

Geometry Div. 1

1. Points A , B , and C lie on a line, in that order, with $AB = 8$ and $BC = 2$. B is rotated 20° counter-clockwise about A to a point B' , tracing out an arc R_1 . C is then rotated 20° clockwise about A to a point C' , tracing out an arc R_2 . What is the area of the region bounded by arc R_1 , segment $B'C$, arc R_2 , and segment $C'B$?
2. In convex quadrilateral $ABCD$, $\angle ADC = 90^\circ + \angle BAC$. Given that $AB = BC = 17$, and $CD = 16$, what is the maximum possible area of the quadrilateral?
3. Let $\triangle ABC$ be a triangle with $AB = 10$ and $AC = 16$, and let I be the intersection of the internal angle bisectors of $\triangle ABC$. Suppose the tangents to the circumcircle of $\triangle BIC$ at B and C intersect at a point P with $PA = 8$. Compute the length of BC .
4. Let $ABCDEF$ be an equilateral hexagon such that $\triangle ACE \cong \triangle DFB$. Given that $AC = 7$, $CE = 8$, and $EA = 9$, what is the side length of this hexagon?
5. Let $\gamma_1, \gamma_2, \gamma_3$ be three circles with radii 3, 4, 9, respectively, such that γ_1 and γ_2 are externally tangent at C , and γ_3 is internally tangent to γ_1 and γ_2 at A and B , respectively. Suppose the tangents to γ_3 at A and B intersect at X . The line through X and C intersect γ_3 at two points, P and Q . Compute the length of PQ .
6. Let circles ω and Γ , centered at O_1 and O_2 and having radii 42 and 54 respectively, intersect at points X, Y , such that $\angle O_1 X O_2 = 105^\circ$. Points A, B lie on ω and Γ respectively such that $\angle O_1 X A = \angle A X B = \angle B X O_2$ and Y lies on both minor arcs XA and XB . Define P to be the intersection of AO_2 and BO_1 . Suppose XP intersects AB at C . What is the value of $\frac{AC}{BC}$?
7. Convex pentagon $ABCDE$ has $\overline{BC} = 17$, $\overline{AB} = 2\overline{CD}$, and $\angle E = 90^\circ$. Additionally, $\overline{BD} - \overline{CD} = \overline{AC}$, and $\overline{BD} + \overline{CD} = 25$. Let M and N be the midpoints of BC and AD respectively. Ray EA is extended out to point P , and a line parallel to AD is drawn through P , intersecting line EM at Q . Let G be the midpoint of AQ . Given that N and G lie on EM and PM respectively, and the perimeter of $\triangle QBC$ is 42, find the length of \overline{EM} .
8. Let ABC be a triangle with $AB < AC$ and ω be a circle through A tangent to both the B -excircle and the C -excircle. Let ω intersect lines AB, AC at X, Y respectively and X, Y lie outside of segments AB, AC . Let O be the center of ω and let OI_C, OI_B intersect line BC at J, K respectively. Suppose $KJ = 4$, $KO = 16$ and $OJ = 13$. Find $\frac{[KI_B I_C]}{[JI_B I_C]}$.