Overview of classical control

Minseok Kim and Hiro Farre

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ML Tokamak subgroup meeting

mseokim@princeton.edu

Plasma Control

Linear Time-Invariant System

- Linear system

Linear combination of input will result in linear combination of output.

$$y(t) = \int_0^t u(\tau)\underbrace{h(t-\tau)}_{\text{Impulse response}}d\tau$$

Sum of impulses by superposition.

C.W.Rowley, Introduction to Feedback Control, Princeton University, 2024

Transfer function

Laplace transform

$$y(t) = \int_0^t u(\tau)h(t-\tau)d\tau \quad \Rightarrow \quad Y(s) = H(s)U(s)$$

$$H(s) = \frac{Y(s)}{U(s)}$$: Transfer function

Transfer function can be obtained from system dynamics.

C.W.Rowley, Introduction to Feedback Control, Princeton University, 2024

Transfer function

Laplace transform

$$T\frac{dy}{dt} + y = Ku(t) \quad \Rightarrow \quad sTY(s) + Y(s) = KU(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{K}{sT+1}$$

$$sT + 1 = 0 \Rightarrow s = -\frac{1}{T}$$

First-order model

Pole of the plant

Plasma Control

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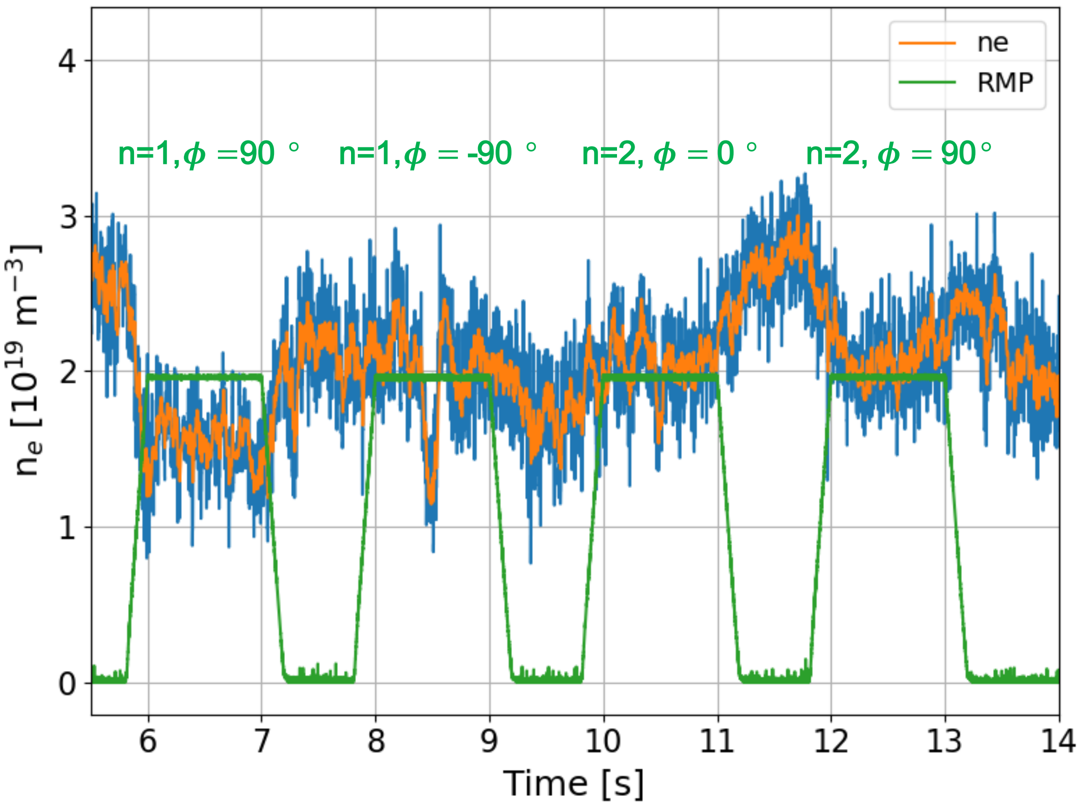
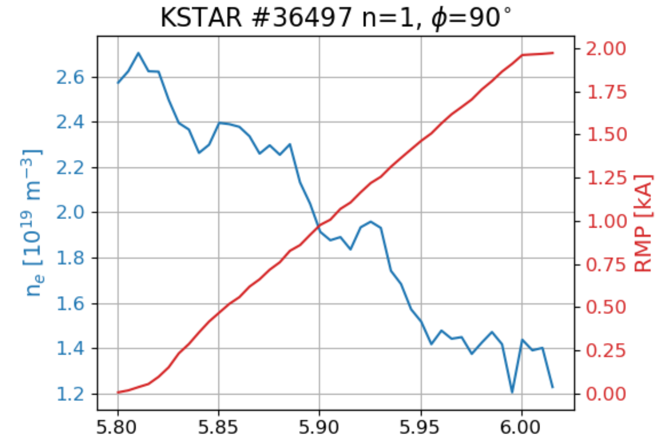
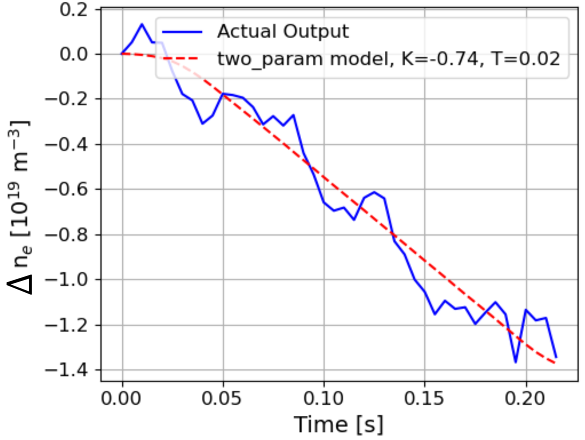
System Identification

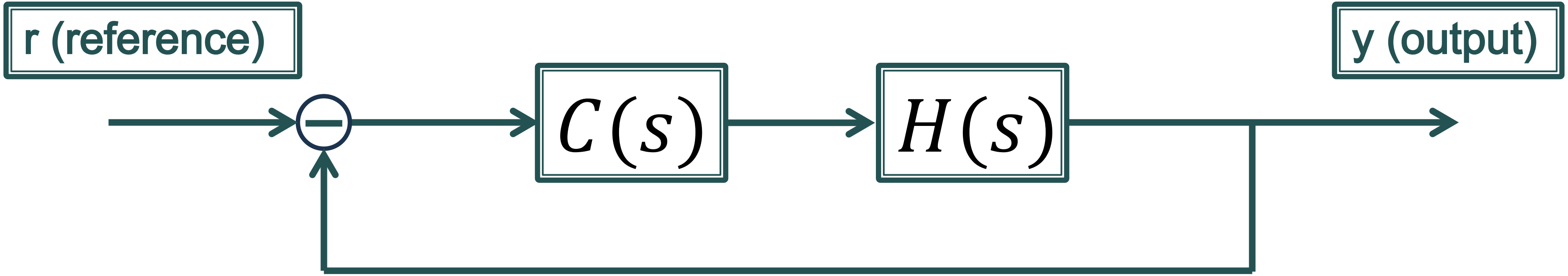
KSTAR @36497







Controller design



$$C(s) = K_p + K_i \frac{1}{s} \quad \Rightarrow \quad Y(s) = \frac{HC}{1 + HC} R(s)$$

G(s):
Closed-loop
transfer function

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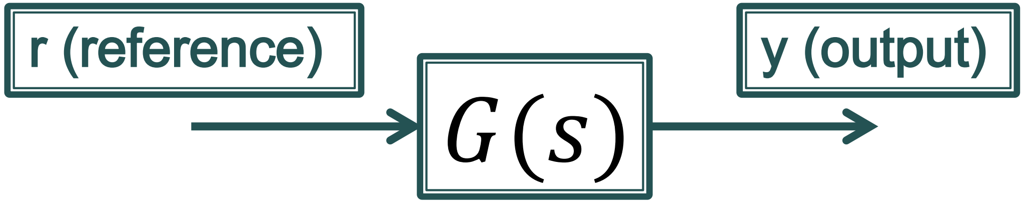
Controller design



$$G(s) = \frac{K/T(K_p s + K_I)}{s^2 + [(K \cdot K_p + 1)/T]s + K \cdot K_I/T}$$

Place the closed-loop pole to be the same value with the plant

$$K_p = \frac{1}{K} \ and \ K_I = \frac{1}{KT}$$

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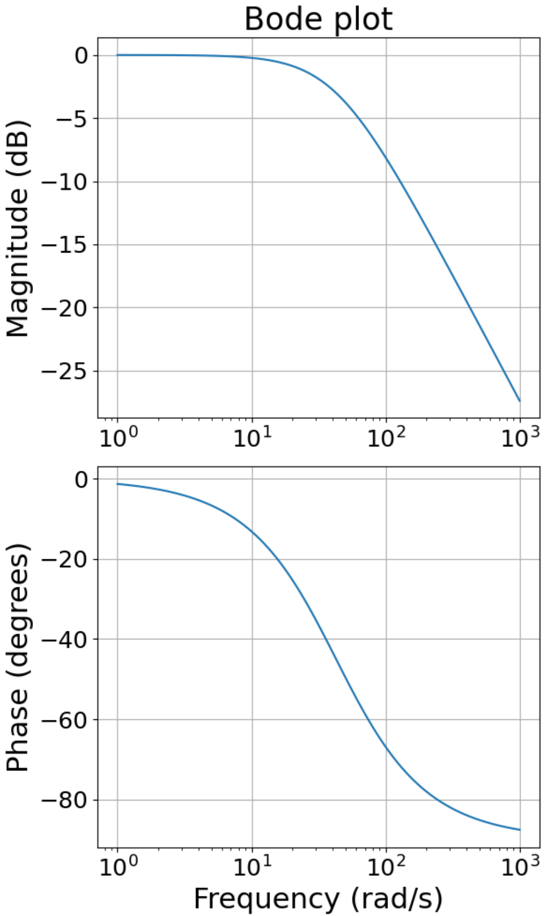
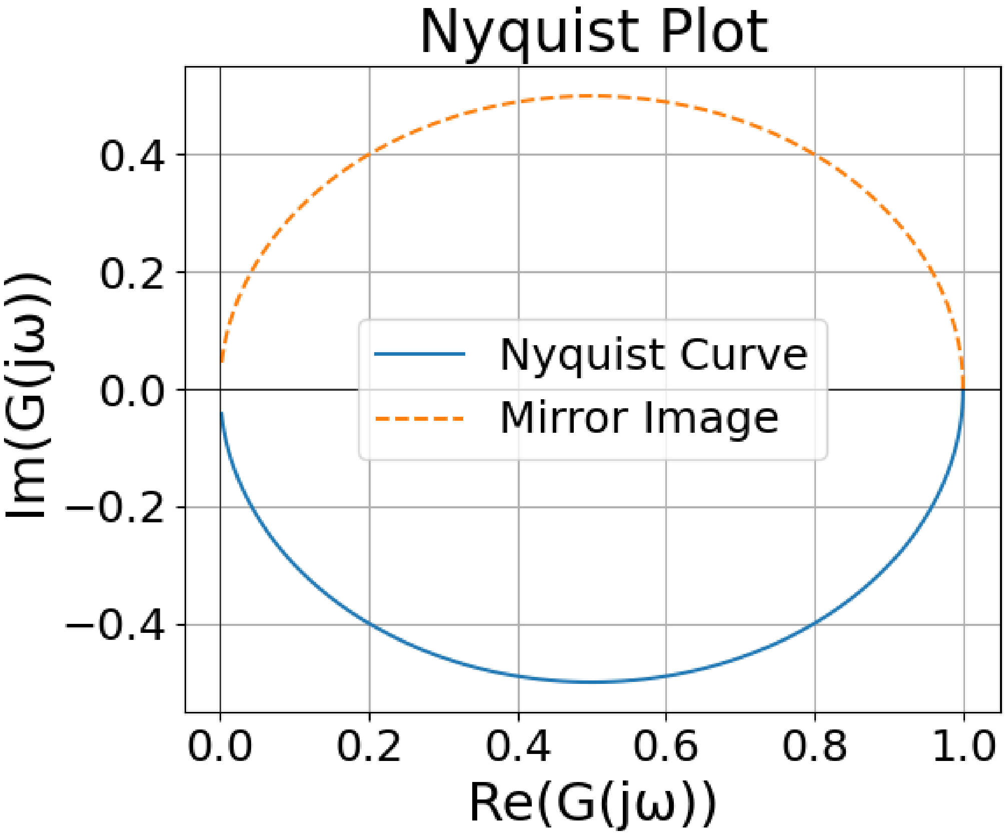
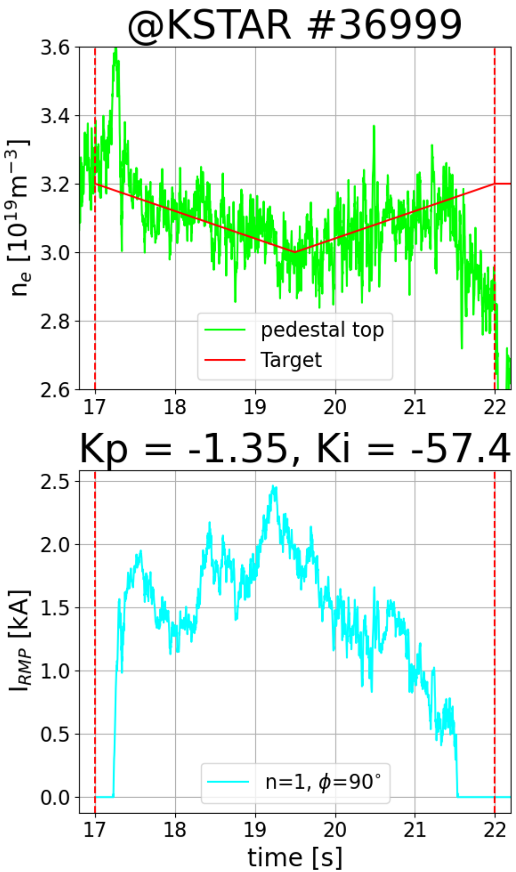
Bode and Nyquist plot



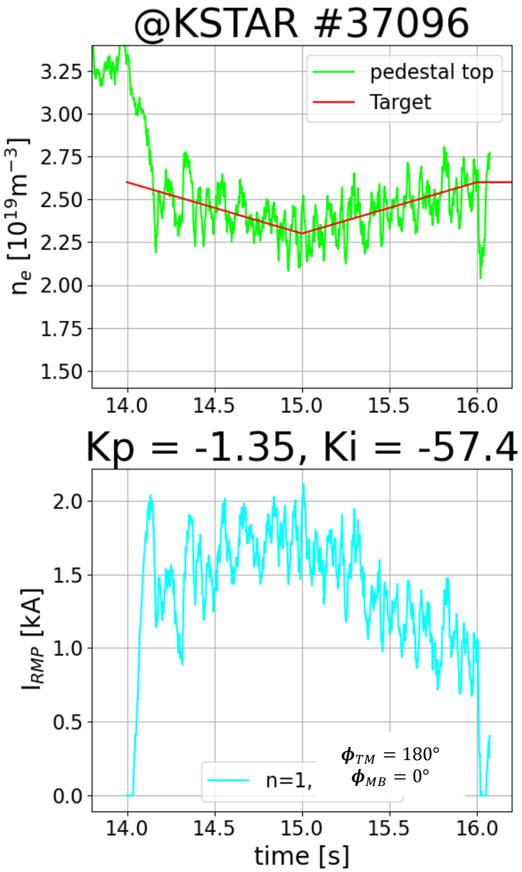


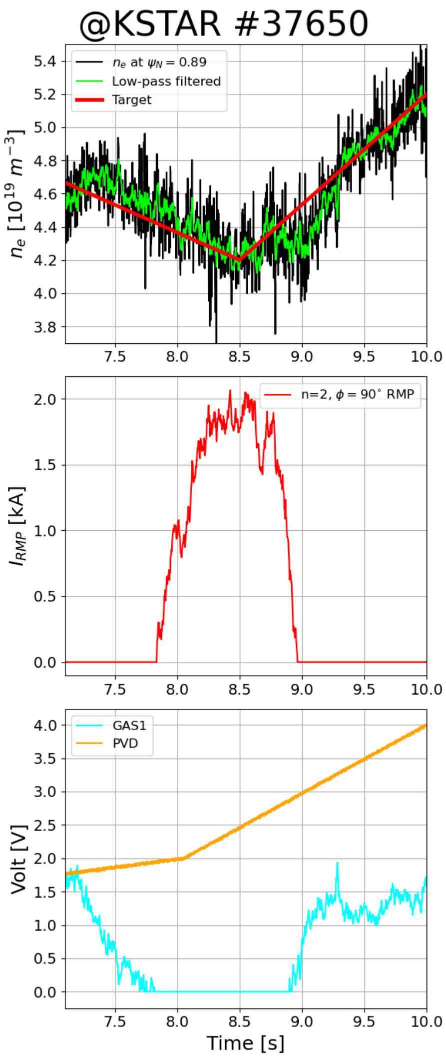
Experimental results

No PVD



PVD D2 0.3V





Modern control

Classical	Modern
Transfer functions	State space
$Y(s) = G(s) \cdot U(s)$	$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ $\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$
Frequency domain	Time domain
Bode, Nyquist plots	Controllability, observability
PID controllers	State feedback, observers
Manual tuning of gains	Systematic methods, optimal control

Table 5.1: Differences between classical and modern control tools.

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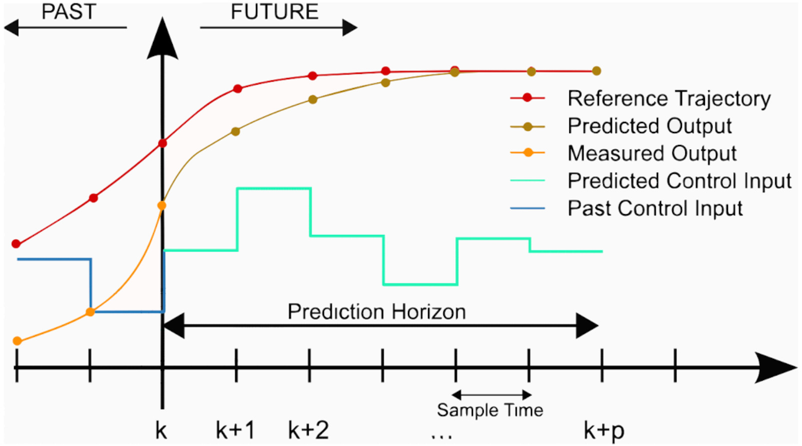
Model Predictive Control

$$x_{t+1} = Ax_t + Bu_t$$

$$Cost = \sum_t (x_{target} - x_t)^T Q (x_{target} - x_t) + u_t{}^T R u_t$$

$$u_{min} < u_t < u_{max} \qquad -\infty < z_t < \infty$$

- Use a model to forecast a t (prediction horizon) timesteps
- Calculate the actuation trajectory t timesteps in future to minimize cost (error) given actuation limits
- Grab only 1st actuation and send command
- Problem can be solved with QP solver in real time for small models
- Attempted for TCV shape ctrl, but preliminary



Appendix

Response to sinusoidal inputs

We can use the solution (2.6) to verify the claim we made in Section 2.1, in particular that for a linear system, the response to a sinusoidal input is sinusoidal. We will consider more generally the complex exponential input

$$u(t) = e^{st}, \qquad s \in \mathbb{C},$$

recognizing that sinusoidal inputs are a special case of this, for $s = i\omega$ (as explained in Section 2.1, page 28). From (2.6), the response is then

$$\begin{aligned} y(t) &= \int_0^\infty h(\tau) e^{s(t-\tau)}\, d\tau \\ &= e^{st} \int_0^\infty e^{-s\tau} h(\tau)\, d\tau \\ &= e^{st} H(s), \end{aligned}$$

where $H(s)$ is a complex number defined by

$$H(s) = \int_0^\infty e^{-s\tau} h(\tau)\, d\tau. \tag{2.7}$$

This function $H(s)$ is thus the transfer function, as defined in Definition 2.3.

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