



Birla Institute of Technology & Science, Pilani

Pilani Campus

INSTRUCTION DIVISION
FIRST SEMESTER 2015-2016
Course Handout Part II

In addition to part-I (General Handout for all courses appended to the time table) this portion gives further specific details regarding the course

Course No. : CS / IS F214
Course Title : Logic in Computer Science

Instructor-in-Charge: Shan Sundar Balasubramaniam (email: sundarb)
Instructors:

Vishal Gupta (email: vishalgupta) – Tut. Sections 2 & 4
Jagat Sesh Chella (email:) – Tut. Sections 1 & 3

1. Scope & Objective:

The objective of this course is to introduce the formal study of Logic for Computer Science undergraduates. Within this context the course covers first order propositional and predicate logics as well as introduces program verification and temporal logics. Natural Deduction as a proof system for propositional and predicate logics is covered. Soundness and Correctness proofs are covered but only for propositional logic. The course also covers some applications in modeling of and reasoning about programs - in particular, model checking based on temporal logics and program verification using Floyd-Hoare logic. The relationship between formal logic and pragmatics of computing is highlighted via a few specialized topics: the **satisfiability** problem in propositional logic and Horn-clause problem solving.

At the end of this course the student shall be able to

- (i) write statements and proofs in first order predicate logic,
- (ii) state the limitations of first order predicate logic, and the need for higher-order logics and temporal logics
- (iii) state and argue soundness and completeness properties of logics
- (iv) write grammars for the syntax of logics and algorithms for verifying satisfiability / validity
- (v) relate problems in logic with problems in computation
- (vi) use logics to formally specify and verify computational properties.

2. Text Book:

T1: Michael Huth and Mark Ryan. Logic in Computer Science – Modelling and Reasoning about Systems. Cambridge University Press. 2nd Edition. 2004.





3. Course Plan:

3.a. Modules

Module #	Topics
I	Introduction to Logics and Proofs. Overview of the interplay of Logic and Computing.
II	Propositional Logic: Natural Deduction, Syntax and Semantics, Soundness and Completeness, Satisfiability and Validity – Forms and Algorithms.
III	Predicate Logic: Syntax and Semantics, Logic Programming, Natural Deduction, Limitations of First Order Logic.
IV	Program Verification: Floyd-Hoare Logic – Pre-conditions, Post-conditions, and Loop Invariants; Verification of imperative programs – Partial and Total correctness.
V	Temporal Logic: LTL, CTL, and Model Checking

3. b. Lecture Schedule:

Lecture #	Module #	Topic	Learning Outcome(s) [The student will be able to:]	Reading
L1	I.a	Why study Logic?	<ul style="list-style-type: none"> state a few reasons to study logics and proofs 	-
L1	I.b	A broad and selective History of Logic and Proofs (HLP) - Part I: Euclid's Formalization of Geometry. Hilbert's Program and the Formalization problem. Russell's efforts in formalization and Russell's paradox.	<ul style="list-style-type: none"> state typical issues and debates in formalizing proofs (and mathematical arguments in general) state the distinction between axioms, statements, and proofs 	-
L2	I.c	HLP – Part II: Brouwer's intuitionism – Intuitionist's Critique on Proof by	<ul style="list-style-type: none"> distinguish between a constructive 	





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		Contradiction and (the use of) Law of Excluded Middle	<ul style="list-style-type: none"> proof and a non-constructive proof state the intuitionist position / critique on non-constructive proofs 	
L2	I.d	HLP – Part III: Godel’s Incompleteness Result and its impact on the formalization of mathematics	<ul style="list-style-type: none"> state soundness and completeness requirements of proof systems and explain – at a high level – the implications of Godel’s incompleteness result 	-
L2	I.e	Logic and Computing (L&C) – Part I: Godel, Church, and Turing: Computability – Systems for defining computability; Church-Turing Thesis.	<ul style="list-style-type: none"> Informally define the notion of “computability” state the equivalence of schemes / mechanisms for computability 	-
L3	I.f	L&C – Part II: Logic and Computing: Non-computable Problems – Example: Halting	<ul style="list-style-type: none"> State the Halting Problem and argue that it is not computable 	
L3	I.g	L&C – Part III: Time Complexity, Complexity Classes, Is $P = NP$?	<ul style="list-style-type: none"> define complexity classes P and NP state what it means to say that $P=NP$ or $P \neq NP$ 	-
L4	I.h	L&C – Part IV: Boolean Satisfiability (SAT) and Horn-Clause Satisfiability problems; their relationship to the classes NP and P ;	<ul style="list-style-type: none"> (by hand) verify satisfiability of Boolean expressions and Horn-style expressions. 	-
L4	I.i	L&C – Part V: Logic and Programming: Logic Programming and Prolog.	<ul style="list-style-type: none"> state the relation between logic and programming at a high level 	-
L5	II.a	Propositional Logic: Natural Deduction as a Proof System.	<ul style="list-style-type: none"> state the notational conventions used in Natural Deduction 	T1 Sec. 1.2.1
L5	II. b	Propositional Logic: Natural Deduction: Conjunction Rules	<ul style="list-style-type: none"> apply proof rules involving conjunction 	T1 Sec. 1.2.1
L5	II.c	Propositional Logic: Natural Deduction: Implication Rules	<ul style="list-style-type: none"> apply proof rules involving implication 	T1 Sec. 1.2.1
L6	II.d	Propositional Logic: Natural Deduction: Disjunction Rules	<ul style="list-style-type: none"> apply proof rules involving disjunction 	T1 Sec. 1.2.1
L6	II.e	Propositional Logic: Natural Deduction: Negation Rules	<ul style="list-style-type: none"> apply proof rules involving negation 	T1 Sec. 1.2.1
L6	II.f	Propositional Logic: Natural	<ul style="list-style-type: none"> apply proof rules involving double 	T1 Sec.





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		Deduction: Double Negation Rules	negation	1.2.1
L6	II.g	Propositional Logic: Natural Deduction: Derived Rules: <i>Modus Tollens</i> , <i>Law of Excluded Middle</i> (LEM), <i>Proof by Contradictio</i> (PbC).	<ul style="list-style-type: none"> apply <i>modus tollens</i>, <i>LEM</i>, and <i>PbC</i> in proofs derive modus tollens, LEM, and PbC from basic proof rules 	T1 Sec. 1.2.2
L7	II.h	Syntax: Context Free Grammars (CFG): Notation and Examples	<ul style="list-style-type: none"> define CFGs for simple constructs 	T1 Sec. 1.3
L7	II.i	Context Free Grammars: Parse Trees: Examples	<ul style="list-style-type: none"> illustrate and explain the internal structure of context-free constructs 	T1 Sec. 1.3
L8	II.j	Propositional Logic: Syntax: Well-formed-formulas and grammar.	<ul style="list-style-type: none"> state the formal syntax of propositional logic as a CFG 	T1 Sec. 1.3
L8	II.k	Propositional Logic: Syntax: Different forms of grammar: Precedence and Order of evaluation: Rules to handle precedence and over-ride precedence	<ul style="list-style-type: none"> write grammar rules to define the syntax of propositional logic with a specific order of evaluation 	T1 Sec. 1.3
L9	I.j	Proof Techniques: Mathematical Induction	<ul style="list-style-type: none"> articulate the generic structure / template of proofs by mathematical induction 	-
L9	I.k	Proof Techniques: Structural Induction and Examples	<ul style="list-style-type: none"> relate mathematical induction to structural induction 	-
L9	I.l	Grammars, Parse Trees and Structural Induction – Approach	<ul style="list-style-type: none"> illustrate the relation between languages and inductive proofs explain structural induction as a proof technique 	T1 Sec. 1.4.2
L9	I.m	Structural Induction - Examples	<ul style="list-style-type: none"> write proofs using structural induction 	T1 Sec. 1.4.2
L10	II.l	Propositional Logic: Semantics: Truth Tables, Soundness and Completeness	<ul style="list-style-type: none"> interpret sentences in propositional logic state the soundness and completeness results w.r.t. propositional logic 	T1 Sec. 1.4.1
L10	II.m	Propositional Logic: Semantics: Equivalence and Normal Forms	<ul style="list-style-type: none"> argue or verify the equivalence (or the lack of it) of given propositional formulas 	T1 Sec. 1.4.1
L10	II.n	Propositional Logic: Syntax: Grammars	<ul style="list-style-type: none"> define a grammar for a given normal 	-





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		for Normal Forms	form	
L11	II.o	Propositional Logic : Soundness	<ul style="list-style-type: none"> state soundness arguments in general and soundness arguments for logics in particular 	T1 Sec. 1.4.3
L11	II.p	Propositional Logic: Soundness Proof	<ul style="list-style-type: none"> argue that proofs in propositional logic are sound 	T1 Sec. 1.4.3
L12	II.q	Propositional Logic: Completeness	<ul style="list-style-type: none"> state completeness arguments in general and completeness arguments for logics in particular 	T1 Sec. 1.4.4
L12	II.r	Propositional Logic: Completeness Proof Overview	<ul style="list-style-type: none"> state completeness arguments for propositional logic 	T1 Sec. 1.4.4
L12	II.s	Propositional Logic: Completeness Proof	<ul style="list-style-type: none"> argue that propositional logic is complete using structural induction 	T1 Sec. 1.4.4
L13	II.t	Propositional Logic: Conjunctive Normal Form (CNF) and Disjunctive Normal Form (DNF)	<ul style="list-style-type: none"> recognize propositional formulas in CNF / DNF rewrite propositional formulas in CNF / DNF 	T1 Sec. 1.5.2
L13	II.u	Propositional Logic: Validity	<ul style="list-style-type: none"> verify whether a propositional formula is valid or not 	T1 Sec. 1.5.1
L13	II.v	Propositional Logic: Validity of specific forms (CNF), Algorithm(s) for validity	<ul style="list-style-type: none"> verify validity of a formula in CNF write a program to verify validity of a formula in CNF 	T1 Sec. 1.5.1
L13	II.w	Propositional Logic: Validity and Satisfiability; Satisfiability of specific forms (DNF)	<ul style="list-style-type: none"> verify whether a propositional formula is satisfiable or not state the relation between validity and satisfiability of propositional formulas 	T1 Sec. 1.5.1
L14	II.x	Propositional Logic: Horn Clauses and Horn Formulas	<ul style="list-style-type: none"> distinguish Horn formulas from general formulas in propositional logic identify when a propositional formula can be rewritten in Horn form 	T1 Sec. 1.5.3
L14	II.y	Propositional Logic: Satisfiability of Horn Formulas. Algorithm for	<ul style="list-style-type: none"> verify whether a Horn formula is satisfiable or not write a program to verify validity 	T1 Sec. 1.5.3





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		Satisfiability of Horn Formulas	of a Horn formula	
L14	II.z	Propositional Logic: Algorithm for Satisfiability of Horn Formulas	<ul style="list-style-type: none"> write a program to verify validity of a Horn formula 	T1 Sec. 1.5.3
L15	III.a	Expressiveness of Propositional Logic – Limitations: Need for predicates	<ul style="list-style-type: none"> illustrate statements and arguments that cannot be expressed in propositional logic explain why predicates are more expressive than propositions 	T1 Sec. 2.1
L15	III.b	Expressiveness of Propositional Logic – Limitations: Need for variables and quantification	<ul style="list-style-type: none"> illustrate statements that cannot be expressed in propositional logic explain how variables and quantification add expressive power 	T1 Sec. 2.1
L16	III.c	Introduction to Predicate Logic: Predicates vs. Functions and Need for Function Terms	<ul style="list-style-type: none"> distinguish between predicates and functions explain when function terms are more expressive than predicates 	T1 Sec. 2.1
L16	III.d	Predicate Logic: Features and Examples	<ul style="list-style-type: none"> write formulas in predicate logic 	T1 Sec. 2.2
L17	III.e	Predicate Logic: Formal Syntax: Grammars and Parse Trees	<ul style="list-style-type: none"> write grammar rules for defining terms write grammar rules for defining formulas in Predicate Logic use the grammar rules to parse formulas in Predicate Logic 	T1 Sec. 2.2.1 to 2.2.2
L17	III.f	Predicate Logic: Theorem Proving: Proof Steps; Automating the proof steps.	<ul style="list-style-type: none"> explain how proofs in predicate logic can be approached. state issues in automating proofs steps 	-
L17	III.g	Horn Clause Programming: Theorem Proving and Logic Programming, Programming with Horn Clauses	<ul style="list-style-type: none"> explain the relation between Theorem Proving and Logic Programming state how Horn Clauses can be used for programming 	-
L18	III.g	Logic Programming and Prolog : Proof	<ul style="list-style-type: none"> perform unification of terms 	-





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		Steps and Term Unification.		
L18	III.h	Logic Programming and Prolog: Proof Search, Backtracking	<ul style="list-style-type: none"> apply search and backtracking to execute a Prolog program 	
L18	III.i	Programming in Prolog: Examples	<ul style="list-style-type: none"> write simple programs in Prolog 	-
L19	III.j	Programming in Prolog: Inductive / Recursive definitions	<ul style="list-style-type: none"> write programs using recursion in Prolog 	-
L19	III.k	Programming in Prolog: Problem Solving in Prolog	<ul style="list-style-type: none"> write reasonably large programs to solve complex problems using Prolog 	-
L20	III.l	Predicate Logic: Proofs and Proof Rules: Introduction: Need for a Substitution Operation	<ul style="list-style-type: none"> explain / illustrate the need for a substitution operation in applying proof rules of Predicate Logic 	T1 Sec. 2.2
L20	III.m	Predicate Logic: Syntax: Free and Bound Variables	<ul style="list-style-type: none"> identify and distinguish between free and bound variables in Predicate Logic formulas 	T1 Sec. 2.2.3
L20	III.n	Predicate Logic: Substitution: Definition and Examples	<ul style="list-style-type: none"> apply substitution on Predicate Logic formulas 	T1 Sec. 2.2.4
L21	III.o	Predicate Logic: Proof Rules: Equality	<ul style="list-style-type: none"> explain the need for rules for equality use the rules for equality 	T1 2.3.1
L21	III.p	Predicate Logic: Natural Deduction: Rules for Universal Quantification	<ul style="list-style-type: none"> apply rules for universal quantification in proofs of predicate logic formulas 	T1 Sec. 2.3.1
L21	III.q	Predicate Logic: Natural Deduction: Rules for Existential Quantification	<ul style="list-style-type: none"> apply rules for existential quantification in proofs of predicate logic formulas 	T1 Sec. 2.3.1
Self Study	III.s	Predicate Logic: Syntactic Equivalence	<ul style="list-style-type: none"> deduce equivalences of formulas in predicate logic 	T1 Sec. 2.3.2
L22	III.r	Predicate Logic: Introduction to Semantics: Models and Interpretation	<ul style="list-style-type: none"> formally define models for different theories interpret formulas in predicate 	T1 Sec. 2.4.1





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		– Examples	logic according to given models	
L23	III.s	Predicate Logic: Semantics: The Model-Checks Relation	<ul style="list-style-type: none"> apply the model-checks relation on predicate logic formulas 	T1 Sec. 2.4.1
L24	III.t	Predicate Logic: Semantics: Semantic Entailment	<ul style="list-style-type: none"> illustrate and explain semantic entailment in predicate logic 	T1 Sec. 2.4.2
L24	III.u	Predicate Logic: Semantics: Validity and Satisfiability	<ul style="list-style-type: none"> state and explain validity and satisfiability in Predicate Logic explain and illustrate issues in verifying satisfiability and validity of formulas in Predicate Logic 	T1 Sec. 2.4.3
L24	III.v	Predicate Logic: Soundness and Completeness	<ul style="list-style-type: none"> state and explain claims regarding Soundness and Completeness of Predicate Logic 	-
L25	I.n	Proof Techniques: Cantor's Diagonalization	<ul style="list-style-type: none"> articulate Cantor's diagonalization technique and apply the same on simple examples 	-
L25	I.o	Review: Undecidability and the Halting Problem	<ul style="list-style-type: none"> state and explain the notion of undecidability state and explain the Halting problem. 	-
L25	III.w	Predicate Logic: <i>Validity is Undecidable</i> : Proof approach using Reduction	<ul style="list-style-type: none"> provide an explanation for why <i>Validity in Predicate Logic is undecidable</i> using the idea of a reduction to a known undecidable problem 	T1 Sec. 2.5
L26	III.x	Predicate Logic: <i>Validity is Undecidable</i> : Proof sketch using Diagonalization	<ul style="list-style-type: none"> provide an proof argument for why <i>Validity in Predicate Logic is undecidable</i> using diagonalization 	-
L27	III.y	Predicate Logic: Expressiveness: Inexpressible Properties: Example and Proof.	<ul style="list-style-type: none"> provide examples of statements that cannot be expressed in First Order Predicate Logic provide an intuitive argument as 	T1 Sec. 2.6





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			<p>to why such statements are inexpressible in First Order Predicate Logic</p> <ul style="list-style-type: none"> provide a rigorous argument as to why <i>reachability</i> in graphs is inexpressible in First Order Predicate Logic 	
L27	III.z	Existential Second Order Logic: Expressiveness: Examples(s)	<ul style="list-style-type: none"> provide examples of statements that require existential quantification over predicates 	T1 Sec. 2.6.1
L27	III.aa	Universal Second Order Logic: Expressiveness: Example(s)	<ul style="list-style-type: none"> provide examples of statements that require universal quantification over predicates 	T1 Sec. 2.6.2
L28	IV.a	Program Verification: Floyd-Hoare Logic: Pre-conditions and Post-Conditions	<ul style="list-style-type: none"> describe what pre-conditions and post-conditions are 	T1 Sec. 4.2.2
L28	IV.b	Program Verification: Floyd-Hoare Logic: Assignment Statements and Sequencing	<ul style="list-style-type: none"> write pre-conditions and post-conditions for assignment statements compose pre-conditions and post-conditions over sequential statements 	T1 Sec. 4.2.2 & 4.3.1
L29	IV.b	Program Verification: Floyd-Hoare Logic: Conditional Statements	<ul style="list-style-type: none"> compose pre-conditions and post-conditions over conditional statements 	T1 Sec. 4.2.2 & 4.3.1
L29	IV.c	Program Verification: Floyd-Hoare Logic: Meta-Rules	-	T1 Sec. 4.2.2 & 4.3.1
L29	IV.d	Program Verification: Verification of Straight-Line programs - Examples	<ul style="list-style-type: none"> apply rules of Floyd-Hoare Logic to verify properties of straight line programs 	T1 Sec. 4.3.1
L30	IV.e	Program Verification: Floyd-Hoare Logic: Program Variables and Logical	<ul style="list-style-type: none"> identify and distinguish between Program Variables and Logical Variables in verification using 	T1 Sec. 4.3.1





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		Variables	Floyd-Hoare Logic <ul style="list-style-type: none"> illustrate the need for Logical Variables with examples 	
L30	IV.f	Program Verification: Partial Correctness and Total Correctness	<ul style="list-style-type: none"> distinguish between partial correctness arguments and total correctness arguments write termination proofs 	T1 Sec. 4.3.1
L30	IV.g	Program Verification: Floyd-Hoare Logic: Loop Invariants	<ul style="list-style-type: none"> illustrate and explain what loop invariants are 	T1 Sec. 4.2.2 & 4.3.1
L30	IV.h	Program Verification: Floyd-Hoare Logic: Verifying correctness of Loops: Partial Correctness using Invariants – Examples	<ul style="list-style-type: none"> write loop invariants given simple loops provide correctness arguments using loop invariants 	T1 Sec. 4.2.2 & 4.3.1
L31	IV.i	Program Verification: Floyd-Hoare Logic: Proof Rules and Verification Examples	<ul style="list-style-type: none"> provide partial correctness proofs for small programs 	T1 Sec. 4.3.1 & 4.3.2
L32	IV.j	Program Verification: Case Study	<ul style="list-style-type: none"> apply Floyd-Hoare logic to verify properties of reasonably large programs 	-
L33	IV.k	Program Verification: Approaches and Limitations	<ul style="list-style-type: none"> describe issues and limitations in proving correctness of programs 	T1 Sec. 3.1
L33	V.a	Dynamic Logics	<ul style="list-style-type: none"> provide examples where “time” needs to be specified explicitly in logical formulas and the form in which it needs to be specified 	T1 Sec. 3.1
L33	V.b	Software Modeling: State Machines	<ul style="list-style-type: none"> illustrate and explain specification using state machines 	T1 Sec. 2.7.1
L34	V.c	Software Modeling: State Machines as Models and Model Checking	<ul style="list-style-type: none"> describe model checking write state machines for simple problems / computations 	T1 Sec. 2.7.1
L34	V.d	Linear-time Temporal Logic (LTL) –	<ul style="list-style-type: none"> describe LTL 	T1 Sec.





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		Introduction		3.2
L35	V.e	LTL: Syntax and Formulas	<ul style="list-style-type: none"> provide examples of formulas in LTL illustrate internal structure of formulas in LTL 	T1 Sec. 3.2.1
L35	V.f	LTL: Semantics: Transitions and Paths	<ul style="list-style-type: none"> interpret LTL formulas 	T1 Sec. 3.2.2
L36	V.g	LTL: Specifications: Examples	<ul style="list-style-type: none"> write LTL specifications for complex scenarios 	T1 Sec. 3.2.3
L37	V.h	Properties expressible in Temporal Logics – Examples	<ul style="list-style-type: none"> define the problem of model checking for a particular case 	T1 Sec. 3.3.1
L37	V.i	Limitations of Linear Time and LTL:	<ul style="list-style-type: none"> state and illustrate the limitations of LTL 	T1 Sec. 3.4
L38	V.j	Branching Time and Branching Time Temporal Logic (CTL) - Introduction	<ul style="list-style-type: none"> read, write, and explain CTL formulas 	T1 Sec. 3.4
L38	V.k	CTL – Semantics of Temporal Operators, Examples.	<ul style="list-style-type: none"> interpret CTL formulas differentiate between interpretation in LTL and that in CTL 	T1 Sec. 3.4
L39	V.l	CTL – Model Checking	<ul style="list-style-type: none"> perform model checking of properties stated in CTL 	T1 Sec. 3.6.1
L40	-	Course Summary	-	-

4. Evaluation

4. a. Evaluation Scheme:

Component	Weight	Date	Remarks
Quizzes (3)	3x18M=54M	In Tutorial / Lecture Sessions	1 session notice (Open Book)





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Assignments (2)	46M (tentative split 15+31)	2 to 3 weeks each.	Take Home (Teams of 2)
Mid-Term Test (90 minutes)	43M	9/10 2:00 - 3:30 PM	(scheduled centrally) Open Book
Comprehensive Exam (120 minutes)	57M	11/12 FN	(scheduled centrally) Open Book
TOTAL	200M	-	-

4. b. Make-up Policy:

- No Make-up will be available for Assignments under any condition.
- Late submission of assignment will incur a penalty of 25% up to 24 hours and a penalty of 50% up to 48 hours from the deadline.
- There will be one make-up (for all three quizzes put together) i.e. a student can take a make-up for at most one quiz out of the three quizzes. The make-up quiz will be conducted after all the regular quizzes are done and the coverage for that will be announced later.
- Prior Permission of the Instructor is usually required to get a make-up for the mid-term test.
- Prior Permission of (Associate) Dean, Instruction is usually required to get a make-up for the comprehensive exam.
- A make-up shall be granted only in genuine cases where - in the Instructor's / Dean's judgment - the student would be physically unable to appear for the test/exam. Instructor's / Dean's decision in this matter would be final.

4.c. Fairness Policy:

- Student teams are expected to work on their own on assignments.
- All students are expected to contribute equally within a team. The instructor's assessment regarding the contributions of team members would be final.
- Any use of unfair means in quizzes, assignment, or test/exam will be reported to the Unfair means committee and will be subject to the severest penalty possible:
 - Unfair means would include copying from other students or from the Web or from other sources of information including electronic devices.
 - All parties involved would be treated equally responsible: allowing others to copy one's work is enabling unfair means and is equally un-acceptable.

5. Consultation Hours: *To be announced.*

6. Notices: All notices concerning this course will be displayed online only. If there is a need email would be used on short notice (12 hours) – only BITS Pilani mail would be used.

Instructor –In- Charge, CS F214.

