

CMM 2022

Geometry Round

INSTRUCTIONS

1. Do not look at the test before the proctor starts the round.
2. This test consists of 10 short-answer problems to be solved in 60 minutes and one estimation question. Each of the short-answer questions is worth points depending on its difficulty, and the estimation question will be used to break ties. **If you do not write an estimate for estimation, you will be placed last in tiebreaking.**
3. Write your name, team name, and team ID on your answer sheet. Circle the subject of the test you are taking.
4. Write your answers in the corresponding boxes on the answer sheets.
5. No computational aids other than pencil/pen are permitted.
6. Answers must be reasonably simplified.
7. If you believe that the test contains an error, submit your protest in writing to Doherty 2302 by the end of lunch.

Geometry

1. An equilateral 12-gon has side length 10 and interior angle measures that alternate between 90° , 90° , and 270° . Compute the area of this 12-gon.
2. A circle has radius 52 and center O . Points A is on the circle, and point P on \overline{OA} satisfies $OP = 28$. Point Q is constructed such that $QA = QP = 15$, and point B is constructed on the circle so that Q is on \overline{OB} . Find QB .
3. Let ABC be an acute triangle with $\angle ABC = 60^\circ$. Suppose points D and E are on lines AB and CB , respectively, such that CDB and AEB are equilateral triangles. Given that the positive difference between the perimeters of CDB and AEB is 60 and $DE = 45$, what is the value of $AB \cdot BC$?
4. Circle Γ has diameter \overline{AB} with $\overline{AB} = 6$. Point C is constructed on \overleftrightarrow{AB} so that $AB = BC$ and $A \neq C$. Let D be on Γ so that \overleftrightarrow{CD} is tangent to Γ . Compute the distance from line \overleftrightarrow{AD} to the circumcenter of $\triangle ADC$.
5. Let ABC be an equilateral triangle of unit side length and suppose D is a point on segment \overline{BC} such that $DB < DC$. Let M and N denote the midpoints of \overline{AB} and \overline{AC} , respectively. Suppose X and Y are the intersections of lines AB and ND , and lines AC and MD , respectively. Given that $XY = 4$, what is the value of $\frac{DB}{DC}$?
6. A triangle $\triangle ABC$ satisfies $AB = 13$, $BC = 14$, and $AC = 15$. Inside $\triangle ABC$ are three points X , Y , and Z such that:
 - Y is the centroid of $\triangle ABX$
 - Z is the centroid of $\triangle BCY$
 - X is the centroid of $\triangle CAZ$

What is the area of $\triangle XYZ$?

7. Let $\Gamma_1, \Gamma_2, \Gamma_3$ be three pairwise externally tangent circles with radii 1, 2, 3, respectively. A circle passes through the centers of Γ_2 and Γ_3 and is externally tangent to Γ_1 at a point P . Suppose A and B are the centers of Γ_2 and Γ_3 , respectively. What is the value of $\frac{PA^2}{PB^2}$?
8. Let A and B be points on circle Γ such that $AB = \sqrt{10}$. Point C is outside Γ such that $\triangle ABC$ is equilateral. Let D be a point on Γ and suppose the line through C and D intersects AB and Γ again at points E and $F \neq D$. It is given that points C, D, E, F are collinear in that order and that $CD = DE = EF$. What is the area of Γ ?