Reconstruction Capabilities in DESC

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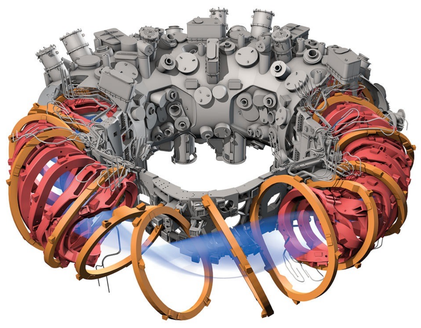
Motivation

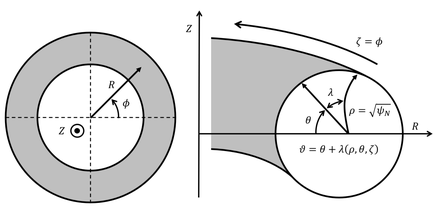
- Forward MHD equilibrium models require as inputs internal quantities such as pressure/current profiles and toroidal flux
- In experiments, must rely mainly on **external diagnostic signals**
- Reconstruction problem is to find model inputs which most closely yield the measured diagnostic signals[2,3]
  - Requires derivative information!

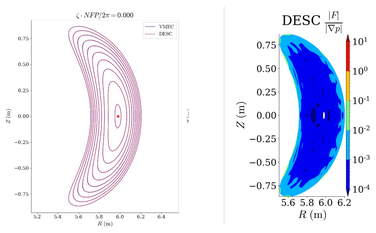


W7-X Schematic Figure[1]

DESC Stellarator Optimization Code

- DESC[1] is a 3D ideal MHD Stellarator Equilibrium and Optimization code[4]
- Written in Python+JAX[2] enables GPU + **Automatic Differentiation** capability
- Can handle optimization of coils + surfaces + equilibria





Reconstruction in DESC

Constraints
- Ideal MHD force balance
- Free Boundary Equilibrium

Objectives
- Difference in synthetic and measured signals,

$$\chi^2 = \sum \frac{(s_{meas} - s_{pred})^2}{\sigma^2}$$

Optimization Algorithm

DESC

Gradient Information

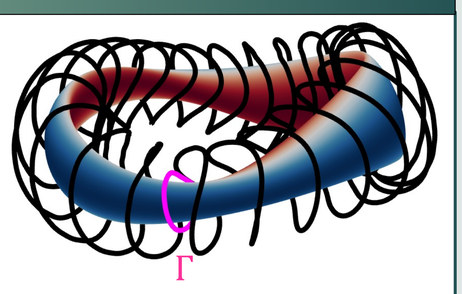
→ Reconstructed Equilibrium

DOFs
Boundary Coefficients: $R^b_{mn} Z^b_{mn}$
Boundary Sheet Current Potential Φ_{mn}
Pressure, Current/Iota profiles
Enclosed toroidal flux

Magnetic Diagnostic Coil Response Calculation

Magnetic diagnostics require **magnetic field** and **vector potential** of coils + plasma
Example: vector potential for flux loops

$$\Phi_{loop} = \oint_\Gamma (\boldsymbol{A}_{plasma} + \boldsymbol{A}_{coils}) \cdot d\boldsymbol{l}$$

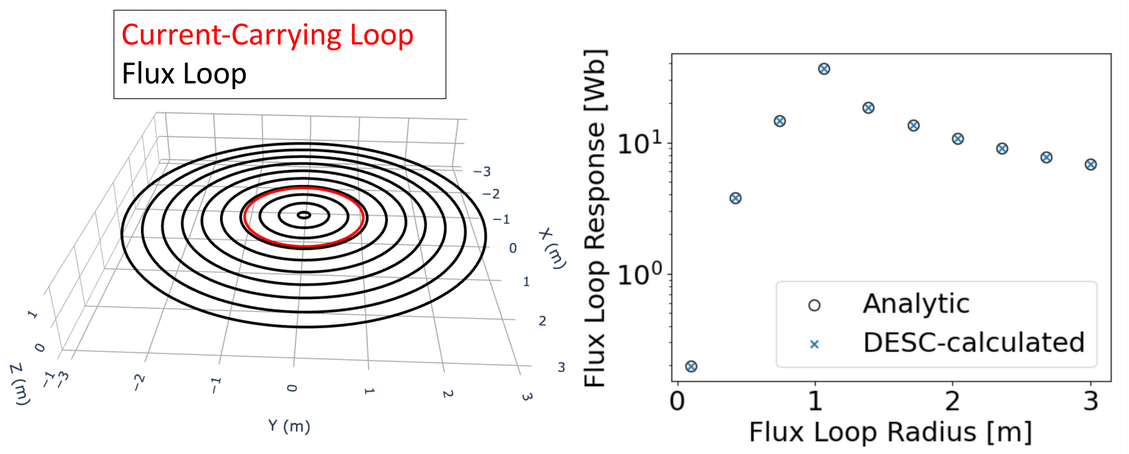


Obtain from 2D Biot-Savart over Virtual Casing Current

Obtain from Biot-Savart over Coils
- Spline/pointwise coil geometries: Hanson-Hirshman analytic formula [Cite HH 2002]
- Fourier-represented coils: quadrature

Analytic Verification of Vacuum Flux Loop Response

- Analytic model for vector potential from current-carrying loop[3,5]
- Excellent agreement (to machine precision) between analytic model and numerically-calculated flux response in DESC for concentric flux loops

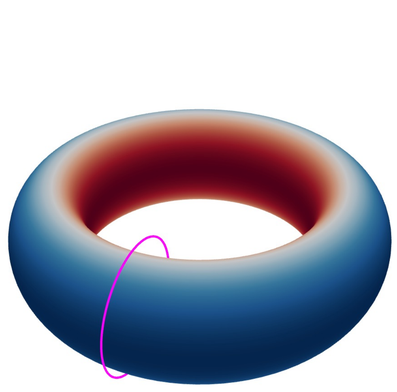


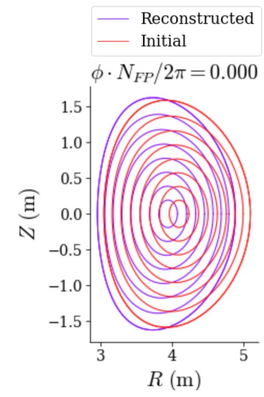
Toy Reconstruction w/ Synthetic Data

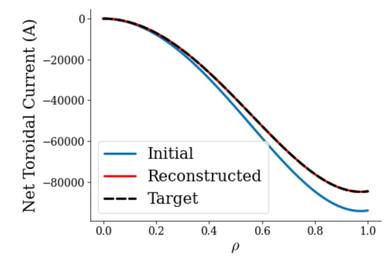
Toy one-parameter reconstruction problem for scale factor a on current in SOLOVEV-like Tokamak

$$I(\rho) = a(c_0 + c_2\rho^2 + c_4\rho^4 + c_6\rho^6 + c_8\rho^8 + c_{10}\rho^{10})$$

Initial a	Final a	Target a
1.0	0.90001	0.9



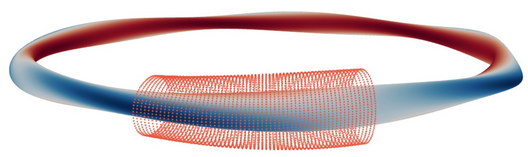


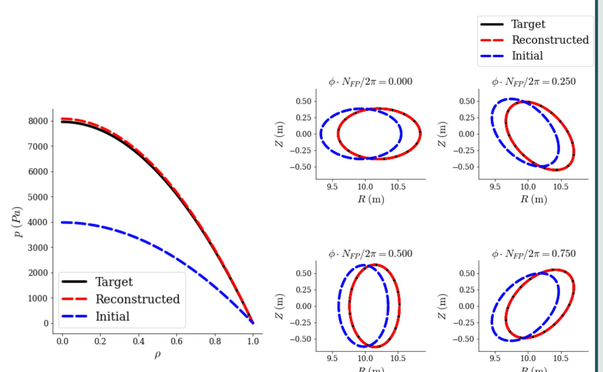


Used single Rogowski loop signal, only a varied to match calculated signal to target synthetic signal, while optimizing with a penalty on free-boundary error and on synthetic vs measured signal difference

Toy one-parameter reconstruction problem for scale factor a on pressure in finite-beta ellipse

$$p(\rho) = a(1 - \rho^2)$$





Used grid of 3-axis magnetic probes, only a varied to match calculated signal to target synthetic signal to within 1%

Discussion

- Treating free-boundary constraint with penalty method like single-stage, can be improved with constrained optimization methods[6]
- Ideally, would strictly enforce free boundary at each step, and use AD to use only one free boundary solve per step

Conclusions, Future Work and References

- Demonstrated initial basic magnetic diagnostic capabilities in DESC code
- Future work:
  - Incorporate uncertainty propagation techniques, a necessity for handling real data and relevant reconstruction problems
  - Expand synthetic diagnostics available for reconstruction
  - Improved handling of free-boundary constraint during reconstruction

[1] D. Clery, *The bizarre reactor that might save nuclear fusion*, (Oct. 2015) http: //www.sciencemag.org/news/2015/10/bizarre-reactor-might-save-nuclear- fusion.
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[6] R. Conlin et al., *Stellarator Optimization with Constraints*. arXiv:2403.11033 2024, Submitted to JPP.
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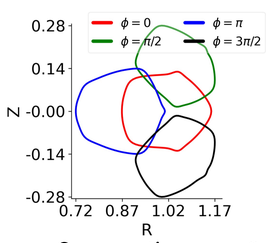
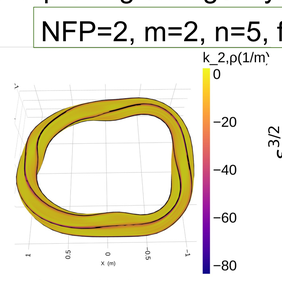
Bonus: Umbilic stellarators – R. Gaur

- By simultaneously optimizing the plasma boundary and a 3D curve, we impose a high curvature along the curve.
- This is done while improving omnigenity. Below are some results.

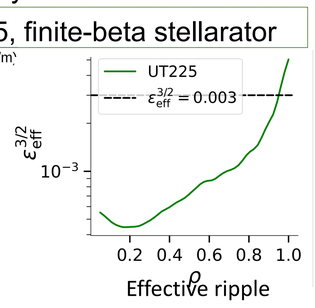
NFP=2, m=2, n=5, finite-beta stellarator



Cross-sections

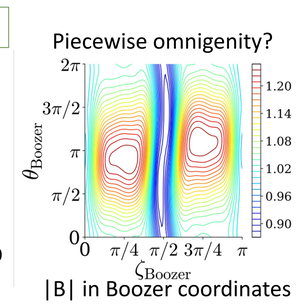


Second principal curvature



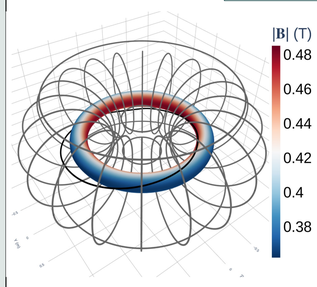
Effective ripple

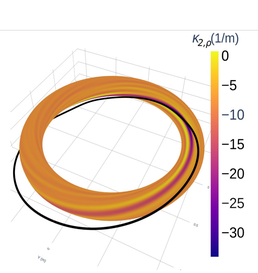
Piecewise omnigenity?

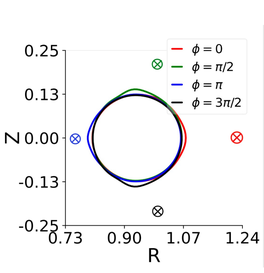


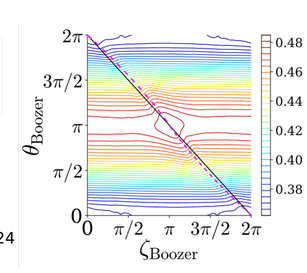
|B| in Boozer coordinates

Columbia HBT-EP -> finite-beta stellarator with m=1, n=1 ridge + coils









Acknowledgements

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