

Geometry Div. 2

1. Triangle ABC has a right angle at A , $AB = 20$, and $AC = 21$. Circles ω_A , ω_B , and ω_C are centered at A , B , and C respectively and pass through the midpoint M of \overline{BC} . ω_A and ω_B intersect at $X \neq M$, and ω_A and ω_C intersect at $Y \neq M$. Find XY .
2. Points A , B , and C lie on a line, in that order, with $AB = 8$ and $BC = 2$. B is rotated 20° counter-clockwise about A to a point B' , tracing out an arc R_1 . C is then rotated 20° clockwise about A to a point C' , tracing out an arc R_2 . What is the area of the region bounded by arc R_1 , segment $B'C$, arc R_2 , and segment $C'B$?
3. Consider trapezoid $[ABCD]$ which has $AB \parallel CD$ with $AB = 5$ and $CD = 9$. Moreover, $\angle C = 15^\circ$ and $\angle D = 75^\circ$. Let M_1 be the midpoint of AB and M_2 be the midpoint of CD . What is the distance M_1M_2 ?
4. A $2\sqrt{5}$ by $4\sqrt{5}$ rectangle is rotated by an angle θ about one of its diagonals. If the total volume swept out by the rotating rectangle is 62π , find the measure of θ in degrees.
5. Emily is at $(0,0)$, chilling, when she sees a spider located at $(1,0)$! Emily runs a continuous path to her home, located at $(\sqrt{2} + 2, 0)$, such that she is always moving away from the spider and toward her home. That is, her distance from the spider always increases whereas her distance to her home always decreases. What is the area of the set of all points that Emily could have visited on her run home?
6. In convex quadrilateral $ABCD$, $\angle ADC = 90^\circ + \angle BAC$. Given that $AB = BC = 17$, and $CD = 16$, what is the maximum possible area of the quadrilateral?
7. Let $\triangle ABC$ be a triangle with $AB = 10$ and $AC = 16$, and let I be the intersection of the internal angle bisectors of $\triangle ABC$. Suppose the tangents to the circumcircle of $\triangle BIC$ at B and C intersect at a point P with $PA = 8$. Compute the length of BC .
8. Let $ABCDEF$ be an equilateral hexagon such that $\triangle ACE \cong \triangle DFB$. Given that $AC = 7$, $CE = 8$, and $EA = 9$, what is the side length of this hexagon?