MTH 151 (47 points) Fall 2022, Final Exam

Name: _____ #( ) December 13, 2022

Read all directions carefully and write your answers in the space provided. **To receive full credit, you must show all of your work.**

1. (3 points) Evaluate the following integral using u-substitution.

$$\int \frac{1}{x(\ln x)^2}\,dx$$

2. (3 points) Evaluate the following integral using integration by parts.

$$\int_0^1 xe^{-2x}\,dx$$

3. (3 points) Estimate the maximum error (i.e. the error bound) involved in approximating $\int_0^{\pi/2} \cos(x)\,dx$ with $n = 6$ subintervals using Midpoint Rule.

4. (4 points) Find the indefinite integral

$$\int 3\sin^3(x)\cos^2(x)\,dx$$

5. (5 points) Compute the arc length of the graph of the given function on the interval given.

$$f(x) = 2(x-1)^{3/2} \text{ on } [1,5]$$

6. Use an appropriate test for sequence convergence to determine whether each of the following sequences converges or diverges.

   (a) (2 points) $\dfrac{n}{2n+1}$

   (b) (2 points) $\dfrac{(-1)^n n}{2n+1}$

   (c) (2 points) $\dfrac{1-2n^2}{3-5n^2}$

7. (4 points) Find the sum of the following geometric series $\displaystyle\sum_{n=k}^{\infty} \left(\frac{1}{3}\right)^{n-k+1}$.

8. (7 points) Determine if the following series converges:

$$\sum_{n=1}^{\infty} \frac{n}{2n+1}$$

.

9. (7 points) Determine if the following series converges and if so, find its sum:

$$\sum_{n=3}^{\infty} \frac{1}{n^2+n-6}$$

.

10. (5 points) Determine whether the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

Hint: Use the **integral test** and question 1. Assume that $f(x) = \dfrac{1}{x(\ln x)^2}$ is a decreasing and positive function.