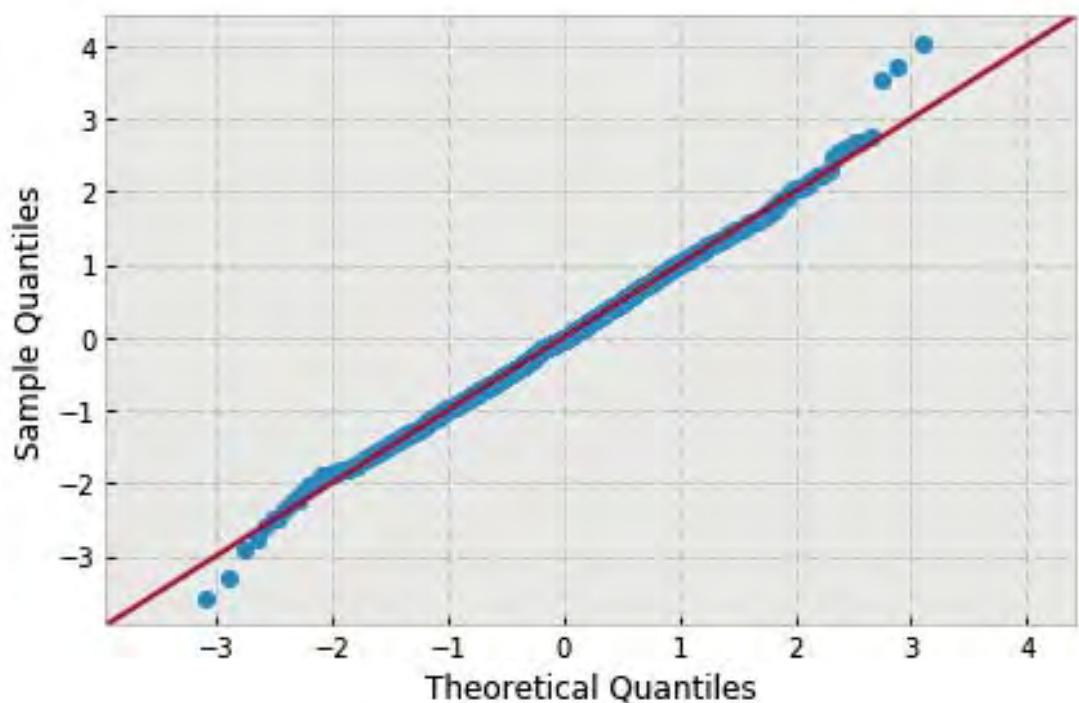


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=0.04404566881487, scale=0.0014273967538693069)
DescribeResult(nobs=1000, minmax=(0.03928724261000002, 0.04884701537000003)
               mean=0.04404566881487001, variance=2.0395009939505851e-06,
               skewness=0.08408540868614646, kurtosis=0.022295578389134985)
```

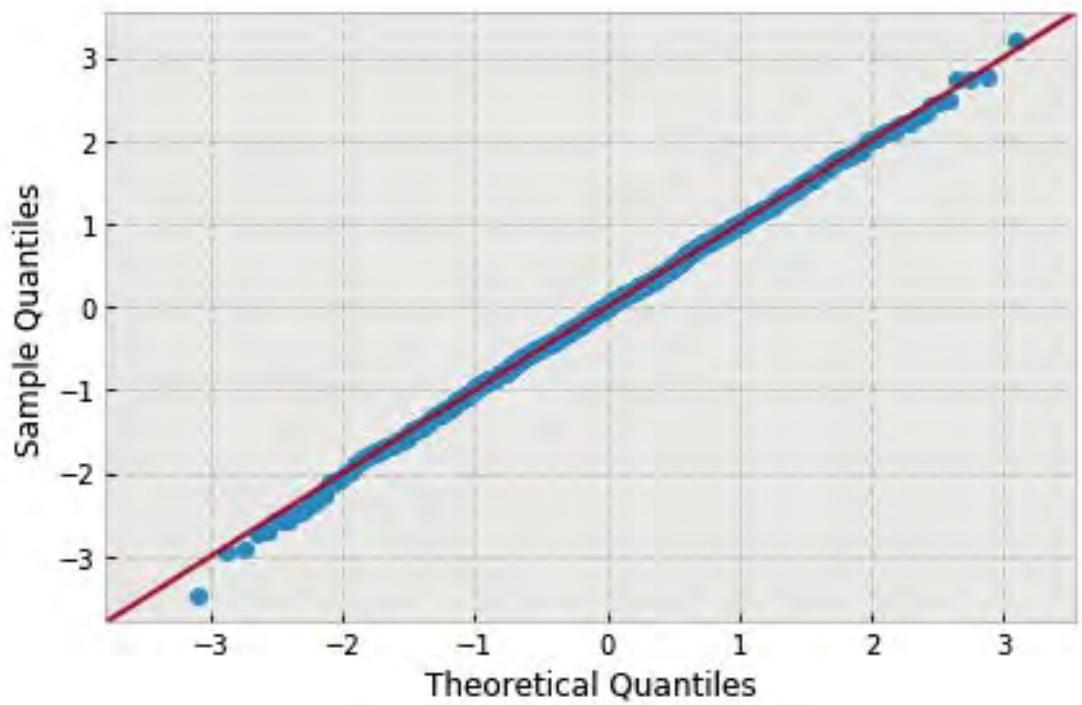
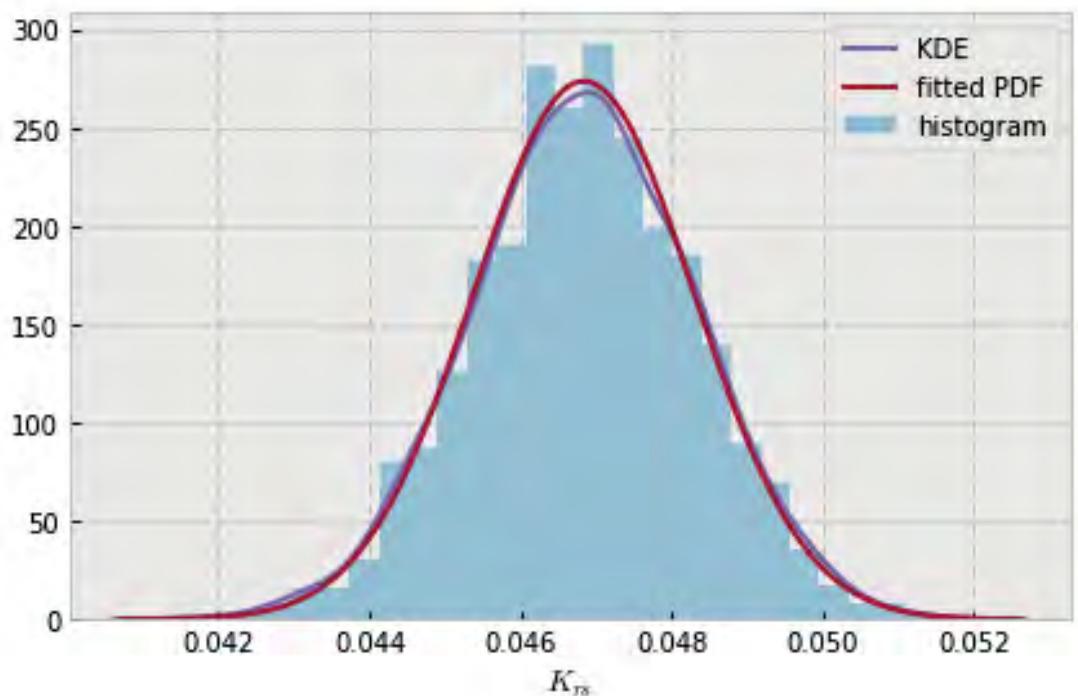


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=0.04694727939101, scale=0.001397682962281167)
DescribeResult(nobs=1000, minmax=(0.04196397505999999, 0.05257290301999999)
               mean=0.046947279391009997, variance=1.9554731361871046e-06,
               skewness=0.1096794320445422, kurtosis=0.28790853088534973)
```

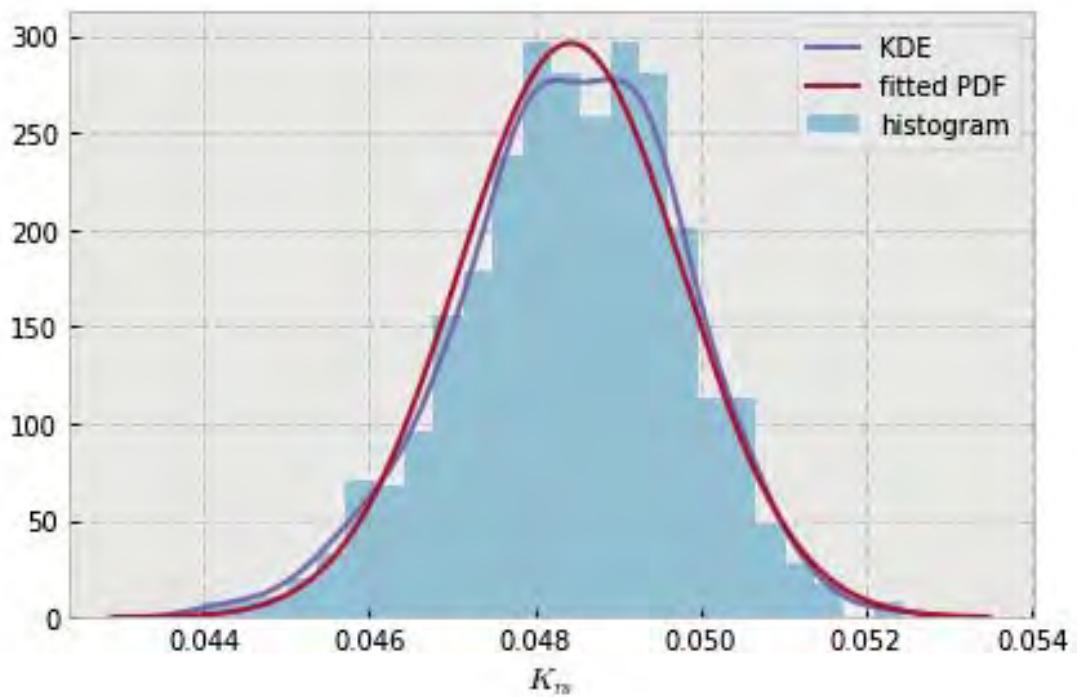


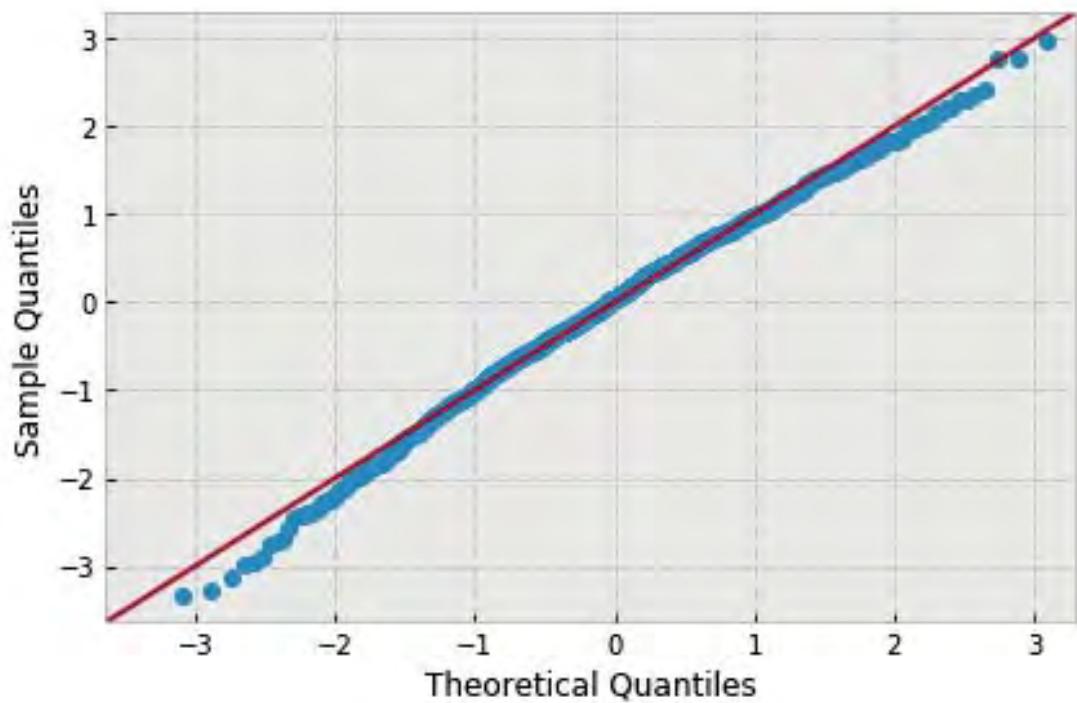


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=0.04683746842252005, scale=0.0014580700032537828)
DescribeResult(nobs=1000, minmax=(0.04179576060000002, 0.05150479758999998)
               mean=0.04683746842252005, variance=2.1280962306191054e-06,
               skewness=-0.05866846334283259, kurtosis=-0.0037584646743673567)
```

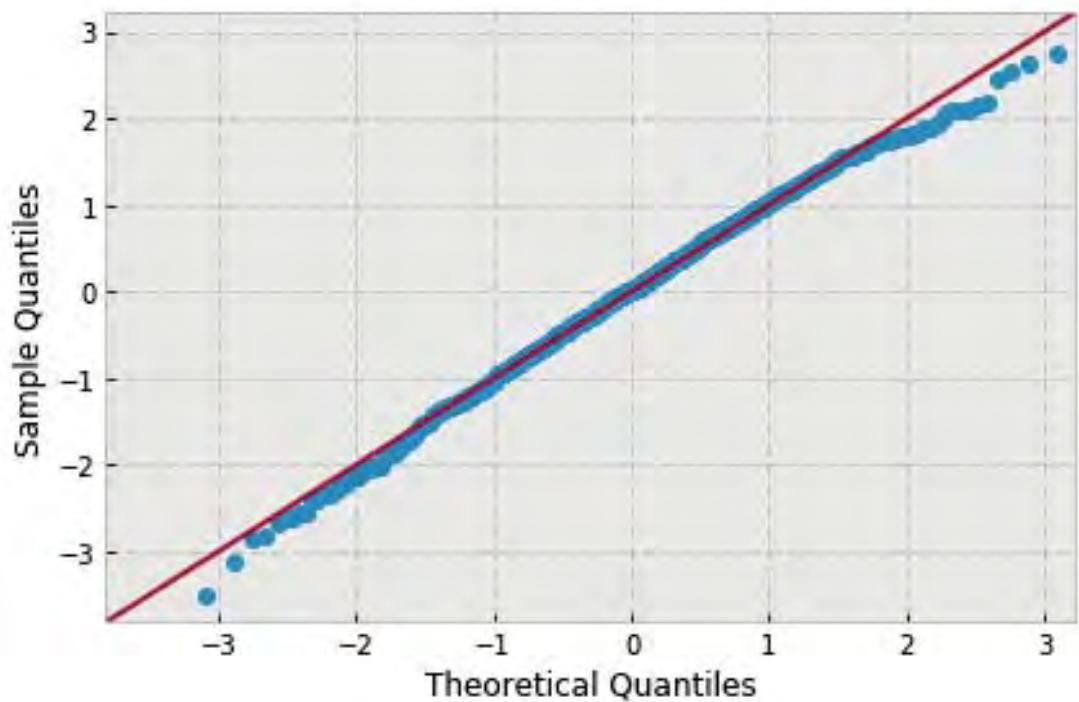
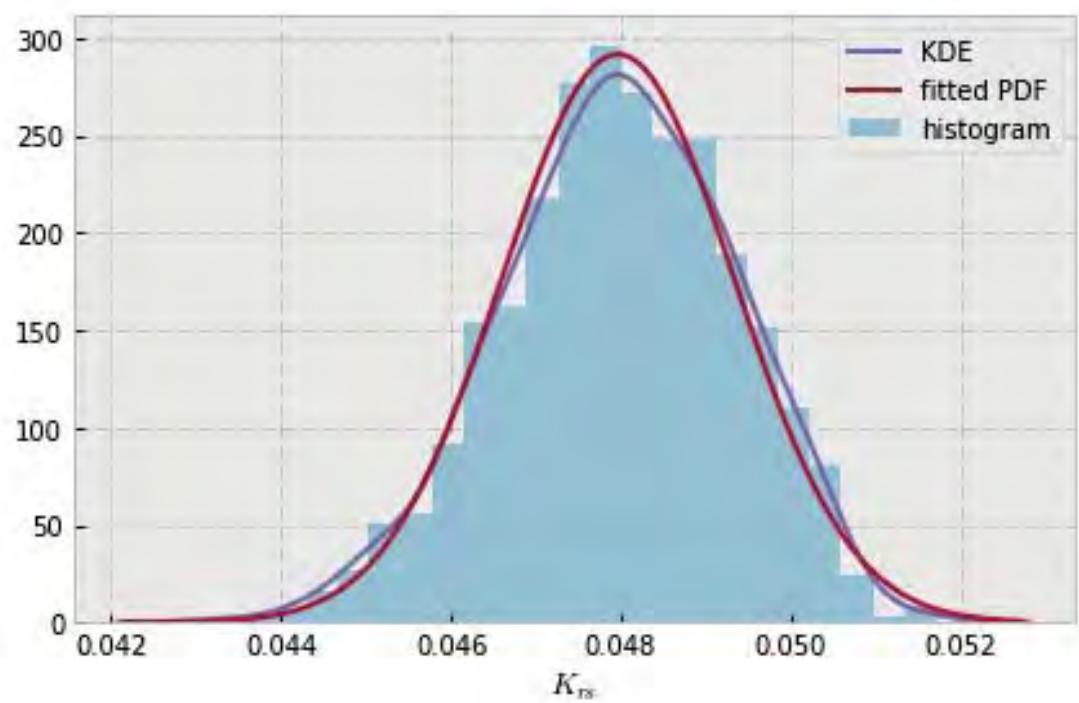


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=0.048431972777789996, scale=0.0013467521645224878)
DescribeResult(nobs=1000, minmax=(0.04395931924000003, 0.05243297587000002)
               mean=0.048431972777789996, variance=1.8155569495956014e-06,
               skewness=-0.28033595169392317, kurtosis=0.09892548243793442)
```

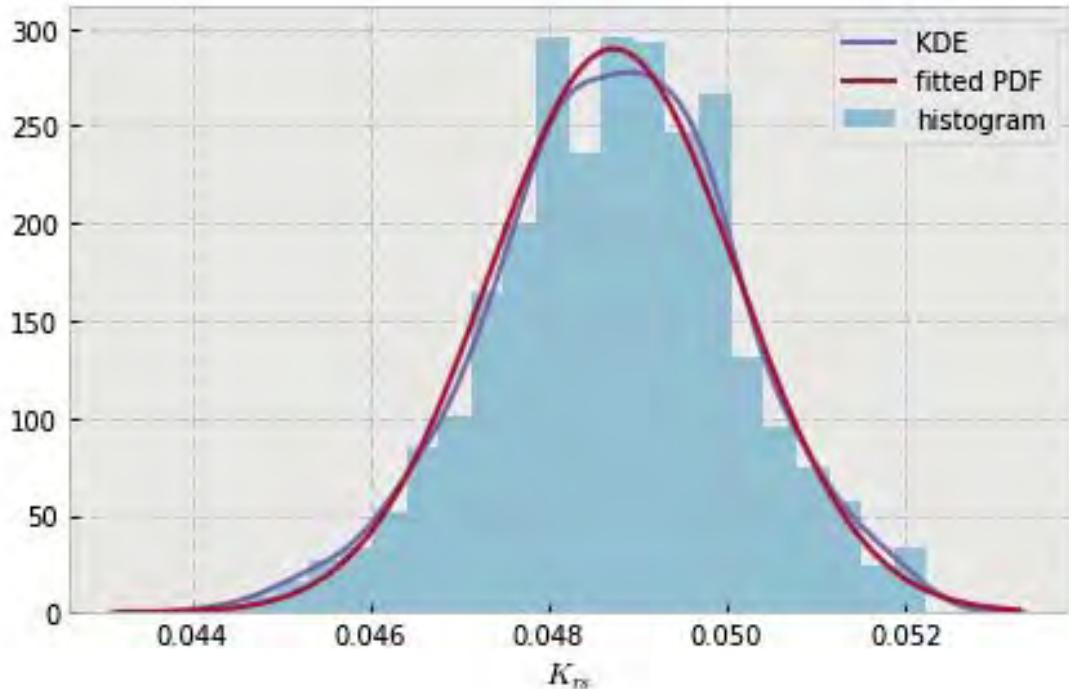


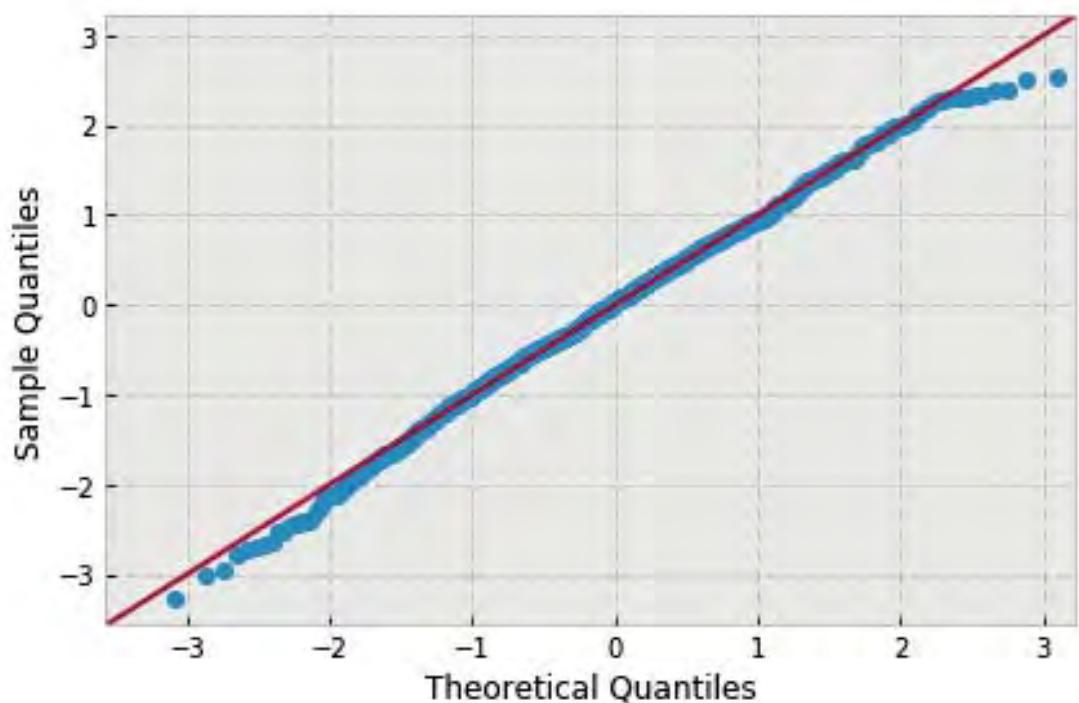


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=0.04796805953233, scale=0.0013674243932333579)
DescribeResult(nobs=1000, minmax=(0.04317930446999999, 0.05171141498000002)
               mean=0.04796805953233, variance=1.8717211924020188e-06,
               skewness=-0.2115815147416378, kurtosis=-0.14638910965036134)
```

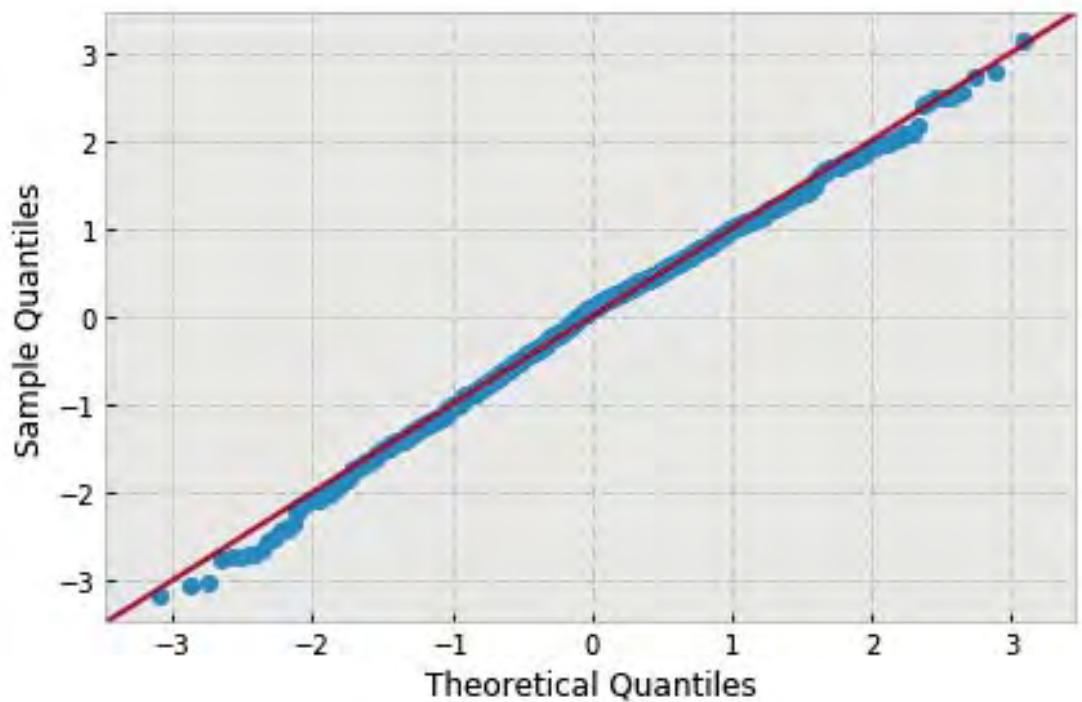
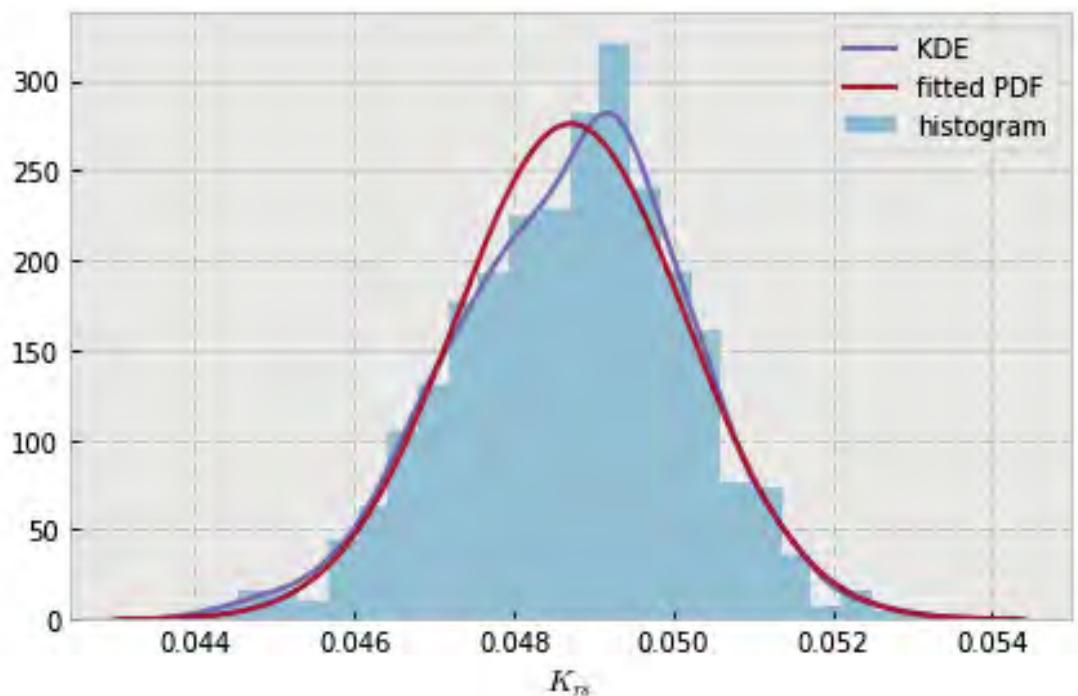


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=0.048727497763000005, scale=0.0013777963154207496)
DescribeResult(nobs=1000, minmax=(0.04421378547999997, 0.05223537066000002)
               mean=0.04872749776300005, variance=1.9002229096966906e-06,
               skewness=-0.16286347118775313, kurtosis=0.008073503783787128)
```

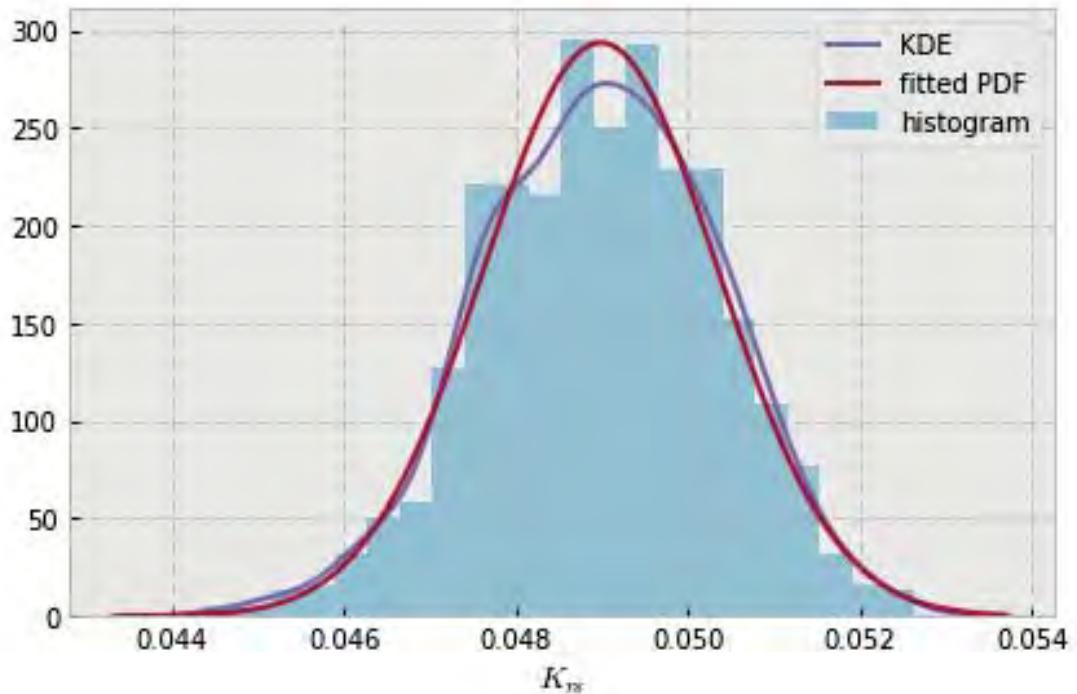


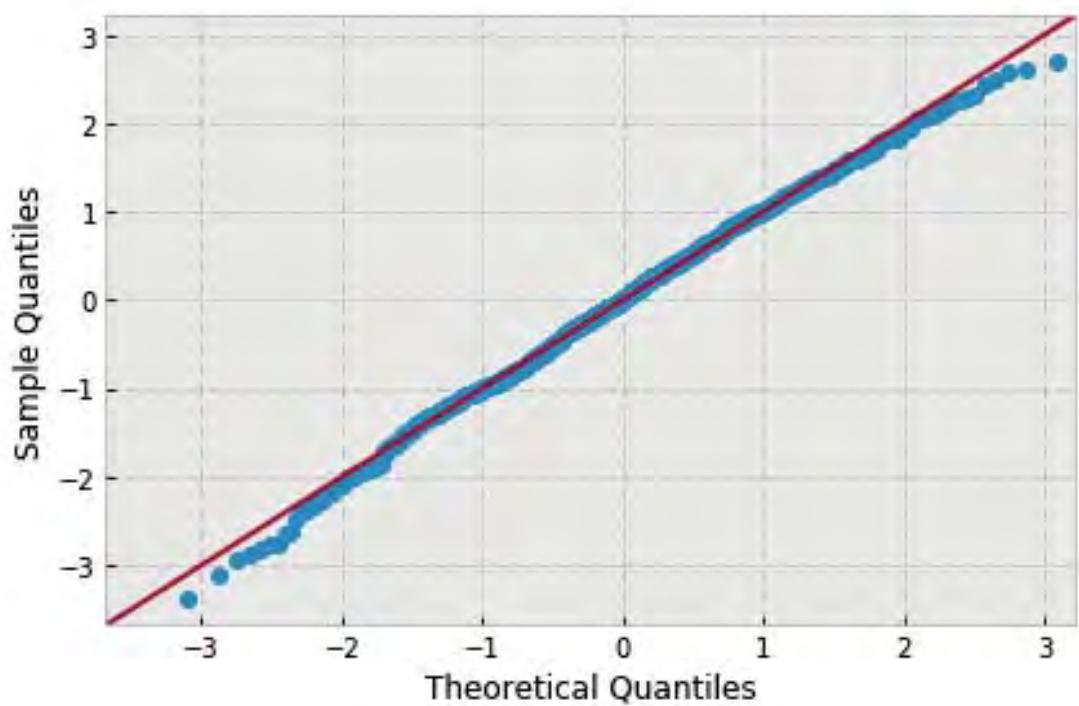


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=0.04871729049382, scale=0.0014450110013147412)
DescribeResult(nobs=1000, minmax=(0.04414728847999998, 0.05325141736999998)
               mean=0.04871729049382002, variance=2.0901469408614928e-06,
               skewness=-0.1922873897290026, kurtosis=0.0035692726234559657)
```

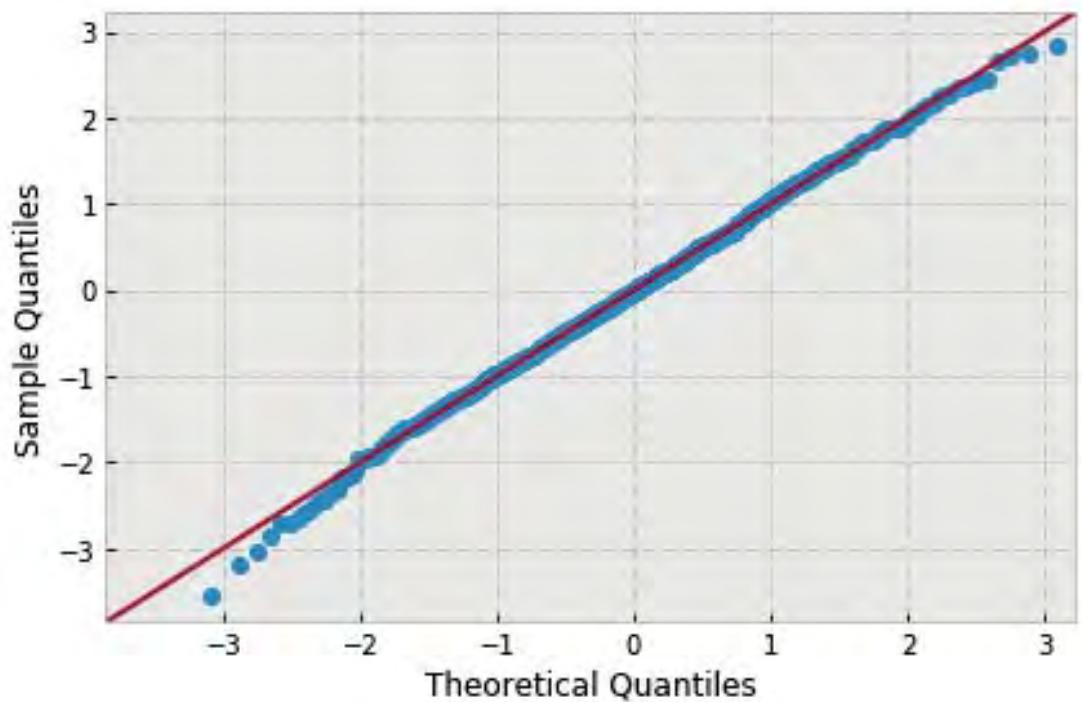
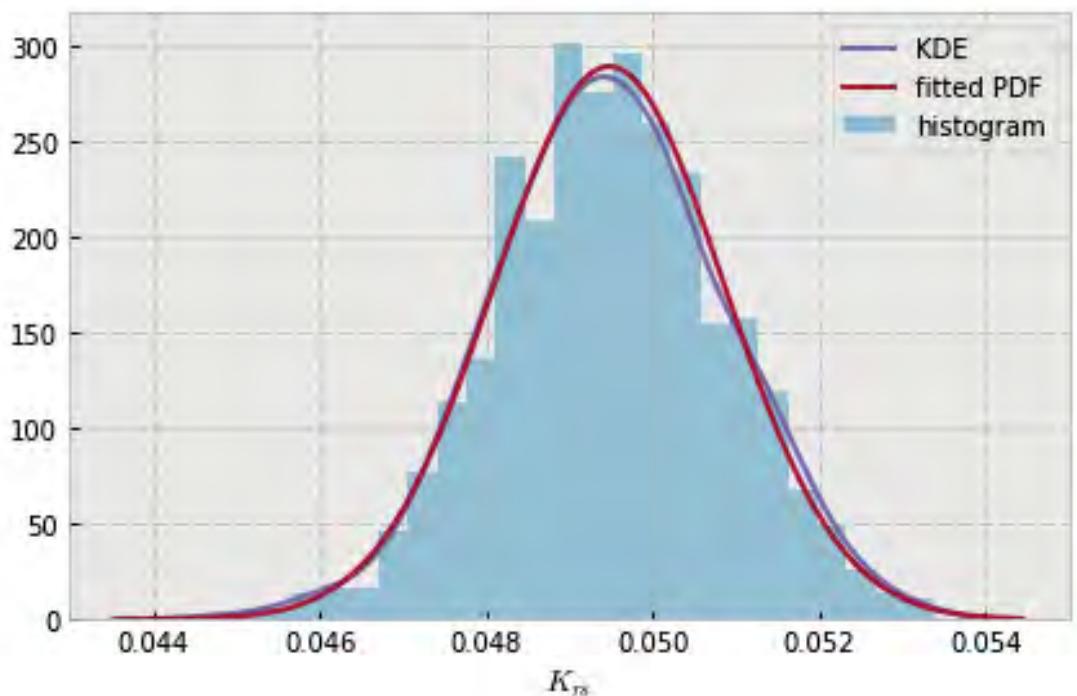


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=0.048983420635879994, scale=0.001358729542614874)
DescribeResult(nobs=1000, minmax=(0.044397978540000001, 0.05264912130999998)
               mean=0.04898342063587994, variance=1.8479939639383629e-06,
               skewness=-0.15256671139761854, kurtosis=-0.10838203683702563)
```

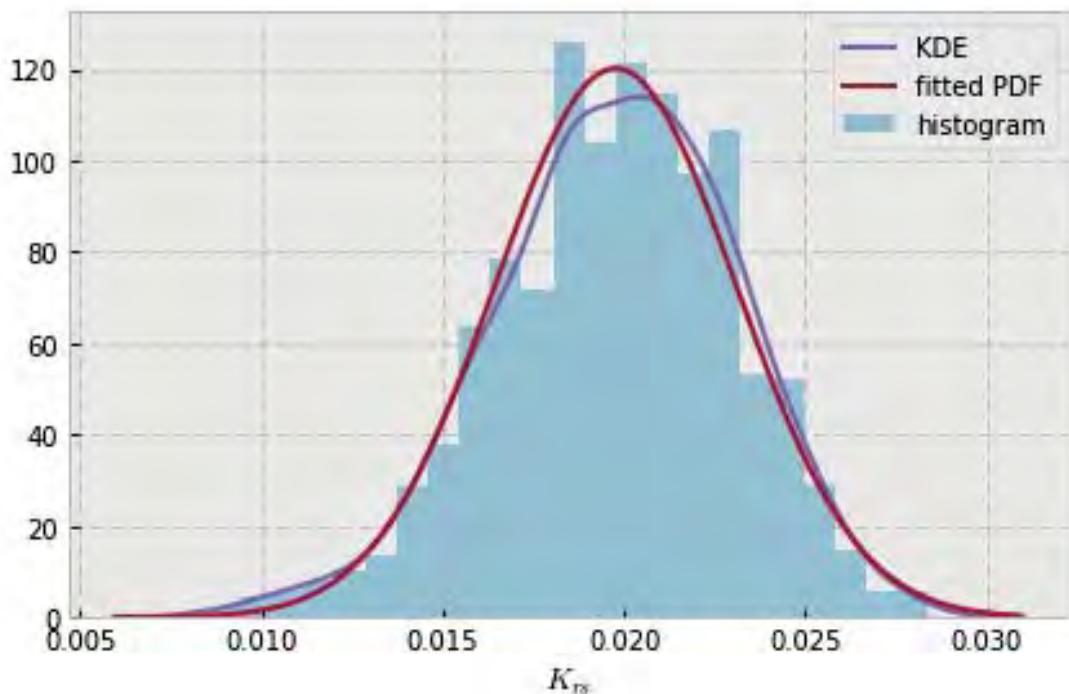


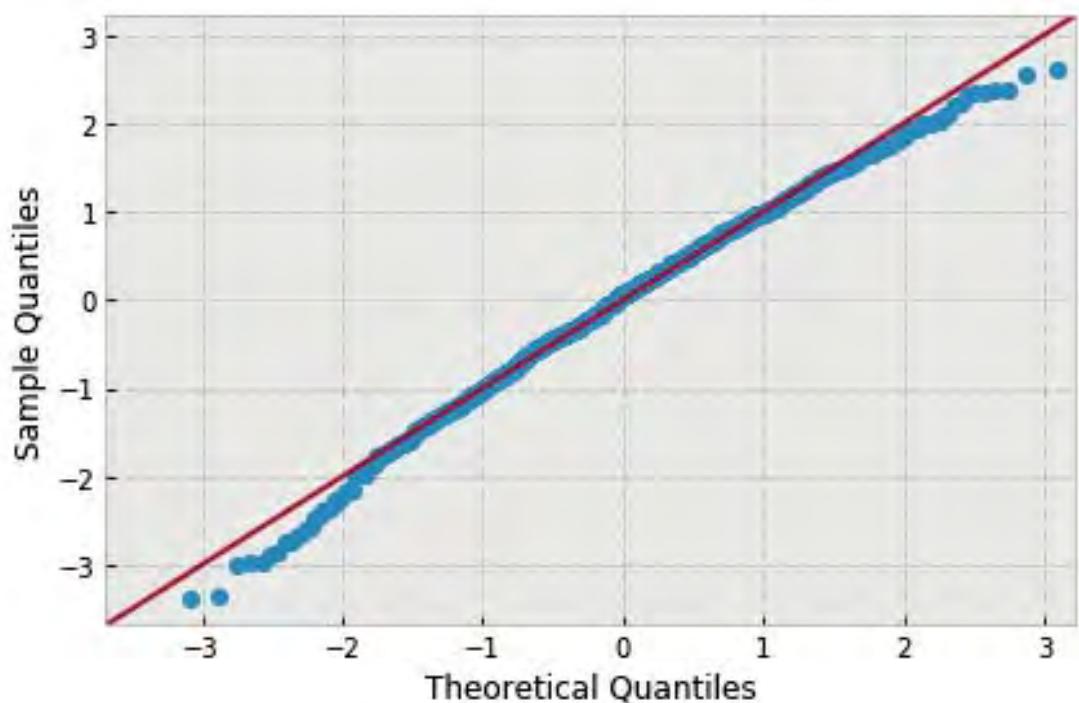


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=0.04948188922531, scale=0.0013801183984140457)
DescribeResult(nobs=1000, minmax=(0.04460332566999999, 0.05338308166999999)
               mean=0.049481889225310001, variance=1.9066334270680188e-06,
               skewness=-0.05172113845110719, kurtosis=0.016644601214522048)
```

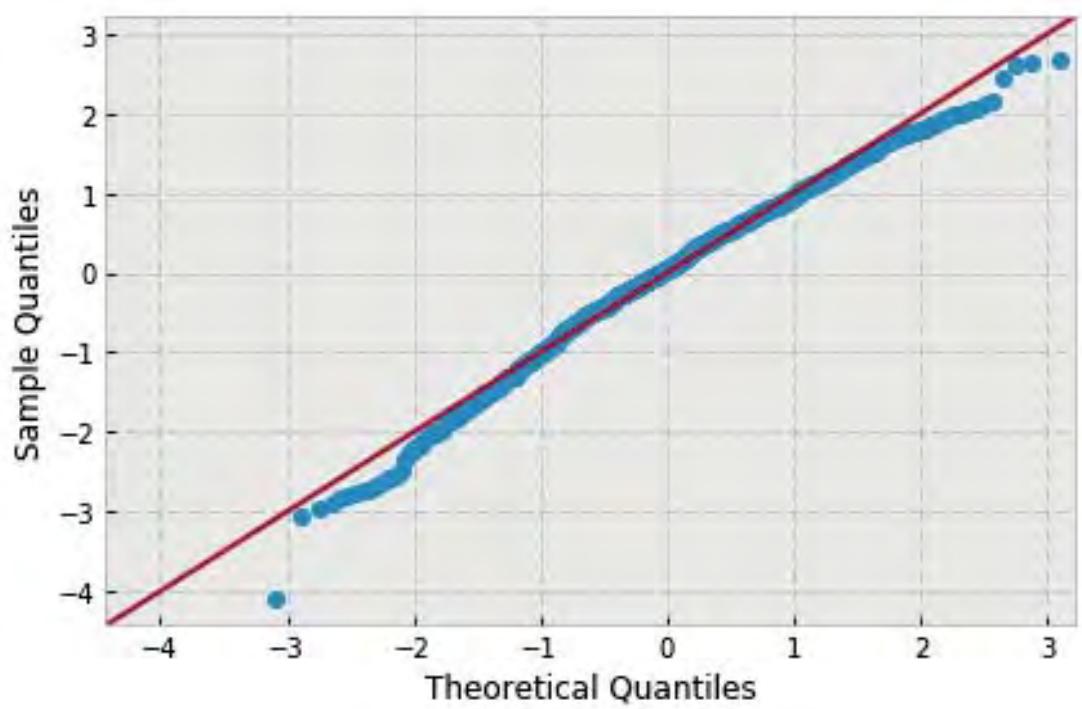
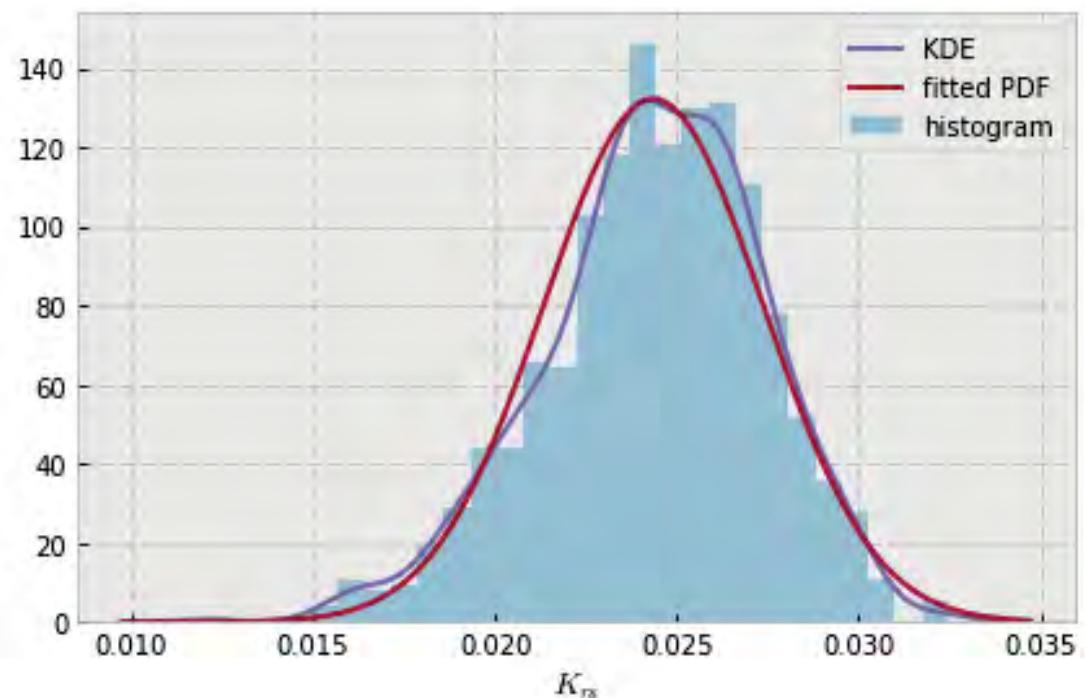


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=0.019766428767599, scale=0.003320106243935119)
DescribeResult(nobs=1000, minmax=(0.008537402500999997, 0.02840450645999999)
               mean=0.019766428767599001, variance=1.1034139610627592e-05,
               skewness=-0.28686696905452375, kurtosis=0.05399637769121712)
```

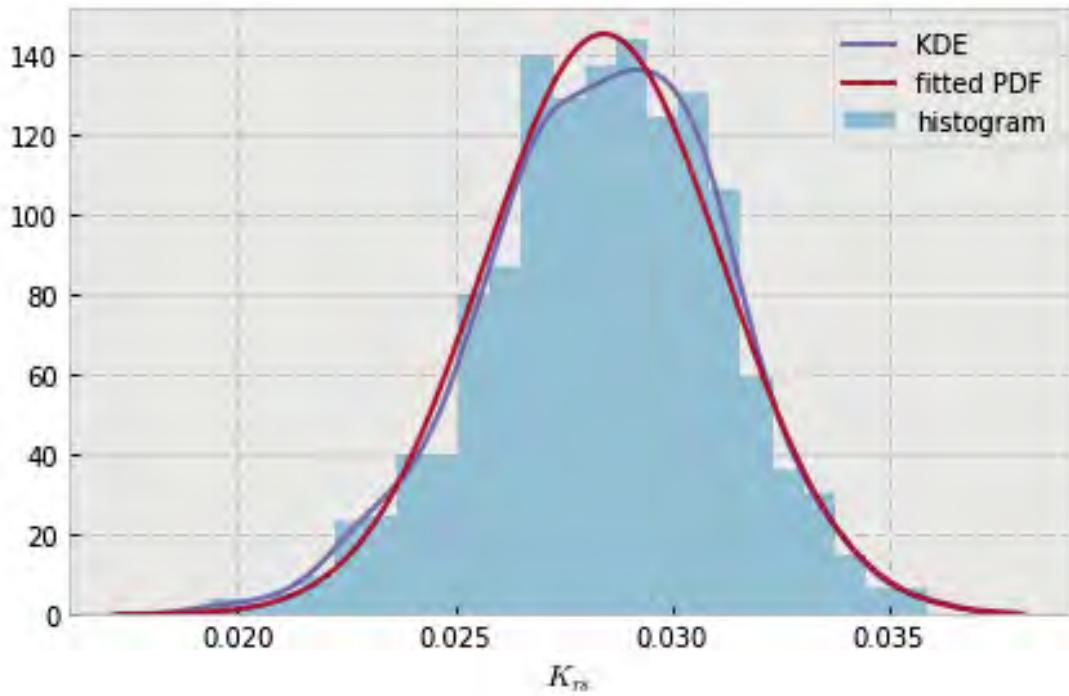


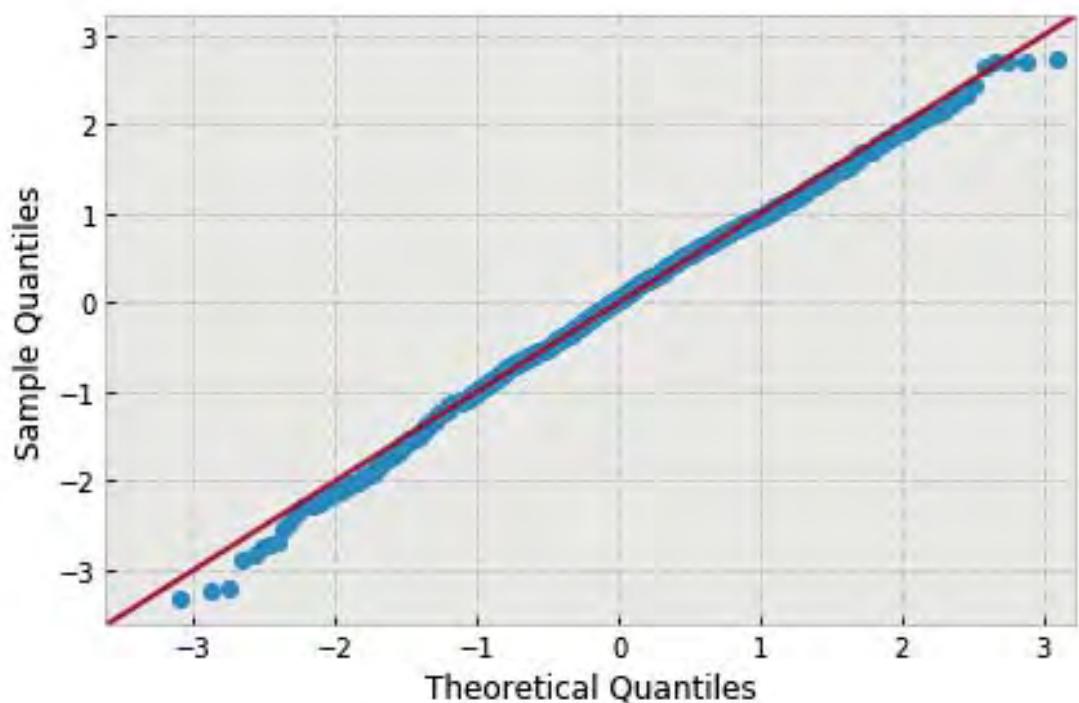


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(), loc=0.02435410454966, scale=0.0030175278078874907)
DescribeResult(nobs=1000, minmax=(0.01201551277999999, 0.032451598550000002)
               mean=0.024354104549659999, variance=9.1145886600343183e-06,
               skewness=-0.39819214516828794, kurtosis=0.21518999705948882)
```

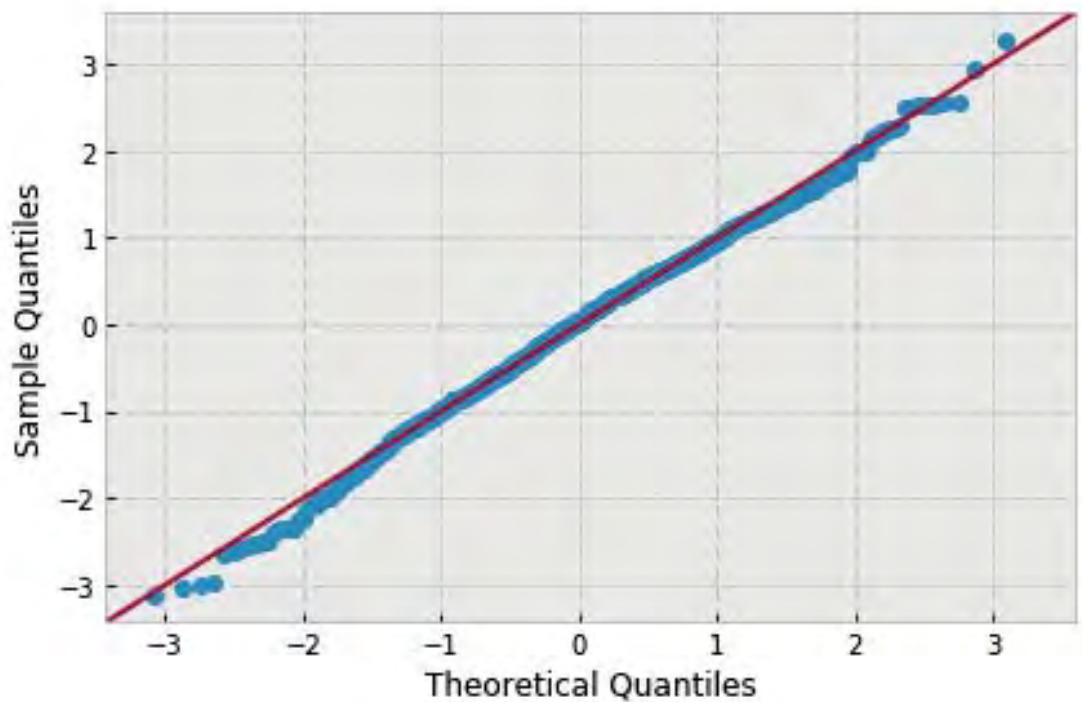
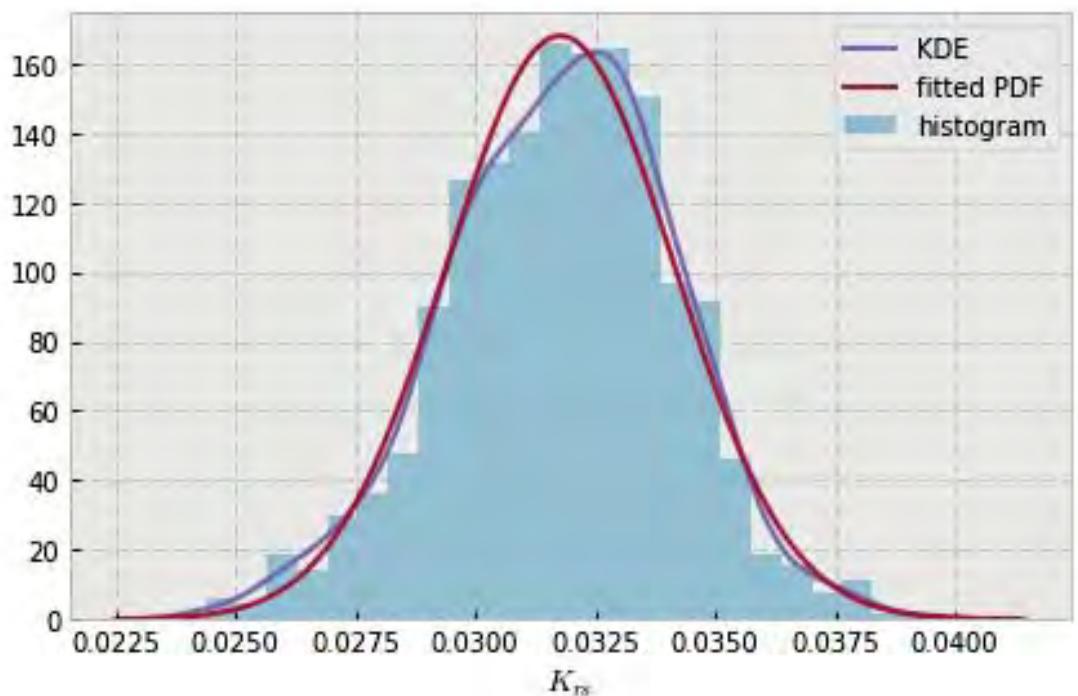


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=0.02843267487277, scale=0.0027462651129600037)
DescribeResult(nobs=1000, minmax=(0.01933098306999998, 0.03591221757000002)
               mean=0.02843267487277, variance=7.5495215922534762e-06,
               skewness=-0.21580061745843662, kurtosis=0.04335260148416653)
```

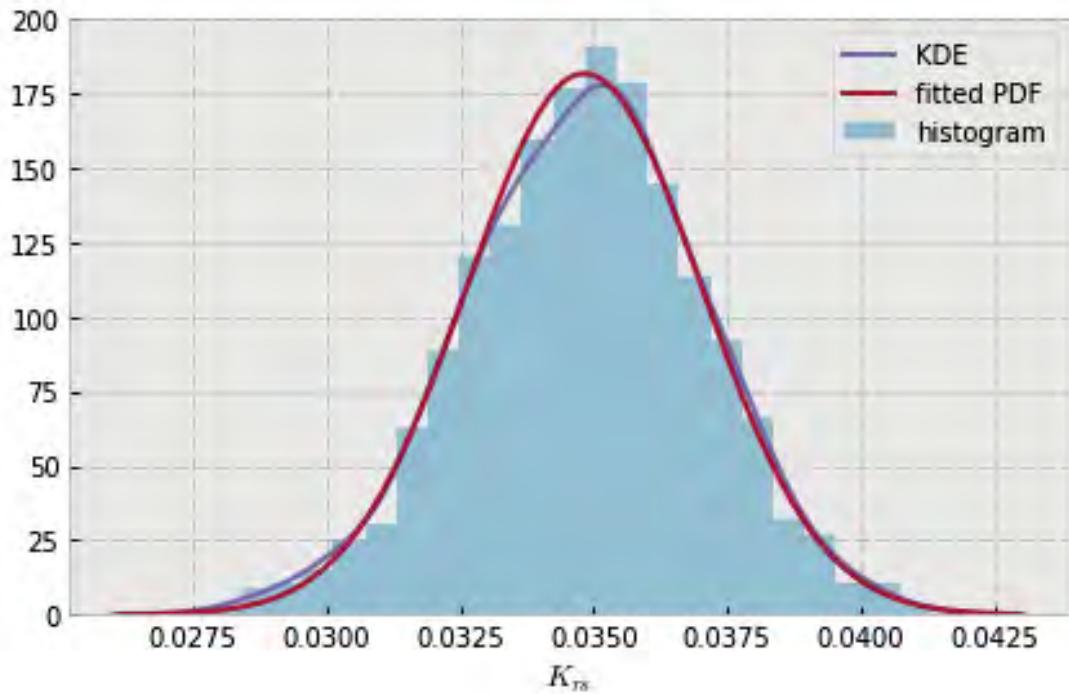


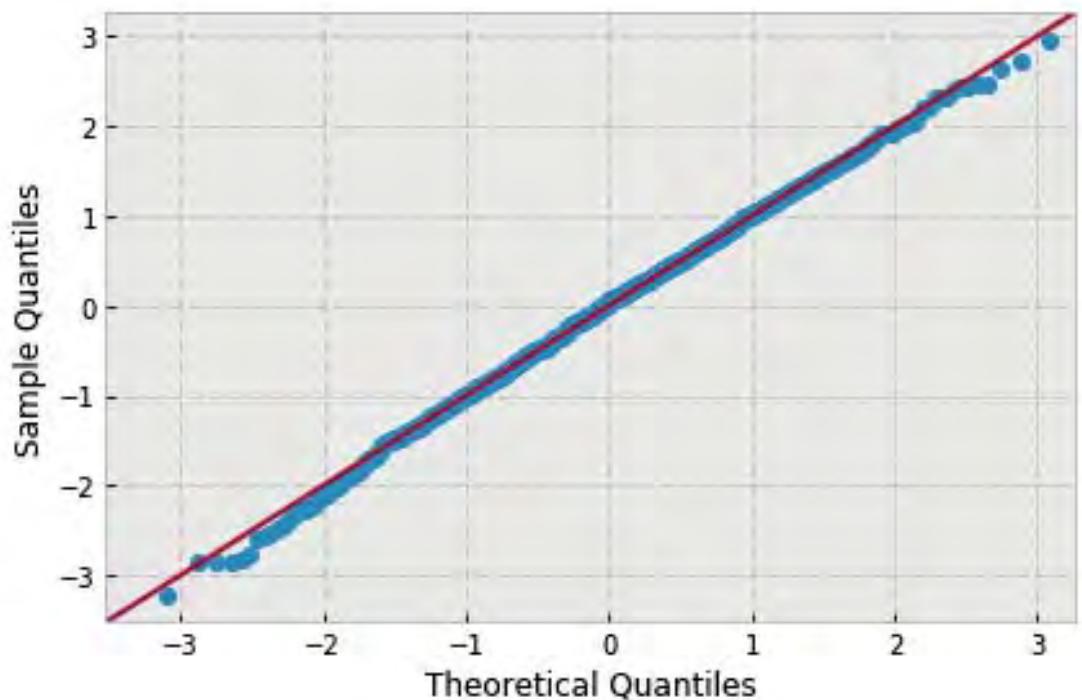


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=0.03177780327101, scale=0.00237564849638438)
DescribeResult(nobs=1000, minmax=(0.02438548057, 0.03955290308)
               mean=0.031777803271009999, variance=5.6493551335068716e-06,
               skewness=-0.17900957310421065, kurtosis=0.08945969276966492)
```

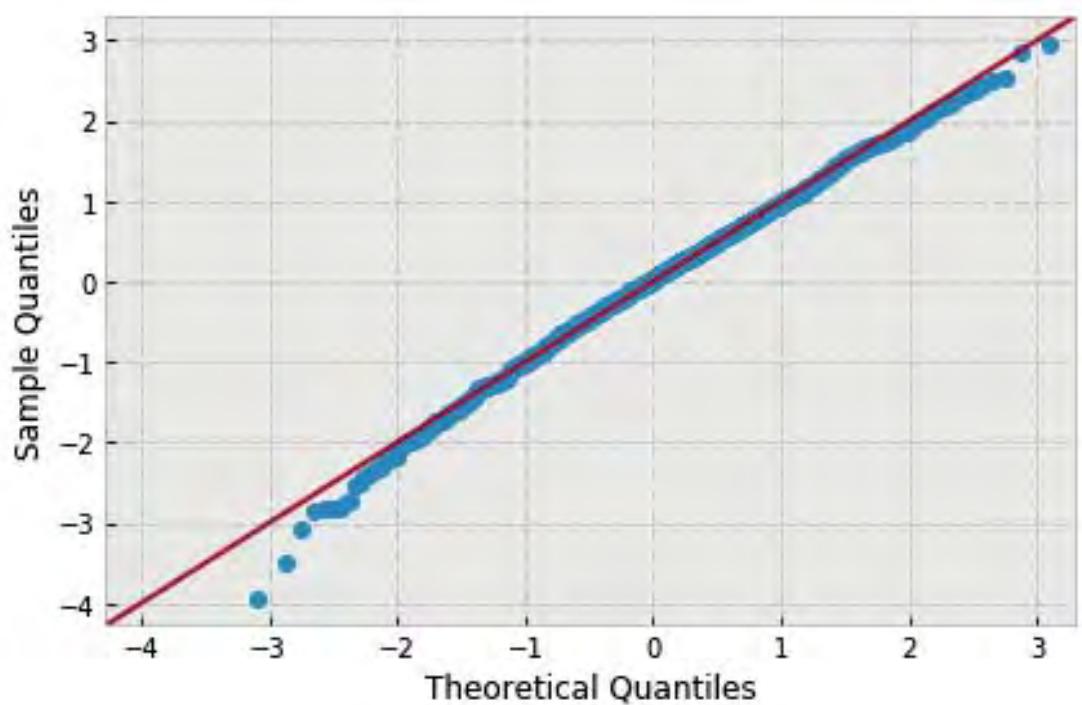
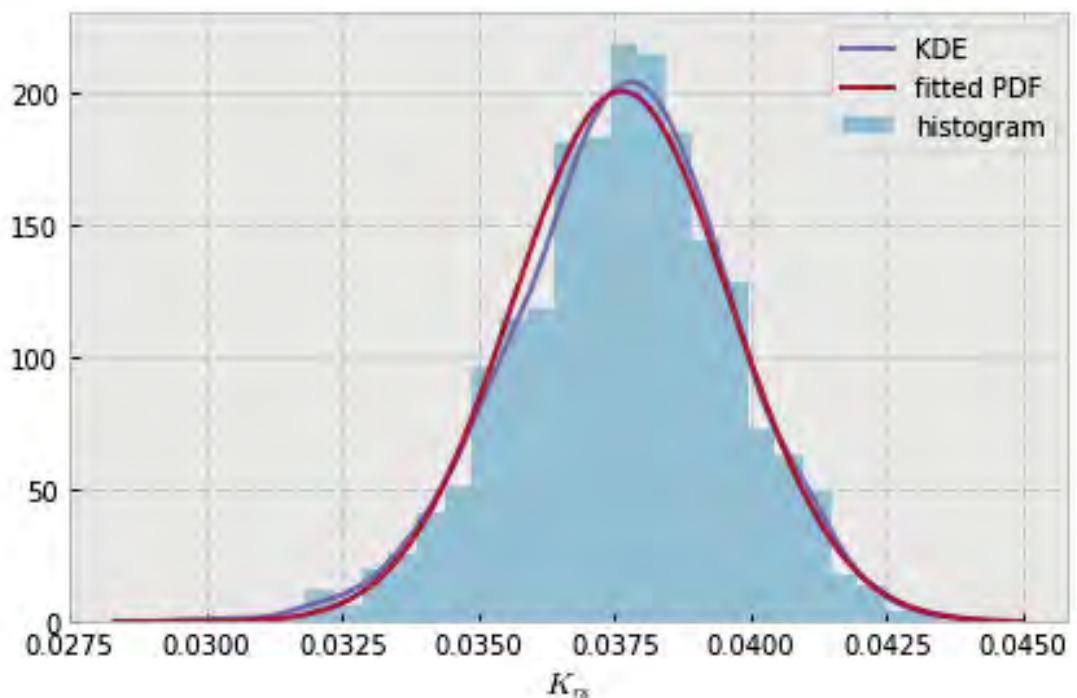


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=0.0348288684718, scale=0.002194110474894911)
DescribeResult(nobs=1000, minmax=(0.02777612715, 0.04129673234999997)
               mean=0.0348288684718003, variance=4.8189397157593302e-06,
               skewness=-0.11213366485183619, kurtosis=-0.04397066188687049)
```

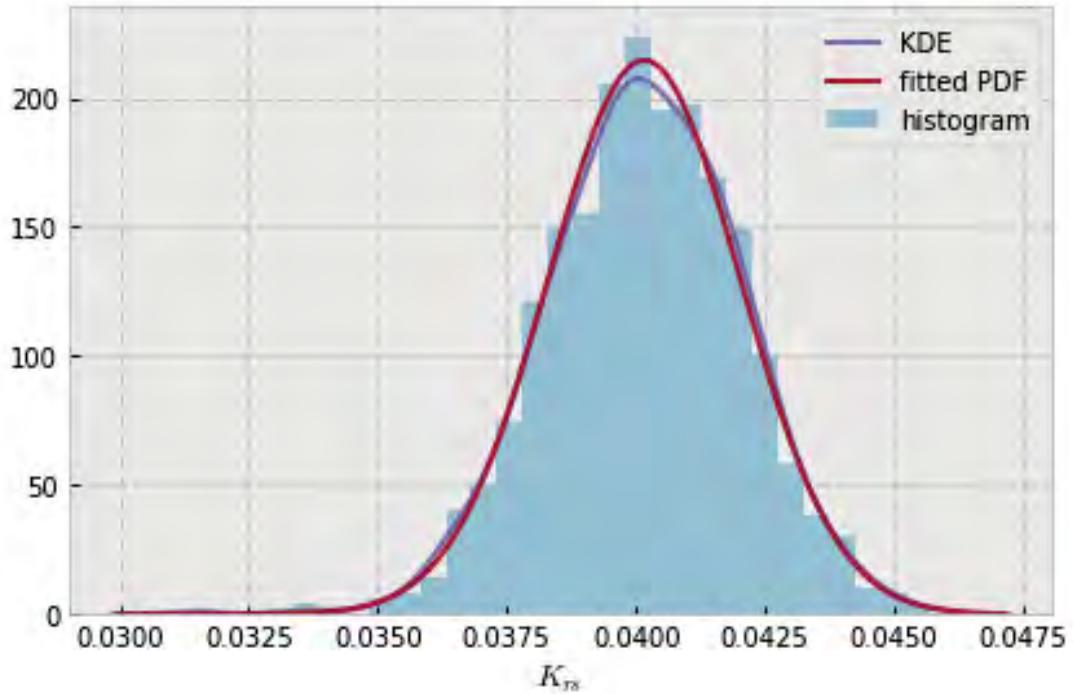


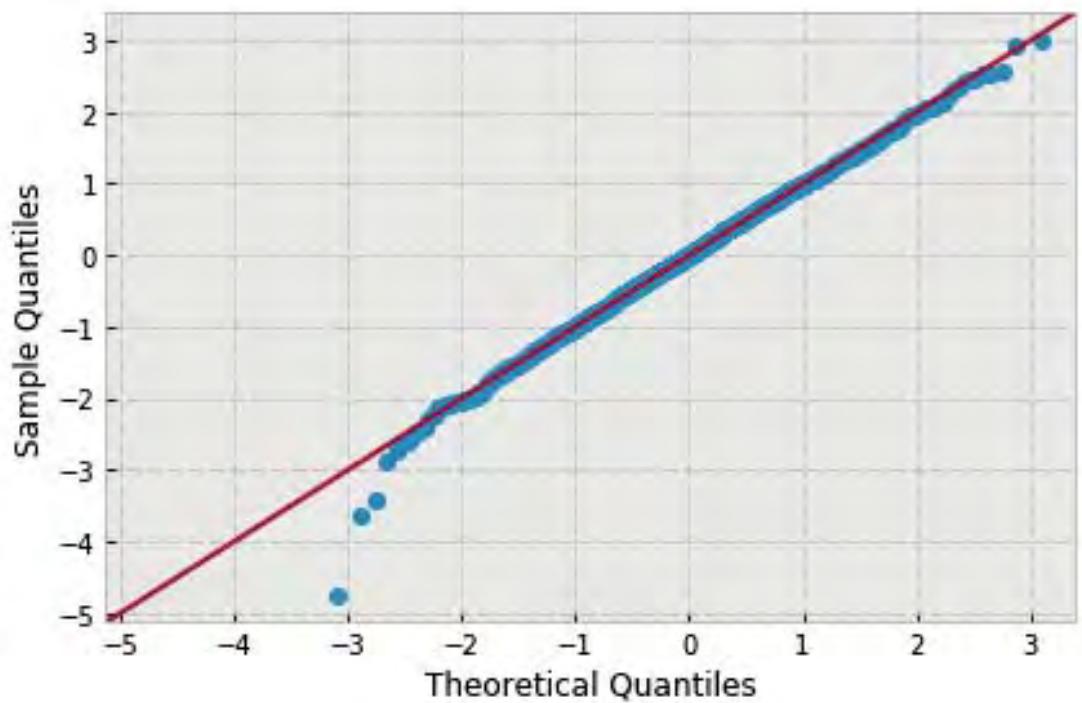


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(), loc=0.037628966105719995, scale=0.0019888909569438645)
DescribeResult(nobs=1000, minmax=(0.029810827580000001, 0.043492050550000001)
               mean=0.037628966105719995, variance=3.9596468854985787e-06,
               skewness=-0.22351976637524332, kurtosis=0.2323554950948279)
```

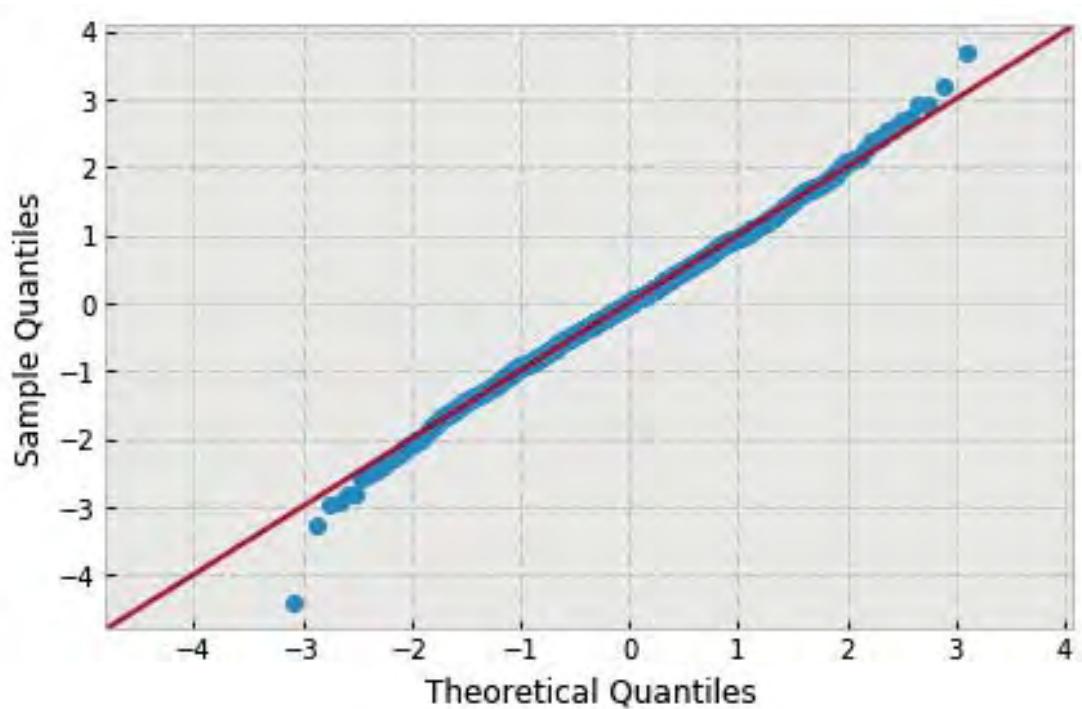
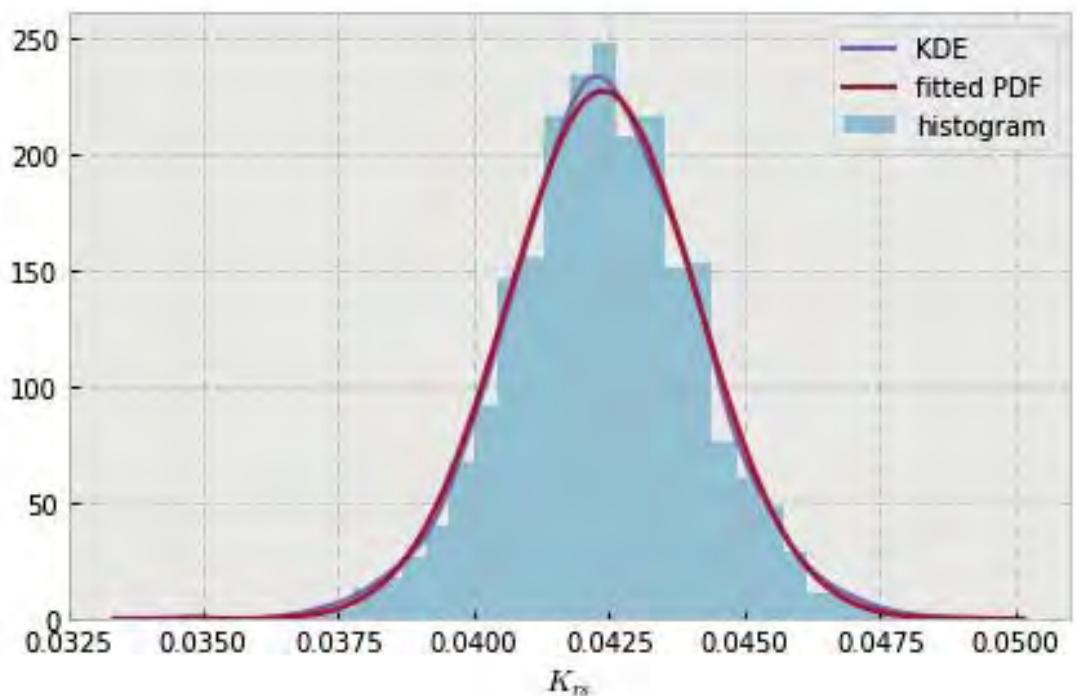


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=0.04016100659769, scale=0.0018590535309880806)
DescribeResult(nobs=1000, minmax=(0.03135672738999998, 0.04572876180000003)
               mean=0.04016100659769001, variance=3.4595395706499003e-06,
               skewness=-0.1830101098118199, kurtosis=0.39228245507128445)
```

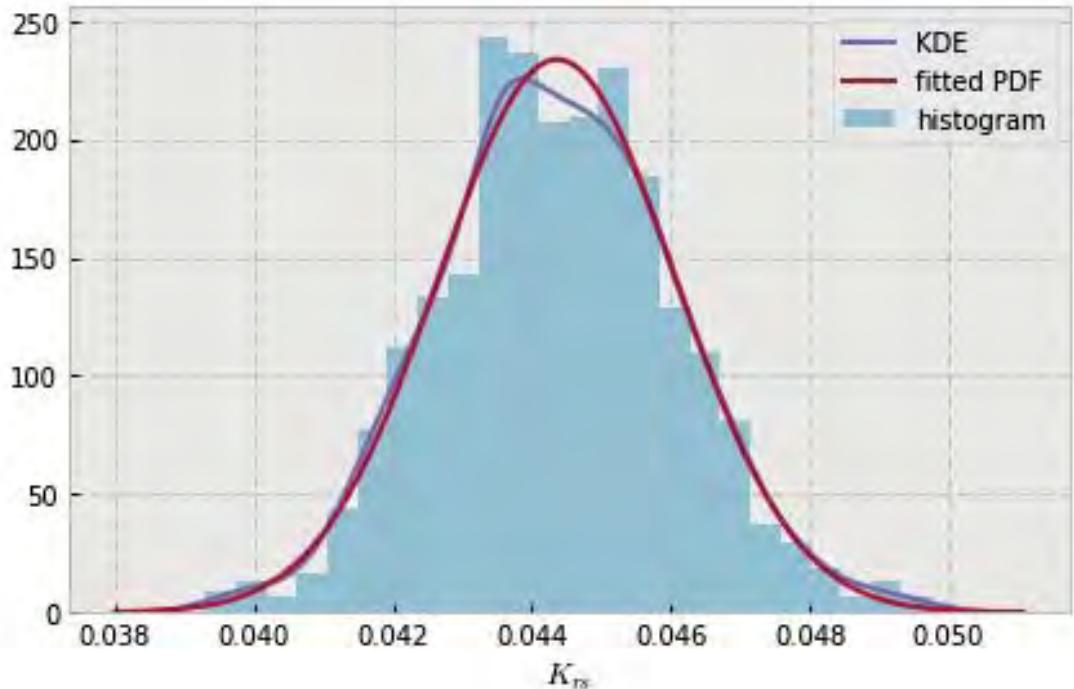


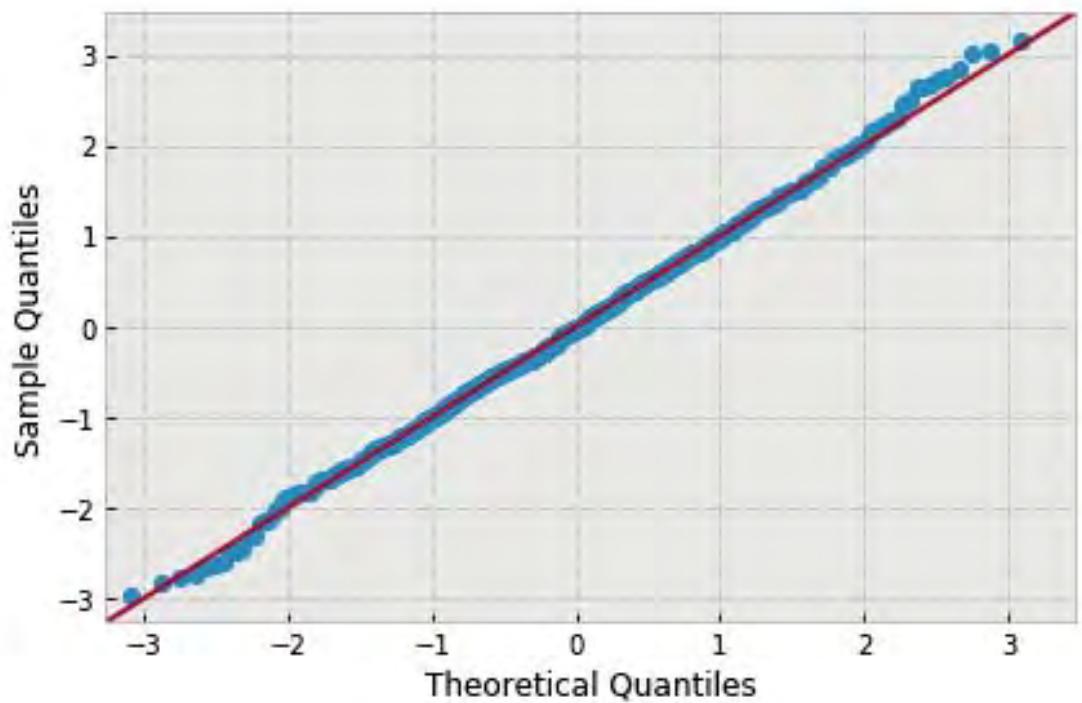


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=0.042383209055770005, scale=0.0017529651665427685)
DescribeResult(nobs=1000, minmax=(0.03467376434000003, 0.04882592613000002)
               mean=0.042383209055770005, variance=3.075962837950266e-06,
               skewness=-0.04164185342259237, kurtosis=0.5520857819926661)
```

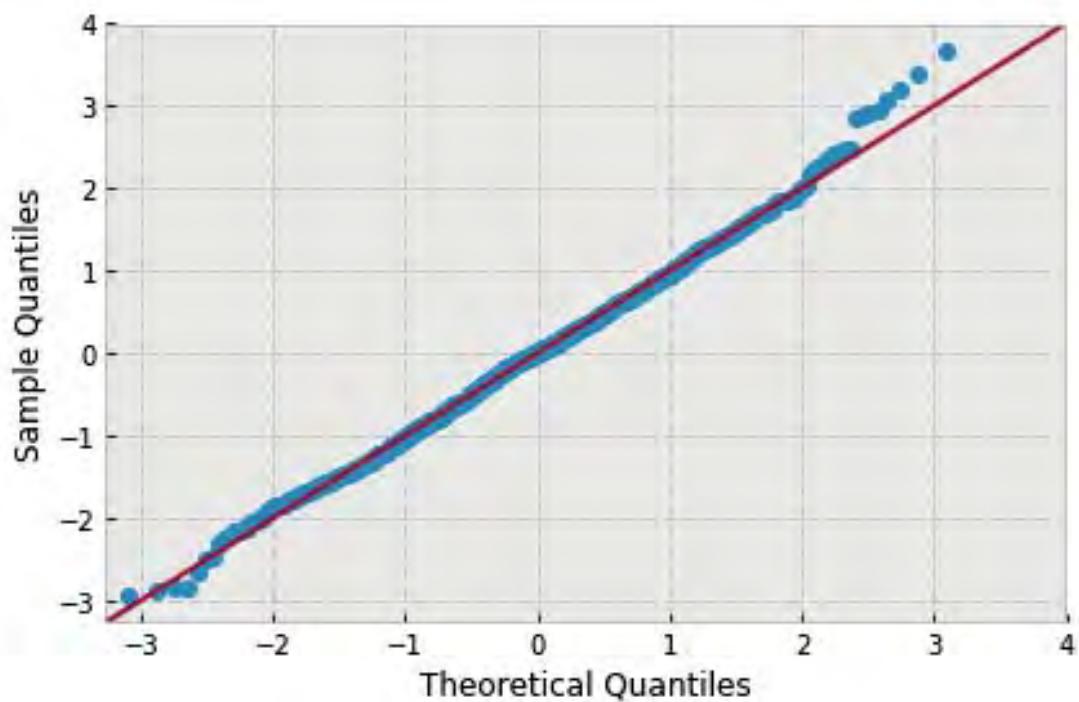
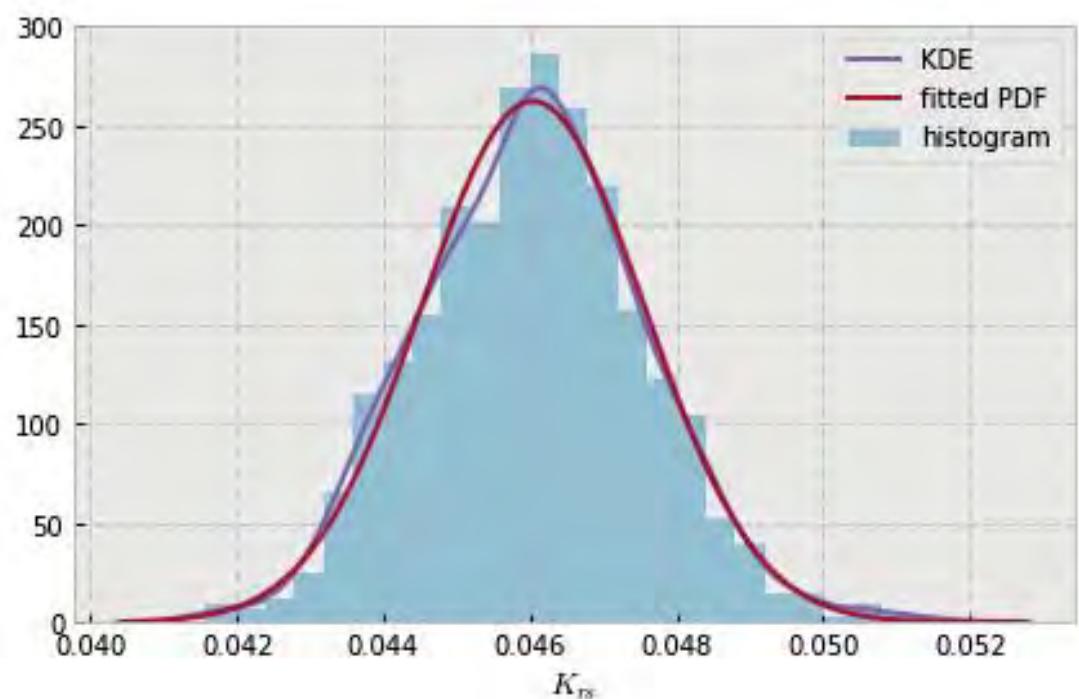


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=0.044360438648279994, scale=0.001707165603976573)
DescribeResult(nobs=1000, minmax=(0.03930548605999999, 0.04974643695)
    mean=0.04436043864827994, variance=2.9173317311318288e-06,
    skewness=0.07878272488293536, kurtosis=0.033687569211443336)
```

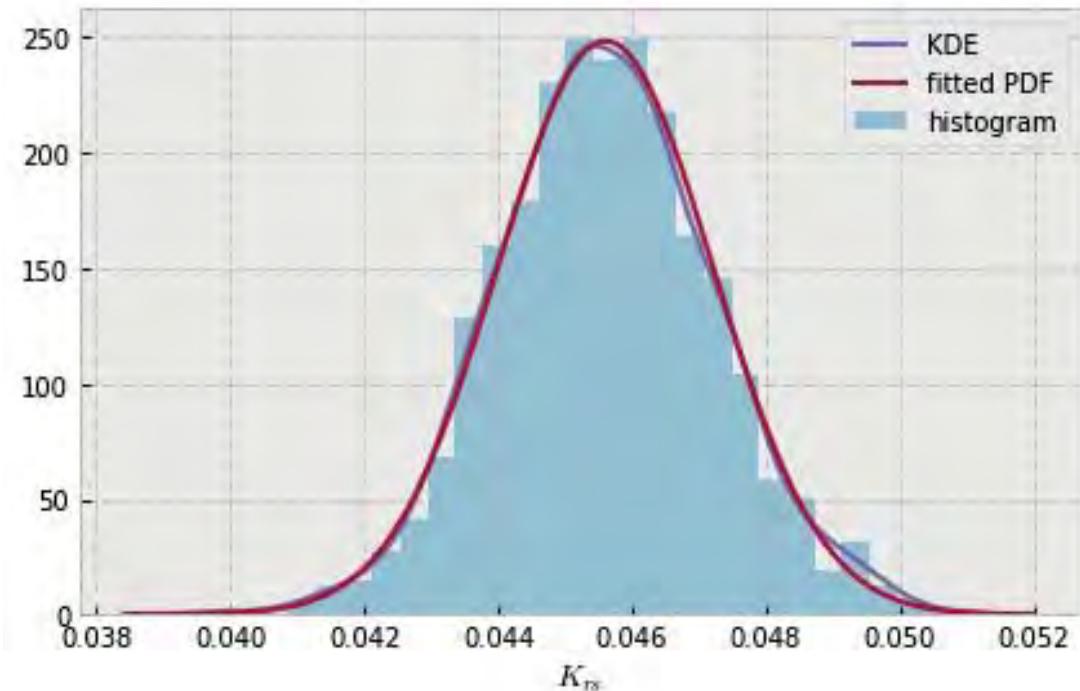


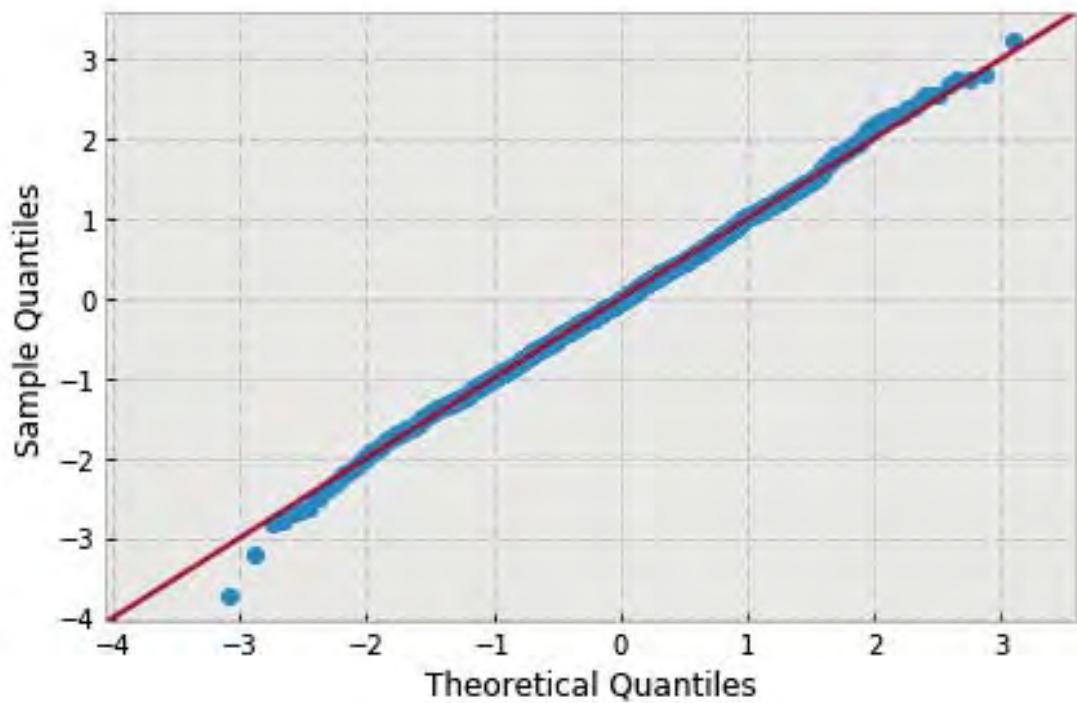


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=0.046041833474580005, scale=0.001520264247419762)
DescribeResult(nobs=1000, minmax=(0.04158306526999997, 0.05160900438999998)
               mean=0.046041833474580005, variance=2.3135168988816572e-06,
               skewness=0.12598704697025995, kurtosis=0.17540026358248495)
```

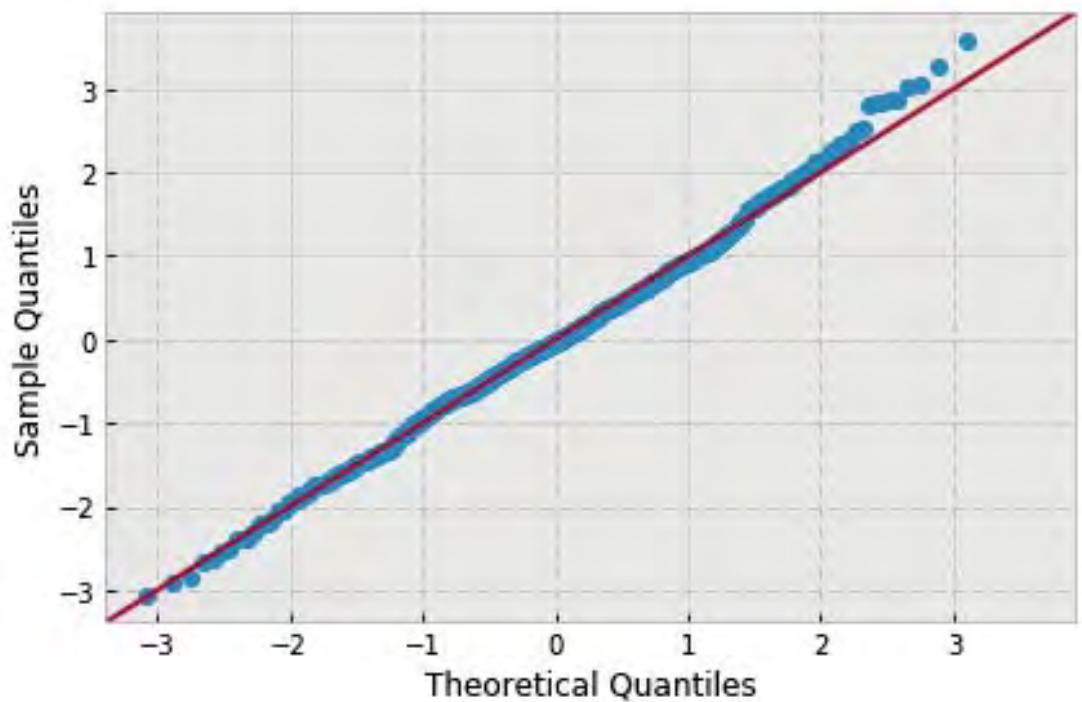
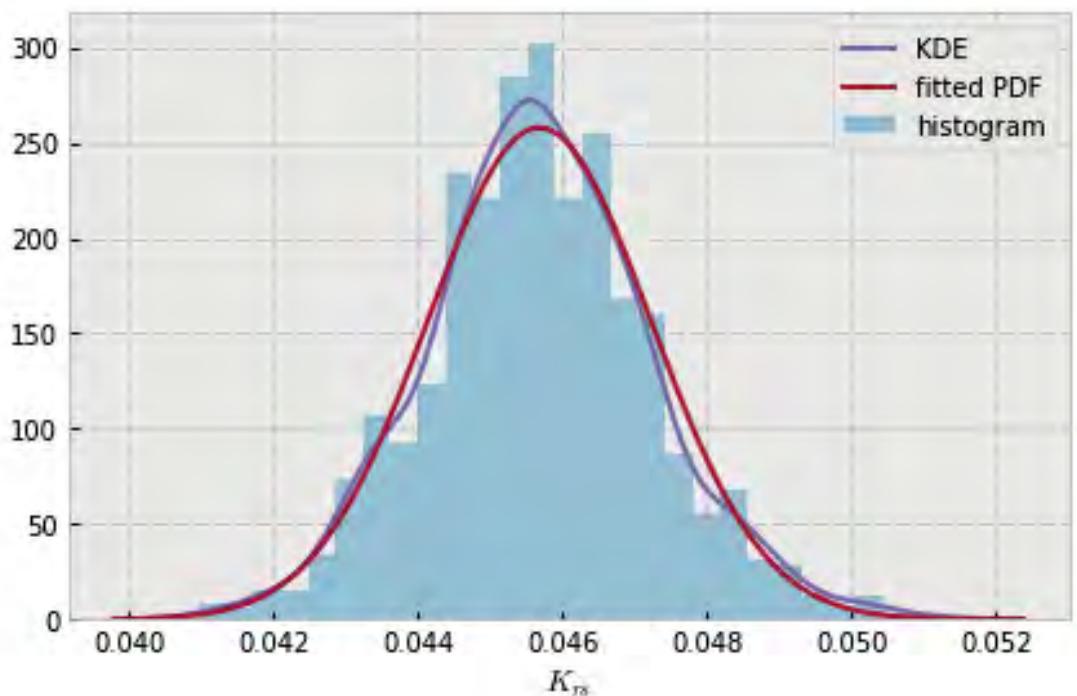


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=0.04560223593544, scale=0.0016048560220399465)
DescribeResult(nobs=1000, minmax=(0.03965187070000001, 0.05078030353000001)
               mean=0.04560223593544001, variance=2.5781409924703514e-06,
               skewness=0.040268507126563303, kurtosis=0.16494342387614713)
```

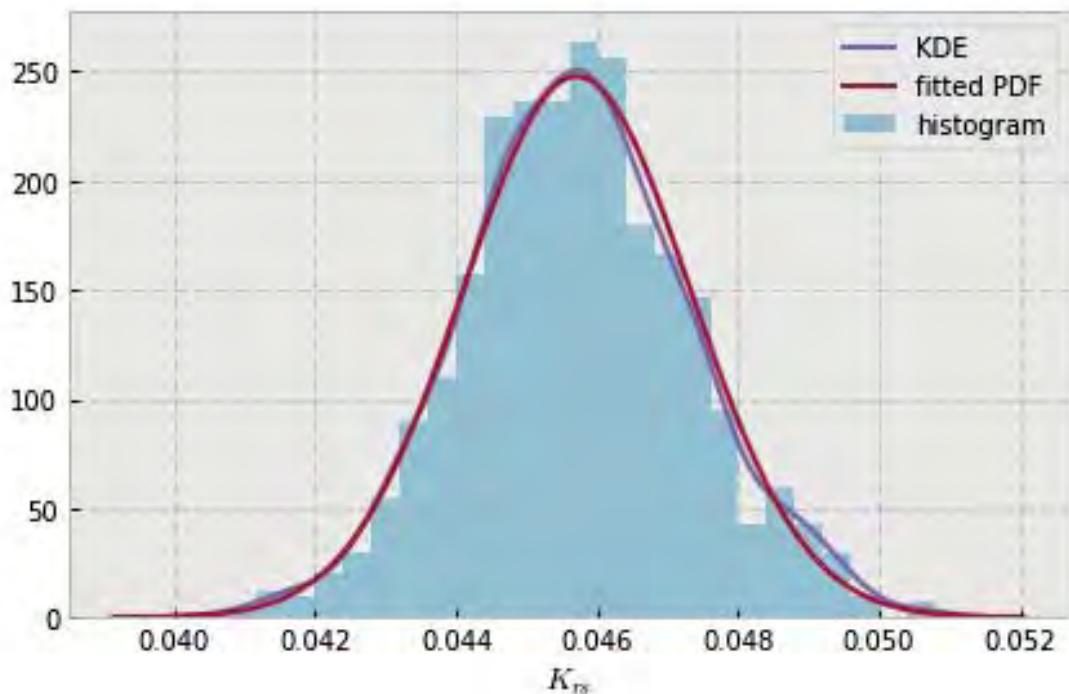


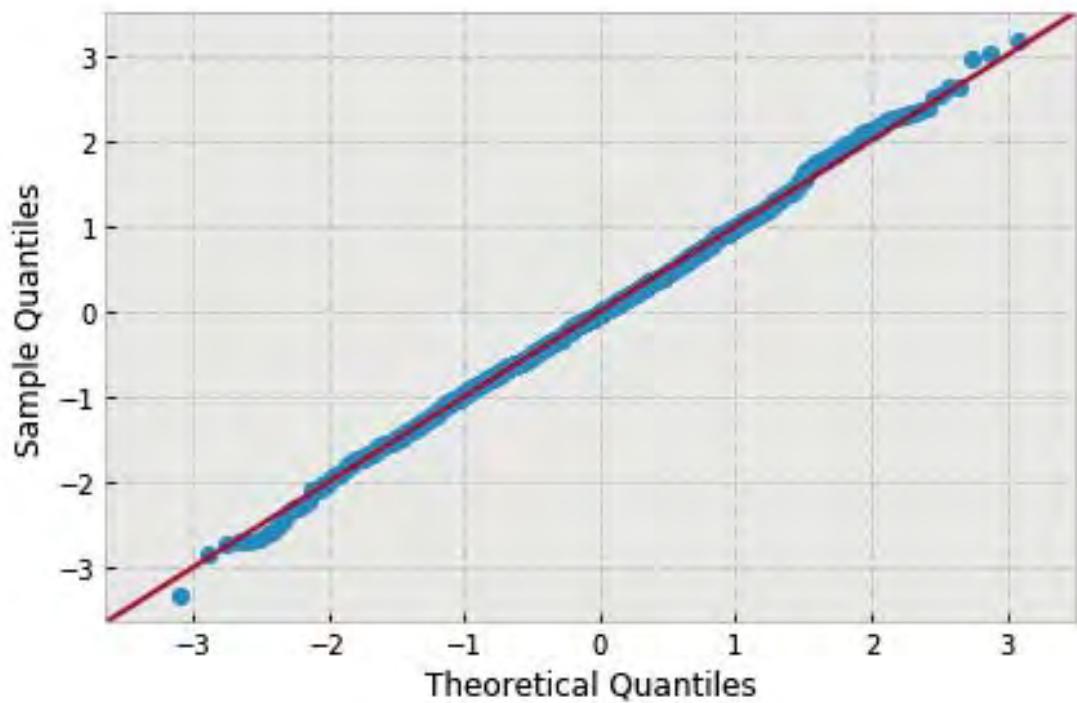


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=0.045676594685729995, scale=0.0015466841029225914)
DescribeResult(nobs=1000, minmax=(0.04095563311, 0.05120781203000001)
               mean=0.045676594685729995, variance=2.3946263405740352e-06,
               skewness=0.15489276123691764, kurtosis=0.3366218308887383)
```

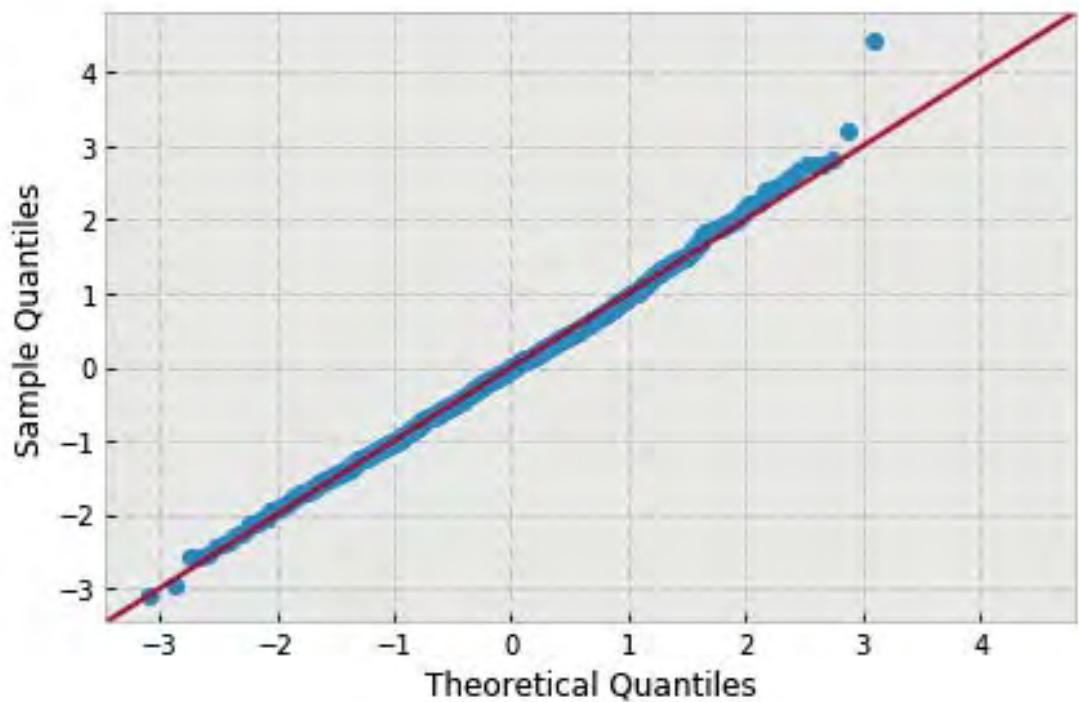
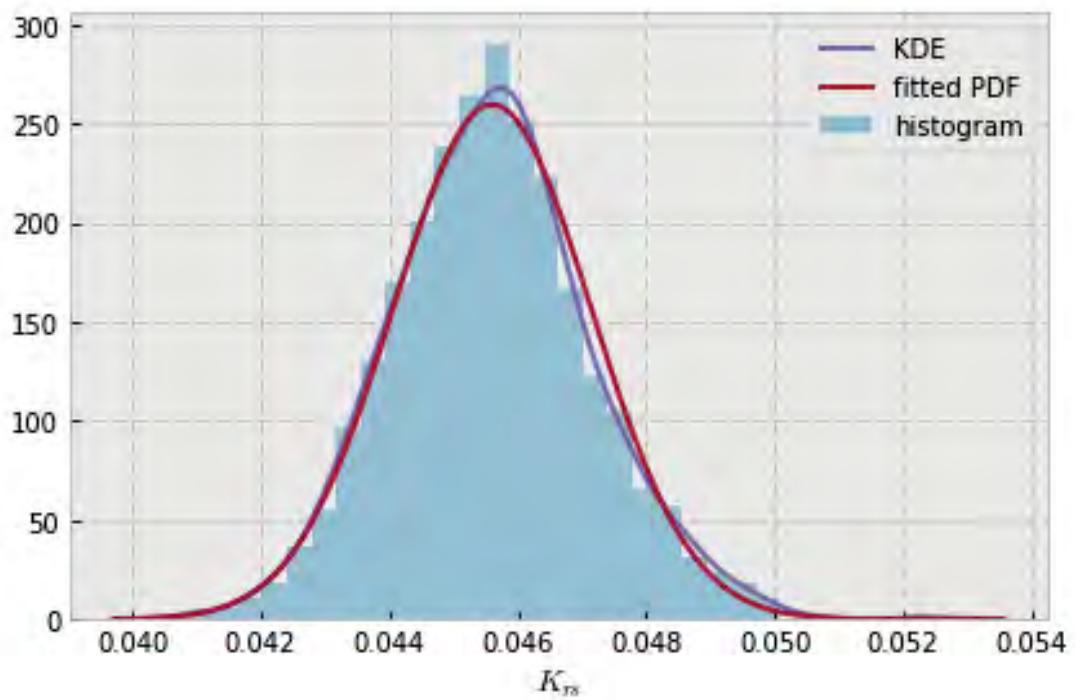


```
variable name: sigma
variable value: 0.03333333333333333
distribution: normal(shape=(), loc=0.045713203396739996, scale=0.0016106211956234647)
DescribeResult(nobs=1000, minmax=(0.040375904210000001, 0.050818988830000002)
               mean=0.045713203396739996, variance=2.5966973331246841e-06,
               skewness=0.07156631288981853, kurtosis=0.09588373608255418)
```

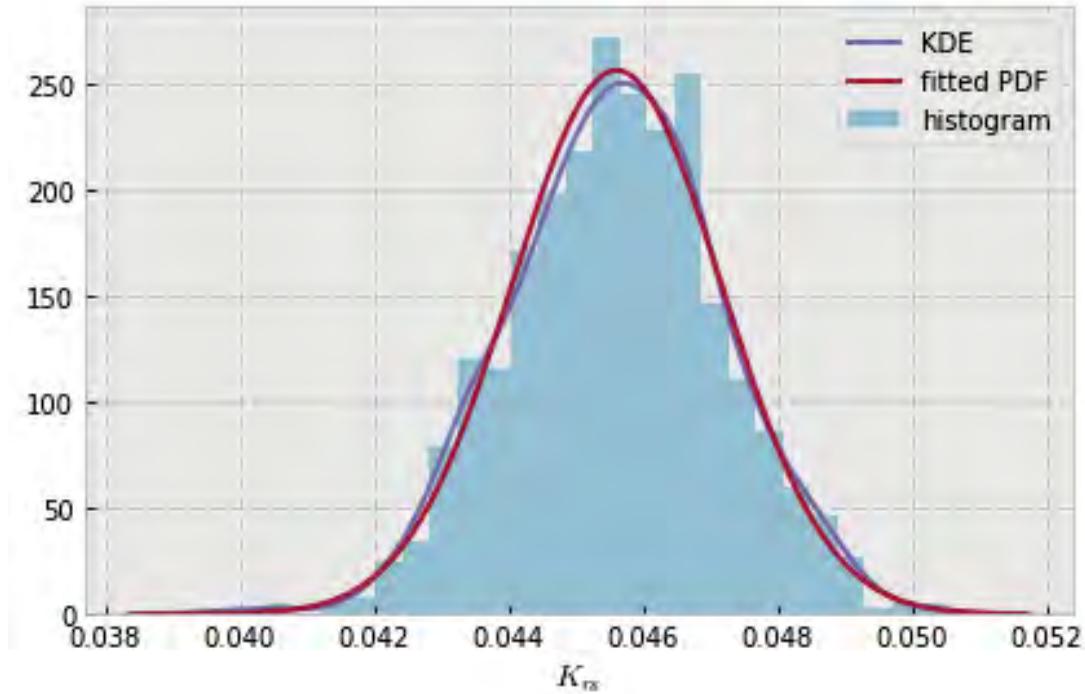


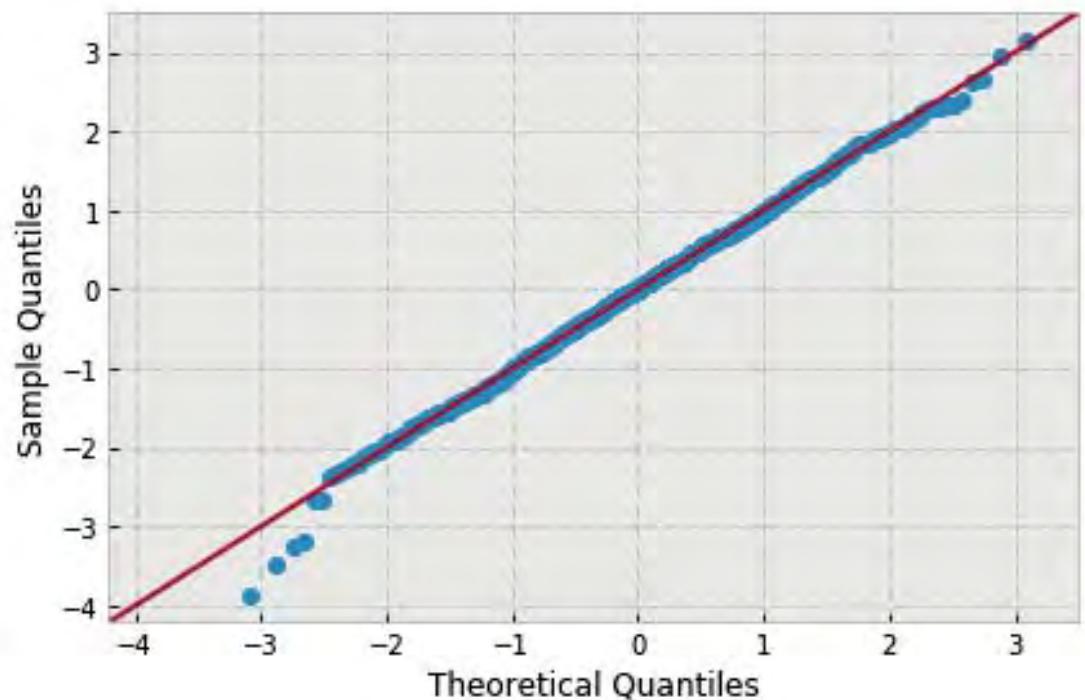


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=0.04560120043687, scale=0.0015365775684101216)
DescribeResult(nobs=1000, minmax=(0.040865455109999997, 0.052402352110000001)
               mean=0.04560120043687, variance=2.3634340577989612e-06,
               skewness=0.21059084917046403, kurtosis=0.3281665983781692)
```

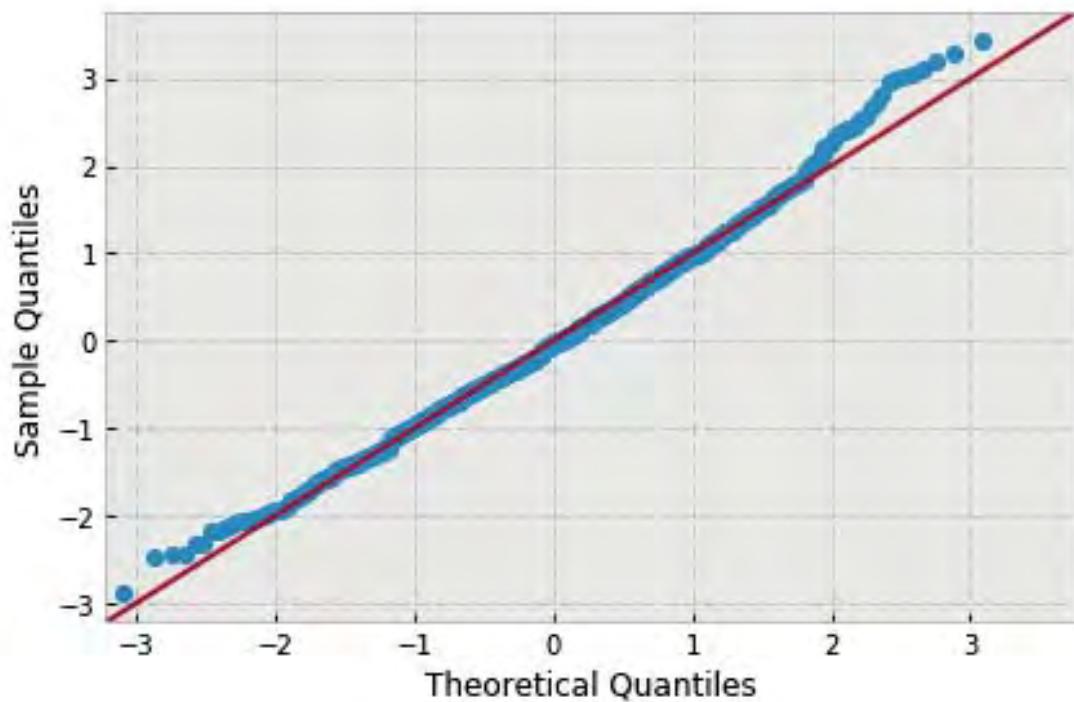
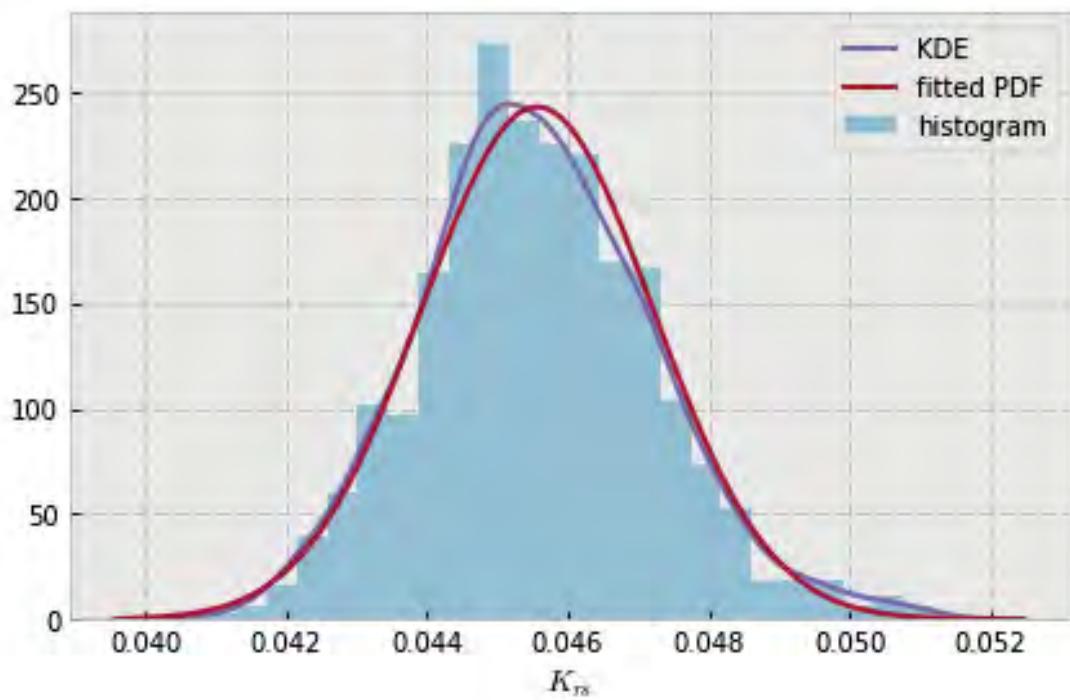


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=0.04560199678325, scale=0.0015564764608756382)
DescribeResult(nobs=1000, minmax=(0.03959715116999997, 0.05049487335999998)
               mean=0.045601996783249998, variance=2.4250440172772296e-06,
               skewness=-0.08253530062371421, kurtosis=0.09736102580779615)
```

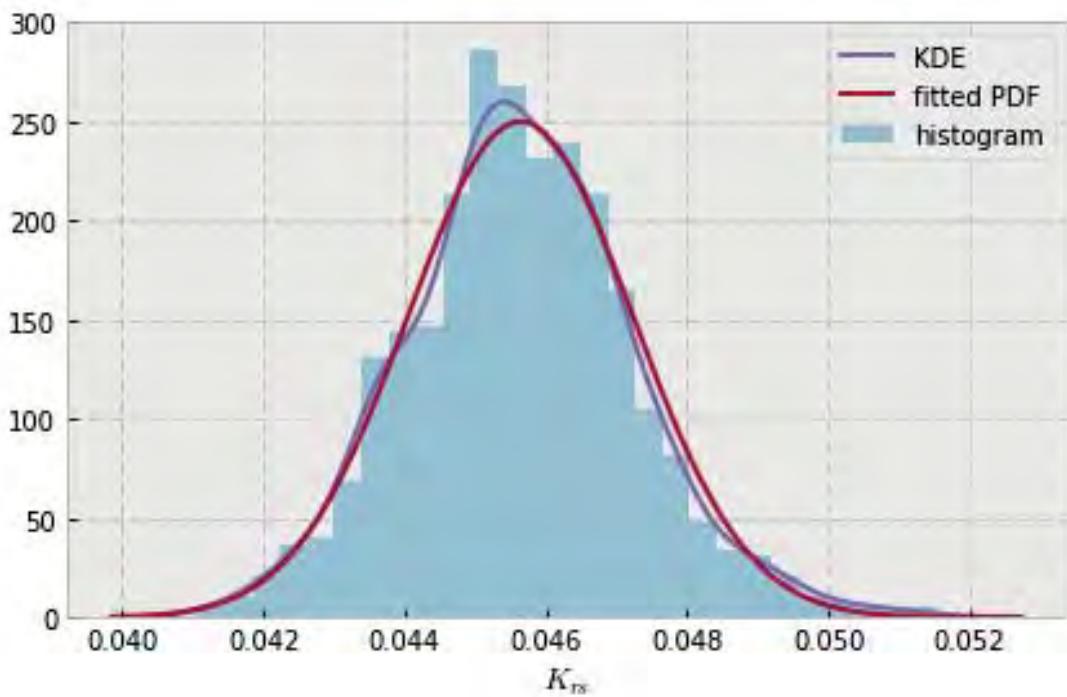


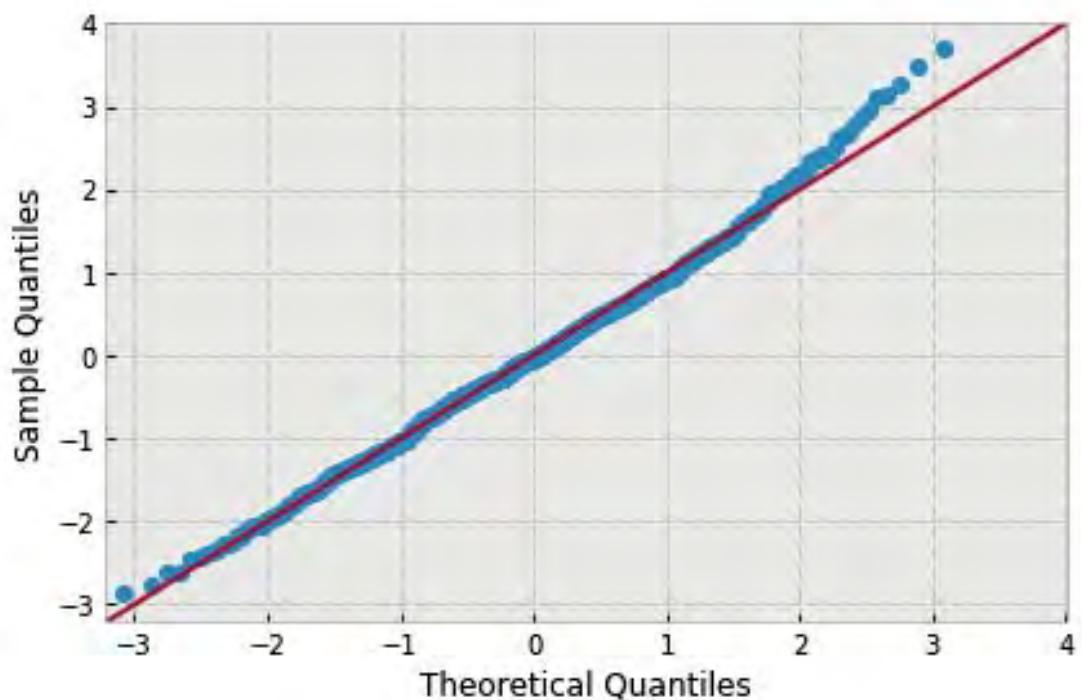


```
variable name: sigma
variable value: 0.066666666666666667
distribution: normal(shape=(), loc=0.04556630042634, scale=0.0016425982418571885)
DescribeResult(nobs=1000, minmax=(0.04085127303000003, 0.05120046560000002)
               mean=0.04556630042634002, variance=2.7008298139662927e-06,
               skewness=0.2992770928247034, kurtosis=0.2323830582272417)
```

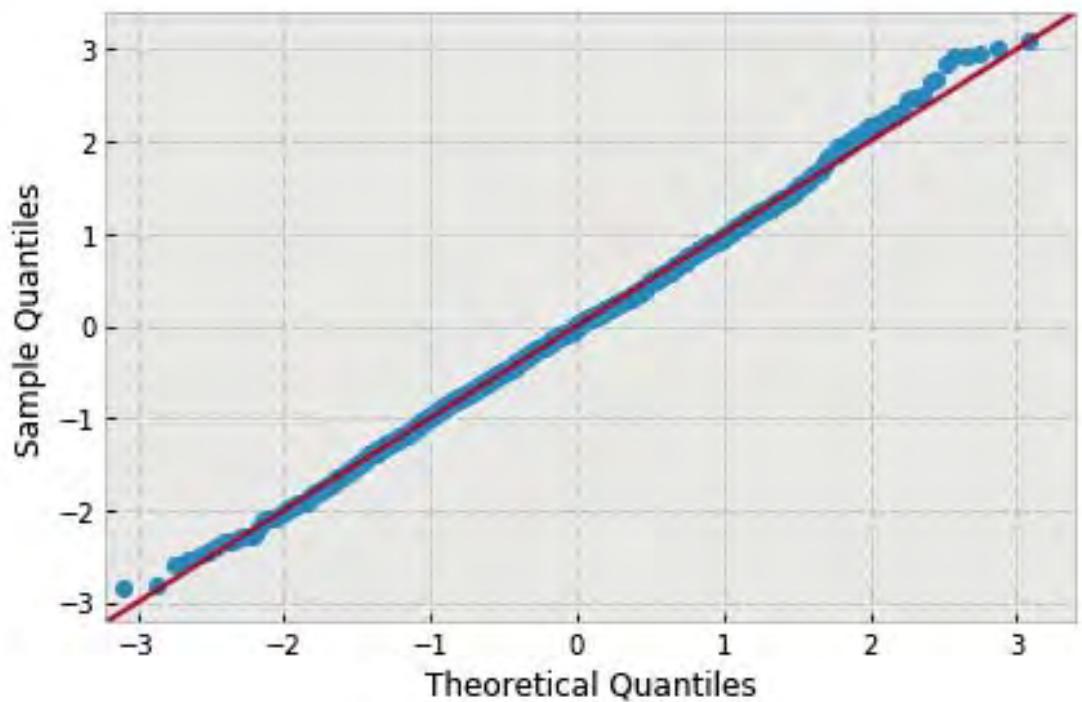
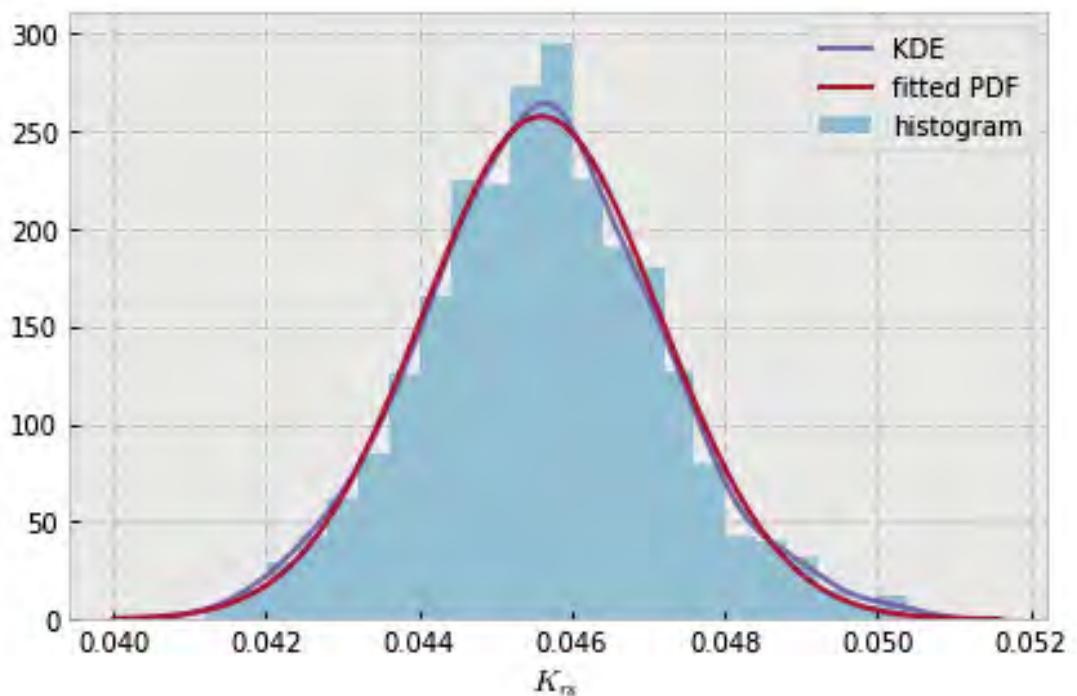


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=0.04563815191813, scale=0.0015958869582076794)
DescribeResult(nobs=1000, minmax=(0.04105566213999997, 0.05152496615999998)
               mean=0.045638151918129997, variance=2.5494045879653244e-06,
               skewness=0.20805200701594617, kurtosis=0.3634958805911461)
```

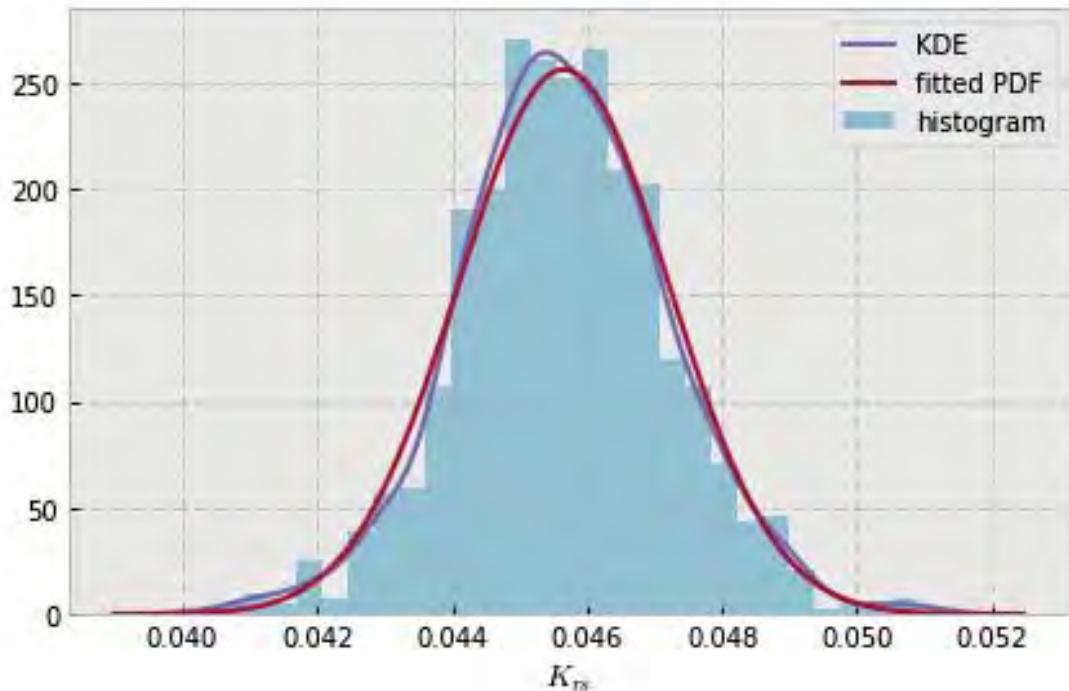


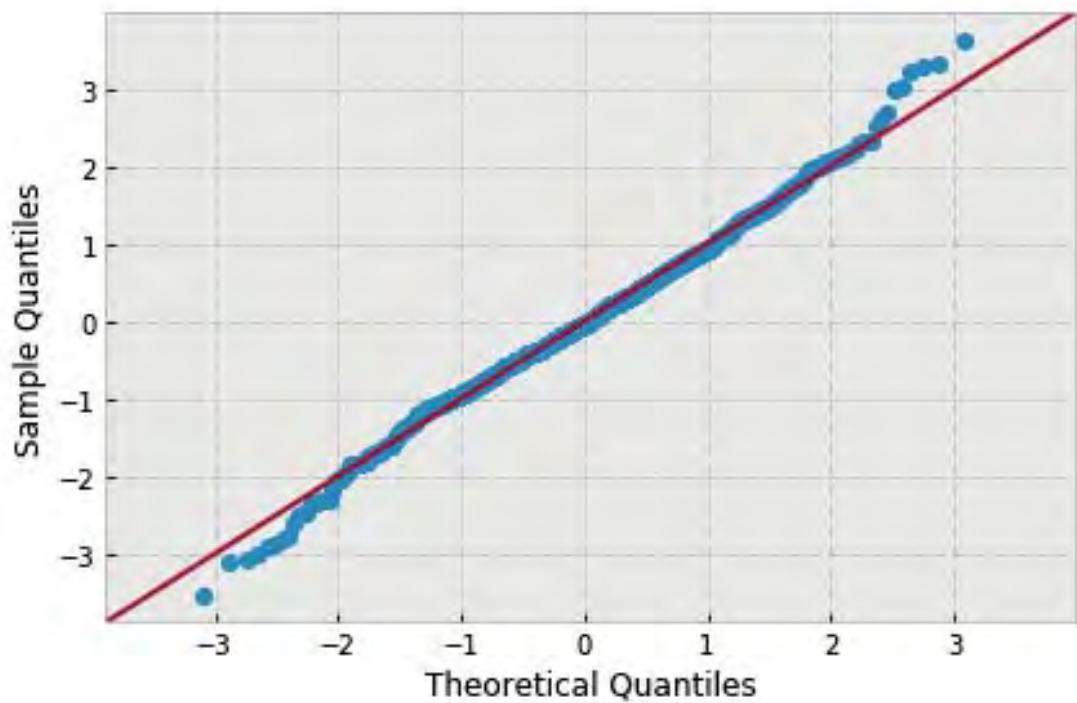


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=0.04560053773108, scale=0.00155083542601896)
DescribeResult(nobs=1000, minmax=(0.04121104682, 0.05040290642999998)
               mean=0.045600537731079999, variance=2.4074980166120214e-06,
               skewness=0.10513399030925294, kurtosis=0.08508438230469961)
```

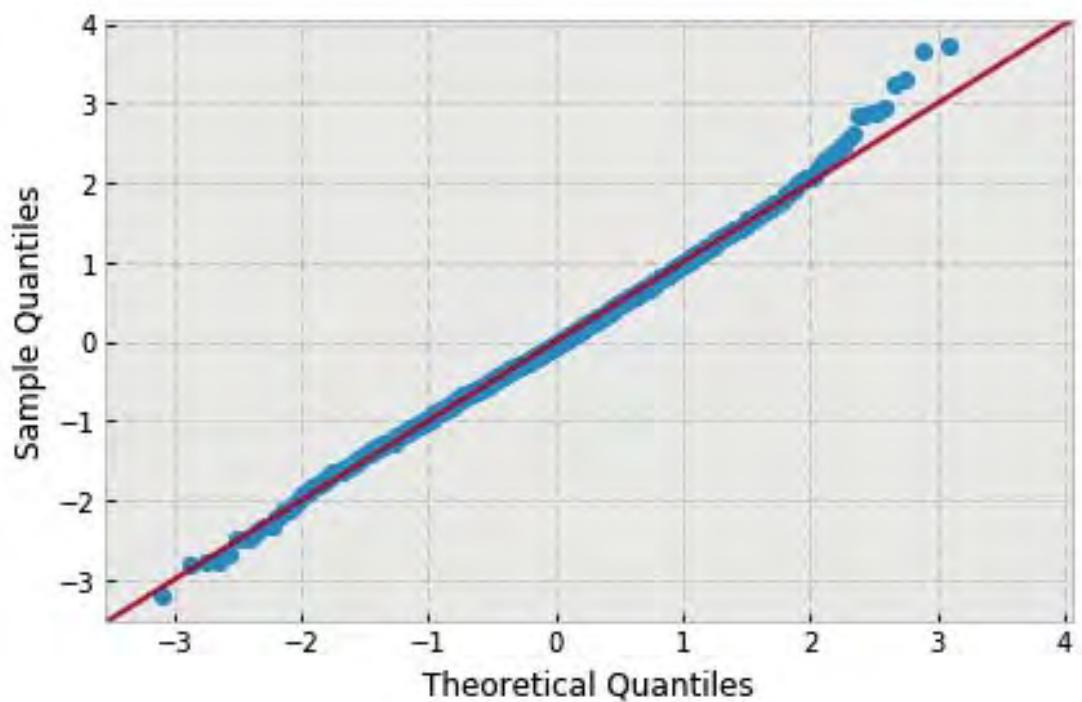
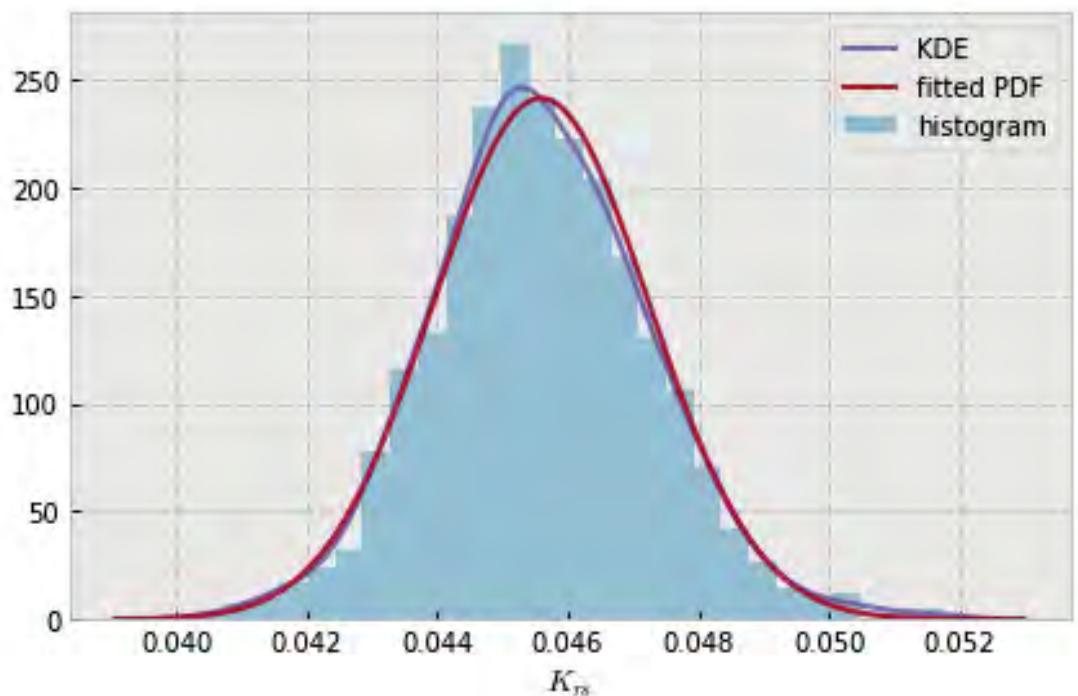


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=0.04565243272583, scale=0.001557751933818464)
DescribeResult(nobs=1000, minmax=(0.04015800432999997, 0.05129085845999998)
               mean=0.04565243272583, variance=2.429020107422587e-06,
               skewness=0.04623001313751583, kurtosis=0.5651416149234079)
```

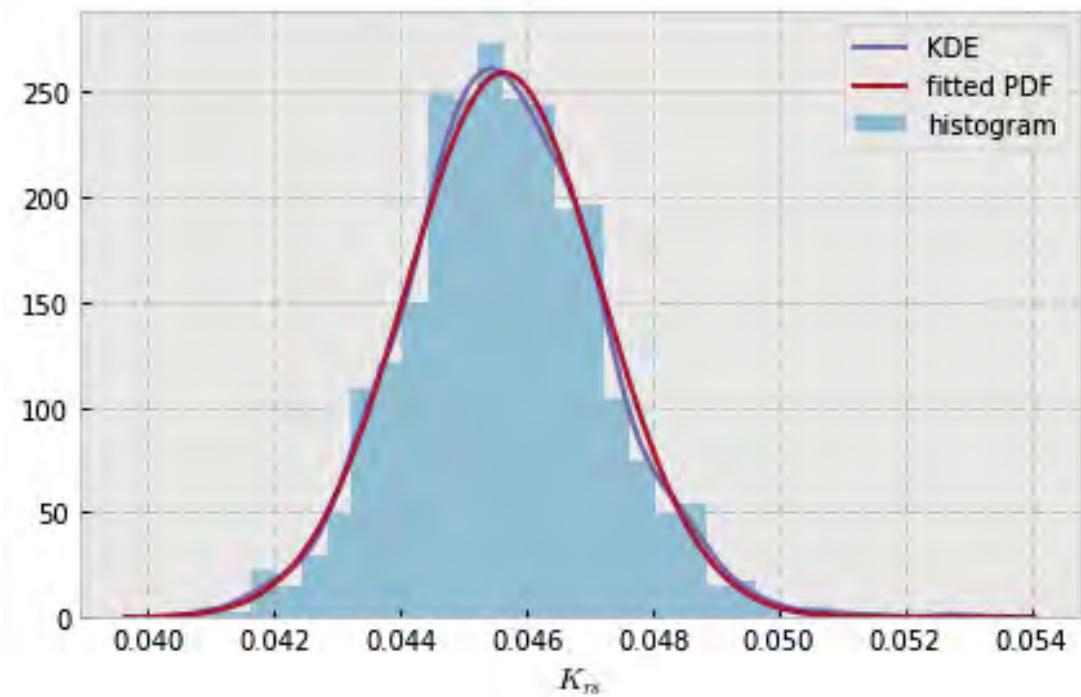


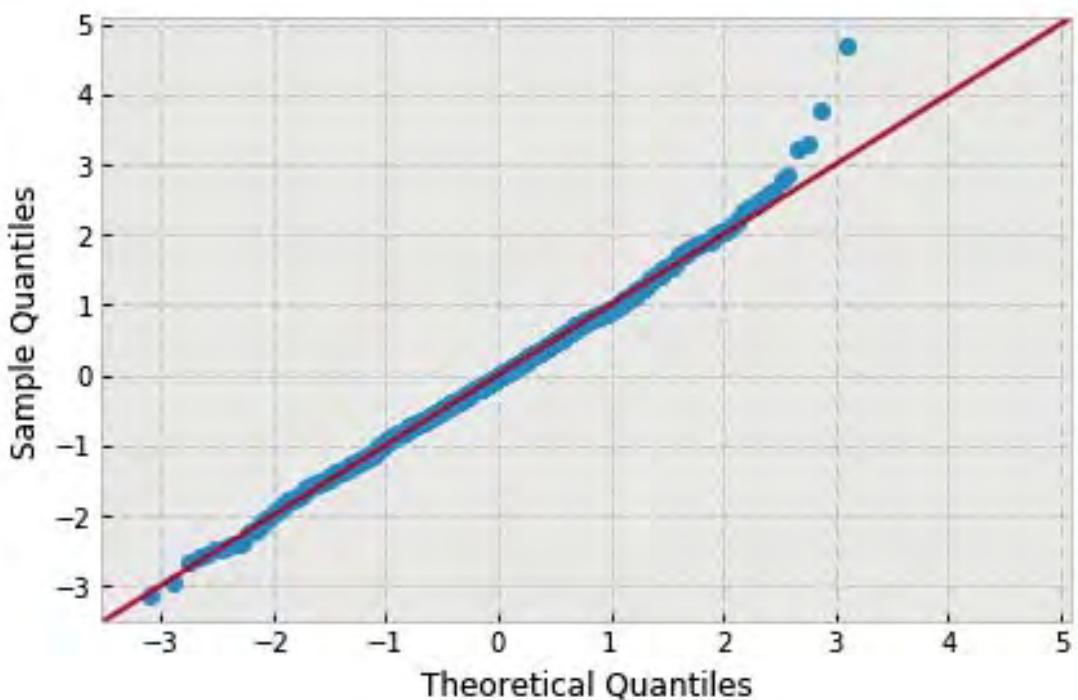


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=0.045588934641989994, scale=0.0016517248851208816)
DescribeResult(nobs=1000, minmax=(0.040323330960000001, 0.05172646311000002)
               mean=0.045588934641989994, variance=2.730926022149739e-06,
               skewness=0.1971492271645529, kurtosis=0.40548856421653534)
```

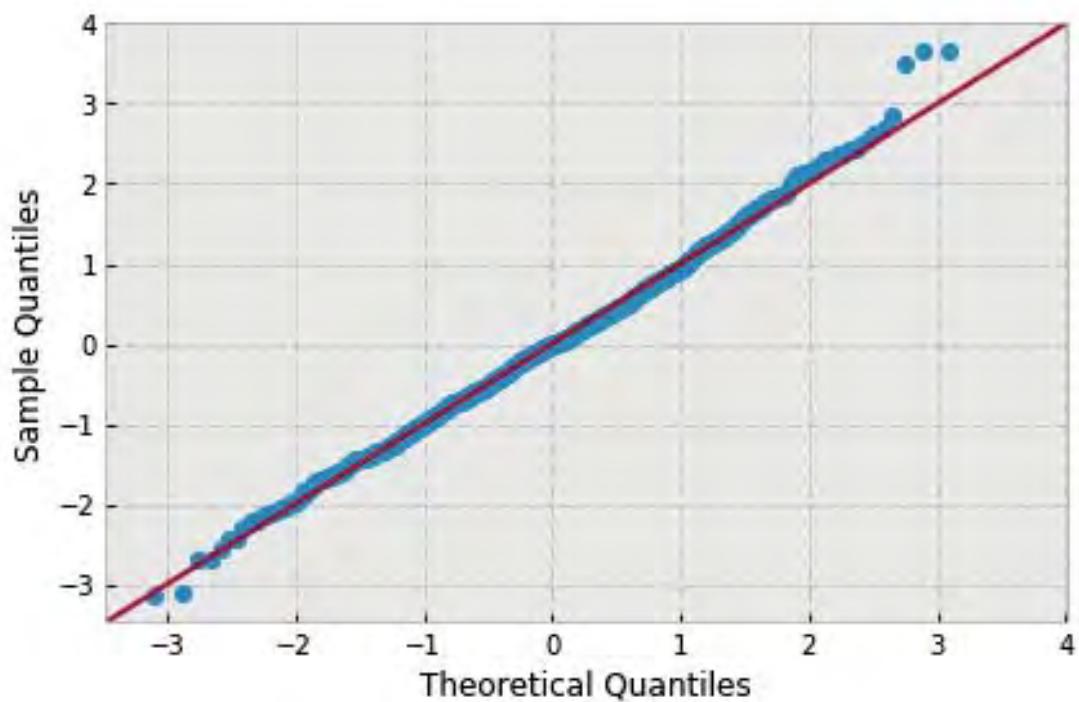
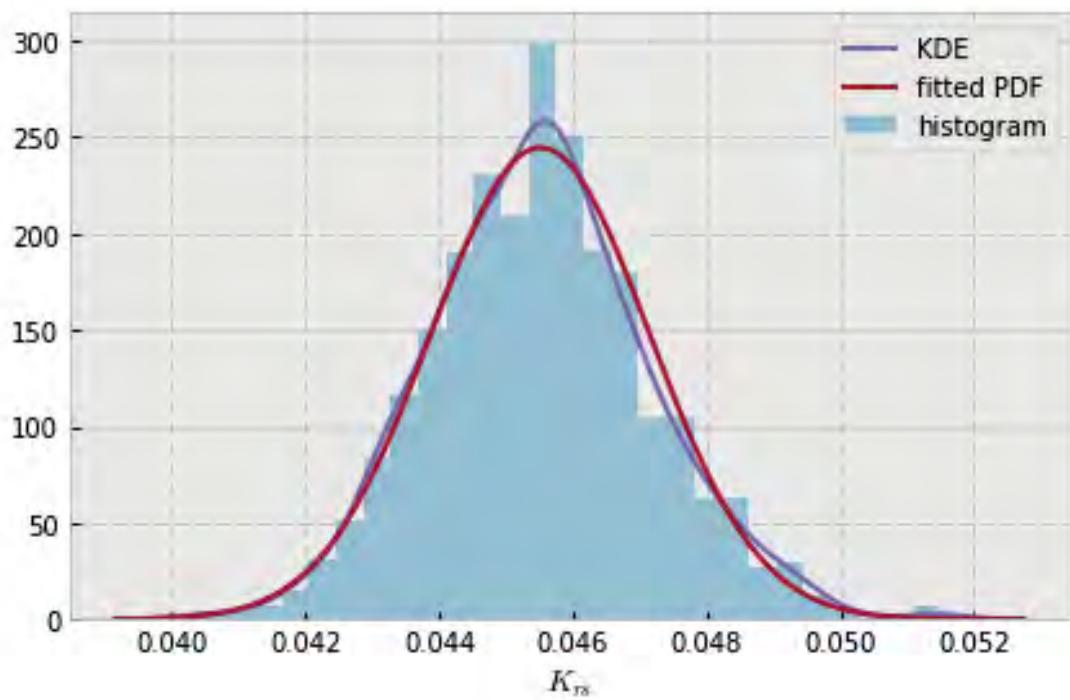


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=0.045637687437840004, scale=0.0015391625901485229)
DescribeResult(nobs=1000, minmax=(0.040826412970000003, 0.052854945200000003)
               mean=0.045637687437840004, variance=2.3713928717844943e-06,
               skewness=0.20407224825322587, kurtosis=0.5892740790141042)
```

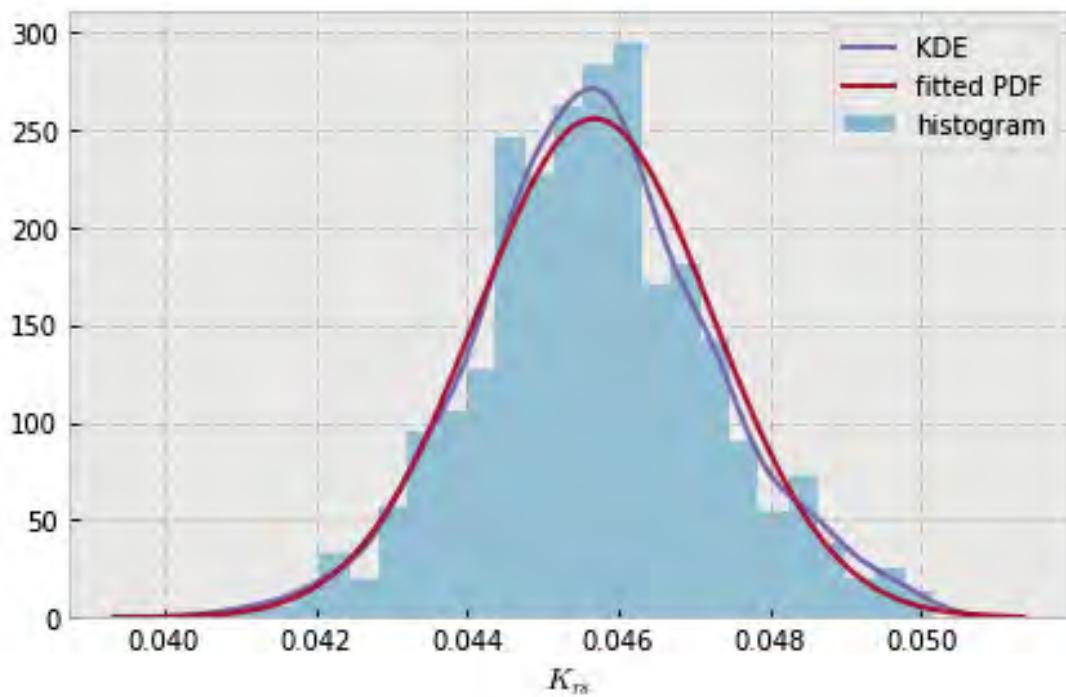


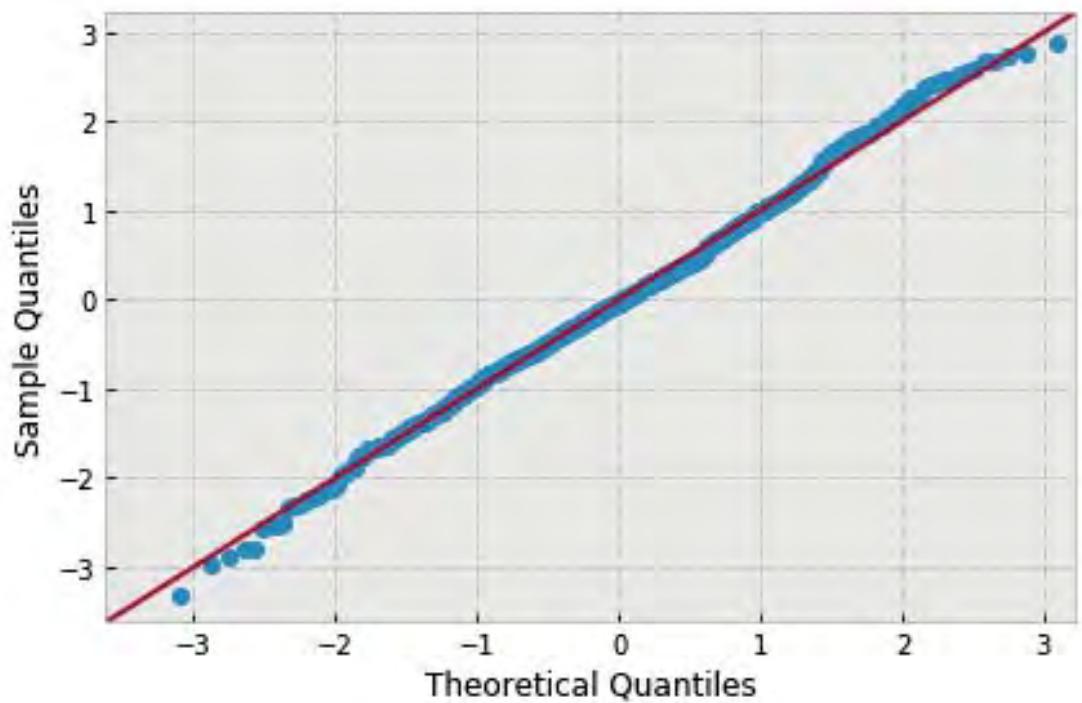


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=0.04551383420288001, scale=0.0016313422567704732)
DescribeResult(nobs=1000, minmax=(0.04040419693999998, 0.05150025919000001)
               mean=0.04551383420288007, variance=2.6639415002252055e-06,
               skewness=0.19734791096598311, kurtosis=0.2758481062309257)
```

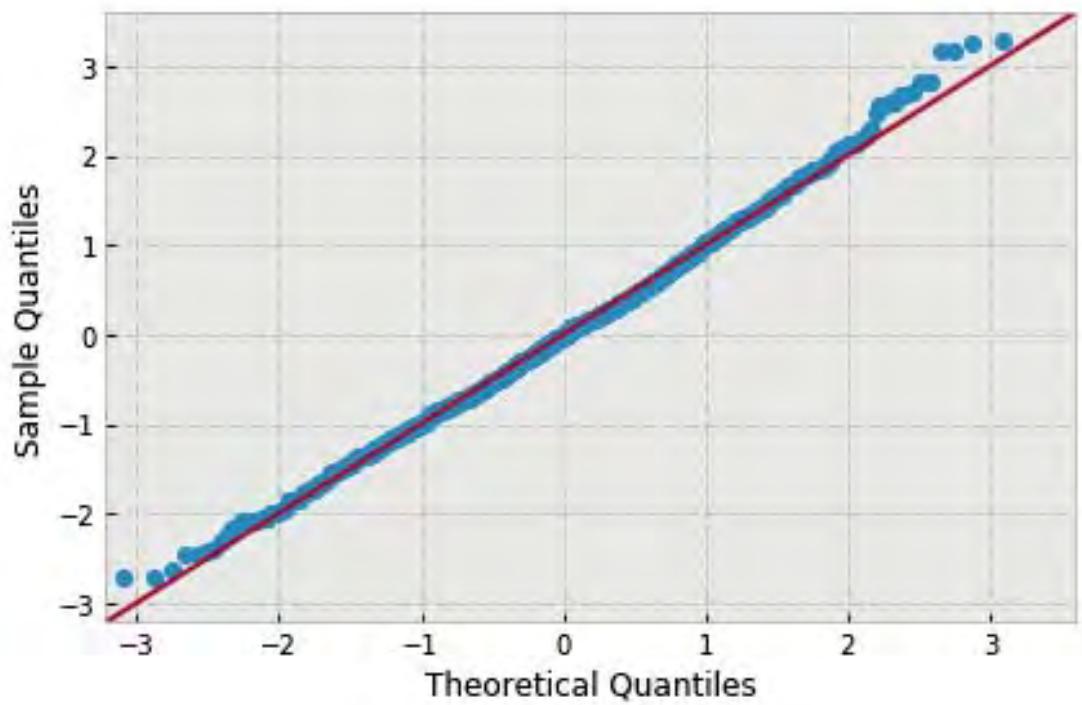
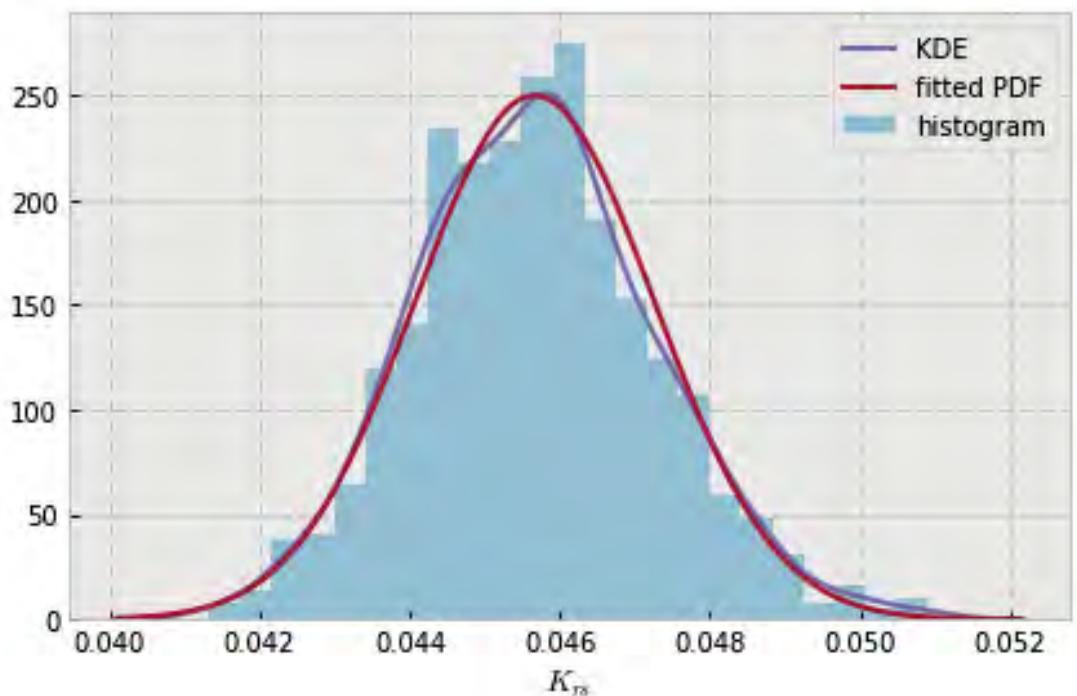


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=0.04568639010633, scale=0.0015607141374191602)
DescribeResult(nobs=1000, minmax=(0.040520640660000003, 0.050169307400000002)
               mean=0.04568639010633001, variance=2.4382668856256588e-06,
               skewness=0.08807508495003107, kurtosis=0.1668767045824553)
```

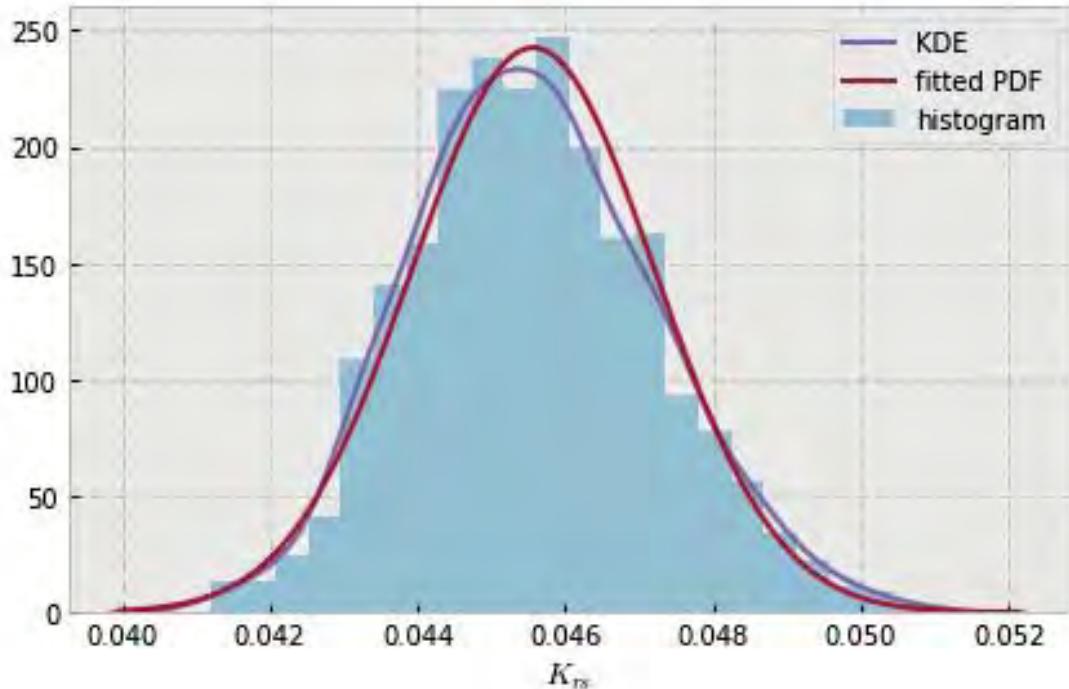


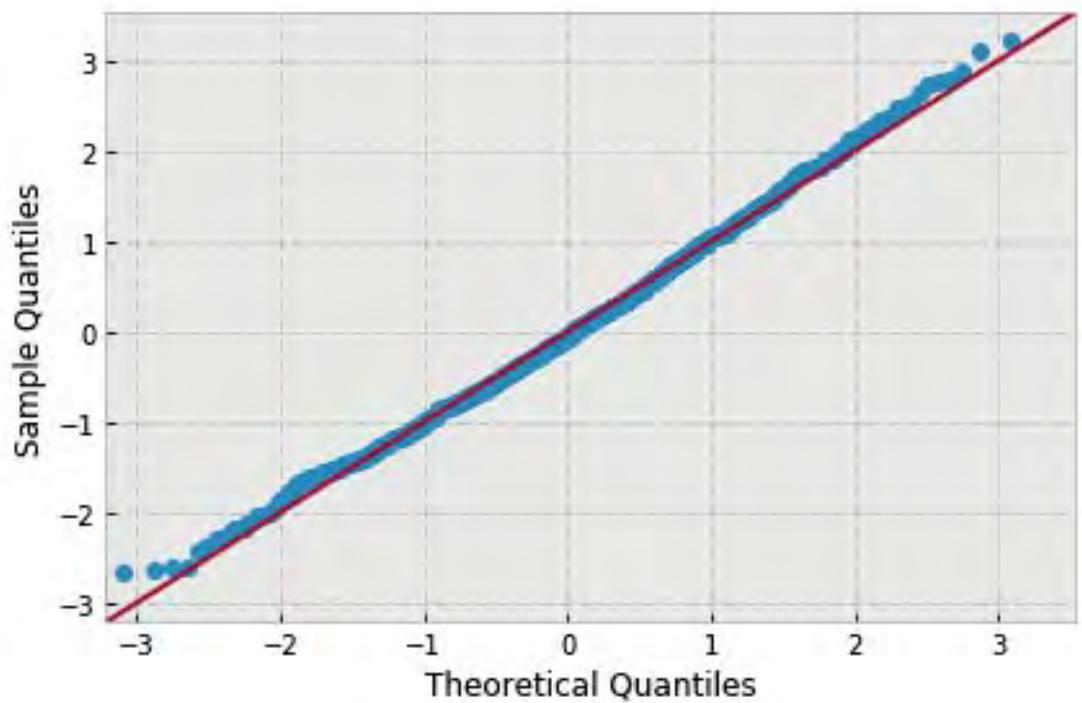


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=0.04566622343753, scale=0.001596060368841776)
DescribeResult(nobs=1000, minmax=(0.04131690646999998, 0.05091811376000001)
               mean=0.045666223437529997, variance=2.5499586596469928e-06,
               skewness=0.21280299992751395, kurtosis=0.09127460595522585)
```

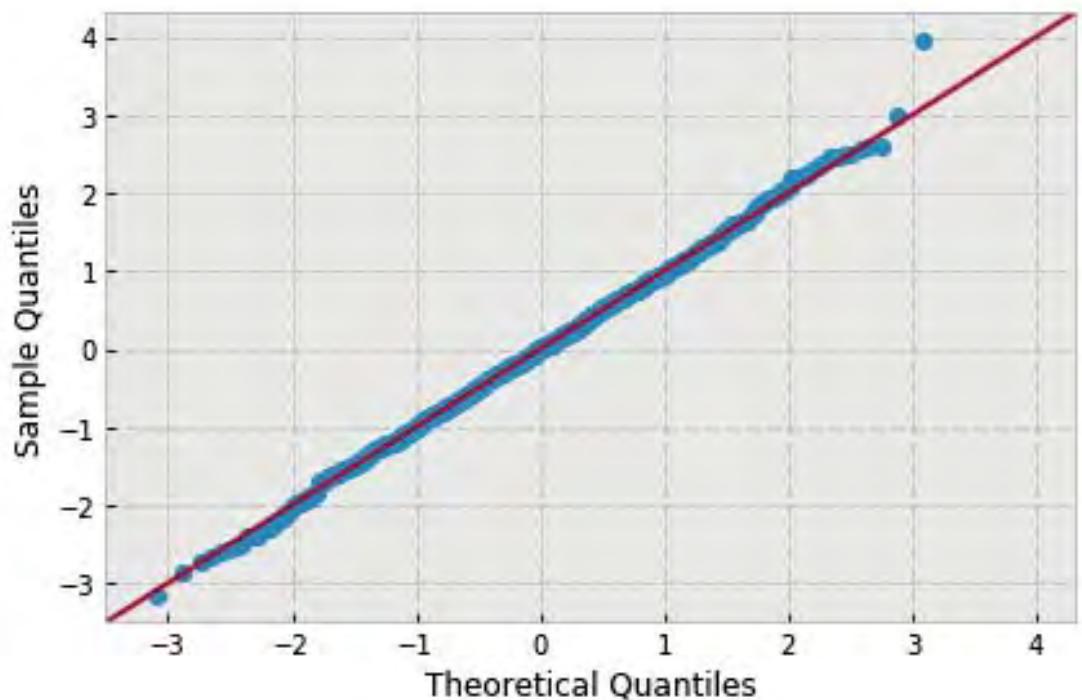
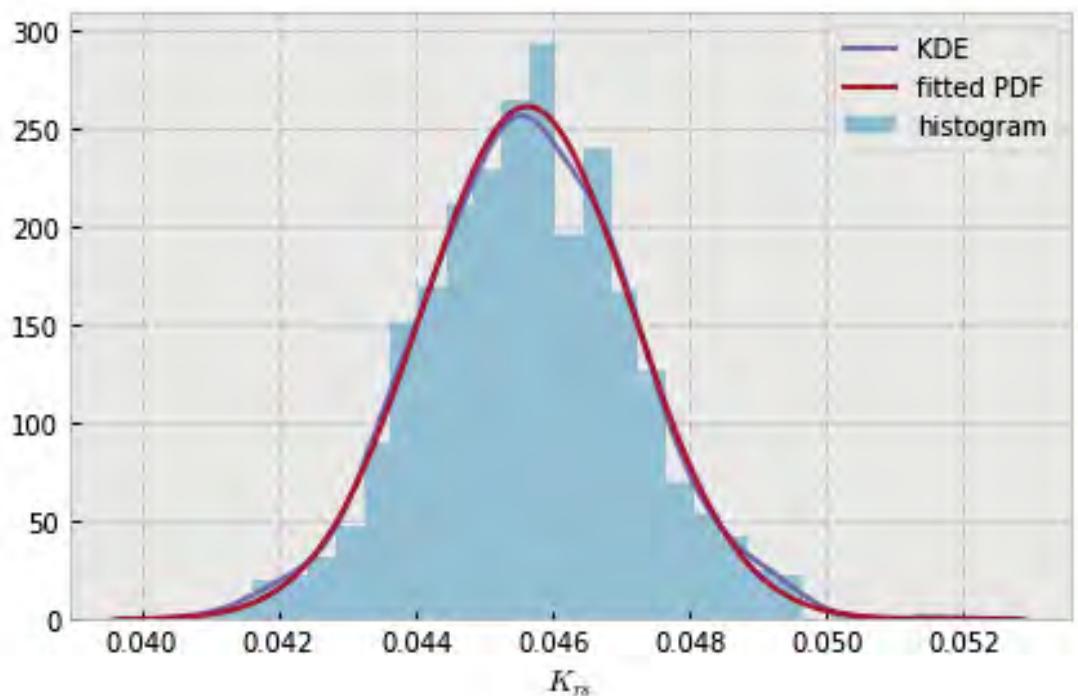


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=0.04556228111021, scale=0.0016447188955687573)
DescribeResult(nobs=1000, minmax=(0.04119806355999998, 0.05088337344999998)
               mean=0.045562281110209997, variance=2.7078080534944072e-06,
               skewness=0.22647696548054438, kurtosis=-0.09612690551014547)
```

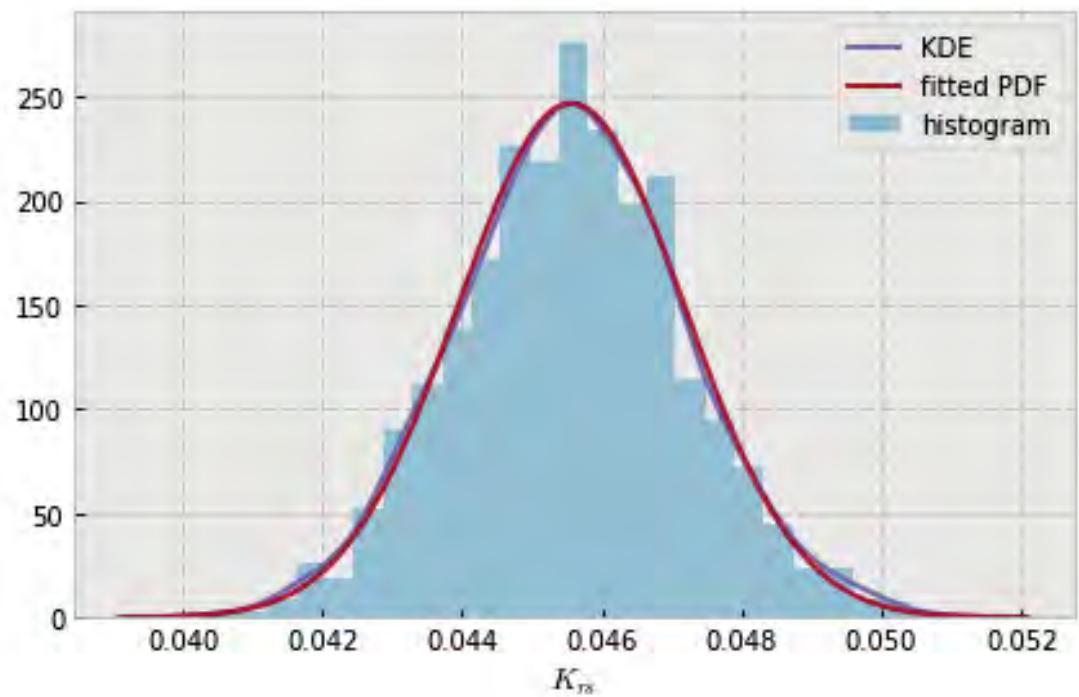


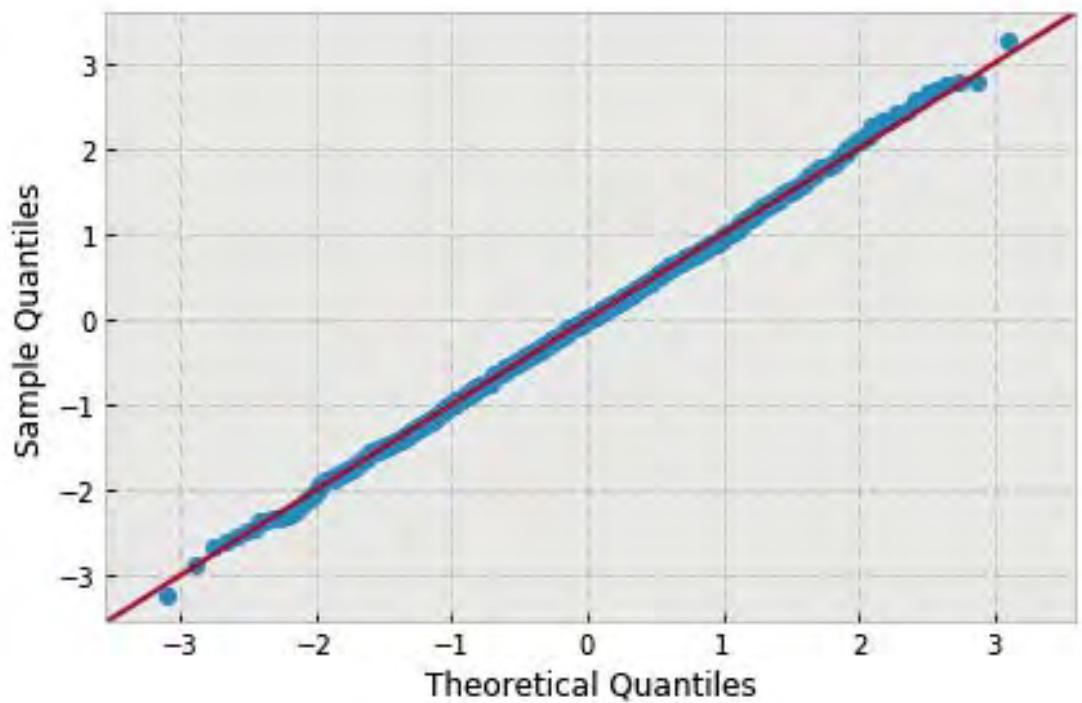


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=0.04562905396137001, scale=0.0015283257475941794)
DescribeResult(nobs=1000, minmax=(0.040823442510000002, 0.051665105660000002)
               mean=0.045629053961370007, variance=2.3381177084677752e-06,
               skewness=0.04981874255618288, kurtosis=0.12117955161884231)
```

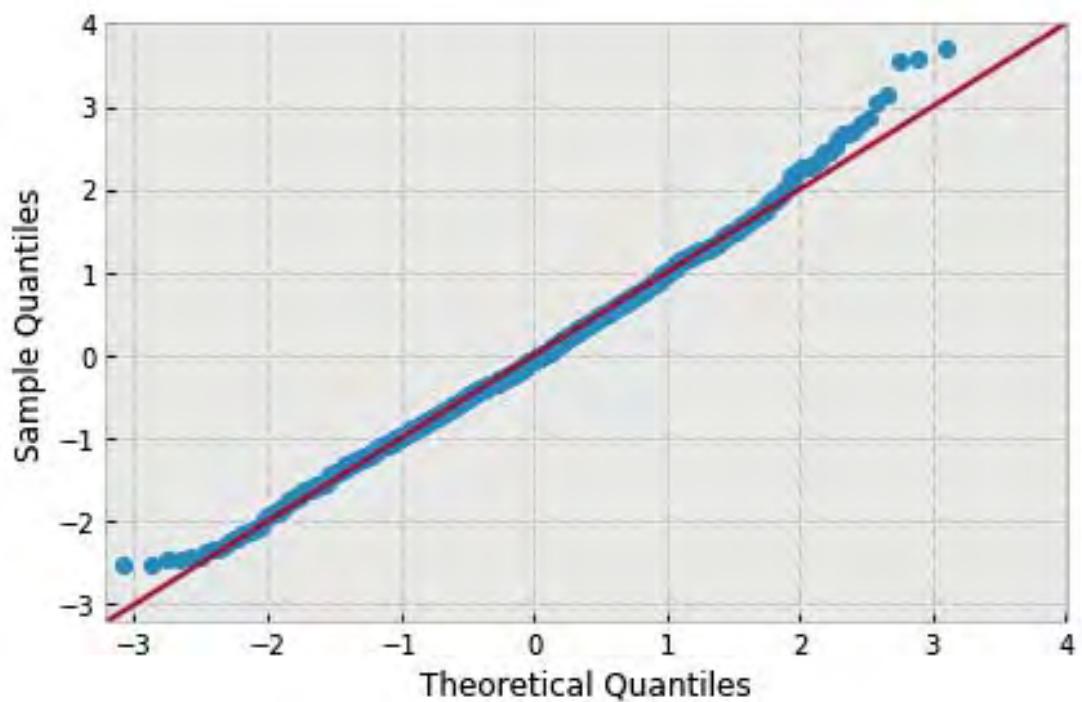
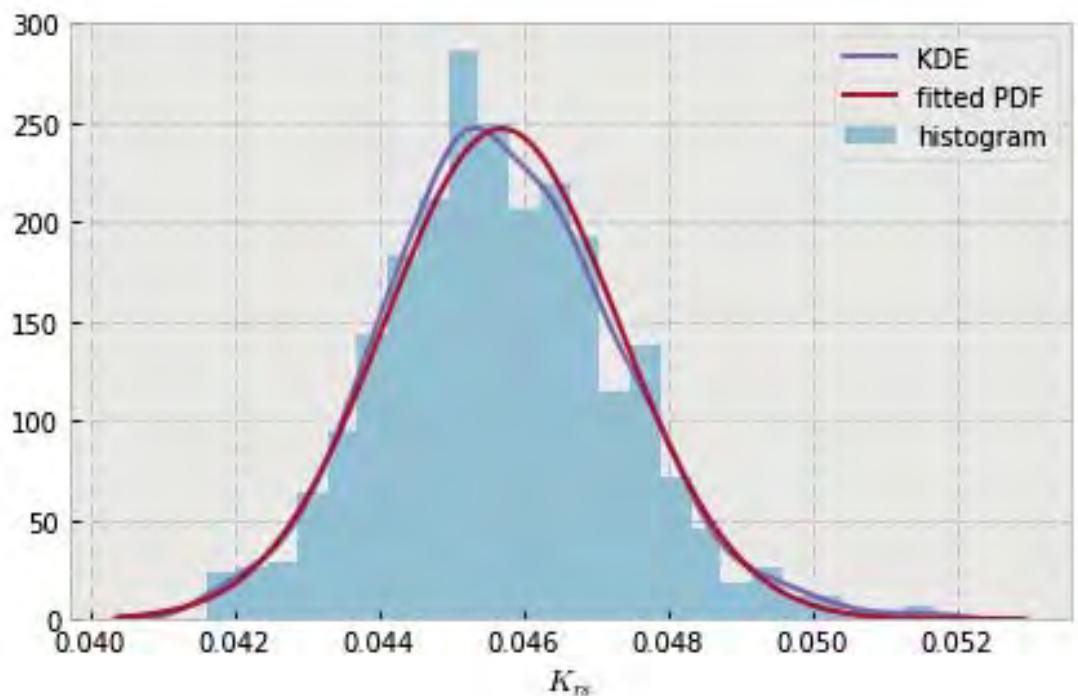


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=0.04556091566024, scale=0.0016174610016566737)
DescribeResult(nobs=1000, minmax=(0.04035204030999997, 0.050837455300000001)
               mean=0.045560915660240002, variance=2.6187988907709811e-06,
               skewness=0.041902746074035, kurtosis=0.018073445344831907)
```

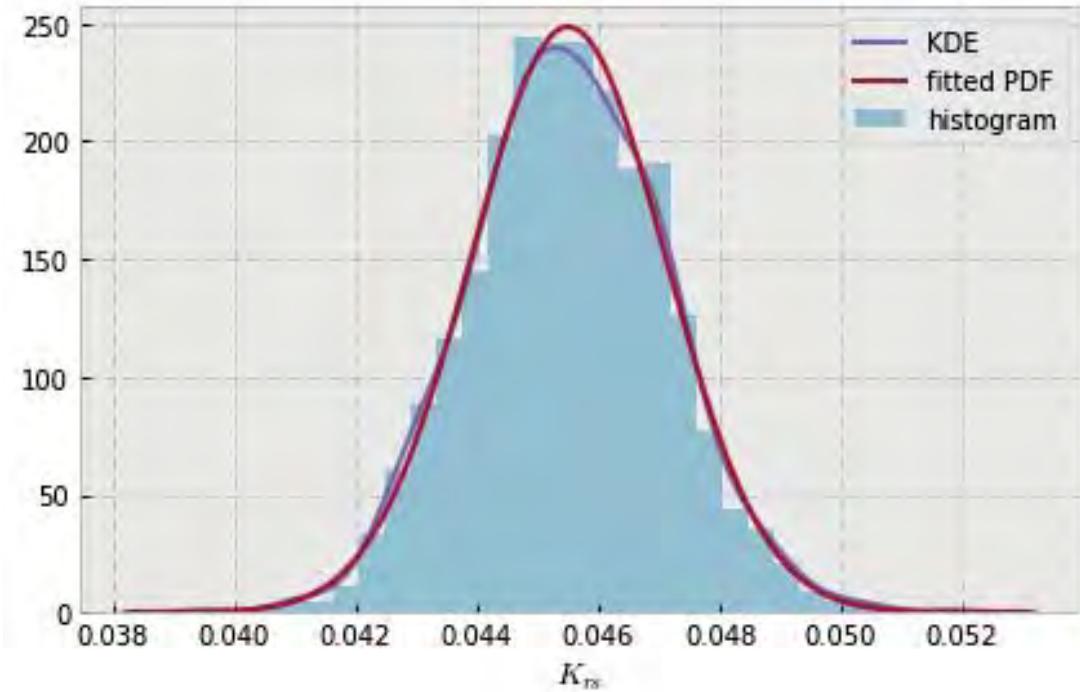


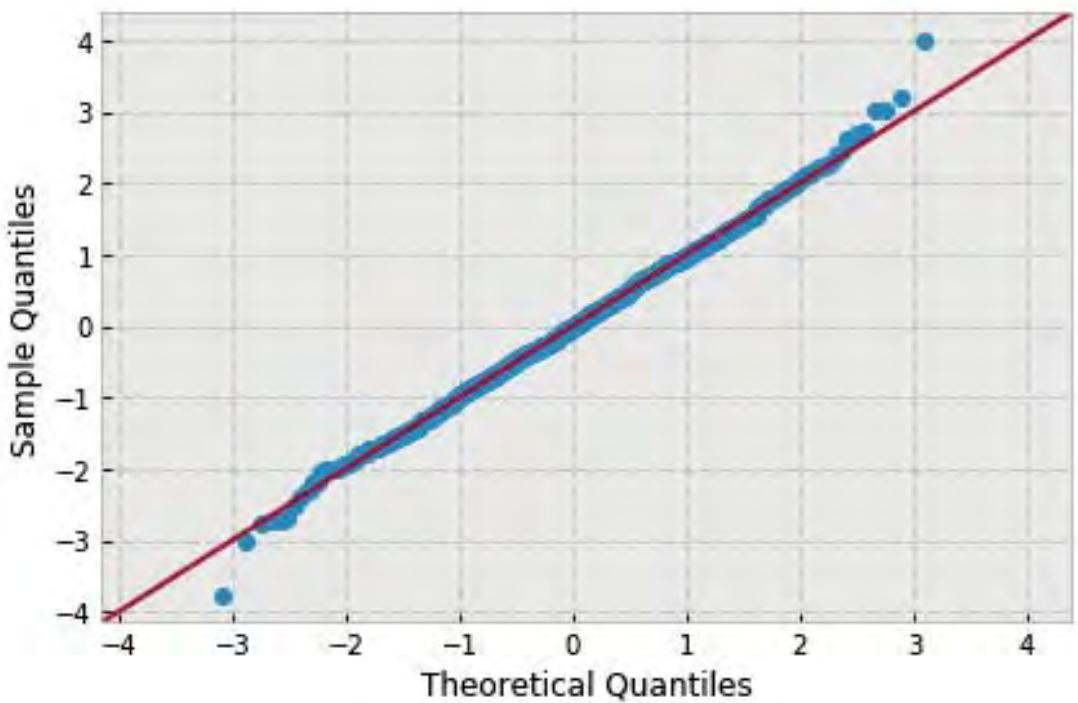


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=0.045694582975960005, scale=0.0016144883866672263)
DescribeResult(nobs=1000, minmax=(0.041604638950000002, 0.05166491264999999)
               mean=0.045694582975960005, variance=2.6091819326159594e-06,
               skewness=0.27725616530948516, kurtosis=0.2884507264288336)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=0.04550134957243, scale=0.0016044984313526131)
DescribeResult(nobs=1000, minmax=(0.03946665989999999, 0.05190718645999997)
               mean=0.04550134957243003, variance=2.5769922084214177e-06,
               skewness=0.06693818806415772, kurtosis=0.2373329946343845)
```

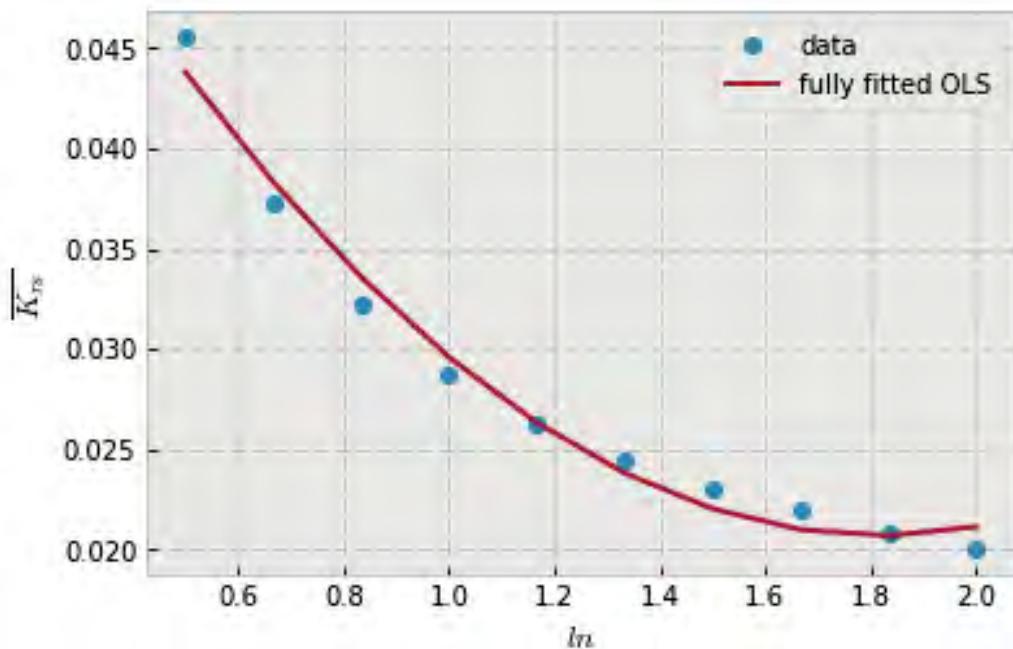




15.2 Parameter regression

```
In [36]: var_name = 'root_system_total_conductance'  
      mean_reg('ln', var_name, 'poly2')  
      mean_reg('theta', var_name, 'poly1')  
      mean_reg('nB', var_name, 'poly2')  
      mean_reg('r', var_name, 'poly2')  
      mean_reg('sigma', var_name, 'poly1')  
      mean_reg('N', var_name, 'poly1')
```

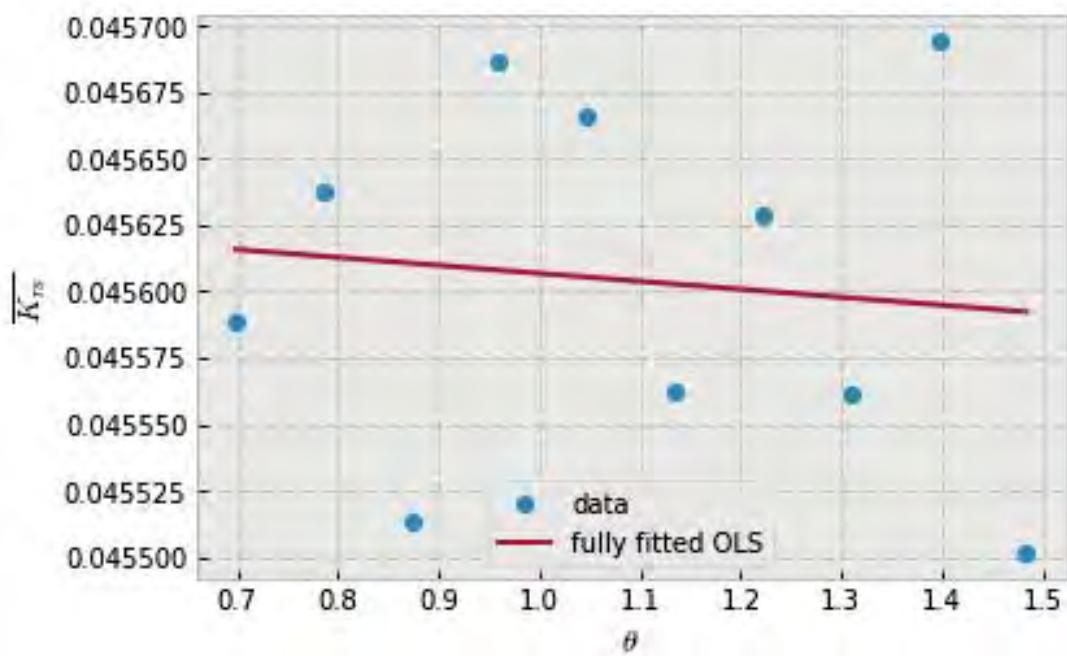
```
variable name: ln  
model: poly2  
r-squared: 0.932452577683079  
explained variance: 0.9325616281341971
```



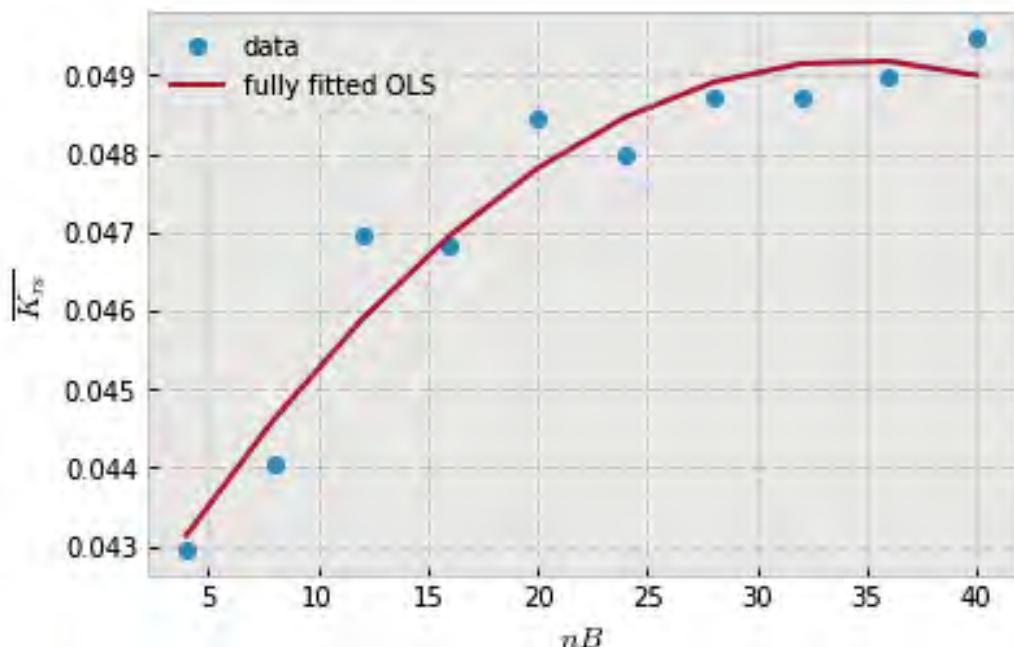
```

variable name: theta
model: poly1
r-squared: -0.6517451632042233
explained variance: -0.6494055309216382

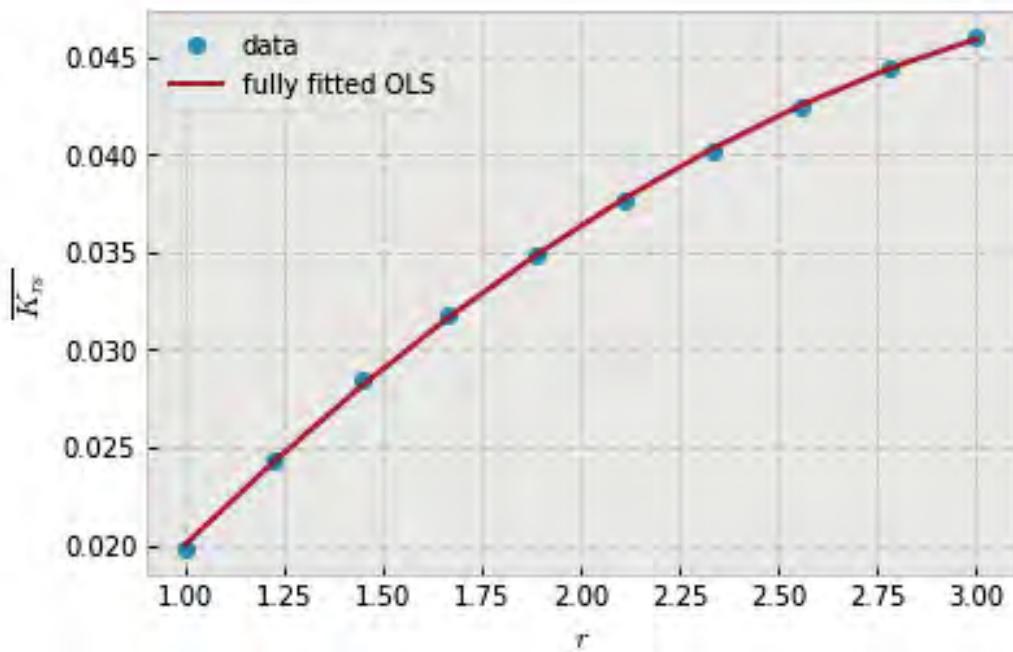
```



```
variable name: nB
model: poly2
r-squared: 0.8697475747245358
explained variance: 0.8699413128268201
```



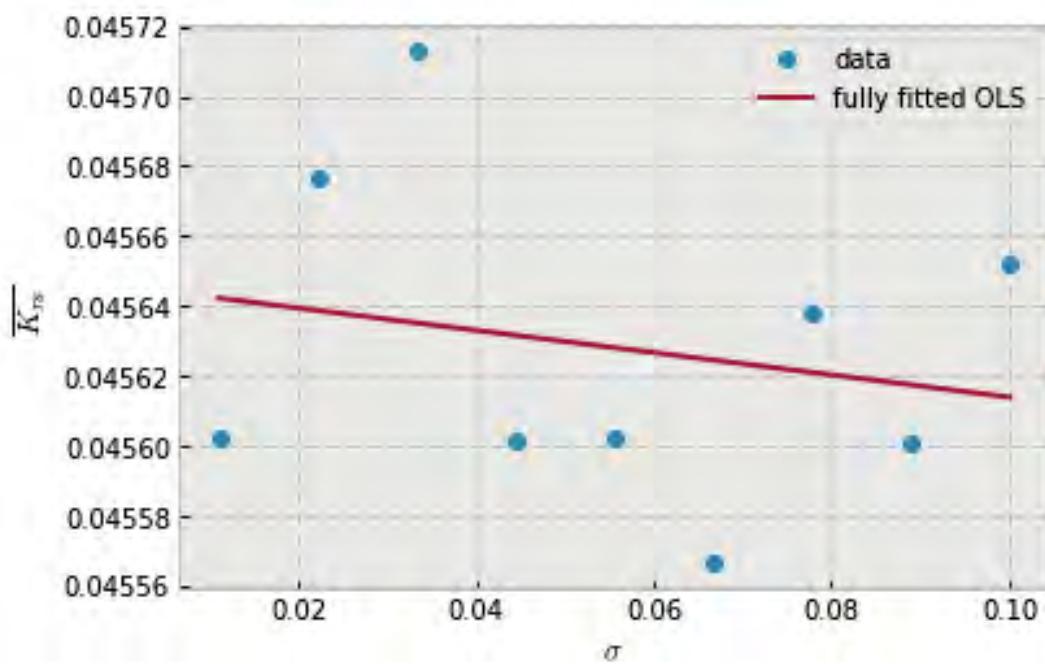
```
variable name: r
model: poly2
r-squared: 0.9987356371805631
explained variance: 0.9987394969820559
```



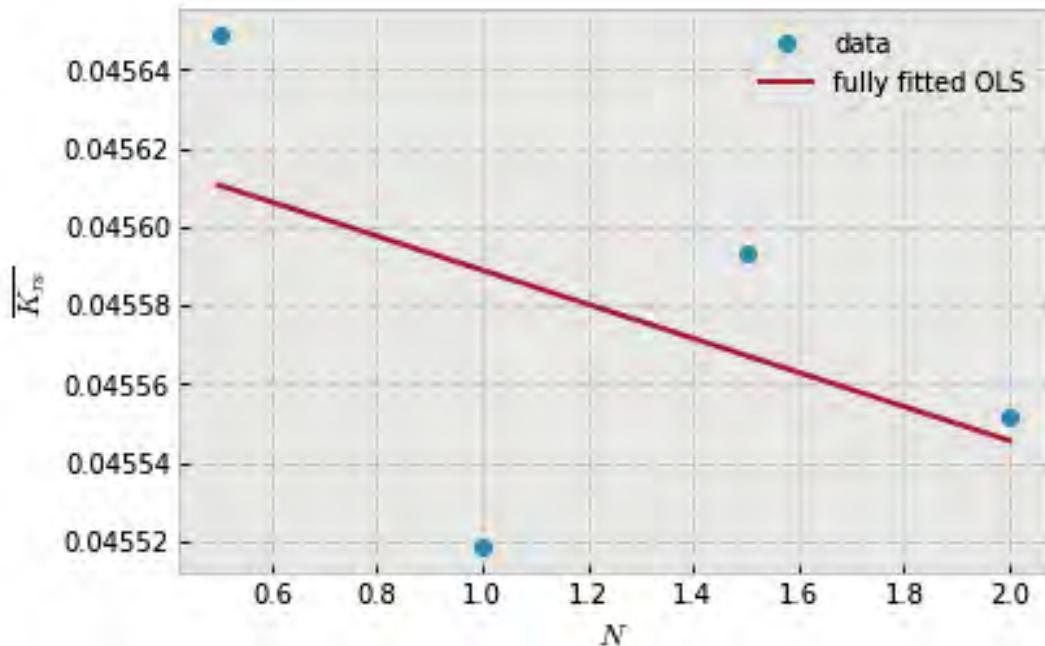
```

variable name: sigma
model: poly1
r-squared: -0.5997367417597548
explained variance: -0.5989631389350947

```



```
variable name: N
model: poly1
r-squared: -1.9779395716877812
explained variance: -1.7887436646216486
```

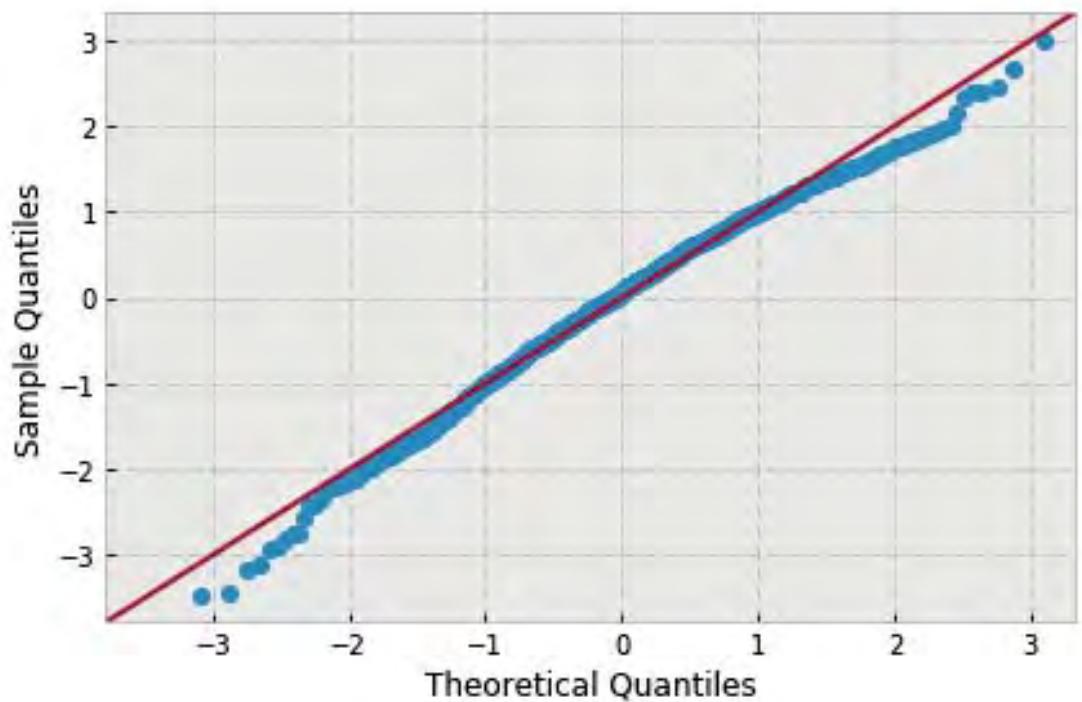
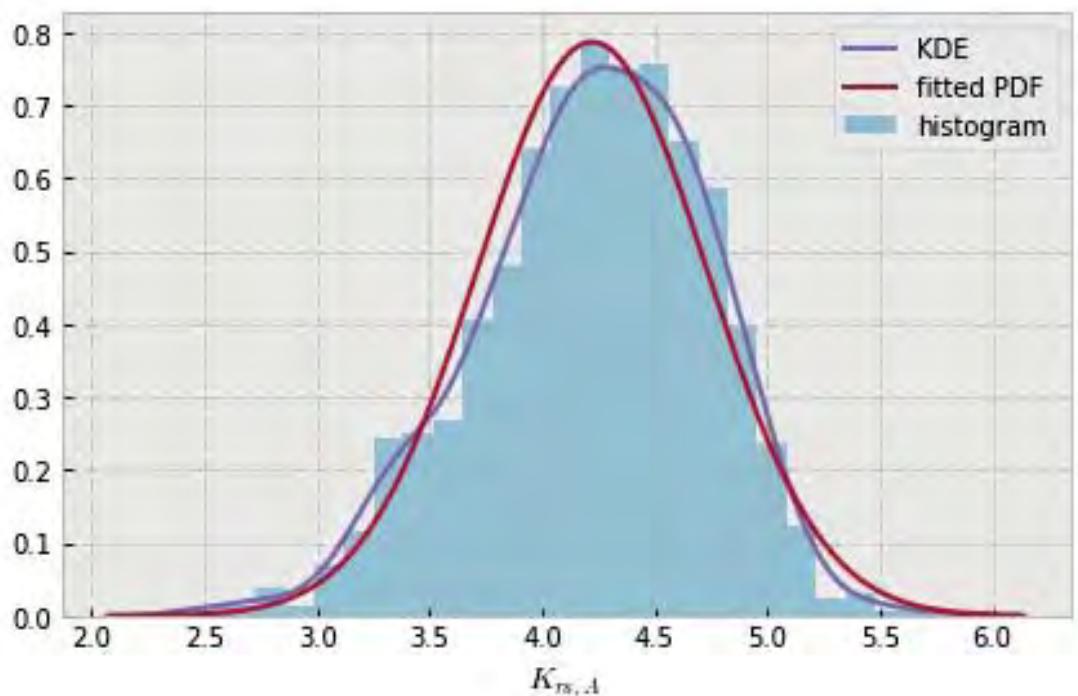


16 Half mean distance between roots

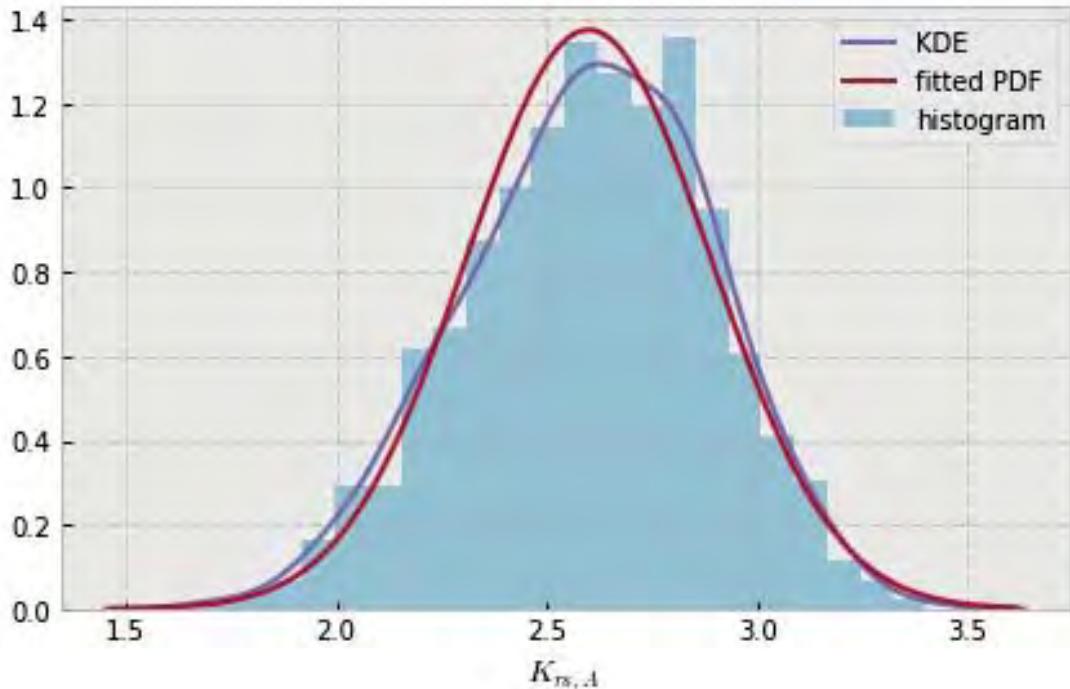
16.1 Probability distributions

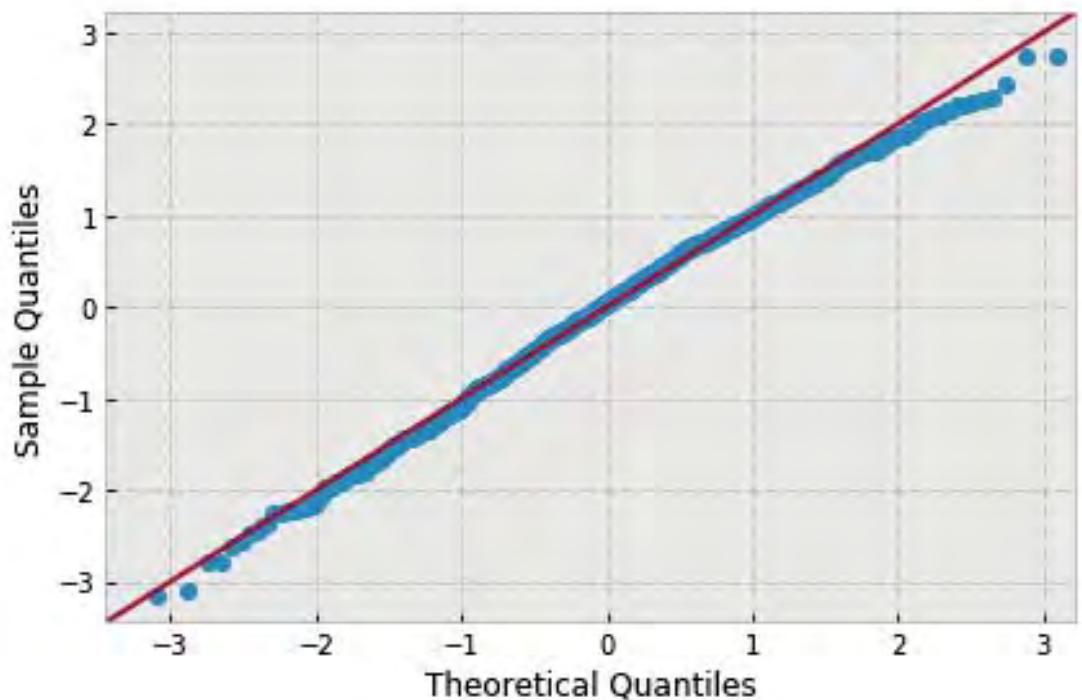
```
In [37]: plot_dist('half_mean_distance_between_roots')

variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=4.22114630996425, scale=0.507067135344299)
DescribeResult(nobs=1000, minmax=(2.4675062026804233, 5.7395870915673814)
               mean=4.2211463099642499, variance=0.25737445420047417,
               skewness=-0.37322431157225433, kurtosis=0.022369759610008977)
```

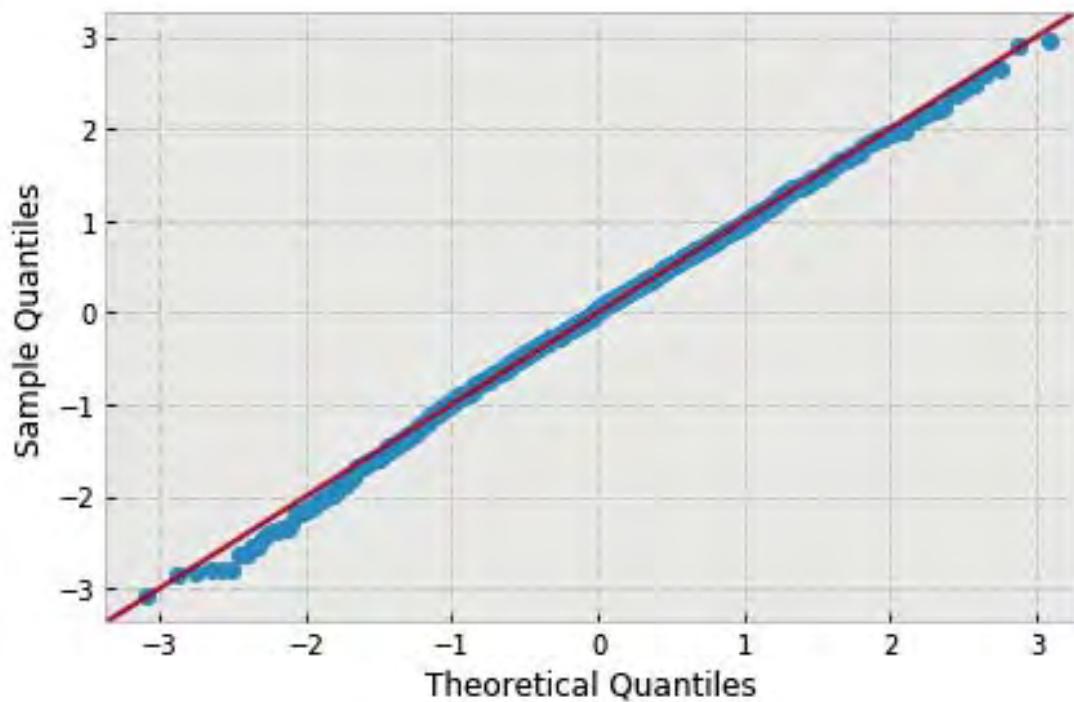
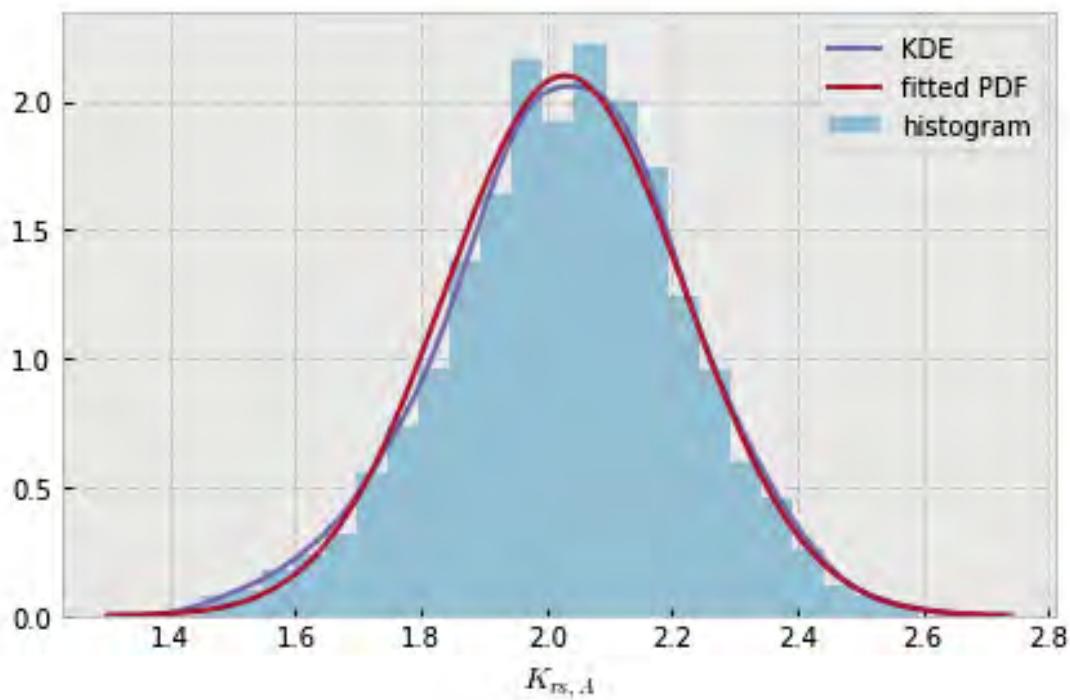


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=2.599485752649209, scale=0.2902010240582045)
DescribeResult(nobs=1000, minmax=(1.6856867315480197, 3.3994216306837743)
               mean=2.5994857526492088, variance=0.084300935299730331,
               skewness=-0.20460023491780407, kurtosis=-0.24728426040572682)
```

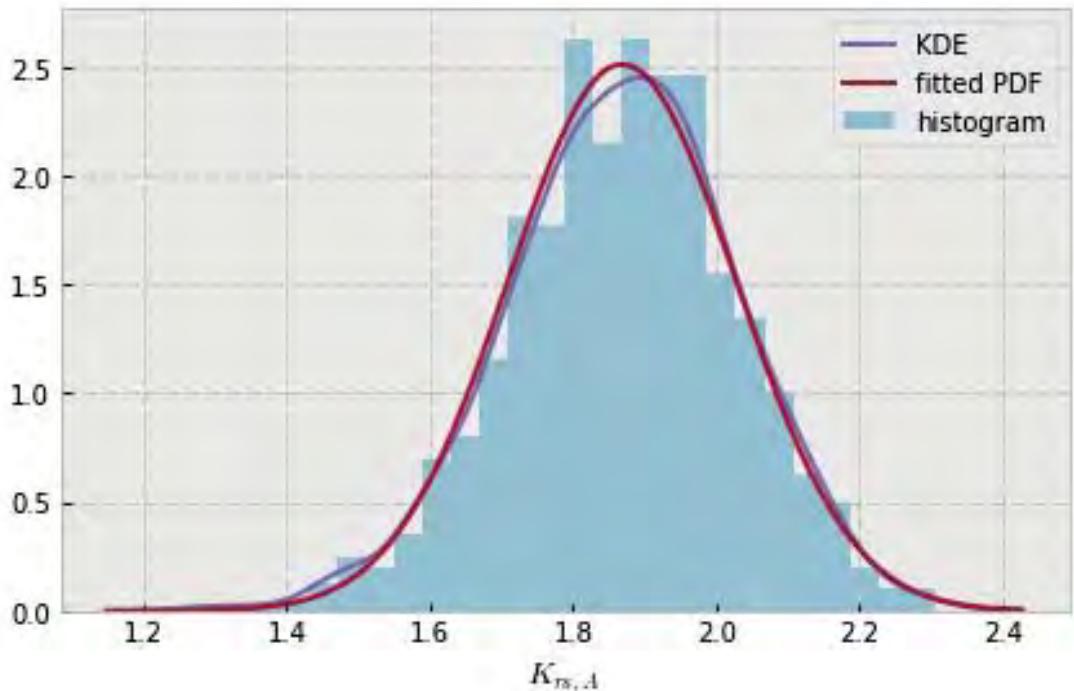


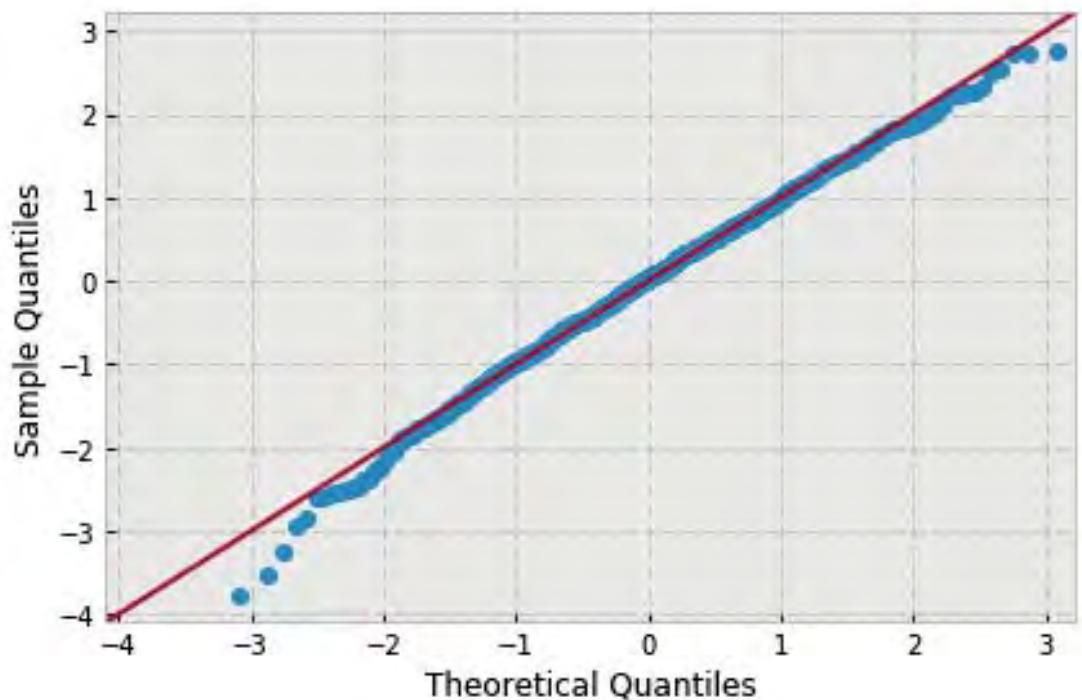


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=2.0295879587479693, scale=0.1900046788095221)
DescribeResult(nobs=1000, minmax=(1.4470735408149125, 2.5913887865649992)
               mean=2.0295879587479693, variance=0.036137915885395049,
               skewness=-0.14754341635305934, kurtosis=0.01844718076865437)
```

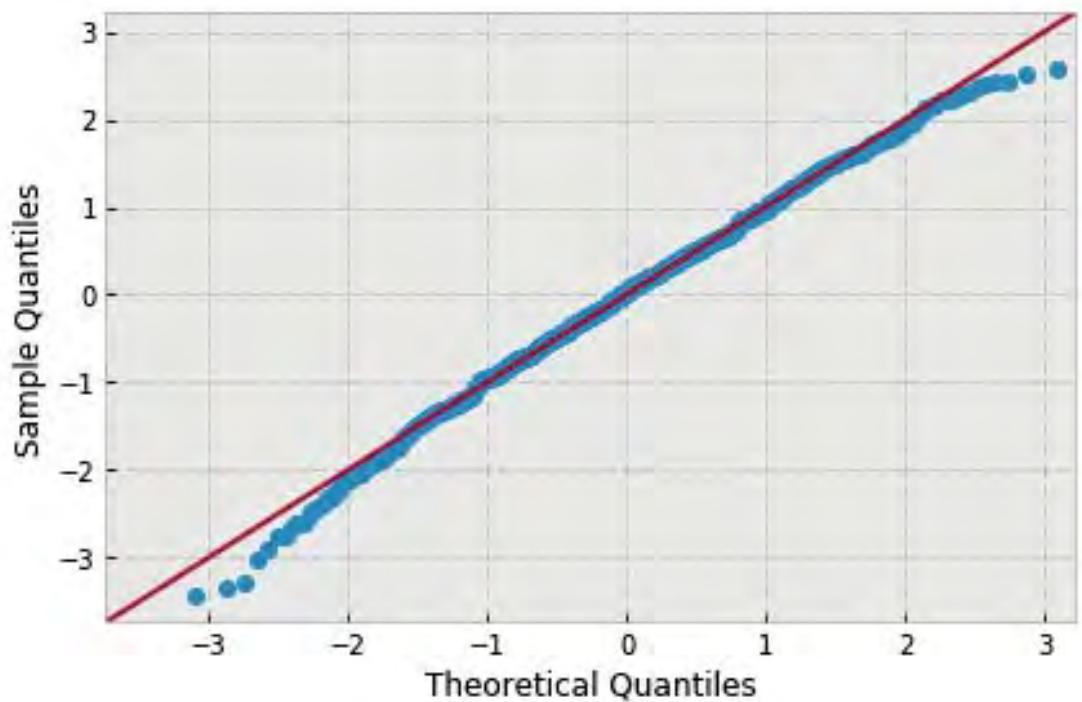
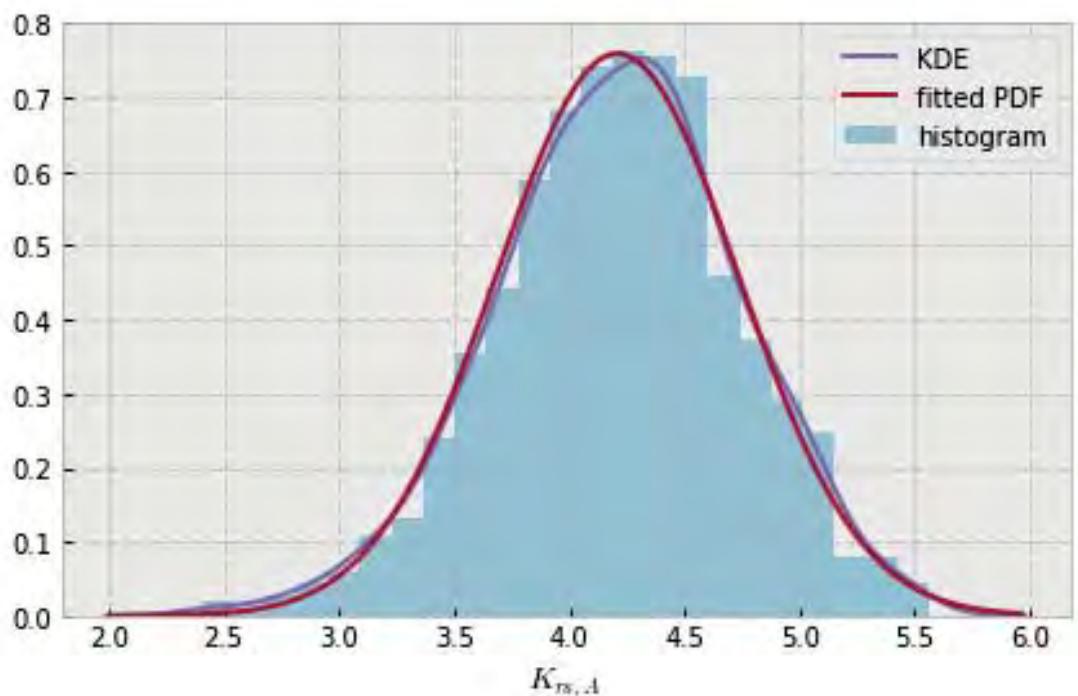


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=1.868453558808494, scale=0.15877276564561696)
DescribeResult(nobs=1000, minmax=(1.2703986993159275, 2.3062842928853287)
               mean=1.8684535588084941, variance=0.025234025135893894,
               skewness=-0.21744262354647687, kurtosis=0.1644553496021528)
```

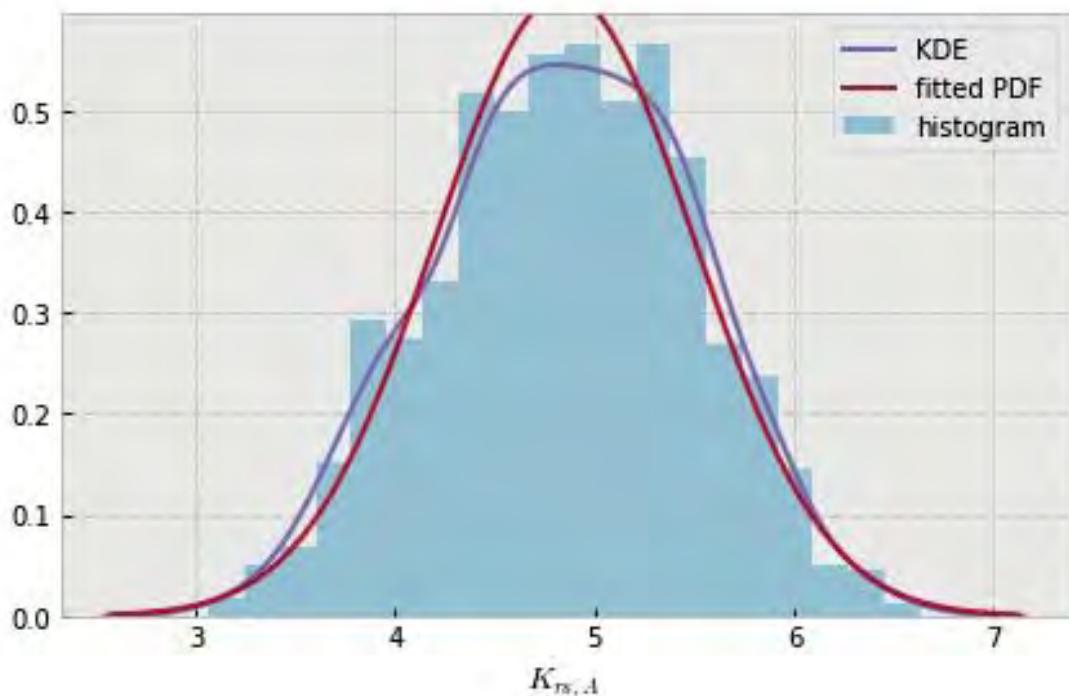


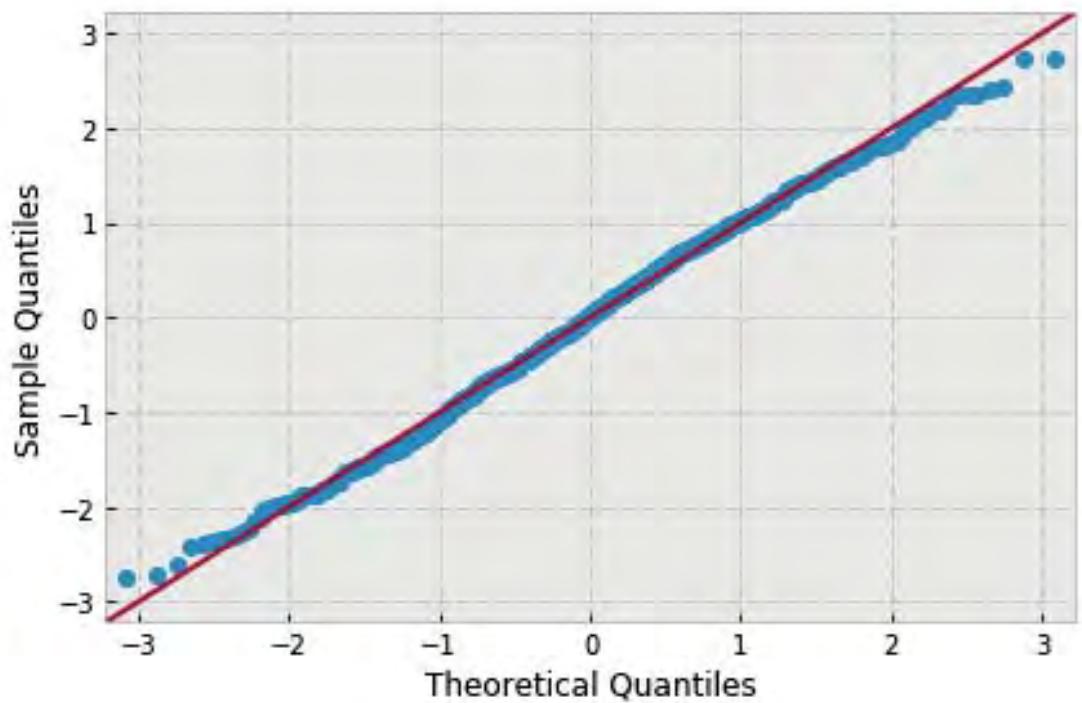


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=4.209383575851395, scale=0.525450622104719)
DescribeResult(nobs=1000, minmax=(2.4032267722366583, 5.5649636045227631)
               mean=4.2093835758513949, variance=0.27637473100123744,
               skewness=-0.2069145931485395, kurtosis=0.08889031999675101)
```

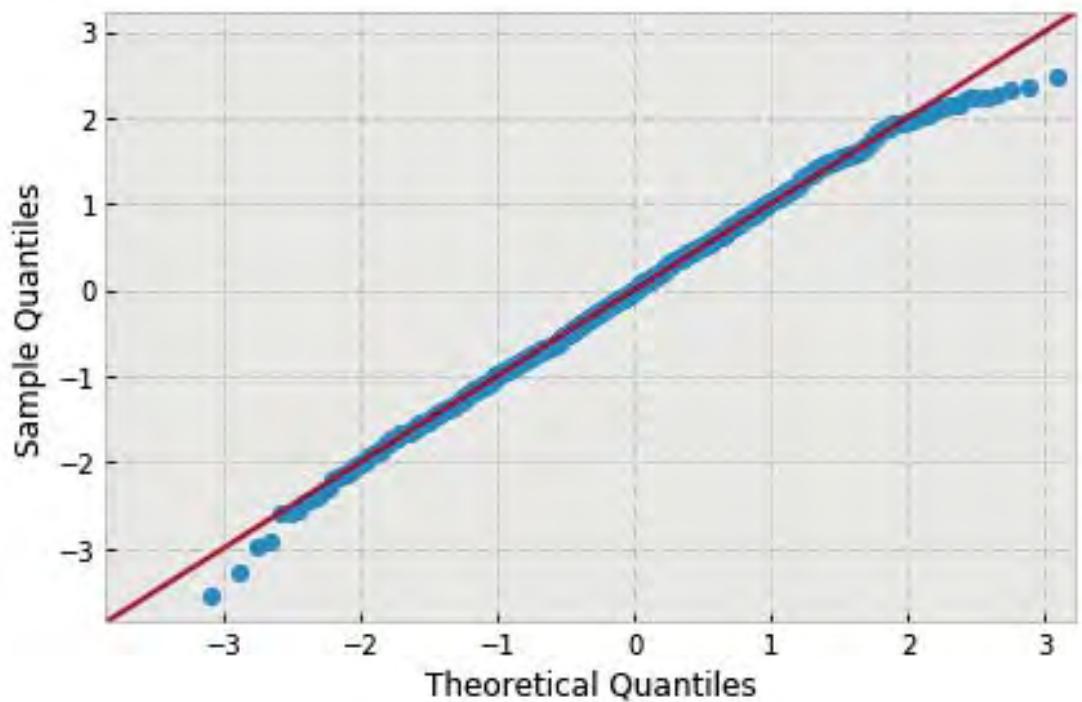
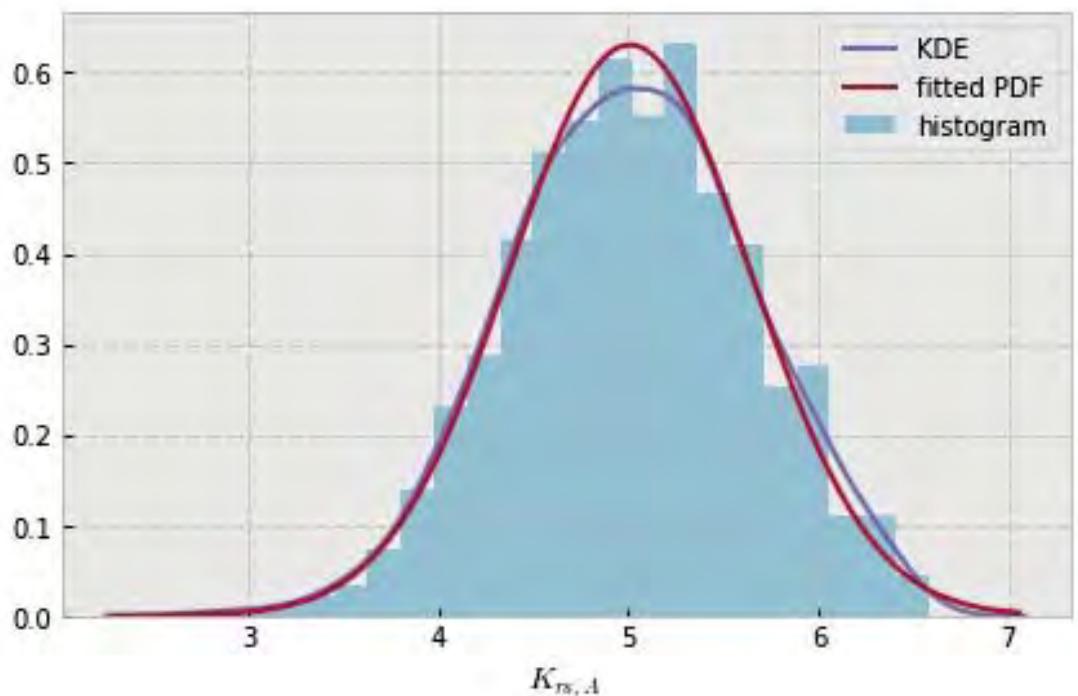


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=4.850365146449756, scale=0.6484199677526701)
DescribeResult(nobs=1000, minmax=(3.0679771118187471, 6.6314105260134824)
               mean=4.850365146449756, variance=0.42086932390427806,
               skewness=-0.0772225933759459, kurtosis=-0.44273557916022055)
```

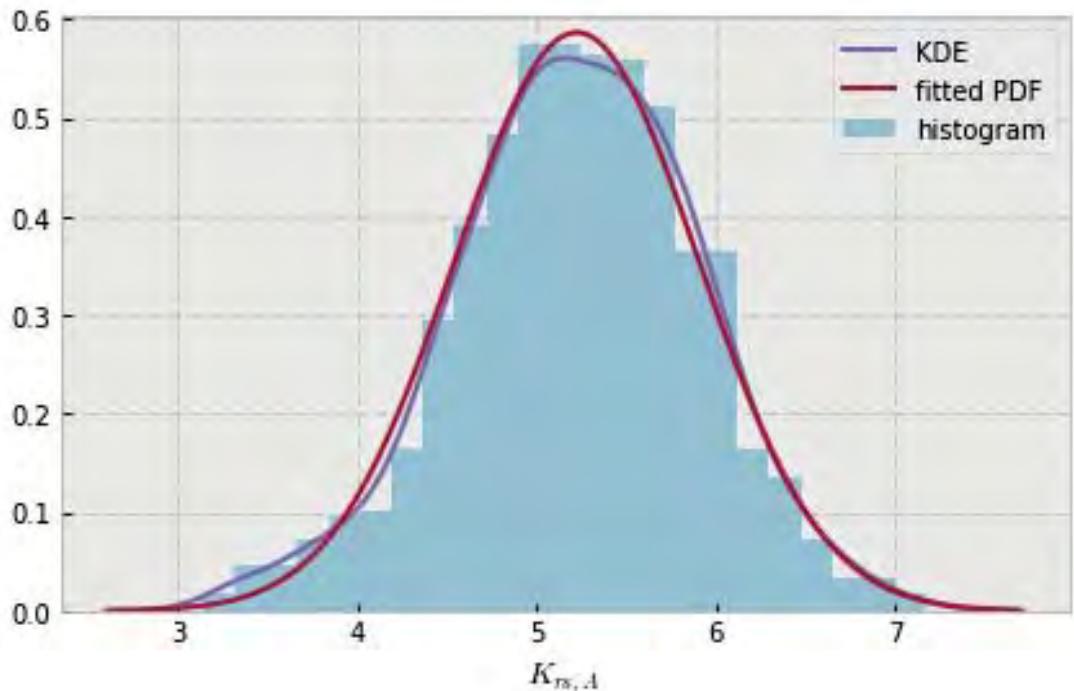


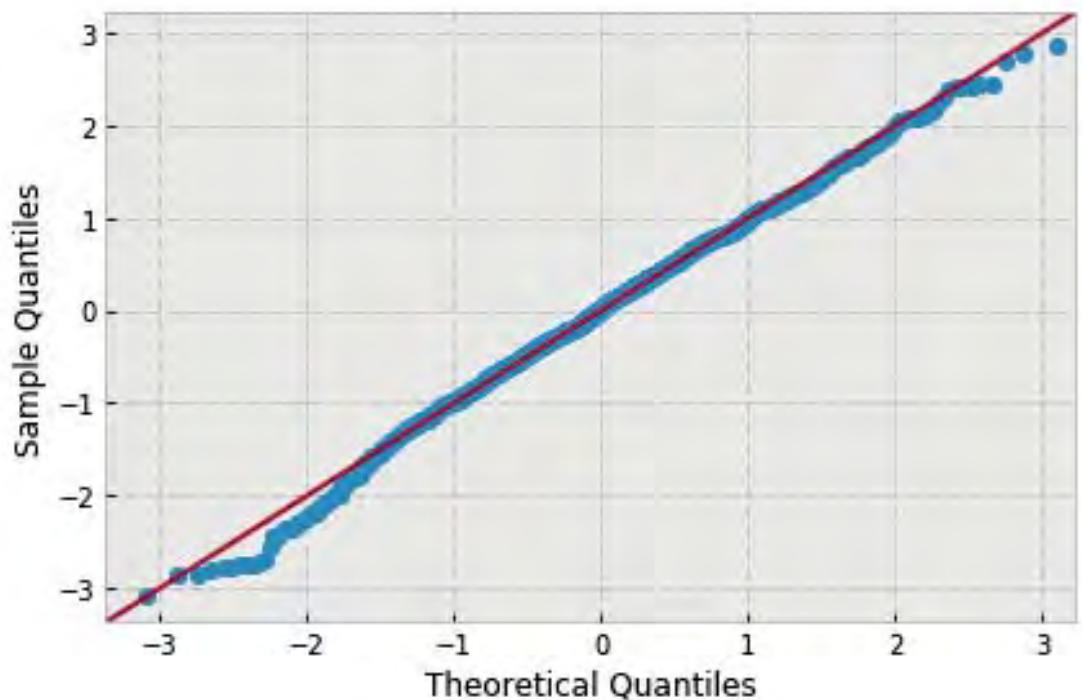


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=5.009237448109501, scale=0.6327855809005478)
DescribeResult(nobs=1000, minmax=(2.7611812691061632, 6.579483255593872)
               mean=5.0092374481095012, variance=0.40081840980544919,
               skewness=-0.10492953750511379, kurtosis=-0.18179643852701455)
```

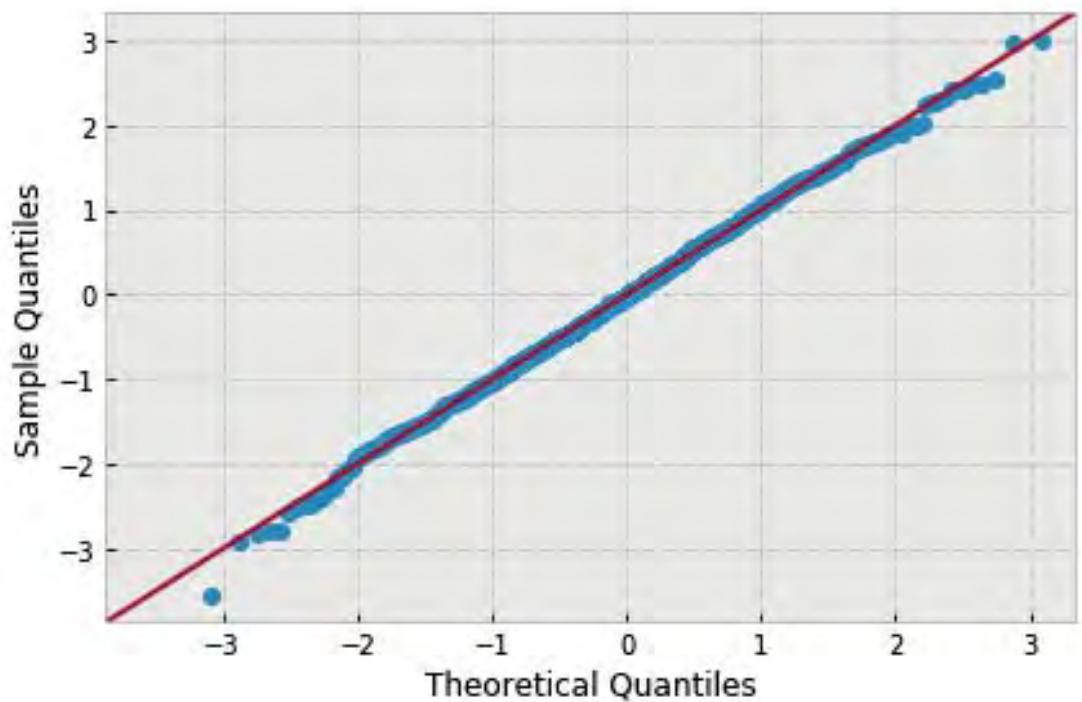
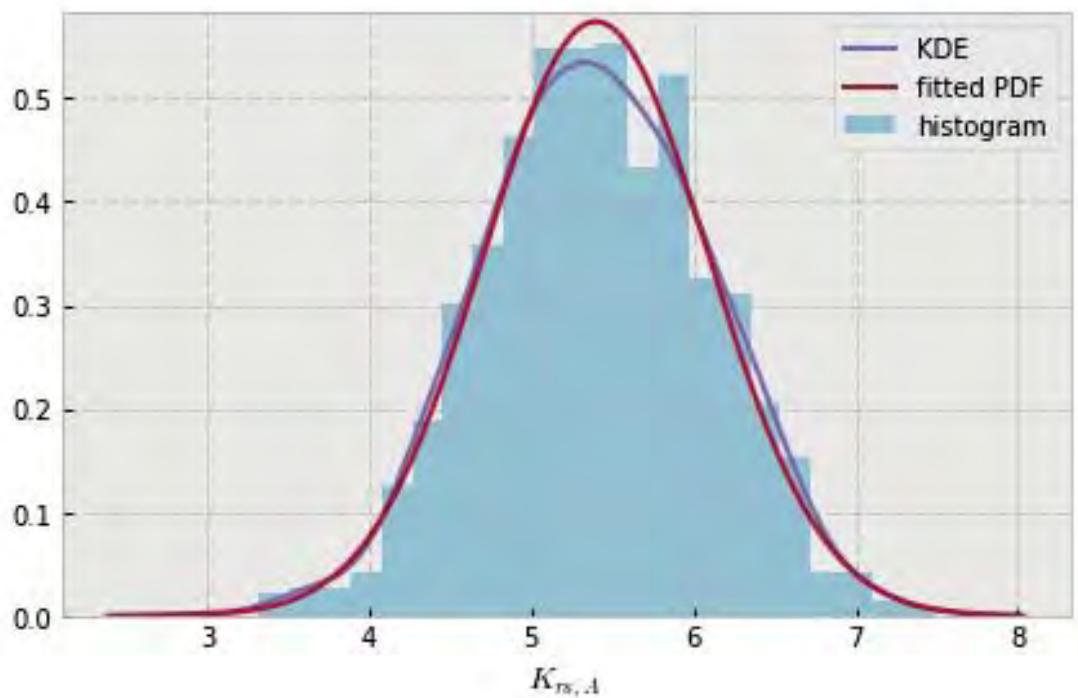


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=5.223531165169826, scale=0.6796283092330648)
DescribeResult(nobs=1000, minmax=(3.1366183213676821, 7.1741853288832393)
               mean=5.2235311651698257, variance=0.46235699570670113,
               skewness=-0.1837494033025925, kurtosis=0.08311508494747155)
```

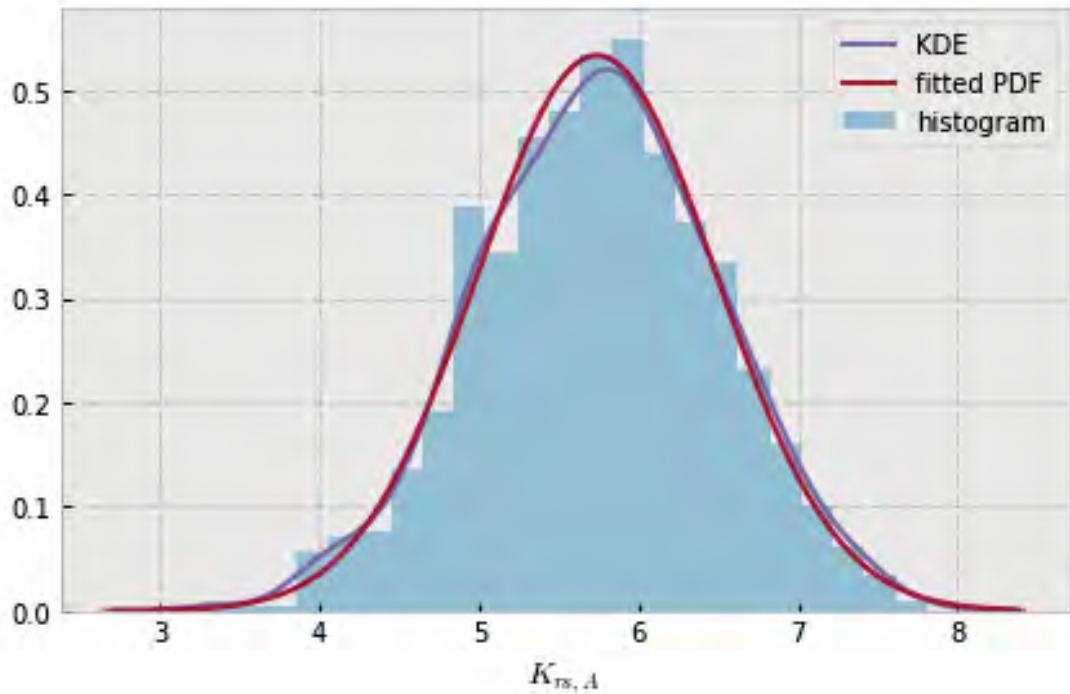


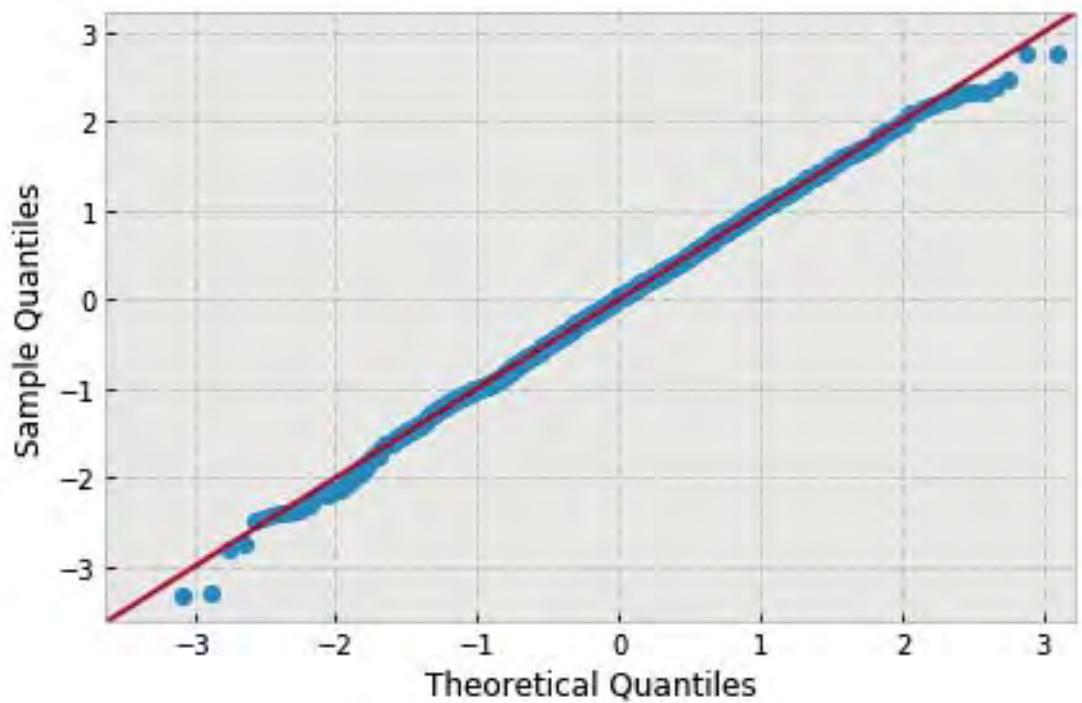


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=5.398794967355525, scale=0.6948533326089944)
DescribeResult(nobs=1000, minmax=(2.9388232709624176, 7.4829108579622936)
               mean=5.3987949673555251, variance=0.4833044582961219,
               skewness=-0.04321462925431739, kurtosis=-0.12957743803774902)
```

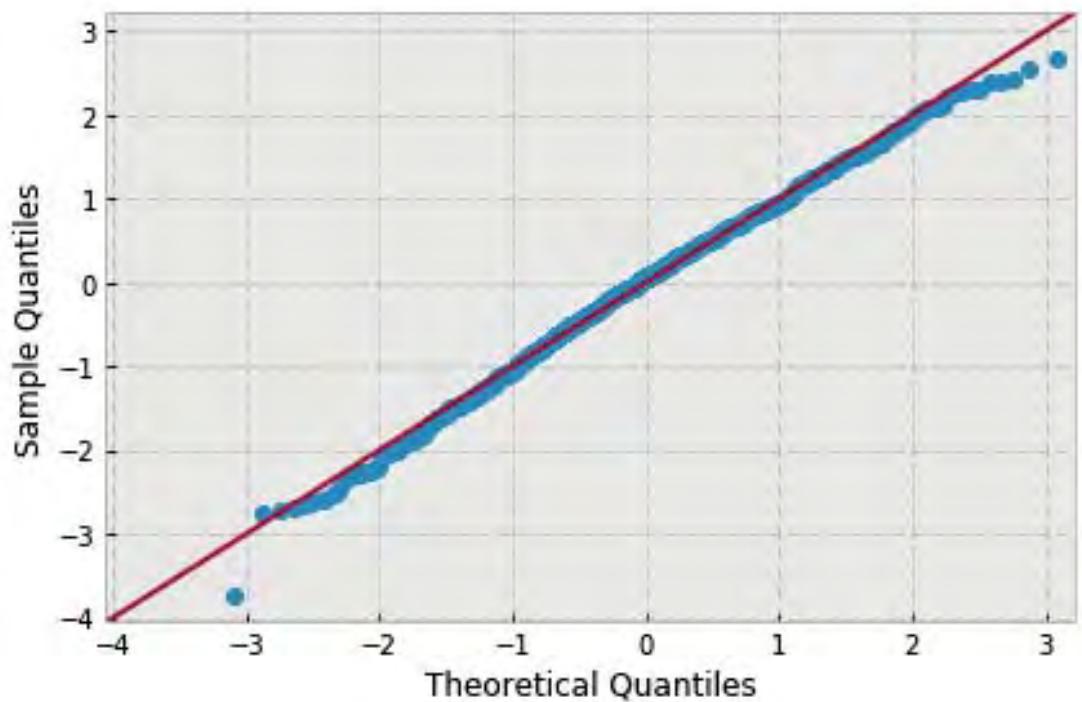
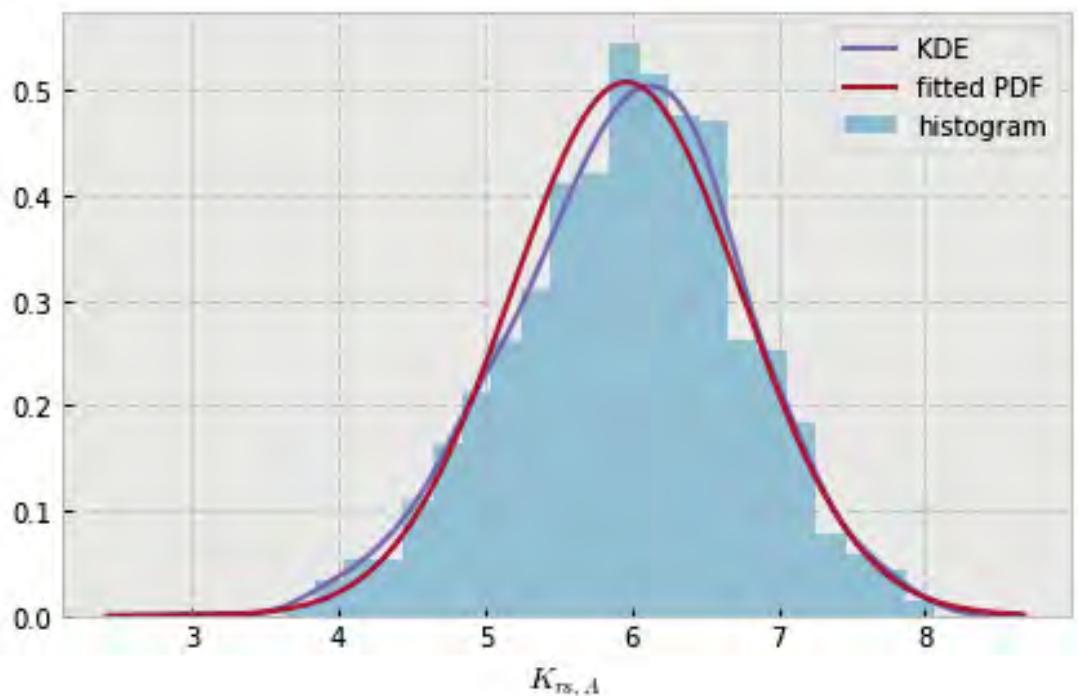


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=5.738321720134851, scale=0.7479695358645678)
DescribeResult(nobs=1000, minmax=(3.2577757509780589, 7.8133182555014793)
               mean=5.7383217201348513, variance=0.56001844502648357,
               skewness=-0.09092976858399848, kurtosis=-0.10851901459810964)
```

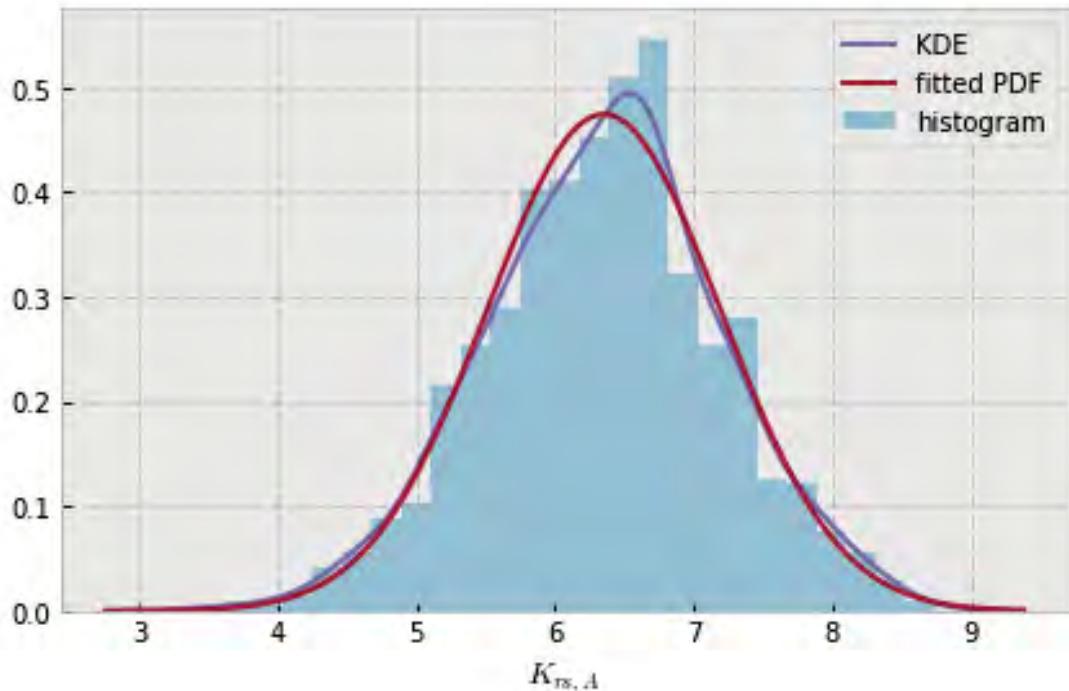


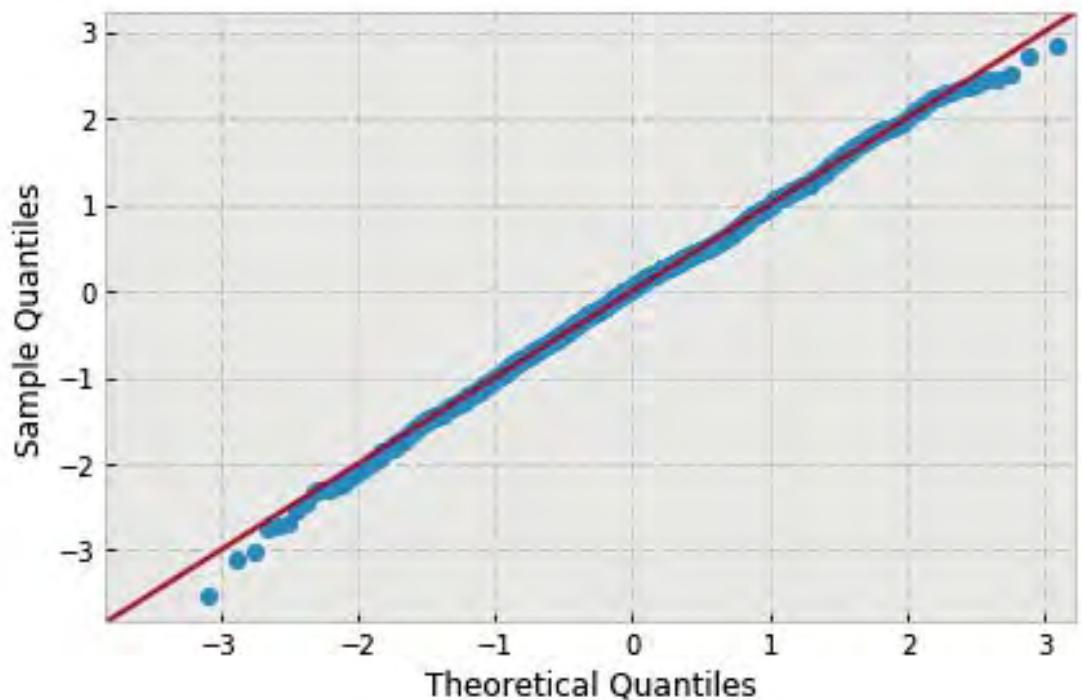


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=5.9623322649492625, scale=0.7852104945194603)
DescribeResult(nobs=1000, minmax=(3.0310270184110104, 8.063115599805462)
               mean=5.9623322649492625, variance=0.61717269339689229,
               skewness=-0.21550701897762536, kurtosis=-0.05250588052310157)
```

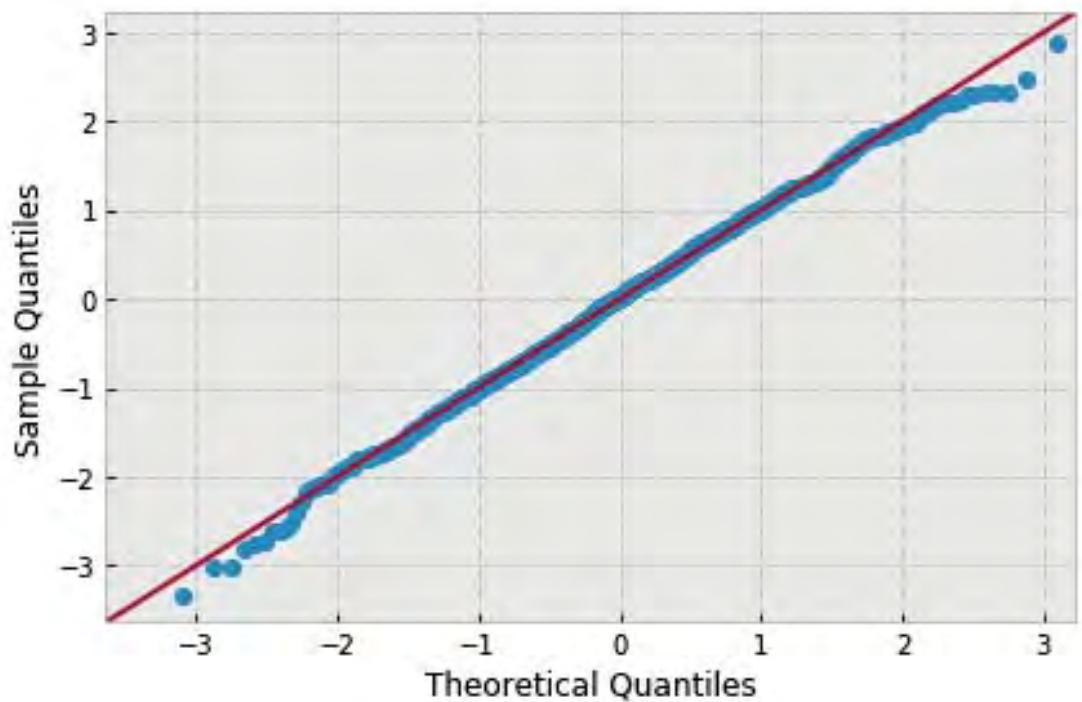
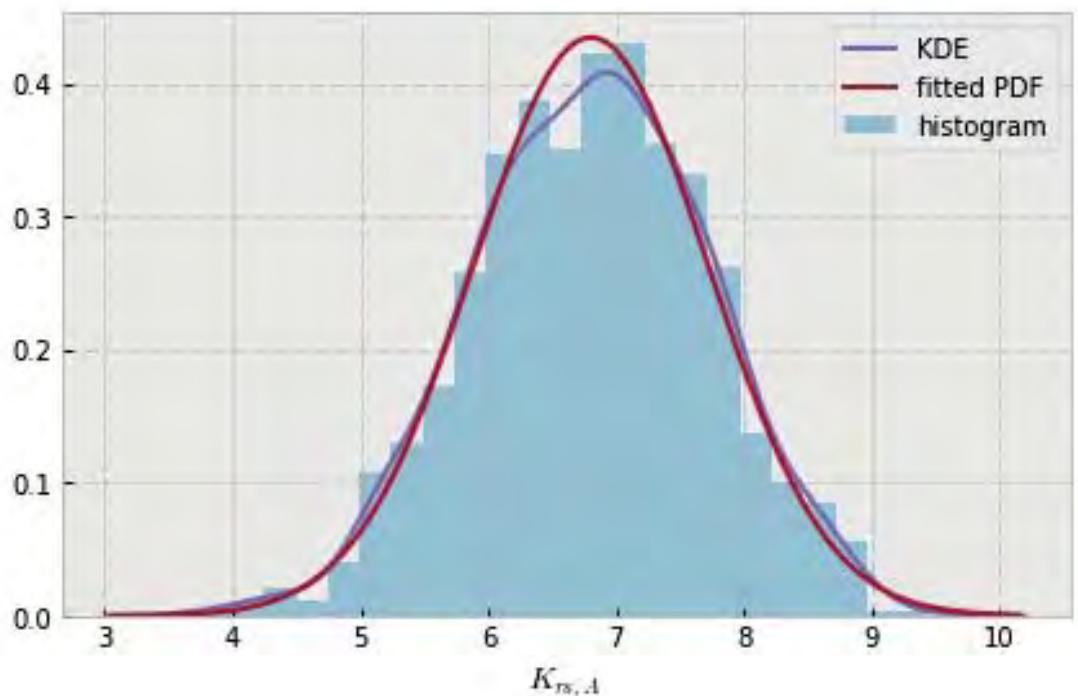


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=6.350196655763222, scale=0.8411761441096779)
DescribeResult(nobs=1000, minmax=(3.3954824771939944, 8.7396907047949597)
               mean=6.3501966557632219, variance=0.7082855910102358,
               skewness=-0.09894653404750207, kurtosis=0.016822147616560645)
```

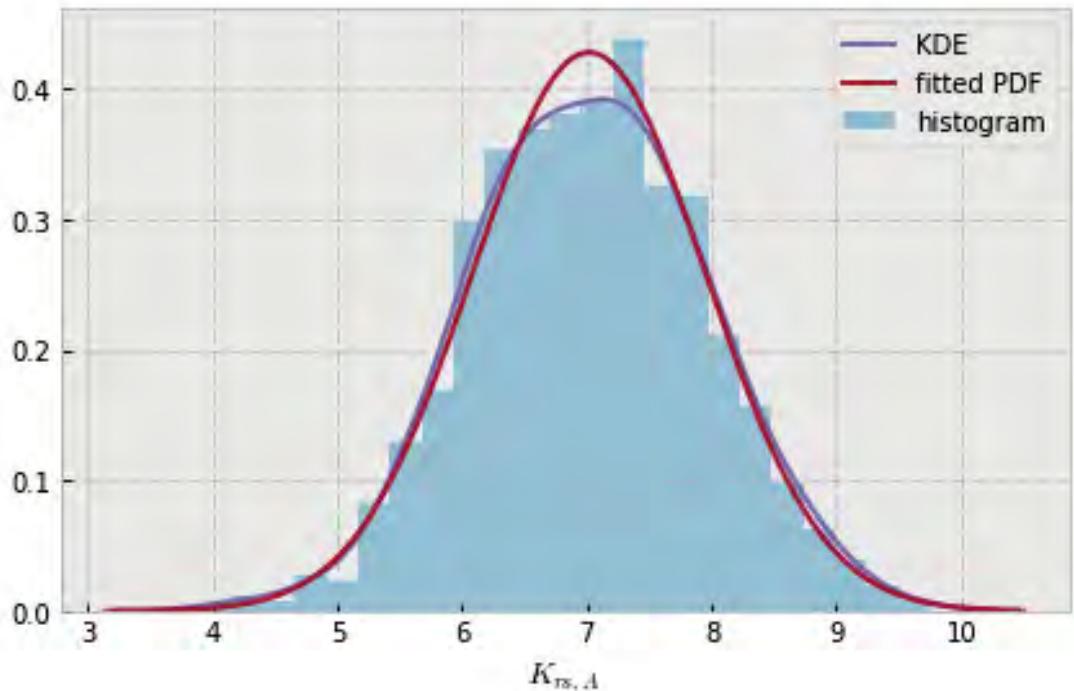


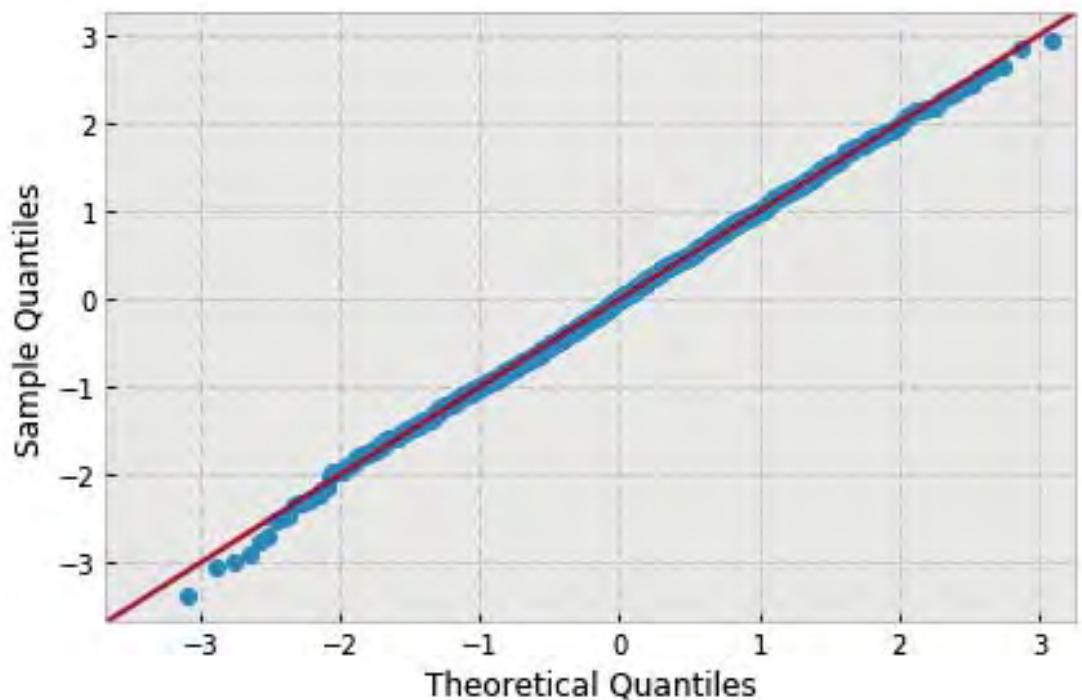


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=6.804886571952806, scale=0.9187529093393955)
DescribeResult(nobs=1000, minmax=(3.7515742807632106, 9.461799803428482)
               mean=6.8048865719528058, variance=0.84495186027988323,
               skewness=-0.11115397422314224, kurtosis=-0.17227647267426516)
```

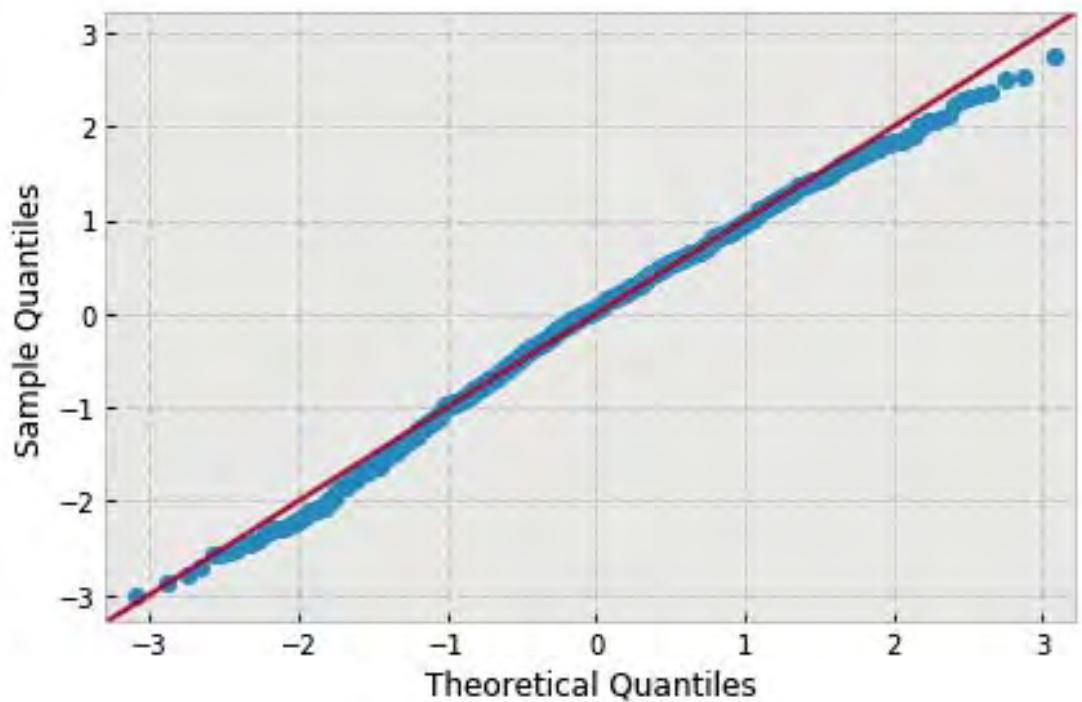
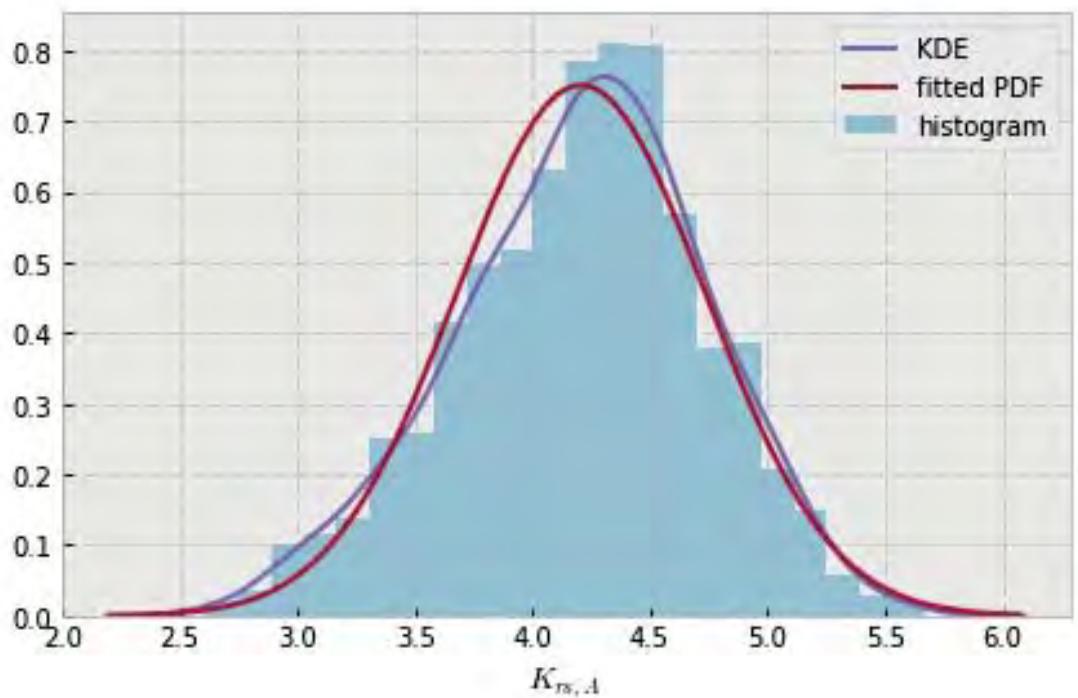


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=7.018910887139297, scale=0.9317163217516226)
DescribeResult(nobs=1000, minmax=(3.8917883212584754, 9.7541579098622542)
               mean=7.0189108871392971, variance=0.86896426848685993,
               skewness=-0.020570019890127324, kurtosis=-0.09656582655519053)
```

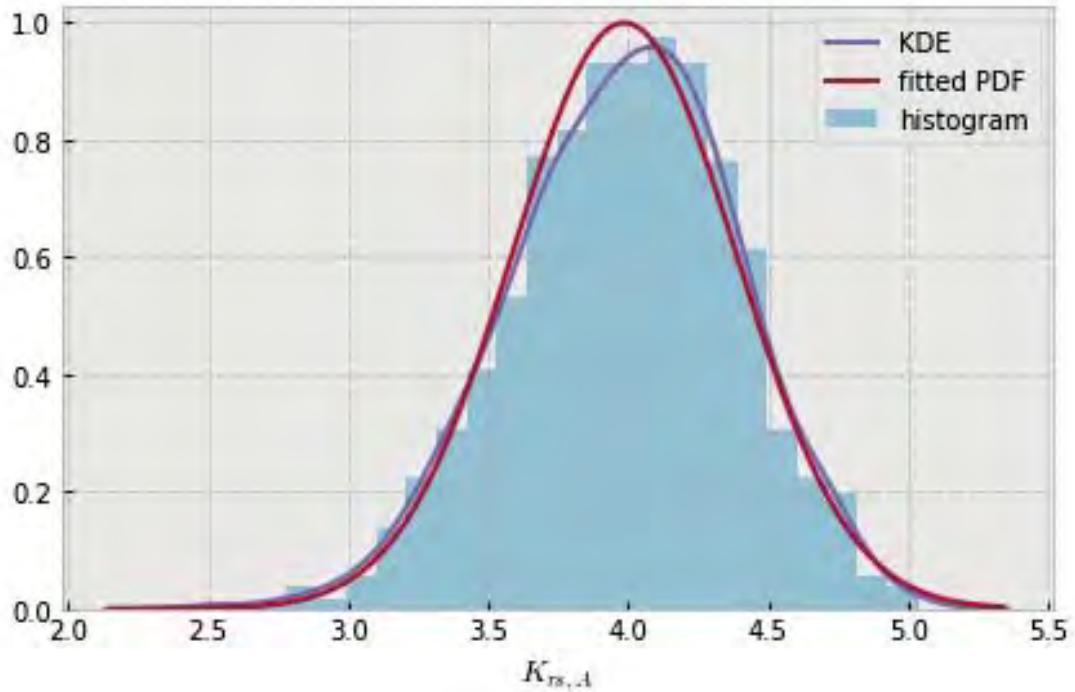


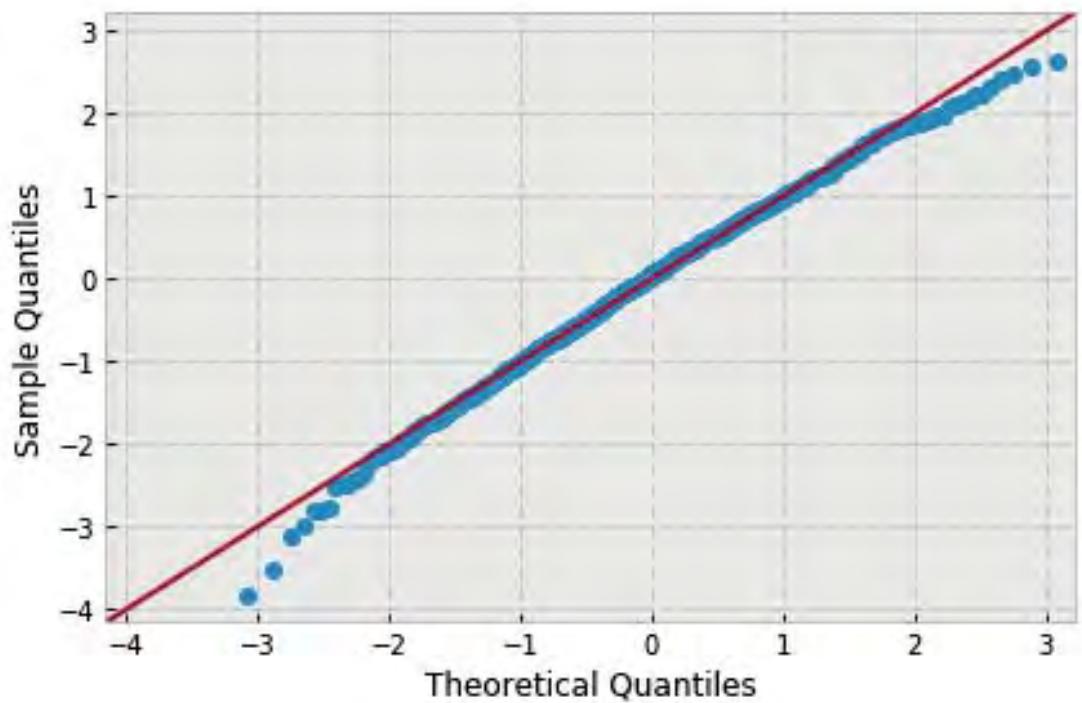


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=4.2058648285736, scale=0.5311314752207128)
DescribeResult(nobs=1000, minmax=(2.6104075662590538, 5.6695496966802006)
               mean=4.2058648285736, variance=0.28238302699712775,
               skewness=-0.2533974467767575, kurtosis=-0.15041114041119874)
```

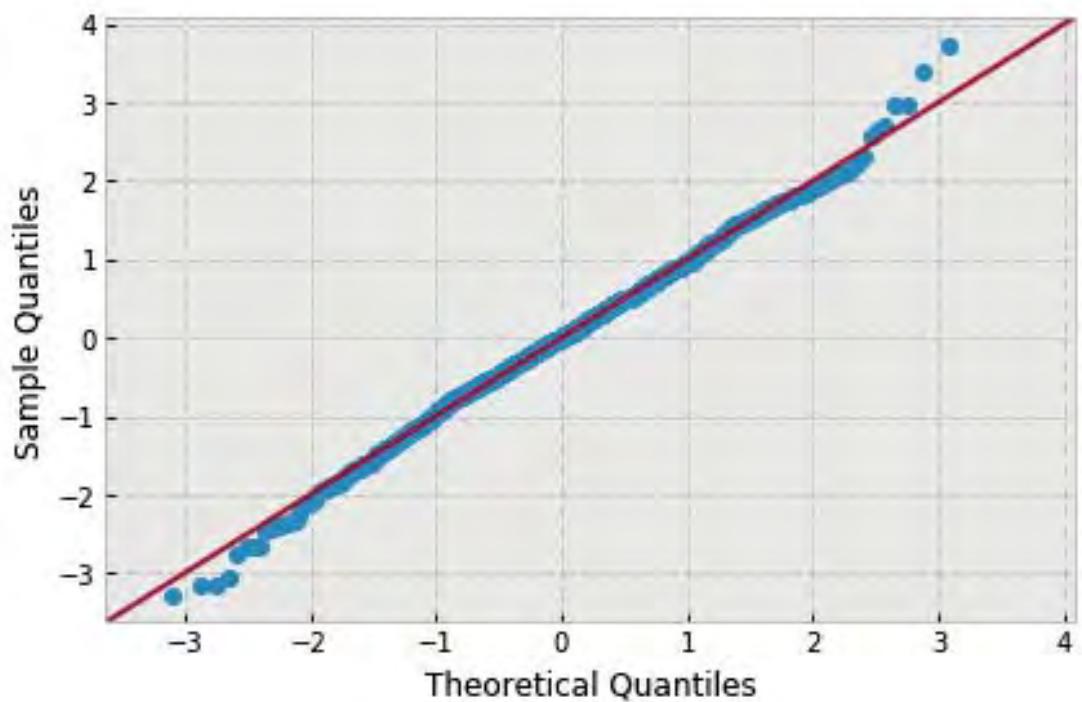
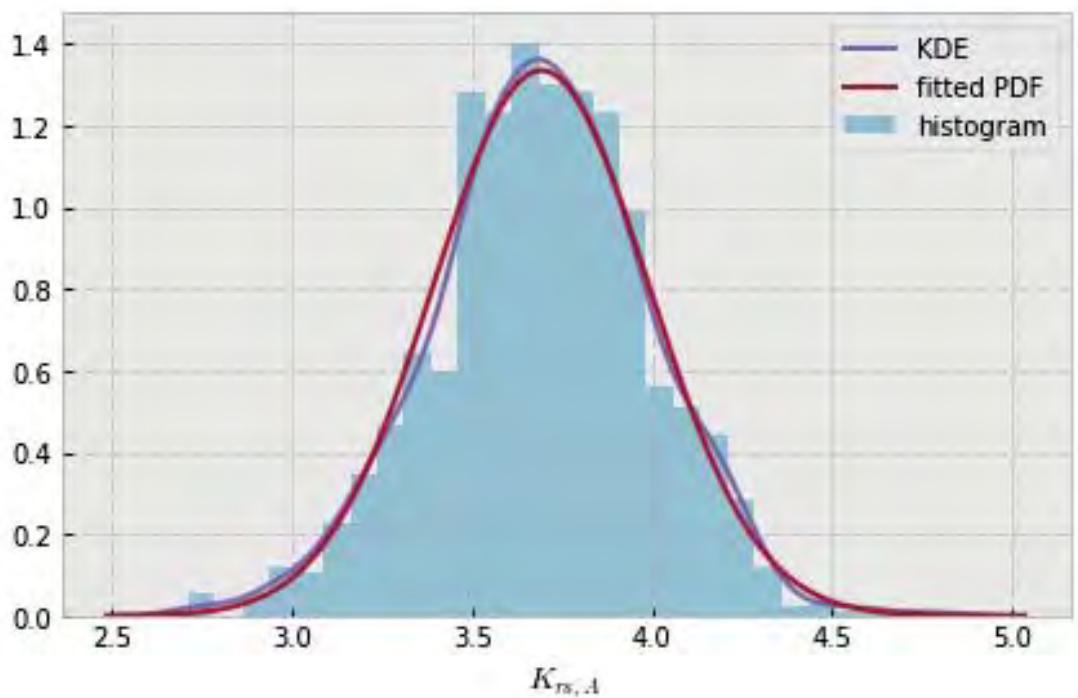


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=3.984758608760525, scale=0.39931507813638084)
DescribeResult(nobs=1000, minmax=(2.4547453524458547, 5.0338735524368641)
               mean=3.9847586087605249, variance=0.15961214377083477,
               skewness=-0.2478729687533784, kurtosis=0.029245646125278135)
```

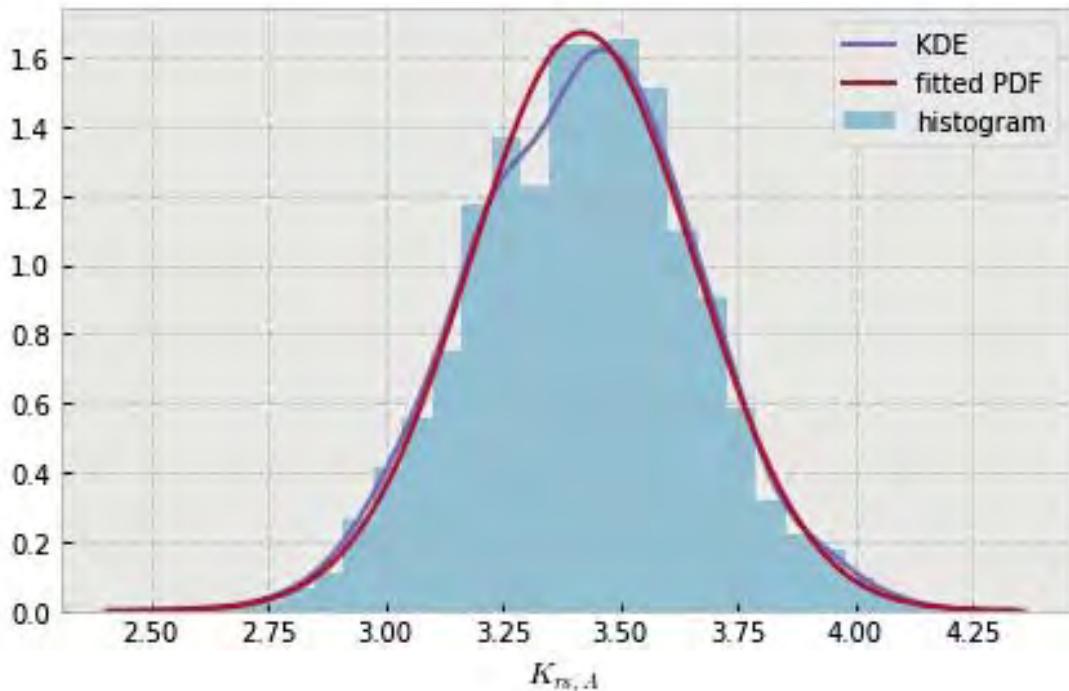


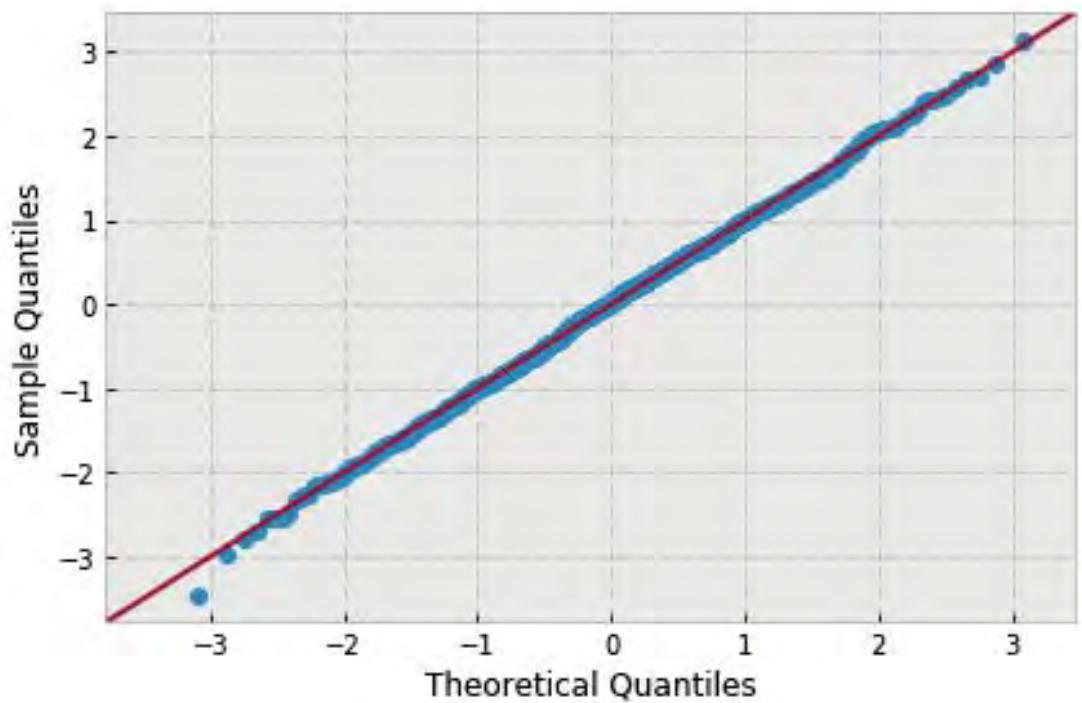


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=3.692268571074507, scale=0.2988142758872937)
DescribeResult(nobs=1000, minmax=(2.7140696289136512, 4.8066193268942028)
               mean=3.6922685710745071, variance=0.089379350824872542,
               skewness=-0.07422035929295794, kurtosis=0.29582458849808946)
```

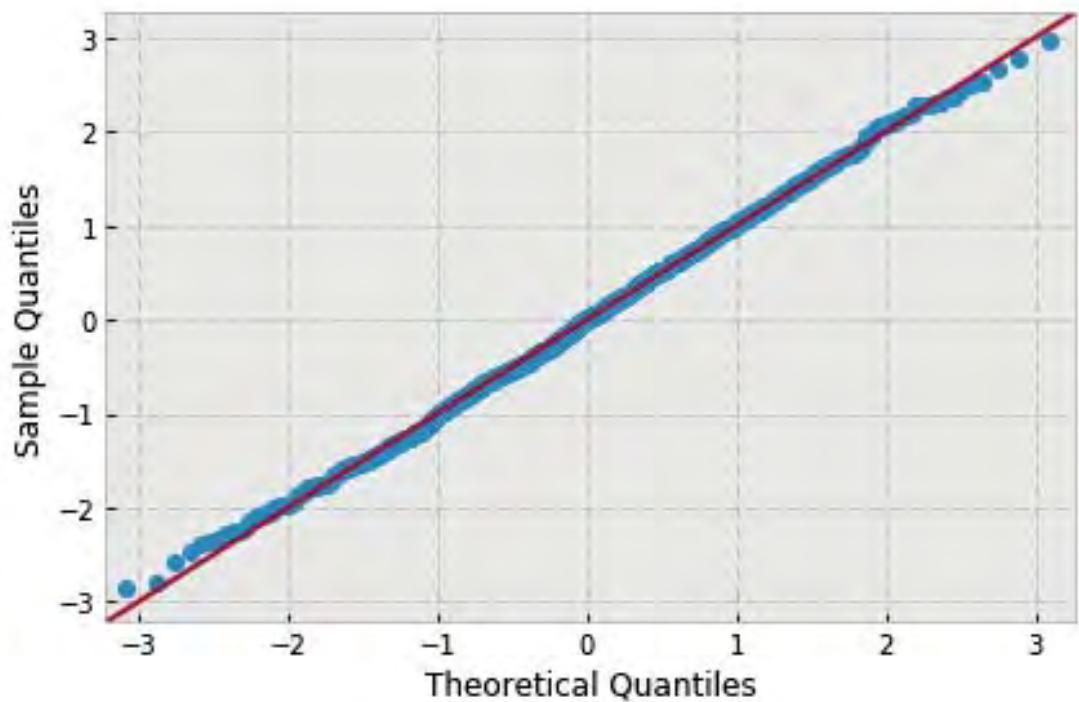
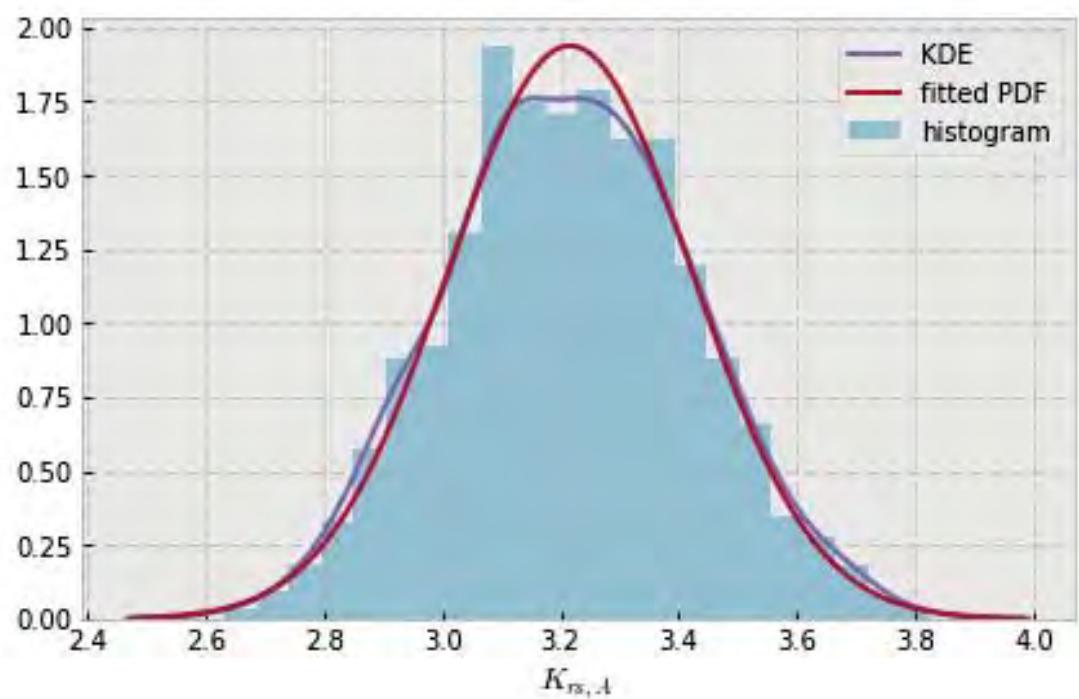


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=3.419399747651255, scale=0.2385369040632482)
DescribeResult(nobs=1000, minmax=(2.5968814518763041, 4.1662736092899824)
               mean=3.419399747651255, variance=0.056956811411490768,
               skewness=-0.04101250320746071, kurtosis=-0.06484791000410972)
```

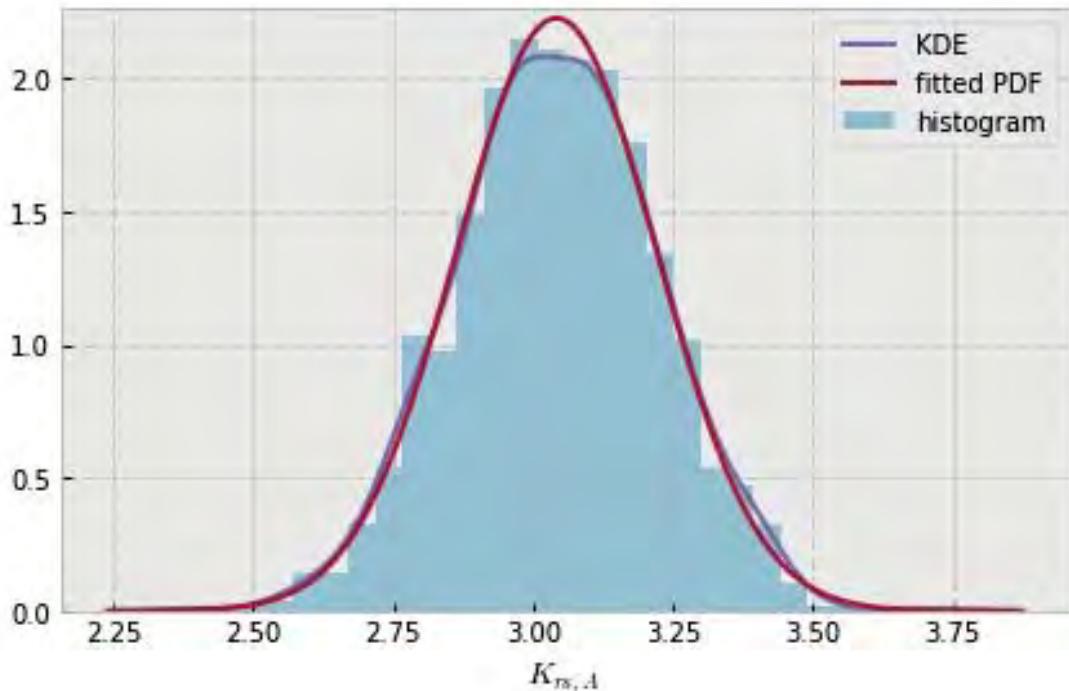


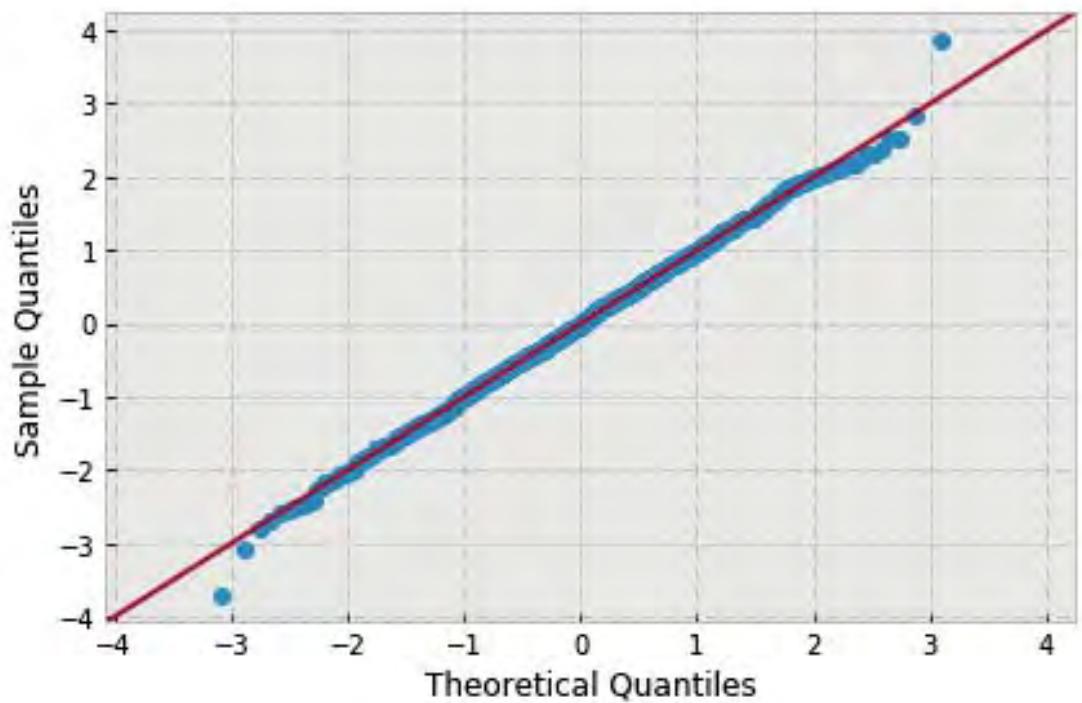


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=3.216105639738268, scale=0.20599864248600597)
DescribeResult(nobs=1000, minmax=(2.6317747983087507, 3.8275723846773984)
               mean=3.2161056397382679, variance=0.042477918624702002,
               skewness=0.05050222816730362, kurtosis=-0.28497655368431873)
```

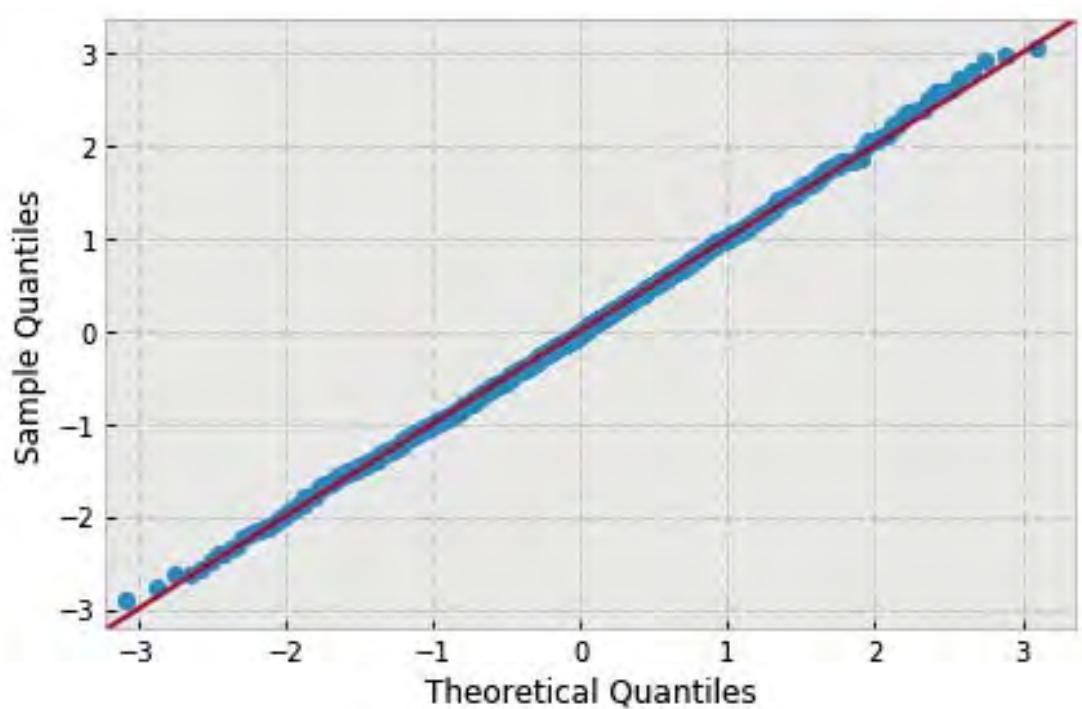
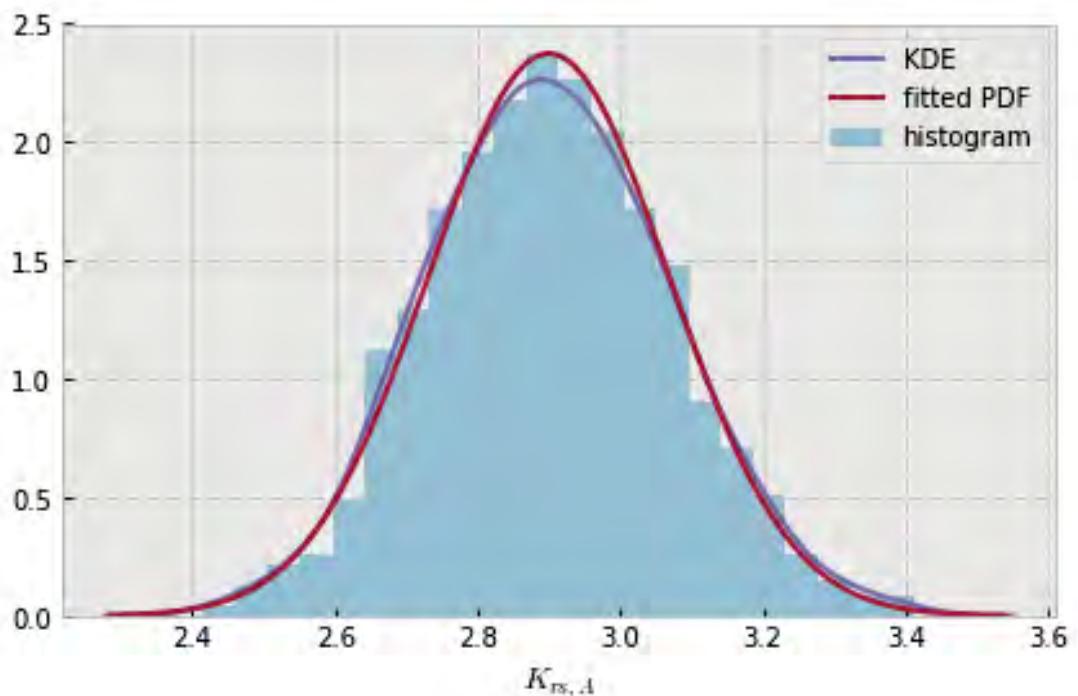


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=3.042002057705741, scale=0.1790845094042891)
DescribeResult(nobs=1000, minmax=(2.3806952359145548, 3.7327885037731843)
               mean=3.0420020577057412, variance=0.032103364873448358,
               skewness=-0.037573477586424306, kurtosis=0.02630169764423318)
```

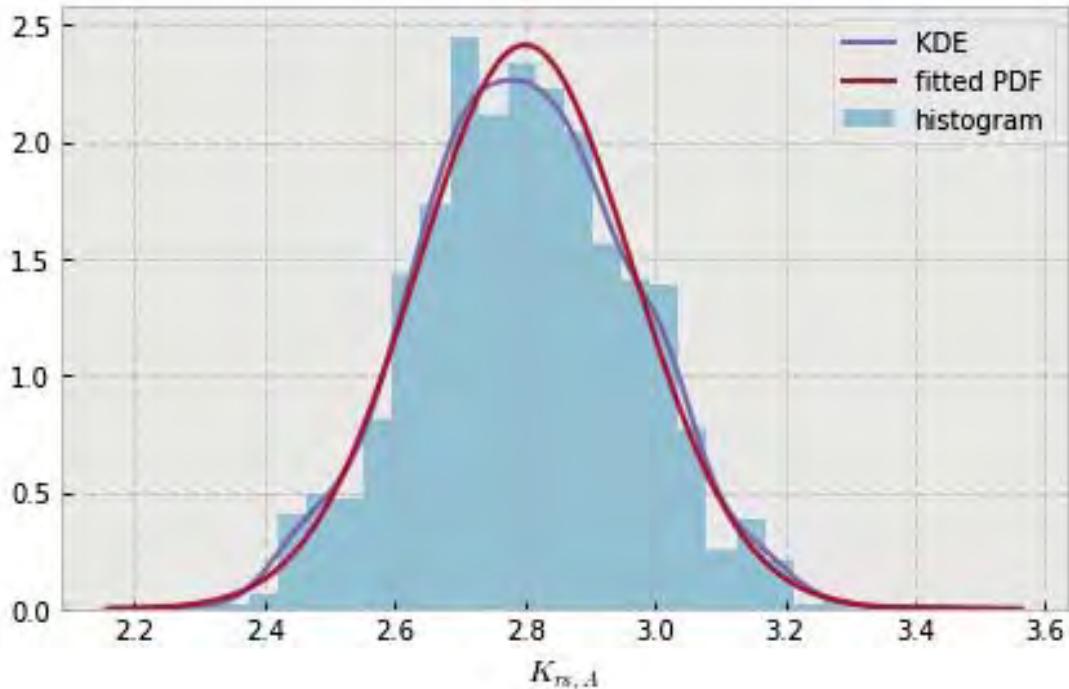


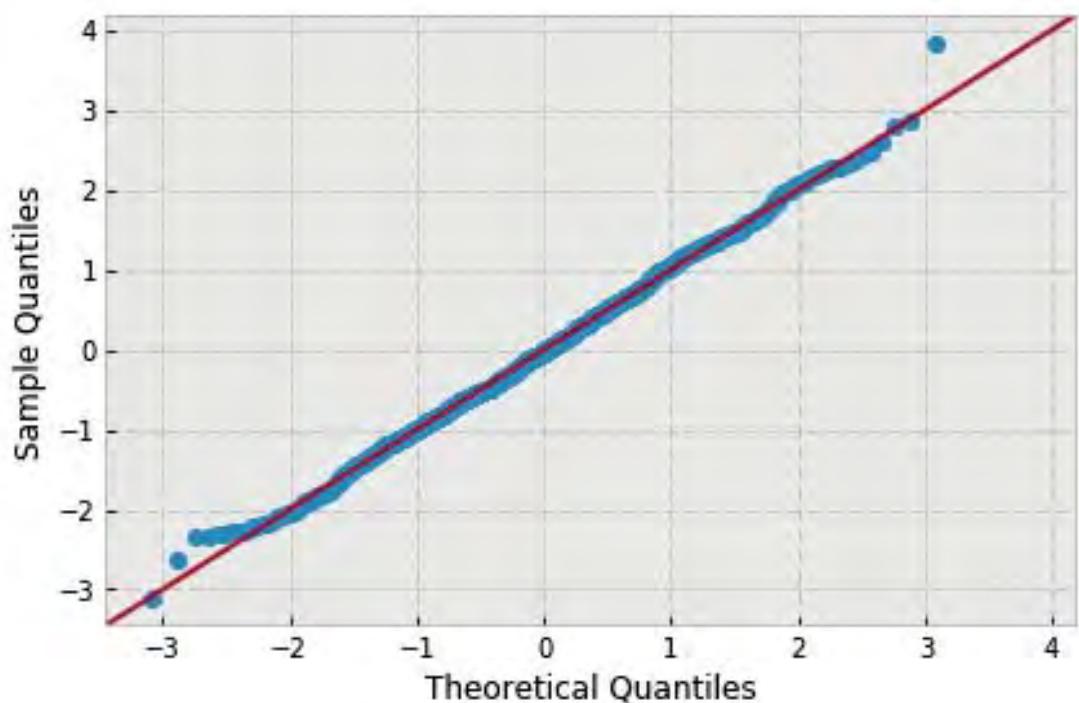


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=2.8995297970117035, scale=0.1681208369443936)
DescribeResult(nobs=1000, minmax=(2.4138995109574402, 3.4126674488236342)
               mean=2.8995297970117035, variance=0.028292908723606982,
               skewness=0.09529075656431303, kurtosis=-0.1051109850697185)
```

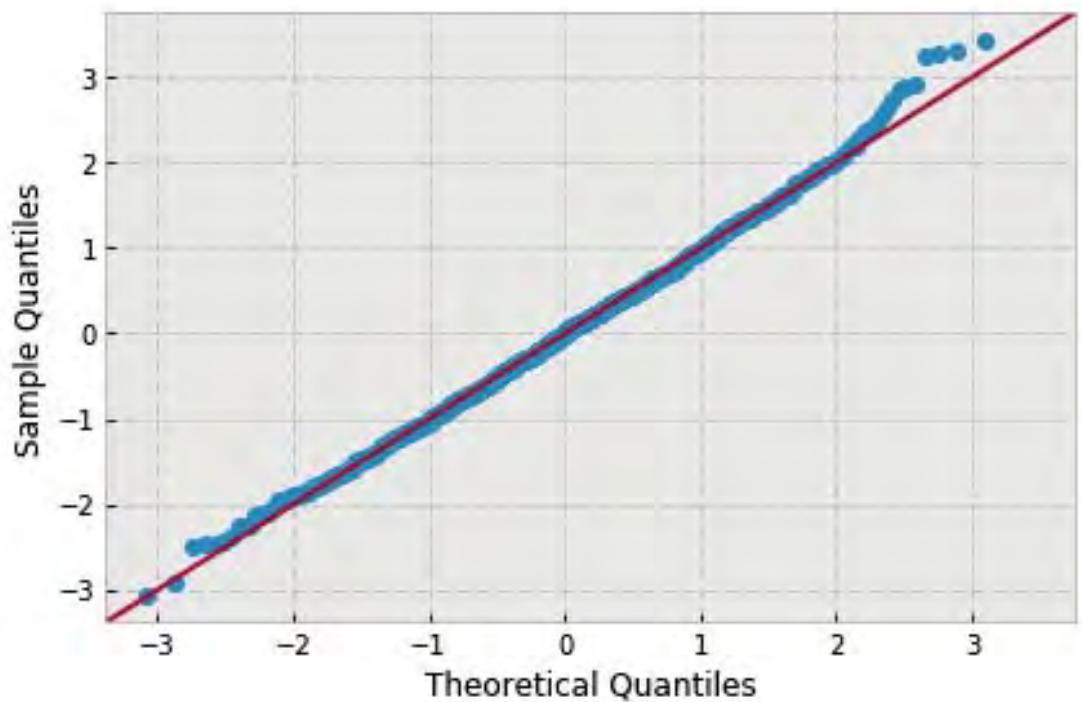
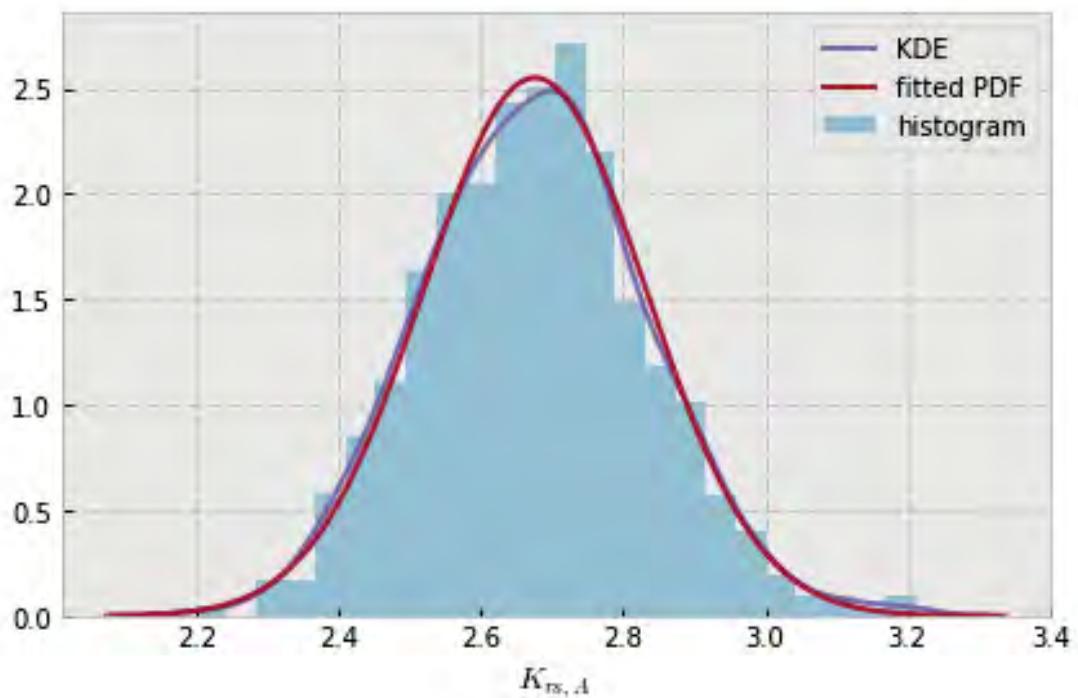


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=2.8005177412772735, scale=0.16532157563057565)
DescribeResult(nobs=1000, minmax=(2.2879762186105701, 3.4328221303590962)
               mean=2.8005177412772735, variance=0.027358581950927066,
               skewness=0.07809368351438514, kurtosis=-0.11593053758597893)
```

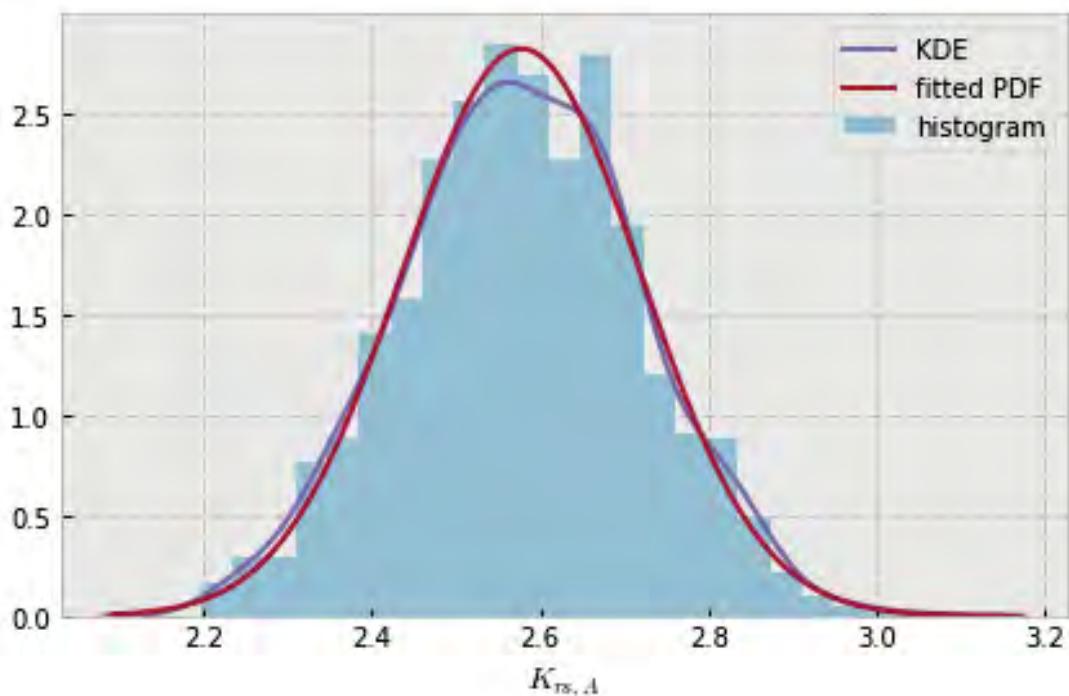


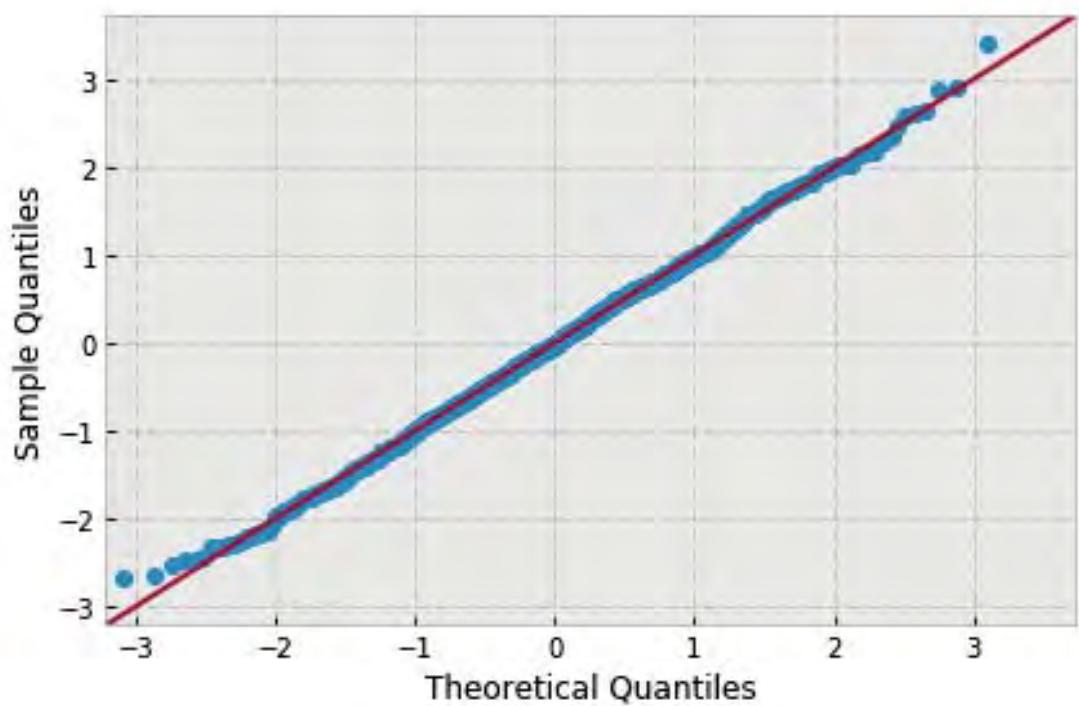


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=2.6755814487129825, scale=0.15643948717236886)
DescribeResult(nobs=1000, minmax=(2.1973599723330439, 3.2118935823527477)
               mean=2.6755814487129825, variance=0.024497810957711471,
               skewness=0.15848028952227952, kurtosis=0.11751847808799232)
```

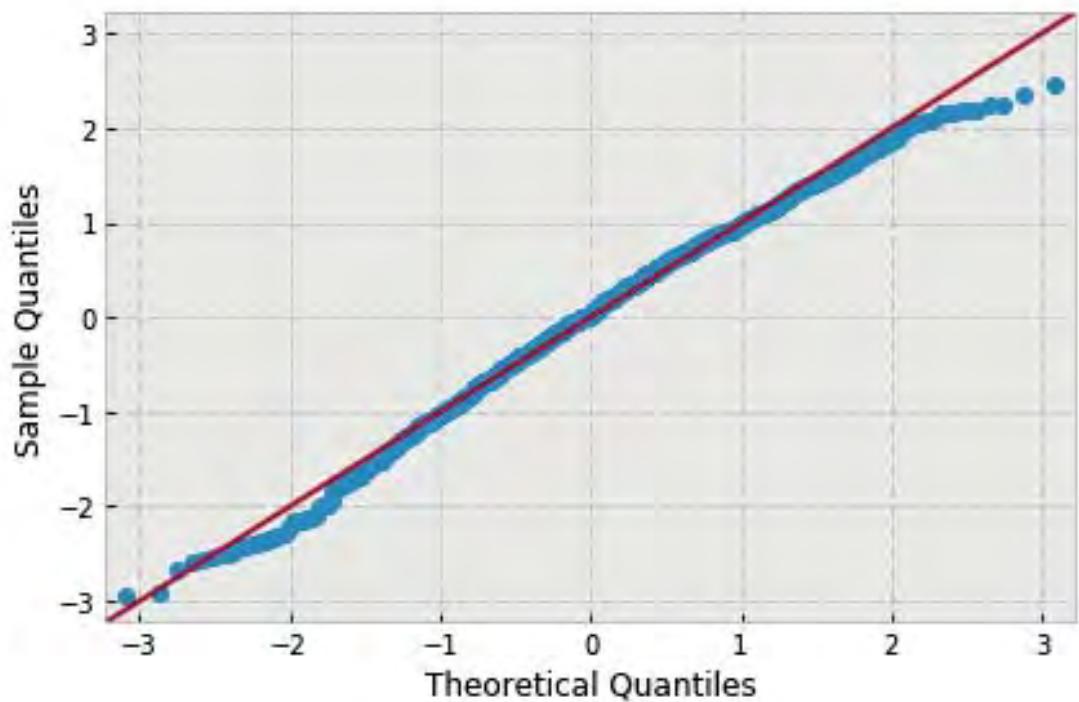
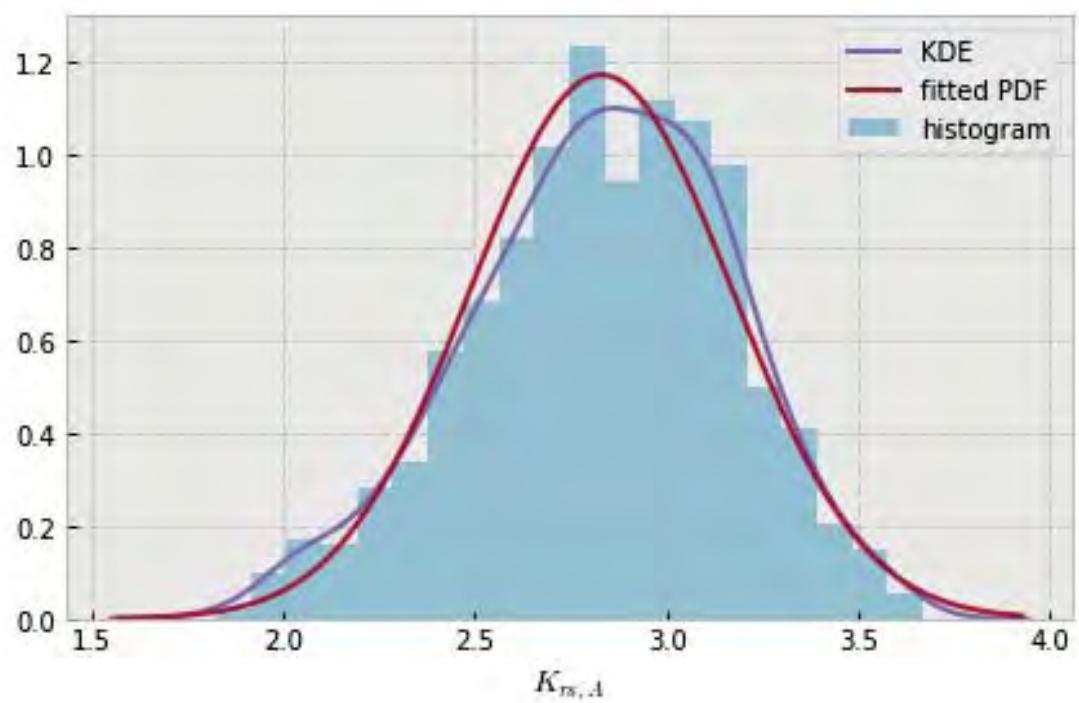


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=2.5775631736196294, scale=0.1415176489478837)
DescribeResult(nobs=1000, minmax=(2.1968579256553231, 3.0608196131683472)
               mean=2.5775631736196294, variance=0.020047292255992447,
               skewness=0.031716188503848844, kurtosis=-0.1579227121614335)
```

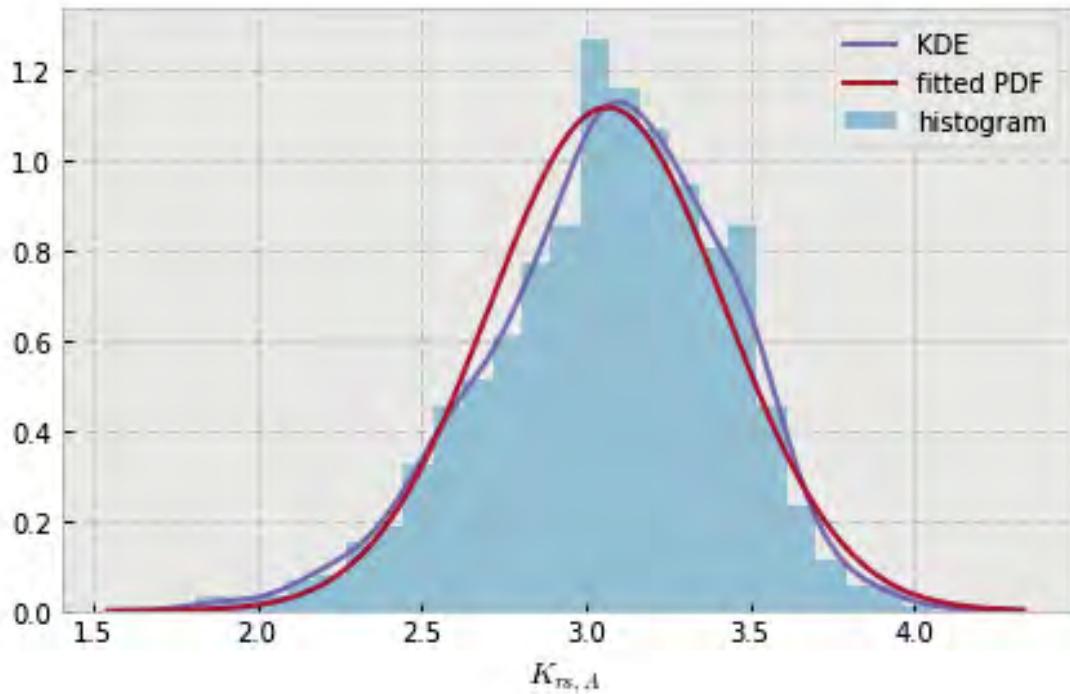


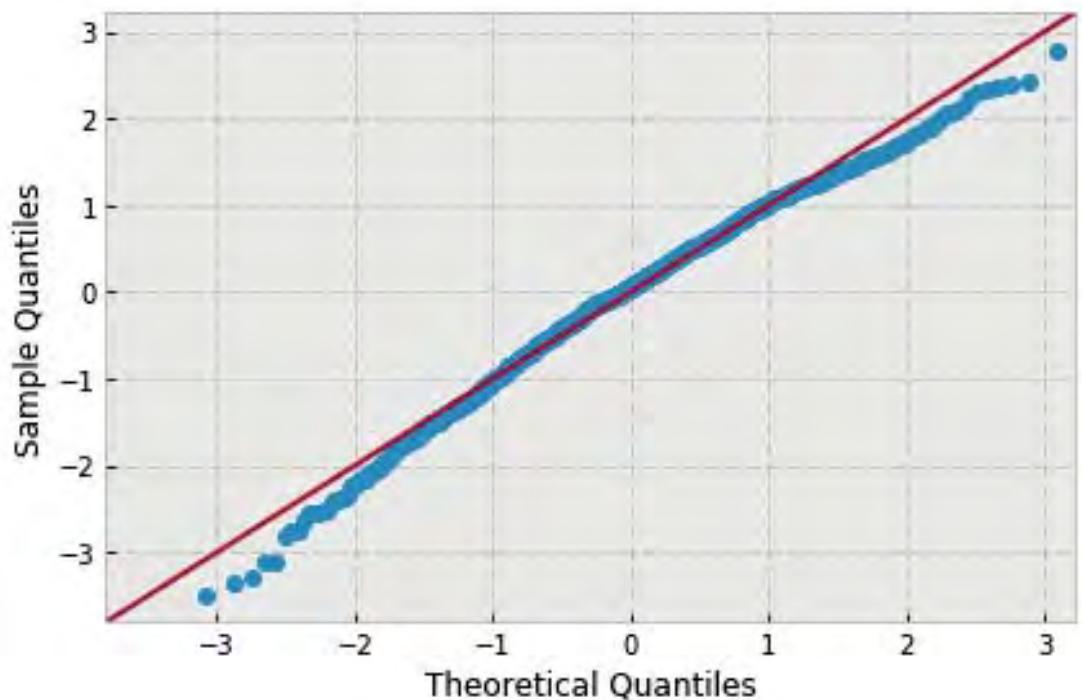


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=2.828959135378004, scale=0.34067513287443596)
DescribeResult(nobs=1000, minmax=(1.8233006407461041, 3.6684692366055591)
               mean=2.8289591353780041, variance=0.11617572188089548,
               skewness=-0.2718367045100893, kurtosis=-0.2220546871952438)
```

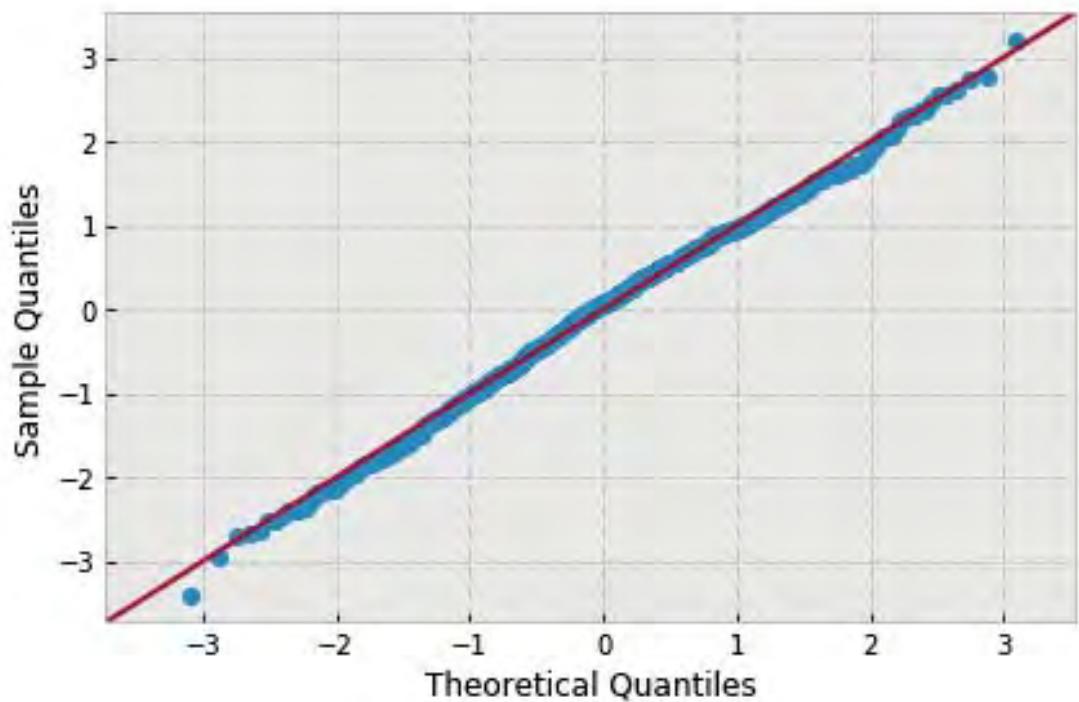
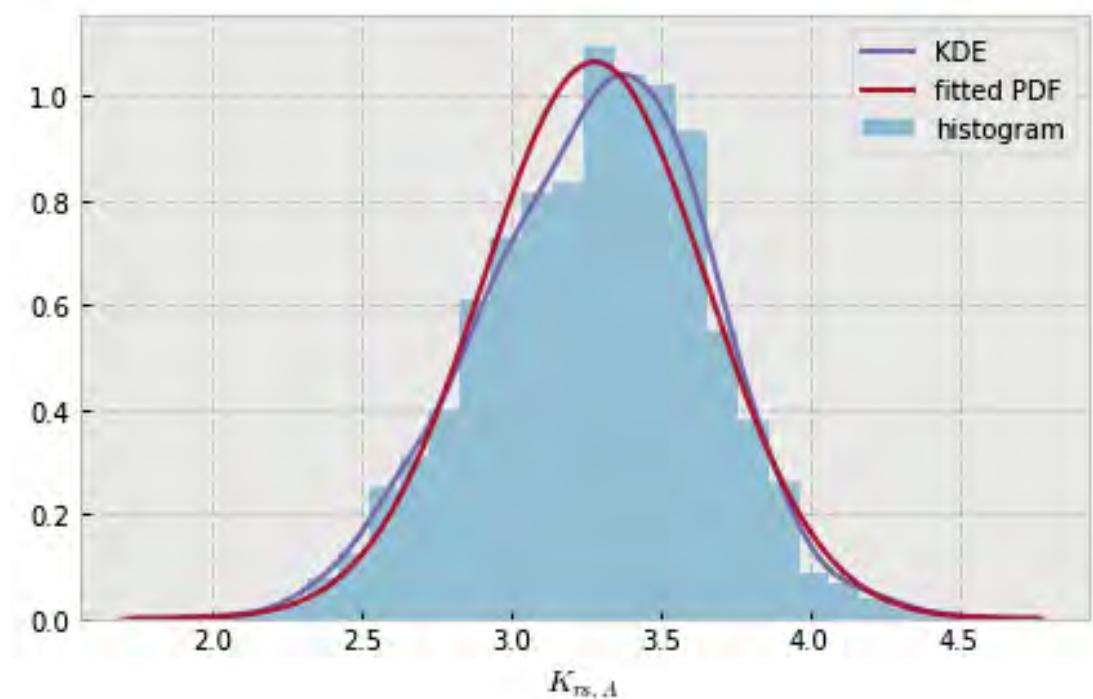


```
variable name: r
variable value: 1.222222222222223
distribution: normal(shape=(), loc=3.0652827462839722, scale=0.35692373507746744)
DescribeResult(nobs=1000, minmax=(1.8191979598993238, 4.0588663664104194)
               mean=3.0652827462839722, variance=0.12752207473638655,
               skewness=-0.39714360173479496, kurtosis=0.10131333657380637)
```

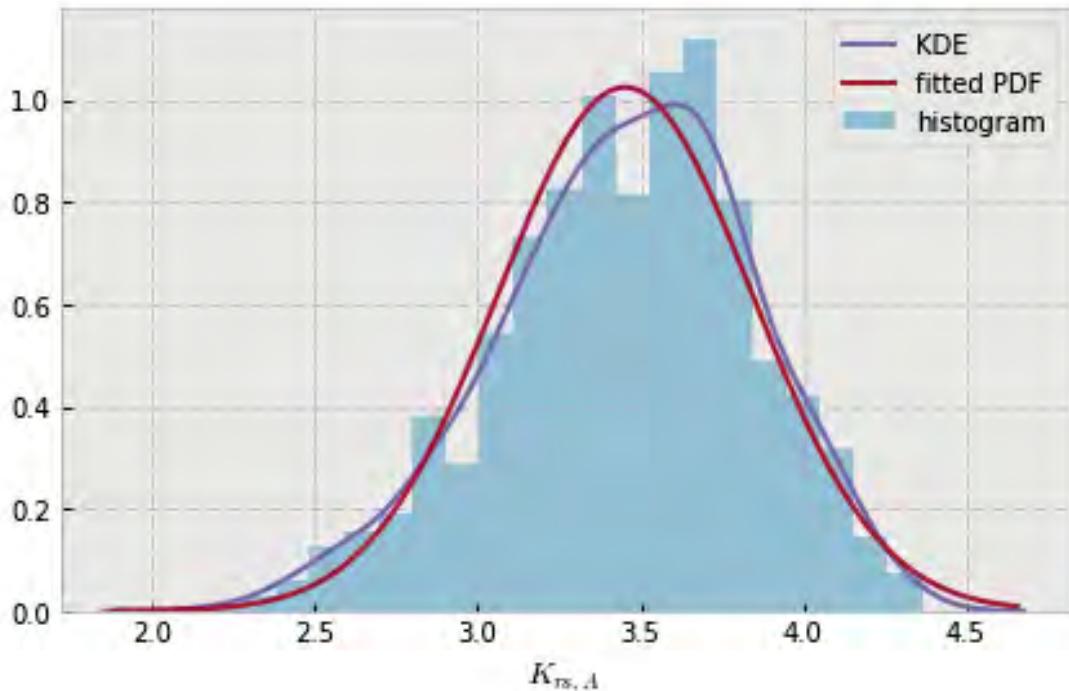


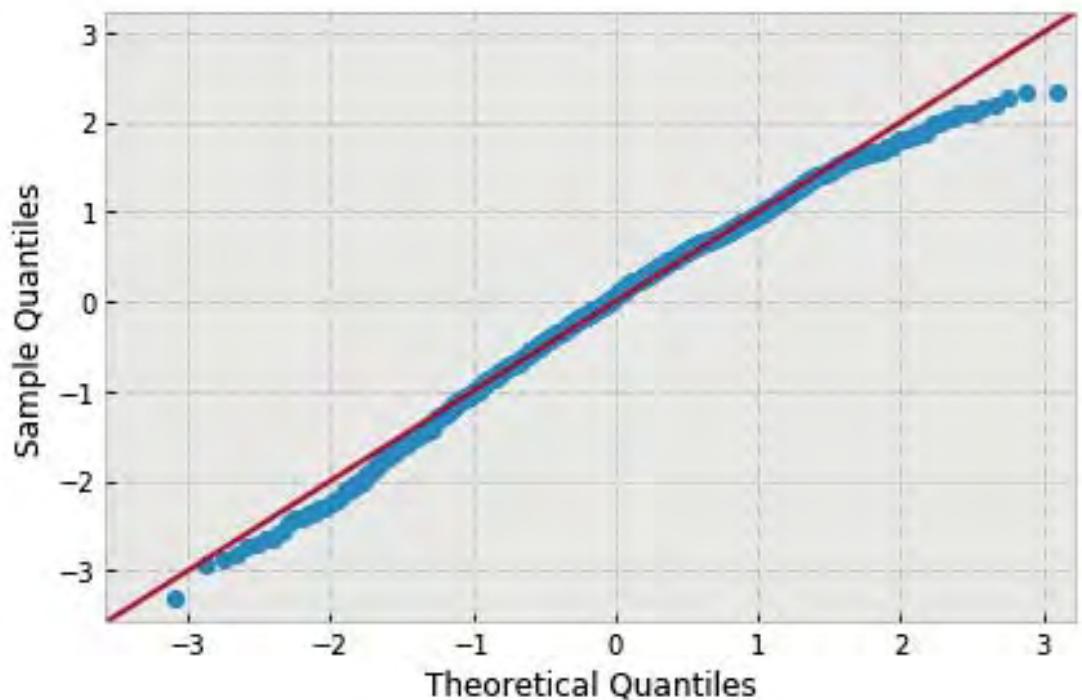


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=3.280088539993492, scale=0.37463268960295887)
DescribeResult(nobs=1000, minmax=(2.0073386067623087, 4.4780108463865584)
               mean=3.2800885399934918, variance=0.14049014226140832,
               skewness=-0.17680265957694985, kurtosis=-0.07346648297369507)
```

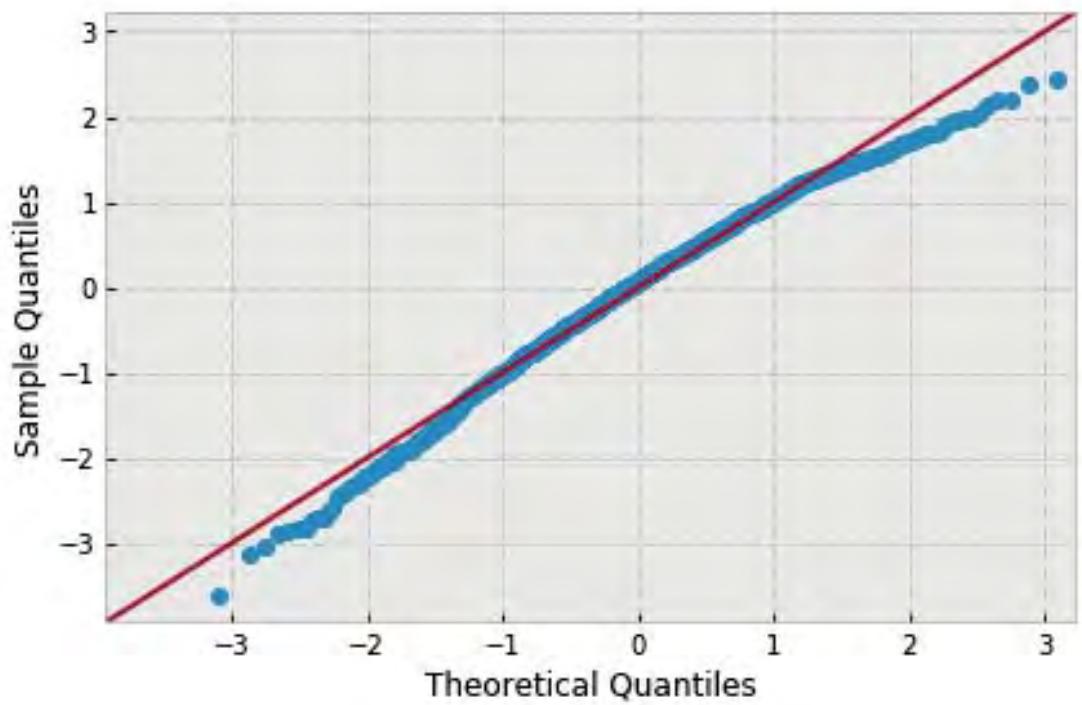
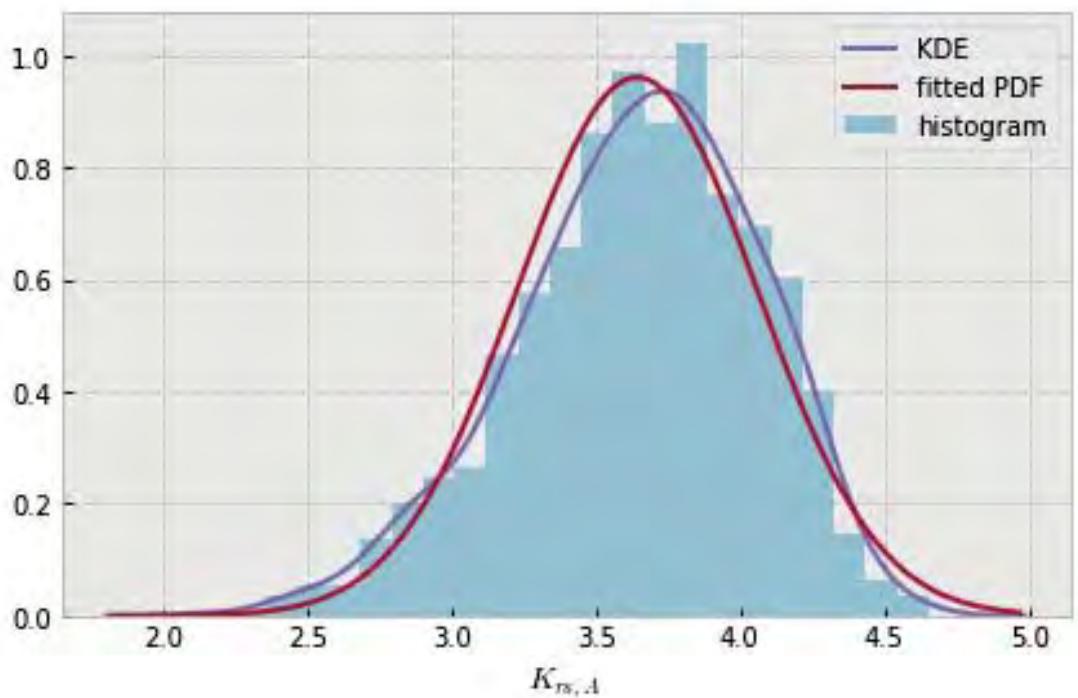


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=3.452087482155345, scale=0.38926647220755545)
DescribeResult(nobs=1000, minmax=(2.1716922527616593, 4.3618696996994295)
               mean=3.4520874821553451, variance=0.15168006645136692,
               skewness=-0.33519547550287565, kurtosis=-0.13162948640336714)
```

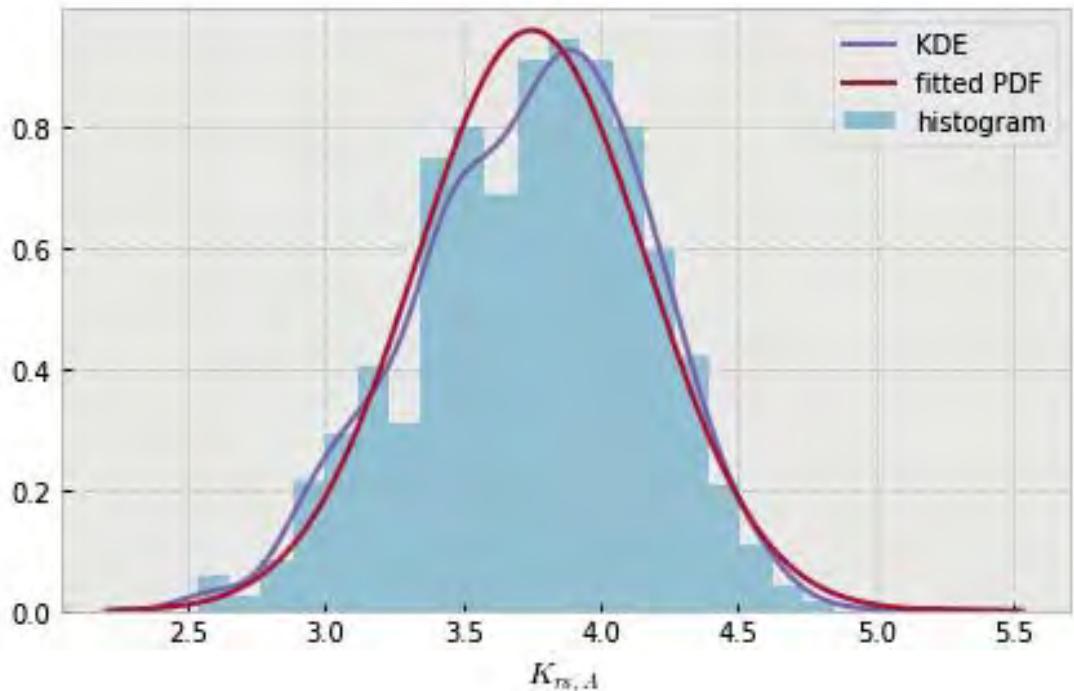


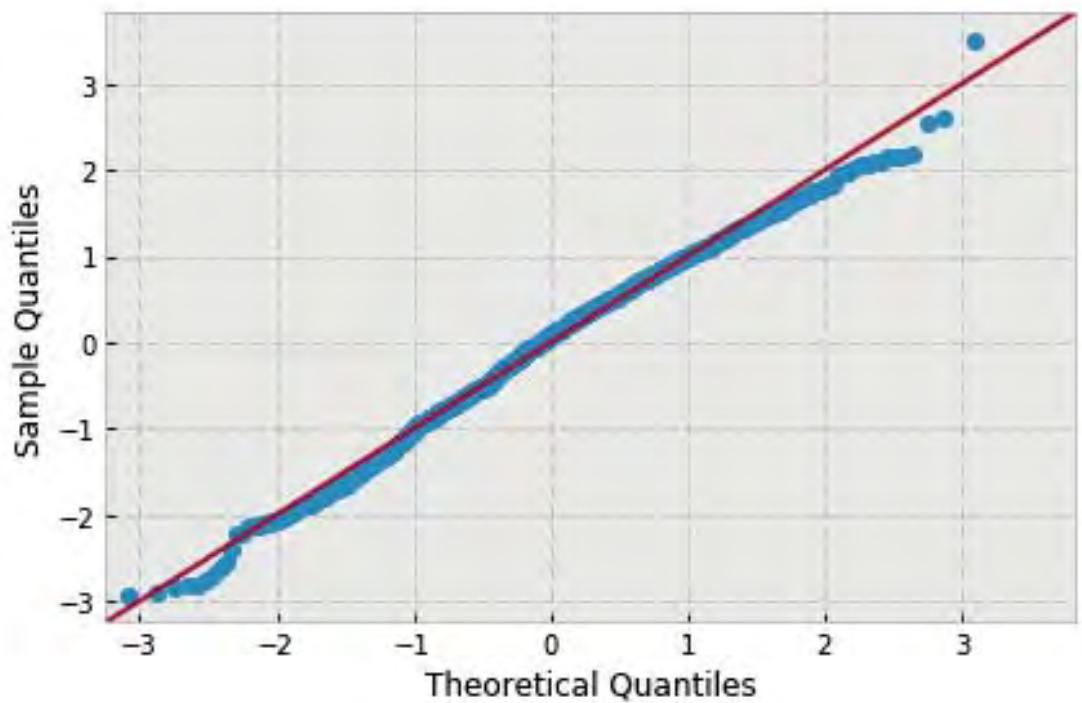


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=3.6421674323810365, scale=0.41523899627294747)
DescribeResult(nobs=1000, minmax=(2.137274689314387, 4.6522864770753989)
               mean=3.6421674323810365, variance=0.17259602004581071,
               skewness=-0.42796064237735915, kurtosis=-0.01953584481199311)
```

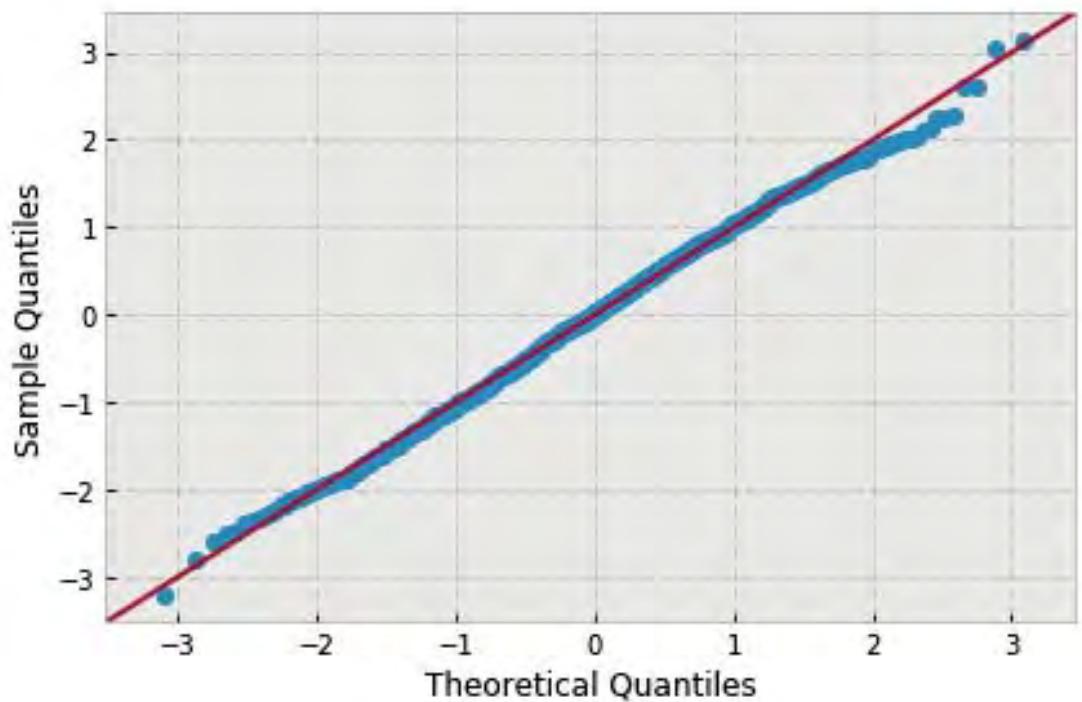
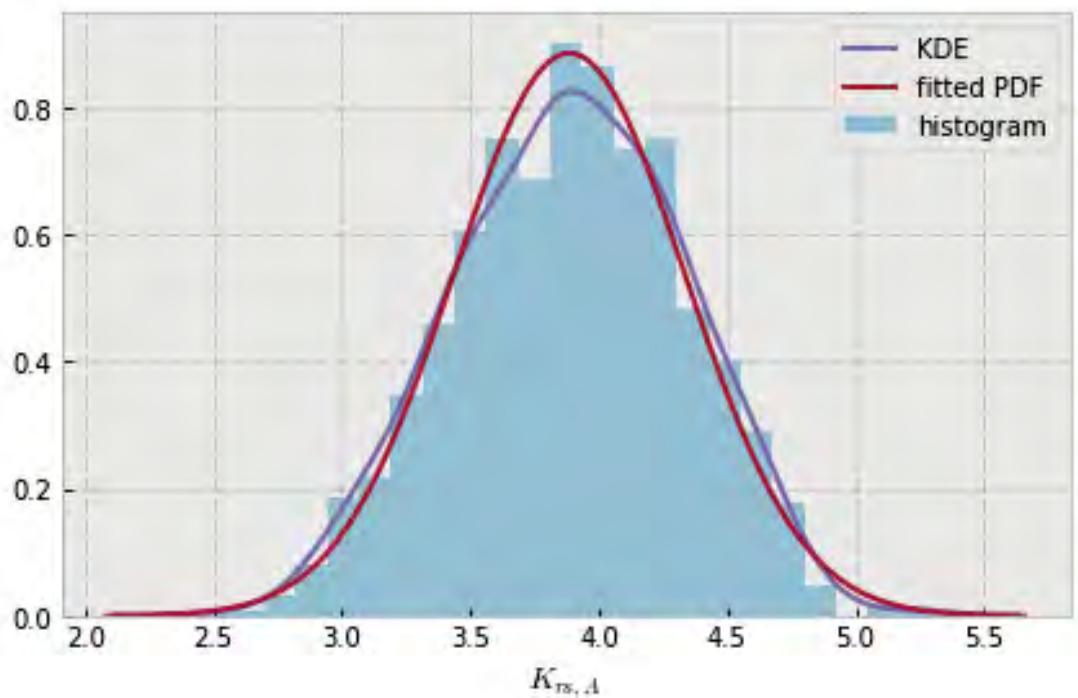


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=3.7485146288967672, scale=0.4154864141806172)
DescribeResult(nobs=1000, minmax=(2.5335778674864606, 5.2034023595666357)
               mean=3.7485146288967672, variance=0.17280176213079818,
               skewness=-0.2344497303059004, kurtosis=-0.20835746729530458)
```

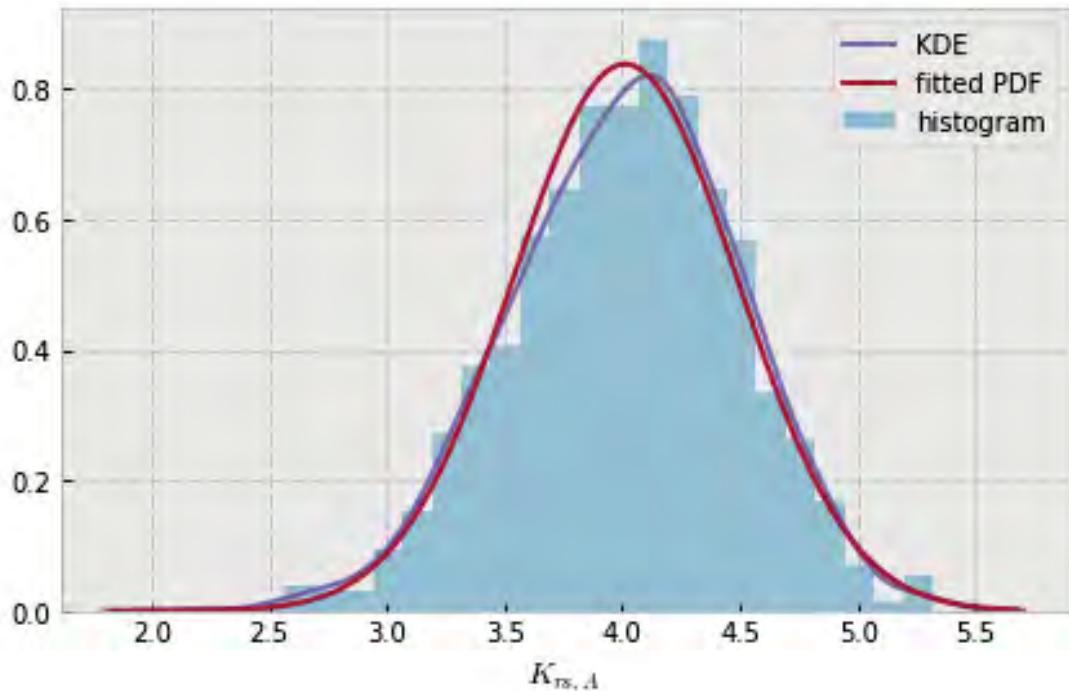


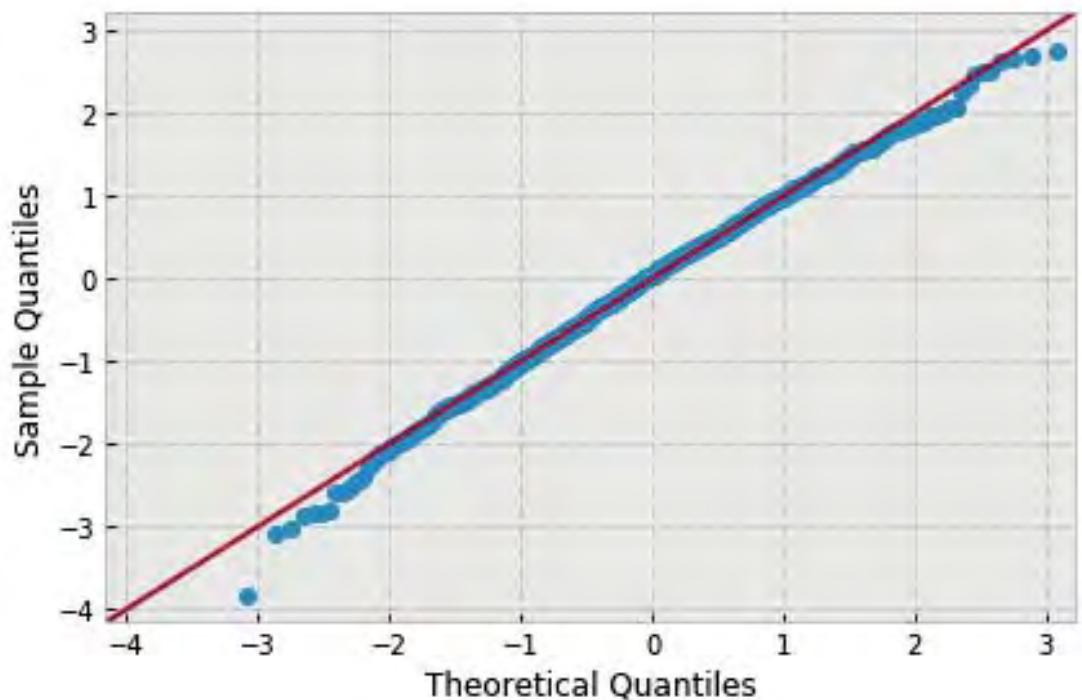


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=3.8857577084777595, scale=0.45024087697055054)
DescribeResult(nobs=1000, minmax=(2.4459228110707829, 5.2957593969412775)
               mean=3.8857577084777595, variance=0.20291976706227269,
               skewness=-0.08280239093025082, kurtosis=-0.32469015916931365)
```

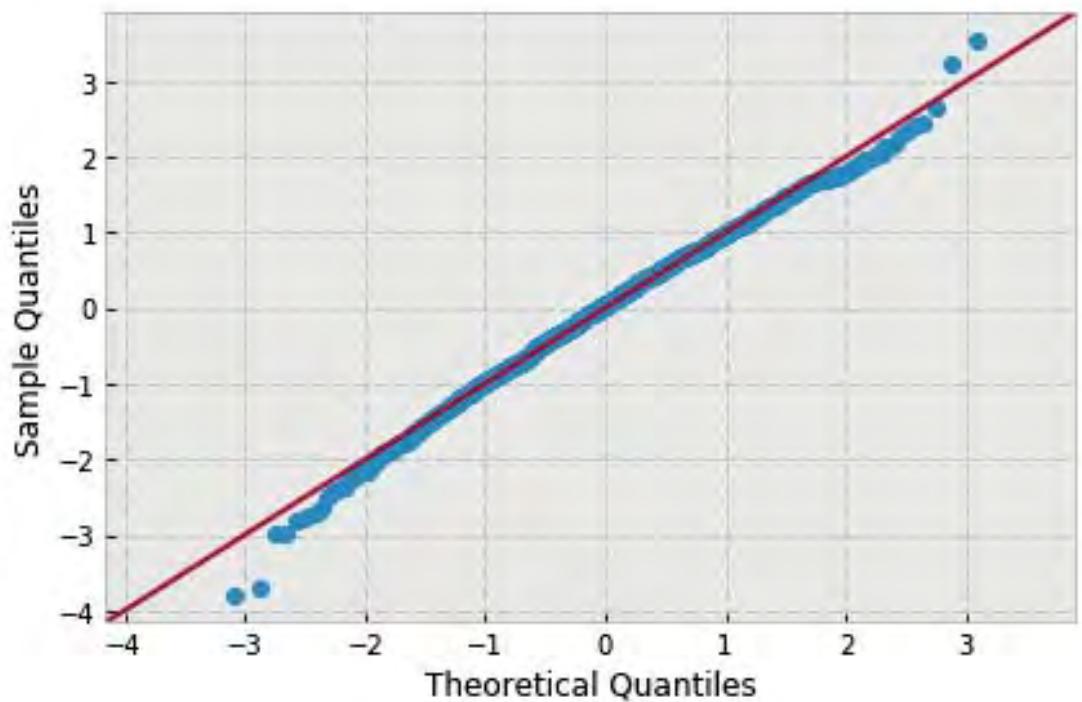
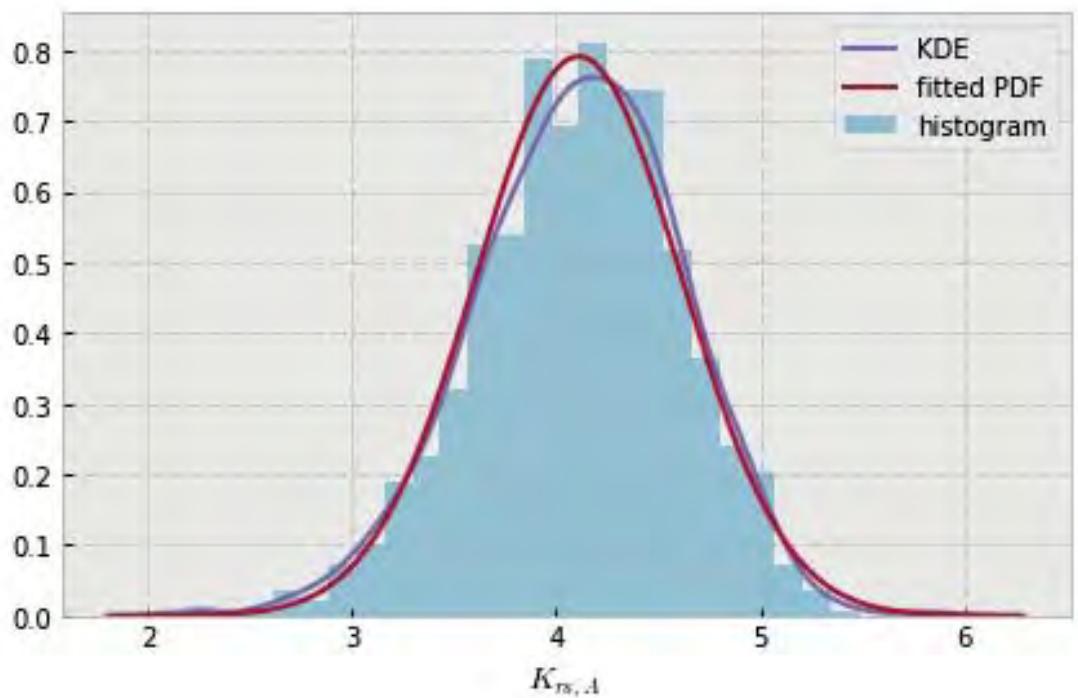


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=4.012346087580092, scale=0.47648608058034325)
DescribeResult(nobs=1000, minmax=(2.1881004623974087, 5.321943500815876)
               mean=4.0123460875800916, variance=0.22726625123805541,
               skewness=-0.2085430718042726, kurtosis=0.030902954893949897)
```

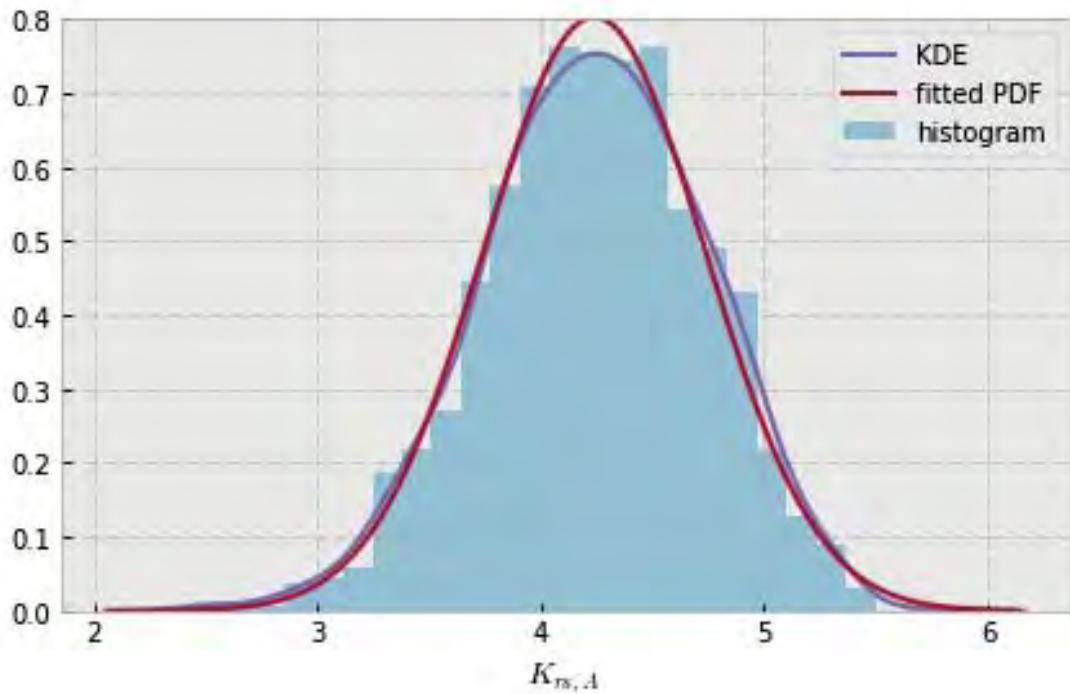


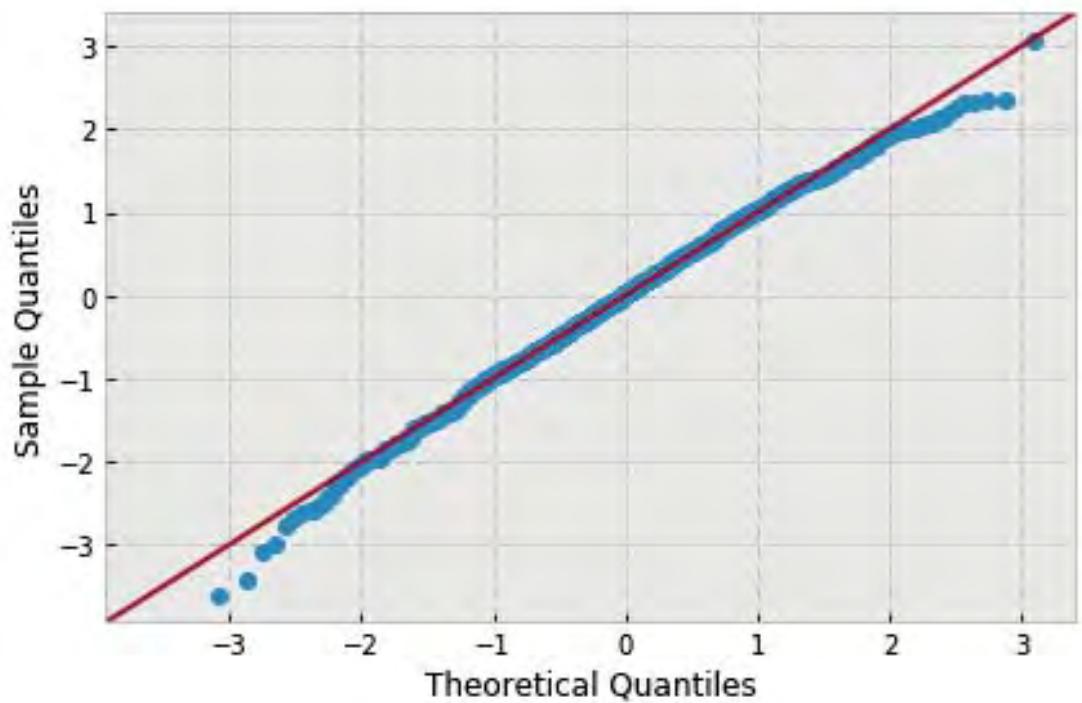


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=4.109188646542356, scale=0.5038320851775705)
DescribeResult(nobs=1000, minmax=(2.2008749740828923, 5.8899077581938064)
               mean=4.1091886465423562, variance=0.25410087092530392,
               skewness=-0.23759759837713704, kurtosis=0.24094537843529196)
```

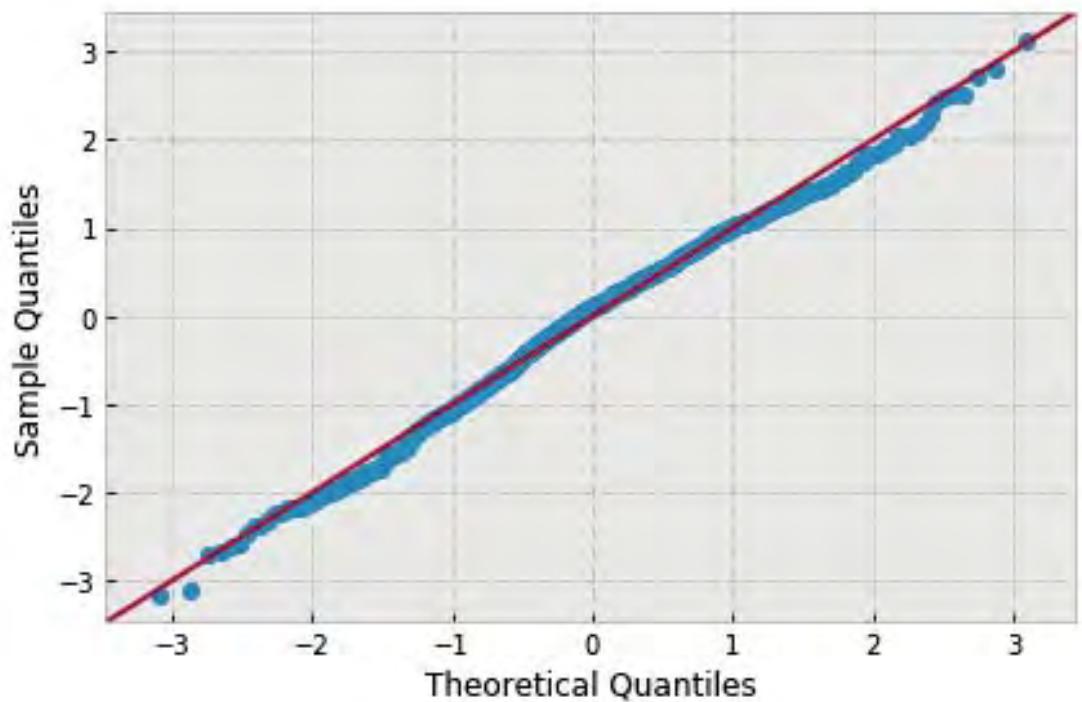
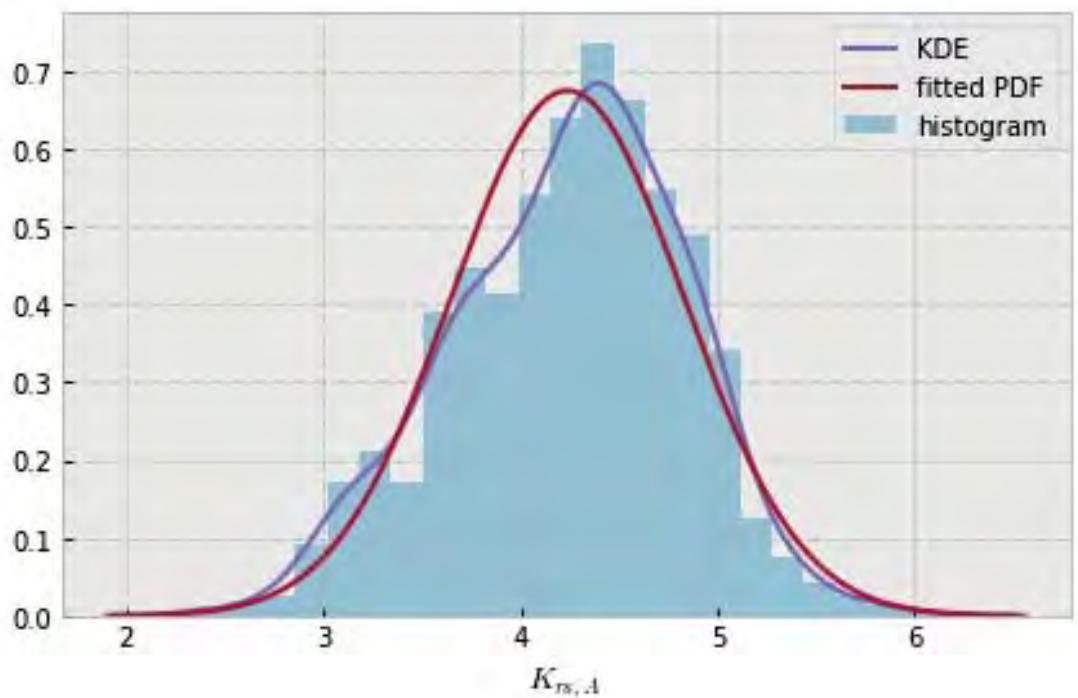


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=4.2398242290347845, scale=0.49662464514969884)
DescribeResult(nobs=1000, minmax=(2.4506260165416367, 5.762799151322386)
               mean=4.2398242290347845, variance=0.24688292109115545,
               skewness=-0.19553167120241333, kurtosis=-0.04127731426119485)
```

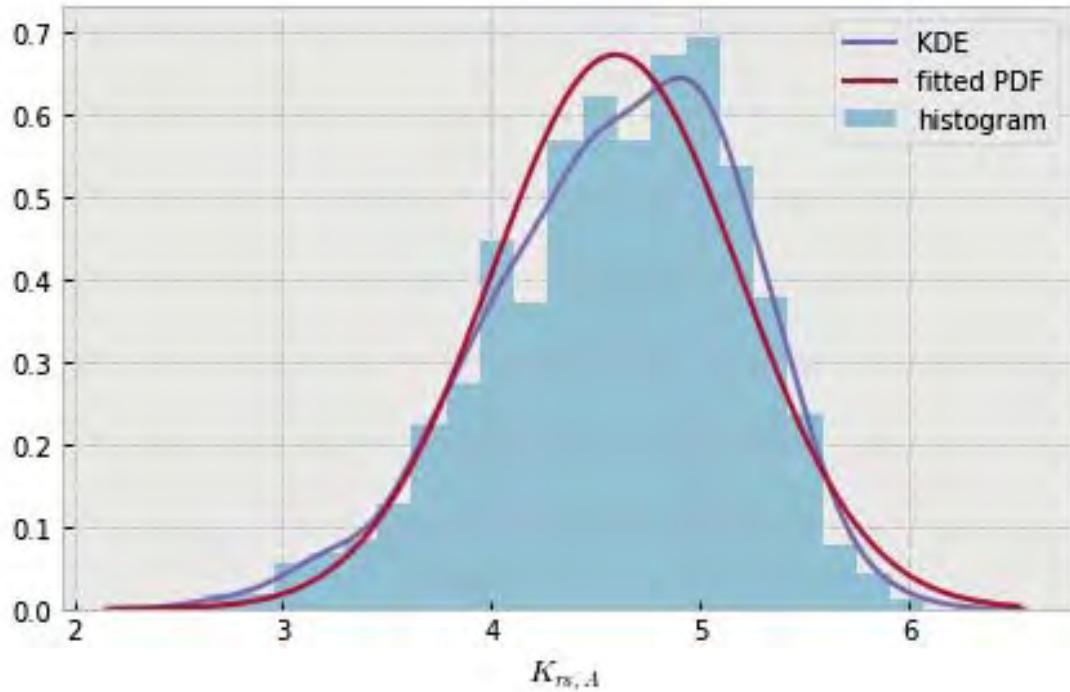


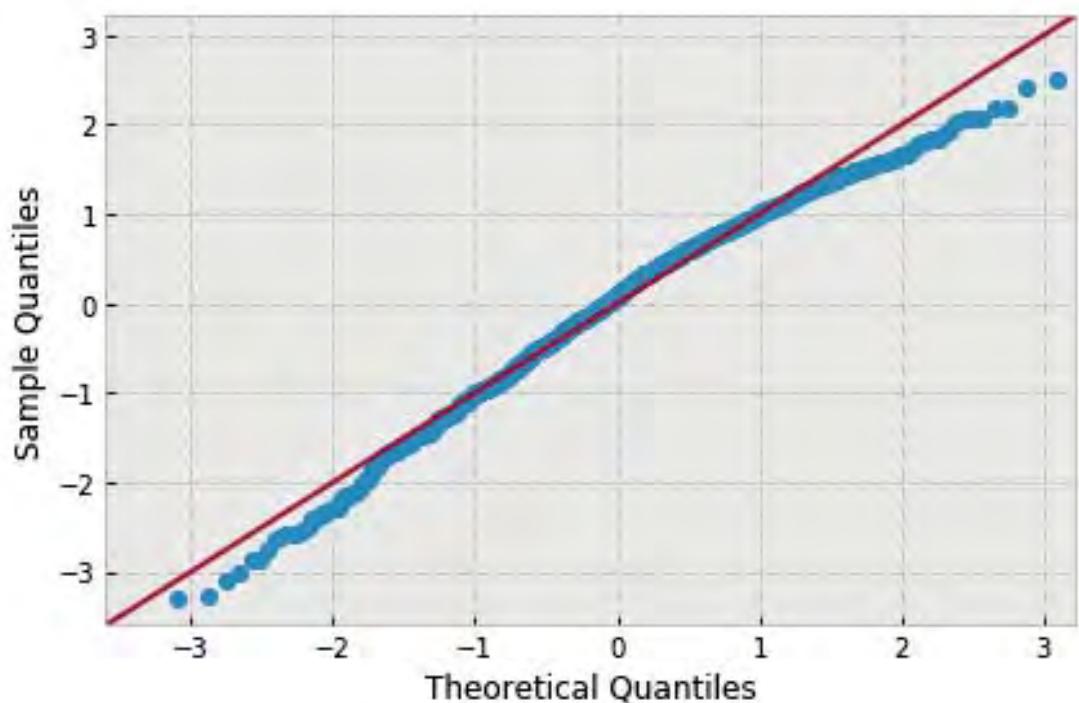


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=4.239426231776814, scale=0.590776327300123)
DescribeResult(nobs=1000, minmax=(2.3751145762946115, 6.0833087600229439)
               mean=4.2394262317768137, variance=0.34936603493315521,
               skewness=-0.2492253973435187, kurtosis=-0.16871022382358092)
```

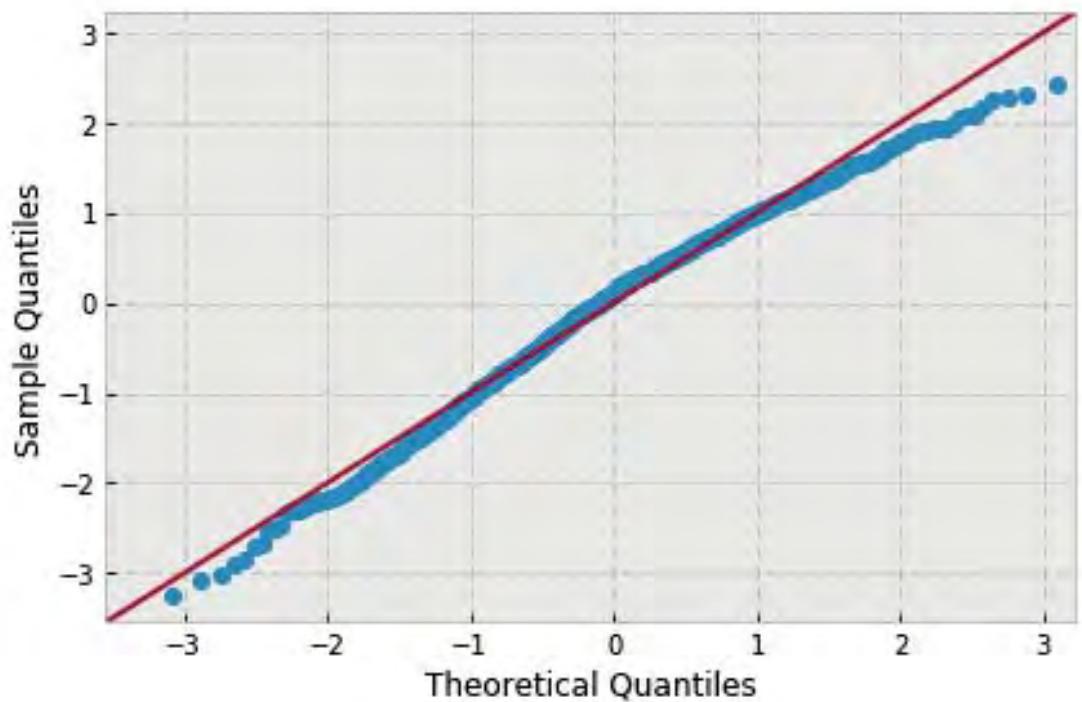
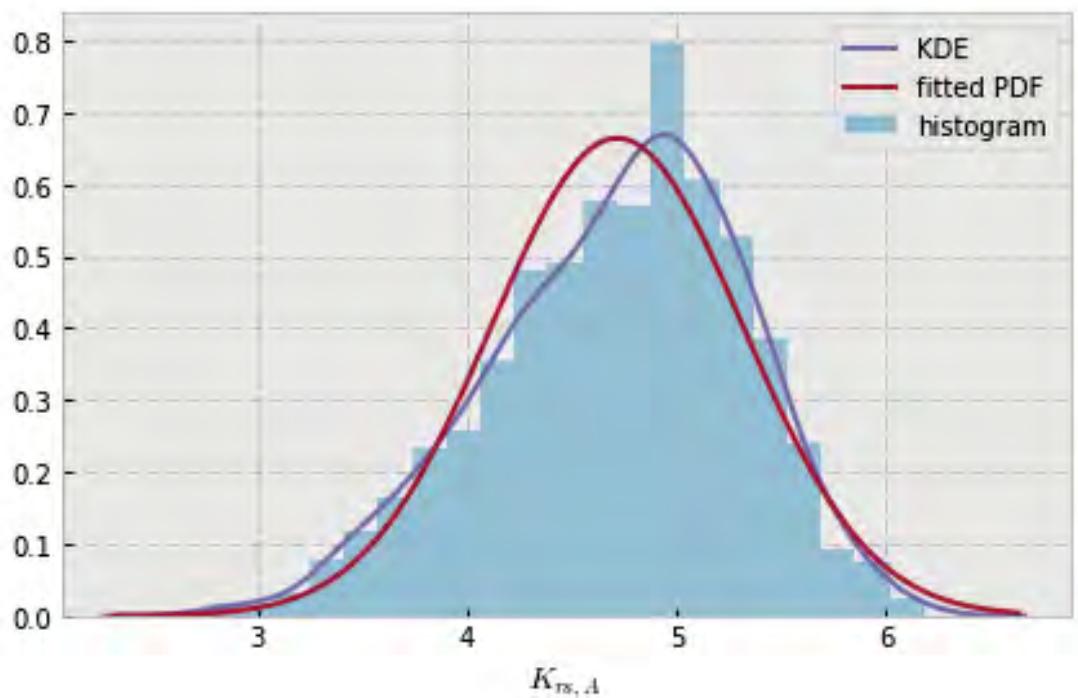


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=4.5948598119520865, scale=0.5938231162763223)
DescribeResult(nobs=1000, minmax=(2.6320717130313231, 6.0775474573695742)
               mean=4.5948598119520865, variance=0.352978872296419,
               skewness=-0.44264372935309443, kurtosis=-0.07997964979085204)
```

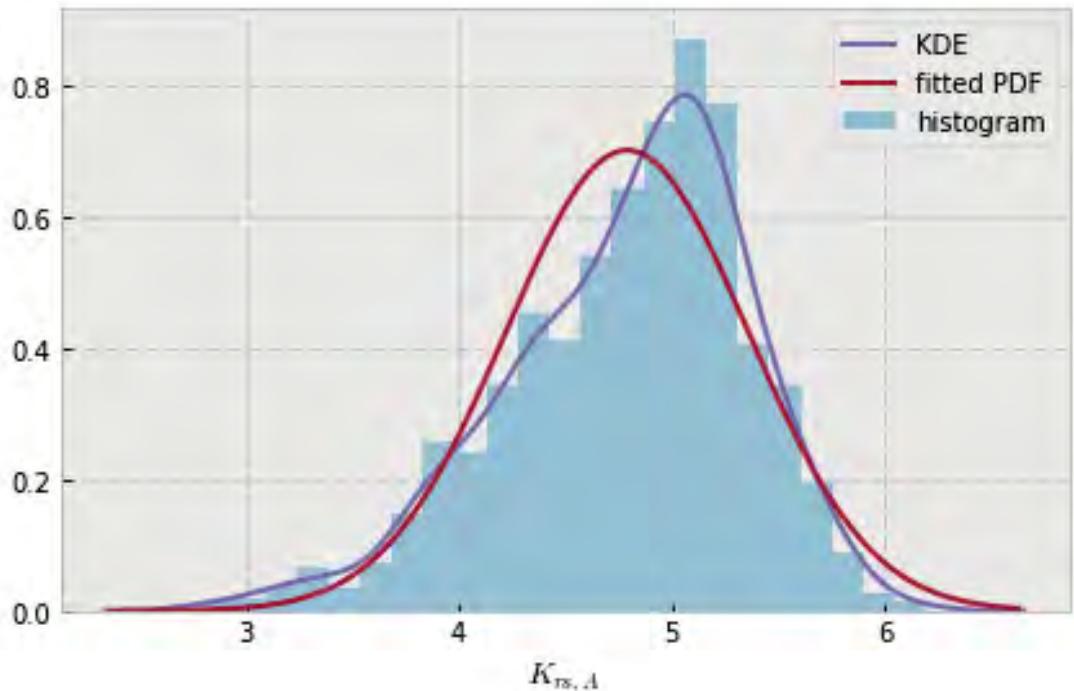


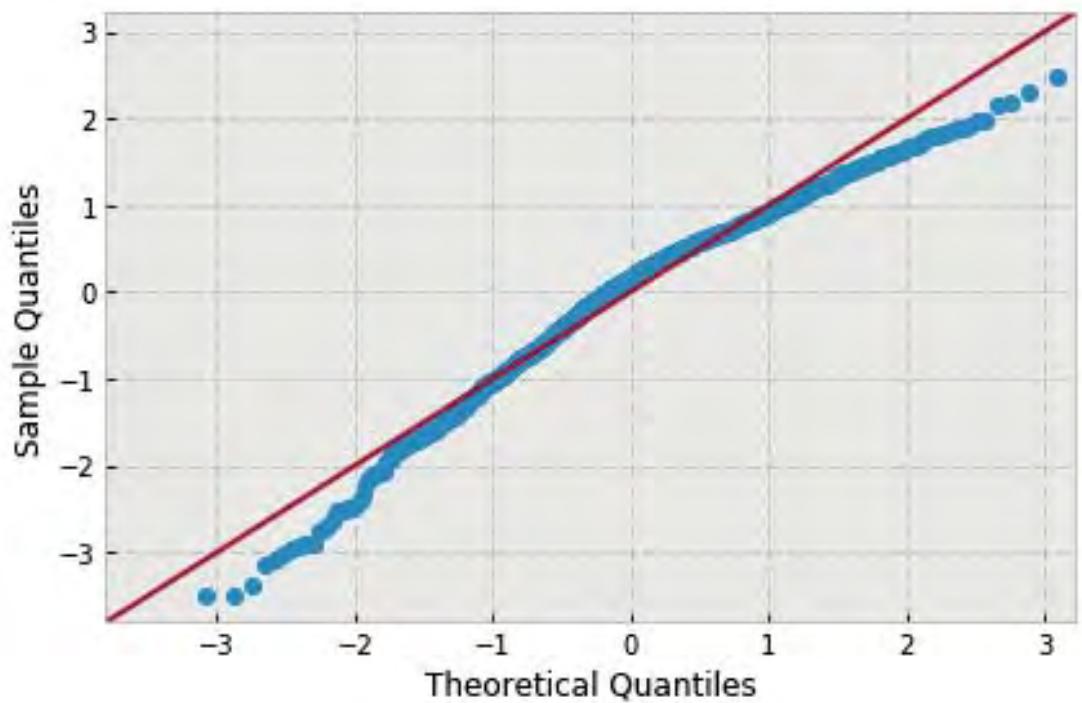


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(), loc=4.716092854372333, scale=0.6005996075563255)
DescribeResult(nobs=1000, minmax=(2.7591522058103704, 6.1793428394592542)
               mean=4.7160928543723326, variance=0.36108096956637858,
               skewness=-0.390338658846545, kurtosis=-0.19869738852230068)
```

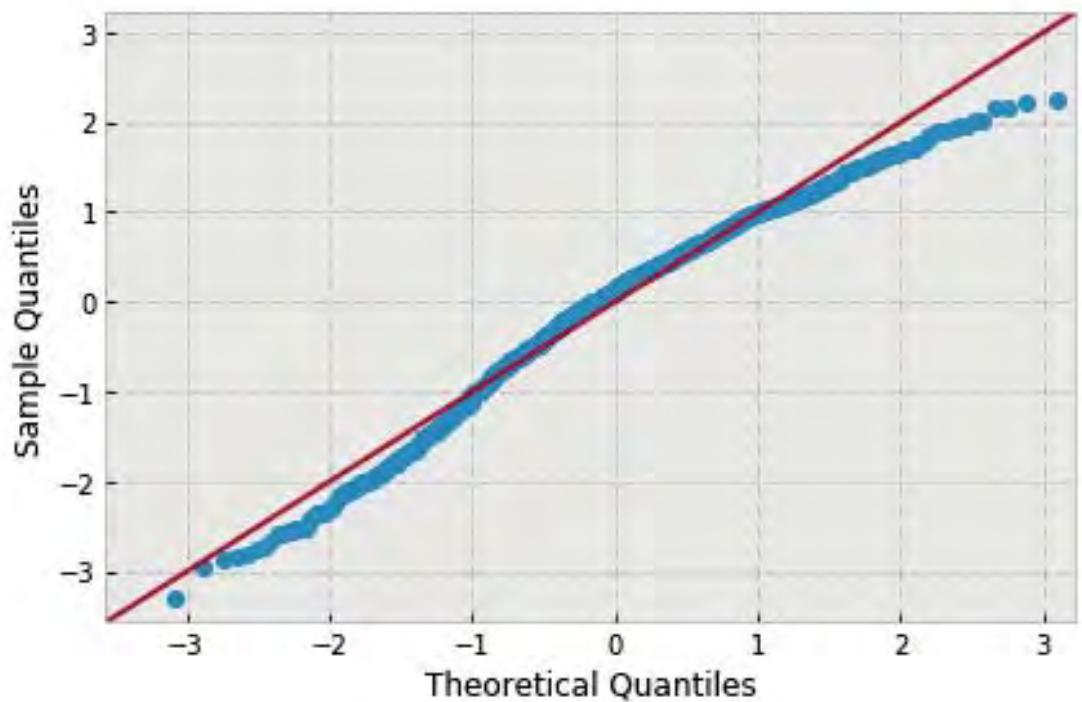
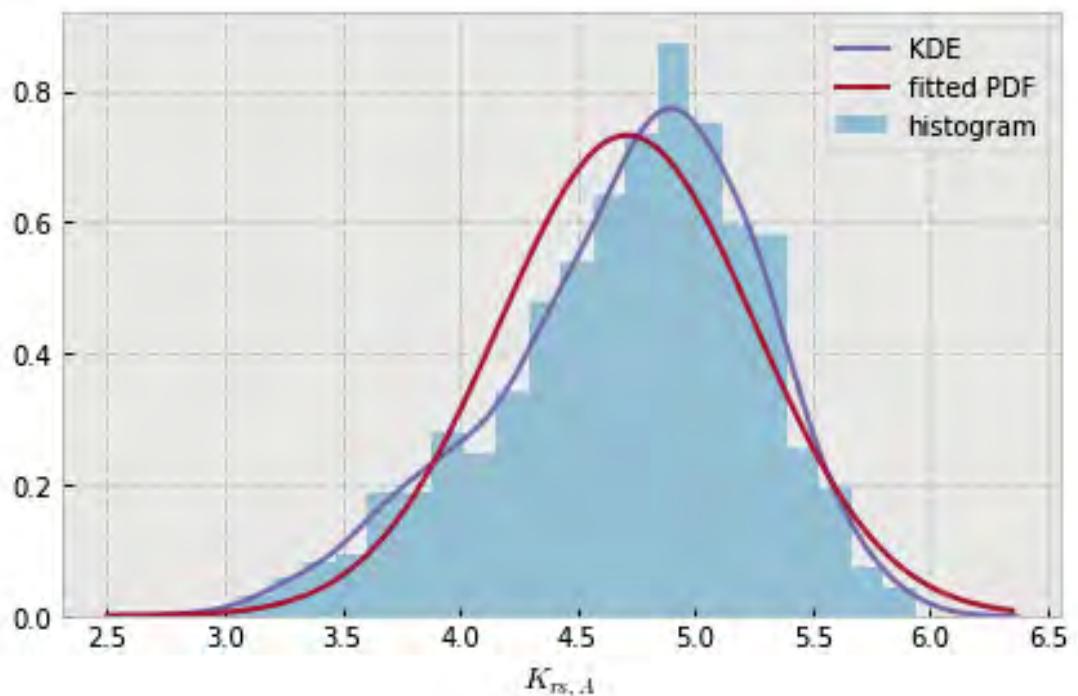


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=4.791409702892852, scale=0.5695929811866995)
DescribeResult(nobs=1000, minmax=(2.7951862439005368, 6.1996835901027882)
               mean=4.791409702892852, variance=0.32476092514229415,
               skewness=-0.6361424826478432, kurtosis=0.29495504148541407)
```

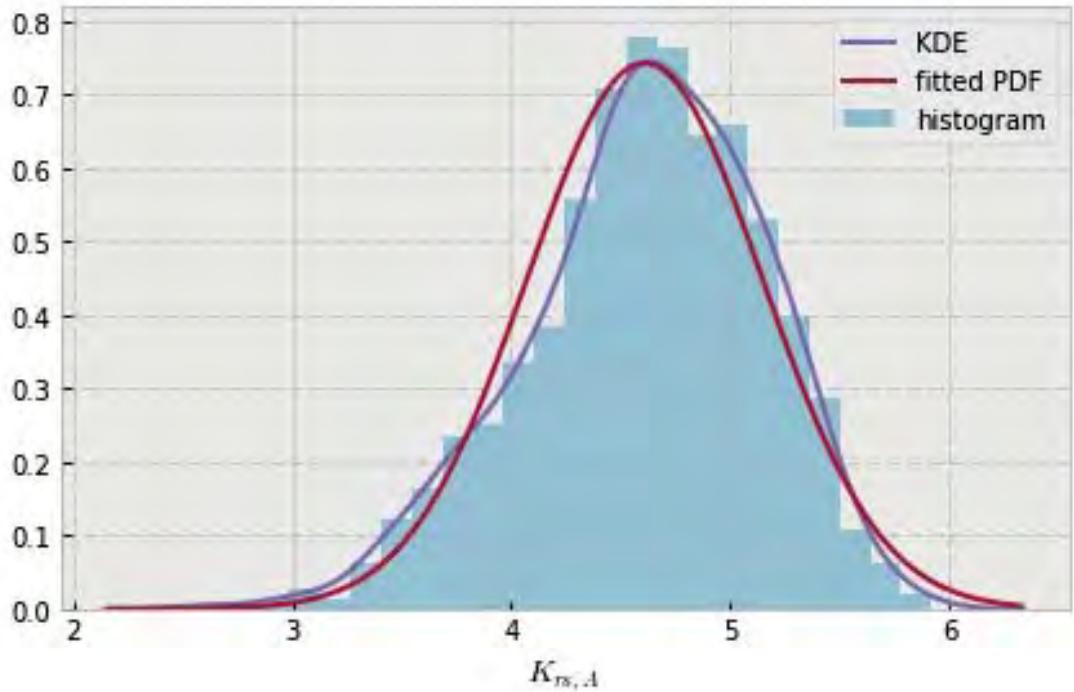


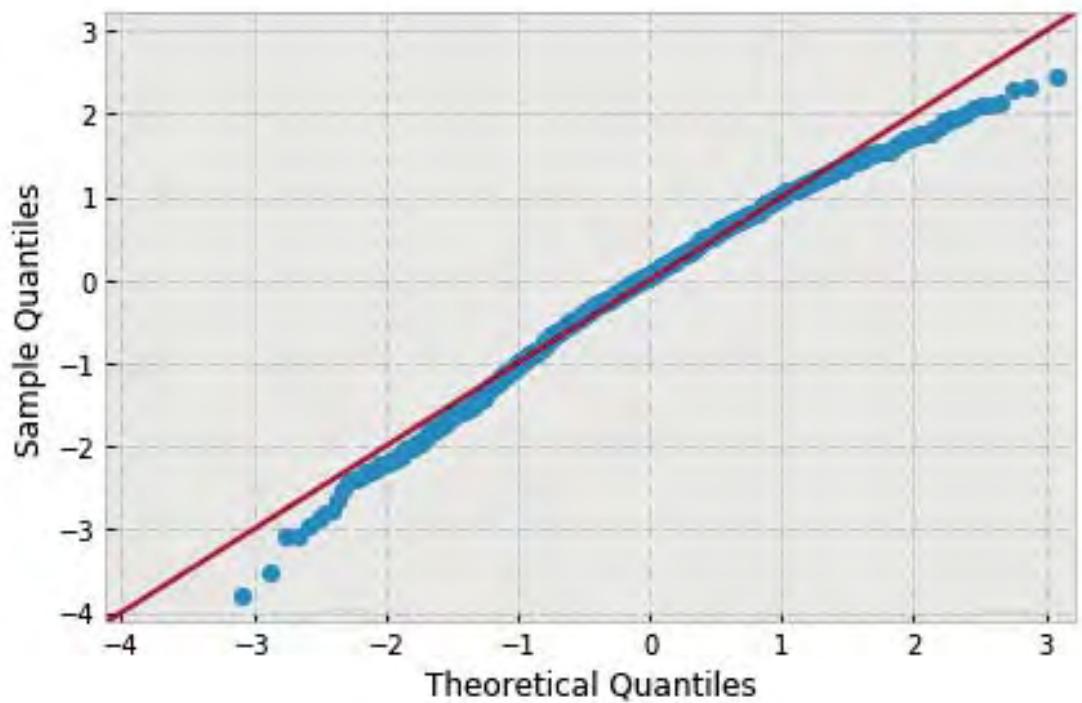


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=4.712342392964504, scale=0.5445385205529809)
DescribeResult(nobs=1000, minmax=(2.9226477421153909, 5.9396211734039666)
               mean=4.7123423929645041, variance=0.29681901938541461,
               skewness=-0.5365663388743355, kurtosis=-0.07489319456747046)
```

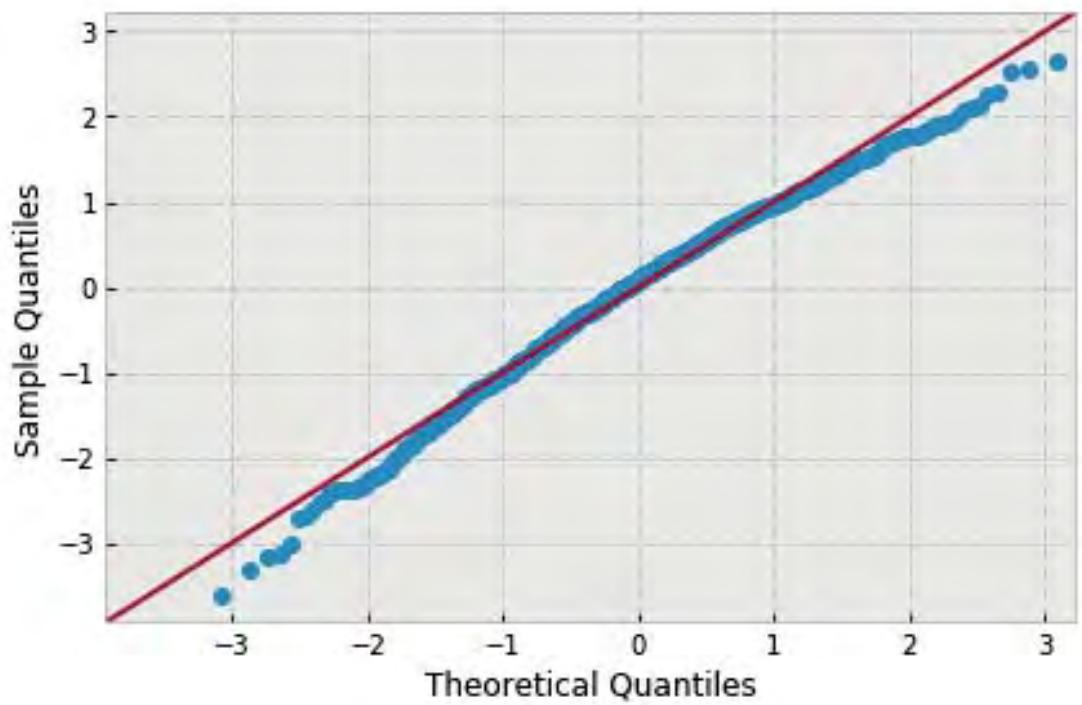
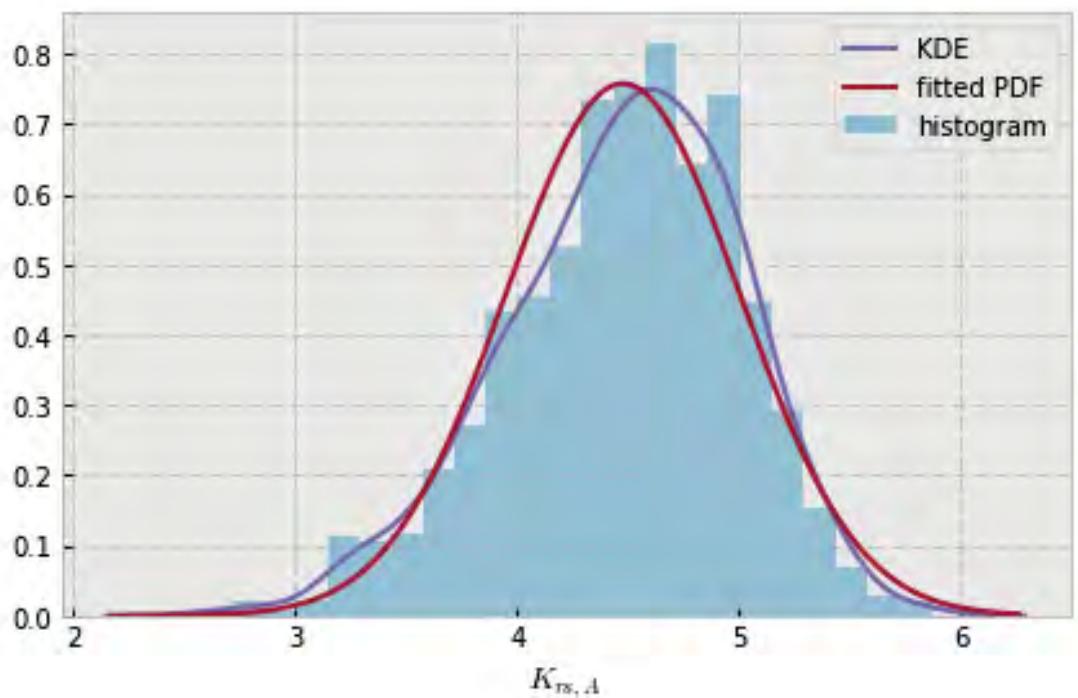


```
variable name: sigma
variable value: 0.066666666666666667
distribution: normal(shape=(), loc=4.609935449419434, scale=0.5370620462156217)
DescribeResult(nobs=1000, minmax=(2.5665889192641593, 5.9207641910148077)
               mean=4.6099354494194342, variance=0.28872436585116179,
               skewness=-0.4501241266825589, kurtosis=0.06663868316031163)
```

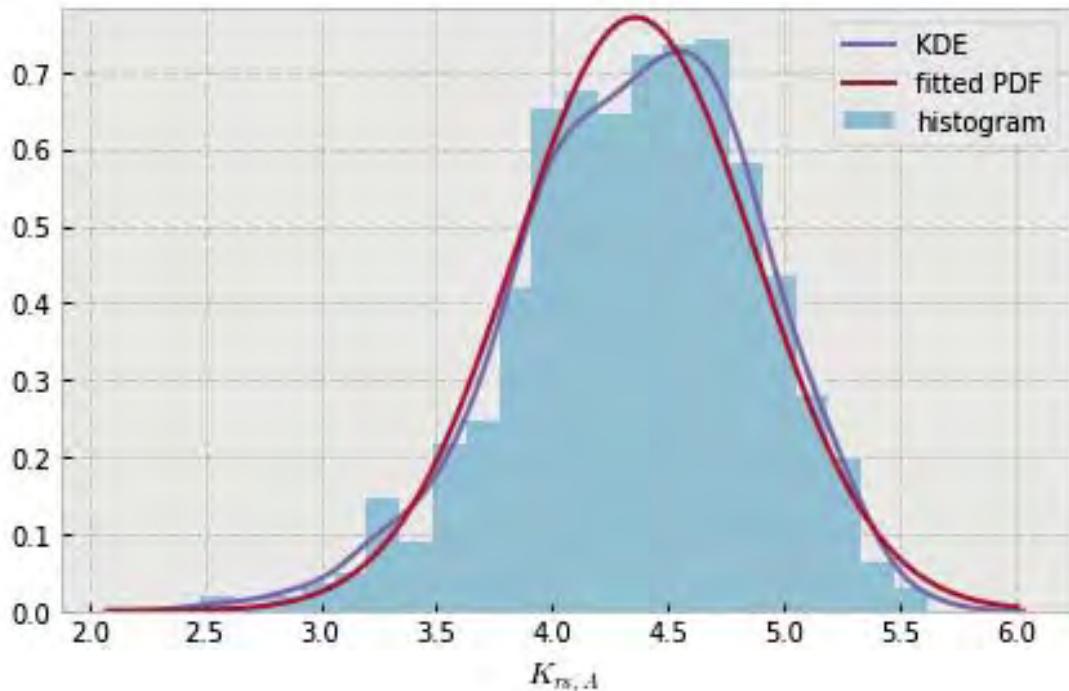


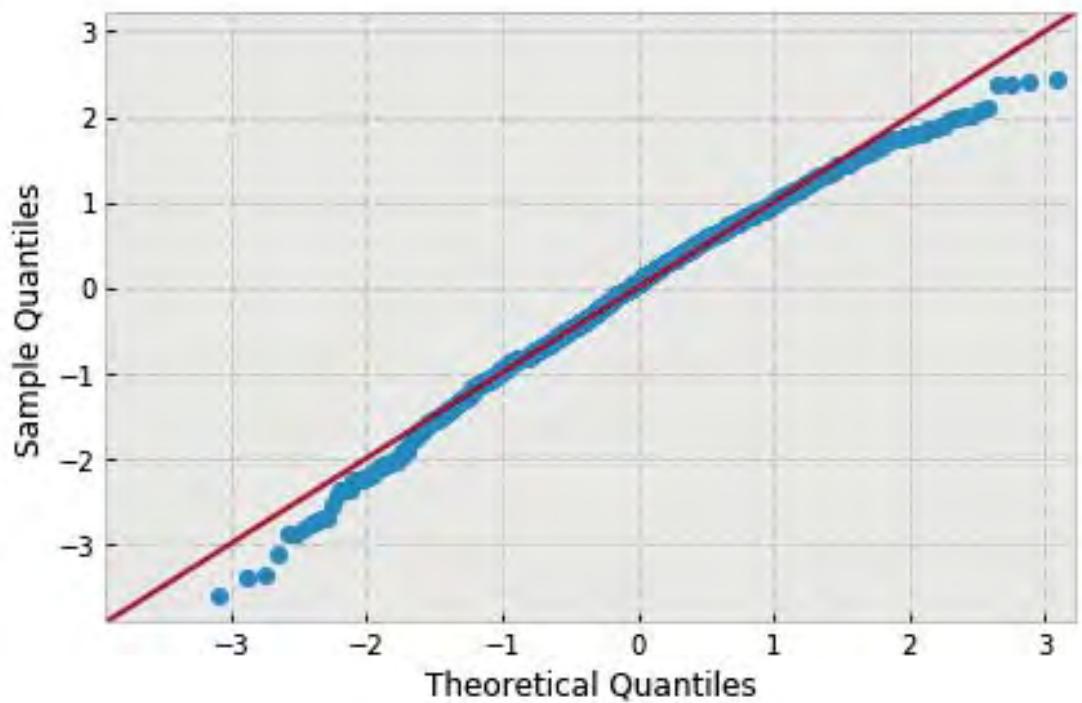


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=4.477988739789233, scale=0.5264685639570786)
DescribeResult(nobs=1000, minmax=(2.5731766207127356, 5.8657779988605743)
               mean=4.477988739789233, variance=0.27744659543045902,
               skewness=-0.41894511401851986, kurtosis=0.06290610308946398)
```

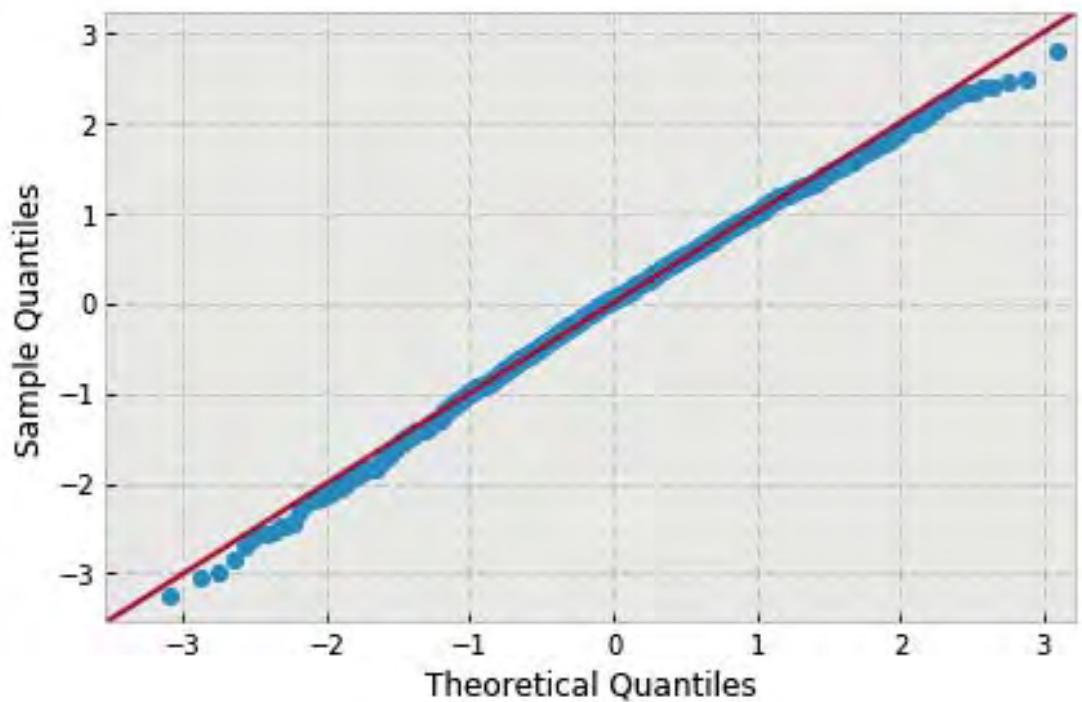
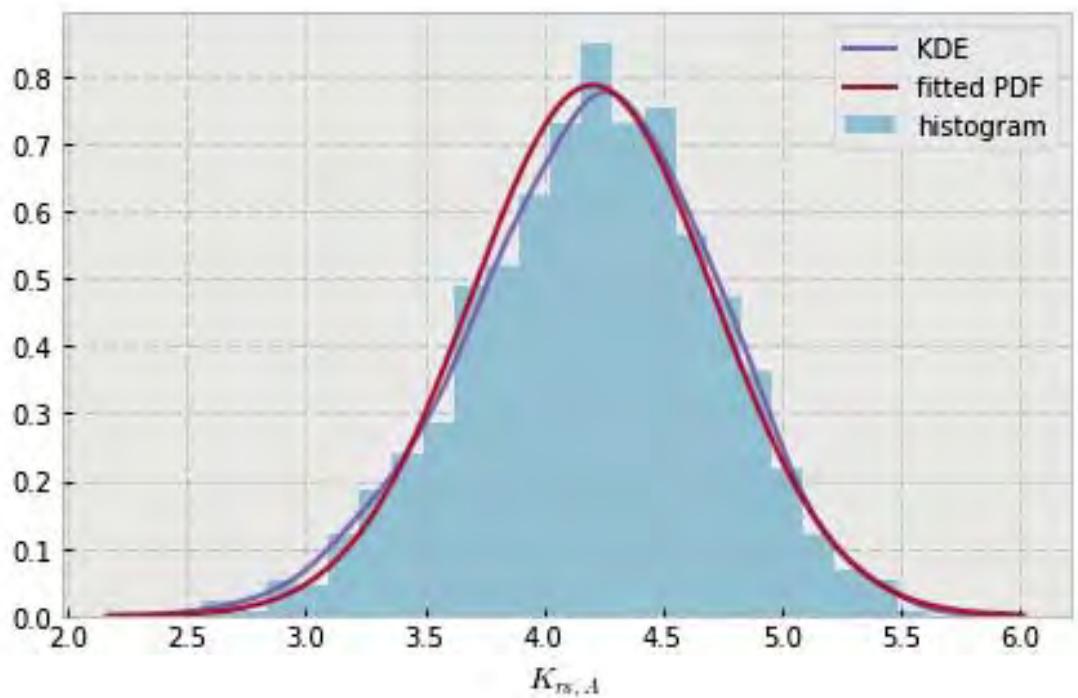


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=4.3588593523458465, scale=0.5174077372806418)
DescribeResult(nobs=1000, minmax=(2.4869611347593259, 5.6190333909485615)
               mean=4.3588593523458465, variance=0.26797874534321681,
               skewness=-0.37708863798008685, kurtosis=0.081263558369713)
```

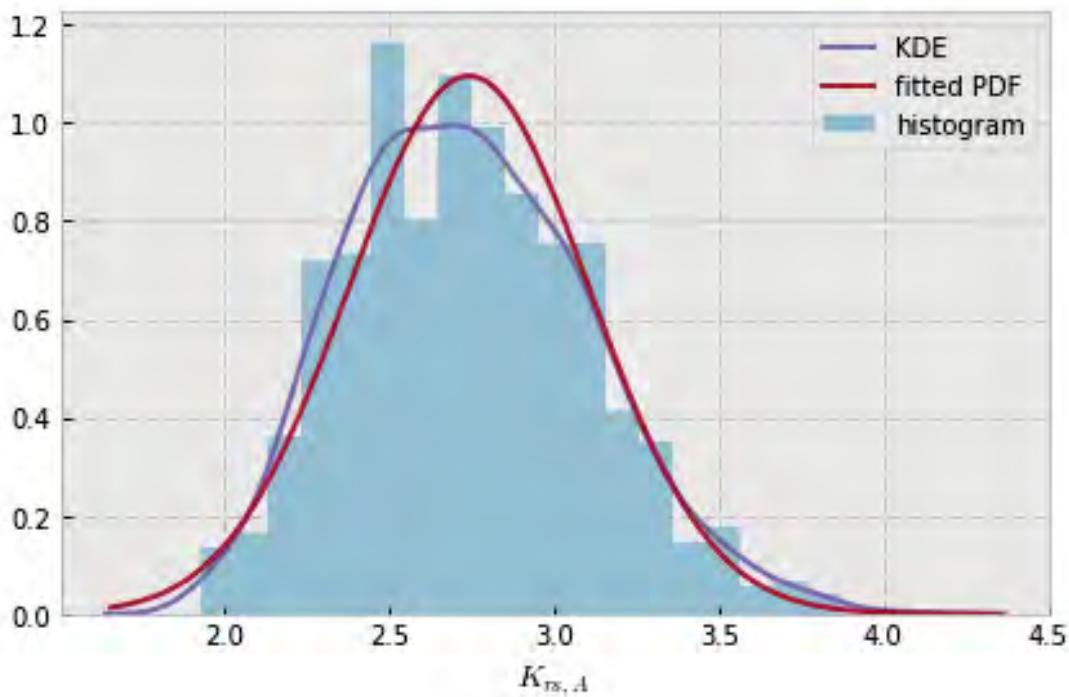


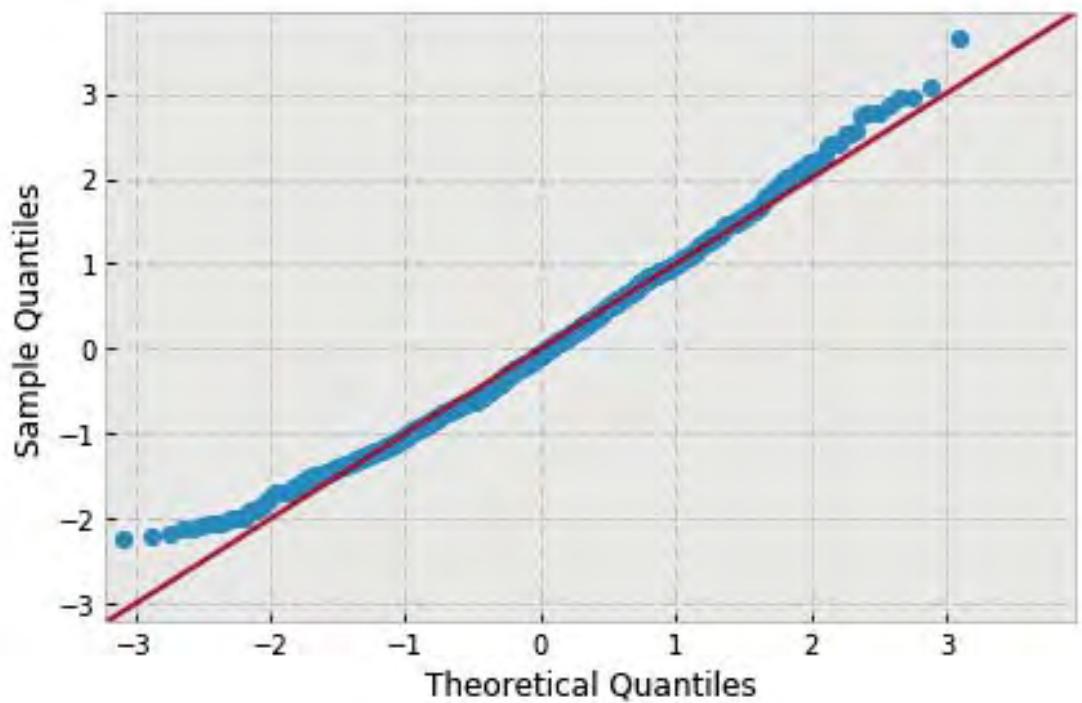


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=4.2048878319442995, scale=0.5072793435082414)
DescribeResult(nobs=1000, minmax=(2.5627221980014592, 5.6180009384264515)
               mean=4.2048878319442995, variance=0.25758992227242483,
               skewness=-0.2109751142974772, kurtosis=-0.09535086246889568)
```

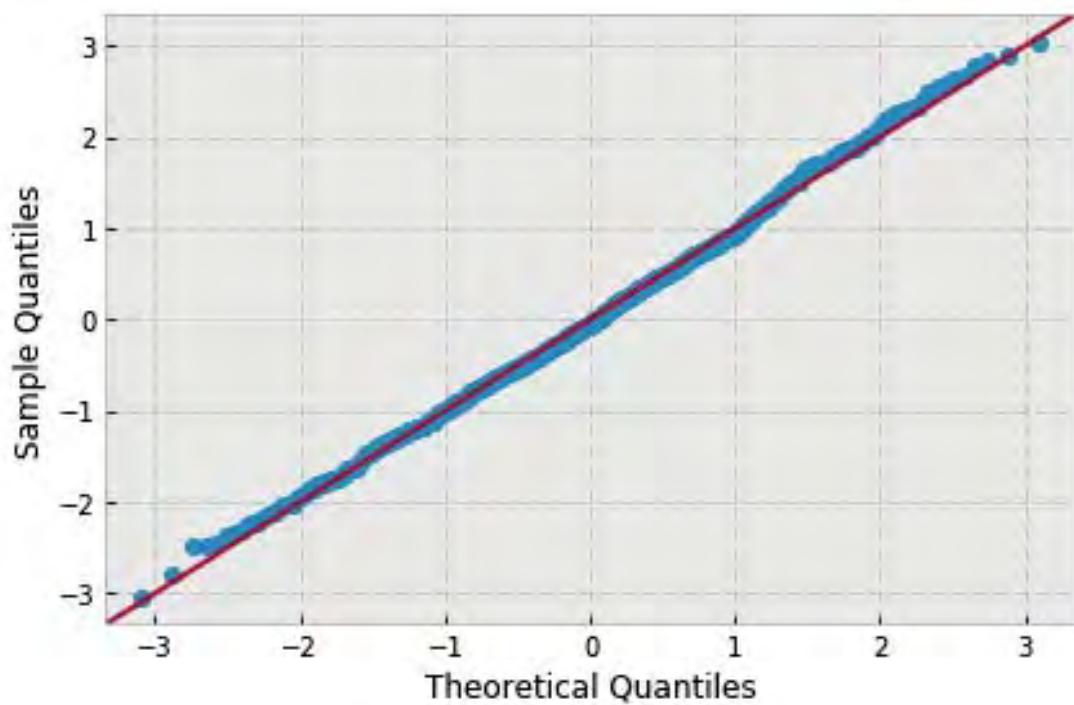
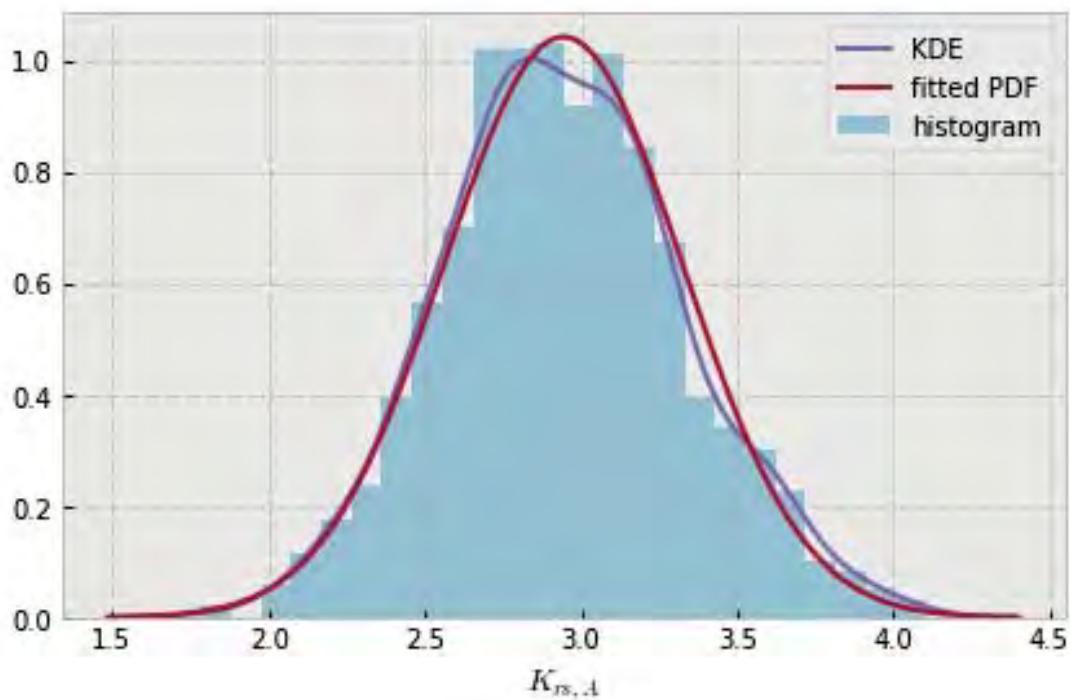


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=2.7440696649160334, scale=0.3641293630125752)
DescribeResult(nobs=1000, minmax=(1.9330966407308812, 4.0741078669767923)
               mean=2.7440696649160334, variance=0.13272291592386765,
               skewness=0.361316500044737, kurtosis=-0.11841480960681272)
```

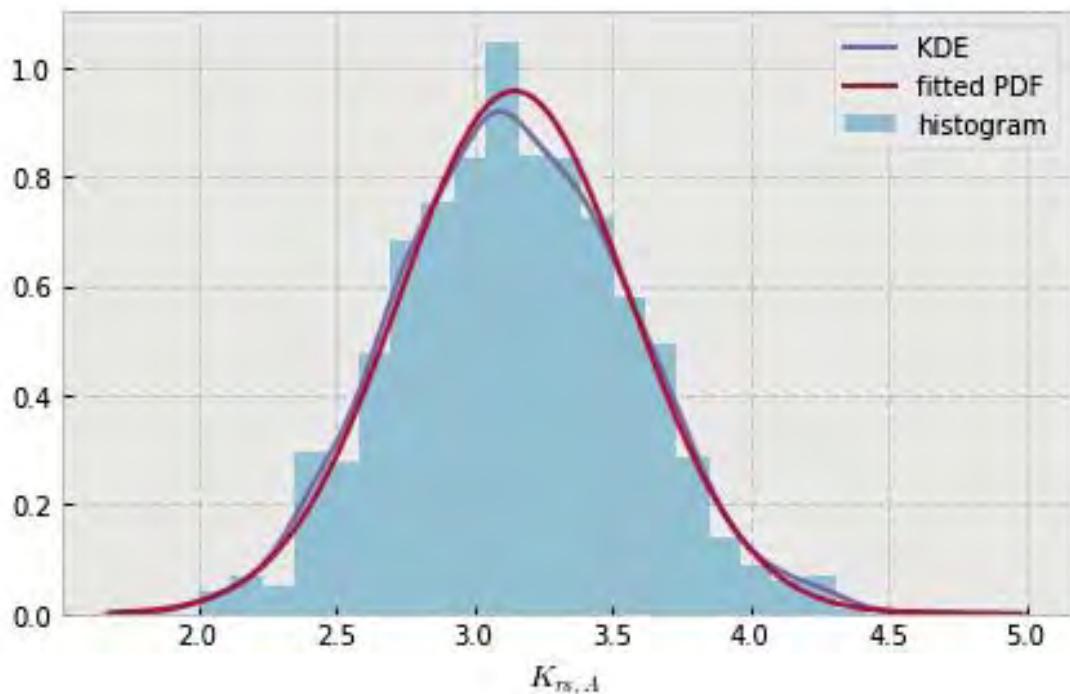


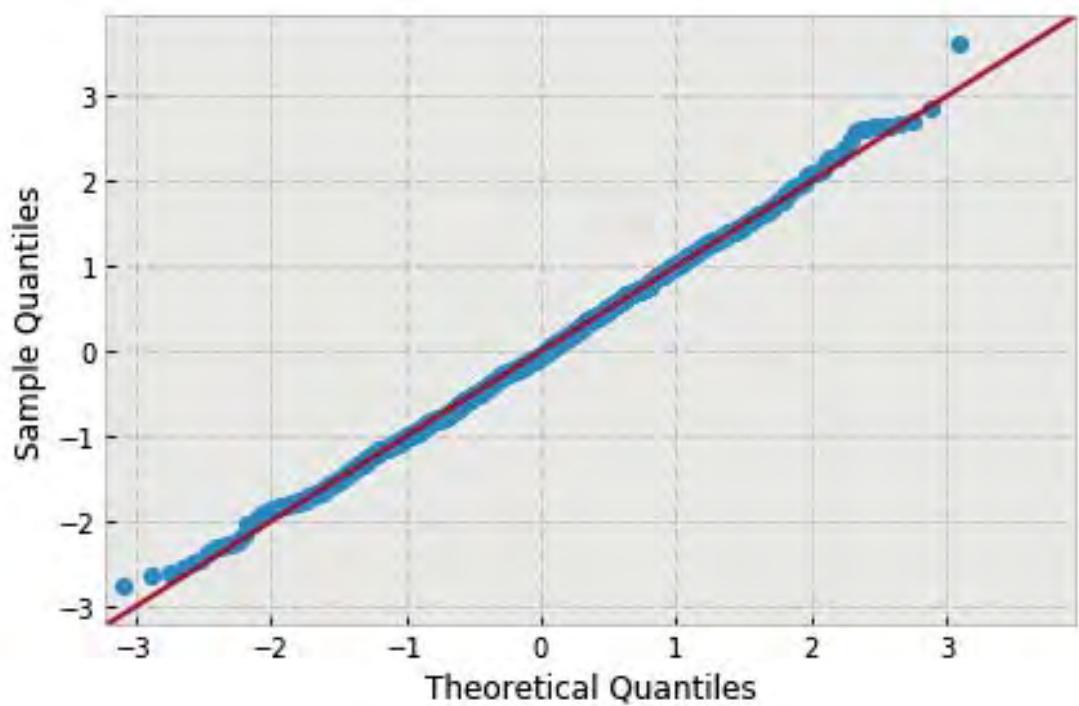


```
variable name: theta
variable value: 0.78539816333974483
distribution: normal(shape=(), loc=2.9432113849905672, scale=0.3829539819502022)
DescribeResult(nobs=1000, minmax=(1.7809391650024851, 4.1051344320372998)
               mean=2.9432113849905672, variance=0.14680055284436014,
               skewness=0.15810059679609995, kurtosis=-0.07336994544843378)
```

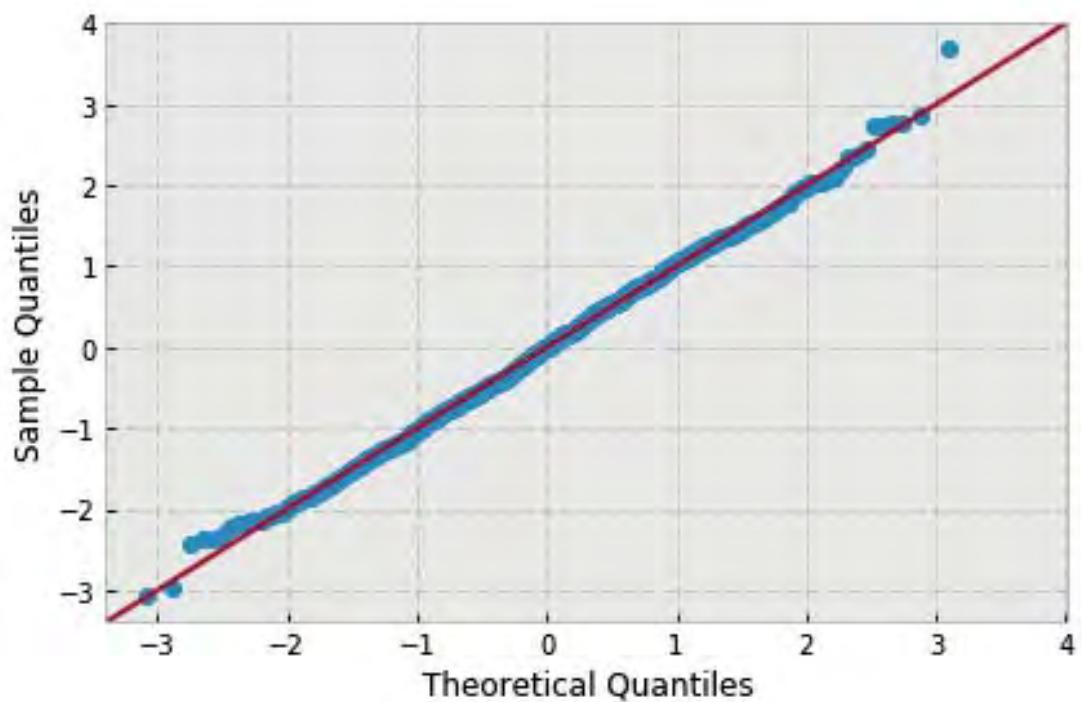
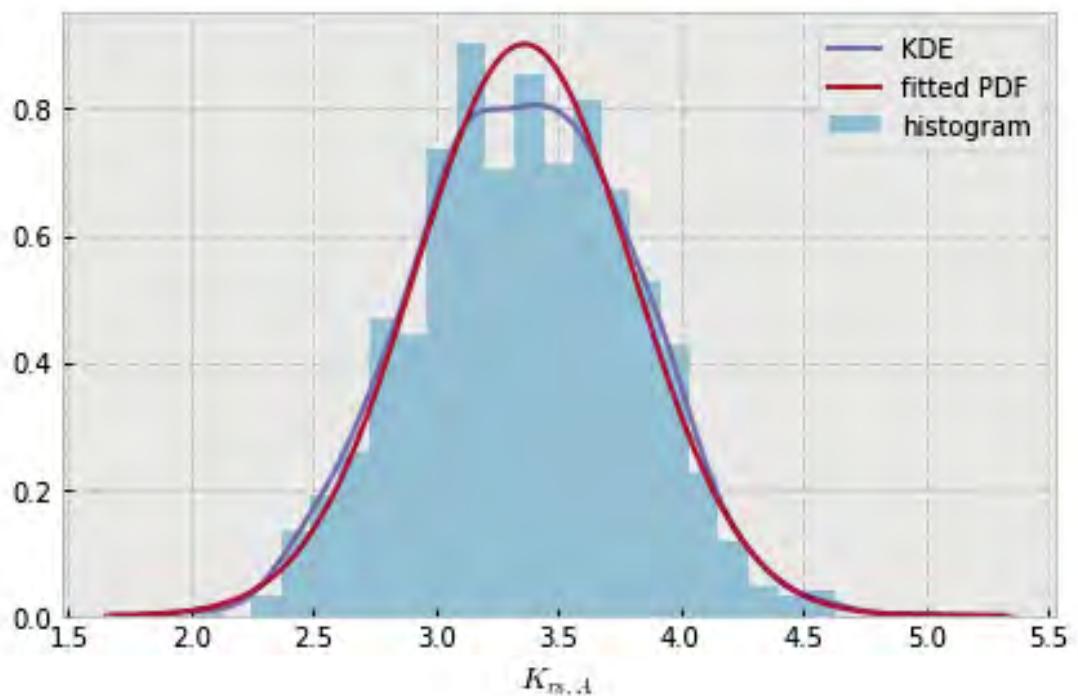


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=3.1481800254337435, scale=0.41731511178818054)
DescribeResult(nobs=1000, minmax=(2.0046476651392471, 4.6601380421992689)
               mean=3.1481800254337435, variance=0.17432622875553716,
               skewness=0.12501889845251435, kurtosis=-0.09970830901798289)
```

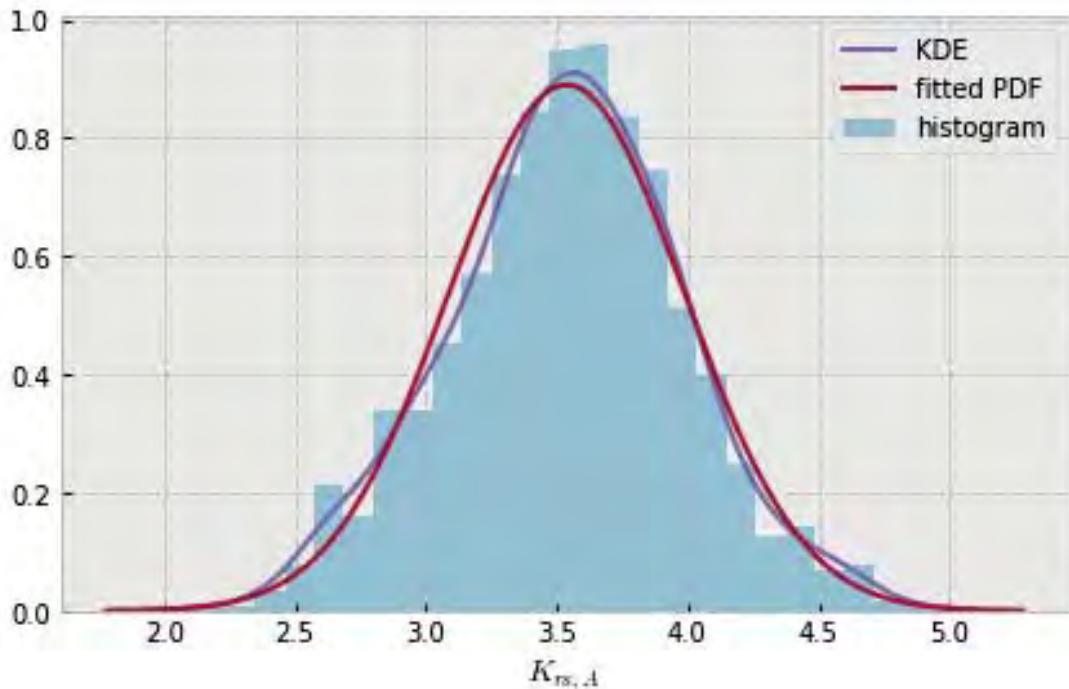


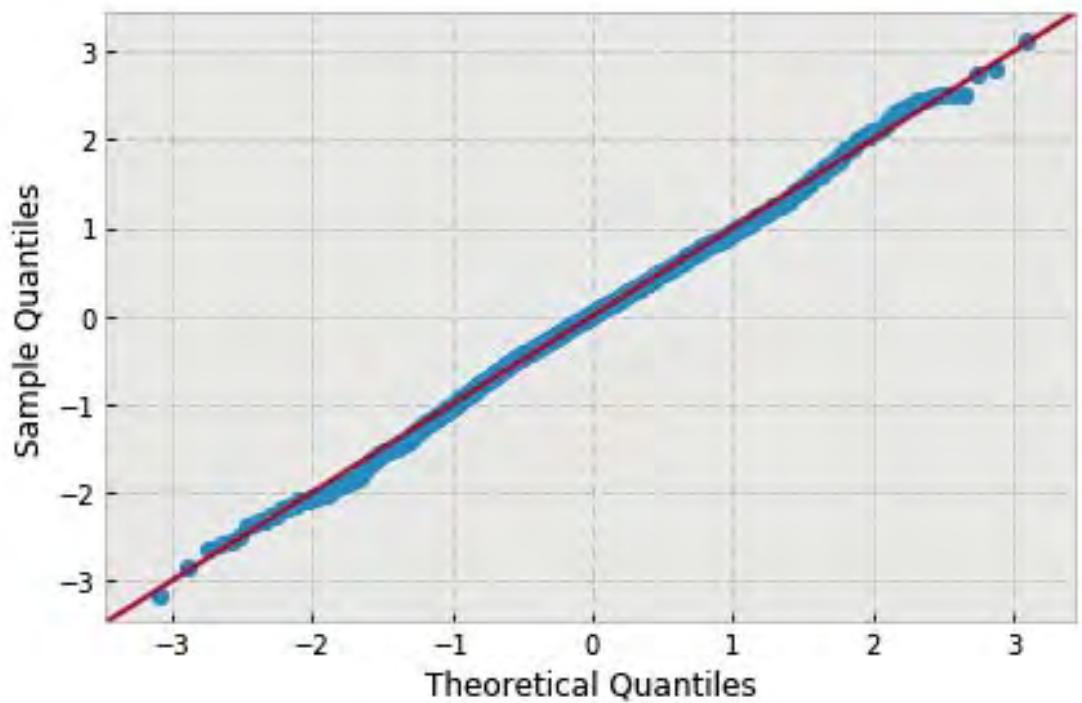


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=3.360037622630085, scale=0.4422142350906332)
DescribeResult(nobs=1000, minmax=(2.0075576911655038, 4.9887622602662143)
               mean=3.360037622630085, variance=0.19574917889568946,
               skewness=0.03730019767240711, kurtosis=-0.18778347218542235)
```

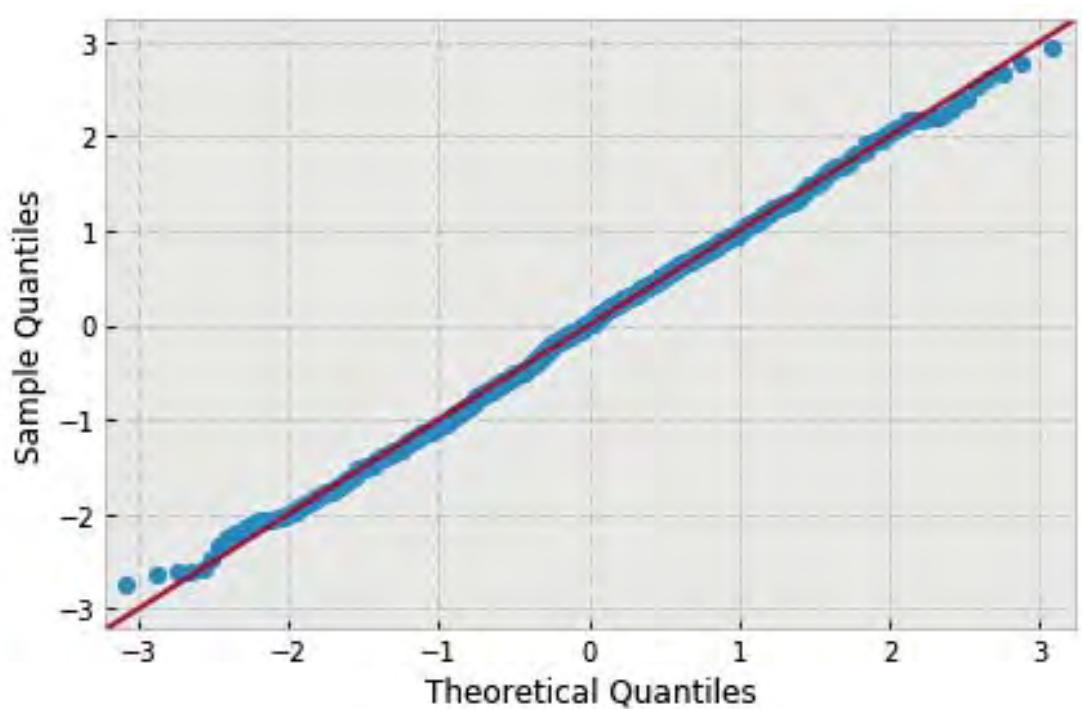
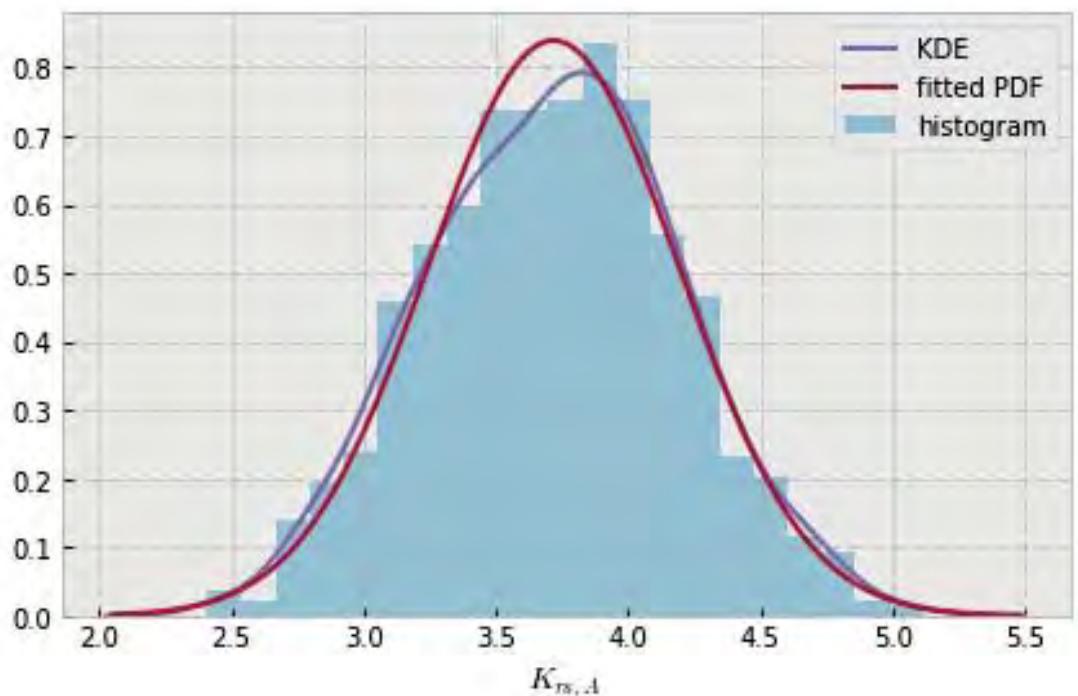


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=3.537594688805644, scale=0.44830893586381815)
DescribeResult(nobs=1000, minmax=(2.1238878559575336, 4.936379174408895)
               mean=3.537594688805644, variance=0.20118208405940843,
               skewness=-0.04651688699522977, kurtosis=0.0070344400591957346)
```

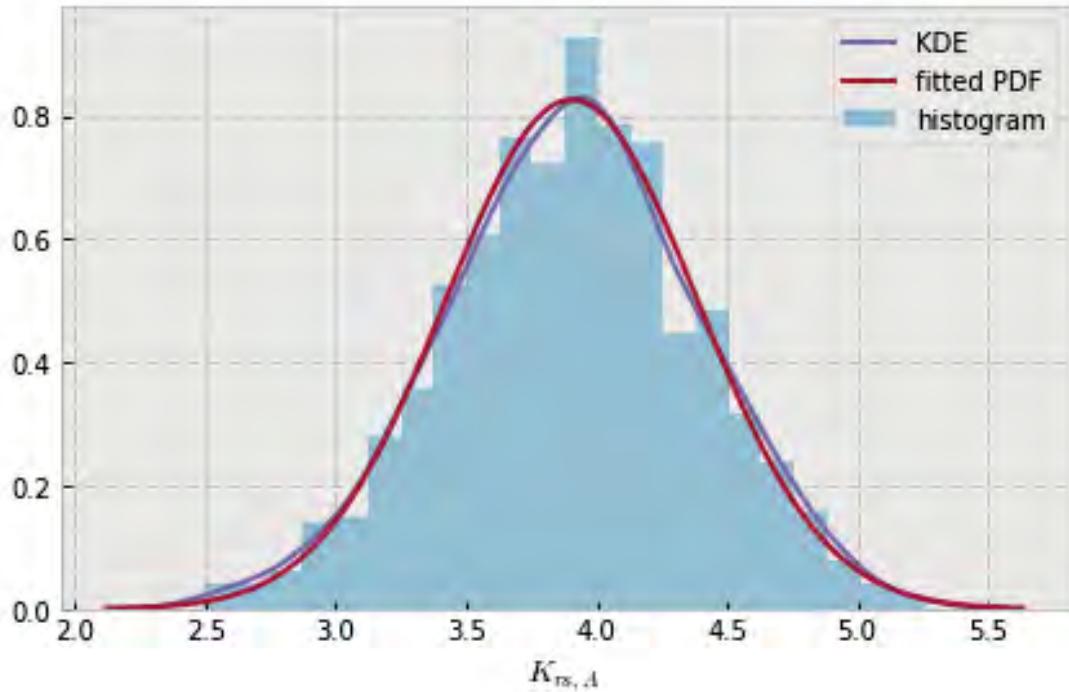


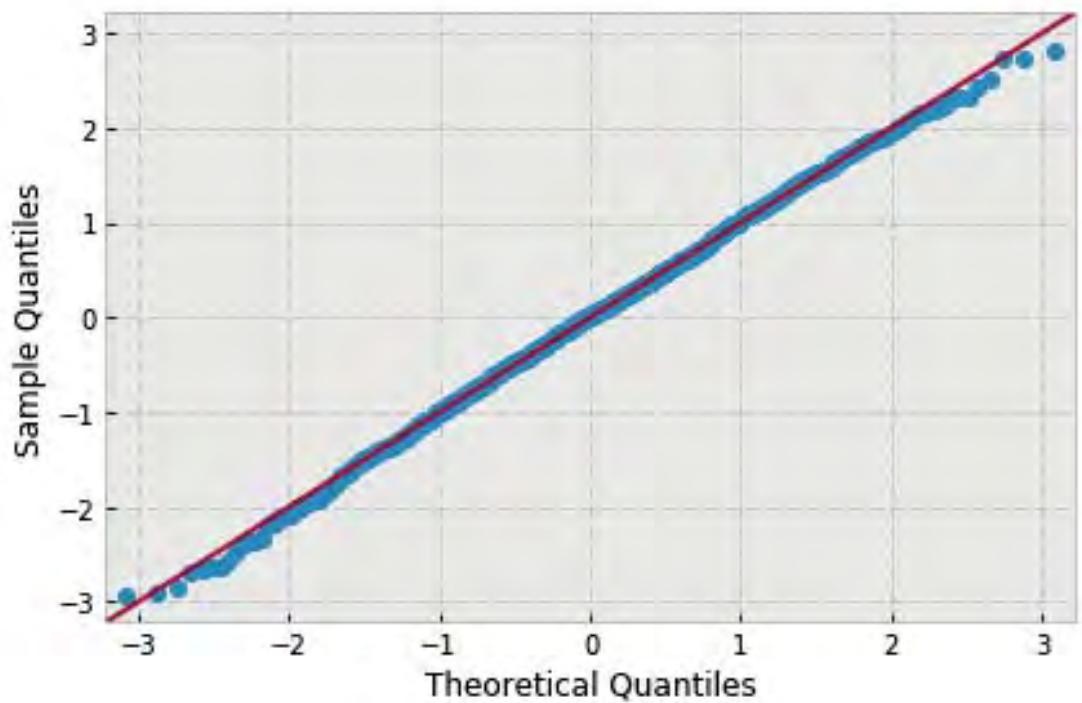


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=3.7165708251974183, scale=0.4752871630396627)
DescribeResult(nobs=1000, minmax=(2.4081430075559633, 5.1164961975679644)
               mean=3.7165708251974183, variance=0.22612401136165253,
               skewness=-0.005464772093466557, kurtosis=-0.3039500006348761)
```

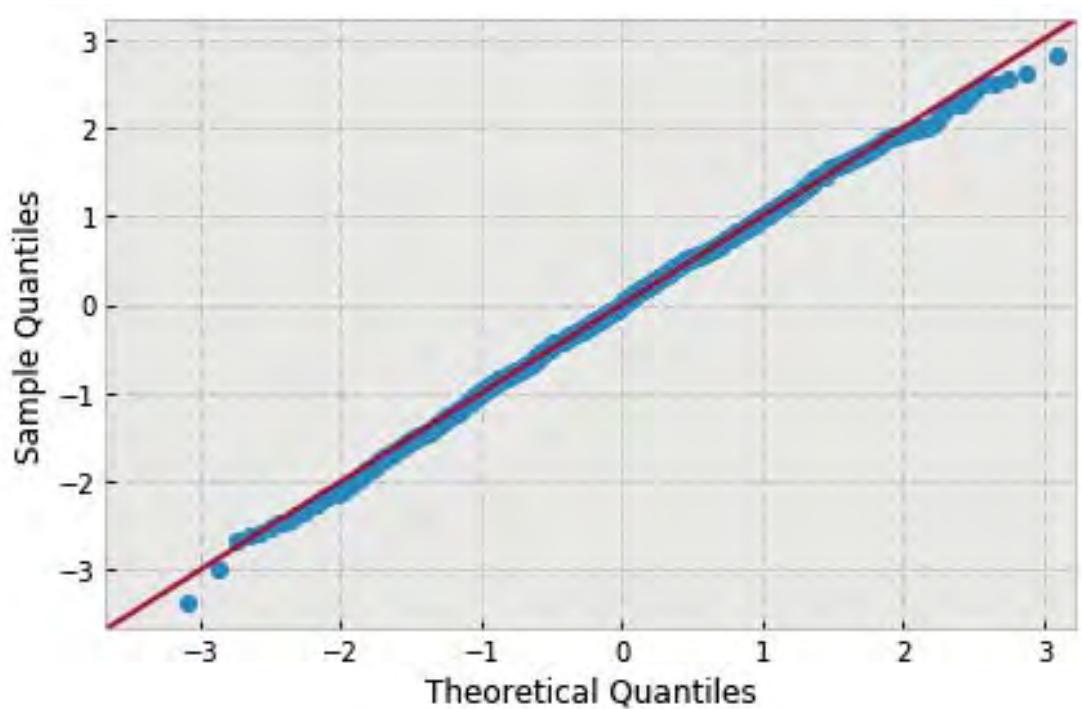
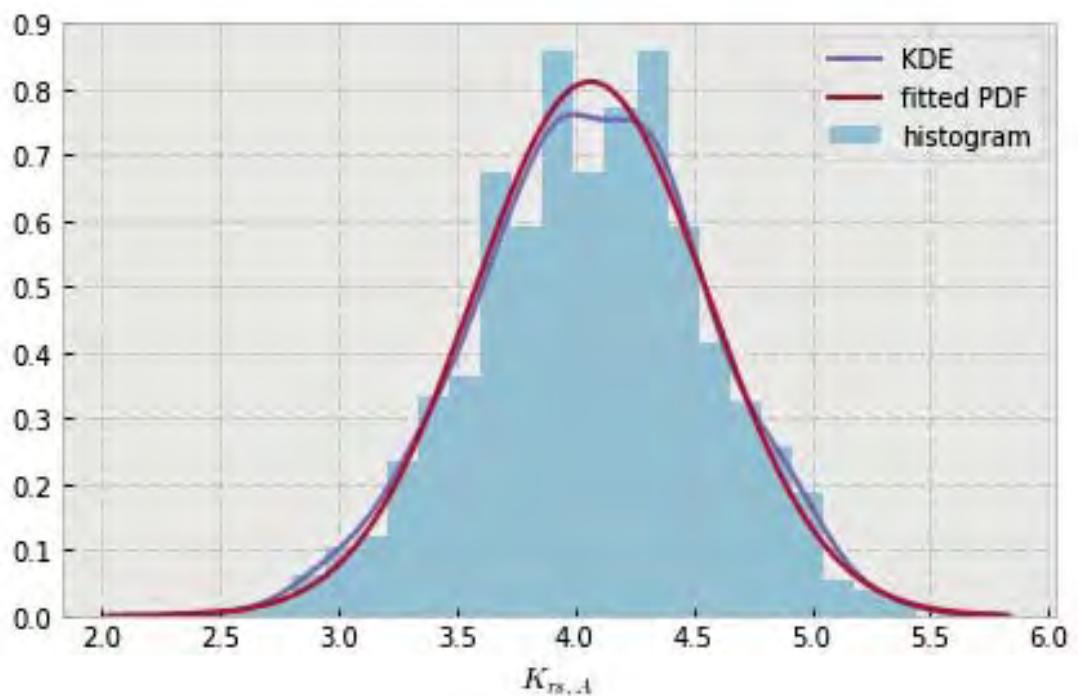


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=3.903636637772367, scale=0.48273581485803485)
DescribeResult(nobs=1000, minmax=(2.4929320323933308, 5.2603022669747022)
               mean=3.903636637772367, variance=0.23326713408073163,
               skewness=-0.07824369944784777, kurtosis=-0.11248232833761884)
```

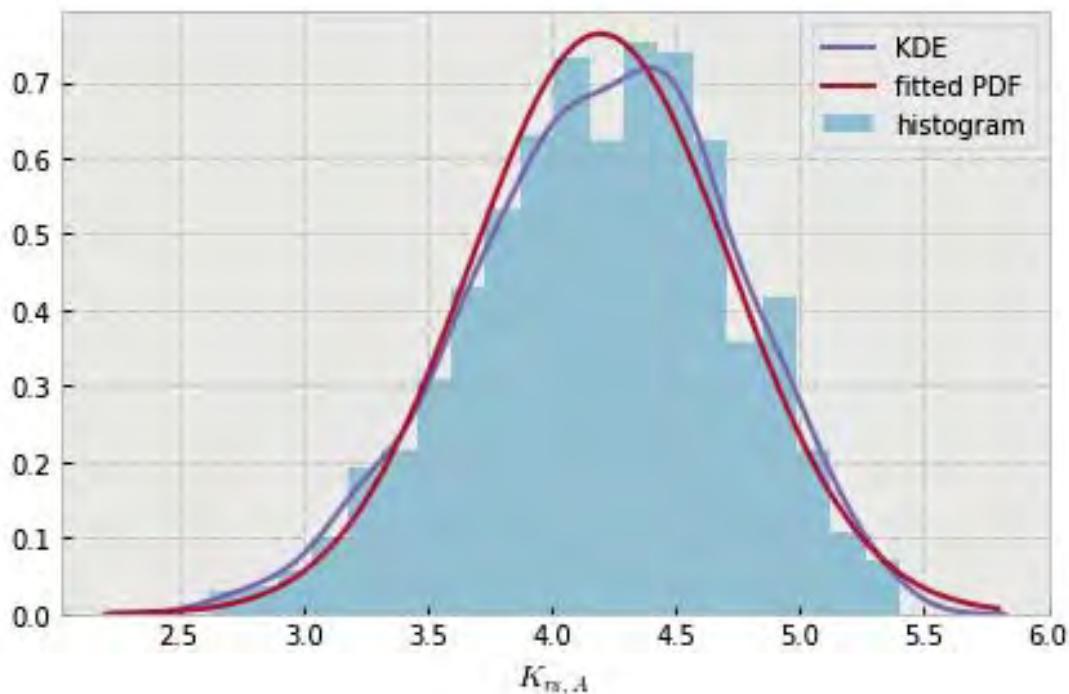


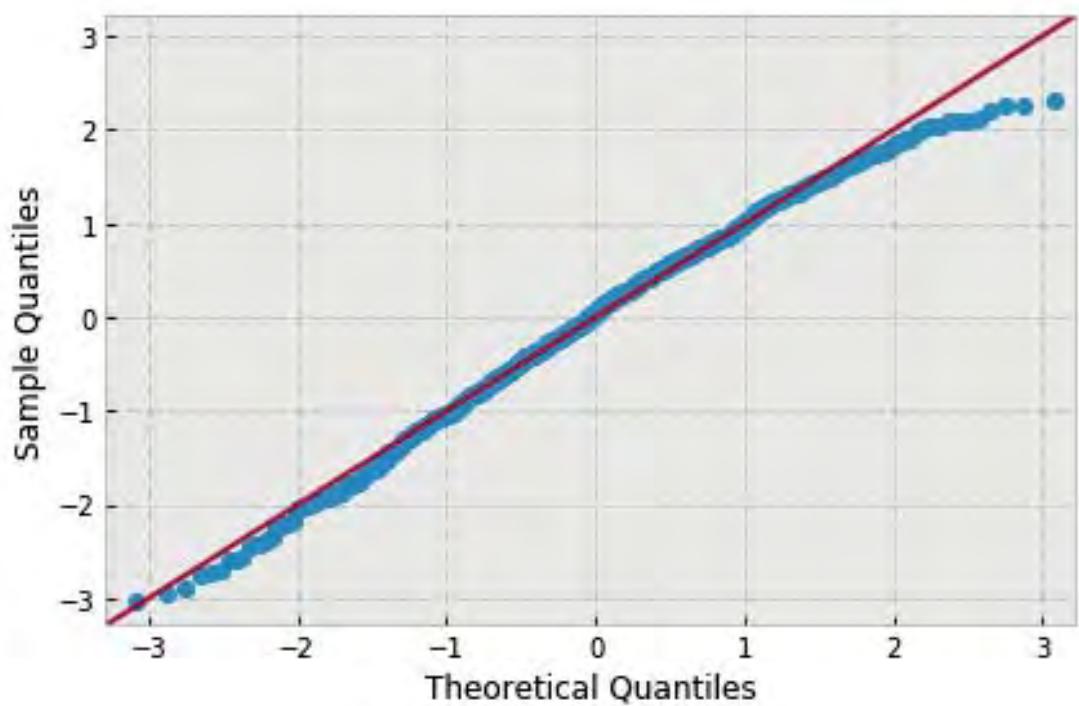


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=4.065272571271387, scale=0.4916028227731875)
DescribeResult(nobs=1000, minmax=(2.4125929342733716, 5.4458974678697416)
               mean=4.0652725712713869, variance=0.24191525060917521,
               skewness=-0.10600189464975605, kurtosis=-0.1501691641811509)
```

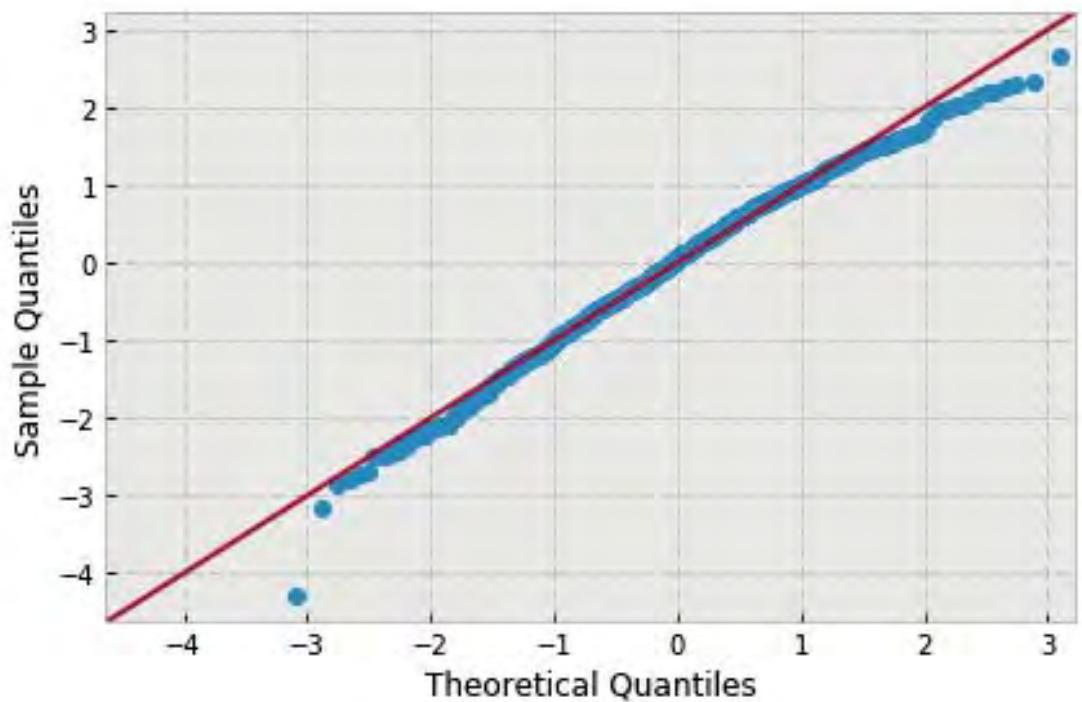
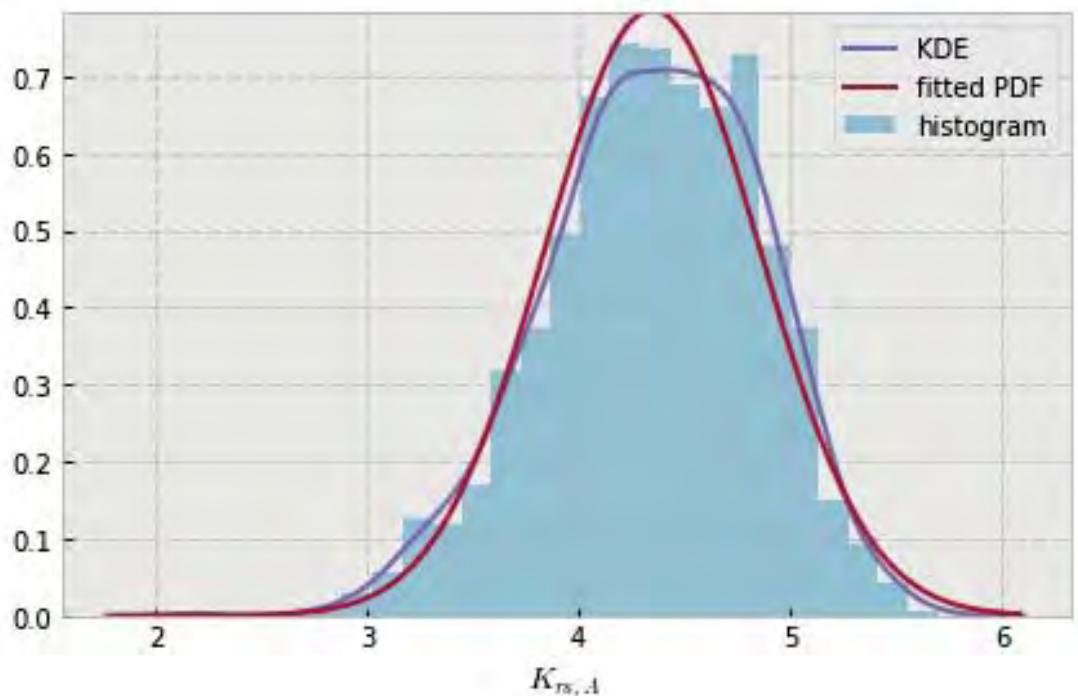


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=4.196971579402141, scale=0.5224667283770031)
DescribeResult(nobs=1000, minmax=(2.6203322121427779, 5.4044689964974246)
               mean=4.1969715794021409, variance=0.27324472698795721,
               skewness=-0.2571249075175762, kurtosis=-0.27170293549299274)
```





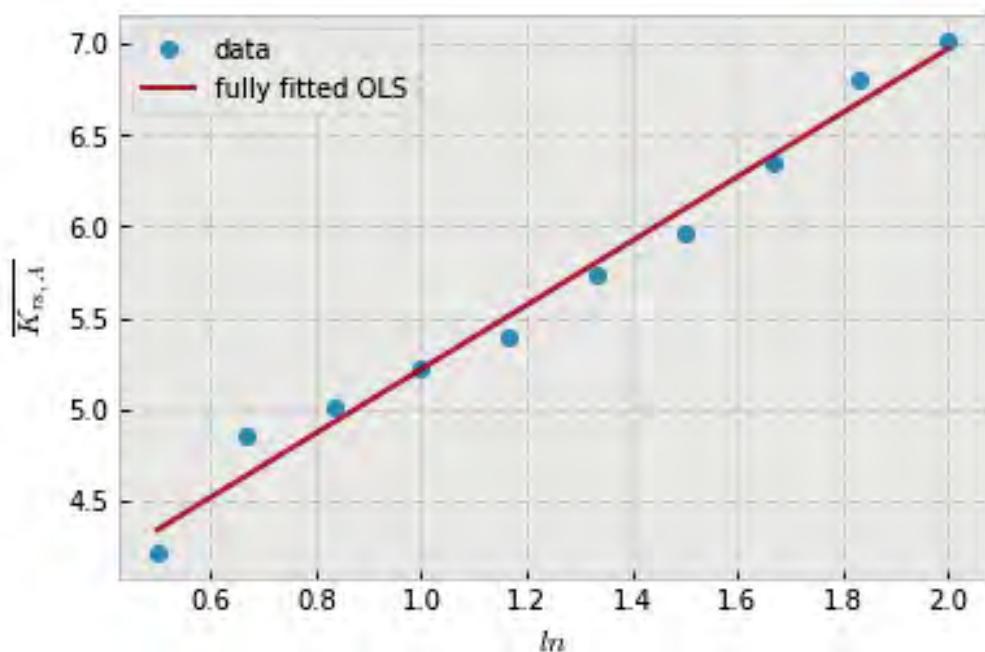
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=4.349139950263933, scale=0.5075636184127669)
DescribeResult(nobs=1000, minmax=(2.1761815858150424, 5.7014681797617861)
               mean=4.349139950263934, variance=0.25787870544170255,
               skewness=-0.33461445442360394, kurtosis=-0.024614048610140493)
```



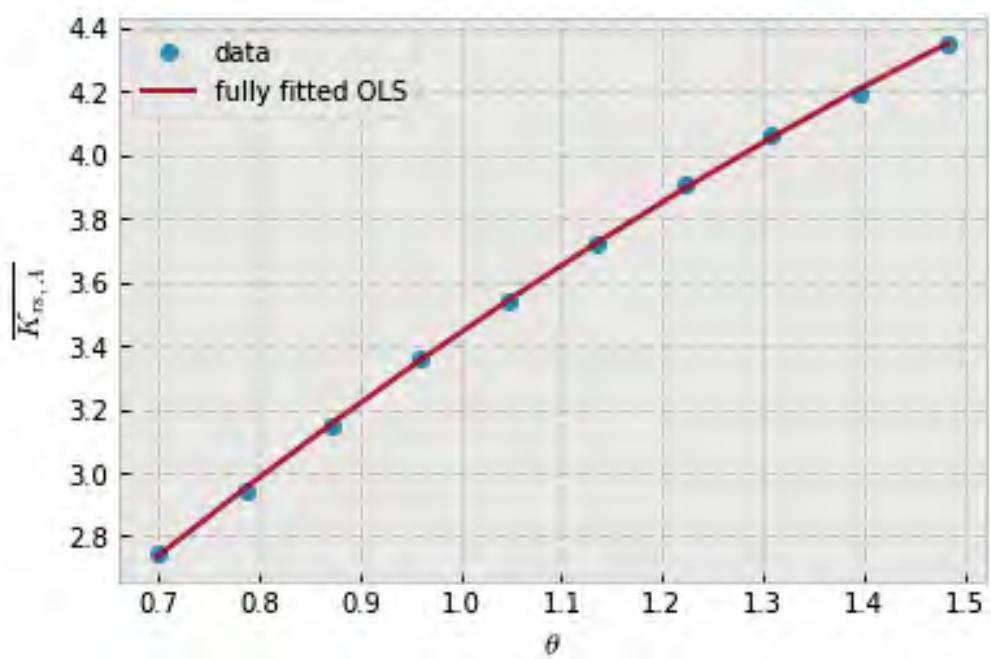
16.2 Parameter regression

```
In [38]: var_name = 'half_mean_distance_between_roots'
mean_reg('ln', var_name, 'poly1')
mean_reg('theta', var_name, 'poly2')
mean_reg('nB', var_name, 'poly2')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly2')
mean_reg('N', var_name, 'poly1')

variable name: ln
model: poly1
r-squared: 0.9703289853093918
explained variance: 0.9703505825275663
```



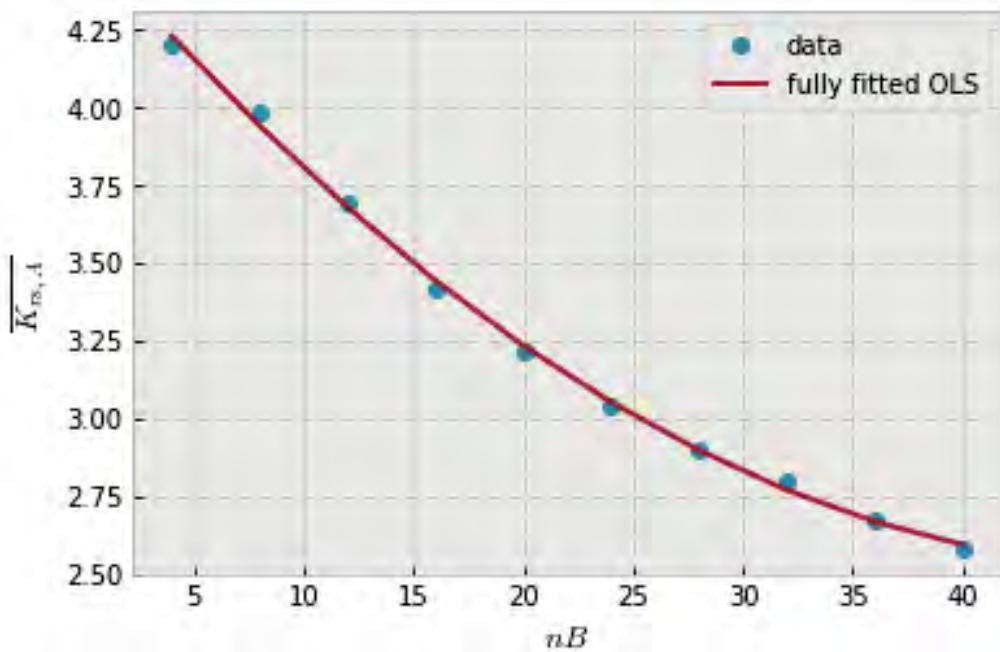
```
variable name: theta
model: poly2
r-squared: 0.9994437688058402
explained variance: 0.9994459390878516
```



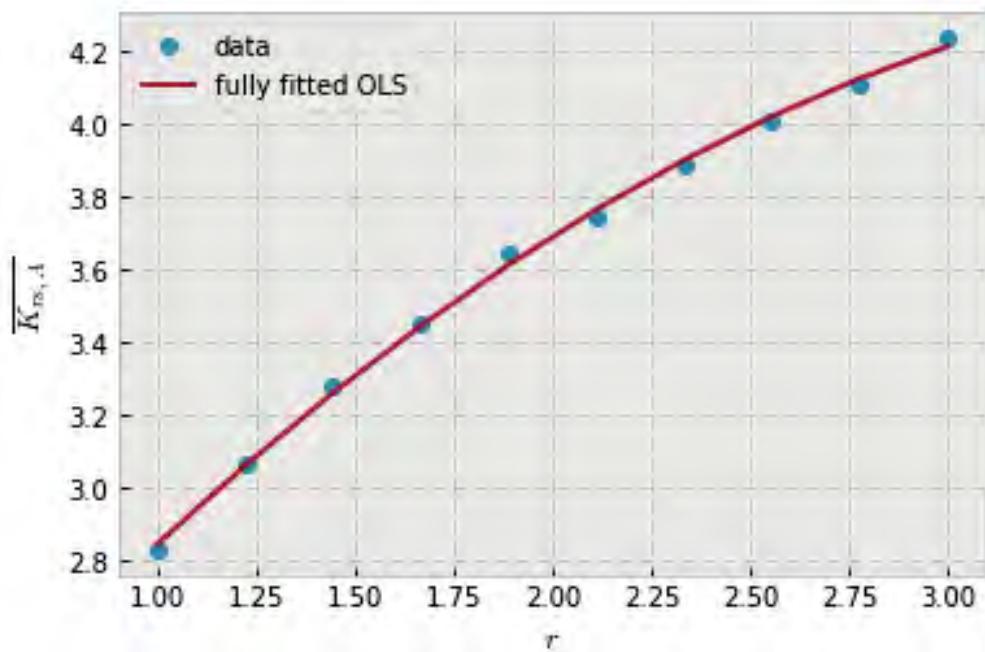
```

variable name: nB
model: poly2
r-squared: 0.9953674902487221
explained variance: 0.9954610008257605

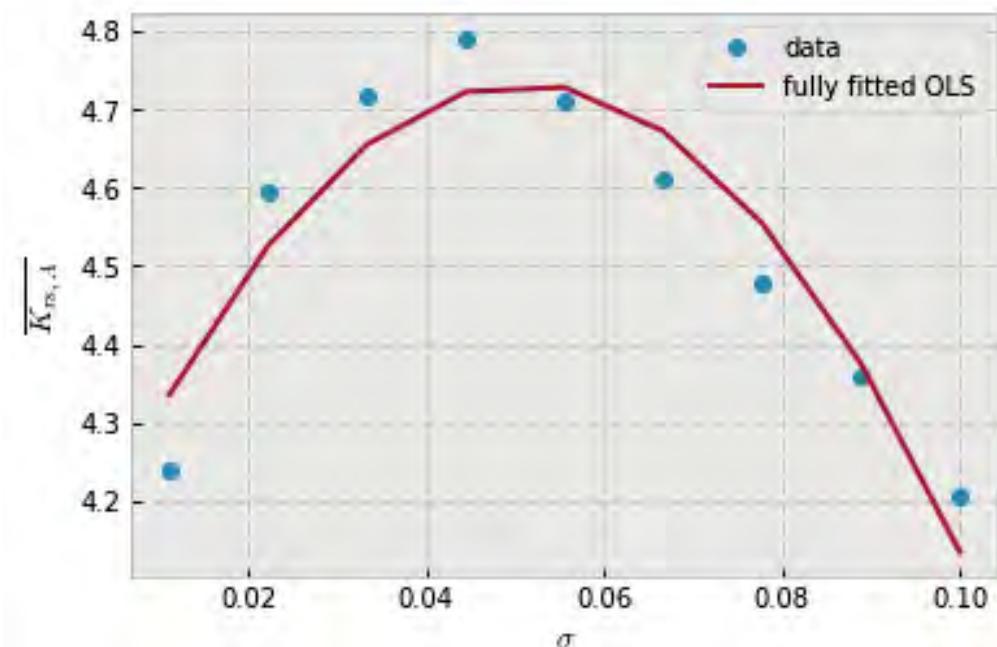
```



```
variable name: r
model: poly2
r-squared: 0.9948123797492481
explained variance: 0.9948161429745568
```



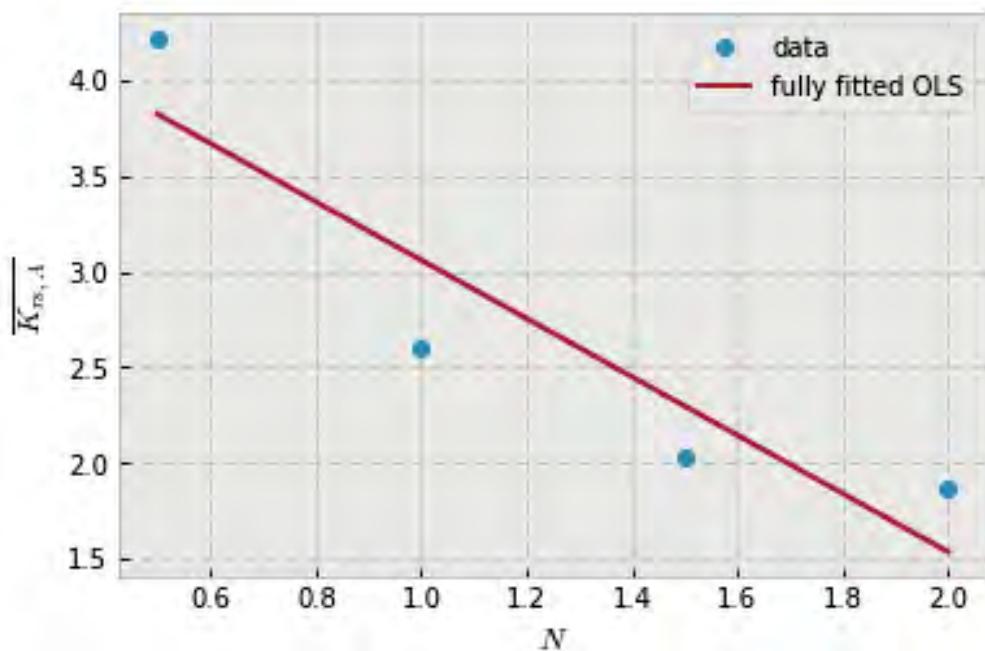
```
variable name: sigma
model: poly2
r-squared: 0.5596462985625497
explained variance: 0.5600695811199357
```



```

variable name: N
model: poly1
r-squared: -0.030185132873665976
explained variance: 0.10948234980061311

```

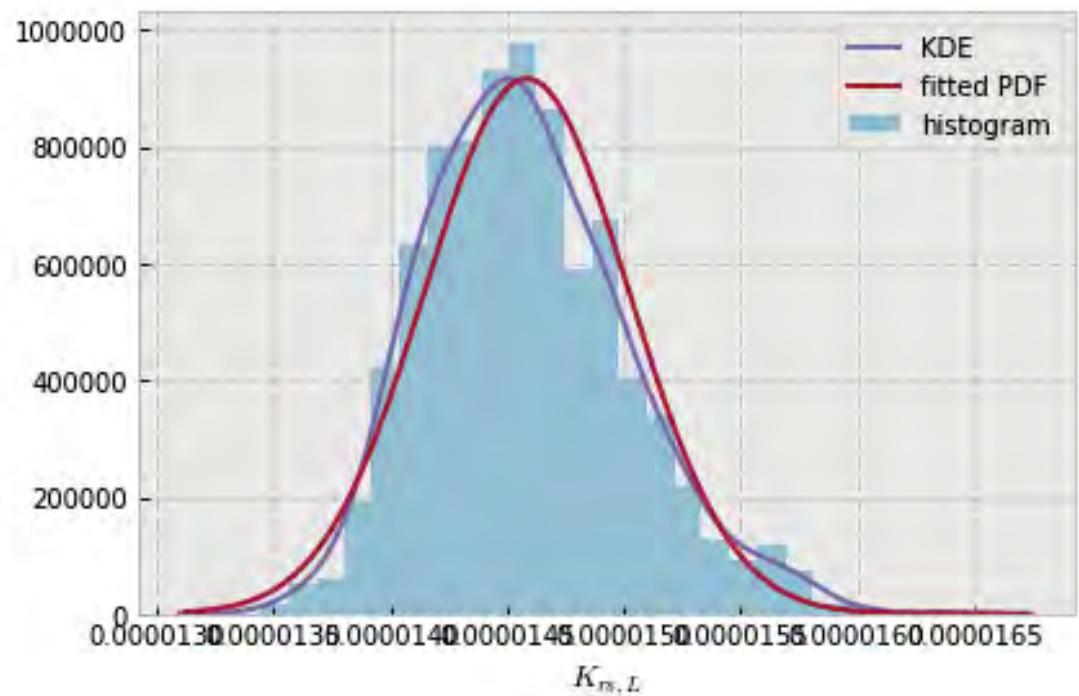


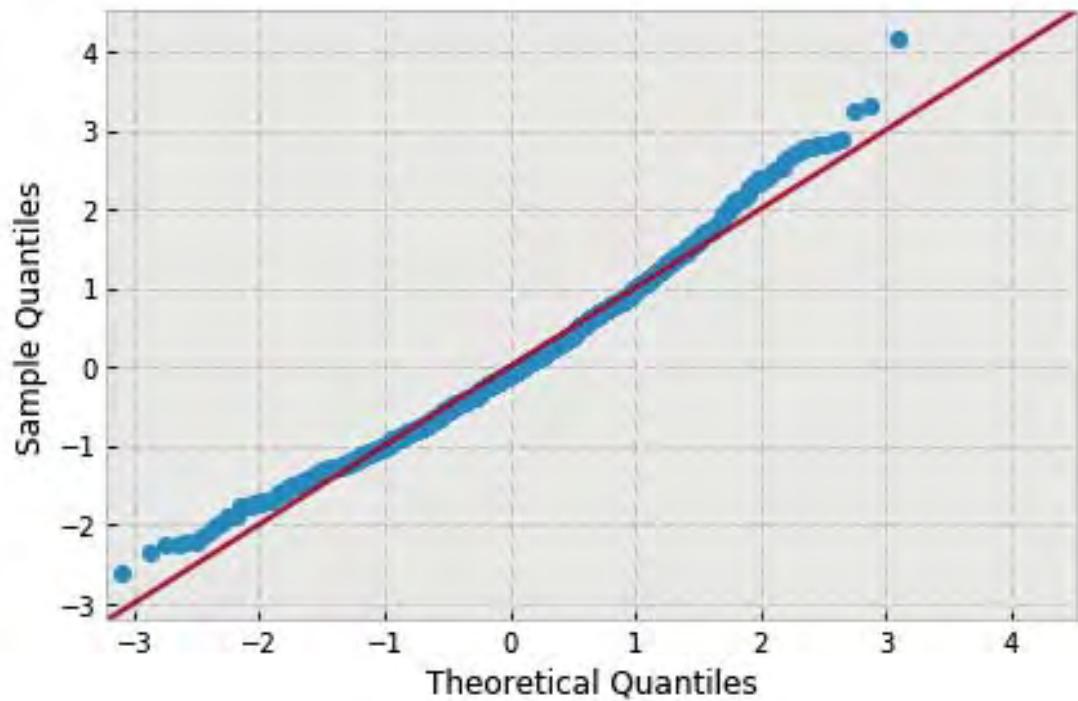
17 Root system conductance per unit of root area

17.1 Probability distributions

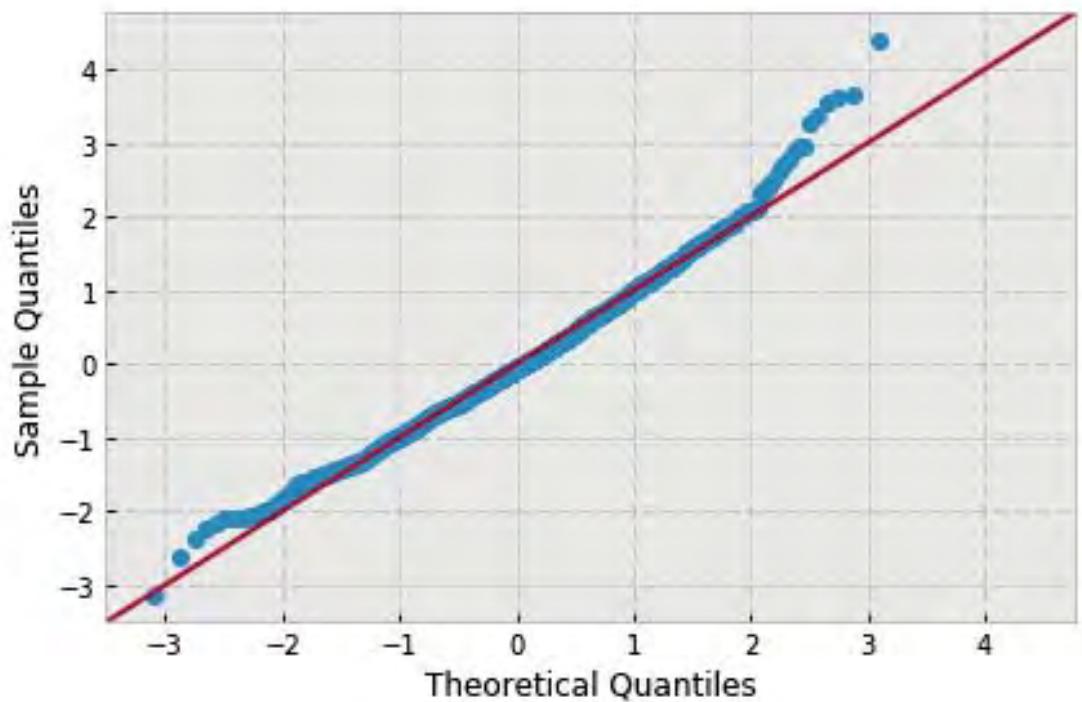
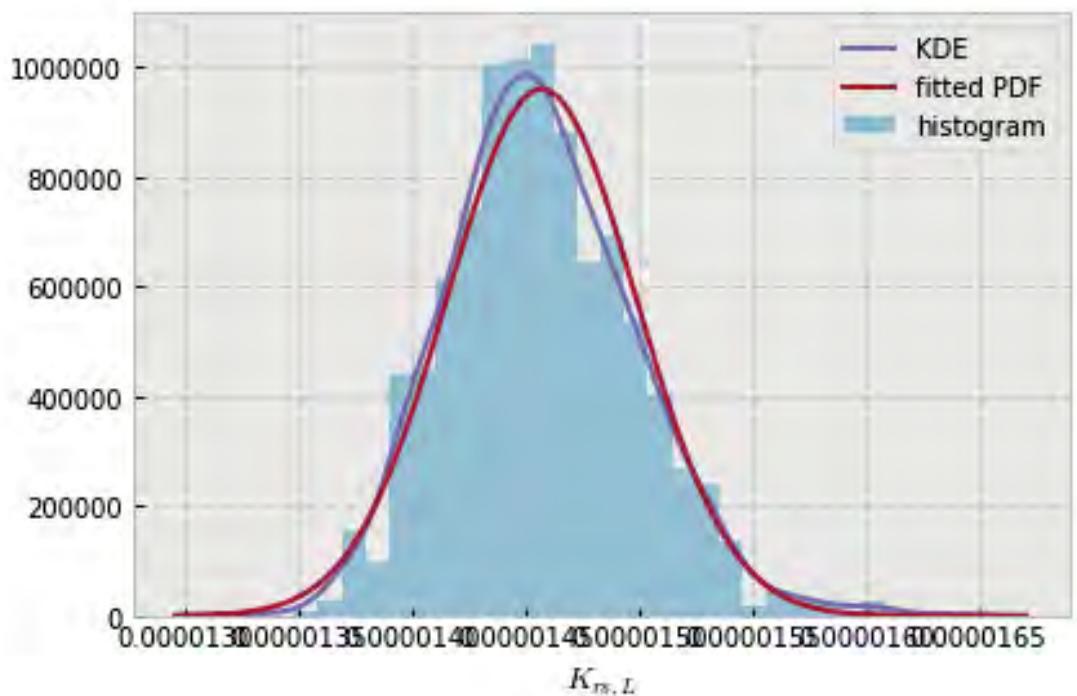
```
In [39]: plot_dist('root_system_conductance_per_unit_of_root_area')

variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=1.4583751258107807e-05, scale=4.352369257630005e-07)
DescribeResult(nobs=1000, minmax=(1.3445007597504146e-05, 1.6399645549551826e-05)
               mean=1.4583751258107807e-05, variance=1.8962080234997757e-13,
               skewness=0.5096976870757304, kurtosis=0.29640765043729456)
```

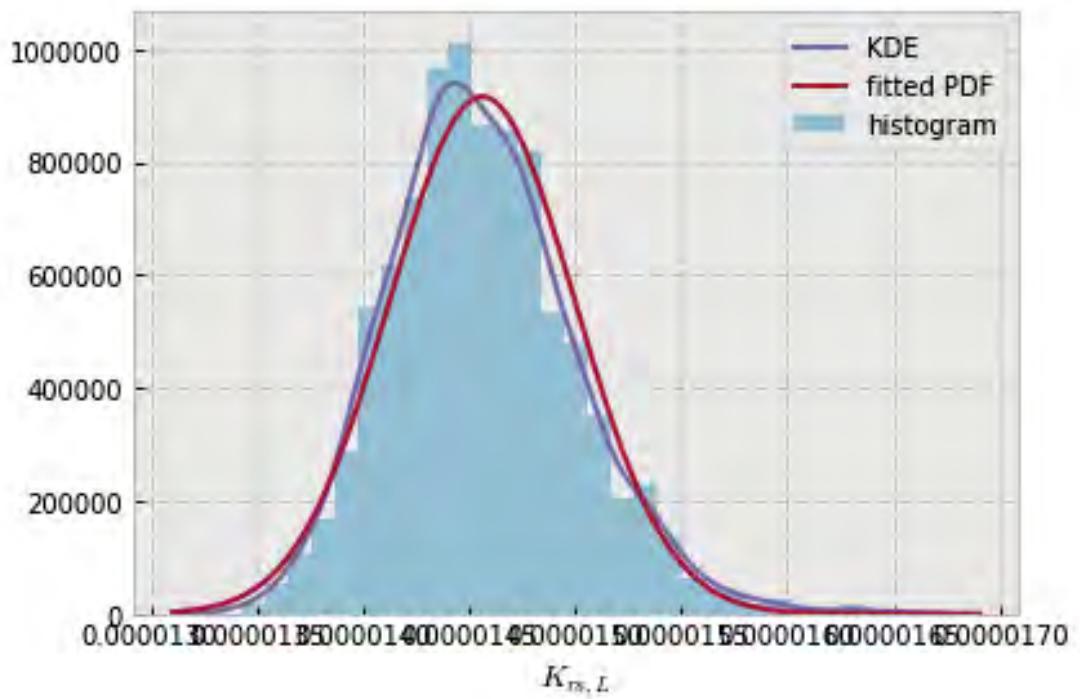


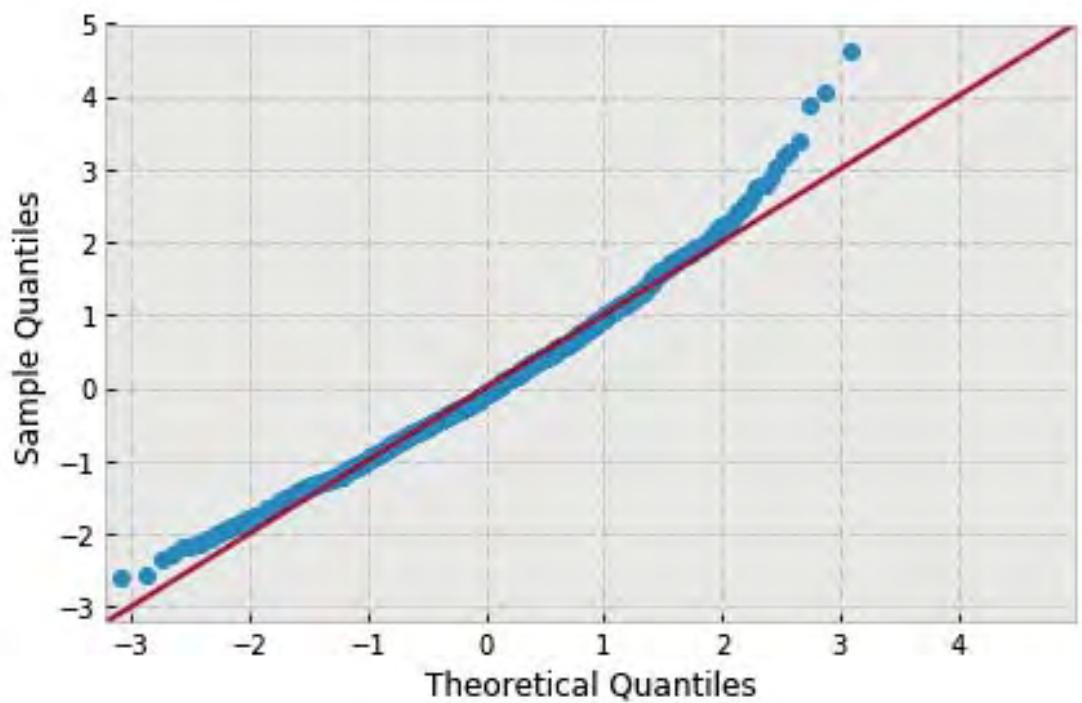


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=1.4570442384417457e-05, scale=4.154249833659862e-07)
DescribeResult(nobs=1000, minmax=(1.3269367767063472e-05, 1.6393413455803865e-05)
               mean=1.4570442384417457e-05, variance=1.7275066747210201e-13,
               skewness=0.4335722315106894, kurtosis=0.6077016636191481)
```

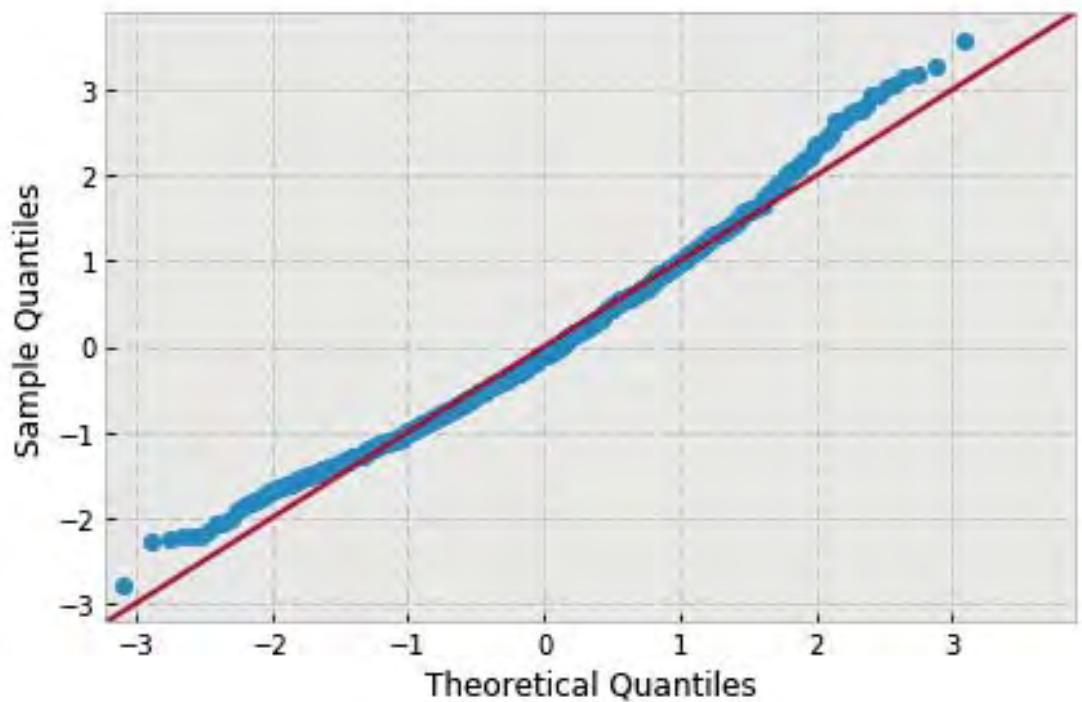
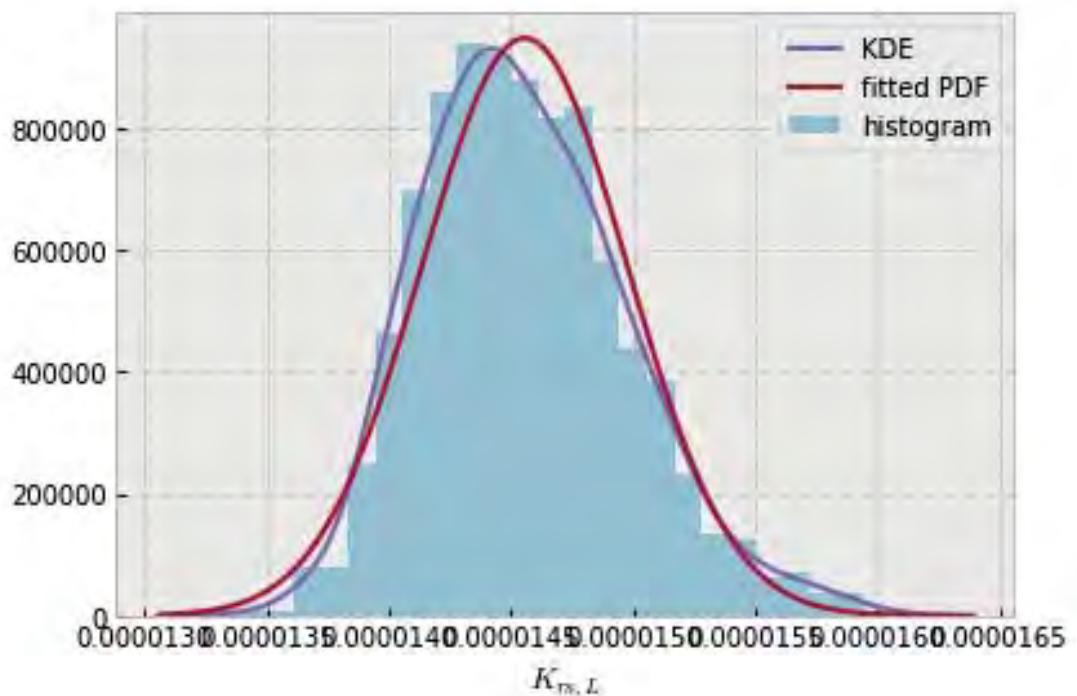


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=1.4563712783707934e-05, scale=4.3530118792947176e-07)
DescribeResult(nobs=1000, minmax=(1.343387252717082e-05, 1.6575858232423418e-05)
               mean=1.4563712783707934e-05, variance=1.8967680101382313e-13,
               skewness=0.5312592971835901, kurtosis=0.7662094570232583)
```

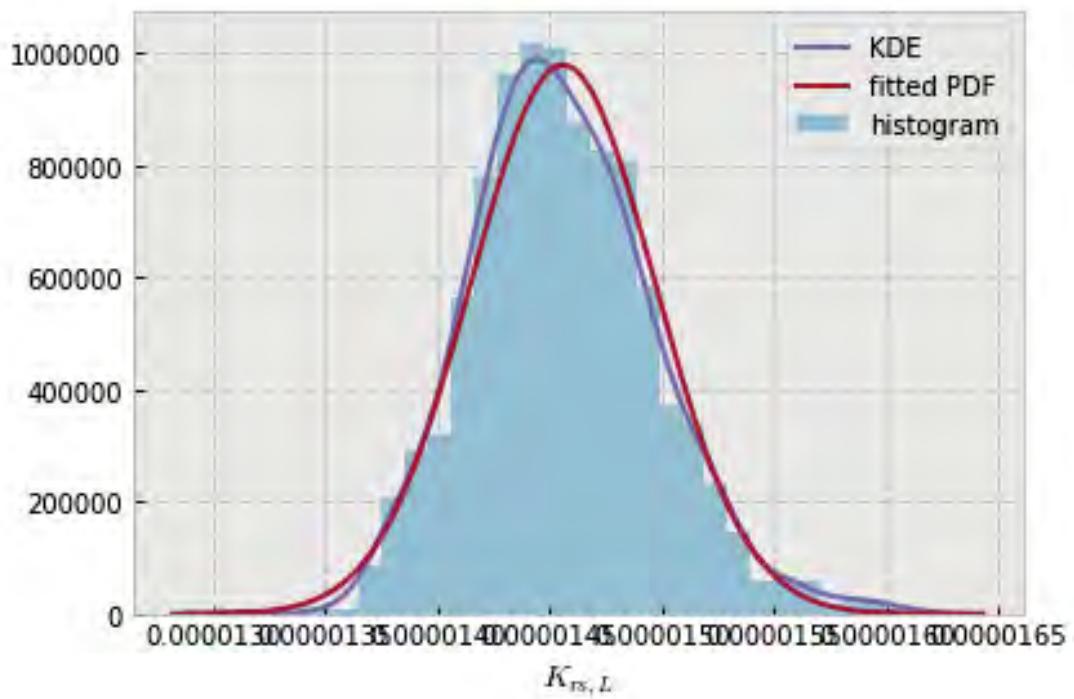


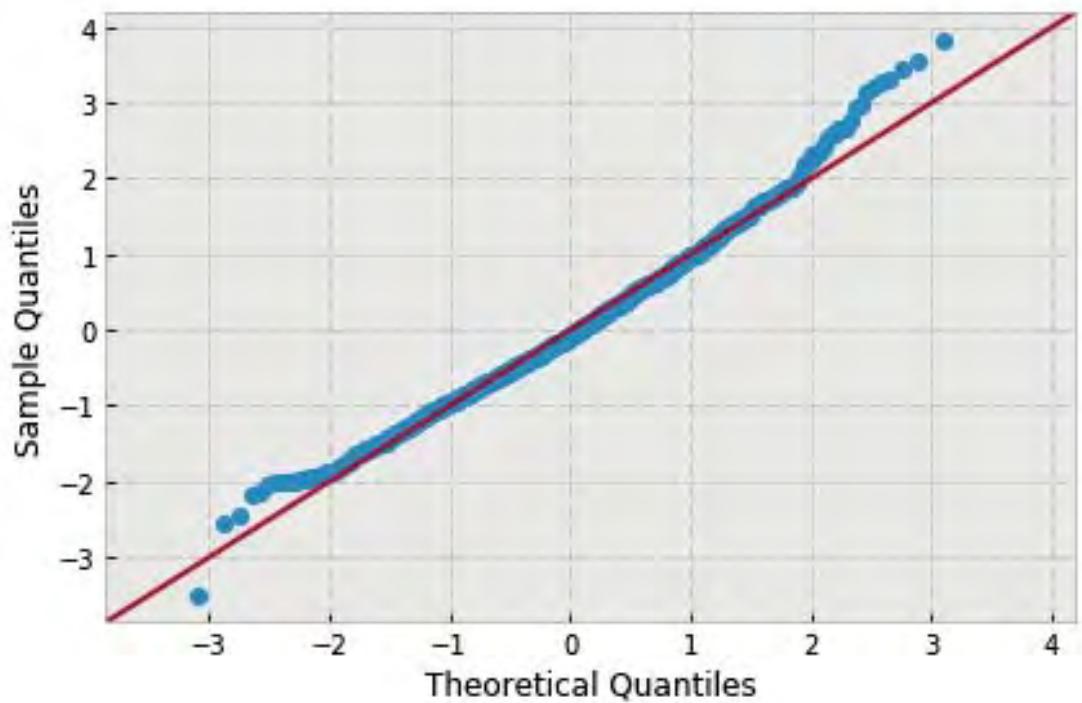


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=1.4555564753934058e-05, scale=4.2071298437845394e-07)
DescribeResult(nobs=1000, minmax=(1.3385323541432731e-05, 1.6059579368484459e-05)
               mean=1.4555564753934058e-05, variance=1.7717659181644168e-13,
               skewness=0.4862153330526284, kurtosis=0.20527511222125527)
```

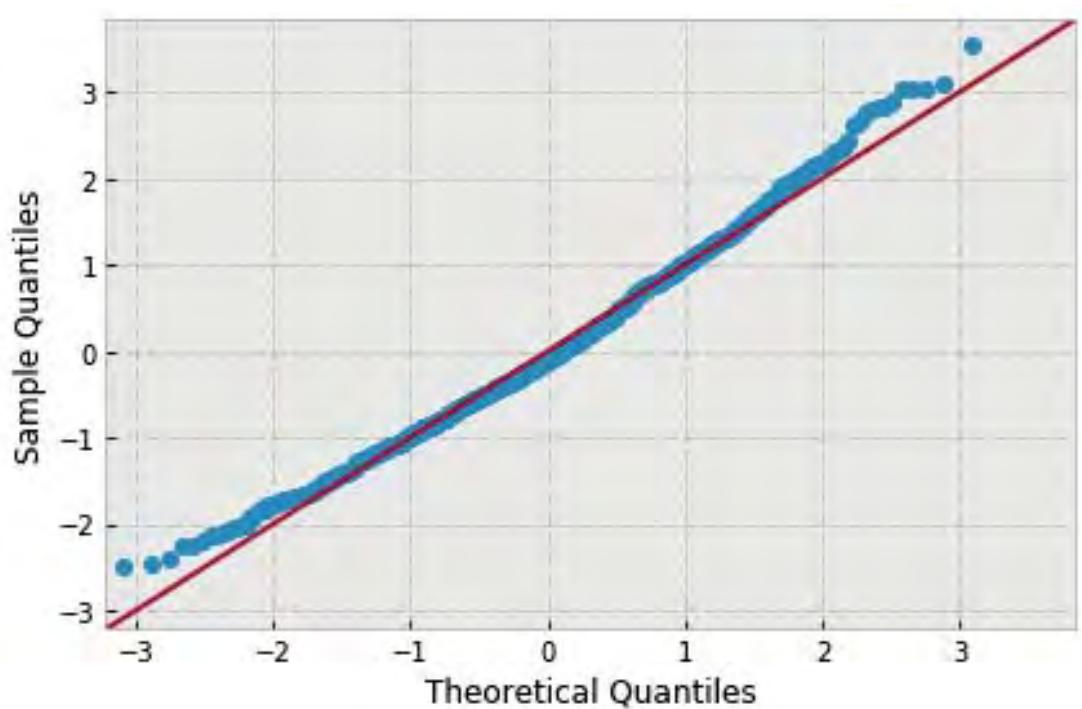
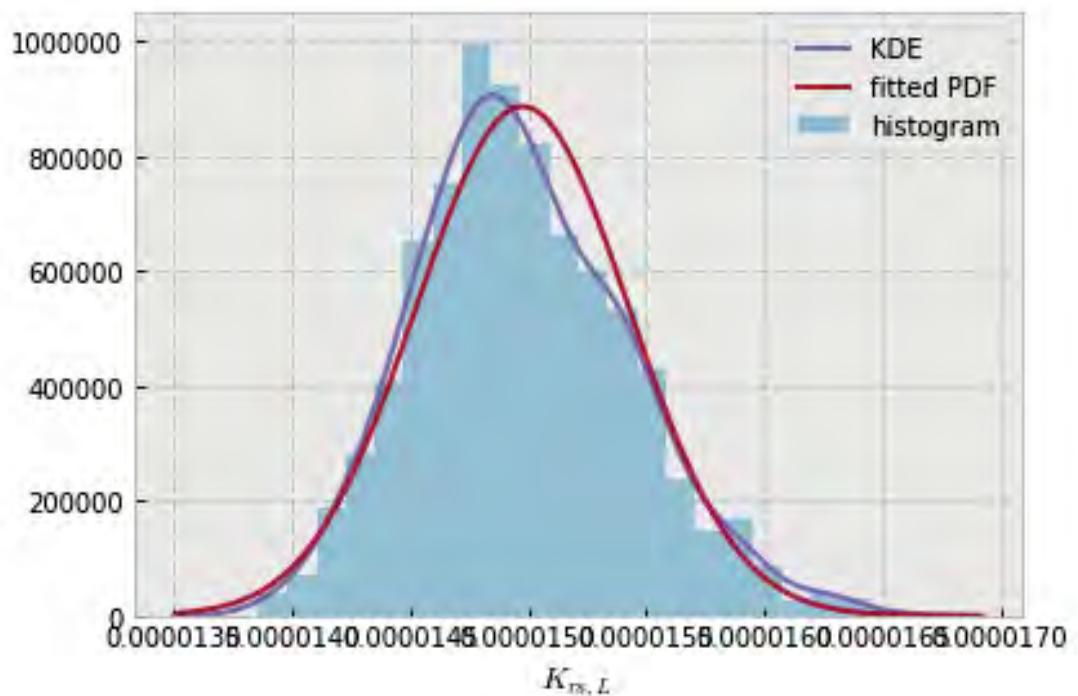


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=1.4558540518194788e-05, scale=4.0781037239270784e-07)
DescribeResult(nobs=1000, minmax=(1.3134705096058307e-05, 1.6115403441963763e-05)
               mean=1.4558540518194788e-05, variance=1.6647577560668571e-13,
               skewness=0.4119069934795062, kurtosis=0.4894682938941859)
```

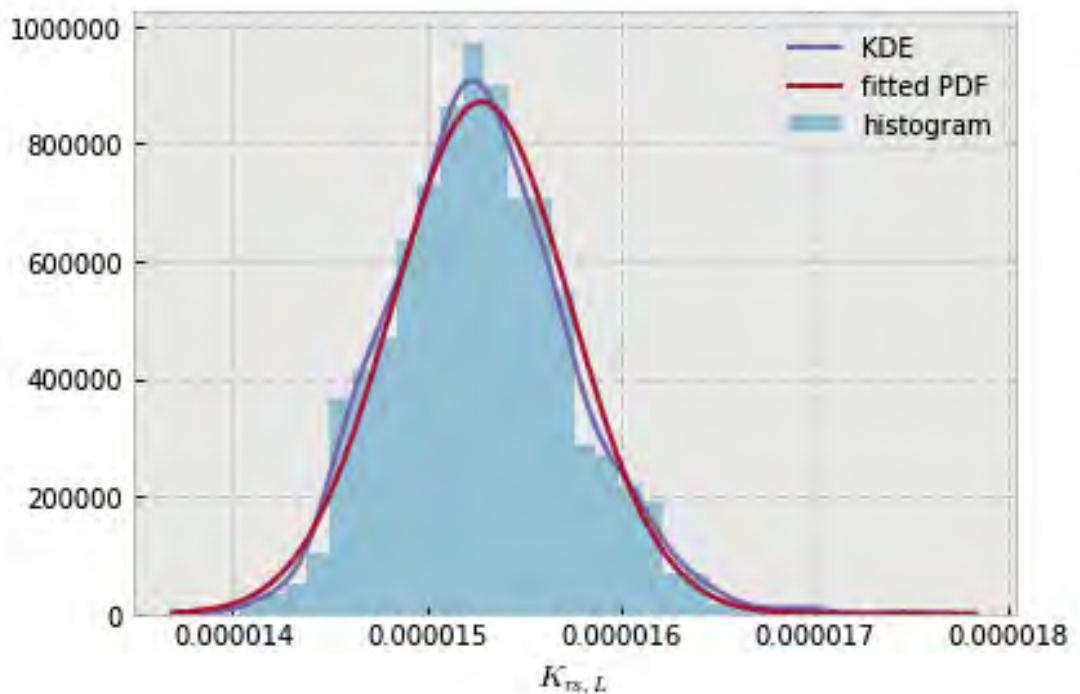


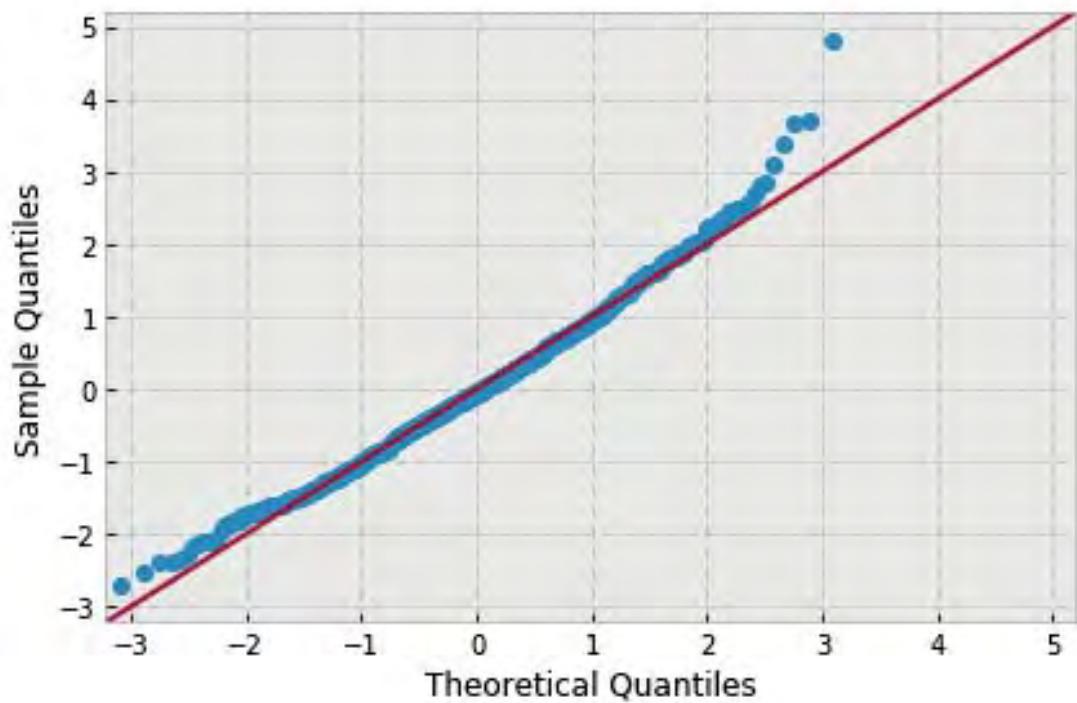


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=1.4975886585840189e-05, scale=4.4996549727879583e-07)
DescribeResult(nobs=1000, minmax=(1.3857804740355526e-05, 1.656598160226226e-05)
               mean=1.4975886585840189e-05, variance=2.0267162036171574e-13,
               skewness=0.3891120116889821, kurtosis=0.06761074278597246)
```

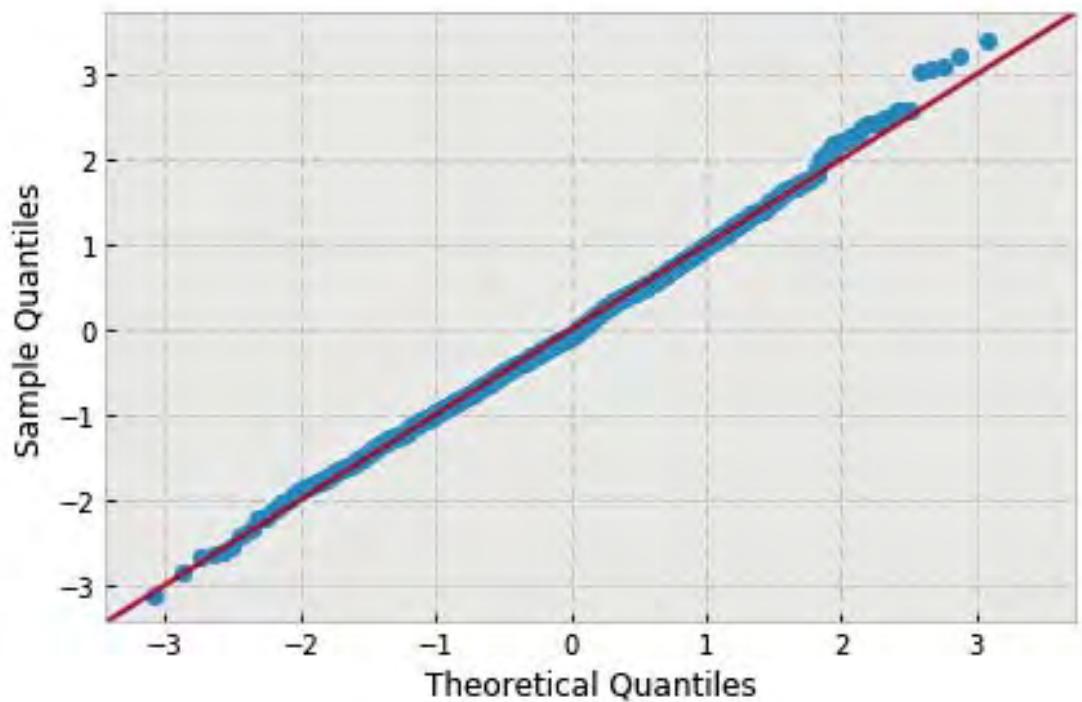
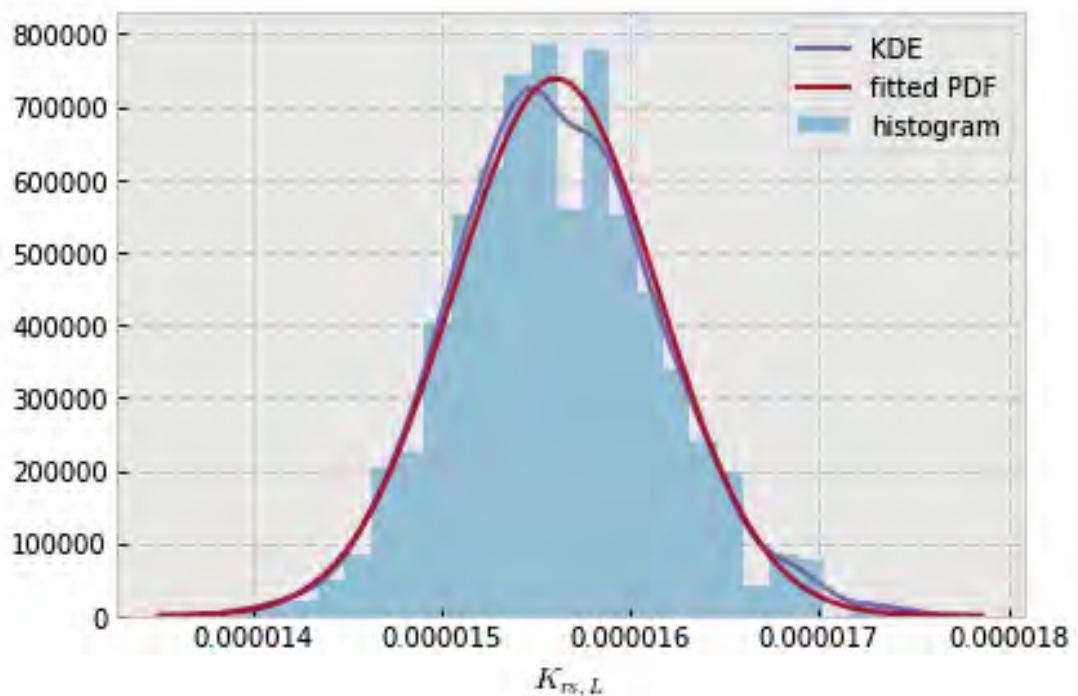


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=1.5282983588134047e-05, scale=4.5787157251854954e-07)
DescribeResult(nobs=1000, minmax=(1.4038597773558001e-05, 1.7483449355499501e-05)
               mean=1.5282983588134047e-05, variance=2.0985623315376312e-13,
               skewness=0.4190600689512046, kurtosis=0.5812945313666269)
```

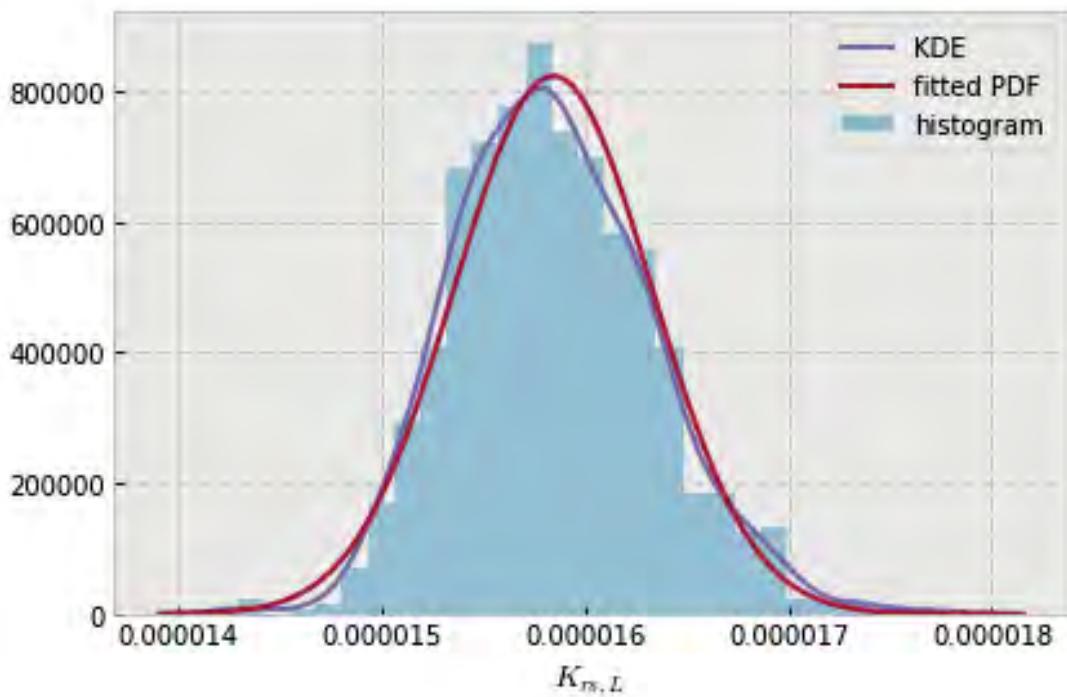


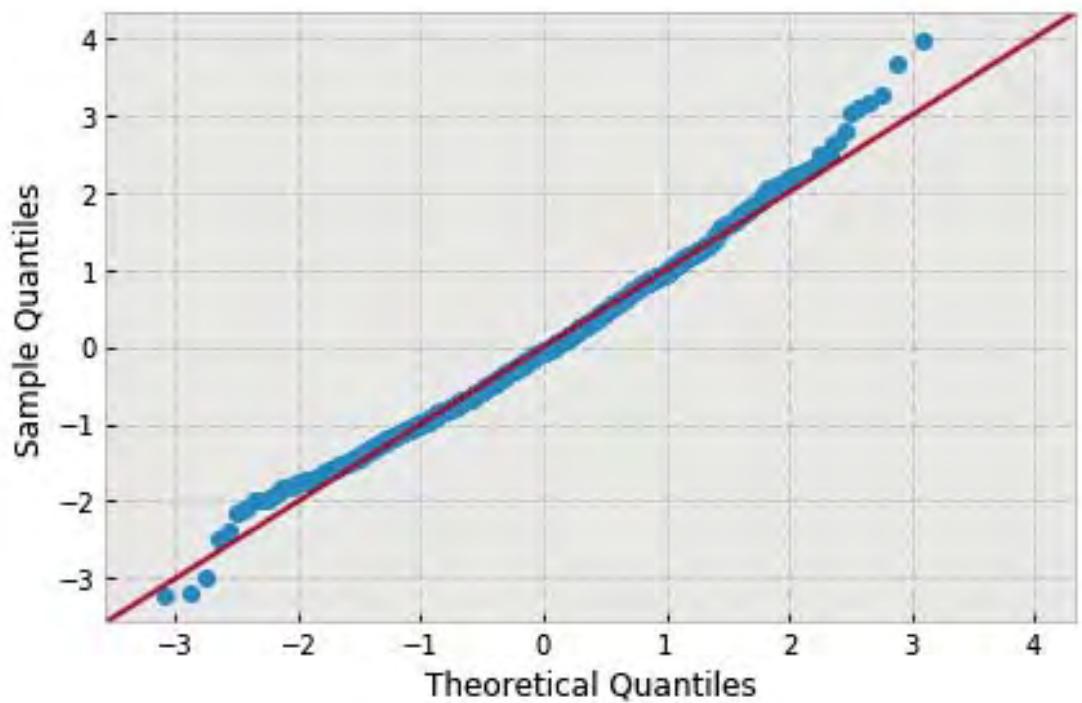


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=1.5606788548440496e-05, scale=5.401568475004497e-07)
DescribeResult(nobs=1000, minmax=(1.3925416971629453e-05, 1.7441808569704265e-05)
               mean=1.5606788548440496e-05, variance=2.9206148138300706e-13,
               skewness=0.1857032072170346, kurtosis=0.13342195211800734)
```

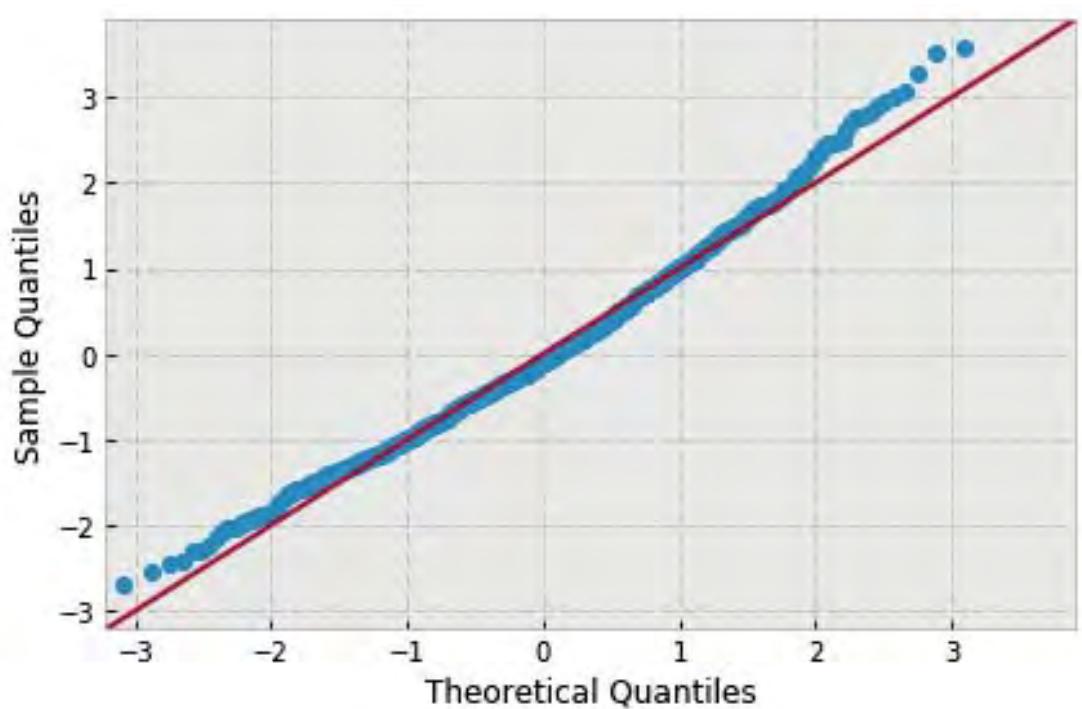
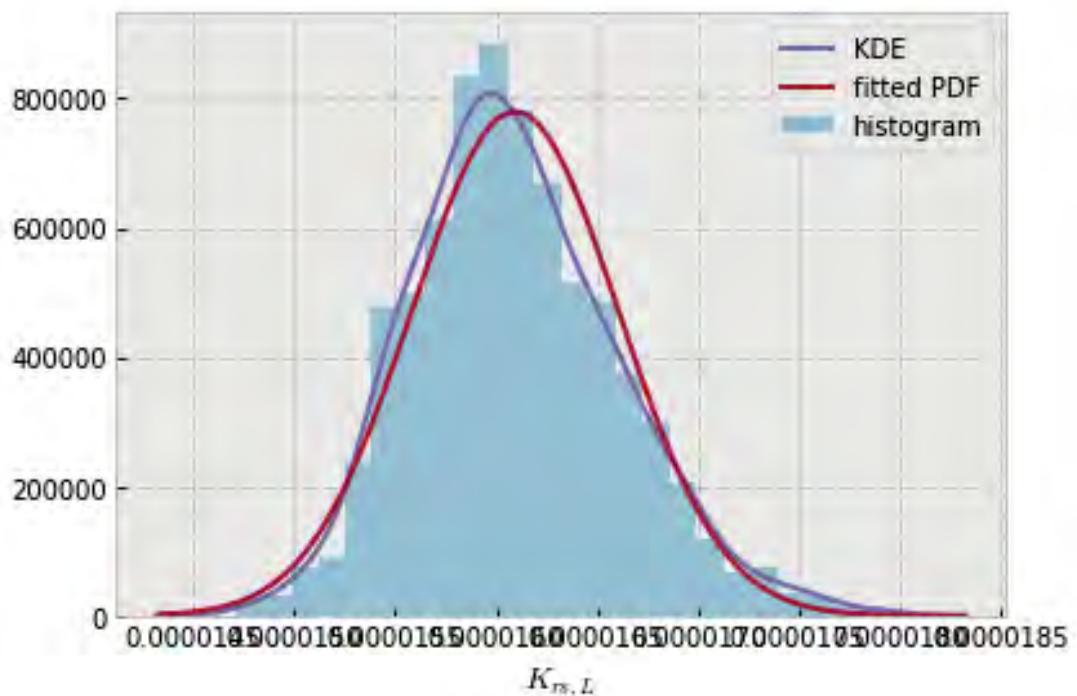


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=1.584607071981206e-05, scale=4.846739020858667e-07)
DescribeResult(nobs=1000, minmax=(1.4290846976689617e-05, 1.7769117373717976e-05)
               mean=1.584607071981206e-05, variance=2.3514393529843875e-13,
               skewness=0.34364275999542093, kurtosis=0.32675011395686004)
```

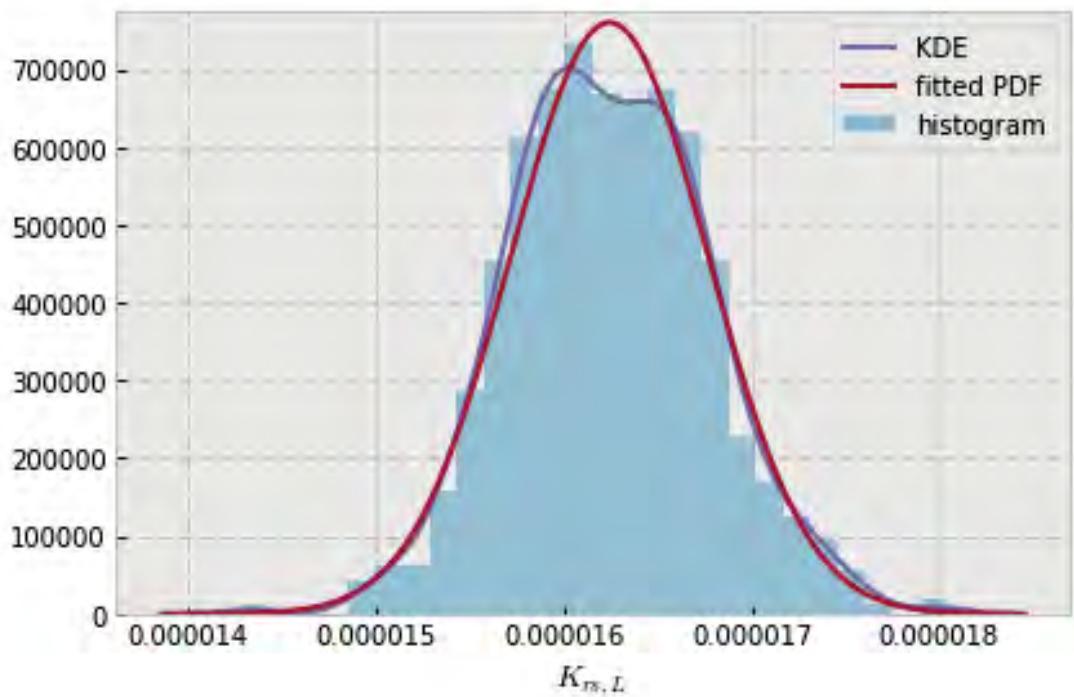


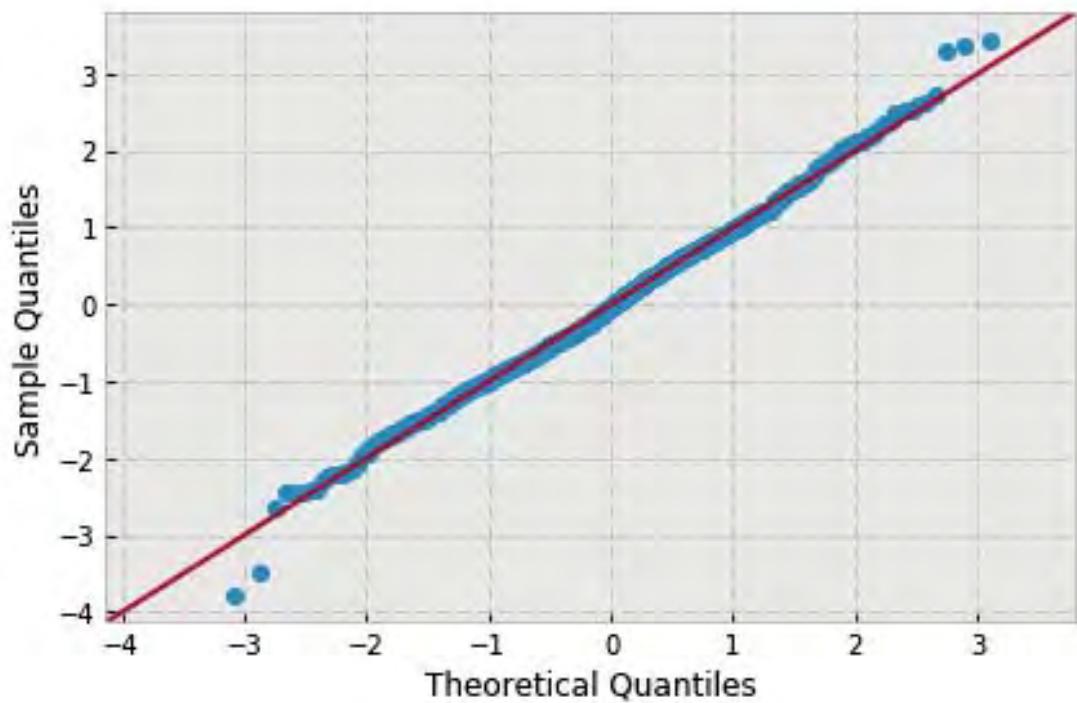


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=1.6094915483236845e-05, scale=5.118855554257729e-07)
DescribeResult(nobs=1000, minmax=(1.4714577253145952e-05, 1.7931963936086677e-05)
               mean=1.6094915483236845e-05, variance=2.6228911096451648e-13,
               skewness=0.43673698210164763, kurtosis=0.25532905480935586)
```

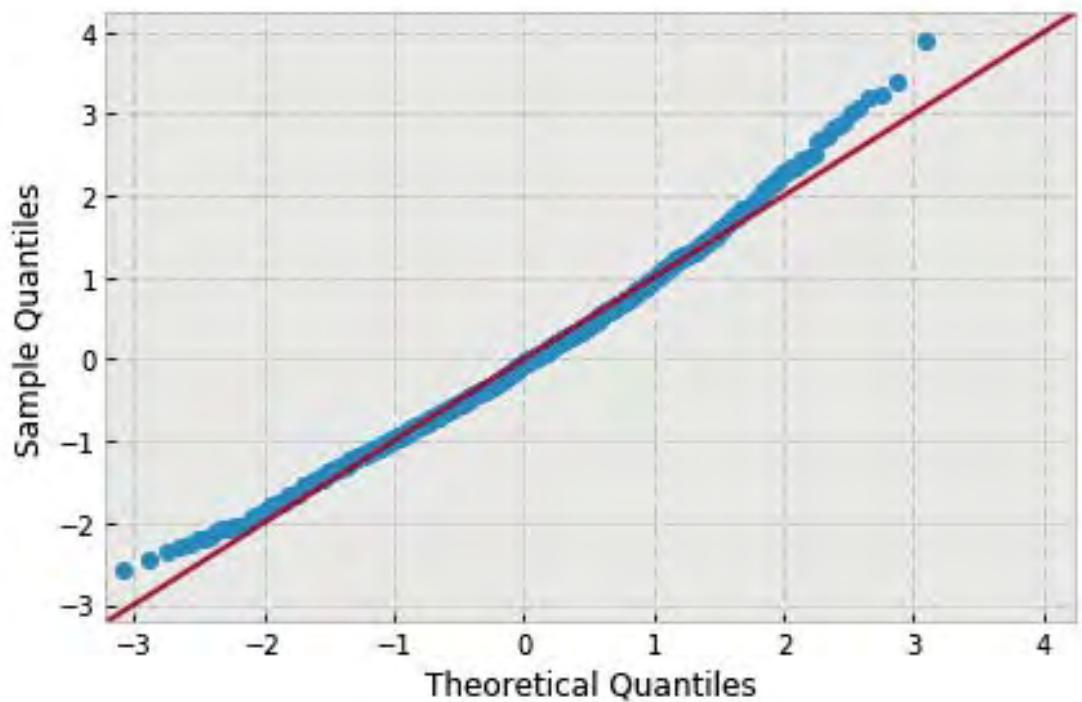
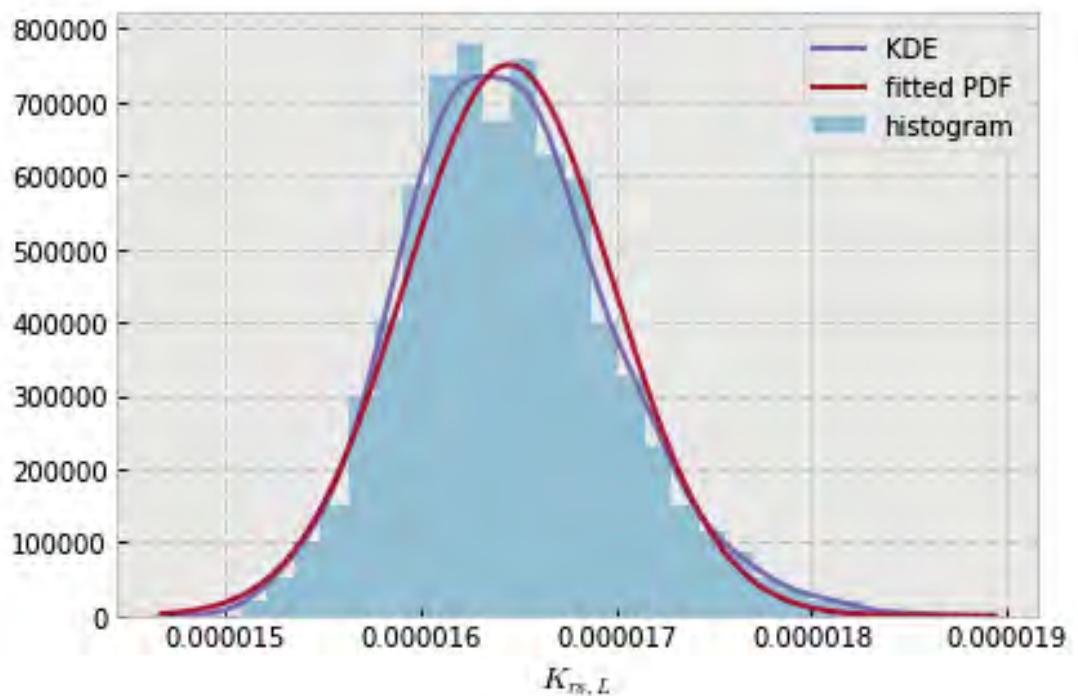


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=1.6243198080592188e-05, scale=5.24940643919321e-07)
DescribeResult(nobs=1000, minmax=(1.4264819896675884e-05, 1.8042666638237614e-05)
               mean=1.6243198080592188e-05, variance=2.7583851815658795e-13,
               skewness=0.11005629977648834, kurtosis=0.1654711585653006)
```

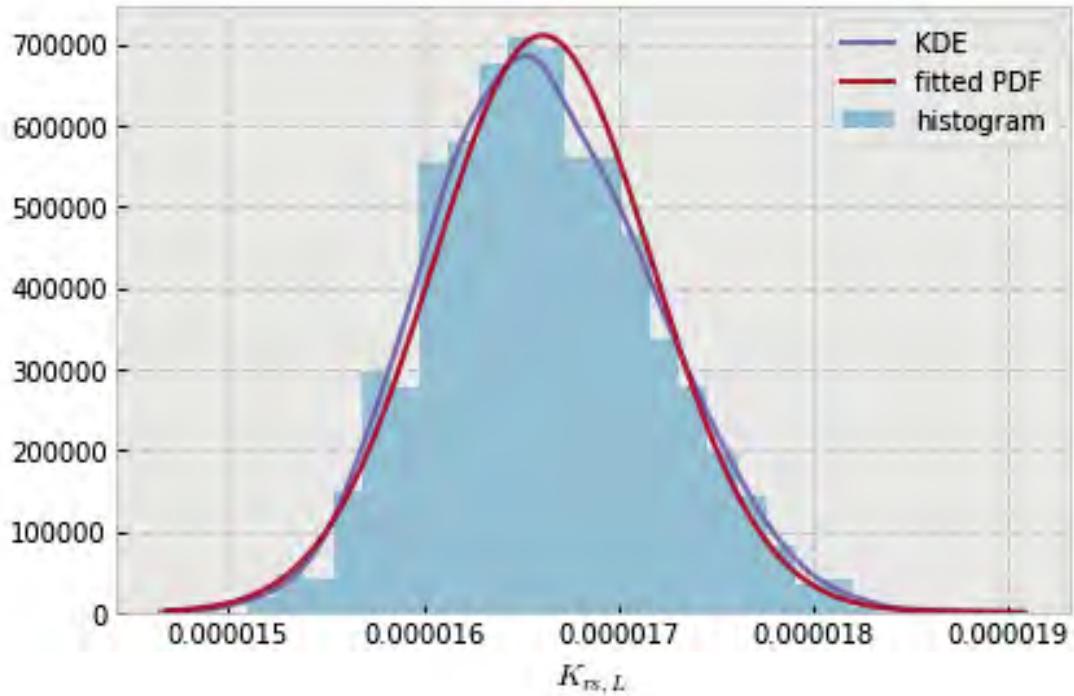


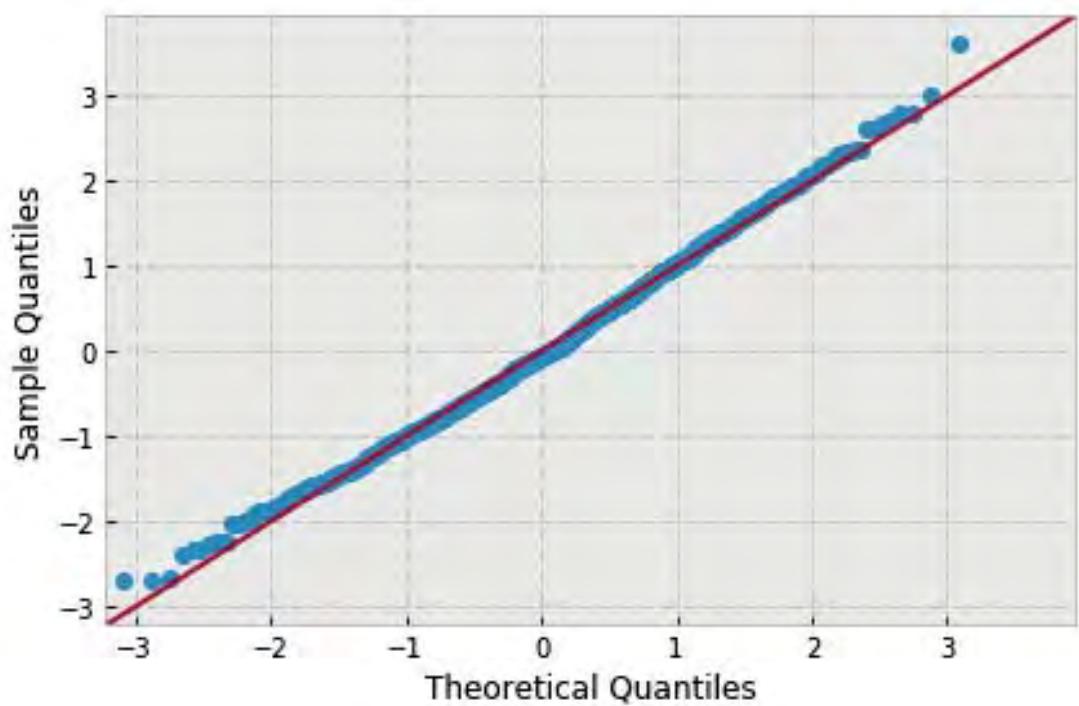


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=1.6452438176315064e-05, scale=5.313741798326965e-07)
DescribeResult(nobs=1000, minmax=(1.5076693747044741e-05, 1.852928147941188e-05)
               mean=1.6452438176315064e-05, variance=2.8264116015302396e-13,
               skewness=0.41227014478100676, kurtosis=0.2910924103237793)
```

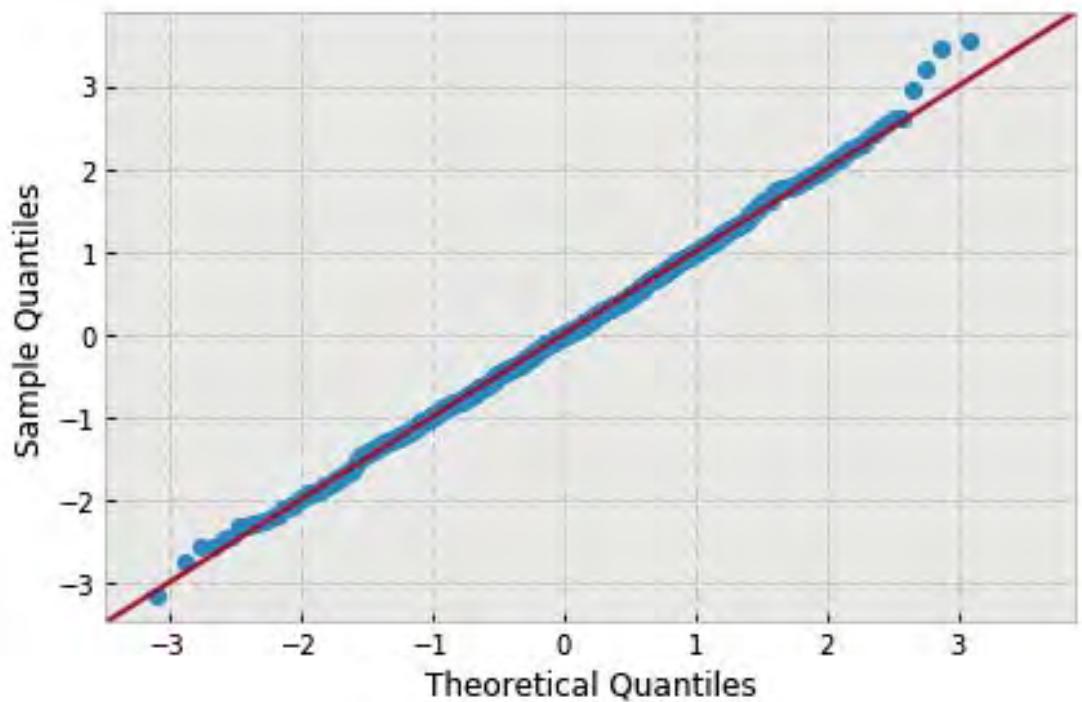
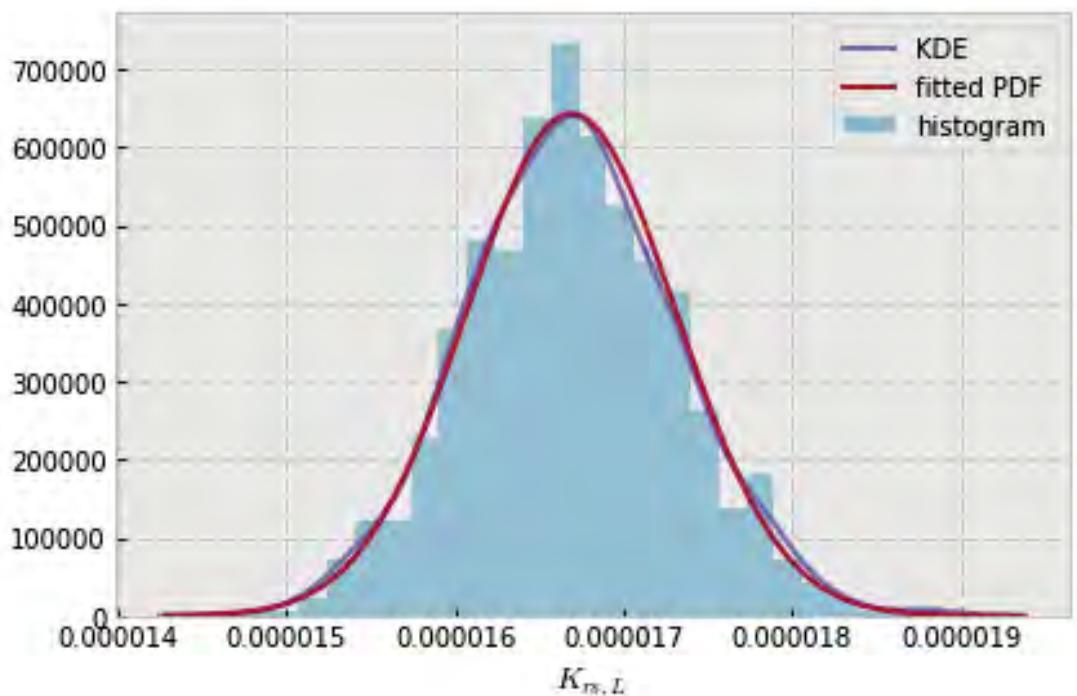


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=1.6613275866416636e-05, scale=5.614310319932939e-07)
DescribeResult(nobs=1000, minmax=(1.5096224464583243e-05, 1.8646218418802566e-05)
               mean=1.6613275866416636e-05, variance=3.1552032400906402e-13,
               skewness=0.22231423943703246, kurtosis=-0.14748420612160773)
```

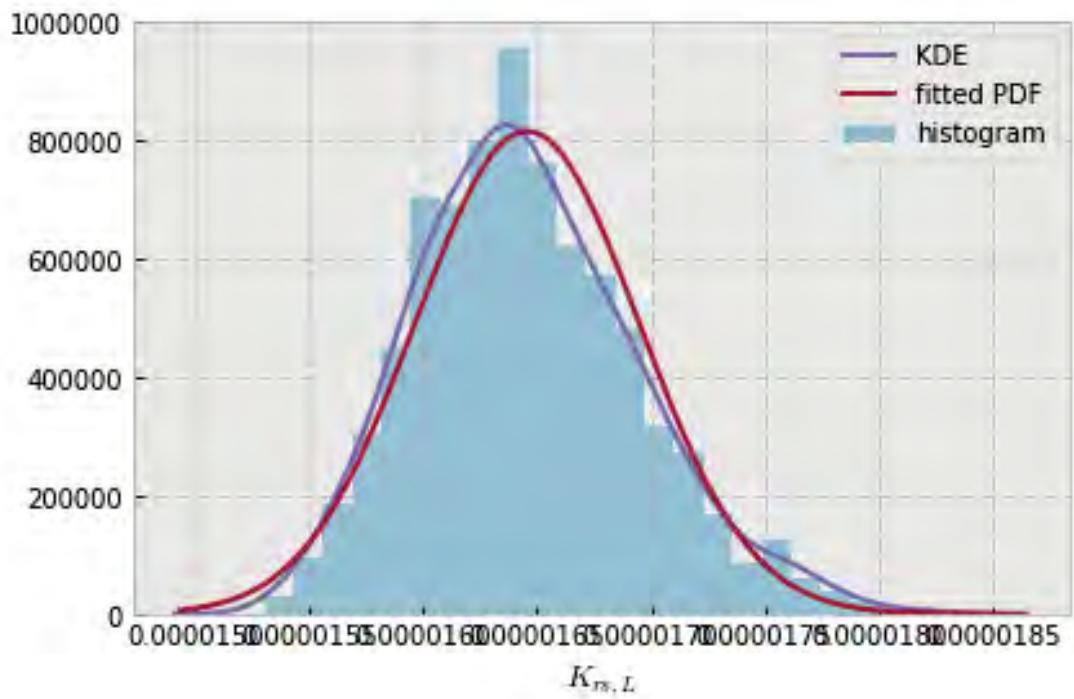


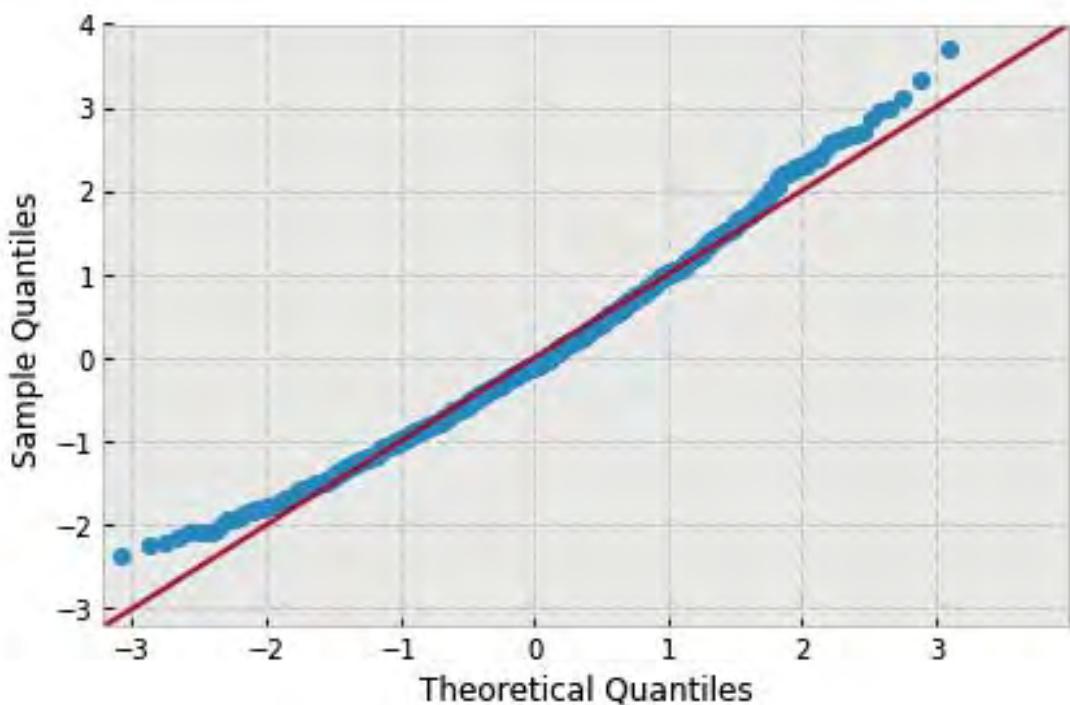


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=1.6691476390858256e-05, scale=6.202358814984679e-07)
DescribeResult(nobs=1000, minmax=(1.4743979992778038e-05, 1.8892155842153047e-05)
               mean=1.6691476390858256e-05, variance=3.8507762632450605e-13,
               skewness=0.12025414509162909, kurtosis=0.07608887204391257)
```

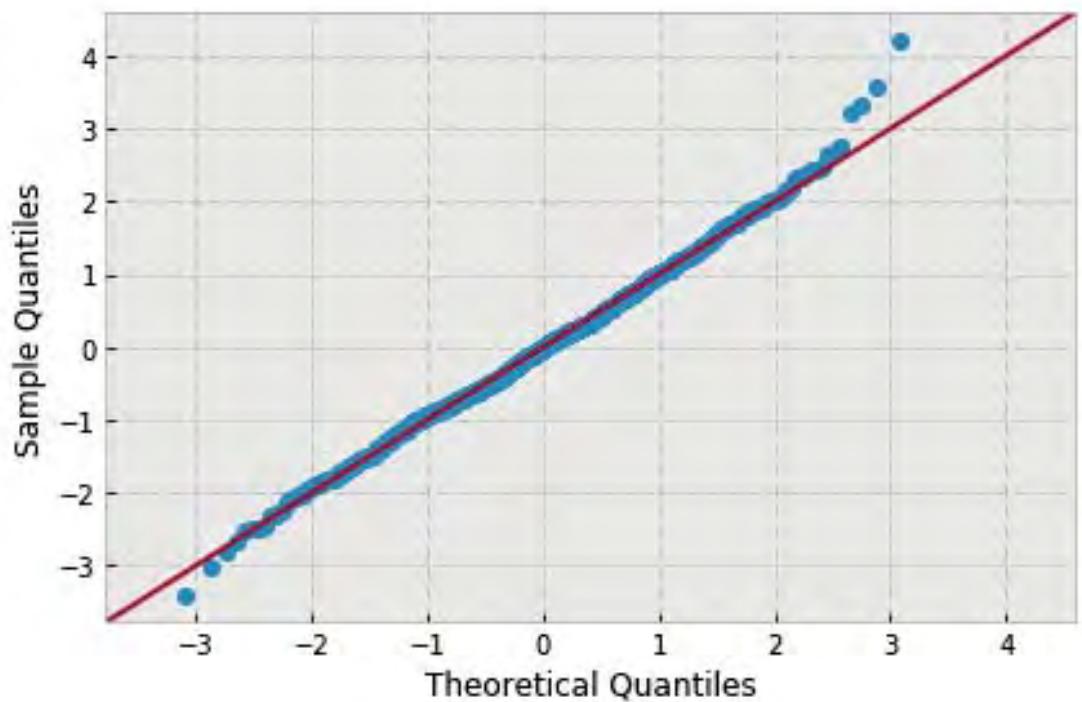
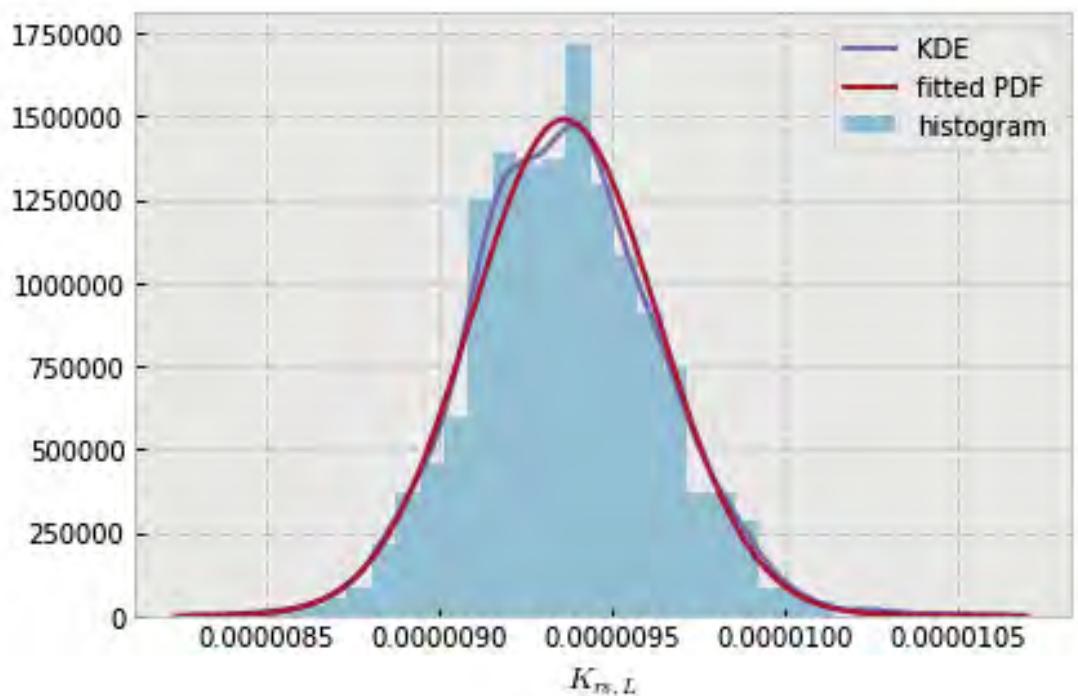


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=1.6456570281519693e-05, scale=4.88754329968327e-07)
DescribeResult(nobs=1000, minmax=(1.5300450423460153e-05, 1.8258780318695676e-05)
               mean=1.6456570281519693e-05, variance=2.391199149777661e-13,
               skewness=0.4441490961230089, kurtosis=0.11813317823265601)
```

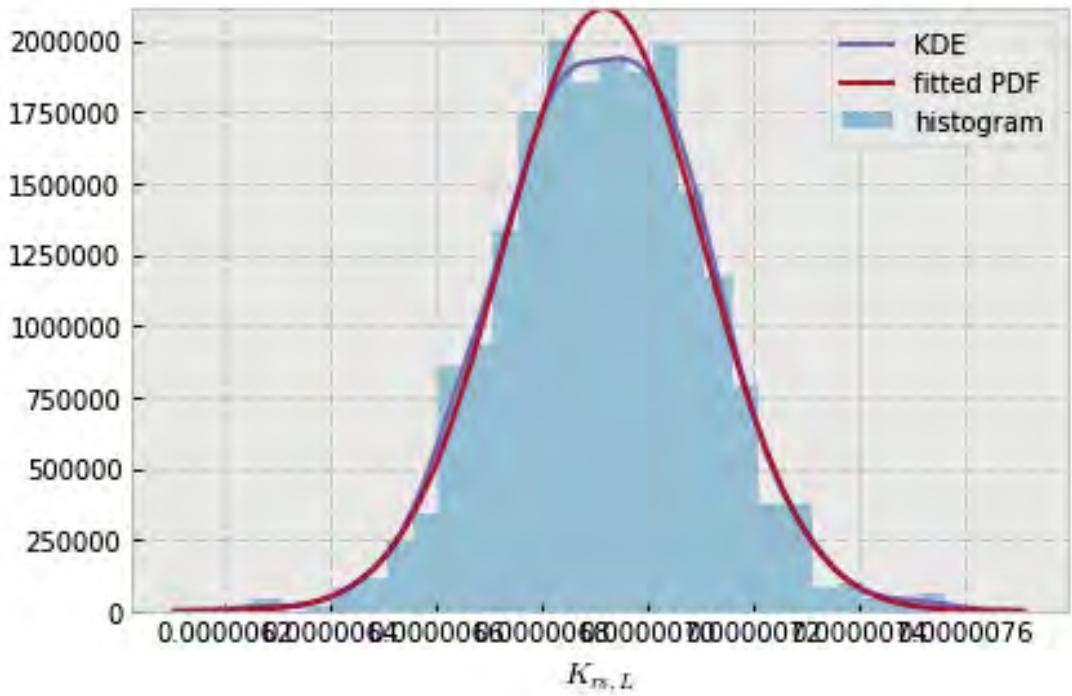


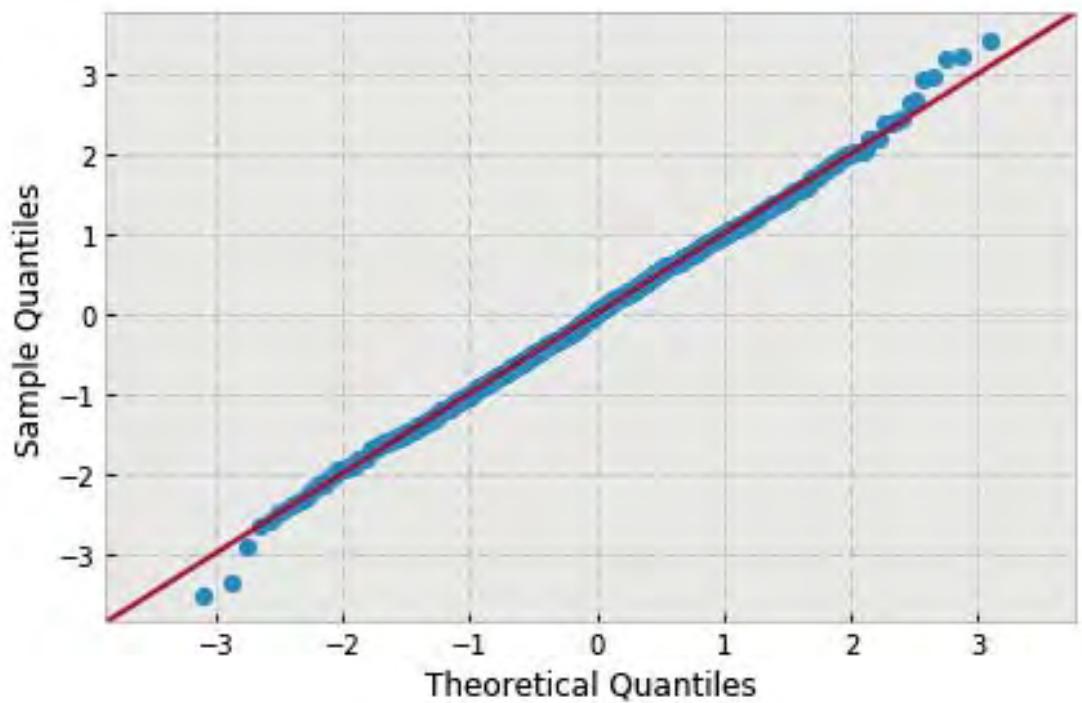


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=9.364647330248162e-06, scale=2.6775829523028276e-07)
DescribeResult(nobs=1000, minmax=(8.4543102620387719e-06, 1.0491290607702414e-05)
               mean=9.3646473302481616e-06, variance=7.1766270935562814e-14,
               skewness=0.17327650600193725, kurtosis=0.37059456595138585)
```

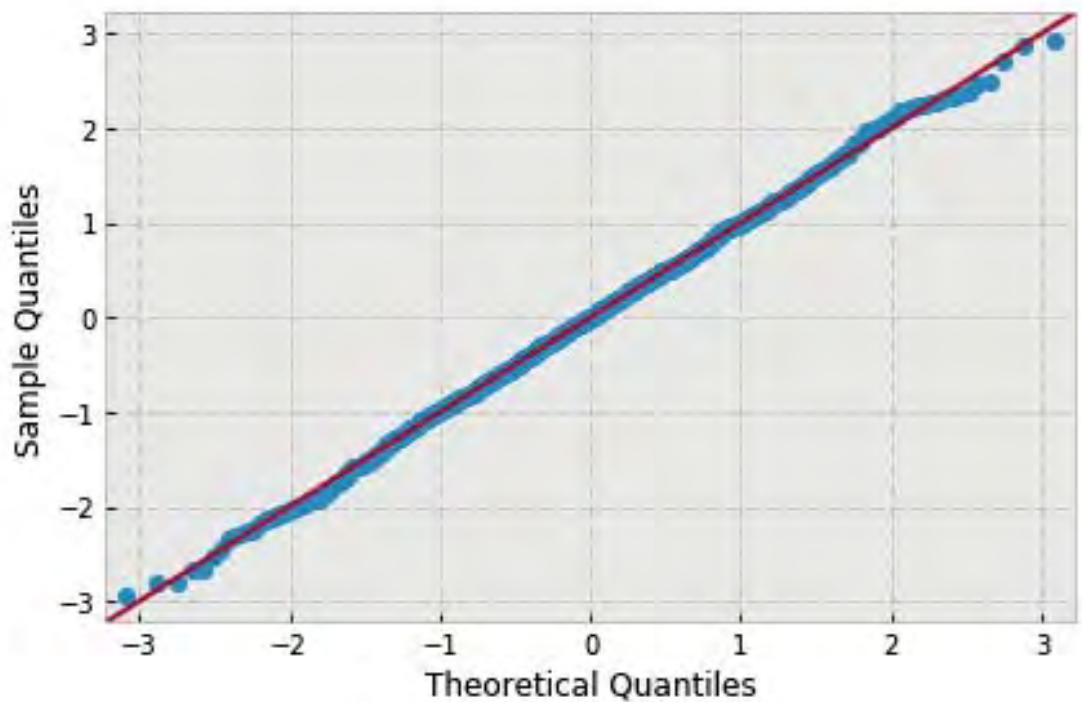
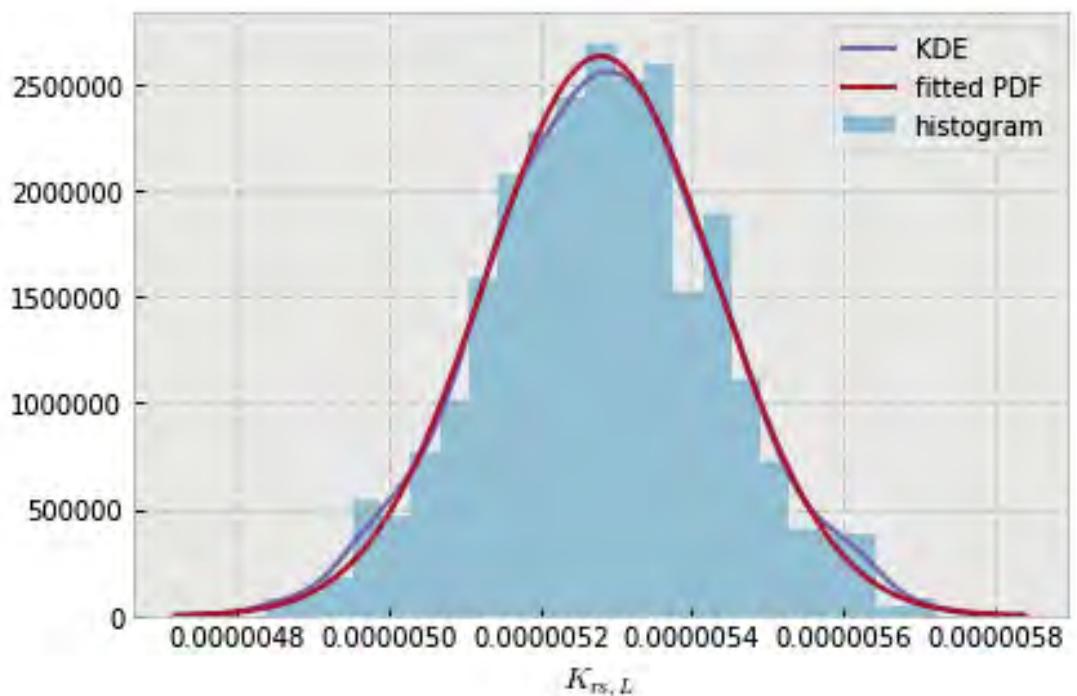


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=6.9184969641955675e-06, scale=1.8878278573184392e-07)
DescribeResult(nobs=1000, minmax=(6.2554313352605653e-06, 7.5622213337299514e-06)
               mean=6.9184969641955675e-06, variance=3.5674614803478779e-14,
               skewness=0.02425490787236573, kurtosis=0.10872789633379609)
```

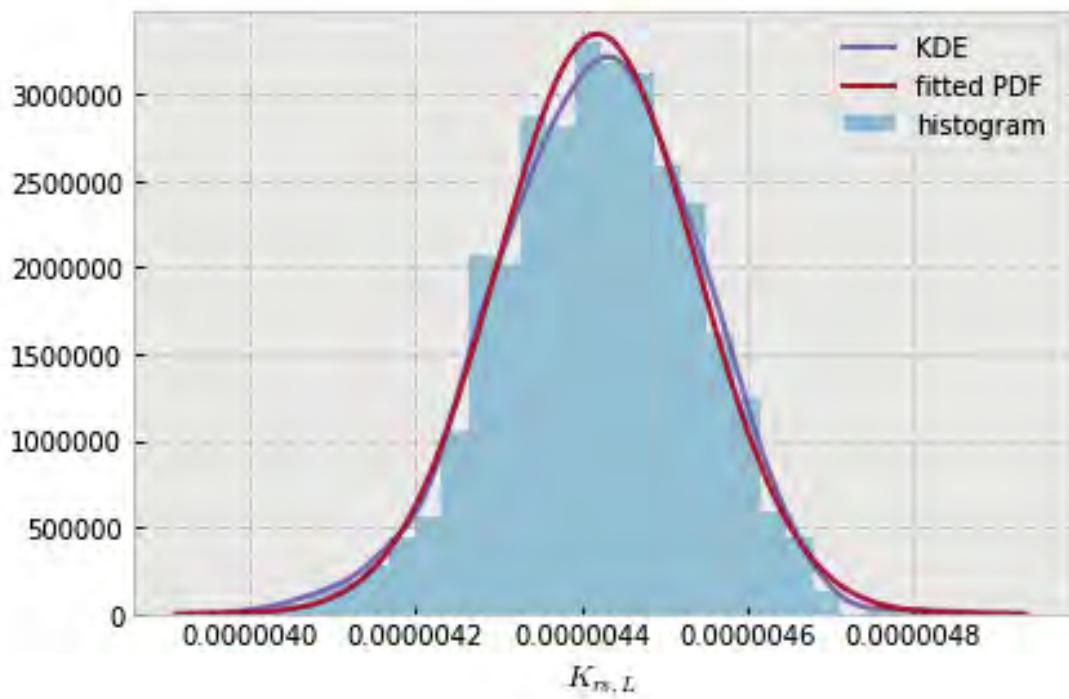


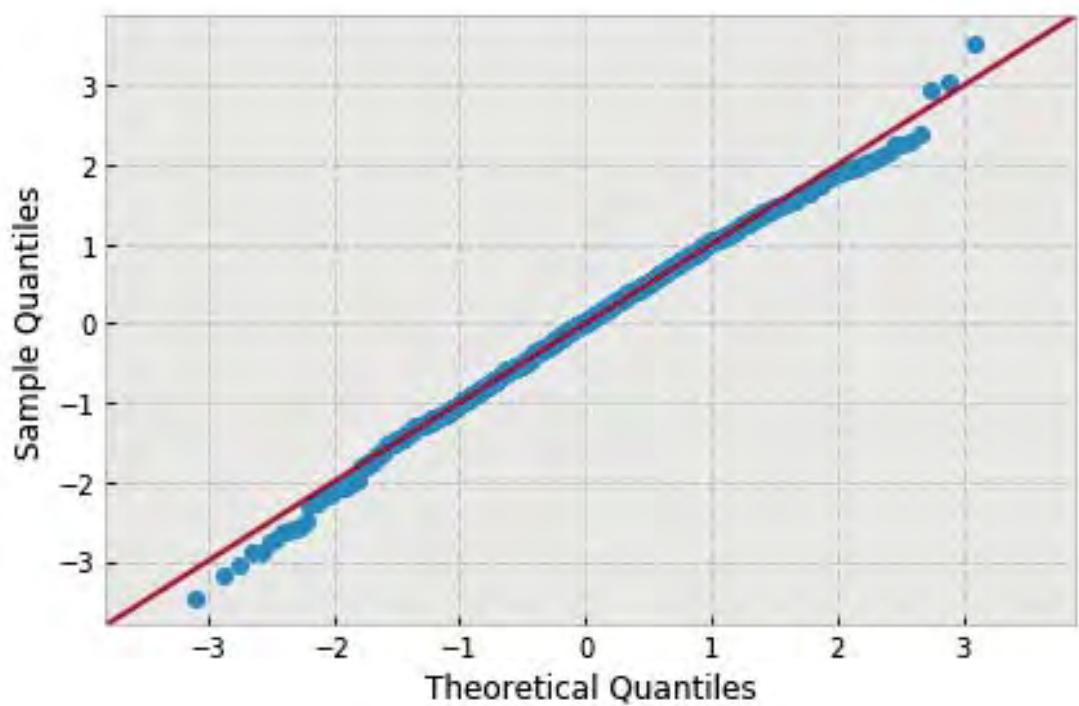


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=5.279699532671825e-06, scale=1.5173723726224346e-07)
DescribeResult(nobs=1000, minmax=(4.8355088572763309e-06, 5.7226379977095728e-06)
               mean=5.2796995326718251e-06, variance=2.3047236408386749e-14,
               skewness=-0.006677422359127417, kurtosis=-0.10580272090157106)
```

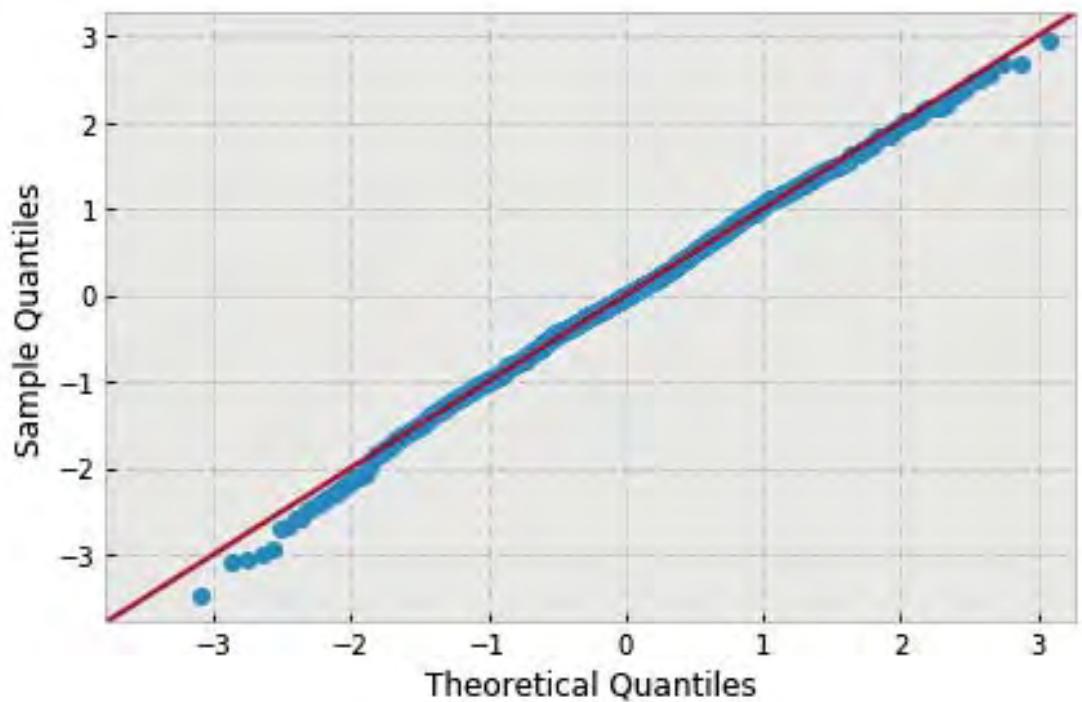
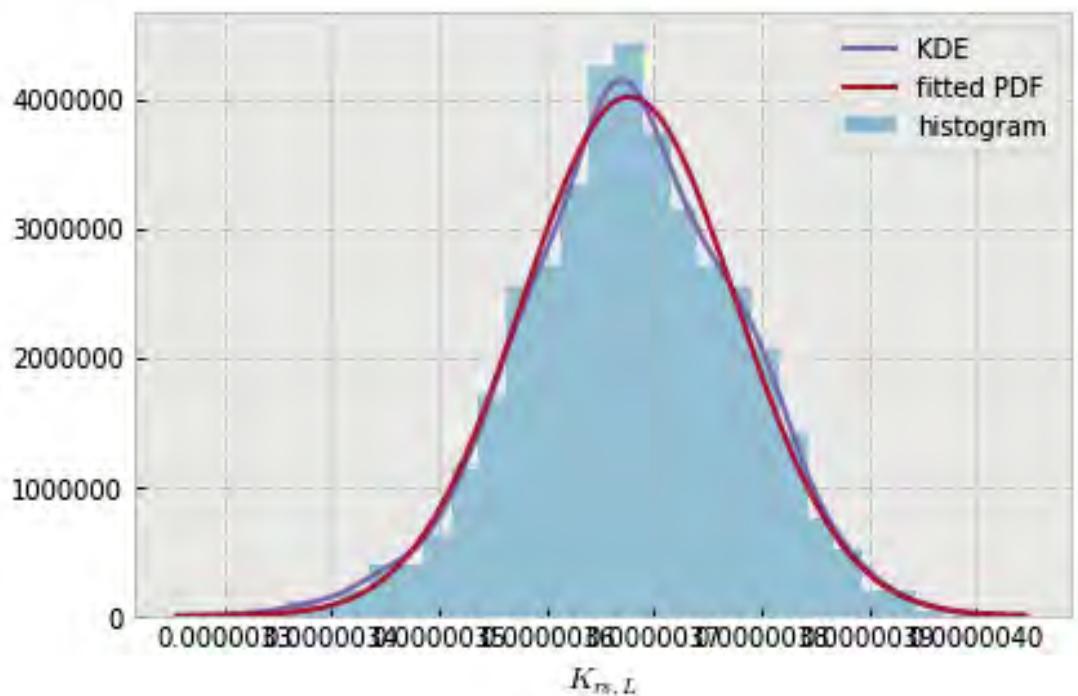


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=4.419154291395491e-06, scale=1.192157426402423e-07)
DescribeResult(nobs=1000, minmax=(4.006853740134043e-06, 4.8402156046159532e-06)
               mean=4.419154291395491e-06, variance=1.4226619913177662e-14,
               skewness=-0.17093636370294063, kurtosis=0.08916583394224586)
```

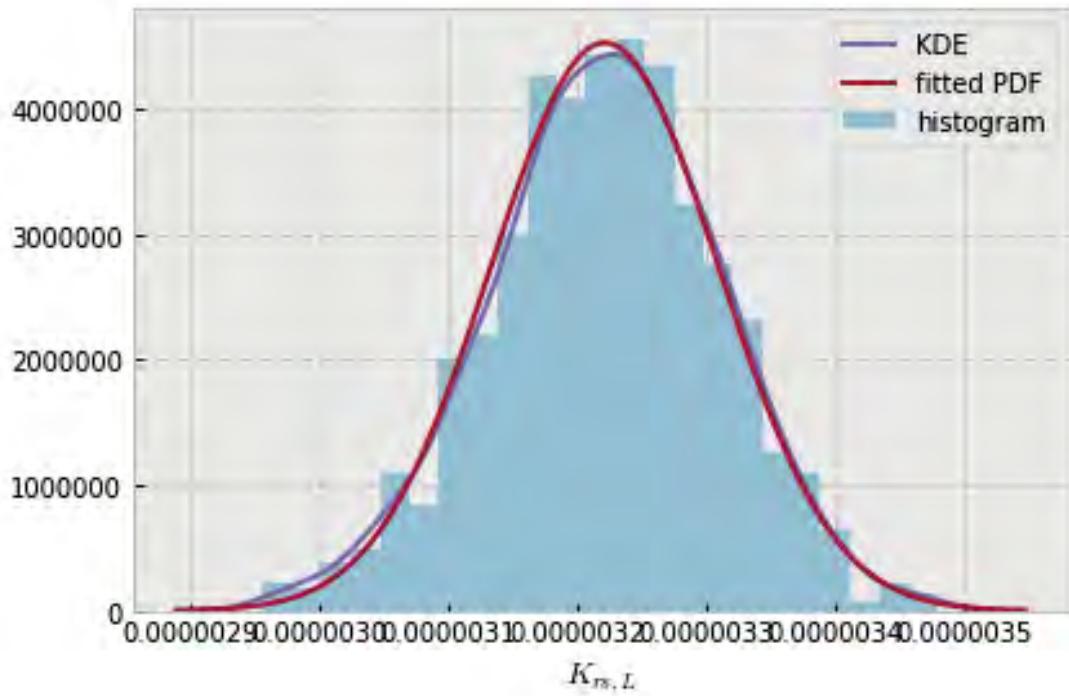


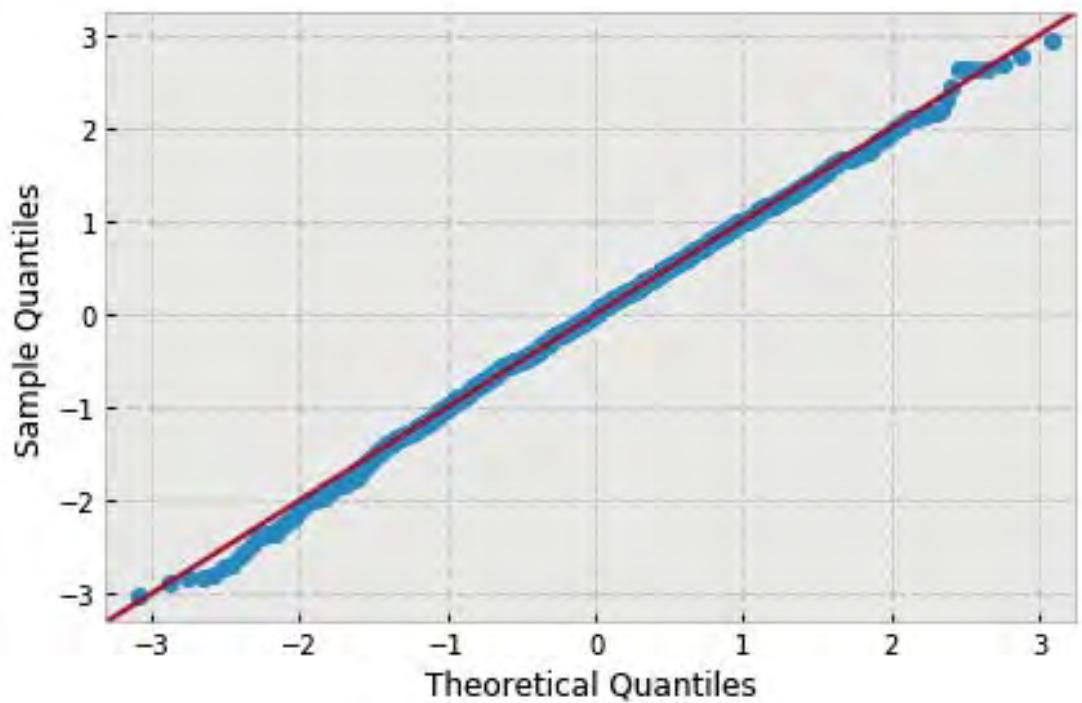


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=3.677186714913479e-06, scale=9.934968556738704e-08)
DescribeResult(nobs=1000, minmax=(3.3333043333243652e-06, 3.9697081858728368e-06)
               mean=3.6771867149134789e-06, variance=9.8802402626012741e-15,
               skewness=-0.13137643039781546, kurtosis=0.061695301947683134)
```

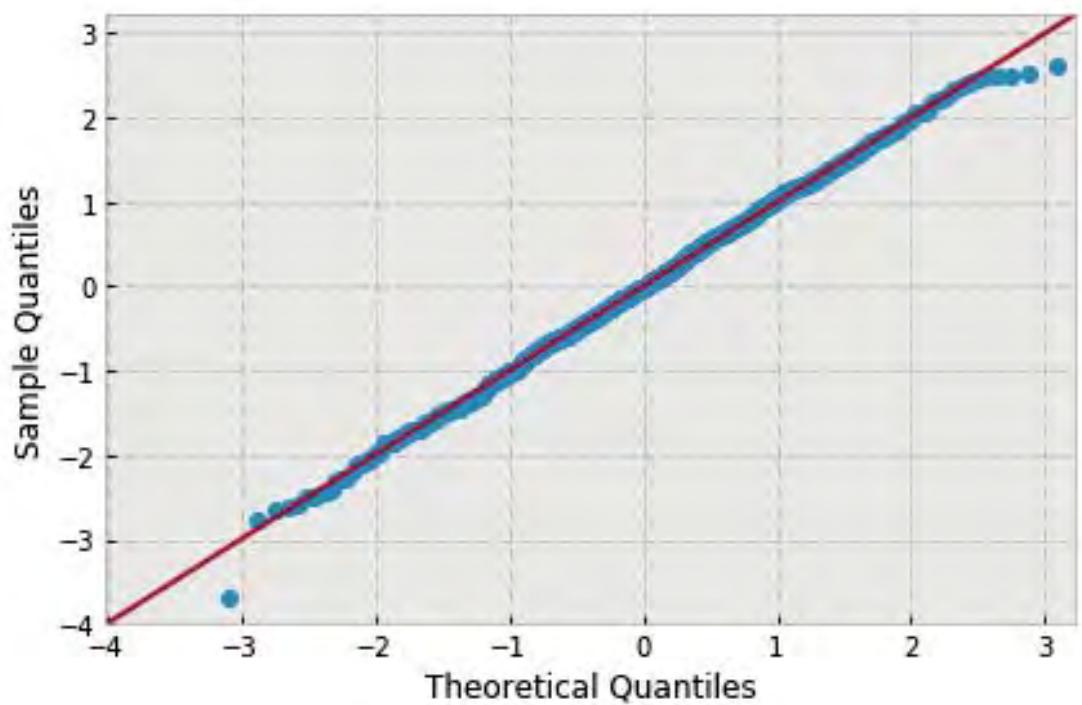
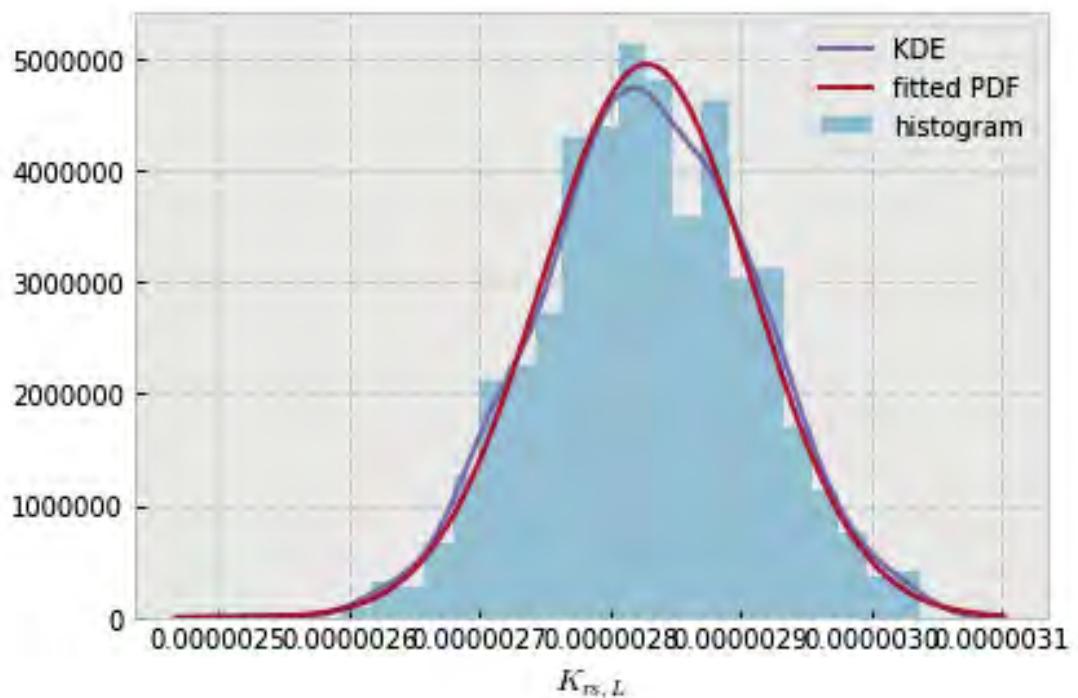


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=3.2215093636416624e-06, scale=8.812134925935197e-08)
DescribeResult(nobs=1000, minmax=(2.9564783587183725e-06, 3.4802628608344811e-06)
               mean=3.2215093636416624e-06, variance=7.7731453406293214e-15,
               skewness=-0.11710045795278202, kurtosis=-0.004846112556284865)
```

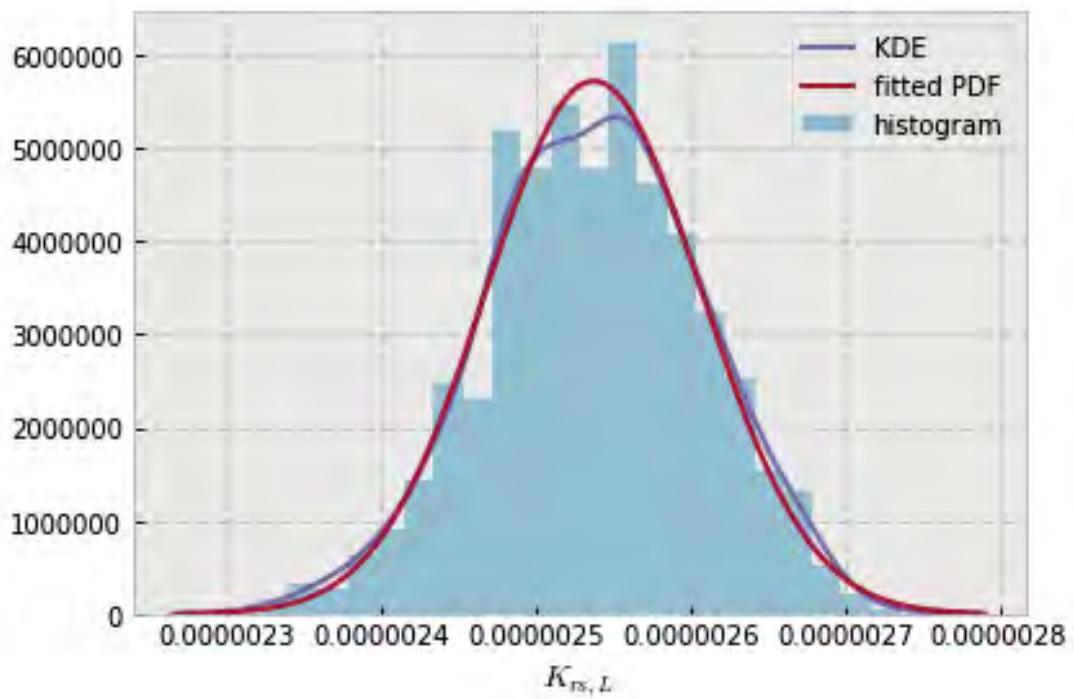


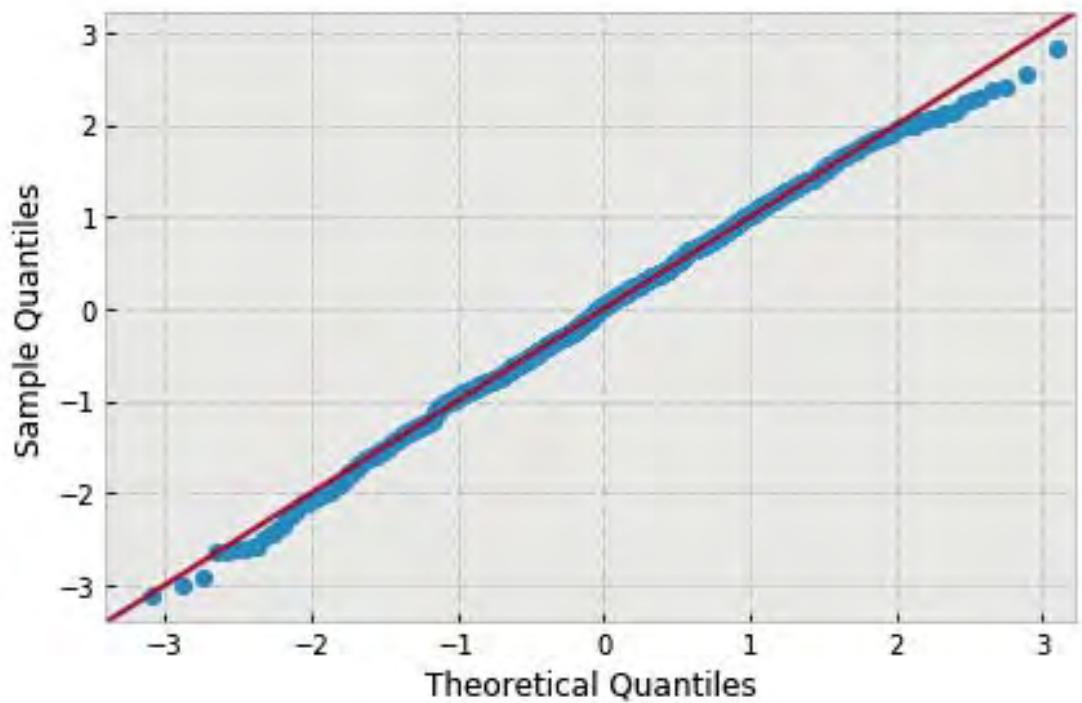


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=2.8283694497793994e-06, scale=8.066277923640715e-08)
DescribeResult(nobs=1000, minmax=(2.5308558927312633e-06, 3.0392802850276334e-06)
               mean=2.8283694497793994e-06, variance=6.5129969510924482e-15,
               skewness=-0.060410683399348054, kurtosis=-0.1702646637193137)
```

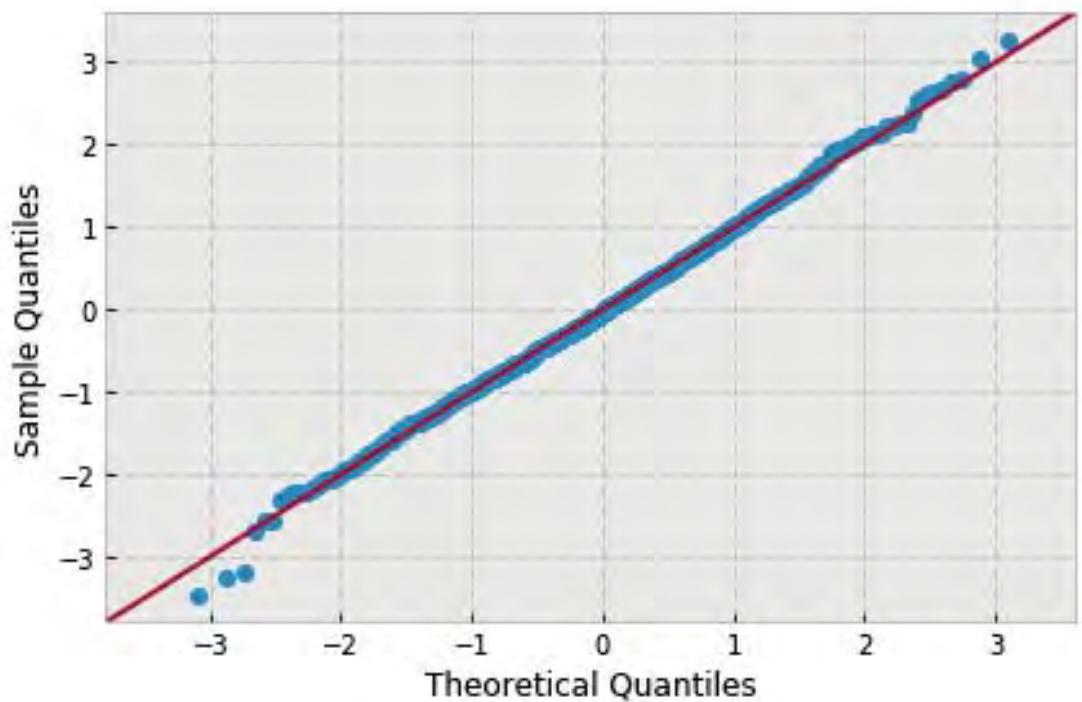
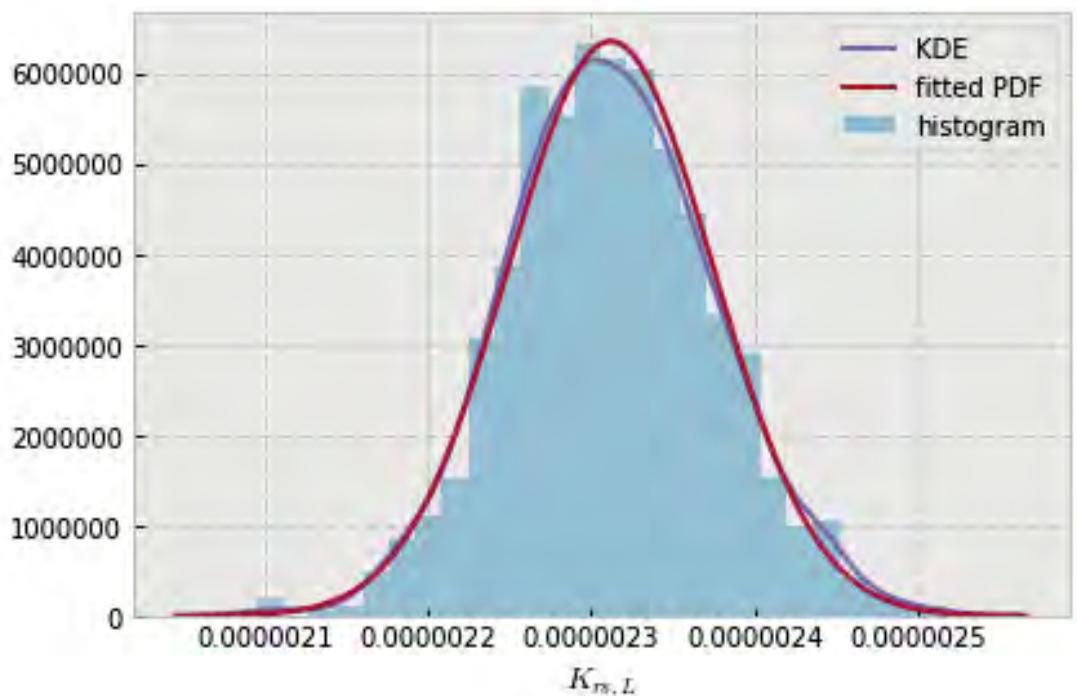


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=2.5375344694350135e-06, scale=6.974611660141559e-08)
DescribeResult(nobs=1000, minmax=(2.3209053207090186e-06, 2.7356742195749606e-06)
               mean=2.5375344694350135e-06, variance=4.8693901711494098e-15,
               skewness=-0.11507885102370406, kurtosis=-0.1754717946356723)
```

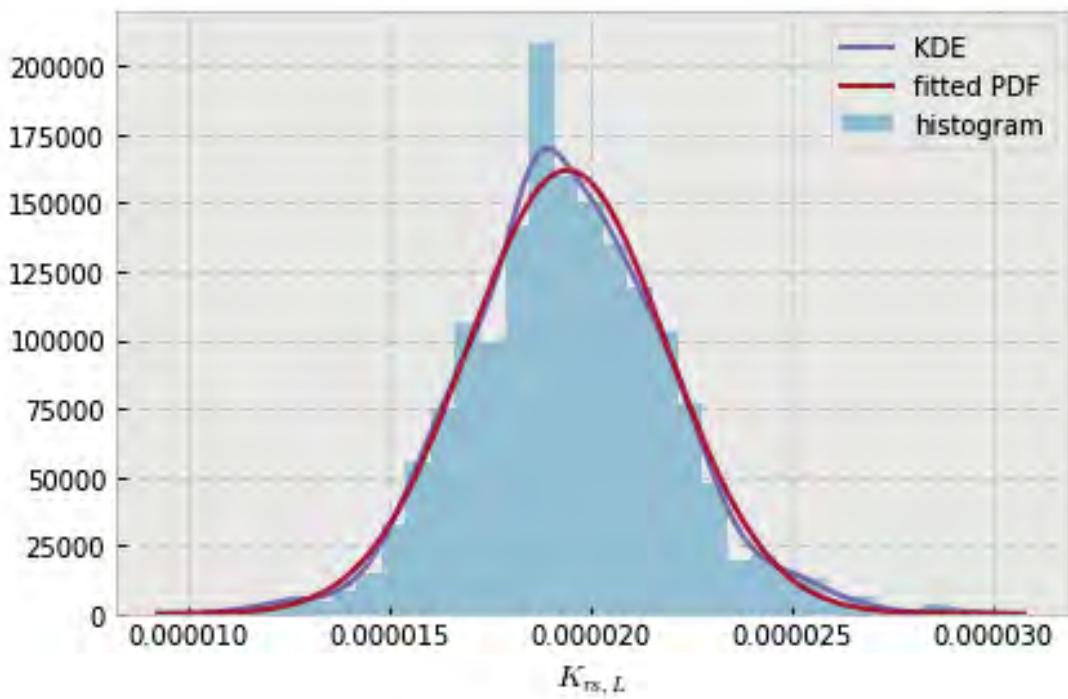


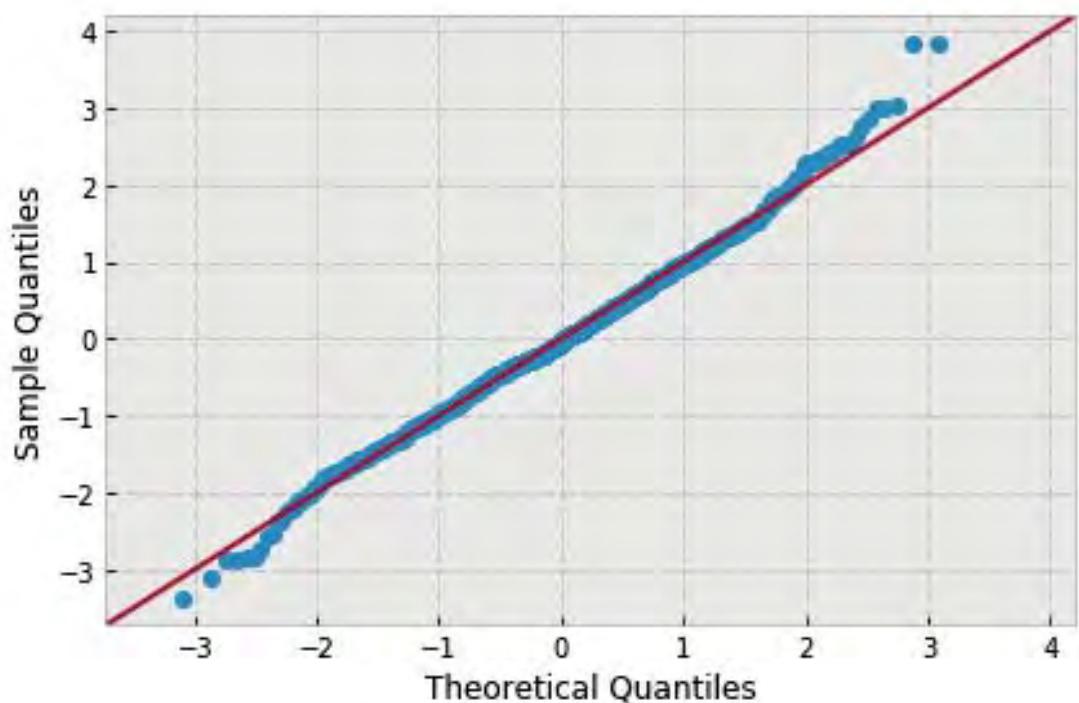


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=2.311932079691834e-06, scale=6.280211536154675e-08)
DescribeResult(nobs=1000, minmax=(2.0947904289842009e-06, 2.5168243054353107e-06)
               mean=2.3119320796918339e-06, variance=3.94805374763266e-15,
               skewness=0.07699548973398283, kurtosis=0.0823435492460014)
```

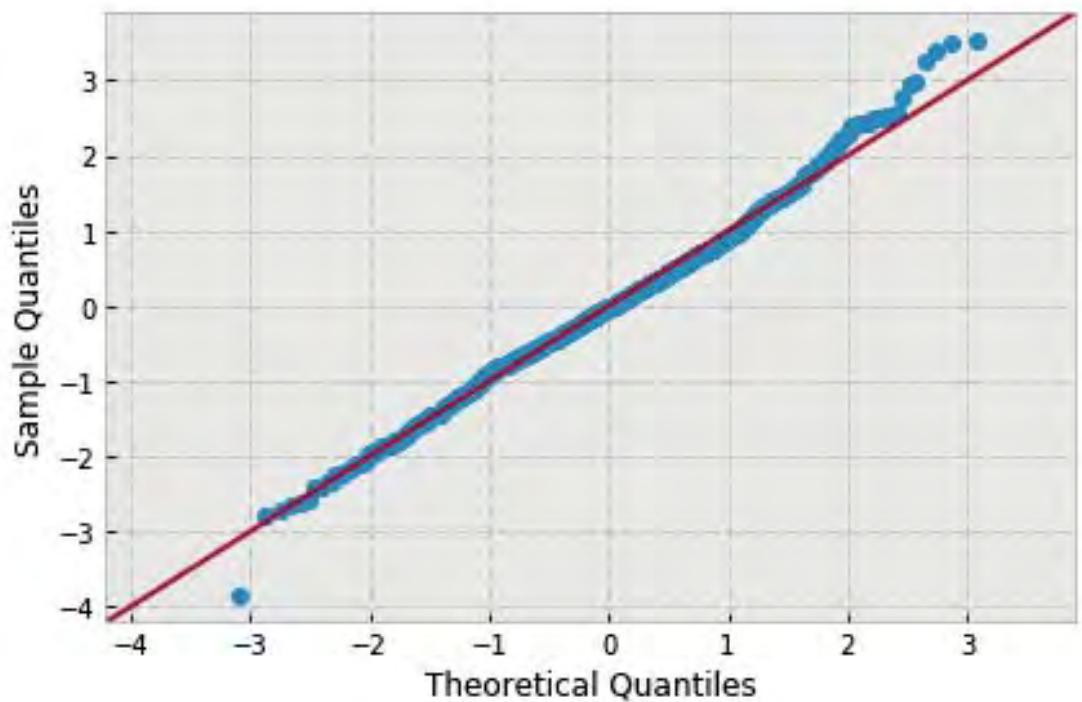
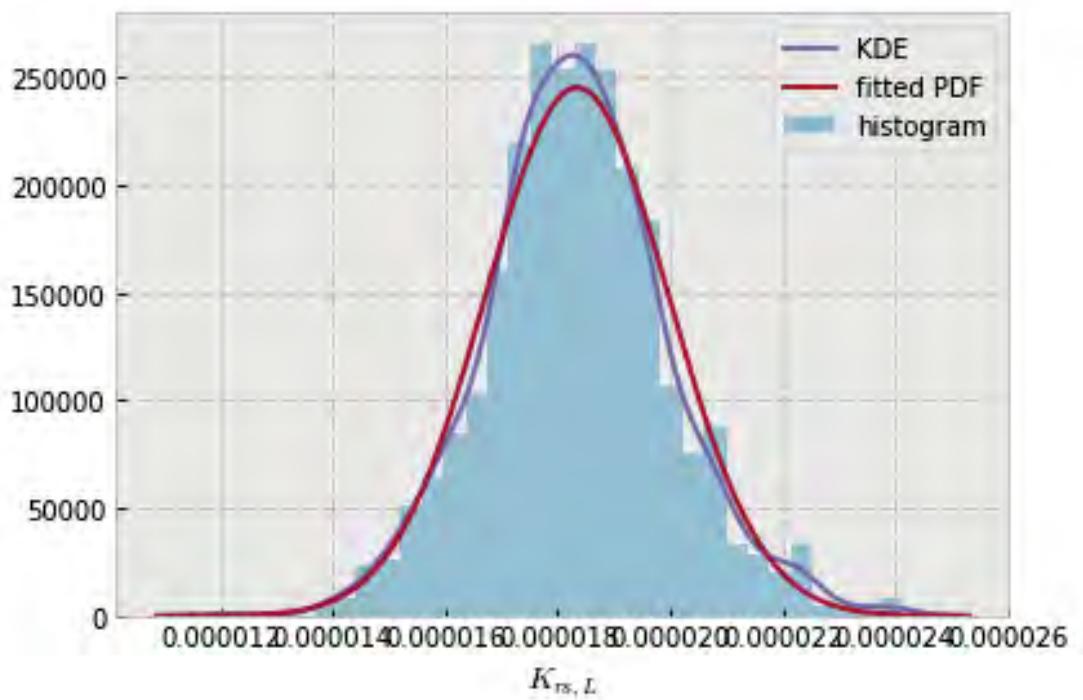


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=1.9435167601354466e-05, scale=2.464182412227445e-06)
DescribeResult(nobs=1000, minmax=(1.1144444322946498e-05, 2.8887147763766002e-05)
               mean=1.9435167601354466e-05, variance=6.0782732339650351e-12,
               skewness=0.15720043849149146, kurtosis=0.5089991224025914)
```

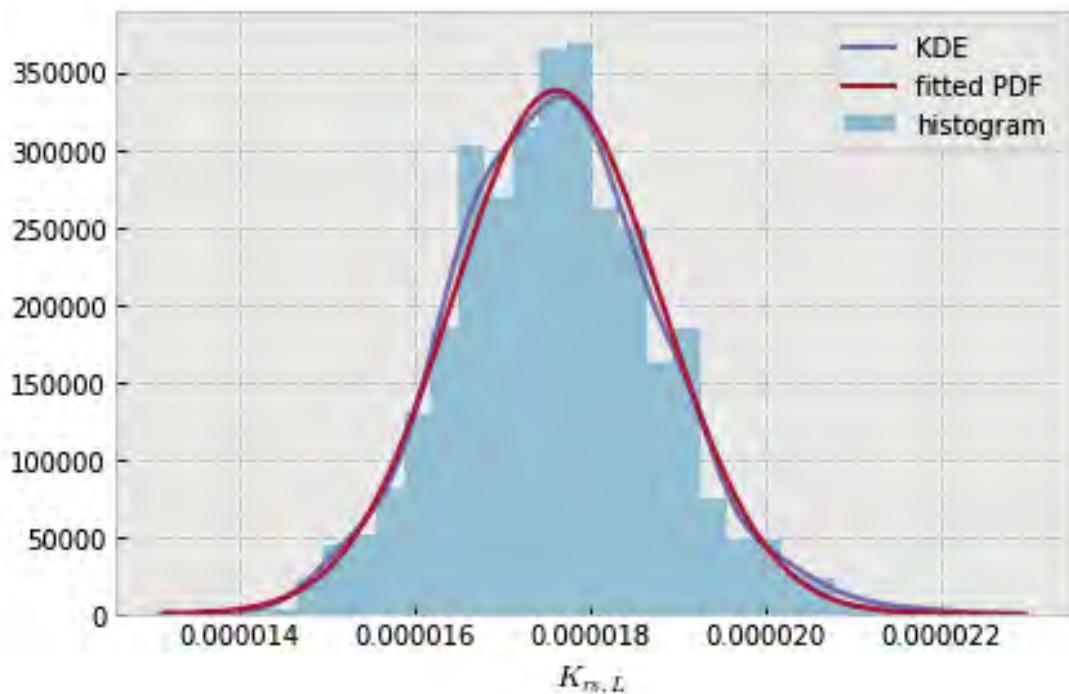


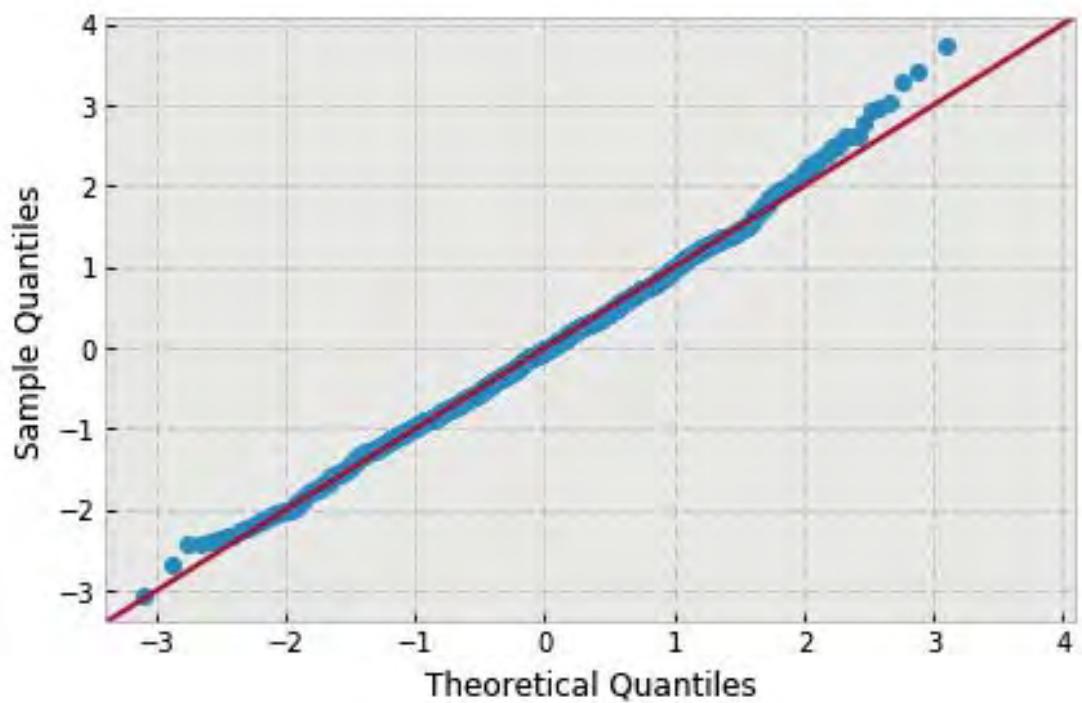


```
variable name: r
variable value: 1.2222222222222223
distribution: normal(shape=(), loc=1.8342327010281072e-05, scale=1.6249440201089315e-06)
DescribeResult(nobs=1000, minmax=(1.210394952587236e-05, 2.4075198342368865e-05)
               mean=1.8342327010281072e-05, variance=2.6430861546424179e-12,
               skewness=0.18848479813709035, kurtosis=0.5888563824123545)
```

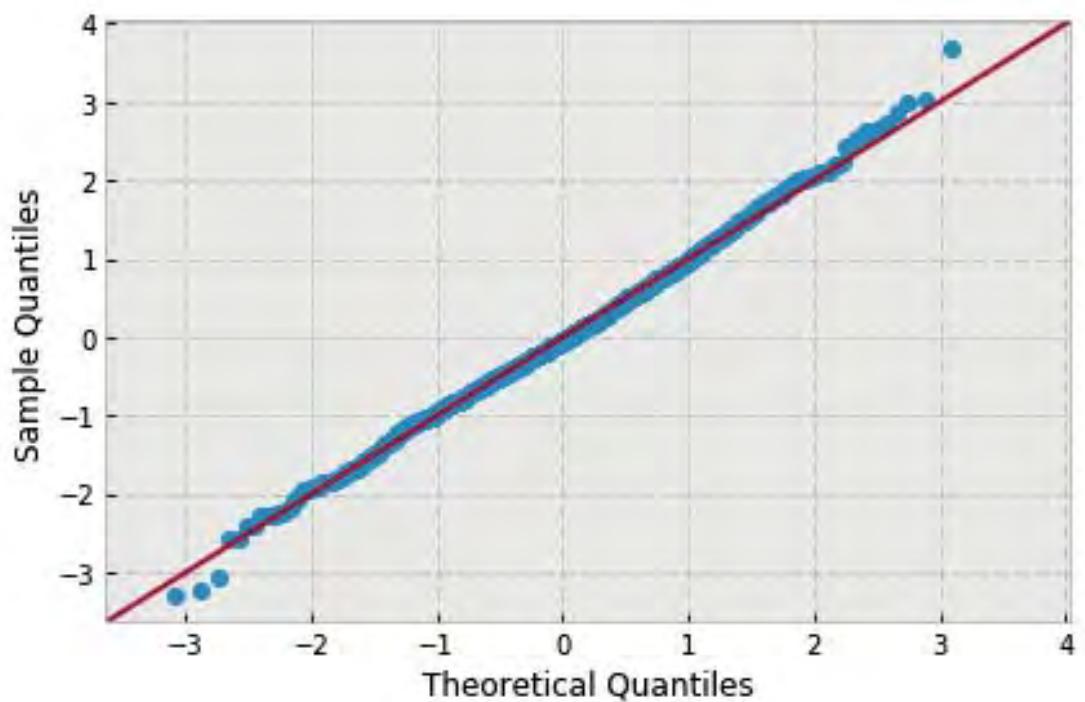
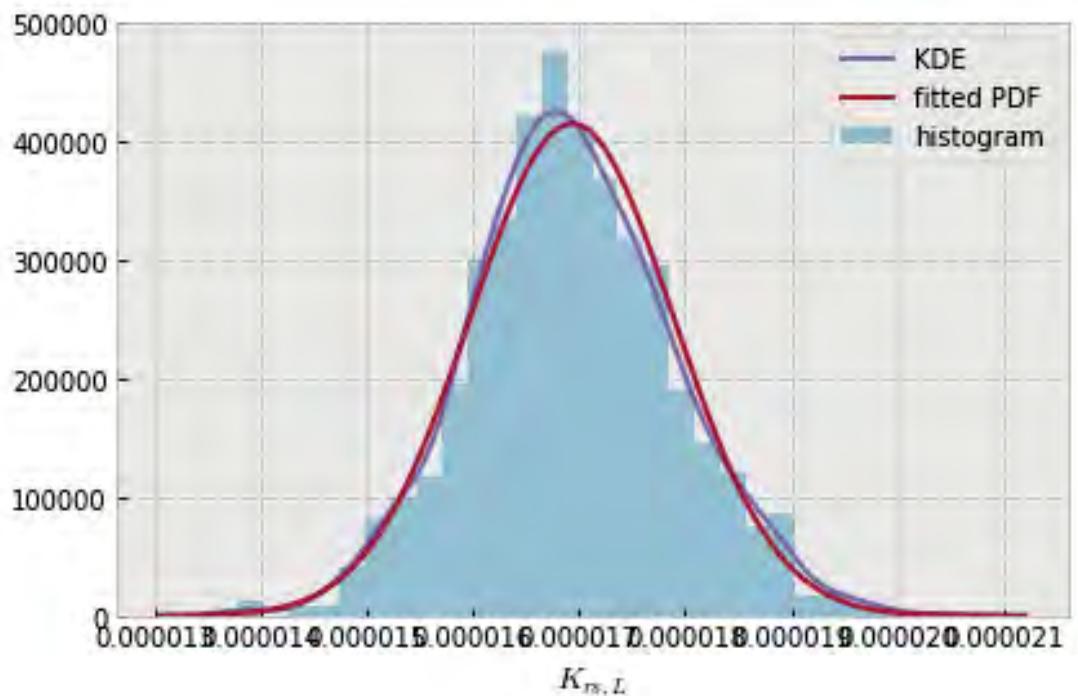


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=1.7618715678530148e-05, scale=1.1794174440142474e-06)
DescribeResult(nobs=1000, minmax=(1.4028344400999934e-05, 2.2033150420892641e-05)
               mean=1.7618715678530148e-05, variance=1.3924179251702704e-12,
               skewness=0.22796895370382697, kurtosis=0.2463166312057785)
```

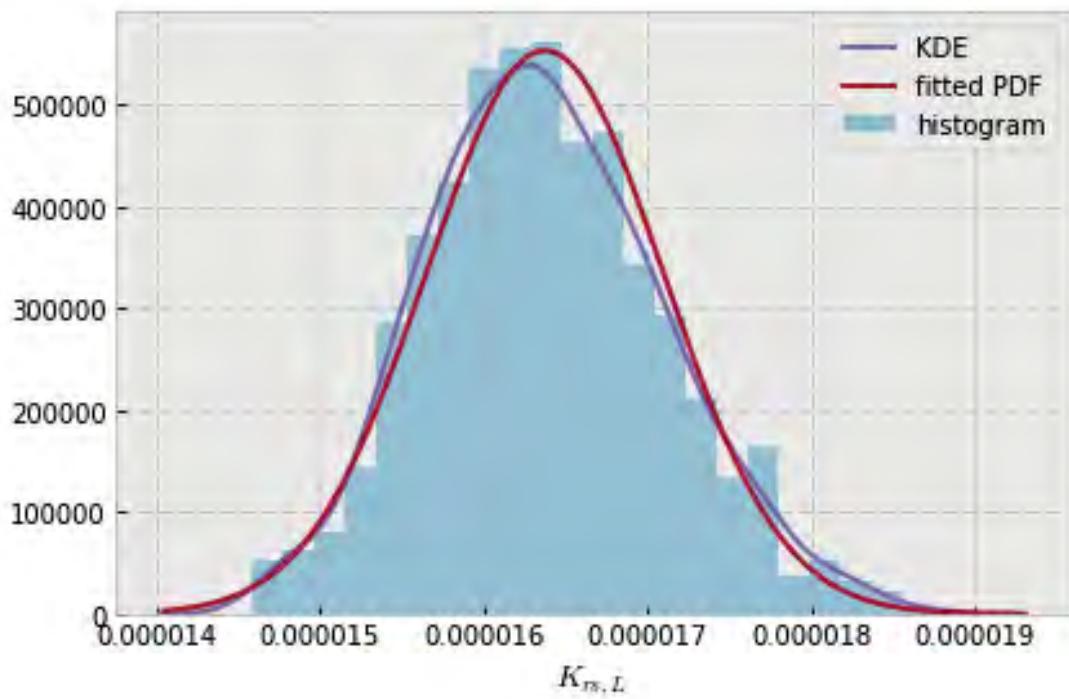


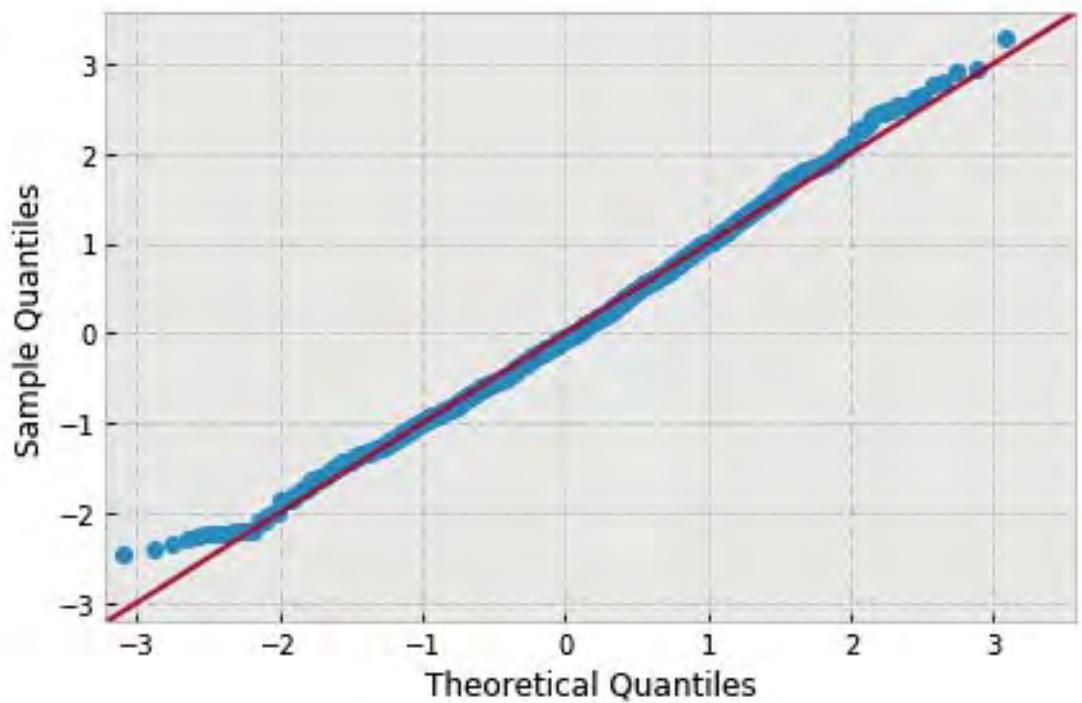


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=1.6930963318255333e-05, scale=9.60293151136992e-07)
DescribeResult(nobs=1000, minmax=(1.3776447392917167e-05, 2.04781475631855e-05)
               mean=1.6930963318255333e-05, variance=9.2308602214275659e-13,
               skewness=0.135538788766632, kurtosis=0.2063336763628727)
```

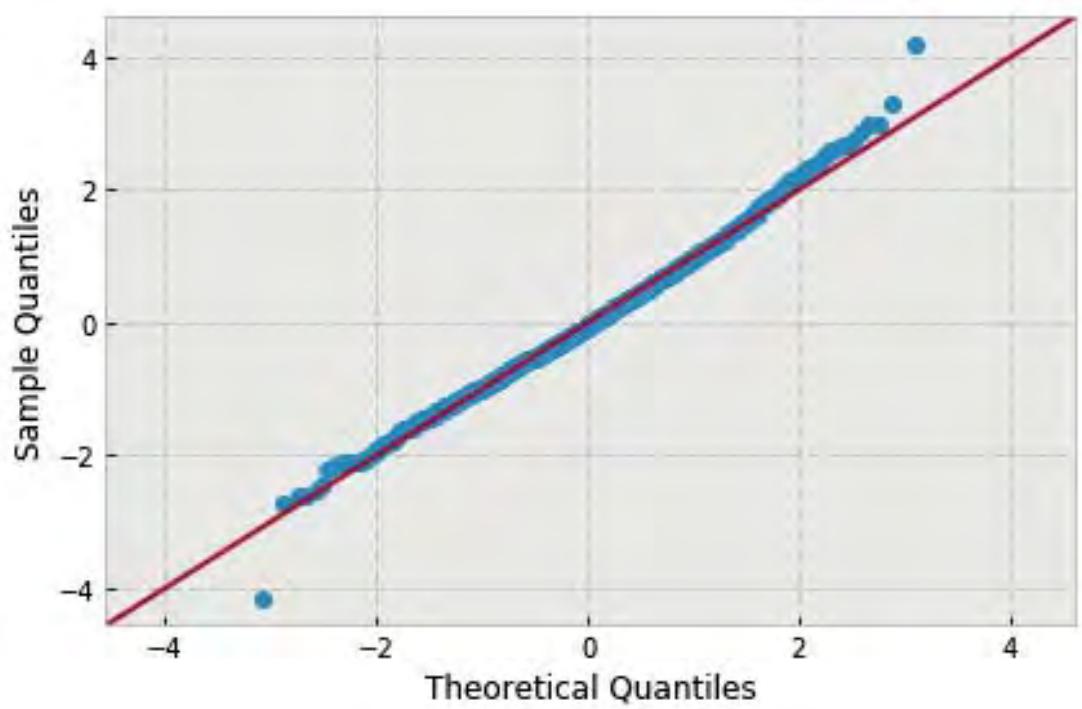
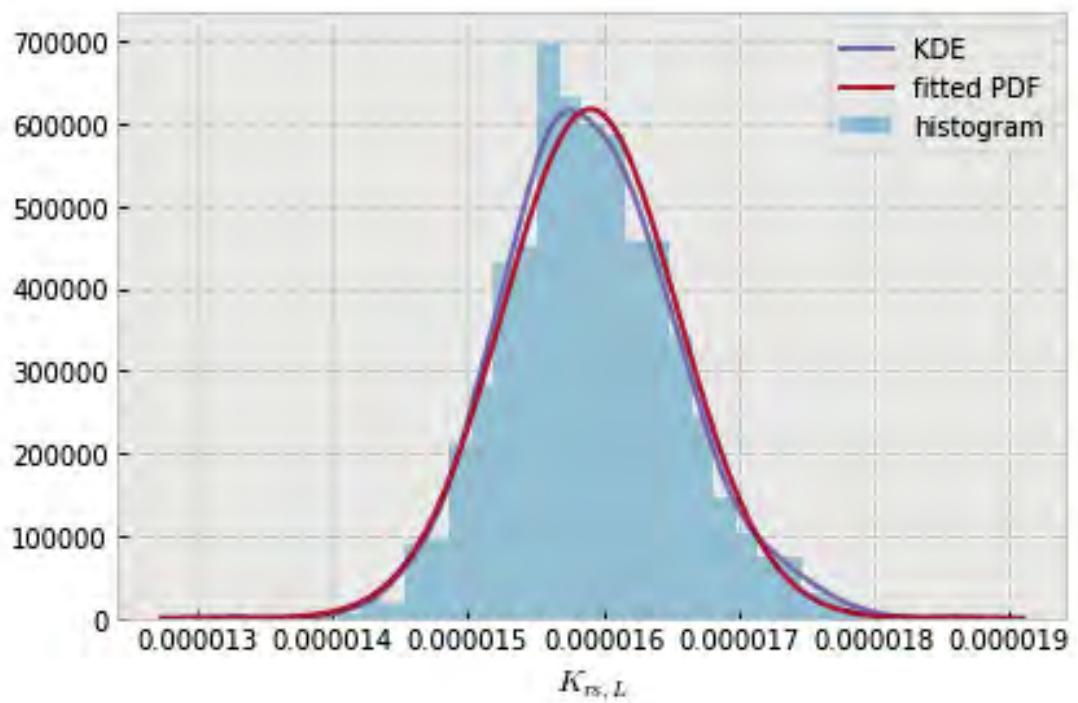


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=1.6370843725478095e-05, scale=7.214787588395908e-07)
DescribeResult(nobs=1000, minmax=(1.4589089379060314e-05, 1.8736028154259917e-05)
               mean=1.6370843725478095e-05, variance=5.2105265210882528e-13,
               skewness=0.2515851788000724, kurtosis=-0.1053407630778671)
```

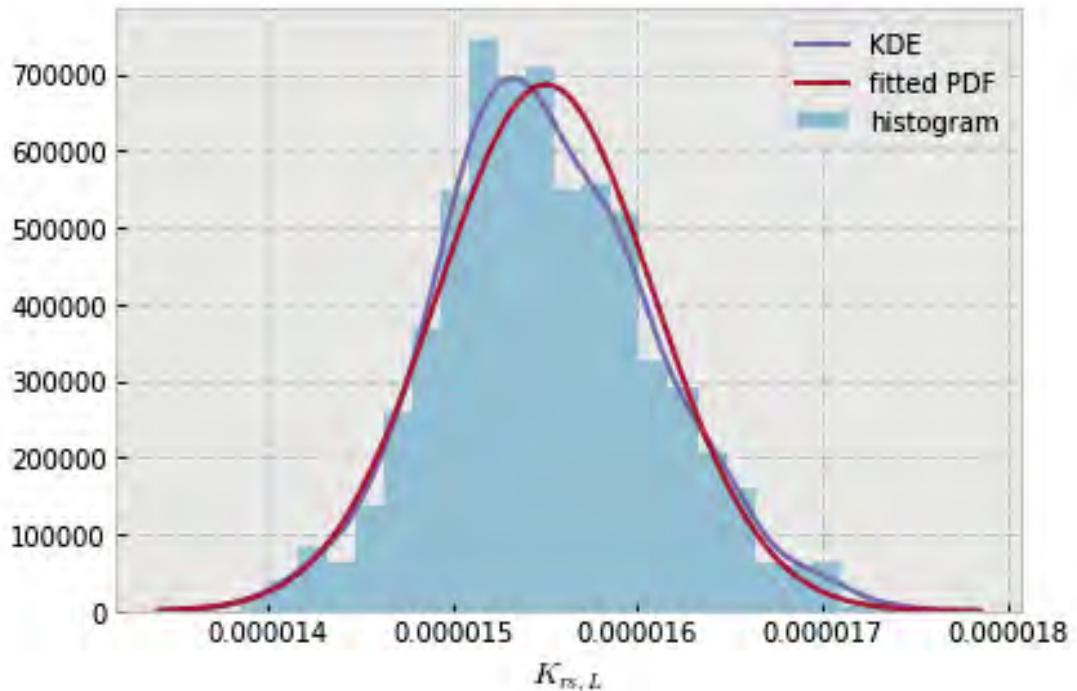


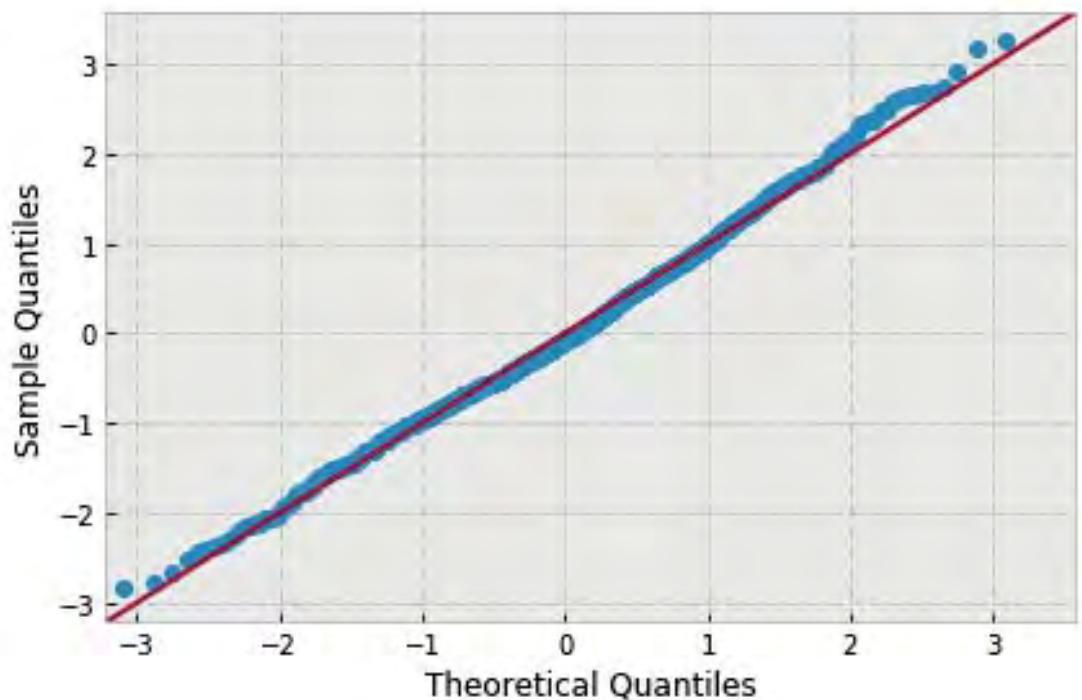


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(), loc=1.590422957073166e-05, scale=6.461804130590507e-07)
DescribeResult(nobs=1000, minmax=(1.3227510341413872e-05, 1.8611244710382243e-05)
               mean=1.5904229570731659e-05, variance=4.1796709331447994e-13,
               skewness=0.24152466857871607, kurtosis=0.47743130253762933)
```

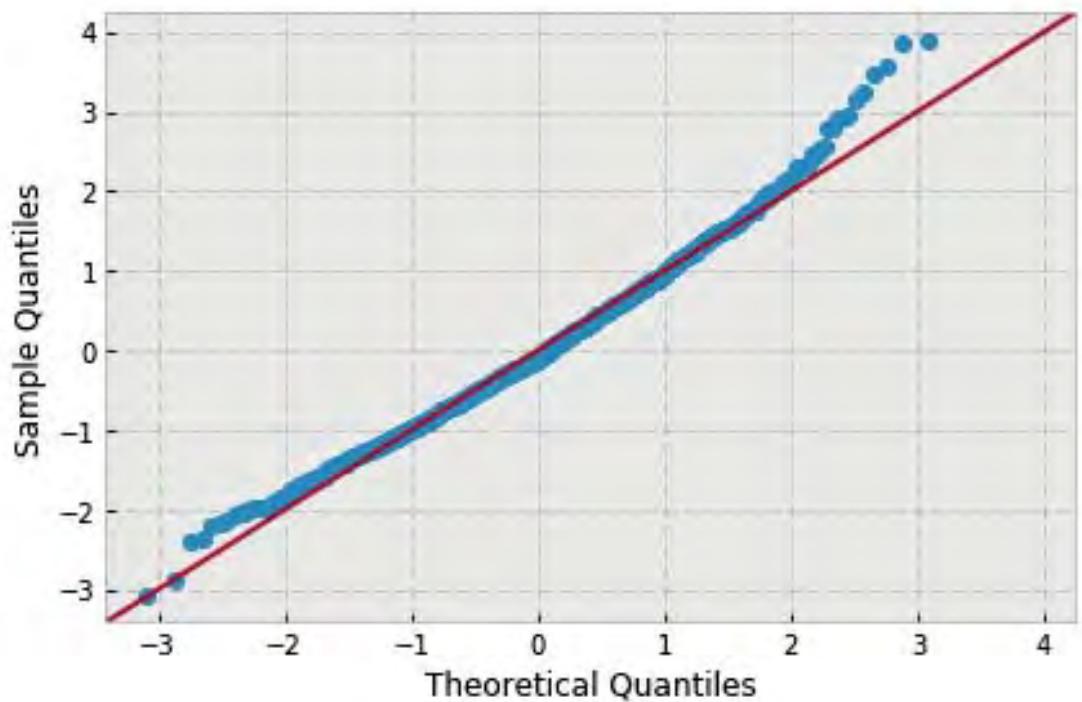
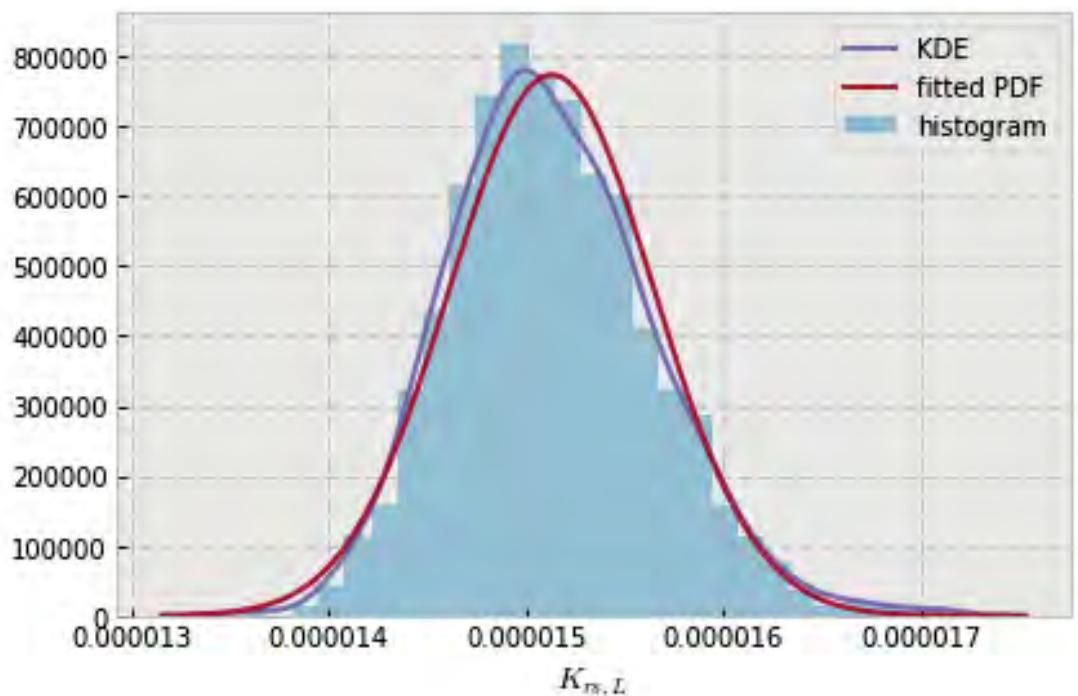


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=1.550295734876567e-05, scale=5.815611675842749e-07)
DescribeResult(nobs=1000, minmax=(1.38594607459055e-05, 1.7400385336298202e-05)
               mean=1.550295734876567e-05, variance=3.3855194358557066e-13,
               skewness=0.23859163585633666, kurtosis=0.053289247139681084)
```

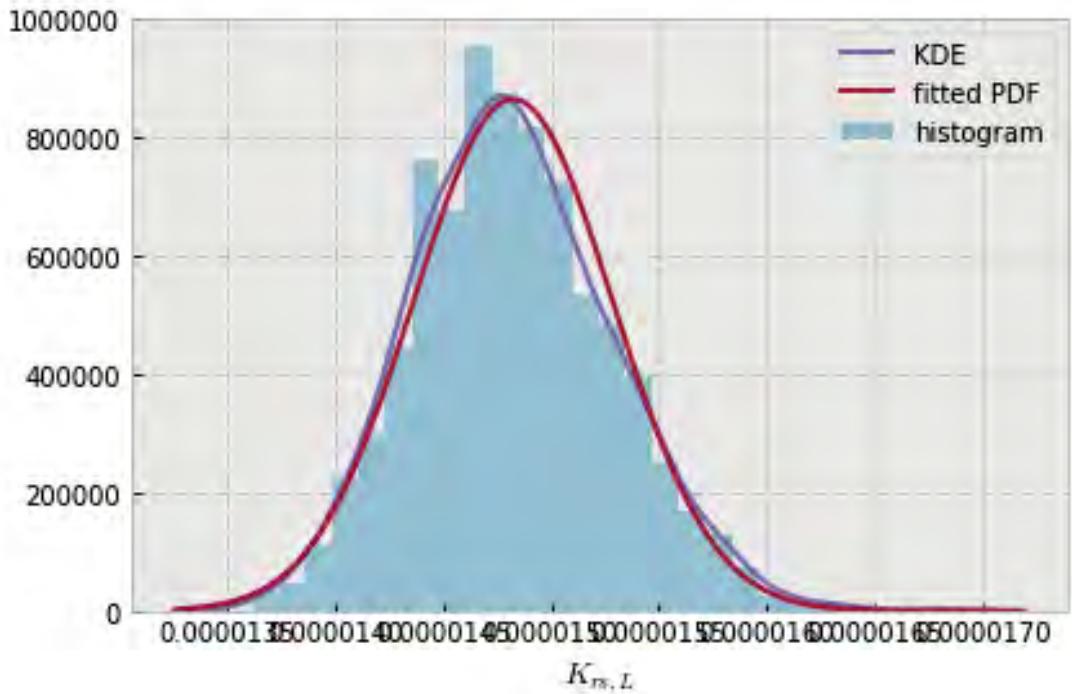


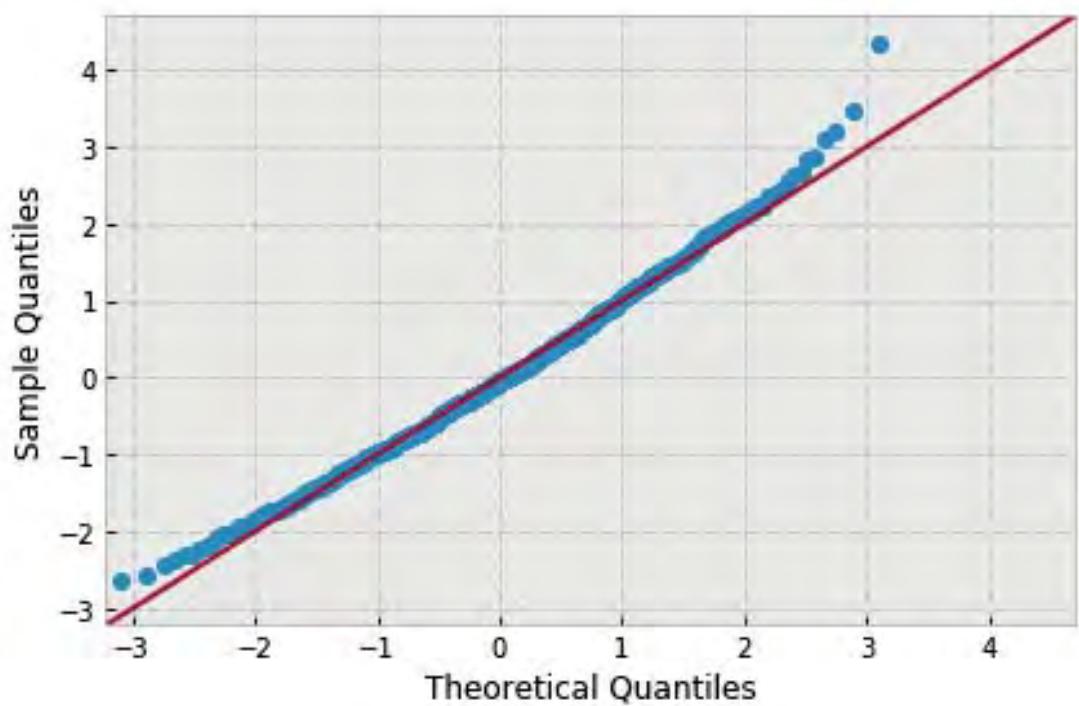


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=1.5130935900019624e-05, scale=5.158632816795269e-07)
DescribeResult(nobs=1000, minmax=(1.3549954613942475e-05, 1.7136887946353615e-05)
               mean=1.5130935900019624e-05, variance=2.6638130669186275e-13,
               skewness=0.43088327892516776, kurtosis=0.48228597342910007)
```

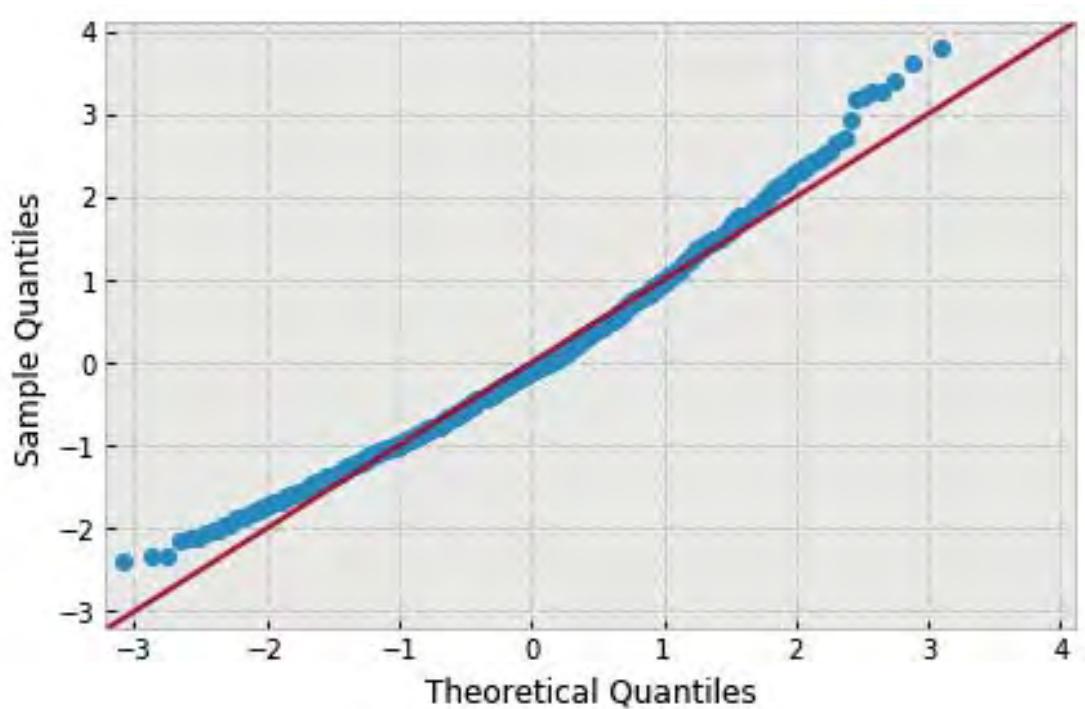
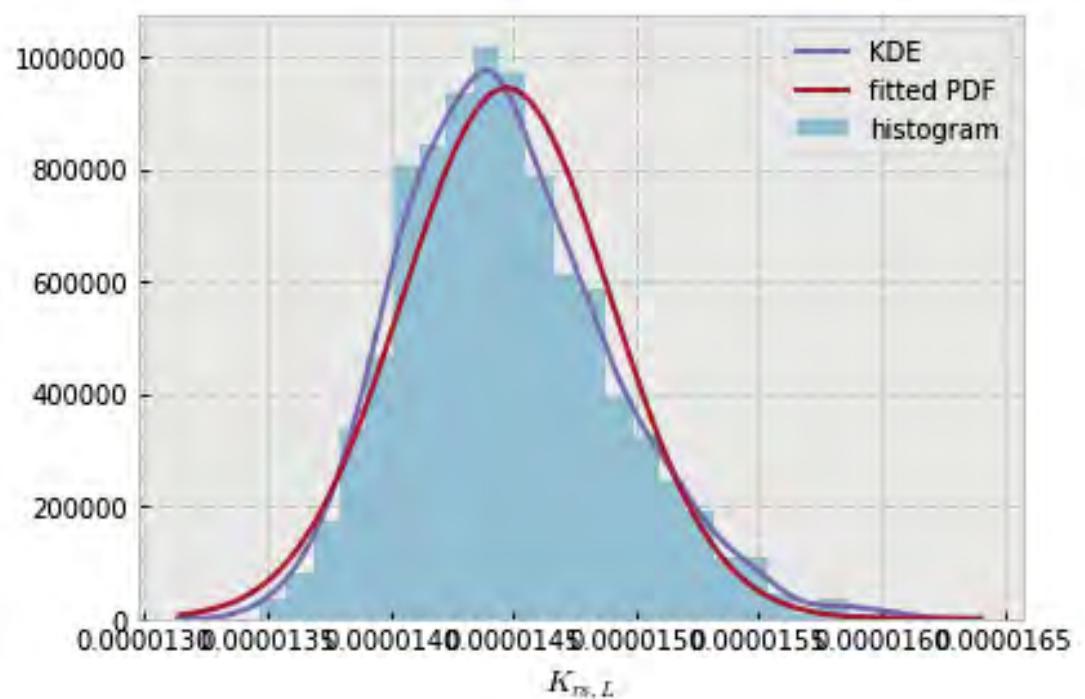


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=1.4825077183699578e-05, scale=4.610067536826433e-07)
DescribeResult(nobs=1000, minmax=(1.3615656915493237e-05, 1.6827846529832957e-05)
               mean=1.4825077183699578e-05, variance=2.1273996690791724e-13,
               skewness=0.3260945419989589, kurtosis=0.20946787347886087)
```

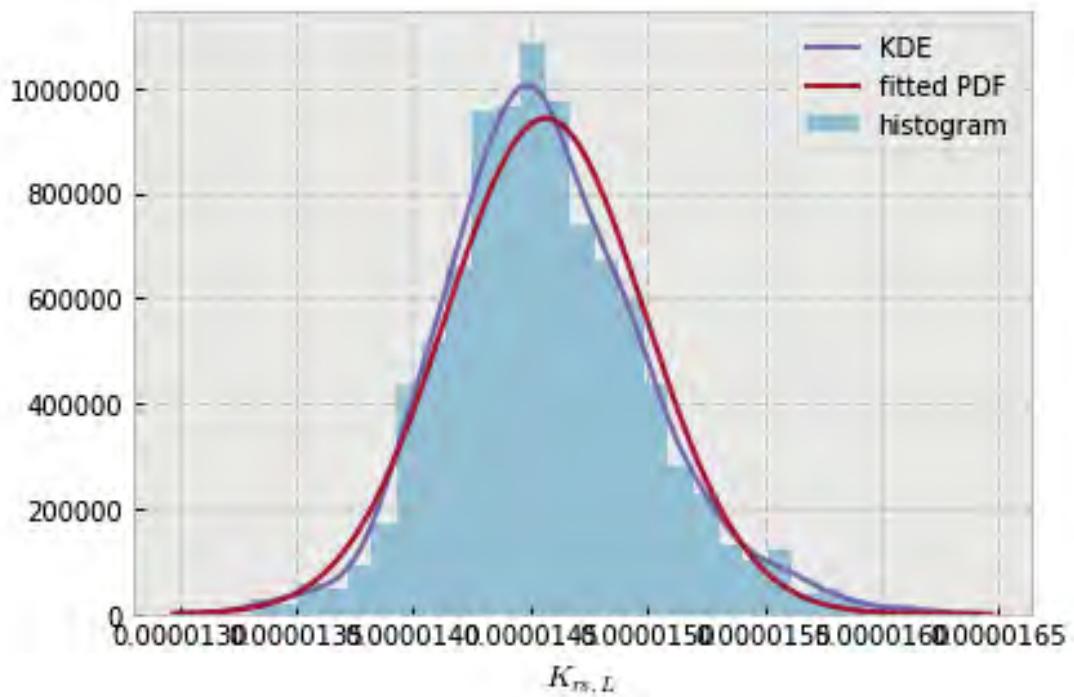


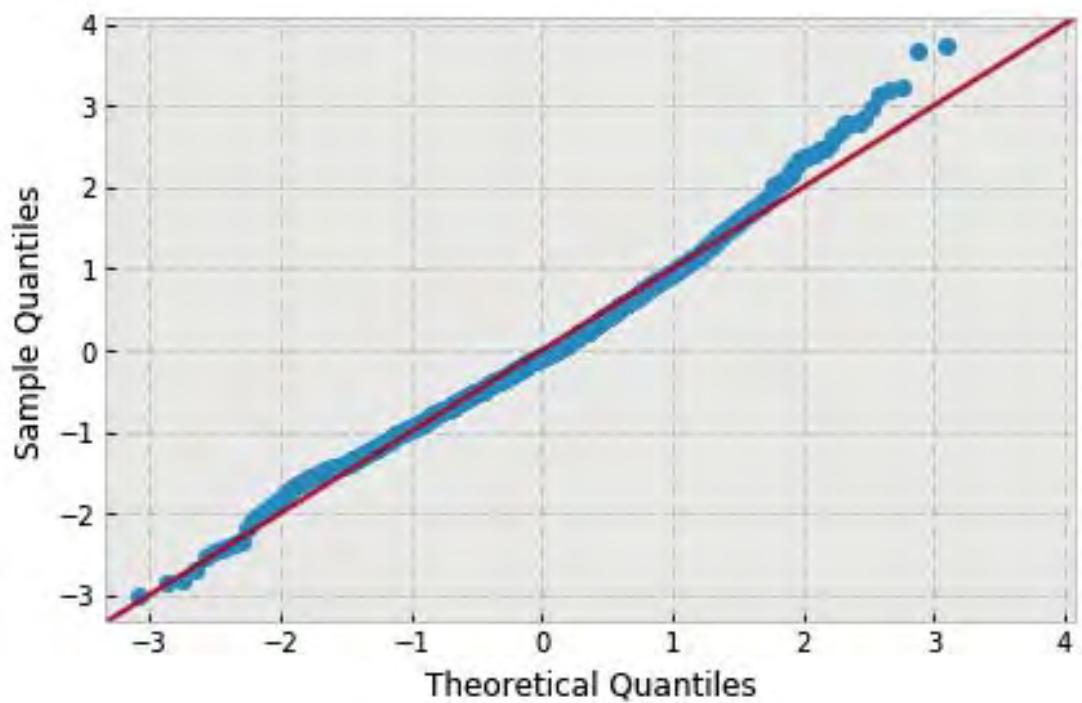


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=1.447285080399581e-05, scale=4.228673915249169e-07)
DescribeResult(nobs=1000, minmax=(1.3465600408982395e-05, 1.6076771747597163e-05)
               mean=1.447285080399581e-05, variance=1.7899582664172908e-13,
               skewness=0.5482546040502364, kurtosis=0.346543951246832)
```

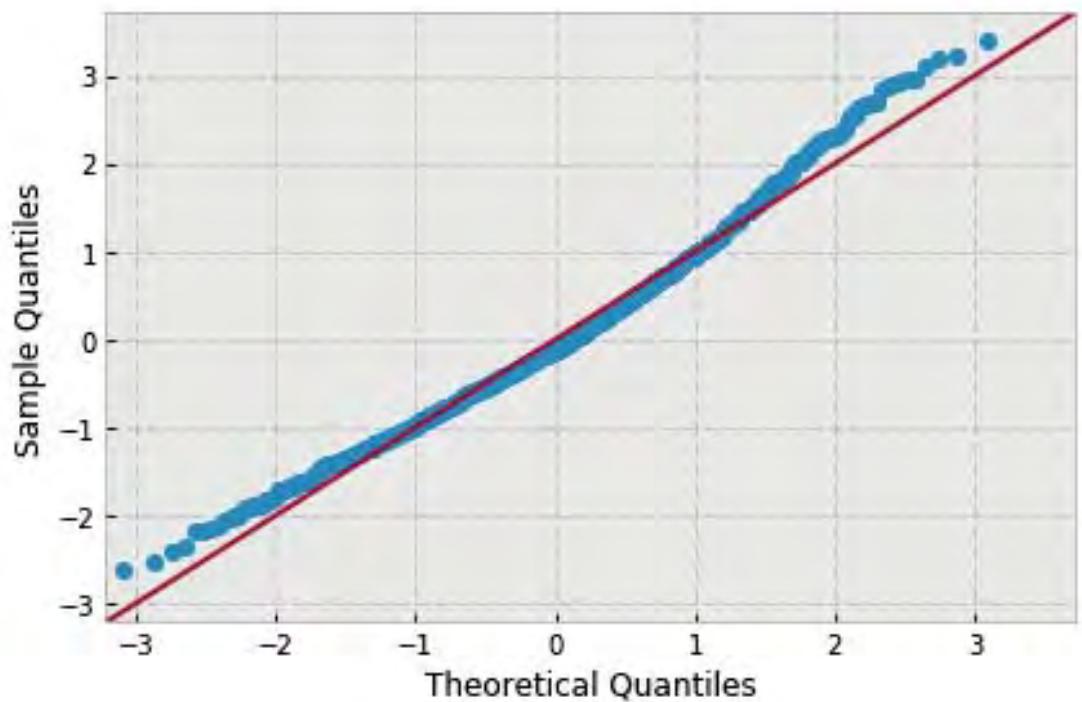
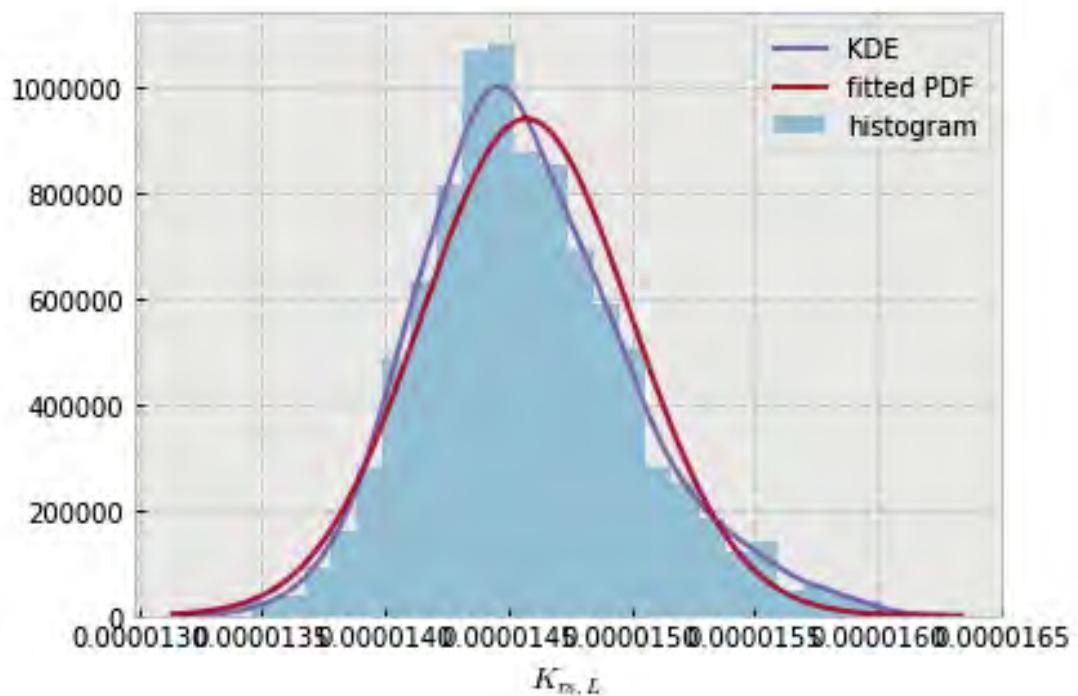


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=1.456657912188311e-05, scale=4.233851761345289e-07)
DescribeResult(nobs=1000, minmax=(1.3296920632768983e-05, 1.6146634056812856e-05)
               mean=1.456657912188311e-05, variance=1.7943444181227836e-13,
               skewness=0.3920177613991987, kurtosis=0.5343323560828952)
```

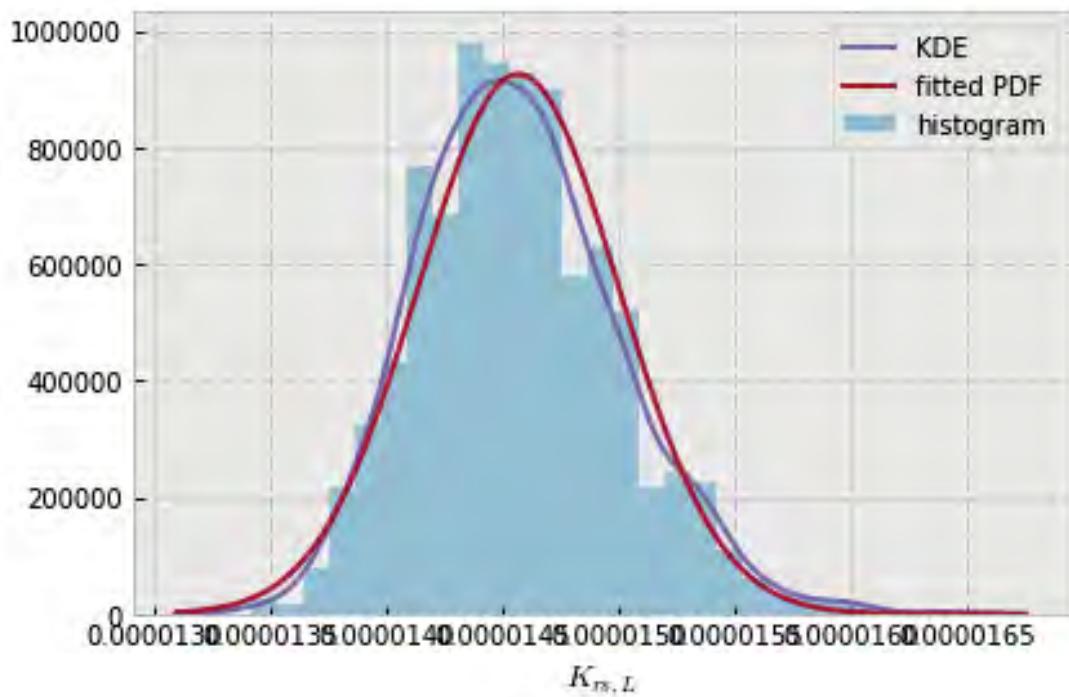


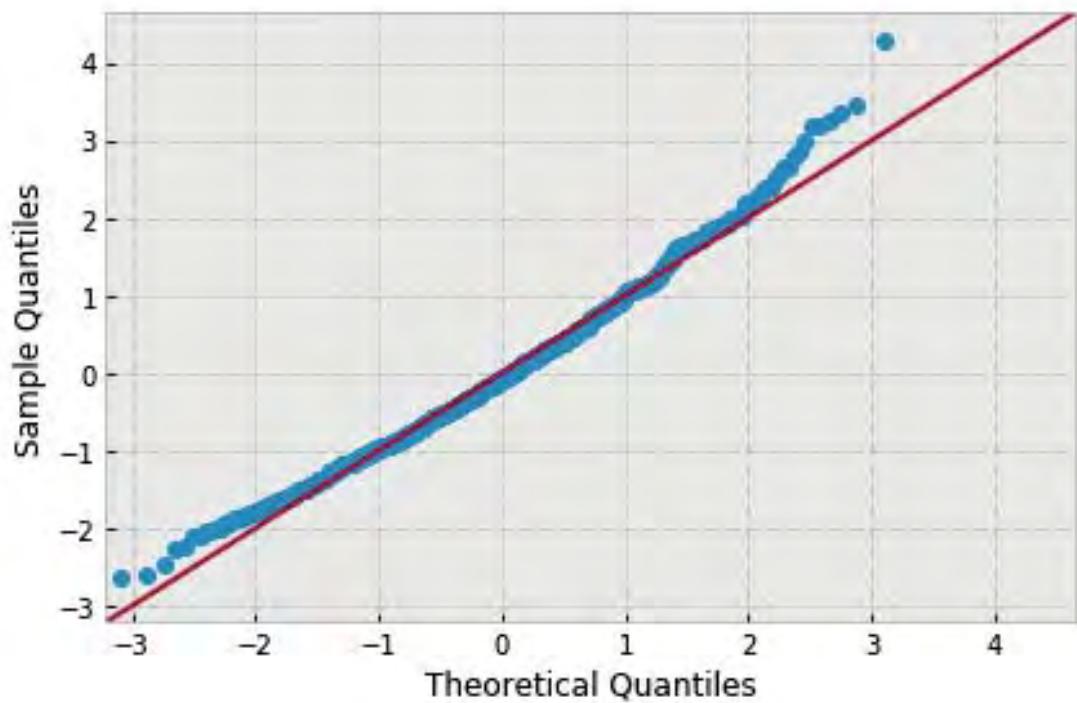


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=1.457406053065869e-05, scale=4.23703094906332e-07)
DescribeResult(nobs=1000, minmax=(1.3458940991236256e-05, 1.6018665316539479e-05)
               mean=1.457406053065869e-05, variance=1.7970401664985406e-13,
               skewness=0.5078446139180889, kurtosis=0.3024178290351913)
```

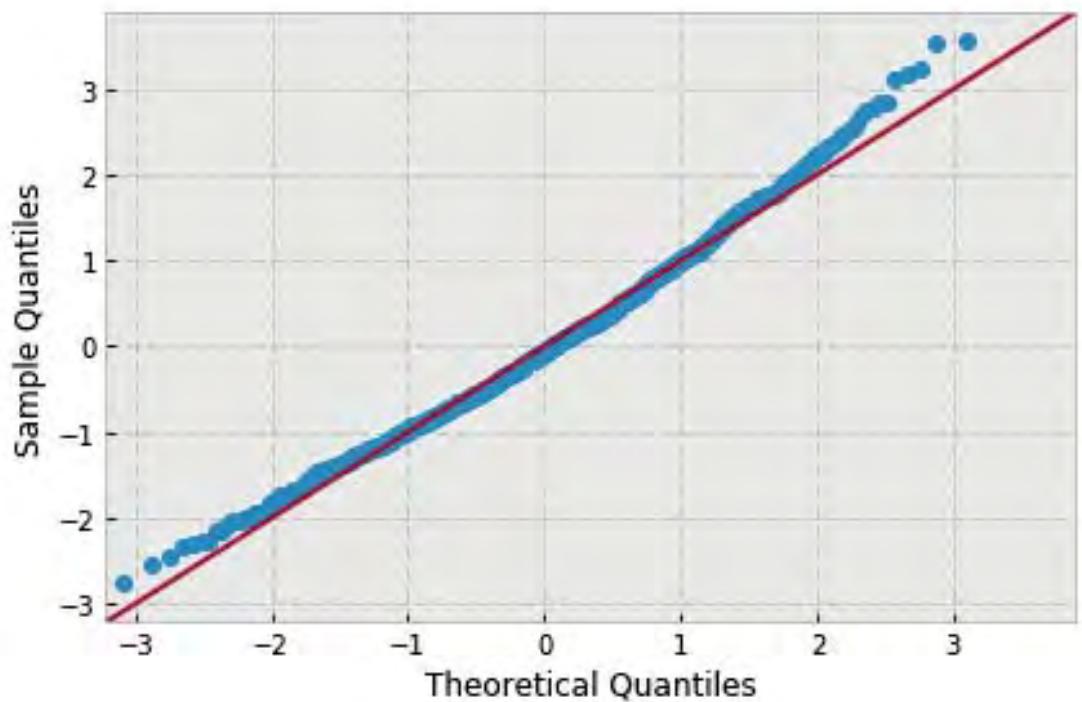
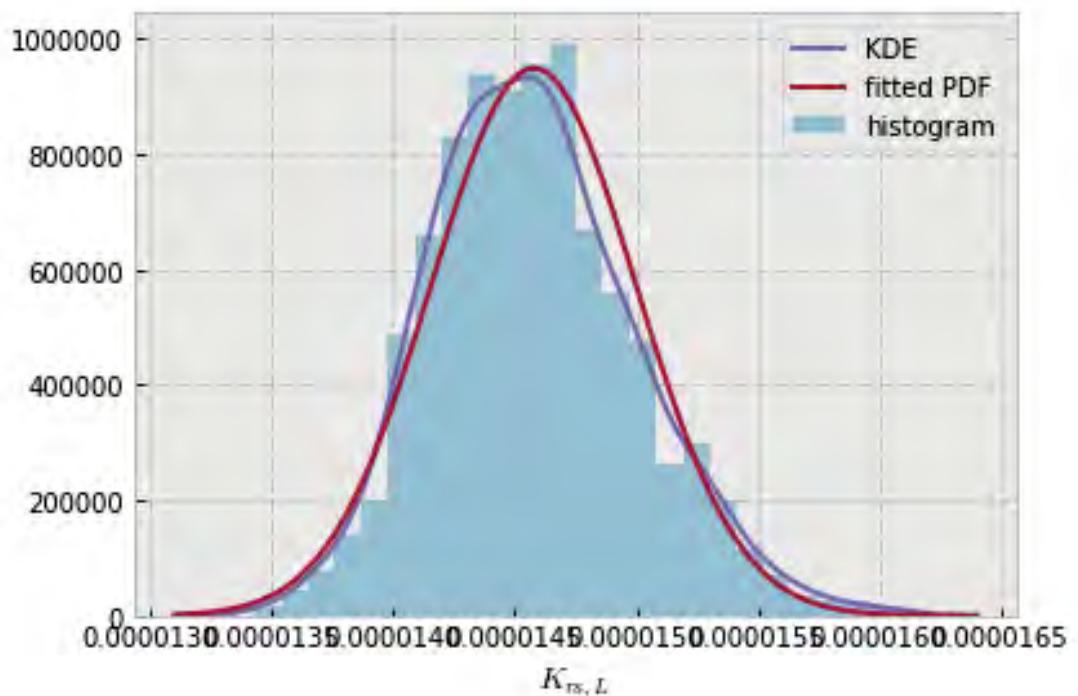


```
variable name: sigma
variable value: 0.0333333333333333
distribution: normal(shape=(), loc=1.4570454718734615e-05, scale=4.309739190090135e-07)
DescribeResult(nobs=1000, minmax=(1.3427203180654178e-05, 1.6420897238629316e-05)
               mean=1.4570454718734615e-05, variance=1.8592444330929701e-13,
               skewness=0.45848361542055527, kurtosis=0.3863029987091404)
```

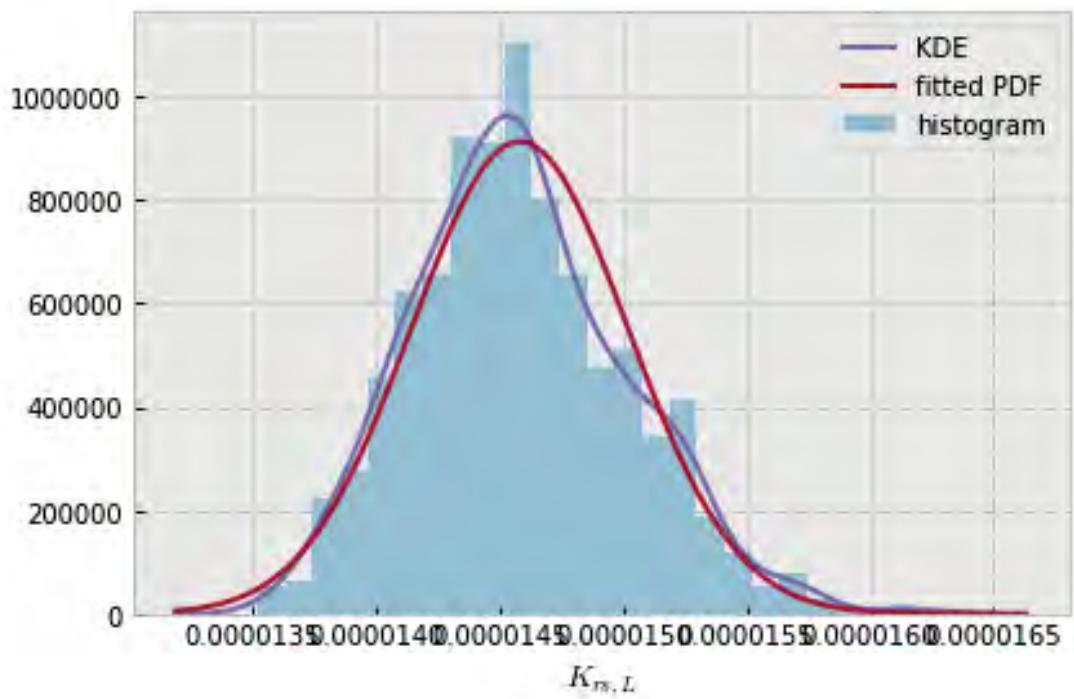


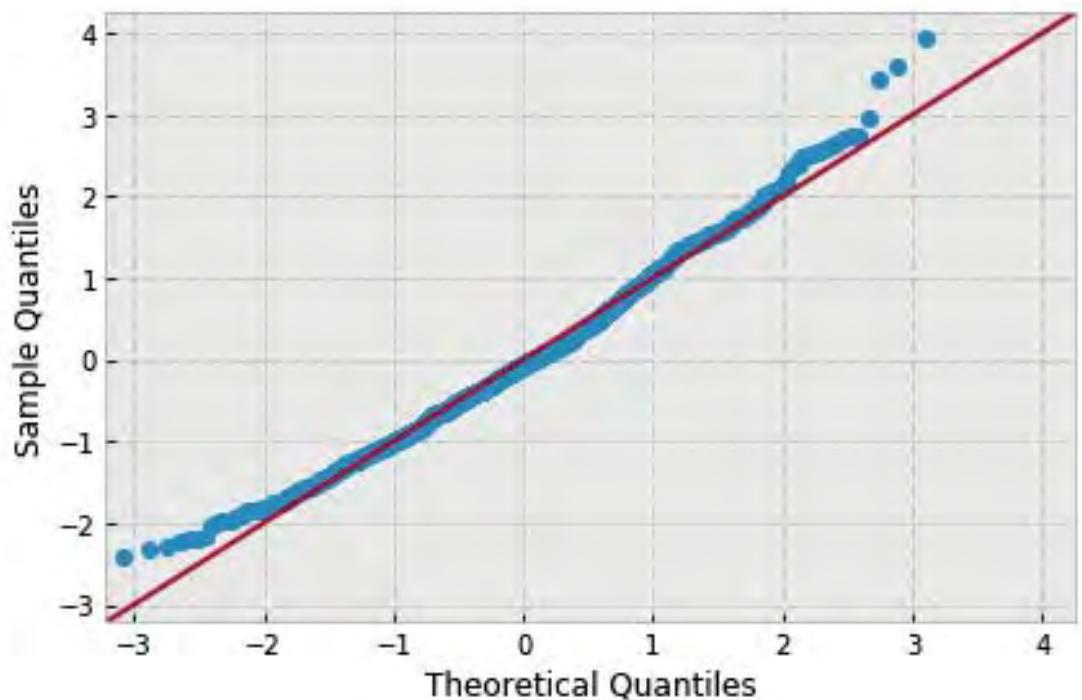


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=1.4577166673013425e-05, scale=4.2043737428631013e-07)
DescribeResult(nobs=1000, minmax=(1.3423394296560741e-05, 1.6077384195074956e-05)
               mean=1.4577166673013425e-05, variance=1.7694453022699384e-13,
               skewness=0.4007318013177871, kurtosis=0.2000736366327187)
```

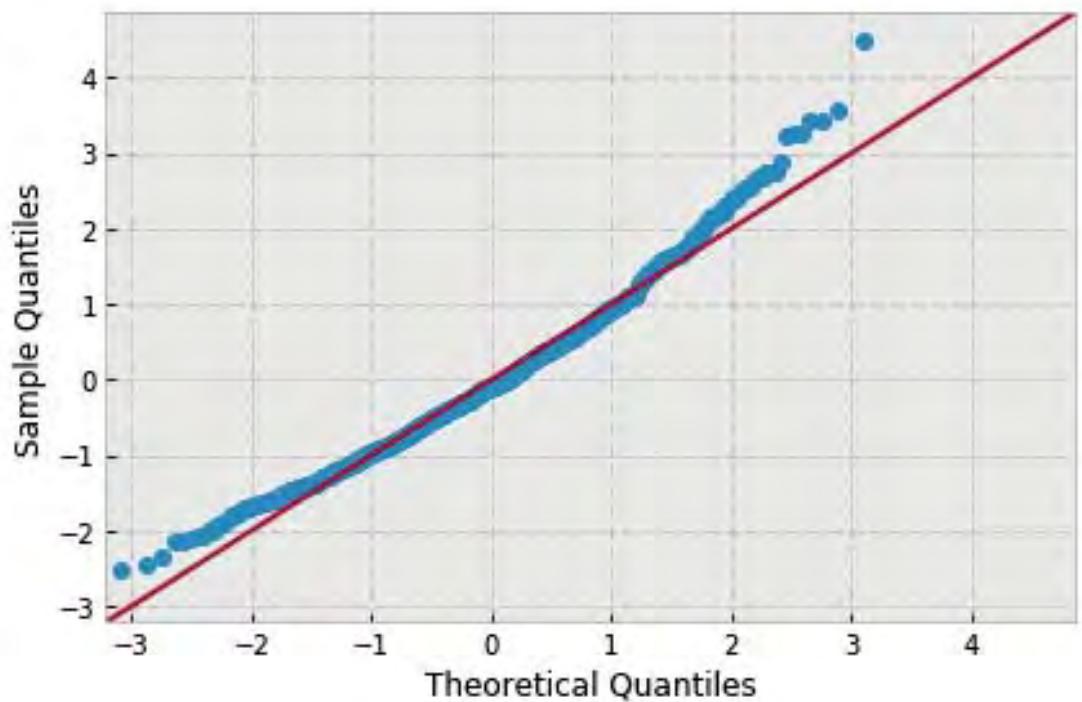
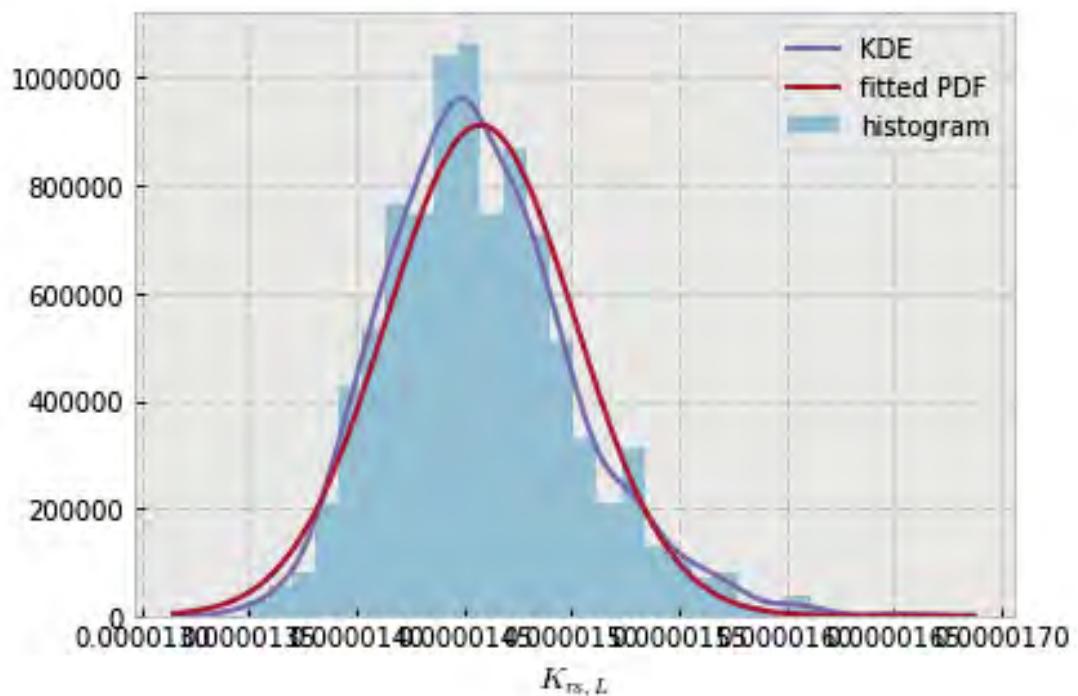


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=1.4582657420418822e-05, scale=4.3702837261810087e-07)
DescribeResult(nobs=1000, minmax=(1.3522086330150511e-05, 1.6298237143370559e-05)
               mean=1.4582657420418822e-05, variance=1.9118498345668227e-13,
               skewness=0.39599321061520326, kurtosis=0.11530269949588812)
```

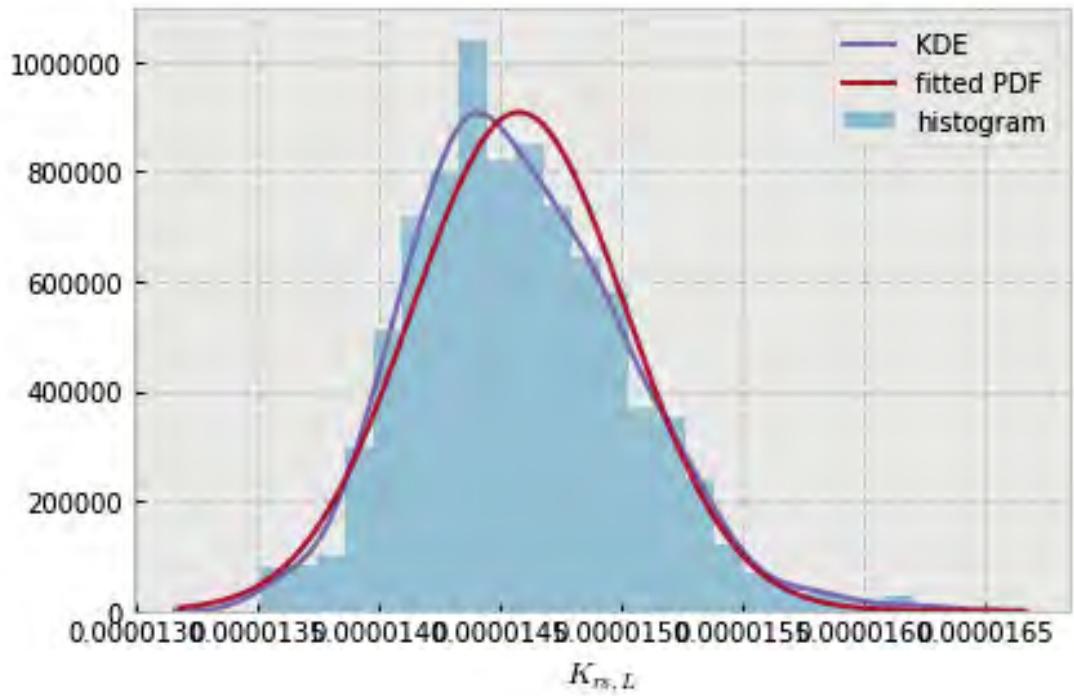


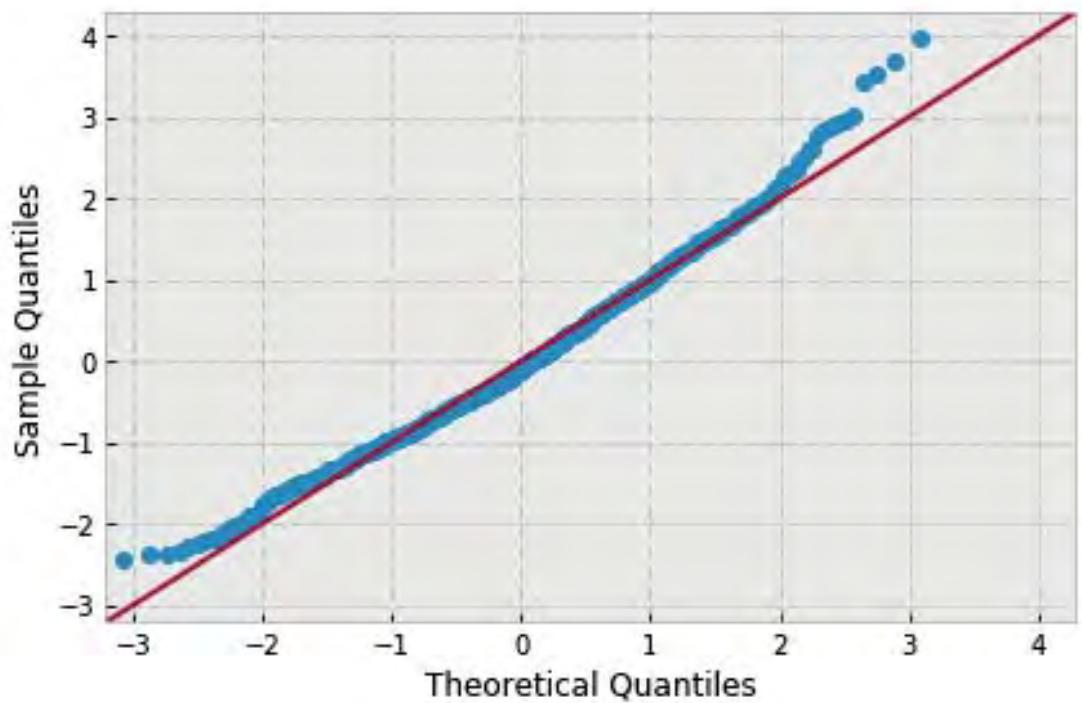


```
variable name: sigma
variable value: 0.066666666666666667
distribution: normal(shape=(), loc=1.4577618647473013e-05, scale=4.373039011538786e-07)
DescribeResult(nobs=1000, minmax=(1.347654235171577e-05, 1.6546895141222211e-05)
               mean=1.4577618647473013e-05, variance=1.9142612809249372e-13,
               skewness=0.6089496026560567, kurtosis=0.6914442298635604)
```

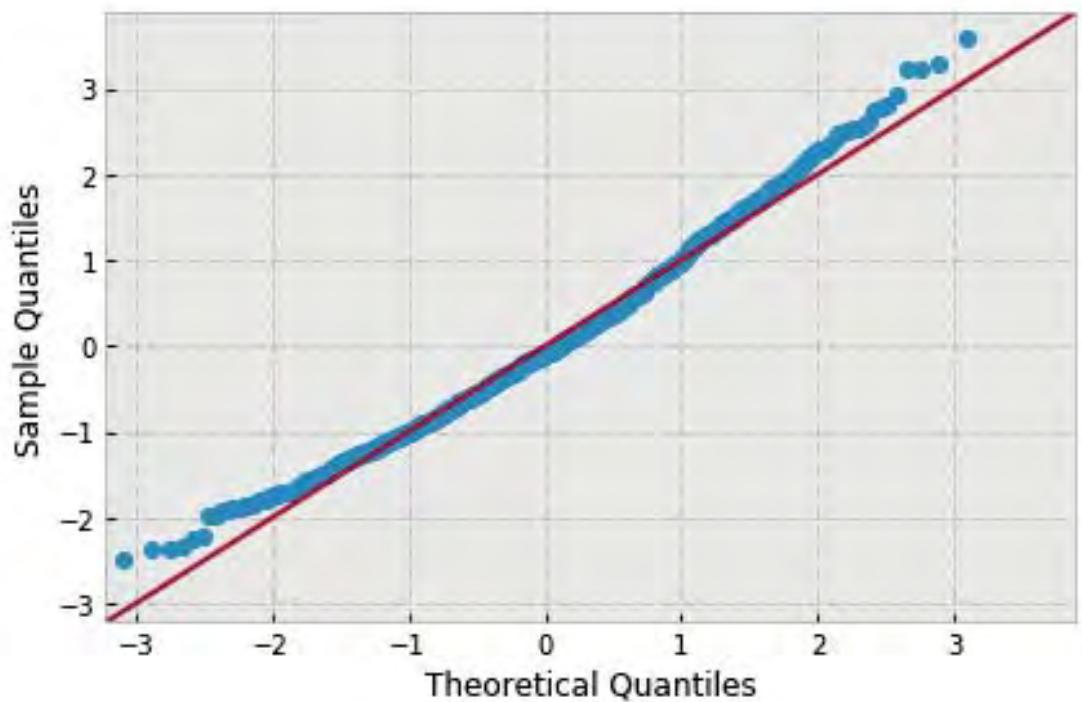
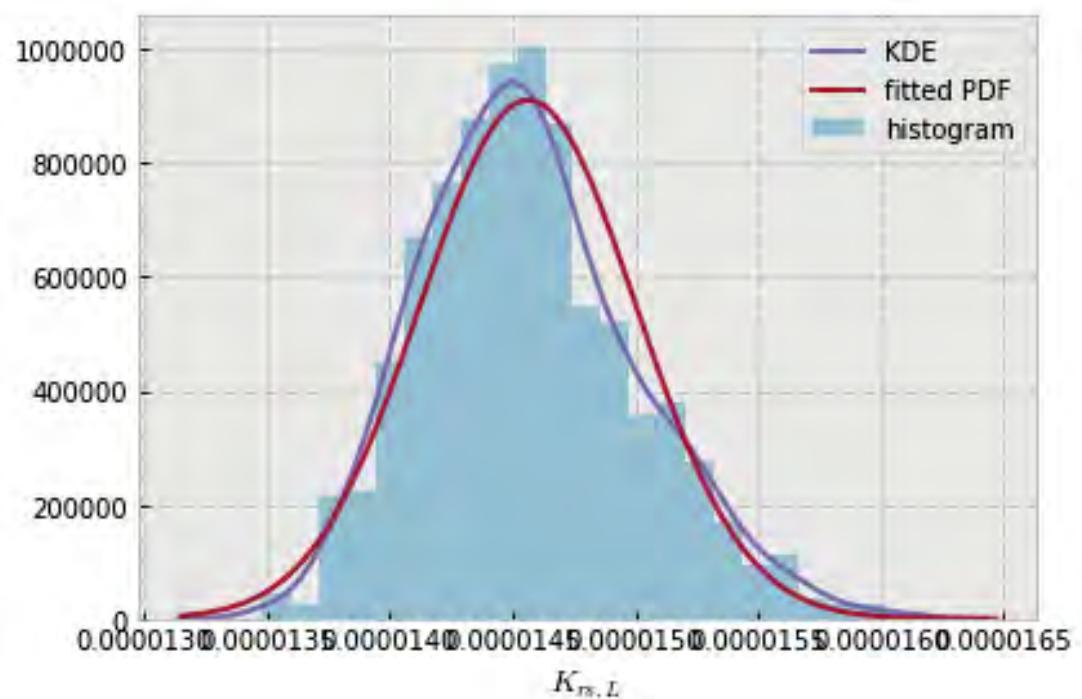


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=1.457609950598749e-05, scale=4.39368888274027e-07)
DescribeResult(nobs=1000, minmax=(1.3509493918028471e-05, 1.6315034389615181e-05)
               mean=1.4576099505987491e-05, variance=1.9323825824139582e-13,
               skewness=0.4529668344967481, kurtosis=0.3203505262039079)
```

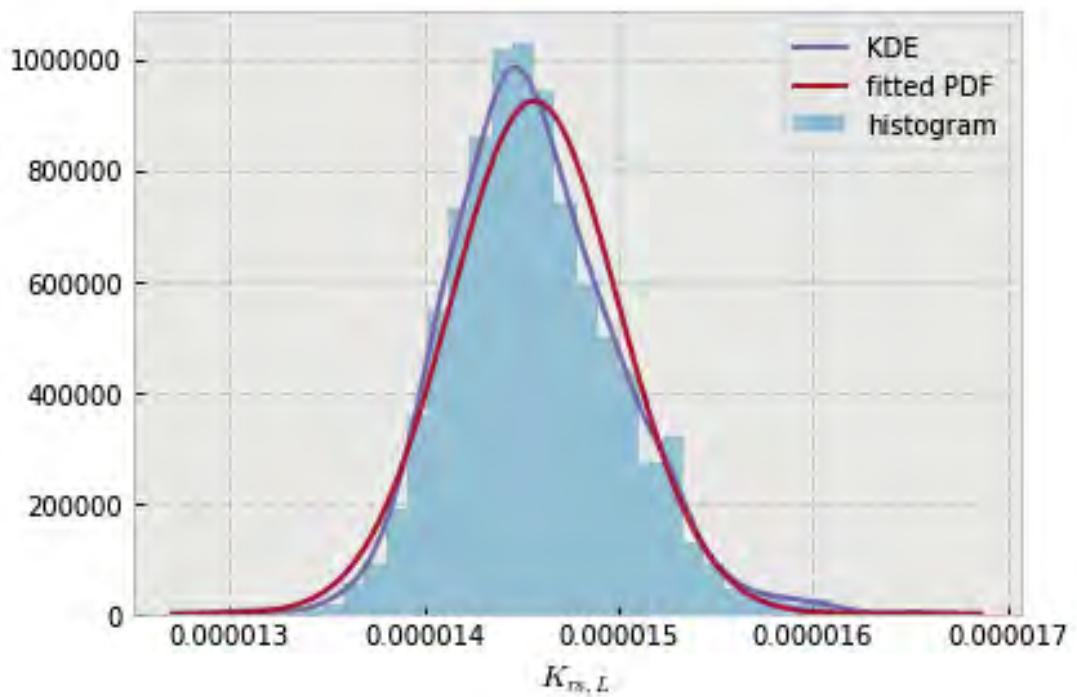


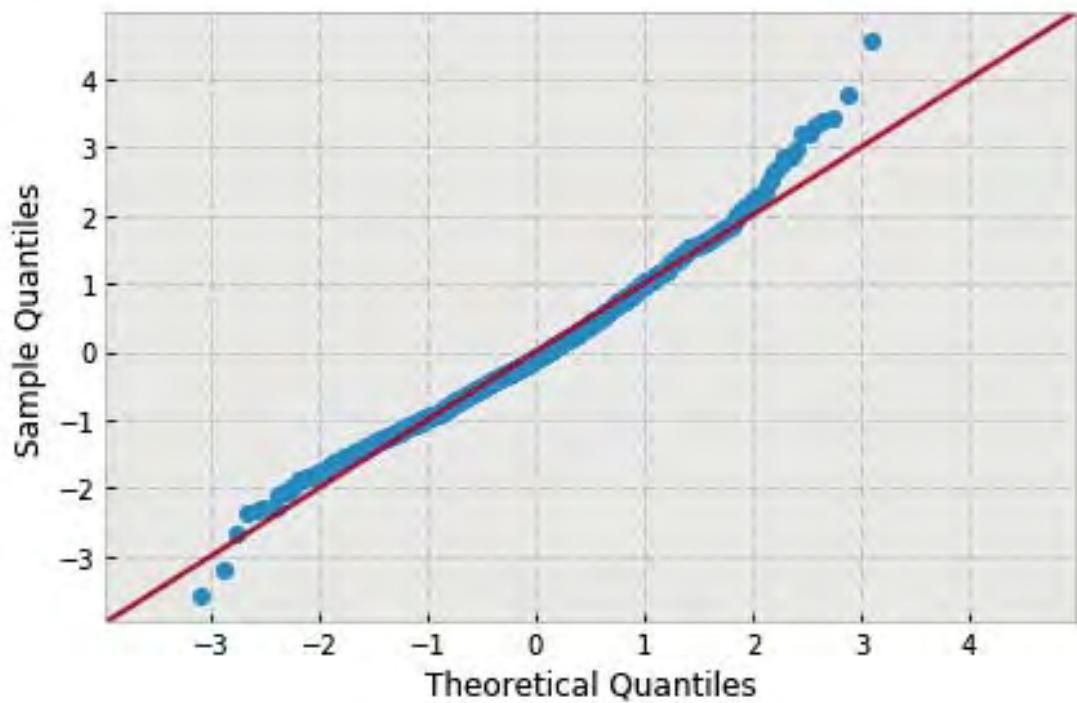


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=1.4563418078542561e-05, scale=4.381116526540486e-07)
DescribeResult(nobs=1000, minmax=(1.348089976197436e-05, 1.6129028198244298e-05)
               mean=1.4563418078542561e-05, variance=1.9213395414540718e-13,
               skewness=0.45548904171730004, kurtosis=0.09359445610310013)
```

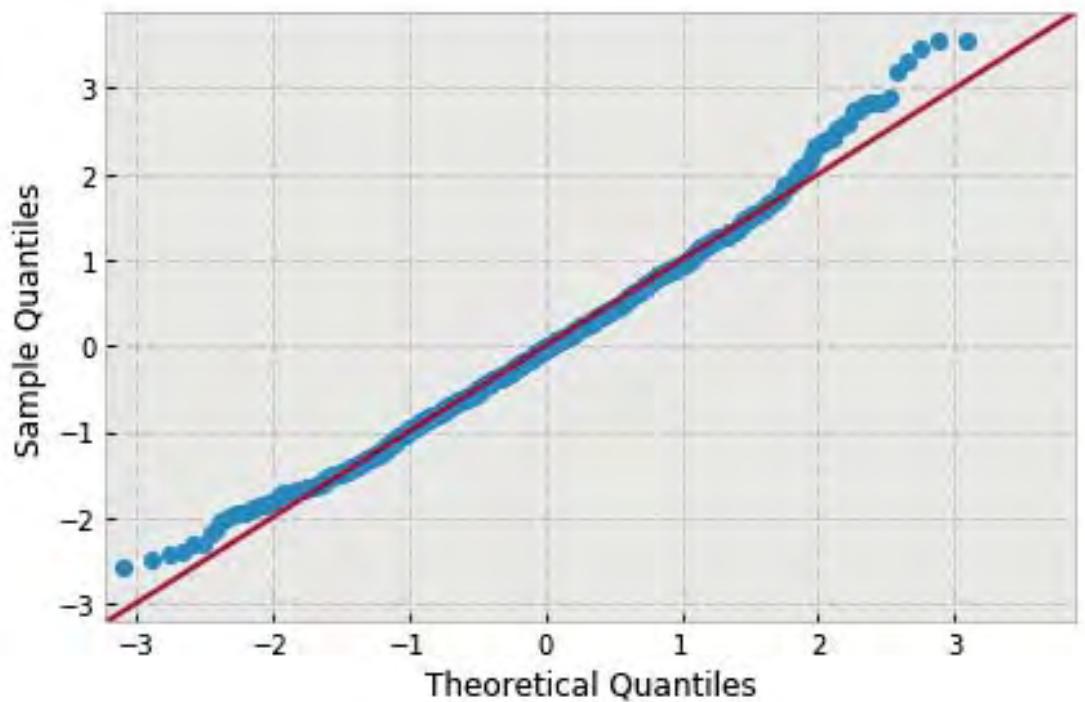
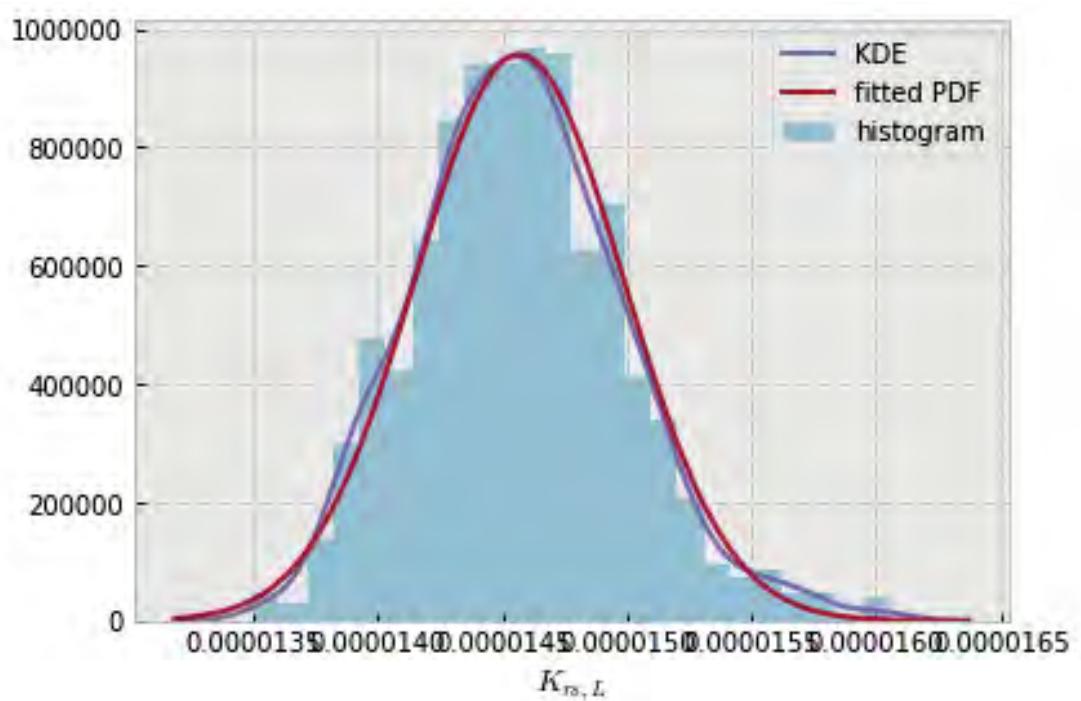


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=1.4566197839323826e-05, scale=4.3127657779171207e-07)
DescribeResult(nobs=1000, minmax=(1.3032730053392861e-05, 1.653287998625723e-05)
               mean=1.4566197839323826e-05, variance=1.8618567222395462e-13,
               skewness=0.4851217245949452, kurtosis=0.8332087369111938)
```

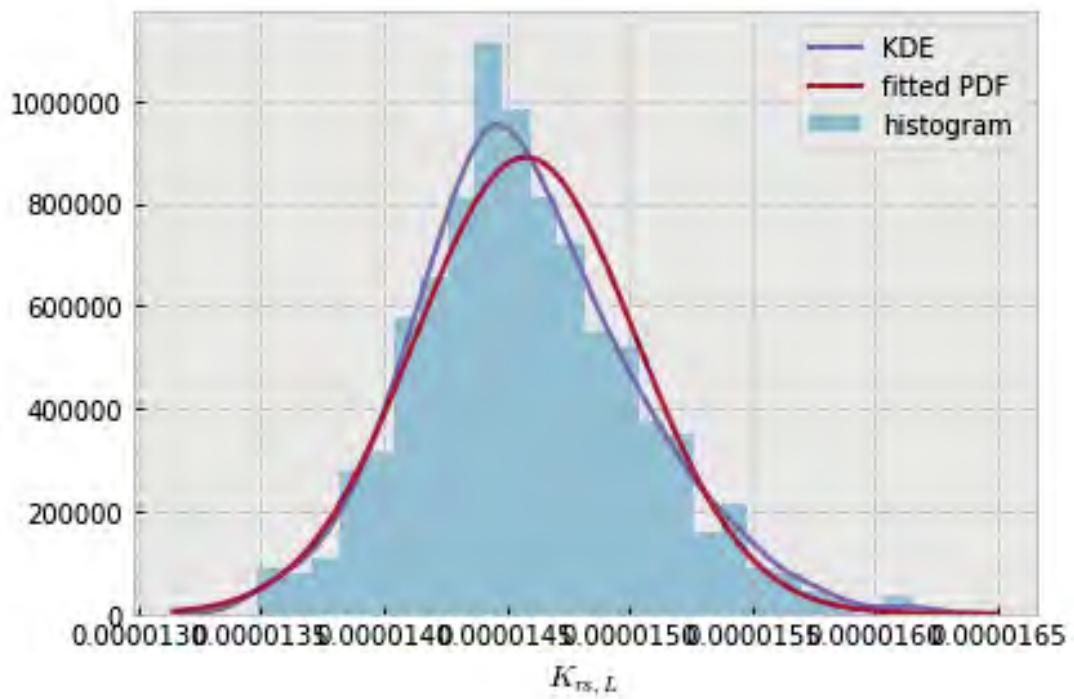


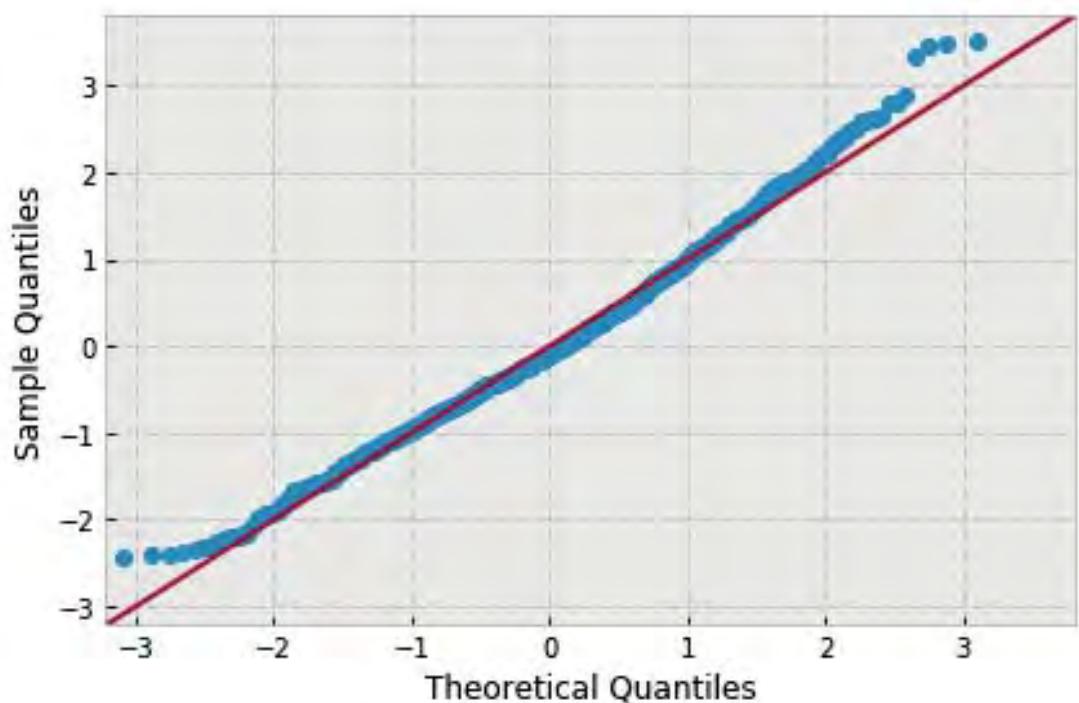


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=1.4566731073761286e-05, scale=4.1673555552502573e-07)
DescribeResult(nobs=1000, minmax=(1.349832190327221e-05, 1.6052638453608122e-05)
               mean=1.4566731073761286e-05, variance=1.7384236560435618e-13,
               skewness=0.3633708829480881, kurtosis=0.3146686478210885)
```

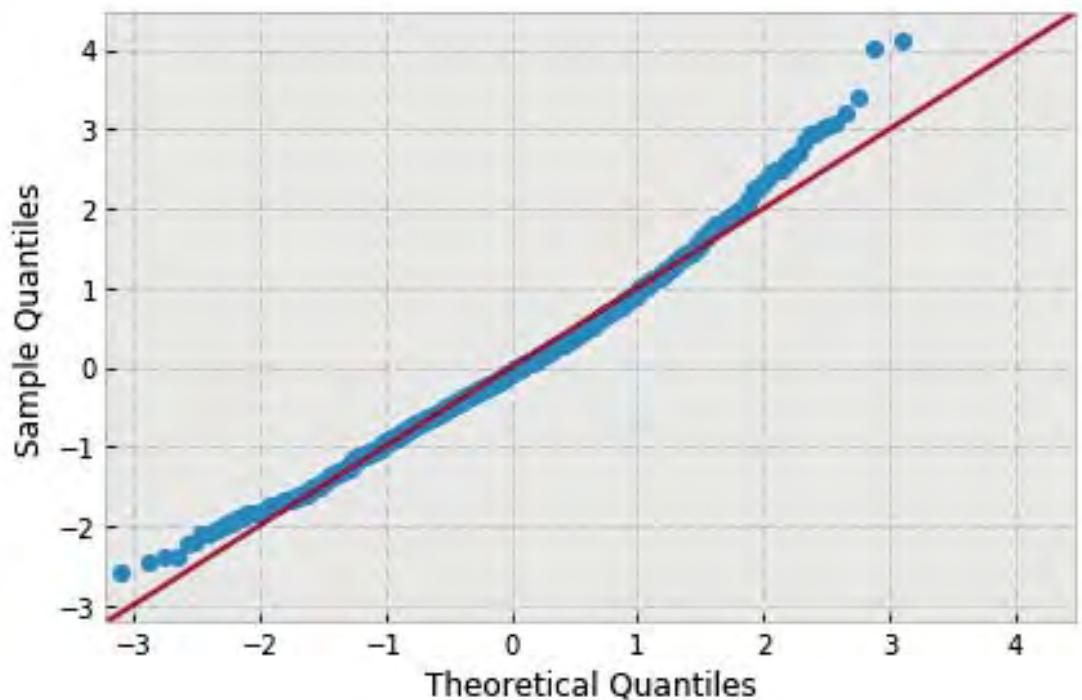
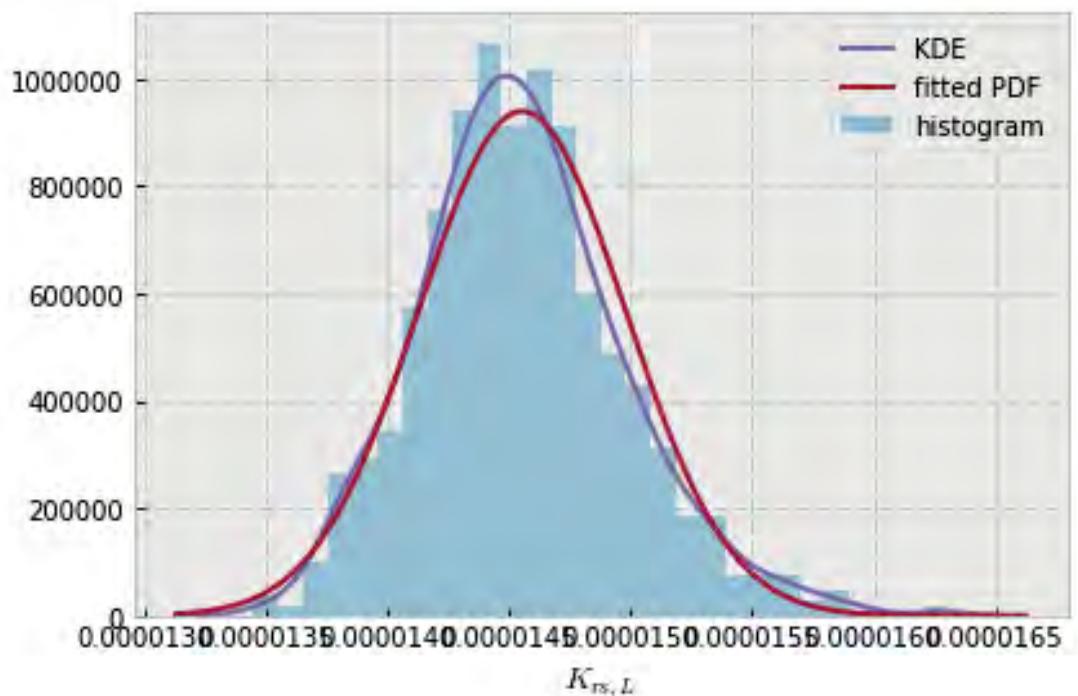


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=1.4578250079662663e-05, scale=4.4815809717361485e-07)
DescribeResult(nobs=1000, minmax=(1.3484142594226503e-05, 1.6148417804135691e-05)
               mean=1.4578250079662663e-05, variance=2.0104672678906428e-13,
               skewness=0.3919089296543544, kurtosis=0.25691796194093364)
```

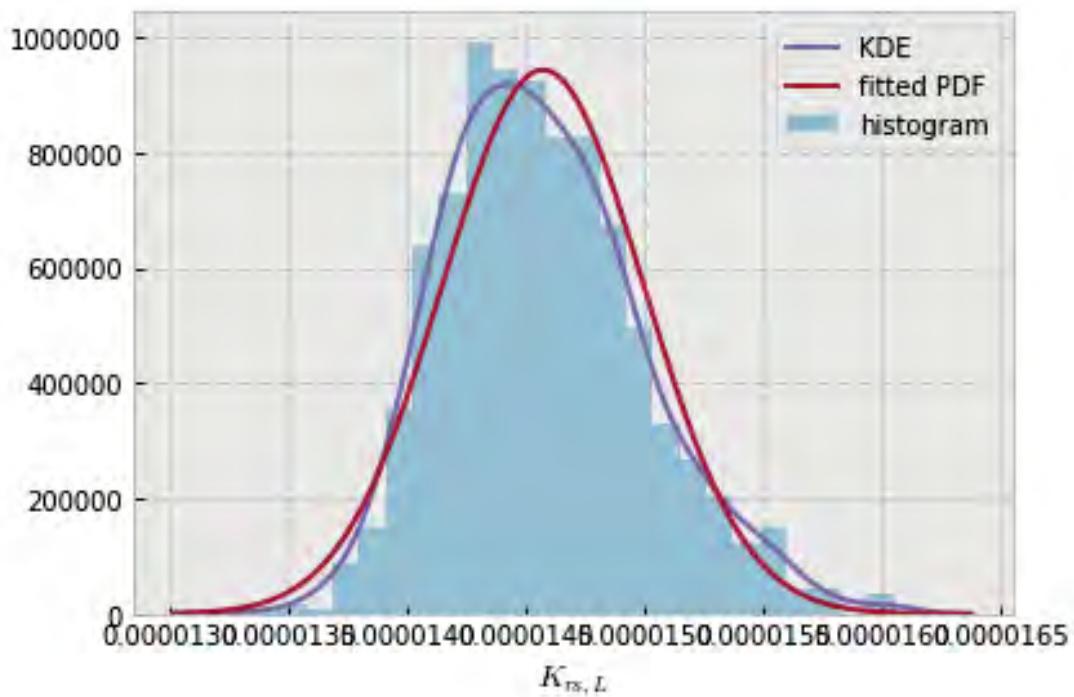


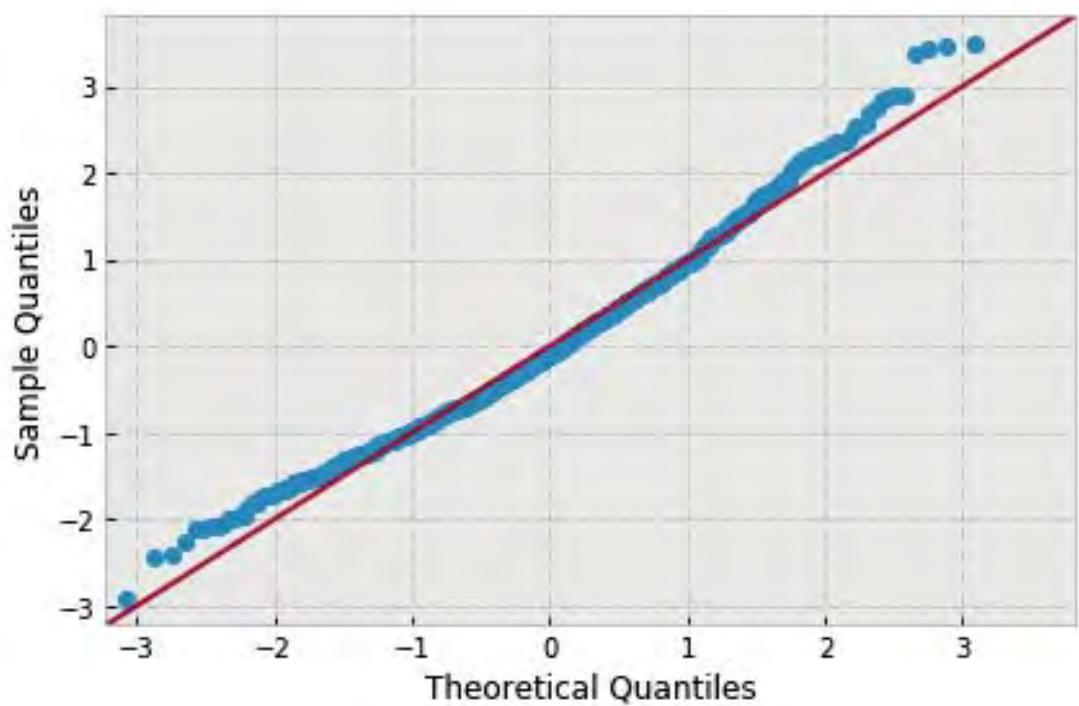


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=1.455327870469303e-05, scale=4.247784238814979e-07)
DescribeResult(nobs=1000, minmax=(1.3449371530534409e-05, 1.6306425835761434e-05)
               mean=1.455327870469303e-05, variance=1.8061732672197146e-13,
               skewness=0.4845745906175721, kurtosis=0.6268595459546042)
```

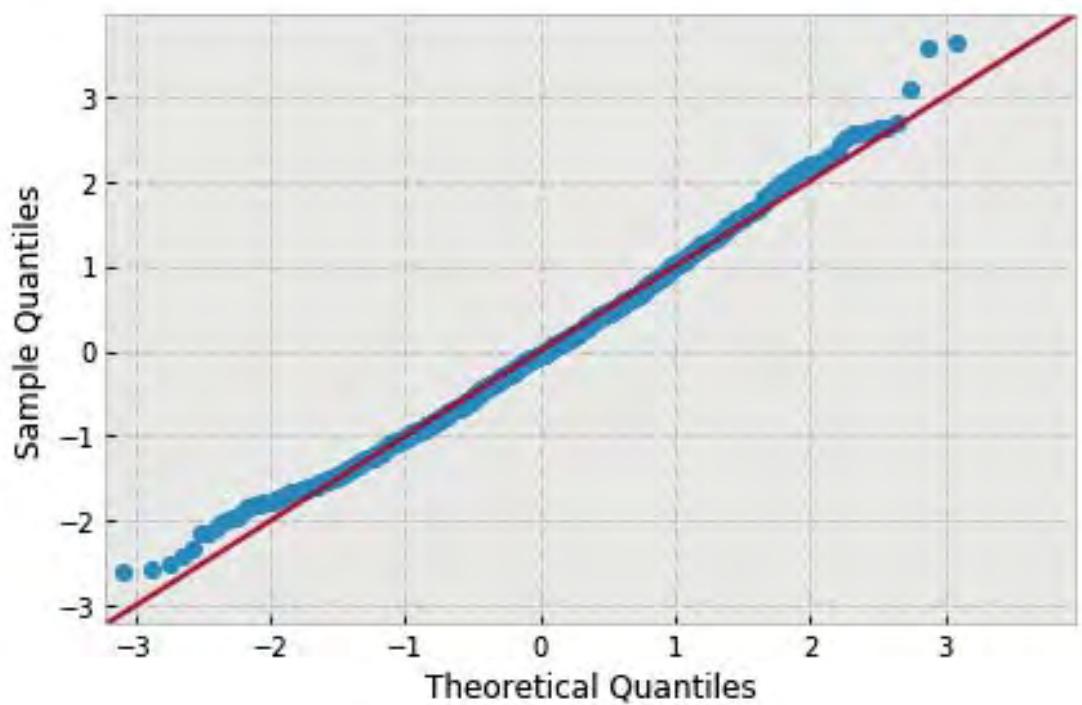
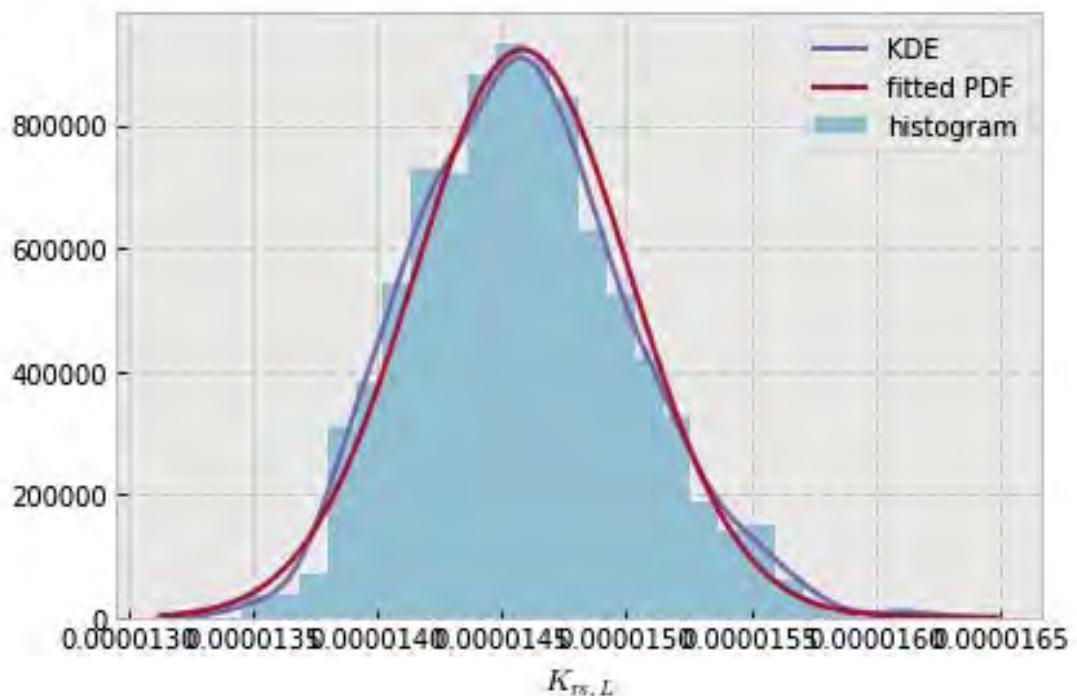


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=1.4575756893158682e-05, scale=4.2313674894724703e-07)
DescribeResult(nobs=1000, minmax=(1.3347250130314593e-05, 1.6053226201163893e-05)
               mean=1.4575756893158682e-05, variance=1.7922393224188744e-13,
               skewness=0.4939766545646995, kurtosis=0.22174189806571798)
```

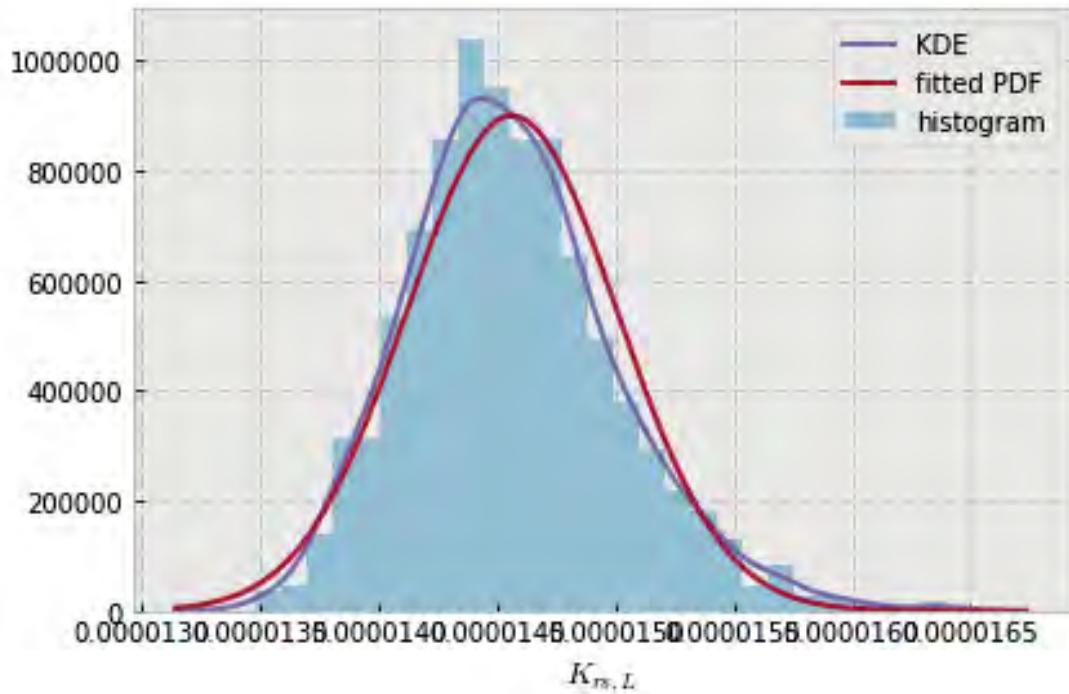


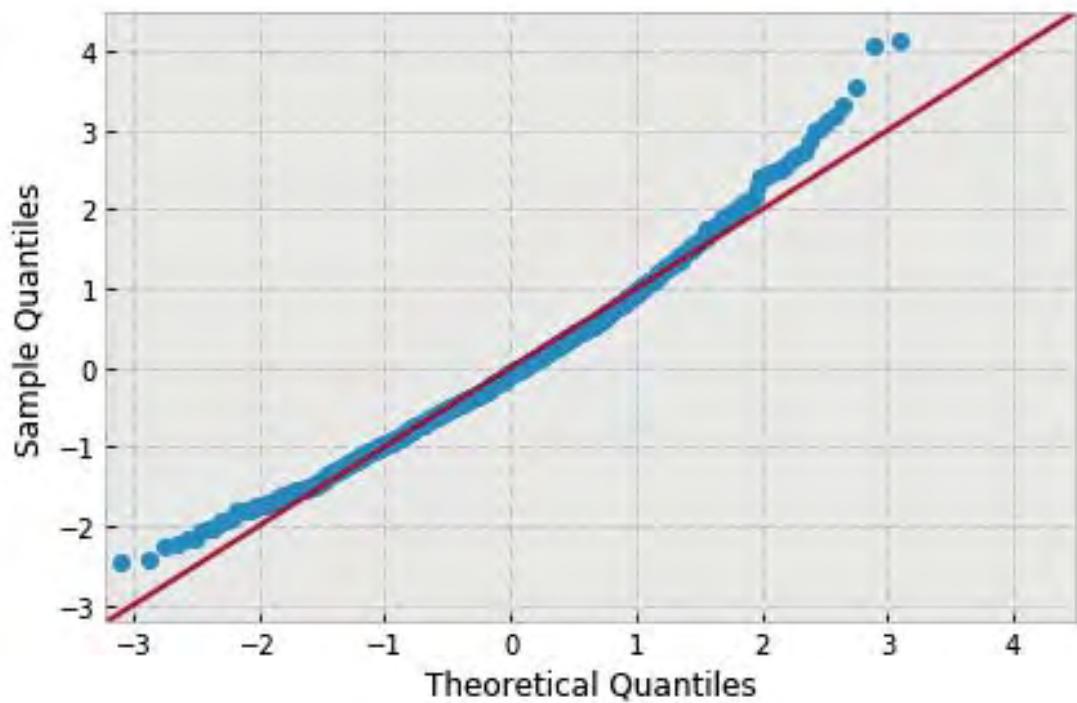


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=1.4582148863396763e-05, scale=4.323638761082828e-07)
DescribeResult(nobs=1000, minmax=(1.3462225160052215e-05, 1.615744683137979e-05)
               mean=1.4582148863396763e-05, variance=1.871256470103889e-13,
               skewness=0.3131439227853409, kurtosis=-0.018648920667504232)
```

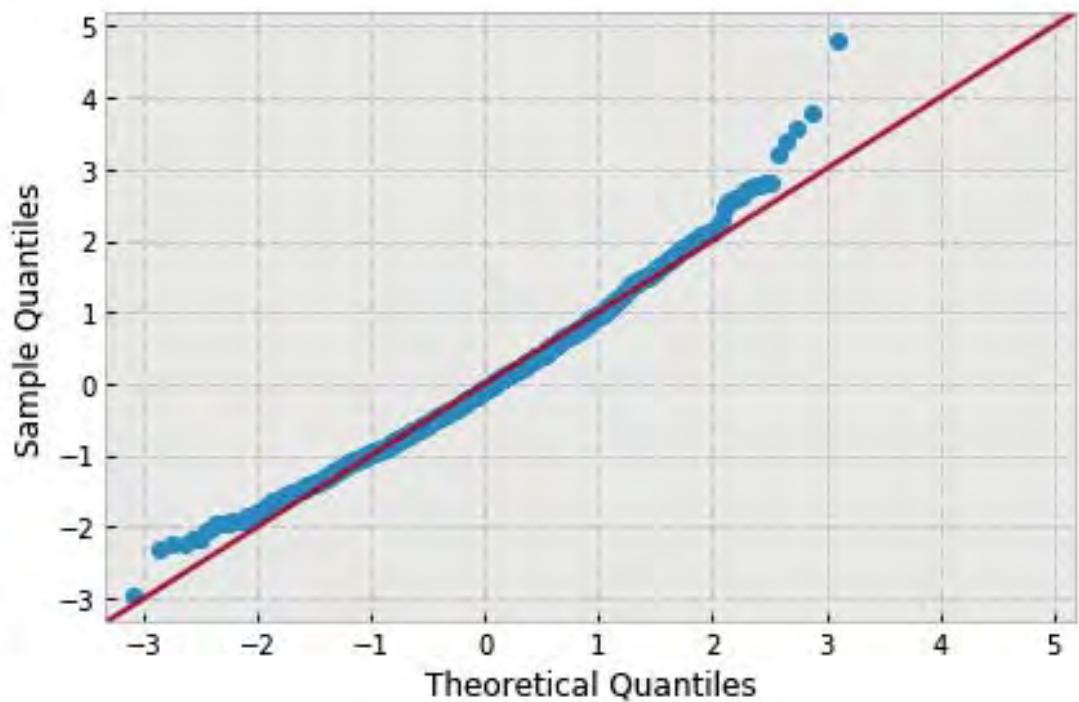
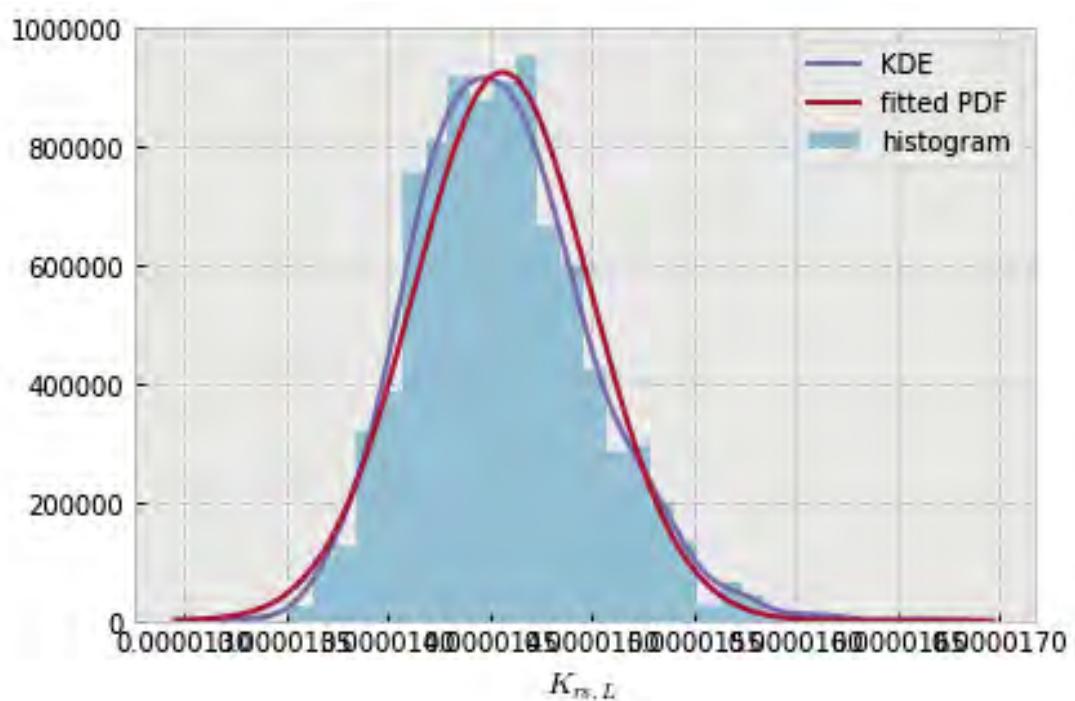


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=1.456323951582273e-05, scale=4.4315929429429894e-07)
DescribeResult(nobs=1000, minmax=(1.3473861853509188e-05, 1.6402298470970632e-05)
               mean=1.4563239515822731e-05, variance=1.9658674686628735e-13,
               skewness=0.5594911768147253, kurtosis=0.6246147438365437)
```

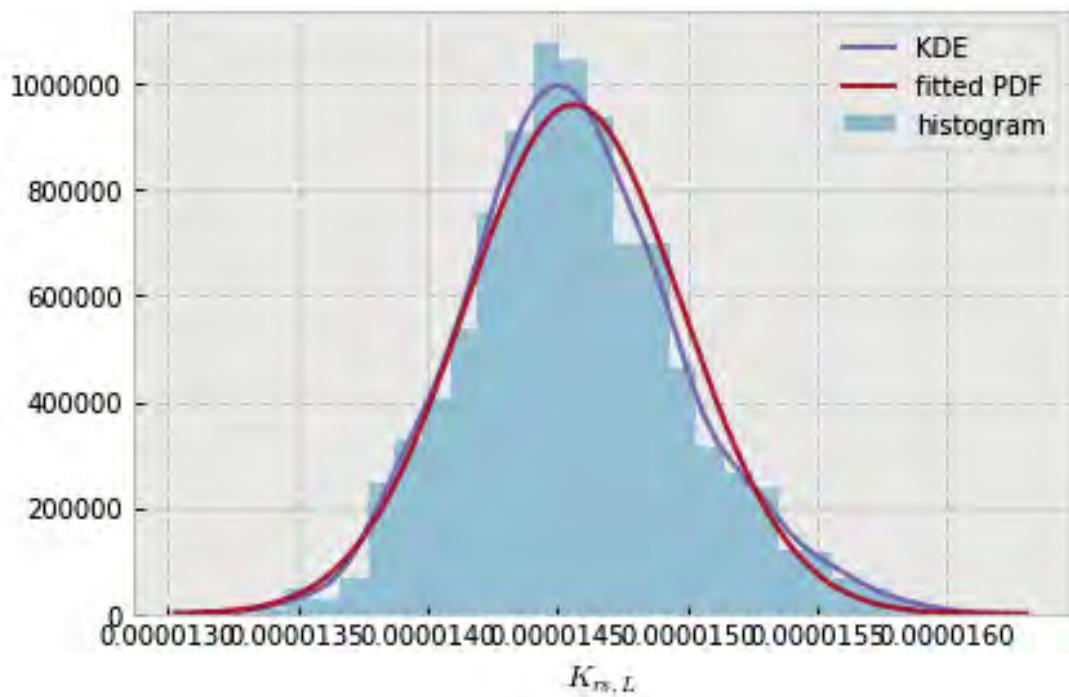


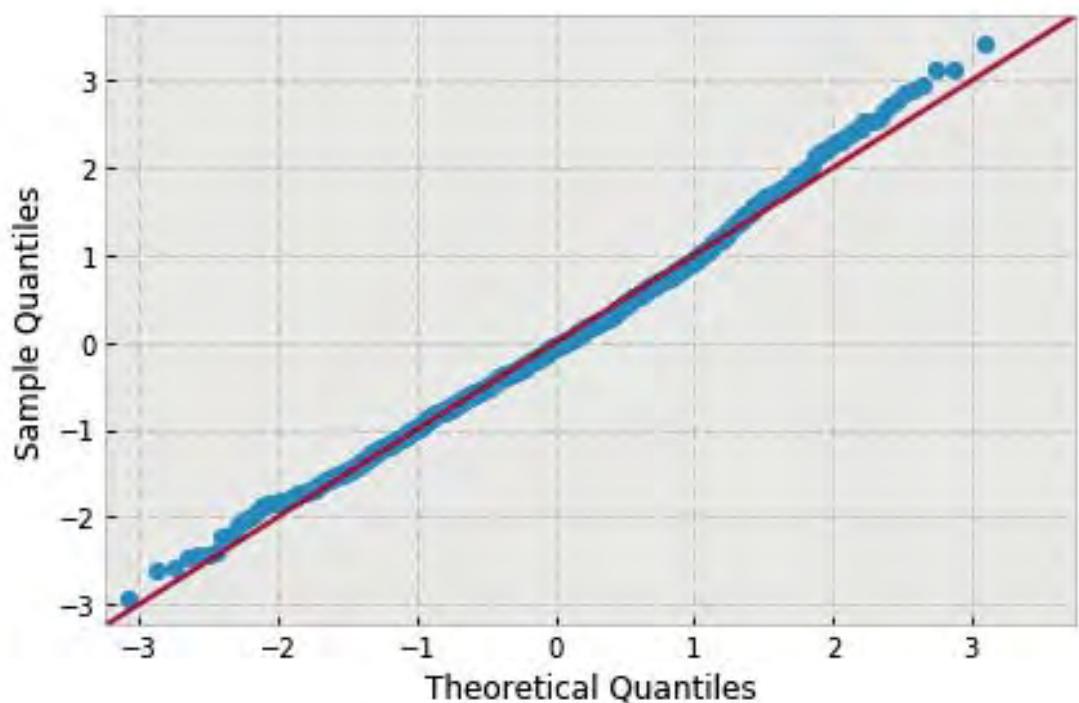


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=1.4562757543972744e-05, scale=4.307511793972551e-07)
DescribeResult(nobs=1000, minmax=(1.3292931626272593e-05, 1.6627378441814238e-05)
               mean=1.4562757543972744e-05, variance=1.8573231086298928e-13,
               skewness=0.5258212647075221, kurtosis=0.6526155187162779)
```

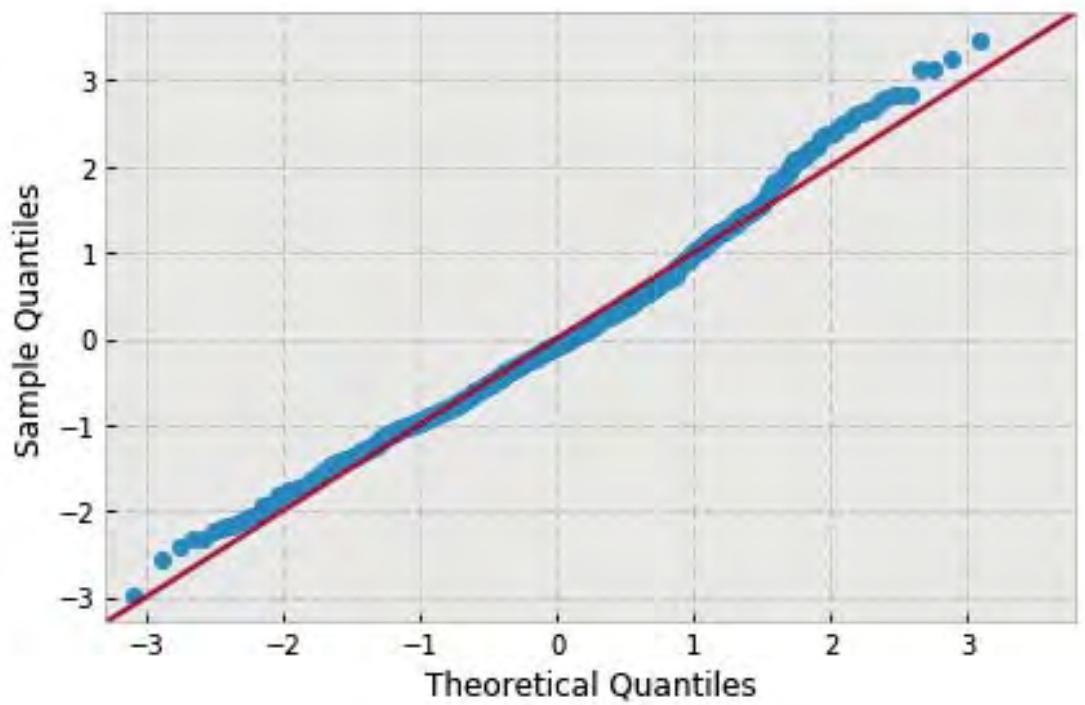
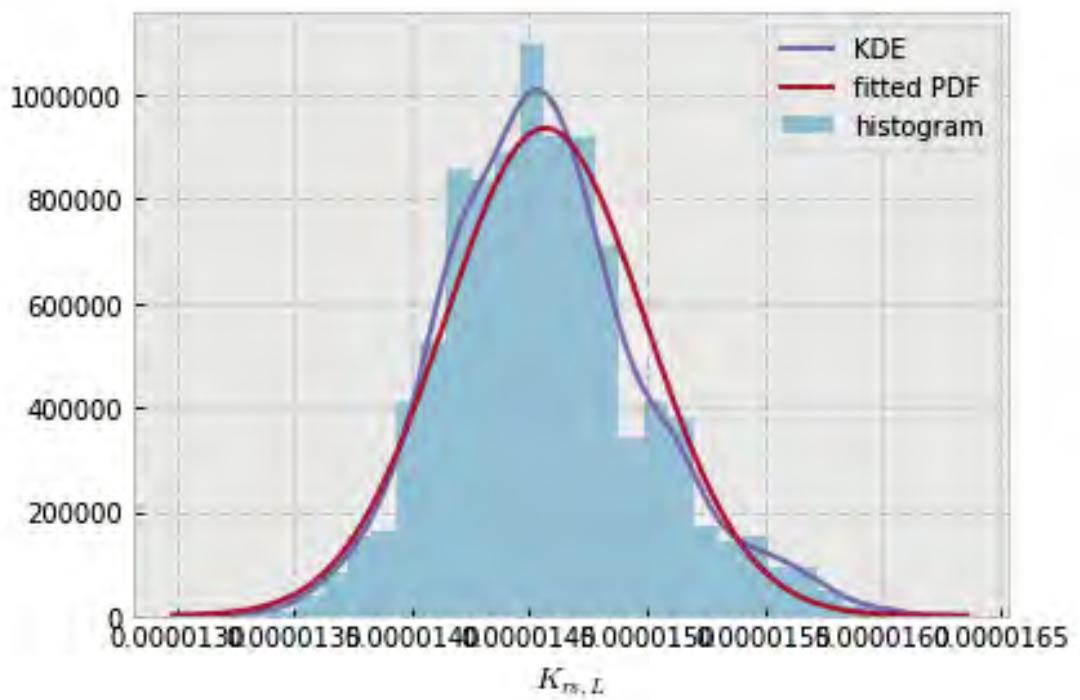


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=1.456278922501739e-05, scale=4.1570617032917763e-07)
DescribeResult(nobs=1000, minmax=(1.3347092089247182e-05, 1.598579622114115e-05)
               mean=1.456278922501739e-05, variance=1.7298460465440568e-13,
               skewness=0.3110945591987144, kurtosis=0.1764296029541237)
```

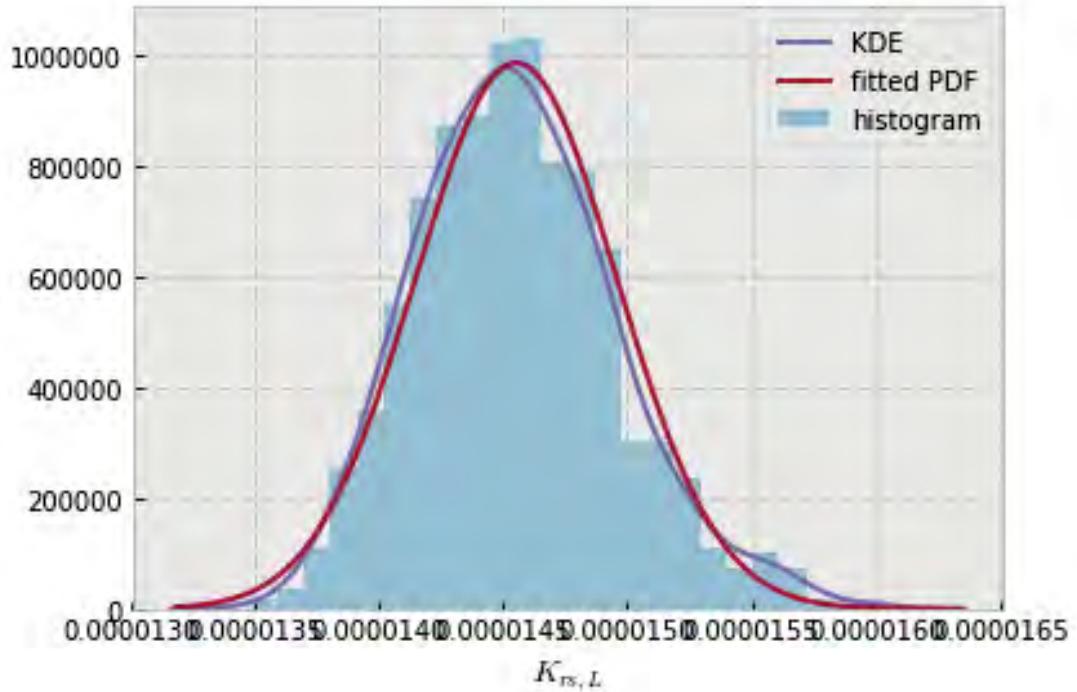


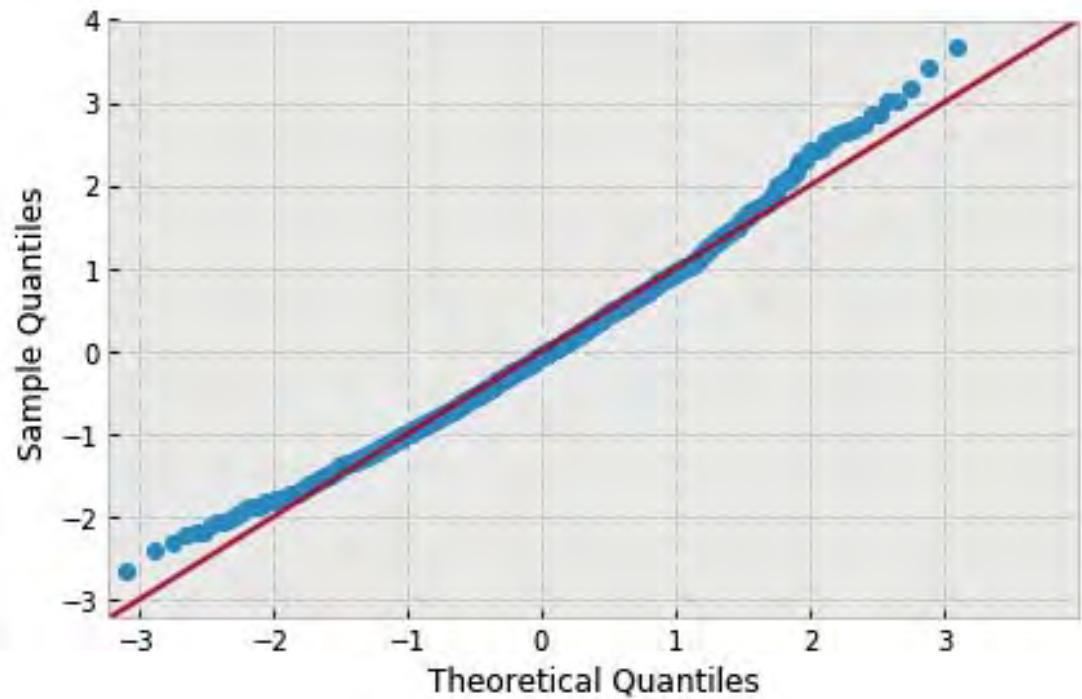


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=1.4567477368375551e-05, scale=4.261057728499397e-07)
DescribeResult(nobs=1000, minmax=(1.3303221402228488e-05, 1.6040149638646671e-05)
               mean=1.4567477368375551e-05, variance=1.81747877533578e-13,
               skewness=0.45407425028374265, kurtosis=0.3478845234640273)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=1.4551050537857814e-05, scale=4.0497374392818366e-07)
DescribeResult(nobs=1000, minmax=(1.3484828726591847e-05, 1.6037841456568082e-05)
               mean=1.4551050537857814e-05, variance=1.6416790117238248e-13,
               skewness=0.42229903686757153, kurtosis=0.23541640689183163)
```

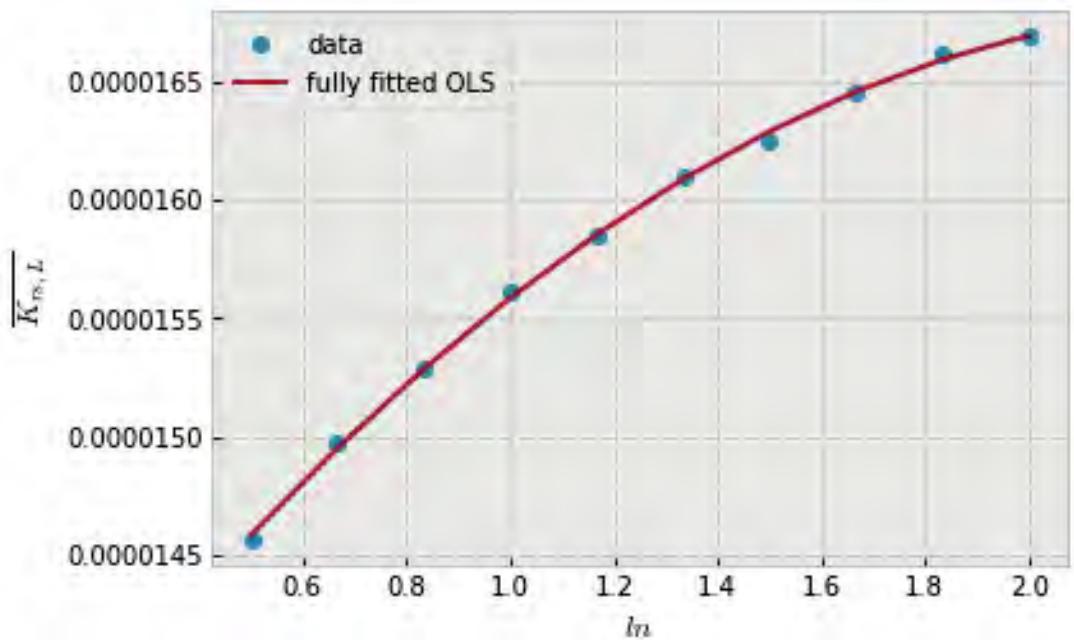




17.2 Parameter regression

```
In [40]: var_name = 'root_system_conductance_per_unit_of_root_area'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly1')
mean_reg('nB', var_name, 'poly2')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly1')
mean_reg('N', var_name, 'poly1')

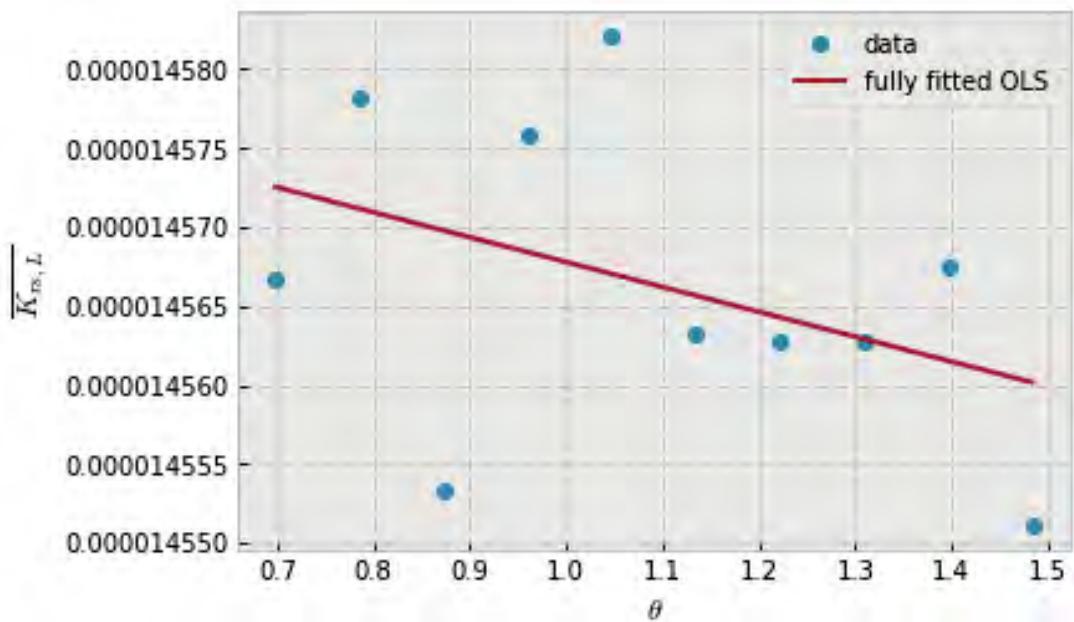
variable name: ln
model: poly2
r-squared: 0.9977542868308985
explained variance: 0.9977713444608118
```



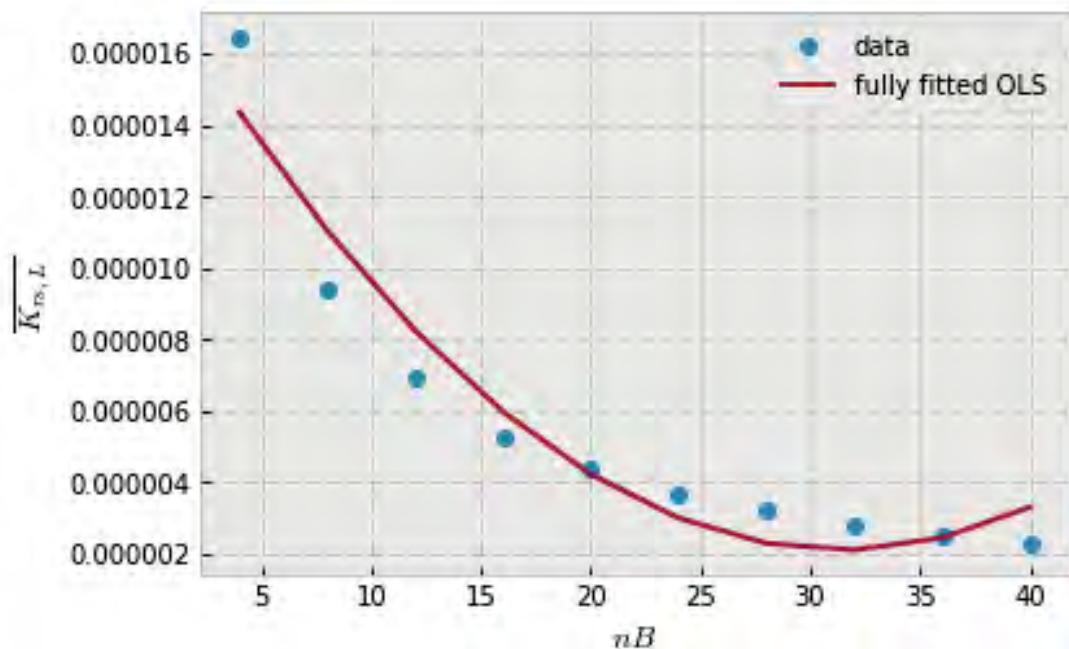
```

variable name: theta
model: poly1
r-squared: -0.29169053129454214
explained variance: -0.2891877681493418

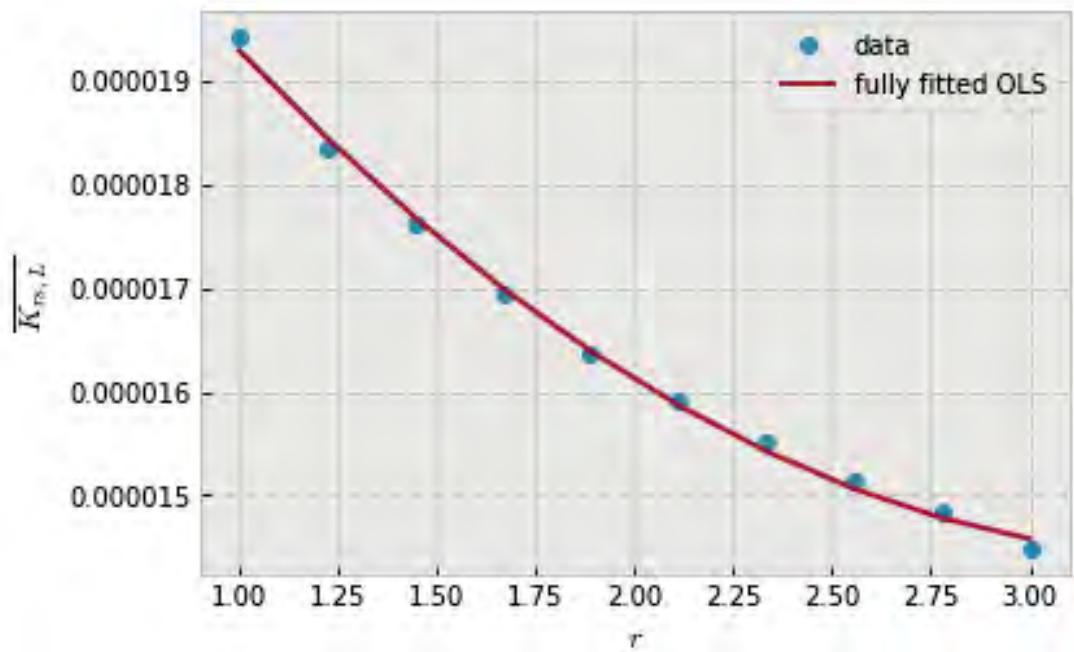
```



```
variable name: nB
model: poly2
r-squared: 0.7161169664203739
explained variance: 0.7171816027791207
```



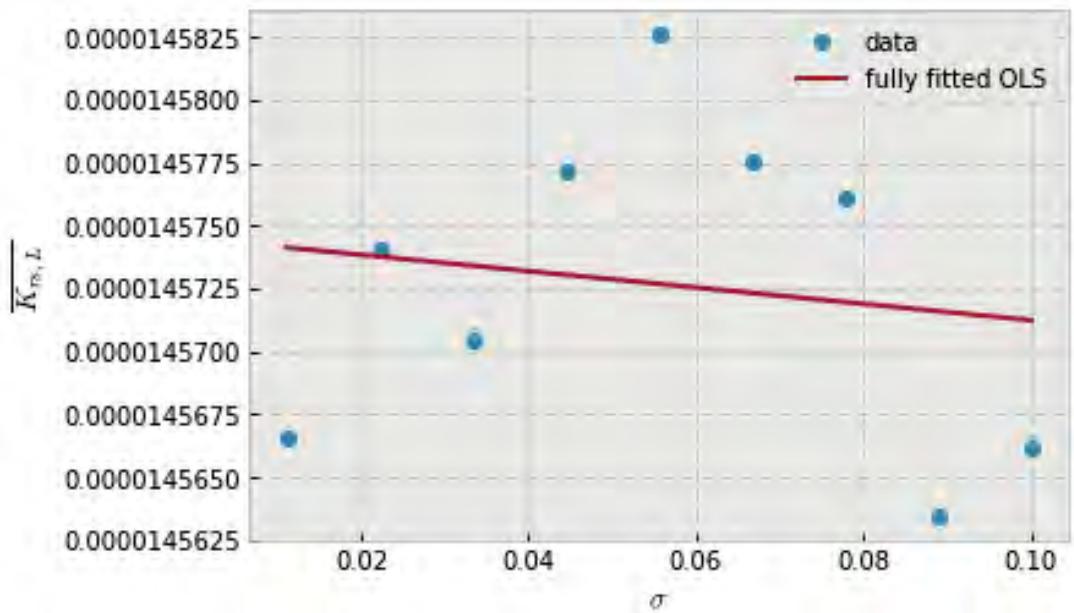
```
variable name: r
model: poly2
r-squared: 0.9885906835425085
explained variance: 0.9885972868340237
```



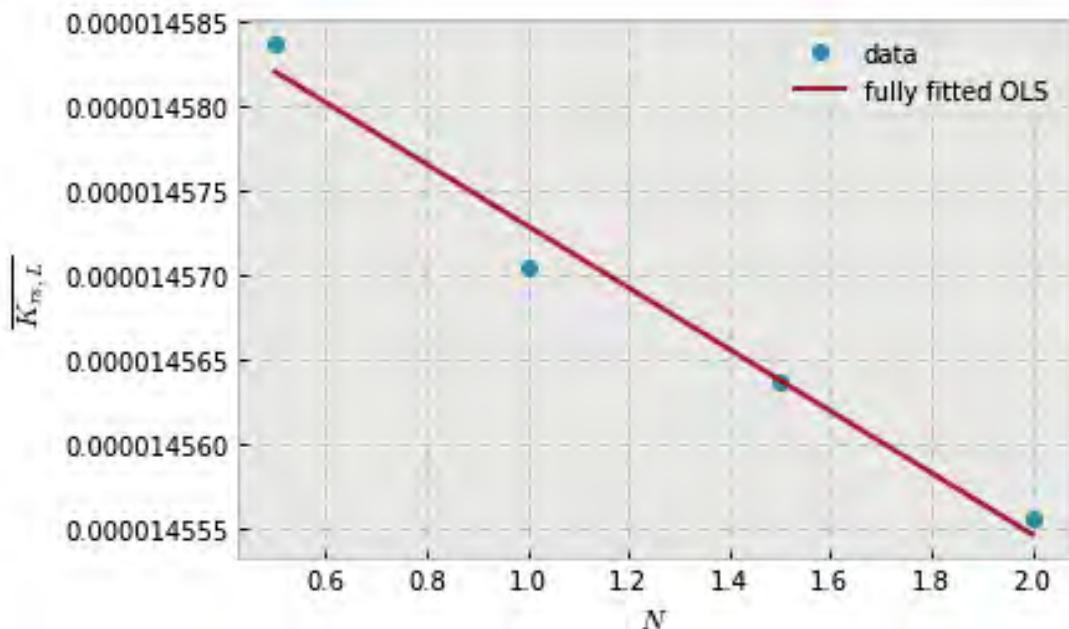
```

variable name: sigma
model: poly1
r-squared: -0.7073216318264504
explained variance: -0.6878392918876544

```



```
variable name: N
model: poly1
r-squared: 0.8752426000002083
explained variance: 0.8894018813789889
```

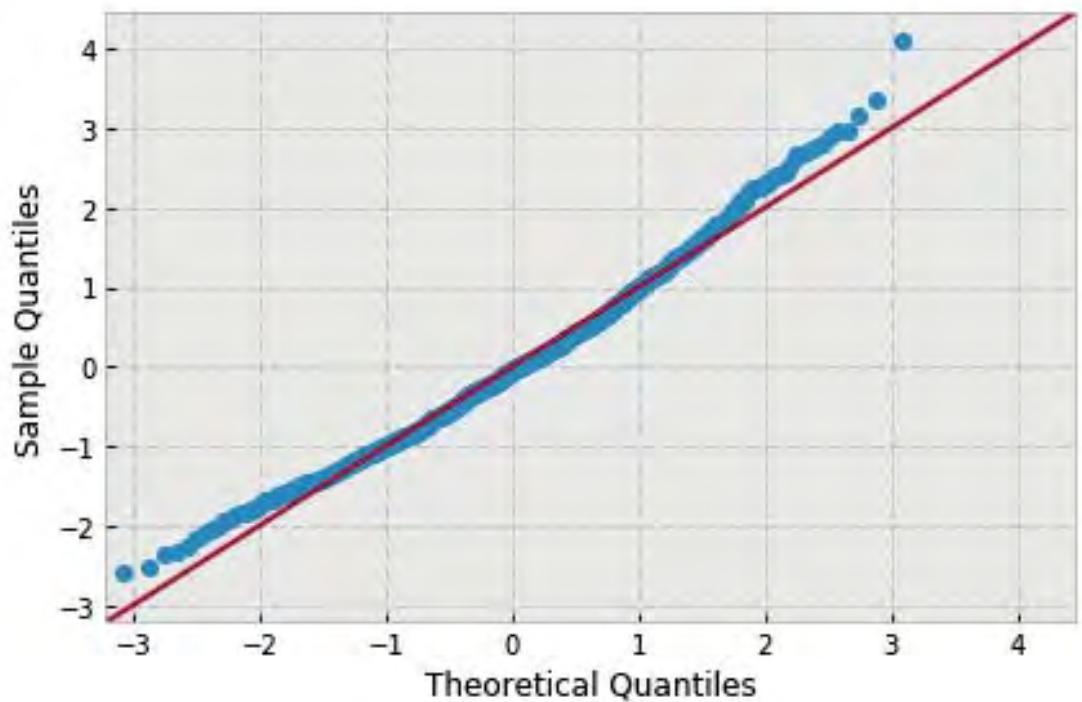
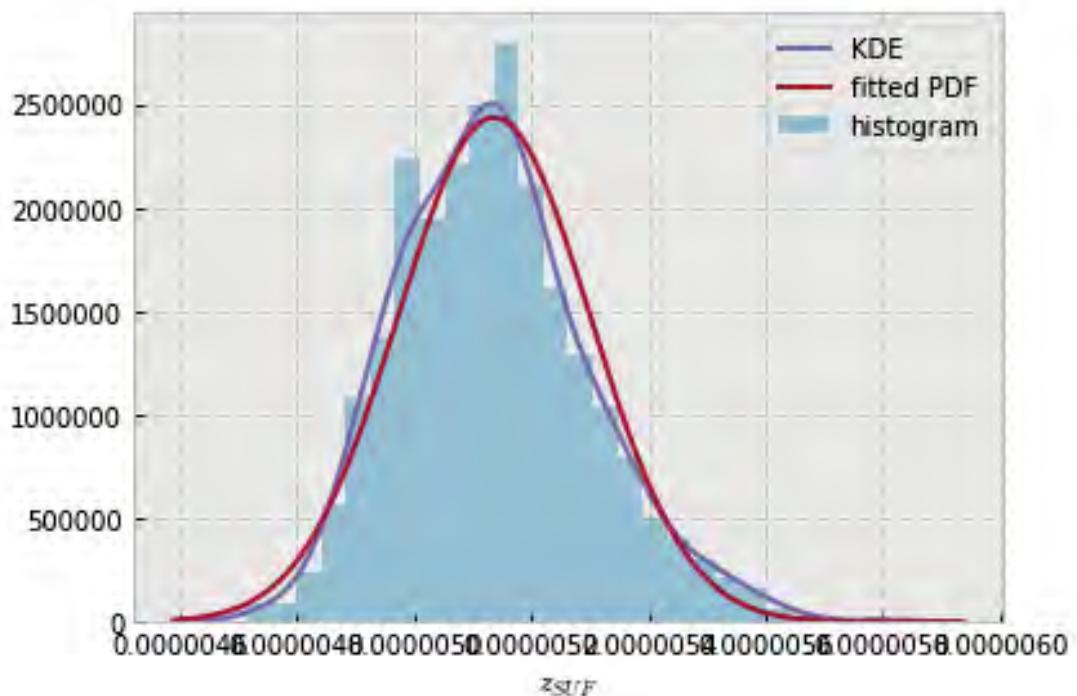


18 Root system conductance per unit of root length

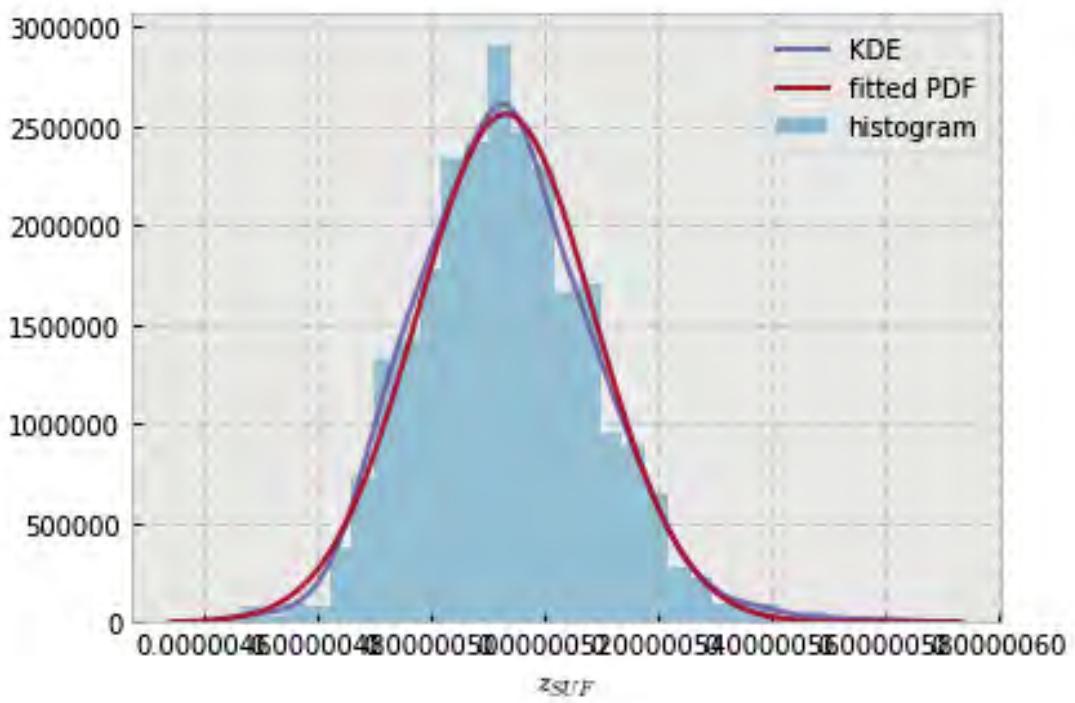
18.1 Probability distributions

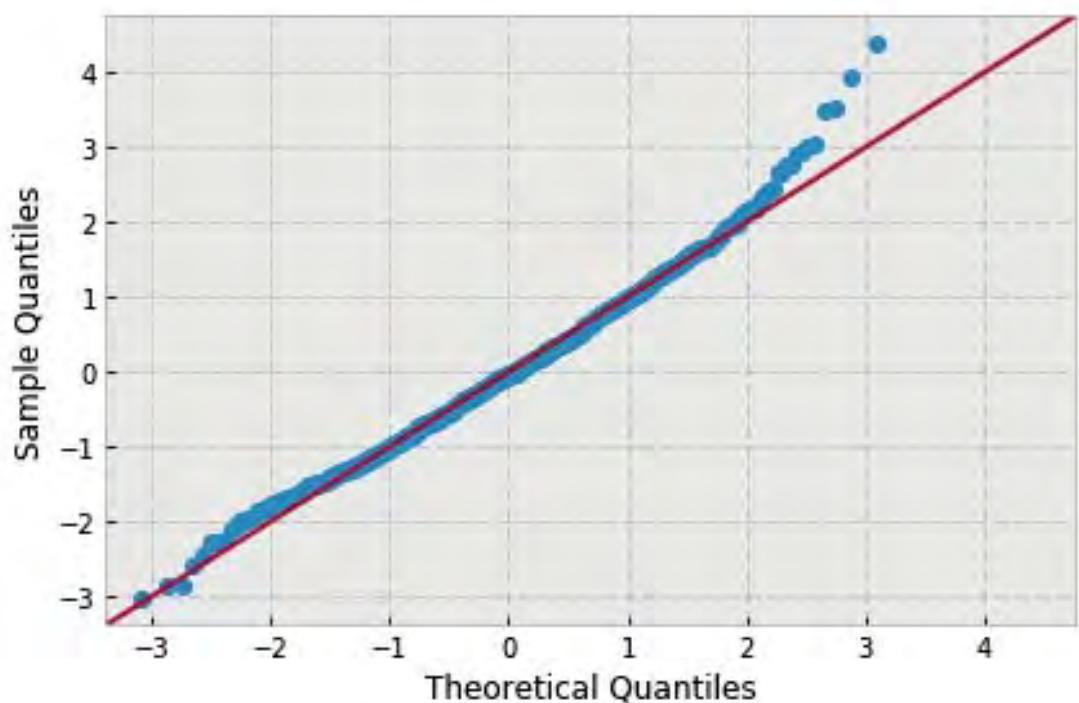
```
In [41]: plot_dist('root_system_conductance_per_unit_of_root_length')

variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=5.136819372335771e-06, scale=1.6377379022419562e-07)
DescribeResult(nobs=1000, minmax=(4.7147925254501379e-06, 5.8109222948429582e-06)
               mean=5.1368193723357707e-06, variance=2.6848703067466301e-14,
               skewness=0.4653443924060871, kurtosis=0.2860396292813454)
```

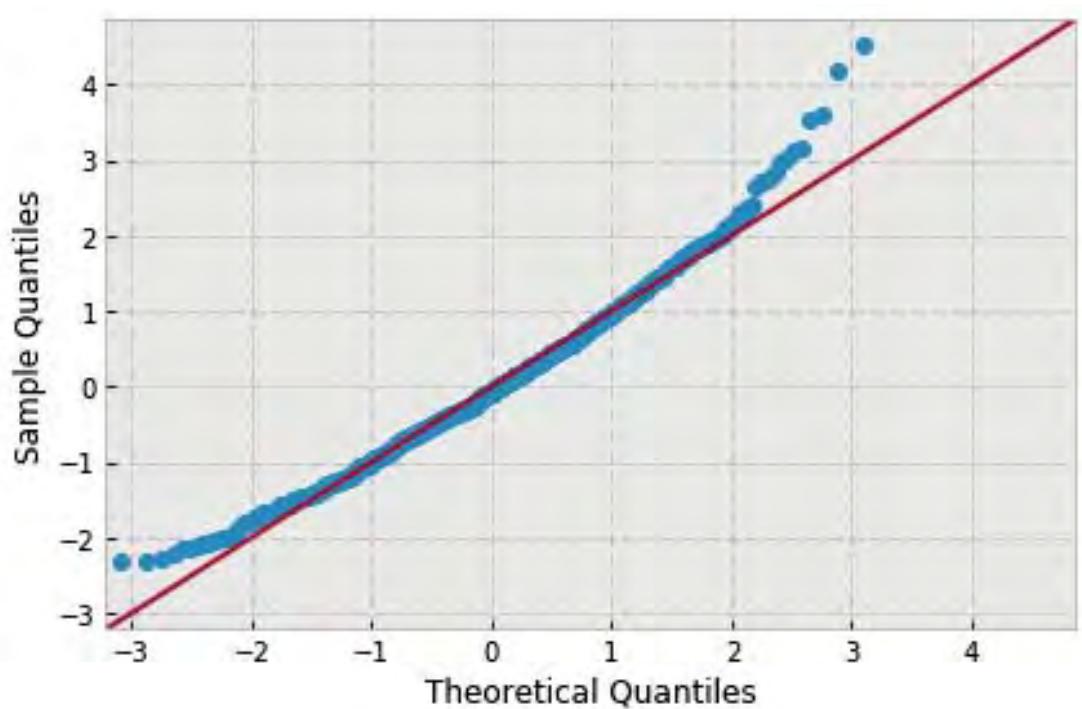
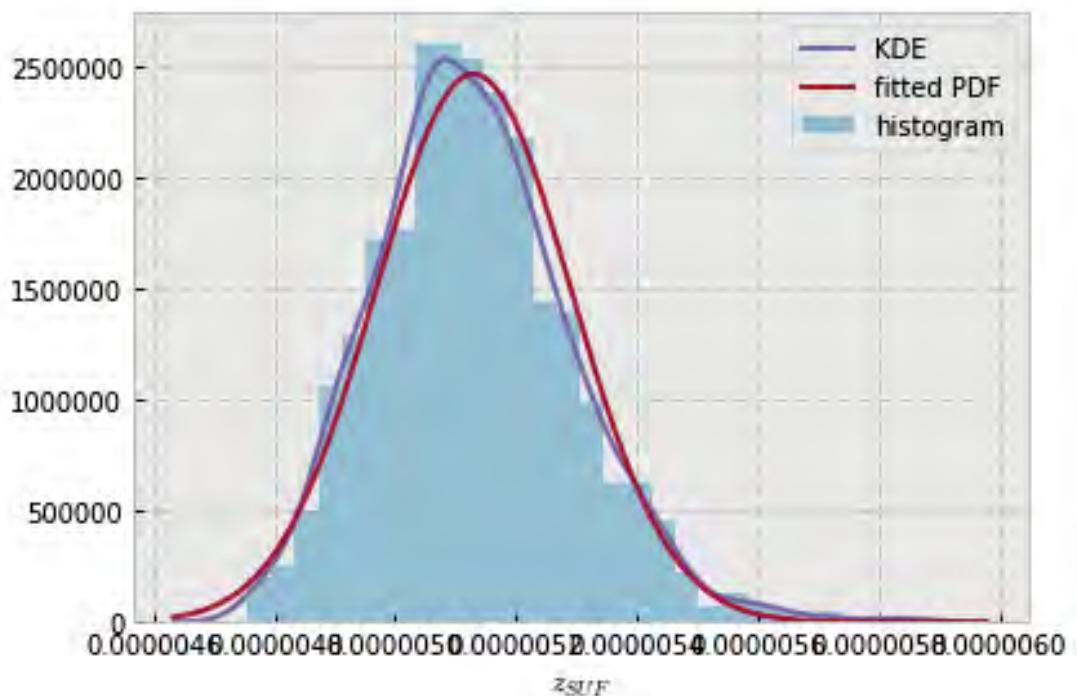


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=5.1321792966375575e-06, scale=1.5579756382277266e-07)
DescribeResult(nobs=1000, minmax=(4.6625274550788545e-06, 5.8148918725387936e-06)
               mean=5.1321792966375575e-06, variance=2.4297178071182105e-14,
               skewness=0.3637355198523635, kurtosis=0.5752954685499696)
```

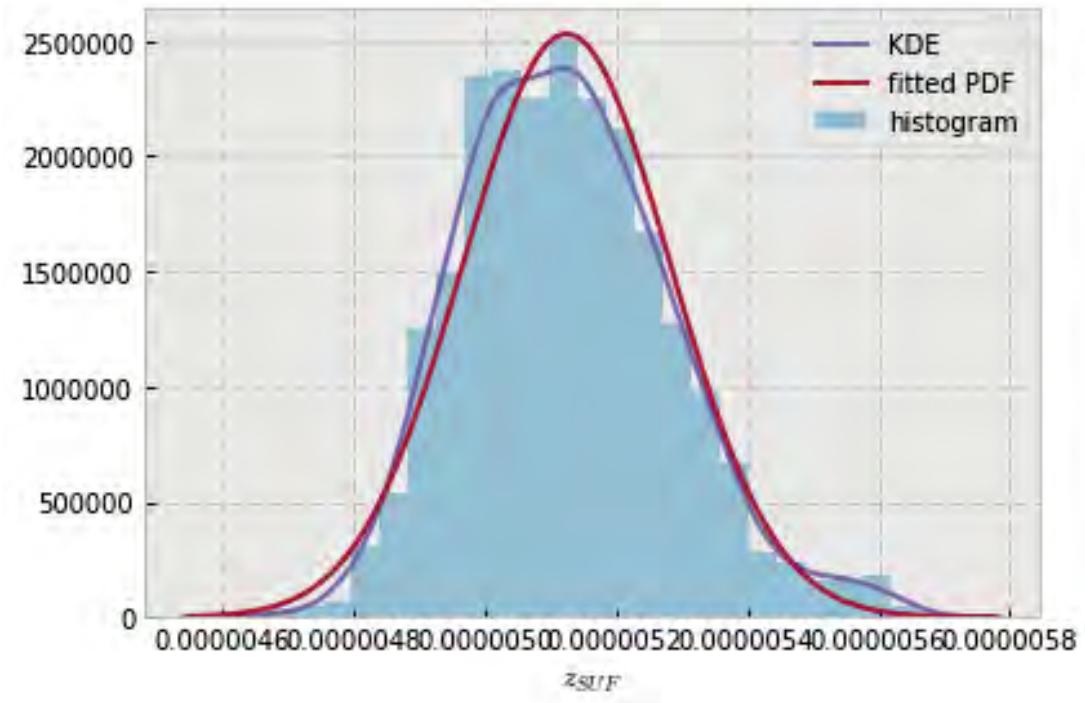


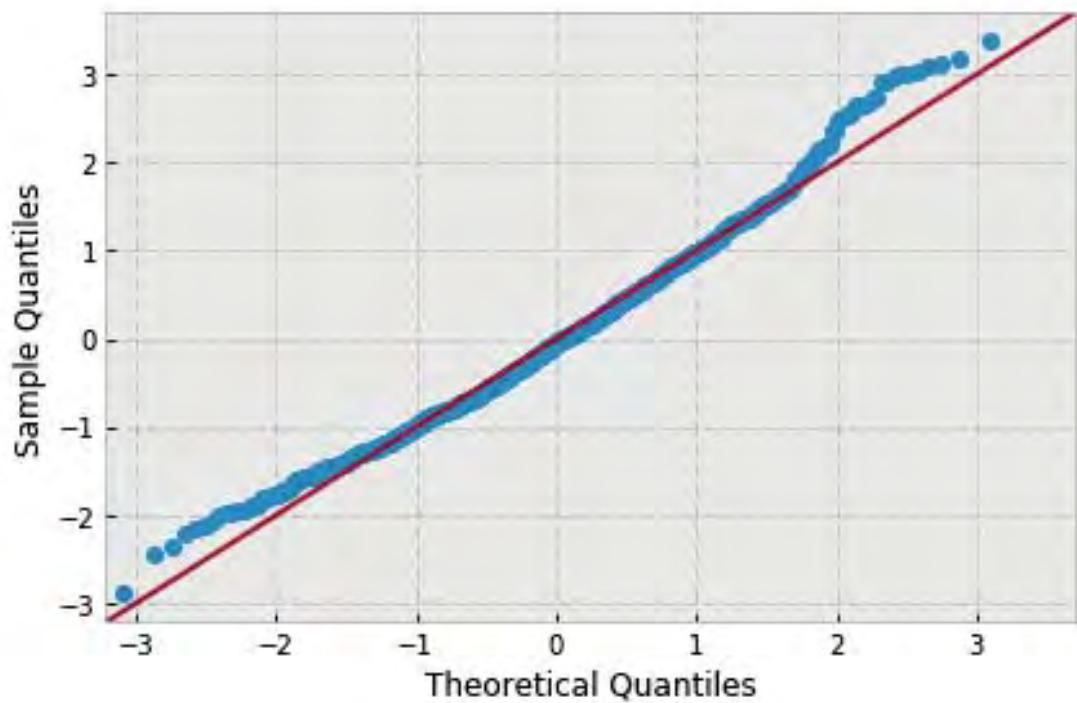


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=5.1294840153056306e-06, scale=1.615435667892132e-07)
DescribeResult(nobs=1000, minmax=(4.7543547503384262e-06, 5.857982585595107e-06)
               mean=5.1294840153056306e-06, variance=2.6122446417398392e-14,
               skewness=0.5109411045429784, kurtosis=0.6796802638568589)
```

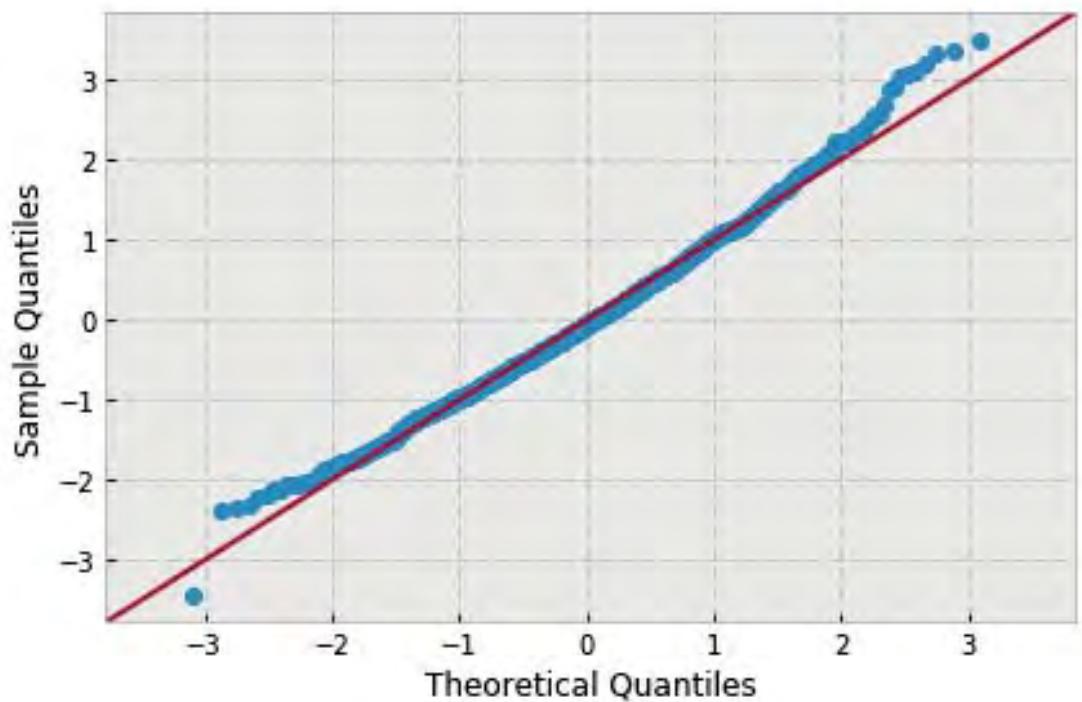
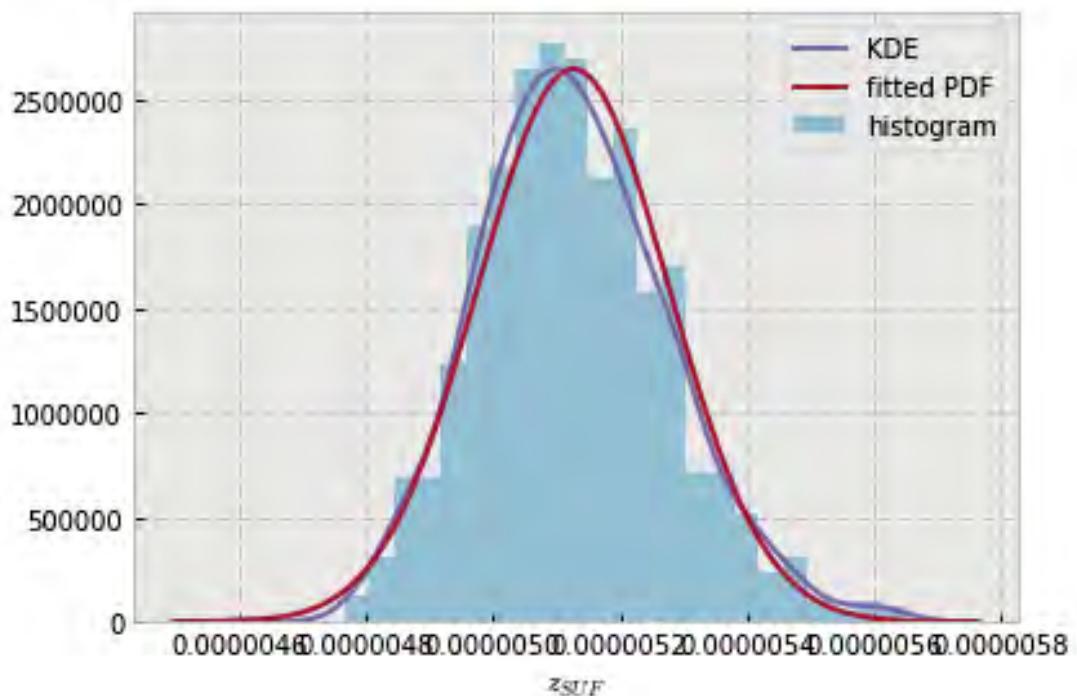


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=5.125696671632045e-06, scale=1.5765948054919972e-07)
DescribeResult(nobs=1000, minmax=(4.6702456767972925e-06, 5.6587561774700072e-06)
               mean=5.1256966716320452e-06, variance=2.4881393200243728e-14,
               skewness=0.4440172442509141, kurtosis=0.21127305592133583)
```

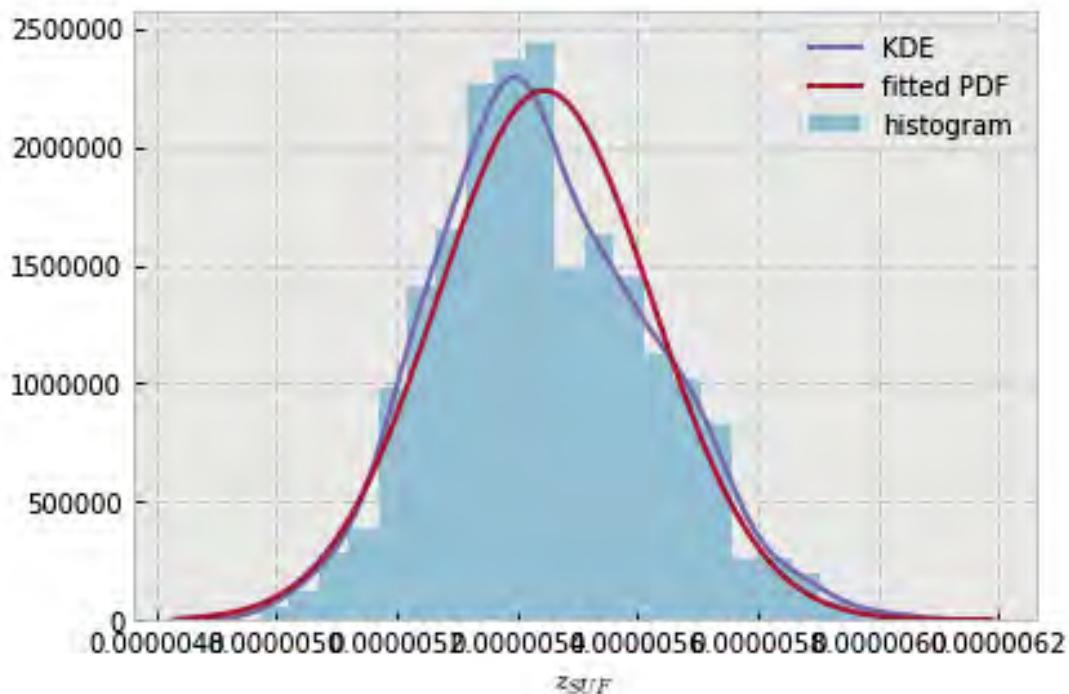


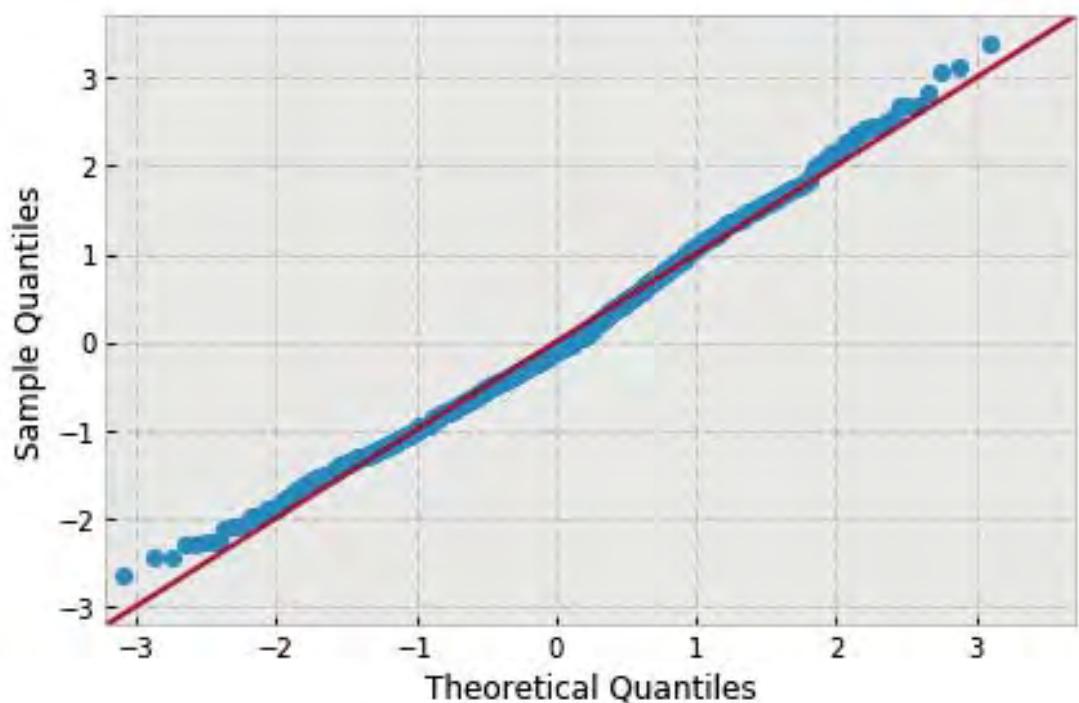


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=5.126280215435601e-06, scale=1.5076991421938794e-07)
DescribeResult(nobs=1000, minmax=(4.609777306795153e-06, 5.6510449214591067e-06)
               mean=5.1262802154356014e-06, variance=2.2754321355076674e-14,
               skewness=0.3437870194693273, kurtosis=0.26538366004230696)
```

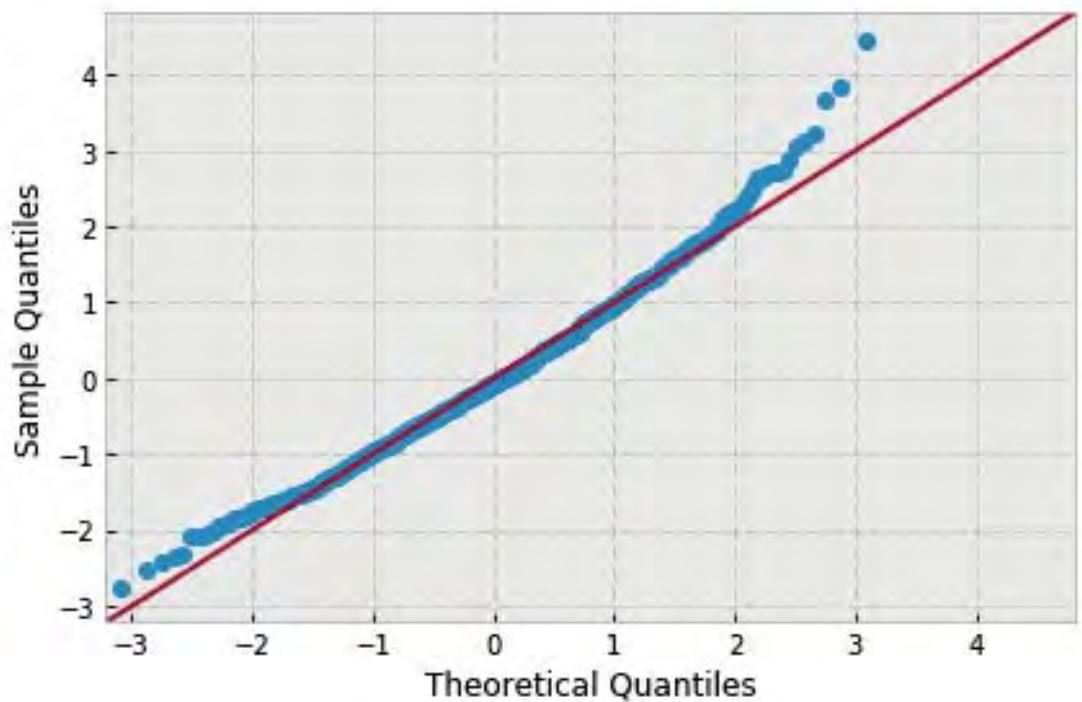
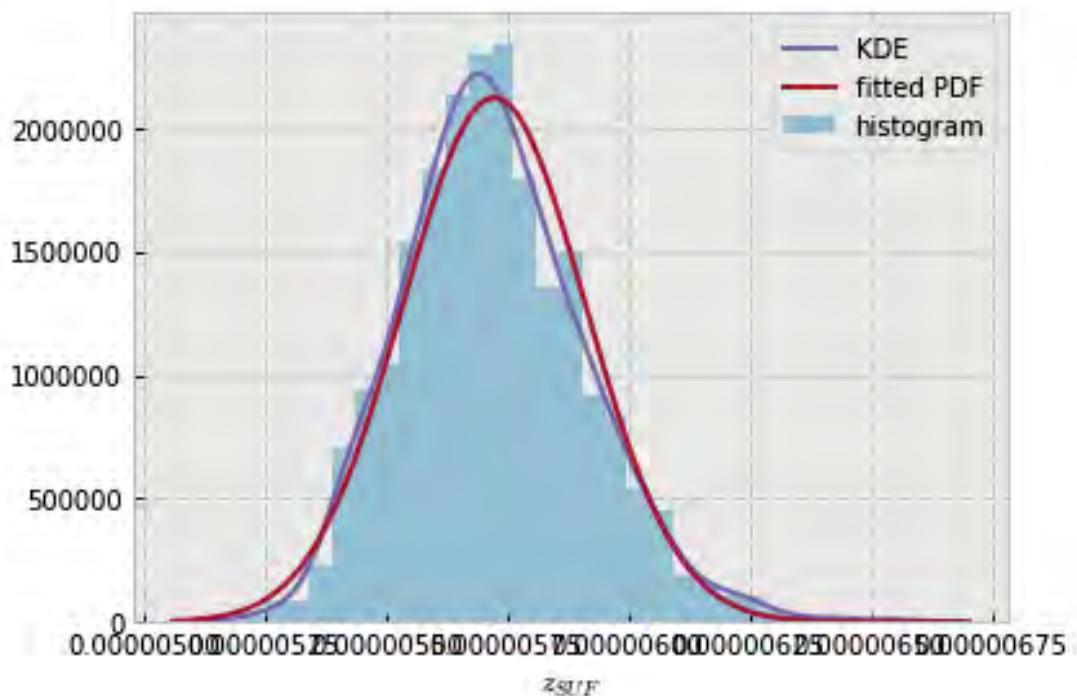


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=5.445544107463372e-06, scale=1.7836801277719195e-07)
DescribeResult(nobs=1000, minmax=(4.9720674710276138e-06, 6.0514157602307875e-06)
               mean=5.4455441074633717e-06, variance=3.1846994977061575e-14,
               skewness=0.3047367942813329, kurtosis=-0.13427034740172195)
```

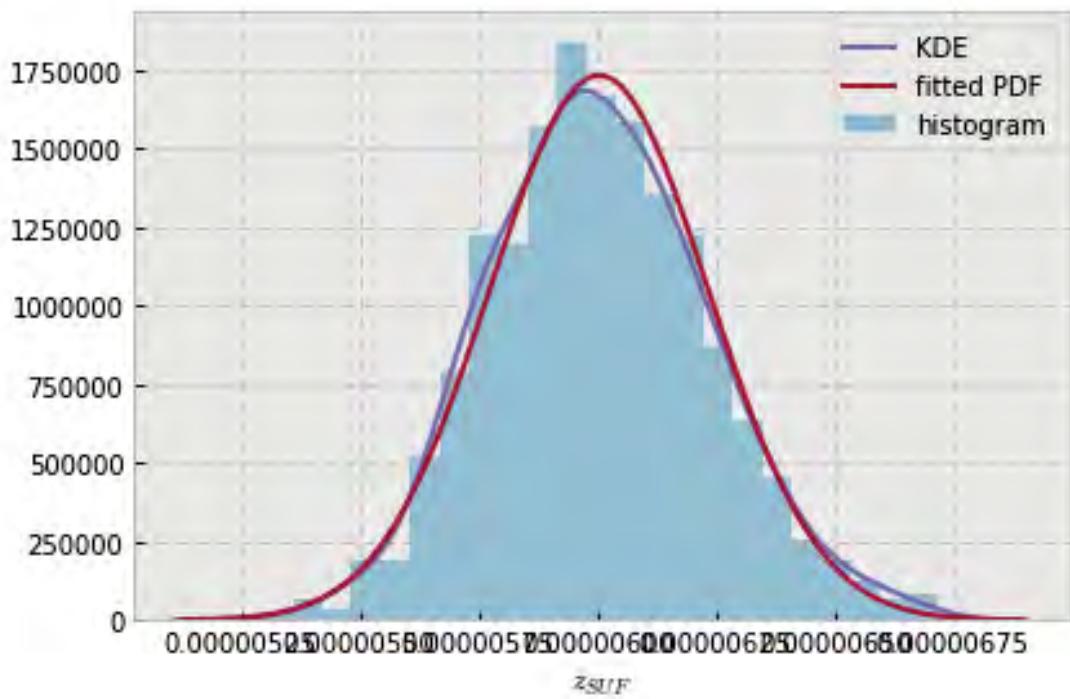


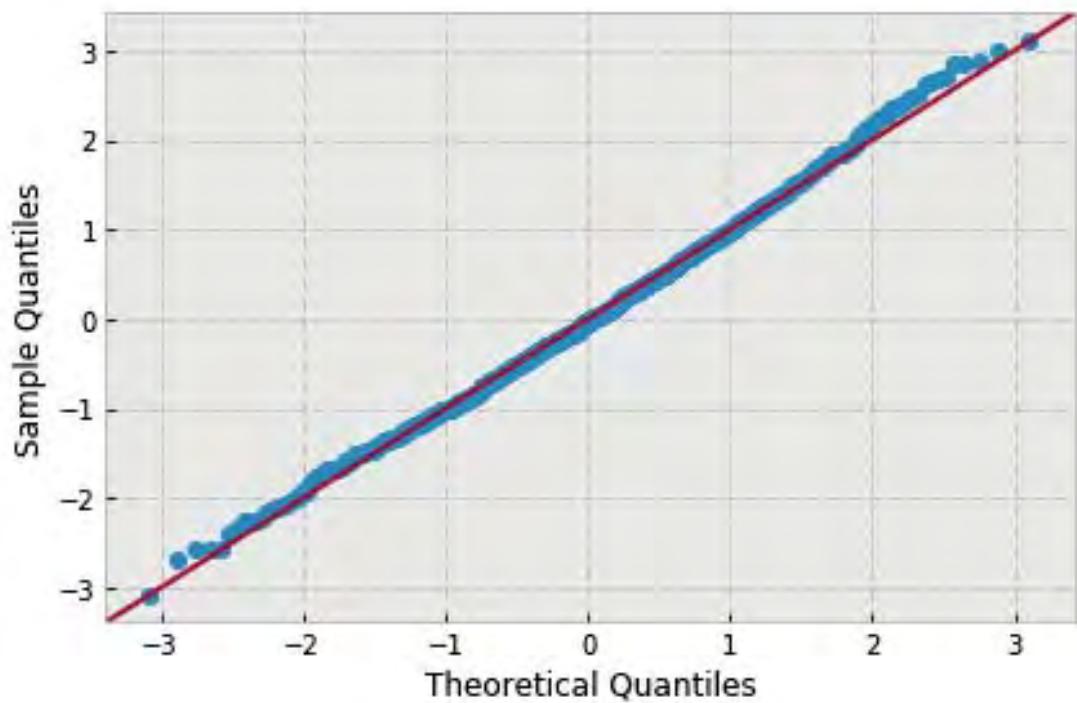


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=5.723841281099328e-06, scale=1.8808181909779862e-07)
DescribeResult(nobs=1000, minmax=(5.20192822350784e-06, 6.5613745885937651e-06)
               mean=5.7238412810993276e-06, variance=3.5410180855993041e-14,
               skewness=0.46842431477947866, kurtosis=0.5703525447754028)
```

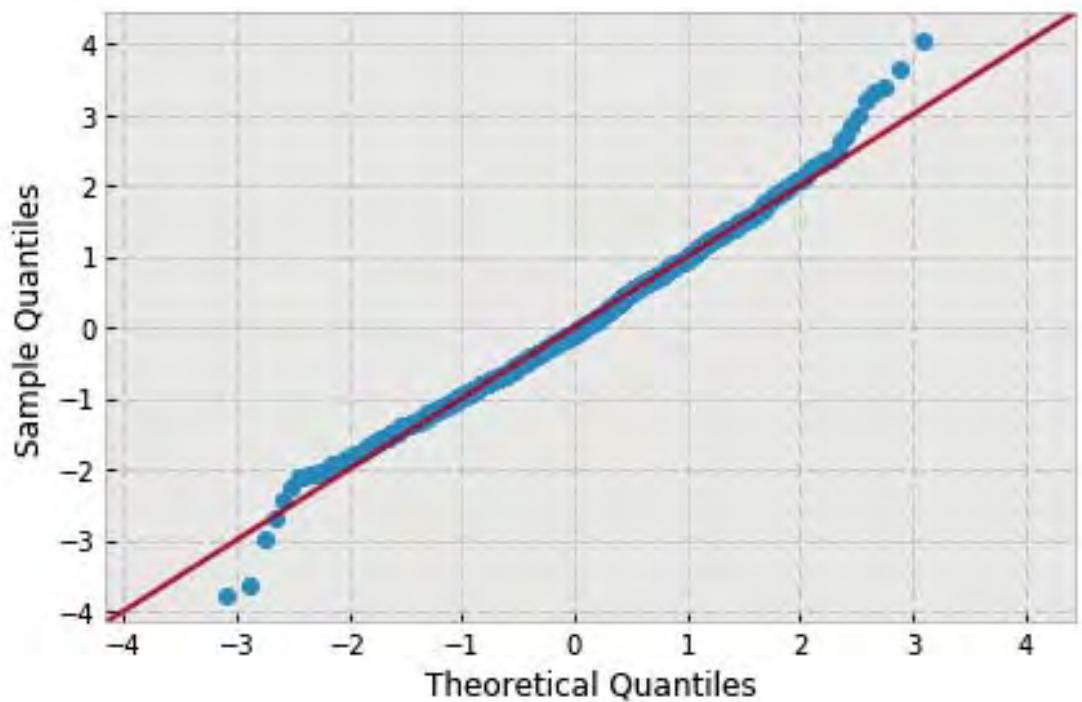
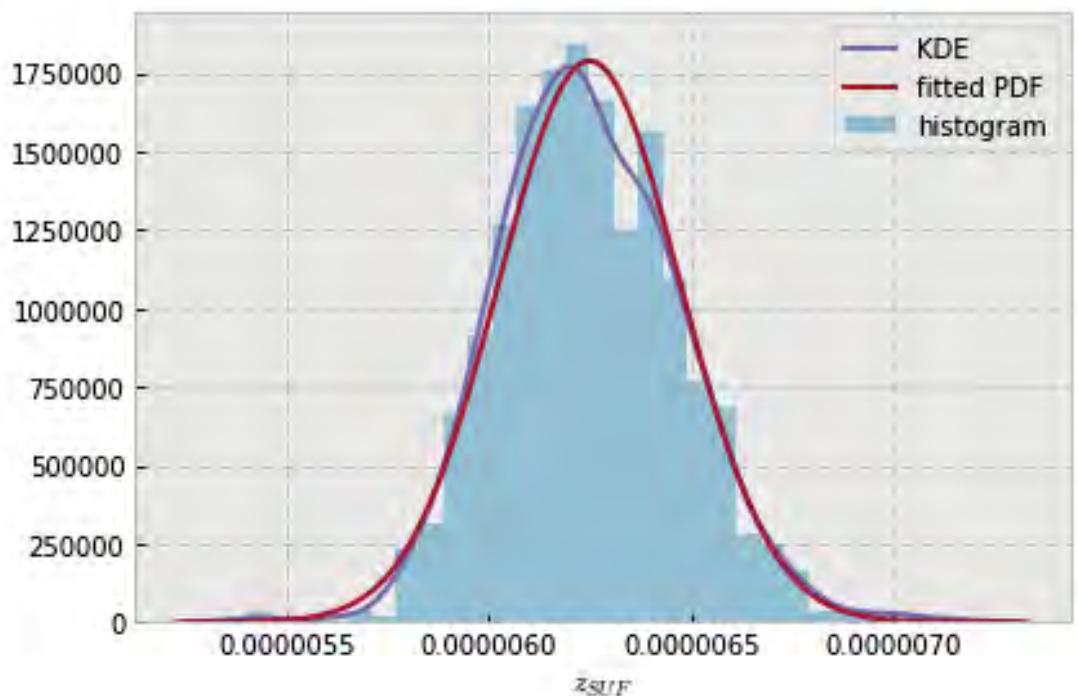


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=6.003883785543642e-06, scale=2.3030117052491595e-07)
DescribeResult(nobs=1000, minmax=(5.296119897026012e-06, 6.7199390866084046e-06)
               mean=6.003883785543642e-06, variance=5.3091720866012433e-14,
               skewness=0.18857461089811367, kurtosis=-0.0196163731468042)
```

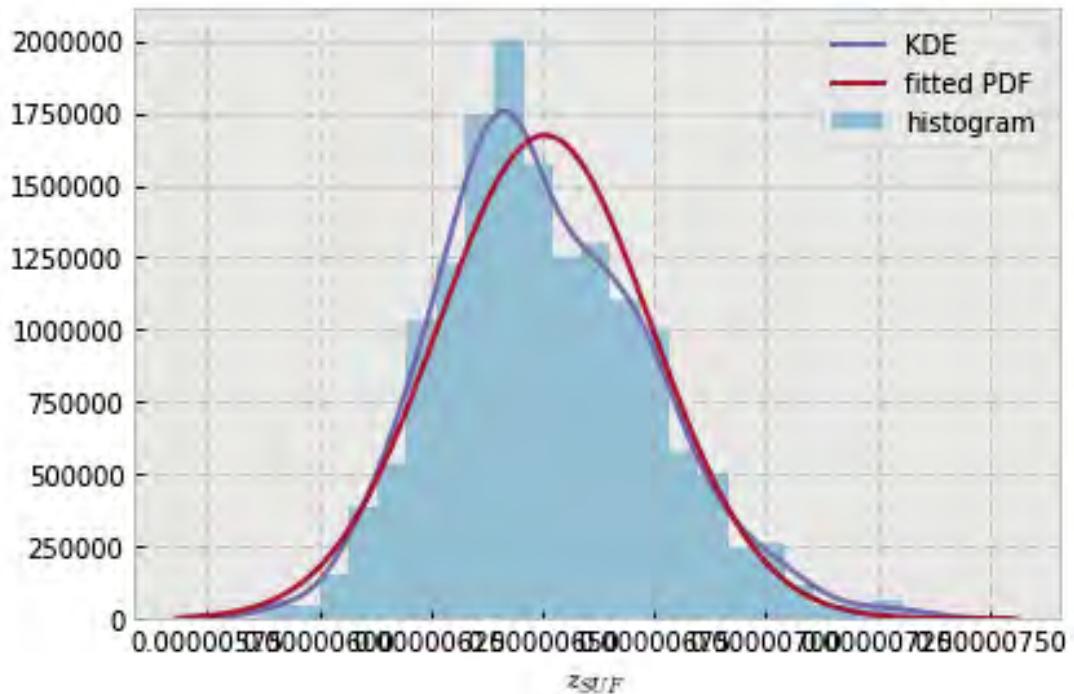


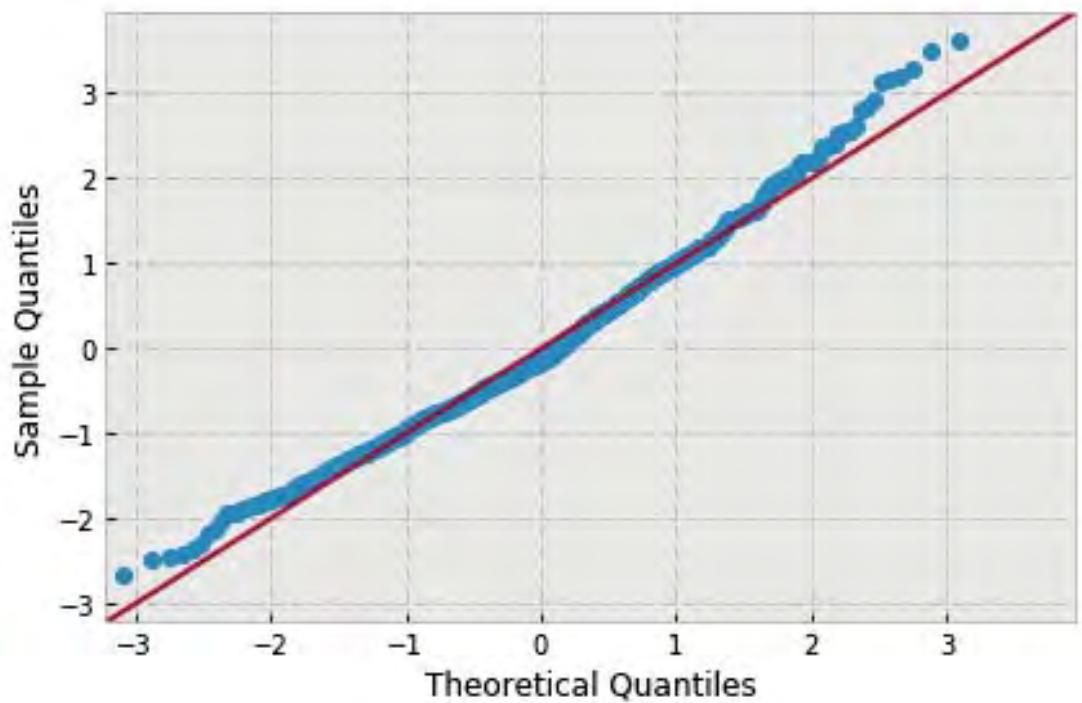


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=6.248825343644636e-06, scale=2.2305403624391032e-07)
DescribeResult(nobs=1000, minmax=(5.4089705029322311e-06, 7.1504662403441401e-06)
               mean=6.2488253436446356e-06, variance=4.980290599069035e-14,
               skewness=0.256360794857108, kurtosis=0.4971887841849303)
```

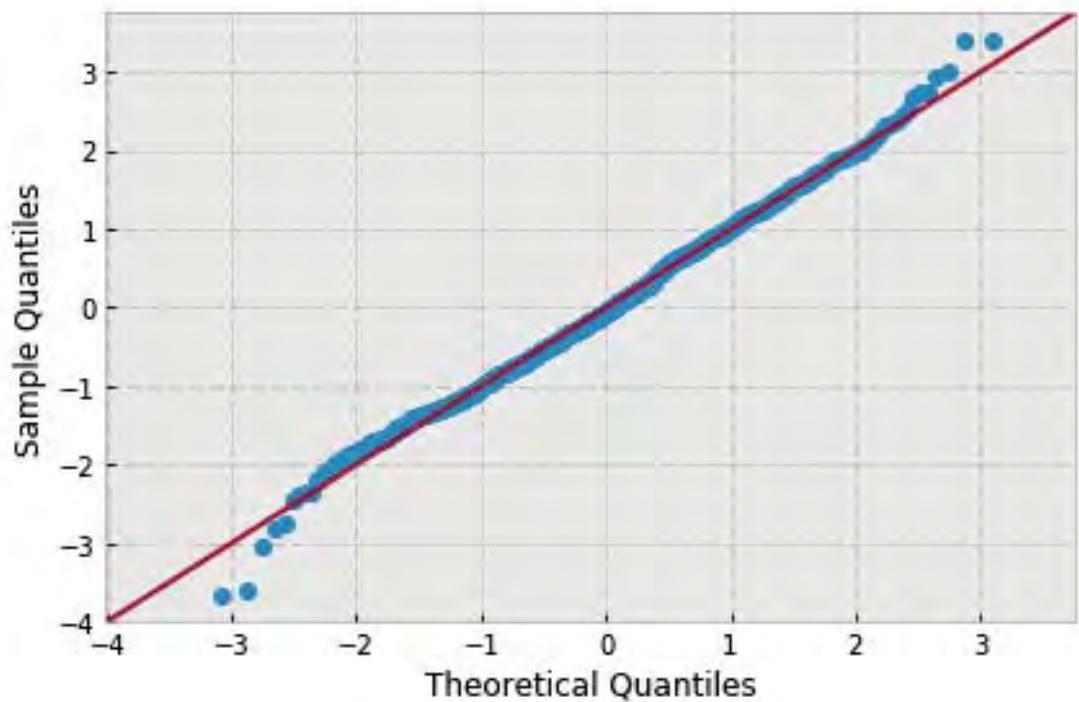
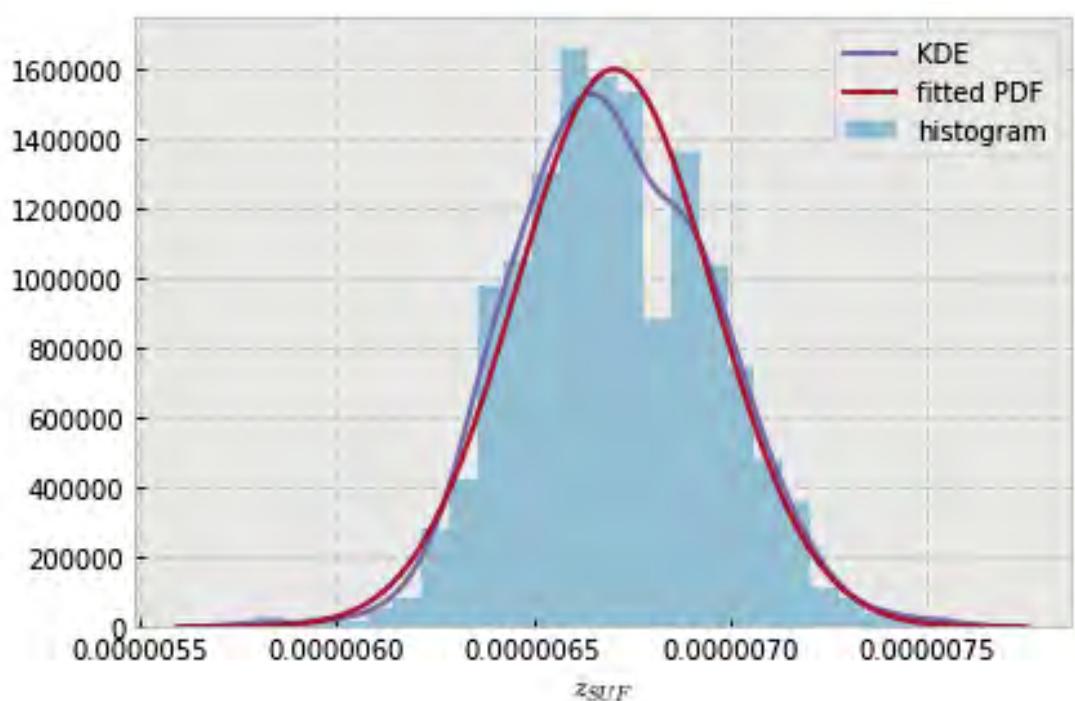


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=6.505282590142195e-06, scale=2.385089253014024e-07)
DescribeResult(nobs=1000, minmax=(5.8701471123396337e-06, 7.3701618054007927e-06)
               mean=6.5052825901421948e-06, variance=5.6943450899329281e-14,
               skewness=0.44618600278633586, kurtosis=0.21435977276076068)
```

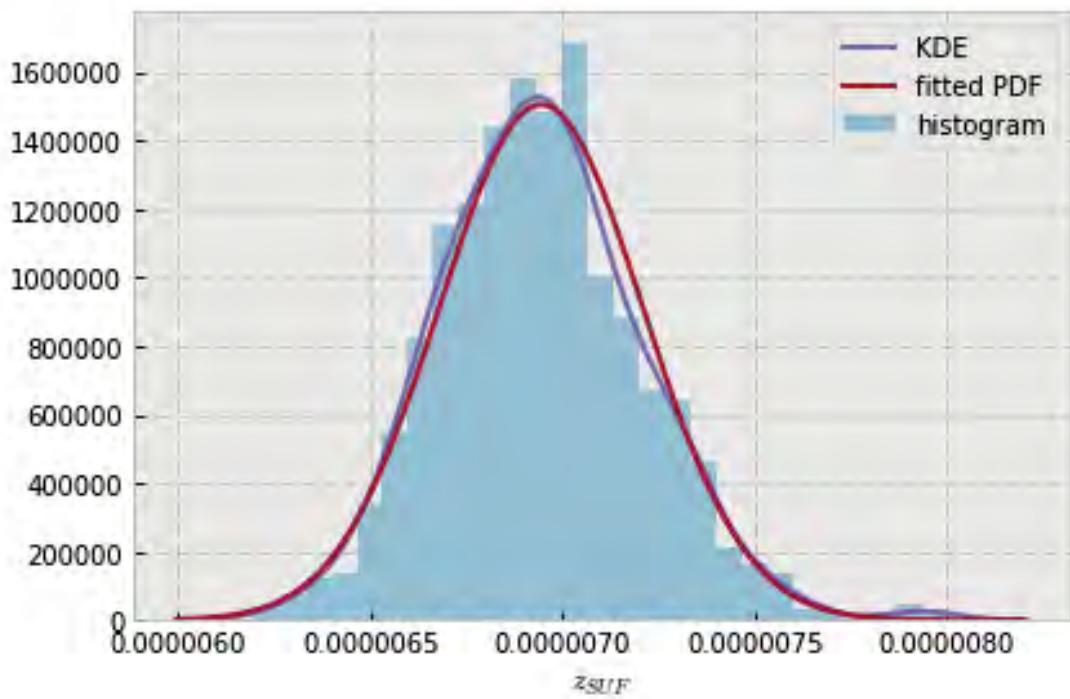


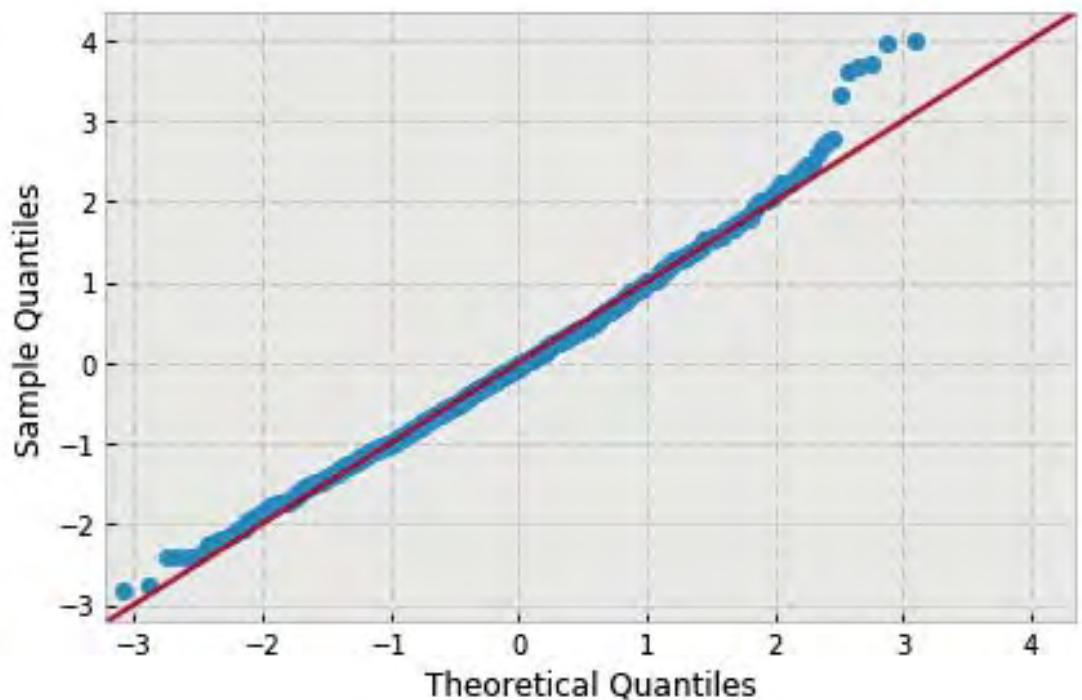


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=6.705925555807249e-06, scale=2.496443507141575e-07)
DescribeResult(nobs=1000, minmax=(5.7938383516648872e-06, 7.554522113520397e-06)
               mean=6.7059255558072487e-06, variance=6.2384686530023308e-14,
               skewness=0.10891599777542205, kurtosis=0.11292301127446347)
```

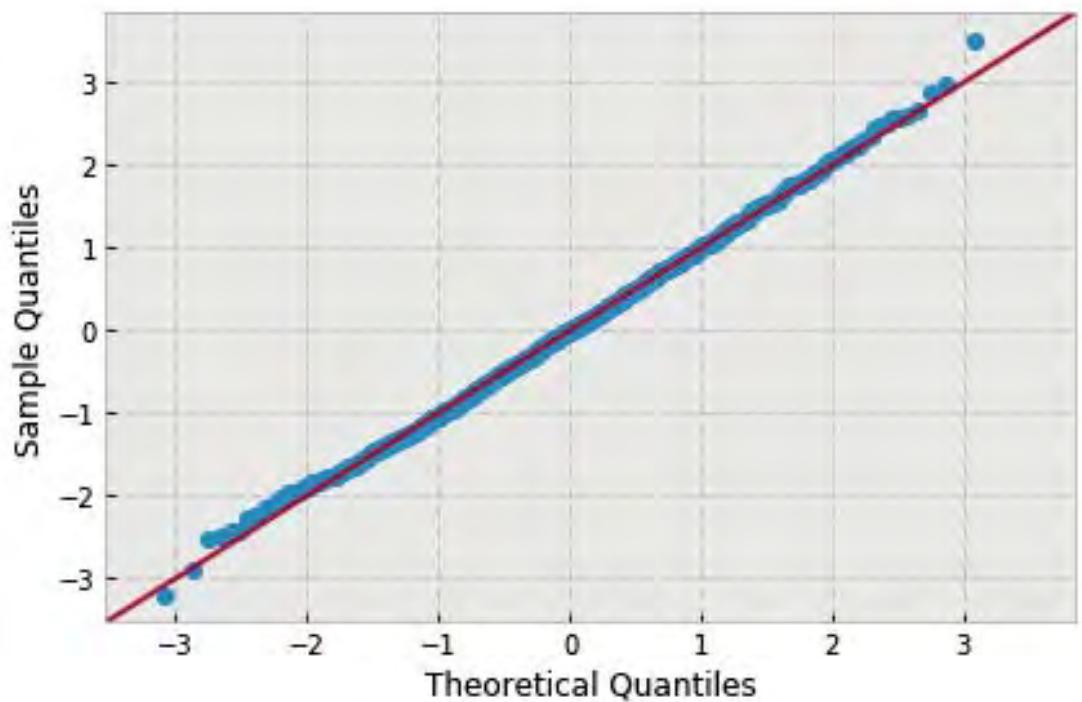
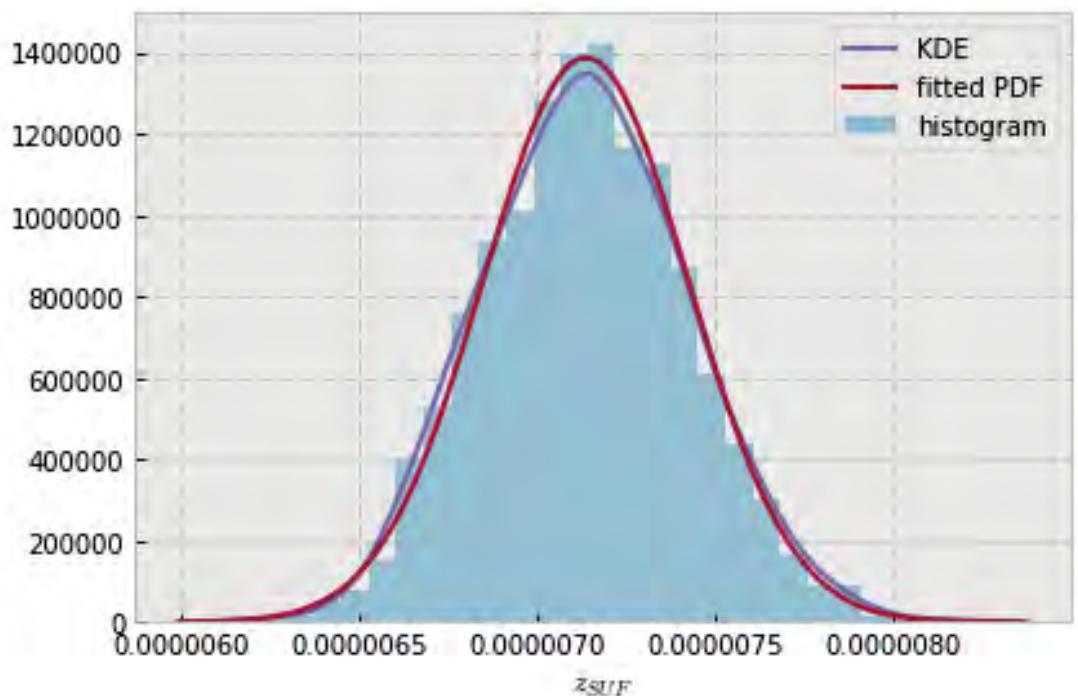


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=6.943948580535977e-06, scale=2.651983051918339e-07)
DescribeResult(nobs=1000, minmax=(6.1969047464951517e-06, 8.0023377462590175e-06)
               mean=6.9439485805359772e-06, variance=7.0400541618239321e-14,
               skewness=0.36017991076565364, kurtosis=0.567285045494788)
```

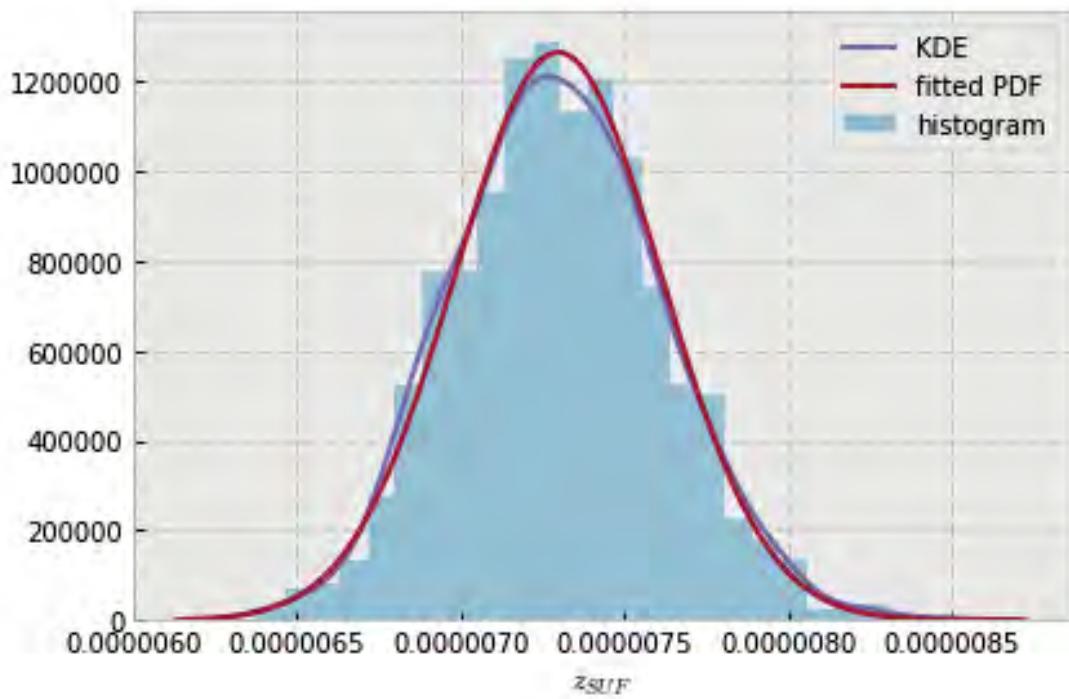


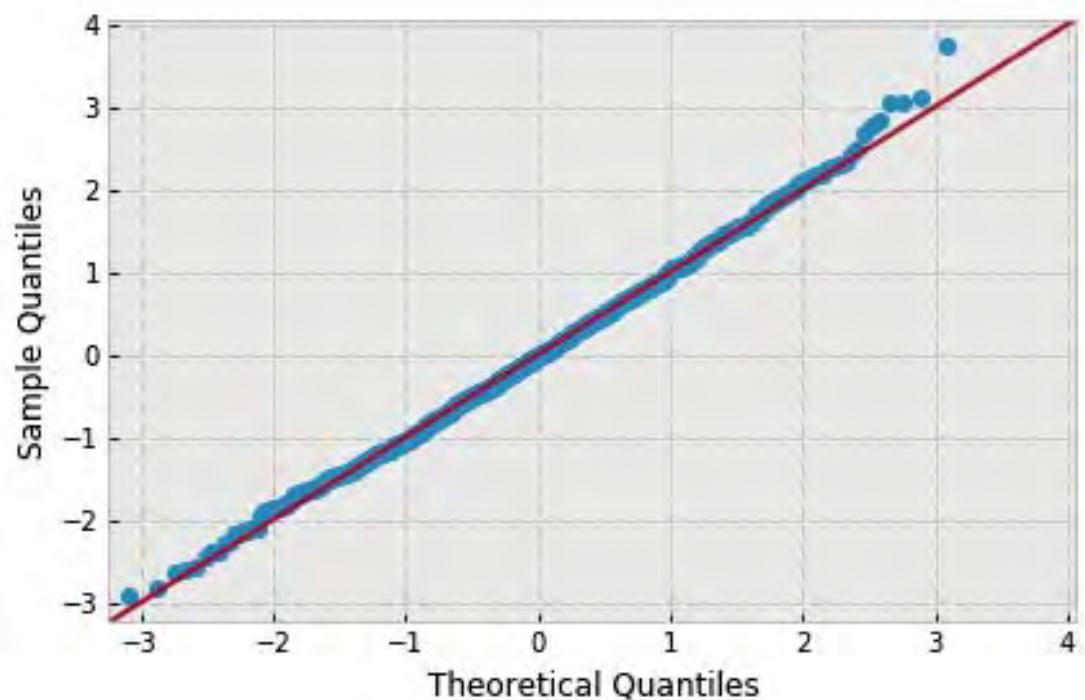


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=7.13603168990032e-06, scale=2.876012366458457e-07)
DescribeResult(nobs=1000, minmax=(6.2167663211898527e-06, 8.1455594425598901e-06)
               mean=7.1360316899003198e-06, variance=8.2797268588808567e-14,
               skewness=0.10635358185202445, kurtosis=-0.10612582241200874)
```

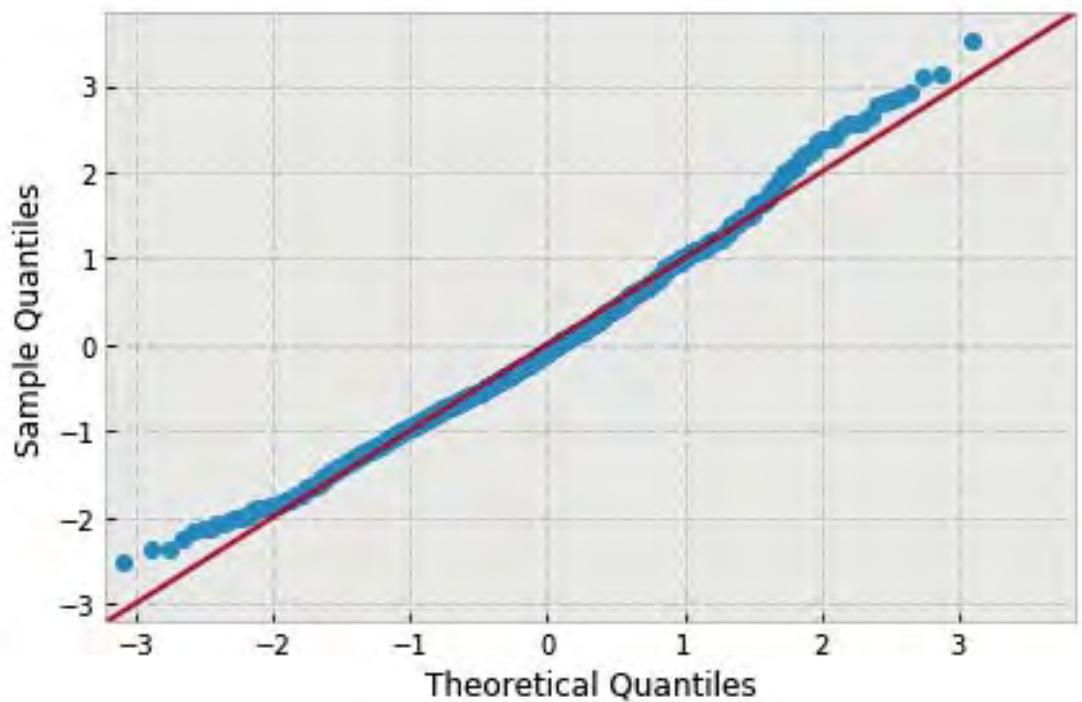
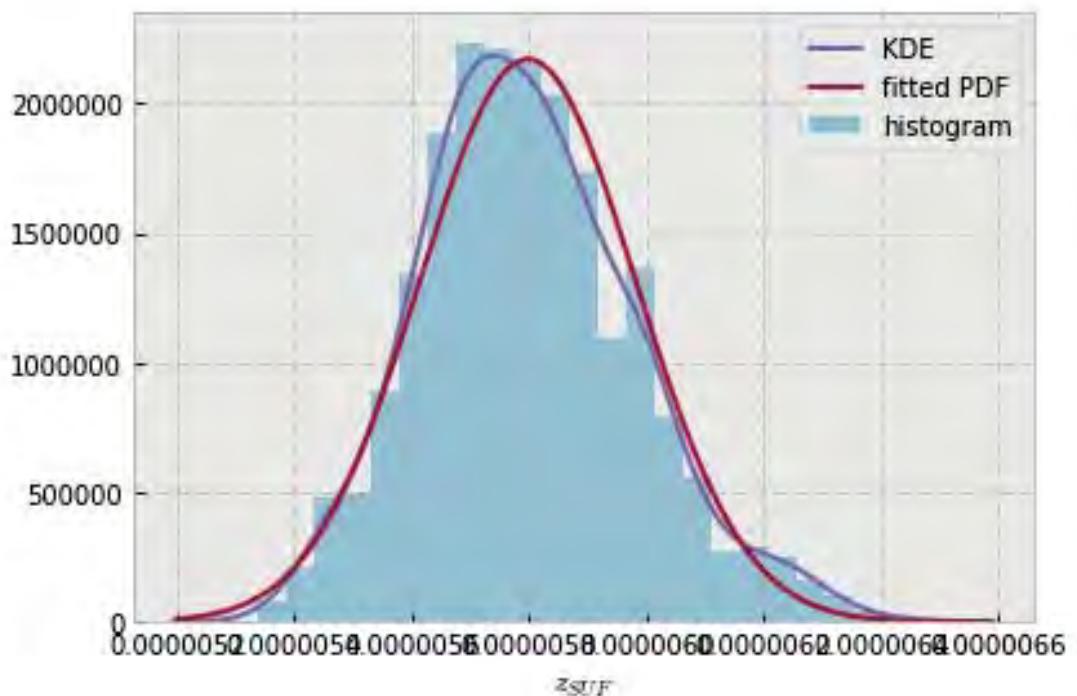


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=7.297555969273288e-06, scale=3.154954243044617e-07)
DescribeResult(nobs=1000, minmax=(6.3809525134716275e-06, 8.474812548415905e-06)
               mean=7.2975559692732876e-06, variance=9.9636999756809142e-14,
               skewness=0.1559354985398343, kurtosis=0.01765592290232698)
```

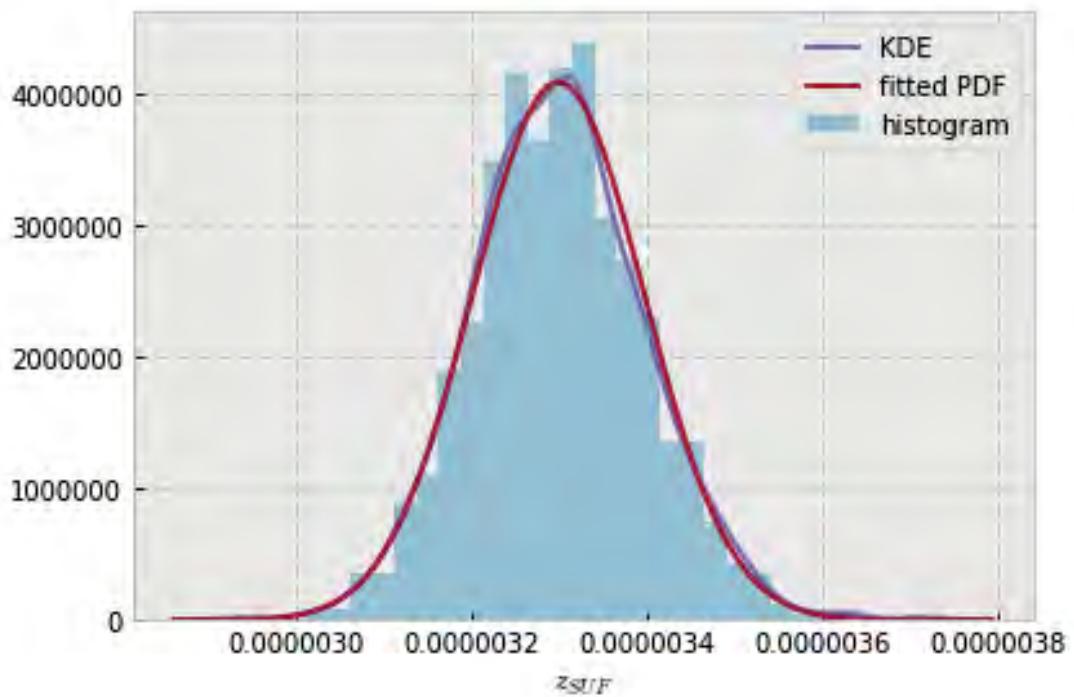


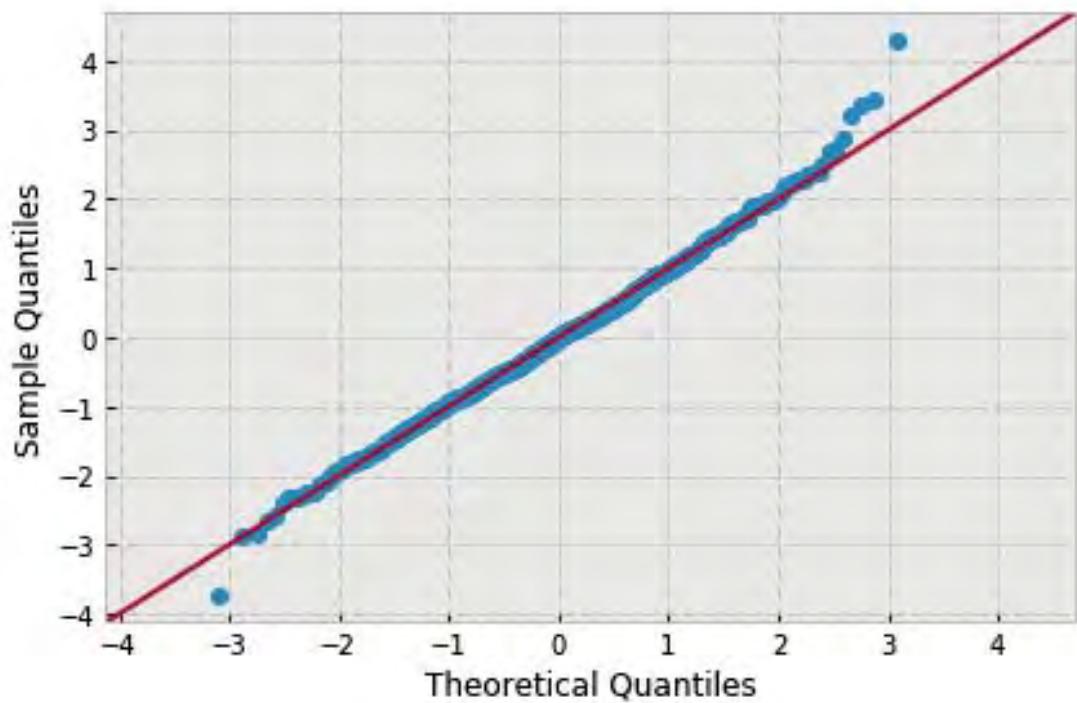


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=5.797803598118172e-06, scale=1.839259546815837e-07)
DescribeResult(nobs=1000, minmax=(5.3374042106269566e-06, 6.4487989518900234e-06)
               mean=5.7978035981181716e-06, variance=3.3862619424956936e-14,
               skewness=0.4014453891168891, kurtosis=0.14576999422186132)
```

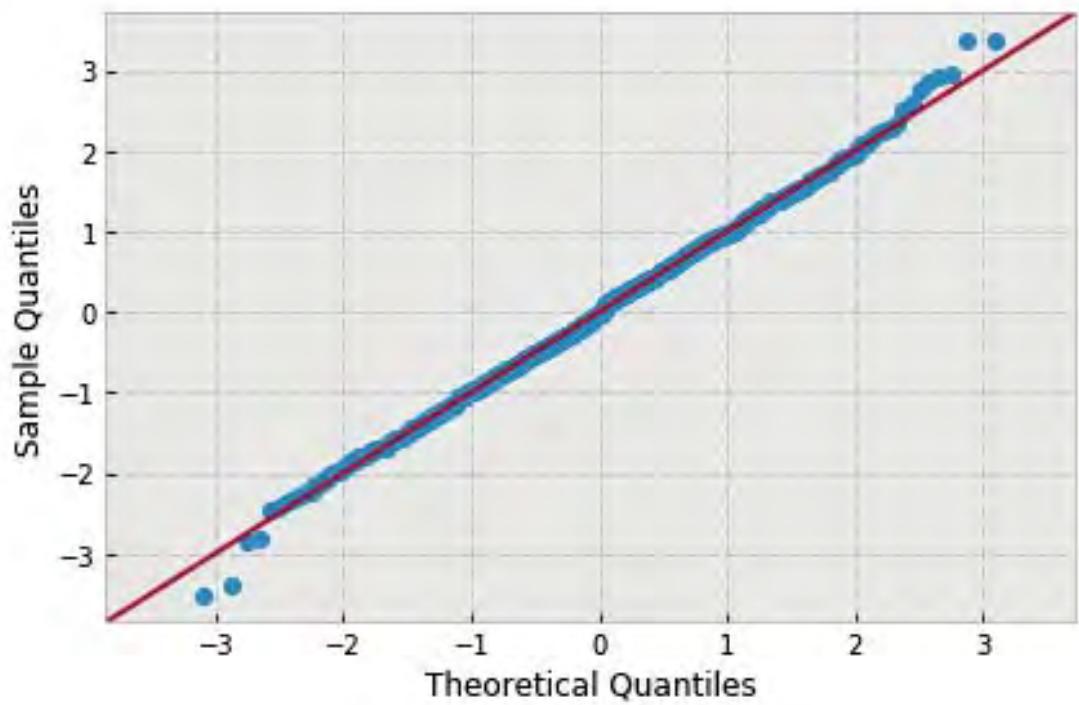
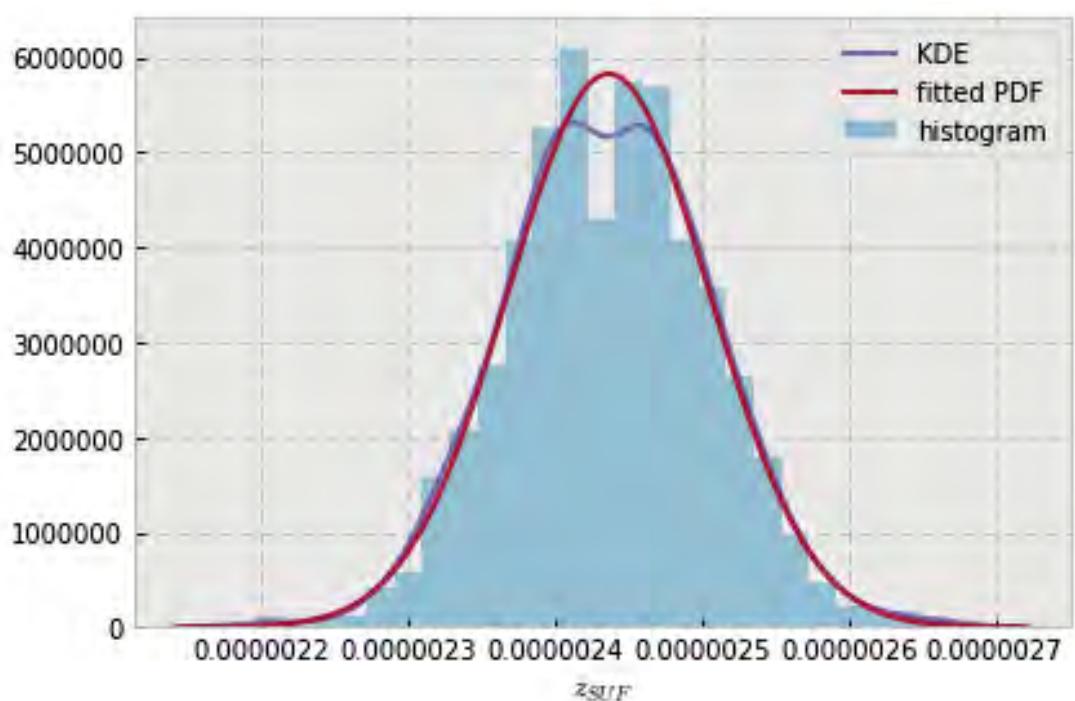


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=3.2986347815434287e-06, scale=9.76776879929178e-08)
DescribeResult(nobs=1000, minmax=(2.9352856384528542e-06, 3.7177123358303756e-06)
               mean=3.2986347815434287e-06, variance=9.5504812128546536e-15,
               skewness=0.17527216328231673, kurtosis=0.442542102323626)
```

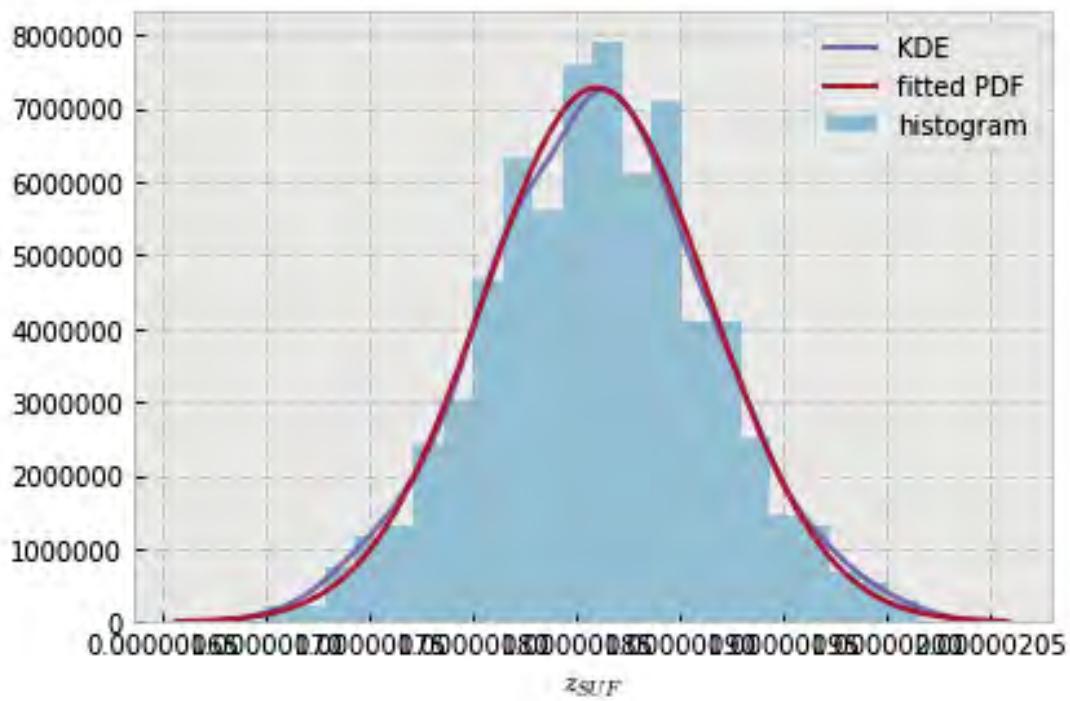


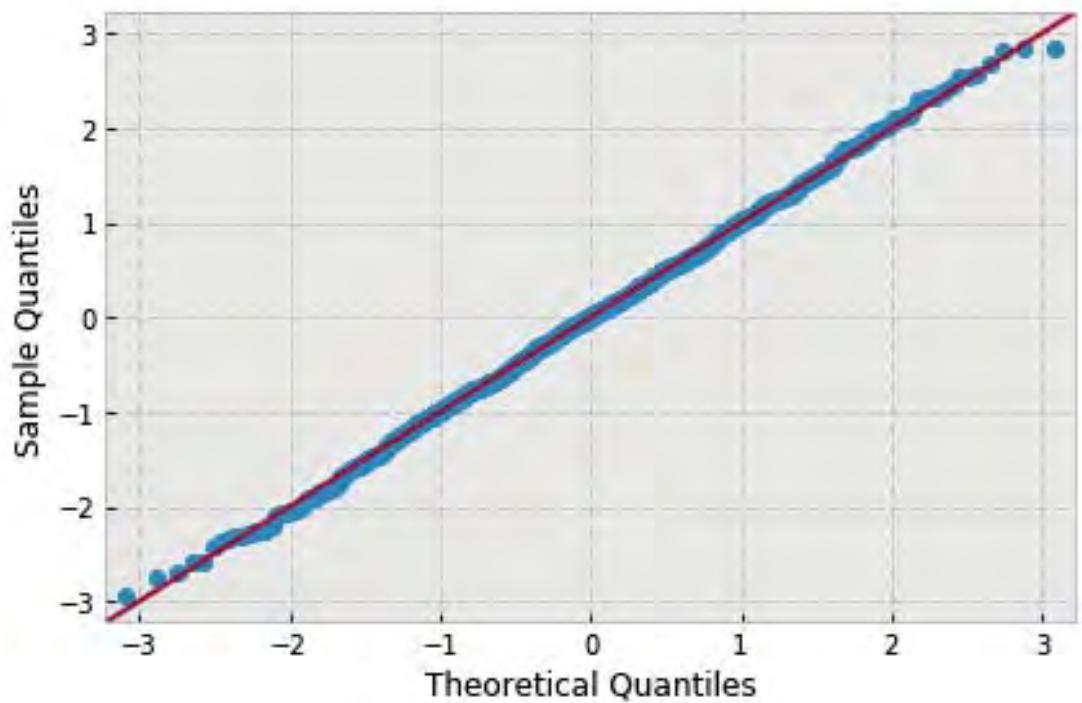


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=2.4367054350359072e-06, scale=6.848815768129505e-08)
DescribeResult(nobs=1000, minmax=(2.1964391938014478e-06, 2.6672168934409144e-06)
               mean=2.4367054350359072e-06, variance=4.6953230656435781e-15,
               skewness=0.03920502276709106, kurtosis=0.0793688229850007)
```

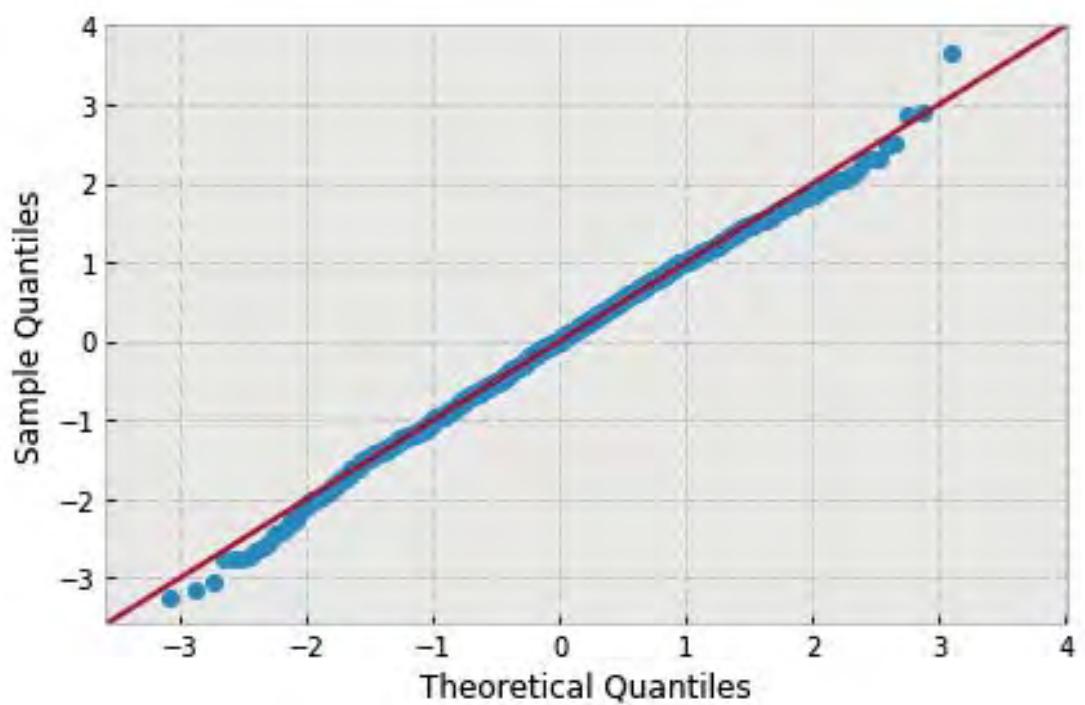
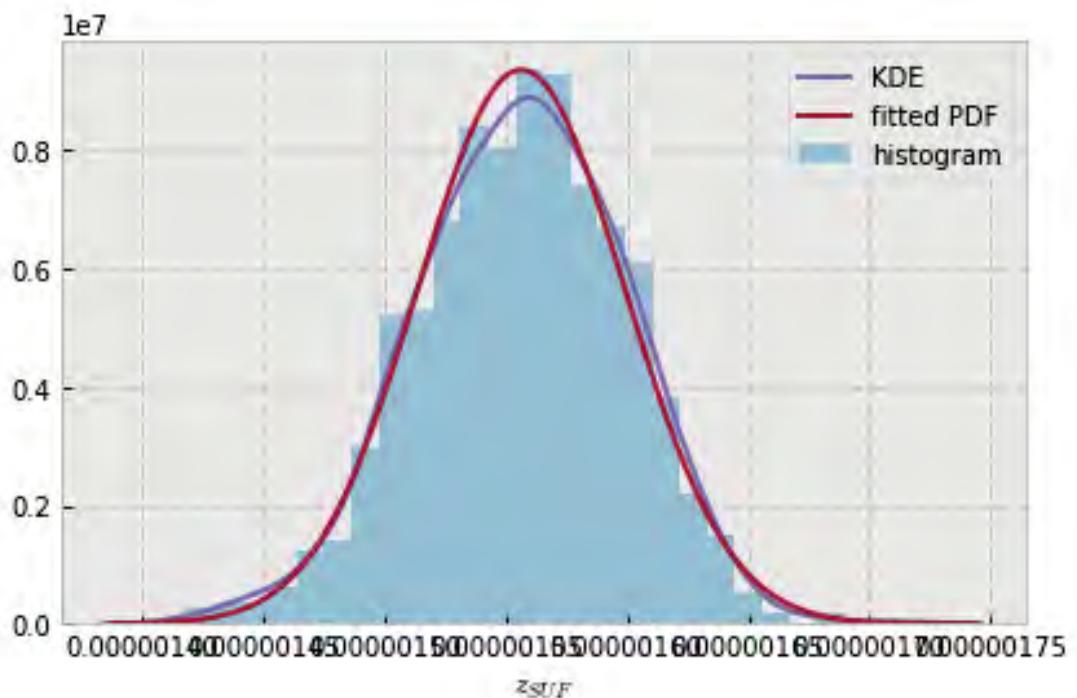


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=1.8596401018114856e-06, scale=5.478675905657108e-08)
DescribeResult(nobs=1000, minmax=(1.6989885636302368e-06, 2.0157938762177184e-06)
               mean=1.8596401018114856e-06, variance=3.0045935614842575e-15,
               skewness=0.013814558262906456, kurtosis=-0.08279598803025623)
```

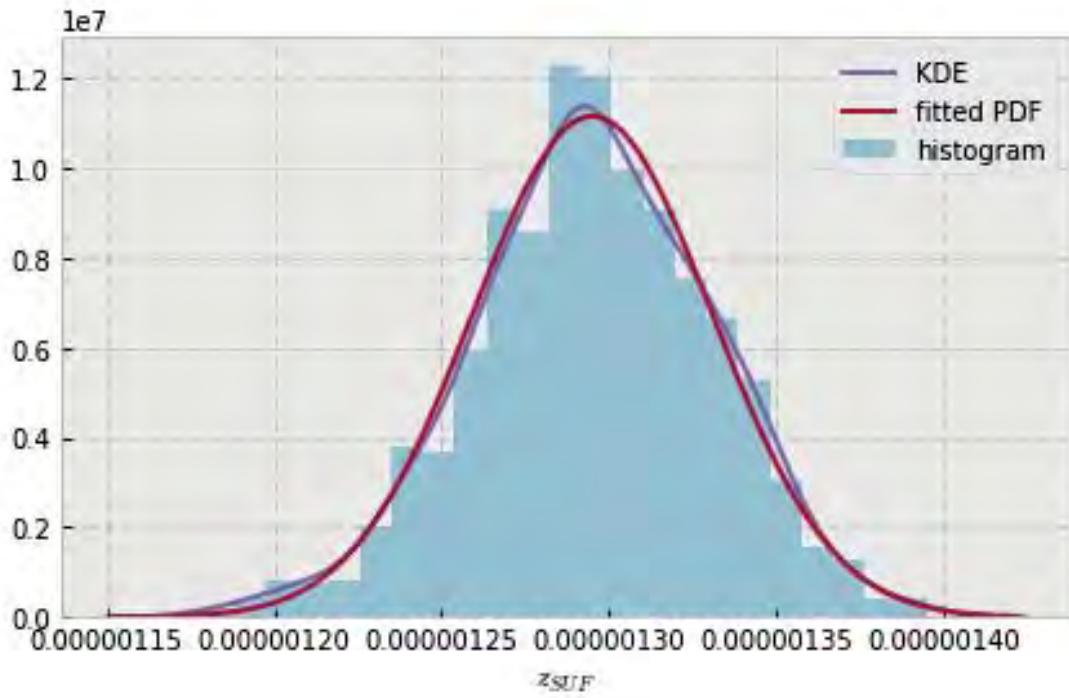


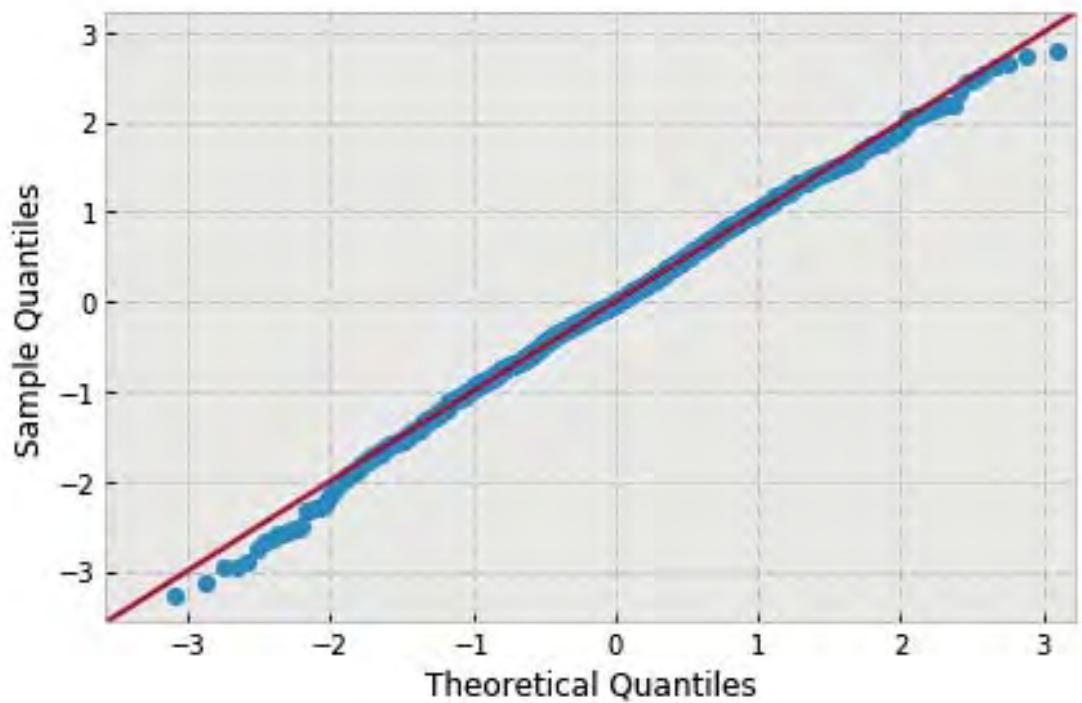


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=1.5563903505975328e-06, scale=4.2608016828887144e-08)
DescribeResult(nobs=1000, minmax=(1.4182249742959054e-06, 1.7123822964778948e-06)
               mean=1.5563903505975328e-06, variance=1.8172603584491793e-15,
               skewness=-0.13810025488012825, kurtosis=0.04237988872575027)
```

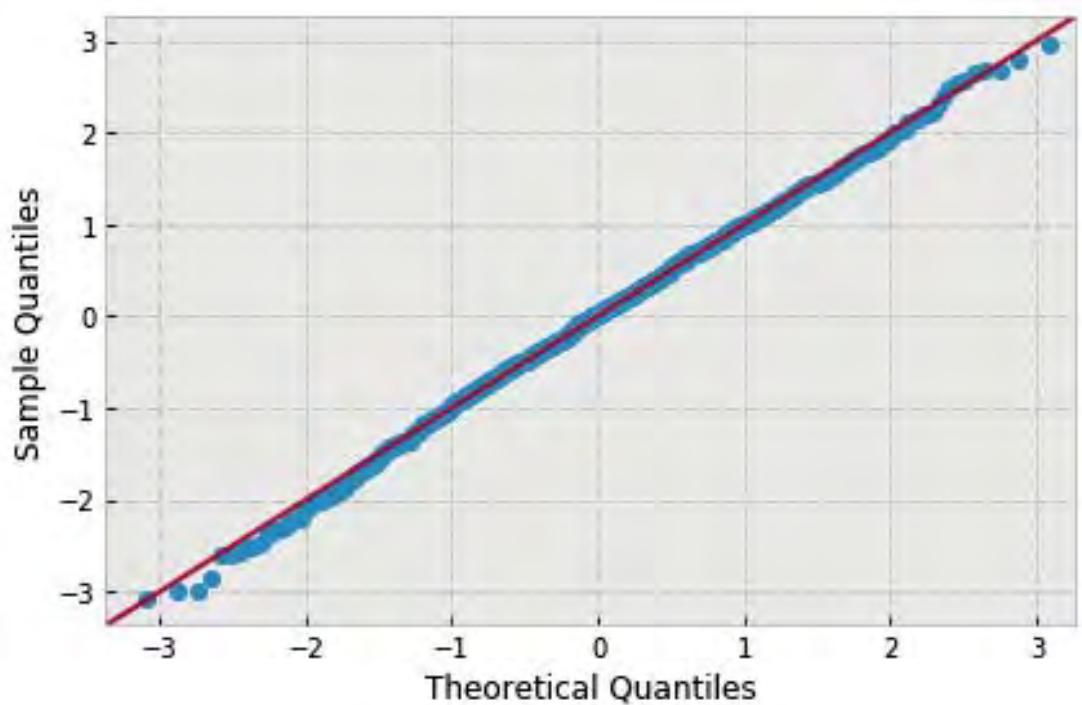
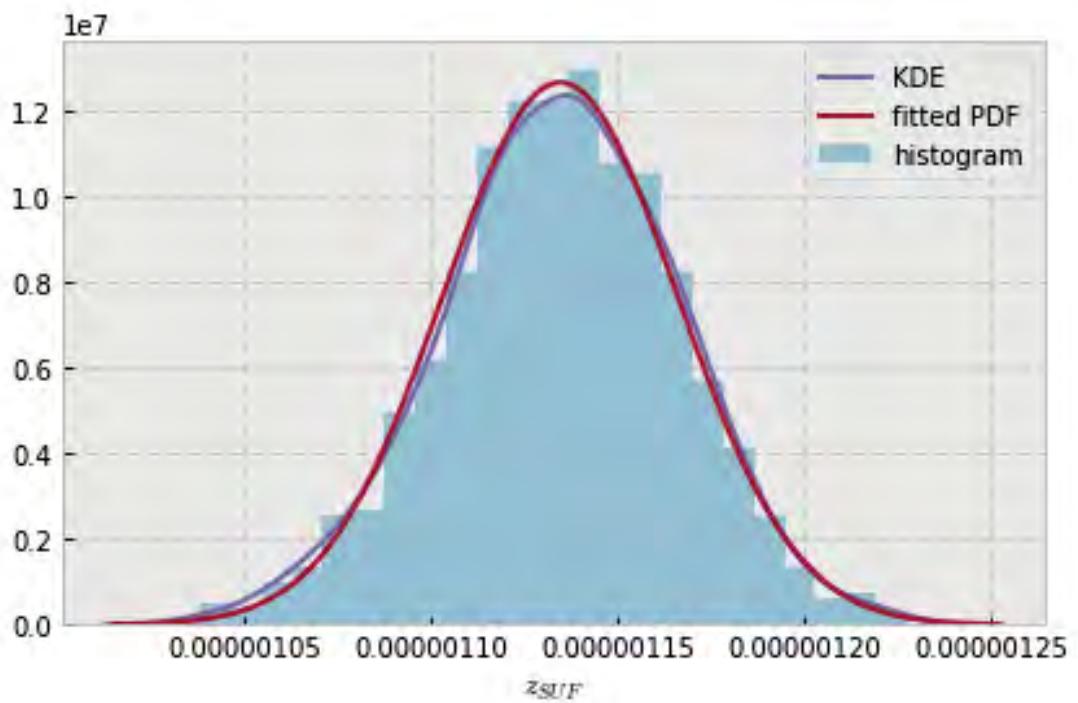


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=1.295112534992594e-06, scale=3.574825227874915e-08)
DescribeResult(nobs=1000, minmax=(1.1783620607663381e-06, 1.3952958734603198e-06)
               mean=1.2951125349925939e-06, variance=1.2792167577428363e-15,
               skewness=-0.15375625748141777, kurtosis=0.04617733553836478)
```

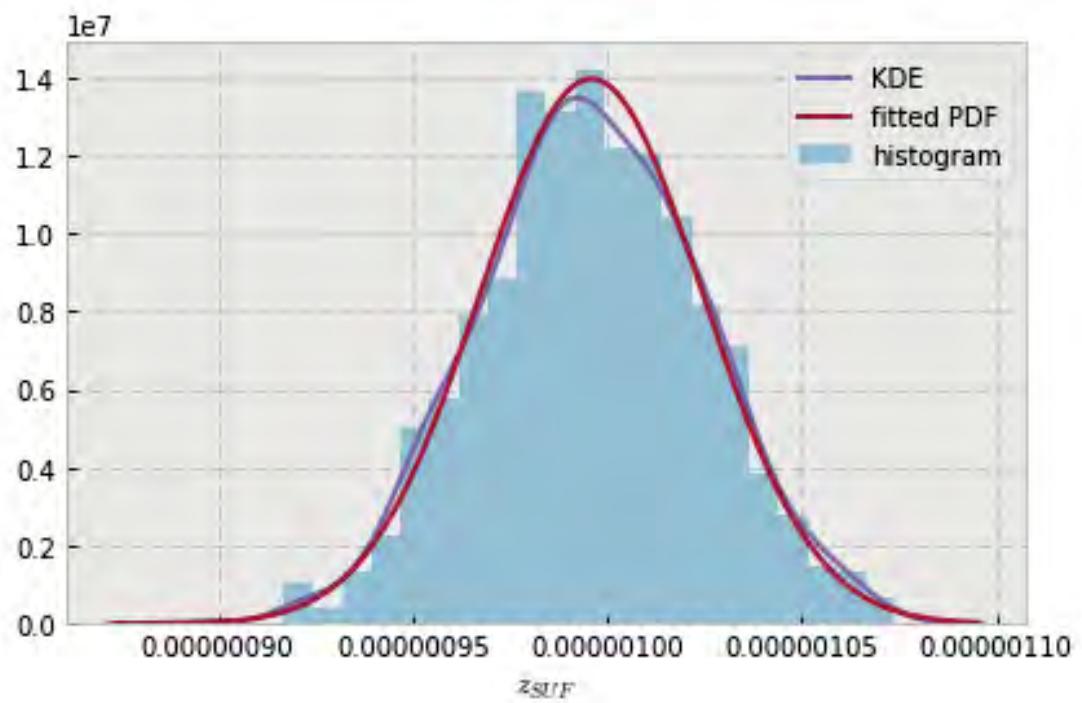


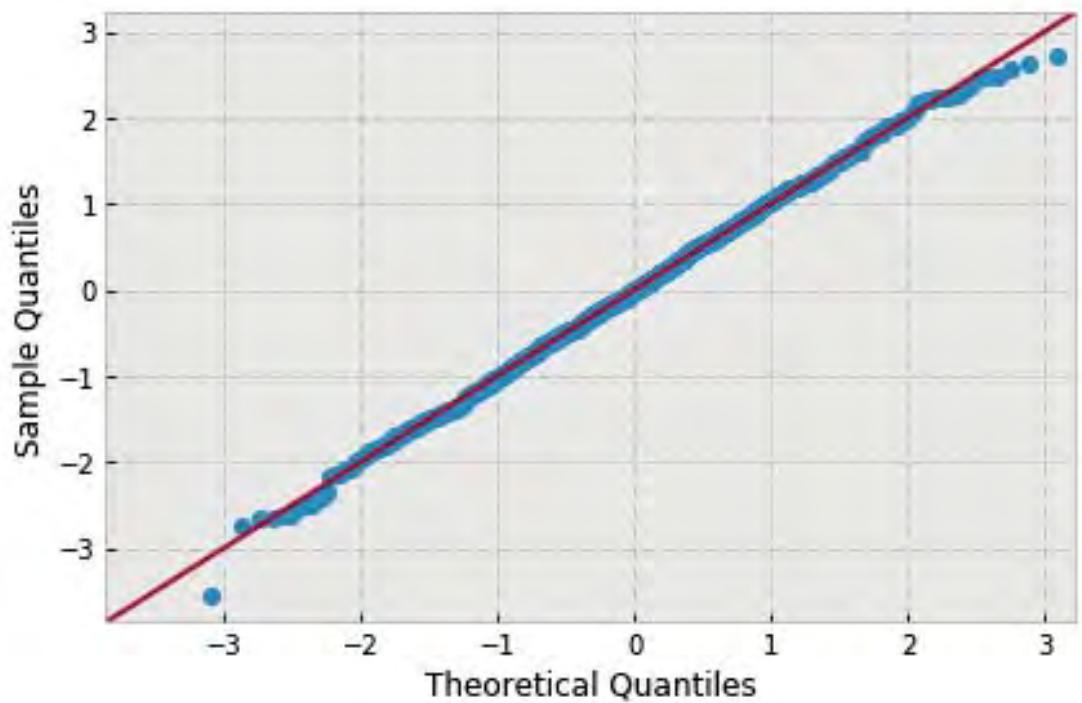


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=1.1344936148075805e-06, scale=3.1514754444688455e-08)
DescribeResult(nobs=1000, minmax=(1.0378584640527932e-06, 1.2277283332242391e-06)
               mean=1.1344936148075805e-06, variance=9.9417392163064158e-16,
               skewness=-0.1170576825240064, kurtosis=0.004399425956741698)
```

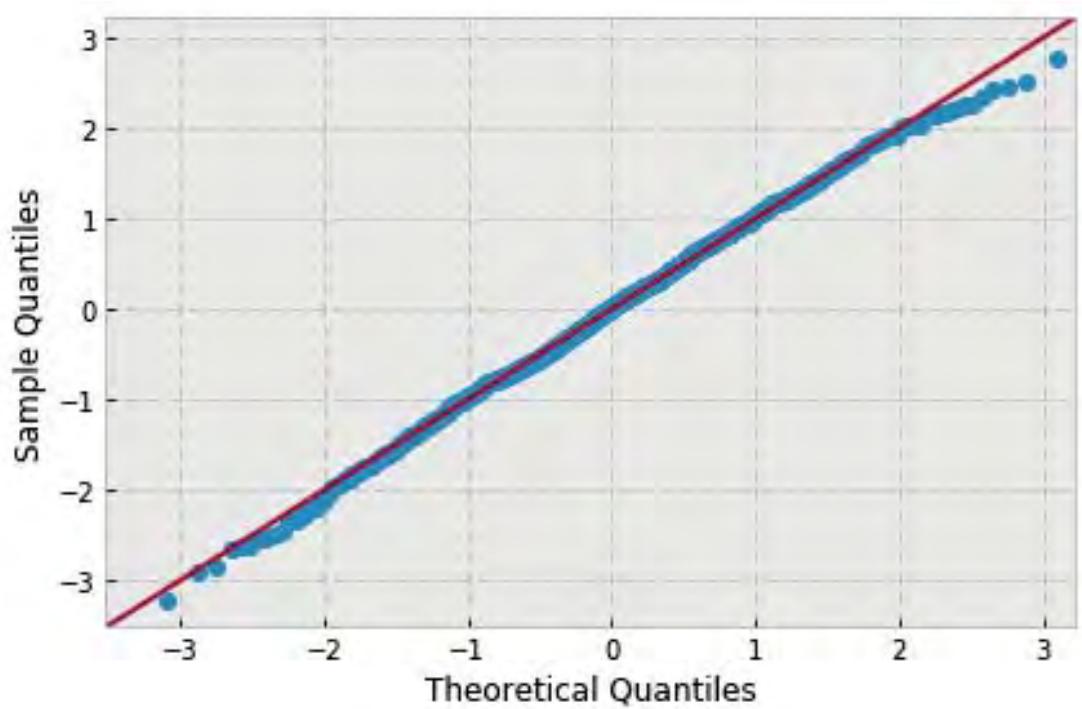
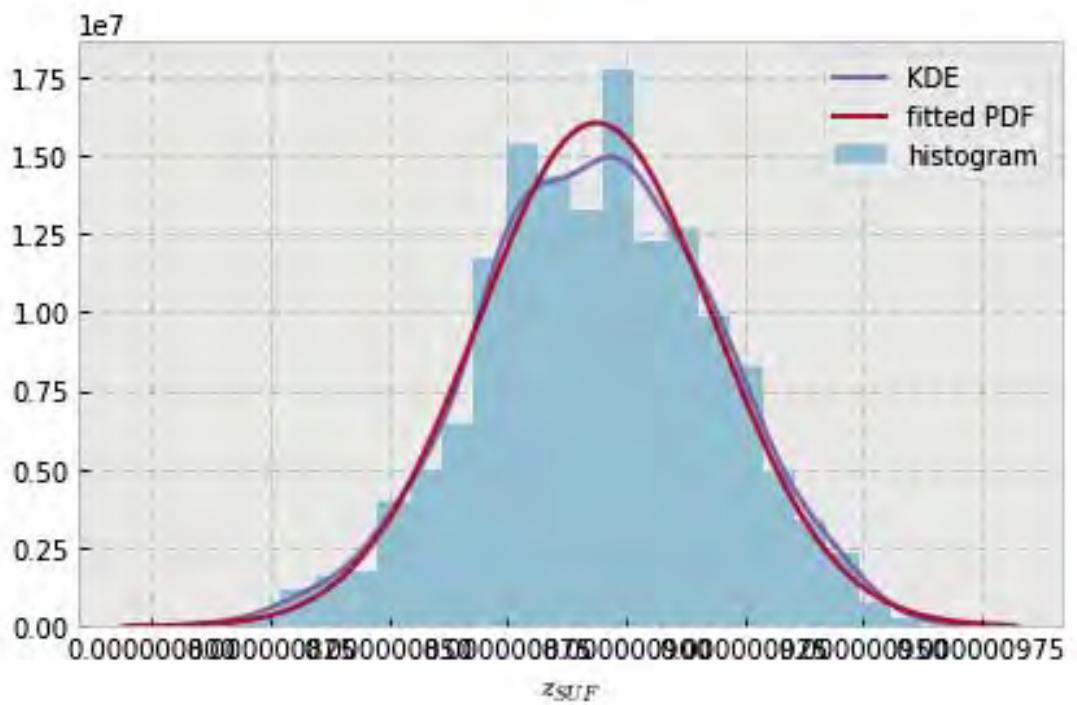


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=9.959376125958946e-07, scale=2.8581520017253244e-08)
DescribeResult(nobs=1000, minmax=(8.945211816988792e-07, 1.073961280525488e-06)
               mean=9.9593761259589459e-07, variance=8.1772100750415209e-16,
               skewness=-0.03769071371273488, kurtosis=-0.15376079538180987)
```

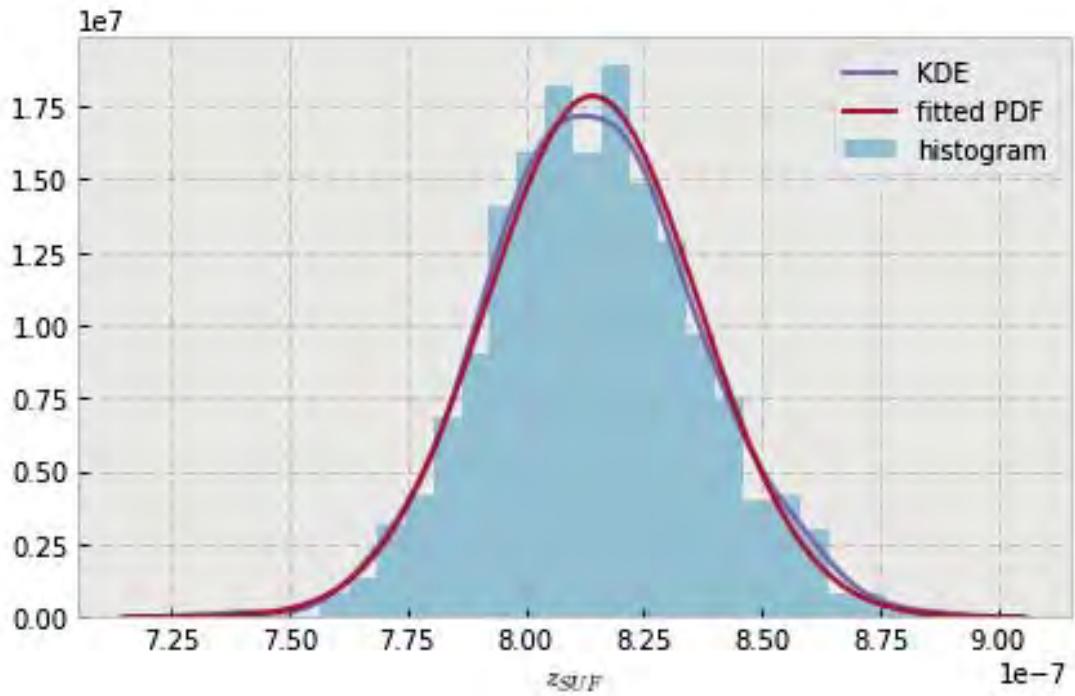


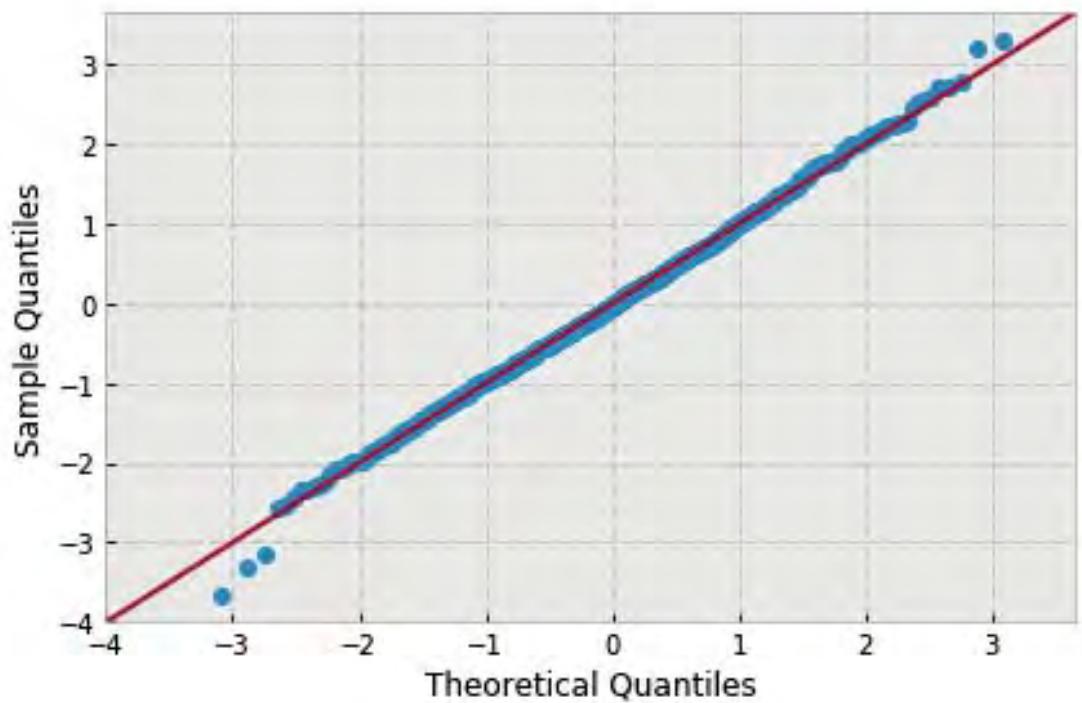


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=8.938106074992762e-07, scale=2.4876954827163646e-08)
DescribeResult(nobs=1000, minmax=(8.138194713762846e-07, 9.627135494300952e-07)
               mean=8.938106074992762e-07, variance=6.1948236383657702e-16,
               skewness=-0.10070300114518142, kurtosis=-0.17976625295737136)
```

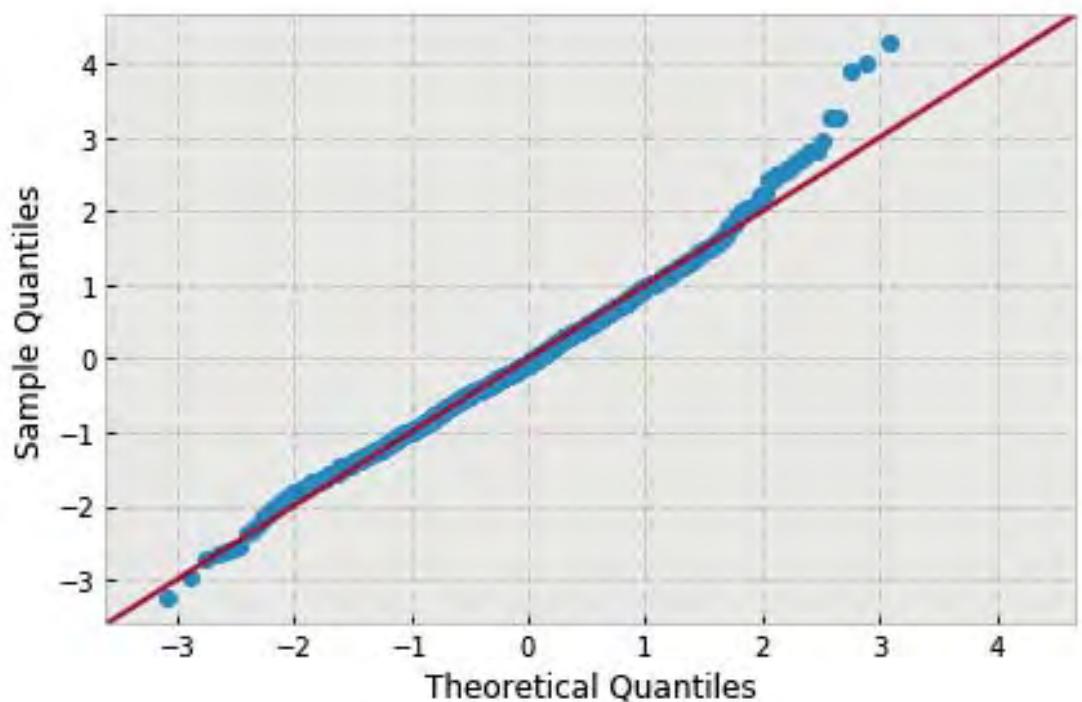
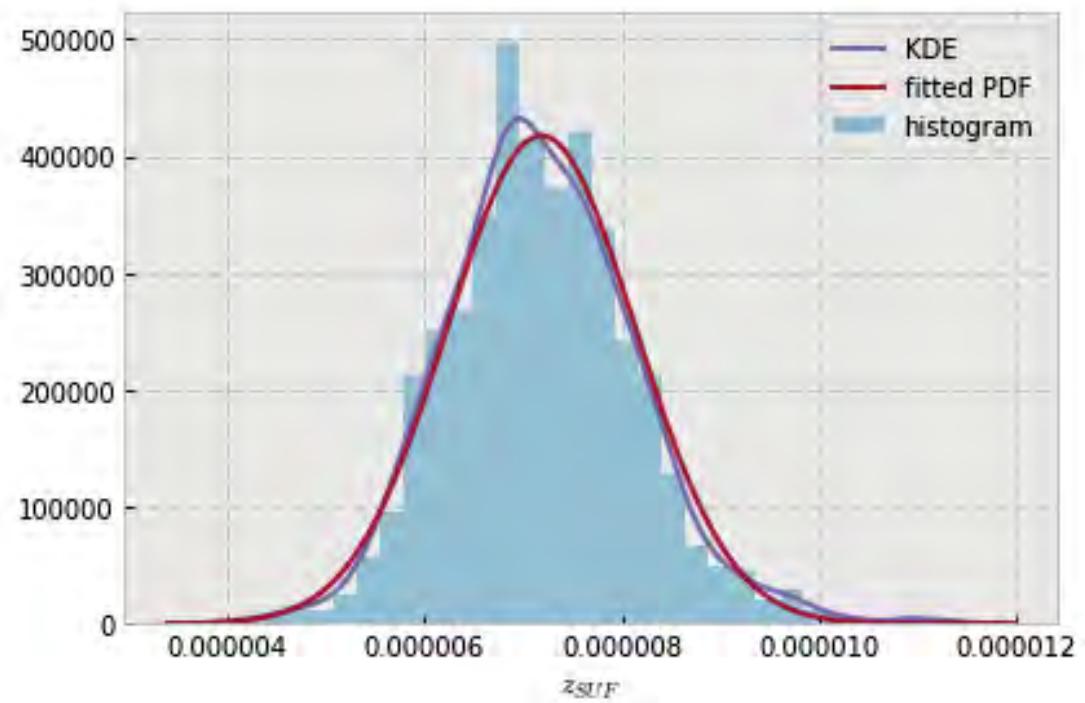


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=8.141833435839741e-07, scale=2.2351457295247898e-08)
DescribeResult(nobs=1000, minmax=(7.3255524385089917e-07, 8.879202601735415e-07)
               mean=8.1418334358397406e-07, variance=5.000877309522428e-16,
               skewness=0.07293929423543259, kurtosis=0.11592271754157935)
```

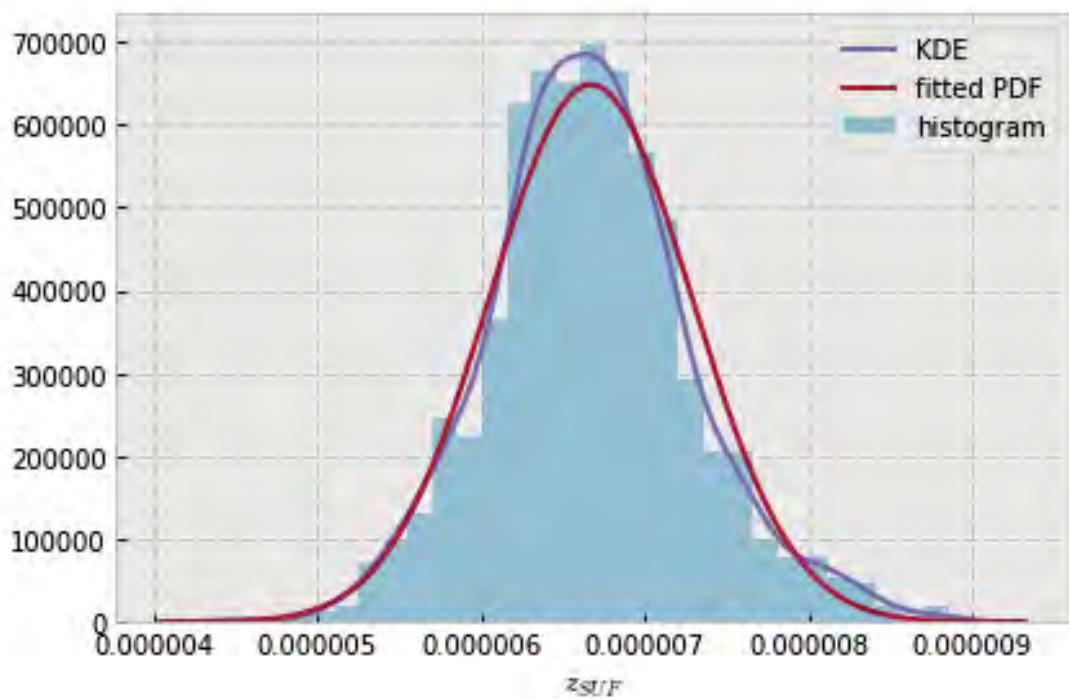


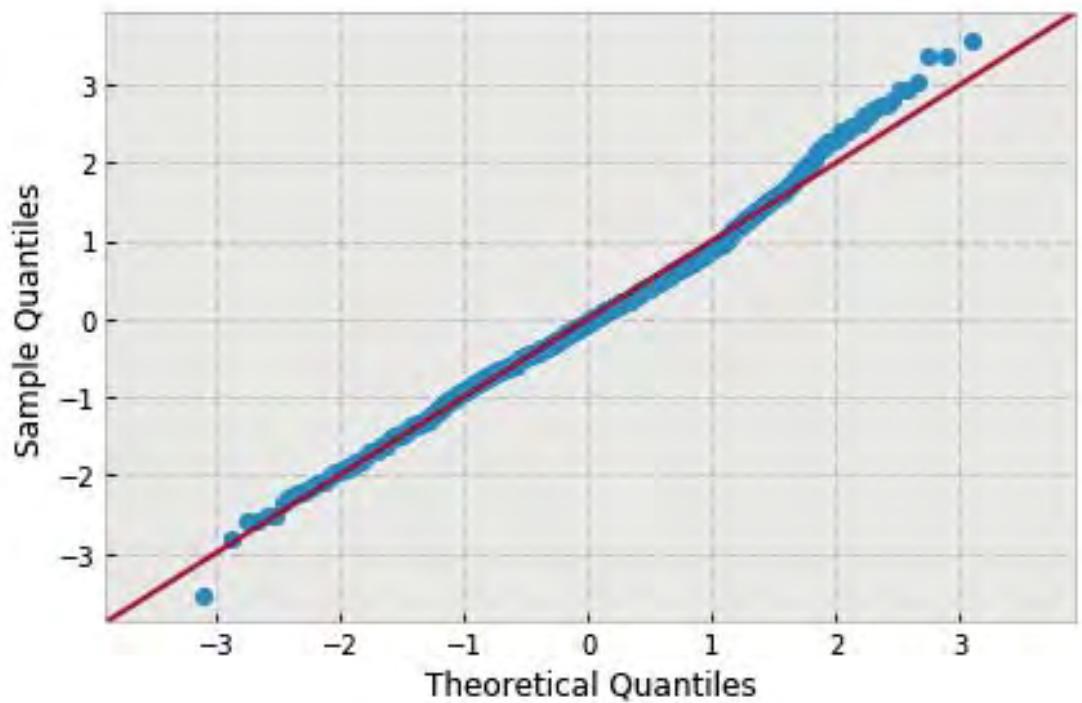


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=7.183911252559954e-06, scale=9.556959315261641e-07)
DescribeResult(nobs=1000, minmax=(4.1013372139816013e-06, 1.1278017242877861e-05)
               mean=7.1839112525599537e-06, variance=9.1426898251818069e-13,
               skewness=0.35534460581342975, kurtosis=0.797758552251183)
```

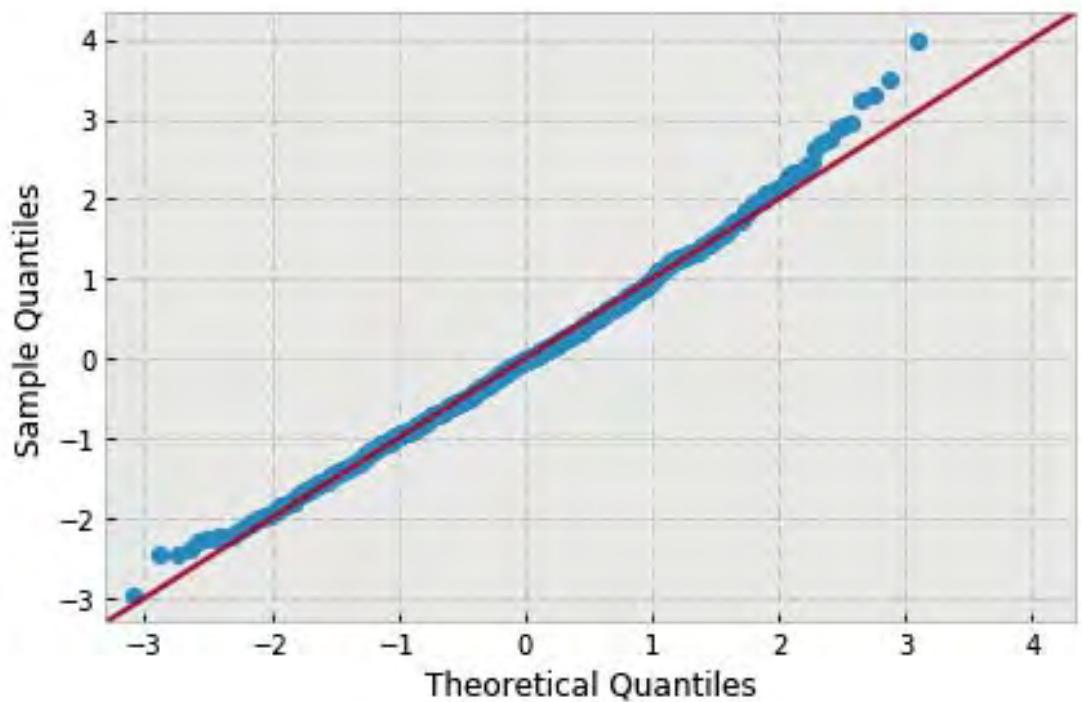
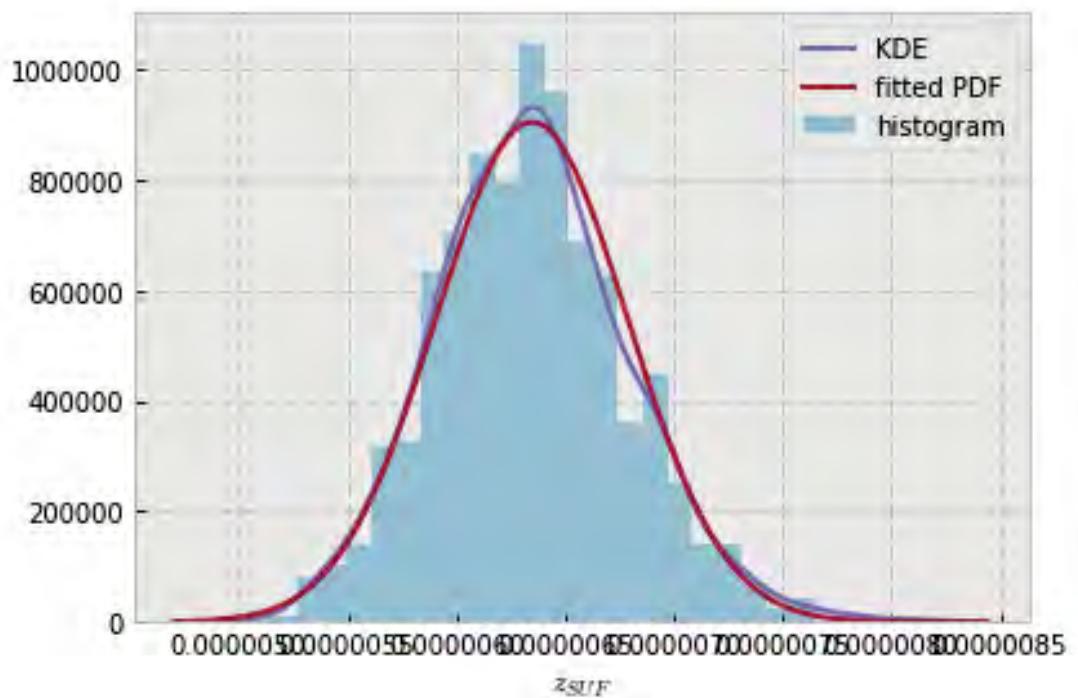


```
variable name: r
variable value: 1.222222222222223
distribution: normal(shape=(), loc=6.6712664879091044e-06, scale=6.156161570230887e-07)
DescribeResult(nobs=1000, minmax=(4.5018257959279097e-06, 8.864187373807161e-06)
               mean=6.6712664879091044e-06, variance=3.793626154032795e-13,
               skewness=0.2790130516653367, kurtosis=0.5393220269803876)
```

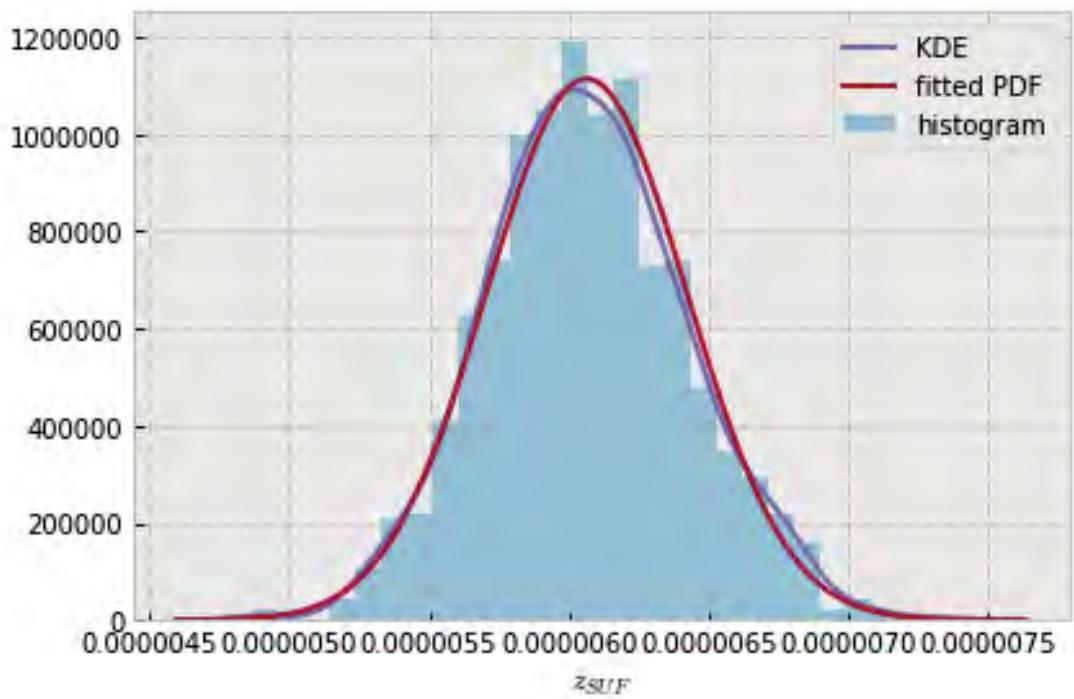


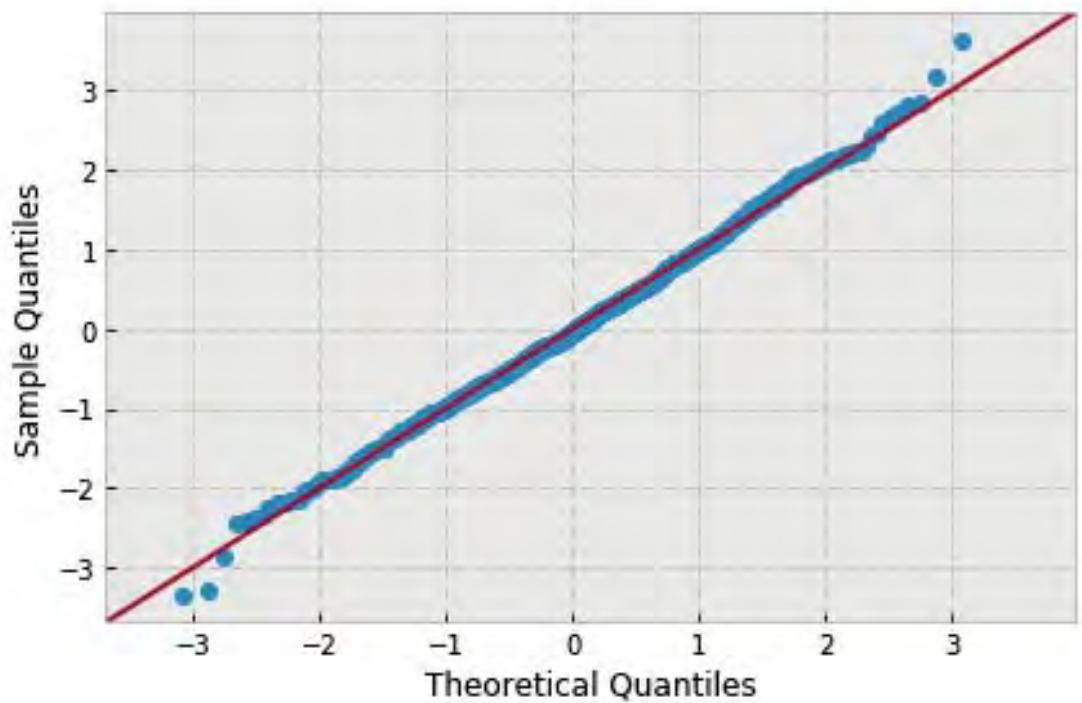


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=6.346944153354838e-06, scale=4.414099273362624e-07)
DescribeResult(nobs=1000, minmax=(5.0409540377210848e-06, 8.1058584962091629e-06)
               mean=6.3469441533548379e-06, variance=1.9503776171271716e-13,
               skewness=0.29144602776183465, kurtosis=0.3068864016444235)
```

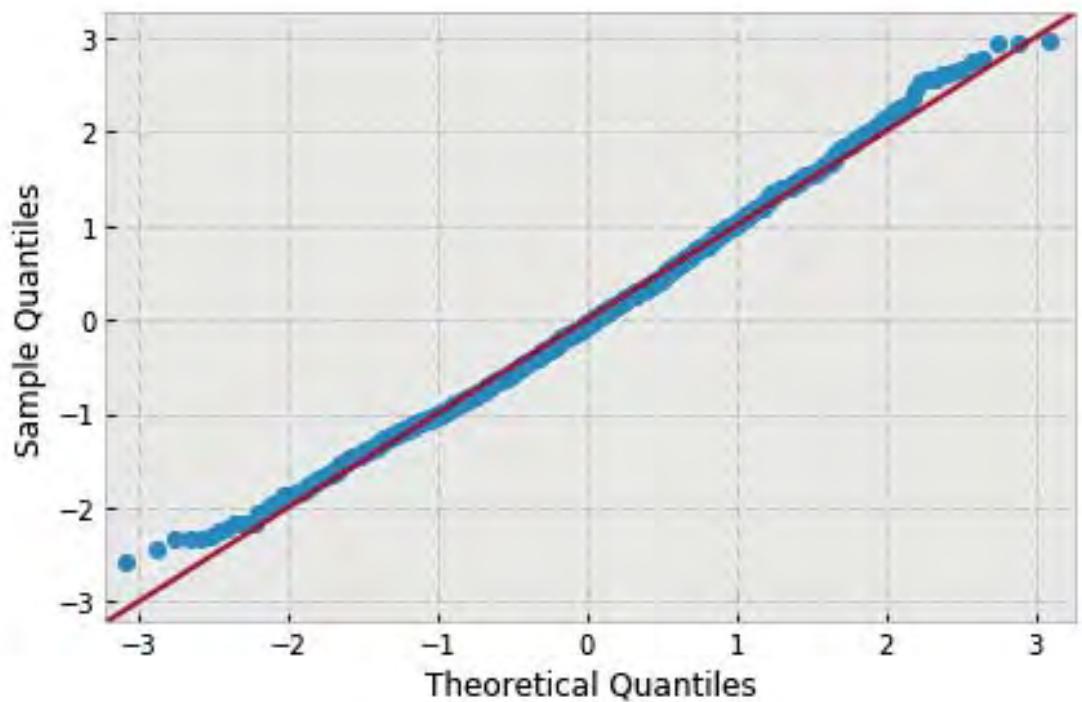
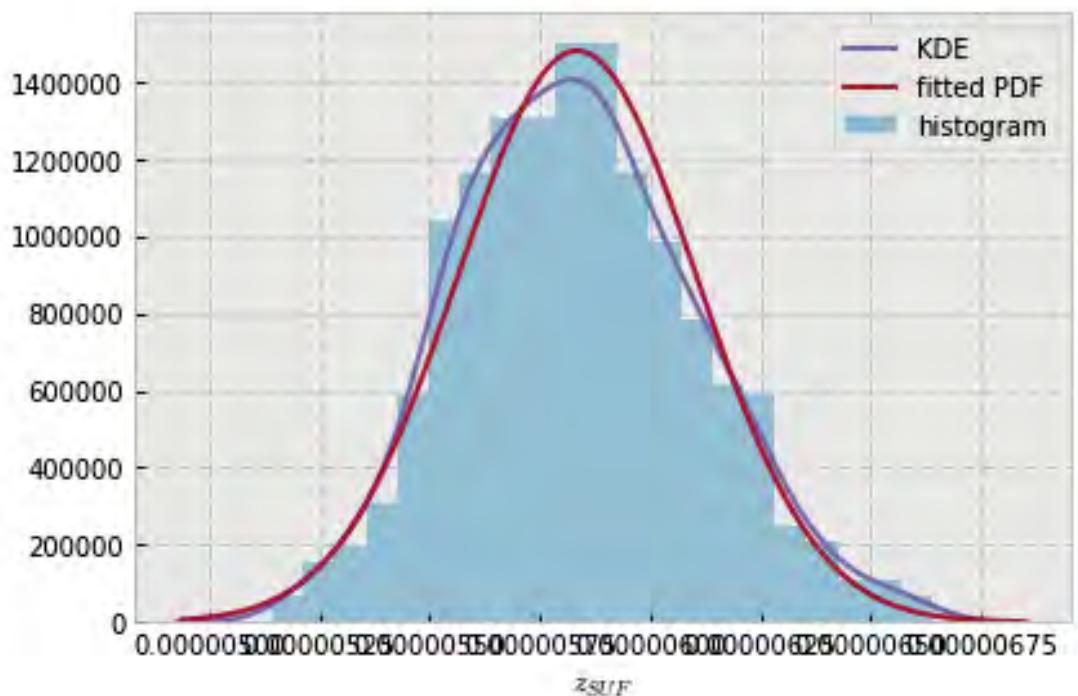


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=6.06416229151387e-06, scale=3.5802137314465457e-07)
DescribeResult(nobs=1000, minmax=(4.8689678961350805e-06, 7.3610261336843045e-06)
               mean=6.0641622915138703e-06, variance=1.283076112396236e-13,
               skewness=0.12653132357236077, kurtosis=0.0949001859755807)
```

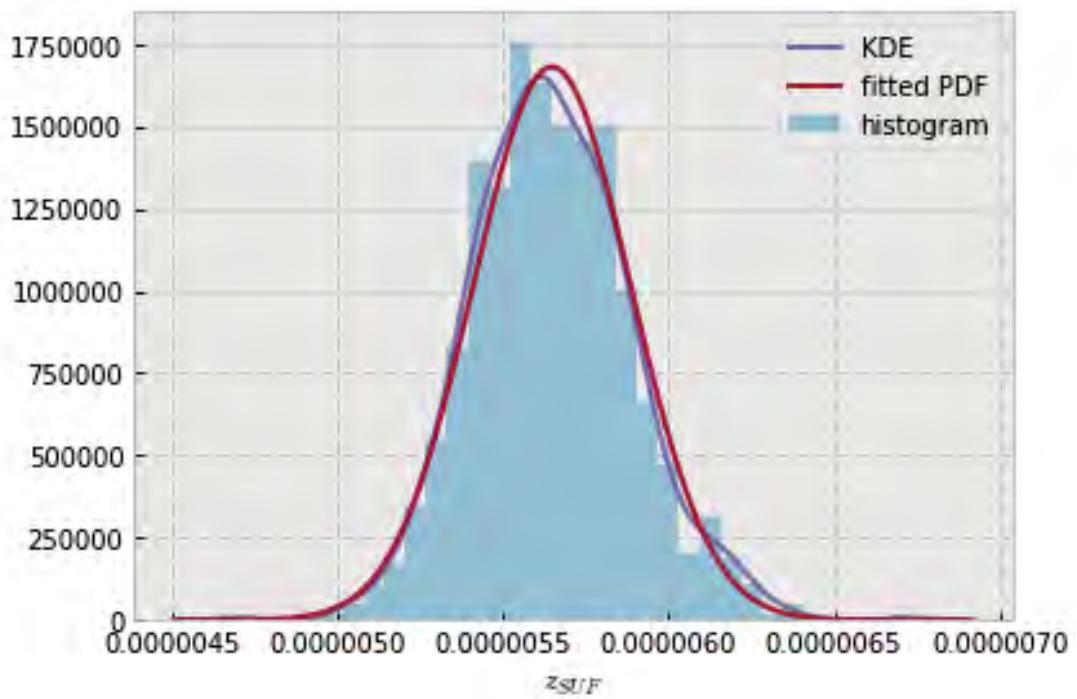


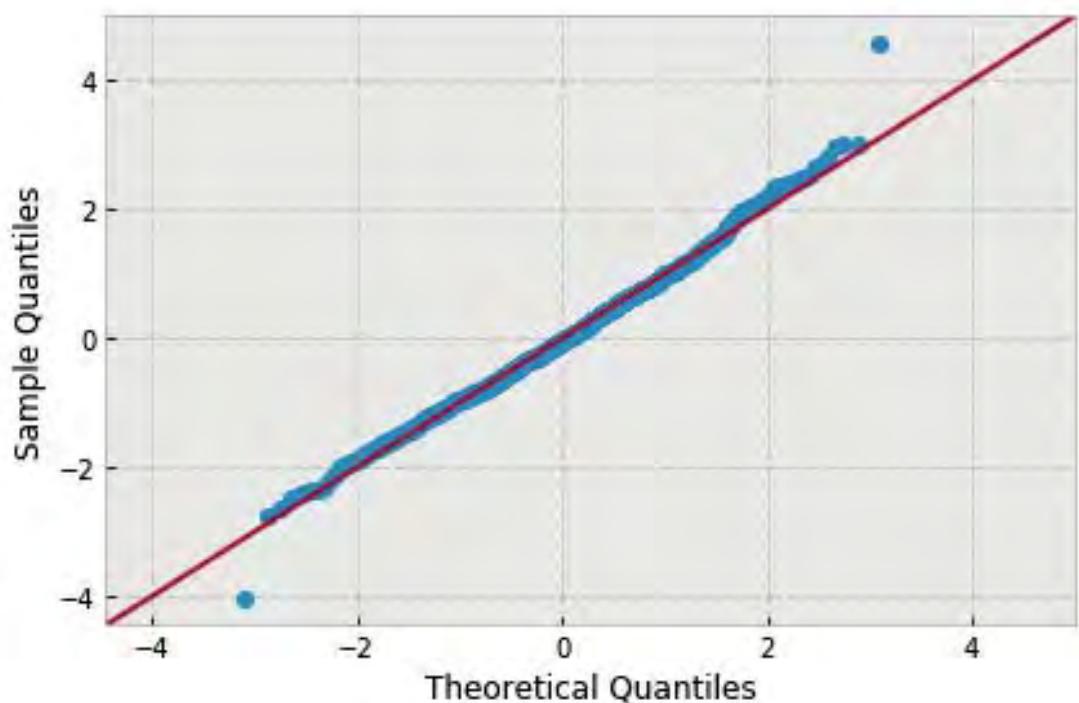


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=5.8345928418283565e-06, scale=2.6975244723590955e-07)
DescribeResult(nobs=1000, minmax=(5.142147094715605e-06, 6.6387083011691872e-06)
               mean=5.8345928418283565e-06, variance=7.2839222011773941e-14,
               skewness=0.24906768127306958, kurtosis=-0.16127280560557855)
```

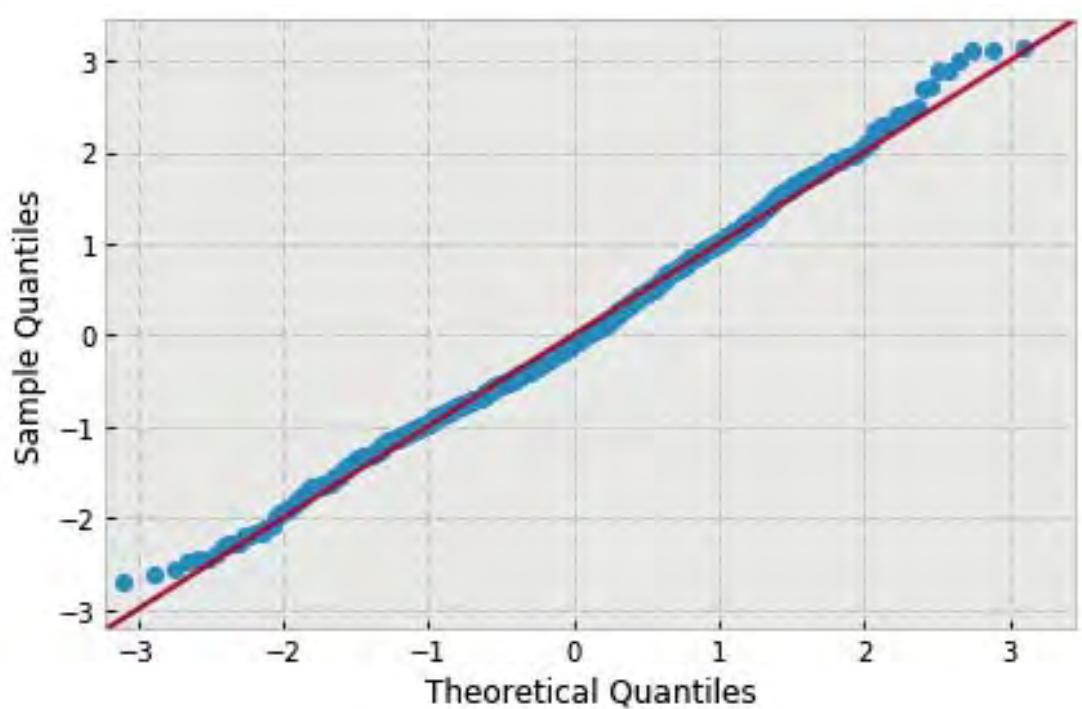
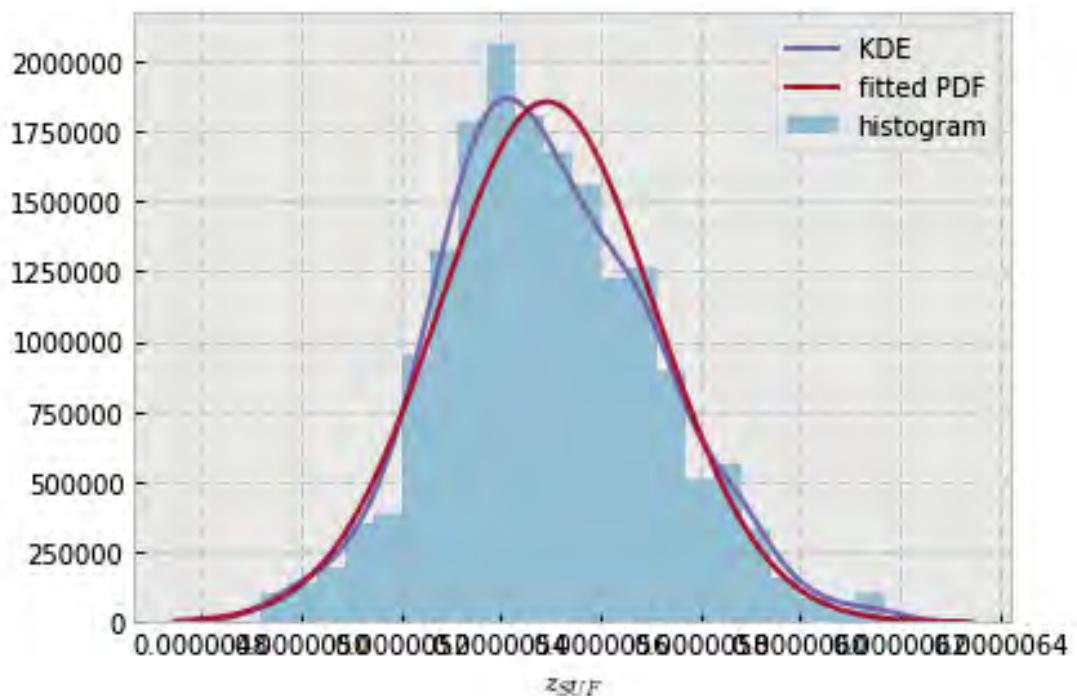


```
variable name: r
variable value: 2.11111111111111
distribution: normal(shape=(), loc=5.648015126207983e-06, scale=2.3743991487597214e-07)
DescribeResult(nobs=1000, minmax=(4.693624448747521e-06, 6.7318772534546503e-06)
               mean=5.648015126207983e-06, variance=5.6434147323632537e-14,
               skewness=0.2444549767391054, kurtosis=0.4888391276493089)
```

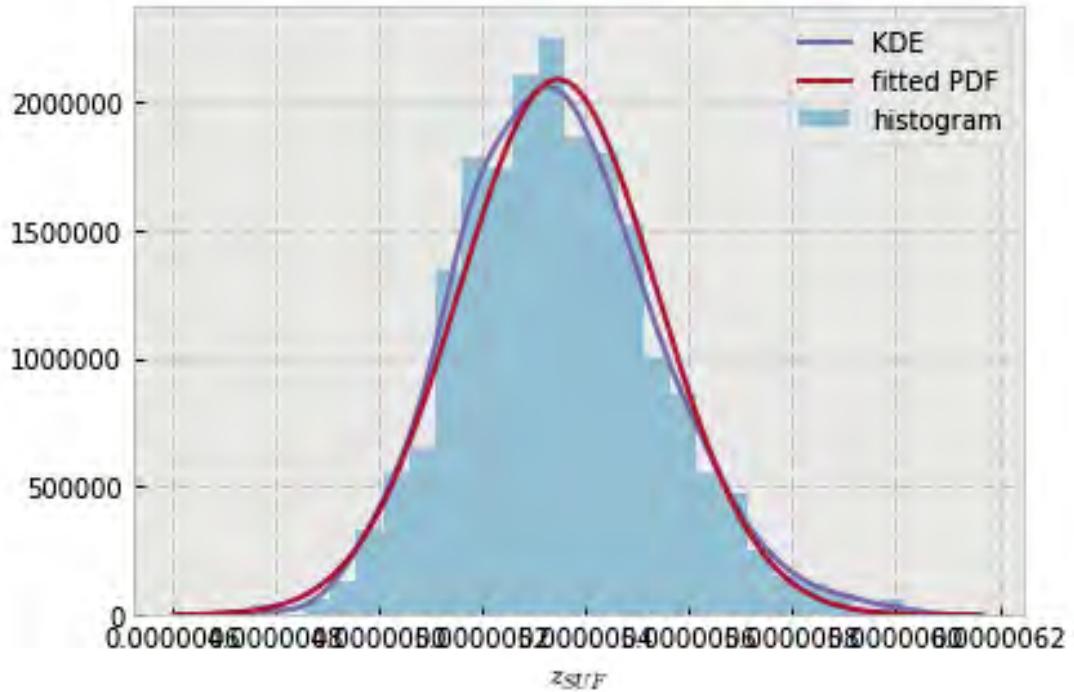


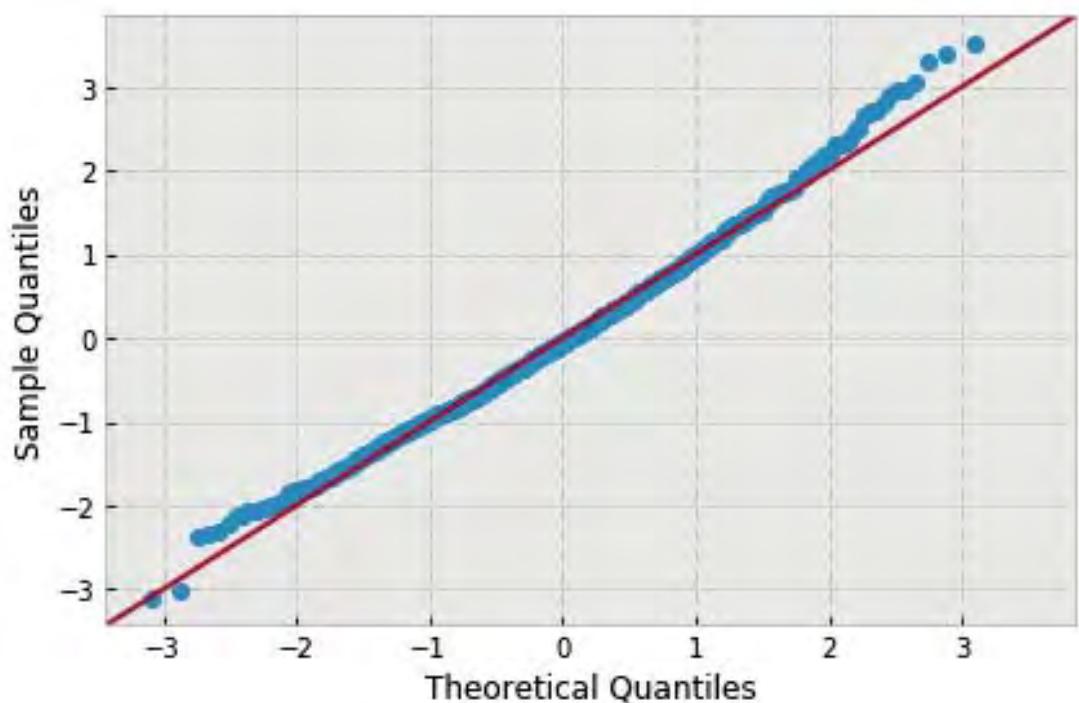


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=5.492303735016215e-06, scale=2.1515615945718632e-07)
DescribeResult(nobs=1000, minmax=(4.914442691104521e-06, 6.1706497984979328e-06)
               mean=5.4923037350162152e-06, variance=4.6338511463830015e-14,
               skewness=0.2544581801892668, kurtosis=-0.0059132591647972355)
```

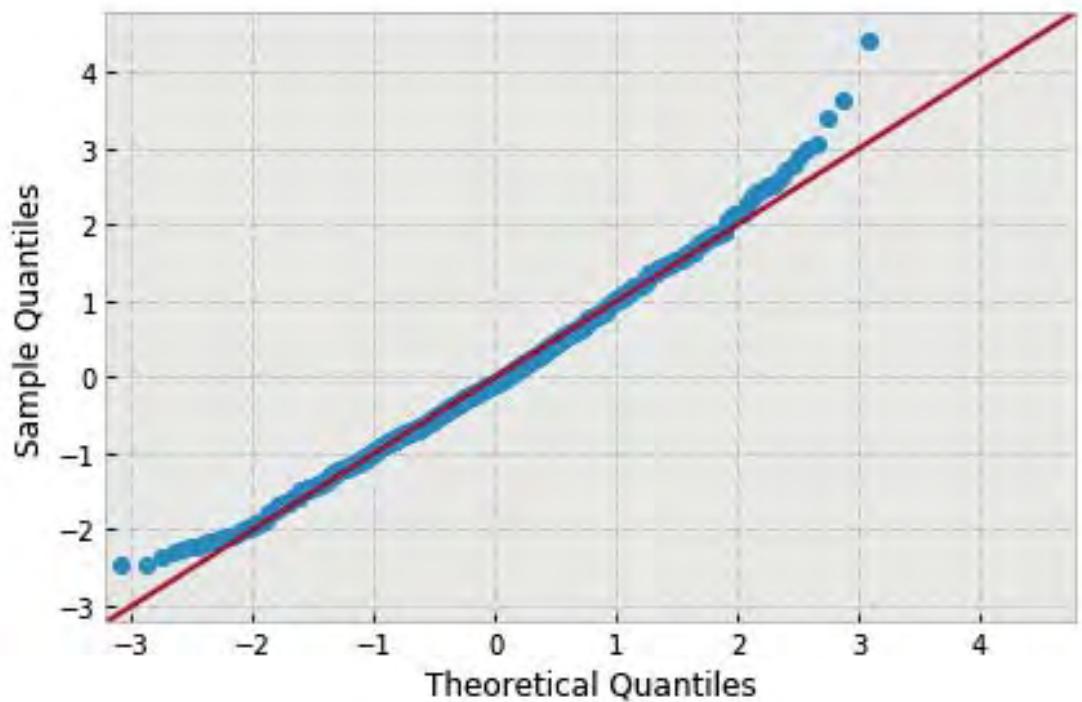
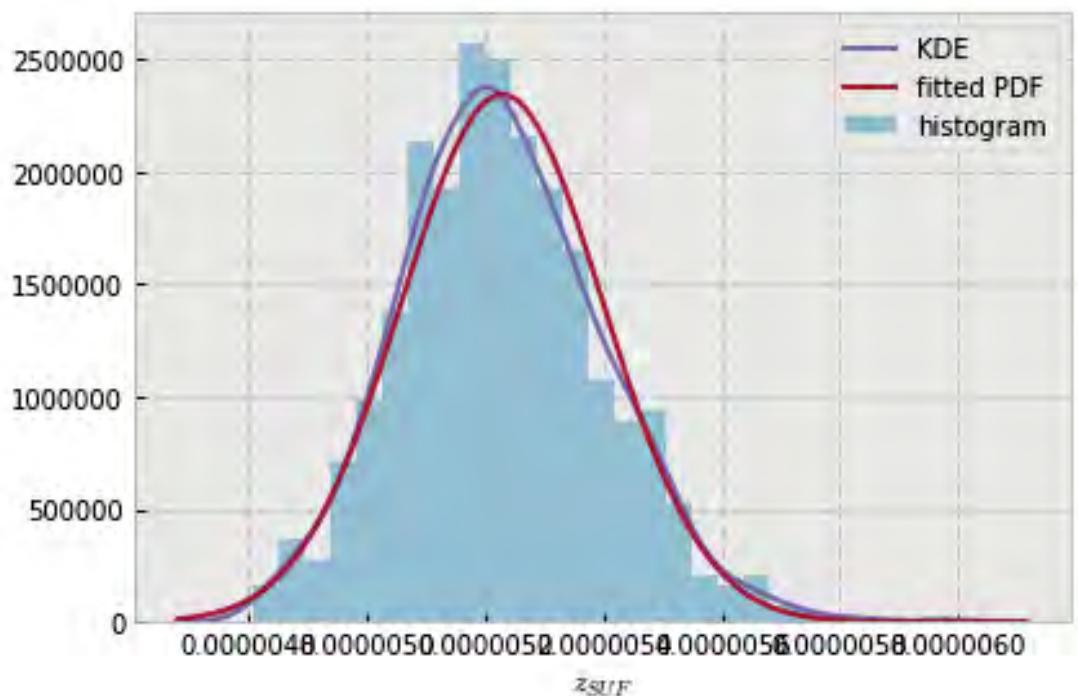


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=5.346032226561337e-06, scale=1.9131547766796356e-07)
DescribeResult(nobs=1000, minmax=(4.7535053489590139e-06, 6.0178754703041498e-06)
               mean=5.3460322265613371e-06, variance=3.6638250245566633e-14,
               skewness=0.33200146748889003, kurtosis=0.22169921410682258)
```

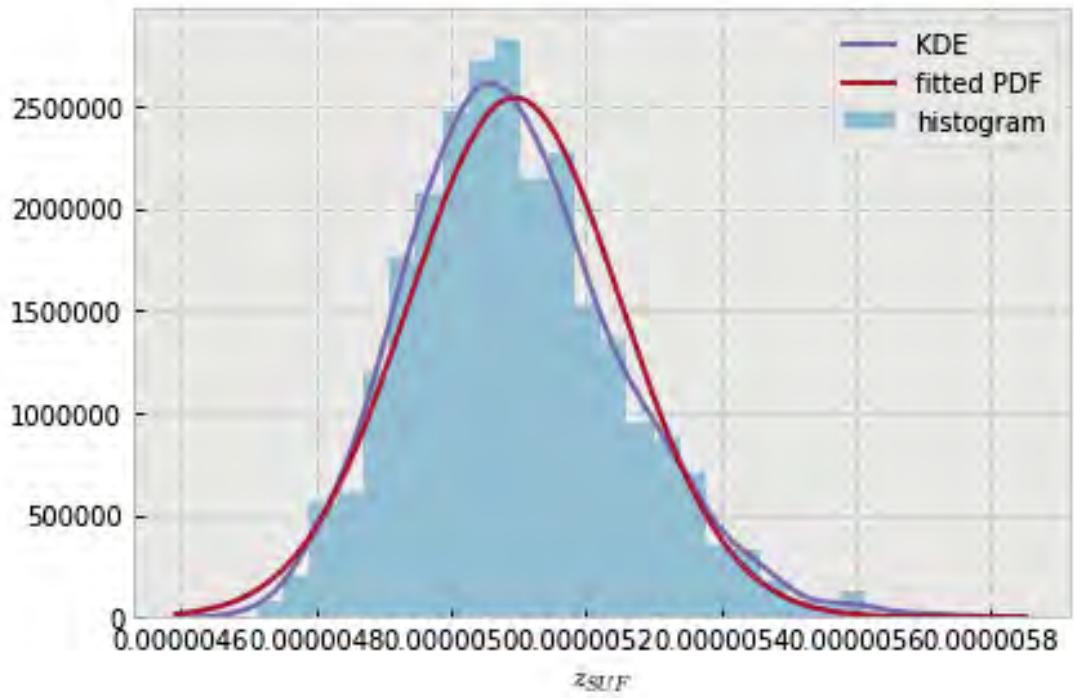


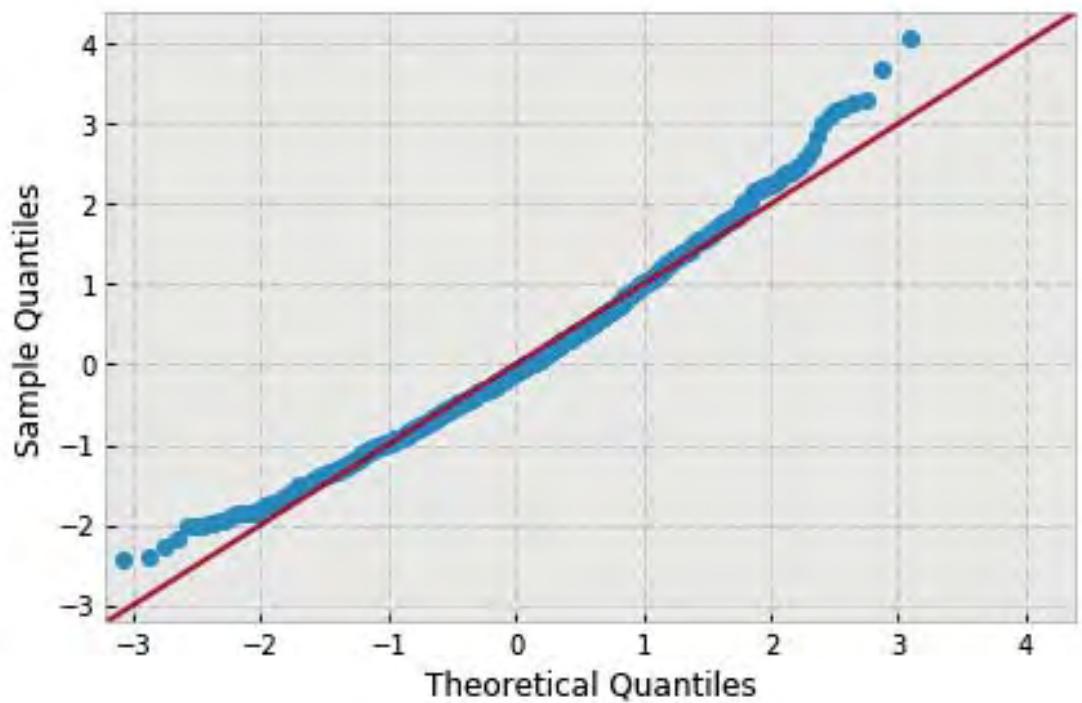


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=5.228691264912813e-06, scale=1.704276260643278e-07)
DescribeResult(nobs=1000, minmax=(4.8097446808118126e-06, 5.9847349683509963e-06)
               mean=5.2286912649128128e-06, variance=2.9074650376298641e-14,
               skewness=0.35292350207305323, kurtosis=0.34878202004110204)
```

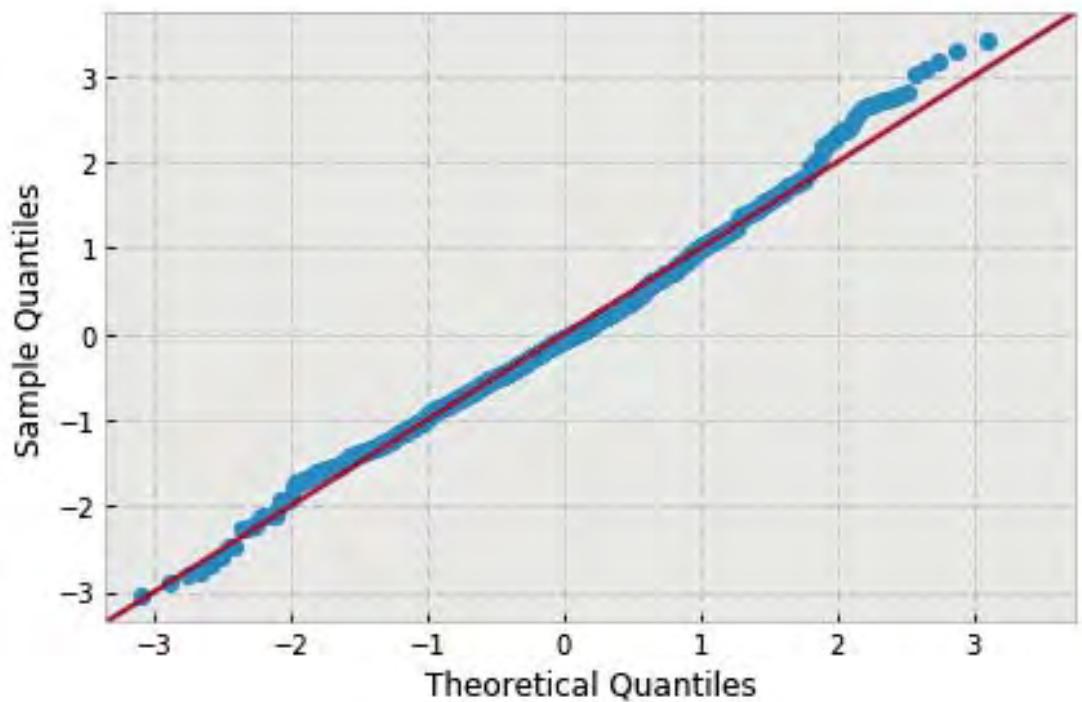
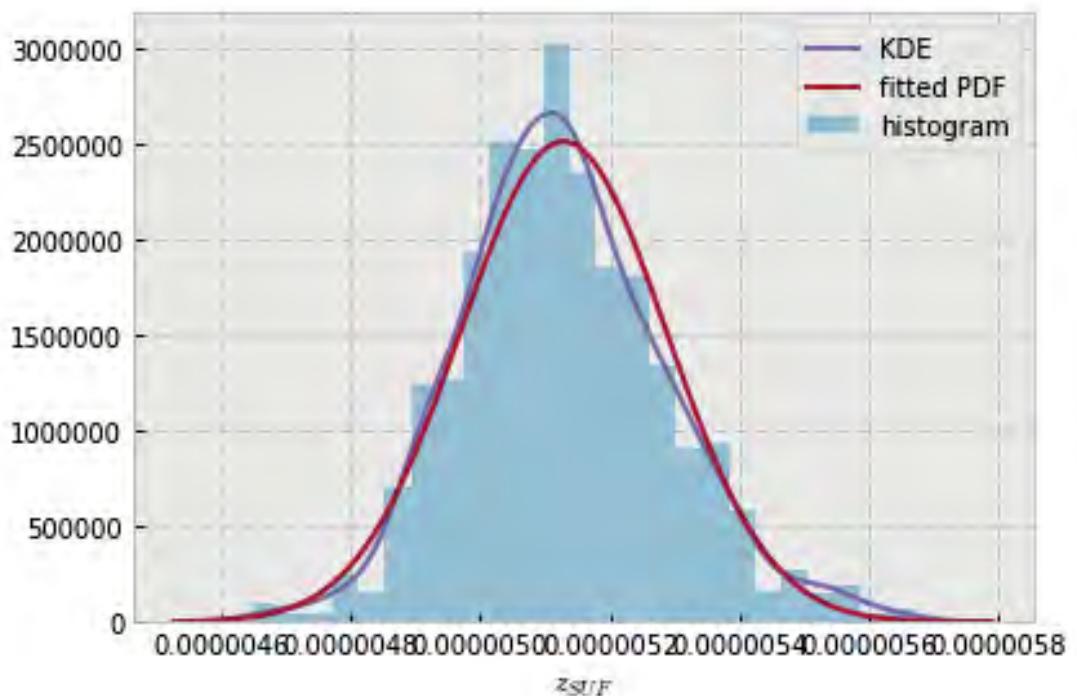


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=5.094536242143038e-06, scale=1.571056628539857e-07)
DescribeResult(nobs=1000, minmax=(4.7122621241924857e-06, 5.7317488119935458e-06)
               mean=5.0945362421430377e-06, variance=2.4706896196987208e-14,
               skewness=0.5163016233047991, kurtosis=0.3798960221614869)
```

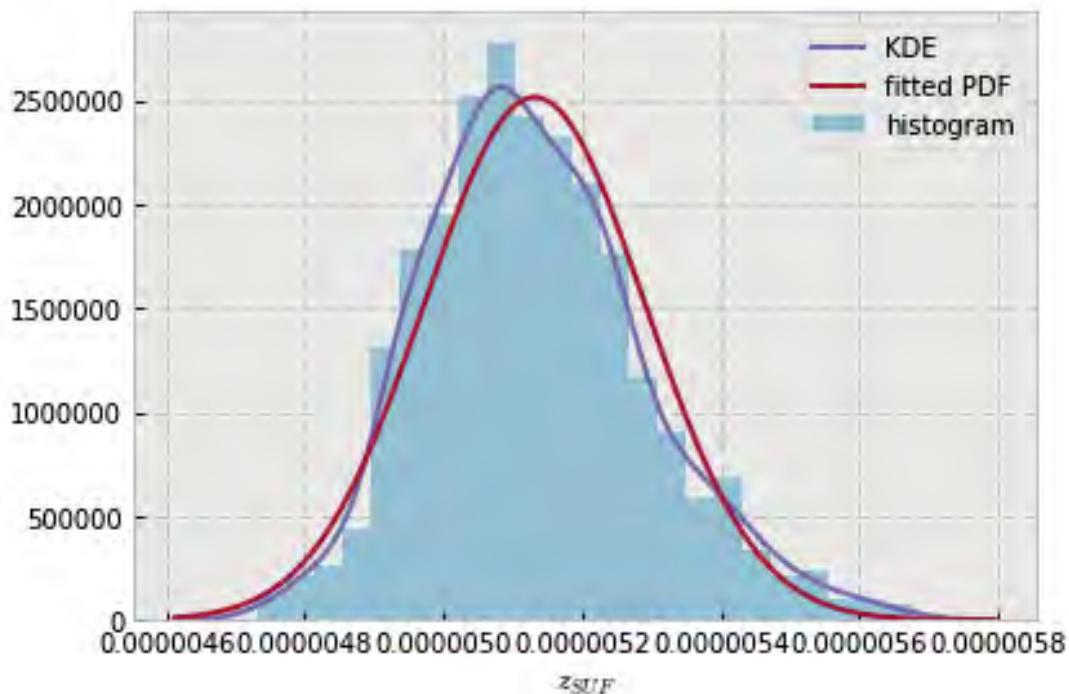


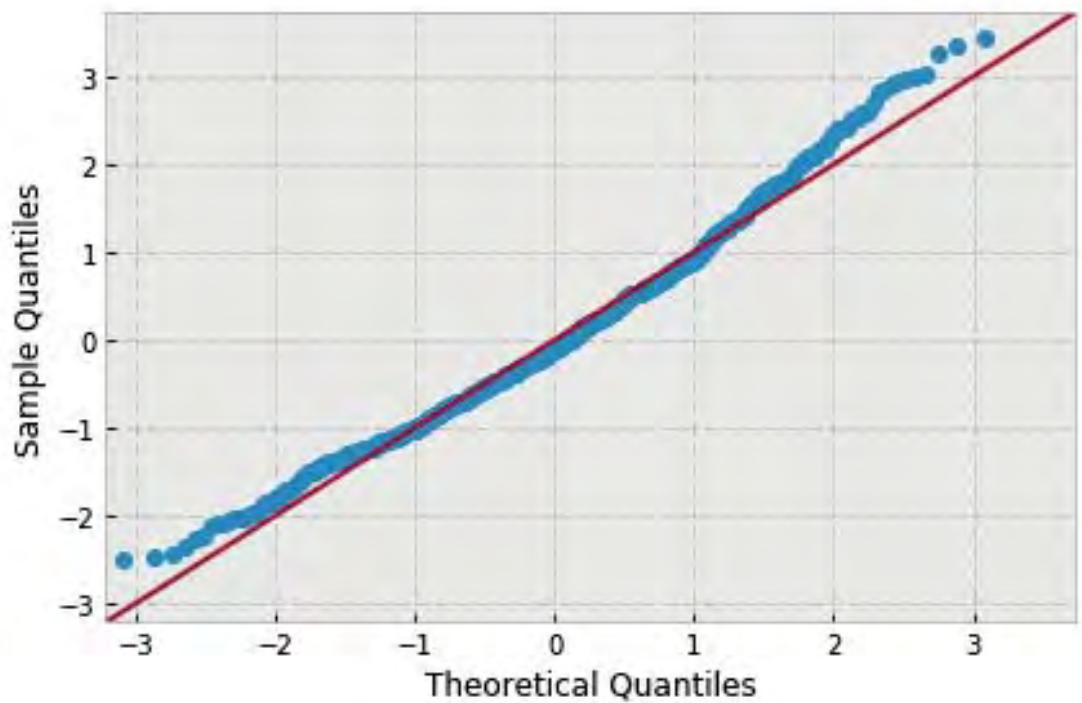


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=5.128432787652644e-06, scale=1.58592220778978e-07)
DescribeResult(nobs=1000, minmax=(4.6482635560268412e-06, 5.6693207561380973e-06)
               mean=5.1284327876526443e-06, variance=2.5176669160768874e-14,
               skewness=0.3110905863933188, kurtosis=0.37849974002845954)
```

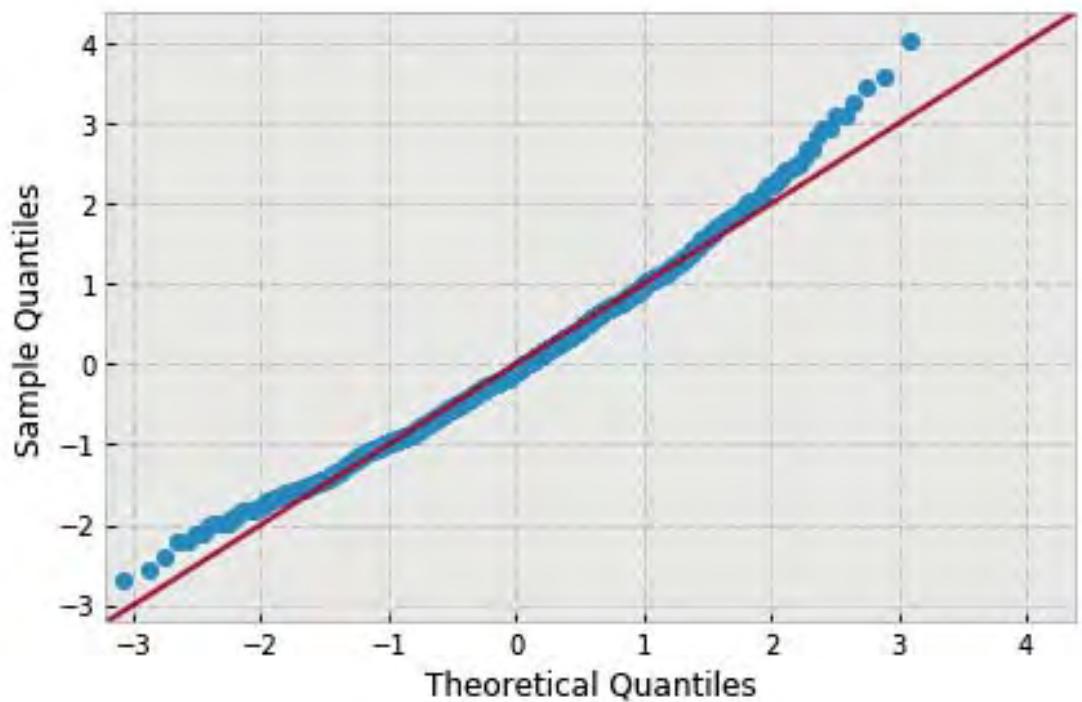
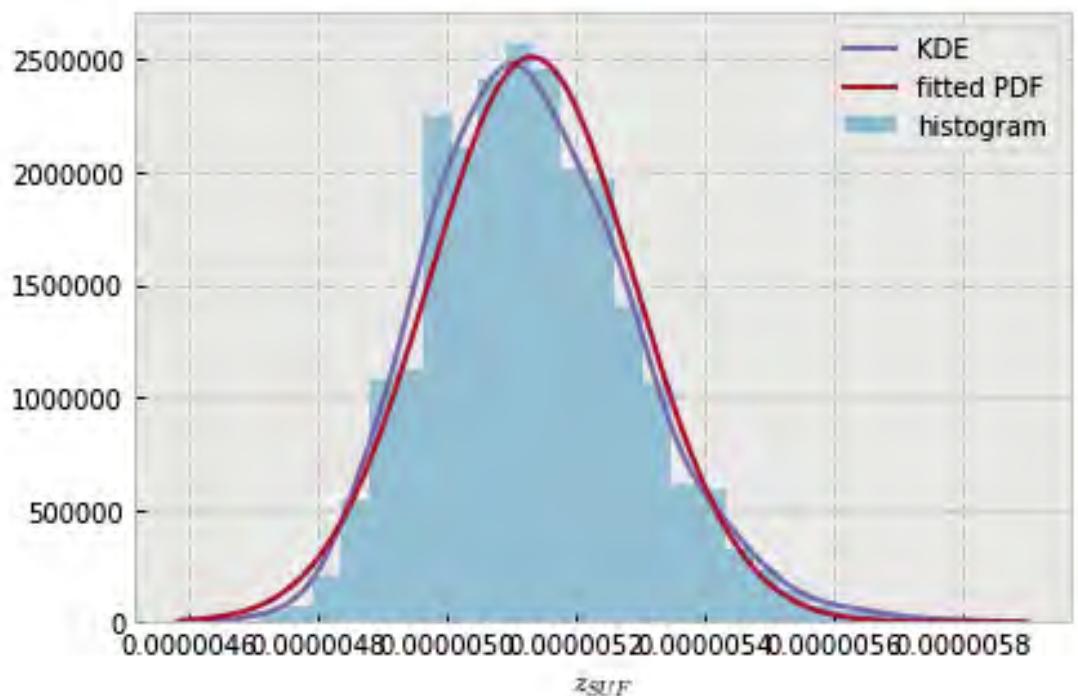


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=5.131585701754121e-06, scale=1.58711625834444e-07)
DescribeResult(nobs=1000, minmax=(4.73332207937374e-06, 5.6751728114165217e-06)
               mean=5.1315857017541209e-06, variance=2.5214594769782333e-14,
               skewness=0.4808872122605223, kurtosis=0.25941279898516534)
```

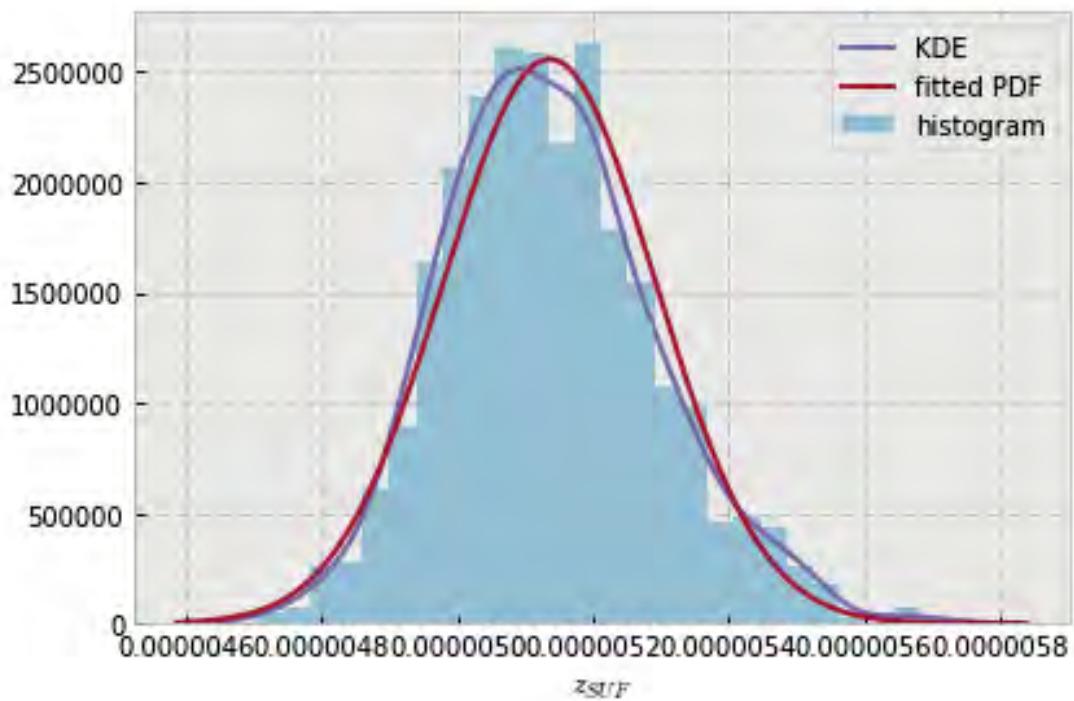


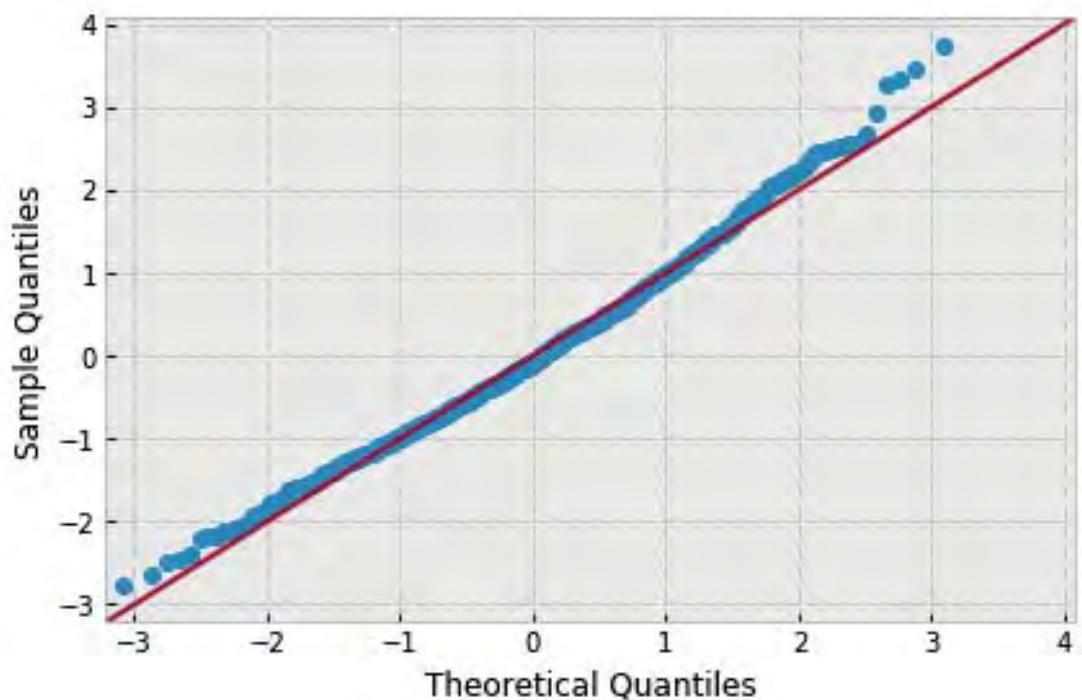


```
variable name: sigma
variable value: 0.03333333333333333
distribution: normal(shape=(), loc=5.131319781452394e-06, scale=1.588368048989729e-07)
DescribeResult(nobs=1000, minmax=(4.7064600954228993e-06, 5.7732524484888106e-06)
               mean=5.131319781452394e-06, variance=2.525438497548987e-14,
               skewness=0.4510188085674612, kurtosis=0.3399975400588744)
```

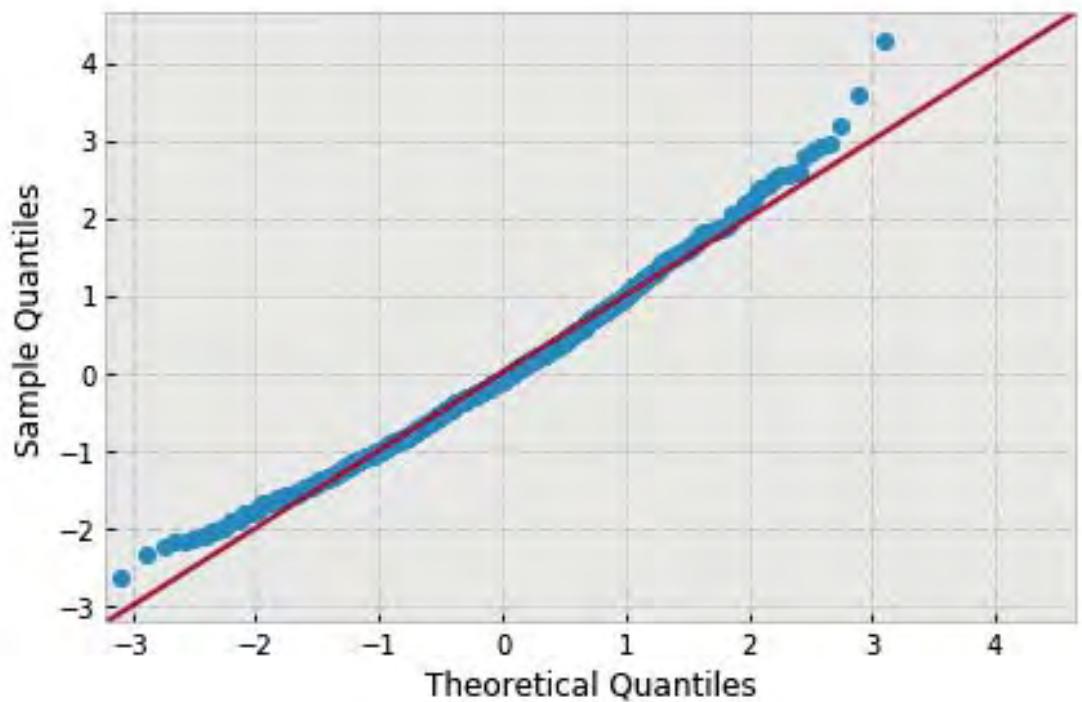
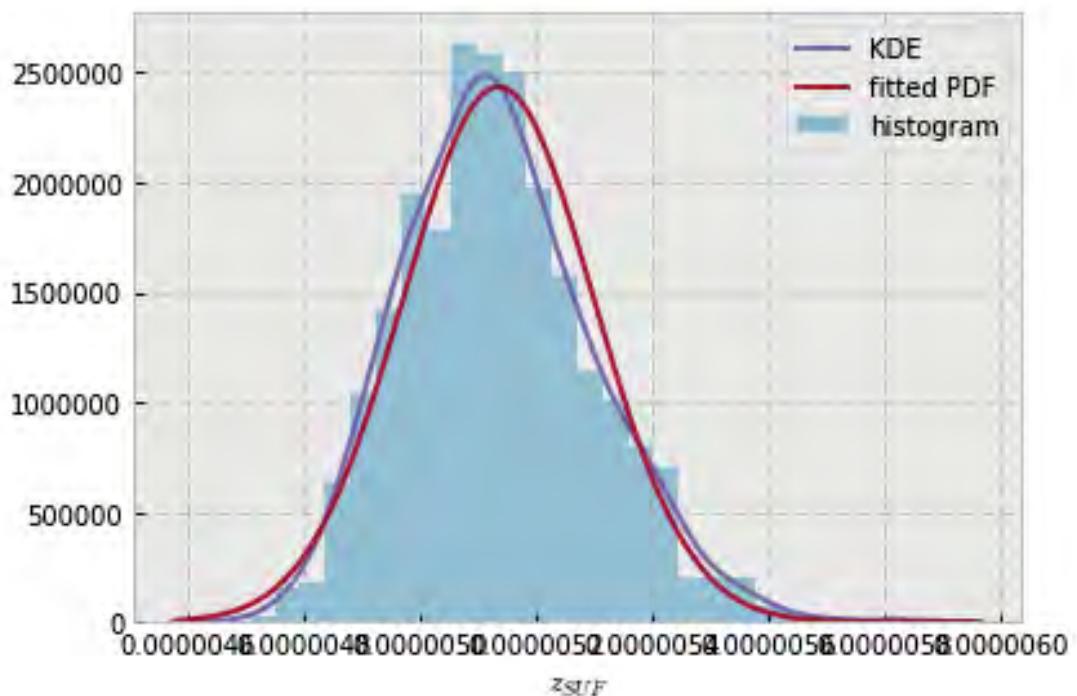


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=5.136175701925101e-06, scale=1.5591729504687543e-07)
DescribeResult(nobs=1000, minmax=(4.7051128440996742e-06, 5.7200411741298577e-06)
               mean=5.1361757019251013e-06, variance=2.4334537432166576e-14,
               skewness=0.3830361890402258, kurtosis=0.22317623351701243)
```

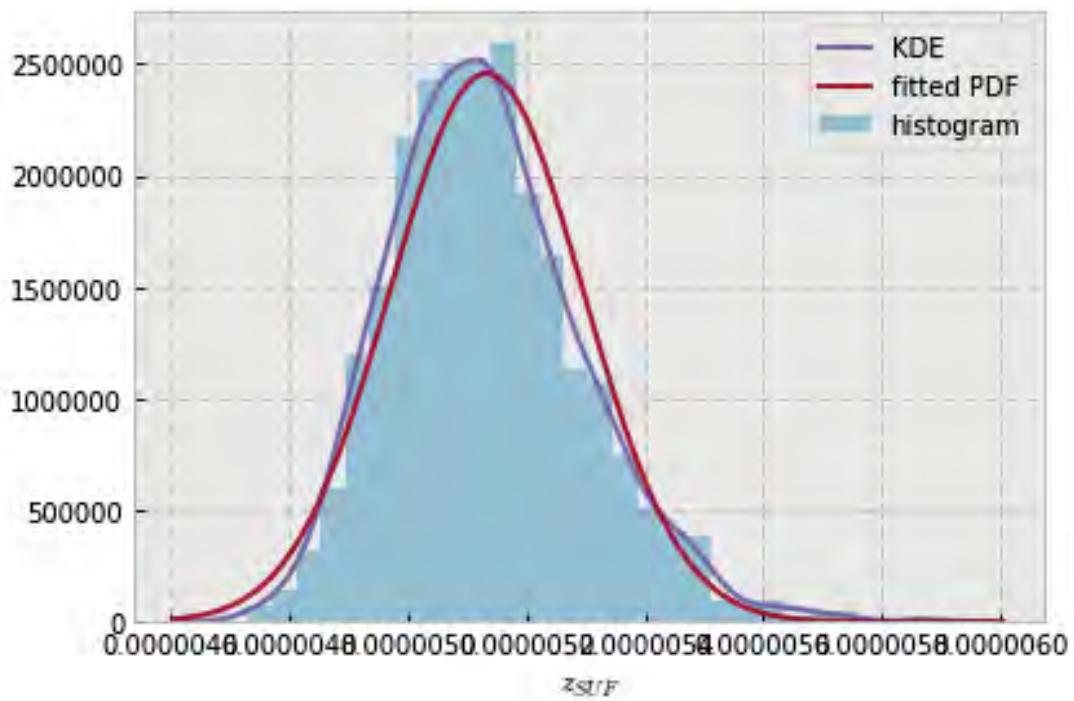


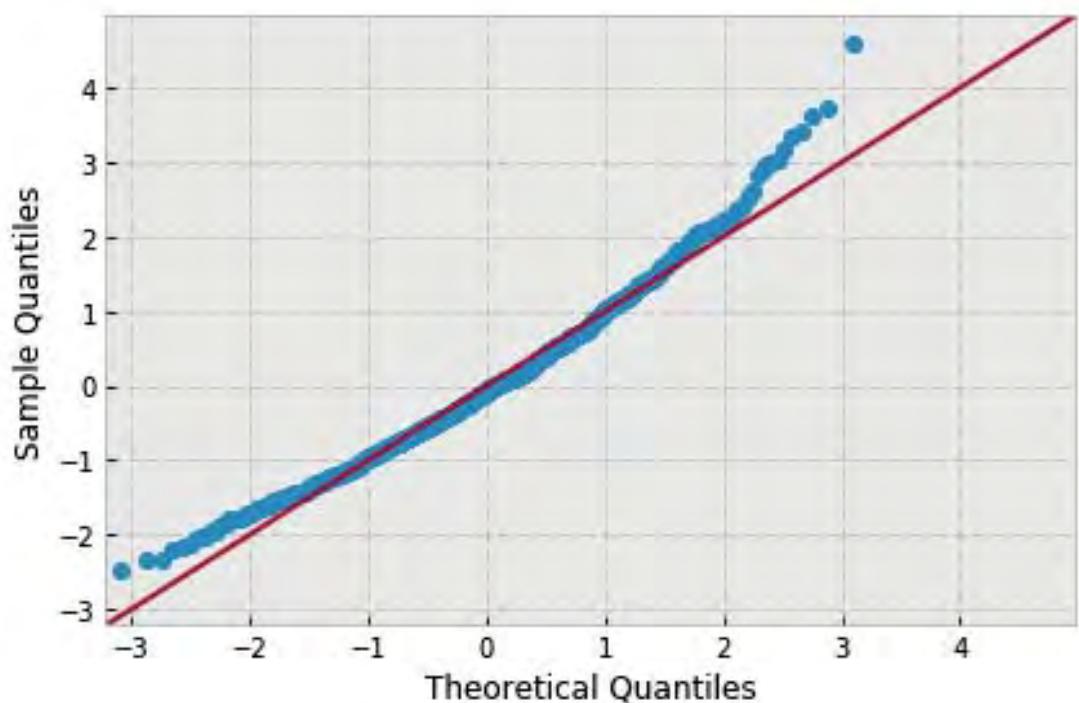


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=5.135795420895345e-06, scale=1.637691791292173e-07)
DescribeResult(nobs=1000, minmax=(4.7054085335299515e-06, 5.8384608617299631e-06)
               mean=5.1357954208953451e-06, variance=2.6847191223881538e-14,
               skewness=0.4320208662975798, kurtosis=0.218805011587468)
```

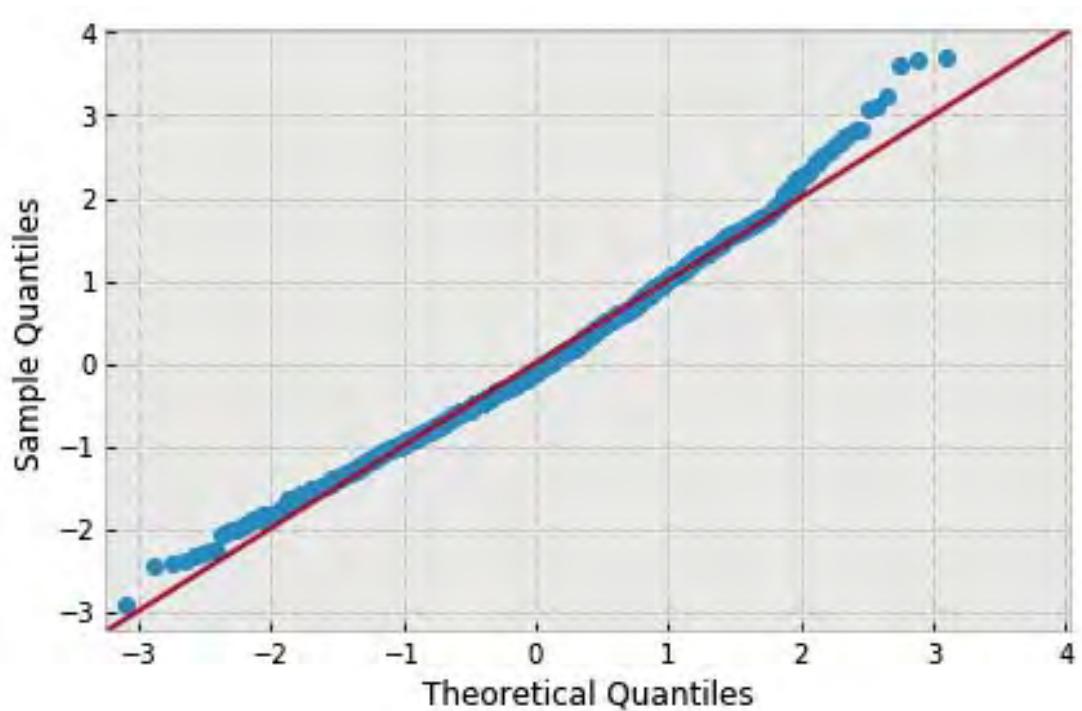
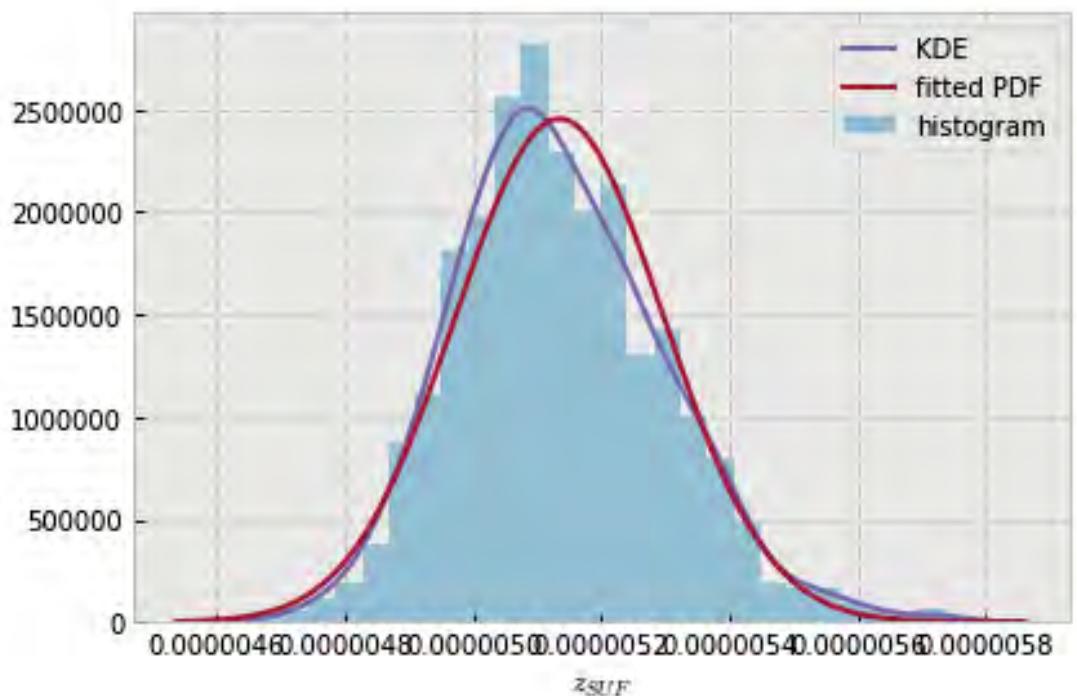


```
variable name: sigma
variable value: 0.06666666666666667
distribution: normal(shape=(), loc=5.132868328025852e-06, scale=1.6234605864501015e-07)
DescribeResult(nobs=1000, minmax=(4.7296243045587678e-06, 5.881207095868826e-06)
               mean=5.1328683280258522e-06, variance=2.6382625382952026e-14,
               skewness=0.5916728785518045, kurtosis=0.6704901032190631)
```

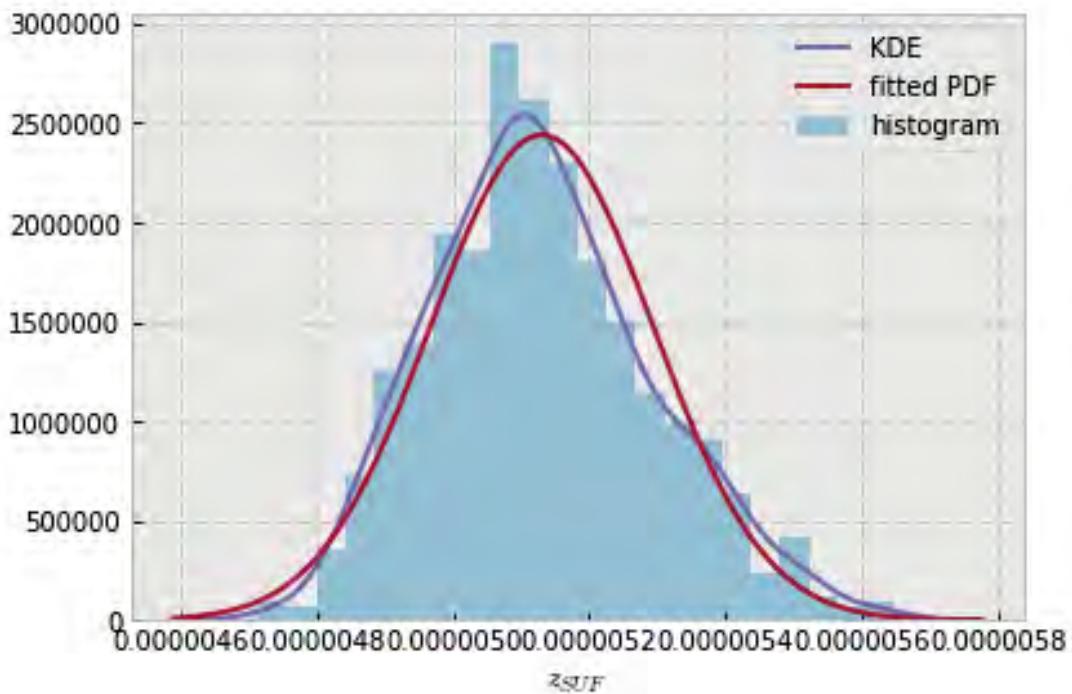


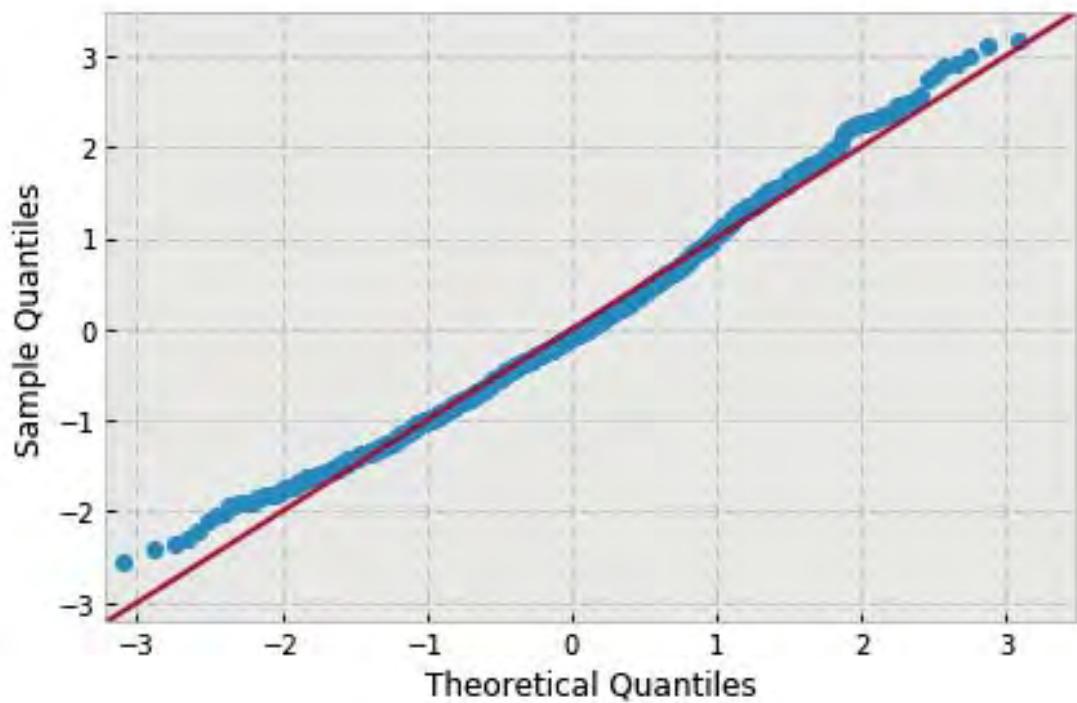


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=5.134276401055037e-06, scale=1.6250519726878384e-07)
DescribeResult(nobs=1000, minmax=(4.6612293331712302e-06, 5.7365653574656909e-06)
               mean=5.1342764010550372e-06, variance=2.6434373512879229e-14,
               skewness=0.4217681466263346, kurtosis=0.3248027893150618)
```

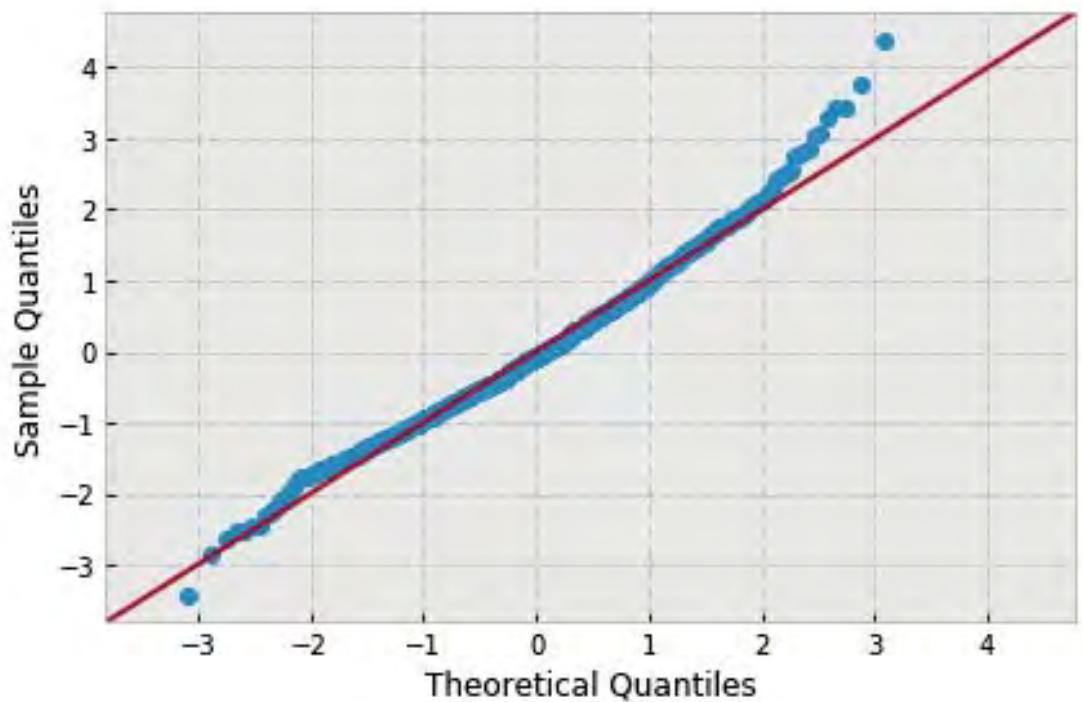
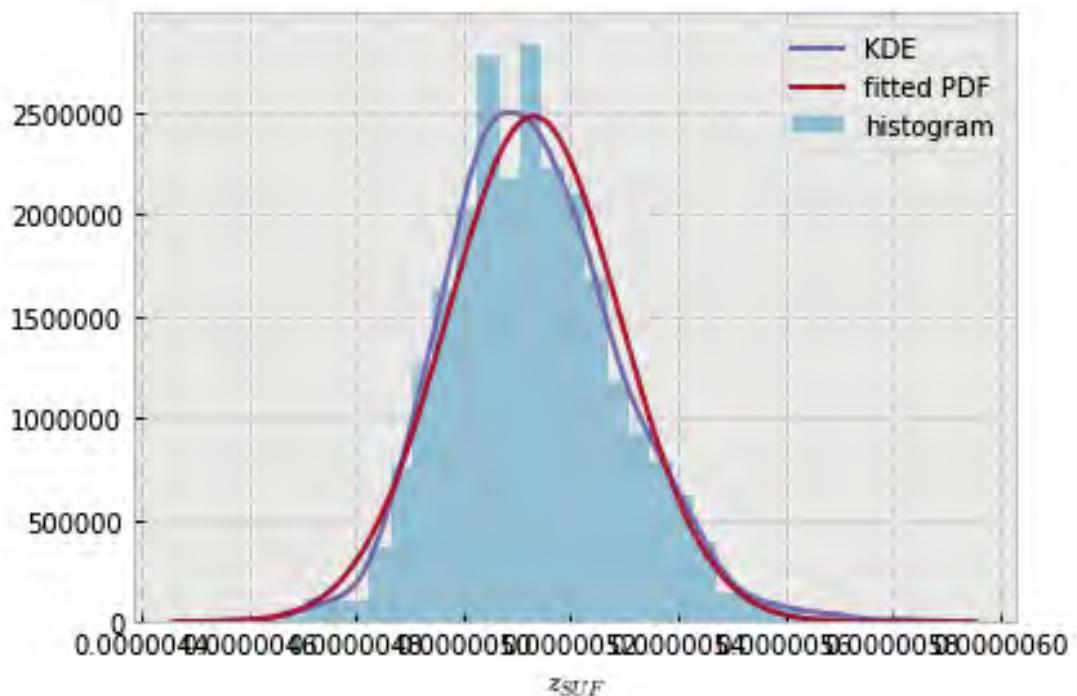


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=5.130336305123606e-06, scale=1.6358642551565547e-07)
DescribeResult(nobs=1000, minmax=(4.7136745411286467e-06, 5.6515584109825988e-06)
               mean=5.1303363051236064e-06, variance=2.6787305918908003e-14,
               skewness=0.3791305167577935, kurtosis=-0.08204513433808502)
```

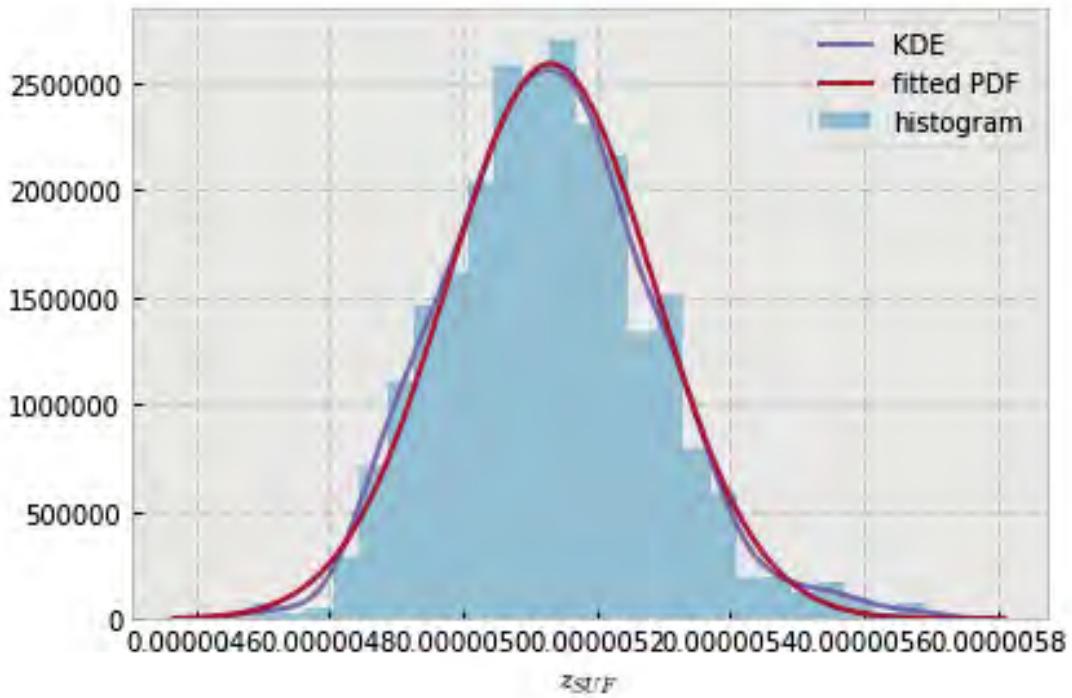


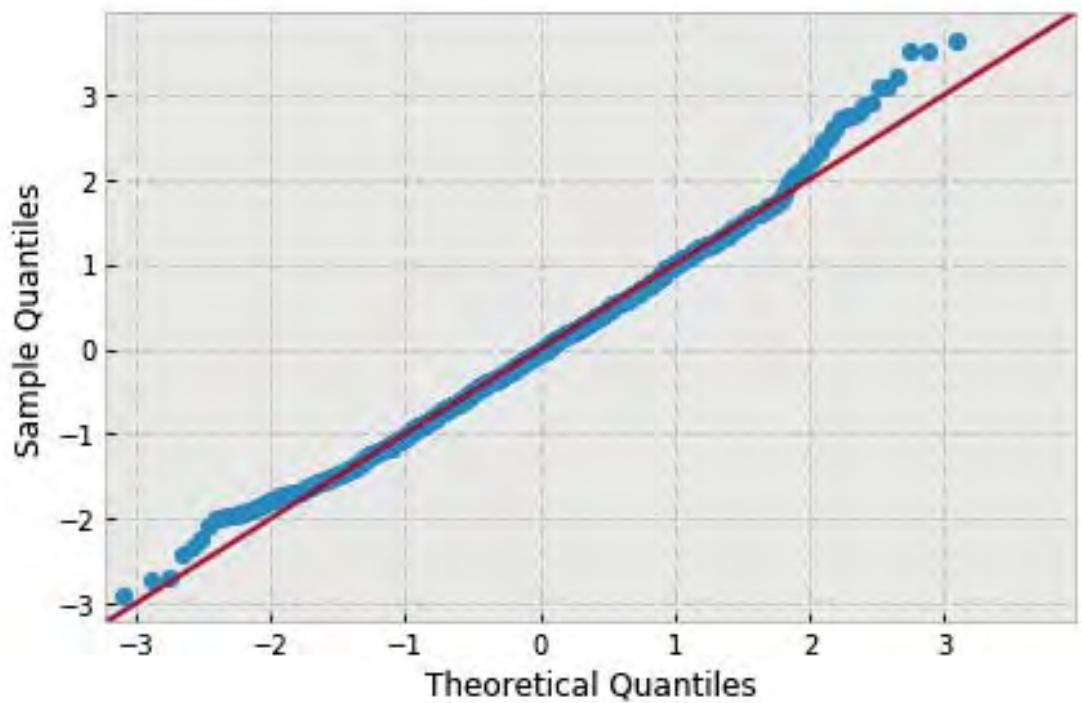


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=5.130635602612961e-06, scale=1.6080247303670128e-07)
DescribeResult(nobs=1000, minmax=(4.5809594817261983e-06, 5.8340084985763946e-06)
               mean=5.1306356026129611e-06, variance=2.5883318653372412e-14,
               skewness=0.42231210169642447, kurtosis=0.6543424235583188)
```

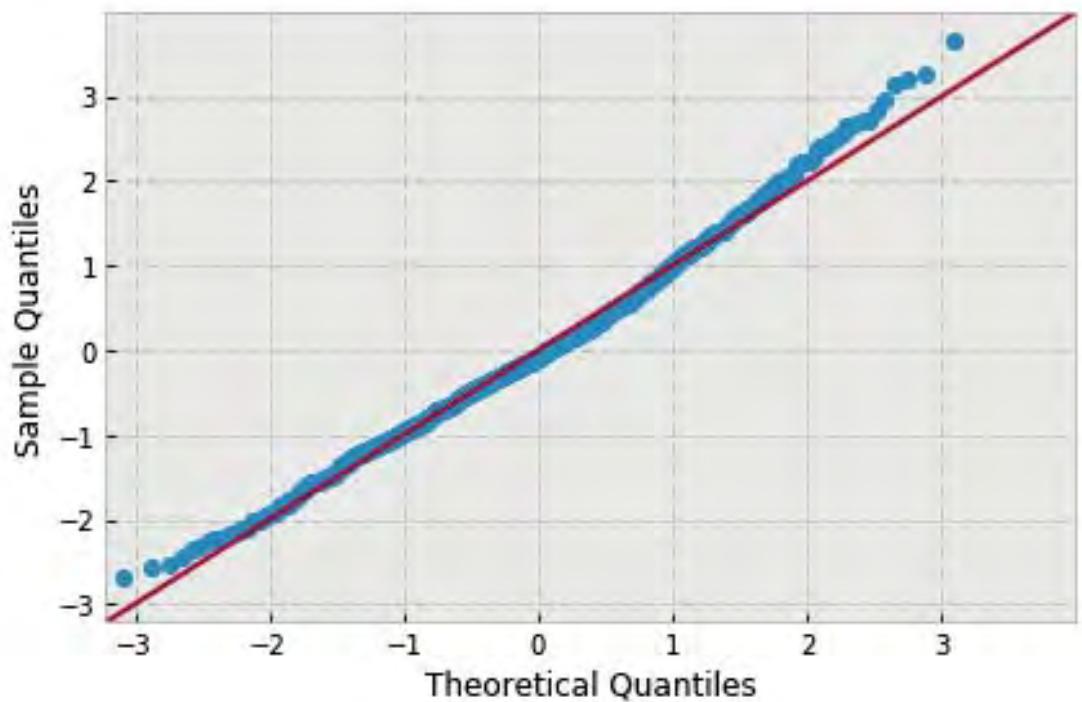
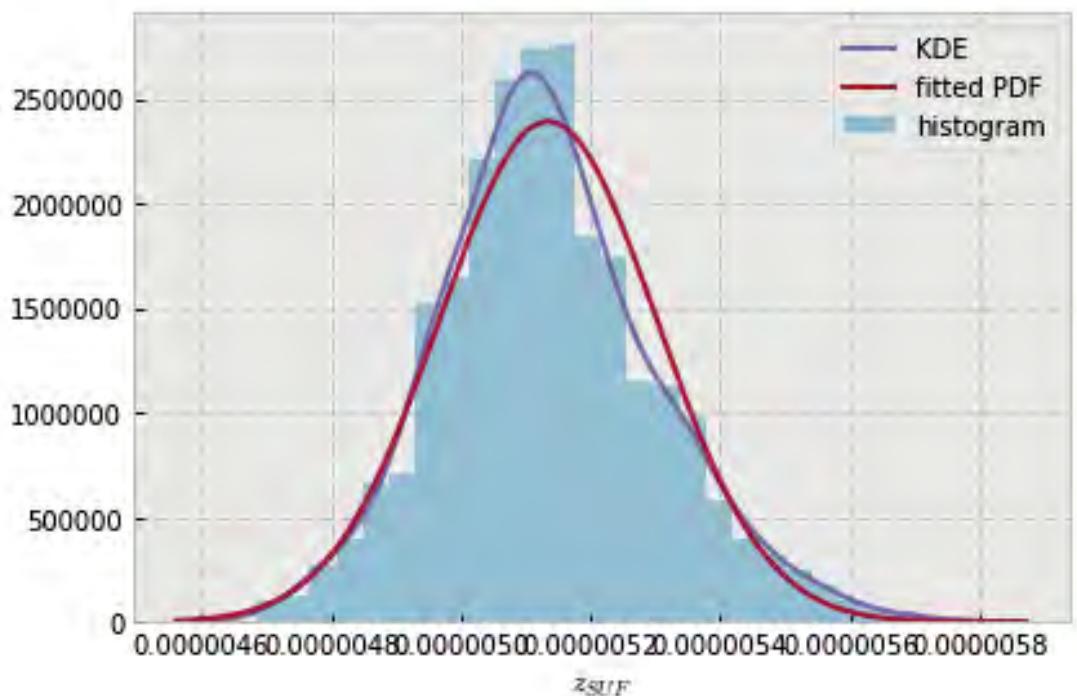


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=5.131362247824867e-06, scale=1.5401951678963124e-07)
DescribeResult(nobs=1000, minmax=(4.6855858245775502e-06, 5.6914172353644209e-06)
               mean=5.1313622478248666e-06, variance=2.3745757309420921e-14,
               skewness=0.3279201549247228, kurtosis=0.3179532178498583)
```

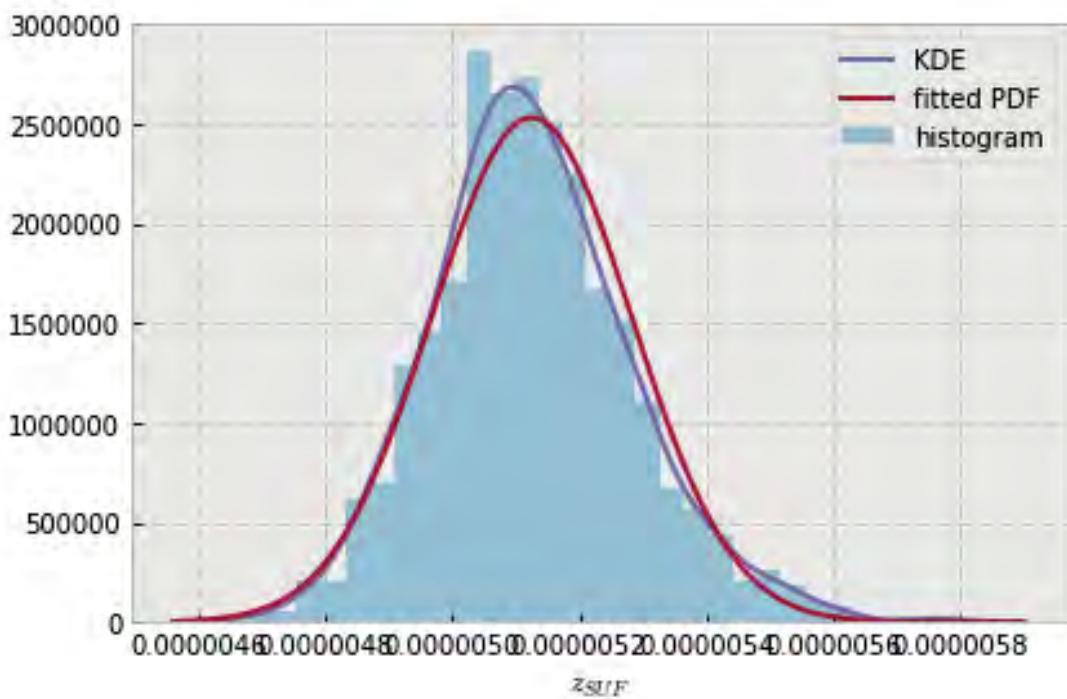


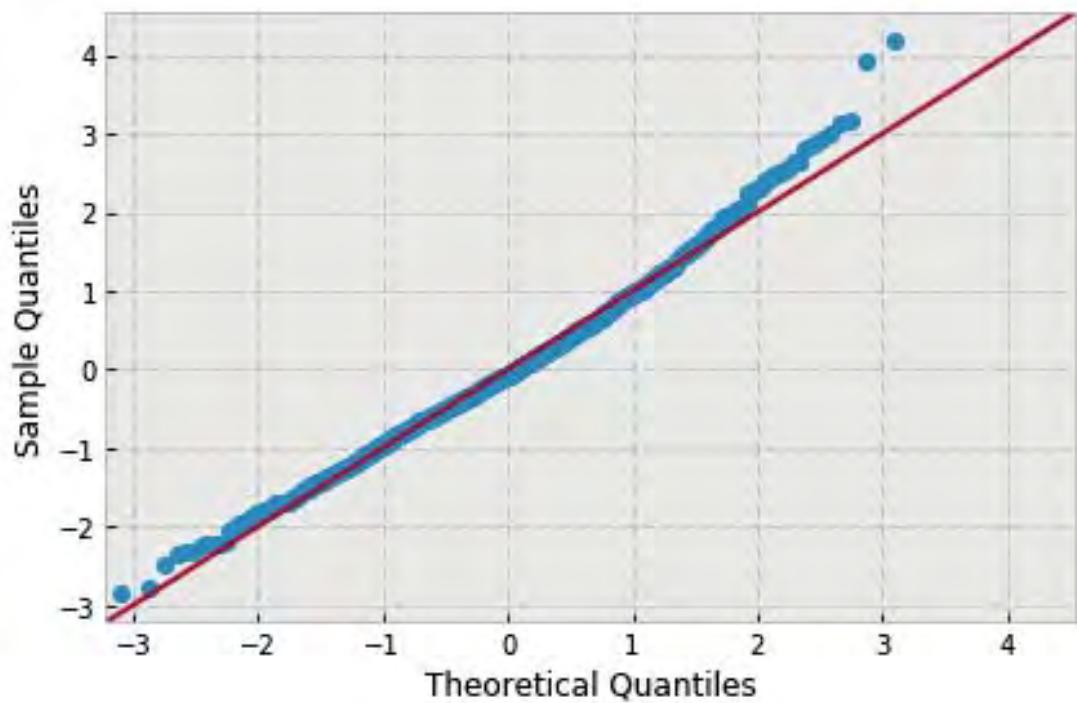


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=5.134893538269826e-06, scale=1.6687664257547272e-07)
DescribeResult(nobs=1000, minmax=(4.6857877096882925e-06, 5.746580458578384e-06)
               mean=5.1348935382698264e-06, variance=2.7875689526788861e-14,
               skewness=0.35173359359854284, kurtosis=0.27922385555497575)
```

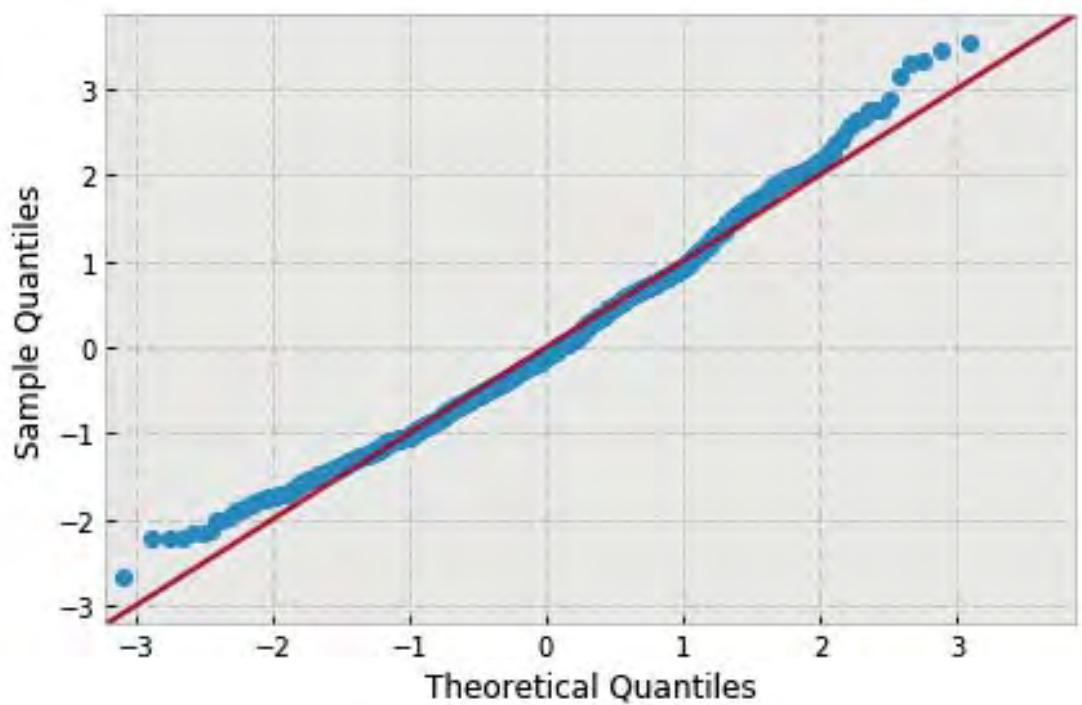
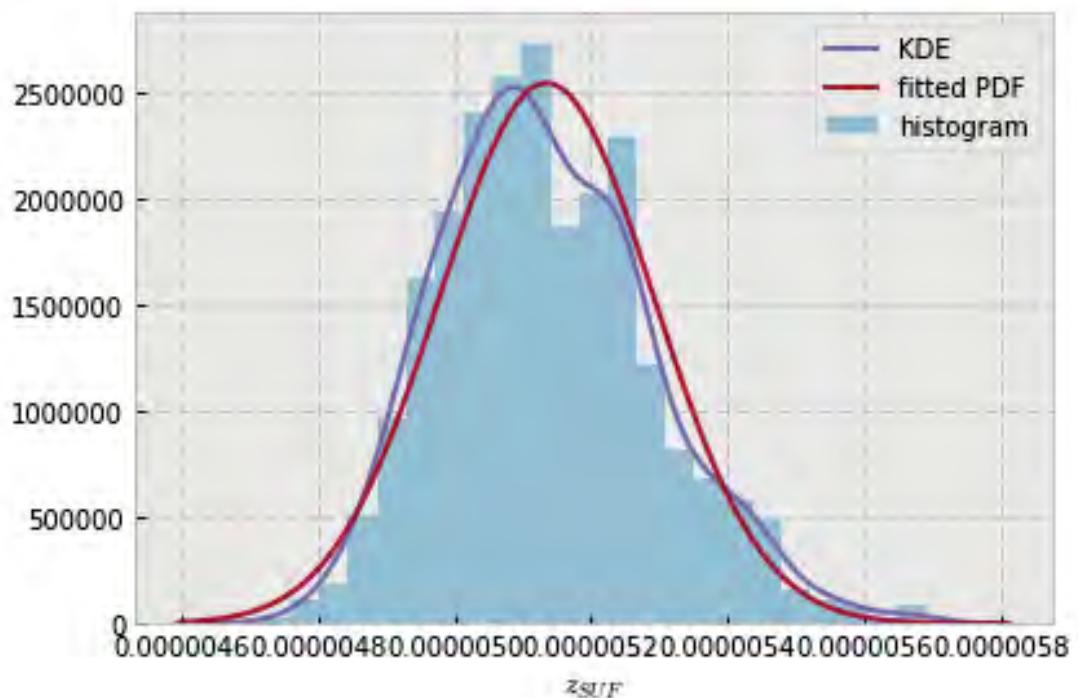


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=5.124951988083768e-06, scale=1.5751293714637465e-07)
DescribeResult(nobs=1000, minmax=(4.6783419707542135e-06, 5.783786749416861e-06)
               mean=5.1249519880837681e-06, variance=2.483516052900678e-14,
               skewness=0.4070655177356903, kurtosis=0.5454292105585967)
```

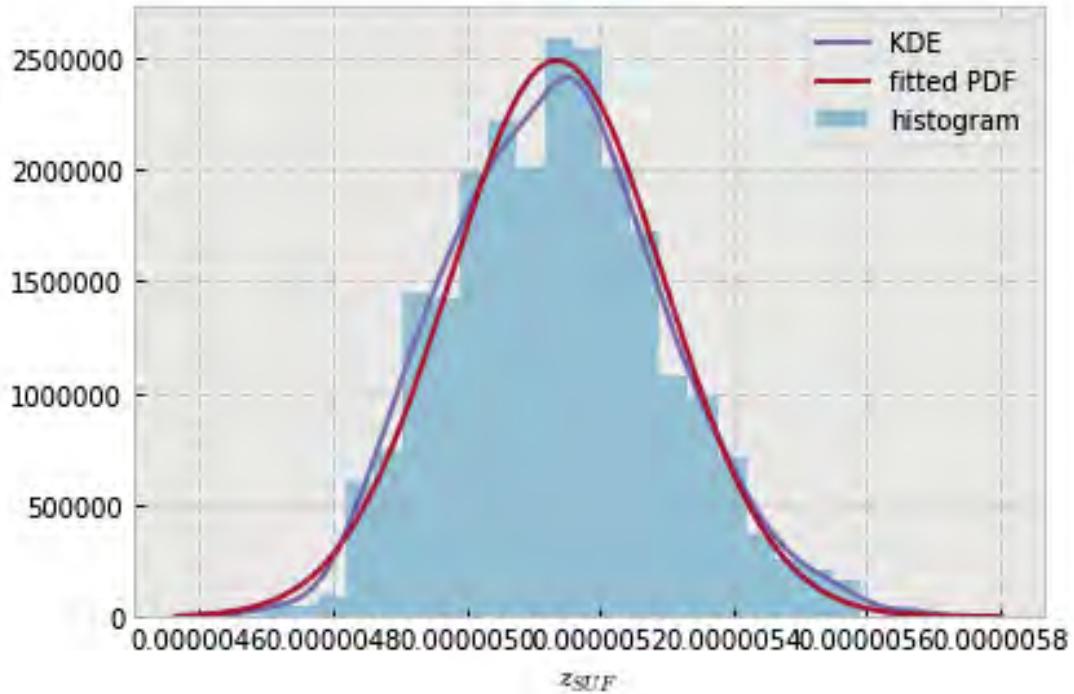


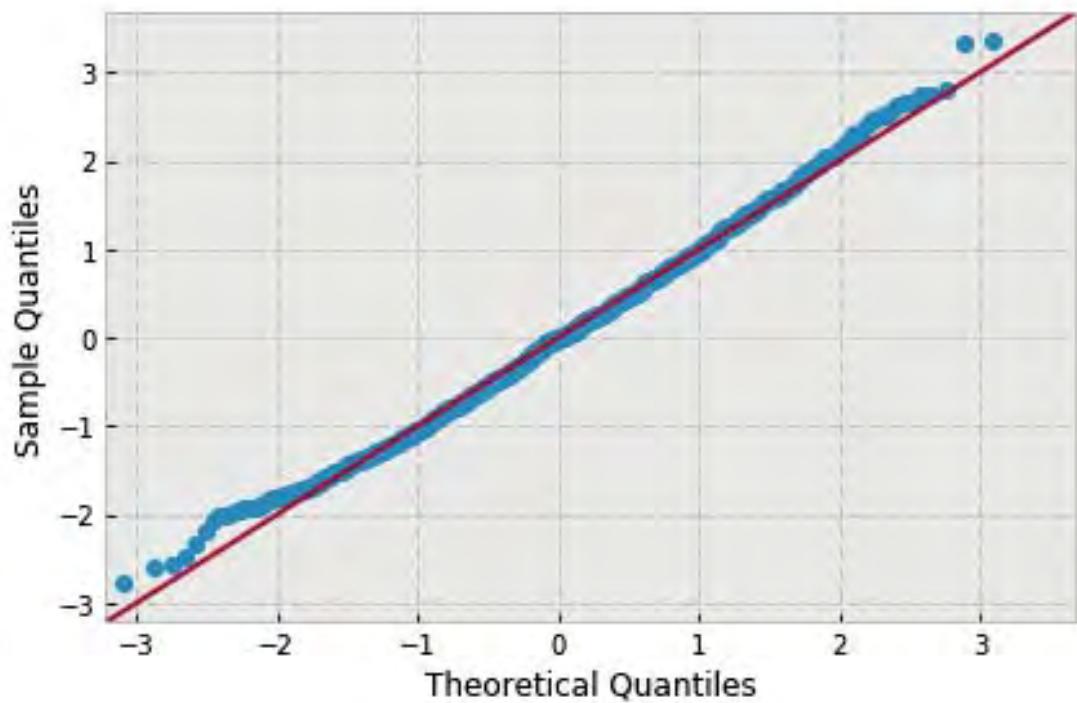


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=5.135004301885885e-06, scale=1.5660492683254174e-07)
DescribeResult(nobs=1000, minmax=(4.7165972298297779e-06, 5.6904081788490417e-06)
               mean=5.1350043018858849e-06, variance=2.4549652760986735e-14,
               skewness=0.4590352577789928, kurtosis=0.13198629998768618)
```

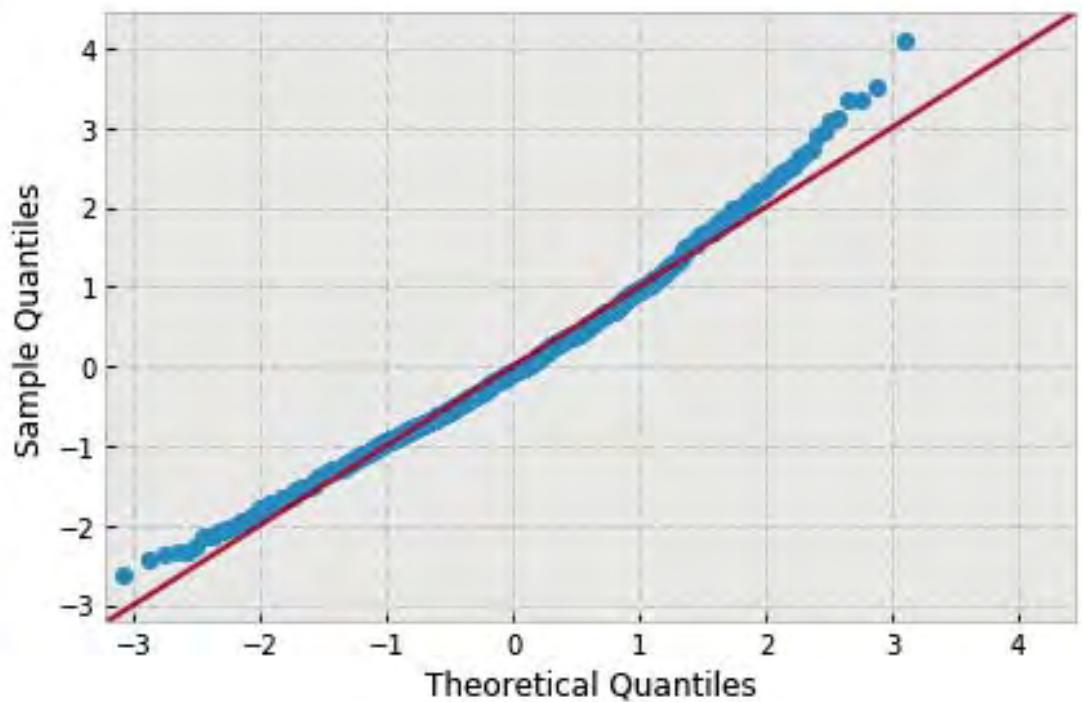
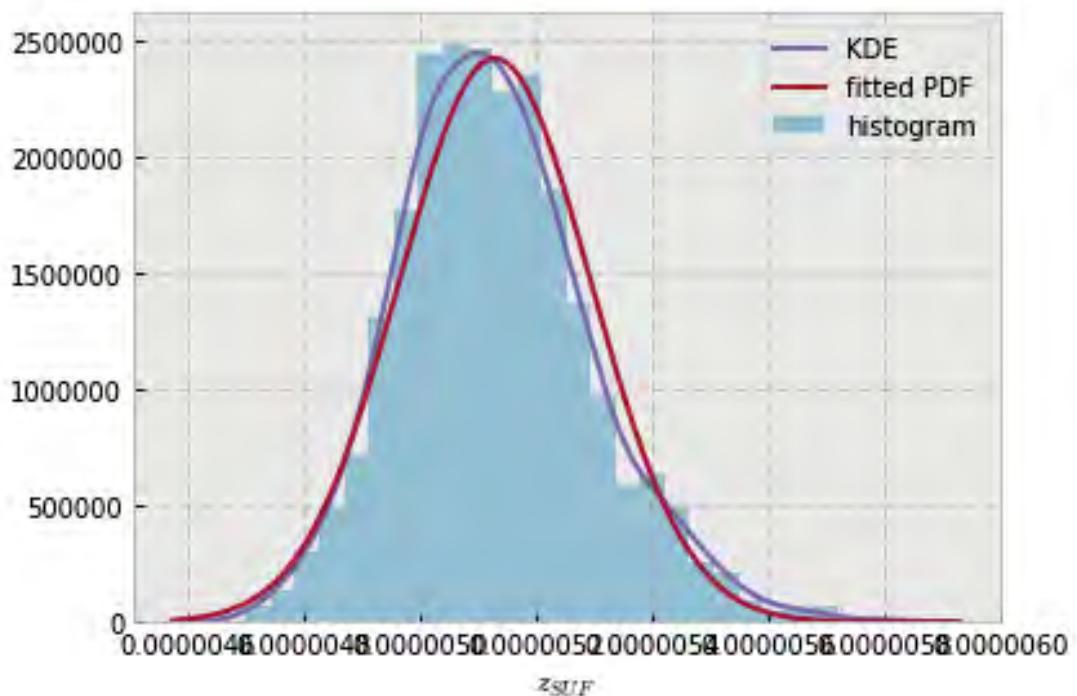


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=5.135918073507662e-06, scale=1.6048125092462085e-07)
DescribeResult(nobs=1000, minmax=(4.6909511248563738e-06, 5.6749972007890238e-06)
               mean=5.1359180735076616e-06, variance=2.5780011910241358e-14,
               skewness=0.23500860698132145, kurtosis=-0.11016418081554136)
```

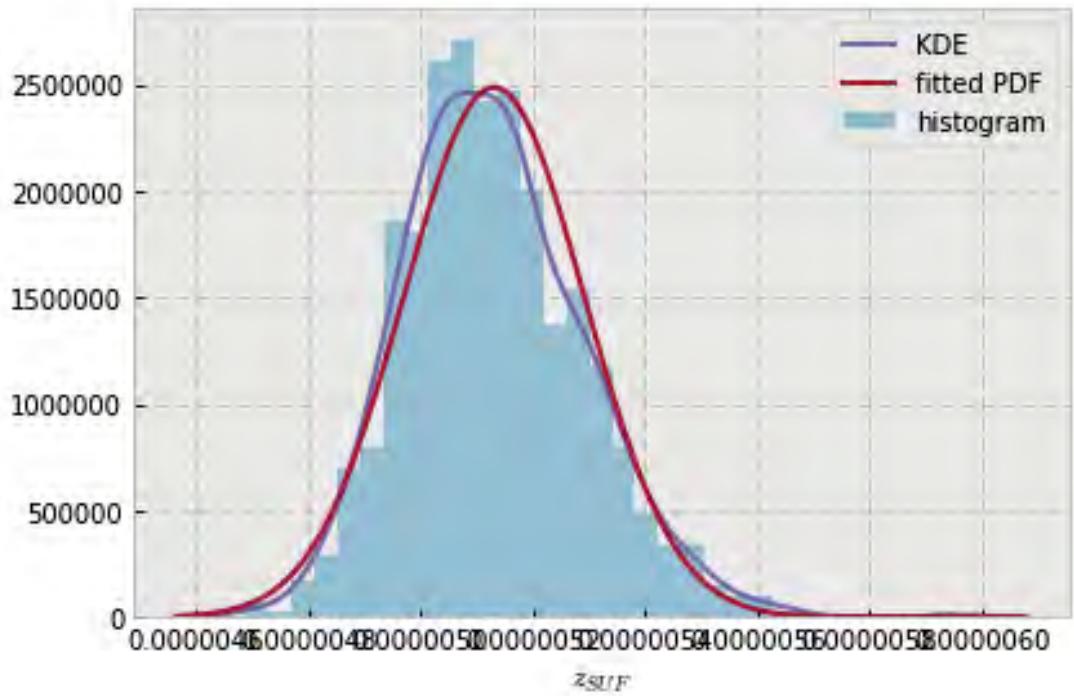


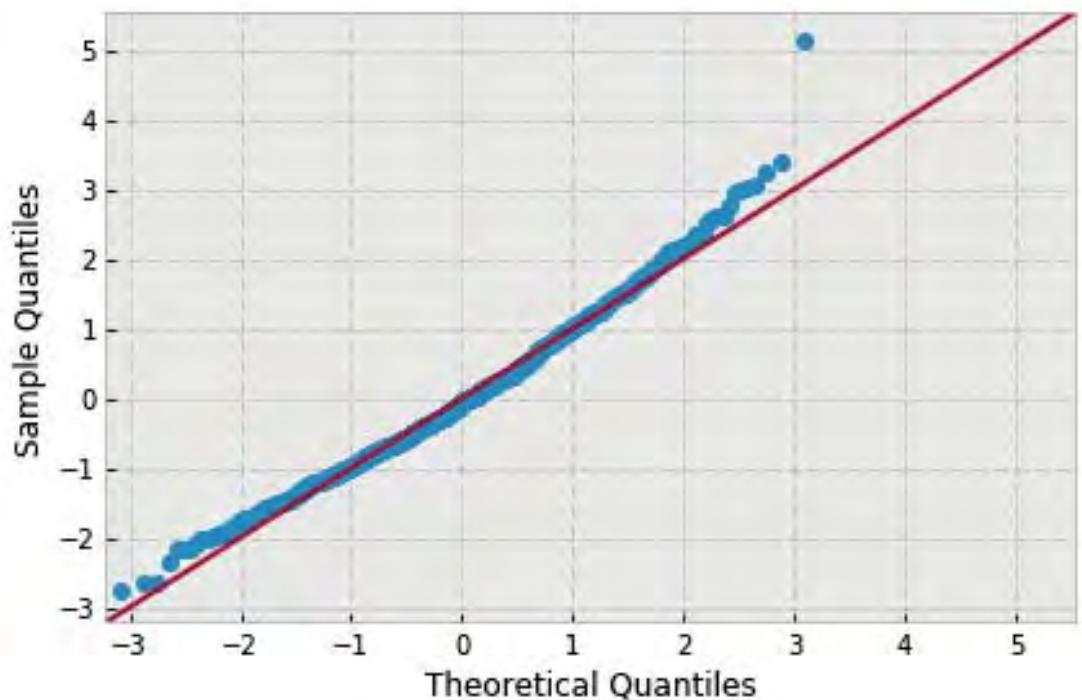


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=5.130105632493544e-06, scale=1.6446070918387043e-07)
DescribeResult(nobs=1000, minmax=(4.7003439627405276e-06, 5.8046157688669483e-06)
               mean=5.130105632493544e-06, variance=2.7074399264526129e-14,
               skewness=0.4695291689437227, kurtosis=0.43735781289341835)
```

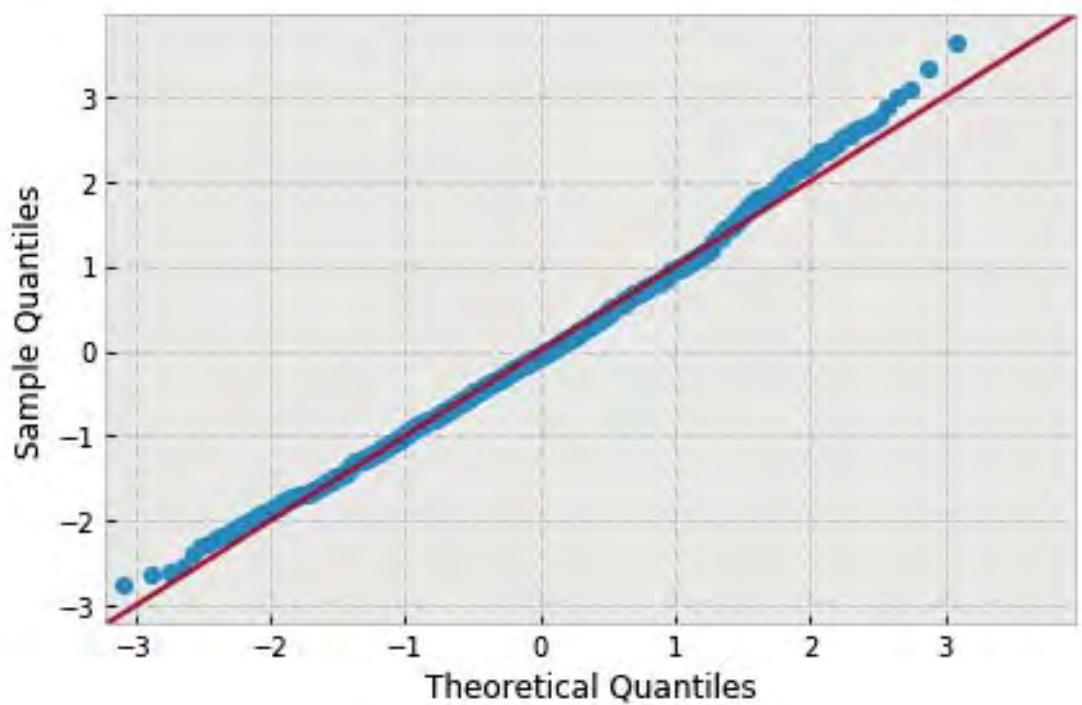
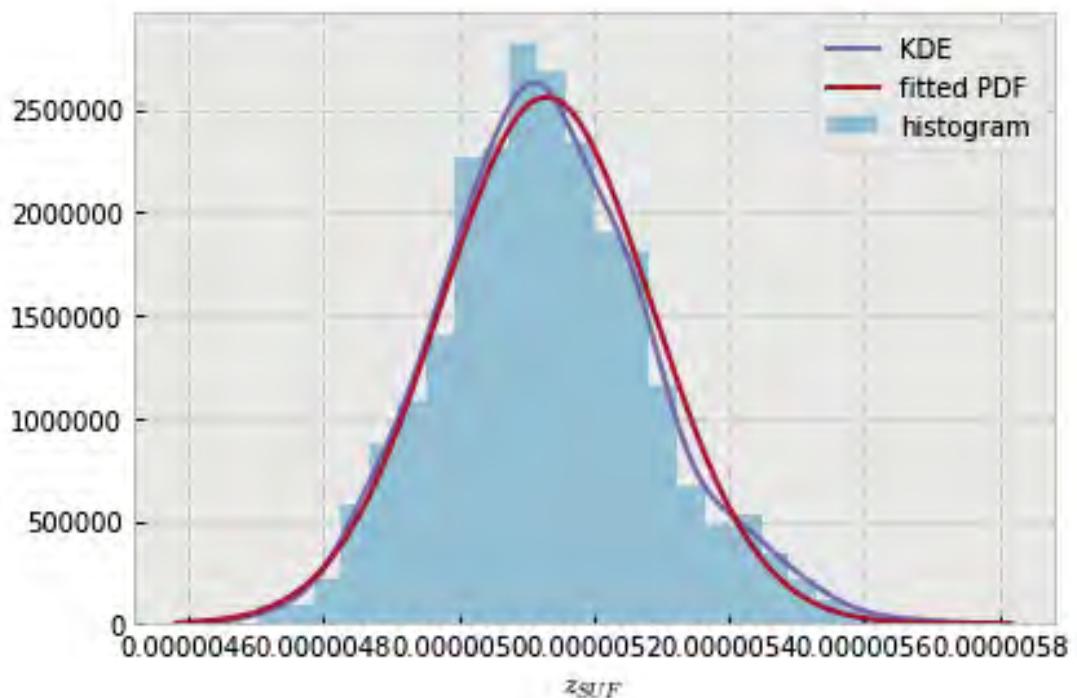


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=5.131092446700991e-06, scale=1.6035314306222574e-07)
DescribeResult(nobs=1000, minmax=(4.6883859959723625e-06, 5.9542355882285265e-06)
               mean=5.1310924467009907e-06, variance=2.5738869359293923e-14,
               skewness=0.47919643689153824, kurtosis=0.6274927993688957)
```

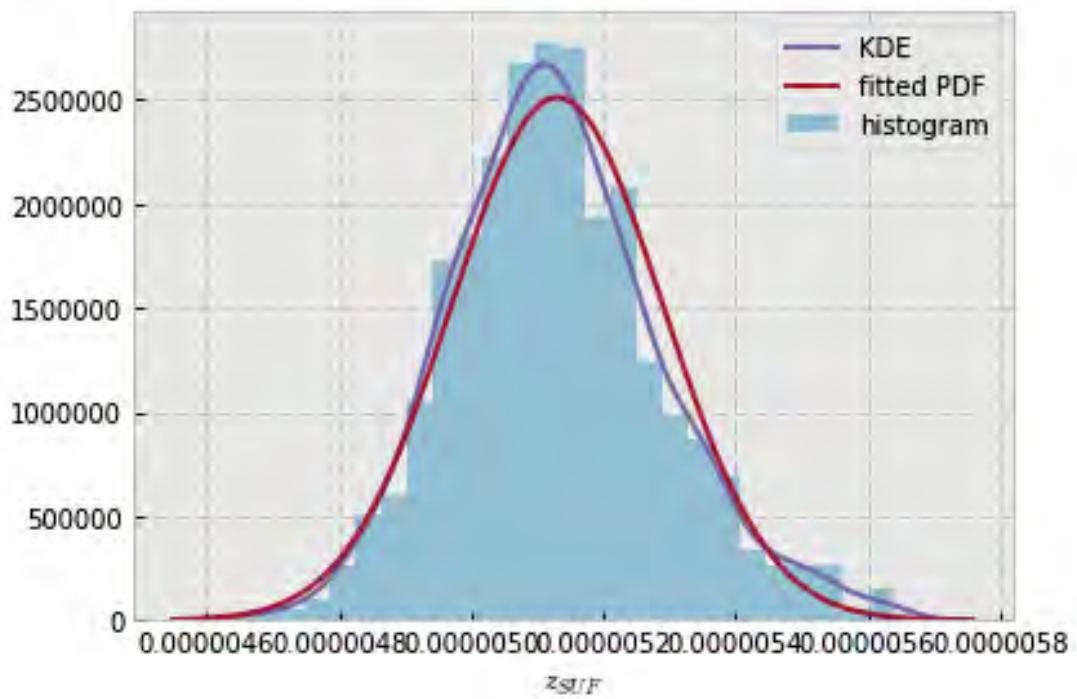


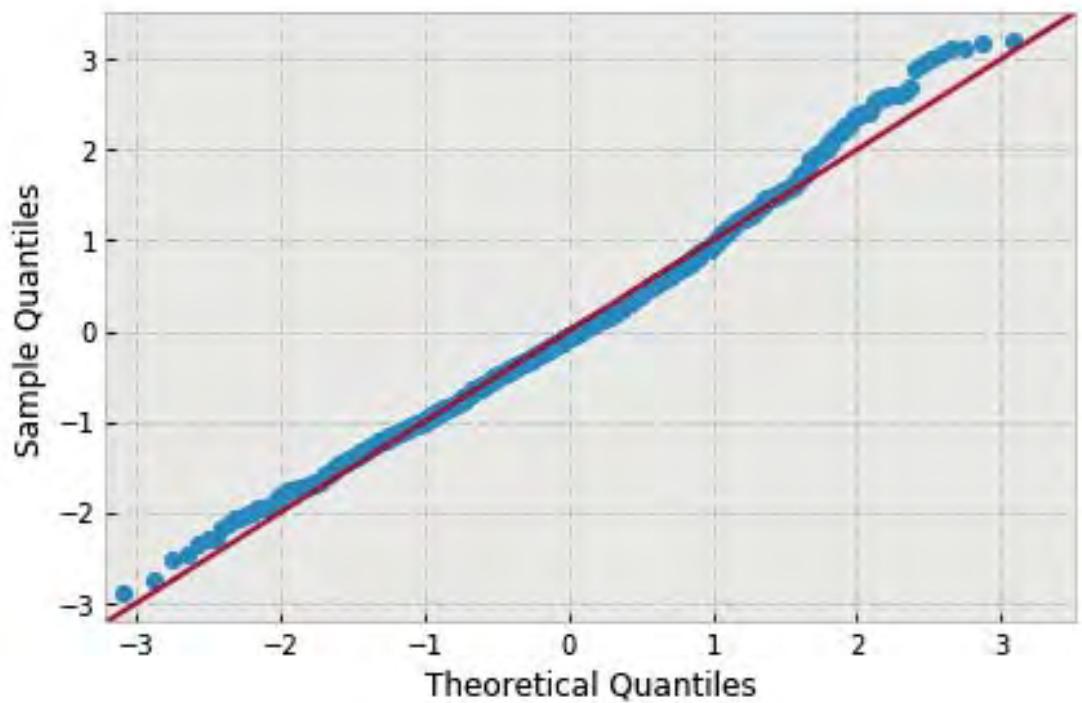


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=5.129821726769263e-06, scale=1.5579976151788255e-07)
DescribeResult(nobs=1000, minmax=(4.7030445653875035e-06, 5.6962924983116942e-06)
               mean=5.129821726769263e-06, variance=2.4297863552581657e-14,
               skewness=0.29555617214177055, kurtosis=0.18439505895697428)
```

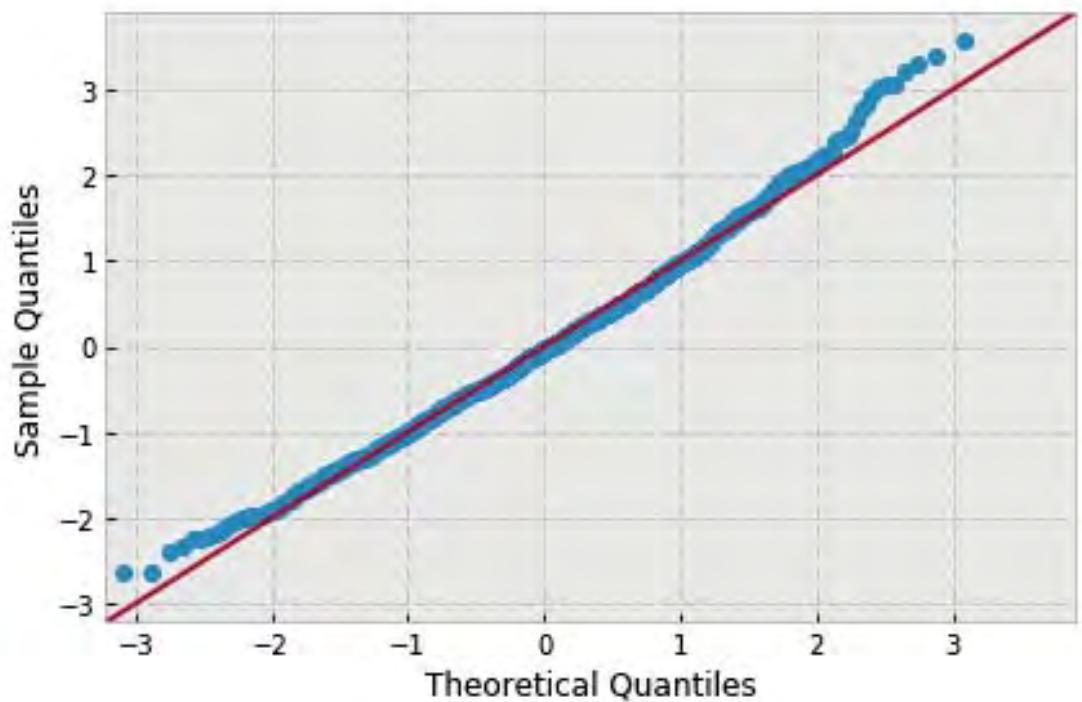
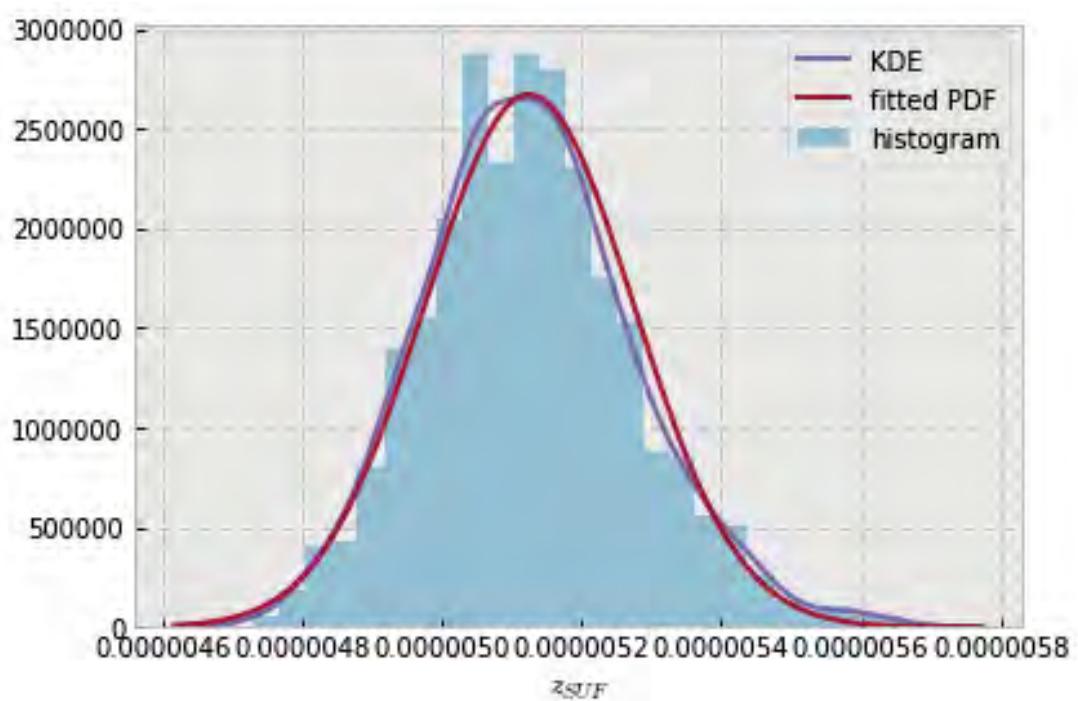


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=5.1304542872400925e-06, scale=1.589226034544062e-07)
DescribeResult(nobs=1000, minmax=(4.6688053481075507e-06, 5.6395460898000944e-06)
               mean=5.1304542872400925e-06, variance=2.5281675564290733e-14,
               skewness=0.38865831190492256, kurtosis=0.29942718238736266)
```





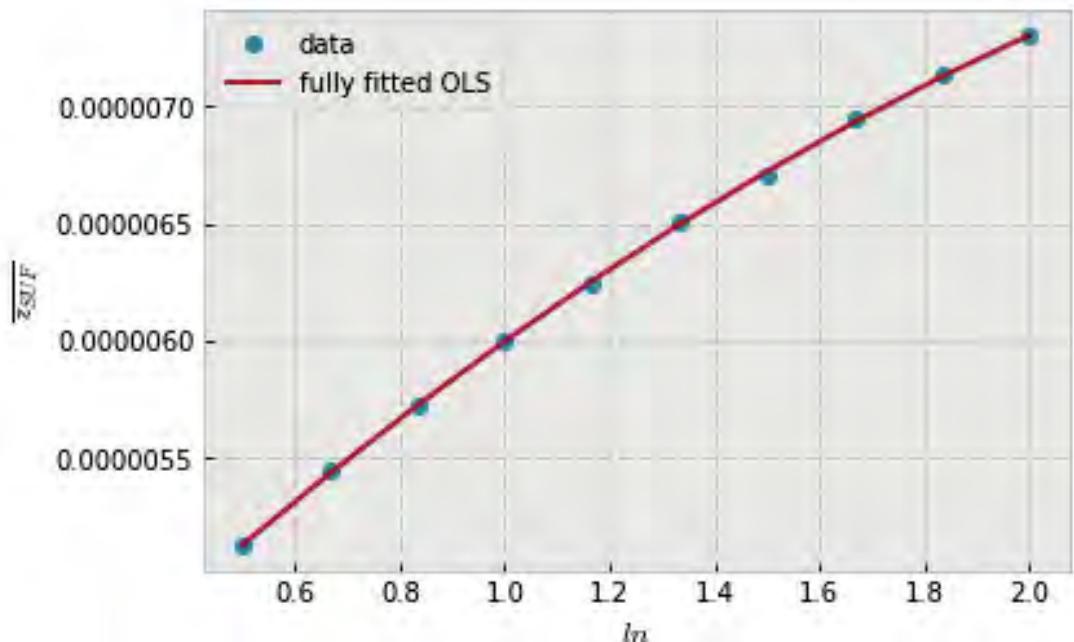
```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=5.12509736112718e-06, scale=1.4944440798822444e-07)
DescribeResult(nobs=1000, minmax=(4.7305362499141158e-06, 5.6602373905725468e-06)
               mean=5.1250973611271798e-06, variance=2.2355987066016896e-14,
               skewness=0.349562472566311, kurtosis=0.25710592458299386)
```



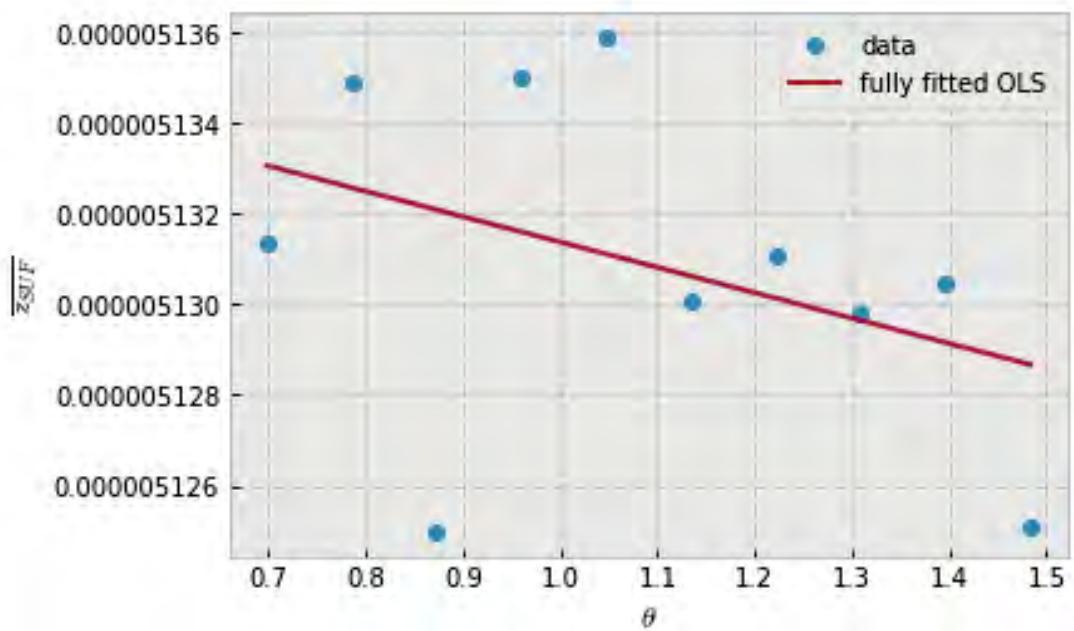
18.2 Parameter regression

```
In [42]: var_name = 'root_system_conductance_per_unit_of_root_length'
mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly1')
mean_reg('nB', var_name, 'poly2')
mean_reg('r', var_name, 'poly2')
mean_reg('sigma', var_name, 'poly1')
mean_reg('N', var_name, 'poly1')

variable name: ln
model: poly2
r-squared: 0.9996480856782238
explained variance: 0.9996522062319211
```



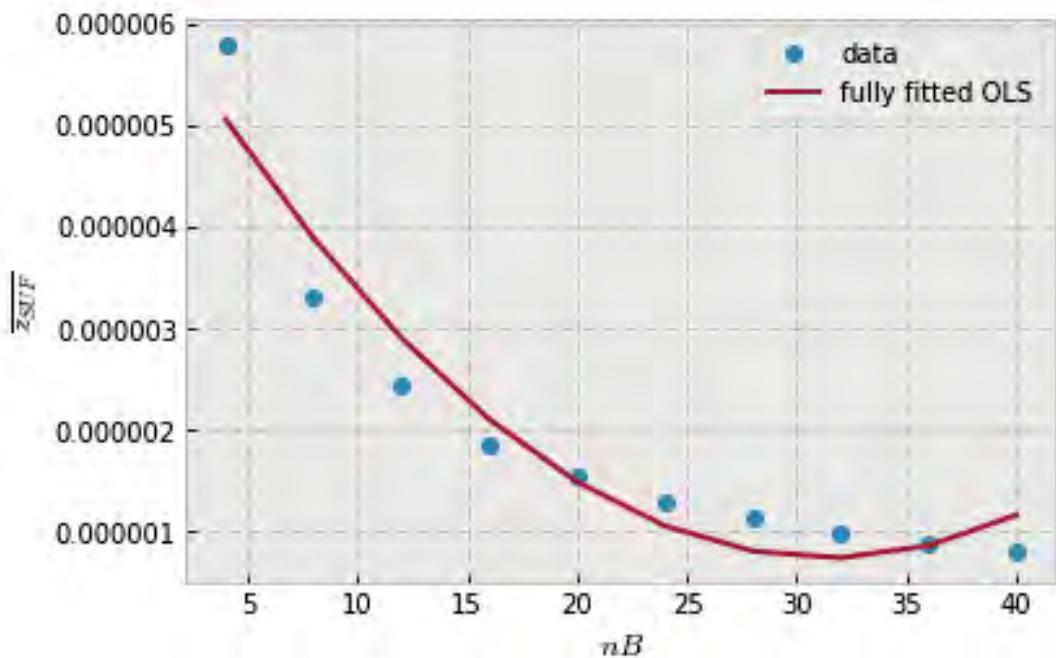
```
variable name: theta
model: poly1
r-squared: -0.3115671908309312
explained variance: -0.30861803539480914
```



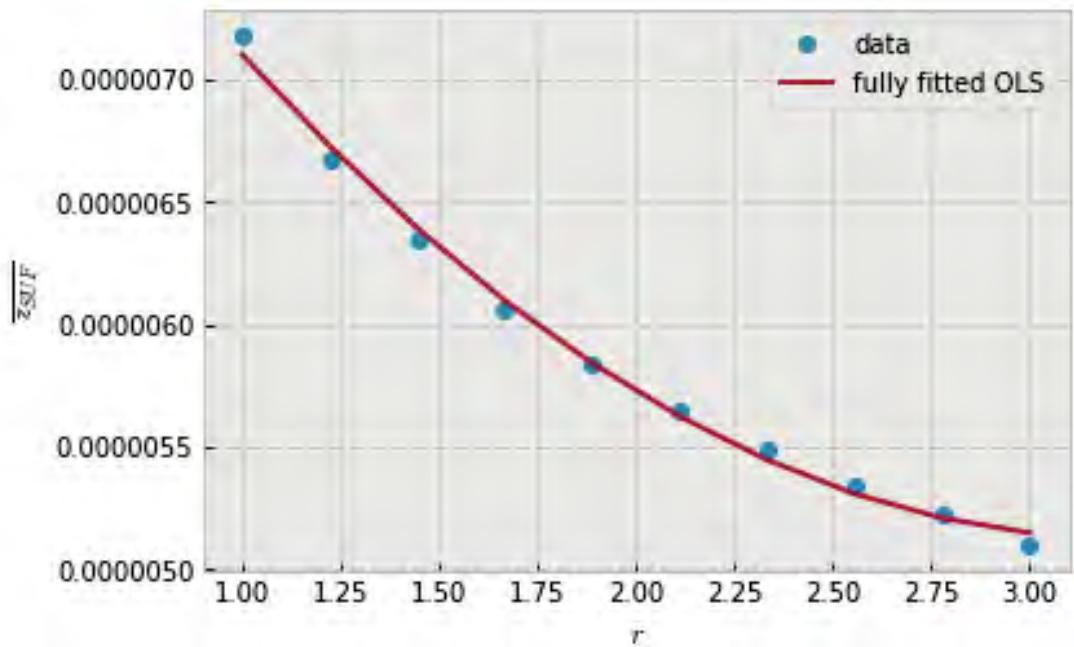
```

variable name: nB
model: poly2
r-squared: 0.7159420665248543
explained variance: 0.7170075054160312

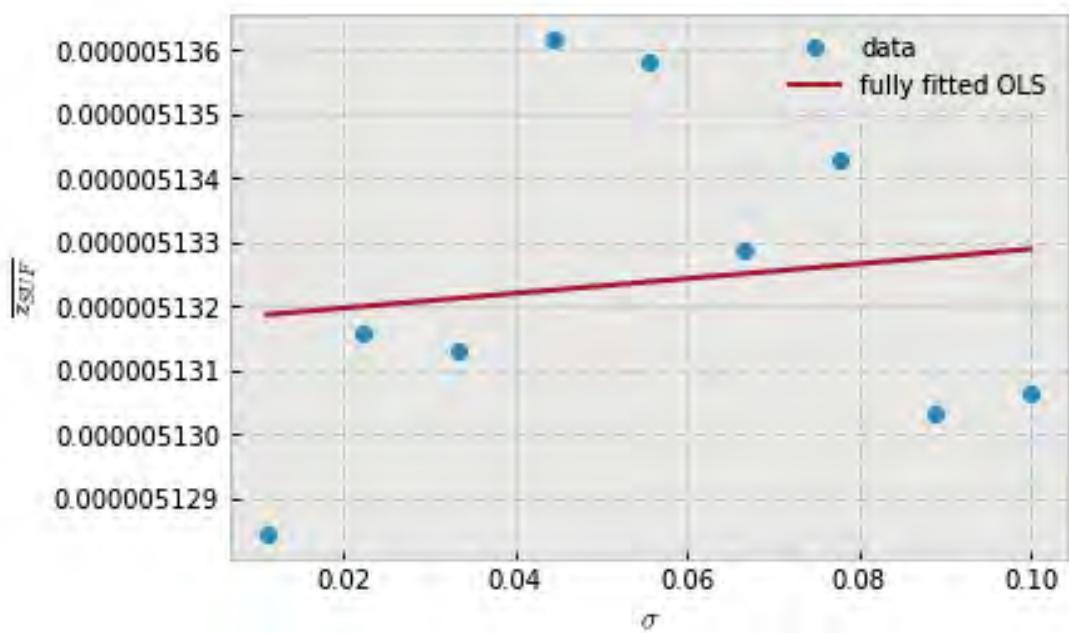
```



```
variable name: r
model: poly2
r-squared: 0.9795107219579947
explained variance: 0.979533425889127
```



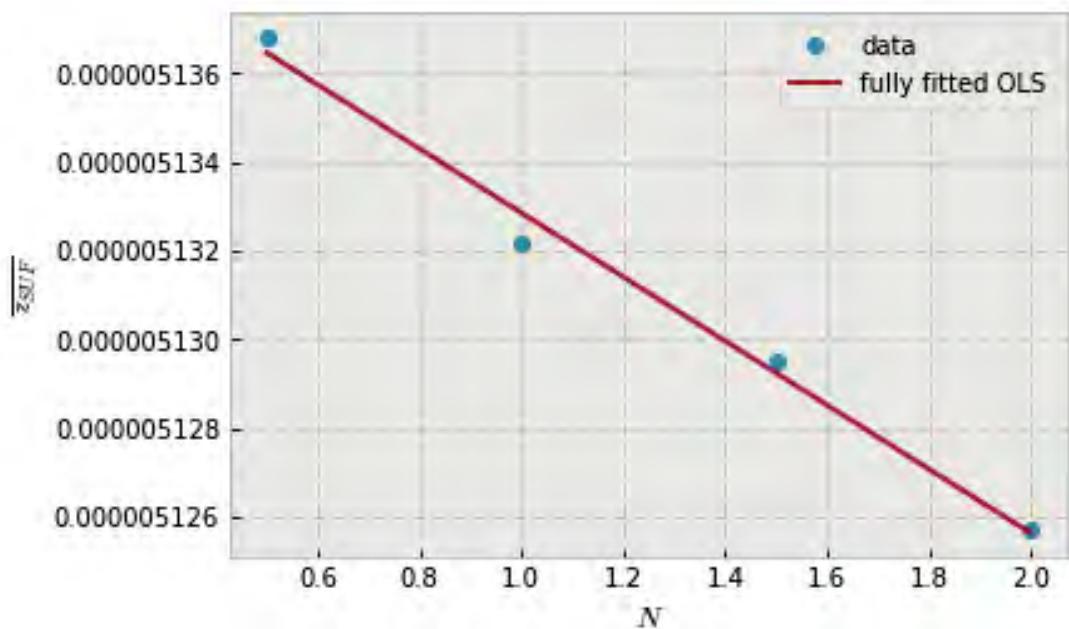
```
variable name: sigma
model: poly1
r-squared: -0.7416633648999937
explained variance: -0.7204438227514063
```



```

variable name: N
model: poly1
r-squared: 0.9610978225775526
explained variance: 0.9636086160001215

```

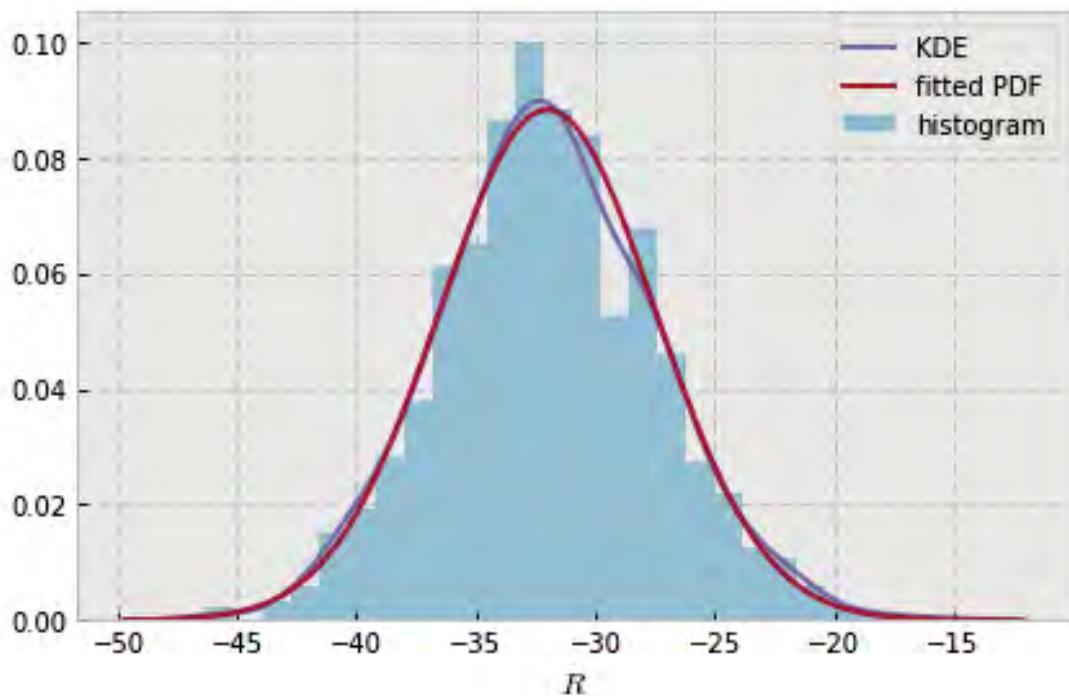


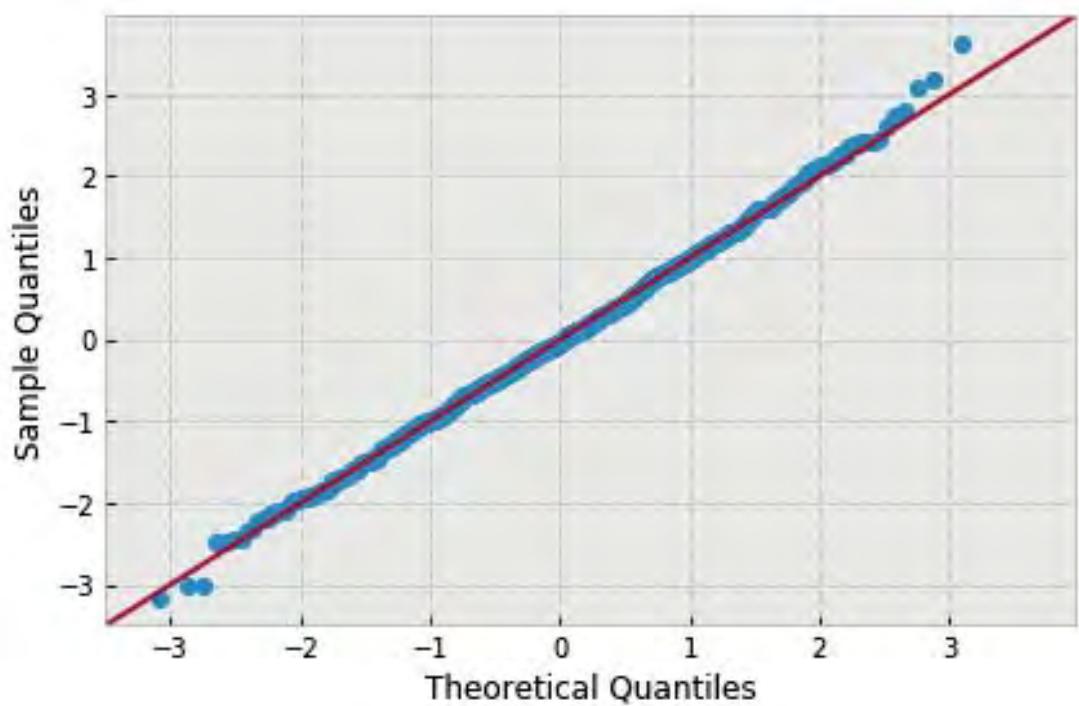
19 Mean depth of standard root water uptake

19.1 Probability distributions

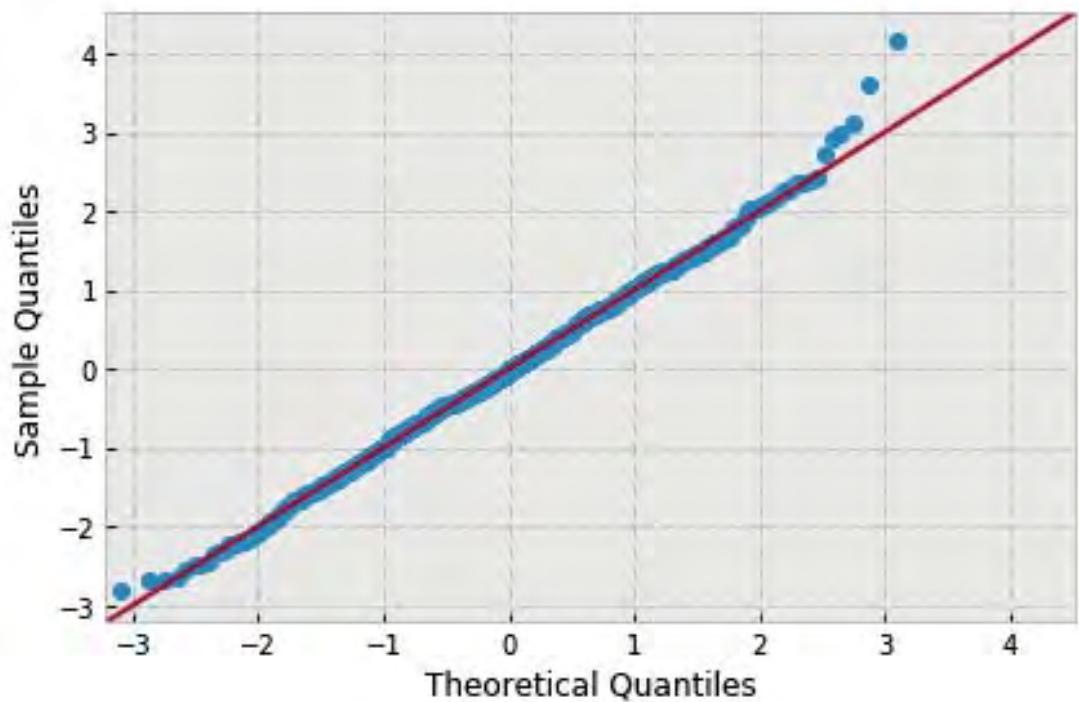
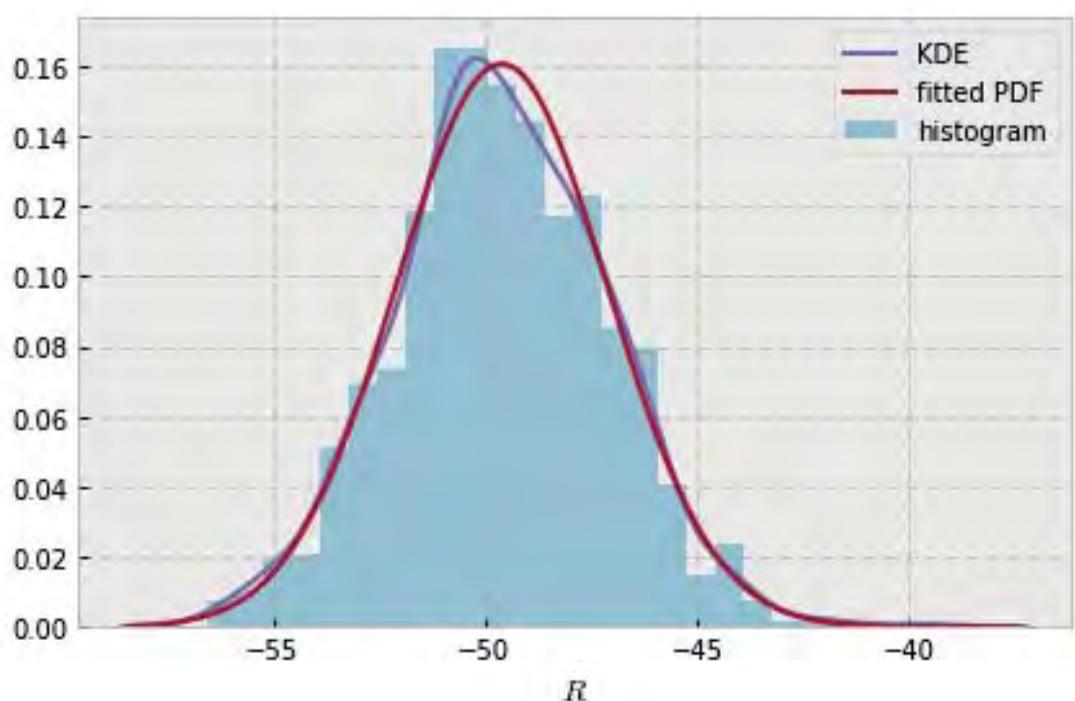
```
In [43]: plot_dist('mean_depth_of_standard_root_water_uptake')
```

```
variable name: N
variable value: 0.5
distribution: normal(shape=(), loc=-32.00072324974, scale=4.518623399915355)
DescribeResult(nobs=1000, minmax=(-46.273955960000002, -15.63054546)
    mean=-32.000723249739998, variance=20.438395826088691,
    skewness=0.10158224881523667, kurtosis=0.11186527355733977)
```

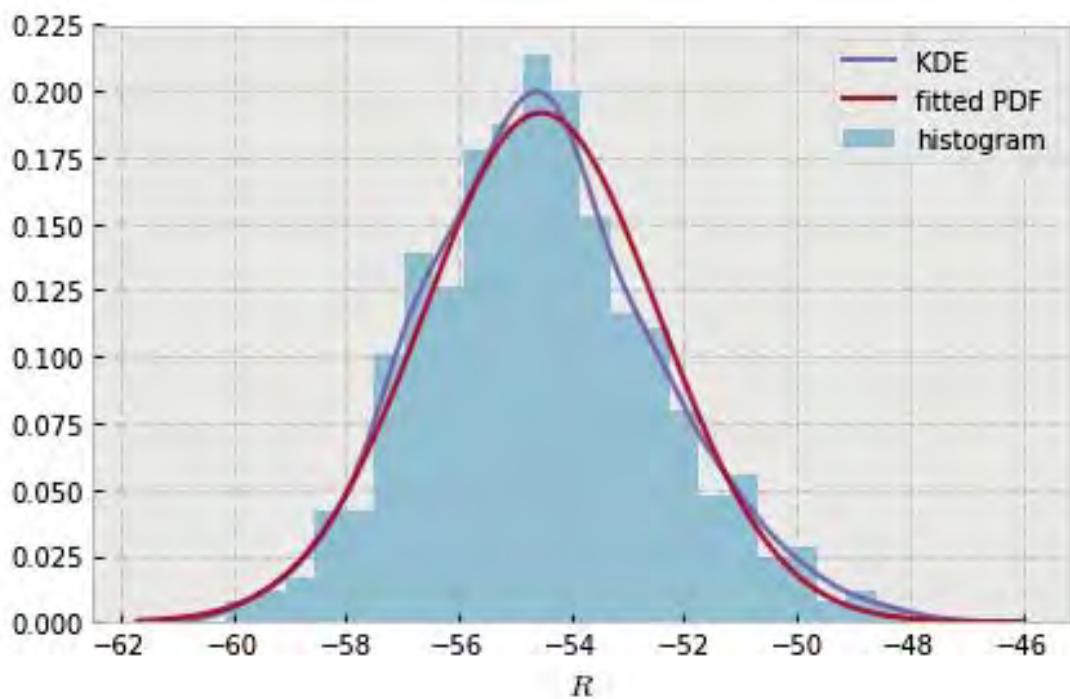


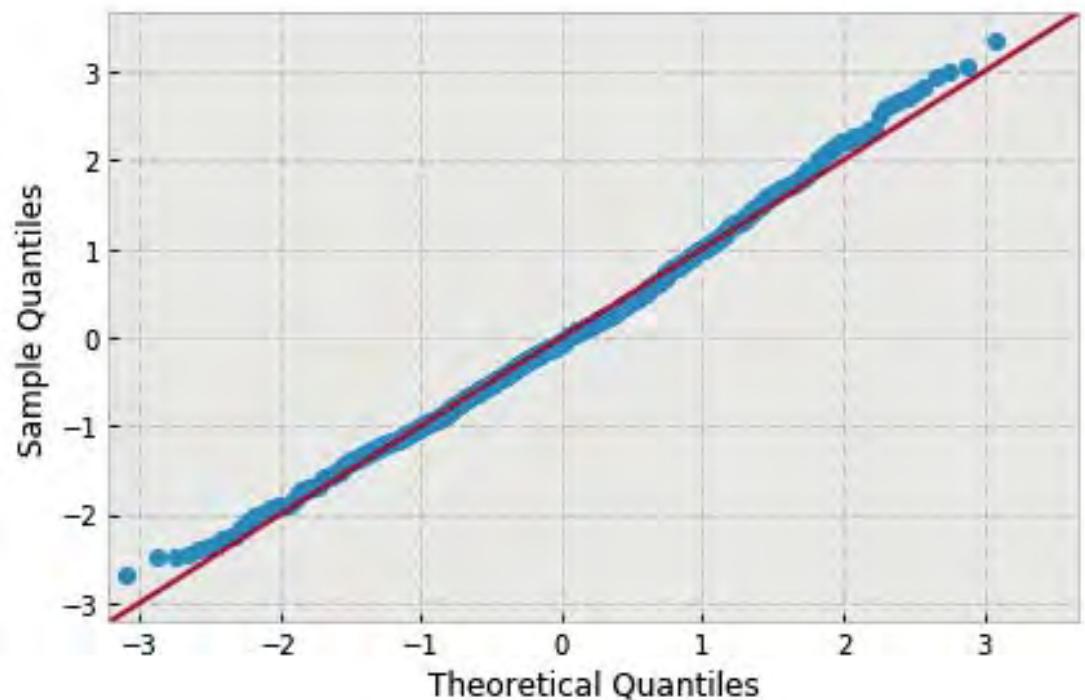


```
variable name: N
variable value: 1.0
distribution: normal(shape=(), loc=-49.60578726716, scale=2.4864198201780274)
DescribeResult(nobs=1000, minmax=(-56.55981414, -39.254587979999997)
               mean=-49.605787267159997, variance=6.1884719941683013,
               skewness=0.10446849274374957, kurtosis=0.23461207425816433)
```

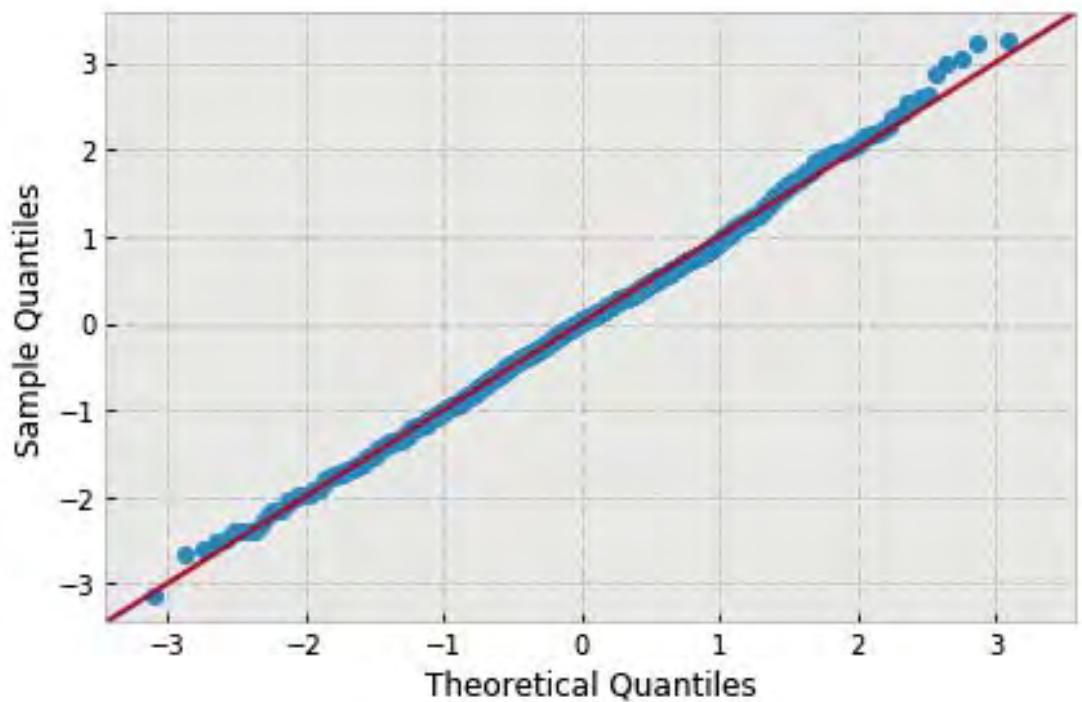
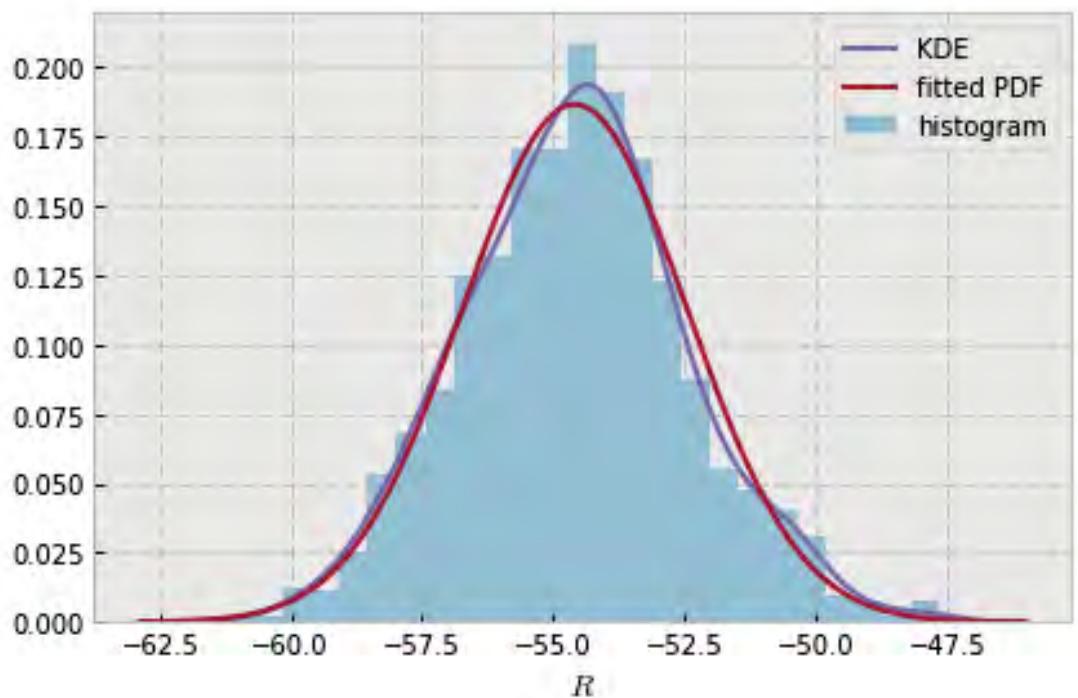


```
variable name: N
variable value: 1.5
distribution: normal(shape=(), loc=-54.533689614550006, scale=2.0808596736902807)
DescribeResult(nobs=1000, minmax=(-60.10773294999999, -47.54344291999997)
               mean=-54.533689614550006, variance=4.3343112928833047,
               skewness=0.2782190641752982, kurtosis=0.04147931191821819)
```

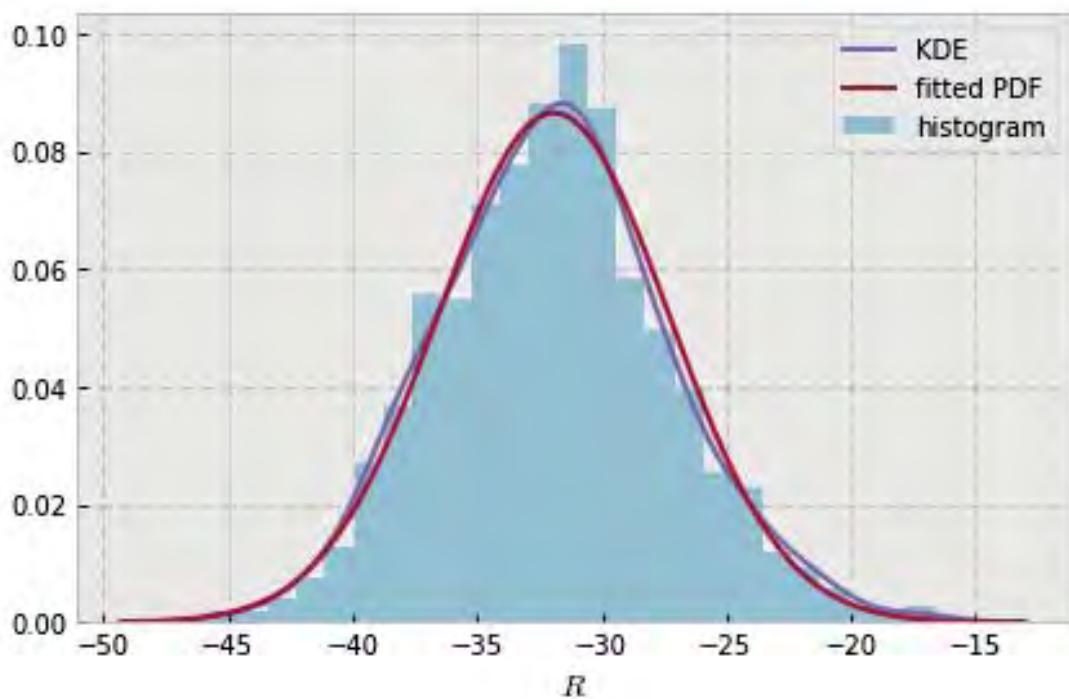


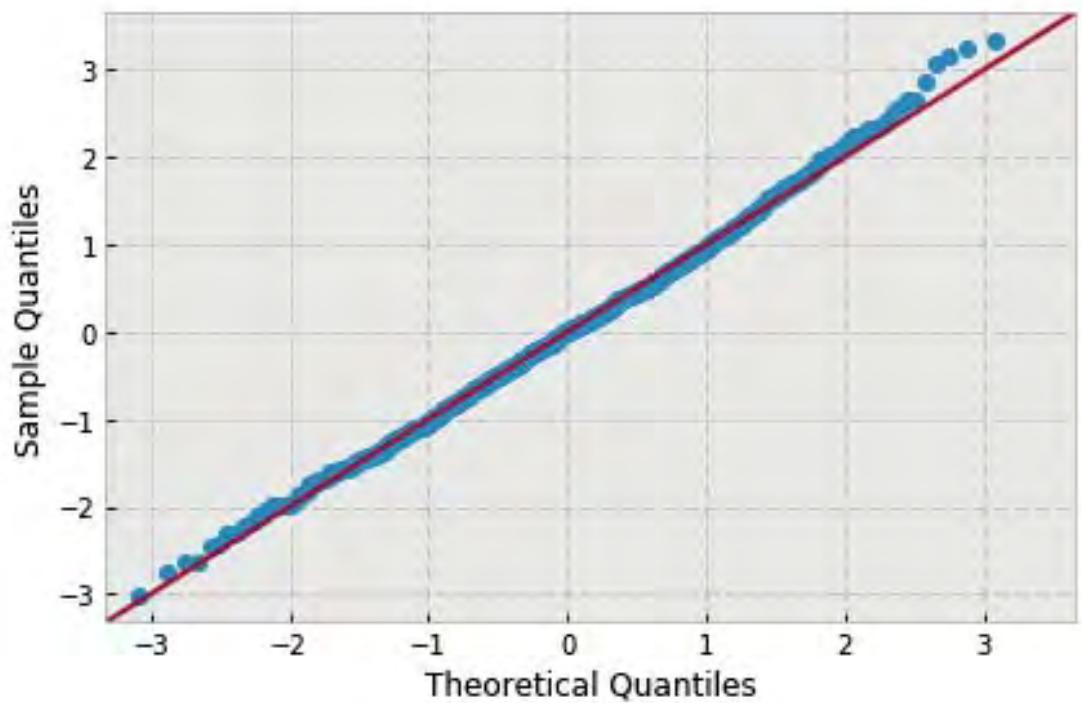


```
variable name: N
variable value: 2.0
distribution: normal(shape=(), loc=-54.587555135230005, scale=2.136542779228207)
DescribeResult(nobs=1000, minmax=(-61.26170943000003, -47.63576238000003)
               mean=-54.58755513523005, variance=4.569384431904095,
               skewness=0.1176381426086149, kurtosis=0.07998417126262547)
```

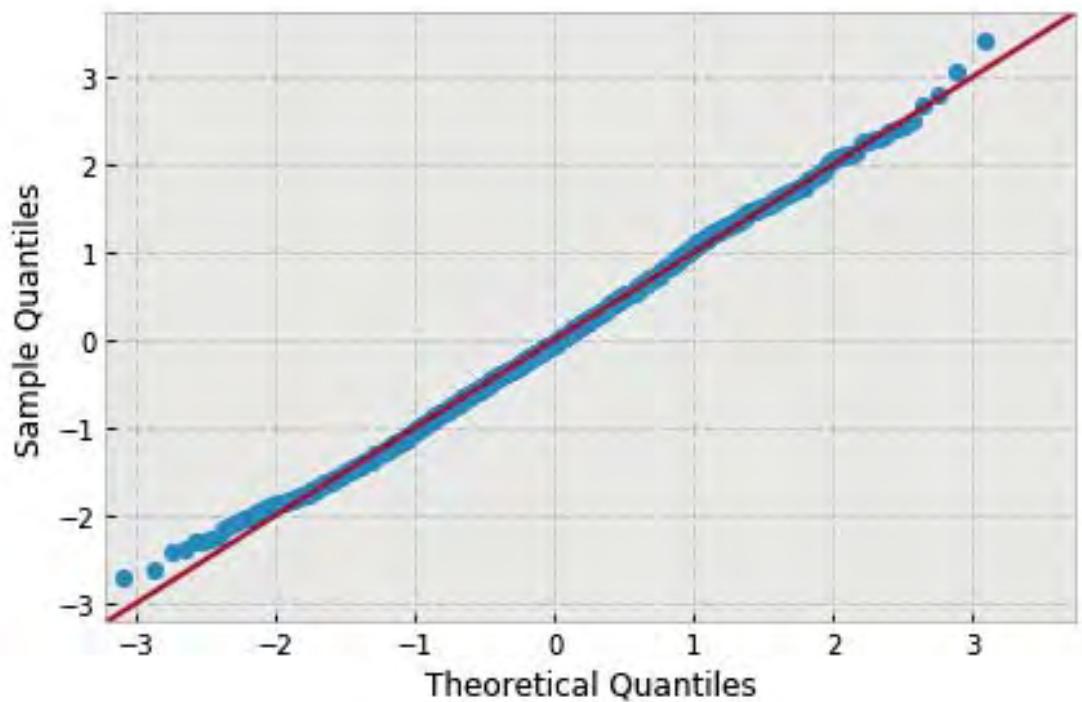
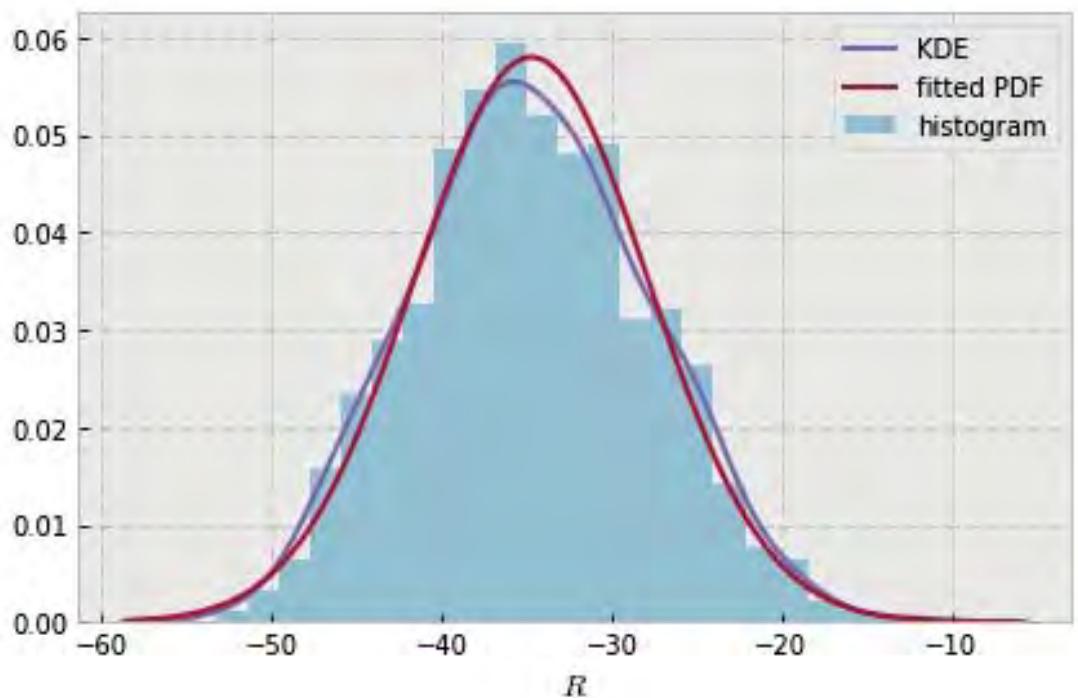


```
variable name: ln
variable value: 0.5
distribution: normal(shape=(), loc=-31.93716019632, scale=4.610519394507342)
DescribeResult(nobs=1000, minmax=(-45.788869560000002, -16.563757160000002)
               mean=-31.937160196320001, variance=21.278167254382733,
               skewness=0.17063086601226113, kurtosis=0.08568222223616662)
```

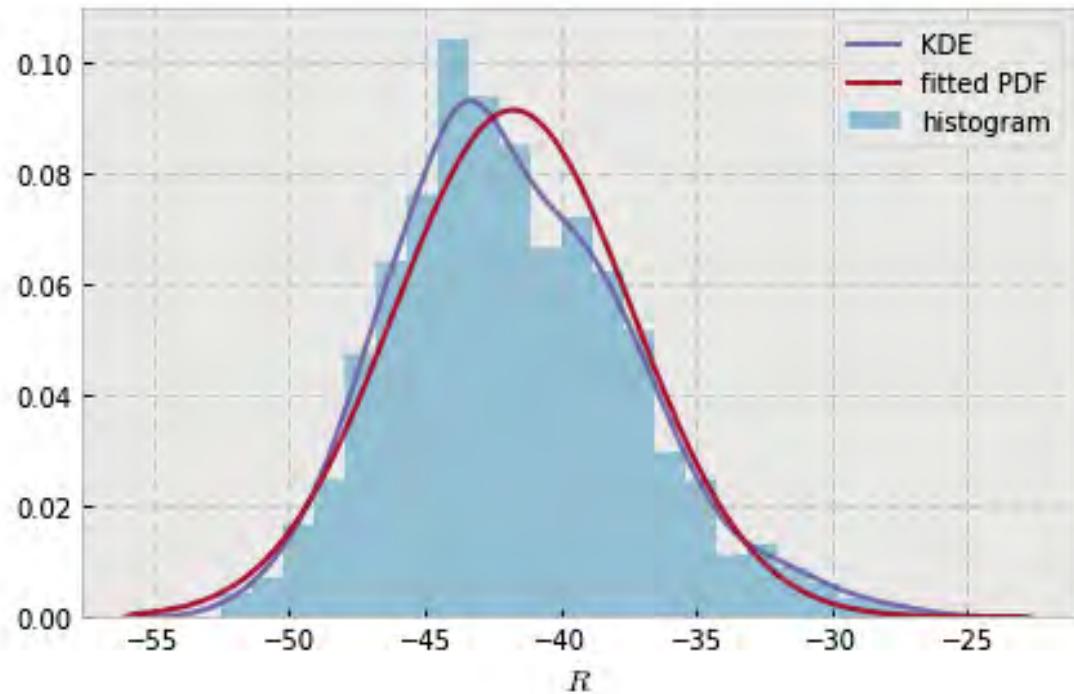


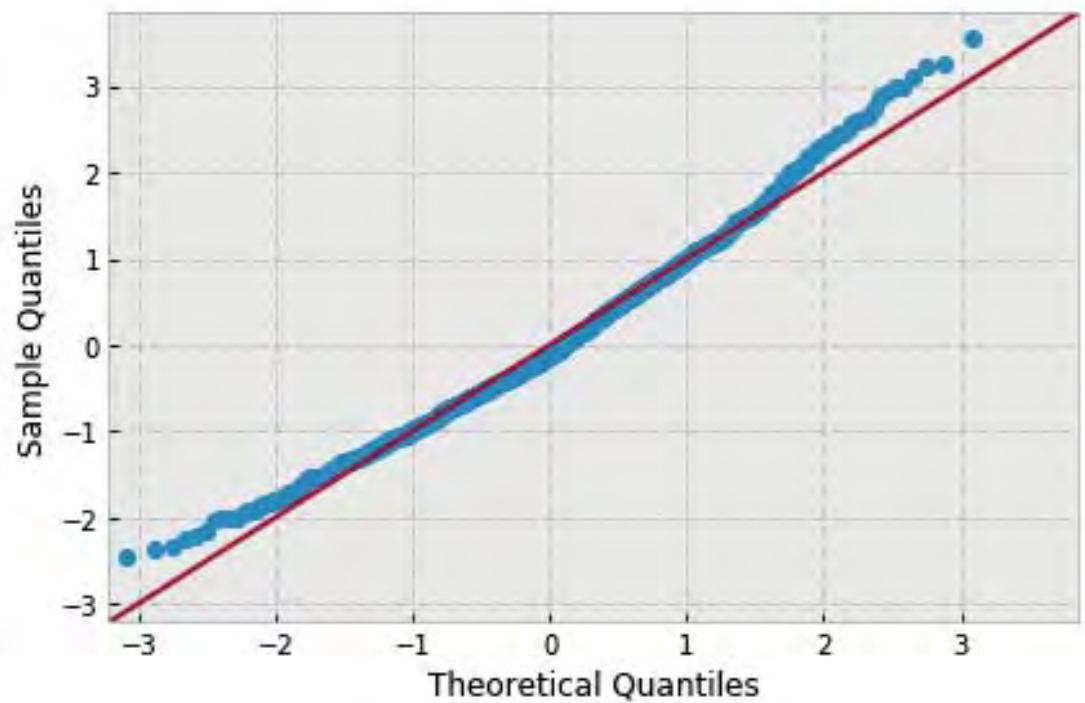


```
variable name: ln
variable value: 0.6666666666666666
distribution: normal(shape=(), loc=-34.72887064347999, scale=6.882555158308726)
DescribeResult(nobs=1000, minmax=(-53.29629284, -11.167890399999999)
               mean=-34.72887064347993, variance=47.416982489651701,
               skewness=0.11495401733947026, kurtosis=-0.27991302437274435)
```

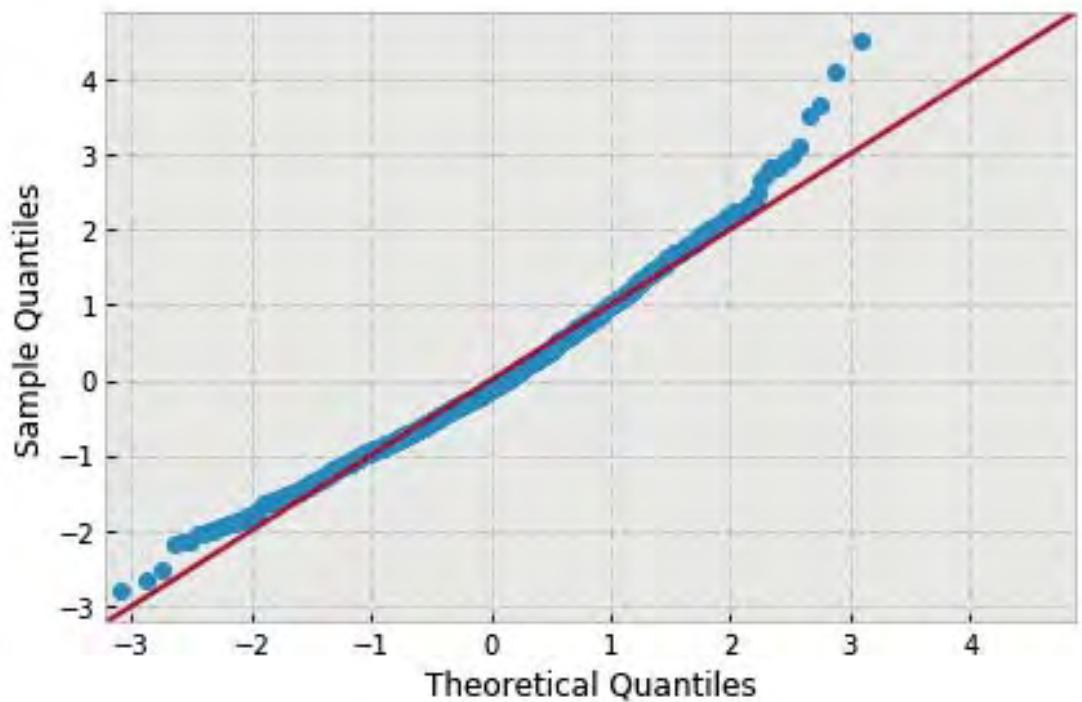
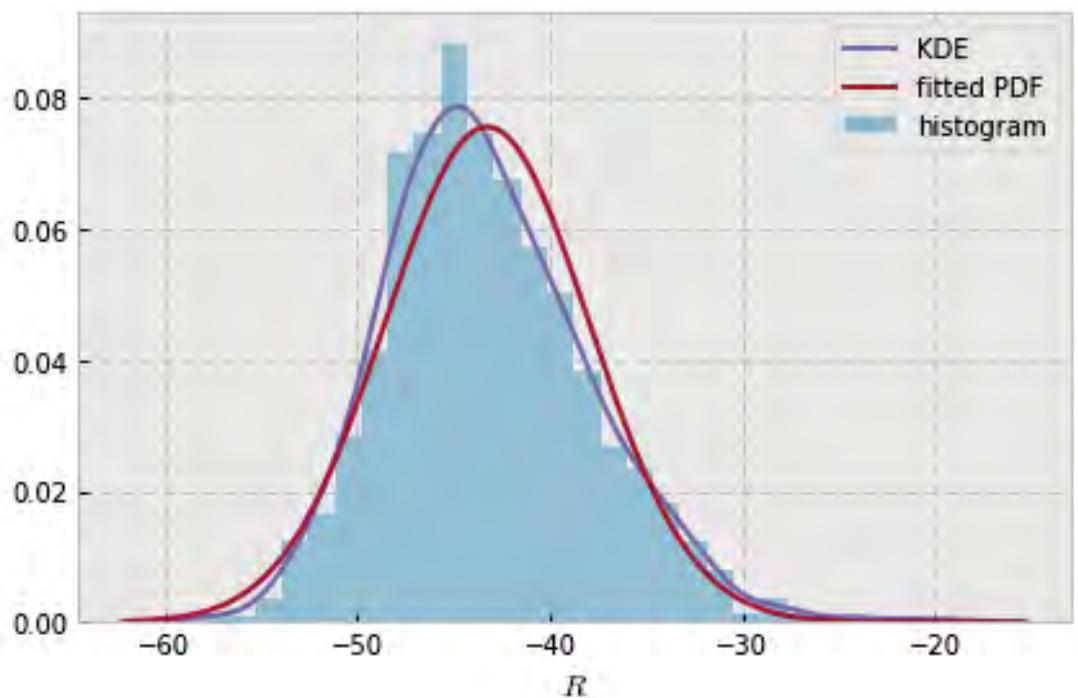


```
variable name: ln
variable value: 0.8333333333333333
distribution: normal(shape=(), loc=-41.736500430300005, scale=4.371467743558859)
DescribeResult(nobs=1000, minmax=(-52.49815478, -26.234480510000001)
               mean=-41.736500430300005, variance=19.128859092067646,
               skewness=0.44860559797561284, kurtosis=0.15056641447328278)
```

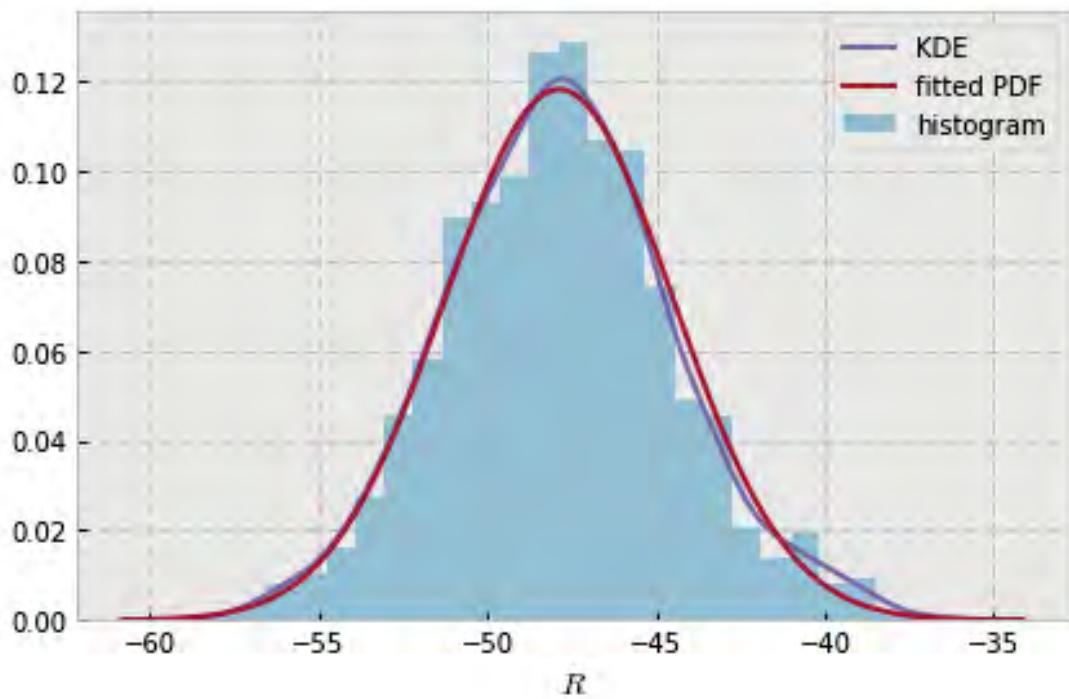


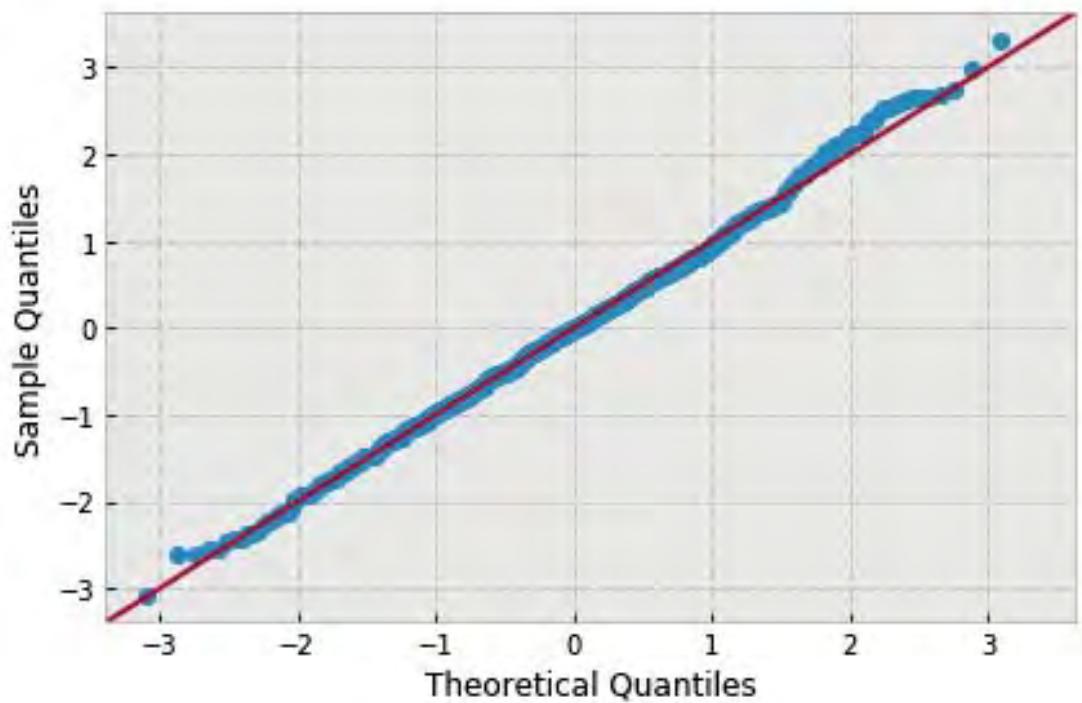


```
variable name: ln
variable value: 1.0
distribution: normal(shape=(), loc=-43.245710245059996, scale=5.277577925815016)
DescribeResult(nobs=1000, minmax=(-58.05687452000001, -19.47186844000001)
               mean=-43.24571024505996, variance=27.880709472522451,
               skewness=0.539240595787454, kurtosis=0.6387154819401646)
```

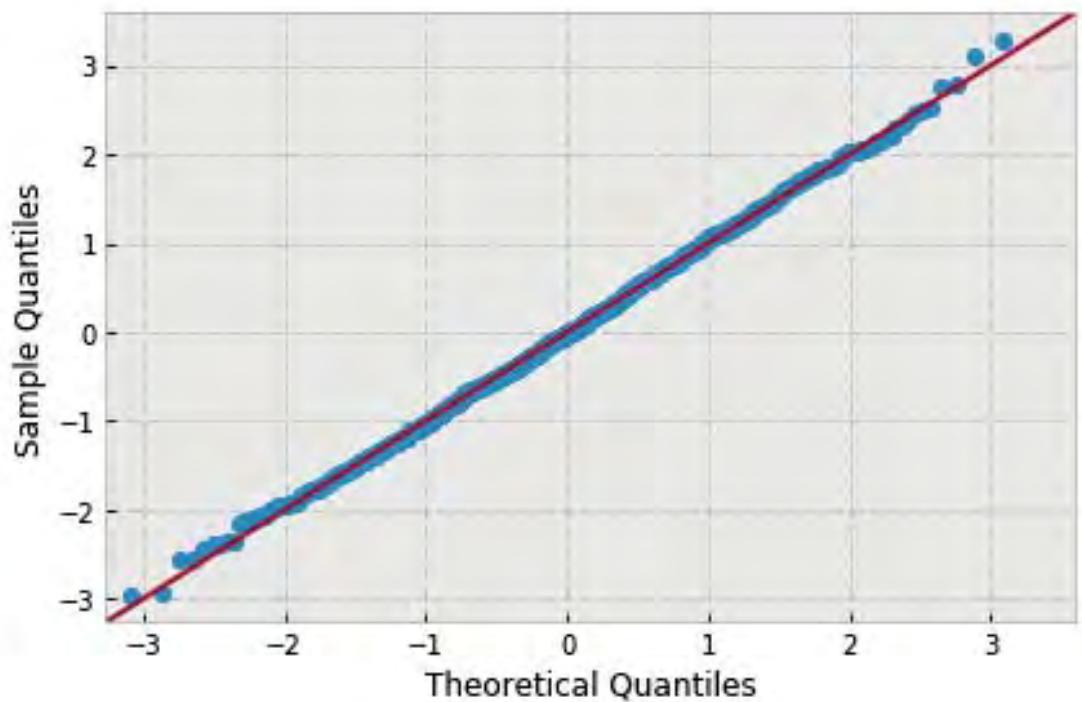
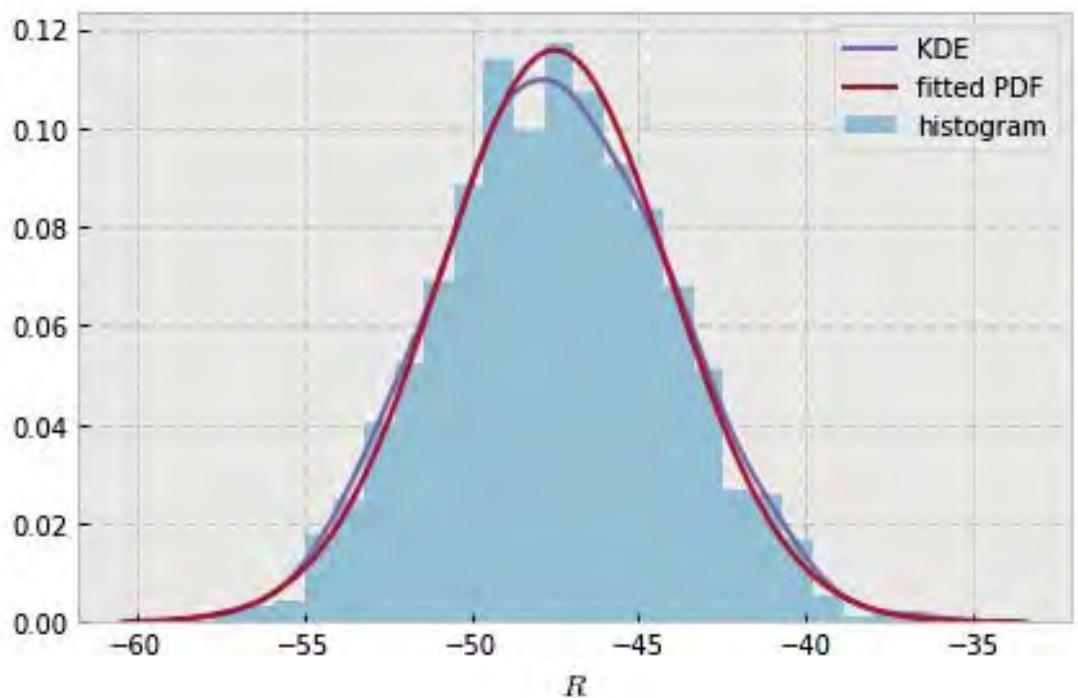


```
variable name: ln
variable value: 1.1666666666666665
distribution: normal(shape=(), loc=-47.87308997536999, scale=3.3738595648496537)
DescribeResult(nobs=1000, minmax=(-58.22432392000003, -36.71528010000001)
               mean=-47.87308997536993, variance=11.394322686013506,
               skewness=0.13279252634302258, kurtosis=0.11371011998257208)
```

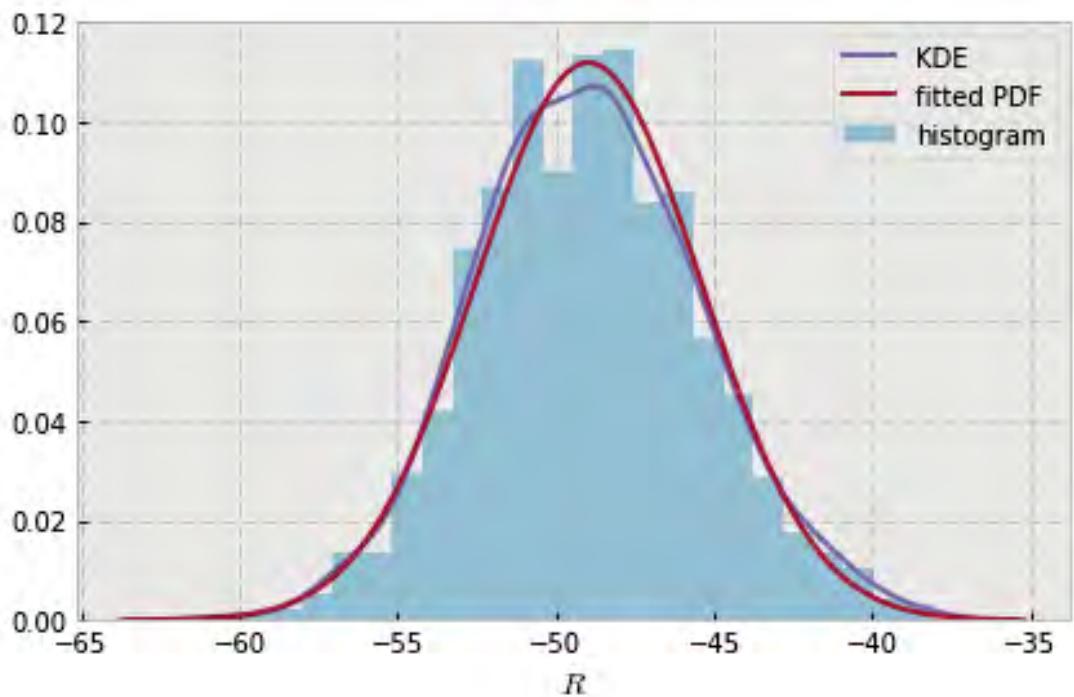


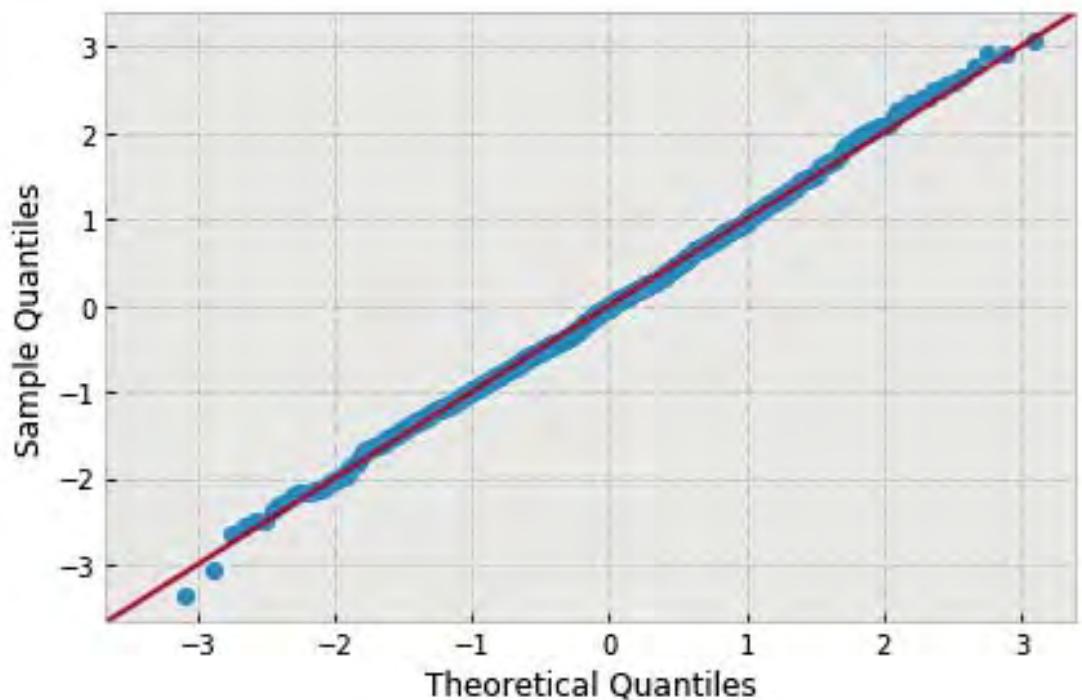


```
variable name: ln
variable value: 1.333333333333333
distribution: normal(shape=(), loc=-47.49725687814, scale=3.445179705631458)
DescribeResult(nobs=1000, minmax=(-57.66962080999998, -36.203440180000001)
               mean=-47.49725687814, variance=11.881144348443305,
               skewness=0.05923717611460461, kurtosis=-0.19124822558917032)
```

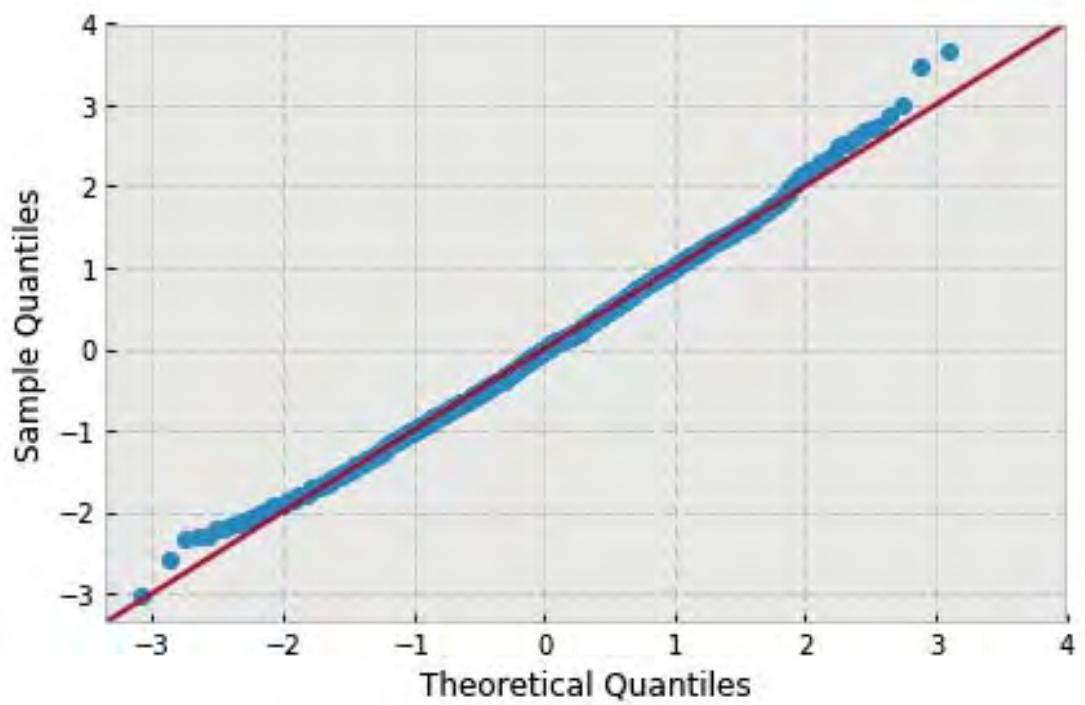
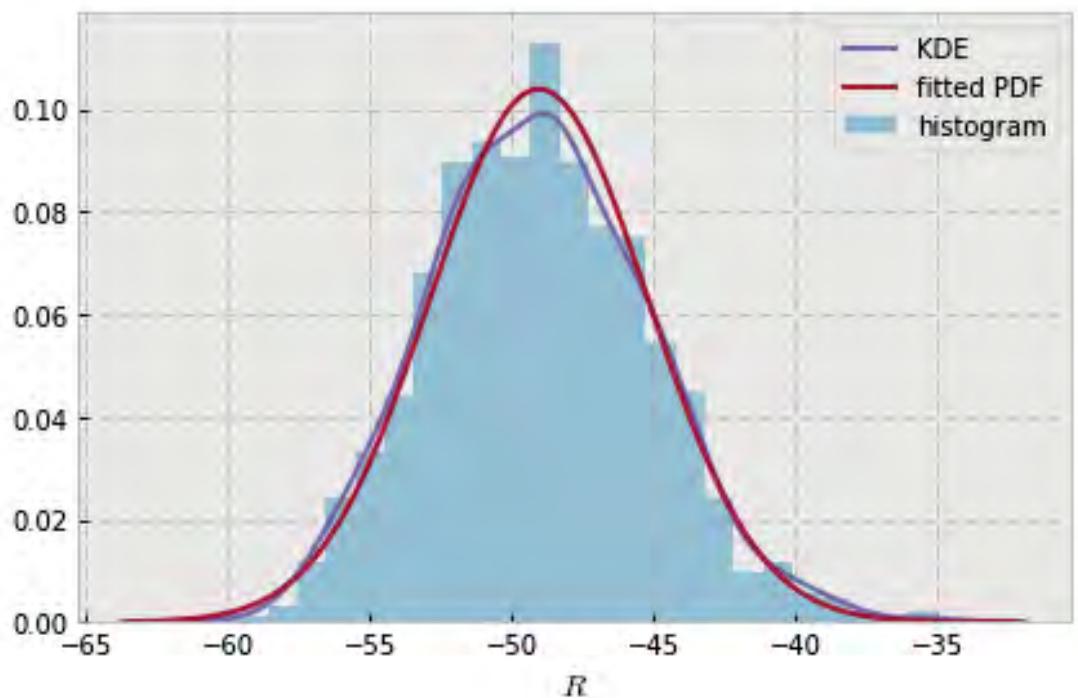


```
variable name: ln
variable value: 1.5
distribution: normal(shape=(), loc=-48.969339005820004, scale=3.5639012788796207)
DescribeResult(nobs=1000, minmax=(-60.88324142999998, -38.04253848999998)
               mean=-48.96933900582004, variance=12.714106432031828,
               skewness=0.11699487956722274, kurtosis=0.0013337491706013616)
```

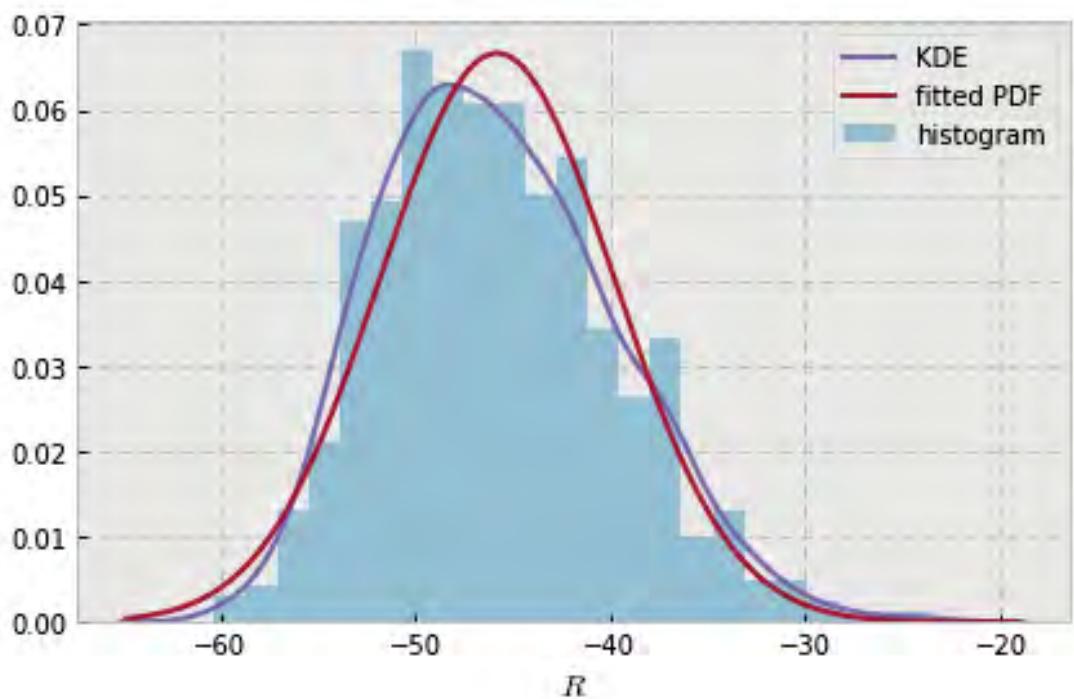


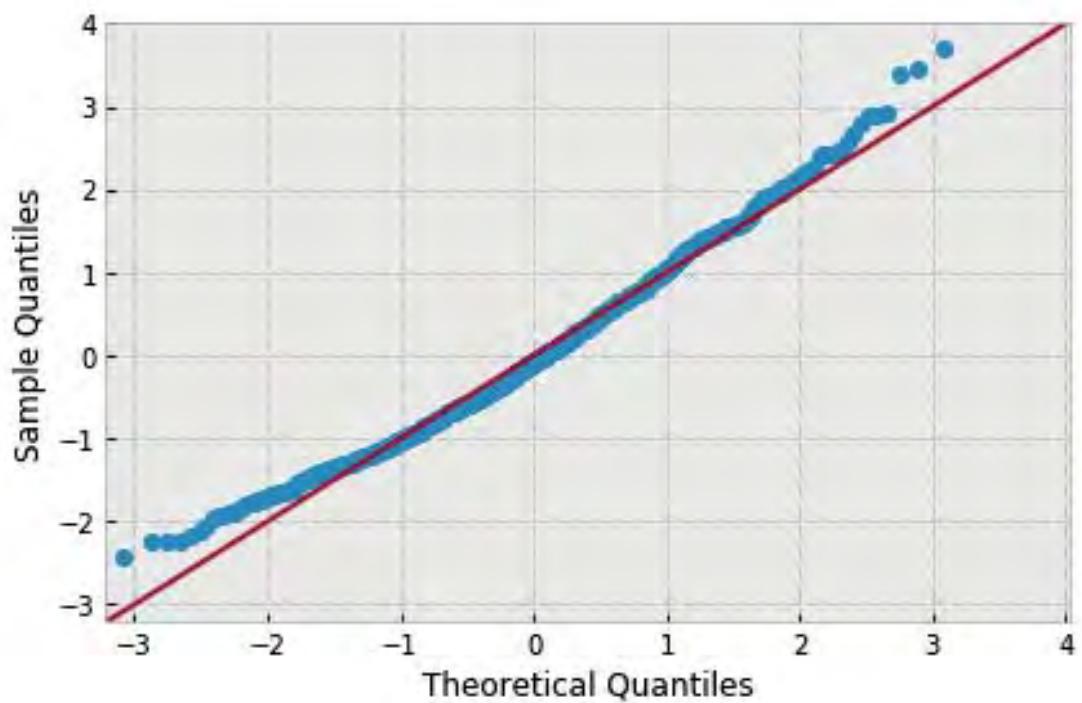


```
variable name: ln
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=-49.03673807033, scale=3.8382004515674044)
DescribeResult(nobs=1000, minmax=(-60.63422706000001, -34.995965349999999)
               mean=-49.03673807033003, variance=14.746529235647875,
               skewness=0.20227227936376874, kurtosis=-0.0038831961380543234)
```

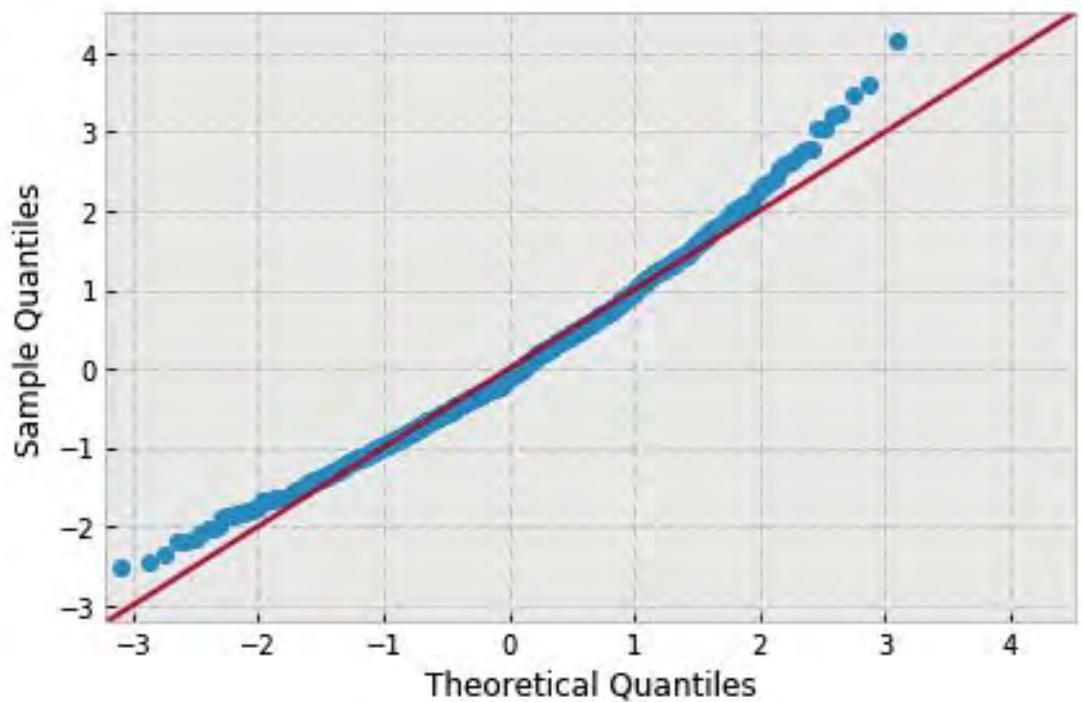
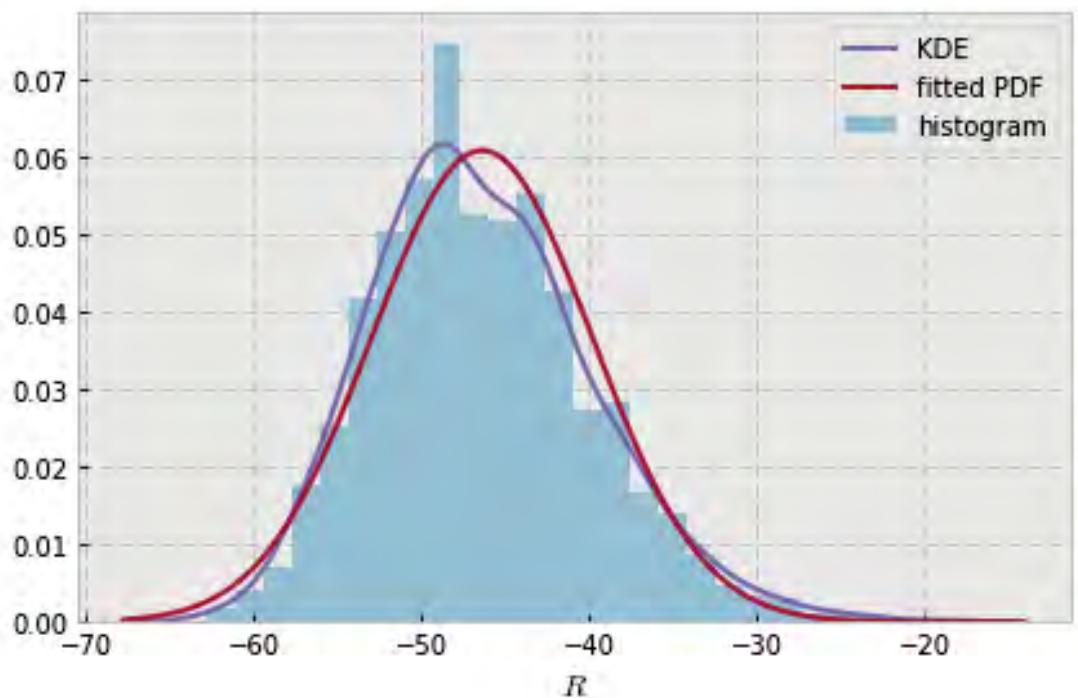


```
variable name: ln
variable value: 1.833333333333333
distribution: normal(shape=(), loc=-45.81068216829001, scale=5.986318499988607)
DescribeResult(nobs=1000, minmax=(-60.304076940000002, -23.562845249999999)
               mean=-45.810682168290008, variance=35.871881064370214,
               skewness=0.4244259645129395, kurtosis=-0.05613301806157622)
```

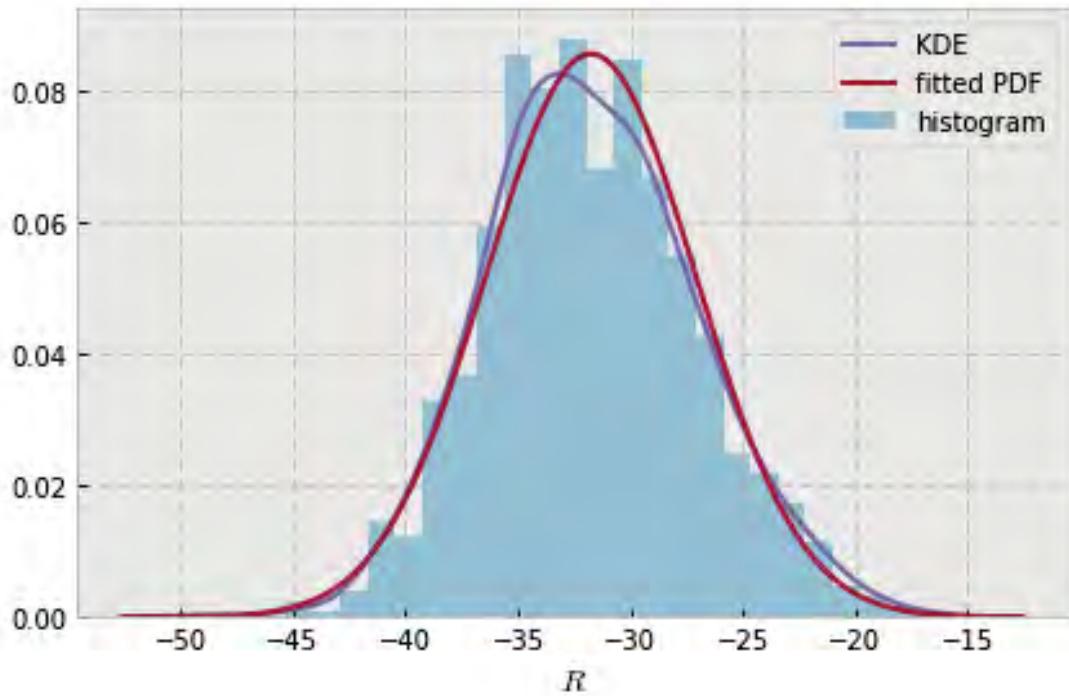


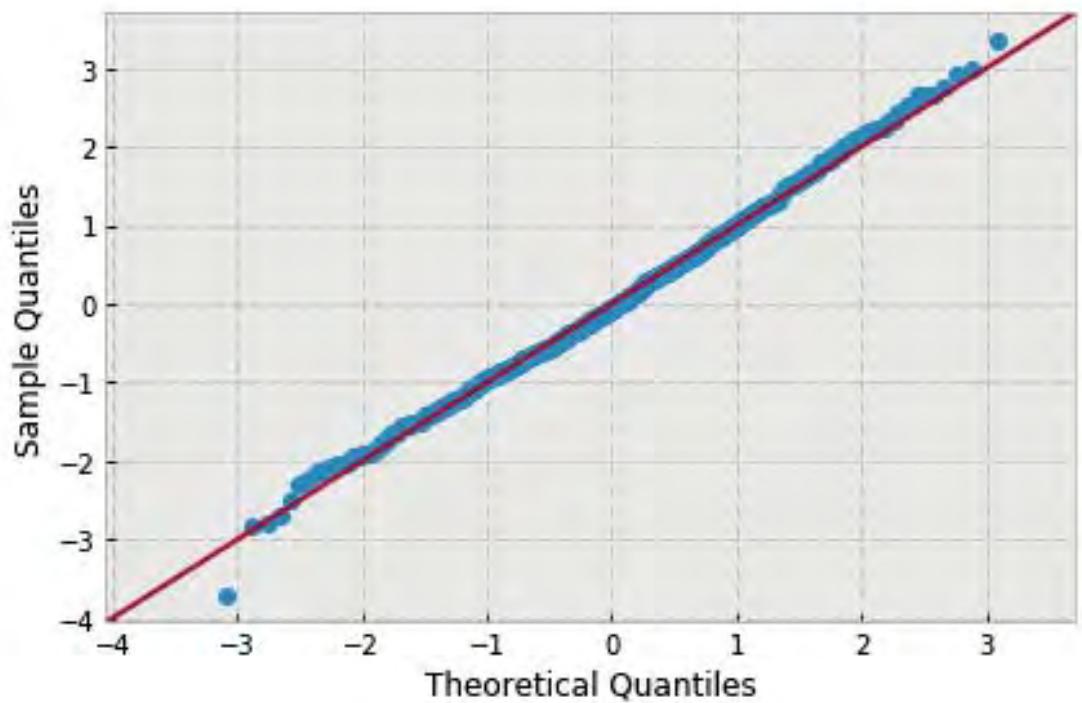


```
variable name: ln
variable value: 2.0
distribution: normal(shape=(), loc=-46.38260209806, scale=6.545446934621923)
DescribeResult(nobs=1000, minmax=(-62.78255621999999, -19.06683759999999)
               mean=-46.382602098059998, variance=42.885761335286816,
               skewness=0.5102803381687228, kurtosis=0.4005693390762337)
```

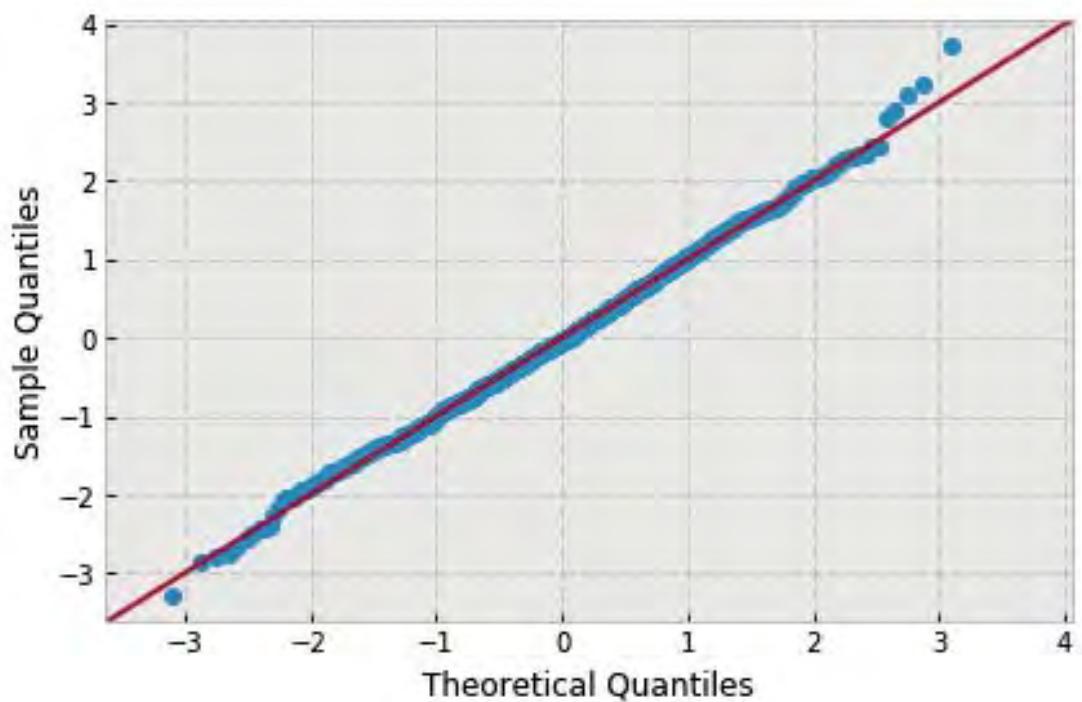
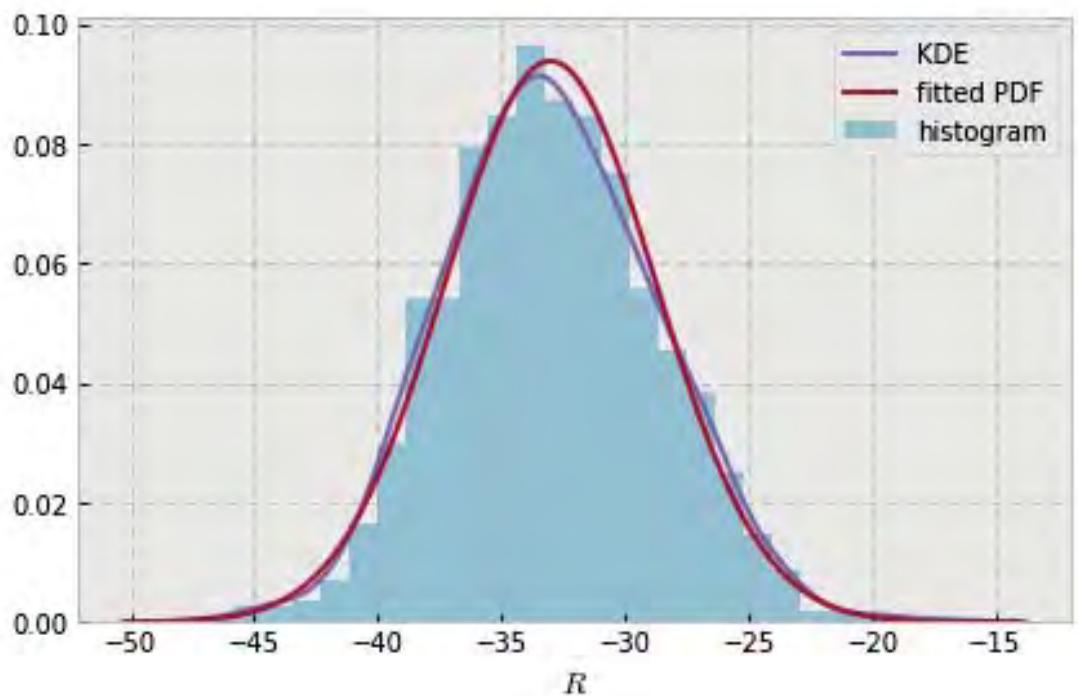


```
variable name: nB
variable value: 4.0
distribution: normal(shape=(), loc=-31.75389260831, scale=4.666240187129613)
DescribeResult(nobs=1000, minmax=(-49.001878720000001, -16.13233889)
               mean=-31.75389260831, variance=21.795593077060474,
               skewness=0.17233270968561915, kurtosis=0.04339168414167549)
```

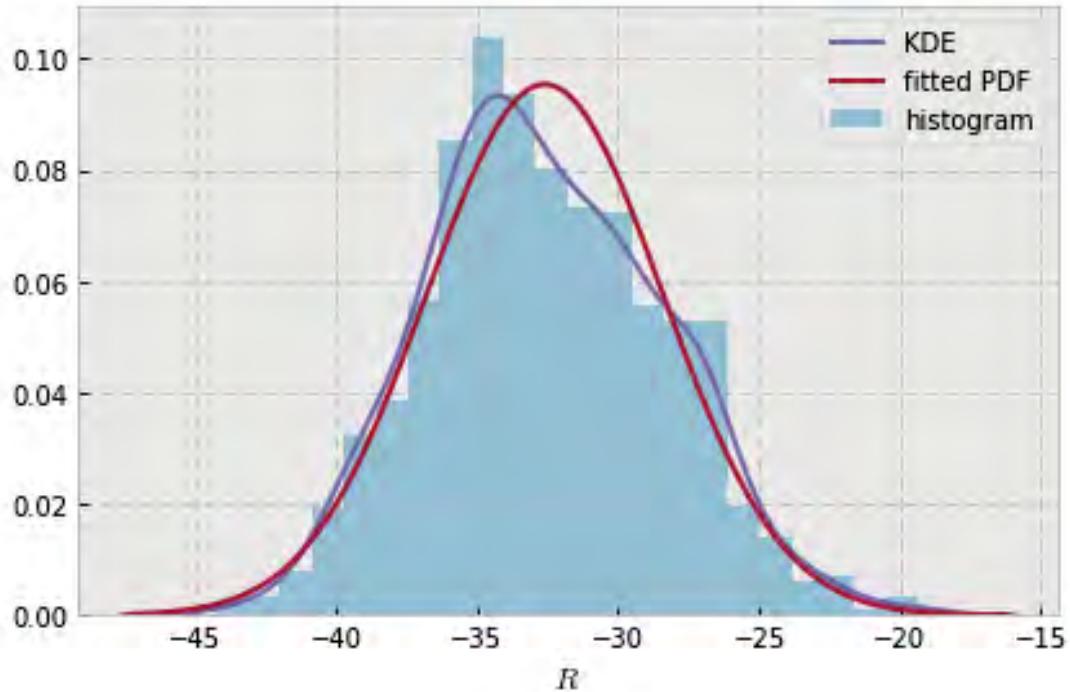


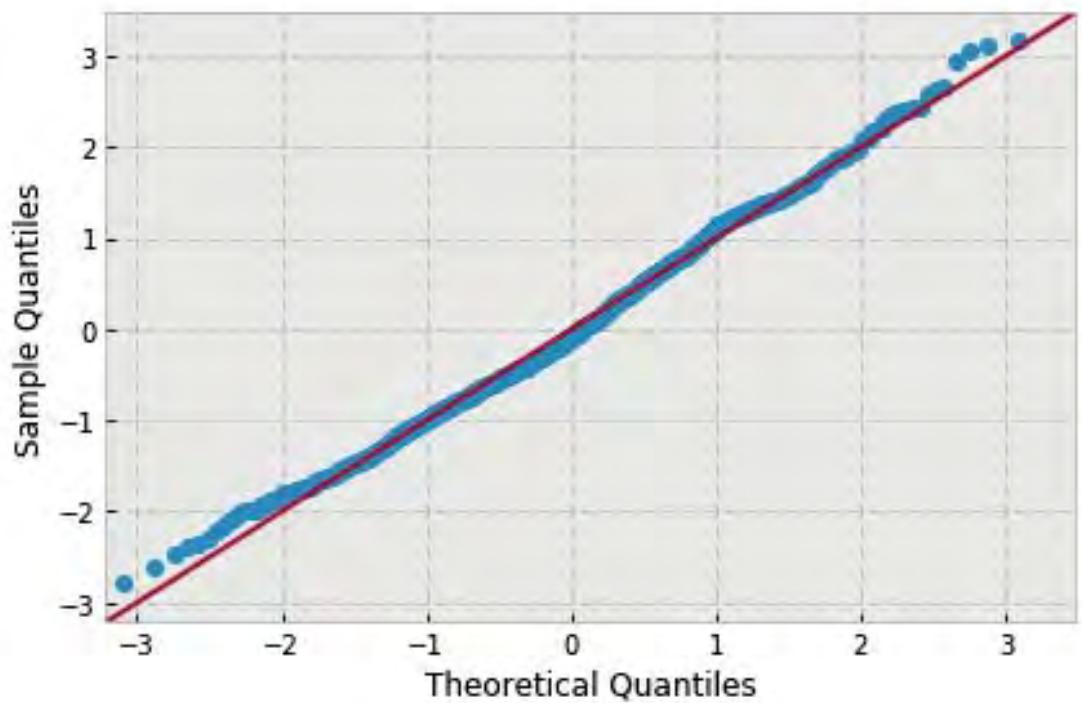


```
variable name: nB
variable value: 8.0
distribution: normal(shape=(), loc=-33.01969339527, scale=4.25332684615452)
DescribeResult(nobs=1000, minmax=(-46.94122398999997, -17.22502179)
               mean=-33.019693395269996, variance=18.108898158377134,
               skewness=0.11742051570916107, kurtosis=0.01962087871371354)
```

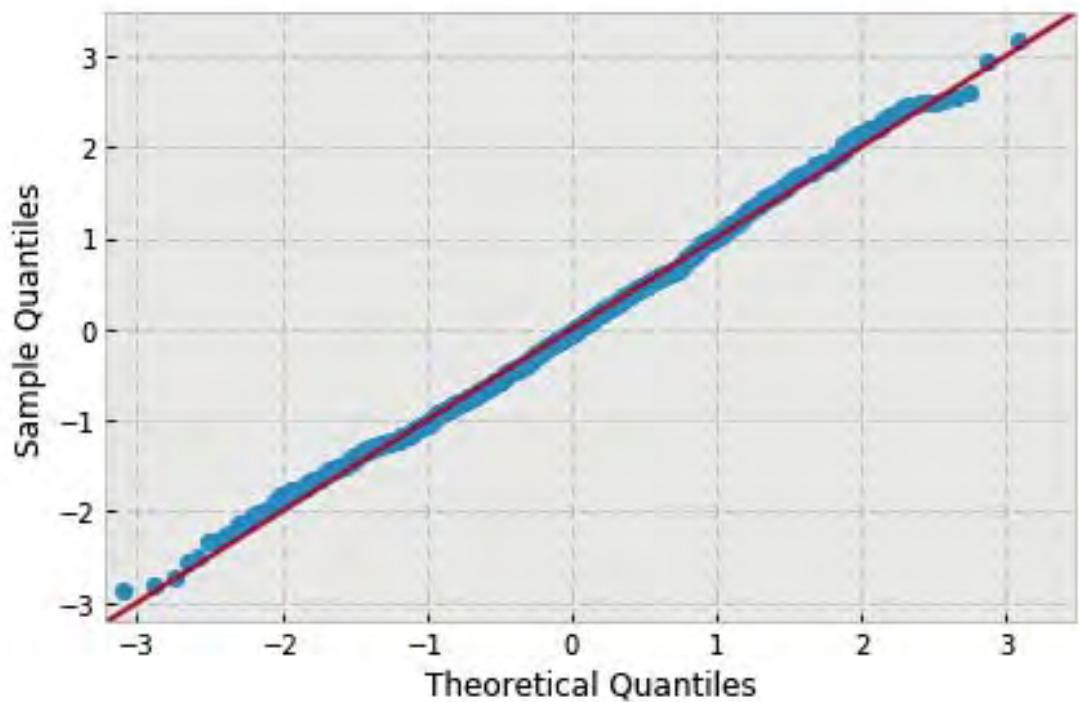
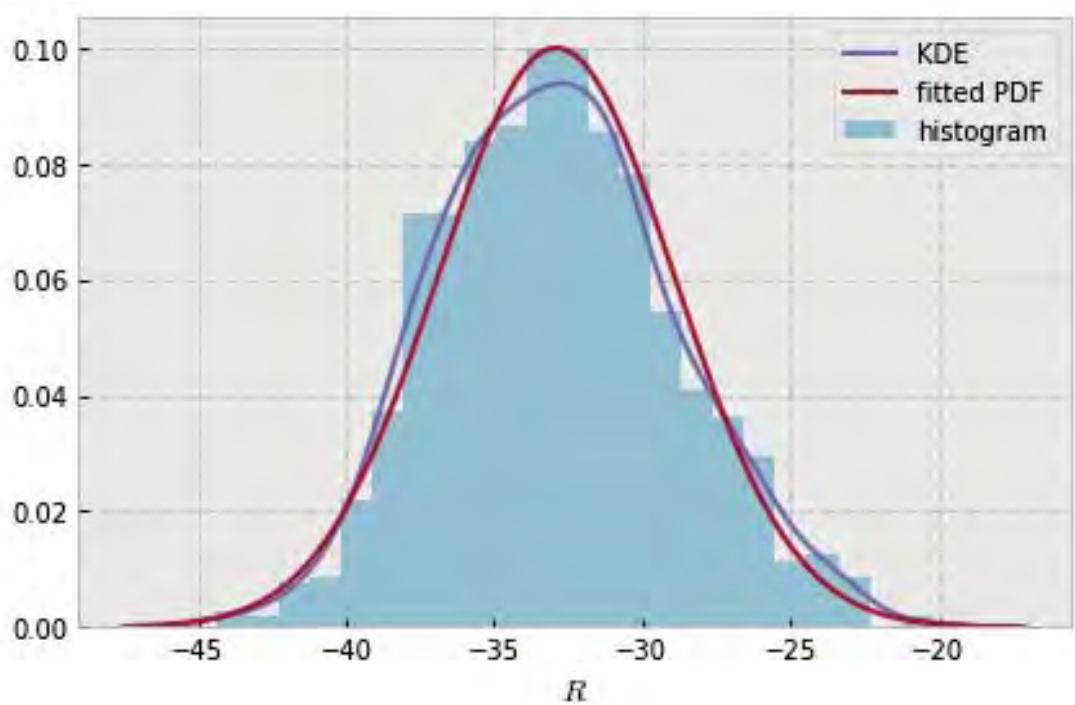


```
variable name: nB
variable value: 12.0
distribution: normal(shape=(), loc=-32.5969170562, scale=4.183909270840132)
DescribeResult(nobs=1000, minmax=(-44.264772710000003, -19.310768209999999)
               mean=-32.596917056199999, variance=17.522619406028042,
               skewness=0.20634218589156164, kurtosis=-0.2514061198564672)
```

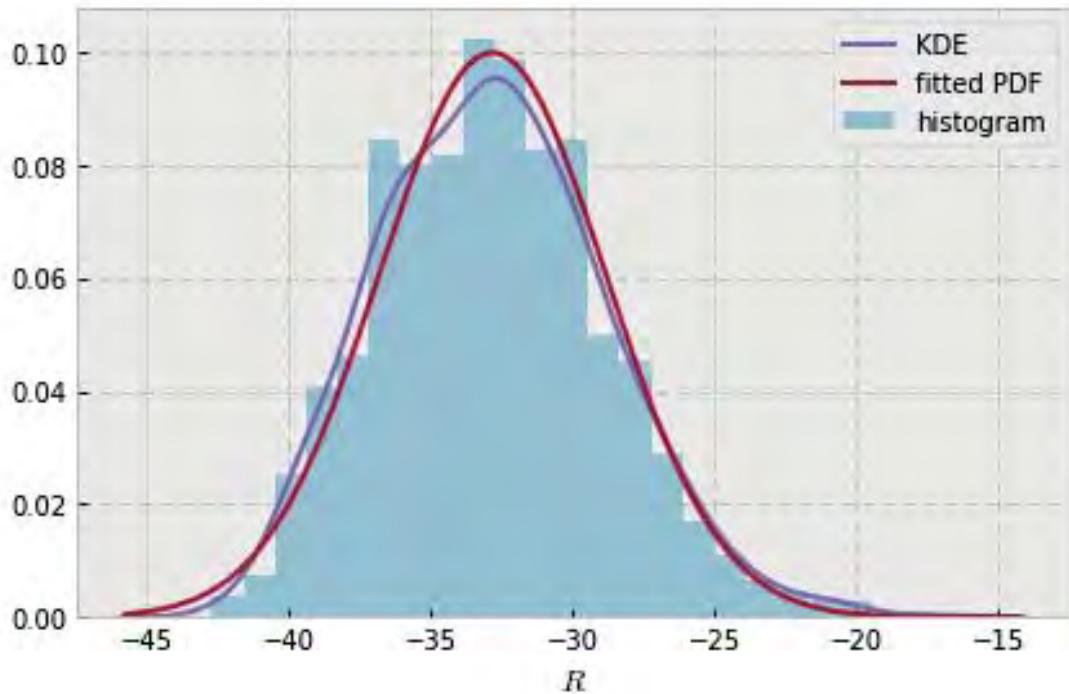


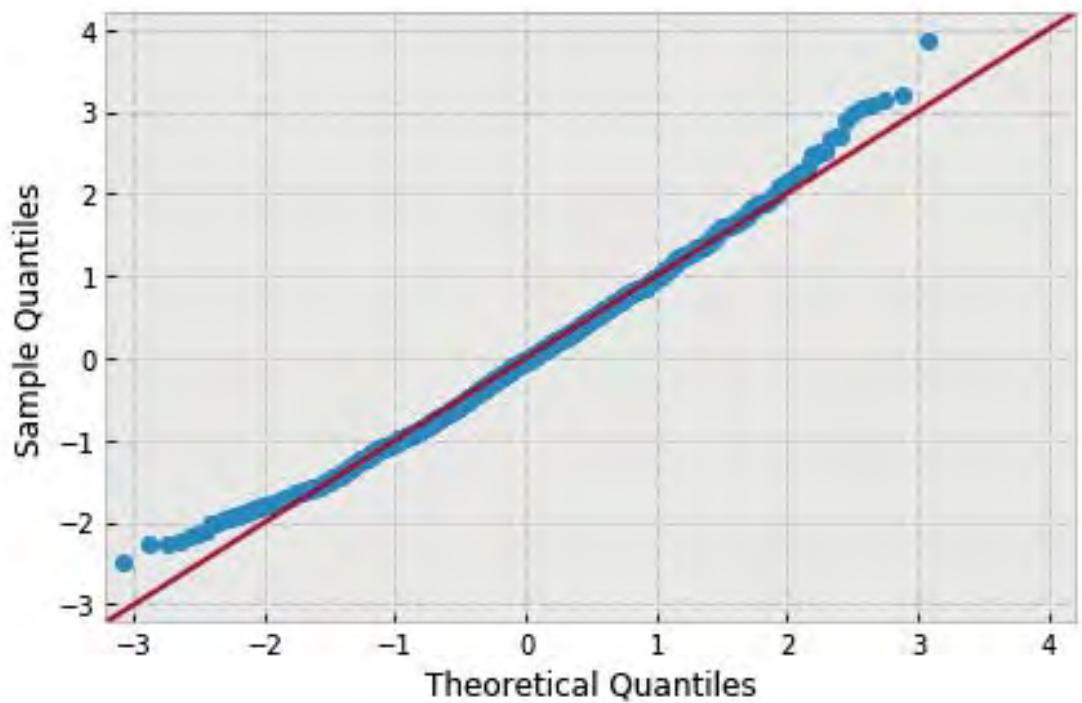


```
variable name: nB
variable value: 16.0
distribution: normal(shape=(), loc=-32.900874763500006, scale=3.989561247154807)
DescribeResult(nobs=1000, minmax=(-44.37048008, -20.254164429999999)
               mean=-32.90087476350006, variance=15.932531476275694,
               skewness=0.18510806908320773, kurtosis=-0.1751401430394548)
```

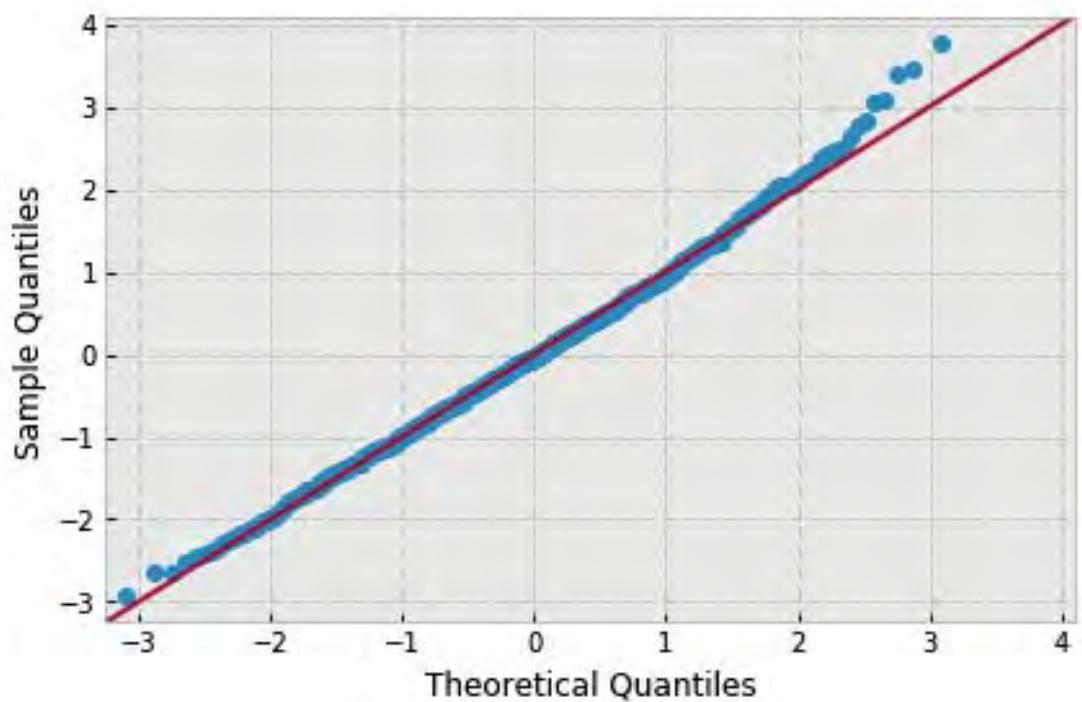
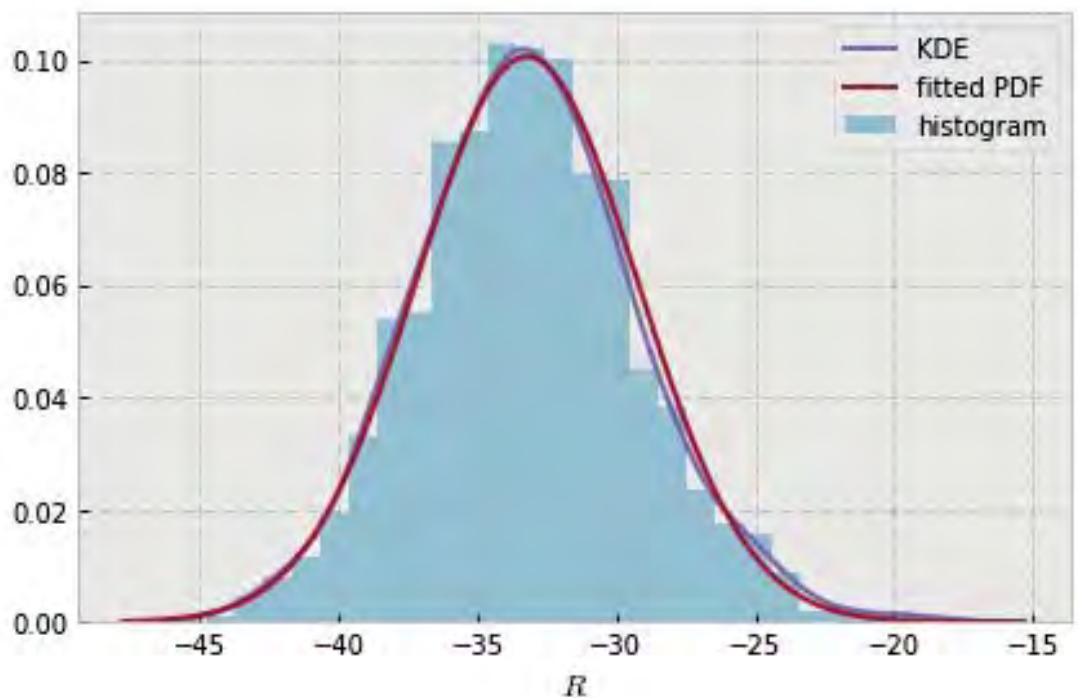


```
variable name: nB
variable value: 20.0
distribution: normal(shape=(), loc=-32.81545656654, scale=3.997116777205032)
DescribeResult(nobs=1000, minmax=(-42.685499020000002, -17.30025771)
               mean=-32.81545656654, variance=15.992935466080022,
               skewness=0.31976609275246265, kurtosis=0.026032858816614546)
```

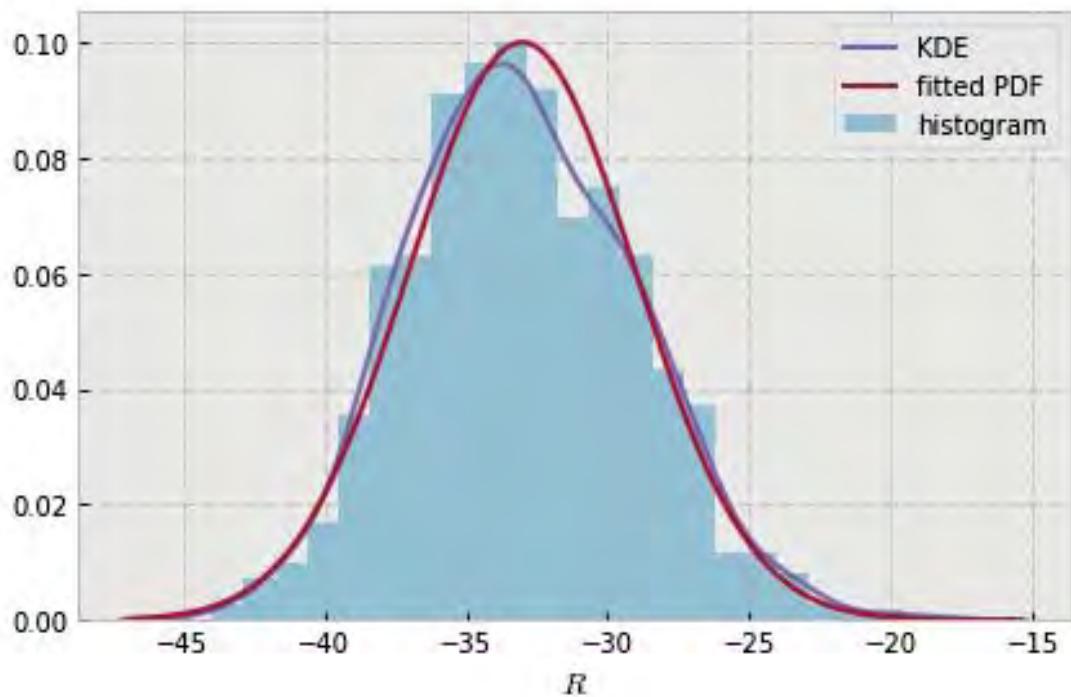


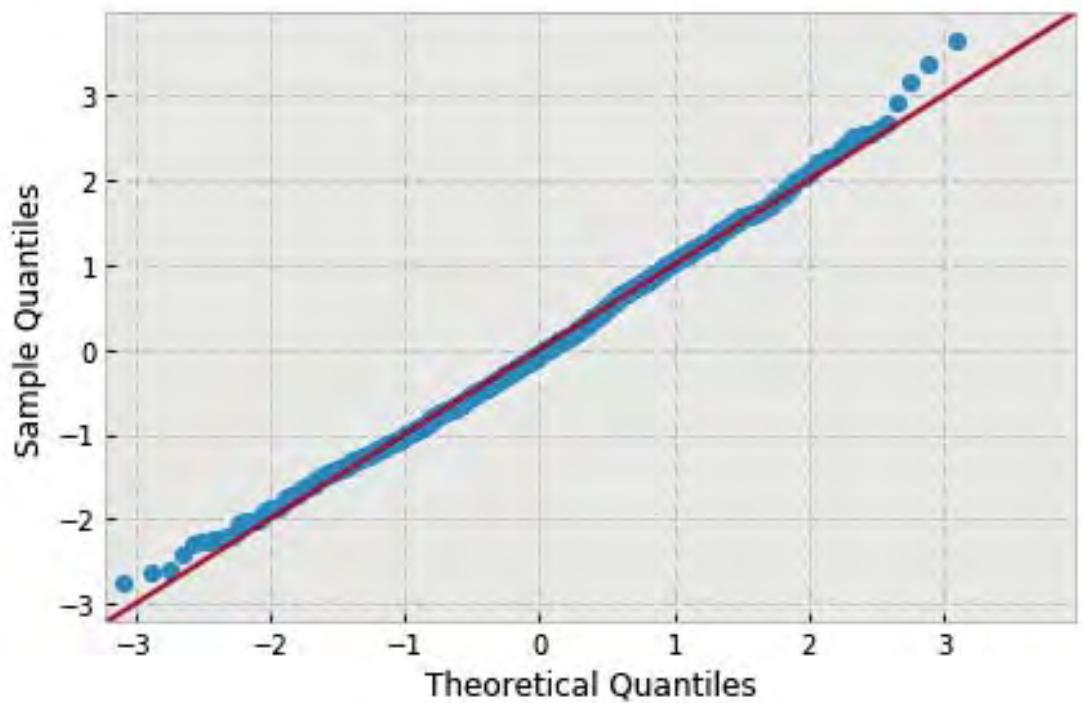


```
variable name: nB
variable value: 24.0
distribution: normal(shape=(), loc=-33.22031188893, scale=3.965836134251512)
DescribeResult(nobs=1000, minmax=(-44.76503457999999, -18.307320860000001)
               mean=-33.220311888929999, variance=15.743599843578554,
               skewness=0.22191107319314668, kurtosis=0.2621536835148768)
```

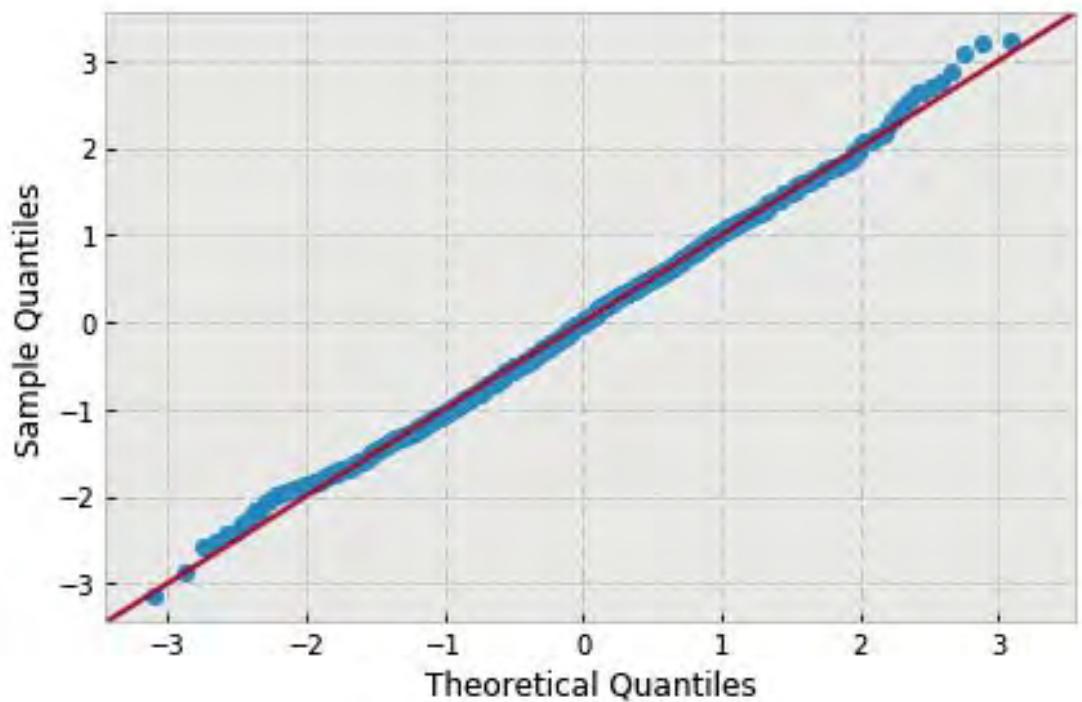
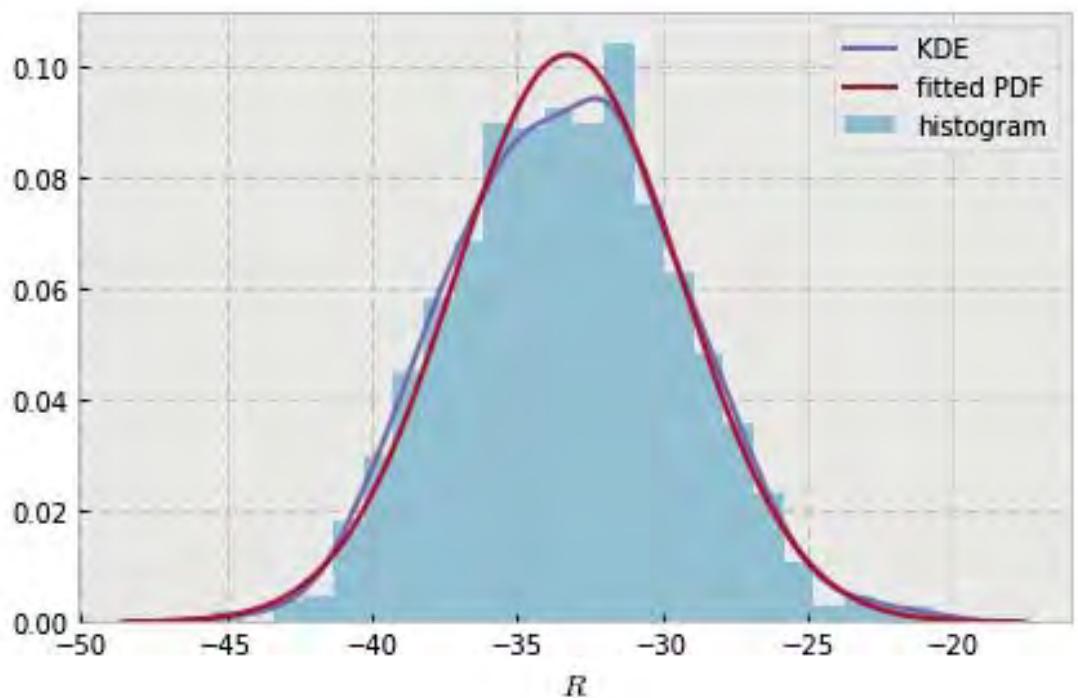


```
variable name: nB
variable value: 28.0
distribution: normal(shape=(), loc=-33.028290165630004, scale=3.9883624114214604)
DescribeResult(nobs=1000, minmax=(-43.94055268999997, -18.47573493999999)
               mean=-33.02829016563004, variance=15.922957682522128,
               skewness=0.22835683301191126, kurtosis=-0.07092944772174059)
```

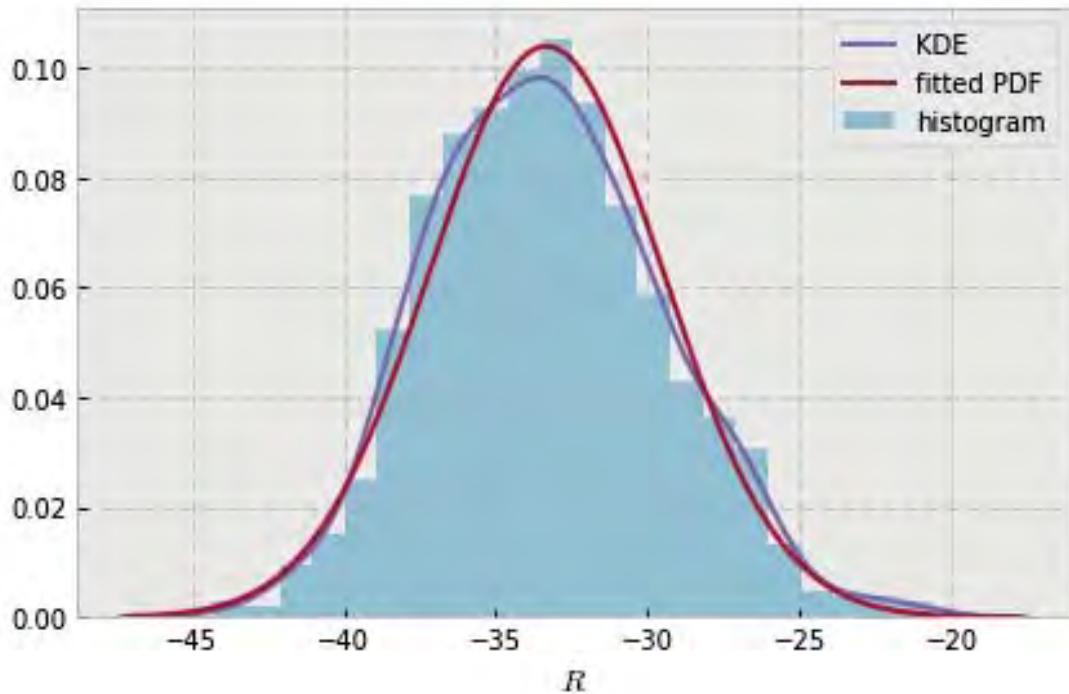


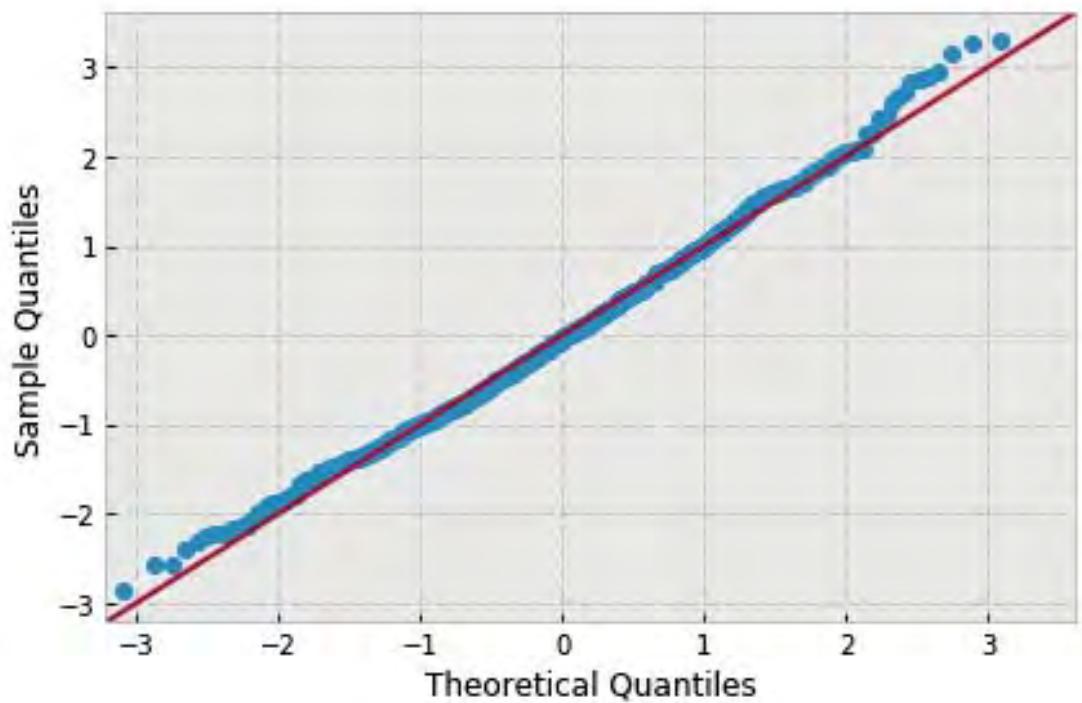


```
variable name: nB
variable value: 32.0
distribution: normal(shape=(), loc=-33.261248829820005, scale=3.902756434007384)
DescribeResult(nobs=1000, minmax=(-45.467207629999997, -20.660573530000001)
               mean=-33.26124882982005, variance=15.246754537723756,
               skewness=0.10351902958500257, kurtosis=-0.14201571824063963)
```

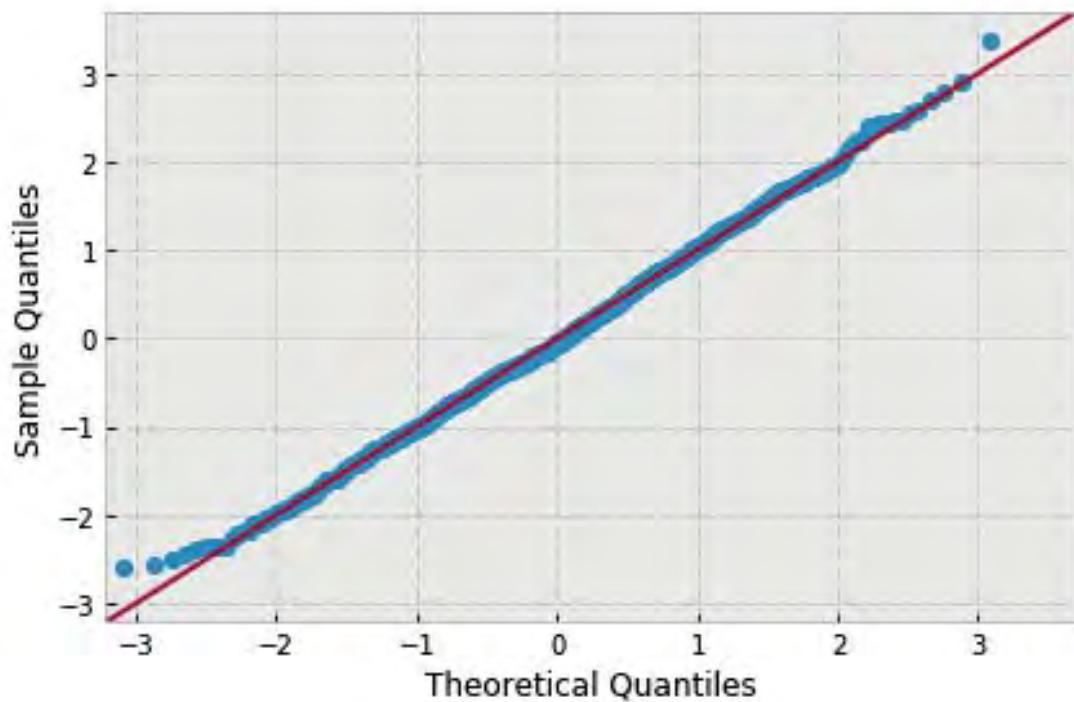
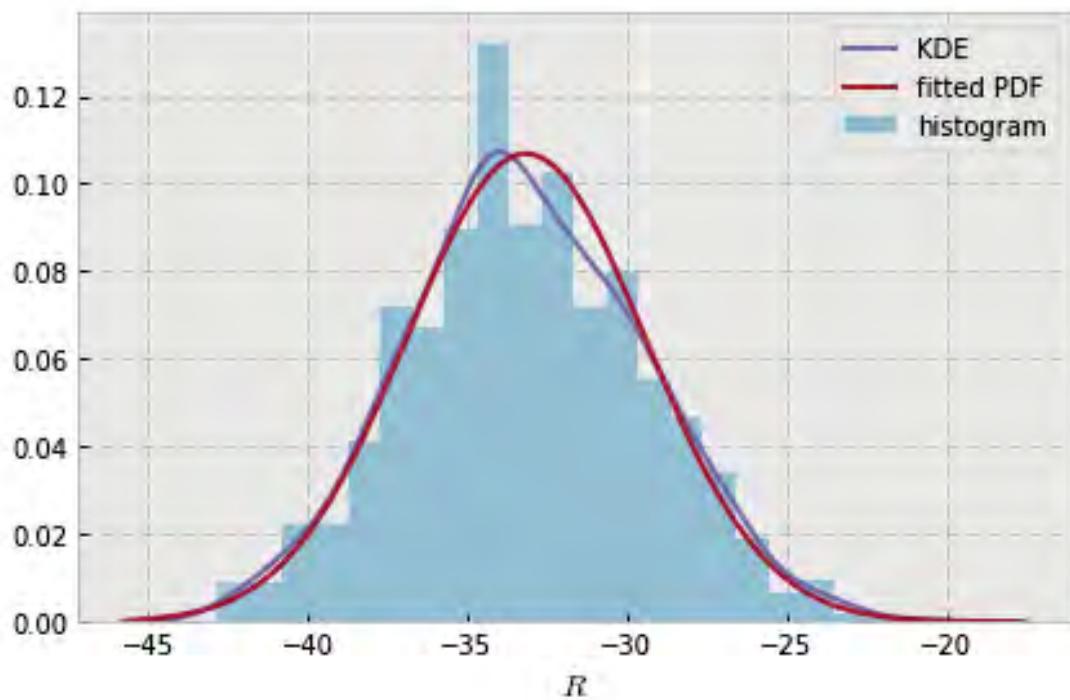


```
variable name: nB
variable value: 36.0
distribution: normal(shape=(), loc=-33.33919935315001, scale=3.8383353085570446)
DescribeResult(nobs=1000, minmax=(-44.262255170000003, -20.682595249999999)
               mean=-33.33919935315008, variance=14.747565506422125,
               skewness=0.2568803719082728, kurtosis=-0.06375946038969049)
```

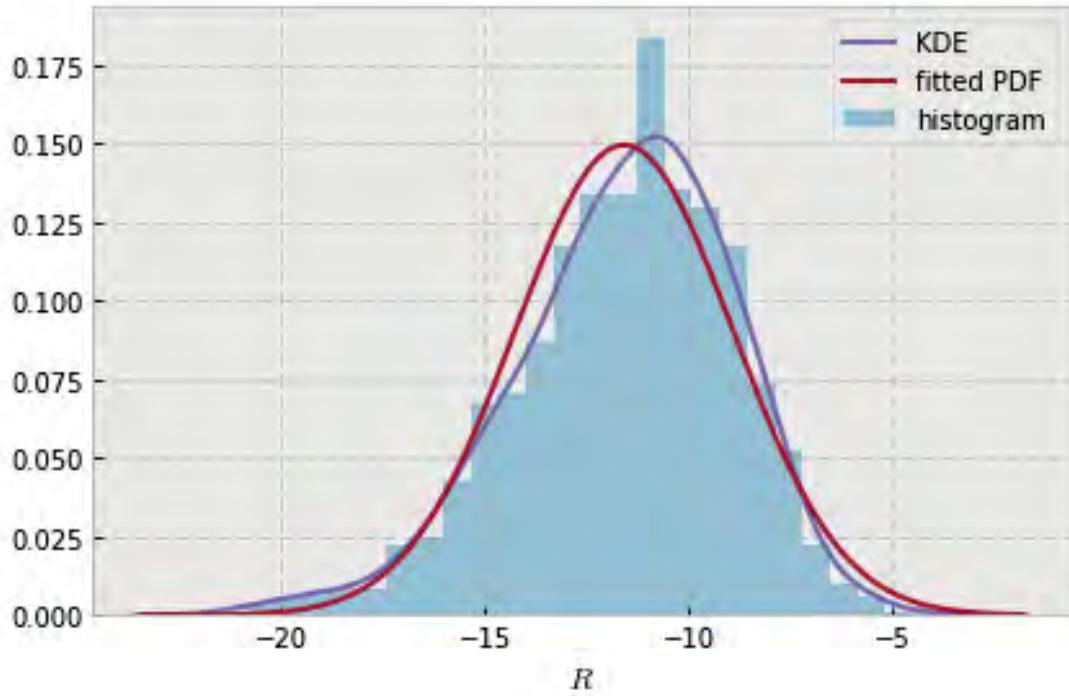


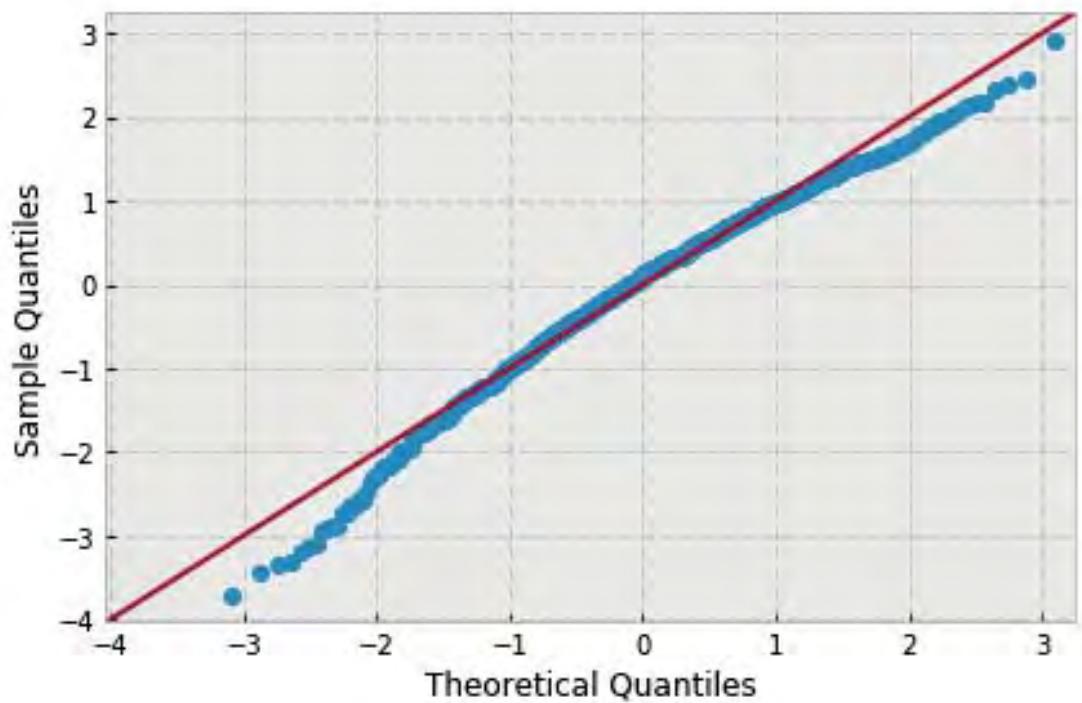


```
variable name: nB
variable value: 40.0
distribution: normal(shape=(), loc=-33.153479231700004, scale=3.7340835813772957)
DescribeResult(nobs=1000, minmax=(-42.79744172000002, -20.514132239999999)
               mean=-33.15347923170004, variance=13.957337530241732,
               skewness=0.11488411063686266, kurtosis=-0.1600684825652472)
```

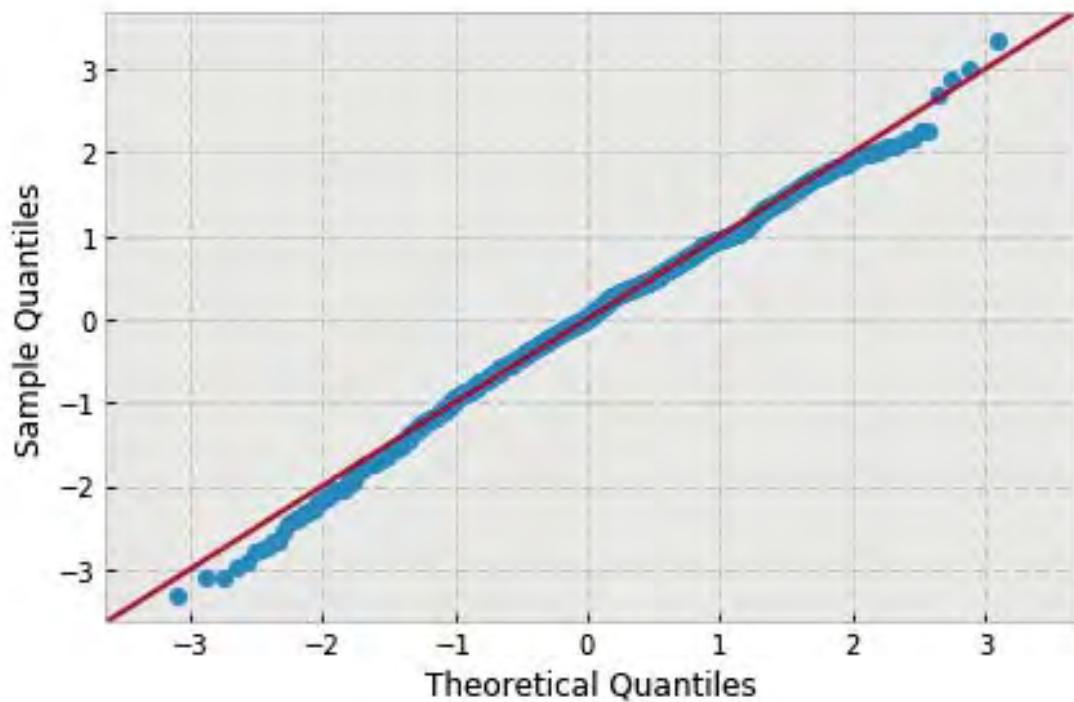
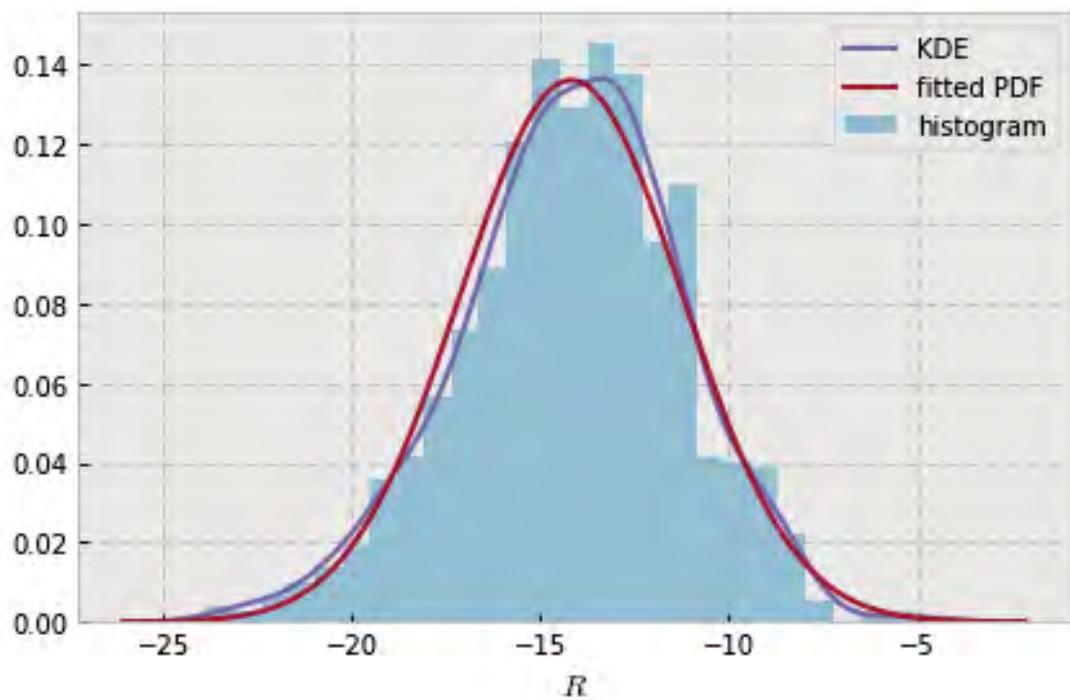


```
variable name: r
variable value: 1.0
distribution: normal(shape=(), loc=-11.591235789775, scale=2.667951157626695)
DescribeResult(nobs=1000, minmax=(-21.471458120000001, -3.821486063)
               mean=-11.591235789775, variance=7.1250884679495723,
               skewness=-0.5166406888683749, kurtosis=0.39168136602036174)
```

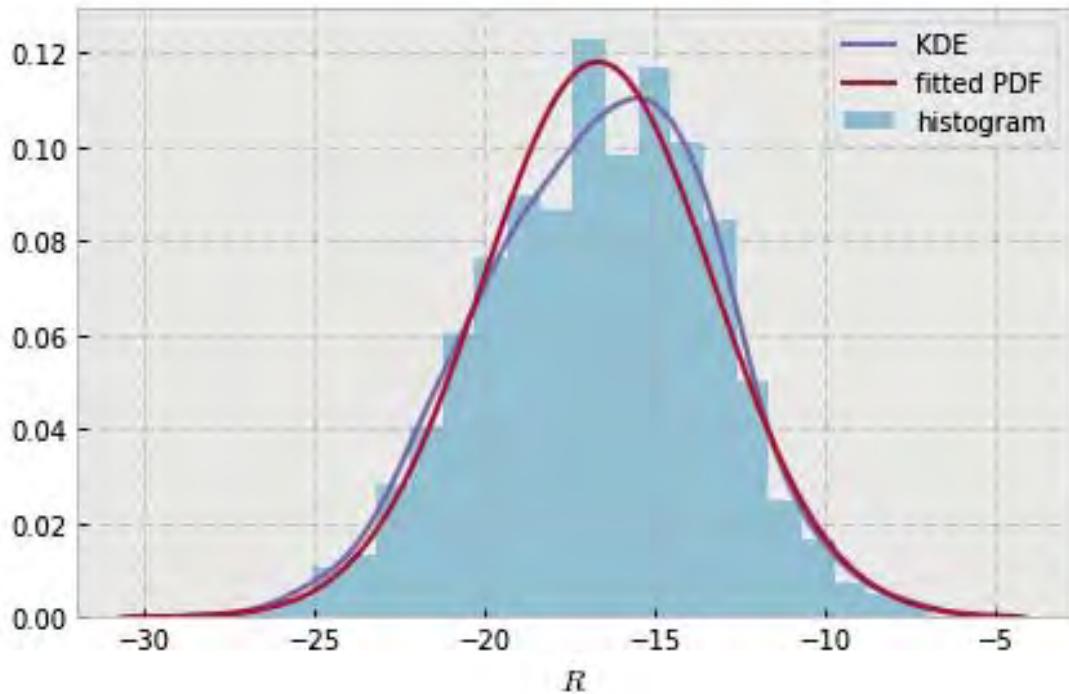


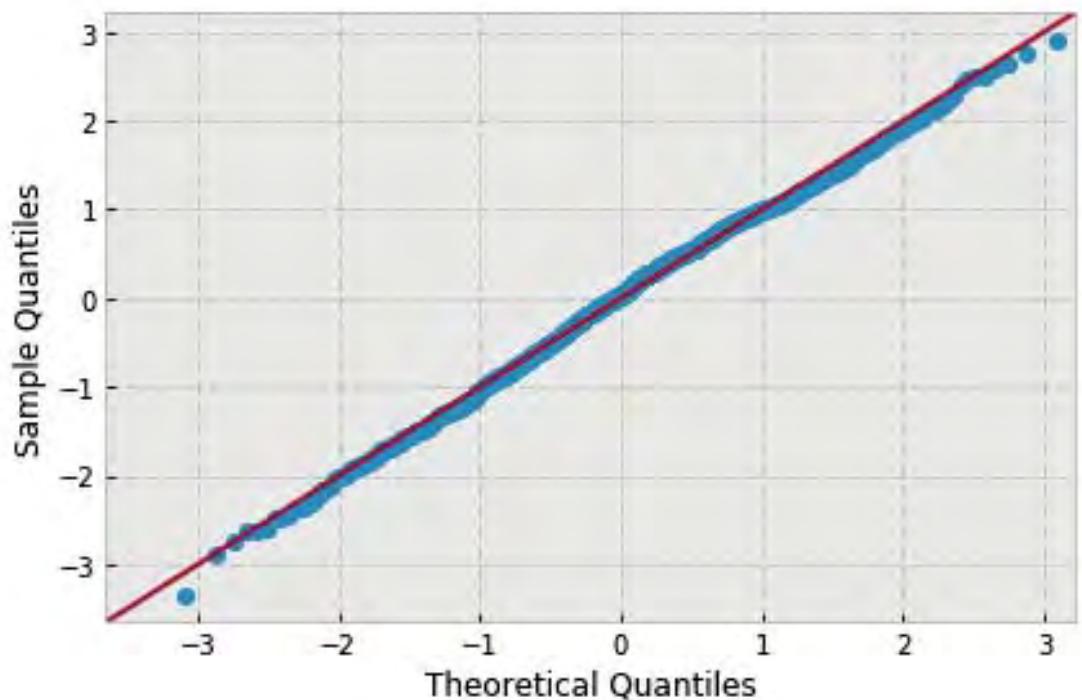


```
variable name: r
variable value: 1.222222222222223
distribution: normal(shape=(), loc=-14.177006214292, scale=2.9321227434692263)
DescribeResult(nobs=1000, minmax=(-23.84744617000001, -4.376216848000003)
               mean=-14.177006214292, variance=8.6059497325020029,
               skewness=-0.22833733056179129, kurtosis=0.1831186710730437)
```

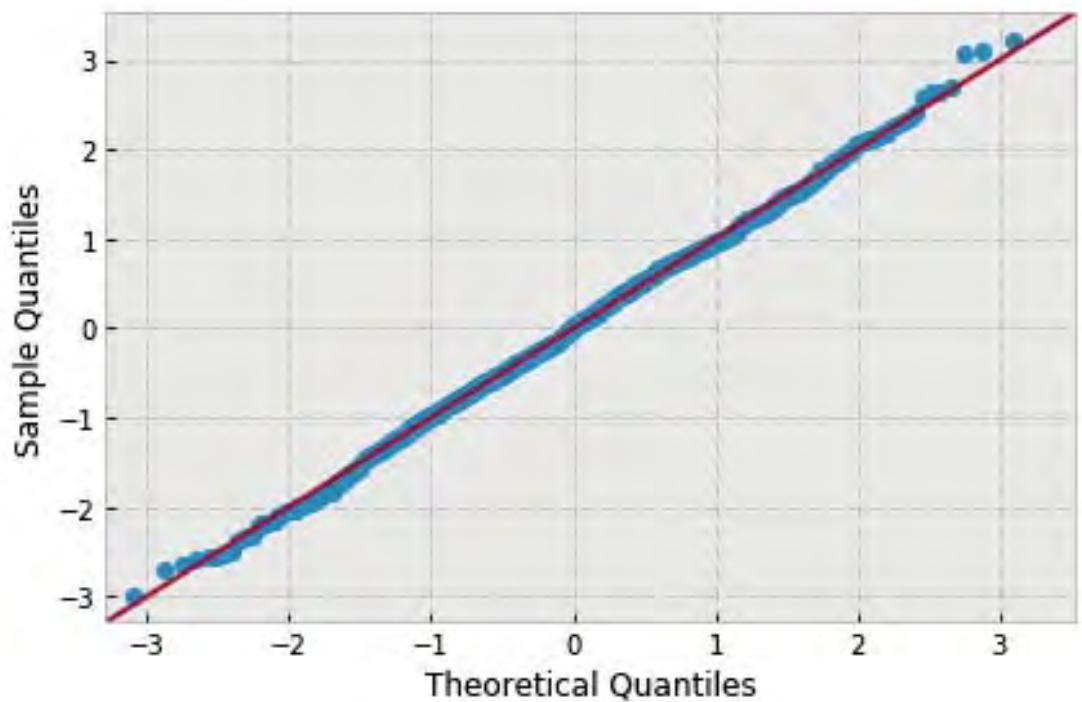
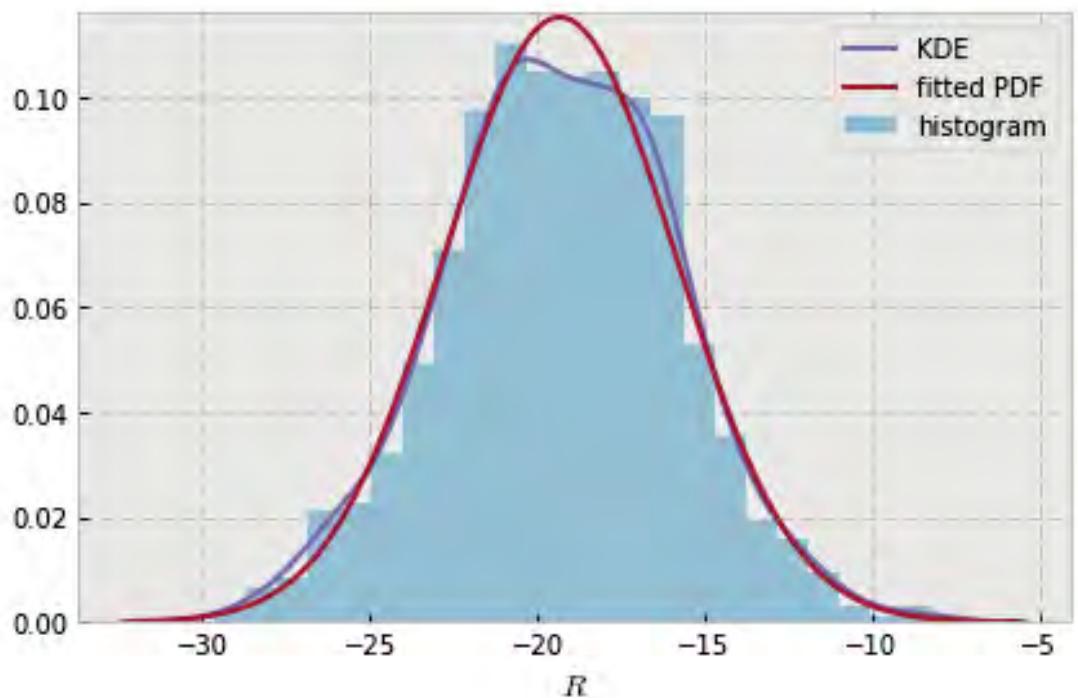


```
variable name: r
variable value: 1.444444444444444
distribution: normal(shape=(), loc=-16.676086386456, scale=3.378796130675062)
DescribeResult(nobs=1000, minmax=(-27.95913043999999, -6.880135683999999)
               mean=-16.67608638645599, variance=11.427690983648418,
               skewness=-0.1532788967344575, kurtosis=-0.19383431631754267)
```

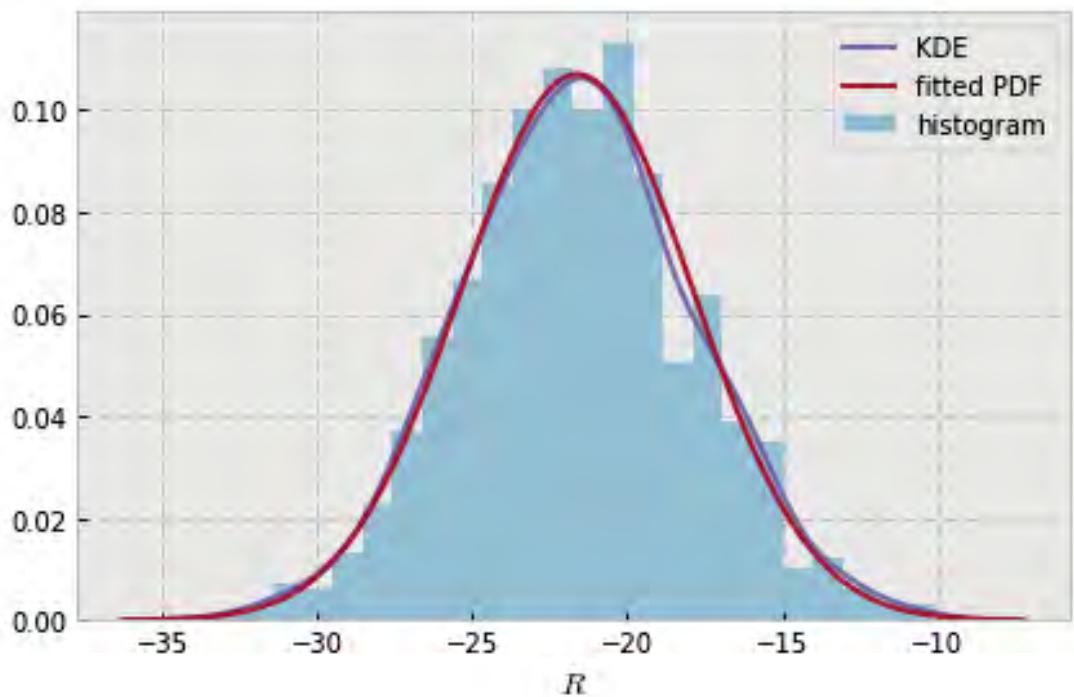


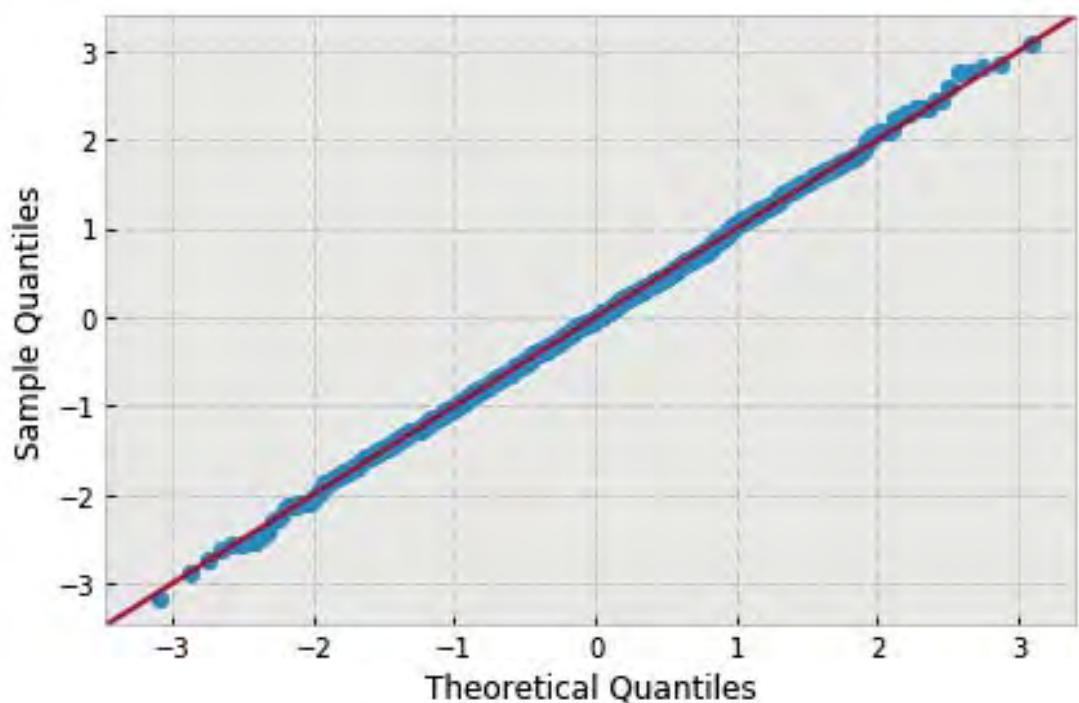


```
variable name: r
variable value: 1.6666666666666665
distribution: normal(shape=(), loc=-19.314520846761, scale=3.4625955628644376)
DescribeResult(nobs=1000, minmax=(-29.626628090000001, -8.181916693999999)
               mean=-19.314520846760999, variance=12.001569601570063,
               skewness=-0.014962072229165884, kurtosis=-0.02913077731933944)
```

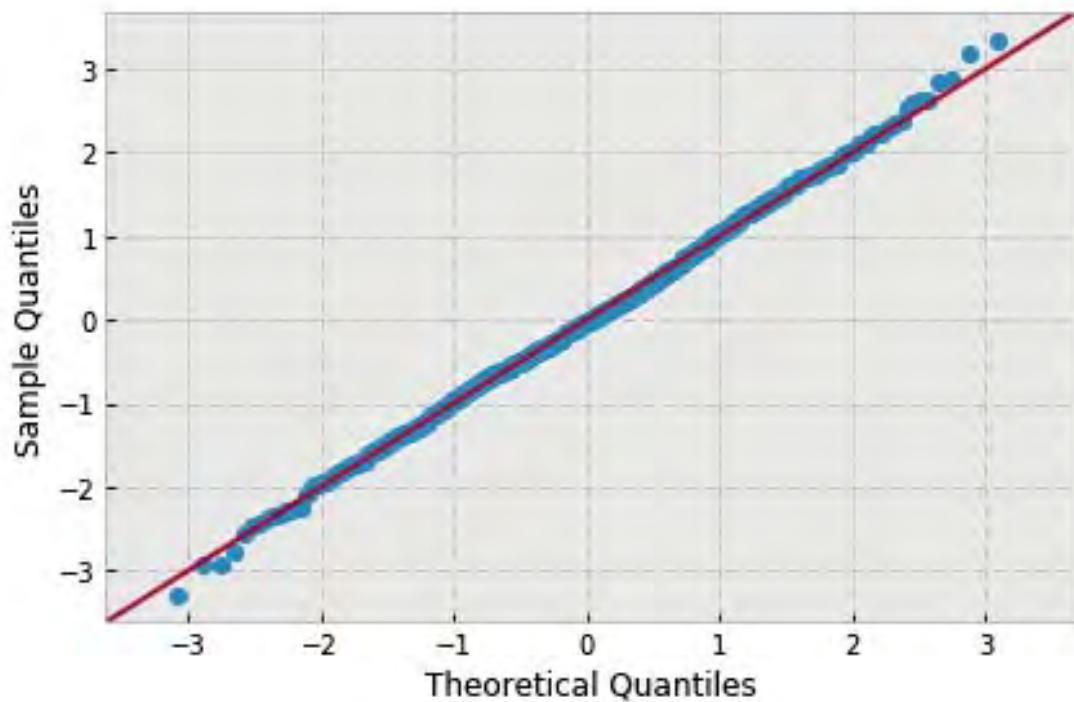
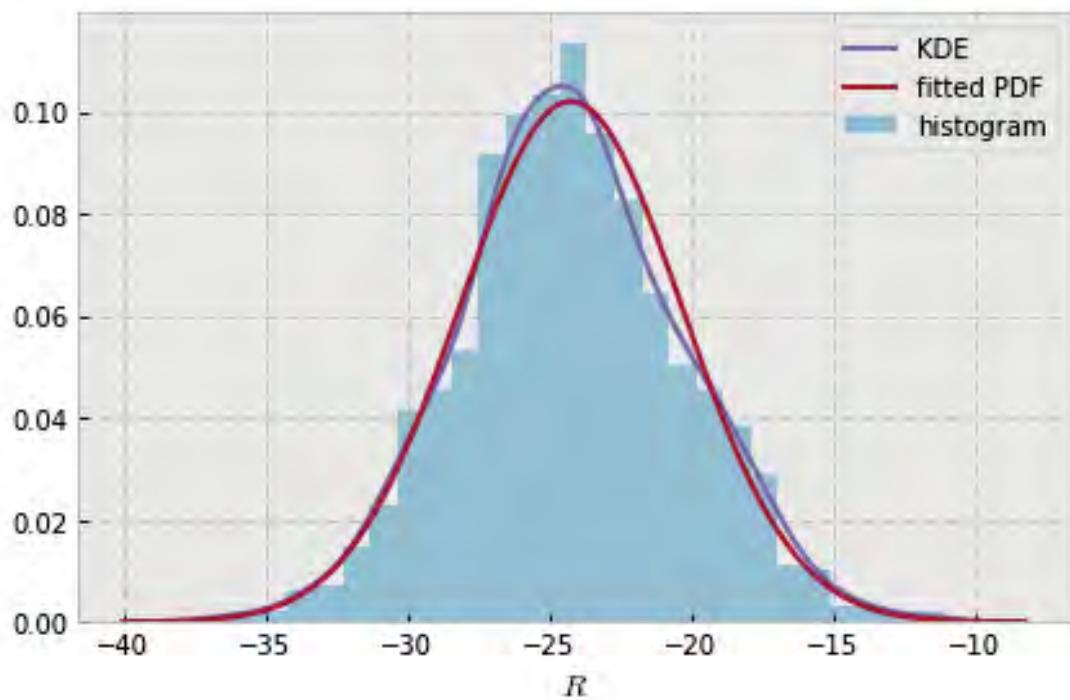


```
variable name: r
variable value: 1.888888888888888
distribution: normal(shape=(), loc=-21.61686044807, scale=3.7276579376789463)
DescribeResult(nobs=1000, minmax=(-33.38736033, -10.13137060999999)
               mean=-21.61686044807, variance=13.909343043384238,
               skewness=0.04425163083091408, kurtosis=-0.04795805496220673)
```

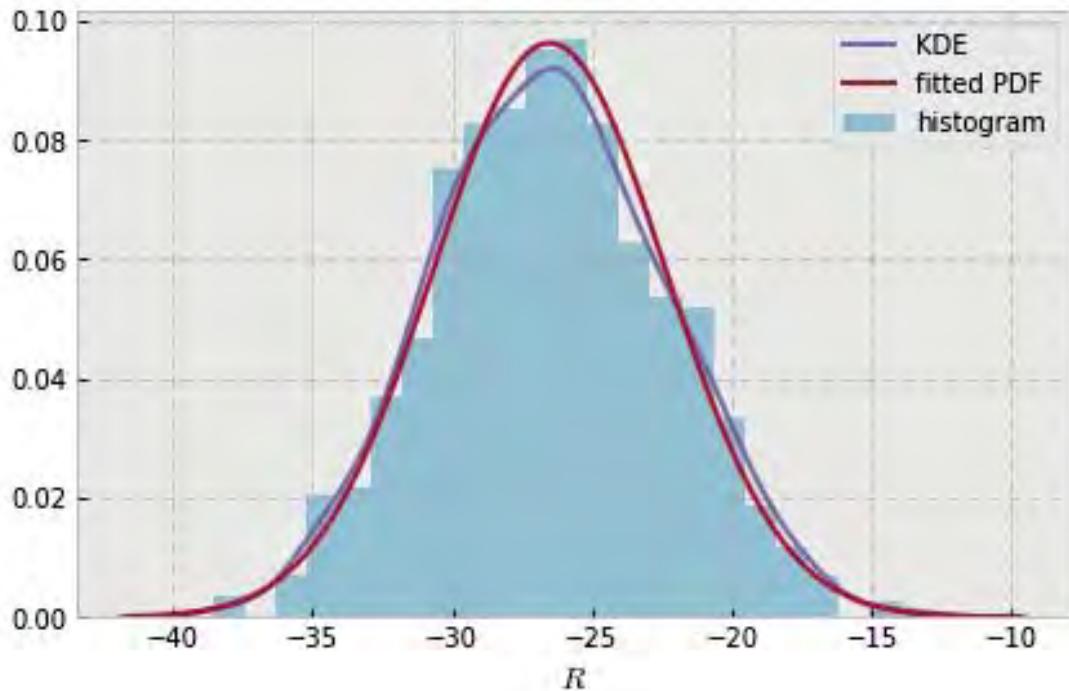


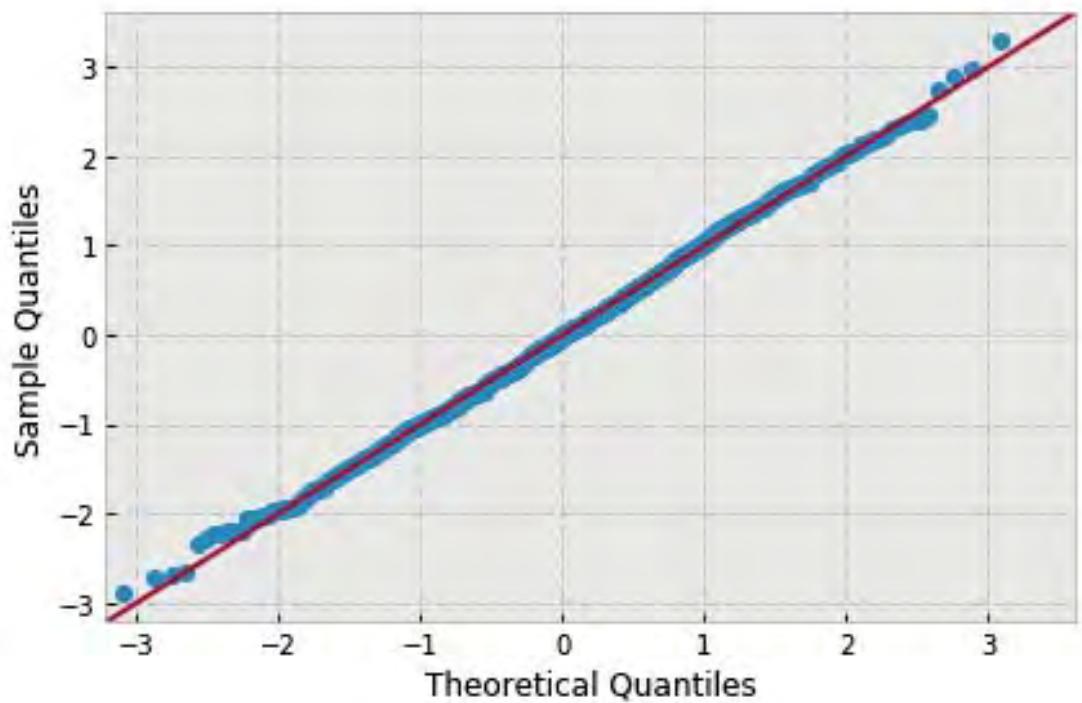


```
variable name: r
variable value: 2.111111111111111
distribution: normal(shape=(), loc=-24.230380311139996, scale=3.910793249868697)
DescribeResult(nobs=1000, minmax=(-37.080680899999997, -11.17739675)
               mean=-24.230380311139996, variance=15.309613456675239,
               skewness=0.08167576381314906, kurtosis=0.06662120135632721)
```

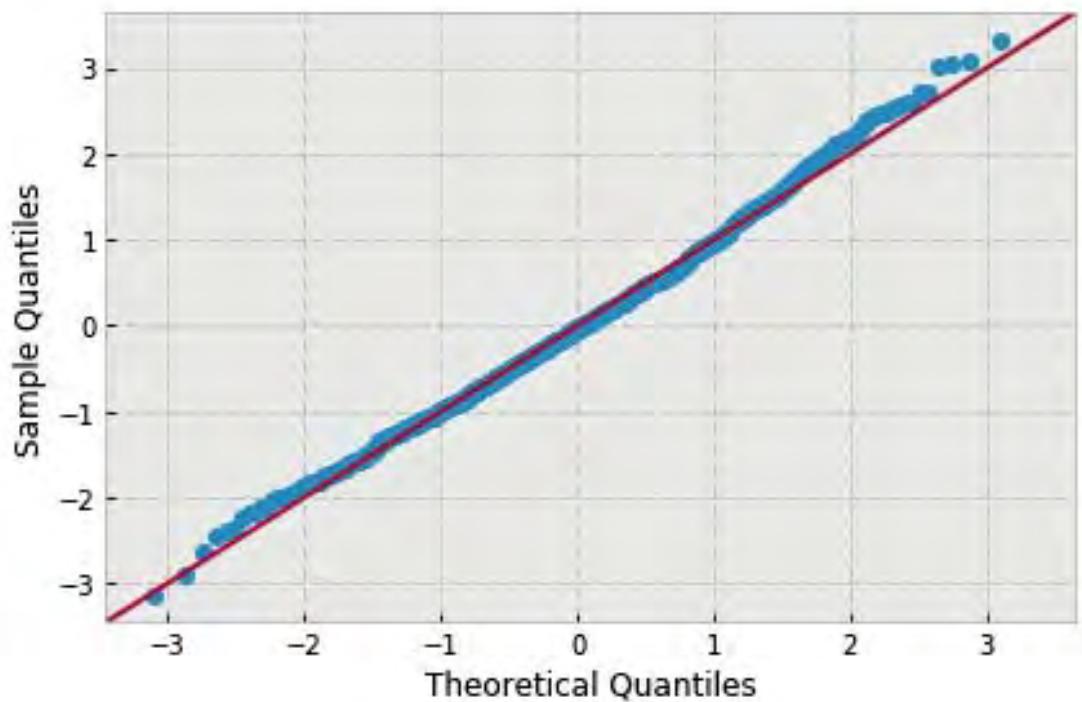
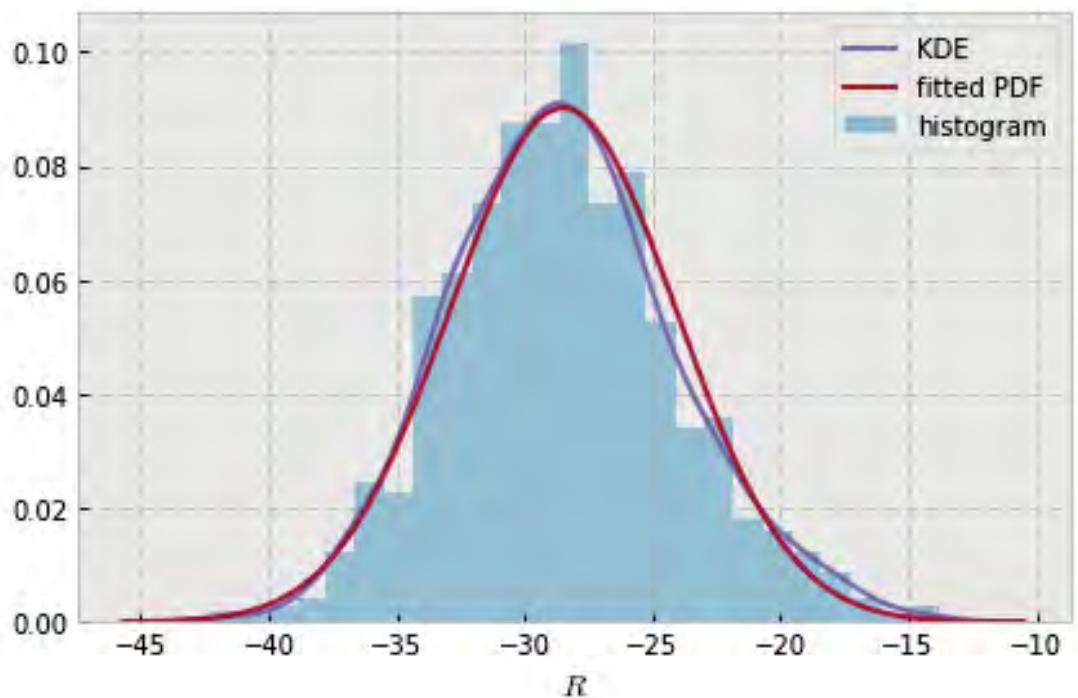


```
variable name: r
variable value: 2.333333333333333
distribution: normal(shape=(), loc=-26.520748238699998, scale=4.1477306468106505)
DescribeResult(nobs=1000, minmax=(-38.528175840000003, -12.865756559999999)
               mean=-26.520748238699998, variance=17.220890408901194,
               skewness=0.08372638701155213, kurtosis=-0.2147989552350409)
```

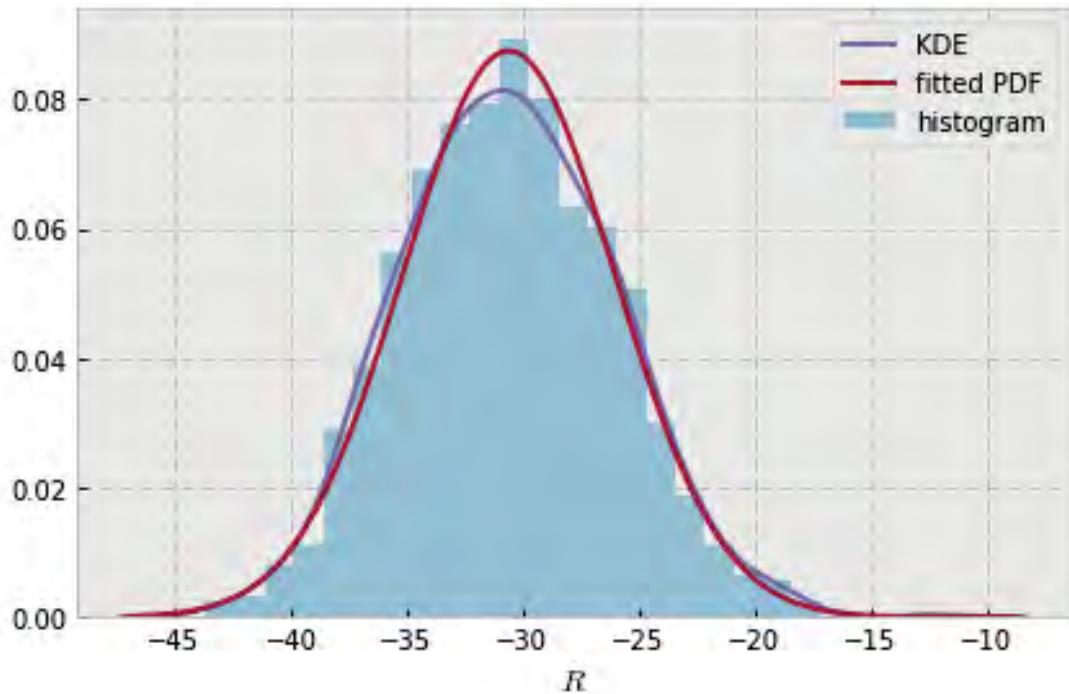


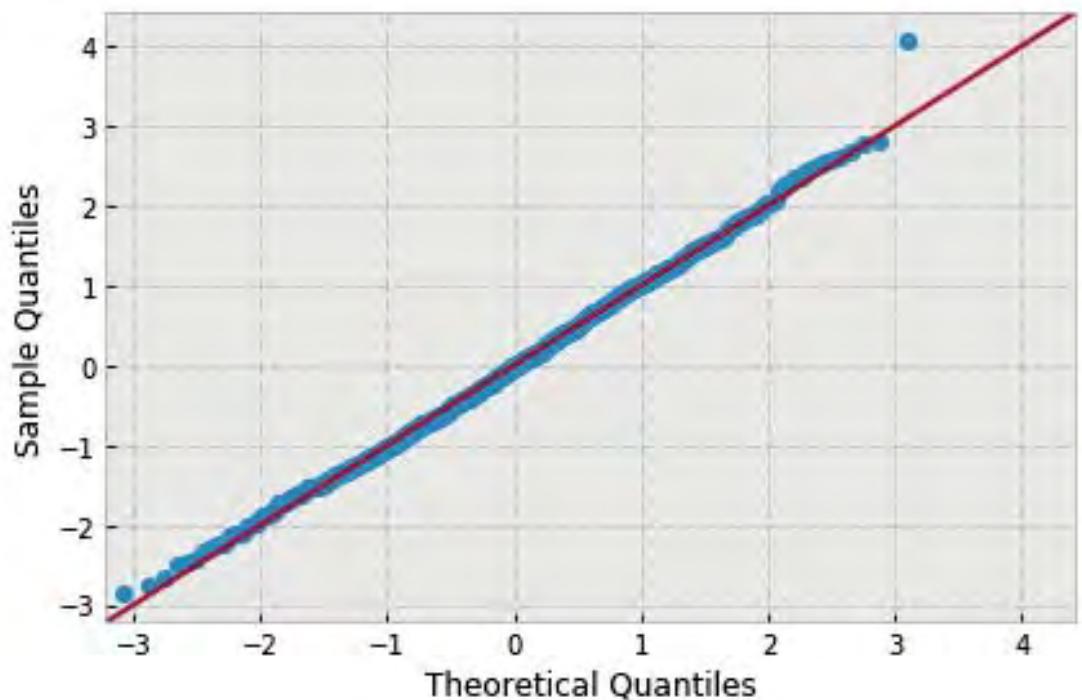


```
variable name: r
variable value: 2.5555555555555554
distribution: normal(shape=(), loc=-28.4976012204, scale=4.423093427954631)
DescribeResult(nobs=1000, minmax=(-42.330466719999997, -13.82919148)
               mean=-28.4976012204, variance=19.583338811226675,
               skewness=0.2529939248973862, kurtosis=0.09971378476488146)
```

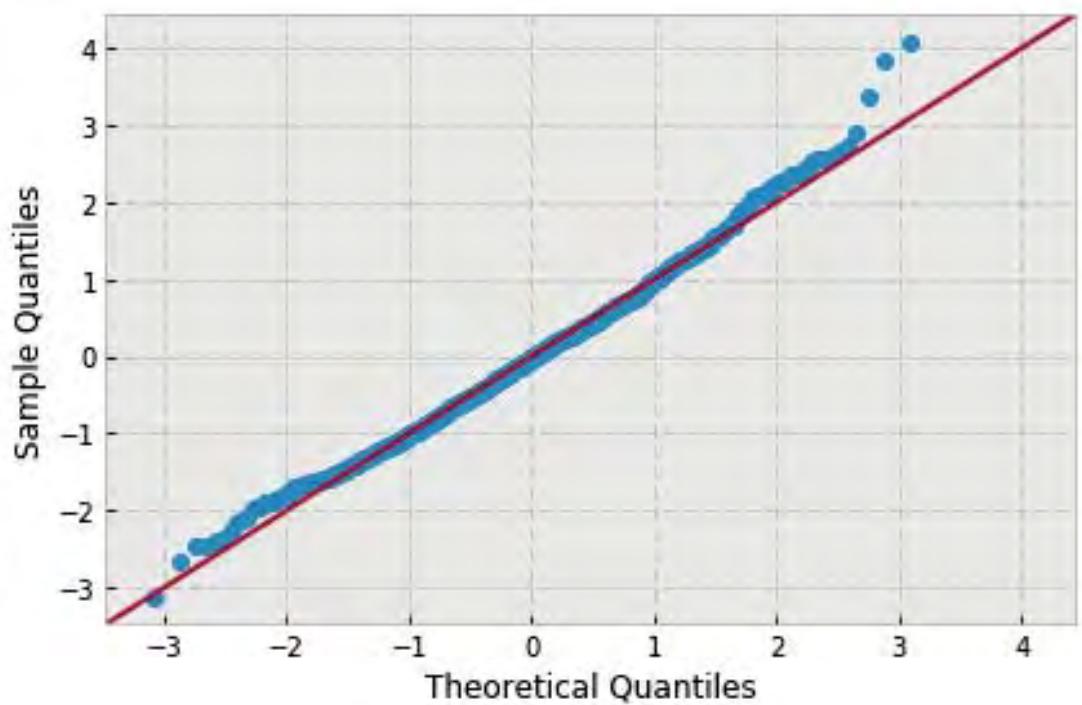
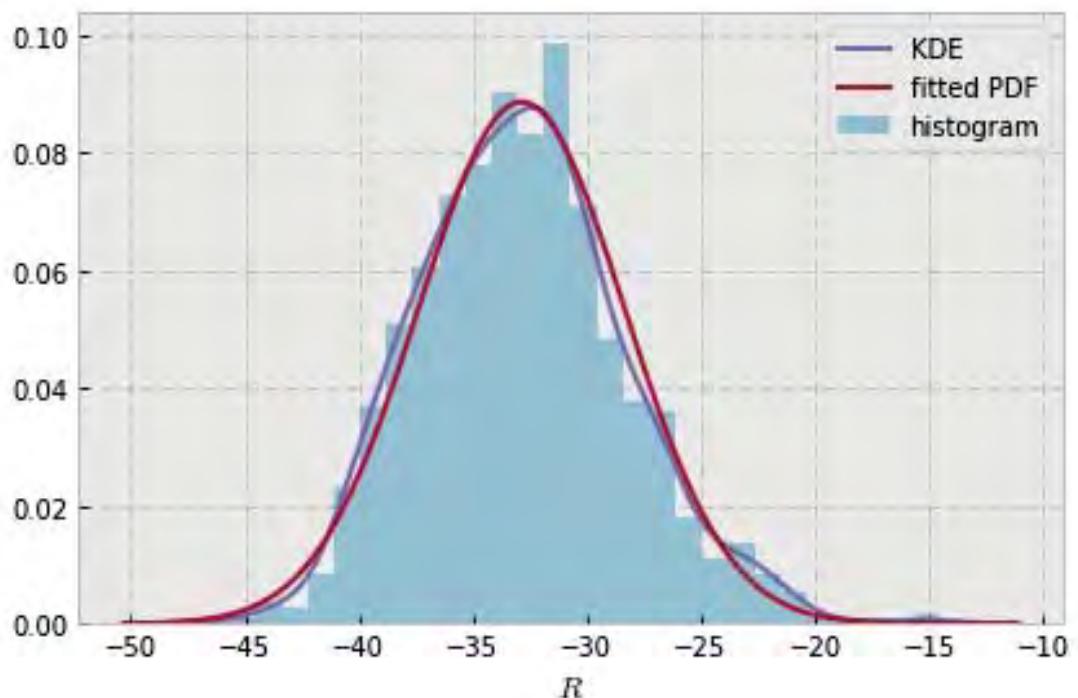


```
variable name: r
variable value: 2.777777777777777
distribution: normal(shape=(), loc=-30.59433547853, scale=4.561200765666767)
DescribeResult(nobs=1000, minmax=(-43.63777383, -12.05399177)
    mean=-30.594335478529999, variance=20.825377802521622,
    skewness=0.13611541549862446, kurtosis=-0.06045781932218075)
```

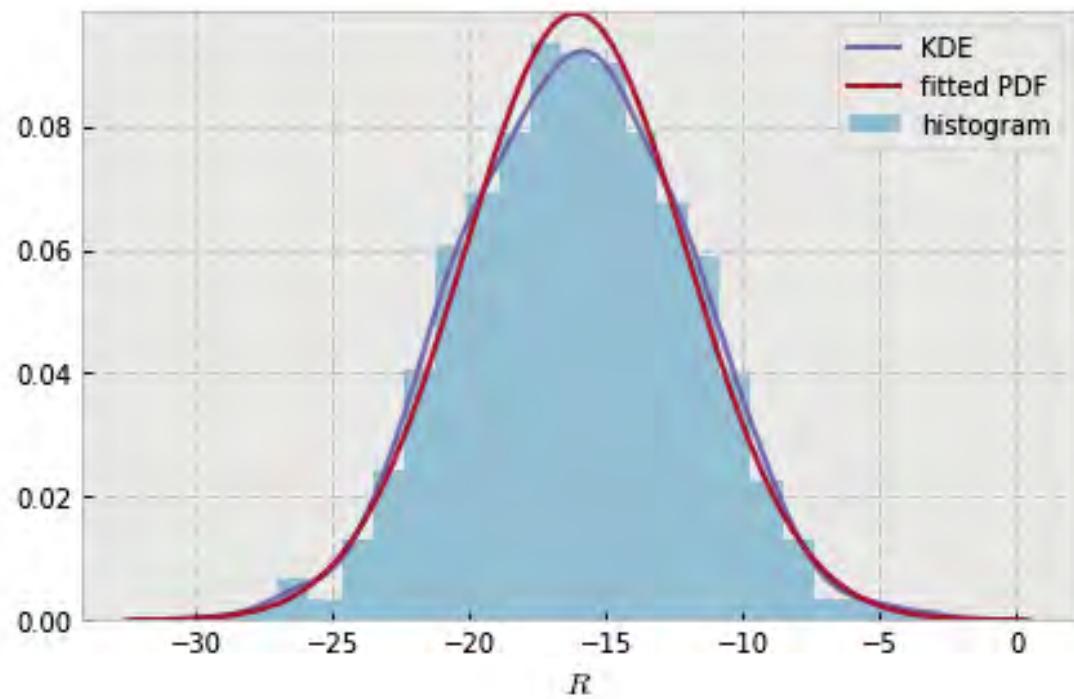


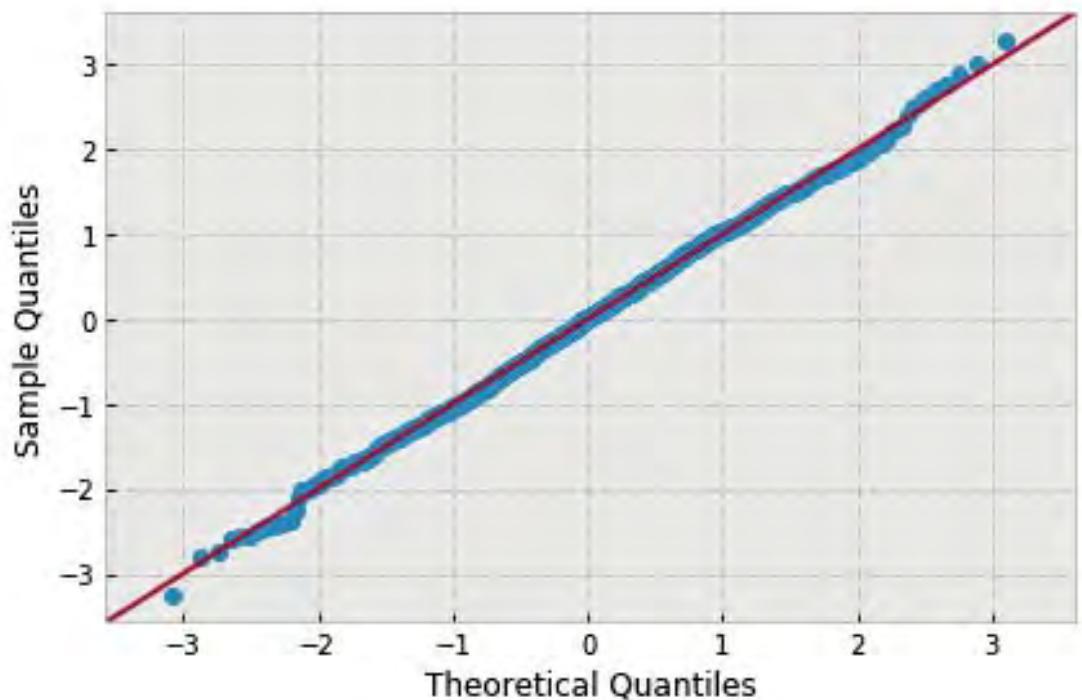


```
variable name: r
variable value: 3.0
distribution: normal(shape=(), loc=-32.86308004628, scale=4.502671186097614)
DescribeResult(nobs=1000, minmax=(-46.87840906999999, -14.549591250000001)
               mean=-32.863080046279997, variance=20.294342152265958,
               skewness=0.3264915170807752, kurtosis=0.27396687401038733)
```

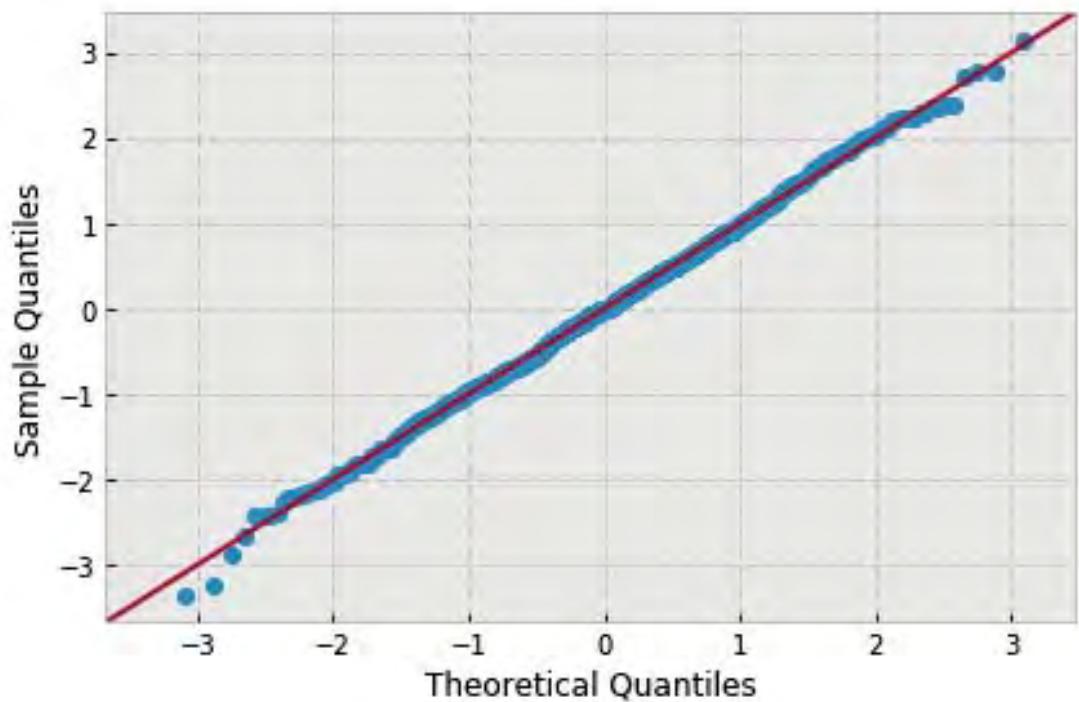
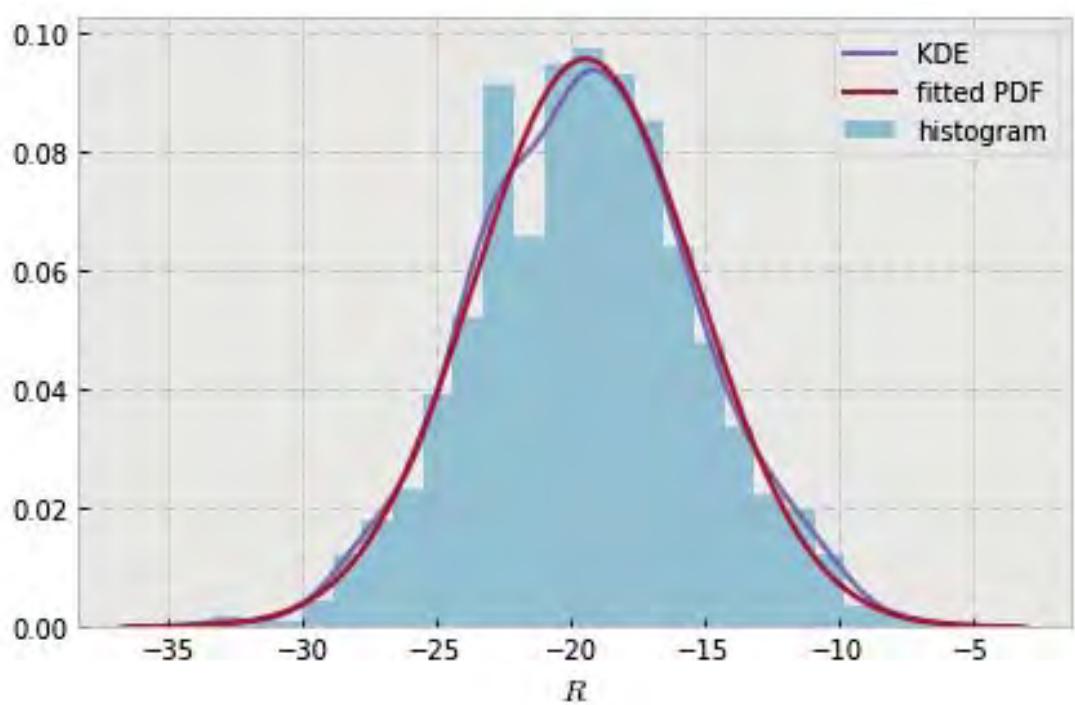


```
variable name: sigma
variable value: 0.01111111111111112
distribution: normal(shape=(), loc=-16.099682727831002, scale=4.059561929485104)
DescribeResult(nobs=1000, minmax=(-29.275856510000001, -2.7818094059999998)
               mean=-16.099682727831002, variance=16.496539598923743,
               skewness=-0.00413562099115514, kurtosis=-0.13193209914433623)
```

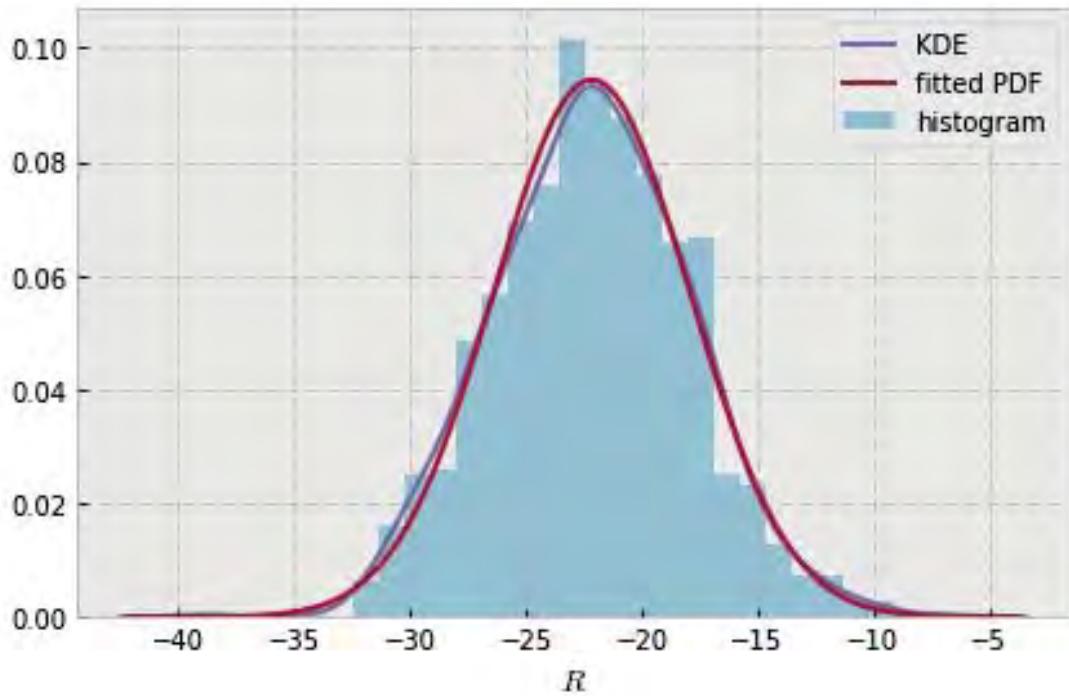


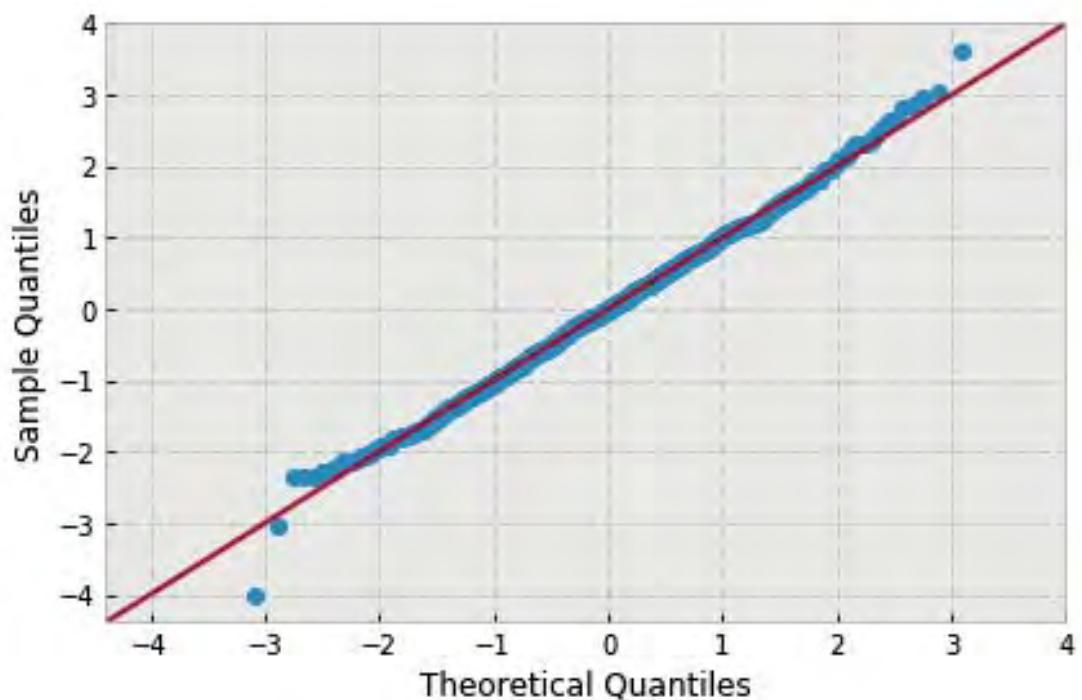


```
variable name: sigma
variable value: 0.02222222222222223
distribution: normal(shape=(), loc=-19.426177801812997, scale=4.167048261569078)
DescribeResult(nobs=1000, minmax=(-33.381373619999998, -6.365472321999996)
               mean=-19.426177801812997, variance=17.381672887133007,
               skewness=0.021337311178422937, kurtosis=-0.04894437990112044)
```

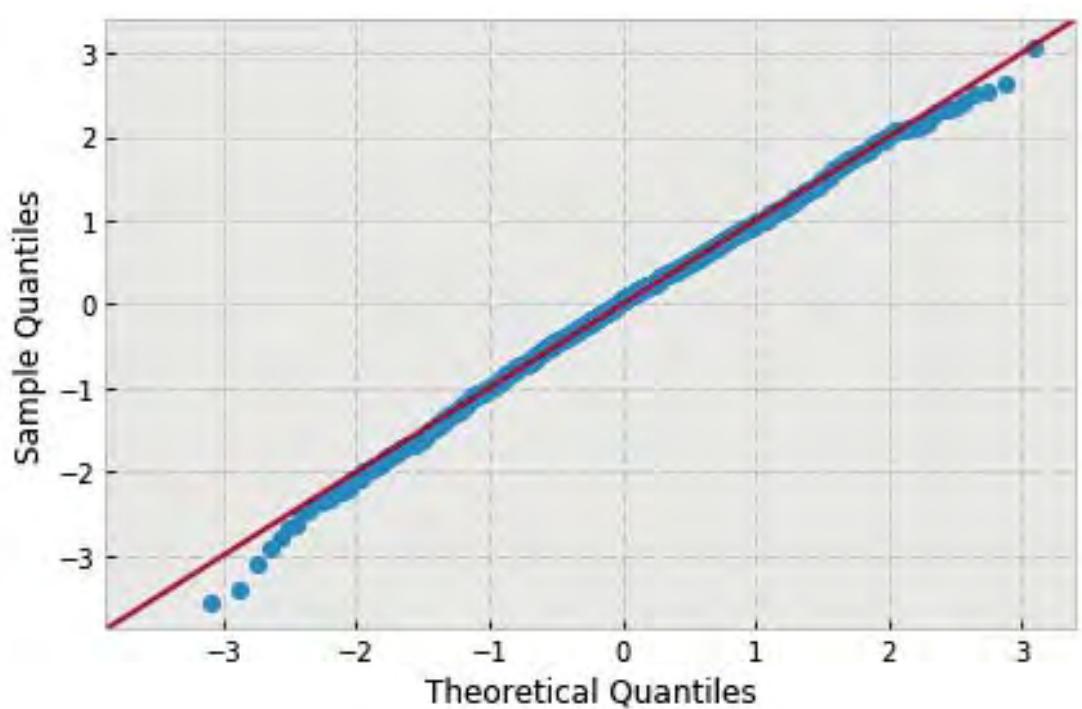
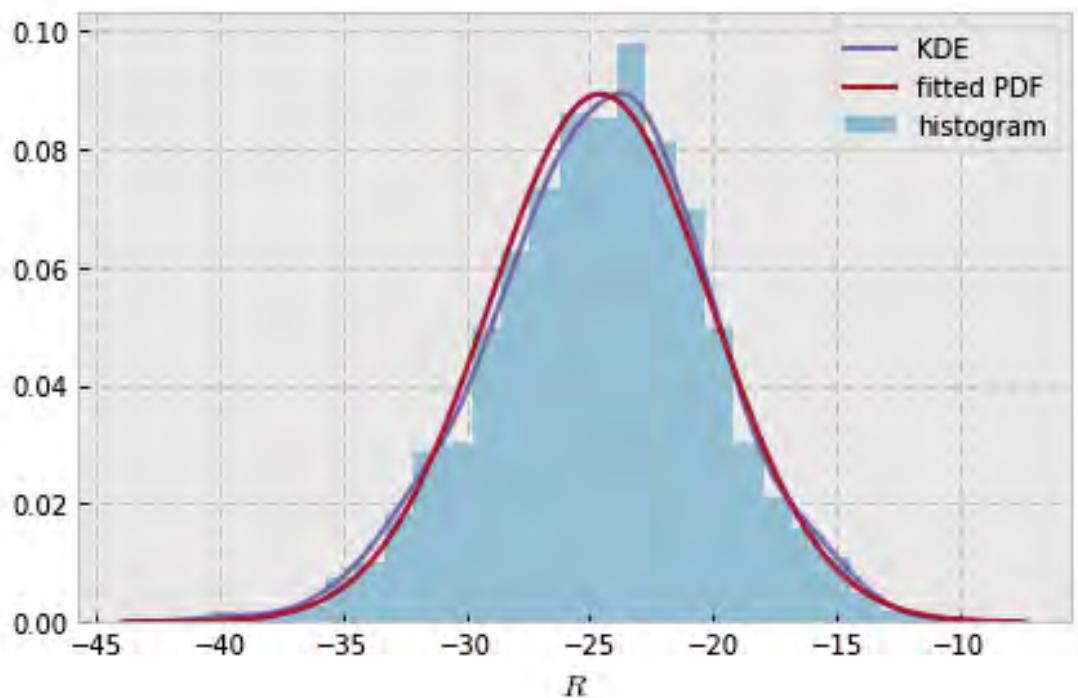


```
variable name: sigma
variable value: 0.03333333333333333
distribution: normal(shape=(), loc=-22.155468374409, scale=4.226201785247434)
DescribeResult(nobs=1000, minmax=(-39.09220673000001, -6.876115021999996)
               mean=-22.155468374409001, variance=17.878660189818419,
               skewness=0.04527251562476141, kurtosis=0.11359207983904396)
```

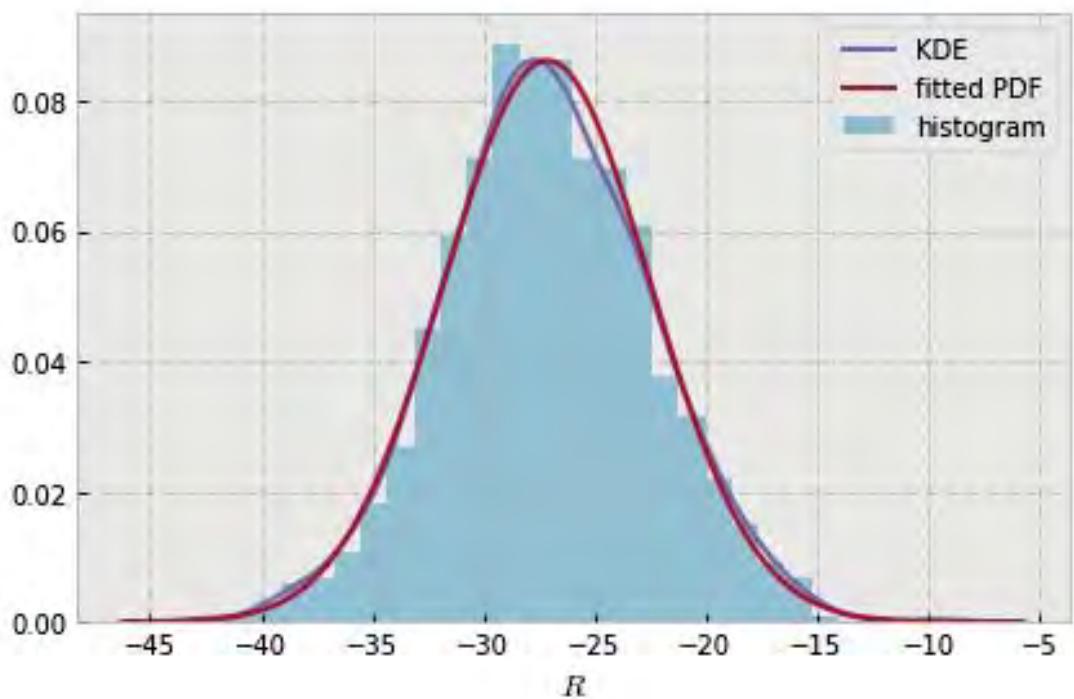


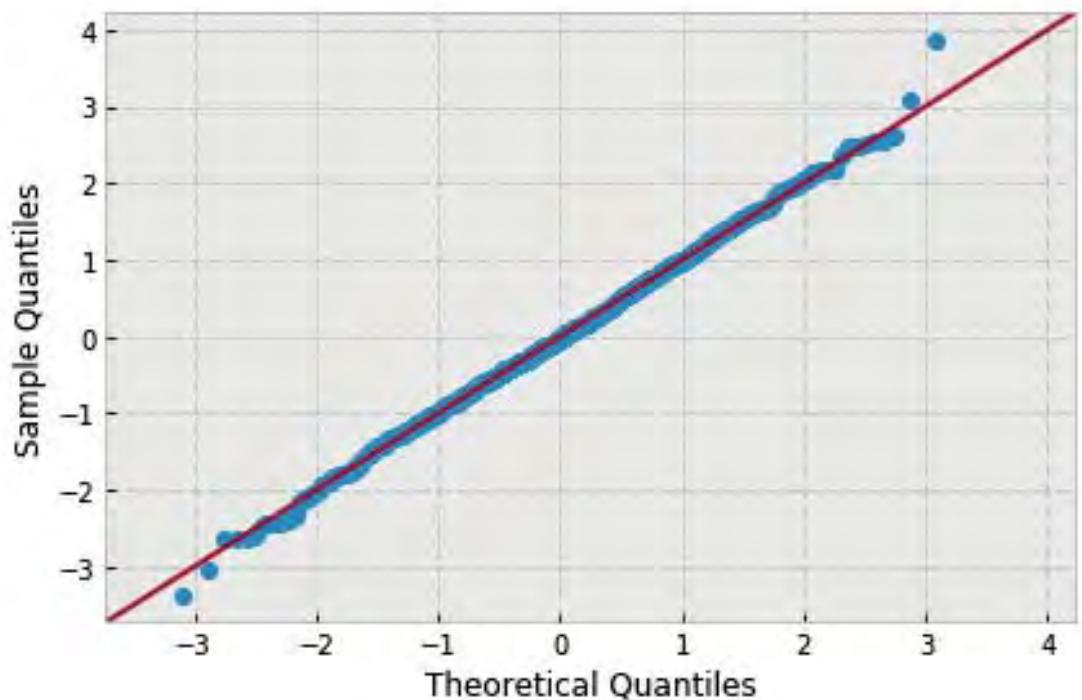


```
variable name: sigma
variable value: 0.04444444444444446
distribution: normal(shape=(), loc=-24.56386907415, scale=4.470164468633611)
DescribeResult(nobs=1000, minmax=(-40.43668779000001, -10.85727041)
               mean=-24.563869074149999, variance=20.002372749383799,
               skewness=-0.16577890992185623, kurtosis=0.09735386521394629)
```

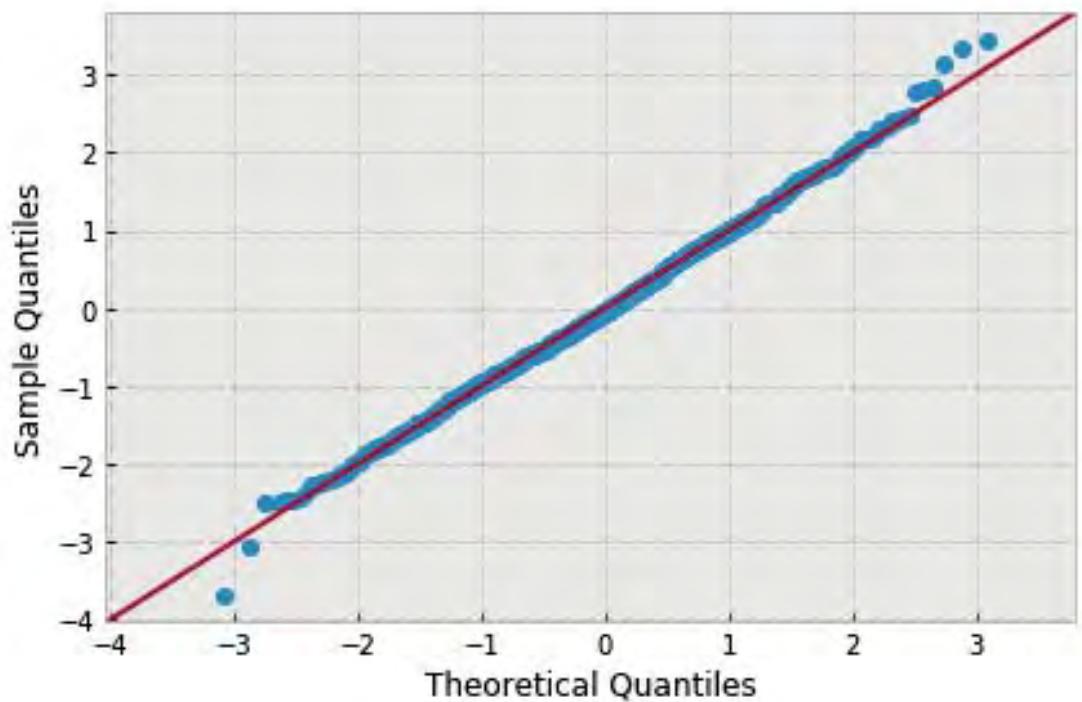
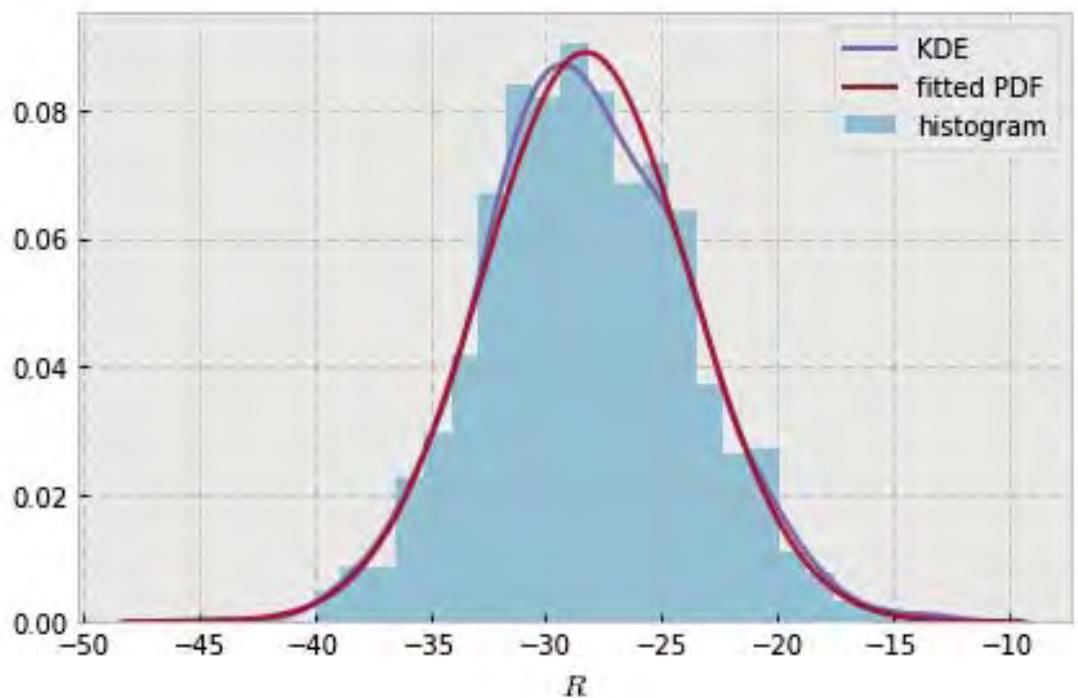


```
variable name: sigma
variable value: 0.05555555555555555
distribution: normal(shape=(), loc=-27.133159017997, scale=4.631496149063794)
DescribeResult(nobs=1000, minmax=(-42.697276090000003, -9.275280636999998)
               mean=-27.133159017996999, variance=21.472228807600352,
               skewness=0.04331079290780273, kurtosis=0.11812920446603048)
```

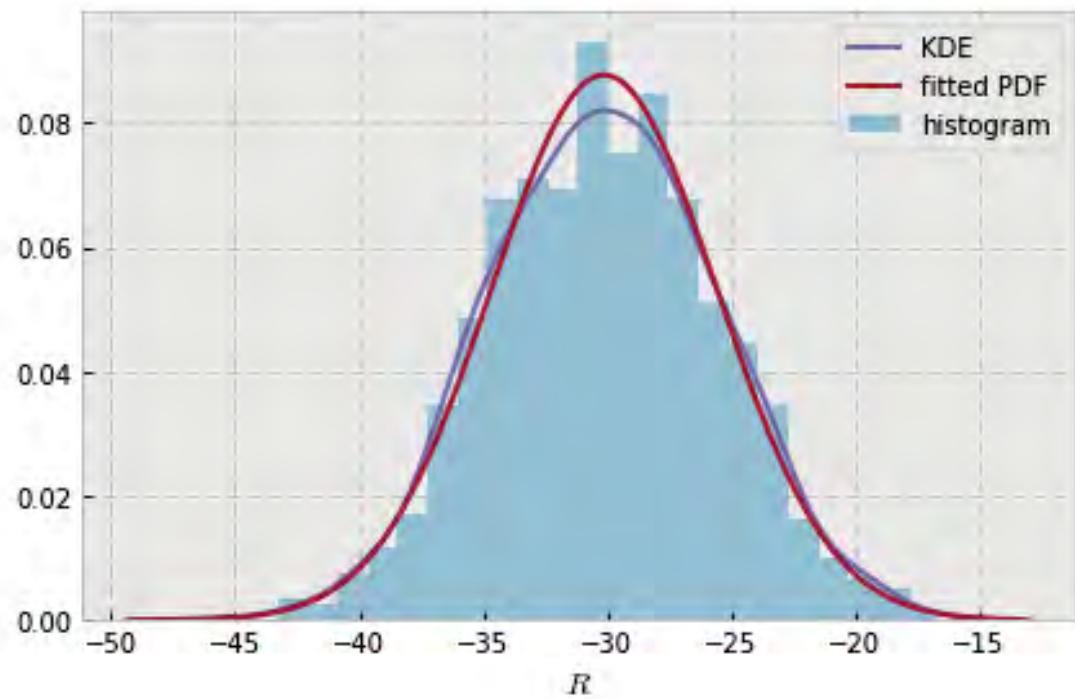


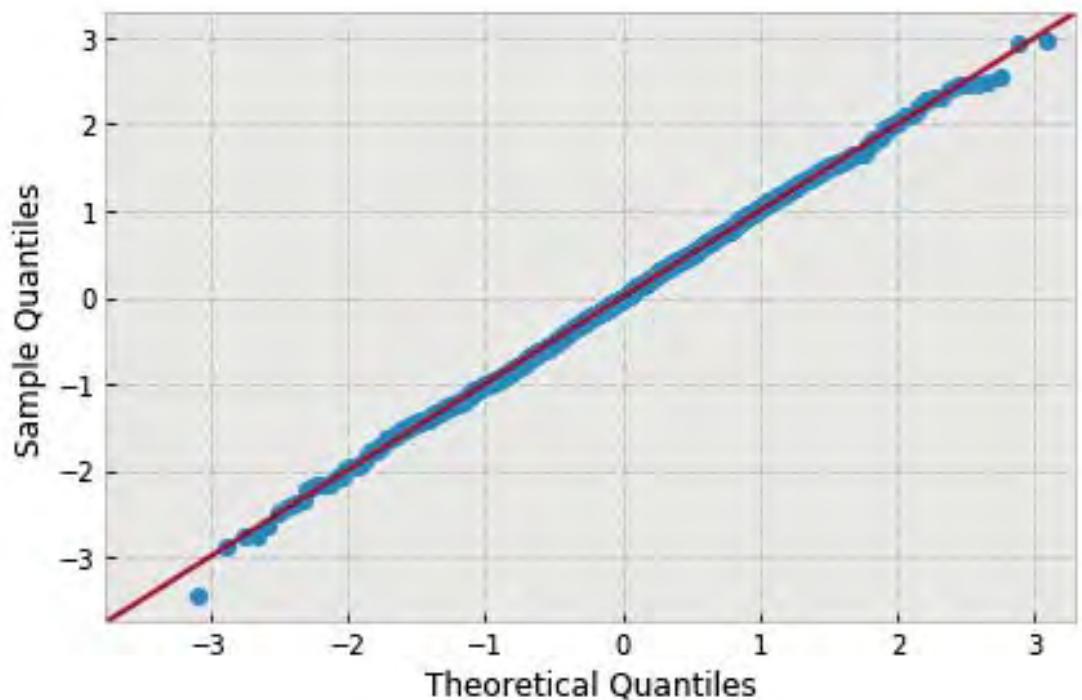


```
variable name: sigma
variable value: 0.06666666666666667
distribution: normal(shape=(), loc=-28.26045647754, scale=4.4794191441547655)
DescribeResult(nobs=1000, minmax=(-44.727282809999998, -12.85536894)
               mean=-28.26045647754, variance=20.085281150170378,
               skewness=0.10480242465546906, kurtosis=0.1175017258164206)
```

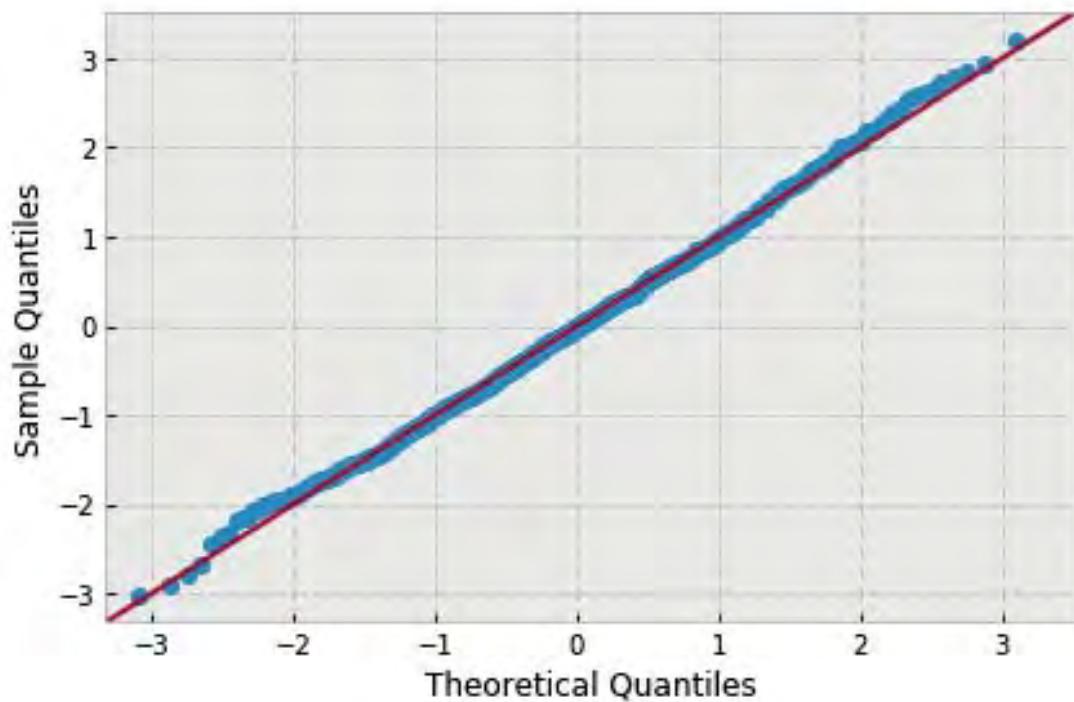
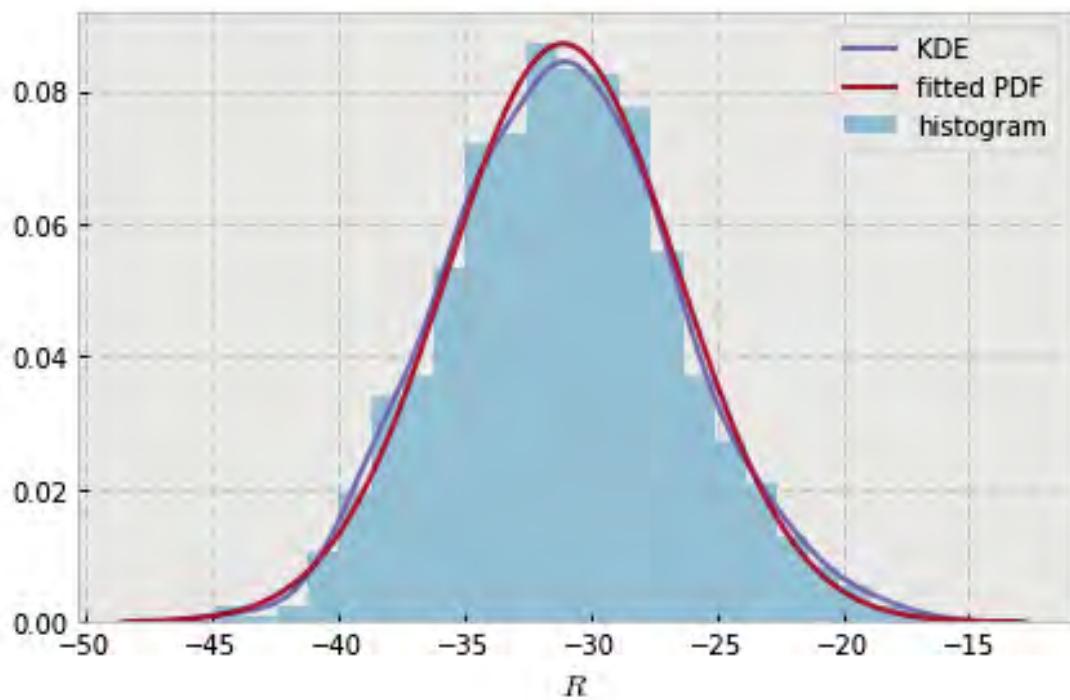


```
variable name: sigma
variable value: 0.07777777777777778
distribution: normal(shape=(), loc=-30.126021530040003, scale=4.549401715337685)
DescribeResult(nobs=1000, minmax=(-45.748461059999997, -16.63371643)
               mean=-30.126021530040003, variance=20.717773741258728,
               skewness=0.00012016768079073046, kurtosis=-0.1501033981761215)
```

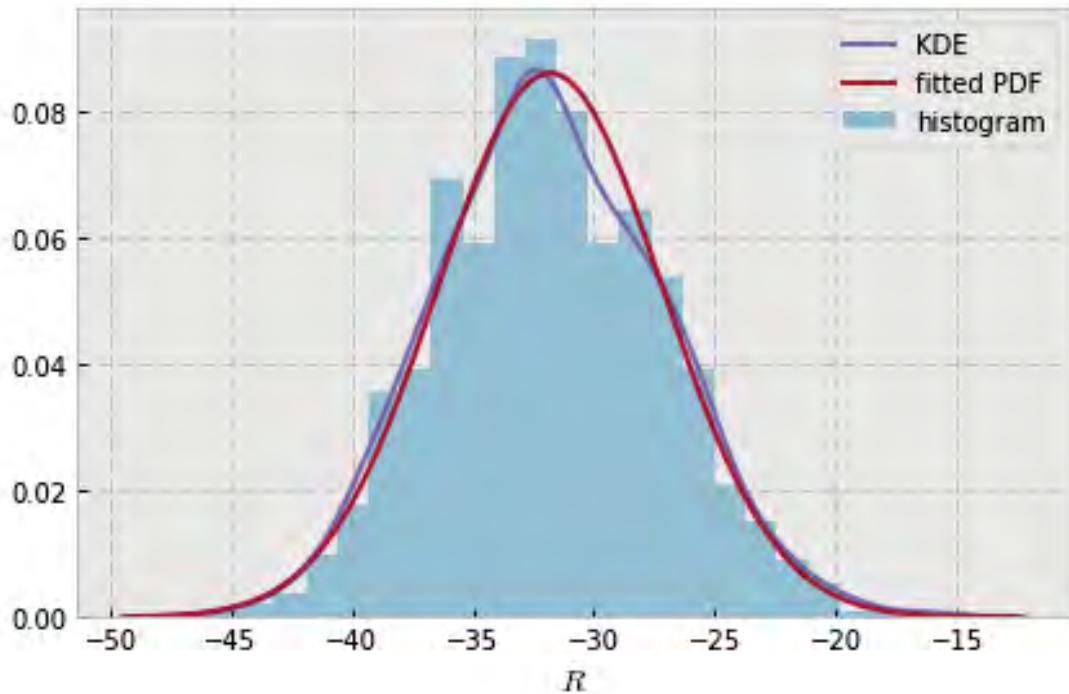


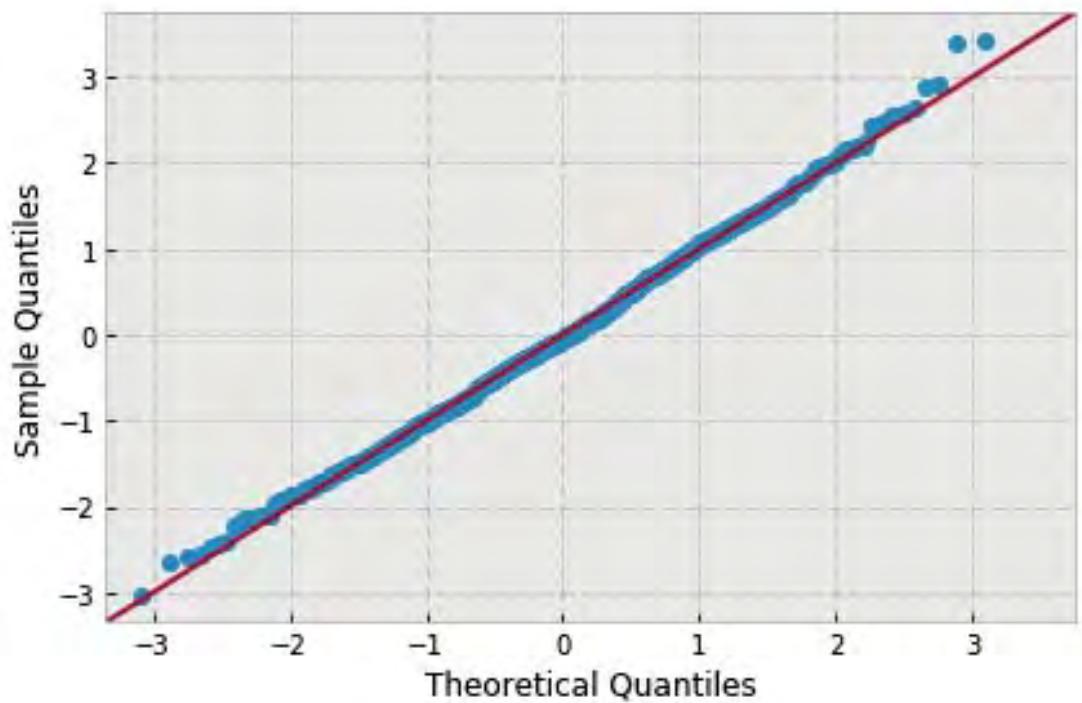


```
variable name: sigma
variable value: 0.08888888888888889
distribution: normal(shape=(), loc=-31.07777140939, scale=4.5767638450916985)
DescribeResult(nobs=1000, minmax=(-44.86401812999998, -16.44575699)
               mean=-31.077771409389999, variance=20.967735028767315,
               skewness=0.1382781012766355, kurtosis=-0.05915320530828527)
```

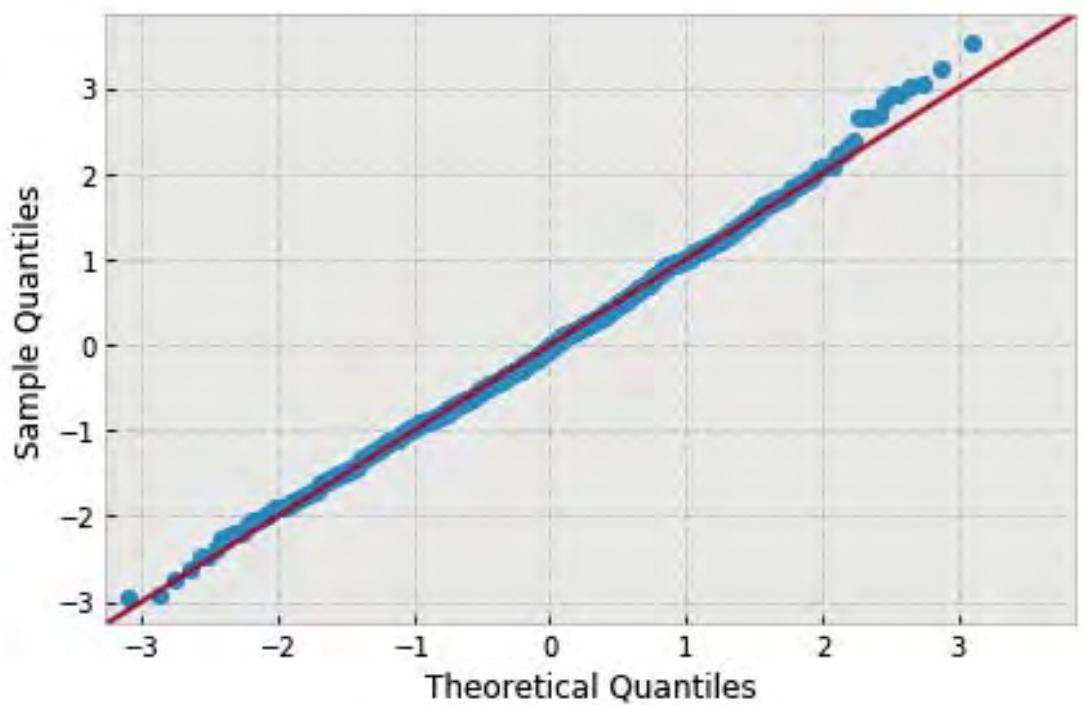
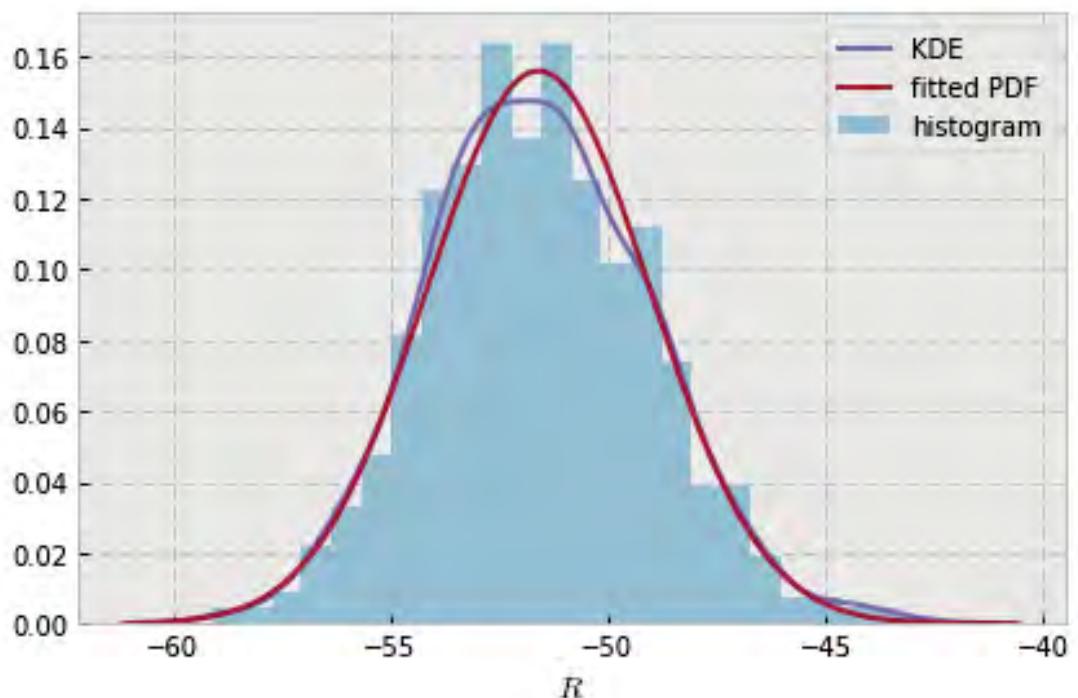


```
variable name: sigma
variable value: 0.1
distribution: normal(shape=(), loc=-31.80918807846, scale=4.64087571087459)
DescribeResult(nobs=1000, minmax=(-45.834588500000002, -15.922513220000001)
               mean=-31.80918807846, variance=21.559286650436164,
               skewness=0.14094883921761642, kurtosis=-0.08179447222226699)
```

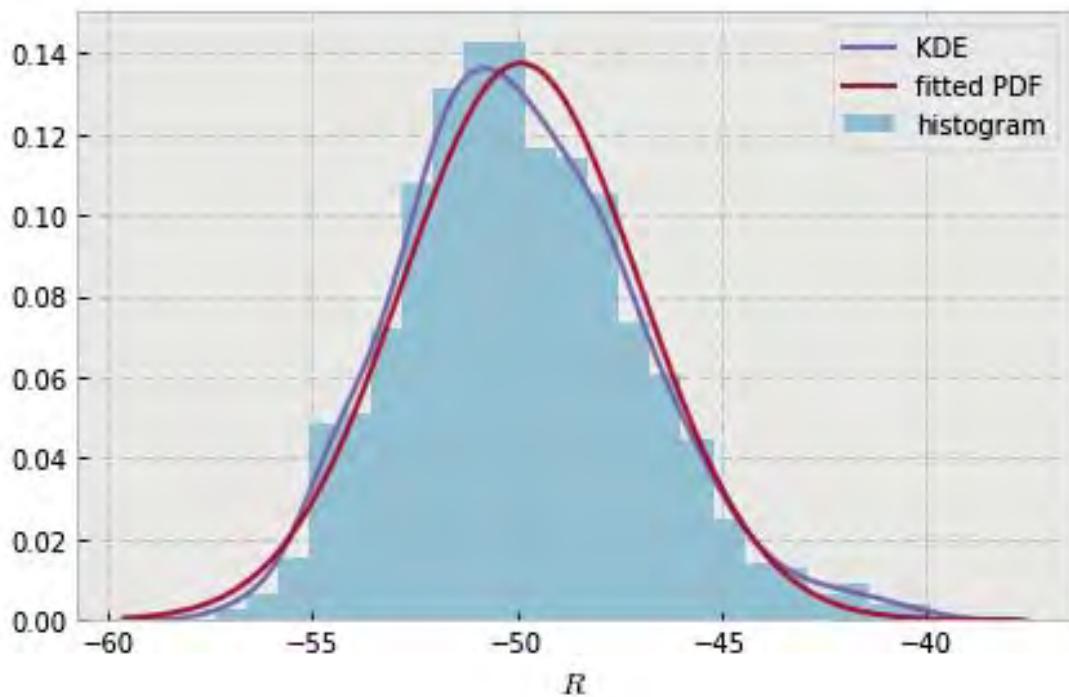


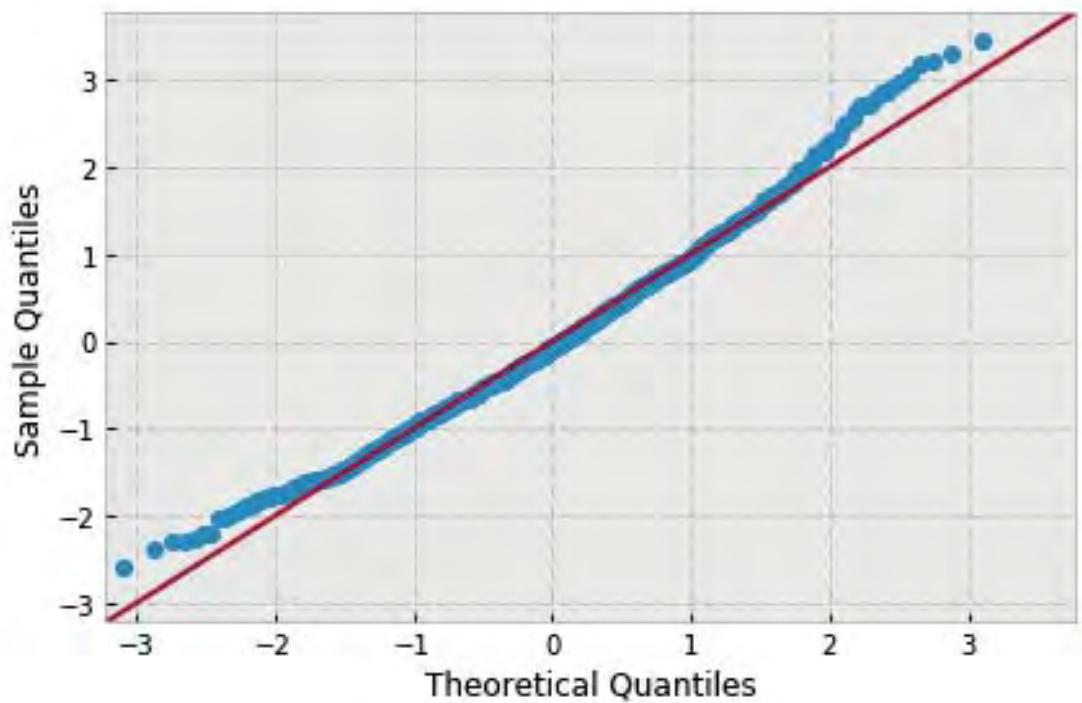


```
variable name: theta
variable value: 0.6981317007977318
distribution: normal(shape=(), loc=-51.59046151224, scale=2.55730862842361)
DescribeResult(nobs=1000, minmax=(-59.10102347000001, -42.573181419999997)
               mean=-51.590461512239997, variance=6.5463737948046496,
               skewness=0.1852340148387277, kurtosis=0.09752971912653274)
```

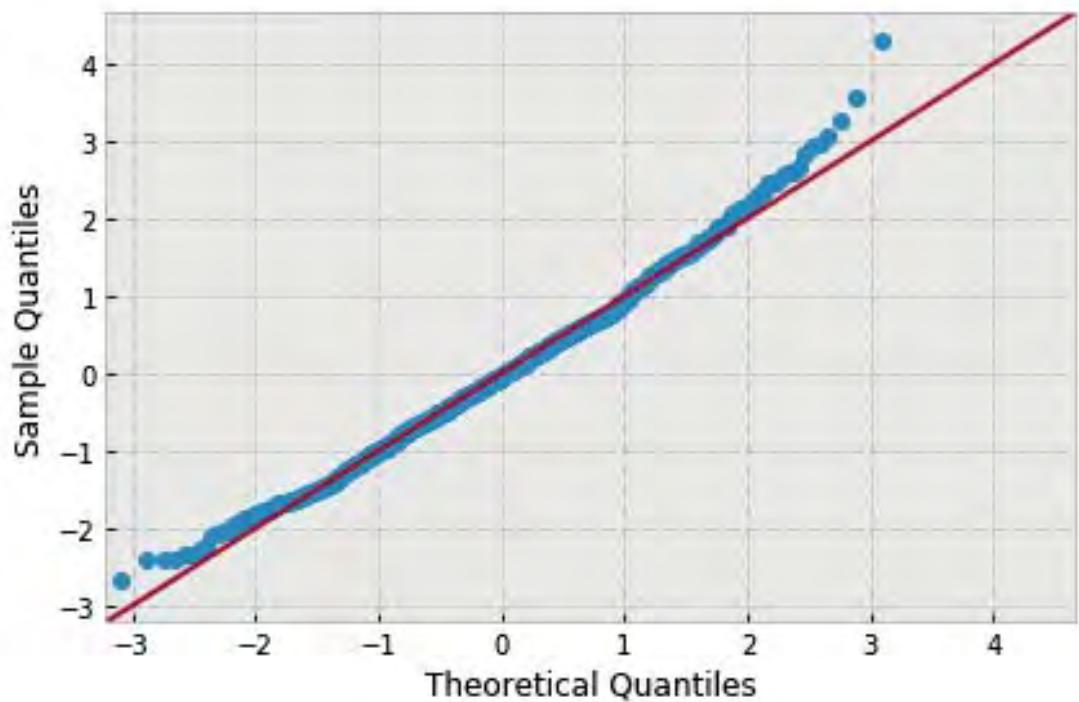
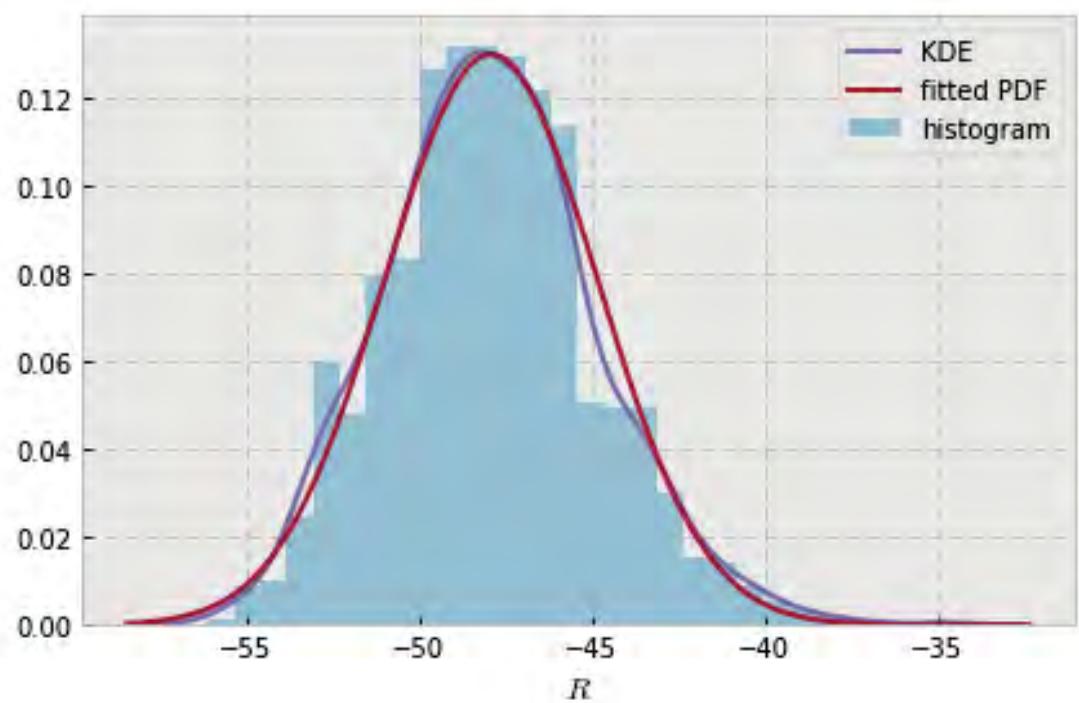


```
variable name: theta
variable value: 0.7853981633974483
distribution: normal(shape=(), loc=-49.900596653250005, scale=2.9007123222156803)
DescribeResult(nobs=1000, minmax=(-57.393424779999997, -39.86552417)
               mean=-49.900596653250005, variance=8.4225545307846712,
               skewness=0.4074481285312719, kurtosis=0.18202623602578472)
```

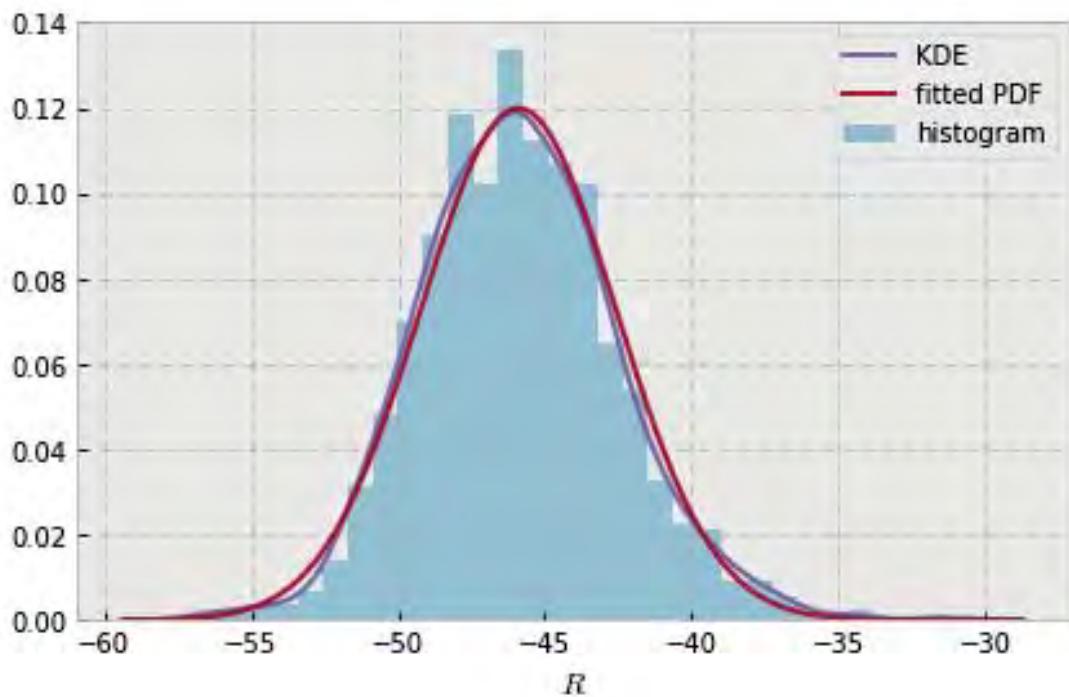


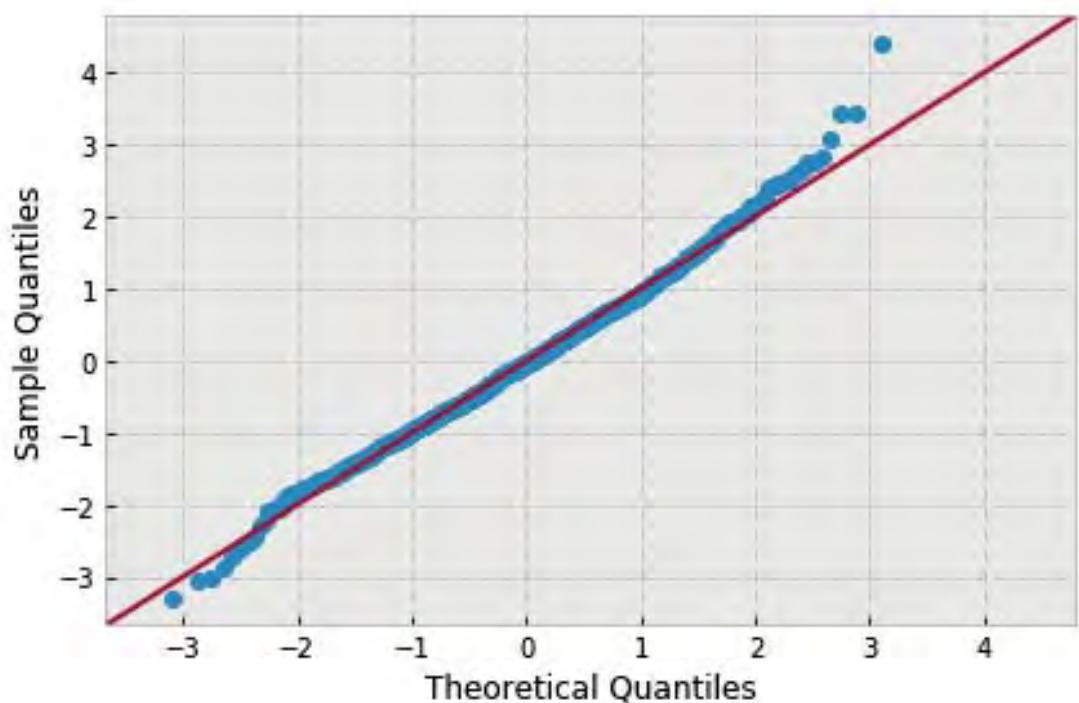


```
variable name: theta
variable value: 0.8726646259971648
distribution: normal(shape=(), loc=-47.90042641672, scale=3.0743290207144667)
DescribeResult(nobs=1000, minmax=(-56.11871732000002, -34.668823109999998)
               mean=-47.90042641672002, variance=9.460959887494667,
               skewness=0.32607262564890727, kurtosis=0.3325835767069938)
```

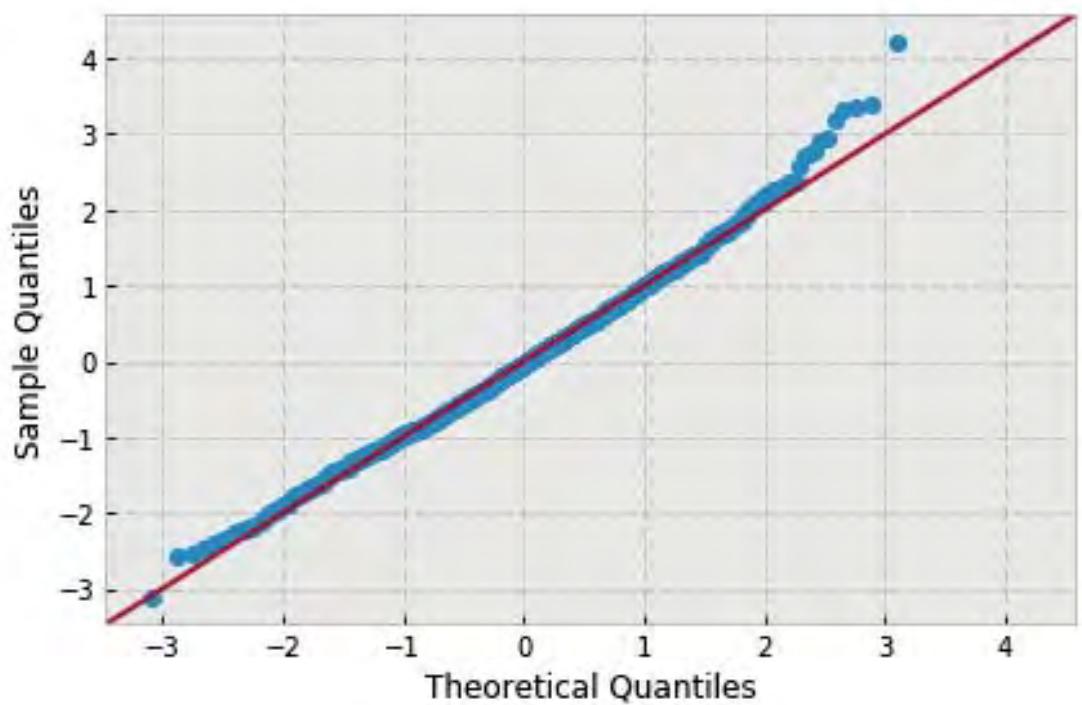
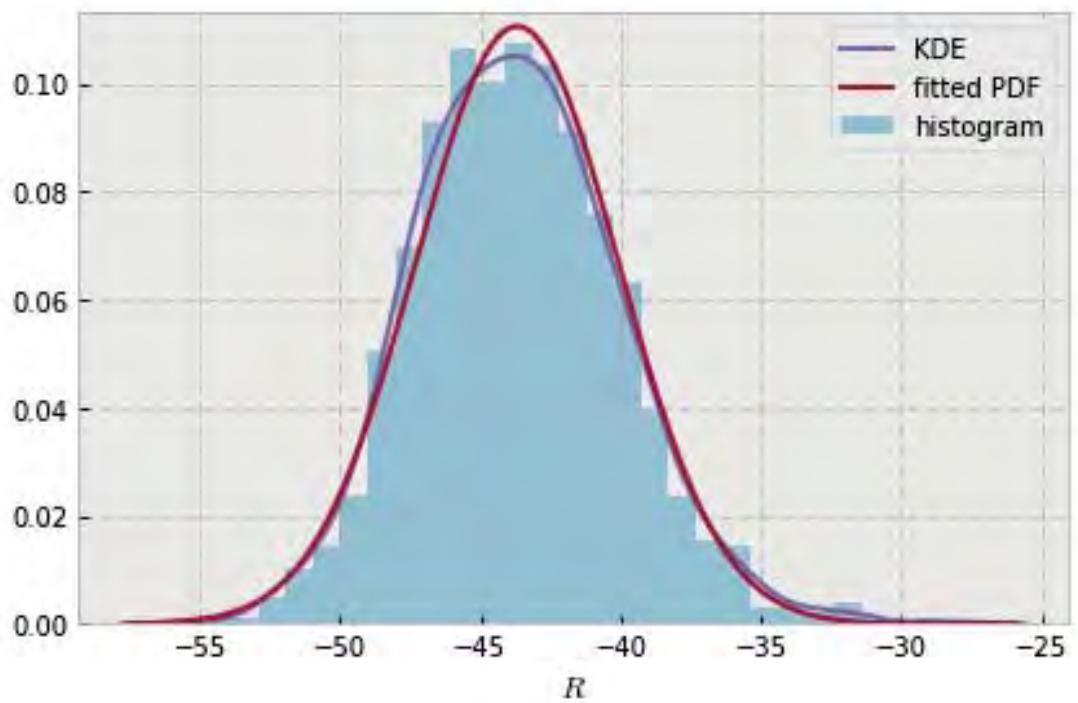


```
variable name: theta
variable value: 0.9599310885968813
distribution: normal(shape=(), loc=-45.900869858830006, scale=3.3269301368465607)
DescribeResult(nobs=1000, minmax=(-56.82767252, -31.2698684)
    mean=-45.900869858830006, variance=11.079543679137011,
    skewness=0.2505095838456253, kurtosis=0.5378100557441057)
```

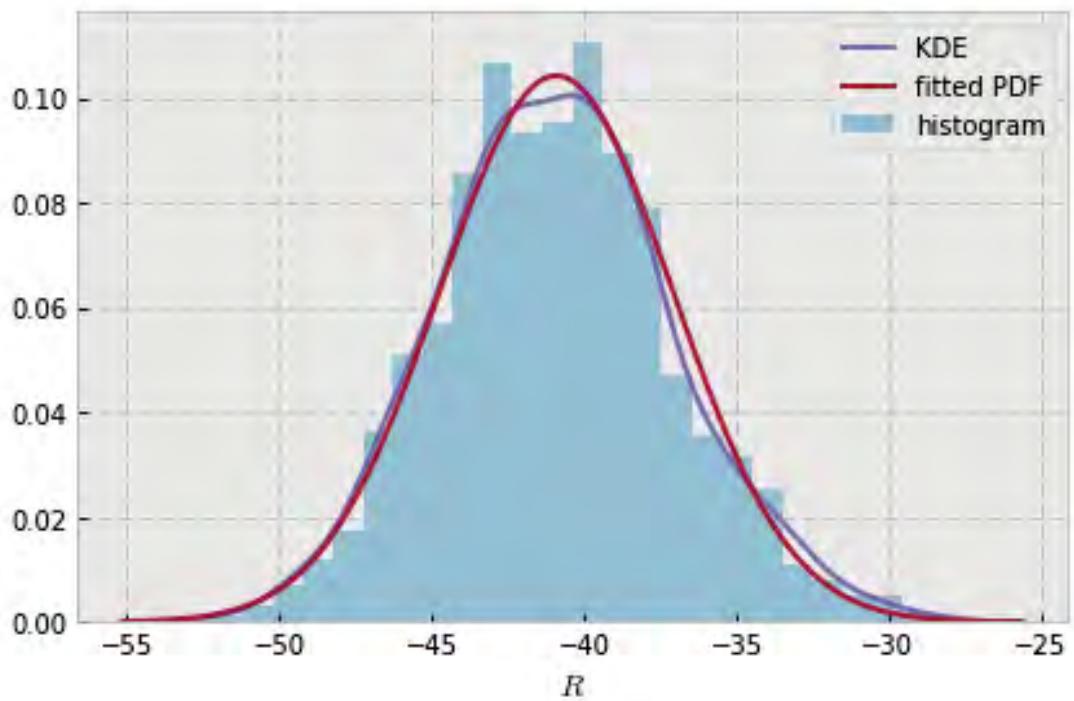


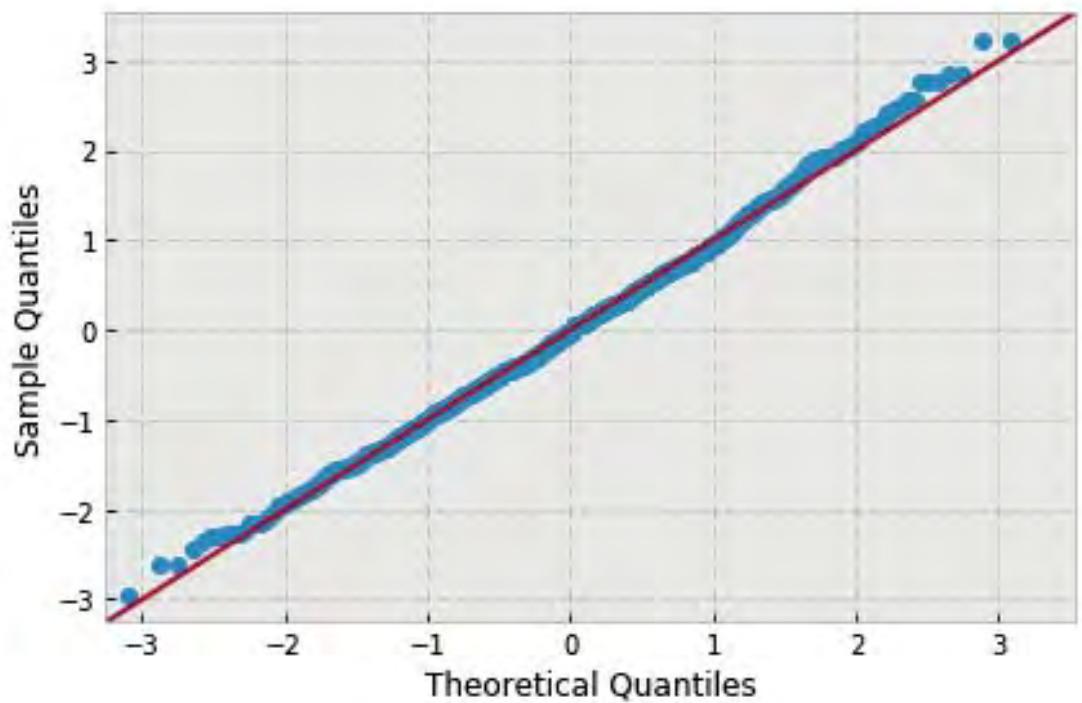


```
variable name: theta
variable value: 1.0471975511965979
distribution: normal(shape=(), loc=-43.70157718175, scale=3.608992819346678)
DescribeResult(nobs=1000, minmax=(-54.88280535999999, -28.53601518)
               mean=-43.701577181749997, variance=13.037867037133017,
               skewness=0.2993064539091108, kurtosis=0.3577749363582612)
```

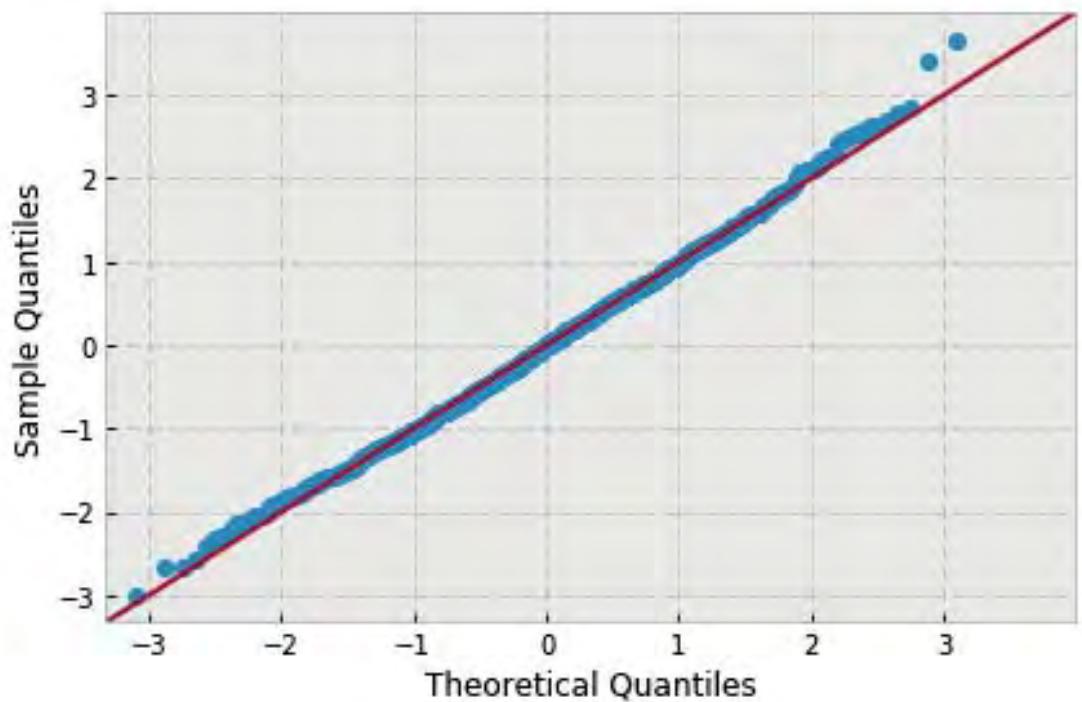
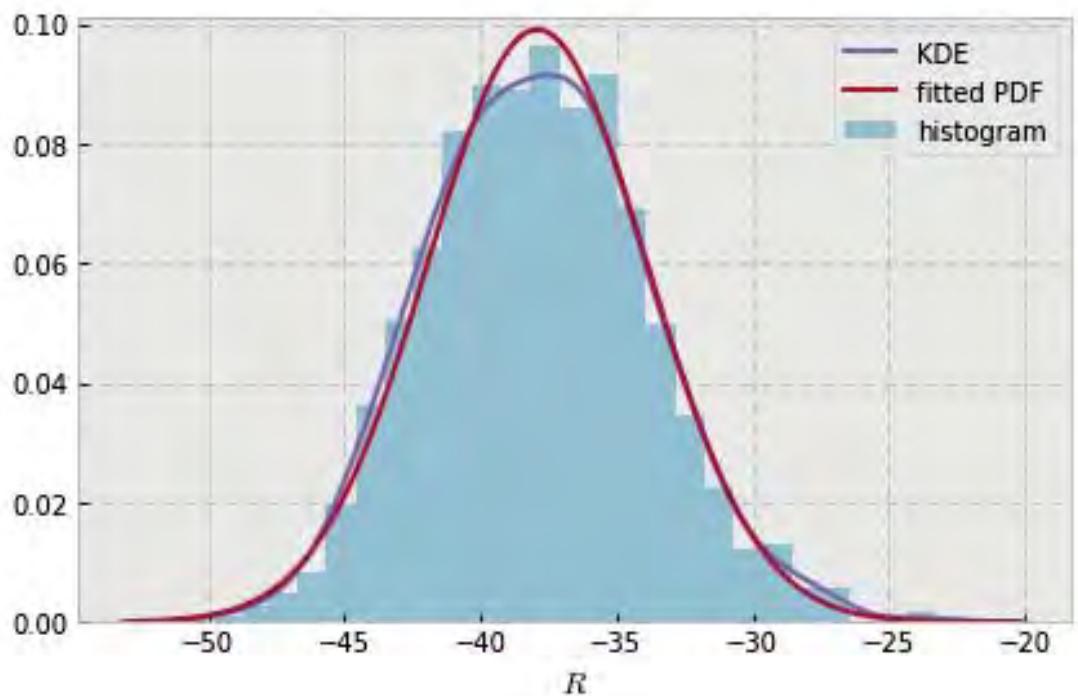


```
variable name: theta
variable value: 1.1344640137963142
distribution: normal(shape=(), loc=-40.904056081019995, scale=3.827872924546665)
DescribeResult(nobs=1000, minmax=(-52.17699687999997, -28.56956164)
               mean=-40.904056081019995, variance=14.667278404882323,
               skewness=0.1874662979884482, kurtosis=0.02296766636265346)
```

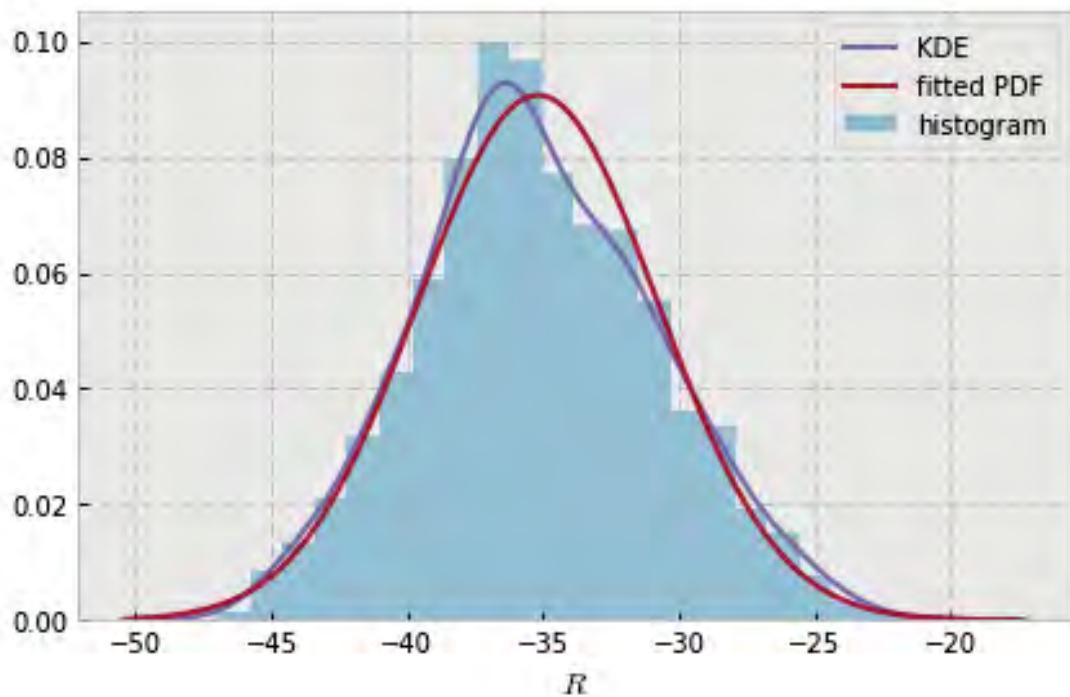


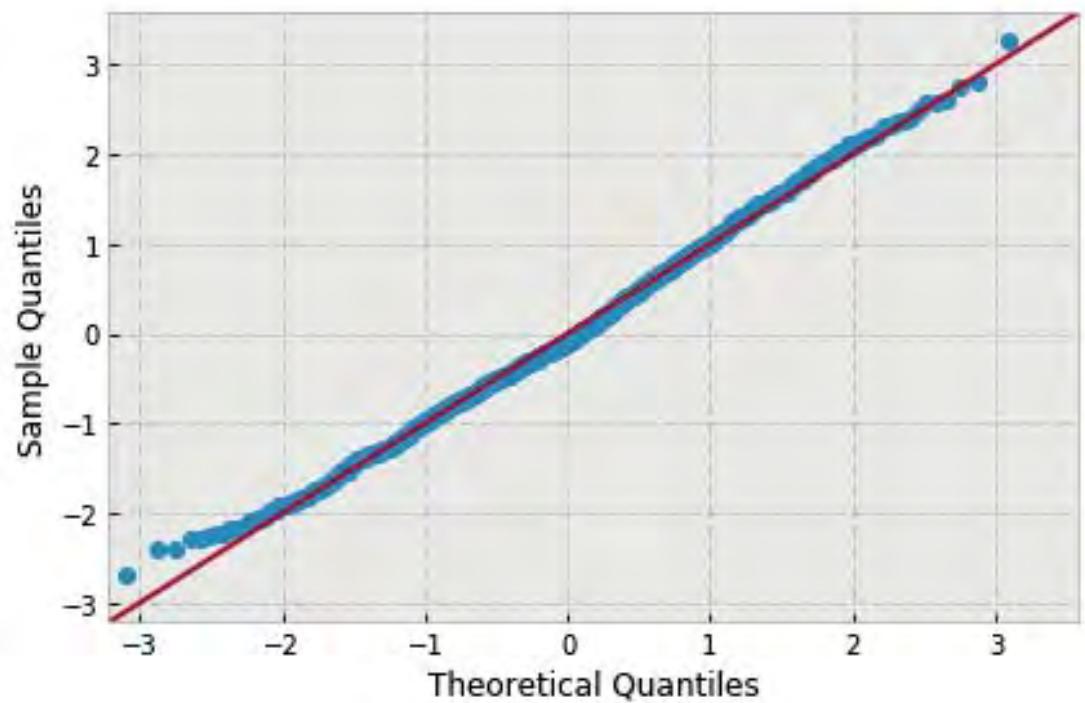


```
variable name: theta
variable value: 1.2217304763960306
distribution: normal(shape=(), loc=-37.905234185610006, scale=4.027593554397524)
DescribeResult(nobs=1000, minmax=(-49.94046841000001, -23.20245439)
               mean=-37.905234185610006, variance=16.237747587011494,
               skewness=0.17273156250240412, kurtosis=-0.012135504993052493)
```

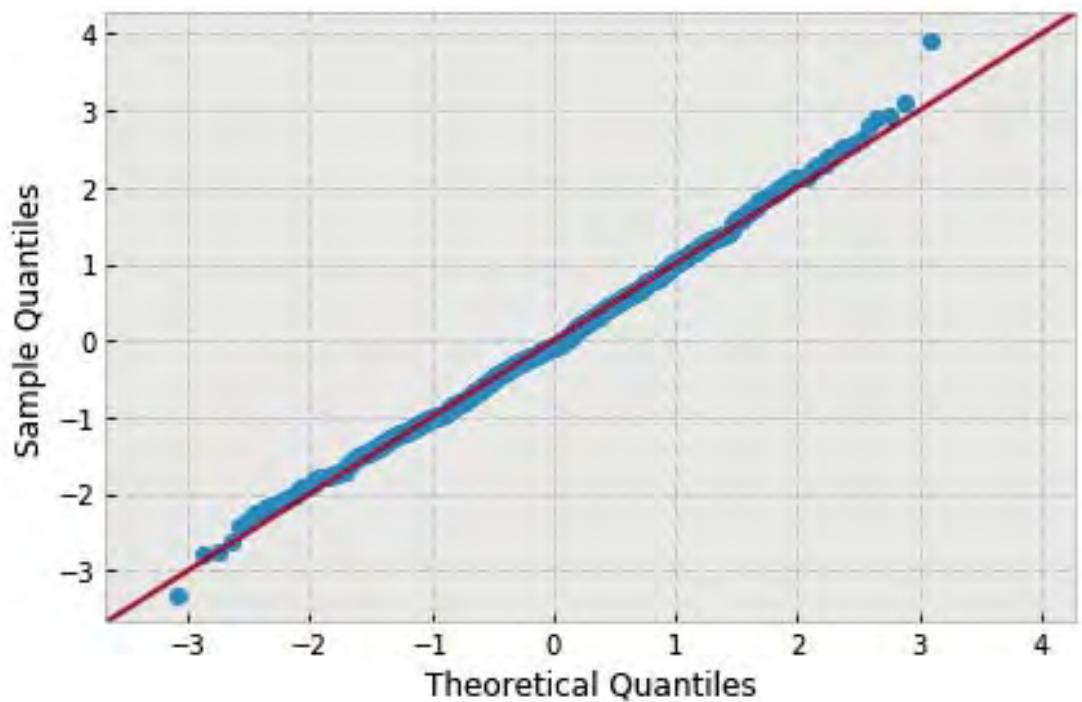
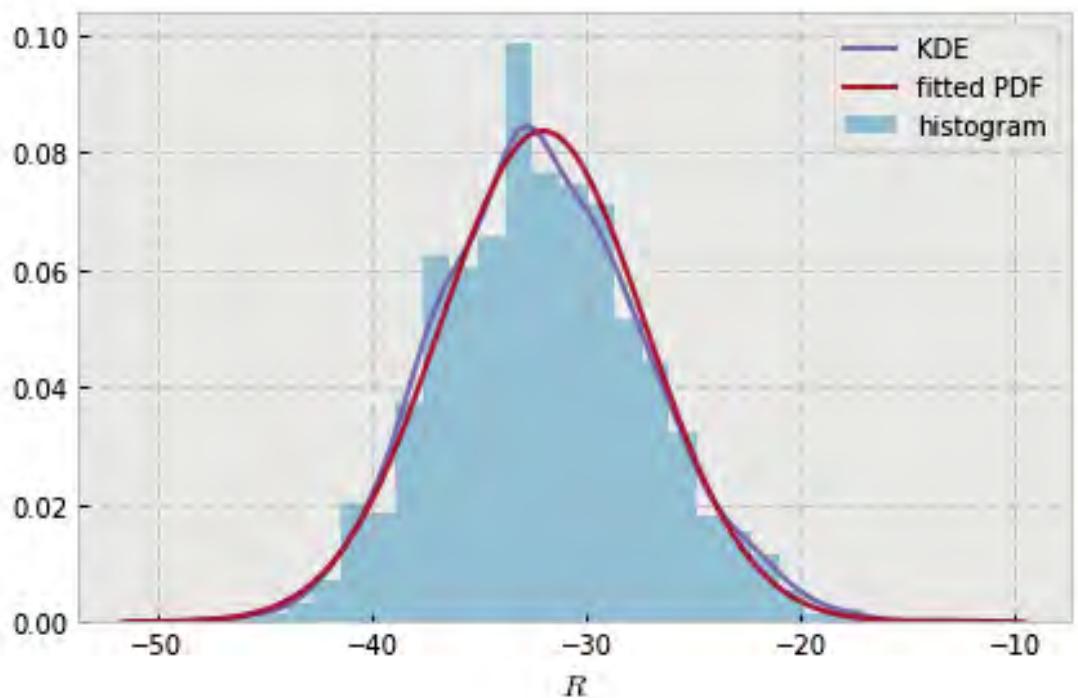


```
variable name: theta
variable value: 1.3089969389957472
distribution: normal(shape=(), loc=-35.19173212767, scale=4.399794001856719)
DescribeResult(nobs=1000, minmax=(-46.963836780000001, -20.787455980000001)
               mean=-35.191732127670001, variance=19.377564823597961,
               skewness=0.18646897107165492, kurtosis=-0.2254974383707018)
```

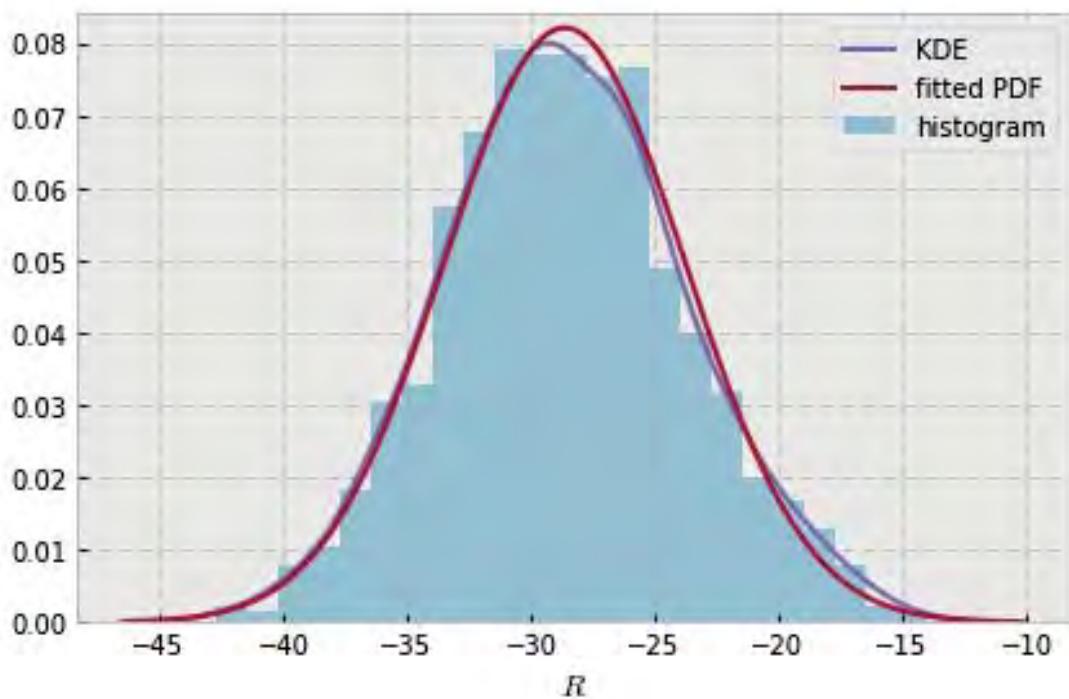


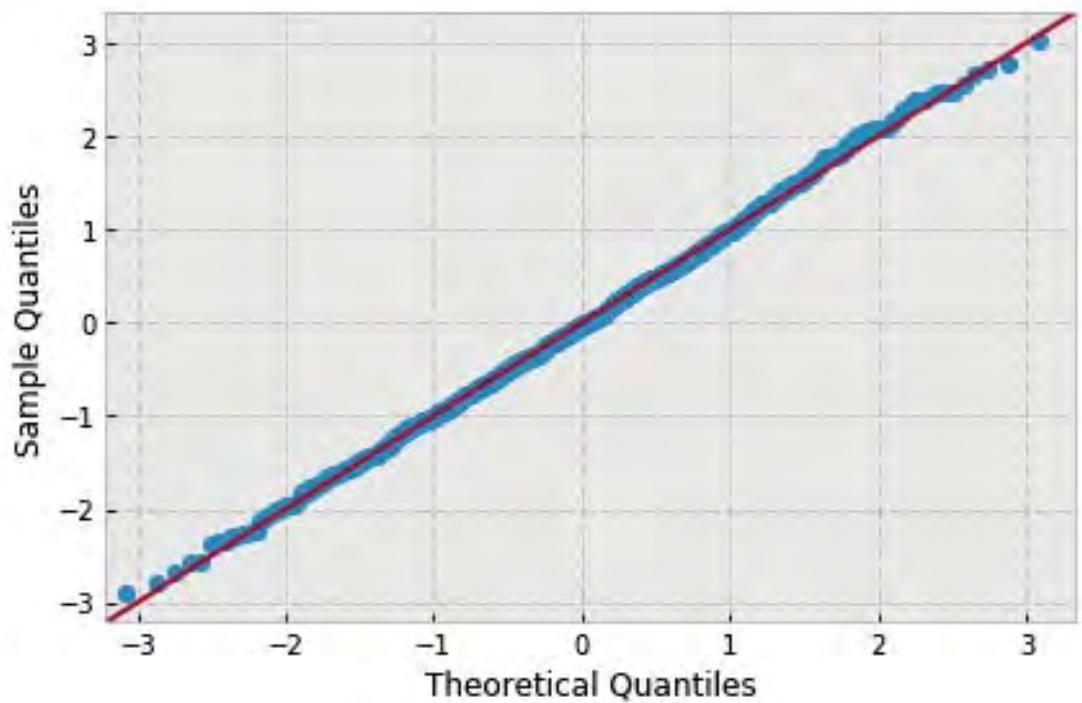


```
variable name: theta
variable value: 1.3962634015954638
distribution: normal(shape=(), loc=-32.00626315716, scale=4.772154878520703)
DescribeResult(nobs=1000, minmax=(-47.80611294999999, -13.34582466)
               mean=-32.006263157159999, variance=22.796258443031981,
               skewness=0.20349136899082923, kurtosis=0.08980464662606069)
```



```
variable name: theta
variable value: 1.4835298641951802
distribution: normal(shape=(), loc=-28.616662349589998, scale=4.858165211851241)
DescribeResult(nobs=1000, minmax=(-42.69451234999997, -13.900715959999999)
               mean=-28.616662349589998, variance=23.625394620261876,
               skewness=0.10202333341882726, kurtosis=-0.10961524860517002)
```



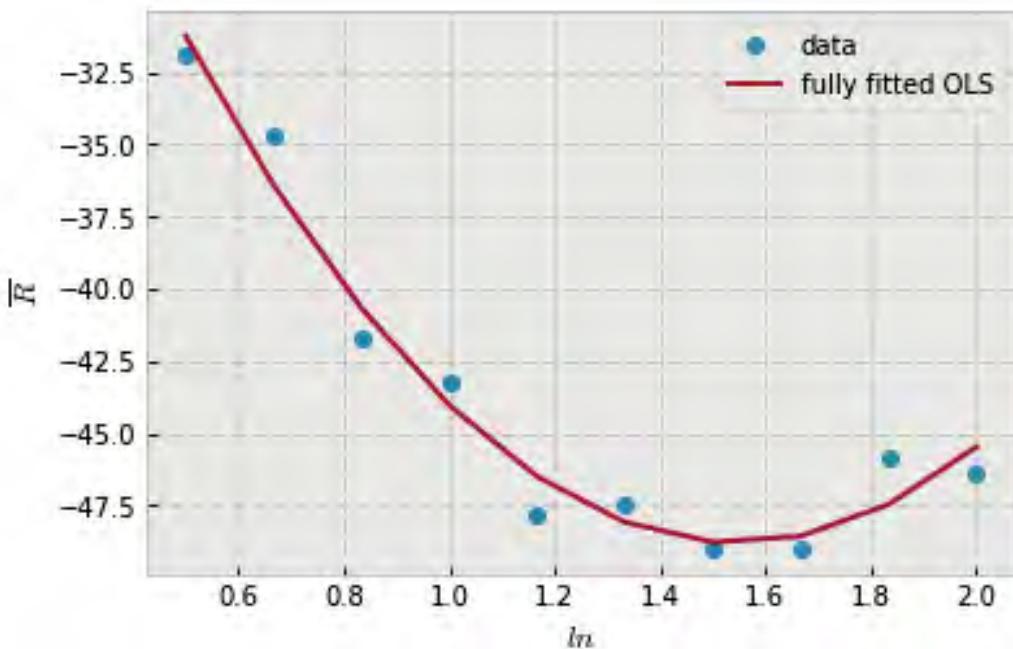


19.2 Parameter regression

```
In [44]: var_name = 'mean_depth_of_standard_root_water_uptake'

mean_reg('ln', var_name, 'poly2')
mean_reg('theta', var_name, 'poly2')
mean_reg('nB', var_name, 'poly1')
mean_reg('r', var_name, 'poly1')
mean_reg('sigma', var_name, 'poly2')
mean_reg('N', var_name, 'poly2')

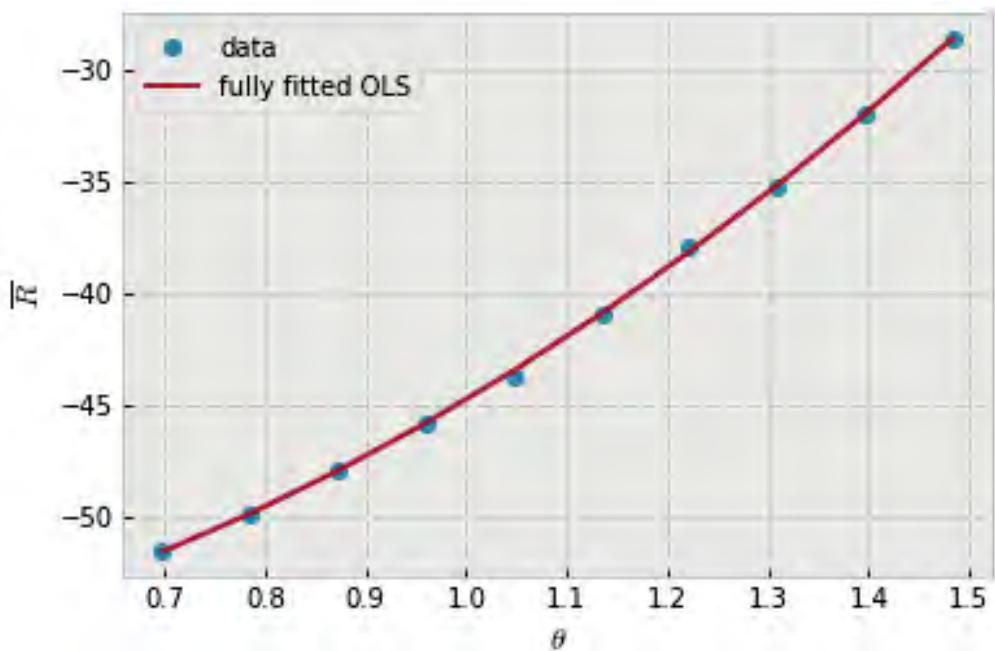
variable name: ln
model: poly2
r-squared: 0.9184665827479074
explained variance: 0.91938717006963
```



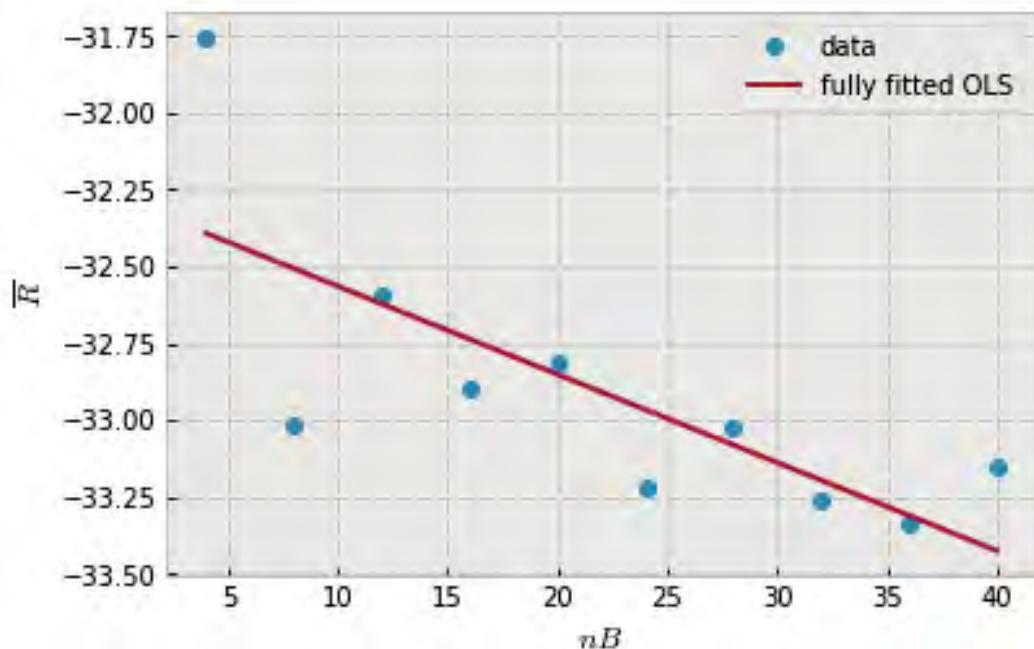
```

variable name: theta
model: poly2
r-squared: 0.9996231518681564
explained variance: 0.9996234211736845

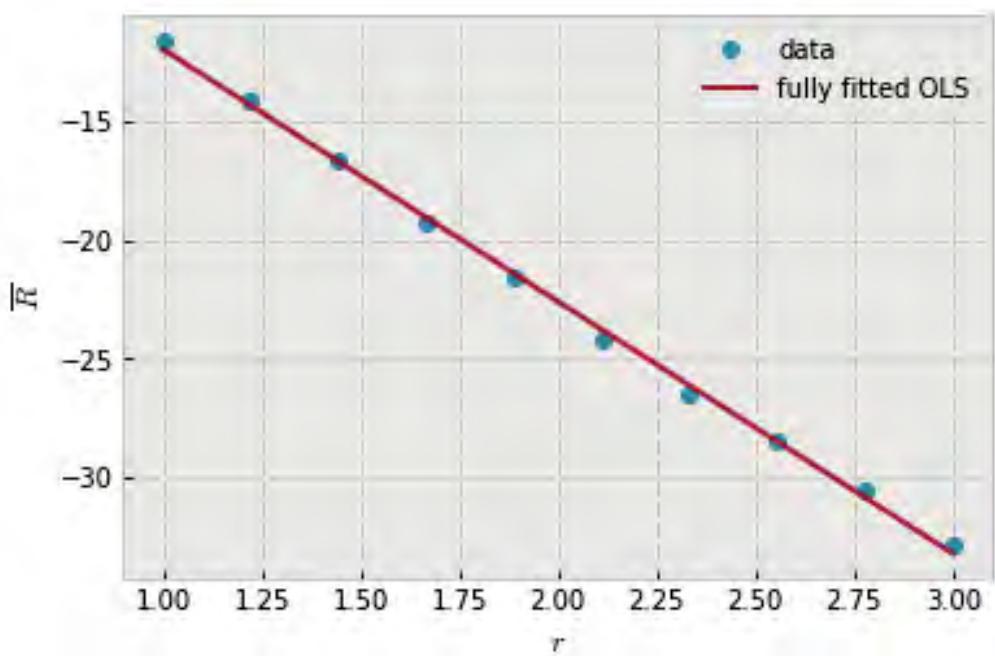
```



```
variable name: nB
model: poly1
r-squared: 0.11072169026650547
explained variance: 0.11400222069589105
```



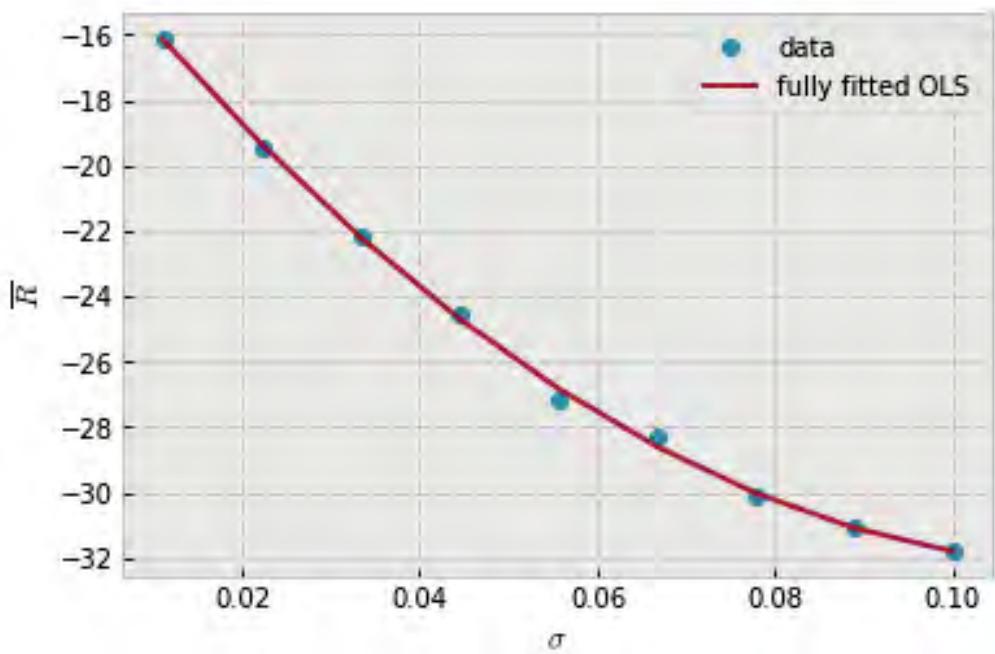
```
variable name: r
model: poly1
r-squared: 0.9969323010758074
explained variance: 0.9969662667438381
```



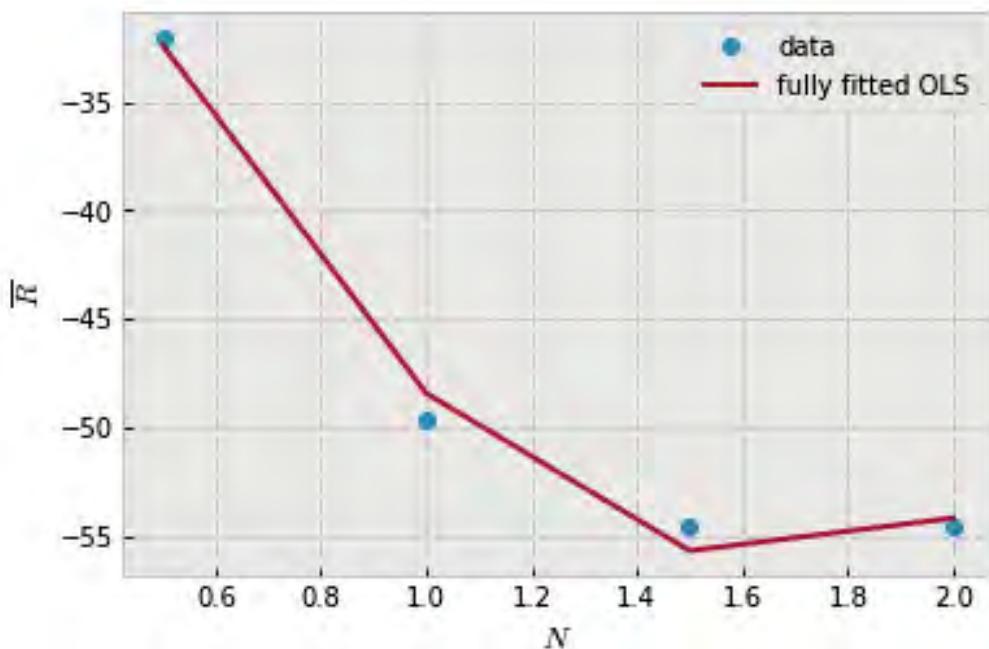
```

variable name: sigma
model: poly2
r-squared: 0.9981443859702446
explained variance: 0.998145196823463

```



```
variable name: N
model: poly2
r-squared: 0.6069311411366995
explained variance: 0.6069311411366995
```



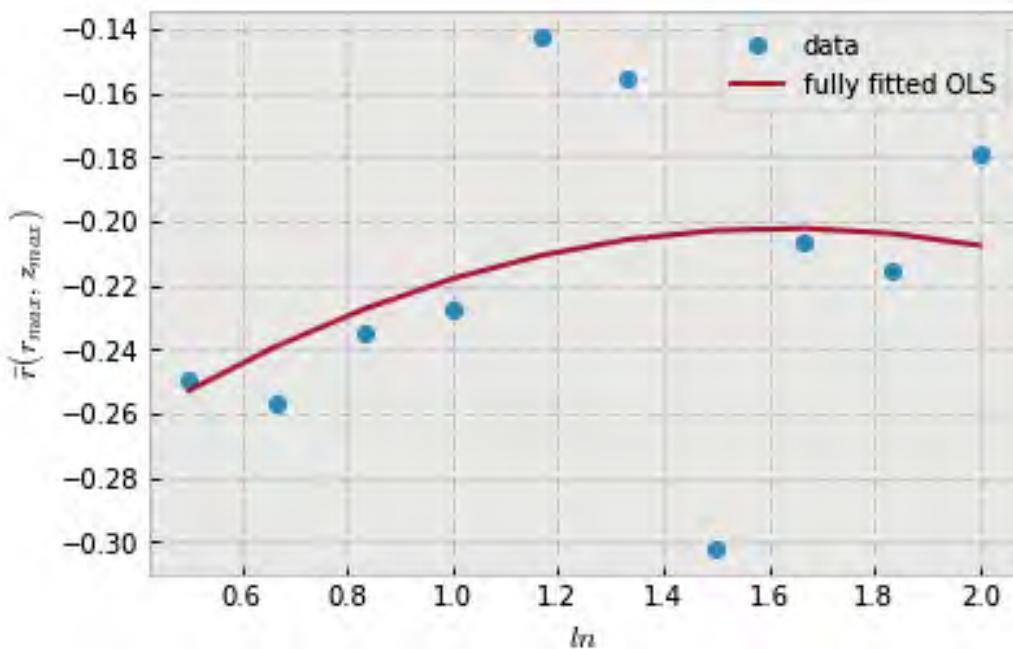
20 Correlation

20.1 Selected root system measures

```
In [45]: y1_name, y2_name = ('maximal_horizontal_spread', 'maximal_rooting_depth')

corr_reg('ln', y1_name, y2_name, 'poly2')
corr_reg('theta', y1_name, y2_name, 'poly2')
corr_reg('nB', y1_name, y2_name, 'poly2')
corr_reg('r', y1_name, y2_name, 'poly1')
corr_reg('sigma', y1_name, y2_name, 'poly2')
corr_reg('N', y1_name, y2_name, 'poly1')

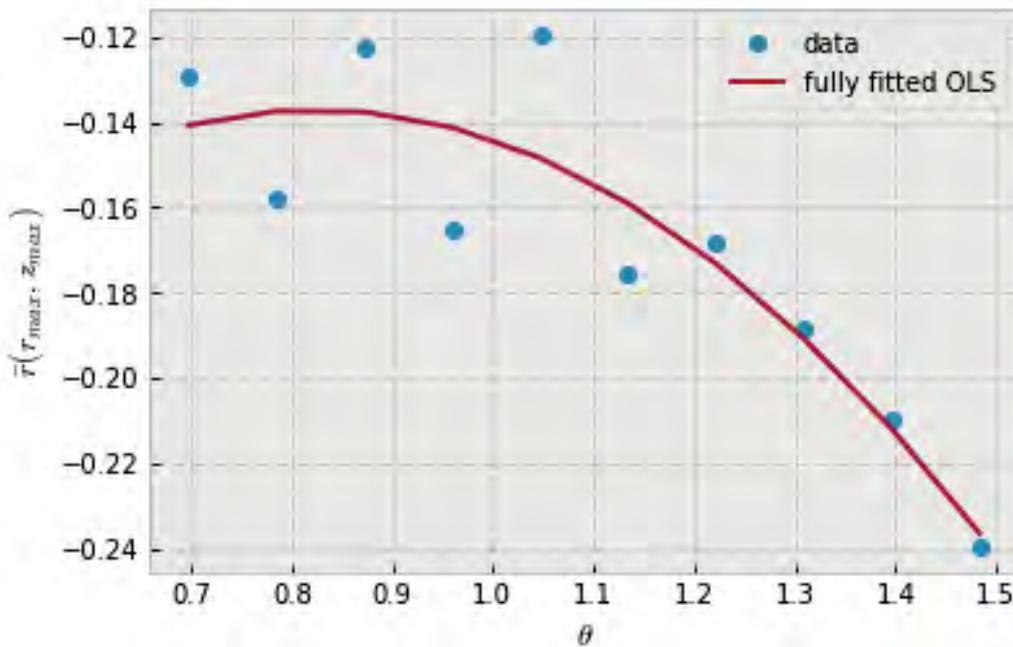
variable name: ln
model: poly2
r-squared: -0.604114798801384
explained variance: -0.5943094122236425
```



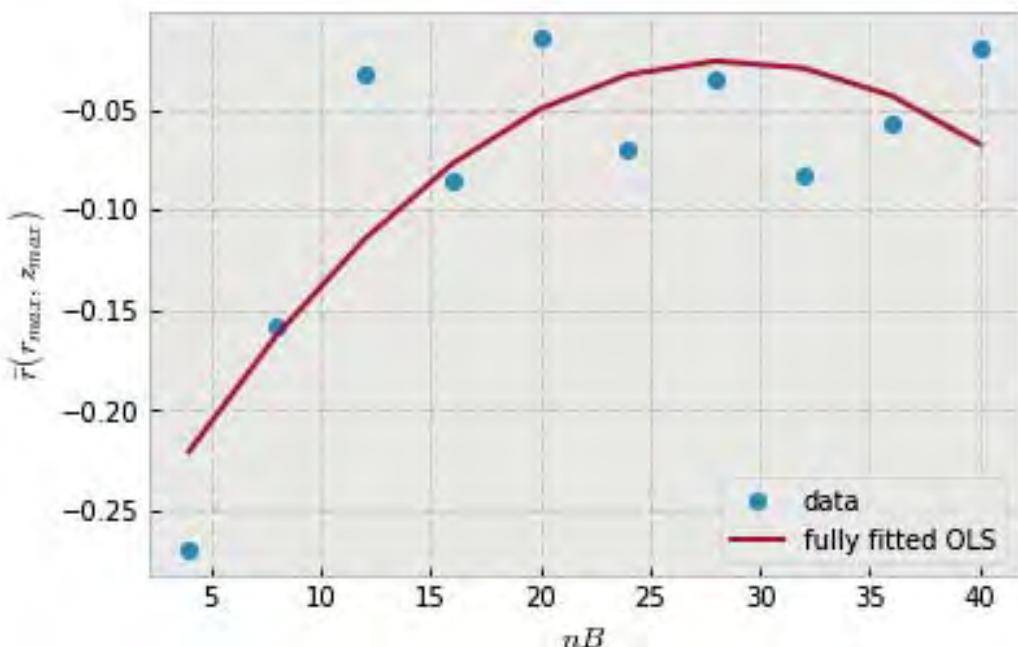
```

variable name: theta
model: poly2
r-squared: 0.6359426037415885
explained variance: 0.6364614300983975

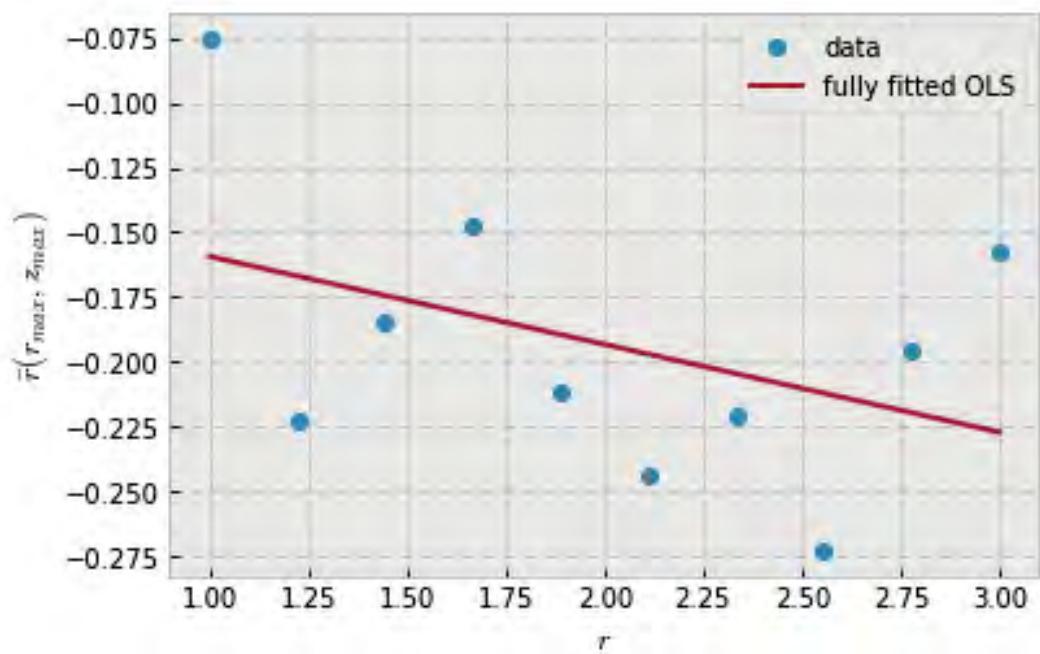
```



```
variable name: nB
model: poly2
r-squared: 0.05964956974923863
explained variance: 0.059679379206102334
```



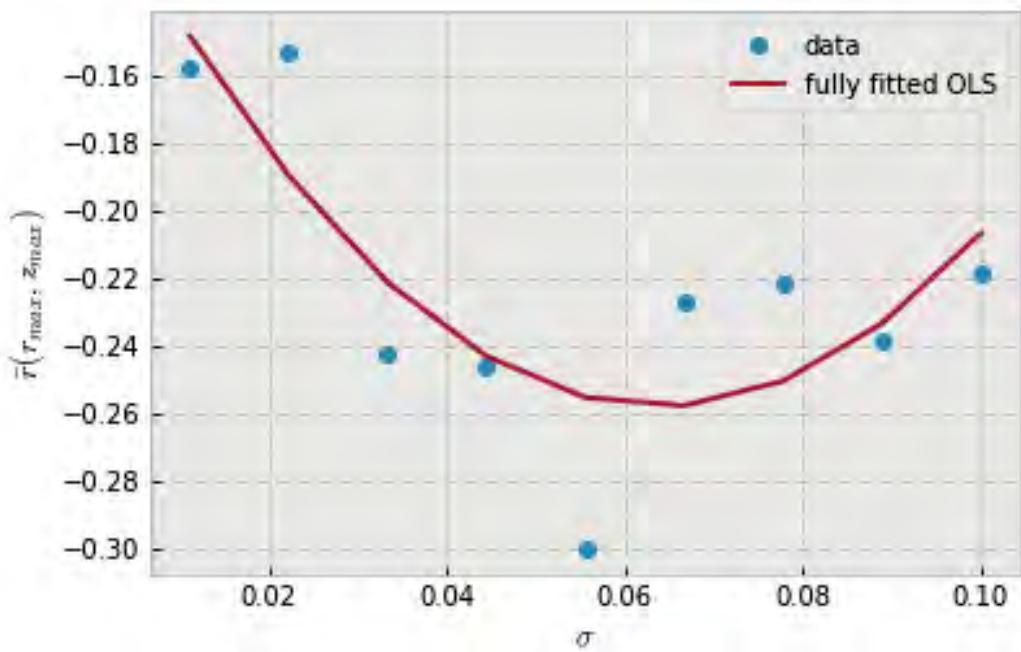
```
variable name: r
model: poly1
r-squared: -0.6034488770230966
explained variance: -0.5944946040967924
```



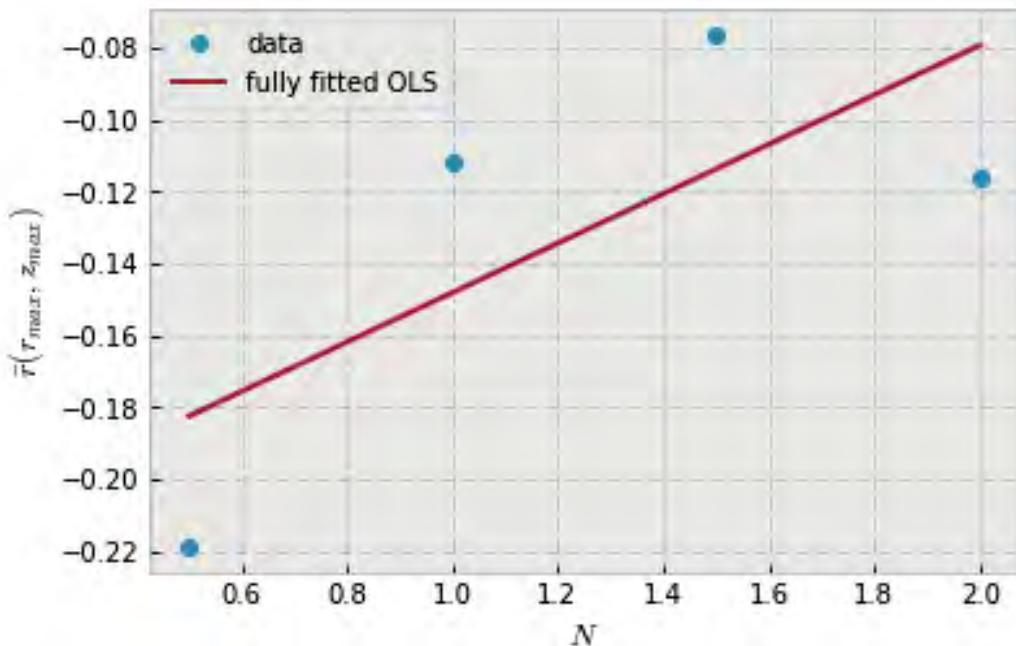
```

variable name: sigma
model: poly2
r-squared: 0.2716030705171668
explained variance: 0.279685713002954

```



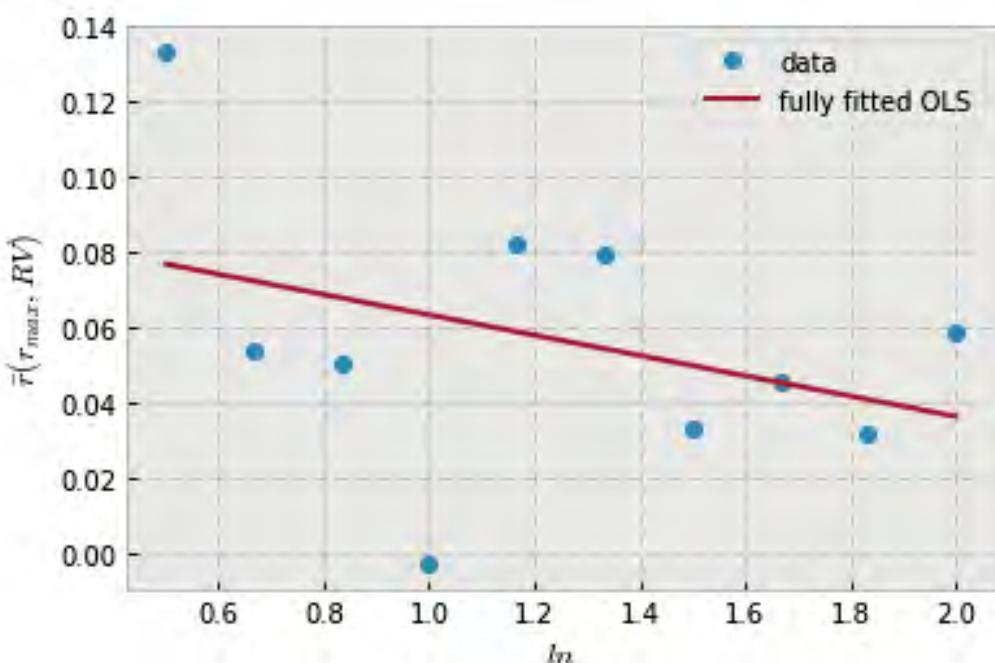
```
variable name: N
model: poly1
r-squared: -2.1512581257376704
explained variance: -1.7166395775561138
```



```
In [46]: y1_name, y2_name = ('maximal_horizontal_spread', 'total_root_volume')
```

```
corr_reg('ln', y1_name, y2_name, 'poly1')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('nB', y1_name, y2_name, 'poly2')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly2')
corr_reg('N', y1_name, y2_name, 'poly1')
```

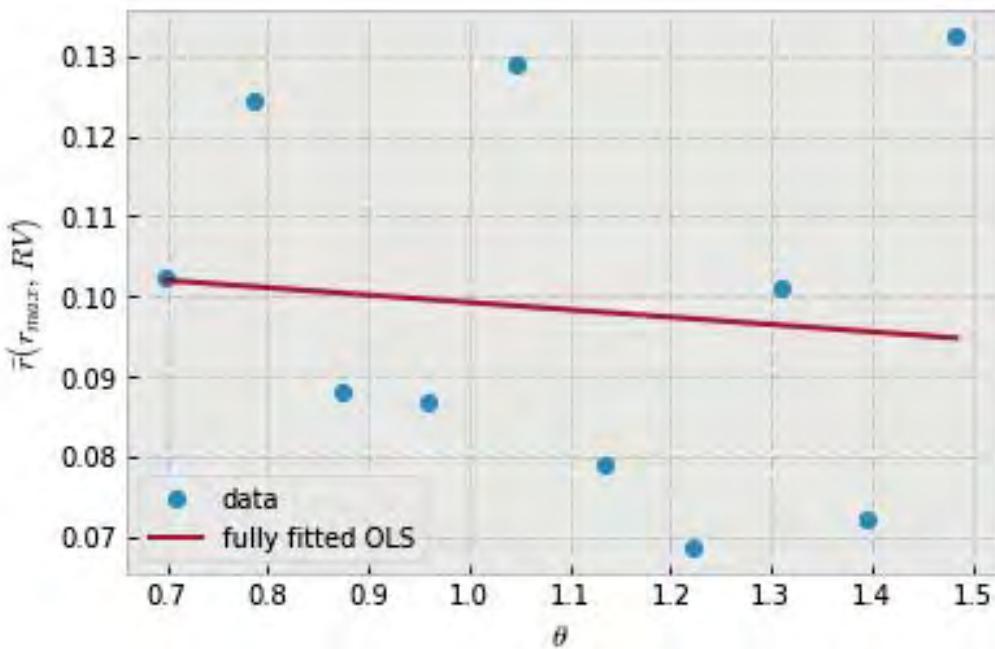
```
variable name: ln
model: poly1
r-squared: -0.45887292223685083
explained variance: -0.454637827367129
```



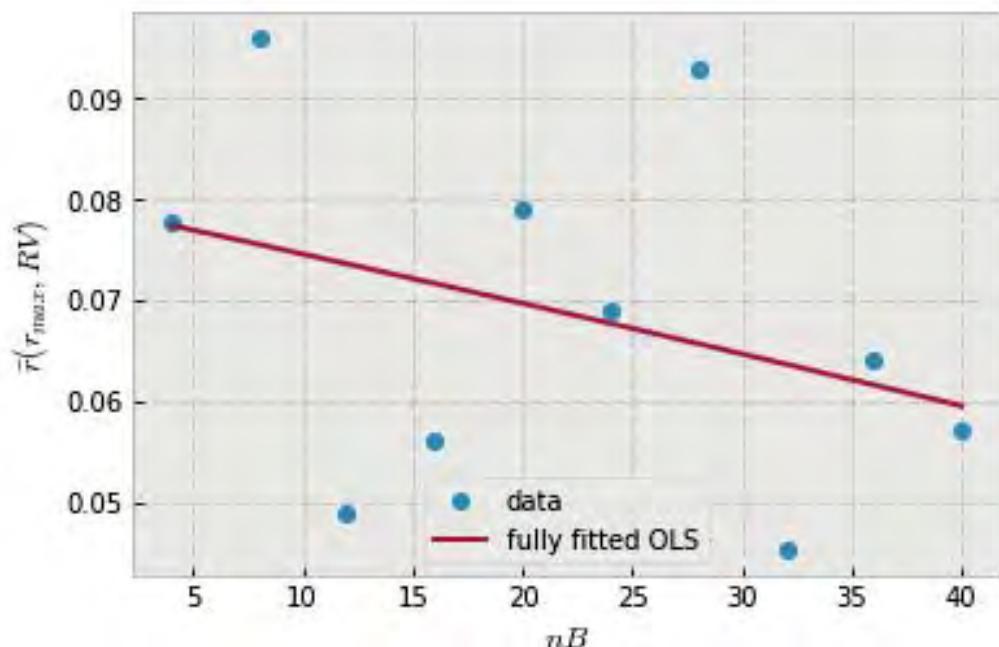
```

variable name: theta
model: poly1
r-squared: -0.6779806636160564
explained variance: -0.6742605384688309

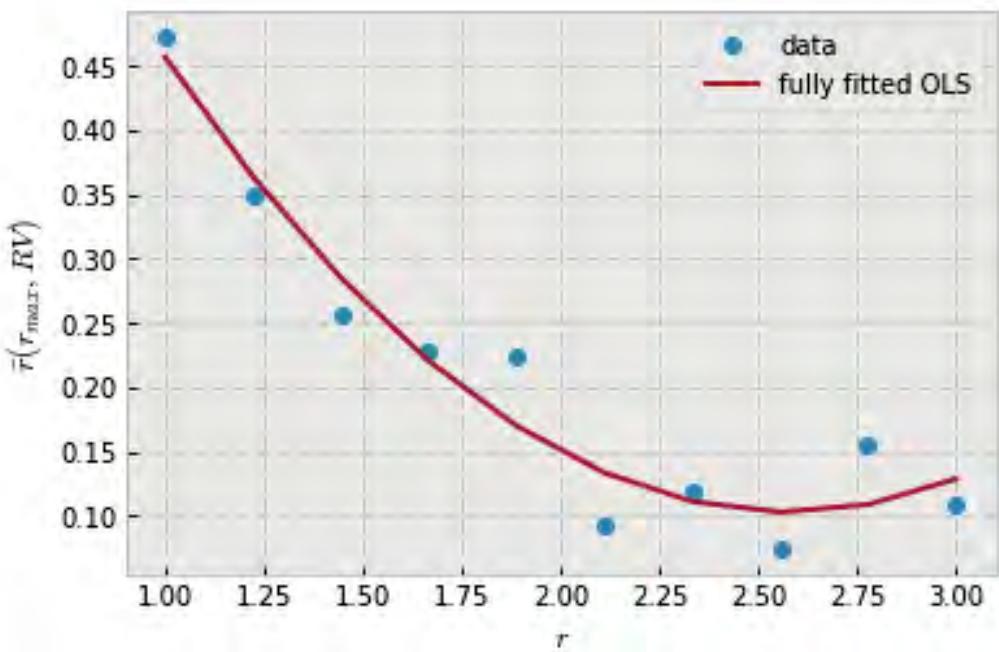
```



```
variable name: nB
model: poly2
r-squared: -0.4208426610977434
explained variance: -0.42072839893121117
```



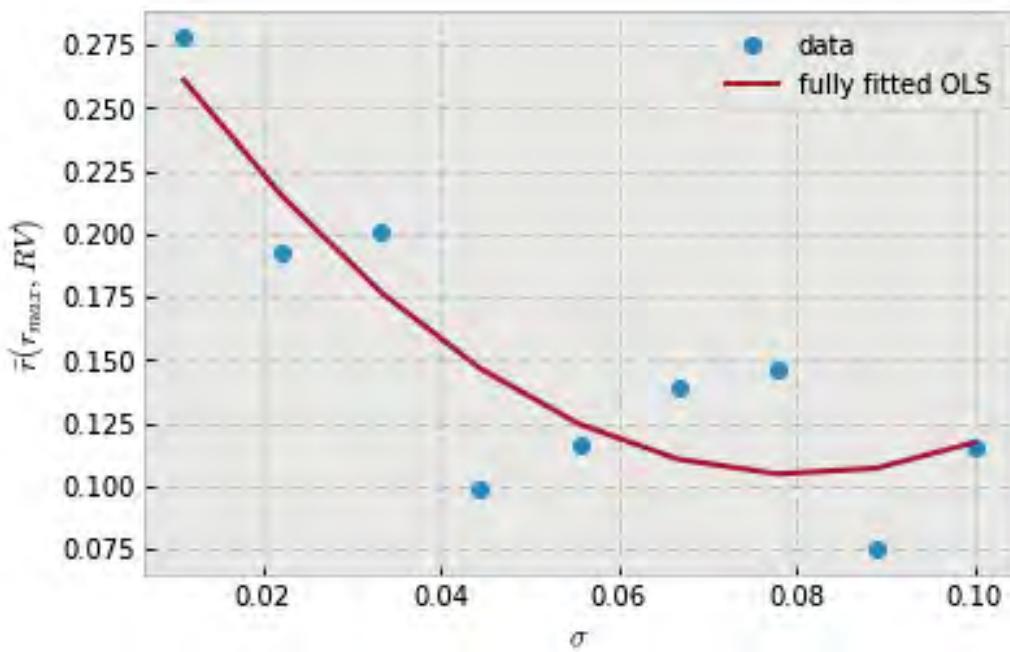
```
variable name: r
model: poly2
r-squared: 0.8654747755706119
explained variance: 0.8654775657884961
```



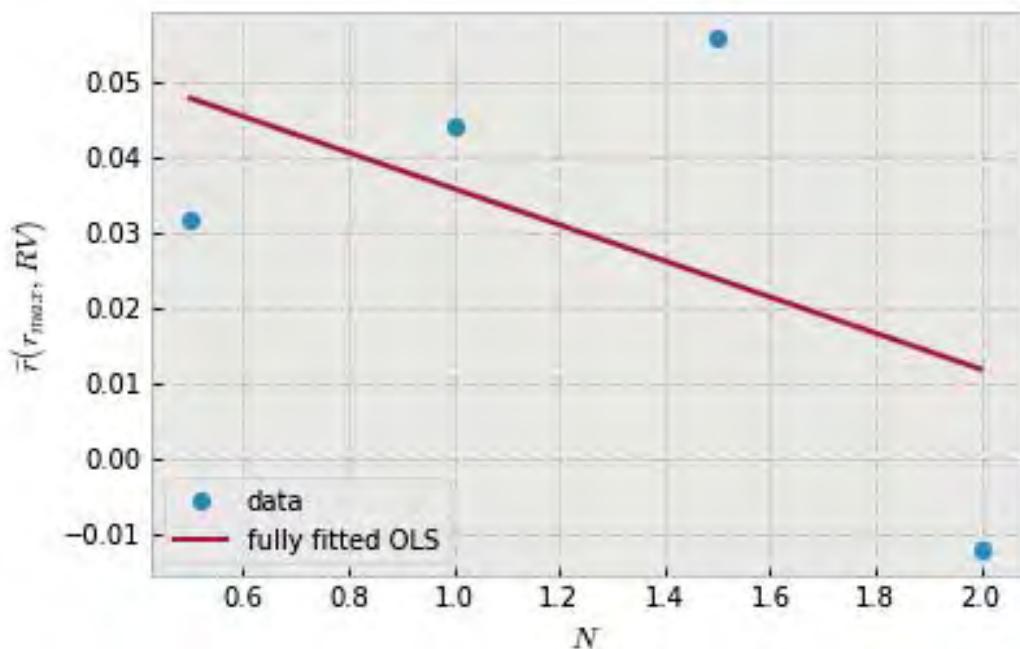
```

variable name: sigma
model: poly2
r-squared: 0.5413233228630014
explained variance: 0.5423141301184169

```



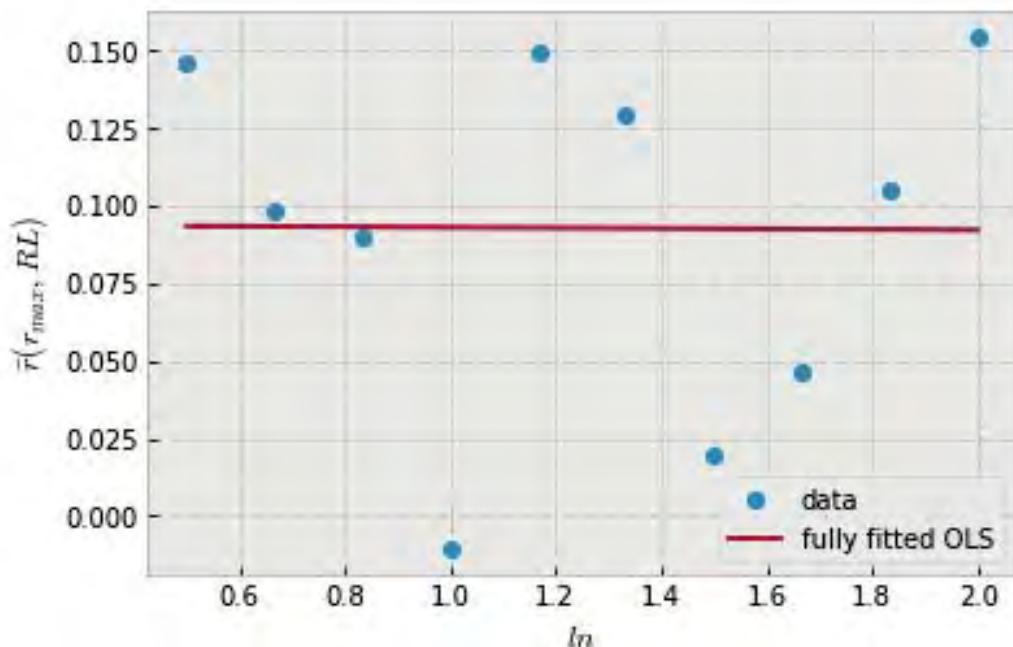
```
variable name: N
model: poly1
r-squared: -3.362009766639968
explained variance: -2.8092117727511154
```



```
In [47]: y1_name, y2_name = ('maximal_horizontal_spread', 'total_root_length')

corr_reg('ln', y1_name, y2_name, 'poly1')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('nB', y1_name, y2_name, 'poly1')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly1')
corr_reg('N', y1_name, y2_name, 'poly1')

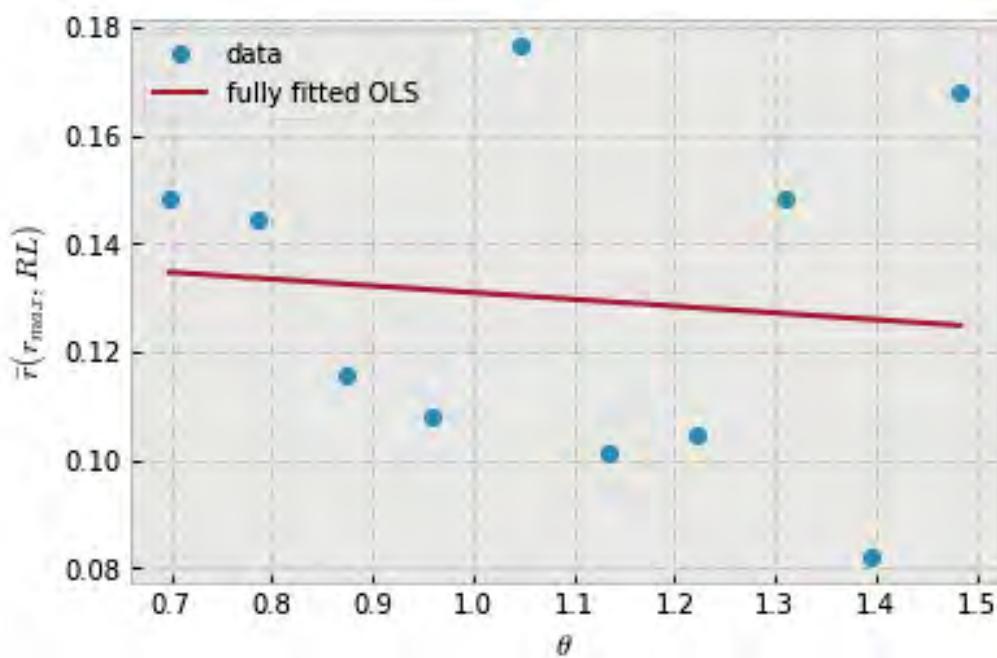
variable name: ln
model: poly1
r-squared: -0.5449297755826337
explained variance: -0.5393028127215396
```



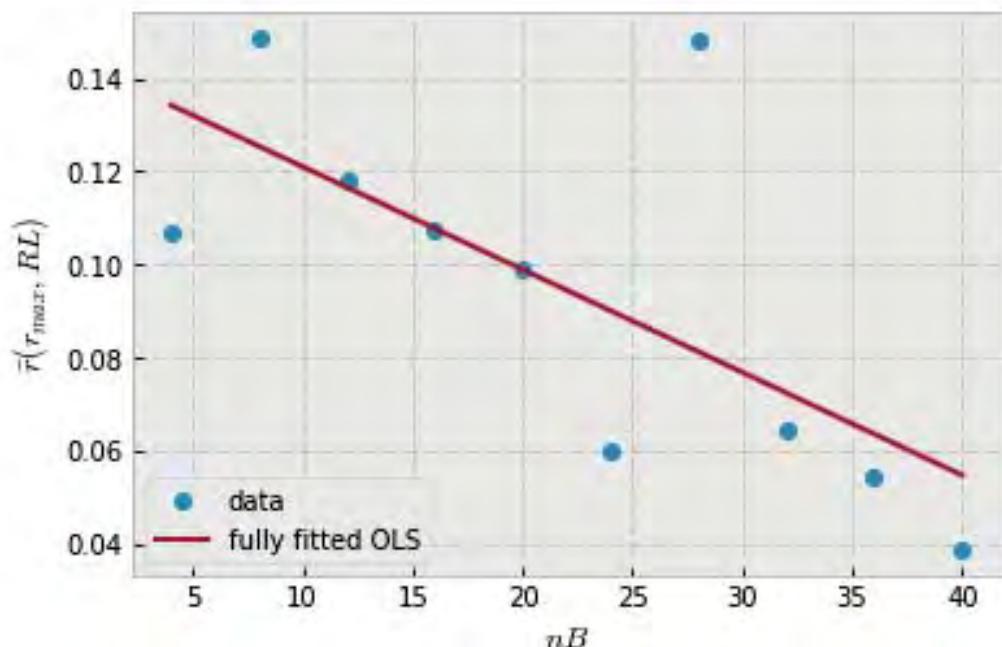
```

variable name: theta
model: poly1
r-squared: -0.6274287061938943
explained variance: -0.6248153887522667

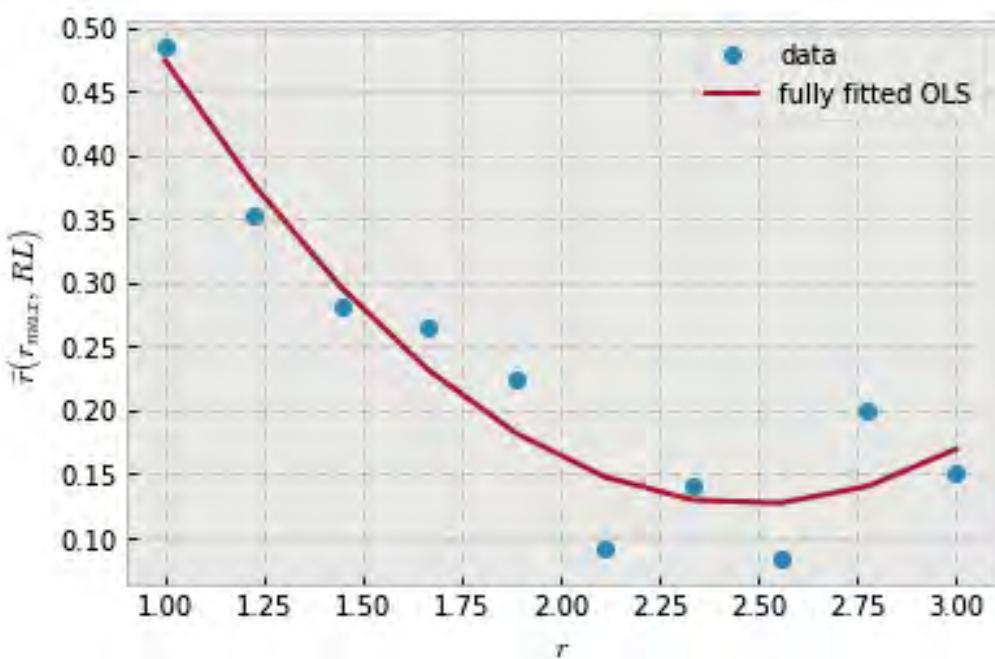
```



```
variable name: nB
model: poly1
r-squared: 0.21539379826381444
explained variance: 0.21675549134847683
```



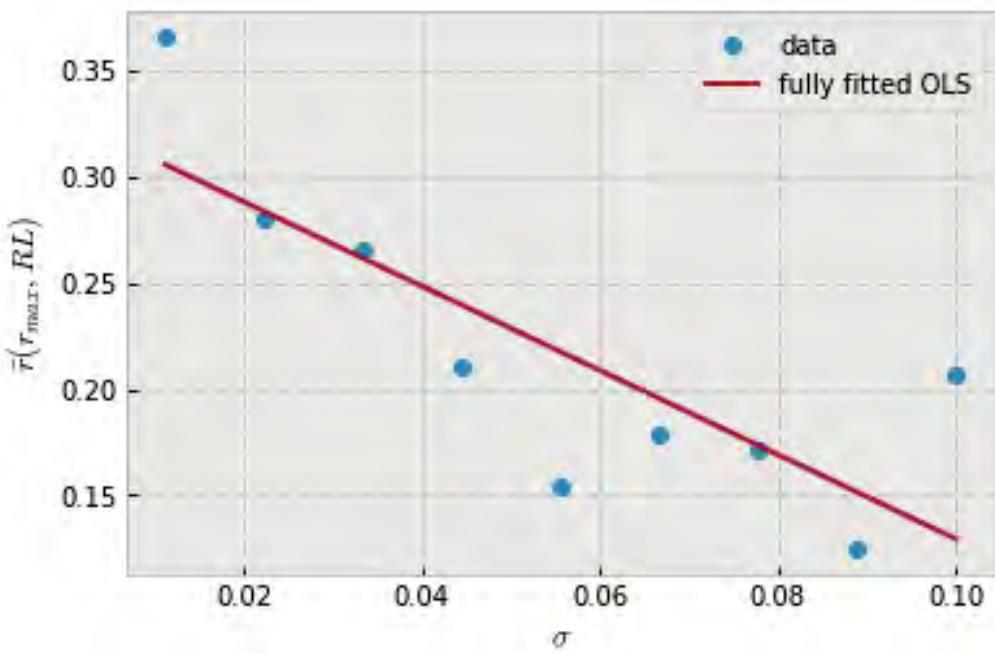
```
variable name: r
model: poly2
r-squared: 0.8219053012426477
explained variance: 0.8219223330216676
```



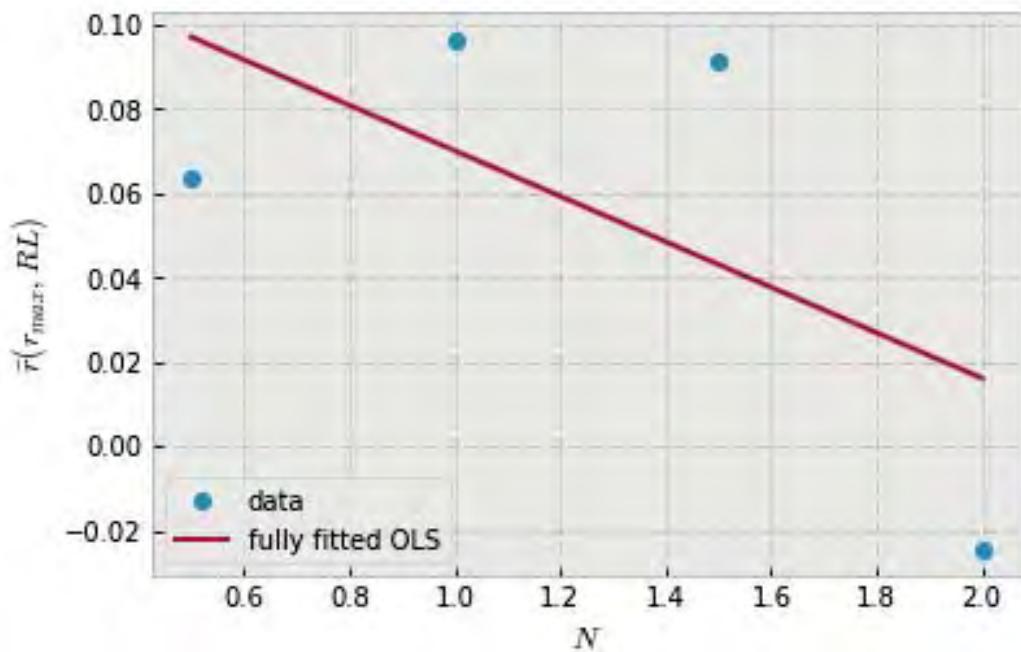
```

variable name: sigma
model: poly1
r-squared: 0.26101812776299105
explained variance: 0.26961209260580266

```



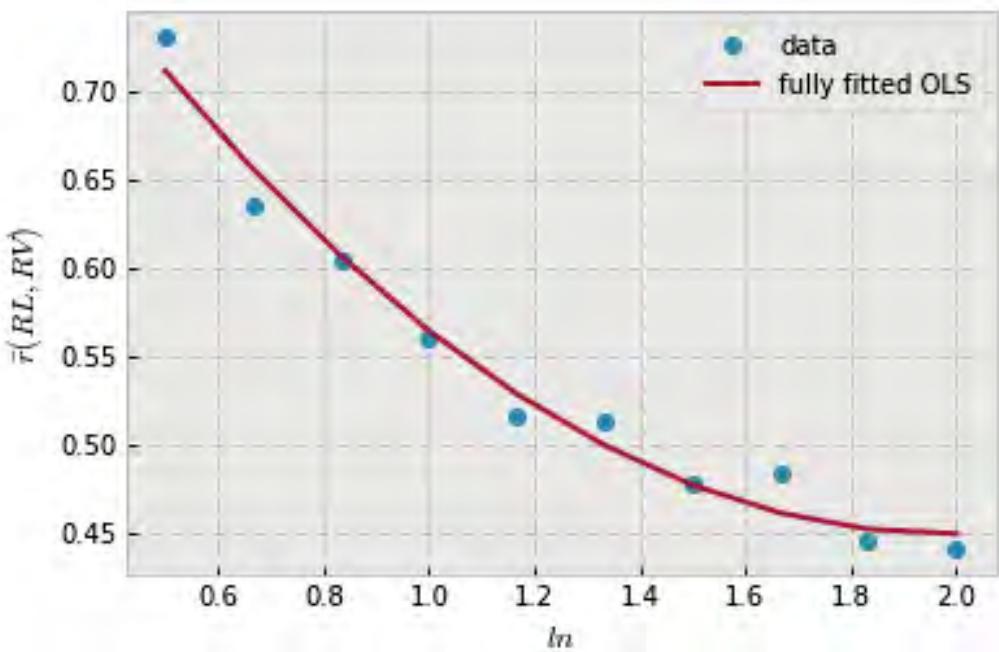
```
variable name: N
model: poly1
r-squared: -2.9524707597056263
explained variance: -2.418553557436616
```



```
In [48]: y1_name, y2_name = ('total_root_length', 'total_root_volume')
```

```
corr_reg('ln', y1_name, y2_name, 'poly2')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('nB', y1_name, y2_name, 'poly1')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly1')
corr_reg('N', y1_name, y2_name, 'poly2')
```

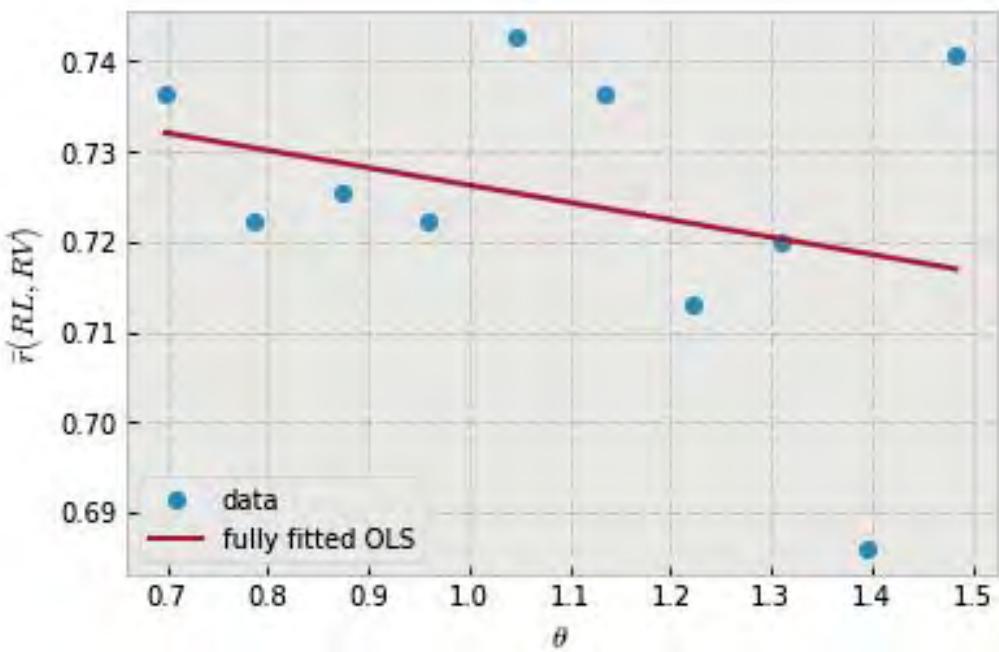
```
variable name: ln
model: poly2
r-squared: 0.9301505998033823
explained variance: 0.93029041826877
```



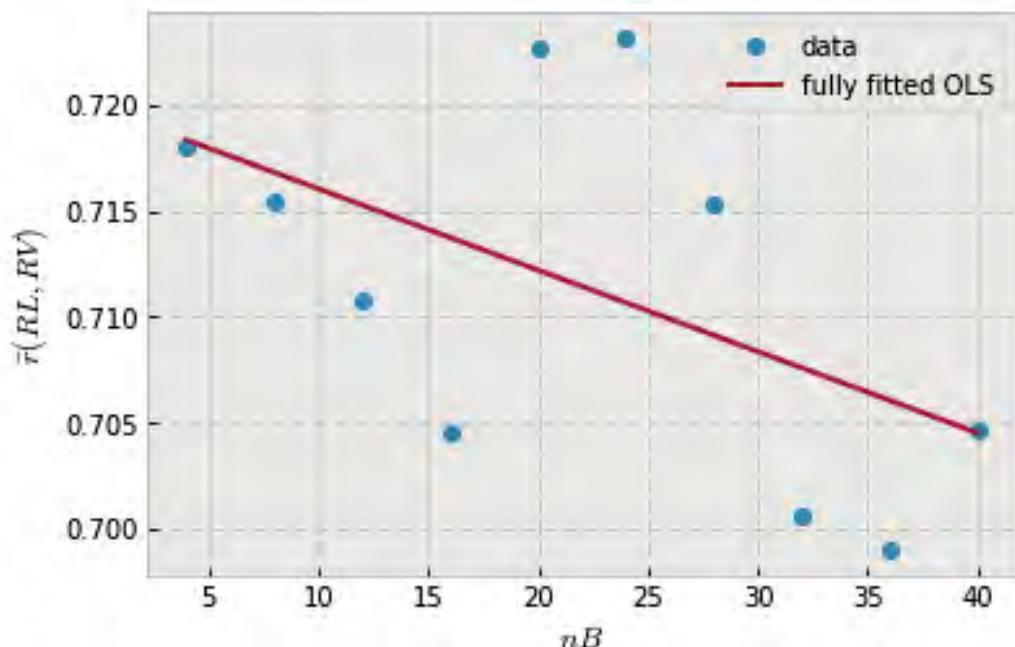
```

variable name: theta
model: poly1
r-squared: -0.621989160240934
explained variance: -0.621829326643456

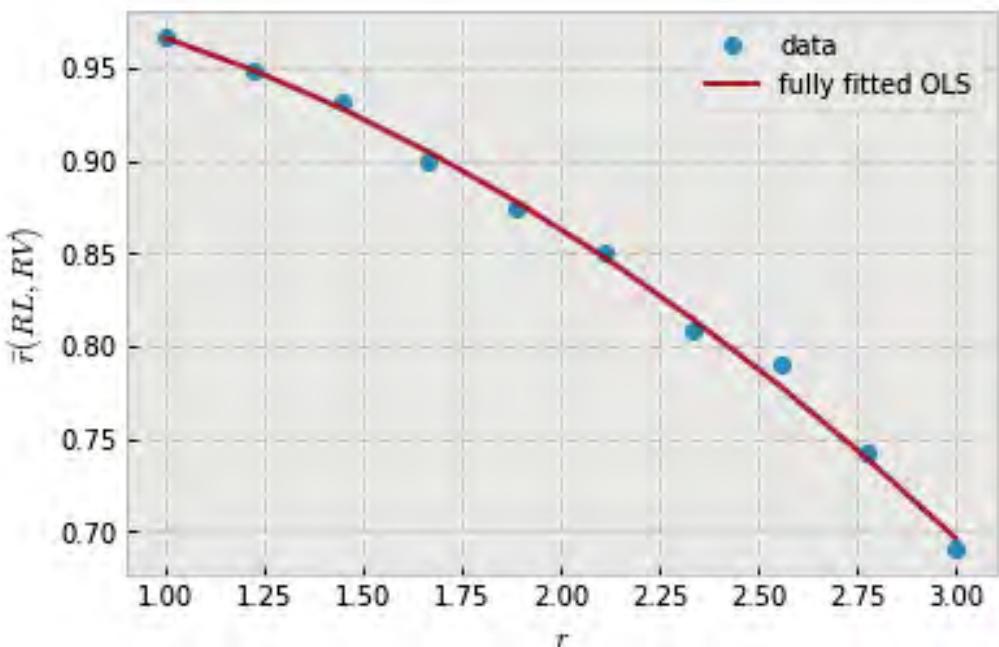
```



```
variable name: nB
model: poly1
r-squared: 0.025801110866067845
explained variance: 0.027185930289603255
```



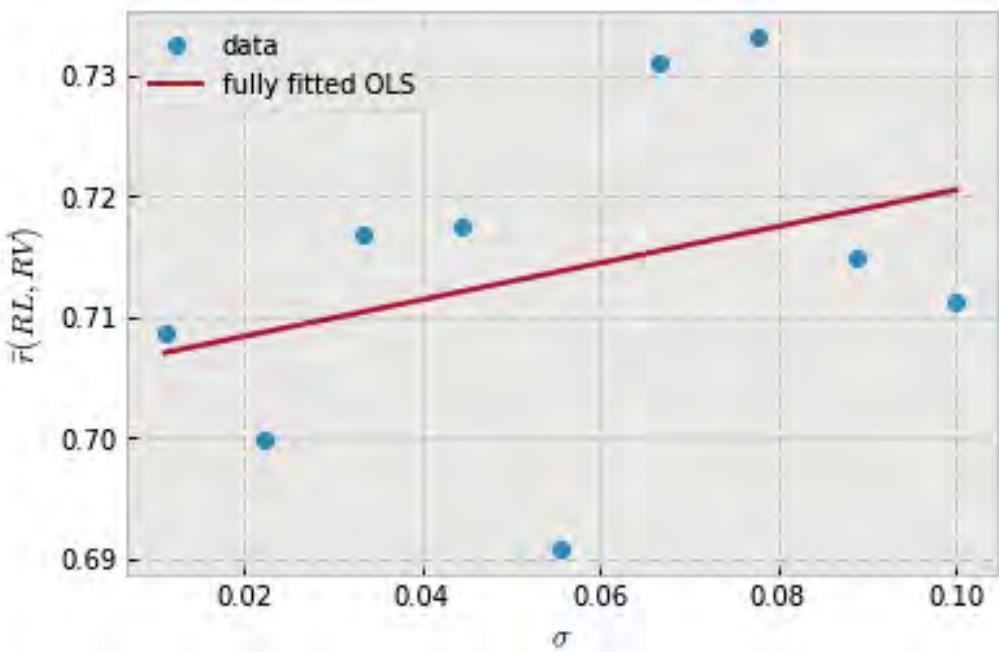
```
variable name: r
model: poly2
r-squared: 0.9914416863479607
explained variance: 0.9915049948580617
```



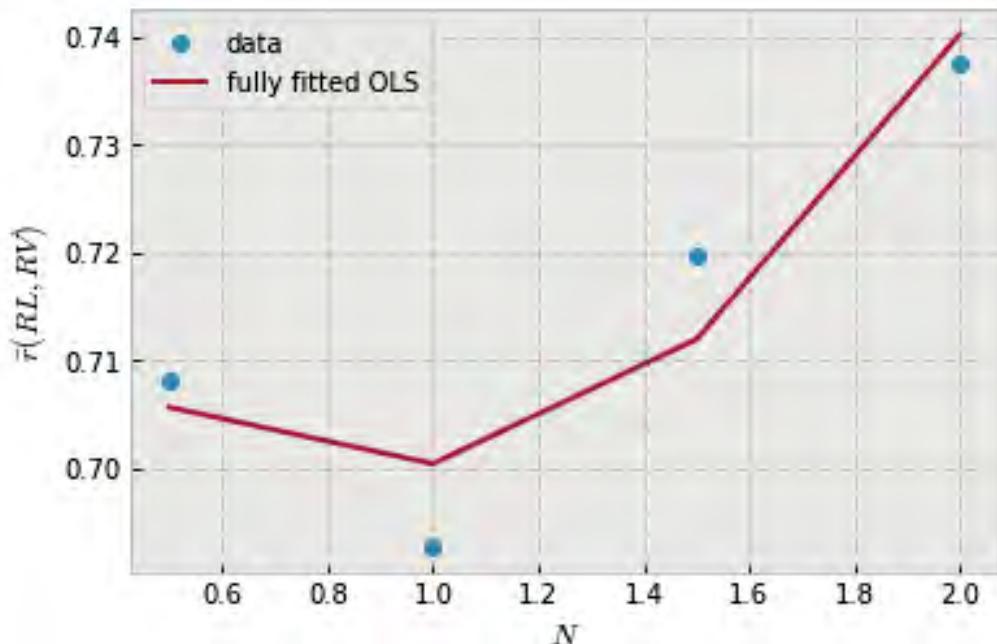
```

variable name: sigma
model: poly1
r-squared: -0.28829000447673403
explained variance: -0.2870178616175325

```



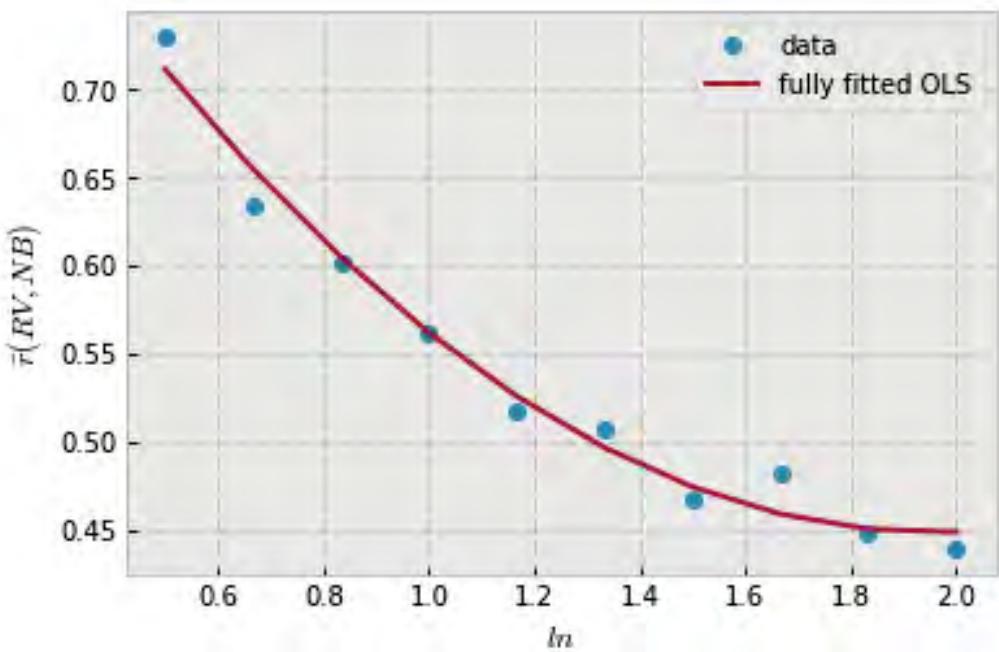
```
variable name: N
model: poly2
r-squared: -4.545107952290249
explained variance: -4.545107952290249
```



```
In [49]: y1_name, y2_name = ('total_root_volume', 'number_of_root_branches')
```

```
corr_reg('ln', y1_name, y2_name, 'poly2')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('nB', y1_name, y2_name, 'poly1')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly2')
corr_reg('N', y1_name, y2_name, 'poly2')
```

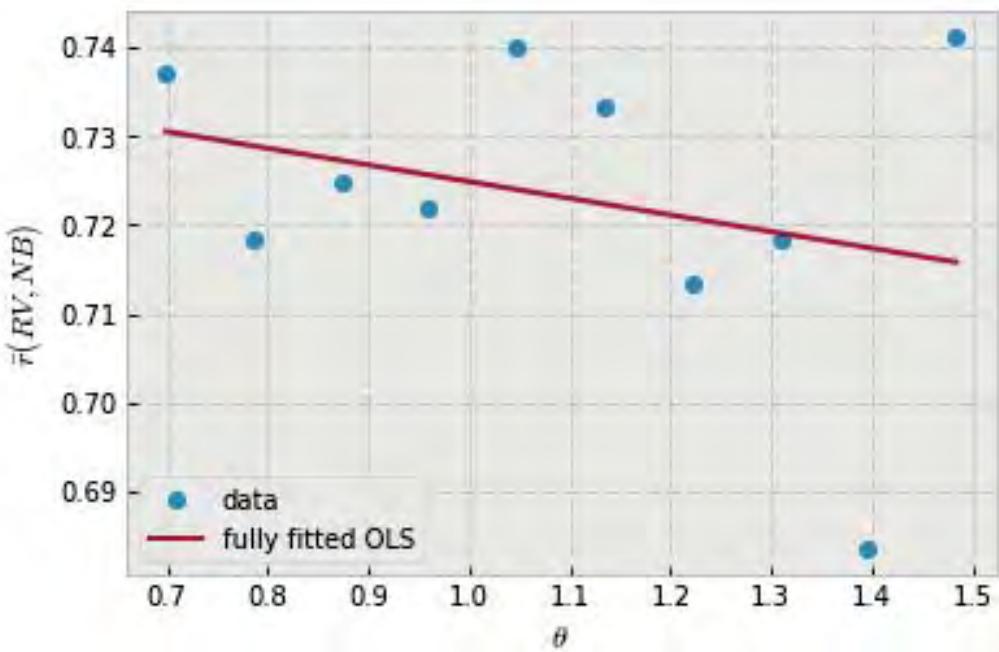
```
variable name: ln
model: poly2
r-squared: 0.9332566538355056
explained variance: 0.9333471469035082
```



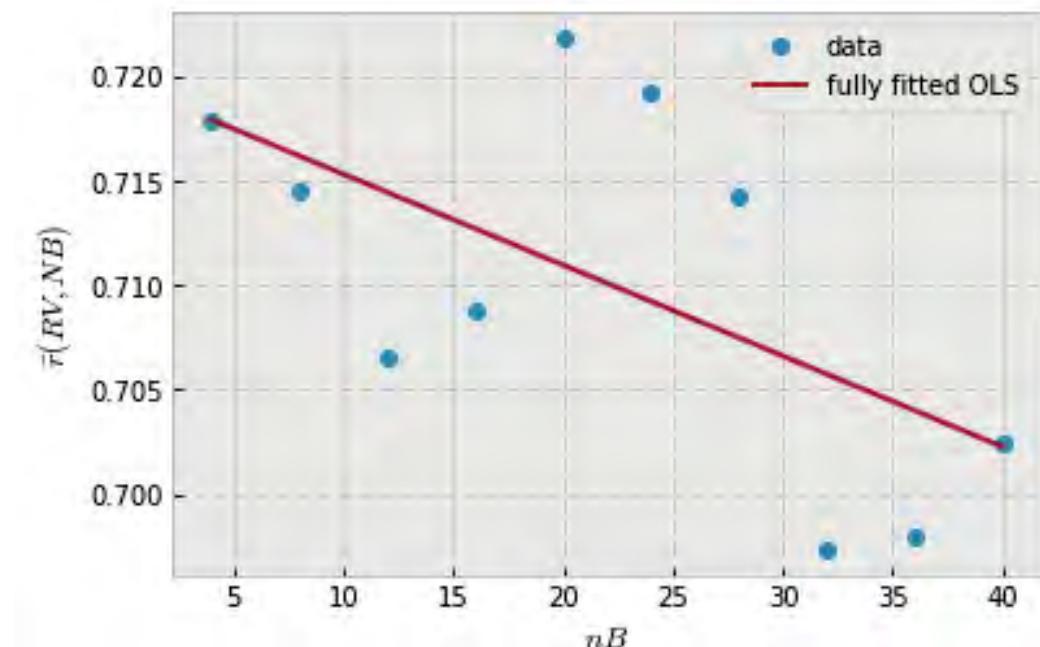
```

variable name: theta
model: poly1
r-squared: -0.6829057874103734
explained variance: -0.6825734441114706

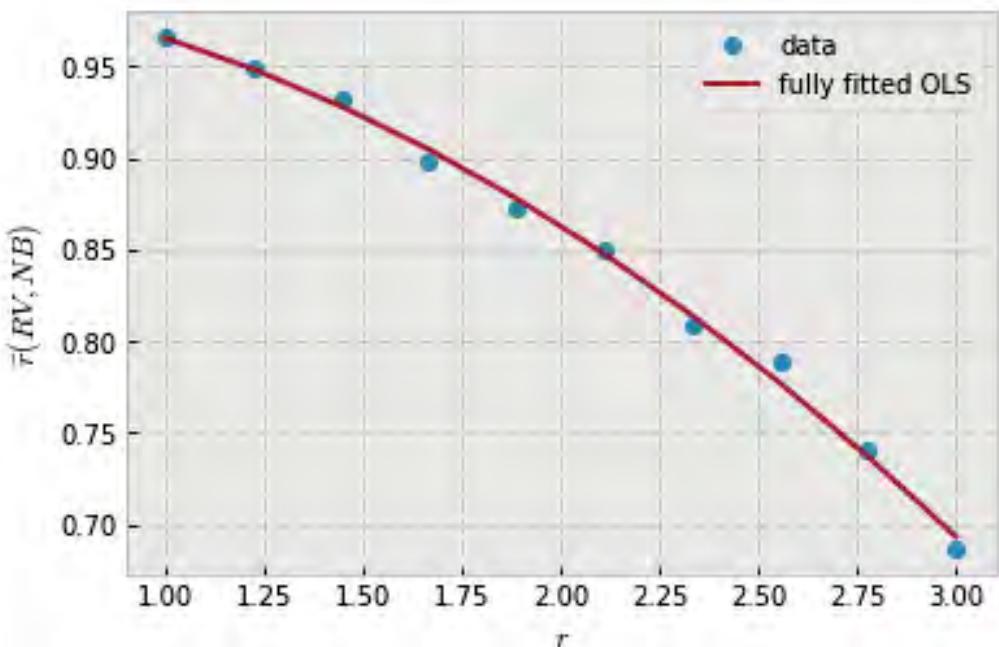
```



```
variable name: nB
model: poly1
r-squared: 0.11970752546138919
explained variance: 0.12110404190084334
```



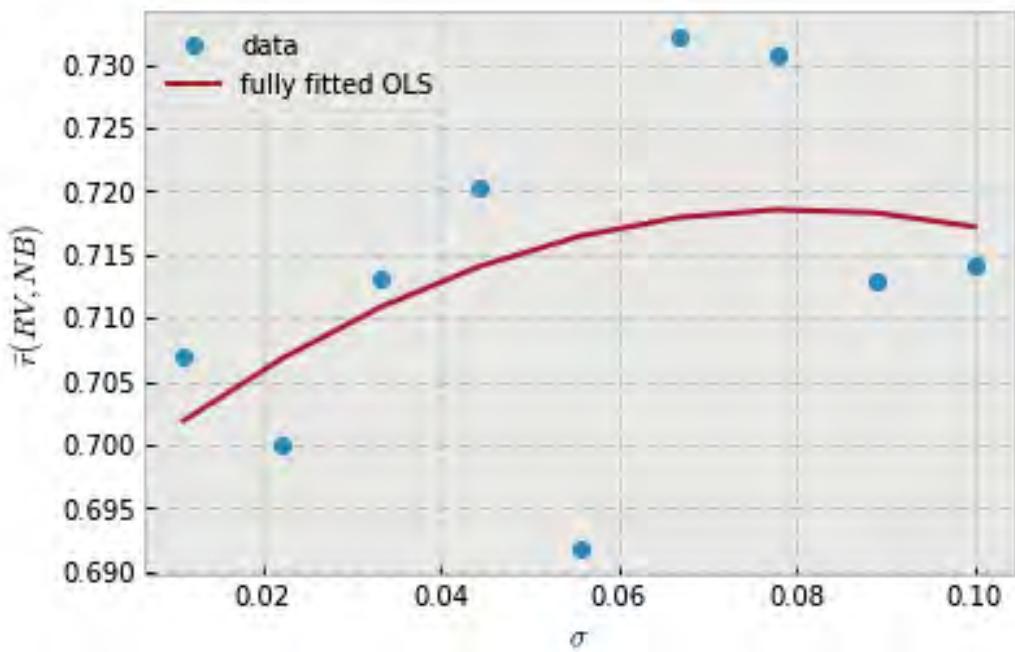
```
variable name: r
model: poly2
r-squared: 0.990005889234084
explained variance: 0.9900933047649824
```



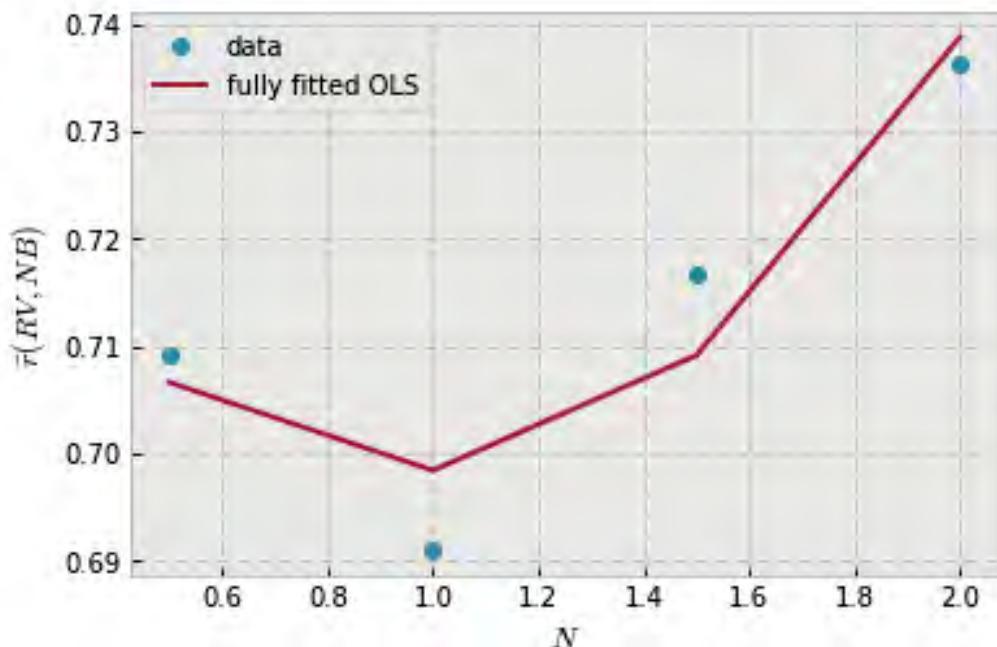
```

variable name: sigma
model: poly2
r-squared: -0.5842527935368456
explained variance: -0.584236033475459

```



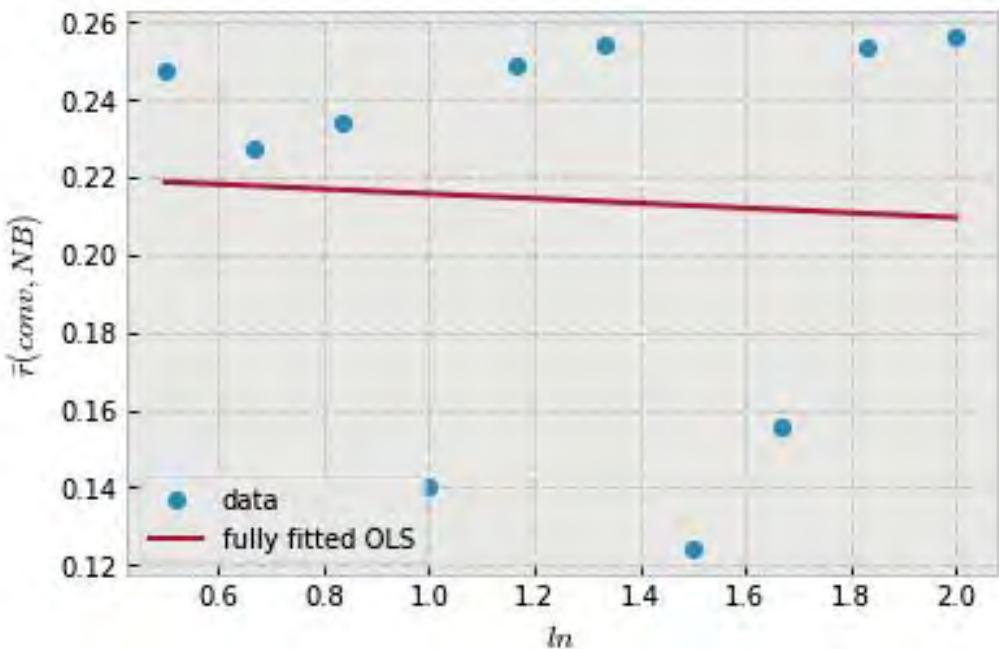
```
variable name: N
model: poly2
r-squared: -4.2319392747508875
explained variance: -4.2319392747508875
```



```
In [50]: y1_name, y2_name = ('convex_hull_volume', 'number_of_root_branches')
```

```
corr_reg('ln', y1_name, y2_name, 'poly1')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('nB', y1_name, y2_name, 'poly1')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly1')
corr_reg('N', y1_name, y2_name, 'poly1')
```

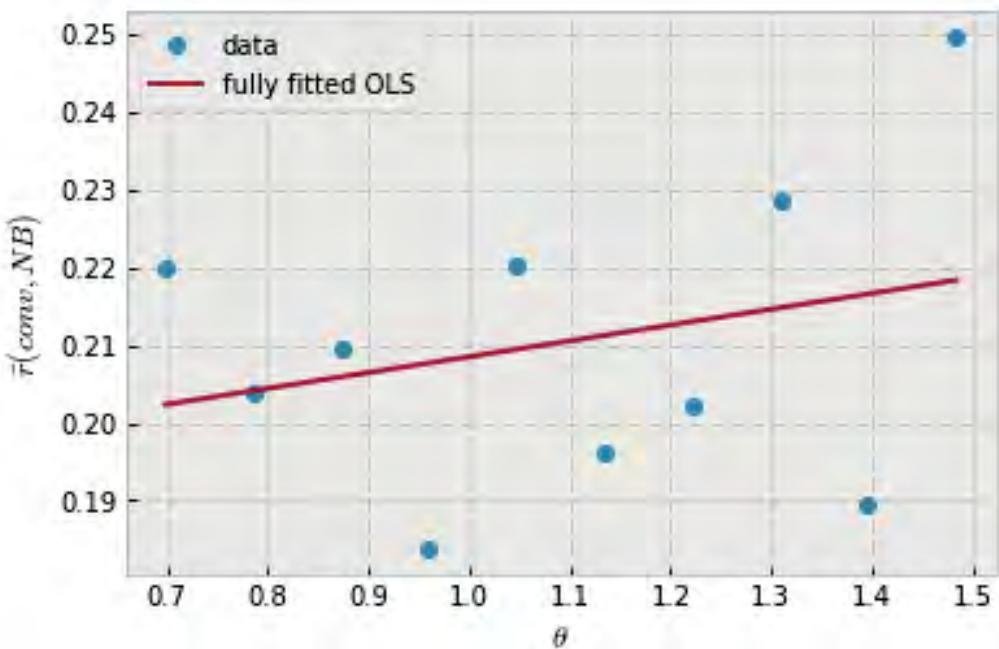
```
variable name: ln
model: poly1
r-squared: -0.48166605640022975
explained variance: -0.47713377220862996
```



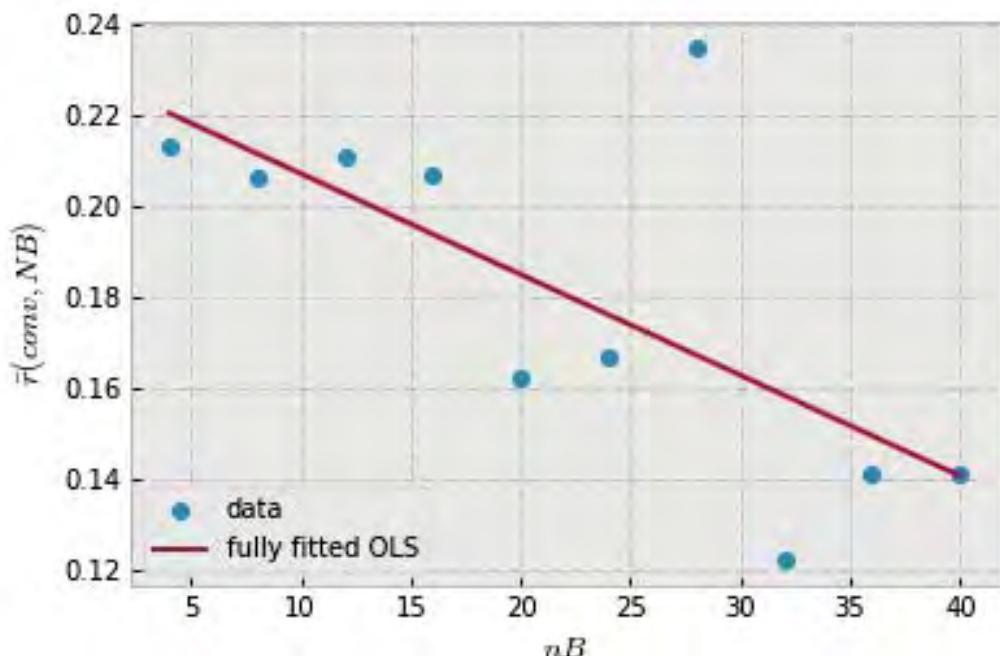
```

variable name: theta
model: poly1
r-squared: -0.6887039729777646
explained variance: -0.6823805537552112

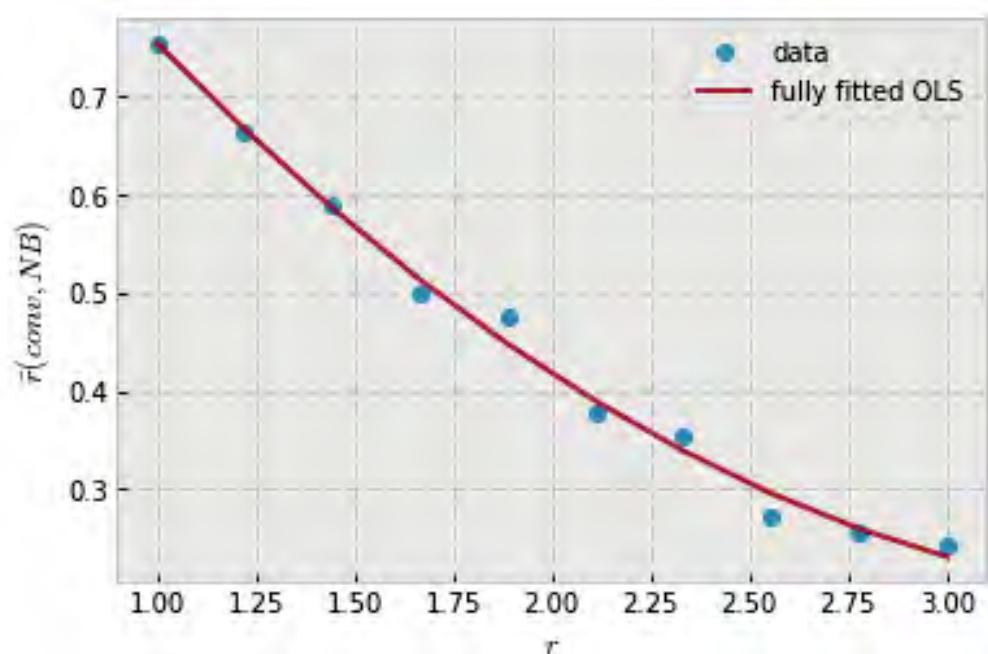
```



```
variable name: nB
model: poly1
r-squared: 0.3021277623715236
explained variance: 0.3024022649629108
```



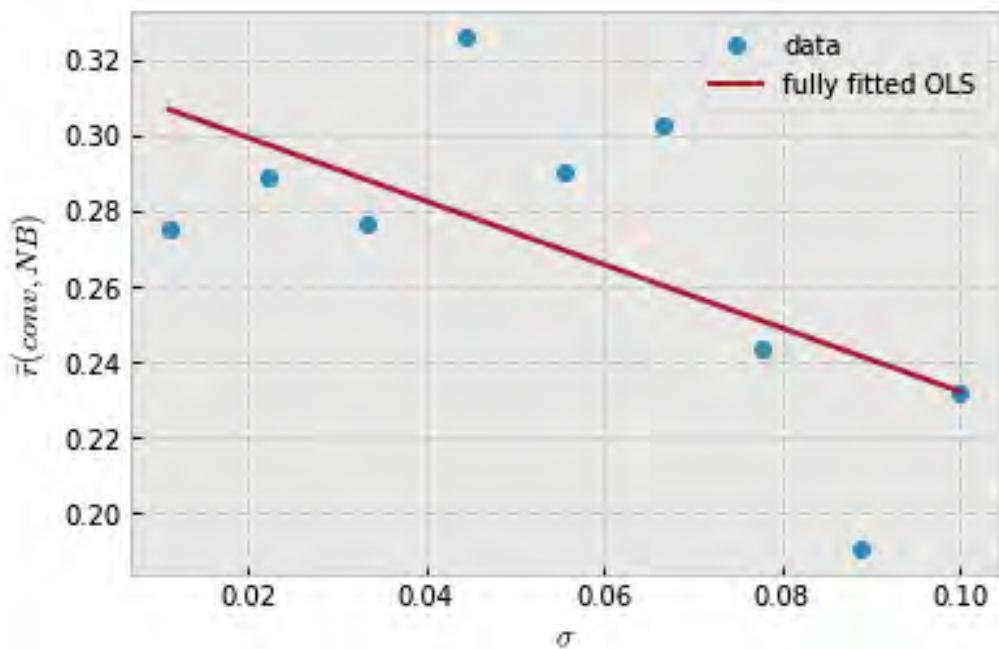
```
variable name: r
model: poly2
r-squared: 0.9867117617601608
explained variance: 0.9867881738016635
```



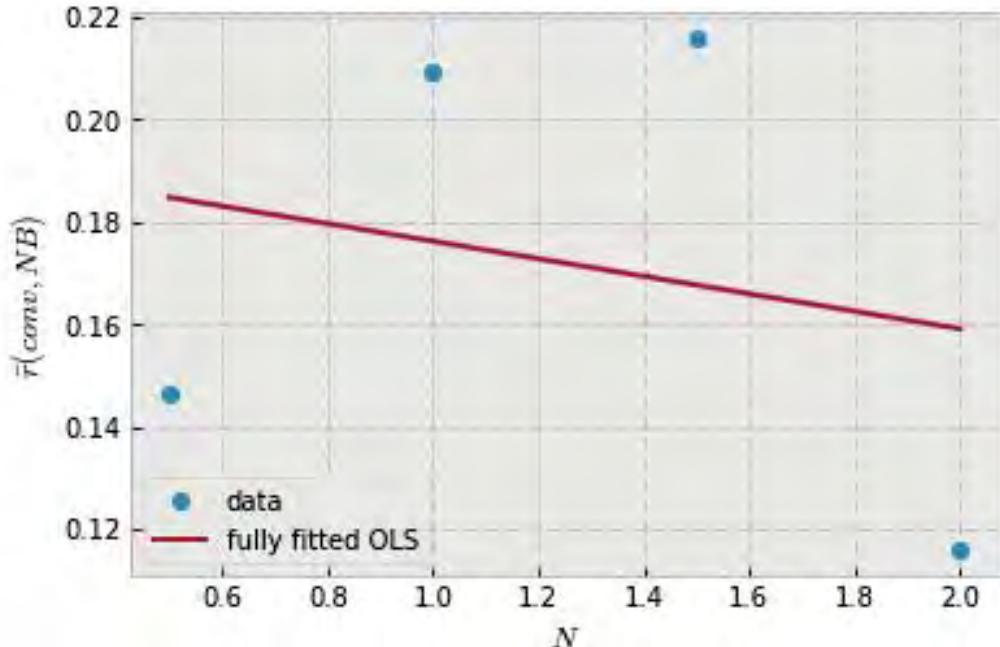
```

variable name: sigma
model: poly1
r-squared: -0.007184676190029604
explained variance: -0.00031197407053928927

```



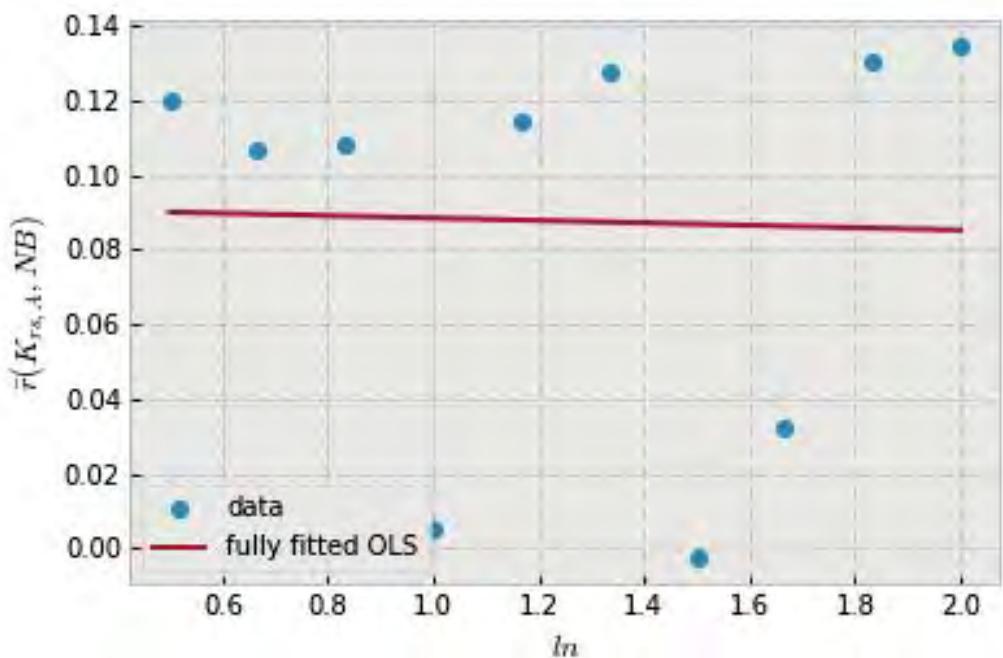
```
variable name: N
model: poly1
r-squared: -5.175951075600234
explained variance: -4.331258398801477
```



```
In [51]: y1_name, y2_name = ('half_mean_distance_between_roots', 'number_of_root_branches')

corr_reg('ln', y1_name, y2_name, 'poly1')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('nB', y1_name, y2_name, 'poly1')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly1')
corr_reg('N', y1_name, y2_name, 'poly1')

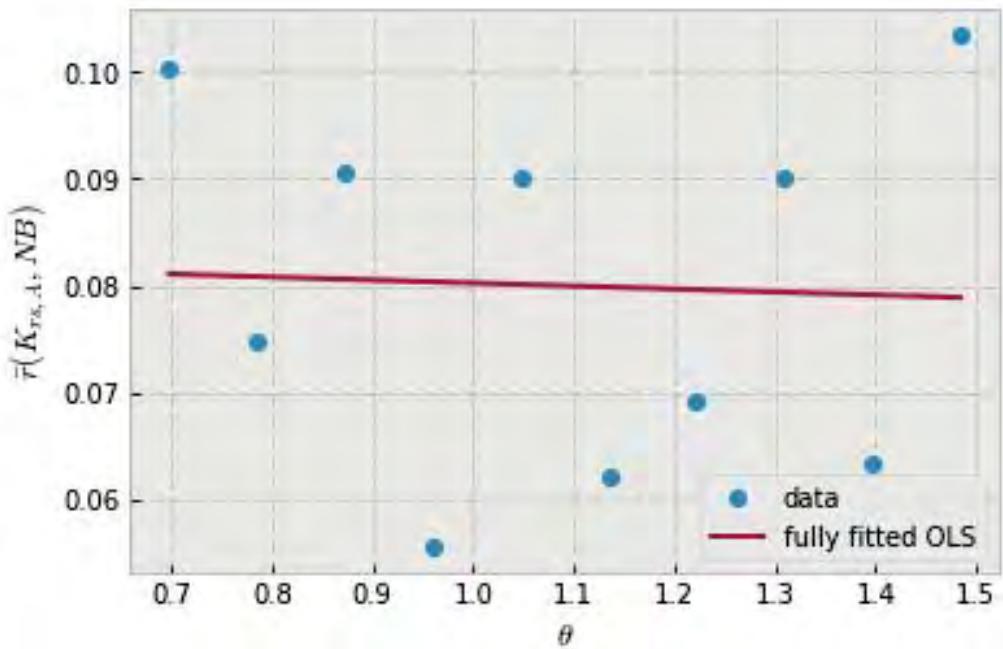
variable name: ln
model: poly1
r-squared: -0.49577111839232835
explained variance: -0.49044967718467336
```



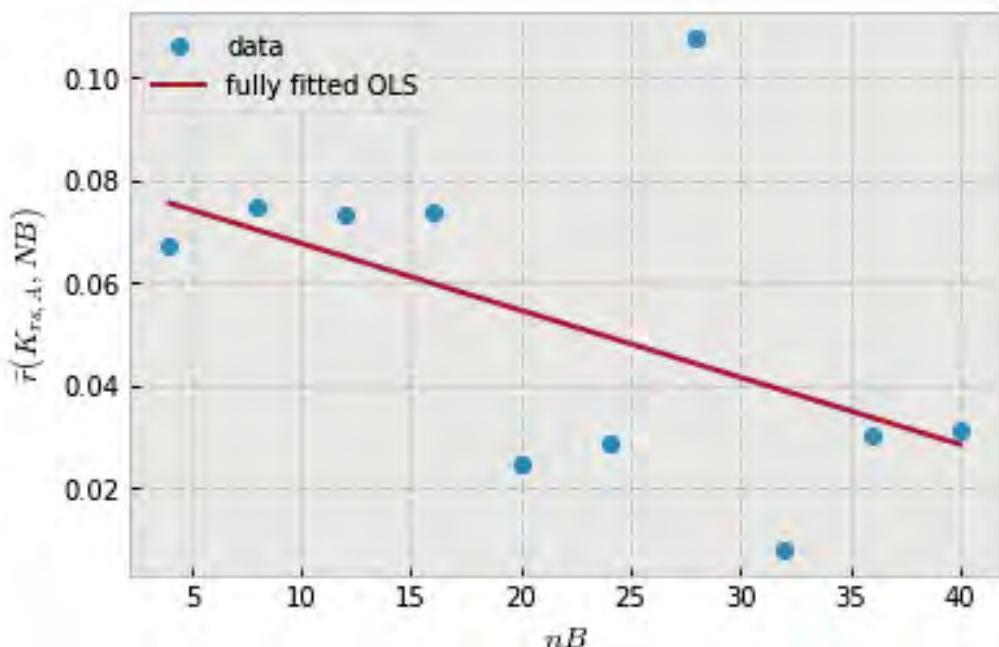
```

variable name: theta
model: poly1
r-squared: -0.742318535665935
explained variance: -0.734464735242847

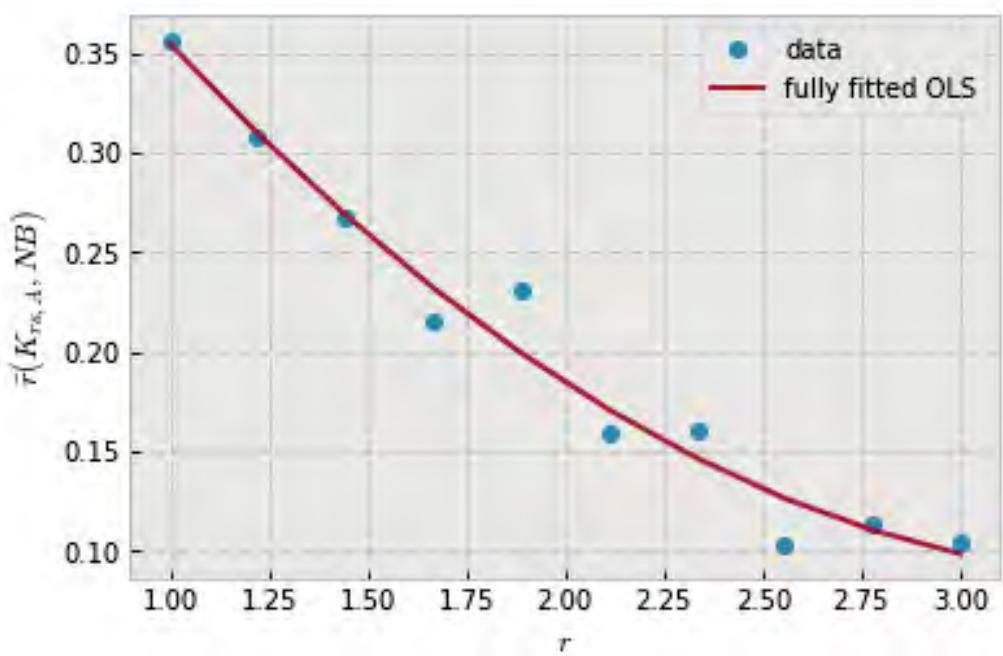
```



```
variable name: nB
model: poly1
r-squared: -0.0022674093433168174
explained variance: -0.002231391051582099
```



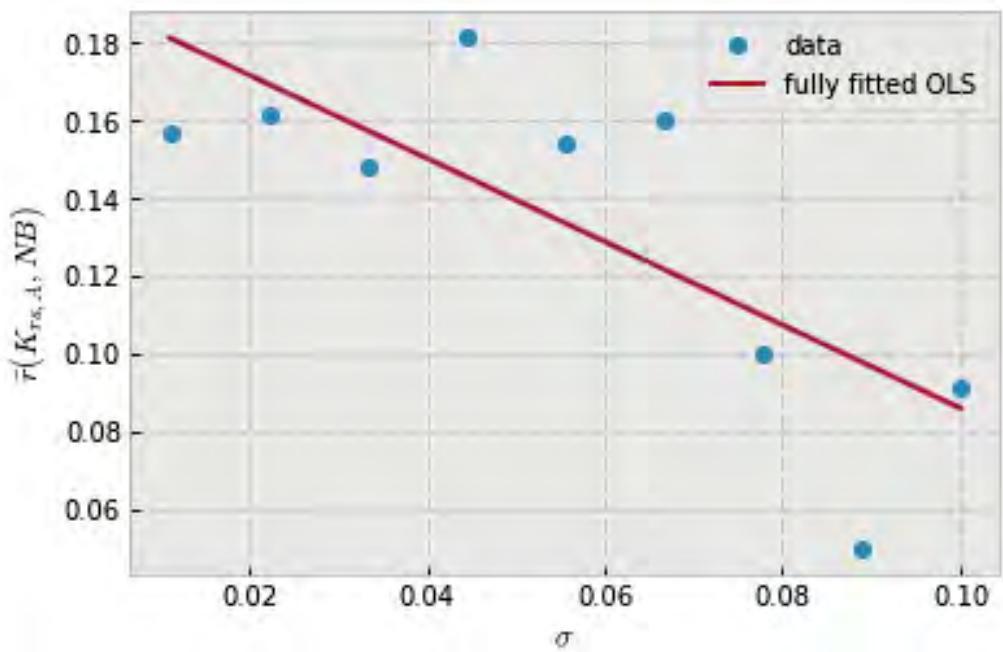
```
variable name: r
model: poly2
r-squared: 0.9484018825139208
explained variance: 0.948580742492806
```



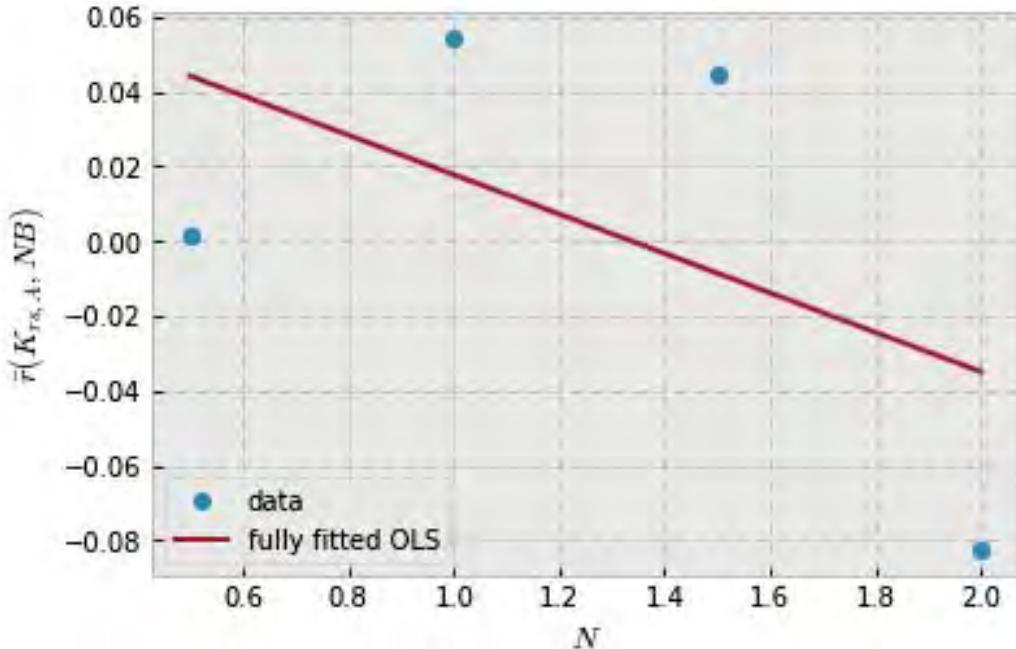
```

variable name: sigma
model: poly1
r-squared: 0.29609087300004244
explained variance: 0.29978998571676185

```



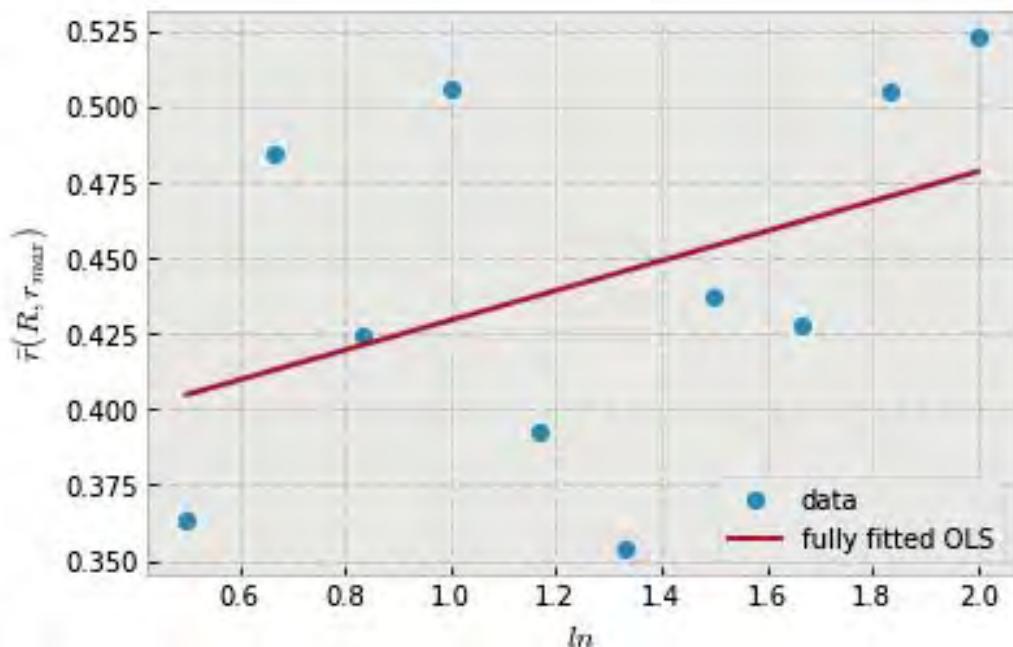
```
variable name: N
model: poly1
r-squared: -3.5865462605351572
explained variance: -2.9593070928546665
```



```
In [52]: y1_name, y2_name = ('mean_depth_of_standard_root_water_uptake', 'maximal_horizontal_spread')

corr_reg('ln', y1_name, y2_name, 'poly1')
corr_reg('theta', y1_name, y2_name, 'poly1')
corr_reg('nB', y1_name, y2_name, 'poly2')
corr_reg('r', y1_name, y2_name, 'poly2')
corr_reg('sigma', y1_name, y2_name, 'poly2')
corr_reg('N', y1_name, y2_name, 'poly1')

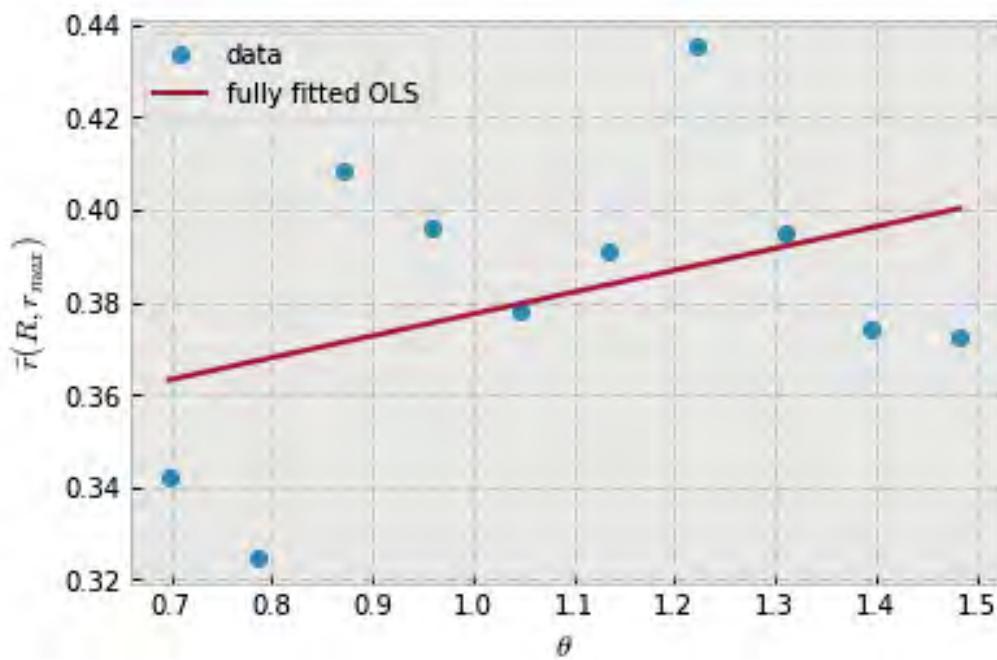
variable name: ln
model: poly1
r-squared: -0.2787434755787208
explained variance: -0.2771542385817307
```



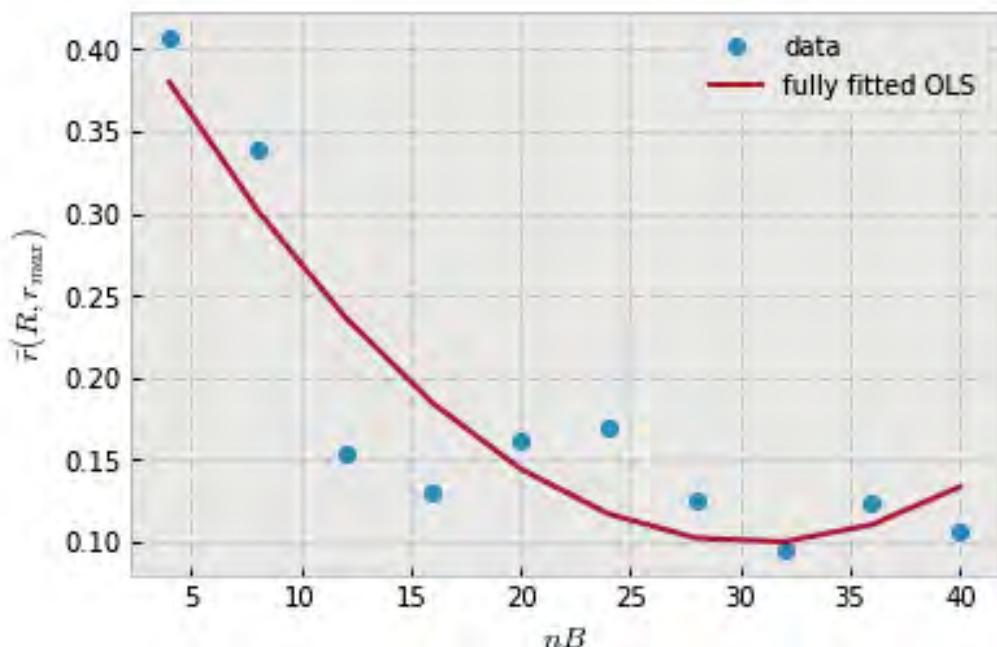
```

variable name: theta
model: poly1
r-squared: -0.3883794734172419
explained variance: -0.37961552908812335

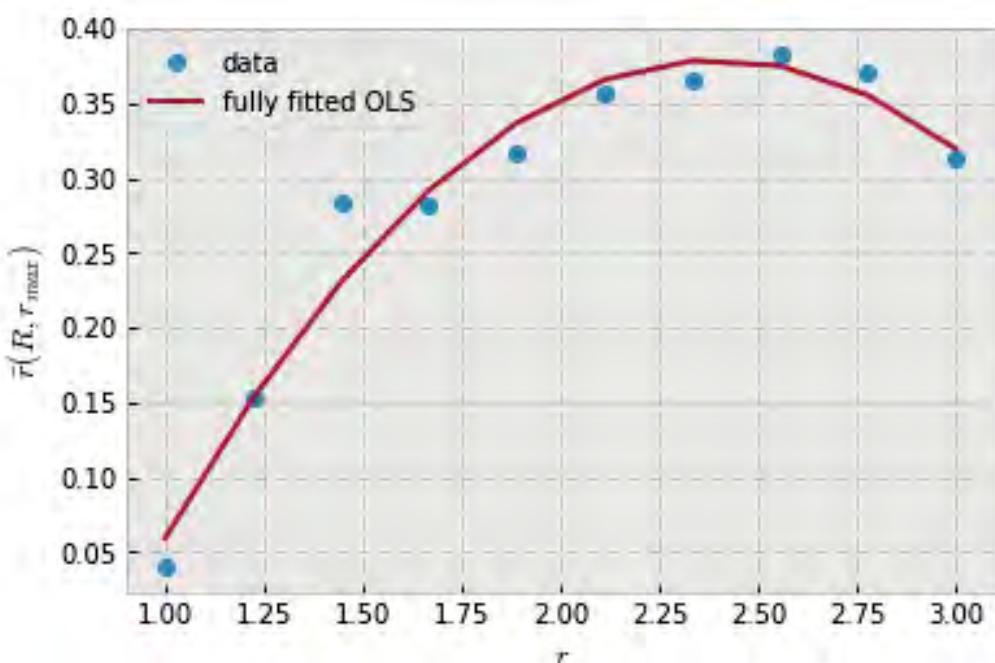
```



```
variable name: nB
model: poly2
r-squared: 0.6555169906205094
explained variance: 0.6556523079204208
```



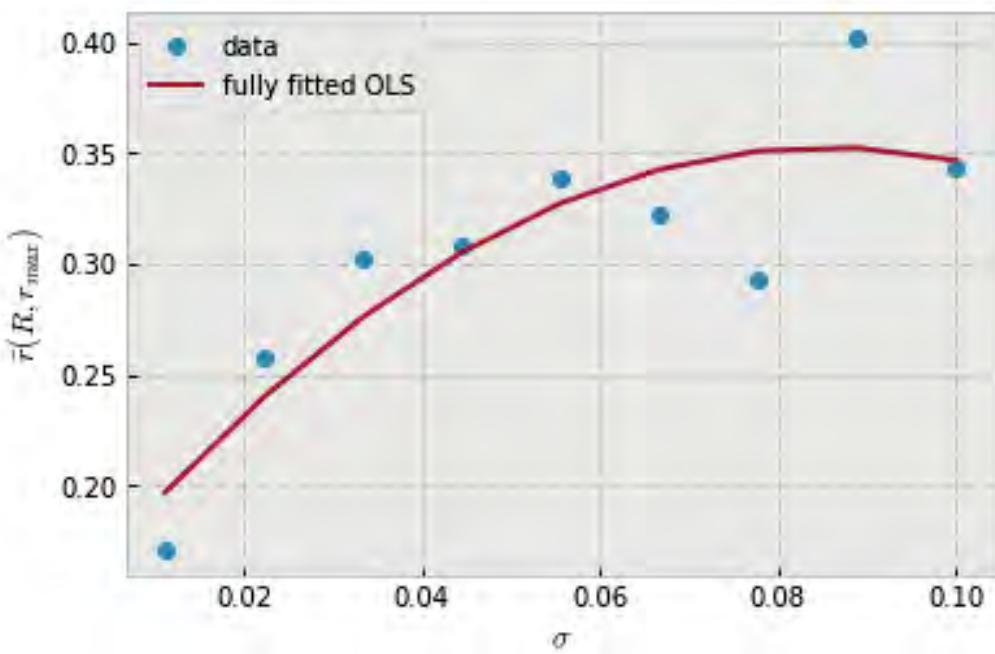
```
variable name: r
model: poly2
r-squared: 0.915747792200097
explained variance: 0.9169572442983475
```



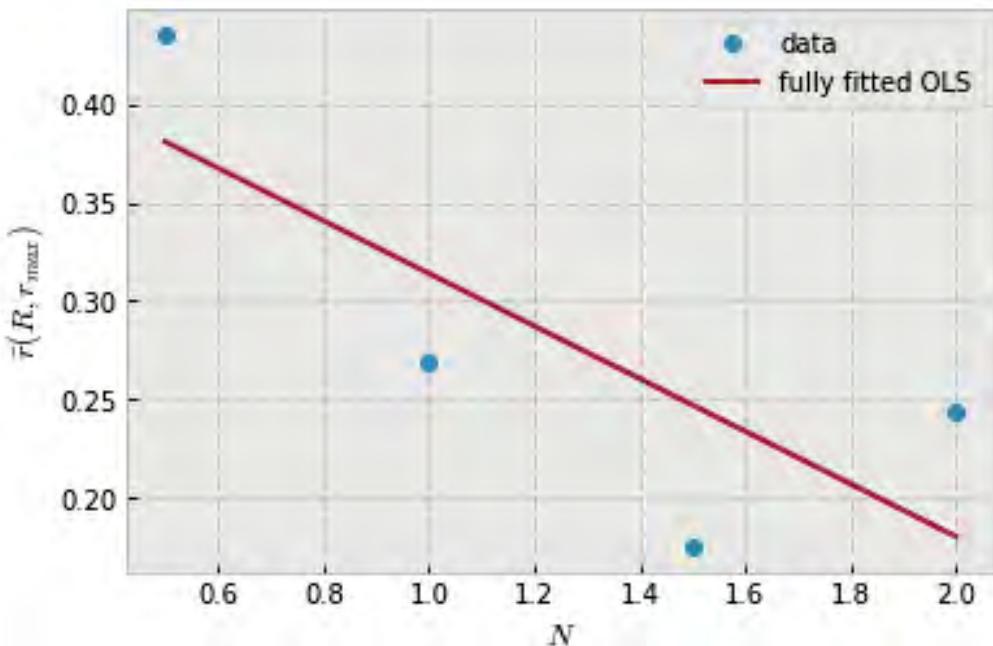
```

variable name: sigma
model: poly2
r-squared: 0.42717222381070297
explained variance: 0.43268569569560955

```



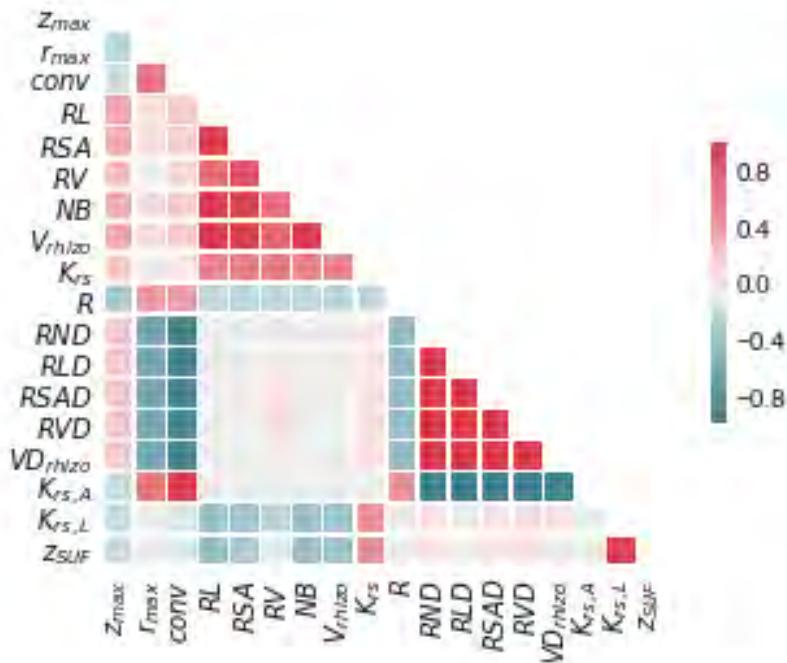
```
variable name: N
model: poly1
r-squared: -1.5361553779928023
explained variance: -1.190748388453935
```



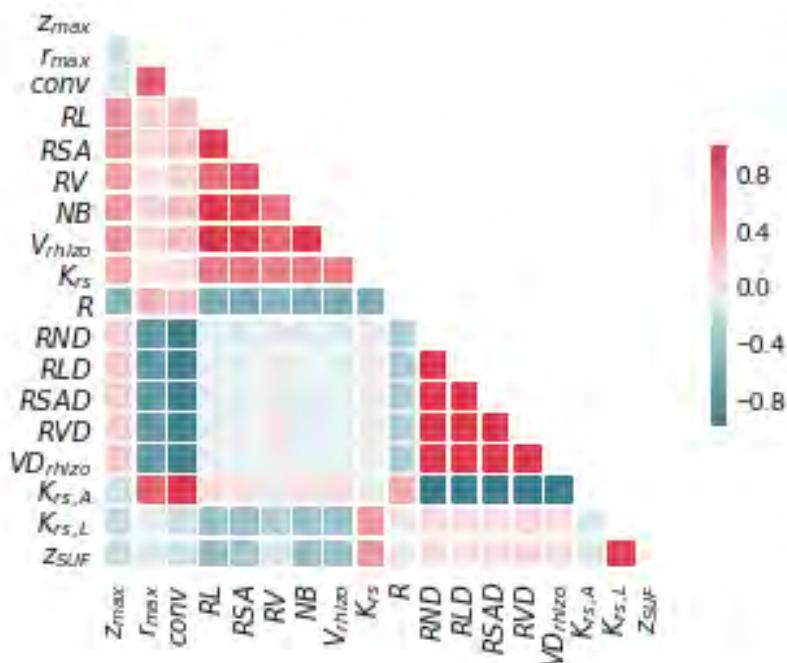
20.2 Correlation matrices

```
In [53]: plot_corr_matrix()
```

```
variable name: N
variable value: 0.5
```



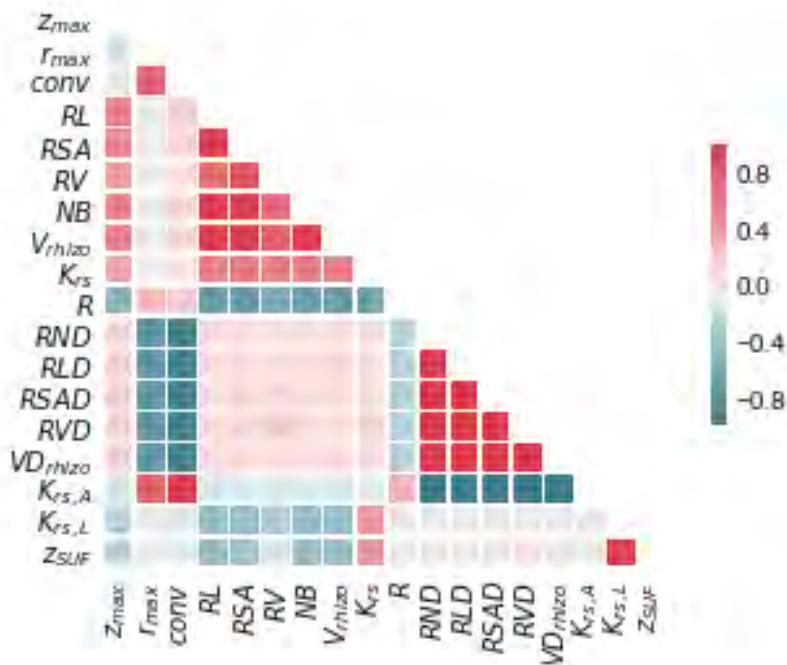
variable name: N
 variable value: 1.0



variable name: N
variable value: 1.5



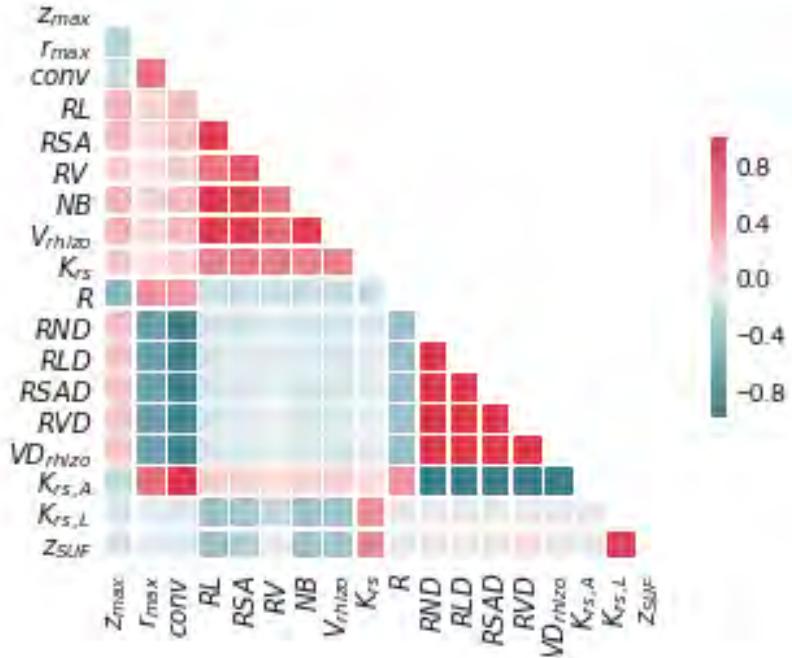
variable name: N
variable value: 2.0



variable name: ln
 variable value: 0.5



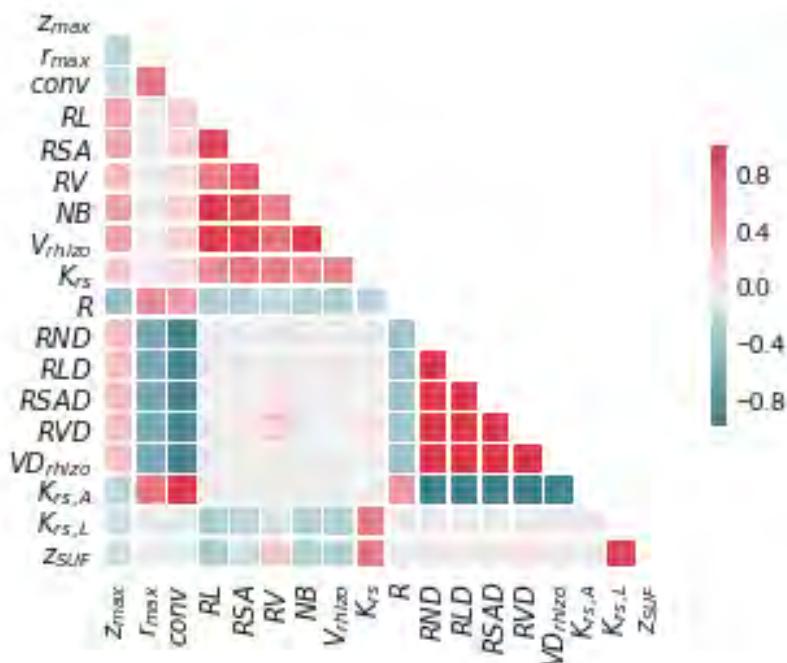
variable name: ln
variable value: 0.6666666666666666



variable name: ln
variable value: 0.8333333333333333



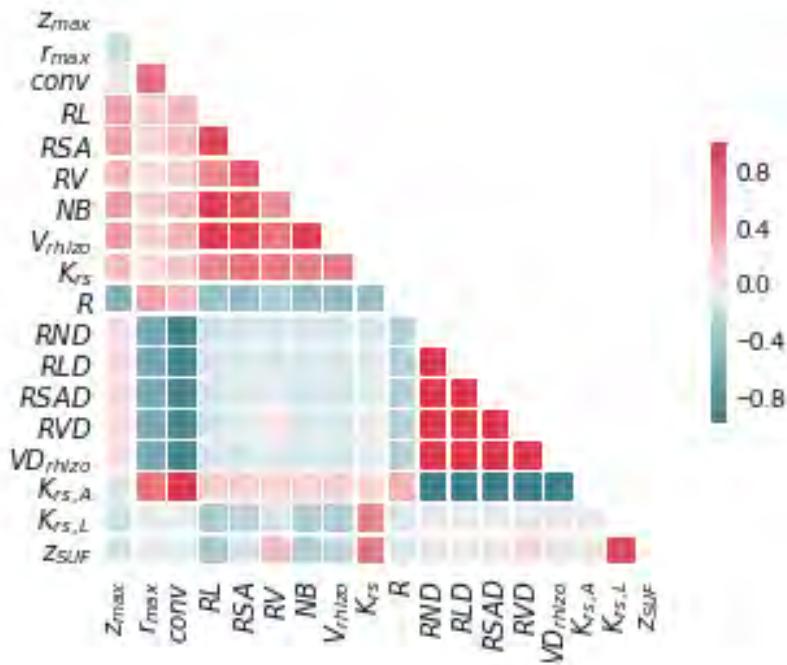
variable name: ln
 variable value: 1.0



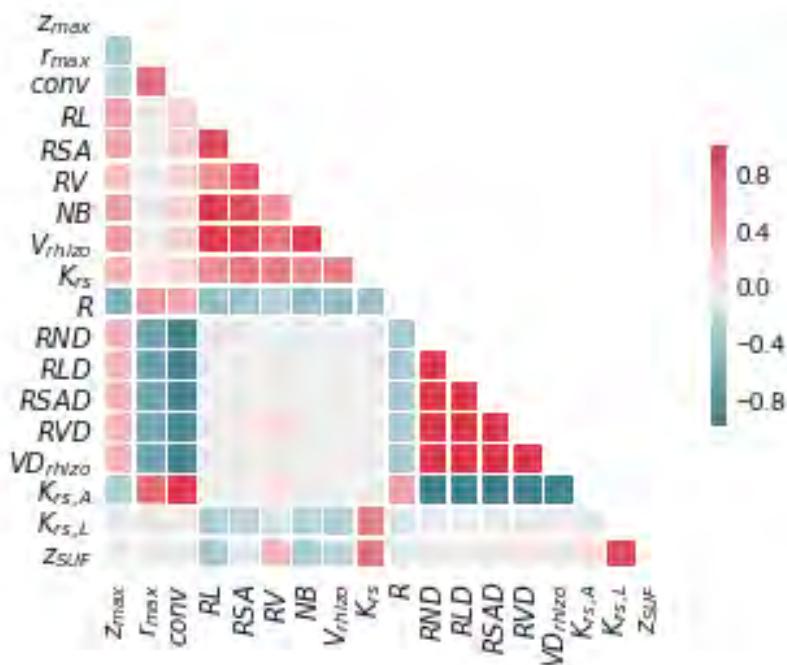
variable name: ln
variable value: 1.1666666666666665



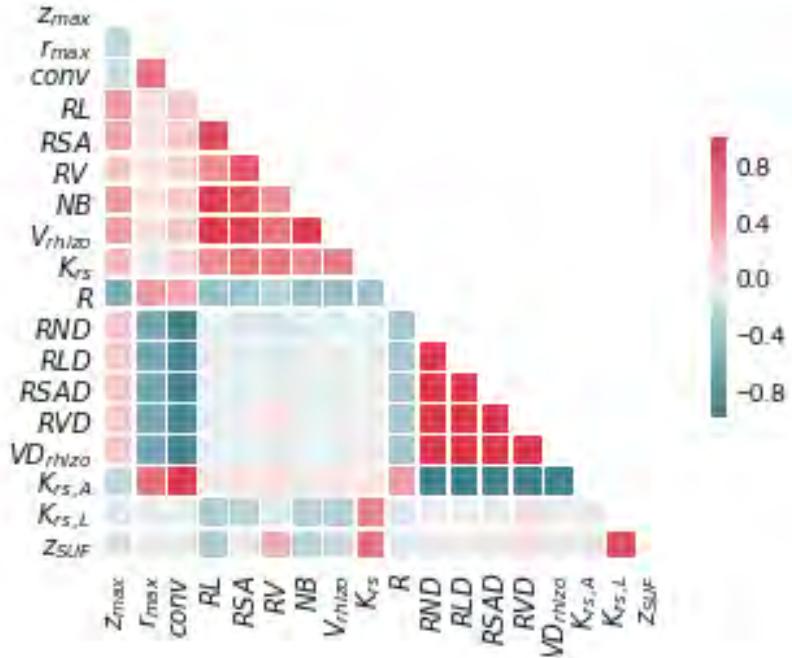
variable name: ln
variable value: 1.3333333333333333



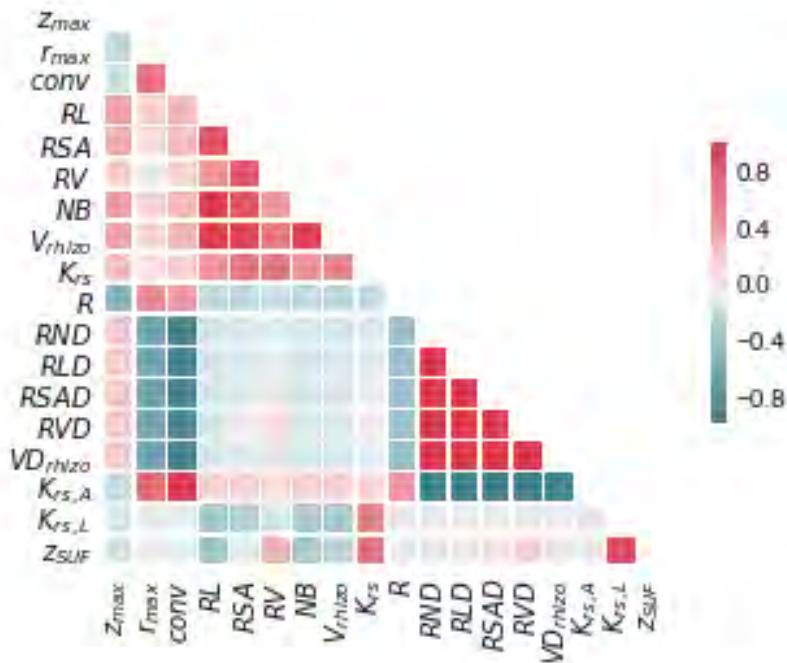
variable name: ln
 variable value: 1.5



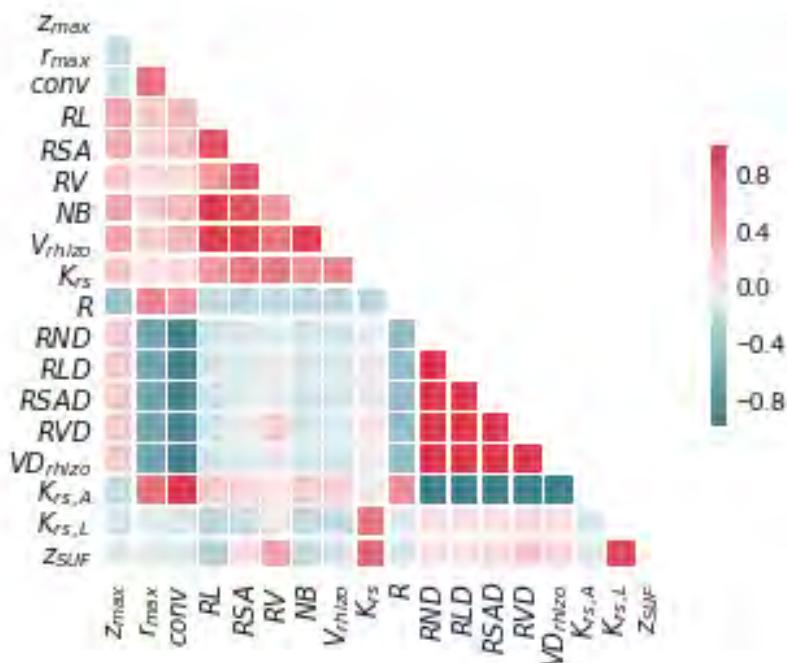
variable name: ln
variable value: 1.6666666666666665



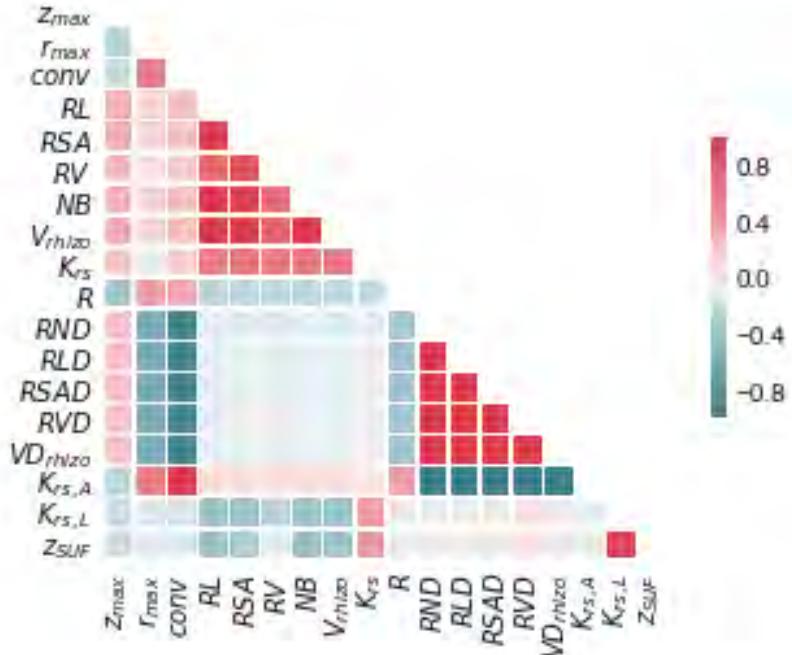
variable name: ln
variable value: 1.833333333333333



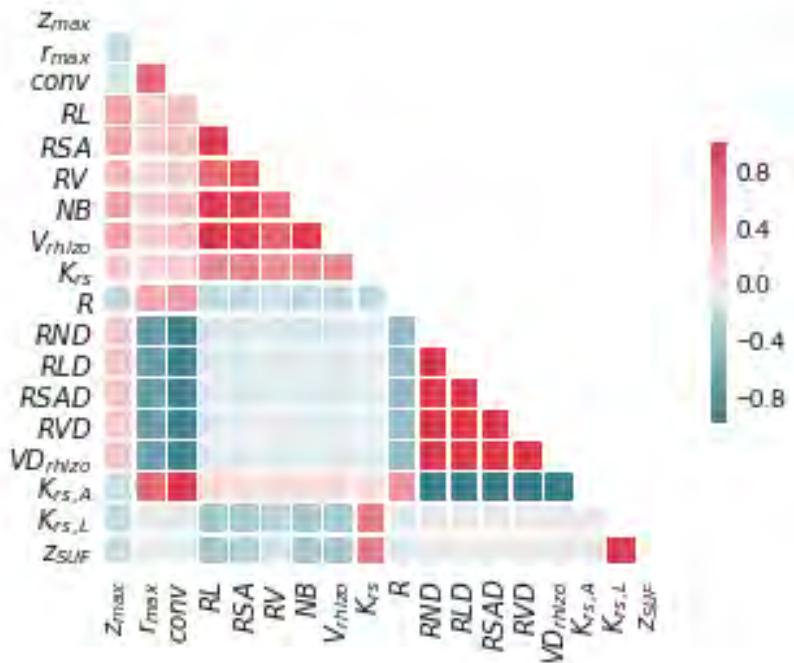
variable name: ln
 variable value: 2.0



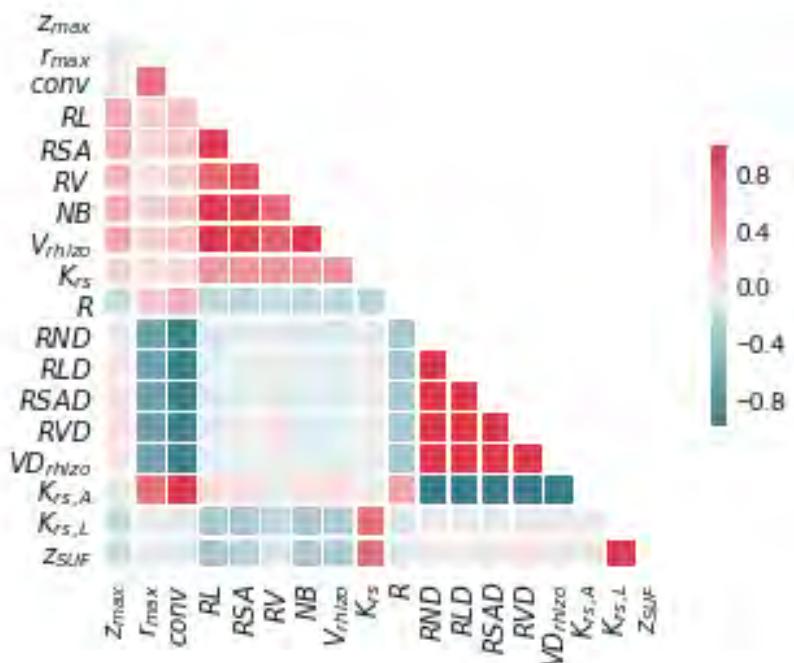
variable name: nB
variable value: 4.0



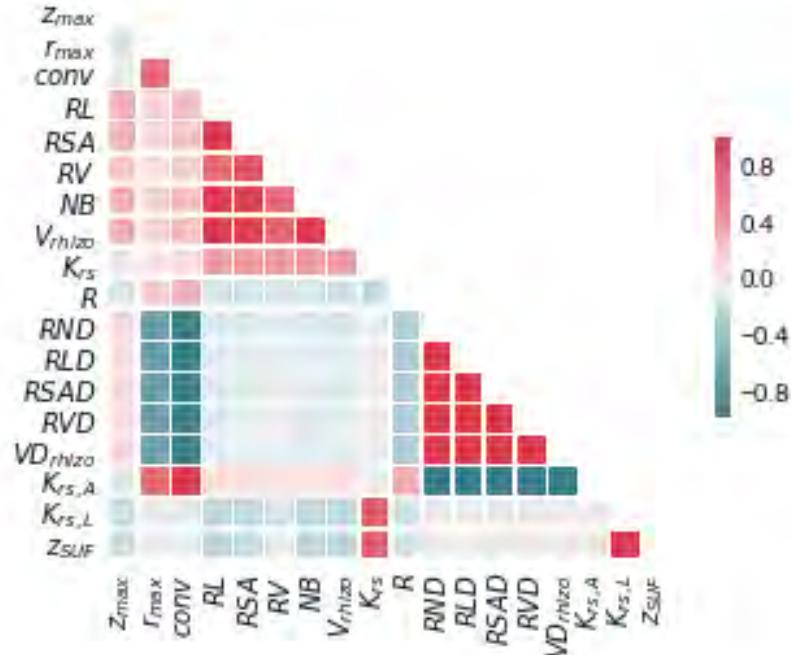
variable name: nB
variable value: 8.0



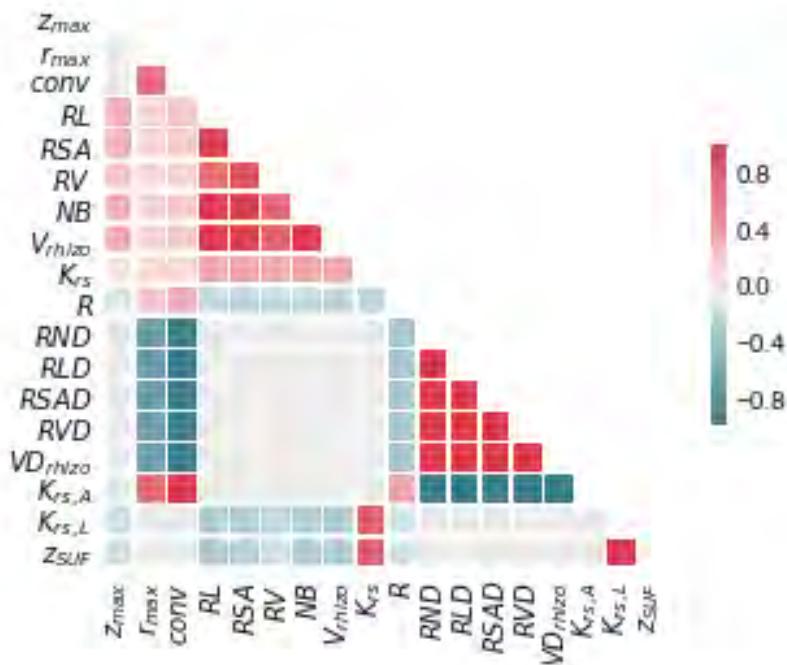
variable name: nB
 variable value: 12.0



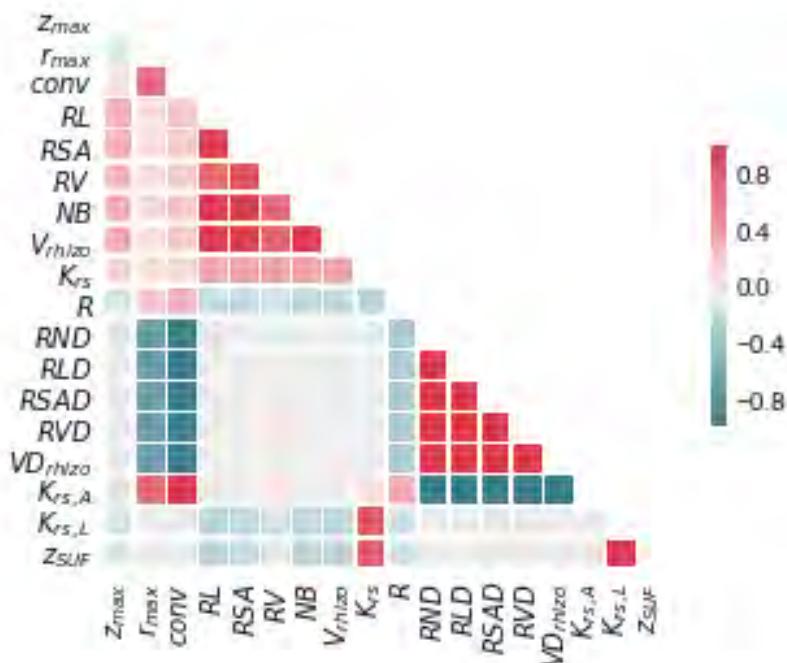
variable name: nB
variable value: 16.0



variable name: nB
variable value: 20.0



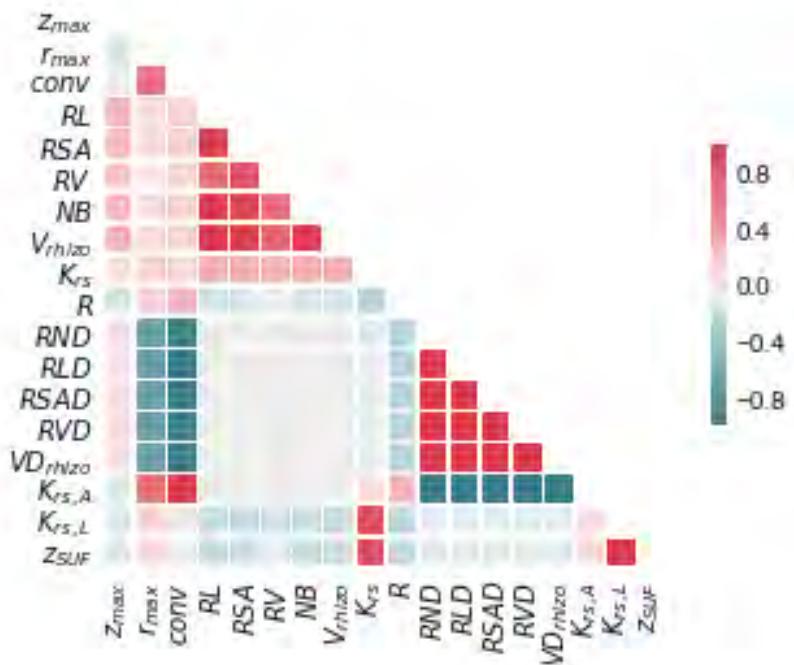
variable name: nB
 variable value: 24.0



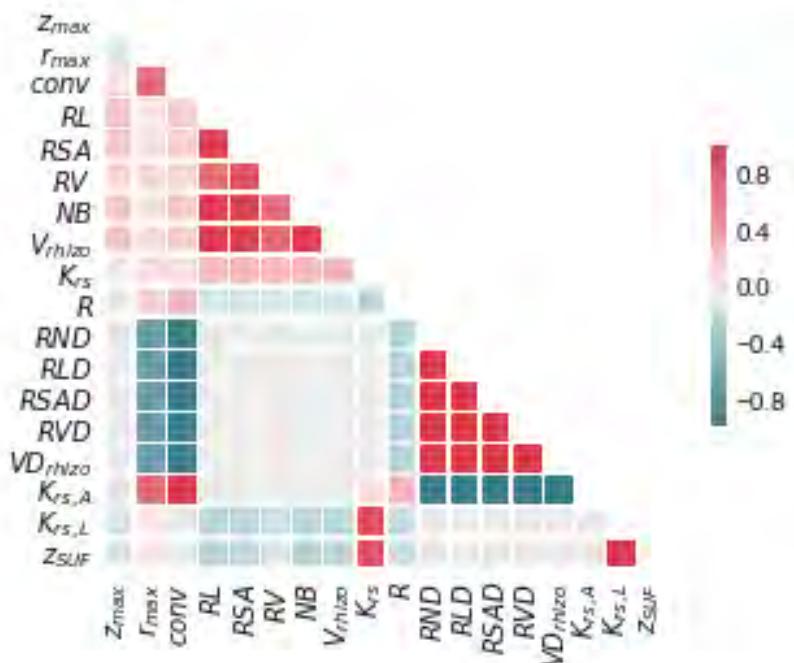
variable name: nB
variable value: 28.0



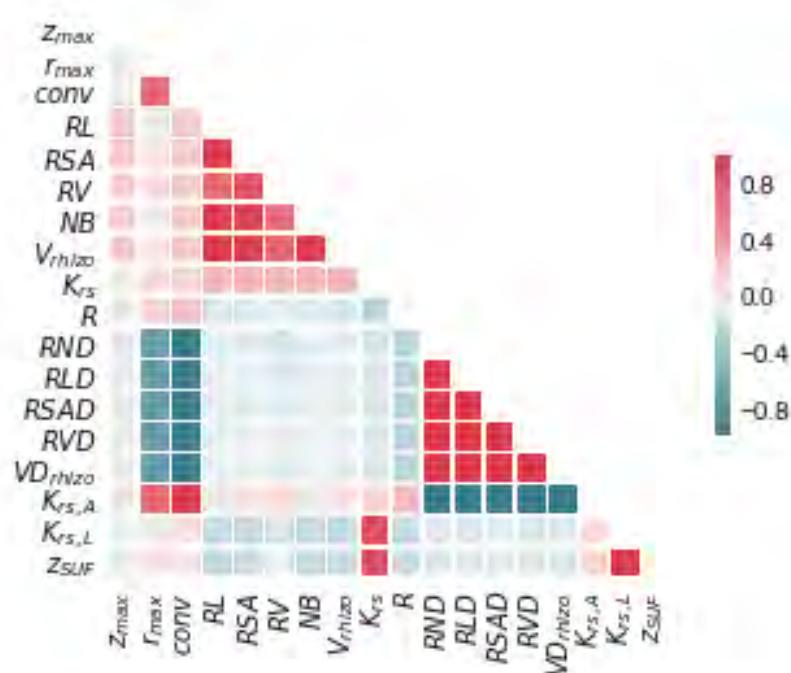
variable name: nB
variable value: 32.0



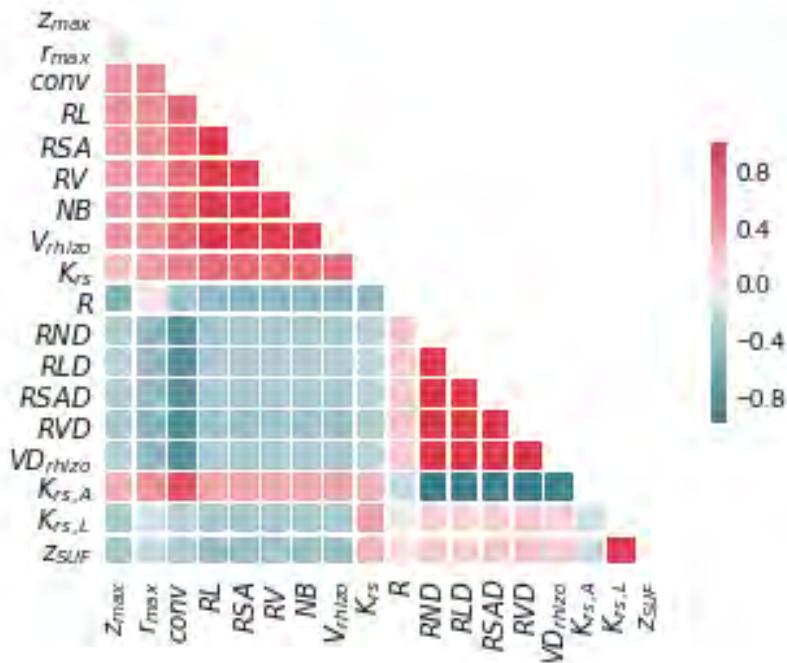
variable name: nB
 variable value: 36.0



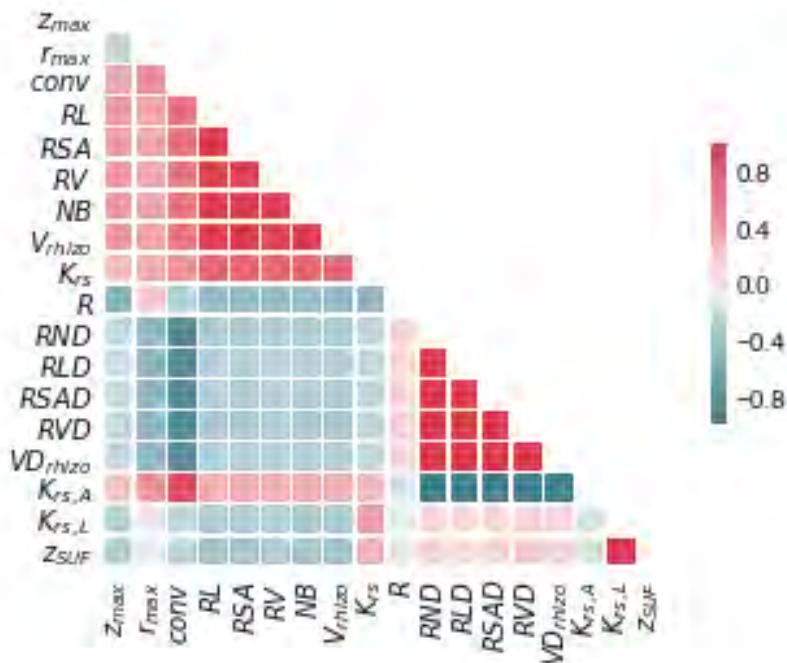
variable name: nB
variable value: 40.0



variable name: r
variable value: 1.0



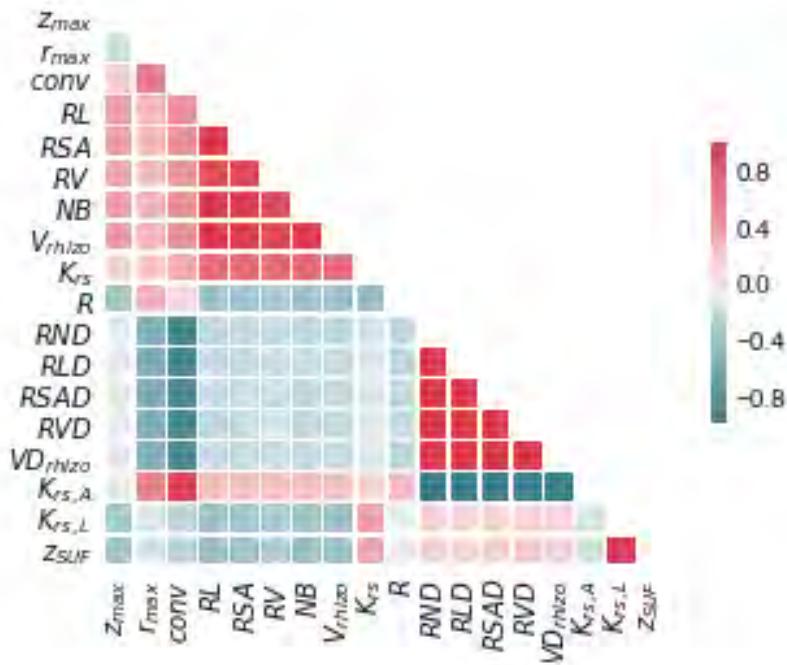
variable name: r
 variable value: 1.222222222222223



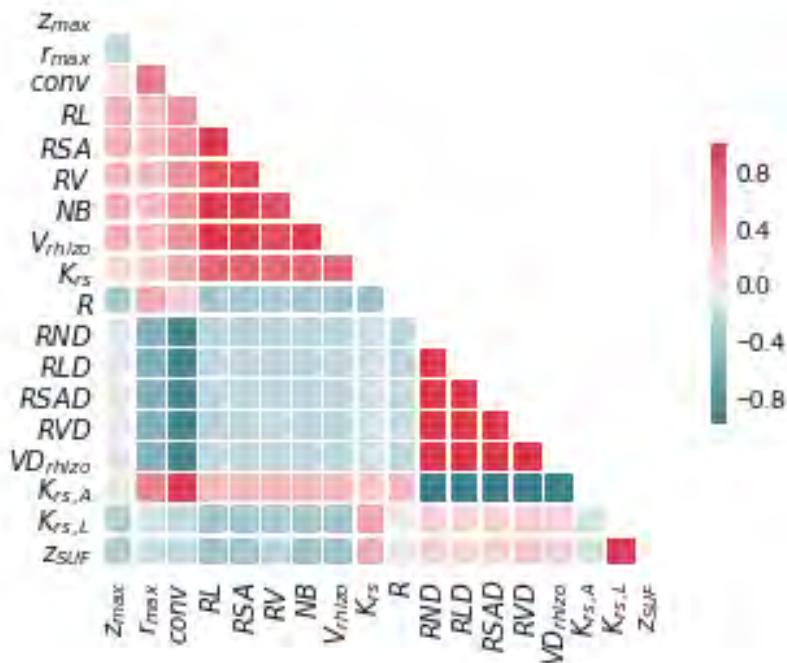
variable name: r
variable value: 1.4444444444444444



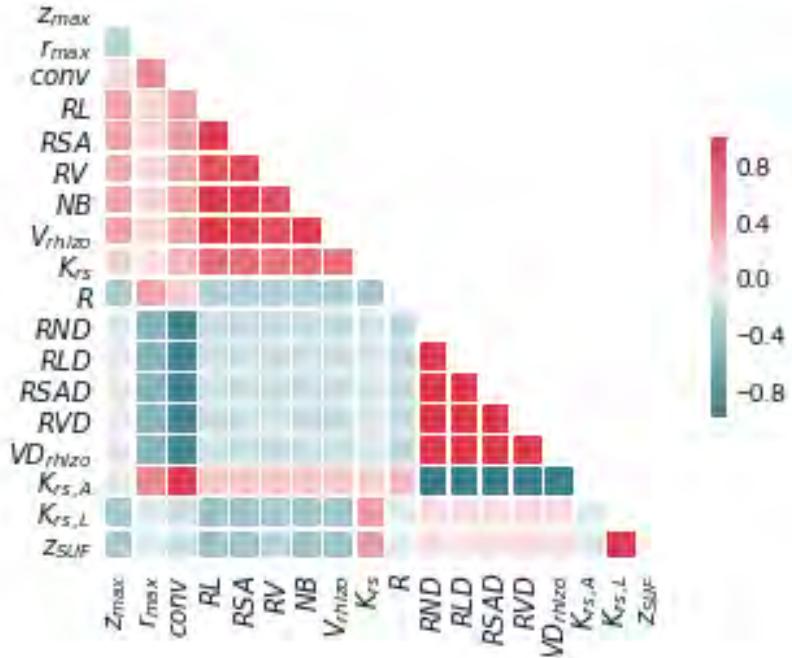
variable name: r
variable value: 1.6666666666666665



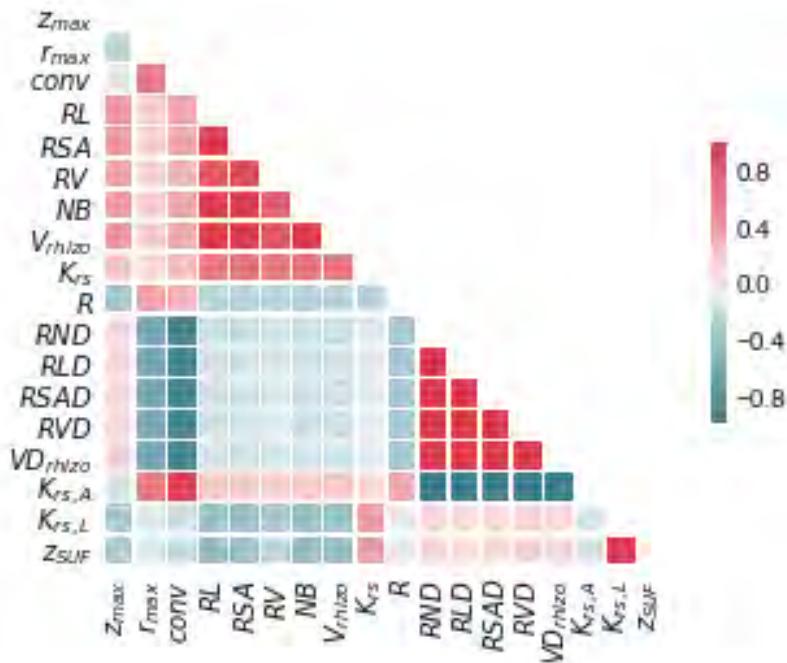
variable name: r
 variable value: 1.8888888888888888



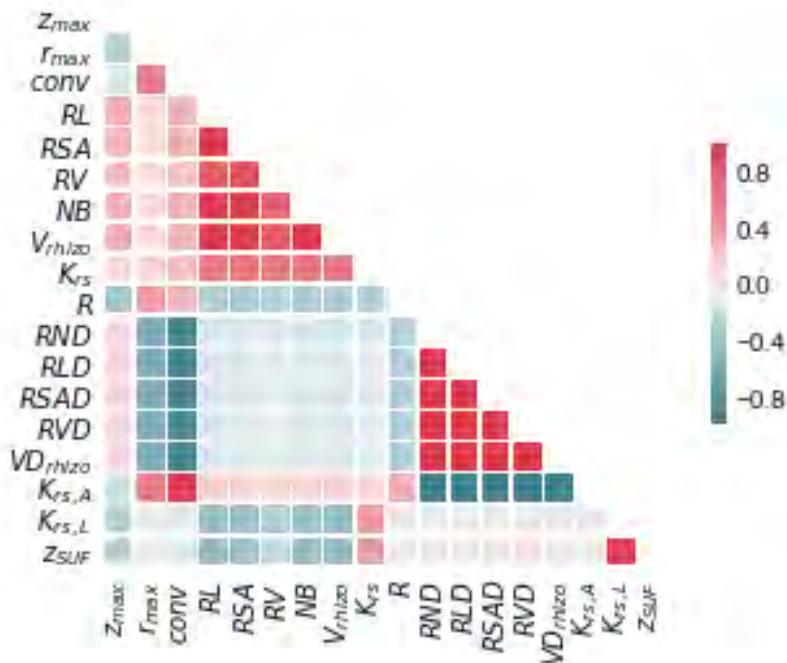
variable name: r
variable value: 2.11111111111111



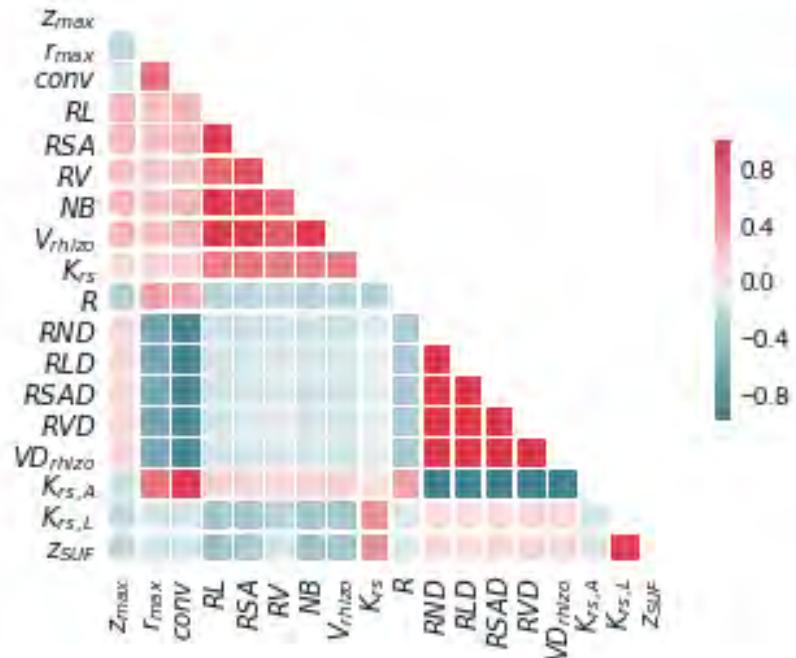
variable name: r
variable value: 2.33333333333333



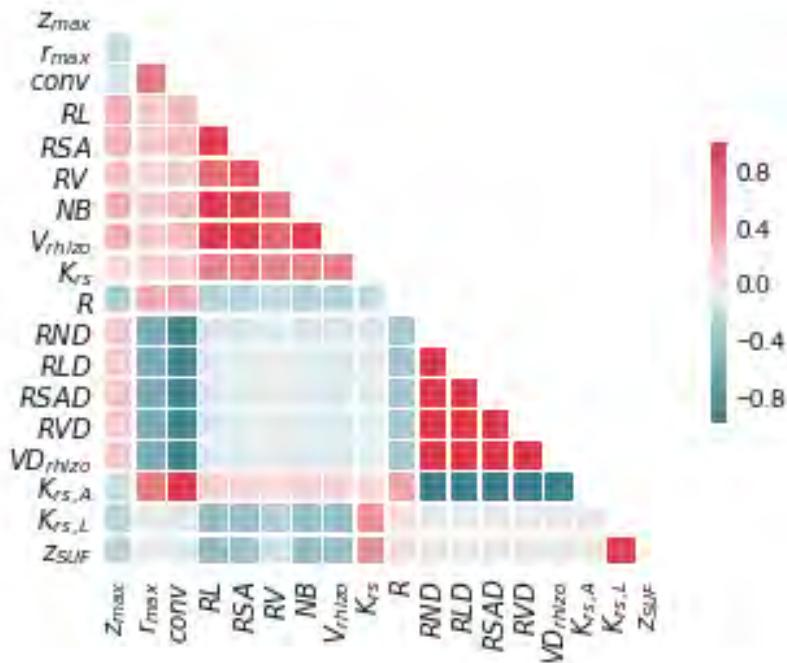
variable name: r
 variable value: 2.5555555555555554



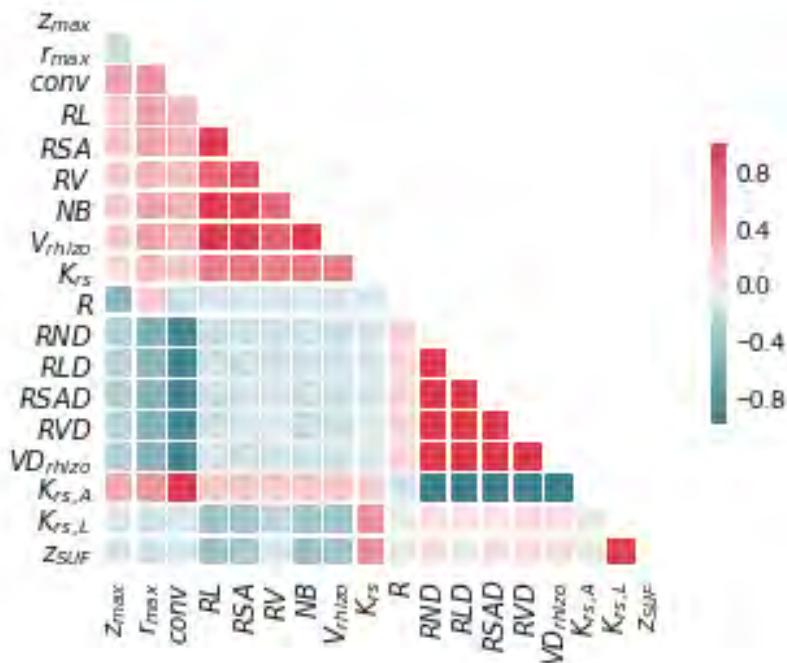
variable name: r
variable value: 2.7777777777777777



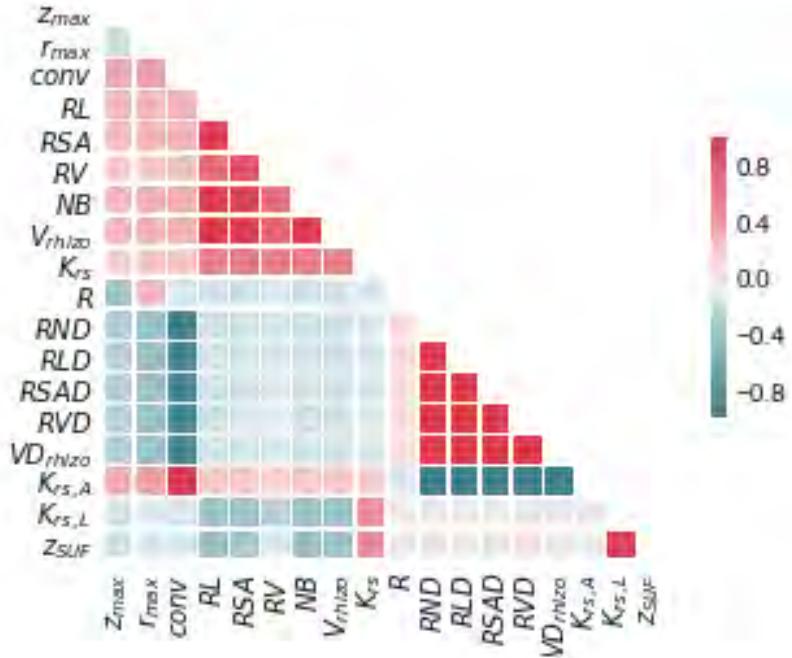
variable name: r
variable value: 3.0



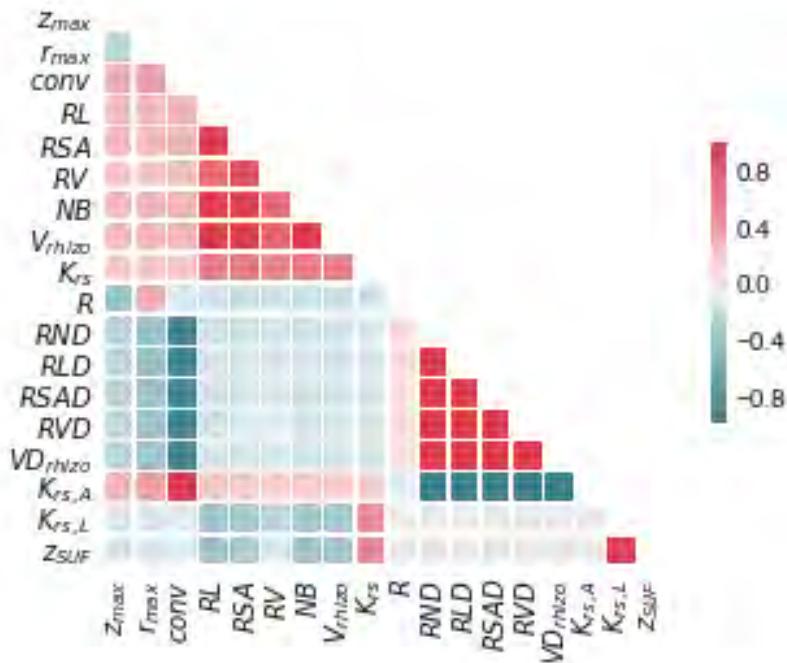
variable name: sigma
 variable value: 0.01111111111111112



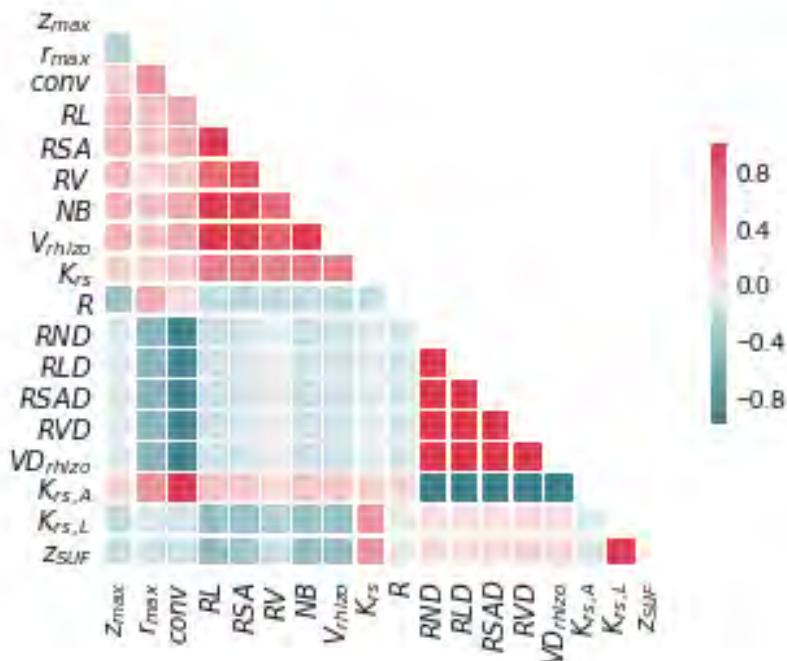
variable name: sigma
variable value: 0.02222222222222223



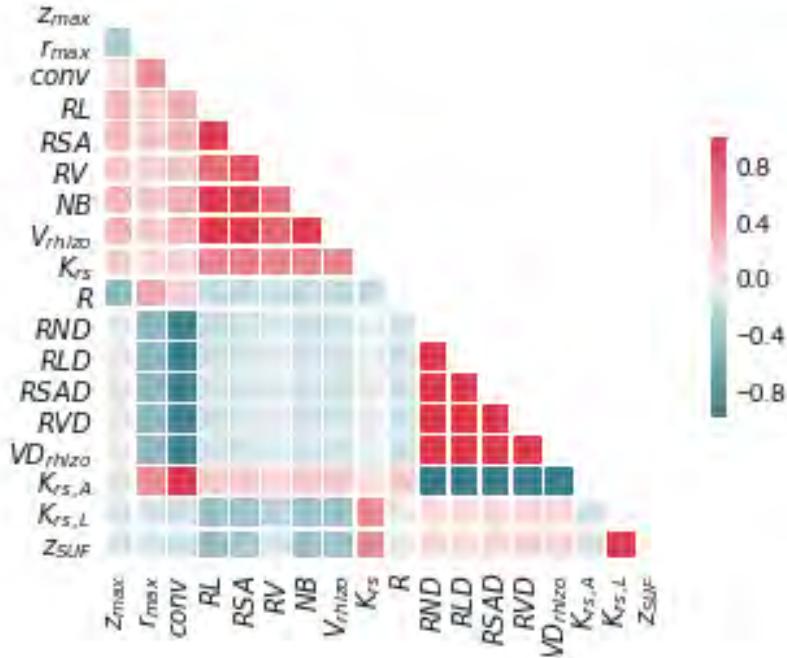
variable name: sigma
variable value: 0.03333333333333333



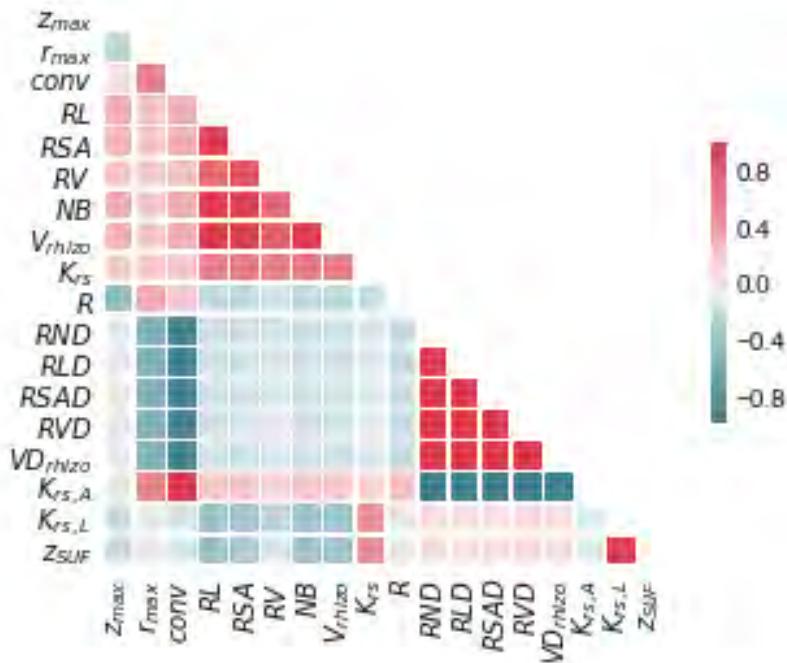
variable name: sigma
 variable value: 0.04444444444444446



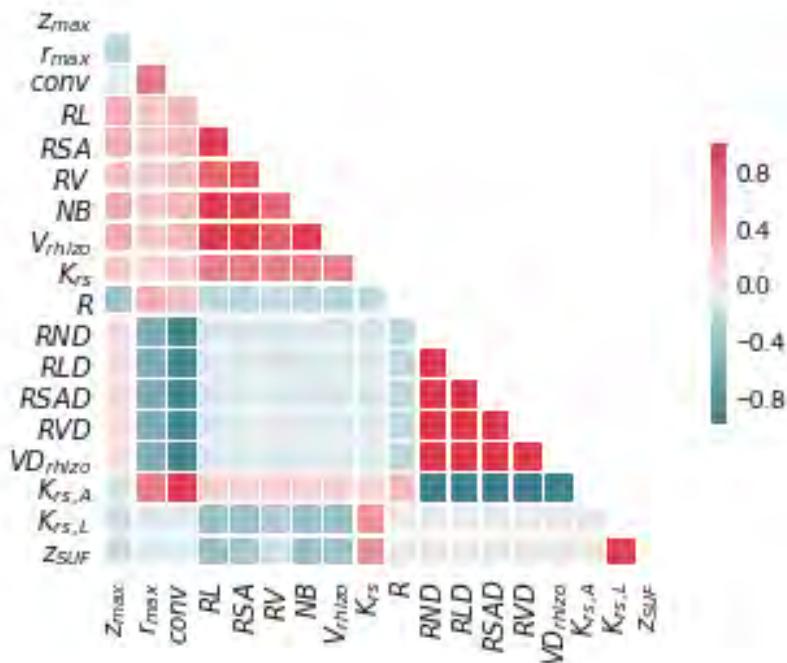
```
variable name: sigma  
variable value: 0.05555555555555555
```



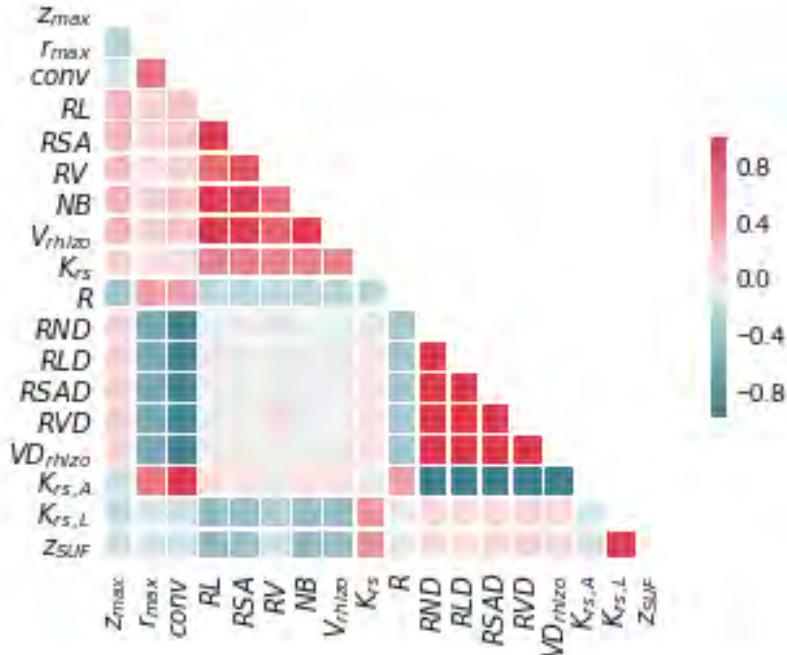
```
variable name: sigma  
variable value: 0.06666666666666667
```



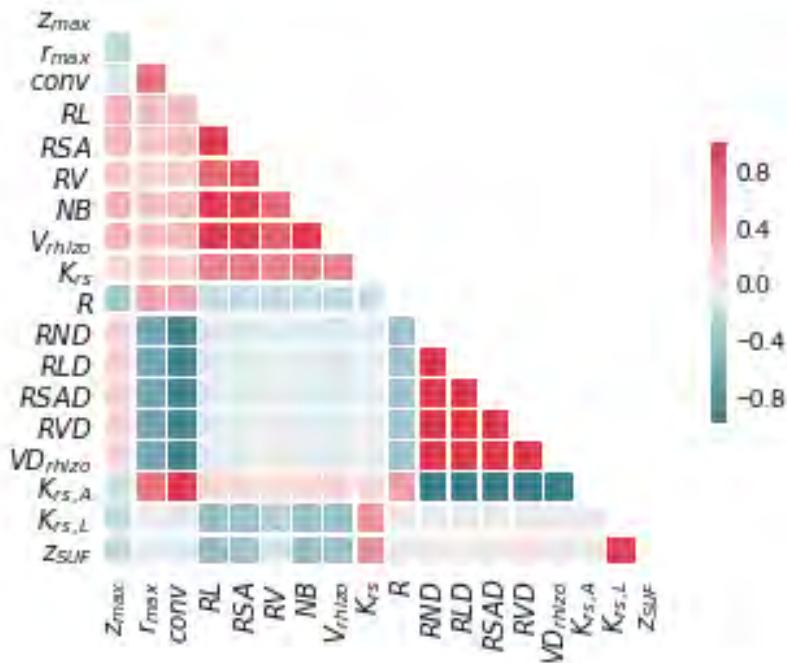
variable name: sigma
 variable value: 0.07777777777777778



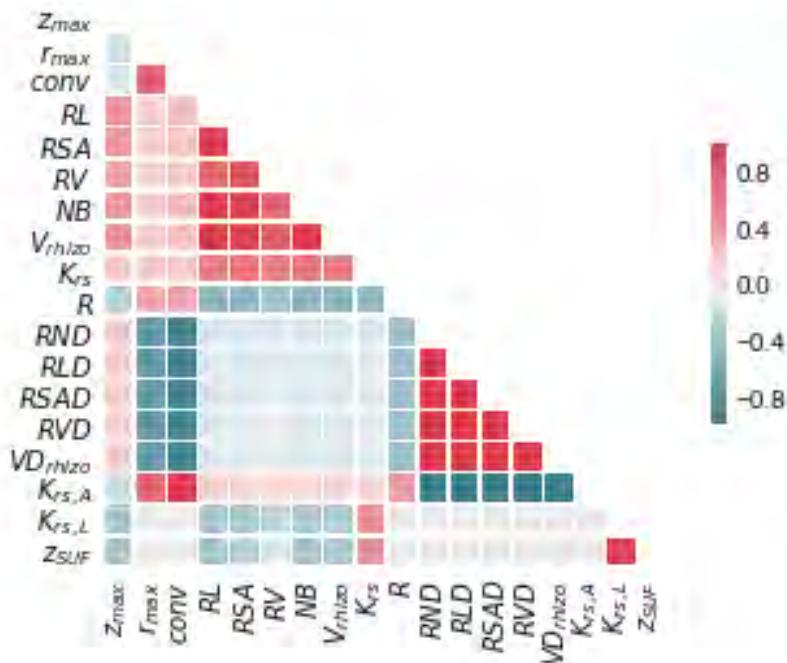
variable name: sigma
variable value: 0.08888888888888889



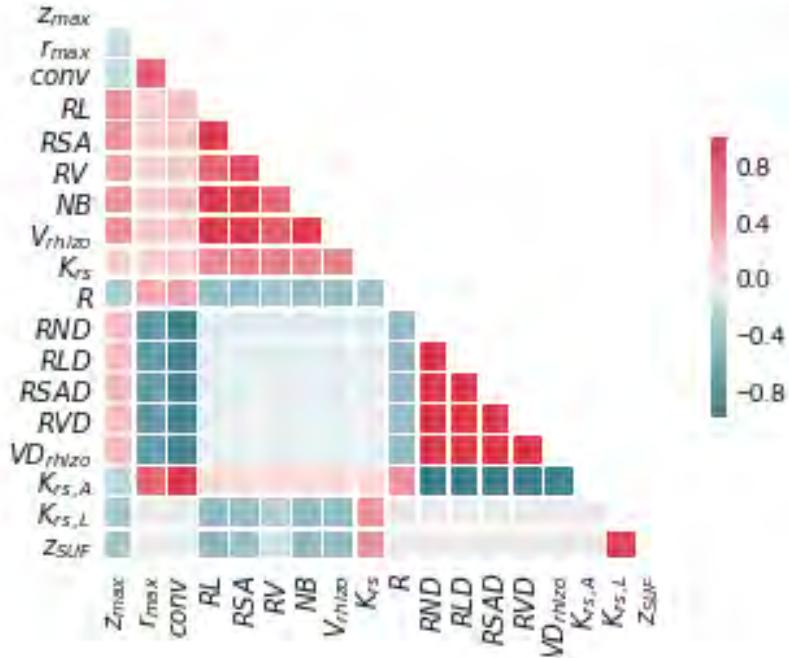
variable name: sigma
variable value: 0.1



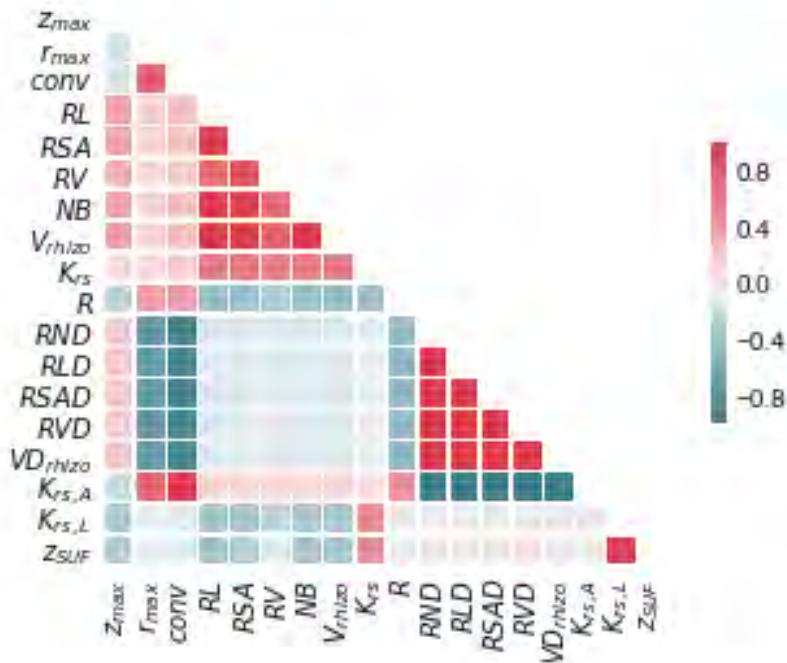
variable name: theta
 variable value: 0.6981317007977318



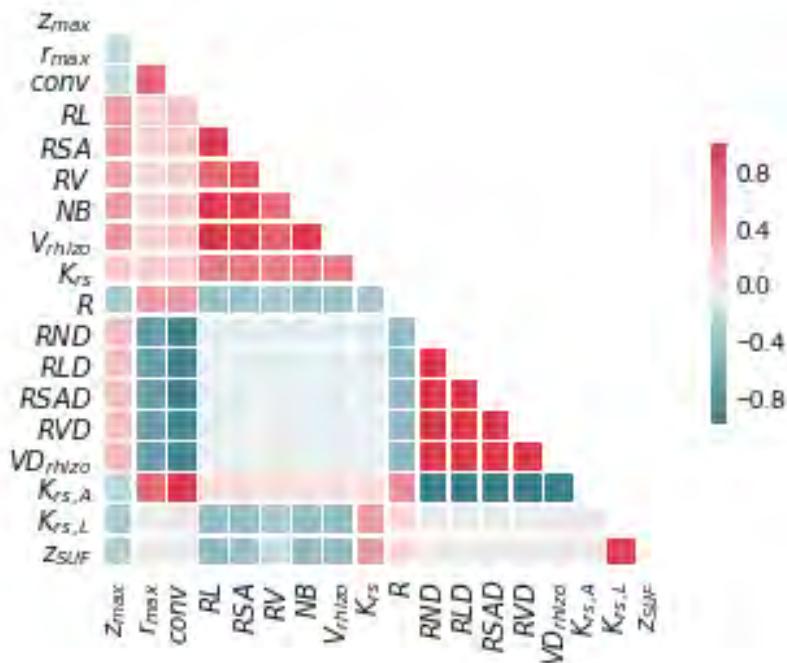
```
variable name: theta  
variable value: 0.7853981633974483
```



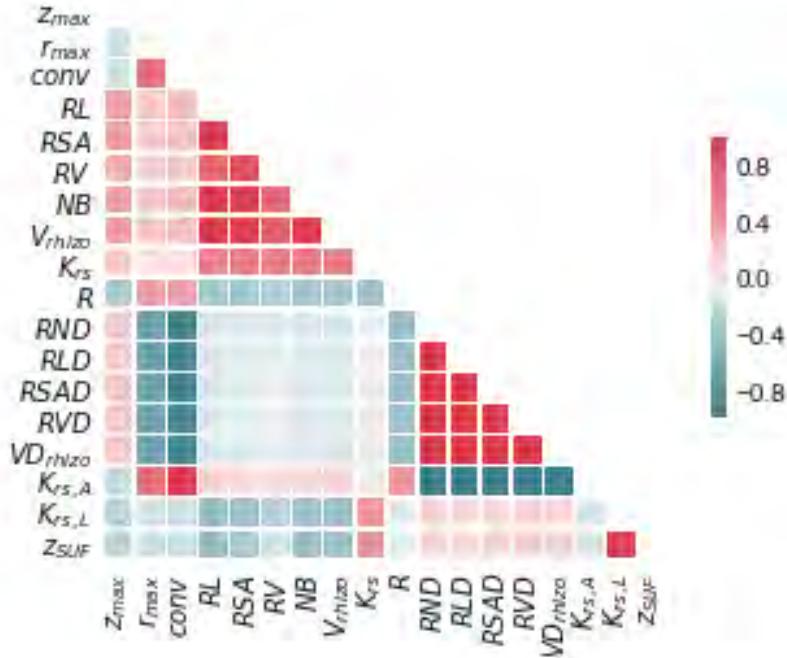
```
variable name: theta  
variable value: 0.8726646259971648
```



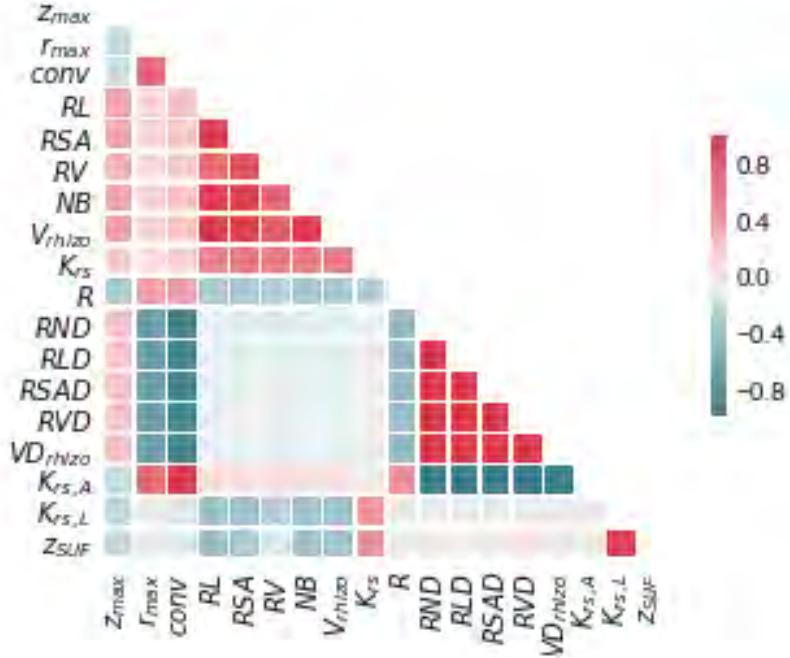
variable name: theta
 variable value: 0.9599310885968813



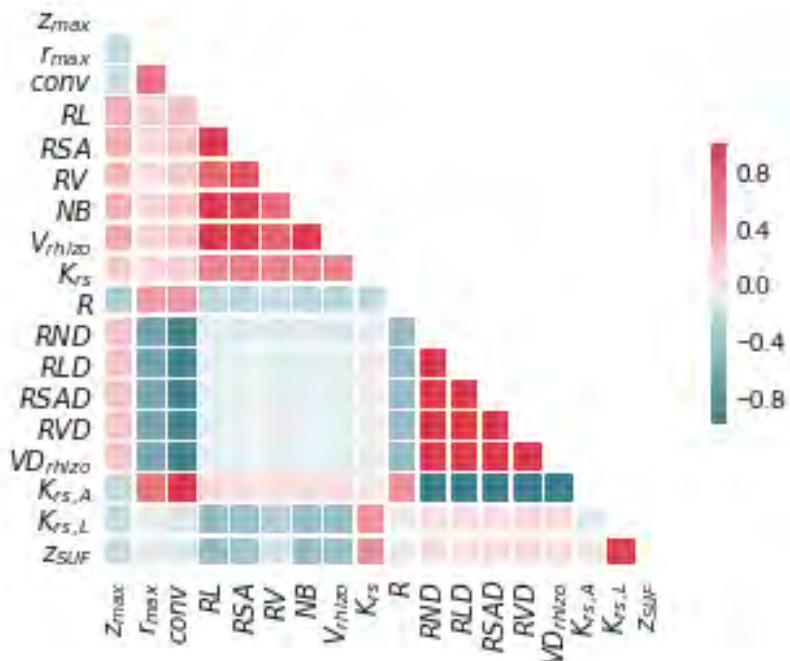
```
variable name: theta  
variable value: 1.0471975511965979
```



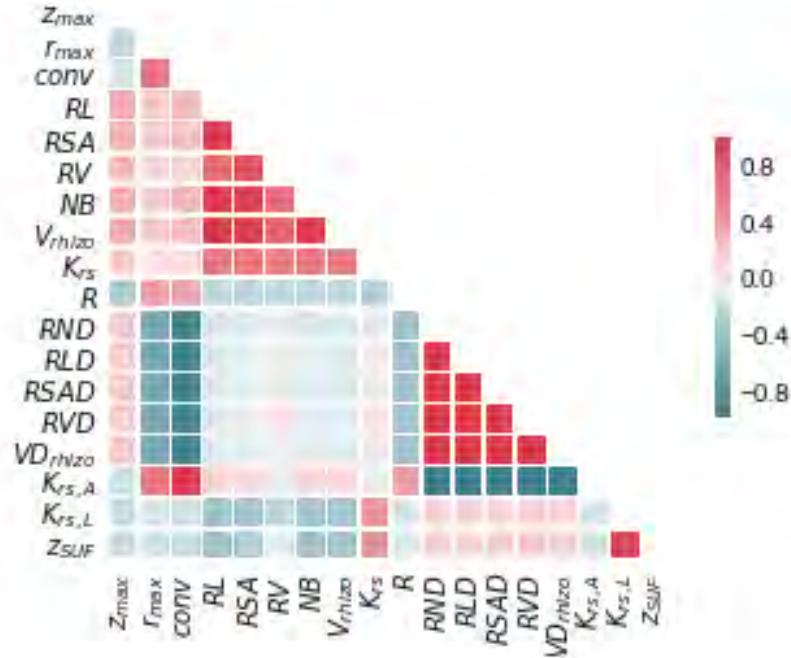
```
variable name: theta  
variable value: 1.1344640137963142
```



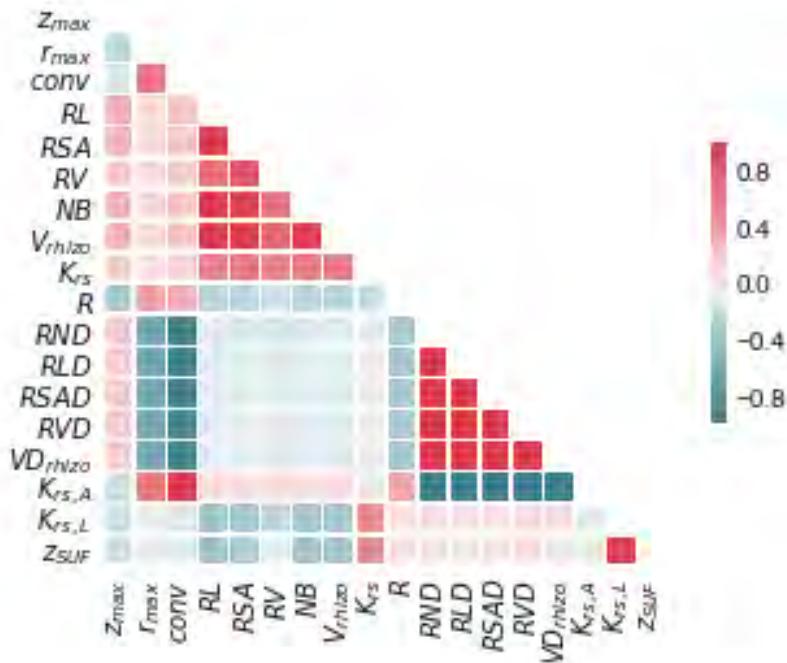
variable name: theta
 variable value: 1.2217304763960306



```
variable name: theta  
variable value: 1.3089969389957472
```



```
variable name: theta  
variable value: 1.3962634015954638
```



variable name: theta
 variable value: 1.4835298641951802

